Computer algebra independent integration tests

6-Hyperbolic-functions/6.7-Miscellaneous/6.7.1-Hyperbolic-functions

Nasser M. Abbasi

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Contents

Chapter 1

Introduction

This report gives the result of running the computer algebra independent integration problems. The listing of the problems are maintained by and can be downloaded from `https://rulebasedintegration.org`

The number of integrals in this report is [1059]. This is test number [185].

1.1 Listing of CAS systems tested

The following systems were tested at this time.

1. Mathematica 12.3 (64 bit) on windows 10.
2. Rubi 4.16.1 in Mathematica 12.1 on windows 10.
3. Maple 2021.1 (64 bit) on windows 10.
4. Maxima 5.44 on Linux. (via sagemath 9.3)
5. Fricas 1.3.7 on Linux (via sagemath 9.3)
6. Giac/Xcas 1.7 on Linux. (via sagemath 9.3)
7. Sympy 1.8 under Python 3.8.8 using Anaconda distribution on Ubuntu.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 under windows 10 (64 bit)

Maxima, Fricas and Giac/Xcas were called from inside SageMath. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	solved	Failed
Rubi	% 100.00 (1059)	% 0.00 (0)
Mathematica	% 99.06 (1049)	% 0.94 (10)
Maple	% 88.39 (936)	% 11.61 (123)
Maxima	% 71.95 (762)	% 28.05 (297)
Fricas	% 92.07 (975)	% 7.93 (84)
Sympy	% 29.56 (313)	% 70.44 (746)
Giac	% 75.73 (802)	% 24.27 (257)
Mupad	% 69.88 (740)	% 30.12 (319)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

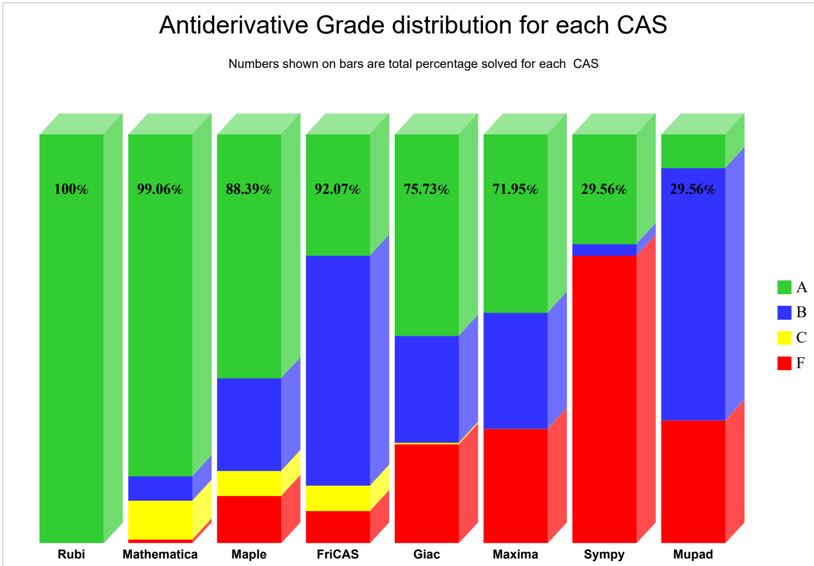
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

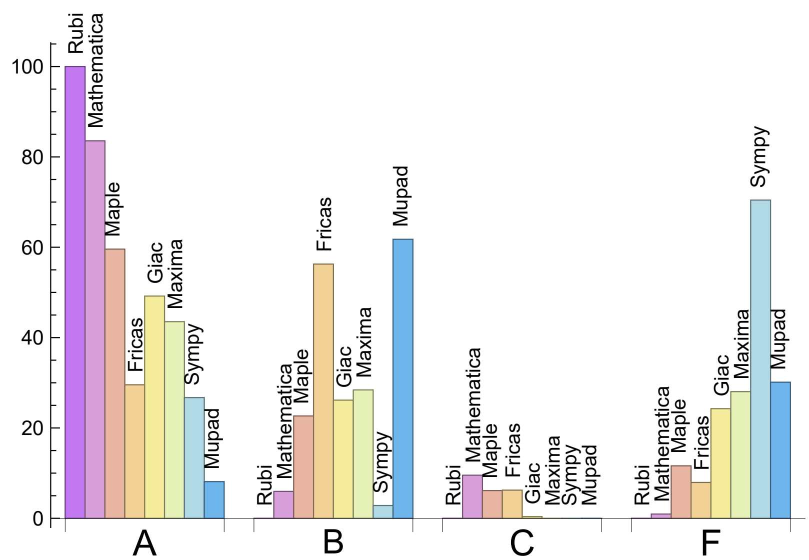
System	% A grade	% B grade	% C grade	% F grade
Rubi	100.00	0.00	0.00	0.00
Mathematica	83.57	5.95	9.54	0.94
Maple	59.58	22.66	6.14	11.61
Maxima	43.53	28.42	0.00	28.05
Fricas	29.56	56.28	6.23	7.93
Sympy	26.72	2.83	0.00	70.44
Giac	49.20	26.16	0.38	24.27
Mupad	8.12	61.76	0.00	30.12

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



The following table shows the distribution of the different types of failure for each CAS. There are 3 types of reasons why it can fail. The first is when CAS returns back the input within the time limit, which means it could not solve it. This the typical normal failure **F** .

The second is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned **F(-1)**.

The third is due to an exception generated. Assigned **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in CAS. This type of error requires more investigations to determine the cause.

System	Number failed	Percentage normal failure	Percentage time-out failure	Percentage exception failure
Rubi	0	0.00 %	0.00 %	0.00 %
Mathematica	10	40.00 %	60.00 %	0.00 %
Maple	123	98.37 %	0.00 %	1.63 %
Maxima	297	61.62 %	0.00 %	38.38 %
Fricas	84	58.33 %	1.19 %	40.48 %
Sympy	746	72.92 %	26.81 %	0.27 %
Giac	257	88.72 %	3.11 %	8.17 %
Mupad	319	96.87 %	3.13 %	0.00 %

Table 1.4: Time and leaf size performance for each CAS

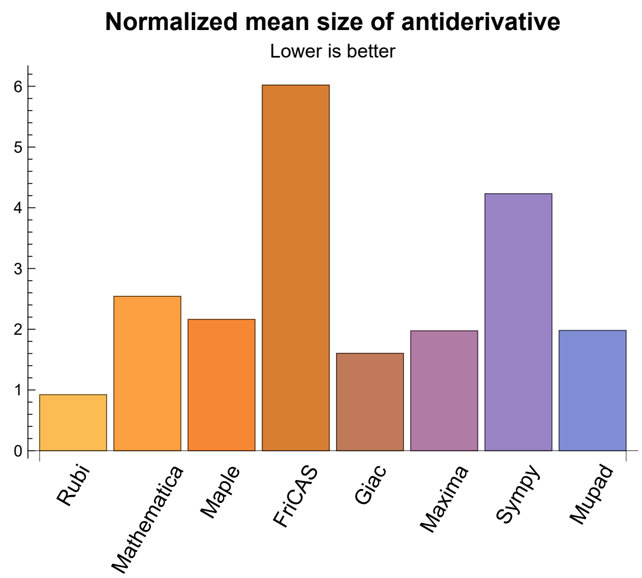
1.3 Performance

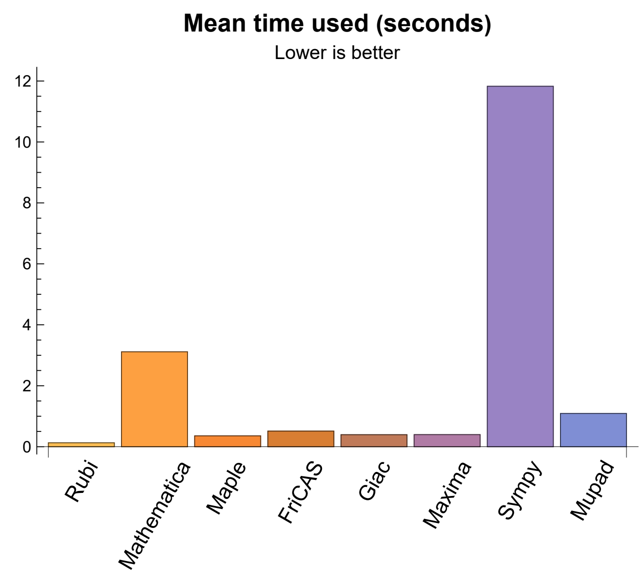
The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.13	67.91	0.92	50.00	1.00
Mathematica	3.11	201.02	2.54	53.00	1.00
Maple	0.36	199.53	2.16	61.00	1.33
Maxima	0.40	90.00	1.97	61.00	1.40
Fricas	0.51	544.02	6.02	159.00	3.31
Sympy	11.83	268.94	4.23	61.00	1.87
Giac	0.39	88.45	1.60	56.00	1.35
Mupad	1.09	96.98	1.98	51.00	1.28

Table 1.5: Time and leaf size performance for each CAS

The following are bar charts for the normalized leafsize and time used columns from the above table.





1.4 list of integrals that has no closed form antiderivative

{334, 339, 340, 341, 346, 347, 348, 353, 354, 355, 360, 361, 362, 367, 368, 369, 374, 375, 376, 381, 382, 383, 388, 389, 390, 395, 396, 397, 402, 403, 404, 409, 410, 411, 416, 417, 424, 429, 430, 431, 436, 437, 438, 443, 444, 445, 450, 451, 452, 457, 458, 459, 464, 465, 466, 471, 472, 473, 478, 479, 480, 485, 486, 487, 492, 493, 494, 499, 500, 501, 505, 506, 507, 512, 513, 514, 519, 520, 521, 526, 527, 870, 1014, 1015, 1016, 1017}

1.5 list of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {}

Mathematica {219, 337, 349, 364, 372, 393, 405, 406, 425, 433, 439, 446, 462, 474, 475, 488, 490, 531, 547, 556, 590, 592, 594, 622, 761, 762, 763, 764, 765, 766, 767, 768, 769, 770, 774, 775, 776, 777, 854, 911, 918, 930, 935, 943, 944, 966}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

Mupad Verification phase not implemented yet.

1.7 Timing

The command `AboluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since these integrals are run in a batch mode, using an automated script, and by using sagemath (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 14 such integrals out of total 705, or about 2 percent. This pecrentage can be higher or lower depending on the specific input test file.

Such integrals can be indentified by looking at the output of the integration in each section for Maxima. The exception message will indicate of the error is due to the interactive question being asked or not.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima `abs_integrate` was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

1.9.2 Important note about FriCAS and Giac/XCAS results

There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error `Exception raised: NotImplementedError`

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi and Maple, the buildin system function `LeafSize` is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special buildin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy and Giac antiderivatives is determined using the following function, thanks to user `slelievre` at `https://`

ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which is called directly from Python, the following code is used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
         leafCount =1
```

1.9.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative, Maple was used to determine the leaf size of Mupad output by post processing.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

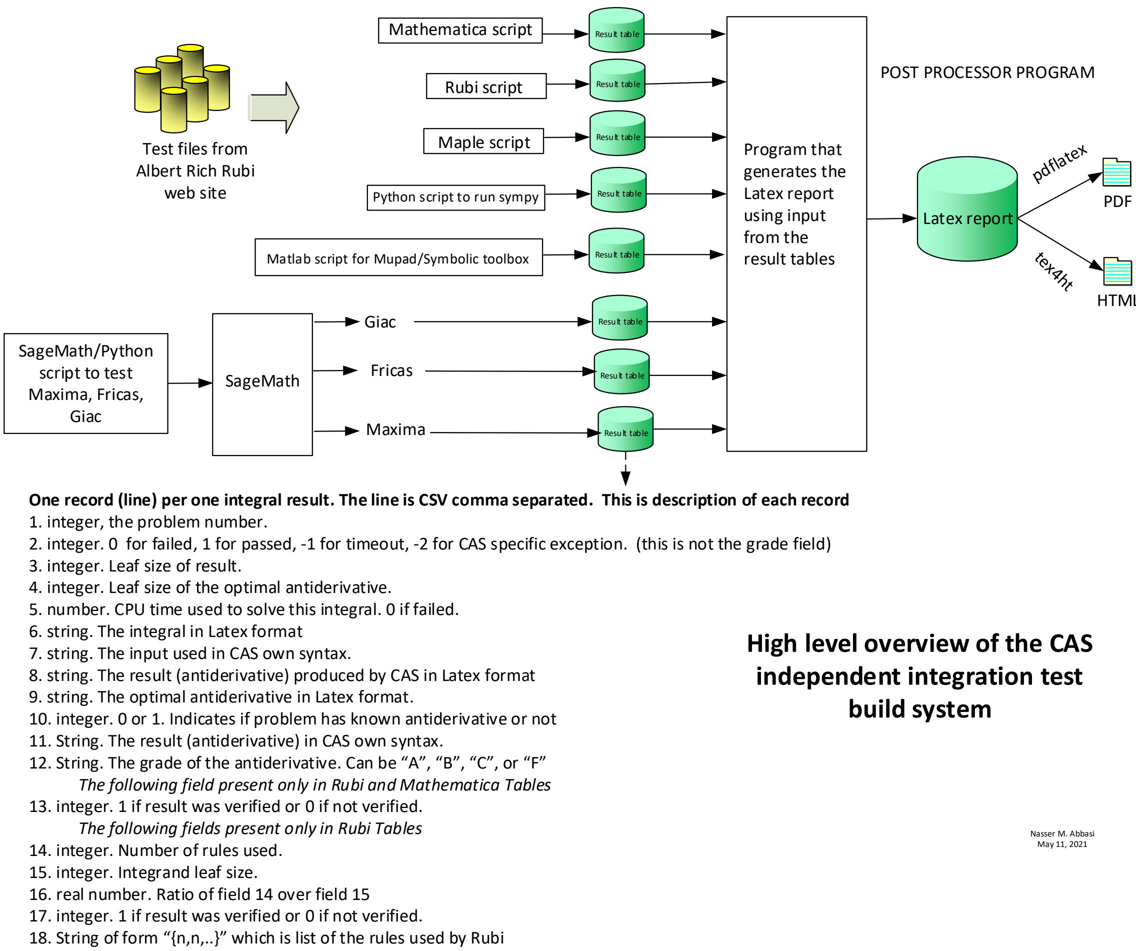
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



Chapter 2

detailed summary tables of results

2.1 List of integrals sorted by grade for each CAS

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 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1016, 1017, 1018, 1019, 1020, 1021, 1022, 1023, 1024, 1025, 1026, 1027, 1028, 1029, 1030, 1031, 1032, 1033, 1034, 1035, 1036, 1037, 1038, 1039, 1040, 1041, 1042, 1043, 1044, 1045, 1046, 1047, 1048, 1049, 1050, 1051, 1052, 1053, 1054, 1055, 1056, 1057, 1058, 1059 }

B grade: { }

C grade: { }

F grade: { }

2.1.2 Mathematica

A grade: { 5, 7, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 25, 26, 27, 28, 30, 32, 34, 35, 36, 37, 38, 40, 42, 44, 45, 46, 47, 48, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 145, 148, 149, 151, 152, 154, 157, 160, 161, 163, 164, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 206, 207, 208, 209, 210, 212, 214, 216, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 234, 235, 237, 240, 242, 243, 244, 245, 246, 248, 250, 251, 252, 253, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 350, 351, 352, 353, 354, 355, 356, 357, 359, 360, 361, 362, 363, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 421, 423, 424, 425, 428, 429, 430, 431, 434, 436, 437, 438, 439, 441, 442, 443, 444, 445, 447, 448, 449, 450, 451, 453, 454, 455, 456, 457, 458, 459, 463, 464, 465, 466, 467, 468, 469, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 483, 484, 485, 486, 487, 488, 489, 490, 492, 493, 494, 497, 498, 499, 500, 501, 502, 503, 505, 506, 507, 510, 511, 512, 513, 514, 515, 517, 518, 519, 520, 522, 523, 524, 525, 526, 527, 528, 530, 532, 533, 534, 535, 536, 537, 538, 539, 541, 543, 544, 546, 548, 549, 550, 551, 552, 553, 554, 555, 557, 559, 560, 561, 562, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 585, 586, 587, 588, 589, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 615, 617, 618, 619, 621, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 657, 658, 659, 660, 661, 662, 663, 665, 667, 668, 669, 671, 673, 674, 675, 676, 678, 679, 680, 681, 682, 683, 684, 685, 686, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700, 701, 703, 705, 706, 707, 708, 709, 710, 711, 712, 713, 714, 715, 716, 717, 718, 719, 720, 721, 722, 723, 724, 725, 726, 727, 728, 729, 730, 731, 732, 733, 734, 735, 736, 737, 738, 739, 740, 741, 742, 743, 744, 746, 747, 748, 751, 753, 754, 755, 756, 757, 758, 759, 780, 781, 782, 783, 784, 785, 786, 787, 788, 789, 790, 791, 792, 793, 794, 795, 796, 797, 798, 799, 800, 801, 802, 803, 804, 805, 806, 807, 808, 809, 810, 811, 812, 813, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 827, 828, 829, 830, 831, 832, 833, 834, 835, 836, 837, 838, 839, 840, 841, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 853, 854, 855, 856, 857, 858, 859, 860, 861, 862, 863, 864, 865, 866, 867, 868, 870, 871, 872, 873, 874, 875, 876, 877, 878, 879, 880, 881, 883, 884, 885, 886, 887, 888, 889, 890, 891, 892, 893, 894, 895, 896, 897, 898, 899, 900, 901, 902, 903, 904, 905, 906, 907, 908, 909, 910, 912, 913, 914, 915, 916, 917, 919, 920, 921, 922, 923, 924, 925, 926, 927, 928, 929, 931, 932, 933, 934, 936, 938, 941, 942, 945, 946, 947, 948, 950, 951, 952, 953, 954, 955, 956, 957, 958, 959, 960, 961, 962, 963, 964, 965, 967, 968, 969, 970, 971, 972, 973, 974, 975, 976, 977, 978, 979, 980, 982, 983, 984, 986, 987, 988, 990, 991, 992, 993, 995, 996, 1003, 1004, 1005, 1006, 1007, 1008, 1009, 1010, 1011, 1012, 1013, 1014, 1015, 1016, 1017, 1018, 1019, 1020, 1021, 1022, 1023, 1024, 1025, 1028, 1029, 1030, 1031, 1032, 1033, 1034, 1035, 1036, 1037, 1038, 1039, 1040, 1041, 1042, 1043, 1044, 1045, 1046, 1047, 1048, 1049, 1050, 1051, 1052, 1054, 1057, 1058, 1059 }

B grade: { 1, 2, 3, 4, 6, 8, 24, 100, 127, 143, 144, 150, 155, 156, 162, 188, 189, 205, 239, 254, 358, 426, 427, 432, 440, 460, 461, 470, 482, 495, 496, 508, 509, 516, 563, 584, 614, 616, 656, 664, 666, 677, 687, 702, 704, 745, 749, 750, 752, 760, 882, 949, 994, 998, 999, 1000, 1001, 1002, 1026, 1027, 1053, 1055, 1056 }

C grade: { 29, 31, 33, 39, 41, 43, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 146, 147, 153, 158, 159, 165, 211, 213, 215, 217, 229, 230, 231, 232, 233, 236, 238, 241, 247, 249, 337, 349, 364, 393, 420, 422, 433, 435, 446, 462, 491, 504, 529, 531, 540, 542, 545, 547, 556, 558, 590, 591, 592, 593, 594, 595, 620, 622, 670, 672, 761, 762, 763, 764, 765, 766, 767, 768, 769, 770, 771, 774, 775, 776, 777, 852, 869, 911, 918, 930, 935, 937, 939, 940, 943, 944, 966 }

F grade: { 452, 521, 772, 773, 778, 779, 981, 985, 989, 997 }

2.1.3 Maple

A grade: { 1, 8, 9, 12, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 86, 87, 88, 90, 93, 94, 95, 96, 97, 98, 99, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 117, 118, 119, 121, 122, 123, 125, 127, 129, 130, 131, 132, 133, 134, 151, 154, 163, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 192, 193, 194, 195, 196, 197, 198, 199, 206, 212, 216, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 245, 246, 253, 254, 255, 256, 257, 258, 261, 262, 263, 264, 265, 266, 267, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 282, 283, 284, 285, 286, 287, 288, 293, 294, 295, 296, 297, 301, 302, 303, 304, 305, 306, 309, 310, 311, 312, 313, 314, 315, 318, 319, 320, 321, 322, 323, 324, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 401, 402, 403, 404, 405, 406, 408, 409, 410, 411, 412, 413, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 428, 429, 430, 431, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 447, 448, 449, 450, 451, 452, 453, 454, 456, 457, 458, 459, 463, 464, 465, 466, 467, 470, 471, 472, 473, 476, 477, 478, 479, 480, 481, 482, 484, 485, 486, 487, 491, 492, 493, 494, 498, 499, 500, 501, 504, 505, 506, 507, 508, 509, 511, 512, 513, 514, 517, 518, 519, 520, 521, 522, 525, 526, 527, 572, 573, 578, 579, 580, 581, 582, 583, 584, 585, 586, 588, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 625, 626, 627, 628, 630, 632, 633, 635, 636, 637, 638, 640, 642, 643, 644, 645, 646, 647, 648, 650, 655, 656, 657, 658, 660, 661, 662, 663, 665, 667, 668, 670, 671, 672, 673, 674, 675, 676, 680, 681, 682, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 698, 699, 701, 702, 706, 707, 708, 709, 710, 711, 712, 713, 714, 715, 716, 717, 718, 719, 720, 721, 722, 723, 725, 726, 728, 729, 730, 731, 732, 735, 737, 738, 739, 740, 741, 742, 746, 747, 748, 749, 750, 751, 752, 754, 755, 756, 762, 764, 771, 777, 784, 786, 787, 788, 803, 805, 806, 848, 855, 856, 857, 864, 866, 870, 872, 873, 874, 875, 876, 877, 878, 879, 880, 885, 886, 887, 893, 894, 897, 898, 901, 902, 903, 904, 905, 906, 907, 908, 909, 910, 911, 912, 913, 914, 915, 916, 917, 918, 919, 920, 921, 922, 923, 924, 925, 926, 927, 928, 929, 930, 931, 932, 933, 934, 935, 936, 945, 946, 947, 948, 952, 953, 954, 955, 959, 960, 961, 962, 967, 968, 969, 970, 971, 972, 973, 974, 975, 976, 977, 978, 979, 982, 993, 996, 1003, 1004, 1005, 1006, 1007, 1008, 1009, 1010, 1011, 1012, 1013, 1014, 1015, 1016, 1017, 1018, 1019, 1020, 1021, 1026, 1027, 1028, 1029, 1030, 1031, 1032, 1033, 1034, 1038, 1039, 1041, 1043, 1044, 1045, 1046, 1049, 1050, 1051, 1055 }

B grade: { 2, 3, 4, 5, 6, 7, 84, 85, 100, 115, 116, 124, 128, 135, 136, 137, 138, 139, 140, 141, 142, 146, 147, 148, 149, 152, 153, 158, 159, 160, 161, 164, 165, 207, 208, 209, 210, 211, 214, 236, 237, 238, 247, 248, 249, 251, 252, 260, 269, 281, 290, 291, 292, 299, 300, 308, 317, 326, 327, 328, 343, 358, 372, 385, 398, 399, 400, 407, 414, 426, 427, 432, 433, 446, 455, 460, 461, 462, 468, 469, 483, 490, 495, 496, 497, 503, 510, 523, 524, 531, 540, 547, 556, 560, 563, 566, 567, 568, 569, 570, 571, 574, 575, 576, 577, 587, 589, 619, 620, 621, 622, 623, 624, 629, 631, 634, 639, 641, 649, 651, 652, 653, 654, 659, 664, 666, 669, 677, 683, 697, 700, 703, 704, 705, 724, 727, 733, 734, 736, 743, 744, 745, 753, 758, 759, 760, 761, 763, 765, 766, 767, 768, 769, 770, 772, 773, 774, 775, 776, 778, 779, 780, 781, 782, 783, 785, 789, 790, 791, 792, 793, 794, 795, 796, 797, 798, 799, 800, 801, 802, 804, 807, 808, 809, 810, 817, 818, 819, 820, 821, 822, 830, 832, 833, 834, 835, 836, 837, 838, 839, 842, 843, 844, 845, 846, 847, 851, 852, 853, 858, 859, 860, 861, 862, 863, 865, 867, 868, 869, 980, 981, 983, 984, 986, 987, 990, 991, 992, 994, 997, 1022, 1023, 1024, 1025, 1035, 1036, 1040, 1054, 1056, 1057 }

C grade: { 13, 89, 92, 120, 126, 143, 144, 145, 150, 155, 156, 157, 162, 200, 201, 202, 203, 204, 213, 215, 217, 218, 219, 220, 240, 241, 242, 243, 244, 344, 386, 757, 811, 812, 813, 814, 815, 816, 823, 824, 825, 826, 827, 828, 829, 831, 840, 841, 937, 938, 939, 940, 941, 942, 943, 944, 985, 988, 989, 995, 1037, 1052, 1053, 1058, 1059 }

F grade: { 10, 11, 14, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 91, 188, 189, 190, 191, 205, 239, 250, 259, 268, 280, 289, 298, 307, 316, 325, 342, 356, 357, 370, 371, 384, 474, 475, 488, 489, 502, 515, 516, 528, 529, 530, 532, 533, 534, 535, 536, 537, 538, 539, 541, 542, 543, 544, 545, 546, 548, 549, 550, 551, 552, 553, 554, 555, 557, 558, 559, 561, 562, 564, 565, 678, 679, 684, 685, 849, 850, 854, 871, 881, 882, 883, 884, 888, 889, 890, 891, 892, 895, 896, 899, 900, 949, 950, 951, 956, 957, 958, 963, 964, 965, 966, 998, 999, 1000, 1001, 1002, 1042, 1047, 1048 }

2.1.4 Maxima

A grade: { 1, 2, 3, 4, 6, 8, 9, 12, 15, 16, 17, 18, 19, 20, 21, 22, 23, 29, 30, 42, 70, 75, 77, 79, 82, 84, 85, 86, 106, 113, 115, 116, 117, 139, 140, 143, 149, 150, 161, 162, 206, 216, 235, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 278, 279, 280, 281, 282, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 321, 322, 323, 324, 325, 326, 327, 328, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 344, 346, 347, 348, 349, 352, 353, 354, 355, 359, 360, 361, 362, 363, 364, 366, 367, 368, 369, 374, 375, 376, 377, 378, 379, 381, 382, 383, 386, 388, 389, 390, 391, 392, 393, 395, 396, 397, 398, 399, 400, 402, 403, 404, 405, 406, 407, 409, 410, 411, 412, 413, 414, 416, 417, 423, 424, 425, 426, 429, 430, 431, 432, 433, 435, 436, 437, 438, 439, 440, 443, 444, 445, 446, 449, 450, 451, 452, 453, 454, 455, 457, 458, 459, 460, 464, 465, 466, 467, 468, 469, 471, 472, 473, 476, 478, 479, 480, 481, 482, 483, 485, 486, 487, 491, 492, 493, 494, 496, 498, 499, 500, 501, 505, 506, 507, 508, 509, 510, 512, 513, 514, 517, 519, 520, 521, 522, 523, 524, 526, 527, 575, 576, 580, 581, 583, 586, 596, 599, 600, 601, 602, 603, 604, 605, 608, 609, 610, 611, 612, 613, 617, 618, 620, 627, 628, 629, 630, 631, 632, 633, 637, 638, 640, 641, 642, 647, 648, 658, 660, 662, 668, 669, 670, 671, 672, 674, 675, 680, 681, 688, 690, 691, 693, 697, 700, 707, 709, 711, 713, 715, 717, 719, 721, 723, 730, 731, 732, 733, 739, 740, 741, 746, 747, 748, 753, 754, 755, 756, 783, 786, 802, 803, 804, 805, 806, 807, 809, 811, 812, 813, 814, 815, 816, 823, 824, 825, 842, 843, 844, 845, 846, 847, 848, 851, 852, 853, 855, 856, 857, 858, 859, 870, 872, 873, 874, 875, 876, 877, 893, 894, 897, 898, 901, 902, 903, 904, 905, 906, 907, 908, 909, 910, 911, 912, 913, 914, 915, 916, 917, 918, 919, 920, 921, 922, 923, 924, 925, 926, 927, 928, 929, 930, 931, 932, 933, 934, 935, 936, 937, 938, 939, 940, 941, 942, 943, 944, 967, 968, 969, 973, 974, 975, 979, 981, 982, 986, 993, 994, 1013, 1014, 1015, 1016, 1017, 1019, 1020, 1026, 1027, 1028, 1031, 1033, 1034, 1037, 1038, 1043, 1044, 1045, 1046, 1050, 1051, 1055, 1056 }

B grade: { 5, 7, 10, 11, 13, 14, 24, 25, 26, 27, 28, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 43, 44, 45, 46, 47, 48, 67, 68, 71, 72, 73, 74, 76, 78, 80, 81, 83, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 107, 108, 109, 110, 111, 112, 114, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 141, 142, 144, 145, 146, 147, 148, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 163, 164, 165, 166, 192, 193, 194, 196, 197, 198, 200, 201, 207, 208, 211, 212, 217, 218, 221, 222, 223, 225, 226, 227, 229, 230, 234, 236, 240, 241, 245, 246, 247, 262, 277, 283, 302, 320, 329, 345, 350, 351, 365, 373, 380, 387, 394, 401, 408, 415, 421, 422, 427, 428, 434, 441, 442, 447, 448, 456, 461, 462, 463, 470, 477, 484, 495, 497, 504, 511, 518, 525, 560, 561, 562, 563, 564, 565, 566, 567, 570, 573, 574, 577, 579, 582, 584, 588, 597, 598, 606, 607, 614, 615, 616, 619, 621, 622, 623, 624, 625, 626, 634, 635, 636, 639, 643, 644, 645, 646, 649, 651, 653, 654, 655, 656, 657, 659, 661, 663, 664, 665, 666, 667, 673, 676, 677, 678, 679, 682, 683, 684, 685, 686, 687, 702, 703, 704, 705, 735, 749, 750, 751, 752, 768, 769, 770, 774, 775, 776, 808, 810, 817, 818, 819, 820, 821, 822, 830, 838, 839, 840, 841, 860, 980, 983, 984, 987, 990, 991, 992, 995, 996, 1003, 1004, 1005, 1006, 1007, 1008, 1009, 1010, 1011, 1012, 1018, 1021, 1022, 1023, 1024, 1025, 1030, 1032, 1040, 1048, 1049, 1053, 1054, 1057, 1058 }

C grade: { }

F grade: { 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 69, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 195, 199, 202, 203, 204, 205, 209, 210, 213, 214, 215, 219, 220, 224, 228, 231, 232, 233, 237, 238, 239, 242, 243, 244, 248, 249, 342, 343, 356, 357, 358, 370, 371, 372, 384, 385, 418, 419, 420, 474, 475, 488, 489, 490, 502, 503, 515, 516, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 568, 569, 571, 572, 578, 585, 587, 589, 590, 591, 592, 593, 594, 595, 650, 652, 689, 692, 694, 695, 696, 698, 699, 701, 706, 708, 710, 712, 714, 716, 718, 720, 722, 724, 725, 726, 727, 728, 729, 734, 736, 737, 738, 742, 743, 744, 745, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 767, 771, 772, 773, 777, 778, 779, 780, 781, 782, 784, 785, 787, 788, 789, 790, 791, 792, 793, 794, 795, 796, 797, 798, 799, 800, 801, 826, 827, 828, 829, 831, 832, 833, 834, 835, 836, 837, 849, 850, 854, 861, 862, 863, 864, 865, 866, 867, 868, 869, 871, 878, 879, 880, 881, 882, 883, 884, 885, 886, 887, 888, 889, 890, 891, 892, 895, 896, 899, 900, 945, 946, 947, 948, 949, 950, 951, 952, 953, 954, 955, 956, 957, 958, 959, 960, 961, 962, 963, 964, 965, 966, 970, 971, 972, 976, 977, 978, 985, 988, 989, 997, 998, 999, 1000, 1001, 1002, 1029, 1035, 1036, 1039, 1041, 1042, 1047, 1052, 1059 }

2.1.5 FriCAS

A grade: { 8, 15, 16, 17, 18, 19, 20, 21, 22, 23, 71, 75, 78, 102, 106, 109, 167, 173, 179, 182, 193, 195, 196, 199, 216, 224, 225, 226, 228, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 264, 265, 266, 267, 268, 269, 270, 271, 273, 274, 276, 278, 279, 280, 281, 282, 283, 285, 286, 287, 288, 289, 293, 294, 295, 298, 299, 300, 301, 303, 306, 307, 308, 309, 310, 312, 313, 315, 316, 317, 318, 319, 321, 322, 324, 325, 326, 330, 334, 339, 340, 341, 346, 347, 348, 353, 354, 355, 360, 361, 362, 367, 368, 369, 374, 375, 376, 381, 382, 383, 387, 388, 389, 390, 395, 396, 397, 402, 403, 404, 409, 410, 411, 416, 417, 424, 429, 430, 431, 436, 437, 438, 442, 443, 444, 445, 450, 451, 452, 457, 458, 459, 464, 465, 466, 471, 472, 473, 478, 479, 480, 485, 486, 487, 492, 493, 494, 499, 500, 501, 505, 506, 507, 512, 513, 514, 519, 520, 521, 526, 527, 572, 580, 581, 585, 596, 597, 599, 600, 603, 604, 605, 606, 608, 609, 610, 612, 613, 617, 625, 627, 628, 629, 630, 632, 635, 637, 638, 639, 640, 642, 647, 657, 658, 660, 667, 668, 669, 670, 675, 681, 688, 691, 694, 695, 724, 727, 730, 732, 733, 736, 739, 740, 741, 742, 746, 747, 748, 780, 781, 782, 783, 784, 785, 786, 787, 789, 792, 795, 798, 802, 803, 804, 805, 806, 807, 811, 812, 813, 814, 815, 816, 823, 824, 825, 838, 839, 842, 855, 856, 857, 870, 872, 879, 880, 886, 887, 893, 894, 901, 903, 910, 915, 920, 922, 927, 948, 955, 967, 968, 969, 970, 971, 972, 973, 974, 975, 976, 977, 978, 989, 1000, 1004, 1005, 1006, 1007, 1008, 1009, 1010, 1011, 1012, 1013, 1014, 1015, 1016, 1017, 1035, 1036, 1043, 1044, 1055 }

B grade: { 1, 2, 3, 4, 5, 6, 7, 9, 10, 11, 12, 13, 14, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 72, 73, 74, 76, 77, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 103, 104, 105, 107, 108, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 168, 169, 170, 171, 172, 174, 175, 176, 177, 178, 180, 181, 183, 184, 185, 186, 187, 192, 194, 197, 198, 200, 201, 202, 203, 204, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 217, 218, 219, 220, 221, 222, 223, 227, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 263, 272, 275, 277, 284, 290, 291, 292, 296, 297, 302, 304, 305, 311, 314, 320, 323, 327, 328, 329, 331, 332, 333, 338, 343, 344, 345, 350, 351, 352, 358, 359, 365, 366, 372, 373, 380, 385, 386, 394, 400, 401, 407, 408, 414, 415, 418, 419, 421, 426, 427, 428, 433, 434, 435, 440, 441, 446, 447, 448, 449, 455, 456, 462, 463, 470, 476, 477, 484, 490, 491, 497, 498, 503, 504, 511, 517, 518, 525, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 573, 574, 575, 576, 577, 578, 579, 582, 583, 584, 586, 587, 588, 589, 598, 601, 602, 607, 611, 614, 615, 616, 618, 619, 620, 621, 622, 623, 624, 626, 631, 633, 634, 636, 641, 643, 644, 645, 646, 648, 649, 650, 651, 652, 653, 654, 655, 656, 659, 661, 662, 663, 664, 665, 666, 671, 672, 673, 674, 676, 677, 678, 679, 680, 682, 683, 684, 685, 686, 687, 689, 690, 692, 693, 696, 697, 698, 699, 700, 701, 702, 703, 704, 705, 706, 707, 708, 709, 710, 711, 712, 713, 714, 715, 716, 717, 718, 719, 720, 721, 722, 723, 725, 726, 728, 729, 731, 734, 735, 737, 738, 743, 744, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 768, 769, 770, 771, 772, 773, 774, 775, 776, 777, 778, 779, 788, 790, 791, 793, 794, 796, 797, 799, 800, 801, 808, 809, 810, 817, 818, 819, 820, 821, 822, 826, 827, 828, 829, 830, 831, 832, 833, 834, 835, 836, 837, 840, 841, 845, 848, 858, 859, 860, 869, 873, 874, 875, 876, 877, 878, 885, 897, 898, 902, 904, 905, 906, 907, 908, 909, 911, 912, 913, 914, 916, 917, 918, 919, 921, 923, 924, 925, 926, 928, 929, 930, 931, 932, 933, 934, 935, 936, 937, 938, 939, 940, 941, 942, 943, 944, 945, 946, 947, 952, 953, 954, 959, 960, 961, 962, 979, 980, 981, 982, 983, 984, 985, 986, 987, 988, 990, 991, 992, 993, 994, 995, 996, 997, 998, 999, 1001, 1002, 1003, 1018, 1019, 1020, 1021, 1022, 1023, 1024, 1025, 1026, 1027, 1028, 1029, 1030, 1031, 1032, 1033, 1034, 1037, 1038, 1039, 1040, 1041, 1045, 1046, 1047, 1048, 1049, 1050, 1051, 1052, 1053, 1054, 1056, 1057, 1058, 1059 }

C grade: { 335, 336, 337, 342, 349, 356, 357, 363, 364, 370, 371, 377, 378, 379, 384, 391, 392, 393, 398, 399, 405, 406, 412, 413, 420, 422, 423, 425, 432, 439, 453, 454, 460, 461, 467, 468, 469, 474, 475, 481, 482, 483, 488, 489, 495, 496, 502, 508, 509, 510, 515, 516, 522, 523, 524, 843, 844, 846, 847, 849, 850, 851, 852, 853, 867, 868 }

F grade: { 188, 189, 190, 191, 205, 239, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 590, 591, 592, 593, 594, 595, 745, 761, 762, 763, 764, 765, 766, 767, 854, 861, 862, 863, 864, 865, 866, 871, 881, 882, 883, 884, 888, 889, 890, 891, 892, 895, 896, 899, 900, 949, 950, 951, 956, 957, 958, 963, 964, 965, 966, 1042 }

2.1.6 Sympy

A grade: { 1, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 81, 82, 83, 85, 87, 88, 96, 97, 105, 113, 131, 132, 133, 134, 167, 168, 169, 170, 171, 173, 174, 175, 176, 177, 179, 180, 181, 182, 183, 184, 185, 186, 193, 194, 195, 196, 197, 198, 199, 222, 223, 224, 225, 226, 227, 228, 251, 252, 253, 254, 260, 261, 262, 263, 269, 270, 271, 272, 281, 282, 283, 284, 290, 291, 292, 293, 299, 300, 301, 302, 308, 309, 310, 311, 317, 318, 319, 320, 326, 327, 328, 329, 334, 339, 340, 341, 346, 347, 353, 354, 355, 360, 361, 367, 368, 374, 375, 381, 382, 388, 389, 395, 396, 397, 402, 403, 409, 410, 416, 417, 429, 430, 436, 437, 443, 444, 450, 451, 457, 458, 464, 466, 471, 472, 473, 478, 479, 480, 485, 486, 487, 492, 493, 494, 499, 500, 501, 505, 506, 507, 512, 513, 514, 519, 520, 521, 526, 527, 566, 567, 568, 569, 580, 596, 597, 598, 599, 600, 601, 602, 605, 606, 607, 608, 609, 610, 611, 619, 621, 623, 688, 691, 697, 700, 703, 705, 715, 724, 727, 730, 731, 732, 733, 736, 739, 740, 741, 746, 747, 748, 749, 755, 756, 802, 803, 804, 805, 806, 807, 838, 839, 855, 856, 857, 870, 872, 873, 874, 878, 879, 880, 885, 886, 887, 893, 894, 897, 898, 901, 902, 903, 907, 908, 909, 914, 915, 919, 920, 921, 925, 926, 927, 932, 933, 945, 946, 947, 948, 953, 954, 955, 961, 962, 967, 969, 973, 975, 1003, 1014, 1015, 1016, 1017, 1018, 1026, 1027, 1028, 1031, 1032, 1043, 1045, 1050, 1051, 1052, 1053, 1054, 1055 }

B grade: { 2, 192, 221, 574, 575, 576, 577, 581, 582, 583, 584, 629, 631, 633, 639, 641, 643, 753, 754, 783, 808, 809, 811, 812, 813, 840, 841, 983, 984, 1044 }

C grade: { }

F grade: { 3, 4, 5, 6, 7, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 84, 86, 89, 90, 91, 92, 93, 94, 95, 98, 99, 100, 101, 102, 103, 104, 106, 107, 108, 109, 110, 111, 112, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 172, 178, 187, 188, 189, 190, 191, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 255, 256, 257, 258, 259, 264, 265, 266, 267, 268, 273, 274, 275, 276, 277, 278, 279, 280, 285, 286, 287, 288, 289, 294, 295, 296, 297, 298, 303, 304, 305, 306, 307, 312, 313, 314, 315, 316, 321, 322, 323, 324, 325, 330, 331, 332, 333, 335, 336, 337, 338, 342, 343, 344, 345, 348, 349, 350, 351, 352, 356, 357, 358, 359, 362, 363, 364, 365, 366, 369, 370, 371, 372, 373, 376, 377, 378, 379, 380, 383, 384, 385, 386, 387, 390, 391, 392, 393, 394, 398, 399, 400, 401, 404, 405, 406, 407, 408, 411, 412, 413, 414, 415, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 431, 432, 433, 434, 435, 438, 439, 440, 441, 442, 445, 446, 447, 448, 449, 452, 453, 454, 455, 456, 459, 460, 461, 462, 463, 465, 467, 468, 469, 470, 474, 475, 476, 477, 481, 482, 483, 484, 488, 489, 490, 491, 495, 496, 497, 498, 502, 503, 504, 508, 509, 510, 511, 515, 516, 517, 518, 522, 523, 524, 525, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 570, 571, 572, 573, 578, 579, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 603, 604, 612, 613, 614, 615, 616, 617, 618, 620, 622, 624, 625, 626, 627, 628, 630, 632, 634, 635, 636, 637, 638, 640, 642, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 686, 687, 689, 690, 692, 693, 694, 695, 696, 698, 699, 701, 702, 704, 706, 707, 708, 709, 710, 711, 712, 713, 714, 716, 717, 718, 719, 720, 721, 722, 723, 725, 726, 728, 729, 734, 735, 737, 738, 742, 743, 744, 745, 750, 751, 752, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 777, 778, 779, 780, 781, 782, 784, 785, 786, 787, 788, 789, 790, 791, 792, 793, 794, 795, 796, 797, 798, 799, 800, 801, 810, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 827, 828, 829, 830, 831, 832, 833, 834, 835, 836, 837, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 853, 854, 858, 859, 860, 861, 862, 863, 864, 865, 866, 867, 868, 869, 871, 875, 876, 877, 881, 882, 883, 884, 888, 889, 890, 891, 892, 895, 896, 899, 900, 904, 905, 906, 910, 911, 912, 913, 916, 917, 918, 922, 923, 924, 928, 929, 930, 931, 934, 935, 936, 937, 938, 939, 940, 941, 942, 943, 944, 949, 950, 951, 952, 956, 957, 958, 959, 960, 963, 964, 965, 966, 968, 970, 971, 972, 974, 976, 977, 978, 979, 980, 981, 982, 985, 986, 987, 988, 989, 990, 991, 992, 993, 994, 995, 996, 997, 998, 999, 1000, 1001, 1002, 1004, 1005, 1006, 1007, 1008, 1009, 1010, 1011, 1012, 1013, 1019, 1020, 1021, 1022, 1023, 1024, 1025, 1029, 1030, 1033, 1034, 1035, 1036, 1037, 1038, 1039, 1040, 1041, 1042, 1046, 1047, 1048, 1049, 1056, 1057, 1058, 1059 }

2.1.7 Giac

A grade: { 1, 2, 15, 16, 17, 18, 19, 20, 21, 22, 23, 30, 32, 33, 40, 42, 43, 47, 48, 70, 72, 75, 77, 79, 87, 88, 90, 93, 94, 95, 97, 101, 103, 106, 110, 118, 119, 121, 122, 123, 143, 144, 145, 149, 150, 151, 152, 154, 155, 156, 157, 161, 162, 163, 164, 166, 170, 171, 176, 177, 182, 183, 184, 197, 202, 203, 204, 206, 209, 216, 217, 219, 222, 235, 237, 241, 243, 251, 252, 253, 255, 256, 257, 258, 260, 261, 262, 264, 265, 266, 267, 269, 270, 271, 273, 274, 275, 276, 281, 282, 283, 285, 286, 287, 288, 290, 291, 292, 293, 294, 295, 296, 297, 299, 300, 301, 303, 304, 305, 306, 308, 309, 310, 312, 313, 314, 315, 317, 318, 319, 321, 322, 323, 324, 326, 327, 328, 330, 331, 332, 333, 334, 339, 340, 341, 346, 347, 348, 353, 354, 355, 359, 360, 361, 362, 366, 367, 368, 369, 374, 375, 376, 381, 382, 383, 387, 388, 389, 390, 395, 396, 397, 402, 403, 404, 408, 409, 410, 411, 416, 417, 424, 429, 430, 431, 435, 436, 437, 438, 443, 444, 445, 450, 451, 452, 457, 458, 459, 464, 465, 466, 471, 472, 473, 478, 479, 480, 485, 486, 487, 492, 494, 498, 499, 500, 501, 505, 506, 507, 512, 513, 514, 519, 520, 521, 526, 527, 566, 567, 568, 569, 572, 573, 575, 576, 578, 579, 585, 586, 587, 588, 597, 598, 599, 600, 601, 602, 603, 604, 606, 607, 608, 609, 610, 611, 612, 613, 615, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 645, 647, 649, 650, 651, 652, 653, 654, 655, 656, 657, 660, 661, 662, 663, 664, 665, 666, 667, 669, 670, 671, 672, 673, 680, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700, 701, 706, 707, 708, 709, 710, 711, 712, 713, 714, 715, 716, 717, 718, 719, 720, 721, 722, 723, 724, 725, 726, 727, 728, 729, 730, 731, 732, 733, 734, 735, 736, 737, 738, 740, 742, 743, 746, 747, 754, 755, 756, 780, 781, 782, 783, 784, 785, 786, 787, 788, 789, 790, 792, 793, 795, 796, 798, 799, 801, 802, 803, 804, 805, 806, 807, 808, 809, 811, 812, 813, 814, 815, 816, 819, 822, 823, 824, 825, 826, 828, 829, 830, 831, 832, 834, 835, 837, 838, 839, 840, 841, 855, 856, 857, 858, 859, 860, 870, 872, 873, 874, 876, 877, 878, 879, 880, 885, 886, 887, 901, 902, 903, 904, 905, 906, 907, 908, 909, 910, 911, 912, 913, 914, 915, 916, 917, 918, 919, 920, 921, 922, 923, 924, 925, 926, 927, 928, 929, 930, 931, 932, 933, 934, 935, 936, 937, 938, 939, 940, 941, 942, 943, 944, 945, 946, 947, 948, 952, 953, 954, 955, 959, 960, 961, 962, 967, 968, 969, 973, 974, 975, 980, 981, 985, 986, 987, 988, 989, 995, 1014, 1015, 1016, 1017, 1020, 1033, 1034, 1035, 1036, 1043, 1044, 1046, 1052, 1053, 1055, 1056, 1058, 1059 }

B grade: { 8, 9, 10, 11, 12, 13, 14, 24, 25, 26, 27, 28, 29, 31, 34, 35, 36, 37, 38, 39, 41, 44, 45, 46, 69, 71, 73, 74, 76, 78, 80, 81, 82, 84, 85, 91, 96, 98, 99, 100, 102, 104, 105, 107, 108, 109, 111, 112, 113, 115, 116, 124, 125, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 146, 147, 148, 153, 158, 159, 160, 165, 167, 168, 169, 172, 173, 174, 175, 178, 179, 180, 181, 185, 186, 187, 192, 193, 194, 195, 196, 198, 199, 200, 201, 207, 208, 210, 211, 212, 213, 214, 215, 218, 220, 221, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 236, 238, 240, 242, 244, 245, 246, 247, 248, 249, 254, 263, 272, 277, 278, 279, 284, 302, 311, 320, 329, 338, 344, 345, 350, 351, 352, 365, 373, 380, 386, 394, 401, 415, 418, 427, 428, 434, 441, 442, 447, 448, 449, 456, 463, 470, 477, 484, 491, 497, 504, 511, 518, 525, 570, 571, 574, 577, 580, 581, 582, 583, 584, 589, 596, 605, 614, 616, 644, 646, 648, 658, 659, 668, 674, 675, 676, 681, 682, 686, 687, 702, 703, 704, 705, 739, 741, 744, 745, 748, 749, 750, 751, 752, 753, 768, 769, 770, 771, 774, 775, 776, 777, 791, 794, 797, 800, 810, 817, 818, 820, 821, 836, 875, 893, 894, 979, 982, 983, 984, 990, 991, 992, 993, 994, 996, 997, 998, 999, 1001, 1002, 1003, 1006, 1007, 1010, 1011, 1013, 1018, 1019, 1021, 1022, 1023, 1024, 1025, 1026, 1027, 1030, 1031, 1032, 1040, 1045, 1047, 1054, 1057 }

C grade: { 897, 898, 1050, 1051 }

F grade: { 3, 4, 5, 6, 7, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 83, 86, 89, 92, 114, 117, 120, 126, 188, 189, 190, 191, 205, 239, 250, 259, 268, 280, 289, 298, 307, 316, 325, 335, 336, 337, 342, 343, 349, 356, 357, 358, 363, 364, 370, 371, 372, 377, 378, 379, 384, 385, 391, 392, 393, 398, 399, 400, 405, 406, 407, 412, 413, 414, 419, 420, 421, 422, 423, 425, 426, 432, 433, 439, 440, 446, 453, 454, 455, 460, 461, 462, 467, 468, 469, 474, 475, 476, 481, 482, 483, 488, 489, 490, 493, 495, 496, 502, 503, 508, 509, 510, 515, 516, 517, 522, 523, 524, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 590, 591, 592, 593, 594, 595, 677, 678, 679, 683, 684, 685, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 767, 772, 773, 778, 779, 827, 833, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 853, 854, 861, 862, 863, 864, 865, 866, 867, 868, 869, 871, 881, 882, 883, 884, 888, 889, 890, 891, 892, 895, 896, 899, 900, 949, 950, 951, 956, 957, 958, 963, 964, 965, 966, 970, 971, 972, 976, 977, 978, 1000, 1004, 1005, 1008, 1009, 1012, 1028, 1029, 1037, 1038, 1039, 1041, 1042, 1048, 1049 }

2.1.8 Mupad

A grade: { 334, 339, 340, 341, 346, 347, 348, 353, 354, 355, 360, 361, 362, 367, 368, 369, 374, 375, 376, 381, 382, 383, 388, 389, 390, 395, 396, 397, 402, 403, 404, 409, 410, 411, 416, 417, 424, 429, 430, 431, 436, 437, 438, 443, 444, 445, 450, 451, 452, 457, 458, 459, 464, 465, 466, 471, 472, 473, 478, 479, 480, 485, 486, 487, 492, 493, 494, 499, 500, 501, 505, 506, 507, 512, 513, 514, 519, 520, 521, 526, 527, 870, 1014, 1015, 1016, 1017 }

B grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 67, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 146, 147, 149, 150, 152, 153, 155, 156, 158, 159, 161, 162, 164, 165, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 251, 252, 253, 254, 260, 261, 262, 263, 269, 270, 271, 272, 281, 282, 283, 284, 290, 291, 292, 293, 299, 300, 301, 302, 308, 309, 310, 311, 317, 318, 319, 320, 326, 327, 328, 329, 338, 344, 345, 350, 351, 352, 359, 365, 366, 373, 380, 386, 387, 394, 401, 408, 415, 418, 427, 428, 434, 435, 441, 442, 447, 448, 449, 456, 463, 470, 477, 484, 491, 497, 498, 504, 511, 518, 525, 560, 563, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 680, 681, 682, 683, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700, 701, 702, 703, 704, 705, 706, 707, 708, 709, 710, 711, 712, 713, 714, 715, 716, 717, 718, 719, 720, 721, 722, 723, 724, 725, 726, 727, 728, 729, 730, 731, 732, 733, 734, 735, 736, 737, 738, 739, 740, 741, 742, 746, 747, 748, 749, 753, 754, 755, 756, 780, 781, 782, 783, 784, 785, 786, 787, 788, 789, 792, 795, 798, 802, 803, 804, 805, 806, 807, 808, 809, 810, 811, 812, 813, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 830, 836, 838, 839, 840, 841, 855, 856, 857, 858, 859, 872, 873, 874, 875, 876, 877, 878, 879, 880, 885, 886, 887, 893, 894, 897, 898, 901, 902, 903, 904, 905, 906, 907, 908, 909, 910, 911, 912, 913, 914, 915, 916, 917, 918, 919, 920, 921, 922, 923, 924, 925, 926, 927, 928, 929, 930, 931, 932, 933, 934, 935, 936, 937, 938, 939, 940, 941, 942, 943, 944, 945, 946, 947, 948, 952, 953, 954, 955, 959, 960, 961, 962, 967, 968, 969, 973, 974, 975, 979, 980, 981, 982, 983, 984, 985, 986, 987, 988, 989, 990, 991, 992, 993, 994, 995, 996, 997, 1003, 1004, 1005, 1006, 1007, 1008, 1009, 1010, 1011, 1012, 1013, 1018, 1019, 1020, 1021, 1022, 1023, 1024, 1025, 1026, 1027, 1028, 1029, 1030, 1031, 1032, 1033, 1034, 1035, 1036, 1040, 1042, 1043, 1044, 1045, 1046, 1047, 1048, 1049, 1052, 1053, 1054, 1055, 1056, 1057, 1058, 1059 }

C grade: { }

F grade: { 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 68, 145, 148, 151, 154, 157, 160, 163, 166, 188, 189, 190, 191, 205, 239, 250, 255, 256, 257, 258, 259, 264, 265, 266, 267, 268, 273, 274, 275, 276, 277, 278, 279, 280, 285, 286, 287, 288, 289, 294, 295, 296, 297, 298, 303, 304, 305, 306, 307, 312, 313, 314, 315, 316, 321, 322, 323, 324, 325, 330, 331, 332, 333, 335, 336, 337, 342, 343, 349, 356, 357, 358, 363, 364, 370, 371, 372, 377, 378, 379, 384, 385, 391, 392, 393, 398, 399, 400, 405, 406, 407, 412, 413, 414, 419, 420, 421, 422, 423, 425, 426, 432, 433, 439, 440, 446, 453, 454, 455, 460, 461, 462, 467, 468, 469, 474, 475, 476, 481, 482, 483, 488, 489, 490, 495, 496, 502, 503, 508, 509, 510, 515, 516, 517, 522, 523, 524, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 561, 562, 564, 565, 590, 591, 592, 593, 594, 595, 621, 622, 623, 651, 652, 653, 678, 679, 684, 685, 743, 744, 745, 750, 751, 752, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 777, 778, 779, 790, 791, 793, 794, 796, 797, 799, 800, 801, 826, 827, 828, 829, 831, 832, 833, 834, 835, 837, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 853, 854, 860, 861, 862, 863, 864, 865, 866, 867, 868, 869, 871, 881, 882, 883, 884, 888, 889, 890, 891, 892, 895, 896, 899, 900, 949, 950, 951, 956, 957, 958, 963, 964, 965, 966, 970, 971, 972, 976, 977, 978, 998, 999, 1000, 1001, 1002, 1037, 1038, 1039, 1041, 1050, 1051 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$

Problem 1	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	47	17	21	34	19	21	21
normalized size	1	1.00	2.14	0.77	0.95	1.55	0.86	0.95	0.95
time (sec)	N/A	0.016	0.075	0.131	0.484	0.410	0.273	0.133	0.119

Problem 2	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	47	156	21	47	185	21	21
normalized size	1	1.00	2.14	7.09	0.95	2.14	8.41	0.95	0.95
time (sec)	N/A	0.021	0.070	0.515	0.434	0.422	7.887	0.148	0.096

Problem 3	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	A	B	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	47	156	21	47	0	0	21
normalized size	1	1.00	2.14	7.09	0.95	2.14	0.00	0.00	0.95
time (sec)	N/A	0.040	0.083	0.604	0.565	0.445	0.000	0.000	1.486

Problem 4	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	A	B	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	47	156	21	47	0	0	21
normalized size	1	1.00	2.14	7.09	0.95	2.14	0.00	0.00	0.95
time (sec)	N/A	0.039	0.076	0.523	0.686	0.394	0.000	0.000	1.529

Problem 5	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	42	44	69	89	0	0	57
normalized size	1	1.00	1.91	2.00	3.14	4.05	0.00	0.00	2.59
time (sec)	N/A	0.039	0.105	0.450	0.410	0.468	0.000	0.000	0.502

Problem 6	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	A	B	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	47	132	21	47	0	0	21
normalized size	1	1.00	2.14	6.00	0.95	2.14	0.00	0.00	0.95
time (sec)	N/A	0.040	0.106	0.537	0.502	0.417	0.000	0.000	0.175

Problem 7	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	42	102	38	89	0	0	57
normalized size	1	1.00	1.91	4.64	1.73	4.05	0.00	0.00	2.59
time (sec)	N/A	0.039	0.111	0.451	0.518	0.448	0.000	0.000	1.882

Problem 8	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	37	14	13	22	19	29	13
normalized size	1	1.00	2.47	0.93	0.87	1.47	1.27	1.93	0.87
time (sec)	N/A	0.012	0.014	0.021	0.578	0.518	0.183	0.132	0.066

Problem 9	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	20	19	68	49	96	19
normalized size	1	1.00	1.00	1.05	1.00	3.58	2.58	5.05	1.00
time (sec)	N/A	0.023	0.009	0.123	0.376	0.498	1.248	0.293	1.548

Problem 10	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	39	0	373	175	638	327	135
normalized size	1	1.00	1.00	0.00	9.56	4.49	16.36	8.38	3.46
time (sec)	N/A	0.045	0.063	1.275	0.819	0.544	8.422	0.323	1.638

Problem 11	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	49	0	686	379	2574	722	255
normalized size	1	1.00	0.83	0.00	11.63	6.42	43.63	12.24	4.32
time (sec)	N/A	0.052	0.200	1.188	0.659	0.483	42.556	0.298	1.750

Problem 12	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	20	19	68	49	94	19
normalized size	1	1.00	1.00	1.05	1.00	3.58	2.58	4.95	1.00
time (sec)	N/A	0.027	0.010	0.067	0.382	0.605	1.257	0.206	1.579

Problem 13	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	44	923	293	189	648	325	132
normalized size	1	1.00	1.10	23.08	7.32	4.72	16.20	8.12	3.30
time (sec)	N/A	0.050	0.135	1.018	0.579	0.425	8.551	0.351	0.179

Problem 14	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	77	0	558	407	2351	721	254
normalized size	1	1.00	1.31	0.00	9.46	6.90	39.85	12.22	4.31
time (sec)	N/A	0.061	0.228	0.975	0.496	0.422	42.814	0.309	1.752

Problem 15	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	23	43	39	40	92	32	18
normalized size	1	1.00	0.50	0.93	0.85	0.87	2.00	0.70	0.39
time (sec)	N/A	0.044	0.032	0.087	0.327	0.381	0.841	0.228	0.113

Problem 16	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	40	61	88	90	136	88	43
normalized size	1	1.00	0.58	0.88	1.28	1.30	1.97	1.28	0.62
time (sec)	N/A	0.074	0.080	0.091	0.338	0.393	2.909	0.201	1.641

Problem 17	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	52	79	110	138	189	116	53
normalized size	1	1.00	0.57	0.86	1.20	1.50	2.05	1.26	0.58
time (sec)	N/A	0.104	0.129	0.091	0.324	0.426	8.318	0.205	1.728

Problem 18	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	40	56	88	90	136	88	42
normalized size	1	1.00	0.60	0.84	1.31	1.34	2.03	1.31	0.63
time (sec)	N/A	0.052	0.061	0.329	0.311	0.389	2.867	0.159	1.601

Problem 19	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	33	74	66	97	189	60	32
normalized size	1	1.00	0.37	0.82	0.73	1.08	2.10	0.67	0.36
time (sec)	N/A	0.084	0.045	0.331	0.311	0.390	8.288	0.235	0.203

Problem 20	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	62	92	132	197	231	144	65
normalized size	1	1.00	0.55	0.81	1.17	1.74	2.04	1.27	0.58
time (sec)	N/A	0.118	0.184	0.299	0.343	0.437	21.040	0.156	1.735

Problem 21	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	52	66	110	138	189	116	53
normalized size	1	1.00	0.59	0.75	1.25	1.57	2.15	1.32	0.60
time (sec)	N/A	0.064	0.098	0.328	0.438	0.389	8.352	0.155	1.677

Problem 22	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	62	84	132	195	231	144	65
normalized size	1	1.00	0.56	0.76	1.19	1.76	2.08	1.30	0.59
time (sec)	N/A	0.099	0.132	0.328	0.376	0.395	21.150	0.176	0.260

Problem 23	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	134	43	102	86	179	277	88	42
normalized size	1	1.00	0.32	0.76	0.64	1.34	2.07	0.66	0.31
time (sec)	N/A	0.132	0.090	0.335	0.338	0.386	47.861	0.239	1.820

Problem 24	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	31	12	50	60	0	41	30
normalized size	1	1.00	2.82	1.09	4.55	5.45	0.00	3.73	2.73
time (sec)	N/A	0.012	0.022	0.090	0.625	0.432	0.000	0.117	0.151

Problem 25	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	26	23	61	155	0	64	52
normalized size	1	1.00	1.13	1.00	2.65	6.74	0.00	2.78	2.26
time (sec)	N/A	0.026	0.034	0.093	0.357	0.432	0.000	0.137	0.089

Problem 26	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	36	26	88	371	0	93	78
normalized size	1	1.00	1.33	0.96	3.26	13.74	0.00	3.44	2.89
time (sec)	N/A	0.024	0.038	0.143	0.417	0.424	0.000	0.136	1.503

Problem 27	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	41	33	108	697	0	88	133
normalized size	1	1.00	1.08	0.87	2.84	18.34	0.00	2.32	3.50
time (sec)	N/A	0.029	0.028	0.162	0.313	0.437	0.000	0.138	1.509

Problem 28	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	46	39	131	1073	0	122	169
normalized size	1	1.00	1.15	0.98	3.28	26.82	0.00	3.05	4.22
time (sec)	N/A	0.033	0.100	0.170	0.414	0.450	0.000	0.154	1.463

Problem 29	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	29	27	43	103	0	54	48
normalized size	1	1.00	1.21	1.12	1.79	4.29	0.00	2.25	2.00
time (sec)	N/A	0.022	0.020	0.098	0.646	0.410	0.000	0.122	1.466

Problem 30	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	13	32	18	81	0	18	18
normalized size	1	1.00	0.57	1.39	0.78	3.52	0.00	0.78	0.78
time (sec)	N/A	0.031	0.016	0.327	0.437	0.409	0.000	0.146	0.084

Problem 31	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	29	52	90	511	0	102	107
normalized size	1	1.00	0.59	1.06	1.84	10.43	0.00	2.08	2.18
time (sec)	N/A	0.041	0.015	0.335	0.824	0.423	0.000	0.161	1.509

Problem 32	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	46	44	94	230	0	60	152
normalized size	1	1.00	1.21	1.16	2.47	6.05	0.00	1.58	4.00
time (sec)	N/A	0.035	0.040	0.305	0.306	0.399	0.000	0.150	1.520

Problem 33	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	29	71	136	1183	0	124	210
normalized size	1	1.00	0.41	1.01	1.94	16.90	0.00	1.77	3.00
time (sec)	N/A	0.043	0.015	0.334	0.409	0.437	0.000	0.155	0.073

Problem 34	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	34	27	91	379	0	93	78
normalized size	1	1.00	1.21	0.96	3.25	13.54	0.00	3.32	2.79
time (sec)	N/A	0.025	0.048	0.168	0.410	0.434	0.000	0.129	0.071

Problem 35	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	68	43	106	709	0	110	111
normalized size	1	1.00	1.39	0.88	2.16	14.47	0.00	2.24	2.27
time (sec)	N/A	0.046	0.034	0.178	0.315	0.389	0.000	0.133	1.459

Problem 36	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	47	48	102	774	0	96	96
normalized size	1	1.00	1.09	1.12	2.37	18.00	0.00	2.23	2.23
time (sec)	N/A	0.044	0.016	0.216	0.412	0.423	0.000	0.139	1.442

Problem 37	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	83	53	149	1573	0	128	192
normalized size	1	1.00	1.26	0.80	2.26	23.83	0.00	1.94	2.91
time (sec)	N/A	0.050	0.034	0.217	0.329	0.429	0.000	0.136	1.437

Problem 38	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	54	61	181	2103	0	171	187
normalized size	1	1.00	0.93	1.05	3.12	36.26	0.00	2.95	3.22
time (sec)	N/A	0.049	0.455	0.219	0.414	0.416	0.000	0.166	0.077

Problem 39	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	33	39	90	515	0	80	129
normalized size	1	1.00	0.89	1.05	2.43	13.92	0.00	2.16	3.49
time (sec)	N/A	0.027	0.017	0.146	0.434	0.430	0.000	0.141	0.076

Problem 40	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	45	50	90	229	0	60	153
normalized size	1	1.00	1.22	1.35	2.43	6.19	0.00	1.62	4.14
time (sec)	N/A	0.037	0.042	0.318	0.313	0.386	0.000	0.137	1.476

Problem 41	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	33	73	132	1176	0	124	187
normalized size	1	1.00	0.50	1.11	2.00	17.82	0.00	1.88	2.83
time (sec)	N/A	0.043	0.016	0.335	0.418	0.440	0.000	0.164	1.558

Problem 42	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	43	62	90	330	0	31	31
normalized size	1	1.00	0.81	1.17	1.70	6.23	0.00	0.58	0.58
time (sec)	N/A	0.040	0.020	0.324	0.310	0.407	0.000	0.147	0.062

Problem 43	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	33	92	178	2092	0	148	291
normalized size	1	1.00	0.37	1.03	2.00	23.51	0.00	1.66	3.27
time (sec)	N/A	0.048	0.019	0.360	0.511	0.424	0.000	0.148	1.465

Problem 44	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	46	39	133	1082	0	122	169
normalized size	1	1.00	1.18	1.00	3.41	27.74	0.00	3.13	4.33
time (sec)	N/A	0.032	0.128	0.174	0.467	0.493	0.000	0.154	0.063

Problem 45	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	105	61	155	1591	0	130	214
normalized size	1	1.00	1.50	0.87	2.21	22.73	0.00	1.86	3.06
time (sec)	N/A	0.050	0.037	0.151	0.603	0.431	0.000	0.157	1.447

Problem 46	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	56	69	179	2114	0	171	187
normalized size	1	1.00	0.97	1.19	3.09	36.45	0.00	2.95	3.22
time (sec)	N/A	0.045	0.200	0.217	0.800	0.446	0.000	0.148	1.472

Problem 47	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	121	71	195	2802	0	152	295
normalized size	1	1.00	1.36	0.80	2.19	31.48	0.00	1.71	3.31
time (sec)	N/A	0.055	0.040	0.222	0.344	0.462	0.000	0.139	0.097

Problem 48	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	77	81	150	2231	0	124	205
normalized size	1	1.00	1.12	1.17	2.17	32.33	0.00	1.80	2.97
time (sec)	N/A	0.054	0.027	0.216	0.902	0.482	0.000	0.166	1.422

Problem 49	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	B	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	59	0	0	997	0	0	-1
normalized size	1	1.00	0.56	0.00	0.00	9.41	0.00	0.00	-0.01
time (sec)	N/A	0.116	0.061	0.216	0.000	0.466	0.000	0.000	0.000

Problem 50	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	B	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	59	0	0	591	0	0	-1
normalized size	1	1.00	0.73	0.00	0.00	7.30	0.00	0.00	-0.01
time (sec)	N/A	0.081	0.049	0.192	0.000	0.447	0.000	0.000	0.000

Problem 51	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	59	0	0	310	0	0	-1
normalized size	1	1.00	0.75	0.00	0.00	3.92	0.00	0.00	-0.01
time (sec)	N/A	0.073	0.053	0.179	0.000	0.461	0.000	0.000	0.000

Problem 52	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	59	0	0	142	0	0	-1
normalized size	1	1.00	1.09	0.00	0.00	2.63	0.00	0.00	-0.02
time (sec)	N/A	0.037	0.035	0.283	0.000	0.424	0.000	0.000	0.000

Problem 53	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	57	0	0	144	0	0	-1
normalized size	1	1.00	1.06	0.00	0.00	2.67	0.00	0.00	-0.02
time (sec)	N/A	0.038	0.025	0.237	0.000	0.421	0.000	0.000	0.000

Problem 54	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	57	0	0	311	0	0	-1
normalized size	1	1.00	0.72	0.00	0.00	3.94	0.00	0.00	-0.01
time (sec)	N/A	0.071	0.032	0.181	0.000	0.424	0.000	0.000	0.000

Problem 55	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	B	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	59	0	0	598	0	0	-1
normalized size	1	1.00	0.73	0.00	0.00	7.38	0.00	0.00	-0.01
time (sec)	N/A	0.073	0.031	0.179	0.000	0.428	0.000	0.000	0.000

Problem 56	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	B	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	59	0	0	1001	0	0	-1
normalized size	1	1.00	0.56	0.00	0.00	9.44	0.00	0.00	-0.01
time (sec)	N/A	0.105	0.040	0.182	0.000	0.473	0.000	0.000	0.000

Problem 57	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	B	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	155	155	59	0	0	1042	0	0	-1
normalized size	1	1.00	0.38	0.00	0.00	6.72	0.00	0.00	-0.01
time (sec)	N/A	0.182	0.057	0.175	0.000	0.585	0.000	0.000	0.000

Problem 58	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	B	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	155	155	59	0	0	751	0	0	-1
normalized size	1	1.00	0.38	0.00	0.00	4.85	0.00	0.00	-0.01
time (sec)	N/A	0.182	0.059	0.160	0.000	0.440	0.000	0.000	0.000

Problem 59	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	243	243	59	0	0	1003	0	0	-1
normalized size	1	1.00	0.24	0.00	0.00	4.13	0.00	0.00	-0.00
time (sec)	N/A	0.244	0.052	0.156	0.000	0.461	0.000	0.000	0.000

Problem 60	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	218	218	59	0	0	727	0	0	-1
normalized size	1	1.00	0.27	0.00	0.00	3.33	0.00	0.00	-0.00
time (sec)	N/A	0.223	0.042	0.165	0.000	0.457	0.000	0.000	0.000

Problem 61	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	128	59	0	0	572	0	0	-1
normalized size	1	1.00	0.46	0.00	0.00	4.47	0.00	0.00	-0.01
time (sec)	N/A	0.092	0.037	0.208	0.000	0.511	0.000	0.000	0.000

Problem 62	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	128	59	0	0	578	0	0	-1
normalized size	1	1.00	0.46	0.00	0.00	4.52	0.00	0.00	-0.01
time (sec)	N/A	0.089	0.026	0.153	0.000	0.463	0.000	0.000	0.000

Problem 63	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	218	218	57	0	0	723	0	0	-1
normalized size	1	1.00	0.26	0.00	0.00	3.32	0.00	0.00	-0.00
time (sec)	N/A	0.201	0.029	0.183	0.000	0.472	0.000	0.000	0.000

Problem 64	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	243	243	57	0	0	1013	0	0	-1
normalized size	1	1.00	0.23	0.00	0.00	4.17	0.00	0.00	-0.00
time (sec)	N/A	0.236	0.030	0.147	0.000	0.454	0.000	0.000	0.000

Problem 65	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	B	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	155	155	59	0	0	749	0	0	-1
normalized size	1	1.00	0.38	0.00	0.00	4.83	0.00	0.00	-0.01
time (sec)	N/A	0.125	0.037	0.145	0.000	0.459	0.000	0.000	0.000

Problem 66	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	B	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	155	155	59	0	0	1056	0	0	-1
normalized size	1	1.00	0.38	0.00	0.00	6.81	0.00	0.00	-0.01
time (sec)	N/A	0.121	0.032	0.143	0.000	0.440	0.000	0.000	0.000

Problem 67	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F(-2)	B	B	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	0	61	93	0	0	6
normalized size	1	1.00	1.00	0.00	3.81	5.81	0.00	0.00	0.38
time (sec)	N/A	0.030	0.011	180.000	0.490	0.433	0.000	0.000	1.540

Problem 68	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F(-2)	B	B	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	0	61	93	0	0	-1
normalized size	1	1.00	1.00	0.00	3.81	5.81	0.00	0.00	-0.06
time (sec)	N/A	0.030	0.017	180.000	0.460	0.412	0.000	0.000	0.000

Problem 69	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	10	7	0	54	0	17	31
normalized size	1	1.00	1.00	0.70	0.00	5.40	0.00	1.70	3.10
time (sec)	N/A	0.026	0.008	0.076	0.000	0.456	0.000	0.125	1.459

Problem 70	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	24	41	86	0	32	49
normalized size	1	1.00	1.00	1.04	1.78	3.74	0.00	1.39	2.13
time (sec)	N/A	0.017	0.013	0.142	0.404	0.549	0.000	0.133	1.471

Problem 71	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	33	54	31	0	46	49
normalized size	1	1.00	1.00	1.57	2.57	1.48	0.00	2.19	2.33
time (sec)	N/A	0.026	0.032	0.127	0.308	0.407	0.000	0.150	1.485

Problem 72	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	48	70	91	463	0	65	107
normalized size	1	1.00	0.98	1.43	1.86	9.45	0.00	1.33	2.18
time (sec)	N/A	0.030	0.055	0.309	0.412	0.421	0.000	0.142	1.464

Problem 73	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	37	51	98	93	0	71	131
normalized size	1	1.00	1.00	1.38	2.65	2.51	0.00	1.92	3.54
time (sec)	N/A	0.033	0.037	0.156	0.311	0.462	0.000	0.152	1.497

Problem 74	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	25	27	56	197	0	57	48
normalized size	1	1.00	0.89	0.96	2.00	7.04	0.00	2.04	1.71
time (sec)	N/A	0.025	0.019	0.139	0.581	0.450	0.000	0.129	0.080

Problem 75	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	31	39	64	54	0	68	50
normalized size	1	1.00	0.78	0.98	1.60	1.35	0.00	1.70	1.25
time (sec)	N/A	0.040	0.113	0.122	0.420	0.463	0.000	0.150	1.475

Problem 76	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	35	47	103	742	0	96	97
normalized size	1	1.00	0.81	1.09	2.40	17.26	0.00	2.23	2.26
time (sec)	N/A	0.044	0.055	0.141	0.408	0.442	0.000	0.180	1.496

Problem 77	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	38	38	71	290	0	63	77
normalized size	1	1.00	1.00	1.00	1.87	7.63	0.00	1.66	2.03
time (sec)	N/A	0.026	0.015	0.138	0.597	0.460	0.000	0.129	0.090

Problem 78	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	40	52	79	63	0	76	78
normalized size	1	1.00	1.05	1.37	2.08	1.66	0.00	2.00	2.05
time (sec)	N/A	0.041	0.033	0.110	0.427	0.524	0.000	0.155	1.539

Problem 79	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	65	92	116	851	0	96	136
normalized size	1	1.00	0.98	1.39	1.76	12.89	0.00	1.45	2.06
time (sec)	N/A	0.042	0.106	0.336	0.418	0.453	0.000	0.181	1.523

Problem 80	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	34	39	81	457	0	86	77
normalized size	1	1.00	0.85	0.98	2.02	11.42	0.00	2.15	1.92
time (sec)	N/A	0.032	0.031	0.137	0.467	0.413	0.000	0.169	0.115

Problem 81	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	12	23	54	17	24	24
normalized size	1	1.00	1.00	1.09	2.09	4.91	1.55	2.18	2.18
time (sec)	N/A	0.013	0.007	0.082	0.304	0.447	0.244	0.116	1.439

Problem 82	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	13	84	22	27	13
normalized size	1	1.00	1.00	0.93	0.87	5.60	1.47	1.80	0.87
time (sec)	N/A	0.020	0.011	0.060	0.301	0.451	0.439	0.124	0.072

Problem 83	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	17	36	115	39	0	31
normalized size	1	1.00	1.00	1.06	2.25	7.19	2.44	0.00	1.94
time (sec)	N/A	0.030	0.040	0.100	0.826	0.402	0.494	0.000	1.477

Problem 84	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	42	13	138	0	31	31
normalized size	1	1.00	1.00	2.80	0.87	9.20	0.00	2.07	2.07
time (sec)	N/A	0.029	0.007	0.321	0.327	0.415	0.000	0.128	0.071

Problem 85	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	34	13	208	44	37	230
normalized size	1	1.00	1.00	2.27	0.87	13.87	2.93	2.47	15.33
time (sec)	N/A	0.030	0.005	0.144	0.305	0.435	1.527	0.143	0.108

Problem 86	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	20	19	69	0	0	42
normalized size	1	1.00	1.00	1.05	1.00	3.63	0.00	0.00	2.21
time (sec)	N/A	0.035	0.020	0.160	0.355	0.422	0.000	0.000	1.531

Problem 87	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	27	34	148	172	41	49	48
normalized size	1	1.00	1.00	1.26	5.48	6.37	1.52	1.81	1.78
time (sec)	N/A	0.024	0.032	0.156	0.448	0.441	0.847	0.156	1.412

Problem 88	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	31	34	214	345	46	52	251
normalized size	1	1.00	1.00	1.10	6.90	11.13	1.48	1.68	8.10
time (sec)	N/A	0.035	0.062	0.131	0.312	0.406	2.719	0.155	1.542

Problem 89	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	32	275	345	219	0	0	101
normalized size	1	1.00	0.89	7.64	9.58	6.08	0.00	0.00	2.81
time (sec)	N/A	0.049	0.133	0.449	0.542	0.448	0.000	0.000	1.569

Problem 90	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	56	52	276	304	0	53	270
normalized size	1	1.00	1.81	1.68	8.90	9.81	0.00	1.71	8.71
time (sec)	N/A	0.033	0.042	0.325	0.393	0.439	0.000	0.137	0.126

Problem 91	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	29	0	352	551	0	148	168
normalized size	1	1.00	0.83	0.00	10.06	15.74	0.00	4.23	4.80
time (sec)	N/A	0.037	0.112	0.714	0.440	0.435	0.000	0.180	1.634

Problem 92	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	73	535	504	180	0	0	115
normalized size	1	1.00	1.82	13.38	12.60	4.50	0.00	0.00	2.88
time (sec)	N/A	0.044	0.925	0.690	0.440	0.443	0.000	0.000	1.608

Problem 93	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	34	49	66	269	0	47	82
normalized size	1	1.00	1.00	1.44	1.94	7.91	0.00	1.38	2.41
time (sec)	N/A	0.024	0.016	0.342	0.471	0.419	0.000	0.132	0.086

Problem 94	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	59	90	112	814	0	71	186
normalized size	1	1.00	1.07	1.64	2.04	14.80	0.00	1.29	3.38
time (sec)	N/A	0.044	0.173	0.330	0.616	0.440	0.000	0.149	1.438

Problem 95	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	55	69	110	808	0	67	215
normalized size	1	1.00	1.00	1.25	2.00	14.69	0.00	1.22	3.91
time (sec)	N/A	0.045	0.023	0.309	0.479	0.469	0.000	0.153	1.459

Problem 96	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	28	191	185	29	35	129
normalized size	1	1.00	1.00	1.33	9.10	8.81	1.38	1.67	6.14
time (sec)	N/A	0.019	0.013	0.077	0.322	0.442	1.466	0.132	1.433

Problem 97	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	28	371	634	34	35	520
normalized size	1	1.00	1.00	1.12	14.84	25.36	1.36	1.40	20.80
time (sec)	N/A	0.029	0.016	0.098	0.347	0.489	22.408	0.115	1.547

Problem 98	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	48	46	85	925	0	73	200
normalized size	1	1.00	1.26	1.21	2.24	24.34	0.00	1.92	5.26
time (sec)	N/A	0.053	0.010	0.328	0.528	0.493	0.000	0.116	0.056

Problem 99	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	36	36	85	925	0	73	206
normalized size	1	1.00	1.00	1.00	2.36	25.69	0.00	2.03	5.72
time (sec)	N/A	0.037	0.009	0.332	0.440	0.405	0.000	0.134	1.445

Problem 100	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	67	72	857	778	0	66	820
normalized size	1	1.00	2.03	2.18	25.97	23.58	0.00	2.00	24.85
time (sec)	N/A	0.030	0.025	0.299	0.380	0.419	0.000	0.119	1.455

Problem 101	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	26	21	59	113	0	44	53
normalized size	1	1.00	1.13	0.91	2.57	4.91	0.00	1.91	2.30
time (sec)	N/A	0.018	0.028	0.109	0.332	0.507	0.000	0.141	0.072

Problem 102	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	33	56	31	0	50	49
normalized size	1	1.00	1.00	1.50	2.55	1.41	0.00	2.27	2.23
time (sec)	N/A	0.022	0.014	0.126	0.347	0.494	0.000	0.143	1.431

Problem 103	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	67	62	108	612	0	79	112
normalized size	1	1.00	1.37	1.27	2.20	12.49	0.00	1.61	2.29
time (sec)	N/A	0.036	0.033	0.312	0.345	0.438	0.000	0.175	0.078

Problem 104	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	37	51	100	89	0	71	131
normalized size	1	1.00	1.00	1.38	2.70	2.41	0.00	1.92	3.54
time (sec)	N/A	0.025	0.022	0.147	0.323	0.461	0.000	0.193	1.454

Problem 105	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	25	26	70	203	116	60	49
normalized size	1	1.00	0.93	0.96	2.59	7.52	4.30	2.22	1.81
time (sec)	N/A	0.024	0.023	0.109	0.358	0.429	11.851	0.144	0.074

Problem 106	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	31	39	66	60	0	67	50
normalized size	1	1.00	0.78	0.98	1.65	1.50	0.00	1.68	1.25
time (sec)	N/A	0.041	0.111	0.112	0.381	0.415	0.000	0.175	0.087

Problem 107	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	35	48	120	743	0	99	97
normalized size	1	1.00	0.81	1.12	2.79	17.28	0.00	2.30	2.26
time (sec)	N/A	0.042	0.051	0.174	0.338	0.455	0.000	0.205	1.458

Problem 108	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	44	31	87	357	0	77	81
normalized size	1	1.00	1.16	0.82	2.29	9.39	0.00	2.03	2.13
time (sec)	N/A	0.030	0.031	0.117	0.318	0.523	0.000	0.158	1.438

Problem 109	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	38	52	79	63	0	76	78
normalized size	1	1.00	1.00	1.37	2.08	1.66	0.00	2.00	2.05
time (sec)	N/A	0.035	0.019	0.112	0.328	0.445	0.000	0.173	0.097

Problem 110	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	85	81	133	1077	0	108	140
normalized size	1	1.00	1.29	1.23	2.02	16.32	0.00	1.64	2.12
time (sec)	N/A	0.048	0.043	0.344	0.359	0.538	0.000	0.218	1.486

Problem 111	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	35	39	95	457	0	87	77
normalized size	1	1.00	0.90	1.00	2.44	11.72	0.00	2.23	1.97
time (sec)	N/A	0.031	0.034	0.116	0.372	0.475	0.000	0.159	0.109

Problem 112	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	12	25	56	0	24	24
normalized size	1	1.00	1.00	1.09	2.27	5.09	0.00	2.18	2.18
time (sec)	N/A	0.011	0.010	0.110	0.398	0.379	0.000	0.135	0.058

Problem 113	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	13	86	22	27	13
normalized size	1	1.00	1.00	0.93	0.87	5.73	1.47	1.80	0.87
time (sec)	N/A	0.020	0.014	0.138	0.354	0.426	2.615	0.144	0.070

Problem 114	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	17	53	115	0	0	31
normalized size	1	1.00	1.00	1.06	3.31	7.19	0.00	0.00	1.94
time (sec)	N/A	0.028	0.022	0.099	0.523	0.483	0.000	0.000	1.482

Problem 115	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	42	13	139	0	31	31
normalized size	1	1.00	1.00	2.80	0.87	9.27	0.00	2.07	2.07
time (sec)	N/A	0.027	0.005	0.341	0.355	0.396	0.000	0.152	1.452

Problem 116	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	34	13	208	0	37	231
normalized size	1	1.00	1.00	2.27	0.87	13.87	0.00	2.47	15.40
time (sec)	N/A	0.029	0.005	0.159	0.387	0.407	0.000	0.217	1.493

Problem 117	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	21	20	70	0	0	43
normalized size	1	1.00	1.00	1.05	1.00	3.50	0.00	0.00	2.15
time (sec)	N/A	0.034	0.024	0.152	0.779	0.443	0.000	0.000	1.483

Problem 118	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	27	34	148	171	0	49	48
normalized size	1	1.00	1.00	1.26	5.48	6.33	0.00	1.81	1.78
time (sec)	N/A	0.021	0.014	0.178	0.321	0.394	0.000	0.148	1.412

Problem 119	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	31	34	214	343	0	52	252
normalized size	1	1.00	1.00	1.10	6.90	11.06	0.00	1.68	8.13
time (sec)	N/A	0.032	0.025	0.158	0.481	0.389	0.000	0.154	1.510

Problem 120	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	34	499	414	216	0	0	100
normalized size	1	1.00	0.92	13.49	11.19	5.84	0.00	0.00	2.70
time (sec)	N/A	0.042	0.074	0.499	0.496	0.446	0.000	0.000	1.550

Problem 121	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	57	45	84	387	0	58	87
normalized size	1	1.00	1.68	1.32	2.47	11.38	0.00	1.71	2.56
time (sec)	N/A	0.030	0.033	0.310	0.338	0.411	0.000	0.133	0.078

Problem 122	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	95	58	129	1109	0	80	219
normalized size	1	1.00	1.73	1.05	2.35	20.16	0.00	1.45	3.98
time (sec)	N/A	0.060	0.043	0.337	0.602	0.535	0.000	0.158	0.109

Problem 123	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	95	74	133	1114	0	86	190
normalized size	1	1.00	1.73	1.35	2.42	20.25	0.00	1.56	3.45
time (sec)	N/A	0.057	0.040	0.344	0.304	0.527	0.000	0.160	1.409

Problem 124	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	27	28	149	164	0	30	144
normalized size	1	1.00	1.59	1.65	8.76	9.65	0.00	1.76	8.47
time (sec)	N/A	0.026	0.024	0.336	0.317	0.484	0.000	0.132	1.461

Problem 125	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	18	139	222	0	29	210
normalized size	1	1.00	1.00	1.06	8.18	13.06	0.00	1.71	12.35
time (sec)	N/A	0.028	0.010	0.095	0.308	0.456	0.000	0.141	1.478

Problem 126	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	30	371	368	114	0	0	87
normalized size	1	1.00	1.15	14.27	14.15	4.38	0.00	0.00	3.35
time (sec)	N/A	0.035	0.091	0.535	0.451	0.465	0.000	0.000	1.541

Problem 127	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	84	46	98	1260	0	71	214
normalized size	1	1.00	2.21	1.21	2.58	33.16	0.00	1.87	5.63
time (sec)	N/A	0.065	0.020	0.337	0.347	0.454	0.000	0.135	1.393

Problem 128	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	47	50	431	430	0	48	413
normalized size	1	1.00	1.88	2.00	17.24	17.20	0.00	1.92	16.52
time (sec)	N/A	0.029	0.028	0.335	0.306	0.385	0.000	0.117	1.394

Problem 129	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	29	38	191	250	0	47	40
normalized size	1	1.00	1.00	1.31	6.59	8.62	0.00	1.62	1.38
time (sec)	N/A	0.019	0.018	0.130	0.538	0.417	0.000	0.125	1.453

Problem 130	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	33	38	435	442	0	54	372
normalized size	1	1.00	1.00	1.15	13.18	13.39	0.00	1.64	11.27
time (sec)	N/A	0.034	0.013	0.108	0.323	0.682	0.000	0.134	1.415

Problem 131	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	26	24	58	87	58	71	23
normalized size	1	1.00	0.96	0.89	2.15	3.22	2.15	2.63	0.85
time (sec)	N/A	0.027	0.034	0.086	0.315	0.459	0.737	0.119	0.156

Problem 132	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	27	24	59	75	61	78	23
normalized size	1	1.00	1.00	0.89	2.19	2.78	2.26	2.89	0.85
time (sec)	N/A	0.027	0.044	0.088	0.316	0.409	0.746	0.115	1.486

Problem 133	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	26	24	58	89	58	69	23
normalized size	1	1.00	0.96	0.89	2.15	3.30	2.15	2.56	0.85
time (sec)	N/A	0.019	0.024	0.172	0.635	0.412	0.738	0.114	1.440

Problem 134	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	26	24	59	77	58	74	23
normalized size	1	1.00	0.96	0.89	2.19	2.85	2.15	2.74	0.85
time (sec)	N/A	0.019	0.024	0.151	0.312	0.418	0.750	0.133	1.485

Problem 135	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	29	151	83	259	0	95	115
normalized size	1	1.00	0.78	4.08	2.24	7.00	0.00	2.57	3.11
time (sec)	N/A	0.070	0.517	0.197	0.426	0.435	0.000	0.138	1.926

Problem 136	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	30	149	87	216	0	86	121
normalized size	1	1.00	0.83	4.14	2.42	6.00	0.00	2.39	3.36
time (sec)	N/A	0.069	0.519	0.234	0.406	0.440	0.000	0.189	1.887

Problem 137	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	29	155	157	259	0	97	115
normalized size	1	1.00	0.78	4.19	4.24	7.00	0.00	2.62	3.11
time (sec)	N/A	0.035	0.505	0.268	0.397	0.453	0.000	0.157	0.491

Problem 138	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	32	153	160	216	0	90	121
normalized size	1	1.00	0.89	4.25	4.44	6.00	0.00	2.50	3.36
time (sec)	N/A	0.036	0.490	0.275	0.382	0.529	0.000	0.144	1.839

Problem 139	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	27	77	68	184	0	79	266
normalized size	1	1.00	0.75	2.14	1.89	5.11	0.00	2.19	7.39
time (sec)	N/A	0.025	0.213	0.155	0.621	0.594	0.000	0.131	2.393

Problem 140	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	27	75	67	156	0	70	268
normalized size	1	1.00	0.82	2.27	2.03	4.73	0.00	2.12	8.12
time (sec)	N/A	0.023	0.202	0.159	0.402	0.424	0.000	0.123	2.367

Problem 141	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	28	79	133	184	0	81	266
normalized size	1	1.00	0.78	2.19	3.69	5.11	0.00	2.25	7.39
time (sec)	N/A	0.024	0.229	0.130	0.320	0.527	0.000	0.123	1.775

Problem 142	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	29	77	129	156	0	74	269
normalized size	1	1.00	0.88	2.33	3.91	4.73	0.00	2.24	8.15
time (sec)	N/A	0.024	0.204	0.125	0.426	0.455	0.000	0.143	0.362

Problem 143	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	86	167	57	327	0	49	133
normalized size	1	1.00	2.97	5.76	1.97	11.28	0.00	1.69	4.59
time (sec)	N/A	0.023	0.063	0.223	0.458	0.543	0.000	0.134	1.866

Problem 144	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	102	205	105	902	0	88	173
normalized size	1	1.00	2.27	4.56	2.33	20.04	0.00	1.96	3.84
time (sec)	N/A	0.055	0.106	0.324	0.423	0.513	0.000	0.151	1.624

Problem 145	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	70	240	149	1737	0	112	-1
normalized size	1	1.00	0.97	3.33	2.07	24.12	0.00	1.56	-0.01
time (sec)	N/A	0.083	0.349	0.326	0.426	0.463	0.000	0.143	0.000

Problem 146	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	93	155	94	439	0	87	139
normalized size	1	1.00	3.21	5.34	3.24	15.14	0.00	3.00	4.79
time (sec)	N/A	0.020	0.059	0.203	0.333	0.427	0.000	0.124	0.154

Problem 147	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	110	197	140	1237	0	121	181
normalized size	1	1.00	2.39	4.28	3.04	26.89	0.00	2.63	3.93
time (sec)	N/A	0.046	0.103	0.229	0.334	0.449	0.000	0.138	1.585

Problem 148	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	70	230	186	2372	0	153	-1
normalized size	1	1.00	0.96	3.15	2.55	32.49	0.00	2.10	-0.01
time (sec)	N/A	0.087	0.357	0.276	0.331	0.462	0.000	0.160	0.000

Problem 149	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	26	148	49	87	0	49	65
normalized size	1	1.00	1.00	5.69	1.88	3.35	0.00	1.88	2.50
time (sec)	N/A	0.016	0.126	0.136	0.318	0.431	0.000	0.111	0.223

Problem 150	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	83	181	70	405	0	68	150
normalized size	1	1.00	2.37	5.17	2.00	11.57	0.00	1.94	4.29
time (sec)	N/A	0.033	0.090	0.257	0.407	0.472	0.000	0.117	1.587

Problem 151	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	35	58	120	246	0	51	-1
normalized size	1	1.00	0.92	1.53	3.16	6.47	0.00	1.34	-0.03
time (sec)	N/A	0.044	0.172	0.183	0.336	0.417	0.000	0.142	0.000

Problem 152	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	26	150	84	86	0	51	65
normalized size	1	1.00	1.00	5.77	3.23	3.31	0.00	1.96	2.50
time (sec)	N/A	0.015	0.119	0.135	0.323	0.435	0.000	0.114	1.573

Problem 153	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	90	172	103	617	0	104	156
normalized size	1	1.00	2.50	4.78	2.86	17.14	0.00	2.89	4.33
time (sec)	N/A	0.032	0.083	0.152	0.325	0.459	0.000	0.140	0.199

Problem 154	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	35	57	131	246	0	53	-1
normalized size	1	1.00	0.90	1.46	3.36	6.31	0.00	1.36	-0.03
time (sec)	N/A	0.044	0.185	0.172	0.327	0.428	0.000	0.119	0.000

Problem 155	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	86	167	59	327	0	53	133
normalized size	1	1.00	2.97	5.76	2.03	11.28	0.00	1.83	4.59
time (sec)	N/A	0.019	0.057	0.257	0.421	0.433	0.000	0.123	0.155

Problem 156	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	102	207	103	902	0	86	173
normalized size	1	1.00	2.27	4.60	2.29	20.04	0.00	1.91	3.84
time (sec)	N/A	0.042	0.099	0.346	0.411	0.446	0.000	0.132	0.194

Problem 157	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	115	238	149	1737	0	114	-1
normalized size	1	1.00	1.60	3.31	2.07	24.12	0.00	1.58	-0.01
time (sec)	N/A	0.079	0.320	0.359	0.431	0.455	0.000	0.144	0.000

Problem 158	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	93	155	90	439	0	85	139
normalized size	1	1.00	3.21	5.34	3.10	15.14	0.00	2.93	4.79
time (sec)	N/A	0.020	0.057	0.221	0.326	0.444	0.000	0.121	0.148

Problem 159	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	110	195	144	1237	0	125	183
normalized size	1	1.00	2.39	4.24	3.13	26.89	0.00	2.72	3.98
time (sec)	N/A	0.043	0.098	0.258	0.360	0.513	0.000	0.147	1.551

Problem 160	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	70	228	184	2372	0	151	-1
normalized size	1	1.00	0.96	3.12	2.52	32.49	0.00	2.07	-0.01
time (sec)	N/A	0.086	0.342	0.265	0.325	0.457	0.000	0.159	0.000

Problem 161	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	26	146	51	86	0	50	64
normalized size	1	1.00	1.00	5.62	1.96	3.31	0.00	1.92	2.46
time (sec)	N/A	0.015	0.114	0.159	0.321	0.513	0.000	0.116	0.204

Problem 162	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	83	183	70	405	0	68	148
normalized size	1	1.00	2.37	5.23	2.00	11.57	0.00	1.94	4.23
time (sec)	N/A	0.031	0.087	0.279	0.416	0.447	0.000	0.138	1.547

Problem 163	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	35	56	119	248	0	49	-1
normalized size	1	1.00	0.92	1.47	3.13	6.53	0.00	1.29	-0.03
time (sec)	N/A	0.042	0.177	0.215	0.317	0.390	0.000	0.118	0.000

Problem 164	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	26	152	80	87	0	50	66
normalized size	1	1.00	1.00	5.85	3.08	3.35	0.00	1.92	2.54
time (sec)	N/A	0.015	0.112	0.180	0.319	0.495	0.000	0.134	1.519

Problem 165	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	90	170	105	617	0	106	156
normalized size	1	1.00	2.50	4.72	2.92	17.14	0.00	2.94	4.33
time (sec)	N/A	0.031	0.069	0.189	0.324	0.502	0.000	0.143	0.174

Problem 166	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	35	59	132	243	0	51	-1
normalized size	1	1.00	0.90	1.51	3.38	6.23	0.00	1.31	-0.03
time (sec)	N/A	0.043	0.179	0.206	0.322	0.434	0.000	0.122	0.000

Problem 167	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	43	40	0	72	153	85	42
normalized size	1	1.00	1.00	0.93	0.00	1.67	3.56	1.98	0.98
time (sec)	N/A	0.042	0.225	0.161	0.000	0.453	1.468	0.116	0.153

Problem 168	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	69	57	0	120	405	120	76
normalized size	1	1.00	1.11	0.92	0.00	1.94	6.53	1.94	1.23
time (sec)	N/A	0.063	0.796	0.067	0.000	0.454	6.354	0.145	0.248

Problem 169	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	86	84	0	218	918	179	182
normalized size	1	1.00	0.95	0.92	0.00	2.40	10.09	1.97	2.00
time (sec)	N/A	0.075	0.485	0.300	0.000	0.439	30.311	0.147	0.512

Problem 170	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	106	83	0	192	1027	156	152
normalized size	1	1.00	1.20	0.94	0.00	2.18	11.67	1.77	1.73
time (sec)	N/A	0.069	0.738	0.328	0.000	0.425	21.616	0.136	1.915

Problem 171	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	144	158	133	0	414	2006	260	337
normalized size	1	1.00	1.10	0.92	0.00	2.88	13.93	1.81	2.34
time (sec)	N/A	0.122	1.603	0.105	0.000	0.436	109.063	0.140	1.936

Problem 172	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	195	195	177	184	0	731	0	373	906
normalized size	1	1.00	0.91	0.94	0.00	3.75	0.00	1.91	4.65
time (sec)	N/A	0.144	1.590	0.544	0.000	0.463	0.000	0.148	2.080

Problem 173	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	43	40	0	71	153	85	42
normalized size	1	1.00	1.00	0.93	0.00	1.65	3.56	1.98	0.98
time (sec)	N/A	0.040	0.184	0.206	0.000	0.428	1.473	0.134	0.142

Problem 174	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	69	57	0	115	408	120	68
normalized size	1	1.00	1.11	0.92	0.00	1.85	6.58	1.94	1.10
time (sec)	N/A	0.055	0.720	0.206	0.000	0.435	6.294	0.125	0.227

Problem 175	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	85	84	0	217	921	179	180
normalized size	1	1.00	0.93	0.92	0.00	2.38	10.12	1.97	1.98
time (sec)	N/A	0.069	0.457	0.329	0.000	0.452	30.704	0.131	1.898

Problem 176	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	105	83	0	192	1027	156	115
normalized size	1	1.00	1.19	0.94	0.00	2.18	11.67	1.77	1.31
time (sec)	N/A	0.067	0.705	0.328	0.000	0.441	21.549	0.145	1.916

Problem 177	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	144	158	133	0	397	2003	260	337
normalized size	1	1.00	1.10	0.92	0.00	2.76	13.91	1.81	2.34
time (sec)	N/A	0.100	1.600	0.386	0.000	0.432	109.365	0.153	1.949

Problem 178	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	195	195	176	184	0	726	0	373	908
normalized size	1	1.00	0.90	0.94	0.00	3.72	0.00	1.91	4.66
time (sec)	N/A	0.133	1.502	0.553	0.000	0.550	0.000	0.146	2.025

Problem 179	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	43	40	0	71	153	85	42
normalized size	1	1.00	1.00	0.93	0.00	1.65	3.56	1.98	0.98
time (sec)	N/A	0.045	0.205	0.064	0.000	0.525	1.477	0.117	0.140

Problem 180	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	69	57	0	119	408	120	68
normalized size	1	1.00	1.11	0.92	0.00	1.92	6.58	1.94	1.10
time (sec)	N/A	0.057	0.710	0.089	0.000	0.467	6.415	0.146	0.230

Problem 181	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	85	84	0	213	921	179	182
normalized size	1	1.00	0.93	0.92	0.00	2.34	10.12	1.97	2.00
time (sec)	N/A	0.082	0.462	0.069	0.000	0.484	31.093	0.129	1.871

Problem 182	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	74	63	0	114	411	124	76
normalized size	1	1.00	1.09	0.93	0.00	1.68	6.04	1.82	1.12
time (sec)	N/A	0.054	0.734	0.208	0.000	0.430	6.535	0.120	1.650

Problem 183	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	107	83	0	192	1027	156	135
normalized size	1	1.00	1.22	0.94	0.00	2.18	11.67	1.77	1.53
time (sec)	N/A	0.066	0.722	0.337	0.000	0.468	21.978	0.129	1.959

Problem 184	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	144	158	133	0	398	2006	260	337
normalized size	1	1.00	1.10	0.92	0.00	2.76	13.93	1.81	2.34
time (sec)	N/A	0.093	1.526	0.388	0.000	0.446	108.484	0.168	1.983

Problem 185	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	90	90	0	243	935	183	183
normalized size	1	1.00	0.93	0.93	0.00	2.51	9.64	1.89	1.89
time (sec)	N/A	0.081	0.515	0.103	0.000	0.523	30.372	0.143	0.529

Problem 186	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	153	127	0	443	2030	256	337
normalized size	1	1.00	1.11	0.92	0.00	3.21	14.71	1.86	2.44
time (sec)	N/A	0.114	1.636	0.099	0.000	0.414	109.986	0.155	2.002

Problem 187	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	195	195	176	184	0	729	0	373	908
normalized size	1	1.00	0.90	0.94	0.00	3.74	0.00	1.91	4.66
time (sec)	N/A	0.153	1.688	0.124	0.000	0.507	0.000	0.148	2.094

Problem 188	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	278	0	0	0	0	0	-1
normalized size	1	1.00	2.30	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.106	14.699	0.205	0.000	0.475	0.000	0.000	0.000

Problem 189	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	240	0	0	0	0	0	-1
normalized size	1	1.00	2.05	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.109	6.919	0.255	0.000	0.456	0.000	0.000	0.000

Problem 190	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	99	0	0	0	0	0	-1
normalized size	1	1.00	0.85	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.106	9.565	0.293	0.000	0.498	0.000	0.000	0.000

Problem 191	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	103	0	0	0	0	0	-1
normalized size	1	1.00	0.86	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.100	11.707	0.240	0.000	0.421	0.000	0.000	0.000

Problem 192	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	15	15	7	27	17	20	25	6
normalized size	1	1.88	1.88	0.88	3.38	2.12	2.50	3.12	0.75
time (sec)	N/A	0.009	0.006	0.092	0.450	0.403	0.444	0.128	0.067

Problem 193	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	27	22	20	27	13
normalized size	1	1.00	1.00	0.82	1.59	1.29	1.18	1.59	0.76
time (sec)	N/A	0.009	0.007	0.190	0.299	0.404	0.414	0.129	0.058

Problem 194	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	27	36	20	27	14
normalized size	1	1.00	1.00	0.82	1.59	2.12	1.18	1.59	0.82
time (sec)	N/A	0.010	0.007	0.162	0.481	0.411	0.408	0.129	0.055

Problem 195	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	25	28	0	42	78	59	26
normalized size	1	1.00	0.71	0.80	0.00	1.20	2.23	1.69	0.74
time (sec)	N/A	0.032	0.043	0.174	0.000	0.408	0.772	0.118	0.091

Problem 196	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	12	27	19	20	25	11
normalized size	1	1.00	1.00	0.80	1.80	1.27	1.33	1.67	0.73
time (sec)	N/A	0.010	0.006	0.133	0.302	0.401	0.448	0.109	1.419

Problem 197	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	27	33	20	26	11
normalized size	1	1.00	1.00	0.82	1.59	1.94	1.18	1.53	0.65
time (sec)	N/A	0.011	0.006	0.162	0.303	0.457	0.407	0.110	0.058

Problem 198	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	27	38	20	27	15
normalized size	1	1.00	1.00	0.82	1.59	2.24	1.18	1.59	0.88
time (sec)	N/A	0.011	0.007	0.196	0.335	0.414	0.411	0.109	1.434

Problem 199	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	25	28	0	42	37	59	26
normalized size	1	1.00	0.71	0.80	0.00	1.20	1.06	1.69	0.74
time (sec)	N/A	0.035	0.042	0.068	0.000	0.495	0.905	0.115	1.441

Problem 200	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	54	53	115	0	36	47
normalized size	1	1.00	1.00	2.84	2.79	6.05	0.00	1.89	2.47
time (sec)	N/A	0.026	0.015	0.259	0.400	0.434	0.000	0.133	1.472

Problem 201	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	60	46	76	0	43	23
normalized size	1	1.00	1.00	3.16	2.42	4.00	0.00	2.26	1.21
time (sec)	N/A	0.043	0.030	0.287	0.438	0.518	0.000	0.137	1.441

Problem 202	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	69	42	0	168	0	71	71
normalized size	1	1.00	1.00	0.61	0.00	2.43	0.00	1.03	1.03
time (sec)	N/A	0.096	0.148	0.321	0.000	0.495	0.000	0.217	0.773

Problem 203	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	81	60	0	237	0	81	82
normalized size	1	1.00	0.93	0.69	0.00	2.72	0.00	0.93	0.94
time (sec)	N/A	0.291	0.199	0.343	0.000	0.457	0.000	0.149	2.665

Problem 204	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	87	84	0	205	0	100	98
normalized size	1	1.00	1.00	0.97	0.00	2.36	0.00	1.15	1.13
time (sec)	N/A	0.259	0.132	0.363	0.000	0.584	0.000	0.149	2.667

Problem 205	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	164	0	0	0	0	0	-1
normalized size	1	1.00	2.02	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.070	0.181	0.170	0.000	0.498	0.000	0.000	0.000

Problem 206	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	10	9	16	42	0	16	16
normalized size	1	1.00	1.00	0.90	1.60	4.20	0.00	1.60	1.60
time (sec)	N/A	0.023	0.009	0.141	0.436	0.440	0.000	0.127	0.052

Problem 207	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	51	49	118	0	36	47
normalized size	1	1.00	1.00	2.55	2.45	5.90	0.00	1.80	2.35
time (sec)	N/A	0.027	0.017	0.217	0.444	0.429	0.000	0.132	0.078

Problem 208	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	28	143	60	128	0	54	52
normalized size	1	1.00	1.00	5.11	2.14	4.57	0.00	1.93	1.86
time (sec)	N/A	0.053	0.032	0.309	0.462	0.502	0.000	0.138	1.428

Problem 209	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	76	246	0	230	0	75	141
normalized size	1	1.00	0.93	3.00	0.00	2.80	0.00	0.91	1.72
time (sec)	N/A	0.187	0.241	0.404	0.000	0.468	0.000	0.160	3.335

Problem 210	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	38	172	0	164	0	68	56
normalized size	1	1.00	1.00	4.53	0.00	4.32	0.00	1.79	1.47
time (sec)	N/A	0.077	0.061	0.338	0.000	0.464	0.000	0.145	1.605

Problem 211	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	155	39	42	35	0	38	35
normalized size	1	1.00	9.69	2.44	2.62	2.19	0.00	2.38	2.19
time (sec)	N/A	0.019	0.297	0.226	0.403	0.487	0.000	0.131	0.093

Problem 212	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	17	26	45	52	0	41	27
normalized size	1	1.00	0.81	1.24	2.14	2.48	0.00	1.95	1.29
time (sec)	N/A	0.030	0.009	0.211	0.437	0.432	0.000	0.113	1.463

Problem 213	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	110	40	0	215	0	115	251
normalized size	1	1.00	1.55	0.56	0.00	3.03	0.00	1.62	3.54
time (sec)	N/A	0.074	0.025	0.249	0.000	0.453	0.000	0.194	1.481

Problem 214	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	57	101	0	182	0	118	100
normalized size	1	1.00	0.92	1.63	0.00	2.94	0.00	1.90	1.61
time (sec)	N/A	0.083	0.085	0.237	0.000	0.440	0.000	0.159	1.519

Problem 215	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	385	78	0	250	0	154	288
normalized size	1	1.00	4.53	0.92	0.00	2.94	0.00	1.81	3.39
time (sec)	N/A	0.079	0.282	0.287	0.000	0.460	0.000	0.149	1.725

Problem 216	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	7	7	7	4	7	6	0	3	3
normalized size	1	1.00	1.00	0.57	1.00	0.86	0.00	0.43	0.43
time (sec)	N/A	0.013	0.003	0.157	0.430	0.425	0.000	0.111	0.046

Problem 217	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	44	40	39	31	0	19	19
normalized size	1	1.00	2.93	2.67	2.60	2.07	0.00	1.27	1.27
time (sec)	N/A	0.036	0.026	0.233	0.435	0.435	0.000	0.110	1.461

Problem 218	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	26	62	50	76	0	44	42
normalized size	1	1.00	1.00	2.38	1.92	2.92	0.00	1.69	1.62
time (sec)	N/A	0.027	0.023	0.224	0.435	0.445	0.000	0.120	0.070

Problem 219	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	84	41	0	171	0	68	282
normalized size	1	1.00	1.12	0.55	0.00	2.28	0.00	0.91	3.76
time (sec)	N/A	0.108	0.111	0.247	0.000	0.475	0.000	0.150	4.259

Problem 220	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	30	92	0	107	0	58	41
normalized size	1	1.00	0.83	2.56	0.00	2.97	0.00	1.61	1.14
time (sec)	N/A	0.045	0.036	0.247	0.000	0.442	0.000	0.144	0.199

Problem 221	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	15	15	7	27	19	20	25	6
normalized size	1	1.88	1.88	0.88	3.38	2.38	2.50	3.12	0.75
time (sec)	N/A	0.011	0.006	0.096	0.336	0.449	0.451	0.132	0.052

Problem 222	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	12	27	33	20	26	11
normalized size	1	1.00	1.00	0.71	1.59	1.94	1.18	1.53	0.65
time (sec)	N/A	0.011	0.006	0.110	0.328	0.474	0.415	0.110	1.453

Problem 223	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	27	36	20	27	14
normalized size	1	1.00	1.00	0.82	1.59	2.12	1.18	1.59	0.82
time (sec)	N/A	0.011	0.006	0.100	0.340	0.423	0.409	0.112	1.443

Problem 224	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	25	28	0	42	42	59	26
normalized size	1	1.00	0.71	0.80	0.00	1.20	1.20	1.69	0.74
time (sec)	N/A	0.033	0.039	0.044	0.000	0.513	0.757	0.138	1.466

Problem 225	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	12	27	17	20	25	9
normalized size	1	1.00	1.00	0.80	1.80	1.13	1.33	1.67	0.60
time (sec)	N/A	0.009	0.006	0.156	0.342	0.433	0.447	0.135	0.059

Problem 226	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	27	20	20	27	20
normalized size	1	1.00	1.00	0.82	1.59	1.18	1.18	1.59	1.18
time (sec)	N/A	0.009	0.006	0.208	0.335	0.393	0.410	0.110	1.453

Problem 227	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	27	34	20	27	15
normalized size	1	1.00	1.00	0.82	1.59	2.00	1.18	1.59	0.88
time (sec)	N/A	0.009	0.005	0.226	0.334	0.409	0.413	0.110	1.445

Problem 228	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	25	28	0	42	56	59	26
normalized size	1	1.00	0.71	0.80	0.00	1.20	1.60	1.69	0.74
time (sec)	N/A	0.029	0.038	0.179	0.000	0.419	0.916	0.114	0.065

Problem 229	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	164	16	52	73	0	45	48
normalized size	1	1.00	8.63	0.84	2.74	3.84	0.00	2.37	2.53
time (sec)	N/A	0.032	0.181	0.066	0.442	0.447	0.000	0.113	1.452

Problem 230	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	55	17	153	82	0	45	53
normalized size	1	1.00	2.75	0.85	7.65	4.10	0.00	2.25	2.65
time (sec)	N/A	0.030	0.061	0.081	0.458	0.414	0.000	0.118	1.463

Problem 231	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	113	66	0	213	0	119	133
normalized size	1	1.00	1.64	0.96	0.00	3.09	0.00	1.72	1.93
time (sec)	N/A	0.087	0.025	0.158	0.000	0.523	0.000	0.214	0.078

Problem 232	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	249	70	0	293	0	127	141
normalized size	1	1.00	3.04	0.85	0.00	3.57	0.00	1.55	1.72
time (sec)	N/A	0.117	0.029	0.167	0.000	0.423	0.000	0.179	0.086

Problem 233	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	395	102	0	258	0	157	170
normalized size	1	1.00	4.54	1.17	0.00	2.97	0.00	1.80	1.95
time (sec)	N/A	0.251	0.306	0.179	0.000	0.480	0.000	0.157	0.097

Problem 234	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	14	9	29	52	0	26	29
normalized size	1	1.00	1.40	0.90	2.90	5.20	0.00	2.60	2.90
time (sec)	N/A	0.026	0.015	0.151	0.355	0.411	0.000	0.111	1.414

Problem 235	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	47	50	57	104	0	55	57
normalized size	1	1.00	1.04	1.11	1.27	2.31	0.00	1.22	1.27
time (sec)	N/A	0.063	0.019	0.244	0.438	0.442	0.000	0.119	0.057

Problem 236	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	192	63	70	101	0	67	71
normalized size	1	1.00	6.86	2.25	2.50	3.61	0.00	2.39	2.54
time (sec)	N/A	0.057	0.240	0.262	0.442	0.485	0.000	0.122	1.446

Problem 237	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	133	190	0	272	0	157	143
normalized size	1	1.00	1.21	1.73	0.00	2.47	0.00	1.43	1.30
time (sec)	N/A	0.176	0.121	0.275	0.000	0.440	0.000	0.155	0.097

Problem 238	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	95	87	0	157	0	89	101
normalized size	1	1.00	2.50	2.29	0.00	4.13	0.00	2.34	2.66
time (sec)	N/A	0.086	0.076	0.289	0.000	0.566	0.000	0.119	0.085

Problem 239	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	156	0	0	0	0	0	-1
normalized size	1	1.00	2.05	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.072	0.179	0.217	0.000	0.431	0.000	0.000	0.000

Problem 240	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	44	43	68	0	39	32
normalized size	1	1.00	1.00	2.93	2.87	4.53	0.00	2.60	2.13
time (sec)	N/A	0.016	0.007	0.224	0.441	0.409	0.000	0.134	0.092

Problem 241	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	48	40	114	31	0	19	19
normalized size	1	1.00	3.20	2.67	7.60	2.07	0.00	1.27	1.27
time (sec)	N/A	0.032	0.023	0.208	0.443	0.472	0.000	0.131	1.494

Problem 242	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	67	40	0	142	0	135	126
normalized size	1	1.00	0.94	0.56	0.00	2.00	0.00	1.90	1.77
time (sec)	N/A	0.039	0.106	0.257	0.000	0.448	0.000	0.218	2.949

Problem 243	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	84	41	0	173	0	68	297
normalized size	1	1.00	1.12	0.55	0.00	2.31	0.00	0.91	3.96
time (sec)	N/A	0.125	0.107	0.277	0.000	0.446	0.000	0.161	4.148

Problem 244	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	81	83	0	156	0	177	206
normalized size	1	1.00	0.95	0.98	0.00	1.84	0.00	2.08	2.42
time (sec)	N/A	0.058	0.091	0.322	0.000	0.463	0.000	0.131	3.526

Problem 245	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	7	7	11	6	19	19	0	16	19
normalized size	1	1.00	1.57	0.86	2.71	2.71	0.00	2.29	2.71
time (sec)	N/A	0.015	0.003	0.150	0.344	0.448	0.000	0.133	0.056

Problem 246	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	24	47	52	0	40	29
normalized size	1	1.00	1.00	1.14	2.24	2.48	0.00	1.90	1.38
time (sec)	N/A	0.031	0.010	0.223	0.435	0.403	0.000	0.112	1.487

Problem 247	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	183	53	60	54	0	57	61
normalized size	1	1.00	7.04	2.04	2.31	2.08	0.00	2.19	2.35
time (sec)	N/A	0.033	0.314	0.220	0.441	0.502	0.000	0.117	0.058

Problem 248	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	57	101	0	180	0	108	104
normalized size	1	1.00	0.92	1.63	0.00	2.90	0.00	1.74	1.68
time (sec)	N/A	0.078	0.062	0.233	0.000	0.434	0.000	0.137	1.513

Problem 249	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	91	77	0	101	0	79	91
normalized size	1	1.00	2.53	2.14	0.00	2.81	0.00	2.19	2.53
time (sec)	N/A	0.052	0.070	0.233	0.000	0.464	0.000	0.124	0.083

Problem 250	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	66	0	59	88	0	0	-1
normalized size	1	1.00	0.94	0.00	0.84	1.26	0.00	0.00	-0.01
time (sec)	N/A	0.118	0.050	0.131	0.417	0.623	0.000	0.000	0.000

Problem 251	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	50	203	86	74	119	73	64
normalized size	1	1.00	0.53	2.16	0.91	0.79	1.27	0.78	0.68
time (sec)	N/A	0.069	0.113	0.044	0.349	0.391	1.726	0.131	1.504

Problem 252	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	39	114	64	62	75	57	46
normalized size	1	1.00	0.61	1.78	1.00	0.97	1.17	0.89	0.72
time (sec)	N/A	0.043	0.075	0.048	0.322	0.437	0.821	0.112	0.081

Problem 253	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	28	53	46	42	56	41	28
normalized size	1	1.00	0.64	1.20	1.05	0.95	1.27	0.93	0.64
time (sec)	N/A	0.023	0.061	0.044	0.349	0.556	0.425	0.115	1.434

Problem 254	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	37	14	13	22	19	29	13
normalized size	1	1.00	2.47	0.93	0.87	1.47	1.27	1.93	0.87
time (sec)	N/A	0.014	0.011	0.016	0.376	0.603	0.183	0.132	1.426

Problem 255	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	25	26	23	37	0	23	-1
normalized size	1	1.00	0.93	0.96	0.85	1.37	0.00	0.85	-0.04
time (sec)	N/A	0.075	0.027	0.115	0.377	0.584	0.000	0.110	0.000

Problem 256	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	42	56	27	65	0	52	-1
normalized size	1	1.00	1.08	1.44	0.69	1.67	0.00	1.33	-0.03
time (sec)	N/A	0.095	0.075	0.134	0.389	0.467	0.000	0.137	0.000

Problem 257	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	61	90	30	104	0	86	-1
normalized size	1	1.00	1.02	1.50	0.50	1.73	0.00	1.43	-0.02
time (sec)	N/A	0.123	0.183	0.125	0.385	0.517	0.000	0.114	0.000

Problem 258	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	77	124	31	115	0	120	-1
normalized size	1	1.00	0.91	1.46	0.36	1.35	0.00	1.41	-0.01
time (sec)	N/A	0.151	0.167	0.125	0.389	0.445	0.000	0.117	0.000

Problem 259	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	134	114	0	113	162	0	0	-1
normalized size	1	1.00	0.85	0.00	0.84	1.21	0.00	0.00	-0.01
time (sec)	N/A	0.196	0.184	0.399	0.426	1.196	0.000	0.000	0.000

Problem 260	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	86	244	160	135	146	140	108
normalized size	1	1.00	0.74	2.09	1.37	1.15	1.25	1.20	0.92
time (sec)	N/A	0.121	0.413	0.316	0.333	0.493	2.900	0.132	1.541

Problem 261	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	65	131	122	105	105	108	69
normalized size	1	1.00	0.78	1.58	1.47	1.27	1.27	1.30	0.83
time (sec)	N/A	0.071	0.211	0.317	0.398	0.518	1.713	0.123	1.512

Problem 262	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	46	56	84	74	61	76	41
normalized size	1	1.00	1.02	1.24	1.87	1.64	1.36	1.69	0.91
time (sec)	N/A	0.031	0.145	0.318	0.318	0.716	0.810	0.119	0.085

Problem 263	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	13	38	20	54	13
normalized size	1	1.00	1.00	0.93	0.87	2.53	1.33	3.60	0.87
time (sec)	N/A	0.021	0.004	0.022	0.305	0.931	0.375	0.139	1.461

Problem 264	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	39	47	42	67	0	42	-1
normalized size	1	1.00	0.83	1.00	0.89	1.43	0.00	0.89	-0.02
time (sec)	N/A	0.141	0.074	0.349	0.532	0.627	0.000	0.116	0.000

Problem 265	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	70	104	50	124	0	90	-1
normalized size	1	1.00	0.88	1.30	0.62	1.55	0.00	1.12	-0.01
time (sec)	N/A	0.186	0.157	0.363	0.545	0.551	0.000	0.138	0.000

Problem 266	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	105	169	58	196	0	156	-1
normalized size	1	1.00	0.88	1.42	0.49	1.65	0.00	1.31	-0.01
time (sec)	N/A	0.249	0.294	0.367	0.408	0.608	0.000	0.138	0.000

Problem 267	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	154	138	234	58	223	0	223	-1
normalized size	1	1.00	0.90	1.52	0.38	1.45	0.00	1.45	-0.01
time (sec)	N/A	0.289	0.362	0.374	0.406	0.505	0.000	0.140	0.000

Problem 268	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	139	110	0	117	172	0	0	-1
normalized size	1	1.00	0.79	0.00	0.84	1.24	0.00	0.00	-0.01
time (sec)	N/A	0.237	0.131	0.425	0.420	0.610	0.000	0.000	0.000

Problem 269	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	155	155	91	304	171	191	226	145	125
normalized size	1	1.00	0.59	1.96	1.10	1.23	1.46	0.94	0.81
time (sec)	N/A	0.141	0.637	0.334	0.328	0.468	5.259	0.144	1.691

Problem 270	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	70	161	127	154	150	113	89
normalized size	1	1.00	0.69	1.59	1.26	1.52	1.49	1.12	0.88
time (sec)	N/A	0.076	0.241	0.330	0.322	0.712	2.922	0.117	0.157

Problem 271	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	50	69	91	108	110	81	57
normalized size	1	1.00	0.77	1.06	1.40	1.66	1.69	1.25	0.88
time (sec)	N/A	0.043	0.160	0.302	0.330	0.620	1.676	0.138	0.116

Problem 272	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	13	54	20	57	13
normalized size	1	1.00	1.00	0.93	0.87	3.60	1.33	3.80	0.87
time (sec)	N/A	0.020	0.003	0.022	0.321	1.017	0.741	0.141	1.491

Problem 273	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	47	50	45	73	0	45	-1
normalized size	1	1.00	0.89	0.94	0.85	1.38	0.00	0.85	-0.02
time (sec)	N/A	0.141	0.084	0.467	0.419	1.133	0.000	0.122	0.000

Problem 274	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	80	110	53	139	0	100	-1
normalized size	1	1.00	0.90	1.24	0.60	1.56	0.00	1.12	-0.01
time (sec)	N/A	0.189	0.208	0.483	0.445	0.846	0.000	0.120	0.000

Problem 275	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	125	112	178	60	227	0	168	-1
normalized size	1	1.00	0.90	1.42	0.48	1.82	0.00	1.34	-0.01
time (sec)	N/A	0.248	0.613	0.520	0.427	0.906	0.000	0.145	0.000

Problem 276	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	169	169	150	246	59	261	0	236	-1
normalized size	1	1.00	0.89	1.46	0.35	1.54	0.00	1.40	-0.01
time (sec)	N/A	0.297	0.525	0.483	0.421	0.878	0.000	0.143	0.000

Problem 277	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	8	7	13	13	0	13	-1
normalized size	1	1.00	1.00	0.88	1.62	1.62	0.00	1.62	-0.12
time (sec)	N/A	0.030	0.006	0.130	0.357	0.846	0.000	0.129	0.000

Problem 278	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	15	15	24	0	30	-1
normalized size	1	1.00	1.00	0.94	0.94	1.50	0.00	1.88	-0.06
time (sec)	N/A	0.047	0.007	0.121	0.364	0.915	0.000	0.115	0.000

Problem 279	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	27	24	13	43	0	48	-1
normalized size	1	1.00	1.00	0.89	0.48	1.59	0.00	1.78	-0.04
time (sec)	N/A	0.063	0.008	0.112	0.345	0.584	0.000	0.114	0.000

Problem 280	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	134	114	0	113	162	0	0	-1
normalized size	1	1.00	0.85	0.00	0.84	1.21	0.00	0.00	-0.01
time (sec)	N/A	0.183	0.230	0.484	0.440	0.782	0.000	0.000	0.000

Problem 281	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	84	244	160	135	146	140	119
normalized size	1	1.00	0.72	2.09	1.37	1.15	1.25	1.20	1.02
time (sec)	N/A	0.145	0.400	0.304	0.333	0.542	2.927	0.122	0.174

Problem 282	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	66	131	122	104	105	108	82
normalized size	1	1.00	0.80	1.58	1.47	1.25	1.27	1.30	0.99
time (sec)	N/A	0.079	0.438	0.331	0.350	0.463	1.696	0.125	1.679

Problem 283	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	38	56	84	76	61	76	44
normalized size	1	1.00	0.84	1.24	1.87	1.69	1.36	1.69	0.98
time (sec)	N/A	0.035	0.142	0.337	0.343	0.404	0.805	0.119	1.676

Problem 284	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	13	32	20	54	13
normalized size	1	1.00	1.00	0.93	0.87	2.13	1.33	3.60	0.87
time (sec)	N/A	0.018	0.004	0.020	0.328	0.585	0.372	0.144	1.635

Problem 285	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	41	47	42	67	0	42	-1
normalized size	1	1.00	0.87	1.00	0.89	1.43	0.00	0.89	-0.02
time (sec)	N/A	0.123	0.073	0.589	0.413	0.625	0.000	0.142	0.000

Problem 286	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	68	104	50	126	0	91	-1
normalized size	1	1.00	0.85	1.30	0.62	1.58	0.00	1.14	-0.01
time (sec)	N/A	0.167	0.217	0.573	0.421	0.573	0.000	0.142	0.000

Problem 287	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	107	169	58	195	0	156	-1
normalized size	1	1.00	0.90	1.42	0.49	1.64	0.00	1.31	-0.01
time (sec)	N/A	0.218	0.258	0.579	0.429	0.616	0.000	0.143	0.000

Problem 288	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	154	138	234	58	224	0	222	-1
normalized size	1	1.00	0.90	1.52	0.38	1.45	0.00	1.44	-0.01
time (sec)	N/A	0.281	0.318	0.582	0.438	0.481	0.000	0.136	0.000

Problem 289	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	76	0	71	122	0	0	-1
normalized size	1	1.00	0.89	0.00	0.84	1.44	0.00	0.00	-0.01
time (sec)	N/A	0.128	0.222	0.427	0.414	0.931	0.000	0.000	0.000

Problem 290	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	58	404	91	140	250	78	70
normalized size	1	1.00	0.73	5.11	1.15	1.77	3.16	0.99	0.89
time (sec)	N/A	0.114	0.210	0.340	0.337	0.552	5.359	0.139	1.799

Problem 291	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	48	241	69	110	204	62	52
normalized size	1	1.00	0.80	4.02	1.15	1.83	3.40	1.03	0.87
time (sec)	N/A	0.101	0.171	0.323	0.342	0.774	3.049	0.121	0.143

Problem 292	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	41	116	51	88	131	46	36
normalized size	1	1.00	1.00	2.83	1.24	2.15	3.20	1.12	0.88
time (sec)	N/A	0.050	0.140	0.066	0.382	0.723	1.706	0.143	1.756

Problem 293	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	23	43	39	40	92	32	18
normalized size	1	1.00	0.50	0.93	0.85	0.87	2.00	0.70	0.39
time (sec)	N/A	0.039	0.022	0.036	0.662	0.649	0.825	0.121	0.066

Problem 294	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	32	30	27	41	0	27	-1
normalized size	1	1.00	0.97	0.91	0.82	1.24	0.00	0.82	-0.03
time (sec)	N/A	0.085	0.101	0.342	0.799	0.731	0.000	0.123	0.000

Problem 295	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	45	61	32	88	0	55	-1
normalized size	1	1.00	0.87	1.17	0.62	1.69	0.00	1.06	-0.02
time (sec)	N/A	0.112	0.095	0.351	0.603	0.625	0.000	0.145	0.000

Problem 296	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	65	95	36	140	0	89	-1
normalized size	1	1.00	0.97	1.42	0.54	2.09	0.00	1.33	-0.01
time (sec)	N/A	0.136	0.110	0.351	0.508	0.456	0.000	0.141	0.000

Problem 297	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	79	129	36	172	0	123	-1
normalized size	1	1.00	0.86	1.40	0.39	1.87	0.00	1.34	-0.01
time (sec)	N/A	0.170	0.196	0.361	0.629	1.109	0.000	0.121	0.000

Problem 298	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	209	209	175	0	171	248	0	0	-1
normalized size	1	1.00	0.84	0.00	0.82	1.19	0.00	0.00	-0.00
time (sec)	N/A	0.287	0.310	0.421	0.938	0.949	0.000	0.000	0.000

Problem 299	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	202	202	125	478	245	274	253	212	167
normalized size	1	1.00	0.62	2.37	1.21	1.36	1.25	1.05	0.83
time (sec)	N/A	0.267	1.105	0.421	0.446	0.419	8.501	0.131	0.505

Problem 300	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	105	278	187	209	182	164	123
normalized size	1	1.00	0.71	1.88	1.26	1.41	1.23	1.11	0.83
time (sec)	N/A	0.180	0.317	0.380	0.382	0.573	5.024	0.126	1.829

Problem 301	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	70	129	129	152	112	116	83
normalized size	1	1.00	0.74	1.37	1.37	1.62	1.19	1.23	0.88
time (sec)	N/A	0.096	0.200	0.334	0.334	1.009	2.824	0.148	1.532

Problem 302	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	27	42	78	64	44	82	26
normalized size	1	1.00	0.87	1.35	2.52	2.06	1.42	2.65	0.84
time (sec)	N/A	0.033	0.062	0.333	0.622	0.836	1.458	0.125	1.447

Problem 303	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	61	71	64	103	0	64	-1
normalized size	1	1.00	0.84	0.97	0.88	1.41	0.00	0.88	-0.01
time (sec)	N/A	0.190	0.110	0.581	0.717	0.499	0.000	0.126	0.000

Problem 304	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	104	158	76	214	0	144	-1
normalized size	1	1.00	0.84	1.27	0.61	1.73	0.00	1.16	-0.01
time (sec)	N/A	0.263	0.365	0.538	0.691	0.669	0.000	0.128	0.000

Problem 305	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	184	184	162	257	88	338	0	243	-1
normalized size	1	1.00	0.88	1.40	0.48	1.84	0.00	1.32	-0.01
time (sec)	N/A	0.344	0.555	0.533	0.838	0.593	0.000	0.130	0.000

Problem 306	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	238	238	212	356	88	397	0	342	-1
normalized size	1	1.00	0.89	1.50	0.37	1.67	0.00	1.44	-0.00
time (sec)	N/A	0.444	0.561	0.553	0.450	0.581	0.000	0.131	0.000

Problem 307	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	141	141	112	0	117	172	0	0	-1
normalized size	1	1.00	0.79	0.00	0.83	1.22	0.00	0.00	-0.01
time (sec)	N/A	0.210	0.141	0.448	0.454	0.769	0.000	0.000	0.000

Problem 308	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	155	155	95	304	171	191	226	145	126
normalized size	1	1.00	0.61	1.96	1.10	1.23	1.46	0.94	0.81
time (sec)	N/A	0.139	0.657	0.362	0.350	0.766	5.247	0.119	0.260

Problem 309	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	72	161	127	154	150	113	89
normalized size	1	1.00	0.71	1.59	1.26	1.52	1.49	1.12	0.88
time (sec)	N/A	0.077	0.233	0.333	0.348	0.477	2.905	0.124	1.608

Problem 310	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	50	69	91	108	110	81	55
normalized size	1	1.00	0.77	1.06	1.40	1.66	1.69	1.25	0.85
time (sec)	N/A	0.043	0.163	0.334	0.330	0.527	1.681	0.124	0.153

Problem 311	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	13	54	20	57	13
normalized size	1	1.00	1.00	0.93	0.87	3.60	1.33	3.80	0.87
time (sec)	N/A	0.021	0.003	0.056	0.316	0.591	0.727	0.153	0.066

Problem 312	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	47	50	45	73	0	45	-1
normalized size	1	1.00	0.89	0.94	0.85	1.38	0.00	0.85	-0.02
time (sec)	N/A	0.133	0.083	0.606	0.432	0.610	0.000	0.122	0.000

Problem 313	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	78	110	53	139	0	100	-1
normalized size	1	1.00	0.88	1.24	0.60	1.56	0.00	1.12	-0.01
time (sec)	N/A	0.176	0.251	0.622	0.441	0.655	0.000	0.146	0.000

Problem 314	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	125	113	178	60	229	0	168	-1
normalized size	1	1.00	0.90	1.42	0.48	1.83	0.00	1.34	-0.01
time (sec)	N/A	0.230	0.562	0.617	0.447	0.567	0.000	0.122	0.000

Problem 315	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	169	169	150	246	61	264	0	236	-1
normalized size	1	1.00	0.89	1.46	0.36	1.56	0.00	1.40	-0.01
time (sec)	N/A	0.287	0.561	0.632	0.441	0.682	0.000	0.123	0.000

Problem 316	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	209	209	174	0	171	248	0	0	-1
normalized size	1	1.00	0.83	0.00	0.82	1.19	0.00	0.00	-0.00
time (sec)	N/A	0.280	0.273	0.289	0.500	0.788	0.000	0.000	0.000

Problem 317	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	202	202	136	439	245	274	253	212	173
normalized size	1	1.00	0.67	2.17	1.21	1.36	1.25	1.05	0.86
time (sec)	N/A	0.262	0.512	0.406	0.336	1.549	8.507	0.142	0.273

Problem 318	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	98	246	187	214	182	164	112
normalized size	1	1.00	0.66	1.66	1.26	1.45	1.23	1.11	0.76
time (sec)	N/A	0.182	0.456	0.295	0.345	0.662	5.062	0.143	0.233

Problem 319	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	70	116	129	153	112	116	71
normalized size	1	1.00	0.74	1.23	1.37	1.63	1.19	1.23	0.76
time (sec)	N/A	0.090	0.193	0.280	0.351	0.634	2.772	0.147	0.145

Problem 320	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	27	34	78	79	44	82	26
normalized size	1	1.00	0.87	1.10	2.52	2.55	1.42	2.65	0.84
time (sec)	N/A	0.035	0.071	0.063	0.329	0.855	1.470	0.151	1.455

Problem 321	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	63	71	64	103	0	64	-1
normalized size	1	1.00	0.86	0.97	0.88	1.41	0.00	0.88	-0.01
time (sec)	N/A	0.182	0.115	0.398	0.453	0.560	0.000	0.127	0.000

Problem 322	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	106	158	76	203	0	140	-1
normalized size	1	1.00	0.85	1.27	0.61	1.64	0.00	1.13	-0.01
time (sec)	N/A	0.250	0.271	0.406	0.467	1.014	0.000	0.147	0.000

Problem 323	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	184	184	164	257	88	336	0	239	-1
normalized size	1	1.00	0.89	1.40	0.48	1.83	0.00	1.30	-0.01
time (sec)	N/A	0.339	0.424	0.418	0.452	0.823	0.000	0.150	0.000

Problem 324	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	238	238	212	356	88	392	0	342	-1
normalized size	1	1.00	0.89	1.50	0.37	1.65	0.00	1.44	-0.00
time (sec)	N/A	0.414	0.514	0.418	0.467	0.556	0.000	0.156	0.000

Problem 325	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	155	155	119	0	117	172	0	0	-1
normalized size	1	1.00	0.77	0.00	0.75	1.11	0.00	0.00	-0.01
time (sec)	N/A	0.243	0.159	0.601	0.468	0.649	0.000	0.000	0.000

Problem 326	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	143	90	499	171	248	314	145	126
normalized size	1	1.00	0.63	3.49	1.20	1.73	2.20	1.01	0.88
time (sec)	N/A	0.197	0.903	0.428	0.353	0.807	14.185	0.150	1.717

Problem 327	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	72	276	127	202	212	113	89
normalized size	1	1.00	0.69	2.63	1.21	1.92	2.02	1.08	0.85
time (sec)	N/A	0.140	0.234	0.331	0.345	0.504	8.366	0.137	0.321

Problem 328	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	50	129	91	148	148	81	55
normalized size	1	1.00	0.75	1.93	1.36	2.21	2.21	1.21	0.82
time (sec)	N/A	0.073	0.150	0.329	0.445	0.652	4.944	0.149	0.233

Problem 329	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	35	34	56	72	42	57	26
normalized size	1	1.00	1.13	1.10	1.81	2.32	1.35	1.84	0.84
time (sec)	N/A	0.037	0.012	0.065	0.341	0.666	2.589	0.150	1.522

Problem 330	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	47	50	45	73	0	45	-1
normalized size	1	1.00	0.89	0.94	0.85	1.38	0.00	0.85	-0.02
time (sec)	N/A	0.157	0.182	0.681	0.534	0.678	0.000	0.127	0.000

Problem 331	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	78	110	53	159	0	100	-1
normalized size	1	1.00	0.88	1.24	0.60	1.79	0.00	1.12	-0.01
time (sec)	N/A	0.196	0.245	0.682	0.481	0.505	0.000	0.129	0.000

Problem 332	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	118	178	61	274	0	168	-1
normalized size	1	1.00	0.90	1.36	0.47	2.09	0.00	1.28	-0.01
time (sec)	N/A	0.257	0.250	0.686	0.441	1.087	0.000	0.133	0.000

Problem 333	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	169	169	150	246	61	315	0	236	-1
normalized size	1	1.00	0.89	1.46	0.36	1.86	0.00	1.40	-0.01
time (sec)	N/A	0.315	0.338	0.689	0.428	0.778	0.000	0.134	0.000

Problem 334	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	13	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.08
time (sec)	N/A	0.017	0.479	0.135	0.000	0.931	0.000	0.000	0.000

Problem 335	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	88	116	84	257	0	0	-1
normalized size	1	1.00	0.97	1.27	0.92	2.82	0.00	0.00	-0.01
time (sec)	N/A	0.160	2.311	0.318	0.482	0.636	0.000	0.000	0.000

Problem 336	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	66	94	63	207	0	0	-1
normalized size	1	1.00	1.02	1.45	0.97	3.18	0.00	0.00	-0.02
time (sec)	N/A	0.135	2.153	0.317	0.405	1.805	0.000	0.000	0.000

Problem 337	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	C	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	149	70	40	141	0	0	-1
normalized size	1	1.00	3.31	1.56	0.89	3.13	0.00	0.00	-0.02
time (sec)	N/A	0.082	3.240	0.306	0.499	0.688	0.000	0.000	0.000

Problem 338	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	13	21	37	0	24	11
normalized size	1	1.00	1.00	1.18	1.91	3.36	0.00	2.18	1.00
time (sec)	N/A	0.006	0.009	0.026	0.307	0.658	0.000	0.131	0.062

Problem 339	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	13	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.08
time (sec)	N/A	0.018	12.307	0.332	0.000	0.488	0.000	0.000	0.000

Problem 340	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	13	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.08
time (sec)	N/A	0.017	19.065	0.334	0.000	0.753	0.000	0.000	0.000

Problem 341	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	19	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.394	22.285	0.101	0.000	0.667	0.000	0.000	0.000

Problem 342	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	130	0	0	670	0	0	-1
normalized size	1	1.00	1.15	0.00	0.00	5.93	0.00	0.00	-0.01
time (sec)	N/A	0.105	2.276	0.557	0.000	0.774	0.000	0.000	0.000

Problem 343	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	125	154	0	474	0	0	-1
normalized size	1	1.00	1.81	2.23	0.00	6.87	0.00	0.00	-0.01
time (sec)	N/A	0.061	0.451	0.327	0.000	0.612	0.000	0.000	0.000

Problem 344	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	32	59	37	116	0	70	49
normalized size	1	1.00	1.33	2.46	1.54	4.83	0.00	2.92	2.04
time (sec)	N/A	0.019	0.048	0.168	0.497	0.485	0.000	0.161	0.068

Problem 345	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	12	23	54	0	23	13
normalized size	1	1.00	1.00	1.09	2.09	4.91	0.00	2.09	1.18
time (sec)	N/A	0.015	0.007	0.023	0.430	0.381	0.000	0.121	0.055

Problem 346	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	19	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.176	7.019	0.360	0.000	0.478	0.000	0.000	0.000

Problem 347	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	19	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.196	8.198	0.347	0.000	0.427	0.000	0.000	0.000

Problem 348	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	21	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.470	36.885	0.112	0.000	0.485	0.000	0.000	0.000

Problem 349	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	C	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	227	121	110	1113	0	0	-1
normalized size	1	1.00	2.73	1.46	1.33	13.41	0.00	0.00	-0.01
time (sec)	N/A	0.183	6.139	0.233	0.426	0.930	0.000	0.000	0.000

Problem 350	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	55	73	94	378	0	142	102
normalized size	1	1.00	1.31	1.74	2.24	9.00	0.00	3.38	2.43
time (sec)	N/A	0.061	0.125	0.154	0.409	0.887	0.000	0.129	1.476

Problem 351	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	43	131	105	0	184	36
normalized size	1	1.00	1.00	1.43	4.37	3.50	0.00	6.13	1.20
time (sec)	N/A	0.030	0.076	0.155	0.346	0.432	0.000	0.146	1.460

Problem 352	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	23	84	0	27	13
normalized size	1	1.00	1.00	0.93	1.53	5.60	0.00	1.80	0.87
time (sec)	N/A	0.021	0.009	0.026	0.321	0.641	0.000	0.143	1.450

Problem 353	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	21	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.182	24.685	0.373	0.000	0.547	0.000	0.000	0.000

Problem 354	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	21	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.215	21.288	0.374	0.000	0.632	0.000	0.000	0.000

Problem 355	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	74	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.106	14.081	0.318	0.000	0.553	0.000	0.000	0.000

Problem 356	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	195	195	211	0	0	609	0	0	-1
normalized size	1	1.00	1.08	0.00	0.00	3.12	0.00	0.00	-0.01
time (sec)	N/A	0.198	1.476	0.671	0.000	0.518	0.000	0.000	0.000

Problem 357	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	153	0	0	477	0	0	-1
normalized size	1	1.00	1.13	0.00	0.00	3.53	0.00	0.00	-0.01
time (sec)	N/A	0.130	1.468	0.744	0.000	0.512	0.000	0.000	0.000

Problem 358	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	213	162	0	322	0	0	-1
normalized size	1	1.00	2.77	2.10	0.00	4.18	0.00	0.00	-0.01
time (sec)	N/A	0.064	0.118	0.388	0.000	0.684	0.000	0.000	0.000

Problem 359	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	24	41	86	0	32	49
normalized size	1	1.00	1.00	1.04	1.78	3.74	0.00	1.39	2.13
time (sec)	N/A	0.015	0.012	0.083	0.450	0.594	0.000	0.136	1.427

Problem 360	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	30	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.086	9.148	0.516	0.000	0.621	0.000	0.000	0.000

Problem 361	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	43	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.107	8.901	0.520	0.000	0.823	0.000	0.000	0.000

Problem 362	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	15	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.07
time (sec)	N/A	0.032	0.617	0.284	0.000	0.537	0.000	0.000	0.000

Problem 363	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	104	125	108	721	0	0	-1
normalized size	1	1.00	1.17	1.40	1.21	8.10	0.00	0.00	-0.01
time (sec)	N/A	0.178	4.647	0.570	0.422	0.711	0.000	0.000	0.000

Problem 364	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	C	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	168	99	84	515	0	0	-1
normalized size	1	1.00	2.58	1.52	1.29	7.92	0.00	0.00	-0.02
time (sec)	N/A	0.118	3.337	0.569	0.421	0.826	0.000	0.000	0.000

Problem 365	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	46	54	95	185	0	95	45
normalized size	1	1.00	1.48	1.74	3.06	5.97	0.00	3.06	1.45
time (sec)	N/A	0.027	0.146	0.501	0.373	1.513	0.000	0.144	0.098

Problem 366	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	23	18	25	33	0	24	20
normalized size	1	1.00	1.77	1.38	1.92	2.54	0.00	1.85	1.54
time (sec)	N/A	0.009	0.007	0.112	0.315	0.738	0.000	0.134	0.072

Problem 367	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	15	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.07
time (sec)	N/A	0.030	18.882	0.589	0.000	0.652	0.000	0.000	0.000

Problem 368	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	15	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.07
time (sec)	N/A	0.030	11.891	0.584	0.000	0.680	0.000	0.000	0.000

Problem 369	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	31	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.067	58.507	0.302	0.000	0.724	0.000	0.000	0.000

Problem 370	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	240	240	245	0	0	2125	0	0	-1
normalized size	1	1.00	1.02	0.00	0.00	8.85	0.00	0.00	-0.00
time (sec)	N/A	0.298	3.419	1.368	0.000	0.898	0.000	0.000	0.000

Problem 371	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	143	180	0	0	1549	0	0	-1
normalized size	1	1.00	1.26	0.00	0.00	10.83	0.00	0.00	-0.01
time (sec)	N/A	0.199	1.652	1.195	0.000	0.726	0.000	0.000	0.000

Problem 372	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	93	178	0	1046	0	0	-1
normalized size	1	1.00	1.02	1.96	0.00	11.49	0.00	0.00	-0.01
time (sec)	N/A	0.102	0.779	0.679	0.000	0.990	0.000	0.000	0.000

Problem 373	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	34	49	66	269	0	76	82
normalized size	1	1.00	1.00	1.44	1.94	7.91	0.00	2.24	2.41
time (sec)	N/A	0.024	0.017	0.342	0.420	0.926	0.000	0.155	1.474

Problem 374	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	31	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.064	16.712	0.884	0.000	0.650	0.000	0.000	0.000

Problem 375	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	31	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.069	13.404	0.926	0.000	0.591	0.000	0.000	0.000

Problem 376	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	85	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.139	22.597	0.431	0.000	0.397	0.000	0.000	0.000

Problem 377	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	185	185	191	189	181	966	0	0	-1
normalized size	1	1.00	1.03	1.02	0.98	5.22	0.00	0.00	-0.01
time (sec)	N/A	0.247	2.991	0.492	0.436	0.597	0.000	0.000	0.000

Problem 378	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	154	152	138	789	0	0	-1
normalized size	1	1.00	1.18	1.17	1.06	6.07	0.00	0.00	-0.01
time (sec)	N/A	0.190	2.734	0.482	0.464	0.790	0.000	0.000	0.000

Problem 379	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	102	110	95	558	0	0	-1
normalized size	1	1.00	1.15	1.24	1.07	6.27	0.00	0.00	-0.01
time (sec)	N/A	0.116	0.234	0.473	0.439	0.733	0.000	0.000	0.000

Problem 380	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	25	27	56	197	0	60	48
normalized size	1	1.00	0.89	0.96	2.00	7.04	0.00	2.14	1.71
time (sec)	N/A	0.025	0.019	0.116	0.651	0.591	0.000	0.148	0.061

Problem 381	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	42	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.103	19.594	0.603	0.000	0.509	0.000	0.000	0.000

Problem 382	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	54	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.131	20.204	0.603	0.000	0.540	0.000	0.000	0.000

Problem 383	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	80	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.167	38.125	0.420	0.000	0.593	0.000	0.000	0.000

Problem 384	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	162	162	196	0	0	1213	0	0	-1
normalized size	1	1.00	1.21	0.00	0.00	7.49	0.00	0.00	-0.01
time (sec)	N/A	0.183	3.354	0.753	0.000	0.514	0.000	0.000	0.000

Problem 385	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	172	205	0	869	0	0	-1
normalized size	1	1.00	1.65	1.97	0.00	8.36	0.00	0.00	-0.01
time (sec)	N/A	0.118	0.507	0.485	0.000	0.727	0.000	0.000	0.000

Problem 386	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	50	94	81	283	0	102	90
normalized size	1	1.00	1.09	2.04	1.76	6.15	0.00	2.22	1.96
time (sec)	N/A	0.051	0.119	0.402	0.801	0.526	0.000	0.186	0.104

Problem 387	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	33	54	31	0	41	22
normalized size	1	1.00	1.00	1.57	2.57	1.48	0.00	1.95	1.05
time (sec)	N/A	0.024	0.029	0.107	0.420	0.701	0.000	0.147	0.074

Problem 388	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	36	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.128	12.095	0.669	0.000	0.679	0.000	0.000	0.000

Problem 389	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	49	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.126	10.232	0.609	0.000	0.870	0.000	0.000	0.000

Problem 390	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	15	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.07
time (sec)	N/A	0.030	1.528	0.480	0.000	0.740	0.000	0.000	0.000

Problem 391	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	183	183	230	234	236	2207	0	0	-1
normalized size	1	1.00	1.26	1.28	1.29	12.06	0.00	0.00	-0.01
time (sec)	N/A	0.326	2.896	0.624	1.070	0.629	0.000	0.000	0.000

Problem 392	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	185	164	183	1649	0	0	-1
normalized size	1	1.00	1.59	1.41	1.58	14.22	0.00	0.00	-0.01
time (sec)	N/A	0.202	2.378	0.614	0.972	0.941	0.000	0.000	0.000

Problem 393	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	C	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	231	111	131	1106	0	0	-1
normalized size	1	1.00	2.82	1.35	1.60	13.49	0.00	0.00	-0.01
time (sec)	N/A	0.117	6.127	0.520	0.466	0.573	0.000	0.000	0.000

Problem 394	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	27	26	61	339	0	65	25
normalized size	1	1.00	1.00	0.96	2.26	12.56	0.00	2.41	0.93
time (sec)	N/A	0.018	0.013	0.131	0.792	0.594	0.000	0.156	1.508

Problem 395	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	15	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.07
time (sec)	N/A	0.028	14.114	1.306	0.000	0.643	0.000	0.000	0.000

Problem 396	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	15	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.07
time (sec)	N/A	0.028	8.403	1.415	0.000	0.688	0.000	0.000	0.000

Problem 397	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	13	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.08
time (sec)	N/A	0.016	7.579	0.211	0.000	0.823	0.000	0.000	0.000

Problem 398	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	91	200	130	216	0	0	-1
normalized size	1	1.00	1.05	2.30	1.49	2.48	0.00	0.00	-0.01
time (sec)	N/A	0.153	0.011	0.283	0.382	0.593	0.000	0.000	0.000

Problem 399	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	66	166	96	168	0	0	-1
normalized size	1	1.00	1.05	2.63	1.52	2.67	0.00	0.00	-0.02
time (sec)	N/A	0.133	0.010	0.272	0.409	1.001	0.000	0.000	0.000

Problem 400	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	47	122	58	112	0	0	-1
normalized size	1	1.00	1.04	2.71	1.29	2.49	0.00	0.00	-0.02
time (sec)	N/A	0.081	0.007	0.264	0.430	0.471	0.000	0.000	0.000

Problem 401	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	19	13	23	37	0	25	11
normalized size	1	1.00	1.73	1.18	2.09	3.36	0.00	2.27	1.00
time (sec)	N/A	0.006	0.015	0.025	0.340	0.532	0.000	0.116	0.065

Problem 402	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	13	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.08
time (sec)	N/A	0.016	0.455	0.403	0.000	0.564	0.000	0.000	0.000

Problem 403	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	13	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.08
time (sec)	N/A	0.017	0.869	0.464	0.000	0.446	0.000	0.000	0.000

Problem 404	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	72	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.091	19.589	0.431	0.000	0.458	0.000	0.000	0.000

Problem 405	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	C	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	165	165	202	246	206	511	0	0	-1
normalized size	1	1.00	1.22	1.49	1.25	3.10	0.00	0.00	-0.01
time (sec)	N/A	0.178	4.178	0.533	0.461	0.799	0.000	0.000	0.000

Problem 406	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	C	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	138	196	152	391	0	0	-1
normalized size	1	1.00	1.20	1.70	1.32	3.40	0.00	0.00	-0.01
time (sec)	N/A	0.123	4.075	0.507	0.426	0.550	0.000	0.000	0.000

Problem 407	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	131	139	94	255	0	0	-1
normalized size	1	1.00	1.98	2.11	1.42	3.86	0.00	0.00	-0.02
time (sec)	N/A	0.061	0.138	0.519	0.427	0.832	0.000	0.000	0.000

Problem 408	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	26	21	59	113	0	44	53
normalized size	1	1.00	1.13	0.91	2.57	4.91	0.00	1.91	2.30
time (sec)	N/A	0.017	0.022	0.112	0.351	0.737	0.000	0.120	0.088

Problem 409	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.080	26.618	0.574	0.000	0.607	0.000	0.000	0.000

Problem 410	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	41	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.099	38.165	0.587	0.000	0.391	0.000	0.000	0.000

Problem 411	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	83	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.131	23.545	0.495	0.000	0.537	0.000	0.000	0.000

Problem 412	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	180	180	236	272	225	876	0	0	-1
normalized size	1	1.00	1.31	1.51	1.25	4.87	0.00	0.00	-0.01
time (sec)	N/A	0.237	2.615	0.603	0.581	0.646	0.000	0.000	0.000

Problem 413	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	178	222	171	697	0	0	-1
normalized size	1	1.00	1.41	1.76	1.36	5.53	0.00	0.00	-0.01
time (sec)	N/A	0.197	2.591	0.615	0.449	0.917	0.000	0.000	0.000

Problem 414	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	82	162	113	488	0	0	-1
normalized size	1	1.00	0.93	1.84	1.28	5.55	0.00	0.00	-0.01
time (sec)	N/A	0.121	0.249	0.609	0.412	1.108	0.000	0.000	0.000

Problem 415	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	25	26	70	203	0	63	49
normalized size	1	1.00	0.93	0.96	2.59	7.52	0.00	2.33	1.81
time (sec)	N/A	0.023	0.022	0.116	0.332	0.960	0.000	0.140	0.061

Problem 416	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	40	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.104	11.414	0.742	0.000	0.981	0.000	0.000	0.000

Problem 417	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	52	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.128	16.850	0.719	0.000	0.491	0.000	0.000	0.000

Problem 418	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	33	48	0	336	0	101	48
normalized size	1	1.00	1.00	1.45	0.00	10.18	0.00	3.06	1.45
time (sec)	N/A	0.054	0.029	0.210	0.000	0.580	0.000	0.118	0.119

Problem 419	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	64	87	0	617	0	0	-1
normalized size	1	1.00	0.88	1.19	0.00	8.45	0.00	0.00	-0.01
time (sec)	N/A	0.158	0.098	0.236	0.000	0.471	0.000	0.000	0.000

Problem 420	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-2)	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	94	117	0	875	0	0	-1
normalized size	1	1.00	0.92	1.15	0.00	8.58	0.00	0.00	-0.01
time (sec)	N/A	0.183	0.152	0.215	0.000	0.546	0.000	0.000	0.000

Problem 421	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	56	82	146	916	0	0	-1
normalized size	1	1.00	0.89	1.30	2.32	14.54	0.00	0.00	-0.02
time (sec)	N/A	0.163	0.080	0.298	0.656	0.466	0.000	0.000	0.000

Problem 422	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	B	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	98	127	174	1512	0	0	-1
normalized size	1	1.00	1.02	1.32	1.81	15.75	0.00	0.00	-0.01
time (sec)	N/A	0.280	0.284	0.322	0.649	0.590	0.000	0.000	0.000

Problem 423	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	158	158	133	184	238	2067	0	0	-1
normalized size	1	1.00	0.84	1.16	1.51	13.08	0.00	0.00	-0.01
time (sec)	N/A	0.386	0.335	0.305	0.807	0.545	0.000	0.000	0.000

Problem 424	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	19	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.380	31.012	0.104	0.000	0.487	0.000	0.000	0.000

Problem 425	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	C	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	167	174	121	551	0	0	-1
normalized size	1	1.00	1.80	1.87	1.30	5.92	0.00	0.00	-0.01
time (sec)	N/A	0.105	7.209	0.224	0.544	0.426	0.000	0.000	0.000

Problem 426	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	A	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	133	134	83	367	0	0	-1
normalized size	1	1.00	2.25	2.27	1.41	6.22	0.00	0.00	-0.02
time (sec)	N/A	0.061	0.787	0.223	0.897	0.493	0.000	0.000	0.000

Problem 427	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	114	54	64	169	0	93	53
normalized size	1	1.00	4.56	2.16	2.56	6.76	0.00	3.72	2.12
time (sec)	N/A	0.019	0.050	0.135	0.516	0.509	0.000	0.151	0.119

Problem 428	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	12	25	56	0	25	13
normalized size	1	1.00	1.00	1.09	2.27	5.09	0.00	2.27	1.18
time (sec)	N/A	0.011	0.010	0.026	0.523	0.453	0.000	0.120	1.419

Problem 429	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	19	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.142	32.048	0.353	0.000	0.504	0.000	0.000	0.000

Problem 430	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	19	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.193	38.185	0.360	0.000	0.512	0.000	0.000	0.000

Problem 431	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	15	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.07
time (sec)	N/A	0.032	9.959	0.281	0.000	0.460	0.000	0.000	0.000

Problem 432	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	A	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	204	198	146	632	0	0	-1
normalized size	1	1.00	2.34	2.28	1.68	7.26	0.00	0.00	-0.01
time (sec)	N/A	0.190	0.601	0.584	0.446	0.499	0.000	0.000	0.000

Problem 433	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	A	B	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	163	156	108	453	0	0	-1
normalized size	1	1.00	2.51	2.40	1.66	6.97	0.00	0.00	-0.02
time (sec)	N/A	0.125	5.059	0.575	0.424	0.536	0.000	0.000	0.000

Problem 434	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	46	54	115	189	0	98	45
normalized size	1	1.00	1.48	1.74	3.71	6.10	0.00	3.16	1.45
time (sec)	N/A	0.028	0.155	0.511	0.371	0.457	0.000	0.155	1.455

Problem 435	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	27	18	25	33	0	24	20
normalized size	1	1.00	2.08	1.38	1.92	2.54	0.00	1.85	1.54
time (sec)	N/A	0.009	0.014	0.125	0.312	0.399	0.000	0.141	0.064

Problem 436	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	15	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.07
time (sec)	N/A	0.030	0.396	0.582	0.000	0.411	0.000	0.000	0.000

Problem 437	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	15	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.07
time (sec)	N/A	0.031	0.918	0.587	0.000	0.411	0.000	0.000	0.000

Problem 438	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	78	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.153	38.878	0.444	0.000	1.021	0.000	0.000	0.000

Problem 439	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	C	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	143	143	225	241	216	1055	0	0	-1
normalized size	1	1.00	1.57	1.69	1.51	7.38	0.00	0.00	-0.01
time (sec)	N/A	0.184	0.342	0.535	0.509	0.432	0.000	0.000	0.000

Problem 440	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	B	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	230	185	157	731	0	0	-1
normalized size	1	1.00	2.42	1.95	1.65	7.69	0.00	0.00	-0.01
time (sec)	N/A	0.118	0.454	0.534	0.424	0.442	0.000	0.000	0.000

Problem 441	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	66	89	109	367	0	144	95
normalized size	1	1.00	1.40	1.89	2.32	7.81	0.00	3.06	2.02
time (sec)	N/A	0.054	0.234	0.451	0.439	0.417	0.000	0.208	0.094

Problem 442	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	33	56	31	0	45	22
normalized size	1	1.00	1.00	1.50	2.55	1.41	0.00	2.05	1.00
time (sec)	N/A	0.022	0.013	0.116	0.311	0.401	0.000	0.145	1.453

Problem 443	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	34	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.105	24.246	0.666	0.000	0.401	0.000	0.000	0.000

Problem 444	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	47	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.137	21.255	0.665	0.000	0.392	0.000	0.000	0.000

Problem 445	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	21	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.471	36.783	0.142	0.000	0.406	0.000	0.000	0.000

Problem 446	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	A	B	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	227	177	130	979	0	0	-1
normalized size	1	1.00	2.73	2.13	1.57	11.80	0.00	0.00	-0.01
time (sec)	N/A	0.163	6.136	0.263	0.477	0.430	0.000	0.000	0.000

Problem 447	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	55	72	107	383	0	139	101
normalized size	1	1.00	1.31	1.71	2.55	9.12	0.00	3.31	2.40
time (sec)	N/A	0.058	0.122	0.189	0.603	0.415	0.000	0.126	1.470

Problem 448	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	43	130	107	0	184	36
normalized size	1	1.00	1.00	1.43	4.33	3.57	0.00	6.13	1.20
time (sec)	N/A	0.030	0.074	0.174	0.389	0.395	0.000	0.125	1.451

Problem 449	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	25	86	0	27	13
normalized size	1	1.00	1.00	0.93	1.67	5.73	0.00	1.80	0.87
time (sec)	N/A	0.021	0.012	0.072	0.429	0.391	0.000	0.147	1.458

Problem 450	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	21	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.162	15.583	0.433	0.000	0.411	0.000	0.000	0.000

Problem 451	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	21	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.236	20.373	0.433	0.000	0.415	0.000	0.000	0.000

Problem 452	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F(-1)	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	29	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.074	180.000	0.305	0.000	0.410	0.000	0.000	0.000

Problem 453	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	201	201	280	340	262	1802	0	0	-1
normalized size	1	1.00	1.39	1.69	1.30	8.97	0.00	0.00	-0.00
time (sec)	N/A	0.360	6.920	0.760	0.543	0.455	0.000	0.000	0.000

Problem 454	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	222	210	197	1311	0	0	-1
normalized size	1	1.00	1.80	1.71	1.60	10.66	0.00	0.00	-0.01
time (sec)	N/A	0.236	4.148	0.718	0.495	0.449	0.000	0.000	0.000

Problem 455	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	144	156	124	842	0	0	-1
normalized size	1	1.00	1.76	1.90	1.51	10.27	0.00	0.00	-0.01
time (sec)	N/A	0.125	2.035	0.658	0.455	0.433	0.000	0.000	0.000

Problem 456	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	57	45	84	387	0	84	87
normalized size	1	1.00	1.68	1.32	2.47	11.38	0.00	2.47	2.56
time (sec)	N/A	0.033	0.035	0.349	0.339	0.401	0.000	0.137	1.500

Problem 457	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	29	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.072	54.005	0.916	0.000	0.423	0.000	0.000	0.000

Problem 458	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	29	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.076	45.880	1.042	0.000	0.405	0.000	0.000	0.000

Problem 459	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	15	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.07
time (sec)	N/A	0.030	122.971	0.467	0.000	0.396	0.000	0.000	0.000

Problem 460	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	A	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	179	179	390	375	302	1985	0	0	-1
normalized size	1	1.00	2.18	2.09	1.69	11.09	0.00	0.00	-0.01
time (sec)	N/A	0.337	3.343	0.597	0.411	0.448	0.000	0.000	0.000

Problem 461	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	295	246	226	1467	0	0	-1
normalized size	1	1.00	2.59	2.16	1.98	12.87	0.00	0.00	-0.01
time (sec)	N/A	0.212	2.212	0.596	0.722	0.433	0.000	0.000	0.000

Problem 462	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	B	B	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	231	164	149	975	0	0	-1
normalized size	1	1.00	2.82	2.00	1.82	11.89	0.00	0.00	-0.01
time (sec)	N/A	0.122	6.128	0.590	0.376	0.431	0.000	0.000	0.000

Problem 463	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	34	26	79	346	0	66	25
normalized size	1	1.00	1.26	0.96	2.93	12.81	0.00	2.44	0.93
time (sec)	N/A	0.018	0.079	0.166	0.433	0.423	0.000	0.145	1.498

Problem 464	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	15	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.07
time (sec)	N/A	0.029	0.748	1.347	0.000	0.401	0.000	0.000	0.000

Problem 465	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	15	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.07
time (sec)	N/A	0.031	0.429	1.355	0.000	0.408	0.000	0.000	0.000

Problem 466	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	19	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.212	9.552	0.204	0.000	0.409	0.000	0.000	0.000

Problem 467	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	150	241	203	448	0	0	-1
normalized size	1	1.00	1.01	1.63	1.37	3.03	0.00	0.00	-0.01
time (sec)	N/A	0.150	4.345	0.447	0.549	0.444	0.000	0.000	0.000

Problem 468	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	108	186	148	351	0	0	-1
normalized size	1	1.00	1.11	1.92	1.53	3.62	0.00	0.00	-0.01
time (sec)	N/A	0.106	4.313	0.448	0.402	0.435	0.000	0.000	0.000

Problem 469	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	110	125	87	224	0	0	-1
normalized size	1	1.00	1.90	2.16	1.50	3.86	0.00	0.00	-0.02
time (sec)	N/A	0.056	0.088	0.436	0.625	0.432	0.000	0.000	0.000

Problem 470	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	31	12	50	60	0	41	30
normalized size	1	1.00	2.82	1.09	4.55	5.45	0.00	3.73	2.73
time (sec)	N/A	0.013	0.013	0.100	0.410	0.412	0.000	0.113	1.446

Problem 471	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	18	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.06
time (sec)	N/A	0.035	17.610	0.507	0.000	0.404	0.000	0.000	0.000

Problem 472	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	18	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.06
time (sec)	N/A	0.037	16.012	0.487	0.000	0.387	0.000	0.000	0.000

Problem 473	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	21	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.391	125.599	0.201	0.000	0.402	0.000	0.000	0.000

Problem 474	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	C	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	226	226	282	0	0	1270	0	0	-1
normalized size	1	1.00	1.25	0.00	0.00	5.62	0.00	0.00	-0.00
time (sec)	N/A	0.337	0.395	1.581	0.000	0.459	0.000	0.000	0.000

Problem 475	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	C	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	146	146	225	0	0	929	0	0	-1
normalized size	1	1.00	1.54	0.00	0.00	6.36	0.00	0.00	-0.01
time (sec)	N/A	0.227	0.646	1.753	0.000	0.455	0.000	0.000	0.000

Problem 476	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	106	95	90	401	0	0	-1
normalized size	1	1.00	1.58	1.42	1.34	5.99	0.00	0.00	-0.01
time (sec)	N/A	0.114	0.290	0.515	0.462	0.413	0.000	0.000	0.000

Problem 477	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	26	23	61	155	0	64	52
normalized size	1	1.00	1.13	1.00	2.65	6.74	0.00	2.78	2.26
time (sec)	N/A	0.026	0.030	0.107	0.338	0.403	0.000	0.912	1.466

Problem 478	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	21	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.214	35.906	0.990	0.000	0.401	0.000	0.000	0.000

Problem 479	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	21	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.254	25.900	1.093	0.000	0.414	0.000	0.000	0.000

Problem 480	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	21	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.475	82.194	0.230	0.000	0.410	0.000	0.000	0.000

Problem 481	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	240	240	462	359	329	3409	0	0	-1
normalized size	1	1.00	1.92	1.50	1.37	14.20	0.00	0.00	-0.00
time (sec)	N/A	0.431	7.393	0.533	0.360	0.509	0.000	0.000	0.000

Problem 482	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	362	256	229	2523	0	0	-1
normalized size	1	1.00	2.45	1.73	1.55	17.05	0.00	0.00	-0.01
time (sec)	N/A	0.251	3.868	0.570	0.358	0.483	0.000	0.000	0.000

Problem 483	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	139	166	142	1543	0	0	-1
normalized size	1	1.00	1.46	1.75	1.49	16.24	0.00	0.00	-0.01
time (sec)	N/A	0.120	0.435	0.416	0.363	0.466	0.000	0.000	0.000

Problem 484	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	36	26	88	371	0	93	78
normalized size	1	1.00	1.33	0.96	3.26	13.74	0.00	3.44	2.89
time (sec)	N/A	0.026	0.033	0.150	0.465	0.429	0.000	0.144	0.069

Problem 485	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	21	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.214	57.727	1.520	0.000	0.443	0.000	0.000	0.000

Problem 486	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	21	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.254	29.651	1.671	0.000	0.497	0.000	0.000	0.000

Problem 487	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	21	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.399	44.676	0.165	0.000	0.426	0.000	0.000	0.000

Problem 488	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	C	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	237	237	333	0	0	1307	0	0	-1
normalized size	1	1.00	1.41	0.00	0.00	5.51	0.00	0.00	-0.00
time (sec)	N/A	0.344	1.814	1.553	0.000	0.483	0.000	0.000	0.000

Problem 489	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	C	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	157	312	0	0	964	0	0	-1
normalized size	1	1.00	1.99	0.00	0.00	6.14	0.00	0.00	-0.01
time (sec)	N/A	0.234	1.931	1.771	0.000	0.471	0.000	0.000	0.000

Problem 490	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	112	179	0	565	0	0	-1
normalized size	1	1.00	1.42	2.27	0.00	7.15	0.00	0.00	-0.01
time (sec)	N/A	0.112	0.833	0.495	0.000	0.442	0.000	0.000	0.000

Problem 491	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	29	27	43	103	0	54	48
normalized size	1	1.00	1.21	1.12	1.79	4.29	0.00	2.25	2.00
time (sec)	N/A	0.025	0.016	0.102	0.515	0.420	0.000	0.170	1.466

Problem 492	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	21	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.162	23.449	1.007	0.000	0.392	0.000	0.000	0.000

Problem 493	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	21	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.211	26.000	1.104	0.000	0.452	0.000	0.000	0.000

Problem 494	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.489	7.380	0.167	0.000	0.432	0.000	0.000	0.000

Problem 495	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	284	263	180	1924	0	0	-1
normalized size	1	1.00	3.34	3.09	2.12	22.64	0.00	0.00	-0.01
time (sec)	N/A	0.238	5.760	0.650	0.336	0.472	0.000	0.000	0.000

Problem 496	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	A	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	216	199	118	1327	0	0	-1
normalized size	1	1.00	3.38	3.11	1.84	20.73	0.00	0.00	-0.02
time (sec)	N/A	0.167	4.189	0.661	0.448	0.476	0.000	0.000	0.000

Problem 497	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	26	62	87	292	0	72	43
normalized size	1	1.00	0.87	2.07	2.90	9.73	0.00	2.40	1.43
time (sec)	N/A	0.057	0.157	0.527	0.324	0.425	0.000	0.140	0.073

Problem 498	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	13	32	18	81	0	18	18
normalized size	1	1.00	0.57	1.39	0.78	3.52	0.00	0.78	0.78
time (sec)	N/A	0.033	0.010	0.344	0.312	0.398	0.000	0.141	0.072

Problem 499	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	20	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.066	23.955	0.526	0.000	0.420	0.000	0.000	0.000

Problem 500	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	20	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.071	21.121	0.531	0.000	0.409	0.000	0.000	0.000

Problem 501	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.578	56.028	0.209	0.000	0.418	0.000	0.000	0.000

Problem 502	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	206	206	397	0	0	3777	0	0	-1
normalized size	1	1.00	1.93	0.00	0.00	18.33	0.00	0.00	-0.00
time (sec)	N/A	0.434	7.421	2.067	0.000	0.581	0.000	0.000	0.000

Problem 503	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	209	232	0	2201	0	0	-1
normalized size	1	1.00	1.74	1.93	0.00	18.34	0.00	0.00	-0.01
time (sec)	N/A	0.166	2.492	0.712	0.000	0.503	0.000	0.000	0.000

Problem 504	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	29	52	90	511	0	102	107
normalized size	1	1.00	0.59	1.06	1.84	10.43	0.00	2.08	2.18
time (sec)	N/A	0.042	0.015	0.343	0.407	0.408	0.000	0.152	0.083

Problem 505	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.240	43.975	1.243	0.000	0.410	0.000	0.000	0.000

Problem 506	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.302	35.315	1.361	0.000	0.404	0.000	0.000	0.000

Problem 507	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	21	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.453	69.578	0.230	0.000	0.403	0.000	0.000	0.000

Problem 508	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	240	240	490	417	352	3394	0	0	-1
normalized size	1	1.00	2.04	1.74	1.47	14.14	0.00	0.00	-0.00
time (sec)	N/A	0.424	7.556	0.520	0.348	0.518	0.000	0.000	0.000

Problem 509	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	369	266	243	2562	0	0	-1
normalized size	1	1.00	2.49	1.80	1.64	17.31	0.00	0.00	-0.01
time (sec)	N/A	0.232	4.468	0.515	0.361	0.493	0.000	0.000	0.000

Problem 510	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	137	170	145	1578	0	0	-1
normalized size	1	1.00	1.44	1.79	1.53	16.61	0.00	0.00	-0.01
time (sec)	N/A	0.121	0.680	0.410	0.342	0.470	0.000	0.000	0.000

Problem 511	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	34	27	91	379	0	93	78
normalized size	1	1.00	1.21	0.96	3.25	13.54	0.00	3.32	2.79
time (sec)	N/A	0.027	0.048	0.175	1.181	0.426	0.000	0.149	1.454

Problem 512	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	21	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.196	57.308	1.501	0.000	0.421	0.000	0.000	0.000

Problem 513	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	21	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.244	27.622	1.730	0.000	0.446	0.000	0.000	0.000

Problem 514	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.601	69.162	0.238	0.000	0.413	0.000	0.000	0.000

Problem 515	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	317	317	433	0	0	5318	0	0	-1
normalized size	1	1.00	1.37	0.00	0.00	16.78	0.00	0.00	-0.00
time (sec)	N/A	1.177	8.425	2.102	0.000	0.578	0.000	0.000	0.000

Problem 516	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	C	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	197	197	441	0	0	3772	0	0	-1
normalized size	1	1.00	2.24	0.00	0.00	19.15	0.00	0.00	-0.01
time (sec)	N/A	0.489	8.067	1.938	0.000	0.545	0.000	0.000	0.000

Problem 517	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	168	148	166	1694	0	0	-1
normalized size	1	1.00	1.54	1.36	1.52	15.54	0.00	0.00	-0.01
time (sec)	N/A	0.165	2.995	0.559	0.462	0.447	0.000	0.000	0.000

Problem 518	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	68	43	106	709	0	110	111
normalized size	1	1.00	1.39	0.88	2.16	14.47	0.00	2.24	2.27
time (sec)	N/A	0.048	0.037	0.201	0.308	0.435	0.000	0.137	0.082

Problem 519	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.277	67.551	1.272	0.000	0.408	0.000	0.000	0.000

Problem 520	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.333	46.122	1.392	0.000	0.418	0.000	0.000	0.000

Problem 521	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F(-1)	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.613	180.001	0.271	0.000	0.409	0.000	0.000	0.000

Problem 522	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	240	240	274	445	381	6764	0	0	-1
normalized size	1	1.00	1.14	1.85	1.59	28.18	0.00	0.00	-0.00
time (sec)	N/A	0.304	7.416	0.486	2.151	0.572	0.000	0.000	0.000

Problem 523	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	149	192	299	273	4779	0	0	-1
normalized size	1	1.00	1.29	2.01	1.83	32.07	0.00	0.00	-0.01
time (sec)	N/A	0.198	6.365	0.467	2.243	0.550	0.000	0.000	0.000

Problem 524	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	148	197	164	3025	0	0	-1
normalized size	1	1.00	1.63	2.16	1.80	33.24	0.00	0.00	-0.01
time (sec)	N/A	0.110	1.361	0.470	0.489	0.491	0.000	0.000	0.000

Problem 525	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	47	48	102	774	0	96	96
normalized size	1	1.00	1.09	1.12	2.37	18.00	0.00	2.23	2.23
time (sec)	N/A	0.044	0.012	0.231	0.445	0.440	0.000	0.158	0.084

Problem 526	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	20	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.068	60.733	1.561	0.000	0.418	0.000	0.000	0.000

Problem 527	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	20	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.073	43.443	1.697	0.000	0.414	0.000	0.000	0.000

Problem 528	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	77	0	0	0	0	0	-1
normalized size	1	1.00	0.89	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.058	0.358	0.177	0.000	0.000	0.000	0.000	0.000

Problem 529	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-2)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	142	0	0	0	0	0	-1
normalized size	1	1.00	2.22	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.043	2.015	0.141	0.000	0.000	0.000	0.000	0.000

Problem 530	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	56	0	0	0	0	0	-1
normalized size	1	1.00	0.88	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.042	0.165	0.108	0.000	0.000	0.000	0.000	0.000

Problem 531	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	190	250	0	0	0	0	-1
normalized size	1	1.00	5.14	6.76	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.030	1.554	0.152	0.000	0.000	0.000	0.000	0.000

Problem 532	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	37	0	0	0	0	0	-1
normalized size	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.029	0.176	0.114	0.000	0.000	0.000	0.000	0.000

Problem 533	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	57	0	0	0	0	0	-1
normalized size	1	1.00	0.89	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.042	0.196	0.113	0.000	0.000	0.000	0.000	0.000

Problem 534	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	57	0	0	0	0	0	-1
normalized size	1	1.00	0.89	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.043	0.247	0.114	0.000	0.000	0.000	0.000	0.000

Problem 535	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	69	0	0	0	0	0	-1
normalized size	1	1.00	0.79	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.053	0.282	0.112	0.000	0.000	0.000	0.000	0.000

Problem 536	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	69	0	0	0	0	0	-1
normalized size	1	1.00	0.64	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.065	0.333	0.172	0.000	0.000	0.000	0.000	0.000

Problem 537	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	65	0	0	0	0	0	-1
normalized size	1	1.00	0.77	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.052	0.251	0.151	0.000	0.000	0.000	0.000	0.000

Problem 538	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	57	0	0	0	0	0	-1
normalized size	1	1.00	0.68	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.052	0.196	0.148	0.000	0.000	0.000	0.000	0.000

Problem 539	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	46	0	0	0	0	0	-1
normalized size	1	1.00	0.81	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.039	0.143	0.152	0.000	0.000	0.000	0.000	0.000

Problem 540	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	100	250	0	0	0	0	-1
normalized size	1	1.00	1.75	4.39	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.040	1.147	0.166	0.000	0.000	0.000	0.000	0.000

Problem 541	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	71	0	0	0	0	0	-1
normalized size	1	1.00	0.85	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.052	0.177	0.141	0.000	0.000	0.000	0.000	0.000

Problem 542	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	125	0	0	0	0	0	-1
normalized size	1	1.00	1.49	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.053	2.431	0.182	0.000	0.000	0.000	0.000	0.000

Problem 543	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	93	0	0	0	0	0	-1
normalized size	1	1.00	0.87	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.069	0.291	0.171	0.000	0.000	0.000	0.000	0.000

Problem 544	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	103	0	0	0	0	0	-1
normalized size	1	1.00	0.85	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.070	0.318	0.142	0.000	0.000	0.000	0.000	0.000

Problem 545	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	143	0	0	0	0	0	-1
normalized size	1	1.00	1.46	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.052	2.357	0.130	0.000	0.000	0.000	0.000	0.000

Problem 546	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	77	0	0	0	0	0	-1
normalized size	1	1.00	0.79	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.053	0.206	0.105	0.000	0.000	0.000	0.000	0.000

Problem 547	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	182	229	0	0	0	0	-1
normalized size	1	1.00	2.56	3.23	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.039	1.773	0.165	0.000	0.000	0.000	0.000	0.000

Problem 548	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	56	0	0	0	0	0	-1
normalized size	1	1.00	0.79	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.040	0.170	0.097	0.000	0.000	0.000	0.000	0.000

Problem 549	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	66	0	0	0	0	0	-1
normalized size	1	1.00	0.67	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.052	0.215	0.112	0.000	0.000	0.000	0.000	0.000

Problem 550	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	67	0	0	0	0	0	-1
normalized size	1	1.00	0.68	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.053	0.281	0.123	0.000	0.000	0.000	0.000	0.000

Problem 551	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	89	0	0	0	0	0	-1
normalized size	1	1.00	0.74	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.066	0.277	0.109	0.000	0.000	0.000	0.000	0.000

Problem 552	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	83	0	0	0	0	0	-1
normalized size	1	1.00	0.69	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.066	0.527	0.237	0.000	0.000	0.000	0.000	0.000

Problem 553	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	75	0	0	0	0	0	-1
normalized size	1	1.00	0.77	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.053	0.304	0.211	0.000	0.000	0.000	0.000	0.000

Problem 554	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	70	0	0	0	0	0	-1
normalized size	1	1.00	0.71	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.051	0.217	0.213	0.000	0.000	0.000	0.000	0.000

Problem 555	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	56	0	0	0	0	0	-1
normalized size	1	1.00	0.79	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.042	0.151	0.211	0.000	0.000	0.000	0.000	0.000

Problem 556	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	183	229	0	0	0	0	-1
normalized size	1	1.00	2.58	3.23	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.041	1.097	0.237	0.000	0.000	0.000	0.000	0.000

Problem 557	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	67	0	0	0	0	0	-1
normalized size	1	1.00	0.68	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.054	0.433	0.145	0.000	0.000	0.000	0.000	0.000

Problem 558	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	111	0	0	0	0	0	-1
normalized size	1	1.00	1.13	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.053	2.054	0.175	0.000	0.000	0.000	0.000	0.000

Problem 559	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	103	0	0	0	0	0	-1
normalized size	1	1.00	0.85	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.069	0.410	0.157	0.000	0.000	0.000	0.000	0.000

Problem 560	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	42	35	53	0	0	35
normalized size	1	1.00	1.00	3.23	2.69	4.08	0.00	0.00	2.69
time (sec)	N/A	0.050	0.066	0.517	0.774	0.436	0.000	0.000	1.622

Problem 561	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	23	0	69	95	0	0	-1
normalized size	1	1.00	0.74	0.00	2.23	3.06	0.00	0.00	-0.03
time (sec)	N/A	0.088	0.070	0.440	0.582	0.411	0.000	0.000	0.000

Problem 562	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	B	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	29	0	103	253	0	0	-1
normalized size	1	1.00	0.58	0.00	2.06	5.06	0.00	0.00	-0.02
time (sec)	N/A	0.118	0.115	0.444	0.689	0.421	0.000	0.000	0.000

Problem 563	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	35	42	54	55	0	0	23
normalized size	1	1.00	2.69	3.23	4.15	4.23	0.00	0.00	1.77
time (sec)	N/A	0.053	0.080	0.521	0.635	0.416	0.000	0.000	1.531

Problem 564	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	B	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	21	0	109	97	0	0	-1
normalized size	1	1.00	0.68	0.00	3.52	3.13	0.00	0.00	-0.03
time (sec)	N/A	0.097	0.051	0.437	0.616	0.422	0.000	0.000	0.000

Problem 565	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	B	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	44	0	163	259	0	0	-1
normalized size	1	1.00	0.88	0.00	3.26	5.18	0.00	0.00	-0.02
time (sec)	N/A	0.132	0.323	0.441	0.722	0.421	0.000	0.000	0.000

Problem 566	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	60	120	122	167	843	87	178
normalized size	1	1.00	1.15	2.31	2.35	3.21	16.21	1.67	3.42
time (sec)	N/A	0.132	0.116	0.135	0.527	0.432	88.924	0.135	1.830

Problem 567	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	62	119	119	174	845	88	177
normalized size	1	1.00	1.17	2.25	2.25	3.28	15.94	1.66	3.34
time (sec)	N/A	0.133	0.113	0.135	0.411	0.421	89.168	0.171	0.320

Problem 568	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	56	127	0	289	840	60	198
normalized size	1	1.00	0.98	2.23	0.00	5.07	14.74	1.05	3.47
time (sec)	N/A	0.133	0.096	0.128	0.000	0.444	30.818	0.120	1.808

Problem 569	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	56	154	0	299	840	62	199
normalized size	1	1.00	0.95	2.61	0.00	5.07	14.24	1.05	3.37
time (sec)	N/A	0.140	0.099	0.133	0.000	0.446	29.673	0.116	1.728

Problem 570	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	58	62	134	0	96	103
normalized size	1	1.00	1.00	2.32	2.48	5.36	0.00	3.84	4.12
time (sec)	N/A	0.058	0.177	0.909	0.706	0.415	0.000	0.156	1.543

Problem 571	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	55	0	129	0	100	105
normalized size	1	1.00	1.00	2.20	0.00	5.16	0.00	4.00	4.20
time (sec)	N/A	0.058	0.125	0.479	0.000	0.437	0.000	0.136	1.528

Problem 572	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	63	89	0	249	0	53	636
normalized size	1	1.00	1.02	1.44	0.00	4.02	0.00	0.85	10.26
time (sec)	N/A	0.157	0.130	0.192	0.000	0.774	0.000	0.139	6.352

Problem 573	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	67	86	141	172	0	90	539
normalized size	1	1.00	1.16	1.48	2.43	2.97	0.00	1.55	9.29
time (sec)	N/A	0.168	0.139	0.184	0.530	0.762	0.000	0.156	3.373

Problem 574	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	24	54	64	70	238	41	56
normalized size	1	1.00	1.26	2.84	3.37	3.68	12.53	2.16	2.95
time (sec)	N/A	0.042	0.032	0.162	0.463	0.425	6.541	0.143	0.132

Problem 575	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	8	34	14	17	41	14	14
normalized size	1	1.00	1.00	4.25	1.75	2.12	5.12	1.75	1.75
time (sec)	N/A	0.042	0.016	0.148	0.342	0.397	1.032	0.135	0.042

Problem 576	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	8	28	14	17	12	14	14
normalized size	1	1.00	1.00	3.50	1.75	2.12	1.50	1.75	1.75
time (sec)	N/A	0.043	0.007	0.142	0.362	0.404	0.906	0.112	0.047

Problem 577	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	24	102	102	70	61	38	56
normalized size	1	1.00	1.26	5.37	5.37	3.68	3.21	2.00	2.95
time (sec)	N/A	0.039	0.031	0.120	0.568	0.438	3.022	0.136	0.110

Problem 578	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	127	112	0	598	0	71	704
normalized size	1	1.00	1.72	1.51	0.00	8.08	0.00	0.96	9.51
time (sec)	N/A	0.245	0.223	0.184	0.000	0.791	0.000	0.138	6.896

Problem 579	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	125	112	158	401	0	109	613
normalized size	1	1.00	1.81	1.62	2.29	5.81	0.00	1.58	8.88
time (sec)	N/A	0.263	0.420	0.196	0.533	0.791	0.000	0.159	3.774

Problem 580	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	9	9	9	10	9	9	8	23	9
normalized size	1	1.00	1.00	1.11	1.00	1.00	0.89	2.56	1.00
time (sec)	N/A	0.009	0.004	0.017	0.354	0.404	0.113	0.129	1.500

Problem 581	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	36	37	46	42	78	74	39
normalized size	1	1.00	0.97	1.00	1.24	1.14	2.11	2.00	1.05
time (sec)	N/A	0.018	0.058	0.146	0.553	0.414	0.194	0.131	1.536

Problem 582	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	63	48	69	97	66	134	53
normalized size	1	1.00	1.80	1.37	1.97	2.77	1.89	3.83	1.51
time (sec)	N/A	0.025	0.136	0.459	0.390	0.406	0.327	0.137	0.095

Problem 583	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	87	90	103	168	265	208	143
normalized size	1	1.00	1.21	1.25	1.43	2.33	3.68	2.89	1.99
time (sec)	N/A	0.037	0.150	0.458	0.569	0.431	0.695	0.120	1.557

Problem 584	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	133	114	191	298	172	344	117
normalized size	1	1.00	2.18	1.87	3.13	4.89	2.82	5.64	1.92
time (sec)	N/A	0.045	0.230	0.417	0.382	0.410	1.177	0.137	0.125

Problem 585	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	46	39	0	148	0	35	35
normalized size	1	1.00	1.21	1.03	0.00	3.89	0.00	0.92	0.92
time (sec)	N/A	0.029	0.047	0.196	0.000	0.424	0.000	0.130	0.102

Problem 586	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	29	29	62	0	26	22
normalized size	1	1.00	1.00	1.71	1.71	3.65	0.00	1.53	1.29
time (sec)	N/A	0.015	0.027	0.239	0.364	0.396	0.000	0.117	1.537

Problem 587	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	96	167	0	1495	0	88	157
normalized size	1	1.00	1.25	2.17	0.00	19.42	0.00	1.14	2.04
time (sec)	N/A	0.047	0.491	0.257	0.000	0.448	0.000	0.142	1.610

Problem 588	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	64	87	498	527	0	53	47
normalized size	1	1.00	0.96	1.30	7.43	7.87	0.00	0.79	0.70
time (sec)	N/A	0.032	0.143	0.284	0.619	0.417	0.000	0.142	1.701

Problem 589	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	147	462	0	6874	0	236	354
normalized size	1	1.00	1.31	4.12	0.00	61.38	0.00	2.11	3.16
time (sec)	N/A	0.069	1.061	0.305	0.000	0.564	0.000	0.161	1.638

Problem 590	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	206	33	0	0	0	0	-1
normalized size	1	1.00	3.17	0.51	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.027	0.758	0.398	0.000	0.417	0.000	0.000	0.000

Problem 591	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	92	171	0	0	0	0	-1
normalized size	1	1.00	0.89	1.66	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.053	0.598	0.600	0.000	0.410	0.000	0.000	0.000

Problem 592	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	193	51	0	0	0	0	-1
normalized size	1	1.00	1.87	0.50	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.050	0.857	0.414	0.000	0.428	0.000	0.000	0.000

Problem 593	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	81	97	0	0	0	0	-1
normalized size	1	1.00	1.25	1.49	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.027	0.097	0.412	0.000	0.420	0.000	0.000	0.000

Problem 594	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	148	33	0	0	0	0	-1
normalized size	1	1.00	1.32	0.29	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.051	0.464	0.343	0.000	0.427	0.000	0.000	0.000

Problem 595	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	133	37	0	0	0	0	-1
normalized size	1	1.00	1.15	0.32	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.048	0.589	0.461	0.000	0.419	0.000	0.000	0.000

Problem 596	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	45	19	23	21	29	56	11
normalized size	1	1.00	1.96	0.83	1.00	0.91	1.26	2.43	0.48
time (sec)	N/A	0.015	0.019	0.018	0.310	0.407	0.156	0.132	0.058

Problem 597	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	25	24	88	43	44	17	17
normalized size	1	1.00	0.96	0.92	3.38	1.65	1.69	0.65	0.65
time (sec)	N/A	0.016	0.054	0.021	0.300	0.409	0.207	0.116	0.066

Problem 598	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	25	24	146	64	83	17	17
normalized size	1	1.00	0.96	0.92	5.62	2.46	3.19	0.65	0.65
time (sec)	N/A	0.015	0.082	0.019	0.306	0.405	0.462	0.137	0.071

Problem 599	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	24	27	18	34	36	20	17
normalized size	1	1.00	0.92	1.04	0.69	1.31	1.38	0.77	0.65
time (sec)	N/A	0.015	0.081	0.019	0.571	0.420	0.187	0.119	1.627

Problem 600	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	24	24	17	23	34	17	17
normalized size	1	1.00	1.00	1.00	0.71	0.96	1.42	0.71	0.71
time (sec)	N/A	0.016	0.042	0.020	0.306	0.399	0.447	0.128	0.070

Problem 601	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	26	24	17	49	66	17	17
normalized size	1	1.00	1.00	0.92	0.65	1.88	2.54	0.65	0.65
time (sec)	N/A	0.016	0.045	0.032	0.326	0.400	0.819	0.115	0.101

Problem 602	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	26	24	17	71	90	17	17
normalized size	1	1.00	1.00	0.92	0.65	2.73	3.46	0.65	0.65
time (sec)	N/A	0.016	0.048	0.016	0.323	0.401	1.418	0.133	1.470

Problem 603	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	24	25	17	24	0	17	15
normalized size	1	1.00	0.92	0.96	0.65	0.92	0.00	0.65	0.58
time (sec)	N/A	0.016	0.024	0.020	0.358	0.379	0.000	0.125	1.585

Problem 604	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	24	25	17	42	0	17	15
normalized size	1	1.00	0.92	0.96	0.65	1.62	0.00	0.65	0.58
time (sec)	N/A	0.017	0.036	0.022	0.361	0.400	0.000	0.132	1.566

Problem 605	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	47	21	24	22	29	56	15
normalized size	1	1.00	1.96	0.88	1.00	0.92	1.21	2.33	0.62
time (sec)	N/A	0.015	0.020	0.018	0.360	0.384	0.151	0.109	0.048

Problem 606	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	27	26	89	43	44	17	17
normalized size	1	1.00	1.00	0.96	3.30	1.59	1.63	0.63	0.63
time (sec)	N/A	0.015	0.030	0.018	0.303	0.383	0.211	0.137	0.060

Problem 607	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	27	26	147	62	83	17	17
normalized size	1	1.00	1.00	0.96	5.44	2.30	3.07	0.63	0.63
time (sec)	N/A	0.015	0.025	0.021	0.308	0.400	0.468	0.112	1.849

Problem 608	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	27	29	20	39	37	23	21
normalized size	1	1.00	0.96	1.04	0.71	1.39	1.32	0.82	0.75
time (sec)	N/A	0.016	0.049	0.024	0.488	0.404	0.195	0.141	1.659

Problem 609	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	22	25	13	20	32	13	13
normalized size	1	1.00	0.92	1.04	0.54	0.83	1.33	0.54	0.54
time (sec)	N/A	0.016	0.008	0.021	0.515	0.409	0.353	0.137	1.533

Problem 610	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	27	26	17	41	65	17	17
normalized size	1	1.00	1.00	0.96	0.63	1.52	2.41	0.63	0.63
time (sec)	N/A	0.015	0.044	0.020	0.346	0.825	0.620	0.116	0.096

Problem 611	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	27	26	17	59	88	17	17
normalized size	1	1.00	1.00	0.96	0.63	2.19	3.26	0.63	0.63
time (sec)	N/A	0.016	0.072	0.020	0.517	0.419	1.123	0.142	0.056

Problem 612	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	26	26	17	24	0	17	18
normalized size	1	1.00	0.96	0.96	0.63	0.89	0.00	0.63	0.67
time (sec)	N/A	0.016	0.022	0.020	0.308	0.430	0.000	0.136	1.551

Problem 613	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	26	26	17	40	0	17	27
normalized size	1	1.00	0.96	0.96	0.63	1.48	0.00	0.63	1.00
time (sec)	N/A	0.017	0.030	0.019	0.353	0.417	0.000	0.118	0.173

Problem 614	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	355	201	279	2040	0	240	495
normalized size	1	1.00	2.86	1.62	2.25	16.45	0.00	1.94	3.99
time (sec)	N/A	0.185	1.959	0.343	0.535	0.456	0.000	0.140	1.918

Problem 615	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	79	94	210	207	0	110	145
normalized size	1	1.00	0.79	0.94	2.10	2.07	0.00	1.10	1.45
time (sec)	N/A	0.209	0.186	0.389	0.385	0.396	0.000	0.128	1.520

Problem 616	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	194	75	120	502	0	117	233
normalized size	1	1.00	3.34	1.29	2.07	8.66	0.00	2.02	4.02
time (sec)	N/A	0.107	1.994	0.367	0.562	0.414	0.000	0.124	1.613

Problem 617	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	26	26	43	42	0	31	33
normalized size	1	1.00	0.90	0.90	1.48	1.45	0.00	1.07	1.14
time (sec)	N/A	0.063	0.050	0.353	0.396	0.386	0.000	0.115	1.566

Problem 618	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	16	12	11	30	0	21	40
normalized size	1	1.00	1.45	1.09	1.00	2.73	0.00	1.91	3.64
time (sec)	N/A	0.010	0.005	0.021	0.315	0.443	0.000	0.110	0.071

Problem 619	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	50	28	27	32	22	25
normalized size	1	1.00	1.00	4.55	2.55	2.45	2.91	2.00	2.27
time (sec)	N/A	0.039	0.006	0.213	0.320	0.410	0.442	0.119	0.080

Problem 620	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	502	119	100	362	0	97	132
normalized size	1	1.00	8.10	1.92	1.61	5.84	0.00	1.56	2.13
time (sec)	N/A	0.127	2.085	0.281	0.428	0.421	0.000	0.138	1.741

Problem 621	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	42	241	117	543	651	75	-1
normalized size	1	1.00	0.88	5.02	2.44	11.31	13.56	1.56	-0.02
time (sec)	N/A	0.080	0.137	0.273	0.484	0.417	2.887	0.131	0.000

Problem 622	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	B	B	F	A	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	146	146	3430	972	375	2978	0	267	-1
normalized size	1	1.00	23.49	6.66	2.57	20.40	0.00	1.83	-0.01
time (sec)	N/A	0.365	6.456	0.308	0.782	0.491	0.000	0.171	0.000

Problem 623	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	83	721	297	2640	2162	152	-1
normalized size	1	1.00	0.87	7.59	3.13	27.79	22.76	1.60	-0.01
time (sec)	N/A	0.116	0.251	0.335	0.359	0.461	16.636	0.165	0.000

Problem 624	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	62	78	235	92	0	34	90
normalized size	1	1.00	1.55	1.95	5.88	2.30	0.00	0.85	2.25
time (sec)	N/A	0.061	0.112	0.431	0.502	0.424	0.000	0.146	0.285

Problem 625	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	74	60	181	52	0	22	65
normalized size	1	1.00	1.95	1.58	4.76	1.37	0.00	0.58	1.71
time (sec)	N/A	0.113	0.135	0.339	0.494	0.404	0.000	0.120	1.632

Problem 626	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	39	41	73	49	0	21	39
normalized size	1	1.00	1.50	1.58	2.81	1.88	0.00	0.81	1.50
time (sec)	N/A	0.054	0.031	0.342	0.715	0.430	0.000	0.119	1.647

Problem 627	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	14	16	25	17	0	12	14
normalized size	1	1.00	0.70	0.80	1.25	0.85	0.00	0.60	0.70
time (sec)	N/A	0.075	0.005	0.357	0.318	0.410	0.000	0.144	1.640

Problem 628	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	17	11	9	11	0	18	14
normalized size	1	1.00	1.55	1.00	0.82	1.00	0.00	1.64	1.27
time (sec)	N/A	0.008	0.004	0.019	0.312	0.425	0.000	0.137	1.595

Problem 629	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	17	33	15	11	22	13	14
normalized size	1	1.00	1.31	2.54	1.15	0.85	1.69	1.00	1.08
time (sec)	N/A	0.029	0.017	0.238	0.344	0.426	0.212	0.137	1.565

Problem 630	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	31	29	14	17	0	12	14
normalized size	1	1.00	1.55	1.45	0.70	0.85	0.00	0.60	0.70
time (sec)	N/A	0.044	0.027	0.283	0.355	0.409	0.000	0.140	1.504

Problem 631	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	40	56	33	49	432	27	41
normalized size	1	1.00	1.43	2.00	1.18	1.75	15.43	0.96	1.46
time (sec)	N/A	0.052	0.039	0.345	0.514	0.420	1.801	0.135	0.187

Problem 632	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	75	41	40	52	0	22	67
normalized size	1	1.00	1.97	1.08	1.05	1.37	0.00	0.58	1.76
time (sec)	N/A	0.077	0.081	0.360	0.422	0.425	0.000	0.137	0.172

Problem 633	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	45	68	60	92	1445	38	94
normalized size	1	1.00	1.07	1.62	1.43	2.19	34.40	0.90	2.24
time (sec)	N/A	0.055	0.108	0.418	0.392	0.416	8.423	0.131	1.679

Problem 634	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	62	78	235	92	0	34	94
normalized size	1	1.00	1.48	1.86	5.60	2.19	0.00	0.81	2.24
time (sec)	N/A	0.059	0.060	0.388	0.607	0.422	0.000	0.121	1.604

Problem 635	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	75	60	181	52	0	22	67
normalized size	1	1.00	1.97	1.58	4.76	1.37	0.00	0.58	1.76
time (sec)	N/A	0.109	0.054	0.357	0.371	0.437	0.000	0.145	0.128

Problem 636	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	39	41	73	49	0	21	41
normalized size	1	1.00	1.39	1.46	2.61	1.75	0.00	0.75	1.46
time (sec)	N/A	0.060	0.029	0.371	0.858	0.409	0.000	0.139	0.155

Problem 637	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	14	16	25	17	0	12	14
normalized size	1	1.00	0.70	0.80	1.25	0.85	0.00	0.60	0.70
time (sec)	N/A	0.074	0.005	0.354	0.406	0.418	0.000	0.118	1.560

Problem 638	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	17	11	9	11	0	18	14
normalized size	1	1.00	1.55	1.00	0.82	1.00	0.00	1.64	1.27
time (sec)	N/A	0.008	0.004	0.019	0.307	0.424	0.000	0.125	1.471

Problem 639	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	17	33	15	11	22	13	14
normalized size	1	1.00	1.55	3.00	1.36	1.00	2.00	1.18	1.27
time (sec)	N/A	0.030	0.017	0.229	0.316	0.435	0.321	0.118	0.128

Problem 640	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	31	29	14	17	0	12	14
normalized size	1	1.00	1.55	1.45	0.70	0.85	0.00	0.60	0.70
time (sec)	N/A	0.047	0.028	0.286	0.503	0.418	0.000	0.117	1.506

Problem 641	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	27	56	33	49	432	27	39
normalized size	1	1.00	1.04	2.15	1.27	1.88	16.62	1.04	1.50
time (sec)	N/A	0.052	0.039	0.409	0.563	0.445	1.789	0.139	0.172

Problem 642	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	74	41	40	52	0	22	65
normalized size	1	1.00	1.95	1.08	1.05	1.37	0.00	0.58	1.71
time (sec)	N/A	0.077	0.066	0.405	0.333	0.416	0.000	0.139	1.611

Problem 643	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	45	68	60	92	1445	38	90
normalized size	1	1.00	1.12	1.70	1.50	2.30	36.12	0.95	2.25
time (sec)	N/A	0.055	0.104	0.401	0.374	0.430	8.385	0.120	1.687

Problem 644	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	244	201	330	2716	0	234	392
normalized size	1	1.00	1.97	1.62	2.66	21.90	0.00	1.89	3.16
time (sec)	N/A	0.243	0.474	0.381	0.389	0.473	0.000	0.138	0.202

Problem 645	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	95	94	214	209	0	112	146
normalized size	1	1.00	0.94	0.93	2.12	2.07	0.00	1.11	1.45
time (sec)	N/A	0.239	0.277	0.349	0.692	0.424	0.000	0.121	1.550

Problem 646	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	99	75	152	674	0	115	169
normalized size	1	1.00	1.68	1.27	2.58	11.42	0.00	1.95	2.86
time (sec)	N/A	0.123	0.253	0.363	0.437	0.424	0.000	0.137	1.540

Problem 647	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	23	27	45	37	0	29	33
normalized size	1	1.00	0.85	1.00	1.67	1.37	0.00	1.07	1.22
time (sec)	N/A	0.068	0.128	0.343	0.447	0.404	0.000	0.134	1.605

Problem 648	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	15	14	13	29	0	33	35
normalized size	1	1.00	1.25	1.17	1.08	2.42	0.00	2.75	2.92
time (sec)	N/A	0.010	0.004	0.018	0.386	0.429	0.000	0.131	0.064

Problem 649	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	51	26	27	0	19	23
normalized size	1	1.00	1.00	4.64	2.36	2.45	0.00	1.73	2.09
time (sec)	N/A	0.046	0.019	0.211	0.311	0.419	0.000	0.151	0.086

Problem 650	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	61	95	0	682	0	68	139
normalized size	1	1.00	0.91	1.42	0.00	10.18	0.00	1.01	2.07
time (sec)	N/A	0.132	0.349	0.217	0.000	0.449	0.000	0.125	1.789

Problem 651	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	77	144	111	521	0	66	-1
normalized size	1	1.00	1.54	2.88	2.22	10.42	0.00	1.32	-0.02
time (sec)	N/A	0.107	0.113	0.218	0.677	0.781	0.000	0.146	0.000

Problem 652	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	159	159	150	507	0	5830	0	242	-1
normalized size	1	1.00	0.94	3.19	0.00	36.67	0.00	1.52	-0.01
time (sec)	N/A	0.373	0.468	0.234	0.000	0.550	0.000	0.183	0.000

Problem 653	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	138	309	285	2564	0	135	-1
normalized size	1	1.00	1.41	3.15	2.91	26.16	0.00	1.38	-0.01
time (sec)	N/A	0.156	0.316	0.236	0.603	0.495	0.000	0.183	0.000

Problem 654	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	53	71	236	270	0	33	81
normalized size	1	1.00	1.89	2.54	8.43	9.64	0.00	1.18	2.89
time (sec)	N/A	0.067	0.091	0.324	0.582	0.432	0.000	0.121	1.522

Problem 655	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	49	183	68	0	22	59
normalized size	1	1.00	1.00	1.63	6.10	2.27	0.00	0.73	1.97
time (sec)	N/A	0.114	0.054	0.371	0.521	0.440	0.000	0.109	0.049

Problem 656	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	41	35	66	91	0	22	33
normalized size	1	1.00	2.28	1.94	3.67	5.06	0.00	1.22	1.83
time (sec)	N/A	0.056	0.053	0.367	0.363	0.425	0.000	0.131	0.046

Problem 657	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	10	13	25	22	0	10	10
normalized size	1	1.00	0.71	0.93	1.79	1.57	0.00	0.71	0.71
time (sec)	N/A	0.082	0.026	0.371	0.314	0.399	0.000	0.131	1.665

Problem 658	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	9	9	11	10	9	13	0	25	11
normalized size	1	1.00	1.22	1.11	1.00	1.44	0.00	2.78	1.22
time (sec)	N/A	0.008	0.003	0.029	0.548	0.399	0.000	0.111	0.036

Problem 659	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	5	5	9	20	11	13	0	11	11
normalized size	1	1.00	1.80	4.00	2.20	2.60	0.00	2.20	2.20
time (sec)	N/A	0.032	0.020	0.209	0.333	0.408	0.000	0.127	0.044

Problem 660	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	10	24	12	20	0	10	10
normalized size	1	1.00	0.83	2.00	1.00	1.67	0.00	0.83	0.83
time (sec)	N/A	0.049	0.021	0.194	0.470	0.396	0.000	0.134	0.177

Problem 661	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	18	28	31	89	0	21	33
normalized size	1	1.00	1.29	2.00	2.21	6.36	0.00	1.50	2.36
time (sec)	N/A	0.057	0.022	0.193	0.443	0.414	0.000	0.115	1.550

Problem 662	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	30	32	38	68	0	22	57
normalized size	1	1.00	1.15	1.23	1.46	2.62	0.00	0.85	2.19
time (sec)	N/A	0.085	0.022	0.206	0.665	0.408	0.000	0.117	1.553

Problem 663	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	32	36	52	266	0	30	79
normalized size	1	1.00	1.45	1.64	2.36	12.09	0.00	1.36	3.59
time (sec)	N/A	0.063	0.021	0.200	0.677	0.407	0.000	0.116	1.613

Problem 664	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	55	73	238	265	0	28	77
normalized size	1	1.00	2.29	3.04	9.92	11.04	0.00	1.17	3.21
time (sec)	N/A	0.063	0.086	0.346	0.349	0.403	0.000	0.131	1.501

Problem 665	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	30	49	183	68	0	22	57
normalized size	1	1.00	1.15	1.88	7.04	2.62	0.00	0.85	2.19
time (sec)	N/A	0.117	0.008	0.369	0.453	0.442	0.000	0.129	1.514

Problem 666	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	43	37	68	88	0	19	31
normalized size	1	1.00	2.69	2.31	4.25	5.50	0.00	1.19	1.94
time (sec)	N/A	0.058	0.055	0.373	0.510	0.399	0.000	0.111	1.536

Problem 667	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	18	13	25	20	0	10	10
normalized size	1	1.00	1.50	1.08	2.08	1.67	0.00	0.83	0.83
time (sec)	N/A	0.084	0.005	0.367	0.397	0.388	0.000	0.112	0.057

Problem 668	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	13	12	11	11	0	25	9
normalized size	1	1.00	1.18	1.09	1.00	1.00	0.00	2.27	0.82
time (sec)	N/A	0.008	0.005	0.023	0.612	0.403	0.000	0.124	1.536

Problem 669	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	9	9	9	23	13	11	0	10	9
normalized size	1	1.00	1.00	2.56	1.44	1.22	0.00	1.11	1.00
time (sec)	N/A	0.034	0.022	0.191	0.371	0.401	0.000	0.132	0.040

Problem 670	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	24	26	12	22	0	10	10
normalized size	1	1.00	1.71	1.86	0.86	1.57	0.00	0.71	0.71
time (sec)	N/A	0.051	0.009	0.200	0.369	0.394	0.000	0.129	1.672

Problem 671	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	18	29	35	90	0	20	31
normalized size	1	1.00	0.90	1.45	1.75	4.50	0.00	1.00	1.55
time (sec)	N/A	0.059	0.025	0.198	0.396	0.436	0.000	0.130	1.511

Problem 672	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	28	34	38	68	0	22	59
normalized size	1	1.00	0.93	1.13	1.27	2.27	0.00	0.73	1.97
time (sec)	N/A	0.083	0.008	0.216	0.370	0.408	0.000	0.139	1.532

Problem 673	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	32	37	58	269	0	31	79
normalized size	1	1.00	1.07	1.23	1.93	8.97	0.00	1.03	2.63
time (sec)	N/A	0.062	0.024	0.213	0.420	0.430	0.000	0.113	1.523

Problem 674	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	10	9	8	53	0	24	27
normalized size	1	1.00	1.25	1.12	1.00	6.62	0.00	3.00	3.38
time (sec)	N/A	0.008	0.004	0.022	0.319	0.403	0.000	0.111	0.045

Problem 675	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	18	15	26	32	0	39	26
normalized size	1	1.00	0.82	0.68	1.18	1.45	0.00	1.77	1.18
time (sec)	N/A	0.025	0.004	0.404	0.362	0.411	0.000	0.113	1.576

Problem 676	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	45	28	67	616	0	62	71
normalized size	1	1.00	1.32	0.82	1.97	18.12	0.00	1.82	2.09
time (sec)	N/A	0.052	0.071	0.470	0.373	0.430	0.000	0.114	0.064

Problem 677	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	35	42	54	55	0	0	13
normalized size	1	1.00	2.69	3.23	4.15	4.23	0.00	0.00	1.00
time (sec)	N/A	0.070	0.070	0.584	0.519	0.406	0.000	0.000	1.539

Problem 678	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	B	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	21	0	109	97	0	0	-1
normalized size	1	1.00	0.68	0.00	3.52	3.13	0.00	0.00	-0.03
time (sec)	N/A	0.122	0.052	0.494	0.542	0.426	0.000	0.000	0.000

Problem 679	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	B	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	44	0	163	259	0	0	-1
normalized size	1	1.00	0.88	0.00	3.26	5.18	0.00	0.00	-0.02
time (sec)	N/A	0.157	0.325	0.481	0.856	0.441	0.000	0.000	0.000

Problem 680	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	14	9	8	42	0	16	16
normalized size	1	1.00	1.75	1.12	1.00	5.25	0.00	2.00	2.00
time (sec)	N/A	0.006	0.005	0.018	0.517	0.474	0.000	0.130	1.491

Problem 681	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	16	13	26	30	0	37	26
normalized size	1	1.00	0.73	0.59	1.18	1.36	0.00	1.68	1.18
time (sec)	N/A	0.025	0.029	0.353	0.344	0.480	0.000	0.145	1.543

Problem 682	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	37	29	56	486	0	66	57
normalized size	1	1.00	1.09	0.85	1.65	14.29	0.00	1.94	1.68
time (sec)	N/A	0.048	0.029	0.459	0.985	0.450	0.000	0.124	0.062

Problem 683	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	43	39	57	0	0	15
normalized size	1	1.00	1.00	3.07	2.79	4.07	0.00	0.00	1.07
time (sec)	N/A	0.057	0.056	0.625	0.881	0.418	0.000	0.000	1.604

Problem 684	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	24	0	77	99	0	0	-1
normalized size	1	1.00	0.73	0.00	2.33	3.00	0.00	0.00	-0.03
time (sec)	N/A	0.097	0.086	0.521	0.798	0.426	0.000	0.000	0.000

Problem 685	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	B	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	30	0	115	257	0	0	-1
normalized size	1	1.00	0.57	0.00	2.17	4.85	0.00	0.00	-0.02
time (sec)	N/A	0.126	0.136	0.536	0.570	0.468	0.000	0.000	0.000

Problem 686	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	24	35	17	39	96	0	43	39
normalized size	1	1.33	1.94	0.94	2.17	5.33	0.00	2.39	2.17
time (sec)	N/A	0.075	0.027	0.201	0.347	0.451	0.000	0.131	0.045

Problem 687	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	24	50	17	40	96	0	43	41
normalized size	1	1.20	2.50	0.85	2.00	4.80	0.00	2.15	2.05
time (sec)	N/A	0.073	0.055	0.202	0.340	0.440	0.000	0.115	0.042

Problem 688	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	29	70	40	43	143	43	30
normalized size	1	1.00	0.74	1.79	1.03	1.10	3.67	1.10	0.77
time (sec)	N/A	0.067	0.075	0.201	0.313	0.435	0.552	0.132	0.087

Problem 689	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	89	93	0	435	0	61	157
normalized size	1	1.00	1.20	1.26	0.00	5.88	0.00	0.82	2.12
time (sec)	N/A	0.083	0.205	0.205	0.000	0.447	0.000	0.119	1.723

Problem 690	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	75	175	87	337	0	114	86
normalized size	1	1.00	0.74	1.73	0.86	3.34	0.00	1.13	0.85
time (sec)	N/A	0.133	0.158	0.219	0.561	0.428	0.000	0.143	1.958

Problem 691	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	29	71	41	42	150	43	29
normalized size	1	1.00	0.74	1.82	1.05	1.08	3.85	1.10	0.74
time (sec)	N/A	0.062	0.052	0.186	0.306	0.431	0.578	0.121	1.515

Problem 692	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	80	93	0	435	0	61	157
normalized size	1	1.00	1.08	1.26	0.00	5.88	0.00	0.82	2.12
time (sec)	N/A	0.079	0.165	0.229	0.000	0.467	0.000	0.126	1.719

Problem 693	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	75	175	86	331	0	111	84
normalized size	1	1.00	0.74	1.73	0.85	3.28	0.00	1.10	0.83
time (sec)	N/A	0.116	0.123	0.232	0.509	0.440	0.000	0.127	1.669

Problem 694	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	60	54	0	200	0	48	164
normalized size	1	1.00	1.20	1.08	0.00	4.00	0.00	0.96	3.28
time (sec)	N/A	0.096	0.135	0.300	0.000	0.468	0.000	0.125	3.533

Problem 695	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	59	53	0	239	0	60	177
normalized size	1	1.00	1.16	1.04	0.00	4.69	0.00	1.18	3.47
time (sec)	N/A	0.100	0.085	0.303	0.000	0.457	0.000	0.118	1.784

Problem 696	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	125	99	0	594	0	72	183
normalized size	1	1.00	1.89	1.50	0.00	9.00	0.00	1.09	2.77
time (sec)	N/A	0.063	0.191	0.265	0.000	0.451	0.000	0.130	1.647

Problem 697	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	61	146	104	348	983	113	108
normalized size	1	1.00	0.90	2.15	1.53	5.12	14.46	1.66	1.59
time (sec)	N/A	0.138	0.278	0.262	0.499	0.441	1.424	0.142	0.471

Problem 698	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	195	301	205	164	0	1633	0	174	255
normalized size	1	1.54	1.05	0.84	0.00	8.37	0.00	0.89	1.31
time (sec)	N/A	1.225	0.457	0.277	0.000	0.488	0.000	0.135	1.792

Problem 699	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	124	98	0	596	0	72	183
normalized size	1	1.00	1.94	1.53	0.00	9.31	0.00	1.12	2.86
time (sec)	N/A	0.055	0.132	0.297	0.000	0.461	0.000	0.127	1.709

Problem 700	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	66	149	104	348	952	114	104
normalized size	1	1.00	0.99	2.22	1.55	5.19	14.21	1.70	1.55
time (sec)	N/A	0.126	0.324	0.262	0.513	0.449	1.488	0.127	0.424

Problem 701	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	193	204	167	0	1645	0	174	255
normalized size	1	1.45	1.53	1.26	0.00	12.37	0.00	1.31	1.92
time (sec)	N/A	0.782	0.370	0.274	0.000	0.477	0.000	0.135	1.855

Problem 702	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	54	31	167	216	0	50	42
normalized size	1	1.00	2.84	1.63	8.79	11.37	0.00	2.63	2.21
time (sec)	N/A	0.032	0.122	0.281	0.458	0.407	0.000	0.129	1.598

Problem 703	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	117	404	289	1268	3813	251	159
normalized size	1	1.00	1.12	3.88	2.78	12.19	36.66	2.41	1.53
time (sec)	N/A	0.238	0.776	0.284	0.453	0.480	4.091	0.142	1.699

Problem 704	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	40	55	167	216	0	48	42
normalized size	1	1.00	2.11	2.89	8.79	11.37	0.00	2.53	2.21
time (sec)	N/A	0.031	0.062	0.252	0.335	0.418	0.000	0.126	1.523

Problem 705	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	119	494	292	1269	3840	251	159
normalized size	1	1.00	1.14	4.75	2.81	12.20	36.92	2.41	1.53
time (sec)	N/A	0.201	1.051	0.286	0.465	0.476	4.388	0.146	1.630

Problem 706	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	79	92	0	427	0	60	157
normalized size	1	1.00	1.10	1.28	0.00	5.93	0.00	0.83	2.18
time (sec)	N/A	0.089	0.209	0.212	0.000	0.492	0.000	0.121	1.686

Problem 707	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	73	145	83	334	0	101	81
normalized size	1	1.00	0.72	1.42	0.81	3.27	0.00	0.99	0.79
time (sec)	N/A	0.160	0.254	0.210	0.331	0.449	0.000	0.115	1.682

Problem 708	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	180	166	0	1861	0	163	261
normalized size	1	1.00	1.31	1.21	0.00	13.58	0.00	1.19	1.91
time (sec)	N/A	0.203	1.150	0.224	0.000	0.484	0.000	0.126	1.780

Problem 709	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	73	146	84	334	0	102	81
normalized size	1	1.00	0.72	1.43	0.82	3.27	0.00	1.00	0.79
time (sec)	N/A	0.163	0.253	0.209	0.498	0.470	0.000	0.117	1.906

Problem 710	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	179	168	0	1847	0	159	260
normalized size	1	1.00	1.47	1.38	0.00	15.14	0.00	1.30	2.13
time (sec)	N/A	0.229	1.011	0.249	0.000	0.480	0.000	0.123	1.755

Problem 711	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	194	194	128	321	153	1158	0	199	127
normalized size	1	1.00	0.66	1.65	0.79	5.97	0.00	1.03	0.65
time (sec)	N/A	0.342	0.636	0.231	0.504	0.460	0.000	0.123	1.867

Problem 712	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	167	200	0	1829	0	163	259
normalized size	1	1.00	1.22	1.46	0.00	13.35	0.00	1.19	1.89
time (sec)	N/A	0.195	1.177	0.213	0.000	0.466	0.000	0.135	1.763

Problem 713	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	194	194	126	322	150	1162	0	202	129
normalized size	1	1.00	0.65	1.66	0.77	5.99	0.00	1.04	0.66
time (sec)	N/A	0.338	0.613	0.218	0.330	0.612	0.000	0.121	1.818

Problem 714	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	212	212	325	344	0	4935	0	325	371
normalized size	1	1.00	1.53	1.62	0.00	23.28	0.00	1.53	1.75
time (sec)	N/A	0.432	2.295	0.224	0.000	0.668	0.000	0.133	2.070

Problem 715	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	60	181	107	376	962	128	98
normalized size	1	1.00	0.65	1.95	1.15	4.04	10.34	1.38	1.05
time (sec)	N/A	0.204	0.236	0.249	0.448	0.607	1.386	0.133	1.931

Problem 716	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	165	165	222	219	0	1819	0	179	397
normalized size	1	1.00	1.35	1.33	0.00	11.02	0.00	1.08	2.41
time (sec)	N/A	0.307	1.098	0.271	0.000	0.553	0.000	0.138	1.819

Problem 717	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	215	215	176	253	241	1655	0	238	127
normalized size	1	1.00	0.82	1.18	1.12	7.70	0.00	1.11	0.59
time (sec)	N/A	0.536	1.031	0.273	0.522	0.576	0.000	0.143	1.823

Problem 718	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	163	163	264	217	0	1805	0	179	397
normalized size	1	1.00	1.62	1.33	0.00	11.07	0.00	1.10	2.44
time (sec)	N/A	0.316	0.901	0.260	0.000	0.467	0.000	0.157	1.822

Problem 719	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	205	205	174	286	244	1726	0	232	132
normalized size	1	1.00	0.85	1.40	1.19	8.42	0.00	1.13	0.64
time (sec)	N/A	0.659	2.047	0.264	0.546	0.549	0.000	0.154	1.854

Problem 720	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	261	261	474	326	0	5061	0	310	592
normalized size	1	1.00	1.82	1.25	0.00	19.39	0.00	1.19	2.27
time (sec)	N/A	1.034	3.937	0.269	0.000	0.628	0.000	0.154	1.948

Problem 721	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	215	215	183	253	240	1661	0	240	127
normalized size	1	1.00	0.85	1.18	1.12	7.73	0.00	1.12	0.59
time (sec)	N/A	0.558	1.318	0.260	0.458	0.519	0.000	0.129	1.834

Problem 722	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	259	259	481	289	0	5031	0	310	590
normalized size	1	1.00	1.86	1.12	0.00	19.42	0.00	1.20	2.28
time (sec)	N/A	0.908	2.173	0.267	0.000	0.693	0.000	0.150	1.968

Problem 723	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	314	314	366	398	384	4001	0	384	173
normalized size	1	1.00	1.17	1.27	1.22	12.74	0.00	1.22	0.55
time (sec)	N/A	1.732	1.487	0.285	0.520	0.603	0.000	0.149	1.922

Problem 724	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	78	181	0	233	367	80	178
normalized size	1	1.00	0.98	2.26	0.00	2.91	4.59	1.00	2.22
time (sec)	N/A	0.082	0.239	0.214	0.000	0.533	53.136	0.117	3.781

Problem 725	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	155	115	0	679	0	83	168
normalized size	1	1.00	1.89	1.40	0.00	8.28	0.00	1.01	2.05
time (sec)	N/A	0.079	0.460	0.259	0.000	0.496	0.000	0.121	1.847

Problem 726	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	134	187	0	1855	0	152	217
normalized size	1	1.00	1.09	1.52	0.00	15.08	0.00	1.24	1.76
time (sec)	N/A	0.127	1.193	0.275	0.000	0.557	0.000	0.128	1.685

Problem 727	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	78	182	0	234	697	80	177
normalized size	1	1.00	0.98	2.28	0.00	2.92	8.71	1.00	2.21
time (sec)	N/A	0.061	0.179	0.213	0.000	0.515	50.440	0.134	3.054

Problem 728	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	151	116	0	680	0	83	168
normalized size	1	1.00	1.94	1.49	0.00	8.72	0.00	1.06	2.15
time (sec)	N/A	0.061	0.300	0.281	0.000	0.526	0.000	0.129	1.793

Problem 729	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	134	214	0	1855	0	152	216
normalized size	1	1.00	1.12	1.78	0.00	15.46	0.00	1.27	1.80
time (sec)	N/A	0.119	1.091	0.273	0.000	0.519	0.000	0.132	1.673

Problem 730	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	17	17	6	16	8	6	6
normalized size	1	1.00	1.55	1.55	0.55	1.45	0.73	0.55	0.55
time (sec)	N/A	0.039	0.004	0.017	0.499	0.526	0.351	0.109	0.063

Problem 731	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	17	17	6	19	10	6	6
normalized size	1	1.00	1.55	1.55	0.55	1.73	0.91	0.55	0.55
time (sec)	N/A	0.034	0.005	0.019	0.404	0.434	0.311	0.111	1.534

Problem 732	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	15	13	10	13	14	13	16
normalized size	1	1.00	1.07	0.93	0.71	0.93	1.00	0.93	1.14
time (sec)	N/A	0.027	0.035	0.166	0.559	0.492	0.124	0.123	0.085

Problem 733	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	43	145	87	60	326	54	53
normalized size	1	1.00	0.81	2.74	1.64	1.13	6.15	1.02	1.00
time (sec)	N/A	0.046	0.124	0.215	0.477	0.526	0.705	0.114	1.562

Problem 734	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	87	152	0	749	0	88	199
normalized size	1	1.00	1.12	1.95	0.00	9.60	0.00	1.13	2.55
time (sec)	N/A	0.072	0.248	0.262	0.000	0.538	0.000	0.121	1.917

Problem 735	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	70	63	337	232	0	70	67
normalized size	1	1.00	0.99	0.89	4.75	3.27	0.00	0.99	0.94
time (sec)	N/A	0.069	0.171	0.280	0.595	0.506	0.000	0.137	1.664

Problem 736	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	90	253	0	264	643	89	302
normalized size	1	1.00	0.98	2.75	0.00	2.87	6.99	0.97	3.28
time (sec)	N/A	0.070	0.268	0.214	0.000	0.534	49.362	0.134	3.807

Problem 737	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	106	167	0	799	0	95	210
normalized size	1	1.00	1.20	1.90	0.00	9.08	0.00	1.08	2.39
time (sec)	N/A	0.069	0.292	0.272	0.000	0.531	0.000	0.140	1.745

Problem 738	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	146	228	0	1931	0	183	224
normalized size	1	1.00	1.08	1.69	0.00	14.30	0.00	1.36	1.66
time (sec)	N/A	0.140	0.841	0.287	0.000	0.523	0.000	0.140	1.687

Problem 739	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	116	110	137	160	196	219	131
normalized size	1	1.00	0.97	0.92	1.15	1.34	1.65	1.84	1.10
time (sec)	N/A	0.132	0.199	0.419	0.311	0.496	0.425	0.136	0.151

Problem 740	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	54	54	63	59	100	83	55
normalized size	1	1.00	0.92	0.92	1.07	1.00	1.69	1.41	0.93
time (sec)	N/A	0.035	0.088	0.148	0.505	0.488	0.212	0.113	1.544

Problem 741	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	13	12	12	12	26	12
normalized size	1	1.00	1.00	1.08	1.00	1.00	1.00	2.17	1.00
time (sec)	N/A	0.009	0.002	0.024	0.304	0.530	0.108	0.113	0.050

Problem 742	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	54	53	0	248	0	46	78
normalized size	1	1.00	1.06	1.04	0.00	4.86	0.00	0.90	1.53
time (sec)	N/A	0.069	0.081	0.191	0.000	0.474	0.000	0.130	0.205

Problem 743	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	105	191	0	1268	0	111	-1
normalized size	1	1.00	1.17	2.12	0.00	14.09	0.00	1.23	-0.01
time (sec)	N/A	0.088	0.277	0.257	0.000	0.473	0.000	0.123	0.000

Problem 744	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	146	146	183	747	0	7379	0	304	-1
normalized size	1	1.00	1.25	5.12	0.00	50.54	0.00	2.08	-0.01
time (sec)	N/A	0.165	0.530	0.279	0.000	0.596	0.000	0.135	0.000

Problem 745	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F(-2)	F(-1)	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	220	220	488	1842	0	0	0	717	-1
normalized size	1	1.00	2.22	8.37	0.00	0.00	0.00	3.26	-0.00
time (sec)	N/A	0.302	0.961	0.356	0.000	0.000	0.000	0.179	0.000

Problem 746	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	112	109	137	144	189	186	131
normalized size	1	1.00	1.07	1.04	1.30	1.37	1.80	1.77	1.25
time (sec)	N/A	0.116	0.166	0.496	0.313	0.442	0.424	0.118	0.147

Problem 747	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	55	55	63	57	100	81	51
normalized size	1	1.00	0.96	0.96	1.11	1.00	1.75	1.42	0.89
time (sec)	N/A	0.035	0.071	0.145	0.407	0.417	0.218	0.134	1.531

Problem 748	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	13	12	12	12	26	12
normalized size	1	1.00	1.00	1.08	1.00	1.00	1.00	2.17	1.00
time (sec)	N/A	0.009	0.002	0.020	0.311	0.440	0.114	0.116	1.486

Problem 749	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	35	14	36	32	17	39	46
normalized size	1	1.00	2.33	0.93	2.40	2.13	1.13	2.60	3.07
time (sec)	N/A	0.019	0.043	0.180	0.313	0.398	0.715	0.126	0.161

Problem 750	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	87	58	86	236	0	84	-1
normalized size	1	1.00	2.02	1.35	2.00	5.49	0.00	1.95	-0.02
time (sec)	N/A	0.041	0.321	0.247	0.532	0.444	0.000	0.138	0.000

Problem 751	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	148	138	248	1504	0	205	-1
normalized size	1	1.00	1.66	1.55	2.79	16.90	0.00	2.30	-0.01
time (sec)	N/A	0.094	0.550	0.270	0.349	0.446	0.000	0.128	0.000

Problem 752	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	300	250	487	4015	0	377	-1
normalized size	1	1.00	2.14	1.79	3.48	28.68	0.00	2.69	-0.01
time (sec)	N/A	0.213	0.634	0.302	0.380	0.520	0.000	0.160	0.000

Problem 753	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	188	188	208	363	277	1293	626	390	361
normalized size	1	1.00	1.11	1.93	1.47	6.88	3.33	2.07	1.92
time (sec)	N/A	0.148	0.514	0.673	0.340	0.449	1.466	0.156	0.451

Problem 754	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	134	182	161	664	298	194	144
normalized size	1	1.00	0.99	1.34	1.18	4.88	2.19	1.43	1.06
time (sec)	N/A	0.090	0.267	0.421	0.449	0.419	0.694	0.146	1.659

Problem 755	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	72	80	79	238	122	96	70
normalized size	1	1.00	0.80	0.89	0.88	2.64	1.36	1.07	0.78
time (sec)	N/A	0.048	0.119	0.158	0.314	0.415	0.287	0.141	1.607

Problem 756	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	24	23	22	61	20	36	22
normalized size	1	1.00	1.00	0.96	0.92	2.54	0.83	1.50	0.92
time (sec)	N/A	0.011	0.008	0.019	0.467	0.402	0.125	0.126	0.065

Problem 757	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F(-2)	B	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	36	596	0	88	0	0	-1
normalized size	1	1.00	1.06	17.53	0.00	2.59	0.00	0.00	-0.03
time (sec)	N/A	0.037	0.084	0.617	0.000	0.399	0.000	0.000	0.000

Problem 758	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	68	217	0	660	0	0	-1
normalized size	1	1.00	0.68	2.17	0.00	6.60	0.00	0.00	-0.01
time (sec)	N/A	0.081	0.165	0.365	0.000	0.425	0.000	0.000	0.000

Problem 759	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	146	146	184	488	0	3035	0	0	-1
normalized size	1	1.00	1.26	3.34	0.00	20.79	0.00	0.00	-0.01
time (sec)	N/A	0.120	0.402	0.470	0.000	0.502	0.000	0.000	0.000

Problem 760	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F(-2)	B	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	198	198	425	828	0	6590	0	0	-1
normalized size	1	1.00	2.15	4.18	0.00	33.28	0.00	0.00	-0.01
time (sec)	N/A	0.178	0.829	0.662	0.000	0.589	0.000	0.000	0.000

Problem 761	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	294	294	3775	1036	0	0	0	0	-1
normalized size	1	1.00	12.84	3.52	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.483	6.355	1.383	0.000	0.439	0.000	0.000	0.000

Problem 762	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	249	249	2292	318	0	0	0	0	-1
normalized size	1	1.00	9.20	1.28	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.270	6.157	0.791	0.000	0.443	0.000	0.000	0.000

Problem 763	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	1401	314	0	0	0	0	-1
normalized size	1	1.00	13.74	3.08	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.071	6.112	0.800	0.000	0.423	0.000	0.000	0.000

Problem 764	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	237	248	0	0	0	0	-1
normalized size	1	1.00	2.32	2.43	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.071	0.520	0.645	0.000	0.417	0.000	0.000	0.000

Problem 765	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	156	156	1522	1430	0	0	0	0	-1
normalized size	1	1.00	9.76	9.17	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.098	6.210	1.738	0.000	0.462	0.000	0.000	0.000

Problem 766	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	322	322	2492	6075	0	0	0	0	-1
normalized size	1	1.00	7.74	18.87	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.340	6.261	2.957	0.000	0.460	0.000	0.000	0.000

Problem 767	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	411	411	4093	57909	0	0	0	0	-1
normalized size	1	1.00	9.96	140.90	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.544	6.567	12.719	0.000	0.473	0.000	0.000	0.000

Problem 768	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	B	B	F(-1)	B	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	10223	518	1783	784	0	657	-1
normalized size	1	1.00	73.02	3.70	12.74	5.60	0.00	4.69	-0.01
time (sec)	N/A	0.123	75.523	1.131	4.419	0.444	0.000	0.328	0.000

Problem 769	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	B	B	F	B	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	10141	190	640	329	0	303	-1
normalized size	1	1.00	110.23	2.07	6.96	3.58	0.00	3.29	-0.01
time (sec)	N/A	0.077	73.414	0.814	0.944	0.445	0.000	0.246	0.000

Problem 770	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	B	B	F	B	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	10054	201	153	143	0	104	-1
normalized size	1	1.00	271.73	5.43	4.14	3.86	0.00	2.81	-0.03
time (sec)	N/A	0.039	74.465	0.809	0.832	0.451	0.000	0.141	0.000

Problem 771	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	211	129	0	681	0	297	-1
normalized size	1	1.00	2.13	1.30	0.00	6.88	0.00	3.00	-0.01
time (sec)	N/A	0.113	32.695	0.627	0.000	0.499	0.000	1.478	0.000

Problem 772	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F(-1)	B	F	B	F	F(-2)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	155	155	0	417	0	1801	0	0	-1
normalized size	1	1.00	0.00	2.69	0.00	11.62	0.00	0.00	-0.01
time (sec)	N/A	0.132	180.002	1.575	0.000	0.512	0.000	0.000	0.000

Problem 773	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F(-1)	B	F	B	F(-1)	F(-2)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	205	205	0	954	0	5297	0	0	-1
normalized size	1	1.00	0.00	4.65	0.00	25.84	0.00	0.00	-0.00
time (sec)	N/A	0.176	180.010	1.898	0.000	0.691	0.000	0.000	0.000

Problem 774	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	B	B	F(-1)	B	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	146	146	9943	288	1789	784	0	657	-1
normalized size	1	1.00	68.10	1.97	12.25	5.37	0.00	4.50	-0.01
time (sec)	N/A	0.122	76.232	1.089	4.384	0.497	0.000	0.314	0.000

Problem 775	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	B	B	F	B	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	9861	190	644	329	0	302	-1
normalized size	1	1.00	102.72	1.98	6.71	3.43	0.00	3.15	-0.01
time (sec)	N/A	0.079	73.737	0.814	0.981	0.442	0.000	0.198	0.000

Problem 776	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	B	B	F	B	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	9771	202	156	143	0	103	-1
normalized size	1	1.00	250.54	5.18	4.00	3.67	0.00	2.64	-0.03
time (sec)	N/A	0.037	74.761	0.891	0.826	0.446	0.000	0.148	0.000

Problem 777	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F	B	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	52609	129	0	680	0	299	-1
normalized size	1	1.00	515.77	1.26	0.00	6.67	0.00	2.93	-0.01
time (sec)	N/A	0.094	31.290	0.583	0.000	0.534	0.000	1.435	0.000

Problem 778	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F(-1)	B	F	B	F	F(-2)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	159	159	0	415	0	2137	0	0	-1
normalized size	1	1.00	0.00	2.61	0.00	13.44	0.00	0.00	-0.01
time (sec)	N/A	0.121	180.001	1.482	0.000	0.590	0.000	0.000	0.000

Problem 779	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F(-1)	B	F	B	F	F(-2)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	211	211	0	984	0	5675	0	0	-1
normalized size	1	1.00	0.00	4.66	0.00	26.90	0.00	0.00	-0.00
time (sec)	N/A	0.167	180.003	1.921	0.000	0.692	0.000	0.000	0.000

Problem 780	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	86	422	0	429	0	106	472
normalized size	1	1.00	0.80	3.94	0.00	4.01	0.00	0.99	4.41
time (sec)	N/A	0.137	0.205	0.243	0.000	0.456	0.000	0.133	6.276

Problem 781	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	86	421	0	438	0	106	324
normalized size	1	1.00	0.76	3.73	0.00	3.88	0.00	0.94	2.87
time (sec)	N/A	0.156	0.225	0.215	0.000	0.460	0.000	0.141	0.704

Problem 782	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	86	429	0	455	0	106	325
normalized size	1	1.00	0.83	4.12	0.00	4.38	0.00	1.02	3.12
time (sec)	N/A	0.115	0.214	0.204	0.000	0.465	0.000	0.119	2.155

Problem 783	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	28	10	19	34	10	10
normalized size	1	1.00	1.00	1.56	0.56	1.06	1.89	0.56	0.56
time (sec)	N/A	0.025	0.046	0.167	0.740	0.408	0.561	0.136	0.046

Problem 784	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	54	53	0	234	0	46	78
normalized size	1	1.00	1.00	0.98	0.00	4.33	0.00	0.85	1.44
time (sec)	N/A	0.082	0.062	0.243	0.000	0.435	0.000	0.112	1.816

Problem 785	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	146	146	96	406	0	486	0	126	1069
normalized size	1	1.00	0.66	2.78	0.00	3.33	0.00	0.86	7.32
time (sec)	N/A	0.481	0.266	0.217	0.000	1.535	0.000	0.139	28.994

Problem 786	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	28	20	11	17	0	13	11
normalized size	1	1.00	1.47	1.05	0.58	0.89	0.00	0.68	0.58
time (sec)	N/A	0.050	0.073	0.199	0.827	0.405	0.000	0.111	0.057

Problem 787	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	54	53	0	244	0	46	78
normalized size	1	1.00	1.08	1.06	0.00	4.88	0.00	0.92	1.56
time (sec)	N/A	0.090	0.047	0.201	0.000	0.454	0.000	0.131	0.212

Problem 788	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	97	180	0	546	0	122	1069
normalized size	1	1.00	0.82	1.53	0.00	4.63	0.00	1.03	9.06
time (sec)	N/A	0.584	0.244	0.201	0.000	1.547	0.000	0.122	7.175

Problem 789	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	104	573	0	505	0	122	377
normalized size	1	1.00	0.87	4.78	0.00	4.21	0.00	1.02	3.14
time (sec)	N/A	0.118	0.258	0.198	0.000	0.481	0.000	0.116	0.753

Problem 790	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	130	287	0	2211	0	179	-1
normalized size	1	1.00	1.20	2.66	0.00	20.47	0.00	1.66	-0.01
time (sec)	N/A	0.116	0.331	0.257	0.000	0.509	0.000	0.121	0.000

Problem 791	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	198	198	373	1091	0	12285	0	625	-1
normalized size	1	1.00	1.88	5.51	0.00	62.05	0.00	3.16	-0.01
time (sec)	N/A	0.291	0.881	0.298	0.000	0.682	0.000	0.188	0.000

Problem 792	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	104	574	0	508	0	122	375
normalized size	1	1.00	0.86	4.74	0.00	4.20	0.00	1.01	3.10
time (sec)	N/A	0.118	0.215	0.198	0.000	0.464	0.000	0.135	0.670

Problem 793	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	125	287	0	2228	0	177	-1
normalized size	1	1.00	1.16	2.66	0.00	20.63	0.00	1.64	-0.01
time (sec)	N/A	0.128	0.275	0.263	0.000	0.480	0.000	0.121	0.000

Problem 794	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	194	194	336	1112	0	12366	0	625	-1
normalized size	1	1.00	1.73	5.73	0.00	63.74	0.00	3.22	-0.01
time (sec)	N/A	0.276	0.729	0.297	0.000	0.696	0.000	0.170	0.000

Problem 795	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	125	107	873	0	583	0	125	376
normalized size	1	1.00	0.86	6.98	0.00	4.66	0.00	1.00	3.01
time (sec)	N/A	0.143	0.349	0.201	0.000	0.475	0.000	0.136	0.700

Problem 796	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	123	287	0	2119	0	179	-1
normalized size	1	1.00	1.14	2.66	0.00	19.62	0.00	1.66	-0.01
time (sec)	N/A	0.126	0.343	0.267	0.000	0.486	0.000	0.121	0.000

Problem 797	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	194	194	319	885	0	10107	0	577	-1
normalized size	1	1.00	1.64	4.56	0.00	52.10	0.00	2.97	-0.01
time (sec)	N/A	0.260	0.753	0.306	0.000	0.650	0.000	0.181	0.000

Problem 798	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	119	1009	0	605	0	136	454
normalized size	1	1.00	0.87	7.36	0.00	4.42	0.00	0.99	3.31
time (sec)	N/A	0.219	0.337	0.203	0.000	0.507	0.000	0.141	2.454

Problem 799	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	143	376	0	2541	0	207	-1
normalized size	1	1.00	1.18	3.11	0.00	21.00	0.00	1.71	-0.01
time (sec)	N/A	0.150	0.420	0.269	0.000	0.499	0.000	0.128	0.000

Problem 800	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	233	233	465	1425	0	13813	0	819	-1
normalized size	1	1.00	2.00	6.12	0.00	59.28	0.00	3.52	-0.00
time (sec)	N/A	0.511	1.101	0.357	0.000	0.703	0.000	0.188	0.000

Problem 801	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	34	73	0	55	0	35	-1
normalized size	1	1.00	1.55	3.32	0.00	2.50	0.00	1.59	-0.05
time (sec)	N/A	0.083	0.106	0.274	0.000	0.453	0.000	0.140	0.000

Problem 802	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	86	136	58	107	753	58	57
normalized size	1	1.00	1.21	1.92	0.82	1.51	10.61	0.82	0.80
time (sec)	N/A	0.056	0.236	0.187	0.323	0.449	5.113	0.136	1.660

Problem 803	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	84	137	57	110	806	58	57
normalized size	1	1.00	1.09	1.78	0.74	1.43	10.47	0.75	0.74
time (sec)	N/A	0.050	0.191	0.205	0.445	0.441	5.155	0.133	1.688

Problem 804	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	103	232	99	134	1321	79	75
normalized size	1	1.00	1.20	2.70	1.15	1.56	15.36	0.92	0.87
time (sec)	N/A	0.083	0.312	0.194	0.323	0.431	5.936	0.117	1.718

Problem 805	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	86	125	65	59	852	49	48
normalized size	1	1.00	1.12	1.62	0.84	0.77	11.06	0.64	0.62
time (sec)	N/A	0.051	0.247	0.193	0.325	0.448	5.256	0.133	0.122

Problem 806	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	86	125	62	56	904	48	47
normalized size	1	1.00	1.10	1.60	0.79	0.72	11.59	0.62	0.60
time (sec)	N/A	0.046	0.164	0.191	0.321	0.440	5.263	0.134	1.587

Problem 807	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	102	213	105	70	1420	69	64
normalized size	1	1.00	1.26	2.63	1.30	0.86	17.53	0.85	0.79
time (sec)	N/A	0.080	0.298	0.207	0.329	0.459	5.973	0.134	1.628

Problem 808	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	3	3	3	116	35	19	172	5	5
normalized size	1	1.00	1.00	38.67	11.67	6.33	57.33	1.67	1.67
time (sec)	N/A	0.017	0.004	0.232	0.412	0.426	7.664	0.125	0.045

Problem 809	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	8	36	8	40	48	10	10
normalized size	1	1.00	0.73	3.27	0.73	3.64	4.36	0.91	0.91
time (sec)	N/A	0.025	0.003	0.192	0.308	0.410	1.368	0.132	1.550

Problem 810	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	22	166	64	304	0	46	28
normalized size	1	1.00	0.85	6.38	2.46	11.69	0.00	1.77	1.08
time (sec)	N/A	0.029	0.007	0.234	0.413	0.427	0.000	0.134	1.557

Problem 811	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1	1	1	8	1	1	10	1	1
normalized size	1	1.00	1.00	8.00	1.00	1.00	10.00	1.00	1.00
time (sec)	N/A	0.015	0.000	0.099	0.306	0.397	0.393	0.112	1.522

Problem 812	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1	1	1	8	1	1	22	1	1
normalized size	1	1.00	1.00	8.00	1.00	1.00	22.00	1.00	1.00
time (sec)	N/A	0.013	0.000	0.105	0.308	0.401	1.110	0.110	0.037

Problem 813	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1	1	1	8	1	1	34	1	1
normalized size	1	1.00	1.00	8.00	1.00	1.00	34.00	1.00	1.00
time (sec)	N/A	0.014	0.000	0.104	0.316	0.393	2.312	0.113	0.020

Problem 814	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1	1	1	8	1	1	0	1	1
normalized size	1	1.00	1.00	8.00	1.00	1.00	0.00	1.00	1.00
time (sec)	N/A	0.013	0.001	0.112	0.307	0.389	0.000	0.111	1.569

Problem 815	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1	1	1	8	1	1	0	1	1
normalized size	1	1.00	1.00	8.00	1.00	1.00	0.00	1.00	1.00
time (sec)	N/A	0.012	0.000	0.133	0.308	0.385	0.000	0.112	1.567

Problem 816	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1	1	1	8	1	1	0	1	1
normalized size	1	1.00	1.00	8.00	1.00	1.00	0.00	1.00	1.00
time (sec)	N/A	0.013	0.000	0.122	0.464	0.399	0.000	0.112	1.506

Problem 817	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	54	64	70	0	41	56
normalized size	1	1.00	1.00	2.84	3.37	3.68	0.00	2.16	2.95
time (sec)	N/A	0.028	0.043	0.219	0.434	0.438	0.000	0.142	0.170

Problem 818	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	42	108	88	266	0	63	78
normalized size	1	1.00	1.35	3.48	2.84	8.58	0.00	2.03	2.52
time (sec)	N/A	0.054	0.121	0.245	0.420	0.439	0.000	0.140	1.613

Problem 819	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	66	140	114	717	0	77	114
normalized size	1	1.00	1.22	2.59	2.11	13.28	0.00	1.43	2.11
time (sec)	N/A	0.064	0.213	0.249	0.442	0.448	0.000	0.123	0.072

Problem 820	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	102	36	67	0	36	54
normalized size	1	1.00	1.00	5.67	2.00	3.72	0.00	2.00	3.00
time (sec)	N/A	0.030	0.042	0.213	0.414	0.451	0.000	0.145	0.150

Problem 821	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	64	129	60	262	0	60	77
normalized size	1	1.00	2.00	4.03	1.88	8.19	0.00	1.88	2.41
time (sec)	N/A	0.047	0.099	0.229	0.426	0.430	0.000	0.117	0.057

Problem 822	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	66	145	84	715	0	72	112
normalized size	1	1.00	1.22	2.69	1.56	13.24	0.00	1.33	2.07
time (sec)	N/A	0.088	0.206	0.231	0.455	0.427	0.000	0.132	1.667

Problem 823	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1	1	1	8	1	1	0	1	1
normalized size	1	1.00	1.00	8.00	1.00	1.00	0.00	1.00	1.00
time (sec)	N/A	0.016	0.001	0.111	0.304	0.394	0.000	0.111	0.066

Problem 824	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1	1	1	8	1	1	0	1	1
normalized size	1	1.00	1.00	8.00	1.00	1.00	0.00	1.00	1.00
time (sec)	N/A	0.015	0.001	0.126	0.318	0.382	0.000	0.110	0.025

Problem 825	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1	1	1	8	1	1	0	1	1
normalized size	1	1.00	1.00	8.00	1.00	1.00	0.00	1.00	1.00
time (sec)	N/A	0.016	0.001	0.125	0.320	0.379	0.000	0.113	1.540

Problem 826	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	271	271	217	74	0	3313	0	1	-1
normalized size	1	1.00	0.80	0.27	0.00	12.23	0.00	0.00	-0.00
time (sec)	N/A	0.915	0.729	0.237	0.000	0.591	0.000	94.702	0.000

Problem 827	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	280	280	244	70	0	3309	0	0	-1
normalized size	1	1.00	0.87	0.25	0.00	11.82	0.00	0.00	-0.00
time (sec)	N/A	0.724	0.586	0.237	0.000	0.585	0.000	0.000	0.000

Problem 828	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	309	309	283	108	0	4943	0	5	-1
normalized size	1	1.00	0.92	0.35	0.00	16.00	0.00	0.02	-0.00
time (sec)	N/A	1.081	0.608	0.250	0.000	0.840	0.000	2.114	0.000

Problem 829	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	363	363	326	144	0	6680	0	24	-1
normalized size	1	1.00	0.90	0.40	0.00	18.40	0.00	0.07	-0.00
time (sec)	N/A	4.677	0.938	0.233	0.000	1.389	0.000	6.459	0.000

Problem 830	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	14	36	225	57	0	28	48
normalized size	1	1.00	1.17	3.00	18.75	4.75	0.00	2.33	4.00
time (sec)	N/A	0.091	0.038	0.171	0.420	0.428	0.000	0.129	1.913

Problem 831	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	300	300	258	79	0	6841	0	1	-1
normalized size	1	1.00	0.86	0.26	0.00	22.80	0.00	0.00	-0.00
time (sec)	N/A	0.758	0.587	0.263	0.000	4.496	0.000	4.277	0.000

Problem 832	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	223	223	198	1264	0	3485	0	1	-1
normalized size	1	1.00	0.89	5.67	0.00	15.63	0.00	0.00	-0.00
time (sec)	N/A	0.629	0.610	0.190	0.000	0.600	0.000	93.578	0.000

Problem 833	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	230	230	227	1262	0	3505	0	0	-1
normalized size	1	1.00	0.99	5.49	0.00	15.24	0.00	0.00	-0.00
time (sec)	N/A	0.583	0.546	0.145	0.000	0.609	0.000	0.000	0.000

Problem 834	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	255	255	264	1957	0	5079	0	5	-1
normalized size	1	1.00	1.04	7.67	0.00	19.92	0.00	0.02	-0.00
time (sec)	N/A	1.291	0.587	0.155	0.000	0.862	0.000	1.180	0.000

Problem 835	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	299	299	309	2530	0	6794	0	24	-1
normalized size	1	1.00	1.03	8.46	0.00	22.72	0.00	0.08	-0.00
time (sec)	N/A	6.389	0.799	0.175	0.000	1.404	0.000	5.384	0.000

Problem 836	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	29	0	54	0	26	51
normalized size	1	1.00	1.00	2.64	0.00	4.91	0.00	2.36	4.64
time (sec)	N/A	0.087	0.056	0.161	0.000	0.418	0.000	0.141	1.926

Problem 837	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	246	246	241	2556	0	6997	0	1	-1
normalized size	1	1.00	0.98	10.39	0.00	28.44	0.00	0.00	-0.00
time (sec)	N/A	0.682	0.479	0.155	0.000	4.586	0.000	4.300	0.000

Problem 838	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	34	414	74	367	243	46	209
normalized size	1	1.00	0.87	10.62	1.90	9.41	6.23	1.18	5.36
time (sec)	N/A	0.151	0.109	0.200	0.433	0.468	1.539	0.120	2.291

Problem 839	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	33	389	74	363	265	45	208
normalized size	1	1.00	0.87	10.24	1.95	9.55	6.97	1.18	5.47
time (sec)	N/A	0.112	0.059	0.212	0.440	0.491	1.694	0.139	1.948

Problem 840	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	40	96	73	95	136	33	25
normalized size	1	1.00	1.05	2.53	1.92	2.50	3.58	0.87	0.66
time (sec)	N/A	0.154	0.139	0.245	0.464	0.431	1.545	0.133	1.745

Problem 841	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	40	96	73	95	136	33	25
normalized size	1	1.00	1.05	2.53	1.92	2.50	3.58	0.87	0.66
time (sec)	N/A	0.092	0.105	0.241	0.415	0.425	1.527	0.114	1.689

Problem 842	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	55	136	36	91	0	0	-1
normalized size	1	1.00	0.93	2.31	0.61	1.54	0.00	0.00	-0.02
time (sec)	N/A	0.701	0.058	0.344	0.438	0.421	0.000	0.000	0.000

Problem 843	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	83	209	60	188	0	0	-1
normalized size	1	1.00	0.80	2.01	0.58	1.81	0.00	0.00	-0.01
time (sec)	N/A	0.813	0.071	0.351	0.445	0.434	0.000	0.000	0.000

Problem 844	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	150	113	281	80	272	0	0	-1
normalized size	1	1.00	0.75	1.87	0.53	1.81	0.00	0.00	-0.01
time (sec)	N/A	0.833	0.090	0.348	0.459	0.433	0.000	0.000	0.000

Problem 845	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	44	175	43	152	0	0	-1
normalized size	1	1.00	0.60	2.40	0.59	2.08	0.00	0.00	-0.01
time (sec)	N/A	0.556	0.051	0.369	0.457	0.447	0.000	0.000	0.000

Problem 846	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	57	253	67	294	0	0	-1
normalized size	1	1.00	0.58	2.58	0.68	3.00	0.00	0.00	-0.01
time (sec)	N/A	0.608	0.052	0.366	1.067	0.475	0.000	0.000	0.000

Problem 847	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	68	329	87	427	0	0	-1
normalized size	1	1.00	0.53	2.55	0.67	3.31	0.00	0.00	-0.01
time (sec)	N/A	0.554	0.060	0.367	0.458	0.476	0.000	0.000	0.000

Problem 848	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	74	150	60	351	0	0	-1
normalized size	1	1.00	0.84	1.70	0.68	3.99	0.00	0.00	-0.01
time (sec)	N/A	0.360	0.061	0.377	0.438	0.489	0.000	0.000	0.000

Problem 849	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	187	187	154	0	0	778	0	0	-1
normalized size	1	1.00	0.82	0.00	0.00	4.16	0.00	0.00	-0.01
time (sec)	N/A	0.512	0.205	0.543	0.000	0.492	0.000	0.000	0.000

Problem 850	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	287	287	249	0	0	1190	0	0	-1
normalized size	1	1.00	0.87	0.00	0.00	4.15	0.00	0.00	-0.00
time (sec)	N/A	0.673	0.660	0.530	0.000	0.541	0.000	0.000	0.000

Problem 851	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	132	71	252	92	1757	0	0	-1
normalized size	1	1.00	0.54	1.91	0.70	13.31	0.00	0.00	-0.01
time (sec)	N/A	0.407	0.230	0.345	0.457	0.526	0.000	0.000	0.000

Problem 852	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	A	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	204	204	120	441	154	3431	0	0	-1
normalized size	1	1.00	0.59	2.16	0.75	16.82	0.00	0.00	-0.00
time (sec)	N/A	0.638	0.646	0.358	0.450	0.837	0.000	0.000	0.000

Problem 853	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	326	326	157	602	207	4629	0	0	-1
normalized size	1	1.00	0.48	1.85	0.63	14.20	0.00	0.00	-0.00
time (sec)	N/A	0.683	1.042	0.367	0.436	0.755	0.000	0.000	0.000

Problem 854	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	162	0	0	0	0	0	-1
normalized size	1	1.00	1.10	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.130	0.689	1.023	0.000	0.552	0.000	0.000	0.000

Problem 855	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	77	106	126	164	190	138	79
normalized size	1	1.00	0.71	0.97	1.16	1.50	1.74	1.27	0.72
time (sec)	N/A	0.099	0.271	0.441	0.331	0.684	3.245	0.119	1.872

Problem 856	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	50	68	63	80	129	81	44
normalized size	1	1.00	0.79	1.08	1.00	1.27	2.05	1.29	0.70
time (sec)	N/A	0.035	0.116	0.426	0.389	0.492	0.928	0.119	0.138

Problem 857	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	38	19	18	31	24	34	18
normalized size	1	1.00	1.90	0.95	0.90	1.55	1.20	1.70	0.90
time (sec)	N/A	0.019	0.009	0.020	0.302	0.556	0.186	0.134	1.645

Problem 858	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	48	207	73	299	0	79	343
normalized size	1	1.00	1.09	4.70	1.66	6.80	0.00	1.80	7.80
time (sec)	N/A	0.123	0.078	0.664	0.417	0.765	0.000	0.407	2.299

Problem 859	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	90	469	150	765	0	140	229
normalized size	1	1.00	1.01	5.27	1.69	8.60	0.00	1.57	2.57
time (sec)	N/A	0.104	0.351	0.804	0.434	0.554	0.000	0.505	2.110

Problem 860	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	143	121	2082	327	2439	0	256	-1
normalized size	1	1.00	0.85	14.56	2.29	17.06	0.00	1.79	-0.01
time (sec)	N/A	0.172	0.664	0.865	0.487	0.482	0.000	0.898	0.000

Problem 861	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	301	301	239	1260	0	0	0	0	-1
normalized size	1	1.00	0.79	4.19	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.392	1.414	1.106	0.000	0.482	0.000	0.000	0.000

Problem 862	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	248	248	202	935	0	0	0	0	-1
normalized size	1	1.00	0.81	3.77	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.253	0.854	1.082	0.000	0.469	0.000	0.000	0.000

Problem 863	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	94	351	0	0	0	0	-1
normalized size	1	1.00	0.98	3.66	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.073	0.117	1.171	0.000	0.461	0.000	0.000	0.000

Problem 864	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	90	181	0	0	0	0	-1
normalized size	1	1.00	0.94	1.89	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.078	0.139	0.895	0.000	0.435	0.000	0.000	0.000

Problem 865	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	158	158	119	630	0	0	0	0	-1
normalized size	1	1.00	0.75	3.99	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.113	0.445	1.086	0.000	0.475	0.000	0.000	0.000

Problem 866	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	325	325	237	641	0	0	0	0	-1
normalized size	1	1.00	0.73	1.97	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.381	1.654	1.859	0.000	0.477	0.000	0.000	0.000

Problem 867	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	386	386	279	687	0	1488	0	0	-1
normalized size	1	1.00	0.72	1.78	0.00	3.85	0.00	0.00	-0.00
time (sec)	N/A	0.604	0.407	0.319	0.000	0.541	0.000	0.000	0.000

Problem 868	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	281	281	210	530	0	1122	0	0	-1
normalized size	1	1.00	0.75	1.89	0.00	3.99	0.00	0.00	-0.00
time (sec)	N/A	0.515	0.282	0.312	0.000	0.539	0.000	0.000	0.000

Problem 869	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	186	186	956	376	0	754	0	0	-1
normalized size	1	1.00	5.14	2.02	0.00	4.05	0.00	0.00	-0.01
time (sec)	N/A	0.303	1.868	0.303	0.000	0.482	0.000	0.000	0.000

Problem 870	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	20	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.094	1.082	0.342	0.000	0.539	0.000	0.000	0.000

Problem 871	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	96	0	0	0	0	0	-1
normalized size	1	1.00	1.01	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.148	0.073	0.114	0.000	0.490	0.000	0.000	0.000

Problem 872	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	50	80	53	105	124	53	45
normalized size	1	1.00	0.76	1.21	0.80	1.59	1.88	0.80	0.68
time (sec)	N/A	0.036	0.034	0.282	0.330	0.490	16.576	0.138	0.276

Problem 873	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	32	61	37	92	139	30	32
normalized size	1	1.00	0.80	1.52	0.92	2.30	3.48	0.75	0.80
time (sec)	N/A	0.035	0.018	0.195	0.306	0.447	4.585	0.132	1.857

Problem 874	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	25	52	26	54	54	23	25
normalized size	1	1.00	0.78	1.62	0.81	1.69	1.69	0.72	0.78
time (sec)	N/A	0.017	0.014	0.154	0.322	0.513	1.075	0.118	0.070

Problem 875	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	23	40	45	53	0	50	39
normalized size	1	1.00	0.88	1.54	1.73	2.04	0.00	1.92	1.50
time (sec)	N/A	0.018	0.015	0.304	0.311	0.509	0.000	0.115	0.081

Problem 876	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	37	43	62	104	0	48	37
normalized size	1	1.00	0.88	1.02	1.48	2.48	0.00	1.14	0.88
time (sec)	N/A	0.043	0.052	0.325	0.321	0.481	0.000	0.154	1.788

Problem 877	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	61	67	88	388	0	78	90
normalized size	1	1.00	0.84	0.92	1.21	5.32	0.00	1.07	1.23
time (sec)	N/A	0.044	0.059	0.342	0.308	0.445	0.000	0.134	1.812

Problem 878	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	139	108	166	0	316	976	84	127
normalized size	1	1.00	0.78	1.19	0.00	2.27	7.02	0.60	0.91
time (sec)	N/A	0.063	0.497	0.326	0.000	0.452	44.607	0.120	2.320

Problem 879	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	58	112	0	149	428	56	105
normalized size	1	1.00	0.66	1.27	0.00	1.69	4.86	0.64	1.19
time (sec)	N/A	0.036	0.158	0.228	0.000	0.450	9.118	0.119	0.245

Problem 880	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	38	78	0	67	201	40	54
normalized size	1	1.00	0.70	1.44	0.00	1.24	3.72	0.74	1.00
time (sec)	N/A	0.019	0.079	0.171	0.000	0.489	2.311	0.136	0.107

Problem 881	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	59	0	0	0	0	0	-1
normalized size	1	1.00	1.18	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.019	0.127	0.271	0.000	0.467	0.000	0.000	0.000

Problem 882	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	131	0	0	0	0	0	-1
normalized size	1	1.00	2.43	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.029	3.368	0.339	0.000	0.497	0.000	0.000	0.000

Problem 883	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	94	0	0	0	0	0	-1
normalized size	1	1.00	0.94	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.049	2.486	0.352	0.000	0.548	0.000	0.000	0.000

Problem 884	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	96	0	0	0	0	0	-1
normalized size	1	1.00	1.01	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.117	0.071	0.190	0.000	0.541	0.000	0.000	0.000

Problem 885	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	139	106	166	0	313	1085	84	125
normalized size	1	1.00	0.76	1.19	0.00	2.25	7.81	0.60	0.90
time (sec)	N/A	0.050	0.507	0.361	0.000	0.556	45.782	0.120	2.207

Problem 886	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	56	112	0	146	432	56	68
normalized size	1	1.00	0.64	1.27	0.00	1.66	4.91	0.64	0.77
time (sec)	N/A	0.031	0.163	0.286	0.000	0.445	9.018	0.117	0.228

Problem 887	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	38	78	0	66	167	40	53
normalized size	1	1.00	0.70	1.44	0.00	1.22	3.09	0.74	0.98
time (sec)	N/A	0.018	0.076	0.191	0.000	0.527	2.095	0.111	0.077

Problem 888	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	51	0	0	0	0	0	-1
normalized size	1	1.00	0.98	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.018	0.020	0.265	0.000	0.533	0.000	0.000	0.000

Problem 889	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	56	0	0	0	0	0	-1
normalized size	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.027	0.018	0.336	0.000	0.407	0.000	0.000	0.000

Problem 890	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	80	0	0	0	0	0	-1
normalized size	1	1.00	0.78	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.050	0.151	0.273	0.000	0.472	0.000	0.000	0.000

Problem 891	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	89	0	0	0	0	0	-1
normalized size	1	1.00	0.99	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.137	0.075	0.328	0.000	0.547	0.000	0.000	0.000

Problem 892	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	90	0	0	0	0	0	-1
normalized size	1	1.00	0.99	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.167	0.104	0.243	0.000	0.529	0.000	0.000	0.000

Problem 893	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	254	254	196	434	189	442	1731	1574	252
normalized size	1	1.00	0.77	1.71	0.74	1.74	6.81	6.20	0.99
time (sec)	N/A	0.407	1.915	0.646	0.348	0.537	119.102	0.298	2.778

Problem 894	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	93	141	88	135	400	899	88
normalized size	1	1.00	0.88	1.33	0.83	1.27	3.77	8.48	0.83
time (sec)	N/A	0.179	0.664	0.179	0.340	0.493	9.895	0.214	1.980

Problem 895	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	104	0	0	0	0	0	-1
normalized size	1	1.00	1.22	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.075	3.736	0.155	0.000	0.482	0.000	0.000	0.000

Problem 896	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	196	196	255	0	0	0	0	0	-1
normalized size	1	1.00	1.30	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.115	3.343	0.594	0.000	0.490	0.000	0.000	0.000

Problem 897	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	251	251	230	426	187	2340	1719	1576	288
normalized size	1	1.00	0.92	1.70	0.75	9.32	6.85	6.28	1.15
time (sec)	N/A	0.313	0.595	0.597	0.369	0.492	61.570	0.287	1.994

Problem 898	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	88	135	87	430	391	900	134
normalized size	1	1.00	0.87	1.34	0.86	4.26	3.87	8.91	1.33
time (sec)	N/A	0.146	0.236	0.225	0.329	0.418	8.458	0.232	1.742

Problem 899	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	61	0	0	0	0	0	-1
normalized size	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.063	0.047	0.191	0.000	0.510	0.000	0.000	0.000

Problem 900	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	151	151	127	0	0	0	0	0	-1
normalized size	1	1.00	0.84	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.102	0.346	0.392	0.000	0.486	0.000	0.000	0.000

Problem 901	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	51	44	56	111	139	52	50
normalized size	1	1.00	0.74	0.64	0.81	1.61	2.01	0.75	0.72
time (sec)	N/A	0.051	0.066	0.156	0.323	0.483	60.112	0.118	0.546

Problem 902	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	45	53	50	95	177	57	43
normalized size	1	1.00	0.79	0.93	0.88	1.67	3.11	1.00	0.75
time (sec)	N/A	0.053	0.064	0.129	0.344	0.434	18.393	0.119	0.279

Problem 903	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	28	26	29	53	76	26	26
normalized size	1	1.00	0.80	0.74	0.83	1.51	2.17	0.74	0.74
time (sec)	N/A	0.027	0.010	0.033	0.341	0.445	4.987	0.113	1.751

Problem 904	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	22	27	38	49	0	32	38
normalized size	1	1.00	0.88	1.08	1.52	1.96	0.00	1.28	1.52
time (sec)	N/A	0.018	0.018	0.163	0.319	0.426	0.000	0.132	0.060

Problem 905	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	34	30	45	103	0	46	36
normalized size	1	1.00	0.83	0.73	1.10	2.51	0.00	1.12	0.88
time (sec)	N/A	0.045	0.052	0.161	0.322	0.434	0.000	0.138	1.770

Problem 906	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	59	55	78	387	0	62	102
normalized size	1	1.00	0.84	0.79	1.11	5.53	0.00	0.89	1.46
time (sec)	N/A	0.060	0.082	0.418	0.322	0.446	0.000	0.122	1.761

Problem 907	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	67	89	77	167	294	81	65
normalized size	1	1.00	0.74	0.98	0.85	1.84	3.23	0.89	0.71
time (sec)	N/A	0.074	0.096	0.175	0.323	0.473	176.089	0.121	0.629

Problem 908	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	40	70	38	90	144	36	36
normalized size	1	1.00	0.82	1.43	0.78	1.84	2.94	0.73	0.73
time (sec)	N/A	0.053	0.028	0.365	0.342	0.435	62.532	0.132	0.519

Problem 909	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	43	53	50	96	175	57	42
normalized size	1	1.00	0.75	0.93	0.88	1.68	3.07	1.00	0.74
time (sec)	N/A	0.047	0.047	0.101	0.325	0.530	18.659	0.118	0.271

Problem 910	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	39	52	50	72	0	39	35
normalized size	1	1.00	0.93	1.24	1.19	1.71	0.00	0.93	0.83
time (sec)	N/A	0.043	0.030	0.199	0.334	0.476	0.000	0.141	1.707

Problem 911	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F(-1)	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	179	48	62	198	0	56	62
normalized size	1	1.00	3.38	0.91	1.17	3.74	0.00	1.06	1.17
time (sec)	N/A	0.038	1.756	0.140	0.321	0.509	0.000	0.124	1.841

Problem 912	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	46	43	69	262	0	59	65
normalized size	1	1.00	0.74	0.69	1.11	4.23	0.00	0.95	1.05
time (sec)	N/A	0.065	0.088	0.171	0.337	0.470	0.000	0.130	1.837

Problem 913	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	51	88	54	154	0	52	50
normalized size	1	1.00	0.74	1.28	0.78	2.23	0.00	0.75	0.72
time (sec)	N/A	0.059	0.039	0.366	0.367	0.448	0.000	0.126	2.103

Problem 914	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	67	84	77	165	325	81	65
normalized size	1	1.00	0.74	0.92	0.85	1.81	3.57	0.89	0.71
time (sec)	N/A	0.079	0.113	0.381	0.328	0.467	170.926	0.140	0.602

Problem 915	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	51	52	56	111	139	52	50
normalized size	1	1.00	0.74	0.75	0.81	1.61	2.01	0.75	0.72
time (sec)	N/A	0.047	0.057	0.372	0.331	0.442	59.475	0.139	0.516

Problem 916	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	68	50	65	170	0	57	66
normalized size	1	1.00	1.15	0.85	1.10	2.88	0.00	0.97	1.12
time (sec)	N/A	0.054	0.079	0.368	0.335	0.539	0.000	0.121	0.111

Problem 917	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	52	67	68	213	0	63	53
normalized size	1	1.00	0.83	1.06	1.08	3.38	0.00	1.00	0.84
time (sec)	N/A	0.065	0.084	0.188	0.344	0.461	0.000	0.147	1.820

Problem 918	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F(-1)	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	286	89	88	459	0	72	97
normalized size	1	1.00	3.53	1.10	1.09	5.67	0.00	0.89	1.20
time (sec)	N/A	0.052	2.535	0.378	0.351	0.489	0.000	0.147	1.791

Problem 919	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	43	89	52	152	235	60	42
normalized size	1	1.00	0.75	1.56	0.91	2.67	4.12	1.05	0.74
time (sec)	N/A	0.054	0.079	0.328	0.343	0.453	58.177	0.122	0.557

Problem 920	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	51	69	53	105	128	55	47
normalized size	1	1.00	0.77	1.05	0.80	1.59	1.94	0.83	0.71
time (sec)	N/A	0.050	0.077	0.224	0.329	0.426	16.530	0.121	0.252

Problem 921	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	25	33	24	91	117	18	18
normalized size	1	1.00	1.09	1.43	1.04	3.96	5.09	0.78	0.78
time (sec)	N/A	0.023	0.014	0.096	0.324	0.468	4.559	0.124	1.811

Problem 922	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	35	38	57	64	0	30	30
normalized size	1	1.00	0.95	1.03	1.54	1.73	0.00	0.81	0.81
time (sec)	N/A	0.032	0.029	0.256	0.321	0.526	0.000	0.113	0.059

Problem 923	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	62	65	76	200	0	55	63
normalized size	1	1.00	1.15	1.20	1.41	3.70	0.00	1.02	1.17
time (sec)	N/A	0.043	0.120	0.224	0.322	0.480	0.000	0.150	1.762

Problem 924	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	47	56	86	262	0	48	66
normalized size	1	1.00	0.75	0.89	1.37	4.16	0.00	0.76	1.05
time (sec)	N/A	0.072	0.079	0.216	0.322	0.520	0.000	0.147	1.861

Problem 925	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	73	108	78	175	197	80	69
normalized size	1	1.00	0.73	1.08	0.78	1.75	1.97	0.80	0.69
time (sec)	N/A	0.078	0.108	0.453	0.321	0.445	163.246	0.150	0.588

Problem 926	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	38	58	42	108	128	43	39
normalized size	1	1.00	0.73	1.12	0.81	2.08	2.46	0.83	0.75
time (sec)	N/A	0.059	0.034	0.260	0.317	0.467	57.103	0.138	0.528

Problem 927	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	54	69	53	105	128	55	47
normalized size	1	1.00	0.82	1.05	0.80	1.59	1.94	0.83	0.71
time (sec)	N/A	0.047	0.047	0.165	0.318	0.541	16.235	0.118	0.244

Problem 928	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	58	54	61	98	0	54	53
normalized size	1	1.00	1.29	1.20	1.36	2.18	0.00	1.20	1.18
time (sec)	N/A	0.034	0.134	0.656	0.323	0.497	0.000	0.117	0.087

Problem 929	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	48	57	86	195	0	56	51
normalized size	1	1.00	0.81	0.97	1.46	3.31	0.00	0.95	0.86
time (sec)	N/A	0.050	0.071	0.596	0.322	0.481	0.000	0.127	1.847

Problem 930	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F(-1)	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	247	78	96	459	0	72	98
normalized size	1	1.00	2.91	0.92	1.13	5.40	0.00	0.85	1.15
time (sec)	N/A	0.068	4.508	0.639	0.326	0.515	0.000	0.127	1.897

Problem 931	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	45	61	52	186	0	60	46
normalized size	1	1.00	0.79	1.07	0.91	3.26	0.00	1.05	0.81
time (sec)	N/A	0.062	0.052	0.199	0.343	0.457	0.000	0.153	1.929

Problem 932	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	73	108	76	176	197	80	69
normalized size	1	1.00	0.73	1.08	0.76	1.76	1.97	0.80	0.69
time (sec)	N/A	0.074	0.108	0.230	0.323	0.460	159.706	0.145	0.596

Problem 933	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	43	89	52	152	233	61	42
normalized size	1	1.00	0.75	1.56	0.91	2.67	4.09	1.07	0.74
time (sec)	N/A	0.052	0.069	0.257	0.326	0.583	52.942	0.124	0.558

Problem 934	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	48	55	70	127	0	52	49
normalized size	1	1.00	0.81	0.93	1.19	2.15	0.00	0.88	0.83
time (sec)	N/A	0.060	0.049	0.661	0.327	0.453	0.000	0.140	0.099

Problem 935	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F(-1)	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	220	79	87	272	0	75	77
normalized size	1	1.00	3.01	1.08	1.19	3.73	0.00	1.03	1.05
time (sec)	N/A	0.054	0.961	0.643	0.321	0.523	0.000	0.145	1.866

Problem 936	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	60	70	106	398	0	70	80
normalized size	1	1.00	0.75	0.88	1.32	4.98	0.00	0.88	1.00
time (sec)	N/A	0.063	0.124	0.688	0.323	0.484	0.000	0.136	0.069

Problem 937	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	42	40	90	164	0	90	91
normalized size	1	1.00	0.37	0.35	0.80	1.45	0.00	0.80	0.81
time (sec)	N/A	0.082	0.030	0.293	0.418	0.506	0.000	0.133	0.280

Problem 938	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	120	44	101	210	0	95	122
normalized size	1	1.00	0.93	0.34	0.78	1.63	0.00	0.74	0.95
time (sec)	N/A	0.096	0.126	0.348	0.417	0.511	0.000	0.135	2.029

Problem 939	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	58	48	105	213	0	99	112
normalized size	1	1.00	0.45	0.37	0.81	1.64	0.00	0.76	0.86
time (sec)	N/A	0.106	0.065	0.352	0.627	0.506	0.000	0.118	2.095

Problem 940	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	149	64	50	115	259	0	103	154
normalized size	1	1.00	0.43	0.34	0.77	1.74	0.00	0.69	1.03
time (sec)	N/A	0.128	0.066	0.368	0.413	0.508	0.000	0.116	2.165

Problem 941	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	31	48	34	202	0	35	38
normalized size	1	1.00	0.91	1.41	1.00	5.94	0.00	1.03	1.12
time (sec)	N/A	0.028	0.058	0.329	0.408	0.535	0.000	0.129	0.203

Problem 942	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	54	54	47	522	0	42	80
normalized size	1	1.00	1.02	1.02	0.89	9.85	0.00	0.79	1.51
time (sec)	N/A	0.042	0.096	0.378	0.579	0.542	0.000	0.117	2.001

Problem 943	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	A	B	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	161	56	47	557	0	42	62
normalized size	1	1.00	2.93	1.02	0.85	10.13	0.00	0.76	1.13
time (sec)	N/A	0.048	3.341	0.447	0.643	0.469	0.000	0.121	1.875

Problem 944	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	A	B	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	310	60	59	992	0	48	114
normalized size	1	1.00	4.13	0.80	0.79	13.23	0.00	0.64	1.52
time (sec)	N/A	0.067	5.542	0.456	0.438	0.532	0.000	0.117	2.181

Problem 945	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	86	202	0	505	1292	93	228
normalized size	1	1.00	0.63	1.47	0.00	3.69	9.43	0.68	1.66
time (sec)	N/A	0.101	1.207	0.364	0.000	0.499	142.133	0.139	1.140

Problem 946	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	80	178	0	379	972	86	126
normalized size	1	1.00	0.63	1.40	0.00	2.98	7.65	0.68	0.99
time (sec)	N/A	0.091	0.985	0.354	0.000	0.429	43.496	0.138	2.269

Problem 947	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	47	102	0	142	304	47	58
normalized size	1	1.00	0.71	1.55	0.00	2.15	4.61	0.71	0.88
time (sec)	N/A	0.047	0.052	0.197	0.000	0.429	8.446	0.131	1.908

Problem 948	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	38	78	0	97	184	40	54
normalized size	1	1.00	0.70	1.44	0.00	1.80	3.41	0.74	1.00
time (sec)	N/A	0.017	0.047	0.223	0.000	0.479	2.097	0.129	1.747

Problem 949	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	120	0	0	0	0	0	-1
normalized size	1	1.00	2.26	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.065	1.943	0.299	0.000	0.433	0.000	0.000	0.000

Problem 950	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	92	0	0	0	0	0	-1
normalized size	1	1.00	0.91	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.250	0.668	0.293	0.000	0.438	0.000	0.000	0.000

Problem 951	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	159	0	0	0	0	0	-1
normalized size	1	1.00	1.41	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.273	1.548	0.356	0.000	0.465	0.000	0.000	0.000

Problem 952	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	195	195	117	278	0	919	0	132	395
normalized size	1	1.00	0.60	1.43	0.00	4.71	0.00	0.68	2.03
time (sec)	N/A	0.136	1.174	0.523	0.000	0.449	0.000	0.150	2.493

Problem 953	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	58	124	0	303	819	58	96
normalized size	1	1.00	0.70	1.49	0.00	3.65	9.87	0.70	1.16
time (sec)	N/A	0.079	0.389	0.290	0.000	0.549	88.850	0.126	2.731

Problem 954	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	80	178	0	381	972	86	126
normalized size	1	1.00	0.63	1.40	0.00	3.00	7.65	0.68	0.99
time (sec)	N/A	0.090	0.563	0.355	0.000	0.445	42.141	0.120	2.179

Problem 955	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	55	124	0	176	432	58	68
normalized size	1	1.00	0.58	1.31	0.00	1.85	4.55	0.61	0.72
time (sec)	N/A	0.036	0.153	0.263	0.000	0.448	8.574	0.117	0.269

Problem 956	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	93	0	0	0	0	0	-1
normalized size	1	1.00	0.90	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.207	0.577	0.648	0.000	0.429	0.000	0.000	0.000

Problem 957	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	145	0	0	0	0	0	-1
normalized size	1	1.00	1.54	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.114	0.900	0.679	0.000	0.505	0.000	0.000	0.000

Problem 958	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	151	151	111	0	0	0	0	0	-1
normalized size	1	1.00	0.74	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.343	1.332	0.746	0.000	0.521	0.000	0.000	0.000

Problem 959	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	113	202	0	676	0	93	182
normalized size	1	1.00	0.82	1.47	0.00	4.93	0.00	0.68	1.33
time (sec)	N/A	0.113	1.015	0.391	0.000	0.476	0.000	0.130	0.956

Problem 960	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	195	195	118	278	0	917	0	132	393
normalized size	1	1.00	0.61	1.43	0.00	4.70	0.00	0.68	2.02
time (sec)	N/A	0.133	1.292	0.539	0.000	0.533	0.000	0.150	2.424

Problem 961	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	86	202	0	501	1295	93	163
normalized size	1	1.00	0.63	1.47	0.00	3.66	9.45	0.68	1.19
time (sec)	N/A	0.094	1.023	0.428	0.000	0.444	143.713	0.126	2.682

Problem 962	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	144	106	178	0	381	1046	86	125
normalized size	1	1.00	0.74	1.24	0.00	2.65	7.26	0.60	0.87
time (sec)	N/A	0.057	0.485	0.401	0.000	0.453	42.808	0.134	2.198

Problem 963	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	125	172	0	0	0	0	0	-1
normalized size	1	1.00	1.38	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.247	1.101	0.830	0.000	0.443	0.000	0.000	0.000

Problem 964	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	160	160	145	0	0	0	0	0	-1
normalized size	1	1.00	0.91	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.306	1.257	0.805	0.000	0.410	0.000	0.000	0.000

Problem 965	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	176	0	0	0	0	0	-1
normalized size	1	1.00	1.30	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.164	3.811	0.885	0.000	0.488	0.000	0.000	0.000

Problem 966	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	155	0	0	0	0	0	-1
normalized size	1	1.00	2.12	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.614	1.321	1.028	0.000	0.000	0.000	0.000	0.000

Problem 967	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	17	16	26	36	30	16
normalized size	1	1.00	1.00	1.00	0.94	1.53	2.12	1.76	0.94
time (sec)	N/A	0.015	0.044	0.021	0.308	0.499	0.434	0.138	0.096

Problem 968	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	39	22	35	0	38	38
normalized size	1	1.00	1.00	1.77	1.00	1.59	0.00	1.73	1.73
time (sec)	N/A	0.017	0.219	3.066	0.309	0.459	0.000	0.128	1.746

Problem 969	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	22	39	22	35	51	38	38
normalized size	1	1.00	0.96	1.70	0.96	1.52	2.22	1.65	1.65
time (sec)	N/A	0.015	0.044	2.830	0.303	0.444	2.448	0.130	1.687

Problem 970	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	17	0	13	0	0	-1
normalized size	1	1.00	1.00	1.31	0.00	1.00	0.00	0.00	-0.08
time (sec)	N/A	0.024	0.064	0.072	0.000	0.422	0.000	0.000	0.000

Problem 971	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	31	0	19	0	0	-1
normalized size	1	1.00	1.00	1.72	0.00	1.06	0.00	0.00	-0.06
time (sec)	N/A	0.023	0.184	0.401	0.000	0.509	0.000	0.000	0.000

Problem 972	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	18	31	0	19	0	0	-1
normalized size	1	1.00	0.95	1.63	0.00	1.00	0.00	0.00	-0.05
time (sec)	N/A	0.024	0.063	0.342	0.000	0.432	0.000	0.000	0.000

Problem 973	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	17	16	26	36	30	16
normalized size	1	1.00	1.00	1.00	0.94	1.53	2.12	1.76	0.94
time (sec)	N/A	0.014	0.016	0.060	0.301	0.432	0.424	0.146	0.068

Problem 974	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	23	39	22	35	0	38	38
normalized size	1	1.00	1.05	1.77	1.00	1.59	0.00	1.73	1.73
time (sec)	N/A	0.015	0.128	2.835	0.304	0.481	0.000	0.157	1.713

Problem 975	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	39	22	35	51	38	38
normalized size	1	1.00	1.00	1.70	0.96	1.52	2.22	1.65	1.65
time (sec)	N/A	0.014	0.042	2.679	0.304	0.417	2.272	0.138	1.692

Problem 976	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	17	0	13	0	0	-1
normalized size	1	1.00	1.00	1.31	0.00	1.00	0.00	0.00	-0.08
time (sec)	N/A	0.023	0.037	0.102	0.000	0.451	0.000	0.000	0.000

Problem 977	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	31	0	19	0	0	-1
normalized size	1	1.00	1.00	1.72	0.00	1.06	0.00	0.00	-0.06
time (sec)	N/A	0.023	0.061	0.402	0.000	0.525	0.000	0.000	0.000

Problem 978	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	18	31	0	19	0	0	-1
normalized size	1	1.00	0.95	1.63	0.00	1.00	0.00	0.00	-0.05
time (sec)	N/A	0.023	0.058	0.401	0.000	0.505	0.000	0.000	0.000

Problem 979	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	20	12	11	42	0	45	50
normalized size	1	1.00	1.82	1.09	1.00	3.82	0.00	4.09	4.55
time (sec)	N/A	0.040	0.063	0.161	0.301	0.432	0.000	0.124	1.770

Problem 980	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	3	3	3	116	35	19	0	5	5
normalized size	1	1.00	1.00	38.67	11.67	6.33	0.00	1.67	1.67
time (sec)	N/A	0.032	0.004	0.232	0.402	0.443	0.000	0.115	0.073

Problem 981	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	B	A	B	F	A	B
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	0	116	11	21	0	11	11
normalized size	1	1.00	0.00	10.55	1.00	1.91	0.00	1.00	1.00
time (sec)	N/A	0.034	0.023	0.295	0.409	0.426	0.000	0.131	1.639

Problem 982	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	18	20	19	69	0	39	54
normalized size	1	1.00	0.95	1.05	1.00	3.63	0.00	2.05	2.84
time (sec)	N/A	0.043	0.191	0.152	0.301	0.434	0.000	0.124	1.791

Problem 983	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	4	4	4	34	12	14	29	12	12
normalized size	1	1.00	1.00	8.50	3.00	3.50	7.25	3.00	3.00
time (sec)	N/A	0.052	0.007	0.174	0.303	0.415	0.811	0.131	1.724

Problem 984	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	4	4	4	34	12	14	29	12	12
normalized size	1	1.00	1.00	8.50	3.00	3.50	7.25	3.00	3.00
time (sec)	N/A	0.074	0.003	0.174	0.315	0.432	0.806	0.130	1.662

Problem 985	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	C	F	B	F	A	B
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	5	5	0	42	0	23	0	9	9
normalized size	1	1.00	0.00	8.40	0.00	4.60	0.00	1.80	1.80
time (sec)	N/A	0.049	0.042	0.245	0.000	0.426	0.000	0.120	1.658

Problem 986	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	11	32	29	77	0	29	23
normalized size	1	1.00	0.73	2.13	1.93	5.13	0.00	1.93	1.53
time (sec)	N/A	0.053	0.034	0.231	0.309	0.450	0.000	0.122	0.087

Problem 987	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	11	32	32	53	0	26	20
normalized size	1	1.00	0.73	2.13	2.13	3.53	0.00	1.73	1.33
time (sec)	N/A	0.058	0.026	0.235	0.304	0.420	0.000	0.119	1.655

Problem 988	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	74	34	0	309	0	1	169
normalized size	1	1.00	0.73	0.33	0.00	3.03	0.00	0.01	1.66
time (sec)	N/A	0.112	0.114	0.240	0.000	0.462	0.000	0.138	3.338

Problem 989	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	C	F	A	F	A	B
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	0	62	0	32	0	19	19
normalized size	1	1.00	0.00	2.82	0.00	1.45	0.00	0.86	0.86
time (sec)	N/A	0.068	0.035	0.253	0.000	0.477	0.000	0.132	0.107

Problem 990	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	54	100	66	172	0	113	297
normalized size	1	1.00	1.93	3.57	2.36	6.14	0.00	4.04	10.61
time (sec)	N/A	0.099	0.361	0.204	0.403	0.437	0.000	0.133	2.155

Problem 991	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	61	251	151	688	0	264	107
normalized size	1	1.00	1.15	4.74	2.85	12.98	0.00	4.98	2.02
time (sec)	N/A	0.158	0.615	0.231	0.416	0.469	0.000	0.128	2.130

Problem 992	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	134	542	276	1975	0	543	1347
normalized size	1	1.00	1.72	6.95	3.54	25.32	0.00	6.96	17.27
time (sec)	N/A	0.163	0.872	0.257	0.422	0.555	0.000	0.154	2.732

Problem 993	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	11	10	73	0	32	32
normalized size	1	1.00	1.00	0.92	0.83	6.08	0.00	2.67	2.67
time (sec)	N/A	0.086	0.043	0.155	0.297	0.424	0.000	0.122	1.799

Problem 994	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	A	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	67	306	25	778	0	66	820
normalized size	1	1.00	2.03	9.27	0.76	23.58	0.00	2.00	24.85
time (sec)	N/A	0.108	0.026	0.644	0.298	0.438	0.000	0.161	1.737

Problem 995	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	27	78	122	50	0	28	47
normalized size	1	1.00	1.04	3.00	4.69	1.92	0.00	1.08	1.81
time (sec)	N/A	0.091	0.225	0.258	0.408	0.436	0.000	0.136	0.504

Problem 996	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	4	4	4	5	12	14	0	12	12
normalized size	1	1.00	1.00	1.25	3.00	3.50	0.00	3.00	3.00
time (sec)	N/A	0.020	0.002	0.371	0.313	0.470	0.000	0.113	1.649

Problem 997	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	B	F	B	F	B	B
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	0	63	0	67	0	35	50
normalized size	1	1.00	0.00	3.15	0.00	3.35	0.00	1.75	2.50
time (sec)	N/A	0.124	0.030	0.326	0.000	0.409	0.000	0.120	1.785

Problem 998	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	9	9	43	0	0	112	0	44	-1
normalized size	1	1.00	4.78	0.00	0.00	12.44	0.00	4.89	-0.11
time (sec)	N/A	0.049	0.043	0.394	0.000	0.423	0.000	0.155	0.000

Problem 999	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	9	9	47	0	0	118	0	44	-1
normalized size	1	1.00	5.22	0.00	0.00	13.11	0.00	4.89	-0.11
time (sec)	N/A	0.050	0.055	0.518	0.000	0.460	0.000	0.156	0.000

Problem 1000	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	46	0	0	1	0	0	-1
normalized size	1	1.00	3.29	0.00	0.00	0.07	0.00	0.00	-0.07
time (sec)	N/A	0.051	0.042	0.518	0.000	0.469	0.000	0.000	0.000

Problem 1001	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	51	0	0	219	0	120	-1
normalized size	1	1.00	2.68	0.00	0.00	11.53	0.00	6.32	-0.05
time (sec)	N/A	0.049	0.226	0.652	0.000	0.443	0.000	0.153	0.000

Problem 1002	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	55	0	0	334	0	145	-1
normalized size	1	1.00	2.29	0.00	0.00	13.92	0.00	6.04	-0.04
time (sec)	N/A	0.043	0.097	0.474	0.000	0.545	0.000	0.139	0.000

Problem 1003	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	25	20	34	340	19	41	375
normalized size	1	1.00	1.47	1.18	2.00	20.00	1.12	2.41	22.06
time (sec)	N/A	0.078	0.017	0.089	0.304	0.424	6.581	0.118	1.761

Problem 1004	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	28	61	104	73	0	0	108
normalized size	1	1.00	0.65	1.42	2.42	1.70	0.00	0.00	2.51
time (sec)	N/A	0.039	0.060	0.369	0.398	0.426	0.000	0.000	1.795

Problem 1005	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	28	61	104	73	0	0	108
normalized size	1	1.00	0.65	1.42	2.42	1.70	0.00	0.00	2.51
time (sec)	N/A	0.040	0.029	0.221	0.399	0.468	0.000	0.000	0.002

Problem 1006	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	36	65	117	91	0	255	127
normalized size	1	1.00	0.56	1.02	1.83	1.42	0.00	3.98	1.98
time (sec)	N/A	0.039	0.064	0.344	0.404	0.435	0.000	0.189	1.793

Problem 1007	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	36	65	117	91	0	255	127
normalized size	1	1.00	0.56	1.02	1.83	1.42	0.00	3.98	1.98
time (sec)	N/A	0.044	0.029	0.196	0.409	0.432	0.000	0.189	0.002

Problem 1008	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	28	59	103	73	0	0	107
normalized size	1	1.00	0.65	1.37	2.40	1.70	0.00	0.00	2.49
time (sec)	N/A	0.042	0.135	0.470	0.407	0.420	0.000	0.000	1.763

Problem 1009	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	28	59	103	73	0	0	107
normalized size	1	1.00	0.65	1.37	2.40	1.70	0.00	0.00	2.49
time (sec)	N/A	0.039	0.033	0.331	0.402	0.416	0.000	0.000	0.002

Problem 1010	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	36	63	117	91	0	254	127
normalized size	1	1.00	0.56	0.98	1.83	1.42	0.00	3.97	1.98
time (sec)	N/A	0.049	0.166	0.437	0.400	0.470	0.000	0.177	1.769

Problem 1011	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	36	63	117	91	0	254	127
normalized size	1	1.00	0.56	0.98	1.83	1.42	0.00	3.97	1.98
time (sec)	N/A	0.042	0.044	0.314	0.412	0.457	0.000	0.175	0.002

Problem 1012	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	9	9	9	8	95	12	0	0	21
normalized size	1	1.00	1.00	0.89	10.56	1.33	0.00	0.00	2.33
time (sec)	N/A	0.026	0.005	0.166	1.480	0.421	0.000	0.000	1.762

Problem 1013	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	9	9	9	8	7	12	0	20	21
normalized size	1	1.00	1.00	0.89	0.78	1.33	0.00	2.22	2.33
time (sec)	N/A	0.024	0.009	0.120	0.302	0.420	0.000	0.134	1.666

Problem 1014	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	21	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.016	0.036	0.108	0.000	0.407	0.000	0.000	0.000

Problem 1015	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	21	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.016	0.041	0.073	0.000	0.422	0.000	0.000	0.000

Problem 1016	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.020	0.084	0.156	0.000	0.410	0.000	0.000	0.000

Problem 1017	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.022	0.079	0.125	0.000	0.411	0.000	0.000	0.000

Problem 1018	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	12	23	87	12	24	26
normalized size	1	1.00	1.00	0.92	1.77	6.69	0.92	1.85	2.00
time (sec)	N/A	0.036	0.008	0.086	0.308	0.406	0.502	0.134	1.861

Problem 1019	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	20	13	12	43	0	46	51
normalized size	1	1.00	1.67	1.08	1.00	3.58	0.00	3.83	4.25
time (sec)	N/A	0.048	0.069	0.128	0.306	0.472	0.000	0.138	0.215

Problem 1020	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	19	21	20	70	0	40	55
normalized size	1	1.00	0.95	1.05	1.00	3.50	0.00	2.00	2.75
time (sec)	N/A	0.048	0.237	0.157	0.306	0.475	0.000	0.140	1.991

Problem 1021	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	4	4	4	5	12	14	0	12	12
normalized size	1	1.00	1.00	1.25	3.00	3.50	0.00	3.00	3.00
time (sec)	N/A	0.020	0.004	0.385	0.309	0.416	0.000	0.131	1.722

Problem 1022	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	4	4	4	32	12	14	0	12	12
normalized size	1	1.00	1.00	8.00	3.00	3.50	0.00	3.00	3.00
time (sec)	N/A	0.062	0.007	0.171	0.308	0.454	0.000	0.128	1.833

Problem 1023	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	56	94	77	174	0	113	297
normalized size	1	1.00	2.00	3.36	2.75	6.21	0.00	4.04	10.61
time (sec)	N/A	0.097	0.347	0.178	0.322	0.476	0.000	0.123	2.160

Problem 1024	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	62	203	177	694	0	265	107
normalized size	1	1.00	1.17	3.83	3.34	13.09	0.00	5.00	2.02
time (sec)	N/A	0.145	0.528	0.226	0.329	0.490	0.000	0.127	2.163

Problem 1025	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	136	378	316	1980	0	544	1346
normalized size	1	1.00	1.74	4.85	4.05	25.38	0.00	6.97	17.26
time (sec)	N/A	0.153	1.280	0.240	0.339	0.499	0.000	0.141	2.702

Problem 1026	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	136	40	39	386	44	224	39
normalized size	1	1.00	3.78	1.11	1.08	10.72	1.22	6.22	1.08
time (sec)	N/A	0.095	0.257	0.033	0.319	0.434	10.716	0.143	1.894

Problem 1027	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	114	40	39	386	44	224	39
normalized size	1	1.00	3.17	1.11	1.08	10.72	1.22	6.22	1.08
time (sec)	N/A	0.094	0.615	0.035	0.302	0.405	10.836	0.133	0.268

Problem 1028	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	16	15	154	46	0	15
normalized size	1	1.00	1.00	0.84	0.79	8.11	2.42	0.00	0.79
time (sec)	N/A	0.062	0.013	0.050	0.302	0.451	1.318	0.000	1.867

Problem 1029	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	27	22	0	53	0	0	21
normalized size	1	1.00	1.00	0.81	0.00	1.96	0.00	0.00	0.78
time (sec)	N/A	0.170	0.035	0.237	0.000	0.458	0.000	0.000	1.880

Problem 1030	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	8	7	17	37	0	17	8
normalized size	1	1.00	1.00	0.88	2.12	4.62	0.00	2.12	1.00
time (sec)	N/A	0.191	0.017	0.157	0.308	0.466	0.000	0.135	1.753

Problem 1031	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	12	7	6	17	7	17	6
normalized size	1	1.00	1.50	0.88	0.75	2.12	0.88	2.12	0.75
time (sec)	N/A	0.014	0.004	0.093	0.306	0.463	0.272	0.115	1.778

Problem 1032	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	8	7	15	37	8	15	8
normalized size	1	1.00	1.00	0.88	1.88	4.62	1.00	1.88	1.00
time (sec)	N/A	0.189	0.017	0.128	0.305	0.415	0.322	0.121	1.732

Problem 1033	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	68	59	75	85	251	0	92	273
normalized size	1	1.31	1.13	1.44	1.63	4.83	0.00	1.77	5.25
time (sec)	N/A	0.162	0.089	0.390	0.420	0.476	0.000	0.154	0.490

Problem 1034	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	68	59	75	85	251	0	92	137
normalized size	1	1.31	1.13	1.44	1.63	4.83	0.00	1.77	2.63
time (sec)	N/A	0.130	0.063	0.340	0.411	0.487	0.000	0.142	0.280

Problem 1035	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	48	92	0	303	0	47	153
normalized size	1	1.00	0.92	1.77	0.00	5.83	0.00	0.90	2.94
time (sec)	N/A	0.134	0.083	0.283	0.000	0.446	0.000	0.140	2.154

Problem 1036	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	50	92	0	297	0	49	120
normalized size	1	1.00	0.96	1.77	0.00	5.71	0.00	0.94	2.31
time (sec)	N/A	0.106	0.062	0.261	0.000	0.566	0.000	0.137	0.264

Problem 1037	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	31	39	18	335	0	0	-1
normalized size	1	1.00	1.03	1.30	0.60	11.17	0.00	0.00	-0.03
time (sec)	N/A	0.039	0.036	0.363	0.438	0.439	0.000	0.000	0.000

Problem 1038	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	49	31	40	174	0	0	-1
normalized size	1	1.00	1.58	1.00	1.29	5.61	0.00	0.00	-0.03
time (sec)	N/A	0.039	0.061	0.342	0.464	0.426	0.000	0.000	0.000

Problem 1039	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	71	68	0	124	0	0	-1
normalized size	1	1.00	1.65	1.58	0.00	2.88	0.00	0.00	-0.02
time (sec)	N/A	0.073	0.032	0.252	0.000	0.438	0.000	0.000	0.000

Problem 1040	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	26	33	91	0	47	25
normalized size	1	1.00	1.00	2.17	2.75	7.58	0.00	3.92	2.08
time (sec)	N/A	0.036	0.020	0.219	0.455	0.419	0.000	0.131	1.735

Problem 1041	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	78	75	0	400	0	0	-1
normalized size	1	1.00	1.47	1.42	0.00	7.55	0.00	0.00	-0.02
time (sec)	N/A	0.132	0.071	0.273	0.000	0.440	0.000	0.000	0.000

Problem 1042	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	17	0	0	0	0	0	51
normalized size	1	1.00	0.85	0.00	0.00	0.00	0.00	0.00	2.55
time (sec)	N/A	0.167	1.139	0.843	0.000	0.000	0.000	0.000	1.902

Problem 1043	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	17	22	29	31	10	10
normalized size	1	1.00	1.00	0.77	1.00	1.32	1.41	0.45	0.45
time (sec)	N/A	0.034	0.004	0.064	0.322	0.429	0.173	0.129	0.062

Problem 1044	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	37	48	29	95	381	27	27
normalized size	1	1.00	0.54	0.70	0.42	1.38	5.52	0.39	0.39
time (sec)	N/A	0.201	0.029	0.207	0.314	0.434	1.183	0.132	1.721

Problem 1045	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	120	194	213	396	241	382	213
normalized size	1	1.00	0.93	1.50	1.65	3.07	1.87	2.96	1.65
time (sec)	N/A	0.140	0.481	0.368	0.345	0.466	68.208	0.135	0.280

Problem 1046	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	33	48	27	95	0	27	27
normalized size	1	1.00	0.85	1.23	0.69	2.44	0.00	0.69	0.69
time (sec)	N/A	0.049	0.027	0.078	0.315	0.416	0.000	0.111	0.056

Problem 1047	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	30	0	0	431	0	46	148
normalized size	1	1.00	1.20	0.00	0.00	17.24	0.00	1.84	5.92
time (sec)	N/A	0.100	0.187	0.390	0.000	0.409	0.000	0.152	2.043

Problem 1048	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	34	0	389	271	0	0	207
normalized size	1	1.00	0.92	0.00	10.51	7.32	0.00	0.00	5.59
time (sec)	N/A	0.106	0.052	0.446	0.422	0.412	0.000	0.000	2.409

Problem 1049	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	4	4	4	5	21	13	0	0	4
normalized size	1	1.00	1.00	1.25	5.25	3.25	0.00	0.00	1.00
time (sec)	N/A	0.140	0.075	0.141	0.620	0.412	0.000	0.000	1.950

Problem 1050	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	33	34	28	70	94	274	-1
normalized size	1	1.00	1.22	1.26	1.04	2.59	3.48	10.15	-0.04
time (sec)	N/A	0.091	0.086	0.023	0.335	0.413	4.800	0.168	0.000

Problem 1051	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	37	37	36	76	92	282	-1
normalized size	1	1.00	1.16	1.16	1.12	2.38	2.88	8.81	-0.03
time (sec)	N/A	0.095	0.073	0.023	0.341	0.411	4.998	0.171	0.000

Problem 1052	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	25	138	0	192	100	34	77
normalized size	1	1.00	0.49	2.71	0.00	3.76	1.96	0.67	1.51
time (sec)	N/A	0.172	0.032	0.779	0.000	0.415	22.708	0.771	0.252

Problem 1053	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	B	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	115	120	93	127	197	37	48
normalized size	1	1.00	2.45	2.55	1.98	2.70	4.19	0.79	1.02
time (sec)	N/A	0.328	1.352	1.122	0.457	0.447	15.236	0.216	1.818

Problem 1054	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	17	148	49	38	56	44	25
normalized size	1	1.00	1.55	13.45	4.45	3.45	5.09	4.00	2.27
time (sec)	N/A	0.062	0.004	0.632	0.452	0.400	2.242	0.161	0.090

Problem 1055	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	65	36	14	40	37	14	14
normalized size	1	1.00	2.95	1.64	0.64	1.82	1.68	0.64	0.64
time (sec)	N/A	0.050	0.024	0.019	0.356	0.408	0.414	0.118	0.096

Problem 1056	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	65	36	14	40	0	14	14
normalized size	1	1.00	4.64	2.57	1.00	2.86	0.00	1.00	1.00
time (sec)	N/A	0.207	0.025	1.080	0.354	0.407	0.000	0.153	0.082

Problem 1057	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	17	148	50	37	0	43	26
normalized size	1	1.00	1.42	12.33	4.17	3.08	0.00	3.58	2.17
time (sec)	N/A	0.263	0.008	0.803	0.440	0.427	0.000	0.169	1.962

Problem 1058	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	52	120	93	127	0	37	48
normalized size	1	1.00	1.11	2.55	1.98	2.70	0.00	0.79	1.02
time (sec)	N/A	0.410	0.331	1.408	0.462	0.413	0.000	0.216	2.156

Problem 1059	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	26	138	0	192	0	34	77
normalized size	1	1.00	0.51	2.71	0.00	3.76	0.00	0.67	1.51
time (sec)	N/A	1.418	0.031	0.875	0.000	0.414	0.000	0.355	2.243

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [423] had the largest ratio of [1.250]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	3	3	1.00	14	0.214
2	A	2	1	1.00	21	0.048
3	A	2	2	1.00	23	0.087
4	A	2	2	1.00	21	0.095
5	A	2	2	1.00	23	0.087
6	A	2	2	1.00	23	0.087
7	A	2	2	1.00	23	0.087
8	A	2	2	1.00	13	0.154
9	A	2	2	1.00	15	0.133
10	A	3	2	1.00	17	0.118

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
11	A	3	2	1.00	17	0.118
12	A	2	2	1.00	15	0.133
13	A	3	2	1.00	17	0.118
14	A	3	2	1.00	17	0.118
15	A	3	3	1.00	17	0.176
16	A	4	3	1.00	17	0.176
17	A	5	3	1.00	17	0.176
18	A	4	3	1.00	17	0.176
19	A	5	3	1.00	17	0.176
20	A	6	3	1.00	17	0.176
21	A	5	3	1.00	17	0.176
22	A	6	3	1.00	17	0.176
23	A	7	3	1.00	17	0.176
24	A	2	2	1.00	13	0.154
25	A	3	3	1.00	15	0.200
26	A	3	2	1.00	15	0.133
27	A	4	3	1.00	15	0.200
28	A	4	3	1.00	15	0.200
29	A	3	3	1.00	15	0.200
30	A	3	2	1.00	17	0.118
31	A	4	4	1.00	17	0.235
32	A	3	2	1.00	17	0.118
33	A	5	4	1.00	17	0.235
34	A	3	2	1.00	15	0.133
35	A	4	4	1.00	17	0.235
36	A	4	3	1.00	17	0.176

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
37	A	5	4	1.00	17	0.235
38	A	4	3	1.00	17	0.176
39	A	4	3	1.00	15	0.200
40	A	3	2	1.00	17	0.118
41	A	5	4	1.00	17	0.235
42	A	3	2	1.00	17	0.118
43	A	6	4	1.00	17	0.235
44	A	4	3	1.00	15	0.200
45	A	5	4	1.00	17	0.235
46	A	4	3	1.00	17	0.176
47	A	6	4	1.00	17	0.235
48	A	4	3	1.00	17	0.176
49	A	6	5	1.00	21	0.238
50	A	5	5	1.00	21	0.238
51	A	5	5	1.00	21	0.238
52	A	4	4	1.00	21	0.190
53	A	4	4	1.00	21	0.190
54	A	5	5	1.00	21	0.238
55	A	5	5	1.00	21	0.238
56	A	6	5	1.00	21	0.238
57	A	9	9	1.00	21	0.429
58	A	9	9	1.00	21	0.429
59	A	12	8	1.00	21	0.381
60	A	11	7	1.00	21	0.333
61	A	8	8	1.00	21	0.381
62	A	8	8	1.00	21	0.381

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
63	A	11	7	1.00	21	0.333
64	A	12	8	1.00	21	0.381
65	A	9	9	1.00	21	0.429
66	A	9	9	1.00	21	0.429
67	A	1	1	1.00	13	0.077
68	A	1	1	1.00	13	0.077
69	A	2	2	1.00	9	0.222
70	A	3	3	1.00	13	0.231
71	A	3	2	1.00	15	0.133
72	A	4	4	1.00	15	0.267
73	A	3	2	1.00	15	0.133
74	A	3	2	1.00	15	0.133
75	A	4	4	1.00	17	0.235
76	A	4	3	1.00	17	0.176
77	A	4	3	1.00	15	0.200
78	A	3	2	1.00	17	0.118
79	A	5	4	1.00	17	0.235
80	A	4	3	1.00	15	0.200
81	A	2	2	1.00	13	0.154
82	A	2	2	1.00	15	0.133
83	A	2	2	1.00	17	0.118
84	A	2	2	1.00	17	0.118
85	A	2	2	1.00	17	0.118
86	A	2	2	1.00	17	0.118
87	A	2	1	1.00	15	0.067
88	A	3	2	1.00	17	0.118

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
89	A	3	2	1.00	19	0.105
90	A	3	2	1.00	17	0.118
91	A	3	2	1.00	19	0.105
92	A	3	2	1.00	17	0.118
93	A	2	2	1.00	15	0.133
94	A	3	2	1.00	15	0.133
95	A	3	3	1.00	17	0.176
96	A	3	2	1.00	7	0.286
97	A	3	2	1.00	9	0.222
98	A	4	3	1.00	9	0.333
99	A	4	3	1.00	9	0.333
100	A	3	2	1.00	9	0.222
101	A	3	3	1.00	13	0.231
102	A	3	2	1.00	15	0.133
103	A	4	4	1.00	15	0.267
104	A	3	2	1.00	15	0.133
105	A	3	2	1.00	15	0.133
106	A	4	4	1.00	17	0.235
107	A	4	3	1.00	17	0.176
108	A	4	3	1.00	15	0.200
109	A	3	2	1.00	17	0.118
110	A	5	4	1.00	17	0.235
111	A	4	3	1.00	15	0.200
112	A	2	2	1.00	13	0.154
113	A	2	2	1.00	15	0.133
114	A	2	2	1.00	17	0.118

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
115	A	2	2	1.00	17	0.118
116	A	2	2	1.00	17	0.118
117	A	2	2	1.00	17	0.118
118	A	2	1	1.00	15	0.067
119	A	3	2	1.00	17	0.118
120	A	3	2	1.00	19	0.105
121	A	2	2	1.00	15	0.133
122	A	3	3	1.00	17	0.176
123	A	3	2	1.00	15	0.133
124	A	3	2	1.00	9	0.222
125	A	3	2	1.00	9	0.222
126	A	3	2	1.00	9	0.222
127	A	4	3	1.00	9	0.333
128	A	3	2	1.00	9	0.222
129	A	3	2	1.00	11	0.182
130	A	3	2	1.00	9	0.222
131	A	3	2	1.00	13	0.154
132	A	3	2	1.00	14	0.143
133	A	3	2	1.00	13	0.154
134	A	3	2	1.00	14	0.143
135	A	4	3	1.00	13	0.231
136	A	4	3	1.00	14	0.214
137	A	4	3	1.00	13	0.231
138	A	4	3	1.00	14	0.214
139	A	3	2	1.00	13	0.154
140	A	3	2	1.00	14	0.143

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
141	A	3	2	1.00	13	0.154
142	A	3	2	1.00	14	0.143
143	A	3	3	1.00	13	0.231
144	A	6	6	1.00	15	0.400
145	A	9	7	1.00	15	0.467
146	A	3	3	1.00	13	0.231
147	A	6	6	1.00	15	0.400
148	A	9	7	1.00	15	0.467
149	A	3	3	1.00	13	0.231
150	A	4	4	1.00	15	0.267
151	A	5	5	1.00	15	0.333
152	A	3	3	1.00	13	0.231
153	A	4	4	1.00	15	0.267
154	A	5	5	1.00	15	0.333
155	A	3	3	1.00	13	0.231
156	A	6	6	1.00	15	0.400
157	A	9	7	1.00	15	0.467
158	A	3	3	1.00	13	0.231
159	A	6	6	1.00	15	0.400
160	A	9	7	1.00	15	0.467
161	A	3	3	1.00	13	0.231
162	A	4	4	1.00	15	0.267
163	A	5	5	1.00	15	0.333
164	A	3	3	1.00	13	0.231
165	A	4	4	1.00	15	0.267
166	A	5	5	1.00	15	0.333

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
167	A	4	2	1.00	13	0.154
168	A	5	2	1.00	15	0.133
169	A	6	2	1.00	15	0.133
170	A	6	2	1.00	17	0.118
171	A	8	2	1.00	17	0.118
172	A	10	2	1.00	17	0.118
173	A	4	2	1.00	13	0.154
174	A	5	2	1.00	15	0.133
175	A	6	2	1.00	15	0.133
176	A	6	2	1.00	17	0.118
177	A	8	2	1.00	17	0.118
178	A	10	2	1.00	17	0.118
179	A	4	2	1.00	13	0.154
180	A	5	2	1.00	15	0.133
181	A	6	2	1.00	15	0.133
182	A	5	2	1.00	15	0.133
183	A	6	2	1.00	17	0.118
184	A	8	2	1.00	17	0.118
185	A	6	2	1.00	15	0.133
186	A	8	2	1.00	17	0.118
187	A	10	2	1.00	17	0.118
188	A	6	3	1.00	13	0.231
189	A	6	3	1.00	13	0.231
190	A	6	3	1.00	13	0.231
191	A	6	3	1.00	13	0.231
192	A	1	1	1.88	7	0.143

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
193	A	1	1	1.00	7	0.143
194	A	1	1	1.00	7	0.143
195	A	4	2	1.00	7	0.286
196	A	1	1	1.00	7	0.143
197	A	1	1	1.00	7	0.143
198	A	1	1	1.00	7	0.143
199	A	4	2	1.00	7	0.286
200	A	4	3	1.00	7	0.429
201	A	5	3	1.00	7	0.429
202	A	6	4	1.00	7	0.571
203	A	9	4	1.00	7	0.571
204	A	10	5	1.00	7	0.714
205	A	6	3	1.00	7	0.429
206	A	3	2	1.00	7	0.286
207	A	3	2	1.00	7	0.286
208	A	6	3	1.00	7	0.429
209	A	6	3	1.00	7	0.429
210	A	7	3	1.00	7	0.429
211	A	2	2	1.00	7	0.286
212	A	5	5	1.00	7	0.714
213	A	4	3	1.00	7	0.429
214	A	7	6	1.00	7	0.857
215	A	7	4	1.00	7	0.571
216	A	2	2	1.00	7	0.286
217	A	2	1	1.00	7	0.143
218	A	4	2	1.00	7	0.286

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
219	A	4	2	1.00	7	0.286
220	A	7	3	1.00	7	0.429
221	A	1	1	1.88	7	0.143
222	A	1	1	1.00	7	0.143
223	A	1	1	1.00	7	0.143
224	A	4	2	1.00	7	0.286
225	A	1	1	1.00	7	0.143
226	A	1	1	1.00	7	0.143
227	A	1	1	1.00	7	0.143
228	A	4	2	1.00	7	0.286
229	A	4	3	1.00	7	0.429
230	A	3	2	1.00	7	0.286
231	A	6	4	1.00	7	0.571
232	A	6	3	1.00	7	0.429
233	A	10	5	1.00	7	0.714
234	A	4	3	1.00	7	0.429
235	A	9	4	1.00	7	0.571
236	A	6	3	1.00	7	0.429
237	A	10	4	1.00	7	0.571
238	A	7	3	1.00	7	0.429
239	A	6	3	1.00	7	0.429
240	A	2	2	1.00	7	0.286
241	A	2	1	1.00	7	0.143
242	A	4	3	1.00	7	0.429
243	A	4	2	1.00	7	0.286
244	A	7	4	1.00	7	0.571

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
245	A	2	2	1.00	7	0.286
246	A	5	5	1.00	7	0.714
247	A	4	2	1.00	7	0.286
248	A	7	6	1.00	7	0.857
249	A	7	3	1.00	7	0.429
250	A	5	4	1.00	16	0.250
251	A	5	5	1.00	16	0.312
252	A	3	3	1.00	16	0.188
253	A	3	3	1.00	14	0.214
254	A	2	2	1.00	13	0.154
255	A	5	5	1.00	16	0.312
256	A	6	6	1.00	16	0.375
257	A	7	6	1.00	16	0.375
258	A	8	6	1.00	16	0.375
259	A	8	3	1.00	18	0.167
260	A	7	5	1.00	18	0.278
261	A	4	4	1.00	18	0.222
262	A	3	2	1.00	16	0.125
263	A	2	2	1.00	15	0.133
264	A	8	4	1.00	18	0.222
265	A	10	5	1.00	18	0.278
266	A	12	5	1.00	18	0.278
267	A	14	5	1.00	18	0.278
268	A	8	3	1.00	18	0.167
269	A	9	5	1.00	18	0.278
270	A	4	3	1.00	18	0.167

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
271	A	4	3	1.00	16	0.188
272	A	2	2	1.00	15	0.133
273	A	8	4	1.00	18	0.222
274	A	10	5	1.00	18	0.278
275	A	12	5	1.00	18	0.278
276	A	14	5	1.00	18	0.278
277	A	3	3	1.00	8	0.375
278	A	4	4	1.00	8	0.500
279	A	5	4	1.00	8	0.500
280	A	8	3	1.00	18	0.167
281	A	7	5	1.00	18	0.278
282	A	4	4	1.00	18	0.222
283	A	3	2	1.00	16	0.125
284	A	2	2	1.00	15	0.133
285	A	8	4	1.00	18	0.222
286	A	10	5	1.00	18	0.278
287	A	12	5	1.00	18	0.278
288	A	14	5	1.00	18	0.278
289	A	5	3	1.00	20	0.150
290	A	6	3	1.00	20	0.150
291	A	5	3	1.00	20	0.150
292	A	4	3	1.00	18	0.167
293	A	3	3	1.00	17	0.176
294	A	5	4	1.00	20	0.200
295	A	6	5	1.00	20	0.250
296	A	7	5	1.00	20	0.250

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
297	A	8	5	1.00	20	0.250
298	A	11	3	1.00	20	0.150
299	A	14	3	1.00	20	0.150
300	A	11	3	1.00	20	0.150
301	A	8	3	1.00	18	0.167
302	A	3	2	1.00	17	0.118
303	A	11	4	1.00	20	0.200
304	A	14	5	1.00	20	0.250
305	A	17	5	1.00	20	0.250
306	A	20	5	1.00	20	0.250
307	A	8	3	1.00	18	0.167
308	A	9	5	1.00	18	0.278
309	A	4	3	1.00	18	0.167
310	A	4	3	1.00	16	0.188
311	A	2	2	1.00	15	0.133
312	A	8	4	1.00	18	0.222
313	A	10	5	1.00	18	0.278
314	A	12	5	1.00	18	0.278
315	A	14	5	1.00	18	0.278
316	A	11	3	1.00	20	0.150
317	A	14	3	1.00	20	0.150
318	A	11	3	1.00	20	0.150
319	A	8	3	1.00	18	0.167
320	A	3	2	1.00	17	0.118
321	A	11	4	1.00	20	0.200
322	A	14	5	1.00	20	0.250

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
323	A	17	5	1.00	20	0.250
324	A	20	5	1.00	20	0.250
325	A	8	3	1.00	20	0.150
326	A	10	3	1.00	20	0.150
327	A	8	3	1.00	20	0.150
328	A	6	3	1.00	18	0.167
329	A	3	2	1.00	17	0.118
330	A	8	4	1.00	20	0.200
331	A	10	5	1.00	20	0.250
332	A	12	5	1.00	20	0.250
333	A	14	5	1.00	20	0.250
334	A	0	0	0.00	0	0.000
335	A	6	6	1.00	10	0.600
336	A	5	5	1.00	10	0.500
337	A	4	4	1.00	8	0.500
338	A	1	1	1.00	6	0.167
339	A	0	0	0.00	0	0.000
340	A	0	0	0.00	0	0.000
341	A	0	0	0.00	0	0.000
342	A	8	5	1.00	16	0.312
343	A	6	4	1.00	16	0.250
344	A	2	2	1.00	14	0.143
345	A	2	2	1.00	13	0.154
346	A	0	0	0.00	0	0.000
347	A	0	0	0.00	0	0.000
348	A	0	0	0.00	0	0.000

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
349	A	6	6	1.00	18	0.333
350	A	3	3	1.00	18	0.167
351	A	3	3	1.00	16	0.188
352	A	2	2	1.00	15	0.133
353	A	0	0	0.00	0	0.000
354	A	0	0	0.00	0	0.000
355	A	0	0	0.00	0	0.000
356	A	14	8	1.00	16	0.500
357	A	11	7	1.00	16	0.438
358	A	8	6	1.00	14	0.429
359	A	3	3	1.00	13	0.231
360	A	0	0	0.00	0	0.000
361	A	0	0	0.00	0	0.000
362	A	0	0	0.00	0	0.000
363	A	7	7	1.00	12	0.583
364	A	6	6	1.00	12	0.500
365	A	3	3	1.00	10	0.300
366	A	2	2	1.00	8	0.250
367	A	0	0	0.00	0	0.000
368	A	0	0	0.00	0	0.000
369	A	0	0	0.00	0	0.000
370	A	25	9	1.00	18	0.500
371	A	17	7	1.00	18	0.389
372	A	12	5	1.00	16	0.312
373	A	2	2	1.00	15	0.133
374	A	0	0	0.00	0	0.000

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
375	A	0	0	0.00	0	0.000
376	A	0	0	0.00	0	0.000
377	A	12	12	1.00	18	0.667
378	A	9	9	1.00	18	0.500
379	A	8	8	1.00	16	0.500
380	A	3	2	1.00	15	0.133
381	A	0	0	0.00	0	0.000
382	A	0	0	0.00	0	0.000
383	A	0	0	0.00	0	0.000
384	A	13	8	1.00	18	0.444
385	A	10	7	1.00	18	0.389
386	A	5	5	1.00	16	0.312
387	A	3	2	1.00	15	0.133
388	A	0	0	0.00	0	0.000
389	A	0	0	0.00	0	0.000
390	A	0	0	0.00	0	0.000
391	A	13	10	1.00	12	0.833
392	A	9	8	1.00	12	0.667
393	A	7	7	1.00	10	0.700
394	A	2	2	1.00	8	0.250
395	A	0	0	0.00	0	0.000
396	A	0	0	0.00	0	0.000
397	A	0	0	0.00	0	0.000
398	A	6	6	1.00	10	0.600
399	A	5	5	1.00	10	0.500
400	A	4	4	1.00	8	0.500

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
401	A	1	1	1.00	6	0.167
402	A	0	0	0.00	0	0.000
403	A	0	0	0.00	0	0.000
404	A	0	0	0.00	0	0.000
405	A	14	8	1.00	16	0.500
406	A	11	7	1.00	16	0.438
407	A	8	6	1.00	14	0.429
408	A	3	3	1.00	13	0.231
409	A	0	0	0.00	0	0.000
410	A	0	0	0.00	0	0.000
411	A	0	0	0.00	0	0.000
412	A	12	12	1.00	18	0.667
413	A	9	9	1.00	18	0.500
414	A	8	8	1.00	16	0.500
415	A	3	2	1.00	15	0.133
416	A	0	0	0.00	0	0.000
417	A	0	0	0.00	0	0.000
418	A	6	5	1.00	10	0.500
419	A	11	10	1.00	12	0.833
420	A	12	10	1.00	12	0.833
421	A	16	10	1.00	10	1.000
422	A	19	11	1.00	12	0.917
423	A	26	15	1.00	12	1.250
424	A	0	0	0.00	0	0.000
425	A	8	5	1.00	16	0.312
426	A	6	4	1.00	16	0.250

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
427	A	2	2	1.00	14	0.143
428	A	2	2	1.00	13	0.154
429	A	0	0	0.00	0	0.000
430	A	0	0	0.00	0	0.000
431	A	0	0	0.00	0	0.000
432	A	7	7	1.00	12	0.583
433	A	6	6	1.00	12	0.500
434	A	3	3	1.00	10	0.300
435	A	2	2	1.00	8	0.250
436	A	0	0	0.00	0	0.000
437	A	0	0	0.00	0	0.000
438	A	0	0	0.00	0	0.000
439	A	13	8	1.00	18	0.444
440	A	10	7	1.00	18	0.389
441	A	5	5	1.00	16	0.312
442	A	3	2	1.00	15	0.133
443	A	0	0	0.00	0	0.000
444	A	0	0	0.00	0	0.000
445	A	0	0	0.00	0	0.000
446	A	6	6	1.00	18	0.333
447	A	3	3	1.00	18	0.167
448	A	3	3	1.00	16	0.188
449	A	2	2	1.00	15	0.133
450	A	0	0	0.00	0	0.000
451	A	0	0	0.00	0	0.000
452	A	0	0	0.00	0	0.000

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
453	A	25	9	1.00	18	0.500
454	A	17	7	1.00	18	0.389
455	A	12	5	1.00	16	0.312
456	A	2	2	1.00	15	0.133
457	A	0	0	0.00	0	0.000
458	A	0	0	0.00	0	0.000
459	A	0	0	0.00	0	0.000
460	A	13	10	1.00	12	0.833
461	A	9	8	1.00	12	0.667
462	A	7	7	1.00	10	0.700
463	A	2	2	1.00	8	0.250
464	A	0	0	0.00	0	0.000
465	A	0	0	0.00	0	0.000
466	A	0	0	0.00	0	0.000
467	A	10	6	1.00	16	0.375
468	A	8	5	1.00	16	0.312
469	A	6	4	1.00	14	0.286
470	A	2	2	1.00	13	0.154
471	A	0	0	0.00	0	0.000
472	A	0	0	0.00	0	0.000
473	A	0	0	0.00	0	0.000
474	A	21	13	1.00	18	0.722
475	A	17	14	1.00	18	0.778
476	A	10	10	1.00	16	0.625
477	A	3	3	1.00	15	0.200
478	A	0	0	0.00	0	0.000

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
479	A	0	0	0.00	0	0.000
480	A	0	0	0.00	0	0.000
481	A	20	16	1.00	18	0.889
482	A	15	12	1.00	18	0.667
483	A	11	10	1.00	16	0.625
484	A	3	2	1.00	15	0.133
485	A	0	0	0.00	0	0.000
486	A	0	0	0.00	0	0.000
487	A	0	0	0.00	0	0.000
488	A	21	13	1.00	18	0.722
489	A	17	14	1.00	18	0.778
490	A	10	10	1.00	16	0.625
491	A	3	3	1.00	15	0.200
492	A	0	0	0.00	0	0.000
493	A	0	0	0.00	0	0.000
494	A	0	0	0.00	0	0.000
495	A	7	7	1.00	20	0.350
496	A	6	6	1.00	20	0.300
497	A	3	3	1.00	18	0.167
498	A	3	2	1.00	17	0.118
499	A	0	0	0.00	0	0.000
500	A	0	0	0.00	0	0.000
501	A	0	0	0.00	0	0.000
502	A	29	18	1.00	20	0.900
503	A	13	12	1.00	18	0.667
504	A	4	4	1.00	17	0.235

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
505	A	0	0	0.00	0	0.000
506	A	0	0	0.00	0	0.000
507	A	0	0	0.00	0	0.000
508	A	20	16	1.00	18	0.889
509	A	15	12	1.00	18	0.667
510	A	11	10	1.00	16	0.625
511	A	3	2	1.00	15	0.133
512	A	0	0	0.00	0	0.000
513	A	0	0	0.00	0	0.000
514	A	0	0	0.00	0	0.000
515	A	40	19	1.00	20	0.950
516	A	29	19	1.00	20	0.950
517	A	13	12	1.00	18	0.667
518	A	4	4	1.00	17	0.235
519	A	0	0	0.00	0	0.000
520	A	0	0	0.00	0	0.000
521	A	0	0	0.00	0	0.000
522	A	16	9	1.00	20	0.450
523	A	10	7	1.00	20	0.350
524	A	7	5	1.00	18	0.278
525	A	4	3	1.00	17	0.176
526	A	0	0	0.00	0	0.000
527	A	0	0	0.00	0	0.000
528	A	4	3	1.00	18	0.167
529	A	3	3	1.00	18	0.167
530	A	3	3	1.00	18	0.167

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
531	A	2	2	1.00	18	0.111
532	A	2	2	1.00	18	0.111
533	A	3	3	1.00	18	0.167
534	A	3	3	1.00	18	0.167
535	A	4	3	1.00	18	0.167
536	A	5	4	1.00	18	0.222
537	A	4	4	1.00	18	0.222
538	A	4	4	1.00	18	0.222
539	A	3	3	1.00	18	0.167
540	A	3	3	1.00	18	0.167
541	A	4	4	1.00	18	0.222
542	A	4	4	1.00	18	0.222
543	A	5	4	1.00	18	0.222
544	A	5	4	1.00	18	0.222
545	A	4	4	1.00	18	0.222
546	A	4	4	1.00	18	0.222
547	A	3	3	1.00	18	0.167
548	A	3	3	1.00	18	0.167
549	A	4	4	1.00	18	0.222
550	A	4	4	1.00	18	0.222
551	A	5	4	1.00	18	0.222
552	A	5	4	1.00	18	0.222
553	A	4	4	1.00	18	0.222
554	A	4	4	1.00	18	0.222
555	A	3	3	1.00	18	0.167
556	A	3	3	1.00	18	0.167

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
557	A	4	4	1.00	18	0.222
558	A	4	4	1.00	18	0.222
559	A	5	4	1.00	18	0.222
560	A	3	3	1.00	9	0.333
561	A	4	4	1.00	9	0.444
562	A	5	5	1.00	9	0.556
563	A	3	3	1.00	9	0.333
564	A	4	4	1.00	9	0.444
565	A	5	5	1.00	9	0.556
566	A	7	6	1.00	14	0.429
567	A	7	6	1.00	15	0.400
568	A	6	5	1.00	14	0.357
569	A	6	5	1.00	15	0.333
570	A	3	3	1.00	17	0.176
571	A	3	3	1.00	16	0.188
572	A	5	5	1.00	15	0.333
573	A	6	6	1.00	15	0.400
574	A	3	3	1.00	17	0.176
575	A	4	4	1.00	17	0.235
576	A	4	4	1.00	17	0.235
577	A	3	3	1.00	17	0.176
578	A	6	6	1.00	17	0.353
579	A	7	7	1.00	17	0.412
580	A	3	2	1.00	9	0.222
581	A	2	2	1.00	11	0.182
582	A	2	1	1.00	11	0.091

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
583	A	3	2	1.00	11	0.182
584	A	3	2	1.00	11	0.182
585	A	2	2	1.00	11	0.182
586	A	1	1	1.00	11	0.091
587	A	3	3	1.00	11	0.273
588	A	2	2	1.00	11	0.182
589	A	4	3	1.00	11	0.273
590	A	2	2	1.00	13	0.154
591	A	3	3	1.00	13	0.231
592	A	3	3	1.00	13	0.231
593	A	2	2	1.00	13	0.154
594	A	3	3	1.00	13	0.231
595	A	3	3	1.00	13	0.231
596	A	3	2	1.00	17	0.118
597	A	1	1	1.00	19	0.053
598	A	1	1	1.00	19	0.053
599	A	1	1	1.00	19	0.053
600	A	1	1	1.00	19	0.053
601	A	1	1	1.00	19	0.053
602	A	1	1	1.00	19	0.053
603	A	1	1	1.00	21	0.048
604	A	1	1	1.00	21	0.048
605	A	3	2	1.00	18	0.111
606	A	1	1	1.00	20	0.050
607	A	1	1	1.00	20	0.050
608	A	1	1	1.00	20	0.050

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
609	A	1	1	1.00	20	0.050
610	A	1	1	1.00	20	0.050
611	A	1	1	1.00	20	0.050
612	A	1	1	1.00	22	0.045
613	A	1	1	1.00	22	0.045
614	A	8	8	1.00	11	0.727
615	A	4	4	1.00	11	0.364
616	A	7	7	1.00	11	0.636
617	A	4	3	1.00	11	0.273
618	A	3	2	1.00	9	0.222
619	A	3	3	1.00	11	0.273
620	A	6	6	1.00	11	0.546
621	A	4	3	1.00	11	0.273
622	A	8	8	1.00	11	0.727
623	A	4	3	1.00	11	0.273
624	A	4	3	1.00	11	0.273
625	A	5	4	1.00	11	0.364
626	A	4	3	1.00	11	0.273
627	A	4	4	1.00	11	0.364
628	A	3	2	1.00	9	0.222
629	A	3	3	1.00	11	0.273
630	A	3	3	1.00	11	0.273
631	A	4	3	1.00	11	0.273
632	A	4	3	1.00	11	0.273
633	A	4	3	1.00	11	0.273
634	A	4	3	1.00	11	0.273

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
635	A	5	4	1.00	11	0.364
636	A	4	3	1.00	11	0.273
637	A	4	4	1.00	11	0.364
638	A	3	2	1.00	9	0.222
639	A	3	3	1.00	11	0.273
640	A	3	3	1.00	11	0.273
641	A	4	3	1.00	11	0.273
642	A	4	3	1.00	11	0.273
643	A	4	3	1.00	11	0.273
644	A	8	8	1.00	11	0.727
645	A	4	4	1.00	11	0.364
646	A	7	7	1.00	11	0.636
647	A	4	3	1.00	11	0.273
648	A	3	2	1.00	9	0.222
649	A	3	3	1.00	11	0.273
650	A	5	5	1.00	11	0.454
651	A	4	3	1.00	11	0.273
652	A	7	7	1.00	11	0.636
653	A	4	3	1.00	11	0.273
654	A	4	3	1.00	7	0.429
655	A	5	4	1.00	7	0.571
656	A	4	3	1.00	7	0.429
657	A	4	4	1.00	7	0.571
658	A	3	2	1.00	5	0.400
659	A	3	3	1.00	7	0.429
660	A	3	3	1.00	7	0.429

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
661	A	4	3	1.00	7	0.429
662	A	4	3	1.00	7	0.429
663	A	4	3	1.00	7	0.429
664	A	4	3	1.00	9	0.333
665	A	5	4	1.00	9	0.444
666	A	4	3	1.00	9	0.333
667	A	4	4	1.00	9	0.444
668	A	3	2	1.00	7	0.286
669	A	3	3	1.00	9	0.333
670	A	3	3	1.00	9	0.333
671	A	4	3	1.00	9	0.333
672	A	4	3	1.00	9	0.333
673	A	4	3	1.00	9	0.333
674	A	3	2	1.00	5	0.400
675	A	4	3	1.00	7	0.429
676	A	6	5	1.00	7	0.714
677	A	4	4	1.00	9	0.444
678	A	5	5	1.00	9	0.556
679	A	6	6	1.00	9	0.667
680	A	3	2	1.00	7	0.286
681	A	4	3	1.00	9	0.333
682	A	6	5	1.00	9	0.556
683	A	3	3	1.00	11	0.273
684	A	4	4	1.00	11	0.364
685	A	5	5	1.00	11	0.454
686	A	6	6	1.33	7	0.857

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
687	A	6	6	1.20	9	0.667
688	A	2	2	1.00	14	0.143
689	A	4	4	1.00	16	0.250
690	A	5	5	1.00	16	0.312
691	A	2	2	1.00	14	0.143
692	A	4	4	1.00	16	0.250
693	A	5	5	1.00	16	0.312
694	A	5	4	1.00	14	0.286
695	A	5	4	1.00	14	0.286
696	A	3	3	1.00	14	0.214
697	A	4	4	1.00	16	0.250
698	A	16	10	1.54	16	0.625
699	A	3	3	1.00	14	0.214
700	A	4	4	1.00	16	0.250
701	A	8	4	1.45	16	0.250
702	A	2	2	1.00	14	0.143
703	A	5	5	1.00	16	0.312
704	A	2	2	1.00	14	0.143
705	A	5	5	1.00	16	0.312
706	A	5	5	1.00	16	0.312
707	A	7	7	1.00	18	0.389
708	A	9	8	1.00	18	0.444
709	A	7	7	1.00	18	0.389
710	A	10	8	1.00	20	0.400
711	A	13	8	1.00	20	0.400
712	A	9	8	1.00	18	0.444

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
713	A	13	9	1.00	20	0.450
714	A	17	9	1.00	20	0.450
715	A	6	5	1.00	16	0.312
716	A	13	8	1.00	18	0.444
717	A	17	13	1.00	18	0.722
718	A	13	8	1.00	18	0.444
719	A	21	10	1.00	20	0.500
720	A	33	12	1.00	20	0.600
721	A	17	13	1.00	18	0.722
722	A	33	12	1.00	20	0.600
723	A	48	12	1.00	20	0.600
724	A	3	3	1.00	18	0.167
725	A	3	3	1.00	18	0.167
726	A	4	4	1.00	18	0.222
727	A	3	3	1.00	18	0.167
728	A	3	3	1.00	18	0.167
729	A	4	4	1.00	18	0.222
730	A	1	1	1.00	15	0.067
731	A	1	1	1.00	15	0.067
732	A	1	1	1.00	21	0.048
733	A	1	1	1.00	21	0.048
734	A	3	3	1.00	21	0.143
735	A	3	3	1.00	21	0.143
736	A	3	3	1.00	22	0.136
737	A	3	3	1.00	22	0.136
738	A	4	4	1.00	22	0.182

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
739	A	5	4	1.00	12	0.333
740	A	4	3	1.00	12	0.250
741	A	3	2	1.00	10	0.200
742	A	3	3	1.00	12	0.250
743	A	5	5	1.00	12	0.417
744	A	5	5	1.00	12	0.417
745	A	6	6	1.00	12	0.500
746	A	5	4	1.00	12	0.333
747	A	4	3	1.00	12	0.250
748	A	3	2	1.00	10	0.200
749	A	2	2	1.00	12	0.167
750	A	4	4	1.00	12	0.333
751	A	4	4	1.00	12	0.333
752	A	5	5	1.00	12	0.417
753	A	6	3	1.00	24	0.125
754	A	5	3	1.00	24	0.125
755	A	4	3	1.00	24	0.125
756	A	3	2	1.00	22	0.091
757	A	1	1	1.00	24	0.042
758	A	2	2	1.00	24	0.083
759	A	3	2	1.00	24	0.083
760	A	4	2	1.00	24	0.083
761	A	7	7	1.00	14	0.500
762	A	6	6	1.00	14	0.429
763	A	2	2	1.00	14	0.143
764	A	2	2	1.00	14	0.143

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
765	A	3	3	1.00	14	0.214
766	A	7	7	1.00	14	0.500
767	A	8	7	1.00	14	0.500
768	A	3	2	1.00	26	0.077
769	A	2	2	1.00	26	0.077
770	A	1	1	1.00	26	0.038
771	A	3	3	1.00	26	0.115
772	A	4	4	1.00	26	0.154
773	A	5	4	1.00	26	0.154
774	A	3	2	1.00	28	0.071
775	A	2	2	1.00	28	0.071
776	A	1	1	1.00	28	0.036
777	A	3	3	1.00	28	0.107
778	A	4	4	1.00	28	0.143
779	A	5	4	1.00	28	0.143
780	A	5	5	1.00	12	0.417
781	A	5	5	1.00	12	0.417
782	A	4	4	1.00	15	0.267
783	A	1	1	1.00	11	0.091
784	A	4	4	1.00	15	0.267
785	A	10	9	1.00	17	0.529
786	A	4	4	1.00	15	0.267
787	A	4	4	1.00	15	0.267
788	A	9	7	1.00	17	0.412
789	A	4	4	1.00	19	0.210
790	A	4	4	1.00	19	0.210

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
791	A	5	5	1.00	19	0.263
792	A	4	4	1.00	19	0.210
793	A	4	4	1.00	19	0.210
794	A	5	5	1.00	19	0.263
795	A	4	4	1.00	22	0.182
796	A	4	4	1.00	22	0.182
797	A	5	5	1.00	22	0.227
798	A	4	4	1.00	23	0.174
799	A	4	4	1.00	23	0.174
800	A	5	5	1.00	23	0.217
801	A	1	1	1.00	32	0.031
802	A	1	1	1.00	19	0.053
803	A	1	1	1.00	19	0.053
804	A	1	1	1.00	23	0.043
805	A	1	1	1.00	20	0.050
806	A	1	1	1.00	20	0.050
807	A	1	1	1.00	24	0.042
808	A	2	1	1.00	11	0.091
809	A	2	1	1.00	11	0.091
810	A	4	3	1.00	11	0.273
811	A	2	2	1.00	13	0.154
812	A	2	2	1.00	13	0.154
813	A	2	2	1.00	13	0.154
814	A	2	2	1.00	11	0.182
815	A	2	2	1.00	11	0.182
816	A	2	2	1.00	11	0.182

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
817	A	4	2	1.00	13	0.154
818	A	6	4	1.00	13	0.308
819	A	6	4	1.00	13	0.308
820	A	4	2	1.00	11	0.182
821	A	6	4	1.00	11	0.364
822	A	6	4	1.00	11	0.364
823	A	2	2	1.00	13	0.154
824	A	2	2	1.00	13	0.154
825	A	2	2	1.00	13	0.154
826	A	7	4	1.00	14	0.286
827	A	8	4	1.00	17	0.235
828	A	9	5	1.00	19	0.263
829	A	10	6	1.00	19	0.316
830	A	3	3	1.00	27	0.111
831	A	7	4	1.00	21	0.190
832	A	5	3	1.00	14	0.214
833	A	6	3	1.00	17	0.176
834	A	7	4	1.00	19	0.210
835	A	8	5	1.00	19	0.263
836	A	3	3	1.00	27	0.111
837	A	5	3	1.00	21	0.143
838	A	4	3	1.00	20	0.150
839	A	4	3	1.00	20	0.150
840	A	6	4	1.00	16	0.250
841	A	6	4	1.00	16	0.250
842	A	6	4	1.00	16	0.250

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
843	A	8	5	1.00	18	0.278
844	A	10	6	1.00	18	0.333
845	A	5	5	1.00	16	0.312
846	A	6	6	1.00	18	0.333
847	A	7	7	1.00	18	0.389
848	A	10	10	1.00	16	0.625
849	A	17	14	1.00	18	0.778
850	A	21	13	1.00	18	0.722
851	A	12	11	1.00	16	0.688
852	A	16	13	1.00	18	0.722
853	A	21	17	1.00	18	0.944
854	A	4	4	1.00	18	0.222
855	A	3	3	1.00	18	0.167
856	A	2	2	1.00	18	0.111
857	A	3	2	1.00	16	0.125
858	A	4	4	1.00	18	0.222
859	A	6	6	1.00	18	0.333
860	A	7	7	1.00	18	0.389
861	A	8	8	1.00	20	0.400
862	A	7	7	1.00	20	0.350
863	A	3	3	1.00	20	0.150
864	A	3	3	1.00	20	0.150
865	A	5	5	1.00	20	0.250
866	A	8	8	1.00	20	0.400
867	A	13	8	1.00	14	0.571
868	A	11	7	1.00	14	0.500

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
869	A	9	6	1.00	12	0.500
870	A	0	0	0.00	0	0.000
871	A	2	2	1.00	18	0.111
872	A	4	3	1.00	18	0.167
873	A	5	4	1.00	18	0.222
874	A	3	2	1.00	16	0.125
875	A	4	4	1.00	16	0.250
876	A	5	4	1.00	18	0.222
877	A	5	4	1.00	18	0.222
878	A	2	2	1.00	16	0.125
879	A	2	2	1.00	16	0.125
880	A	1	1	1.00	14	0.071
881	A	1	1	1.00	14	0.071
882	A	1	1	1.00	16	0.062
883	A	2	2	1.00	16	0.125
884	A	2	2	1.00	18	0.111
885	A	2	2	1.00	16	0.125
886	A	2	2	1.00	16	0.125
887	A	1	1	1.00	14	0.071
888	A	1	1	1.00	14	0.071
889	A	1	1	1.00	16	0.062
890	A	2	2	1.00	16	0.125
891	A	2	2	1.00	18	0.111
892	A	2	2	1.00	18	0.111
893	A	8	6	1.00	25	0.240
894	A	6	5	1.00	23	0.217

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
895	A	2	2	1.00	25	0.080
896	A	3	3	1.00	25	0.120
897	A	8	6	1.00	22	0.273
898	A	6	5	1.00	20	0.250
899	A	2	2	1.00	22	0.091
900	A	3	3	1.00	22	0.136
901	A	4	3	1.00	22	0.136
902	A	5	4	1.00	22	0.182
903	A	4	3	1.00	20	0.150
904	A	3	3	1.00	14	0.214
905	A	5	4	1.00	20	0.200
906	A	5	5	1.00	22	0.227
907	A	5	4	1.00	24	0.167
908	A	4	3	1.00	24	0.125
909	A	5	4	1.00	22	0.182
910	A	5	4	1.00	20	0.200
911	A	5	4	1.00	16	0.250
912	A	5	4	1.00	22	0.182
913	A	4	3	1.00	24	0.125
914	A	5	4	1.00	24	0.167
915	A	4	3	1.00	22	0.136
916	A	5	4	1.00	22	0.182
917	A	5	4	1.00	22	0.182
918	A	7	6	1.00	16	0.375
919	A	5	4	1.00	24	0.167
920	A	4	3	1.00	24	0.125

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
921	A	4	3	1.00	22	0.136
922	A	4	3	1.00	16	0.188
923	A	5	5	1.00	22	0.227
924	A	5	4	1.00	24	0.167
925	A	4	3	1.00	26	0.115
926	A	4	3	1.00	26	0.115
927	A	4	3	1.00	24	0.125
928	A	5	4	1.00	22	0.182
929	A	4	3	1.00	18	0.167
930	A	6	6	1.00	24	0.250
931	A	5	4	1.00	26	0.154
932	A	4	3	1.00	26	0.115
933	A	5	4	1.00	24	0.167
934	A	5	4	1.00	24	0.167
935	A	6	5	1.00	24	0.208
936	A	4	3	1.00	18	0.167
937	A	12	9	1.00	12	0.750
938	A	13	10	1.00	14	0.714
939	A	13	10	1.00	14	0.714
940	A	14	11	1.00	16	0.688
941	A	6	6	1.00	12	0.500
942	A	7	7	1.00	14	0.500
943	A	7	7	1.00	14	0.500
944	A	8	8	1.00	16	0.500
945	A	4	2	1.00	22	0.091
946	A	4	2	1.00	22	0.091

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
947	A	3	3	1.00	20	0.150
948	A	1	1	1.00	14	0.071
949	A	4	3	1.00	14	0.214
950	A	4	2	1.00	20	0.100
951	A	4	2	1.00	22	0.091
952	A	5	2	1.00	24	0.083
953	A	4	3	1.00	24	0.125
954	A	4	2	1.00	22	0.091
955	A	2	2	1.00	16	0.125
956	A	6	4	1.00	20	0.200
957	A	5	3	1.00	16	0.188
958	A	5	2	1.00	22	0.091
959	A	4	2	1.00	24	0.083
960	A	5	2	1.00	24	0.083
961	A	4	2	1.00	22	0.091
962	A	2	2	1.00	16	0.125
963	A	8	4	1.00	22	0.182
964	A	7	4	1.00	22	0.182
965	A	6	3	1.00	16	0.188
966	A	10	5	1.00	56	0.089
967	A	2	2	1.00	17	0.118
968	A	2	2	1.00	22	0.091
969	A	2	2	1.00	22	0.091
970	A	2	2	1.00	17	0.118
971	A	2	2	1.00	22	0.091
972	A	2	2	1.00	22	0.091

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
973	A	2	2	1.00	17	0.118
974	A	2	2	1.00	22	0.091
975	A	2	2	1.00	22	0.091
976	A	2	2	1.00	17	0.118
977	A	2	2	1.00	22	0.091
978	A	2	2	1.00	22	0.091
979	A	2	2	1.00	13	0.154
980	A	2	2	1.00	13	0.154
981	A	2	2	1.00	13	0.154
982	A	2	2	1.00	13	0.154
983	A	3	1	1.00	17	0.059
984	A	4	3	1.00	23	0.130
985	A	3	3	1.00	17	0.176
986	A	3	2	1.00	16	0.125
987	A	3	2	1.00	18	0.111
988	A	7	7	1.00	15	0.467
989	A	3	3	1.00	19	0.158
990	A	3	2	1.00	19	0.105
991	A	3	2	1.00	21	0.095
992	A	3	2	1.00	21	0.095
993	A	2	2	1.00	17	0.118
994	A	4	3	1.00	19	0.158
995	A	5	5	1.00	19	0.263
996	A	2	2	1.00	11	0.182
997	A	3	2	1.00	17	0.118
998	A	2	2	1.00	17	0.118

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
999	A	2	2	1.00	17	0.118
1000	A	3	3	1.00	15	0.200
1001	A	3	3	1.00	15	0.200
1002	A	3	3	1.00	15	0.200
1003	A	4	3	1.00	15	0.200
1004	A	4	3	1.00	20	0.150
1005	A	4	3	1.00	19	0.158
1006	A	4	3	1.00	24	0.125
1007	A	4	3	1.00	21	0.143
1008	A	4	3	1.00	20	0.150
1009	A	4	3	1.00	19	0.158
1010	A	4	3	1.00	24	0.125
1011	A	4	3	1.00	21	0.143
1012	A	1	3	1.00	8	0.375
1013	A	1	2	1.00	8	0.250
1014	A	0	0	0.00	0	0.000
1015	A	0	0	0.00	0	0.000
1016	A	0	0	0.00	0	0.000
1017	A	0	0	0.00	0	0.000
1018	A	3	2	1.00	13	0.154
1019	A	2	2	1.00	13	0.154
1020	A	2	2	1.00	13	0.154
1021	A	2	2	1.00	11	0.182
1022	A	3	2	1.00	19	0.105
1023	A	3	2	1.00	19	0.105
1024	A	3	2	1.00	21	0.095

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1025	A	3	2	1.00	21	0.095
1026	A	4	3	1.00	17	0.176
1027	A	4	3	1.00	17	0.176
1028	A	2	2	1.00	17	0.118
1029	A	3	3	1.00	16	0.188
1030	A	3	3	1.00	18	0.167
1031	A	1	1	1.00	18	0.056
1032	A	3	3	1.00	18	0.167
1033	A	9	7	1.31	15	0.467
1034	A	8	7	1.31	15	0.467
1035	A	4	2	1.00	15	0.133
1036	A	4	2	1.00	15	0.133
1037	A	3	3	1.00	21	0.143
1038	A	3	3	1.00	21	0.143
1039	A	12	7	1.00	8	0.875
1040	A	5	4	1.00	10	0.400
1041	A	19	6	1.00	10	0.600
1042	A	8	5	1.00	18	0.278
1043	A	6	5	1.00	8	0.625
1044	A	5	3	1.00	15	0.200
1045	A	7	4	1.00	22	0.182
1046	A	4	3	1.00	12	0.250
1047	A	6	5	1.00	15	0.333
1048	A	5	4	1.00	15	0.267
1049	A	3	2	1.00	13	0.154
1050	A	4	4	1.00	23	0.174

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1051	A	4	4	1.00	25	0.160
1052	A	6	3	1.00	39	0.077
1053	A	5	3	1.00	39	0.077
1054	A	3	2	1.00	39	0.051
1055	A	1	1	1.00	31	0.032
1056	A	2	1	1.00	31	0.032
1057	A	2	1	1.00	39	0.026
1058	A	5	3	1.00	39	0.077
1059	A	6	3	1.00	39	0.077

Chapter 3

Listing of integrals

3.1 $\int \frac{2}{-1+3\cosh(4+6x)}\,dx$

Optimal. Leaf size=22

$$\frac{\tan^{-1}\left(\sqrt{2}\,\tanh(3x+2)\right)}{3\sqrt{2}}$$

[Out] 1/6*arctan(2^(1/2)*tanh(2+3*x))*2^(1/2)

Rubi [A] time = 0.02, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {12, 2659, 206}

$$\frac{\tan^{-1}\left(\sqrt{2}\,\tanh(3x+2)\right)}{3\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Int[2/(-1 + 3*Cosh[4 + 6*x]),x]

[Out] ArcTan[Sqrt[2]*Tanh[2 + 3*x]]/(3*Sqrt[2])

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{2}{-1+3\cosh(4+6x)}\,dx &= 2\int \frac{1}{-1+3\cosh(4+6x)}\,dx\\ &= -\left(\frac{2}{3}i\,\text{Subst}\left(\int \frac{1}{2-4x^2}\,dx, x, \tan\left(\frac{1}{2}(4i+6ix)\right)\right)\right)\\ &= \frac{\tan^{-1}\left(\sqrt{2}\,\tanh(2+3x)\right)}{3\sqrt{2}}\end{aligned}$$

Mathematica [B] time = 0.08, size = 47, normalized size = 2.14

$$\frac{\tan^{-1}\left(\frac{\left(3+2e^4+3e^8\right)\tanh(3x)+3\left(e^8-1\right)}{4\sqrt{2}e^4}\right)}{3\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[2/(-1 + 3*Cosh[4 + 6*x]),x]
```

```
[Out] ArcTan[(3*(-1 + E^8) + (3 + 2*E^4 + 3*E^8)*Tanh[3*x])/(4*Sqrt[2]*E^4)]/(3*S
qrt[2])
```

fricas [B] time = 0.41, size = 34, normalized size = 1.55

$$\frac{1}{6}\sqrt{2}\arctan\left(\frac{3}{4}\sqrt{2}\cosh\left(6x+4\right)+\frac{3}{4}\sqrt{2}\sinh\left(6x+4\right)-\frac{1}{4}\sqrt{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2/(-1+3*cosh(4+6*x)),x, algorithm="fricas")
```

```
[Out] 1/6*sqrt(2)*arctan(3/4*sqrt(2)*cosh(6*x + 4) + 3/4*sqrt(2)*sinh(6*x + 4) -
1/4*sqrt(2))
```

giac [A] time = 0.13, size = 21, normalized size = 0.95

$$\frac{1}{6}\,\sqrt{2}\,\arctan\left(\frac{1}{4}\,\sqrt{2}\left(3\,e^{(6\,x+4)}-1\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2/(-1+3*cosh(4+6*x)),x, algorithm="giac")
[Out] 1/6*sqrt(2)*arctan(1/4*sqrt(2)*(3*e^(6*x + 4) - 1))
```

maple [A] time = 0.13, size = 17, normalized size = 0.77

$$\frac{\arctan\left(\sqrt{2}\,\tanh\left(2+3x\right)\right)\sqrt{2}}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(2/(-1+3*cosh(4+6*x)),x)
[Out] 1/6*arctan(2^(1/2)*tanh(2+3*x))*2^(1/2)
```

maxima [A] time = 0.48, size = 21, normalized size = 0.95

$$-\frac{1}{6}\,\sqrt{2}\,\arctan\left(\frac{1}{4}\,\sqrt{2}\left(3\,e^{(-6\,x-4)}-1\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2/(-1+3*cosh(4+6*x)),x, algorithm="maxima")
[Out] -1/6*sqrt(2)*arctan(1/4*sqrt(2)*(3*e^(-6*x - 4) - 1))
```

mupad [B] time = 0.12, size = 21, normalized size = 0.95

$$\frac{\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,\left(3\,\mathrm{e}^{6\,x+4}-1\right)}{4}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(2/(3*cosh(6*x + 4) - 1),x)
[Out] (2^(1/2)*atan((2^(1/2)*(3*exp(6*x + 4) - 1))/4))/6
```

sympy [A] time = 0.27, size = 19, normalized size = 0.86

$$\frac{\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,\tanh\left(3x+2\right)\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2/(-1+3*cosh(4+6*x)),x)
```

```
[Out] sqrt(2)*atan(sqrt(2)*tanh(3*x + 2))/6
```

3.2 $\int \frac{1}{\cosh^2(2+3x)+2\sinh^2(2+3x)}\,dx$

Optimal. Leaf size=22

$$\frac{\tan^{-1}\left(\sqrt{2}\tanh(3x+2)\right)}{3\sqrt{2}}$$

```
[Out] 1/6*arctan(2^(1/2)*tanh(2+3*x))*2^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {203}

$$\frac{\tan^{-1}\left(\sqrt{2}\tanh(3x+2)\right)}{3\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cosh[2 + 3*x]^2 + 2*Sinh[2 + 3*x]^2)^(-1),x]
```

```
[Out] ArcTan[Sqrt[2]*Tanh[2 + 3*x]]/(3*Sqrt[2])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\cosh^2(2+3x)+2\sinh^2(2+3x)}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{1}{1+2x^2}\,dx, x, \tanh(2+3x)\right)\\ &= \frac{\tan^{-1}\left(\sqrt{2}\tanh(2+3x)\right)}{3\sqrt{2}}\end{aligned}$$

Mathematica [B] time = 0.07, size = 47, normalized size = 2.14

$$\frac{\tan^{-1}\left(\frac{(3+2e^4+3e^8)\tanh(3x)+3(e^8-1)}{4\sqrt{2}e^4}\right)}{3\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cosh[2 + 3*x]^2 + 2*Sinh[2 + 3*x]^2)^(-1),x]
```

```
[Out] ArcTan[(3*(-1 + E^8) + (3 + 2*E^4 + 3*E^8)*Tanh[3*x])/(4*Sqrt[2]*E^4)]/(3*S
qrt[2])
```

fricas [B] time = 0.42, size = 47, normalized size = 2.14

$$-\frac{1}{6}\sqrt{2}\arctan\left(-\frac{\sqrt{2}\cosh\left(3x+2\right)+2\sqrt{2}\sinh\left(3x+2\right)}{2\left(\cosh\left(3x+2\right)-\sinh\left(3x+2\right)\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cosh(2+3*x)^2+2*sinh(2+3*x)^2),x, algorithm="fricas")
```

```
[Out] -1/6*sqrt(2)*arctan(-1/2*(sqrt(2)*cosh(3*x + 2) + 2*sqrt(2)*sinh(3*x + 2))/
(cosh(3*x + 2) - sinh(3*x + 2)))
```

giac [A] time = 0.15, size = 21, normalized size = 0.95

$$\frac{1}{6}\sqrt{2}\arctan\left(\frac{1}{4}\sqrt{2}\left(3e^{(6x+4)}-1\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cosh(2+3*x)^2+2*sinh(2+3*x)^2),x, algorithm="giac")
```

```
[Out] 1/6*sqrt(2)*arctan(1/4*sqrt(2)*(3*e^(6*x + 4) - 1))
```

maple [B] time = 0.52, size = 156, normalized size = 7.09

$$\frac{\sqrt{6}\arctan\left(\frac{2\tanh\left(1+\frac{3x}{2}\right)}{2\sqrt{3}-2\sqrt{2}}\right)}{6\sqrt{3}-6\sqrt{2}}-\frac{2\arctan\left(\frac{2\tanh\left(1+\frac{3x}{2}\right)}{2\sqrt{3}-2\sqrt{2}}\right)}{3\left(2\sqrt{3}-2\sqrt{2}\right)}-\frac{\sqrt{6}\arctan\left(\frac{2\tanh\left(1+\frac{3x}{2}\right)}{2\sqrt{3}+2\sqrt{2}}\right)}{3\left(2\sqrt{3}+2\sqrt{2}\right)}-\frac{2\arctan\left(\frac{2\tanh\left(1+\frac{3x}{2}\right)}{2\sqrt{3}+2\sqrt{2}}\right)}{3\left(2\sqrt{3}+2\sqrt{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(2+3*x)^2+2*sinh(2+3*x)^2),x)
```

```
[Out] 1/3*6^(1/2)/(2*3^(1/2)-2*2^(1/2))*arctan(2*tanh(1+3/2*x)/(2*3^(1/2)-2*2^(1/
2)))-2/3/(2*3^(1/2)-2*2^(1/2))*arctan(2*tanh(1+3/2*x)/(2*3^(1/2)-2*2^(1/2))
)-1/3*6^(1/2)/(2*3^(1/2)+2*2^(1/2))*arctan(2*tanh(1+3/2*x)/(2*3^(1/2)+2*2^(
1/2)))-2/3/(2*3^(1/2)+2*2^(1/2))*arctan(2*tanh(1+3/2*x)/(2*3^(1/2)+2*2^(1/2
)))
```

maxima [A] time = 0.43, size = 21, normalized size = 0.95

$$-\frac{1}{6}\sqrt{2}\arctan\left(\frac{1}{4}\sqrt{2}\left(3e^{(-6x-4)}-1\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cosh(2+3*x)^2+2*sinh(2+3*x)^2),x, algorithm="maxima")
```

```
[Out] -1/6*sqrt(2)*arctan(1/4*sqrt(2)*(3*e^(-6*x - 4) - 1))
```

mupad [B] time = 0.10, size = 21, normalized size = 0.95

$$\frac{\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,\left(3\,\mathrm{e}^{6\,x+4}-1\right)}{4}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*sinh(3*x + 2)^2 + cosh(3*x + 2)^2),x)
```

```
[Out] (2^(1/2)*atan((2^(1/2)*(3*exp(6*x + 4) - 1))/4))/6
```

sympy [B] time = 7.89, size = 185, normalized size = 8.41

$$\frac{2093258\sqrt{5-2\sqrt{6}}\ \mathrm{atan}\left(\frac{\tanh\left(\frac{3x}{2}+1\right)}{\sqrt{5-2\sqrt{6}}}\right)}{1152360\sqrt{6}+2822694}+\frac{854569\sqrt{6}\sqrt{5-2\sqrt{6}}\ \mathrm{atan}\left(\frac{\tanh\left(\frac{3x}{2}+1\right)}{\sqrt{5-2\sqrt{6}}}\right)}{1152360\sqrt{6}+2822694}-\frac{86329\sqrt{6}\sqrt{2\sqrt{6}+5}\ \mathrm{atan}\left(\ldots\right)}{1152360\sqrt{6}+28226\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cosh(2+3*x)**2+2*sinh(2+3*x)**2),x)
```

```
[Out] 2093258*sqrt(5 - 2*sqrt(6))*atan(tanh(3*x/2 + 1)/sqrt(5 - 2*sqrt(6)))/(1152
360*sqrt(6) + 2822694) + 854569*sqrt(6)*sqrt(5 - 2*sqrt(6))*atan(tanh(3*x/2
 + 1)/sqrt(5 - 2*sqrt(6)))/(1152360*sqrt(6) + 2822694) - 86329*sqrt(6)*sqrt
(2*sqrt(6) + 5)*atan(tanh(3*x/2 + 1)/sqrt(2*sqrt(6) + 5))/(1152360*sqrt(6)
+ 2822694) - 211462*sqrt(2*sqrt(6) + 5)*atan(tanh(3*x/2 + 1)/sqrt(2*sqrt(6)
 + 5))/(1152360*sqrt(6) + 2822694)
```

3.3 $\int \frac{\text{sech}^2(2+3x)}{1+2\tanh^2(2+3x)} \, dx$

Optimal. Leaf size=22

$$\frac{\tan^{-1}\left(\sqrt{2}\tanh(3x+2)\right)}{3\sqrt{2}}$$

```
[Out] 1/6*arctan(2^(1/2)*tanh(2+3*x))*2^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {3675, 203}

$$\frac{\tan^{-1}\left(\sqrt{2}\tanh(3x+2)\right)}{3\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[2 + 3*x]^2/(1 + 2*Tanh[2 + 3*x]^2),x]
```

```
[Out] ArcTan[Sqrt[2]*Tanh[2 + 3*x]]/(3*Sqrt[2])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3675

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/(c^(m - 1)*f), Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^p
, x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && In
tegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
|| EqQ[n^2, 16])
```

Rubi steps

$$\int \frac{\operatorname{sech}^2(2+3x)}{1+2\tanh^2(2+3x)}\,dx = \frac{1}{3}\operatorname{Subst}\left(\int \frac{1}{1+2x^2}\,dx, x, \tanh(2+3x)\right)$$
$$= \frac{\tan^{-1}\left(\sqrt{2}\,\tanh(2+3x)\right)}{3\sqrt{2}}$$

Mathematica [B] time = 0.08, size = 47, normalized size = 2.14

$$\frac{\tan^{-1}\left(\frac{(3+2e^4+3e^8)\tanh(3x)+3(e^8-1)}{4\sqrt{2}\,e^4}\right)}{3\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[2 + 3*x]^2/(1 + 2*Tanh[2 + 3*x]^2),x]
```

```
[Out] ArcTan[(3*(-1 + E^8) + (3 + 2*E^4 + 3*E^8)*Tanh[3*x])/(4*Sqrt[2]*E^4)]/(3*S
qrt[2])
```

fricas [B] time = 0.44, size = 47, normalized size = 2.14

$$-\frac{1}{6}\,\sqrt{2}\,\arctan\left(-\frac{\sqrt{2}\,\cosh\left(3\,x+2\right)+2\,\sqrt{2}\,\sinh\left(3\,x+2\right)}{2\,\left(\cosh\left(3\,x+2\right)-\sinh\left(3\,x+2\right)\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(2+3*x)^2/(1+2*tanh(2+3*x)^2),x, algorithm="fricas")
```

```
[Out] -1/6*sqrt(2)*arctan(-1/2*(sqrt(2)*cosh(3*x + 2) + 2*sqrt(2)*sinh(3*x + 2))/
(cosh(3*x + 2) - sinh(3*x + 2)))
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(2+3*x)^2/(1+2*tanh(2+3*x)^2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Evaluation time: 190.91Not invertible Error:
 Bad Argument Value
```

maple [B] time = 0.60, size = 156, normalized size = 7.09

$$\frac{\sqrt{6}\ \arctan\left(\frac{2\tanh\left(1+\frac{3x}{2}\right)}{2\sqrt{3}-2\sqrt{2}}\right)}{6\sqrt{3}-6\sqrt{2}}-\frac{2\arctan\left(\frac{2\tanh\left(1+\frac{3x}{2}\right)}{2\sqrt{3}-2\sqrt{2}}\right)}{3\left(2\sqrt{3}-2\sqrt{2}\right)}-\frac{\sqrt{6}\ \arctan\left(\frac{2\tanh\left(1+\frac{3x}{2}\right)}{2\sqrt{3}+2\sqrt{2}}\right)}{3\left(2\sqrt{3}+2\sqrt{2}\right)}-\frac{2\arctan\left(\frac{2\tanh\left(1+\frac{3x}{2}\right)}{2\sqrt{3}+2\sqrt{2}}\right)}{3\left(2\sqrt{3}+2\sqrt{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(2+3*x)^2/(1+2*tanh(2+3*x)^2),x)
```

```
[Out] 1/3*6^(1/2)/(2*3^(1/2)-2*2^(1/2))*arctan(2*tanh(1+3/2*x)/(2*3^(1/2)-2*2^(1/
2)))-2/3/(2*3^(1/2)-2*2^(1/2))*arctan(2*tanh(1+3/2*x)/(2*3^(1/2)-2*2^(1/2))
)-1/3*6^(1/2)/(2*3^(1/2)+2*2^(1/2))*arctan(2*tanh(1+3/2*x)/(2*3^(1/2)+2*2^(
1/2)))-2/3/(2*3^(1/2)+2*2^(1/2))*arctan(2*tanh(1+3/2*x)/(2*3^(1/2)+2*2^(1/2
)))
```

maxima [A] time = 0.56, size = 21, normalized size = 0.95

$$-\frac{1}{6}\sqrt{2}\arctan\left(\frac{1}{4}\sqrt{2}\left(3e^{(-6x-4)}-1\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(2+3*x)^2/(1+2*tanh(2+3*x)^2),x, algorithm="maxima")
```

```
[Out] -1/6*sqrt(2)*arctan(1/4*sqrt(2)*(3*e^(-6*x - 4) - 1))
```

mupad [B] time = 1.49, size = 21, normalized size = 0.95

$$\frac{\sqrt{2}\ \mathrm{atan}\left(\frac{\sqrt{2}\left(3\,e^{6x+4}-1\right)}{4}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(3*x + 2)^2*(2*tanh(3*x + 2)^2 + 1)),x)
```

```
[Out] (2^(1/2)*atan((2^(1/2)*(3*exp(6*x + 4) - 1))/4))/6
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\operatorname{sech}^2(3x+2)}{2\tanh^2(3x+2)+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(2+3*x)**2/(1+2*tanh(2+3*x)**2),x)
```

```
[Out] Integral(sech(3*x + 2)**2/(2*tanh(3*x + 2)**2 + 1), x)
```

3.4 $\int \frac{\operatorname{csch}^2(2+3x)}{2+\coth^2(2+3x)}\,dx$

Optimal. Leaf size=22

$$\frac{\tan^{-1}\left(\sqrt{2}\tanh(3x+2)\right)}{3\sqrt{2}}$$

```
[Out] 1/6*arctan(2^(1/2)*tanh(2+3*x))*2^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3675, 203}

$$\frac{\tan^{-1}\left(\sqrt{2}\tanh(3x+2)\right)}{3\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[2 + 3*x]^2/(2 + Coth[2 + 3*x]^2),x]
```

```
[Out] ArcTan[Sqrt[2]*Tanh[2 + 3*x]]/(3*Sqrt[2])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3675

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/(c^(m - 1)*f), Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^p
, x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && In
tegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
|| EqQ[n^2, 16])
```

Rubi steps

$$\int \frac{\operatorname{csch}^2(2+3x)}{2+\coth^2(2+3x)}\,dx = -\left(\frac{1}{3}\,\text{Subst}\left(\int \frac{1}{2+x^2}\,dx, x, \coth(2+3x)\right)\right)$$
$$= \frac{\tan^{-1}\left(\sqrt{2}\,\tanh(2+3x)\right)}{3\sqrt{2}}$$

Mathematica [B] time = 0.08, size = 47, normalized size = 2.14

$$\frac{\tan^{-1}\left(\frac{\left(3+2e^4+3e^8\right)\tanh(3x)+3\left(e^8-1\right)}{4\sqrt{2}e^4}\right)}{3\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[2 + 3*x]^2/(2 + Coth[2 + 3*x]^2),x]
```

```
[Out] ArcTan[(3*(-1 + E^8) + (3 + 2*E^4 + 3*E^8)*Tanh[3*x])/(4*Sqrt[2]*E^4)]/(3*S
qrt[2])
```

fricas [B] time = 0.39, size = 47, normalized size = 2.14

$$-\frac{1}{6}\sqrt{2}\arctan\left(-\frac{\sqrt{2}\cosh\left(3x+2\right)+2\sqrt{2}\sinh\left(3x+2\right)}{2\left(\cosh\left(3x+2\right)-\sinh\left(3x+2\right)\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(2+3*x)^2/(2+coth(2+3*x)^2),x, algorithm="fricas")
```

```
[Out] -1/6*sqrt(2)*arctan(-1/2*(sqrt(2)*cosh(3*x + 2) + 2*sqrt(2)*sinh(3*x + 2))/
(cosh(3*x + 2) - sinh(3*x + 2)))
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(2+3*x)^2/(2+coth(2+3*x)^2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Evaluation time: 190.94Not invertible Error:
 Bad Argument Value
```

maple [B] time = 0.52, size = 156, normalized size = 7.09

$$\frac{\sqrt{6}\,\arctan\left(\frac{2\tanh\left(1+\frac{3x}{2}\right)}{2\sqrt{3}-2\sqrt{2}}\right)}{6\sqrt{3}-6\sqrt{2}}-\frac{2\arctan\left(\frac{2\tanh\left(1+\frac{3x}{2}\right)}{2\sqrt{3}-2\sqrt{2}}\right)}{3\left(2\sqrt{3}-2\sqrt{2}\right)}-\frac{\sqrt{6}\,\arctan\left(\frac{2\tanh\left(1+\frac{3x}{2}\right)}{2\sqrt{3}+2\sqrt{2}}\right)}{3\left(2\sqrt{3}+2\sqrt{2}\right)}-\frac{2\arctan\left(\frac{2\tanh\left(1+\frac{3x}{2}\right)}{2\sqrt{3}+2\sqrt{2}}\right)}{3\left(2\sqrt{3}+2\sqrt{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(2+3*x)^2/(2+coth(2+3*x)^2),x)
```

```
[Out] 1/3*6^(1/2)/(2*3^(1/2)-2*2^(1/2))*arctan(2*tanh(1+3/2*x)/(2*3^(1/2)-2*2^(1/
2)))-2/3/(2*3^(1/2)-2*2^(1/2))*arctan(2*tanh(1+3/2*x)/(2*3^(1/2)-2*2^(1/2))
)-1/3*6^(1/2)/(2*3^(1/2)+2*2^(1/2))*arctan(2*tanh(1+3/2*x)/(2*3^(1/2)+2*2^(
1/2)))-2/3/(2*3^(1/2)+2*2^(1/2))*arctan(2*tanh(1+3/2*x)/(2*3^(1/2)+2*2^(1/2
)))
```

maxima [A] time = 0.69, size = 21, normalized size = 0.95

$$-\frac{1}{6}\sqrt{2}\arctan\left(\frac{1}{4}\sqrt{2}\left(3e^{(-6x-4)}-1\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(2+3*x)^2/(2+coth(2+3*x)^2),x, algorithm="maxima")
```

```
[Out] -1/6*sqrt(2)*arctan(1/4*sqrt(2)*(3*e^(-6*x - 4) - 1))
```

mupad [B] time = 1.53, size = 21, normalized size = 0.95

$$\frac{\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\left(3\,e^{6x+4}-1\right)}{4}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sinh(3*x + 2)^2*(coth(3*x + 2)^2 + 2)),x)
```

```
[Out] (2^(1/2)*atan((2^(1/2)*(3*exp(6*x + 4) - 1))/4))/6
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\operatorname{csch}^2(3x+2)}{\coth^2(3x+2)+2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(2+3*x)**2/(2+coth(2+3*x)**2),x)
```

```
[Out] Integral(csch(3*x + 2)**2/(coth(3*x + 2)**2 + 2), x)
```

3.5 $\int \frac{\text{csch}^2(2+3x)}{2-\coth^2(2+3x)}\,dx$

Optimal. Leaf size=22

$$-\frac{\tanh^{-1}\left(\sqrt{2}\tanh(3x+2)\right)}{3\sqrt{2}}$$

```
[Out] -1/6*arctanh(2^(1/2)*tanh(2+3*x))*2^(1/2)
```

Rubi [**A**] time = 0.04, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {3675, 206}

$$-\frac{\tanh^{-1}\left(\sqrt{2}\tanh(3x+2)\right)}{3\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[2 + 3*x]^2/(2 - Coth[2 + 3*x]^2),x]
```

```
[Out] -ArcTanh[Sqrt[2]*Tanh[2 + 3*x]]/(3*Sqrt[2])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3675

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/(c^(m - 1)*f), Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^p
, x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && In
tegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
|| EqQ[n^2, 16])
```

Rubi steps

$$\int \frac{\operatorname{csch}^2(2+3x)}{2-\coth^2(2+3x)}\,dx = -\left(\frac{1}{3}\operatorname{Subst}\left(\int \frac{1}{2-x^2}\,dx, x, \coth(2+3x)\right)\right)$$
$$= -\frac{\tanh^{-1}\left(\sqrt{2}\tanh(2+3x)\right)}{3\sqrt{2}}$$

Mathematica [A] time = 0.11, size = 42, normalized size = 1.91

$$-\frac{\tanh^{-1}\left(\frac{(1+6e^4+e^8)\tanh(3x)+e^8-1}{4\sqrt{2}e^4}\right)}{3\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[2 + 3*x]^2/(2 - Coth[2 + 3*x]^2),x]
```

```
[Out] -1/3*ArcTanh[(-1 + E^8 + (1 + 6*E^4 + E^8)*Tanh[3*x])/(4*Sqrt[2]*E^4)]/Sqrt
[2]
```

fricas [B] time = 0.47, size = 89, normalized size = 4.05

$$\frac{1}{12}\sqrt{2}\log\left(\frac{3\left(2\sqrt{2}+3\right)\cosh\left(3x+2\right)^2 - 4\left(3\sqrt{2}+4\right)\cosh\left(3x+2\right)\sinh\left(3x+2\right) + 3\left(2\sqrt{2}+3\right)\sinh\left(3x+\right.}{\cosh\left(3x+2\right)^2 + \sinh\left(3x+2\right)^2 - 3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(2+3*x)^2/(2-coth(2+3*x)^2),x, algorithm="fricas")
```

```
[Out] 1/12*sqrt(2)*log((3*(2*sqrt(2) + 3)*cosh(3*x + 2)^2 - 4*(3*sqrt(2) + 4)*cos
h(3*x + 2)*sinh(3*x + 2) + 3*(2*sqrt(2) + 3)*sinh(3*x + 2)^2 - 2*sqrt(2) -
3)/(cosh(3*x + 2)^2 + sinh(3*x + 2)^2 - 3))
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(2+3*x)^2/(2-coth(2+3*x)^2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.45, size = 44, normalized size = 2.00

$$-\frac{\sqrt{2}\ \operatorname{arctanh}\left(\frac{\left(-2+2\tanh\left(1+\frac{3x}{2}\right)\right)\sqrt{2}}{4}\right)}{6}-\frac{\sqrt{2}\ \operatorname{arctanh}\left(\frac{\left(2\tanh\left(1+\frac{3x}{2}\right)+2\right)\sqrt{2}}{4}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(2+3*x)^2/(2-coth(2+3*x)^2),x)
```

```
[Out] -1/6*2^(1/2)*arctanh(1/4*(-2+2*tanh(1+3/2*x))*2^(1/2))-1/6*2^(1/2)*arctanh(
1/4*(2*tanh(1+3/2*x)+2)*2^(1/2))
```

maxima [B] time = 0.41, size = 69, normalized size = 3.14

$$-\frac{1}{12}\sqrt{2}\log\left(-\frac{\sqrt{2}-e^{(-3x-2)}+1}{\sqrt{2}+e^{(-3x-2)}-1}\right)+\frac{1}{12}\sqrt{2}\log\left(-\frac{\sqrt{2}-e^{(-3x-2)}-1}{\sqrt{2}+e^{(-3x-2)}+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(2+3*x)^2/(2-coth(2+3*x)^2),x, algorithm="maxima")
```

```
[Out] -1/12*sqrt(2)*log(-(sqrt(2) - e^(-3*x - 2) + 1)/(sqrt(2) + e^(-3*x - 2) - 1
)) + 1/12*sqrt(2)*log(-(sqrt(2) - e^(-3*x - 2) - 1)/(sqrt(2) + e^(-3*x - 2)
 + 1))
```

mupad [B] time = 0.50, size = 57, normalized size = 2.59

$$\frac{\sqrt{2}\ \left(\ln\left(\sqrt{2}\ \left(3\,\mathrm{e}^{6\,x+4}-1\right)-4\,\mathrm{e}^{6\,x+4}\right)-\ln\left(-4\,\mathrm{e}^{6\,x+4}-\sqrt{2}\ \left(3\,\mathrm{e}^{6\,x+4}-1\right)\right)\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/(sinh(3*x + 2)^2*(coth(3*x + 2)^2 - 2)),x)
```

```
[Out] (2^(1/2)*(log(2^(1/2)*(3*exp(6*x + 4) - 1) - 4*exp(6*x + 4)) - log(- 4*exp(
6*x + 4) - 2^(1/2)*(3*exp(6*x + 4) - 1))))/12
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int\frac{\operatorname{csch}^{2}(3x+2)}{\coth^{2}(3x+2)-2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(2+3*x)**2/(2-coth(2+3*x)**2),x)
```

```
[Out] -Integral(csch(3*x + 2)**2/(coth(3*x + 2)**2 - 2), x)
```

3.6 $\int \frac{\operatorname{csch}^2(2+3x)}{1+2\coth^2(2+3x)}\,dx$

Optimal. Leaf size=22

$$\frac{\tan^{-1}\left(\frac{\tanh(3x+2)}{\sqrt{2}}\right)}{3\sqrt{2}}$$

```
[Out] 1/6*arctan(1/2*2^(1/2)*tanh(2+3*x))*2^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {3675, 203}

$$\frac{\tan^{-1}\left(\frac{\tanh(3x+2)}{\sqrt{2}}\right)}{3\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[2 + 3*x]^2/(1 + 2*Coth[2 + 3*x]^2),x]
```

```
[Out] ArcTan[Tanh[2 + 3*x]/Sqrt[2]]/(3*Sqrt[2])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3675

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/(c^(m - 1)*f), Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^p
, x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && In
tegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
|| EqQ[n^2, 16])
```

Rubi steps

$$\int \frac{\operatorname{csch}^2(2+3x)}{1+2\coth^2(2+3x)}\,dx = -\left(\frac{1}{3}\operatorname{Subst}\left(\int \frac{1}{1+2x^2}\,dx, x, \coth(2+3x)\right)\right)$$
$$= \frac{\tan^{-1}\left(\frac{\tanh(2+3x)}{\sqrt{2}}\right)}{3\sqrt{2}}$$

Mathematica [B] time = 0.11, size = 47, normalized size = 2.14

$$\frac{\tan^{-1}\left(\frac{(3-2e^4+3e^8)\tanh(3x)+3(e^8-1)}{4\sqrt{2}e^4}\right)}{3\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[2 + 3*x]^2/(1 + 2*Coth[2 + 3*x]^2),x]
```

```
[Out] ArcTan[(3*(-1 + E^8) + (3 - 2*E^4 + 3*E^8)*Tanh[3*x])/(4*Sqrt[2]*E^4)]/(3*S
qrt[2])
```

fricas [B] time = 0.42, size = 47, normalized size = 2.14

$$-\frac{1}{6}\sqrt{2}\arctan\left(-\frac{2\sqrt{2}\cosh(3x+2)+\sqrt{2}\sinh(3x+2)}{2(\cosh(3x+2)-\sinh(3x+2))}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(2+3*x)^2/(1+2*coth(2+3*x)^2),x, algorithm="fricas")
```

```
[Out] -1/6*sqrt(2)*arctan(-1/2*(2*sqrt(2)*cosh(3*x + 2) + sqrt(2)*sinh(3*x + 2))/
(cosh(3*x + 2) - sinh(3*x + 2)))
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(2+3*x)^2/(1+2*coth(2+3*x)^2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.54, size = 132, normalized size = 6.00

$$\frac{\sqrt{3}\ \arctan\left(\frac{2\tanh\left(1+\frac{3x}{2}\right)}{\sqrt{6}-\sqrt{2}}\right)}{3\sqrt{6}\ -3\sqrt{2}}-\frac{\arctan\left(\frac{2\tanh\left(1+\frac{3x}{2}\right)}{\sqrt{6}-\sqrt{2}}\right)}{3\left(\sqrt{6}\ -\sqrt{2}\right)}-\frac{\sqrt{3}\ \arctan\left(\frac{2\tanh\left(1+\frac{3x}{2}\right)}{\sqrt{6}+\sqrt{2}}\right)}{3\left(\sqrt{6}\ +\sqrt{2}\right)}-\frac{\arctan\left(\frac{2\tanh\left(1+\frac{3x}{2}\right)}{\sqrt{6}+\sqrt{2}}\right)}{3\left(\sqrt{6}\ +\sqrt{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(2+3*x)^2/(1+2*coth(2+3*x)^2),x)
```

```
[Out] 1/3*3^(1/2)/(6^(1/2)-2^(1/2))*arctan(2*tanh(1+3/2*x)/(6^(1/2)-2^(1/2)))-1/3
/(6^(1/2)-2^(1/2))*arctan(2*tanh(1+3/2*x)/(6^(1/2)-2^(1/2)))-1/3*3^(1/2)/(6
^(1/2)+2^(1/2))*arctan(2*tanh(1+3/2*x)/(6^(1/2)+2^(1/2)))-1/3/(6^(1/2)+2^(1
/2))*arctan(2*tanh(1+3/2*x)/(6^(1/2)+2^(1/2)))
```

maxima [A] time = 0.50, size = 21, normalized size = 0.95

$$-\frac{1}{6}\sqrt{2}\arctan\left(\frac{1}{4}\sqrt{2}\left(3e^{(-6x-4)}+1\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(2+3*x)^2/(1+2*coth(2+3*x)^2),x, algorithm="maxima")
```

```
[Out] -1/6*sqrt(2)*arctan(1/4*sqrt(2)*(3*e^(-6*x - 4) + 1))
```

mupad [B] time = 0.18, size = 21, normalized size = 0.95

$$\frac{\sqrt{2}\ \mathrm{atan}\left(\frac{\sqrt{2}\ \left(3\,\mathrm{e}^{6\,x+4}+1\right)}{4}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sinh(3*x + 2)^2*(2*coth(3*x + 2)^2 + 1)),x)
```

```
[Out] (2^(1/2)*atan((2^(1/2)*(3*exp(6*x + 4) + 1))/4))/6
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}^2(3x+2)}{2\coth^2(3x+2)+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(2+3*x)**2/(1+2*coth(2+3*x)**2),x)
```

```
[Out] Integral(csch(3*x + 2)**2/(2*coth(3*x + 2)**2 + 1), x)
```

3.7 $\int \frac{\operatorname{csch}^2(2+3x)}{1-2\coth^2(2+3x)}\,dx$

Optimal. Leaf size=22

$$-\frac{\tanh^{-1}\left(\frac{\tanh(3x+2)}{\sqrt{2}}\right)}{3\sqrt{2}}$$

```
[Out] -1/6*arctanh(1/2*2^(1/2)*tanh(2+3*x))*2^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {3675, 206}

$$-\frac{\tanh^{-1}\left(\frac{\tanh(3x+2)}{\sqrt{2}}\right)}{3\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[2 + 3*x]^2/(1 - 2*Coth[2 + 3*x]^2),x]
```

```
[Out] -ArcTanh[Tanh[2 + 3*x]/Sqrt[2]]/(3*Sqrt[2])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3675

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/(c^(m - 1)*f), Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^p
, x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && In
tegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
|| EqQ[n^2, 16])
```

Rubi steps

$$\int \frac{\operatorname{csch}^2(2+3x)}{1-2\coth^2(2+3x)}\,dx = -\left(\frac{1}{3}\operatorname{Subst}\left(\int \frac{1}{1-2x^2}\,dx, x, \coth(2+3x)\right)\right)$$
$$= -\frac{\tanh^{-1}\left(\frac{\tanh(2+3x)}{\sqrt{2}}\right)}{3\sqrt{2}}$$

Mathematica [A] time = 0.11, size = 42, normalized size = 1.91

$$\frac{\tanh^{-1}\left(\frac{\left(1-6e^4+e^8\right)\tanh(3x)+e^8-1}{4\sqrt{2}e^4}\right)}{3\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[2 + 3*x]^2/(1 - 2*Coth[2 + 3*x]^2),x]
```

```
[Out] ArcTanh[(-1 + E^8 + (1 - 6*E^4 + E^8)*Tanh[3*x])/(4*Sqrt[2]*E^4)]/(3*Sqrt[2
])
```

fricas [B] time = 0.45, size = 89, normalized size = 4.05

$$\frac{1}{12}\sqrt{2}\log\left(\frac{3\left(2\sqrt{2}+3\right)\cosh\left(3x+2\right)^2-4\left(3\sqrt{2}+4\right)\cosh\left(3x+2\right)\sinh\left(3x+2\right)+3\left(2\sqrt{2}+3\right)\sinh\left(3x+\right.}{\cosh\left(3x+2\right)^2+\sinh\left(3x+2\right)^2+3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(2+3*x)^2/(1-2*coth(2+3*x)^2),x, algorithm="fricas")
```

```
[Out] 1/12*sqrt(2)*log((3*(2*sqrt(2) + 3)*cosh(3*x + 2)^2 - 4*(3*sqrt(2) + 4)*cos
h(3*x + 2)*sinh(3*x + 2) + 3*(2*sqrt(2) + 3)*sinh(3*x + 2)^2 + 2*sqrt(2) +
3)/(cosh(3*x + 2)^2 + sinh(3*x + 2)^2 + 3))
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(2+3*x)^2/(1-2*coth(2+3*x)^2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.45, size = 102, normalized size = 4.64

$$-\frac{\sqrt{2}\,\ln\left(\frac{\tanh^{2}\left(1+\frac{3x}{2}\right)+\sqrt{2}\,\tanh\left(1+\frac{3x}{2}\right)+1}{\tanh^{2}\left(1+\frac{3x}{2}\right)-\sqrt{2}\,\tanh\left(1+\frac{3x}{2}\right)+1}\right)}{24}+\frac{\sqrt{2}\,\ln\left(\frac{\tanh^{2}\left(1+\frac{3x}{2}\right)-\sqrt{2}\,\tanh\left(1+\frac{3x}{2}\right)+1}{\tanh^{2}\left(1+\frac{3x}{2}\right)+\sqrt{2}\,\tanh\left(1+\frac{3x}{2}\right)+1}\right)}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(2+3*x)^2/(1-2*coth(2+3*x)^2),x)
```

```
[Out] -1/24*2^(1/2)*ln((tanh(1+3/2*x)^2+2^(1/2)*tanh(1+3/2*x)+1)/(tanh(1+3/2*x)^2
-2^(1/2)*tanh(1+3/2*x)+1))+1/24*2^(1/2)*ln((tanh(1+3/2*x)^2-2^(1/2)*tanh(1+
3/2*x)+1)/(tanh(1+3/2*x)^2+2^(1/2)*tanh(1+3/2*x)+1))
```

maxima [B] time = 0.52, size = 38, normalized size = 1.73

$$\frac{1}{12}\,\sqrt{2}\,\log\left(-\frac{2\,\sqrt{2}\,-e^{(-6\,x-4)}-3}{2\,\sqrt{2}\,+e^{(-6\,x-4)}+3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(2+3*x)^2/(1-2*coth(2+3*x)^2),x, algorithm="maxima")
```

```
[Out] 1/12*sqrt(2)*log(-(2*sqrt(2) - e^(-6*x - 4) - 3)/(2*sqrt(2) + e^(-6*x - 4)
+ 3))
```

mupad [B] time = 1.88, size = 57, normalized size = 2.59

$$-\frac{\sqrt{2}\,\left(\ln\left(4\,\mathrm{e}^{6\,x+4}+\sqrt{2}\,\left(3\,\mathrm{e}^{6\,x+4}+1\right)\right)-\ln\left(4\,\mathrm{e}^{6\,x+4}-\sqrt{2}\,\left(3\,\mathrm{e}^{6\,x+4}+1\right)\right)\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/(sinh(3*x + 2)^2*(2*coth(3*x + 2)^2 - 1)),x)
```

```
[Out] -(2^(1/2)*(log(4*exp(6*x + 4) + 2^(1/2)*(3*exp(6*x + 4) + 1)) - log(4*exp(6
*x + 4) - 2^(1/2)*(3*exp(6*x + 4) + 1))))/12
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int\frac{\operatorname{csch}^{2}(3x+2)}{2\coth^{2}(3x+2)-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(2+3*x)**2/(1-2*coth(2+3*x)**2),x)
```

```
[Out] -Integral(csch(3*x + 2)**2/(2*coth(3*x + 2)**2 - 1), x)
```

3.8 $\int \cosh(a+bx)\sinh(a+bx)\,dx$

Optimal. Leaf size=15

$$\frac{\sinh^2(a+bx)}{2b}$$

```
[Out] 1/2*sinh(b*x+a)^2/b
```

Rubi [A] time = 0.01, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2564, 30}

$$\frac{\sinh^2(a+bx)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[a + b*x]*Sinh[a + b*x],x]
```

```
[Out] Sinh[a + b*x]^2/(2*b)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] && !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rubi steps

$$\begin{aligned}\int \cosh(a+bx)\sinh(a+bx)\,dx &= -\frac{\text{Subst}\left(\int x\,dx, x, i\sinh(a+bx)\right)}{b}\\ &= \frac{\sinh^2(a+bx)}{2b}\end{aligned}$$

Mathematica [B] time = 0.01, size = 37, normalized size = 2.47

$$\frac{1}{2}\left(\frac{\sinh(2a)\sinh(2bx)}{2b}+\frac{\cosh(2a)\cosh(2bx)}{2b}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[a + b*x]*Sinh[a + b*x],x]
```

```
[Out] ((Cosh[2*a]*Cosh[2*b*x])/(2*b) + (Sinh[2*a]*Sinh[2*b*x])/(2*b))/2
```

fricas [A] time = 0.52, size = 22, normalized size = 1.47

$$\frac{\cosh\left(bx+a\right)^{2}+\sinh\left(bx+a\right)^{2}}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*sinh(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/4*(cosh(b*x + a)^2 + sinh(b*x + a)^2)/b
```

giac [B] time = 0.13, size = 29, normalized size = 1.93

$$\frac{e^{(2\,bx+2\,a)}}{8\,b}+\frac{e^{(-2\,bx-2\,a)}}{8\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*sinh(b*x+a),x, algorithm="giac")
```

```
[Out] 1/8*e^(2*b*x + 2*a)/b + 1/8*e^(-2*b*x - 2*a)/b
```

maple [A] time = 0.02, size = 14, normalized size = 0.93

$$\frac{\cosh^{2}\left(bx+a\right)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)*sinh(b*x+a),x)
```

```
[Out] 1/2*cosh(b*x+a)^2/b
```

maxima [A] time = 0.58, size = 13, normalized size = 0.87

$$\frac{\cosh\left(bx+a\right)^{2}}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*sinh(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/2*cosh(b*x + a)^2/b
```

mupad [**B**] time = 0.07, size = 13, normalized size = 0.87

$$\frac{\cosh{(a+b\,x)}^2}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a + b*x)*sinh(a + b*x),x)
```

```
[Out] cosh(a + b*x)^2/(2*b)
```

sympy [**A**] time = 0.18, size = 19, normalized size = 1.27

$$\begin{cases} \frac{\sinh^2{(a+bx)}}{2b} & \text{for } b \neq 0 \\ x\sinh{(a)}\cosh{(a)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*sinh(b*x+a),x)
```

```
[Out] Piecewise((sinh(a + b*x)**2/(2*b), Ne(b, 0)), (x*sinh(a)*cosh(a), True))
```

3.9 $\int \cosh(a+bx)\sinh^n(a+bx)\,dx$

Optimal. Leaf size=19

$$\frac{\sinh^{n+1}(a+bx)}{b(n+1)}$$

```
[Out] sinh(b*x+a)^(1+n)/b/(1+n)
```

Rubi [A] time = 0.02, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2564, 30}

$$\frac{\sinh^{n+1}(a+bx)}{b(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[a + b*x]*Sinh[a + b*x]^n,x]
```

```
[Out] Sinh[a + b*x]^(1 + n)/(b*(1 + n))
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] && !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rubi steps

$$\begin{aligned}\int \cosh(a+bx)\sinh^n(a+bx)\,dx &= \frac{\text{Subst}\left(\int x^n\,dx, x, \sinh(a+bx)\right)}{b}\\ &= \frac{\sinh^{1+n}(a+bx)}{b(1+n)}\end{aligned}$$

Mathematica [A] time = 0.01, size = 19, normalized size = 1.00

$$\frac{\sinh^{n+1}(a+bx)}{b(n+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[a + b*x]*Sinh[a + b*x]^n,x]
```

```
[Out] Sinh[a + b*x]^(1 + n)/(b*(1 + n))
```

fricas [B] time = 0.50, size = 68, normalized size = 3.58

$$\frac{\cosh\left(n\log\left(\sinh\left(bx+a\right)\right)\right)\sinh\left(bx+a\right)+\sinh\left(bx+a\right)\sinh\left(n\log\left(\sinh\left(bx+a\right)\right)\right)}{\left(bn+b\right)\cosh\left(bx+a\right)^{2}-\left(bn+b\right)\sinh\left(bx+a\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*sinh(b*x+a)^n,x, algorithm="fricas")
```

```
[Out] (cosh(n*log(sinh(b*x + a)))*sinh(b*x + a) + sinh(b*x + a)*sinh(n*log(sinh(b
*x + a))))/((b*n + b)*cosh(b*x + a)^2 - (b*n + b)*sinh(b*x + a)^2)
```

giac [B] time = 0.29, size = 96, normalized size = 5.05

$$\frac{e^{\left(3\,bx+n\log\left(\frac{1}{2}\,\left(e^{\left(2\,bx+2\,a\right)}-1\right)e^{\left(-bx-a\right)}\right)+3\,a\right)}-e^{\left(bx+n\log\left(\frac{1}{2}\,\left(e^{\left(2\,bx+2\,a\right)}-1\right)e^{\left(-bx-a\right)}\right)+a\right)}}{2\,\left(bne^{\left(2\,bx+2\,a\right)}+be^{\left(2\,bx+2\,a\right)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*sinh(b*x+a)^n,x, algorithm="giac")
```

```
[Out] 1/2*(e^(3*b*x + n*log(1/2*(e^(2*b*x + 2*a) - 1)*e^(-b*x - a)) + 3*a) - e^(b
*x + n*log(1/2*(e^(2*b*x + 2*a) - 1)*e^(-b*x - a)) + a))/(b*n*e^(2*b*x + 2*
a) + b*e^(2*b*x + 2*a))
```

maple [A] time = 0.12, size = 20, normalized size = 1.05

$$\frac{\sinh^{n+1}\left(bx+a\right)}{b\left(n+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)*sinh(b*x+a)^n,x)
```

```
[Out] sinh(b*x+a)^(n+1)/b/(n+1)
```

maxima [A] time = 0.38, size = 19, normalized size = 1.00

$$\frac{\sinh\left(bx+a\right)^{n+1}}{b(n+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*sinh(b*x+a)^n,x, algorithm="maxima")
```

```
[Out] sinh(b*x + a)^(n + 1)/(b*(n + 1))
```

mupad [B] time = 1.55, size = 19, normalized size = 1.00

$$\frac{\sinh\left(a+b\,x\right)^{n+1}}{b\,\left(n+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a + b*x)*sinh(a + b*x)^n,x)
```

```
[Out] sinh(a + b*x)^(n + 1)/(b*(n + 1))
```

sympy [A] time = 1.25, size = 49, normalized size = 2.58

$$\begin{cases} \frac{x\cosh\left(a\right)}{\sinh\left(a\right)} & \text{for}\: b=0 \wedge n=-1 \\ x\sinh^{n}\left(a\right)\cosh\left(a\right) & \text{for}\: b=0 \\ \frac{\log\left(\sinh\left(a+bx\right)\right)}{b} & \text{for}\: n=-1 \\ \frac{\sinh\left(a+bx\right)\sinh^{n}\left(a+bx\right)}{bn+b} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*sinh(b*x+a)**n,x)
```

```
[Out] Piecewise((x*cosh(a)/sinh(a), Eq(b, 0) & Eq(n, -1)), (x*sinh(a)**n*cosh(a),
 Eq(b, 0)), (log(sinh(a + b*x))/b, Eq(n, -1)), (sinh(a + b*x)*sinh(a + b*x)
**n/(b*n + b), True))
```

3.10 $\int \cosh^3(a+bx)\sinh^n(a+bx)\,dx$

Optimal. Leaf size=39

$$\frac{\sinh^{n+1}(a+bx)}{b(n+1)}+\frac{\sinh^{n+3}(a+bx)}{b(n+3)}$$

```
[Out] sinh(b*x+a)^(1+n)/b/(1+n)+sinh(b*x+a)^(3+n)/b/(3+n)
```

Rubi [A] time = 0.04, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2564, 14}

$$\frac{\sinh^{n+1}(a+bx)}{b(n+1)}+\frac{\sinh^{n+3}(a+bx)}{b(n+3)}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[a + b*x]^3*Sinh[a + b*x]^n,x]
```

```
[Out] Sinh[a + b*x]^(1 + n)/(b*(1 + n)) + Sinh[a + b*x]^(3 + n)/(b*(3 + n))
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rubi steps

$$\begin{aligned}\int \cosh^3(a+bx)\sinh^n(a+bx)\,dx &= \frac{\text{Subst}\left(\int x^n\left(1+x^2\right)\,dx,x,\sinh(a+bx)\right)}{b}\\ &= \frac{\text{Subst}\left(\int \left(x^n+x^{2+n}\right)\,dx,x,\sinh(a+bx)\right)}{b}\\ &= \frac{\sinh^{1+n}(a+bx)}{b(1+n)}+\frac{\sinh^{3+n}(a+bx)}{b(3+n)}\end{aligned}$$

Mathematica [A] time = 0.06, size = 39, normalized size = 1.00

$$\frac{\sinh^{n+1}(a+bx)}{b(n+1)}+\frac{\sinh^{n+3}(a+bx)}{b(n+3)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[a + b*x]^3*Sinh[a + b*x]^n,x]
```

```
[Out] Sinh[a + b*x]^(1 + n)/(b*(1 + n)) + Sinh[a + b*x]^(3 + n)/(b*(3 + n))
```

fricas [B] time = 0.54, size = 175, normalized size = 4.49

$$\frac{\left((n+1)\sinh(bx+a)^3+\left(3(n+1)\cosh(bx+a)^2+n+9\right)\sinh(bx+a)\right)\cosh\left(n\log(\sinh(bx+a))\right)+\left((n+1)\right.}{4\left(\left(bn^2+4bn+3b\right)\cosh(bx+a)^4-2\left(bn^2+4bn+3b\right)\cosh(bx+a\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^3*sinh(b*x+a)^n,x, algorithm="fricas")
```

```
[Out] 1/4*(((n + 1)*sinh(b*x + a)^3 + (3*(n + 1)*cosh(b*x + a)^2 + n + 9)*sinh(b*
x + a))*cosh(n*log(sinh(b*x + a))) + ((n + 1)*sinh(b*x + a)^3 + (3*(n + 1)*
cosh(b*x + a)^2 + n + 9)*sinh(b*x + a))*sinh(n*log(sinh(b*x + a))))/((b*n^2
 + 4*b*n + 3*b)*cosh(b*x + a)^4 - 2*(b*n^2 + 4*b*n + 3*b)*cosh(b*x + a)^2*s
inh(b*x + a)^2 + (b*n^2 + 4*b*n + 3*b)*sinh(b*x + a)^4)
```

giac [B] time = 0.32, size = 327, normalized size = 8.38

$$\frac{ne^{\left(7\,bx+n\log\left(\frac{1}{2}\left(e^{(2\,bx+2\,a)}-1\right)e^{(-bx-a)}\right)+7\,a\right)}+ne^{\left(5\,bx+n\log\left(\frac{1}{2}\left(e^{(2\,bx+2\,a)}-1\right)e^{(-bx-a)}\right)+5\,a\right)}-ne^{\left(3\,bx+n\log\left(\frac{1}{2}\left(e^{(2\,bx+2\,a)}-1\right)e^{(-bx-a)}\right)+3\,a\right)}-n}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^3*sinh(b*x+a)^n,x, algorithm="giac")
```

```
[Out] 1/8*(n*e^(7*b*x + n*log(1/2*(e^(2*b*x + 2*a) - 1)*e^(-b*x - a)) + 7*a) + n*
e^(5*b*x + n*log(1/2*(e^(2*b*x + 2*a) - 1)*e^(-b*x - a)) + 5*a) - n*e^(3*b*
x + n*log(1/2*(e^(2*b*x + 2*a) - 1)*e^(-b*x - a)) + 3*a) - n*e^(b*x + n*log
(1/2*(e^(2*b*x + 2*a) - 1)*e^(-b*x - a)) + a) + e^(7*b*x + n*log(1/2*(e^(2*
b*x + 2*a) - 1)*e^(-b*x - a)) + 7*a) + 9*e^(5*b*x + n*log(1/2*(e^(2*b*x + 2
*a) - 1)*e^(-b*x - a)) + 5*a) - 9*e^(3*b*x + n*log(1/2*(e^(2*b*x + 2*a) - 1
)*e^(-b*x - a)) + 3*a) - e^(b*x + n*log(1/2*(e^(2*b*x + 2*a) - 1)*e^(-b*x -
 a)) + a))/(b*n^2*e^(4*b*x + 4*a) + 4*b*n*e^(4*b*x + 4*a) + 3*b*e^(4*b*x +
4*a))
```

maple [F] time = 1.28, size = 0, normalized size = 0.00

$$\int \left(\cosh^3(bx+a)\right)\left(\sinh^n(bx+a)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)^3*sinh(b*x+a)^n,x)
```

```
[Out] int(cosh(b*x+a)^3*sinh(b*x+a)^n,x)
```

maxima [B] time = 0.82, size = 373, normalized size = 9.56

$$\frac{ne^{\left((bx+a)n+3\,bx+n\log\left(e^{(-bx-a)}+1\right)+n\log\left(-e^{(-bx-a)}+1\right)+3\,a\right)}}{8\left(2^n n^2+2^{n+2}n+3\cdot 2^n\right)b}+\frac{(n+9)e^{\left((bx+a)n+bx+n\log\left(e^{(-bx-a)}+1\right)+n\log\left(-e^{(-bx-a)}+1\right)+a\right)}}{8\left(2^n n^2+2^{n+2}n+3\cdot 2^n\right)b}-\frac{(n+9)e^{(}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^3*sinh(b*x+a)^n,x, algorithm="maxima")
```

```
[Out] 1/8*n*e^((b*x + a)*n + 3*b*x + n*log(e^(-b*x - a) + 1) + n*log(-e^(-b*x - a
) + 1) + 3*a)/((2^n*n^2 + 2^(n + 2)*n + 3*2^n)*b) + 1/8*(n + 9)*e^((b*x + a
)*n + b*x + n*log(e^(-b*x - a) + 1) + n*log(-e^(-b*x - a) + 1) + a)/((2^n*n
^2 + 2^(n + 2)*n + 3*2^n)*b) - 1/8*(n + 9)*e^((b*x + a)*n - b*x + n*log(e^(
-b*x - a) + 1) + n*log(-e^(-b*x - a) + 1) - a)/((2^n*n^2 + 2^(n + 2)*n + 3*
2^n)*b) - 1/8*(n + 1)*e^((b*x + a)*n - 3*b*x + n*log(e^(-b*x - a) + 1) + n*
log(-e^(-b*x - a) + 1) - 3*a)/((2^n*n^2 + 2^(n + 2)*n + 3*2^n)*b) + 1/8*e^(
(b*x + a)*n + 3*b*x + n*log(e^(-b*x - a) + 1) + n*log(-e^(-b*x - a) + 1) +
3*a)/((2^n*n^2 + 2^(n + 2)*n + 3*2^n)*b)
```

mupad [B] time = 1.64, size = 135, normalized size = 3.46

$$-\left(\frac{1}{2}\right)^n e^{-3\,a-3\,b\,x}\left(e^{a+b\,x}-e^{-a-b\,x}\right)^n\left(\frac{\frac{n}{8}+\frac{1}{8}}{b\left(n^2+4\,n+3\right)}+\frac{e^{2\,a+2\,b\,x}\,(n+9)}{8\,b\left(n^2+4\,n+3\right)}-\frac{e^{6\,a+6\,b\,x}\,(n+1)}{8\,b\left(n^2+4\,n+3\right)}-\frac{e^{4\,a+4\,b\,x}\,(n+}{8\,b\left(n^2+4\,n\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a + b*x)^3*sinh(a + b*x)^n,x)
```

```
[Out] -(1/2)^n*exp(- 3*a - 3*b*x)*(exp(a + b*x) - exp(- a - b*x))^n*((n/8 + 1/8)/
(b*(4*n + n^2 + 3)) + (exp(2*a + 2*b*x)*(n + 9))/(8*b*(4*n + n^2 + 3)) - (e
xp(6*a + 6*b*x)*(n + 1))/(8*b*(4*n + n^2 + 3)) - (exp(4*a + 4*b*x)*(n + 9))
/(8*b*(4*n + n^2 + 3)))
```

<u>**sympy**</u> **[A]** time = 8.42, size = 638, normalized size = 16.36

$$\begin{cases} x\sinh^{n}(a)\cosh^{3}(a) \\ \frac{\log(\sinh(a+bx))}{b} - \frac{\cosh^{2}(a+bx)}{2b\sinh^{2}(a+bx)} \\ \frac{bx\tanh^{4}\left(\frac{a}{2}+\frac{bx}{2}\right)}{b\tanh^{4}\left(\frac{a}{2}+\frac{bx}{2}\right)-2b\tanh^{2}\left(\frac{a}{2}+\frac{bx}{2}\right)+b} - \frac{2bx\tanh^{2}\left(\frac{a}{2}+\frac{bx}{2}\right)}{b\tanh^{4}\left(\frac{a}{2}+\frac{bx}{2}\right)-2b\tanh^{2}\left(\frac{a}{2}+\frac{bx}{2}\right)+b} + \frac{bx}{b\tanh^{4}\left(\frac{a}{2}+\frac{bx}{2}\right)-2b\tanh^{2}\left(\frac{a}{2}+\frac{bx}{2}\right)+b} - \frac{2\log\left(\tanh\left(\frac{a}{2}+\frac{bx}{2}\right)+1\right)\mathrm{t}}{b\tanh^{4}\left(\frac{a}{2}+\frac{bx}{2}\right)-2b\,\mathrm{tanl}} \\ \frac{n\sinh(a+bx)\sinh^{n}(a+bx)\cosh^{2}(a+bx)}{bn^{2}+4bn+3b} - \frac{2\sinh^{3}(a+bx)\sinh^{n}(a+bx)}{bn^{2}+4bn+3b} + \frac{3\sinh(a+bx)\sinh^{n}(a+bx)\cosh^{2}(a+bx)}{bn^{2}+4bn+3b} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)**3*sinh(b*x+a)**n,x)
```

```
[Out] Piecewise((x*sinh(a)**n*cosh(a)**3, Eq(b, 0)), (log(sinh(a + b*x))/b - cosh
(a + b*x)**2/(2*b*sinh(a + b*x)**2), Eq(n, -3)), (b*x*tanh(a/2 + b*x/2)**4/
(b*tanh(a/2 + b*x/2)**4 - 2*b*tanh(a/2 + b*x/2)**2 + b) - 2*b*x*tanh(a/2 +
b*x/2)**2/(b*tanh(a/2 + b*x/2)**4 - 2*b*tanh(a/2 + b*x/2)**2 + b) + b*x/(b*
tanh(a/2 + b*x/2)**4 - 2*b*tanh(a/2 + b*x/2)**2 + b) - 2*log(tanh(a/2 + b*x
/2) + 1)*tanh(a/2 + b*x/2)**4/(b*tanh(a/2 + b*x/2)**4 - 2*b*tanh(a/2 + b*x/
2)**2 + b) + 4*log(tanh(a/2 + b*x/2) + 1)*tanh(a/2 + b*x/2)**2/(b*tanh(a/2
+ b*x/2)**4 - 2*b*tanh(a/2 + b*x/2)**2 + b) - 2*log(tanh(a/2 + b*x/2) + 1)/
(b*tanh(a/2 + b*x/2)**4 - 2*b*tanh(a/2 + b*x/2)**2 + b) + log(tanh(a/2 + b*
x/2))*tanh(a/2 + b*x/2)**4/(b*tanh(a/2 + b*x/2)**4 - 2*b*tanh(a/2 + b*x/2)*
*2 + b) - 2*log(tanh(a/2 + b*x/2))*tanh(a/2 + b*x/2)**2/(b*tanh(a/2 + b*x/2
)**4 - 2*b*tanh(a/2 + b*x/2)**2 + b) + log(tanh(a/2 + b*x/2))/(b*tanh(a/2 +
 b*x/2)**4 - 2*b*tanh(a/2 + b*x/2)**2 + b) + 2*tanh(a/2 + b*x/2)**2/(b*tanh
(a/2 + b*x/2)**4 - 2*b*tanh(a/2 + b*x/2)**2 + b), Eq(n, -1)), (n*sinh(a + b
*x)*sinh(a + b*x)**n*cosh(a + b*x)**2/(b*n**2 + 4*b*n + 3*b) - 2*sinh(a + b
*x)**3*sinh(a + b*x)**n/(b*n**2 + 4*b*n + 3*b) + 3*sinh(a + b*x)*sinh(a + b
*x)**n*cosh(a + b*x)**2/(b*n**2 + 4*b*n + 3*b), True))
```

3.11 $\int \cosh^5(a+bx)\sinh^n(a+bx)\,dx$

Optimal. Leaf size=59

$$\frac{\sinh^{n+1}(a+bx)}{b(n+1)}+\frac{2\sinh^{n+3}(a+bx)}{b(n+3)}+\frac{\sinh^{n+5}(a+bx)}{b(n+5)}$$

```
[Out] sinh(b*x+a)^(1+n)/b/(1+n)+2*sinh(b*x+a)^(3+n)/b/(3+n)+sinh(b*x+a)^(5+n)/b/(
5+n)
```

Rubi **[A]** time = 0.05, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2564, 270}

$$\frac{\sinh^{n+1}(a+bx)}{b(n+1)}+\frac{2\sinh^{n+3}(a+bx)}{b(n+3)}+\frac{\sinh^{n+5}(a+bx)}{b(n+5)}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[a + b*x]^5*Sinh[a + b*x]^n,x]
```

```
[Out] Sinh[a + b*x]^(1 + n)/(b*(1 + n)) + (2*Sinh[a + b*x]^(3 + n))/(b*(3 + n)) +
 Sinh[a + b*x]^(5 + n)/(b*(5 + n))
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rubi steps

$$\int \cosh^5(a+bx)\sinh^n(a+bx)\,dx = \frac{\text{Subst}\left(\int x^n\left(1+x^2\right)^2\,dx, x, \sinh(a+bx)\right)}{b}$$
$$= \frac{\text{Subst}\left(\int \left(x^n+2x^{2+n}+x^{4+n}\right)\,dx, x, \sinh(a+bx)\right)}{b}$$
$$= \frac{\sinh^{1+n}(a+bx)}{b(1+n)} + \frac{2\sinh^{3+n}(a+bx)}{b(3+n)} + \frac{\sinh^{5+n}(a+bx)}{b(5+n)}$$

Mathematica [**A**] time = 0.20, size = 49, normalized size = 0.83

$$\frac{\sinh^{n+1}(a+bx)\left(\frac{\sinh^4(a+bx)}{n+5}+\frac{2\sinh^2(a+bx)}{n+3}+\frac{1}{n+1}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[a + b*x]^5*Sinh[a + b*x]^n,x]
```

```
[Out] (Sinh[a + b*x]^(1 + n)*((1 + n)^(-1) + (2*Sinh[a + b*x]^2)/(3 + n) + Sinh[a
 + b*x]^4/(5 + n)))/b
```

fricas [**B**] time = 0.48, size = 379, normalized size = 6.42

$$\frac{\left(\left(n^2+4\,n+3\right)\sinh\left(bx+a\right)^5+\left(10\left(n^2+4\,n+3\right)\cosh\left(bx+a\right)^2+3\,n^2+28\,n+25\right)\sinh\left(bx+a\right)^3+\left(5\left(n^2+4\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^5*sinh(b*x+a)^n,x, algorithm="fricas")
```

```
[Out] 1/16*(((n^2 + 4*n + 3)*sinh(b*x + a)^5 + (10*(n^2 + 4*n + 3)*cosh(b*x + a)^
2 + 3*n^2 + 28*n + 25)*sinh(b*x + a)^3 + (5*(n^2 + 4*n + 3)*cosh(b*x + a)^4
 + 3*(3*n^2 + 28*n + 25)*cosh(b*x + a)^2 + 2*n^2 + 24*n + 150)*sinh(b*x + a
))*cosh(n*log(sinh(b*x + a))) + ((n^2 + 4*n + 3)*sinh(b*x + a)^5 + (10*(n^2
 + 4*n + 3)*cosh(b*x + a)^2 + 3*n^2 + 28*n + 25)*sinh(b*x + a)^3 + (5*(n^2
+ 4*n + 3)*cosh(b*x + a)^4 + 3*(3*n^2 + 28*n + 25)*cosh(b*x + a)^2 + 2*n^2
+ 24*n + 150)*sinh(b*x + a))*sinh(n*log(sinh(b*x + a))))/((b*n^3 + 9*b*n^2
+ 23*b*n + 15*b)*cosh(b*x + a)^6 - 3*(b*n^3 + 9*b*n^2 + 23*b*n + 15*b)*cosh
(b*x + a)^4*sinh(b*x + a)^2 + 3*(b*n^3 + 9*b*n^2 + 23*b*n + 15*b)*cosh(b*x
+ a)^2*sinh(b*x + a)^4 - (b*n^3 + 9*b*n^2 + 23*b*n + 15*b)*sinh(b*x + a)^6)
```

giac [**B**] time = 0.30, size = 722, normalized size = 12.24

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^5*sinh(b*x+a)^n,x, algorithm="giac")
```

```
[Out] 1/32*(n^2*e^(11*b*x + n*log(1/2*(e^(2*b*x + 2*a) - 1)*e^(-b*x - a)) + 11*a)
 + 3*n^2*e^(9*b*x + n*log(1/2*(e^(2*b*x + 2*a) - 1)*e^(-b*x - a)) + 9*a) +
2*n^2*e^(7*b*x + n*log(1/2*(e^(2*b*x + 2*a) - 1)*e^(-b*x - a)) + 7*a) - 2*n
^2*e^(5*b*x + n*log(1/2*(e^(2*b*x + 2*a) - 1)*e^(-b*x - a)) + 5*a) - 3*n^2*
e^(3*b*x + n*log(1/2*(e^(2*b*x + 2*a) - 1)*e^(-b*x - a)) + 3*a) - n^2*e^(b*
x + n*log(1/2*(e^(2*b*x + 2*a) - 1)*e^(-b*x - a)) + a) + 4*n*e^(11*b*x + n*
log(1/2*(e^(2*b*x + 2*a) - 1)*e^(-b*x - a)) + 11*a) + 28*n*e^(9*b*x + n*log
(1/2*(e^(2*b*x + 2*a) - 1)*e^(-b*x - a)) + 9*a) + 24*n*e^(7*b*x + n*log(1/2
*(e^(2*b*x + 2*a) - 1)*e^(-b*x - a)) + 7*a) - 24*n*e^(5*b*x + n*log(1/2*(e^
(2*b*x + 2*a) - 1)*e^(-b*x - a)) + 5*a) - 28*n*e^(3*b*x + n*log(1/2*(e^(2*b
*x + 2*a) - 1)*e^(-b*x - a)) + 3*a) - 4*n*e^(b*x + n*log(1/2*(e^(2*b*x + 2*
a) - 1)*e^(-b*x - a)) + a) + 3*e^(11*b*x + n*log(1/2*(e^(2*b*x + 2*a) - 1)*
e^(-b*x - a)) + 11*a) + 25*e^(9*b*x + n*log(1/2*(e^(2*b*x + 2*a) - 1)*e^(-b
*x - a)) + 9*a) + 150*e^(7*b*x + n*log(1/2*(e^(2*b*x + 2*a) - 1)*e^(-b*x -
a)) + 7*a) - 150*e^(5*b*x + n*log(1/2*(e^(2*b*x + 2*a) - 1)*e^(-b*x - a)) +
 5*a) - 25*e^(3*b*x + n*log(1/2*(e^(2*b*x + 2*a) - 1)*e^(-b*x - a)) + 3*a)
- 3*e^(b*x + n*log(1/2*(e^(2*b*x + 2*a) - 1)*e^(-b*x - a)) + a))/(b*n^3*e^(
6*b*x + 6*a) + 9*b*n^2*e^(6*b*x + 6*a) + 23*b*n*e^(6*b*x + 6*a) + 15*b*e^(6
*b*x + 6*a))
```

maple [F] time = 1.19, size = 0, normalized size = 0.00

$$\int \left(\cosh^5(bx+a)\right)\left(\sinh^n(bx+a)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)^5*sinh(b*x+a)^n,x)
```

```
[Out] int(cosh(b*x+a)^5*sinh(b*x+a)^n,x)
```

maxima [B] time = 0.66, size = 686, normalized size = 11.63

$$\frac{n^2 e^{\left((bx+a)n+5\,bx+n\log\left(e^{(-bx-a)}+1\right)+n\log\left(-e^{(-bx-a)}+1\right)+5\,a\right)}}{32\left(2^n n^3+9\cdot 2^n n^2+23\cdot 2^n n+15\cdot 2^n\right)b}+\frac{n e^{\left((bx+a)n+5\,bx+n\log\left(e^{(-bx-a)}+1\right)+n\log\left(-e^{(-bx-a)}+1\right)+5\,a\right)}}{8\left(2^n n^3+9\cdot 2^n n^2+23\cdot 2^n n+15\cdot 2^n\right)b}+\frac{\left(3\,n^2+28\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^5*sinh(b*x+a)^n,x, algorithm="maxima")
```

```
[Out] 1/32*n^2*e^((b*x + a)*n + 5*b*x + n*log(e^(-b*x - a) + 1) + n*log(-e^(-b*x
- a) + 1) + 5*a)/((2^n*n^3 + 9*2^n*n^2 + 23*2^n*n + 15*2^n)*b) + 1/8*n*e^((
b*x + a)*n + 5*b*x + n*log(e^(-b*x - a) + 1) + n*log(-e^(-b*x - a) + 1) + 5
```

```
*a)/((2^n*n^3 + 9*2^n*n^2 + 23*2^n*n + 15*2^n)*b) + 1/32*(3*n^2 + 28*n + 25
)*e^((b*x + a)*n + 3*b*x + n*log(e^(-b*x - a) + 1) + n*log(-e^(-b*x - a) +
1) + 3*a)/((2^n*n^3 + 9*2^n*n^2 + 23*2^n*n + 15*2^n)*b) + 1/16*(n^2 + 12*n
+ 75)*e^((b*x + a)*n + b*x + n*log(e^(-b*x - a) + 1) + n*log(-e^(-b*x - a)
+ 1) + a)/((2^n*n^3 + 9*2^n*n^2 + 23*2^n*n + 15*2^n)*b) - 1/16*(n^2 + 12*n
+ 75)*e^((b*x + a)*n - b*x + n*log(e^(-b*x - a) + 1) + n*log(-e^(-b*x - a)
+ 1) - a)/((2^n*n^3 + 9*2^n*n^2 + 23*2^n*n + 15*2^n)*b) - 1/32*(3*n^2 + 28*
n + 25)*e^((b*x + a)*n - 3*b*x + n*log(e^(-b*x - a) + 1) + n*log(-e^(-b*x -
 a) + 1) - 3*a)/((2^n*n^3 + 9*2^n*n^2 + 23*2^n*n + 15*2^n)*b) - 1/32*(n^2 +
 4*n + 3)*e^((b*x + a)*n - 5*b*x + n*log(e^(-b*x - a) + 1) + n*log(-e^(-b*x
 - a) + 1) - 5*a)/((2^n*n^3 + 9*2^n*n^2 + 23*2^n*n + 15*2^n)*b) + 3/32*e^((
b*x + a)*n + 5*b*x + n*log(e^(-b*x - a) + 1) + n*log(-e^(-b*x - a) + 1) + 5
*a)/((2^n*n^3 + 9*2^n*n^2 + 23*2^n*n + 15*2^n)*b)
```

mupad [B] time = 1.75, size = 255, normalized size = 4.32

$$-\mathrm{e}^{-5\,a-5\,b\,x}\left(\frac{\mathrm{e}^{a+b\,x}}{2}-\frac{\mathrm{e}^{-a-b\,x}}{2}\right)^n\left(\frac{n^2+4\,n+3}{32\,b\,\left(n^3+9\,n^2+23\,n+15\right)}-\frac{\mathrm{e}^{10\,a+10\,b\,x}\,\left(n^2+4\,n+3\right)}{32\,b\,\left(n^3+9\,n^2+23\,n+15\right)}+\frac{\mathrm{e}^{2\,a+2\,b\,x}\,\left(3\,n^2+2\right.}{32\,b\,\left(n^3+9\,n^2+\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a + b*x)^5*sinh(a + b*x)^n,x)
```

```
[Out] -exp(- 5*a - 5*b*x)*(exp(a + b*x)/2 - exp(- a - b*x)/2)^n*((4*n + n^2 + 3)/
(32*b*(23*n + 9*n^2 + n^3 + 15)) - (exp(10*a + 10*b*x)*(4*n + n^2 + 3))/(32
*b*(23*n + 9*n^2 + n^3 + 15)) + (exp(2*a + 2*b*x)*(28*n + 3*n^2 + 25))/(32*
b*(23*n + 9*n^2 + n^3 + 15)) - (exp(8*a + 8*b*x)*(28*n + 3*n^2 + 25))/(32*b
*(23*n + 9*n^2 + n^3 + 15)) + (exp(4*a + 4*b*x)*(24*n + 2*n^2 + 150))/(32*b
*(23*n + 9*n^2 + n^3 + 15)) - (exp(6*a + 6*b*x)*(24*n + 2*n^2 + 150))/(32*b
*(23*n + 9*n^2 + n^3 + 15)))
```

sympy [A] time = 42.56, size = 2574, normalized size = 43.63

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)**5*sinh(b*x+a)**n,x)
```

```
[Out] Piecewise((x*sinh(a)**n*cosh(a)**5, Eq(b, 0)), (log(sinh(a + b*x))/b - cosh
(a + b*x)**2/(2*b*sinh(a + b*x)**2) - cosh(a + b*x)**4/(4*b*sinh(a + b*x)**
4), Eq(n, -5)), (16*b*x*tanh(a/2 + b*x/2)**6/(8*b*tanh(a/2 + b*x/2)**6 - 16
*b*tanh(a/2 + b*x/2)**4 + 8*b*tanh(a/2 + b*x/2)**2) - 32*b*x*tanh(a/2 + b*x
/2)**4/(8*b*tanh(a/2 + b*x/2)**6 - 16*b*tanh(a/2 + b*x/2)**4 + 8*b*tanh(a/2
 + b*x/2)**2) + 16*b*x*tanh(a/2 + b*x/2)**2/(8*b*tanh(a/2 + b*x/2)**6 - 16*
b*tanh(a/2 + b*x/2)**4 + 8*b*tanh(a/2 + b*x/2)**2) - 32*log(tanh(a/2 + b*x/
```

```
2) + 1)*tanh(a/2 + b*x/2)**6/(8*b*tanh(a/2 + b*x/2)**6 - 16*b*tanh(a/2 + b*
x/2)**4 + 8*b*tanh(a/2 + b*x/2)**2) + 64*log(tanh(a/2 + b*x/2) + 1)*tanh(a/
2 + b*x/2)**4/(8*b*tanh(a/2 + b*x/2)**6 - 16*b*tanh(a/2 + b*x/2)**4 + 8*b*t
anh(a/2 + b*x/2)**2) - 32*log(tanh(a/2 + b*x/2) + 1)*tanh(a/2 + b*x/2)**2/(
8*b*tanh(a/2 + b*x/2)**6 - 16*b*tanh(a/2 + b*x/2)**4 + 8*b*tanh(a/2 + b*x/2
)**2) + 16*log(tanh(a/2 + b*x/2))*tanh(a/2 + b*x/2)**6/(8*b*tanh(a/2 + b*x/
2)**6 - 16*b*tanh(a/2 + b*x/2)**4 + 8*b*tanh(a/2 + b*x/2)**2) - 32*log(tanh
(a/2 + b*x/2))*tanh(a/2 + b*x/2)**4/(8*b*tanh(a/2 + b*x/2)**6 - 16*b*tanh(a
/2 + b*x/2)**4 + 8*b*tanh(a/2 + b*x/2)**2) + 16*log(tanh(a/2 + b*x/2))*tanh
(a/2 + b*x/2)**2/(8*b*tanh(a/2 + b*x/2)**6 - 16*b*tanh(a/2 + b*x/2)**4 + 8*
b*tanh(a/2 + b*x/2)**2) - tanh(a/2 + b*x/2)**8/(8*b*tanh(a/2 + b*x/2)**6 -
16*b*tanh(a/2 + b*x/2)**4 + 8*b*tanh(a/2 + b*x/2)**2) + 18*tanh(a/2 + b*x/2
)**4/(8*b*tanh(a/2 + b*x/2)**6 - 16*b*tanh(a/2 + b*x/2)**4 + 8*b*tanh(a/2 +
 b*x/2)**2) - 1/(8*b*tanh(a/2 + b*x/2)**6 - 16*b*tanh(a/2 + b*x/2)**4 + 8*b
*tanh(a/2 + b*x/2)**2), Eq(n, -3)), (b*x*tanh(a/2 + b*x/2)**8/(b*tanh(a/2 +
 b*x/2)**8 - 4*b*tanh(a/2 + b*x/2)**6 + 6*b*tanh(a/2 + b*x/2)**4 - 4*b*tanh
(a/2 + b*x/2)**2 + b) - 4*b*x*tanh(a/2 + b*x/2)**6/(b*tanh(a/2 + b*x/2)**8
- 4*b*tanh(a/2 + b*x/2)**6 + 6*b*tanh(a/2 + b*x/2)**4 - 4*b*tanh(a/2 + b*x/
2)**2 + b) + 6*b*x*tanh(a/2 + b*x/2)**4/(b*tanh(a/2 + b*x/2)**8 - 4*b*tanh(
a/2 + b*x/2)**6 + 6*b*tanh(a/2 + b*x/2)**4 - 4*b*tanh(a/2 + b*x/2)**2 + b)
- 4*b*x*tanh(a/2 + b*x/2)**2/(b*tanh(a/2 + b*x/2)**8 - 4*b*tanh(a/2 + b*x/2
)**6 + 6*b*tanh(a/2 + b*x/2)**4 - 4*b*tanh(a/2 + b*x/2)**2 + b) + b*x/(b*ta
nh(a/2 + b*x/2)**8 - 4*b*tanh(a/2 + b*x/2)**6 + 6*b*tanh(a/2 + b*x/2)**4 -
4*b*tanh(a/2 + b*x/2)**2 + b) - 2*log(tanh(a/2 + b*x/2) + 1)*tanh(a/2 + b*x
/2)**8/(b*tanh(a/2 + b*x/2)**8 - 4*b*tanh(a/2 + b*x/2)**6 + 6*b*tanh(a/2 +
b*x/2)**4 - 4*b*tanh(a/2 + b*x/2)**2 + b) + 8*log(tanh(a/2 + b*x/2) + 1)*ta
nh(a/2 + b*x/2)**6/(b*tanh(a/2 + b*x/2)**8 - 4*b*tanh(a/2 + b*x/2)**6 + 6*b
*tanh(a/2 + b*x/2)**4 - 4*b*tanh(a/2 + b*x/2)**2 + b) - 12*log(tanh(a/2 + b
*x/2) + 1)*tanh(a/2 + b*x/2)**4/(b*tanh(a/2 + b*x/2)**8 - 4*b*tanh(a/2 + b*
x/2)**6 + 6*b*tanh(a/2 + b*x/2)**4 - 4*b*tanh(a/2 + b*x/2)**2 + b) + 8*log(
tanh(a/2 + b*x/2) + 1)*tanh(a/2 + b*x/2)**2/(b*tanh(a/2 + b*x/2)**8 - 4*b*t
anh(a/2 + b*x/2)**6 + 6*b*tanh(a/2 + b*x/2)**4 - 4*b*tanh(a/2 + b*x/2)**2 +
 b) - 2*log(tanh(a/2 + b*x/2) + 1)/(b*tanh(a/2 + b*x/2)**8 - 4*b*tanh(a/2 +
 b*x/2)**6 + 6*b*tanh(a/2 + b*x/2)**4 - 4*b*tanh(a/2 + b*x/2)**2 + b) + log
(tanh(a/2 + b*x/2))*tanh(a/2 + b*x/2)**8/(b*tanh(a/2 + b*x/2)**8 - 4*b*tanh
(a/2 + b*x/2)**6 + 6*b*tanh(a/2 + b*x/2)**4 - 4*b*tanh(a/2 + b*x/2)**2 + b)
 - 4*log(tanh(a/2 + b*x/2))*tanh(a/2 + b*x/2)**6/(b*tanh(a/2 + b*x/2)**8 -
4*b*tanh(a/2 + b*x/2)**6 + 6*b*tanh(a/2 + b*x/2)**4 - 4*b*tanh(a/2 + b*x/2)
**2 + b) + 6*log(tanh(a/2 + b*x/2))*tanh(a/2 + b*x/2)**4/(b*tanh(a/2 + b*x/
2)**8 - 4*b*tanh(a/2 + b*x/2)**6 + 6*b*tanh(a/2 + b*x/2)**4 - 4*b*tanh(a/2
+ b*x/2)**2 + b) - 4*log(tanh(a/2 + b*x/2))*tanh(a/2 + b*x/2)**2/(b*tanh(a/
2 + b*x/2)**8 - 4*b*tanh(a/2 + b*x/2)**6 + 6*b*tanh(a/2 + b*x/2)**4 - 4*b*t
anh(a/2 + b*x/2)**2 + b) + log(tanh(a/2 + b*x/2))/(b*tanh(a/2 + b*x/2)**8 -
 4*b*tanh(a/2 + b*x/2)**6 + 6*b*tanh(a/2 + b*x/2)**4 - 4*b*tanh(a/2 + b*x/2
)**2 + b) + 4*tanh(a/2 + b*x/2)**6/(b*tanh(a/2 + b*x/2)**8 - 4*b*tanh(a/2 +
```

```
 b*x/2)**6 + 6*b*tanh(a/2 + b*x/2)**4 - 4*b*tanh(a/2 + b*x/2)**2 + b) - 4*t
anh(a/2 + b*x/2)**4/(b*tanh(a/2 + b*x/2)**8 - 4*b*tanh(a/2 + b*x/2)**6 + 6*
b*tanh(a/2 + b*x/2)**4 - 4*b*tanh(a/2 + b*x/2)**2 + b) + 4*tanh(a/2 + b*x/2
)**2/(b*tanh(a/2 + b*x/2)**8 - 4*b*tanh(a/2 + b*x/2)**6 + 6*b*tanh(a/2 + b*
x/2)**4 - 4*b*tanh(a/2 + b*x/2)**2 + b), Eq(n, -1)), (n**2*sinh(a + b*x)*si
nh(a + b*x)**n*cosh(a + b*x)**4/(b*n**3 + 9*b*n**2 + 23*b*n + 15*b) - 4*n*s
inh(a + b*x)**3*sinh(a + b*x)**n*cosh(a + b*x)**2/(b*n**3 + 9*b*n**2 + 23*b
*n + 15*b) + 8*n*sinh(a + b*x)*sinh(a + b*x)**n*cosh(a + b*x)**4/(b*n**3 +
9*b*n**2 + 23*b*n + 15*b) + 8*sinh(a + b*x)**5*sinh(a + b*x)**n/(b*n**3 + 9
*b*n**2 + 23*b*n + 15*b) - 20*sinh(a + b*x)**3*sinh(a + b*x)**n*cosh(a + b*
x)**2/(b*n**3 + 9*b*n**2 + 23*b*n + 15*b) + 15*sinh(a + b*x)*sinh(a + b*x)*
*n*cosh(a + b*x)**4/(b*n**3 + 9*b*n**2 + 23*b*n + 15*b), True))
```

3.12 $\int \cosh^m(a+bx)\sinh(a+bx)\,dx$

Optimal. Leaf size=19

$$\frac{\cosh^{m+1}(a+bx)}{b(m+1)}$$

```
[Out] cosh(b*x+a)^(1+m)/b/(1+m)
```

Rubi [A] time = 0.03, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2565, 30}

$$\frac{\cosh^{m+1}(a+bx)}{b(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[a + b*x]^m*Sinh[a + b*x],x]
```

```
[Out] Cosh[a + b*x]^(1 + m)/(b*(1 + m))
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rubi steps

$$\begin{aligned}\int \cosh^m(a+bx)\sinh(a+bx)\,dx &= \frac{\text{Subst}\left(\int x^m\,dx, x, \cosh(a+bx)\right)}{b}\\ &= \frac{\cosh^{1+m}(a+bx)}{b(1+m)}\end{aligned}$$

Mathematica [A] time = 0.01, size = 19, normalized size = 1.00

$$\frac{\cosh^{m+1}(a+bx)}{b(m+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[a + b*x]^m*Sinh[a + b*x],x]
```

```
[Out] Cosh[a + b*x]^(1 + m)/(b*(1 + m))
```

fricas [B] time = 0.61, size = 68, normalized size = 3.58

$$\frac{\cosh(bx+a)\cosh\left(m\log(\cosh(bx+a))\right)+\cosh(bx+a)\sinh\left(m\log(\cosh(bx+a))\right)}{(bm+b)\cosh(bx+a)^2-(bm+b)\sinh(bx+a)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^m*sinh(b*x+a),x, algorithm="fricas")
```

```
[Out] (cosh(b*x + a)*cosh(m*log(cosh(b*x + a))) + cosh(b*x + a)*sinh(m*log(cosh(b
*x + a))))/((b*m + b)*cosh(b*x + a)^2 - (b*m + b)*sinh(b*x + a)^2)
```

giac [B] time = 0.21, size = 94, normalized size = 4.95

$$\frac{e^{\left(3\,bx+m\log\left(\frac{1}{2}\left(e^{(2\,bx+2\,a)}+1\right)e^{(-bx-a)}\right)+3\,a\right)}+e^{\left(bx+m\log\left(\frac{1}{2}\left(e^{(2\,bx+2\,a)}+1\right)e^{(-bx-a)}\right)+a\right)}}{2\left(bme^{(2\,bx+2\,a)}+be^{(2\,bx+2\,a)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^m*sinh(b*x+a),x, algorithm="giac")
```

```
[Out] 1/2*(e^(3*b*x + m*log(1/2*(e^(2*b*x + 2*a) + 1)*e^(-b*x - a)) + 3*a) + e^(b
*x + m*log(1/2*(e^(2*b*x + 2*a) + 1)*e^(-b*x - a)) + a))/(b*m*e^(2*b*x + 2*
a) + b*e^(2*b*x + 2*a))
```

maple [A] time = 0.07, size = 20, normalized size = 1.05

$$\frac{\cosh^{1+m}(bx+a)}{b(1+m)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)^m*sinh(b*x+a),x)
```

```
[Out] cosh(b*x+a)^(1+m)/b/(1+m)
```

maxima [A] time = 0.38, size = 19, normalized size = 1.00

$$\frac{\cosh(bx+a)^{m+1}}{b(m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^m*sinh(b*x+a),x, algorithm="maxima")
```

```
[Out] cosh(b*x + a)^(m + 1)/(b*(m + 1))
```

mupad [B] time = 1.58, size = 19, normalized size = 1.00

$$\frac{\cosh(a+b\,x)^{m+1}}{b\,(m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a + b*x)^m*sinh(a + b*x),x)
```

```
[Out] cosh(a + b*x)^(m + 1)/(b*(m + 1))
```

sympy [A] time = 1.26, size = 49, normalized size = 2.58

$$\begin{cases} \frac{x\sinh(a)}{\cosh(a)} & \text{for } b = 0 \wedge m = -1 \\ x\sinh(a)\cosh^{m}(a) & \text{for } b = 0 \\ \frac{\log(\cosh(a+bx))}{b} & \text{for } m = -1 \\ \frac{\cosh(a+bx)\cosh^{m}(a+bx)}{bm+b} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)**m*sinh(b*x+a),x)
```

```
[Out] Piecewise((x*sinh(a)/cosh(a), Eq(b, 0) & Eq(m, -1)), (x*sinh(a)*cosh(a)**m,
 Eq(b, 0)), (log(cosh(a + b*x))/b, Eq(m, -1)), (cosh(a + b*x)*cosh(a + b*x)
**m/(b*m + b), True))
```

3.13 $\int \cosh^m(a+bx)\sinh^3(a+bx)\,dx$

Optimal. Leaf size=40

$$\frac{\cosh^{m+3}(a+bx)}{b(m+3)}-\frac{\cosh^{m+1}(a+bx)}{b(m+1)}$$

```
[Out] -cosh(b*x+a)^(1+m)/b/(1+m)+cosh(b*x+a)^(3+m)/b/(3+m)
```

Rubi [A] time = 0.05, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2565, 14}

$$\frac{\cosh^{m+3}(a+bx)}{b(m+3)}-\frac{\cosh^{m+1}(a+bx)}{b(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[a + b*x]^m*Sinh[a + b*x]^3,x]
```

```
[Out] -(Cosh[a + b*x]^(1 + m)/(b*(1 + m))) + Cosh[a + b*x]^(3 + m)/(b*(3 + m))
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rubi steps

$$\begin{aligned}\int \cosh^m(a+bx)\sinh^3(a+bx)\,dx &= -\frac{\text{Subst}\left(\int x^m\left(1-x^2\right)\,dx,x,\cosh(a+bx)\right)}{b}\\ &= -\frac{\text{Subst}\left(\int \left(x^m-x^{2+m}\right)\,dx,x,\cosh(a+bx)\right)}{b}\\ &= -\frac{\cosh^{1+m}(a+bx)}{b(1+m)}+\frac{\cosh^{3+m}(a+bx)}{b(3+m)}\end{aligned}$$

Mathematica [A] time = 0.13, size = 44, normalized size = 1.10

$$\frac{\cosh^{m+1}(a+bx)((m+1)\cosh(2(a+bx))-m-5)}{2b(m+1)(m+3)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[a + b*x]^m*Sinh[a + b*x]^3,x]
```

```
[Out] (Cosh[a + b*x]^(1 + m)*(-5 - m + (1 + m)*Cosh[2*(a + b*x)]))/(2*b*(1 + m)*(
3 + m))
```

fricas [B] time = 0.43, size = 189, normalized size = 4.72

$$\frac{\left((m+1)\cosh\left(bx+a\right)^{3}+3\,(m+1)\cosh\left(bx+a\right)\sinh\left(bx+a\right)^{2}-(m+9)\cosh\left(bx+a\right)\right)\cosh\left(m\log\left(\cosh\left(bx\right.\right.\right.}{4\left(\left(bm^{2}+4\,bm+3\,b\right)\cosh\left(bx+a\right)^{4}-2\left(bm^{2}+4\,bm+\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^m*sinh(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/4*(((m + 1)*cosh(b*x + a)^3 + 3*(m + 1)*cosh(b*x + a)*sinh(b*x + a)^2 - (
m + 9)*cosh(b*x + a))*cosh(m*log(cosh(b*x + a))) + ((m + 1)*cosh(b*x + a)^3
 + 3*(m + 1)*cosh(b*x + a)*sinh(b*x + a)^2 - (m + 9)*cosh(b*x + a))*sinh(m*
log(cosh(b*x + a))))/((b*m^2 + 4*b*m + 3*b)*cosh(b*x + a)^4 - 2*(b*m^2 + 4*
b*m + 3*b)*cosh(b*x + a)^2*sinh(b*x + a)^2 + (b*m^2 + 4*b*m + 3*b)*sinh(b*x
 + a)^4)
```

giac [B] time = 0.35, size = 325, normalized size = 8.12

$$\frac{me^{\left(7\,bx+m\log\left(\frac{1}{2}\left(e^{(2\,bx+2\,a)}+1\right)e^{(-bx-a)}\right)+7\,a\right)}-me^{\left(5\,bx+m\log\left(\frac{1}{2}\left(e^{(2\,bx+2\,a)}+1\right)e^{(-bx-a)}\right)+5\,a\right)}-me^{\left(3\,bx+m\log\left(\frac{1}{2}\left(e^{(2\,bx+2\,a)}+1\right)e^{(-bx-a)}\right)+3\,a\right.}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^m*sinh(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/8*(m*e^(7*b*x + m*log(1/2*(e^(2*b*x + 2*a) + 1)*e^(-b*x - a)) + 7*a) - m*
e^(5*b*x + m*log(1/2*(e^(2*b*x + 2*a) + 1)*e^(-b*x - a)) + 5*a) - m*e^(3*b*
x + m*log(1/2*(e^(2*b*x + 2*a) + 1)*e^(-b*x - a)) + 3*a) + m*e^(b*x + m*log
(1/2*(e^(2*b*x + 2*a) + 1)*e^(-b*x - a)) + a) + e^(7*b*x + m*log(1/2*(e^(2*
b*x + 2*a) + 1)*e^(-b*x - a)) + 7*a) - 9*e^(5*b*x + m*log(1/2*(e^(2*b*x + 2
*a) + 1)*e^(-b*x - a)) + 5*a) - 9*e^(3*b*x + m*log(1/2*(e^(2*b*x + 2*a) + 1
)*e^(-b*x - a)) + 3*a) + e^(b*x + m*log(1/2*(e^(2*b*x + 2*a) + 1)*e^(-b*x -
```

```
 a)) + a))/(b*m^2*e^(4*b*x + 4*a) + 4*b*m*e^(4*b*x + 4*a) + 3*b*e^(4*b*x +
4*a))
```

maple [C] time = 1.02, size = 923, normalized size = 23.08

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)^m*sinh(b*x+a)^3,x)
```

```
[Out] 1/8/b/(3+m)*(1+exp(2*b*x+2*a))^m*(1/2)^m*exp(b*x+a)^(-m)*exp(-3*b*x-3*a)*ex
p(-1/2*I*Pi*m*csgn(I*(1+exp(2*b*x+2*a)))*csgn(I*exp(-b*x-a))*csgn(I*exp(-b*
x-a)*(1+exp(2*b*x+2*a))))*exp(1/2*I*Pi*m*csgn(I*(1+exp(2*b*x+2*a)))*csgn(I*
exp(-b*x-a)*(1+exp(2*b*x+2*a)))^2)*exp(1/2*I*Pi*m*csgn(I*exp(-b*x-a))*csgn(
I*exp(-b*x-a)*(1+exp(2*b*x+2*a)))^2)*exp(-1/2*I*Pi*m*csgn(I*exp(-b*x-a)*(1+
exp(2*b*x+2*a)))^3)+1/8*exp(b*x+a)^(-m)*(1/2)^m*(1+exp(2*b*x+2*a))^m/b/(3+m
)*exp(3*b*x+3*a)*exp(-1/2*I*Pi*m*csgn(I*(1+exp(2*b*x+2*a)))*csgn(I*exp(-b*x
-a))*csgn(I*exp(-b*x-a)*(1+exp(2*b*x+2*a))))*exp(1/2*I*Pi*m*csgn(I*(1+exp(2
*b*x+2*a)))*csgn(I*exp(-b*x-a)*(1+exp(2*b*x+2*a)))^2)*exp(1/2*I*Pi*m*csgn(I
*exp(-b*x-a))*csgn(I*exp(-b*x-a)*(1+exp(2*b*x+2*a)))^2)*exp(-1/2*I*Pi*m*csg
n(I*exp(-b*x-a)*(1+exp(2*b*x+2*a)))^3)-1/8*exp(b*x+a)^(-m)*(1/2)^m*(1+exp(2
*b*x+2*a))^m*(m+9)/b/(m^2+4*m+3)*exp(-b*x-a)*exp(-1/2*I*Pi*m*csgn(I*(1+exp(
2*b*x+2*a)))*csgn(I*exp(-b*x-a))*csgn(I*exp(-b*x-a)*(1+exp(2*b*x+2*a))))*ex
p(1/2*I*Pi*m*csgn(I*(1+exp(2*b*x+2*a)))*csgn(I*exp(-b*x-a)*(1+exp(2*b*x+2*a
)))^2)*exp(1/2*I*Pi*m*csgn(I*exp(-b*x-a))*csgn(I*exp(-b*x-a)*(1+exp(2*b*x+2
*a)))^2)*exp(-1/2*I*Pi*m*csgn(I*exp(-b*x-a)*(1+exp(2*b*x+2*a)))^3)-1/8*exp(
b*x+a)^(-m)*(1/2)^m*(1+exp(2*b*x+2*a))^m*(m+9)/b/(m^2+4*m+3)*exp(b*x+a)*exp
(-1/2*I*Pi*m*csgn(I*(1+exp(2*b*x+2*a)))*csgn(I*exp(-b*x-a))*csgn(I*exp(-b*x
-a)*(1+exp(2*b*x+2*a))))*exp(1/2*I*Pi*m*csgn(I*(1+exp(2*b*x+2*a)))*csgn(I*e
xp(-b*x-a)*(1+exp(2*b*x+2*a)))^2)*exp(1/2*I*Pi*m*csgn(I*exp(-b*x-a))*csgn(I
*exp(-b*x-a)*(1+exp(2*b*x+2*a)))^2)*exp(-1/2*I*Pi*m*csgn(I*exp(-b*x-a)*(1+e
xp(2*b*x+2*a)))^3)
```

maxima [B] time = 0.58, size = 293, normalized size = 7.32

$$\frac{me^{\left((bx+a)m+3\,bx+m\log\left(e^{(-2\,bx-2\,a)}+1\right)+3\,a\right)}}{8\left(2^m m^2+2^{m+2}m+3\cdot 2^m\right)b}-\frac{(m+9)e^{\left((bx+a)m+bx+m\log\left(e^{(-2\,bx-2\,a)}+1\right)+a\right)}}{8\left(2^m m^2+2^{m+2}m+3\cdot 2^m\right)b}-\frac{(m+9)e^{\left((bx+a)m-bx+m\log\left(e^{(-2\,bx-2\,a)}+1\right.}}{8\left(2^m m^2+2^{m+2}m+3\cdot 2^m\right)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^m*sinh(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/8*m*e^((b*x + a)*m + 3*b*x + m*log(e^(-2*b*x - 2*a) + 1) + 3*a)/((2^m*m^2
 + 2^(m + 2)*m + 3*2^m)*b) - 1/8*(m + 9)*e^((b*x + a)*m + b*x + m*log(e^(-2
*b*x - 2*a) + 1) + a)/((2^m*m^2 + 2^(m + 2)*m + 3*2^m)*b) - 1/8*(m + 9)*e^(
```

```
(b*x + a)*m - b*x + m*log(e^(-2*b*x - 2*a) + 1) - a)/((2^m*m^2 + 2^(m + 2)*
m + 3*2^m)*b) + 1/8*(m + 1)*e^((b*x + a)*m - 3*b*x + m*log(e^(-2*b*x - 2*a)
 + 1) - 3*a)/((2^m*m^2 + 2^(m + 2)*m + 3*2^m)*b) + 1/8*e^((b*x + a)*m + 3*b
*x + m*log(e^(-2*b*x - 2*a) + 1) + 3*a)/((2^m*m^2 + 2^(m + 2)*m + 3*2^m)*b)
```

mupad [B] time = 0.18, size = 132, normalized size = 3.30

$$\left(\frac{1}{2}\right)^m e^{-3a-3bx}\left(e^{a+bx}+e^{-a-bx}\right)^m\left(\frac{\frac{m}{8}+\frac{1}{8}}{b\left(m^2+4m+3\right)}-\frac{e^{2a+2bx}\,(m+9)}{8b\left(m^2+4m+3\right)}+\frac{e^{6a+6bx}\,(m+1)}{8b\left(m^2+4m+3\right)}-\frac{e^{4a+4bx}\,(}{8b\left(m^2+4\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a + b*x)^m*sinh(a + b*x)^3,x)
```

```
[Out] (1/2)^m*exp(- 3*a - 3*b*x)*(exp(a + b*x) + exp(- a - b*x))^m*((m/8 + 1/8)/(
b*(4*m + m^2 + 3)) - (exp(2*a + 2*b*x)*(m + 9))/(8*b*(4*m + m^2 + 3)) + (ex
p(6*a + 6*b*x)*(m + 1))/(8*b*(4*m + m^2 + 3)) - (exp(4*a + 4*b*x)*(m + 9))/
(8*b*(4*m + m^2 + 3)))
```

sympy [A] time = 8.55, size = 648, normalized size = 16.20

$$\begin{cases} x\sinh^3(a)\cosh^m(a) \\ \frac{\log(\cosh(a+bx))}{b}-\frac{\sinh^2(a+bx)}{2b\cosh^2(a+bx)} \\ -\frac{bx\tanh^4\left(\frac{a}{2}+\frac{bx}{2}\right)}{b\tanh^4\left(\frac{a}{2}+\frac{bx}{2}\right)-2b\tanh^2\left(\frac{a}{2}+\frac{bx}{2}\right)+b}+\frac{2bx\tanh^2\left(\frac{a}{2}+\frac{bx}{2}\right)}{b\tanh^4\left(\frac{a}{2}+\frac{bx}{2}\right)-2b\tanh^2\left(\frac{a}{2}+\frac{bx}{2}\right)+b}-\frac{bx}{b\tanh^4\left(\frac{a}{2}+\frac{bx}{2}\right)-2b\tanh^2\left(\frac{a}{2}+\frac{bx}{2}\right)+b}+\frac{2\log\left(\tanh\left(\frac{a}{2}+\frac{bx}{2}\right)\right.}{b\tanh^4\left(\frac{a}{2}+\frac{bx}{2}\right)-2b} \\ \frac{m\sinh^2(a+bx)\cosh(a+bx)\cosh^m(a+bx)}{bm^2+4bm+3b}+\frac{3\sinh^2(a+bx)\cosh(a+bx)\cosh^m(a+bx)}{bm^2+4bm+3b}-\frac{2\cosh^3(a+bx)\cosh^m(a+bx)}{bm^2+4bm+3b} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)**m*sinh(b*x+a)**3,x)
```

```
[Out] Piecewise((x*sinh(a)**3*cosh(a)**m, Eq(b, 0)), (log(cosh(a + b*x))/b - sinh
(a + b*x)**2/(2*b*cosh(a + b*x)**2), Eq(m, -3)), (-b*x*tanh(a/2 + b*x/2)**4
/(b*tanh(a/2 + b*x/2)**4 - 2*b*tanh(a/2 + b*x/2)**2 + b) + 2*b*x*tanh(a/2 +
 b*x/2)**2/(b*tanh(a/2 + b*x/2)**4 - 2*b*tanh(a/2 + b*x/2)**2 + b) - b*x/(b
*tanh(a/2 + b*x/2)**4 - 2*b*tanh(a/2 + b*x/2)**2 + b) + 2*log(tanh(a/2 + b*
x/2) + 1)*tanh(a/2 + b*x/2)**4/(b*tanh(a/2 + b*x/2)**4 - 2*b*tanh(a/2 + b*x
/2)**2 + b) - 4*log(tanh(a/2 + b*x/2) + 1)*tanh(a/2 + b*x/2)**2/(b*tanh(a/2
 + b*x/2)**4 - 2*b*tanh(a/2 + b*x/2)**2 + b) + 2*log(tanh(a/2 + b*x/2) + 1)
/(b*tanh(a/2 + b*x/2)**4 - 2*b*tanh(a/2 + b*x/2)**2 + b) - log(tanh(a/2 + b
*x/2)**2 + 1)*tanh(a/2 + b*x/2)**4/(b*tanh(a/2 + b*x/2)**4 - 2*b*tanh(a/2 +
```

```
 b*x/2)**2 + b) + 2*log(tanh(a/2 + b*x/2)**2 + 1)*tanh(a/2 + b*x/2)**2/(b*t
anh(a/2 + b*x/2)**4 - 2*b*tanh(a/2 + b*x/2)**2 + b) - log(tanh(a/2 + b*x/2)
**2 + 1)/(b*tanh(a/2 + b*x/2)**4 - 2*b*tanh(a/2 + b*x/2)**2 + b) + 2*tanh(a
/2 + b*x/2)**2/(b*tanh(a/2 + b*x/2)**4 - 2*b*tanh(a/2 + b*x/2)**2 + b), Eq(
m, -1)), (m*sinh(a + b*x)**2*cosh(a + b*x)*cosh(a + b*x)**m/(b*m**2 + 4*b*m
 + 3*b) + 3*sinh(a + b*x)**2*cosh(a + b*x)*cosh(a + b*x)**m/(b*m**2 + 4*b*m
 + 3*b) - 2*cosh(a + b*x)**3*cosh(a + b*x)**m/(b*m**2 + 4*b*m + 3*b), True)
)
```

3.14 $\int \cosh^m(a+bx)\sinh^5(a+bx)\,dx$

Optimal. Leaf size=59

$$\frac{\cosh^{m+1}(a+bx)}{b(m+1)}-\frac{2\cosh^{m+3}(a+bx)}{b(m+3)}+\frac{\cosh^{m+5}(a+bx)}{b(m+5)}$$

```
[Out] cosh(b*x+a)^(1+m)/b/(1+m)-2*cosh(b*x+a)^(3+m)/b/(3+m)+cosh(b*x+a)^(5+m)/b/(
5+m)
```

Rubi [A] time = 0.06, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2565, 270}

$$\frac{\cosh^{m+1}(a+bx)}{b(m+1)}-\frac{2\cosh^{m+3}(a+bx)}{b(m+3)}+\frac{\cosh^{m+5}(a+bx)}{b(m+5)}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[a + b*x]^m*Sinh[a + b*x]^5,x]
```

```
[Out] Cosh[a + b*x]^(1 + m)/(b*(1 + m)) - (2*Cosh[a + b*x]^(3 + m))/(b*(3 + m)) +
 Cosh[a + b*x]^(5 + m)/(b*(5 + m))
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rubi steps

$$\int \cosh^m(a+bx)\sinh^5(a+bx)\,dx = \frac{\text{Subst}\left(\int x^m\left(1-x^2\right)^2\,dx,x,\cosh(a+bx)\right)}{b}$$
$$= \frac{\text{Subst}\left(\int \left(x^m-2x^{2+m}+x^{4+m}\right)\,dx,x,\cosh(a+bx)\right)}{b}$$
$$= \frac{\cosh^{1+m}(a+bx)}{b(1+m)} - \frac{2\cosh^{3+m}(a+bx)}{b(3+m)} + \frac{\cosh^{5+m}(a+bx)}{b(5+m)}$$

Mathematica [A] time = 0.23, size = 77, normalized size = 1.31

$$\frac{\cosh^{m+1}(a+bx)\left(-4\left(m^2+8m+7\right)\cosh(2(a+bx))+\left(m^2+4m+3\right)\cosh(4(a+bx))+3m^2+28m+89\right)}{8b(m+1)(m+3)(m+5)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[a + b*x]^m*Sinh[a + b*x]^5,x]
```

```
[Out] (Cosh[a + b*x]^(1 + m)*(89 + 28*m + 3*m^2 - 4*(7 + 8*m + m^2)*Cosh[2*(a + b
*x)] + (3 + 4*m + m^2)*Cosh[4*(a + b*x)]))/(8*b*(1 + m)*(3 + m)*(5 + m))
```

fricas [B] time = 0.42, size = 407, normalized size = 6.90

$$\frac{\left(\left(m^2+4\,m+3\right)\cosh\left(bx+a\right)^5+5\left(m^2+4\,m+3\right)\cosh\left(bx+a\right)\sinh\left(bx+a\right)^4-\left(3\,m^2+28\,m+25\right)\cosh\left(bx+\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^m*sinh(b*x+a)^5,x, algorithm="fricas")
```

```
[Out] 1/16*(((m^2 + 4*m + 3)*cosh(b*x + a)^5 + 5*(m^2 + 4*m + 3)*cosh(b*x + a)*si
nh(b*x + a)^4 - (3*m^2 + 28*m + 25)*cosh(b*x + a)^3 + (10*(m^2 + 4*m + 3)*c
osh(b*x + a)^3 - 3*(3*m^2 + 28*m + 25)*cosh(b*x + a))*sinh(b*x + a)^2 + 2*(
m^2 + 12*m + 75)*cosh(b*x + a))*cosh(m*log(cosh(b*x + a))) + ((m^2 + 4*m +
3)*cosh(b*x + a)^5 + 5*(m^2 + 4*m + 3)*cosh(b*x + a)*sinh(b*x + a)^4 - (3*m
^2 + 28*m + 25)*cosh(b*x + a)^3 + (10*(m^2 + 4*m + 3)*cosh(b*x + a)^3 - 3*(
3*m^2 + 28*m + 25)*cosh(b*x + a))*sinh(b*x + a)^2 + 2*(m^2 + 12*m + 75)*cos
h(b*x + a))*sinh(m*log(cosh(b*x + a))))/((b*m^3 + 9*b*m^2 + 23*b*m + 15*b)*
cosh(b*x + a)^6 - 3*(b*m^3 + 9*b*m^2 + 23*b*m + 15*b)*cosh(b*x + a)^4*sinh(
b*x + a)^2 + 3*(b*m^3 + 9*b*m^2 + 23*b*m + 15*b)*cosh(b*x + a)^2*sinh(b*x +
 a)^4 - (b*m^3 + 9*b*m^2 + 23*b*m + 15*b)*sinh(b*x + a)^6)
```

giac [B] time = 0.31, size = 721, normalized size = 12.22

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^m*sinh(b*x+a)^5,x, algorithm="giac")
```

```
[Out] 1/32*(m^2*e^(11*b*x + m*log(1/2*(e^(2*b*x + 2*a) + 1)*e^(-b*x - a)) + 11*a)
 - 3*m^2*e^(9*b*x + m*log(1/2*(e^(2*b*x + 2*a) + 1)*e^(-b*x - a)) + 9*a) +
2*m^2*e^(7*b*x + m*log(1/2*(e^(2*b*x + 2*a) + 1)*e^(-b*x - a)) + 7*a) + 2*m
^2*e^(5*b*x + m*log(1/2*(e^(2*b*x + 2*a) + 1)*e^(-b*x - a)) + 5*a) - 3*m^2*
e^(3*b*x + m*log(1/2*(e^(2*b*x + 2*a) + 1)*e^(-b*x - a)) + 3*a) + m^2*e^(b*
x + m*log(1/2*(e^(2*b*x + 2*a) + 1)*e^(-b*x - a)) + a) + 4*m*e^(11*b*x + m*
log(1/2*(e^(2*b*x + 2*a) + 1)*e^(-b*x - a)) + 11*a) - 28*m*e^(9*b*x + m*log
(1/2*(e^(2*b*x + 2*a) + 1)*e^(-b*x - a)) + 9*a) + 24*m*e^(7*b*x + m*log(1/2
*(e^(2*b*x + 2*a) + 1)*e^(-b*x - a)) + 7*a) + 24*m*e^(5*b*x + m*log(1/2*(e^
(2*b*x + 2*a) + 1)*e^(-b*x - a)) + 5*a) - 28*m*e^(3*b*x + m*log(1/2*(e^(2*b
*x + 2*a) + 1)*e^(-b*x - a)) + 3*a) + 4*m*e^(b*x + m*log(1/2*(e^(2*b*x + 2*
a) + 1)*e^(-b*x - a)) + a) + 3*e^(11*b*x + m*log(1/2*(e^(2*b*x + 2*a) + 1)*
e^(-b*x - a)) + 11*a) - 25*e^(9*b*x + m*log(1/2*(e^(2*b*x + 2*a) + 1)*e^(-b
*x - a)) + 9*a) + 150*e^(7*b*x + m*log(1/2*(e^(2*b*x + 2*a) + 1)*e^(-b*x -
a)) + 7*a) + 150*e^(5*b*x + m*log(1/2*(e^(2*b*x + 2*a) + 1)*e^(-b*x - a)) +
 5*a) - 25*e^(3*b*x + m*log(1/2*(e^(2*b*x + 2*a) + 1)*e^(-b*x - a)) + 3*a)
+ 3*e^(b*x + m*log(1/2*(e^(2*b*x + 2*a) + 1)*e^(-b*x - a)) + a))/(b*m^3*e^(
6*b*x + 6*a) + 9*b*m^2*e^(6*b*x + 6*a) + 23*b*m*e^(6*b*x + 6*a) + 15*b*e^(6
*b*x + 6*a))
```

maple [F] time = 0.98, size = 0, normalized size = 0.00

$$\int \left(\cosh^{m}(bx+a)\right)\left(\sinh^{5}(bx+a)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)^m*sinh(b*x+a)^5,x)
```

```
[Out] int(cosh(b*x+a)^m*sinh(b*x+a)^5,x)
```

maxima [B] time = 0.50, size = 558, normalized size = 9.46

$$\frac{m^2 e^{\left((bx+a)m+5\,bx+m\log\left(e^{(-2\,bx-2\,a)}+1\right)+5\,a\right)}}{32\left(2^m m^3+9\cdot 2^m m^2+23\cdot 2^m m+15\cdot 2^m\right)b}+\frac{m e^{\left((bx+a)m+5\,bx+m\log\left(e^{(-2\,bx-2\,a)}+1\right)+5\,a\right)}}{8\left(2^m m^3+9\cdot 2^m m^2+23\cdot 2^m m+15\cdot 2^m\right)b}-\frac{\left(3\,m^2+28\,m+25\right.}{32\left(2^m m^3+9\cdot\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^m*sinh(b*x+a)^5,x, algorithm="maxima")
```

```
[Out] 1/32*m^2*e^((b*x + a)*m + 5*b*x + m*log(e^(-2*b*x - 2*a) + 1) + 5*a)/((2^m*
m^3 + 9*2^m*m^2 + 23*2^m*m + 15*2^m)*b) + 1/8*m*e^((b*x + a)*m + 5*b*x + m*
log(e^(-2*b*x - 2*a) + 1) + 5*a)/((2^m*m^3 + 9*2^m*m^2 + 23*2^m*m + 15*2^m)
*b) - 1/32*(3*m^2 + 28*m + 25)*e^((b*x + a)*m + 3*b*x + m*log(e^(-2*b*x - 2
*a) + 1) + 3*a)/((2^m*m^3 + 9*2^m*m^2 + 23*2^m*m + 15*2^m)*b) + 1/16*(m^2 +
 12*m + 75)*e^((b*x + a)*m + b*x + m*log(e^(-2*b*x - 2*a) + 1) + a)/((2^m*m
^3 + 9*2^m*m^2 + 23*2^m*m + 15*2^m)*b) + 1/16*(m^2 + 12*m + 75)*e^((b*x + a
)*m - b*x + m*log(e^(-2*b*x - 2*a) + 1) - a)/((2^m*m^3 + 9*2^m*m^2 + 23*2^m
*m + 15*2^m)*b) - 1/32*(3*m^2 + 28*m + 25)*e^((b*x + a)*m - 3*b*x + m*log(e
^(-2*b*x - 2*a) + 1) - 3*a)/((2^m*m^3 + 9*2^m*m^2 + 23*2^m*m + 15*2^m)*b) +
 1/32*(m^2 + 4*m + 3)*e^((b*x + a)*m - 5*b*x + m*log(e^(-2*b*x - 2*a) + 1)
- 5*a)/((2^m*m^3 + 9*2^m*m^2 + 23*2^m*m + 15*2^m)*b) + 3/32*e^((b*x + a)*m
+ 5*b*x + m*log(e^(-2*b*x - 2*a) + 1) + 5*a)/((2^m*m^3 + 9*2^m*m^2 + 23*2^m
*m + 15*2^m)*b)
```

mupad [B] time = 1.75, size = 254, normalized size = 4.31

$$e^{-5\,a-5\,b\,x}\left(\frac{e^{a+b\,x}}{2}+\frac{e^{-a-b\,x}}{2}\right)^m\left(\frac{m^2+4\,m+3}{32\,b\,\left(m^3+9\,m^2+23\,m+15\right)}+\frac{e^{10\,a+10\,b\,x}\,\left(m^2+4\,m+3\right)}{32\,b\,\left(m^3+9\,m^2+23\,m+15\right)}-\frac{e^{2\,a+2\,b\,x}\,\left(3\,m^2\right.}{32\,b\,\left(m^3+9\,m^2\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a + b*x)^m*sinh(a + b*x)^5,x)
```

```
[Out] exp(- 5*a - 5*b*x)*(exp(a + b*x)/2 + exp(- a - b*x)/2)^m*((4*m + m^2 + 3)/(
32*b*(23*m + 9*m^2 + m^3 + 15)) + (exp(10*a + 10*b*x)*(4*m + m^2 + 3))/(32*
b*(23*m + 9*m^2 + m^3 + 15)) - (exp(2*a + 2*b*x)*(28*m + 3*m^2 + 25))/(32*b
*(23*m + 9*m^2 + m^3 + 15)) - (exp(8*a + 8*b*x)*(28*m + 3*m^2 + 25))/(32*b*
(23*m + 9*m^2 + m^3 + 15)) + (exp(4*a + 4*b*x)*(24*m + 2*m^2 + 150))/(32*b*
(23*m + 9*m^2 + m^3 + 15)) + (exp(6*a + 6*b*x)*(24*m + 2*m^2 + 150))/(32*b*
(23*m + 9*m^2 + m^3 + 15)))
```

sympy [A] time = 42.81, size = 2351, normalized size = 39.85

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)**m*sinh(b*x+a)**5,x)
```

```
[Out] Piecewise((x*sinh(a)**5*cosh(a)**m, Eq(b, 0)), (log(cosh(a + b*x))/b - sinh
(a + b*x)**4/(4*b*cosh(a + b*x)**4) - sinh(a + b*x)**2/(2*b*cosh(a + b*x)**
2), Eq(m, -5)), (-2*b*x*tanh(a/2 + b*x/2)**8/(b*tanh(a/2 + b*x/2)**8 - 2*b*
tanh(a/2 + b*x/2)**4 + b) + 4*b*x*tanh(a/2 + b*x/2)**4/(b*tanh(a/2 + b*x/2)
```

```
**8 - 2*b*tanh(a/2 + b*x/2)**4 + b) - 2*b*x/(b*tanh(a/2 + b*x/2)**8 - 2*b*t
anh(a/2 + b*x/2)**4 + b) + 4*log(tanh(a/2 + b*x/2) + 1)*tanh(a/2 + b*x/2)**
8/(b*tanh(a/2 + b*x/2)**8 - 2*b*tanh(a/2 + b*x/2)**4 + b) - 8*log(tanh(a/2
+ b*x/2) + 1)*tanh(a/2 + b*x/2)**4/(b*tanh(a/2 + b*x/2)**8 - 2*b*tanh(a/2 +
 b*x/2)**4 + b) + 4*log(tanh(a/2 + b*x/2) + 1)/(b*tanh(a/2 + b*x/2)**8 - 2*
b*tanh(a/2 + b*x/2)**4 + b) - 2*log(tanh(a/2 + b*x/2)**2 + 1)*tanh(a/2 + b*
x/2)**8/(b*tanh(a/2 + b*x/2)**8 - 2*b*tanh(a/2 + b*x/2)**4 + b) + 4*log(tan
h(a/2 + b*x/2)**2 + 1)*tanh(a/2 + b*x/2)**4/(b*tanh(a/2 + b*x/2)**8 - 2*b*t
anh(a/2 + b*x/2)**4 + b) - 2*log(tanh(a/2 + b*x/2)**2 + 1)/(b*tanh(a/2 + b*
x/2)**8 - 2*b*tanh(a/2 + b*x/2)**4 + b) + 4*tanh(a/2 + b*x/2)**6/(b*tanh(a/
2 + b*x/2)**8 - 2*b*tanh(a/2 + b*x/2)**4 + b) + 4*tanh(a/2 + b*x/2)**2/(b*t
anh(a/2 + b*x/2)**8 - 2*b*tanh(a/2 + b*x/2)**4 + b), Eq(m, -3)), (b*x*tanh(
a/2 + b*x/2)**8/(b*tanh(a/2 + b*x/2)**8 - 4*b*tanh(a/2 + b*x/2)**6 + 6*b*ta
nh(a/2 + b*x/2)**4 - 4*b*tanh(a/2 + b*x/2)**2 + b) - 4*b*x*tanh(a/2 + b*x/2
)**6/(b*tanh(a/2 + b*x/2)**8 - 4*b*tanh(a/2 + b*x/2)**6 + 6*b*tanh(a/2 + b*
x/2)**4 - 4*b*tanh(a/2 + b*x/2)**2 + b) + 6*b*x*tanh(a/2 + b*x/2)**4/(b*tan
h(a/2 + b*x/2)**8 - 4*b*tanh(a/2 + b*x/2)**6 + 6*b*tanh(a/2 + b*x/2)**4 - 4
*b*tanh(a/2 + b*x/2)**2 + b) - 4*b*x*tanh(a/2 + b*x/2)**2/(b*tanh(a/2 + b*x
/2)**8 - 4*b*tanh(a/2 + b*x/2)**6 + 6*b*tanh(a/2 + b*x/2)**4 - 4*b*tanh(a/2
 + b*x/2)**2 + b) + b*x/(b*tanh(a/2 + b*x/2)**8 - 4*b*tanh(a/2 + b*x/2)**6
+ 6*b*tanh(a/2 + b*x/2)**4 - 4*b*tanh(a/2 + b*x/2)**2 + b) - 2*log(tanh(a/2
 + b*x/2) + 1)*tanh(a/2 + b*x/2)**8/(b*tanh(a/2 + b*x/2)**8 - 4*b*tanh(a/2
+ b*x/2)**6 + 6*b*tanh(a/2 + b*x/2)**4 - 4*b*tanh(a/2 + b*x/2)**2 + b) + 8*
log(tanh(a/2 + b*x/2) + 1)*tanh(a/2 + b*x/2)**6/(b*tanh(a/2 + b*x/2)**8 - 4
*b*tanh(a/2 + b*x/2)**6 + 6*b*tanh(a/2 + b*x/2)**4 - 4*b*tanh(a/2 + b*x/2)*
*2 + b) - 12*log(tanh(a/2 + b*x/2) + 1)*tanh(a/2 + b*x/2)**4/(b*tanh(a/2 +
b*x/2)**8 - 4*b*tanh(a/2 + b*x/2)**6 + 6*b*tanh(a/2 + b*x/2)**4 - 4*b*tanh(
a/2 + b*x/2)**2 + b) + 8*log(tanh(a/2 + b*x/2) + 1)*tanh(a/2 + b*x/2)**2/(b
*tanh(a/2 + b*x/2)**8 - 4*b*tanh(a/2 + b*x/2)**6 + 6*b*tanh(a/2 + b*x/2)**4
 - 4*b*tanh(a/2 + b*x/2)**2 + b) - 2*log(tanh(a/2 + b*x/2) + 1)/(b*tanh(a/2
 + b*x/2)**8 - 4*b*tanh(a/2 + b*x/2)**6 + 6*b*tanh(a/2 + b*x/2)**4 - 4*b*ta
nh(a/2 + b*x/2)**2 + b) + log(tanh(a/2 + b*x/2)**2 + 1)*tanh(a/2 + b*x/2)**
8/(b*tanh(a/2 + b*x/2)**8 - 4*b*tanh(a/2 + b*x/2)**6 + 6*b*tanh(a/2 + b*x/2
)**4 - 4*b*tanh(a/2 + b*x/2)**2 + b) - 4*log(tanh(a/2 + b*x/2)**2 + 1)*tanh
(a/2 + b*x/2)**6/(b*tanh(a/2 + b*x/2)**8 - 4*b*tanh(a/2 + b*x/2)**6 + 6*b*t
anh(a/2 + b*x/2)**4 - 4*b*tanh(a/2 + b*x/2)**2 + b) + 6*log(tanh(a/2 + b*x/
2)**2 + 1)*tanh(a/2 + b*x/2)**4/(b*tanh(a/2 + b*x/2)**8 - 4*b*tanh(a/2 + b*
x/2)**6 + 6*b*tanh(a/2 + b*x/2)**4 - 4*b*tanh(a/2 + b*x/2)**2 + b) - 4*log(
tanh(a/2 + b*x/2)**2 + 1)*tanh(a/2 + b*x/2)**2/(b*tanh(a/2 + b*x/2)**8 - 4*
b*tanh(a/2 + b*x/2)**6 + 6*b*tanh(a/2 + b*x/2)**4 - 4*b*tanh(a/2 + b*x/2)**
2 + b) + log(tanh(a/2 + b*x/2)**2 + 1)/(b*tanh(a/2 + b*x/2)**8 - 4*b*tanh(a
/2 + b*x/2)**6 + 6*b*tanh(a/2 + b*x/2)**4 - 4*b*tanh(a/2 + b*x/2)**2 + b) -
 2*tanh(a/2 + b*x/2)**6/(b*tanh(a/2 + b*x/2)**8 - 4*b*tanh(a/2 + b*x/2)**6
+ 6*b*tanh(a/2 + b*x/2)**4 - 4*b*tanh(a/2 + b*x/2)**2 + b) + 8*tanh(a/2 + b
*x/2)**4/(b*tanh(a/2 + b*x/2)**8 - 4*b*tanh(a/2 + b*x/2)**6 + 6*b*tanh(a/2
```

```
+ b*x/2)**4 - 4*b*tanh(a/2 + b*x/2)**2 + b) - 2*tanh(a/2 + b*x/2)**2/(b*tan
h(a/2 + b*x/2)**8 - 4*b*tanh(a/2 + b*x/2)**6 + 6*b*tanh(a/2 + b*x/2)**4 - 4
*b*tanh(a/2 + b*x/2)**2 + b), Eq(m, -1)), (m**2*sinh(a + b*x)**4*cosh(a + b
*x)*cosh(a + b*x)**m/(b*m**3 + 9*b*m**2 + 23*b*m + 15*b) + 8*m*sinh(a + b*x
)**4*cosh(a + b*x)*cosh(a + b*x)**m/(b*m**3 + 9*b*m**2 + 23*b*m + 15*b) - 4
*m*sinh(a + b*x)**2*cosh(a + b*x)**3*cosh(a + b*x)**m/(b*m**3 + 9*b*m**2 +
23*b*m + 15*b) + 15*sinh(a + b*x)**4*cosh(a + b*x)*cosh(a + b*x)**m/(b*m**3
 + 9*b*m**2 + 23*b*m + 15*b) - 20*sinh(a + b*x)**2*cosh(a + b*x)**3*cosh(a
+ b*x)**m/(b*m**3 + 9*b*m**2 + 23*b*m + 15*b) + 8*cosh(a + b*x)**5*cosh(a +
 b*x)**m/(b*m**3 + 9*b*m**2 + 23*b*m + 15*b), True))
```

3.15 $\int \cosh^2(a+bx)\sinh^2(a+bx)\,dx$

Optimal. Leaf size=46

$$\frac{\sinh(a+bx)\cosh^3(a+bx)}{4b}-\frac{\sinh(a+bx)\cosh(a+bx)}{8b}-\frac{x}{8}$$

```
[Out] -1/8*x-1/8*cosh(b*x+a)*sinh(b*x+a)/b+1/4*cosh(b*x+a)^3*sinh(b*x+a)/b
```

Rubi [A] time = 0.04, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {2568, 2635, 8}

$$\frac{\sinh(a+bx)\cosh^3(a+bx)}{4b}-\frac{\sinh(a+bx)\cosh(a+bx)}{8b}-\frac{x}{8}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[a + b*x]^2*Sinh[a + b*x]^2,x]
```

```
[Out] -x/8 - (Cosh[a + b*x]*Sinh[a + b*x])/(8*b) + (Cosh[a + b*x]^3*Sinh[a + b*x]
)/(4*b)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2568

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m - 1)
)/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Cos[e + f*x])^n*(a
*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rubi steps

$$\int \cosh^2(a+bx)\sinh^2(a+bx)\,dx = \frac{\cosh^3(a+bx)\sinh(a+bx)}{4b} - \frac{1}{4}\int \cosh^2(a+bx)\,dx$$
$$= -\frac{\cosh(a+bx)\sinh(a+bx)}{8b} + \frac{\cosh^3(a+bx)\sinh(a+bx)}{4b} - \frac{\int 1\,dx}{8}$$
$$= -\frac{x}{8} - \frac{\cosh(a+bx)\sinh(a+bx)}{8b} + \frac{\cosh^3(a+bx)\sinh(a+bx)}{4b}$$

Mathematica [A] time = 0.03, size = 23, normalized size = 0.50

$$\frac{\sinh(4(a+bx)) - 4(a+bx)}{32b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[a + b*x]^2*Sinh[a + b*x]^2,x]
```

```
[Out] (-4*(a + b*x) + Sinh[4*(a + b*x)])/(32*b)
```

fricas [A] time = 0.38, size = 40, normalized size = 0.87

$$\frac{\cosh(bx+a)^3 \sinh(bx+a) + \cosh(bx+a)\sinh(bx+a)^3 - bx}{8\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^2*sinh(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/8*(cosh(b*x + a)^3*sinh(b*x + a) + cosh(b*x + a)*sinh(b*x + a)^3 - b*x)/b
```

giac [A] time = 0.23, size = 32, normalized size = 0.70

$$-\frac{1}{8}\,x + \frac{e^{(4\,bx+4\,a)}}{64\,b} - \frac{e^{(-4\,bx-4\,a)}}{64\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^2*sinh(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -1/8*x + 1/64*e^(4*b*x + 4*a)/b - 1/64*e^(-4*b*x - 4*a)/b
```

maple [A] time = 0.09, size = 43, normalized size = 0.93

$$\frac{\frac{\left(\cosh^3(bx+a)\right)\sinh(bx+a)}{4} - \frac{\cosh(bx+a)\sinh(bx+a)}{8} - \frac{bx}{8} - \frac{a}{8}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)^2*sinh(b*x+a)^2,x)
```

```
[Out] 1/b*(1/4*cosh(b*x+a)^3*sinh(b*x+a)-1/8*cosh(b*x+a)*sinh(b*x+a)-1/8*b*x-1/8*
a)
```

maxima [A] time = 0.33, size = 39, normalized size = 0.85

$$-\frac{bx+a}{8\,b}+\frac{e^{(4\,bx+4\,a)}}{64\,b}-\frac{e^{(-4\,bx-4\,a)}}{64\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^2*sinh(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -1/8*(b*x + a)/b + 1/64*e^(4*b*x + 4*a)/b - 1/64*e^(-4*b*x - 4*a)/b
```

mupad [B] time = 0.11, size = 18, normalized size = 0.39

$$\frac{\sinh\left(4\,a+4\,b\,x\right)}{32\,b}-\frac{x}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a + b*x)^2*sinh(a + b*x)^2,x)
```

```
[Out] sinh(4*a + 4*b*x)/(32*b) - x/8
```

sympy [A] time = 0.84, size = 92, normalized size = 2.00

$$\begin{cases} -\frac{x\sinh^{4}\left(a+bx\right)}{8}+\frac{x\sinh^{2}\left(a+bx\right)\cosh^{2}\left(a+bx\right)}{4}-\frac{x\cosh^{4}\left(a+bx\right)}{8}+\frac{\sinh^{3}\left(a+bx\right)\cosh\left(a+bx\right)}{8b}+\frac{\sinh\left(a+bx\right)\cosh^{3}\left(a+bx\right)}{8b} & \text{for}\ b\neq 0 \\ x\sinh^{2}\left(a\right)\cosh^{2}\left(a\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)**2*sinh(b*x+a)**2,x)
```

```
[Out] Piecewise((-x*sinh(a + b*x)**4/8 + x*sinh(a + b*x)**2*cosh(a + b*x)**2/4 -
x*cosh(a + b*x)**4/8 + sinh(a + b*x)**3*cosh(a + b*x)/(8*b) + sinh(a + b*x)
*cosh(a + b*x)**3/(8*b), Ne(b, 0)), (x*sinh(a)**2*cosh(a)**2, True))
```

3.16 $\int \cosh^2(a+bx)\sinh^4(a+bx)\,dx$

Optimal. Leaf size=69

$$\frac{\sinh^3(a+bx)\cosh^3(a+bx)}{6b}-\frac{\sinh(a+bx)\cosh^3(a+bx)}{8b}+\frac{\sinh(a+bx)\cosh(a+bx)}{16b}+\frac{x}{16}$$

```
[Out] 1/16*x+1/16*cosh(b*x+a)*sinh(b*x+a)/b-1/8*cosh(b*x+a)^3*sinh(b*x+a)/b+1/6*c
osh(b*x+a)^3*sinh(b*x+a)^3/b
```

Rubi [A] time = 0.07, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {2568, 2635, 8}

$$\frac{\sinh^3(a+bx)\cosh^3(a+bx)}{6b}-\frac{\sinh(a+bx)\cosh^3(a+bx)}{8b}+\frac{\sinh(a+bx)\cosh(a+bx)}{16b}+\frac{x}{16}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[a + b*x]^2*Sinh[a + b*x]^4,x]
```

```
[Out] x/16 + (Cosh[a + b*x]*Sinh[a + b*x])/(16*b) - (Cosh[a + b*x]^3*Sinh[a + b*x
])/(8*b) + (Cosh[a + b*x]^3*Sinh[a + b*x]^3)/(6*b)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2568

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m - 1)
)/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Cos[e + f*x])^n*(a
*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rubi steps

$$\int \cosh^2(a+bx)\sinh^4(a+bx)\,dx = \frac{\cosh^3(a+bx)\sinh^3(a+bx)}{6b} - \frac{1}{2}\int \cosh^2(a+bx)\sinh^2(a+bx)\,dx$$
$$= -\frac{\cosh^3(a+bx)\sinh(a+bx)}{8b} + \frac{\cosh^3(a+bx)\sinh^3(a+bx)}{6b} + \frac{1}{8}\int \cosh^2$$
$$= \frac{\cosh(a+bx)\sinh(a+bx)}{16b} - \frac{\cosh^3(a+bx)\sinh(a+bx)}{8b} + \frac{\cosh^3(a+bx)}{6}$$
$$= \frac{x}{16} + \frac{\cosh(a+bx)\sinh(a+bx)}{16b} - \frac{\cosh^3(a+bx)\sinh(a+bx)}{8b} + \frac{\cosh^3(a}{}$$

Mathematica [A] time = 0.08, size = 40, normalized size = 0.58

$$\frac{-3\sinh(2(a+bx)) - 3\sinh(4(a+bx)) + \sinh(6(a+bx)) + 12bx}{192b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[a + b*x]^2*Sinh[a + b*x]^4,x]
```

```
[Out] (12*b*x - 3*Sinh[2*(a + b*x)] - 3*Sinh[4*(a + b*x)] + Sinh[6*(a + b*x)])/(1
92*b)
```

fricas [A] time = 0.39, size = 90, normalized size = 1.30

$$\frac{3\,\cosh(bx+a)\sinh(bx+a)^5 + 2\left(5\,\cosh(bx+a)^3 - 3\,\cosh(bx+a)\right)\sinh(bx+a)^3 + 6\,bx + 3\left(\cosh(bx+a\right.}{96\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^2*sinh(b*x+a)^4,x, algorithm="fricas")
```

```
[Out] 1/96*(3*cosh(b*x + a)*sinh(b*x + a)^5 + 2*(5*cosh(b*x + a)^3 - 3*cosh(b*x +
 a))*sinh(b*x + a)^3 + 6*b*x + 3*(cosh(b*x + a)^5 - 2*cosh(b*x + a)^3 - cos
h(b*x + a))*sinh(b*x + a))/b
```

giac [A] time = 0.20, size = 88, normalized size = 1.28

$$\frac{1}{16}\,x + \frac{e^{(6\,bx+6\,a)}}{384\,b} - \frac{e^{(4\,bx+4\,a)}}{128\,b} - \frac{e^{(2\,bx+2\,a)}}{128\,b} + \frac{e^{(-2\,bx-2\,a)}}{128\,b} + \frac{e^{(-4\,bx-4\,a)}}{128\,b} - \frac{e^{(-6\,bx-6\,a)}}{384\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^2*sinh(b*x+a)^4,x, algorithm="giac")
```

```
[Out] 1/16*x + 1/384*e^(6*b*x + 6*a)/b - 1/128*e^(4*b*x + 4*a)/b - 1/128*e^(2*b*x
 + 2*a)/b + 1/128*e^(-2*b*x - 2*a)/b + 1/128*e^(-4*b*x - 4*a)/b - 1/384*e^(
-6*b*x - 6*a)/b
```

maple [A] time = 0.09, size = 61, normalized size = 0.88

$$\frac{\frac{\left(\cosh^3(bx+a)\right)\left(\sinh^3(bx+a)\right)}{6}-\frac{\left(\cosh^3(bx+a)\right)\sinh(bx+a)}{8}+\frac{\cosh(bx+a)\sinh(bx+a)}{16}+\frac{bx}{16}+\frac{a}{16}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)^2*sinh(b*x+a)^4,x)
```

```
[Out] 1/b*(1/6*cosh(b*x+a)^3*sinh(b*x+a)^3-1/8*cosh(b*x+a)^3*sinh(b*x+a)+1/16*cos
h(b*x+a)*sinh(b*x+a)+1/16*b*x+1/16*a)
```

maxima [A] time = 0.34, size = 88, normalized size = 1.28

$$-\frac{\left(3\,e^{(-2\,bx-2\,a)}+3\,e^{(-4\,bx-4\,a)}-1\right)e^{(6\,bx+6\,a)}}{384\,b}+\frac{bx+a}{16\,b}+\frac{3\,e^{(-2\,bx-2\,a)}+3\,e^{(-4\,bx-4\,a)}-e^{(-6\,bx-6\,a)}}{384\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^2*sinh(b*x+a)^4,x, algorithm="maxima")
```

```
[Out] -1/384*(3*e^(-2*b*x - 2*a) + 3*e^(-4*b*x - 4*a) - 1)*e^(6*b*x + 6*a)/b + 1/
16*(b*x + a)/b + 1/384*(3*e^(-2*b*x - 2*a) + 3*e^(-4*b*x - 4*a) - e^(-6*b*x
 - 6*a))/b
```

mupad [B] time = 1.64, size = 43, normalized size = 0.62

$$\frac{x}{16}-\frac{\frac{\sinh(2\,a+2\,b\,x)}{64}+\frac{\sinh(4\,a+4\,b\,x)}{64}-\frac{\sinh(6\,a+6\,b\,x)}{192}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a + b*x)^2*sinh(a + b*x)^4,x)
```

```
[Out] x/16 - (sinh(2*a + 2*b*x)/64 + sinh(4*a + 4*b*x)/64 - sinh(6*a + 6*b*x)/192
)/b
```

sympy [A] time = 2.91, size = 136, normalized size = 1.97

$$\begin{cases} -\frac{x\sinh^6(a+bx)}{16}+\frac{3x\sinh^4(a+bx)\cosh^2(a+bx)}{16}-\frac{3x\sinh^2(a+bx)\cosh^4(a+bx)}{16}+\frac{x\cosh^6(a+bx)}{16}+\frac{\sinh^5(a+bx)\cosh(a+bx)}{16b}+\frac{\sinh^3(a+bx \\ x\sinh^4(a)\cosh^2(a) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)**2*sinh(b*x+a)**4,x)
```

```
[Out] Piecewise((-x*sinh(a + b*x)**6/16 + 3*x*sinh(a + b*x)**4*cosh(a + b*x)**2/1
6 - 3*x*sinh(a + b*x)**2*cosh(a + b*x)**4/16 + x*cosh(a + b*x)**6/16 + sinh
(a + b*x)**5*cosh(a + b*x)/(16*b) + sinh(a + b*x)**3*cosh(a + b*x)**3/(6*b)
 - sinh(a + b*x)*cosh(a + b*x)**5/(16*b), Ne(b, 0)), (x*sinh(a)**4*cosh(a)*
*2, True))
```

3.17 $\int \cosh^2(a+bx)\sinh^6(a+bx)\,dx$

Optimal. Leaf size=92

$$\frac{\sinh^5(a+bx)\cosh^3(a+bx)}{8b}-\frac{5\sinh^3(a+bx)\cosh^3(a+bx)}{48b}+\frac{5\sinh(a+bx)\cosh^3(a+bx)}{64b}-\frac{5\sinh(a+bx)\cosh}{128b}$$

```
[Out] -5/128*x-5/128*cosh(b*x+a)*sinh(b*x+a)/b+5/64*cosh(b*x+a)^3*sinh(b*x+a)/b-5
/48*cosh(b*x+a)^3*sinh(b*x+a)^3/b+1/8*cosh(b*x+a)^3*sinh(b*x+a)^5/b
```

Rubi [A] time = 0.10, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {2568, 2635, 8}

$$\frac{\sinh^5(a+bx)\cosh^3(a+bx)}{8b}-\frac{5\sinh^3(a+bx)\cosh^3(a+bx)}{48b}+\frac{5\sinh(a+bx)\cosh^3(a+bx)}{64b}-\frac{5\sinh(a+bx)\cosh}{128b}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[a + b*x]^2*Sinh[a + b*x]^6,x]
```

```
[Out] (-5*x)/128 - (5*Cosh[a + b*x]*Sinh[a + b*x])/(128*b) + (5*Cosh[a + b*x]^3*S
inh[a + b*x])/(64*b) - (5*Cosh[a + b*x]^3*Sinh[a + b*x]^3)/(48*b) + (Cosh[a
 + b*x]^3*Sinh[a + b*x]^5)/(8*b)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2568

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m - 1)
)/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Cos[e + f*x])^n*(a
*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rubi steps

$$\int \cosh^2(a+bx)\sinh^6(a+bx)\,dx = \frac{\cosh^3(a+bx)\sinh^5(a+bx)}{8b} - \frac{5}{8}\int \cosh^2(a+bx)\sinh^4(a+bx)\,dx$$
$$= -\frac{5\cosh^3(a+bx)\sinh^3(a+bx)}{48b} + \frac{\cosh^3(a+bx)\sinh^5(a+bx)}{8b} + \frac{5}{16}\int \mathrm{co}$$
$$= \frac{5\cosh^3(a+bx)\sinh(a+bx)}{64b} - \frac{5\cosh^3(a+bx)\sinh^3(a+bx)}{48b} + \frac{\cosh^3(a}{}$$
$$= -\frac{5\cosh(a+bx)\sinh(a+bx)}{128b} + \frac{5\cosh^3(a+bx)\sinh(a+bx)}{64b} - \frac{5\cosh^3(a}{}$$
$$= -\frac{5x}{128} - \frac{5\cosh(a+bx)\sinh(a+bx)}{128b} + \frac{5\cosh^3(a+bx)\sinh(a+bx)}{64b} - \frac{5\,\mathrm{c}}{}$$

Mathematica [A] time = 0.13, size = 52, normalized size = 0.57

$$\frac{48\sinh(2(a+bx)) + 24\sinh(4(a+bx)) - 16\sinh(6(a+bx)) + 3\sinh(8(a+bx)) - 120bx}{3072b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[a + b*x]^2*Sinh[a + b*x]^6,x]
```

```
[Out] (-120*b*x + 48*Sinh[2*(a + b*x)] + 24*Sinh[4*(a + b*x)] - 16*Sinh[6*(a + b*
x)] + 3*Sinh[8*(a + b*x)])/(3072*b)
```

fricas [A] time = 0.43, size = 138, normalized size = 1.50

$$\frac{3\cosh(bx+a)\sinh(bx+a)^7 + 3\left(7\cosh(bx+a)^3 - 4\cosh(bx+a)\right)\sinh(bx+a)^5 + \left(21\cosh(bx+a)^5 - 4\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^2*sinh(b*x+a)^6,x, algorithm="fricas")
```

```
[Out] 1/384*(3*cosh(b*x + a)*sinh(b*x + a)^7 + 3*(7*cosh(b*x + a)^3 - 4*cosh(b*x
+ a))*sinh(b*x + a)^5 + (21*cosh(b*x + a)^5 - 40*cosh(b*x + a)^3 + 12*cosh(
b*x + a))*sinh(b*x + a)^3 - 15*b*x + 3*(cosh(b*x + a)^7 - 4*cosh(b*x + a)^5
 + 4*cosh(b*x + a)^3 + 4*cosh(b*x + a))*sinh(b*x + a))/b
```

giac [A] time = 0.21, size = 116, normalized size = 1.26

$$-\frac{5}{128}x + \frac{e^{(8bx+8a)}}{2048b} - \frac{e^{(6bx+6a)}}{384b} + \frac{e^{(4bx+4a)}}{256b} + \frac{e^{(2bx+2a)}}{128b} - \frac{e^{(-2bx-2a)}}{128b} - \frac{e^{(-4bx-4a)}}{256b} + \frac{e^{(-6bx-6a)}}{384b} - \frac{e^{(-8bx-8a)}}{2048b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^2*sinh(b*x+a)^6,x, algorithm="giac")
```

```
[Out] -5/128*x + 1/2048*e^(8*b*x + 8*a)/b - 1/384*e^(6*b*x + 6*a)/b + 1/256*e^(4*
b*x + 4*a)/b + 1/128*e^(2*b*x + 2*a)/b - 1/128*e^(-2*b*x - 2*a)/b - 1/256*e
^(-4*b*x - 4*a)/b + 1/384*e^(-6*b*x - 6*a)/b - 1/2048*e^(-8*b*x - 8*a)/b
```

maple [A] time = 0.09, size = 79, normalized size = 0.86

$$\frac{\frac{\left(\sinh^5(bx+a)\right)\left(\cosh^3(bx+a)\right)}{8} - \frac{5\left(\cosh^3(bx+a)\right)\left(\sinh^3(bx+a)\right)}{48} + \frac{5\left(\cosh^3(bx+a)\right)\sinh(bx+a)}{64} - \frac{5\cosh(bx+a)\sinh(bx+a)}{128} - \frac{5bx}{128} - \frac{5a}{128}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)^2*sinh(b*x+a)^6,x)
```

```
[Out] 1/b*(1/8*sinh(b*x+a)^5*cosh(b*x+a)^3-5/48*cosh(b*x+a)^3*sinh(b*x+a)^3+5/64*
cosh(b*x+a)^3*sinh(b*x+a)-5/128*cosh(b*x+a)*sinh(b*x+a)-5/128*b*x-5/128*a)
```

maxima [A] time = 0.32, size = 110, normalized size = 1.20

$$-\frac{\left(16\,e^{(-2\,bx-2\,a)} - 24\,e^{(-4\,bx-4\,a)} - 48\,e^{(-6\,bx-6\,a)} - 3\right)e^{(8\,bx+8\,a)}}{6144\,b} - \frac{5\,(bx+a)}{128\,b} - \frac{48\,e^{(-2\,bx-2\,a)} + 24\,e^{(-4\,bx-4\,a)} - 16\,e^{(-6\,bx-6}}{6144\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^2*sinh(b*x+a)^6,x, algorithm="maxima")
```

```
[Out] -1/6144*(16*e^(-2*b*x - 2*a) - 24*e^(-4*b*x - 4*a) - 48*e^(-6*b*x - 6*a) -
3)*e^(8*b*x + 8*a)/b - 5/128*(b*x + a)/b - 1/6144*(48*e^(-2*b*x - 2*a) + 24
*e^(-4*b*x - 4*a) - 16*e^(-6*b*x - 6*a) + 3*e^(-8*b*x - 8*a))/b
```

mupad [B] time = 1.73, size = 53, normalized size = 0.58

$$\frac{\frac{\sinh(2\,a+2\,b\,x)}{64} + \frac{\sinh(4\,a+4\,b\,x)}{128} - \frac{\sinh(6\,a+6\,b\,x)}{192} + \frac{\sinh(8\,a+8\,b\,x)}{1024}}{b} - \frac{5\,x}{128}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a + b*x)^2*sinh(a + b*x)^6,x)
```

```
[Out] (sinh(2*a + 2*b*x)/64 + sinh(4*a + 4*b*x)/128 - sinh(6*a + 6*b*x)/192 + sin
h(8*a + 8*b*x)/1024)/b - (5*x)/128
```

sympy [A] time = 8.32, size = 189, normalized size = 2.05

$$\begin{cases} -\frac{5x\sinh^8(a+bx)}{128} + \frac{5x\sinh^6(a+bx)\cosh^2(a+bx)}{32} - \frac{15x\sinh^4(a+bx)\cosh^4(a+bx)}{64} + \frac{5x\sinh^2(a+bx)\cosh^6(a+bx)}{32} - \frac{5x\cosh^8(a+bx)}{128} + \frac{5\sin \\ x\sinh^6(a)\cosh^2(a) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)**2*sinh(b*x+a)**6,x)
```

```
[Out] Piecewise((-5*x*sinh(a + b*x)**8/128 + 5*x*sinh(a + b*x)**6*cosh(a + b*x)**
2/32 - 15*x*sinh(a + b*x)**4*cosh(a + b*x)**4/64 + 5*x*sinh(a + b*x)**2*cos
h(a + b*x)**6/32 - 5*x*cosh(a + b*x)**8/128 + 5*sinh(a + b*x)**7*cosh(a + b
*x)/(128*b) + 73*sinh(a + b*x)**5*cosh(a + b*x)**3/(384*b) - 55*sinh(a + b*
x)**3*cosh(a + b*x)**5/(384*b) + 5*sinh(a + b*x)*cosh(a + b*x)**7/(128*b),
Ne(b, 0)), (x*sinh(a)**6*cosh(a)**2, True))
```

3.18 $\int \cosh^4(a+bx)\sinh^2(a+bx)\,dx$

Optimal. Leaf size=67

$$\frac{\sinh(a+bx)\cosh^5(a+bx)}{6b}-\frac{\sinh(a+bx)\cosh^3(a+bx)}{24b}-\frac{\sinh(a+bx)\cosh(a+bx)}{16b}-\frac{x}{16}$$

```
[Out] -1/16*x-1/16*cosh(b*x+a)*sinh(b*x+a)/b-1/24*cosh(b*x+a)^3*sinh(b*x+a)/b+1/6
*cosh(b*x+a)^5*sinh(b*x+a)/b
```

Rubi [A] time = 0.05, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {2568, 2635, 8}

$$\frac{\sinh(a+bx)\cosh^5(a+bx)}{6b}-\frac{\sinh(a+bx)\cosh^3(a+bx)}{24b}-\frac{\sinh(a+bx)\cosh(a+bx)}{16b}-\frac{x}{16}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[a + b*x]^4*Sinh[a + b*x]^2,x]
```

```
[Out] -x/16 - (Cosh[a + b*x]*Sinh[a + b*x])/(16*b) - (Cosh[a + b*x]^3*Sinh[a + b*
x])/(24*b) + (Cosh[a + b*x]^5*Sinh[a + b*x])/(6*b)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2568

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m - 1)
)/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Cos[e + f*x])^n*(a
*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rubi steps

$$\int \cosh^4(a+bx)\sinh^2(a+bx)\,dx = \frac{\cosh^5(a+bx)\sinh(a+bx)}{6b} - \frac{1}{6}\int \cosh^4(a+bx)\,dx$$
$$= -\frac{\cosh^3(a+bx)\sinh(a+bx)}{24b} + \frac{\cosh^5(a+bx)\sinh(a+bx)}{6b} - \frac{1}{8}\int \cosh^2($$
$$= -\frac{\cosh(a+bx)\sinh(a+bx)}{16b} - \frac{\cosh^3(a+bx)\sinh(a+bx)}{24b} + \frac{\cosh^5(a+bx}{6}$$
$$= -\frac{x}{16} - \frac{\cosh(a+bx)\sinh(a+bx)}{16b} - \frac{\cosh^3(a+bx)\sinh(a+bx)}{24b} + \frac{\cosh^5(}{}$$

Mathematica [A] time = 0.06, size = 40, normalized size = 0.60

$$\frac{-3\sinh(2(a+bx)) + 3\sinh(4(a+bx)) + \sinh(6(a+bx)) - 12bx}{192b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[a + b*x]^4*Sinh[a + b*x]^2,x]
```

```
[Out] (-12*b*x - 3*Sinh[2*(a + b*x)] + 3*Sinh[4*(a + b*x)] + Sinh[6*(a + b*x)])/(
192*b)
```

fricas [A] time = 0.39, size = 90, normalized size = 1.34

$$\frac{3\,\cosh\left(bx+a\right)\sinh\left(bx+a\right)^{5} + 2\left(5\,\cosh\left(bx+a\right)^{3} + 3\,\cosh\left(bx+a\right)\right)\sinh\left(bx+a\right)^{3} - 6\,bx + 3\left(\cosh\left(bx+a\right)\right.}{96\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^4*sinh(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/96*(3*cosh(b*x + a)*sinh(b*x + a)^5 + 2*(5*cosh(b*x + a)^3 + 3*cosh(b*x +
 a))*sinh(b*x + a)^3 - 6*b*x + 3*(cosh(b*x + a)^5 + 2*cosh(b*x + a)^3 - cos
h(b*x + a))*sinh(b*x + a))/b
```

giac [A] time = 0.16, size = 88, normalized size = 1.31

$$-\frac{1}{16}\,x + \frac{e^{(6\,bx+6\,a)}}{384\,b} + \frac{e^{(4\,bx+4\,a)}}{128\,b} - \frac{e^{(2\,bx+2\,a)}}{128\,b} + \frac{e^{(-2\,bx-2\,a)}}{128\,b} - \frac{e^{(-4\,bx-4\,a)}}{128\,b} - \frac{e^{(-6\,bx-6\,a)}}{384\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^4*sinh(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -1/16*x + 1/384*e^(6*b*x + 6*a)/b + 1/128*e^(4*b*x + 4*a)/b - 1/128*e^(2*b*
x + 2*a)/b + 1/128*e^(-2*b*x - 2*a)/b - 1/128*e^(-4*b*x - 4*a)/b - 1/384*e^
(-6*b*x - 6*a)/b
```

maple [A] time = 0.33, size = 56, normalized size = 0.84

$$\frac{\frac{\sinh(bx+a)\left(\cosh^5(bx+a)\right)}{6}-\frac{\left(\frac{\left(\cosh^3(bx+a)\right)}{4}+\frac{3\cosh(bx+a)}{8}\right)\sinh(bx+a)}{6}-\frac{bx}{16}-\frac{a}{16}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)^4*sinh(b*x+a)^2,x)
```

```
[Out] 1/b*(1/6*sinh(b*x+a)*cosh(b*x+a)^5-1/6*(1/4*cosh(b*x+a)^3+3/8*cosh(b*x+a))*
sinh(b*x+a)-1/16*b*x-1/16*a)
```

maxima [A] time = 0.31, size = 88, normalized size = 1.31

$$\frac{\left(3\,e^{(-2\,bx-2\,a)}-3\,e^{(-4\,bx-4\,a)}+1\right)e^{(6\,bx+6\,a)}}{384\,b}-\frac{bx+a}{16\,b}+\frac{3\,e^{(-2\,bx-2\,a)}-3\,e^{(-4\,bx-4\,a)}-e^{(-6\,bx-6\,a)}}{384\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^4*sinh(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/384*(3*e^(-2*b*x - 2*a) - 3*e^(-4*b*x - 4*a) + 1)*e^(6*b*x + 6*a)/b - 1/1
6*(b*x + a)/b + 1/384*(3*e^(-2*b*x - 2*a) - 3*e^(-4*b*x - 4*a) - e^(-6*b*x
- 6*a))/b
```

mupad [B] time = 1.60, size = 42, normalized size = 0.63

$$\frac{\frac{\sinh(4\,a+4\,b\,x)}{64}-\frac{\sinh(2\,a+2\,b\,x)}{64}+\frac{\sinh(6\,a+6\,b\,x)}{192}}{b}-\frac{x}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a + b*x)^4*sinh(a + b*x)^2,x)
```

```
[Out] (sinh(4*a + 4*b*x)/64 - sinh(2*a + 2*b*x)/64 + sinh(6*a + 6*b*x)/192)/b - x
/16
```

sympy [A] time = 2.87, size = 136, normalized size = 2.03

$$\begin{cases} \frac{x\sinh^6(a+bx)}{16}-\frac{3x\sinh^4(a+bx)\cosh^2(a+bx)}{16}+\frac{3x\sinh^2(a+bx)\cosh^4(a+bx)}{16}-\frac{x\cosh^6(a+bx)}{16}-\frac{\sinh^5(a+bx)\cosh(a+bx)}{16b}+\frac{\sinh^3(a+bx)\,c}{6b} \\ x\sinh^2(a)\cosh^4(a) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)**4*sinh(b*x+a)**2,x)
```

```
[Out] Piecewise((x*sinh(a + b*x)**6/16 - 3*x*sinh(a + b*x)**4*cosh(a + b*x)**2/16
 + 3*x*sinh(a + b*x)**2*cosh(a + b*x)**4/16 - x*cosh(a + b*x)**6/16 - sinh(
a + b*x)**5*cosh(a + b*x)/(16*b) + sinh(a + b*x)**3*cosh(a + b*x)**3/(6*b)
+ sinh(a + b*x)*cosh(a + b*x)**5/(16*b), Ne(b, 0)), (x*sinh(a)**2*cosh(a)**
4, True))
```

3.19 $\int \cosh^4(a+bx)\sinh^4(a+bx)\,dx$

Optimal. Leaf size=90

$$\frac{\sinh^3(a+bx)\cosh^5(a+bx)}{8b}-\frac{\sinh(a+bx)\cosh^5(a+bx)}{16b}+\frac{\sinh(a+bx)\cosh^3(a+bx)}{64b}+\frac{3\sinh(a+bx)\cosh(a+$$

[Out] 3/128*x+3/128*cosh(b*x+a)*sinh(b*x+a)/b+1/64*cosh(b*x+a)^3*sinh(b*x+a)/b-1/16*cosh(b*x+a)^5*sinh(b*x+a)/b+1/8*cosh(b*x+a)^5*sinh(b*x+a)^3/b

Rubi [A] time = 0.08, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {2568, 2635, 8}

$$\frac{\sinh^3(a+bx)\cosh^5(a+bx)}{8b}-\frac{\sinh(a+bx)\cosh^5(a+bx)}{16b}+\frac{\sinh(a+bx)\cosh^3(a+bx)}{64b}+\frac{3\sinh(a+bx)\cosh(a+$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[a + b*x]^4*Sinh[a + b*x]^4,x]
```

```
[Out] (3*x)/128 + (3*Cosh[a + b*x]*Sinh[a + b*x])/(128*b) + (Cosh[a + b*x]^3*Sinh
[a + b*x])/(64*b) - (Cosh[a + b*x]^5*Sinh[a + b*x])/(16*b) + (Cosh[a + b*x]
^5*Sinh[a + b*x]^3)/(8*b)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2568

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m - 1)
)/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Cos[e + f*x])^n*(a
*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rubi steps

$$\int \cosh^4(a+bx)\sinh^4(a+bx)\,dx = \frac{\cosh^5(a+bx)\sinh^3(a+bx)}{8b} - \frac{3}{8}\int \cosh^4(a+bx)\sinh^2(a+bx)\,dx$$
$$= -\frac{\cosh^5(a+bx)\sinh(a+bx)}{16b} + \frac{\cosh^5(a+bx)\sinh^3(a+bx)}{8b} + \frac{1}{16}\int \cosh$$
$$= \frac{\cosh^3(a+bx)\sinh(a+bx)}{64b} - \frac{\cosh^5(a+bx)\sinh(a+bx)}{16b} + \frac{\cosh^5(a+bx}{8}$$
$$= \frac{3\cosh(a+bx)\sinh(a+bx)}{128b} + \frac{\cosh^3(a+bx)\sinh(a+bx)}{64b} - \frac{\cosh^5(a+bx}{1}$$
$$= \frac{3x}{128} + \frac{3\cosh(a+bx)\sinh(a+bx)}{128b} + \frac{\cosh^3(a+bx)\sinh(a+bx)}{64b} - \cosh^5$$

Mathematica [A] time = 0.05, size = 33, normalized size = 0.37

$$\frac{24(a+bx) - 8\sinh(4(a+bx)) + \sinh(8(a+bx))}{1024b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[a + b*x]^4*Sinh[a + b*x]^4,x]
```

```
[Out] (24*(a + b*x) - 8*Sinh[4*(a + b*x)] + Sinh[8*(a + b*x)])/(1024*b)
```

fricas [A] time = 0.39, size = 97, normalized size = 1.08

$$\frac{7\cosh(bx+a)^3\sinh(bx+a)^5 + \cosh(bx+a)\sinh(bx+a)^7 + \left(7\cosh(bx+a)^5 - 4\cosh(bx+a)\right)\sinh(bx}{128\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^4*sinh(b*x+a)^4,x, algorithm="fricas")
```

```
[Out] 1/128*(7*cosh(b*x + a)^3*sinh(b*x + a)^5 + cosh(b*x + a)*sinh(b*x + a)^7 +
(7*cosh(b*x + a)^5 - 4*cosh(b*x + a))*sinh(b*x + a)^3 + 3*b*x + (cosh(b*x +
 a)^7 - 4*cosh(b*x + a)^3)*sinh(b*x + a))/b
```

giac [A] time = 0.23, size = 60, normalized size = 0.67

$$\frac{3}{128}x + \frac{e^{(8bx+8a)}}{2048b} - \frac{e^{(4bx+4a)}}{256b} + \frac{e^{(-4bx-4a)}}{256b} - \frac{e^{(-8bx-8a)}}{2048b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^4*sinh(b*x+a)^4,x, algorithm="giac")
```

```
[Out] 3/128*x + 1/2048*e^(8*b*x + 8*a)/b - 1/256*e^(4*b*x + 4*a)/b + 1/256*e^(-4*
b*x - 4*a)/b - 1/2048*e^(-8*b*x - 8*a)/b
```

maple [A] time = 0.33, size = 74, normalized size = 0.82

$$\frac{\frac{\left(\sinh^3(bx+a)\right)\left(\cosh^5(bx+a)\right)}{8} - \frac{\sinh(bx+a)\left(\cosh^5(bx+a)\right)}{16} + \frac{\left(\frac{\left(\cosh^3(bx+a)\right)}{4} + \frac{3\cosh(bx+a)}{8}\right)\sinh(bx+a)}{16} + \frac{3bx}{128} + \frac{3a}{128}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)^4*sinh(b*x+a)^4,x)
```

```
[Out] 1/b*(1/8*sinh(b*x+a)^3*cosh(b*x+a)^5-1/16*sinh(b*x+a)*cosh(b*x+a)^5+1/16*(1
/4*cosh(b*x+a)^3+3/8*cosh(b*x+a))*sinh(b*x+a)+3/128*b*x+3/128*a)
```

maxima [A] time = 0.31, size = 66, normalized size = 0.73

$$-\frac{\left(8\, e^{(-4\, bx - 4\, a)} - 1\right)e^{(8\, bx + 8\, a)}}{2048\, b} + \frac{3\,(bx + a)}{128\, b} + \frac{8\, e^{(-4\, bx - 4\, a)} - e^{(-8\, bx - 8\, a)}}{2048\, b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^4*sinh(b*x+a)^4,x, algorithm="maxima")
```

```
[Out] -1/2048*(8*e^(-4*b*x - 4*a) - 1)*e^(8*b*x + 8*a)/b + 3/128*(b*x + a)/b + 1/
2048*(8*e^(-4*b*x - 4*a) - e^(-8*b*x - 8*a))/b
```

mupad [B] time = 0.20, size = 32, normalized size = 0.36

$$\frac{3\,x}{128} - \frac{\frac{\sinh(4\,a+4\,b\,x)}{128} - \frac{\sinh(8\,a+8\,b\,x)}{1024}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a + b*x)^4*sinh(a + b*x)^4,x)
```

```
[Out] (3*x)/128 - (sinh(4*a + 4*b*x)/128 - sinh(8*a + 8*b*x)/1024)/b
```

sympy [A] time = 8.29, size = 189, normalized size = 2.10

$$\begin{cases} \frac{3x\sinh^8(a+bx)}{128} - \frac{3x\sinh^6(a+bx)\cosh^2(a+bx)}{32} + \frac{9x\sinh^4(a+bx)\cosh^4(a+bx)}{64} - \frac{3x\sinh^2(a+bx)\cosh^6(a+bx)}{32} + \frac{3x\cosh^8(a+bx)}{128} - \frac{3\sinh^7(}{} \\ x\sinh^4(a)\cosh^4(a) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)**4*sinh(b*x+a)**4,x)
```

```
[Out] Piecewise((3*x*sinh(a + b*x)**8/128 - 3*x*sinh(a + b*x)**6*cosh(a + b*x)**2
/32 + 9*x*sinh(a + b*x)**4*cosh(a + b*x)**4/64 - 3*x*sinh(a + b*x)**2*cosh(
a + b*x)**6/32 + 3*x*cosh(a + b*x)**8/128 - 3*sinh(a + b*x)**7*cosh(a + b*x
)/(128*b) + 11*sinh(a + b*x)**5*cosh(a + b*x)**3/(128*b) + 11*sinh(a + b*x)
**3*cosh(a + b*x)**5/(128*b) - 3*sinh(a + b*x)*cosh(a + b*x)**7/(128*b), Ne
(b, 0)), (x*sinh(a)**4*cosh(a)**4, True))
```

3.20 $\int \cosh^4(a+bx)\sinh^6(a+bx)\,dx$

Optimal. Leaf size=113

$$\frac{\sinh^5(a+bx)\cosh^5(a+bx)}{10b}-\frac{\sinh^3(a+bx)\cosh^5(a+bx)}{16b}+\frac{\sinh(a+bx)\cosh^5(a+bx)}{32b}-\frac{\sinh(a+bx)\cosh^3(a+}{128b}$$

```
[Out] -3/256*x-3/256*cosh(b*x+a)*sinh(b*x+a)/b-1/128*cosh(b*x+a)^3*sinh(b*x+a)/b+
1/32*cosh(b*x+a)^5*sinh(b*x+a)/b-1/16*cosh(b*x+a)^5*sinh(b*x+a)^3/b+1/10*co
sh(b*x+a)^5*sinh(b*x+a)^5/b
```

Rubi [A] time = 0.12, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {2568, 2635, 8}

$$\frac{\sinh^5(a+bx)\cosh^5(a+bx)}{10b}-\frac{\sinh^3(a+bx)\cosh^5(a+bx)}{16b}+\frac{\sinh(a+bx)\cosh^5(a+bx)}{32b}-\frac{\sinh(a+bx)\cosh^3(a+}{128b}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[a + b*x]^4*Sinh[a + b*x]^6,x]
```

```
[Out] (-3*x)/256 - (3*Cosh[a + b*x]*Sinh[a + b*x])/(256*b) - (Cosh[a + b*x]^3*Sin
h[a + b*x])/(128*b) + (Cosh[a + b*x]^5*Sinh[a + b*x])/(32*b) - (Cosh[a + b*
x]^5*Sinh[a + b*x]^3)/(16*b) + (Cosh[a + b*x]^5*Sinh[a + b*x]^5)/(10*b)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2568

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m - 1)
)/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Cos[e + f*x])^n*(a
*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rubi steps

$$\int \cosh^4(a+bx)\sinh^6(a+bx)\,dx = \frac{\cosh^5(a+bx)\sinh^5(a+bx)}{10b} - \frac{1}{2}\int \cosh^4(a+bx)\sinh^4(a+bx)\,dx$$
$$= -\frac{\cosh^5(a+bx)\sinh^3(a+bx)}{16b} + \frac{\cosh^5(a+bx)\sinh^5(a+bx)}{10b} + \frac{3}{16}\int \cos$$
$$= \frac{\cosh^5(a+bx)\sinh(a+bx)}{32b} - \frac{\cosh^5(a+bx)\sinh^3(a+bx)}{16b} + \frac{\cosh^5(a+b}{}$$
$$= -\frac{\cosh^3(a+bx)\sinh(a+bx)}{128b} + \frac{\cosh^5(a+bx)\sinh(a+bx)}{32b} - \frac{\cosh^5(a+b}{}$$
$$= -\frac{3\cosh(a+bx)\sinh(a+bx)}{256b} - \frac{\cosh^3(a+bx)\sinh(a+bx)}{128b} + \frac{\cosh^5(a+}{}$$
$$= -\frac{3x}{256} - \frac{3\cosh(a+bx)\sinh(a+bx)}{256b} - \frac{\cosh^3(a+bx)\sinh(a+bx)}{128b} + \frac{\cosh}{}$$

Mathematica [A] time = 0.18, size = 62, normalized size = 0.55

$$\frac{20\sinh(2(a+bx)) + 40\sinh(4(a+bx)) - 10\sinh(6(a+bx)) - 5\sinh(8(a+bx)) + 2\sinh(10(a+bx)) - 120bx}{10240b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[a + b*x]^4*Sinh[a + b*x]^6,x]
```

```
[Out] (-120*b*x + 20*Sinh[2*(a + b*x)] + 40*Sinh[4*(a + b*x)] - 10*Sinh[6*(a + b*
x)] - 5*Sinh[8*(a + b*x)] + 2*Sinh[10*(a + b*x)])/(10240*b)
```

fricas [A] time = 0.44, size = 197, normalized size = 1.74

$$\frac{5\cosh(bx+a)\sinh(bx+a)^9 + 10\left(6\cosh(bx+a)^3 - \cosh(bx+a)\right)\sinh(bx+a)^7 + \left(126\cosh(bx+a)^5 - 7\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^4*sinh(b*x+a)^6,x, algorithm="fricas")
```

```
[Out] 1/2560*(5*cosh(b*x + a)*sinh(b*x + a)^9 + 10*(6*cosh(b*x + a)^3 - cosh(b*x
+ a))*sinh(b*x + a)^7 + (126*cosh(b*x + a)^5 - 70*cosh(b*x + a)^3 - 15*cosh
(b*x + a))*sinh(b*x + a)^5 + 10*(6*cosh(b*x + a)^7 - 7*cosh(b*x + a)^5 - 5*
cosh(b*x + a)^3 + 4*cosh(b*x + a))*sinh(b*x + a)^3 - 30*b*x + 5*(cosh(b*x +
 a)^9 - 2*cosh(b*x + a)^7 - 3*cosh(b*x + a)^5 + 8*cosh(b*x + a)^3 + 2*cosh(
b*x + a))*sinh(b*x + a))/b
```

giac [A] time = 0.16, size = 144, normalized size = 1.27

$$-\frac{3}{256}x+\frac{e^{(10bx+10a)}}{10240b}-\frac{e^{(8bx+8a)}}{4096b}-\frac{e^{(6bx+6a)}}{2048b}+\frac{e^{(4bx+4a)}}{512b}+\frac{e^{(2bx+2a)}}{1024b}-\frac{e^{(-2bx-2a)}}{1024b}-\frac{e^{(-4bx-4a)}}{512b}+\frac{e^{(-6bx-6a)}}{2048b}+\frac{e^{(-8bx-8a)}}{4096b}-\frac{e^{(-}}{1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^4*sinh(b*x+a)^6,x, algorithm="giac")
```

```
[Out] -3/256*x + 1/10240*e^(10*b*x + 10*a)/b - 1/4096*e^(8*b*x + 8*a)/b - 1/2048*
e^(6*b*x + 6*a)/b + 1/512*e^(4*b*x + 4*a)/b + 1/1024*e^(2*b*x + 2*a)/b - 1/
1024*e^(-2*b*x - 2*a)/b - 1/512*e^(-4*b*x - 4*a)/b + 1/2048*e^(-6*b*x - 6*a
)/b + 1/4096*e^(-8*b*x - 8*a)/b - 1/10240*e^(-10*b*x - 10*a)/b
```

maple [A] time = 0.30, size = 92, normalized size = 0.81

$$\frac{\frac{(\sinh^5(bx+a))(\cosh^5(bx+a))}{10}-\frac{(\sinh^3(bx+a))(\cosh^5(bx+a))}{16}+\frac{\sinh(bx+a)(\cosh^5(bx+a))}{32}-\frac{\left(\frac{(\cosh^3(bx+a))}{4}+\frac{3\cosh(bx+a)}{8}\right)\sinh(bx+a)}{32}-\frac{3bx}{256}-\frac{3}{2}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)^4*sinh(b*x+a)^6,x)
```

```
[Out] 1/b*(1/10*sinh(b*x+a)^5*cosh(b*x+a)^5-1/16*sinh(b*x+a)^3*cosh(b*x+a)^5+1/32
*sinh(b*x+a)*cosh(b*x+a)^5-1/32*(1/4*cosh(b*x+a)^3+3/8*cosh(b*x+a))*sinh(b*
x+a)-3/256*b*x-3/256*a)
```

maxima [A] time = 0.34, size = 132, normalized size = 1.17

$$-\frac{\left(5e^{(-2bx-2a)}+10e^{(-4bx-4a)}-40e^{(-6bx-6a)}-20e^{(-8bx-8a)}-2\right)e^{(10bx+10a)}}{20480b}-\frac{3(bx+a)}{256b}-\frac{20e^{(-2bx-2a)}+40e^{(-4bx-}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^4*sinh(b*x+a)^6,x, algorithm="maxima")
```

```
[Out] -1/20480*(5*e^(-2*b*x - 2*a) + 10*e^(-4*b*x - 4*a) - 40*e^(-6*b*x - 6*a) -
20*e^(-8*b*x - 8*a) - 2)*e^(10*b*x + 10*a)/b - 3/256*(b*x + a)/b - 1/20480*
(20*e^(-2*b*x - 2*a) + 40*e^(-4*b*x - 4*a) - 10*e^(-6*b*x - 6*a) - 5*e^(-8*
b*x - 8*a) + 2*e^(-10*b*x - 10*a))/b
```

mupad [B] time = 1.74, size = 65, normalized size = 0.58

$$\frac{20\sinh(2a+2bx)+40\sinh(4a+4bx)-10\sinh(6a+6bx)-5\sinh(8a+8bx)+2\sinh(10a+10bx)-1}{10240b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a + b*x)^4*sinh(a + b*x)^6,x)
```

```
[Out] (20*sinh(2*a + 2*b*x) + 40*sinh(4*a + 4*b*x) - 10*sinh(6*a + 6*b*x) - 5*sin
h(8*a + 8*b*x) + 2*sinh(10*a + 10*b*x) - 120*b*x)/(10240*b)
```

sympy [A] time = 21.04, size = 231, normalized size = 2.04

$$\begin{cases} \frac{3x\sinh^{10}(a+bx)}{256} - \frac{15x\sinh^{8}(a+bx)\cosh^{2}(a+bx)}{256} + \frac{15x\sinh^{6}(a+bx)\cosh^{4}(a+bx)}{128} - \frac{15x\sinh^{4}(a+bx)\cosh^{6}(a+bx)}{128} + \frac{15x\sinh^{2}(a+bx)\cosh}{256} \\ x\sinh^{6}(a)\cosh^{4}(a) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)**4*sinh(b*x+a)**6,x)
```

```
[Out] Piecewise((3*x*sinh(a + b*x)**10/256 - 15*x*sinh(a + b*x)**8*cosh(a + b*x)*
*2/256 + 15*x*sinh(a + b*x)**6*cosh(a + b*x)**4/128 - 15*x*sinh(a + b*x)**4
*cosh(a + b*x)**6/128 + 15*x*sinh(a + b*x)**2*cosh(a + b*x)**8/256 - 3*x*co
sh(a + b*x)**10/256 - 3*sinh(a + b*x)**9*cosh(a + b*x)/(256*b) + 7*sinh(a +
 b*x)**7*cosh(a + b*x)**3/(128*b) + sinh(a + b*x)**5*cosh(a + b*x)**5/(10*b
) - 7*sinh(a + b*x)**3*cosh(a + b*x)**7/(128*b) + 3*sinh(a + b*x)*cosh(a +
b*x)**9/(256*b), Ne(b, 0)), (x*sinh(a)**6*cosh(a)**4, True))
```

3.21 $\int \cosh^6(a+bx)\sinh^2(a+bx)\,dx$

Optimal. Leaf size=88

$$\frac{\sinh(a+bx)\cosh^7(a+bx)}{8b}-\frac{\sinh(a+bx)\cosh^5(a+bx)}{48b}-\frac{5\sinh(a+bx)\cosh^3(a+bx)}{192b}-\frac{5\sinh(a+bx)\cosh(a+}{128b}$$

```
[Out] -5/128*x-5/128*cosh(b*x+a)*sinh(b*x+a)/b-5/192*cosh(b*x+a)^3*sinh(b*x+a)/b-
1/48*cosh(b*x+a)^5*sinh(b*x+a)/b+1/8*cosh(b*x+a)^7*sinh(b*x+a)/b
```

Rubi [A] time = 0.06, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {2568, 2635, 8}

$$\frac{\sinh(a+bx)\cosh^7(a+bx)}{8b}-\frac{\sinh(a+bx)\cosh^5(a+bx)}{48b}-\frac{5\sinh(a+bx)\cosh^3(a+bx)}{192b}-\frac{5\sinh(a+bx)\cosh(a+}{128b}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[a + b*x]^6*Sinh[a + b*x]^2,x]
```

```
[Out] (-5*x)/128 - (5*Cosh[a + b*x]*Sinh[a + b*x])/(128*b) - (5*Cosh[a + b*x]^3*S
inh[a + b*x])/(192*b) - (Cosh[a + b*x]^5*Sinh[a + b*x])/(48*b) + (Cosh[a +
b*x]^7*Sinh[a + b*x])/(8*b)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2568

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m - 1)
)/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Cos[e + f*x])^n*(a
*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rubi steps

$$\int \cosh^6(a+bx)\sinh^2(a+bx)\,dx = \frac{\cosh^7(a+bx)\sinh(a+bx)}{8b} - \frac{1}{8}\int \cosh^6(a+bx)\,dx$$
$$= -\frac{\cosh^5(a+bx)\sinh(a+bx)}{48b} + \frac{\cosh^7(a+bx)\sinh(a+bx)}{8b} - \frac{5}{48}\int \cosh^4$$
$$= -\frac{5\cosh^3(a+bx)\sinh(a+bx)}{192b} - \frac{\cosh^5(a+bx)\sinh(a+bx)}{48b} + \frac{\cosh^7(a+$$
$$= -\frac{5\cosh(a+bx)\sinh(a+bx)}{128b} - \frac{5\cosh^3(a+bx)\sinh(a+bx)}{192b} - \frac{\cosh^5(a+$$
$$= -\frac{5x}{128} - \frac{5\cosh(a+bx)\sinh(a+bx)}{128b} - \frac{5\cosh^3(a+bx)\sinh(a+bx)}{192b} - \cos$$

Mathematica [A] time = 0.10, size = 52, normalized size = 0.59

$$\frac{-48\sinh(2(a+bx)) + 24\sinh(4(a+bx)) + 16\sinh(6(a+bx)) + 3\sinh(8(a+bx)) - 120bx}{3072b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[a + b*x]^6*Sinh[a + b*x]^2,x]
```

```
[Out] (-120*b*x - 48*Sinh[2*(a + b*x)] + 24*Sinh[4*(a + b*x)] + 16*Sinh[6*(a + b*
x)] + 3*Sinh[8*(a + b*x)])/(3072*b)
```

fricas [A] time = 0.39, size = 138, normalized size = 1.57

$$3\cosh(bx+a)\sinh(bx+a)^7 + 3\left(7\cosh(bx+a)^3 + 4\cosh(bx+a)\right)\sinh(bx+a)^5 + \left(21\cosh(bx+a)^5 + 4\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^6*sinh(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/384*(3*cosh(b*x + a)*sinh(b*x + a)^7 + 3*(7*cosh(b*x + a)^3 + 4*cosh(b*x
+ a))*sinh(b*x + a)^5 + (21*cosh(b*x + a)^5 + 40*cosh(b*x + a)^3 + 12*cosh(
b*x + a))*sinh(b*x + a)^3 - 15*b*x + 3*(cosh(b*x + a)^7 + 4*cosh(b*x + a)^5
 + 4*cosh(b*x + a)^3 - 4*cosh(b*x + a))*sinh(b*x + a))/b
```

giac [A] time = 0.15, size = 116, normalized size = 1.32

$$-\frac{5}{128}x + \frac{e^{(8bx+8a)}}{2048b} + \frac{e^{(6bx+6a)}}{384b} + \frac{e^{(4bx+4a)}}{256b} - \frac{e^{(2bx+2a)}}{128b} + \frac{e^{(-2bx-2a)}}{128b} - \frac{e^{(-4bx-4a)}}{256b} - \frac{e^{(-6bx-6a)}}{384b} - \frac{e^{(-8bx-8a)}}{2048b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^6*sinh(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -5/128*x + 1/2048*e^(8*b*x + 8*a)/b + 1/384*e^(6*b*x + 6*a)/b + 1/256*e^(4*
b*x + 4*a)/b - 1/128*e^(2*b*x + 2*a)/b + 1/128*e^(-2*b*x - 2*a)/b - 1/256*e
^(-4*b*x - 4*a)/b - 1/384*e^(-6*b*x - 6*a)/b - 1/2048*e^(-8*b*x - 8*a)/b
```

maple [A] time = 0.33, size = 66, normalized size = 0.75

$$\frac{\frac{\sinh(bx+a)\left(\cosh^7(bx+a)\right)}{8}-\frac{\left(\frac{\left(\cosh^5(bx+a)\right)}{6}+\frac{5\left(\cosh^3(bx+a)\right)}{24}+\frac{5\cosh(bx+a)}{16}\right)\sinh(bx+a)}{8}-\frac{5bx}{128}-\frac{5a}{128}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)^6*sinh(b*x+a)^2,x)
```

```
[Out] 1/b*(1/8*sinh(b*x+a)*cosh(b*x+a)^7-1/8*(1/6*cosh(b*x+a)^5+5/24*cosh(b*x+a)^
3+5/16*cosh(b*x+a))*sinh(b*x+a)-5/128*b*x-5/128*a)
```

maxima [A] time = 0.44, size = 110, normalized size = 1.25

$$\frac{\left(16\,e^{(-2\,bx-2\,a)}+24\,e^{(-4\,bx-4\,a)}-48\,e^{(-6\,bx-6\,a)}+3\right)e^{(8\,bx+8\,a)}}{6144\,b}-\frac{5\,(bx+a)}{128\,b}+\frac{48\,e^{(-2\,bx-2\,a)}-24\,e^{(-4\,bx-4\,a)}-16\,e^{(-6\,bx-6\,a}}{6144\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^6*sinh(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/6144*(16*e^(-2*b*x - 2*a) + 24*e^(-4*b*x - 4*a) - 48*e^(-6*b*x - 6*a) + 3
)*e^(8*b*x + 8*a)/b - 5/128*(b*x + a)/b + 1/6144*(48*e^(-2*b*x - 2*a) - 24*
e^(-4*b*x - 4*a) - 16*e^(-6*b*x - 6*a) - 3*e^(-8*b*x - 8*a))/b
```

mupad [B] time = 1.68, size = 53, normalized size = 0.60

$$\frac{\frac{\sinh(4\,a+4\,b\,x)}{128}-\frac{\sinh(2\,a+2\,b\,x)}{64}+\frac{\sinh(6\,a+6\,b\,x)}{192}+\frac{\sinh(8\,a+8\,b\,x)}{1024}}{b}-\frac{5\,x}{128}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a + b*x)^6*sinh(a + b*x)^2,x)
```

```
[Out] (sinh(4*a + 4*b*x)/128 - sinh(2*a + 2*b*x)/64 + sinh(6*a + 6*b*x)/192 + sin
h(8*a + 8*b*x)/1024)/b - (5*x)/128
```

sympy [A] time = 8.35, size = 189, normalized size = 2.15

$$\begin{cases} -\frac{5x\sinh^8(a+bx)}{128}+\frac{5x\sinh^6(a+bx)\cosh^2(a+bx)}{32}-\frac{15x\sinh^4(a+bx)\cosh^4(a+bx)}{64}+\frac{5x\sinh^2(a+bx)\cosh^6(a+bx)}{32}-\frac{5x\cosh^8(a+bx)}{128}+\frac{5\sin}{} \\ x\sinh^2(a)\cosh^6(a) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)**6*sinh(b*x+a)**2,x)
```

```
[Out] Piecewise((-5*x*sinh(a + b*x)**8/128 + 5*x*sinh(a + b*x)**6*cosh(a + b*x)**
2/32 - 15*x*sinh(a + b*x)**4*cosh(a + b*x)**4/64 + 5*x*sinh(a + b*x)**2*cos
h(a + b*x)**6/32 - 5*x*cosh(a + b*x)**8/128 + 5*sinh(a + b*x)**7*cosh(a + b
*x)/(128*b) - 55*sinh(a + b*x)**5*cosh(a + b*x)**3/(384*b) + 73*sinh(a + b*
x)**3*cosh(a + b*x)**5/(384*b) + 5*sinh(a + b*x)*cosh(a + b*x)**7/(128*b),
Ne(b, 0)), (x*sinh(a)**2*cosh(a)**6, True))
```

3.22 $\int \cosh^6(a+bx)\sinh^4(a+bx)\,dx$

Optimal. Leaf size=111

$$\frac{\sinh^3(a+bx)\cosh^7(a+bx)}{10b}-\frac{3\sinh(a+bx)\cosh^7(a+bx)}{80b}+\frac{\sinh(a+bx)\cosh^5(a+bx)}{160b}+\frac{\sinh(a+bx)\cosh^3(a\cdots}{128b}$$

```
[Out] 3/256*x+3/256*cosh(b*x+a)*sinh(b*x+a)/b+1/128*cosh(b*x+a)^3*sinh(b*x+a)/b+1
/160*cosh(b*x+a)^5*sinh(b*x+a)/b-3/80*cosh(b*x+a)^7*sinh(b*x+a)/b+1/10*cosh
(b*x+a)^7*sinh(b*x+a)^3/b
```

Rubi [A] time = 0.10, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {2568, 2635, 8}

$$\frac{\sinh^3(a+bx)\cosh^7(a+bx)}{10b}-\frac{3\sinh(a+bx)\cosh^7(a+bx)}{80b}+\frac{\sinh(a+bx)\cosh^5(a+bx)}{160b}+\frac{\sinh(a+bx)\cosh^3(a\cdots}{128b}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[a + b*x]^6*Sinh[a + b*x]^4,x]
```

```
[Out] (3*x)/256 + (3*Cosh[a + b*x]*Sinh[a + b*x])/(256*b) + (Cosh[a + b*x]^3*Sinh
[a + b*x])/(128*b) + (Cosh[a + b*x]^5*Sinh[a + b*x])/(160*b) - (3*Cosh[a +
b*x]^7*Sinh[a + b*x])/(80*b) + (Cosh[a + b*x]^7*Sinh[a + b*x]^3)/(10*b)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2568

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m - 1)
)/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Cos[e + f*x])^n*(a
*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rubi steps

$$\int \cosh^6(a+bx)\sinh^4(a+bx)\,dx = \frac{\cosh^7(a+bx)\sinh^3(a+bx)}{10b} - \frac{3}{10}\int \cosh^6(a+bx)\sinh^2(a+bx)\,dx$$
$$= -\frac{3\cosh^7(a+bx)\sinh(a+bx)}{80b} + \frac{\cosh^7(a+bx)\sinh^3(a+bx)}{10b} + \frac{3}{80}\int \text{cos}$$
$$= \frac{\cosh^5(a+bx)\sinh(a+bx)}{160b} - \frac{3\cosh^7(a+bx)\sinh(a+bx)}{80b} + \frac{\cosh^7(a+b}{}$$
$$= \frac{\cosh^3(a+bx)\sinh(a+bx)}{128b} + \frac{\cosh^5(a+bx)\sinh(a+bx)}{160b} - \frac{3\cosh^7(a+b}{8}$$
$$= \frac{3\cosh(a+bx)\sinh(a+bx)}{256b} + \frac{\cosh^3(a+bx)\sinh(a+bx)}{128b} + \frac{\cosh^5(a+bx}{1}$$
$$= \frac{3x}{256} + \frac{3\cosh(a+bx)\sinh(a+bx)}{256b} + \frac{\cosh^3(a+bx)\sinh(a+bx)}{128b} + \frac{\cosh^5}{}$$

Mathematica [A] time = 0.13, size = 62, normalized size = 0.56

$$\frac{20\sinh(2(a+bx)) - 40\sinh(4(a+bx)) - 10\sinh(6(a+bx)) + 5\sinh(8(a+bx)) + 2\sinh(10(a+bx)) + 120bx}{10240b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[a + b*x]^6*Sinh[a + b*x]^4,x]
```

```
[Out] (120*b*x + 20*Sinh[2*(a + b*x)] - 40*Sinh[4*(a + b*x)] - 10*Sinh[6*(a + b*x
)] + 5*Sinh[8*(a + b*x)] + 2*Sinh[10*(a + b*x)])/(10240*b)
```

fricas [A] time = 0.39, size = 195, normalized size = 1.76

$$\frac{5\cosh(bx+a)\sinh(bx+a)^9 + 10\left(6\cosh(bx+a)^3 + \cosh(bx+a)\right)\sinh(bx+a)^7 + \left(126\cosh(bx+a)^5 + 7\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^6*sinh(b*x+a)^4,x, algorithm="fricas")
```

```
[Out] 1/2560*(5*cosh(b*x + a)*sinh(b*x + a)^9 + 10*(6*cosh(b*x + a)^3 + cosh(b*x
+ a))*sinh(b*x + a)^7 + (126*cosh(b*x + a)^5 + 70*cosh(b*x + a)^3 - 15*cosh
(b*x + a))*sinh(b*x + a)^5 + 10*(6*cosh(b*x + a)^7 + 7*cosh(b*x + a)^5 - 5*
cosh(b*x + a)^3 - 4*cosh(b*x + a))*sinh(b*x + a)^3 + 30*b*x + 5*(cosh(b*x +
 a)^9 + 2*cosh(b*x + a)^7 - 3*cosh(b*x + a)^5 - 8*cosh(b*x + a)^3 + 2*cosh(
b*x + a))*sinh(b*x + a))/b
```

giac [A] time = 0.18, size = 144, normalized size = 1.30

$$\frac{3}{256}\,x+\frac{e^{(10\,bx+10\,a)}}{10240\,b}+\frac{e^{(8\,bx+8\,a)}}{4096\,b}-\frac{e^{(6\,bx+6\,a)}}{2048\,b}-\frac{e^{(4\,bx+4\,a)}}{512\,b}+\frac{e^{(2\,bx+2\,a)}}{1024\,b}-\frac{e^{(-2\,bx-2\,a)}}{1024\,b}+\frac{e^{(-4\,bx-4\,a)}}{512\,b}+\frac{e^{(-6\,bx-6\,a)}}{2048\,b}-\frac{e^{(-8\,bx-8\,a)}}{4096\,b}-\frac{e^{(-1}}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^6*sinh(b*x+a)^4,x, algorithm="giac")
```

```
[Out] 3/256*x + 1/10240*e^(10*b*x + 10*a)/b + 1/4096*e^(8*b*x + 8*a)/b - 1/2048*e
^(6*b*x + 6*a)/b - 1/512*e^(4*b*x + 4*a)/b + 1/1024*e^(2*b*x + 2*a)/b - 1/1
024*e^(-2*b*x - 2*a)/b + 1/512*e^(-4*b*x - 4*a)/b + 1/2048*e^(-6*b*x - 6*a)
/b - 1/4096*e^(-8*b*x - 8*a)/b - 1/10240*e^(-10*b*x - 10*a)/b
```

maple [A] time = 0.33, size = 84, normalized size = 0.76

$$\frac{\frac{\left(\sinh^{3}(bx+a)\right)\left(\cosh^{7}(bx+a)\right)}{10}-\frac{3\sinh(bx+a)\left(\cosh^{7}(bx+a)\right)}{80}+\frac{3\left(\frac{\left(\cosh^{5}(bx+a)\right)}{6}+\frac{5\left(\cosh^{3}(bx+a)\right)}{24}+\frac{5\cosh(bx+a)}{16}\right)\sinh(bx+a)}{80}+\frac{3bx}{256}+\frac{3a}{256}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)^6*sinh(b*x+a)^4,x)
```

```
[Out] 1/b*(1/10*sinh(b*x+a)^3*cosh(b*x+a)^7-3/80*sinh(b*x+a)*cosh(b*x+a)^7+3/80*(
1/6*cosh(b*x+a)^5+5/24*cosh(b*x+a)^3+5/16*cosh(b*x+a))*sinh(b*x+a)+3/256*b*
x+3/256*a)
```

maxima [A] time = 0.38, size = 132, normalized size = 1.19

$$\frac{\left(5\,e^{(-2\,bx-2\,a)}-10\,e^{(-4\,bx-4\,a)}-40\,e^{(-6\,bx-6\,a)}+20\,e^{(-8\,bx-8\,a)}+2\right)e^{(10\,bx+10\,a)}}{20480\,b}+\frac{3\,(bx+a)}{256\,b}-\frac{20\,e^{(-2\,bx-2\,a)}-40\,e^{(-4\,bx-4}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^6*sinh(b*x+a)^4,x, algorithm="maxima")
```

```
[Out] 1/20480*(5*e^(-2*b*x - 2*a) - 10*e^(-4*b*x - 4*a) - 40*e^(-6*b*x - 6*a) + 2
0*e^(-8*b*x - 8*a) + 2)*e^(10*b*x + 10*a)/b + 3/256*(b*x + a)/b - 1/20480*(
20*e^(-2*b*x - 2*a) - 40*e^(-4*b*x - 4*a) - 10*e^(-6*b*x - 6*a) + 5*e^(-8*b
*x - 8*a) + 2*e^(-10*b*x - 10*a))/b
```

mupad [B] time = 0.26, size = 65, normalized size = 0.59

$$\frac{20\sinh(2\,a+2\,b\,x)-40\sinh(4\,a+4\,b\,x)-10\sinh(6\,a+6\,b\,x)+5\sinh(8\,a+8\,b\,x)+2\sinh(10\,a+10\,b\,x)+1}{10240\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a + b*x)^6*sinh(a + b*x)^4,x)
```

```
[Out] (20*sinh(2*a + 2*b*x) - 40*sinh(4*a + 4*b*x) - 10*sinh(6*a + 6*b*x) + 5*sin
h(8*a + 8*b*x) + 2*sinh(10*a + 10*b*x) + 120*b*x)/(10240*b)
```

sympy [A] time = 21.15, size = 231, normalized size = 2.08

$$\begin{cases} -\frac{3x\sinh^{10}(a+bx)}{256} + \frac{15x\sinh^{8}(a+bx)\cosh^{2}(a+bx)}{256} - \frac{15x\sinh^{6}(a+bx)\cosh^{4}(a+bx)}{128} + \frac{15x\sinh^{4}(a+bx)\cosh^{6}(a+bx)}{128} - \frac{15x\sinh^{2}(a+bx)\mathrm{co}}{256} \\ x\sinh^{4}(a)\cosh^{6}(a) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)**6*sinh(b*x+a)**4,x)
```

```
[Out] Piecewise((-3*x*sinh(a + b*x)**10/256 + 15*x*sinh(a + b*x)**8*cosh(a + b*x)
**2/256 - 15*x*sinh(a + b*x)**6*cosh(a + b*x)**4/128 + 15*x*sinh(a + b*x)**
4*cosh(a + b*x)**6/128 - 15*x*sinh(a + b*x)**2*cosh(a + b*x)**8/256 + 3*x*c
osh(a + b*x)**10/256 + 3*sinh(a + b*x)**9*cosh(a + b*x)/(256*b) - 7*sinh(a
+ b*x)**7*cosh(a + b*x)**3/(128*b) + sinh(a + b*x)**5*cosh(a + b*x)**5/(10*
b) + 7*sinh(a + b*x)**3*cosh(a + b*x)**7/(128*b) - 3*sinh(a + b*x)*cosh(a +
 b*x)**9/(256*b), Ne(b, 0)), (x*sinh(a)**4*cosh(a)**6, True))
```

3.23 $\int \cosh^6(a+bx)\sinh^6(a+bx)\,dx$

Optimal. Leaf size=134

$$\frac{\sinh^5(a+bx)\cosh^7(a+bx)}{12b}-\frac{\sinh^3(a+bx)\cosh^7(a+bx)}{24b}+\frac{\sinh(a+bx)\cosh^7(a+bx)}{64b}-\frac{\sinh(a+bx)\cosh^5(a+}{384b}$$

```
[Out] -5/1024*x-5/1024*cosh(b*x+a)*sinh(b*x+a)/b-5/1536*cosh(b*x+a)^3*sinh(b*x+a)
/b-1/384*cosh(b*x+a)^5*sinh(b*x+a)/b+1/64*cosh(b*x+a)^7*sinh(b*x+a)/b-1/24*
cosh(b*x+a)^7*sinh(b*x+a)^3/b+1/12*cosh(b*x+a)^7*sinh(b*x+a)^5/b
```

Rubi [A] time = 0.13, antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {2568, 2635, 8}

$$\frac{\sinh^5(a+bx)\cosh^7(a+bx)}{12b}-\frac{\sinh^3(a+bx)\cosh^7(a+bx)}{24b}+\frac{\sinh(a+bx)\cosh^7(a+bx)}{64b}-\frac{\sinh(a+bx)\cosh^5(a+}{384b}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[a + b*x]^6*Sinh[a + b*x]^6,x]
```

```
[Out] (-5*x)/1024 - (5*Cosh[a + b*x]*Sinh[a + b*x])/(1024*b) - (5*Cosh[a + b*x]^3
*Sinh[a + b*x])/(1536*b) - (Cosh[a + b*x]^5*Sinh[a + b*x])/(384*b) + (Cosh[
a + b*x]^7*Sinh[a + b*x])/(64*b) - (Cosh[a + b*x]^7*Sinh[a + b*x]^3)/(24*b)
 + (Cosh[a + b*x]^7*Sinh[a + b*x]^5)/(12*b)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2568

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m - 1)
)/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Cos[e + f*x])^n*(a
*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rubi steps

$$\int \cosh^6(a+bx)\sinh^6(a+bx)\,dx = \frac{\cosh^7(a+bx)\sinh^5(a+bx)}{12b} - \frac{5}{12}\int \cosh^6(a+bx)\sinh^4(a+bx)\,dx$$
$$= -\frac{\cosh^7(a+bx)\sinh^3(a+bx)}{24b} + \frac{\cosh^7(a+bx)\sinh^5(a+bx)}{12b} + \frac{1}{8}\int \cosh$$
$$= \frac{\cosh^7(a+bx)\sinh(a+bx)}{64b} - \frac{\cosh^7(a+bx)\sinh^3(a+bx)}{24b} + \cosh^7(a+b$$
$$= -\frac{\cosh^5(a+bx)\sinh(a+bx)}{384b} + \frac{\cosh^7(a+bx)\sinh(a+bx)}{64b} - \frac{\cosh^7(a+b}{}$$
$$= -\frac{5\cosh^3(a+bx)\sinh(a+bx)}{1536b} - \frac{\cosh^5(a+bx)\sinh(a+bx)}{384b} + \frac{\cosh^7(a+}{}$$
$$= -\frac{5\cosh(a+bx)\sinh(a+bx)}{1024b} - \frac{5\cosh^3(a+bx)\sinh(a+bx)}{1536b} - \frac{\cosh^5(a+}{}$$
$$= -\frac{5x}{1024} - \frac{5\cosh(a+bx)\sinh(a+bx)}{1024b} - \frac{5\cosh^3(a+bx)\sinh(a+bx)}{1536b} - \frac{\mathrm{co}}{}$$

Mathematica [A] time = 0.09, size = 43, normalized size = 0.32

$$\frac{45\sinh(4(a+bx)) - 9\sinh(8(a+bx)) + \sinh(12(a+bx)) - 120a - 120bx}{24576b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[a + b*x]^6*Sinh[a + b*x]^6,x]
```

```
[Out] (-120*a - 120*b*x + 45*Sinh[4*(a + b*x)] - 9*Sinh[8*(a + b*x)] + Sinh[12*(a
 + b*x)])/(24576*b)
```

fricas [A] time = 0.39, size = 179, normalized size = 1.34

$$\frac{55\cosh(bx+a)^3\sinh(bx+a)^9 + 3\cosh(bx+a)\sinh(bx+a)^{11} + 18\left(11\cosh(bx+a)^5 - \cosh(bx+a)\right)\sin}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^6*sinh(b*x+a)^6,x, algorithm="fricas")
```

```
[Out] 1/6144*(55*cosh(b*x + a)^3*sinh(b*x + a)^9 + 3*cosh(b*x + a)*sinh(b*x + a)^
11 + 18*(11*cosh(b*x + a)^5 - cosh(b*x + a))*sinh(b*x + a)^7 + 18*(11*cosh(
b*x + a)^7 - 7*cosh(b*x + a)^3)*sinh(b*x + a)^5 + (55*cosh(b*x + a)^9 - 126
```

```
*cosh(b*x + a)^5 + 45*cosh(b*x + a))*sinh(b*x + a)^3 - 30*b*x + 3*(cosh(b*x
 + a)^11 - 6*cosh(b*x + a)^7 + 15*cosh(b*x + a)^3)*sinh(b*x + a))/b
```

giac [A] time = 0.24, size = 88, normalized size = 0.66

$$-\frac{5}{1024}\,x + \frac{e^{(12\,bx+12\,a)}}{49152\,b} - \frac{3\,e^{(8\,bx+8\,a)}}{16384\,b} + \frac{15\,e^{(4\,bx+4\,a)}}{16384\,b} - \frac{15\,e^{(-4\,bx-4\,a)}}{16384\,b} + \frac{3\,e^{(-8\,bx-8\,a)}}{16384\,b} - \frac{e^{(-12\,bx-12\,a)}}{49152\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^6*sinh(b*x+a)^6,x, algorithm="giac")
```

```
[Out] -5/1024*x + 1/49152*e^(12*b*x + 12*a)/b - 3/16384*e^(8*b*x + 8*a)/b + 15/16
384*e^(4*b*x + 4*a)/b - 15/16384*e^(-4*b*x - 4*a)/b + 3/16384*e^(-8*b*x - 8
*a)/b - 1/49152*e^(-12*b*x - 12*a)/b
```

maple [A] time = 0.34, size = 102, normalized size = 0.76

$$\frac{\frac{\left(\sinh^{5}(bx+a)\right)\left(\cosh^{7}(bx+a)\right)}{12} - \frac{\left(\sinh^{3}(bx+a)\right)\left(\cosh^{7}(bx+a)\right)}{24} + \frac{\sinh(bx+a)\left(\cosh^{7}(bx+a)\right)}{64} - \frac{\left(\frac{\left(\cosh^{5}(bx+a)\right)}{6} + \frac{5\left(\cosh^{3}(bx+a)\right)}{24} + \frac{5\cosh(bx+a)}{16}\right)\sinh(bx+}{64}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)^6*sinh(b*x+a)^6,x)
```

```
[Out] 1/b*(1/12*sinh(b*x+a)^5*cosh(b*x+a)^7-1/24*sinh(b*x+a)^3*cosh(b*x+a)^7+1/64
*sinh(b*x+a)*cosh(b*x+a)^7-1/64*(1/6*cosh(b*x+a)^5+5/24*cosh(b*x+a)^3+5/16*
cosh(b*x+a))*sinh(b*x+a)-5/1024*b*x-5/1024*a)
```

maxima [A] time = 0.34, size = 86, normalized size = 0.64

$$-\frac{\left(9\,e^{(-4\,bx-4\,a)} - 45\,e^{(-8\,bx-8\,a)} - 1\right)e^{(12\,bx+12\,a)}}{49152\,b} - \frac{5\,(bx+a)}{1024\,b} - \frac{45\,e^{(-4\,bx-4\,a)} - 9\,e^{(-8\,bx-8\,a)} + e^{(-12\,bx-12\,a)}}{49152\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^6*sinh(b*x+a)^6,x, algorithm="maxima")
```

```
[Out] -1/49152*(9*e^(-4*b*x - 4*a) - 45*e^(-8*b*x - 8*a) - 1)*e^(12*b*x + 12*a)/b
 - 5/1024*(b*x + a)/b - 1/49152*(45*e^(-4*b*x - 4*a) - 9*e^(-8*b*x - 8*a) +
 e^(-12*b*x - 12*a))/b
```

mupad [B] time = 1.82, size = 42, normalized size = 0.31

$$\frac{\frac{15\sinh(4\,a+4\,b\,x)}{8192} - \frac{3\sinh(8\,a+8\,b\,x)}{8192} + \frac{\sinh(12\,a+12\,b\,x)}{24576}}{b} - \frac{5\,x}{1024}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a + b*x)^6*sinh(a + b*x)^6,x)
```

```
[Out] ((15*sinh(4*a + 4*b*x))/8192 - (3*sinh(8*a + 8*b*x))/8192 + sinh(12*a + 12*
b*x)/24576)/b - (5*x)/1024
```

sympy [A] time = 47.86, size = 277, normalized size = 2.07

$$\begin{cases} -\frac{5x\sinh^{12}(a+bx)}{1024} + \frac{15x\sinh^{10}(a+bx)\cosh^{2}(a+bx)}{512} - \frac{75x\sinh^{8}(a+bx)\cosh^{4}(a+bx)}{1024} + \frac{25x\sinh^{6}(a+bx)\cosh^{6}(a+bx)}{256} - \frac{75x\sinh^{4}(a+bx)c}{1024} \\ x\sinh^{6}(a)\cosh^{6}(a) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)**6*sinh(b*x+a)**6,x)
```

```
[Out] Piecewise((-5*x*sinh(a + b*x)**12/1024 + 15*x*sinh(a + b*x)**10*cosh(a + b*
x)**2/512 - 75*x*sinh(a + b*x)**8*cosh(a + b*x)**4/1024 + 25*x*sinh(a + b*x
)**6*cosh(a + b*x)**6/256 - 75*x*sinh(a + b*x)**4*cosh(a + b*x)**8/1024 + 1
5*x*sinh(a + b*x)**2*cosh(a + b*x)**10/512 - 5*x*cosh(a + b*x)**12/1024 + 5
*sinh(a + b*x)**11*cosh(a + b*x)/(1024*b) - 85*sinh(a + b*x)**9*cosh(a + b*
x)**3/(3072*b) + 33*sinh(a + b*x)**7*cosh(a + b*x)**5/(512*b) + 33*sinh(a +
 b*x)**5*cosh(a + b*x)**7/(512*b) - 85*sinh(a + b*x)**3*cosh(a + b*x)**9/(3
072*b) + 5*sinh(a + b*x)*cosh(a + b*x)**11/(1024*b), Ne(b, 0)), (x*sinh(a)*
*6*cosh(a)**6, True))
```

3.24 $\int \mathrm{csch}(a+bx)\mathrm{sech}(a+bx)\,dx$

Optimal. Leaf size=11

$$\frac{\log(\tanh(a+bx))}{b}$$

[Out] ln(tanh(b*x+a))/b

Rubi [A] time = 0.01, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2620, 29}

$$\frac{\log(\tanh(a+bx))}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[a + b*x]*Sech[a + b*x],x]
```

```
[Out] Log[Tanh[a + b*x]]/b
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 2620

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rubi steps

$$\begin{aligned}\int \mathrm{csch}(a+bx)\mathrm{sech}(a+bx)\,dx &= \frac{\mathrm{Subst}\left(\int \frac{1}{x}\,dx, x, i\tanh(a+bx)\right)}{b}\\ &= \frac{\log(\tanh(a+bx))}{b}\end{aligned}$$

Mathematica [B] time = 0.02, size = 31, normalized size = 2.82

$$2\left(\frac{\log(\sinh(a+bx))}{2b}-\frac{\log(\cosh(a+bx))}{2b}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[a + b*x]*Sech[a + b*x],x]
```

```
[Out] 2*(-1/2*Log[Cosh[a + b*x]]/b + Log[Sinh[a + b*x]]/(2*b))
```

fricas [B] time = 0.43, size = 60, normalized size = 5.45

$$-\frac{\log\left(\frac{2\cosh(bx+a)}{\cosh(bx+a)-\sinh(bx+a)}\right)-\log\left(\frac{2\sinh(bx+a)}{\cosh(bx+a)-\sinh(bx+a)}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)*sech(b*x+a),x, algorithm="fricas")
```

```
[Out] -(log(2*cosh(b*x + a)/(cosh(b*x + a) - sinh(b*x + a))) - log(2*sinh(b*x + a
)/(cosh(b*x + a) - sinh(b*x + a))))/b
```

giac [B] time = 0.12, size = 41, normalized size = 3.73

$$-\frac{\log\left(e^{(2\,bx+2\,a)}+1\right)-\log\left(e^{(bx+a)}+1\right)-\log\left(\left|e^{(bx+a)}-1\right|\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)*sech(b*x+a),x, algorithm="giac")
```

```
[Out] -(log(e^(2*b*x + 2*a) + 1) - log(e^(b*x + a) + 1) - log(abs(e^(b*x + a) - 1
)))/b
```

maple [A] time = 0.09, size = 12, normalized size = 1.09

$$\frac{\ln\left(\tanh\left(bx+a\right)\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(b*x+a)*sech(b*x+a),x)
```

```
[Out] ln(tanh(b*x+a))/b
```

maxima [B] time = 0.62, size = 50, normalized size = 4.55

$$\frac{\log\left(e^{(-bx-a)}+1\right)}{b}+\frac{\log\left(e^{(-bx-a)}-1\right)}{b}-\frac{\log\left(e^{(-2\,bx-2\,a)}+1\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)*sech(b*x+a),x, algorithm="maxima")
```

```
[Out] log(e^(-b*x - a) + 1)/b + log(e^(-b*x - a) - 1)/b - log(e^(-2*b*x - 2*a) +
1)/b
```

mupad [B] time = 0.15, size = 30, normalized size = 2.73

$$-\frac{2\,\mathrm{atan}\left(\frac{\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}\,\sqrt{-b^2}}{b}\right)}{\sqrt{-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(a + b*x)*sinh(a + b*x)),x)
```

```
[Out] -(2*atan((exp(2*a)*exp(2*b*x)*(-b^2)^(1/2))/b))/(-b^2)^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{csch}(a+bx)\operatorname{sech}(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)*sech(b*x+a),x)
```

```
[Out] Integral(csch(a + b*x)*sech(a + b*x), x)
```

3.25 $\int \operatorname{csch}(a+bx)\operatorname{sech}^2(a+bx)\,dx$

Optimal. Leaf size=23

$$\frac{\operatorname{sech}(a+bx)}{b}-\frac{\tanh^{-1}(\cosh(a+bx))}{b}$$

```
[Out] -arctanh(cosh(b*x+a))/b+sech(b*x+a)/b
```

Rubi [A] time = 0.03, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2622, 321, 207}

$$\frac{\operatorname{sech}(a+bx)}{b}-\frac{\tanh^{-1}(\cosh(a+bx))}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[a + b*x]*Sech[a + b*x]^2,x]
```

```
[Out] -(ArcTanh[Cosh[a + b*x]]/b) + Sech[a + b*x]/b
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rubi steps

$$\int \operatorname{csch}(a+bx)\operatorname{sech}^2(a+bx)\,dx = \frac{\operatorname{Subst}\left(\int \frac{x^2}{-1+x^2}\,dx, x, \operatorname{sech}(a+bx)\right)}{b}$$
$$= \frac{\operatorname{sech}(a+bx)}{b} + \frac{\operatorname{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \operatorname{sech}(a+bx)\right)}{b}$$
$$= -\frac{\tanh^{-1}(\cosh(a+bx))}{b} + \frac{\operatorname{sech}(a+bx)}{b}$$

Mathematica [A] time = 0.03, size = 26, normalized size = 1.13

$$\frac{\operatorname{sech}(a+bx)}{b} + \frac{\log\left(\tanh\left(\frac{1}{2}(a+bx)\right)\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[a + b*x]*Sech[a + b*x]^2,x]
```

```
[Out] Log[Tanh[(a + b*x)/2]]/b + Sech[a + b*x]/b
```

fricas [B] time = 0.43, size = 155, normalized size = 6.74

$$-\frac{\left(\cosh\left(bx+a\right)^2 + 2\,\cosh\left(bx+a\right)\sinh\left(bx+a\right) + \sinh\left(bx+a\right)^2 + 1\right)\log\left(\cosh\left(bx+a\right) + \sinh\left(bx+a\right) + 1\right) -}{b\cosh\left(bx+a\right)^2 + 2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)*sech(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -((cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 + 1)*l
og(cosh(b*x + a) + sinh(b*x + a) + 1) - (cosh(b*x + a)^2 + 2*cosh(b*x + a)*
sinh(b*x + a) + sinh(b*x + a)^2 + 1)*log(cosh(b*x + a) + sinh(b*x + a) - 1)
 - 2*cosh(b*x + a) - 2*sinh(b*x + a))/(b*cosh(b*x + a)^2 + 2*b*cosh(b*x + a
)*sinh(b*x + a) + b*sinh(b*x + a)^2 + b)
```

giac [B] time = 0.14, size = 64, normalized size = 2.78

$$\frac{\frac{4}{e^{(bx+a)}+e^{(-bx-a)}} - \log\left(e^{(bx+a)} + e^{(-bx-a)} + 2\right) + \log\left(e^{(bx+a)} + e^{(-bx-a)} - 2\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)*sech(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*(4/(e^(b*x + a) + e^(-b*x - a)) - log(e^(b*x + a) + e^(-b*x - a) + 2) +
 log(e^(b*x + a) + e^(-b*x - a) - 2))/b
```

maple [A] time = 0.09, size = 23, normalized size = 1.00

$$\frac{\frac{1}{\cosh(bx+a)} - 2\,\mathrm{arctanh}\left(e^{bx+a}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(b*x+a)*sech(b*x+a)^2,x)
```

```
[Out] 1/b*(1/cosh(b*x+a)-2*arctanh(exp(b*x+a)))
```

maxima [B] time = 0.36, size = 61, normalized size = 2.65

$$-\frac{\log\left(e^{(-bx-a)}+1\right)}{b} + \frac{\log\left(e^{(-bx-a)}-1\right)}{b} + \frac{2\,e^{(-bx-a)}}{b\left(e^{(-2\,bx-2\,a)}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)*sech(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -log(e^(-b*x - a) + 1)/b + log(e^(-b*x - a) - 1)/b + 2*e^(-b*x - a)/(b*(e^(
-2*b*x - 2*a) + 1))
```

mupad [B] time = 0.09, size = 52, normalized size = 2.26

$$\frac{2\,e^{a+b\,x}}{b\,\left(e^{2\,a+2\,b\,x}+1\right)} - \frac{2\,\mathrm{atan}\left(\frac{e^{b\,x}\,e^{a}\,\sqrt{-b^2}}{b}\right)}{\sqrt{-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(a + b*x)^2*sinh(a + b*x)),x)
```

```
[Out] (2*exp(a + b*x))/(b*(exp(2*a + 2*b*x) + 1)) - (2*atan((exp(b*x)*exp(a)*(-b^
2)^(1/2))/b))/(-b^2)^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{csch}(a+bx)\operatorname{sech}^2(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)*sech(b*x+a)**2,x)
[Out] Integral(csch(a + b*x)*sech(a + b*x)**2, x)
```

3.26 $\int \text{csch}(a+bx)\text{sech}^3(a+bx)\,dx$

Optimal. Leaf size=27

$$\frac{\log(\tanh(a+bx))}{b}-\frac{\tanh^2(a+bx)}{2b}$$

```
[Out] ln(tanh(b*x+a))/b-1/2*tanh(b*x+a)^2/b
```

Rubi [A] time = 0.02, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2620, 14}

$$\frac{\log(\tanh(a+bx))}{b}-\frac{\tanh^2(a+bx)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[a + b*x]*Sech[a + b*x]^3,x]
```

```
[Out] Log[Tanh[a + b*x]]/b - Tanh[a + b*x]^2/(2*b)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2620

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rubi steps

$$\begin{aligned}\int \text{csch}(a+bx)\text{sech}^3(a+bx)\,dx &= \frac{\text{Subst}\left(\int \frac{1+x^2}{x}\,dx, x, i\tanh(a+bx)\right)}{b}\\ &= \frac{\text{Subst}\left(\int \left(\frac{1}{x}+x\right)dx, x, i\tanh(a+bx)\right)}{b}\\ &= \frac{\log(\tanh(a+bx))}{b}-\frac{\tanh^2(a+bx)}{2b}\end{aligned}$$

Mathematica [A] time = 0.04, size = 36, normalized size = 1.33

$$-\frac{-\operatorname{sech}^2(a+bx)-2\log(\sinh(a+bx))+2\log(\cosh(a+bx))}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[a + b*x]*Sech[a + b*x]^3,x]
```

```
[Out] -1/2*(2*Log[Cosh[a + b*x]] - 2*Log[Sinh[a + b*x]] - Sech[a + b*x]^2)/b
```

fricas [B] time = 0.42, size = 371, normalized size = 13.74

$$2\cosh(bx+a)^2-\left(\cosh(bx+a)^4+4\cosh(bx+a)\sinh(bx+a)^3+\sinh(bx+a)^4+2\left(3\cosh(bx+a)^2+1\right)\mathrm{s}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)*sech(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] (2*cosh(b*x + a)^2 - (cosh(b*x + a)^4 + 4*cosh(b*x + a)*sinh(b*x + a)^3 + s
inh(b*x + a)^4 + 2*(3*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^2 + 2*cosh(b*x + a
)^2 + 4*(cosh(b*x + a)^3 + cosh(b*x + a))*sinh(b*x + a) + 1)*log(2*cosh(b*x
 + a)/(cosh(b*x + a) - sinh(b*x + a))) + (cosh(b*x + a)^4 + 4*cosh(b*x + a)
*sinh(b*x + a)^3 + sinh(b*x + a)^4 + 2*(3*cosh(b*x + a)^2 + 1)*sinh(b*x + a
)^2 + 2*cosh(b*x + a)^2 + 4*(cosh(b*x + a)^3 + cosh(b*x + a))*sinh(b*x + a)
 + 1)*log(2*sinh(b*x + a)/(cosh(b*x + a) - sinh(b*x + a))) + 4*cosh(b*x + a
)*sinh(b*x + a) + 2*sinh(b*x + a)^2)/(b*cosh(b*x + a)^4 + 4*b*cosh(b*x + a)
*sinh(b*x + a)^3 + b*sinh(b*x + a)^4 + 2*b*cosh(b*x + a)^2 + 2*(3*b*cosh(b*
x + a)^2 + b)*sinh(b*x + a)^2 + 4*(b*cosh(b*x + a)^3 + b*cosh(b*x + a))*sin
h(b*x + a) + b)
```

giac [B] time = 0.14, size = 93, normalized size = 3.44

$$\frac{\frac{e^{(2bx+2a)}+e^{(-2bx-2a)}+6}{e^{(2bx+2a)}+e^{(-2bx-2a)}+2}-\log\left(e^{(2bx+2a)}+e^{(-2bx-2a)}+2\right)+\log\left(e^{(2bx+2a)}+e^{(-2bx-2a)}-2\right)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)*sech(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/2*((e^(2*b*x + 2*a) + e^(-2*b*x - 2*a) + 6)/(e^(2*b*x + 2*a) + e^(-2*b*x
- 2*a) + 2) - log(e^(2*b*x + 2*a) + e^(-2*b*x - 2*a) + 2) + log(e^(2*b*x +
2*a) + e^(-2*b*x - 2*a) - 2))/b
```

maple [A] time = 0.14, size = 26, normalized size = 0.96

$$\frac{1}{2b\cosh\left(bx+a\right)^{2}}+\frac{\ln\left(\tanh\left(bx+a\right)\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(b*x+a)*sech(b*x+a)^3,x)
```

```
[Out] 1/2/b/cosh(b*x+a)^2+ln(tanh(b*x+a))/b
```

maxima [B] time = 0.42, size = 88, normalized size = 3.26

$$\frac{\log\left(e^{(-bx-a)}+1\right)}{b}+\frac{\log\left(e^{(-bx-a)}-1\right)}{b}-\frac{\log\left(e^{(-2\,bx-2\,a)}+1\right)}{b}+\frac{2\,e^{(-2\,bx-2\,a)}}{b\left(2\,e^{(-2\,bx-2\,a)}+e^{(-4\,bx-4\,a)}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)*sech(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] log(e^(-b*x - a) + 1)/b + log(e^(-b*x - a) - 1)/b - log(e^(-2*b*x - 2*a) +
1)/b + 2*e^(-2*b*x - 2*a)/(b*(2*e^(-2*b*x - 2*a) + e^(-4*b*x - 4*a) + 1))
```

mupad [B] time = 1.50, size = 78, normalized size = 2.89

$$\frac{2}{b\,\left(\mathrm{e}^{2\,a+2\,b\,x}+1\right)}-\frac{2\,\mathrm{atan}\left(\frac{\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}\,\sqrt{-b^2}}{b}\right)}{\sqrt{-b^2}}-\frac{2}{b\,\left(2\,\mathrm{e}^{2\,a+2\,b\,x}+\mathrm{e}^{4\,a+4\,b\,x}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(a + b*x)^3*sinh(a + b*x)),x)
```

```
[Out] 2/(b*(exp(2*a + 2*b*x) + 1)) - (2*atan((exp(2*a)*exp(2*b*x)*(-b^2)^(1/2))/b
))/(-b^2)^(1/2) - 2/(b*(2*exp(2*a + 2*b*x) + exp(4*a + 4*b*x) + 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\operatorname{csch}\left(a+bx\right)\operatorname{sech}^{3}\left(a+bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)*sech(b*x+a)**3,x)
```

```
[Out] Integral(csch(a + b*x)*sech(a + b*x)**3, x)
```

3.27 $\int \operatorname{csch}(a+bx)\operatorname{sech}^4(a+bx)\,dx$

Optimal. Leaf size=38

$$\frac{\operatorname{sech}^3(a+bx)}{3b}+\frac{\operatorname{sech}(a+bx)}{b}-\frac{\tanh^{-1}(\cosh(a+bx))}{b}$$

```
[Out] -arctanh(cosh(b*x+a))/b+sech(b*x+a)/b+1/3*sech(b*x+a)^3/b
```

Rubi [A] time = 0.03, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2622, 302, 207}

$$\frac{\operatorname{sech}^3(a+bx)}{3b}+\frac{\operatorname{sech}(a+bx)}{b}-\frac{\tanh^{-1}(\cosh(a+bx))}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[a + b*x]*Sech[a + b*x]^4,x]
```

```
[Out] -(ArcTanh[Cosh[a + b*x]]/b) + Sech[a + b*x]/b + Sech[a + b*x]^3/(3*b)
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rubi steps

$$\int \operatorname{csch}(a+bx)\operatorname{sech}^4(a+bx)\,dx = \frac{\text{Subst}\left(\int \frac{x^4}{-1+x^2}\,dx, x, \operatorname{sech}(a+bx)\right)}{b}$$
$$= \frac{\text{Subst}\left(\int \left(1+x^2+\frac{1}{-1+x^2}\right)dx, x, \operatorname{sech}(a+bx)\right)}{b}$$
$$= \frac{\operatorname{sech}(a+bx)}{b} + \frac{\operatorname{sech}^3(a+bx)}{3b} + \frac{\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \operatorname{sech}(a+bx)\right)}{b}$$
$$= -\frac{\tanh^{-1}(\cosh(a+bx))}{b} + \frac{\operatorname{sech}(a+bx)}{b} + \frac{\operatorname{sech}^3(a+bx)}{3b}$$

Mathematica [A] time = 0.03, size = 41, normalized size = 1.08

$$\frac{\operatorname{sech}^3(a+bx)}{3b} + \frac{\operatorname{sech}(a+bx)}{b} + \frac{\log\left(\tanh\left(\frac{1}{2}(a+bx)\right)\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[a + b*x]*Sech[a + b*x]^4,x]
[Out] Log[Tanh[(a + b*x)/2]]/b + Sech[a + b*x]/b + Sech[a + b*x]^3/(3*b)
```

fricas [B] time = 0.44, size = 697, normalized size = 18.34

$$\frac{6\cosh(bx+a)^5 + 30\cosh(bx+a)\sinh(bx+a)^4 + 6\sinh(bx+a)^5 + 20\left(3\cosh(bx+a)^2+1\right)\sinh(bx+a)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)*sech(b*x+a)^4,x, algorithm="fricas")
[Out] 1/3*(6*cosh(b*x + a)^5 + 30*cosh(b*x + a)*sinh(b*x + a)^4 + 6*sinh(b*x + a)
^5 + 20*(3*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^3 + 20*cosh(b*x + a)^3 + 60*(
cosh(b*x + a)^3 + cosh(b*x + a))*sinh(b*x + a)^2 - 3*(cosh(b*x + a)^6 + 6*c
osh(b*x + a)*sinh(b*x + a)^5 + sinh(b*x + a)^6 + 3*(5*cosh(b*x + a)^2 + 1)*
sinh(b*x + a)^4 + 3*cosh(b*x + a)^4 + 4*(5*cosh(b*x + a)^3 + 3*cosh(b*x + a
))*sinh(b*x + a)^3 + 3*(5*cosh(b*x + a)^4 + 6*cosh(b*x + a)^2 + 1)*sinh(b*x
 + a)^2 + 3*cosh(b*x + a)^2 + 6*(cosh(b*x + a)^5 + 2*cosh(b*x + a)^3 + cosh
(b*x + a))*sinh(b*x + a) + 1)*log(cosh(b*x + a) + sinh(b*x + a) + 1) + 3*(c
osh(b*x + a)^6 + 6*cosh(b*x + a)*sinh(b*x + a)^5 + sinh(b*x + a)^6 + 3*(5*c
osh(b*x + a)^2 + 1)*sinh(b*x + a)^4 + 3*cosh(b*x + a)^4 + 4*(5*cosh(b*x + a
```

```
)^3 + 3*cosh(b*x + a))*sinh(b*x + a)^3 + 3*(5*cosh(b*x + a)^4 + 6*cosh(b*x
+ a)^2 + 1)*sinh(b*x + a)^2 + 3*cosh(b*x + a)^2 + 6*(cosh(b*x + a)^5 + 2*co
sh(b*x + a)^3 + cosh(b*x + a))*sinh(b*x + a) + 1)*log(cosh(b*x + a) + sinh(
b*x + a) - 1) + 6*(5*cosh(b*x + a)^4 + 10*cosh(b*x + a)^2 + 1)*sinh(b*x + a
) + 6*cosh(b*x + a))/(b*cosh(b*x + a)^6 + 6*b*cosh(b*x + a)*sinh(b*x + a)^5
 + b*sinh(b*x + a)^6 + 3*b*cosh(b*x + a)^4 + 3*(5*b*cosh(b*x + a)^2 + b)*si
nh(b*x + a)^4 + 4*(5*b*cosh(b*x + a)^3 + 3*b*cosh(b*x + a))*sinh(b*x + a)^3
 + 3*b*cosh(b*x + a)^2 + 3*(5*b*cosh(b*x + a)^4 + 6*b*cosh(b*x + a)^2 + b)*
sinh(b*x + a)^2 + 6*(b*cosh(b*x + a)^5 + 2*b*cosh(b*x + a)^3 + b*cosh(b*x +
 a))*sinh(b*x + a) + b)
```

giac [B] time = 0.14, size = 88, normalized size = 2.32

$$\frac{\frac{4\left(3\left(e^{(bx+a)}+e^{(-bx-a)}\right)^2+4\right)}{\left(e^{(bx+a)}+e^{(-bx-a)}\right)^3}-3\log\left(e^{(bx+a)}+e^{(-bx-a)}+2\right)+3\log\left(e^{(bx+a)}+e^{(-bx-a)}-2\right)}{6\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)*sech(b*x+a)^4,x, algorithm="giac")
```

```
[Out] 1/6*(4*(3*(e^(b*x + a) + e^(-b*x - a))^2 + 4)/(e^(b*x + a) + e^(-b*x - a))^
3 - 3*log(e^(b*x + a) + e^(-b*x - a) + 2) + 3*log(e^(b*x + a) + e^(-b*x - a
) - 2))/b
```

maple [A] time = 0.16, size = 33, normalized size = 0.87

$$\frac{\frac{1}{3\cosh(bx+a)^3}+\frac{1}{\cosh(bx+a)}-2\operatorname{arctanh}\left(e^{bx+a}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(b*x+a)*sech(b*x+a)^4,x)
```

```
[Out] 1/b*(1/3/cosh(b*x+a)^3+1/cosh(b*x+a)-2*arctanh(exp(b*x+a)))
```

maxima [B] time = 0.31, size = 108, normalized size = 2.84

$$-\frac{\log\left(e^{(-bx-a)}+1\right)}{b}+\frac{\log\left(e^{(-bx-a)}-1\right)}{b}+\frac{2\left(3\,e^{(-bx-a)}+10\,e^{(-3\,bx-3\,a)}+3\,e^{(-5\,bx-5\,a)}\right)}{3\,b\left(3\,e^{(-2\,bx-2\,a)}+3\,e^{(-4\,bx-4\,a)}+e^{(-6\,bx-6\,a)}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)*sech(b*x+a)^4,x, algorithm="maxima")
```

```
[Out] -log(e^(-b*x - a) + 1)/b + log(e^(-b*x - a) - 1)/b + 2/3*(3*e^(-b*x - a) +
10*e^(-3*b*x - 3*a) + 3*e^(-5*b*x - 5*a))/(b*(3*e^(-2*b*x - 2*a) + 3*e^(-4*
b*x - 4*a) + e^(-6*b*x - 6*a) + 1))
```

mupad [B] time = 1.51, size = 133, normalized size = 3.50

$$\frac{8\,e^{a+b\,x}}{3\,b\,\left(2\,e^{2\,a+2\,b\,x}+e^{4\,a+4\,b\,x}+1\right)}-\frac{2\,\mathrm{atan}\left(\frac{e^{b\,x}\,e^{a}\,\sqrt{-b^2}}{b}\right)}{\sqrt{-b^2}}-\frac{8\,e^{a+b\,x}}{3\,b\,\left(3\,e^{2\,a+2\,b\,x}+3\,e^{4\,a+4\,b\,x}+e^{6\,a+6\,b\,x}+1\right)}+\frac{2\,e^{a+b\,x}}{b\,\left(e^{2\,a+2\,b\,x}+1\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(a + b*x)^4*sinh(a + b*x)),x)
```

```
[Out] (8*exp(a + b*x))/(3*b*(2*exp(2*a + 2*b*x) + exp(4*a + 4*b*x) + 1)) - (2*ata
n((exp(b*x)*exp(a)*(-b^2)^(1/2))/b))/(-b^2)^(1/2) - (8*exp(a + b*x))/(3*b*(
3*exp(2*a + 2*b*x) + 3*exp(4*a + 4*b*x) + exp(6*a + 6*b*x) + 1)) + (2*exp(a
 + b*x))/(b*(exp(2*a + 2*b*x) + 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{csch}(a+bx)\operatorname{sech}^4(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)*sech(b*x+a)**4,x)
```

```
[Out] Integral(csch(a + b*x)*sech(a + b*x)**4, x)
```

3.28 $\int \operatorname{csch}(a+bx)\operatorname{sech}^5(a+bx)\,dx$

Optimal. Leaf size=40

$$\frac{\tanh^4(a+bx)}{4b}-\frac{\tanh^2(a+bx)}{b}+\frac{\log(\tanh(a+bx))}{b}$$

```
[Out] ln(tanh(b*x+a))/b-tanh(b*x+a)^2/b+1/4*tanh(b*x+a)^4/b
```

Rubi [A] time = 0.03, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2620, 266, 43}

$$\frac{\tanh^4(a+bx)}{4b}-\frac{\tanh^2(a+bx)}{b}+\frac{\log(\tanh(a+bx))}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[a + b*x]*Sech[a + b*x]^5,x]
```

```
[Out] Log[Tanh[a + b*x]]/b - Tanh[a + b*x]^2/b + Tanh[a + b*x]^4/(4*b)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2620

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rubi steps

$$\int \operatorname{csch}(a+bx)\operatorname{sech}^5(a+bx)\,dx = \frac{\operatorname{Subst}\left(\int \frac{\left(1+x^2\right)^2}{x}\,dx, x, i\tanh(a+bx)\right)}{b}$$
$$= \frac{\operatorname{Subst}\left(\int \frac{(1+x)^2}{x}\,dx, x, -\tanh^2(a+bx)\right)}{2b}$$
$$= \frac{\operatorname{Subst}\left(\int \left(2+\frac{1}{x}+x\right)dx, x, -\tanh^2(a+bx)\right)}{2b}$$
$$= \frac{\log(\tanh(a+bx))}{b} - \frac{\tanh^2(a+bx)}{b} + \frac{\tanh^4(a+bx)}{4b}$$

Mathematica [A] time = 0.10, size = 46, normalized size = 1.15

$$-\frac{-\operatorname{sech}^4(a+bx) - 2\operatorname{sech}^2(a+bx) - 4\log(\sinh(a+bx)) + 4\log(\cosh(a+bx))}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[a + b*x]*Sech[a + b*x]^5,x]
```

```
[Out] -1/4*(4*Log[Cosh[a + b*x]] - 4*Log[Sinh[a + b*x]] - 2*Sech[a + b*x]^2 - Sec
h[a + b*x]^4)/b
```

fricas [B] time = 0.45, size = 1073, normalized size = 26.82

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)*sech(b*x+a)^5,x, algorithm="fricas")
```

```
[Out] (2*cosh(b*x + a)^6 + 12*cosh(b*x + a)*sinh(b*x + a)^5 + 2*sinh(b*x + a)^6 +
 2*(15*cosh(b*x + a)^2 + 4)*sinh(b*x + a)^4 + 8*cosh(b*x + a)^4 + 8*(5*cosh
(b*x + a)^3 + 4*cosh(b*x + a))*sinh(b*x + a)^3 + 2*(15*cosh(b*x + a)^4 + 24
*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^2 + 2*cosh(b*x + a)^2 - (cosh(b*x + a)^
8 + 8*cosh(b*x + a)*sinh(b*x + a)^7 + sinh(b*x + a)^8 + 4*(7*cosh(b*x + a)^
2 + 1)*sinh(b*x + a)^6 + 4*cosh(b*x + a)^6 + 8*(7*cosh(b*x + a)^3 + 3*cosh(
b*x + a))*sinh(b*x + a)^5 + 2*(35*cosh(b*x + a)^4 + 30*cosh(b*x + a)^2 + 3)
*sinh(b*x + a)^4 + 6*cosh(b*x + a)^4 + 8*(7*cosh(b*x + a)^5 + 10*cosh(b*x +
 a)^3 + 3*cosh(b*x + a))*sinh(b*x + a)^3 + 4*(7*cosh(b*x + a)^6 + 15*cosh(b
*x + a)^4 + 9*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^2 + 4*cosh(b*x + a)^2 + 8*
(cosh(b*x + a)^7 + 3*cosh(b*x + a)^5 + 3*cosh(b*x + a)^3 + cosh(b*x + a))*s
```

```
inh(b*x + a) + 1)*log(2*cosh(b*x + a)/(cosh(b*x + a) - sinh(b*x + a))) + (c
osh(b*x + a)^8 + 8*cosh(b*x + a)*sinh(b*x + a)^7 + sinh(b*x + a)^8 + 4*(7*c
osh(b*x + a)^2 + 1)*sinh(b*x + a)^6 + 4*cosh(b*x + a)^6 + 8*(7*cosh(b*x + a
)^3 + 3*cosh(b*x + a))*sinh(b*x + a)^5 + 2*(35*cosh(b*x + a)^4 + 30*cosh(b*
x + a)^2 + 3)*sinh(b*x + a)^4 + 6*cosh(b*x + a)^4 + 8*(7*cosh(b*x + a)^5 +
10*cosh(b*x + a)^3 + 3*cosh(b*x + a))*sinh(b*x + a)^3 + 4*(7*cosh(b*x + a)^
6 + 15*cosh(b*x + a)^4 + 9*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^2 + 4*cosh(b*
x + a)^2 + 8*(cosh(b*x + a)^7 + 3*cosh(b*x + a)^5 + 3*cosh(b*x + a)^3 + cos
h(b*x + a))*sinh(b*x + a) + 1)*log(2*sinh(b*x + a)/(cosh(b*x + a) - sinh(b*
x + a))) + 4*(3*cosh(b*x + a)^5 + 8*cosh(b*x + a)^3 + cosh(b*x + a))*sinh(b
*x + a))/(b*cosh(b*x + a)^8 + 8*b*cosh(b*x + a)*sinh(b*x + a)^7 + b*sinh(b*
x + a)^8 + 4*b*cosh(b*x + a)^6 + 4*(7*b*cosh(b*x + a)^2 + b)*sinh(b*x + a)^
6 + 8*(7*b*cosh(b*x + a)^3 + 3*b*cosh(b*x + a))*sinh(b*x + a)^5 + 6*b*cosh(
b*x + a)^4 + 2*(35*b*cosh(b*x + a)^4 + 30*b*cosh(b*x + a)^2 + 3*b)*sinh(b*x
 + a)^4 + 8*(7*b*cosh(b*x + a)^5 + 10*b*cosh(b*x + a)^3 + 3*b*cosh(b*x + a)
)*sinh(b*x + a)^3 + 4*b*cosh(b*x + a)^2 + 4*(7*b*cosh(b*x + a)^6 + 15*b*cos
h(b*x + a)^4 + 9*b*cosh(b*x + a)^2 + b)*sinh(b*x + a)^2 + 8*(b*cosh(b*x + a
)^7 + 3*b*cosh(b*x + a)^5 + 3*b*cosh(b*x + a)^3 + b*cosh(b*x + a))*sinh(b*x
 + a) + b)
```

giac [B] time = 0.15, size = 122, normalized size = 3.05

$$\frac{\frac{3\left(e^{(2bx+2a)}+e^{(-2bx-2a)}\right)^{2}+20e^{(2bx+2a)}+20e^{(-2bx-2a)}+44}{\left(e^{(2bx+2a)}+e^{(-2bx-2a)}+2\right)^{2}}-2\log\left(e^{(2bx+2a)}+e^{(-2bx-2a)}+2\right)+2\log\left(e^{(2bx+2a)}+e^{(-2bx-2a)}-\right.}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)*sech(b*x+a)^5,x, algorithm="giac")
```

```
[Out] 1/4*((3*(e^(2*b*x + 2*a) + e^(-2*b*x - 2*a))^2 + 20*e^(2*b*x + 2*a) + 20*e^
(-2*b*x - 2*a) + 44)/(e^(2*b*x + 2*a) + e^(-2*b*x - 2*a) + 2)^2 - 2*log(e^(
2*b*x + 2*a) + e^(-2*b*x - 2*a) + 2) + 2*log(e^(2*b*x + 2*a) + e^(-2*b*x -
2*a) - 2))/b
```

maple [A] time = 0.17, size = 39, normalized size = 0.98

$$\frac{1}{4b\cosh\left(bx+a\right)^{4}}+\frac{1}{2b\cosh\left(bx+a\right)^{2}}+\frac{\ln\left(\tanh\left(bx+a\right)\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(b*x+a)*sech(b*x+a)^5,x)
```

```
[Out] 1/4/b/cosh(b*x+a)^4+1/2/b/cosh(b*x+a)^2+ln(tanh(b*x+a))/b
```

maxima [B] time = 0.41, size = 131, normalized size = 3.28

$$\frac{\log\left(e^{(-bx-a)}+1\right)}{b}+\frac{\log\left(e^{(-bx-a)}-1\right)}{b}-\frac{\log\left(e^{(-2\,bx-2\,a)}+1\right)}{b}+\frac{2\left(e^{(-2\,bx-2\,a)}+4\,e^{(-4\,bx-4\,a)}+e^{(-6\,bx-6\,a)}\right)}{b\left(4\,e^{(-2\,bx-2\,a)}+6\,e^{(-4\,bx-4\,a)}+4\,e^{(-6\,bx-6\,a)}+e^{(-8\,bx-8}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)*sech(b*x+a)^5,x, algorithm="maxima")
```

```
[Out] log(e^(-b*x - a) + 1)/b + log(e^(-b*x - a) - 1)/b - log(e^(-2*b*x - 2*a) +
1)/b + 2*(e^(-2*b*x - 2*a) + 4*e^(-4*b*x - 4*a) + e^(-6*b*x - 6*a))/(b*(4*e
^(-2*b*x - 2*a) + 6*e^(-4*b*x - 4*a) + 4*e^(-6*b*x - 6*a) + e^(-8*b*x - 8*a
) + 1))
```

mupad [B] time = 1.46, size = 169, normalized size = 4.22

$$\frac{2}{b\left(e^{2\,a+2\,b\,x}+1\right)}-\frac{2\,\mathrm{atan}\left(\frac{e^{2\,a}\,e^{2\,b\,x}\,\sqrt{-b^2}}{b}\right)}{\sqrt{-b^2}}+\frac{2}{b\left(2\,e^{2\,a+2\,b\,x}+e^{4\,a+4\,b\,x}+1\right)}-\frac{8}{b\left(3\,e^{2\,a+2\,b\,x}+3\,e^{4\,a+4\,b\,x}+e^{6\,a+6\,b\,x}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(a + b*x)^5*sinh(a + b*x)),x)
```

```
[Out] 2/(b*(exp(2*a + 2*b*x) + 1)) - (2*atan((exp(2*a)*exp(2*b*x)*(-b^2)^(1/2))/b
))/(-b^2)^(1/2) + 2/(b*(2*exp(2*a + 2*b*x) + exp(4*a + 4*b*x) + 1)) - 8/(b*
(3*exp(2*a + 2*b*x) + 3*exp(4*a + 4*b*x) + exp(6*a + 6*b*x) + 1)) + 4/(b*(4
*exp(2*a + 2*b*x) + 6*exp(4*a + 4*b*x) + 4*exp(6*a + 6*b*x) + exp(8*a + 8*b
*x) + 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{csch}(a+bx)\operatorname{sech}^5(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)*sech(b*x+a)**5,x)
```

```
[Out] Integral(csch(a + b*x)*sech(a + b*x)**5, x)
```

3.29 $\int \operatorname{csch}^2(a+bx)\operatorname{sech}(a+bx)\,dx$

Optimal. Leaf size=24

$$-\frac{\operatorname{csch}(a+bx)}{b}-\frac{\tan^{-1}(\sinh(a+bx))}{b}$$

```
[Out] -arctan(sinh(b*x+a))/b-csch(b*x+a)/b
```

Rubi [A] time = 0.02, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2621, 321, 207}

$$-\frac{\operatorname{csch}(a+bx)}{b}-\frac{\tan^{-1}(\sinh(a+bx))}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[a + b*x]^2*Sech[a + b*x],x]
```

```
[Out] -(ArcTan[Sinh[a + b*x]]/b) - Csch[a + b*x]/b
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2621

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> -Dist[(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rubi steps

$$\int \operatorname{csch}^2(a+bx)\operatorname{sech}(a+bx)\,dx = -\frac{i\,\text{Subst}\left(\int \frac{x^2}{-1+x^2}\,dx, x, -i\operatorname{csch}(a+bx)\right)}{b}$$
$$= -\frac{\operatorname{csch}(a+bx)}{b} - \frac{i\,\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, -i\operatorname{csch}(a+bx)\right)}{b}$$
$$= -\frac{\tan^{-1}(\sinh(a+bx))}{b} - \frac{\operatorname{csch}(a+bx)}{b}$$

Mathematica [C] time = 0.02, size = 29, normalized size = 1.21

$$-\frac{\operatorname{csch}(a+bx)\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};-\sinh^2(a+bx)\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[a + b*x]^2*Sech[a + b*x],x]
[Out] -((Csch[a + b*x]*Hypergeometric2F1[-1/2, 1, 1/2, -Sinh[a + b*x]^2])/b)
```

fricas [B] time = 0.41, size = 103, normalized size = 4.29

$$-\frac{2\left(\left(\cosh(bx+a)^2 + 2\cosh(bx+a)\sinh(bx+a) + \sinh(bx+a)^2 - 1\right)\arctan(\cosh(bx+a) + \sinh(bx+a)\right)}{b\cosh(bx+a)^2 + 2\,b\cosh(bx+a)\sinh(bx+a) + b\sinh(bx+a)^2 - b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^2*sech(b*x+a),x, algorithm="fricas")
[Out] -2*((cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 - 1)
*arctan(cosh(b*x + a) + sinh(b*x + a)) + cosh(b*x + a) + sinh(b*x + a))/(b*
cosh(b*x + a)^2 + 2*b*cosh(b*x + a)*sinh(b*x + a) + b*sinh(b*x + a)^2 - b)
```

giac [B] time = 0.12, size = 54, normalized size = 2.25

$$-\frac{\pi + \frac{4}{e^{(bx+a)} - e^{(-bx-a)}} + 2\arctan\left(\frac{1}{2}\left(e^{(2\,bx+2\,a)} - 1\right)e^{(-bx-a)}\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^2*sech(b*x+a),x, algorithm="giac")
```

```
[Out] -1/2*(pi + 4/(e^(b*x + a) - e^(-b*x - a)) + 2*arctan(1/2*(e^(2*b*x + 2*a) -
 1)*e^(-b*x - a)))/b
```

maple [A] time = 0.10, size = 27, normalized size = 1.12

$$-\frac{1}{b\sinh\left(bx+a\right)}-\frac{2\arctan\left(\mathrm{e}^{bx+a}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(b*x+a)^2*sech(b*x+a),x)
```

```
[Out] -1/b/sinh(b*x+a)-2*arctan(exp(b*x+a))/b
```

maxima [A] time = 0.65, size = 43, normalized size = 1.79

$$\frac{2\arctan\left(e^{(-bx-a)}\right)}{b}+\frac{2\,e^{(-bx-a)}}{b\left(e^{(-2\,bx-2\,a)}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^2*sech(b*x+a),x, algorithm="maxima")
```

```
[Out] 2*arctan(e^(-b*x - a))/b + 2*e^(-b*x - a)/(b*(e^(-2*b*x - 2*a) - 1))
```

mupad [B] time = 1.47, size = 48, normalized size = 2.00

$$-\frac{2\,\mathrm{atan}\left(\frac{\mathrm{e}^{b\,x}\,\mathrm{e}^{a}\,\sqrt{b^2}}{b}\right)}{\sqrt{b^2}}-\frac{2\,\mathrm{e}^{a+b\,x}}{b\,\left(\mathrm{e}^{2\,a+2\,b\,x}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(a + b*x)*sinh(a + b*x)^2),x)
```

```
[Out] - (2*atan((exp(b*x)*exp(a)*(b^2)^(1/2))/b))/(b^2)^(1/2) - (2*exp(a + b*x))/
(b*(exp(2*a + 2*b*x) - 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\operatorname{csch}^{2}\left(a+bx\right)\operatorname{sech}\left(a+bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)**2*sech(b*x+a),x)
```

```
[Out] Integral(csch(a + b*x)**2*sech(a + b*x), x)
```

3.30 $\int \operatorname{csch}^2(a+bx)\operatorname{sech}^2(a+bx)\,dx$

Optimal. Leaf size=23

$$-\frac{\tanh(a+bx)}{b}-\frac{\coth(a+bx)}{b}$$

```
[Out] -coth(b*x+a)/b-tanh(b*x+a)/b
```

Rubi [A] time = 0.03, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2620, 14}

$$-\frac{\tanh(a+bx)}{b}-\frac{\coth(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[a + b*x]^2*Sech[a + b*x]^2,x]
```

```
[Out] -(Coth[a + b*x]/b) - Tanh[a + b*x]/b
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2620

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rubi steps

$$\begin{aligned}\int \operatorname{csch}^2(a+bx)\operatorname{sech}^2(a+bx)\,dx &= \frac{i\,\text{Subst}\left(\int \frac{1+x^2}{x^2}\,dx, x, i\tanh(a+bx)\right)}{b}\\ &= \frac{i\,\text{Subst}\left(\int \left(1+\frac{1}{x^2}\right)dx, x, i\tanh(a+bx)\right)}{b}\\ &= -\frac{\coth(a+bx)}{b}-\frac{\tanh(a+bx)}{b}\end{aligned}$$

Mathematica [A] time = 0.02, size = 13, normalized size = 0.57

$$-\frac{2\coth(2(a+bx))}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[a + b*x]^2*Sech[a + b*x]^2,x]
```

```
[Out] (-2*Coth[2*(a + b*x)])/b
```

fricas [B] time = 0.41, size = 81, normalized size = 3.52

$$-\frac{4}{b\cosh\left(bx+a\right)^{4}+4\,b\cosh\left(bx+a\right)^{3}\sinh\left(bx+a\right)+6\,b\cosh\left(bx+a\right)^{2}\sinh\left(bx+a\right)^{2}+4\,b\cosh\left(bx+a\right)\sinh\left(\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^2*sech(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -4/(b*cosh(b*x + a)^4 + 4*b*cosh(b*x + a)^3*sinh(b*x + a) + 6*b*cosh(b*x +
a)^2*sinh(b*x + a)^2 + 4*b*cosh(b*x + a)*sinh(b*x + a)^3 + b*sinh(b*x + a)^
4 - b)
```

giac [A] time = 0.15, size = 18, normalized size = 0.78

$$-\frac{4}{b\left(e^{(4\,bx+4\,a)}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^2*sech(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -4/(b*(e^(4*b*x + 4*a) - 1))
```

maple [A] time = 0.33, size = 32, normalized size = 1.39

$$\frac{-\frac{1}{\sinh(bx+a)\cosh(bx+a)}-2\tanh\left(bx+a\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(b*x+a)^2*sech(b*x+a)^2,x)
```

```
[Out] 1/b*(-1/sinh(b*x+a)/cosh(b*x+a)-2*tanh(b*x+a))
```

maxima [A] time = 0.44, size = 18, normalized size = 0.78

$$\frac{4}{b\left(e^{(-4\,bx-4\,a)}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^2*sech(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 4/(b*(e^(-4*b*x - 4*a) - 1))
```

mupad [B] time = 0.08, size = 18, normalized size = 0.78

$$-\frac{4}{b\,\left(\mathrm{e}^{4\,a+4\,b\,x}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(a + b*x)^2*sinh(a + b*x)^2),x)
```

```
[Out] -4/(b*(exp(4*a + 4*b*x) - 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{csch}^2(a+bx)\operatorname{sech}^2(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)**2*sech(b*x+a)**2,x)
```

```
[Out] Integral(csch(a + b*x)**2*sech(a + b*x)**2, x)
```

3.31 $\int \operatorname{csch}^2(a+bx)\operatorname{sech}^3(a+bx)\,dx$

Optimal. Leaf size=49

$$-\frac{3\operatorname{csch}(a+bx)}{2b}-\frac{3\tan^{-1}(\sinh(a+bx))}{2b}+\frac{\operatorname{csch}(a+bx)\operatorname{sech}^2(a+bx)}{2b}$$

```
[Out] -3/2*arctan(sinh(b*x+a))/b-3/2*csch(b*x+a)/b+1/2*csch(b*x+a)*sech(b*x+a)^2/
b
```

Rubi [A] time = 0.04, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {2621, 288, 321, 207}

$$-\frac{3\operatorname{csch}(a+bx)}{2b}-\frac{3\tan^{-1}(\sinh(a+bx))}{2b}+\frac{\operatorname{csch}(a+bx)\operatorname{sech}^2(a+bx)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[a + b*x]^2*Sech[a + b*x]^3,x]
```

```
[Out] (-3*ArcTan[Sinh[a + b*x]])/(2*b) - (3*Csch[a + b*x])/(2*b) + (Csch[a + b*x]
*Sech[a + b*x]^2)/(2*b)
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2621

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> -Dist[(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] && !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rubi steps

$$\int \operatorname{csch}^2(a+bx)\operatorname{sech}^3(a+bx)\,dx = -\frac{i\,\text{Subst}\left(\int \frac{x^4}{\left(-1+x^2\right)^2}\,dx, x, -i\operatorname{csch}(a+bx)\right)}{b}$$
$$= \frac{\operatorname{csch}(a+bx)\operatorname{sech}^2(a+bx)}{2b} - \frac{(3i)\,\text{Subst}\left(\int \frac{x^2}{-1+x^2}\,dx, x, -i\operatorname{csch}(a+bx)\right)}{2b}$$
$$= -\frac{3\operatorname{csch}(a+bx)}{2b} + \frac{\operatorname{csch}(a+bx)\operatorname{sech}^2(a+bx)}{2b} - \frac{(3i)\,\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, -\right.}{2b}$$
$$= -\frac{3\tan^{-1}(\sinh(a+bx))}{2b} - \frac{3\operatorname{csch}(a+bx)}{2b} + \frac{\operatorname{csch}(a+bx)\operatorname{sech}^2(a+bx)}{2b}$$

Mathematica [C] time = 0.02, size = 29, normalized size = 0.59

$$-\frac{\operatorname{csch}(a+bx)\,{}_2F_1\left(-\frac{1}{2},2;\frac{1}{2};-\sinh^2(a+bx)\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[a + b*x]^2*Sech[a + b*x]^3,x]
```

```
[Out] -((Csch[a + b*x]*Hypergeometric2F1[-1/2, 2, 1/2, -Sinh[a + b*x]^2])/b)
```

fricas [B] time = 0.42, size = 511, normalized size = 10.43

$$-\frac{3\cosh(bx+a)^5 + 15\cosh(bx+a)\sinh(bx+a)^4 + 3\sinh(bx+a)^5 + 2\left(15\cosh(bx+a)^2+1\right)\sinh(bx+a}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^2*sech(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -(3*cosh(b*x + a)^5 + 15*cosh(b*x + a)*sinh(b*x + a)^4 + 3*sinh(b*x + a)^5
+ 2*(15*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^3 + 2*cosh(b*x + a)^3 + 6*(5*cos
```

```
h(b*x + a)^3 + cosh(b*x + a))*sinh(b*x + a)^2 + 3*(cosh(b*x + a)^6 + 6*cosh
(b*x + a)*sinh(b*x + a)^5 + sinh(b*x + a)^6 + (15*cosh(b*x + a)^2 + 1)*sinh
(b*x + a)^4 + cosh(b*x + a)^4 + 4*(5*cosh(b*x + a)^3 + cosh(b*x + a))*sinh(
b*x + a)^3 + (15*cosh(b*x + a)^4 + 6*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^2 -
 cosh(b*x + a)^2 + 2*(3*cosh(b*x + a)^5 + 2*cosh(b*x + a)^3 - cosh(b*x + a)
)*sinh(b*x + a) - 1)*arctan(cosh(b*x + a) + sinh(b*x + a)) + 3*(5*cosh(b*x
+ a)^4 + 2*cosh(b*x + a)^2 + 1)*sinh(b*x + a) + 3*cosh(b*x + a))/(b*cosh(b*
x + a)^6 + 6*b*cosh(b*x + a)*sinh(b*x + a)^5 + b*sinh(b*x + a)^6 + b*cosh(b
*x + a)^4 + (15*b*cosh(b*x + a)^2 + b)*sinh(b*x + a)^4 + 4*(5*b*cosh(b*x +
a)^3 + b*cosh(b*x + a))*sinh(b*x + a)^3 - b*cosh(b*x + a)^2 + (15*b*cosh(b*
x + a)^4 + 6*b*cosh(b*x + a)^2 - b)*sinh(b*x + a)^2 + 2*(3*b*cosh(b*x + a)^
5 + 2*b*cosh(b*x + a)^3 - b*cosh(b*x + a))*sinh(b*x + a) - b)
```

giac [B] time = 0.16, size = 102, normalized size = 2.08

$$-\frac{3\,\pi + \frac{4\left(3\left(e^{(bx+a)}-e^{(-bx-a)}\right)^{2}+8\right)}{\left(e^{(bx+a)}-e^{(-bx-a)}\right)^{3}+4\,e^{(bx+a)}-4\,e^{(-bx-a)}} + 6\,\arctan\left(\frac{1}{2}\left(e^{(2\,bx+2\,a)}-1\right)e^{(-bx-a)}\right)}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^2*sech(b*x+a)^3,x, algorithm="giac")
```

```
[Out] -1/4*(3*pi + 4*(3*(e^(b*x + a) - e^(-b*x - a))^2 + 8)/((e^(b*x + a) - e^(-b
*x - a))^3 + 4*e^(b*x + a) - 4*e^(-b*x - a)) + 6*arctan(1/2*(e^(2*b*x + 2*a
) - 1)*e^(-b*x - a)))/b
```

maple [A] time = 0.34, size = 52, normalized size = 1.06

$$-\frac{1}{b\sinh\left(bx+a\right)\cosh\left(bx+a\right)^{2}}-\frac{3\,\mathrm{sech}\left(bx+a\right)\tanh\left(bx+a\right)}{2b}-\frac{3\arctan\left(\mathrm{e}^{bx+a}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(b*x+a)^2*sech(b*x+a)^3,x)
```

```
[Out] -1/b/sinh(b*x+a)/cosh(b*x+a)^2-3/2*sech(b*x+a)*tanh(b*x+a)/b-3*arctan(exp(b
*x+a))/b
```

maxima [B] time = 0.82, size = 90, normalized size = 1.84

$$\frac{3\,\arctan\left(e^{(-bx-a)}\right)}{b}-\frac{3\,e^{(-bx-a)}+2\,e^{(-3\,bx-3\,a)}+3\,e^{(-5\,bx-5\,a)}}{b\left(e^{(-2\,bx-2\,a)}-e^{(-4\,bx-4\,a)}-e^{(-6\,bx-6\,a)}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^2*sech(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 3*arctan(e^(-b*x - a))/b - (3*e^(-b*x - a) + 2*e^(-3*b*x - 3*a) + 3*e^(-5*b
*x - 5*a))/(b*(e^(-2*b*x - 2*a) - e^(-4*b*x - 4*a) - e^(-6*b*x - 6*a) + 1))
```

mupad [B] time = 1.51, size = 107, normalized size = 2.18

$$\frac{2\,e^{a+b\,x}}{b\,\left(2\,e^{2\,a+2\,b\,x}+e^{4\,a+4\,b\,x}+1\right)}-\frac{3\,\mathrm{atan}\left(\frac{e^{b\,x}\,e^{a}\,\sqrt{b^2}}{b}\right)}{\sqrt{b^2}}-\frac{2\,e^{a+b\,x}}{b\,\left(e^{2\,a+2\,b\,x}-1\right)}-\frac{e^{a+b\,x}}{b\,\left(e^{2\,a+2\,b\,x}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(a + b*x)^3*sinh(a + b*x)^2),x)
```

```
[Out] (2*exp(a + b*x))/(b*(2*exp(2*a + 2*b*x) + exp(4*a + 4*b*x) + 1)) - (3*atan(
(exp(b*x)*exp(a)*(b^2)^(1/2))/b))/(b^2)^(1/2) - (2*exp(a + b*x))/(b*(exp(2*
a + 2*b*x) - 1)) - exp(a + b*x)/(b*(exp(2*a + 2*b*x) + 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{csch}^{2}{\left(a + bx \right)} \operatorname{sech}^{3}{\left(a + bx \right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)**2*sech(b*x+a)**3,x)
```

```
[Out] Integral(csch(a + b*x)**2*sech(a + b*x)**3, x)
```

3.32 $\int \operatorname{csch}^2(a+bx)\operatorname{sech}^4(a+bx)\,dx$

Optimal. Leaf size=38

$$\frac{\tanh^3(a+bx)}{3b}-\frac{2\tanh(a+bx)}{b}-\frac{\coth(a+bx)}{b}$$

```
[Out] -coth(b*x+a)/b-2*tanh(b*x+a)/b+1/3*tanh(b*x+a)^3/b
```

Rubi [A] time = 0.04, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2620, 270}

$$\frac{\tanh^3(a+bx)}{3b}-\frac{2\tanh(a+bx)}{b}-\frac{\coth(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[a + b*x]^2*Sech[a + b*x]^4,x]
```

```
[Out] -(Coth[a + b*x]/b) - (2*Tanh[a + b*x])/b + Tanh[a + b*x]^3/(3*b)
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2620

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rubi steps

$$\begin{aligned}\int \operatorname{csch}^2(a+bx)\operatorname{sech}^4(a+bx)\,dx &= \frac{i\,\text{Subst}\left(\int \frac{(1+x^2)^2}{x^2}\,dx,x,i\tanh(a+bx)\right)}{b}\\ &= \frac{i\,\text{Subst}\left(\int \left(2+\frac{1}{x^2}+x^2\right)dx,x,i\tanh(a+bx)\right)}{b}\\ &= -\frac{\coth(a+bx)}{b}-\frac{2\tanh(a+bx)}{b}+\frac{\tanh^3(a+bx)}{3b}\end{aligned}$$

Mathematica [A] time = 0.04, size = 46, normalized size = 1.21

$$-\frac{5\tanh(a+bx)}{3b}-\frac{\coth(a+bx)}{b}-\frac{\tanh(a+bx)\text{sech}^2(a+bx)}{3b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[a + b*x]^2*Sech[a + b*x]^4,x]
```

```
[Out] -(Coth[a + b*x]/b) - (5*Tanh[a + b*x])/(3*b) - (Sech[a + b*x]^2*Tanh[a + b*
x])/(3*b)
```

fricas [B] time = 0.40, size = 230, normalized size = 6.05

$$-\frac{}{3\left(b\cosh(bx+a)^7+7\,b\cosh(bx+a)\sinh(bx+a)^6+b\sinh(bx+a)^7+2\,b\cosh(bx+a)^5+\left(21\,b\cosh(bx\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^2*sech(b*x+a)^4,x, algorithm="fricas")
```

```
[Out] -16/3*(3*cosh(b*x + a) + sinh(b*x + a))/(b*cosh(b*x + a)^7 + 7*b*cosh(b*x +
 a)*sinh(b*x + a)^6 + b*sinh(b*x + a)^7 + 2*b*cosh(b*x + a)^5 + (21*b*cosh(
b*x + a)^2 + 2*b)*sinh(b*x + a)^5 + 5*(7*b*cosh(b*x + a)^3 + 2*b*cosh(b*x +
 a))*sinh(b*x + a)^4 + 5*(7*b*cosh(b*x + a)^4 + 4*b*cosh(b*x + a)^2)*sinh(b
*x + a)^3 + (21*b*cosh(b*x + a)^5 + 20*b*cosh(b*x + a)^3)*sinh(b*x + a)^2 -
 3*b*cosh(b*x + a) + (7*b*cosh(b*x + a)^6 + 10*b*cosh(b*x + a)^4 - b)*sinh(
b*x + a))
```

giac [A] time = 0.15, size = 60, normalized size = 1.58

$$-\frac{2\left(\frac{3}{e^{(2\,bx+2\,a)}-1}-\frac{3\,e^{(4\,bx+4\,a)}+12\,e^{(2\,bx+2\,a)}+5}{\left(e^{(2\,bx+2\,a)}+1\right)^3}\right)}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^2*sech(b*x+a)^4,x, algorithm="giac")
```

```
[Out] -2/3*(3/(e^(2*b*x + 2*a) - 1) - (3*e^(4*b*x + 4*a) + 12*e^(2*b*x + 2*a) + 5
)/(e^(2*b*x + 2*a) + 1)^3)/b
```

maple [A] time = 0.30, size = 44, normalized size = 1.16

$$\frac{-\frac{1}{\sinh(bx+a)\cosh(bx+a)^3}-4\left(\frac{2}{3}+\frac{\text{sech}(bx+a)^2}{3}\right)\tanh(bx+a)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(b*x+a)^2*sech(b*x+a)^4,x)
```

```
[Out] 1/b*(-1/sinh(b*x+a)/cosh(b*x+a)^3-4*(2/3+1/3*sech(b*x+a)^2)*tanh(b*x+a))
```

maxima [B] time = 0.31, size = 94, normalized size = 2.47

$$-\frac{32\,e^{(-2\,bx-2\,a)}}{3\,b\left(2\,e^{(-2\,bx-2\,a)}-2\,e^{(-6\,bx-6\,a)}-e^{(-8\,bx-8\,a)}+1\right)}-\frac{16}{3\,b\left(2\,e^{(-2\,bx-2\,a)}-2\,e^{(-6\,bx-6\,a)}-e^{(-8\,bx-8\,a)}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^2*sech(b*x+a)^4,x, algorithm="maxima")
```

```
[Out] -32/3*e^(-2*b*x - 2*a)/(b*(2*e^(-2*b*x - 2*a) - 2*e^(-6*b*x - 6*a) - e^(-8*
b*x - 8*a) + 1)) - 16/3/(b*(2*e^(-2*b*x - 2*a) - 2*e^(-6*b*x - 6*a) - e^(-8
*b*x - 8*a) + 1))
```

mupad [B] time = 1.52, size = 152, normalized size = 4.00

$$\frac{\frac{2}{3\,b}+\frac{4\,e^{2\,a+2\,b\,x}}{b}+\frac{2\,e^{4\,a+4\,b\,x}}{3\,b}}{3\,e^{2\,a+2\,b\,x}+3\,e^{4\,a+4\,b\,x}+e^{6\,a+6\,b\,x}+1}+\frac{\frac{2}{b}+\frac{2\,e^{2\,a+2\,b\,x}}{3\,b}}{2\,e^{2\,a+2\,b\,x}+e^{4\,a+4\,b\,x}+1}-\frac{2}{b\,\left(e^{2\,a+2\,b\,x}-1\right)}+\frac{2}{3\,b\,\left(e^{2\,a+2\,b\,x}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(a + b*x)^4*sinh(a + b*x)^2),x)
```

```
[Out] (2/(3*b) + (4*exp(2*a + 2*b*x))/b + (2*exp(4*a + 4*b*x))/(3*b))/(3*exp(2*a
+ 2*b*x) + 3*exp(4*a + 4*b*x) + exp(6*a + 6*b*x) + 1) + (2/b + (2*exp(2*a +
 2*b*x))/(3*b))/(2*exp(2*a + 2*b*x) + exp(4*a + 4*b*x) + 1) - 2/(b*(exp(2*a
 + 2*b*x) - 1)) + 2/(3*b*(exp(2*a + 2*b*x) + 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{csch}^{2}(a+bx)\operatorname{sech}^{4}(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)**2*sech(b*x+a)**4,x)
```

```
[Out] Integral(csch(a + b*x)**2*sech(a + b*x)**4, x)
```

3.33 $\int \operatorname{csch}^2(a+bx)\operatorname{sech}^5(a+bx)\,dx$

Optimal. Leaf size=70

$$-\frac{15\operatorname{csch}(a+bx)}{8b}-\frac{15\tan^{-1}(\sinh(a+bx))}{8b}+\frac{\operatorname{csch}(a+bx)\operatorname{sech}^4(a+bx)}{4b}+\frac{5\operatorname{csch}(a+bx)\operatorname{sech}^2(a+bx)}{8b}$$

```
[Out] -15/8*arctan(sinh(b*x+a))/b-15/8*csch(b*x+a)/b+5/8*csch(b*x+a)*sech(b*x+a)^
2/b+1/4*csch(b*x+a)*sech(b*x+a)^4/b
```

Rubi [A] time = 0.04, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {2621, 288, 321, 207}

$$-\frac{15\operatorname{csch}(a+bx)}{8b}-\frac{15\tan^{-1}(\sinh(a+bx))}{8b}+\frac{\operatorname{csch}(a+bx)\operatorname{sech}^4(a+bx)}{4b}+\frac{5\operatorname{csch}(a+bx)\operatorname{sech}^2(a+bx)}{8b}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[a + b*x]^2*Sech[a + b*x]^5,x]
```

```
[Out] (-15*ArcTan[Sinh[a + b*x]])/(8*b) - (15*Csch[a + b*x])/(8*b) + (5*Csch[a +
b*x]*Sech[a + b*x]^2)/(8*b) + (Csch[a + b*x]*Sech[a + b*x]^4)/(4*b)
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2621

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> -Dist[(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] && !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rubi steps

$$\int \operatorname{csch}^2(a+bx)\operatorname{sech}^5(a+bx)\,dx = -\frac{i\,\text{Subst}\left(\int \frac{x^6}{\left(-1+x^2\right)^3}\,dx, x, -i\operatorname{csch}(a+bx)\right)}{b}$$

$$= \frac{\operatorname{csch}(a+bx)\operatorname{sech}^4(a+bx)}{4b} - \frac{(5i)\,\text{Subst}\left(\int \frac{x^4}{\left(-1+x^2\right)^2}\,dx, x, -i\operatorname{csch}(a+bx)\right)}{4b}$$

$$= \frac{5\operatorname{csch}(a+bx)\operatorname{sech}^2(a+bx)}{8b} + \frac{\operatorname{csch}(a+bx)\operatorname{sech}^4(a+bx)}{4b} - \frac{(15i)\,\text{Subst}\left(\int\right.}{}$$

$$= -\frac{15\operatorname{csch}(a+bx)}{8b} + \frac{5\operatorname{csch}(a+bx)\operatorname{sech}^2(a+bx)}{8b} + \frac{\operatorname{csch}(a+bx)\operatorname{sech}^4(a+bx}{4b}$$

$$= -\frac{15\tan^{-1}(\sinh(a+bx))}{8b} - \frac{15\operatorname{csch}(a+bx)}{8b} + \frac{5\operatorname{csch}(a+bx)\operatorname{sech}^2(a+bx)}{8b} +$$

Mathematica [C] time = 0.02, size = 29, normalized size = 0.41

$$-\frac{\operatorname{csch}(a+bx)\,{}_2F_1\left(-\frac{1}{2},3;\frac{1}{2};-\sinh^2(a+bx)\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[a + b*x]^2*Sech[a + b*x]^5,x]
[Out] -((Csch[a + b*x]*Hypergeometric2F1[-1/2, 3, 1/2, -Sinh[a + b*x]^2])/b)
```

fricas [B] time = 0.44, size = 1183, normalized size = 16.90

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^2*sech(b*x+a)^5,x, algorithm="fricas")
```

```
[Out] -1/4*(15*cosh(b*x + a)^9 + 135*cosh(b*x + a)*sinh(b*x + a)^8 + 15*sinh(b*x
+ a)^9 + 20*(27*cosh(b*x + a)^2 + 2)*sinh(b*x + a)^7 + 40*cosh(b*x + a)^7 +
 140*(9*cosh(b*x + a)^3 + 2*cosh(b*x + a))*sinh(b*x + a)^6 + 6*(315*cosh(b*
x + a)^4 + 140*cosh(b*x + a)^2 + 3)*sinh(b*x + a)^5 + 18*cosh(b*x + a)^5 +
10*(189*cosh(b*x + a)^5 + 140*cosh(b*x + a)^3 + 9*cosh(b*x + a))*sinh(b*x +
 a)^4 + 20*(63*cosh(b*x + a)^6 + 70*cosh(b*x + a)^4 + 9*cosh(b*x + a)^2 + 2
)*sinh(b*x + a)^3 + 40*cosh(b*x + a)^3 + 60*(9*cosh(b*x + a)^7 + 14*cosh(b*
x + a)^5 + 3*cosh(b*x + a)^3 + 2*cosh(b*x + a))*sinh(b*x + a)^2 + 15*(cosh(
b*x + a)^10 + 10*cosh(b*x + a)*sinh(b*x + a)^9 + sinh(b*x + a)^10 + 3*(15*c
osh(b*x + a)^2 + 1)*sinh(b*x + a)^8 + 3*cosh(b*x + a)^8 + 24*(5*cosh(b*x +
a)^3 + cosh(b*x + a))*sinh(b*x + a)^7 + 2*(105*cosh(b*x + a)^4 + 42*cosh(b*
x + a)^2 + 1)*sinh(b*x + a)^6 + 2*cosh(b*x + a)^6 + 12*(21*cosh(b*x + a)^5
+ 14*cosh(b*x + a)^3 + cosh(b*x + a))*sinh(b*x + a)^5 + 2*(105*cosh(b*x + a
)^6 + 105*cosh(b*x + a)^4 + 15*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^4 - 2*cos
h(b*x + a)^4 + 8*(15*cosh(b*x + a)^7 + 21*cosh(b*x + a)^5 + 5*cosh(b*x + a)
^3 - cosh(b*x + a))*sinh(b*x + a)^3 + 3*(15*cosh(b*x + a)^8 + 28*cosh(b*x +
 a)^6 + 10*cosh(b*x + a)^4 - 4*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^2 - 3*cos
h(b*x + a)^2 + 2*(5*cosh(b*x + a)^9 + 12*cosh(b*x + a)^7 + 6*cosh(b*x + a)^
5 - 4*cosh(b*x + a)^3 - 3*cosh(b*x + a))*sinh(b*x + a) - 1)*arctan(cosh(b*x
 + a) + sinh(b*x + a)) + 5*(27*cosh(b*x + a)^8 + 56*cosh(b*x + a)^6 + 18*co
sh(b*x + a)^4 + 24*cosh(b*x + a)^2 + 3)*sinh(b*x + a) + 15*cosh(b*x + a))/(
b*cosh(b*x + a)^10 + 10*b*cosh(b*x + a)*sinh(b*x + a)^9 + b*sinh(b*x + a)^1
0 + 3*b*cosh(b*x + a)^8 + 3*(15*b*cosh(b*x + a)^2 + b)*sinh(b*x + a)^8 + 24
*(5*b*cosh(b*x + a)^3 + b*cosh(b*x + a))*sinh(b*x + a)^7 + 2*b*cosh(b*x + a
)^6 + 2*(105*b*cosh(b*x + a)^4 + 42*b*cosh(b*x + a)^2 + b)*sinh(b*x + a)^6
+ 12*(21*b*cosh(b*x + a)^5 + 14*b*cosh(b*x + a)^3 + b*cosh(b*x + a))*sinh(b
*x + a)^5 - 2*b*cosh(b*x + a)^4 + 2*(105*b*cosh(b*x + a)^6 + 105*b*cosh(b*x
 + a)^4 + 15*b*cosh(b*x + a)^2 - b)*sinh(b*x + a)^4 + 8*(15*b*cosh(b*x + a)
^7 + 21*b*cosh(b*x + a)^5 + 5*b*cosh(b*x + a)^3 - b*cosh(b*x + a))*sinh(b*x
 + a)^3 - 3*b*cosh(b*x + a)^2 + 3*(15*b*cosh(b*x + a)^8 + 28*b*cosh(b*x + a
)^6 + 10*b*cosh(b*x + a)^4 - 4*b*cosh(b*x + a)^2 - b)*sinh(b*x + a)^2 + 2*(
5*b*cosh(b*x + a)^9 + 12*b*cosh(b*x + a)^7 + 6*b*cosh(b*x + a)^5 - 4*b*cosh
(b*x + a)^3 - 3*b*cosh(b*x + a))*sinh(b*x + a) - b)
```

giac [A] time = 0.16, size = 124, normalized size = 1.77

$$-\frac{15\,\pi + \frac{4\left(7\left(e^{(bx+a)}-e^{(-bx-a)}\right)^{3}+36\,e^{(bx+a)}-36\,e^{(-bx-a)}\right)}{\left(\left(e^{(bx+a)}-e^{(-bx-a)}\right)^{2}+4\right)^{2}} + \frac{32}{e^{(bx+a)}-e^{(-bx-a)}} + 30\,\arctan\left(\frac{1}{2}\left(e^{(2\,bx+2\,a)}-1\right)e^{(-bx-a)}\right)}{16\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^2*sech(b*x+a)^5,x, algorithm="giac")
```

```
[Out] -1/16*(15*pi + 4*(7*(e^(b*x + a) - e^(-b*x - a))^3 + 36*e^(b*x + a) - 36*e^
```

```
(-b*x - a))/((e^(b*x + a) - e^(-b*x - a))^2 + 4)^2 + 32/(e^(b*x + a) - e^(-
b*x - a)) + 30*arctan(1/2*(e^(2*b*x + 2*a) - 1)*e^(-b*x - a)))/b
```

maple [A] time = 0.33, size = 71, normalized size = 1.01

$$-\frac{1}{b\sinh\left(bx+a\right)\cosh\left(bx+a\right)^{4}}-\frac{5\mathrm{sech}\left(bx+a\right)^{3}\tanh\left(bx+a\right)}{4b}-\frac{15\,\mathrm{sech}\left(bx+a\right)\tanh\left(bx+a\right)}{8b}-\frac{15\arctan\left(\mathrm{e}^{bx+a}\right)}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(b*x+a)^2*sech(b*x+a)^5,x)
```

```
[Out] -1/b/sinh(b*x+a)/cosh(b*x+a)^4-5/4*sech(b*x+a)^3*tanh(b*x+a)/b-15/8*sech(b*
x+a)*tanh(b*x+a)/b-15/4*arctan(exp(b*x+a))/b
```

maxima [B] time = 0.41, size = 136, normalized size = 1.94

$$\frac{15\arctan\left(e^{(-bx-a)}\right)}{4\,b}-\frac{15\,e^{(-bx-a)}+40\,e^{(-3\,bx-3\,a)}+18\,e^{(-5\,bx-5\,a)}+40\,e^{(-7\,bx-7\,a)}+15\,e^{(-9\,bx-9\,a)}}{4\,b\left(3\,e^{(-2\,bx-2\,a)}+2\,e^{(-4\,bx-4\,a)}-2\,e^{(-6\,bx-6\,a)}-3\,e^{(-8\,bx-8\,a)}-e^{(-10\,bx-10\,a)}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^2*sech(b*x+a)^5,x, algorithm="maxima")
```

```
[Out] 15/4*arctan(e^(-b*x - a))/b - 1/4*(15*e^(-b*x - a) + 40*e^(-3*b*x - 3*a) +
18*e^(-5*b*x - 5*a) + 40*e^(-7*b*x - 7*a) + 15*e^(-9*b*x - 9*a))/(b*(3*e^(-
2*b*x - 2*a) + 2*e^(-4*b*x - 4*a) - 2*e^(-6*b*x - 6*a) - 3*e^(-8*b*x - 8*a)
 - e^(-10*b*x - 10*a) + 1))
```

mupad [B] time = 0.07, size = 210, normalized size = 3.00

$$\frac{3\,\mathrm{e}^{a+b\,x}}{2\,b\,\left(2\,\mathrm{e}^{2\,a+2\,b\,x}+\mathrm{e}^{4\,a+4\,b\,x}+1\right)}-\frac{15\,\mathrm{atan}\left(\frac{\mathrm{e}^{b\,x}\,\mathrm{e}^{a}\,\sqrt{b^2}}{b}\right)}{4\,\sqrt{b^2}}+\frac{6\,\mathrm{e}^{a+b\,x}}{b\,\left(3\,\mathrm{e}^{2\,a+2\,b\,x}+3\,\mathrm{e}^{4\,a+4\,b\,x}+\mathrm{e}^{6\,a+6\,b\,x}+1\right)}-\frac{}{b\,\left(4\,\mathrm{e}^{2\,a+2\,b\,x}+6\,\mathrm{e}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(a + b*x)^5*sinh(a + b*x)^2),x)
```

```
[Out] (3*exp(a + b*x))/(2*b*(2*exp(2*a + 2*b*x) + exp(4*a + 4*b*x) + 1)) - (15*at
an((exp(b*x)*exp(a)*(b^2)^(1/2))/b))/(4*(b^2)^(1/2)) + (6*exp(a + b*x))/(b*
(3*exp(2*a + 2*b*x) + 3*exp(4*a + 4*b*x) + exp(6*a + 6*b*x) + 1)) - (4*exp(
a + b*x))/(b*(4*exp(2*a + 2*b*x) + 6*exp(4*a + 4*b*x) + 4*exp(6*a + 6*b*x)
+ exp(8*a + 8*b*x) + 1)) - (2*exp(a + b*x))/(b*(exp(2*a + 2*b*x) - 1)) - (7
*exp(a + b*x))/(4*b*(exp(2*a + 2*b*x) + 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{csch}^2(a+bx)\operatorname{sech}^5(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)**2*sech(b*x+a)**5,x)
```

```
[Out] Integral(csch(a + b*x)**2*sech(a + b*x)**5, x)
```

3.34 $\int \operatorname{csch}^3(a+bx)\operatorname{sech}(a+bx)\,dx$

Optimal. Leaf size=28

$$-\frac{\coth^2(a+bx)}{2b}-\frac{\log(\tanh(a+bx))}{b}$$

```
[Out] -1/2*coth(b*x+a)^2/b-ln(tanh(b*x+a))/b
```

Rubi [A] time = 0.02, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2620, 14}

$$-\frac{\coth^2(a+bx)}{2b}-\frac{\log(\tanh(a+bx))}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[a + b*x]^3*Sech[a + b*x],x]
```

```
[Out] -Coth[a + b*x]^2/(2*b) - Log[Tanh[a + b*x]]/b
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2620

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rubi steps

$$\begin{aligned}\int \operatorname{csch}^3(a+bx)\operatorname{sech}(a+bx)\,dx &= -\frac{\operatorname{Subst}\left(\int \frac{1+x^2}{x^3}\,dx,x,i\tanh(a+bx)\right)}{b}\\ &= -\frac{\operatorname{Subst}\left(\int \left(\frac{1}{x^3}+\frac{1}{x}\right)\,dx,x,i\tanh(a+bx)\right)}{b}\\ &= -\frac{\coth^2(a+bx)}{2b}-\frac{\log(\tanh(a+bx))}{b}\end{aligned}$$

Mathematica [A] time = 0.05, size = 34, normalized size = 1.21

$$-\frac{\text{csch}^2(a+bx)+2\log(\sinh(a+bx))-2\log(\cosh(a+bx))}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[a + b*x]^3*Sech[a + b*x],x]
```

```
[Out] -1/2*(Csch[a + b*x]^2 - 2*Log[Cosh[a + b*x]] + 2*Log[Sinh[a + b*x]])/b
```

fricas [B] time = 0.43, size = 379, normalized size = 13.54

$$-\frac{2\cosh(bx+a)^2-\left(\cosh(bx+a)^4+4\cosh(bx+a)\sinh(bx+a)^3+\sinh(bx+a)^4+2\left(3\cosh(bx+a)^2-1\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^3*sech(b*x+a),x, algorithm="fricas")
```

```
[Out] -(2*cosh(b*x + a)^2 - (cosh(b*x + a)^4 + 4*cosh(b*x + a)*sinh(b*x + a)^3 +
sinh(b*x + a)^4 + 2*(3*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^2 - 2*cosh(b*x +
a)^2 + 4*(cosh(b*x + a)^3 - cosh(b*x + a))*sinh(b*x + a) + 1)*log(2*cosh(b*
x + a)/(cosh(b*x + a) - sinh(b*x + a))) + (cosh(b*x + a)^4 + 4*cosh(b*x + a
)*sinh(b*x + a)^3 + sinh(b*x + a)^4 + 2*(3*cosh(b*x + a)^2 - 1)*sinh(b*x +
a)^2 - 2*cosh(b*x + a)^2 + 4*(cosh(b*x + a)^3 - cosh(b*x + a))*sinh(b*x + a
) + 1)*log(2*sinh(b*x + a)/(cosh(b*x + a) - sinh(b*x + a))) + 4*cosh(b*x +
a)*sinh(b*x + a) + 2*sinh(b*x + a)^2)/(b*cosh(b*x + a)^4 + 4*b*cosh(b*x + a
)*sinh(b*x + a)^3 + b*sinh(b*x + a)^4 - 2*b*cosh(b*x + a)^2 + 2*(3*b*cosh(b
*x + a)^2 - b)*sinh(b*x + a)^2 + 4*(b*cosh(b*x + a)^3 - b*cosh(b*x + a))*si
nh(b*x + a) + b)
```

giac [B] time = 0.13, size = 93, normalized size = 3.32

$$\frac{\frac{e^{(2bx+2a)}+e^{(-2bx-2a)}-6}{e^{(2bx+2a)}+e^{(-2bx-2a)}-2}+\log\left(e^{(2bx+2a)}+e^{(-2bx-2a)}+2\right)-\log\left(e^{(2bx+2a)}+e^{(-2bx-2a)}-2\right)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^3*sech(b*x+a),x, algorithm="giac")
```

```
[Out] 1/2*((e^(2*b*x + 2*a) + e^(-2*b*x - 2*a) - 6)/(e^(2*b*x + 2*a) + e^(-2*b*x
- 2*a) - 2) + log(e^(2*b*x + 2*a) + e^(-2*b*x - 2*a) + 2) - log(e^(2*b*x +
2*a) + e^(-2*b*x - 2*a) - 2))/b
```

maple [A] time = 0.17, size = 27, normalized size = 0.96

$$-\frac{1}{2b\sinh\left(bx+a\right)^{2}}-\frac{\ln\left(\tanh\left(bx+a\right)\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(b*x+a)^3*sech(b*x+a),x)
```

```
[Out] -1/2/b/sinh(b*x+a)^2-ln(tanh(b*x+a))/b
```

maxima [B] time = 0.41, size = 91, normalized size = 3.25

$$-\frac{\log\left(e^{(-bx-a)}+1\right)}{b}-\frac{\log\left(e^{(-bx-a)}-1\right)}{b}+\frac{\log\left(e^{(-2\,bx-2\,a)}+1\right)}{b}+\frac{2\,e^{(-2\,bx-2\,a)}}{b\left(2\,e^{(-2\,bx-2\,a)}-e^{(-4\,bx-4\,a)}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^3*sech(b*x+a),x, algorithm="maxima")
```

```
[Out] -log(e^(-b*x - a) + 1)/b - log(e^(-b*x - a) - 1)/b + log(e^(-2*b*x - 2*a) +
 1)/b + 2*e^(-2*b*x - 2*a)/(b*(2*e^(-2*b*x - 2*a) - e^(-4*b*x - 4*a) - 1))
```

mupad [B] time = 0.07, size = 78, normalized size = 2.79

$$\frac{2\,\mathrm{atan}\left(\frac{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}\,\sqrt{-b^2}}{b}\right)}{\sqrt{-b^2}}-\frac{2}{b\,\left({\mathrm{e}}^{2\,a+2\,b\,x}-1\right)}-\frac{2}{b\,\left({\mathrm{e}}^{4\,a+4\,b\,x}-2\,{\mathrm{e}}^{2\,a+2\,b\,x}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(a + b*x)*sinh(a + b*x)^3),x)
```

```
[Out] (2*atan((exp(2*a)*exp(2*b*x)*(-b^2)^(1/2))/b))/(-b^2)^(1/2) - 2/(b*(exp(2*a
 + 2*b*x) - 1)) - 2/(b*(exp(4*a + 4*b*x) - 2*exp(2*a + 2*b*x) + 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{csch}^{3}\left(a+bx\right)\operatorname{sech}\left(a+bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)**3*sech(b*x+a),x)
```

```
[Out] Integral(csch(a + b*x)**3*sech(a + b*x), x)
```

3.35 $\int \operatorname{csch}^3(a+bx)\operatorname{sech}^2(a+bx)\,dx$

Optimal. Leaf size=49

$$-\frac{3\operatorname{sech}(a+bx)}{2b}+\frac{3\tanh^{-1}(\cosh(a+bx))}{2b}-\frac{\operatorname{csch}^2(a+bx)\operatorname{sech}(a+bx)}{2b}$$

```
[Out] 3/2*arctanh(cosh(b*x+a))/b-3/2*sech(b*x+a)/b-1/2*csch(b*x+a)^2*sech(b*x+a)/
b
```

Rubi [A] time = 0.05, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {2622, 288, 321, 207}

$$-\frac{3\operatorname{sech}(a+bx)}{2b}+\frac{3\tanh^{-1}(\cosh(a+bx))}{2b}-\frac{\operatorname{csch}^2(a+bx)\operatorname{sech}(a+bx)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[a + b*x]^3*Sech[a + b*x]^2,x]
```

```
[Out] (3*ArcTanh[Cosh[a + b*x]])/(2*b) - (3*Sech[a + b*x])/(2*b) - (Csch[a + b*x]
^2*Sech[a + b*x])/(2*b)
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rubi steps

$$\int \text{csch}^3(a+bx)\text{sech}^2(a+bx)\,dx = -\frac{\text{Subst}\left(\int \frac{x^4}{(-1+x^2)^2}\,dx, x, \text{sech}(a+bx)\right)}{b}$$
$$= -\frac{\text{csch}^2(a+bx)\text{sech}(a+bx)}{2b} - \frac{3\,\text{Subst}\left(\int \frac{x^2}{-1+x^2}\,dx, x, \text{sech}(a+bx)\right)}{2b}$$
$$= -\frac{3\text{sech}(a+bx)}{2b} - \frac{\text{csch}^2(a+bx)\text{sech}(a+bx)}{2b} - \frac{3\,\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \text{sech}\right.}{2b}$$
$$= \frac{3\tanh^{-1}(\cosh(a+bx))}{2b} - \frac{3\text{sech}(a+bx)}{2b} - \frac{\text{csch}^2(a+bx)\text{sech}(a+bx)}{2b}$$

Mathematica [A] time = 0.03, size = 68, normalized size = 1.39

$$-\frac{\text{csch}^2\left(\frac{1}{2}(a+bx)\right)}{8b} - \frac{\text{sech}^2\left(\frac{1}{2}(a+bx)\right)}{8b} - \frac{\text{sech}(a+bx)}{b} - \frac{3\log\left(\tanh\left(\frac{1}{2}(a+bx)\right)\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[a + b*x]^3*Sech[a + b*x]^2,x]
```

```
[Out] -1/8*Csch[(a + b*x)/2]^2/b - (3*Log[Tanh[(a + b*x)/2]])/(2*b) - Sech[(a + b
*x)/2]^2/(8*b) - Sech[a + b*x]/b
```

fricas [B] time = 0.39, size = 709, normalized size = 14.47

$$-\frac{6\cosh(bx+a)^5 + 30\cosh(bx+a)\sinh(bx+a)^4 + 6\sinh(bx+a)^5 + 4\left(15\cosh(bx+a)^2 - 1\right)\sinh(bx+a)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^3*sech(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/2*(6*cosh(b*x + a)^5 + 30*cosh(b*x + a)*sinh(b*x + a)^4 + 6*sinh(b*x + a
)^5 + 4*(15*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^3 - 4*cosh(b*x + a)^3 + 12*(
5*cosh(b*x + a)^3 - cosh(b*x + a))*sinh(b*x + a)^2 - 3*(cosh(b*x + a)^6 + 6
*cosh(b*x + a)*sinh(b*x + a)^5 + sinh(b*x + a)^6 + (15*cosh(b*x + a)^2 - 1)
*sinh(b*x + a)^4 - cosh(b*x + a)^4 + 4*(5*cosh(b*x + a)^3 - cosh(b*x + a))*
sinh(b*x + a)^3 + (15*cosh(b*x + a)^4 - 6*cosh(b*x + a)^2 - 1)*sinh(b*x + a
)^2 - cosh(b*x + a)^2 + 2*(3*cosh(b*x + a)^5 - 2*cosh(b*x + a)^3 - cosh(b*x
 + a))*sinh(b*x + a) + 1)*log(cosh(b*x + a) + sinh(b*x + a) + 1) + 3*(cosh(
b*x + a)^6 + 6*cosh(b*x + a)*sinh(b*x + a)^5 + sinh(b*x + a)^6 + (15*cosh(b
*x + a)^2 - 1)*sinh(b*x + a)^4 - cosh(b*x + a)^4 + 4*(5*cosh(b*x + a)^3 - c
osh(b*x + a))*sinh(b*x + a)^3 + (15*cosh(b*x + a)^4 - 6*cosh(b*x + a)^2 - 1
)*sinh(b*x + a)^2 - cosh(b*x + a)^2 + 2*(3*cosh(b*x + a)^5 - 2*cosh(b*x + a
)^3 - cosh(b*x + a))*sinh(b*x + a) + 1)*log(cosh(b*x + a) + sinh(b*x + a) -
 1) + 6*(5*cosh(b*x + a)^4 - 2*cosh(b*x + a)^2 + 1)*sinh(b*x + a) + 6*cosh(
b*x + a))/(b*cosh(b*x + a)^6 + 6*b*cosh(b*x + a)*sinh(b*x + a)^5 + b*sinh(b
*x + a)^6 - b*cosh(b*x + a)^4 + (15*b*cosh(b*x + a)^2 - b)*sinh(b*x + a)^4
+ 4*(5*b*cosh(b*x + a)^3 - b*cosh(b*x + a))*sinh(b*x + a)^3 - b*cosh(b*x +
a)^2 + (15*b*cosh(b*x + a)^4 - 6*b*cosh(b*x + a)^2 - b)*sinh(b*x + a)^2 + 2
*(3*b*cosh(b*x + a)^5 - 2*b*cosh(b*x + a)^3 - b*cosh(b*x + a))*sinh(b*x + a
) + b)
```

giac [B] time = 0.13, size = 110, normalized size = 2.24

$$-\frac{\frac{4\left(3\left(e^{(bx+a)}+e^{(-bx-a)}\right)^{2}-8\right)}{\left(e^{(bx+a)}+e^{(-bx-a)}\right)^{3}-4\,e^{(bx+a)}-4\,e^{(-bx-a)}}-3\,\log\left(e^{(bx+a)}+e^{(-bx-a)}+2\right)+3\,\log\left(e^{(bx+a)}+e^{(-bx-a)}-2\right)}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^3*sech(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -1/4*(4*(3*(e^(b*x + a) + e^(-b*x - a))^2 - 8)/((e^(b*x + a) + e^(-b*x - a)
)^3 - 4*e^(b*x + a) - 4*e^(-b*x - a)) - 3*log(e^(b*x + a) + e^(-b*x - a) +
2) + 3*log(e^(b*x + a) + e^(-b*x - a) - 2))/b
```

maple [A] time = 0.18, size = 43, normalized size = 0.88

$$\frac{-\frac{1}{2\sinh(bx+a)^{2}\cosh(bx+a)}-\frac{3}{2\cosh(bx+a)}+3\,\mathrm{arctanh}\left(\mathrm{e}^{bx+a}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(b*x+a)^3*sech(b*x+a)^2,x)
```

```
[Out] 1/b*(-1/2/sinh(b*x+a)^2/cosh(b*x+a)-3/2/cosh(b*x+a)+3*arctanh(exp(b*x+a)))
```

maxima [B] time = 0.32, size = 106, normalized size = 2.16

$$\frac{3 \log \left(e^{(-bx-a)}+1\right)}{2 b}-\frac{3 \log \left(e^{(-bx-a)}-1\right)}{2 b}+\frac{3 e^{(-bx-a)}-2 e^{(-3 bx-3 a)}+3 e^{(-5 bx-5 a)}}{b\left(e^{(-2 bx-2 a)}+e^{(-4 bx-4 a)}-e^{(-6 bx-6 a)}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^3*sech(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 3/2*log(e^(-b*x - a) + 1)/b - 3/2*log(e^(-b*x - a) - 1)/b + (3*e^(-b*x - a)
 - 2*e^(-3*b*x - 3*a) + 3*e^(-5*b*x - 5*a))/(b*(e^(-2*b*x - 2*a) + e^(-4*b*
x - 4*a) - e^(-6*b*x - 6*a) - 1))
```

mupad [B] time = 1.46, size = 111, normalized size = 2.27

$$\frac{3 \operatorname{atan}\left(\frac{e^{b x} e^{a} \sqrt{-b^2}}{b}\right)}{\sqrt{-b^2}}-\frac{2 e^{a+b x}}{b \left(e^{4 a+4 b x}-2 e^{2 a+2 b x}+1\right)}-\frac{e^{a+b x}}{b \left(e^{2 a+2 b x}-1\right)}-\frac{2 e^{a+b x}}{b \left(e^{2 a+2 b x}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(a + b*x)^2*sinh(a + b*x)^3),x)
```

```
[Out] (3*atan((exp(b*x)*exp(a)*(-b^2)^(1/2))/b))/(-b^2)^(1/2) - (2*exp(a + b*x))/
(b*(exp(4*a + 4*b*x) - 2*exp(2*a + 2*b*x) + 1)) - exp(a + b*x)/(b*(exp(2*a
+ 2*b*x) - 1)) - (2*exp(a + b*x))/(b*(exp(2*a + 2*b*x) + 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{csch}^3(a+bx) \operatorname{sech}^2(a+bx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)**3*sech(b*x+a)**2,x)
```

```
[Out] Integral(csch(a + b*x)**3*sech(a + b*x)**2, x)
```

3.36 $\int \operatorname{csch}^3(a+bx)\operatorname{sech}^3(a+bx)\,dx$

Optimal. Leaf size=43

$$\frac{\tanh^2(a+bx)}{2b}-\frac{\coth^2(a+bx)}{2b}-\frac{2\log(\tanh(a+bx))}{b}$$

```
[Out] -1/2*coth(b*x+a)^2/b-2*ln(tanh(b*x+a))/b+1/2*tanh(b*x+a)^2/b
```

Rubi [A] time = 0.04, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {2620, 266, 43}

$$\frac{\tanh^2(a+bx)}{2b}-\frac{\coth^2(a+bx)}{2b}-\frac{2\log(\tanh(a+bx))}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[a + b*x]^3*Sech[a + b*x]^3,x]
```

```
[Out] -Coth[a + b*x]^2/(2*b) - (2*Log[Tanh[a + b*x]])/b + Tanh[a + b*x]^2/(2*b)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2620

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rubi steps

$$\int \operatorname{csch}^3(a+bx)\operatorname{sech}^3(a+bx)\,dx = -\frac{\operatorname{Subst}\left(\int \frac{\left(1+x^2\right)^2}{x^3}\,dx, x, i\tanh(a+bx)\right)}{b}$$
$$= -\frac{\operatorname{Subst}\left(\int \frac{(1+x)^2}{x^2}\,dx, x, -\tanh^2(a+bx)\right)}{2b}$$
$$= -\frac{\operatorname{Subst}\left(\int \left(1+\frac{1}{x^2}+\frac{2}{x}\right)dx, x, -\tanh^2(a+bx)\right)}{2b}$$
$$= -\frac{\coth^2(a+bx)}{2b} - \frac{2\log(\tanh(a+bx))}{b} + \frac{\tanh^2(a+bx)}{2b}$$

Mathematica [A] time = 0.02, size = 47, normalized size = 1.09

$$8\left(-\frac{\operatorname{csch}^2(a+bx)}{16b} - \frac{\operatorname{sech}^2(a+bx)}{16b} - \frac{\log(\tanh(a+bx))}{4b}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[a + b*x]^3*Sech[a + b*x]^3,x]
```

```
[Out] 8*(-1/16*Csch[a + b*x]^2/b - Log[Tanh[a + b*x]]/(4*b) - Sech[a + b*x]^2/(16
*b))
```

fricas [B] time = 0.42, size = 774, normalized size = 18.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^3*sech(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -2*(2*cosh(b*x + a)^6 + 40*cosh(b*x + a)^3*sinh(b*x + a)^3 + 30*cosh(b*x +
a)^2*sinh(b*x + a)^4 + 12*cosh(b*x + a)*sinh(b*x + a)^5 + 2*sinh(b*x + a)^6
 + 2*(15*cosh(b*x + a)^4 + 1)*sinh(b*x + a)^2 + 2*cosh(b*x + a)^2 - (cosh(b
*x + a)^8 + 56*cosh(b*x + a)^3*sinh(b*x + a)^5 + 28*cosh(b*x + a)^2*sinh(b*
x + a)^6 + 8*cosh(b*x + a)*sinh(b*x + a)^7 + sinh(b*x + a)^8 + 2*(35*cosh(b
*x + a)^4 - 1)*sinh(b*x + a)^4 - 2*cosh(b*x + a)^4 + 8*(7*cosh(b*x + a)^5 -
 cosh(b*x + a))*sinh(b*x + a)^3 + 4*(7*cosh(b*x + a)^6 - 3*cosh(b*x + a)^2)
*sinh(b*x + a)^2 + 8*(cosh(b*x + a)^7 - cosh(b*x + a)^3)*sinh(b*x + a) + 1)
*log(2*cosh(b*x + a)/(cosh(b*x + a) - sinh(b*x + a))) + (cosh(b*x + a)^8 +
56*cosh(b*x + a)^3*sinh(b*x + a)^5 + 28*cosh(b*x + a)^2*sinh(b*x + a)^6 + 8
```

```
*cosh(b*x + a)*sinh(b*x + a)^7 + sinh(b*x + a)^8 + 2*(35*cosh(b*x + a)^4 -
1)*sinh(b*x + a)^4 - 2*cosh(b*x + a)^4 + 8*(7*cosh(b*x + a)^5 - cosh(b*x +
a))*sinh(b*x + a)^3 + 4*(7*cosh(b*x + a)^6 - 3*cosh(b*x + a)^2)*sinh(b*x +
a)^2 + 8*(cosh(b*x + a)^7 - cosh(b*x + a)^3)*sinh(b*x + a) + 1)*log(2*sinh(
b*x + a)/(cosh(b*x + a) - sinh(b*x + a))) + 4*(3*cosh(b*x + a)^5 + cosh(b*x
 + a))*sinh(b*x + a))/(b*cosh(b*x + a)^8 + 56*b*cosh(b*x + a)^3*sinh(b*x +
a)^5 + 28*b*cosh(b*x + a)^2*sinh(b*x + a)^6 + 8*b*cosh(b*x + a)*sinh(b*x +
a)^7 + b*sinh(b*x + a)^8 - 2*b*cosh(b*x + a)^4 + 2*(35*b*cosh(b*x + a)^4 -
b)*sinh(b*x + a)^4 + 8*(7*b*cosh(b*x + a)^5 - b*cosh(b*x + a))*sinh(b*x + a
)^3 + 4*(7*b*cosh(b*x + a)^6 - 3*b*cosh(b*x + a)^2)*sinh(b*x + a)^2 + 8*(b*
cosh(b*x + a)^7 - b*cosh(b*x + a)^3)*sinh(b*x + a) + b)
```

giac [B] time = 0.14, size = 96, normalized size = 2.23

$$-\frac{\frac{4\left(e^{(2\,bx+2\,a)}+e^{(-2\,bx-2\,a)}\right)}{\left(e^{(2\,bx+2\,a)}+e^{(-2\,bx-2\,a)}\right)^2-4}-\log\left(e^{(2\,bx+2\,a)}+e^{(-2\,bx-2\,a)}+2\right)+\log\left(e^{(2\,bx+2\,a)}+e^{(-2\,bx-2\,a)}-2\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^3*sech(b*x+a)^3,x, algorithm="giac")
```

```
[Out] -(4*(e^(2*b*x + 2*a) + e^(-2*b*x - 2*a))/((e^(2*b*x + 2*a) + e^(-2*b*x - 2*
a))^2 - 4) - log(e^(2*b*x + 2*a) + e^(-2*b*x - 2*a) + 2) + log(e^(2*b*x + 2
*a) + e^(-2*b*x - 2*a) - 2))/b
```

maple [A] time = 0.22, size = 48, normalized size = 1.12

$$-\frac{1}{2b\sinh\left(bx+a\right)^2\cosh\left(bx+a\right)^2}-\frac{1}{b\cosh\left(bx+a\right)^2}-\frac{2\ln\left(\tanh\left(bx+a\right)\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(b*x+a)^3*sech(b*x+a)^3,x)
```

```
[Out] -1/2/b/sinh(b*x+a)^2/cosh(b*x+a)^2-1/b/cosh(b*x+a)^2-2*ln(tanh(b*x+a))/b
```

maxima [B] time = 0.41, size = 102, normalized size = 2.37

$$-\frac{2\,\log\left(e^{(-bx-a)}+1\right)}{b}-\frac{2\,\log\left(e^{(-bx-a)}-1\right)}{b}+\frac{2\,\log\left(e^{(-2\,bx-2\,a)}+1\right)}{b}+\frac{4\left(e^{(-2\,bx-2\,a)}+e^{(-6\,bx-6\,a)}\right)}{b\left(2\,e^{(-4\,bx-4\,a)}-e^{(-8\,bx-8\,a)}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^3*sech(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -2*log(e^(-b*x - a) + 1)/b - 2*log(e^(-b*x - a) - 1)/b + 2*log(e^(-2*b*x -
2*a) + 1)/b + 4*(e^(-2*b*x - 2*a) + e^(-6*b*x - 6*a))/(b*(2*e^(-4*b*x - 4*a
) - e^(-8*b*x - 8*a) - 1))
```

mupad [B] time = 1.44, size = 96, normalized size = 2.23

$$\frac{4\,\mathrm{atan}\left(\frac{\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}\,\sqrt{-b^2}}{b}\right)}{\sqrt{-b^2}}-\frac{4\,\mathrm{e}^{2\,a+2\,b\,x}}{b\,\left(\mathrm{e}^{4\,a+4\,b\,x}-1\right)}-\frac{8\,\mathrm{e}^{2\,a+2\,b\,x}}{b\,\left(\mathrm{e}^{8\,a+8\,b\,x}-2\,\mathrm{e}^{4\,a+4\,b\,x}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(a + b*x)^3*sinh(a + b*x)^3),x)
```

```
[Out] (4*atan((exp(2*a)*exp(2*b*x)*(-b^2)^(1/2))/b))/(-b^2)^(1/2) - (4*exp(2*a +
2*b*x))/(b*(exp(4*a + 4*b*x) - 1)) - (8*exp(2*a + 2*b*x))/(b*(exp(8*a + 8*b
*x) - 2*exp(4*a + 4*b*x) + 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{csch}^3(a+bx)\operatorname{sech}^3(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)**3*sech(b*x+a)**3,x)
```

```
[Out] Integral(csch(a + b*x)**3*sech(a + b*x)**3, x)
```

3.37 $\int \operatorname{csch}^3(a+bx)\operatorname{sech}^4(a+bx)\,dx$

Optimal. Leaf size=66

$$-\frac{5\operatorname{sech}^3(a+bx)}{6b}-\frac{5\operatorname{sech}(a+bx)}{2b}+\frac{5\tanh^{-1}(\cosh(a+bx))}{2b}-\frac{\operatorname{csch}^2(a+bx)\operatorname{sech}^3(a+bx)}{2b}$$

```
[Out] 5/2*arctanh(cosh(b*x+a))/b-5/2*sech(b*x+a)/b-5/6*sech(b*x+a)^3/b-1/2*csch(b
*x+a)^2*sech(b*x+a)^3/b
```

Rubi [A] time = 0.05, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {2622, 288, 302, 207}

$$-\frac{5\operatorname{sech}^3(a+bx)}{6b}-\frac{5\operatorname{sech}(a+bx)}{2b}+\frac{5\tanh^{-1}(\cosh(a+bx))}{2b}-\frac{\operatorname{csch}^2(a+bx)\operatorname{sech}^3(a+bx)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[a + b*x]^3*Sech[a + b*x]^4,x]
```

```
[Out] (5*ArcTanh[Cosh[a + b*x]])/(2*b) - (5*Sech[a + b*x])/(2*b) - (5*Sech[a + b*
x]^3)/(6*b) - (Csch[a + b*x]^2*Sech[a + b*x]^3)/(2*b)
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rubi steps

$$\int \operatorname{csch}^3(a+bx)\operatorname{sech}^4(a+bx)\,dx = -\frac{\operatorname{Subst}\left(\int \frac{x^6}{\left(-1+x^2\right)^2}\,dx, x, \operatorname{sech}(a+bx)\right)}{b}$$
$$= -\frac{\operatorname{csch}^2(a+bx)\operatorname{sech}^3(a+bx)}{2b} - \frac{5\operatorname{Subst}\left(\int \frac{x^4}{-1+x^2}\,dx, x, \operatorname{sech}(a+bx)\right)}{2b}$$
$$= -\frac{\operatorname{csch}^2(a+bx)\operatorname{sech}^3(a+bx)}{2b} - \frac{5\operatorname{Subst}\left(\int \left(1+x^2+\frac{1}{-1+x^2}\right)dx, x, \operatorname{sech}(a+
$$
$$= -\frac{5\operatorname{sech}(a+bx)}{2b} - \frac{5\operatorname{sech}^3(a+bx)}{6b} - \frac{\operatorname{csch}^2(a+bx)\operatorname{sech}^3(a+bx)}{2b} - \frac{5\operatorname{Subst}\left(\right.}{}$$
$$= \frac{5\tanh^{-1}(\cosh(a+bx))}{2b} - \frac{5\operatorname{sech}(a+bx)}{2b} - \frac{5\operatorname{sech}^3(a+bx)}{6b} - \frac{\operatorname{csch}^2(a+bx)\mathrm{s}}{2}$$

Mathematica [A] time = 0.03, size = 83, normalized size = 1.26

$$-\frac{\operatorname{csch}^2\left(\frac{1}{2}(a+bx)\right)}{8b}-\frac{\operatorname{sech}^3(a+bx)}{3b}-\frac{\operatorname{sech}^2\left(\frac{1}{2}(a+bx)\right)}{8b}-\frac{2\operatorname{sech}(a+bx)}{b}-\frac{5\log\left(\tanh\left(\frac{1}{2}(a+bx)\right)\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[a + b*x]^3*Sech[a + b*x]^4,x]
```

```
[Out] -1/8*Csch[(a + b*x)/2]^2/b - (5*Log[Tanh[(a + b*x)/2]])/(2*b) - Sech[(a + b
*x)/2]^2/(8*b) - (2*Sech[a + b*x])/b - Sech[a + b*x]^3/(3*b)
```

fricas [B] time = 0.43, size = 1573, normalized size = 23.83

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^3*sech(b*x+a)^4,x, algorithm="fricas")
```

```
[Out] -1/6*(30*cosh(b*x + a)^9 + 270*cosh(b*x + a)*sinh(b*x + a)^8 + 30*sinh(b*x
+ a)^9 + 40*(27*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^7 + 40*cosh(b*x + a)^7 +
 280*(9*cosh(b*x + a)^3 + cosh(b*x + a))*sinh(b*x + a)^6 + 4*(945*cosh(b*x
+ a)^4 + 210*cosh(b*x + a)^2 - 11)*sinh(b*x + a)^5 - 44*cosh(b*x + a)^5 + 2
0*(189*cosh(b*x + a)^5 + 70*cosh(b*x + a)^3 - 11*cosh(b*x + a))*sinh(b*x +
a)^4 + 40*(63*cosh(b*x + a)^6 + 35*cosh(b*x + a)^4 - 11*cosh(b*x + a)^2 + 1
)*sinh(b*x + a)^3 + 40*cosh(b*x + a)^3 + 40*(27*cosh(b*x + a)^7 + 21*cosh(b
*x + a)^5 - 11*cosh(b*x + a)^3 + 3*cosh(b*x + a))*sinh(b*x + a)^2 - 15*(cos
h(b*x + a)^10 + 10*cosh(b*x + a)*sinh(b*x + a)^9 + sinh(b*x + a)^10 + (45*c
osh(b*x + a)^2 + 1)*sinh(b*x + a)^8 + cosh(b*x + a)^8 + 8*(15*cosh(b*x + a)
^3 + cosh(b*x + a))*sinh(b*x + a)^7 + 2*(105*cosh(b*x + a)^4 + 14*cosh(b*x
+ a)^2 - 1)*sinh(b*x + a)^6 - 2*cosh(b*x + a)^6 + 4*(63*cosh(b*x + a)^5 + 1
4*cosh(b*x + a)^3 - 3*cosh(b*x + a))*sinh(b*x + a)^5 + 2*(105*cosh(b*x + a)
^6 + 35*cosh(b*x + a)^4 - 15*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^4 - 2*cosh(
b*x + a)^4 + 8*(15*cosh(b*x + a)^7 + 7*cosh(b*x + a)^5 - 5*cosh(b*x + a)^3
- cosh(b*x + a))*sinh(b*x + a)^3 + (45*cosh(b*x + a)^8 + 28*cosh(b*x + a)^6
 - 30*cosh(b*x + a)^4 - 12*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^2 + cosh(b*x
+ a)^2 + 2*(5*cosh(b*x + a)^9 + 4*cosh(b*x + a)^7 - 6*cosh(b*x + a)^5 - 4*c
osh(b*x + a)^3 + cosh(b*x + a))*sinh(b*x + a) + 1)*log(cosh(b*x + a) + sinh
(b*x + a) + 1) + 15*(cosh(b*x + a)^10 + 10*cosh(b*x + a)*sinh(b*x + a)^9 +
sinh(b*x + a)^10 + (45*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^8 + cosh(b*x + a)
^8 + 8*(15*cosh(b*x + a)^3 + cosh(b*x + a))*sinh(b*x + a)^7 + 2*(105*cosh(b
*x + a)^4 + 14*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^6 - 2*cosh(b*x + a)^6 + 4
*(63*cosh(b*x + a)^5 + 14*cosh(b*x + a)^3 - 3*cosh(b*x + a))*sinh(b*x + a)^
5 + 2*(105*cosh(b*x + a)^6 + 35*cosh(b*x + a)^4 - 15*cosh(b*x + a)^2 - 1)*s
inh(b*x + a)^4 - 2*cosh(b*x + a)^4 + 8*(15*cosh(b*x + a)^7 + 7*cosh(b*x + a
)^5 - 5*cosh(b*x + a)^3 - cosh(b*x + a))*sinh(b*x + a)^3 + (45*cosh(b*x + a
)^8 + 28*cosh(b*x + a)^6 - 30*cosh(b*x + a)^4 - 12*cosh(b*x + a)^2 + 1)*sin
h(b*x + a)^2 + cosh(b*x + a)^2 + 2*(5*cosh(b*x + a)^9 + 4*cosh(b*x + a)^7 -
 6*cosh(b*x + a)^5 - 4*cosh(b*x + a)^3 + cosh(b*x + a))*sinh(b*x + a) + 1)*
log(cosh(b*x + a) + sinh(b*x + a) - 1) + 10*(27*cosh(b*x + a)^8 + 28*cosh(b
*x + a)^6 - 22*cosh(b*x + a)^4 + 12*cosh(b*x + a)^2 + 3)*sinh(b*x + a) + 30
*cosh(b*x + a))/(b*cosh(b*x + a)^10 + 10*b*cosh(b*x + a)*sinh(b*x + a)^9 +
b*sinh(b*x + a)^10 + b*cosh(b*x + a)^8 + (45*b*cosh(b*x + a)^2 + b)*sinh(b*
x + a)^8 + 8*(15*b*cosh(b*x + a)^3 + b*cosh(b*x + a))*sinh(b*x + a)^7 - 2*b
*cosh(b*x + a)^6 + 2*(105*b*cosh(b*x + a)^4 + 14*b*cosh(b*x + a)^2 - b)*sin
h(b*x + a)^6 + 4*(63*b*cosh(b*x + a)^5 + 14*b*cosh(b*x + a)^3 - 3*b*cosh(b*
x + a))*sinh(b*x + a)^5 - 2*b*cosh(b*x + a)^4 + 2*(105*b*cosh(b*x + a)^6 +
35*b*cosh(b*x + a)^4 - 15*b*cosh(b*x + a)^2 - b)*sinh(b*x + a)^4 + 8*(15*b*
cosh(b*x + a)^7 + 7*b*cosh(b*x + a)^5 - 5*b*cosh(b*x + a)^3 - b*cosh(b*x +
a))*sinh(b*x + a)^3 + b*cosh(b*x + a)^2 + (45*b*cosh(b*x + a)^8 + 28*b*cosh
(b*x + a)^6 - 30*b*cosh(b*x + a)^4 - 12*b*cosh(b*x + a)^2 + b)*sinh(b*x + a
)^2 + 2*(5*b*cosh(b*x + a)^9 + 4*b*cosh(b*x + a)^7 - 6*b*cosh(b*x + a)^5 -
4*b*cosh(b*x + a)^3 + b*cosh(b*x + a))*sinh(b*x + a) + b)
```

giac [B] time = 0.14, size = 128, normalized size = 1.94

$$-\frac{\frac{12\left(e^{(bx+a)}+e^{(-bx-a)}\right)}{\left(e^{(bx+a)}+e^{(-bx-a)}\right)^2-4}+\frac{16\left(3\left(e^{(bx+a)}+e^{(-bx-a)}\right)^2+2\right)}{\left(e^{(bx+a)}+e^{(-bx-a)}\right)^3}-15\log\left(e^{(bx+a)}+e^{(-bx-a)}+2\right)+15\log\left(e^{(bx+a)}+e^{(-bx-a)}-2\right)}{12\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^3*sech(b*x+a)^4,x, algorithm="giac")
```

```
[Out] -1/12*(12*(e^(b*x + a) + e^(-b*x - a))/((e^(b*x + a) + e^(-b*x - a))^2 - 4)
 + 16*(3*(e^(b*x + a) + e^(-b*x - a))^2 + 2)/(e^(b*x + a) + e^(-b*x - a))^3
 - 15*log(e^(b*x + a) + e^(-b*x - a) + 2) + 15*log(e^(b*x + a) + e^(-b*x -
a) - 2))/b
```

maple [A] time = 0.22, size = 53, normalized size = 0.80

$$\frac{-\frac{1}{2\sinh(bx+a)^2\cosh(bx+a)^3}-\frac{5}{6\cosh(bx+a)^3}-\frac{5}{2\cosh(bx+a)}+5\,\mathrm{arctanh}\left(e^{bx+a}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(b*x+a)^3*sech(b*x+a)^4,x)
```

```
[Out] 1/b*(-1/2/sinh(b*x+a)^2/cosh(b*x+a)^3-5/6/cosh(b*x+a)^3-5/2/cosh(b*x+a)+5*a
rctanh(exp(b*x+a)))
```

maxima [B] time = 0.33, size = 149, normalized size = 2.26

$$\frac{5\log\left(e^{(-bx-a)}+1\right)}{2\,b}-\frac{5\log\left(e^{(-bx-a)}-1\right)}{2\,b}-\frac{15\,e^{(-bx-a)}+20\,e^{(-3\,bx-3\,a)}-22\,e^{(-5\,bx-5\,a)}+20\,e^{(-7\,bx-7\,a)}+15\,e^{(-9\,bx-9\,a}}{3\,b\left(e^{(-2\,bx-2\,a)}-2\,e^{(-4\,bx-4\,a)}-2\,e^{(-6\,bx-6\,a)}+e^{(-8\,bx-8\,a)}+e^{(-10\,bx-10\,a)}+1\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^3*sech(b*x+a)^4,x, algorithm="maxima")
```

```
[Out] 5/2*log(e^(-b*x - a) + 1)/b - 5/2*log(e^(-b*x - a) - 1)/b - 1/3*(15*e^(-b*x
 - a) + 20*e^(-3*b*x - 3*a) - 22*e^(-5*b*x - 5*a) + 20*e^(-7*b*x - 7*a) + 1
5*e^(-9*b*x - 9*a))/(b*(e^(-2*b*x - 2*a) - 2*e^(-4*b*x - 4*a) - 2*e^(-6*b*x
 - 6*a) + e^(-8*b*x - 8*a) + e^(-10*b*x - 10*a) + 1))
```

mupad [B] time = 1.44, size = 192, normalized size = 2.91

$$\frac{5\,\mathrm{atan}\left(\frac{e^{b\,x}\,e^{a}\,\sqrt{-b^2}}{b}\right)}{\sqrt{-b^2}}-\frac{2\,e^{a+b\,x}}{b\,\left(e^{4\,a+4\,b\,x}-2\,e^{2\,a+2\,b\,x}+1\right)}-\frac{8\,e^{a+b\,x}}{3\,b\,\left(2\,e^{2\,a+2\,b\,x}+e^{4\,a+4\,b\,x}+1\right)}+\frac{8\,e^{a+b\,x}}{3\,b\,\left(3\,e^{2\,a+2\,b\,x}+3\,e^{4\,a+4\,b\,x}+e\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(a + b*x)^4*sinh(a + b*x)^3),x)
```

```
[Out] (5*atan((exp(b*x)*exp(a)*(-b^2)^(1/2))/b))/(-b^2)^(1/2) - (2*exp(a + b*x))/
(b*(exp(4*a + 4*b*x) - 2*exp(2*a + 2*b*x) + 1)) - (8*exp(a + b*x))/(3*b*(2*
exp(2*a + 2*b*x) + exp(4*a + 4*b*x) + 1)) + (8*exp(a + b*x))/(3*b*(3*exp(2*
a + 2*b*x) + 3*exp(4*a + 4*b*x) + exp(6*a + 6*b*x) + 1)) - exp(a + b*x)/(b*
(exp(2*a + 2*b*x) - 1)) - (4*exp(a + b*x))/(b*(exp(2*a + 2*b*x) + 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{csch}^3(a+bx)\operatorname{sech}^4(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)**3*sech(b*x+a)**4,x)
```

```
[Out] Integral(csch(a + b*x)**3*sech(a + b*x)**4, x)
```

3.38 $\int \text{csch}^3(a+bx)\text{sech}^5(a+bx)\,dx$

Optimal. Leaf size=58

$$-\frac{\tanh^4(a+bx)}{4b}+\frac{3\tanh^2(a+bx)}{2b}-\frac{\coth^2(a+bx)}{2b}-\frac{3\log(\tanh(a+bx))}{b}$$

```
[Out] -1/2*coth(b*x+a)^2/b-3*ln(tanh(b*x+a))/b+3/2*tanh(b*x+a)^2/b-1/4*tanh(b*x+a
)^4/b
```

Rubi [A] time = 0.05, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {2620, 266, 43}

$$-\frac{\tanh^4(a+bx)}{4b}+\frac{3\tanh^2(a+bx)}{2b}-\frac{\coth^2(a+bx)}{2b}-\frac{3\log(\tanh(a+bx))}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[a + b*x]^3*Sech[a + b*x]^5,x]
```

```
[Out] -Coth[a + b*x]^2/(2*b) - (3*Log[Tanh[a + b*x]])/b + (3*Tanh[a + b*x]^2)/(2*
b) - Tanh[a + b*x]^4/(4*b)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2620

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rubi steps

$$\int \operatorname{csch}^3(a+bx)\operatorname{sech}^5(a+bx)\,dx = -\frac{\operatorname{Subst}\left(\int \frac{(1+x^2)^3}{x^3}\,dx, x, i\tanh(a+bx)\right)}{b}$$
$$= -\frac{\operatorname{Subst}\left(\int \frac{(1+x)^3}{x^2}\,dx, x, -\tanh^2(a+bx)\right)}{2b}$$
$$= -\frac{\operatorname{Subst}\left(\int \left(3+\frac{1}{x^2}+\frac{3}{x}+x\right)dx, x, -\tanh^2(a+bx)\right)}{2b}$$
$$= -\frac{\coth^2(a+bx)}{2b} - \frac{3\log(\tanh(a+bx))}{b} + \frac{3\tanh^2(a+bx)}{2b} - \frac{\tanh^4(a+bx)}{4b}$$

Mathematica [A] time = 0.45, size = 54, normalized size = 0.93

$$-\frac{2\operatorname{csch}^2(a+bx)+\operatorname{sech}^4(a+bx)+4\operatorname{sech}^2(a+bx)+12\log(\sinh(a+bx))-12\log(\cosh(a+bx))}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[a + b*x]^3*Sech[a + b*x]^5,x]
```

```
[Out] -1/4*(2*Csch[a + b*x]^2 - 12*Log[Cosh[a + b*x]] + 12*Log[Sinh[a + b*x]] + 4
*Sech[a + b*x]^2 + Sech[a + b*x]^4)/b
```

fricas [B] time = 0.42, size = 2103, normalized size = 36.26

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^3*sech(b*x+a)^5,x, algorithm="fricas")
```

```
[Out] -(6*cosh(b*x + a)^10 + 60*cosh(b*x + a)*sinh(b*x + a)^9 + 6*sinh(b*x + a)^1
0 + 6*(45*cosh(b*x + a)^2 + 2)*sinh(b*x + a)^8 + 12*cosh(b*x + a)^8 + 48*(1
5*cosh(b*x + a)^3 + 2*cosh(b*x + a))*sinh(b*x + a)^7 + 4*(315*cosh(b*x + a)
^4 + 84*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^6 - 4*cosh(b*x + a)^6 + 24*(63*c
osh(b*x + a)^5 + 28*cosh(b*x + a)^3 - cosh(b*x + a))*sinh(b*x + a)^5 + 12*(
105*cosh(b*x + a)^6 + 70*cosh(b*x + a)^4 - 5*cosh(b*x + a)^2 + 1)*sinh(b*x
+ a)^4 + 12*cosh(b*x + a)^4 + 16*(45*cosh(b*x + a)^7 + 42*cosh(b*x + a)^5 -
 5*cosh(b*x + a)^3 + 3*cosh(b*x + a))*sinh(b*x + a)^3 + 6*(45*cosh(b*x + a)
^8 + 56*cosh(b*x + a)^6 - 10*cosh(b*x + a)^4 + 12*cosh(b*x + a)^2 + 1)*sinh
(b*x + a)^2 + 6*cosh(b*x + a)^2 - 3*(cosh(b*x + a)^12 + 12*cosh(b*x + a)*si
nh(b*x + a)^11 + sinh(b*x + a)^12 + 2*(33*cosh(b*x + a)^2 + 1)*sinh(b*x + a
```

)^10 + 2*cosh(b*x + a)^10 + 20*(11*cosh(b*x + a)^3 + cosh(b*x + a))*sinh(b*x + a)^9 + (495*cosh(b*x + a)^4 + 90*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^8 - cosh(b*x + a)^8 + 8*(99*cosh(b*x + a)^5 + 30*cosh(b*x + a)^3 - cosh(b*x + a))*sinh(b*x + a)^7 + 4*(231*cosh(b*x + a)^6 + 105*cosh(b*x + a)^4 - 7*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^6 - 4*cosh(b*x + a)^6 + 8*(99*cosh(b*x + a)^7 + 63*cosh(b*x + a)^5 - 7*cosh(b*x + a)^3 - 3*cosh(b*x + a))*sinh(b*x + a)^5 + (495*cosh(b*x + a)^8 + 420*cosh(b*x + a)^6 - 70*cosh(b*x + a)^4 - 60*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^4 - cosh(b*x + a)^4 + 4*(55*cosh(b*x + a)^9 + 60*cosh(b*x + a)^7 - 14*cosh(b*x + a)^5 - 20*cosh(b*x + a)^3 - cosh(b*x + a))*sinh(b*x + a)^3 + 2*(33*cosh(b*x + a)^10 + 45*cosh(b*x + a)^8 - 14*cosh(b*x + a)^6 - 30*cosh(b*x + a)^4 - 3*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^2 + 2*cosh(b*x + a)^2 + 4*(3*cosh(b*x + a)^11 + 5*cosh(b*x + a)^9 - 2*cosh(b*x + a)^7 - 6*cosh(b*x + a)^5 - cosh(b*x + a)^3 + cosh(b*x + a))*sinh(b*x + a) + 1)*log(2*cosh(b*x + a)/(cosh(b*x + a) - sinh(b*x + a))) + 3*(cosh(b*x + a)^12 + 12*cosh(b*x + a)*sinh(b*x + a)^11 + sinh(b*x + a)^12 + 2*(33*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^10 + 2*cosh(b*x + a)^10 + 20*(11*cosh(b*x + a)^3 + cosh(b*x + a))*sinh(b*x + a)^9 + (495*cosh(b*x + a)^4 + 90*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^8 - cosh(b*x + a)^8 + 8*(99*cosh(b*x + a)^5 + 30*cosh(b*x + a)^3 - cosh(b*x + a))*sinh(b*x + a)^7 + 4*(231*cosh(b*x + a)^6 + 105*cosh(b*x + a)^4 - 7*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^6 - 4*cosh(b*x + a)^6 + 8*(99*cosh(b*x + a)^7 + 63*cosh(b*x + a)^5 - 7*cosh(b*x + a)^3 - 3*cosh(b*x + a))*sinh(b*x + a)^5 + (495*cosh(b*x + a)^8 + 420*cosh(b*x + a)^6 - 70*cosh(b*x + a)^4 - 60*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^4 - cosh(b*x + a)^4 + 4*(55*cosh(b*x + a)^9 + 60*cosh(b*x + a)^7 - 14*cosh(b*x + a)^5 - 20*cosh(b*x + a)^3 - cosh(b*x + a))*sinh(b*x + a)^3 + 2*(33*cosh(b*x + a)^10 + 45*cosh(b*x + a)^8 - 14*cosh(b*x + a)^6 - 30*cosh(b*x + a)^4 - 3*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^2 + 2*cosh(b*x + a)^2 + 4*(3*cosh(b*x + a)^11 + 5*cosh(b*x + a)^9 - 2*cosh(b*x + a)^7 - 6*cosh(b*x + a)^5 - cosh(b*x + a)^3 + cosh(b*x + a))*sinh(b*x + a) + 1)*log(2*sinh(b*x + a)/(cosh(b*x + a) - sinh(b*x + a))) + 12*(5*cosh(b*x + a)^9 + 8*cosh(b*x + a)^7 - 2*cosh(b*x + a)^5 + 4*cosh(b*x + a)^3 + cosh(b*x + a))*sinh(b*x + a))/(b*cosh(b*x + a)^12 + 12*b*cosh(b*x + a)*sinh(b*x + a)^11 + b*sinh(b*x + a)^12 + 2*b*cosh(b*x + a)^10 + 2*(33*b*cosh(b*x + a)^2 + b)*sinh(b*x + a)^10 + 20*(11*b*cosh(b*x + a)^3 + b*cosh(b*x + a))*sinh(b*x + a)^9 - b*cosh(b*x + a)^8 + (495*b*cosh(b*x + a)^4 + 90*b*cosh(b*x + a)^2 - b)*sinh(b*x + a)^8 + 8*(99*b*cosh(b*x + a)^5 + 30*b*cosh(b*x + a)^3 - b*cosh(b*x + a))*sinh(b*x + a)^7 - 4*b*cosh(b*x + a)^6 + 4*(231*b*cosh(b*x + a)^6 + 105*b*cosh(b*x + a)^4 - 7*b*cosh(b*x + a)^2 - b)*sinh(b*x + a)^6 + 8*(99*b*cosh(b*x + a)^7 + 63*b*cosh(b*x + a)^5 - 7*b*cosh(b*x + a)^3 - 3*b*cosh(b*x + a))*sinh(b*x + a)^5 - b*cosh(b*x + a)^4 + (495*b*cosh(b*x + a)^8 + 420*b*cosh(b*x + a)^6 - 70*b*cosh(b*x + a)^4 - 60*b*cosh(b*x + a)^2 - b)*sinh(b*x + a)^4 + 4*(55*b*cosh(b*x + a)^9 + 60*b*cosh(b*x + a)^7 - 14*b*cosh(b*x + a)^5 - 20*b*cosh(b*x + a)^3 - b*cosh(b*x + a))*sinh(b*x + a)^3 + 2*b*cosh(b*x + a)^2 + 2*(33*b*cosh(b*x + a)^10 + 45*b*cosh(b*x + a)^8 - 14*b*cosh(b*x + a)^6 - 30*b*cosh(b*x + a)^4 - 3*b*cosh(b*x + a)^2 + b)*sinh(b*x + a)^2 + 4*(3*b*cosh(b*x + a)^11 +

```
5*b*cosh(b*x + a)^9 - 2*b*cosh(b*x + a)^7 - 6*b*cosh(b*x + a)^5 - b*cosh(b
*x + a)^3 + b*cosh(b*x + a))*sinh(b*x + a) + b)
```

giac [B] time = 0.17, size = 171, normalized size = 2.95

$$\frac{\frac{2\left(3\,e^{(2\,bx+2\,a)}+3\,e^{(-2\,bx-2\,a)}-10\right)}{e^{(2\,bx+2\,a)}+e^{(-2\,bx-2\,a)}-2}-\frac{9\left(e^{(2\,bx+2\,a)}+e^{(-2\,bx-2\,a)}\right)^2+52\,e^{(2\,bx+2\,a)}+52\,e^{(-2\,bx-2\,a)}+84}{\left(e^{(2\,bx+2\,a)}+e^{(-2\,bx-2\,a)}+2\right)^2}+6\,\log\left(e^{(2\,bx+2\,a)}+e^{(-2\,bx-2\,a)}+2\right)-6}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^3*sech(b*x+a)^5,x, algorithm="giac")
```

```
[Out] 1/4*(2*(3*e^(2*b*x + 2*a) + 3*e^(-2*b*x - 2*a) - 10)/(e^(2*b*x + 2*a) + e^(
-2*b*x - 2*a) - 2) - (9*(e^(2*b*x + 2*a) + e^(-2*b*x - 2*a))^2 + 52*e^(2*b*
x + 2*a) + 52*e^(-2*b*x - 2*a) + 84)/(e^(2*b*x + 2*a) + e^(-2*b*x - 2*a) +
2)^2 + 6*log(e^(2*b*x + 2*a) + e^(-2*b*x - 2*a) + 2) - 6*log(e^(2*b*x + 2*a
) + e^(-2*b*x - 2*a) - 2))/b
```

maple [A] time = 0.22, size = 61, normalized size = 1.05

$$-\frac{1}{2b\sinh\left(bx+a\right)^2\cosh\left(bx+a\right)^4}-\frac{3}{4b\cosh\left(bx+a\right)^4}-\frac{3}{2b\cosh\left(bx+a\right)^2}-\frac{3\ln\left(\tanh\left(bx+a\right)\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(b*x+a)^3*sech(b*x+a)^5,x)
```

```
[Out] -1/2/b/sinh(b*x+a)^2/cosh(b*x+a)^4-3/4/b/cosh(b*x+a)^4-3/2/b/cosh(b*x+a)^2-
3*ln(tanh(b*x+a))/b
```

maxima [B] time = 0.41, size = 181, normalized size = 3.12

$$-\frac{3\,\log\left(e^{(-bx-a)}+1\right)}{b}-\frac{3\,\log\left(e^{(-bx-a)}-1\right)}{b}+\frac{3\,\log\left(e^{(-2\,bx-2\,a)}+1\right)}{b}-\frac{2\left(3\,e^{(-2\,bx-2\,a)}+6\,e^{(-4\,bx-4\,a)}-2\,e^{(-6\,b}\right.}{b\left(2\,e^{(-2\,bx-2\,a)}-e^{(-4\,bx-4\,a)}-4\,e^{(-6\,bx-6\,a)}-e^{(-}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^3*sech(b*x+a)^5,x, algorithm="maxima")
```

```
[Out] -3*log(e^(-b*x - a) + 1)/b - 3*log(e^(-b*x - a) - 1)/b + 3*log(e^(-2*b*x -
2*a) + 1)/b - 2*(3*e^(-2*b*x - 2*a) + 6*e^(-4*b*x - 4*a) - 2*e^(-6*b*x - 6*
a) + 6*e^(-8*b*x - 8*a) + 3*e^(-10*b*x - 10*a))/(b*(2*e^(-2*b*x - 2*a) - e^
(-4*b*x - 4*a) - 4*e^(-6*b*x - 6*a) - e^(-8*b*x - 8*a) + 2*e^(-10*b*x - 10*
a) + e^(-12*b*x - 12*a) + 1))
```

mupad [B] time = 0.08, size = 187, normalized size = 3.22

$$\frac{6\,\mathrm{atan}\left(\frac{\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}\,\sqrt{-b^2}}{b}\right)}{\sqrt{-b^2}}-\frac{4}{b\,\left(\mathrm{e}^{2\,a+2\,b\,x}+1\right)}-\frac{2}{b\,\left(\mathrm{e}^{2\,a+2\,b\,x}-1\right)}-\frac{2}{b\,\left(\mathrm{e}^{4\,a+4\,b\,x}-2\,\mathrm{e}^{2\,a+2\,b\,x}+1\right)}+\frac{8}{b\,\left(3\,\mathrm{e}^{2\,a+2\,b\,x}+3\,\mathrm{e}^{4\,a+4\,b\,x}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(a + b*x)^5*sinh(a + b*x)^3),x)
```

```
[Out] (6*atan((exp(2*a)*exp(2*b*x)*(-b^2)^(1/2))/b))/(-b^2)^(1/2) - 4/(b*(exp(2*a
 + 2*b*x) + 1)) - 2/(b*(exp(2*a + 2*b*x) - 1)) - 2/(b*(exp(4*a + 4*b*x) - 2
*exp(2*a + 2*b*x) + 1)) + 8/(b*(3*exp(2*a + 2*b*x) + 3*exp(4*a + 4*b*x) + e
xp(6*a + 6*b*x) + 1)) - 4/(b*(4*exp(2*a + 2*b*x) + 6*exp(4*a + 4*b*x) + 4*e
xp(6*a + 6*b*x) + exp(8*a + 8*b*x) + 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{csch}^{3}(a+bx)\operatorname{sech}^{5}(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)**3*sech(b*x+a)**5,x)
```

```
[Out] Integral(csch(a + b*x)**3*sech(a + b*x)**5, x)
```

3.39 $\int \mathrm{csch}^4(a+bx)\mathrm{sech}(a+bx)\,dx$

Optimal. Leaf size=37

$$-\frac{\mathrm{csch}^3(a+bx)}{3b}+\frac{\mathrm{csch}(a+bx)}{b}+\frac{\tan^{-1}(\sinh(a+bx))}{b}$$

```
[Out] arctan(sinh(b*x+a))/b+csch(b*x+a)/b-1/3*csch(b*x+a)^3/b
```

Rubi [A] time = 0.03, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2621, 302, 207}

$$-\frac{\mathrm{csch}^3(a+bx)}{3b}+\frac{\mathrm{csch}(a+bx)}{b}+\frac{\tan^{-1}(\sinh(a+bx))}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[a + b*x]^4*Sech[a + b*x],x]
```

```
[Out] ArcTan[Sinh[a + b*x]]/b + Csch[a + b*x]/b - Csch[a + b*x]^3/(3*b)
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 2621

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> -Dist[(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] && !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rubi steps

$$\int \operatorname{csch}^4(a+bx)\operatorname{sech}(a+bx)\,dx = \frac{i\,\text{Subst}\left(\int \frac{x^4}{-1+x^2}\,dx, x, -i\operatorname{csch}(a+bx)\right)}{b}$$
$$= \frac{i\,\text{Subst}\left(\int \left(1+x^2+\frac{1}{-1+x^2}\right)dx, x, -i\operatorname{csch}(a+bx)\right)}{b}$$
$$= \frac{\operatorname{csch}(a+bx)}{b} - \frac{\operatorname{csch}^3(a+bx)}{3b} + \frac{i\,\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, -i\operatorname{csch}(a+bx)\right)}{b}$$
$$= \frac{\tan^{-1}(\sinh(a+bx))}{b} + \frac{\operatorname{csch}(a+bx)}{b} - \frac{\operatorname{csch}^3(a+bx)}{3b}$$

Mathematica [C] time = 0.02, size = 33, normalized size = 0.89

$$-\frac{\operatorname{csch}^3(a+bx)\,{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};-\sinh^2(a+bx)\right)}{3b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[a + b*x]^4*Sech[a + b*x],x]
```

```
[Out] -1/3*(Csch[a + b*x]^3*Hypergeometric2F1[-3/2, 1, -1/2, -Sinh[a + b*x]^2])/b
```

fricas [B] time = 0.43, size = 515, normalized size = 13.92

$$\frac{2\left(3\cosh(bx+a)^5 + 15\cosh(bx+a)\sinh(bx+a)^4 + 3\sinh(bx+a)^5 + 10\left(3\cosh(bx+a)^2-1\right)\sinh(bx+a\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^4*sech(b*x+a),x, algorithm="fricas")
```

```
[Out] 2/3*(3*cosh(b*x + a)^5 + 15*cosh(b*x + a)*sinh(b*x + a)^4 + 3*sinh(b*x + a)
^5 + 10*(3*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^3 - 10*cosh(b*x + a)^3 + 30*(
cosh(b*x + a)^3 - cosh(b*x + a))*sinh(b*x + a)^2 + 3*(cosh(b*x + a)^6 + 6*c
osh(b*x + a)*sinh(b*x + a)^5 + sinh(b*x + a)^6 + 3*(5*cosh(b*x + a)^2 - 1)*
sinh(b*x + a)^4 - 3*cosh(b*x + a)^4 + 4*(5*cosh(b*x + a)^3 - 3*cosh(b*x + a
))*sinh(b*x + a)^3 + 3*(5*cosh(b*x + a)^4 - 6*cosh(b*x + a)^2 + 1)*sinh(b*x
 + a)^2 + 3*cosh(b*x + a)^2 + 6*(cosh(b*x + a)^5 - 2*cosh(b*x + a)^3 + cosh
(b*x + a))*sinh(b*x + a) - 1)*arctan(cosh(b*x + a) + sinh(b*x + a)) + 3*(5*
cosh(b*x + a)^4 - 10*cosh(b*x + a)^2 + 1)*sinh(b*x + a) + 3*cosh(b*x + a))/
(b*cosh(b*x + a)^6 + 6*b*cosh(b*x + a)*sinh(b*x + a)^5 + b*sinh(b*x + a)^6
```

```
- 3*b*cosh(b*x + a)^4 + 3*(5*b*cosh(b*x + a)^2 - b)*sinh(b*x + a)^4 + 4*(5*
b*cosh(b*x + a)^3 - 3*b*cosh(b*x + a))*sinh(b*x + a)^3 + 3*b*cosh(b*x + a)^
2 + 3*(5*b*cosh(b*x + a)^4 - 6*b*cosh(b*x + a)^2 + b)*sinh(b*x + a)^2 + 6*(
b*cosh(b*x + a)^5 - 2*b*cosh(b*x + a)^3 + b*cosh(b*x + a))*sinh(b*x + a) -
b)
```

giac [B] time = 0.14, size = 80, normalized size = 2.16

$$\frac{3\pi + \frac{4\left(3\left(e^{(bx+a)}-e^{(-bx-a)}\right)^2-4\right)}{\left(e^{(bx+a)}-e^{(-bx-a)}\right)^3} + 6\arctan\left(\frac{1}{2}\left(e^{(2bx+2a)}-1\right)e^{(-bx-a)}\right)}{6b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^4*sech(b*x+a),x, algorithm="giac")
```

```
[Out] 1/6*(3*pi + 4*(3*(e^(b*x + a) - e^(-b*x - a))^2 - 4)/(e^(b*x + a) - e^(-b*x
 - a))^3 + 6*arctan(1/2*(e^(2*b*x + 2*a) - 1)*e^(-b*x - a)))/b
```

maple [A] time = 0.15, size = 39, normalized size = 1.05

$$-\frac{1}{3b\sinh(bx+a)^3} + \frac{1}{b\sinh(bx+a)} + \frac{2\arctan\left(e^{bx+a}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(b*x+a)^4*sech(b*x+a),x)
```

```
[Out] -1/3/b/sinh(b*x+a)^3+1/b/sinh(b*x+a)+2*arctan(exp(b*x+a))/b
```

maxima [B] time = 0.43, size = 90, normalized size = 2.43

$$-\frac{2\arctan\left(e^{(-bx-a)}\right)}{b} - \frac{2\left(3e^{(-bx-a)} - 10e^{(-3bx-3a)} + 3e^{(-5bx-5a)}\right)}{3b\left(3e^{(-2bx-2a)} - 3e^{(-4bx-4a)} + e^{(-6bx-6a)} - 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^4*sech(b*x+a),x, algorithm="maxima")
```

```
[Out] -2*arctan(e^(-b*x - a))/b - 2/3*(3*e^(-b*x - a) - 10*e^(-3*b*x - 3*a) + 3*e
^(-5*b*x - 5*a))/(b*(3*e^(-2*b*x - 2*a) - 3*e^(-4*b*x - 4*a) + e^(-6*b*x -
6*a) - 1))
```

mupad [B] time = 0.08, size = 129, normalized size = 3.49

$$\frac{2\,\mathrm{atan}\left(\frac{e^{bx}\,e^{a}\,\sqrt{b^2}}{b}\right)}{\sqrt{b^2}} - \frac{8\,e^{a+bx}}{3b\left(e^{4a+4bx} - 2e^{2a+2bx} + 1\right)} - \frac{8\,e^{a+bx}}{3b\left(3e^{2a+2bx} - 3e^{4a+4bx} + e^{6a+6bx} - 1\right)} + \frac{2\,e^{a+bx}}{b\left(e^{2a+2bx} - 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(a + b*x)*sinh(a + b*x)^4),x)
```

```
[Out] (2*atan((exp(b*x)*exp(a)*(b^2)^(1/2))/b))/(b^2)^(1/2) - (8*exp(a + b*x))/(3
*b*(exp(4*a + 4*b*x) - 2*exp(2*a + 2*b*x) + 1)) - (8*exp(a + b*x))/(3*b*(3*
exp(2*a + 2*b*x) - 3*exp(4*a + 4*b*x) + exp(6*a + 6*b*x) - 1)) + (2*exp(a +
 b*x))/(b*(exp(2*a + 2*b*x) - 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{csch}^4(a+bx)\operatorname{sech}(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)**4*sech(b*x+a),x)
```

```
[Out] Integral(csch(a + b*x)**4*sech(a + b*x), x)
```

3.40 $\int \operatorname{csch}^4(a+bx)\operatorname{sech}^2(a+bx)\,dx$

Optimal. Leaf size=37

$$\frac{\tanh(a+bx)}{b}-\frac{\coth^3(a+bx)}{3b}+\frac{2\coth(a+bx)}{b}$$

```
[Out] 2*coth(b*x+a)/b-1/3*coth(b*x+a)^3/b+tanh(b*x+a)/b
```

Rubi [A] time = 0.04, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2620, 270}

$$\frac{\tanh(a+bx)}{b}-\frac{\coth^3(a+bx)}{3b}+\frac{2\coth(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[a + b*x]^4*Sech[a + b*x]^2,x]
```

```
[Out] (2*Coth[a + b*x])/b - Coth[a + b*x]^3/(3*b) + Tanh[a + b*x]/b
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2620

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rubi steps

$$\begin{aligned}\int \operatorname{csch}^4(a+bx)\operatorname{sech}^2(a+bx)\,dx &= -\frac{i\,\text{Subst}\left(\int \frac{\left(1+x^2\right)^2}{x^4}\,dx,x,i\tanh(a+bx)\right)}{b}\\ &= -\frac{i\,\text{Subst}\left(\int \left(1+\frac{1}{x^4}+\frac{2}{x^2}\right)dx,x,i\tanh(a+bx)\right)}{b}\\ &= \frac{2\coth(a+bx)}{b}-\frac{\coth^3(a+bx)}{3b}+\frac{\tanh(a+bx)}{b}\end{aligned}$$

Mathematica [A] time = 0.04, size = 45, normalized size = 1.22

$$\frac{\tanh(a+bx)}{b}+\frac{5\coth(a+bx)}{3b}-\frac{\coth(a+bx)\operatorname{csch}^2(a+bx)}{3b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[a + b*x]^4*Sech[a + b*x]^2,x]
```

```
[Out] (5*Coth[a + b*x])/(3*b) - (Coth[a + b*x]*Csch[a + b*x]^2)/(3*b) + Tanh[a +
b*x]/b
```

fricas [B] time = 0.39, size = 229, normalized size = 6.19

$$-\frac{}{3\left(b\cosh\left(bx+a\right)^{7}+7\,b\cosh\left(bx+a\right)\sinh\left(bx+a\right)^{6}+b\sinh\left(bx+a\right)^{7}-2\,b\cosh\left(bx+a\right)^{5}+\left(21\,b\cosh\left(bx+\right.\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^4*sech(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -16/3*(cosh(b*x + a) + 3*sinh(b*x + a))/(b*cosh(b*x + a)^7 + 7*b*cosh(b*x +
 a)*sinh(b*x + a)^6 + b*sinh(b*x + a)^7 - 2*b*cosh(b*x + a)^5 + (21*b*cosh(
b*x + a)^2 - 2*b)*sinh(b*x + a)^5 + 5*(7*b*cosh(b*x + a)^3 - 2*b*cosh(b*x +
 a))*sinh(b*x + a)^4 + 5*(7*b*cosh(b*x + a)^4 - 4*b*cosh(b*x + a)^2)*sinh(b
*x + a)^3 + (21*b*cosh(b*x + a)^5 - 20*b*cosh(b*x + a)^3)*sinh(b*x + a)^2 +
 b*cosh(b*x + a) + (7*b*cosh(b*x + a)^6 - 10*b*cosh(b*x + a)^4 + 3*b)*sinh(
b*x + a))
```

giac [A] time = 0.14, size = 60, normalized size = 1.62

$$-\frac{2\left(\frac{3}{e^{(2\,bx+2\,a)}+1}-\frac{3\,e^{(4\,bx+4\,a)}-12\,e^{(2\,bx+2\,a)}+5}{\left(e^{(2\,bx+2\,a)}-1\right)^{3}}\right)}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^4*sech(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -2/3*(3/(e^(2*b*x + 2*a) + 1) - (3*e^(4*b*x + 4*a) - 12*e^(2*b*x + 2*a) + 5
)/(e^(2*b*x + 2*a) - 1)^3)/b
```

maple [A] time = 0.32, size = 50, normalized size = 1.35

$$\frac{-\frac{1}{3\sinh(bx+a)^{3}\cosh(bx+a)}+\frac{4}{3\sinh(bx+a)\cosh(bx+a)}+\frac{8\tanh(bx+a)}{3}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(b*x+a)^4*sech(b*x+a)^2,x)
```

```
[Out] 1/b*(-1/3/sinh(b*x+a)^3/cosh(b*x+a)+4/3/sinh(b*x+a)/cosh(b*x+a)+8/3*tanh(b*
x+a))
```

maxima [B] time = 0.31, size = 90, normalized size = 2.43

$$\frac{32\,e^{(-2\,bx-2\,a)}}{3\,b\left(2\,e^{(-2\,bx-2\,a)}-2\,e^{(-6\,bx-6\,a)}+e^{(-8\,bx-8\,a)}-1\right)}-\frac{16}{3\,b\left(2\,e^{(-2\,bx-2\,a)}-2\,e^{(-6\,bx-6\,a)}+e^{(-8\,bx-8\,a)}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^4*sech(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 32/3*e^(-2*b*x - 2*a)/(b*(2*e^(-2*b*x - 2*a) - 2*e^(-6*b*x - 6*a) + e^(-8*b
*x - 8*a) - 1)) - 16/3/(b*(2*e^(-2*b*x - 2*a) - 2*e^(-6*b*x - 6*a) + e^(-8*
b*x - 8*a) - 1))
```

mupad [B] time = 1.48, size = 153, normalized size = 4.14

$$\frac{\frac{2}{3\,b}-\frac{4\,\mathrm{e}^{2\,a+2\,b\,x}}{b}+\frac{2\,\mathrm{e}^{4\,a+4\,b\,x}}{3\,b}}{3\,\mathrm{e}^{2\,a+2\,b\,x}-3\,\mathrm{e}^{4\,a+4\,b\,x}+\mathrm{e}^{6\,a+6\,b\,x}-1}-\frac{\frac{2}{b}-\frac{2\,\mathrm{e}^{2\,a+2\,b\,x}}{3\,b}}{\mathrm{e}^{4\,a+4\,b\,x}-2\,\mathrm{e}^{2\,a+2\,b\,x}+1}+\frac{2}{3\,b\,\left(\mathrm{e}^{2\,a+2\,b\,x}-1\right)}-\frac{2}{b\,\left(\mathrm{e}^{2\,a+2\,b\,x}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(a + b*x)^2*sinh(a + b*x)^4),x)
```

```
[Out] (2/(3*b) - (4*exp(2*a + 2*b*x))/b + (2*exp(4*a + 4*b*x))/(3*b))/(3*exp(2*a
+ 2*b*x) - 3*exp(4*a + 4*b*x) + exp(6*a + 6*b*x) - 1) - (2/b - (2*exp(2*a +
 2*b*x))/(3*b))/(exp(4*a + 4*b*x) - 2*exp(2*a + 2*b*x) + 1) + 2/(3*b*(exp(2
*a + 2*b*x) - 1)) - 2/(b*(exp(2*a + 2*b*x) + 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{csch}^{4}\left(a+bx\right)\operatorname{sech}^{2}\left(a+bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)**4*sech(b*x+a)**2,x)
```

```
[Out] Integral(csch(a + b*x)**4*sech(a + b*x)**2, x)
```

3.41 $\int \operatorname{csch}^4(a+bx)\operatorname{sech}^3(a+bx)\,dx$

Optimal. Leaf size=66

$$-\frac{5\operatorname{csch}^3(a+bx)}{6b}+\frac{5\operatorname{csch}(a+bx)}{2b}+\frac{5\tan^{-1}(\sinh(a+bx))}{2b}+\frac{\operatorname{csch}^3(a+bx)\operatorname{sech}^2(a+bx)}{2b}$$

```
[Out] 5/2*arctan(sinh(b*x+a))/b+5/2*csch(b*x+a)/b-5/6*csch(b*x+a)^3/b+1/2*csch(b*
x+a)^3*sech(b*x+a)^2/b
```

Rubi [A] time = 0.04, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {2621, 288, 302, 207}

$$-\frac{5\operatorname{csch}^3(a+bx)}{6b}+\frac{5\operatorname{csch}(a+bx)}{2b}+\frac{5\tan^{-1}(\sinh(a+bx))}{2b}+\frac{\operatorname{csch}^3(a+bx)\operatorname{sech}^2(a+bx)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[a + b*x]^4*Sech[a + b*x]^3,x]
```

```
[Out] (5*ArcTan[Sinh[a + b*x]])/(2*b) + (5*Csch[a + b*x])/(2*b) - (5*Csch[a + b*x
]^3)/(6*b) + (Csch[a + b*x]^3*Sech[a + b*x]^2)/(2*b)
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 2621

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> -Dist[(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rubi steps

$$\int \operatorname{csch}^4(a+bx)\operatorname{sech}^3(a+bx)\,dx = \frac{i\,\text{Subst}\left(\int \frac{x^6}{\left(-1+x^2\right)^2}\,dx, x, -i\operatorname{csch}(a+bx)\right)}{b}$$

$$= \frac{\operatorname{csch}^3(a+bx)\operatorname{sech}^2(a+bx)}{2b} + \frac{(5i)\,\text{Subst}\left(\int \frac{x^4}{-1+x^2}\,dx, x, -i\operatorname{csch}(a+bx)\right)}{2b}$$

$$= \frac{\operatorname{csch}^3(a+bx)\operatorname{sech}^2(a+bx)}{2b} + \frac{(5i)\,\text{Subst}\left(\int \left(1+x^2+\frac{1}{-1+x^2}\right) dx, x, -i\operatorname{csch}\right.}{2b}$$

$$= \frac{5\operatorname{csch}(a+bx)}{2b} - \frac{5\operatorname{csch}^3(a+bx)}{6b} + \frac{\operatorname{csch}^3(a+bx)\operatorname{sech}^2(a+bx)}{2b} + \frac{(5i)\,\text{Sub}}{}$$

$$= \frac{5\tan^{-1}(\sinh(a+bx))}{2b} + \frac{5\operatorname{csch}(a+bx)}{2b} - \frac{5\operatorname{csch}^3(a+bx)}{6b} + \frac{\operatorname{csch}^3(a+bx)\text{s}}{2}$$

Mathematica [C] time = 0.02, size = 33, normalized size = 0.50

$$-\frac{\operatorname{csch}^3(a+bx)\,{}_2F_1\left(-\frac{3}{2},2;-\frac{1}{2};-\sinh^2(a+bx)\right)}{3b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[a + b*x]^4*Sech[a + b*x]^3,x]
```

```
[Out] -1/3*(Csch[a + b*x]^3*Hypergeometric2F1[-3/2, 2, -1/2, -Sinh[a + b*x]^2])/b
```

fricas [B] time = 0.44, size = 1176, normalized size = 17.82

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^4*sech(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/3*(15*cosh(b*x + a)^9 + 135*cosh(b*x + a)*sinh(b*x + a)^8 + 15*sinh(b*x +
 a)^9 + 20*(27*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^7 - 20*cosh(b*x + a)^7 +
```

```
140*(9*cosh(b*x + a)^3 - cosh(b*x + a))*sinh(b*x + a)^6 + 2*(945*cosh(b*x +
 a)^4 - 210*cosh(b*x + a)^2 - 11)*sinh(b*x + a)^5 - 22*cosh(b*x + a)^5 + 10
*(189*cosh(b*x + a)^5 - 70*cosh(b*x + a)^3 - 11*cosh(b*x + a))*sinh(b*x + a
)^4 + 20*(63*cosh(b*x + a)^6 - 35*cosh(b*x + a)^4 - 11*cosh(b*x + a)^2 - 1)
*sinh(b*x + a)^3 - 20*cosh(b*x + a)^3 + 20*(27*cosh(b*x + a)^7 - 21*cosh(b*
x + a)^5 - 11*cosh(b*x + a)^3 - 3*cosh(b*x + a))*sinh(b*x + a)^2 + 15*(cosh
(b*x + a)^10 + 10*cosh(b*x + a)*sinh(b*x + a)^9 + sinh(b*x + a)^10 + (45*co
sh(b*x + a)^2 - 1)*sinh(b*x + a)^8 - cosh(b*x + a)^8 + 8*(15*cosh(b*x + a)^
3 - cosh(b*x + a))*sinh(b*x + a)^7 + 2*(105*cosh(b*x + a)^4 - 14*cosh(b*x +
 a)^2 - 1)*sinh(b*x + a)^6 - 2*cosh(b*x + a)^6 + 4*(63*cosh(b*x + a)^5 - 14
*cosh(b*x + a)^3 - 3*cosh(b*x + a))*sinh(b*x + a)^5 + 2*(105*cosh(b*x + a)^
6 - 35*cosh(b*x + a)^4 - 15*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^4 + 2*cosh(b
*x + a)^4 + 8*(15*cosh(b*x + a)^7 - 7*cosh(b*x + a)^5 - 5*cosh(b*x + a)^3 +
 cosh(b*x + a))*sinh(b*x + a)^3 + (45*cosh(b*x + a)^8 - 28*cosh(b*x + a)^6
- 30*cosh(b*x + a)^4 + 12*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^2 + cosh(b*x +
 a)^2 + 2*(5*cosh(b*x + a)^9 - 4*cosh(b*x + a)^7 - 6*cosh(b*x + a)^5 + 4*co
sh(b*x + a)^3 + cosh(b*x + a))*sinh(b*x + a) - 1)*arctan(cosh(b*x + a) + si
nh(b*x + a)) + 5*(27*cosh(b*x + a)^8 - 28*cosh(b*x + a)^6 - 22*cosh(b*x + a
)^4 - 12*cosh(b*x + a)^2 + 3)*sinh(b*x + a) + 15*cosh(b*x + a))/(b*cosh(b*x
 + a)^10 + 10*b*cosh(b*x + a)*sinh(b*x + a)^9 + b*sinh(b*x + a)^10 - b*cosh
(b*x + a)^8 + (45*b*cosh(b*x + a)^2 - b)*sinh(b*x + a)^8 + 8*(15*b*cosh(b*x
 + a)^3 - b*cosh(b*x + a))*sinh(b*x + a)^7 - 2*b*cosh(b*x + a)^6 + 2*(105*b
*cosh(b*x + a)^4 - 14*b*cosh(b*x + a)^2 - b)*sinh(b*x + a)^6 + 4*(63*b*cosh
(b*x + a)^5 - 14*b*cosh(b*x + a)^3 - 3*b*cosh(b*x + a))*sinh(b*x + a)^5 + 2
*b*cosh(b*x + a)^4 + 2*(105*b*cosh(b*x + a)^6 - 35*b*cosh(b*x + a)^4 - 15*b
*cosh(b*x + a)^2 + b)*sinh(b*x + a)^4 + 8*(15*b*cosh(b*x + a)^7 - 7*b*cosh(
b*x + a)^5 - 5*b*cosh(b*x + a)^3 + b*cosh(b*x + a))*sinh(b*x + a)^3 + b*cos
h(b*x + a)^2 + (45*b*cosh(b*x + a)^8 - 28*b*cosh(b*x + a)^6 - 30*b*cosh(b*x
 + a)^4 + 12*b*cosh(b*x + a)^2 + b)*sinh(b*x + a)^2 + 2*(5*b*cosh(b*x + a)^
9 - 4*b*cosh(b*x + a)^7 - 6*b*cosh(b*x + a)^5 + 4*b*cosh(b*x + a)^3 + b*cos
h(b*x + a))*sinh(b*x + a) - b)
```

giac [B] time = 0.16, size = 124, normalized size = 1.88

$$\frac{15\,\pi + \frac{12\left(e^{(bx+a)}-e^{(-bx-a)}\right)}{\left(e^{(bx+a)}-e^{(-bx-a)}\right)^{2}+4} + \frac{16\left(3\left(e^{(bx+a)}-e^{(-bx-a)}\right)^{2}-2\right)}{\left(e^{(bx+a)}-e^{(-bx-a)}\right)^{3}} + 30\arctan\left(\frac{1}{2}\left(e^{(2\,bx+2\,a)}-1\right)e^{(-bx-a)}\right)}{12\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^4*sech(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/12*(15*pi + 12*(e^(b*x + a) - e^(-b*x - a))/((e^(b*x + a) - e^(-b*x - a))
^2 + 4) + 16*(3*(e^(b*x + a) - e^(-b*x - a))^2 - 2)/(e^(b*x + a) - e^(-b*x
- a))^3 + 30*arctan(1/2*(e^(2*b*x + 2*a) - 1)*e^(-b*x - a)))/b
```

maple [A] time = 0.34, size = 73, normalized size = 1.11

$$-\frac{1}{3b\sinh\left(bx+a\right)^{3}\cosh\left(bx+a\right)^{2}}+\frac{5}{3b\sinh\left(bx+a\right)\cosh\left(bx+a\right)^{2}}+\frac{5\,\mathrm{sech}\left(bx+a\right)\tanh\left(bx+a\right)}{2b}+\frac{5\arctan\left(\mathrm{e}^{b}\right.}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(b*x+a)^4*sech(b*x+a)^3,x)
```

```
[Out] -1/3/b/sinh(b*x+a)^3/cosh(b*x+a)^2+5/3/b/sinh(b*x+a)/cosh(b*x+a)^2+5/2*sech
(b*x+a)*tanh(b*x+a)/b+5*arctan(exp(b*x+a))/b
```

maxima [B] time = 0.42, size = 132, normalized size = 2.00

$$-\frac{5\arctan\left(e^{(-bx-a)}\right)}{b}-\frac{15\,e^{(-bx-a)}-20\,e^{(-3\,bx-3\,a)}-22\,e^{(-5\,bx-5\,a)}-20\,e^{(-7\,bx-7\,a)}+15\,e^{(-9\,bx-9\,a)}}{3\,b\left(e^{(-2\,bx-2\,a)}+2\,e^{(-4\,bx-4\,a)}-2\,e^{(-6\,bx-6\,a)}-e^{(-8\,bx-8\,a)}+e^{(-10\,bx-10\,a)}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^4*sech(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -5*arctan(e^(-b*x - a))/b - 1/3*(15*e^(-b*x - a) - 20*e^(-3*b*x - 3*a) - 22
*e^(-5*b*x - 5*a) - 20*e^(-7*b*x - 7*a) + 15*e^(-9*b*x - 9*a))/(b*(e^(-2*b*
x - 2*a) + 2*e^(-4*b*x - 4*a) - 2*e^(-6*b*x - 6*a) - e^(-8*b*x - 8*a) + e^(
-10*b*x - 10*a) - 1))
```

mupad [B] time = 1.56, size = 187, normalized size = 2.83

$$\frac{5\,\mathrm{atan}\left(\frac{\mathrm{e}^{b\,x}\,\mathrm{e}^{a}\,\sqrt{b^2}}{b}\right)}{\sqrt{b^2}}-\frac{8\,\mathrm{e}^{a+b\,x}}{3\,b\,\left(\mathrm{e}^{4\,a+4\,b\,x}-2\,\mathrm{e}^{2\,a+2\,b\,x}+1\right)}-\frac{2\,\mathrm{e}^{a+b\,x}}{b\,\left(2\,\mathrm{e}^{2\,a+2\,b\,x}+\mathrm{e}^{4\,a+4\,b\,x}+1\right)}-\frac{8\,\mathrm{e}^{a+b\,x}}{3\,b\,\left(3\,\mathrm{e}^{2\,a+2\,b\,x}-3\,\mathrm{e}^{4\,a+4\,b\,x}+\mathrm{e}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(a + b*x)^3*sinh(a + b*x)^4),x)
```

```
[Out] (5*atan((exp(b*x)*exp(a)*(b^2)^(1/2))/b))/(b^2)^(1/2) - (8*exp(a + b*x))/(3
*b*(exp(4*a + 4*b*x) - 2*exp(2*a + 2*b*x) + 1)) - (2*exp(a + b*x))/(b*(2*ex
p(2*a + 2*b*x) + exp(4*a + 4*b*x) + 1)) - (8*exp(a + b*x))/(3*b*(3*exp(2*a
+ 2*b*x) - 3*exp(4*a + 4*b*x) + exp(6*a + 6*b*x) - 1)) + (4*exp(a + b*x))/(
b*(exp(2*a + 2*b*x) - 1)) + exp(a + b*x)/(b*(exp(2*a + 2*b*x) + 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{csch}^{4}\left(a+bx\right)\operatorname{sech}^{3}\left(a+bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)**4*sech(b*x+a)**3,x)
```

```
[Out] Integral(csch(a + b*x)**4*sech(a + b*x)**3, x)
```

3.42 $\int \operatorname{csch}^4(a+bx)\operatorname{sech}^4(a+bx)\,dx$

Optimal. Leaf size=53

$$-\frac{\tanh^3(a+bx)}{3b}+\frac{3\tanh(a+bx)}{b}-\frac{\coth^3(a+bx)}{3b}+\frac{3\coth(a+bx)}{b}$$

```
[Out] 3*coth(b*x+a)/b-1/3*coth(b*x+a)^3/b+3*tanh(b*x+a)/b-1/3*tanh(b*x+a)^3/b
```

Rubi [A] time = 0.04, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2620, 270}

$$-\frac{\tanh^3(a+bx)}{3b}+\frac{3\tanh(a+bx)}{b}-\frac{\coth^3(a+bx)}{3b}+\frac{3\coth(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[a + b*x]^4*Sech[a + b*x]^4,x]
```

```
[Out] (3*Coth[a + b*x])/b - Coth[a + b*x]^3/(3*b) + (3*Tanh[a + b*x])/b - Tanh[a
+ b*x]^3/(3*b)
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2620

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rubi steps

$$\int \operatorname{csch}^4(a+bx)\operatorname{sech}^4(a+bx)\,dx = -\frac{i\,\text{Subst}\left(\int \frac{(1+x^2)^3}{x^4}\,dx, x, i\tanh(a+bx)\right)}{b}$$
$$= -\frac{i\,\text{Subst}\left(\int \left(3+\frac{1}{x^4}+\frac{3}{x^2}+x^2\right)dx, x, i\tanh(a+bx)\right)}{b}$$
$$= \frac{3\coth(a+bx)}{b} - \frac{\coth^3(a+bx)}{3b} + \frac{3\tanh(a+bx)}{b} - \frac{\tanh^3(a+bx)}{3b}$$

Mathematica [A] time = 0.02, size = 43, normalized size = 0.81

$$16\left(\frac{\coth(2(a+bx))}{3b} - \frac{\coth(2(a+bx))\operatorname{csch}^2(2(a+bx))}{6b}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[a + b*x]^4*Sech[a + b*x]^4,x]
```

```
[Out] 16*(Coth[2*(a + b*x)]/(3*b) - (Coth[2*(a + b*x)]*Csch[2*(a + b*x)]^2)/(6*b)
)
```

fricas [B] time = 0.41, size = 330, normalized size = 6.23

$$-\frac{}{3\left(b\cosh(bx+a)^{10} + 120\,b\cosh(bx+a)^3\sinh(bx+a)^7 + 45\,b\cosh(bx+a)^2\sinh(bx+a)^8 + 10\,b\cosh(bx+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^4*sech(b*x+a)^4,x, algorithm="fricas")
```

```
[Out] -64/3*(cosh(b*x + a)^2 + 4*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2)/(
b*cosh(b*x + a)^10 + 120*b*cosh(b*x + a)^3*sinh(b*x + a)^7 + 45*b*cosh(b*x
+ a)^2*sinh(b*x + a)^8 + 10*b*cosh(b*x + a)*sinh(b*x + a)^9 + b*sinh(b*x +
a)^10 - 3*b*cosh(b*x + a)^6 + 3*(70*b*cosh(b*x + a)^4 - b)*sinh(b*x + a)^6
+ 18*(14*b*cosh(b*x + a)^5 - b*cosh(b*x + a))*sinh(b*x + a)^5 + 15*(14*b*co
sh(b*x + a)^6 - 3*b*cosh(b*x + a)^2)*sinh(b*x + a)^4 + 60*(2*b*cosh(b*x + a
)^7 - b*cosh(b*x + a)^3)*sinh(b*x + a)^3 + 2*b*cosh(b*x + a)^2 + (45*b*cosh
(b*x + a)^8 - 45*b*cosh(b*x + a)^4 + 2*b)*sinh(b*x + a)^2 + 2*(5*b*cosh(b*x
 + a)^9 - 9*b*cosh(b*x + a)^5 + 4*b*cosh(b*x + a))*sinh(b*x + a))
```

giac [A] time = 0.15, size = 31, normalized size = 0.58

$$-\frac{32\left(3\,e^{(4\,bx+4\,a)}-1\right)}{3\,b\left(e^{(4\,bx+4\,a)}-1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^4*sech(b*x+a)^4,x, algorithm="giac")
```

```
[Out] -32/3*(3*e^(4*b*x + 4*a) - 1)/(b*(e^(4*b*x + 4*a) - 1)^3)
```

maple [A] time = 0.32, size = 62, normalized size = 1.17

$$\frac{-\frac{1}{3\sinh(bx+a)^{3}\cosh(bx+a)^{3}}+\frac{2}{\sinh(bx+a)\cosh(bx+a)^{3}}+8\left(\frac{2}{3}+\frac{\operatorname{sech}(bx+a)^{2}}{3}\right)\tanh(bx+a)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(b*x+a)^4*sech(b*x+a)^4,x)
```

```
[Out] 1/b*(-1/3/sinh(b*x+a)^3/cosh(b*x+a)^3+2/sinh(b*x+a)/cosh(b*x+a)^3+8*(2/3+1/
3*sech(b*x+a)^2)*tanh(b*x+a))
```

maxima [A] time = 0.31, size = 90, normalized size = 1.70

$$\frac{32\,e^{(-4\,bx-4\,a)}}{b\left(3\,e^{(-4\,bx-4\,a)}-3\,e^{(-8\,bx-8\,a)}+e^{(-12\,bx-12\,a)}-1\right)}-\frac{32}{3\,b\left(3\,e^{(-4\,bx-4\,a)}-3\,e^{(-8\,bx-8\,a)}+e^{(-12\,bx-12\,a)}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^4*sech(b*x+a)^4,x, algorithm="maxima")
```

```
[Out] 32*e^(-4*b*x - 4*a)/(b*(3*e^(-4*b*x - 4*a) - 3*e^(-8*b*x - 8*a) + e^(-12*b*
x - 12*a) - 1)) - 32/3/(b*(3*e^(-4*b*x - 4*a) - 3*e^(-8*b*x - 8*a) + e^(-12
*b*x - 12*a) - 1))
```

mupad [B] time = 0.06, size = 31, normalized size = 0.58

$$-\frac{32\,\left(3\,\mathrm{e}^{4\,a+4\,b\,x}-1\right)}{3\,b\,{\left(\mathrm{e}^{4\,a+4\,b\,x}-1\right)}^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(a + b*x)^4*sinh(a + b*x)^4),x)
```

```
[Out] -(32*(3*exp(4*a + 4*b*x) - 1))/(3*b*(exp(4*a + 4*b*x) - 1)^3)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{csch}^{4}\left(a+bx\right)\operatorname{sech}^{4}\left(a+bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)**4*sech(b*x+a)**4,x)
```

```
[Out] Integral(csch(a + b*x)**4*sech(a + b*x)**4, x)
```

3.43 $\int \operatorname{csch}^4(a+bx)\operatorname{sech}^5(a+bx)\,dx$

Optimal. Leaf size=89

$$-\frac{35\operatorname{csch}^3(a+bx)}{24b}+\frac{35\operatorname{csch}(a+bx)}{8b}+\frac{35\tan^{-1}(\sinh(a+bx))}{8b}+\frac{\operatorname{csch}^3(a+bx)\operatorname{sech}^4(a+bx)}{4b}+\frac{7\operatorname{csch}^3(a+bx)\operatorname{sech}}{8b}$$

```
[Out] 35/8*arctan(sinh(b*x+a))/b+35/8*csch(b*x+a)/b-35/24*csch(b*x+a)^3/b+7/8*csc
h(b*x+a)^3*sech(b*x+a)^2/b+1/4*csch(b*x+a)^3*sech(b*x+a)^4/b
```

Rubi [A] time = 0.05, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {2621, 288, 302, 207}

$$-\frac{35\operatorname{csch}^3(a+bx)}{24b}+\frac{35\operatorname{csch}(a+bx)}{8b}+\frac{35\tan^{-1}(\sinh(a+bx))}{8b}+\frac{\operatorname{csch}^3(a+bx)\operatorname{sech}^4(a+bx)}{4b}+\frac{7\operatorname{csch}^3(a+bx)\operatorname{sech}}{8b}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[a + b*x]^4*Sech[a + b*x]^5,x]
```

```
[Out] (35*ArcTan[Sinh[a + b*x]])/(8*b) + (35*Csch[a + b*x])/(8*b) - (35*Csch[a +
b*x]^3)/(24*b) + (7*Csch[a + b*x]^3*Sech[a + b*x]^2)/(8*b) + (Csch[a + b*x]
^3*Sech[a + b*x]^4)/(4*b)
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 2621

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> -Dist[(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rubi steps

$$\int \operatorname{csch}^4(a+bx)\operatorname{sech}^5(a+bx)\,dx = \frac{i\,\text{Subst}\left(\int \frac{x^8}{\left(-1+x^2\right)^3}\,dx, x, -i\operatorname{csch}(a+bx)\right)}{b}$$

$$= \frac{\operatorname{csch}^3(a+bx)\operatorname{sech}^4(a+bx)}{4b} + \frac{(7i)\,\text{Subst}\left(\int \frac{x^6}{\left(-1+x^2\right)^2}\,dx, x, -i\operatorname{csch}(a+bx)\right)}{4b}$$

$$= \frac{7\operatorname{csch}^3(a+bx)\operatorname{sech}^2(a+bx)}{8b} + \frac{\operatorname{csch}^3(a+bx)\operatorname{sech}^4(a+bx)}{4b} + \frac{(35i)\,\text{Subst}\Big(}{}$$

$$= \frac{7\operatorname{csch}^3(a+bx)\operatorname{sech}^2(a+bx)}{8b} + \frac{\operatorname{csch}^3(a+bx)\operatorname{sech}^4(a+bx)}{4b} + \frac{(35i)\,\text{Subst}\Big(}{}$$

$$= \frac{35\operatorname{csch}(a+bx)}{8b} - \frac{35\operatorname{csch}^3(a+bx)}{24b} + \frac{7\operatorname{csch}^3(a+bx)\operatorname{sech}^2(a+bx)}{8b} + \frac{\operatorname{csch}^3(}{}$$

$$= \frac{35\tan^{-1}(\sinh(a+bx))}{8b} + \frac{35\operatorname{csch}(a+bx)}{8b} - \frac{35\operatorname{csch}^3(a+bx)}{24b} + \frac{7\operatorname{csch}^3(a+}{}$$

Mathematica [C] time = 0.02, size = 33, normalized size = 0.37

$$-\frac{\operatorname{csch}^3(a+bx)\,{}_2F_1\left(-\frac{3}{2},3;-\frac{1}{2};-\sinh^2(a+bx)\right)}{3b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[a + b*x]^4*Sech[a + b*x]^5,x]
```

```
[Out] -1/3*(Csch[a + b*x]^3*Hypergeometric2F1[-3/2, 3, -1/2, -Sinh[a + b*x]^2])/b
```

fricas [B] time = 0.42, size = 2092, normalized size = 23.51

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^4*sech(b*x+a)^5,x, algorithm="fricas")

[Out] 1/12*(105*cosh(b*x + a)^13 + 1365*cosh(b*x + a)*sinh(b*x + a)^12 + 105*sinh
(b*x + a)^13 + 70*(117*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^11 + 70*cosh(b*x
+ a)^11 + 770*(39*cosh(b*x + a)^3 + cosh(b*x + a))*sinh(b*x + a)^10 + 7*(10
725*cosh(b*x + a)^4 + 550*cosh(b*x + a)^2 - 47)*sinh(b*x + a)^9 - 329*cosh(
b*x + a)^9 + 21*(6435*cosh(b*x + a)^5 + 550*cosh(b*x + a)^3 - 141*cosh(b*x
+ a))*sinh(b*x + a)^8 + 12*(15015*cosh(b*x + a)^6 + 1925*cosh(b*x + a)^4 -
987*cosh(b*x + a)^2 - 17)*sinh(b*x + a)^7 - 204*cosh(b*x + a)^7 + 84*(2145*
cosh(b*x + a)^7 + 385*cosh(b*x + a)^5 - 329*cosh(b*x + a)^3 - 17*cosh(b*x +
 a))*sinh(b*x + a)^6 + 7*(19305*cosh(b*x + a)^8 + 4620*cosh(b*x + a)^6 - 59
22*cosh(b*x + a)^4 - 612*cosh(b*x + a)^2 - 47)*sinh(b*x + a)^5 - 329*cosh(b
*x + a)^5 + 7*(10725*cosh(b*x + a)^9 + 3300*cosh(b*x + a)^7 - 5922*cosh(b*x
 + a)^5 - 1020*cosh(b*x + a)^3 - 235*cosh(b*x + a))*sinh(b*x + a)^4 + 14*(2
145*cosh(b*x + a)^10 + 825*cosh(b*x + a)^8 - 1974*cosh(b*x + a)^6 - 510*cos
h(b*x + a)^4 - 235*cosh(b*x + a)^2 + 5)*sinh(b*x + a)^3 + 70*cosh(b*x + a)^
3 + 14*(585*cosh(b*x + a)^11 + 275*cosh(b*x + a)^9 - 846*cosh(b*x + a)^7 -
306*cosh(b*x + a)^5 - 235*cosh(b*x + a)^3 + 15*cosh(b*x + a))*sinh(b*x + a)
^2 + 105*(cosh(b*x + a)^14 + 14*cosh(b*x + a)*sinh(b*x + a)^13 + sinh(b*x +
 a)^14 + (91*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^12 + cosh(b*x + a)^12 + 4*(
91*cosh(b*x + a)^3 + 3*cosh(b*x + a))*sinh(b*x + a)^11 + (1001*cosh(b*x + a
)^4 + 66*cosh(b*x + a)^2 - 3)*sinh(b*x + a)^10 - 3*cosh(b*x + a)^10 + 2*(10
01*cosh(b*x + a)^5 + 110*cosh(b*x + a)^3 - 15*cosh(b*x + a))*sinh(b*x + a)^
9 + 3*(1001*cosh(b*x + a)^6 + 165*cosh(b*x + a)^4 - 45*cosh(b*x + a)^2 - 1)
*sinh(b*x + a)^8 - 3*cosh(b*x + a)^8 + 24*(143*cosh(b*x + a)^7 + 33*cosh(b*
x + a)^5 - 15*cosh(b*x + a)^3 - cosh(b*x + a))*sinh(b*x + a)^7 + 3*(1001*co
sh(b*x + a)^8 + 308*cosh(b*x + a)^6 - 210*cosh(b*x + a)^4 - 28*cosh(b*x + a
)^2 + 1)*sinh(b*x + a)^6 + 3*cosh(b*x + a)^6 + 2*(1001*cosh(b*x + a)^9 + 39
6*cosh(b*x + a)^7 - 378*cosh(b*x + a)^5 - 84*cosh(b*x + a)^3 + 9*cosh(b*x +
 a))*sinh(b*x + a)^5 + (1001*cosh(b*x + a)^10 + 495*cosh(b*x + a)^8 - 630*c
osh(b*x + a)^6 - 210*cosh(b*x + a)^4 + 45*cosh(b*x + a)^2 + 3)*sinh(b*x + a
)^4 + 3*cosh(b*x + a)^4 + 4*(91*cosh(b*x + a)^11 + 55*cosh(b*x + a)^9 - 90*
cosh(b*x + a)^7 - 42*cosh(b*x + a)^5 + 15*cosh(b*x + a)^3 + 3*cosh(b*x + a)
)*sinh(b*x + a)^3 + (91*cosh(b*x + a)^12 + 66*cosh(b*x + a)^10 - 135*cosh(b
*x + a)^8 - 84*cosh(b*x + a)^6 + 45*cosh(b*x + a)^4 + 18*cosh(b*x + a)^2 -
1)*sinh(b*x + a)^2 - cosh(b*x + a)^2 + 2*(7*cosh(b*x + a)^13 + 6*cosh(b*x +
 a)^11 - 15*cosh(b*x + a)^9 - 12*cosh(b*x + a)^7 + 9*cosh(b*x + a)^5 + 6*co
sh(b*x + a)^3 - cosh(b*x + a))*sinh(b*x + a) - 1)*arctan(cosh(b*x + a) + si
nh(b*x + a)) + 7*(195*cosh(b*x + a)^12 + 110*cosh(b*x + a)^10 - 423*cosh(b*
x + a)^8 - 204*cosh(b*x + a)^6 - 235*cosh(b*x + a)^4 + 30*cosh(b*x + a)^2 +
 15)*sinh(b*x + a) + 105*cosh(b*x + a))/(b*cosh(b*x + a)^14 + 14*b*cosh(b*x
 + a)*sinh(b*x + a)^13 + b*sinh(b*x + a)^14 + b*cosh(b*x + a)^12 + (91*b*co
sh(b*x + a)^2 + b)*sinh(b*x + a)^12 + 4*(91*b*cosh(b*x + a)^3 + 3*b*cosh(b*
x + a))*sinh(b*x + a)^11 - 3*b*cosh(b*x + a)^10 + (1001*b*cosh(b*x + a)^4 +
 66*b*cosh(b*x + a)^2 - 3*b)*sinh(b*x + a)^10 + 2*(1001*b*cosh(b*x + a)^5 +
```

```
110*b*cosh(b*x + a)^3 - 15*b*cosh(b*x + a))*sinh(b*x + a)^9 - 3*b*cosh(b*x
 + a)^8 + 3*(1001*b*cosh(b*x + a)^6 + 165*b*cosh(b*x + a)^4 - 45*b*cosh(b*x
 + a)^2 - b)*sinh(b*x + a)^8 + 24*(143*b*cosh(b*x + a)^7 + 33*b*cosh(b*x +
a)^5 - 15*b*cosh(b*x + a)^3 - b*cosh(b*x + a))*sinh(b*x + a)^7 + 3*b*cosh(b
*x + a)^6 + 3*(1001*b*cosh(b*x + a)^8 + 308*b*cosh(b*x + a)^6 - 210*b*cosh(
b*x + a)^4 - 28*b*cosh(b*x + a)^2 + b)*sinh(b*x + a)^6 + 2*(1001*b*cosh(b*x
 + a)^9 + 396*b*cosh(b*x + a)^7 - 378*b*cosh(b*x + a)^5 - 84*b*cosh(b*x + a
)^3 + 9*b*cosh(b*x + a))*sinh(b*x + a)^5 + 3*b*cosh(b*x + a)^4 + (1001*b*co
sh(b*x + a)^10 + 495*b*cosh(b*x + a)^8 - 630*b*cosh(b*x + a)^6 - 210*b*cosh
(b*x + a)^4 + 45*b*cosh(b*x + a)^2 + 3*b)*sinh(b*x + a)^4 + 4*(91*b*cosh(b*
x + a)^11 + 55*b*cosh(b*x + a)^9 - 90*b*cosh(b*x + a)^7 - 42*b*cosh(b*x + a
)^5 + 15*b*cosh(b*x + a)^3 + 3*b*cosh(b*x + a))*sinh(b*x + a)^3 - b*cosh(b*
x + a)^2 + (91*b*cosh(b*x + a)^12 + 66*b*cosh(b*x + a)^10 - 135*b*cosh(b*x
+ a)^8 - 84*b*cosh(b*x + a)^6 + 45*b*cosh(b*x + a)^4 + 18*b*cosh(b*x + a)^2
 - b)*sinh(b*x + a)^2 + 2*(7*b*cosh(b*x + a)^13 + 6*b*cosh(b*x + a)^11 - 15
*b*cosh(b*x + a)^9 - 12*b*cosh(b*x + a)^7 + 9*b*cosh(b*x + a)^5 + 6*b*cosh(
b*x + a)^3 - b*cosh(b*x + a))*sinh(b*x + a) - b)
```

giac [A] time = 0.15, size = 148, normalized size = 1.66

$$\frac{105\,\pi + \frac{12\left(11\left(e^{(bx+a)}-e^{(-bx-a)}\right)^3+52\,e^{(bx+a)}-52\,e^{(-bx-a)}\right)}{\left(\left(e^{(bx+a)}-e^{(-bx-a)}\right)^2+4\right)^2} + \frac{32\left(9\left(e^{(bx+a)}-e^{(-bx-a)}\right)^2-4\right)}{\left(e^{(bx+a)}-e^{(-bx-a)}\right)^3} + 210\arctan\left(\frac{1}{2}\left(e^{(2\,bx+2\,a)}-1\right)e^{(-bx-a)}\right)}{48\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^4*sech(b*x+a)^5,x, algorithm="giac")
```

```
[Out] 1/48*(105*pi + 12*(11*(e^(b*x + a) - e^(-b*x - a))^3 + 52*e^(b*x + a) - 52*
e^(-b*x - a))/((e^(b*x + a) - e^(-b*x - a))^2 + 4)^2 + 32*(9*(e^(b*x + a) -
 e^(-b*x - a))^2 - 4)/(e^(b*x + a) - e^(-b*x - a))^3 + 210*arctan(1/2*(e^(2
*b*x + 2*a) - 1)*e^(-b*x - a)))/b
```

maple [A] time = 0.36, size = 92, normalized size = 1.03

$$-\frac{1}{3b\sinh(bx+a)^3\cosh(bx+a)^4}+\frac{7}{3b\sinh(bx+a)\cosh(bx+a)^4}+\frac{35\mathrm{sech}(bx+a)^3\tanh(bx+a)}{12b}+\frac{35\,\mathrm{sech}\,(bx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(b*x+a)^4*sech(b*x+a)^5,x)
```

```
[Out] -1/3/b/sinh(b*x+a)^3/cosh(b*x+a)^4+7/3/b/sinh(b*x+a)/cosh(b*x+a)^4+35/12*se
ch(b*x+a)^3*tanh(b*x+a)/b+35/8*sech(b*x+a)*tanh(b*x+a)/b+35/4*arctan(exp(b*
x+a))/b
```

maxima [B] time = 0.51, size = 178, normalized size = 2.00

$$-\frac{35 \arctan\left(e^{(-bx-a)}\right)}{4\,b}+\frac{105\,e^{(-bx-a)}+70\,e^{(-3\,bx-3\,a)}-329\,e^{(-5\,bx-5\,a)}-204\,e^{(-7\,bx-7\,a)}-329\,e^{(-9\,bx-9\,a)}+70\,e^{(-11\,bx}}{12\,b\left(e^{(-2\,bx-2\,a)}-3\,e^{(-4\,bx-4\,a)}-3\,e^{(-6\,bx-6\,a)}+3\,e^{(-8\,bx-8\,a)}+3\,e^{(-10\,bx-10\,a)}-e^{(-12\,bx-12}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^4*sech(b*x+a)^5,x, algorithm="maxima")
```

```
[Out] -35/4*arctan(e^(-b*x - a))/b + 1/12*(105*e^(-b*x - a) + 70*e^(-3*b*x - 3*a)
 - 329*e^(-5*b*x - 5*a) - 204*e^(-7*b*x - 7*a) - 329*e^(-9*b*x - 9*a) + 70*
e^(-11*b*x - 11*a) + 105*e^(-13*b*x - 13*a))/(b*(e^(-2*b*x - 2*a) - 3*e^(-4
*b*x - 4*a) - 3*e^(-6*b*x - 6*a) + 3*e^(-8*b*x - 8*a) + 3*e^(-10*b*x - 10*a
) - e^(-12*b*x - 12*a) - e^(-14*b*x - 14*a) + 1))
```

mupad [B] time = 1.47, size = 291, normalized size = 3.27

$$\frac{35\,\mathrm{atan}\left(\frac{\mathrm{e}^{b\,x}\,\mathrm{e}^{a}\,\sqrt{b^2}}{b}\right)}{4\,\sqrt{b^2}}-\frac{8\,\mathrm{e}^{a+b\,x}}{3\,b\,\left(\mathrm{e}^{4\,a+4\,b\,x}-2\,\mathrm{e}^{2\,a+2\,b\,x}+1\right)}-\frac{7\,\mathrm{e}^{a+b\,x}}{2\,b\,\left(2\,\mathrm{e}^{2\,a+2\,b\,x}+\mathrm{e}^{4\,a+4\,b\,x}+1\right)}-\frac{8\,\mathrm{e}^{a+b\,x}}{3\,b\,\left(3\,\mathrm{e}^{2\,a+2\,b\,x}-3\,\mathrm{e}^{4\,a+4\,b\,x}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(a + b*x)^5*sinh(a + b*x)^4),x)
```

```
[Out] (35*atan((exp(b*x)*exp(a)*(b^2)^(1/2))/b))/(4*(b^2)^(1/2)) - (8*exp(a + b*x
))/(3*b*(exp(4*a + 4*b*x) - 2*exp(2*a + 2*b*x) + 1)) - (7*exp(a + b*x))/(2*
b*(2*exp(2*a + 2*b*x) + exp(4*a + 4*b*x) + 1)) - (8*exp(a + b*x))/(3*b*(3*e
xp(2*a + 2*b*x) - 3*exp(4*a + 4*b*x) + exp(6*a + 6*b*x) - 1)) - (6*exp(a +
b*x))/(b*(3*exp(2*a + 2*b*x) + 3*exp(4*a + 4*b*x) + exp(6*a + 6*b*x) + 1))
+ (4*exp(a + b*x))/(b*(4*exp(2*a + 2*b*x) + 6*exp(4*a + 4*b*x) + 4*exp(6*a
+ 6*b*x) + exp(8*a + 8*b*x) + 1)) + (6*exp(a + b*x))/(b*(exp(2*a + 2*b*x) -
 1)) + (11*exp(a + b*x))/(4*b*(exp(2*a + 2*b*x) + 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{csch}^{4}(a+bx)\operatorname{sech}^{5}(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)**4*sech(b*x+a)**5,x)
```

```
[Out] Integral(csch(a + b*x)**4*sech(a + b*x)**5, x)
```

3.44 $\int \operatorname{csch}^5(a+bx)\operatorname{sech}(a+bx)\,dx$

Optimal. Leaf size=39

$$-\frac{\coth^4(a+bx)}{4b}+\frac{\coth^2(a+bx)}{b}+\frac{\log(\tanh(a+bx))}{b}$$

```
[Out] coth(b*x+a)^2/b-1/4*coth(b*x+a)^4/b+ln(tanh(b*x+a))/b
```

Rubi [A] time = 0.03, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2620, 266, 43}

$$-\frac{\coth^4(a+bx)}{4b}+\frac{\coth^2(a+bx)}{b}+\frac{\log(\tanh(a+bx))}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[a + b*x]^5*Sech[a + b*x],x]
```

```
[Out] Coth[a + b*x]^2/b - Coth[a + b*x]^4/(4*b) + Log[Tanh[a + b*x]]/b
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2620

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rubi steps

$$\int \operatorname{csch}^5(a+bx)\operatorname{sech}(a+bx)\,dx = \frac{\text{Subst}\left(\int \frac{(1+x^2)^2}{x^5}\,dx, x, i\tanh(a+bx)\right)}{b}$$
$$= \frac{\text{Subst}\left(\int \frac{(1+x)^2}{x^3}\,dx, x, -\tanh^2(a+bx)\right)}{2b}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{1}{x^3}+\frac{2}{x^2}+\frac{1}{x}\right)dx, x, -\tanh^2(a+bx)\right)}{2b}$$
$$= \frac{\coth^2(a+bx)}{b} - \frac{\coth^4(a+bx)}{4b} + \frac{\log(\tanh(a+bx))}{b}$$

Mathematica [A] time = 0.13, size = 46, normalized size = 1.18

$$\frac{-\operatorname{csch}^4(a+bx)+2\operatorname{csch}^2(a+bx)+4\log(\sinh(a+bx))-4\log(\cosh(a+bx))}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[a + b*x]^5*Sech[a + b*x],x]
```

```
[Out] (2*Csch[a + b*x]^2 - Csch[a + b*x]^4 - 4*Log[Cosh[a + b*x]] + 4*Log[Sinh[a
+ b*x]])/(4*b)
```

fricas [B] time = 0.49, size = 1082, normalized size = 27.74

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^5*sech(b*x+a),x, algorithm="fricas")
```

```
[Out] (2*cosh(b*x + a)^6 + 12*cosh(b*x + a)*sinh(b*x + a)^5 + 2*sinh(b*x + a)^6 +
 2*(15*cosh(b*x + a)^2 - 4)*sinh(b*x + a)^4 - 8*cosh(b*x + a)^4 + 8*(5*cosh
(b*x + a)^3 - 4*cosh(b*x + a))*sinh(b*x + a)^3 + 2*(15*cosh(b*x + a)^4 - 24
*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^2 + 2*cosh(b*x + a)^2 - (cosh(b*x + a)^
8 + 8*cosh(b*x + a)*sinh(b*x + a)^7 + sinh(b*x + a)^8 + 4*(7*cosh(b*x + a)^
2 - 1)*sinh(b*x + a)^6 - 4*cosh(b*x + a)^6 + 8*(7*cosh(b*x + a)^3 - 3*cosh(
b*x + a))*sinh(b*x + a)^5 + 2*(35*cosh(b*x + a)^4 - 30*cosh(b*x + a)^2 + 3)
*sinh(b*x + a)^4 + 6*cosh(b*x + a)^4 + 8*(7*cosh(b*x + a)^5 - 10*cosh(b*x +
 a)^3 + 3*cosh(b*x + a))*sinh(b*x + a)^3 + 4*(7*cosh(b*x + a)^6 - 15*cosh(b
*x + a)^4 + 9*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^2 - 4*cosh(b*x + a)^2 + 8*
(cosh(b*x + a)^7 - 3*cosh(b*x + a)^5 + 3*cosh(b*x + a)^3 - cosh(b*x + a))*s
```

```
inh(b*x + a) + 1)*log(2*cosh(b*x + a)/(cosh(b*x + a) - sinh(b*x + a))) + (c
osh(b*x + a)^8 + 8*cosh(b*x + a)*sinh(b*x + a)^7 + sinh(b*x + a)^8 + 4*(7*c
osh(b*x + a)^2 - 1)*sinh(b*x + a)^6 - 4*cosh(b*x + a)^6 + 8*(7*cosh(b*x + a
)^3 - 3*cosh(b*x + a))*sinh(b*x + a)^5 + 2*(35*cosh(b*x + a)^4 - 30*cosh(b*
x + a)^2 + 3)*sinh(b*x + a)^4 + 6*cosh(b*x + a)^4 + 8*(7*cosh(b*x + a)^5 -
10*cosh(b*x + a)^3 + 3*cosh(b*x + a))*sinh(b*x + a)^3 + 4*(7*cosh(b*x + a)^
6 - 15*cosh(b*x + a)^4 + 9*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^2 - 4*cosh(b*
x + a)^2 + 8*(cosh(b*x + a)^7 - 3*cosh(b*x + a)^5 + 3*cosh(b*x + a)^3 - cos
h(b*x + a))*sinh(b*x + a) + 1)*log(2*sinh(b*x + a)/(cosh(b*x + a) - sinh(b*
x + a))) + 4*(3*cosh(b*x + a)^5 - 8*cosh(b*x + a)^3 + cosh(b*x + a))*sinh(b
*x + a))/(b*cosh(b*x + a)^8 + 8*b*cosh(b*x + a)*sinh(b*x + a)^7 + b*sinh(b*
x + a)^8 - 4*b*cosh(b*x + a)^6 + 4*(7*b*cosh(b*x + a)^2 - b)*sinh(b*x + a)^
6 + 8*(7*b*cosh(b*x + a)^3 - 3*b*cosh(b*x + a))*sinh(b*x + a)^5 + 6*b*cosh(
b*x + a)^4 + 2*(35*b*cosh(b*x + a)^4 - 30*b*cosh(b*x + a)^2 + 3*b)*sinh(b*x
 + a)^4 + 8*(7*b*cosh(b*x + a)^5 - 10*b*cosh(b*x + a)^3 + 3*b*cosh(b*x + a)
)*sinh(b*x + a)^3 - 4*b*cosh(b*x + a)^2 + 4*(7*b*cosh(b*x + a)^6 - 15*b*cos
h(b*x + a)^4 + 9*b*cosh(b*x + a)^2 - b)*sinh(b*x + a)^2 + 8*(b*cosh(b*x + a
)^7 - 3*b*cosh(b*x + a)^5 + 3*b*cosh(b*x + a)^3 - b*cosh(b*x + a))*sinh(b*x
 + a) + b)
```

giac [B] time = 0.15, size = 122, normalized size = 3.13

$$-\frac{\frac{3\left(e^{(2bx+2a)}+e^{(-2bx-2a)}\right)^2-20e^{(2bx+2a)}-20e^{(-2bx-2a)}+44}{\left(e^{(2bx+2a)}+e^{(-2bx-2a)}-2\right)^2}+2\log\left(e^{(2bx+2a)}+e^{(-2bx-2a)}+2\right)-2\log\left(e^{(2bx+2a)}+e^{(-2bx-2a)}\right.}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^5*sech(b*x+a),x, algorithm="giac")
```

```
[Out] -1/4*((3*(e^(2*b*x + 2*a) + e^(-2*b*x - 2*a))^2 - 20*e^(2*b*x + 2*a) - 20*e
^(-2*b*x - 2*a) + 44)/(e^(2*b*x + 2*a) + e^(-2*b*x - 2*a) - 2)^2 + 2*log(e^
(2*b*x + 2*a) + e^(-2*b*x - 2*a) + 2) - 2*log(e^(2*b*x + 2*a) + e^(-2*b*x -
 2*a) - 2))/b
```

maple [A] time = 0.17, size = 39, normalized size = 1.00

$$-\frac{1}{4b\sinh(bx+a)^4}+\frac{1}{2b\sinh(bx+a)^2}+\frac{\ln(\tanh(bx+a))}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(b*x+a)^5*sech(b*x+a),x)
```

```
[Out] -1/4/b/sinh(b*x+a)^4+1/2/b/sinh(b*x+a)^2+ln(tanh(b*x+a))/b
```

maxima [B] time = 0.47, size = 133, normalized size = 3.41

$$\frac{\log\left(e^{(-bx-a)}+1\right)}{b}+\frac{\log\left(e^{(-bx-a)}-1\right)}{b}-\frac{\log\left(e^{(-2\,bx-2\,a)}+1\right)}{b}-\frac{2\left(e^{(-2\,bx-2\,a)}-4\,e^{(-4\,bx-4\,a)}+e^{(-6\,bx-6\,a)}\right)}{b\left(4\,e^{(-2\,bx-2\,a)}-6\,e^{(-4\,bx-4\,a)}+4\,e^{(-6\,bx-6\,a)}-e^{(-8\,bx-8\,a)}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^5*sech(b*x+a),x, algorithm="maxima")
```

```
[Out] log(e^(-b*x - a) + 1)/b + log(e^(-b*x - a) - 1)/b - log(e^(-2*b*x - 2*a) +
1)/b - 2*(e^(-2*b*x - 2*a) - 4*e^(-4*b*x - 4*a) + e^(-6*b*x - 6*a))/(b*(4*e
^(-2*b*x - 2*a) - 6*e^(-4*b*x - 4*a) + 4*e^(-6*b*x - 6*a) - e^(-8*b*x - 8*a
) - 1))
```

mupad [B] time = 0.06, size = 169, normalized size = 4.33

$$\frac{2}{b\,\left(\mathrm{e}^{2\,a+2\,b\,x}-1\right)}-\frac{2\,\mathrm{atan}\left(\frac{\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}\,\sqrt{-b^2}}{b}\right)}{\sqrt{-b^2}}-\frac{2}{b\,\left(\mathrm{e}^{4\,a+4\,b\,x}-2\,\mathrm{e}^{2\,a+2\,b\,x}+1\right)}-\frac{8}{b\,\left(3\,\mathrm{e}^{2\,a+2\,b\,x}-3\,\mathrm{e}^{4\,a+4\,b\,x}+\mathrm{e}^{6\,a+6\,b\,x}-1\right)}\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(a + b*x)*sinh(a + b*x)^5),x)
```

```
[Out] 2/(b*(exp(2*a + 2*b*x) - 1)) - (2*atan((exp(2*a)*exp(2*b*x)*(-b^2)^(1/2))/b
))/(-b^2)^(1/2) - 2/(b*(exp(4*a + 4*b*x) - 2*exp(2*a + 2*b*x) + 1)) - 8/(b*
(3*exp(2*a + 2*b*x) - 3*exp(4*a + 4*b*x) + exp(6*a + 6*b*x) - 1)) - 4/(b*(6
*exp(4*a + 4*b*x) - 4*exp(2*a + 2*b*x) - 4*exp(6*a + 6*b*x) + exp(8*a + 8*b
*x) + 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{csch}^5(a+bx)\operatorname{sech}(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)**5*sech(b*x+a),x)
```

```
[Out] Integral(csch(a + b*x)**5*sech(a + b*x), x)
```

3.45 $\int \operatorname{csch}^5(a+bx)\operatorname{sech}^2(a+bx)\,dx$

Optimal. Leaf size=70

$$\frac{15\operatorname{sech}(a+bx)}{8b}-\frac{15\tanh^{-1}(\cosh(a+bx))}{8b}-\frac{\operatorname{csch}^4(a+bx)\operatorname{sech}(a+bx)}{4b}+\frac{5\operatorname{csch}^2(a+bx)\operatorname{sech}(a+bx)}{8b}$$

```
[Out] -15/8*arctanh(cosh(b*x+a))/b+15/8*sech(b*x+a)/b+5/8*csch(b*x+a)^2*sech(b*x+
a)/b-1/4*csch(b*x+a)^4*sech(b*x+a)/b
```

Rubi [A] time = 0.05, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {2622, 288, 321, 207}

$$\frac{15\operatorname{sech}(a+bx)}{8b}-\frac{15\tanh^{-1}(\cosh(a+bx))}{8b}-\frac{\operatorname{csch}^4(a+bx)\operatorname{sech}(a+bx)}{4b}+\frac{5\operatorname{csch}^2(a+bx)\operatorname{sech}(a+bx)}{8b}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[a + b*x]^5*Sech[a + b*x]^2,x]
```

```
[Out] (-15*ArcTanh[Cosh[a + b*x]])/(8*b) + (15*Sech[a + b*x])/(8*b) + (5*Csch[a +
 b*x]^2*Sech[a + b*x])/(8*b) - (Csch[a + b*x]^4*Sech[a + b*x])/(4*b)
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rubi steps

$$\int \operatorname{csch}^5(a+bx)\operatorname{sech}^2(a+bx)\,dx = \frac{\operatorname{Subst}\left(\int \frac{x^6}{(-1+x^2)^3}\,dx, x, \operatorname{sech}(a+bx)\right)}{b}$$

$$= -\frac{\operatorname{csch}^4(a+bx)\operatorname{sech}(a+bx)}{4b} + \frac{5\operatorname{Subst}\left(\int \frac{x^4}{(-1+x^2)^2}\,dx, x, \operatorname{sech}(a+bx)\right)}{4b}$$

$$= \frac{5\operatorname{csch}^2(a+bx)\operatorname{sech}(a+bx)}{8b} - \frac{\operatorname{csch}^4(a+bx)\operatorname{sech}(a+bx)}{4b} + \frac{15\operatorname{Subst}\left(\int -\right.}{}$$

$$= \frac{15\operatorname{sech}(a+bx)}{8b} + \frac{5\operatorname{csch}^2(a+bx)\operatorname{sech}(a+bx)}{8b} - \frac{\operatorname{csch}^4(a+bx)\operatorname{sech}(a+b}{4b}$$

$$= -\frac{15\tanh^{-1}(\cosh(a+bx))}{8b} + \frac{15\operatorname{sech}(a+bx)}{8b} + \frac{5\operatorname{csch}^2(a+bx)\operatorname{sech}(a+bx}{8b}$$

Mathematica [A] time = 0.04, size = 105, normalized size = 1.50

$$-\frac{\operatorname{csch}^4\left(\frac{1}{2}(a+bx)\right)}{64b}+\frac{7\operatorname{csch}^2\left(\frac{1}{2}(a+bx)\right)}{32b}+\frac{\operatorname{sech}^4\left(\frac{1}{2}(a+bx)\right)}{64b}+\frac{7\operatorname{sech}^2\left(\frac{1}{2}(a+bx)\right)}{32b}+\frac{\operatorname{sech}(a+bx)}{b}+\frac{15\log\left(\tanh\right.}{8}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[a + b*x]^5*Sech[a + b*x]^2,x]
```

```
[Out] (7*Csch[(a + b*x)/2]^2)/(32*b) - Csch[(a + b*x)/2]^4/(64*b) + (15*Log[Tanh[
(a + b*x)/2]])/(8*b) + (7*Sech[(a + b*x)/2]^2)/(32*b) + Sech[(a + b*x)/2]^4
/(64*b) + Sech[a + b*x]/b
```

fricas [B] time = 0.43, size = 1591, normalized size = 22.73

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^5*sech(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/8*(30*cosh(b*x + a)^9 + 270*cosh(b*x + a)*sinh(b*x + a)^8 + 30*sinh(b*x +
 a)^9 + 40*(27*cosh(b*x + a)^2 - 2)*sinh(b*x + a)^7 - 80*cosh(b*x + a)^7 +
280*(9*cosh(b*x + a)^3 - 2*cosh(b*x + a))*sinh(b*x + a)^6 + 12*(315*cosh(b*
x + a)^4 - 140*cosh(b*x + a)^2 + 3)*sinh(b*x + a)^5 + 36*cosh(b*x + a)^5 +
20*(189*cosh(b*x + a)^5 - 140*cosh(b*x + a)^3 + 9*cosh(b*x + a))*sinh(b*x +
 a)^4 + 40*(63*cosh(b*x + a)^6 - 70*cosh(b*x + a)^4 + 9*cosh(b*x + a)^2 - 2
)*sinh(b*x + a)^3 - 80*cosh(b*x + a)^3 + 120*(9*cosh(b*x + a)^7 - 14*cosh(b
*x + a)^5 + 3*cosh(b*x + a)^3 - 2*cosh(b*x + a))*sinh(b*x + a)^2 - 15*(cosh
(b*x + a)^10 + 10*cosh(b*x + a)*sinh(b*x + a)^9 + sinh(b*x + a)^10 + 3*(15*
cosh(b*x + a)^2 - 1)*sinh(b*x + a)^8 - 3*cosh(b*x + a)^8 + 24*(5*cosh(b*x +
 a)^3 - cosh(b*x + a))*sinh(b*x + a)^7 + 2*(105*cosh(b*x + a)^4 - 42*cosh(b
*x + a)^2 + 1)*sinh(b*x + a)^6 + 2*cosh(b*x + a)^6 + 12*(21*cosh(b*x + a)^5
 - 14*cosh(b*x + a)^3 + cosh(b*x + a))*sinh(b*x + a)^5 + 2*(105*cosh(b*x +
a)^6 - 105*cosh(b*x + a)^4 + 15*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^4 + 2*co
sh(b*x + a)^4 + 8*(15*cosh(b*x + a)^7 - 21*cosh(b*x + a)^5 + 5*cosh(b*x + a
)^3 + cosh(b*x + a))*sinh(b*x + a)^3 + 3*(15*cosh(b*x + a)^8 - 28*cosh(b*x
+ a)^6 + 10*cosh(b*x + a)^4 + 4*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^2 - 3*co
sh(b*x + a)^2 + 2*(5*cosh(b*x + a)^9 - 12*cosh(b*x + a)^7 + 6*cosh(b*x + a)
^5 + 4*cosh(b*x + a)^3 - 3*cosh(b*x + a))*sinh(b*x + a) + 1)*log(cosh(b*x +
 a) + sinh(b*x + a) + 1) + 15*(cosh(b*x + a)^10 + 10*cosh(b*x + a)*sinh(b*x
 + a)^9 + sinh(b*x + a)^10 + 3*(15*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^8 - 3
*cosh(b*x + a)^8 + 24*(5*cosh(b*x + a)^3 - cosh(b*x + a))*sinh(b*x + a)^7 +
 2*(105*cosh(b*x + a)^4 - 42*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^6 + 2*cosh(
b*x + a)^6 + 12*(21*cosh(b*x + a)^5 - 14*cosh(b*x + a)^3 + cosh(b*x + a))*s
inh(b*x + a)^5 + 2*(105*cosh(b*x + a)^6 - 105*cosh(b*x + a)^4 + 15*cosh(b*x
 + a)^2 + 1)*sinh(b*x + a)^4 + 2*cosh(b*x + a)^4 + 8*(15*cosh(b*x + a)^7 -
21*cosh(b*x + a)^5 + 5*cosh(b*x + a)^3 + cosh(b*x + a))*sinh(b*x + a)^3 + 3
*(15*cosh(b*x + a)^8 - 28*cosh(b*x + a)^6 + 10*cosh(b*x + a)^4 + 4*cosh(b*x
 + a)^2 - 1)*sinh(b*x + a)^2 - 3*cosh(b*x + a)^2 + 2*(5*cosh(b*x + a)^9 - 1
2*cosh(b*x + a)^7 + 6*cosh(b*x + a)^5 + 4*cosh(b*x + a)^3 - 3*cosh(b*x + a)
)*sinh(b*x + a) + 1)*log(cosh(b*x + a) + sinh(b*x + a) - 1) + 10*(27*cosh(b
*x + a)^8 - 56*cosh(b*x + a)^6 + 18*cosh(b*x + a)^4 - 24*cosh(b*x + a)^2 +
3)*sinh(b*x + a) + 30*cosh(b*x + a))/(b*cosh(b*x + a)^10 + 10*b*cosh(b*x +
a)*sinh(b*x + a)^9 + b*sinh(b*x + a)^10 - 3*b*cosh(b*x + a)^8 + 3*(15*b*cos
h(b*x + a)^2 - b)*sinh(b*x + a)^8 + 24*(5*b*cosh(b*x + a)^3 - b*cosh(b*x +
a))*sinh(b*x + a)^7 + 2*b*cosh(b*x + a)^6 + 2*(105*b*cosh(b*x + a)^4 - 42*b
*cosh(b*x + a)^2 + b)*sinh(b*x + a)^6 + 12*(21*b*cosh(b*x + a)^5 - 14*b*cos
h(b*x + a)^3 + b*cosh(b*x + a))*sinh(b*x + a)^5 + 2*b*cosh(b*x + a)^4 + 2*(
105*b*cosh(b*x + a)^6 - 105*b*cosh(b*x + a)^4 + 15*b*cosh(b*x + a)^2 + b)*s
inh(b*x + a)^4 + 8*(15*b*cosh(b*x + a)^7 - 21*b*cosh(b*x + a)^5 + 5*b*cosh(
b*x + a)^3 + b*cosh(b*x + a))*sinh(b*x + a)^3 - 3*b*cosh(b*x + a)^2 + 3*(15
*b*cosh(b*x + a)^8 - 28*b*cosh(b*x + a)^6 + 10*b*cosh(b*x + a)^4 + 4*b*cosh
(b*x + a)^2 - b)*sinh(b*x + a)^2 + 2*(5*b*cosh(b*x + a)^9 - 12*b*cosh(b*x +
```

```
 a)^7 + 6*b*cosh(b*x + a)^5 + 4*b*cosh(b*x + a)^3 - 3*b*cosh(b*x + a))*sinh
(b*x + a) + b)
```

giac [B] time = 0.16, size = 130, normalized size = 1.86

$$\frac{\frac{4\left(7\left(e^{(bx+a)}+e^{(-bx-a)}\right)^3-36\,e^{(bx+a)}-36\,e^{(-bx-a)}\right)}{\left(\left(e^{(bx+a)}+e^{(-bx-a)}\right)^2-4\right)^2}+\frac{32}{e^{(bx+a)}+e^{(-bx-a)}}-15\,\log\left(e^{(bx+a)}+e^{(-bx-a)}+2\right)+15\,\log\left(e^{(bx+a)}+e^{(-bx-a)}-\right.}{16\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^5*sech(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/16*(4*(7*(e^(b*x + a) + e^(-b*x - a))^3 - 36*e^(b*x + a) - 36*e^(-b*x - a
))/((e^(b*x + a) + e^(-b*x - a))^2 - 4)^2 + 32/(e^(b*x + a) + e^(-b*x - a))
 - 15*log(e^(b*x + a) + e^(-b*x - a) + 2) + 15*log(e^(b*x + a) + e^(-b*x -
a) - 2))/b
```

maple [A] time = 0.15, size = 61, normalized size = 0.87

$$\frac{-\frac{1}{4\sinh(bx+a)^4\cosh(bx+a)}+\frac{5}{8\sinh(bx+a)^2\cosh(bx+a)}+\frac{15}{8\cosh(bx+a)}-\frac{15\,\mathrm{arctanh}\left(\mathrm{e}^{bx+a}\right)}{4}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(b*x+a)^5*sech(b*x+a)^2,x)
```

```
[Out] 1/b*(-1/4/sinh(b*x+a)^4/cosh(b*x+a)+5/8/sinh(b*x+a)^2/cosh(b*x+a)+15/8/cosh
(b*x+a)-15/4*arctanh(exp(b*x+a)))
```

maxima [B] time = 0.60, size = 155, normalized size = 2.21

$$-\frac{15\,\log\left(e^{(-bx-a)}+1\right)}{8\,b}+\frac{15\,\log\left(e^{(-bx-a)}-1\right)}{8\,b}-\frac{15\,e^{(-bx-a)}-40\,e^{(-3\,bx-3\,a)}+18\,e^{(-5\,bx-5\,a)}-40\,e^{(-7\,bx-7\,a)}+15\,e^{(-}}{4\,b\left(3\,e^{(-2\,bx-2\,a)}-2\,e^{(-4\,bx-4\,a)}-2\,e^{(-6\,bx-6\,a)}+3\,e^{(-8\,bx-8\,a)}-e^{(-10\,b}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^5*sech(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -15/8*log(e^(-b*x - a) + 1)/b + 15/8*log(e^(-b*x - a) - 1)/b - 1/4*(15*e^(-
b*x - a) - 40*e^(-3*b*x - 3*a) + 18*e^(-5*b*x - 5*a) - 40*e^(-7*b*x - 7*a)
+ 15*e^(-9*b*x - 9*a))/(b*(3*e^(-2*b*x - 2*a) - 2*e^(-4*b*x - 4*a) - 2*e^(-
6*b*x - 6*a) + 3*e^(-8*b*x - 8*a) - e^(-10*b*x - 10*a) - 1))
```

mupad [B] time = 1.45, size = 214, normalized size = 3.06

$$\frac{3\,\mathrm{e}^{a+b\,x}}{2\,b\,\left(\mathrm{e}^{4\,a+4\,b\,x}-2\,\mathrm{e}^{2\,a+2\,b\,x}+1\right)}-\frac{15\,\mathrm{atan}\left(\frac{\mathrm{e}^{b\,x}\,\mathrm{e}^{a}\,\sqrt{-b^2}}{b}\right)}{4\,\sqrt{-b^2}}-\frac{6\,\mathrm{e}^{a+b\,x}}{b\,\left(3\,\mathrm{e}^{2\,a+2\,b\,x}-3\,\mathrm{e}^{4\,a+4\,b\,x}+\mathrm{e}^{6\,a+6\,b\,x}-1\right)}-\frac{}{b\,\left(6\,\mathrm{e}^{4\,a+4\,b\,x}-4\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(a + b*x)^2*sinh(a + b*x)^5),x)
```

```
[Out] (3*exp(a + b*x))/(2*b*(exp(4*a + 4*b*x) - 2*exp(2*a + 2*b*x) + 1)) - (15*at
an((exp(b*x)*exp(a)*(-b^2)^(1/2))/b))/(4*(-b^2)^(1/2)) - (6*exp(a + b*x))/(
b*(3*exp(2*a + 2*b*x) - 3*exp(4*a + 4*b*x) + exp(6*a + 6*b*x) - 1)) - (4*ex
p(a + b*x))/(b*(6*exp(4*a + 4*b*x) - 4*exp(2*a + 2*b*x) - 4*exp(6*a + 6*b*x
) + exp(8*a + 8*b*x) + 1)) + (7*exp(a + b*x))/(4*b*(exp(2*a + 2*b*x) - 1))
+ (2*exp(a + b*x))/(b*(exp(2*a + 2*b*x) + 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{csch}^{5}\left(a+bx\right)\operatorname{sech}^{2}\left(a+bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)**5*sech(b*x+a)**2,x)
```

```
[Out] Integral(csch(a + b*x)**5*sech(a + b*x)**2, x)
```

3.46 $\int \text{csch}^5(a+bx)\text{sech}^3(a+bx)\,dx$

Optimal. Leaf size=58

$$-\frac{\tanh^2(a+bx)}{2b}-\frac{\coth^4(a+bx)}{4b}+\frac{3\coth^2(a+bx)}{2b}+\frac{3\log(\tanh(a+bx))}{b}$$

```
[Out] 3/2*coth(b*x+a)^2/b-1/4*coth(b*x+a)^4/b+3*ln(tanh(b*x+a))/b-1/2*tanh(b*x+a)
^2/b
```

Rubi [A] time = 0.05, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {2620, 266, 43}

$$-\frac{\tanh^2(a+bx)}{2b}-\frac{\coth^4(a+bx)}{4b}+\frac{3\coth^2(a+bx)}{2b}+\frac{3\log(\tanh(a+bx))}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[a + b*x]^5*Sech[a + b*x]^3,x]
```

```
[Out] (3*Coth[a + b*x]^2)/(2*b) - Coth[a + b*x]^4/(4*b) + (3*Log[Tanh[a + b*x]])/
b - Tanh[a + b*x]^2/(2*b)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2620

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rubi steps

$$\int \operatorname{csch}^5(a+bx)\operatorname{sech}^3(a+bx)\,dx = \frac{\text{Subst}\left(\int \frac{(1+x^2)^3}{x^5}\,dx, x, i\tanh(a+bx)\right)}{b}$$
$$= \frac{\text{Subst}\left(\int \frac{(1+x)^3}{x^3}\,dx, x, -\tanh^2(a+bx)\right)}{2b}$$
$$= \frac{\text{Subst}\left(\int \left(1+\frac{1}{x^3}+\frac{3}{x^2}+\frac{3}{x}\right)dx, x, -\tanh^2(a+bx)\right)}{2b}$$
$$= \frac{3\coth^2(a+bx)}{2b} - \frac{\coth^4(a+bx)}{4b} + \frac{3\log(\tanh(a+bx))}{b} - \frac{\tanh^2(a+bx)}{2b}$$

Mathematica [A] time = 0.20, size = 56, normalized size = 0.97

$$\frac{-\operatorname{csch}^4(a+bx)+4\operatorname{csch}^2(a+bx)+2\operatorname{sech}^2(a+bx)+12\log(\sinh(a+bx))-12\log(\cosh(a+bx))}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[a + b*x]^5*Sech[a + b*x]^3,x]
```

```
[Out] (4*Csch[a + b*x]^2 - Csch[a + b*x]^4 - 12*Log[Cosh[a + b*x]] + 12*Log[Sinh[
a + b*x]] + 2*Sech[a + b*x]^2)/(4*b)
```

fricas [B] time = 0.45, size = 2114, normalized size = 36.45

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^5*sech(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] (6*cosh(b*x + a)^10 + 60*cosh(b*x + a)*sinh(b*x + a)^9 + 6*sinh(b*x + a)^10
 + 6*(45*cosh(b*x + a)^2 - 2)*sinh(b*x + a)^8 - 12*cosh(b*x + a)^8 + 48*(15
*cosh(b*x + a)^3 - 2*cosh(b*x + a))*sinh(b*x + a)^7 + 4*(315*cosh(b*x + a)^
4 - 84*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^6 - 4*cosh(b*x + a)^6 + 24*(63*co
sh(b*x + a)^5 - 28*cosh(b*x + a)^3 - cosh(b*x + a))*sinh(b*x + a)^5 + 12*(1
05*cosh(b*x + a)^6 - 70*cosh(b*x + a)^4 - 5*cosh(b*x + a)^2 - 1)*sinh(b*x +
 a)^4 - 12*cosh(b*x + a)^4 + 16*(45*cosh(b*x + a)^7 - 42*cosh(b*x + a)^5 -
5*cosh(b*x + a)^3 - 3*cosh(b*x + a))*sinh(b*x + a)^3 + 6*(45*cosh(b*x + a)^
8 - 56*cosh(b*x + a)^6 - 10*cosh(b*x + a)^4 - 12*cosh(b*x + a)^2 + 1)*sinh(
b*x + a)^2 + 6*cosh(b*x + a)^2 - 3*(cosh(b*x + a)^12 + 12*cosh(b*x + a)*sin
h(b*x + a)^11 + sinh(b*x + a)^12 + 2*(33*cosh(b*x + a)^2 - 1)*sinh(b*x + a)
```

```
^10 - 2*cosh(b*x + a)^10 + 20*(11*cosh(b*x + a)^3 - cosh(b*x + a))*sinh(b*x
 + a)^9 + (495*cosh(b*x + a)^4 - 90*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^8 -
cosh(b*x + a)^8 + 8*(99*cosh(b*x + a)^5 - 30*cosh(b*x + a)^3 - cosh(b*x + a
))*sinh(b*x + a)^7 + 4*(231*cosh(b*x + a)^6 - 105*cosh(b*x + a)^4 - 7*cosh(
b*x + a)^2 + 1)*sinh(b*x + a)^6 + 4*cosh(b*x + a)^6 + 8*(99*cosh(b*x + a)^7
 - 63*cosh(b*x + a)^5 - 7*cosh(b*x + a)^3 + 3*cosh(b*x + a))*sinh(b*x + a)^
5 + (495*cosh(b*x + a)^8 - 420*cosh(b*x + a)^6 - 70*cosh(b*x + a)^4 + 60*co
sh(b*x + a)^2 - 1)*sinh(b*x + a)^4 - cosh(b*x + a)^4 + 4*(55*cosh(b*x + a)^
9 - 60*cosh(b*x + a)^7 - 14*cosh(b*x + a)^5 + 20*cosh(b*x + a)^3 - cosh(b*x
 + a))*sinh(b*x + a)^3 + 2*(33*cosh(b*x + a)^10 - 45*cosh(b*x + a)^8 - 14*c
osh(b*x + a)^6 + 30*cosh(b*x + a)^4 - 3*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^
2 - 2*cosh(b*x + a)^2 + 4*(3*cosh(b*x + a)^11 - 5*cosh(b*x + a)^9 - 2*cosh(
b*x + a)^7 + 6*cosh(b*x + a)^5 - cosh(b*x + a)^3 - cosh(b*x + a))*sinh(b*x
+ a) + 1)*log(2*cosh(b*x + a)/(cosh(b*x + a) - sinh(b*x + a))) + 3*(cosh(b*
x + a)^12 + 12*cosh(b*x + a)*sinh(b*x + a)^11 + sinh(b*x + a)^12 + 2*(33*co
sh(b*x + a)^2 - 1)*sinh(b*x + a)^10 - 2*cosh(b*x + a)^10 + 20*(11*cosh(b*x
+ a)^3 - cosh(b*x + a))*sinh(b*x + a)^9 + (495*cosh(b*x + a)^4 - 90*cosh(b*
x + a)^2 - 1)*sinh(b*x + a)^8 - cosh(b*x + a)^8 + 8*(99*cosh(b*x + a)^5 - 3
0*cosh(b*x + a)^3 - cosh(b*x + a))*sinh(b*x + a)^7 + 4*(231*cosh(b*x + a)^6
 - 105*cosh(b*x + a)^4 - 7*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^6 + 4*cosh(b*
x + a)^6 + 8*(99*cosh(b*x + a)^7 - 63*cosh(b*x + a)^5 - 7*cosh(b*x + a)^3 +
 3*cosh(b*x + a))*sinh(b*x + a)^5 + (495*cosh(b*x + a)^8 - 420*cosh(b*x + a
)^6 - 70*cosh(b*x + a)^4 + 60*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^4 - cosh(b
*x + a)^4 + 4*(55*cosh(b*x + a)^9 - 60*cosh(b*x + a)^7 - 14*cosh(b*x + a)^5
 + 20*cosh(b*x + a)^3 - cosh(b*x + a))*sinh(b*x + a)^3 + 2*(33*cosh(b*x + a
)^10 - 45*cosh(b*x + a)^8 - 14*cosh(b*x + a)^6 + 30*cosh(b*x + a)^4 - 3*cos
h(b*x + a)^2 - 1)*sinh(b*x + a)^2 - 2*cosh(b*x + a)^2 + 4*(3*cosh(b*x + a)^
11 - 5*cosh(b*x + a)^9 - 2*cosh(b*x + a)^7 + 6*cosh(b*x + a)^5 - cosh(b*x +
 a)^3 - cosh(b*x + a))*sinh(b*x + a) + 1)*log(2*sinh(b*x + a)/(cosh(b*x + a
) - sinh(b*x + a))) + 12*(5*cosh(b*x + a)^9 - 8*cosh(b*x + a)^7 - 2*cosh(b*
x + a)^5 - 4*cosh(b*x + a)^3 + cosh(b*x + a))*sinh(b*x + a))/(b*cosh(b*x +
a)^12 + 12*b*cosh(b*x + a)*sinh(b*x + a)^11 + b*sinh(b*x + a)^12 - 2*b*cosh
(b*x + a)^10 + 2*(33*b*cosh(b*x + a)^2 - b)*sinh(b*x + a)^10 + 20*(11*b*cos
h(b*x + a)^3 - b*cosh(b*x + a))*sinh(b*x + a)^9 - b*cosh(b*x + a)^8 + (495*
b*cosh(b*x + a)^4 - 90*b*cosh(b*x + a)^2 - b)*sinh(b*x + a)^8 + 8*(99*b*cos
h(b*x + a)^5 - 30*b*cosh(b*x + a)^3 - b*cosh(b*x + a))*sinh(b*x + a)^7 + 4*
b*cosh(b*x + a)^6 + 4*(231*b*cosh(b*x + a)^6 - 105*b*cosh(b*x + a)^4 - 7*b*
cosh(b*x + a)^2 + b)*sinh(b*x + a)^6 + 8*(99*b*cosh(b*x + a)^7 - 63*b*cosh(
b*x + a)^5 - 7*b*cosh(b*x + a)^3 + 3*b*cosh(b*x + a))*sinh(b*x + a)^5 - b*c
osh(b*x + a)^4 + (495*b*cosh(b*x + a)^8 - 420*b*cosh(b*x + a)^6 - 70*b*cosh
(b*x + a)^4 + 60*b*cosh(b*x + a)^2 - b)*sinh(b*x + a)^4 + 4*(55*b*cosh(b*x
+ a)^9 - 60*b*cosh(b*x + a)^7 - 14*b*cosh(b*x + a)^5 + 20*b*cosh(b*x + a)^3
 - b*cosh(b*x + a))*sinh(b*x + a)^3 - 2*b*cosh(b*x + a)^2 + 2*(33*b*cosh(b*
x + a)^10 - 45*b*cosh(b*x + a)^8 - 14*b*cosh(b*x + a)^6 + 30*b*cosh(b*x + a
)^4 - 3*b*cosh(b*x + a)^2 - b)*sinh(b*x + a)^2 + 4*(3*b*cosh(b*x + a)^11 -
```

```
5*b*cosh(b*x + a)^9 - 2*b*cosh(b*x + a)^7 + 6*b*cosh(b*x + a)^5 - b*cosh(b*
x + a)^3 - b*cosh(b*x + a))*sinh(b*x + a) + b)
```

giac [B] time = 0.15, size = 171, normalized size = 2.95

$$\frac{\frac{2\left(3\,e^{(2\,bx+2\,a)}+3\,e^{(-2\,bx-2\,a)}+10\right)}{e^{(2\,bx+2\,a)}+e^{(-2\,bx-2\,a)}+2}-\frac{9\left(e^{(2\,bx+2\,a)}+e^{(-2\,bx-2\,a)}\right)^2-52\,e^{(2\,bx+2\,a)}-52\,e^{(-2\,bx-2\,a)}+84}{\left(e^{(2\,bx+2\,a)}+e^{(-2\,bx-2\,a)}-2\right)^2}-6\,\log\left(e^{(2\,bx+2\,a)}+e^{(-2\,bx-2\,a)}+2\right)+6\,\mathrm{l}}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^5*sech(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/4*(2*(3*e^(2*b*x + 2*a) + 3*e^(-2*b*x - 2*a) + 10)/(e^(2*b*x + 2*a) + e^(
-2*b*x - 2*a) + 2) - (9*(e^(2*b*x + 2*a) + e^(-2*b*x - 2*a))^2 - 52*e^(2*b*
x + 2*a) - 52*e^(-2*b*x - 2*a) + 84)/(e^(2*b*x + 2*a) + e^(-2*b*x - 2*a) -
2)^2 - 6*log(e^(2*b*x + 2*a) + e^(-2*b*x - 2*a) + 2) + 6*log(e^(2*b*x + 2*a
) + e^(-2*b*x - 2*a) - 2))/b
```

maple [A] time = 0.22, size = 69, normalized size = 1.19

$$-\frac{1}{4b\sinh(bx+a)^4\cosh(bx+a)^2}+\frac{3}{4b\sinh(bx+a)^2\cosh(bx+a)^2}+\frac{3}{2b\cosh(bx+a)^2}+\frac{3\ln(\tanh(bx+a))}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(b*x+a)^5*sech(b*x+a)^3,x)
```

```
[Out] -1/4/b/sinh(b*x+a)^4/cosh(b*x+a)^2+3/4/b/sinh(b*x+a)^2/cosh(b*x+a)^2+3/2/b/
cosh(b*x+a)^2+3*ln(tanh(b*x+a))/b
```

maxima [B] time = 0.80, size = 179, normalized size = 3.09

$$\frac{3\,\log\left(e^{(-bx-a)}+1\right)}{b}+\frac{3\,\log\left(e^{(-bx-a)}-1\right)}{b}-\frac{3\,\log\left(e^{(-2\,bx-2\,a)}+1\right)}{b}-\frac{2\left(3\,e^{(-2\,bx-2\,a)}-6\,e^{(-4\,bx-4\,a)}-2\,e^{(-6\,bx-6\,a}\right.}{b\left(2\,e^{(-2\,bx-2\,a)}+e^{(-4\,bx-4\,a)}-4\,e^{(-6\,bx-6\,a)}+e^{(-8\,b}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^5*sech(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 3*log(e^(-b*x - a) + 1)/b + 3*log(e^(-b*x - a) - 1)/b - 3*log(e^(-2*b*x - 2
*a) + 1)/b - 2*(3*e^(-2*b*x - 2*a) - 6*e^(-4*b*x - 4*a) - 2*e^(-6*b*x - 6*a
) - 6*e^(-8*b*x - 8*a) + 3*e^(-10*b*x - 10*a))/(b*(2*e^(-2*b*x - 2*a) + e^(
-4*b*x - 4*a) - 4*e^(-6*b*x - 6*a) + e^(-8*b*x - 8*a) + 2*e^(-10*b*x - 10*a
) - e^(-12*b*x - 12*a) - 1))
```

mupad [B] time = 1.47, size = 187, normalized size = 3.22

$$\frac{4}{b\left(e^{2a+2bx}-1\right)}+\frac{2}{b\left(e^{2a+2bx}+1\right)}-\frac{6\,\mathrm{atan}\left(\frac{e^{2a}\,e^{2bx}\,\sqrt{-b^2}}{b}\right)}{\sqrt{-b^2}}-\frac{2}{b\left(2\,e^{2a+2bx}+e^{4a+4bx}+1\right)}-\frac{8}{b\left(3\,e^{2a+2bx}-3\,e^{4a+4}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(a + b*x)^3*sinh(a + b*x)^5),x)
```

```
[Out] 4/(b*(exp(2*a + 2*b*x) - 1)) + 2/(b*(exp(2*a + 2*b*x) + 1)) - (6*atan((exp(
2*a)*exp(2*b*x)*(-b^2)^(1/2))/b))/(-b^2)^(1/2) - 2/(b*(2*exp(2*a + 2*b*x) +
 exp(4*a + 4*b*x) + 1)) - 8/(b*(3*exp(2*a + 2*b*x) - 3*exp(4*a + 4*b*x) + e
xp(6*a + 6*b*x) - 1)) - 4/(b*(6*exp(4*a + 4*b*x) - 4*exp(2*a + 2*b*x) - 4*e
xp(6*a + 6*b*x) + exp(8*a + 8*b*x) + 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{csch}^5(a+bx)\,\operatorname{sech}^3(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)**5*sech(b*x+a)**3,x)
```

```
[Out] Integral(csch(a + b*x)**5*sech(a + b*x)**3, x)
```

3.47 $\int \mathrm{csch}^5(a+bx)\mathrm{sech}^4(a+bx)\,dx$

Optimal. Leaf size=89

$$\frac{35\mathrm{sech}^3(a+bx)}{24b}+\frac{35\mathrm{sech}(a+bx)}{8b}-\frac{35\tanh^{-1}(\cosh(a+bx))}{8b}-\frac{\mathrm{csch}^4(a+bx)\mathrm{sech}^3(a+bx)}{4b}+\frac{7\mathrm{csch}^2(a+bx)\mathrm{sech}^3}{8b}$$

```
[Out] -35/8*arctanh(cosh(b*x+a))/b+35/8*sech(b*x+a)/b+35/24*sech(b*x+a)^3/b+7/8*c
sch(b*x+a)^2*sech(b*x+a)^3/b-1/4*csch(b*x+a)^4*sech(b*x+a)^3/b
```

Rubi [A] time = 0.06, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {2622, 288, 302, 207}

$$\frac{35\mathrm{sech}^3(a+bx)}{24b}+\frac{35\mathrm{sech}(a+bx)}{8b}-\frac{35\tanh^{-1}(\cosh(a+bx))}{8b}-\frac{\mathrm{csch}^4(a+bx)\mathrm{sech}^3(a+bx)}{4b}+\frac{7\mathrm{csch}^2(a+bx)\mathrm{sech}^3}{8b}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[a + b*x]^5*Sech[a + b*x]^4,x]
```

```
[Out] (-35*ArcTanh[Cosh[a + b*x]])/(8*b) + (35*Sech[a + b*x])/(8*b) + (35*Sech[a
+ b*x]^3)/(24*b) + (7*Csch[a + b*x]^2*Sech[a + b*x]^3)/(8*b) - (Csch[a + b*
x]^4*Sech[a + b*x]^3)/(4*b)
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rubi steps

$$\int \operatorname{csch}^5(a+bx)\operatorname{sech}^4(a+bx)\,dx = \frac{\operatorname{Subst}\left(\int \frac{x^8}{\left(-1+x^2\right)^3}\,dx, x, \operatorname{sech}(a+bx)\right)}{b}$$

$$= -\frac{\operatorname{csch}^4(a+bx)\operatorname{sech}^3(a+bx)}{4b} + \frac{7\operatorname{Subst}\left(\int \frac{x^6}{\left(-1+x^2\right)^2}\,dx, x, \operatorname{sech}(a+bx)\right)}{4b}$$

$$= \frac{7\operatorname{csch}^2(a+bx)\operatorname{sech}^3(a+bx)}{8b} - \frac{\operatorname{csch}^4(a+bx)\operatorname{sech}^3(a+bx)}{4b} + \frac{35\operatorname{Subst}\left(\int\right.}{}$$

$$= \frac{7\operatorname{csch}^2(a+bx)\operatorname{sech}^3(a+bx)}{8b} - \frac{\operatorname{csch}^4(a+bx)\operatorname{sech}^3(a+bx)}{4b} + \frac{35\operatorname{Subst}\left(\int\right.}{}$$

$$= \frac{35\operatorname{sech}(a+bx)}{8b} + \frac{35\operatorname{sech}^3(a+bx)}{24b} + \frac{7\operatorname{csch}^2(a+bx)\operatorname{sech}^3(a+bx)}{8b} - \operatorname{csch}$$

$$= -\frac{35\tanh^{-1}(\cosh(a+bx))}{8b} + \frac{35\operatorname{sech}(a+bx)}{8b} + \frac{35\operatorname{sech}^3(a+bx)}{24b} + 7\operatorname{csch}^2$$

Mathematica [A] time = 0.04, size = 121, normalized size = 1.36

$$-\frac{\operatorname{csch}^4\left(\frac{1}{2}(a+bx)\right)}{64b}+\frac{11\operatorname{csch}^2\left(\frac{1}{2}(a+bx)\right)}{32b}+\frac{\operatorname{sech}^4\left(\frac{1}{2}(a+bx)\right)}{64b}+\frac{\operatorname{sech}^3(a+bx)}{3b}+\frac{11\operatorname{sech}^2\left(\frac{1}{2}(a+bx)\right)}{32b}+\frac{3\operatorname{sech}(a-}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[a + b*x]^5*Sech[a + b*x]^4,x]
```

```
[Out] (11*Csch[(a + b*x)/2]^2)/(32*b) - Csch[(a + b*x)/2]^4/(64*b) + (35*Log[Tanh
[(a + b*x)/2]])/(8*b) + (11*Sech[(a + b*x)/2]^2)/(32*b) + Sech[(a + b*x)/2]
^4/(64*b) + (3*Sech[a + b*x])/b + Sech[a + b*x]^3/(3*b)
```

fricas [B] time = 0.46, size = 2802, normalized size = 31.48

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^5*sech(b*x+a)^4,x, algorithm="fricas")
```

```
[Out] 1/24*(210*cosh(b*x + a)^13 + 2730*cosh(b*x + a)*sinh(b*x + a)^12 + 210*sinh
(b*x + a)^13 + 140*(117*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^11 - 140*cosh(b*
x + a)^11 + 1540*(39*cosh(b*x + a)^3 - cosh(b*x + a))*sinh(b*x + a)^10 + 14
*(10725*cosh(b*x + a)^4 - 550*cosh(b*x + a)^2 - 47)*sinh(b*x + a)^9 - 658*c
osh(b*x + a)^9 + 42*(6435*cosh(b*x + a)^5 - 550*cosh(b*x + a)^3 - 141*cosh(
b*x + a))*sinh(b*x + a)^8 + 24*(15015*cosh(b*x + a)^6 - 1925*cosh(b*x + a)^
4 - 987*cosh(b*x + a)^2 + 17)*sinh(b*x + a)^7 + 408*cosh(b*x + a)^7 + 168*(
2145*cosh(b*x + a)^7 - 385*cosh(b*x + a)^5 - 329*cosh(b*x + a)^3 + 17*cosh(
b*x + a))*sinh(b*x + a)^6 + 14*(19305*cosh(b*x + a)^8 - 4620*cosh(b*x + a)^
6 - 5922*cosh(b*x + a)^4 + 612*cosh(b*x + a)^2 - 47)*sinh(b*x + a)^5 - 658*
cosh(b*x + a)^5 + 14*(10725*cosh(b*x + a)^9 - 3300*cosh(b*x + a)^7 - 5922*c
osh(b*x + a)^5 + 1020*cosh(b*x + a)^3 - 235*cosh(b*x + a))*sinh(b*x + a)^4
+ 28*(2145*cosh(b*x + a)^10 - 825*cosh(b*x + a)^8 - 1974*cosh(b*x + a)^6 +
510*cosh(b*x + a)^4 - 235*cosh(b*x + a)^2 - 5)*sinh(b*x + a)^3 - 140*cosh(b
*x + a)^3 + 28*(585*cosh(b*x + a)^11 - 275*cosh(b*x + a)^9 - 846*cosh(b*x +
 a)^7 + 306*cosh(b*x + a)^5 - 235*cosh(b*x + a)^3 - 15*cosh(b*x + a))*sinh(
b*x + a)^2 - 105*(cosh(b*x + a)^14 + 14*cosh(b*x + a)*sinh(b*x + a)^13 + si
nh(b*x + a)^14 + (91*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^12 - cosh(b*x + a)^
12 + 4*(91*cosh(b*x + a)^3 - 3*cosh(b*x + a))*sinh(b*x + a)^11 + (1001*cosh
(b*x + a)^4 - 66*cosh(b*x + a)^2 - 3)*sinh(b*x + a)^10 - 3*cosh(b*x + a)^10
 + 2*(1001*cosh(b*x + a)^5 - 110*cosh(b*x + a)^3 - 15*cosh(b*x + a))*sinh(b
*x + a)^9 + 3*(1001*cosh(b*x + a)^6 - 165*cosh(b*x + a)^4 - 45*cosh(b*x + a
)^2 + 1)*sinh(b*x + a)^8 + 3*cosh(b*x + a)^8 + 24*(143*cosh(b*x + a)^7 - 33
*cosh(b*x + a)^5 - 15*cosh(b*x + a)^3 + cosh(b*x + a))*sinh(b*x + a)^7 + 3*
(1001*cosh(b*x + a)^8 - 308*cosh(b*x + a)^6 - 210*cosh(b*x + a)^4 + 28*cosh
(b*x + a)^2 + 1)*sinh(b*x + a)^6 + 3*cosh(b*x + a)^6 + 2*(1001*cosh(b*x + a
)^9 - 396*cosh(b*x + a)^7 - 378*cosh(b*x + a)^5 + 84*cosh(b*x + a)^3 + 9*co
sh(b*x + a))*sinh(b*x + a)^5 + (1001*cosh(b*x + a)^10 - 495*cosh(b*x + a)^8
 - 630*cosh(b*x + a)^6 + 210*cosh(b*x + a)^4 + 45*cosh(b*x + a)^2 - 3)*sinh
(b*x + a)^4 - 3*cosh(b*x + a)^4 + 4*(91*cosh(b*x + a)^11 - 55*cosh(b*x + a)
^9 - 90*cosh(b*x + a)^7 + 42*cosh(b*x + a)^5 + 15*cosh(b*x + a)^3 - 3*cosh(
b*x + a))*sinh(b*x + a)^3 + (91*cosh(b*x + a)^12 - 66*cosh(b*x + a)^10 - 13
5*cosh(b*x + a)^8 + 84*cosh(b*x + a)^6 + 45*cosh(b*x + a)^4 - 18*cosh(b*x +
 a)^2 - 1)*sinh(b*x + a)^2 - cosh(b*x + a)^2 + 2*(7*cosh(b*x + a)^13 - 6*co
sh(b*x + a)^11 - 15*cosh(b*x + a)^9 + 12*cosh(b*x + a)^7 + 9*cosh(b*x + a)^
5 - 6*cosh(b*x + a)^3 - cosh(b*x + a))*sinh(b*x + a) + 1)*log(cosh(b*x + a)
 + sinh(b*x + a) + 1) + 105*(cosh(b*x + a)^14 + 14*cosh(b*x + a)*sinh(b*x +
 a)^13 + sinh(b*x + a)^14 + (91*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^12 - cos
h(b*x + a)^12 + 4*(91*cosh(b*x + a)^3 - 3*cosh(b*x + a))*sinh(b*x + a)^11 +
 (1001*cosh(b*x + a)^4 - 66*cosh(b*x + a)^2 - 3)*sinh(b*x + a)^10 - 3*cosh(
b*x + a)^10 + 2*(1001*cosh(b*x + a)^5 - 110*cosh(b*x + a)^3 - 15*cosh(b*x +
```

```
 a))*sinh(b*x + a)^9 + 3*(1001*cosh(b*x + a)^6 - 165*cosh(b*x + a)^4 - 45*c
osh(b*x + a)^2 + 1)*sinh(b*x + a)^8 + 3*cosh(b*x + a)^8 + 24*(143*cosh(b*x
+ a)^7 - 33*cosh(b*x + a)^5 - 15*cosh(b*x + a)^3 + cosh(b*x + a))*sinh(b*x
+ a)^7 + 3*(1001*cosh(b*x + a)^8 - 308*cosh(b*x + a)^6 - 210*cosh(b*x + a)^
4 + 28*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^6 + 3*cosh(b*x + a)^6 + 2*(1001*c
osh(b*x + a)^9 - 396*cosh(b*x + a)^7 - 378*cosh(b*x + a)^5 + 84*cosh(b*x +
a)^3 + 9*cosh(b*x + a))*sinh(b*x + a)^5 + (1001*cosh(b*x + a)^10 - 495*cosh
(b*x + a)^8 - 630*cosh(b*x + a)^6 + 210*cosh(b*x + a)^4 + 45*cosh(b*x + a)^
2 - 3)*sinh(b*x + a)^4 - 3*cosh(b*x + a)^4 + 4*(91*cosh(b*x + a)^11 - 55*co
sh(b*x + a)^9 - 90*cosh(b*x + a)^7 + 42*cosh(b*x + a)^5 + 15*cosh(b*x + a)^
3 - 3*cosh(b*x + a))*sinh(b*x + a)^3 + (91*cosh(b*x + a)^12 - 66*cosh(b*x +
 a)^10 - 135*cosh(b*x + a)^8 + 84*cosh(b*x + a)^6 + 45*cosh(b*x + a)^4 - 18
*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^2 - cosh(b*x + a)^2 + 2*(7*cosh(b*x + a
)^13 - 6*cosh(b*x + a)^11 - 15*cosh(b*x + a)^9 + 12*cosh(b*x + a)^7 + 9*cos
h(b*x + a)^5 - 6*cosh(b*x + a)^3 - cosh(b*x + a))*sinh(b*x + a) + 1)*log(co
sh(b*x + a) + sinh(b*x + a) - 1) + 14*(195*cosh(b*x + a)^12 - 110*cosh(b*x
+ a)^10 - 423*cosh(b*x + a)^8 + 204*cosh(b*x + a)^6 - 235*cosh(b*x + a)^4 -
 30*cosh(b*x + a)^2 + 15)*sinh(b*x + a) + 210*cosh(b*x + a))/(b*cosh(b*x +
a)^14 + 14*b*cosh(b*x + a)*sinh(b*x + a)^13 + b*sinh(b*x + a)^14 - b*cosh(b
*x + a)^12 + (91*b*cosh(b*x + a)^2 - b)*sinh(b*x + a)^12 + 4*(91*b*cosh(b*x
 + a)^3 - 3*b*cosh(b*x + a))*sinh(b*x + a)^11 - 3*b*cosh(b*x + a)^10 + (100
1*b*cosh(b*x + a)^4 - 66*b*cosh(b*x + a)^2 - 3*b)*sinh(b*x + a)^10 + 2*(100
1*b*cosh(b*x + a)^5 - 110*b*cosh(b*x + a)^3 - 15*b*cosh(b*x + a))*sinh(b*x
+ a)^9 + 3*b*cosh(b*x + a)^8 + 3*(1001*b*cosh(b*x + a)^6 - 165*b*cosh(b*x +
 a)^4 - 45*b*cosh(b*x + a)^2 + b)*sinh(b*x + a)^8 + 24*(143*b*cosh(b*x + a)
^7 - 33*b*cosh(b*x + a)^5 - 15*b*cosh(b*x + a)^3 + b*cosh(b*x + a))*sinh(b*
x + a)^7 + 3*b*cosh(b*x + a)^6 + 3*(1001*b*cosh(b*x + a)^8 - 308*b*cosh(b*x
 + a)^6 - 210*b*cosh(b*x + a)^4 + 28*b*cosh(b*x + a)^2 + b)*sinh(b*x + a)^6
 + 2*(1001*b*cosh(b*x + a)^9 - 396*b*cosh(b*x + a)^7 - 378*b*cosh(b*x + a)^
5 + 84*b*cosh(b*x + a)^3 + 9*b*cosh(b*x + a))*sinh(b*x + a)^5 - 3*b*cosh(b*
x + a)^4 + (1001*b*cosh(b*x + a)^10 - 495*b*cosh(b*x + a)^8 - 630*b*cosh(b*
x + a)^6 + 210*b*cosh(b*x + a)^4 + 45*b*cosh(b*x + a)^2 - 3*b)*sinh(b*x + a
)^4 + 4*(91*b*cosh(b*x + a)^11 - 55*b*cosh(b*x + a)^9 - 90*b*cosh(b*x + a)^
7 + 42*b*cosh(b*x + a)^5 + 15*b*cosh(b*x + a)^3 - 3*b*cosh(b*x + a))*sinh(b
*x + a)^3 - b*cosh(b*x + a)^2 + (91*b*cosh(b*x + a)^12 - 66*b*cosh(b*x + a)
^10 - 135*b*cosh(b*x + a)^8 + 84*b*cosh(b*x + a)^6 + 45*b*cosh(b*x + a)^4 -
 18*b*cosh(b*x + a)^2 - b)*sinh(b*x + a)^2 + 2*(7*b*cosh(b*x + a)^13 - 6*b*
cosh(b*x + a)^11 - 15*b*cosh(b*x + a)^9 + 12*b*cosh(b*x + a)^7 + 9*b*cosh(b
*x + a)^5 - 6*b*cosh(b*x + a)^3 - b*cosh(b*x + a))*sinh(b*x + a) + b)
```

giac [A] time = 0.14, size = 152, normalized size = 1.71

$$\frac{\frac{12\left(11\left(e^{(bx+a)}+e^{(-bx-a)}\right)^{3}-52\,e^{(bx+a)}-52\,e^{(-bx-a)}\right)}{\left(\left(e^{(bx+a)}+e^{(-bx-a)}\right)^{2}-4\right)^{2}}+\frac{32\left(9\left(e^{(bx+a)}+e^{(-bx-a)}\right)^{2}+4\right)}{\left(e^{(bx+a)}+e^{(-bx-a)}\right)^{3}}-105\,\log\left(e^{(bx+a)}+e^{(-bx-a)}+2\right)+105\,\log\left(e^{(bx+}\right.}{48\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^5*sech(b*x+a)^4,x, algorithm="giac")
```

```
[Out] 1/48*(12*(11*(e^(b*x + a) + e^(-b*x - a))^3 - 52*e^(b*x + a) - 52*e^(-b*x -
 a))/((e^(b*x + a) + e^(-b*x - a))^2 - 4)^2 + 32*(9*(e^(b*x + a) + e^(-b*x
- a))^2 + 4)/(e^(b*x + a) + e^(-b*x - a))^3 - 105*log(e^(b*x + a) + e^(-b*x
 - a) + 2) + 105*log(e^(b*x + a) + e^(-b*x - a) - 2))/b
```

maple [A]   time = 0.22, size = 71, normalized size = 0.80

$$\frac{-\frac{1}{4\sinh(bx+a)^4\cosh(bx+a)^3}+\frac{7}{8\sinh(bx+a)^2\cosh(bx+a)^3}+\frac{35}{24\cosh(bx+a)^3}+\frac{35}{8\cosh(bx+a)}-\frac{35\,\mathrm{arctanh}\left(e^{bx+a}\right)}{4}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(b*x+a)^5*sech(b*x+a)^4,x)
```

```
[Out] 1/b*(-1/4/sinh(b*x+a)^4/cosh(b*x+a)^3+7/8/sinh(b*x+a)^2/cosh(b*x+a)^3+35/24
/cosh(b*x+a)^3+35/8/cosh(b*x+a)-35/4*arctanh(exp(b*x+a)))
```

maxima [B]   time = 0.34, size = 195, normalized size = 2.19

$$-\frac{35\,\log\left(e^{(-bx-a)}+1\right)}{8\,b}+\frac{35\,\log\left(e^{(-bx-a)}-1\right)}{8\,b}-\frac{105\,e^{(-bx-a)}-70\,e^{(-3\,bx-3\,a)}-329\,e^{(-5\,bx-5\,a)}+204\,e^{(-7\,bx-7\,a)}-329\,e}{12\,b\left(e^{(-2\,bx-2\,a)}+3\,e^{(-4\,bx-4\,a)}-3\,e^{(-6\,bx-6\,a)}-3\,e^{(-8\,bx-8\,a)}+3\,e^{(-1}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^5*sech(b*x+a)^4,x, algorithm="maxima")
```

```
[Out] -35/8*log(e^(-b*x - a) + 1)/b + 35/8*log(e^(-b*x - a) - 1)/b - 1/12*(105*e^
(-b*x - a) - 70*e^(-3*b*x - 3*a) - 329*e^(-5*b*x - 5*a) + 204*e^(-7*b*x - 7
*a) - 329*e^(-9*b*x - 9*a) - 70*e^(-11*b*x - 11*a) + 105*e^(-13*b*x - 13*a)
)/(b*(e^(-2*b*x - 2*a) + 3*e^(-4*b*x - 4*a) - 3*e^(-6*b*x - 6*a) - 3*e^(-8*
b*x - 8*a) + 3*e^(-10*b*x - 10*a) + e^(-12*b*x - 12*a) - e^(-14*b*x - 14*a)
 - 1))
```

mupad [B]   time = 0.10, size = 295, normalized size = 3.31

$$\frac{7\,\mathrm{e}^{a+b\,x}}{2\,b\,\left(\mathrm{e}^{4\,a+4\,b\,x}-2\,\mathrm{e}^{2\,a+2\,b\,x}+1\right)}-\frac{35\,\mathrm{atan}\left(\frac{\mathrm{e}^{b\,x}\,\mathrm{e}^{a}\,\sqrt{-b^2}}{b}\right)}{4\,\sqrt{-b^2}}+\frac{8\,\mathrm{e}^{a+b\,x}}{3\,b\,\left(2\,\mathrm{e}^{2\,a+2\,b\,x}+\mathrm{e}^{4\,a+4\,b\,x}+1\right)}-\frac{6\,\mathrm{e}^{a+b\,x}}{b\,\left(3\,\mathrm{e}^{2\,a+2\,b\,x}-3\,\mathrm{e}^{4\,a+4\,b\,x}+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(a + b*x)^4*sinh(a + b*x)^5),x)
```

```
[Out] (7*exp(a + b*x))/(2*b*(exp(4*a + 4*b*x) - 2*exp(2*a + 2*b*x) + 1)) - (35*at
an((exp(b*x)*exp(a)*(-b^2)^(1/2))/b))/(4*(-b^2)^(1/2)) + (8*exp(a + b*x))/(
3*b*(2*exp(2*a + 2*b*x) + exp(4*a + 4*b*x) + 1)) - (6*exp(a + b*x))/(b*(3*e
xp(2*a + 2*b*x) - 3*exp(4*a + 4*b*x) + exp(6*a + 6*b*x) - 1)) - (8*exp(a +
b*x))/(3*b*(3*exp(2*a + 2*b*x) + 3*exp(4*a + 4*b*x) + exp(6*a + 6*b*x) + 1)
) - (4*exp(a + b*x))/(b*(6*exp(4*a + 4*b*x) - 4*exp(2*a + 2*b*x) - 4*exp(6*
a + 6*b*x) + exp(8*a + 8*b*x) + 1)) + (11*exp(a + b*x))/(4*b*(exp(2*a + 2*b
*x) - 1)) + (6*exp(a + b*x))/(b*(exp(2*a + 2*b*x) + 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{csch}^5(a+bx)\operatorname{sech}^4(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)**5*sech(b*x+a)**4,x)
```

```
[Out] Integral(csch(a + b*x)**5*sech(a + b*x)**4, x)
```

3.48 $\int \operatorname{csch}^5(a+bx)\operatorname{sech}^5(a+bx)\,dx$

Optimal. Leaf size=69

$$\frac{\tanh^4(a+bx)}{4b}-\frac{2\tanh^2(a+bx)}{b}-\frac{\coth^4(a+bx)}{4b}+\frac{2\coth^2(a+bx)}{b}+\frac{6\log(\tanh(a+bx))}{b}$$

```
[Out] 2*coth(b*x+a)^2/b-1/4*coth(b*x+a)^4/b+6*ln(tanh(b*x+a))/b-2*tanh(b*x+a)^2/b
+1/4*tanh(b*x+a)^4/b
```

Rubi [A] time = 0.05, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {2620, 266, 43}

$$\frac{\tanh^4(a+bx)}{4b}-\frac{2\tanh^2(a+bx)}{b}-\frac{\coth^4(a+bx)}{4b}+\frac{2\coth^2(a+bx)}{b}+\frac{6\log(\tanh(a+bx))}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[a + b*x]^5*Sech[a + b*x]^5,x]
```

```
[Out] (2*Coth[a + b*x]^2)/b - Coth[a + b*x]^4/(4*b) + (6*Log[Tanh[a + b*x]])/b -
(2*Tanh[a + b*x]^2)/b + Tanh[a + b*x]^4/(4*b)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2620

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rubi steps

$$\int \operatorname{csch}^5(a+bx)\operatorname{sech}^5(a+bx)\,dx = \frac{\text{Subst}\left(\int \frac{(1+x^2)^4}{x^5}\,dx, x, i\tanh(a+bx)\right)}{b}$$
$$= \frac{\text{Subst}\left(\int \frac{(1+x)^4}{x^3}\,dx, x, -\tanh^2(a+bx)\right)}{2b}$$
$$= \frac{\text{Subst}\left(\int \left(4+\frac{1}{x^3}+\frac{4}{x^2}+\frac{6}{x}+x\right)dx, x, -\tanh^2(a+bx)\right)}{2b}$$
$$= \frac{2\coth^2(a+bx)}{b} - \frac{\coth^4(a+bx)}{4b} + \frac{6\log(\tanh(a+bx))}{b} - \frac{2\tanh^2(a+bx)}{b}$$

Mathematica [A] time = 0.03, size = 77, normalized size = 1.12

$$32\left(-\frac{\operatorname{csch}^4(a+bx)}{128b}+\frac{3\operatorname{csch}^2(a+bx)}{64b}+\frac{\operatorname{sech}^4(a+bx)}{128b}+\frac{3\operatorname{sech}^2(a+bx)}{64b}+\frac{3\log(\tanh(a+bx))}{16b}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[a + b*x]^5*Sech[a + b*x]^5,x]
```

```
[Out] 32*((3*Csch[a + b*x]^2)/(64*b) - Csch[a + b*x]^4/(128*b) + (3*Log[Tanh[a +
b*x]])/(16*b) + (3*Sech[a + b*x]^2)/(64*b) + Sech[a + b*x]^4/(128*b))
```

fricas [B] time = 0.48, size = 2231, normalized size = 32.33

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^5*sech(b*x+a)^5,x, algorithm="fricas")
```

```
[Out] 2*(6*cosh(b*x + a)^14 + 2184*cosh(b*x + a)^3*sinh(b*x + a)^11 + 546*cosh(b*
x + a)^2*sinh(b*x + a)^12 + 84*cosh(b*x + a)*sinh(b*x + a)^13 + 6*sinh(b*x
+ a)^14 + 22*(273*cosh(b*x + a)^4 - 1)*sinh(b*x + a)^10 - 22*cosh(b*x + a)^
10 + 44*(273*cosh(b*x + a)^5 - 5*cosh(b*x + a))*sinh(b*x + a)^9 + 198*(91*c
osh(b*x + a)^6 - 5*cosh(b*x + a)^2)*sinh(b*x + a)^8 + 528*(39*cosh(b*x + a)
^7 - 5*cosh(b*x + a)^3)*sinh(b*x + a)^7 + 22*(819*cosh(b*x + a)^8 - 210*cos
h(b*x + a)^4 - 1)*sinh(b*x + a)^6 - 22*cosh(b*x + a)^6 + 132*(91*cosh(b*x +
 a)^9 - 42*cosh(b*x + a)^5 - cosh(b*x + a))*sinh(b*x + a)^5 + 66*(91*cosh(b
*x + a)^10 - 70*cosh(b*x + a)^6 - 5*cosh(b*x + a)^2)*sinh(b*x + a)^4 + 8*(2
73*cosh(b*x + a)^11 - 330*cosh(b*x + a)^7 - 55*cosh(b*x + a)^3)*sinh(b*x +
```

```
a)^3 + 6*(91*cosh(b*x + a)^12 - 165*cosh(b*x + a)^8 - 55*cosh(b*x + a)^4 +
1)*sinh(b*x + a)^2 + 6*cosh(b*x + a)^2 - 3*(cosh(b*x + a)^16 + 560*cosh(b*x
 + a)^3*sinh(b*x + a)^13 + 120*cosh(b*x + a)^2*sinh(b*x + a)^14 + 16*cosh(b
*x + a)*sinh(b*x + a)^15 + sinh(b*x + a)^16 + 4*(455*cosh(b*x + a)^4 - 1)*s
inh(b*x + a)^12 - 4*cosh(b*x + a)^12 + 48*(91*cosh(b*x + a)^5 - cosh(b*x +
a))*sinh(b*x + a)^11 + 88*(91*cosh(b*x + a)^6 - 3*cosh(b*x + a)^2)*sinh(b*x
 + a)^10 + 880*(13*cosh(b*x + a)^7 - cosh(b*x + a)^3)*sinh(b*x + a)^9 + 6*(
2145*cosh(b*x + a)^8 - 330*cosh(b*x + a)^4 + 1)*sinh(b*x + a)^8 + 6*cosh(b*
x + a)^8 + 16*(715*cosh(b*x + a)^9 - 198*cosh(b*x + a)^5 + 3*cosh(b*x + a))
*sinh(b*x + a)^7 + 56*(143*cosh(b*x + a)^10 - 66*cosh(b*x + a)^6 + 3*cosh(b
*x + a)^2)*sinh(b*x + a)^6 + 48*(91*cosh(b*x + a)^11 - 66*cosh(b*x + a)^7 +
 7*cosh(b*x + a)^3)*sinh(b*x + a)^5 + 4*(455*cosh(b*x + a)^12 - 495*cosh(b*
x + a)^8 + 105*cosh(b*x + a)^4 - 1)*sinh(b*x + a)^4 - 4*cosh(b*x + a)^4 + 1
6*(35*cosh(b*x + a)^13 - 55*cosh(b*x + a)^9 + 21*cosh(b*x + a)^5 - cosh(b*x
 + a))*sinh(b*x + a)^3 + 24*(5*cosh(b*x + a)^14 - 11*cosh(b*x + a)^10 + 7*c
osh(b*x + a)^6 - cosh(b*x + a)^2)*sinh(b*x + a)^2 + 16*(cosh(b*x + a)^15 -
3*cosh(b*x + a)^11 + 3*cosh(b*x + a)^7 - cosh(b*x + a)^3)*sinh(b*x + a) + 1
)*log(2*cosh(b*x + a)/(cosh(b*x + a) - sinh(b*x + a))) + 3*(cosh(b*x + a)^1
6 + 560*cosh(b*x + a)^3*sinh(b*x + a)^13 + 120*cosh(b*x + a)^2*sinh(b*x + a
)^14 + 16*cosh(b*x + a)*sinh(b*x + a)^15 + sinh(b*x + a)^16 + 4*(455*cosh(b
*x + a)^4 - 1)*sinh(b*x + a)^12 - 4*cosh(b*x + a)^12 + 48*(91*cosh(b*x + a)
^5 - cosh(b*x + a))*sinh(b*x + a)^11 + 88*(91*cosh(b*x + a)^6 - 3*cosh(b*x
+ a)^2)*sinh(b*x + a)^10 + 880*(13*cosh(b*x + a)^7 - cosh(b*x + a)^3)*sinh(
b*x + a)^9 + 6*(2145*cosh(b*x + a)^8 - 330*cosh(b*x + a)^4 + 1)*sinh(b*x +
a)^8 + 6*cosh(b*x + a)^8 + 16*(715*cosh(b*x + a)^9 - 198*cosh(b*x + a)^5 +
3*cosh(b*x + a))*sinh(b*x + a)^7 + 56*(143*cosh(b*x + a)^10 - 66*cosh(b*x +
 a)^6 + 3*cosh(b*x + a)^2)*sinh(b*x + a)^6 + 48*(91*cosh(b*x + a)^11 - 66*c
osh(b*x + a)^7 + 7*cosh(b*x + a)^3)*sinh(b*x + a)^5 + 4*(455*cosh(b*x + a)^
12 - 495*cosh(b*x + a)^8 + 105*cosh(b*x + a)^4 - 1)*sinh(b*x + a)^4 - 4*cos
h(b*x + a)^4 + 16*(35*cosh(b*x + a)^13 - 55*cosh(b*x + a)^9 + 21*cosh(b*x +
 a)^5 - cosh(b*x + a))*sinh(b*x + a)^3 + 24*(5*cosh(b*x + a)^14 - 11*cosh(b
*x + a)^10 + 7*cosh(b*x + a)^6 - cosh(b*x + a)^2)*sinh(b*x + a)^2 + 16*(cos
h(b*x + a)^15 - 3*cosh(b*x + a)^11 + 3*cosh(b*x + a)^7 - cosh(b*x + a)^3)*s
inh(b*x + a) + 1)*log(2*sinh(b*x + a)/(cosh(b*x + a) - sinh(b*x + a))) + 4*
(21*cosh(b*x + a)^13 - 55*cosh(b*x + a)^9 - 33*cosh(b*x + a)^5 + 3*cosh(b*x
 + a))*sinh(b*x + a))/(b*cosh(b*x + a)^16 + 560*b*cosh(b*x + a)^3*sinh(b*x
+ a)^13 + 120*b*cosh(b*x + a)^2*sinh(b*x + a)^14 + 16*b*cosh(b*x + a)*sinh(
b*x + a)^15 + b*sinh(b*x + a)^16 - 4*b*cosh(b*x + a)^12 + 4*(455*b*cosh(b*x
 + a)^4 - b)*sinh(b*x + a)^12 + 48*(91*b*cosh(b*x + a)^5 - b*cosh(b*x + a))
*sinh(b*x + a)^11 + 88*(91*b*cosh(b*x + a)^6 - 3*b*cosh(b*x + a)^2)*sinh(b*
x + a)^10 + 880*(13*b*cosh(b*x + a)^7 - b*cosh(b*x + a)^3)*sinh(b*x + a)^9
+ 6*b*cosh(b*x + a)^8 + 6*(2145*b*cosh(b*x + a)^8 - 330*b*cosh(b*x + a)^4 +
 b)*sinh(b*x + a)^8 + 16*(715*b*cosh(b*x + a)^9 - 198*b*cosh(b*x + a)^5 + 3
*b*cosh(b*x + a))*sinh(b*x + a)^7 + 56*(143*b*cosh(b*x + a)^10 - 66*b*cosh(
b*x + a)^6 + 3*b*cosh(b*x + a)^2)*sinh(b*x + a)^6 + 48*(91*b*cosh(b*x + a)^
```

```
11 - 66*b*cosh(b*x + a)^7 + 7*b*cosh(b*x + a)^3)*sinh(b*x + a)^5 - 4*b*cosh
(b*x + a)^4 + 4*(455*b*cosh(b*x + a)^12 - 495*b*cosh(b*x + a)^8 + 105*b*cos
h(b*x + a)^4 - b)*sinh(b*x + a)^4 + 16*(35*b*cosh(b*x + a)^13 - 55*b*cosh(b
*x + a)^9 + 21*b*cosh(b*x + a)^5 - b*cosh(b*x + a))*sinh(b*x + a)^3 + 24*(5
*b*cosh(b*x + a)^14 - 11*b*cosh(b*x + a)^10 + 7*b*cosh(b*x + a)^6 - b*cosh(
b*x + a)^2)*sinh(b*x + a)^2 + 16*(b*cosh(b*x + a)^15 - 3*b*cosh(b*x + a)^11
 + 3*b*cosh(b*x + a)^7 - b*cosh(b*x + a)^3)*sinh(b*x + a) + b)
```

giac [A] time = 0.17, size = 124, normalized size = 1.80

$$\frac{\frac{4\left(3\left(e^{(2bx+2a)}+e^{(-2bx-2a)}\right)^3-20e^{(2bx+2a)}-20e^{(-2bx-2a)}\right)}{\left(\left(e^{(2bx+2a)}+e^{(-2bx-2a)}\right)^2-4\right)^2}-3\log\left(e^{(2bx+2a)}+e^{(-2bx-2a)}+2\right)+3\log\left(e^{(2bx+2a)}+e^{(-2bx-2a)}\right.}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^5*sech(b*x+a)^5,x, algorithm="giac")
```

```
[Out] (4*(3*(e^(2*b*x + 2*a) + e^(-2*b*x - 2*a))^3 - 20*e^(2*b*x + 2*a) - 20*e^(-
2*b*x - 2*a))/((e^(2*b*x + 2*a) + e^(-2*b*x - 2*a))^2 - 4)^2 - 3*log(e^(2*b
*x + 2*a) + e^(-2*b*x - 2*a) + 2) + 3*log(e^(2*b*x + 2*a) + e^(-2*b*x - 2*a
) - 2))/b
```

maple [A] time = 0.22, size = 81, normalized size = 1.17

$$-\frac{1}{4b\sinh(bx+a)^4\cosh(bx+a)^4}+\frac{1}{b\sinh(bx+a)^2\cosh(bx+a)^4}+\frac{3}{2b\cosh(bx+a)^4}+\frac{3}{b\cosh(bx+a)^2}+\frac{6\ln(}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(b*x+a)^5*sech(b*x+a)^5,x)
```

```
[Out] -1/4/b/sinh(b*x+a)^4/cosh(b*x+a)^4+1/b/sinh(b*x+a)^2/cosh(b*x+a)^4+3/2/b/co
sh(b*x+a)^4+3/b/cosh(b*x+a)^2+6*ln(tanh(b*x+a))/b
```

maxima [B] time = 0.90, size = 150, normalized size = 2.17

$$\frac{6\log\left(e^{(-bx-a)}+1\right)}{b}+\frac{6\log\left(e^{(-bx-a)}-1\right)}{b}-\frac{6\log\left(e^{(-2bx-2a)}+1\right)}{b}-\frac{4\left(3e^{(-2bx-2a)}-11e^{(-6bx-6a)}-11e^{(-10bx-10a)}\right.}{b\left(4e^{(-4bx-4a)}-6e^{(-8bx-8a)}+4e^{(-12bx-12a)}-\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^5*sech(b*x+a)^5,x, algorithm="maxima")
```

```
[Out] 6*log(e^(-b*x - a) + 1)/b + 6*log(e^(-b*x - a) - 1)/b - 6*log(e^(-2*b*x - 2
*a) + 1)/b - 4*(3*e^(-2*b*x - 2*a) - 11*e^(-6*b*x - 6*a) - 11*e^(-10*b*x -
```

```
10*a) + 3*e^(-14*b*x - 14*a))/(b*(4*e^(-4*b*x - 4*a) - 6*e^(-8*b*x - 8*a) +
 4*e^(-12*b*x - 12*a) - e^(-16*b*x - 16*a) - 1))
```

mupad [B] time = 1.42, size = 205, normalized size = 2.97

$$\frac{12\,\mathrm{e}^{2\,a+2\,b\,x}}{b\,\left(\mathrm{e}^{4\,a+4\,b\,x}-1\right)}-\frac{12\,\mathrm{atan}\left(\frac{\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}\,\sqrt{-b^2}}{b}\right)}{\sqrt{-b^2}}-\frac{8\,\mathrm{e}^{2\,a+2\,b\,x}}{b\,\left(\mathrm{e}^{8\,a+8\,b\,x}-2\,\mathrm{e}^{4\,a+4\,b\,x}+1\right)}-\frac{32\,\mathrm{e}^{2\,a+2\,b\,x}}{b\,\left(3\,\mathrm{e}^{4\,a+4\,b\,x}-3\,\mathrm{e}^{8\,a+8\,b\,x}+\mathrm{e}^{12\,a+12\,b\,x}-1\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(a + b*x)^5*sinh(a + b*x)^5),x)
```

```
[Out] (12*exp(2*a + 2*b*x))/(b*(exp(4*a + 4*b*x) - 1)) - (12*atan((exp(2*a)*exp(2
*b*x)*(-b^2)^(1/2))/b))/(-b^2)^(1/2) - (8*exp(2*a + 2*b*x))/(b*(exp(8*a + 8
*b*x) - 2*exp(4*a + 4*b*x) + 1)) - (32*exp(2*a + 2*b*x))/(b*(3*exp(4*a + 4*
b*x) - 3*exp(8*a + 8*b*x) + exp(12*a + 12*b*x) - 1)) - (64*exp(6*a + 6*b*x)
)/(b*(6*exp(8*a + 8*b*x) - 4*exp(4*a + 4*b*x) - 4*exp(12*a + 12*b*x) + exp(
16*a + 16*b*x) + 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{csch}^5(a+bx)\operatorname{sech}^5(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)**5*sech(b*x+a)**5,x)
```

```
[Out] Integral(csch(a + b*x)**5*sech(a + b*x)**5, x)
```

3.49 $\int \frac{\sinh^{\frac{7}{2}}(a+bx)}{\cosh^{\frac{7}{2}}(a+bx)}\,dx$

Optimal. Leaf size=106

$$-\frac{2\sinh^{\frac{5}{2}}(a+bx)}{5b\cosh^{\frac{5}{2}}(a+bx)}-\frac{2\sqrt{\sinh(a+bx)}}{b\sqrt{\cosh(a+bx)}}-\frac{\tan^{-1}\left(\frac{\sqrt{\cosh(a+bx)}}{\sqrt{\sinh(a+bx)}}\right)}{b}+\frac{\tanh^{-1}\left(\frac{\sqrt{\cosh(a+bx)}}{\sqrt{\sinh(a+bx)}}\right)}{b}$$

```
[Out] -arctan(cosh(b*x+a)^(1/2)/sinh(b*x+a)^(1/2))/b+arctanh(cosh(b*x+a)^(1/2)/si
nh(b*x+a)^(1/2))/b-2/5*sinh(b*x+a)^(5/2)/b/cosh(b*x+a)^(5/2)-2*sinh(b*x+a)^
(1/2)/b/cosh(b*x+a)^(1/2)
```

Rubi [A] time = 0.12, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {2566, 2575, 298, 203, 206}

$$-\frac{2\sinh^{\frac{5}{2}}(a+bx)}{5b\cosh^{\frac{5}{2}}(a+bx)}-\frac{2\sqrt{\sinh(a+bx)}}{b\sqrt{\cosh(a+bx)}}-\frac{\tan^{-1}\left(\frac{\sqrt{\cosh(a+bx)}}{\sqrt{\sinh(a+bx)}}\right)}{b}+\frac{\tanh^{-1}\left(\frac{\sqrt{\cosh(a+bx)}}{\sqrt{\sinh(a+bx)}}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[a + b*x]^(7/2)/Cosh[a + b*x]^(7/2),x]
```

```
[Out] -(ArcTan[Sqrt[Cosh[a + b*x]]/Sqrt[Sinh[a + b*x]]]/b) + ArcTanh[Sqrt[Cosh[a
+ b*x]]/Sqrt[Sinh[a + b*x]]]/b - (2*Sqrt[Sinh[a + b*x]])/(b*Sqrt[Cosh[a + b
*x]]) - (2*Sinh[a + b*x]^(5/2))/(5*b*Cosh[a + b*x]^(5/2))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
```

```
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 2566

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(a*Sin[e + f*x])^(m - 1)*(b*Cos[e + f*x])^(n + 1)
)/(b*f*(n + 1)), x] + Dist[(a^2*(m - 1))/(b^2*(n + 1)), Int[(a*Sin[e + f*x]
)^(m - 2)*(b*Cos[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ
[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

Rule 2575

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> With[{k = Denominator[m]}, -Dist[(k*a*b)/f, Subst[Int[x^(k
*(m + 1) - 1)/(a^2 + b^2*x^(2*k)), x], x, (a*Cos[e + f*x])^(1/k)/(b*Sin[e +
 f*x])^(1/k)], x]] /; FreeQ[{a, b, e, f}, x] && EqQ[m + n, 0] && GtQ[m, 0]
&& LtQ[m, 1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sinh^{\frac{7}{2}}(a+bx)}{\cosh^{\frac{7}{2}}(a+bx)}\,dx &= -\frac{2\sinh^{\frac{5}{2}}(a+bx)}{5b\cosh^{\frac{5}{2}}(a+bx)} + \int \frac{\sinh^{\frac{3}{2}}(a+bx)}{\cosh^{\frac{3}{2}}(a+bx)}\,dx \\
&= -\frac{2\sqrt{\sinh(a+bx)}}{b\sqrt{\cosh(a+bx)}} - \frac{2\sinh^{\frac{5}{2}}(a+bx)}{5b\cosh^{\frac{5}{2}}(a+bx)} + \int \frac{\sqrt{\cosh(a+bx)}}{\sqrt{\sinh(a+bx)}}\,dx \\
&= -\frac{2\sqrt{\sinh(a+bx)}}{b\sqrt{\cosh(a+bx)}} - \frac{2\sinh^{\frac{5}{2}}(a+bx)}{5b\cosh^{\frac{5}{2}}(a+bx)} + \frac{2\,\mathrm{Subst}\left(\int \frac{x^2}{1-x^4}\,dx, x, \frac{\sqrt{\cosh(a+bx)}}{\sqrt{\sinh(a+bx)}}\right)}{b} \\
&= -\frac{2\sqrt{\sinh(a+bx)}}{b\sqrt{\cosh(a+bx)}} - \frac{2\sinh^{\frac{5}{2}}(a+bx)}{5b\cosh^{\frac{5}{2}}(a+bx)} + \frac{\mathrm{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{\sqrt{\cosh(a+bx)}}{\sqrt{\sinh(a+bx)}}\right)}{b} - \frac{\mathrm{Subst}\left(\int \frac{1}{1+\dots}\right.}{} \\
&= -\frac{\tan^{-1}\left(\frac{\sqrt{\cosh(a+bx)}}{\sqrt{\sinh(a+bx)}}\right)}{b} + \frac{\tanh^{-1}\left(\frac{\sqrt{\cosh(a+bx)}}{\sqrt{\sinh(a+bx)}}\right)}{b} - \frac{2\sqrt{\sinh(a+bx)}}{b\sqrt{\cosh(a+bx)}} - \frac{2\sinh^{\frac{5}{2}}(a+bx)}{5b\cosh^{\frac{5}{2}}(a+bx)}
\end{aligned}$$

Mathematica [C] time = 0.06, size = 59, normalized size = 0.56

$$\frac{2\sinh^{\frac{9}{2}}(a+bx)\sqrt[4]{\cosh^2(a+bx)}\,{}_2F_1\left(\frac{9}{4},\frac{9}{4};\frac{13}{4};-\sinh^2(a+bx)\right)}{9b\sqrt{\cosh(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[a + b*x]^(7/2)/Cosh[a + b*x]^(7/2),x]
```

```
[Out] (2*(Cosh[a + b*x]^2)^(1/4)*Hypergeometric2F1[9/4, 9/4, 13/4, -Sinh[a + b*x]
^2]*Sinh[a + b*x]^(9/2))/(9*b*Sqrt[Cosh[a + b*x]])
```

fricas [B] time = 0.47, size = 997, normalized size = 9.41

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)^(7/2)/cosh(b*x+a)^(7/2),x, algorithm="fricas")
```

```
[Out] -1/10*(24*cosh(b*x + a)^6 + 144*cosh(b*x + a)*sinh(b*x + a)^5 + 24*sinh(b*x
 + a)^6 + 72*(5*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^4 + 72*cosh(b*x + a)^4 +
 96*(5*cosh(b*x + a)^3 + 3*cosh(b*x + a))*sinh(b*x + a)^3 + 72*(5*cosh(b*x
+ a)^4 + 6*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^2 - 10*(cosh(b*x + a)^6 + 6*c
osh(b*x + a)*sinh(b*x + a)^5 + sinh(b*x + a)^6 + 3*(5*cosh(b*x + a)^2 + 1)*
sinh(b*x + a)^4 + 3*cosh(b*x + a)^4 + 4*(5*cosh(b*x + a)^3 + 3*cosh(b*x + a
))*sinh(b*x + a)^3 + 3*(5*cosh(b*x + a)^4 + 6*cosh(b*x + a)^2 + 1)*sinh(b*x
 + a)^2 + 3*cosh(b*x + a)^2 + 6*(cosh(b*x + a)^5 + 2*cosh(b*x + a)^3 + cosh
(b*x + a))*sinh(b*x + a) + 1)*arctan(-cosh(b*x + a)^2 + 2*(cosh(b*x + a) +
sinh(b*x + a))*sqrt(cosh(b*x + a))*sqrt(sinh(b*x + a)) - 2*cosh(b*x + a)*si
nh(b*x + a) - sinh(b*x + a)^2) + 72*cosh(b*x + a)^2 + 5*(cosh(b*x + a)^6 +
6*cosh(b*x + a)*sinh(b*x + a)^5 + sinh(b*x + a)^6 + 3*(5*cosh(b*x + a)^2 +
1)*sinh(b*x + a)^4 + 3*cosh(b*x + a)^4 + 4*(5*cosh(b*x + a)^3 + 3*cosh(b*x
+ a))*sinh(b*x + a)^3 + 3*(5*cosh(b*x + a)^4 + 6*cosh(b*x + a)^2 + 1)*sinh(
b*x + a)^2 + 3*cosh(b*x + a)^2 + 6*(cosh(b*x + a)^5 + 2*cosh(b*x + a)^3 + c
osh(b*x + a))*sinh(b*x + a) + 1)*log(-cosh(b*x + a)^2 + 2*(cosh(b*x + a) +
sinh(b*x + a))*sqrt(cosh(b*x + a))*sqrt(sinh(b*x + a)) - 2*cosh(b*x + a)*si
nh(b*x + a) - sinh(b*x + a)^2) + 16*(3*cosh(b*x + a)^5 + 15*cosh(b*x + a)*s
inh(b*x + a)^4 + 3*sinh(b*x + a)^5 + 2*(15*cosh(b*x + a)^2 + 2)*sinh(b*x +
a)^3 + 4*cosh(b*x + a)^3 + 6*(5*cosh(b*x + a)^3 + 2*cosh(b*x + a))*sinh(b*x
 + a)^2 + 3*(5*cosh(b*x + a)^4 + 4*cosh(b*x + a)^2 + 1)*sinh(b*x + a) + 3*c
osh(b*x + a))*sqrt(cosh(b*x + a))*sqrt(sinh(b*x + a)) + 144*(cosh(b*x + a)^
5 + 2*cosh(b*x + a)^3 + cosh(b*x + a))*sinh(b*x + a) + 24)/(b*cosh(b*x + a)
^6 + 6*b*cosh(b*x + a)*sinh(b*x + a)^5 + b*sinh(b*x + a)^6 + 3*b*cosh(b*x +
 a)^4 + 3*(5*b*cosh(b*x + a)^2 + b)*sinh(b*x + a)^4 + 4*(5*b*cosh(b*x + a)^
```

```
3 + 3*b*cosh(b*x + a))*sinh(b*x + a)^3 + 3*b*cosh(b*x + a)^2 + 3*(5*b*cosh(
b*x + a)^4 + 6*b*cosh(b*x + a)^2 + b)*sinh(b*x + a)^2 + 6*(b*cosh(b*x + a)^
5 + 2*b*cosh(b*x + a)^3 + b*cosh(b*x + a))*sinh(b*x + a) + b)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh (bx+a)^{\frac{7}{2}}}{\cosh (bx+a)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)^(7/2)/cosh(b*x+a)^(7/2),x, algorithm="giac")
[Out] integrate(sinh(b*x + a)^(7/2)/cosh(b*x + a)^(7/2), x)
```

maple [F] time = 0.22, size = 0, normalized size = 0.00

$$\int \frac{\sinh^{\frac{7}{2}} (bx+a)}{\cosh (bx+a)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(b*x+a)^(7/2)/cosh(b*x+a)^(7/2),x)
[Out] int(sinh(b*x+a)^(7/2)/cosh(b*x+a)^(7/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh (bx+a)^{\frac{7}{2}}}{\cosh (bx+a)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)^(7/2)/cosh(b*x+a)^(7/2),x, algorithm="maxima")
[Out] integrate(sinh(b*x + a)^(7/2)/cosh(b*x + a)^(7/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\sinh \left(a+b\,x\right)}^{7/2}}{{\cosh \left(a+b\,x\right)}^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a + b*x)^(7/2)/cosh(a + b*x)^(7/2),x)
```

```
[Out] int(sinh(a + b*x)^(7/2)/cosh(a + b*x)^(7/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)**(7/2)/cosh(b*x+a)**(7/2),x)
```

```
[Out] Timed out
```

3.50 $\int \frac{\sinh^{\frac{5}{2}}(a+bx)}{\cosh^{\frac{5}{2}}(a+bx)}\,dx$

Optimal. Leaf size=81

$$-\frac{2\sinh^{\frac{3}{2}}(a+bx)}{3b\cosh^{\frac{3}{2}}(a+bx)}-\frac{\tan^{-1}\left(\frac{\sqrt{\sinh(a+bx)}}{\sqrt{\cosh(a+bx)}}\right)}{b}+\frac{\tanh^{-1}\left(\frac{\sqrt{\sinh(a+bx)}}{\sqrt{\cosh(a+bx)}}\right)}{b}$$

```
[Out] -arctan(sinh(b*x+a)^(1/2)/cosh(b*x+a)^(1/2))/b+arctanh(sinh(b*x+a)^(1/2)/co
sh(b*x+a)^(1/2))/b-2/3*sinh(b*x+a)^(3/2)/b/cosh(b*x+a)^(3/2)
```

Rubi [A] time = 0.08, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {2566, 2574, 298, 203, 206}

$$-\frac{2\sinh^{\frac{3}{2}}(a+bx)}{3b\cosh^{\frac{3}{2}}(a+bx)}-\frac{\tan^{-1}\left(\frac{\sqrt{\sinh(a+bx)}}{\sqrt{\cosh(a+bx)}}\right)}{b}+\frac{\tanh^{-1}\left(\frac{\sqrt{\sinh(a+bx)}}{\sqrt{\cosh(a+bx)}}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[a + b*x]^(5/2)/Cosh[a + b*x]^(5/2),x]
```

```
[Out] -(ArcTan[Sqrt[Sinh[a + b*x]]/Sqrt[Cosh[a + b*x]]]/b) + ArcTanh[Sqrt[Sinh[a
+ b*x]]/Sqrt[Cosh[a + b*x]]]/b - (2*Sinh[a + b*x]^(3/2))/(3*b*Cosh[a + b*x]
^(3/2))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
```

```
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 2566

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(a*Sin[e + f*x])^(m - 1)*(b*Cos[e + f*x])^(n + 1)
)/(b*f*(n + 1)), x] + Dist[(a^2*(m - 1))/(b^2*(n + 1)), Int[(a*Sin[e + f*x]
)^(m - 2)*(b*Cos[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ
[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

Rule 2574

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> With[{k = Denominator[m]}, Dist[(k*a*b)/f, Subst[Int[x^(k*
(m + 1) - 1)/(a^2 + b^2*x^(2*k)), x], x, (a*Sin[e + f*x])^(1/k)/(b*Cos[e +
f*x])^(1/k)], x]] /; FreeQ[{a, b, e, f}, x] && EqQ[m + n, 0] && GtQ[m, 0] &
& LtQ[m, 1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sinh^{\frac{5}{2}}(a+bx)}{\cosh^{\frac{5}{2}}(a+bx)}\,dx &= -\frac{2\sinh^{\frac{3}{2}}(a+bx)}{3b\cosh^{\frac{3}{2}}(a+bx)} + \int \frac{\sqrt{\sinh(a+bx)}}{\sqrt{\cosh(a+bx)}}\,dx \\
&= -\frac{2\sinh^{\frac{3}{2}}(a+bx)}{3b\cosh^{\frac{3}{2}}(a+bx)} - \frac{2\,\text{Subst}\left(\int \frac{x^2}{-1+x^4}\,dx, x, \frac{\sqrt{\sinh(a+bx)}}{\sqrt{\cosh(a+bx)}}\right)}{b} \\
&= -\frac{2\sinh^{\frac{3}{2}}(a+bx)}{3b\cosh^{\frac{3}{2}}(a+bx)} + \frac{\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{\sqrt{\sinh(a+bx)}}{\sqrt{\cosh(a+bx)}}\right)}{b} - \frac{\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \frac{\sqrt{\sinh(a+bx)}}{\sqrt{\cosh(a+bx)}}\right)}{b} \\
&= -\frac{\tan^{-1}\left(\frac{\sqrt{\sinh(a+bx)}}{\sqrt{\cosh(a+bx)}}\right)}{b} + \frac{\tanh^{-1}\left(\frac{\sqrt{\sinh(a+bx)}}{\sqrt{\cosh(a+bx)}}\right)}{b} - \frac{2\sinh^{\frac{3}{2}}(a+bx)}{3b\cosh^{\frac{3}{2}}(a+bx)}
\end{aligned}$$

Mathematica [C] time = 0.05, size = 59, normalized size = 0.73

$$\frac{2\sinh^{\frac{7}{2}}(a+bx)\cosh^2(a+bx)^{3/4}\,{}_2F_1\left(\frac{7}{4},\frac{7}{4};\frac{11}{4};-\sinh^2(a+bx)\right)}{7b\cosh^{\frac{3}{2}}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[a + b*x]^(5/2)/Cosh[a + b*x]^(5/2),x]
```

```
[Out] (2*(Cosh[a + b*x]^2)^(3/4)*Hypergeometric2F1[7/4, 7/4, 11/4, -Sinh[a + b*x]
^2]*Sinh[a + b*x]^(7/2))/(7*b*Cosh[a + b*x]^(3/2))
```

fricas [B] time = 0.45, size = 591, normalized size = 7.30

$$-\frac{4 \cosh(bx+a)^4 + 16 \cosh(bx+a)\sinh(bx+a)^3 + 4 \sinh(bx+a)^4 + 8\left(3 \cosh(bx+a)^2 + 1\right)\sinh(bx+a)^2}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)^(5/2)/cosh(b*x+a)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/6*(4*cosh(b*x + a)^4 + 16*cosh(b*x + a)*sinh(b*x + a)^3 + 4*sinh(b*x + a
)^4 + 8*(3*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^2 + 6*(cosh(b*x + a)^4 + 4*co
sh(b*x + a)*sinh(b*x + a)^3 + sinh(b*x + a)^4 + 2*(3*cosh(b*x + a)^2 + 1)*s
inh(b*x + a)^2 + 2*cosh(b*x + a)^2 + 4*(cosh(b*x + a)^3 + cosh(b*x + a))*si
nh(b*x + a) + 1)*arctan(-cosh(b*x + a)^2 + 2*(cosh(b*x + a) + sinh(b*x + a)
)*sqrt(cosh(b*x + a))*sqrt(sinh(b*x + a)) - 2*cosh(b*x + a)*sinh(b*x + a) -
 sinh(b*x + a)^2) + 8*cosh(b*x + a)^2 + 3*(cosh(b*x + a)^4 + 4*cosh(b*x + a
)*sinh(b*x + a)^3 + sinh(b*x + a)^4 + 2*(3*cosh(b*x + a)^2 + 1)*sinh(b*x +
a)^2 + 2*cosh(b*x + a)^2 + 4*(cosh(b*x + a)^3 + cosh(b*x + a))*sinh(b*x + a
) + 1)*log(-cosh(b*x + a)^2 + 2*(cosh(b*x + a) + sinh(b*x + a))*sqrt(cosh(b
*x + a))*sqrt(sinh(b*x + a)) - 2*cosh(b*x + a)*sinh(b*x + a) - sinh(b*x + a
)^2) + 8*(cosh(b*x + a)^3 + 3*cosh(b*x + a)*sinh(b*x + a)^2 + sinh(b*x + a)
^3 + (3*cosh(b*x + a)^2 - 1)*sinh(b*x + a) - cosh(b*x + a))*sqrt(cosh(b*x +
 a))*sqrt(sinh(b*x + a)) + 16*(cosh(b*x + a)^3 + cosh(b*x + a))*sinh(b*x +
a) + 4)/(b*cosh(b*x + a)^4 + 4*b*cosh(b*x + a)*sinh(b*x + a)^3 + b*sinh(b*x
 + a)^4 + 2*b*cosh(b*x + a)^2 + 2*(3*b*cosh(b*x + a)^2 + b)*sinh(b*x + a)^2
 + 4*(b*cosh(b*x + a)^3 + b*cosh(b*x + a))*sinh(b*x + a) + b)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh(bx+a)^{\frac{5}{2}}}{\cosh(bx+a)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)^(5/2)/cosh(b*x+a)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sinh(b*x + a)^(5/2)/cosh(b*x + a)^(5/2), x)
```

maple [F] time = 0.19, size = 0, normalized size = 0.00

$$\int \frac{\sinh^{\frac{5}{2}}(bx+a)}{\cosh(bx+a)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(b*x+a)^(5/2)/cosh(b*x+a)^(5/2),x)
```

```
[Out] int(sinh(b*x+a)^(5/2)/cosh(b*x+a)^(5/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh(bx+a)^{\frac{5}{2}}}{\cosh(bx+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)^(5/2)/cosh(b*x+a)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(sinh(b*x + a)^(5/2)/cosh(b*x + a)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sinh(a+b\,x)^{5/2}}{\cosh(a+b\,x)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a + b*x)^(5/2)/cosh(a + b*x)^(5/2),x)
```

```
[Out] int(sinh(a + b*x)^(5/2)/cosh(a + b*x)^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)**(5/2)/cosh(b*x+a)**(5/2),x)
```

```
[Out] Timed out
```

3.51 $\int \frac{\sinh^{\frac{3}{2}}(a+bx)}{\cosh^{\frac{3}{2}}(a+bx)}\,dx$

Optimal. Leaf size=79

$$-\frac{2\sqrt{\sinh(a+bx)}}{b\sqrt{\cosh(a+bx)}}-\frac{\tan^{-1}\left(\frac{\sqrt{\cosh(a+bx)}}{\sqrt{\sinh(a+bx)}}\right)}{b}+\frac{\tanh^{-1}\left(\frac{\sqrt{\cosh(a+bx)}}{\sqrt{\sinh(a+bx)}}\right)}{b}$$

```
[Out] -arctan(cosh(b*x+a)^(1/2)/sinh(b*x+a)^(1/2))/b+arctanh(cosh(b*x+a)^(1/2)/si
nh(b*x+a)^(1/2))/b-2*sinh(b*x+a)^(1/2)/b/cosh(b*x+a)^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {2566, 2575, 298, 203, 206}

$$-\frac{2\sqrt{\sinh(a+bx)}}{b\sqrt{\cosh(a+bx)}}-\frac{\tan^{-1}\left(\frac{\sqrt{\cosh(a+bx)}}{\sqrt{\sinh(a+bx)}}\right)}{b}+\frac{\tanh^{-1}\left(\frac{\sqrt{\cosh(a+bx)}}{\sqrt{\sinh(a+bx)}}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[a + b*x]^(3/2)/Cosh[a + b*x]^(3/2),x]
```

```
[Out] -(ArcTan[Sqrt[Cosh[a + b*x]]/Sqrt[Sinh[a + b*x]]]/b) + ArcTanh[Sqrt[Cosh[a
+ b*x]]/Sqrt[Sinh[a + b*x]]]/b - (2*Sqrt[Sinh[a + b*x]])/(b*Sqrt[Cosh[a + b
*x]])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
```

```
tQ[a/b, 0]
```

Rule 2566

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(a*Sin[e + f*x])^(m - 1)*(b*Cos[e + f*x])^(n + 1)
)/(b*f*(n + 1)), x] + Dist[(a^2*(m - 1))/(b^2*(n + 1)), Int[(a*Sin[e + f*x]
)^(m - 2)*(b*Cos[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ
[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

Rule 2575

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> With[{k = Denominator[m]}, -Dist[(k*a*b)/f, Subst[Int[x^(k
*(m + 1) - 1)/(a^2 + b^2*x^(2*k)), x], x, (a*Cos[e + f*x])^(1/k)/(b*Sin[e +
 f*x])^(1/k)], x]] /; FreeQ[{a, b, e, f}, x] && EqQ[m + n, 0] && GtQ[m, 0]
&& LtQ[m, 1]
```

Rubi steps

$$\begin{aligned}\int \frac{\sinh^{\frac{3}{2}}(a+bx)}{\cosh^{\frac{3}{2}}(a+bx)}\,dx &= -\frac{2\sqrt{\sinh(a+bx)}}{b\sqrt{\cosh(a+bx)}}+\int \frac{\sqrt{\cosh(a+bx)}}{\sqrt{\sinh(a+bx)}}\,dx\\ &= -\frac{2\sqrt{\sinh(a+bx)}}{b\sqrt{\cosh(a+bx)}}+\frac{2\,\text{Subst}\left(\int \frac{x^2}{1-x^4}\,dx,x,\frac{\sqrt{\cosh(a+bx)}}{\sqrt{\sinh(a+bx)}}\right)}{b}\\ &= -\frac{2\sqrt{\sinh(a+bx)}}{b\sqrt{\cosh(a+bx)}}+\frac{\text{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,\frac{\sqrt{\cosh(a+bx)}}{\sqrt{\sinh(a+bx)}}\right)}{b}-\frac{\text{Subst}\left(\int \frac{1}{1+x^2}\,dx,x,\frac{\sqrt{\cosh(a+bx)}}{\sqrt{\sinh(a+bx)}}\right)}{b}\\ &= -\frac{\tan^{-1}\left(\frac{\sqrt{\cosh(a+bx)}}{\sqrt{\sinh(a+bx)}}\right)}{b}+\frac{\tanh^{-1}\left(\frac{\sqrt{\cosh(a+bx)}}{\sqrt{\sinh(a+bx)}}\right)}{b}-\frac{2\sqrt{\sinh(a+bx)}}{b\sqrt{\cosh(a+bx)}}\end{aligned}$$

Mathematica [C] time = 0.05, size = 59, normalized size = 0.75

$$\frac{2\sinh^{\frac{5}{2}}(a+bx)\sqrt[4]{\cosh^2(a+bx)}\,{}_2F_1\left(\frac{5}{4},\frac{5}{4};\frac{9}{4};-\sinh^2(a+bx)\right)}{5b\sqrt{\cosh(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[a + b*x]^(3/2)/Cosh[a + b*x]^(3/2),x]
```

```
[Out] (2*(Cosh[a + b*x]^2)^(1/4)*Hypergeometric2F1[5/4, 5/4, 9/4, -Sinh[a + b*x]^
2]*Sinh[a + b*x]^(5/2))/(5*b*Sqrt[Cosh[a + b*x]])
```

fricas [B] time = 0.46, size = 310, normalized size = 3.92

$$\frac{2\left(\cosh\left(bx+a\right)^{2}+2\,\cosh\left(bx+a\right)\sinh\left(bx+a\right)+\sinh\left(bx+a\right)^{2}+1\right)\arctan\left(-\cosh\left(bx+a\right)^{2}+2\left(\cosh\left(bx+\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)^(3/2)/cosh(b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/2*(2*(cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 +
 1)*arctan(-cosh(b*x + a)^2 + 2*(cosh(b*x + a) + sinh(b*x + a))*sqrt(cosh(b
*x + a))*sqrt(sinh(b*x + a)) - 2*cosh(b*x + a)*sinh(b*x + a) - sinh(b*x + a
)^2) - 4*cosh(b*x + a)^2 - (cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a)
 + sinh(b*x + a)^2 + 1)*log(-cosh(b*x + a)^2 + 2*(cosh(b*x + a) + sinh(b*x
+ a))*sqrt(cosh(b*x + a))*sqrt(sinh(b*x + a)) - 2*cosh(b*x + a)*sinh(b*x +
a) - sinh(b*x + a)^2) - 8*(cosh(b*x + a) + sinh(b*x + a))*sqrt(cosh(b*x + a
))*sqrt(sinh(b*x + a)) - 8*cosh(b*x + a)*sinh(b*x + a) - 4*sinh(b*x + a)^2
- 4)/(b*cosh(b*x + a)^2 + 2*b*cosh(b*x + a)*sinh(b*x + a) + b*sinh(b*x + a)
^2 + b)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh\left(bx+a\right)^{\frac{3}{2}}}{\cosh\left(bx+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)^(3/2)/cosh(b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sinh(b*x + a)^(3/2)/cosh(b*x + a)^(3/2), x)
```

maple [F] time = 0.18, size = 0, normalized size = 0.00

$$\int \frac{\sinh^{\frac{3}{2}}\left(bx+a\right)}{\cosh\left(bx+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(b*x+a)^(3/2)/cosh(b*x+a)^(3/2),x)
```

```
[Out] int(sinh(b*x+a)^(3/2)/cosh(b*x+a)^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh(bx+a)^{\frac{3}{2}}}{\cosh(bx+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)^(3/2)/cosh(b*x+a)^(3/2),x, algorithm="maxima")
[Out] integrate(sinh(b*x + a)^(3/2)/cosh(b*x + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\sinh\left(a+b\,x\right)}^{3/2}}{{\cosh\left(a+b\,x\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a + b*x)^(3/2)/cosh(a + b*x)^(3/2),x)
[Out] int(sinh(a + b*x)^(3/2)/cosh(a + b*x)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh^{\frac{3}{2}}(a+bx)}{\cosh^{\frac{3}{2}}(a+bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)**(3/2)/cosh(b*x+a)**(3/2),x)
[Out] Integral(sinh(a + b*x)**(3/2)/cosh(a + b*x)**(3/2), x)
```

3.52 $\int \frac{\sqrt{\sinh(a+bx)}}{\sqrt{\cosh(a+bx)}}\,dx$

Optimal. Leaf size=54

$$\frac{\tanh^{-1}\left(\frac{\sqrt{\sinh(a+bx)}}{\sqrt{\cosh(a+bx)}}\right)}{b}-\frac{\tan^{-1}\left(\frac{\sqrt{\sinh(a+bx)}}{\sqrt{\cosh(a+bx)}}\right)}{b}$$

```
[Out] -arctan(sinh(b*x+a)^(1/2)/cosh(b*x+a)^(1/2))/b+arctanh(sinh(b*x+a)^(1/2)/co
sh(b*x+a)^(1/2))/b
```

Rubi [A] time = 0.04, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {2574, 298, 203, 206}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{\sinh(a+bx)}}{\sqrt{\cosh(a+bx)}}\right)}{b}-\frac{\tan^{-1}\left(\frac{\sqrt{\sinh(a+bx)}}{\sqrt{\cosh(a+bx)}}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Sinh[a + b*x]]/Sqrt[Cosh[a + b*x]],x]
```

```
[Out] -(ArcTan[Sqrt[Sinh[a + b*x]]/Sqrt[Cosh[a + b*x]]]/b) + ArcTanh[Sqrt[Sinh[a
+ b*x]]/Sqrt[Cosh[a + b*x]]]/b
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 2574

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> With[{k = Denominator[m]}, Dist[(k*a*b)/f, Subst[Int[x^(k*
(m + 1) - 1)/(a^2 + b^2*x^(2*k)), x], x, (a*Sin[e + f*x])^(1/k)/(b*Cos[e +
f*x])^(1/k)], x]] /; FreeQ[{a, b, e, f}, x] && EqQ[m + n, 0] && GtQ[m, 0] &
& LtQ[m, 1]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{\sinh(a+bx)}}{\sqrt{\cosh(a+bx)}}\,dx &= -\frac{2\,\mathrm{Subst}\left(\int \frac{x^2}{-1+x^4}\,dx, x, \frac{\sqrt{\sinh(a+bx)}}{\sqrt{\cosh(a+bx)}}\right)}{b}\\ &= \frac{\mathrm{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{\sqrt{\sinh(a+bx)}}{\sqrt{\cosh(a+bx)}}\right)}{b} - \frac{\mathrm{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \frac{\sqrt{\sinh(a+bx)}}{\sqrt{\cosh(a+bx)}}\right)}{b}\\ &= -\frac{\tan^{-1}\left(\frac{\sqrt{\sinh(a+bx)}}{\sqrt{\cosh(a+bx)}}\right)}{b} + \frac{\tanh^{-1}\left(\frac{\sqrt{\sinh(a+bx)}}{\sqrt{\cosh(a+bx)}}\right)}{b}\end{aligned}$$

Mathematica [C] time = 0.04, size = 59, normalized size = 1.09

$$\frac{2\sinh^{\frac{3}{2}}(a+bx)\cosh^2(a+bx)^{3/4}\,{}_2F_1\left(\frac{3}{4},\frac{3}{4};\frac{7}{4};-\sinh^2(a+bx)\right)}{3b\cosh^{\frac{3}{2}}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Sinh[a + b*x]]/Sqrt[Cosh[a + b*x]],x]
```

```
[Out] (2*(Cosh[a + b*x]^2)^(3/4)*Hypergeometric2F1[3/4, 3/4, 7/4, -Sinh[a + b*x]^
2]*Sinh[a + b*x]^(3/2))/(3*b*Cosh[a + b*x]^(3/2))
```

fricas [B] time = 0.42, size = 142, normalized size = 2.63

$$-\frac{2\arctan\left(-\cosh(bx+a)^2+2\left(\cosh(bx+a)+\sinh(bx+a)\right)\sqrt{\cosh(bx+a)}\,\sqrt{\sinh(bx+a)}\,-2\cosh(bx+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)^(1/2)/cosh(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/2*(2*arctan(-cosh(b*x + a)^2 + 2*(cosh(b*x + a) + sinh(b*x + a))*sqrt(co
sh(b*x + a))*sqrt(sinh(b*x + a)) - 2*cosh(b*x + a)*sinh(b*x + a) - sinh(b*x
```

```
 + a)^2) + log(-cosh(b*x + a)^2 + 2*(cosh(b*x + a) + sinh(b*x + a))*sqrt(co
sh(b*x + a))*sqrt(sinh(b*x + a)) - 2*cosh(b*x + a)*sinh(b*x + a) - sinh(b*x
 + a)^2))/b
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\sinh(bx+a)}}{\sqrt{\cosh(bx+a)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)^(1/2)/cosh(b*x+a)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(sinh(b*x + a))/sqrt(cosh(b*x + a)), x)
```

maple [F] time = 0.28, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\sinh}\,(bx+a)}{\sqrt{\cosh(bx+a)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(b*x+a)^(1/2)/cosh(b*x+a)^(1/2),x)
[Out] int(sinh(b*x+a)^(1/2)/cosh(b*x+a)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\sinh(bx+a)}}{\sqrt{\cosh(bx+a)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)^(1/2)/cosh(b*x+a)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(sinh(b*x + a))/sqrt(cosh(b*x + a)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{\sinh(a+b\,x)}}{\sqrt{\cosh(a+b\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a + b*x)^(1/2)/cosh(a + b*x)^(1/2),x)
[Out] int(sinh(a + b*x)^(1/2)/cosh(a + b*x)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\sinh(a+bx)}}{\sqrt{\cosh(a+bx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)**(1/2)/cosh(b*x+a)**(1/2),x)
```

```
[Out] Integral(sqrt(sinh(a + b*x))/sqrt(cosh(a + b*x)), x)
```

3.53 $\int \frac{\sqrt{\cosh(a+bx)}}{\sqrt{\sinh(a+bx)}}\,dx$

Optimal. Leaf size=54

$$\frac{\tanh^{-1}\left(\frac{\sqrt{\cosh(a+bx)}}{\sqrt{\sinh(a+bx)}}\right)}{b}-\frac{\tan^{-1}\left(\frac{\sqrt{\cosh(a+bx)}}{\sqrt{\sinh(a+bx)}}\right)}{b}$$

```
[Out] -arctan(cosh(b*x+a)^(1/2)/sinh(b*x+a)^(1/2))/b+arctanh(cosh(b*x+a)^(1/2)/si
nh(b*x+a)^(1/2))/b
```

Rubi [A] time = 0.04, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {2575, 298, 203, 206}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{\cosh(a+bx)}}{\sqrt{\sinh(a+bx)}}\right)}{b}-\frac{\tan^{-1}\left(\frac{\sqrt{\cosh(a+bx)}}{\sqrt{\sinh(a+bx)}}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cosh[a + b*x]]/Sqrt[Sinh[a + b*x]],x]
```

```
[Out] -(ArcTan[Sqrt[Cosh[a + b*x]]/Sqrt[Sinh[a + b*x]]]/b) + ArcTanh[Sqrt[Cosh[a
+ b*x]]/Sqrt[Sinh[a + b*x]]]/b
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 2575

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> With[{k = Denominator[m]}, -Dist[(k*a*b)/f, Subst[Int[x^(k
*(m + 1) - 1)/(a^2 + b^2*x^(2*k)), x], x, (a*Cos[e + f*x])^(1/k)/(b*Sin[e +
 f*x])^(1/k)], x]] /; FreeQ[{a, b, e, f}, x] && EqQ[m + n, 0] && GtQ[m, 0]
&& LtQ[m, 1]
```

Rubi steps

$$\int \frac{\sqrt{\cosh(a+bx)}}{\sqrt{\sinh(a+bx)}}\,dx = \frac{2\,\mathrm{Subst}\left(\int \frac{x^2}{1-x^4}\,dx, x, \frac{\sqrt{\cosh(a+bx)}}{\sqrt{\sinh(a+bx)}}\right)}{b}$$

$$= \frac{\mathrm{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{\sqrt{\cosh(a+bx)}}{\sqrt{\sinh(a+bx)}}\right)}{b} - \frac{\mathrm{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \frac{\sqrt{\cosh(a+bx)}}{\sqrt{\sinh(a+bx)}}\right)}{b}$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt{\cosh(a+bx)}}{\sqrt{\sinh(a+bx)}}\right)}{b} + \frac{\tanh^{-1}\left(\frac{\sqrt{\cosh(a+bx)}}{\sqrt{\sinh(a+bx)}}\right)}{b}$$

Mathematica [C] time = 0.02, size = 57, normalized size = 1.06

$$\frac{2\sqrt{\sinh(a+bx)}\,\sqrt[4]{\cosh^2(a+bx)}\,{}_2F_1\left(\frac{1}{4},\frac{1}{4};\frac{5}{4};-\sinh^2(a+bx)\right)}{b\sqrt{\cosh(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Cosh[a + b*x]]/Sqrt[Sinh[a + b*x]],x]
```

```
[Out] (2*(Cosh[a + b*x]^2)^(1/4)*Hypergeometric2F1[1/4, 1/4, 5/4, -Sinh[a + b*x]^
2]*Sqrt[Sinh[a + b*x]])/(b*Sqrt[Cosh[a + b*x]])
```

fricas [B] time = 0.42, size = 144, normalized size = 2.67

$$\frac{2\,\arctan\left(-\cosh\left(bx+a\right)^2 + 2\left(\cosh\left(bx+a\right)+\sinh\left(bx+a\right)\right)\sqrt{\cosh\left(bx+a\right)}\,\sqrt{\sinh\left(bx+a\right)} - 2\,\cosh\left(bx+a\right)\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^(1/2)/sinh(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*(2*arctan(-cosh(b*x + a)^2 + 2*(cosh(b*x + a) + sinh(b*x + a))*sqrt(cos
h(b*x + a))*sqrt(sinh(b*x + a)) - 2*cosh(b*x + a)*sinh(b*x + a) - sinh(b*x
```

```
+ a)^2) - log(-cosh(b*x + a)^2 + 2*(cosh(b*x + a) + sinh(b*x + a))*sqrt(cos
h(b*x + a))*sqrt(sinh(b*x + a)) - 2*cosh(b*x + a)*sinh(b*x + a) - sinh(b*x
+ a)^2))/b
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\cosh(bx+a)}}{\sqrt{\sinh(bx+a)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^(1/2)/sinh(b*x+a)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(cosh(b*x + a))/sqrt(sinh(b*x + a)), x)
```

maple [F] time = 0.24, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\cosh}\,(bx+a)}{\sqrt{\sinh(bx+a)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)^(1/2)/sinh(b*x+a)^(1/2),x)
[Out] int(cosh(b*x+a)^(1/2)/sinh(b*x+a)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\cosh(bx+a)}}{\sqrt{\sinh(bx+a)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^(1/2)/sinh(b*x+a)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(cosh(b*x + a))/sqrt(sinh(b*x + a)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{\cosh(a+b\,x)}}{\sqrt{\sinh(a+b\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a + b*x)^(1/2)/sinh(a + b*x)^(1/2),x)
[Out] int(cosh(a + b*x)^(1/2)/sinh(a + b*x)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\cosh\left(a+bx\right)}}{\sqrt{\sinh\left(a+bx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)**(1/2)/sinh(b*x+a)**(1/2),x)
```

```
[Out] Integral(sqrt(cosh(a + b*x))/sqrt(sinh(a + b*x)), x)
```

3.54 $\int \frac{\cosh^{\frac{3}{2}}(a+bx)}{\sinh^{\frac{3}{2}}(a+bx)}\,dx$

Optimal. Leaf size=79

$$-\frac{2\sqrt{\cosh(a+bx)}}{b\sqrt{\sinh(a+bx)}}-\frac{\tan^{-1}\left(\frac{\sqrt{\sinh(a+bx)}}{\sqrt{\cosh(a+bx)}}\right)}{b}+\frac{\tanh^{-1}\left(\frac{\sqrt{\sinh(a+bx)}}{\sqrt{\cosh(a+bx)}}\right)}{b}$$

```
[Out] -arctan(sinh(b*x+a)^(1/2)/cosh(b*x+a)^(1/2))/b+arctanh(sinh(b*x+a)^(1/2)/co
sh(b*x+a)^(1/2))/b-2*cosh(b*x+a)^(1/2)/b/sinh(b*x+a)^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {2567, 2574, 298, 203, 206}

$$-\frac{2\sqrt{\cosh(a+bx)}}{b\sqrt{\sinh(a+bx)}}-\frac{\tan^{-1}\left(\frac{\sqrt{\sinh(a+bx)}}{\sqrt{\cosh(a+bx)}}\right)}{b}+\frac{\tanh^{-1}\left(\frac{\sqrt{\sinh(a+bx)}}{\sqrt{\cosh(a+bx)}}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[a + b*x]^(3/2)/Sinh[a + b*x]^(3/2),x]
```

```
[Out] -(ArcTan[Sqrt[Sinh[a + b*x]]/Sqrt[Cosh[a + b*x]]]/b) + ArcTanh[Sqrt[Sinh[a
+ b*x]]/Sqrt[Cosh[a + b*x]]]/b - (2*Sqrt[Cosh[a + b*x]])/(b*Sqrt[Sinh[a + b
*x]])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
```

```
tQ[a/b, 0]
```

Rule 2567

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(a*(a*Cos[e + f*x])^(m - 1)*(b*Sin[e + f*x])^(n + 1))
/(b*f*(n + 1)), x] + Dist[(a^2*(m - 1))/(b^2*(n + 1)), Int[(a*Cos[e + f*x])
^(m - 2)*(b*Sin[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ[
m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

Rule 2574

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> With[{k = Denominator[m]}, Dist[(k*a*b)/f, Subst[Int[x^(k*
(m + 1) - 1)/(a^2 + b^2*x^(2*k)), x], x, (a*Sin[e + f*x])^(1/k)/(b*Cos[e +
f*x])^(1/k)], x]] /; FreeQ[{a, b, e, f}, x] && EqQ[m + n, 0] && GtQ[m, 0] &
& LtQ[m, 1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cosh^{\frac{3}{2}}(a+bx)}{\sinh^{\frac{3}{2}}(a+bx)}\,dx &= -\frac{2\sqrt{\cosh(a+bx)}}{b\sqrt{\sinh(a+bx)}} + \int \frac{\sqrt{\sinh(a+bx)}}{\sqrt{\cosh(a+bx)}}\,dx \\
&= -\frac{2\sqrt{\cosh(a+bx)}}{b\sqrt{\sinh(a+bx)}} - \frac{2\,\text{Subst}\left(\int \frac{x^2}{-1+x^4}\,dx, x, \frac{\sqrt{\sinh(a+bx)}}{\sqrt{\cosh(a+bx)}}\right)}{b} \\
&= -\frac{2\sqrt{\cosh(a+bx)}}{b\sqrt{\sinh(a+bx)}} + \frac{\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{\sqrt{\sinh(a+bx)}}{\sqrt{\cosh(a+bx)}}\right)}{b} - \frac{\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \frac{\sqrt{\sinh(a+bx)}}{\sqrt{\cosh(a+bx)}}\right)}{b} \\
&= -\frac{\tan^{-1}\left(\frac{\sqrt{\sinh(a+bx)}}{\sqrt{\cosh(a+bx)}}\right)}{b} + \frac{\tanh^{-1}\left(\frac{\sqrt{\sinh(a+bx)}}{\sqrt{\cosh(a+bx)}}\right)}{b} - \frac{2\sqrt{\cosh(a+bx)}}{b\sqrt{\sinh(a+bx)}}
\end{aligned}
$$

Mathematica [C] time = 0.03, size = 57, normalized size = 0.72

$$
-\frac{2\cosh^2(a+bx)^{3/4}\,{}_2F_1\left(-\frac{1}{4},-\frac{1}{4};\frac{3}{4};-\sinh^2(a+bx)\right)}{b\sqrt{\sinh(a+bx)}\,\cosh^{\frac{3}{2}}(a+bx)}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[a + b*x]^(3/2)/Sinh[a + b*x]^(3/2),x]
```

```
[Out] (-2*(Cosh[a + b*x]^2)^(3/4)*Hypergeometric2F1[-1/4, -1/4, 3/4, -Sinh[a + b*
x]^2])/(b*Cosh[a + b*x]^(3/2)*Sqrt[Sinh[a + b*x]])
```

fricas [B] time = 0.42, size = 311, normalized size = 3.94

$$-\frac{2\left(\cosh\left(bx+a\right)^{2}+2\,\cosh\left(bx+a\right)\sinh\left(bx+a\right)+\sinh\left(bx+a\right)^{2}-1\right)\arctan\left(-\cosh\left(bx+a\right)^{2}+2\left(\cosh\left(bx+\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^(3/2)/sinh(b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/2*(2*(cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2
- 1)*arctan(-cosh(b*x + a)^2 + 2*(cosh(b*x + a) + sinh(b*x + a))*sqrt(cosh(
b*x + a))*sqrt(sinh(b*x + a)) - 2*cosh(b*x + a)*sinh(b*x + a) - sinh(b*x +
a)^2) + 4*cosh(b*x + a)^2 + (cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a
) + sinh(b*x + a)^2 - 1)*log(-cosh(b*x + a)^2 + 2*(cosh(b*x + a) + sinh(b*x
 + a))*sqrt(cosh(b*x + a))*sqrt(sinh(b*x + a)) - 2*cosh(b*x + a)*sinh(b*x +
 a) - sinh(b*x + a)^2) + 8*(cosh(b*x + a) + sinh(b*x + a))*sqrt(cosh(b*x +
a))*sqrt(sinh(b*x + a)) + 8*cosh(b*x + a)*sinh(b*x + a) + 4*sinh(b*x + a)^2
 - 4)/(b*cosh(b*x + a)^2 + 2*b*cosh(b*x + a)*sinh(b*x + a) + b*sinh(b*x + a
)^2 - b)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh\left(bx+a\right)^{\frac{3}{2}}}{\sinh\left(bx+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^(3/2)/sinh(b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(cosh(b*x + a)^(3/2)/sinh(b*x + a)^(3/2), x)
```

maple [F] time = 0.18, size = 0, normalized size = 0.00

$$\int \frac{\cosh^{\frac{3}{2}}\left(bx+a\right)}{\sinh\left(bx+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)^(3/2)/sinh(b*x+a)^(3/2),x)
```

```
[Out] int(cosh(b*x+a)^(3/2)/sinh(b*x+a)^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh(bx+a)^{\frac{3}{2}}}{\sinh(bx+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^(3/2)/sinh(b*x+a)^(3/2),x, algorithm="maxima")
[Out] integrate(cosh(b*x + a)^(3/2)/sinh(b*x + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cosh(a+b\,x)^{3/2}}{\sinh(a+b\,x)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a + b*x)^(3/2)/sinh(a + b*x)^(3/2),x)
[Out] int(cosh(a + b*x)^(3/2)/sinh(a + b*x)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh^{\frac{3}{2}}(a+bx)}{\sinh^{\frac{3}{2}}(a+bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)**(3/2)/sinh(b*x+a)**(3/2),x)
[Out] Integral(cosh(a + b*x)**(3/2)/sinh(a + b*x)**(3/2), x)
```

3.55 $\int \frac{\cosh^{\frac{5}{2}}(a+bx)}{\sinh^{\frac{5}{2}}(a+bx)}\,dx$

Optimal. Leaf size=81

$$-\frac{2\cosh^{\frac{3}{2}}(a+bx)}{3b\sinh^{\frac{3}{2}}(a+bx)}-\frac{\tan^{-1}\left(\frac{\sqrt{\cosh(a+bx)}}{\sqrt{\sinh(a+bx)}}\right)}{b}+\frac{\tanh^{-1}\left(\frac{\sqrt{\cosh(a+bx)}}{\sqrt{\sinh(a+bx)}}\right)}{b}$$

```
[Out] -arctan(cosh(b*x+a)^(1/2)/sinh(b*x+a)^(1/2))/b+arctanh(cosh(b*x+a)^(1/2)/si
nh(b*x+a)^(1/2))/b-2/3*cosh(b*x+a)^(3/2)/b/sinh(b*x+a)^(3/2)
```

Rubi [A] time = 0.07, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {2567, 2575, 298, 203, 206}

$$-\frac{2\cosh^{\frac{3}{2}}(a+bx)}{3b\sinh^{\frac{3}{2}}(a+bx)}-\frac{\tan^{-1}\left(\frac{\sqrt{\cosh(a+bx)}}{\sqrt{\sinh(a+bx)}}\right)}{b}+\frac{\tanh^{-1}\left(\frac{\sqrt{\cosh(a+bx)}}{\sqrt{\sinh(a+bx)}}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[a + b*x]^(5/2)/Sinh[a + b*x]^(5/2),x]
```

```
[Out] -(ArcTan[Sqrt[Cosh[a + b*x]]/Sqrt[Sinh[a + b*x]]]/b) + ArcTanh[Sqrt[Cosh[a
 + b*x]]/Sqrt[Sinh[a + b*x]]]/b - (2*Cosh[a + b*x]^(3/2))/(3*b*Sinh[a + b*x]
^(3/2))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
```

```
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 2567

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(a*(a*Cos[e + f*x])^(m - 1)*(b*Sin[e + f*x])^(n + 1))
/(b*f*(n + 1)), x] + Dist[(a^2*(m - 1))/(b^2*(n + 1)), Int[(a*Cos[e + f*x])
^(m - 2)*(b*Sin[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ[
m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

Rule 2575

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> With[{k = Denominator[m]}, -Dist[(k*a*b)/f, Subst[Int[x^(k
*(m + 1) - 1)/(a^2 + b^2*x^(2*k)), x], x, (a*Cos[e + f*x])^(1/k)/(b*Sin[e +
 f*x])^(1/k)], x]] /; FreeQ[{a, b, e, f}, x] && EqQ[m + n, 0] && GtQ[m, 0]
&& LtQ[m, 1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cosh^{\frac{5}{2}}(a+bx)}{\sinh^{\frac{5}{2}}(a+bx)}\,dx &= -\frac{2\cosh^{\frac{3}{2}}(a+bx)}{3b\sinh^{\frac{3}{2}}(a+bx)} + \int \frac{\sqrt{\cosh(a+bx)}}{\sqrt{\sinh(a+bx)}}\,dx \\
&= -\frac{2\cosh^{\frac{3}{2}}(a+bx)}{3b\sinh^{\frac{3}{2}}(a+bx)} + \frac{2\,\text{Subst}\left(\int \frac{x^2}{1-x^4}\,dx, x, \frac{\sqrt{\cosh(a+bx)}}{\sqrt{\sinh(a+bx)}}\right)}{b} \\
&= -\frac{2\cosh^{\frac{3}{2}}(a+bx)}{3b\sinh^{\frac{3}{2}}(a+bx)} + \frac{\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{\sqrt{\cosh(a+bx)}}{\sqrt{\sinh(a+bx)}}\right)}{b} - \frac{\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \frac{\sqrt{\cosh(a+bx)}}{\sqrt{\sinh(a+bx)}}\right)}{b} \\
&= -\frac{\tan^{-1}\left(\frac{\sqrt{\cosh(a+bx)}}{\sqrt{\sinh(a+bx)}}\right)}{b} + \frac{\tanh^{-1}\left(\frac{\sqrt{\cosh(a+bx)}}{\sqrt{\sinh(a+bx)}}\right)}{b} - \frac{2\cosh^{\frac{3}{2}}(a+bx)}{3b\sinh^{\frac{3}{2}}(a+bx)}
\end{aligned}
$$

Mathematica [C] time = 0.03, size = 59, normalized size = 0.73

$$
-\frac{2\sqrt[4]{\cosh^2(a+bx)}\,{}_2F_1\left(-\frac{3}{4},-\frac{3}{4};\frac{1}{4};-\sinh^2(a+bx)\right)}{3b\sinh^{\frac{3}{2}}(a+bx)\sqrt{\cosh(a+bx)}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[a + b*x]^(5/2)/Sinh[a + b*x]^(5/2),x]
```

```
[Out] (-2*(Cosh[a + b*x]^2)^(1/4)*Hypergeometric2F1[-3/4, -3/4, 1/4, -Sinh[a + b*
x]^2])/(3*b*Sqrt[Cosh[a + b*x]]*Sinh[a + b*x]^(3/2))
```

fricas [B] time = 0.43, size = 598, normalized size = 7.38

$$-\frac{4 \cosh(bx+a)^4 + 16 \cosh(bx+a)\sinh(bx+a)^3 + 4 \sinh(bx+a)^4 + 8\left(3 \cosh(bx+a)^2 - 1\right)\sinh(bx+a)^2}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^(5/2)/sinh(b*x+a)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/6*(4*cosh(b*x + a)^4 + 16*cosh(b*x + a)*sinh(b*x + a)^3 + 4*sinh(b*x + a
)^4 + 8*(3*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^2 - 6*(cosh(b*x + a)^4 + 4*co
sh(b*x + a)*sinh(b*x + a)^3 + sinh(b*x + a)^4 + 2*(3*cosh(b*x + a)^2 - 1)*s
inh(b*x + a)^2 - 2*cosh(b*x + a)^2 + 4*(cosh(b*x + a)^3 - cosh(b*x + a))*si
nh(b*x + a) + 1)*arctan(-cosh(b*x + a)^2 + 2*(cosh(b*x + a) + sinh(b*x + a)
)*sqrt(cosh(b*x + a))*sqrt(sinh(b*x + a)) - 2*cosh(b*x + a)*sinh(b*x + a) -
 sinh(b*x + a)^2) - 8*cosh(b*x + a)^2 + 3*(cosh(b*x + a)^4 + 4*cosh(b*x + a
)*sinh(b*x + a)^3 + sinh(b*x + a)^4 + 2*(3*cosh(b*x + a)^2 - 1)*sinh(b*x +
a)^2 - 2*cosh(b*x + a)^2 + 4*(cosh(b*x + a)^3 - cosh(b*x + a))*sinh(b*x + a
) + 1)*log(-cosh(b*x + a)^2 + 2*(cosh(b*x + a) + sinh(b*x + a))*sqrt(cosh(b
*x + a))*sqrt(sinh(b*x + a)) - 2*cosh(b*x + a)*sinh(b*x + a) - sinh(b*x + a
)^2) + 8*(cosh(b*x + a)^3 + 3*cosh(b*x + a)*sinh(b*x + a)^2 + sinh(b*x + a)
^3 + (3*cosh(b*x + a)^2 + 1)*sinh(b*x + a) + cosh(b*x + a))*sqrt(cosh(b*x +
 a))*sqrt(sinh(b*x + a)) + 16*(cosh(b*x + a)^3 - cosh(b*x + a))*sinh(b*x +
a) + 4)/(b*cosh(b*x + a)^4 + 4*b*cosh(b*x + a)*sinh(b*x + a)^3 + b*sinh(b*x
 + a)^4 - 2*b*cosh(b*x + a)^2 + 2*(3*b*cosh(b*x + a)^2 - b)*sinh(b*x + a)^2
 + 4*(b*cosh(b*x + a)^3 - b*cosh(b*x + a))*sinh(b*x + a) + b)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh(bx+a)^{\frac{5}{2}}}{\sinh(bx+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^(5/2)/sinh(b*x+a)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(cosh(b*x + a)^(5/2)/sinh(b*x + a)^(5/2), x)
```

maple [F] time = 0.18, size = 0, normalized size = 0.00

$$\int \frac{\cosh^{\frac{5}{2}}(bx+a)}{\sinh(bx+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)^(5/2)/sinh(b*x+a)^(5/2),x)
[Out] int(cosh(b*x+a)^(5/2)/sinh(b*x+a)^(5/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh(bx+a)^{\frac{5}{2}}}{\sinh(bx+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^(5/2)/sinh(b*x+a)^(5/2),x, algorithm="maxima")
[Out] integrate(cosh(b*x + a)^(5/2)/sinh(b*x + a)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cosh(a+b\,x)^{5/2}}{\sinh(a+b\,x)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a + b*x)^(5/2)/sinh(a + b*x)^(5/2),x)
[Out] int(cosh(a + b*x)^(5/2)/sinh(a + b*x)^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)**(5/2)/sinh(b*x+a)**(5/2),x)
[Out] Timed out
```

3.56 $\int \frac{\cosh^{\frac{7}{2}}(a+bx)}{\sinh^{\frac{7}{2}}(a+bx)}\,dx$

Optimal. Leaf size=106

$$-\frac{2\cosh^{\frac{5}{2}}(a+bx)}{5b\sinh^{\frac{5}{2}}(a+bx)}-\frac{2\sqrt{\cosh(a+bx)}}{b\sqrt{\sinh(a+bx)}}-\frac{\tan^{-1}\left(\frac{\sqrt{\sinh(a+bx)}}{\sqrt{\cosh(a+bx)}}\right)}{b}+\frac{\tanh^{-1}\left(\frac{\sqrt{\sinh(a+bx)}}{\sqrt{\cosh(a+bx)}}\right)}{b}$$

```
[Out] -arctan(sinh(b*x+a)^(1/2)/cosh(b*x+a)^(1/2))/b+arctanh(sinh(b*x+a)^(1/2)/co
sh(b*x+a)^(1/2))/b-2/5*cosh(b*x+a)^(5/2)/b/sinh(b*x+a)^(5/2)-2*cosh(b*x+a)^
(1/2)/b/sinh(b*x+a)^(1/2)
```

Rubi [A] time = 0.11, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {2567, 2574, 298, 203, 206}

$$-\frac{2\cosh^{\frac{5}{2}}(a+bx)}{5b\sinh^{\frac{5}{2}}(a+bx)}-\frac{2\sqrt{\cosh(a+bx)}}{b\sqrt{\sinh(a+bx)}}-\frac{\tan^{-1}\left(\frac{\sqrt{\sinh(a+bx)}}{\sqrt{\cosh(a+bx)}}\right)}{b}+\frac{\tanh^{-1}\left(\frac{\sqrt{\sinh(a+bx)}}{\sqrt{\cosh(a+bx)}}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[a + b*x]^(7/2)/Sinh[a + b*x]^(7/2),x]
```

```
[Out] -(ArcTan[Sqrt[Sinh[a + b*x]]/Sqrt[Cosh[a + b*x]]]/b) + ArcTanh[Sqrt[Sinh[a
+ b*x]]/Sqrt[Cosh[a + b*x]]]/b - (2*Cosh[a + b*x]^(5/2))/(5*b*Sinh[a + b*x]
^(5/2)) - (2*Sqrt[Cosh[a + b*x]])/(b*Sqrt[Sinh[a + b*x]])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
```

```
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 2567

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(a*(a*Cos[e + f*x])^(m - 1)*(b*Sin[e + f*x])^(n + 1))
/(b*f*(n + 1)), x] + Dist[(a^2*(m - 1))/(b^2*(n + 1)), Int[(a*Cos[e + f*x])
^(m - 2)*(b*Sin[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ[
m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

Rule 2574

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> With[{k = Denominator[m]}, Dist[(k*a*b)/f, Subst[Int[x^(k*
(m + 1) - 1)/(a^2 + b^2*x^(2*k)), x], x, (a*Sin[e + f*x])^(1/k)/(b*Cos[e +
f*x])^(1/k)], x]] /; FreeQ[{a, b, e, f}, x] && EqQ[m + n, 0] && GtQ[m, 0] &
& LtQ[m, 1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cosh^{\frac{7}{2}}(a+bx)}{\sinh^{\frac{7}{2}}(a+bx)}\,dx &= -\frac{2\cosh^{\frac{5}{2}}(a+bx)}{5b\sinh^{\frac{5}{2}}(a+bx)}+\int \frac{\cosh^{\frac{3}{2}}(a+bx)}{\sinh^{\frac{3}{2}}(a+bx)}\,dx\\
&= -\frac{2\cosh^{\frac{5}{2}}(a+bx)}{5b\sinh^{\frac{5}{2}}(a+bx)}-\frac{2\sqrt{\cosh(a+bx)}}{b\sqrt{\sinh(a+bx)}}+\int \frac{\sqrt{\sinh(a+bx)}}{\sqrt{\cosh(a+bx)}}\,dx\\
&= -\frac{2\cosh^{\frac{5}{2}}(a+bx)}{5b\sinh^{\frac{5}{2}}(a+bx)}-\frac{2\sqrt{\cosh(a+bx)}}{b\sqrt{\sinh(a+bx)}}-\frac{2\,\text{Subst}\left(\int \frac{x^2}{-1+x^4}\,dx,x,\frac{\sqrt{\sinh(a+bx)}}{\sqrt{\cosh(a+bx)}}\right)}{b}\\
&= -\frac{2\cosh^{\frac{5}{2}}(a+bx)}{5b\sinh^{\frac{5}{2}}(a+bx)}-\frac{2\sqrt{\cosh(a+bx)}}{b\sqrt{\sinh(a+bx)}}+\frac{\text{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,\frac{\sqrt{\sinh(a+bx)}}{\sqrt{\cosh(a+bx)}}\right)}{b}-\frac{\text{Subst}\left(\int \frac{1}{1\ldots}\right.}{\ldots}\\
&= -\frac{\tan^{-1}\left(\frac{\sqrt{\sinh(a+bx)}}{\sqrt{\cosh(a+bx)}}\right)}{b}+\frac{\tanh^{-1}\left(\frac{\sqrt{\sinh(a+bx)}}{\sqrt{\cosh(a+bx)}}\right)}{b}-\frac{2\cosh^{\frac{5}{2}}(a+bx)}{5b\sinh^{\frac{5}{2}}(a+bx)}-\frac{2\sqrt{\cosh(a+bx)}}{b\sqrt{\sinh(a+bx)}}
\end{aligned}
$$

Mathematica [C] time = 0.04, size = 59, normalized size = 0.56

$$-\frac{2\cosh^2(a+bx)^{3/4}\,{}_2F_1\left(-\frac{5}{4},-\frac{5}{4};-\frac{1}{4};-\sinh^2(a+bx)\right)}{5b\sinh^{\frac{5}{2}}(a+bx)\cosh^{\frac{3}{2}}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[a + b*x]^(7/2)/Sinh[a + b*x]^(7/2),x]
```

```
[Out] (-2*(Cosh[a + b*x]^2)^(3/4)*Hypergeometric2F1[-5/4, -5/4, -1/4, -Sinh[a + b
*x]^2])/(5*b*Cosh[a + b*x]^(3/2)*Sinh[a + b*x]^(5/2))
```

fricas [B] time = 0.47, size = 1001, normalized size = 9.44

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^(7/2)/sinh(b*x+a)^(7/2),x, algorithm="fricas")
```

```
[Out] -1/10*(24*cosh(b*x + a)^6 + 144*cosh(b*x + a)*sinh(b*x + a)^5 + 24*sinh(b*x
 + a)^6 + 72*(5*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^4 - 72*cosh(b*x + a)^4 +
 96*(5*cosh(b*x + a)^3 - 3*cosh(b*x + a))*sinh(b*x + a)^3 + 72*(5*cosh(b*x
+ a)^4 - 6*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^2 + 10*(cosh(b*x + a)^6 + 6*c
osh(b*x + a)*sinh(b*x + a)^5 + sinh(b*x + a)^6 + 3*(5*cosh(b*x + a)^2 - 1)*
sinh(b*x + a)^4 - 3*cosh(b*x + a)^4 + 4*(5*cosh(b*x + a)^3 - 3*cosh(b*x + a
))*sinh(b*x + a)^3 + 3*(5*cosh(b*x + a)^4 - 6*cosh(b*x + a)^2 + 1)*sinh(b*x
 + a)^2 + 3*cosh(b*x + a)^2 + 6*(cosh(b*x + a)^5 - 2*cosh(b*x + a)^3 + cosh
(b*x + a))*sinh(b*x + a) - 1)*arctan(-cosh(b*x + a)^2 + 2*(cosh(b*x + a) +
sinh(b*x + a))*sqrt(cosh(b*x + a))*sqrt(sinh(b*x + a)) - 2*cosh(b*x + a)*si
nh(b*x + a) - sinh(b*x + a)^2) + 72*cosh(b*x + a)^2 + 5*(cosh(b*x + a)^6 +
6*cosh(b*x + a)*sinh(b*x + a)^5 + sinh(b*x + a)^6 + 3*(5*cosh(b*x + a)^2 -
1)*sinh(b*x + a)^4 - 3*cosh(b*x + a)^4 + 4*(5*cosh(b*x + a)^3 - 3*cosh(b*x
+ a))*sinh(b*x + a)^3 + 3*(5*cosh(b*x + a)^4 - 6*cosh(b*x + a)^2 + 1)*sinh(
b*x + a)^2 + 3*cosh(b*x + a)^2 + 6*(cosh(b*x + a)^5 - 2*cosh(b*x + a)^3 + c
osh(b*x + a))*sinh(b*x + a) - 1)*log(-cosh(b*x + a)^2 + 2*(cosh(b*x + a) +
sinh(b*x + a))*sqrt(cosh(b*x + a))*sqrt(sinh(b*x + a)) - 2*cosh(b*x + a)*si
nh(b*x + a) - sinh(b*x + a)^2) + 16*(3*cosh(b*x + a)^5 + 15*cosh(b*x + a)*s
inh(b*x + a)^4 + 3*sinh(b*x + a)^5 + 2*(15*cosh(b*x + a)^2 - 2)*sinh(b*x +
a)^3 - 4*cosh(b*x + a)^3 + 6*(5*cosh(b*x + a)^3 - 2*cosh(b*x + a))*sinh(b*x
 + a)^2 + 3*(5*cosh(b*x + a)^4 - 4*cosh(b*x + a)^2 + 1)*sinh(b*x + a) + 3*c
osh(b*x + a))*sqrt(cosh(b*x + a))*sqrt(sinh(b*x + a)) + 144*(cosh(b*x + a)^
5 - 2*cosh(b*x + a)^3 + cosh(b*x + a))*sinh(b*x + a) - 24)/(b*cosh(b*x + a)
^6 + 6*b*cosh(b*x + a)*sinh(b*x + a)^5 + b*sinh(b*x + a)^6 - 3*b*cosh(b*x +
 a)^4 + 3*(5*b*cosh(b*x + a)^2 - b)*sinh(b*x + a)^4 + 4*(5*b*cosh(b*x + a)^
```

```
3 - 3*b*cosh(b*x + a))*sinh(b*x + a)^3 + 3*b*cosh(b*x + a)^2 + 3*(5*b*cosh(
b*x + a)^4 - 6*b*cosh(b*x + a)^2 + b)*sinh(b*x + a)^2 + 6*(b*cosh(b*x + a)^
5 - 2*b*cosh(b*x + a)^3 + b*cosh(b*x + a))*sinh(b*x + a) - b)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh(bx+a)^{\frac{7}{2}}}{\sinh(bx+a)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^(7/2)/sinh(b*x+a)^(7/2),x, algorithm="giac")
[Out] integrate(cosh(b*x + a)^(7/2)/sinh(b*x + a)^(7/2), x)
```

maple [F] time = 0.18, size = 0, normalized size = 0.00

$$\int \frac{\cosh^{\frac{7}{2}}(bx+a)}{\sinh(bx+a)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)^(7/2)/sinh(b*x+a)^(7/2),x)
[Out] int(cosh(b*x+a)^(7/2)/sinh(b*x+a)^(7/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh(bx+a)^{\frac{7}{2}}}{\sinh(bx+a)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^(7/2)/sinh(b*x+a)^(7/2),x, algorithm="maxima")
[Out] integrate(cosh(b*x + a)^(7/2)/sinh(b*x + a)^(7/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\cosh(a+b\,x)}^{7/2}}{{\sinh(a+b\,x)}^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a + b*x)^(7/2)/sinh(a + b*x)^(7/2),x)
```

```
[Out] int(cosh(a + b*x)^(7/2)/sinh(a + b*x)^(7/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)**(7/2)/sinh(b*x+a)**(7/2),x)
```

```
[Out] Timed out
```

3.57 $\int \frac{\sinh^{\frac{7}{3}}(a+bx)}{\cosh^{\frac{7}{3}}(a+bx)}\,dx$

Optimal. Leaf size=155

$$-\frac{3\sinh^{\frac{4}{3}}(a+bx)}{4b\cosh^{\frac{4}{3}}(a+bx)}-\frac{\log\left(1-\frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}\right)}{2b}+\frac{\log\left(\frac{\sinh^{\frac{4}{3}}(a+bx)}{\cosh^{\frac{4}{3}}(a+bx)}+\frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}+1\right)}{4b}-\frac{\sqrt{3}\tan^{-1}\left(\frac{\frac{2\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}+1}{\sqrt{3}}\right)}{2b}$$

```
[Out] -1/2*ln(1-sinh(b*x+a)^(2/3)/cosh(b*x+a)^(2/3))/b+1/4*ln(1+sinh(b*x+a)^(2/3)
/cosh(b*x+a)^(2/3)+sinh(b*x+a)^(4/3)/cosh(b*x+a)^(4/3))/b-3/4*sinh(b*x+a)^(
4/3)/b/cosh(b*x+a)^(4/3)-1/2*arctan(1/3*(1+2*sinh(b*x+a)^(2/3)/cosh(b*x+a)^
(2/3))*3^(1/2))*3^(1/2)/b
```

Rubi [A] time = 0.18, antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {2566, 2574, 275, 292, 31, 634, 618, 204, 628}

$$-\frac{3\sinh^{\frac{4}{3}}(a+bx)}{4b\cosh^{\frac{4}{3}}(a+bx)}-\frac{\log\left(1-\frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}\right)}{2b}+\frac{\log\left(\frac{\sinh^{\frac{4}{3}}(a+bx)}{\cosh^{\frac{4}{3}}(a+bx)}+\frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}+1\right)}{4b}-\frac{\sqrt{3}\tan^{-1}\left(\frac{\frac{2\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}+1}{\sqrt{3}}\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[a + b*x]^(7/3)/Cosh[a + b*x]^(7/3),x]
```

```
[Out] -(Sqrt[3]*ArcTan[(1 + (2*Sinh[a + b*x]^(2/3))/Cosh[a + b*x]^(2/3))/Sqrt[3]]
)/(2*b) - Log[1 - Sinh[a + b*x]^(2/3)/Cosh[a + b*x]^(2/3)]/(2*b) + Log[1 +
Sinh[a + b*x]^(2/3)/Cosh[a + b*x]^(2/3) + Sinh[a + b*x]^(4/3)/Cosh[a + b*x]
^(4/3)]/(4*b) - (3*Sinh[a + b*x]^(4/3))/(4*b*Cosh[a + b*x]^(4/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2566

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(a*Sin[e + f*x])^(m - 1)*(b*Cos[e + f*x])^(n + 1)
)/(b*f*(n + 1)), x] + Dist[(a^2*(m - 1))/(b^2*(n + 1)), Int[(a*Sin[e + f*x]
)^(m - 2)*(b*Cos[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ
[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

Rule 2574

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> With[{k = Denominator[m]}, Dist[(k*a*b)/f, Subst[Int[x^(k*
(m + 1) - 1)/(a^2 + b^2*x^(2*k)), x], x, (a*Sin[e + f*x])^(1/k)/(b*Cos[e +
f*x])^(1/k)], x]] /; FreeQ[{a, b, e, f}, x] && EqQ[m + n, 0] && GtQ[m, 0] &
& LtQ[m, 1]
```

Rubi steps

$$\int \frac{\sinh^{\frac{7}{3}}(a+bx)}{\cosh^{\frac{7}{3}}(a+bx)}\,dx = -\frac{3\sinh^{\frac{4}{3}}(a+bx)}{4b\cosh^{\frac{4}{3}}(a+bx)} + \int \frac{\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}\,dx$$

$$= -\frac{3\sinh^{\frac{4}{3}}(a+bx)}{4b\cosh^{\frac{4}{3}}(a+bx)} - \frac{3\,\text{Subst}\left(\int \frac{x^3}{-1+x^6}\,dx, x, \frac{\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}\right)}{b}$$

$$= -\frac{3\sinh^{\frac{4}{3}}(a+bx)}{4b\cosh^{\frac{4}{3}}(a+bx)} - \frac{3\,\text{Subst}\left(\int \frac{x}{-1+x^3}\,dx, x, \frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}\right)}{2b}$$

$$= -\frac{3\sinh^{\frac{4}{3}}(a+bx)}{4b\cosh^{\frac{4}{3}}(a+bx)} - \frac{\text{Subst}\left(\int \frac{1}{-1+x}\,dx, x, \frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}\right)}{2b} + \frac{\text{Subst}\left(\int \frac{-1+x}{1+x+x^2}\,dx, x, \frac{\sinh^{\frac{2}{3}}(a+}{\cosh^{\frac{2}{3}}(a+}\right.}{2b}$$

$$= -\frac{\log\left(1-\frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}\right)}{2b} - \frac{3\sinh^{\frac{4}{3}}(a+bx)}{4b\cosh^{\frac{4}{3}}(a+bx)} + \frac{\text{Subst}\left(\int \frac{1+2x}{1+x+x^2}\,dx, x, \frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}\right)}{4b} - \frac{3\,\text{Sul}}{}$$

$$= -\frac{\log\left(1-\frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}\right)}{2b} + \frac{\log\left(1+\frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}+\frac{\sinh^{\frac{4}{3}}(a+bx)}{\cosh^{\frac{4}{3}}(a+bx)}\right)}{4b} - \frac{3\sinh^{\frac{4}{3}}(a+bx)}{4b\cosh^{\frac{4}{3}}(a+bx)} + \frac{3\,\text{Sul}}{}$$

$$= -\frac{\sqrt{3}\tan^{-1}\left(\frac{1+\frac{2\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}}{\sqrt{3}}\right)}{2b} - \frac{\log\left(1-\frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}\right)}{2b} + \frac{\log\left(1+\frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}+\frac{\sinh^{\frac{4}{3}}(a+bx)}{\cosh^{\frac{4}{3}}(a+bx)}\right)}{4b}$$

Mathematica [C] time = 0.06, size = 59, normalized size = 0.38

$$\frac{3\sinh^{\frac{10}{3}}(a+bx)\cosh^2(a+bx)^{2/3}\,{}_2F_1\left(\frac{5}{3},\frac{5}{3};\frac{8}{3};-\sinh^2(a+bx)\right)}{10b\cosh^{\frac{4}{3}}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[a + b*x]^(7/3)/Cosh[a + b*x]^(7/3),x]
```

```
[Out] (3*(Cosh[a + b*x]^2)^(2/3)*Hypergeometric2F1[5/3, 5/3, 8/3, -Sinh[a + b*x]^
2]*Sinh[a + b*x]^(10/3))/(10*b*Cosh[a + b*x]^(4/3))
```

fricas [B] time = 0.59, size = 1042, normalized size = 6.72

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)^(7/3)/cosh(b*x+a)^(7/3),x, algorithm="fricas")
```

```
[Out] -1/4*(2*(sqrt(3)*cosh(b*x + a)^4 + 4*sqrt(3)*cosh(b*x + a)*sinh(b*x + a)^3
+ sqrt(3)*sinh(b*x + a)^4 + 2*(3*sqrt(3)*cosh(b*x + a)^2 + sqrt(3))*sinh(b*
x + a)^2 + 2*sqrt(3)*cosh(b*x + a)^2 + 4*(sqrt(3)*cosh(b*x + a)^3 + sqrt(3)
*cosh(b*x + a))*sinh(b*x + a) + sqrt(3))*arctan(1/3*(sqrt(3)*cosh(b*x + a)^
2 + 2*sqrt(3)*cosh(b*x + a)*sinh(b*x + a) + sqrt(3)*sinh(b*x + a)^2 + 4*(sq
rt(3)*cosh(b*x + a) + sqrt(3)*sinh(b*x + a))*cosh(b*x + a)^(1/3)*sinh(b*x +
 a)^(2/3) + sqrt(3))/(cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sin
h(b*x + a)^2 + 1)) - (cosh(b*x + a)^4 + 4*cosh(b*x + a)*sinh(b*x + a)^3 + s
inh(b*x + a)^4 + 2*(3*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^2 + 2*cosh(b*x + a
)^2 + 4*(cosh(b*x + a)^3 + cosh(b*x + a))*sinh(b*x + a) + 1)*log((cosh(b*x
+ a)^4 + 4*cosh(b*x + a)*sinh(b*x + a)^3 + sinh(b*x + a)^4 + 2*(3*cosh(b*x
+ a)^2 + 1)*sinh(b*x + a)^2 + 2*cosh(b*x + a)^2 + 2*(cosh(b*x + a)^3 + 3*co
sh(b*x + a)*sinh(b*x + a)^2 + sinh(b*x + a)^3 + (3*cosh(b*x + a)^2 - 1)*sin
h(b*x + a) - cosh(b*x + a))*cosh(b*x + a)^(2/3)*sinh(b*x + a)^(1/3) + 2*(co
sh(b*x + a)^3 + 3*cosh(b*x + a)*sinh(b*x + a)^2 + sinh(b*x + a)^3 + (3*cosh
(b*x + a)^2 + 1)*sinh(b*x + a) + cosh(b*x + a))*cosh(b*x + a)^(1/3)*sinh(b*
x + a)^(2/3) + 4*(cosh(b*x + a)^3 + cosh(b*x + a))*sinh(b*x + a) + 1)/(cosh
(b*x + a)^4 + 4*cosh(b*x + a)*sinh(b*x + a)^3 + sinh(b*x + a)^4 + 2*(3*cosh
(b*x + a)^2 + 1)*sinh(b*x + a)^2 + 2*cosh(b*x + a)^2 + 4*(cosh(b*x + a)^3 +
 cosh(b*x + a))*sinh(b*x + a) + 1)) + 2*(cosh(b*x + a)^4 + 4*cosh(b*x + a)*
sinh(b*x + a)^3 + sinh(b*x + a)^4 + 2*(3*cosh(b*x + a)^2 + 1)*sinh(b*x + a)
^2 + 2*cosh(b*x + a)^2 + 4*(cosh(b*x + a)^3 + cosh(b*x + a))*sinh(b*x + a)
+ 1)*log(-(cosh(b*x + a)^2 - 2*(cosh(b*x + a) + sinh(b*x + a))*cosh(b*x + a
)^(1/3)*sinh(b*x + a)^(2/3) + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)
^2 + 1)/(cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2
```

```
+ 1)) + 6*(cosh(b*x + a)^3 + 3*cosh(b*x + a)*sinh(b*x + a)^2 + sinh(b*x + a
)^3 + (3*cosh(b*x + a)^2 - 1)*sinh(b*x + a) - cosh(b*x + a))*cosh(b*x + a)^
(2/3)*sinh(b*x + a)^(1/3))/(b*cosh(b*x + a)^4 + 4*b*cosh(b*x + a)*sinh(b*x
+ a)^3 + b*sinh(b*x + a)^4 + 2*b*cosh(b*x + a)^2 + 2*(3*b*cosh(b*x + a)^2 +
 b)*sinh(b*x + a)^2 + 4*(b*cosh(b*x + a)^3 + b*cosh(b*x + a))*sinh(b*x + a)
 + b)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh\left(bx + a\right)^{\frac{7}{3}}}{\cosh\left(bx + a\right)^{\frac{7}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)^(7/3)/cosh(b*x+a)^(7/3),x, algorithm="giac")
[Out] integrate(sinh(b*x + a)^(7/3)/cosh(b*x + a)^(7/3), x)
```

maple [F] time = 0.18, size = 0, normalized size = 0.00

$$\int \frac{\sinh^{\frac{7}{3}}\left(bx + a\right)}{\cosh\left(bx + a\right)^{\frac{7}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(b*x+a)^(7/3)/cosh(b*x+a)^(7/3),x)
[Out] int(sinh(b*x+a)^(7/3)/cosh(b*x+a)^(7/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh\left(bx + a\right)^{\frac{7}{3}}}{\cosh\left(bx + a\right)^{\frac{7}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)^(7/3)/cosh(b*x+a)^(7/3),x, algorithm="maxima")
[Out] integrate(sinh(b*x + a)^(7/3)/cosh(b*x + a)^(7/3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\mathrm{sinh}\left(a+b\,x\right)}^{7/3}}{{\mathrm{cosh}\left(a+b\,x\right)}^{7/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a + b*x)^(7/3)/cosh(a + b*x)^(7/3),x)
```

```
[Out] int(sinh(a + b*x)^(7/3)/cosh(a + b*x)^(7/3), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)**(7/3)/cosh(b*x+a)**(7/3),x)
```

```
[Out] Timed out
```

3.58 $\int \frac{\sinh^{\frac{5}{3}}(a+bx)}{\cosh^{\frac{5}{3}}(a+bx)} dx$

Optimal. Leaf size=155

$$-\frac{3\sinh^{\frac{2}{3}}(a+bx)}{2b\cosh^{\frac{2}{3}}(a+bx)}-\frac{\log\left(1-\frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}\right)}{2b}+\frac{\log\left(\frac{\cosh^{\frac{4}{3}}(a+bx)}{\sinh^{\frac{4}{3}}(a+bx)}+\frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}+1\right)}{4b}-\frac{\sqrt{3}\tan^{-1}\left(\frac{\frac{2\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}+1}{\sqrt{3}}\right)}{2b}$$

```
[Out] -1/2*ln(1-cosh(b*x+a)^(2/3)/sinh(b*x+a)^(2/3))/b+1/4*ln(1+cosh(b*x+a)^(4/3)
/sinh(b*x+a)^(4/3)+cosh(b*x+a)^(2/3)/sinh(b*x+a)^(2/3))/b-3/2*sinh(b*x+a)^(
2/3)/b/cosh(b*x+a)^(2/3)-1/2*arctan(1/3*(1+2*cosh(b*x+a)^(2/3)/sinh(b*x+a)^
(2/3))*3^(1/2))*3^(1/2)/b
```

Rubi [A] time = 0.18, antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {2566, 2575, 275, 292, 31, 634, 618, 204, 628}

$$-\frac{3\sinh^{\frac{2}{3}}(a+bx)}{2b\cosh^{\frac{2}{3}}(a+bx)}-\frac{\log\left(1-\frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}\right)}{2b}+\frac{\log\left(\frac{\cosh^{\frac{4}{3}}(a+bx)}{\sinh^{\frac{4}{3}}(a+bx)}+\frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}+1\right)}{4b}-\frac{\sqrt{3}\tan^{-1}\left(\frac{\frac{2\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}+1}{\sqrt{3}}\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[a + b*x]^(5/3)/Cosh[a + b*x]^(5/3),x]
```

```
[Out] -(Sqrt[3]*ArcTan[(1 + (2*Cosh[a + b*x]^(2/3))/Sinh[a + b*x]^(2/3))/Sqrt[3]]
)/(2*b) - Log[1 - Cosh[a + b*x]^(2/3)/Sinh[a + b*x]^(2/3)]/(2*b) + Log[1 +
Cosh[a + b*x]^(4/3)/Sinh[a + b*x]^(4/3) + Cosh[a + b*x]^(2/3)/Sinh[a + b*x]
^(2/3)]/(4*b) - (3*Sinh[a + b*x]^(2/3))/(2*b*Cosh[a + b*x]^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2566

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(a*Sin[e + f*x])^(m - 1)*(b*Cos[e + f*x])^(n + 1)
)/(b*f*(n + 1)), x] + Dist[(a^2*(m - 1))/(b^2*(n + 1)), Int[(a*Sin[e + f*x]
)^(m - 2)*(b*Cos[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ
[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

Rule 2575

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> With[{k = Denominator[m]}, -Dist[(k*a*b)/f, Subst[Int[x^(k
*(m + 1) - 1)/(a^2 + b^2*x^(2*k)), x], x, (a*Cos[e + f*x])^(1/k)/(b*Sin[e +
 f*x])^(1/k)], x]] /; FreeQ[{a, b, e, f}, x] && EqQ[m + n, 0] && GtQ[m, 0]
&& LtQ[m, 1]
```

Rubi steps

$$\int \frac{\sinh^{\frac{5}{3}}(a+bx)}{\cosh^{\frac{5}{3}}(a+bx)}\,dx = -\frac{3\sinh^{\frac{2}{3}}(a+bx)}{2b\cosh^{\frac{2}{3}}(a+bx)} + \int \frac{\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}\,dx$$

$$= -\frac{3\sinh^{\frac{2}{3}}(a+bx)}{2b\cosh^{\frac{2}{3}}(a+bx)} + \frac{3\,\mathrm{Subst}\left(\int \frac{x^3}{1-x^6}\,dx, x, \frac{\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}\right)}{b}$$

$$= -\frac{3\sinh^{\frac{2}{3}}(a+bx)}{2b\cosh^{\frac{2}{3}}(a+bx)} + \frac{3\,\mathrm{Subst}\left(\int \frac{x}{1-x^3}\,dx, x, \frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}\right)}{2b}$$

$$= -\frac{3\sinh^{\frac{2}{3}}(a+bx)}{2b\cosh^{\frac{2}{3}}(a+bx)} + \frac{\mathrm{Subst}\left(\int \frac{1}{1-x}\,dx, x, \frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}\right)}{2b} - \frac{\mathrm{Subst}\left(\int \frac{1-x}{1+x+x^2}\,dx, x, \frac{\cosh^{\frac{2}{3}}(a+b}{\sinh^{\frac{2}{3}}(a+b}\right.}{2b}$$

$$= -\frac{\log\left(1-\frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}\right)}{2b} - \frac{3\sinh^{\frac{2}{3}}(a+bx)}{2b\cosh^{\frac{2}{3}}(a+bx)} + \frac{\mathrm{Subst}\left(\int \frac{1+2x}{1+x+x^2}\,dx, x, \frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}\right)}{4b} - \frac{3\,\mathrm{Sub}}{}$$

$$= -\frac{\log\left(1-\frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}\right)}{2b} + \frac{\log\left(1+\frac{\cosh^{\frac{4}{3}}(a+bx)}{\sinh^{\frac{4}{3}}(a+bx)}+\frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}\right)}{4b} - \frac{3\sinh^{\frac{2}{3}}(a+bx)}{2b\cosh^{\frac{2}{3}}(a+bx)} + \frac{3\,\mathrm{Sub}}{}$$

$$= -\frac{\sqrt{3}\tan^{-1}\left(\frac{1+\frac{2\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}}{\sqrt{3}}\right)}{2b} - \frac{\log\left(1-\frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}\right)}{2b} + \frac{\log\left(1+\frac{\cosh^{\frac{4}{3}}(a+bx)}{\sinh^{\frac{4}{3}}(a+bx)}+\frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}\right)}{4b}$$

Mathematica [C] time = 0.06, size = 59, normalized size = 0.38

$$\frac{3\sinh^{\frac{8}{3}}(a+bx)\sqrt[3]{\cosh^2(a+bx)}\;{}_2F_1\left(\frac{4}{3},\frac{4}{3};\frac{7}{3};-\sinh^2(a+bx)\right)}{8b\cosh^{\frac{2}{3}}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[a + b*x]^(5/3)/Cosh[a + b*x]^(5/3),x]
```

```
[Out] (3*(Cosh[a + b*x]^2)^(1/3)*Hypergeometric2F1[4/3, 4/3, 7/3, -Sinh[a + b*x]^
2]*Sinh[a + b*x]^(8/3))/(8*b*Cosh[a + b*x]^(2/3))
```

fricas [B] time = 0.44, size = 751, normalized size = 4.85

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)^(5/3)/cosh(b*x+a)^(5/3),x, algorithm="fricas")
```

```
[Out] -1/4*(2*(sqrt(3)*cosh(b*x + a)^2 + 2*sqrt(3)*cosh(b*x + a)*sinh(b*x + a) +
sqrt(3)*sinh(b*x + a)^2 + sqrt(3))*arctan(1/3*(sqrt(3)*cosh(b*x + a)^2 + 2*
sqrt(3)*cosh(b*x + a)*sinh(b*x + a) + sqrt(3)*sinh(b*x + a)^2 + 4*(sqrt(3)*
cosh(b*x + a) + sqrt(3)*sinh(b*x + a))*cosh(b*x + a)^(2/3)*sinh(b*x + a)^(1
/3) - sqrt(3))/(cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x
+ a)^2 - 1)) - (cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x
+ a)^2 + 1)*log((cosh(b*x + a)^4 + 4*cosh(b*x + a)*sinh(b*x + a)^3 + sinh(b
*x + a)^4 + 2*(3*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^2 - 2*cosh(b*x + a)^2 +
 2*(cosh(b*x + a)^3 + 3*cosh(b*x + a)*sinh(b*x + a)^2 + sinh(b*x + a)^3 + (
3*cosh(b*x + a)^2 - 1)*sinh(b*x + a) - cosh(b*x + a))*cosh(b*x + a)^(2/3)*s
inh(b*x + a)^(1/3) + 2*(cosh(b*x + a)^3 + 3*cosh(b*x + a)*sinh(b*x + a)^2 +
 sinh(b*x + a)^3 + (3*cosh(b*x + a)^2 + 1)*sinh(b*x + a) + cosh(b*x + a))*c
osh(b*x + a)^(1/3)*sinh(b*x + a)^(2/3) + 4*(cosh(b*x + a)^3 - cosh(b*x + a)
)*sinh(b*x + a) + 1)/(cosh(b*x + a)^4 + 4*cosh(b*x + a)*sinh(b*x + a)^3 + s
inh(b*x + a)^4 + 2*(3*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^2 - 2*cosh(b*x + a
)^2 + 4*(cosh(b*x + a)^3 - cosh(b*x + a))*sinh(b*x + a) + 1)) + 2*(cosh(b*x
 + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 + 1)*log(-(cosh(b
*x + a)^2 - 2*(cosh(b*x + a) + sinh(b*x + a))*cosh(b*x + a)^(2/3)*sinh(b*x
+ a)^(1/3) + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 - 1)/(cosh(b*x
 + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 - 1)) + 12*(cosh(
b*x + a) + sinh(b*x + a))*cosh(b*x + a)^(1/3)*sinh(b*x + a)^(2/3))/(b*cosh(
b*x + a)^2 + 2*b*cosh(b*x + a)*sinh(b*x + a) + b*sinh(b*x + a)^2 + b)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh(bx+a)^{\frac{5}{3}}}{\cosh(bx+a)^{\frac{5}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)^(5/3)/cosh(b*x+a)^(5/3),x, algorithm="giac")
[Out] integrate(sinh(b*x + a)^(5/3)/cosh(b*x + a)^(5/3), x)
```

maple [F] time = 0.16, size = 0, normalized size = 0.00

$$\int \frac{\sinh^{\frac{5}{3}}(bx+a)}{\cosh(bx+a)^{\frac{5}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(b*x+a)^(5/3)/cosh(b*x+a)^(5/3),x)
[Out] int(sinh(b*x+a)^(5/3)/cosh(b*x+a)^(5/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh(bx+a)^{\frac{5}{3}}}{\cosh(bx+a)^{\frac{5}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)^(5/3)/cosh(b*x+a)^(5/3),x, algorithm="maxima")
[Out] integrate(sinh(b*x + a)^(5/3)/cosh(b*x + a)^(5/3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\sinh\left(a+b\,x\right)}^{5/3}}{{\cosh\left(a+b\,x\right)}^{5/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a + b*x)^(5/3)/cosh(a + b*x)^(5/3),x)
[Out] int(sinh(a + b*x)^(5/3)/cosh(a + b*x)^(5/3), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)**(5/3)/cosh(b*x+a)**(5/3),x)
```

```
[Out] Timed out
```

3.59 $\int \frac{\sinh^{\frac{4}{3}}(a+bx)}{\cosh^{\frac{4}{3}}(a+bx)}\,dx$

Optimal. Leaf size=243

$$-\frac{3\sqrt[3]{\sinh(a+bx)}}{b\sqrt[3]{\cosh(a+bx)}}-\frac{\log\left(\frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}-\frac{\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}+1\right)}{4b}+\frac{\log\left(\frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}+\frac{\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}+1\right)}{4b}+\frac{\sqrt{3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{\cosh}}{\sqrt[3]{\sinh}}}{\sqrt{3}}\right)}{2b}$$

[Out] arctanh(cosh(b*x+a)^(1/3)/sinh(b*x+a)^(1/3))/b-1/4*ln(1+cosh(b*x+a)^(2/3)/sinh(b*x+a)^(2/3)-cosh(b*x+a)^(1/3)/sinh(b*x+a)^(1/3))/b+1/4*ln(1+cosh(b*x+a)^(2/3)/sinh(b*x+a)^(2/3)+cosh(b*x+a)^(1/3)/sinh(b*x+a)^(1/3))/b-3*sinh(b*x+a)^(1/3)/b/cosh(b*x+a)^(1/3)+1/2*arctan(1/3*(1-2*cosh(b*x+a)^(1/3)/sinh(b*x+a)^(1/3))*3^(1/2))*3^(1/2)/b-1/2*arctan(1/3*(1+2*cosh(b*x+a)^(1/3)/sinh(b*x+a)^(1/3))*3^(1/2))*3^(1/2)/b

Rubi [A] time = 0.24, antiderivative size = 243, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {2566, 2575, 296, 634, 618, 204, 628, 206}

$$-\frac{3\sqrt[3]{\sinh(a+bx)}}{b\sqrt[3]{\cosh(a+bx)}}-\frac{\log\left(\frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}-\frac{\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}+1\right)}{4b}+\frac{\log\left(\frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}+\frac{\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}+1\right)}{4b}+\frac{\sqrt{3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{\cosh}}{\sqrt[3]{\sinh}}}{\sqrt{3}}\right)}{2b}$$

Antiderivative was successfully verified.

[In] Int[Sinh[a + b*x]^(4/3)/Cosh[a + b*x]^(4/3),x]

[Out] (Sqrt[3]*ArcTan[(1 - (2*Cosh[a + b*x]^(1/3))/Sinh[a + b*x]^(1/3))/Sqrt[3]])/(2*b) - (Sqrt[3]*ArcTan[(1 + (2*Cosh[a + b*x]^(1/3))/Sinh[a + b*x]^(1/3))/Sqrt[3]])/(2*b) + ArcTanh[Cosh[a + b*x]^(1/3)/Sinh[a + b*x]^(1/3)]/b - Log[1 + Cosh[a + b*x]^(2/3)/Sinh[a + b*x]^(2/3) - Cosh[a + b*x]^(1/3)/Sinh[a + b*x]^(1/3)]/(4*b) + Log[1 + Cosh[a + b*x]^(2/3)/Sinh[a + b*x]^(2/3) + Cosh[a + b*x]^(1/3)/Sinh[a + b*x]^(1/3)]/(4*b) - (3*Sinh[a + b*x]^(1/3))/(b*Cosh[a + b*x]^(1/3))

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 296

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[-(a/b), n]], s = Denominator[Rt[-(a/b), n]], k, u}, Simp[u = Int[(r*Cos
[(2*k*m*Pi)/n] - s*Cos[(2*k*(m + 1)*Pi)/n]*x)/(r^2 - 2*r*s*Cos[(2*k*Pi)/n]*
x + s^2*x^2), x] + Int[(r*Cos[(2*k*m*Pi)/n] + s*Cos[(2*k*(m + 1)*Pi)/n]*x)/
(r^2 + 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x]; (2*r^(m + 2)*Int[1/(r^2 - s^
2*x^2), x])/(a*n*s^m) + Dist[(2*r^(m + 1))/(a*n*s^m), Sum[u, {k, 1, (n - 2)
/4}], x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && IGtQ[m, 0] && Lt
Q[m, n - 1] && NegQ[a/b]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2566

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(a*Sin[e + f*x])^(m - 1)*(b*Cos[e + f*x])^(n + 1)
)/(b*f*(n + 1)), x] + Dist[(a^2*(m - 1))/(b^2*(n + 1)), Int[(a*Sin[e + f*x]
)^(m - 2)*(b*Cos[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ
[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

Rule 2575

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> With[{k = Denominator[m]}, -Dist[(k*a*b)/f, Subst[Int[x^(k
*(m + 1) - 1)/(a^2 + b^2*x^(2*k)), x], x, (a*Cos[e + f*x])^(1/k)/(b*Sin[e +
 f*x])^(1/k)], x]] /; FreeQ[{a, b, e, f}, x] && EqQ[m + n, 0] && GtQ[m, 0]
&& LtQ[m, 1]
```

Rubi steps

$$\int \frac{\sinh^{\frac{4}{3}}(a+bx)}{\cosh^{\frac{4}{3}}(a+bx)}\,dx = -\frac{3\sqrt[3]{\sinh(a+bx)}}{b\sqrt[3]{\cosh(a+bx)}} + \int \frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}\,dx$$

$$= -\frac{3\sqrt[3]{\sinh(a+bx)}}{b\sqrt[3]{\cosh(a+bx)}} + \frac{3\,\text{Subst}\left(\int \frac{x^4}{1-x^6}\,dx, x, \frac{\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}\right)}{b}$$

$$= -\frac{3\sqrt[3]{\sinh(a+bx)}}{b\sqrt[3]{\cosh(a+bx)}} + \frac{\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}\right)}{b} + \frac{\text{Subst}\left(\int \frac{-\frac{1}{2}-\frac{x}{2}}{1-x+x^2}\,dx, x, \frac{\sqrt[3]{\cosh(a+}}{\sqrt[3]{\sinh(a+}}\right.}{b}$$

$$= \frac{\tanh^{-1}\left(\frac{\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}\right)}{b} - \frac{3\sqrt[3]{\sinh(a+bx)}}{b\sqrt[3]{\cosh(a+bx)}} - \frac{\text{Subst}\left(\int \frac{-1+2x}{1-x+x^2}\,dx, x, \frac{\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}\right)}{4b} + \frac{\text{Subst}}{}$$

$$= \frac{\tanh^{-1}\left(\frac{\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}\right)}{b} - \frac{\log\left(1+\frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}-\frac{\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}\right)}{4b} + \frac{\log\left(1+\frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}+\frac{\sqrt[3]{\cos}}{\sqrt[3]{\sin}}\right.}{4b}$$

$$= \frac{\sqrt{3}\,\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}}{\sqrt{3}}\right)}{2b} - \frac{\sqrt{3}\,\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}}{\sqrt{3}}\right)}{2b} + \frac{\tanh^{-1}\left(\frac{\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}\right)}{b} - \frac{\log\left(1+\right.}{}$$

Mathematica [C] time = 0.05, size = 59, normalized size = 0.24

$$\frac{3\sinh^{\frac{7}{3}}(a+bx)\sqrt[6]{\cosh^2(a+bx)}\;{}_2F_1\left(\frac{7}{6},\frac{7}{6};\frac{13}{6};-\sinh^2(a+bx)\right)}{7b\sqrt[3]{\cosh(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[a + b*x]^(4/3)/Cosh[a + b*x]^(4/3),x]
```

```
[Out] (3*(Cosh[a + b*x]^2)^(1/6)*Hypergeometric2F1[7/6, 7/6, 13/6, -Sinh[a + b*x]
^2]*Sinh[a + b*x]^(7/3))/(7*b*Cosh[a + b*x]^(1/3))
```

fricas [B] time = 0.46, size = 1003, normalized size = 4.13

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)^(4/3)/cosh(b*x+a)^(4/3),x, algorithm="fricas")
```

```
[Out] 1/4*(2*(sqrt(3)*cosh(b*x + a)^2 + 2*sqrt(3)*cosh(b*x + a)*sinh(b*x + a) + s
qrt(3)*sinh(b*x + a)^2 + sqrt(3))*arctan(1/3*(sqrt(3)*cosh(b*x + a)^2 + 2*s
qrt(3)*cosh(b*x + a)*sinh(b*x + a) + sqrt(3)*sinh(b*x + a)^2 + 4*(sqrt(3)*c
osh(b*x + a) + sqrt(3)*sinh(b*x + a))*cosh(b*x + a)^(2/3)*sinh(b*x + a)^(1/
3) + sqrt(3))/(cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x +
 a)^2 + 1)) + 2*(sqrt(3)*cosh(b*x + a)^2 + 2*sqrt(3)*cosh(b*x + a)*sinh(b*x
 + a) + sqrt(3)*sinh(b*x + a)^2 + sqrt(3))*arctan(-1/3*(sqrt(3)*cosh(b*x +
a)^2 + 2*sqrt(3)*cosh(b*x + a)*sinh(b*x + a) + sqrt(3)*sinh(b*x + a)^2 - 4*
(sqrt(3)*cosh(b*x + a) + sqrt(3)*sinh(b*x + a))*cosh(b*x + a)^(2/3)*sinh(b*
x + a)^(1/3) + sqrt(3))/(cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) +
sinh(b*x + a)^2 + 1)) + (cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) +
sinh(b*x + a)^2 + 1)*log((cosh(b*x + a)^2 + 2*(cosh(b*x + a) + sinh(b*x + a
))*cosh(b*x + a)^(2/3)*sinh(b*x + a)^(1/3) + 2*(cosh(b*x + a) + sinh(b*x +
a))*cosh(b*x + a)^(1/3)*sinh(b*x + a)^(2/3) + 2*cosh(b*x + a)*sinh(b*x + a)
 + sinh(b*x + a)^2 + 1)/(cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) +
sinh(b*x + a)^2 + 1)) + 2*(cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a)
+ sinh(b*x + a)^2 + 1)*log((cosh(b*x + a)^2 + 2*(cosh(b*x + a) + sinh(b*x +
 a))*cosh(b*x + a)^(2/3)*sinh(b*x + a)^(1/3) + 2*cosh(b*x + a)*sinh(b*x + a
) + sinh(b*x + a)^2 + 1)/(cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) +
 sinh(b*x + a)^2 + 1)) - (cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) +
 sinh(b*x + a)^2 + 1)*log((cosh(b*x + a)^2 - 2*(cosh(b*x + a) + sinh(b*x +
a))*cosh(b*x + a)^(2/3)*sinh(b*x + a)^(1/3) + 2*(cosh(b*x + a) + sinh(b*x +
 a))*cosh(b*x + a)^(1/3)*sinh(b*x + a)^(2/3) + 2*cosh(b*x + a)*sinh(b*x + a
) + sinh(b*x + a)^2 + 1)/(cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) +
 sinh(b*x + a)^2 + 1)) - 2*(cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a)
 + sinh(b*x + a)^2 + 1)*log(-(cosh(b*x + a)^2 - 2*(cosh(b*x + a) + sinh(b*x
 + a))*cosh(b*x + a)^(2/3)*sinh(b*x + a)^(1/3) + 2*cosh(b*x + a)*sinh(b*x +
 a) + sinh(b*x + a)^2 + 1)/(cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a)
 + sinh(b*x + a)^2 + 1)) - 24*(cosh(b*x + a) + sinh(b*x + a))*cosh(b*x + a)
^(2/3)*sinh(b*x + a)^(1/3))/(b*cosh(b*x + a)^2 + 2*b*cosh(b*x + a)*sinh(b*x
 + a) + b*sinh(b*x + a)^2 + b)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh(bx+a)^{\frac{4}{3}}}{\cosh(bx+a)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)^(4/3)/cosh(b*x+a)^(4/3),x, algorithm="giac")
[Out] integrate(sinh(b*x + a)^(4/3)/cosh(b*x + a)^(4/3), x)
```

maple [F] time = 0.16, size = 0, normalized size = 0.00

$$\int \frac{\sinh^{\frac{4}{3}}(bx+a)}{\cosh(bx+a)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(b*x+a)^(4/3)/cosh(b*x+a)^(4/3),x)
[Out] int(sinh(b*x+a)^(4/3)/cosh(b*x+a)^(4/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh(bx+a)^{\frac{4}{3}}}{\cosh(bx+a)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)^(4/3)/cosh(b*x+a)^(4/3),x, algorithm="maxima")
[Out] integrate(sinh(b*x + a)^(4/3)/cosh(b*x + a)^(4/3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sinh(a+b\,x)^{4/3}}{\cosh(a+b\,x)^{4/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a + b*x)^(4/3)/cosh(a + b*x)^(4/3),x)
[Out] int(sinh(a + b*x)^(4/3)/cosh(a + b*x)^(4/3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh^{\frac{4}{3}}(a+bx)}{\cosh^{\frac{4}{3}}(a+bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)**(4/3)/cosh(b*x+a)**(4/3),x)
[Out] Integral(sinh(a + b*x)**(4/3)/cosh(a + b*x)**(4/3), x)
```

3.60 $\int \frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}\,dx$

Optimal. Leaf size=218

$$-\frac{\log\left(\frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}-\frac{\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}+1\right)}{4b}+\frac{\log\left(\frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}+\frac{\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}+1\right)}{4b}+\frac{\sqrt{3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}}{\sqrt{3}}\right)}{2b}-\frac{\sqrt{3}\tan^{-1}\left(\cdots\right.}{2\cdots}$$

```
[Out] arctanh(sinh(b*x+a)^(1/3)/cosh(b*x+a)^(1/3))/b-1/4*ln(1-sinh(b*x+a)^(1/3)/c
osh(b*x+a)^(1/3)+sinh(b*x+a)^(2/3)/cosh(b*x+a)^(2/3))/b+1/4*ln(1+sinh(b*x+a
)^(1/3)/cosh(b*x+a)^(1/3)+sinh(b*x+a)^(2/3)/cosh(b*x+a)^(2/3))/b+1/2*arctan
(1/3*(1-2*sinh(b*x+a)^(1/3)/cosh(b*x+a)^(1/3))*3^(1/2))*3^(1/2)/b-1/2*arcta
n(1/3*(1+2*sinh(b*x+a)^(1/3)/cosh(b*x+a)^(1/3))*3^(1/2))*3^(1/2)/b
```

Rubi [A] time = 0.22, antiderivative size = 218, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2574, 296, 634, 618, 204, 628, 206}

$$-\frac{\log\left(\frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}-\frac{\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}+1\right)}{4b}+\frac{\log\left(\frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}+\frac{\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}+1\right)}{4b}+\frac{\sqrt{3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}}{\sqrt{3}}\right)}{2b}-\frac{\sqrt{3}\tan^{-1}\left(\cdots\right.}{2\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[a + b*x]^(2/3)/Cosh[a + b*x]^(2/3),x]
```

```
[Out] (Sqrt[3]*ArcTan[(1 - (2*Sinh[a + b*x]^(1/3))/Cosh[a + b*x]^(1/3))/Sqrt[3]])
/(2*b) - (Sqrt[3]*ArcTan[(1 + (2*Sinh[a + b*x]^(1/3))/Cosh[a + b*x]^(1/3))/
Sqrt[3]])/(2*b) + ArcTanh[Sinh[a + b*x]^(1/3)/Cosh[a + b*x]^(1/3)]/b - Log[
1 - Sinh[a + b*x]^(1/3)/Cosh[a + b*x]^(1/3) + Sinh[a + b*x]^(2/3)/Cosh[a +
b*x]^(2/3)]/(4*b) + Log[1 + Sinh[a + b*x]^(1/3)/Cosh[a + b*x]^(1/3) + Sinh[
a + b*x]^(2/3)/Cosh[a + b*x]^(2/3)]/(4*b)
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 296

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[-(a/b), n]], s = Denominator[Rt[-(a/b), n]], k, u}, Simp[u = Int[(r*Cos
[(2*k*m*Pi)/n] - s*Cos[(2*k*(m + 1)*Pi)/n]*x)/(r^2 - 2*r*s*Cos[(2*k*Pi)/n]*
x + s^2*x^2), x] + Int[(r*Cos[(2*k*m*Pi)/n] + s*Cos[(2*k*(m + 1)*Pi)/n]*x)/
(r^2 + 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x]; (2*r^(m + 2)*Int[1/(r^2 - s^
2*x^2), x])/(a*n*s^m) + Dist[(2*r^(m + 1))/(a*n*s^m), Sum[u, {k, 1, (n - 2)
/4}], x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && IGtQ[m, 0] && Lt
Q[m, n - 1] && NegQ[a/b]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2574

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> With[{k = Denominator[m]}, Dist[(k*a*b)/f, Subst[Int[x^(k*
(m + 1) - 1)/(a^2 + b^2*x^(2*k)), x], x, (a*Sin[e + f*x])^(1/k)/(b*Cos[e +
f*x])^(1/k)], x]] /; FreeQ[{a, b, e, f}, x] && EqQ[m + n, 0] && GtQ[m, 0] &
& LtQ[m, 1]
```

Rubi steps

$$\int \frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}\,dx = -\frac{3\,\text{Subst}\left(\int \frac{x^4}{-1+x^6}\,dx, x, \frac{\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}\right)}{b}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}\right)}{b} + \frac{\text{Subst}\left(\int \frac{-\frac{1}{2}-\frac{x}{2}}{1-x+x^2}\,dx, x, \frac{\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}\right)}{b} + \frac{\text{Subst}\left(\int \frac{-\frac{1}{2}+\frac{x}{2}}{1+x+x^2}\right.}{}$$

$$= \frac{\tanh^{-1}\left(\frac{\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}\right)}{b} - \frac{\text{Subst}\left(\int \frac{-1+2x}{1-x+x^2}\,dx, x, \frac{\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}\right)}{4b} + \frac{\text{Subst}\left(\int \frac{1+2x}{1+x+x^2}\,dx, x, \frac{\sqrt[3]{\sinh}}{\sqrt[3]{\cosh}}\right.}{4b}$$

$$= \frac{\tanh^{-1}\left(\frac{\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}\right)}{b} - \frac{\log\left(1-\frac{\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}+\frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}\right)}{4b} + \frac{\log\left(1+\frac{\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}+\frac{\sinh^{\frac{2}{3}}}{\cosh^{\frac{2}{3}}}\right.}{4b}$$

$$= \frac{\sqrt{3}\,\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}}{\sqrt{3}}\right)}{2b} - \frac{\sqrt{3}\,\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}}{\sqrt{3}}\right)}{2b} + \frac{\tanh^{-1}\left(\frac{\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}\right)}{b} - \frac{\log\left(1-\right.}{}$$

Mathematica [C] time = 0.04, size = 59, normalized size = 0.27

$$\frac{3\sinh^{\frac{5}{3}}(a+bx)\cosh^2(a+bx)^{5/6}\,{}_2F_1\left(\frac{5}{6},\frac{5}{6};\frac{11}{6};-\sinh^2(a+bx)\right)}{5b\cosh^{\frac{5}{3}}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[a + b*x]^(2/3)/Cosh[a + b*x]^(2/3),x]
```

```
[Out] (3*(Cosh[a + b*x]^2)^(5/6)*Hypergeometric2F1[5/6, 5/6, 11/6, -Sinh[a + b*x]
^2]*Sinh[a + b*x]^(5/3))/(5*b*Cosh[a + b*x]^(5/3))
```

fricas [B] time = 0.46, size = 727, normalized size = 3.33

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)^(2/3)/cosh(b*x+a)^(2/3),x, algorithm="fricas")
```

```
[Out] 1/4*(2*sqrt(3)*arctan(1/3*(sqrt(3)*cosh(b*x + a)^2 + 2*sqrt(3)*cosh(b*x + a
)*sinh(b*x + a) + sqrt(3)*sinh(b*x + a)^2 + 4*(sqrt(3)*cosh(b*x + a) + sqrt
(3)*sinh(b*x + a))*cosh(b*x + a)^(1/3)*sinh(b*x + a)^(2/3) - sqrt(3))/(cosh
```

```
(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 - 1)) + 2*sqr
t(3)*arctan(-1/3*(sqrt(3)*cosh(b*x + a)^2 + 2*sqrt(3)*cosh(b*x + a)*sinh(b*
x + a) + sqrt(3)*sinh(b*x + a)^2 - 4*(sqrt(3)*cosh(b*x + a) + sqrt(3)*sinh(
b*x + a))*cosh(b*x + a)^(1/3)*sinh(b*x + a)^(2/3) - sqrt(3))/(cosh(b*x + a)
^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 - 1)) + log((cosh(b*x
+ a)^2 + 2*(cosh(b*x + a) + sinh(b*x + a))*cosh(b*x + a)^(2/3)*sinh(b*x + a
)^(1/3) + 2*(cosh(b*x + a) + sinh(b*x + a))*cosh(b*x + a)^(1/3)*sinh(b*x +
a)^(2/3) + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 - 1)/(cosh(b*x +
 a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 - 1)) - log((cosh(b
*x + a)^2 + 2*(cosh(b*x + a) + sinh(b*x + a))*cosh(b*x + a)^(2/3)*sinh(b*x
+ a)^(1/3) - 2*(cosh(b*x + a) + sinh(b*x + a))*cosh(b*x + a)^(1/3)*sinh(b*x
 + a)^(2/3) + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 - 1)/(cosh(b*
x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 - 1)) + 2*log((c
osh(b*x + a)^2 + 2*(cosh(b*x + a) + sinh(b*x + a))*cosh(b*x + a)^(1/3)*sinh
(b*x + a)^(2/3) + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 - 1)/(cos
h(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 - 1)) - 2*lo
g(-(cosh(b*x + a)^2 - 2*(cosh(b*x + a) + sinh(b*x + a))*cosh(b*x + a)^(1/3)
*sinh(b*x + a)^(2/3) + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 - 1)
/(cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 - 1)))/
b
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh\left(bx+a\right)^{\frac{2}{3}}}{\cosh\left(bx+a\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)^(2/3)/cosh(b*x+a)^(2/3),x, algorithm="giac")
[Out] integrate(sinh(b*x + a)^(2/3)/cosh(b*x + a)^(2/3), x)
```

maple [F] time = 0.16, size = 0, normalized size = 0.00

$$\int \frac{\sinh^{\frac{2}{3}}\left(bx+a\right)}{\cosh\left(bx+a\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(b*x+a)^(2/3)/cosh(b*x+a)^(2/3),x)
[Out] int(sinh(b*x+a)^(2/3)/cosh(b*x+a)^(2/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh\left(bx+a\right)^{\frac{2}{3}}}{\cosh\left(bx+a\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)^(2/3)/cosh(b*x+a)^(2/3),x, algorithm="maxima")
[Out] integrate(sinh(b*x + a)^(2/3)/cosh(b*x + a)^(2/3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\sinh\left(a+b\,x\right)}^{2/3}}{{\cosh\left(a+b\,x\right)}^{2/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a + b*x)^(2/3)/cosh(a + b*x)^(2/3),x)
[Out] int(sinh(a + b*x)^(2/3)/cosh(a + b*x)^(2/3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh^{\frac{2}{3}}\left(a+bx\right)}{\cosh^{\frac{2}{3}}\left(a+bx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)**(2/3)/cosh(b*x+a)**(2/3),x)
[Out] Integral(sinh(a + b*x)**(2/3)/cosh(a + b*x)**(2/3), x)
```

3.61 $\int \frac{\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}\,dx$

Optimal. Leaf size=128

$$-\frac{\log\left(1-\frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}\right)}{2b}+\frac{\log\left(\frac{\sinh^{\frac{4}{3}}(a+bx)}{\cosh^{\frac{4}{3}}(a+bx)}+\frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}+1\right)}{4b}-\frac{\sqrt{3}\tan^{-1}\left(\frac{\frac{2\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}+1}{\sqrt{3}}\right)}{2b}$$

```
[Out] -1/2*ln(1-sinh(b*x+a)^(2/3)/cosh(b*x+a)^(2/3))/b+1/4*ln(1+sinh(b*x+a)^(2/3)
/cosh(b*x+a)^(2/3)+sinh(b*x+a)^(4/3)/cosh(b*x+a)^(4/3))/b-1/2*arctan(1/3*(1
+2*sinh(b*x+a)^(2/3)/cosh(b*x+a)^(2/3))*3^(1/2))*3^(1/2)/b
```

Rubi [A] time = 0.09, antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {2574, 275, 292, 31, 634, 618, 204, 628}

$$-\frac{\log\left(1-\frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}\right)}{2b}+\frac{\log\left(\frac{\sinh^{\frac{4}{3}}(a+bx)}{\cosh^{\frac{4}{3}}(a+bx)}+\frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}+1\right)}{4b}-\frac{\sqrt{3}\tan^{-1}\left(\frac{\frac{2\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}+1}{\sqrt{3}}\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[a + b*x]^(1/3)/Cosh[a + b*x]^(1/3),x]
```

```
[Out] -(Sqrt[3]*ArcTan[(1 + (2*Sinh[a + b*x]^(2/3))/Cosh[a + b*x]^(2/3))/Sqrt[3]]
)/(2*b) - Log[1 - Sinh[a + b*x]^(2/3)/Cosh[a + b*x]^(2/3)]/(2*b) + Log[1 +
Sinh[a + b*x]^(2/3)/Cosh[a + b*x]^(2/3) + Sinh[a + b*x]^(4/3)/Cosh[a + b*x]
^(4/3)]/(4*b)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2574

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> With[{k = Denominator[m]}, Dist[(k*a*b)/f, Subst[Int[x^(k*
(m + 1) - 1)/(a^2 + b^2*x^(2*k)), x], x, (a*Sin[e + f*x])^(1/k)/(b*Cos[e +
f*x])^(1/k)], x]] /; FreeQ[{a, b, e, f}, x] && EqQ[m + n, 0] && GtQ[m, 0] &
& LtQ[m, 1]
```

Rubi steps

$$\int \frac{\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}\,dx = -\frac{3\,\text{Subst}\left(\int \frac{x^3}{-1+x^6}\,dx, x, \frac{\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}\right)}{b}$$

$$= -\frac{3\,\text{Subst}\left(\int \frac{x}{-1+x^3}\,dx, x, \frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}\right)}{2b}$$

$$= -\frac{\text{Subst}\left(\int \frac{1}{-1+x}\,dx, x, \frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}\right)}{2b} + \frac{\text{Subst}\left(\int \frac{-1+x}{1+x+x^2}\,dx, x, \frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}\right)}{2b}$$

$$= -\frac{\log\left(1-\frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}\right)}{2b} + \frac{\text{Subst}\left(\int \frac{1+2x}{1+x+x^2}\,dx, x, \frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}\right)}{4b} - \frac{3\,\text{Subst}\left(\int \frac{1}{1+x+x^2}\,dx, x,\right.}{4b}$$

$$= -\frac{\log\left(1-\frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}\right)}{2b} + \frac{\log\left(1+\frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}+\frac{\sinh^{\frac{4}{3}}(a+bx)}{\cosh^{\frac{4}{3}}(a+bx)}\right)}{4b} + \frac{3\,\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1\right.}{2b}$$

$$= -\frac{\sqrt{3}\,\tan^{-1}\left(\frac{1+\frac{2\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}}{\sqrt{3}}\right)}{2b} - \frac{\log\left(1-\frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}\right)}{2b} + \frac{\log\left(1+\frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}+\frac{\sinh^{\frac{4}{3}}(a+bx)}{\cosh^{\frac{4}{3}}(a+bx)}\right)}{4b}$$

Mathematica [C] time = 0.04, size = 59, normalized size = 0.46

$$\frac{3\sinh^{\frac{4}{3}}(a+bx)\cosh^2(a+bx)^{2/3}\,{}_2F_1\left(\frac{2}{3},\frac{2}{3};\frac{5}{3};-\sinh^2(a+bx)\right)}{4b\cosh^{\frac{4}{3}}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[a + b*x]^(1/3)/Cosh[a + b*x]^(1/3),x]
```

```
[Out] (3*(Cosh[a + b*x]^2)^(2/3)*Hypergeometric2F1[2/3, 2/3, 5/3, -Sinh[a + b*x]^
2]*Sinh[a + b*x]^(4/3))/(4*b*Cosh[a + b*x]^(4/3))
```

fricas [B] time = 0.51, size = 572, normalized size = 4.47

$$-\frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}\cosh(bx+a)^2+2\sqrt{3}\cosh(bx+a)\sinh(bx+a)+\sqrt{3}\sinh(bx+a)^2+4\left(\sqrt{3}\cosh(bx+a)+\sqrt{3}\sinh(bx+a)\right)\cosh(bx+a)^{\frac{1}{3}}\sinh(bx+}{3\left(\cosh(bx+a)^2+2\cosh(bx+a)\sinh(bx+a)+\sinh(bx+a)^2+1\right)}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)^(1/3)/cosh(b*x+a)^(1/3),x, algorithm="fricas")
```

```
[Out] -1/4*(2*sqrt(3)*arctan(1/3*(sqrt(3)*cosh(b*x + a)^2 + 2*sqrt(3)*cosh(b*x +
a)*sinh(b*x + a) + sqrt(3)*sinh(b*x + a)^2 + 4*(sqrt(3)*cosh(b*x + a) + sqr
t(3)*sinh(b*x + a))*cosh(b*x + a)^(1/3)*sinh(b*x + a)^(2/3) + sqrt(3))/(cos
h(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 + 1)) - log(
(cosh(b*x + a)^4 + 4*cosh(b*x + a)*sinh(b*x + a)^3 + sinh(b*x + a)^4 + 2*(3
*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^2 + 2*cosh(b*x + a)^2 + 2*(cosh(b*x + a
)^3 + 3*cosh(b*x + a)*sinh(b*x + a)^2 + sinh(b*x + a)^3 + (3*cosh(b*x + a)^
2 - 1)*sinh(b*x + a) - cosh(b*x + a))*cosh(b*x + a)^(2/3)*sinh(b*x + a)^(1/
3) + 2*(cosh(b*x + a)^3 + 3*cosh(b*x + a)*sinh(b*x + a)^2 + sinh(b*x + a)^3
 + (3*cosh(b*x + a)^2 + 1)*sinh(b*x + a) + cosh(b*x + a))*cosh(b*x + a)^(1/
3)*sinh(b*x + a)^(2/3) + 4*(cosh(b*x + a)^3 + cosh(b*x + a))*sinh(b*x + a)
+ 1)/(cosh(b*x + a)^4 + 4*cosh(b*x + a)*sinh(b*x + a)^3 + sinh(b*x + a)^4 +
 2*(3*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^2 + 2*cosh(b*x + a)^2 + 4*(cosh(b*
x + a)^3 + cosh(b*x + a))*sinh(b*x + a) + 1)) + 2*log(-(cosh(b*x + a)^2 - 2
*(cosh(b*x + a) + sinh(b*x + a))*cosh(b*x + a)^(1/3)*sinh(b*x + a)^(2/3) +
2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 + 1)/(cosh(b*x + a)^2 + 2*c
osh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 + 1)))/b
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh\left(bx+a\right)^{\frac{1}{3}}}{\cosh\left(bx+a\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)^(1/3)/cosh(b*x+a)^(1/3),x, algorithm="giac")
```

```
[Out] integrate(sinh(b*x + a)^(1/3)/cosh(b*x + a)^(1/3), x)
```

maple [F] time = 0.21, size = 0, normalized size = 0.00

$$\int \frac{\sinh^{\frac{1}{3}}\left(bx+a\right)}{\cosh\left(bx+a\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(b*x+a)^(1/3)/cosh(b*x+a)^(1/3),x)
```

```
[Out] int(sinh(b*x+a)^(1/3)/cosh(b*x+a)^(1/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh\left(bx+a\right)^{\frac{1}{3}}}{\cosh\left(bx+a\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)^(1/3)/cosh(b*x+a)^(1/3),x, algorithm="maxima")
[Out] integrate(sinh(b*x + a)^(1/3)/cosh(b*x + a)^(1/3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\mathrm{sinh}\left(a+b\,x\right)}^{1/3}}{{\mathrm{cosh}\left(a+b\,x\right)}^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a + b*x)^(1/3)/cosh(a + b*x)^(1/3),x)
[Out] int(sinh(a + b*x)^(1/3)/cosh(a + b*x)^(1/3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[3]{\sinh\left(a+bx\right)}}{\sqrt[3]{\cosh\left(a+bx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)**(1/3)/cosh(b*x+a)**(1/3),x)
[Out] Integral(sinh(a + b*x)**(1/3)/cosh(a + b*x)**(1/3), x)
```

3.62 $\int \frac{\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}\,dx$

Optimal. Leaf size=128

$$-\frac{\log\left(1-\frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}\right)}{2b}+\frac{\log\left(\frac{\cosh^{\frac{4}{3}}(a+bx)}{\sinh^{\frac{4}{3}}(a+bx)}+\frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}+1\right)}{4b}-\frac{\sqrt{3}\tan^{-1}\left(\frac{\frac{2\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}+1}{\sqrt{3}}\right)}{2b}$$

```
[Out] -1/2*ln(1-cosh(b*x+a)^(2/3)/sinh(b*x+a)^(2/3))/b+1/4*ln(1+cosh(b*x+a)^(4/3)
/sinh(b*x+a)^(4/3)+cosh(b*x+a)^(2/3)/sinh(b*x+a)^(2/3))/b-1/2*arctan(1/3*(1
+2*cosh(b*x+a)^(2/3)/sinh(b*x+a)^(2/3))*3^(1/2))*3^(1/2)/b
```

Rubi [A] time = 0.09, antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {2575, 275, 292, 31, 634, 618, 204, 628}

$$-\frac{\log\left(1-\frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}\right)}{2b}+\frac{\log\left(\frac{\cosh^{\frac{4}{3}}(a+bx)}{\sinh^{\frac{4}{3}}(a+bx)}+\frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}+1\right)}{4b}-\frac{\sqrt{3}\tan^{-1}\left(\frac{\frac{2\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}+1}{\sqrt{3}}\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[a + b*x]^(1/3)/Sinh[a + b*x]^(1/3),x]
```

```
[Out] -(Sqrt[3]*ArcTan[(1 + (2*Cosh[a + b*x]^(2/3))/Sinh[a + b*x]^(2/3))/Sqrt[3]]
)/(2*b) - Log[1 - Cosh[a + b*x]^(2/3)/Sinh[a + b*x]^(2/3)]/(2*b) + Log[1 +
Cosh[a + b*x]^(4/3)/Sinh[a + b*x]^(4/3) + Cosh[a + b*x]^(2/3)/Sinh[a + b*x]
^(2/3)]/(4*b)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2575

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> With[{k = Denominator[m]}, -Dist[(k*a*b)/f, Subst[Int[x^(k
*(m + 1) - 1)/(a^2 + b^2*x^(2*k)), x], x, (a*Cos[e + f*x])^(1/k)/(b*Sin[e +
 f*x])^(1/k)], x]] /; FreeQ[{a, b, e, f}, x] && EqQ[m + n, 0] && GtQ[m, 0]
&& LtQ[m, 1]
```

Rubi steps

$$\int \frac{\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}\,dx = \frac{3\,\text{Subst}\left(\int \frac{x^3}{1-x^6}\,dx, x, \frac{\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}\right)}{b}$$
$$= \frac{3\,\text{Subst}\left(\int \frac{x}{1-x^3}\,dx, x, \frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}\right)}{2b}$$
$$= \frac{\text{Subst}\left(\int \frac{1}{1-x}\,dx, x, \frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}\right)}{2b} - \frac{\text{Subst}\left(\int \frac{1-x}{1+x+x^2}\,dx, x, \frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}\right)}{2b}$$
$$= -\frac{\log\left(1-\frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}\right)}{2b} + \frac{\text{Subst}\left(\int \frac{1+2x}{1+x+x^2}\,dx, x, \frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}\right)}{4b} - \frac{3\,\text{Subst}\left(\int \frac{1}{1+x+x^2}\,dx, x, \frac{\text{co}}{\text{si}}\right.}{4b}$$
$$= -\frac{\log\left(1-\frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}\right)}{2b} + \frac{\log\left(1+\frac{\cosh^{\frac{4}{3}}(a+bx)}{\sinh^{\frac{4}{3}}(a+bx)}+\frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}\right)}{4b} + \frac{3\,\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+\right.}{2b}$$
$$= -\frac{\sqrt{3}\tan^{-1}\left(\frac{1+\frac{2\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}}{\sqrt{3}}\right)}{2b} - \frac{\log\left(1-\frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}\right)}{2b} + \frac{\log\left(1+\frac{\cosh^{\frac{4}{3}}(a+bx)}{\sinh^{\frac{4}{3}}(a+bx)}+\frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}\right)}{4b}$$

Mathematica [**C**] time = 0.03, size = 59, normalized size = 0.46

$$\frac{3\sinh^{\frac{2}{3}}(a+bx)\sqrt[3]{\cosh^2(a+bx)}\,{}_2F_1\left(\frac{1}{3},\frac{1}{3};\frac{4}{3};-\sinh^2(a+bx)\right)}{2b\cosh^{\frac{2}{3}}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[a + b*x]^(1/3)/Sinh[a + b*x]^(1/3),x]
```

```
[Out] (3*(Cosh[a + b*x]^2)^(1/3)*Hypergeometric2F1[1/3, 1/3, 4/3, -Sinh[a + b*x]^
2]*Sinh[a + b*x]^(2/3))/(2*b*Cosh[a + b*x]^(2/3))
```

fricas [**B**] time = 0.46, size = 578, normalized size = 4.52

$$-\frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}\cosh(bx+a)^2+2\sqrt{3}\cosh(bx+a)\sinh(bx+a)+\sqrt{3}\sinh(bx+a)^2+4\left(\sqrt{3}\cosh(bx+a)+\sqrt{3}\sinh(bx+a)\right)\cosh(bx+a)^{\frac{2}{3}}\sinh(bx+a)^{\frac{1}{3}}}{3\left(\cosh(bx+a)^2+2\cosh(bx+a)\sinh(bx+a)+\sinh(bx+a)^2-1\right)}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(b*x+a)^(1/3)/sinh(b*x+a)^(1/3),x, algorithm="fricas")

[Out] -1/4*(2*sqrt(3)*arctan(1/3*(sqrt(3)*cosh(b*x + a)^2 + 2*sqrt(3)*cosh(b*x + a)*sinh(b*x + a) + sqrt(3)*sinh(b*x + a)^2 + 4*(sqrt(3)*cosh(b*x + a) + sqrt(3)*sinh(b*x + a))*cosh(b*x + a)^(2/3)*sinh(b*x + a)^(1/3) - sqrt(3))/(cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 - 1)) - log((cosh(b*x + a)^4 + 4*cosh(b*x + a)*sinh(b*x + a)^3 + sinh(b*x + a)^4 + 2*(3*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^2 - 2*cosh(b*x + a)^2 + 2*(cosh(b*x + a)^3 + 3*cosh(b*x + a)*sinh(b*x + a)^2 + sinh(b*x + a)^3 + (3*cosh(b*x + a)^2 - 1)*sinh(b*x + a) - cosh(b*x + a))*cosh(b*x + a)^(2/3)*sinh(b*x + a)^(1/3) + 2*(cosh(b*x + a)^3 + 3*cosh(b*x + a)*sinh(b*x + a)^2 + sinh(b*x + a)^3 + (3*cosh(b*x + a)^2 + 1)*sinh(b*x + a) + cosh(b*x + a))*cosh(b*x + a)^(1/3)*sinh(b*x + a)^(2/3) + 4*(cosh(b*x + a)^3 - cosh(b*x + a))*sinh(b*x + a) + 1)/(cosh(b*x + a)^4 + 4*cosh(b*x + a)*sinh(b*x + a)^3 + sinh(b*x + a)^4 + 2*(3*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^2 - 2*cosh(b*x + a)^2 + 4*(cosh(b*x + a)^3 - cosh(b*x + a))*sinh(b*x + a) + 1)) + 2*log(-(cosh(b*x + a)^2 - 2*(cosh(b*x + a) + sinh(b*x + a))*cosh(b*x + a)^(2/3)*sinh(b*x + a)^(1/3) + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 - 1)/(cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 - 1)))/b

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh(bx+a)^{\frac{1}{3}}}{\sinh(bx+a)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(b*x+a)^(1/3)/sinh(b*x+a)^(1/3),x, algorithm="giac")

[Out] integrate(cosh(b*x + a)^(1/3)/sinh(b*x + a)^(1/3), x)

maple [F] time = 0.15, size = 0, normalized size = 0.00

$$\int \frac{\cosh^{\frac{1}{3}}(bx+a)}{\sinh(bx+a)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cosh(b*x+a)^(1/3)/sinh(b*x+a)^(1/3),x)

[Out] int(cosh(b*x+a)^(1/3)/sinh(b*x+a)^(1/3),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh(bx+a)^{\frac{1}{3}}}{\sinh(bx+a)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^(1/3)/sinh(b*x+a)^(1/3),x, algorithm="maxima")
[Out] integrate(cosh(b*x + a)^(1/3)/sinh(b*x + a)^(1/3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cosh(a+b\,x)^{1/3}}{\sinh(a+b\,x)^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a + b*x)^(1/3)/sinh(a + b*x)^(1/3),x)
[Out] int(cosh(a + b*x)^(1/3)/sinh(a + b*x)^(1/3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)**(1/3)/sinh(b*x+a)**(1/3),x)
[Out] Integral(cosh(a + b*x)**(1/3)/sinh(a + b*x)**(1/3), x)
```

3.63 $\int \frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}\,dx$

Optimal. Leaf size=218

$$-\frac{\log\left(\frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}-\frac{\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}+1\right)}{4b}+\frac{\log\left(\frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}+\frac{\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}+1\right)}{4b}+\frac{\sqrt{3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}}{\sqrt{3}}\right)}{2b}-\frac{\sqrt{3}\tan^{-1}}{}$$

```
[Out] arctanh(cosh(b*x+a)^(1/3)/sinh(b*x+a)^(1/3))/b-1/4*ln(1+cosh(b*x+a)^(2/3)/s
inh(b*x+a)^(2/3)-cosh(b*x+a)^(1/3)/sinh(b*x+a)^(1/3))/b+1/4*ln(1+cosh(b*x+a
)^(2/3)/sinh(b*x+a)^(2/3)+cosh(b*x+a)^(1/3)/sinh(b*x+a)^(1/3))/b+1/2*arctan
(1/3*(1-2*cosh(b*x+a)^(1/3)/sinh(b*x+a)^(1/3))*3^(1/2))*3^(1/2)/b-1/2*arcta
n(1/3*(1+2*cosh(b*x+a)^(1/3)/sinh(b*x+a)^(1/3))*3^(1/2))*3^(1/2)/b
```

Rubi [A] time = 0.20, antiderivative size = 218, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2575, 296, 634, 618, 204, 628, 206}

$$-\frac{\log\left(\frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}-\frac{\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}+1\right)}{4b}+\frac{\log\left(\frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}+\frac{\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}+1\right)}{4b}+\frac{\sqrt{3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}}{\sqrt{3}}\right)}{2b}-\frac{\sqrt{3}\tan^{-1}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[a + b*x]^(2/3)/Sinh[a + b*x]^(2/3),x]
```

```
[Out] (Sqrt[3]*ArcTan[(1 - (2*Cosh[a + b*x]^(1/3))/Sinh[a + b*x]^(1/3))/Sqrt[3]])
/(2*b) - (Sqrt[3]*ArcTan[(1 + (2*Cosh[a + b*x]^(1/3))/Sinh[a + b*x]^(1/3))/
Sqrt[3]])/(2*b) + ArcTanh[Cosh[a + b*x]^(1/3)/Sinh[a + b*x]^(1/3)]/b - Log[
1 + Cosh[a + b*x]^(2/3)/Sinh[a + b*x]^(2/3) - Cosh[a + b*x]^(1/3)/Sinh[a +
b*x]^(1/3)]/(4*b) + Log[1 + Cosh[a + b*x]^(2/3)/Sinh[a + b*x]^(2/3) + Cosh[
a + b*x]^(1/3)/Sinh[a + b*x]^(1/3)]/(4*b)
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 296

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[-(a/b), n]], s = Denominator[Rt[-(a/b), n]], k, u}, Simp[u = Int[(r*Cos
[(2*k*m*Pi)/n] - s*Cos[(2*k*(m + 1)*Pi)/n]*x)/(r^2 - 2*r*s*Cos[(2*k*Pi)/n]*
x + s^2*x^2), x] + Int[(r*Cos[(2*k*m*Pi)/n] + s*Cos[(2*k*(m + 1)*Pi)/n]*x)/
(r^2 + 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x]; (2*r^(m + 2)*Int[1/(r^2 - s^
2*x^2), x])/(a*n*s^m) + Dist[(2*r^(m + 1))/(a*n*s^m), Sum[u, {k, 1, (n - 2)
/4}], x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && IGtQ[m, 0] && Lt
Q[m, n - 1] && NegQ[a/b]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2575

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> With[{k = Denominator[m]}, -Dist[(k*a*b)/f, Subst[Int[x^(k
*(m + 1) - 1)/(a^2 + b^2*x^(2*k)), x], x, (a*Cos[e + f*x])^(1/k)/(b*Sin[e +
 f*x])^(1/k)], x]] /; FreeQ[{a, b, e, f}, x] && EqQ[m + n, 0] && GtQ[m, 0]
&& LtQ[m, 1]
```

Rubi steps

$$\int \frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}\,dx = \frac{3\,\text{Subst}\left(\int \frac{x^4}{1-x^6}\,dx, x, \frac{\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}\right)}{b}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}\right)}{b} + \frac{\text{Subst}\left(\int \frac{-\frac{1}{2}-\frac{x}{2}}{1-x+x^2}\,dx, x, \frac{\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}\right)}{b} + \frac{\text{Subst}\left(\int \frac{-\frac{1}{2}+}{1+x+}\right.}{}$$

$$= \frac{\tanh^{-1}\left(\frac{\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}\right)}{b} - \frac{\text{Subst}\left(\int \frac{-1+2x}{1-x+x^2}\,dx, x, \frac{\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}\right)}{4b} + \frac{\text{Subst}\left(\int \frac{1+2x}{1+x+x^2}\,dx, x, \frac{\sqrt[3]{\text{co}}}{\sqrt[3]{\text{si}}}\right.}{4b}$$

$$= \frac{\tanh^{-1}\left(\frac{\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}\right)}{b} - \frac{\log\left(1+\frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}-\frac{\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}\right)}{4b} + \frac{\log\left(1+\frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}+\frac{\sqrt[3]{\text{cos}}}{\sqrt[3]{\text{sin}}}\right.}{4b}$$

$$= \frac{\sqrt{3}\,\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}}{\sqrt{3}}\right)}{2b} - \frac{\sqrt{3}\,\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}}{\sqrt{3}}\right)}{2b} + \frac{\tanh^{-1}\left(\frac{\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}\right)}{b} - \frac{\log\left(1+\right.}{}$$

Mathematica [C] time = 0.03, size = 57, normalized size = 0.26

$$\frac{3\sqrt[3]{\sinh(a+bx)}\,\sqrt[6]{\cosh^2(a+bx)}\,{}_2F_1\left(\frac{1}{6},\frac{1}{6};\frac{7}{6};-\sinh^2(a+bx)\right)}{b\sqrt[3]{\cosh(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[a + b*x]^(2/3)/Sinh[a + b*x]^(2/3),x]
```

```
[Out] (3*(Cosh[a + b*x]^2)^(1/6)*Hypergeometric2F1[1/6, 1/6, 7/6, -Sinh[a + b*x]^
2]*Sinh[a + b*x]^(1/3))/(b*Cosh[a + b*x]^(1/3))
```

fricas [B] time = 0.47, size = 723, normalized size = 3.32

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^(2/3)/sinh(b*x+a)^(2/3),x, algorithm="fricas")
```

```
[Out] 1/4*(2*sqrt(3)*arctan(1/3*(sqrt(3)*cosh(b*x + a)^2 + 2*sqrt(3)*cosh(b*x + a
)*sinh(b*x + a) + sqrt(3)*sinh(b*x + a)^2 + 4*(sqrt(3)*cosh(b*x + a) + sqrt
(3)*sinh(b*x + a))*cosh(b*x + a)^(2/3)*sinh(b*x + a)^(1/3) + sqrt(3))/(cosh
```

```
(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 + 1)) + 2*sqr
t(3)*arctan(-1/3*(sqrt(3)*cosh(b*x + a)^2 + 2*sqrt(3)*cosh(b*x + a)*sinh(b*
x + a) + sqrt(3)*sinh(b*x + a)^2 - 4*(sqrt(3)*cosh(b*x + a) + sqrt(3)*sinh(
b*x + a))*cosh(b*x + a)^(2/3)*sinh(b*x + a)^(1/3) + sqrt(3))/(cosh(b*x + a)
^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 + 1)) + log((cosh(b*x
+ a)^2 + 2*(cosh(b*x + a) + sinh(b*x + a))*cosh(b*x + a)^(2/3)*sinh(b*x + a
)^(1/3) + 2*(cosh(b*x + a) + sinh(b*x + a))*cosh(b*x + a)^(1/3)*sinh(b*x +
a)^(2/3) + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 + 1)/(cosh(b*x +
 a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 + 1)) + 2*log((cosh
(b*x + a)^2 + 2*(cosh(b*x + a) + sinh(b*x + a))*cosh(b*x + a)^(2/3)*sinh(b*
x + a)^(1/3) + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 + 1)/(cosh(b
*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 + 1)) - log((co
sh(b*x + a)^2 - 2*(cosh(b*x + a) + sinh(b*x + a))*cosh(b*x + a)^(2/3)*sinh(
b*x + a)^(1/3) + 2*(cosh(b*x + a) + sinh(b*x + a))*cosh(b*x + a)^(1/3)*sinh
(b*x + a)^(2/3) + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 + 1)/(cos
h(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 + 1)) - 2*lo
g(-(cosh(b*x + a)^2 - 2*(cosh(b*x + a) + sinh(b*x + a))*cosh(b*x + a)^(2/3)
*sinh(b*x + a)^(1/3) + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 + 1)
/(cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 + 1)))/
b
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh(bx+a)^{\frac{2}{3}}}{\sinh(bx+a)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^(2/3)/sinh(b*x+a)^(2/3),x, algorithm="giac")
```

```
[Out] integrate(cosh(b*x + a)^(2/3)/sinh(b*x + a)^(2/3), x)
```

maple [F] time = 0.18, size = 0, normalized size = 0.00

$$\int \frac{\cosh^{\frac{2}{3}}(bx+a)}{\sinh(bx+a)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)^(2/3)/sinh(b*x+a)^(2/3),x)
```

```
[Out] int(cosh(b*x+a)^(2/3)/sinh(b*x+a)^(2/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh\left(bx+a\right)^{\frac{2}{3}}}{\sinh\left(bx+a\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^(2/3)/sinh(b*x+a)^(2/3),x, algorithm="maxima")
[Out] integrate(cosh(b*x + a)^(2/3)/sinh(b*x + a)^(2/3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\cosh\left(a+b\,x\right)}^{2/3}}{{\sinh\left(a+b\,x\right)}^{2/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a + b*x)^(2/3)/sinh(a + b*x)^(2/3),x)
[Out] int(cosh(a + b*x)^(2/3)/sinh(a + b*x)^(2/3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh^{\frac{2}{3}}\left(a+bx\right)}{\sinh^{\frac{2}{3}}\left(a+bx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)**(2/3)/sinh(b*x+a)**(2/3),x)
[Out] Integral(cosh(a + b*x)**(2/3)/sinh(a + b*x)**(2/3), x)
```

3.64 $\int \frac{\cosh^{\frac{4}{3}}(a+bx)}{\sinh^{\frac{4}{3}}(a+bx)}\,dx$

Optimal. Leaf size=243

$$-\frac{3\sqrt[3]{\cosh(a+bx)}}{b\sqrt[3]{\sinh(a+bx)}}-\frac{\log\left(\frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}-\frac{\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}+1\right)}{4b}+\frac{\log\left(\frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}+\frac{\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}+1\right)}{4b}+\frac{\sqrt{3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{\sinh(a}}{\sqrt[3]{\cosh(a}}}{\sqrt{3}}\right.}{2b}$$

```
[Out] arctanh(sinh(b*x+a)^(1/3)/cosh(b*x+a)^(1/3))/b-1/4*ln(1-sinh(b*x+a)^(1/3)/c
osh(b*x+a)^(1/3)+sinh(b*x+a)^(2/3)/cosh(b*x+a)^(2/3))/b+1/4*ln(1+sinh(b*x+a
)^(1/3)/cosh(b*x+a)^(1/3)+sinh(b*x+a)^(2/3)/cosh(b*x+a)^(2/3))/b-3*cosh(b*x
+a)^(1/3)/b/sinh(b*x+a)^(1/3)+1/2*arctan(1/3*(1-2*sinh(b*x+a)^(1/3)/cosh(b*
x+a)^(1/3))*3^(1/2))*3^(1/2)/b-1/2*arctan(1/3*(1+2*sinh(b*x+a)^(1/3)/cosh(b
*x+a)^(1/3))*3^(1/2))*3^(1/2)/b
```

Rubi [A] time = 0.24, antiderivative size = 243, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {2567, 2574, 296, 634, 618, 204, 628, 206}

$$-\frac{3\sqrt[3]{\cosh(a+bx)}}{b\sqrt[3]{\sinh(a+bx)}}-\frac{\log\left(\frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}-\frac{\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}+1\right)}{4b}+\frac{\log\left(\frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}+\frac{\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}+1\right)}{4b}+\frac{\sqrt{3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{\sinh(a}}{\sqrt[3]{\cosh(a}}}{\sqrt{3}}\right.}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[a + b*x]^(4/3)/Sinh[a + b*x]^(4/3),x]
```

```
[Out] (Sqrt[3]*ArcTan[(1 - (2*Sinh[a + b*x]^(1/3))/Cosh[a + b*x]^(1/3))/Sqrt[3]])
/(2*b) - (Sqrt[3]*ArcTan[(1 + (2*Sinh[a + b*x]^(1/3))/Cosh[a + b*x]^(1/3))/
Sqrt[3]])/(2*b) + ArcTanh[Sinh[a + b*x]^(1/3)/Cosh[a + b*x]^(1/3)]/b - Log[
1 - Sinh[a + b*x]^(1/3)/Cosh[a + b*x]^(1/3) + Sinh[a + b*x]^(2/3)/Cosh[a +
b*x]^(2/3)]/(4*b) + Log[1 + Sinh[a + b*x]^(1/3)/Cosh[a + b*x]^(1/3) + Sinh[
a + b*x]^(2/3)/Cosh[a + b*x]^(2/3)]/(4*b) - (3*Cosh[a + b*x]^(1/3))/(b*Sinh
[a + b*x]^(1/3))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 296

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[-(a/b), n]], s = Denominator[Rt[-(a/b), n]], k, u}, Simp[u = Int[(r*Cos
[(2*k*m*Pi)/n] - s*Cos[(2*k*(m + 1)*Pi)/n]*x)/(r^2 - 2*r*s*Cos[(2*k*Pi)/n]*
x + s^2*x^2), x] + Int[(r*Cos[(2*k*m*Pi)/n] + s*Cos[(2*k*(m + 1)*Pi)/n]*x)/
(r^2 + 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x]; (2*r^(m + 2)*Int[1/(r^2 - s^
2*x^2), x])/(a*n*s^m) + Dist[(2*r^(m + 1))/(a*n*s^m), Sum[u, {k, 1, (n - 2)
/4}], x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && IGtQ[m, 0] && Lt
Q[m, n - 1] && NegQ[a/b]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2567

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(a*(a*Cos[e + f*x])^(m - 1)*(b*Sin[e + f*x])^(n + 1))
/(b*f*(n + 1)), x] + Dist[(a^2*(m - 1))/(b^2*(n + 1)), Int[(a*Cos[e + f*x])
^(m - 2)*(b*Sin[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ[
m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

Rule 2574

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> With[{k = Denominator[m]}, Dist[(k*a*b)/f, Subst[Int[x^(k*
(m + 1) - 1)/(a^2 + b^2*x^(2*k)), x], x, (a*Sin[e + f*x])^(1/k)/(b*Cos[e +
f*x])^(1/k)], x]] /; FreeQ[{a, b, e, f}, x] && EqQ[m + n, 0] && GtQ[m, 0] &
& LtQ[m, 1]
```

Rubi steps

$$\int \frac{\cosh^{\frac{4}{3}}(a+bx)}{\sinh^{\frac{4}{3}}(a+bx)}\,dx = -\frac{3\sqrt[3]{\cosh(a+bx)}}{b\sqrt[3]{\sinh(a+bx)}} + \int \frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}\,dx$$

$$= -\frac{3\sqrt[3]{\cosh(a+bx)}}{b\sqrt[3]{\sinh(a+bx)}} - \frac{3\,\text{Subst}\left(\int \frac{x^4}{-1+x^6}\,dx, x, \frac{\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}\right)}{b}$$

$$= -\frac{3\sqrt[3]{\cosh(a+bx)}}{b\sqrt[3]{\sinh(a+bx)}} + \frac{\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}\right)}{b} + \frac{\text{Subst}\left(\int \frac{-\frac{1}{2}-\frac{x}{2}}{1-x+x^2}\,dx, x, \frac{\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}\right)}{b}$$

$$= \frac{\tanh^{-1}\left(\frac{\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}\right)}{b} - \frac{3\sqrt[3]{\cosh(a+bx)}}{b\sqrt[3]{\sinh(a+bx)}} - \frac{\text{Subst}\left(\int \frac{-1+2x}{1-x+x^2}\,dx, x, \frac{\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}\right)}{4b} + \frac{\text{Subst}\left(\right.}{}$$

$$= \frac{\tanh^{-1}\left(\frac{\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}\right)}{b} - \frac{\log\left(1-\frac{\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}+\frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}\right)}{4b} + \frac{\log\left(1+\frac{\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}+\frac{\sinh^{\frac{2}{3}}}{\cosh^{\frac{2}{3}}}\right.}{4b}$$

$$= \frac{\sqrt{3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}}{\sqrt{3}}\right)}{2b} - \frac{\sqrt{3}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}}{\sqrt{3}}\right)}{2b} + \frac{\tanh^{-1}\left(\frac{\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}\right)}{b} - \frac{\log\left(1-\right.}{}$$

Mathematica [C] time = 0.03, size = 57, normalized size = 0.23

$$-\frac{3\cosh^2(a+bx)^{5/6}\,{}_2F_1\left(-\frac{1}{6},-\frac{1}{6};\frac{5}{6};-\sinh^2(a+bx)\right)}{b\sqrt[3]{\sinh(a+bx)}\cosh^{\frac{5}{3}}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[a + b*x]^(4/3)/Sinh[a + b*x]^(4/3),x]
```

```
[Out] (-3*(Cosh[a + b*x]^2)^(5/6)*Hypergeometric2F1[-1/6, -1/6, 5/6, -Sinh[a + b*
x]^2])/(b*Cosh[a + b*x]^(5/3)*Sinh[a + b*x]^(1/3))
```

fricas [B] time = 0.45, size = 1013, normalized size = 4.17

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^(4/3)/sinh(b*x+a)^(4/3),x, algorithm="fricas")
```

```
[Out] 1/4*(2*(sqrt(3)*cosh(b*x + a)^2 + 2*sqrt(3)*cosh(b*x + a)*sinh(b*x + a) + s
qrt(3)*sinh(b*x + a)^2 - sqrt(3))*arctan(1/3*(sqrt(3)*cosh(b*x + a)^2 + 2*s
qrt(3)*cosh(b*x + a)*sinh(b*x + a) + sqrt(3)*sinh(b*x + a)^2 + 4*(sqrt(3)*c
osh(b*x + a) + sqrt(3)*sinh(b*x + a))*cosh(b*x + a)^(1/3)*sinh(b*x + a)^(2/
3) - sqrt(3))/(cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x +
 a)^2 - 1)) + 2*(sqrt(3)*cosh(b*x + a)^2 + 2*sqrt(3)*cosh(b*x + a)*sinh(b*x
 + a) + sqrt(3)*sinh(b*x + a)^2 - sqrt(3))*arctan(-1/3*(sqrt(3)*cosh(b*x +
a)^2 + 2*sqrt(3)*cosh(b*x + a)*sinh(b*x + a) + sqrt(3)*sinh(b*x + a)^2 - 4*
(sqrt(3)*cosh(b*x + a) + sqrt(3)*sinh(b*x + a))*cosh(b*x + a)^(1/3)*sinh(b*
x + a)^(2/3) - sqrt(3))/(cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) +
sinh(b*x + a)^2 - 1)) + (cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) +
sinh(b*x + a)^2 - 1)*log((cosh(b*x + a)^2 + 2*(cosh(b*x + a) + sinh(b*x + a
))*cosh(b*x + a)^(2/3)*sinh(b*x + a)^(1/3) + 2*(cosh(b*x + a) + sinh(b*x +
a))*cosh(b*x + a)^(1/3)*sinh(b*x + a)^(2/3) + 2*cosh(b*x + a)*sinh(b*x + a)
 + sinh(b*x + a)^2 - 1)/(cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) +
sinh(b*x + a)^2 - 1)) - (cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) +
sinh(b*x + a)^2 - 1)*log((cosh(b*x + a)^2 + 2*(cosh(b*x + a) + sinh(b*x + a
))*cosh(b*x + a)^(2/3)*sinh(b*x + a)^(1/3) - 2*(cosh(b*x + a) + sinh(b*x +
a))*cosh(b*x + a)^(1/3)*sinh(b*x + a)^(2/3) + 2*cosh(b*x + a)*sinh(b*x + a)
 + sinh(b*x + a)^2 - 1)/(cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) +
sinh(b*x + a)^2 - 1)) + 2*(cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a)
+ sinh(b*x + a)^2 - 1)*log((cosh(b*x + a)^2 + 2*(cosh(b*x + a) + sinh(b*x +
 a))*cosh(b*x + a)^(1/3)*sinh(b*x + a)^(2/3) + 2*cosh(b*x + a)*sinh(b*x + a
) + sinh(b*x + a)^2 - 1)/(cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) +
 sinh(b*x + a)^2 - 1)) - 2*(cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a)
 + sinh(b*x + a)^2 - 1)*log(-(cosh(b*x + a)^2 - 2*(cosh(b*x + a) + sinh(b*x
 + a))*cosh(b*x + a)^(1/3)*sinh(b*x + a)^(2/3) + 2*cosh(b*x + a)*sinh(b*x +
 a) + sinh(b*x + a)^2 - 1)/(cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a)
 + sinh(b*x + a)^2 - 1)) - 24*(cosh(b*x + a) + sinh(b*x + a))*cosh(b*x + a)
^(1/3)*sinh(b*x + a)^(2/3))/(b*cosh(b*x + a)^2 + 2*b*cosh(b*x + a)*sinh(b*x
 + a) + b*sinh(b*x + a)^2 - b)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh(bx+a)^{\frac{4}{3}}}{\sinh(bx+a)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^(4/3)/sinh(b*x+a)^(4/3),x, algorithm="giac")
[Out] integrate(cosh(b*x + a)^(4/3)/sinh(b*x + a)^(4/3), x)
```

maple [F] time = 0.15, size = 0, normalized size = 0.00

$$\int \frac{\cosh^{\frac{4}{3}}(bx+a)}{\sinh(bx+a)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)^(4/3)/sinh(b*x+a)^(4/3),x)
[Out] int(cosh(b*x+a)^(4/3)/sinh(b*x+a)^(4/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh(bx+a)^{\frac{4}{3}}}{\sinh(bx+a)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^(4/3)/sinh(b*x+a)^(4/3),x, algorithm="maxima")
[Out] integrate(cosh(b*x + a)^(4/3)/sinh(b*x + a)^(4/3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cosh(a+b\,x)^{4/3}}{\sinh(a+b\,x)^{4/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a + b*x)^(4/3)/sinh(a + b*x)^(4/3),x)
[Out] int(cosh(a + b*x)^(4/3)/sinh(a + b*x)^(4/3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh^{\frac{4}{3}}(a+bx)}{\sinh^{\frac{4}{3}}(a+bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)**(4/3)/sinh(b*x+a)**(4/3),x)
```

```
[Out] Integral(cosh(a + b*x)**(4/3)/sinh(a + b*x)**(4/3), x)
```

3.65 $\int \frac{\cosh^{\frac{5}{3}}(a+bx)}{\sinh^{\frac{5}{3}}(a+bx)}\,dx$

Optimal. Leaf size=155

$$-\frac{3\cosh^{\frac{2}{3}}(a+bx)}{2b\sinh^{\frac{2}{3}}(a+bx)}-\frac{\log\left(1-\frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}\right)}{2b}+\frac{\log\left(\frac{\sinh^{\frac{4}{3}}(a+bx)}{\cosh^{\frac{4}{3}}(a+bx)}+\frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}+1\right)}{4b}-\frac{\sqrt{3}\tan^{-1}\left(\frac{\frac{2\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}+1}{\sqrt{3}}\right)}{2b}$$

```
[Out] -1/2*ln(1-sinh(b*x+a)^(2/3)/cosh(b*x+a)^(2/3))/b+1/4*ln(1+sinh(b*x+a)^(2/3)
/cosh(b*x+a)^(2/3)+sinh(b*x+a)^(4/3)/cosh(b*x+a)^(4/3))/b-3/2*cosh(b*x+a)^(
2/3)/b/sinh(b*x+a)^(2/3)-1/2*arctan(1/3*(1+2*sinh(b*x+a)^(2/3)/cosh(b*x+a)^
(2/3))*3^(1/2))*3^(1/2)/b
```

Rubi [A] time = 0.12, antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {2567, 2574, 275, 292, 31, 634, 618, 204, 628}

$$-\frac{3\cosh^{\frac{2}{3}}(a+bx)}{2b\sinh^{\frac{2}{3}}(a+bx)}-\frac{\log\left(1-\frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}\right)}{2b}+\frac{\log\left(\frac{\sinh^{\frac{4}{3}}(a+bx)}{\cosh^{\frac{4}{3}}(a+bx)}+\frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}+1\right)}{4b}-\frac{\sqrt{3}\tan^{-1}\left(\frac{\frac{2\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}+1}{\sqrt{3}}\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[a + b*x]^(5/3)/Sinh[a + b*x]^(5/3),x]
```

```
[Out] -(Sqrt[3]*ArcTan[(1 + (2*Sinh[a + b*x]^(2/3))/Cosh[a + b*x]^(2/3))/Sqrt[3]]
)/(2*b) - Log[1 - Sinh[a + b*x]^(2/3)/Cosh[a + b*x]^(2/3)]/(2*b) + Log[1 +
Sinh[a + b*x]^(2/3)/Cosh[a + b*x]^(2/3) + Sinh[a + b*x]^(4/3)/Cosh[a + b*x]
^(4/3)]/(4*b) - (3*Cosh[a + b*x]^(2/3))/(2*b*Sinh[a + b*x]^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2567

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(a*(a*Cos[e + f*x])^(m - 1)*(b*Sin[e + f*x])^(n + 1))
/(b*f*(n + 1)), x] + Dist[(a^2*(m - 1))/(b^2*(n + 1)), Int[(a*Cos[e + f*x])
^(m - 2)*(b*Sin[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ[
m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

Rule 2574

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> With[{k = Denominator[m]}, Dist[(k*a*b)/f, Subst[Int[x^(k*
(m + 1) - 1)/(a^2 + b^2*x^(2*k)), x], x, (a*Sin[e + f*x])^(1/k)/(b*Cos[e +
f*x])^(1/k)], x]] /; FreeQ[{a, b, e, f}, x] && EqQ[m + n, 0] && GtQ[m, 0] &
& LtQ[m, 1]
```

Rubi steps

$$\int \frac{\cosh^{\frac{5}{3}}(a+bx)}{\sinh^{\frac{5}{3}}(a+bx)}\,dx = -\frac{3\cosh^{\frac{2}{3}}(a+bx)}{2b\sinh^{\frac{2}{3}}(a+bx)} + \int \frac{\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}\,dx$$

$$= -\frac{3\cosh^{\frac{2}{3}}(a+bx)}{2b\sinh^{\frac{2}{3}}(a+bx)} - \frac{3\,\text{Subst}\left(\int \frac{x^3}{-1+x^6}\,dx, x, \frac{\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}\right)}{b}$$

$$= -\frac{3\cosh^{\frac{2}{3}}(a+bx)}{2b\sinh^{\frac{2}{3}}(a+bx)} - \frac{3\,\text{Subst}\left(\int \frac{x}{-1+x^3}\,dx, x, \frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}\right)}{2b}$$

$$= -\frac{3\cosh^{\frac{2}{3}}(a+bx)}{2b\sinh^{\frac{2}{3}}(a+bx)} - \frac{\text{Subst}\left(\int \frac{1}{-1+x}\,dx, x, \frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}\right)}{2b} + \frac{\text{Subst}\left(\int \frac{-1+x}{1+x+x^2}\,dx, x, \frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx}\right.}{2b}$$

$$= -\frac{\log\left(1-\frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}\right)}{2b} - \frac{3\cosh^{\frac{2}{3}}(a+bx)}{2b\sinh^{\frac{2}{3}}(a+bx)} + \frac{\text{Subst}\left(\int \frac{1+2x}{1+x+x^2}\,dx, x, \frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}\right)}{4b} - \frac{3\,\text{Subs}}{}$$

$$= -\frac{\log\left(1-\frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}\right)}{2b} + \frac{\log\left(1+\frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}+\frac{\sinh^{\frac{4}{3}}(a+bx)}{\cosh^{\frac{4}{3}}(a+bx)}\right)}{4b} - \frac{3\cosh^{\frac{2}{3}}(a+bx)}{2b\sinh^{\frac{2}{3}}(a+bx)} + \frac{3\,\text{Subs}}{}$$

$$= -\frac{\sqrt{3}\tan^{-1}\left(\frac{1+\frac{2\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}}{\sqrt{3}}\right)}{2b} - \frac{\log\left(1-\frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}\right)}{2b} + \frac{\log\left(1+\frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}+\frac{\sinh^{\frac{4}{3}}(a+bx)}{\cosh^{\frac{4}{3}}(a+bx)}\right)}{4b} -$$

Mathematica [C] time = 0.04, size = 59, normalized size = 0.38

$$-\frac{3\cosh^2(a+bx)^{2/3}\,{}_2F_1\left(-\frac{1}{3},-\frac{1}{3};\frac{2}{3};-\sinh^2(a+bx)\right)}{2b\sinh^{\frac{2}{3}}(a+bx)\cosh^{\frac{4}{3}}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[a + b*x]^(5/3)/Sinh[a + b*x]^(5/3),x]
```

```
[Out] (-3*(Cosh[a + b*x]^2)^(2/3)*Hypergeometric2F1[-1/3, -1/3, 2/3, -Sinh[a + b*
x]^2])/(2*b*Cosh[a + b*x]^(4/3)*Sinh[a + b*x]^(2/3))
```

fricas [B] time = 0.46, size = 749, normalized size = 4.83

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^(5/3)/sinh(b*x+a)^(5/3),x, algorithm="fricas")
```

```
[Out] -1/4*(2*(sqrt(3)*cosh(b*x + a)^2 + 2*sqrt(3)*cosh(b*x + a)*sinh(b*x + a) +
sqrt(3)*sinh(b*x + a)^2 - sqrt(3))*arctan(1/3*(sqrt(3)*cosh(b*x + a)^2 + 2*
sqrt(3)*cosh(b*x + a)*sinh(b*x + a) + sqrt(3)*sinh(b*x + a)^2 + 4*(sqrt(3)*
cosh(b*x + a) + sqrt(3)*sinh(b*x + a))*cosh(b*x + a)^(1/3)*sinh(b*x + a)^(2
/3) + sqrt(3))/(cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x
+ a)^2 + 1)) - (cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x
+ a)^2 - 1)*log((cosh(b*x + a)^4 + 4*cosh(b*x + a)*sinh(b*x + a)^3 + sinh(b
*x + a)^4 + 2*(3*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^2 + 2*cosh(b*x + a)^2 +
 2*(cosh(b*x + a)^3 + 3*cosh(b*x + a)*sinh(b*x + a)^2 + sinh(b*x + a)^3 + (
3*cosh(b*x + a)^2 - 1)*sinh(b*x + a) - cosh(b*x + a))*cosh(b*x + a)^(2/3)*s
inh(b*x + a)^(1/3) + 2*(cosh(b*x + a)^3 + 3*cosh(b*x + a)*sinh(b*x + a)^2 +
 sinh(b*x + a)^3 + (3*cosh(b*x + a)^2 + 1)*sinh(b*x + a) + cosh(b*x + a))*c
osh(b*x + a)^(1/3)*sinh(b*x + a)^(2/3) + 4*(cosh(b*x + a)^3 + cosh(b*x + a)
)*sinh(b*x + a) + 1)/(cosh(b*x + a)^4 + 4*cosh(b*x + a)*sinh(b*x + a)^3 + s
inh(b*x + a)^4 + 2*(3*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^2 + 2*cosh(b*x + a
)^2 + 4*(cosh(b*x + a)^3 + cosh(b*x + a))*sinh(b*x + a) + 1)) + 2*(cosh(b*x
 + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 - 1)*log(-(cosh(b
*x + a)^2 - 2*(cosh(b*x + a) + sinh(b*x + a))*cosh(b*x + a)^(1/3)*sinh(b*x
+ a)^(2/3) + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 + 1)/(cosh(b*x
 + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 + 1)) + 12*(cosh(
b*x + a) + sinh(b*x + a))*cosh(b*x + a)^(2/3)*sinh(b*x + a)^(1/3))/(b*cosh(
b*x + a)^2 + 2*b*cosh(b*x + a)*sinh(b*x + a) + b*sinh(b*x + a)^2 - b)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh(bx+a)^{\frac{5}{3}}}{\sinh(bx+a)^{\frac{5}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^(5/3)/sinh(b*x+a)^(5/3),x, algorithm="giac")
[Out] integrate(cosh(b*x + a)^(5/3)/sinh(b*x + a)^(5/3), x)
```

maple [F] time = 0.14, size = 0, normalized size = 0.00

$$\int \frac{\cosh^{\frac{5}{3}}(bx+a)}{\sinh(bx+a)^{\frac{5}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)^(5/3)/sinh(b*x+a)^(5/3),x)
[Out] int(cosh(b*x+a)^(5/3)/sinh(b*x+a)^(5/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh(bx+a)^{\frac{5}{3}}}{\sinh(bx+a)^{\frac{5}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^(5/3)/sinh(b*x+a)^(5/3),x, algorithm="maxima")
[Out] integrate(cosh(b*x + a)^(5/3)/sinh(b*x + a)^(5/3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\cosh\left(a+b\,x\right)}^{5/3}}{{\sinh\left(a+b\,x\right)}^{5/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a + b*x)^(5/3)/sinh(a + b*x)^(5/3),x)
[Out] int(cosh(a + b*x)^(5/3)/sinh(a + b*x)^(5/3), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)**(5/3)/sinh(b*x+a)**(5/3),x)
```

```
[Out] Timed out
```

3.66 $\int \frac{\cosh^{\frac{7}{3}}(a+bx)}{\sinh^{\frac{7}{3}}(a+bx)}\,dx$

Optimal. Leaf size=155

$$-\frac{3\cosh^{\frac{4}{3}}(a+bx)}{4b\sinh^{\frac{4}{3}}(a+bx)}-\frac{\log\left(1-\frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}\right)}{2b}+\frac{\log\left(\frac{\cosh^{\frac{4}{3}}(a+bx)}{\sinh^{\frac{4}{3}}(a+bx)}+\frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}+1\right)}{4b}-\frac{\sqrt{3}\tan^{-1}\left(\frac{\frac{2\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}+1}{\sqrt{3}}\right)}{2b}$$

```
[Out] -1/2*ln(1-cosh(b*x+a)^(2/3)/sinh(b*x+a)^(2/3))/b+1/4*ln(1+cosh(b*x+a)^(4/3)
/sinh(b*x+a)^(4/3)+cosh(b*x+a)^(2/3)/sinh(b*x+a)^(2/3))/b-3/4*cosh(b*x+a)^(
4/3)/b/sinh(b*x+a)^(4/3)-1/2*arctan(1/3*(1+2*cosh(b*x+a)^(2/3)/sinh(b*x+a)^
(2/3))*3^(1/2))*3^(1/2)/b
```

Rubi [A] time = 0.12, antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {2567, 2575, 275, 292, 31, 634, 618, 204, 628}

$$-\frac{3\cosh^{\frac{4}{3}}(a+bx)}{4b\sinh^{\frac{4}{3}}(a+bx)}-\frac{\log\left(1-\frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}\right)}{2b}+\frac{\log\left(\frac{\cosh^{\frac{4}{3}}(a+bx)}{\sinh^{\frac{4}{3}}(a+bx)}+\frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}+1\right)}{4b}-\frac{\sqrt{3}\tan^{-1}\left(\frac{\frac{2\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}+1}{\sqrt{3}}\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[a + b*x]^(7/3)/Sinh[a + b*x]^(7/3),x]
```

```
[Out] -(Sqrt[3]*ArcTan[(1 + (2*Cosh[a + b*x]^(2/3))/Sinh[a + b*x]^(2/3))/Sqrt[3]]
)/(2*b) - Log[1 - Cosh[a + b*x]^(2/3)/Sinh[a + b*x]^(2/3)]/(2*b) + Log[1 +
Cosh[a + b*x]^(4/3)/Sinh[a + b*x]^(4/3) + Cosh[a + b*x]^(2/3)/Sinh[a + b*x]
^(2/3)]/(4*b) - (3*Cosh[a + b*x]^(4/3))/(4*b*Sinh[a + b*x]^(4/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2567

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(a*(a*Cos[e + f*x])^(m - 1)*(b*Sin[e + f*x])^(n + 1))
/(b*f*(n + 1)), x] + Dist[(a^2*(m - 1))/(b^2*(n + 1)), Int[(a*Cos[e + f*x])
^(m - 2)*(b*Sin[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ[
m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

Rule 2575

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> With[{k = Denominator[m]}, -Dist[(k*a*b)/f, Subst[Int[x^(k
*(m + 1) - 1)/(a^2 + b^2*x^(2*k)), x], x, (a*Cos[e + f*x])^(1/k)/(b*Sin[e +
 f*x])^(1/k)], x]] /; FreeQ[{a, b, e, f}, x] && EqQ[m + n, 0] && GtQ[m, 0]
&& LtQ[m, 1]
```

Rubi steps

$$\int \frac{\cosh^{\frac{7}{3}}(a+bx)}{\sinh^{\frac{7}{3}}(a+bx)}\,dx = -\frac{3\cosh^{\frac{4}{3}}(a+bx)}{4b\sinh^{\frac{4}{3}}(a+bx)} + \int \frac{\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}\,dx$$

$$= -\frac{3\cosh^{\frac{4}{3}}(a+bx)}{4b\sinh^{\frac{4}{3}}(a+bx)} + \frac{3\,\mathrm{Subst}\left(\int \frac{x^3}{1-x^6}\,dx, x, \frac{\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}\right)}{b}$$

$$= -\frac{3\cosh^{\frac{4}{3}}(a+bx)}{4b\sinh^{\frac{4}{3}}(a+bx)} + \frac{3\,\mathrm{Subst}\left(\int \frac{x}{1-x^3}\,dx, x, \frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}\right)}{2b}$$

$$= -\frac{3\cosh^{\frac{4}{3}}(a+bx)}{4b\sinh^{\frac{4}{3}}(a+bx)} + \frac{\mathrm{Subst}\left(\int \frac{1}{1-x}\,dx, x, \frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}\right)}{2b} - \frac{\mathrm{Subst}\left(\int \frac{1-x}{1+x+x^2}\,dx, x, \frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}\right)}{2b}$$

$$= -\frac{\log\left(1-\frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}\right)}{2b} - \frac{3\cosh^{\frac{4}{3}}(a+bx)}{4b\sinh^{\frac{4}{3}}(a+bx)} + \frac{\mathrm{Subst}\left(\int \frac{1+2x}{1+x+x^2}\,dx, x, \frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}\right)}{4b} - \frac{3\,\mathrm{Subs}}{}$$

$$= -\frac{\log\left(1-\frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}\right)}{2b} + \frac{\log\left(1+\frac{\cosh^{\frac{4}{3}}(a+bx)}{\sinh^{\frac{4}{3}}(a+bx)}+\frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}\right)}{4b} - \frac{3\cosh^{\frac{4}{3}}(a+bx)}{4b\sinh^{\frac{4}{3}}(a+bx)} + \frac{3\,\mathrm{Subs}}{}$$

$$= -\frac{\sqrt{3}\,\tan^{-1}\left(\frac{1+\frac{2\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}}{\sqrt{3}}\right)}{2b} - \frac{\log\left(1-\frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}\right)}{2b} + \frac{\log\left(1+\frac{\cosh^{\frac{4}{3}}(a+bx)}{\sinh^{\frac{4}{3}}(a+bx)}+\frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}\right)}{4b} -$$

Mathematica [C] time = 0.03, size = 59, normalized size = 0.38

$$-\frac{3\sqrt[3]{\cosh^2(a+bx)}\,{}_2F_1\left(-\frac{2}{3},-\frac{2}{3};\frac{1}{3};-\sinh^2(a+bx)\right)}{4b\sinh^{\frac{4}{3}}(a+bx)\cosh^{\frac{2}{3}}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[a + b*x]^(7/3)/Sinh[a + b*x]^(7/3),x]
```

```
[Out] (-3*(Cosh[a + b*x]^2)^(1/3)*Hypergeometric2F1[-2/3, -2/3, 1/3, -Sinh[a + b*
x]^2])/(4*b*Cosh[a + b*x]^(2/3)*Sinh[a + b*x]^(4/3))
```

fricas [B] time = 0.44, size = 1056, normalized size = 6.81

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^(7/3)/sinh(b*x+a)^(7/3),x, algorithm="fricas")
```

```
[Out] -1/4*(2*(sqrt(3)*cosh(b*x + a)^4 + 4*sqrt(3)*cosh(b*x + a)*sinh(b*x + a)^3
+ sqrt(3)*sinh(b*x + a)^4 + 2*(3*sqrt(3)*cosh(b*x + a)^2 - sqrt(3))*sinh(b*
x + a)^2 - 2*sqrt(3)*cosh(b*x + a)^2 + 4*(sqrt(3)*cosh(b*x + a)^3 - sqrt(3)
*cosh(b*x + a))*sinh(b*x + a) + sqrt(3))*arctan(1/3*(sqrt(3)*cosh(b*x + a)^
2 + 2*sqrt(3)*cosh(b*x + a)*sinh(b*x + a) + sqrt(3)*sinh(b*x + a)^2 + 4*(sq
rt(3)*cosh(b*x + a) + sqrt(3)*sinh(b*x + a))*cosh(b*x + a)^(2/3)*sinh(b*x +
 a)^(1/3) - sqrt(3))/(cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sin
h(b*x + a)^2 - 1)) - (cosh(b*x + a)^4 + 4*cosh(b*x + a)*sinh(b*x + a)^3 + s
inh(b*x + a)^4 + 2*(3*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^2 - 2*cosh(b*x + a
)^2 + 4*(cosh(b*x + a)^3 - cosh(b*x + a))*sinh(b*x + a) + 1)*log((cosh(b*x
+ a)^4 + 4*cosh(b*x + a)*sinh(b*x + a)^3 + sinh(b*x + a)^4 + 2*(3*cosh(b*x
+ a)^2 - 1)*sinh(b*x + a)^2 - 2*cosh(b*x + a)^2 + 2*(cosh(b*x + a)^3 + 3*co
sh(b*x + a)*sinh(b*x + a)^2 + sinh(b*x + a)^3 + (3*cosh(b*x + a)^2 - 1)*sin
h(b*x + a) - cosh(b*x + a))*cosh(b*x + a)^(2/3)*sinh(b*x + a)^(1/3) + 2*(co
sh(b*x + a)^3 + 3*cosh(b*x + a)*sinh(b*x + a)^2 + sinh(b*x + a)^3 + (3*cosh
(b*x + a)^2 + 1)*sinh(b*x + a) + cosh(b*x + a))*cosh(b*x + a)^(1/3)*sinh(b*
x + a)^(2/3) + 4*(cosh(b*x + a)^3 - cosh(b*x + a))*sinh(b*x + a) + 1)/(cosh
(b*x + a)^4 + 4*cosh(b*x + a)*sinh(b*x + a)^3 + sinh(b*x + a)^4 + 2*(3*cosh
(b*x + a)^2 - 1)*sinh(b*x + a)^2 - 2*cosh(b*x + a)^2 + 4*(cosh(b*x + a)^3 -
 cosh(b*x + a))*sinh(b*x + a) + 1)) + 2*(cosh(b*x + a)^4 + 4*cosh(b*x + a)*
sinh(b*x + a)^3 + sinh(b*x + a)^4 + 2*(3*cosh(b*x + a)^2 - 1)*sinh(b*x + a)
^2 - 2*cosh(b*x + a)^2 + 4*(cosh(b*x + a)^3 - cosh(b*x + a))*sinh(b*x + a)
+ 1)*log(-(cosh(b*x + a)^2 - 2*(cosh(b*x + a) + sinh(b*x + a))*cosh(b*x + a
)^(2/3)*sinh(b*x + a)^(1/3) + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)
^2 - 1)/(cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2
```

```
- 1)) + 6*(cosh(b*x + a)^3 + 3*cosh(b*x + a)*sinh(b*x + a)^2 + sinh(b*x + a
)^3 + (3*cosh(b*x + a)^2 + 1)*sinh(b*x + a) + cosh(b*x + a))*cosh(b*x + a)^
(1/3)*sinh(b*x + a)^(2/3))/(b*cosh(b*x + a)^4 + 4*b*cosh(b*x + a)*sinh(b*x
+ a)^3 + b*sinh(b*x + a)^4 - 2*b*cosh(b*x + a)^2 + 2*(3*b*cosh(b*x + a)^2 -
 b)*sinh(b*x + a)^2 + 4*(b*cosh(b*x + a)^3 - b*cosh(b*x + a))*sinh(b*x + a)
 + b)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh(bx+a)^{\frac{7}{3}}}{\sinh(bx+a)^{\frac{7}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^(7/3)/sinh(b*x+a)^(7/3),x, algorithm="giac")
[Out] integrate(cosh(b*x + a)^(7/3)/sinh(b*x + a)^(7/3), x)
```

maple [F] time = 0.14, size = 0, normalized size = 0.00

$$\int \frac{\cosh^{\frac{7}{3}}(bx+a)}{\sinh(bx+a)^{\frac{7}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)^(7/3)/sinh(b*x+a)^(7/3),x)
[Out] int(cosh(b*x+a)^(7/3)/sinh(b*x+a)^(7/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh(bx+a)^{\frac{7}{3}}}{\sinh(bx+a)^{\frac{7}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^(7/3)/sinh(b*x+a)^(7/3),x, algorithm="maxima")
[Out] integrate(cosh(b*x + a)^(7/3)/sinh(b*x + a)^(7/3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\cosh(a+b\,x)}^{7/3}}{{\sinh(a+b\,x)}^{7/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a + b*x)^(7/3)/sinh(a + b*x)^(7/3),x)
```

```
[Out] int(cosh(a + b*x)^(7/3)/sinh(a + b*x)^(7/3), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)**(7/3)/sinh(b*x+a)**(7/3),x)
```

```
[Out] Timed out
```

3.67 $\int \frac{\cosh^{\frac{2}{3}}(x)}{\sinh^{\frac{8}{3}}(x)}\,dx$

Optimal. Leaf size=16

$$-\frac{3\cosh^{\frac{5}{3}}(x)}{5\sinh^{\frac{5}{3}}(x)}$$

[Out] -3/5*cosh(x)^(5/3)/sinh(x)^(5/3)

Rubi [A] time = 0.03, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {2563}

$$-\frac{3\cosh^{\frac{5}{3}}(x)}{5\sinh^{\frac{5}{3}}(x)}$$

Antiderivative was successfully verified.

[In] Int[Cosh[x]^(2/3)/Sinh[x]^(8/3),x]

[Out] (-3*Cosh[x]^(5/3))/(5*Sinh[x]^(5/3))

Rule 2563

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(
m_.), x_Symbol] :> Simp[((a*Sin[e + f*x])^(m + 1)*(b*Cos[e + f*x])^(n + 1))
/(a*b*f*(m + 1)), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n + 2, 0] &
& NeQ[m, -1]
```

Rubi steps

$$\int \frac{\cosh^{\frac{2}{3}}(x)}{\sinh^{\frac{8}{3}}(x)}\,dx = -\frac{3\cosh^{\frac{5}{3}}(x)}{5\sinh^{\frac{5}{3}}(x)}$$

Mathematica [A] time = 0.01, size = 16, normalized size = 1.00

$$-\frac{3\cosh^{\frac{5}{3}}(x)}{5\sinh^{\frac{5}{3}}(x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]^(2/3)/Sinh[x]^(8/3),x]
```

```
[Out] (-3*Cosh[x]^(5/3))/(5*Sinh[x]^(5/3))
```

fricas [B] time = 0.43, size = 93, normalized size = 5.81

$$-\frac{6\left(\cosh(x)^3+3\,\cosh(x)\sinh(x)^2+\sinh(x)^3+\left(3\,\cosh(x)^2+1\right)\sinh(x)+\cosh(x)\right)\cosh(x)^{\frac{2}{3}}\sinh}{5\left(\cosh(x)^4+4\,\cosh(x)\sinh(x)^3+\sinh(x)^4+2\left(3\,\cosh(x)^2-1\right)\sinh(x)^2-2\,\cosh(x)^2+4\left(\cosh(x)^3-\cosh\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^(2/3)/sinh(x)^(8/3),x, algorithm="fricas")
```

```
[Out] -6/5*(cosh(x)^3 + 3*cosh(x)*sinh(x)^2 + sinh(x)^3 + (3*cosh(x)^2 + 1)*sinh(
x) + cosh(x))*cosh(x)^(2/3)*sinh(x)^(1/3)/(cosh(x)^4 + 4*cosh(x)*sinh(x)^3
+ sinh(x)^4 + 2*(3*cosh(x)^2 - 1)*sinh(x)^2 - 2*cosh(x)^2 + 4*(cosh(x)^3 -
cosh(x))*sinh(x) + 1)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh(x)^{\frac{2}{3}}}{\sinh(x)^{\frac{8}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^(2/3)/sinh(x)^(8/3),x, algorithm="giac")
```

```
[Out] integrate(cosh(x)^(2/3)/sinh(x)^(8/3), x)
```

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh^{\frac{2}{3}}(x)}{\sinh(x)^{\frac{8}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^(2/3)/sinh(x)^(8/3),x)
```

```
[Out] int(cosh(x)^(2/3)/sinh(x)^(8/3),x)
```

maxima [B] time = 0.49, size = 61, normalized size = 3.81

$$\frac{3\left(e^{(-2\,x)}+1\right)^{\frac{2}{3}}e^{(-4\,x)}}{5\left(e^{(-x)}+1\right)^{\frac{8}{3}}\left(-e^{(-x)}+1\right)^{\frac{8}{3}}}-\frac{3\left(e^{(-2\,x)}+1\right)^{\frac{2}{3}}}{5\left(e^{(-x)}+1\right)^{\frac{8}{3}}\left(-e^{(-x)}+1\right)^{\frac{8}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^(2/3)/sinh(x)^(8/3),x, algorithm="maxima")
```

```
[Out] 3/5*(e^(-2*x) + 1)^(2/3)*e^(-4*x)/((e^(-x) + 1)^(8/3)*(-e^(-x) + 1)^(8/3))
- 3/5*(e^(-2*x) + 1)^(2/3)/((e^(-x) + 1)^(8/3)*(-e^(-x) + 1)^(8/3))
```

mupad [B] time = 1.54, size = 6, normalized size = 0.38

$$-\frac{3\coth(x)^{5/3}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^(2/3)/sinh(x)^(8/3),x)
```

```
[Out] -(3*coth(x)^(5/3))/5
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)**(2/3)/sinh(x)**(8/3),x)
```

```
[Out] Timed out
```

3.68 $\int \frac{\sinh^{\frac{2}{3}}(x)}{\cosh^{\frac{8}{3}}(x)} dx$

Optimal. Leaf size=16

$$\frac{3\sinh^{\frac{5}{3}}(x)}{5\cosh^{\frac{5}{3}}(x)}$$

```
[Out] 3/5*sinh(x)^(5/3)/cosh(x)^(5/3)
```

Rubi [A] time = 0.03, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {2563}

$$\frac{3\sinh^{\frac{5}{3}}(x)}{5\cosh^{\frac{5}{3}}(x)}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[x]^(2/3)/Cosh[x]^(8/3),x]
```

```
[Out] (3*Sinh[x]^(5/3))/(5*Cosh[x]^(5/3))
```

Rule 2563

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(
m_.), x_Symbol] :> Simp[((a*Sin[e + f*x])^(m + 1)*(b*Cos[e + f*x])^(n + 1))
/(a*b*f*(m + 1)), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n + 2, 0] &
& NeQ[m, -1]
```

Rubi steps

$$\int \frac{\sinh^{\frac{2}{3}}(x)}{\cosh^{\frac{8}{3}}(x)} dx = \frac{3\sinh^{\frac{5}{3}}(x)}{5\cosh^{\frac{5}{3}}(x)}$$

Mathematica [A] time = 0.02, size = 16, normalized size = 1.00

$$\frac{3\sinh^{\frac{5}{3}}(x)}{5\cosh^{\frac{5}{3}}(x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[x]^(2/3)/Cosh[x]^(8/3),x]
```

```
[Out] (3*Sinh[x]^(5/3))/(5*Cosh[x]^(5/3))
```

fricas [B] time = 0.41, size = 93, normalized size = 5.81

$$\frac{6\left(\cosh(x)^3 + 3\cosh(x)\sinh(x)^2 + \sinh(x)^3 + \left(3\cosh(x)^2 - 1\right)\sinh(x) - \cosh(x)\right)\cosh(x)^{\frac{1}{3}}\sinh(x}{5\left(\cosh(x)^4 + 4\cosh(x)\sinh(x)^3 + \sinh(x)^4 + 2\left(3\cosh(x)^2 + 1\right)\sinh(x)^2 + 2\cosh(x)^2 + 4\left(\cosh(x)^3 + \cosh(x\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^(2/3)/cosh(x)^(8/3),x, algorithm="fricas")
```

```
[Out] 6/5*(cosh(x)^3 + 3*cosh(x)*sinh(x)^2 + sinh(x)^3 + (3*cosh(x)^2 - 1)*sinh(x
) - cosh(x))*cosh(x)^(1/3)*sinh(x)^(2/3)/(cosh(x)^4 + 4*cosh(x)*sinh(x)^3 +
 sinh(x)^4 + 2*(3*cosh(x)^2 + 1)*sinh(x)^2 + 2*cosh(x)^2 + 4*(cosh(x)^3 + c
osh(x))*sinh(x) + 1)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh(x)^{\frac{2}{3}}}{\cosh(x)^{\frac{8}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^(2/3)/cosh(x)^(8/3),x, algorithm="giac")
```

```
[Out] integrate(sinh(x)^(2/3)/cosh(x)^(8/3), x)
```

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh^{\frac{2}{3}}(x)}{\cosh(x)^{\frac{8}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)^(2/3)/cosh(x)^(8/3),x)
```

```
[Out] int(sinh(x)^(2/3)/cosh(x)^(8/3),x)
```

maxima [B] time = 0.46, size = 61, normalized size = 3.81

$$-\frac{3\left(e^{(-x)}+1\right)^{\frac{2}{3}}\left(-e^{(-x)}+1\right)^{\frac{2}{3}}e^{(-4x)}}{5\left(e^{(-2x)}+1\right)^{\frac{8}{3}}} + \frac{3\left(e^{(-x)}+1\right)^{\frac{2}{3}}\left(-e^{(-x)}+1\right)^{\frac{2}{3}}}{5\left(e^{(-2x)}+1\right)^{\frac{8}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^(2/3)/cosh(x)^(8/3),x, algorithm="maxima")
```

```
[Out] -3/5*(e^(-x) + 1)^(2/3)*(-e^(-x) + 1)^(2/3)*e^(-4*x)/(e^(-2*x) + 1)^(8/3) +
 3/5*(e^(-x) + 1)^(2/3)*(-e^(-x) + 1)^(2/3)/(e^(-2*x) + 1)^(8/3)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.06

$$\int \frac{\sinh(x)^{2/3}}{\cosh(x)^{8/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)^(2/3)/cosh(x)^(8/3),x)
```

```
[Out] int(sinh(x)^(2/3)/cosh(x)^(8/3), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)**(2/3)/cosh(x)**(8/3),x)
```

```
[Out] Timed out
```

3.69 $\int \cosh(x)\operatorname{csch}^{\frac{7}{3}}(x)\,dx$

Optimal. Leaf size=10

$$-\frac{3}{4}\operatorname{csch}^{\frac{4}{3}}(x)$$

```
[Out] -3/4*csch(x)^(4/3)
```

Rubi [A] time = 0.03, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2621, 30}

$$-\frac{3}{4}\operatorname{csch}^{\frac{4}{3}}(x)$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]*Csch[x]^(7/3),x]
```

```
[Out] (-3*Csch[x]^(4/3))/4
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2621

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> -Dist[(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rubi steps

$$\begin{aligned}\int \cosh(x)\operatorname{csch}^{\frac{7}{3}}(x)\,dx &= -\operatorname{Subst}\left(\int \sqrt[3]{x}\,dx, x, \operatorname{csch}(x)\right)\\ &= -\frac{3}{4}\operatorname{csch}^{\frac{4}{3}}(x)\end{aligned}$$

Mathematica [A] time = 0.01, size = 10, normalized size = 1.00

$$-\frac{3}{4}\operatorname{csch}^{\frac{4}{3}}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]*Csch[x]^(7/3),x]
```

```
[Out] (-3*Csch[x]^(4/3))/4
```

fricas [B] time = 0.46, size = 54, normalized size = 5.40

$$-\frac{3 \cdot 2^{\frac{1}{3}} \left(\frac{\cosh(x)+\sinh(x)}{\cosh(x)^{2}+2 \cosh(x)\sinh(x)+\sinh(x)^{2}-1}\right)^{\frac{1}{3}} (\cosh(x)+\sinh(x))}{2 \left(\cosh(x)^{2}+2 \cosh(x)\sinh(x)+\sinh(x)^{2}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*csch(x)^(7/3),x, algorithm="fricas")
```

```
[Out] -3/2*2^(1/3)*((cosh(x) + sinh(x))/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^
2 - 1))^(1/3)*(cosh(x) + sinh(x))/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^
2 - 1)
```

giac [B] time = 0.13, size = 17, normalized size = 1.70

$$-\frac{3 \cdot 2^{\frac{1}{3}} e^{\left(\frac{4}{3} x\right)}}{2 \left(e^{(2 x)}-1\right)^{\frac{4}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*csch(x)^(7/3),x, algorithm="giac")
```

```
[Out] -3/2*2^(1/3)*e^(4/3*x)/(e^(2*x) - 1)^(4/3)
```

maple [A] time = 0.08, size = 7, normalized size = 0.70

$$-\frac{3\operatorname{csch}(x)^{\frac{4}{3}}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)*csch(x)^(7/3),x)
```

```
[Out] -3/4*csch(x)^(4/3)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \cosh(x)\operatorname{csch}(x)^{\frac{7}{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*csch(x)^(7/3),x, algorithm="maxima")
[Out] integrate(cosh(x)*csch(x)^(7/3), x)
```

mupad [B] time = 1.46, size = 31, normalized size = 3.10

$$-\frac{3\,e^x\left(-\frac{1}{\frac{e^{-x}}{2}-\frac{e^x}{2}}\right)^{1/3}}{2\,\left(e^{2\,x}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)*(1/sinh(x))^(7/3),x)
[Out] -(3*exp(x)*(-1/(exp(-x)/2 - exp(x)/2))^(1/3))/(2*(exp(2*x) - 1))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*csch(x)**(7/3),x)
[Out] Timed out
```

3.70 $\int \sinh(a+bx)\tanh(a+bx)\,dx$

Optimal. Leaf size=23

$$\frac{\sinh(a+bx)}{b}-\frac{\tan^{-1}(\sinh(a+bx))}{b}$$

[Out] -arctan(sinh(b*x+a))/b+sinh(b*x+a)/b

Rubi [A] time = 0.02, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {2592, 321, 203}

$$\frac{\sinh(a+bx)}{b}-\frac{\tan^{-1}(\sinh(a+bx))}{b}$$

Antiderivative was successfully verified.

[In] Int[Sinh[a + b*x]*Tanh[a + b*x],x]

[Out] -(ArcTan[Sinh[a + b*x]]/b) + Sinh[a + b*x]/b

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2592

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, (a*Sin[e + f*x])/ff], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rubi steps

$$\int \sinh(a+bx)\tanh(a+bx)\,dx = \frac{\text{Subst}\left(\int \frac{x^2}{1+x^2}\,dx, x, \sinh(a+bx)\right)}{b}$$
$$= \frac{\sinh(a+bx)}{b} - \frac{\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sinh(a+bx)\right)}{b}$$
$$= -\frac{\tan^{-1}(\sinh(a+bx))}{b} + \frac{\sinh(a+bx)}{b}$$

Mathematica [A] time = 0.01, size = 23, normalized size = 1.00

$$\frac{\sinh(a+bx)}{b} - \frac{\tan^{-1}(\sinh(a+bx))}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[a + b*x]*Tanh[a + b*x],x]
```

```
[Out] -(ArcTan[Sinh[a + b*x]]/b) + Sinh[a + b*x]/b
```

fricas [B] time = 0.55, size = 86, normalized size = 3.74

$$-\frac{4\left(\cosh\left(bx+a\right)+\sinh\left(bx+a\right)\right)\arctan\left(\cosh\left(bx+a\right)+\sinh\left(bx+a\right)\right)-\cosh\left(bx+a\right)^2-2\cosh\left(bx+a\right)\sinh}{2\left(b\cosh\left(bx+a\right)+b\sinh\left(bx+a\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)*tanh(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/2*(4*(cosh(b*x + a) + sinh(b*x + a))*arctan(cosh(b*x + a) + sinh(b*x + a
)) - cosh(b*x + a)^2 - 2*cosh(b*x + a)*sinh(b*x + a) - sinh(b*x + a)^2 + 1)
/(b*cosh(b*x + a) + b*sinh(b*x + a))
```

giac [A] time = 0.13, size = 32, normalized size = 1.39

$$-\frac{4\arctan\left(e^{(bx+a)}\right)-e^{(bx+a)}+e^{(-bx-a)}}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)*tanh(b*x+a),x, algorithm="giac")
```

```
[Out] -1/2*(4*arctan(e^(b*x + a)) - e^(b*x + a) + e^(-b*x - a))/b
```

maple [A] time = 0.14, size = 24, normalized size = 1.04

$$\frac{\sinh(bx+a)}{b}-\frac{2\arctan\left(e^{bx+a}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(b*x+a)*tanh(b*x+a),x)
```

```
[Out] sinh(b*x+a)/b-2*arctan(exp(b*x+a))/b
```

maxima [A] time = 0.40, size = 41, normalized size = 1.78

$$\frac{2\arctan\left(e^{(-bx-a)}\right)}{b}+\frac{e^{(bx+a)}}{2b}-\frac{e^{(-bx-a)}}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)*tanh(b*x+a),x, algorithm="maxima")
```

```
[Out] 2*arctan(e^(-b*x - a))/b + 1/2*e^(b*x + a)/b - 1/2*e^(-b*x - a)/b
```

mupad [B] time = 1.47, size = 49, normalized size = 2.13

$$\frac{e^{a+bx}}{2b}-\frac{2\,\mathrm{atan}\left(\frac{e^{bx}\,e^{a}\,\sqrt{b^2}}{b}\right)}{\sqrt{b^2}}-\frac{e^{-a-bx}}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a + b*x)*tanh(a + b*x),x)
```

```
[Out] exp(a + b*x)/(2*b) - (2*atan((exp(b*x)*exp(a)*(b^2)^(1/2))/b))/(b^2)^(1/2)
- exp(- a - b*x)/(2*b)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sinh(a+bx)\tanh(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)*tanh(b*x+a),x)
```

```
[Out] Integral(sinh(a + b*x)*tanh(a + b*x), x)
```

3.71 $\int \sinh(a+bx)\tanh^2(a+bx)\,dx$

Optimal. Leaf size=21

$$\frac{\cosh(a+bx)}{b}+\frac{\operatorname{sech}(a+bx)}{b}$$

```
[Out] cosh(b*x+a)/b+sech(b*x+a)/b
```

Rubi [A] time = 0.03, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2590, 14}

$$\frac{\cosh(a+bx)}{b}+\frac{\operatorname{sech}(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[a + b*x]*Tanh[a + b*x]^2,x]
```

```
[Out] Cosh[a + b*x]/b + Sech[a + b*x]/b
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2590

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> -Dist[f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rubi steps

$$\begin{aligned}\int \sinh(a+bx)\tanh^2(a+bx)\,dx &= -\frac{\text{Subst}\left(\int \frac{1-x^2}{x^2}\,dx, x, \cosh(a+bx)\right)}{b}\\ &= -\frac{\text{Subst}\left(\int \left(-1+\frac{1}{x^2}\right)dx, x, \cosh(a+bx)\right)}{b}\\ &= \frac{\cosh(a+bx)}{b}+\frac{\operatorname{sech}(a+bx)}{b}\end{aligned}$$

Mathematica [A] time = 0.03, size = 21, normalized size = 1.00

$$\frac{\cosh(a+bx)}{b}+\frac{\operatorname{sech}(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[a + b*x]*Tanh[a + b*x]^2,x]
```

```
[Out] Cosh[a + b*x]/b + Sech[a + b*x]/b
```

fricas [A] time = 0.41, size = 31, normalized size = 1.48

$$\frac{\cosh(bx+a)^2+\sinh(bx+a)^2+3}{2\,b\cosh(bx+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)*tanh(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(cosh(b*x + a)^2 + sinh(b*x + a)^2 + 3)/(b*cosh(b*x + a))
```

giac [B] time = 0.15, size = 46, normalized size = 2.19

$$\frac{\frac{\left(5\,e^{(2\,bx+2\,a)}+1\right)e^{(-a)}}{e^{(3\,bx+2\,a)}+e^{(bx)}}+e^{(bx+a)}}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)*tanh(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*((5*e^(2*b*x + 2*a) + 1)*e^(-a)/(e^(3*b*x + 2*a) + e^(b*x)) + e^(b*x +
a))/b
```

maple [A] time = 0.13, size = 33, normalized size = 1.57

$$\frac{\frac{\sinh^2(bx+a)}{\cosh(bx+a)}+\frac{2}{\cosh(bx+a)}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(b*x+a)*tanh(b*x+a)^2,x)
```

```
[Out] 1/b*(sinh(b*x+a)^2/cosh(b*x+a)+2/cosh(b*x+a))
```

maxima [B] time = 0.31, size = 54, normalized size = 2.57

$$\frac{e^{(-bx-a)}}{2\,b}+\frac{5\,e^{(-2\,bx-2\,a)}+1}{2\,b\left(e^{(-bx-a)}+e^{(-3\,bx-3\,a)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)*tanh(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*e^(-b*x - a)/b + 1/2*(5*e^(-2*b*x - 2*a) + 1)/(b*(e^(-b*x - a) + e^(-3*
b*x - 3*a)))
```

mupad [B] time = 1.49, size = 49, normalized size = 2.33

$$\frac{e^{-a-b\,x}\,\left(6\,e^{2\,a+2\,b\,x}+e^{4\,a+4\,b\,x}+1\right)}{2\,b\,\left(e^{2\,a+2\,b\,x}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a + b*x)*tanh(a + b*x)^2,x)
```

```
[Out] (exp(- a - b*x)*(6*exp(2*a + 2*b*x) + exp(4*a + 4*b*x) + 1))/(2*b*(exp(2*a
+ 2*b*x) + 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sinh(a+bx)\tanh^2(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)*tanh(b*x+a)**2,x)
```

```
[Out] Integral(sinh(a + b*x)*tanh(a + b*x)**2, x)
```

3.72 $\int \sinh(a+bx)\tanh^3(a+bx)\,dx$

Optimal. Leaf size=49

$$\frac{3\sinh(a+bx)}{2b}-\frac{3\tan^{-1}(\sinh(a+bx))}{2b}-\frac{\sinh(a+bx)\tanh^2(a+bx)}{2b}$$

```
[Out] -3/2*arctan(sinh(b*x+a))/b+3/2*sinh(b*x+a)/b-1/2*sinh(b*x+a)*tanh(b*x+a)^2/
b
```

Rubi [A] time = 0.03, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {2592, 288, 321, 203}

$$\frac{3\sinh(a+bx)}{2b}-\frac{3\tan^{-1}(\sinh(a+bx))}{2b}-\frac{\sinh(a+bx)\tanh^2(a+bx)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[a + b*x]*Tanh[a + b*x]^3,x]
```

```
[Out] (-3*ArcTan[Sinh[a + b*x]])/(2*b) + (3*Sinh[a + b*x])/(2*b) - (Sinh[a + b*x]
*Tanh[a + b*x]^2)/(2*b)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2592

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, (a*Sin[e + f*x])/ff], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rubi steps

$$\int \sinh(a+bx)\tanh^3(a+bx)\,dx = \frac{\text{Subst}\left(\int \frac{x^4}{\left(1+x^2\right)^2}\,dx, x, \sinh(a+bx)\right)}{b}$$
$$= -\frac{\sinh(a+bx)\tanh^2(a+bx)}{2b} + \frac{3\,\text{Subst}\left(\int \frac{x^2}{1+x^2}\,dx, x, \sinh(a+bx)\right)}{2b}$$
$$= \frac{3\sinh(a+bx)}{2b} - \frac{\sinh(a+bx)\tanh^2(a+bx)}{2b} - \frac{3\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sinh(a\right.}{2b}$$
$$= -\frac{3\tan^{-1}(\sinh(a+bx))}{2b} + \frac{3\sinh(a+bx)}{2b} - \frac{\sinh(a+bx)\tanh^2(a+bx)}{2b}$$

Mathematica [A] time = 0.05, size = 48, normalized size = 0.98

$$\frac{\sinh(a+bx)\tanh^2(a+bx)}{b} - \frac{3\left(\tan^{-1}(\sinh(a+bx)) - \tanh(a+bx)\text{sech}(a+bx)\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[a + b*x]*Tanh[a + b*x]^3,x]
```

```
[Out] (Sinh[a + b*x]*Tanh[a + b*x]^2)/b - (3*(ArcTan[Sinh[a + b*x]] - Sech[a + b*
x]*Tanh[a + b*x]))/(2*b)
```

fricas [B] time = 0.42, size = 463, normalized size = 9.45

$$\frac{\cosh(bx+a)^6 + 6\cosh(bx+a)\sinh(bx+a)^5 + \sinh(bx+a)^6 + 3\left(5\cosh(bx+a)^2+1\right)\sinh(bx+a)^4 + 3\,\text{cc}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)*tanh(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/2*(cosh(b*x + a)^6 + 6*cosh(b*x + a)*sinh(b*x + a)^5 + sinh(b*x + a)^6 +
3*(5*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^4 + 3*cosh(b*x + a)^4 + 4*(5*cosh(b
*x + a)^3 + 3*cosh(b*x + a))*sinh(b*x + a)^3 + 3*(5*cosh(b*x + a)^4 + 6*cos
h(b*x + a)^2 - 1)*sinh(b*x + a)^2 - 6*(cosh(b*x + a)^5 + 5*cosh(b*x + a)*si
nh(b*x + a)^4 + sinh(b*x + a)^5 + 2*(5*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^3
 + 2*cosh(b*x + a)^3 + 2*(5*cosh(b*x + a)^3 + 3*cosh(b*x + a))*sinh(b*x + a
)^2 + (5*cosh(b*x + a)^4 + 6*cosh(b*x + a)^2 + 1)*sinh(b*x + a) + cosh(b*x
+ a))*arctan(cosh(b*x + a) + sinh(b*x + a)) - 3*cosh(b*x + a)^2 + 6*(cosh(b
*x + a)^5 + 2*cosh(b*x + a)^3 - cosh(b*x + a))*sinh(b*x + a) - 1)/(b*cosh(b
*x + a)^5 + 5*b*cosh(b*x + a)*sinh(b*x + a)^4 + b*sinh(b*x + a)^5 + 2*b*cos
h(b*x + a)^3 + 2*(5*b*cosh(b*x + a)^2 + b)*sinh(b*x + a)^3 + 2*(5*b*cosh(b*
x + a)^3 + 3*b*cosh(b*x + a))*sinh(b*x + a)^2 + b*cosh(b*x + a) + (5*b*cosh
(b*x + a)^4 + 6*b*cosh(b*x + a)^2 + b)*sinh(b*x + a))
```

giac [A] time = 0.14, size = 65, normalized size = 1.33

$$\frac{\frac{2\left(e^{(3\,bx+3\,a)}-e^{(bx+a)}\right)}{\left(e^{(2\,bx+2\,a)}+1\right)^{2}} - 6\,\arctan\left(e^{(bx+a)}\right) + e^{(bx+a)} - e^{(-bx-a)}}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)*tanh(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/2*(2*(e^(3*b*x + 3*a) - e^(b*x + a))/(e^(2*b*x + 2*a) + 1)^2 - 6*arctan(e
^(b*x + a)) + e^(b*x + a) - e^(-b*x - a))/b
```

maple [A] time = 0.31, size = 70, normalized size = 1.43

$$\frac{\sinh^{3}(bx+a)}{b\cosh(bx+a)^{2}} + \frac{3\sinh(bx+a)}{b\cosh(bx+a)^{2}} - \frac{3\,\mathrm{sech}(bx+a)\tanh(bx+a)}{2b} - \frac{3\arctan\left(e^{bx+a}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(b*x+a)*tanh(b*x+a)^3,x)
```

```
[Out] 1/b*sinh(b*x+a)^3/cosh(b*x+a)^2+3/b*sinh(b*x+a)/cosh(b*x+a)^2-3/2*sech(b*x+
a)*tanh(b*x+a)/b-3*arctan(exp(b*x+a))/b
```

maxima [B] time = 0.41, size = 91, normalized size = 1.86

$$\frac{3\,\arctan\left(e^{(-bx-a)}\right)}{b} - \frac{e^{(-bx-a)}}{2\,b} + \frac{4\,e^{(-2\,bx-2\,a)} - e^{(-4\,bx-4\,a)} + 1}{2\,b\left(e^{(-bx-a)} + 2\,e^{(-3\,bx-3\,a)} + e^{(-5\,bx-5\,a)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)*tanh(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 3*arctan(e^(-b*x - a))/b - 1/2*e^(-b*x - a)/b + 1/2*(4*e^(-2*b*x - 2*a) - e
^(-4*b*x - 4*a) + 1)/(b*(e^(-b*x - a) + 2*e^(-3*b*x - 3*a) + e^(-5*b*x - 5*
a)))
```

mupad [B] time = 1.46, size = 107, normalized size = 2.18

$$\frac{e^{a+b\,x}}{2\,b}-\frac{3\,\mathrm{atan}\left(\frac{e^{b\,x}\,e^{a}\,\sqrt{b^2}}{b}\right)}{\sqrt{b^2}}-\frac{e^{-a-b\,x}}{2\,b}-\frac{2\,e^{a+b\,x}}{b\,\left(2\,e^{2\,a+2\,b\,x}+e^{4\,a+4\,b\,x}+1\right)}+\frac{e^{a+b\,x}}{b\,\left(e^{2\,a+2\,b\,x}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a + b*x)*tanh(a + b*x)^3,x)
```

```
[Out] exp(a + b*x)/(2*b) - (3*atan((exp(b*x)*exp(a)*(b^2)^(1/2))/b))/(b^2)^(1/2)
- exp(- a - b*x)/(2*b) - (2*exp(a + b*x))/(b*(2*exp(2*a + 2*b*x) + exp(4*a
+ 4*b*x) + 1)) + exp(a + b*x)/(b*(exp(2*a + 2*b*x) + 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sinh \left(a + bx\right) \tanh^{3}\left(a + bx\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)*tanh(b*x+a)**3,x)
```

```
[Out] Integral(sinh(a + b*x)*tanh(a + b*x)**3, x)
```

3.73 $\int \sinh(a+bx)\tanh^4(a+bx)\,dx$

Optimal. Leaf size=37

$$\frac{\cosh(a+bx)}{b}-\frac{\operatorname{sech}^3(a+bx)}{3b}+\frac{2\operatorname{sech}(a+bx)}{b}$$

```
[Out] cosh(b*x+a)/b+2*sech(b*x+a)/b-1/3*sech(b*x+a)^3/b
```

Rubi [A] time = 0.03, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2590, 270}

$$\frac{\cosh(a+bx)}{b}-\frac{\operatorname{sech}^3(a+bx)}{3b}+\frac{2\operatorname{sech}(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[a + b*x]*Tanh[a + b*x]^4,x]
```

```
[Out] Cosh[a + b*x]/b + (2*Sech[a + b*x])/b - Sech[a + b*x]^3/(3*b)
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2590

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> -Dist[f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rubi steps

$$\begin{aligned}\int \sinh(a+bx)\tanh^4(a+bx)\,dx&=\frac{\operatorname{Subst}\left(\int \frac{\left(1-x^2\right)^2}{x^4}\,dx,x,\cosh(a+bx)\right)}{b}\\&=\frac{\operatorname{Subst}\left(\int \left(1+\frac{1}{x^4}-\frac{2}{x^2}\right)\,dx,x,\cosh(a+bx)\right)}{b}\\&=\frac{\cosh(a+bx)}{b}+\frac{2\operatorname{sech}(a+bx)}{b}-\frac{\operatorname{sech}^3(a+bx)}{3b}\end{aligned}$$

Mathematica [A] time = 0.04, size = 37, normalized size = 1.00

$$\frac{\cosh(a+bx)}{b}-\frac{\operatorname{sech}^3(a+bx)}{3b}+\frac{2\operatorname{sech}(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[a + b*x]*Tanh[a + b*x]^4,x]
```

```
[Out] Cosh[a + b*x]/b + (2*Sech[a + b*x])/b - Sech[a + b*x]^3/(3*b)
```

fricas [B] time = 0.46, size = 93, normalized size = 2.51

$$\frac{3\,\cosh\left(bx+a\right)^4+3\,\sinh\left(bx+a\right)^4+18\left(\cosh\left(bx+a\right)^2+2\right)\sinh\left(bx+a\right)^2+36\,\cosh\left(bx+a\right)^2+25}{6\left(b\cosh\left(bx+a\right)^3+3\,b\cosh\left(bx+a\right)\sinh\left(bx+a\right)^2+3\,b\cosh\left(bx+a\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)*tanh(b*x+a)^4,x, algorithm="fricas")
```

```
[Out] 1/6*(3*cosh(b*x + a)^4 + 3*sinh(b*x + a)^4 + 18*(cosh(b*x + a)^2 + 2)*sinh(
b*x + a)^2 + 36*cosh(b*x + a)^2 + 25)/(b*cosh(b*x + a)^3 + 3*b*cosh(b*x + a
)*sinh(b*x + a)^2 + 3*b*cosh(b*x + a))
```

giac [B] time = 0.15, size = 71, normalized size = 1.92

$$\frac{\frac{8\left(3\,e^{(5\,bx+5\,a)}+4\,e^{(3\,bx+3\,a)}+3\,e^{(bx+a)}\right)}{\left(e^{(2\,bx+2\,a)}+1\right)^3}+3\,e^{(bx+a)}+3\,e^{(-bx-a)}}{6\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)*tanh(b*x+a)^4,x, algorithm="giac")
```

```
[Out] 1/6*(8*(3*e^(5*b*x + 5*a) + 4*e^(3*b*x + 3*a) + 3*e^(b*x + a))/(e^(2*b*x +
2*a) + 1)^3 + 3*e^(b*x + a) + 3*e^(-b*x - a))/b
```

maple [A] time = 0.16, size = 51, normalized size = 1.38

$$\frac{\frac{\sinh^4(bx+a)}{\cosh(bx+a)^3}+\frac{4\left(\sinh^2(bx+a)\right)}{\cosh(bx+a)^3}+\frac{8}{3\cosh(bx+a)^3}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(b*x+a)*tanh(b*x+a)^4,x)
```

```
[Out] 1/b*(sinh(b*x+a)^4/cosh(b*x+a)^3+4*sinh(b*x+a)^2/cosh(b*x+a)^3+8/3/cosh(b*x
+a)^3)
```

maxima [B] time = 0.31, size = 98, normalized size = 2.65

$$\frac{e^{(-bx-a)}}{2\,b} + \frac{33\,e^{(-2\,bx-2\,a)} + 41\,e^{(-4\,bx-4\,a)} + 27\,e^{(-6\,bx-6\,a)} + 3}{6\,b\left(e^{(-bx-a)} + 3\,e^{(-3\,bx-3\,a)} + 3\,e^{(-5\,bx-5\,a)} + e^{(-7\,bx-7\,a)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)*tanh(b*x+a)^4,x, algorithm="maxima")
```

```
[Out] 1/2*e^(-b*x - a)/b + 1/6*(33*e^(-2*b*x - 2*a) + 41*e^(-4*b*x - 4*a) + 27*e^
(-6*b*x - 6*a) + 3)/(b*(e^(-b*x - a) + 3*e^(-3*b*x - 3*a) + 3*e^(-5*b*x - 5
*a) + e^(-7*b*x - 7*a)))
```

mupad [B] time = 1.50, size = 131, normalized size = 3.54

$$\frac{e^{a+b\,x}}{2\,b}+\frac{e^{-a-b\,x}}{2\,b}-\frac{8\,e^{a+b\,x}}{3\,b\,\left(2\,e^{2\,a+2\,b\,x}+e^{4\,a+4\,b\,x}+1\right)}+\frac{8\,e^{a+b\,x}}{3\,b\,\left(3\,e^{2\,a+2\,b\,x}+3\,e^{4\,a+4\,b\,x}+e^{6\,a+6\,b\,x}+1\right)}+\frac{4\,e^{a+b\,x}}{b\,\left(e^{2\,a+2\,b\,x}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a + b*x)*tanh(a + b*x)^4,x)
```

```
[Out] exp(a + b*x)/(2*b) + exp(- a - b*x)/(2*b) - (8*exp(a + b*x))/(3*b*(2*exp(2*
a + 2*b*x) + exp(4*a + 4*b*x) + 1)) + (8*exp(a + b*x))/(3*b*(3*exp(2*a + 2*
b*x) + 3*exp(4*a + 4*b*x) + exp(6*a + 6*b*x) + 1)) + (4*exp(a + b*x))/(b*(e
xp(2*a + 2*b*x) + 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sinh{\left(a + bx \right)} \tanh^{4}{\left(a + bx \right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)*tanh(b*x+a)**4,x)
```

```
[Out] Integral(sinh(a + b*x)*tanh(a + b*x)**4, x)
```

3.74 $\int \sinh^2(a+bx)\tanh(a+bx)\,dx$

Optimal. Leaf size=28

$$\frac{\cosh^2(a+bx)}{2b}-\frac{\log(\cosh(a+bx))}{b}$$

```
[Out] 1/2*cosh(b*x+a)^2/b-ln(cosh(b*x+a))/b
```

Rubi [A] time = 0.03, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2590, 14}

$$\frac{\cosh^2(a+bx)}{2b}-\frac{\log(\cosh(a+bx))}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[a + b*x]^2*Tanh[a + b*x],x]
```

```
[Out] Cosh[a + b*x]^2/(2*b) - Log[Cosh[a + b*x]]/b
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2590

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> -Dist[f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rubi steps

$$\begin{aligned}\int \sinh^2(a+bx)\tanh(a+bx)\,dx &= -\frac{\text{Subst}\left(\int \frac{1-x^2}{x}\,dx,x,\cosh(a+bx)\right)}{b}\\ &= -\frac{\text{Subst}\left(\int \left(\frac{1}{x}-x\right)\,dx,x,\cosh(a+bx)\right)}{b}\\ &= \frac{\cosh^2(a+bx)}{2b}-\frac{\log(\cosh(a+bx))}{b}\end{aligned}$$

Mathematica [A] time = 0.02, size = 25, normalized size = 0.89

$$-\frac{\log(\cosh(a+bx))-\frac{1}{2}\cosh^2(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[a + b*x]^2*Tanh[a + b*x],x]
```

```
[Out] -((-1/2*Cosh[a + b*x]^2 + Log[Cosh[a + b*x]])/b)
```

fricas [B] time = 0.45, size = 197, normalized size = 7.04

$$\frac{8\,bx\cosh\left(bx+a\right)^2+\cosh\left(bx+a\right)^4+4\,\cosh\left(bx+a\right)\sinh\left(bx+a\right)^3+\sinh\left(bx+a\right)^4+2\left(4\,bx+3\,\cosh\left(bx+\right.\right.}{8\left(b\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)^2*tanh(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/8*(8*b*x*cosh(b*x + a)^2 + cosh(b*x + a)^4 + 4*cosh(b*x + a)*sinh(b*x + a
)^3 + sinh(b*x + a)^4 + 2*(4*b*x + 3*cosh(b*x + a)^2)*sinh(b*x + a)^2 - 8*(
cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2)*log(2*co
sh(b*x + a)/(cosh(b*x + a) - sinh(b*x + a))) + 4*(4*b*x*cosh(b*x + a) + cos
h(b*x + a)^3)*sinh(b*x + a) + 1)/(b*cosh(b*x + a)^2 + 2*b*cosh(b*x + a)*sin
h(b*x + a) + b*sinh(b*x + a)^2)
```

giac [B] time = 0.13, size = 57, normalized size = 2.04

$$\frac{8\,bx-\left(4\,e^{(2\,bx+2\,a)}-1\right)e^{(-2\,bx-2\,a)}+e^{(2\,bx+2\,a)}-8\,\log\left(e^{(2\,bx+2\,a)}+1\right)}{8\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)^2*tanh(b*x+a),x, algorithm="giac")
```

```
[Out] 1/8*(8*b*x - (4*e^(2*b*x + 2*a) - 1)*e^(-2*b*x - 2*a) + e^(2*b*x + 2*a) - 8
*log(e^(2*b*x + 2*a) + 1))/b
```

maple [A] time = 0.14, size = 27, normalized size = 0.96

$$\frac{\sinh^2\left(bx+a\right)}{2b}-\frac{\ln\left(\cosh\left(bx+a\right)\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(b*x+a)^2*tanh(b*x+a),x)
```

```
[Out] 1/2*sinh(b*x+a)^2/b-ln(cosh(b*x+a))/b
```

maxima [B] time = 0.58, size = 56, normalized size = 2.00

$$-\frac{bx+a}{b}+\frac{e^{(2\,bx+2\,a)}}{8\,b}+\frac{e^{(-2\,bx-2\,a)}}{8\,b}-\frac{\log\left(e^{(-2\,bx-2\,a)}+1\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)^2*tanh(b*x+a),x, algorithm="maxima")
```

```
[Out] -(b*x + a)/b + 1/8*e^(2*b*x + 2*a)/b + 1/8*e^(-2*b*x - 2*a)/b - log(e^(-2*b
*x - 2*a) + 1)/b
```

mupad [B] time = 0.08, size = 48, normalized size = 1.71

$$x-\frac{\ln\left(e^{2\,a}\,e^{2\,b\,x}+1\right)}{b}+\frac{e^{-2\,a-2\,b\,x}}{8\,b}+\frac{e^{2\,a+2\,b\,x}}{8\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a + b*x)^2*tanh(a + b*x),x)
```

```
[Out] x - log(exp(2*a)*exp(2*b*x) + 1)/b + exp(- 2*a - 2*b*x)/(8*b) + exp(2*a + 2
*b*x)/(8*b)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sinh^{2}\left(a+bx\right)\tanh\left(a+bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)**2*tanh(b*x+a),x)
```

```
[Out] Integral(sinh(a + b*x)**2*tanh(a + b*x), x)
```

3.75 $\int \sinh^2(a+bx)\tanh^2(a+bx)\,dx$

Optimal. Leaf size=40

$$\frac{3\tanh(a+bx)}{2b}+\frac{\sinh^2(a+bx)\tanh(a+bx)}{2b}-\frac{3x}{2}$$

```
[Out] -3/2*x+3/2*tanh(b*x+a)/b+1/2*sinh(b*x+a)^2*tanh(b*x+a)/b
```

Rubi [A] time = 0.04, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {2591, 288, 321, 206}

$$\frac{3\tanh(a+bx)}{2b}+\frac{\sinh^2(a+bx)\tanh(a+bx)}{2b}-\frac{3x}{2}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[a + b*x]^2*Tanh[a + b*x]^2,x]
```

```
[Out] (-3*x)/2 + (3*Tanh[a + b*x])/(2*b) + (Sinh[a + b*x]^2*Tanh[a + b*x])/(2*b)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2591

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff)/f, Subst[In
t[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, (b*Tan[e + f*x])/ff], x
]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

Rubi steps

$$\int \sinh^2(a+bx)\tanh^2(a+bx)\,dx = \frac{\text{Subst}\left(\int \frac{x^4}{(1-x^2)^2}\,dx, x, \tanh(a+bx)\right)}{b}$$
$$= \frac{\sinh^2(a+bx)\tanh(a+bx)}{2b} - \frac{3\,\text{Subst}\left(\int \frac{x^2}{1-x^2}\,dx, x, \tanh(a+bx)\right)}{2b}$$
$$= \frac{3\tanh(a+bx)}{2b} + \frac{\sinh^2(a+bx)\tanh(a+bx)}{2b} - \frac{3\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \tanh(\right.}{2b}$$
$$= -\frac{3x}{2} + \frac{3\tanh(a+bx)}{2b} + \frac{\sinh^2(a+bx)\tanh(a+bx)}{2b}$$

Mathematica [A] time = 0.11, size = 31, normalized size = 0.78

$$\frac{-6(a+bx)+\sinh(2(a+bx))+4\tanh(a+bx)}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[a + b*x]^2*Tanh[a + b*x]^2,x]
[Out] (-6*(a + b*x) + Sinh[2*(a + b*x)] + 4*Tanh[a + b*x])/(4*b)
```

fricas [A] time = 0.46, size = 54, normalized size = 1.35

$$\frac{\sinh(bx+a)^3 - 4(3bx+2)\cosh(bx+a) + 3\left(\cosh(bx+a)^2+3\right)\sinh(bx+a)}{8b\cosh(bx+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)^2*tanh(b*x+a)^2,x, algorithm="fricas")
[Out] 1/8*(sinh(b*x + a)^3 - 4*(3*b*x + 2)*cosh(b*x + a) + 3*(cosh(b*x + a)^2 + 3
)*sinh(b*x + a))/(b*cosh(b*x + a))
```

giac [A] time = 0.15, size = 68, normalized size = 1.70

$$-\frac{12\,bx - \frac{\left(3\,e^{(4\,bx+4\,a)} - 14\,e^{(2\,bx+2\,a)} - 1\right)e^{(-2\,a)}}{e^{(2\,bx)} + e^{(4\,bx+2\,a)}} - e^{(2\,bx+2\,a)}}{8\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)^2*tanh(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -1/8*(12*b*x - (3*e^(4*b*x + 4*a) - 14*e^(2*b*x + 2*a) - 1)*e^(-2*a)/(e^(2*
b*x) + e^(4*b*x + 2*a)) - e^(2*b*x + 2*a))/b
```

maple [A] time = 0.12, size = 39, normalized size = 0.98

$$\frac{\frac{\sinh^3(bx+a)}{2\cosh(bx+a)} - \frac{3bx}{2} - \frac{3a}{2} + \frac{3\tanh(bx+a)}{2}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(b*x+a)^2*tanh(b*x+a)^2,x)
```

```
[Out] 1/b*(1/2*sinh(b*x+a)^3/cosh(b*x+a)-3/2*b*x-3/2*a+3/2*tanh(b*x+a))
```

maxima [A] time = 0.42, size = 64, normalized size = 1.60

$$-\frac{3\,(bx+a)}{2\,b} - \frac{e^{(-2\,bx-2\,a)}}{8\,b} + \frac{17\,e^{(-2\,bx-2\,a)} + 1}{8\,b\left(e^{(-2\,bx-2\,a)} + e^{(-4\,bx-4\,a)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)^2*tanh(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -3/2*(b*x + a)/b - 1/8*e^(-2*b*x - 2*a)/b + 1/8*(17*e^(-2*b*x - 2*a) + 1)/(
b*(e^(-2*b*x - 2*a) + e^(-4*b*x - 4*a)))
```

mupad [B] time = 1.47, size = 50, normalized size = 1.25

$$\frac{e^{2\,a+2\,b\,x}}{8\,b} - \frac{2}{b\,\left(e^{2\,a+2\,b\,x}+1\right)} - \frac{e^{-2\,a-2\,b\,x}}{8\,b} - \frac{3\,x}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a + b*x)^2*tanh(a + b*x)^2,x)
```

```
[Out] exp(2*a + 2*b*x)/(8*b) - 2/(b*(exp(2*a + 2*b*x) + 1)) - exp(- 2*a - 2*b*x)/
(8*b) - (3*x)/2
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sinh^2(a+bx)\tanh^2(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)**2*tanh(b*x+a)**2,x)
```

```
[Out] Integral(sinh(a + b*x)**2*tanh(a + b*x)**2, x)
```

3.76 $\int \sinh^2(a+bx)\tanh^3(a+bx)\,dx$

Optimal. Leaf size=43

$$\frac{\cosh^2(a+bx)}{2b}-\frac{\operatorname{sech}^2(a+bx)}{2b}-\frac{2\log(\cosh(a+bx))}{b}$$

```
[Out] 1/2*cosh(b*x+a)^2/b-2*ln(cosh(b*x+a))/b-1/2*sech(b*x+a)^2/b
```

Rubi [A] time = 0.04, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {2590, 266, 43}

$$\frac{\cosh^2(a+bx)}{2b}-\frac{\operatorname{sech}^2(a+bx)}{2b}-\frac{2\log(\cosh(a+bx))}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[a + b*x]^2*Tanh[a + b*x]^3,x]
```

```
[Out] Cosh[a + b*x]^2/(2*b) - (2*Log[Cosh[a + b*x]])/b - Sech[a + b*x]^2/(2*b)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2590

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> -Dist[f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rubi steps

$$\int \sinh^2(a+bx)\tanh^3(a+bx)\,dx = \frac{\text{Subst}\left(\int \frac{(1-x^2)^2}{x^3}\,dx, x, \cosh(a+bx)\right)}{b}$$
$$= \frac{\text{Subst}\left(\int \frac{(1-x)^2}{x^2}\,dx, x, \cosh^2(a+bx)\right)}{2b}$$
$$= \frac{\text{Subst}\left(\int \left(1+\frac{1}{x^2}-\frac{2}{x}\right)dx, x, \cosh^2(a+bx)\right)}{2b}$$
$$= \frac{\cosh^2(a+bx)}{2b} - \frac{2\log(\cosh(a+bx))}{b} - \frac{\operatorname{sech}^2(a+bx)}{2b}$$

Mathematica [**A**] time = 0.05, size = 35, normalized size = 0.81

$$-\frac{-\sinh^2(a+bx)+\operatorname{sech}^2(a+bx)+4\log(\cosh(a+bx))}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[a + b*x]^2*Tanh[a + b*x]^3,x]
```

```
[Out] -1/2*(4*Log[Cosh[a + b*x]] + Sech[a + b*x]^2 - Sinh[a + b*x]^2)/b
```

fricas [**B**] time = 0.44, size = 742, normalized size = 17.26

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)^2*tanh(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/8*(cosh(b*x + a)^8 + 8*cosh(b*x + a)*sinh(b*x + a)^7 + sinh(b*x + a)^8 +
2*(8*b*x + 1)*cosh(b*x + a)^6 + 2*(8*b*x + 14*cosh(b*x + a)^2 + 1)*sinh(b*x
 + a)^6 + 4*(14*cosh(b*x + a)^3 + 3*(8*b*x + 1)*cosh(b*x + a))*sinh(b*x + a
)^5 + 2*(16*b*x - 7)*cosh(b*x + a)^4 + 2*(35*cosh(b*x + a)^4 + 15*(8*b*x +
1)*cosh(b*x + a)^2 + 16*b*x - 7)*sinh(b*x + a)^4 + 8*(7*cosh(b*x + a)^5 + 5
*(8*b*x + 1)*cosh(b*x + a)^3 + (16*b*x - 7)*cosh(b*x + a))*sinh(b*x + a)^3
+ 2*(8*b*x + 1)*cosh(b*x + a)^2 + 2*(14*cosh(b*x + a)^6 + 15*(8*b*x + 1)*co
sh(b*x + a)^4 + 6*(16*b*x - 7)*cosh(b*x + a)^2 + 8*b*x + 1)*sinh(b*x + a)^2
 - 16*(cosh(b*x + a)^6 + 6*cosh(b*x + a)*sinh(b*x + a)^5 + sinh(b*x + a)^6
+ (15*cosh(b*x + a)^2 + 2)*sinh(b*x + a)^4 + 2*cosh(b*x + a)^4 + 4*(5*cosh(
b*x + a)^3 + 2*cosh(b*x + a))*sinh(b*x + a)^3 + (15*cosh(b*x + a)^4 + 12*co
sh(b*x + a)^2 + 1)*sinh(b*x + a)^2 + cosh(b*x + a)^2 + 2*(3*cosh(b*x + a)^5
```

```
 + 4*cosh(b*x + a)^3 + cosh(b*x + a))*sinh(b*x + a))*log(2*cosh(b*x + a)/(c
osh(b*x + a) - sinh(b*x + a))) + 4*(2*cosh(b*x + a)^7 + 3*(8*b*x + 1)*cosh(
b*x + a)^5 + 2*(16*b*x - 7)*cosh(b*x + a)^3 + (8*b*x + 1)*cosh(b*x + a))*si
nh(b*x + a) + 1)/(b*cosh(b*x + a)^6 + 6*b*cosh(b*x + a)*sinh(b*x + a)^5 + b
*sinh(b*x + a)^6 + 2*b*cosh(b*x + a)^4 + (15*b*cosh(b*x + a)^2 + 2*b)*sinh(
b*x + a)^4 + 4*(5*b*cosh(b*x + a)^3 + 2*b*cosh(b*x + a))*sinh(b*x + a)^3 +
b*cosh(b*x + a)^2 + (15*b*cosh(b*x + a)^4 + 12*b*cosh(b*x + a)^2 + b)*sinh(
b*x + a)^2 + 2*(3*b*cosh(b*x + a)^5 + 4*b*cosh(b*x + a)^3 + b*cosh(b*x + a)
)*sinh(b*x + a))
```

giac [B] time = 0.18, size = 96, normalized size = 2.23

$$\frac{16\,bx-\left(8\,e^{(2\,bx+2\,a)}-1\right)e^{(-2\,bx-2\,a)}+\frac{8\left(3\,e^{(4\,bx+4\,a)}+4\,e^{(2\,bx+2\,a)}+3\right)}{\left(e^{(2\,bx+2\,a)}+1\right)^2}+e^{(2\,bx+2\,a)}-16\,\log\left(e^{(2\,bx+2\,a)}+1\right)}{8\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)^2*tanh(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/8*(16*b*x - (8*e^(2*b*x + 2*a) - 1)*e^(-2*b*x - 2*a) + 8*(3*e^(4*b*x + 4*
a) + 4*e^(2*b*x + 2*a) + 3)/(e^(2*b*x + 2*a) + 1)^2 + e^(2*b*x + 2*a) - 16*
log(e^(2*b*x + 2*a) + 1))/b
```

maple [A] time = 0.14, size = 47, normalized size = 1.09

$$\frac{\sinh^4(bx+a)}{2b\cosh(bx+a)^2}-\frac{2\ln(\cosh(bx+a))}{b}+\frac{\tanh^2(bx+a)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(b*x+a)^2*tanh(b*x+a)^3,x)
```

```
[Out] 1/2/b*sinh(b*x+a)^4/cosh(b*x+a)^2-2*ln(cosh(b*x+a))/b+tanh(b*x+a)^2/b
```

maxima [B] time = 0.41, size = 103, normalized size = 2.40

$$-\frac{2\,(bx+a)}{b}+\frac{e^{(-2\,bx-2\,a)}}{8\,b}-\frac{2\,\log\left(e^{(-2\,bx-2\,a)}+1\right)}{b}+\frac{2\,e^{(-2\,bx-2\,a)}-15\,e^{(-4\,bx-4\,a)}+1}{8\,b\left(e^{(-2\,bx-2\,a)}+2\,e^{(-4\,bx-4\,a)}+e^{(-6\,bx-6\,a)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)^2*tanh(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -2*(b*x + a)/b + 1/8*e^(-2*b*x - 2*a)/b - 2*log(e^(-2*b*x - 2*a) + 1)/b + 1
/8*(2*e^(-2*b*x - 2*a) - 15*e^(-4*b*x - 4*a) + 1)/(b*(e^(-2*b*x - 2*a) + 2*
e^(-4*b*x - 4*a) + e^(-6*b*x - 6*a)))
```

mupad [B] time = 1.50, size = 97, normalized size = 2.26

$$2\,x-\frac{2\,\ln\left({\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1\right)}{b}-\frac{2}{b\,\left({\mathrm{e}}^{2\,a+2\,b\,x}+1\right)}+\frac{2}{b\,\left(2\,{\mathrm{e}}^{2\,a+2\,b\,x}+{\mathrm{e}}^{4\,a+4\,b\,x}+1\right)}+\frac{{\mathrm{e}}^{-2\,a-2\,b\,x}}{8\,b}+\frac{{\mathrm{e}}^{2\,a+2\,b\,x}}{8\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a + b*x)^2*tanh(a + b*x)^3,x)
```

```
[Out] 2*x - (2*log(exp(2*a)*exp(2*b*x) + 1))/b - 2/(b*(exp(2*a + 2*b*x) + 1)) + 2
/(b*(2*exp(2*a + 2*b*x) + exp(4*a + 4*b*x) + 1)) + exp(- 2*a - 2*b*x)/(8*b)
 + exp(2*a + 2*b*x)/(8*b)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sinh^{2}{\left(a + bx \right)} \tanh^{3}{\left(a + bx \right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)**2*tanh(b*x+a)**3,x)
```

```
[Out] Integral(sinh(a + b*x)**2*tanh(a + b*x)**3, x)
```

3.77 $\int \sinh^3(a+bx)\tanh(a+bx)\,dx$

Optimal. Leaf size=38

$$\frac{\sinh^3(a+bx)}{3b}-\frac{\sinh(a+bx)}{b}+\frac{\tan^{-1}(\sinh(a+bx))}{b}$$

```
[Out] arctan(sinh(b*x+a))/b-sinh(b*x+a)/b+1/3*sinh(b*x+a)^3/b
```

Rubi [A] time = 0.03, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2592, 302, 203}

$$\frac{\sinh^3(a+bx)}{3b}-\frac{\sinh(a+bx)}{b}+\frac{\tan^{-1}(\sinh(a+bx))}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[a + b*x]^3*Tanh[a + b*x],x]
```

```
[Out] ArcTan[Sinh[a + b*x]]/b - Sinh[a + b*x]/b + Sinh[a + b*x]^3/(3*b)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 2592

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, (a*Sin[e + f*x])/ff], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rubi steps

$$\int \sinh^3(a+bx)\tanh(a+bx)\,dx = \frac{\text{Subst}\left(\int \frac{x^4}{1+x^2}\,dx, x, \sinh(a+bx)\right)}{b}$$
$$= \frac{\text{Subst}\left(\int \left(-1+x^2+\frac{1}{1+x^2}\right)dx, x, \sinh(a+bx)\right)}{b}$$
$$= -\frac{\sinh(a+bx)}{b} + \frac{\sinh^3(a+bx)}{3b} + \frac{\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sinh(a+bx)\right)}{b}$$
$$= \frac{\tan^{-1}(\sinh(a+bx))}{b} - \frac{\sinh(a+bx)}{b} + \frac{\sinh^3(a+bx)}{3b}$$

Mathematica [A] time = 0.01, size = 38, normalized size = 1.00

$$\frac{\sinh^3(a+bx)}{3b} - \frac{\sinh(a+bx)}{b} + \frac{\tan^{-1}(\sinh(a+bx))}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[a + b*x]^3*Tanh[a + b*x],x]
```

```
[Out] ArcTan[Sinh[a + b*x]]/b - Sinh[a + b*x]/b + Sinh[a + b*x]^3/(3*b)
```

fricas [B] time = 0.46, size = 290, normalized size = 7.63

$$\frac{\cosh(bx+a)^6 + 6\cosh(bx+a)\sinh(bx+a)^5 + \sinh(bx+a)^6 + 15\left(\cosh(bx+a)^2-1\right)\sinh(bx+a)^4 - 15\,\mathrm{co}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)^3*tanh(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/24*(cosh(b*x + a)^6 + 6*cosh(b*x + a)*sinh(b*x + a)^5 + sinh(b*x + a)^6 +
 15*(cosh(b*x + a)^2 - 1)*sinh(b*x + a)^4 - 15*cosh(b*x + a)^4 + 20*(cosh(b
*x + a)^3 - 3*cosh(b*x + a))*sinh(b*x + a)^3 + 15*(cosh(b*x + a)^4 - 6*cosh
(b*x + a)^2 + 1)*sinh(b*x + a)^2 + 48*(cosh(b*x + a)^3 + 3*cosh(b*x + a)^2*
sinh(b*x + a) + 3*cosh(b*x + a)*sinh(b*x + a)^2 + sinh(b*x + a)^3)*arctan(c
osh(b*x + a) + sinh(b*x + a)) + 15*cosh(b*x + a)^2 + 6*(cosh(b*x + a)^5 - 1
0*cosh(b*x + a)^3 + 5*cosh(b*x + a))*sinh(b*x + a) - 1)/(b*cosh(b*x + a)^3
+ 3*b*cosh(b*x + a)^2*sinh(b*x + a) + 3*b*cosh(b*x + a)*sinh(b*x + a)^2 + b
*sinh(b*x + a)^3)
```

giac [A] time = 0.13, size = 63, normalized size = 1.66

$$\frac{\left(15\,e^{(2\,bx+2\,a)}-1\right)e^{(-3\,bx-3\,a)}+\left(e^{(3\,bx+18\,a)}-15\,e^{(bx+16\,a)}\right)e^{(-15\,a)}+48\,\arctan\left(e^{(bx+a)}\right)}{24\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)^3*tanh(b*x+a),x, algorithm="giac")
```

```
[Out] 1/24*((15*e^(2*b*x + 2*a) - 1)*e^(-3*b*x - 3*a) + (e^(3*b*x + 18*a) - 15*e^
(b*x + 16*a))*e^(-15*a) + 48*arctan(e^(b*x + a)))/b
```

maple [A] time = 0.14, size = 38, normalized size = 1.00

$$\frac{\sinh^3(bx+a)}{3b}-\frac{\sinh(bx+a)}{b}+\frac{2\arctan\left(e^{bx+a}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(b*x+a)^3*tanh(b*x+a),x)
```

```
[Out] 1/3*sinh(b*x+a)^3/b-sinh(b*x+a)/b+2*arctan(exp(b*x+a))/b
```

maxima [A] time = 0.60, size = 71, normalized size = 1.87

$$-\frac{\left(15\,e^{(-2\,bx-2\,a)}-1\right)e^{(3\,bx+3\,a)}}{24\,b}+\frac{15\,e^{(-bx-a)}-e^{(-3\,bx-3\,a)}}{24\,b}-\frac{2\,\arctan\left(e^{(-bx-a)}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)^3*tanh(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/24*(15*e^(-2*b*x - 2*a) - 1)*e^(3*b*x + 3*a)/b + 1/24*(15*e^(-b*x - a) -
 e^(-3*b*x - 3*a))/b - 2*arctan(e^(-b*x - a))/b
```

mupad [B] time = 0.09, size = 77, normalized size = 2.03

$$\frac{2\,\mathrm{atan}\left(\frac{e^{b\,x}\,e^{a}\,\sqrt{b^2}}{b}\right)}{\sqrt{b^2}}-\frac{5\,e^{a+b\,x}}{8\,b}+\frac{5\,e^{-a-b\,x}}{8\,b}-\frac{e^{-3\,a-3\,b\,x}}{24\,b}+\frac{e^{3\,a+3\,b\,x}}{24\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a + b*x)^3*tanh(a + b*x),x)
```

```
[Out] (2*atan((exp(b*x)*exp(a)*(b^2)^(1/2))/b))/(b^2)^(1/2) - (5*exp(a + b*x))/(8
*b) + (5*exp(- a - b*x))/(8*b) - exp(- 3*a - 3*b*x)/(24*b) + exp(3*a + 3*b*
x)/(24*b)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sinh^3(a+bx)\tanh(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)**3*tanh(b*x+a),x)
```

```
[Out] Integral(sinh(a + b*x)**3*tanh(a + b*x), x)
```

3.78 $\int \sinh^3(a+bx)\tanh^2(a+bx)\,dx$

Optimal. Leaf size=38

$$\frac{\cosh^3(a+bx)}{3b}-\frac{2\cosh(a+bx)}{b}-\frac{\operatorname{sech}(a+bx)}{b}$$

```
[Out] -2*cosh(b*x+a)/b+1/3*cosh(b*x+a)^3/b-sech(b*x+a)/b
```

Rubi [A] time = 0.04, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2590, 270}

$$\frac{\cosh^3(a+bx)}{3b}-\frac{2\cosh(a+bx)}{b}-\frac{\operatorname{sech}(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[a + b*x]^3*Tanh[a + b*x]^2,x]
```

```
[Out] (-2*Cosh[a + b*x])/b + Cosh[a + b*x]^3/(3*b) - Sech[a + b*x]/b
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2590

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> -Dist[f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rubi steps

$$\begin{aligned}\int \sinh^3(a+bx)\tanh^2(a+bx)\,dx &= \frac{\text{Subst}\left(\int \frac{\left(1-x^2\right)^2}{x^2}\,dx,x,\cosh(a+bx)\right)}{b}\\ &= \frac{\text{Subst}\left(\int \left(-2+\frac{1}{x^2}+x^2\right)dx,x,\cosh(a+bx)\right)}{b}\\ &= -\frac{2\cosh(a+bx)}{b}+\frac{\cosh^3(a+bx)}{3b}-\frac{\operatorname{sech}(a+bx)}{b}\end{aligned}$$

Mathematica [A] time = 0.03, size = 40, normalized size = 1.05

$$-\frac{7\cosh(a+bx)}{4b}+\frac{\cosh(3(a+bx))}{12b}-\frac{\operatorname{sech}(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[a + b*x]^3*Tanh[a + b*x]^2,x]
[Out] (-7*Cosh[a + b*x])/(4*b) + Cosh[3*(a + b*x)]/(12*b) - Sech[a + b*x]/b
```

fricas [A] time = 0.52, size = 63, normalized size = 1.66

$$\frac{\cosh\left(bx+a\right)^{4}+\sinh\left(bx+a\right)^{4}+2\left(3\cosh\left(bx+a\right)^{2}-10\right)\sinh\left(bx+a\right)^{2}-20\cosh\left(bx+a\right)^{2}-45}{24\,b\cosh\left(bx+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)^3*tanh(b*x+a)^2,x, algorithm="fricas")
[Out] 1/24*(cosh(b*x + a)^4 + sinh(b*x + a)^4 + 2*(3*cosh(b*x + a)^2 - 10)*sinh(b
*x + a)^2 - 20*cosh(b*x + a)^2 - 45)/(b*cosh(b*x + a))
```

giac [B] time = 0.15, size = 76, normalized size = 2.00

$$-\frac{\left(21\,e^{(2\,bx+2\,a)}-1\right)e^{(-3\,bx-3\,a)}-\left(e^{(3\,bx+24\,a)}-21\,e^{(bx+22\,a)}\right)e^{(-21\,a)}+\frac{48\,e^{(bx+a)}}{e^{(2\,bx+2\,a)}+1}}{24\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)^3*tanh(b*x+a)^2,x, algorithm="giac")
[Out] -1/24*((21*e^(2*b*x + 2*a) - 1)*e^(-3*b*x - 3*a) - (e^(3*b*x + 24*a) - 21*e
^(b*x + 22*a))*e^(-21*a) + 48*e^(b*x + a)/(e^(2*b*x + 2*a) + 1))/b
```

maple [A] time = 0.11, size = 52, normalized size = 1.37

$$\frac{\frac{\sinh^{4}(bx+a)}{3\cosh(bx+a)}-\frac{4\left(\sinh^{2}(bx+a)\right)}{3\cosh(bx+a)}-\frac{8}{3\cosh(bx+a)}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(b*x+a)^3*tanh(b*x+a)^2,x)
[Out] 1/b*(1/3*sinh(b*x+a)^4/cosh(b*x+a)-4/3*sinh(b*x+a)^2/cosh(b*x+a)-8/3/cosh(b
*x+a))
```

maxima [B] time = 0.43, size = 79, normalized size = 2.08

$$-\frac{21\,e^{(-bx-a)}-e^{(-3\,bx-3\,a)}}{24\,b}-\frac{20\,e^{(-2\,bx-2\,a)}+69\,e^{(-4\,bx-4\,a)}-1}{24\,b\left(e^{(-3\,bx-3\,a)}+e^{(-5\,bx-5\,a)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)^3*tanh(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -1/24*(21*e^(-b*x - a) - e^(-3*b*x - 3*a))/b - 1/24*(20*e^(-2*b*x - 2*a) +
69*e^(-4*b*x - 4*a) - 1)/(b*(e^(-3*b*x - 3*a) + e^(-5*b*x - 5*a)))
```

mupad [B] time = 1.54, size = 78, normalized size = 2.05

$$\frac{\mathrm{e}^{-3\,a-3\,b\,x}}{24\,b}-\frac{7\,\mathrm{e}^{-a-b\,x}}{8\,b}-\frac{7\,\mathrm{e}^{a+b\,x}}{8\,b}+\frac{\mathrm{e}^{3\,a+3\,b\,x}}{24\,b}-\frac{2\,\mathrm{e}^{a+b\,x}}{b\,\left(\mathrm{e}^{2\,a+2\,b\,x}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a + b*x)^3*tanh(a + b*x)^2,x)
```

```
[Out] exp(- 3*a - 3*b*x)/(24*b) - (7*exp(- a - b*x))/(8*b) - (7*exp(a + b*x))/(8*
b) + exp(3*a + 3*b*x)/(24*b) - (2*exp(a + b*x))/(b*(exp(2*a + 2*b*x) + 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sinh^{3}{\left(a+bx \right)}\tanh^{2}{\left(a+bx \right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)**3*tanh(b*x+a)**2,x)
```

```
[Out] Integral(sinh(a + b*x)**3*tanh(a + b*x)**2, x)
```

3.79 $\int \sinh^3(a+bx)\tanh^3(a+bx)\,dx$

Optimal. Leaf size=66

$$\frac{5\sinh^3(a+bx)}{6b}-\frac{5\sinh(a+bx)}{2b}-\frac{\sinh^3(a+bx)\tanh^2(a+bx)}{2b}+\frac{5\tan^{-1}(\sinh(a+bx))}{2b}$$

```
[Out] 5/2*arctan(sinh(b*x+a))/b-5/2*sinh(b*x+a)/b+5/6*sinh(b*x+a)^3/b-1/2*sinh(b*
x+a)^3*tanh(b*x+a)^2/b
```

Rubi [A] time = 0.04, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {2592, 288, 302, 203}

$$\frac{5\sinh^3(a+bx)}{6b}-\frac{5\sinh(a+bx)}{2b}-\frac{\sinh^3(a+bx)\tanh^2(a+bx)}{2b}+\frac{5\tan^{-1}(\sinh(a+bx))}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[a + b*x]^3*Tanh[a + b*x]^3,x]
```

```
[Out] (5*ArcTan[Sinh[a + b*x]])/(2*b) - (5*Sinh[a + b*x])/(2*b) + (5*Sinh[a + b*x
]^3)/(6*b) - (Sinh[a + b*x]^3*Tanh[a + b*x]^2)/(2*b)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 2592

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, (a*Sin[e + f*x])/ff], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rubi steps

$$\int \sinh^3(a+bx)\tanh^3(a+bx)\,dx = \frac{\text{Subst}\left(\int \frac{x^6}{\left(1+x^2\right)^2}\,dx,x,\sinh(a+bx)\right)}{b}$$

$$= -\frac{\sinh^3(a+bx)\tanh^2(a+bx)}{2b} + \frac{5\,\text{Subst}\left(\int \frac{x^4}{1+x^2}\,dx,x,\sinh(a+bx)\right)}{2b}$$

$$= -\frac{\sinh^3(a+bx)\tanh^2(a+bx)}{2b} + \frac{5\,\text{Subst}\left(\int \left(-1+x^2+\frac{1}{1+x^2}\right)dx,x,\sinh(a$$

$$= -\frac{5\sinh(a+bx)}{2b} + \frac{5\sinh^3(a+bx)}{6b} - \frac{\sinh^3(a+bx)\tanh^2(a+bx)}{2b} + \frac{5\,\text{Sub}$$

$$= \frac{5\tan^{-1}(\sinh(a+bx))}{2b} - \frac{5\sinh(a+bx)}{2b} + \frac{5\sinh^3(a+bx)}{6b} - \frac{\sinh^3(a+bx)}{2}$$

Mathematica [A] time = 0.11, size = 65, normalized size = 0.98

$$\frac{2\sinh^3(a+bx)\tanh^2(a+bx)+15\tan^{-1}(\sinh(a+bx))-10\sinh(a+bx)\tanh^2(a+bx)-15\tanh(a+bx)\text{sech}}{6b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[a + b*x]^3*Tanh[a + b*x]^3,x]
```

```
[Out] (15*ArcTan[Sinh[a + b*x]] - 15*Sech[a + b*x]*Tanh[a + b*x] - 10*Sinh[a + b*
x]*Tanh[a + b*x]^2 + 2*Sinh[a + b*x]^3*Tanh[a + b*x]^2)/(6*b)
```

fricas [B] time = 0.45, size = 851, normalized size = 12.89

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)^3*tanh(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/24*(cosh(b*x + a)^10 + 10*cosh(b*x + a)*sinh(b*x + a)^9 + sinh(b*x + a)^1
0 + 5*(9*cosh(b*x + a)^2 - 5)*sinh(b*x + a)^8 - 25*cosh(b*x + a)^8 + 40*(3*
```

```
cosh(b*x + a)^3 - 5*cosh(b*x + a))*sinh(b*x + a)^7 + 10*(21*cosh(b*x + a)^4
 - 70*cosh(b*x + a)^2 - 5)*sinh(b*x + a)^6 - 50*cosh(b*x + a)^6 + 4*(63*cos
h(b*x + a)^5 - 350*cosh(b*x + a)^3 - 75*cosh(b*x + a))*sinh(b*x + a)^5 + 10
*(21*cosh(b*x + a)^6 - 175*cosh(b*x + a)^4 - 75*cosh(b*x + a)^2 + 5)*sinh(b
*x + a)^4 + 50*cosh(b*x + a)^4 + 40*(3*cosh(b*x + a)^7 - 35*cosh(b*x + a)^5
 - 25*cosh(b*x + a)^3 + 5*cosh(b*x + a))*sinh(b*x + a)^3 + 5*(9*cosh(b*x +
a)^8 - 140*cosh(b*x + a)^6 - 150*cosh(b*x + a)^4 + 60*cosh(b*x + a)^2 + 5)*
sinh(b*x + a)^2 + 120*(cosh(b*x + a)^7 + 7*cosh(b*x + a)*sinh(b*x + a)^6 +
sinh(b*x + a)^7 + (21*cosh(b*x + a)^2 + 2)*sinh(b*x + a)^5 + 2*cosh(b*x + a
)^5 + 5*(7*cosh(b*x + a)^3 + 2*cosh(b*x + a))*sinh(b*x + a)^4 + (35*cosh(b*
x + a)^4 + 20*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^3 + cosh(b*x + a)^3 + (21*
cosh(b*x + a)^5 + 20*cosh(b*x + a)^3 + 3*cosh(b*x + a))*sinh(b*x + a)^2 + (
7*cosh(b*x + a)^6 + 10*cosh(b*x + a)^4 + 3*cosh(b*x + a)^2)*sinh(b*x + a))*
arctan(cosh(b*x + a) + sinh(b*x + a)) + 25*cosh(b*x + a)^2 + 10*(cosh(b*x +
 a)^9 - 20*cosh(b*x + a)^7 - 30*cosh(b*x + a)^5 + 20*cosh(b*x + a)^3 + 5*co
sh(b*x + a))*sinh(b*x + a) - 1)/(b*cosh(b*x + a)^7 + 7*b*cosh(b*x + a)*sinh
(b*x + a)^6 + b*sinh(b*x + a)^7 + 2*b*cosh(b*x + a)^5 + (21*b*cosh(b*x + a)
^2 + 2*b)*sinh(b*x + a)^5 + 5*(7*b*cosh(b*x + a)^3 + 2*b*cosh(b*x + a))*sin
h(b*x + a)^4 + b*cosh(b*x + a)^3 + (35*b*cosh(b*x + a)^4 + 20*b*cosh(b*x +
a)^2 + b)*sinh(b*x + a)^3 + (21*b*cosh(b*x + a)^5 + 20*b*cosh(b*x + a)^3 +
3*b*cosh(b*x + a))*sinh(b*x + a)^2 + (7*b*cosh(b*x + a)^6 + 10*b*cosh(b*x +
 a)^4 + 3*b*cosh(b*x + a)^2)*sinh(b*x + a))
```

giac [A] time = 0.18, size = 96, normalized size = 1.45

$$\frac{\left(27\,e^{(2\,bx+2\,a)}-1\right)e^{(-3\,bx-3\,a)}+\left(e^{(3\,bx+30\,a)}-27\,e^{(bx+28\,a)}\right)e^{(-27\,a)}-\frac{24\left(e^{(3\,bx+3\,a)}-e^{(bx+a)}\right)}{\left(e^{(2\,bx+2\,a)}+1\right)^2}+120\,\arctan\left(e^{(bx+a)}\right)}{24\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)^3*tanh(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/24*((27*e^(2*b*x + 2*a) - 1)*e^(-3*b*x - 3*a) + (e^(3*b*x + 30*a) - 27*e^
(b*x + 28*a))*e^(-27*a) - 24*(e^(3*b*x + 3*a) - e^(b*x + a))/(e^(2*b*x + 2*
a) + 1)^2 + 120*arctan(e^(b*x + a)))/b
```

maple [A] time = 0.34, size = 92, normalized size = 1.39

$$\frac{\sinh^5(bx+a)}{3b\cosh(bx+a)^2}-\frac{5\left(\sinh^3(bx+a)\right)}{3b\cosh(bx+a)^2}-\frac{5\sinh(bx+a)}{b\cosh(bx+a)^2}+\frac{5\,\mathrm{sech}(bx+a)\tanh(bx+a)}{2b}+\frac{5\arctan\left(e^{bx+a}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(b*x+a)^3*tanh(b*x+a)^3,x)
```

[Out] 1/3/b*sinh(b*x+a)^5/cosh(b*x+a)^2-5/3/b*sinh(b*x+a)^3/cosh(b*x+a)^2-5/b*sinh(b*x+a)/cosh(b*x+a)^2+5/2*sech(b*x+a)*tanh(b*x+a)/b+5*arctan(exp(b*x+a))/b

maxima [A] time = 0.42, size = 116, normalized size = 1.76

$$\frac{27\,e^{(-bx-a)}-e^{(-3\,bx-3\,a)}}{24\,b}-\frac{5\,\arctan\left(e^{(-bx-a)}\right)}{b}-\frac{25\,e^{(-2\,bx-2\,a)}+77\,e^{(-4\,bx-4\,a)}+3\,e^{(-6\,bx-6\,a)}-1}{24\,b\left(e^{(-3\,bx-3\,a)}+2\,e^{(-5\,bx-5\,a)}+e^{(-7\,bx-7\,a)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(b*x+a)^3*tanh(b*x+a)^3,x, algorithm="maxima")

[Out] 1/24*(27*e^(-b*x - a) - e^(-3*b*x - 3*a))/b - 5*arctan(e^(-b*x - a))/b - 1/24*(25*e^(-2*b*x - 2*a) + 77*e^(-4*b*x - 4*a) + 3*e^(-6*b*x - 6*a) - 1)/(b*(e^(-3*b*x - 3*a) + 2*e^(-5*b*x - 5*a) + e^(-7*b*x - 7*a)))

mupad [B] time = 1.52, size = 136, normalized size = 2.06

$$\frac{5\,\mathrm{atan}\left(\frac{\mathrm{e}^{b\,x}\,\mathrm{e}^{a}\,\sqrt{b^2}}{b}\right)}{\sqrt{b^2}}-\frac{9\,\mathrm{e}^{a+b\,x}}{8\,b}+\frac{9\,\mathrm{e}^{-a-b\,x}}{8\,b}-\frac{\mathrm{e}^{-3\,a-3\,b\,x}}{24\,b}+\frac{\mathrm{e}^{3\,a+3\,b\,x}}{24\,b}+\frac{2\,\mathrm{e}^{a+b\,x}}{b\,\left(2\,\mathrm{e}^{2\,a+2\,b\,x}+\mathrm{e}^{4\,a+4\,b\,x}+1\right)}-\frac{\mathrm{e}^{a+b\,x}}{b\,\left(\mathrm{e}^{2\,a+2\,b\,x}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sinh(a + b*x)^3*tanh(a + b*x)^3,x)

[Out] (5*atan((exp(b*x)*exp(a)*(b^2)^(1/2))/b))/(b^2)^(1/2) - (9*exp(a + b*x))/(8*b) + (9*exp(- a - b*x))/(8*b) - exp(- 3*a - 3*b*x)/(24*b) + exp(3*a + 3*b*x)/(24*b) + (2*exp(a + b*x))/(b*(2*exp(2*a + 2*b*x) + exp(4*a + 4*b*x) + 1)) - exp(a + b*x)/(b*(exp(2*a + 2*b*x) + 1))

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sinh^3(a+bx)\tanh^3(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(b*x+a)**3*tanh(b*x+a)**3,x)

[Out] Integral(sinh(a + b*x)**3*tanh(a + b*x)**3, x)

3.80 $\int \sinh^4(a+bx)\tanh(a+bx)\,dx$

Optimal. Leaf size=40

$$\frac{\cosh^4(a+bx)}{4b}-\frac{\cosh^2(a+bx)}{b}+\frac{\log(\cosh(a+bx))}{b}$$

```
[Out] -cosh(b*x+a)^2/b+1/4*cosh(b*x+a)^4/b+ln(cosh(b*x+a))/b
```

Rubi [A] time = 0.03, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2590, 266, 43}

$$\frac{\cosh^4(a+bx)}{4b}-\frac{\cosh^2(a+bx)}{b}+\frac{\log(\cosh(a+bx))}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[a + b*x]^4*Tanh[a + b*x],x]
```

```
[Out] -(Cosh[a + b*x]^2/b) + Cosh[a + b*x]^4/(4*b) + Log[Cosh[a + b*x]]/b
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2590

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> -Dist[f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rubi steps

$$\int \sinh^4(a+bx)\tanh(a+bx)\,dx = \frac{\text{Subst}\left(\int \frac{(1-x^2)^2}{x}\,dx, x, \cosh(a+bx)\right)}{b}$$
$$= \frac{\text{Subst}\left(\int \frac{(1-x)^2}{x}\,dx, x, \cosh^2(a+bx)\right)}{2b}$$
$$= \frac{\text{Subst}\left(\int \left(-2+\frac{1}{x}+x\right)\,dx, x, \cosh^2(a+bx)\right)}{2b}$$
$$= -\frac{\cosh^2(a+bx)}{b} + \frac{\cosh^4(a+bx)}{4b} + \frac{\log(\cosh(a+bx))}{b}$$

Mathematica [A] time = 0.03, size = 34, normalized size = 0.85

$$\frac{\frac{1}{4}\cosh^4(a+bx) - \cosh^2(a+bx) + \log(\cosh(a+bx))}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[a + b*x]^4*Tanh[a + b*x],x]
```

```
[Out] (-Cosh[a + b*x]^2 + Cosh[a + b*x]^4/4 + Log[Cosh[a + b*x]])/b
```

fricas [B] time = 0.41, size = 457, normalized size = 11.42

$$\frac{\cosh(bx+a)^8 + 8\cosh(bx+a)\sinh(bx+a)^7 + \sinh(bx+a)^8 + 4\left(7\cosh(bx+a)^2-3\right)\sinh(bx+a)^6 - 64}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)^4*tanh(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/64*(cosh(b*x + a)^8 + 8*cosh(b*x + a)*sinh(b*x + a)^7 + sinh(b*x + a)^8 +
 4*(7*cosh(b*x + a)^2 - 3)*sinh(b*x + a)^6 - 64*b*x*cosh(b*x + a)^4 - 12*co
sh(b*x + a)^6 + 8*(7*cosh(b*x + a)^3 - 9*cosh(b*x + a))*sinh(b*x + a)^5 + 2
*(35*cosh(b*x + a)^4 - 32*b*x - 90*cosh(b*x + a)^2)*sinh(b*x + a)^4 + 8*(7*
cosh(b*x + a)^5 - 32*b*x*cosh(b*x + a) - 30*cosh(b*x + a)^3)*sinh(b*x + a)^
3 + 4*(7*cosh(b*x + a)^6 - 96*b*x*cosh(b*x + a)^2 - 45*cosh(b*x + a)^4 - 3)
*sinh(b*x + a)^2 - 12*cosh(b*x + a)^2 + 64*(cosh(b*x + a)^4 + 4*cosh(b*x +
a)^3*sinh(b*x + a) + 6*cosh(b*x + a)^2*sinh(b*x + a)^2 + 4*cosh(b*x + a)*si
nh(b*x + a)^3 + sinh(b*x + a)^4)*log(2*cosh(b*x + a)/(cosh(b*x + a) - sinh(
```

```
b*x + a))) + 8*(cosh(b*x + a)^7 - 32*b*x*cosh(b*x + a)^3 - 9*cosh(b*x + a)^
5 - 3*cosh(b*x + a))*sinh(b*x + a) + 1)/(b*cosh(b*x + a)^4 + 4*b*cosh(b*x +
 a)^3*sinh(b*x + a) + 6*b*cosh(b*x + a)^2*sinh(b*x + a)^2 + 4*b*cosh(b*x +
a)*sinh(b*x + a)^3 + b*sinh(b*x + a)^4)
```

giac [B] time = 0.17, size = 86, normalized size = 2.15

$$-\frac{64\,bx-\left(48\,e^{(4\,bx+4\,a)}-12\,e^{(2\,bx+2\,a)}+1\right)e^{(-4\,bx-4\,a)}-\left(e^{(4\,bx+16\,a)}-12\,e^{(2\,bx+14\,a)}\right)e^{(-12\,a)}-64\,\log\left(e^{(2\,bx+2\,a)}+1\right)}{64\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)^4*tanh(b*x+a),x, algorithm="giac")
```

```
[Out] -1/64*(64*b*x - (48*e^(4*b*x + 4*a) - 12*e^(2*b*x + 2*a) + 1)*e^(-4*b*x - 4
*a) - (e^(4*b*x + 16*a) - 12*e^(2*b*x + 14*a))*e^(-12*a) - 64*log(e^(2*b*x
+ 2*a) + 1))/b
```

maple [A] time = 0.14, size = 39, normalized size = 0.98

$$\frac{\sinh^4(bx+a)}{4b}-\frac{\sinh^2(bx+a)}{2b}+\frac{\ln(\cosh(bx+a))}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(b*x+a)^4*tanh(b*x+a),x)
```

```
[Out] 1/4*sinh(b*x+a)^4/b-1/2*sinh(b*x+a)^2/b+ln(cosh(b*x+a))/b
```

maxima [B] time = 0.47, size = 81, normalized size = 2.02

$$-\frac{\left(12\,e^{(-2\,bx-2\,a)}-1\right)e^{(4\,bx+4\,a)}}{64\,b}+\frac{bx+a}{b}-\frac{12\,e^{(-2\,bx-2\,a)}-e^{(-4\,bx-4\,a)}}{64\,b}+\frac{\log\left(e^{(-2\,bx-2\,a)}+1\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)^4*tanh(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/64*(12*e^(-2*b*x - 2*a) - 1)*e^(4*b*x + 4*a)/b + (b*x + a)/b - 1/64*(12*
e^(-2*b*x - 2*a) - e^(-4*b*x - 4*a))/b + log(e^(-2*b*x - 2*a) + 1)/b
```

mupad [B] time = 0.12, size = 77, normalized size = 1.92

$$\frac{\ln\left(\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}+1\right)}{b}-x-\frac{3\,\mathrm{e}^{-2\,a-2\,b\,x}}{16\,b}-\frac{3\,\mathrm{e}^{2\,a+2\,b\,x}}{16\,b}+\frac{\mathrm{e}^{-4\,a-4\,b\,x}}{64\,b}+\frac{\mathrm{e}^{4\,a+4\,b\,x}}{64\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a + b*x)^4*tanh(a + b*x),x)
```

```
[Out] log(exp(2*a)*exp(2*b*x) + 1)/b - x - (3*exp(- 2*a - 2*b*x))/(16*b) - (3*exp
(2*a + 2*b*x))/(16*b) + exp(- 4*a - 4*b*x)/(64*b) + exp(4*a + 4*b*x)/(64*b)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sinh^4(a+bx)\tanh(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)**4*tanh(b*x+a),x)
```

```
[Out] Integral(sinh(a + b*x)**4*tanh(a + b*x), x)
```

3.81 $\int \operatorname{sech}(a+bx)\tanh(a+bx)\,dx$

Optimal. Leaf size=11

$$-\frac{\operatorname{sech}(a+bx)}{b}$$

```
[Out] -sech(b*x+a)/b
```

Rubi [A] time = 0.01, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2606, 8}

$$-\frac{\operatorname{sech}(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[a + b*x]*Tanh[a + b*x],x]
```

```
[Out] -(Sech[a + b*x]/b)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rubi steps

$$\begin{aligned}\int \operatorname{sech}(a+bx)\tanh(a+bx)\,dx &= -\frac{\operatorname{Subst}\left(\int 1\,dx, x, \operatorname{sech}(a+bx)\right)}{b}\\ &= -\frac{\operatorname{sech}(a+bx)}{b}\end{aligned}$$

Mathematica [A] time = 0.01, size = 11, normalized size = 1.00

$$-\frac{\operatorname{sech}(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[a + b*x]*Tanh[a + b*x],x]
```

```
[Out] -(Sech[a + b*x]/b)
```

fricas [B] time = 0.45, size = 54, normalized size = 4.91

$$-\frac{2\left(\cosh\left(bx+a\right)+\sinh\left(bx+a\right)\right)}{b\cosh\left(bx+a\right)^{2}+2\,b\cosh\left(bx+a\right)\sinh\left(bx+a\right)+b\sinh\left(bx+a\right)^{2}+b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)*tanh(b*x+a),x, algorithm="fricas")
```

```
[Out] -2*(cosh(b*x + a) + sinh(b*x + a))/(b*cosh(b*x + a)^2 + 2*b*cosh(b*x + a)*s
inh(b*x + a) + b*sinh(b*x + a)^2 + b)
```

giac [B] time = 0.12, size = 24, normalized size = 2.18

$$-\frac{2\,e^{(bx+a)}}{b\left(e^{(2\,bx+2\,a)}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)*tanh(b*x+a),x, algorithm="giac")
```

```
[Out] -2*e^(b*x + a)/(b*(e^(2*b*x + 2*a) + 1))
```

maple [A] time = 0.08, size = 12, normalized size = 1.09

$$-\frac{\operatorname{sech}\left(bx+a\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(b*x+a)*tanh(b*x+a),x)
```

```
[Out] -sech(b*x+a)/b
```

maxima [B] time = 0.30, size = 23, normalized size = 2.09

$$-\frac{2}{b\left(e^{(bx+a)}+e^{(-bx-a)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)*tanh(b*x+a),x, algorithm="maxima")
```

```
[Out] -2/(b*(e^(b*x + a) + e^(-b*x - a)))
```

mupad [B] time = 1.44, size = 24, normalized size = 2.18

$$-\frac{2\,e^{a+b\,x}}{b\,\left(e^{2\,a+2\,b\,x}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(a + b*x)/cosh(a + b*x),x)
[Out] -(2*exp(a + b*x))/(b*(exp(2*a + 2*b*x) + 1))
```

sympy [A] time = 0.24, size = 17, normalized size = 1.55

$$\begin{cases} -\frac{\operatorname{sech}\left(a+bx\right)}{b} & \text{for}\: b \neq 0 \\ x \tanh\left(a\right) \operatorname{sech}\left(a\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)*tanh(b*x+a),x)
[Out] Piecewise((-sech(a + b*x)/b, Ne(b, 0)), (x*tanh(a)*sech(a), True))
```

3.82 $\int \operatorname{sech}^2(a+bx)\tanh(a+bx)\,dx$

Optimal. Leaf size=15

$$-\frac{\operatorname{sech}^2(a+bx)}{2b}$$

```
[Out] -1/2*sech(b*x+a)^2/b
```

Rubi [A] time = 0.02, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2606, 30}

$$-\frac{\operatorname{sech}^2(a+bx)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[a + b*x]^2*Tanh[a + b*x],x]
```

```
[Out] -Sech[a + b*x]^2/(2*b)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rubi steps

$$\begin{aligned}\int \operatorname{sech}^2(a+bx)\tanh(a+bx)\,dx &= -\frac{\operatorname{Subst}\left(\int x\,dx, x, \operatorname{sech}(a+bx)\right)}{b}\\ &= -\frac{\operatorname{sech}^2(a+bx)}{2b}\end{aligned}$$

Mathematica [A] time = 0.01, size = 15, normalized size = 1.00

$$-\frac{\operatorname{sech}^2(a+bx)}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[a + b*x]^2*Tanh[a + b*x],x]
```

```
[Out] -1/2*Sech[a + b*x]^2/b
```

fricas [B] time = 0.45, size = 84, normalized size = 5.60

$$-\frac{2\left(\cosh\left(bx+a\right)+\sinh\left(bx+a\right)\right)}{b\cosh\left(bx+a\right)^{3}+3\,b\cosh\left(bx+a\right)\sinh\left(bx+a\right)^{2}+b\sinh\left(bx+a\right)^{3}+3\,b\cosh\left(bx+a\right)+\left(3\,b\cosh\left(bx+a\right)^{2}+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)^2*tanh(b*x+a),x, algorithm="fricas")
```

```
[Out] -2*(cosh(b*x + a) + sinh(b*x + a))/(b*cosh(b*x + a)^3 + 3*b*cosh(b*x + a)*s
inh(b*x + a)^2 + b*sinh(b*x + a)^3 + 3*b*cosh(b*x + a) + (3*b*cosh(b*x + a)
^2 + b)*sinh(b*x + a))
```

giac [B] time = 0.12, size = 27, normalized size = 1.80

$$-\frac{2\,e^{\left(2\,bx+2\,a\right)}}{b\left(e^{\left(2\,bx+2\,a\right)}+1\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)^2*tanh(b*x+a),x, algorithm="giac")
```

```
[Out] -2*e^(2*b*x + 2*a)/(b*(e^(2*b*x + 2*a) + 1)^2)
```

maple [A] time = 0.06, size = 14, normalized size = 0.93

$$-\frac{\operatorname{sech}\left(bx+a\right)^{2}}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(b*x+a)^2*tanh(b*x+a),x)
```

```
[Out] -1/2*sech(b*x+a)^2/b
```

maxima [A] time = 0.30, size = 13, normalized size = 0.87

$$\frac{\tanh\left(bx+a\right)^{2}}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)^2*tanh(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/2*tanh(b*x + a)^2/b
```

mupad [B] time = 0.07, size = 13, normalized size = 0.87

$$-\frac{1}{2\,b\cosh\left(a+b\,x\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(a + b*x)/cosh(a + b*x)^2,x)
```

```
[Out] -1/(2*b*cosh(a + b*x)^2)
```

sympy [A] time = 0.44, size = 22, normalized size = 1.47

$$\begin{cases} -\frac{\operatorname{sech}^{2}{\left(a+bx \right)}}{2b} & \text{for}\: b \neq 0 \\ x \tanh{\left(a \right)} \operatorname{sech}^{2}{\left(a \right)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)**2*tanh(b*x+a),x)
```

```
[Out] Piecewise((-sech(a + b*x)**2/(2*b), Ne(b, 0)), (x*tanh(a)*sech(a)**2, True)
)
```

3.83 $\int \text{sech}^{1+n}(a+bx)\sinh(a+bx)\,dx$

Optimal. Leaf size=16

$$-\frac{\text{sech}^n(a+bx)}{bn}$$

```
[Out] -sech(b*x+a)^n/b/n
```

Rubi [A] time = 0.03, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2622, 30}

$$-\frac{\text{sech}^n(a+bx)}{bn}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[a + b*x]^(1 + n)*Sinh[a + b*x],x]
```

```
[Out] -(Sech[a + b*x]^n/(b*n))
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] && !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rubi steps

$$\begin{aligned}\int \text{sech}^{1+n}(a+bx)\sinh(a+bx)\,dx &= -\frac{\text{Subst}\left(\int x^{-1+n}\,dx, x, \text{sech}(a+bx)\right)}{b}\\ &= -\frac{\text{sech}^n(a+bx)}{bn}\end{aligned}$$

Mathematica [A] time = 0.04, size = 16, normalized size = 1.00

$$-\frac{\text{sech}^n(a+bx)}{bn}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[a + b*x]^(1 + n)*Sinh[a + b*x],x]
```

```
[Out] -(Sech[a + b*x]^n/(b*n))
```

fricas [B] time = 0.40, size = 115, normalized size = 7.19

$$-\frac{\cosh\left(n\log\left(\frac{2\left(\cosh(bx+a)+\sinh(bx+a)\right)}{\cosh(bx+a)^2+2\cosh(bx+a)\sinh(bx+a)+\sinh(bx+a)^2+1}\right)\right)+\sinh\left(n\log\left(\frac{2\left(\cosh(bx+a)+\sinh(bx+a)\right)}{\cosh(bx+a)^2+2\cosh(bx+a)\sinh(bx+a)+\sinh(b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)^n*tanh(b*x+a),x, algorithm="fricas")
```

```
[Out] -(cosh(n*log(2*(cosh(b*x + a) + sinh(b*x + a))/(cosh(b*x + a)^2 + 2*cosh(b*
x + a)*sinh(b*x + a) + sinh(b*x + a)^2 + 1))) + sinh(n*log(2*(cosh(b*x + a)
 + sinh(b*x + a))/(cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b
*x + a)^2 + 1))))/(b*n)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{sech}\left(bx+a\right)^n \tanh\left(bx+a\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)^n*tanh(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(sech(b*x + a)^n*tanh(b*x + a), x)
```

maple [A] time = 0.10, size = 17, normalized size = 1.06

$$-\frac{\operatorname{sech}\left(bx+a\right)^n}{bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(b*x+a)^n*tanh(b*x+a),x)
```

```
[Out] -sech(b*x+a)^n/b/n
```

maxima [B] time = 0.83, size = 36, normalized size = 2.25

$$-\frac{2^n e^{\left(-(bx+a)n-n\log\left(e^{(-2\,bx-2\,a)}+1\right)\right)}}{bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)^n*tanh(b*x+a),x, algorithm="maxima")
```

```
[Out] -2^n*e^(-(b*x + a)*n - n*log(e^(-2*b*x - 2*a) + 1))/(b*n)
```

mupad [B] time = 1.48, size = 31, normalized size = 1.94

$$-\frac{\left(\frac{2\,\mathrm{e}^{a+b\,x}}{\mathrm{e}^{2\,a+2\,b\,x}+1}\right)^n}{b\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(a + b*x)*(1/cosh(a + b*x))^n,x)
```

```
[Out] -((2*exp(a + b*x))/(exp(2*a + 2*b*x) + 1))^n/(b*n)
```

sympy [A] time = 0.49, size = 39, normalized size = 2.44

$$\begin{cases} x\tanh{\left(a \right)} & \text{for}\: b = 0 \wedge n = 0 \\ x\tanh{\left(a \right)}\operatorname{sech}^{n}{\left(a \right)} & \text{for}\: b = 0 \\ x - \frac{\log{\left(\tanh{\left(a + bx \right)} + 1 \right)}}{b} & \text{for}\: n = 0 \\ - \frac{\operatorname{sech}^{n}{\left(a + bx \right)}}{bn} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)**n*tanh(b*x+a),x)
```

```
[Out] Piecewise((x*tanh(a), Eq(b, 0) & Eq(n, 0)), (x*tanh(a)*sech(a)**n, Eq(b, 0)
), (x - log(tanh(a + b*x) + 1)/b, Eq(n, 0)), (-sech(a + b*x)**n/(b*n), True
))
```

3.84 $\int \operatorname{sech}^2(a+bx)\tanh^2(a+bx)\,dx$

Optimal. Leaf size=15

$$\frac{\tanh^3(a+bx)}{3b}$$

[Out] 1/3*tanh(b*x+a)^3/b

Rubi [A] time = 0.03, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2607, 30}

$$\frac{\tanh^3(a+bx)}{3b}$$

Antiderivative was successfully verified.

[In] Int[Sech[a + b*x]^2*Tanh[a + b*x]^2,x]

[Out] Tanh[a + b*x]^3/(3*b)

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rubi steps

$$\begin{aligned}\int \operatorname{sech}^2(a+bx)\tanh^2(a+bx)\,dx &= \frac{i\,\text{Subst}\left(\int x^2\,dx, x, i\tanh(a+bx)\right)}{b}\\ &= \frac{\tanh^3(a+bx)}{3b}\end{aligned}$$

Mathematica [A] time = 0.01, size = 15, normalized size = 1.00

$$\frac{\tanh^3(a+bx)}{3b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[a + b*x]^2*Tanh[a + b*x]^2,x]
```

```
[Out] Tanh[a + b*x]^3/(3*b)
```

fricas [B] time = 0.42, size = 138, normalized size = 9.20

$$-\frac{8\left(\cosh\left(bx+a\right)^{2}+\cosh\left(bx+a\right)\sinh\left(bx+a\right)+\sinh\left(bx+a\right)^{2}\right)}{3\left(b\cosh\left(bx+a\right)^{4}+4\,b\cosh\left(bx+a\right)\sinh\left(bx+a\right)^{3}+b\sinh\left(bx+a\right)^{4}+4\,b\cosh\left(bx+a\right)^{2}+2\left(3\,b\cosh\left(bx+a\right)^{2}+2\,b\right)\sinh\left(bx+a\right)^{2}+4\left(b\cosh\left(bx+a\right)^{3}+b\cosh\left(bx+a\right)\right)\sinh\left(bx+a\right)+3\,b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)^2*tanh(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -8/3*(cosh(b*x + a)^2 + cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2)/(b*c
osh(b*x + a)^4 + 4*b*cosh(b*x + a)*sinh(b*x + a)^3 + b*sinh(b*x + a)^4 + 4*
b*cosh(b*x + a)^2 + 2*(3*b*cosh(b*x + a)^2 + 2*b)*sinh(b*x + a)^2 + 4*(b*co
sh(b*x + a)^3 + b*cosh(b*x + a))*sinh(b*x + a) + 3*b)
```

giac [B] time = 0.13, size = 31, normalized size = 2.07

$$-\frac{2\left(3\,e^{\left(4\,bx+4\,a\right)}+1\right)}{3\,b\left(e^{\left(2\,bx+2\,a\right)}+1\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)^2*tanh(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -2/3*(3*e^(4*b*x + 4*a) + 1)/(b*(e^(2*b*x + 2*a) + 1)^3)
```

maple [B] time = 0.32, size = 42, normalized size = 2.80

$$\frac{-\frac{\sinh(bx+a)}{2\cosh(bx+a)^{3}}+\frac{\left(\frac{2}{3}+\frac{\mathrm{sech}(bx+a)^{2}}{3}\right)\tanh(bx+a)}{2}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(b*x+a)^2*tanh(b*x+a)^2,x)
```

```
[Out] 1/b*(-1/2*sinh(b*x+a)/cosh(b*x+a)^3+1/2*(2/3+1/3*sech(b*x+a)^2)*tanh(b*x+a)
)
```

maxima [A] time = 0.33, size = 13, normalized size = 0.87

$$\frac{\tanh\left(bx+a\right)^{3}}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)^2*tanh(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/3*tanh(b*x + a)^3/b
```

mupad [B] time = 0.07, size = 31, normalized size = 2.07

$$-\frac{2\left(3\,\mathrm{e}^{4\,a+4\,b\,x}+1\right)}{3\,b\,{\left(\mathrm{e}^{2\,a+2\,b\,x}+1\right)}^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(a + b*x)^2/cosh(a + b*x)^2,x)
```

```
[Out] -(2*(3*exp(4*a + 4*b*x) + 1))/(3*b*(exp(2*a + 2*b*x) + 1)^3)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \tanh^2(a+bx)\,\mathrm{sech}^2(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)**2*tanh(b*x+a)**2,x)
```

```
[Out] Integral(tanh(a + b*x)**2*sech(a + b*x)**2, x)
```

3.85 $\int \operatorname{sech}^2(a+bx)\tanh^3(a+bx)\,dx$

Optimal. Leaf size=15

$$\frac{\tanh^4(a+bx)}{4b}$$

[Out] 1/4*tanh(b*x+a)^4/b

Rubi [A] time = 0.03, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2607, 30}

$$\frac{\tanh^4(a+bx)}{4b}$$

Antiderivative was successfully verified.

[In] Int[Sech[a + b*x]^2*Tanh[a + b*x]^3,x]

[Out] Tanh[a + b*x]^4/(4*b)

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rubi steps

$$\begin{aligned}\int \operatorname{sech}^2(a+bx)\tanh^3(a+bx)\,dx &= \frac{\operatorname{Subst}\left(\int x^3\,dx, x, i\tanh(a+bx)\right)}{b}\\ &= \frac{\tanh^4(a+bx)}{4b}\end{aligned}$$

Mathematica [A] time = 0.01, size = 15, normalized size = 1.00

$$\frac{\tanh^4(a+bx)}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[a + b*x]^2*Tanh[a + b*x]^3,x]
```

```
[Out] Tanh[a + b*x]^4/(4*b)
```

fricas [B] time = 0.44, size = 208, normalized size = 13.87

$$-\frac{2\left(\cosh\left(bx+a\right)^{3}+3\,\cosh\left(bx+a\right.\right.}{b\cosh\left(bx+a\right)^{5}+5\,b\cosh\left(bx+a\right)\sinh\left(bx+a\right)^{4}+b\sinh\left(bx+a\right)^{5}+5\,b\cosh\left(bx+a\right)^{3}+\left(10\,b\cosh\left(bx+a\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)^2*tanh(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -2*(cosh(b*x + a)^3 + 3*cosh(b*x + a)*sinh(b*x + a)^2 + sinh(b*x + a)^3 + (
3*cosh(b*x + a)^2 - 1)*sinh(b*x + a) + cosh(b*x + a))/(b*cosh(b*x + a)^5 +
5*b*cosh(b*x + a)*sinh(b*x + a)^4 + b*sinh(b*x + a)^5 + 5*b*cosh(b*x + a)^3
 + (10*b*cosh(b*x + a)^2 + 3*b)*sinh(b*x + a)^3 + 5*(2*b*cosh(b*x + a)^3 +
3*b*cosh(b*x + a))*sinh(b*x + a)^2 + 10*b*cosh(b*x + a) + (5*b*cosh(b*x + a
)^4 + 9*b*cosh(b*x + a)^2 + 2*b)*sinh(b*x + a))
```

giac [B] time = 0.14, size = 37, normalized size = 2.47

$$-\frac{2\left(e^{\left(6\,bx+6\,a\right)}+e^{\left(2\,bx+2\,a\right)}\right)}{b\left(e^{\left(2\,bx+2\,a\right)}+1\right)^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)^2*tanh(b*x+a)^3,x, algorithm="giac")
```

```
[Out] -2*(e^(6*b*x + 6*a) + e^(2*b*x + 2*a))/(b*(e^(2*b*x + 2*a) + 1)^4)
```

maple [B] time = 0.14, size = 34, normalized size = 2.27

$$\frac{-\frac{\sinh^{2}\left(bx+a\right)}{2\cosh\left(bx+a\right)^{4}}-\frac{1}{4\cosh\left(bx+a\right)^{4}}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(b*x+a)^2*tanh(b*x+a)^3,x)
```

```
[Out] 1/b*(-1/2/cosh(b*x+a)^4*sinh(b*x+a)^2-1/4/cosh(b*x+a)^4)
```

maxima [A] time = 0.31, size = 13, normalized size = 0.87

$$\frac{\tanh\left(bx+a\right)^{4}}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)^2*tanh(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/4*tanh(b*x + a)^4/b
```

mupad [B] time = 0.11, size = 230, normalized size = 15.33

$$\frac{\frac{1}{2b}-\frac{3e^{2a+2bx}}{2b}+\frac{3e^{4a+4bx}}{2b}-\frac{e^{6a+6bx}}{2b}}{4e^{2a+2bx}+6e^{4a+4bx}+4e^{6a+6bx}+e^{8a+8bx}+1}-\frac{\frac{1}{2b}-\frac{e^{2a+2bx}}{b}+\frac{e^{4a+4bx}}{2b}}{3e^{2a+2bx}+3e^{4a+4bx}+e^{6a+6bx}+1}+\frac{\frac{1}{2b}-\frac{e^{2a+2bx}}{2b}}{2e^{2a+2bx}+e^{4a+4bx}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(a + b*x)^3/cosh(a + b*x)^2,x)
```

```
[Out] (1/(2*b) - (3*exp(2*a + 2*b*x))/(2*b) + (3*exp(4*a + 4*b*x))/(2*b) - exp(6*
a + 6*b*x)/(2*b))/(4*exp(2*a + 2*b*x) + 6*exp(4*a + 4*b*x) + 4*exp(6*a + 6*
b*x) + exp(8*a + 8*b*x) + 1) - (1/(2*b) - exp(2*a + 2*b*x)/b + exp(4*a + 4*
b*x)/(2*b))/(3*exp(2*a + 2*b*x) + 3*exp(4*a + 4*b*x) + exp(6*a + 6*b*x) + 1
) + (1/(2*b) - exp(2*a + 2*b*x)/(2*b))/(2*exp(2*a + 2*b*x) + exp(4*a + 4*b*
x) + 1) - 1/(2*b*(exp(2*a + 2*b*x) + 1))
```

sympy [A] time = 1.53, size = 44, normalized size = 2.93

$$\begin{cases} -\frac{\tanh^{2}{\left(a+bx \right)}\operatorname{sech}^{2}{\left(a+bx \right)}}{4b}-\frac{\operatorname{sech}^{2}{\left(a+bx \right)}}{4b} & \text{for}\: b \neq 0 \\ x\tanh^{3}{\left(a \right)}\operatorname{sech}^{2}{\left(a \right)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)**2*tanh(b*x+a)**3,x)
```

```
[Out] Piecewise((-tanh(a + b*x)**2*sech(a + b*x)**2/(4*b) - sech(a + b*x)**2/(4*b
), Ne(b, 0)), (x*tanh(a)**3*sech(a)**2, True))
```

3.86 $\int \text{sech}^2(a+bx)\tanh^n(a+bx)\,dx$

Optimal. Leaf size=19

$$\frac{\tanh^{n+1}(a+bx)}{b(n+1)}$$

```
[Out] tanh(b*x+a)^(1+n)/b/(1+n)
```

Rubi [A] time = 0.04, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2607, 32}

$$\frac{\tanh^{n+1}(a+bx)}{b(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[a + b*x]^2*Tanh[a + b*x]^n,x]
```

```
[Out] Tanh[a + b*x]^(1 + n)/(b*(1 + n))
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rubi steps

$$\begin{aligned}\int \text{sech}^2(a+bx)\tanh^n(a+bx)\,dx &= -\frac{i\,\text{Subst}\left(\int(-ix)^n\,dx,x,i\tanh(a+bx)\right)}{b}\\ &= \frac{\tanh^{1+n}(a+bx)}{b(1+n)}\end{aligned}$$

Mathematica [A] time = 0.02, size = 19, normalized size = 1.00

$$\frac{\tanh^{n+1}(a+bx)}{b(n+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[a + b*x]^2*Tanh[a + b*x]^n,x]
```

```
[Out] Tanh[a + b*x]^(1 + n)/(b*(1 + n))
```

fricas [B] time = 0.42, size = 69, normalized size = 3.63

$$\frac{\cosh\left(n\log\left(\frac{\sinh(bx+a)}{\cosh(bx+a)}\right)\right)\sinh\left(bx+a\right)+\sinh\left(bx+a\right)\sinh\left(n\log\left(\frac{\sinh(bx+a)}{\cosh(bx+a)}\right)\right)}{(bn+b)\cosh\left(bx+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)^2*tanh(b*x+a)^n,x, algorithm="fricas")
```

```
[Out] (cosh(n*log(sinh(b*x + a)/cosh(b*x + a)))*sinh(b*x + a) + sinh(b*x + a)*sin
h(n*log(sinh(b*x + a)/cosh(b*x + a))))/((b*n + b)*cosh(b*x + a))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \tanh\left(bx+a\right)^{n}\operatorname{sech}\left(bx+a\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)^2*tanh(b*x+a)^n,x, algorithm="giac")
```

```
[Out] integrate(tanh(b*x + a)^n*sech(b*x + a)^2, x)
```

maple [A] time = 0.16, size = 20, normalized size = 1.05

$$\frac{\tanh^{n+1}\left(bx+a\right)}{b\left(n+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(b*x+a)^2*tanh(b*x+a)^n,x)
```

```
[Out] tanh(b*x+a)^(n+1)/b/(n+1)
```

maxima [A] time = 0.36, size = 19, normalized size = 1.00

$$\frac{\tanh\left(bx+a\right)^{n+1}}{b(n+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)^2*tanh(b*x+a)^n,x, algorithm="maxima")
```

```
[Out] tanh(b*x + a)^(n + 1)/(b*(n + 1))
```

mupad [B] time = 1.53, size = 42, normalized size = 2.21

$$\frac{\tanh\left(a+b\,x\right)\,{\left(\frac{{\mathrm{e}}^{2\,a+2\,b\,x}-1}{{\mathrm{e}}^{2\,a+2\,b\,x}+1}\right)}^n}{b\,\left(n+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(a + b*x)^n/cosh(a + b*x)^2,x)
```

```
[Out] (tanh(a + b*x)*((exp(2*a + 2*b*x) - 1)/(exp(2*a + 2*b*x) + 1))^n)/(b*(n + 1
))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \tanh^{n}{\left(a + bx \right)} \operatorname{sech}^{2}{\left(a + bx \right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)**2*tanh(b*x+a)**n,x)
```

```
[Out] Integral(tanh(a + b*x)**n*sech(a + b*x)**2, x)
```

3.87 $\int \operatorname{sech}(a+bx)\tanh^3(a+bx)\,dx$

Optimal. Leaf size=27

$$\frac{\operatorname{sech}^3(a+bx)}{3b}-\frac{\operatorname{sech}(a+bx)}{b}$$

```
[Out] -sech(b*x+a)/b+1/3*sech(b*x+a)^3/b
```

Rubi [A] time = 0.02, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {2606}

$$\frac{\operatorname{sech}^3(a+bx)}{3b}-\frac{\operatorname{sech}(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[a + b*x]*Tanh[a + b*x]^3,x]
```

```
[Out] -(Sech[a + b*x]/b) + Sech[a + b*x]^3/(3*b)
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 && !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rubi steps

$$\begin{aligned}\int \operatorname{sech}(a+bx)\tanh^3(a+bx)\,dx &= \frac{\operatorname{Subst}\left(\int \left(-1+x^2\right)\,dx,x,\operatorname{sech}(a+bx)\right)}{b}\\ &= -\frac{\operatorname{sech}(a+bx)}{b}+\frac{\operatorname{sech}^3(a+bx)}{3b}\end{aligned}$$

Mathematica [A] time = 0.03, size = 27, normalized size = 1.00

$$\frac{\operatorname{sech}^3(a+bx)}{3b}-\frac{\operatorname{sech}(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[a + b*x]*Tanh[a + b*x]^3,x]
```

```
[Out] -(Sech[a + b*x]/b) + Sech[a + b*x]^3/(3*b)
```

fricas [B] time = 0.44, size = 172, normalized size = 6.37

$$-\frac{2\left(3\cosh\left(bx+a\right)^{3}+9\cosh\left(bx+a\right)\sinh\left(bx+a\right)^{2}+3\sinh\left(bx+a\right)^{3}+\left(9\right.\right.}{3\left(b\cosh\left(bx+a\right)^{4}+4\,b\cosh\left(bx+a\right)\sinh\left(bx+a\right)^{3}+b\sinh\left(bx+a\right)^{4}+4\,b\cosh\left(bx+a\right)^{2}+2\left(3\,b\cosh\left(bx\right.\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)*tanh(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -2/3*(3*cosh(b*x + a)^3 + 9*cosh(b*x + a)*sinh(b*x + a)^2 + 3*sinh(b*x + a)
^3 + (9*cosh(b*x + a)^2 - 1)*sinh(b*x + a) + 5*cosh(b*x + a))/(b*cosh(b*x +
 a)^4 + 4*b*cosh(b*x + a)*sinh(b*x + a)^3 + b*sinh(b*x + a)^4 + 4*b*cosh(b*
x + a)^2 + 2*(3*b*cosh(b*x + a)^2 + 2*b)*sinh(b*x + a)^2 + 4*(b*cosh(b*x +
a)^3 + b*cosh(b*x + a))*sinh(b*x + a) + 3*b)
```

giac [A] time = 0.16, size = 49, normalized size = 1.81

$$-\frac{2\left(3\,e^{(5\,bx+5\,a)}+2\,e^{(3\,bx+3\,a)}+3\,e^{(bx+a)}\right)}{3\,b\left(e^{(2\,bx+2\,a)}+1\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)*tanh(b*x+a)^3,x, algorithm="giac")
```

```
[Out] -2/3*(3*e^(5*b*x + 5*a) + 2*e^(3*b*x + 3*a) + 3*e^(b*x + a))/(b*(e^(2*b*x +
 2*a) + 1)^3)
```

maple [A] time = 0.16, size = 34, normalized size = 1.26

$$\frac{-\frac{\sinh^{2}(bx+a)}{\cosh(bx+a)^{3}}-\frac{2}{3\cosh(bx+a)^{3}}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(b*x+a)*tanh(b*x+a)^3,x)
```

```
[Out] 1/b*(-sinh(b*x+a)^2/cosh(b*x+a)^3-2/3/cosh(b*x+a)^3)
```

maxima [B] time = 0.45, size = 148, normalized size = 5.48

$$-\frac{2\,e^{(-bx-a)}}{b\left(3\,e^{(-2\,bx-2\,a)}+3\,e^{(-4\,bx-4\,a)}+e^{(-6\,bx-6\,a)}+1\right)}-\frac{4\,e^{(-3\,bx-3\,a)}}{3\,b\left(3\,e^{(-2\,bx-2\,a)}+3\,e^{(-4\,bx-4\,a)}+e^{(-6\,bx-6\,a)}+1\right)}-\frac{}{b\left(3\,e^{(-2\,bx-2\,a)}+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)*tanh(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -2*e^(-b*x - a)/(b*(3*e^(-2*b*x - 2*a) + 3*e^(-4*b*x - 4*a) + e^(-6*b*x - 6
*a) + 1)) - 4/3*e^(-3*b*x - 3*a)/(b*(3*e^(-2*b*x - 2*a) + 3*e^(-4*b*x - 4*a
) + e^(-6*b*x - 6*a) + 1)) - 2*e^(-5*b*x - 5*a)/(b*(3*e^(-2*b*x - 2*a) + 3*
e^(-4*b*x - 4*a) + e^(-6*b*x - 6*a) + 1))
```

mupad [B] time = 1.41, size = 48, normalized size = 1.78

$$-\frac{2\,e^{a+b\,x}\,\left(2\,e^{2\,a+2\,b\,x}+3\,e^{4\,a+4\,b\,x}+3\right)}{3\,b\,{\left(e^{2\,a+2\,b\,x}+1\right)}^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(a + b*x)^3/cosh(a + b*x),x)
```

```
[Out] -(2*exp(a + b*x)*(2*exp(2*a + 2*b*x) + 3*exp(4*a + 4*b*x) + 3))/(3*b*(exp(2
*a + 2*b*x) + 1)^3)
```

sympy [A] time = 0.85, size = 41, normalized size = 1.52

$$\begin{cases} -\frac{\tanh^{2}{\left(a+bx\right)}\operatorname{sech}{\left(a+bx\right)}}{3b} - \frac{2\operatorname{sech}{\left(a+bx\right)}}{3b} & \text{for}\: b \neq 0 \\ x\tanh^{3}{\left(a\right)}\operatorname{sech}{\left(a\right)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)*tanh(b*x+a)**3,x)
```

```
[Out] Piecewise((-tanh(a + b*x)**2*sech(a + b*x)/(3*b) - 2*sech(a + b*x)/(3*b), N
e(b, 0)), (x*tanh(a)**3*sech(a), True))
```

3.88 $\int \text{sech}^3(a+bx)\tanh^3(a+bx)\,dx$

Optimal. Leaf size=31

$$\frac{\text{sech}^5(a+bx)}{5b}-\frac{\text{sech}^3(a+bx)}{3b}$$

```
[Out] -1/3*sech(b*x+a)^3/b+1/5*sech(b*x+a)^5/b
```

Rubi [A] time = 0.04, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2606, 14}

$$\frac{\text{sech}^5(a+bx)}{5b}-\frac{\text{sech}^3(a+bx)}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[a + b*x]^3*Tanh[a + b*x]^3,x]
```

```
[Out] -Sech[a + b*x]^3/(3*b) + Sech[a + b*x]^5/(5*b)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 && !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rubi steps

$$\begin{aligned}\int \text{sech}^3(a+bx)\tanh^3(a+bx)\,dx&=\frac{\text{Subst}\left(\int x^2\left(-1+x^2\right)\,dx,x,\text{sech}(a+bx)\right)}{b}\\&=\frac{\text{Subst}\left(\int \left(-x^2+x^4\right)\,dx,x,\text{sech}(a+bx)\right)}{b}\\&=-\frac{\text{sech}^3(a+bx)}{3b}+\frac{\text{sech}^5(a+bx)}{5b}\end{aligned}$$

Mathematica [A] time = 0.06, size = 31, normalized size = 1.00

$$\frac{\operatorname{sech}^5(a+bx)}{5b}-\frac{\operatorname{sech}^3(a+bx)}{3b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[a + b*x]^3*Tanh[a + b*x]^3,x]
```

```
[Out] -1/3*Sech[a + b*x]^3/b + Sech[a + b*x]^5/(5*b)
```

fricas [B] time = 0.41, size = 345, normalized size = 11.13

$$-\frac{}{15\left(b\cosh\left(bx+a\right)^7+7\,b\cosh\left(bx+a\right)\sinh\left(bx+a\right)^6+b\sinh\left(bx+a\right)^7+5\,b\cosh\left(bx+a\right)^5+\left(21\,b\cosh\left(bx+\right.\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)^3*tanh(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -8/15*(5*cosh(b*x + a)^4 + 20*cosh(b*x + a)*sinh(b*x + a)^3 + 5*sinh(b*x +
a)^4 + 2*(15*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^2 - 2*cosh(b*x + a)^2 + 4*(
5*cosh(b*x + a)^3 - cosh(b*x + a))*sinh(b*x + a) + 5)/(b*cosh(b*x + a)^7 +
7*b*cosh(b*x + a)*sinh(b*x + a)^6 + b*sinh(b*x + a)^7 + 5*b*cosh(b*x + a)^5
 + (21*b*cosh(b*x + a)^2 + 5*b)*sinh(b*x + a)^5 + 5*(7*b*cosh(b*x + a)^3 +
5*b*cosh(b*x + a))*sinh(b*x + a)^4 + 11*b*cosh(b*x + a)^3 + (35*b*cosh(b*x
+ a)^4 + 50*b*cosh(b*x + a)^2 + 9*b)*sinh(b*x + a)^3 + (21*b*cosh(b*x + a)^
5 + 50*b*cosh(b*x + a)^3 + 33*b*cosh(b*x + a))*sinh(b*x + a)^2 + 15*b*cosh(
b*x + a) + (7*b*cosh(b*x + a)^6 + 25*b*cosh(b*x + a)^4 + 27*b*cosh(b*x + a)
^2 + 5*b)*sinh(b*x + a))
```

giac [A] time = 0.15, size = 52, normalized size = 1.68

$$-\frac{8\left(5\,e^{(7\,bx+7\,a)}-2\,e^{(5\,bx+5\,a)}+5\,e^{(3\,bx+3\,a)}\right)}{15\,b\left(e^{(2\,bx+2\,a)}+1\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)^3*tanh(b*x+a)^3,x, algorithm="giac")
```

```
[Out] -8/15*(5*e^(7*b*x + 7*a) - 2*e^(5*b*x + 5*a) + 5*e^(3*b*x + 3*a))/(b*(e^(2*
b*x + 2*a) + 1)^5)
```

maple [A] time = 0.13, size = 34, normalized size = 1.10

$$\frac{-\frac{\sinh^2(bx+a)}{3\cosh(bx+a)^5}-\frac{2}{15\cosh(bx+a)^5}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(b*x+a)^3*tanh(b*x+a)^3,x)
```

```
[Out] 1/b*(-1/3*sinh(b*x+a)^2/cosh(b*x+a)^5-2/15/cosh(b*x+a)^5)
```

maxima [B] time = 0.31, size = 214, normalized size = 6.90

$$-\frac{8\,e^{(-3\,bx-3\,a)}}{3\,b\left(5\,e^{(-2\,bx-2\,a)}+10\,e^{(-4\,bx-4\,a)}+10\,e^{(-6\,bx-6\,a)}+5\,e^{(-8\,bx-8\,a)}+e^{(-10\,bx-10\,a)}+1\right)}+\frac{}{15\,b\left(5\,e^{(-2\,bx-2\,a)}+10\,e^{(-4\,bx-4}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)^3*tanh(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -8/3*e^(-3*b*x - 3*a)/(b*(5*e^(-2*b*x - 2*a) + 10*e^(-4*b*x - 4*a) + 10*e^(
-6*b*x - 6*a) + 5*e^(-8*b*x - 8*a) + e^(-10*b*x - 10*a) + 1)) + 16/15*e^(-5
*b*x - 5*a)/(b*(5*e^(-2*b*x - 2*a) + 10*e^(-4*b*x - 4*a) + 10*e^(-6*b*x - 6
*a) + 5*e^(-8*b*x - 8*a) + e^(-10*b*x - 10*a) + 1)) - 8/3*e^(-7*b*x - 7*a)/
(b*(5*e^(-2*b*x - 2*a) + 10*e^(-4*b*x - 4*a) + 10*e^(-6*b*x - 6*a) + 5*e^(-
8*b*x - 8*a) + e^(-10*b*x - 10*a) + 1))
```

mupad [B] time = 1.54, size = 251, normalized size = 8.10

$$\frac{\frac{4\,e^{a+b\,x}}{5\,b}-\frac{12\,e^{3\,a+3\,b\,x}}{5\,b}+\frac{12\,e^{5\,a+5\,b\,x}}{5\,b}-\frac{4\,e^{7\,a+7\,b\,x}}{5\,b}}{5\,e^{2\,a+2\,b\,x}+10\,e^{4\,a+4\,b\,x}+10\,e^{6\,a+6\,b\,x}+5\,e^{8\,a+8\,b\,x}+e^{10\,a+10\,b\,x}+1}-\frac{28\,e^{a+b\,x}}{15\,b\,\left(2\,e^{2\,a+2\,b\,x}+e^{4\,a+4\,b\,x}+1\right)}+\frac{}{15\,b\,\left(3\,e^{2\,a}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(a + b*x)^3/cosh(a + b*x)^3,x)
```

```
[Out] ((4*exp(a + b*x))/(5*b) - (12*exp(3*a + 3*b*x))/(5*b) + (12*exp(5*a + 5*b*x
))/(5*b) - (4*exp(7*a + 7*b*x))/(5*b))/(5*exp(2*a + 2*b*x) + 10*exp(4*a + 4
*b*x) + 10*exp(6*a + 6*b*x) + 5*exp(8*a + 8*b*x) + exp(10*a + 10*b*x) + 1)
- (28*exp(a + b*x))/(15*b*(2*exp(2*a + 2*b*x) + exp(4*a + 4*b*x) + 1)) + (6
4*exp(a + b*x))/(15*b*(3*exp(2*a + 2*b*x) + 3*exp(4*a + 4*b*x) + exp(6*a +
6*b*x) + 1)) - (16*exp(a + b*x))/(5*b*(4*exp(2*a + 2*b*x) + 6*exp(4*a + 4*b
*x) + 4*exp(6*a + 6*b*x) + exp(8*a + 8*b*x) + 1))
```

sympy [A] time = 2.72, size = 46, normalized size = 1.48

$$\begin{cases} -\frac{\tanh^{2}{\left(a+bx \right)}\operatorname{sech}^{3}{\left(a+bx \right)}}{5b}-\frac{2\operatorname{sech}^{3}{\left(a+bx \right)}}{15b} & \text{for}\: b \neq 0 \\ x\tanh^{3}{\left(a \right)}\operatorname{sech}^{3}{\left(a \right)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)**3*tanh(b*x+a)**3,x)
```

```
[Out] Piecewise((-tanh(a + b*x)**2*sech(a + b*x)**3/(5*b) - 2*sech(a + b*x)**3/(1
5*b), Ne(b, 0)), (x*tanh(a)**3*sech(a)**3, True))
```

3.89 $\int \operatorname{sech}^{3+n}(a+bx)\sinh^3(a+bx)\,dx$

Optimal. Leaf size=36

$$\frac{\operatorname{sech}^{n+2}(a+bx)}{b(n+2)}-\frac{\operatorname{sech}^n(a+bx)}{bn}$$

```
[Out] -sech(b*x+a)^n/b/n+sech(b*x+a)^(2+n)/b/(2+n)
```

Rubi [A] time = 0.05, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2622, 14}

$$\frac{\operatorname{sech}^{n+2}(a+bx)}{b(n+2)}-\frac{\operatorname{sech}^n(a+bx)}{bn}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[a + b*x]^(3 + n)*Sinh[a + b*x]^3,x]
```

```
[Out] -(Sech[a + b*x]^n/(b*n)) + Sech[a + b*x]^(2 + n)/(b*(2 + n))
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rubi steps

$$\begin{aligned}\int \operatorname{sech}^{3+n}(a+bx)\sinh^3(a+bx)\,dx &= \frac{\operatorname{Subst}\left(\int x^{-1+n}\left(-1+x^2\right)\,dx,x,\operatorname{sech}(a+bx)\right)}{b}\\ &= \frac{\operatorname{Subst}\left(\int \left(-x^{-1+n}+x^{1+n}\right)\,dx,x,\operatorname{sech}(a+bx)\right)}{b}\\ &= -\frac{\operatorname{sech}^n(a+bx)}{bn}+\frac{\operatorname{sech}^{2+n}(a+bx)}{b(2+n)}\end{aligned}$$

Mathematica [A] time = 0.13, size = 32, normalized size = 0.89

$$\frac{\operatorname{sech}^n(a+bx)\left(\frac{\operatorname{sech}^2(a+bx)}{n+2}-\frac{1}{n}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[a + b*x]^(3 + n)*Sinh[a + b*x]^3,x]
[Out] (Sech[a + b*x]^n*(-n^(-1) + Sech[a + b*x]^2/(2 + n)))/b
```

fricas [B] time = 0.45, size = 219, normalized size = 6.08

$$-\frac{\left((n+2)\cosh(bx+a)^2+(n+2)\sinh(bx+a)^2-n+2\right)\cosh\left(n\log\left(\frac{2(\cosh(bx+a)+\sinh(bx+a))}{\cosh(bx+a)^2+2\cosh(bx+a)\sinh(bx+a)+\sinh(bx+a)^2+}\right.\right.}{bn^2+\left(bn^2+2bn\right)\cosh(bx+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)^n*tanh(b*x+a)^3,x, algorithm="fricas")
[Out] -(((n + 2)*cosh(b*x + a)^2 + (n + 2)*sinh(b*x + a)^2 - n + 2)*cosh(n*log(2*
(cosh(b*x + a) + sinh(b*x + a))/(cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x
 + a) + sinh(b*x + a)^2 + 1))) + ((n + 2)*cosh(b*x + a)^2 + (n + 2)*sinh(b*
x + a)^2 - n + 2)*sinh(n*log(2*(cosh(b*x + a) + sinh(b*x + a))/(cosh(b*x +
a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 + 1))))/(b*n^2 + (b*
n^2 + 2*b*n)*cosh(b*x + a)^2 + (b*n^2 + 2*b*n)*sinh(b*x + a)^2 + 2*b*n)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{sech}(bx+a)^n \tanh(bx+a)^3\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)^n*tanh(b*x+a)^3,x, algorithm="giac")
[Out] integrate(sech(b*x + a)^n*tanh(b*x + a)^3, x)
```

maple [C] time = 0.45, size = 275, normalized size = 7.64

$$-\frac{\left(n\,e^{4bx+4a}+2\,e^{4bx+4a}-2\,e^{2bx+2a}n+4\,e^{2bx+2a}+n+2\right)e^{\frac{n\left(-i\pi\operatorname{csgn}\left(\frac{ie^{bx+a}}{1+e^{2bx+2a}}\right)^3+i\pi\operatorname{csgn}\left(\frac{ie^{bx+a}}{1+e^{2bx+2a}}\right)^2\operatorname{csgn}\left(ie^{bx+a}\right)+i\pi\operatorname{csgn}\left(\frac{ie^{bx+a}}{1+e^{2bx+2a}}\right)^2\right.}{}}}{bn(n+2)\left(1+e^{2bx+2a}\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(b*x+a)^n*tanh(b*x+a)^3,x)
```

```
[Out] -(n*exp(4*b*x+4*a)+2*exp(4*b*x+4*a)-2*exp(2*b*x+2*a)*n+4*exp(2*b*x+2*a)+n+2
)/b/n/(n+2)/(1+exp(2*b*x+2*a))^2*exp(1/2*n*(-I*Pi*csgn(I*exp(b*x+a)/(1+exp(
2*b*x+2*a)))^3+I*Pi*csgn(I*exp(b*x+a)/(1+exp(2*b*x+2*a)))^2*csgn(I*exp(b*x+
a))+I*Pi*csgn(I*exp(b*x+a)/(1+exp(2*b*x+2*a)))^2*csgn(I/(1+exp(2*b*x+2*a)))
-I*Pi*csgn(I*exp(b*x+a)/(1+exp(2*b*x+2*a)))*csgn(I*exp(b*x+a))*csgn(I/(1+ex
p(2*b*x+2*a)))-2*ln(1+exp(2*b*x+2*a))+2*ln(2)+2*ln(exp(b*x+a))))
```

maxima [B] time = 0.54, size = 345, normalized size = 9.58

$$-\frac{2^{n} n e^{\left(-(bx+a)n-n\log\left(e^{(-2\,bx-2\,a)}+1\right)\right)}}{\left(n^2+2\left(n^2+2\,n\right)e^{(-2\,bx-2\,a)}+\left(n^2+2\,n\right)e^{(-4\,bx-4\,a)}+2\,n\right)b}+\frac{\left(2^{n+1}n-2^{n+2}\right)e^{\left(-(bx+a)n-2\,bx-n\log\left(e^{(-2\,bx-2\,a)}+1\right)\right.}}{\left(n^2+2\left(n^2+2\,n\right)e^{(-2\,bx-2\,a)}+\left(n^2+2\,n\right)e^{(-4\,bx-4\,a}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)^n*tanh(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -2^n*n*e^(-(b*x + a)*n - n*log(e^(-2*b*x - 2*a) + 1))/((n^2 + 2*(n^2 + 2*n)
*e^(-2*b*x - 2*a) + (n^2 + 2*n)*e^(-4*b*x - 4*a) + 2*n)*b) + (2^(n + 1)*n -
 2^(n + 2))*e^(-(b*x + a)*n - 2*b*x - n*log(e^(-2*b*x - 2*a) + 1) - 2*a)/((
n^2 + 2*(n^2 + 2*n)*e^(-2*b*x - 2*a) + (n^2 + 2*n)*e^(-4*b*x - 4*a) + 2*n)*
b) - (2^n*n + 2^(n + 1))*e^(-(b*x + a)*n - 4*b*x - n*log(e^(-2*b*x - 2*a) +
 1) - 4*a)/((n^2 + 2*(n^2 + 2*n)*e^(-2*b*x - 2*a) + (n^2 + 2*n)*e^(-4*b*x -
 4*a) + 2*n)*b) - 2^(n + 1)*e^(-(b*x + a)*n - n*log(e^(-2*b*x - 2*a) + 1))/
((n^2 + 2*(n^2 + 2*n)*e^(-2*b*x - 2*a) + (n^2 + 2*n)*e^(-4*b*x - 4*a) + 2*n
)*b)
```

mupad [B] time = 1.57, size = 101, normalized size = 2.81

$$-\frac{{\left(\frac{1}{\frac{e^{a+b\,x}}{2}+\frac{e^{-a-b\,x}}{2}}\right)}^n\,\left(\frac{1}{b\,n}+\frac{e^{4\,a+4\,b\,x}}{b\,n}-\frac{e^{2\,a+2\,b\,x}\,\left(2\,n-4\right)}{b\,n\,\left(n+2\right)}\right)}{2\,e^{2\,a+2\,b\,x}+e^{4\,a+4\,b\,x}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(a + b*x)^3*(1/cosh(a + b*x))^n,x)
```

```
[Out] -((1/(exp(a + b*x)/2 + exp(- a - b*x)/2))^n*(1/(b*n) + exp(4*a + 4*b*x)/(b*
n) - (exp(2*a + 2*b*x)*(2*n - 4))/(b*n*(n + 2))))/(2*exp(2*a + 2*b*x) + exp
(4*a + 4*b*x) + 1)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} x\tanh^3{(a)}\operatorname{sech}^n{(a)} & \text{for}\: b = 0 \\ \int \frac{\tanh^3{(a+bx)}}{\operatorname{sech}^2{(a+bx)}}\, dx & \text{for}\: n = -2 \\ x - \frac{\log{(\tanh{(a+bx)}+1)}}{b} - \frac{\tanh^2{(a+bx)}}{2b} & \text{for}\: n = 0 \\ -\frac{n\tanh^2{(a+bx)}\operatorname{sech}^n{(a+bx)}}{bn^2+2bn} - \frac{2\operatorname{sech}^n{(a+bx)}}{bn^2+2bn} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)**n*tanh(b*x+a)**3,x)
```

```
[Out] Piecewise((x*tanh(a)**3*sech(a)**n, Eq(b, 0)), (Integral(tanh(a + b*x)**3/s
ech(a + b*x)**2, x), Eq(n, -2)), (x - log(tanh(a + b*x) + 1)/b - tanh(a + b
*x)**2/(2*b), Eq(n, 0)), (-n*tanh(a + b*x)**2*sech(a + b*x)**n/(b*n**2 + 2*
b*n) - 2*sech(a + b*x)**n/(b*n**2 + 2*b*n), True))
```

3.90 $\int \operatorname{sech}^4(a+bx)\tanh^2(a+bx)\,dx$

Optimal. Leaf size=31

$$\frac{\tanh^3(a+bx)}{3b}-\frac{\tanh^5(a+bx)}{5b}$$

```
[Out] 1/3*tanh(b*x+a)^3/b-1/5*tanh(b*x+a)^5/b
```

Rubi [A] time = 0.03, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2607, 14}

$$\frac{\tanh^3(a+bx)}{3b}-\frac{\tanh^5(a+bx)}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[a + b*x]^4*Tanh[a + b*x]^2,x]
```

```
[Out] Tanh[a + b*x]^3/(3*b) - Tanh[a + b*x]^5/(5*b)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rubi steps

$$\begin{aligned}
\int \operatorname{sech}^4(a+bx)\tanh^2(a+bx)\,dx &= \frac{i\,\text{Subst}\left(\int x^2\left(1+x^2\right)\,dx,x,i\tanh(a+bx)\right)}{b}\\
&= \frac{i\,\text{Subst}\left(\int \left(x^2+x^4\right)\,dx,x,i\tanh(a+bx)\right)}{b}\\
&= \frac{\tanh^3(a+bx)}{3b}-\frac{\tanh^5(a+bx)}{5b}
\end{aligned}$$

Mathematica [A] time = 0.04, size = 56, normalized size = 1.81

$$\frac{2\tanh(a+bx)}{15b}-\frac{\tanh(a+bx)\operatorname{sech}^4(a+bx)}{5b}+\frac{\tanh(a+bx)\operatorname{sech}^2(a+bx)}{15b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[a + b*x]^4*Tanh[a + b*x]^2,x]
```

```
[Out] (2*Tanh[a + b*x])/(15*b) + (Sech[a + b*x]^2*Tanh[a + b*x])/(15*b) - (Sech[a
 + b*x]^4*Tanh[a + b*x])/(5*b)
```

fricas [B] time = 0.44, size = 304, normalized size = 9.81

$$-\frac{}{15\left(b\cosh(bx+a)^7+7\,b\cosh(bx+a)\sinh(bx+a)^6+b\sinh(bx+a)^7+5\,b\cosh(bx+a)^5+\left(21\,b\cosh(bx+\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)^4*tanh(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -8/15*(8*cosh(b*x + a)^3 + 24*cosh(b*x + a)*sinh(b*x + a)^2 + 7*sinh(b*x +
a)^3 + (21*cosh(b*x + a)^2 - 5)*sinh(b*x + a))/(b*cosh(b*x + a)^7 + 7*b*cos
h(b*x + a)*sinh(b*x + a)^6 + b*sinh(b*x + a)^7 + 5*b*cosh(b*x + a)^5 + (21*
b*cosh(b*x + a)^2 + 5*b)*sinh(b*x + a)^5 + 5*(7*b*cosh(b*x + a)^3 + 5*b*cos
h(b*x + a))*sinh(b*x + a)^4 + 11*b*cosh(b*x + a)^3 + (35*b*cosh(b*x + a)^4
+ 50*b*cosh(b*x + a)^2 + 9*b)*sinh(b*x + a)^3 + (21*b*cosh(b*x + a)^5 + 50*
b*cosh(b*x + a)^3 + 33*b*cosh(b*x + a))*sinh(b*x + a)^2 + 15*b*cosh(b*x + a
) + (7*b*cosh(b*x + a)^6 + 25*b*cosh(b*x + a)^4 + 27*b*cosh(b*x + a)^2 + 5*
b)*sinh(b*x + a))
```

giac [A] time = 0.14, size = 53, normalized size = 1.71

$$-\frac{4\left(15\,e^{(6\,bx+6\,a)}-5\,e^{(4\,bx+4\,a)}+5\,e^{(2\,bx+2\,a)}+1\right)}{15\,b\left(e^{(2\,bx+2\,a)}+1\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)^4*tanh(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -4/15*(15*e^(6*b*x + 6*a) - 5*e^(4*b*x + 4*a) + 5*e^(2*b*x + 2*a) + 1)/(b*(
e^(2*b*x + 2*a) + 1)^5)
```

maple [A] time = 0.32, size = 52, normalized size = 1.68

$$\frac{-\frac{\sinh(bx+a)}{4\cosh(bx+a)^5}+\frac{\left(\frac{8}{15}+\frac{\operatorname{sech}(bx+a)^4}{5}+\frac{4\operatorname{sech}(bx+a)^2}{15}\right)\tanh(bx+a)}{4}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(b*x+a)^4*tanh(b*x+a)^2,x)
```

```
[Out] 1/b*(-1/4*sinh(b*x+a)/cosh(b*x+a)^5+1/4*(8/15+1/5*sech(b*x+a)^4+4/15*sech(b
*x+a)^2)*tanh(b*x+a))
```

maxima [B] time = 0.39, size = 276, normalized size = 8.90

$$\frac{4\,e^{(-2\,bx-2\,a)}}{3\,b\left(5\,e^{(-2\,bx-2\,a)}+10\,e^{(-4\,bx-4\,a)}+10\,e^{(-6\,bx-6\,a)}+5\,e^{(-8\,bx-8\,a)}+e^{(-10\,bx-10\,a)}+1\right)}-\frac{}{3\,b\left(5\,e^{(-2\,bx-2\,a)}+10\,e^{(-4\,bx-4\,a)}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)^4*tanh(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 4/3*e^(-2*b*x - 2*a)/(b*(5*e^(-2*b*x - 2*a) + 10*e^(-4*b*x - 4*a) + 10*e^(-
6*b*x - 6*a) + 5*e^(-8*b*x - 8*a) + e^(-10*b*x - 10*a) + 1)) - 4/3*e^(-4*b*
x - 4*a)/(b*(5*e^(-2*b*x - 2*a) + 10*e^(-4*b*x - 4*a) + 10*e^(-6*b*x - 6*a)
 + 5*e^(-8*b*x - 8*a) + e^(-10*b*x - 10*a) + 1)) + 4*e^(-6*b*x - 6*a)/(b*(5
*e^(-2*b*x - 2*a) + 10*e^(-4*b*x - 4*a) + 10*e^(-6*b*x - 6*a) + 5*e^(-8*b*x
 - 8*a) + e^(-10*b*x - 10*a) + 1)) + 4/15/(b*(5*e^(-2*b*x - 2*a) + 10*e^(-4
*b*x - 4*a) + 10*e^(-6*b*x - 6*a) + 5*e^(-8*b*x - 8*a) + e^(-10*b*x - 10*a)
 + 1))
```

mupad [B] time = 0.13, size = 270, normalized size = 8.71

$$\frac{\frac{8}{15\,b}-\frac{4\,e^{2\,a+2\,b\,x}}{5\,b}}{3\,e^{2\,a+2\,b\,x}+3\,e^{4\,a+4\,b\,x}+e^{6\,a+6\,b\,x}+1}-\frac{\frac{2}{5\,b}-\frac{8\,e^{2\,a+2\,b\,x}}{5\,b}+\frac{6\,e^{4\,a+4\,b\,x}}{5\,b}}{4\,e^{2\,a+2\,b\,x}+6\,e^{4\,a+4\,b\,x}+4\,e^{6\,a+6\,b\,x}+e^{8\,a+8\,b\,x}+1}-\frac{\frac{8\,e^{2\,a+}}{5\,b}}{5\,e^{2\,a+2\,b\,x}+10\,e^{4\,a+4\,b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(a + b*x)^2/cosh(a + b*x)^4,x)
```

```
[Out] (8/(15*b) - (4*exp(2*a + 2*b*x))/(5*b))/(3*exp(2*a + 2*b*x) + 3*exp(4*a + 4
*b*x) + exp(6*a + 6*b*x) + 1) - (2/(5*b) - (8*exp(2*a + 2*b*x))/(5*b) + (6*
exp(4*a + 4*b*x))/(5*b))/(4*exp(2*a + 2*b*x) + 6*exp(4*a + 4*b*x) + 4*exp(6
*a + 6*b*x) + exp(8*a + 8*b*x) + 1) - ((8*exp(2*a + 2*b*x))/(5*b) - (16*exp
(4*a + 4*b*x))/(5*b) + (8*exp(6*a + 6*b*x))/(5*b))/(5*exp(2*a + 2*b*x) + 10
*exp(4*a + 4*b*x) + 10*exp(6*a + 6*b*x) + 5*exp(8*a + 8*b*x) + exp(10*a + 1
0*b*x) + 1) - 2/(5*b*(2*exp(2*a + 2*b*x) + exp(4*a + 4*b*x) + 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \tanh^2(a+bx)\operatorname{sech}^4(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)**4*tanh(b*x+a)**2,x)
```

```
[Out] Integral(tanh(a + b*x)**2*sech(a + b*x)**4, x)
```

3.91 $\int \operatorname{sech}^4(a+bx)\sqrt{\tanh(a+bx)}\,dx$

Optimal. Leaf size=35

$$\frac{2\tanh^{\frac{3}{2}}(a+bx)}{3b}-\frac{2\tanh^{\frac{7}{2}}(a+bx)}{7b}$$

```
[Out] 2/3*tanh(b*x+a)^(3/2)/b-2/7*tanh(b*x+a)^(7/2)/b
```

Rubi [A] time = 0.04, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2607, 14}

$$\frac{2\tanh^{\frac{3}{2}}(a+bx)}{3b}-\frac{2\tanh^{\frac{7}{2}}(a+bx)}{7b}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[a + b*x]^4*Sqrt[Tanh[a + b*x]],x]
```

```
[Out] (2*Tanh[a + b*x]^(3/2))/(3*b) - (2*Tanh[a + b*x]^(7/2))/(7*b)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] && !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rubi steps

$$\begin{aligned}\int \operatorname{sech}^4(a+bx)\sqrt{\tanh(a+bx)}\,dx &= -\frac{i\,\text{Subst}\left(\int \sqrt{-ix}\,\left(1+x^2\right)\,dx,x,i\tanh(a+bx)\right)}{b}\\ &= -\frac{i\,\text{Subst}\left(\int \left(\sqrt{-ix}-(-ix)^{5/2}\right)\,dx,x,i\tanh(a+bx)\right)}{b}\\ &= \frac{2\tanh^{\frac{3}{2}}(a+bx)}{3b}-\frac{2\tanh^{\frac{7}{2}}(a+bx)}{7b}\end{aligned}$$

Mathematica [A] time = 0.11, size = 29, normalized size = 0.83

$$\frac{2\tanh^{\frac{3}{2}}(a+bx)\left(3\mathrm{sech}^2(a+bx)+4\right)}{21b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[a + b*x]^4*Sqrt[Tanh[a + b*x]],x]
```

```
[Out] (2*(4 + 3*Sech[a + b*x]^2)*Tanh[a + b*x]^(3/2))/(21*b)
```

fricas [B] time = 0.44, size = 551, normalized size = 15.74

$$8\left(\cosh(bx+a)^6+6\cosh(bx+a)\sinh(bx+a)^5+\sinh(bx+a)^6+3\left(5\cosh(bx+a)^2+1\right)\sinh(bx+a)^4+3\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)^4*tanh(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] 8/21*(cosh(b*x + a)^6 + 6*cosh(b*x + a)*sinh(b*x + a)^5 + sinh(b*x + a)^6 +
 3*(5*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^4 + 3*cosh(b*x + a)^4 + 4*(5*cosh(
b*x + a)^3 + 3*cosh(b*x + a))*sinh(b*x + a)^3 + 3*(5*cosh(b*x + a)^4 + 6*co
sh(b*x + a)^2 + 1)*sinh(b*x + a)^2 + 3*cosh(b*x + a)^2 + 6*(cosh(b*x + a)^5
 + 2*cosh(b*x + a)^3 + cosh(b*x + a))*sinh(b*x + a) + (cosh(b*x + a)^6 + 6*
cosh(b*x + a)*sinh(b*x + a)^5 + sinh(b*x + a)^6 + (15*cosh(b*x + a)^2 + 4)*
sinh(b*x + a)^4 + 4*cosh(b*x + a)^4 + 4*(5*cosh(b*x + a)^3 + 4*cosh(b*x + a
))*sinh(b*x + a)^3 + (15*cosh(b*x + a)^4 + 24*cosh(b*x + a)^2 - 4)*sinh(b*x
 + a)^2 - 4*cosh(b*x + a)^2 + 2*(3*cosh(b*x + a)^5 + 8*cosh(b*x + a)^3 - 4*
cosh(b*x + a))*sinh(b*x + a) - 1)*sqrt(sinh(b*x + a)/cosh(b*x + a)) + 1)/(b
*cosh(b*x + a)^6 + 6*b*cosh(b*x + a)*sinh(b*x + a)^5 + b*sinh(b*x + a)^6 +
3*b*cosh(b*x + a)^4 + 3*(5*b*cosh(b*x + a)^2 + b)*sinh(b*x + a)^4 + 4*(5*b*
cosh(b*x + a)^3 + 3*b*cosh(b*x + a))*sinh(b*x + a)^3 + 3*b*cosh(b*x + a)^2
+ 3*(5*b*cosh(b*x + a)^4 + 6*b*cosh(b*x + a)^2 + b)*sinh(b*x + a)^2 + 6*(b*
cosh(b*x + a)^5 + 2*b*cosh(b*x + a)^3 + b*cosh(b*x + a))*sinh(b*x + a) + b)
```

giac [B] time = 0.18, size = 148, normalized size = 4.23

$$\frac{16\left(21\left(\sqrt{e^{(4bx+4a)}-1}-e^{(2bx+2a)}\right)^5-7\left(\sqrt{e^{(4bx+4a)}-1}-e^{(2bx+2a)}\right)^4+28\left(\sqrt{e^{(4bx+4a)}-1}-e^{(2bx+2a)}\right)^3+7\sqrt{e^{(4\cdots}}\right.}{21\,b\left(\sqrt{e^{(4bx+4a)}-1}-e^{(2bx+2a)}-1\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)^4*tanh(b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] 16/21*(21*(sqrt(e^(4*b*x + 4*a) - 1) - e^(2*b*x + 2*a))^5 - 7*(sqrt(e^(4*b*
x + 4*a) - 1) - e^(2*b*x + 2*a))^4 + 28*(sqrt(e^(4*b*x + 4*a) - 1) - e^(2*b
*x + 2*a))^3 + 7*sqrt(e^(4*b*x + 4*a) - 1) - 7*e^(2*b*x + 2*a) - 1)/(b*(sqr
t(e^(4*b*x + 4*a) - 1) - e^(2*b*x + 2*a) - 1)^7)
```

maple [F] time = 0.71, size = 0, normalized size = 0.00

$$\int \operatorname{sech}\left(bx+a\right)^{4} \left(\sqrt{\tanh}\,\left(bx+a\right)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(b*x+a)^4*tanh(b*x+a)^(1/2),x)
```

```
[Out] int(sech(b*x+a)^4*tanh(b*x+a)^(1/2),x)
```

maxima [B] time = 0.44, size = 352, normalized size = 10.06

$$\frac{32\,\sqrt{e^{(-bx-a)}+1}\,\sqrt{-e^{(-bx-a)}+1}\,e^{(-2\,bx-2\,a)}}{21\,b\left(3\,e^{(-2\,bx-2\,a)}+3\,e^{(-4\,bx-4\,a)}+e^{(-6\,bx-6\,a)}+1\right)\sqrt{e^{(-2\,bx-2\,a)}+1}}-\frac{32\,\sqrt{e^{(-bx-a)}+1}\,\sqrt{-e^{(-bx-a)}+1}\,e^{(-4}}{21\,b\left(3\,e^{(-2\,bx-2\,a)}+3\,e^{(-4\,bx-4\,a)}+e^{(-6\,bx-6\,a)}+1\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)^4*tanh(b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 32/21*sqrt(e^(-b*x - a) + 1)*sqrt(-e^(-b*x - a) + 1)*e^(-2*b*x - 2*a)/(b*(3
*e^(-2*b*x - 2*a) + 3*e^(-4*b*x - 4*a) + e^(-6*b*x - 6*a) + 1)*sqrt(e^(-2*b
*x - 2*a) + 1)) - 32/21*sqrt(e^(-b*x - a) + 1)*sqrt(-e^(-b*x - a) + 1)*e^(-
4*b*x - 4*a)/(b*(3*e^(-2*b*x - 2*a) + 3*e^(-4*b*x - 4*a) + e^(-6*b*x - 6*a)
 + 1)*sqrt(e^(-2*b*x - 2*a) + 1)) - 8/21*sqrt(e^(-b*x - a) + 1)*sqrt(-e^(-b
*x - a) + 1)*e^(-6*b*x - 6*a)/(b*(3*e^(-2*b*x - 2*a) + 3*e^(-4*b*x - 4*a) +
 e^(-6*b*x - 6*a) + 1)*sqrt(e^(-2*b*x - 2*a) + 1)) + 8/21*sqrt(e^(-b*x - a)
 + 1)*sqrt(-e^(-b*x - a) + 1)/(b*(3*e^(-2*b*x - 2*a) + 3*e^(-4*b*x - 4*a) +
 e^(-6*b*x - 6*a) + 1)*sqrt(e^(-2*b*x - 2*a) + 1))
```

mupad [B] time = 1.63, size = 168, normalized size = 4.80

$$\frac{8\,\sqrt{\frac{{\mathrm{e}}^{2\,a+2\,b\,x}-1}{{\mathrm{e}}^{2\,a+2\,b\,x}+1}}}{21\,b}+\frac{8\,\sqrt{\frac{{\mathrm{e}}^{2\,a+2\,b\,x}-1}{{\mathrm{e}}^{2\,a+2\,b\,x}+1}}}{21\,b\,\left({\mathrm{e}}^{2\,a+2\,b\,x}+1\right)}-\frac{24\,\sqrt{\frac{{\mathrm{e}}^{2\,a+2\,b\,x}-1}{{\mathrm{e}}^{2\,a+2\,b\,x}+1}}}{7\,b\,{\left({\mathrm{e}}^{2\,a+2\,b\,x}+1\right)}^2}+\frac{16\,\sqrt{\frac{{\mathrm{e}}^{2\,a+2\,b\,x}-1}{{\mathrm{e}}^{2\,a+2\,b\,x}+1}}}{7\,b\,{\left({\mathrm{e}}^{2\,a+2\,b\,x}+1\right)}^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(a + b*x)^(1/2)/cosh(a + b*x)^4,x)
```

```
[Out] (8*((exp(2*a + 2*b*x) - 1)/(exp(2*a + 2*b*x) + 1))^(1/2))/(21*b) + (8*((exp
(2*a + 2*b*x) - 1)/(exp(2*a + 2*b*x) + 1))^(1/2))/(21*b*(exp(2*a + 2*b*x) +
 1)) - (24*((exp(2*a + 2*b*x) - 1)/(exp(2*a + 2*b*x) + 1))^(1/2))/(7*b*(exp
(2*a + 2*b*x) + 1)^2) + (16*((exp(2*a + 2*b*x) - 1)/(exp(2*a + 2*b*x) + 1))
^(1/2))/(7*b*(exp(2*a + 2*b*x) + 1)^3)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\tanh (a + bx)} \operatorname{sech}^4 (a + bx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)**4*tanh(b*x+a)**(1/2),x)
```

```
[Out] Integral(sqrt(tanh(a + b*x))*sech(a + b*x)**4, x)
```

3.92 $\int \operatorname{sech}^4(a+bx)\tanh^n(a+bx)\,dx$

Optimal. Leaf size=40

$$\frac{\tanh^{n+1}(a+bx)}{b(n+1)}-\frac{\tanh^{n+3}(a+bx)}{b(n+3)}$$

```
[Out] tanh(b*x+a)^(1+n)/b/(1+n)-tanh(b*x+a)^(3+n)/b/(3+n)
```

Rubi [A] time = 0.04, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2607, 14}

$$\frac{\tanh^{n+1}(a+bx)}{b(n+1)}-\frac{\tanh^{n+3}(a+bx)}{b(n+3)}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[a + b*x]^4*Tanh[a + b*x]^n,x]
```

```
[Out] Tanh[a + b*x]^(1 + n)/(b*(1 + n)) - Tanh[a + b*x]^(3 + n)/(b*(3 + n))
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] && !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rubi steps

$$\begin{aligned}
\int \operatorname{sech}^4(a+bx)\tanh^n(a+bx)\,dx &= -\frac{i\,\text{Subst}\left(\int (-ix)^n\left(1+x^2\right)\,dx,x,i\tanh(a+bx)\right)}{b}\\
&= -\frac{i\,\text{Subst}\left(\int \left((-ix)^n-(-ix)^{2+n}\right)\,dx,x,i\tanh(a+bx)\right)}{b}\\
&= \frac{\tanh^{1+n}(a+bx)}{b(1+n)}-\frac{\tanh^{3+n}(a+bx)}{b(3+n)}
\end{aligned}$$

Mathematica [A] time = 0.93, size = 73, normalized size = 1.82

$$\frac{\tanh^{n-1}(a+bx)\left(\tanh^2(a+bx)\text{sech}^2(a+bx)(\cosh(2(a+bx))+n+2)-2\tanh^2(a+bx)^{\frac{1-n}{2}}\right)}{b(n+1)(n+3)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[a + b*x]^4*Tanh[a + b*x]^n,x]
```

```
[Out] (Tanh[a + b*x]^(-1 + n)*((2 + n + Cosh[2*(a + b*x)])*Sech[a + b*x]^2*Tanh[a
 + b*x]^2 - 2*(Tanh[a + b*x]^2)^((1 - n)/2)))/(b*(1 + n)*(3 + n))
```

fricas [B] time = 0.44, size = 180, normalized size = 4.50

$$\frac{2\left(\left(\sinh(bx+a)^3+\left(3\cosh(bx+a)^2+2n+3\right)\sinh(bx+a)\right)\cosh\left(n\log\left(\frac{\sinh(bx+a)}{\cosh(bx+a)}\right)\right)+\left(\sinh(bx+a)^3+\left(3\right.\right.}{\left(bn^2+4bn+3b\right)\cosh(bx+a)^3+3\left(bn^2+4bn+3b\right)\cosh(bx+a)\sinh(bx+a)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)^4*tanh(b*x+a)^n,x, algorithm="fricas")
```

```
[Out] 2*((sinh(b*x + a)^3 + (3*cosh(b*x + a)^2 + 2*n + 3)*sinh(b*x + a))*cosh(n*l
og(sinh(b*x + a)/cosh(b*x + a))) + (sinh(b*x + a)^3 + (3*cosh(b*x + a)^2 +
2*n + 3)*sinh(b*x + a))*sinh(n*log(sinh(b*x + a)/cosh(b*x + a))))/((b*n^2 +
 4*b*n + 3*b)*cosh(b*x + a)^3 + 3*(b*n^2 + 4*b*n + 3*b)*cosh(b*x + a)*sinh(
b*x + a)^2 + 3*(b*n^2 + 4*b*n + 3*b)*cosh(b*x + a))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \tanh(bx+a)^n \operatorname{sech}(bx+a)^4\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)^4*tanh(b*x+a)^n,x, algorithm="giac")
```

```
[Out] integrate(tanh(b*x + a)^n*sech(b*x + a)^4, x)
```

maple [C] time = 0.69, size = 535, normalized size = 13.38

$$\frac{2\left(e^{6bx+6a}+2n\,e^{4bx+4a}+3\,e^{4bx+4a}-2\,e^{2bx+2a}n-3\,e^{2bx+2a}-1\right)e^{\frac{n\left(-i\pi\text{csgn}\left(\frac{i\left(e^{bx+a}-1\right)}{1+e^{2bx+2a}}\right)^3+i\pi\text{csgn}\left(\frac{i\left(e^{bx+a}-1\right)}{1+e^{2bx+2a}}\right)^2\text{csgn}\left(i\left(e^{bx+a}-1\right)\right)+i\pi\text{csgn}\right.}{}}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(b*x+a)^4*tanh(b*x+a)^n,x)
```

```
[Out] 2*(exp(6*b*x+6*a)+2*n*exp(4*b*x+4*a)+3*exp(4*b*x+4*a)-2*exp(2*b*x+2*a)*n-3*
exp(2*b*x+2*a)-1)/b/(n+1)/(n+3)/(1+exp(2*b*x+2*a))^3*exp(1/2*n*(-I*Pi*csgn(
I*(exp(b*x+a)-1)/(1+exp(2*b*x+2*a)))^3+I*Pi*csgn(I*(exp(b*x+a)-1)/(1+exp(2*
b*x+2*a)))^2*csgn(I*(exp(b*x+a)-1))+I*Pi*csgn(I*(exp(b*x+a)-1)/(1+exp(2*b*x
+2*a)))^2*csgn(I/(1+exp(2*b*x+2*a)))-I*Pi*csgn(I*(exp(b*x+a)-1)/(1+exp(2*b*
x+2*a)))*csgn(I*(exp(b*x+a)-1))*csgn(I/(1+exp(2*b*x+2*a)))+I*Pi*csgn(I*(exp
(b*x+a)-1)/(1+exp(2*b*x+2*a)))*csgn(I*(1+exp(b*x+a))/(1+exp(2*b*x+2*a))*(ex
p(b*x+a)-1))^2-I*Pi*csgn(I*(exp(b*x+a)-1)/(1+exp(2*b*x+2*a)))*csgn(I*(1+exp
(b*x+a))/(1+exp(2*b*x+2*a))*(exp(b*x+a)-1))*csgn(I*(1+exp(b*x+a)))-I*Pi*csg
n(I*(1+exp(b*x+a))/(1+exp(2*b*x+2*a))*(exp(b*x+a)-1))^3+I*Pi*csgn(I*(1+exp(
b*x+a))/(1+exp(2*b*x+2*a))*(exp(b*x+a)-1))^2*csgn(I*(1+exp(b*x+a)))+2*ln(ex
p(b*x+a)-1)-2*ln(1+exp(2*b*x+2*a))+2*ln(1+exp(b*x+a))))
```

maxima [B] time = 0.44, size = 504, normalized size = 12.60

$$\frac{2\,(2\,n+3)e^{\left(-2\,bx+n\log\left(e^{(-bx-a)}+1\right)+n\log\left(-e^{(-bx-a)}+1\right)-n\log\left(e^{(-2\,bx-2\,a)}+1\right)-2\,a\right)}}{\left(n^2+3\left(n^2+4\,n+3\right)e^{(-2\,bx-2\,a)}+3\left(n^2+4\,n+3\right)e^{(-4\,bx-4\,a)}+\left(n^2+4\,n+3\right)e^{(-6\,bx-6\,a)}+4\,n+3\right)b}-\frac{}{\left(n^2+3\left(n^2\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)^4*tanh(b*x+a)^n,x, algorithm="maxima")
```

```
[Out] 2*(2*n + 3)*e^(-2*b*x + n*log(e^(-b*x - a) + 1) + n*log(-e^(-b*x - a) + 1)
- n*log(e^(-2*b*x - 2*a) + 1) - 2*a)/((n^2 + 3*(n^2 + 4*n + 3)*e^(-2*b*x -
2*a) + 3*(n^2 + 4*n + 3)*e^(-4*b*x - 4*a) + (n^2 + 4*n + 3)*e^(-6*b*x - 6*a
) + 4*n + 3)*b) - 2*(2*n + 3)*e^(-4*b*x + n*log(e^(-b*x - a) + 1) + n*log(-
e^(-b*x - a) + 1) - n*log(e^(-2*b*x - 2*a) + 1) - 4*a)/((n^2 + 3*(n^2 + 4*n
 + 3)*e^(-2*b*x - 2*a) + 3*(n^2 + 4*n + 3)*e^(-4*b*x - 4*a) + (n^2 + 4*n +
3)*e^(-6*b*x - 6*a) + 4*n + 3)*b) - 2*e^(-6*b*x + n*log(e^(-b*x - a) + 1) +
 n*log(-e^(-b*x - a) + 1) - n*log(e^(-2*b*x - 2*a) + 1) - 6*a)/((n^2 + 3*(n
^2 + 4*n + 3)*e^(-2*b*x - 2*a) + 3*(n^2 + 4*n + 3)*e^(-4*b*x - 4*a) + (n^2
+ 4*n + 3)*e^(-6*b*x - 6*a) + 4*n + 3)*b) + 2*e^(n*log(e^(-b*x - a) + 1) +
n*log(-e^(-b*x - a) + 1) - n*log(e^(-2*b*x - 2*a) + 1))/((n^2 + 3*(n^2 + 4*
n + 3)*e^(-2*b*x - 2*a) + 3*(n^2 + 4*n + 3)*e^(-4*b*x - 4*a) + (n^2 + 4*n +
 3)*e^(-6*b*x - 6*a) + 4*n + 3)*b)
```

mupad [B] time = 1.61, size = 115, normalized size = 2.88

$$\frac{\mathrm{e}^{-3\,a-3\,b\,x}\,\left(\frac{4\,\mathrm{e}^{3\,a+3\,b\,x}\,\mathrm{sinh}(3\,a+3\,b\,x)}{b\,\left(n^2+4\,n+3\right)}+\frac{2\,\mathrm{e}^{3\,a+3\,b\,x}\,\mathrm{sinh}(a+b\,x)\,(4\,n+6)}{b\,\left(n^2+4\,n+3\right)}\right)\,{\left(\frac{\mathrm{e}^{2\,a+2\,b\,x}-1}{\mathrm{e}^{2\,a+2\,b\,x}+1}\right)}^n}{8\,\mathrm{cosh}\left(a+b\,x\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(a + b*x)^n/cosh(a + b*x)^4,x)
```

```
[Out] (exp(- 3*a - 3*b*x)*((4*exp(3*a + 3*b*x)*sinh(3*a + 3*b*x))/(b*(4*n + n^2 +
 3)) + (2*exp(3*a + 3*b*x)*sinh(a + b*x)*(4*n + 6))/(b*(4*n + n^2 + 3)))*((
exp(2*a + 2*b*x) - 1)/(exp(2*a + 2*b*x) + 1))^n)/(8*cosh(a + b*x)^3)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \tanh^{n}(a+bx)\operatorname{sech}^{4}(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)**4*tanh(b*x+a)**n,x)
```

```
[Out] Integral(tanh(a + b*x)**n*sech(a + b*x)**4, x)
```

3.93 $\int \text{sech}(a+bx)\tanh^2(a+bx)\,dx$

Optimal. Leaf size=34

$$\frac{\tan^{-1}(\sinh(a+bx))}{2b}-\frac{\tanh(a+bx)\text{sech}(a+bx)}{2b}$$

```
[Out] 1/2*arctan(sinh(b*x+a))/b-1/2*sech(b*x+a)*tanh(b*x+a)/b
```

Rubi [A] time = 0.02, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2611, 3770}

$$\frac{\tan^{-1}(\sinh(a+bx))}{2b}-\frac{\tanh(a+bx)\text{sech}(a+bx)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[a + b*x]*Tanh[a + b*x]^2,x]
```

```
[Out] ArcTan[Sinh[a + b*x]]/(2*b) - (Sech[a + b*x]*Tanh[a + b*x])/(2*b)
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \text{sech}(a+bx)\tanh^2(a+bx)\,dx &= -\frac{\text{sech}(a+bx)\tanh(a+bx)}{2b}+\frac{1}{2}\int \text{sech}(a+bx)\,dx\\ &= \frac{\tan^{-1}(\sinh(a+bx))}{2b}-\frac{\text{sech}(a+bx)\tanh(a+bx)}{2b}\end{aligned}$$

Mathematica [A] time = 0.02, size = 34, normalized size = 1.00

$$\frac{\tan^{-1}(\sinh(a+bx))}{2b}-\frac{\tanh(a+bx)\text{sech}(a+bx)}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[a + b*x]*Tanh[a + b*x]^2,x]
```

```
[Out] ArcTan[Sinh[a + b*x]]/(2*b) - (Sech[a + b*x]*Tanh[a + b*x])/(2*b)
```

fricas [B] time = 0.42, size = 269, normalized size = 7.91

$$-\frac{\cosh(bx+a)^3+3\,\cosh(bx+a)\sinh(bx+a)^2+\sinh(bx+a)^3-\left(\cosh(bx+a)^4+4\,\cosh(bx+a)\sinh(bx+\right.}{b\cosh(bx+a)^4+4\,b\cosh(bx+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)*tanh(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -(cosh(b*x + a)^3 + 3*cosh(b*x + a)*sinh(b*x + a)^2 + sinh(b*x + a)^3 - (co
sh(b*x + a)^4 + 4*cosh(b*x + a)*sinh(b*x + a)^3 + sinh(b*x + a)^4 + 2*(3*co
sh(b*x + a)^2 + 1)*sinh(b*x + a)^2 + 2*cosh(b*x + a)^2 + 4*(cosh(b*x + a)^3
 + cosh(b*x + a))*sinh(b*x + a) + 1)*arctan(cosh(b*x + a) + sinh(b*x + a))
+ (3*cosh(b*x + a)^2 - 1)*sinh(b*x + a) - cosh(b*x + a))/(b*cosh(b*x + a)^4
 + 4*b*cosh(b*x + a)*sinh(b*x + a)^3 + b*sinh(b*x + a)^4 + 2*b*cosh(b*x + a
)^2 + 2*(3*b*cosh(b*x + a)^2 + b)*sinh(b*x + a)^2 + 4*(b*cosh(b*x + a)^3 +
b*cosh(b*x + a))*sinh(b*x + a) + b)
```

giac [A] time = 0.13, size = 47, normalized size = 1.38

$$-\frac{\frac{e^{(3\,bx+3\,a)}-e^{(bx+a)}}{\left(e^{(2\,bx+2\,a)}+1\right)^2}-\arctan\left(e^{(bx+a)}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)*tanh(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -((e^(3*b*x + 3*a) - e^(b*x + a))/(e^(2*b*x + 2*a) + 1)^2 - arctan(e^(b*x +
 a)))/b
```

maple [A] time = 0.34, size = 49, normalized size = 1.44

$$-\frac{\sinh(bx+a)}{b\cosh(bx+a)^2}+\frac{\text{sech}(bx+a)\tanh(bx+a)}{2b}+\frac{\arctan\left(e^{bx+a}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(b*x+a)*tanh(b*x+a)^2,x)
```

```
[Out] -1/b*sinh(b*x+a)/cosh(b*x+a)^2+1/2*sech(b*x+a)*tanh(b*x+a)/b+arctan(exp(b*x
+a))/b
```

maxima [B] time = 0.47, size = 66, normalized size = 1.94

$$-\frac{\arctan\left(e^{(-bx-a)}\right)}{b}-\frac{e^{(-bx-a)}-e^{(-3\,bx-3\,a)}}{b\left(2\,e^{(-2\,bx-2\,a)}+e^{(-4\,bx-4\,a)}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)*tanh(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -arctan(e^(-b*x - a))/b - (e^(-b*x - a) - e^(-3*b*x - 3*a))/(b*(2*e^(-2*b*x
 - 2*a) + e^(-4*b*x - 4*a) + 1))
```

mupad [B] time = 0.09, size = 82, normalized size = 2.41

$$\frac{\mathrm{atan}\left(\frac{\mathrm{e}^{b\,x}\,\mathrm{e}^{a}\,\sqrt{b^2}}{b}\right)}{\sqrt{b^2}}+\frac{2\,\mathrm{e}^{a+b\,x}}{b\,\left(2\,\mathrm{e}^{2\,a+2\,b\,x}+\mathrm{e}^{4\,a+4\,b\,x}+1\right)}-\frac{\mathrm{e}^{a+b\,x}}{b\,\left(\mathrm{e}^{2\,a+2\,b\,x}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(a + b*x)^2/cosh(a + b*x),x)
```

```
[Out] atan((exp(b*x)*exp(a)*(b^2)^(1/2))/b)/(b^2)^(1/2) + (2*exp(a + b*x))/(b*(2*
exp(2*a + 2*b*x) + exp(4*a + 4*b*x) + 1)) - exp(a + b*x)/(b*(exp(2*a + 2*b*
x) + 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \tanh^{2}{\left(a + bx \right)}\operatorname{sech}{\left(a + bx \right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)*tanh(b*x+a)**2,x)
```

```
[Out] Integral(tanh(a + b*x)**2*sech(a + b*x), x)
```

3.94 $\int \operatorname{sech}(a+bx)\tanh^4(a+bx)\,dx$

Optimal. Leaf size=55

$$\frac{3\tan^{-1}(\sinh(a+bx))}{8b}-\frac{\tanh^3(a+bx)\operatorname{sech}(a+bx)}{4b}-\frac{3\tanh(a+bx)\operatorname{sech}(a+bx)}{8b}$$

```
[Out] 3/8*arctan(sinh(b*x+a))/b-3/8*sech(b*x+a)*tanh(b*x+a)/b-1/4*sech(b*x+a)*tan
h(b*x+a)^3/b
```

Rubi [A] time = 0.04, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2611, 3770}

$$\frac{3\tan^{-1}(\sinh(a+bx))}{8b}-\frac{\tanh^3(a+bx)\operatorname{sech}(a+bx)}{4b}-\frac{3\tanh(a+bx)\operatorname{sech}(a+bx)}{8b}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[a + b*x]*Tanh[a + b*x]^4,x]
```

```
[Out] (3*ArcTan[Sinh[a + b*x]])/(8*b) - (3*Sech[a + b*x]*Tanh[a + b*x])/(8*b) - (
Sech[a + b*x]*Tanh[a + b*x]^3)/(4*b)
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \operatorname{sech}(a+bx)\tanh^4(a+bx)\,dx = -\frac{\operatorname{sech}(a+bx)\tanh^3(a+bx)}{4b} + \frac{3}{4}\int \operatorname{sech}(a+bx)\tanh^2(a+bx)\,dx$$
$$= -\frac{3\operatorname{sech}(a+bx)\tanh(a+bx)}{8b} - \frac{\operatorname{sech}(a+bx)\tanh^3(a+bx)}{4b} + \frac{3}{8}\int \operatorname{sech}(a$$
$$= \frac{3\tan^{-1}(\sinh(a+bx))}{8b} - \frac{3\operatorname{sech}(a+bx)\tanh(a+bx)}{8b} - \frac{\operatorname{sech}(a+bx)\tanh^3(}{4b}$$

Mathematica [A] time = 0.17, size = 59, normalized size = 1.07

$$\frac{3\tan^{-1}(\sinh(a+bx)) - 6\tanh(a+bx)\operatorname{sech}^3(a+bx) + \left(3\tanh(a+bx) - 8\tanh^3(a+bx)\right)\operatorname{sech}(a+bx)}{8b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[a + b*x]*Tanh[a + b*x]^4,x]
```

```
[Out] (3*ArcTan[Sinh[a + b*x]] - 6*Sech[a + b*x]^3*Tanh[a + b*x] + Sech[a + b*x]*
(3*Tanh[a + b*x] - 8*Tanh[a + b*x]^3))/(8*b)
```

fricas [B] time = 0.44, size = 814, normalized size = 14.80

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)*tanh(b*x+a)^4,x, algorithm="fricas")
```

```
[Out] -1/4*(5*cosh(b*x + a)^7 + 35*cosh(b*x + a)*sinh(b*x + a)^6 + 5*sinh(b*x + a
)^7 + 3*(35*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^5 - 3*cosh(b*x + a)^5 + 5*(3
5*cosh(b*x + a)^3 - 3*cosh(b*x + a))*sinh(b*x + a)^4 + (175*cosh(b*x + a)^4
 - 30*cosh(b*x + a)^2 + 3)*sinh(b*x + a)^3 + 3*cosh(b*x + a)^3 + 3*(35*cosh
(b*x + a)^5 - 10*cosh(b*x + a)^3 + 3*cosh(b*x + a))*sinh(b*x + a)^2 - 3*(co
sh(b*x + a)^8 + 8*cosh(b*x + a)*sinh(b*x + a)^7 + sinh(b*x + a)^8 + 4*(7*co
sh(b*x + a)^2 + 1)*sinh(b*x + a)^6 + 4*cosh(b*x + a)^6 + 8*(7*cosh(b*x + a)
^3 + 3*cosh(b*x + a))*sinh(b*x + a)^5 + 2*(35*cosh(b*x + a)^4 + 30*cosh(b*x
 + a)^2 + 3)*sinh(b*x + a)^4 + 6*cosh(b*x + a)^4 + 8*(7*cosh(b*x + a)^5 + 1
0*cosh(b*x + a)^3 + 3*cosh(b*x + a))*sinh(b*x + a)^3 + 4*(7*cosh(b*x + a)^6
 + 15*cosh(b*x + a)^4 + 9*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^2 + 4*cosh(b*x
 + a)^2 + 8*(cosh(b*x + a)^7 + 3*cosh(b*x + a)^5 + 3*cosh(b*x + a)^3 + cosh
(b*x + a))*sinh(b*x + a) + 1)*arctan(cosh(b*x + a) + sinh(b*x + a)) + (35*c
osh(b*x + a)^6 - 15*cosh(b*x + a)^4 + 9*cosh(b*x + a)^2 - 5)*sinh(b*x + a)
- 5*cosh(b*x + a))/(b*cosh(b*x + a)^8 + 8*b*cosh(b*x + a)*sinh(b*x + a)^7 +
```

```
 b*sinh(b*x + a)^8 + 4*b*cosh(b*x + a)^6 + 4*(7*b*cosh(b*x + a)^2 + b)*sinh
(b*x + a)^6 + 8*(7*b*cosh(b*x + a)^3 + 3*b*cosh(b*x + a))*sinh(b*x + a)^5 +
 6*b*cosh(b*x + a)^4 + 2*(35*b*cosh(b*x + a)^4 + 30*b*cosh(b*x + a)^2 + 3*b
)*sinh(b*x + a)^4 + 8*(7*b*cosh(b*x + a)^5 + 10*b*cosh(b*x + a)^3 + 3*b*cos
h(b*x + a))*sinh(b*x + a)^3 + 4*b*cosh(b*x + a)^2 + 4*(7*b*cosh(b*x + a)^6
+ 15*b*cosh(b*x + a)^4 + 9*b*cosh(b*x + a)^2 + b)*sinh(b*x + a)^2 + 8*(b*co
sh(b*x + a)^7 + 3*b*cosh(b*x + a)^5 + 3*b*cosh(b*x + a)^3 + b*cosh(b*x + a)
)*sinh(b*x + a) + b)
```

giac [A] time = 0.15, size = 71, normalized size = 1.29

$$-\frac{\frac{5\,e^{(7\,bx+7\,a)}-3\,e^{(5\,bx+5\,a)}+3\,e^{(3\,bx+3\,a)}-5\,e^{(bx+a)}}{\left(e^{(2\,bx+2\,a)}+1\right)^{4}}-3\,\arctan\left(e^{(bx+a)}\right)}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)*tanh(b*x+a)^4,x, algorithm="giac")
```

```
[Out] -1/4*((5*e^(7*b*x + 7*a) - 3*e^(5*b*x + 5*a) + 3*e^(3*b*x + 3*a) - 5*e^(b*x
 + a))/(e^(2*b*x + 2*a) + 1)^4 - 3*arctan(e^(b*x + a)))/b
```

maple [A] time = 0.33, size = 90, normalized size = 1.64

$$-\frac{\sinh^{3}(bx+a)}{b\cosh(bx+a)^{4}}-\frac{\sinh(bx+a)}{b\cosh(bx+a)^{4}}+\frac{\operatorname{sech}(bx+a)^{3}\tanh(bx+a)}{4b}+\frac{3\operatorname{sech}(bx+a)\tanh(bx+a)}{8b}+\frac{3\arctan\left(e^{bx+}\right.}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(b*x+a)*tanh(b*x+a)^4,x)
```

```
[Out] -1/b*sinh(b*x+a)^3/cosh(b*x+a)^4-1/b*sinh(b*x+a)/cosh(b*x+a)^4+1/4*sech(b*x
+a)^3*tanh(b*x+a)/b+3/8*sech(b*x+a)*tanh(b*x+a)/b+3/4*arctan(exp(b*x+a))/b
```

maxima [B] time = 0.62, size = 112, normalized size = 2.04

$$-\frac{3\,\arctan\left(e^{(-bx-a)}\right)}{4\,b}-\frac{5\,e^{(-bx-a)}-3\,e^{(-3\,bx-3\,a)}+3\,e^{(-5\,bx-5\,a)}-5\,e^{(-7\,bx-7\,a)}}{4\,b\left(4\,e^{(-2\,bx-2\,a)}+6\,e^{(-4\,bx-4\,a)}+4\,e^{(-6\,bx-6\,a)}+e^{(-8\,bx-8\,a)}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)*tanh(b*x+a)^4,x, algorithm="maxima")
```

```
[Out] -3/4*arctan(e^(-b*x - a))/b - 1/4*(5*e^(-b*x - a) - 3*e^(-3*b*x - 3*a) + 3*
e^(-5*b*x - 5*a) - 5*e^(-7*b*x - 7*a))/(b*(4*e^(-2*b*x - 2*a) + 6*e^(-4*b*x
 - 4*a) + 4*e^(-6*b*x - 6*a) + e^(-8*b*x - 8*a) + 1))
```

mupad [B] time = 1.44, size = 186, normalized size = 3.38

$$\frac{3\,\mathrm{atan}\left(\frac{\mathrm{e}^{b\,x}\,\mathrm{e}^{a}\,\sqrt{b^2}}{b}\right)}{4\,\sqrt{b^2}}+\frac{9\,\mathrm{e}^{a+b\,x}}{2\,b\,\left(2\,\mathrm{e}^{2\,a+2\,b\,x}+\mathrm{e}^{4\,a+4\,b\,x}+1\right)}-\frac{6\,\mathrm{e}^{a+b\,x}}{b\,\left(3\,\mathrm{e}^{2\,a+2\,b\,x}+3\,\mathrm{e}^{4\,a+4\,b\,x}+\mathrm{e}^{6\,a+6\,b\,x}+1\right)}+\frac{}{b\,\left(4\,\mathrm{e}^{2\,a+2\,b\,x}+6\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(a + b*x)^4/cosh(a + b*x),x)
```

```
[Out] (3*atan((exp(b*x)*exp(a)*(b^2)^(1/2))/b))/(4*(b^2)^(1/2)) + (9*exp(a + b*x)
)/(2*b*(2*exp(2*a + 2*b*x) + exp(4*a + 4*b*x) + 1)) - (6*exp(a + b*x))/(b*(
3*exp(2*a + 2*b*x) + 3*exp(4*a + 4*b*x) + exp(6*a + 6*b*x) + 1)) + (4*exp(a
 + b*x))/(b*(4*exp(2*a + 2*b*x) + 6*exp(4*a + 4*b*x) + 4*exp(6*a + 6*b*x) +
 exp(8*a + 8*b*x) + 1)) - (5*exp(a + b*x))/(4*b*(exp(2*a + 2*b*x) + 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \tanh^{4}\left(a+bx\right)\operatorname{sech}\left(a+bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)*tanh(b*x+a)**4,x)
```

```
[Out] Integral(tanh(a + b*x)**4*sech(a + b*x), x)
```

3.95 $\int \operatorname{sech}^3(a+bx)\tanh^2(a+bx)\,dx$

Optimal. Leaf size=55

$$\frac{\tan^{-1}(\sinh(a+bx))}{8b}-\frac{\tanh(a+bx)\operatorname{sech}^3(a+bx)}{4b}+\frac{\tanh(a+bx)\operatorname{sech}(a+bx)}{8b}$$

```
[Out] 1/8*arctan(sinh(b*x+a))/b+1/8*sech(b*x+a)*tanh(b*x+a)/b-1/4*sech(b*x+a)^3*t
anh(b*x+a)/b
```

Rubi [A] time = 0.05, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {2611, 3768, 3770}

$$\frac{\tan^{-1}(\sinh(a+bx))}{8b}-\frac{\tanh(a+bx)\operatorname{sech}^3(a+bx)}{4b}+\frac{\tanh(a+bx)\operatorname{sech}(a+bx)}{8b}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[a + b*x]^3*Tanh[a + b*x]^2,x]
```

```
[Out] ArcTan[Sinh[a + b*x]]/(8*b) + (Sech[a + b*x]*Tanh[a + b*x])/(8*b) - (Sech[a
 + b*x]^3*Tanh[a + b*x])/(4*b)
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \operatorname{sech}^3(a+bx)\tanh^2(a+bx)\,dx = -\frac{\operatorname{sech}^3(a+bx)\tanh(a+bx)}{4b} + \frac{1}{4}\int \operatorname{sech}^3(a+bx)\,dx$$
$$= \frac{\operatorname{sech}(a+bx)\tanh(a+bx)}{8b} - \frac{\operatorname{sech}^3(a+bx)\tanh(a+bx)}{4b} + \frac{1}{8}\int \operatorname{sech}(a+$$
$$= \frac{\tan^{-1}(\sinh(a+bx))}{8b} + \frac{\operatorname{sech}(a+bx)\tanh(a+bx)}{8b} - \frac{\operatorname{sech}^3(a+bx)\tanh(a}{4b}$$

Mathematica [A] time = 0.02, size = 55, normalized size = 1.00

$$\frac{\tan^{-1}(\sinh(a+bx))}{8b} - \frac{\tanh(a+bx)\operatorname{sech}^3(a+bx)}{4b} + \frac{\tanh(a+bx)\operatorname{sech}(a+bx)}{8b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[a + b*x]^3*Tanh[a + b*x]^2,x]
```

```
[Out] ArcTan[Sinh[a + b*x]]/(8*b) + (Sech[a + b*x]*Tanh[a + b*x])/(8*b) - (Sech[a
 + b*x]^3*Tanh[a + b*x])/(4*b)
```

fricas [B] time = 0.47, size = 808, normalized size = 14.69

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)^3*tanh(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/4*(cosh(b*x + a)^7 + 7*cosh(b*x + a)*sinh(b*x + a)^6 + sinh(b*x + a)^7 +
7*(3*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^5 - 7*cosh(b*x + a)^5 + 35*(cosh(b*
x + a)^3 - cosh(b*x + a))*sinh(b*x + a)^4 + 7*(5*cosh(b*x + a)^4 - 10*cosh(
b*x + a)^2 + 1)*sinh(b*x + a)^3 + 7*cosh(b*x + a)^3 + 7*(3*cosh(b*x + a)^5
- 10*cosh(b*x + a)^3 + 3*cosh(b*x + a))*sinh(b*x + a)^2 + (cosh(b*x + a)^8
+ 8*cosh(b*x + a)*sinh(b*x + a)^7 + sinh(b*x + a)^8 + 4*(7*cosh(b*x + a)^2
+ 1)*sinh(b*x + a)^6 + 4*cosh(b*x + a)^6 + 8*(7*cosh(b*x + a)^3 + 3*cosh(b*
x + a))*sinh(b*x + a)^5 + 2*(35*cosh(b*x + a)^4 + 30*cosh(b*x + a)^2 + 3)*s
inh(b*x + a)^4 + 6*cosh(b*x + a)^4 + 8*(7*cosh(b*x + a)^5 + 10*cosh(b*x + a
)^3 + 3*cosh(b*x + a))*sinh(b*x + a)^3 + 4*(7*cosh(b*x + a)^6 + 15*cosh(b*x
 + a)^4 + 9*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^2 + 4*cosh(b*x + a)^2 + 8*(c
osh(b*x + a)^7 + 3*cosh(b*x + a)^5 + 3*cosh(b*x + a)^3 + cosh(b*x + a))*sin
h(b*x + a) + 1)*arctan(cosh(b*x + a) + sinh(b*x + a)) + (7*cosh(b*x + a)^6
- 35*cosh(b*x + a)^4 + 21*cosh(b*x + a)^2 - 1)*sinh(b*x + a) - cosh(b*x + a
))/(b*cosh(b*x + a)^8 + 8*b*cosh(b*x + a)*sinh(b*x + a)^7 + b*sinh(b*x + a)
```

```
^8 + 4*b*cosh(b*x + a)^6 + 4*(7*b*cosh(b*x + a)^2 + b)*sinh(b*x + a)^6 + 8*
(7*b*cosh(b*x + a)^3 + 3*b*cosh(b*x + a))*sinh(b*x + a)^5 + 6*b*cosh(b*x +
a)^4 + 2*(35*b*cosh(b*x + a)^4 + 30*b*cosh(b*x + a)^2 + 3*b)*sinh(b*x + a)^
4 + 8*(7*b*cosh(b*x + a)^5 + 10*b*cosh(b*x + a)^3 + 3*b*cosh(b*x + a))*sinh
(b*x + a)^3 + 4*b*cosh(b*x + a)^2 + 4*(7*b*cosh(b*x + a)^6 + 15*b*cosh(b*x
+ a)^4 + 9*b*cosh(b*x + a)^2 + b)*sinh(b*x + a)^2 + 8*(b*cosh(b*x + a)^7 +
3*b*cosh(b*x + a)^5 + 3*b*cosh(b*x + a)^3 + b*cosh(b*x + a))*sinh(b*x + a)
+ b)
```

giac [A] time = 0.15, size = 67, normalized size = 1.22

$$\frac{\frac{e^{(7\,bx+7\,a)}-7\,e^{(5\,bx+5\,a)}+7\,e^{(3\,bx+3\,a)}-e^{(bx+a)}}{\left(e^{(2\,bx+2\,a)}+1\right)^4}+\arctan\left(e^{(bx+a)}\right)}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)^3*tanh(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/4*((e^(7*b*x + 7*a) - 7*e^(5*b*x + 5*a) + 7*e^(3*b*x + 3*a) - e^(b*x + a)
)/(e^(2*b*x + 2*a) + 1)^4 + arctan(e^(b*x + a)))/b
```

maple [A] time = 0.31, size = 69, normalized size = 1.25

$$-\frac{\sinh\left(bx+a\right)}{3b\cosh\left(bx+a\right)^4}+\frac{\mathrm{sech}\left(bx+a\right)^3\tanh\left(bx+a\right)}{12b}+\frac{\mathrm{sech}\left(bx+a\right)\tanh\left(bx+a\right)}{8b}+\frac{\arctan\left(e^{bx+a}\right)}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(b*x+a)^3*tanh(b*x+a)^2,x)
```

```
[Out] -1/3/b*sinh(b*x+a)/cosh(b*x+a)^4+1/12*sech(b*x+a)^3*tanh(b*x+a)/b+1/8*sech(
b*x+a)*tanh(b*x+a)/b+1/4*arctan(exp(b*x+a))/b
```

maxima [B] time = 0.48, size = 110, normalized size = 2.00

$$-\frac{\arctan\left(e^{(-bx-a)}\right)}{4\,b}+\frac{e^{(-bx-a)}-7\,e^{(-3\,bx-3\,a)}+7\,e^{(-5\,bx-5\,a)}-e^{(-7\,bx-7\,a)}}{4\,b\left(4\,e^{(-2\,bx-2\,a)}+6\,e^{(-4\,bx-4\,a)}+4\,e^{(-6\,bx-6\,a)}+e^{(-8\,bx-8\,a)}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)^3*tanh(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -1/4*arctan(e^(-b*x - a))/b + 1/4*(e^(-b*x - a) - 7*e^(-3*b*x - 3*a) + 7*e^
(-5*b*x - 5*a) - e^(-7*b*x - 7*a))/(b*(4*e^(-2*b*x - 2*a) + 6*e^(-4*b*x - 4
*a) + 4*e^(-6*b*x - 6*a) + e^(-8*b*x - 8*a) + 1))
```

mupad [B] time = 1.46, size = 215, normalized size = 3.91

$$\frac{\operatorname{atan}\left(\frac{e^{bx}\,e^{a}\,\sqrt{b^2}}{b}\right)}{4\sqrt{b^2}}-\frac{\frac{e^{a+b\,x}}{b}-\frac{2\,e^{3\,a+3\,b\,x}}{b}+\frac{e^{5\,a+5\,b\,x}}{b}}{4\,e^{2\,a+2\,b\,x}+6\,e^{4\,a+4\,b\,x}+4\,e^{6\,a+6\,b\,x}+e^{8\,a+8\,b\,x}+1}-\frac{3\,e^{a+b\,x}}{2\,b\,\left(2\,e^{2\,a+2\,b\,x}+e^{4\,a+4\,b\,x}+1\right)}+\frac{}{b\,\left(3\,e^{2\,a+2}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(a + b*x)^2/cosh(a + b*x)^3,x)
```

```
[Out] atan((exp(b*x)*exp(a)*(b^2)^(1/2))/b)/(4*(b^2)^(1/2)) - (exp(a + b*x)/b - (
2*exp(3*a + 3*b*x))/b + exp(5*a + 5*b*x)/b)/(4*exp(2*a + 2*b*x) + 6*exp(4*a
 + 4*b*x) + 4*exp(6*a + 6*b*x) + exp(8*a + 8*b*x) + 1) - (3*exp(a + b*x))/(
2*b*(2*exp(2*a + 2*b*x) + exp(4*a + 4*b*x) + 1)) + (2*exp(a + b*x))/(b*(3*e
xp(2*a + 2*b*x) + 3*exp(4*a + 4*b*x) + exp(6*a + 6*b*x) + 1)) + exp(a + b*x
)/(4*b*(exp(2*a + 2*b*x) + 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \tanh^2(a+bx)\operatorname{sech}^3(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)**3*tanh(b*x+a)**2,x)
```

```
[Out] Integral(tanh(a + b*x)**2*sech(a + b*x)**3, x)
```

3.96 $\int \operatorname{sech}(x)\tanh^5(x)\,dx$

Optimal. Leaf size=21

$$-\frac{1}{5}\operatorname{sech}^5(x)+\frac{2\operatorname{sech}^3(x)}{3}-\operatorname{sech}(x)$$

```
[Out] -sech(x)+2/3*sech(x)^3-1/5*sech(x)^5
```

Rubi [A] time = 0.02, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2606, 194}

$$-\frac{1}{5}\operatorname{sech}^5(x)+\frac{2\operatorname{sech}^3(x)}{3}-\operatorname{sech}(x)$$

Antiderivative was successfully verified.

```
[In] Int[Sech[x]*Tanh[x]^5,x]
```

```
[Out] -Sech[x] + (2*Sech[x]^3)/3 - Sech[x]^5/5
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rubi steps

$$\begin{aligned}\int \operatorname{sech}(x)\tanh^5(x)\,dx &= -\operatorname{Subst}\left(\int \left(-1+x^2\right)^2\,dx,x,\operatorname{sech}(x)\right)\\ &= -\operatorname{Subst}\left(\int \left(1-2x^2+x^4\right)\,dx,x,\operatorname{sech}(x)\right)\\ &= -\operatorname{sech}(x)+\frac{2\operatorname{sech}^3(x)}{3}-\frac{\operatorname{sech}^5(x)}{5}\end{aligned}$$

Mathematica [A] time = 0.01, size = 21, normalized size = 1.00

$$-\frac{1}{5}\text{sech}^5(x)+\frac{2\text{sech}^3(x)}{3}-\text{sech}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[x]*Tanh[x]^5,x]
```

```
[Out] -Sech[x] + (2*Sech[x]^3)/3 - Sech[x]^5/5
```

fricas [B] time = 0.44, size = 185, normalized size = 8.81

$$-\frac{2\left(15\cosh(x)^5+75\cosh(x)\sinh(x)^4+15\sinh(x)^5+5\left(30\cosh(x)^2+1\right)\sinh(x)^3+\right.}{15\left(\cosh(x)^6+6\cosh(x)\sinh(x)^5+\sinh(x)^6+3\left(5\cosh(x)^2+2\right)\sinh(x)^4+6\cosh(x)^4+4\left(5\cosh(x)^3+\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)*tanh(x)^5,x, algorithm="fricas")
```

```
[Out] -2/15*(15*cosh(x)^5 + 75*cosh(x)*sinh(x)^4 + 15*sinh(x)^5 + 5*(30*cosh(x)^2
 + 1)*sinh(x)^3 + 35*cosh(x)^3 + 15*(10*cosh(x)^3 + 7*cosh(x))*sinh(x)^2 +
(75*cosh(x)^4 + 15*cosh(x)^2 + 38)*sinh(x) + 78*cosh(x))/(cosh(x)^6 + 6*cos
h(x)*sinh(x)^5 + sinh(x)^6 + 3*(5*cosh(x)^2 + 2)*sinh(x)^4 + 6*cosh(x)^4 +
4*(5*cosh(x)^3 + 4*cosh(x))*sinh(x)^3 + 3*(5*cosh(x)^4 + 12*cosh(x)^2 + 5)*
sinh(x)^2 + 15*cosh(x)^2 + 2*(3*cosh(x)^5 + 8*cosh(x)^3 + 5*cosh(x))*sinh(x
) + 10)
```

giac [B] time = 0.13, size = 35, normalized size = 1.67

$$-\frac{2\left(15\left(e^{(-x)}+e^x\right)^4-40\left(e^{(-x)}+e^x\right)^2+48\right)}{15\left(e^{(-x)}+e^x\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)*tanh(x)^5,x, algorithm="giac")
```

```
[Out] -2/15*(15*(e^(-x) + e^x)^4 - 40*(e^(-x) + e^x)^2 + 48)/(e^(-x) + e^x)^5
```

maple [A] time = 0.08, size = 28, normalized size = 1.33

$$-\frac{\sinh^4(x)}{\cosh(x)^5}-\frac{4\left(\sinh^2(x)\right)}{3\cosh(x)^5}-\frac{8}{15\cosh(x)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(x)*tanh(x)^5,x)
```

```
[Out] -sinh(x)^4/cosh(x)^5-4/3*sinh(x)^2/cosh(x)^5-8/15/cosh(x)^5
```

maxima [B] time = 0.32, size = 191, normalized size = 9.10

$$-\frac{2\,e^{(-x)}}{5\,e^{(-2\,x)}+10\,e^{(-4\,x)}+10\,e^{(-6\,x)}+5\,e^{(-8\,x)}+e^{(-10\,x)}+1}-\frac{8\,e^{(-3\,x)}}{3\left(5\,e^{(-2\,x)}+10\,e^{(-4\,x)}+10\,e^{(-6\,x)}+5\,e^{(-8\,x)}+e^{(-10\,x)}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)*tanh(x)^5,x, algorithm="maxima")
```

```
[Out] -2*e^(-x)/(5*e^(-2*x) + 10*e^(-4*x) + 10*e^(-6*x) + 5*e^(-8*x) + e^(-10*x)
+ 1) - 8/3*e^(-3*x)/(5*e^(-2*x) + 10*e^(-4*x) + 10*e^(-6*x) + 5*e^(-8*x) +
e^(-10*x) + 1) - 116/15*e^(-5*x)/(5*e^(-2*x) + 10*e^(-4*x) + 10*e^(-6*x) +
5*e^(-8*x) + e^(-10*x) + 1) - 8/3*e^(-7*x)/(5*e^(-2*x) + 10*e^(-4*x) + 10*e
^(-6*x) + 5*e^(-8*x) + e^(-10*x) + 1) - 2*e^(-9*x)/(5*e^(-2*x) + 10*e^(-4*x
) + 10*e^(-6*x) + 5*e^(-8*x) + e^(-10*x) + 1)
```

mupad [B] time = 1.43, size = 129, normalized size = 6.14

$$\frac{64\,e^{x}}{5\left(4\,e^{2\,x}+6\,e^{4\,x}+4\,e^{6\,x}+e^{8\,x}+1\right)}-\frac{2\,e^{x}}{e^{2\,x}+1}-\frac{176\,e^{x}}{15\left(3\,e^{2\,x}+3\,e^{4\,x}+e^{6\,x}+1\right)}-\frac{32\,e^{x}}{5\left(5\,e^{2\,x}+10\,e^{4\,x}+10\,e^{6\,x}+5\,e^{8\,x}+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^5/cosh(x),x)
```

```
[Out] (64*exp(x))/(5*(4*exp(2*x) + 6*exp(4*x) + 4*exp(6*x) + exp(8*x) + 1)) - (2*
exp(x))/(exp(2*x) + 1) - (176*exp(x))/(15*(3*exp(2*x) + 3*exp(4*x) + exp(6*
x) + 1)) - (32*exp(x))/(5*(5*exp(2*x) + 10*exp(4*x) + 10*exp(6*x) + 5*exp(8
*x) + exp(10*x) + 1)) + (16*exp(x))/(3*(2*exp(2*x) + exp(4*x) + 1))
```

sympy [A] time = 1.47, size = 29, normalized size = 1.38

$$-\frac{\tanh^{4}(x)\operatorname{sech}(x)}{5}-\frac{4\tanh^{2}(x)\operatorname{sech}(x)}{15}-\frac{8\operatorname{sech}(x)}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)*tanh(x)**5,x)
```

```
[Out] -tanh(x)**4*sech(x)/5 - 4*tanh(x)**2*sech(x)/15 - 8*sech(x)/15
```

3.97 $\int \text{sech}^7(x)\tanh^5(x)\,dx$

Optimal. Leaf size=25

$$-\frac{1}{11}\text{sech}^{11}(x)+\frac{2\text{sech}^9(x)}{9}-\frac{\text{sech}^7(x)}{7}$$

```
[Out] -1/7*sech(x)^7+2/9*sech(x)^9-1/11*sech(x)^11
```

Rubi [A] time = 0.03, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2606, 270}

$$-\frac{1}{11}\text{sech}^{11}(x)+\frac{2\text{sech}^9(x)}{9}-\frac{\text{sech}^7(x)}{7}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[x]^7*Tanh[x]^5,x]
```

```
[Out] -Sech[x]^7/7 + (2*Sech[x]^9)/9 - Sech[x]^11/11
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rubi steps

$$\begin{aligned}\int \text{sech}^7(x)\tanh^5(x)\,dx &= -\text{Subst}\left(\int x^6\left(-1+x^2\right)^2\,dx,x,\text{sech}(x)\right)\\ &= -\text{Subst}\left(\int \left(x^6-2x^8+x^{10}\right)dx,x,\text{sech}(x)\right)\\ &= -\frac{1}{7}\text{sech}^7(x)+\frac{2\text{sech}^9(x)}{9}-\frac{\text{sech}^{11}(x)}{11}\end{aligned}$$

Mathematica [A] time = 0.02, size = 25, normalized size = 1.00

$$-\frac{1}{11}\text{sech}^{11}(x)+\frac{2\text{sech}^9(x)}{9}-\frac{\text{sech}^7(x)}{7}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[x]^7*Tanh[x]^5,x]
```

```
[Out] -1/7*Sech[x]^7 + (2*Sech[x]^9)/9 - Sech[x]^11/11
```

fricas [B] time = 0.49, size = 634, normalized size = 25.36

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^7*tanh(x)^5,x, algorithm="fricas")
```

```
[Out] -128/693*(99*cosh(x)^8 + 792*cosh(x)*sinh(x)^7 + 99*sinh(x)^8 + 44*(63*cosh
(x)^2 - 5)*sinh(x)^6 - 220*cosh(x)^6 + 264*(21*cosh(x)^3 - 5*cosh(x))*sinh(
x)^5 + 10*(693*cosh(x)^4 - 330*cosh(x)^2 + 37)*sinh(x)^4 + 370*cosh(x)^4 +
8*(693*cosh(x)^5 - 550*cosh(x)^3 + 185*cosh(x))*sinh(x)^3 + 4*(693*cosh(x)^
6 - 825*cosh(x)^4 + 555*cosh(x)^2 - 55)*sinh(x)^2 - 220*cosh(x)^2 + 8*(99*c
osh(x)^7 - 165*cosh(x)^5 + 185*cosh(x)^3 - 55*cosh(x))*sinh(x) + 99)/(cosh(
x)^15 + 15*cosh(x)*sinh(x)^14 + sinh(x)^15 + (105*cosh(x)^2 + 11)*sinh(x)^1
3 + 11*cosh(x)^13 + 13*(35*cosh(x)^3 + 11*cosh(x))*sinh(x)^12 + (1365*cosh(
x)^4 + 858*cosh(x)^2 + 55)*sinh(x)^11 + 55*cosh(x)^11 + 11*(273*cosh(x)^5 +
 286*cosh(x)^3 + 55*cosh(x))*sinh(x)^10 + 55*(91*cosh(x)^6 + 143*cosh(x)^4
+ 55*cosh(x)^2 + 3)*sinh(x)^9 + 165*cosh(x)^9 + 33*(195*cosh(x)^7 + 429*cos
h(x)^5 + 275*cosh(x)^3 + 45*cosh(x))*sinh(x)^8 + (6435*cosh(x)^8 + 18876*co
sh(x)^6 + 18150*cosh(x)^4 + 5940*cosh(x)^2 + 329)*sinh(x)^7 + 331*cosh(x)^7
 + (5005*cosh(x)^9 + 18876*cosh(x)^7 + 25410*cosh(x)^5 + 13860*cosh(x)^3 +
2317*cosh(x))*sinh(x)^6 + (3003*cosh(x)^10 + 14157*cosh(x)^8 + 25410*cosh(x
)^6 + 20790*cosh(x)^4 + 6909*cosh(x)^2 + 451)*sinh(x)^5 + 473*cosh(x)^5 + 5
*(273*cosh(x)^11 + 1573*cosh(x)^9 + 3630*cosh(x)^7 + 4158*cosh(x)^5 + 2317*
cosh(x)^3 + 473*cosh(x))*sinh(x)^4 + (455*cosh(x)^12 + 3146*cosh(x)^10 + 90
75*cosh(x)^8 + 13860*cosh(x)^6 + 11515*cosh(x)^4 + 4510*cosh(x)^2 + 407)*si
nh(x)^3 + 517*cosh(x)^3 + (105*cosh(x)^13 + 858*cosh(x)^11 + 3025*cosh(x)^9
 + 5940*cosh(x)^7 + 6951*cosh(x)^5 + 4730*cosh(x)^3 + 1551*cosh(x))*sinh(x)
^2 + (15*cosh(x)^14 + 143*cosh(x)^12 + 605*cosh(x)^10 + 1485*cosh(x)^8 + 23
03*cosh(x)^6 + 2255*cosh(x)^4 + 1221*cosh(x)^2 + 165)*sinh(x) + 495*cosh(x)
)
```

giac [A] time = 0.12, size = 35, normalized size = 1.40

$$-\frac{128\left(99\left(e^{(-x)}+e^{x}\right)^{4}-616\left(e^{(-x)}+e^{x}\right)^{2}+1008\right)}{693\left(e^{(-x)}+e^{x}\right)^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^7*tanh(x)^5,x, algorithm="giac")
```

```
[Out] -128/693*(99*(e^(-x) + e^x)^4 - 616*(e^(-x) + e^x)^2 + 1008)/(e^(-x) + e^x)
^11
```

maple [A] time = 0.10, size = 28, normalized size = 1.12

$$-\frac{\sinh^{4}(x)}{7\cosh(x)^{11}}-\frac{4\left(\sinh^{2}(x)\right)}{63\cosh(x)^{11}}-\frac{8}{693\cosh(x)^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(x)^7*tanh(x)^5,x)
```

```
[Out] -1/7*sinh(x)^4/cosh(x)^11-4/63*sinh(x)^2/cosh(x)^11-8/693/cosh(x)^11
```

maxima [B] time = 0.35, size = 371, normalized size = 14.84

$$-\frac{128\,e^{(-7\,x)}}{7\left(11\,e^{(-2\,x)}+55\,e^{(-4\,x)}+165\,e^{(-6\,x)}+330\,e^{(-8\,x)}+462\,e^{(-10\,x)}+462\,e^{(-12\,x)}+330\,e^{(-14\,x)}+165\,e^{(-16\,x)}+55\,e^{(-18}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^7*tanh(x)^5,x, algorithm="maxima")
```

```
[Out] -128/7*e^(-7*x)/(11*e^(-2*x) + 55*e^(-4*x) + 165*e^(-6*x) + 330*e^(-8*x) +
462*e^(-10*x) + 462*e^(-12*x) + 330*e^(-14*x) + 165*e^(-16*x) + 55*e^(-18*x
) + 11*e^(-20*x) + e^(-22*x) + 1) + 2560/63*e^(-9*x)/(11*e^(-2*x) + 55*e^(-
4*x) + 165*e^(-6*x) + 330*e^(-8*x) + 462*e^(-10*x) + 462*e^(-12*x) + 330*e^
(-14*x) + 165*e^(-16*x) + 55*e^(-18*x) + 11*e^(-20*x) + e^(-22*x) + 1) - 47
360/693*e^(-11*x)/(11*e^(-2*x) + 55*e^(-4*x) + 165*e^(-6*x) + 330*e^(-8*x)
+ 462*e^(-10*x) + 462*e^(-12*x) + 330*e^(-14*x) + 165*e^(-16*x) + 55*e^(-18
*x) + 11*e^(-20*x) + e^(-22*x) + 1) + 2560/63*e^(-13*x)/(11*e^(-2*x) + 55*e
^(-4*x) + 165*e^(-6*x) + 330*e^(-8*x) + 462*e^(-10*x) + 462*e^(-12*x) + 330
*e^(-14*x) + 165*e^(-16*x) + 55*e^(-18*x) + 11*e^(-20*x) + e^(-22*x) + 1) -
 128/7*e^(-15*x)/(11*e^(-2*x) + 55*e^(-4*x) + 165*e^(-6*x) + 330*e^(-8*x) +
 462*e^(-10*x) + 462*e^(-12*x) + 330*e^(-14*x) + 165*e^(-16*x) + 55*e^(-18*
x) + 11*e^(-20*x) + e^(-22*x) + 1)
```

mupad [B] time = 1.55, size = 520, normalized size = 20.80

$$\frac{\frac{64\,e^{5\,x}}{11}-\frac{320\,e^{7\,x}}{11}+\frac{640\,e^{9\,x}}{11}-\frac{640\,e^{11\,x}}{11}+\frac{320\,e^{13\,x}}{11}-\frac{64\,e^{15\,x}}{11}}{11\,e^{2\,x}+55\,e^{4\,x}+165\,e^{6\,x}+330\,e^{8\,x}+462\,e^{10\,x}+462\,e^{12\,x}+330\,e^{14\,x}+165\,e^{16\,x}+55\,e^{18\,x}+11\,e^{20\,x}+e^{22\,x}+1}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^5/cosh(x)^7,x)
```

```
[Out] ((64*exp(5*x))/11 - (320*exp(7*x))/11 + (640*exp(9*x))/11 - (640*exp(11*x))
/11 + (320*exp(13*x))/11 - (64*exp(15*x))/11)/(11*exp(2*x) + 55*exp(4*x) +
165*exp(6*x) + 330*exp(8*x) + 462*exp(10*x) + 462*exp(12*x) + 330*exp(14*x)
 + 165*exp(16*x) + 55*exp(18*x) + 11*exp(20*x) + exp(22*x) + 1) - (38464*ex
p(x))/(693*(6*exp(2*x) + 15*exp(4*x) + 20*exp(6*x) + 15*exp(8*x) + 6*exp(10
*x) + exp(12*x) + 1)) - (640*exp(x))/(33*(8*exp(2*x) + 28*exp(4*x) + 56*exp
(6*x) + 70*exp(8*x) + 56*exp(10*x) + 28*exp(12*x) + 8*exp(14*x) + exp(16*x)
 + 1)) - (104*exp(x))/(21*(4*exp(2*x) + 6*exp(4*x) + 4*exp(6*x) + exp(8*x)
+ 1)) + (1664*exp(x))/(63*(5*exp(2*x) + 10*exp(4*x) + 10*exp(6*x) + 5*exp(8
*x) + exp(10*x) + 1)) + (4096*exp(x))/(77*(7*exp(2*x) + 21*exp(4*x) + 35*ex
p(6*x) + 35*exp(8*x) + 21*exp(10*x) + 7*exp(12*x) + exp(14*x) + 1)) + ((16*
exp(3*x))/11 - (112*exp(5*x))/11 + (288*exp(7*x))/11 - 32*exp(9*x) + (208*e
xp(11*x))/11 - (48*exp(13*x))/11)/(10*exp(2*x) + 45*exp(4*x) + 120*exp(6*x)
 + 210*exp(8*x) + 252*exp(10*x) + 210*exp(12*x) + 120*exp(14*x) + 45*exp(16
*x) + 10*exp(18*x) + exp(20*x) + 1) - ((280*exp(3*x))/99 - (112*exp(5*x))/1
1 + 16*exp(7*x) - (104*exp(9*x))/9 + (104*exp(11*x))/33 - (8*exp(x))/33)/(9
*exp(2*x) + 36*exp(4*x) + 84*exp(6*x) + 126*exp(8*x) + 126*exp(10*x) + 84*e
xp(12*x) + 36*exp(14*x) + 9*exp(16*x) + exp(18*x) + 1)
```

sympy [A] time = 22.41, size = 34, normalized size = 1.36

$$-\frac{\tanh^4(x)\operatorname{sech}^7(x)}{11}-\frac{4\tanh^2(x)\operatorname{sech}^7(x)}{99}-\frac{8\operatorname{sech}^7(x)}{693}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)**7*tanh(x)**5,x)
```

```
[Out] -tanh(x)**4*sech(x)**7/11 - 4*tanh(x)**2*sech(x)**7/99 - 8*sech(x)**7/693
```

3.98 $\int \mathrm{sech}^3(x) \tanh^4(x)\, dx$

Optimal. Leaf size=38

$$\frac{1}{16}\tan^{-1}(\sinh(x)) - \frac{1}{6}\tanh^3(x)\mathrm{sech}^3(x) - \frac{1}{8}\tanh(x)\mathrm{sech}^3(x) + \frac{1}{16}\tanh(x)\mathrm{sech}(x)$$

```
[Out] 1/16*arctan(sinh(x))+1/16*sech(x)*tanh(x)-1/8*sech(x)^3*tanh(x)-1/6*sech(x)
^3*tanh(x)^3
```

Rubi [A] time = 0.05, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2611, 3768, 3770}

$$\frac{1}{16}\tan^{-1}(\sinh(x)) - \frac{1}{6}\tanh^3(x)\mathrm{sech}^3(x) - \frac{1}{8}\tanh(x)\mathrm{sech}^3(x) + \frac{1}{16}\tanh(x)\mathrm{sech}(x)$$

Antiderivative was successfully verified.

```
[In] Int[Sech[x]^3*Tanh[x]^4,x]
```

```
[Out] ArcTan[Sinh[x]]/16 + (Sech[x]*Tanh[x])/16 - (Sech[x]^3*Tanh[x])/8 - (Sech[x
]^3*Tanh[x]^3)/6
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \operatorname{sech}^3(x)\tanh^4(x)\,dx &= -\frac{1}{6}\operatorname{sech}^3(x)\tanh^3(x) + \frac{1}{2}\int \operatorname{sech}^3(x)\tanh^2(x)\,dx \\ &= -\frac{1}{8}\operatorname{sech}^3(x)\tanh(x) - \frac{1}{6}\operatorname{sech}^3(x)\tanh^3(x) + \frac{1}{8}\int \operatorname{sech}^3(x)\,dx \\ &= \frac{1}{16}\operatorname{sech}(x)\tanh(x) - \frac{1}{8}\operatorname{sech}^3(x)\tanh(x) - \frac{1}{6}\operatorname{sech}^3(x)\tanh^3(x) + \frac{1}{16}\int \operatorname{sech}(x)\,dx \\ &= \frac{1}{16}\tan^{-1}(\sinh(x)) + \frac{1}{16}\operatorname{sech}(x)\tanh(x) - \frac{1}{8}\operatorname{sech}^3(x)\tanh(x) - \frac{1}{6}\operatorname{sech}^3(x)\tanh^3(x)\end{aligned}$$

Mathematica [A] time = 0.01, size = 48, normalized size = 1.26

$$\frac{1}{16}\tan^{-1}(\sinh(x)) - \frac{1}{6}\tanh(x)\operatorname{sech}^5(x) - \frac{1}{3}\tanh^3(x)\operatorname{sech}^3(x) + \frac{1}{24}\tanh(x)\operatorname{sech}^3(x) + \frac{1}{16}\tanh(x)\operatorname{sech}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[x]^3*Tanh[x]^4,x]
```

```
[Out] ArcTan[Sinh[x]]/16 + (Sech[x]*Tanh[x])/16 + (Sech[x]^3*Tanh[x])/24 - (Sech[
x]^5*Tanh[x])/6 - (Sech[x]^3*Tanh[x]^3)/3
```

fricas [B] time = 0.49, size = 925, normalized size = 24.34

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^3*tanh(x)^4,x, algorithm="fricas")
```

```
[Out] 1/24*(3*cosh(x)^11 + 33*cosh(x)*sinh(x)^10 + 3*sinh(x)^11 + (165*cosh(x)^2
- 47)*sinh(x)^9 - 47*cosh(x)^9 + 9*(55*cosh(x)^3 - 47*cosh(x))*sinh(x)^8 +
6*(165*cosh(x)^4 - 282*cosh(x)^2 + 13)*sinh(x)^7 + 78*cosh(x)^7 + 42*(33*co
sh(x)^5 - 94*cosh(x)^3 + 13*cosh(x))*sinh(x)^6 + 6*(231*cosh(x)^6 - 987*cos
h(x)^4 + 273*cosh(x)^2 - 13)*sinh(x)^5 - 78*cosh(x)^5 + 6*(165*cosh(x)^7 -
987*cosh(x)^5 + 455*cosh(x)^3 - 65*cosh(x))*sinh(x)^4 + (495*cosh(x)^8 - 39
48*cosh(x)^6 + 2730*cosh(x)^4 - 780*cosh(x)^2 + 47)*sinh(x)^3 + 47*cosh(x)^
3 + 3*(55*cosh(x)^9 - 564*cosh(x)^7 + 546*cosh(x)^5 - 260*cosh(x)^3 + 47*co
sh(x))*sinh(x)^2 + 3*(cosh(x)^12 + 12*cosh(x)*sinh(x)^11 + sinh(x)^12 + 6*(
11*cosh(x)^2 + 1)*sinh(x)^10 + 6*cosh(x)^10 + 20*(11*cosh(x)^3 + 3*cosh(x))
*sinh(x)^9 + 15*(33*cosh(x)^4 + 18*cosh(x)^2 + 1)*sinh(x)^8 + 15*cosh(x)^8
+ 24*(33*cosh(x)^5 + 30*cosh(x)^3 + 5*cosh(x))*sinh(x)^7 + 4*(231*cosh(x)^6
 + 315*cosh(x)^4 + 105*cosh(x)^2 + 5)*sinh(x)^6 + 20*cosh(x)^6 + 24*(33*cos
h(x)^7 + 63*cosh(x)^5 + 35*cosh(x)^3 + 5*cosh(x))*sinh(x)^5 + 15*(33*cosh(x
```

```
)^8 + 84*cosh(x)^6 + 70*cosh(x)^4 + 20*cosh(x)^2 + 1)*sinh(x)^4 + 15*cosh(x
)^4 + 20*(11*cosh(x)^9 + 36*cosh(x)^7 + 42*cosh(x)^5 + 20*cosh(x)^3 + 3*cos
h(x))*sinh(x)^3 + 6*(11*cosh(x)^10 + 45*cosh(x)^8 + 70*cosh(x)^6 + 50*cosh(
x)^4 + 15*cosh(x)^2 + 1)*sinh(x)^2 + 6*cosh(x)^2 + 12*(cosh(x)^11 + 5*cosh(
x)^9 + 10*cosh(x)^7 + 10*cosh(x)^5 + 5*cosh(x)^3 + cosh(x))*sinh(x) + 1)*ar
ctan(cosh(x) + sinh(x)) + 3*(11*cosh(x)^10 - 141*cosh(x)^8 + 182*cosh(x)^6
- 130*cosh(x)^4 + 47*cosh(x)^2 - 1)*sinh(x) - 3*cosh(x))/(cosh(x)^12 + 12*c
osh(x)*sinh(x)^11 + sinh(x)^12 + 6*(11*cosh(x)^2 + 1)*sinh(x)^10 + 6*cosh(x
)^10 + 20*(11*cosh(x)^3 + 3*cosh(x))*sinh(x)^9 + 15*(33*cosh(x)^4 + 18*cosh
(x)^2 + 1)*sinh(x)^8 + 15*cosh(x)^8 + 24*(33*cosh(x)^5 + 30*cosh(x)^3 + 5*c
osh(x))*sinh(x)^7 + 4*(231*cosh(x)^6 + 315*cosh(x)^4 + 105*cosh(x)^2 + 5)*s
inh(x)^6 + 20*cosh(x)^6 + 24*(33*cosh(x)^7 + 63*cosh(x)^5 + 35*cosh(x)^3 +
5*cosh(x))*sinh(x)^5 + 15*(33*cosh(x)^8 + 84*cosh(x)^6 + 70*cosh(x)^4 + 20*
cosh(x)^2 + 1)*sinh(x)^4 + 15*cosh(x)^4 + 20*(11*cosh(x)^9 + 36*cosh(x)^7 +
 42*cosh(x)^5 + 20*cosh(x)^3 + 3*cosh(x))*sinh(x)^3 + 6*(11*cosh(x)^10 + 45
*cosh(x)^8 + 70*cosh(x)^6 + 50*cosh(x)^4 + 15*cosh(x)^2 + 1)*sinh(x)^2 + 6*
cosh(x)^2 + 12*(cosh(x)^11 + 5*cosh(x)^9 + 10*cosh(x)^7 + 10*cosh(x)^5 + 5*
cosh(x)^3 + cosh(x))*sinh(x) + 1)
```

giac [B] time = 0.12, size = 73, normalized size = 1.92

$$\frac{1}{32}\,\pi - \frac{3\left(e^{(-x)} - e^{x}\right)^{5} - 32\left(e^{(-x)} - e^{x}\right)^{3} - 48\,e^{(-x)} + 48\,e^{x}}{24\left(\left(e^{(-x)} - e^{x}\right)^{2} + 4\right)^{3}} + \frac{1}{16}\,\arctan\left(\frac{1}{2}\left(e^{(2x)} - 1\right)e^{(-x)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^3*tanh(x)^4,x, algorithm="giac")
```

```
[Out] 1/32*pi - 1/24*(3*(e^(-x) - e^x)^5 - 32*(e^(-x) - e^x)^3 - 48*e^(-x) + 48*e
^x)/((e^(-x) - e^x)^2 + 4)^3 + 1/16*arctan(1/2*(e^(2*x) - 1)*e^(-x))
```

maple [A] time = 0.33, size = 46, normalized size = 1.21

$$-\frac{\sinh^{3}(x)}{3\cosh(x)^{6}} - \frac{\sinh(x)}{5\cosh(x)^{6}} + \frac{\left(\frac{\mathrm{sech}(x)^{5}}{6} + \frac{5\mathrm{sech}(x)^{3}}{24} + \frac{5\,\mathrm{sech}(x)}{16}\right)\tanh(x)}{5} + \frac{\arctan\left(\mathrm{e}^{x}\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(x)^3*tanh(x)^4,x)
```

```
[Out] -1/3*sinh(x)^3/cosh(x)^6-1/5*sinh(x)/cosh(x)^6+1/5*(1/6*sech(x)^5+5/24*sech
(x)^3+5/16*sech(x))*tanh(x)+1/8*arctan(exp(x))
```

maxima [B] time = 0.53, size = 85, normalized size = 2.24

$$\frac{3\,e^{(-x)} - 47\,e^{(-3\,x)} + 78\,e^{(-5\,x)} - 78\,e^{(-7\,x)} + 47\,e^{(-9\,x)} - 3\,e^{(-11\,x)}}{24\left(6\,e^{(-2\,x)} + 15\,e^{(-4\,x)} + 20\,e^{(-6\,x)} + 15\,e^{(-8\,x)} + 6\,e^{(-10\,x)} + e^{(-12\,x)} + 1\right)} - \frac{1}{8}\arctan\left(e^{(-x)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^3*tanh(x)^4,x, algorithm="maxima")
```

```
[Out] 1/24*(3*e^(-x) - 47*e^(-3*x) + 78*e^(-5*x) - 78*e^(-7*x) + 47*e^(-9*x) - 3*
e^(-11*x))/(6*e^(-2*x) + 15*e^(-4*x) + 20*e^(-6*x) + 15*e^(-8*x) + 6*e^(-10
*x) + e^(-12*x) + 1) - 1/8*arctan(e^(-x))
```

mupad [B] time = 0.06, size = 200, normalized size = 5.26

$$\frac{\operatorname{atan}\left(e^{x}\right)}{8}-\frac{10\,e^{x}}{4\,e^{2\,x}+6\,e^{4\,x}+4\,e^{6\,x}+e^{8\,x}+1}+\frac{e^{x}}{8\,\left(e^{2\,x}+1\right)}+\frac{7\,e^{x}}{3\,e^{2\,x}+3\,e^{4\,x}+e^{6\,x}+1}-\frac{4\,e^{5\,x}-\frac{8\,e^{3\,x}}{3}-\frac{8\,e^{7\,x}}{3}}{6\,e^{2\,x}+15\,e^{4\,x}+20\,e^{6\,x}+15\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^4/cosh(x)^3,x)
```

```
[Out] atan(exp(x))/8 - (10*exp(x))/(4*exp(2*x) + 6*exp(4*x) + 4*exp(6*x) + exp(8*
x) + 1) + exp(x)/(8*(exp(2*x) + 1)) + (7*exp(x))/(3*exp(2*x) + 3*exp(4*x) +
 exp(6*x) + 1) - (4*exp(5*x) - (8*exp(3*x))/3 - (8*exp(7*x))/3 + (2*exp(9*x
))/3 + (2*exp(x))/3)/(6*exp(2*x) + 15*exp(4*x) + 20*exp(6*x) + 15*exp(8*x)
+ 6*exp(10*x) + exp(12*x) + 1) + (16*exp(x))/(3*(5*exp(2*x) + 10*exp(4*x) +
 10*exp(6*x) + 5*exp(8*x) + exp(10*x) + 1)) - (23*exp(x))/(12*(2*exp(2*x) +
 exp(4*x) + 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \tanh^{4}\left(x\right)\operatorname{sech}^{3}\left(x\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)**3*tanh(x)**4,x)
```

```
[Out] Integral(tanh(x)**4*sech(x)**3, x)
```

3.99 $\int \mathrm{sech}^5(x) \tanh^2(x) \, dx$

Optimal. Leaf size=36

$$\frac{1}{16} \tan^{-1}(\sinh(x)) - \frac{1}{6} \tanh(x)\mathrm{sech}^5(x) + \frac{1}{24} \tanh(x)\mathrm{sech}^3(x) + \frac{1}{16} \tanh(x)\mathrm{sech}(x)$$

```
[Out] 1/16*arctan(sinh(x))+1/16*sech(x)*tanh(x)+1/24*sech(x)^3*tanh(x)-1/6*sech(x
)^5*tanh(x)
```

Rubi [A] time = 0.04, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2611, 3768, 3770}

$$\frac{1}{16} \tan^{-1}(\sinh(x)) - \frac{1}{6} \tanh(x)\mathrm{sech}^5(x) + \frac{1}{24} \tanh(x)\mathrm{sech}^3(x) + \frac{1}{16} \tanh(x)\mathrm{sech}(x)$$

Antiderivative was successfully verified.

```
[In] Int[Sech[x]^5*Tanh[x]^2,x]
```

```
[Out] ArcTan[Sinh[x]]/16 + (Sech[x]*Tanh[x])/16 + (Sech[x]^3*Tanh[x])/24 - (Sech[
x]^5*Tanh[x])/6
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \operatorname{sech}^5(x)\tanh^2(x)\,dx &= -\frac{1}{6}\operatorname{sech}^5(x)\tanh(x) + \frac{1}{6}\int \operatorname{sech}^5(x)\,dx \\ &= \frac{1}{24}\operatorname{sech}^3(x)\tanh(x) - \frac{1}{6}\operatorname{sech}^5(x)\tanh(x) + \frac{1}{8}\int \operatorname{sech}^3(x)\,dx \\ &= \frac{1}{16}\operatorname{sech}(x)\tanh(x) + \frac{1}{24}\operatorname{sech}^3(x)\tanh(x) - \frac{1}{6}\operatorname{sech}^5(x)\tanh(x) + \frac{1}{16}\int \operatorname{sech}(x)\,dx \\ &= \frac{1}{16}\tan^{-1}(\sinh(x)) + \frac{1}{16}\operatorname{sech}(x)\tanh(x) + \frac{1}{24}\operatorname{sech}^3(x)\tanh(x) - \frac{1}{6}\operatorname{sech}^5(x)\tanh(x)\end{aligned}$$

Mathematica [A] time = 0.01, size = 36, normalized size = 1.00

$$\frac{1}{16}\tan^{-1}(\sinh(x)) - \frac{1}{6}\tanh(x)\operatorname{sech}^5(x) + \frac{1}{24}\tanh(x)\operatorname{sech}^3(x) + \frac{1}{16}\tanh(x)\operatorname{sech}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[x]^5*Tanh[x]^2,x]
```

```
[Out] ArcTan[Sinh[x]]/16 + (Sech[x]*Tanh[x])/16 + (Sech[x]^3*Tanh[x])/24 - (Sech[
x]^5*Tanh[x])/6
```

fricas [B] time = 0.41, size = 925, normalized size = 25.69

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^5*tanh(x)^2,x, algorithm="fricas")
```

```
[Out] 1/24*(3*cosh(x)^11 + 33*cosh(x)*sinh(x)^10 + 3*sinh(x)^11 + (165*cosh(x)^2
+ 17)*sinh(x)^9 + 17*cosh(x)^9 + 9*(55*cosh(x)^3 + 17*cosh(x))*sinh(x)^8 +
6*(165*cosh(x)^4 + 102*cosh(x)^2 - 19)*sinh(x)^7 - 114*cosh(x)^7 + 42*(33*c
osh(x)^5 + 34*cosh(x)^3 - 19*cosh(x))*sinh(x)^6 + 6*(231*cosh(x)^6 + 357*co
sh(x)^4 - 399*cosh(x)^2 + 19)*sinh(x)^5 + 114*cosh(x)^5 + 6*(165*cosh(x)^7
+ 357*cosh(x)^5 - 665*cosh(x)^3 + 95*cosh(x))*sinh(x)^4 + (495*cosh(x)^8 +
1428*cosh(x)^6 - 3990*cosh(x)^4 + 1140*cosh(x)^2 - 17)*sinh(x)^3 - 17*cosh(
x)^3 + 3*(55*cosh(x)^9 + 204*cosh(x)^7 - 798*cosh(x)^5 + 380*cosh(x)^3 - 17
*cosh(x))*sinh(x)^2 + 3*(cosh(x)^12 + 12*cosh(x)*sinh(x)^11 + sinh(x)^12 +
6*(11*cosh(x)^2 + 1)*sinh(x)^10 + 6*cosh(x)^10 + 20*(11*cosh(x)^3 + 3*cosh(
x))*sinh(x)^9 + 15*(33*cosh(x)^4 + 18*cosh(x)^2 + 1)*sinh(x)^8 + 15*cosh(x)
^8 + 24*(33*cosh(x)^5 + 30*cosh(x)^3 + 5*cosh(x))*sinh(x)^7 + 4*(231*cosh(x
)^6 + 315*cosh(x)^4 + 105*cosh(x)^2 + 5)*sinh(x)^6 + 20*cosh(x)^6 + 24*(33*
cosh(x)^7 + 63*cosh(x)^5 + 35*cosh(x)^3 + 5*cosh(x))*sinh(x)^5 + 15*(33*cos
```

```
h(x)^8 + 84*cosh(x)^6 + 70*cosh(x)^4 + 20*cosh(x)^2 + 1)*sinh(x)^4 + 15*cos
h(x)^4 + 20*(11*cosh(x)^9 + 36*cosh(x)^7 + 42*cosh(x)^5 + 20*cosh(x)^3 + 3*
cosh(x))*sinh(x)^3 + 6*(11*cosh(x)^10 + 45*cosh(x)^8 + 70*cosh(x)^6 + 50*co
sh(x)^4 + 15*cosh(x)^2 + 1)*sinh(x)^2 + 6*cosh(x)^2 + 12*(cosh(x)^11 + 5*co
sh(x)^9 + 10*cosh(x)^7 + 10*cosh(x)^5 + 5*cosh(x)^3 + cosh(x))*sinh(x) + 1)
*arctan(cosh(x) + sinh(x)) + 3*(11*cosh(x)^10 + 51*cosh(x)^8 - 266*cosh(x)^
6 + 190*cosh(x)^4 - 17*cosh(x)^2 - 1)*sinh(x) - 3*cosh(x))/(cosh(x)^12 + 12
*cosh(x)*sinh(x)^11 + sinh(x)^12 + 6*(11*cosh(x)^2 + 1)*sinh(x)^10 + 6*cosh
(x)^10 + 20*(11*cosh(x)^3 + 3*cosh(x))*sinh(x)^9 + 15*(33*cosh(x)^4 + 18*co
sh(x)^2 + 1)*sinh(x)^8 + 15*cosh(x)^8 + 24*(33*cosh(x)^5 + 30*cosh(x)^3 + 5
*cosh(x))*sinh(x)^7 + 4*(231*cosh(x)^6 + 315*cosh(x)^4 + 105*cosh(x)^2 + 5)
*sinh(x)^6 + 20*cosh(x)^6 + 24*(33*cosh(x)^7 + 63*cosh(x)^5 + 35*cosh(x)^3
+ 5*cosh(x))*sinh(x)^5 + 15*(33*cosh(x)^8 + 84*cosh(x)^6 + 70*cosh(x)^4 + 2
0*cosh(x)^2 + 1)*sinh(x)^4 + 15*cosh(x)^4 + 20*(11*cosh(x)^9 + 36*cosh(x)^7
 + 42*cosh(x)^5 + 20*cosh(x)^3 + 3*cosh(x))*sinh(x)^3 + 6*(11*cosh(x)^10 +
45*cosh(x)^8 + 70*cosh(x)^6 + 50*cosh(x)^4 + 15*cosh(x)^2 + 1)*sinh(x)^2 +
6*cosh(x)^2 + 12*(cosh(x)^11 + 5*cosh(x)^9 + 10*cosh(x)^7 + 10*cosh(x)^5 +
5*cosh(x)^3 + cosh(x))*sinh(x) + 1)
```

giac [B] time = 0.13, size = 73, normalized size = 2.03

$$\frac{1}{32}\,\pi - \frac{3\,{\left(e^{(-x)} - e^{x}\right)}^{5} + 32\,{\left(e^{(-x)} - e^{x}\right)}^{3} - 48\,e^{(-x)} + 48\,e^{x}}{24\,{\left({\left(e^{(-x)} - e^{x}\right)}^{2} + 4\right)}^{3}} + \frac{1}{16}\,\arctan\left(\frac{1}{2}\,{\left(e^{(2\,x)} - 1\right)}e^{(-x)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^5*tanh(x)^2,x, algorithm="giac")
```

```
[Out] 1/32*pi - 1/24*(3*(e^(-x) - e^x)^5 + 32*(e^(-x) - e^x)^3 - 48*e^(-x) + 48*e
^x)/((e^(-x) - e^x)^2 + 4)^3 + 1/16*arctan(1/2*(e^(2*x) - 1)*e^(-x))
```

maple [A] time = 0.33, size = 36, normalized size = 1.00

$$-\frac{\sinh(x)}{5\cosh(x)^{6}} + \frac{\left(\frac{\mathrm{sech}(x)^{5}}{6} + \frac{5\mathrm{sech}(x)^{3}}{24} + \frac{5\,\mathrm{sech}(x)}{16}\right)\tanh(x)}{5} + \frac{\arctan\left(\mathrm{e}^{x}\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(x)^5*tanh(x)^2,x)
```

```
[Out] -1/5*sinh(x)/cosh(x)^6+1/5*(1/6*sech(x)^5+5/24*sech(x)^3+5/16*sech(x))*tanh
(x)+1/8*arctan(exp(x))
```

maxima [B] time = 0.44, size = 85, normalized size = 2.36

$$\frac{3\,e^{(-x)}+17\,e^{(-3\,x)}-114\,e^{(-5\,x)}+114\,e^{(-7\,x)}-17\,e^{(-9\,x)}-3\,e^{(-11\,x)}}{24\left(6\,e^{(-2\,x)}+15\,e^{(-4\,x)}+20\,e^{(-6\,x)}+15\,e^{(-8\,x)}+6\,e^{(-10\,x)}+e^{(-12\,x)}+1\right)}-\frac{1}{8}\,\arctan\left(e^{(-x)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^5*tanh(x)^2,x, algorithm="maxima")
```

```
[Out] 1/24*(3*e^(-x) + 17*e^(-3*x) - 114*e^(-5*x) + 114*e^(-7*x) - 17*e^(-9*x) -
3*e^(-11*x))/(6*e^(-2*x) + 15*e^(-4*x) + 20*e^(-6*x) + 15*e^(-8*x) + 6*e^(-
10*x) + e^(-12*x) + 1) - 1/8*arctan(e^(-x))
```

mupad [B] time = 1.44, size = 206, normalized size = 5.72

$$\frac{\mathrm{atan}\left(\mathrm{e}^x\right)}{8}+\frac{34\,\mathrm{e}^x}{15\,\left(4\,\mathrm{e}^{2\,x}+6\,\mathrm{e}^{4\,x}+4\,\mathrm{e}^{6\,x}+\mathrm{e}^{8\,x}+1\right)}+\frac{\mathrm{e}^x}{8\,\left(\mathrm{e}^{2\,x}+1\right)}-\frac{9\,\mathrm{e}^x}{5\,\left(3\,\mathrm{e}^{2\,x}+3\,\mathrm{e}^{4\,x}+\mathrm{e}^{6\,x}+1\right)}-\frac{\frac{8\,\mathrm{e}^{3\,x}}{3}}{6\,\mathrm{e}^{2\,x}+15\,\mathrm{e}^{4\,x}+20\,\mathrm{e}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^2/cosh(x)^5,x)
```

```
[Out] atan(exp(x))/8 + (34*exp(x))/(15*(4*exp(2*x) + 6*exp(4*x) + 4*exp(6*x) + ex
p(8*x) + 1)) + exp(x)/(8*(exp(2*x) + 1)) - (9*exp(x))/(5*(3*exp(2*x) + 3*ex
p(4*x) + exp(6*x) + 1)) - ((8*exp(3*x))/3 - (16*exp(5*x))/3 + (8*exp(7*x))/
3)/(6*exp(2*x) + 15*exp(4*x) + 20*exp(6*x) + 15*exp(8*x) + 6*exp(10*x) + ex
p(12*x) + 1) - ((28*exp(5*x))/15 - (8*exp(3*x))/3 + (4*exp(x))/5)/(5*exp(2*
x) + 10*exp(4*x) + 10*exp(6*x) + 5*exp(8*x) + exp(10*x) + 1) + exp(x)/(12*(
2*exp(2*x) + exp(4*x) + 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \tanh^2(x)\,\mathrm{sech}^5(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)**5*tanh(x)**2,x)
```

```
[Out] Integral(tanh(x)**2*sech(x)**5, x)
```

3.100 $\int \mathbf{sech}^8(x) \tanh^6(x)\, dx$

Optimal. Leaf size=33

$$-\frac{1}{13}\tanh^{13}(x)+\frac{3\tanh^{11}(x)}{11}-\frac{\tanh^9(x)}{3}+\frac{\tanh^7(x)}{7}$$

```
[Out] 1/7*tanh(x)^7-1/3*tanh(x)^9+3/11*tanh(x)^11-1/13*tanh(x)^13
```

Rubi [A] time = 0.03, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2607, 270}

$$-\frac{1}{13}\tanh^{13}(x)+\frac{3\tanh^{11}(x)}{11}-\frac{\tanh^9(x)}{3}+\frac{\tanh^7(x)}{7}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[x]^8*Tanh[x]^6,x]
```

```
[Out] Tanh[x]^7/7 - Tanh[x]^9/3 + (3*Tanh[x]^11)/11 - Tanh[x]^13/13
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rubi steps

$$\begin{aligned}
\int \operatorname{sech}^8(x)\tanh^6(x)\,dx &= i\,\text{Subst}\left(\int x^6\left(1+x^2\right)^3\,dx,x,i\tanh(x)\right)\\
&= i\,\text{Subst}\left(\int \left(x^6+3x^8+3x^{10}+x^{12}\right)dx,x,i\tanh(x)\right)\\
&= \frac{\tanh^7(x)}{7}-\frac{\tanh^9(x)}{3}+\frac{3\tanh^{11}(x)}{11}-\frac{\tanh^{13}(x)}{13}
\end{aligned}$$

Mathematica [B] time = 0.03, size = 67, normalized size = 2.03

$$\frac{16\tanh(x)}{3003}-\frac{1}{13}\tanh(x)\text{sech}^{12}(x)+\frac{27}{143}\tanh(x)\text{sech}^{10}(x)-\frac{53}{429}\tanh(x)\text{sech}^{8}(x)+\frac{5\tanh(x)\text{sech}^{6}(x)}{3003}+\frac{2\tanh(x)\text{s}}{100}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[x]^8*Tanh[x]^6,x]
```

```
[Out] (16*Tanh[x])/3003 + (8*Sech[x]^2*Tanh[x])/3003 + (2*Sech[x]^4*Tanh[x])/1001
 + (5*Sech[x]^6*Tanh[x])/3003 - (53*Sech[x]^8*Tanh[x])/429 + (27*Sech[x]^10
*Tanh[x])/143 - (Sech[x]^12*Tanh[x])/13
```

fricas [B] time = 0.42, size = 778, normalized size = 23.58

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^8*tanh(x)^6,x, algorithm="fricas")
```

```
[Out] -64/3003*(1502*cosh(x)^9 + 13518*cosh(x)*sinh(x)^8 + 1501*sinh(x)^9 + (5403
6*cosh(x)^2 - 4511)*sinh(x)^7 - 4498*cosh(x)^7 + 14*(9012*cosh(x)^3 - 2249*
cosh(x))*sinh(x)^6 + 3*(63042*cosh(x)^4 - 31577*cosh(x)^2 + 2990)*sinh(x)^5
 + 9048*cosh(x)^5 + 2*(94626*cosh(x)^5 - 78715*cosh(x)^3 + 22620*cosh(x))*s
inh(x)^4 + (126084*cosh(x)^6 - 157885*cosh(x)^4 + 89700*cosh(x)^2 - 8294)*s
inh(x)^3 - 8008*cosh(x)^3 + 6*(9012*cosh(x)^7 - 15743*cosh(x)^5 + 15080*cos
h(x)^3 - 4004*cosh(x))*sinh(x)^2 + (13509*cosh(x)^8 - 31577*cosh(x)^6 + 448
50*cosh(x)^4 - 24882*cosh(x)^2 + 6292)*sinh(x) + 4004*cosh(x))/(cosh(x)^17
+ 17*cosh(x)*sinh(x)^16 + sinh(x)^17 + (136*cosh(x)^2 + 13)*sinh(x)^15 + 13
*cosh(x)^15 + 5*(136*cosh(x)^3 + 39*cosh(x))*sinh(x)^14 + (2380*cosh(x)^4 +
 1365*cosh(x)^2 + 78)*sinh(x)^13 + 78*cosh(x)^13 + 13*(476*cosh(x)^5 + 455*
cosh(x)^3 + 78*cosh(x))*sinh(x)^12 + 13*(952*cosh(x)^6 + 1365*cosh(x)^4 + 4
68*cosh(x)^2 + 22)*sinh(x)^11 + 286*cosh(x)^11 + 143*(136*cosh(x)^7 + 273*c
osh(x)^5 + 156*cosh(x)^3 + 22*cosh(x))*sinh(x)^10 + (24310*cosh(x)^8 + 6506
5*cosh(x)^6 + 55770*cosh(x)^4 + 15730*cosh(x)^2 + 714)*sinh(x)^9 + 716*cosh
(x)^9 + (24310*cosh(x)^9 + 83655*cosh(x)^7 + 100386*cosh(x)^5 + 47190*cosh(
x)^3 + 6444*cosh(x))*sinh(x)^8 + (19448*cosh(x)^10 + 83655*cosh(x)^8 + 1338
48*cosh(x)^6 + 94380*cosh(x)^4 + 25704*cosh(x)^2 + 1274)*sinh(x)^7 + 1300*c
osh(x)^7 + (12376*cosh(x)^11 + 65065*cosh(x)^9 + 133848*cosh(x)^7 + 132132*
cosh(x)^5 + 60144*cosh(x)^3 + 9100*cosh(x))*sinh(x)^6 + (6188*cosh(x)^12 +
39039*cosh(x)^10 + 100386*cosh(x)^8 + 132132*cosh(x)^6 + 89964*cosh(x)^4 +
26754*cosh(x)^2 + 1638)*sinh(x)^5 + 1794*cosh(x)^5 + (2380*cosh(x)^13 + 177
45*cosh(x)^11 + 55770*cosh(x)^9 + 94380*cosh(x)^7 + 90216*cosh(x)^5 + 45500
*cosh(x)^3 + 8970*cosh(x))*sinh(x)^4 + (680*cosh(x)^14 + 5915*cosh(x)^12 +
22308*cosh(x)^10 + 47190*cosh(x)^8 + 59976*cosh(x)^6 + 44590*cosh(x)^4 + 16
```

```
380*cosh(x)^2 + 1430)*sinh(x)^3 + 2002*cosh(x)^3 + (136*cosh(x)^15 + 1365*c
osh(x)^13 + 6084*cosh(x)^11 + 15730*cosh(x)^9 + 25776*cosh(x)^7 + 27300*cos
h(x)^5 + 17940*cosh(x)^3 + 6006*cosh(x))*sinh(x)^2 + (17*cosh(x)^16 + 195*c
osh(x)^14 + 1014*cosh(x)^12 + 3146*cosh(x)^10 + 6426*cosh(x)^8 + 8918*cosh(
x)^6 + 8190*cosh(x)^4 + 4290*cosh(x)^2 + 572)*sinh(x) + 2002*cosh(x))
```

giac [B] time = 0.12, size = 66, normalized size = 2.00

$$-\frac{32\left(3003\,e^{(18\,x)}-9009\,e^{(16\,x)}+18018\,e^{(14\,x)}-16302\,e^{(12\,x)}+10296\,e^{(10\,x)}-2288\,e^{(8\,x)}+286\,e^{(6\,x)}+78\,e^{(4\,x)}+13\right.}{3003\left(e^{(2\,x)}+1\right)^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^8*tanh(x)^6,x, algorithm="giac")
```

```
[Out] -32/3003*(3003*e^(18*x) - 9009*e^(16*x) + 18018*e^(14*x) - 16302*e^(12*x) +
 10296*e^(10*x) - 2288*e^(8*x) + 286*e^(6*x) + 78*e^(4*x) + 13*e^(2*x) + 1)
/(e^(2*x) + 1)^13
```

maple [B] time = 0.30, size = 72, normalized size = 2.18

$$-\frac{\sinh^{5}(x)}{8\cosh(x)^{13}}-\frac{\sinh^{3}(x)}{16\cosh(x)^{13}}-\frac{\sinh(x)}{64\cosh(x)^{13}}+\frac{\left(\frac{1024}{3003}+\frac{\mathrm{sech}(x)^{12}}{13}+\frac{12\mathrm{sech}(x)^{10}}{143}+\frac{40\mathrm{sech}(x)^{8}}{429}+\frac{320\mathrm{sech}(x)^{6}}{3003}+\frac{128\mathrm{sech}(x)^{4}}{1001}+\right.}{64}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(x)^8*tanh(x)^6,x)
```

```
[Out] -1/8*sinh(x)^5/cosh(x)^13-1/16*sinh(x)^3/cosh(x)^13-1/64*sinh(x)/cosh(x)^13
+1/64*(1024/3003+1/13*sech(x)^12+12/143*sech(x)^10+40/429*sech(x)^8+320/300
3*sech(x)^6+128/1001*sech(x)^4+512/3003*sech(x)^2)*tanh(x)
```

maxima [B] time = 0.38, size = 857, normalized size = 25.97

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^8*tanh(x)^6,x, algorithm="maxima")
```

```
[Out] 32/231*e^(-2*x)/(13*e^(-2*x) + 78*e^(-4*x) + 286*e^(-6*x) + 715*e^(-8*x) +
1287*e^(-10*x) + 1716*e^(-12*x) + 1716*e^(-14*x) + 1287*e^(-16*x) + 715*e^(
-18*x) + 286*e^(-20*x) + 78*e^(-22*x) + 13*e^(-24*x) + e^(-26*x) + 1) + 64/
77*e^(-4*x)/(13*e^(-2*x) + 78*e^(-4*x) + 286*e^(-6*x) + 715*e^(-8*x) + 1287
*e^(-10*x) + 1716*e^(-12*x) + 1716*e^(-14*x) + 1287*e^(-16*x) + 715*e^(-18*
```

```
x) + 286*e^(-20*x) + 78*e^(-22*x) + 13*e^(-24*x) + e^(-26*x) + 1) + 64/21*e
^(-6*x)/(13*e^(-2*x) + 78*e^(-4*x) + 286*e^(-6*x) + 715*e^(-8*x) + 1287*e^(
-10*x) + 1716*e^(-12*x) + 1716*e^(-14*x) + 1287*e^(-16*x) + 715*e^(-18*x) +
 286*e^(-20*x) + 78*e^(-22*x) + 13*e^(-24*x) + e^(-26*x) + 1) - 512/21*e^(-
8*x)/(13*e^(-2*x) + 78*e^(-4*x) + 286*e^(-6*x) + 715*e^(-8*x) + 1287*e^(-10
*x) + 1716*e^(-12*x) + 1716*e^(-14*x) + 1287*e^(-16*x) + 715*e^(-18*x) + 28
6*e^(-20*x) + 78*e^(-22*x) + 13*e^(-24*x) + e^(-26*x) + 1) + 768/7*e^(-10*x
)/(13*e^(-2*x) + 78*e^(-4*x) + 286*e^(-6*x) + 715*e^(-8*x) + 1287*e^(-10*x)
 + 1716*e^(-12*x) + 1716*e^(-14*x) + 1287*e^(-16*x) + 715*e^(-18*x) + 286*e
^(-20*x) + 78*e^(-22*x) + 13*e^(-24*x) + e^(-26*x) + 1) - 1216/7*e^(-12*x)/
(13*e^(-2*x) + 78*e^(-4*x) + 286*e^(-6*x) + 715*e^(-8*x) + 1287*e^(-10*x) +
 1716*e^(-12*x) + 1716*e^(-14*x) + 1287*e^(-16*x) + 715*e^(-18*x) + 286*e^(
-20*x) + 78*e^(-22*x) + 13*e^(-24*x) + e^(-26*x) + 1) + 192*e^(-14*x)/(13*e
^(-2*x) + 78*e^(-4*x) + 286*e^(-6*x) + 715*e^(-8*x) + 1287*e^(-10*x) + 1716
*e^(-12*x) + 1716*e^(-14*x) + 1287*e^(-16*x) + 715*e^(-18*x) + 286*e^(-20*x
) + 78*e^(-22*x) + 13*e^(-24*x) + e^(-26*x) + 1) - 96*e^(-16*x)/(13*e^(-2*x
) + 78*e^(-4*x) + 286*e^(-6*x) + 715*e^(-8*x) + 1287*e^(-10*x) + 1716*e^(-1
2*x) + 1716*e^(-14*x) + 1287*e^(-16*x) + 715*e^(-18*x) + 286*e^(-20*x) + 78
*e^(-22*x) + 13*e^(-24*x) + e^(-26*x) + 1) + 32*e^(-18*x)/(13*e^(-2*x) + 78
*e^(-4*x) + 286*e^(-6*x) + 715*e^(-8*x) + 1287*e^(-10*x) + 1716*e^(-12*x) +
 1716*e^(-14*x) + 1287*e^(-16*x) + 715*e^(-18*x) + 286*e^(-20*x) + 78*e^(-2
2*x) + 13*e^(-24*x) + e^(-26*x) + 1) + 32/3003/(13*e^(-2*x) + 78*e^(-4*x) +
 286*e^(-6*x) + 715*e^(-8*x) + 1287*e^(-10*x) + 1716*e^(-12*x) + 1716*e^(-1
4*x) + 1287*e^(-16*x) + 715*e^(-18*x) + 286*e^(-20*x) + 78*e^(-22*x) + 13*e
^(-24*x) + e^(-26*x) + 1)
```

mupad [B] time = 1.45, size = 820, normalized size = 24.85

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^6/cosh(x)^8,x)
```

```
[Out] - ((64*exp(4*x))/143 - (256*exp(2*x))/429 + 80/429)/(6*exp(2*x) + 15*exp(4*
x) + 20*exp(6*x) + 15*exp(8*x) + 6*exp(10*x) + exp(12*x) + 1) - ((64*exp(2*
x))/143 - (768*exp(4*x))/143 + (3200*exp(6*x))/143 - (6400*exp(8*x))/143 +
(6720*exp(10*x))/143 - (3584*exp(12*x))/143 + (768*exp(14*x))/143)/(11*exp(
2*x) + 55*exp(4*x) + 165*exp(6*x) + 330*exp(8*x) + 462*exp(10*x) + 462*exp(
12*x) + 330*exp(14*x) + 165*exp(16*x) + 55*exp(18*x) + 11*exp(20*x) + exp(2
2*x) + 1) - ((160*exp(2*x))/143 - (256*exp(4*x))/143 + (128*exp(6*x))/143 -
 640/3003)/(7*exp(2*x) + 21*exp(4*x) + 35*exp(6*x) + 35*exp(8*x) + 21*exp(1
0*x) + 7*exp(12*x) + exp(14*x) + 1) - ((128*exp(6*x))/13 - (768*exp(8*x))/1
3 + (1920*exp(10*x))/13 - (2560*exp(12*x))/13 + (1920*exp(14*x))/13 - (768*
exp(16*x))/13 + (128*exp(18*x))/13)/(13*exp(2*x) + 78*exp(4*x) + 286*exp(6*
x) + 715*exp(8*x) + 1287*exp(10*x) + 1716*exp(12*x) + 1716*exp(14*x) + 1287
```

```
*exp(16*x) + 715*exp(18*x) + 286*exp(20*x) + 78*exp(22*x) + 13*exp(24*x) +
exp(26*x) + 1) - ((560*exp(4*x))/143 - (640*exp(2*x))/429 - (1792*exp(6*x))
/429 + (224*exp(8*x))/143 + 80/429)/(8*exp(2*x) + 28*exp(4*x) + 56*exp(6*x)
 + 70*exp(8*x) + 56*exp(10*x) + 28*exp(12*x) + 8*exp(14*x) + exp(16*x) + 1)
 - ((640*exp(2*x))/429 - (2560*exp(4*x))/429 + (4480*exp(6*x))/429 - (3584*
exp(8*x))/429 + (1792*exp(10*x))/715 - 256/2145)/(9*exp(2*x) + 36*exp(4*x)
+ 84*exp(6*x) + 126*exp(8*x) + 126*exp(10*x) + 84*exp(12*x) + 36*exp(14*x)
+ 9*exp(16*x) + exp(18*x) + 1) - ((32*exp(4*x))/13 - (256*exp(6*x))/13 + (8
00*exp(8*x))/13 - (1280*exp(10*x))/13 + (1120*exp(12*x))/13 - (512*exp(14*x
))/13 + (96*exp(16*x))/13)/(12*exp(2*x) + 66*exp(4*x) + 220*exp(6*x) + 495*
exp(8*x) + 792*exp(10*x) + 924*exp(12*x) + 792*exp(14*x) + 495*exp(16*x) +
220*exp(18*x) + 66*exp(20*x) + 12*exp(22*x) + exp(24*x) + 1) - ((128*exp(2*
x))/715 - 256/2145)/(5*exp(2*x) + 10*exp(4*x) + 10*exp(6*x) + 5*exp(8*x) +
exp(10*x) + 1) - 32/(715*(4*exp(2*x) + 6*exp(4*x) + 4*exp(6*x) + exp(8*x) +
 1)) - ((960*exp(4*x))/143 - (768*exp(2*x))/715 - (2560*exp(6*x))/143 + (33
60*exp(8*x))/143 - (10752*exp(10*x))/715 + (2688*exp(12*x))/715 + 32/715)/(
10*exp(2*x) + 45*exp(4*x) + 120*exp(6*x) + 210*exp(8*x) + 252*exp(10*x) + 2
10*exp(12*x) + 120*exp(14*x) + 45*exp(16*x) + 10*exp(18*x) + exp(20*x) + 1)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \tanh^6(x)\operatorname{sech}^8(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)**8*tanh(x)**6,x)
```

```
[Out] Integral(tanh(x)**6*sech(x)**8, x)
```

3.101 $\int \cosh(a+bx)\coth(a+bx)\,dx$

Optimal. Leaf size=23

$$\frac{\cosh(a+bx)}{b}-\frac{\tanh^{-1}(\cosh(a+bx))}{b}$$

```
[Out] -arctanh(cosh(b*x+a))/b+cosh(b*x+a)/b
```

Rubi [A] time = 0.02, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {2592, 321, 206}

$$\frac{\cosh(a+bx)}{b}-\frac{\tanh^{-1}(\cosh(a+bx))}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[a + b*x]*Coth[a + b*x],x]
```

```
[Out] -(ArcTanh[Cosh[a + b*x]]/b) + Cosh[a + b*x]/b
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2592

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, (a*Sin[e + f*x])/ff], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rubi steps

$$\int \cosh(a+bx)\coth(a+bx)\,dx = -\frac{\text{Subst}\left(\int \frac{x^2}{1-x^2}\,dx, x, \cosh(a+bx)\right)}{b}$$
$$= \frac{\cosh(a+bx)}{b} - \frac{\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \cosh(a+bx)\right)}{b}$$
$$= -\frac{\tanh^{-1}(\cosh(a+bx))}{b} + \frac{\cosh(a+bx)}{b}$$

Mathematica [A] time = 0.03, size = 26, normalized size = 1.13

$$\frac{\cosh(a+bx)}{b} + \frac{\log\left(\tanh\left(\frac{1}{2}(a+bx)\right)\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[a + b*x]*Coth[a + b*x],x]
```

```
[Out] Cosh[a + b*x]/b + Log[Tanh[(a + b*x)/2]]/b
```

fricas [B] time = 0.51, size = 113, normalized size = 4.91

$$\frac{\cosh(bx+a)^2 - 2(\cosh(bx+a)+\sinh(bx+a))\log(\cosh(bx+a)+\sinh(bx+a)+1) + 2(\cosh(bx+a)+\text{s}}{2(b\cosh(bx+a)+b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*coth(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/2*(cosh(b*x + a)^2 - 2*(cosh(b*x + a) + sinh(b*x + a))*log(cosh(b*x + a)
+ sinh(b*x + a) + 1) + 2*(cosh(b*x + a) + sinh(b*x + a))*log(cosh(b*x + a)
+ sinh(b*x + a) - 1) + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 + 1)
/(b*cosh(b*x + a) + b*sinh(b*x + a))
```

giac [A] time = 0.14, size = 44, normalized size = 1.91

$$\frac{e^{(bx+a)} + e^{(-bx-a)} - 2\log\left(e^{(bx+a)}+1\right) + 2\log\left(\left|e^{(bx+a)}-1\right|\right)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*coth(b*x+a),x, algorithm="giac")
```

```
[Out] 1/2*(e^(b*x + a) + e^(-b*x - a) - 2*log(e^(b*x + a) + 1) + 2*log(abs(e^(b*x
 + a) - 1)))/b
```

maple [A] time = 0.11, size = 21, normalized size = 0.91

$$\frac{\cosh(bx+a) - 2\,\mathrm{arctanh}\left(\mathrm{e}^{bx+a}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)*coth(b*x+a),x)
```

```
[Out] 1/b*(cosh(b*x+a)-2*arctanh(exp(b*x+a)))
```

maxima [B] time = 0.33, size = 59, normalized size = 2.57

$$\frac{e^{(bx+a)}}{2\,b} + \frac{e^{(-bx-a)}}{2\,b} - \frac{\log\left(e^{(-bx-a)} + 1\right)}{b} + \frac{\log\left(e^{(-bx-a)} - 1\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*coth(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/2*e^(b*x + a)/b + 1/2*e^(-b*x - a)/b - log(e^(-b*x - a) + 1)/b + log(e^(-
b*x - a) - 1)/b
```

mupad [B] time = 0.07, size = 53, normalized size = 2.30

$$\frac{\mathrm{e}^{a+b\,x}}{2\,b} - \frac{2\,\mathrm{atan}\left(\frac{\mathrm{e}^{b\,x}\,\mathrm{e}^{a}\,\sqrt{-b^2}}{b}\right)}{\sqrt{-b^2}} + \frac{\mathrm{e}^{-a-b\,x}}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a + b*x)*coth(a + b*x),x)
```

```
[Out] exp(a + b*x)/(2*b) - (2*atan((exp(b*x)*exp(a)*(-b^2)^(1/2))/b))/(-b^2)^(1/2
) + exp(- a - b*x)/(2*b)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \cosh(a+bx)\coth(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*coth(b*x+a),x)
```

```
[Out] Integral(cosh(a + b*x)*coth(a + b*x), x)
```

3.102 $\int \cosh(a+bx)\coth^2(a+bx)\,dx$

Optimal. Leaf size=22

$$\frac{\sinh(a+bx)}{b}-\frac{\operatorname{csch}(a+bx)}{b}$$

```
[Out] -csch(b*x+a)/b+sinh(b*x+a)/b
```

Rubi [A] time = 0.02, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2590, 14}

$$\frac{\sinh(a+bx)}{b}-\frac{\operatorname{csch}(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[a + b*x]*Coth[a + b*x]^2,x]
```

```
[Out] -(Csch[a + b*x]/b) + Sinh[a + b*x]/b
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2590

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> -Dist[f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rubi steps

$$\begin{aligned}\int \cosh(a+bx)\coth^2(a+bx)\,dx &= -\frac{i\,\text{Subst}\left(\int \frac{1-x^2}{x^2}\,dx, x, -i\sinh(a+bx)\right)}{b}\\ &= -\frac{i\,\text{Subst}\left(\int \left(-1+\frac{1}{x^2}\right)dx, x, -i\sinh(a+bx)\right)}{b}\\ &= -\frac{\operatorname{csch}(a+bx)}{b}+\frac{\sinh(a+bx)}{b}\end{aligned}$$

Mathematica [A] time = 0.01, size = 22, normalized size = 1.00

$$\frac{\sinh(a+bx)}{b}-\frac{\operatorname{csch}(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[a + b*x]*Coth[a + b*x]^2,x]
[Out] -(Csch[a + b*x]/b) + Sinh[a + b*x]/b
```

fricas [A] time = 0.49, size = 31, normalized size = 1.41

$$\frac{\cosh\left(bx+a\right)^{2}+\sinh\left(bx+a\right)^{2}-3}{2\,b\sinh\left(bx+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*coth(b*x+a)^2,x, algorithm="fricas")
[Out] 1/2*(cosh(b*x + a)^2 + sinh(b*x + a)^2 - 3)/(b*sinh(b*x + a))
```

giac [B] time = 0.14, size = 50, normalized size = 2.27

$$-\frac{\frac{\left(5\,e^{(2\,bx+2\,a)}-1\right)e^{(-a)}}{e^{(3\,bx+2\,a)}-e^{(bx)}}-e^{(bx+a)}}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*coth(b*x+a)^2,x, algorithm="giac")
[Out] -1/2*((5*e^(2*b*x + 2*a) - 1)*e^(-a)/(e^(3*b*x + 2*a) - e^(b*x)) - e^(b*x +
 a))/b
```

maple [A] time = 0.13, size = 33, normalized size = 1.50

$$\frac{\frac{\cosh^{2}(bx+a)}{\sinh(bx+a)}-\frac{2}{\sinh(bx+a)}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)*coth(b*x+a)^2,x)
[Out] 1/b*(cosh(b*x+a)^2/sinh(b*x+a)-2/sinh(b*x+a))
```

maxima [B] time = 0.35, size = 56, normalized size = 2.55

$$-\frac{e^{(-bx-a)}}{2\,b}-\frac{5\,e^{(-2\,bx-2\,a)}-1}{2\,b\left(e^{(-bx-a)}-e^{(-3\,bx-3\,a)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*coth(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*e^(-b*x - a)/b - 1/2*(5*e^(-2*b*x - 2*a) - 1)/(b*(e^(-b*x - a) - e^(-3
*b*x - 3*a)))
```

mupad [B] time = 1.43, size = 49, normalized size = 2.23

$$\frac{e^{-a-b\,x}\,\left(e^{4\,a+4\,b\,x}-6\,e^{2\,a+2\,b\,x}+1\right)}{2\,b\,\left(e^{2\,a+2\,b\,x}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a + b*x)*coth(a + b*x)^2,x)
```

```
[Out] (exp(- a - b*x)*(exp(4*a + 4*b*x) - 6*exp(2*a + 2*b*x) + 1))/(2*b*(exp(2*a
+ 2*b*x) - 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \cosh\left(a+bx\right)\coth^{2}\left(a+bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*coth(b*x+a)**2,x)
```

```
[Out] Integral(cosh(a + b*x)*coth(a + b*x)**2, x)
```

3.103 $\int \cosh(a+bx)\coth^3(a+bx)\,dx$

Optimal. Leaf size=49

$$\frac{3\cosh(a+bx)}{2b}-\frac{3\tanh^{-1}(\cosh(a+bx))}{2b}-\frac{\cosh(a+bx)\coth^2(a+bx)}{2b}$$

```
[Out] -3/2*arctanh(cosh(b*x+a))/b+3/2*cosh(b*x+a)/b-1/2*cosh(b*x+a)*coth(b*x+a)^2
/b
```

Rubi [A] time = 0.04, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {2592, 288, 321, 206}

$$\frac{3\cosh(a+bx)}{2b}-\frac{3\tanh^{-1}(\cosh(a+bx))}{2b}-\frac{\cosh(a+bx)\coth^2(a+bx)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[a + b*x]*Coth[a + b*x]^3,x]
```

```
[Out] (-3*ArcTanh[Cosh[a + b*x]])/(2*b) + (3*Cosh[a + b*x])/(2*b) - (Cosh[a + b*x
]*Coth[a + b*x]^2)/(2*b)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2592

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, (a*Sin[e + f*x])/ff], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rubi steps

$$\int \cosh(a+bx)\coth^3(a+bx)\,dx = \frac{\text{Subst}\left(\int \frac{x^4}{(1-x^2)^2}\,dx, x, \cosh(a+bx)\right)}{b}$$
$$= -\frac{\cosh(a+bx)\coth^2(a+bx)}{2b} - \frac{3\,\text{Subst}\left(\int \frac{x^2}{1-x^2}\,dx, x, \cosh(a+bx)\right)}{2b}$$
$$= \frac{3\cosh(a+bx)}{2b} - \frac{\cosh(a+bx)\coth^2(a+bx)}{2b} - \frac{3\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \cosh\right.}{2b}$$
$$= -\frac{3\tanh^{-1}(\cosh(a+bx))}{2b} + \frac{3\cosh(a+bx)}{2b} - \frac{\cosh(a+bx)\coth^2(a+bx)}{2b}$$

Mathematica [A] time = 0.03, size = 67, normalized size = 1.37

$$\frac{\cosh(a+bx)}{b} - \frac{\text{csch}^2\left(\frac{1}{2}(a+bx)\right)}{8b} - \frac{\text{sech}^2\left(\frac{1}{2}(a+bx)\right)}{8b} + \frac{3\log\left(\tanh\left(\frac{1}{2}(a+bx)\right)\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[a + b*x]*Coth[a + b*x]^3,x]
```

```
[Out] Cosh[a + b*x]/b - Csch[(a + b*x)/2]^2/(8*b) + (3*Log[Tanh[(a + b*x)/2]])/(2
*b) - Sech[(a + b*x)/2]^2/(8*b)
```

fricas [B] time = 0.44, size = 612, normalized size = 12.49

$$\frac{\cosh(bx+a)^6 + 6\cosh(bx+a)\sinh(bx+a)^5 + \sinh(bx+a)^6 + 3\left(5\cosh(bx+a)^2 - 1\right)\sinh(bx+a)^4 - 3\,\text{c}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*coth(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/2*(cosh(b*x + a)^6 + 6*cosh(b*x + a)*sinh(b*x + a)^5 + sinh(b*x + a)^6 +
3*(5*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^4 - 3*cosh(b*x + a)^4 + 4*(5*cosh(b
*x + a)^3 - 3*cosh(b*x + a))*sinh(b*x + a)^3 + 3*(5*cosh(b*x + a)^4 - 6*cos
h(b*x + a)^2 - 1)*sinh(b*x + a)^2 - 3*cosh(b*x + a)^2 - 3*(cosh(b*x + a)^5
+ 5*cosh(b*x + a)*sinh(b*x + a)^4 + sinh(b*x + a)^5 + 2*(5*cosh(b*x + a)^2
- 1)*sinh(b*x + a)^3 - 2*cosh(b*x + a)^3 + 2*(5*cosh(b*x + a)^3 - 3*cosh(b*
x + a))*sinh(b*x + a)^2 + (5*cosh(b*x + a)^4 - 6*cosh(b*x + a)^2 + 1)*sinh(
b*x + a) + cosh(b*x + a))*log(cosh(b*x + a) + sinh(b*x + a) + 1) + 3*(cosh(
b*x + a)^5 + 5*cosh(b*x + a)*sinh(b*x + a)^4 + sinh(b*x + a)^5 + 2*(5*cosh(
b*x + a)^2 - 1)*sinh(b*x + a)^3 - 2*cosh(b*x + a)^3 + 2*(5*cosh(b*x + a)^3
- 3*cosh(b*x + a))*sinh(b*x + a)^2 + (5*cosh(b*x + a)^4 - 6*cosh(b*x + a)^2
 + 1)*sinh(b*x + a) + cosh(b*x + a))*log(cosh(b*x + a) + sinh(b*x + a) - 1)
 + 6*(cosh(b*x + a)^5 - 2*cosh(b*x + a)^3 - cosh(b*x + a))*sinh(b*x + a) +
1)/(b*cosh(b*x + a)^5 + 5*b*cosh(b*x + a)*sinh(b*x + a)^4 + b*sinh(b*x + a)
^5 - 2*b*cosh(b*x + a)^3 + 2*(5*b*cosh(b*x + a)^2 - b)*sinh(b*x + a)^3 + 2*
(5*b*cosh(b*x + a)^3 - 3*b*cosh(b*x + a))*sinh(b*x + a)^2 + b*cosh(b*x + a)
 + (5*b*cosh(b*x + a)^4 - 6*b*cosh(b*x + a)^2 + b)*sinh(b*x + a))
```

giac [A] time = 0.17, size = 79, normalized size = 1.61

$$-\frac{\frac{2\left(e^{(3bx+3a)}+e^{(bx+a)}\right)}{\left(e^{(2bx+2a)}-1\right)^2}-e^{(bx+a)}-e^{(-bx-a)}+3\log\left(e^{(bx+a)}+1\right)-3\log\left(\left|e^{(bx+a)}-1\right|\right)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*coth(b*x+a)^3,x, algorithm="giac")
```

```
[Out] -1/2*(2*(e^(3*b*x + 3*a) + e^(b*x + a))/(e^(2*b*x + 2*a) - 1)^2 - e^(b*x +
a) - e^(-b*x - a) + 3*log(e^(b*x + a) + 1) - 3*log(abs(e^(b*x + a) - 1)))/b
```

maple [A] time = 0.31, size = 62, normalized size = 1.27

$$\frac{\frac{\cosh^3(bx+a)}{\sinh(bx+a)^2}-\frac{3\cosh(bx+a)}{\sinh(bx+a)^2}+\frac{3\operatorname{csch}(bx+a)\coth(bx+a)}{2}-3\operatorname{arctanh}\left(e^{bx+a}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)*coth(b*x+a)^3,x)
```

```
[Out] 1/b*(cosh(b*x+a)^3/sinh(b*x+a)^2-3/sinh(b*x+a)^2*cosh(b*x+a)+3/2*csch(b*x+a
)*coth(b*x+a)-3*arctanh(exp(b*x+a)))
```

maxima [B] time = 0.35, size = 108, normalized size = 2.20

$$\frac{e^{(-bx-a)}}{2b}-\frac{3\log\left(e^{(-bx-a)}+1\right)}{2b}+\frac{3\log\left(e^{(-bx-a)}-1\right)}{2b}-\frac{4e^{(-2bx-2a)}+e^{(-4bx-4a)}-1}{2b\left(e^{(-bx-a)}-2e^{(-3bx-3a)}+e^{(-5bx-5a)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*coth(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/2*e^(-b*x - a)/b - 3/2*log(e^(-b*x - a) + 1)/b + 3/2*log(e^(-b*x - a) - 1
)/b - 1/2*(4*e^(-2*b*x - 2*a) + e^(-4*b*x - 4*a) - 1)/(b*(e^(-b*x - a) - 2*
e^(-3*b*x - 3*a) + e^(-5*b*x - 5*a)))
```

mupad [B] time = 0.08, size = 112, normalized size = 2.29

$$\frac{e^{a+b\,x}}{2\,b}-\frac{3\,\mathrm{atan}\left(\frac{e^{b\,x}\,e^{a}\,\sqrt{-b^2}}{b}\right)}{\sqrt{-b^2}}+\frac{e^{-a-b\,x}}{2\,b}-\frac{2\,e^{a+b\,x}}{b\,\left(e^{4\,a+4\,b\,x}-2\,e^{2\,a+2\,b\,x}+1\right)}-\frac{e^{a+b\,x}}{b\,\left(e^{2\,a+2\,b\,x}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a + b*x)*coth(a + b*x)^3,x)
```

```
[Out] exp(a + b*x)/(2*b) - (3*atan((exp(b*x)*exp(a)*(-b^2)^(1/2))/b))/(-b^2)^(1/2
) + exp(- a - b*x)/(2*b) - (2*exp(a + b*x))/(b*(exp(4*a + 4*b*x) - 2*exp(2*
a + 2*b*x) + 1)) - exp(a + b*x)/(b*(exp(2*a + 2*b*x) - 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \cosh(a+bx)\coth^3(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*coth(b*x+a)**3,x)
```

```
[Out] Integral(cosh(a + b*x)*coth(a + b*x)**3, x)
```

3.104 $\int \cosh(a+bx)\coth^4(a+bx)\,dx$

Optimal. Leaf size=37

$$\frac{\sinh(a+bx)}{b}-\frac{\text{csch}^3(a+bx)}{3b}-\frac{2\text{csch}(a+bx)}{b}$$

```
[Out] -2*csch(b*x+a)/b-1/3*csch(b*x+a)^3/b+sinh(b*x+a)/b
```

Rubi [A] time = 0.03, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2590, 270}

$$\frac{\sinh(a+bx)}{b}-\frac{\text{csch}^3(a+bx)}{3b}-\frac{2\text{csch}(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[a + b*x]*Coth[a + b*x]^4,x]
```

```
[Out] (-2*Csch[a + b*x])/b - Csch[a + b*x]^3/(3*b) + Sinh[a + b*x]/b
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2590

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> -Dist[f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rubi steps

$$\begin{aligned}\int \cosh(a+bx)\coth^4(a+bx)\,dx &= \frac{i\,\text{Subst}\left(\int \frac{(1-x^2)^2}{x^4}\,dx, x, -i\sinh(a+bx)\right)}{b}\\ &= \frac{i\,\text{Subst}\left(\int \left(1+\frac{1}{x^4}-\frac{2}{x^2}\right)dx, x, -i\sinh(a+bx)\right)}{b}\\ &= -\frac{2\text{csch}(a+bx)}{b}-\frac{\text{csch}^3(a+bx)}{3b}+\frac{\sinh(a+bx)}{b}\end{aligned}$$

Mathematica [A] time = 0.02, size = 37, normalized size = 1.00

$$\frac{\sinh(a+bx)}{b}-\frac{\operatorname{csch}^3(a+bx)}{3b}-\frac{2\operatorname{csch}(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[a + b*x]*Coth[a + b*x]^4,x]
```

```
[Out] (-2*Csch[a + b*x])/b - Csch[a + b*x]^3/(3*b) + Sinh[a + b*x]/b
```

fricas [B] time = 0.46, size = 89, normalized size = 2.41

$$\frac{3\cosh(bx+a)^4+3\sinh(bx+a)^4+18\left(\cosh(bx+a)^2-2\right)\sinh(bx+a)^2-36\cosh(bx+a)^2+25}{6\left(b\sinh(bx+a)^3+3\left(b\cosh(bx+a)^2-b\right)\sinh(bx+a)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*coth(b*x+a)^4,x, algorithm="fricas")
```

```
[Out] 1/6*(3*cosh(b*x + a)^4 + 3*sinh(b*x + a)^4 + 18*(cosh(b*x + a)^2 - 2)*sinh(
b*x + a)^2 - 36*cosh(b*x + a)^2 + 25)/(b*sinh(b*x + a)^3 + 3*(b*cosh(b*x +
a)^2 - b)*sinh(b*x + a))
```

giac [B] time = 0.19, size = 71, normalized size = 1.92

$$-\frac{\frac{8\left(3e^{(5bx+5a)}-4e^{(3bx+3a)}+3e^{(bx+a)}\right)}{\left(e^{(2bx+2a)}-1\right)^3}-3e^{(bx+a)}+3e^{(-bx-a)}}{6b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*coth(b*x+a)^4,x, algorithm="giac")
```

```
[Out] -1/6*(8*(3*e^(5*b*x + 5*a) - 4*e^(3*b*x + 3*a) + 3*e^(b*x + a))/(e^(2*b*x +
 2*a) - 1)^3 - 3*e^(b*x + a) + 3*e^(-b*x - a))/b
```

maple [A] time = 0.15, size = 51, normalized size = 1.38

$$\frac{\frac{\cosh^4(bx+a)}{\sinh(bx+a)^3}-\frac{4\left(\cosh^2(bx+a)\right)}{\sinh(bx+a)^3}+\frac{8}{3\sinh(bx+a)^3}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)*coth(b*x+a)^4,x)
```

```
[Out] 1/b*(cosh(b*x+a)^4/sinh(b*x+a)^3-4/sinh(b*x+a)^3*cosh(b*x+a)^2+8/3/sinh(b*x
+a)^3)
```

maxima [B] time = 0.32, size = 100, normalized size = 2.70

$$-\frac{e^{(-bx-a)}}{2\,b}-\frac{33\,e^{(-2\,bx-2\,a)}-41\,e^{(-4\,bx-4\,a)}+27\,e^{(-6\,bx-6\,a)}-3}{6\,b\left(e^{(-bx-a)}-3\,e^{(-3\,bx-3\,a)}+3\,e^{(-5\,bx-5\,a)}-e^{(-7\,bx-7\,a)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*coth(b*x+a)^4,x, algorithm="maxima")
```

```
[Out] -1/2*e^(-b*x - a)/b - 1/6*(33*e^(-2*b*x - 2*a) - 41*e^(-4*b*x - 4*a) + 27*e
^(-6*b*x - 6*a) - 3)/(b*(e^(-b*x - a) - 3*e^(-3*b*x - 3*a) + 3*e^(-5*b*x -
5*a) - e^(-7*b*x - 7*a)))
```

mupad [B] time = 1.45, size = 131, normalized size = 3.54

$$\frac{e^{a+b\,x}}{2\,b}-\frac{e^{-a-b\,x}}{2\,b}-\frac{8\,e^{a+b\,x}}{3\,b\,\left(e^{4\,a+4\,b\,x}-2\,e^{2\,a+2\,b\,x}+1\right)}-\frac{8\,e^{a+b\,x}}{3\,b\,\left(3\,e^{2\,a+2\,b\,x}-3\,e^{4\,a+4\,b\,x}+e^{6\,a+6\,b\,x}-1\right)}-\frac{4\,e^{a+b\,x}}{b\,\left(e^{2\,a+2\,b\,x}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a + b*x)*coth(a + b*x)^4,x)
```

```
[Out] exp(a + b*x)/(2*b) - exp(- a - b*x)/(2*b) - (8*exp(a + b*x))/(3*b*(exp(4*a
+ 4*b*x) - 2*exp(2*a + 2*b*x) + 1)) - (8*exp(a + b*x))/(3*b*(3*exp(2*a + 2*
b*x) - 3*exp(4*a + 4*b*x) + exp(6*a + 6*b*x) - 1)) - (4*exp(a + b*x))/(b*(e
xp(2*a + 2*b*x) - 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \cosh{\left(a+bx \right)}\coth^{4}{\left(a+bx \right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*coth(b*x+a)**4,x)
```

```
[Out] Integral(cosh(a + b*x)*coth(a + b*x)**4, x)
```

3.105 $\int \cosh^2(a+bx)\coth(a+bx)\,dx$

Optimal. Leaf size=27

$$\frac{\sinh^2(a+bx)}{2b}+\frac{\log(\sinh(a+bx))}{b}$$

```
[Out] ln(sinh(b*x+a))/b+1/2*sinh(b*x+a)^2/b
```

Rubi [A] time = 0.02, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2590, 14}

$$\frac{\sinh^2(a+bx)}{2b}+\frac{\log(\sinh(a+bx))}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[a + b*x]^2*Coth[a + b*x],x]
```

```
[Out] Log[Sinh[a + b*x]]/b + Sinh[a + b*x]^2/(2*b)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2590

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> -Dist[f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rubi steps

$$\begin{aligned}\int \cosh^2(a+bx)\coth(a+bx)\,dx &= \frac{\text{Subst}\left(\int \frac{1-x^2}{x}\,dx,x,-i\sinh(a+bx)\right)}{b}\\ &= \frac{\text{Subst}\left(\int \left(\frac{1}{x}-x\right)dx,x,-i\sinh(a+bx)\right)}{b}\\ &= \frac{\log(\sinh(a+bx))}{b}+\frac{\sinh^2(a+bx)}{2b}\end{aligned}$$

Mathematica [A] time = 0.02, size = 25, normalized size = 0.93

$$\frac{\sinh^2(a+bx)+2\log(\sinh(a+bx))}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[a + b*x]^2*Coth[a + b*x],x]
```

```
[Out] (2*Log[Sinh[a + b*x]] + Sinh[a + b*x]^2)/(2*b)
```

fricas [B] time = 0.43, size = 203, normalized size = 7.52

$$-\frac{8\,bx\cosh\left(bx+a\right)^{2}-\cosh\left(bx+a\right)^{4}-4\,\cosh\left(bx+a\right)\sinh\left(bx+a\right)^{3}-\sinh\left(bx+a\right)^{4}+2\left(4\,bx-3\,\cosh\left(bx+\right.\right.}{8\left(b\,c\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^2*coth(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/8*(8*b*x*cosh(b*x + a)^2 - cosh(b*x + a)^4 - 4*cosh(b*x + a)*sinh(b*x +
a)^3 - sinh(b*x + a)^4 + 2*(4*b*x - 3*cosh(b*x + a)^2)*sinh(b*x + a)^2 - 8*
(cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2)*log(2*s
inh(b*x + a)/(cosh(b*x + a) - sinh(b*x + a))) + 4*(4*b*x*cosh(b*x + a) - co
sh(b*x + a)^3)*sinh(b*x + a) - 1)/(b*cosh(b*x + a)^2 + 2*b*cosh(b*x + a)*si
nh(b*x + a) + b*sinh(b*x + a)^2)
```

giac [B] time = 0.14, size = 60, normalized size = 2.22

$$-\frac{8\,bx-\left(4\,e^{\left(2\,bx+2\,a\right)}+1\right)e^{\left(-2\,bx-2\,a\right)}-e^{\left(2\,bx+2\,a\right)}-8\,\log\left(\left|e^{\left(2\,bx+2\,a\right)}-1\right|\right)}{8\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^2*coth(b*x+a),x, algorithm="giac")
```

```
[Out] -1/8*(8*b*x - (4*e^(2*b*x + 2*a) + 1)*e^(-2*b*x - 2*a) - e^(2*b*x + 2*a) -
8*log(abs(e^(2*b*x + 2*a) - 1)))/b
```

maple [A] time = 0.11, size = 26, normalized size = 0.96

$$\frac{\cosh^2\left(bx+a\right)}{2b}+\frac{\ln\left(\sinh\left(bx+a\right)\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)^2*coth(b*x+a),x)
```

```
[Out] 1/2*cosh(b*x+a)^2/b+ln(sinh(b*x+a))/b
```

maxima [B] time = 0.36, size = 70, normalized size = 2.59

$$\frac{bx+a}{b}+\frac{e^{(2\,bx+2\,a)}}{8\,b}+\frac{e^{(-2\,bx-2\,a)}}{8\,b}+\frac{\log\left(e^{(-bx-a)}+1\right)}{b}+\frac{\log\left(e^{(-bx-a)}-1\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^2*coth(b*x+a),x, algorithm="maxima")
```

```
[Out] (b*x + a)/b + 1/8*e^(2*b*x + 2*a)/b + 1/8*e^(-2*b*x - 2*a)/b + log(e^(-b*x
 - a) + 1)/b + log(e^(-b*x - a) - 1)/b
```

mupad [B] time = 0.07, size = 49, normalized size = 1.81

$$\frac{\ln\left(\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}-1\right)}{b}-x+\frac{\mathrm{e}^{-2\,a-2\,b\,x}}{8\,b}+\frac{\mathrm{e}^{2\,a+2\,b\,x}}{8\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a + b*x)^2*coth(a + b*x),x)
```

```
[Out] log(exp(2*a)*exp(2*b*x) - 1)/b - x + exp(- 2*a - 2*b*x)/(8*b) + exp(2*a + 2
*b*x)/(8*b)
```

sympy [A] time = 11.85, size = 116, normalized size = 4.30

$$\begin{cases} x\cosh^2(a)\coth(a) & \text{fo} \\ \tilde{\infty}x & \text{fo} \\ -\frac{x\sinh^2(a+bx)\coth(a+bx)}{2}+\frac{x\cosh^2(a+bx)\coth(a+bx)}{2}-\frac{x\cosh(a+bx)}{2\sinh(a+bx)}+\frac{\log(\sinh(a+bx))}{b}+\frac{\sinh(a+bx)\cosh(a+bx)\coth(a+bx)}{2b} & \text{ot} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)**2*coth(b*x+a),x)
```

```
[Out] Piecewise((x*cosh(a)**2*coth(a), Eq(b, 0)), (zoo*x, Eq(a, log(exp(-b*x))) |
 Eq(a, log(-exp(-b*x)))), (-x*sinh(a + b*x)**2*coth(a + b*x)/2 + x*cosh(a +
 b*x)**2*coth(a + b*x)/2 - x*cosh(a + b*x)/(2*sinh(a + b*x)) + log(sinh(a +
 b*x))/b + sinh(a + b*x)*cosh(a + b*x)*coth(a + b*x)/(2*b), True))
```

3.106 $\int \cosh^2(a+bx)\coth^2(a+bx)\,dx$

Optimal. Leaf size=40

$$-\frac{3\coth(a+bx)}{2b}+\frac{\cosh^2(a+bx)\coth(a+bx)}{2b}+\frac{3x}{2}$$

[Out] 3/2*x-3/2*coth(b*x+a)/b+1/2*cosh(b*x+a)^2*coth(b*x+a)/b

Rubi [A] time = 0.04, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {2591, 288, 321, 206}

$$-\frac{3\coth(a+bx)}{2b}+\frac{\cosh^2(a+bx)\coth(a+bx)}{2b}+\frac{3x}{2}$$

Antiderivative was successfully verified.

[In] Int[Cosh[a + b*x]^2*Coth[a + b*x]^2,x]

[Out] (3*x)/2 - (3*Coth[a + b*x])/(2*b) + (Cosh[a + b*x]^2*Coth[a + b*x])/(2*b)

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2591

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff)/f, Subst[In
t[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, (b*Tan[e + f*x])/ff], x
]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

Rubi steps

$$\int \cosh^2(a+bx)\coth^2(a+bx)\,dx = -\frac{\text{Subst}\left(\int \frac{x^4}{\left(1-x^2\right)^2}\,dx, x, \coth(a+bx)\right)}{b}$$
$$= \frac{\cosh^2(a+bx)\coth(a+bx)}{2b} + \frac{3\,\text{Subst}\left(\int \frac{x^2}{1-x^2}\,dx, x, \coth(a+bx)\right)}{2b}$$
$$= -\frac{3\coth(a+bx)}{2b} + \frac{\cosh^2(a+bx)\coth(a+bx)}{2b} + \frac{3\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \text{co}\right.}{2b}$$
$$= \frac{3x}{2} - \frac{3\coth(a+bx)}{2b} + \frac{\cosh^2(a+bx)\coth(a+bx)}{2b}$$

Mathematica [A] time = 0.11, size = 31, normalized size = 0.78

$$\frac{6(a+bx)+\sinh(2(a+bx))-4\coth(a+bx)}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[a + b*x]^2*Coth[a + b*x]^2,x]
```

```
[Out] (6*(a + b*x) - 4*Coth[a + b*x] + Sinh[2*(a + b*x)])/(4*b)
```

fricas [A] time = 0.42, size = 60, normalized size = 1.50

$$\frac{\cosh\left(bx+a\right)^3+3\,\cosh\left(bx+a\right)\sinh\left(bx+a\right)^2+4\left(3\,bx+2\right)\sinh\left(bx+a\right)-9\,\cosh\left(bx+a\right)}{8\,b\sinh\left(bx+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^2*coth(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/8*(cosh(b*x + a)^3 + 3*cosh(b*x + a)*sinh(b*x + a)^2 + 4*(3*b*x + 2)*sinh
(b*x + a) - 9*cosh(b*x + a))/(b*sinh(b*x + a))
```

giac [A] time = 0.17, size = 67, normalized size = 1.68

$$\frac{12\,bx + \frac{\left(3\,e^{(4\,bx+4\,a)}+14\,e^{(2\,bx+2\,a)}-1\right)e^{(-2\,a)}}{e^{(2\,bx)}-e^{(4\,bx+2\,a)}} + e^{(2\,bx+2\,a)}}{8\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^2*coth(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/8*(12*b*x + (3*e^(4*b*x + 4*a) + 14*e^(2*b*x + 2*a) - 1)*e^(-2*a)/(e^(2*b
*x) - e^(4*b*x + 2*a)) + e^(2*b*x + 2*a))/b
```

maple [A] time = 0.11, size = 39, normalized size = 0.98

$$\frac{\frac{\cosh^3(bx+a)}{2\sinh(bx+a)} + \frac{3bx}{2} + \frac{3a}{2} - \frac{3\coth(bx+a)}{2}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)^2*coth(b*x+a)^2,x)
```

```
[Out] 1/b*(1/2*cosh(b*x+a)^3/sinh(b*x+a)+3/2*b*x+3/2*a-3/2*coth(b*x+a))
```

maxima [A] time = 0.38, size = 66, normalized size = 1.65

$$\frac{3\,(bx+a)}{2\,b} - \frac{e^{(-2\,bx-2\,a)}}{8\,b} - \frac{17\,e^{(-2\,bx-2\,a)} - 1}{8\,b\left(e^{(-2\,bx-2\,a)} - e^{(-4\,bx-4\,a)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^2*coth(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 3/2*(b*x + a)/b - 1/8*e^(-2*b*x - 2*a)/b - 1/8*(17*e^(-2*b*x - 2*a) - 1)/(b
*(e^(-2*b*x - 2*a) - e^(-4*b*x - 4*a)))
```

mupad [B] time = 0.09, size = 50, normalized size = 1.25

$$\frac{3\,x}{2} - \frac{2}{b\,\left(e^{2\,a+2\,b\,x}-1\right)} - \frac{e^{-2\,a-2\,b\,x}}{8\,b} + \frac{e^{2\,a+2\,b\,x}}{8\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a + b*x)^2*coth(a + b*x)^2,x)
```

```
[Out] (3*x)/2 - 2/(b*(exp(2*a + 2*b*x) - 1)) - exp(- 2*a - 2*b*x)/(8*b) + exp(2*a
 + 2*b*x)/(8*b)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \cosh^2(a+bx)\coth^2(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)**2*coth(b*x+a)**2,x)
```

```
[Out] Integral(cosh(a + b*x)**2*coth(a + b*x)**2, x)
```

3.107 $\int \cosh^2(a+bx)\coth^3(a+bx)\,dx$

Optimal. Leaf size=43

$$\frac{\sinh^2(a+bx)}{2b}-\frac{\operatorname{csch}^2(a+bx)}{2b}+\frac{2\log(\sinh(a+bx))}{b}$$

```
[Out] -1/2*csch(b*x+a)^2/b+2*ln(sinh(b*x+a))/b+1/2*sinh(b*x+a)^2/b
```

Rubi [A] time = 0.04, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {2590, 266, 43}

$$\frac{\sinh^2(a+bx)}{2b}-\frac{\operatorname{csch}^2(a+bx)}{2b}+\frac{2\log(\sinh(a+bx))}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[a + b*x]^2*Coth[a + b*x]^3,x]
```

```
[Out] -Csch[a + b*x]^2/(2*b) + (2*Log[Sinh[a + b*x]])/b + Sinh[a + b*x]^2/(2*b)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2590

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> -Dist[f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rubi steps

$$\int \cosh^2(a+bx)\coth^3(a+bx)\,dx = -\frac{\text{Subst}\left(\int \frac{(1-x^2)^2}{x^3}\,dx, x, -i\sinh(a+bx)\right)}{b}$$
$$= -\frac{\text{Subst}\left(\int \frac{(1-x)^2}{x^2}\,dx, x, -\sinh^2(a+bx)\right)}{2b}$$
$$= -\frac{\text{Subst}\left(\int \left(1+\frac{1}{x^2}-\frac{2}{x}\right)dx, x, -\sinh^2(a+bx)\right)}{2b}$$
$$= -\frac{\operatorname{csch}^2(a+bx)}{2b} + \frac{2\log(\sinh(a+bx))}{b} + \frac{\sinh^2(a+bx)}{2b}$$

Mathematica [A] time = 0.05, size = 35, normalized size = 0.81

$$-\frac{-\sinh^2(a+bx)+\operatorname{csch}^2(a+bx)-4\log(\sinh(a+bx))}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[a + b*x]^2*Coth[a + b*x]^3,x]
```

```
[Out] -1/2*(Csch[a + b*x]^2 - 4*Log[Sinh[a + b*x]] - Sinh[a + b*x]^2)/b
```

fricas [B] time = 0.45, size = 743, normalized size = 17.28

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^2*coth(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/8*(cosh(b*x + a)^8 + 8*cosh(b*x + a)*sinh(b*x + a)^7 + sinh(b*x + a)^8 -
2*(8*b*x + 1)*cosh(b*x + a)^6 - 2*(8*b*x - 14*cosh(b*x + a)^2 + 1)*sinh(b*x
 + a)^6 + 4*(14*cosh(b*x + a)^3 - 3*(8*b*x + 1)*cosh(b*x + a))*sinh(b*x + a
)^5 + 2*(16*b*x - 7)*cosh(b*x + a)^4 + 2*(35*cosh(b*x + a)^4 - 15*(8*b*x +
1)*cosh(b*x + a)^2 + 16*b*x - 7)*sinh(b*x + a)^4 + 8*(7*cosh(b*x + a)^5 - 5
*(8*b*x + 1)*cosh(b*x + a)^3 + (16*b*x - 7)*cosh(b*x + a))*sinh(b*x + a)^3
- 2*(8*b*x + 1)*cosh(b*x + a)^2 + 2*(14*cosh(b*x + a)^6 - 15*(8*b*x + 1)*co
sh(b*x + a)^4 + 6*(16*b*x - 7)*cosh(b*x + a)^2 - 8*b*x - 1)*sinh(b*x + a)^2
 + 16*(cosh(b*x + a)^6 + 6*cosh(b*x + a)*sinh(b*x + a)^5 + sinh(b*x + a)^6
+ (15*cosh(b*x + a)^2 - 2)*sinh(b*x + a)^4 - 2*cosh(b*x + a)^4 + 4*(5*cosh(
b*x + a)^3 - 2*cosh(b*x + a))*sinh(b*x + a)^3 + (15*cosh(b*x + a)^4 - 12*co
sh(b*x + a)^2 + 1)*sinh(b*x + a)^2 + cosh(b*x + a)^2 + 2*(3*cosh(b*x + a)^5
```

```
 - 4*cosh(b*x + a)^3 + cosh(b*x + a))*sinh(b*x + a))*log(2*sinh(b*x + a)/(c
osh(b*x + a) - sinh(b*x + a))) + 4*(2*cosh(b*x + a)^7 - 3*(8*b*x + 1)*cosh(
b*x + a)^5 + 2*(16*b*x - 7)*cosh(b*x + a)^3 - (8*b*x + 1)*cosh(b*x + a))*si
nh(b*x + a) + 1)/(b*cosh(b*x + a)^6 + 6*b*cosh(b*x + a)*sinh(b*x + a)^5 + b
*sinh(b*x + a)^6 - 2*b*cosh(b*x + a)^4 + (15*b*cosh(b*x + a)^2 - 2*b)*sinh(
b*x + a)^4 + 4*(5*b*cosh(b*x + a)^3 - 2*b*cosh(b*x + a))*sinh(b*x + a)^3 +
b*cosh(b*x + a)^2 + (15*b*cosh(b*x + a)^4 - 12*b*cosh(b*x + a)^2 + b)*sinh(
b*x + a)^2 + 2*(3*b*cosh(b*x + a)^5 - 4*b*cosh(b*x + a)^3 + b*cosh(b*x + a)
)*sinh(b*x + a))
```

giac [B] time = 0.21, size = 99, normalized size = 2.30

$$-\frac{16\,bx-\left(8\,e^{(2\,bx+2\,a)}+1\right)e^{(-2\,bx-2\,a)}+\frac{8\left(3\,e^{(4\,bx+4\,a)}-4\,e^{(2\,bx+2\,a)}+3\right)}{\left(e^{(2\,bx+2\,a)}-1\right)^2}-e^{(2\,bx+2\,a)}-16\,\log\left(\left|e^{(2\,bx+2\,a)}-1\right|\right)}{8\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^2*coth(b*x+a)^3,x, algorithm="giac")
```

```
[Out] -1/8*(16*b*x - (8*e^(2*b*x + 2*a) + 1)*e^(-2*b*x - 2*a) + 8*(3*e^(4*b*x + 4
*a) - 4*e^(2*b*x + 2*a) + 3)/(e^(2*b*x + 2*a) - 1)^2 - e^(2*b*x + 2*a) - 16
*log(abs(e^(2*b*x + 2*a) - 1)))/b
```

maple [A] time = 0.17, size = 48, normalized size = 1.12

$$\frac{\cosh^4(bx+a)}{2b\sinh(bx+a)^2}+\frac{2\ln(\sinh(bx+a))}{b}-\frac{\coth^2(bx+a)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)^2*coth(b*x+a)^3,x)
```

```
[Out] 1/2/b*cosh(b*x+a)^4/sinh(b*x+a)^2+2*ln(sinh(b*x+a))/b-coth(b*x+a)^2/b
```

maxima [B] time = 0.34, size = 120, normalized size = 2.79

$$\frac{2\,(bx+a)}{b}+\frac{e^{(-2\,bx-2\,a)}}{8\,b}+\frac{2\,\log\left(e^{(-bx-a)}+1\right)}{b}+\frac{2\,\log\left(e^{(-bx-a)}-1\right)}{b}-\frac{2\,e^{(-2\,bx-2\,a)}+15\,e^{(-4\,bx-4\,a)}-1}{8\,b\left(e^{(-2\,bx-2\,a)}-2\,e^{(-4\,bx-4\,a)}+e^{(-6\,bx-6\,a)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^2*coth(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 2*(b*x + a)/b + 1/8*e^(-2*b*x - 2*a)/b + 2*log(e^(-b*x - a) + 1)/b + 2*log(
e^(-b*x - a) - 1)/b - 1/8*(2*e^(-2*b*x - 2*a) + 15*e^(-4*b*x - 4*a) - 1)/(b
*(e^(-2*b*x - 2*a) - 2*e^(-4*b*x - 4*a) + e^(-6*b*x - 6*a)))
```

mupad [B] time = 1.46, size = 97, normalized size = 2.26

$$\frac{2\ln\left(e^{2a}\,e^{2bx}-1\right)}{b}-2x-\frac{2}{b\left(e^{2a+2bx}-1\right)}-\frac{2}{b\left(e^{4a+4bx}-2\,e^{2a+2bx}+1\right)}+\frac{e^{-2a-2bx}}{8b}+\frac{e^{2a+2bx}}{8b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a + b*x)^2*coth(a + b*x)^3,x)
```

```
[Out] (2*log(exp(2*a)*exp(2*b*x) - 1))/b - 2*x - 2/(b*(exp(2*a + 2*b*x) - 1)) - 2
/(b*(exp(4*a + 4*b*x) - 2*exp(2*a + 2*b*x) + 1)) + exp(- 2*a - 2*b*x)/(8*b)
 + exp(2*a + 2*b*x)/(8*b)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\cosh^2{(a+bx)}\coth^3{(a+bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)**2*coth(b*x+a)**3,x)
```

```
[Out] Integral(cosh(a + b*x)**2*coth(a + b*x)**3, x)
```

3.108 $\int \cosh^3(a+bx)\coth(a+bx)\,dx$

Optimal. Leaf size=38

$$\frac{\cosh^3(a+bx)}{3b}+\frac{\cosh(a+bx)}{b}-\frac{\tanh^{-1}(\cosh(a+bx))}{b}$$

```
[Out] -arctanh(cosh(b*x+a))/b+cosh(b*x+a)/b+1/3*cosh(b*x+a)^3/b
```

Rubi [A] time = 0.03, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2592, 302, 206}

$$\frac{\cosh^3(a+bx)}{3b}+\frac{\cosh(a+bx)}{b}-\frac{\tanh^{-1}(\cosh(a+bx))}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[a + b*x]^3*Coth[a + b*x],x]
```

```
[Out] -(ArcTanh[Cosh[a + b*x]]/b) + Cosh[a + b*x]/b + Cosh[a + b*x]^3/(3*b)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 2592

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, (a*Sin[e + f*x])/ff], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rubi steps

$$\int \cosh^3(a+bx)\coth(a+bx)\,dx = -\frac{\text{Subst}\left(\int \frac{x^4}{1-x^2}\,dx, x, \cosh(a+bx)\right)}{b}$$
$$= -\frac{\text{Subst}\left(\int \left(-1-x^2+\frac{1}{1-x^2}\right)dx, x, \cosh(a+bx)\right)}{b}$$
$$= \frac{\cosh(a+bx)}{b} + \frac{\cosh^3(a+bx)}{3b} - \frac{\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \cosh(a+bx)\right)}{b}$$
$$= -\frac{\tanh^{-1}(\cosh(a+bx))}{b} + \frac{\cosh(a+bx)}{b} + \frac{\cosh^3(a+bx)}{3b}$$

Mathematica [A] time = 0.03, size = 44, normalized size = 1.16

$$\frac{5\cosh(a+bx)}{4b} + \frac{\cosh(3(a+bx))}{12b} + \frac{\log\left(\tanh\left(\frac{1}{2}(a+bx)\right)\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[a + b*x]^3*Coth[a + b*x],x]
```

```
[Out] (5*Cosh[a + b*x])/(4*b) + Cosh[3*(a + b*x)]/(12*b) + Log[Tanh[(a + b*x)/2]]
/b
```

fricas [B] time = 0.52, size = 357, normalized size = 9.39

$$\frac{\cosh(bx+a)^6 + 6\cosh(bx+a)\sinh(bx+a)^5 + \sinh(bx+a)^6 + 15\left(\cosh(bx+a)^2+1\right)\sinh(bx+a)^4 + 15\,c}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^3*coth(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/24*(cosh(b*x + a)^6 + 6*cosh(b*x + a)*sinh(b*x + a)^5 + sinh(b*x + a)^6 +
 15*(cosh(b*x + a)^2 + 1)*sinh(b*x + a)^4 + 15*cosh(b*x + a)^4 + 20*(cosh(b
*x + a)^3 + 3*cosh(b*x + a))*sinh(b*x + a)^3 + 15*(cosh(b*x + a)^4 + 6*cosh
(b*x + a)^2 + 1)*sinh(b*x + a)^2 + 15*cosh(b*x + a)^2 - 24*(cosh(b*x + a)^3
 + 3*cosh(b*x + a)^2*sinh(b*x + a) + 3*cosh(b*x + a)*sinh(b*x + a)^2 + sinh
(b*x + a)^3)*log(cosh(b*x + a) + sinh(b*x + a) + 1) + 24*(cosh(b*x + a)^3 +
 3*cosh(b*x + a)^2*sinh(b*x + a) + 3*cosh(b*x + a)*sinh(b*x + a)^2 + sinh(b
*x + a)^3)*log(cosh(b*x + a) + sinh(b*x + a) - 1) + 6*(cosh(b*x + a)^5 + 10
*cosh(b*x + a)^3 + 5*cosh(b*x + a))*sinh(b*x + a) + 1)/(b*cosh(b*x + a)^3 +
```

```
3*b*cosh(b*x + a)^2*sinh(b*x + a) + 3*b*cosh(b*x + a)*sinh(b*x + a)^2 + b*
sinh(b*x + a)^3)
```

giac [B] time = 0.16, size = 77, normalized size = 2.03

$$\frac{\left(15\,e^{(2\,bx+2\,a)}+1\right)e^{(-3\,bx-3\,a)}+\left(e^{(3\,bx+18\,a)}+15\,e^{(bx+16\,a)}\right)e^{(-15\,a)}-24\,\log\left(e^{(bx+a)}+1\right)+24\,\log\left(\left|e^{(bx+a)}-1\right|\right)}{24\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^3*coth(b*x+a),x, algorithm="giac")
```

```
[Out] 1/24*((15*e^(2*b*x + 2*a) + 1)*e^(-3*b*x - 3*a) + (e^(3*b*x + 18*a) + 15*e^
(b*x + 16*a))*e^(-15*a) - 24*log(e^(b*x + a) + 1) + 24*log(abs(e^(b*x + a)
- 1)))/b
```

maple [A] time = 0.12, size = 31, normalized size = 0.82

$$\frac{\frac{\left(\cosh^3(bx+a)\right)}{3}+\cosh\left(bx+a\right)-2\,\mathrm{arctanh}\left(\mathrm{e}^{bx+a}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)^3*coth(b*x+a),x)
```

```
[Out] 1/b*(1/3*cosh(b*x+a)^3+cosh(b*x+a)-2*arctanh(exp(b*x+a)))
```

maxima [B] time = 0.32, size = 87, normalized size = 2.29

$$\frac{\left(15\,e^{(-2\,bx-2\,a)}+1\right)e^{(3\,bx+3\,a)}}{24\,b}+\frac{15\,e^{(-bx-a)}+e^{(-3\,bx-3\,a)}}{24\,b}-\frac{\log\left(e^{(-bx-a)}+1\right)}{b}+\frac{\log\left(e^{(-bx-a)}-1\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^3*coth(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/24*(15*e^(-2*b*x - 2*a) + 1)*e^(3*b*x + 3*a)/b + 1/24*(15*e^(-b*x - a) +
e^(-3*b*x - 3*a))/b - log(e^(-b*x - a) + 1)/b + log(e^(-b*x - a) - 1)/b
```

mupad [B] time = 1.44, size = 81, normalized size = 2.13

$$\frac{5\,\mathrm{e}^{a+b\,x}}{8\,b}-\frac{2\,\mathrm{atan}\left(\frac{\mathrm{e}^{b\,x}\,\mathrm{e}^{a}\,\sqrt{-b^2}}{b}\right)}{\sqrt{-b^2}}+\frac{5\,\mathrm{e}^{-a-b\,x}}{8\,b}+\frac{\mathrm{e}^{-3\,a-3\,b\,x}}{24\,b}+\frac{\mathrm{e}^{3\,a+3\,b\,x}}{24\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a + b*x)^3*coth(a + b*x),x)
```

```
[Out] (5*exp(a + b*x))/(8*b) - (2*atan((exp(b*x)*exp(a)*(-b^2)^(1/2))/b))/(-b^2)^
(1/2) + (5*exp(- a - b*x))/(8*b) + exp(- 3*a - 3*b*x)/(24*b) + exp(3*a + 3*
b*x)/(24*b)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \cosh^3(a+bx)\coth(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)**3*coth(b*x+a),x)
```

```
[Out] Integral(cosh(a + b*x)**3*coth(a + b*x), x)
```

3.109 $\int \cosh^3(a+bx)\coth^2(a+bx)\,dx$

Optimal. Leaf size=38

$$\frac{\sinh^3(a+bx)}{3b}+\frac{2\sinh(a+bx)}{b}-\frac{\operatorname{csch}(a+bx)}{b}$$

```
[Out] -csch(b*x+a)/b+2*sinh(b*x+a)/b+1/3*sinh(b*x+a)^3/b
```

Rubi [A] time = 0.04, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2590, 270}

$$\frac{\sinh^3(a+bx)}{3b}+\frac{2\sinh(a+bx)}{b}-\frac{\operatorname{csch}(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[a + b*x]^3*Coth[a + b*x]^2,x]
```

```
[Out] -(Csch[a + b*x]/b) + (2*Sinh[a + b*x])/b + Sinh[a + b*x]^3/(3*b)
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2590

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> -Dist[f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rubi steps

$$\begin{aligned}
\int \cosh^3(a+bx)\coth^2(a+bx)\,dx &= -\frac{i\,\text{Subst}\left(\int \frac{(1-x^2)^2}{x^2}\,dx,x,-i\sinh(a+bx)\right)}{b}\\
&= -\frac{i\,\text{Subst}\left(\int \left(-2+\frac{1}{x^2}+x^2\right)dx,x,-i\sinh(a+bx)\right)}{b}\\
&= -\frac{\operatorname{csch}(a+bx)}{b}+\frac{2\sinh(a+bx)}{b}+\frac{\sinh^3(a+bx)}{3b}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 38, normalized size = 1.00

$$\frac{\sinh^3(a+bx)}{3b}+\frac{2\sinh(a+bx)}{b}-\frac{\operatorname{csch}(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[a + b*x]^3*Coth[a + b*x]^2,x]
```

```
[Out] -(Csch[a + b*x]/b) + (2*Sinh[a + b*x])/b + Sinh[a + b*x]^3/(3*b)
```

fricas [A] time = 0.44, size = 63, normalized size = 1.66

$$\frac{\cosh(bx+a)^4+\sinh(bx+a)^4+2\left(3\cosh(bx+a)^2+10\right)\sinh(bx+a)^2+20\cosh(bx+a)^2-45}{24\,b\sinh(bx+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^3*coth(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/24*(cosh(b*x + a)^4 + sinh(b*x + a)^4 + 2*(3*cosh(b*x + a)^2 + 10)*sinh(b
*x + a)^2 + 20*cosh(b*x + a)^2 - 45)/(b*sinh(b*x + a))
```

giac [B] time = 0.17, size = 76, normalized size = 2.00

$$-\frac{\left(21\,e^{(2\,bx+2\,a)}+1\right)e^{(-3\,bx-3\,a)}-\left(e^{(3\,bx+24\,a)}+21\,e^{(bx+22\,a)}\right)e^{(-21\,a)}+\frac{48\,e^{(bx+a)}}{e^{(2\,bx+2\,a)}-1}}{24\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^3*coth(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -1/24*((21*e^(2*b*x + 2*a) + 1)*e^(-3*b*x - 3*a) - (e^(3*b*x + 24*a) + 21*e
^(b*x + 22*a))*e^(-21*a) + 48*e^(b*x + a)/(e^(2*b*x + 2*a) - 1))/b
```

maple [A] time = 0.11, size = 52, normalized size = 1.37

$$\frac{\frac{\cosh^4(bx+a)}{3\sinh(bx+a)}+\frac{4\left(\cosh^2(bx+a)\right)}{3\sinh(bx+a)}-\frac{8}{3\sinh(bx+a)}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)^3*coth(b*x+a)^2,x)
```

```
[Out] 1/b*(1/3*cosh(b*x+a)^4/sinh(b*x+a)+4/3*cosh(b*x+a)^2/sinh(b*x+a)-8/3/sinh(b
*x+a))
```

maxima [B] time = 0.33, size = 79, normalized size = 2.08

$$-\frac{21\,e^{(-bx-a)}+e^{(-3\,bx-3\,a)}}{24\,b}+\frac{20\,e^{(-2\,bx-2\,a)}-69\,e^{(-4\,bx-4\,a)}+1}{24\,b\left(e^{(-3\,bx-3\,a)}-e^{(-5\,bx-5\,a)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^3*coth(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -1/24*(21*e^(-b*x - a) + e^(-3*b*x - 3*a))/b + 1/24*(20*e^(-2*b*x - 2*a) -
69*e^(-4*b*x - 4*a) + 1)/(b*(e^(-3*b*x - 3*a) - e^(-5*b*x - 5*a)))
```

mupad [B] time = 0.10, size = 78, normalized size = 2.05

$$\frac{7\,e^{a+b\,x}}{8\,b}-\frac{7\,e^{-a-b\,x}}{8\,b}-\frac{e^{-3\,a-3\,b\,x}}{24\,b}+\frac{e^{3\,a+3\,b\,x}}{24\,b}-\frac{2\,e^{a+b\,x}}{b\,\left(e^{2\,a+2\,b\,x}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a + b*x)^3*coth(a + b*x)^2,x)
```

```
[Out] (7*exp(a + b*x))/(8*b) - (7*exp(- a - b*x))/(8*b) - exp(- 3*a - 3*b*x)/(24*
b) + exp(3*a + 3*b*x)/(24*b) - (2*exp(a + b*x))/(b*(exp(2*a + 2*b*x) - 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \cosh^3(a+bx)\coth^2(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)**3*coth(b*x+a)**2,x)
```

```
[Out] Integral(cosh(a + b*x)**3*coth(a + b*x)**2, x)
```

3.110 $\int \cosh^3(a+bx)\coth^3(a+bx)\,dx$

Optimal. Leaf size=66

$$\frac{5\cosh^3(a+bx)}{6b}+\frac{5\cosh(a+bx)}{2b}-\frac{\cosh^3(a+bx)\coth^2(a+bx)}{2b}-\frac{5\tanh^{-1}(\cosh(a+bx))}{2b}$$

```
[Out] -5/2*arctanh(cosh(b*x+a))/b+5/2*cosh(b*x+a)/b+5/6*cosh(b*x+a)^3/b-1/2*cosh(
b*x+a)^3*coth(b*x+a)^2/b
```

Rubi [A] time = 0.05, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {2592, 288, 302, 206}

$$\frac{5\cosh^3(a+bx)}{6b}+\frac{5\cosh(a+bx)}{2b}-\frac{\cosh^3(a+bx)\coth^2(a+bx)}{2b}-\frac{5\tanh^{-1}(\cosh(a+bx))}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[a + b*x]^3*Coth[a + b*x]^3,x]
```

```
[Out] (-5*ArcTanh[Cosh[a + b*x]])/(2*b) + (5*Cosh[a + b*x])/(2*b) + (5*Cosh[a + b
*x]^3)/(6*b) - (Cosh[a + b*x]^3*Coth[a + b*x]^2)/(2*b)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 2592

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, (a*Sin[e + f*x])/ff], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rubi steps

$$\int \cosh^3(a+bx)\coth^3(a+bx)\,dx = \frac{\text{Subst}\left(\int \frac{x^6}{(1-x^2)^2}\,dx, x, \cosh(a+bx)\right)}{b}$$

$$= -\frac{\cosh^3(a+bx)\coth^2(a+bx)}{2b} - \frac{5\,\text{Subst}\left(\int \frac{x^4}{1-x^2}\,dx, x, \cosh(a+bx)\right)}{2b}$$

$$= -\frac{\cosh^3(a+bx)\coth^2(a+bx)}{2b} - \frac{5\,\text{Subst}\left(\int \left(-1-x^2+\frac{1}{1-x^2}\right)dx, x, \cosh(a+\right.}{2b}$$

$$= \frac{5\cosh(a+bx)}{2b} + \frac{5\cosh^3(a+bx)}{6b} - \frac{\cosh^3(a+bx)\coth^2(a+bx)}{2b} - \frac{5\,\text{Subst}}{}$$

$$= -\frac{5\tanh^{-1}(\cosh(a+bx))}{2b} + \frac{5\cosh(a+bx)}{2b} + \frac{5\cosh^3(a+bx)}{6b} - \frac{\cosh^3(a+}{}$$

Mathematica [A] time = 0.04, size = 85, normalized size = 1.29

$$\frac{9\cosh(a+bx)}{4b}+\frac{\cosh(3(a+bx))}{12b}-\frac{\text{csch}^2\left(\frac{1}{2}(a+bx)\right)}{8b}-\frac{\text{sech}^2\left(\frac{1}{2}(a+bx)\right)}{8b}+\frac{5\log\left(\tanh\left(\frac{1}{2}(a+bx)\right)\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[a + b*x]^3*Coth[a + b*x]^3,x]
```

```
[Out] (9*Cosh[a + b*x])/(4*b) + Cosh[3*(a + b*x)]/(12*b) - Csch[(a + b*x)/2]^2/(8
*b) + (5*Log[Tanh[(a + b*x)/2]])/(2*b) - Sech[(a + b*x)/2]^2/(8*b)
```

fricas [B] time = 0.54, size = 1077, normalized size = 16.32

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^3*coth(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/24*(cosh(b*x + a)^10 + 10*cosh(b*x + a)*sinh(b*x + a)^9 + sinh(b*x + a)^1
0 + 5*(9*cosh(b*x + a)^2 + 5)*sinh(b*x + a)^8 + 25*cosh(b*x + a)^8 + 40*(3*
cosh(b*x + a)^3 + 5*cosh(b*x + a))*sinh(b*x + a)^7 + 10*(21*cosh(b*x + a)^4
 + 70*cosh(b*x + a)^2 - 5)*sinh(b*x + a)^6 - 50*cosh(b*x + a)^6 + 4*(63*cos
h(b*x + a)^5 + 350*cosh(b*x + a)^3 - 75*cosh(b*x + a))*sinh(b*x + a)^5 + 10
*(21*cosh(b*x + a)^6 + 175*cosh(b*x + a)^4 - 75*cosh(b*x + a)^2 - 5)*sinh(b
*x + a)^4 - 50*cosh(b*x + a)^4 + 40*(3*cosh(b*x + a)^7 + 35*cosh(b*x + a)^5
 - 25*cosh(b*x + a)^3 - 5*cosh(b*x + a))*sinh(b*x + a)^3 + 5*(9*cosh(b*x +
a)^8 + 140*cosh(b*x + a)^6 - 150*cosh(b*x + a)^4 - 60*cosh(b*x + a)^2 + 5)*
sinh(b*x + a)^2 + 25*cosh(b*x + a)^2 - 60*(cosh(b*x + a)^7 + 7*cosh(b*x + a
)*sinh(b*x + a)^6 + sinh(b*x + a)^7 + (21*cosh(b*x + a)^2 - 2)*sinh(b*x + a
)^5 - 2*cosh(b*x + a)^5 + 5*(7*cosh(b*x + a)^3 - 2*cosh(b*x + a))*sinh(b*x
+ a)^4 + (35*cosh(b*x + a)^4 - 20*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^3 + co
sh(b*x + a)^3 + (21*cosh(b*x + a)^5 - 20*cosh(b*x + a)^3 + 3*cosh(b*x + a))
*sinh(b*x + a)^2 + (7*cosh(b*x + a)^6 - 10*cosh(b*x + a)^4 + 3*cosh(b*x + a
)^2)*sinh(b*x + a))*log(cosh(b*x + a) + sinh(b*x + a) + 1) + 60*(cosh(b*x +
 a)^7 + 7*cosh(b*x + a)*sinh(b*x + a)^6 + sinh(b*x + a)^7 + (21*cosh(b*x +
a)^2 - 2)*sinh(b*x + a)^5 - 2*cosh(b*x + a)^5 + 5*(7*cosh(b*x + a)^3 - 2*co
sh(b*x + a))*sinh(b*x + a)^4 + (35*cosh(b*x + a)^4 - 20*cosh(b*x + a)^2 + 1
)*sinh(b*x + a)^3 + cosh(b*x + a)^3 + (21*cosh(b*x + a)^5 - 20*cosh(b*x + a
)^3 + 3*cosh(b*x + a))*sinh(b*x + a)^2 + (7*cosh(b*x + a)^6 - 10*cosh(b*x +
 a)^4 + 3*cosh(b*x + a)^2)*sinh(b*x + a))*log(cosh(b*x + a) + sinh(b*x + a)
 - 1) + 10*(cosh(b*x + a)^9 + 20*cosh(b*x + a)^7 - 30*cosh(b*x + a)^5 - 20*
cosh(b*x + a)^3 + 5*cosh(b*x + a))*sinh(b*x + a) + 1)/(b*cosh(b*x + a)^7 +
7*b*cosh(b*x + a)*sinh(b*x + a)^6 + b*sinh(b*x + a)^7 - 2*b*cosh(b*x + a)^5
 + (21*b*cosh(b*x + a)^2 - 2*b)*sinh(b*x + a)^5 + 5*(7*b*cosh(b*x + a)^3 -
2*b*cosh(b*x + a))*sinh(b*x + a)^4 + b*cosh(b*x + a)^3 + (35*b*cosh(b*x + a
)^4 - 20*b*cosh(b*x + a)^2 + b)*sinh(b*x + a)^3 + (21*b*cosh(b*x + a)^5 - 2
0*b*cosh(b*x + a)^3 + 3*b*cosh(b*x + a))*sinh(b*x + a)^2 + (7*b*cosh(b*x +
a)^6 - 10*b*cosh(b*x + a)^4 + 3*b*cosh(b*x + a)^2)*sinh(b*x + a))
```

giac [A] time = 0.22, size = 108, normalized size = 1.64

$$\frac{\left(27\,e^{(2\,bx+2\,a)}+1\right)e^{(-3\,bx-3\,a)}+\left(e^{(3\,bx+30\,a)}+27\,e^{(bx+28\,a)}\right)e^{(-27\,a)}-\frac{24\left(e^{(3\,bx+3\,a)}+e^{(bx+a)}\right)}{\left(e^{(2\,bx+2\,a)}-1\right)^{2}}-60\,\log\left(e^{(bx+a)}+1\right)+60\,\mathrm{lo}}{24\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^3*coth(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/24*((27*e^(2*b*x + 2*a) + 1)*e^(-3*b*x - 3*a) + (e^(3*b*x + 30*a) + 27*e^
(b*x + 28*a))*e^(-27*a) - 24*(e^(3*b*x + 3*a) + e^(b*x + a))/(e^(2*b*x + 2*
a) - 1)^2 - 60*log(e^(b*x + a) + 1) + 60*log(abs(e^(b*x + a) - 1)))/b
```

maple [A] time = 0.34, size = 81, normalized size = 1.23

$$\frac{\frac{\cosh^5(bx+a)}{3\sinh(bx+a)^2}+\frac{5\left(\cosh^3(bx+a)\right)}{3\sinh(bx+a)^2}-\frac{5\cosh(bx+a)}{\sinh(bx+a)^2}+\frac{5\,\mathrm{csch}(bx+a)\coth(bx+a)}{2}-5\,\mathrm{arctanh}\left(\mathrm{e}^{bx+a}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)^3*coth(b*x+a)^3,x)
```

```
[Out] 1/b*(1/3*cosh(b*x+a)^5/sinh(b*x+a)^2+5/3*cosh(b*x+a)^3/sinh(b*x+a)^2-5/sinh
(b*x+a)^2*cosh(b*x+a)+5/2*csch(b*x+a)*coth(b*x+a)-5*arctanh(exp(b*x+a)))
```

maxima [B] time = 0.36, size = 133, normalized size = 2.02

$$\frac{27\,e^{(-bx-a)}+e^{(-3\,bx-3\,a)}}{24\,b}-\frac{5\,\log\left(e^{(-bx-a)}+1\right)}{2\,b}+\frac{5\,\log\left(e^{(-bx-a)}-1\right)}{2\,b}+\frac{25\,e^{(-2\,bx-2\,a)}-77\,e^{(-4\,bx-4\,a)}+3\,e^{(-6\,bx-6\,a)}+1}{24\,b\left(e^{(-3\,bx-3\,a)}-2\,e^{(-5\,bx-5\,a)}+e^{(-7\,bx-7\,a)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^3*coth(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/24*(27*e^(-b*x - a) + e^(-3*b*x - 3*a))/b - 5/2*log(e^(-b*x - a) + 1)/b +
 5/2*log(e^(-b*x - a) - 1)/b + 1/24*(25*e^(-2*b*x - 2*a) - 77*e^(-4*b*x - 4
*a) + 3*e^(-6*b*x - 6*a) + 1)/(b*(e^(-3*b*x - 3*a) - 2*e^(-5*b*x - 5*a) + e
^(-7*b*x - 7*a)))
```

mupad [B] time = 1.49, size = 140, normalized size = 2.12

$$\frac{9\,\mathrm{e}^{a+b\,x}}{8\,b}-\frac{5\,\mathrm{atan}\left(\frac{\mathrm{e}^{b\,x}\,\mathrm{e}^{a}\,\sqrt{-b^2}}{b}\right)}{\sqrt{-b^2}}+\frac{9\,\mathrm{e}^{-a-b\,x}}{8\,b}+\frac{\mathrm{e}^{-3\,a-3\,b\,x}}{24\,b}+\frac{\mathrm{e}^{3\,a+3\,b\,x}}{24\,b}-\frac{2\,\mathrm{e}^{a+b\,x}}{b\,\left(\mathrm{e}^{4\,a+4\,b\,x}-2\,\mathrm{e}^{2\,a+2\,b\,x}+1\right)}-\frac{\mathrm{e}^{a+b\,x}}{b\,\left(\mathrm{e}^{2\,a+2\,b\,x}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a + b*x)^3*coth(a + b*x)^3,x)
```

```
[Out] (9*exp(a + b*x))/(8*b) - (5*atan((exp(b*x)*exp(a)*(-b^2)^(1/2))/b))/(-b^2)^
(1/2) + (9*exp(- a - b*x))/(8*b) + exp(- 3*a - 3*b*x)/(24*b) + exp(3*a + 3*
b*x)/(24*b) - (2*exp(a + b*x))/(b*(exp(4*a + 4*b*x) - 2*exp(2*a + 2*b*x) +
1)) - exp(a + b*x)/(b*(exp(2*a + 2*b*x) - 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\cosh^3(a+bx)\coth^3(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)**3*coth(b*x+a)**3,x)
```

```
[Out] Integral(cosh(a + b*x)**3*coth(a + b*x)**3, x)
```

3.111 $\int \cosh^4(a+bx)\coth(a+bx)\,dx$

Optimal. Leaf size=39

$$\frac{\sinh^4(a+bx)}{4b}+\frac{\sinh^2(a+bx)}{b}+\frac{\log(\sinh(a+bx))}{b}$$

```
[Out] ln(sinh(b*x+a))/b+sinh(b*x+a)^2/b+1/4*sinh(b*x+a)^4/b
```

Rubi [A] time = 0.03, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2590, 266, 43}

$$\frac{\sinh^4(a+bx)}{4b}+\frac{\sinh^2(a+bx)}{b}+\frac{\log(\sinh(a+bx))}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[a + b*x]^4*Coth[a + b*x],x]
```

```
[Out] Log[Sinh[a + b*x]]/b + Sinh[a + b*x]^2/b + Sinh[a + b*x]^4/(4*b)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2590

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> -Dist[f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rubi steps

$$\int \cosh^4(a+bx)\coth(a+bx)\,dx = \frac{\text{Subst}\left(\int \frac{\left(1-x^2\right)^2}{x}\,dx, x, -i\sinh(a+bx)\right)}{b}$$
$$= \frac{\text{Subst}\left(\int \frac{(1-x)^2}{x}\,dx, x, -\sinh^2(a+bx)\right)}{2b}$$
$$= \frac{\text{Subst}\left(\int \left(-2+\frac{1}{x}+x\right)dx, x, -\sinh^2(a+bx)\right)}{2b}$$
$$= \frac{\log(\sinh(a+bx))}{b} + \frac{\sinh^2(a+bx)}{b} + \frac{\sinh^4(a+bx)}{4b}$$

Mathematica [A] time = 0.03, size = 35, normalized size = 0.90

$$\frac{\sinh^4(a+bx)+4\sinh^2(a+bx)+4\log(\sinh(a+bx))}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[a + b*x]^4*Coth[a + b*x],x]
```

```
[Out] (4*Log[Sinh[a + b*x]] + 4*Sinh[a + b*x]^2 + Sinh[a + b*x]^4)/(4*b)
```

fricas [B] time = 0.47, size = 457, normalized size = 11.72

$$\frac{\cosh(bx+a)^8 + 8\cosh(bx+a)\sinh(bx+a)^7 + \sinh(bx+a)^8 + 4\left(7\cosh(bx+a)^2+3\right)\sinh(bx+a)^6 - 64}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^4*coth(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/64*(cosh(b*x + a)^8 + 8*cosh(b*x + a)*sinh(b*x + a)^7 + sinh(b*x + a)^8 +
 4*(7*cosh(b*x + a)^2 + 3)*sinh(b*x + a)^6 - 64*b*x*cosh(b*x + a)^4 + 12*co
sh(b*x + a)^6 + 8*(7*cosh(b*x + a)^3 + 9*cosh(b*x + a))*sinh(b*x + a)^5 + 2
*(35*cosh(b*x + a)^4 - 32*b*x + 90*cosh(b*x + a)^2)*sinh(b*x + a)^4 + 8*(7*
cosh(b*x + a)^5 - 32*b*x*cosh(b*x + a) + 30*cosh(b*x + a)^3)*sinh(b*x + a)^
3 + 4*(7*cosh(b*x + a)^6 - 96*b*x*cosh(b*x + a)^2 + 45*cosh(b*x + a)^4 + 3)
*sinh(b*x + a)^2 + 12*cosh(b*x + a)^2 + 64*(cosh(b*x + a)^4 + 4*cosh(b*x +
a)^3*sinh(b*x + a) + 6*cosh(b*x + a)^2*sinh(b*x + a)^2 + 4*cosh(b*x + a)*si
nh(b*x + a)^3 + sinh(b*x + a)^4)*log(2*sinh(b*x + a)/(cosh(b*x + a) - sinh(
```

```
b*x + a))) + 8*(cosh(b*x + a)^7 - 32*b*x*cosh(b*x + a)^3 + 9*cosh(b*x + a)^
5 + 3*cosh(b*x + a))*sinh(b*x + a) + 1)/(b*cosh(b*x + a)^4 + 4*b*cosh(b*x +
 a)^3*sinh(b*x + a) + 6*b*cosh(b*x + a)^2*sinh(b*x + a)^2 + 4*b*cosh(b*x +
a)*sinh(b*x + a)^3 + b*sinh(b*x + a)^4)
```

giac [B] time = 0.16, size = 87, normalized size = 2.23

$$-\frac{64\,bx - \left(48\,e^{(4\,bx+4\,a)} + 12\,e^{(2\,bx+2\,a)} + 1\right)e^{(-4\,bx-4\,a)} - \left(e^{(4\,bx+16\,a)} + 12\,e^{(2\,bx+14\,a)}\right)e^{(-12\,a)} - 64\,\log\left(\left|e^{(2\,bx+2\,a)} - 1\right|\right)}{64\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^4*coth(b*x+a),x, algorithm="giac")
```

```
[Out] -1/64*(64*b*x - (48*e^(4*b*x + 4*a) + 12*e^(2*b*x + 2*a) + 1)*e^(-4*b*x - 4
*a) - (e^(4*b*x + 16*a) + 12*e^(2*b*x + 14*a))*e^(-12*a) - 64*log(abs(e^(2*
b*x + 2*a) - 1)))/b
```

maple [A] time = 0.12, size = 39, normalized size = 1.00

$$\frac{\cosh^4(bx+a)}{4b} + \frac{\cosh^2(bx+a)}{2b} + \frac{\ln(\sinh(bx+a))}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)^4*coth(b*x+a),x)
```

```
[Out] 1/4*cosh(b*x+a)^4/b+1/2*cosh(b*x+a)^2/b+ln(sinh(b*x+a))/b
```

maxima [B] time = 0.37, size = 95, normalized size = 2.44

$$\frac{\left(12\,e^{(-2\,bx-2\,a)} + 1\right)e^{(4\,bx+4\,a)}}{64\,b} + \frac{bx+a}{b} + \frac{12\,e^{(-2\,bx-2\,a)} + e^{(-4\,bx-4\,a)}}{64\,b} + \frac{\log\left(e^{(-bx-a)} + 1\right)}{b} + \frac{\log\left(e^{(-bx-a)} - 1\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^4*coth(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/64*(12*e^(-2*b*x - 2*a) + 1)*e^(4*b*x + 4*a)/b + (b*x + a)/b + 1/64*(12*e
^(-2*b*x - 2*a) + e^(-4*b*x - 4*a))/b + log(e^(-b*x - a) + 1)/b + log(e^(-b
*x - a) - 1)/b
```

mupad [B] time = 0.11, size = 77, normalized size = 1.97

$$\frac{\ln\left(\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x} - 1\right)}{b} - x + \frac{3\,\mathrm{e}^{-2\,a-2\,b\,x}}{16\,b} + \frac{3\,\mathrm{e}^{2\,a+2\,b\,x}}{16\,b} + \frac{\mathrm{e}^{-4\,a-4\,b\,x}}{64\,b} + \frac{\mathrm{e}^{4\,a+4\,b\,x}}{64\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a + b*x)^4*coth(a + b*x),x)
```

```
[Out] log(exp(2*a)*exp(2*b*x) - 1)/b - x + (3*exp(- 2*a - 2*b*x))/(16*b) + (3*exp
(2*a + 2*b*x))/(16*b) + exp(- 4*a - 4*b*x)/(64*b) + exp(4*a + 4*b*x)/(64*b)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \cosh^4(a+bx)\coth(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)**4*coth(b*x+a),x)
```

```
[Out] Integral(cosh(a + b*x)**4*coth(a + b*x), x)
```

3.112 $\int \coth(a+bx)\mathbf{csch}(a+bx)\,dx$

Optimal. Leaf size=11

$$-\frac{\operatorname{csch}(a+bx)}{b}$$

```
[Out] -csch(b*x+a)/b
```

Rubi [A] time = 0.01, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2606, 8}

$$-\frac{\operatorname{csch}(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[a + b*x]*Csch[a + b*x],x]
```

```
[Out] -(Csch[a + b*x]/b)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rubi steps

$$\begin{aligned}\int \coth(a+bx)\operatorname{csch}(a+bx)\,dx &= -\frac{i\,\text{Subst}\left(\int 1\,dx, x, -i\operatorname{csch}(a+bx)\right)}{b}\\ &= -\frac{\operatorname{csch}(a+bx)}{b}\end{aligned}$$

Mathematica [A] time = 0.01, size = 11, normalized size = 1.00

$$-\frac{\operatorname{csch}(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[a + b*x]*Csch[a + b*x],x]
```

```
[Out] -(Csch[a + b*x]/b)
```

fricas [B] time = 0.38, size = 56, normalized size = 5.09

$$-\frac{2\left(\cosh\left(bx+a\right)+\sinh\left(bx+a\right)\right)}{b\cosh\left(bx+a\right)^{2}+2\,b\cosh\left(bx+a\right)\sinh\left(bx+a\right)+b\sinh\left(bx+a\right)^{2}-b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(b*x+a)*csch(b*x+a),x, algorithm="fricas")
```

```
[Out] -2*(cosh(b*x + a) + sinh(b*x + a))/(b*cosh(b*x + a)^2 + 2*b*cosh(b*x + a)*s
inh(b*x + a) + b*sinh(b*x + a)^2 - b)
```

giac [B] time = 0.13, size = 24, normalized size = 2.18

$$-\frac{2\,e^{\left(bx+a\right)}}{b\left(e^{\left(2\,bx+2\,a\right)}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(b*x+a)*csch(b*x+a),x, algorithm="giac")
```

```
[Out] -2*e^(b*x + a)/(b*(e^(2*b*x + 2*a) - 1))
```

maple [A] time = 0.11, size = 12, normalized size = 1.09

$$-\frac{\operatorname{csch}\left(bx+a\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(b*x+a)*coth(b*x+a),x)
```

```
[Out] -csch(b*x+a)/b
```

maxima [B] time = 0.40, size = 25, normalized size = 2.27

$$-\frac{2}{b\left(e^{\left(bx+a\right)}-e^{\left(-bx-a\right)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(b*x+a)*csch(b*x+a),x, algorithm="maxima")
```

```
[Out] -2/(b*(e^(b*x + a) - e^(-b*x - a)))
```

mupad [B] time = 0.06, size = 24, normalized size = 2.18

$$-\frac{2\,{\mathrm{e}}^{a+b\,x}}{b\,\left({\mathrm{e}}^{2\,a+2\,b\,x}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(a + b*x)/sinh(a + b*x),x)
```

```
[Out] -(2*exp(a + b*x))/(b*(exp(2*a + 2*b*x) - 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \coth{\left (a + b x \right)} \operatorname{csch}{\left (a + b x \right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(b*x+a)*csch(b*x+a),x)
```

```
[Out] Integral(coth(a + b*x)*csch(a + b*x), x)
```

3.113 $\int \coth(a+bx)\mathbf{csch}^2(a+bx)\,dx$

Optimal. Leaf size=15

$$-\frac{\operatorname{csch}^2(a+bx)}{2b}$$

```
[Out] -1/2*csch(b*x+a)^2/b
```

Rubi [A] time = 0.02, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2606, 30}

$$-\frac{\operatorname{csch}^2(a+bx)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[a + b*x]*Csch[a + b*x]^2,x]
```

```
[Out] -Csch[a + b*x]^2/(2*b)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rubi steps

$$\begin{aligned}\int \coth(a+bx)\operatorname{csch}^2(a+bx)\,dx &= \frac{\operatorname{Subst}\left(\int x\,dx, x, -i\operatorname{csch}(a+bx)\right)}{b}\\ &= -\frac{\operatorname{csch}^2(a+bx)}{2b}\end{aligned}$$

Mathematica [A] time = 0.01, size = 15, normalized size = 1.00

$$-\frac{\operatorname{csch}^2(a+bx)}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[a + b*x]*Csch[a + b*x]^2,x]
```

```
[Out] -1/2*Csch[a + b*x]^2/b
```

fricas [B] time = 0.43, size = 86, normalized size = 5.73

$$-\frac{2 \left(\cosh\left(bx+a\right)+\sinh\left(bx+a\right)\right)}{b \cosh\left(bx+a\right)^{3}+3 \, b \cosh\left(bx+a\right) \sinh\left(bx+a\right)^{2}+b \sinh\left(bx+a\right)^{3}-b \cosh\left(bx+a\right)+3 \left(b \cosh\left(bx+a\right)^{2}-b\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(b*x+a)*csch(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -2*(cosh(b*x + a) + sinh(b*x + a))/(b*cosh(b*x + a)^3 + 3*b*cosh(b*x + a)*s
inh(b*x + a)^2 + b*sinh(b*x + a)^3 - b*cosh(b*x + a) + 3*(b*cosh(b*x + a)^2
 - b)*sinh(b*x + a))
```

giac [B] time = 0.14, size = 27, normalized size = 1.80

$$-\frac{2 \, e^{\left(2 \, bx+2 \, a\right)}}{b {\left(e^{\left(2 \, bx+2 \, a\right)}-1\right)}^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(b*x+a)*csch(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -2*e^(2*b*x + 2*a)/(b*(e^(2*b*x + 2*a) - 1)^2)
```

maple [A] time = 0.14, size = 14, normalized size = 0.93

$$-\frac{\operatorname{csch}\left(bx+a\right)^{2}}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(b*x+a)*csch(b*x+a)^2,x)
```

```
[Out] -1/2*csch(b*x+a)^2/b
```

maxima [A] time = 0.35, size = 13, normalized size = 0.87

$$-\frac{\coth\left(bx+a\right)^{2}}{2 \, b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(b*x+a)*csch(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*coth(b*x + a)^2/b
```

mupad [B] time = 0.07, size = 13, normalized size = 0.87

$$-\frac{1}{2\,b\,\sinh\left(a+b\,x\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(a + b*x)/sinh(a + b*x)^2,x)
```

```
[Out] -1/(2*b*sinh(a + b*x)^2)
```

sympy [A] time = 2.62, size = 22, normalized size = 1.47

$$\begin{cases} -\frac{\operatorname{csch}^{2}{\left(a+bx \right)}}{2b} & \text{for}\: b \neq 0 \\ x \coth{\left(a \right)} \operatorname{csch}^{2}{\left(a \right)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(b*x+a)*csch(b*x+a)**2,x)
```

```
[Out] Piecewise((-csch(a + b*x)**2/(2*b), Ne(b, 0)), (x*coth(a)*csch(a)**2, True)
)
```

3.114 $\int \cosh(a+bx)\mathbf{csch}^{1+n}(a+bx)\,dx$

Optimal. Leaf size=16

$$-\frac{\operatorname{csch}^n(a+bx)}{bn}$$

```
[Out] -csch(b*x+a)^n/b/n
```

Rubi [A] time = 0.03, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2621, 30}

$$-\frac{\operatorname{csch}^n(a+bx)}{bn}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[a + b*x]*Csch[a + b*x]^(1 + n),x]
```

```
[Out] -(Csch[a + b*x]^n/(b*n))
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2621

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> -Dist[(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] && !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rubi steps

$$\begin{aligned}\int \cosh(a+bx)\operatorname{csch}^{1+n}(a+bx)\,dx &= -\frac{\operatorname{Subst}\left(\int x^{-1+n}\,dx, x, \operatorname{csch}(a+bx)\right)}{b}\\ &= -\frac{\operatorname{csch}^n(a+bx)}{bn}\end{aligned}$$

Mathematica [A] time = 0.02, size = 16, normalized size = 1.00

$$-\frac{\operatorname{csch}^n(a+bx)}{bn}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[a + b*x]*Csch[a + b*x]^(1 + n),x]
```

```
[Out] -(Csch[a + b*x]^n/(b*n))
```

fricas [B] time = 0.48, size = 115, normalized size = 7.19

$$-\frac{\cosh\left(n\log\left(\frac{2\,(\cosh(bx+a)+\sinh(bx+a))}{\cosh(bx+a)^2+2\,\cosh(bx+a)\sinh(bx+a)+\sinh(bx+a)^2-1}\right)\right)+\sinh\left(n\log\left(\frac{2\,(\cosh(bx+a)+\sinh(bx+a))}{\cosh(bx+a)^2+2\,\cosh(bx+a)\sinh(bx+a)+\sinh(bx+a)^2-1}\right)\right)}{bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(b*x+a)*csch(b*x+a)^n,x, algorithm="fricas")
```

```
[Out] -(cosh(n*log(2*(cosh(b*x + a) + sinh(b*x + a))/(cosh(b*x + a)^2 + 2*cosh(b*
x + a)*sinh(b*x + a) + sinh(b*x + a)^2 - 1))) + sinh(n*log(2*(cosh(b*x + a)
 + sinh(b*x + a))/(cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b
*x + a)^2 - 1))))/(b*n)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{csch}\left(bx+a\right)^{n}\coth\left(bx+a\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(b*x+a)*csch(b*x+a)^n,x, algorithm="giac")
```

```
[Out] integrate(csch(b*x + a)^n*coth(b*x + a), x)
```

maple [A] time = 0.10, size = 17, normalized size = 1.06

$$-\frac{\operatorname{csch}\left(bx+a\right)^{n}}{bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(b*x+a)*csch(b*x+a)^n,x)
```

```
[Out] -csch(b*x+a)^n/b/n
```

maxima [B] time = 0.52, size = 53, normalized size = 3.31

$$-\frac{2^{n}e^{\left(-(bx+a)n-n\log\left(e^{(-bx-a)}+1\right)-n\log\left(-e^{(-bx-a)}+1\right)\right)}}{bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(b*x+a)*csch(b*x+a)^n,x, algorithm="maxima")
```

```
[Out] -2^n*e^(-(b*x + a)*n - n*log(e^(-b*x - a) + 1) - n*log(-e^(-b*x - a) + 1))/
(b*n)
```

mupad [B] time = 1.48, size = 31, normalized size = 1.94

$$-\frac{\left(\frac{2\,\mathrm{e}^{a+b\,x}}{\mathrm{e}^{2\,a+2\,b\,x}-1}\right)^n}{b\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(a + b*x)*(1/sinh(a + b*x))^n,x)
```

```
[Out] -((2*exp(a + b*x))/(exp(2*a + 2*b*x) - 1))^n/(b*n)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \coth\left(a+bx\right)\operatorname{csch}^n\left(a+bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(b*x+a)*csch(b*x+a)**n,x)
```

```
[Out] Integral(coth(a + b*x)*csch(a + b*x)**n, x)
```

3.115 $\int \coth^2(a+bx)\mathbf{csch}^2(a+bx)\,dx$

Optimal. Leaf size=15

$$-\frac{\coth^3(a+bx)}{3b}$$

```
[Out] -1/3*coth(b*x+a)^3/b
```

Rubi [A] time = 0.03, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2607, 30}

$$-\frac{\coth^3(a+bx)}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[a + b*x]^2*Csch[a + b*x]^2,x]
```

```
[Out] -Coth[a + b*x]^3/(3*b)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rubi steps

$$\begin{aligned}\int \coth^2(a+bx)\text{csch}^2(a+bx)\,dx &= -\frac{i\,\text{Subst}\left(\int x^2\,dx, x, i\coth(a+bx)\right)}{b}\\ &= -\frac{\coth^3(a+bx)}{3b}\end{aligned}$$

Mathematica [A] time = 0.00, size = 15, normalized size = 1.00

$$-\frac{\coth^3(a+bx)}{3b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[a + b*x]^2*Csch[a + b*x]^2,x]
```

```
[Out] -1/3*Coth[a + b*x]^3/b
```

fricas [B] time = 0.40, size = 139, normalized size = 9.27

$$-\frac{8\left(\cosh\left(bx+a\right)^{2}+\cosh\left(bx+a\right)\sinh\left(bx+a\right)\right.}{3\left(b\cosh\left(bx+a\right)^{4}+4\,b\cosh\left(bx+a\right)\sinh\left(bx+a\right)^{3}+b\sinh\left(bx+a\right)^{4}-4\,b\cosh\left(bx+a\right)^{2}+2\left(3\,b\cosh\left(bx+\right.\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(b*x+a)^2*csch(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -8/3*(cosh(b*x + a)^2 + cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2)/(b*c
osh(b*x + a)^4 + 4*b*cosh(b*x + a)*sinh(b*x + a)^3 + b*sinh(b*x + a)^4 - 4*
b*cosh(b*x + a)^2 + 2*(3*b*cosh(b*x + a)^2 - 2*b)*sinh(b*x + a)^2 + 4*(b*co
sh(b*x + a)^3 - b*cosh(b*x + a))*sinh(b*x + a) + 3*b)
```

giac [B] time = 0.15, size = 31, normalized size = 2.07

$$-\frac{2\left(3\,e^{(4\,bx+4\,a)}+1\right)}{3\,b\left(e^{(2\,bx+2\,a)}-1\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(b*x+a)^2*csch(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -2/3*(3*e^(4*b*x + 4*a) + 1)/(b*(e^(2*b*x + 2*a) - 1)^3)
```

maple [B] time = 0.34, size = 42, normalized size = 2.80

$$\frac{-\frac{\cosh(bx+a)}{2\sinh(bx+a)^{3}}-\frac{\left(\frac{2}{3}-\frac{\operatorname{csch}(bx+a)^{2}}{3}\right)\coth(bx+a)}{2}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(b*x+a)^2*csch(b*x+a)^2,x)
```

```
[Out] 1/b*(-1/2/sinh(b*x+a)^3*cosh(b*x+a)-1/2*(2/3-1/3*csch(b*x+a)^2)*coth(b*x+a)
)
```

maxima [A] time = 0.35, size = 13, normalized size = 0.87

$$-\frac{\coth\left(bx+a\right)^{3}}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(b*x+a)^2*csch(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -1/3*coth(b*x + a)^3/b
```

mupad [B] time = 1.45, size = 31, normalized size = 2.07

$$-\frac{2\left(3\,{\mathrm{e}}^{4\,a+4\,b\,x}+1\right)}{3\,b\,{\left({\mathrm{e}}^{2\,a+2\,b\,x}-1\right)}^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(a + b*x)^2/sinh(a + b*x)^2,x)
```

```
[Out] -(2*(3*exp(4*a + 4*b*x) + 1))/(3*b*(exp(2*a + 2*b*x) - 1)^3)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \coth^2(a+bx)\operatorname{csch}^2(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(b*x+a)**2*csch(b*x+a)**2,x)
```

```
[Out] Integral(coth(a + b*x)**2*csch(a + b*x)**2, x)
```

3.116 $\int \coth^3(a+bx)\mathbf{csch}^2(a+bx)\,dx$

Optimal. Leaf size=15

$$-\frac{\coth^4(a+bx)}{4b}$$

```
[Out] -1/4*coth(b*x+a)^4/b
```

Rubi [A] time = 0.03, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2607, 30}

$$-\frac{\coth^4(a+bx)}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[a + b*x]^3*Csch[a + b*x]^2,x]
```

```
[Out] -Coth[a + b*x]^4/(4*b)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rubi steps

$$\begin{aligned}\int \coth^3(a+bx)\text{csch}^2(a+bx)\,dx &= -\frac{\text{Subst}\left(\int x^3\,dx, x, i\coth(a+bx)\right)}{b}\\ &= -\frac{\coth^4(a+bx)}{4b}\end{aligned}$$

Mathematica [A] time = 0.01, size = 15, normalized size = 1.00

$$-\frac{\coth^4(a+bx)}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[a + b*x]^3*Csch[a + b*x]^2,x]
```

```
[Out] -1/4*Coth[a + b*x]^4/b
```

fricas [B] time = 0.41, size = 208, normalized size = 13.87

$$-\frac{2\left(\cosh\left(bx+a\right)^{3}+3\,\cosh\left(bx+a\right)\right.}{b\cosh\left(bx+a\right)^{5}+5\,b\cosh\left(bx+a\right)\sinh\left(bx+a\right)^{4}+b\sinh\left(bx+a\right)^{5}-3\,b\cosh\left(bx+a\right)^{3}+5\left(2\,b\cosh\left(bx+\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(b*x+a)^3*csch(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -2*(cosh(b*x + a)^3 + 3*cosh(b*x + a)*sinh(b*x + a)^2 + sinh(b*x + a)^3 + (
3*cosh(b*x + a)^2 - 1)*sinh(b*x + a) + cosh(b*x + a))/(b*cosh(b*x + a)^5 +
5*b*cosh(b*x + a)*sinh(b*x + a)^4 + b*sinh(b*x + a)^5 - 3*b*cosh(b*x + a)^3
 + 5*(2*b*cosh(b*x + a)^2 - b)*sinh(b*x + a)^3 + (10*b*cosh(b*x + a)^3 - 9*
b*cosh(b*x + a))*sinh(b*x + a)^2 + 2*b*cosh(b*x + a) + 5*(b*cosh(b*x + a)^4
 - 3*b*cosh(b*x + a)^2 + 2*b)*sinh(b*x + a))
```

giac [B] time = 0.22, size = 37, normalized size = 2.47

$$-\frac{2\left(e^{\left(6\,bx+6\,a\right)}+e^{\left(2\,bx+2\,a\right)}\right)}{b\left(e^{\left(2\,bx+2\,a\right)}-1\right)^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(b*x+a)^3*csch(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -2*(e^(6*b*x + 6*a) + e^(2*b*x + 2*a))/(b*(e^(2*b*x + 2*a) - 1)^4)
```

maple [B] time = 0.16, size = 34, normalized size = 2.27

$$\frac{-\frac{\cosh^{2}\left(bx+a\right)}{2\sinh\left(bx+a\right)^{4}}+\frac{1}{4\sinh\left(bx+a\right)^{4}}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(b*x+a)^3*csch(b*x+a)^2,x)
```

```
[Out] 1/b*(-1/2*cosh(b*x+a)^2/sinh(b*x+a)^4+1/4/sinh(b*x+a)^4)
```

maxima [A] time = 0.39, size = 13, normalized size = 0.87

$$-\frac{\coth\left(bx+a\right)^{4}}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(b*x+a)^3*csch(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -1/4*coth(b*x + a)^4/b
```

mupad [B] time = 1.49, size = 231, normalized size = 15.40

$$-\frac{\frac{1}{2b}+\frac{3e^{2a+2bx}}{2b}+\frac{3e^{4a+4bx}}{2b}+\frac{e^{6a+6bx}}{2b}}{6e^{4a+4bx}-4e^{2a+2bx}-4e^{6a+6bx}+e^{8a+8bx}+1}-\frac{\frac{1}{2b}+\frac{e^{2a+2bx}}{b}+\frac{e^{4a+4bx}}{2b}}{3e^{2a+2bx}-3e^{4a+4bx}+e^{6a+6bx}-1}-\frac{\frac{1}{2b}+\frac{e^{2a+2bx}}{2b}}{e^{4a+4bx}-2e^{2a+2bx}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(a + b*x)^3/sinh(a + b*x)^2,x)
```

```
[Out] - (1/(2*b) + (3*exp(2*a + 2*b*x))/(2*b) + (3*exp(4*a + 4*b*x))/(2*b) + exp(
6*a + 6*b*x)/(2*b))/(6*exp(4*a + 4*b*x) - 4*exp(2*a + 2*b*x) - 4*exp(6*a +
6*b*x) + exp(8*a + 8*b*x) + 1) - (1/(2*b) + exp(2*a + 2*b*x)/b + exp(4*a +
4*b*x)/(2*b))/(3*exp(2*a + 2*b*x) - 3*exp(4*a + 4*b*x) + exp(6*a + 6*b*x) -
 1) - (1/(2*b) + exp(2*a + 2*b*x)/(2*b))/(exp(4*a + 4*b*x) - 2*exp(2*a + 2*
b*x) + 1) - 1/(2*b*(exp(2*a + 2*b*x) - 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \coth^3(a+bx)\operatorname{csch}^2(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(b*x+a)**3*csch(b*x+a)**2,x)
```

```
[Out] Integral(coth(a + b*x)**3*csch(a + b*x)**2, x)
```

3.117 $\int \coth^n(a+bx)\mathrm{csch}^2(a+bx)\,dx$

Optimal. Leaf size=20

$$-\frac{\coth^{n+1}(a+bx)}{b(n+1)}$$

```
[Out] -coth(b*x+a)^(1+n)/b/(1+n)
```

Rubi [A] time = 0.03, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2607, 32}

$$-\frac{\coth^{n+1}(a+bx)}{b(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[a + b*x]^n*Csch[a + b*x]^2,x]
```

```
[Out] -(Coth[a + b*x]^(1 + n)/(b*(1 + n)))
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m + 1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] && !(IntegerQ[(n - 1)/2] && LtQ[0, n, m - 1])
```

Rubi steps

$$\begin{aligned}\int \coth^n(a+bx)\mathrm{csch}^2(a+bx)\,dx &= \frac{i\,\mathrm{Subst}\left(\int (-ix)^n\,dx, x, i\coth(a+bx)\right)}{b}\\ &= -\frac{\coth^{1+n}(a+bx)}{b(1+n)}\end{aligned}$$

Mathematica [A] time = 0.02, size = 20, normalized size = 1.00

$$-\frac{\coth^{n+1}(a+bx)}{b(n+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[a + b*x]^n*Csch[a + b*x]^2,x]
```

```
[Out] -(Coth[a + b*x]^(1 + n)/(b*(1 + n)))
```

fricas [B] time = 0.44, size = 70, normalized size = 3.50

$$-\frac{\cosh(bx+a)\cosh\left(n\log\left(\frac{\cosh(bx+a)}{\sinh(bx+a)}\right)\right)+\cosh(bx+a)\sinh\left(n\log\left(\frac{\cosh(bx+a)}{\sinh(bx+a)}\right)\right)}{(bn+b)\sinh(bx+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(b*x+a)^n*csch(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -(cosh(b*x + a)*cosh(n*log(cosh(b*x + a)/sinh(b*x + a))) + cosh(b*x + a)*si
nh(n*log(cosh(b*x + a)/sinh(b*x + a))))/((b*n + b)*sinh(b*x + a))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \coth(bx+a)^{n}\operatorname{csch}(bx+a)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(b*x+a)^n*csch(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate(coth(b*x + a)^n*csch(b*x + a)^2, x)
```

maple [A] time = 0.15, size = 21, normalized size = 1.05

$$-\frac{\coth^{n+1}(bx+a)}{b(n+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(b*x+a)^n*csch(b*x+a)^2,x)
```

```
[Out] -coth(b*x+a)^(n+1)/b/(n+1)
```

maxima [A] time = 0.78, size = 20, normalized size = 1.00

$$-\frac{\coth(bx+a)^{n+1}}{b(n+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(b*x+a)^n*csch(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -coth(b*x + a)^(n + 1)/(b*(n + 1))
```

mupad [B] time = 1.48, size = 43, normalized size = 2.15

$$-\frac{\coth\left(a+b\,x\right)\,{\left(\frac{{\mathrm{e}}^{2\,a+2\,b\,x}+1}{{\mathrm{e}}^{2\,a+2\,b\,x}-1}\right)}^n}{b\,\left(n+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(a + b*x)^n/sinh(a + b*x)^2,x)
```

```
[Out] -(coth(a + b*x)*((exp(2*a + 2*b*x) + 1)/(exp(2*a + 2*b*x) - 1))^n)/(b*(n +
1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \coth^{n}\left(a+bx\right)\operatorname{csch}^{2}\left(a+bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(b*x+a)**n*csch(b*x+a)**2,x)
```

```
[Out] Integral(coth(a + b*x)**n*csch(a + b*x)**2, x)
```

3.118 $\int \coth^3(a+bx)\mathbf{csch}(a+bx)\,dx$

Optimal. Leaf size=27

$$-\frac{\operatorname{csch}^3(a+bx)}{3b}-\frac{\operatorname{csch}(a+bx)}{b}$$

```
[Out] -csch(b*x+a)/b-1/3*csch(b*x+a)^3/b
```

Rubi [A] time = 0.02, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {2606}

$$-\frac{\operatorname{csch}^3(a+bx)}{3b}-\frac{\operatorname{csch}(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[a + b*x]^3*Csch[a + b*x],x]
```

```
[Out] -(Csch[a + b*x]/b) - Csch[a + b*x]^3/(3*b)
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rubi steps

$$\begin{aligned}\int \coth^3(a+bx)\operatorname{csch}(a+bx)\,dx &= \frac{i\,\text{Subst}\left(\int \left(-1+x^2\right)\,dx,x,-i\operatorname{csch}(a+bx)\right)}{b}\\ &= -\frac{\operatorname{csch}(a+bx)}{b}-\frac{\operatorname{csch}^3(a+bx)}{3b}\end{aligned}$$

Mathematica [A] time = 0.01, size = 27, normalized size = 1.00

$$-\frac{\operatorname{csch}^3(a+bx)}{3b}-\frac{\operatorname{csch}(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[a + b*x]^3*Csch[a + b*x],x]
```

[Out] -(Csch[a + b*x]/b) - Csch[a + b*x]^3/(3*b)

fricas [B] time = 0.39, size = 171, normalized size = 6.33

$$-\frac{2\left(3\,\cosh\left(bx+a\right)^{3}+9\,\cosh\left(bx+a\right)\sinh\left(bx+a\right)^{2}+3\,\sinh\left(bx+a\right)^{3}+(9\right.}{3\left(b\cosh\left(bx+a\right)^{4}+4\,b\cosh\left(bx+a\right)\sinh\left(bx+a\right)^{3}+b\sinh\left(bx+a\right)^{4}-4\,b\cosh\left(bx+a\right)^{2}+2\left(3\,b\cosh\left(bx\right.\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(b*x+a)^3*csch(b*x+a),x, algorithm="fricas")
```

```
[Out] -2/3*(3*cosh(b*x + a)^3 + 9*cosh(b*x + a)*sinh(b*x + a)^2 + 3*sinh(b*x + a)
^3 + (9*cosh(b*x + a)^2 - 5)*sinh(b*x + a) + cosh(b*x + a))/(b*cosh(b*x + a
)^4 + 4*b*cosh(b*x + a)*sinh(b*x + a)^3 + b*sinh(b*x + a)^4 - 4*b*cosh(b*x
+ a)^2 + 2*(3*b*cosh(b*x + a)^2 - 2*b)*sinh(b*x + a)^2 + 4*(b*cosh(b*x + a)
^3 - b*cosh(b*x + a))*sinh(b*x + a) + 3*b)
```

giac [A] time = 0.15, size = 49, normalized size = 1.81

$$-\frac{2\left(3\,e^{(5\,bx+5\,a)}-2\,e^{(3\,bx+3\,a)}+3\,e^{(bx+a)}\right)}{3\,b\left(e^{(2\,bx+2\,a)}-1\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(b*x+a)^3*csch(b*x+a),x, algorithm="giac")
```

```
[Out] -2/3*(3*e^(5*b*x + 5*a) - 2*e^(3*b*x + 3*a) + 3*e^(b*x + a))/(b*(e^(2*b*x +
 2*a) - 1)^3)
```

maple [A] time = 0.18, size = 34, normalized size = 1.26

$$\frac{-\frac{\cosh^{2}(bx+a)}{\sinh(bx+a)^{3}}+\frac{2}{3\sinh(bx+a)^{3}}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(b*x+a)^3*csch(b*x+a),x)
```

```
[Out] 1/b*(-1/sinh(b*x+a)^3*cosh(b*x+a)^2+2/3/sinh(b*x+a)^3)
```

maxima [B] time = 0.32, size = 148, normalized size = 5.48

$$\frac{2\,e^{(-bx-a)}}{b\left(3\,e^{(-2\,bx-2\,a)}-3\,e^{(-4\,bx-4\,a)}+e^{(-6\,bx-6\,a)}-1\right)}-\frac{4\,e^{(-3\,bx-3\,a)}}{3\,b\left(3\,e^{(-2\,bx-2\,a)}-3\,e^{(-4\,bx-4\,a)}+e^{(-6\,bx-6\,a)}-1\right)}+\frac{2}{b\left(3\,e^{(-2\,bx-2\,a)}-3\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(b*x+a)^3*csch(b*x+a),x, algorithm="maxima")
```

```
[Out] 2*e^(-b*x - a)/(b*(3*e^(-2*b*x - 2*a) - 3*e^(-4*b*x - 4*a) + e^(-6*b*x - 6*
a) - 1)) - 4/3*e^(-3*b*x - 3*a)/(b*(3*e^(-2*b*x - 2*a) - 3*e^(-4*b*x - 4*a)
 + e^(-6*b*x - 6*a) - 1)) + 2*e^(-5*b*x - 5*a)/(b*(3*e^(-2*b*x - 2*a) - 3*e
^(-4*b*x - 4*a) + e^(-6*b*x - 6*a) - 1))
```

mupad [B] time = 1.41, size = 48, normalized size = 1.78

$$-\frac{2\,e^{a+b\,x}\,\left(3\,e^{4\,a+4\,b\,x}-2\,e^{2\,a+2\,b\,x}+3\right)}{3\,b\,{\left(e^{2\,a+2\,b\,x}-1\right)}^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(a + b*x)^3/sinh(a + b*x),x)
```

```
[Out] -(2*exp(a + b*x)*(3*exp(4*a + 4*b*x) - 2*exp(2*a + 2*b*x) + 3))/(3*b*(exp(2
*a + 2*b*x) - 1)^3)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \coth^{3}{\left (a + b x \right)} \operatorname{csch}{\left (a + b x \right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(b*x+a)**3*csch(b*x+a),x)
```

```
[Out] Integral(coth(a + b*x)**3*csch(a + b*x), x)
```

3.119 $\int \coth^3(a+bx)\operatorname{csch}^3(a+bx)\,dx$

Optimal. Leaf size=31

$$-\frac{\operatorname{csch}^5(a+bx)}{5b}-\frac{\operatorname{csch}^3(a+bx)}{3b}$$

```
[Out] -1/3*csch(b*x+a)^3/b-1/5*csch(b*x+a)^5/b
```

Rubi [A] time = 0.03, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2606, 14}

$$-\frac{\operatorname{csch}^5(a+bx)}{5b}-\frac{\operatorname{csch}^3(a+bx)}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[a + b*x]^3*Csch[a + b*x]^3,x]
```

```
[Out] -Csch[a + b*x]^3/(3*b) - Csch[a + b*x]^5/(5*b)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rubi steps

$$\begin{aligned}
\int \coth^3(a+bx)\operatorname{csch}^3(a+bx)\,dx &= -\frac{i\,\text{Subst}\left(\int x^2\left(-1+x^2\right)\,dx,x,-i\operatorname{csch}(a+bx)\right)}{b}\\
&= -\frac{i\,\text{Subst}\left(\int \left(-x^2+x^4\right)\,dx,x,-i\operatorname{csch}(a+bx)\right)}{b}\\
&= -\frac{\operatorname{csch}^3(a+bx)}{3b}-\frac{\operatorname{csch}^5(a+bx)}{5b}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 31, normalized size = 1.00

$$-\frac{\operatorname{csch}^5(a+bx)}{5b}-\frac{\operatorname{csch}^3(a+bx)}{3b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[a + b*x]^3*Csch[a + b*x]^3,x]
```

```
[Out] -1/3*Csch[a + b*x]^3/b - Csch[a + b*x]^5/(5*b)
```

fricas [B] time = 0.39, size = 343, normalized size = 11.06

$$-\frac{}{15\left(b\cosh\left(bx+a\right)^7+7\,b\cosh\left(bx+a\right)\sinh\left(bx+a\right)^6+b\sinh\left(bx+a\right)^7-5\,b\cosh\left(bx+a\right)^5+\left(21\,b\cosh\left(bx+\right.\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(b*x+a)^3*csch(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -8/15*(5*cosh(b*x + a)^4 + 20*cosh(b*x + a)*sinh(b*x + a)^3 + 5*sinh(b*x +
a)^4 + 2*(15*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^2 + 2*cosh(b*x + a)^2 + 4*(
5*cosh(b*x + a)^3 + cosh(b*x + a))*sinh(b*x + a) + 5)/(b*cosh(b*x + a)^7 +
7*b*cosh(b*x + a)*sinh(b*x + a)^6 + b*sinh(b*x + a)^7 - 5*b*cosh(b*x + a)^5
 + (21*b*cosh(b*x + a)^2 - 5*b)*sinh(b*x + a)^5 + 5*(7*b*cosh(b*x + a)^3 -
5*b*cosh(b*x + a))*sinh(b*x + a)^4 + 9*b*cosh(b*x + a)^3 + (35*b*cosh(b*x +
 a)^4 - 50*b*cosh(b*x + a)^2 + 11*b)*sinh(b*x + a)^3 + (21*b*cosh(b*x + a)^
5 - 50*b*cosh(b*x + a)^3 + 27*b*cosh(b*x + a))*sinh(b*x + a)^2 - 5*b*cosh(b
*x + a) + (7*b*cosh(b*x + a)^6 - 25*b*cosh(b*x + a)^4 + 33*b*cosh(b*x + a)^
2 - 15*b)*sinh(b*x + a))
```

giac [A] time = 0.15, size = 52, normalized size = 1.68

$$-\frac{8\left(5\,e^{(7\,bx+7\,a)}+2\,e^{(5\,bx+5\,a)}+5\,e^{(3\,bx+3\,a)}\right)}{15\,b\left(e^{(2\,bx+2\,a)}-1\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(b*x+a)^3*csch(b*x+a)^3,x, algorithm="giac")
```

```
[Out] -8/15*(5*e^(7*b*x + 7*a) + 2*e^(5*b*x + 5*a) + 5*e^(3*b*x + 3*a))/(b*(e^(2*
b*x + 2*a) - 1)^5)
```

maple [A] time = 0.16, size = 34, normalized size = 1.10

$$\frac{-\frac{\cosh^2(bx+a)}{3\sinh(bx+a)^5}+\frac{2}{15\sinh(bx+a)^5}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(b*x+a)^3*csch(b*x+a)^3,x)
```

```
[Out] 1/b*(-1/3/sinh(b*x+a)^5*cosh(b*x+a)^2+2/15/sinh(b*x+a)^5)
```

maxima [B] time = 0.48, size = 214, normalized size = 6.90

$$\frac{8\,e^{(-3\,bx-3\,a)}}{3\,b\left(5\,e^{(-2\,bx-2\,a)}-10\,e^{(-4\,bx-4\,a)}+10\,e^{(-6\,bx-6\,a)}-5\,e^{(-8\,bx-8\,a)}+e^{(-10\,bx-10\,a)}-1\right)}+\frac{}{15\,b\left(5\,e^{(-2\,bx-2\,a)}-10\,e^{(-4\,bx-4\,a)}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(b*x+a)^3*csch(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 8/3*e^(-3*b*x - 3*a)/(b*(5*e^(-2*b*x - 2*a) - 10*e^(-4*b*x - 4*a) + 10*e^(-
6*b*x - 6*a) - 5*e^(-8*b*x - 8*a) + e^(-10*b*x - 10*a) - 1)) + 16/15*e^(-5*
b*x - 5*a)/(b*(5*e^(-2*b*x - 2*a) - 10*e^(-4*b*x - 4*a) + 10*e^(-6*b*x - 6*
a) - 5*e^(-8*b*x - 8*a) + e^(-10*b*x - 10*a) - 1)) + 8/3*e^(-7*b*x - 7*a)/(
b*(5*e^(-2*b*x - 2*a) - 10*e^(-4*b*x - 4*a) + 10*e^(-6*b*x - 6*a) - 5*e^(-8
*b*x - 8*a) + e^(-10*b*x - 10*a) - 1))
```

mupad [B] time = 1.51, size = 252, normalized size = 8.13

$$-\frac{\frac{4\,e^{a+b\,x}}{5\,b}+\frac{12\,e^{3\,a+3\,b\,x}}{5\,b}+\frac{12\,e^{5\,a+5\,b\,x}}{5\,b}+\frac{4\,e^{7\,a+7\,b\,x}}{5\,b}}{5\,e^{2\,a+2\,b\,x}-10\,e^{4\,a+4\,b\,x}+10\,e^{6\,a+6\,b\,x}-5\,e^{8\,a+8\,b\,x}+e^{10\,a+10\,b\,x}-1}-\frac{28\,e^{a+b\,x}}{15\,b\,\left(e^{4\,a+4\,b\,x}-2\,e^{2\,a+2\,b\,x}+1\right)}-\frac{}{15\,b\,\left(3\,e^{2\,a}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(a + b*x)^3/sinh(a + b*x)^3,x)
```

```
[Out] - ((4*exp(a + b*x))/(5*b) + (12*exp(3*a + 3*b*x))/(5*b) + (12*exp(5*a + 5*b
*x))/(5*b) + (4*exp(7*a + 7*b*x))/(5*b))/(5*exp(2*a + 2*b*x) - 10*exp(4*a +
 4*b*x) + 10*exp(6*a + 6*b*x) - 5*exp(8*a + 8*b*x) + exp(10*a + 10*b*x) - 1
) - (28*exp(a + b*x))/(15*b*(exp(4*a + 4*b*x) - 2*exp(2*a + 2*b*x) + 1)) -
(64*exp(a + b*x))/(15*b*(3*exp(2*a + 2*b*x) - 3*exp(4*a + 4*b*x) + exp(6*a
+ 6*b*x) - 1)) - (16*exp(a + b*x))/(5*b*(6*exp(4*a + 4*b*x) - 4*exp(2*a + 2
*b*x) - 4*exp(6*a + 6*b*x) + exp(8*a + 8*b*x) + 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \coth^{3}{\left(a + bx \right)} \operatorname{csch}^{3}{\left(a + bx \right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(b*x+a)**3*csch(b*x+a)**3,x)
```

```
[Out] Integral(coth(a + b*x)**3*csch(a + b*x)**3, x)
```

3.120 $\int \cosh^3(a+bx)\mathbf{csch}^{3+n}(a+bx)\,dx$

Optimal. Leaf size=37

$$-\frac{\text{csch}^{n+2}(a+bx)}{b(n+2)}-\frac{\text{csch}^n(a+bx)}{bn}$$

```
[Out] -csch(b*x+a)^n/b/n-csch(b*x+a)^(2+n)/b/(2+n)
```

Rubi [A] time = 0.04, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2621, 14}

$$-\frac{\text{csch}^n(a+bx)}{bn}-\frac{\text{csch}^{n+2}(a+bx)}{b(n+2)}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[a + b*x]^3*Csch[a + b*x]^(3 + n),x]
```

```
[Out] -(Csch[a + b*x]^n/(b*n)) - Csch[a + b*x]^(2 + n)/(b*(2 + n))
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2621

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> -Dist[(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] && !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rubi steps

$$\begin{aligned}\int \cosh^3(a+bx)\text{csch}^{3+n}(a+bx)\,dx &= \frac{\text{Subst}\left(\int x^{-1+n}\left(-1-x^2\right)\,dx,x,\text{csch}(a+bx)\right)}{b}\\ &= \frac{\text{Subst}\left(\int \left(-x^{-1+n}-x^{1+n}\right)\,dx,x,\text{csch}(a+bx)\right)}{b}\\ &= -\frac{\text{csch}^n(a+bx)}{bn}-\frac{\text{csch}^{2+n}(a+bx)}{b(2+n)}\end{aligned}$$

Mathematica [A] time = 0.07, size = 34, normalized size = 0.92

$$-\frac{\operatorname{csch}^n(a+bx)\left(n\operatorname{csch}^2(a+bx)+n+2\right)}{bn(n+2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[a + b*x]^3*Csch[a + b*x]^(3 + n),x]
```

```
[Out] -((Csch[a + b*x]^n*(2 + n + n*Csch[a + b*x]^2))/(b*n*(2 + n)))
```

fricas [B] time = 0.45, size = 216, normalized size = 5.84

$$\frac{\left((n+2)\cosh(bx+a)^2+(n+2)\sinh(bx+a)^2+n-2\right)\cosh\left(n\log\left(\frac{2\,(\cosh(bx+a)+\sinh(bx+a))}{\cosh(bx+a)^2+2\,\cosh(bx+a)\sinh(bx+a)+\sinh(bx+a)^2-}\right.\right.}{bn^2-\left(bn^2+2\,bn\right)\cosh(bx+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(b*x+a)^3*csch(b*x+a)^n,x, algorithm="fricas")
```

```
[Out] (((n + 2)*cosh(b*x + a)^2 + (n + 2)*sinh(b*x + a)^2 + n - 2)*cosh(n*log(2*(
cosh(b*x + a) + sinh(b*x + a))/(cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x
+ a) + sinh(b*x + a)^2 - 1))) + ((n + 2)*cosh(b*x + a)^2 + (n + 2)*sinh(b*x
 + a)^2 + n - 2)*sinh(n*log(2*(cosh(b*x + a) + sinh(b*x + a))/(cosh(b*x + a
)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 - 1))))/(b*n^2 - (b*n
^2 + 2*b*n)*cosh(b*x + a)^2 - (b*n^2 + 2*b*n)*sinh(b*x + a)^2 + 2*b*n)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{csch}(bx+a)^n \coth(bx+a)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(b*x+a)^3*csch(b*x+a)^n,x, algorithm="giac")
```

```
[Out] integrate(csch(b*x + a)^n*coth(b*x + a)^3, x)
```

maple [C] time = 0.50, size = 499, normalized size = 13.49

$$-\frac{\left(n\,\mathrm{e}^{4bx+4a}+2\,\mathrm{e}^{4bx+4a}+2\,\mathrm{e}^{2bx+2a}n-4\,\mathrm{e}^{2bx+2a}+n+2\right)\mathrm{e}^{\frac{n\left(-i\operatorname{csgn}\left(\frac{i}{\left(1+\mathrm{e}^{bx+a}\right)\left(\mathrm{e}^{bx+a}-1\right)}\right)^3\pi+i\operatorname{csgn}\left(\frac{i}{\left(1+\mathrm{e}^{bx+a}\right)\left(\mathrm{e}^{bx+a}-1\right)}\right)^2\operatorname{csgn}\left(\frac{i}{1+\mathrm{e}^{bx+a}}\right.\right.}{}}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(b*x+a)^3*csch(b*x+a)^n,x)
```

```
[Out] -(n*exp(4*b*x+4*a)+2*exp(4*b*x+4*a)+2*exp(2*b*x+2*a)*n-4*exp(2*b*x+2*a)+n+2
)/b/n/(n+2)/(exp(2*b*x+2*a)-1)^2*exp(1/2*n*(-I*csgn(I/(1+exp(b*x+a))/(exp(b
*x+a)-1))^3*Pi+I*csgn(I/(1+exp(b*x+a))/(exp(b*x+a)-1))^2*csgn(I/(1+exp(b*x+
a)))*Pi+I*csgn(I/(1+exp(b*x+a))/(exp(b*x+a)-1))^2*csgn(I/(exp(b*x+a)-1))*Pi
-I*csgn(I/(1+exp(b*x+a))/(exp(b*x+a)-1))*csgn(I/(1+exp(b*x+a)))*csgn(I/(exp
(b*x+a)-1))*Pi+I*csgn(I/(1+exp(b*x+a))/(exp(b*x+a)-1))*csgn(I*exp(b*x+a)/(1
+exp(b*x+a))/(exp(b*x+a)-1))^2*Pi-I*csgn(I/(1+exp(b*x+a))/(exp(b*x+a)-1))*c
sgn(I*exp(b*x+a)/(1+exp(b*x+a))/(exp(b*x+a)-1))*csgn(I*exp(b*x+a))*Pi-I*csg
n(I*exp(b*x+a)/(1+exp(b*x+a))/(exp(b*x+a)-1))^3*Pi+I*csgn(I*exp(b*x+a)/(1+e
xp(b*x+a))/(exp(b*x+a)-1))^2*csgn(I*exp(b*x+a))*Pi+2*ln(exp(b*x+a))-2*ln(ex
p(b*x+a)-1)+2*ln(2)-2*ln(1+exp(b*x+a))))
```

maxima [B] time = 0.50, size = 414, normalized size = 11.19

$$-\frac{2^{n}ne^{\left(-(bx+a)n-n\log\left(e^{(-bx-a)}+1\right)-n\log\left(-e^{(-bx-a)}+1\right)\right)}}{\left(n^{2}-2\left(n^{2}+2n\right)e^{(-2bx-2a)}+\left(n^{2}+2n\right)e^{(-4bx-4a)}+2n\right)b}-\frac{\left(2^{n+1}n-2^{n+2}\right)e^{\left(-(bx+a)n-2bx-n\log\left(e^{(-bx-a)}+1\right)-n\log\left(-e^{(-\right.\right.}}{\left(n^{2}-2\left(n^{2}+2n\right)e^{(-2bx-2a)}+\left(n^{2}+2n\right)e^{(-4bx-4a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(b*x+a)^3*csch(b*x+a)^n,x, algorithm="maxima")
```

```
[Out] -2^n*n*e^(-(b*x + a)*n - n*log(e^(-b*x - a) + 1) - n*log(-e^(-b*x - a) + 1)
)/((n^2 - 2*(n^2 + 2*n)*e^(-2*b*x - 2*a) + (n^2 + 2*n)*e^(-4*b*x - 4*a) + 2
*n)*b) - (2^(n + 1)*n - 2^(n + 2))*e^(-(b*x + a)*n - 2*b*x - n*log(e^(-b*x
- a) + 1) - n*log(-e^(-b*x - a) + 1) - 2*a)/((n^2 - 2*(n^2 + 2*n)*e^(-2*b*x
 - 2*a) + (n^2 + 2*n)*e^(-4*b*x - 4*a) + 2*n)*b) - (2^n*n + 2^(n + 1))*e^(-
(b*x + a)*n - 4*b*x - n*log(e^(-b*x - a) + 1) - n*log(-e^(-b*x - a) + 1) -
4*a)/((n^2 - 2*(n^2 + 2*n)*e^(-2*b*x - 2*a) + (n^2 + 2*n)*e^(-4*b*x - 4*a)
+ 2*n)*b) - 2^(n + 1)*e^(-(b*x + a)*n - n*log(e^(-b*x - a) + 1) - n*log(-e^
(-b*x - a) + 1))/((n^2 - 2*(n^2 + 2*n)*e^(-2*b*x - 2*a) + (n^2 + 2*n)*e^(-4
*b*x - 4*a) + 2*n)*b)
```

mupad [B] time = 1.55, size = 100, normalized size = 2.70

$$-\frac{\left(\frac{1}{\frac{e^{a+b\,x}}{2}-\frac{e^{-a-b\,x}}{2}}\right)^{n}\left(\frac{1}{b\,n}+\frac{e^{4\,a+4\,b\,x}}{b\,n}+\frac{e^{2\,a+2\,b\,x}\,(2\,n-4)}{b\,n\,(n+2)}\right)}{e^{4\,a+4\,b\,x}-2\,e^{2\,a+2\,b\,x}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(a + b*x)^3*(1/sinh(a + b*x))^n,x)
```

```
[Out] -((1/(exp(a + b*x)/2 - exp(- a - b*x)/2))^n*(1/(b*n) + exp(4*a + 4*b*x)/(b*
n) + (exp(2*a + 2*b*x)*(2*n - 4))/(b*n*(n + 2))))/(exp(4*a + 4*b*x) - 2*exp
(2*a + 2*b*x) + 1)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \coth^3(a+bx)\operatorname{csch}^n(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(b*x+a)**3*csch(b*x+a)**n,x)
```

```
[Out] Integral(coth(a + b*x)**3*csch(a + b*x)**n, x)
```

3.121 $\int \coth^2(a+bx)\mathbf{csch}(a+bx)\,dx$

Optimal. Leaf size=34

$$-\frac{\tanh^{-1}(\cosh(a+bx))}{2b}-\frac{\coth(a+bx)\operatorname{csch}(a+bx)}{2b}$$

```
[Out] -1/2*arctanh(cosh(b*x+a))/b-1/2*coth(b*x+a)*csch(b*x+a)/b
```

Rubi [A] time = 0.03, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2611, 3770}

$$-\frac{\tanh^{-1}(\cosh(a+bx))}{2b}-\frac{\coth(a+bx)\operatorname{csch}(a+bx)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[a + b*x]^2*Csch[a + b*x],x]
```

```
[Out] -ArcTanh[Cosh[a + b*x]]/(2*b) - (Coth[a + b*x]*Csch[a + b*x])/(2*b)
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \coth^2(a+bx)\operatorname{csch}(a+bx)\,dx &= -\frac{\coth(a+bx)\operatorname{csch}(a+bx)}{2b}+\frac{1}{2}\int \operatorname{csch}(a+bx)\,dx\\ &= -\frac{\tanh^{-1}(\cosh(a+bx))}{2b}-\frac{\coth(a+bx)\operatorname{csch}(a+bx)}{2b}\end{aligned}$$

Mathematica [A] time = 0.03, size = 57, normalized size = 1.68

$$-\frac{\operatorname{csch}^2\left(\frac{1}{2}(a+bx)\right)}{8b}-\frac{\operatorname{sech}^2\left(\frac{1}{2}(a+bx)\right)}{8b}+\frac{\log\left(\tanh\left(\frac{1}{2}(a+bx)\right)\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[a + b*x]^2*Csch[a + b*x],x]
```

```
[Out] -1/8*Csch[(a + b*x)/2]^2/b + Log[Tanh[(a + b*x)/2]]/(2*b) - Sech[(a + b*x)/
2]^2/(8*b)
```

fricas [B] time = 0.41, size = 387, normalized size = 11.38

$$-\frac{2\cosh(bx+a)^3+6\cosh(bx+a)\sinh(bx+a)^2+2\sinh(bx+a)^3+\left(\cosh(bx+a)^4+4\cosh(bx+a)\sinh\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(b*x+a)^2*csch(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/2*(2*cosh(b*x + a)^3 + 6*cosh(b*x + a)*sinh(b*x + a)^2 + 2*sinh(b*x + a)
^3 + (cosh(b*x + a)^4 + 4*cosh(b*x + a)*sinh(b*x + a)^3 + sinh(b*x + a)^4 +
 2*(3*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^2 - 2*cosh(b*x + a)^2 + 4*(cosh(b*
x + a)^3 - cosh(b*x + a))*sinh(b*x + a) + 1)*log(cosh(b*x + a) + sinh(b*x +
 a) + 1) - (cosh(b*x + a)^4 + 4*cosh(b*x + a)*sinh(b*x + a)^3 + sinh(b*x +
a)^4 + 2*(3*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^2 - 2*cosh(b*x + a)^2 + 4*(c
osh(b*x + a)^3 - cosh(b*x + a))*sinh(b*x + a) + 1)*log(cosh(b*x + a) + sinh
(b*x + a) - 1) + 2*(3*cosh(b*x + a)^2 + 1)*sinh(b*x + a) + 2*cosh(b*x + a))
/(b*cosh(b*x + a)^4 + 4*b*cosh(b*x + a)*sinh(b*x + a)^3 + b*sinh(b*x + a)^4
 - 2*b*cosh(b*x + a)^2 + 2*(3*b*cosh(b*x + a)^2 - b)*sinh(b*x + a)^2 + 4*(b
*cosh(b*x + a)^3 - b*cosh(b*x + a))*sinh(b*x + a) + b)
```

giac [A] time = 0.13, size = 58, normalized size = 1.71

$$-\frac{\frac{2\left(e^{(3bx+3a)}+e^{(bx+a)}\right)}{\left(e^{(2bx+2a)}-1\right)^2}+\log\left(e^{(bx+a)}+1\right)-\log\left(\left|e^{(bx+a)}-1\right|\right)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(b*x+a)^2*csch(b*x+a),x, algorithm="giac")
```

```
[Out] -1/2*(2*(e^(3*b*x + 3*a) + e^(b*x + a))/(e^(2*b*x + 2*a) - 1)^2 + log(e^(b*
x + a) + 1) - log(abs(e^(b*x + a) - 1)))/b
```

maple [A] time = 0.31, size = 45, normalized size = 1.32

$$\frac{-\frac{\cosh(bx+a)}{\sinh(bx+a)^2}+\frac{\operatorname{csch}(bx+a)\coth(bx+a)}{2}-\operatorname{arctanh}\left(e^{bx+a}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(b*x+a)^2*csch(b*x+a),x)
```

```
[Out] 1/b*(-1/sinh(b*x+a)^2*cosh(b*x+a)+1/2*csch(b*x+a)*coth(b*x+a)-arctanh(exp(b
*x+a)))
```

maxima [B] time = 0.34, size = 84, normalized size = 2.47

$$-\frac{\log\left(e^{(-bx-a)}+1\right)}{2\,b}+\frac{\log\left(e^{(-bx-a)}-1\right)}{2\,b}+\frac{e^{(-bx-a)}+e^{(-3\,bx-3\,a)}}{b\left(2\,e^{(-2\,bx-2\,a)}-e^{(-4\,bx-4\,a)}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(b*x+a)^2*csch(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/2*log(e^(-b*x - a) + 1)/b + 1/2*log(e^(-b*x - a) - 1)/b + (e^(-b*x - a)
+ e^(-3*b*x - 3*a))/(b*(2*e^(-2*b*x - 2*a) - e^(-4*b*x - 4*a) - 1))
```

mupad [B] time = 0.08, size = 87, normalized size = 2.56

$$-\frac{\operatorname{atan}\left(\frac{e^{b\,x}\,e^{a}\,\sqrt{-b^2}}{b}\right)}{\sqrt{-b^2}}-\frac{2\,e^{a+b\,x}}{b\,\left(e^{4\,a+4\,b\,x}-2\,e^{2\,a+2\,b\,x}+1\right)}-\frac{e^{a+b\,x}}{b\,\left(e^{2\,a+2\,b\,x}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(a + b*x)^2/sinh(a + b*x),x)
```

```
[Out] - atan((exp(b*x)*exp(a)*(-b^2)^(1/2))/b)/(-b^2)^(1/2) - (2*exp(a + b*x))/(b
*(exp(4*a + 4*b*x) - 2*exp(2*a + 2*b*x) + 1)) - exp(a + b*x)/(b*(exp(2*a +
2*b*x) - 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\coth^2(a+bx)\operatorname{csch}(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(b*x+a)**2*csch(b*x+a),x)
```

```
[Out] Integral(coth(a + b*x)**2*csch(a + b*x), x)
```

3.122 $\int \coth^2(a+bx)\mathbf{csch}^3(a+bx)\,dx$

Optimal. Leaf size=55

$$\frac{\tanh^{-1}(\cosh(a+bx))}{8b}-\frac{\coth(a+bx)\text{csch}^3(a+bx)}{4b}-\frac{\coth(a+bx)\text{csch}(a+bx)}{8b}$$

```
[Out] 1/8*arctanh(cosh(b*x+a))/b-1/8*coth(b*x+a)*csch(b*x+a)/b-1/4*coth(b*x+a)*cs
ch(b*x+a)^3/b
```

Rubi [A] time = 0.06, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {2611, 3768, 3770}

$$\frac{\tanh^{-1}(\cosh(a+bx))}{8b}-\frac{\coth(a+bx)\text{csch}^3(a+bx)}{4b}-\frac{\coth(a+bx)\text{csch}(a+bx)}{8b}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[a + b*x]^2*Csch[a + b*x]^3,x]
```

```
[Out] ArcTanh[Cosh[a + b*x]]/(8*b) - (Coth[a + b*x]*Csch[a + b*x])/(8*b) - (Coth[
a + b*x]*Csch[a + b*x]^3)/(4*b)
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \coth^2(a+bx)\text{csch}^3(a+bx)\,dx = -\frac{\coth(a+bx)\text{csch}^3(a+bx)}{4b} + \frac{1}{4}\int \text{csch}^3(a+bx)\,dx$$
$$= -\frac{\coth(a+bx)\text{csch}(a+bx)}{8b} - \frac{\coth(a+bx)\text{csch}^3(a+bx)}{4b} - \frac{1}{8}\int \text{csch}(a+bx$$
$$= \frac{\tanh^{-1}(\cosh(a+bx))}{8b} - \frac{\coth(a+bx)\text{csch}(a+bx)}{8b} - \frac{\coth(a+bx)\text{csch}^3(a+}{4b}$$

Mathematica [A] time = 0.04, size = 95, normalized size = 1.73

$$-\frac{\text{csch}^4\left(\frac{1}{2}(a+bx)\right)}{64b}-\frac{\text{csch}^2\left(\frac{1}{2}(a+bx)\right)}{32b}+\frac{\text{sech}^4\left(\frac{1}{2}(a+bx)\right)}{64b}-\frac{\text{sech}^2\left(\frac{1}{2}(a+bx)\right)}{32b}-\frac{\log\left(\tanh\left(\frac{1}{2}(a+bx)\right)\right)}{8b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[a + b*x]^2*Csch[a + b*x]^3,x]
```

```
[Out] -1/32*Csch[(a + b*x)/2]^2/b - Csch[(a + b*x)/2]^4/(64*b) - Log[Tanh[(a + b*
x)/2]]/(8*b) - Sech[(a + b*x)/2]^2/(32*b) + Sech[(a + b*x)/2]^4/(64*b)
```

fricas [B] time = 0.53, size = 1109, normalized size = 20.16

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(b*x+a)^2*csch(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -1/8*(2*cosh(b*x + a)^7 + 14*cosh(b*x + a)*sinh(b*x + a)^6 + 2*sinh(b*x + a
)^7 + 14*(3*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^5 + 14*cosh(b*x + a)^5 + 70*
(cosh(b*x + a)^3 + cosh(b*x + a))*sinh(b*x + a)^4 + 14*(5*cosh(b*x + a)^4 +
 10*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^3 + 14*cosh(b*x + a)^3 + 14*(3*cosh(
b*x + a)^5 + 10*cosh(b*x + a)^3 + 3*cosh(b*x + a))*sinh(b*x + a)^2 - (cosh(
b*x + a)^8 + 8*cosh(b*x + a)*sinh(b*x + a)^7 + sinh(b*x + a)^8 + 4*(7*cosh(
b*x + a)^2 - 1)*sinh(b*x + a)^6 - 4*cosh(b*x + a)^6 + 8*(7*cosh(b*x + a)^3
- 3*cosh(b*x + a))*sinh(b*x + a)^5 + 2*(35*cosh(b*x + a)^4 - 30*cosh(b*x +
a)^2 + 3)*sinh(b*x + a)^4 + 6*cosh(b*x + a)^4 + 8*(7*cosh(b*x + a)^5 - 10*c
osh(b*x + a)^3 + 3*cosh(b*x + a))*sinh(b*x + a)^3 + 4*(7*cosh(b*x + a)^6 -
15*cosh(b*x + a)^4 + 9*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^2 - 4*cosh(b*x +
a)^2 + 8*(cosh(b*x + a)^7 - 3*cosh(b*x + a)^5 + 3*cosh(b*x + a)^3 - cosh(b*
x + a))*sinh(b*x + a) + 1)*log(cosh(b*x + a) + sinh(b*x + a) + 1) + (cosh(b
*x + a)^8 + 8*cosh(b*x + a)*sinh(b*x + a)^7 + sinh(b*x + a)^8 + 4*(7*cosh(b
*x + a)^2 - 1)*sinh(b*x + a)^6 - 4*cosh(b*x + a)^6 + 8*(7*cosh(b*x + a)^3 -
```

```
3*cosh(b*x + a))*sinh(b*x + a)^5 + 2*(35*cosh(b*x + a)^4 - 30*cosh(b*x + a
)^2 + 3)*sinh(b*x + a)^4 + 6*cosh(b*x + a)^4 + 8*(7*cosh(b*x + a)^5 - 10*co
sh(b*x + a)^3 + 3*cosh(b*x + a))*sinh(b*x + a)^3 + 4*(7*cosh(b*x + a)^6 - 1
5*cosh(b*x + a)^4 + 9*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^2 - 4*cosh(b*x + a
)^2 + 8*(cosh(b*x + a)^7 - 3*cosh(b*x + a)^5 + 3*cosh(b*x + a)^3 - cosh(b*x
 + a))*sinh(b*x + a) + 1)*log(cosh(b*x + a) + sinh(b*x + a) - 1) + 2*(7*cos
h(b*x + a)^6 + 35*cosh(b*x + a)^4 + 21*cosh(b*x + a)^2 + 1)*sinh(b*x + a) +
 2*cosh(b*x + a))/(b*cosh(b*x + a)^8 + 8*b*cosh(b*x + a)*sinh(b*x + a)^7 +
b*sinh(b*x + a)^8 - 4*b*cosh(b*x + a)^6 + 4*(7*b*cosh(b*x + a)^2 - b)*sinh(
b*x + a)^6 + 8*(7*b*cosh(b*x + a)^3 - 3*b*cosh(b*x + a))*sinh(b*x + a)^5 +
6*b*cosh(b*x + a)^4 + 2*(35*b*cosh(b*x + a)^4 - 30*b*cosh(b*x + a)^2 + 3*b)
*sinh(b*x + a)^4 + 8*(7*b*cosh(b*x + a)^5 - 10*b*cosh(b*x + a)^3 + 3*b*cosh
(b*x + a))*sinh(b*x + a)^3 - 4*b*cosh(b*x + a)^2 + 4*(7*b*cosh(b*x + a)^6 -
 15*b*cosh(b*x + a)^4 + 9*b*cosh(b*x + a)^2 - b)*sinh(b*x + a)^2 + 8*(b*cos
h(b*x + a)^7 - 3*b*cosh(b*x + a)^5 + 3*b*cosh(b*x + a)^3 - b*cosh(b*x + a))
*sinh(b*x + a) + b)
```

giac [A] time = 0.16, size = 80, normalized size = 1.45

$$-\frac{\frac{2\left(e^{(7\,bx+7\,a)}+7\,e^{(5\,bx+5\,a)}+7\,e^{(3\,bx+3\,a)}+e^{(bx+a)}\right)}{\left(e^{(2\,bx+2\,a)}-1\right)^4}-\log\left(e^{(bx+a)}+1\right)+\log\left(\left|e^{(bx+a)}-1\right|\right)}{8\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(b*x+a)^2*csch(b*x+a)^3,x, algorithm="giac")
```

```
[Out] -1/8*(2*(e^(7*b*x + 7*a) + 7*e^(5*b*x + 5*a) + 7*e^(3*b*x + 3*a) + e^(b*x +
 a))/(e^(2*b*x + 2*a) - 1)^4 - log(e^(b*x + a) + 1) + log(abs(e^(b*x + a) -
 1)))/b
```

maple [A] time = 0.34, size = 58, normalized size = 1.05

$$\frac{-\frac{\cosh(bx+a)}{3\sinh(bx+a)^4}-\frac{\left(-\frac{\mathrm{csch}(bx+a)^3}{4}+\frac{3\,\mathrm{csch}(bx+a)}{8}\right)\coth(bx+a)}{3}+\frac{\mathrm{arctanh}\left(\mathrm{e}^{bx+a}\right)}{4}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(b*x+a)^2*csch(b*x+a)^3,x)
```

```
[Out] 1/b*(-1/3/sinh(b*x+a)^4*cosh(b*x+a)-1/3*(-1/4*csch(b*x+a)^3+3/8*csch(b*x+a)
)*coth(b*x+a)+1/4*arctanh(exp(b*x+a)))
```

maxima [B] time = 0.60, size = 129, normalized size = 2.35

$$\frac{\log\left(e^{(-bx-a)}+1\right)}{8\,b}-\frac{\log\left(e^{(-bx-a)}-1\right)}{8\,b}+\frac{e^{(-bx-a)}+7\,e^{(-3\,bx-3\,a)}+7\,e^{(-5\,bx-5\,a)}+e^{(-7\,bx-7\,a)}}{4\,b\left(4\,e^{(-2\,bx-2\,a)}-6\,e^{(-4\,bx-4\,a)}+4\,e^{(-6\,bx-6\,a)}-e^{(-8\,bx-8\,a)}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(b*x+a)^2*csch(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/8*log(e^(-b*x - a) + 1)/b - 1/8*log(e^(-b*x - a) - 1)/b + 1/4*(e^(-b*x -
a) + 7*e^(-3*b*x - 3*a) + 7*e^(-5*b*x - 5*a) + e^(-7*b*x - 7*a))/(b*(4*e^(-
2*b*x - 2*a) - 6*e^(-4*b*x - 4*a) + 4*e^(-6*b*x - 6*a) - e^(-8*b*x - 8*a) -
 1))
```

mupad [B] time = 0.11, size = 219, normalized size = 3.98

$$\frac{\operatorname{atan}\left(\frac{e^{b\,x}\,e^{a}\,\sqrt{-b^2}}{b}\right)}{4\,\sqrt{-b^2}}-\frac{\frac{e^{a+b\,x}}{b}+\frac{2\,e^{3\,a+3\,b\,x}}{b}+\frac{e^{5\,a+5\,b\,x}}{b}}{6\,e^{4\,a+4\,b\,x}-4\,e^{2\,a+2\,b\,x}-4\,e^{6\,a+6\,b\,x}+e^{8\,a+8\,b\,x}+1}-\frac{3\,e^{a+b\,x}}{2\,b\,\left(e^{4\,a+4\,b\,x}-2\,e^{2\,a+2\,b\,x}+1\right)}-\frac{}{b\,\left(3\,e^{2\,a+2\,b}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(a + b*x)^2/sinh(a + b*x)^3,x)
```

```
[Out] atan((exp(b*x)*exp(a)*(-b^2)^(1/2))/b)/(4*(-b^2)^(1/2)) - (exp(a + b*x)/b +
 (2*exp(3*a + 3*b*x))/b + exp(5*a + 5*b*x)/b)/(6*exp(4*a + 4*b*x) - 4*exp(2
*a + 2*b*x) - 4*exp(6*a + 6*b*x) + exp(8*a + 8*b*x) + 1) - (3*exp(a + b*x))
/(2*b*(exp(4*a + 4*b*x) - 2*exp(2*a + 2*b*x) + 1)) - (2*exp(a + b*x))/(b*(3
*exp(2*a + 2*b*x) - 3*exp(4*a + 4*b*x) + exp(6*a + 6*b*x) - 1)) - exp(a + b
*x)/(4*b*(exp(2*a + 2*b*x) - 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \coth^2(a+bx)\operatorname{csch}^3(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(b*x+a)**2*csch(b*x+a)**3,x)
```

```
[Out] Integral(coth(a + b*x)**2*csch(a + b*x)**3, x)
```

3.123 $\int \coth^4(a+bx)\mathbf{csch}(a+bx)\,dx$

Optimal. Leaf size=55

$$-\frac{3\tanh^{-1}(\cosh(a+bx))}{8b}-\frac{\coth^3(a+bx)\text{csch}(a+bx)}{4b}-\frac{3\coth(a+bx)\text{csch}(a+bx)}{8b}$$

```
[Out] -3/8*arctanh(cosh(b*x+a))/b-3/8*coth(b*x+a)*csch(b*x+a)/b-1/4*coth(b*x+a)^3
*csch(b*x+a)/b
```

Rubi **[A]** time = 0.06, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2611, 3770}

$$-\frac{3\tanh^{-1}(\cosh(a+bx))}{8b}-\frac{\coth^3(a+bx)\text{csch}(a+bx)}{4b}-\frac{3\coth(a+bx)\text{csch}(a+bx)}{8b}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[a + b*x]^4*Csch[a + b*x],x]
```

```
[Out] (-3*ArcTanh[Cosh[a + b*x]])/(8*b) - (3*Coth[a + b*x]*Csch[a + b*x])/(8*b) -
 (Coth[a + b*x]^3*Csch[a + b*x])/(4*b)
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \coth^4(a+bx)\text{csch}(a+bx)\,dx = -\frac{\coth^3(a+bx)\text{csch}(a+bx)}{4b} + \frac{3}{4}\int \coth^2(a+bx)\text{csch}(a+bx)\,dx$$
$$= -\frac{3\coth(a+bx)\text{csch}(a+bx)}{8b} - \frac{\coth^3(a+bx)\text{csch}(a+bx)}{4b} + \frac{3}{8}\int \text{csch}(a+b$$
$$= -\frac{3\tanh^{-1}(\cosh(a+bx))}{8b} - \frac{3\coth(a+bx)\text{csch}(a+bx)}{8b} - \frac{\coth^3(a+bx)\text{csch}(}{4b}$$

Mathematica [**A**] time = 0.04, size = 95, normalized size = 1.73

$$-\frac{\text{csch}^4\left(\frac{1}{2}(a+bx)\right)}{64b} - \frac{5\text{csch}^2\left(\frac{1}{2}(a+bx)\right)}{32b} + \frac{\text{sech}^4\left(\frac{1}{2}(a+bx)\right)}{64b} - \frac{5\text{sech}^2\left(\frac{1}{2}(a+bx)\right)}{32b} + \frac{3\log\left(\tanh\left(\frac{1}{2}(a+bx)\right)\right)}{8b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[a + b*x]^4*Csch[a + b*x],x]
```

```
[Out] (-5*Csch[(a + b*x)/2]^2)/(32*b) - Csch[(a + b*x)/2]^4/(64*b) + (3*Log[Tanh[
(a + b*x)/2]])/(8*b) - (5*Sech[(a + b*x)/2]^2)/(32*b) + Sech[(a + b*x)/2]^4
/(64*b)
```

fricas [**B**] time = 0.53, size = 1114, normalized size = 20.25

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(b*x+a)^4*csch(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/8*(10*cosh(b*x + a)^7 + 70*cosh(b*x + a)*sinh(b*x + a)^6 + 10*sinh(b*x +
 a)^7 + 6*(35*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^5 + 6*cosh(b*x + a)^5 + 10
*(35*cosh(b*x + a)^3 + 3*cosh(b*x + a))*sinh(b*x + a)^4 + 2*(175*cosh(b*x +
 a)^4 + 30*cosh(b*x + a)^2 + 3)*sinh(b*x + a)^3 + 6*cosh(b*x + a)^3 + 6*(35
*cosh(b*x + a)^5 + 10*cosh(b*x + a)^3 + 3*cosh(b*x + a))*sinh(b*x + a)^2 +
3*(cosh(b*x + a)^8 + 8*cosh(b*x + a)*sinh(b*x + a)^7 + sinh(b*x + a)^8 + 4*
(7*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^6 - 4*cosh(b*x + a)^6 + 8*(7*cosh(b*x
 + a)^3 - 3*cosh(b*x + a))*sinh(b*x + a)^5 + 2*(35*cosh(b*x + a)^4 - 30*cos
h(b*x + a)^2 + 3)*sinh(b*x + a)^4 + 6*cosh(b*x + a)^4 + 8*(7*cosh(b*x + a)^
5 - 10*cosh(b*x + a)^3 + 3*cosh(b*x + a))*sinh(b*x + a)^3 + 4*(7*cosh(b*x +
 a)^6 - 15*cosh(b*x + a)^4 + 9*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^2 - 4*cos
h(b*x + a)^2 + 8*(cosh(b*x + a)^7 - 3*cosh(b*x + a)^5 + 3*cosh(b*x + a)^3 -
 cosh(b*x + a))*sinh(b*x + a) + 1)*log(cosh(b*x + a) + sinh(b*x + a) + 1) -
 3*(cosh(b*x + a)^8 + 8*cosh(b*x + a)*sinh(b*x + a)^7 + sinh(b*x + a)^8 + 4
```

```
*(7*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^6 - 4*cosh(b*x + a)^6 + 8*(7*cosh(b*
x + a)^3 - 3*cosh(b*x + a))*sinh(b*x + a)^5 + 2*(35*cosh(b*x + a)^4 - 30*co
sh(b*x + a)^2 + 3)*sinh(b*x + a)^4 + 6*cosh(b*x + a)^4 + 8*(7*cosh(b*x + a)
^5 - 10*cosh(b*x + a)^3 + 3*cosh(b*x + a))*sinh(b*x + a)^3 + 4*(7*cosh(b*x
+ a)^6 - 15*cosh(b*x + a)^4 + 9*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^2 - 4*co
sh(b*x + a)^2 + 8*(cosh(b*x + a)^7 - 3*cosh(b*x + a)^5 + 3*cosh(b*x + a)^3
- cosh(b*x + a))*sinh(b*x + a) + 1)*log(cosh(b*x + a) + sinh(b*x + a) - 1)
+ 2*(35*cosh(b*x + a)^6 + 15*cosh(b*x + a)^4 + 9*cosh(b*x + a)^2 + 5)*sinh(
b*x + a) + 10*cosh(b*x + a))/(b*cosh(b*x + a)^8 + 8*b*cosh(b*x + a)*sinh(b*
x + a)^7 + b*sinh(b*x + a)^8 - 4*b*cosh(b*x + a)^6 + 4*(7*b*cosh(b*x + a)^2
 - b)*sinh(b*x + a)^6 + 8*(7*b*cosh(b*x + a)^3 - 3*b*cosh(b*x + a))*sinh(b*
x + a)^5 + 6*b*cosh(b*x + a)^4 + 2*(35*b*cosh(b*x + a)^4 - 30*b*cosh(b*x +
a)^2 + 3*b)*sinh(b*x + a)^4 + 8*(7*b*cosh(b*x + a)^5 - 10*b*cosh(b*x + a)^3
 + 3*b*cosh(b*x + a))*sinh(b*x + a)^3 - 4*b*cosh(b*x + a)^2 + 4*(7*b*cosh(b
*x + a)^6 - 15*b*cosh(b*x + a)^4 + 9*b*cosh(b*x + a)^2 - b)*sinh(b*x + a)^2
 + 8*(b*cosh(b*x + a)^7 - 3*b*cosh(b*x + a)^5 + 3*b*cosh(b*x + a)^3 - b*cos
h(b*x + a))*sinh(b*x + a) + b)
```

giac [A] time = 0.16, size = 86, normalized size = 1.56

$$-\frac{\frac{2\left(5\,e^{(7\,bx+7\,a)}+3\,e^{(5\,bx+5\,a)}+3\,e^{(3\,bx+3\,a)}+5\,e^{(bx+a)}\right)}{\left(e^{(2\,bx+2\,a)}-1\right)^{4}}+3\,\log\left(e^{(bx+a)}+1\right)-3\,\log\left(\left|e^{(bx+a)}-1\right|\right)}{8\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(b*x+a)^4*csch(b*x+a),x, algorithm="giac")
```

```
[Out] -1/8*(2*(5*e^(7*b*x + 7*a) + 3*e^(5*b*x + 5*a) + 3*e^(3*b*x + 3*a) + 5*e^(b
*x + a))/(e^(2*b*x + 2*a) - 1)^4 + 3*log(e^(b*x + a) + 1) - 3*log(abs(e^(b*
x + a) - 1)))/b
```

maple [A] time = 0.34, size = 74, normalized size = 1.35

$$\frac{-\frac{\cosh^{3}(bx+a)}{\sinh(bx+a)^{4}}+\frac{\cosh(bx+a)}{\sinh(bx+a)^{4}}+\left(-\frac{\operatorname{csch}(bx+a)^{3}}{4}+\frac{3\operatorname{csch}(bx+a)}{8}\right)\coth(bx+a)-\frac{3\operatorname{arctanh}\left(e^{bx+a}\right)}{4}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(b*x+a)^4*csch(b*x+a),x)
```

```
[Out] 1/b*(-1/sinh(b*x+a)^4*cosh(b*x+a)^3+1/sinh(b*x+a)^4*cosh(b*x+a)+(-1/4*csch(
b*x+a)^3+3/8*csch(b*x+a))*coth(b*x+a)-3/4*arctanh(exp(b*x+a)))
```

maxima [B] time = 0.30, size = 133, normalized size = 2.42

$$-\frac{3 \log\left(e^{(-bx-a)}+1\right)}{8 b}+\frac{3 \log\left(e^{(-bx-a)}-1\right)}{8 b}+\frac{5 e^{(-bx-a)}+3 e^{(-3 bx-3 a)}+3 e^{(-5 bx-5 a)}+5 e^{(-7 bx-7 a)}}{4 b\left(4 e^{(-2 bx-2 a)}-6 e^{(-4 bx-4 a)}+4 e^{(-6 bx-6 a)}-e^{(-8 bx-8 a)}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(b*x+a)^4*csch(b*x+a),x, algorithm="maxima")
```

```
[Out] -3/8*log(e^(-b*x - a) + 1)/b + 3/8*log(e^(-b*x - a) - 1)/b + 1/4*(5*e^(-b*x
 - a) + 3*e^(-3*b*x - 3*a) + 3*e^(-5*b*x - 5*a) + 5*e^(-7*b*x - 7*a))/(b*(4
*e^(-2*b*x - 2*a) - 6*e^(-4*b*x - 4*a) + 4*e^(-6*b*x - 6*a) - e^(-8*b*x - 8
*a) - 1))
```

mupad [B] time = 1.41, size = 190, normalized size = 3.45

$$-\frac{3\,\mathrm{atan}\left(\frac{\mathrm{e}^{b\,x}\,\mathrm{e}^{a}\,\sqrt{-b^2}}{b}\right)}{4\,\sqrt{-b^2}}-\frac{9\,\mathrm{e}^{a+b\,x}}{2\,b\,\left(\mathrm{e}^{4\,a+4\,b\,x}-2\,\mathrm{e}^{2\,a+2\,b\,x}+1\right)}-\frac{6\,\mathrm{e}^{a+b\,x}}{b\,\left(3\,\mathrm{e}^{2\,a+2\,b\,x}-3\,\mathrm{e}^{4\,a+4\,b\,x}+\mathrm{e}^{6\,a+6\,b\,x}-1\right)}-\frac{}{b\,\left(6\,\mathrm{e}^{4\,a+4\,b\,x}-4\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(a + b*x)^4/sinh(a + b*x),x)
```

```
[Out] - (3*atan((exp(b*x)*exp(a)*(-b^2)^(1/2))/b))/(4*(-b^2)^(1/2)) - (9*exp(a +
b*x))/(2*b*(exp(4*a + 4*b*x) - 2*exp(2*a + 2*b*x) + 1)) - (6*exp(a + b*x))/
(b*(3*exp(2*a + 2*b*x) - 3*exp(4*a + 4*b*x) + exp(6*a + 6*b*x) - 1)) - (4*e
xp(a + b*x))/(b*(6*exp(4*a + 4*b*x) - 4*exp(2*a + 2*b*x) - 4*exp(6*a + 6*b*
x) + exp(8*a + 8*b*x) + 1)) - (5*exp(a + b*x))/(4*b*(exp(2*a + 2*b*x) - 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \coth^4(a+bx)\operatorname{csch}(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(b*x+a)**4*csch(b*x+a),x)
```

```
[Out] Integral(coth(a + b*x)**4*csch(a + b*x), x)
```

3.124 $\int \coth^2(x)\mathbf{csch}^4(x)\,dx$

Optimal. Leaf size=17

$$\frac{\coth^3(x)}{3}-\frac{\coth^5(x)}{5}$$

[Out] 1/3*coth(x)^3-1/5*coth(x)^5

Rubi [A] time = 0.03, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2607, 14}

$$\frac{\coth^3(x)}{3}-\frac{\coth^5(x)}{5}$$

Antiderivative was successfully verified.

[In] Int[Coth[x]^2*Csch[x]^4,x]

[Out] Coth[x]^3/3 - Coth[x]^5/5

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rubi steps

$$\begin{aligned}\int \coth^2(x)\text{csch}^4(x)\,dx&=i\,\text{Subst}\left(\int x^2\left(1+x^2\right)\,dx,x,i\coth(x)\right)\\&=i\,\text{Subst}\left(\int\left(x^2+x^4\right)\,dx,x,i\coth(x)\right)\\&=\frac{\coth^3(x)}{3}-\frac{\coth^5(x)}{5}\end{aligned}$$

Mathematica [A] time = 0.02, size = 27, normalized size = 1.59

$$\frac{2\coth(x)}{15} - \frac{1}{5}\coth(x)\operatorname{csch}^4(x) - \frac{1}{15}\coth(x)\operatorname{csch}^2(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]^2*Csch[x]^4,x]
[Out] (2*Coth[x])/15 - (Coth[x]*Csch[x]^2)/15 - (Coth[x]*Csch[x]^4)/5
```

fricas [B] time = 0.48, size = 164, normalized size = 9.65

$$-\frac{}{15\left(\cosh(x)^7 + 7\cosh(x)\sinh(x)^6 + \sinh(x)^7 + \left(21\cosh(x)^2 - 5\right)\sinh(x)^5 - 5\cosh(x)^5 + 5\left(7\cosh(x)^3 - 5\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^2*csch(x)^4,x, algorithm="fricas")
[Out] -8/15*(7*cosh(x)^3 + 24*cosh(x)^2*sinh(x) + 21*cosh(x)*sinh(x)^2 + 8*sinh(x
)^3 + 5*cosh(x))/(cosh(x)^7 + 7*cosh(x)*sinh(x)^6 + sinh(x)^7 + (21*cosh(x)
^2 - 5)*sinh(x)^5 - 5*cosh(x)^5 + 5*(7*cosh(x)^3 - 5*cosh(x))*sinh(x)^4 + (
35*cosh(x)^4 - 50*cosh(x)^2 + 11)*sinh(x)^3 + 9*cosh(x)^3 + (21*cosh(x)^5 -
 50*cosh(x)^3 + 27*cosh(x))*sinh(x)^2 + (7*cosh(x)^6 - 25*cosh(x)^4 + 33*co
sh(x)^2 - 15)*sinh(x) - 5*cosh(x))
```

giac [B] time = 0.13, size = 30, normalized size = 1.76

$$-\frac{4\left(15e^{(6x)} + 5e^{(4x)} + 5e^{(2x)} - 1\right)}{15\left(e^{(2x)} - 1\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^2*csch(x)^4,x, algorithm="giac")
[Out] -4/15*(15*e^(6*x) + 5*e^(4*x) + 5*e^(2*x) - 1)/(e^(2*x) - 1)^5
```

maple [B] time = 0.34, size = 28, normalized size = 1.65

$$-\frac{\cosh(x)}{4\sinh(x)^5} - \frac{\left(-\frac{8}{15} - \frac{\operatorname{csch}(x)^4}{5} + \frac{4\operatorname{csch}(x)^2}{15}\right)\coth(x)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^2*csch(x)^4,x)
```

```
[Out] -1/4/sinh(x)^5*cosh(x)-1/4*(-8/15-1/5*csch(x)^4+4/15*csch(x)^2)*coth(x)
```

maxima [B] time = 0.32, size = 149, normalized size = 8.76

$$\frac{4\,e^{(-2\,x)}}{3\left(5\,e^{(-2\,x)}-10\,e^{(-4\,x)}+10\,e^{(-6\,x)}-5\,e^{(-8\,x)}+e^{(-10\,x)}-1\right)}+\frac{4\,e^{(-4\,x)}}{3\left(5\,e^{(-2\,x)}-10\,e^{(-4\,x)}+10\,e^{(-6\,x)}-5\,e^{(-8\,x)}+e^{(-10\,x)}-\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^2*csch(x)^4,x, algorithm="maxima")
```

```
[Out] 4/3*e^(-2*x)/(5*e^(-2*x) - 10*e^(-4*x) + 10*e^(-6*x) - 5*e^(-8*x) + e^(-10*
x) - 1) + 4/3*e^(-4*x)/(5*e^(-2*x) - 10*e^(-4*x) + 10*e^(-6*x) - 5*e^(-8*x)
 + e^(-10*x) - 1) + 4*e^(-6*x)/(5*e^(-2*x) - 10*e^(-4*x) + 10*e^(-6*x) - 5*
e^(-8*x) + e^(-10*x) - 1) - 4/15/(5*e^(-2*x) - 10*e^(-4*x) + 10*e^(-6*x) -
5*e^(-8*x) + e^(-10*x) - 1)
```

mupad [B] time = 1.46, size = 144, normalized size = 8.47

$$-\frac{\frac{8\,e^{2\,x}}{5}+\frac{16\,e^{4\,x}}{5}+\frac{8\,e^{6\,x}}{5}}{5\,e^{2\,x}-10\,e^{4\,x}+10\,e^{6\,x}-5\,e^{8\,x}+e^{10\,x}-1}-\frac{\frac{4\,e^{2\,x}}{5}+\frac{8}{15}}{3\,e^{2\,x}-3\,e^{4\,x}+e^{6\,x}-1}-\frac{2}{5\left(e^{4\,x}-2\,e^{2\,x}+1\right)}-\frac{\frac{8\,e^{2\,x}}{5}+\frac{6\,e^{4\,x}}{5}}{6\,e^{4\,x}-4\,e^{2\,x}-4\,e^{6\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^2/sinh(x)^4,x)
```

```
[Out] - ((8*exp(2*x))/5 + (16*exp(4*x))/5 + (8*exp(6*x))/5)/(5*exp(2*x) - 10*exp(
4*x) + 10*exp(6*x) - 5*exp(8*x) + exp(10*x) - 1) - ((4*exp(2*x))/5 + 8/15)/
(3*exp(2*x) - 3*exp(4*x) + exp(6*x) - 1) - 2/(5*(exp(4*x) - 2*exp(2*x) + 1)
) - ((8*exp(2*x))/5 + (6*exp(4*x))/5 + 2/5)/(6*exp(4*x) - 4*exp(2*x) - 4*ex
p(6*x) + exp(8*x) + 1)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \coth^2(x)\,\mathrm{csch}^4(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)**2*csch(x)**4,x)
```

```
[Out] Integral(coth(x)**2*csch(x)**4, x)
```

3.125 $\int \coth^3(x)\mathbf{csch}^4(x)\,dx$

Optimal. Leaf size=17

$$-\frac{1}{6}\text{csch}^6(x)-\frac{\text{csch}^4(x)}{4}$$

```
[Out] -1/4*csch(x)^4-1/6*csch(x)^6
```

Rubi [A] time = 0.03, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2606, 14}

$$-\frac{1}{6}\text{csch}^6(x)-\frac{\text{csch}^4(x)}{4}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]^3*Csch[x]^4,x]
```

```
[Out] -Csch[x]^4/4 - Csch[x]^6/6
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rubi steps

$$\begin{aligned}\int \coth^3(x)\text{csch}^4(x)\,dx&=\text{Subst}\left(\int x^3\left(-1+x^2\right)\,dx,x,-i\text{csch}(x)\right)\\&=\text{Subst}\left(\int \left(-x^3+x^5\right)\,dx,x,-i\text{csch}(x)\right)\\&=-\frac{1}{4}\text{csch}^4(x)-\frac{\text{csch}^6(x)}{6}\end{aligned}$$

Mathematica [A] time = 0.01, size = 17, normalized size = 1.00

$$-\frac{1}{6}\operatorname{csch}^6(x)-\frac{\operatorname{csch}^4(x)}{4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]^3*Csch[x]^4,x]
```

```
[Out] -1/4*Csch[x]^4 - Csch[x]^6/6
```

fricas [B] time = 0.46, size = 222, normalized size = 13.06

$$-\frac{}{3\left(\cosh(x)^8+8\cosh(x)\sinh(x)^7+\sinh(x)^8+2\left(14\cosh(x)^2-3\right)\sinh(x)^6-6\cosh(x)^6+4\left(14\cosh(x)^3-\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^3*csch(x)^4,x, algorithm="fricas")
```

```
[Out] -4/3*(3*cosh(x)^4 + 12*cosh(x)*sinh(x)^3 + 3*sinh(x)^4 + 2*(9*cosh(x)^2 + 1
)*sinh(x)^2 + 2*cosh(x)^2 + 4*(3*cosh(x)^3 + cosh(x))*sinh(x) + 3)/(cosh(x)
^8 + 8*cosh(x)*sinh(x)^7 + sinh(x)^8 + 2*(14*cosh(x)^2 - 3)*sinh(x)^6 - 6*c
osh(x)^6 + 4*(14*cosh(x)^3 - 9*cosh(x))*sinh(x)^5 + 2*(35*cosh(x)^4 - 45*co
sh(x)^2 + 8)*sinh(x)^4 + 16*cosh(x)^4 + 8*(7*cosh(x)^5 - 15*cosh(x)^3 + 7*c
osh(x))*sinh(x)^3 + 2*(14*cosh(x)^6 - 45*cosh(x)^4 + 48*cosh(x)^2 - 13)*sin
h(x)^2 - 26*cosh(x)^2 + 4*(2*cosh(x)^7 - 9*cosh(x)^5 + 14*cosh(x)^3 - 7*cos
h(x))*sinh(x) + 15)
```

giac [B] time = 0.14, size = 29, normalized size = 1.71

$$-\frac{4\left(3\,e^{(8\,x)}+2\,e^{(6\,x)}+3\,e^{(4\,x)}\right)}{3\left(e^{(2\,x)}-1\right)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^3*csch(x)^4,x, algorithm="giac")
```

```
[Out] -4/3*(3*e^(8*x) + 2*e^(6*x) + 3*e^(4*x))/(e^(2*x) - 1)^6
```

maple [A] time = 0.10, size = 18, normalized size = 1.06

$$-\frac{\cosh^2(x)}{4\sinh(x)^6}+\frac{1}{12\sinh(x)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^3*csch(x)^4,x)
```

```
[Out] -1/4*cosh(x)^2/sinh(x)^6+1/12/sinh(x)^6
```

maxima [B] time = 0.31, size = 139, normalized size = 8.18

$$\frac{4\,e^{(-4\,x)}}{6\,e^{(-2\,x)} - 15\,e^{(-4\,x)} + 20\,e^{(-6\,x)} - 15\,e^{(-8\,x)} + 6\,e^{(-10\,x)} - e^{(-12\,x)} - 1} + \frac{8\,e^{(-6\,x)}}{3\left(6\,e^{(-2\,x)} - 15\,e^{(-4\,x)} + 20\,e^{(-6\,x)} - 15\,e^{(-8\,x)} + 6 \right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^3*csch(x)^4,x, algorithm="maxima")
```

```
[Out] 4*e^(-4*x)/(6*e^(-2*x) - 15*e^(-4*x) + 20*e^(-6*x) - 15*e^(-8*x) + 6*e^(-10
*x) - e^(-12*x) - 1) + 8/3*e^(-6*x)/(6*e^(-2*x) - 15*e^(-4*x) + 20*e^(-6*x)
 - 15*e^(-8*x) + 6*e^(-10*x) - e^(-12*x) - 1) + 4*e^(-8*x)/(6*e^(-2*x) - 15
*e^(-4*x) + 20*e^(-6*x) - 15*e^(-8*x) + 6*e^(-10*x) - e^(-12*x) - 1)
```

mupad [B] time = 1.48, size = 210, normalized size = 12.35

$$-\frac{\frac{8\,e^{2\,x}}{5} + \frac{12\,e^{4\,x}}{5} + \frac{16\,e^{6\,x}}{15} + \frac{4}{15}}{5\,e^{2\,x} - 10\,e^{4\,x} + 10\,e^{6\,x} - 5\,e^{8\,x} + e^{10\,x} - 1} - \frac{\frac{4\,e^{2\,x}}{3} + 4\,e^{4\,x} + 4\,e^{6\,x} + \frac{4\,e^{8\,x}}{3}}{15\,e^{4\,x} - 6\,e^{2\,x} - 20\,e^{6\,x} + 15\,e^{8\,x} - 6\,e^{10\,x} + e^{12\,x} + 1} - \frac{\frac{8\,e^{2\,x}}{15} + }{3\,e^{2\,x} - 3\,e^{4\,x} + }$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^3/sinh(x)^4,x)
```

```
[Out] - ((8*exp(2*x))/5 + (12*exp(4*x))/5 + (16*exp(6*x))/15 + 4/15)/(5*exp(2*x)
- 10*exp(4*x) + 10*exp(6*x) - 5*exp(8*x) + exp(10*x) - 1) - ((4*exp(2*x))/3
 + 4*exp(4*x) + 4*exp(6*x) + (4*exp(8*x))/3)/(15*exp(4*x) - 6*exp(2*x) - 20
*exp(6*x) + 15*exp(8*x) - 6*exp(10*x) + exp(12*x) + 1) - ((8*exp(2*x))/15 +
 2/5)/(3*exp(2*x) - 3*exp(4*x) + exp(6*x) - 1) - 4/(15*(exp(4*x) - 2*exp(2*
x) + 1)) - ((6*exp(2*x))/5 + (4*exp(4*x))/5 + 2/5)/(6*exp(4*x) - 4*exp(2*x)
 - 4*exp(6*x) + exp(8*x) + 1)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \coth^{3}(x) \operatorname{csch}^{4}(x)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)**3*csch(x)**4,x)
```

```
[Out] Integral(coth(x)**3*csch(x)**4, x)
```

3.126 $\int \coth^n(x)\mathbf{csch}^4(x)\,dx$

Optimal. Leaf size=26

$$\frac{\coth^{n+1}(x)}{n+1}-\frac{\coth^{n+3}(x)}{n+3}$$

```
[Out] coth(x)^(1+n)/(1+n)-coth(x)^(3+n)/(3+n)
```

Rubi [A] time = 0.04, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2607, 14}

$$\frac{\coth^{n+1}(x)}{n+1}-\frac{\coth^{n+3}(x)}{n+3}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]^n*Csch[x]^4,x]
```

```
[Out] Coth[x]^(1 + n)/(1 + n) - Coth[x]^(3 + n)/(3 + n)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rubi steps

$$\begin{aligned}\int \coth^n(x)\text{csch}^4(x)\,dx &= -\left(i\,\text{Subst}\left(\int(-ix)^n\left(1+x^2\right)\,dx,x,i\coth(x)\right)\right)\\ &= -\left(i\,\text{Subst}\left(\int\left((-ix)^n-(-ix)^{2+n}\right)\,dx,x,i\coth(x)\right)\right)\\ &= \frac{\coth^{1+n}(x)}{1+n}-\frac{\coth^{3+n}(x)}{3+n}\end{aligned}$$

Mathematica [A] time = 0.09, size = 30, normalized size = 1.15

$$\frac{\operatorname{csch}^2(x)(-n+\cosh(2x)-2)\coth^{n+1}(x)}{(n+1)(n+3)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]^n*Csch[x]^4,x]
[Out] ((-2 - n + Cosh[2*x])*Coth[x]^(1 + n)*Csch[x]^2)/((1 + n)*(3 + n))
```

fricas [B] time = 0.46, size = 114, normalized size = 4.38

$$\frac{2\left(\left(\cosh(x)^3+3\cosh(x)\sinh(x)^2-(2n+3)\cosh(x)\right)\cosh\left(n\log\left(\frac{\cosh(x)}{\sinh(x)}\right)\right)+\left(\cosh(x)^3+3\cosh(x)\sinh(x)^2-\right.}{\left(n^2+4n+3\right)\sinh(x)^3+3\left(\left(n^2+4n+3\right)\cosh(x)^2-n^2-4n-3\right)\sinh}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^n*csch(x)^4,x, algorithm="fricas")
[Out] 2*((cosh(x)^3 + 3*cosh(x)*sinh(x)^2 - (2*n + 3)*cosh(x))*cosh(n*log(cosh(x)
/sinh(x))) + (cosh(x)^3 + 3*cosh(x)*sinh(x)^2 - (2*n + 3)*cosh(x))*sinh(n*l
og(cosh(x)/sinh(x))))/((n^2 + 4*n + 3)*sinh(x)^3 + 3*((n^2 + 4*n + 3)*cosh(
x)^2 - n^2 - 4*n - 3)*sinh(x))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \coth(x)^n \operatorname{csch}(x)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^n*csch(x)^4,x, algorithm="giac")
[Out] integrate(coth(x)^n*csch(x)^4, x)
```

maple [C] time = 0.54, size = 371, normalized size = 14.27

$$-\frac{2\left(-e^{6x}+2n\,e^{4x}+3\,e^{4x}+2n\,e^{2x}+3\,e^{2x}-1\right)e^{\frac{n\left(-i\pi\operatorname{csgn}\left(\frac{i\left(1+e^{2x}\right)}{e^{x}-1}\right)^3+i\pi\operatorname{csgn}\left(\frac{i\left(1+e^{2x}\right)}{e^{x}-1}\right)^2\operatorname{csgn}\left(\frac{i}{e^{x}-1}\right)+i\pi\operatorname{csgn}\left(\frac{i\left(1+e^{2x}\right)}{e^{x}-1}\right)^2\operatorname{csgn}\left(i\left(1+e^{2x}\right)\right)-i\pi\operatorname{csg}\cdots\right.}{}}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^n*csch(x)^4,x)
```

```
[Out] -2*(-exp(6*x)+2*n*exp(4*x)+3*exp(4*x)+2*n*exp(2*x)+3*exp(2*x)-1)/(n+1)/(n+3
)/(exp(2*x)-1)^3*exp(1/2*n*(-I*Pi*csgn(I/(exp(x)-1)*(1+exp(2*x)))^3+I*Pi*cs
gn(I/(exp(x)-1)*(1+exp(2*x)))^2*csgn(I/(exp(x)-1))+I*Pi*csgn(I/(exp(x)-1)*(
1+exp(2*x)))^2*csgn(I*(1+exp(2*x)))-I*Pi*csgn(I/(exp(x)-1)*(1+exp(2*x)))*cs
gn(I/(exp(x)-1))*csgn(I*(1+exp(2*x)))+I*Pi*csgn(I/(exp(x)-1)*(1+exp(2*x)))*
csgn(I/(exp(x)+1)*(1+exp(2*x))/(exp(x)-1))^2-I*Pi*csgn(I/(exp(x)-1)*(1+exp(
2*x)))*csgn(I/(exp(x)+1)*(1+exp(2*x))/(exp(x)-1))*csgn(I/(exp(x)+1))-I*Pi*c
sgn(I/(exp(x)+1)*(1+exp(2*x))/(exp(x)-1))^3+I*Pi*csgn(I/(exp(x)+1)*(1+exp(2
*x))/(exp(x)-1))^2*csgn(I/(exp(x)+1))-2*ln(exp(x)+1)+2*ln(1+exp(2*x))-2*ln(
exp(x)-1)))
```

maxima [B] time = 0.45, size = 368, normalized size = 14.15

$$-\frac{2\,(2\,n+3)e^{\left(-n\log\left(e^{(-x)}+1\right)-n\log\left(-e^{(-x)}+1\right)+n\log\left(e^{(-2\,x)}+1\right)-2\,x\right)}}{n^2-3\left(n^2+4\,n+3\right)e^{(-2\,x)}+3\left(n^2+4\,n+3\right)e^{(-4\,x)}-\left(n^2+4\,n+3\right)e^{(-6\,x)}+4\,n+3}-\frac{2\,(2\,n+3)}{n^2-3\left(n^2+4\,n+3\right)e^{(-2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^n*csch(x)^4,x, algorithm="maxima")
```

```
[Out] -2*(2*n + 3)*e^(-n*log(e^(-x) + 1) - n*log(-e^(-x) + 1) + n*log(e^(-2*x) +
1) - 2*x)/(n^2 - 3*(n^2 + 4*n + 3)*e^(-2*x) + 3*(n^2 + 4*n + 3)*e^(-4*x) -
(n^2 + 4*n + 3)*e^(-6*x) + 4*n + 3) - 2*(2*n + 3)*e^(-n*log(e^(-x) + 1) - n
*log(-e^(-x) + 1) + n*log(e^(-2*x) + 1) - 4*x)/(n^2 - 3*(n^2 + 4*n + 3)*e^(
-2*x) + 3*(n^2 + 4*n + 3)*e^(-4*x) - (n^2 + 4*n + 3)*e^(-6*x) + 4*n + 3) +
2*e^(-n*log(e^(-x) + 1) - n*log(-e^(-x) + 1) + n*log(e^(-2*x) + 1) - 6*x)/(
n^2 - 3*(n^2 + 4*n + 3)*e^(-2*x) + 3*(n^2 + 4*n + 3)*e^(-4*x) - (n^2 + 4*n
+ 3)*e^(-6*x) + 4*n + 3) + 2*e^(-n*log(e^(-x) + 1) - n*log(-e^(-x) + 1) + n
*log(e^(-2*x) + 1))/(n^2 - 3*(n^2 + 4*n + 3)*e^(-2*x) + 3*(n^2 + 4*n + 3)*e
^(-4*x) - (n^2 + 4*n + 3)*e^(-6*x) + 4*n + 3)
```

mupad [B] time = 1.54, size = 87, normalized size = 3.35

$$\frac{\left(\frac{4\cosh(3\,x)}{n^2+4\,n+3}-\frac{2\cosh(x)\,(4\,n+6)}{n^2+4\,n+3}\right)\left(\frac{e^{2\,x}+1}{e^{2\,x}-1}\right)^n}{2\sinh\left(3\,x\right)-\frac{2\sinh(x)\left(3\,n^2+12\,n+9\right)}{n^2+4\,n+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^n/sinh(x)^4,x)
```

```
[Out] (((4*cosh(3*x))/(4*n + n^2 + 3) - (2*cosh(x)*(4*n + 6))/(4*n + n^2 + 3))*((
exp(2*x) + 1)/(exp(2*x) - 1))^n)/(2*sinh(3*x) - (2*sinh(x)*(12*n + 3*n^2 +
9))/(4*n + n^2 + 3))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \coth^n(x)\operatorname{csch}^4(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)**n*csch(x)**4,x)
```

```
[Out] Integral(coth(x)**n*csch(x)**4, x)
```

3.127 $\int \coth^4(x)\operatorname{csch}^3(x)\,dx$

Optimal. Leaf size=38

$$\frac{1}{16}\tanh^{-1}(\cosh(x))-\frac{1}{6}\coth^3(x)\operatorname{csch}^3(x)-\frac{1}{8}\coth(x)\operatorname{csch}^3(x)-\frac{1}{16}\coth(x)\operatorname{csch}(x)$$

```
[Out] 1/16*arctanh(cosh(x))-1/16*coth(x)*csch(x)-1/8*coth(x)*csch(x)^3-1/6*coth(x
)^3*csch(x)^3
```

Rubi [A] time = 0.06, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2611, 3768, 3770}

$$\frac{1}{16}\tanh^{-1}(\cosh(x))-\frac{1}{6}\coth^3(x)\operatorname{csch}^3(x)-\frac{1}{8}\coth(x)\operatorname{csch}^3(x)-\frac{1}{16}\coth(x)\operatorname{csch}(x)$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]^4*Csch[x]^3,x]
```

```
[Out] ArcTanh[Cosh[x]]/16 - (Coth[x]*Csch[x])/16 - (Coth[x]*Csch[x]^3)/8 - (Coth[
x]^3*Csch[x]^3)/6
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \coth^4(x)\text{csch}^3(x)\,dx &= -\frac{1}{6}\coth^3(x)\text{csch}^3(x)+\frac{1}{2}\int \coth^2(x)\text{csch}^3(x)\,dx\\ &= -\frac{1}{8}\coth(x)\text{csch}^3(x)-\frac{1}{6}\coth^3(x)\text{csch}^3(x)+\frac{1}{8}\int \text{csch}^3(x)\,dx\\ &= -\frac{1}{16}\coth(x)\text{csch}(x)-\frac{1}{8}\coth(x)\text{csch}^3(x)-\frac{1}{6}\coth^3(x)\text{csch}^3(x)-\frac{1}{16}\int \text{csch}(x)\,dx\\ &= \frac{1}{16}\tanh^{-1}(\cosh(x))-\frac{1}{16}\coth(x)\text{csch}(x)-\frac{1}{8}\coth(x)\text{csch}^3(x)-\frac{1}{6}\coth^3(x)\text{csch}^3(x)\end{aligned}$$

Mathematica [B] time = 0.02, size = 84, normalized size = 2.21

$$-\frac{1}{384}\text{csch}^6\left(\frac{x}{2}\right)-\frac{1}{64}\text{csch}^4\left(\frac{x}{2}\right)-\frac{1}{64}\text{csch}^2\left(\frac{x}{2}\right)-\frac{1}{384}\text{sech}^6\left(\frac{x}{2}\right)+\frac{1}{64}\text{sech}^4\left(\frac{x}{2}\right)-\frac{1}{64}\text{sech}^2\left(\frac{x}{2}\right)-\frac{1}{16}\log\left(\tanh\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]^4*Csch[x]^3,x]
```

```
[Out] -1/64*Csch[x/2]^2 - Csch[x/2]^4/64 - Csch[x/2]^6/384 - Log[Tanh[x/2]]/16 -
Sech[x/2]^2/64 + Sech[x/2]^4/64 - Sech[x/2]^6/384
```

fricas [B] time = 0.45, size = 1260, normalized size = 33.16

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^4*csch(x)^3,x, algorithm="fricas")
```

```
[Out] -1/48*(6*cosh(x)^11 + 66*cosh(x)*sinh(x)^10 + 6*sinh(x)^11 + 2*(165*cosh(x)
^2 + 47)*sinh(x)^9 + 94*cosh(x)^9 + 18*(55*cosh(x)^3 + 47*cosh(x))*sinh(x)^
8 + 12*(165*cosh(x)^4 + 282*cosh(x)^2 + 13)*sinh(x)^7 + 156*cosh(x)^7 + 84*
(33*cosh(x)^5 + 94*cosh(x)^3 + 13*cosh(x))*sinh(x)^6 + 12*(231*cosh(x)^6 +
987*cosh(x)^4 + 273*cosh(x)^2 + 13)*sinh(x)^5 + 156*cosh(x)^5 + 12*(165*cos
h(x)^7 + 987*cosh(x)^5 + 455*cosh(x)^3 + 65*cosh(x))*sinh(x)^4 + 2*(495*cos
h(x)^8 + 3948*cosh(x)^6 + 2730*cosh(x)^4 + 780*cosh(x)^2 + 47)*sinh(x)^3 +
94*cosh(x)^3 + 6*(55*cosh(x)^9 + 564*cosh(x)^7 + 546*cosh(x)^5 + 260*cosh(x
)^3 + 47*cosh(x))*sinh(x)^2 - 3*(cosh(x)^12 + 12*cosh(x)*sinh(x)^11 + sinh(
x)^12 + 6*(11*cosh(x)^2 - 1)*sinh(x)^10 - 6*cosh(x)^10 + 20*(11*cosh(x)^3 -
 3*cosh(x))*sinh(x)^9 + 15*(33*cosh(x)^4 - 18*cosh(x)^2 + 1)*sinh(x)^8 + 15
*cosh(x)^8 + 24*(33*cosh(x)^5 - 30*cosh(x)^3 + 5*cosh(x))*sinh(x)^7 + 4*(23
1*cosh(x)^6 - 315*cosh(x)^4 + 105*cosh(x)^2 - 5)*sinh(x)^6 - 20*cosh(x)^6 +
 24*(33*cosh(x)^7 - 63*cosh(x)^5 + 35*cosh(x)^3 - 5*cosh(x))*sinh(x)^5 + 15
```

```
*(33*cosh(x)^8 - 84*cosh(x)^6 + 70*cosh(x)^4 - 20*cosh(x)^2 + 1)*sinh(x)^4
+ 15*cosh(x)^4 + 20*(11*cosh(x)^9 - 36*cosh(x)^7 + 42*cosh(x)^5 - 20*cosh(x
)^3 + 3*cosh(x))*sinh(x)^3 + 6*(11*cosh(x)^10 - 45*cosh(x)^8 + 70*cosh(x)^6
 - 50*cosh(x)^4 + 15*cosh(x)^2 - 1)*sinh(x)^2 - 6*cosh(x)^2 + 12*(cosh(x)^1
1 - 5*cosh(x)^9 + 10*cosh(x)^7 - 10*cosh(x)^5 + 5*cosh(x)^3 - cosh(x))*sinh
(x) + 1)*log(cosh(x) + sinh(x) + 1) + 3*(cosh(x)^12 + 12*cosh(x)*sinh(x)^11
 + sinh(x)^12 + 6*(11*cosh(x)^2 - 1)*sinh(x)^10 - 6*cosh(x)^10 + 20*(11*cos
h(x)^3 - 3*cosh(x))*sinh(x)^9 + 15*(33*cosh(x)^4 - 18*cosh(x)^2 + 1)*sinh(x
)^8 + 15*cosh(x)^8 + 24*(33*cosh(x)^5 - 30*cosh(x)^3 + 5*cosh(x))*sinh(x)^7
 + 4*(231*cosh(x)^6 - 315*cosh(x)^4 + 105*cosh(x)^2 - 5)*sinh(x)^6 - 20*cos
h(x)^6 + 24*(33*cosh(x)^7 - 63*cosh(x)^5 + 35*cosh(x)^3 - 5*cosh(x))*sinh(x
)^5 + 15*(33*cosh(x)^8 - 84*cosh(x)^6 + 70*cosh(x)^4 - 20*cosh(x)^2 + 1)*si
nh(x)^4 + 15*cosh(x)^4 + 20*(11*cosh(x)^9 - 36*cosh(x)^7 + 42*cosh(x)^5 - 2
0*cosh(x)^3 + 3*cosh(x))*sinh(x)^3 + 6*(11*cosh(x)^10 - 45*cosh(x)^8 + 70*c
osh(x)^6 - 50*cosh(x)^4 + 15*cosh(x)^2 - 1)*sinh(x)^2 - 6*cosh(x)^2 + 12*(c
osh(x)^11 - 5*cosh(x)^9 + 10*cosh(x)^7 - 10*cosh(x)^5 + 5*cosh(x)^3 - cosh(
x))*sinh(x) + 1)*log(cosh(x) + sinh(x) - 1) + 6*(11*cosh(x)^10 + 141*cosh(x
)^8 + 182*cosh(x)^6 + 130*cosh(x)^4 + 47*cosh(x)^2 + 1)*sinh(x) + 6*cosh(x)
)/(cosh(x)^12 + 12*cosh(x)*sinh(x)^11 + sinh(x)^12 + 6*(11*cosh(x)^2 - 1)*s
inh(x)^10 - 6*cosh(x)^10 + 20*(11*cosh(x)^3 - 3*cosh(x))*sinh(x)^9 + 15*(33
*cosh(x)^4 - 18*cosh(x)^2 + 1)*sinh(x)^8 + 15*cosh(x)^8 + 24*(33*cosh(x)^5
- 30*cosh(x)^3 + 5*cosh(x))*sinh(x)^7 + 4*(231*cosh(x)^6 - 315*cosh(x)^4 +
105*cosh(x)^2 - 5)*sinh(x)^6 - 20*cosh(x)^6 + 24*(33*cosh(x)^7 - 63*cosh(x)
^5 + 35*cosh(x)^3 - 5*cosh(x))*sinh(x)^5 + 15*(33*cosh(x)^8 - 84*cosh(x)^6
+ 70*cosh(x)^4 - 20*cosh(x)^2 + 1)*sinh(x)^4 + 15*cosh(x)^4 + 20*(11*cosh(x
)^9 - 36*cosh(x)^7 + 42*cosh(x)^5 - 20*cosh(x)^3 + 3*cosh(x))*sinh(x)^3 + 6
*(11*cosh(x)^10 - 45*cosh(x)^8 + 70*cosh(x)^6 - 50*cosh(x)^4 + 15*cosh(x)^2
 - 1)*sinh(x)^2 - 6*cosh(x)^2 + 12*(cosh(x)^11 - 5*cosh(x)^9 + 10*cosh(x)^7
 - 10*cosh(x)^5 + 5*cosh(x)^3 - cosh(x))*sinh(x) + 1)
```

giac [B] time = 0.13, size = 71, normalized size = 1.87

$$-\frac{3\left(e^{(-x)}+e^{x}\right)^{5}+32\left(e^{(-x)}+e^{x}\right)^{3}-48\,e^{(-x)}-48\,e^{x}}{24\left(\left(e^{(-x)}+e^{x}\right)^{2}-4\right)^{3}}+\frac{1}{32}\,\log\left(e^{(-x)}+e^{x}+2\right)-\frac{1}{32}\,\log\left(e^{(-x)}+e^{x}-2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^4*csch(x)^3,x, algorithm="giac")
```

```
[Out] -1/24*(3*(e^(-x) + e^x)^5 + 32*(e^(-x) + e^x)^3 - 48*e^(-x) - 48*e^x)/((e^(
-x) + e^x)^2 - 4)^3 + 1/32*log(e^(-x) + e^x + 2) - 1/32*log(e^(-x) + e^x -
2)
```

maple [A] time = 0.34, size = 46, normalized size = 1.21

$$-\frac{\cosh^3(x)}{3\sinh(x)^6}+\frac{\cosh(x)}{5\sinh(x)^6}+\frac{\left(-\frac{\mathrm{csch}(x)^5}{6}+\frac{5\mathrm{csch}(x)^3}{24}-\frac{5\,\mathrm{csch}(x)}{16}\right)\coth(x)}{5}+\frac{\mathrm{arctanh}\left(\mathrm{e}^x\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^4*csch(x)^3,x)
```

```
[Out] -1/3/sinh(x)^6*cosh(x)^3+1/5/sinh(x)^6*cosh(x)+1/5*(-1/6*csch(x)^5+5/24*csc
h(x)^3-5/16*csch(x))*coth(x)+1/8*arctanh(exp(x))
```

maxima [B] time = 0.35, size = 98, normalized size = 2.58

$$\frac{3\,e^{(-x)}+47\,e^{(-3\,x)}+78\,e^{(-5\,x)}+78\,e^{(-7\,x)}+47\,e^{(-9\,x)}+3\,e^{(-11\,x)}}{24\left(6\,e^{(-2\,x)}-15\,e^{(-4\,x)}+20\,e^{(-6\,x)}-15\,e^{(-8\,x)}+6\,e^{(-10\,x)}-e^{(-12\,x)}-1\right)}+\frac{1}{16}\,\log\left(e^{(-x)}+1\right)-\frac{1}{16}\,\log\left(e^{(-x)}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^4*csch(x)^3,x, algorithm="maxima")
```

```
[Out] 1/24*(3*e^(-x) + 47*e^(-3*x) + 78*e^(-5*x) + 78*e^(-7*x) + 47*e^(-9*x) + 3*
e^(-11*x))/(6*e^(-2*x) - 15*e^(-4*x) + 20*e^(-6*x) - 15*e^(-8*x) + 6*e^(-10
*x) - e^(-12*x) - 1) + 1/16*log(e^(-x) + 1) - 1/16*log(e^(-x) - 1)
```

mupad [B] time = 1.39, size = 214, normalized size = 5.63

$$\frac{\ln\left(\frac{\mathrm{e}^x}{8}+\frac{1}{8}\right)}{16}-\frac{\ln\left(\frac{\mathrm{e}^x}{8}-\frac{1}{8}\right)}{16}-\frac{10\,\mathrm{e}^x}{6\,\mathrm{e}^{4\,x}-4\,\mathrm{e}^{2\,x}-4\,\mathrm{e}^{6\,x}+\mathrm{e}^{8\,x}+1}-\frac{\mathrm{e}^x}{8\,\left(\mathrm{e}^{2\,x}-1\right)}-\frac{7\,\mathrm{e}^x}{3\,\mathrm{e}^{2\,x}-3\,\mathrm{e}^{4\,x}+\mathrm{e}^{6\,x}-1}-\frac{\frac{8\,\mathrm{e}^{3\,x}}{3}+}{15\,\mathrm{e}^{4\,x}-6\,\mathrm{e}^{2\,x}-}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^4/sinh(x)^3,x)
```

```
[Out] log(exp(x)/8 + 1/8)/16 - log(exp(x)/8 - 1/8)/16 - (10*exp(x))/(6*exp(4*x) -
 4*exp(2*x) - 4*exp(6*x) + exp(8*x) + 1) - exp(x)/(8*(exp(2*x) - 1)) - (7*e
xp(x))/(3*exp(2*x) - 3*exp(4*x) + exp(6*x) - 1) - ((8*exp(3*x))/3 + 4*exp(5
*x) + (8*exp(7*x))/3 + (2*exp(9*x))/3 + (2*exp(x))/3)/(15*exp(4*x) - 6*exp(
2*x) - 20*exp(6*x) + 15*exp(8*x) - 6*exp(10*x) + exp(12*x) + 1) - (16*exp(x
))/(3*(5*exp(2*x) - 10*exp(4*x) + 10*exp(6*x) - 5*exp(8*x) + exp(10*x) - 1)
) - (23*exp(x))/(12*(exp(4*x) - 2*exp(2*x) + 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \coth^4(x)\,\mathrm{csch}^3(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)**4*csch(x)**3,x)
```

```
[Out] Integral(coth(x)**4*csch(x)**3, x)
```

3.128 $\int \coth^4(x)\mathbf{csch}^6(x)\,dx$

Optimal. Leaf size=25

$$-\frac{1}{9}\coth^9(x)+\frac{2\coth^7(x)}{7}-\frac{\coth^5(x)}{5}$$

```
[Out] -1/5*coth(x)^5+2/7*coth(x)^7-1/9*coth(x)^9
```

Rubi [A] time = 0.03, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2607, 270}

$$-\frac{1}{9}\coth^9(x)+\frac{2\coth^7(x)}{7}-\frac{\coth^5(x)}{5}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]^4*Csch[x]^6,x]
```

```
[Out] -Coth[x]^5/5 + (2*Coth[x]^7)/7 - Coth[x]^9/9
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] && !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rubi steps

$$\begin{aligned}\int \coth^4(x)\mathrm{csch}^6(x)\,dx &= i\,\mathrm{Subst}\left(\int x^4\left(1+x^2\right)^2\,dx,x,i\coth(x)\right)\\ &= i\,\mathrm{Subst}\left(\int \left(x^4+2x^6+x^8\right)\,dx,x,i\coth(x)\right)\\ &= -\frac{1}{5}\coth^5(x)+\frac{2\coth^7(x)}{7}-\frac{\coth^9(x)}{9}\end{aligned}$$

Mathematica [A] time = 0.03, size = 47, normalized size = 1.88

$$-\frac{8\coth(x)}{315}-\frac{1}{9}\coth(x)\text{csch}^8(x)-\frac{10}{63}\coth(x)\text{csch}^6(x)-\frac{1}{105}\coth(x)\text{csch}^4(x)+\frac{4}{315}\coth(x)\text{csch}^2(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]^4*Csch[x]^6,x]
```

```
[Out] (-8*Coth[x])/315 + (4*Coth[x]*Csch[x]^2)/315 - (Coth[x]*Csch[x]^4)/105 - (1
0*Coth[x]*Csch[x]^6)/63 - (Coth[x]*Csch[x]^8)/9
```

fricas [B] time = 0.38, size = 430, normalized size = 17.20

$$-\frac{}{315\left(\cosh(x)^{12}+12\cosh(x)\sinh(x)^{11}+\sinh(x)^{12}+3\left(22\cosh(x)^2-3\right)\sinh(x)^{10}-9\cosh(x)^{10}+10\left(22\ \text{co}\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^4*csch(x)^6,x, algorithm="fricas")
```

```
[Out] -16/315*(211*cosh(x)^6 + 1254*cosh(x)*sinh(x)^5 + 211*sinh(x)^6 + 3*(1055*c
osh(x)^2 + 102)*sinh(x)^4 + 306*cosh(x)^4 + 4*(1045*cosh(x)^3 + 324*cosh(x)
)*sinh(x)^3 + 3*(1055*cosh(x)^4 + 612*cosh(x)^2 + 159)*sinh(x)^2 + 477*cosh
(x)^2 + 6*(209*cosh(x)^5 + 216*cosh(x)^3 + 135*cosh(x))*sinh(x) + 126)/(cos
h(x)^12 + 12*cosh(x)*sinh(x)^11 + sinh(x)^12 + 3*(22*cosh(x)^2 - 3)*sinh(x)
^10 - 9*cosh(x)^10 + 10*(22*cosh(x)^3 - 9*cosh(x))*sinh(x)^9 + 9*(55*cosh(x
)^4 - 45*cosh(x)^2 + 4)*sinh(x)^8 + 36*cosh(x)^8 + 72*(11*cosh(x)^5 - 15*co
sh(x)^3 + 4*cosh(x))*sinh(x)^7 + (924*cosh(x)^6 - 1890*cosh(x)^4 + 1008*cos
h(x)^2 - 85)*sinh(x)^6 - 85*cosh(x)^6 + 6*(132*cosh(x)^7 - 378*cosh(x)^5 +
336*cosh(x)^3 - 83*cosh(x))*sinh(x)^5 + 15*(33*cosh(x)^8 - 126*cosh(x)^6 +
168*cosh(x)^4 - 85*cosh(x)^2 + 9)*sinh(x)^4 + 135*cosh(x)^4 + 4*(55*cosh(x)
^9 - 270*cosh(x)^7 + 504*cosh(x)^5 - 415*cosh(x)^3 + 117*cosh(x))*sinh(x)^3
 + 3*(22*cosh(x)^10 - 135*cosh(x)^8 + 336*cosh(x)^6 - 425*cosh(x)^4 + 270*c
osh(x)^2 - 54)*sinh(x)^2 - 162*cosh(x)^2 + 6*(2*cosh(x)^11 - 15*cosh(x)^9 +
 48*cosh(x)^7 - 83*cosh(x)^5 + 78*cosh(x)^3 - 30*cosh(x))*sinh(x) + 84)
```

giac [B] time = 0.12, size = 48, normalized size = 1.92

$$-\frac{16\left(210\,e^{(12\,x)}+315\,e^{(10\,x)}+441\,e^{(8\,x)}+126\,e^{(6\,x)}+36\,e^{(4\,x)}-9\,e^{(2\,x)}+1\right)}{315\left(e^{(2\,x)}-1\right)^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^4*csch(x)^6,x, algorithm="giac")
```

```
[Out] -16/315*(210*e^(12*x) + 315*e^(10*x) + 441*e^(8*x) + 126*e^(6*x) + 36*e^(4*
x) - 9*e^(2*x) + 1)/(e^(2*x) - 1)^9
```

maple [B] time = 0.34, size = 50, normalized size = 2.00

$$-\frac{\cosh^3(x)}{6\sinh(x)^9}+\frac{\cosh(x)}{16\sinh(x)^9}+\frac{\left(-\frac{128}{315}-\frac{\operatorname{csch}(x)^8}{9}+\frac{8\operatorname{csch}(x)^6}{63}-\frac{16\operatorname{csch}(x)^4}{105}+\frac{64\operatorname{csch}(x)^2}{315}\right)\coth(x)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^4*csch(x)^6,x)
```

```
[Out] -1/6/sinh(x)^9*cosh(x)^3+1/16/sinh(x)^9*cosh(x)+1/16*(-128/315-1/9*csch(x)^
8+8/63*csch(x)^6-16/105*csch(x)^4+64/315*csch(x)^2)*coth(x)
```

maxima [B] time = 0.31, size = 431, normalized size = 17.24

$$-\frac{16\,e^{(-2\,x)}}{35\left(9\,e^{(-2\,x)}-36\,e^{(-4\,x)}+84\,e^{(-6\,x)}-126\,e^{(-8\,x)}+126\,e^{(-10\,x)}-84\,e^{(-12\,x)}+36\,e^{(-14\,x)}-9\,e^{(-16\,x)}+e^{(-18\,x)}-1\right)}+\frac{}{35}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^4*csch(x)^6,x, algorithm="maxima")
```

```
[Out] -16/35*e^(-2*x)/(9*e^(-2*x) - 36*e^(-4*x) + 84*e^(-6*x) - 126*e^(-8*x) + 12
6*e^(-10*x) - 84*e^(-12*x) + 36*e^(-14*x) - 9*e^(-16*x) + e^(-18*x) - 1) +
64/35*e^(-4*x)/(9*e^(-2*x) - 36*e^(-4*x) + 84*e^(-6*x) - 126*e^(-8*x) + 126
*e^(-10*x) - 84*e^(-12*x) + 36*e^(-14*x) - 9*e^(-16*x) + e^(-18*x) - 1) + 3
2/5*e^(-6*x)/(9*e^(-2*x) - 36*e^(-4*x) + 84*e^(-6*x) - 126*e^(-8*x) + 126*e
^(-10*x) - 84*e^(-12*x) + 36*e^(-14*x) - 9*e^(-16*x) + e^(-18*x) - 1) + 112
/5*e^(-8*x)/(9*e^(-2*x) - 36*e^(-4*x) + 84*e^(-6*x) - 126*e^(-8*x) + 126*e^
(-10*x) - 84*e^(-12*x) + 36*e^(-14*x) - 9*e^(-16*x) + e^(-18*x) - 1) + 16*e
^(-10*x)/(9*e^(-2*x) - 36*e^(-4*x) + 84*e^(-6*x) - 126*e^(-8*x) + 126*e^(-1
0*x) - 84*e^(-12*x) + 36*e^(-14*x) - 9*e^(-16*x) + e^(-18*x) - 1) + 32/3*e^
(-12*x)/(9*e^(-2*x) - 36*e^(-4*x) + 84*e^(-6*x) - 126*e^(-8*x) + 126*e^(-10
*x) - 84*e^(-12*x) + 36*e^(-14*x) - 9*e^(-16*x) + e^(-18*x) - 1) + 16/315/(
9*e^(-2*x) - 36*e^(-4*x) + 84*e^(-6*x) - 126*e^(-8*x) + 126*e^(-10*x) - 84*
e^(-12*x) + 36*e^(-14*x) - 9*e^(-16*x) + e^(-18*x) - 1)
```

mupad [B] time = 1.39, size = 413, normalized size = 16.52

$$-\frac{\frac{8\,e^{2\,x}}{9}+\frac{16\,e^{4\,x}}{3}+\frac{32\,e^{6\,x}}{3}+\frac{80\,e^{8\,x}}{9}+\frac{8\,e^{10\,x}}{3}}{28\,e^{4\,x}-8\,e^{2\,x}-56\,e^{6\,x}+70\,e^{8\,x}-56\,e^{10\,x}+28\,e^{12\,x}-8\,e^{14\,x}+e^{16\,x}+1}-\frac{\frac{8\,e^{2\,x}}{21}+\frac{16}{63}}{6\,e^{4\,x}-4\,e^{2\,x}-4\,e^{6\,x}+e^{8\,x}+1}-\frac{}{63\,(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^4/sinh(x)^6,x)
```

```
[Out] - ((8*exp(2*x))/9 + (16*exp(4*x))/3 + (32*exp(6*x))/3 + (80*exp(8*x))/9 + (
8*exp(10*x))/3)/(28*exp(4*x) - 8*exp(2*x) - 56*exp(6*x) + 70*exp(8*x) - 56*
exp(10*x) + 28*exp(12*x) - 8*exp(14*x) + exp(16*x) + 1) - ((8*exp(2*x))/21
+ 16/63)/(6*exp(4*x) - 4*exp(2*x) - 4*exp(6*x) + exp(8*x) + 1) - 8/(63*(3*e
xp(2*x) - 3*exp(4*x) + exp(6*x) - 1)) - ((64*exp(2*x))/63 + (16*exp(4*x))/2
1 + 32/105)/(5*exp(2*x) - 10*exp(4*x) + 10*exp(6*x) - 5*exp(8*x) + exp(10*x
) - 1) - ((32*exp(2*x))/21 + (160*exp(4*x))/63 + (80*exp(6*x))/63 + 16/63)/
(15*exp(4*x) - 6*exp(2*x) - 20*exp(6*x) + 15*exp(8*x) - 6*exp(10*x) + exp(1
2*x) + 1) - ((32*exp(4*x))/9 + (128*exp(6*x))/9 + (64*exp(8*x))/3 + (128*ex
p(10*x))/9 + (32*exp(12*x))/9)/(9*exp(2*x) - 36*exp(4*x) + 84*exp(6*x) - 12
6*exp(8*x) + 126*exp(10*x) - 84*exp(12*x) + 36*exp(14*x) - 9*exp(16*x) + ex
p(18*x) - 1) - ((32*exp(2*x))/21 + (32*exp(4*x))/7 + (320*exp(6*x))/63 + (4
0*exp(8*x))/21 + 8/63)/(7*exp(2*x) - 21*exp(4*x) + 35*exp(6*x) - 35*exp(8*x
) + 21*exp(10*x) - 7*exp(12*x) + exp(14*x) - 1)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \coth^4(x)\,\text{csch}^6(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)**4*csch(x)**6,x)
```

```
[Out] Integral(coth(x)**4*csch(x)**6, x)
```

3.129 $\int \coth^5(6x)\mathbf{csch}(6x)\,dx$

Optimal. Leaf size=29

$$-\frac{1}{30}\text{csch}^5(6x)-\frac{1}{9}\text{csch}^3(6x)-\frac{1}{6}\text{csch}(6x)$$

```
[Out] -1/6*csch(6*x)-1/9*csch(6*x)^3-1/30*csch(6*x)^5
```

Rubi [A] time = 0.02, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2606, 194}

$$-\frac{1}{30}\text{csch}^5(6x)-\frac{1}{9}\text{csch}^3(6x)-\frac{1}{6}\text{csch}(6x)$$

Antiderivative was successfully verified.

```
[In] Int[Coth[6*x]^5*Csch[6*x],x]
```

```
[Out] -Csch[6*x]/6 - Csch[6*x]^3/9 - Csch[6*x]^5/30
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rubi steps

$$\begin{aligned}\int \coth^5(6x)\text{csch}(6x)\,dx &= -\left(\frac{1}{6}i\,\text{Subst}\left(\int \left(-1+x^2\right)^2\,dx,x,-i\text{csch}(6x)\right)\right)\\ &= -\left(\frac{1}{6}i\,\text{Subst}\left(\int \left(1-2x^2+x^4\right)\,dx,x,-i\text{csch}(6x)\right)\right)\\ &= -\frac{1}{6}\text{csch}(6x)-\frac{1}{9}\text{csch}^3(6x)-\frac{1}{30}\text{csch}^5(6x)\end{aligned}$$

Mathematica [A] time = 0.02, size = 29, normalized size = 1.00

$$-\frac{1}{30}\text{csch}^5(6x)-\frac{1}{9}\text{csch}^3(6x)-\frac{1}{6}\text{csch}(6x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[6*x]^5*Csch[6*x],x]
```

```
[Out] -1/6*Csch[6*x] - Csch[6*x]^3/9 - Csch[6*x]^5/30
```

fricas [B] time = 0.42, size = 250, normalized size = 8.62

$$-\frac{15\cosh(6x)^5+75\cosh(6x)\sinh(6x)^4+15\sinh(6x)^5+5\left(30\cosh(6x)^2-\right.}{45\left(\cosh(6x)^6+6\cosh(6x)\sinh(6x)^5+\sinh(6x)^6+3\left(5\cosh(6x)^2-2\right)\sinh(6x)^4-6\cosh(6x)^4+4\left(\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(6*x)^5*csch(6*x),x, algorithm="fricas")
```

```
[Out] -1/45*(15*cosh(6*x)^5 + 75*cosh(6*x)*sinh(6*x)^4 + 15*sinh(6*x)^5 + 5*(30*c
osh(6*x)^2 - 7)*sinh(6*x)^3 - 5*cosh(6*x)^3 + 15*(10*cosh(6*x)^3 - cosh(6*x
))*sinh(6*x)^2 + 3*(25*cosh(6*x)^4 - 35*cosh(6*x)^2 + 26)*sinh(6*x) + 38*co
sh(6*x))/(cosh(6*x)^6 + 6*cosh(6*x)*sinh(6*x)^5 + sinh(6*x)^6 + 3*(5*cosh(6
*x)^2 - 2)*sinh(6*x)^4 - 6*cosh(6*x)^4 + 4*(5*cosh(6*x)^3 - 4*cosh(6*x))*si
nh(6*x)^3 + 3*(5*cosh(6*x)^4 - 12*cosh(6*x)^2 + 5)*sinh(6*x)^2 + 15*cosh(6*
x)^2 + 2*(3*cosh(6*x)^5 - 8*cosh(6*x)^3 + 5*cosh(6*x))*sinh(6*x) - 10)
```

giac [B] time = 0.13, size = 47, normalized size = 1.62

$$-\frac{15\left(e^{(6x)}-e^{(-6x)}\right)^4+40\left(e^{(6x)}-e^{(-6x)}\right)^2+48}{45\left(e^{(6x)}-e^{(-6x)}\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(6*x)^5*csch(6*x),x, algorithm="giac")
```

```
[Out] -1/45*(15*(e^(6*x) - e^(-6*x))^4 + 40*(e^(6*x) - e^(-6*x))^2 + 48)/(e^(6*x)
 - e^(-6*x))^5
```

maple [A] time = 0.13, size = 38, normalized size = 1.31

$$-\frac{\cosh^4(6x)}{6\sinh(6x)^5}+\frac{2\left(\cosh^2(6x)\right)}{9\sinh(6x)^5}-\frac{4}{45\sinh(6x)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(6*x)^5*csch(6*x),x)
```

```
[Out] -1/6/sinh(6*x)^5*cosh(6*x)^4+2/9/sinh(6*x)^5*cosh(6*x)^2-4/45/sinh(6*x)^5
```

maxima [B] time = 0.54, size = 191, normalized size = 6.59

$$\frac{e^{(-6x)}}{3\left(5e^{(-12x)} - 10e^{(-24x)} + 10e^{(-36x)} - 5e^{(-48x)} + e^{(-60x)} - 1\right)} - \frac{4e^{(-18x)}}{9\left(5e^{(-12x)} - 10e^{(-24x)} + 10e^{(-36x)} - 5e^{(-48x)} + e^{(-6}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(6*x)^5*csch(6*x),x, algorithm="maxima")
```

```
[Out] 1/3*e^(-6*x)/(5*e^(-12*x) - 10*e^(-24*x) + 10*e^(-36*x) - 5*e^(-48*x) + e^(
-60*x) - 1) - 4/9*e^(-18*x)/(5*e^(-12*x) - 10*e^(-24*x) + 10*e^(-36*x) - 5*
e^(-48*x) + e^(-60*x) - 1) + 58/45*e^(-30*x)/(5*e^(-12*x) - 10*e^(-24*x) +
10*e^(-36*x) - 5*e^(-48*x) + e^(-60*x) - 1) - 4/9*e^(-42*x)/(5*e^(-12*x) -
10*e^(-24*x) + 10*e^(-36*x) - 5*e^(-48*x) + e^(-60*x) - 1) + 1/3*e^(-54*x)/
(5*e^(-12*x) - 10*e^(-24*x) + 10*e^(-36*x) - 5*e^(-48*x) + e^(-60*x) - 1)
```

mupad [B] time = 1.45, size = 40, normalized size = 1.38

$$-\frac{e^{6x}\left(58e^{24x} - 20e^{12x} - 20e^{36x} + 15e^{48x} + 15\right)}{45\left(e^{12x} - 1\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(6*x)^5/sinh(6*x),x)
```

```
[Out] -(exp(6*x)*(58*exp(24*x) - 20*exp(12*x) - 20*exp(36*x) + 15*exp(48*x) + 15)
)/(45*(exp(12*x) - 1)^5)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \coth^5(6x)\operatorname{csch}(6x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(6*x)**5*csch(6*x),x)
```

```
[Out] Integral(coth(6*x)**5*csch(6*x), x)
```

3.130 $\int \coth^7(x)\mathbf{csch}^3(x)\,dx$

Optimal. Leaf size=33

$$-\frac{1}{9}\text{csch}^9(x)-\frac{3\text{csch}^7(x)}{7}-\frac{3\text{csch}^5(x)}{5}-\frac{\text{csch}^3(x)}{3}$$

```
[Out] -1/3*csch(x)^3-3/5*csch(x)^5-3/7*csch(x)^7-1/9*csch(x)^9
```

Rubi [A] time = 0.03, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2606, 270}

$$-\frac{1}{9}\text{csch}^9(x)-\frac{3\text{csch}^7(x)}{7}-\frac{3\text{csch}^5(x)}{5}-\frac{\text{csch}^3(x)}{3}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]^7*Csch[x]^3,x]
```

```
[Out] -Csch[x]^3/3 - (3*Csch[x]^5)/5 - (3*Csch[x]^7)/7 - Csch[x]^9/9
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rubi steps

$$\begin{aligned}\int \coth^7(x)\text{csch}^3(x)\,dx &= -\left(i\,\text{Subst}\left(\int x^2\left(-1+x^2\right)^3\,dx,x,-i\text{csch}(x)\right)\right)\\ &= -\left(i\,\text{Subst}\left(\int \left(-x^2+3x^4-3x^6+x^8\right)\,dx,x,-i\text{csch}(x)\right)\right)\\ &= -\frac{1}{3}\text{csch}^3(x)-\frac{3\text{csch}^5(x)}{5}-\frac{3\text{csch}^7(x)}{7}-\frac{\text{csch}^9(x)}{9}\end{aligned}$$

Mathematica [A] time = 0.01, size = 33, normalized size = 1.00

$$-\frac{1}{9}\text{csch}^9(x)-\frac{3\text{csch}^7(x)}{7}-\frac{3\text{csch}^5(x)}{5}-\frac{\text{csch}^3(x)}{3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]^7*Csch[x]^3,x]
```

```
[Out] -1/3*Csch[x]^3 - (3*Csch[x]^5)/5 - (3*Csch[x]^7)/7 - Csch[x]^9/9
```

fricas [B] time = 0.68, size = 442, normalized size = 13.39

$$-\frac{\quad}{315\left(\cosh(x)^{11}+11\cosh(x)\sinh(x)^{10}+\sinh(x)^{11}+\left(55\cosh(x)^2-9\right)\sinh(x)^9-9\cosh(x)^9+3\left(55\cosh(x)\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^7*csch(x)^3,x, algorithm="fricas")
```

```
[Out] -8/315*(105*cosh(x)^8 + 840*cosh(x)*sinh(x)^7 + 105*sinh(x)^8 + 42*(70*cosh
(x)^2 + 3)*sinh(x)^6 + 126*cosh(x)^6 + 84*(70*cosh(x)^3 + 9*cosh(x))*sinh(x
)^5 + 6*(1225*cosh(x)^4 + 315*cosh(x)^2 + 136)*sinh(x)^4 + 816*cosh(x)^4 +
24*(245*cosh(x)^5 + 105*cosh(x)^3 + 101*cosh(x))*sinh(x)^3 + 2*(1470*cosh(x
)^6 + 945*cosh(x)^4 + 2448*cosh(x)^2 + 241)*sinh(x)^2 + 482*cosh(x)^2 + 4*(
210*cosh(x)^7 + 189*cosh(x)^5 + 606*cosh(x)^3 + 115*cosh(x))*sinh(x) + 711)
/(cosh(x)^11 + 11*cosh(x)*sinh(x)^10 + sinh(x)^11 + (55*cosh(x)^2 - 9)*sinh
(x)^9 - 9*cosh(x)^9 + 3*(55*cosh(x)^3 - 27*cosh(x))*sinh(x)^8 + (330*cosh(x
)^4 - 324*cosh(x)^2 + 37)*sinh(x)^7 + 35*cosh(x)^7 + 7*(66*cosh(x)^5 - 108*
cosh(x)^3 + 35*cosh(x))*sinh(x)^6 + 3*(154*cosh(x)^6 - 378*cosh(x)^4 + 259*
cosh(x)^2 - 31)*sinh(x)^5 - 75*cosh(x)^5 + (330*cosh(x)^7 - 1134*cosh(x)^5
+ 1225*cosh(x)^3 - 375*cosh(x))*sinh(x)^4 + (165*cosh(x)^8 - 756*cosh(x)^6
+ 1295*cosh(x)^4 - 930*cosh(x)^2 + 162)*sinh(x)^3 + 90*cosh(x)^3 + (55*cosh
(x)^9 - 324*cosh(x)^7 + 735*cosh(x)^5 - 750*cosh(x)^3 + 270*cosh(x))*sinh(x
)^2 + (11*cosh(x)^10 - 81*cosh(x)^8 + 259*cosh(x)^6 - 465*cosh(x)^4 + 486*c
osh(x)^2 - 210)*sinh(x) - 42*cosh(x))
```

giac [B] time = 0.13, size = 54, normalized size = 1.64

$$\frac{8\left(105\left(e^{(-x)}-e^x\right)^6+756\left(e^{(-x)}-e^x\right)^4+2160\left(e^{(-x)}-e^x\right)^2+2240\right)}{315\left(e^{(-x)}-e^x\right)^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^7*csch(x)^3,x, algorithm="giac")
```

```
[Out] 8/315*(105*(e^(-x) - e^x)^6 + 756*(e^(-x) - e^x)^4 + 2160*(e^(-x) - e^x)^2
+ 2240)/(e^(-x) - e^x)^9
```

maple [A] time = 0.11, size = 38, normalized size = 1.15

$$-\frac{\cosh^6(x)}{3\sinh(x)^9}+\frac{2\left(\cosh^4(x)\right)}{5\sinh(x)^9}-\frac{8\left(\cosh^2(x)\right)}{35\sinh(x)^9}+\frac{16}{315\sinh(x)^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^7*csch(x)^3,x)
```

```
[Out] -1/3/sinh(x)^9*cosh(x)^6+2/5/sinh(x)^9*cosh(x)^4-8/35/sinh(x)^9*cosh(x)^2+1
6/315/sinh(x)^9
```

maxima [B] time = 0.32, size = 435, normalized size = 13.18

$$\frac{8\,e^{(-3\,x)}}{3\left(9\,e^{(-2\,x)}-36\,e^{(-4\,x)}+84\,e^{(-6\,x)}-126\,e^{(-8\,x)}+126\,e^{(-10\,x)}-84\,e^{(-12\,x)}+36\,e^{(-14\,x)}-9\,e^{(-16\,x)}+e^{(-18\,x)}-1\right)}+\frac{}{5\,(\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^7*csch(x)^3,x, algorithm="maxima")
```

```
[Out] 8/3*e^(-3*x)/(9*e^(-2*x) - 36*e^(-4*x) + 84*e^(-6*x) - 126*e^(-8*x) + 126*e
^(-10*x) - 84*e^(-12*x) + 36*e^(-14*x) - 9*e^(-16*x) + e^(-18*x) - 1) + 16/
5*e^(-5*x)/(9*e^(-2*x) - 36*e^(-4*x) + 84*e^(-6*x) - 126*e^(-8*x) + 126*e^(
-10*x) - 84*e^(-12*x) + 36*e^(-14*x) - 9*e^(-16*x) + e^(-18*x) - 1) + 632/3
5*e^(-7*x)/(9*e^(-2*x) - 36*e^(-4*x) + 84*e^(-6*x) - 126*e^(-8*x) + 126*e^(
-10*x) - 84*e^(-12*x) + 36*e^(-14*x) - 9*e^(-16*x) + e^(-18*x) - 1) + 2848/
315*e^(-9*x)/(9*e^(-2*x) - 36*e^(-4*x) + 84*e^(-6*x) - 126*e^(-8*x) + 126*e
^(-10*x) - 84*e^(-12*x) + 36*e^(-14*x) - 9*e^(-16*x) + e^(-18*x) - 1) + 632
/35*e^(-11*x)/(9*e^(-2*x) - 36*e^(-4*x) + 84*e^(-6*x) - 126*e^(-8*x) + 126*
e^(-10*x) - 84*e^(-12*x) + 36*e^(-14*x) - 9*e^(-16*x) + e^(-18*x) - 1) + 16
/5*e^(-13*x)/(9*e^(-2*x) - 36*e^(-4*x) + 84*e^(-6*x) - 126*e^(-8*x) + 126*e
^(-10*x) - 84*e^(-12*x) + 36*e^(-14*x) - 9*e^(-16*x) + e^(-18*x) - 1) + 8/3
*e^(-15*x)/(9*e^(-2*x) - 36*e^(-4*x) + 84*e^(-6*x) - 126*e^(-8*x) + 126*e^(
-10*x) - 84*e^(-12*x) + 36*e^(-14*x) - 9*e^(-16*x) + e^(-18*x) - 1)
```

mupad [B] time = 1.41, size = 372, normalized size = 11.27

$$-\frac{5872\,e^{x}}{105\left(6\,e^{4\,x}-4\,e^{2\,x}-4\,e^{6\,x}+e^{8\,x}+1\right)}-\frac{\frac{28\,e^{3\,x}}{9}+\frac{28\,e^{5\,x}}{3}+\frac{140\,e^{7\,x}}{9}+\frac{140\,e^{9\,x}}{9}+\frac{28\,e^{11\,x}}{3}+\frac{28\,e^{13\,x}}{9}+\frac{4\,e^{15\,x}}{9}+\cdots}{9\,e^{2\,x}-36\,e^{4\,x}+84\,e^{6\,x}-126\,e^{8\,x}+126\,e^{10\,x}-84\,e^{12\,x}+36\,e^{14\,x}-9\,e^{1\cdots}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^7/sinh(x)^3,x)
```

```
[Out] - (5872*exp(x))/(105*(6*exp(4*x) - 4*exp(2*x) - 4*exp(6*x) + exp(8*x) + 1))
 - ((28*exp(3*x))/9 + (28*exp(5*x))/3 + (140*exp(7*x))/9 + (140*exp(9*x))/9
 + (28*exp(11*x))/3 + (28*exp(13*x))/9 + (4*exp(15*x))/9 + (4*exp(x))/9)/(9
*exp(2*x) - 36*exp(4*x) + 84*exp(6*x) - 126*exp(8*x) + 126*exp(10*x) - 84*e
xp(12*x) + 36*exp(14*x) - 9*exp(16*x) + exp(18*x) - 1) - (3008*exp(x))/(21*
(15*exp(4*x) - 6*exp(2*x) - 20*exp(6*x) + 15*exp(8*x) - 6*exp(10*x) + exp(1
2*x) + 1)) - (704*exp(x))/(45*(3*exp(2*x) - 3*exp(4*x) + exp(6*x) - 1)) - (
256*exp(x))/(9*(28*exp(4*x) - 8*exp(2*x) - 56*exp(6*x) + 70*exp(8*x) - 56*e
xp(10*x) + 28*exp(12*x) - 8*exp(14*x) + exp(16*x) + 1)) - (36608*exp(x))/(3
15*(5*exp(2*x) - 10*exp(4*x) + 10*exp(6*x) - 5*exp(8*x) + exp(10*x) - 1)) -
 (20*exp(x))/(9*(exp(4*x) - 2*exp(2*x) + 1)) - (2048*exp(x))/(21*(7*exp(2*x
) - 21*exp(4*x) + 35*exp(6*x) - 35*exp(8*x) + 21*exp(10*x) - 7*exp(12*x) +
exp(14*x) - 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \coth^7(x)\operatorname{csch}^3(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)**7*csch(x)**3,x)
```

```
[Out] Integral(coth(x)**7*csch(x)**3, x)
```

3.131 $\int \sinh(a+bx)\sinh(c+bx)\,dx$

Optimal. Leaf size=27

$$\frac{\sinh(a+2bx+c)}{4b}-\frac{1}{2}x\cosh(a-c)$$

```
[Out] -1/2*x*cosh(a-c)+1/4*sinh(2*b*x+a+c)/b
```

Rubi [A] time = 0.03, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {5613, 2637}

$$\frac{\sinh(a+2bx+c)}{4b}-\frac{1}{2}x\cosh(a-c)$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[a + b*x]*Sinh[c + b*x],x]
```

```
[Out] -(x*Cosh[a - c])/2 + Sinh[a + c + 2*b*x]/(4*b)
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 5613

```
Int[Sinh[v_]^(p_.)*Sinh[w_]^(q_.), x_Symbol] :> Int[ExpandTrigReduce[Sinh[v
]^p*Sinh[w]^q, x], x] /; IGtQ[p, 0] && IGtQ[q, 0] && ((PolynomialQ[v, x] &&
 PolynomialQ[w, x]) || (BinomialQ[{v, w}, x] && IndependentQ[Cancel[v/w], x
]))
```

Rubi steps

$$\begin{aligned}
\int \sinh(a+bx)\sinh(c+bx)\,dx &= \int \left(-\frac{1}{2}\cosh(a-c)+\frac{1}{2}\cosh(a+c+2bx)\right)dx\\
&= -\frac{1}{2}x\cosh(a-c)+\frac{1}{2}\int \cosh(a+c+2bx)\,dx\\
&= -\frac{1}{2}x\cosh(a-c)+\frac{\sinh(a+c+2bx)}{4b}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 26, normalized size = 0.96

$$\frac{\sinh(a+2bx+c)-2bx\cosh(a-c)}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[a + b*x]*Sinh[c + b*x],x]
```

```
[Out] (-2*b*x*Cosh[a - c] + Sinh[a + c + 2*b*x])/(4*b)
```

fricas [B] time = 0.46, size = 87, normalized size = 3.22

$$-\frac{2\,bx\cosh\left(-a+c\right)-2\,\cosh\left(bx+c\right)\cosh\left(-a+c\right)\sinh\left(bx+c\right)+\cosh\left(bx+c\right)^{2}\sinh\left(-a+c\right)+\sinh\left(bx+c\right)^{2}}{4\left(b\cosh\left(-a+c\right)^{2}-b\sinh\left(-a+c\right)^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)*sinh(b*x+c),x, algorithm="fricas")
```

```
[Out] -1/4*(2*b*x*cosh(-a + c) - 2*cosh(b*x + c)*cosh(-a + c)*sinh(b*x + c) + cos
h(b*x + c)^2*sinh(-a + c) + sinh(b*x + c)^2*sinh(-a + c))/(b*cosh(-a + c)^2
 - b*sinh(-a + c)^2)
```

giac [B] time = 0.12, size = 71, normalized size = 2.63

$$-\frac{2\,bx\left(e^{(2\,a)}+e^{(2\,c)}\right)e^{(-a-c)}-\left(e^{(2\,bx+2\,a)}+e^{(2\,bx+2\,c)}-1\right)e^{(-2\,bx-a-c)}-e^{(2\,bx+a+c)}}{8\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)*sinh(b*x+c),x, algorithm="giac")
```

```
[Out] -1/8*(2*b*x*(e^(2*a) + e^(2*c))*e^(-a - c) - (e^(2*b*x + 2*a) + e^(2*b*x +
2*c) - 1)*e^(-2*b*x - a - c) - e^(2*b*x + a + c))/b
```

maple [A] time = 0.09, size = 24, normalized size = 0.89

$$-\frac{x\cosh\left(a-c\right)}{2}+\frac{\sinh\left(2bx+a+c\right)}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(b*x+a)*sinh(b*x+c),x)
```

```
[Out] -1/2*x*cosh(a-c)+1/4*sinh(2*b*x+a+c)/b
```

maxima [B] time = 0.32, size = 58, normalized size = 2.15

$$-\frac{(bx+a)\left(e^{(2a)}+e^{(2c)}\right)e^{(-a-c)}}{4b}+\frac{e^{(2bx+a+c)}}{8b}-\frac{e^{(-2bx-a-c)}}{8b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)*sinh(b*x+c),x, algorithm="maxima")
```

```
[Out] -1/4*(b*x + a)*(e^(2*a) + e^(2*c))*e^(-a - c)/b + 1/8*e^(2*b*x + a + c)/b -
 1/8*e^(-2*b*x - a - c)/b
```

mupad [B] time = 0.16, size = 23, normalized size = 0.85

$$\frac{\sinh(a+c+2bx)}{4b}-\frac{x\cosh(a-c)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a + b*x)*sinh(c + b*x),x)
```

```
[Out] sinh(a + c + 2*b*x)/(4*b) - (x*cosh(a - c))/2
```

sympy [A] time = 0.74, size = 58, normalized size = 2.15

$$\begin{cases} \frac{x\sinh(a+bx)\sinh(bx+c)}{2}-\frac{x\cosh(a+bx)\cosh(bx+c)}{2}+\frac{\sinh(a+bx)\cosh(bx+c)}{2b} & \text{for } b\neq 0 \\ x\sinh(a)\sinh(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)*sinh(b*x+c),x)
```

```
[Out] Piecewise((x*sinh(a + b*x)*sinh(b*x + c)/2 - x*cosh(a + b*x)*cosh(b*x + c)/
2 + sinh(a + b*x)*cosh(b*x + c)/(2*b), Ne(b, 0)), (x*sinh(a)*sinh(c), True)
)
```

3.132 $\int \sinh(c-bx)\sinh(a+bx)\,dx$

Optimal. Leaf size=27

$$\frac{1}{2}x\cosh(a+c)-\frac{\sinh(a+2bx-c)}{4b}$$

```
[Out] 1/2*x*cosh(a+c)-1/4*sinh(2*b*x+a-c)/b
```

Rubi [A] time = 0.03, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {5613, 2637}

$$\frac{1}{2}x\cosh(a+c)-\frac{\sinh(a+2bx-c)}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c - b*x]*Sinh[a + b*x],x]
```

```
[Out] (x*Cosh[a + c])/2 - Sinh[a - c + 2*b*x]/(4*b)
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 5613

```
Int[Sinh[v_]^(p_.)*Sinh[w_]^(q_.), x_Symbol] :> Int[ExpandTrigReduce[Sinh[v
]^p*Sinh[w]^q, x], x] /; IGtQ[p, 0] && IGtQ[q, 0] && ((PolynomialQ[v, x] &&
 PolynomialQ[w, x]) || (BinomialQ[{v, w}, x] && IndependentQ[Cancel[v/w], x
]))
```

Rubi steps

$$\begin{aligned}\int \sinh(c-bx)\sinh(a+bx)\,dx &= \int \left(\frac{1}{2}\cosh(a+c)-\frac{1}{2}\cosh(a-c+2bx)\right)dx\\ &= \frac{1}{2}x\cosh(a+c)-\frac{1}{2}\int \cosh(a-c+2bx)\,dx\\ &= \frac{1}{2}x\cosh(a+c)-\frac{\sinh(a-c+2bx)}{4b}\end{aligned}$$

Mathematica [A] time = 0.04, size = 27, normalized size = 1.00

$$\frac{1}{2}x\cosh(a+c)-\frac{\sinh(a+2bx-c)}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c - b*x]*Sinh[a + b*x],x]
```

```
[Out] (x*Cosh[a + c])/2 - Sinh[a - c + 2*b*x]/(4*b)
```

fricas [B] time = 0.41, size = 75, normalized size = 2.78

$$\frac{2\,bx\cosh\left(a+c\right)-2\,\cosh\left(bx+a\right)\cosh\left(a+c\right)\sinh\left(bx+a\right)+\cosh\left(bx+a\right)^{2}\sinh\left(a+c\right)+\sinh\left(bx+a\right)^{2}\sinh\left(a+c\right)}{4\left(b\cosh\left(a+c\right)^{2}-b\sinh\left(a+c\right)^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-sinh(b*x-c)*sinh(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/4*(2*b*x*cosh(a + c) - 2*cosh(b*x + a)*cosh(a + c)*sinh(b*x + a) + cosh(b
*x + a)^2*sinh(a + c) + sinh(b*x + a)^2*sinh(a + c))/(b*cosh(a + c)^2 - b*s
inh(a + c)^2)
```

giac [B] time = 0.11, size = 78, normalized size = 2.89

$$\frac{2\,bx\left(e^{(2\,a+2\,c)}+1\right)e^{(-a-c)}-\left(e^{(2\,bx)}+e^{(2\,bx+2\,a+2\,c)}-e^{(2\,c)}\right)e^{(-2\,bx-a-c)}-e^{(2\,bx+a-c)}}{8\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-sinh(b*x-c)*sinh(b*x+a),x, algorithm="giac")
```

```
[Out] 1/8*(2*b*x*(e^(2*a + 2*c) + 1)*e^(-a - c) - (e^(2*b*x) + e^(2*b*x + 2*a + 2
*c) - e^(2*c))*e^(-2*b*x - a - c) - e^(2*b*x + a - c))/b
```

maple [A] time = 0.09, size = 24, normalized size = 0.89

$$\frac{x\cosh\left(a+c\right)}{2}-\frac{\sinh\left(2bx+a-c\right)}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-sinh(b*x-c)*sinh(b*x+a),x)
```

```
[Out] 1/2*x*cosh(a+c)-1/4*sinh(2*b*x+a-c)/b
```

maxima [B] time = 0.32, size = 59, normalized size = 2.19

$$\frac{(bx+a)\left(e^{(2a+2c)}+1\right)e^{(-a-c)}}{4b}-\frac{e^{(2bx+a-c)}}{8b}+\frac{e^{(-2bx-a+c)}}{8b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-sinh(b*x-c)*sinh(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/4*(b*x + a)*(e^(2*a + 2*c) + 1)*e^(-a - c)/b - 1/8*e^(2*b*x + a - c)/b +
1/8*e^(-2*b*x - a + c)/b
```

mupad [B] time = 1.49, size = 23, normalized size = 0.85

$$\frac{x\cosh(a+c)}{2}-\frac{\sinh(a-c+2bx)}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a + b*x)*sinh(c - b*x),x)
```

```
[Out] (x*cosh(a + c))/2 - sinh(a - c + 2*b*x)/(4*b)
```

sympy [A] time = 0.75, size = 61, normalized size = 2.26

$$-\begin{cases}\frac{x\sinh(a+bx)\sinh(bx-c)}{2}-\frac{x\cosh(a+bx)\cosh(bx-c)}{2}+\frac{\sinh(bx-c)\cosh(a+bx)}{2b} & \text{for } b\neq 0\\ -x\sinh(a)\sinh(c) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-sinh(b*x-c)*sinh(b*x+a),x)
```

```
[Out] -Piecewise((x*sinh(a + b*x)*sinh(b*x - c)/2 - x*cosh(a + b*x)*cosh(b*x - c)
/2 + sinh(b*x - c)*cosh(a + b*x)/(2*b), Ne(b, 0)), (-x*sinh(a)*sinh(c), Tru
e))
```

3.133 $\int \cosh(a+bx)\cosh(c+bx)\,dx$

Optimal. Leaf size=27

$$\frac{\sinh(a+2bx+c)}{4b}+\frac{1}{2}x\cosh(a-c)$$

```
[Out] 1/2*x*cosh(a-c)+1/4*sinh(2*b*x+a+c)/b
```

Rubi [A] time = 0.02, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {5614, 2637}

$$\frac{\sinh(a+2bx+c)}{4b}+\frac{1}{2}x\cosh(a-c)$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[a + b*x]*Cosh[c + b*x],x]
```

```
[Out] (x*Cosh[a - c])/2 + Sinh[a + c + 2*b*x]/(4*b)
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 5614

```
Int[Cosh[v_]^(p_.)*Cosh[w_]^(q_.), x_Symbol] :> Int[ExpandTrigReduce[Cosh[v
]^p*Cosh[w]^q, x], x] /; IGtQ[p, 0] && IGtQ[q, 0] && ((PolynomialQ[v, x] &&
 PolynomialQ[w, x]) || (BinomialQ[{v, w}, x] && IndependentQ[Cancel[v/w], x
]))
```

Rubi steps

$$\begin{aligned}\int \cosh(a+bx)\cosh(c+bx)\,dx&=\int\left(\frac{1}{2}\cosh(a-c)+\frac{1}{2}\cosh(a+c+2bx)\right)dx\\&=\frac{1}{2}x\cosh(a-c)+\frac{1}{2}\int\cosh(a+c+2bx)\,dx\\&=\frac{1}{2}x\cosh(a-c)+\frac{\sinh(a+c+2bx)}{4b}\end{aligned}$$

Mathematica [A] time = 0.02, size = 26, normalized size = 0.96

$$\frac{\sinh(a+2bx+c)+2bx\cosh(a-c)}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[a + b*x]*Cosh[c + b*x],x]
```

```
[Out] (2*b*x*Cosh[a - c] + Sinh[a + c + 2*b*x])/(4*b)
```

fricas [B] time = 0.41, size = 89, normalized size = 3.30

$$\frac{2\,bx\cosh\left(-a+c\right)+2\,\cosh\left(bx+c\right)\cosh\left(-a+c\right)\sinh\left(bx+c\right)-\cosh\left(bx+c\right)^{2}\sinh\left(-a+c\right)-\sinh\left(bx+c\right)^{2}\,\mathrm{si}}{4\left(b\cosh\left(-a+c\right)^{2}-b\sinh\left(-a+c\right)^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*cosh(b*x+c),x, algorithm="fricas")
```

```
[Out] 1/4*(2*b*x*cosh(-a + c) + 2*cosh(b*x + c)*cosh(-a + c)*sinh(b*x + c) - cosh
(b*x + c)^2*sinh(-a + c) - sinh(b*x + c)^2*sinh(-a + c))/(b*cosh(-a + c)^2
- b*sinh(-a + c)^2)
```

giac [B] time = 0.11, size = 69, normalized size = 2.56

$$\frac{2\,bx\left(e^{(2\,a)}+e^{(2\,c)}\right)e^{(-a-c)}-\left(e^{(2\,bx+2\,a)}+e^{(2\,bx+2\,c)}+1\right)e^{(-2\,bx-a-c)}+e^{(2\,bx+a+c)}}{8\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*cosh(b*x+c),x, algorithm="giac")
```

```
[Out] 1/8*(2*b*x*(e^(2*a) + e^(2*c))*e^(-a - c) - (e^(2*b*x + 2*a) + e^(2*b*x + 2
*c) + 1)*e^(-2*b*x - a - c) + e^(2*b*x + a + c))/b
```

maple [A] time = 0.17, size = 24, normalized size = 0.89

$$\frac{x\cosh\left(a-c\right)}{2}+\frac{\sinh\left(2bx+a+c\right)}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)*cosh(b*x+c),x)
```

```
[Out] 1/2*x*cosh(a-c)+1/4*sinh(2*b*x+a+c)/b
```

maxima [B] time = 0.64, size = 58, normalized size = 2.15

$$\frac{(bx+a)\left(e^{(2a)}+e^{(2c)}\right)e^{(-a-c)}}{4b}+\frac{e^{(2bx+a+c)}}{8b}-\frac{e^{(-2bx-a-c)}}{8b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*cosh(b*x+c),x, algorithm="maxima")
```

```
[Out] 1/4*(b*x + a)*(e^(2*a) + e^(2*c))*e^(-a - c)/b + 1/8*e^(2*b*x + a + c)/b -
1/8*e^(-2*b*x - a - c)/b
```

mupad [B] time = 1.44, size = 23, normalized size = 0.85

$$\frac{x\cosh(a-c)}{2}+\frac{\sinh(a+c+2bx)}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a + b*x)*cosh(c + b*x),x)
```

```
[Out] (x*cosh(a - c))/2 + sinh(a + c + 2*b*x)/(4*b)
```

sympy [A] time = 0.74, size = 58, normalized size = 2.15

$$\begin{cases} -\frac{x\sinh(a+bx)\sinh(bx+c)}{2}+\frac{x\cosh(a+bx)\cosh(bx+c)}{2}+\frac{\sinh(a+bx)\cosh(bx+c)}{2b} & \text{for } b\neq 0 \\ x\cosh(a)\cosh(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*cosh(b*x+c),x)
```

```
[Out] Piecewise((-x*sinh(a + b*x)*sinh(b*x + c)/2 + x*cosh(a + b*x)*cosh(b*x + c)
/2 + sinh(a + b*x)*cosh(b*x + c)/(2*b), Ne(b, 0)), (x*cosh(a)*cosh(c), True
))
```

3.134 $\int \cosh(c-bx)\cosh(a+bx)\,dx$

Optimal. Leaf size=27

$$\frac{\sinh(a+2bx-c)}{4b}+\frac{1}{2}x\cosh(a+c)$$

```
[Out] 1/2*x*cosh(a+c)+1/4*sinh(2*b*x+a-c)/b
```

Rubi [A] time = 0.02, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {5614, 2637}

$$\frac{\sinh(a+2bx-c)}{4b}+\frac{1}{2}x\cosh(a+c)$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c - b*x]*Cosh[a + b*x],x]
```

```
[Out] (x*Cosh[a + c])/2 + Sinh[a - c + 2*b*x]/(4*b)
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 5614

```
Int[Cosh[v_]^(p_.)*Cosh[w_]^(q_.), x_Symbol] :> Int[ExpandTrigReduce[Cosh[v
]^p*Cosh[w]^q, x], x] /; IGtQ[p, 0] && IGtQ[q, 0] && ((PolynomialQ[v, x] &&
 PolynomialQ[w, x]) || (BinomialQ[{v, w}, x] && IndependentQ[Cancel[v/w], x
]))
```

Rubi steps

$$\begin{aligned}\int \cosh(c-bx)\cosh(a+bx)\,dx&=\int\left(\frac{1}{2}\cosh(a+c)+\frac{1}{2}\cosh(a-c+2bx)\right)dx\\&=\frac{1}{2}x\cosh(a+c)+\frac{1}{2}\int\cosh(a-c+2bx)\,dx\\&=\frac{1}{2}x\cosh(a+c)+\frac{\sinh(a-c+2bx)}{4b}\end{aligned}$$

Mathematica [A] time = 0.02, size = 26, normalized size = 0.96

$$\frac{\sinh(a+2bx-c)+2bx\cosh(a+c)}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c - b*x]*Cosh[a + b*x],x]
```

```
[Out] (2*b*x*Cosh[a + c] + Sinh[a - c + 2*b*x])/(4*b)
```

fricas [B] time = 0.42, size = 77, normalized size = 2.85

$$\frac{2\,bx\cosh\left(a+c\right)+2\,\cosh\left(bx+a\right)\cosh\left(a+c\right)\sinh\left(bx+a\right)-\cosh\left(bx+a\right)^{2}\sinh\left(a+c\right)-\sinh\left(bx+a\right)^{2}\sinh}{4\left(b\cosh\left(a+c\right)^{2}-b\sinh\left(a+c\right)^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x-c)*cosh(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/4*(2*b*x*cosh(a + c) + 2*cosh(b*x + a)*cosh(a + c)*sinh(b*x + a) - cosh(b
*x + a)^2*sinh(a + c) - sinh(b*x + a)^2*sinh(a + c))/(b*cosh(a + c)^2 - b*s
inh(a + c)^2)
```

giac [B] time = 0.13, size = 74, normalized size = 2.74

$$\frac{2\,bx\left(e^{(2\,a+2\,c)}+1\right)e^{(-a-c)}-\left(e^{(2\,bx)}+e^{(2\,bx+2\,a+2\,c)}+e^{(2\,c)}\right)e^{(-2\,bx-a-c)}+e^{(2\,bx+a-c)}}{8\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x-c)*cosh(b*x+a),x, algorithm="giac")
```

```
[Out] 1/8*(2*b*x*(e^(2*a + 2*c) + 1)*e^(-a - c) - (e^(2*b*x) + e^(2*b*x + 2*a + 2
*c) + e^(2*c))*e^(-2*b*x - a - c) + e^(2*b*x + a - c))/b
```

maple [A] time = 0.15, size = 24, normalized size = 0.89

$$\frac{x\cosh\left(a+c\right)}{2}+\frac{\sinh\left(2bx+a-c\right)}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x-c)*cosh(b*x+a),x)
```

```
[Out] 1/2*x*cosh(a+c)+1/4*sinh(2*b*x+a-c)/b
```

maxima [B] time = 0.31, size = 59, normalized size = 2.19

$$\frac{(bx+a)\left(e^{(2a+2c)}+1\right)e^{(-a-c)}}{4b}+\frac{e^{(2bx+a-c)}}{8b}-\frac{e^{(-2bx-a+c)}}{8b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x-c)*cosh(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/4*(b*x + a)*(e^(2*a + 2*c) + 1)*e^(-a - c)/b + 1/8*e^(2*b*x + a - c)/b -
1/8*e^(-2*b*x - a + c)/b
```

mupad [B] time = 1.49, size = 23, normalized size = 0.85

$$\frac{\sinh(a-c+2bx)}{4b}+\frac{x\cosh(a+c)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a + b*x)*cosh(c - b*x),x)
```

```
[Out] sinh(a - c + 2*b*x)/(4*b) + (x*cosh(a + c))/2
```

sympy [A] time = 0.75, size = 58, normalized size = 2.15

$$\begin{cases} -\frac{x\sinh(a+bx)\sinh(bx-c)}{2}+\frac{x\cosh(a+bx)\cosh(bx-c)}{2}+\frac{\sinh(bx-c)\cosh(a+bx)}{2b} & \text{for } b\neq 0 \\ x\cosh(a)\cosh(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x-c)*cosh(b*x+a),x)
```

```
[Out] Piecewise((-x*sinh(a + b*x)*sinh(b*x - c)/2 + x*cosh(a + b*x)*cosh(b*x - c)
/2 + sinh(b*x - c)*cosh(a + b*x)/(2*b), Ne(b, 0)), (x*cosh(a)*cosh(c), True
))
```

3.135 $\int \tanh(a+bx)\tanh(c+bx)\,dx$

Optimal. Leaf size=37

$$-\frac{\coth(a-c)\log(\cosh(a+bx))}{b}+\frac{\coth(a-c)\log(\cosh(bx+c))}{b}+x$$

```
[Out] x-coth(a-c)*ln(cosh(b*x+a))/b+coth(a-c)*ln(cosh(b*x+c))/b
```

Rubi [A] time = 0.07, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {5646, 5644, 3475}

$$-\frac{\coth(a-c)\log(\cosh(a+bx))}{b}+\frac{\coth(a-c)\log(\cosh(bx+c))}{b}+x$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[a + b*x]*Tanh[c + b*x],x]
```

```
[Out] x - (Coth[a - c]*Log[Cosh[a + b*x]])/b + (Coth[a - c]*Log[Cosh[c + b*x]])/b
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 5644

```
Int[Sech[(a_.) + (b_.)*(x_)]*Sech[(c_) + (d_.)*(x_)], x_Symbol] :> -Dist[Cs
ch[(b*c - a*d)/d], Int[Tanh[a + b*x], x], x] + Dist[Csch[(b*c - a*d)/b], In
t[Tanh[c + d*x], x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b^2 - d^2, 0] && N
eQ[b*c - a*d, 0]
```

Rule 5646

```
Int[Tanh[(a_.) + (b_.)*(x_)]*Tanh[(c_) + (d_.)*(x_)], x_Symbol] :> Simp[(b*
x)/d, x] - Dist[(b*Cosh[(b*c - a*d)/d])/d, Int[Sech[a + b*x]*Sech[c + d*x],
 x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b^2 - d^2, 0] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\int \tanh(a+bx)\tanh(c+bx)\,dx = x-\cosh(a-c)\int \operatorname{sech}(a+bx)\operatorname{sech}(c+bx)\,dx$$
$$= x-\coth(a-c)\int \tanh(a+bx)\,dx+\coth(a-c)\int \tanh(c+bx)\,dx$$
$$= x-\frac{\coth(a-c)\log(\cosh(a+bx))}{b}+\frac{\coth(a-c)\log(\cosh(c+bx))}{b}$$

Mathematica [A] time = 0.52, size = 29, normalized size = 0.78

$$\frac{\coth(a-c)(\log(\cosh(bx+c))-\log(\cosh(a+bx)))}{b}+x$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[a + b*x]*Tanh[c + b*x],x]
```

```
[Out] x + (Coth[a - c]*(-Log[Cosh[a + b*x]] + Log[Cosh[c + b*x]]))/b
```

fricas [B] time = 0.43, size = 259, normalized size = 7.00

$$bx\cosh(-a+c)^2-2\,bx\cosh(-a+c)\sinh(-a+c)+bx\sinh(-a+c)^2-bx-\left(\cosh(-a+c)^2-2\,\cosh(-a+c\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(b*x+a)*tanh(b*x+c),x, algorithm="fricas")
```

```
[Out] (b*x*cosh(-a + c)^2 - 2*b*x*cosh(-a + c)*sinh(-a + c) + b*x*sinh(-a + c)^2
- b*x - (cosh(-a + c)^2 - 2*cosh(-a + c)*sinh(-a + c) + sinh(-a + c)^2 + 1)
*log(2*(cosh(b*x + c)*cosh(-a + c) - sinh(b*x + c)*sinh(-a + c))/(cosh(b*x
+ c)*cosh(-a + c) - (cosh(-a + c) + sinh(-a + c))*sinh(b*x + c) + cosh(b*x
+ c)*sinh(-a + c))) + (cosh(-a + c)^2 - 2*cosh(-a + c)*sinh(-a + c) + sinh(
-a + c)^2 + 1)*log(2*cosh(b*x + c)/(cosh(b*x + c) - sinh(b*x + c))))/(b*cos
h(-a + c)^2 - 2*b*cosh(-a + c)*sinh(-a + c) + b*sinh(-a + c)^2 - b)
```

giac [B] time = 0.14, size = 95, normalized size = 2.57

$$\frac{bx-\frac{\left(e^{(4a)}+e^{(2a+2c)}\right)\log\left(e^{(2bx+2a)}+1\right)}{e^{(4a)}-e^{(2a+2c)}}+\frac{\left(e^{(2a+2c)}+e^{(4c)}\right)\log\left(e^{(2bx+2c)}+1\right)}{e^{(2a+2c)}-e^{(4c)}}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(b*x+a)*tanh(b*x+c),x, algorithm="giac")
```

```
[Out] (b*x - (e^(4*a) + e^(2*a + 2*c))*log(e^(2*b*x + 2*a) + 1)/(e^(4*a) - e^(2*a
 + 2*c)) + (e^(2*a + 2*c) + e^(4*c))*log(e^(2*b*x + 2*c) + 1)/(e^(2*a + 2*c
) - e^(4*c)))/b
```

maple [B] time = 0.20, size = 151, normalized size = 4.08

$$x-\frac{\ln\left(1+e^{2bx+2a}\right)e^{2a}}{b\left(e^{2a}-e^{2c}\right)}-\frac{\ln\left(1+e^{2bx+2a}\right)e^{2c}}{b\left(e^{2a}-e^{2c}\right)}+\frac{\ln\left(e^{2bx+2a}+e^{2a-2c}\right)e^{2a}}{b\left(e^{2a}-e^{2c}\right)}+\frac{\ln\left(e^{2bx+2a}+e^{2a-2c}\right)e^{2c}}{b\left(e^{2a}-e^{2c}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(b*x+a)*tanh(b*x+c),x)
```

```
[Out] x-1/b/(exp(2*a)-exp(2*c))*ln(1+exp(2*b*x+2*a))*exp(2*a)-1/b/(exp(2*a)-exp(2
*c))*ln(1+exp(2*b*x+2*a))*exp(2*c)+1/b/(exp(2*a)-exp(2*c))*ln(exp(2*b*x+2*a
)+exp(2*a-2*c))*exp(2*a)+1/b/(exp(2*a)-exp(2*c))*ln(exp(2*b*x+2*a)+exp(2*a-
2*c))*exp(2*c)
```

maxima [B] time = 0.43, size = 83, normalized size = 2.24

$$x+\frac{a}{b}-\frac{\left(e^{(2a)}+e^{(2c)}\right)\log\left(e^{(-2bx-2a)}+1\right)}{b\left(e^{(2a)}-e^{(2c)}\right)}+\frac{\left(e^{(2a)}+e^{(2c)}\right)\log\left(e^{(-2bx)}+e^{(2c)}\right)}{b\left(e^{(2a)}-e^{(2c)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(b*x+a)*tanh(b*x+c),x, algorithm="maxima")
```

```
[Out] x + a/b - (e^(2*a) + e^(2*c))*log(e^(-2*b*x - 2*a) + 1)/(b*(e^(2*a) - e^(2*
c))) + (e^(2*a) + e^(2*c))*log(e^(-2*b*x) + e^(2*c))/(b*(e^(2*a) - e^(2*c))
)
```

mupad [B] time = 1.93, size = 115, normalized size = 3.11

$$x-\frac{\ln\left(4\,e^{4\,a}+4\,e^{6\,a}\,e^{2\,b\,x}+4\,e^{2\,a}\,e^{2\,c}+4\,e^{4\,a}\,e^{2\,c}\,e^{2\,b\,x}\right)\,\coth\left(a-c\right)}{b}+\frac{\ln\left(4\,e^{4\,a}+4\,e^{2\,a}\,e^{2\,c}+4\,e^{2\,a}\,e^{4\,c}\,e^{2\,b\,x}+4\,e^{4}\right.}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(a + b*x)*tanh(c + b*x),x)
```

```
[Out] x - (log(4*exp(4*a) + 4*exp(6*a)*exp(2*b*x) + 4*exp(2*a)*exp(2*c) + 4*exp(4
*a)*exp(2*c)*exp(2*b*x))*coth(a - c))/b + (log(4*exp(4*a) + 4*exp(2*a)*exp(
2*c) + 4*exp(2*a)*exp(4*c)*exp(2*b*x) + 4*exp(4*a)*exp(2*c)*exp(2*b*x))*cot
h(a - c))/b
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \tanh(a + bx) \tanh(bx + c) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(b*x+a)*tanh(b*x+c),x)
```

```
[Out] Integral(tanh(a + b*x)*tanh(b*x + c), x)
```

3.136 $\int \tanh(c - bx) \tanh(a + bx) \, dx$

Optimal. Leaf size=36

$$-\frac{\coth(a+c)\log(\cosh(c-bx))}{b}+\frac{\coth(a+c)\log(\cosh(a+bx))}{b}-x$$

```
[Out] -x-coth(a+c)*ln(cosh(b*x-c))/b+coth(a+c)*ln(cosh(b*x+a))/b
```

Rubi [A] time = 0.07, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {5646, 5644, 3475}

$$-\frac{\coth(a+c)\log(\cosh(c-bx))}{b}+\frac{\coth(a+c)\log(\cosh(a+bx))}{b}-x$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[c - b*x]*Tanh[a + b*x],x]
```

```
[Out] -x - (Coth[a + c]*Log[Cosh[c - b*x]])/b + (Coth[a + c]*Log[Cosh[a + b*x]])/
b
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 5644

```
Int[Sech[(a_.) + (b_.)*(x_)]*Sech[(c_) + (d_.)*(x_)], x_Symbol] :> -Dist[Cs
ch[(b*c - a*d)/d], Int[Tanh[a + b*x], x], x] + Dist[Csch[(b*c - a*d)/b], In
t[Tanh[c + d*x], x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b^2 - d^2, 0] && N
eQ[b*c - a*d, 0]
```

Rule 5646

```
Int[Tanh[(a_.) + (b_.)*(x_)]*Tanh[(c_) + (d_.)*(x_)], x_Symbol] :> Simp[(b*
x)/d, x] - Dist[(b*Cosh[(b*c - a*d)/d])/d, Int[Sech[a + b*x]*Sech[c + d*x],
 x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b^2 - d^2, 0] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\int \tanh(c-bx)\tanh(a+bx)\,dx = -x+\cosh(a+c)\int \operatorname{sech}(c-bx)\operatorname{sech}(a+bx)\,dx$$
$$= -x+\coth(a+c)\int \tanh(c-bx)\,dx+\coth(a+c)\int \tanh(a+bx)\,dx$$
$$= -x-\frac{\coth(a+c)\log(\cosh(c-bx))}{b}+\frac{\coth(a+c)\log(\cosh(a+bx))}{b}$$

Mathematica [A] time = 0.52, size = 30, normalized size = 0.83

$$\frac{\coth(a+c)(\log(\cosh(a+bx))-\log(\cosh(c-bx)))}{b}-x$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[c - b*x]*Tanh[a + b*x],x]
```

```
[Out] -x + (Coth[a + c]*(-Log[Cosh[c - b*x]] + Log[Cosh[a + b*x]]))/b
```

fricas [B] time = 0.44, size = 216, normalized size = 6.00

$$-\frac{bx\cosh(a+c)^2-2\,bx\cosh(a+c)\sinh(a+c)+bx\sinh(a+c)^2-bx-\left(\cosh(a+c)^2-2\,\cosh(a+c)\sinh(a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-tanh(b*x-c)*tanh(b*x+a),x, algorithm="fricas")
```

```
[Out] -(b*x*cosh(a + c)^2 - 2*b*x*cosh(a + c)*sinh(a + c) + b*x*sinh(a + c)^2 - b
*x - (cosh(a + c)^2 - 2*cosh(a + c)*sinh(a + c) + sinh(a + c)^2 + 1)*log(2*
(cosh(b*x + a)*cosh(a + c) - sinh(b*x + a)*sinh(a + c))/(cosh(b*x + a)*cosh
(a + c) - (cosh(a + c) + sinh(a + c))*sinh(b*x + a) + cosh(b*x + a)*sinh(a
+ c))) + (cosh(a + c)^2 - 2*cosh(a + c)*sinh(a + c) + sinh(a + c)^2 + 1)*lo
g(2*cosh(b*x + a)/(cosh(b*x + a) - sinh(b*x + a))))/(b*cosh(a + c)^2 - 2*b*
cosh(a + c)*sinh(a + c) + b*sinh(a + c)^2 - b)
```

giac [B] time = 0.19, size = 86, normalized size = 2.39

$$-\frac{bx+\frac{\left(e^{(2\,a+2\,c)}+1\right)\log\left(e^{(2\,bx)}+e^{(2\,c)}\right)}{e^{(2\,a+2\,c)}-1}+\frac{\left(e^{(2\,a)}+e^{(4\,a+2\,c)}\right)\log\left(e^{(2\,bx+2\,a)}+1\right)}{e^{(2\,a)}-e^{(4\,a+2\,c)}}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-tanh(b*x-c)*tanh(b*x+a),x, algorithm="giac")
```

```
[Out] -(b*x + (e^(2*a + 2*c) + 1)*log(e^(2*b*x) + e^(2*c))/(e^(2*a + 2*c) - 1) +
(e^(2*a) + e^(4*a + 2*c))*log(e^(2*b*x + 2*a) + 1)/(e^(2*a) - e^(4*a + 2*c)
))/b
```

maple [B] time = 0.23, size = 149, normalized size = 4.14

$$-x-\frac{\ln\left(e^{2a+2c}+e^{2bx+2a}\right)e^{2a+2c}}{b\left(e^{2a+2c}-1\right)}-\frac{\ln\left(e^{2a+2c}+e^{2bx+2a}\right)}{b\left(e^{2a+2c}-1\right)}+\frac{\ln\left(1+e^{2bx+2a}\right)e^{2a+2c}}{b\left(e^{2a+2c}-1\right)}+\frac{\ln\left(1+e^{2bx+2a}\right)}{b\left(e^{2a+2c}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-tanh(b*x-c)*tanh(b*x+a),x)
```

```
[Out] -x-1/b/(exp(2*a+2*c)-1)*ln(exp(2*a+2*c)+exp(2*b*x+2*a))*exp(2*a+2*c)-1/b/(e
xp(2*a+2*c)-1)*ln(exp(2*a+2*c)+exp(2*b*x+2*a))+1/b/(exp(2*a+2*c)-1)*ln(1+ex
p(2*b*x+2*a))*exp(2*a+2*c)+1/b/(exp(2*a+2*c)-1)*ln(1+exp(2*b*x+2*a))
```

maxima [B] time = 0.41, size = 87, normalized size = 2.42

$$-x-\frac{a}{b}+\frac{\left(e^{(2\,a+2\,c)}+1\right)\log\left(e^{(-2\,bx-2\,a)}+1\right)}{b\left(e^{(2\,a+2\,c)}-1\right)}-\frac{\left(e^{(2\,a+2\,c)}+1\right)\log\left(e^{(-2\,bx+2\,c)}+1\right)}{b\left(e^{(2\,a+2\,c)}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-tanh(b*x-c)*tanh(b*x+a),x, algorithm="maxima")
```

```
[Out] -x - a/b + (e^(2*a + 2*c) + 1)*log(e^(-2*b*x - 2*a) + 1)/(b*(e^(2*a + 2*c)
- 1)) - (e^(2*a + 2*c) + 1)*log(e^(-2*b*x + 2*c) + 1)/(b*(e^(2*a + 2*c) - 1
))
```

mupad [B] time = 1.89, size = 121, normalized size = 3.36

$$\frac{\coth(a+c)\,\ln\left(4\,e^{2\,a}\,e^{2\,c}+4\,e^{4\,a}\,e^{4\,c}+4\,e^{4\,a}\,e^{2\,c}\,e^{2\,b\,x}+4\,e^{6\,a}\,e^{4\,c}\,e^{2\,b\,x}\right)}{b}-\frac{\coth(a+c)\,\ln\left(4\,e^{2\,a}\,e^{2\,b\,x}+4\,e^{2\,a}\,e^{2\,c}\right.}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(a + b*x)*tanh(c - b*x),x)
```

```
[Out] (coth(a + c)*log(4*exp(2*a)*exp(2*c) + 4*exp(4*a)*exp(4*c) + 4*exp(4*a)*exp
(2*c)*exp(2*b*x) + 4*exp(6*a)*exp(4*c)*exp(2*b*x)))/b - (coth(a + c)*log(4*
exp(2*a)*exp(2*b*x) + 4*exp(2*a)*exp(2*c) + 4*exp(4*a)*exp(4*c) + 4*exp(4*a
)*exp(2*c)*exp(2*b*x)))/b - x
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int\tanh(a+bx)\tanh(bx-c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-tanh(b*x-c)*tanh(b*x+a),x)
```

```
[Out] -Integral(tanh(a + b*x)*tanh(b*x - c), x)
```

3.137 $\int \coth(a+bx)\coth(c+bx)\,dx$

Optimal. Leaf size=37

$$-\frac{\coth(a-c)\log(\sinh(a+bx))}{b}+\frac{\coth(a-c)\log(\sinh(bx+c))}{b}+x$$

[Out] x-coth(a-c)*ln(sinh(b*x+a))/b+coth(a-c)*ln(sinh(b*x+c))/b

Rubi [A] time = 0.03, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {5647, 5645, 3475}

$$-\frac{\coth(a-c)\log(\sinh(a+bx))}{b}+\frac{\coth(a-c)\log(\sinh(bx+c))}{b}+x$$

Antiderivative was successfully verified.

[In] Int[Coth[a + b*x]*Coth[c + b*x],x]

[Out] x - (Coth[a - c]*Log[Sinh[a + b*x]])/b + (Coth[a - c]*Log[Sinh[c + b*x]])/b

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 5645

```
Int[Csch[(a_.) + (b_.)*(x_)]*Csch[(c_) + (d_.)*(x_)], x_Symbol] :> Dist[Csc
h[(b*c - a*d)/b], Int[Coth[a + b*x], x], x] - Dist[Csch[(b*c - a*d)/d], Int
[Coth[c + d*x], x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b^2 - d^2, 0] && Ne
Q[b*c - a*d, 0]
```

Rule 5647

```
Int[Coth[(a_.) + (b_.)*(x_)]*Coth[(c_) + (d_.)*(x_)], x_Symbol] :> Simp[(b*
x)/d, x] + Dist[Cosh[(b*c - a*d)/d], Int[Csch[a + b*x]*Csch[c + d*x], x], x
] /; FreeQ[{a, b, c, d}, x] && EqQ[b^2 - d^2, 0] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\int \coth(a+bx)\coth(c+bx)\,dx = x+\cosh(a-c)\int \operatorname{csch}(a+bx)\operatorname{csch}(c+bx)\,dx$$
$$= x-\coth(a-c)\int \coth(a+bx)\,dx+\coth(a-c)\int \coth(c+bx)\,dx$$
$$= x-\frac{\coth(a-c)\log(\sinh(a+bx))}{b}+\frac{\coth(a-c)\log(\sinh(c+bx))}{b}$$

Mathematica [A] time = 0.50, size = 29, normalized size = 0.78

$$\frac{\coth(a-c)(\log(\sinh(bx+c))-\log(\sinh(a+bx)))}{b}+x$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[a + b*x]*Coth[c + b*x],x]
```

```
[Out] x + (Coth[a - c]*(-Log[Sinh[a + b*x]] + Log[Sinh[c + b*x]]))/b
```

fricas [B] time = 0.45, size = 259, normalized size = 7.00

$$\frac{bx\cosh(-a+c)^2-2\,bx\cosh(-a+c)\sinh(-a+c)+bx\sinh(-a+c)^2-bx-\left(\cosh(-a+c)^2-2\,\cosh(-a+c\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(b*x+a)*coth(b*x+c),x, algorithm="fricas")
```

```
[Out] (b*x*cosh(-a + c)^2 - 2*b*x*cosh(-a + c)*sinh(-a + c) + b*x*sinh(-a + c)^2
- b*x - (cosh(-a + c)^2 - 2*cosh(-a + c)*sinh(-a + c) + sinh(-a + c)^2 + 1)
*log(2*(cosh(-a + c)*sinh(b*x + c) - cosh(b*x + c)*sinh(-a + c))/(cosh(b*x
+ c)*cosh(-a + c) - (cosh(-a + c) + sinh(-a + c))*sinh(b*x + c) + cosh(b*x
+ c)*sinh(-a + c))) + (cosh(-a + c)^2 - 2*cosh(-a + c)*sinh(-a + c) + sinh(
-a + c)^2 + 1)*log(2*sinh(b*x + c)/(cosh(b*x + c) - sinh(b*x + c))))/(b*cos
h(-a + c)^2 - 2*b*cosh(-a + c)*sinh(-a + c) + b*sinh(-a + c)^2 - b)
```

giac [B] time = 0.16, size = 97, normalized size = 2.62

$$\frac{bx-\frac{\left(e^{(4a)}+e^{(2a+2c)}\right)\log\left(\left|e^{(2bx+2a)}-1\right|\right)}{e^{(4a)}-e^{(2a+2c)}}+\frac{\left(e^{(2a+2c)}+e^{(4c)}\right)\log\left(\left|e^{(2bx+2c)}-1\right|\right)}{e^{(2a+2c)}-e^{(4c)}}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(b*x+a)*coth(b*x+c),x, algorithm="giac")
```

```
[Out] (b*x - (e^(4*a) + e^(2*a + 2*c))*log(abs(e^(2*b*x + 2*a) - 1))/(e^(4*a) - e
^(2*a + 2*c)) + (e^(2*a + 2*c) + e^(4*c))*log(abs(e^(2*b*x + 2*c) - 1))/(e^
(2*a + 2*c) - e^(4*c)))/b
```

maple [B] time = 0.27, size = 155, normalized size = 4.19

$$x-\frac{\ln\left(e^{2bx+2a}-1\right)e^{2a}}{b\left(e^{2a}-e^{2c}\right)}-\frac{\ln\left(e^{2bx+2a}-1\right)e^{2c}}{b\left(e^{2a}-e^{2c}\right)}+\frac{\ln\left(e^{2bx+2a}-e^{2a-2c}\right)e^{2a}}{b\left(e^{2a}-e^{2c}\right)}+\frac{\ln\left(e^{2bx+2a}-e^{2a-2c}\right)e^{2c}}{b\left(e^{2a}-e^{2c}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(b*x+a)*coth(b*x+c),x)
```

```
[Out] x-1/b/(exp(2*a)-exp(2*c))*ln(exp(2*b*x+2*a)-1)*exp(2*a)-1/b/(exp(2*a)-exp(2
*c))*ln(exp(2*b*x+2*a)-1)*exp(2*c)+1/b/(exp(2*a)-exp(2*c))*ln(exp(2*b*x+2*a
)-exp(2*a-2*c))*exp(2*a)+1/b/(exp(2*a)-exp(2*c))*ln(exp(2*b*x+2*a)-exp(2*a-
2*c))*exp(2*c)
```

maxima [B] time = 0.40, size = 157, normalized size = 4.24

$$x+\frac{a}{b}-\frac{\left(e^{(2a)}+e^{(2c)}\right)\log\left(e^{(-bx-a)}+1\right)}{b\left(e^{(2a)}-e^{(2c)}\right)}-\frac{\left(e^{(2a)}+e^{(2c)}\right)\log\left(e^{(-bx-a)}-1\right)}{b\left(e^{(2a)}-e^{(2c)}\right)}+\frac{\left(e^{(2a)}+e^{(2c)}\right)\log\left(e^{(-bx)}+e^{c}\right)}{b\left(e^{(2a)}-e^{(2c)}\right)}+\frac{\left(e^{(2a)}+\right.}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(b*x+a)*coth(b*x+c),x, algorithm="maxima")
```

```
[Out] x + a/b - (e^(2*a) + e^(2*c))*log(e^(-b*x - a) + 1)/(b*(e^(2*a) - e^(2*c)))
 - (e^(2*a) + e^(2*c))*log(e^(-b*x - a) - 1)/(b*(e^(2*a) - e^(2*c))) + (e^(
2*a) + e^(2*c))*log(e^(-b*x) + e^c)/(b*(e^(2*a) - e^(2*c))) + (e^(2*a) + e^
(2*c))*log(e^(-b*x) - e^c)/(b*(e^(2*a) - e^(2*c)))
```

mupad [B] time = 0.49, size = 115, normalized size = 3.11

$$x-\frac{\ln\left(4\,e^{4\,a}-4\,e^{6\,a}\,e^{2\,b\,x}+4\,e^{2\,a}\,e^{2\,c}-4\,e^{4\,a}\,e^{2\,c}\,e^{2\,b\,x}\right)\,\coth\left(a-c\right)}{b}+\frac{\ln\left(4\,e^{4\,a}+4\,e^{2\,a}\,e^{2\,c}-4\,e^{2\,a}\,e^{4\,c}\,e^{2\,b\,x}-4\,e^{4\,a}\right.}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(a + b*x)*coth(c + b*x),x)
```

```
[Out] x - (log(4*exp(4*a) - 4*exp(6*a)*exp(2*b*x) + 4*exp(2*a)*exp(2*c) - 4*exp(4
*a)*exp(2*c)*exp(2*b*x))*coth(a - c))/b + (log(4*exp(4*a) + 4*exp(2*a)*exp(
2*c) - 4*exp(2*a)*exp(4*c)*exp(2*b*x) - 4*exp(4*a)*exp(2*c)*exp(2*b*x))*cot
h(a - c))/b
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \coth(a+bx)\coth(bx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(b*x+a)*coth(b*x+c),x)
```

```
[Out] Integral(coth(a + b*x)*coth(b*x + c), x)
```

3.138 $\int \coth(c-bx)\coth(a+bx)\,dx$

Optimal. Leaf size=36

$$-\frac{\coth(a+c)\log(\sinh(c-bx))}{b}+\frac{\coth(a+c)\log(\sinh(a+bx))}{b}-x$$

```
[Out] -x-coth(a+c)*ln(-sinh(b*x-c))/b+coth(a+c)*ln(sinh(b*x+a))/b
```

Rubi [A] time = 0.04, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {5647, 5645, 3475}

$$-\frac{\coth(a+c)\log(\sinh(c-bx))}{b}+\frac{\coth(a+c)\log(\sinh(a+bx))}{b}-x$$

Antiderivative was successfully verified.

```
[In] Int[Coth[c - b*x]*Coth[a + b*x],x]
```

```
[Out] -x - (Coth[a + c]*Log[Sinh[c - b*x]])/b + (Coth[a + c]*Log[Sinh[a + b*x]])/
b
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 5645

```
Int[Csch[(a_.) + (b_.)*(x_)]*Csch[(c_) + (d_.)*(x_)], x_Symbol] :> Dist[Csc
h[(b*c - a*d)/b], Int[Coth[a + b*x], x], x] - Dist[Csch[(b*c - a*d)/d], Int
[Coth[c + d*x], x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b^2 - d^2, 0] && Ne
Q[b*c - a*d, 0]
```

Rule 5647

```
Int[Coth[(a_.) + (b_.)*(x_)]*Coth[(c_) + (d_.)*(x_)], x_Symbol] :> Simp[(b*
x)/d, x] + Dist[Cosh[(b*c - a*d)/d], Int[Csch[a + b*x]*Csch[c + d*x], x], x
] /; FreeQ[{a, b, c, d}, x] && EqQ[b^2 - d^2, 0] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\int \coth(c-bx)\coth(a+bx)\,dx = -x+\cosh(a+c)\int \operatorname{csch}(c-bx)\operatorname{csch}(a+bx)\,dx$$
$$= -x+\coth(a+c)\int \coth(c-bx)\,dx+\coth(a+c)\int \coth(a+bx)\,dx$$
$$= -x-\frac{\coth(a+c)\log(\sinh(c-bx))}{b}+\frac{\coth(a+c)\log(\sinh(a+bx))}{b}$$

Mathematica [A] time = 0.49, size = 32, normalized size = 0.89

$$\frac{\coth(a+c)(\log(-\sinh(a+bx))-\log(\sinh(c-bx)))}{b}-x$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[c - b*x]*Coth[a + b*x],x]
```

```
[Out] -x + (Coth[a + c]*(-Log[Sinh[c - b*x]] + Log[-Sinh[a + b*x]]))/b
```

fricas [B] time = 0.53, size = 216, normalized size = 6.00

$$-\frac{bx\cosh(a+c)^2-2\,bx\cosh(a+c)\sinh(a+c)+bx\sinh(a+c)^2-bx-\left(\cosh(a+c)^2-2\,\cosh(a+c)\sinh(a\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-coth(b*x-c)*coth(b*x+a),x, algorithm="fricas")
```

```
[Out] -(b*x*cosh(a + c)^2 - 2*b*x*cosh(a + c)*sinh(a + c) + b*x*sinh(a + c)^2 - b
*x - (cosh(a + c)^2 - 2*cosh(a + c)*sinh(a + c) + sinh(a + c)^2 + 1)*log(2*
(cosh(a + c)*sinh(b*x + a) - cosh(b*x + a)*sinh(a + c))/(cosh(b*x + a)*cosh
(a + c) - (cosh(a + c) + sinh(a + c))*sinh(b*x + a) + cosh(b*x + a)*sinh(a
+ c))) + (cosh(a + c)^2 - 2*cosh(a + c)*sinh(a + c) + sinh(a + c)^2 + 1)*lo
g(2*sinh(b*x + a)/(cosh(b*x + a) - sinh(b*x + a))))/(b*cosh(a + c)^2 - 2*b*
cosh(a + c)*sinh(a + c) + b*sinh(a + c)^2 - b)
```

giac [B] time = 0.14, size = 90, normalized size = 2.50

$$-\frac{bx+\frac{\left(e^{(2a+2c)}+1\right)\log\left(\left|e^{(2bx)}-e^{(2c)}\right|\right)}{e^{(2a+2c)}-1}+\frac{\left(e^{(2a)}+e^{(4a+2c)}\right)\log\left(\left|e^{(2bx+2a)}-1\right|\right)}{e^{(2a)}-e^{(4a+2c)}}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-coth(b*x-c)*coth(b*x+a),x, algorithm="giac")
```

```
[Out] -(b*x + (e^(2*a + 2*c) + 1)*log(abs(e^(2*b*x) - e^(2*c)))/(e^(2*a + 2*c) -
1) + (e^(2*a) + e^(4*a + 2*c))*log(abs(e^(2*b*x + 2*a) - 1))/(e^(2*a) - e^(
4*a + 2*c)))/b
```

maple [B] time = 0.28, size = 153, normalized size = 4.25

$$-x-\frac{\ln\left(-e^{2a+2c}+e^{2bx+2a}\right)e^{2a+2c}}{b\left(e^{2a+2c}-1\right)}-\frac{\ln\left(-e^{2a+2c}+e^{2bx+2a}\right)}{b\left(e^{2a+2c}-1\right)}+\frac{\ln\left(e^{2bx+2a}-1\right)e^{2a+2c}}{b\left(e^{2a+2c}-1\right)}+\frac{\ln\left(e^{2bx+2a}-1\right)}{b\left(e^{2a+2c}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-coth(b*x-c)*coth(b*x+a),x)
```

```
[Out] -x-1/b/(exp(2*a+2*c)-1)*ln(-exp(2*a+2*c)+exp(2*b*x+2*a))*exp(2*a+2*c)-1/b/(
exp(2*a+2*c)-1)*ln(-exp(2*a+2*c)+exp(2*b*x+2*a))+1/b/(exp(2*a+2*c)-1)*ln(ex
p(2*b*x+2*a)-1)*exp(2*a+2*c)+1/b/(exp(2*a+2*c)-1)*ln(exp(2*b*x+2*a)-1)
```

maxima [B] time = 0.38, size = 160, normalized size = 4.44

$$-x-\frac{a}{b}+\frac{\left(e^{(2a+2c)}+1\right)\log\left(e^{(-bx-a)}+1\right)}{b\left(e^{(2a+2c)}-1\right)}+\frac{\left(e^{(2a+2c)}+1\right)\log\left(e^{(-bx-a)}-1\right)}{b\left(e^{(2a+2c)}-1\right)}-\frac{\left(e^{(2a+2c)}+1\right)\log\left(e^{(-bx+c)}+1\right)}{b\left(e^{(2a+2c)}-1\right)}-\frac{\left(e^{(2a+}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-coth(b*x-c)*coth(b*x+a),x, algorithm="maxima")
```

```
[Out] -x - a/b + (e^(2*a + 2*c) + 1)*log(e^(-b*x - a) + 1)/(b*(e^(2*a + 2*c) - 1)
) + (e^(2*a + 2*c) + 1)*log(e^(-b*x - a) - 1)/(b*(e^(2*a + 2*c) - 1)) - (e^
(2*a + 2*c) + 1)*log(e^(-b*x + c) + 1)/(b*(e^(2*a + 2*c) - 1)) - (e^(2*a +
2*c) + 1)*log(e^(-b*x + c) - 1)/(b*(e^(2*a + 2*c) - 1))
```

mupad [B] time = 1.84, size = 121, normalized size = 3.36

$$\frac{\coth(a+c)\,\ln\left(4\,e^{2a}\,e^{2c}+4\,e^{4a}\,e^{4c}-4\,e^{4a}\,e^{2c}\,e^{2bx}-4\,e^{6a}\,e^{4c}\,e^{2bx}\right)}{b}-\frac{\coth(a+c)\,\ln\left(4\,e^{2a}\,e^{2bx}-4\,e^{2a}\,e^{2c}-\right.}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(a + b*x)*coth(c - b*x),x)
```

```
[Out] (coth(a + c)*log(4*exp(2*a)*exp(2*c) + 4*exp(4*a)*exp(4*c) - 4*exp(4*a)*exp
(2*c)*exp(2*b*x) - 4*exp(6*a)*exp(4*c)*exp(2*b*x)))/b - (coth(a + c)*log(4*
exp(2*a)*exp(2*b*x) - 4*exp(2*a)*exp(2*c) - 4*exp(4*a)*exp(4*c) + 4*exp(4*a
)*exp(2*c)*exp(2*b*x)))/b - x
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \coth(a+bx)\coth(bx-c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-coth(b*x-c)*coth(b*x+a),x)
[Out] -Integral(coth(a + b*x)*coth(b*x - c), x)
```

3.139 $\int \mathbf{sech}(a+bx)\mathbf{sech}(c+bx)\,dx$

Optimal. Leaf size=36

$$\frac{\operatorname{csch}(a-c)\log(\cosh(a+bx))}{b}-\frac{\operatorname{csch}(a-c)\log(\cosh(bx+c))}{b}$$

```
[Out] csch(a-c)*ln(cosh(b*x+a))/b-csch(a-c)*ln(cosh(b*x+c))/b
```

Rubi [A] time = 0.02, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {5644, 3475}

$$\frac{\operatorname{csch}(a-c)\log(\cosh(a+bx))}{b}-\frac{\operatorname{csch}(a-c)\log(\cosh(bx+c))}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[a + b*x]*Sech[c + b*x],x]
```

```
[Out] (Csch[a - c]*Log[Cosh[a + b*x]])/b - (Csch[a - c]*Log[Cosh[c + b*x]])/b
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 5644

```
Int[Sech[(a_.) + (b_.)*(x_)]*Sech[(c_) + (d_.)*(x_)], x_Symbol] :> -Dist[Cs
ch[(b*c - a*d)/d], Int[Tanh[a + b*x], x], x] + Dist[Csch[(b*c - a*d)/b], In
t[Tanh[c + d*x], x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b^2 - d^2, 0] && N
eQ[b*c - a*d, 0]
```

Rubi steps

$$\begin{aligned}\int \operatorname{sech}(a+bx)\operatorname{sech}(c+bx)\,dx&=\operatorname{csch}(a-c)\int \tanh(a+bx)\,dx-\operatorname{csch}(a-c)\int \tanh(c+bx)\,dx\\&=\frac{\operatorname{csch}(a-c)\log(\cosh(a+bx))}{b}-\frac{\operatorname{csch}(a-c)\log(\cosh(c+bx))}{b}\end{aligned}$$

Mathematica [A] time = 0.21, size = 27, normalized size = 0.75

$$\frac{\operatorname{csch}(a-c)(\log(\cosh(a+bx))-\log(\cosh(bx+c)))}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[a + b*x]*Sech[c + b*x],x]
```

```
[Out] (Csch[a - c]*(Log[Cosh[a + b*x]] - Log[Cosh[c + b*x]]))/b
```

fricas [B] time = 0.59, size = 184, normalized size = 5.11

$$\frac{2\left(\left(\cosh\left(-a+c\right)-\sinh\left(-a+c\right)\right)\log\left(\frac{2\left(\cosh(bx+c)\cosh(-a+c)-\sinh(bx+c)\sinh(-a+c)\right)}{\cosh(bx+c)\cosh(-a+c)-(\cosh(-a+c)+\sinh(-a+c))\sinh(bx+c)+\cosh(bx+c)\sinh(-a+c)}\right)-(\mathrm{co}\right.}{b\cosh\left(-a+c\right)^{2}-2\,b\cosh\left(-a+c\right)\sinh\left(-a+c\right)+b\sinh\left(-a+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)*sech(b*x+c),x, algorithm="fricas")
```

```
[Out] 2*((cosh(-a + c) - sinh(-a + c))*log(2*(cosh(b*x + c)*cosh(-a + c) - sinh(b
*x + c)*sinh(-a + c))/(cosh(b*x + c)*cosh(-a + c) - (cosh(-a + c) + sinh(-a
 + c))*sinh(b*x + c) + cosh(b*x + c)*sinh(-a + c))) - (cosh(-a + c) - sinh(
-a + c))*log(2*cosh(b*x + c)/(cosh(b*x + c) - sinh(b*x + c))))/(b*cosh(-a +
 c)^2 - 2*b*cosh(-a + c)*sinh(-a + c) + b*sinh(-a + c)^2 - b)
```

giac [B] time = 0.13, size = 79, normalized size = 2.19

$$\frac{2\left(\frac{e^{(3a+c)}\log\left(e^{(2bx+2a)}+1\right)}{e^{(4a)}-e^{(2a+2c)}}-\frac{e^{(a+3c)}\log\left(e^{(2bx+2c)}+1\right)}{e^{(2a+2c)}-e^{(4c)}}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)*sech(b*x+c),x, algorithm="giac")
```

```
[Out] 2*(e^(3*a + c)*log(e^(2*b*x + 2*a) + 1)/(e^(4*a) - e^(2*a + 2*c)) - e^(a +
3*c)*log(e^(2*b*x + 2*c) + 1)/(e^(2*a + 2*c) - e^(4*c)))/b
```

maple [B] time = 0.16, size = 77, normalized size = 2.14

$$\frac{2\ln\left(1+\mathrm{e}^{2bx+2a}\right)\mathrm{e}^{a+c}}{\left(\mathrm{e}^{2a}-\mathrm{e}^{2c}\right)b}-\frac{2\ln\left(\mathrm{e}^{2bx+2a}+\mathrm{e}^{2a-2c}\right)\mathrm{e}^{a+c}}{\left(\mathrm{e}^{2a}-\mathrm{e}^{2c}\right)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(b*x+a)*sech(b*x+c),x)
```

```
[Out] 2*ln(1+exp(2*b*x+2*a))/(exp(2*a)-exp(2*c))/b*exp(a+c)-2*ln(exp(2*b*x+2*a)+e
xp(2*a-2*c))/(exp(2*a)-exp(2*c))/b*exp(a+c)
```

maxima [A] time = 0.62, size = 68, normalized size = 1.89

$$\frac{2\,e^{(a+c)}\log\left(e^{(-2\,bx-2\,a)}+1\right)}{b\left(e^{(2\,a)}-e^{(2\,c)}\right)}-\frac{2\,e^{(a+c)}\log\left(e^{(-2\,bx)}+e^{(2\,c)}\right)}{b\left(e^{(2\,a)}-e^{(2\,c)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)*sech(b*x+c),x, algorithm="maxima")
```

```
[Out] 2*e^(a + c)*log(e^(-2*b*x - 2*a) + 1)/(b*(e^(2*a) - e^(2*c))) - 2*e^(a + c)
*log(e^(-2*b*x) + e^(2*c))/(b*(e^(2*a) - e^(2*c)))
```

mupad [B] time = 2.39, size = 266, normalized size = 7.39

$$\frac{4\sqrt{e^{2\,a-2\,c}}\ \mathrm{atan}\left(\frac{b\left(e^{-a}\,e^{c}+e^{-3\,a}\,e^{3\,c}\right)\left(e^{2\,a}\,e^{-2\,c}\right)^{3/2}}{\sqrt{-b^2\left(e^{2\,a}\,e^{-2\,c}-1\right)^2}}+\frac{e^{2\,a}\,e^{2\,b\,x}\left(\frac{2\,e^{-c}\,e^{a}}{b\left(e^{2\,a}\,e^{-2\,c}\right)^{3/2}}+\frac{2\left(e^{-a}\,e^{c}+e^{-3\,a}\,e^{3\,c}\right)\left(b\,\sqrt{e^{2\,a}\,e^{-2\,c}}+b\left(e^{2\,a}\,e^{-2\,c}\right)^{3/2}\right)}{\sqrt{-b^2\left(e^{2\,a}\,e^{-2\,c}-1\right)^2}\ \sqrt{2\,b^2\,e^{2\,a}\,e^{-2\,c}-b^2-b^2\,e^{4\,a}\,e^{-4\,c}}}\right)\sqrt{2\,b^2\,e^{2\,a}\,e}}{4}\right)}{\sqrt{2\,b^2\,e^{2\,a-2\,c}-b^2\,e^{4\,a-4\,c}-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(a + b*x)*cosh(c + b*x)),x)
```

```
[Out] (4*exp(2*a - 2*c)^(1/2)*atan((b*(exp(-a)*exp(c) + exp(-3*a)*exp(3*c))*(exp(
2*a)*exp(-2*c))^(3/2))/(-b^2*(exp(2*a)*exp(-2*c) - 1)^2)^(1/2) + (exp(2*a)*
exp(2*b*x)*((2*exp(-c)*exp(a))/(b*(exp(2*a)*exp(-2*c))^(3/2)) + (2*(exp(-a)
*exp(c) + exp(-3*a)*exp(3*c))*(b*(exp(2*a)*exp(-2*c))^(1/2) + b*(exp(2*a)*e
xp(-2*c))^(3/2)))/((-b^2*(exp(2*a)*exp(-2*c) - 1)^2)^(1/2)*(2*b^2*exp(2*a)*
exp(-2*c) - b^2 - b^2*exp(4*a)*exp(-4*c))^(1/2)))*(2*b^2*exp(2*a)*exp(-2*c)
 - b^2 - b^2*exp(4*a)*exp(-4*c))^(1/2))/4))/(2*b^2*exp(2*a - 2*c) - b^2*exp
(4*a - 4*c) - b^2)^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{sech}\left(a+bx\right)\operatorname{sech}\left(bx+c\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)*sech(b*x+c),x)
```

```
[Out] Integral(sech(a + b*x)*sech(b*x + c), x)
```

3.140 $\int \operatorname{sech}(c-bx)\operatorname{sech}(a+bx)\,dx$

Optimal. Leaf size=33

$$\frac{\operatorname{csch}(a+c)\log(\cosh(a+bx))}{b}-\frac{\operatorname{csch}(a+c)\log(\cosh(c-bx))}{b}$$

```
[Out] -csch(a+c)*ln(cosh(b*x-c))/b+csch(a+c)*ln(cosh(b*x+a))/b
```

Rubi [A] time = 0.02, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {5644, 3475}

$$\frac{\operatorname{csch}(a+c)\log(\cosh(a+bx))}{b}-\frac{\operatorname{csch}(a+c)\log(\cosh(c-bx))}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[c - b*x]*Sech[a + b*x],x]
[Out] -((Csch[a + c]*Log[Cosh[c - b*x]])/b) + (Csch[a + c]*Log[Cosh[a + b*x]])/b
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 5644

```
Int[Sech[(a_.) + (b_.)*(x_)]*Sech[(c_) + (d_.)*(x_)], x_Symbol] :> -Dist[Cs
ch[(b*c - a*d)/d], Int[Tanh[a + b*x], x], x] + Dist[Csch[(b*c - a*d)/b], In
t[Tanh[c + d*x], x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b^2 - d^2, 0] && N
eQ[b*c - a*d, 0]
```

Rubi steps

$$\begin{aligned}\int \operatorname{sech}(c-bx)\operatorname{sech}(a+bx)\,dx &= \operatorname{csch}(a+c)\int \tanh(c-bx)\,dx+\operatorname{csch}(a+c)\int \tanh(a+bx)\,dx\\ &= -\frac{\operatorname{csch}(a+c)\log(\cosh(c-bx))}{b}+\frac{\operatorname{csch}(a+c)\log(\cosh(a+bx))}{b}\end{aligned}$$

Mathematica [A] time = 0.20, size = 27, normalized size = 0.82

$$-\frac{\operatorname{csch}(a+c)(\log(\cosh(c-bx))-\log(\cosh(a+bx)))}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[c - b*x]*Sech[a + b*x],x]
```

```
[Out] -((Csch[a + c]*(Log[Cosh[c - b*x]] - Log[Cosh[a + b*x]]))/b)
```

fricas [B] time = 0.42, size = 156, normalized size = 4.73

$$\frac{2\left((\cosh(a+c)-\sinh(a+c))\log\left(\frac{2\,(\cosh(bx+a)\cosh(a+c)-\sinh(bx+a)\sinh(a+c))}{\cosh(bx+a)\cosh(a+c)-(\cosh(a+c)+\sinh(a+c))\sinh(bx+a)+\cosh(bx+a)\sinh(a+c)}\right)-(\cosh(a\right.}{b\cosh(a+c)^2-2\,b\cosh(a+c)\sinh(a+c)+b\sinh(a+c)^2-b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x-c)*sech(b*x+a),x, algorithm="fricas")
```

```
[Out] 2*((cosh(a + c) - sinh(a + c))*log(2*(cosh(b*x + a)*cosh(a + c) - sinh(b*x
+ a)*sinh(a + c))/(cosh(b*x + a)*cosh(a + c) - (cosh(a + c) + sinh(a + c))*
sinh(b*x + a) + cosh(b*x + a)*sinh(a + c))) - (cosh(a + c) - sinh(a + c))*l
og(2*cosh(b*x + a)/(cosh(b*x + a) - sinh(b*x + a))))/(b*cosh(a + c)^2 - 2*b
*cosh(a + c)*sinh(a + c) + b*sinh(a + c)^2 - b)
```

giac [B] time = 0.12, size = 70, normalized size = 2.12

$$-\frac{2\left(\frac{e^{(a+c)}\log\left(e^{(2\,bx)}+e^{(2\,c)}\right)}{e^{(2\,a+2\,c)}-1}+\frac{e^{(3\,a+c)}\log\left(e^{(2\,bx+2\,a)}+1\right)}{e^{(2\,a)}-e^{(4\,a+2\,c)}}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x-c)*sech(b*x+a),x, algorithm="giac")
```

```
[Out] -2*(e^(a + c)*log(e^(2*b*x) + e^(2*c))/(e^(2*a + 2*c) - 1) + e^(3*a + c)*lo
g(e^(2*b*x + 2*a) + 1)/(e^(2*a) - e^(4*a + 2*c)))/b
```

maple [B] time = 0.16, size = 75, normalized size = 2.27

$$\frac{2\ln\left(1+e^{2bx+2a}\right)e^{a+c}}{b\left(e^{2a+2c}-1\right)}-\frac{2\ln\left(e^{2a+2c}+e^{2bx+2a}\right)e^{a+c}}{b\left(e^{2a+2c}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(b*x-c)*sech(b*x+a),x)
```

```
[Out] 2/b/(exp(2*a+2*c)-1)*ln(1+exp(2*b*x+2*a))*exp(a+c)-2/b/(exp(2*a+2*c)-1)*ln(
exp(2*a+2*c)+exp(2*b*x+2*a))*exp(a+c)
```

maxima [A] time = 0.40, size = 67, normalized size = 2.03

$$\frac{2\, e^{(a+c)} \log\left(e^{(-2\, bx - 2\, a)} + 1\right)}{b\left(e^{(2\, a + 2\, c)} - 1\right)} - \frac{2\, e^{(a+c)} \log\left(e^{(-2\, bx + 2\, c)} + 1\right)}{b\left(e^{(2\, a + 2\, c)} - 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x-c)*sech(b*x+a),x, algorithm="maxima")
```

```
[Out] 2*e^(a + c)*log(e^(-2*b*x - 2*a) + 1)/(b*(e^(2*a + 2*c) - 1)) - 2*e^(a + c)
*log(e^(-2*b*x + 2*c) + 1)/(b*(e^(2*a + 2*c) - 1))
```

mupad [B] time = 2.37, size = 268, normalized size = 8.12

$$\frac{4\,\mathrm{atan}\left(\frac{e^{2\,a}\, e^{2\,b\,x}\left(\frac{2\, e^{a}\, e^{c}}{b\left(e^{2\,a}\, e^{2\,c}\right)^{3/2}} + \frac{2\, e^{-3\,a}\, e^{-3\,c}\left(e^{2\,a}\, e^{2\,c}+1\right)\left(b\,\sqrt{e^{2\,a}\, e^{2\,c}} + b\left(e^{2\,a}\, e^{2\,c}\right)^{3/2}\right)}{\sqrt{-b^2\left(e^{2\,a}\, e^{2\,c}-1\right)^2}\,\sqrt{2\,b^2\, e^{2\,a}\, e^{2\,c} - b^2 - b^2\, e^{4\,a}\, e^{4\,c}}}\right)\sqrt{2\,b^2\, e^{2\,a}\, e^{2\,c} - b^2 - b^2\, e^{4\,a}\, e^{4\,c}}}{4} + \frac{b\, e^{-3\,a}\, e^{-3\,c}\left(e^{2\,a}\, e^{2\,c}+1\right)\left(e^{2\,a}\, e^{2 \cdots}\right.}{\sqrt{-b^2\left(e^{2\,a}\, e^{2\,c}-1\right)^2}} \cdots\right.}{\sqrt{2\,b^2\, e^{2\,a+2\,c} - b^2\, e^{4\,a+4\,c} - b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(a + b*x)*cosh(c - b*x)),x)
```

```
[Out] (4*atan((exp(2*a)*exp(2*b*x)*((2*exp(a)*exp(c))/(b*(exp(2*a)*exp(2*c))^(3/2
)) + (2*exp(-3*a)*exp(-3*c)*(exp(2*a)*exp(2*c) + 1)*(b*(exp(2*a)*exp(2*c))^
(1/2) + b*(exp(2*a)*exp(2*c))^(3/2)))/((-b^2*(exp(2*a)*exp(2*c) - 1)^2)^(1/
2)*(2*b^2*exp(2*a)*exp(2*c) - b^2 - b^2*exp(4*a)*exp(4*c))^(1/2)))*(2*b^2*e
xp(2*a)*exp(2*c) - b^2 - b^2*exp(4*a)*exp(4*c))^(1/2))/4 + (b*exp(-3*a)*exp
(-3*c)*(exp(2*a)*exp(2*c) + 1)*(exp(2*a)*exp(2*c))^(3/2))/(-b^2*(exp(2*a)*e
xp(2*c) - 1)^2)^(1/2))*exp(2*a + 2*c)^(1/2))/(2*b^2*exp(2*a + 2*c) - b^2*ex
p(4*a + 4*c) - b^2)^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{sech}(a + bx)\operatorname{sech}(bx - c)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x-c)*sech(b*x+a),x)
```

```
[Out] Integral(sech(a + b*x)*sech(b*x - c), x)
```

3.141 $\int \text{csch}(a+bx)\text{csch}(c+bx)\,dx$

Optimal. Leaf size=36

$$\frac{\text{csch}(a-c)\log(\sinh(bx+c))}{b}-\frac{\text{csch}(a-c)\log(\sinh(a+bx))}{b}$$

```
[Out] -csch(a-c)*ln(sinh(b*x+a))/b+csch(a-c)*ln(sinh(b*x+c))/b
```

Rubi [A] time = 0.02, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {5645, 3475}

$$\frac{\text{csch}(a-c)\log(\sinh(bx+c))}{b}-\frac{\text{csch}(a-c)\log(\sinh(a+bx))}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[a + b*x]*Csch[c + b*x],x]
```

```
[Out] -((Csch[a - c]*Log[Sinh[a + b*x]])/b) + (Csch[a - c]*Log[Sinh[c + b*x]])/b
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 5645

```
Int[Csch[(a_.) + (b_.)*(x_)]*Csch[(c_) + (d_.)*(x_)], x_Symbol] :> Dist[Csc
h[(b*c - a*d)/b], Int[Coth[a + b*x], x], x] - Dist[Csch[(b*c - a*d)/d], Int
[Coth[c + d*x], x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b^2 - d^2, 0] && Ne
Q[b*c - a*d, 0]
```

Rubi steps

$$\begin{aligned}\int \text{csch}(a+bx)\text{csch}(c+bx)\,dx &= -\left(\text{csch}(a-c)\int \coth(a+bx)\,dx\right)+\text{csch}(a-c)\int \coth(c+bx)\,dx\\ &= -\frac{\text{csch}(a-c)\log(\sinh(a+bx))}{b}+\frac{\text{csch}(a-c)\log(\sinh(c+bx))}{b}\end{aligned}$$

Mathematica [A] time = 0.23, size = 28, normalized size = 0.78

$$-\frac{\text{csch}(a-c)(\log(\sinh(a+bx))-\log(\sinh(bx+c)))}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[a + b*x]*Csch[c + b*x],x]
```

```
[Out] -((Csch[a - c]*(Log[Sinh[a + b*x]] - Log[Sinh[c + b*x]]))/b)
```

fricas [B] time = 0.53, size = 184, normalized size = 5.11

$$-\frac{2\left((\cosh(-a+c)-\sinh(-a+c))\log\left(\frac{2(\cosh(-a+c)\sinh(bx+c)-\cosh(bx+c)\sinh(-a+c))}{\cosh(bx+c)\cosh(-a+c)-(\cosh(-a+c)+\sinh(-a+c))\sinh(bx+c)+\cosh(bx+c)\sinh(-a+c)}\right)-(c\right.}{b\cosh(-a+c)^2-2b\cosh(-a+c)\sinh(-a+c)+b\sinh(-a+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)*csch(b*x+c),x, algorithm="fricas")
```

```
[Out] -2*((cosh(-a + c) - sinh(-a + c))*log(2*(cosh(-a + c)*sinh(b*x + c) - cosh(
b*x + c)*sinh(-a + c))/(cosh(b*x + c)*cosh(-a + c) - (cosh(-a + c) + sinh(-
a + c))*sinh(b*x + c) + cosh(b*x + c)*sinh(-a + c))) - (cosh(-a + c) - sinh
(-a + c))*log(2*sinh(b*x + c)/(cosh(b*x + c) - sinh(b*x + c))))/(b*cosh(-a
+ c)^2 - 2*b*cosh(-a + c)*sinh(-a + c) + b*sinh(-a + c)^2 - b)
```

giac [B] time = 0.12, size = 81, normalized size = 2.25

$$-\frac{2\left(\frac{e^{(3a+c)}\log\left(\left|e^{(2bx+2a)}-1\right|\right)}{e^{(4a)}-e^{(2a+2c)}}-\frac{e^{(a+3c)}\log\left(\left|e^{(2bx+2c)}-1\right|\right)}{e^{(2a+2c)}-e^{(4c)}}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)*csch(b*x+c),x, algorithm="giac")
```

```
[Out] -2*(e^(3*a + c)*log(abs(e^(2*b*x + 2*a) - 1))/(e^(4*a) - e^(2*a + 2*c)) - e
^(a + 3*c)*log(abs(e^(2*b*x + 2*c) - 1))/(e^(2*a + 2*c) - e^(4*c)))/b
```

maple [B] time = 0.13, size = 79, normalized size = 2.19

$$\frac{2\ln\left(e^{2bx+2a}-e^{2a-2c}\right)e^{a+c}}{\left(e^{2a}-e^{2c}\right)b}-\frac{2\ln\left(e^{2bx+2a}-1\right)e^{a+c}}{\left(e^{2a}-e^{2c}\right)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(b*x+a)*csch(b*x+c),x)
```

```
[Out] 2*ln(exp(2*b*x+2*a)-exp(2*a-2*c))/(exp(2*a)-exp(2*c))/b*exp(a+c)-2*ln(exp(2
*b*x+2*a)-1)/(exp(2*a)-exp(2*c))/b*exp(a+c)
```

maxima [B] time = 0.32, size = 133, normalized size = 3.69

$$-\frac{2\,e^{(a+c)}\log\left(e^{(-bx-a)}+1\right)}{b\left(e^{(2\,a)}-e^{(2\,c)}\right)}-\frac{2\,e^{(a+c)}\log\left(e^{(-bx-a)}-1\right)}{b\left(e^{(2\,a)}-e^{(2\,c)}\right)}+\frac{2\,e^{(a+c)}\log\left(e^{(-bx)}+e^{c}\right)}{b\left(e^{(2\,a)}-e^{(2\,c)}\right)}+\frac{2\,e^{(a+c)}\log\left(e^{(-bx)}-e^{c}\right)}{b\left(e^{(2\,a)}-e^{(2\,c)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)*csch(b*x+c),x, algorithm="maxima")
```

```
[Out] -2*e^(a + c)*log(e^(-b*x - a) + 1)/(b*(e^(2*a) - e^(2*c))) - 2*e^(a + c)*lo
g(e^(-b*x - a) - 1)/(b*(e^(2*a) - e^(2*c))) + 2*e^(a + c)*log(e^(-b*x) + e^
c)/(b*(e^(2*a) - e^(2*c))) + 2*e^(a + c)*log(e^(-b*x) - e^c)/(b*(e^(2*a) -
e^(2*c)))
```

mupad [B] time = 1.77, size = 266, normalized size = 7.39

$$-\frac{4\sqrt{e^{2\,a-2\,c}}\,\mathrm{atan}\left(\frac{b\left(e^{-a}\,e^{c}+e^{-3\,a}\,e^{3\,c}\right)\left(e^{2\,a}\,e^{-2\,c}\right)^{3/2}}{\sqrt{-b^2\left(e^{2\,a}\,e^{-2\,c}-1\right)^2}}-\frac{e^{2\,a}\,e^{2\,b\,x}\left(\frac{2\,e^{-c}\,e^{a}}{b\left(e^{2\,a}\,e^{-2\,c}\right)^{3/2}}+\frac{2\left(e^{-a}\,e^{c}+e^{-3\,a}\,e^{3\,c}\right)\left(b\,\sqrt{e^{2\,a}\,e^{-2\,c}}+b\left(e^{2\,a}\,e^{-2\,c}\right)^{3/2}\right)}{\sqrt{-b^2\left(e^{2\,a}\,e^{-2\,c}-1\right)^2}\,\sqrt{2\,b^2\,e^{2\,a}\,e^{-2\,c}-b^2-b^2\,e^{4\,a}\,e^{-4\,c}}}\right)\sqrt{2\,b^2\,e^{2\,a}}}{4}\right)}{\sqrt{2\,b^2\,e^{2\,a-2\,c}-b^2\,e^{4\,a-4\,c}-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sinh(a + b*x)*sinh(c + b*x)),x)
```

```
[Out] -(4*exp(2*a - 2*c)^(1/2)*atan((b*(exp(-a)*exp(c) + exp(-3*a)*exp(3*c))*(exp
(2*a)*exp(-2*c))^(3/2))/(-b^2*(exp(2*a)*exp(-2*c) - 1)^2)^(1/2) - (exp(2*a)
*exp(2*b*x)*((2*exp(-c)*exp(a))/(b*(exp(2*a)*exp(-2*c))^(3/2)) + (2*(exp(-a
)*exp(c) + exp(-3*a)*exp(3*c))*(b*(exp(2*a)*exp(-2*c))^(1/2) + b*(exp(2*a)*
exp(-2*c))^(3/2)))/((-b^2*(exp(2*a)*exp(-2*c) - 1)^2)^(1/2)*(2*b^2*exp(2*a)
*exp(-2*c) - b^2 - b^2*exp(4*a)*exp(-4*c))^(1/2)))*(2*b^2*exp(2*a)*exp(-2*c
) - b^2 - b^2*exp(4*a)*exp(-4*c))^(1/2))/4))/(2*b^2*exp(2*a - 2*c) - b^2*ex
p(4*a - 4*c) - b^2)^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{csch}(a+bx)\operatorname{csch}(bx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)*csch(b*x+c),x)
```

```
[Out] Integral(csch(a + b*x)*csch(b*x + c), x)
```

3.142 $\int \operatorname{csch}(c-bx)\operatorname{csch}(a+bx)\,dx$

Optimal. Leaf size=33

$$\frac{\operatorname{csch}(a+c)\log(\sinh(a+bx))}{b}-\frac{\operatorname{csch}(a+c)\log(\sinh(c-bx))}{b}$$

```
[Out] -csch(a+c)*ln(-sinh(b*x-c))/b+csch(a+c)*ln(sinh(b*x+a))/b
```

Rubi [A] time = 0.02, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {5645, 3475}

$$\frac{\operatorname{csch}(a+c)\log(\sinh(a+bx))}{b}-\frac{\operatorname{csch}(a+c)\log(\sinh(c-bx))}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c - b*x]*Csch[a + b*x],x]
```

```
[Out] -((Csch[a + c]*Log[Sinh[c - b*x]])/b) + (Csch[a + c]*Log[Sinh[a + b*x]])/b
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 5645

```
Int[Csch[(a_.) + (b_.)*(x_)]*Csch[(c_) + (d_.)*(x_)], x_Symbol] :> Dist[Csc
h[(b*c - a*d)/b], Int[Coth[a + b*x], x], x] - Dist[Csch[(b*c - a*d)/d], Int
[Coth[c + d*x], x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b^2 - d^2, 0] && Ne
Q[b*c - a*d, 0]
```

Rubi steps

$$\begin{aligned}\int \operatorname{csch}(c-bx)\operatorname{csch}(a+bx)\,dx&=\operatorname{csch}(a+c)\int \coth(c-bx)\,dx+\operatorname{csch}(a+c)\int \coth(a+bx)\,dx\\&=-\frac{\operatorname{csch}(a+c)\log(\sinh(c-bx))}{b}+\frac{\operatorname{csch}(a+c)\log(\sinh(a+bx))}{b}\end{aligned}$$

Mathematica [A] time = 0.20, size = 29, normalized size = 0.88

$$-\frac{\operatorname{csch}(a+c)(\log(\sinh(c-bx))-\log(-\sinh(a+bx)))}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c - b*x]*Csch[a + b*x],x]
```

```
[Out] -((Csch[a + c]*(Log[Sinh[c - b*x]] - Log[-Sinh[a + b*x]]))/b)
```

fricas [B] time = 0.46, size = 156, normalized size = 4.73

$$\frac{2\left((\cosh(a+c)-\sinh(a+c))\log\left(\frac{2(\cosh(a+c)\sinh(bx+a)-\cosh(bx+a)\sinh(a+c))}{\cosh(bx+a)\cosh(a+c)-(\cosh(a+c)+\sinh(a+c))\sinh(bx+a)+\cosh(bx+a)\sinh(a+c)}\right)-(\cosh(a\right.}{b\cosh(a+c)^2-2b\cosh(a+c)\sinh(a+c)+b\sinh(a+c)^2-b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-csch(b*x-c)*csch(b*x+a),x, algorithm="fricas")
```

```
[Out] 2*((cosh(a + c) - sinh(a + c))*log(2*(cosh(a + c)*sinh(b*x + a) - cosh(b*x
+ a)*sinh(a + c))/(cosh(b*x + a)*cosh(a + c) - (cosh(a + c) + sinh(a + c))*
sinh(b*x + a) + cosh(b*x + a)*sinh(a + c))) - (cosh(a + c) - sinh(a + c))*l
og(2*sinh(b*x + a)/(cosh(b*x + a) - sinh(b*x + a))))/(b*cosh(a + c)^2 - 2*b
*cosh(a + c)*sinh(a + c) + b*sinh(a + c)^2 - b)
```

giac [B] time = 0.14, size = 74, normalized size = 2.24

$$-\frac{2\left(\frac{e^{(a+c)}\log\left(\left|e^{(2bx)}-e^{(2c)}\right|\right)}{e^{(2a+2c)}-1}+\frac{e^{(3a+c)}\log\left(\left|e^{(2bx+2a)}-1\right|\right)}{e^{(2a)}-e^{(4a+2c)}}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-csch(b*x-c)*csch(b*x+a),x, algorithm="giac")
```

```
[Out] -2*(e^(a + c)*log(abs(e^(2*b*x) - e^(2*c)))/(e^(2*a + 2*c) - 1) + e^(3*a +
c)*log(abs(e^(2*b*x + 2*a) - 1))/(e^(2*a) - e^(4*a + 2*c)))/b
```

maple [B] time = 0.12, size = 77, normalized size = 2.33

$$\frac{2\ln\left(e^{2bx+2a}-1\right)e^{a+c}}{b\left(e^{2a+2c}-1\right)}-\frac{2\ln\left(-e^{2a+2c}+e^{2bx+2a}\right)e^{a+c}}{b\left(e^{2a+2c}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-csch(b*x-c)*csch(b*x+a),x)
```

```
[Out] 2/b/(exp(2*a+2*c)-1)*ln(exp(2*b*x+2*a)-1)*exp(a+c)-2/b/(exp(2*a+2*c)-1)*ln(
-exp(2*a+2*c)+exp(2*b*x+2*a))*exp(a+c)
```

maxima [B] time = 0.43, size = 129, normalized size = 3.91

$$\frac{2\,e^{(a+c)}\log\left(e^{(-bx-a)}+1\right)}{b\left(e^{(2\,a+2\,c)}-1\right)}+\frac{2\,e^{(a+c)}\log\left(e^{(-bx-a)}-1\right)}{b\left(e^{(2\,a+2\,c)}-1\right)}-\frac{2\,e^{(a+c)}\log\left(e^{(-bx+c)}+1\right)}{b\left(e^{(2\,a+2\,c)}-1\right)}-\frac{2\,e^{(a+c)}\log\left(e^{(-bx+c)}-1\right)}{b\left(e^{(2\,a+2\,c)}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-csch(b*x-c)*csch(b*x+a),x, algorithm="maxima")
```

```
[Out] 2*e^(a + c)*log(e^(-b*x - a) + 1)/(b*(e^(2*a + 2*c) - 1)) + 2*e^(a + c)*log
(e^(-b*x - a) - 1)/(b*(e^(2*a + 2*c) - 1)) - 2*e^(a + c)*log(e^(-b*x + c) +
 1)/(b*(e^(2*a + 2*c) - 1)) - 2*e^(a + c)*log(e^(-b*x + c) - 1)/(b*(e^(2*a
+ 2*c) - 1))
```

mupad [B] time = 0.36, size = 269, normalized size = 8.15

$$-\frac{4\,\mathrm{atan}\left(\frac{e^{2\,a}\,e^{2\,b\,x}\left(\frac{2\,e^{a}\,e^{c}}{b\left(e^{2\,a}\,e^{2\,c}\right)^{3/2}}+\frac{2\,e^{-3\,a}\,e^{-3\,c}\left(e^{2\,a}\,e^{2\,c}+1\right)\left(b\,\sqrt{e^{2\,a}\,e^{2\,c}}+b\left(e^{2\,a}\,e^{2\,c}\right)^{3/2}\right)}{\sqrt{-b^2\left(e^{2\,a}\,e^{2\,c}-1\right)^2}\,\sqrt{2\,b^2\,e^{2\,a}\,e^{2\,c}-b^2-b^2\,e^{4\,a}\,e^{4\,c}}}\right)\sqrt{2\,b^2\,e^{2\,a}\,e^{2\,c}-b^2-b^2\,e^{4\,a}\,e^{4\,c}}}{4}-\frac{b\,e^{-3\,a}\,e^{-3\,c}\left(e^{2\,a}\,e^{2\,c}+1\right)\left(e^{2\,a}\cdots\right.}{\sqrt{-b^2\left(e^{2\,a}\,e^{2\,c}-1\right)^2}\cdots}\right.}{\sqrt{2\,b^2\,e^{2\,a+2\,c}-b^2\,e^{4\,a+4\,c}-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sinh(a + b*x)*sinh(c - b*x)),x)
```

```
[Out] -(4*atan((exp(2*a)*exp(2*b*x)*((2*exp(a)*exp(c))/(b*(exp(2*a)*exp(2*c))^(3/
2)) + (2*exp(-3*a)*exp(-3*c)*(exp(2*a)*exp(2*c) + 1)*(b*(exp(2*a)*exp(2*c))
^(1/2) + b*(exp(2*a)*exp(2*c))^(3/2)))/((-b^2*(exp(2*a)*exp(2*c) - 1)^2)^(1
/2)*(2*b^2*exp(2*a)*exp(2*c) - b^2 - b^2*exp(4*a)*exp(4*c))^(1/2)))*(2*b^2*
exp(2*a)*exp(2*c) - b^2 - b^2*exp(4*a)*exp(4*c))^(1/2))/4 - (b*exp(-3*a)*ex
p(-3*c)*(exp(2*a)*exp(2*c) + 1)*(exp(2*a)*exp(2*c))^(3/2))/(-b^2*(exp(2*a)*
exp(2*c) - 1)^2)^(1/2))*exp(2*a + 2*c)^(1/2))/(2*b^2*exp(2*a + 2*c) - b^2*e
xp(4*a + 4*c) - b^2)^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \operatorname{csch}\left(a+bx\right)\operatorname{csch}\left(bx-c\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-csch(b*x-c)*csch(b*x+a),x)
```

```
[Out] -Integral(csch(a + b*x)*csch(b*x - c), x)
```

3.143 $\int \sinh(a+bx)\tanh(c+bx)\,dx$

Optimal. Leaf size=29

$$\frac{\sinh(a+bx)}{b}-\frac{\cosh(a-c)\tan^{-1}(\sinh(bx+c))}{b}$$

```
[Out] -arctan(sinh(b*x+c))*cosh(a-c)/b+sinh(b*x+a)/b
```

Rubi [A] time = 0.02, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {5620, 2637, 3770}

$$\frac{\sinh(a+bx)}{b}-\frac{\cosh(a-c)\tan^{-1}(\sinh(bx+c))}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[a + b*x]*Tanh[c + b*x],x]
```

```
[Out] -((ArcTan[Sinh[c + b*x]]*Cosh[a - c])/b) + Sinh[a + b*x]/b
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 5620

```
Int[Sinh[v_]*Tanh[w_]^(n_.), x_Symbol] :> Int[Cosh[v]*Tanh[w]^(n - 1), x] -
 Dist[Cosh[v - w], Int[Sech[w]*Tanh[w]^(n - 1), x], x] /; GtQ[n, 0] && NeQ[
w, v] && FreeQ[v - w, x]
```

Rubi steps

$$\begin{aligned}\int \sinh(a+bx)\tanh(c+bx)\,dx &= -\left(\cosh(a-c)\int \operatorname{sech}(c+bx)\,dx\right)+\int \cosh(a+bx)\,dx\\ &= -\frac{\tan^{-1}(\sinh(c+bx))\cosh(a-c)}{b}+\frac{\sinh(a+bx)}{b}\end{aligned}$$

Mathematica [B] time = 0.06, size = 86, normalized size = 2.97

$$-\frac{2\cosh(a-c)\tan^{-1}\left(\frac{(\cosh(c)-\sinh(c))\left(\sinh(c)\cosh\left(\frac{bx}{2}\right)+\cosh(c)\sinh\left(\frac{bx}{2}\right)\right)}{\cosh(c)\cosh\left(\frac{bx}{2}\right)-\sinh(c)\cosh\left(\frac{bx}{2}\right)}\right)}{b}+\frac{\sinh(a)\cosh(bx)}{b}+\frac{\cosh(a)\sinh(bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[a + b*x]*Tanh[c + b*x],x]
```

```
[Out] (-2*ArcTan[((Cosh[c] - Sinh[c])*(Cosh[(b*x)/2]*Sinh[c] + Cosh[c]*Sinh[(b*x)
/2]))/(Cosh[c]*Cosh[(b*x)/2] - Cosh[(b*x)/2]*Sinh[c])]*Cosh[a - c])/b + (Co
sh[b*x]*Sinh[a])/b + (Cosh[a]*Sinh[b*x])/b
```

fricas [B] time = 0.54, size = 327, normalized size = 11.28

$$\frac{\cosh(bx+c)^2\cosh(-a+c)^2-2\cosh(bx+c)^2\cosh(-a+c)\sinh(-a+c)+\cosh(bx+c)^2\sinh(-a+c)^2+\left(c\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)*tanh(b*x+c),x, algorithm="fricas")
```

```
[Out] 1/2*(cosh(b*x + c)^2*cosh(-a + c)^2 - 2*cosh(b*x + c)^2*cosh(-a + c)*sinh(-
a + c) + cosh(b*x + c)^2*sinh(-a + c)^2 + (cosh(-a + c)^2 - 2*cosh(-a + c)*
sinh(-a + c) + sinh(-a + c)^2)*sinh(b*x + c)^2 + 2*(2*cosh(b*x + c)*cosh(-a
 + c)*sinh(-a + c) - cosh(b*x + c)*sinh(-a + c)^2 - (cosh(-a + c)^2 + 1)*co
sh(b*x + c) - (cosh(-a + c)^2 - 2*cosh(-a + c)*sinh(-a + c) + sinh(-a + c)^
2 + 1)*sinh(b*x + c))*arctan(cosh(b*x + c) + sinh(b*x + c)) + 2*(cosh(b*x +
 c)*cosh(-a + c)^2 - 2*cosh(b*x + c)*cosh(-a + c)*sinh(-a + c) + cosh(b*x +
 c)*sinh(-a + c)^2)*sinh(b*x + c) - 1)/(b*cosh(b*x + c)*cosh(-a + c) - b*co
sh(b*x + c)*sinh(-a + c) + (b*cosh(-a + c) - b*sinh(-a + c))*sinh(b*x + c))
```

giac [A] time = 0.13, size = 49, normalized size = 1.69

$$-\frac{2\left(e^{(2a)}+e^{(2c)}\right)\arctan\left(e^{(bx+c)}\right)e^{(-a-c)}-e^{(bx+a)}+e^{(-bx-a)}}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)*tanh(b*x+c),x, algorithm="giac")
```

```
[Out] -1/2*(2*(e^(2*a) + e^(2*c))*arctan(e^(b*x + c))*e^(-a - c) - e^(b*x + a) +
e^(-b*x - a))/b
```

maple [C] time = 0.22, size = 167, normalized size = 5.76

$$\frac{e^{bx+a}}{2b}-\frac{e^{-bx-a}}{2b}+\frac{i\ln\left(e^{bx+a}-ie^{a-c}\right)e^{-a-c}e^{2a}}{2b}+\frac{i\ln\left(e^{bx+a}-ie^{a-c}\right)e^{-a-c}e^{2c}}{2b}-\frac{i\ln\left(e^{bx+a}+ie^{a-c}\right)e^{-a-c}e^{2a}}{2b}-\frac{i\ln\left(e^{bx+a}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(b*x+a)*tanh(b*x+c),x)
```

```
[Out] 1/2*exp(b*x+a)/b-1/2*exp(-b*x-a)/b+1/2*I*ln(exp(b*x+a)-I*exp(a-c))/b*exp(-a
-c)*exp(2*a)+1/2*I*ln(exp(b*x+a)-I*exp(a-c))/b*exp(-a-c)*exp(2*c)-1/2*I*ln(
exp(b*x+a)+I*exp(a-c))/b*exp(-a-c)*exp(2*a)-1/2*I*ln(exp(b*x+a)+I*exp(a-c))
/b*exp(-a-c)*exp(2*c)
```

maxima [A] time = 0.46, size = 57, normalized size = 1.97

$$\frac{\left(e^{(2a)}+e^{(2c)}\right)\arctan\left(e^{(-bx-c)}\right)e^{(-a-c)}}{b}+\frac{e^{(bx+a)}}{2b}-\frac{e^{(-bx-a)}}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)*tanh(b*x+c),x, algorithm="maxima")
```

```
[Out] (e^(2*a) + e^(2*c))*arctan(e^(-b*x - c))*e^(-a - c)/b + 1/2*e^(b*x + a)/b -
 1/2*e^(-b*x - a)/b
```

mupad [B] time = 1.87, size = 133, normalized size = 4.59

$$\frac{e^{a+bx}}{2b}-\frac{e^{-a-bx}}{2b}-\frac{\operatorname{atan}\left(\frac{e^{-a}\,e^{2c}\,e^{bx}\left(\sqrt{b^2}+e^{2a}\,e^{-2c}\,\sqrt{b^2}\right)}{b\,\sqrt{e^{-2a}\,e^{2c}\left(2\,e^{2a}\,e^{-2c}+e^{4a}\,e^{-4c}+1\right)}}\right)\sqrt{e^{2c-2a}\,\left(2\,e^{2a-2c}+e^{4a-4c}+1\right)}}{\sqrt{b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a + b*x)*tanh(c + b*x),x)
```

```
[Out] exp(a + b*x)/(2*b) - exp(- a - b*x)/(2*b) - (atan((exp(-a)*exp(2*c)*exp(b*x
)*((b^2)^(1/2) + exp(2*a)*exp(-2*c)*(b^2)^(1/2)))/(b*(exp(-2*a)*exp(2*c)*(2
*exp(2*a)*exp(-2*c) + exp(4*a)*exp(-4*c) + 1))^(1/2)))*(exp(2*c - 2*a)*(2*e
xp(2*a - 2*c) + exp(4*a - 4*c) + 1))^(1/2))/(b^2)^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sinh(a+bx)\tanh(bx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)*tanh(b*x+c),x)
[Out] Integral(sinh(a + b*x)*tanh(b*x + c), x)
```

3.144 $\int \sinh(a+bx)\tanh^2(c+bx)\,dx$

Optimal. Leaf size=45

$$-\frac{\sinh(a-c)\tan^{-1}(\sinh(bx+c))}{b}+\frac{\cosh(a-c)\operatorname{sech}(bx+c)}{b}+\frac{\cosh(a+bx)}{b}$$

```
[Out] cosh(b*x+a)/b+cosh(a-c)*sech(b*x+c)/b-arctan(sinh(b*x+c))*sinh(a-c)/b
```

Rubi [A] time = 0.06, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {5620, 5623, 2638, 3770, 2606, 8}

$$-\frac{\sinh(a-c)\tan^{-1}(\sinh(bx+c))}{b}+\frac{\cosh(a-c)\operatorname{sech}(bx+c)}{b}+\frac{\cosh(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[a + b*x]*Tanh[c + b*x]^2,x]
```

```
[Out] Cosh[a + b*x]/b + (Cosh[a - c]*Sech[c + b*x])/b - (ArcTan[Sinh[c + b*x]]*Si
nh[a - c])/b
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 5620

```
Int[Sinh[v_]*Tanh[w_]^(n_.), x_Symbol] :> Int[Cosh[v]*Tanh[w]^(n - 1), x] -
 Dist[Cosh[v - w], Int[Sech[w]*Tanh[w]^(n - 1), x], x] /; GtQ[n, 0] && NeQ[
w, v] && FreeQ[v - w, x]
```

Rule 5623

```
Int[Cosh[v_]*Tanh[w_]^(n_.), x_Symbol] :> Int[Sinh[v]*Tanh[w]^(n - 1), x] -
 Dist[Sinh[v - w], Int[Sech[w]*Tanh[w]^(n - 1), x], x] /; GtQ[n, 0] && NeQ[
w, v] && FreeQ[v - w, x]
```

Rubi steps

$$\int \sinh(a+bx)\tanh^2(c+bx)\,dx = -\left(\cosh(a-c)\int \operatorname{sech}(c+bx)\tanh(c+bx)\,dx\right) + \int \cosh(a+bx)\tanh(c+b$$

$$= \frac{\cosh(a-c)\operatorname{Subst}\left(\int 1\,dx, x, \operatorname{sech}(c+bx)\right)}{b} - \sinh(a-c)\int \operatorname{sech}(c+bx)\,dx +$$

$$= \frac{\cosh(a+bx)}{b} + \frac{\cosh(a-c)\operatorname{sech}(c+bx)}{b} - \frac{\tan^{-1}(\sinh(c+bx))\sinh(a-c)}{b}$$

Mathematica [B] time = 0.11, size = 102, normalized size = 2.27

$$\frac{\cosh(a-c)\operatorname{sech}(bx+c)}{b} - \frac{2\sinh(a-c)\tan^{-1}\left(\frac{(\cosh(c)-\sinh(c))\left(\sinh(c)\cosh\left(\frac{bx}{2}\right)+\cosh(c)\sinh\left(\frac{bx}{2}\right)\right)}{\cosh(c)\cosh\left(\frac{bx}{2}\right)-\sinh(c)\cosh\left(\frac{bx}{2}\right)}\right)}{b} + \frac{\sinh(a)\sinh(bx)}{b} + \frac{c\text{o}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[a + b*x]*Tanh[c + b*x]^2,x]
```

```
[Out] (Cosh[a]*Cosh[b*x])/b + (Cosh[a - c]*Sech[c + b*x])/b - (2*ArcTan[((Cosh[c]
 - Sinh[c])*(Cosh[(b*x)/2]*Sinh[c] + Cosh[c]*Sinh[(b*x)/2]))/(Cosh[c]*Cosh[
(b*x)/2] - Cosh[(b*x)/2]*Sinh[c])]*Sinh[a - c])/b + (Sinh[a]*Sinh[b*x])/b
```

fricas [B] time = 0.51, size = 902, normalized size = 20.04

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)*tanh(b*x+c)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(cosh(b*x + c)^4*cosh(-a + c)^2 + (cosh(-a + c)^2 - 2*cosh(-a + c)*sinh
(-a + c) + sinh(-a + c)^2)*sinh(b*x + c)^4 + 4*(cosh(b*x + c)*cosh(-a + c)^
```

```
2 - 2*cosh(b*x + c)*cosh(-a + c)*sinh(-a + c) + cosh(b*x + c)*sinh(-a + c)^
2)*sinh(b*x + c)^3 + 3*(cosh(-a + c)^2 + 1)*cosh(b*x + c)^2 + 3*(2*cosh(b*x
 + c)^2*cosh(-a + c)^2 + (2*cosh(b*x + c)^2 + 1)*sinh(-a + c)^2 + cosh(-a +
 c)^2 - 2*(2*cosh(b*x + c)^2*cosh(-a + c) + cosh(-a + c))*sinh(-a + c) + 1)
*sinh(b*x + c)^2 + (cosh(b*x + c)^4 + 3*cosh(b*x + c)^2)*sinh(-a + c)^2 - 2
*((cosh(-a + c)^2 - 1)*cosh(b*x + c)^3 + (cosh(-a + c)^2 - 2*cosh(-a + c)*s
inh(-a + c) + sinh(-a + c)^2 - 1)*sinh(b*x + c)^3 - 3*(2*cosh(b*x + c)*cosh
(-a + c)*sinh(-a + c) - cosh(b*x + c)*sinh(-a + c)^2 - (cosh(-a + c)^2 - 1)
*cosh(b*x + c))*sinh(b*x + c)^2 + (cosh(b*x + c)^3 + cosh(b*x + c))*sinh(-a
 + c)^2 + (cosh(-a + c)^2 - 1)*cosh(b*x + c) + (3*(cosh(-a + c)^2 - 1)*cosh
(b*x + c)^2 + (3*cosh(b*x + c)^2 + 1)*sinh(-a + c)^2 + cosh(-a + c)^2 - 2*(
3*cosh(b*x + c)^2*cosh(-a + c) + cosh(-a + c))*sinh(-a + c) - 1)*sinh(b*x +
 c) - 2*(cosh(b*x + c)^3*cosh(-a + c) + cosh(b*x + c)*cosh(-a + c))*sinh(-a
 + c))*arctan(cosh(b*x + c) + sinh(b*x + c)) + 2*(2*cosh(b*x + c)^3*cosh(-a
 + c)^2 + (2*cosh(b*x + c)^3 + 3*cosh(b*x + c))*sinh(-a + c)^2 + 3*(cosh(-a
 + c)^2 + 1)*cosh(b*x + c) - 2*(2*cosh(b*x + c)^3*cosh(-a + c) + 3*cosh(b*x
 + c)*cosh(-a + c))*sinh(-a + c))*sinh(b*x + c) - 2*(cosh(b*x + c)^4*cosh(-
a + c) + 3*cosh(b*x + c)^2*cosh(-a + c))*sinh(-a + c) + 1)/(b*cosh(b*x + c)
^3*cosh(-a + c) + (b*cosh(-a + c) - b*sinh(-a + c))*sinh(b*x + c)^3 + b*cos
h(b*x + c)*cosh(-a + c) + 3*(b*cosh(b*x + c)*cosh(-a + c) - b*cosh(b*x + c)
*sinh(-a + c))*sinh(b*x + c)^2 + (3*b*cosh(b*x + c)^2*cosh(-a + c) + b*cosh
(-a + c) - (3*b*cosh(b*x + c)^2 + b)*sinh(-a + c))*sinh(b*x + c) - (b*cosh(
b*x + c)^3 + b*cosh(b*x + c))*sinh(-a + c))
```

giac [A] time = 0.15, size = 88, normalized size = 1.96

$$-\frac{2\left(e^{(2a)}-e^{(2c)}\right)\arctan\left(e^{(bx+c)}\right)e^{(-a-c)}-\frac{\left(2e^{(2bx+2a)}+3e^{(2bx+2c)}+1\right)e^{(-a)}}{e^{(3bx+2c)}+e^{(bx)}}-e^{(bx+a)}}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)*tanh(b*x+c)^2,x, algorithm="giac")
```

```
[Out] -1/2*(2*(e^(2*a) - e^(2*c))*arctan(e^(b*x + c))*e^(-a - c) - (2*e^(2*b*x +
2*a) + 3*e^(2*b*x + 2*c) + 1)*e^(-a)/(e^(3*b*x + 2*c) + e^(b*x)) - e^(b*x +
 a))/b
```

maple [C] time = 0.32, size = 205, normalized size = 4.56

$$\frac{e^{bx+a}}{2b}+\frac{e^{-bx-a}}{2b}+\frac{e^{bx+a}\left(e^{2a}+e^{2c}\right)}{b\left(e^{2bx+2a+2c}+e^{2a}\right)}+\frac{i\ln\left(e^{bx+a}-ie^{a-c}\right)e^{-a-c}e^{2a}}{2b}-\frac{i\ln\left(e^{bx+a}-ie^{a-c}\right)e^{-a-c}e^{2c}}{2b}-\frac{i\ln\left(e^{bx+a}+ie^{a-}\right.}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(b*x+a)*tanh(b*x+c)^2,x)
```

```
[Out] 1/2*exp(b*x+a)/b+1/2*exp(-b*x-a)/b+1/b*exp(b*x+a)*(exp(2*a)+exp(2*c))/(exp(
2*b*x+2*a+2*c)+exp(2*a))+1/2*I*ln(exp(b*x+a)-I*exp(a-c))/b*exp(-a-c)*exp(a)
^2-1/2*I*ln(exp(b*x+a)-I*exp(a-c))/b*exp(-a-c)*exp(c)^2-1/2*I*ln(exp(b*x+a)
+I*exp(a-c))/b*exp(-a-c)*exp(a)^2+1/2*I*ln(exp(b*x+a)+I*exp(a-c))/b*exp(-a-
c)*exp(c)^2
```

maxima [B] time = 0.42, size = 105, normalized size = 2.33

$$\frac{\left(e^{(2a)} - e^{(2c)}\right)\arctan\left(e^{(-bx-c)}\right)e^{(-a-c)}}{b} + \frac{e^{(-bx-a)}}{2b} + \frac{\left(3e^{(2a)} + 2e^{(2c)}\right)e^{(-2bx-2a)} + e^{(2c)}}{2b\left(e^{(-bx-a+2c)} + e^{(-3bx-a)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)*tanh(b*x+c)^2,x, algorithm="maxima")
```

```
[Out] (e^(2*a) - e^(2*c))*arctan(e^(-b*x - c))*e^(-a - c)/b + 1/2*e^(-b*x - a)/b
+ 1/2*((3*e^(2*a) + 2*e^(2*c))*e^(-2*b*x - 2*a) + e^(2*c))/(b*(e^(-b*x - a
+ 2*c) + e^(-3*b*x - a)))
```

mupad [B] time = 1.62, size = 173, normalized size = 3.84

$$\frac{e^{a+bx}}{2b}+\frac{e^{-a-bx}}{2b}+\frac{\operatorname{atan}\left(\frac{e^{-a}\,e^{2c}\,e^{bx}\left(\sqrt{b^2}-e^{2a}\,e^{-2c}\,\sqrt{b^2}\right)}{b\,\sqrt{e^{-2a}\,e^{2c}\left(e^{4a}\,e^{-4c}-2\,e^{2a}\,e^{-2c}+1\right)}}\right)\sqrt{e^{2c-2a}\,\left(e^{4a-4c}-2\,e^{2a-2c}+1\right)}}{\sqrt{b^2}}+\frac{e^{a+bx}\,\left(e^{2a-2c}+1\right)}{b\,\left(e^{2a-2c}+e^{2a+2bx}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a + b*x)*tanh(c + b*x)^2,x)
```

```
[Out] exp(a + b*x)/(2*b) + exp(- a - b*x)/(2*b) + (atan((exp(-a)*exp(2*c)*exp(b*x
)*((b^2)^(1/2) - exp(2*a)*exp(-2*c)*(b^2)^(1/2)))/(b*(exp(-2*a)*exp(2*c)*(e
xp(4*a)*exp(-4*c) - 2*exp(2*a)*exp(-2*c) + 1))^(1/2)))*(exp(2*c - 2*a)*(exp
(4*a - 4*c) - 2*exp(2*a - 2*c) + 1))^(1/2))/(b^2)^(1/2) + (exp(a + b*x)*(ex
p(2*a - 2*c) + 1))/(b*(exp(2*a - 2*c) + exp(2*a + 2*b*x)))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sinh(a+bx)\tanh^2(bx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)*tanh(b*x+c)**2,x)
```

```
[Out] Integral(sinh(a + b*x)*tanh(b*x + c)**2, x)
```

3.145 $\int \sinh(a+bx)\tanh^3(c+bx)\,dx$

Optimal. Leaf size=72

$$\frac{\sinh(a-c)\text{sech}(bx+c)}{b}-\frac{3\cosh(a-c)\tan^{-1}(\sinh(bx+c))}{2b}+\frac{\cosh(a-c)\tanh(bx+c)\text{sech}(bx+c)}{2b}+\frac{\sinh(a+b}{b}$$

```
[Out] -3/2*arctan(sinh(b*x+c))*cosh(a-c)/b+sech(b*x+c)*sinh(a-c)/b+sinh(b*x+a)/b+
1/2*cosh(a-c)*sech(b*x+c)*tanh(b*x+c)/b
```

Rubi [A] time = 0.08, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.467, Rules used = {5620, 5623, 2637, 3770, 2606, 8, 2611}

$$\frac{\sinh(a-c)\text{sech}(bx+c)}{b}-\frac{3\cosh(a-c)\tan^{-1}(\sinh(bx+c))}{2b}+\frac{\cosh(a-c)\tanh(bx+c)\text{sech}(bx+c)}{2b}+\frac{\sinh(a+b}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[a + b*x]*Tanh[c + b*x]^3,x]
```

```
[Out] (-3*ArcTan[Sinh[c + b*x]]*Cosh[a - c])/(2*b) + (Sech[c + b*x]*Sinh[a - c])/
b + Sinh[a + b*x]/b + (Cosh[a - c]*Sech[c + b*x]*Tanh[c + b*x])/(2*b)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 && !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 5620

```
Int[Sinh[v_]*Tanh[w_]^(n_.), x_Symbol] :> Int[Cosh[v]*Tanh[w]^(n - 1), x] -
 Dist[Cosh[v - w], Int[Sech[w]*Tanh[w]^(n - 1), x], x] /; GtQ[n, 0] && NeQ[
w, v] && FreeQ[v - w, x]
```

Rule 5623

```
Int[Cosh[v_]*Tanh[w_]^(n_.), x_Symbol] :> Int[Sinh[v]*Tanh[w]^(n - 1), x] -
 Dist[Sinh[v - w], Int[Sech[w]*Tanh[w]^(n - 1), x], x] /; GtQ[n, 0] && NeQ[
w, v] && FreeQ[v - w, x]
```

Rubi steps

$$\int \sinh(a+bx)\tanh^3(c+bx)\,dx = -\left(\cosh(a-c)\int \mathrm{sech}(c+bx)\tanh^2(c+bx)\,dx\right) + \int \cosh(a+bx)\tanh^2(c+$$

$$= \frac{\cosh(a-c)\mathrm{sech}(c+bx)\tanh(c+bx)}{2b} - \frac{1}{2}\cosh(a-c)\int \mathrm{sech}(c+bx)\,dx - \mathrm{si}$$

$$= -\frac{\tan^{-1}(\sinh(c+bx))\cosh(a-c)}{2b} + \frac{\cosh(a-c)\mathrm{sech}(c+bx)\tanh(c+bx)}{2b} -$$

$$= -\frac{3\tan^{-1}(\sinh(c+bx))\cosh(a-c)}{2b} + \frac{\mathrm{sech}(c+bx)\sinh(a-c)}{b} + \frac{\sinh(a+b}{b}$$

Mathematica [A] time = 0.35, size = 70, normalized size = 0.97

$$\frac{\mathrm{sech}^2(bx+c)(2\sinh(a-bx-2c)+\sinh(a+3bx+2c)+5\sinh(a+bx)) - 12\cosh(a-c)\tan^{-1}\left(\cosh(c)\tanh\left(\right.\right.}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[a + b*x]*Tanh[c + b*x]^3,x]
```

```
[Out] (-12*ArcTan[Sinh[c] + Cosh[c]*Tanh[(b*x)/2]]*Cosh[a - c] + Sech[c + b*x]^2*
(2*Sinh[a - 2*c - b*x] + 5*Sinh[a + b*x] + Sinh[a + 2*c + 3*b*x]))/(4*b)
```

fricas [B] time = 0.46, size = 1737, normalized size = 24.12

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)*tanh(b*x+c)^3,x, algorithm="fricas")
```

```
[Out] 1/2*(cosh(b*x + c)^6*cosh(-a + c)^2 + (cosh(-a + c)^2 - 2*cosh(-a + c)*sinh
(-a + c) + sinh(-a + c)^2)*sinh(b*x + c)^6 + 6*(cosh(b*x + c)*cosh(-a + c)^
2 - 2*cosh(b*x + c)*cosh(-a + c)*sinh(-a + c) + cosh(b*x + c)*sinh(-a + c)^
2)*sinh(b*x + c)^5 + (5*cosh(-a + c)^2 - 2)*cosh(b*x + c)^4 + (15*cosh(b*x
+ c)^2*cosh(-a + c)^2 + 5*(3*cosh(b*x + c)^2 + 1)*sinh(-a + c)^2 + 5*cosh(-
a + c)^2 - 10*(3*cosh(b*x + c)^2*cosh(-a + c) + cosh(-a + c))*sinh(-a + c)
- 2)*sinh(b*x + c)^4 + 4*(5*cosh(b*x + c)^3*cosh(-a + c)^2 + 5*(cosh(b*x +
c)^3 + cosh(b*x + c))*sinh(-a + c)^2 + (5*cosh(-a + c)^2 - 2)*cosh(b*x + c)
 - 10*(cosh(b*x + c)^3*cosh(-a + c) + cosh(b*x + c)*cosh(-a + c))*sinh(-a +
 c))*sinh(b*x + c)^3 + (2*cosh(-a + c)^2 - 5)*cosh(b*x + c)^2 + (15*cosh(b*
x + c)^4*cosh(-a + c)^2 + 6*(5*cosh(-a + c)^2 - 2)*cosh(b*x + c)^2 + (15*co
sh(b*x + c)^4 + 30*cosh(b*x + c)^2 + 2)*sinh(-a + c)^2 + 2*cosh(-a + c)^2 -
 2*(15*cosh(b*x + c)^4*cosh(-a + c) + 30*cosh(b*x + c)^2*cosh(-a + c) + 2*c
osh(-a + c))*sinh(-a + c) - 5)*sinh(b*x + c)^2 + (cosh(b*x + c)^6 + 5*cosh(
b*x + c)^4 + 2*cosh(b*x + c)^2)*sinh(-a + c)^2 - 3*((cosh(-a + c)^2 + 1)*co
sh(b*x + c)^5 + (cosh(-a + c)^2 - 2*cosh(-a + c)*sinh(-a + c) + sinh(-a + c
)^2 + 1)*sinh(b*x + c)^5 - 5*(2*cosh(b*x + c)*cosh(-a + c)*sinh(-a + c) - c
osh(b*x + c)*sinh(-a + c)^2 - (cosh(-a + c)^2 + 1)*cosh(b*x + c))*sinh(b*x
+ c)^4 + 2*(cosh(-a + c)^2 + 1)*cosh(b*x + c)^3 + 2*(5*(cosh(-a + c)^2 + 1)
*cosh(b*x + c)^2 + (5*cosh(b*x + c)^2 + 1)*sinh(-a + c)^2 + cosh(-a + c)^2
- 2*(5*cosh(b*x + c)^2*cosh(-a + c) + cosh(-a + c))*sinh(-a + c) + 1)*sinh(
b*x + c)^3 + 2*(5*(cosh(-a + c)^2 + 1)*cosh(b*x + c)^3 + (5*cosh(b*x + c)^3
 + 3*cosh(b*x + c))*sinh(-a + c)^2 + 3*(cosh(-a + c)^2 + 1)*cosh(b*x + c) -
 2*(5*cosh(b*x + c)^3*cosh(-a + c) + 3*cosh(b*x + c)*cosh(-a + c))*sinh(-a
+ c))*sinh(b*x + c)^2 + (cosh(b*x + c)^5 + 2*cosh(b*x + c)^3 + cosh(b*x + c
))*sinh(-a + c)^2 + (cosh(-a + c)^2 + 1)*cosh(b*x + c) + (5*(cosh(-a + c)^2
 + 1)*cosh(b*x + c)^4 + 6*(cosh(-a + c)^2 + 1)*cosh(b*x + c)^2 + (5*cosh(b*
x + c)^4 + 6*cosh(b*x + c)^2 + 1)*sinh(-a + c)^2 + cosh(-a + c)^2 - 2*(5*co
sh(b*x + c)^4*cosh(-a + c) + 6*cosh(b*x + c)^2*cosh(-a + c) + cosh(-a + c))
*sinh(-a + c) + 1)*sinh(b*x + c) - 2*(cosh(b*x + c)^5*cosh(-a + c) + 2*cosh
(b*x + c)^3*cosh(-a + c) + cosh(b*x + c)*cosh(-a + c))*sinh(-a + c))*arctan
(cosh(b*x + c) + sinh(b*x + c)) + 2*(3*cosh(b*x + c)^5*cosh(-a + c)^2 + 2*(
5*cosh(-a + c)^2 - 2)*cosh(b*x + c)^3 + (3*cosh(b*x + c)^5 + 10*cosh(b*x +
c)^3 + 2*cosh(b*x + c))*sinh(-a + c)^2 + (2*cosh(-a + c)^2 - 5)*cosh(b*x +
c) - 2*(3*cosh(b*x + c)^5*cosh(-a + c) + 10*cosh(b*x + c)^3*cosh(-a + c) +
2*cosh(b*x + c)*cosh(-a + c))*sinh(-a + c))*sinh(b*x + c) - 2*(cosh(b*x + c
)^6*cosh(-a + c) + 5*cosh(b*x + c)^4*cosh(-a + c) + 2*cosh(b*x + c)^2*cosh(
```

```
-a + c))*sinh(-a + c) - 1)/(b*cosh(b*x + c)^5*cosh(-a + c) + (b*cosh(-a + c
) - b*sinh(-a + c))*sinh(b*x + c)^5 + 2*b*cosh(b*x + c)^3*cosh(-a + c) + 5*
(b*cosh(b*x + c)*cosh(-a + c) - b*cosh(b*x + c)*sinh(-a + c))*sinh(b*x + c)
^4 + 2*(5*b*cosh(b*x + c)^2*cosh(-a + c) + b*cosh(-a + c) - (5*b*cosh(b*x +
 c)^2 + b)*sinh(-a + c))*sinh(b*x + c)^3 + b*cosh(b*x + c)*cosh(-a + c) + 2
*(5*b*cosh(b*x + c)^3*cosh(-a + c) + 3*b*cosh(b*x + c)*cosh(-a + c) - (5*b*
cosh(b*x + c)^3 + 3*b*cosh(b*x + c))*sinh(-a + c))*sinh(b*x + c)^2 + (5*b*c
osh(b*x + c)^4*cosh(-a + c) + 6*b*cosh(b*x + c)^2*cosh(-a + c) + b*cosh(-a
+ c) - (5*b*cosh(b*x + c)^4 + 6*b*cosh(b*x + c)^2 + b)*sinh(-a + c))*sinh(b
*x + c) - (b*cosh(b*x + c)^5 + 2*b*cosh(b*x + c)^3 + b*cosh(b*x + c))*sinh(
-a + c))
```

giac [A] time = 0.14, size = 112, normalized size = 1.56

$$-\frac{3\left(e^{(2a)}+e^{(2c)}\right)\arctan\left(e^{(bx+c)}\right)e^{(-a-c)}-\frac{\left(3e^{(3bx+2a+2c)}-e^{(3bx+4c)}+e^{(bx+2a)}-3e^{(bx+2c)}\right)e^{(-a)}}{\left(e^{(2bx+2c)}+1\right)^2}-e^{(bx+a)}+e^{(-bx-a)}}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)*tanh(b*x+c)^3,x, algorithm="giac")
```

```
[Out] -1/2*(3*(e^(2*a) + e^(2*c))*arctan(e^(b*x + c))*e^(-a - c) - (3*e^(3*b*x +
2*a + 2*c) - e^(3*b*x + 4*c) + e^(b*x + 2*a) - 3*e^(b*x + 2*c))*e^(-a)/(e^(
2*b*x + 2*c) + 1)^2 - e^(b*x + a) + e^(-b*x - a))/b
```

maple [C] time = 0.33, size = 240, normalized size = 3.33

$$\frac{e^{bx+a}}{2b}-\frac{e^{-bx-a}}{2b}+\frac{e^{bx+a}\left(3e^{2bx+4a+2c}-e^{2bx+2a+4c}+e^{4a}-3e^{2a+2c}\right)}{2b\left(e^{2bx+2a+2c}+e^{2a}\right)^2}+\frac{3i\ln\left(e^{bx+a}-ie^{a-c}\right)e^{-a-c}e^{2a}}{4b}+\frac{3i\ln\left(e^{bx+a}-ie^{a-}\right.}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(b*x+a)*tanh(b*x+c)^3,x)
```

```
[Out] 1/2*exp(b*x+a)/b-1/2*exp(-b*x-a)/b+1/2*exp(b*x+a)*(3*exp(2*b*x+4*a+2*c)-exp
(2*b*x+2*a+4*c)+exp(4*a)-3*exp(2*a+2*c))/b/(exp(2*b*x+2*a+2*c)+exp(2*a))^2+
3/4*I*ln(exp(b*x+a)-I*exp(a-c))/b*exp(-a-c)*exp(2*a)+3/4*I*ln(exp(b*x+a)-I*
exp(a-c))/b*exp(-a-c)*exp(2*c)-3/4*I*ln(exp(b*x+a)+I*exp(a-c))/b*exp(-a-c)*
exp(2*a)-3/4*I*ln(exp(b*x+a)+I*exp(a-c))/b*exp(-a-c)*exp(2*c)
```

maxima [B] time = 0.43, size = 149, normalized size = 2.07

$$\frac{3\left(e^{(2a)}+e^{(2c)}\right)\arctan\left(e^{(-bx-c)}\right)e^{(-a-c)}}{2b}-\frac{e^{(-bx-a)}}{2b}+\frac{\left(5e^{(2a+2c)}-e^{(4c)}\right)e^{(-2bx-2a)}+\left(2e^{(4a)}-3e^{(2a+2c)}\right)e^{(-4bx-4a)}+e}{2b\left(e^{(-bx-a+4c)}+2e^{(-3bx-a+2c)}+e^{(-5bx-a)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)*tanh(b*x+c)^3,x, algorithm="maxima")
```

```
[Out] 3/2*(e^(2*a) + e^(2*c))*arctan(e^(-b*x - c))*e^(-a - c)/b - 1/2*e^(-b*x - a
)/b + 1/2*((5*e^(2*a + 2*c) - e^(4*c))*e^(-2*b*x - 2*a) + (2*e^(4*a) - 3*e^
(2*a + 2*c))*e^(-4*b*x - 4*a) + e^(4*c))/(b*(e^(-b*x - a + 4*c) + 2*e^(-3*b
*x - a + 2*c) + e^(-5*b*x - a)))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sinh(a + b\,x)\,\tanh(c + b\,x)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a + b*x)*tanh(c + b*x)^3,x)
```

```
[Out] int(sinh(a + b*x)*tanh(c + b*x)^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sinh(a + bx)\tanh^3(bx + c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)*tanh(b*x+c)**3,x)
```

```
[Out] Integral(sinh(a + b*x)*tanh(b*x + c)**3, x)
```

3.146 $\int \coth(c+bx)\sinh(a+bx)\,dx$

Optimal. Leaf size=29

$$\frac{\sinh(a+bx)}{b}-\frac{\sinh(a-c)\tanh^{-1}(\cosh(bx+c))}{b}$$

```
[Out] -arctanh(cosh(b*x+c))*sinh(a-c)/b+sinh(b*x+a)/b
```

Rubi [A] time = 0.02, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {5622, 2637, 3770}

$$\frac{\sinh(a+bx)}{b}-\frac{\sinh(a-c)\tanh^{-1}(\cosh(bx+c))}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[c + b*x]*Sinh[a + b*x],x]
```

```
[Out] -((ArcTanh[Cosh[c + b*x]]*Sinh[a - c])/b) + Sinh[a + b*x]/b
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 5622

```
Int[Coth[w_]^(n_.)*Sinh[v_], x_Symbol] :> Int[Cosh[v]*Coth[w]^(n - 1), x] +
 Dist[Sinh[v - w], Int[Csch[w]*Coth[w]^(n - 1), x], x] /; GtQ[n, 0] && NeQ[
w, v] && FreeQ[v - w, x]
```

Rubi steps

$$\begin{aligned}\int \coth(c+bx)\sinh(a+bx)\,dx &= \sinh(a-c)\int \operatorname{csch}(c+bx)\,dx+\int \cosh(a+bx)\,dx\\ &= -\frac{\tanh^{-1}(\cosh(c+bx))\sinh(a-c)}{b}+\frac{\sinh(a+bx)}{b}\end{aligned}$$

Mathematica [C] time = 0.06, size = 93, normalized size = 3.21

$$-\frac{2i\sinh(a-c)\tan^{-1}\left(\frac{(\cosh(c)-\sinh(c))\left(\sinh(c)\sinh\left(\frac{bx}{2}\right)+\cosh(c)\cosh\left(\frac{bx}{2}\right)\right)}{i\cosh(c)\cosh\left(\frac{bx}{2}\right)-i\sinh(c)\cosh\left(\frac{bx}{2}\right)}\right)}{b}+\frac{\sinh(a)\cosh(bx)}{b}+\frac{\cosh(a)\sinh(bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[c + b*x]*Sinh[a + b*x],x]
```

```
[Out] (Cosh[b*x]*Sinh[a])/b - ((2*I)*ArcTan[((Cosh[c] - Sinh[c])*(Cosh[c]*Cosh[(b
*x)/2] + Sinh[c]*Sinh[(b*x)/2]))/(I*Cosh[c]*Cosh[(b*x)/2] - I*Cosh[(b*x)/2]
*Sinh[c])]*Sinh[a - c])/b + (Cosh[a]*Sinh[b*x])/b
```

fricas [B] time = 0.43, size = 439, normalized size = 15.14

$$\frac{\cosh(bx+c)^2\cosh(-a+c)^2-2\cosh(bx+c)^2\cosh(-a+c)\sinh(-a+c)+\cosh(bx+c)^2\sinh(-a+c)^2+(}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(b*x+c)*sinh(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/2*(cosh(b*x + c)^2*cosh(-a + c)^2 - 2*cosh(b*x + c)^2*cosh(-a + c)*sinh(-
a + c) + cosh(b*x + c)^2*sinh(-a + c)^2 + (cosh(-a + c)^2 - 2*cosh(-a + c)*
sinh(-a + c) + sinh(-a + c)^2)*sinh(b*x + c)^2 + (2*cosh(b*x + c)*cosh(-a +
 c)*sinh(-a + c) - cosh(b*x + c)*sinh(-a + c)^2 - (cosh(-a + c)^2 - 1)*cosh
(b*x + c) - (cosh(-a + c)^2 - 2*cosh(-a + c)*sinh(-a + c) + sinh(-a + c)^2
- 1)*sinh(b*x + c))*log(cosh(b*x + c) + sinh(b*x + c) + 1) - (2*cosh(b*x +
c)*cosh(-a + c)*sinh(-a + c) - cosh(b*x + c)*sinh(-a + c)^2 - (cosh(-a + c)
^2 - 1)*cosh(b*x + c) - (cosh(-a + c)^2 - 2*cosh(-a + c)*sinh(-a + c) + sin
h(-a + c)^2 - 1)*sinh(b*x + c))*log(cosh(b*x + c) + sinh(b*x + c) - 1) + 2*
(cosh(b*x + c)*cosh(-a + c)^2 - 2*cosh(b*x + c)*cosh(-a + c)*sinh(-a + c) +
 cosh(b*x + c)*sinh(-a + c)^2)*sinh(b*x + c) - 1)/(b*cosh(b*x + c)*cosh(-a
+ c) - b*cosh(b*x + c)*sinh(-a + c) + (b*cosh(-a + c) - b*sinh(-a + c))*sin
h(b*x + c))
```

giac [B] time = 0.12, size = 87, normalized size = 3.00

$$-\frac{\left(e^{(2a+c)}-e^{(3c)}\right)e^{(-a-2c)}\log\left(e^{(bx+c)}+1\right)-\left(e^{(2a+c)}-e^{(3c)}\right)e^{(-a-2c)}\log\left(\left|e^{(bx+c)}-1\right|\right)-e^{(bx+a)}+e^{(-bx-a)}}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(b*x+c)*sinh(b*x+a),x, algorithm="giac")
```

```
[Out] -1/2*((e^(2*a + c) - e^(3*c))*e^(-a - 2*c)*log(e^(b*x + c) + 1) - (e^(2*a +
 c) - e^(3*c))*e^(-a - 2*c)*log(abs(e^(b*x + c) - 1)) - e^(b*x + a) + e^(-b
*x - a))/b
```

maple [B] time = 0.20, size = 155, normalized size = 5.34

$$\frac{e^{bx+a}}{2b}-\frac{e^{-bx-a}}{2b}+\frac{\ln\left(e^{bx+a}-e^{a-c}\right)e^{-a-c}e^{2a}}{2b}-\frac{\ln\left(e^{bx+a}-e^{a-c}\right)e^{-a-c}e^{2c}}{2b}-\frac{\ln\left(e^{bx+a}+e^{a-c}\right)e^{-a-c}e^{2a}}{2b}+\frac{\ln\left(e^{bx+a}+e^{a-c}\right)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(b*x+c)*sinh(b*x+a),x)
```

```
[Out] 1/2*exp(b*x+a)/b-1/2*exp(-b*x-a)/b+1/2*ln(exp(b*x+a)-exp(a-c))/b*exp(-a-c)*
exp(2*a)-1/2*ln(exp(b*x+a)-exp(a-c))/b*exp(-a-c)*exp(2*c)-1/2*ln(exp(b*x+a)
+exp(a-c))/b*exp(-a-c)*exp(2*a)+1/2*ln(exp(b*x+a)+exp(a-c))/b*exp(-a-c)*exp
(2*c)
```

maxima [B] time = 0.33, size = 94, normalized size = 3.24

$$-\frac{\left(e^{(2a)}-e^{(2c)}\right)e^{(-a-c)}\log\left(e^{(-bx)}+e^{c}\right)}{2b}+\frac{\left(e^{(2a)}-e^{(2c)}\right)e^{(-a-c)}\log\left(e^{(-bx)}-e^{c}\right)}{2b}+\frac{e^{(bx+a)}}{2b}-\frac{e^{(-bx-a)}}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(b*x+c)*sinh(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/2*(e^(2*a) - e^(2*c))*e^(-a - c)*log(e^(-b*x) + e^c)/b + 1/2*(e^(2*a) -
e^(2*c))*e^(-a - c)*log(e^(-b*x) - e^c)/b + 1/2*e^(b*x + a)/b - 1/2*e^(-b*x
 - a)/b
```

mupad [B] time = 0.15, size = 139, normalized size = 4.79

$$\frac{e^{a+b\,x}}{2\,b}-\frac{e^{-a-b\,x}}{2\,b}+\frac{\mathrm{atan}\left(\frac{e^{-a}\,e^{2\,c}\,e^{b\,x}\,\left(\sqrt{-b^2}-e^{2\,a}\,e^{-2\,c}\,\sqrt{-b^2}\right)}{b\,\sqrt{e^{-2\,a}\,e^{2\,c}\,\left(e^{4\,a}\,e^{-4\,c}-2\,e^{2\,a}\,e^{-2\,c}+1\right)}}\right)\,\sqrt{e^{2\,c-2\,a}\,\left(e^{4\,a-4\,c}-2\,e^{2\,a-2\,c}+1\right)}}{\sqrt{-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(c + b*x)*sinh(a + b*x),x)
```

```
[Out] exp(a + b*x)/(2*b) - exp(- a - b*x)/(2*b) + (atan((exp(-a)*exp(2*c)*exp(b*x
)*((-b^2)^(1/2) - exp(2*a)*exp(-2*c)*(-b^2)^(1/2)))/(b*(exp(-2*a)*exp(2*c)*
(exp(4*a)*exp(-4*c) - 2*exp(2*a)*exp(-2*c) + 1))^(1/2)))*(exp(2*c - 2*a)*(e
xp(4*a - 4*c) - 2*exp(2*a - 2*c) + 1))^(1/2))/(-b^2)^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sinh(a+bx)\coth(bx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(b*x+c)*sinh(b*x+a),x)
```

```
[Out] Integral(sinh(a + b*x)*coth(b*x + c), x)
```

3.147 $\int \coth^2(c+bx)\sinh(a+bx)\,dx$

Optimal. Leaf size=46

$$-\frac{\cosh(a-c)\tanh^{-1}(\cosh(bx+c))}{b}-\frac{\sinh(a-c)\text{csch}(bx+c)}{b}+\frac{\cosh(a+bx)}{b}$$

```
[Out] -arctanh(cosh(b*x+c))*cosh(a-c)/b+cosh(b*x+a)/b-csch(b*x+c)*sinh(a-c)/b
```

Rubi [A] time = 0.05, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {5622, 5621, 2638, 3770, 2606, 8}

$$-\frac{\cosh(a-c)\tanh^{-1}(\cosh(bx+c))}{b}-\frac{\sinh(a-c)\text{csch}(bx+c)}{b}+\frac{\cosh(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[c + b*x]^2*Sinh[a + b*x],x]
```

```
[Out] -((ArcTanh[Cosh[c + b*x]]*Cosh[a - c])/b) + Cosh[a + b*x]/b - (Csch[c + b*x
]*Sinh[a - c])/b
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 5621

```
Int[Cosh[v_]*Coth[w_]^(n_.), x_Symbol] :> Int[Sinh[v]*Coth[w]^(n - 1), x] +
 Dist[Cosh[v - w], Int[Csch[w]*Coth[w]^(n - 1), x], x] /; GtQ[n, 0] && NeQ[
w, v] && FreeQ[v - w, x]
```

Rule 5622

```
Int[Coth[w_]^(n_.)*Sinh[v_], x_Symbol] :> Int[Cosh[v]*Coth[w]^(n - 1), x] +
 Dist[Sinh[v - w], Int[Csch[w]*Coth[w]^(n - 1), x], x] /; GtQ[n, 0] && NeQ[
w, v] && FreeQ[v - w, x]
```

Rubi steps

$$\int \coth^2(c+bx)\sinh(a+bx)\,dx = \sinh(a-c)\int \coth(c+bx)\text{csch}(c+bx)\,dx + \int \cosh(a+bx)\coth(c+bx)\,d$$
$$= \cosh(a-c)\int \text{csch}(c+bx)\,dx - \frac{(i\sinh(a-c))\,\text{Subst}(\int 1\,dx, x, -i\text{csch}(c+}{b}$$
$$= -\frac{\tanh^{-1}(\cosh(c+bx))\cosh(a-c)}{b} + \frac{\cosh(a+bx)}{b} - \frac{\text{csch}(c+bx)\sinh(a-}{b}$$

Mathematica [C] time = 0.10, size = 110, normalized size = 2.39

$$-\frac{\sinh(a-c)\text{csch}(bx+c)}{b} - \frac{2i\cosh(a-c)\tan^{-1}\left(\frac{(\cosh(c)-\sinh(c))\left(\sinh(c)\sinh\left(\frac{bx}{2}\right)+\cosh(c)\cosh\left(\frac{bx}{2}\right)\right)}{i\cosh(c)\cosh\left(\frac{bx}{2}\right)-i\sinh(c)\cosh\left(\frac{bx}{2}\right)}\right)}{b} + \frac{\sinh(a)\sinh(bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[c + b*x]^2*Sinh[a + b*x],x]
```

```
[Out] ((-2*I)*ArcTan[((Cosh[c] - Sinh[c])*(Cosh[c]*Cosh[(b*x)/2] + Sinh[c]*Sinh[(
b*x)/2]))/(I*Cosh[c]*Cosh[(b*x)/2] - I*Cosh[(b*x)/2]*Sinh[c])]*Cosh[a - c])
/b + (Cosh[a]*Cosh[b*x])/b - (Csch[c + b*x]*Sinh[a - c])/b + (Sinh[a]*Sinh[
b*x])/b
```

fricas [B] time = 0.45, size = 1237, normalized size = 26.89

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(b*x+c)^2*sinh(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/2*(cosh(b*x + c)^4*cosh(-a + c)^2 + (cosh(-a + c)^2 - 2*cosh(-a + c)*sinh
(-a + c) + sinh(-a + c)^2)*sinh(b*x + c)^4 + 4*(cosh(b*x + c)*cosh(-a + c)^
2 - 2*cosh(b*x + c)*cosh(-a + c)*sinh(-a + c) + cosh(b*x + c)*sinh(-a + c)^
2)*sinh(b*x + c)^3 - 3*(cosh(-a + c)^2 - 1)*cosh(b*x + c)^2 + 3*(2*cosh(b*x
 + c)^2*cosh(-a + c)^2 + (2*cosh(b*x + c)^2 - 1)*sinh(-a + c)^2 - cosh(-a +
 c)^2 - 2*(2*cosh(b*x + c)^2*cosh(-a + c) - cosh(-a + c))*sinh(-a + c) + 1)
*sinh(b*x + c)^2 + (cosh(b*x + c)^4 - 3*cosh(b*x + c)^2)*sinh(-a + c)^2 - (
(cosh(-a + c)^2 + 1)*cosh(b*x + c)^3 + (cosh(-a + c)^2 - 2*cosh(-a + c)*sin
h(-a + c) + sinh(-a + c)^2 + 1)*sinh(b*x + c)^3 - 3*(2*cosh(b*x + c)*cosh(-
a + c)*sinh(-a + c) - cosh(b*x + c)*sinh(-a + c)^2 - (cosh(-a + c)^2 + 1)*c
osh(b*x + c))*sinh(b*x + c)^2 + (cosh(b*x + c)^3 - cosh(b*x + c))*sinh(-a +
 c)^2 - (cosh(-a + c)^2 + 1)*cosh(b*x + c) + (3*(cosh(-a + c)^2 + 1)*cosh(b
*x + c)^2 + (3*cosh(b*x + c)^2 - 1)*sinh(-a + c)^2 - cosh(-a + c)^2 - 2*(3*
cosh(b*x + c)^2*cosh(-a + c) - cosh(-a + c))*sinh(-a + c) - 1)*sinh(b*x + c
) - 2*(cosh(b*x + c)^3*cosh(-a + c) - cosh(b*x + c)*cosh(-a + c))*sinh(-a +
 c))*log(cosh(b*x + c) + sinh(b*x + c) + 1) + ((cosh(-a + c)^2 + 1)*cosh(b*
x + c)^3 + (cosh(-a + c)^2 - 2*cosh(-a + c)*sinh(-a + c) + sinh(-a + c)^2 +
 1)*sinh(b*x + c)^3 - 3*(2*cosh(b*x + c)*cosh(-a + c)*sinh(-a + c) - cosh(b
*x + c)*sinh(-a + c)^2 - (cosh(-a + c)^2 + 1)*cosh(b*x + c))*sinh(b*x + c)^
2 + (cosh(b*x + c)^3 - cosh(b*x + c))*sinh(-a + c)^2 - (cosh(-a + c)^2 + 1)
*cosh(b*x + c) + (3*(cosh(-a + c)^2 + 1)*cosh(b*x + c)^2 + (3*cosh(b*x + c)
^2 - 1)*sinh(-a + c)^2 - cosh(-a + c)^2 - 2*(3*cosh(b*x + c)^2*cosh(-a + c)
 - cosh(-a + c))*sinh(-a + c) - 1)*sinh(b*x + c) - 2*(cosh(b*x + c)^3*cosh(
-a + c) - cosh(b*x + c)*cosh(-a + c))*sinh(-a + c))*log(cosh(b*x + c) + sin
h(b*x + c) - 1) + 2*(2*cosh(b*x + c)^3*cosh(-a + c)^2 + (2*cosh(b*x + c)^3
- 3*cosh(b*x + c))*sinh(-a + c)^2 - 3*(cosh(-a + c)^2 - 1)*cosh(b*x + c) -
2*(2*cosh(b*x + c)^3*cosh(-a + c) - 3*cosh(b*x + c)*cosh(-a + c))*sinh(-a +
 c))*sinh(b*x + c) - 2*(cosh(b*x + c)^4*cosh(-a + c) - 3*cosh(b*x + c)^2*co
sh(-a + c))*sinh(-a + c) - 1)/(b*cosh(b*x + c)^3*cosh(-a + c) + (b*cosh(-a
+ c) - b*sinh(-a + c))*sinh(b*x + c)^3 - b*cosh(b*x + c)*cosh(-a + c) + 3*(
b*cosh(b*x + c)*cosh(-a + c) - b*cosh(b*x + c)*sinh(-a + c))*sinh(b*x + c)^
2 + (3*b*cosh(b*x + c)^2*cosh(-a + c) - b*cosh(-a + c) - (3*b*cosh(b*x + c)
^2 - b)*sinh(-a + c))*sinh(b*x + c) - (b*cosh(b*x + c)^3 - b*cosh(b*x + c))
*sinh(-a + c))
```

giac [B] time = 0.14, size = 121, normalized size = 2.63

$$-\frac{\left(e^{(2\,a+c)}+e^{(3\,c)}\right)e^{(-a-2\,c)}\log\left(e^{(bx+c)}+1\right)-\left(e^{(2\,a+c)}+e^{(3\,c)}\right)e^{(-a-2\,c)}\log\left(\left|e^{(bx+c)}-1\right|\right)+\frac{\left(2\,e^{(2\,bx+2\,a)}-3\,e^{(2\,bx+2\,c)}+1\right)e^{(-a)}}{e^{(3\,bx+2\,c)}-e^{(bx)}}-}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(b*x+c)^2*sinh(b*x+a),x, algorithm="giac")
```

```
[Out] -1/2*((e^(2*a + c) + e^(3*c))*e^(-a - 2*c)*log(e^(b*x + c) + 1) - (e^(2*a +
```

```
 c) + e^(3*c))*e^(-a - 2*c)*log(abs(e^(b*x + c) - 1)) + (2*e^(2*b*x + 2*a)
- 3*e^(2*b*x + 2*c) + 1)*e^(-a)/(e^(3*b*x + 2*c) - e^(b*x)) - e^(b*x + a))/
b
```

maple [B] time = 0.23, size = 197, normalized size = 4.28

$$\frac{e^{bx+a}}{2b}+\frac{e^{-bx-a}}{2b}+\frac{e^{bx+a}\left(e^{2a}-e^{2c}\right)}{b\left(-e^{2bx+2a+2c}+e^{2a}\right)}+\frac{\ln\left(e^{bx+a}-e^{a-c}\right)e^{-a-c}e^{2a}}{2b}+\frac{\ln\left(e^{bx+a}-e^{a-c}\right)e^{-a-c}e^{2c}}{2b}-\frac{\ln\left(e^{bx+a}+e^{a-c}\right)e^{-}}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(b*x+c)^2*sinh(b*x+a),x)
```

```
[Out] 1/2*exp(b*x+a)/b+1/2*exp(-b*x-a)/b+1/b*exp(b*x+a)*(exp(2*a)-exp(2*c))/(-exp
(2*b*x+2*a+2*c)+exp(2*a))+1/2*ln(exp(b*x+a)-exp(a-c))/b*exp(-a-c)*exp(2*a)+
1/2*ln(exp(b*x+a)-exp(a-c))/b*exp(-a-c)*exp(2*c)-1/2*ln(exp(b*x+a)+exp(a-c)
)/b*exp(-a-c)*exp(2*a)-1/2*ln(exp(b*x+a)+exp(a-c))/b*exp(-a-c)*exp(2*c)
```

maxima [B] time = 0.33, size = 140, normalized size = 3.04

$$-\frac{\left(e^{(2a)}+e^{(2c)}\right)e^{(-a-c)}\log\left(e^{(-bx)}+e^{c}\right)}{2b}+\frac{\left(e^{(2a)}+e^{(2c)}\right)e^{(-a-c)}\log\left(e^{(-bx)}-e^{c}\right)}{2b}+\frac{e^{(-bx-a)}}{2b}-\frac{\left(3e^{(2a)}-2e^{(2c)}\right)e^{(-2bx-2a}}{2b\left(e^{(-bx-a+2c)}-e^{(-3bx}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(b*x+c)^2*sinh(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/2*(e^(2*a) + e^(2*c))*e^(-a - c)*log(e^(-b*x) + e^c)/b + 1/2*(e^(2*a) +
e^(2*c))*e^(-a - c)*log(e^(-b*x) - e^c)/b + 1/2*e^(-b*x - a)/b - 1/2*((3*e^
(2*a) - 2*e^(2*c))*e^(-2*b*x - 2*a) - e^(2*c))/(b*(e^(-b*x - a + 2*c) - e^(
-3*b*x - a)))
```

mupad [B] time = 1.59, size = 181, normalized size = 3.93

$$\frac{e^{a+bx}}{2b}+\frac{e^{-a-bx}}{2b}-\frac{\mathrm{atan}\left(\frac{e^{-a}e^{2c}e^{bx}\left(\sqrt{-b^2}+e^{2a}e^{-2c}\sqrt{-b^2}\right)}{b\sqrt{e^{-2a}e^{2c}\left(2e^{2a}e^{-2c}+e^{4a}e^{-4c}+1\right)}}\right)\sqrt{e^{2c-2a}\left(2e^{2a-2c}+e^{4a-4c}+1\right)}}{\sqrt{-b^2}}+\frac{e^{a+bx}\left(e^{2a-2c}-1\right)}{b\left(e^{2a-2c}-e^{2a+2bx}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(c + b*x)^2*sinh(a + b*x),x)
```

```
[Out] exp(a + b*x)/(2*b) + exp(- a - b*x)/(2*b) - (atan((exp(-a)*exp(2*c)*exp(b*x
)*((-b^2)^(1/2) + exp(2*a)*exp(-2*c)*(-b^2)^(1/2)))/(b*(exp(-2*a)*exp(2*c)*
(2*exp(2*a)*exp(-2*c) + exp(4*a)*exp(-4*c) + 1))^(1/2)))*(exp(2*c - 2*a)*(2
```

```
*exp(2*a - 2*c) + exp(4*a - 4*c) + 1))^(1/2))/(-b^2)^(1/2) + (exp(a + b*x)*
(exp(2*a - 2*c) - 1))/(b*(exp(2*a - 2*c) - exp(2*a + 2*b*x)))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sinh(a+bx)\coth^2(bx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(b*x+c)**2*sinh(b*x+a),x)
```

```
[Out] Integral(sinh(a + b*x)*coth(b*x + c)**2, x)
```

3.148 $\int \coth^3(c+bx)\sinh(a+bx)\,dx$

Optimal. Leaf size=73

$$-\frac{\cosh(a-c)\text{csch}(bx+c)}{b}-\frac{3\sinh(a-c)\tanh^{-1}(\cosh(bx+c))}{2b}-\frac{\sinh(a-c)\coth(bx+c)\text{csch}(bx+c)}{2b}+\frac{\sinh(a-}{b}$$

```
[Out] -cosh(a-c)*csch(b*x+c)/b-3/2*arctanh(cosh(b*x+c))*sinh(a-c)/b-1/2*coth(b*x+
c)*csch(b*x+c)*sinh(a-c)/b+sinh(b*x+a)/b
```

Rubi [**A**] time = 0.09, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.467, Rules used = {5622, 5621, 2637, 3770, 2606, 8, 2611}

$$-\frac{\cosh(a-c)\text{csch}(bx+c)}{b}-\frac{3\sinh(a-c)\tanh^{-1}(\cosh(bx+c))}{2b}-\frac{\sinh(a-c)\coth(bx+c)\text{csch}(bx+c)}{2b}+\frac{\sinh(a-}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[c + b*x]^3*Sinh[a + b*x],x]
```

```
[Out] -((Cosh[a - c]*Csch[c + b*x])/b) - (3*ArcTanh[Cosh[c + b*x]]*Sinh[a - c])/(
2*b) - (Coth[c + b*x]*Csch[c + b*x]*Sinh[a - c])/(2*b) + Sinh[a + b*x]/b
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 5621

```
Int[Cosh[v_]*Coth[w_]^(n_.), x_Symbol] :> Int[Sinh[v]*Coth[w]^(n - 1), x] +
 Dist[Cosh[v - w], Int[Csch[w]*Coth[w]^(n - 1), x], x] /; GtQ[n, 0] && NeQ[
w, v] && FreeQ[v - w, x]
```

Rule 5622

```
Int[Coth[w_]^(n_.)*Sinh[v_], x_Symbol] :> Int[Cosh[v]*Coth[w]^(n - 1), x] +
 Dist[Sinh[v - w], Int[Csch[w]*Coth[w]^(n - 1), x], x] /; GtQ[n, 0] && NeQ[
w, v] && FreeQ[v - w, x]
```

Rubi steps

$$\int \coth^3(c+bx)\sinh(a+bx)\,dx = \sinh(a-c)\int \coth^2(c+bx)\text{csch}(c+bx)\,dx + \int \cosh(a+bx)\coth^2(c+bx)\,$$
$$= -\frac{\coth(c+bx)\text{csch}(c+bx)\sinh(a-c)}{2b} + \cosh(a-c)\int \coth(c+bx)\text{csch}(c+$$
$$= -\frac{\tanh^{-1}(\cosh(c+bx))\sinh(a-c)}{2b} - \frac{\coth(c+bx)\text{csch}(c+bx)\sinh(a-c)}{2b} -$$
$$= -\frac{\cosh(a-c)\text{csch}(c+bx)}{b} - \frac{3\tanh^{-1}(\cosh(c+bx))\sinh(a-c)}{2b} - \frac{\coth(c+b}{}$$

Mathematica [A] time = 0.36, size = 70, normalized size = 0.96

$$\frac{\text{csch}^2(bx+c)(2\sinh(a-bx-2c)+\sinh(a+3bx+2c)-5\sinh(a+bx))-12\sinh(a-c)\tanh^{-1}\left(\sinh(c)\tanh\left(\right.\right.}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[c + b*x]^3*Sinh[a + b*x],x]
```

```
[Out] (-12*ArcTanh[Cosh[c] + Sinh[c]*Tanh[(b*x)/2]]*Sinh[a - c] + Csch[c + b*x]^2
*(2*Sinh[a - 2*c - b*x] - 5*Sinh[a + b*x] + Sinh[a + 2*c + 3*b*x]))/(4*b)
```

fricas [B] time = 0.46, size = 2372, normalized size = 32.49

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(b*x+c)^3*sinh(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/4*(2*cosh(b*x + c)^6*cosh(-a + c)^2 + 2*(cosh(-a + c)^2 - 2*cosh(-a + c)*
sinh(-a + c) + sinh(-a + c)^2)*sinh(b*x + c)^6 + 12*(cosh(b*x + c)*cosh(-a
+ c)^2 - 2*cosh(b*x + c)*cosh(-a + c)*sinh(-a + c) + cosh(b*x + c)*sinh(-a
+ c)^2)*sinh(b*x + c)^5 - 2*(5*cosh(-a + c)^2 + 2)*cosh(b*x + c)^4 + 2*(15*
cosh(b*x + c)^2*cosh(-a + c)^2 + 5*(3*cosh(b*x + c)^2 - 1)*sinh(-a + c)^2 -
 5*cosh(-a + c)^2 - 10*(3*cosh(b*x + c)^2*cosh(-a + c) - cosh(-a + c))*sinh
(-a + c) - 2)*sinh(b*x + c)^4 + 8*(5*cosh(b*x + c)^3*cosh(-a + c)^2 + 5*(co
sh(b*x + c)^3 - cosh(b*x + c))*sinh(-a + c)^2 - (5*cosh(-a + c)^2 + 2)*cosh
(b*x + c) - 10*(cosh(b*x + c)^3*cosh(-a + c) - cosh(b*x + c)*cosh(-a + c))*
sinh(-a + c))*sinh(b*x + c)^3 + 2*(2*cosh(-a + c)^2 + 5)*cosh(b*x + c)^2 +
2*(15*cosh(b*x + c)^4*cosh(-a + c)^2 - 6*(5*cosh(-a + c)^2 + 2)*cosh(b*x +
c)^2 + (15*cosh(b*x + c)^4 - 30*cosh(b*x + c)^2 + 2)*sinh(-a + c)^2 + 2*cos
h(-a + c)^2 - 2*(15*cosh(b*x + c)^4*cosh(-a + c) - 30*cosh(b*x + c)^2*cosh(
-a + c) + 2*cosh(-a + c))*sinh(-a + c) + 5)*sinh(b*x + c)^2 + 2*(cosh(b*x +
 c)^6 - 5*cosh(b*x + c)^4 + 2*cosh(b*x + c)^2)*sinh(-a + c)^2 - 3*((cosh(-a
 + c)^2 - 1)*cosh(b*x + c)^5 + (cosh(-a + c)^2 - 2*cosh(-a + c)*sinh(-a + c
) + sinh(-a + c)^2 - 1)*sinh(b*x + c)^5 - 5*(2*cosh(b*x + c)*cosh(-a + c)*s
inh(-a + c) - cosh(b*x + c)*sinh(-a + c)^2 - (cosh(-a + c)^2 - 1)*cosh(b*x
+ c))*sinh(b*x + c)^4 - 2*(cosh(-a + c)^2 - 1)*cosh(b*x + c)^3 + 2*(5*(cosh
(-a + c)^2 - 1)*cosh(b*x + c)^2 + (5*cosh(b*x + c)^2 - 1)*sinh(-a + c)^2 -
cosh(-a + c)^2 - 2*(5*cosh(b*x + c)^2*cosh(-a + c) - cosh(-a + c))*sinh(-a
+ c) + 1)*sinh(b*x + c)^3 + 2*(5*(cosh(-a + c)^2 - 1)*cosh(b*x + c)^3 + (5*
cosh(b*x + c)^3 - 3*cosh(b*x + c))*sinh(-a + c)^2 - 3*(cosh(-a + c)^2 - 1)*
cosh(b*x + c) - 2*(5*cosh(b*x + c)^3*cosh(-a + c) - 3*cosh(b*x + c)*cosh(-a
 + c))*sinh(-a + c))*sinh(b*x + c)^2 + (cosh(b*x + c)^5 - 2*cosh(b*x + c)^3
 + cosh(b*x + c))*sinh(-a + c)^2 + (cosh(-a + c)^2 - 1)*cosh(b*x + c) + (5*
(cosh(-a + c)^2 - 1)*cosh(b*x + c)^4 - 6*(cosh(-a + c)^2 - 1)*cosh(b*x + c)
^2 + (5*cosh(b*x + c)^4 - 6*cosh(b*x + c)^2 + 1)*sinh(-a + c)^2 + cosh(-a +
 c)^2 - 2*(5*cosh(b*x + c)^4*cosh(-a + c) - 6*cosh(b*x + c)^2*cosh(-a + c)
+ cosh(-a + c))*sinh(-a + c) - 1)*sinh(b*x + c) - 2*(cosh(b*x + c)^5*cosh(-
a + c) - 2*cosh(b*x + c)^3*cosh(-a + c) + cosh(b*x + c)*cosh(-a + c))*sinh(
-a + c))*log(cosh(b*x + c) + sinh(b*x + c) + 1) + 3*((cosh(-a + c)^2 - 1)*c
osh(b*x + c)^5 + (cosh(-a + c)^2 - 2*cosh(-a + c)*sinh(-a + c) + sinh(-a +
c)^2 - 1)*sinh(b*x + c)^5 - 5*(2*cosh(b*x + c)*cosh(-a + c)*sinh(-a + c) -
cosh(b*x + c)*sinh(-a + c)^2 - (cosh(-a + c)^2 - 1)*cosh(b*x + c))*sinh(b*x
 + c)^4 - 2*(cosh(-a + c)^2 - 1)*cosh(b*x + c)^3 + 2*(5*(cosh(-a + c)^2 - 1
)*cosh(b*x + c)^2 + (5*cosh(b*x + c)^2 - 1)*sinh(-a + c)^2 - cosh(-a + c)^2
```

```
 - 2*(5*cosh(b*x + c)^2*cosh(-a + c) - cosh(-a + c))*sinh(-a + c) + 1)*sinh
(b*x + c)^3 + 2*(5*(cosh(-a + c)^2 - 1)*cosh(b*x + c)^3 + (5*cosh(b*x + c)^
3 - 3*cosh(b*x + c))*sinh(-a + c)^2 - 3*(cosh(-a + c)^2 - 1)*cosh(b*x + c)
- 2*(5*cosh(b*x + c)^3*cosh(-a + c) - 3*cosh(b*x + c)*cosh(-a + c))*sinh(-a
 + c))*sinh(b*x + c)^2 + (cosh(b*x + c)^5 - 2*cosh(b*x + c)^3 + cosh(b*x +
c))*sinh(-a + c)^2 + (cosh(-a + c)^2 - 1)*cosh(b*x + c) + (5*(cosh(-a + c)^
2 - 1)*cosh(b*x + c)^4 - 6*(cosh(-a + c)^2 - 1)*cosh(b*x + c)^2 + (5*cosh(b
*x + c)^4 - 6*cosh(b*x + c)^2 + 1)*sinh(-a + c)^2 + cosh(-a + c)^2 - 2*(5*c
osh(b*x + c)^4*cosh(-a + c) - 6*cosh(b*x + c)^2*cosh(-a + c) + cosh(-a + c)
)*sinh(-a + c) - 1)*sinh(b*x + c) - 2*(cosh(b*x + c)^5*cosh(-a + c) - 2*cos
h(b*x + c)^3*cosh(-a + c) + cosh(b*x + c)*cosh(-a + c))*sinh(-a + c))*log(c
osh(b*x + c) + sinh(b*x + c) - 1) + 4*(3*cosh(b*x + c)^5*cosh(-a + c)^2 - 2
*(5*cosh(-a + c)^2 + 2)*cosh(b*x + c)^3 + (3*cosh(b*x + c)^5 - 10*cosh(b*x
+ c)^3 + 2*cosh(b*x + c))*sinh(-a + c)^2 + (2*cosh(-a + c)^2 + 5)*cosh(b*x
+ c) - 2*(3*cosh(b*x + c)^5*cosh(-a + c) - 10*cosh(b*x + c)^3*cosh(-a + c)
+ 2*cosh(b*x + c)*cosh(-a + c))*sinh(-a + c))*sinh(b*x + c) - 4*(cosh(b*x +
 c)^6*cosh(-a + c) - 5*cosh(b*x + c)^4*cosh(-a + c) + 2*cosh(b*x + c)^2*cos
h(-a + c))*sinh(-a + c) - 2)/(b*cosh(b*x + c)^5*cosh(-a + c) + (b*cosh(-a +
 c) - b*sinh(-a + c))*sinh(b*x + c)^5 - 2*b*cosh(b*x + c)^3*cosh(-a + c) +
5*(b*cosh(b*x + c)*cosh(-a + c) - b*cosh(b*x + c)*sinh(-a + c))*sinh(b*x +
c)^4 + 2*(5*b*cosh(b*x + c)^2*cosh(-a + c) - b*cosh(-a + c) - (5*b*cosh(b*x
 + c)^2 - b)*sinh(-a + c))*sinh(b*x + c)^3 + b*cosh(b*x + c)*cosh(-a + c) +
 2*(5*b*cosh(b*x + c)^3*cosh(-a + c) - 3*b*cosh(b*x + c)*cosh(-a + c) - (5*
b*cosh(b*x + c)^3 - 3*b*cosh(b*x + c))*sinh(-a + c))*sinh(b*x + c)^2 + (5*b
*cosh(b*x + c)^4*cosh(-a + c) - 6*b*cosh(b*x + c)^2*cosh(-a + c) + b*cosh(-
a + c) - (5*b*cosh(b*x + c)^4 - 6*b*cosh(b*x + c)^2 + b)*sinh(-a + c))*sinh
(b*x + c) - (b*cosh(b*x + c)^5 - 2*b*cosh(b*x + c)^3 + b*cosh(b*x + c))*sin
h(-a + c))
```

giac [B] time = 0.16, size = 153, normalized size = 2.10

$$-\frac{3\left(e^{(2a+c)}-e^{(3c)}\right)e^{(-a-2c)}\log\left(e^{(bx+c)}+1\right)-3\left(e^{(2a+c)}-e^{(3c)}\right)e^{(-a-2c)}\log\left(\left|e^{(bx+c)}-1\right|\right)+\frac{2\left(3e^{(3bx+2a+2c)}+e^{(3bx+4c)}-e^{(b}\right.}{\left(e^{(2bx+2c)}-1\right.}}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(b*x+c)^3*sinh(b*x+a),x, algorithm="giac")
```

```
[Out] -1/4*(3*(e^(2*a + c) - e^(3*c))*e^(-a - 2*c)*log(e^(b*x + c) + 1) - 3*(e^(2
*a + c) - e^(3*c))*e^(-a - 2*c)*log(abs(e^(b*x + c) - 1)) + 2*(3*e^(3*b*x +
 2*a + 2*c) + e^(3*b*x + 4*c) - e^(b*x + 2*a) - 3*e^(b*x + 2*c))*e^(-a)/(e^
(2*b*x + 2*c) - 1)^2 - 2*e^(b*x + a) + 2*e^(-b*x - a))/b
```

maple [B] time = 0.28, size = 230, normalized size = 3.15

$$\frac{e^{bx+a}}{2b}-\frac{e^{-bx-a}}{2b}+\frac{e^{bx+a}\left(-3\,e^{2bx+4a+2c}-e^{2bx+2a+4c}+e^{4a}+3\,e^{2a+2c}\right)}{2b\left(-e^{2bx+2a+2c}+e^{2a}\right)^{2}}-\frac{3\ln\left(e^{bx+a}+e^{a-c}\right)e^{-a-c}e^{2a}}{4b}+\frac{3\ln\left(e^{bx+a}+e^{a}\right.}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(b*x+c)^3*sinh(b*x+a),x)
```

```
[Out] 1/2*exp(b*x+a)/b-1/2*exp(-b*x-a)/b+1/2*exp(b*x+a)*(-3*exp(2*b*x+4*a+2*c)-ex
p(2*b*x+2*a+4*c)+exp(4*a)+3*exp(2*a+2*c))/b/(-exp(2*b*x+2*a+2*c)+exp(2*a))^
2-3/4*ln(exp(b*x+a)+exp(a-c))/b*exp(-a-c)*exp(2*a)+3/4*ln(exp(b*x+a)+exp(a-
c))/b*exp(-a-c)*exp(2*c)+3/4*ln(exp(b*x+a)-exp(a-c))/b*exp(-a-c)*exp(2*a)-3
/4*ln(exp(b*x+a)-exp(a-c))/b*exp(-a-c)*exp(2*c)
```

maxima [B] time = 0.33, size = 186, normalized size = 2.55

$$-\frac{3\left(e^{(2a)}-e^{(2c)}\right)e^{(-a-c)}\log\left(e^{(-bx)}+e^{c}\right)}{4b}+\frac{3\left(e^{(2a)}-e^{(2c)}\right)e^{(-a-c)}\log\left(e^{(-bx)}-e^{c}\right)}{4b}-\frac{e^{(-bx-a)}}{2b}-\frac{\left(5\,e^{(2a+2c)}+e^{(4c)}\right)e^{(-2}}{2b\left(e^{(-bx-a}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(b*x+c)^3*sinh(b*x+a),x, algorithm="maxima")
```

```
[Out] -3/4*(e^(2*a) - e^(2*c))*e^(-a - c)*log(e^(-b*x) + e^c)/b + 3/4*(e^(2*a) -
e^(2*c))*e^(-a - c)*log(e^(-b*x) - e^c)/b - 1/2*e^(-b*x - a)/b - 1/2*((5*e^
(2*a + 2*c) + e^(4*c))*e^(-2*b*x - 2*a) - (2*e^(4*a) + 3*e^(2*a + 2*c))*e^(
-4*b*x - 4*a) - e^(4*c))/(b*(e^(-b*x - a + 4*c) - 2*e^(-3*b*x - a + 2*c) +
e^(-5*b*x - a)))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \coth\left(c+b\,x\right)^3\,\sinh\left(a+b\,x\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(c + b*x)^3*sinh(a + b*x),x)
```

```
[Out] int(coth(c + b*x)^3*sinh(a + b*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sinh\left(a+bx\right)\coth^{3}\left(bx+c\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(b*x+c)**3*sinh(b*x+a),x)
[Out] Integral(sinh(a + b*x)*coth(b*x + c)**3, x)
```

3.149 $\int \operatorname{sech}(c+bx)\sinh(a+bx)\,dx$

Optimal. Leaf size=26

$$\frac{\cosh(a-c)\log(\cosh(bx+c))}{b}+x\sinh(a-c)$$

```
[Out] cosh(a-c)*ln(cosh(b*x+c))/b+x*sinh(a-c)
```

Rubi [A] time = 0.02, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {5624, 3475, 8}

$$\frac{\cosh(a-c)\log(\cosh(bx+c))}{b}+x\sinh(a-c)$$

Antiderivative was successfully verified.

```
[In] Int[Sech[c + b*x]*Sinh[a + b*x],x]
```

```
[Out] (Cosh[a - c]*Log[Cosh[c + b*x]])/b + x*Sinh[a - c]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 5624

```
Int[Sech[w_]^(n_.)*Sinh[v_], x_Symbol] :> Dist[Cosh[v - w], Int[Tanh[w]*Sec
h[w]^(n - 1), x], x] + Dist[Sinh[v - w], Int[Sech[w]^(n - 1), x], x] /; GtQ
[n, 0] && NeQ[w, v] && FreeQ[v - w, x]
```

Rubi steps

$$\begin{aligned}\int \operatorname{sech}(c+bx)\sinh(a+bx)\,dx &= \cosh(a-c)\int \tanh(c+bx)\,dx+\sinh(a-c)\int 1\,dx\\ &= \frac{\cosh(a-c)\log(\cosh(c+bx))}{b}+x\sinh(a-c)\end{aligned}$$

Mathematica [A] time = 0.13, size = 26, normalized size = 1.00

$$\frac{\cosh(a-c)\log(\cosh(bx+c))}{b}+x\sinh(a-c)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[c + b*x]*Sinh[a + b*x],x]
```

```
[Out] (Cosh[a - c]*Log[Cosh[c + b*x]])/b + x*Sinh[a - c]
```

fricas [B] time = 0.43, size = 87, normalized size = 3.35

$$-\frac{2\,bx-\left(\cosh\left(-a+c\right)^{2}-2\,\cosh\left(-a+c\right)\sinh\left(-a+c\right)+\sinh\left(-a+c\right)^{2}+1\right)\log\left(\frac{2\,\cosh(bx+c)}{\cosh(bx+c)-\sinh(bx+c)}\right)}{2\left(b\cosh\left(-a+c\right)-b\sinh\left(-a+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+c)*sinh(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/2*(2*b*x - (cosh(-a + c)^2 - 2*cosh(-a + c)*sinh(-a + c) + sinh(-a + c)^
2 + 1)*log(2*cosh(b*x + c)/(cosh(b*x + c) - sinh(b*x + c))))/(b*cosh(-a + c
) - b*sinh(-a + c))
```

giac [A] time = 0.11, size = 49, normalized size = 1.88

$$-\frac{2\,bxe^{(-a+c)}-\left(e^{(2\,a+c)}+e^{(3\,c)}\right)e^{(-a-2\,c)}\log\left(e^{(2\,bx+2\,c)}+1\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+c)*sinh(b*x+a),x, algorithm="giac")
```

```
[Out] -1/2*(2*b*x*e^(-a + c) - (e^(2*a + c) + e^(3*c))*e^(-a - 2*c)*log(e^(2*b*x
+ 2*c) + 1))/b
```

maple [B] time = 0.14, size = 148, normalized size = 5.69

$$x\,e^{a-c}-e^{-a-c}e^{2a}x-e^{-a-c}e^{2c}x-\frac{e^{-a-c}e^{2a}a}{b}-\frac{e^{-a-c}e^{2c}a}{b}+\frac{\ln\left(e^{2bx+2a}+e^{2a-2c}\right)e^{-a-c}e^{2a}}{2b}+\frac{\ln\left(e^{2bx+2a}+e^{2a-2c}\right)e^{-a-c}e^{2c}}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(b*x+c)*sinh(b*x+a),x)
```

```
[Out] x*exp(a-c)-exp(-a-c)*exp(2*a)*x-exp(-a-c)*exp(2*c)*x-1/b*exp(-a-c)*exp(2*a)
*a-1/b*exp(-a-c)*exp(2*c)*a+1/2*ln(exp(2*b*x+2*a)+exp(2*a-2*c))/b*exp(-a-c)
*exp(2*a)+1/2*ln(exp(2*b*x+2*a)+exp(2*a-2*c))/b*exp(-a-c)*exp(2*c)
```

maxima [A] time = 0.32, size = 49, normalized size = 1.88

$$\frac{\left(e^{(2a)} + e^{(2c)}\right)e^{(-a-c)}\log\left(e^{(-2bx)} + e^{(2c)}\right)}{2b} + \frac{(bx+a)e^{(a-c)}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+c)*sinh(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/2*(e^(2*a) + e^(2*c))*e^(-a - c)*log(e^(-2*b*x) + e^(2*c))/b + (b*x + a)*
e^(a - c)/b
```

mupad [B] time = 0.22, size = 65, normalized size = 2.50

$$\frac{e^{2c-2a}\ln\left(e^{2a}\,e^{2bx} + e^{2a}\,e^{-2c}\right)\left(2b\,e^{3a-3c} + 2b\,e^{a-c}\right)}{4b^2} - x\,e^{c-a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a + b*x)/cosh(c + b*x),x)
```

```
[Out] (exp(2*c - 2*a)*log(exp(2*a)*exp(2*b*x) + exp(2*a)*exp(-2*c))*(2*b*exp(3*a
- 3*c) + 2*b*exp(a - c)))/(4*b^2) - x*exp(c - a)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sinh(a+bx)\operatorname{sech}(bx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+c)*sinh(b*x+a),x)
```

```
[Out] Integral(sinh(a + b*x)*sech(b*x + c), x)
```

3.150 $\int \operatorname{sech}^2(c+bx)\sinh(a+bx)\,dx$

Optimal. Leaf size=35

$$\frac{\sinh(a-c)\tan^{-1}(\sinh(bx+c))}{b}-\frac{\cosh(a-c)\operatorname{sech}(bx+c)}{b}$$

```
[Out] -cosh(a-c)*sech(b*x+c)/b+arctan(sinh(b*x+c))*sinh(a-c)/b
```

Rubi [A] time = 0.03, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {5624, 2606, 8, 3770}

$$\frac{\sinh(a-c)\tan^{-1}(\sinh(bx+c))}{b}-\frac{\cosh(a-c)\operatorname{sech}(bx+c)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[c + b*x]^2*Sinh[a + b*x],x]
```

```
[Out] -((Cosh[a - c]*Sech[c + b*x])/b) + (ArcTan[Sinh[c + b*x]]*Sinh[a - c])/b
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 5624

```
Int[Sech[w_]^(n_.)*Sinh[v_], x_Symbol] :> Dist[Cosh[v - w], Int[Tanh[w]*Sec
h[w]^(n - 1), x], x] + Dist[Sinh[v - w], Int[Sech[w]^(n - 1), x], x] /; GtQ
[n, 0] && NeQ[w, v] && FreeQ[v - w, x]
```

Rubi steps

$$\int \operatorname{sech}^2(c+bx)\sinh(a+bx)\,dx = \cosh(a-c)\int \operatorname{sech}(c+bx)\tanh(c+bx)\,dx + \sinh(a-c)\int \operatorname{sech}(c+bx)\,dx$$
$$= \frac{\tan^{-1}(\sinh(c+bx))\sinh(a-c)}{b} - \frac{\cosh(a-c)\operatorname{Subst}(\int 1\,dx, x, \operatorname{sech}(c+bx))}{b}$$
$$= -\frac{\cosh(a-c)\operatorname{sech}(c+bx)}{b} + \frac{\tan^{-1}(\sinh(c+bx))\sinh(a-c)}{b}$$

Mathematica [B] time = 0.09, size = 83, normalized size = 2.37

$$\frac{2\sinh(a-c)\tan^{-1}\left(\frac{(\cosh(c)-\sinh(c))\left(\sinh(c)\cosh\left(\frac{bx}{2}\right)+\cosh(c)\sinh\left(\frac{bx}{2}\right)\right)}{\cosh(c)\cosh\left(\frac{bx}{2}\right)-\sinh(c)\cosh\left(\frac{bx}{2}\right)}\right)}{b} - \frac{\cosh(a-c)\operatorname{sech}(bx+c)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[c + b*x]^2*Sinh[a + b*x],x]
```

```
[Out] -((Cosh[a - c]*Sech[c + b*x])/b) + (2*ArcTan[((Cosh[c] - Sinh[c])*(Cosh[(b*
x)/2]*Sinh[c] + Cosh[c]*Sinh[(b*x)/2]))/(Cosh[c]*Cosh[(b*x)/2] - Cosh[(b*x)
/2]*Sinh[c])]*Sinh[a - c])/b
```

fricas [B] time = 0.47, size = 405, normalized size = 11.57

$$\frac{2\cosh(bx+c)\cosh(-a+c)\sinh(-a+c) - \cosh(bx+c)\sinh(-a+c)^2 + \left(\left(\cosh(-a+c)^2-1\right)\cosh(bx+c)\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+c)^2*sinh(b*x+a),x, algorithm="fricas")
```

```
[Out] (2*cosh(b*x + c)*cosh(-a + c)*sinh(-a + c) - cosh(b*x + c)*sinh(-a + c)^2 +
 ((cosh(-a + c)^2 - 1)*cosh(b*x + c)^2 + (cosh(-a + c)^2 - 2*cosh(-a + c)*s
inh(-a + c) + sinh(-a + c)^2 - 1)*sinh(b*x + c)^2 + (cosh(b*x + c)^2 + 1)*s
inh(-a + c)^2 + cosh(-a + c)^2 - 2*(2*cosh(b*x + c)*cosh(-a + c)*sinh(-a +
c) - cosh(b*x + c)*sinh(-a + c)^2 - (cosh(-a + c)^2 - 1)*cosh(b*x + c))*sin
h(b*x + c) - 2*(cosh(b*x + c)^2*cosh(-a + c) + cosh(-a + c))*sinh(-a + c) -
 1)*arctan(cosh(b*x + c) + sinh(b*x + c)) - (cosh(-a + c)^2 + 1)*cosh(b*x +
 c) - (cosh(-a + c)^2 - 2*cosh(-a + c)*sinh(-a + c) + sinh(-a + c)^2 + 1)*s
inh(b*x + c))/(b*cosh(b*x + c)^2*cosh(-a + c) + (b*cosh(-a + c) - b*sinh(-a
 + c))*sinh(b*x + c)^2 + b*cosh(-a + c) + 2*(b*cosh(b*x + c)*cosh(-a + c) -
 b*cosh(b*x + c)*sinh(-a + c))*sinh(b*x + c) - (b*cosh(b*x + c)^2 + b)*sinh
(-a + c))
```

giac [A] time = 0.12, size = 68, normalized size = 1.94

$$\frac{\left(e^{(2a)}-e^{(2c)}\right)\arctan\left(e^{(bx+c)}\right)e^{(-a-c)}-\frac{\left(e^{(bx+2a)}+e^{(bx+2c)}\right)e^{(-a)}}{e^{(2bx+2c)}+1}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+c)^2*sinh(b*x+a),x, algorithm="giac")
```

```
[Out] ((e^(2*a) - e^(2*c))*arctan(e^(b*x + c))*e^(-a - c) - (e^(b*x + 2*a) + e^(b
*x + 2*c))*e^(-a)/(e^(2*b*x + 2*c) + 1))/b
```

maple [C] time = 0.26, size = 181, normalized size = 5.17

$$-\frac{e^{bx+a}\left(e^{2a}+e^{2c}\right)}{b\left(e^{2bx+2a+2c}+e^{2a}\right)}+\frac{i\ln\left(e^{bx+a}+ie^{a-c}\right)e^{-a-c}e^{2a}}{2b}-\frac{i\ln\left(e^{bx+a}+ie^{a-c}\right)e^{-a-c}e^{2c}}{2b}-\frac{i\ln\left(e^{bx+a}-ie^{a-c}\right)e^{-a-c}e^{2a}}{2b}+\frac{i\ln}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(b*x+c)^2*sinh(b*x+a),x)
```

```
[Out] -1/b*exp(b*x+a)*(exp(2*a)+exp(2*c))/(exp(2*b*x+2*a+2*c)+exp(2*a))+1/2*I*ln(
exp(b*x+a)+I*exp(a-c))/b*exp(-a-c)*exp(a)^2-1/2*I*ln(exp(b*x+a)+I*exp(a-c))
/b*exp(-a-c)*exp(c)^2-1/2*I*ln(exp(b*x+a)-I*exp(a-c))/b*exp(-a-c)*exp(a)^2+
1/2*I*ln(exp(b*x+a)-I*exp(a-c))/b*exp(-a-c)*exp(c)^2
```

maxima [A] time = 0.41, size = 70, normalized size = 2.00

$$-\frac{\left(e^{(2a)}-e^{(2c)}\right)\arctan\left(e^{(-bx-c)}\right)e^{(-a-c)}}{b}-\frac{\left(e^{(2a)}+e^{(2c)}\right)e^{(-bx-a)}}{b\left(e^{(-2bx)}+e^{(2c)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+c)^2*sinh(b*x+a),x, algorithm="maxima")
```

```
[Out] -(e^(2*a) - e^(2*c))*arctan(e^(-b*x - c))*e^(-a - c)/b - (e^(2*a) + e^(2*c)
)*e^(-b*x - a)/(b*(e^(-2*b*x) + e^(2*c)))
```

mupad [B] time = 1.59, size = 150, normalized size = 4.29

$$-\frac{\mathrm{atan}\left(\frac{e^{-a}\,e^{2c}\,e^{bx}\left(\sqrt{b^2}-e^{2a}\,e^{-2c}\sqrt{b^2}\right)}{b\sqrt{e^{-2a}\,e^{2c}\left(e^{4a}\,e^{-4c}-2\,e^{2a}\,e^{-2c}+1\right)}}\right)\sqrt{e^{2c-2a}\left(e^{4a-4c}-2\,e^{2a-2c}+1\right)}}{\sqrt{b^2}}-\frac{e^{a+bx}\left(e^{2a-2c}+1\right)}{b\left(e^{2a-2c}+e^{2a+2bx}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a + b*x)/cosh(c + b*x)^2,x)
```

```
[Out] - (atan((exp(-a)*exp(2*c)*exp(b*x)*((b^2)^(1/2) - exp(2*a)*exp(-2*c)*(b^2)^
(1/2)))/(b*(exp(-2*a)*exp(2*c)*(exp(4*a)*exp(-4*c) - 2*exp(2*a)*exp(-2*c) +
 1))^(1/2)))*(exp(2*c - 2*a)*(exp(4*a - 4*c) - 2*exp(2*a - 2*c) + 1))^(1/2)
)/(b^2)^(1/2) - (exp(a + b*x)*(exp(2*a - 2*c) + 1))/(b*(exp(2*a - 2*c) + ex
p(2*a + 2*b*x)))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sinh(a+bx)\operatorname{sech}^2(bx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+c)**2*sinh(b*x+a),x)
```

```
[Out] Integral(sinh(a + b*x)*sech(b*x + c)**2, x)
```

3.151 $\int \operatorname{sech}^3(c+bx)\sinh(a+bx)\,dx$

Optimal. Leaf size=38

$$\frac{\sinh(a-c)\tanh(bx+c)}{b}-\frac{\cosh(a-c)\operatorname{sech}^2(bx+c)}{2b}$$

```
[Out] -1/2*cosh(a-c)*sech(b*x+c)^2/b+sinh(a-c)*tanh(b*x+c)/b
```

Rubi [A] time = 0.04, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5624, 2606, 30, 3767, 8}

$$\frac{\sinh(a-c)\tanh(bx+c)}{b}-\frac{\cosh(a-c)\operatorname{sech}^2(bx+c)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[c + b*x]^3*Sinh[a + b*x],x]
```

```
[Out] -(Cosh[a - c]*Sech[c + b*x]^2)/(2*b) + (Sinh[a - c]*Tanh[c + b*x])/b
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 5624

```
Int[Sech[w_]^(n_.)*Sinh[v_], x_Symbol] :> Dist[Cosh[v - w], Int[Tanh[w]*Sec
h[w]^(n - 1), x], x] + Dist[Sinh[v - w], Int[Sech[w]^(n - 1), x], x] /; GtQ
[n, 0] && NeQ[w, v] && FreeQ[v - w, x]
```

Rubi steps

$$\int \text{sech}^3(c+bx)\sinh(a+bx)\,dx = \cosh(a-c)\int \text{sech}^2(c+bx)\tanh(c+bx)\,dx + \sinh(a-c)\int \text{sech}^2(c+bx)$$
$$= -\frac{\cosh(a-c)\,\text{Subst}\left(\int x\,dx, x, \text{sech}(c+bx)\right)}{b} + \frac{(i\sinh(a-c))\,\text{Subst}\left(\int 1\,dx,\right.}{b}$$
$$= -\frac{\cosh(a-c)\text{sech}^2(c+bx)}{2b} + \frac{\sinh(a-c)\tanh(c+bx)}{b}$$

Mathematica [A] time = 0.17, size = 35, normalized size = 0.92

$$-\frac{\text{sech}(c)\text{sech}^2(bx+c)(\cosh(a)-\sinh(a-c)\sinh(2bx+c))}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[c + b*x]^3*Sinh[a + b*x],x]
[Out] -1/2*(Sech[c]*Sech[c + b*x]^2*(Cosh[a] - Sinh[a - c]*Sinh[c + 2*b*x]))/b
```

fricas [B] time = 0.42, size = 246, normalized size = 6.47

$$-\frac{}{b\cosh(bx+c)^3\cosh(-a+c)^2 + 3\,b\cosh(bx+c)\cosh(-a+c)^2 + \left(b\cosh(-a+c)^2 - b\sinh(-a+c)^2\right)\sinh}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+c)^3*sinh(b*x+a),x, algorithm="fricas")
[Out] -2*(cosh(b*x + c)*cosh(-a + c) + cosh(-a + c)*sinh(b*x + c) - 2*cosh(b*x +
c)*sinh(-a + c))/(b*cosh(b*x + c)^3*cosh(-a + c)^2 + 3*b*cosh(b*x + c)*cosh
(-a + c)^2 + (b*cosh(-a + c)^2 - b*sinh(-a + c)^2)*sinh(b*x + c)^3 + 3*(b*c
osh(b*x + c)*cosh(-a + c)^2 - b*cosh(b*x + c)*sinh(-a + c)^2)*sinh(b*x + c)
^2 - (b*cosh(b*x + c)^3 + 3*b*cosh(b*x + c))*sinh(-a + c)^2 + (3*b*cosh(b*x
 + c)^2*cosh(-a + c)^2 + b*cosh(-a + c)^2 - (3*b*cosh(b*x + c)^2 + b)*sinh(
-a + c)^2)*sinh(b*x + c))
```

giac [A] time = 0.14, size = 51, normalized size = 1.34

$$-\frac{\left(2\,e^{(2\,bx+2\,a+2\,c)} + e^{(2\,a)} - e^{(2\,c)}\right)e^{(-a-c)}}{b\left(e^{(2\,bx+2\,c)}+1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+c)^3*sinh(b*x+a),x, algorithm="giac")
```

```
[Out] -(2*e^(2*b*x + 2*a + 2*c) + e^(2*a) - e^(2*c))*e^(-a - c)/(b*(e^(2*b*x + 2*
c) + 1)^2)
```

maple [A] time = 0.18, size = 58, normalized size = 1.53

$$-\frac{\left(2\,\mathrm{e}^{2bx+2a+2c}+\mathrm{e}^{2a}-\mathrm{e}^{2c}\right)\mathrm{e}^{3a-c}}{\left(\mathrm{e}^{2bx+2a+2c}+\mathrm{e}^{2a}\right)^{2}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(b*x+c)^3*sinh(b*x+a),x)
```

```
[Out] -1/(exp(2*b*x+2*a+2*c)+exp(2*a))^2/b*(2*exp(2*b*x+2*a+2*c)+exp(2*a)-exp(2*c
))*exp(3*a-c)
```

maxima [B] time = 0.34, size = 120, normalized size = 3.16

$$-\frac{2\,e^{(-2\,bx+3\,c)}}{b\left(2\,e^{(-2\,bx+a+2\,c)}+e^{(-4\,bx+a)}+e^{(a+4\,c)}\right)}+\frac{e^{(2\,a+3\,c)}}{b\left(2\,e^{(-2\,bx+a+2\,c)}+e^{(-4\,bx+a)}+e^{(a+4\,c)}\right)}-\frac{e^{(5\,c)}}{b\left(2\,e^{(-2\,bx+a+2\,c)}+e^{(-4\,bx+a)}+e^{(a+4}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+c)^3*sinh(b*x+a),x, algorithm="maxima")
```

```
[Out] -2*e^(-2*b*x + 3*c)/(b*(2*e^(-2*b*x + a + 2*c) + e^(-4*b*x + a) + e^(a + 4*
c))) + e^(2*a + 3*c)/(b*(2*e^(-2*b*x + a + 2*c) + e^(-4*b*x + a) + e^(a + 4
*c))) - e^(5*c)/(b*(2*e^(-2*b*x + a + 2*c) + e^(-4*b*x + a) + e^(a + 4*c)))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\sinh\left(a+b\,x\right)}{{\cosh\left(c+b\,x\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a + b*x)/cosh(c + b*x)^3,x)
```

```
[Out] int(sinh(a + b*x)/cosh(c + b*x)^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sinh{\left(a+bx\right)}\operatorname{sech}^{3}{\left(bx+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+c)**3*sinh(b*x+a),x)
```

```
[Out] Integral(sinh(a + b*x)*sech(b*x + c)**3, x)
```

3.152 $\int \operatorname{csch}(c+bx)\sinh(a+bx)\,dx$

Optimal. Leaf size=26

$$\frac{\sinh(a-c)\log(\sinh(bx+c))}{b}+x\cosh(a-c)$$

```
[Out] x*cosh(a-c)+ln(sinh(b*x+c))*sinh(a-c)/b
```

Rubi [A] time = 0.01, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {5626, 3475, 8}

$$\frac{\sinh(a-c)\log(\sinh(bx+c))}{b}+x\cosh(a-c)$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + b*x]*Sinh[a + b*x],x]
```

```
[Out] x*Cosh[a - c] + (Log[Sinh[c + b*x]]*Sinh[a - c])/b
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 5626

```
Int[Csch[w_]^(n_.)*Sinh[v_], x_Symbol] :> Dist[Sinh[v - w], Int[Coth[w]*Csc
h[w]^(n - 1), x], x] + Dist[Cosh[v - w], Int[Csch[w]^(n - 1), x], x] /; GtQ
[n, 0] && NeQ[w, v] && FreeQ[v - w, x]
```

Rubi steps

$$\begin{aligned}\int \operatorname{csch}(c+bx)\sinh(a+bx)\,dx &= \cosh(a-c)\int 1\,dx+\sinh(a-c)\int \coth(c+bx)\,dx\\ &= x\cosh(a-c)+\frac{\log(\sinh(c+bx))\sinh(a-c)}{b}\end{aligned}$$

Mathematica [A] time = 0.12, size = 26, normalized size = 1.00

$$\frac{\sinh(a-c)\log(\sinh(bx+c))}{b}+x\cosh(a-c)$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + b*x]*Sinh[a + b*x],x]
```

```
[Out] x*Cosh[a - c] + (Log[Sinh[c + b*x]]*Sinh[a - c])/b
```

fricas [B] time = 0.43, size = 86, normalized size = 3.31

$$\frac{2\,bx+\left(\cosh\left(-a+c\right)^{2}-2\,\cosh\left(-a+c\right)\sinh\left(-a+c\right)+\sinh\left(-a+c\right)^{2}-1\right)\log\left(\frac{2\,\sinh(bx+c)}{\cosh(bx+c)-\sinh(bx+c)}\right)}{2\left(b\cosh\left(-a+c\right)-b\sinh\left(-a+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+c)*sinh(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/2*(2*b*x + (cosh(-a + c)^2 - 2*cosh(-a + c)*sinh(-a + c) + sinh(-a + c)^2
 - 1)*log(2*sinh(b*x + c)/(cosh(b*x + c) - sinh(b*x + c))))/(b*cosh(-a + c)
 - b*sinh(-a + c))
```

giac [A] time = 0.11, size = 51, normalized size = 1.96

$$\frac{2\,bxe^{(-a+c)}+\left(e^{(2\,a+c)}-e^{(3\,c)}\right)e^{(-a-2\,c)}\log\left(\left|e^{(2\,bx+2\,c)}-1\right|\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+c)*sinh(b*x+a),x, algorithm="giac")
```

```
[Out] 1/2*(2*b*x*e^(-a + c) + (e^(2*a + c) - e^(3*c))*e^(-a - 2*c)*log(abs(e^(2*b
*x + 2*c) - 1)))/b
```

maple [B] time = 0.14, size = 150, normalized size = 5.77

$$x\,\mathrm{e}^{a-c}+\mathrm{e}^{-a-c}\mathrm{e}^{2c}x-\mathrm{e}^{-a-c}\mathrm{e}^{2a}x+\frac{\mathrm{e}^{-a-c}\mathrm{e}^{2c}a}{b}-\frac{\mathrm{e}^{-a-c}\mathrm{e}^{2a}a}{b}+\frac{\ln\left(\mathrm{e}^{2bx+2a}-\mathrm{e}^{2a-2c}\right)\mathrm{e}^{-a-c}\mathrm{e}^{2a}}{2b}-\frac{\ln\left(\mathrm{e}^{2bx+2a}-\mathrm{e}^{2a-2c}\right)\mathrm{e}^{-a-c}\mathrm{e}^{2c}}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(b*x+c)*sinh(b*x+a),x)
```

```
[Out] x*exp(a-c)+exp(-a-c)*exp(2*c)*x-exp(-a-c)*exp(2*a)*x+1/b*exp(-a-c)*exp(2*c)
*a-1/b*exp(-a-c)*exp(2*a)*a+1/2*ln(exp(2*b*x+2*a)-exp(2*a-2*c))/b*exp(-a-c)
*exp(2*a)-1/2*ln(exp(2*b*x+2*a)-exp(2*a-2*c))/b*exp(-a-c)*exp(2*c)
```

maxima [B] time = 0.32, size = 84, normalized size = 3.23

$$\frac{\left(e^{(2a)} - e^{(2c)}\right)e^{(-a-c)}\log\left(e^{(-bx)} + e^{c}\right)}{2b} + \frac{\left(e^{(2a)} - e^{(2c)}\right)e^{(-a-c)}\log\left(e^{(-bx)} - e^{c}\right)}{2b} + \frac{(bx+a)e^{(a-c)}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+c)*sinh(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/2*(e^(2*a) - e^(2*c))*e^(-a - c)*log(e^(-b*x) + e^c)/b + 1/2*(e^(2*a) - e
^(2*c))*e^(-a - c)*log(e^(-b*x) - e^c)/b + (b*x + a)*e^(a - c)/b
```

mupad [B] time = 1.57, size = 65, normalized size = 2.50

$$x\,\mathrm{e}^{c-a} + \frac{\mathrm{e}^{2c-2a}\,\ln\left(\mathrm{e}^{2a}\,\mathrm{e}^{2bx} - \mathrm{e}^{2a}\,\mathrm{e}^{-2c}\right)\,\left(2\,b\,\mathrm{e}^{3a-3c} - 2\,b\,\mathrm{e}^{a-c}\right)}{4\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a + b*x)/sinh(c + b*x),x)
```

```
[Out] x*exp(c - a) + (exp(2*c - 2*a)*log(exp(2*a)*exp(2*b*x) - exp(2*a)*exp(-2*c)
)*(2*b*exp(3*a - 3*c) - 2*b*exp(a - c)))/(4*b^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sinh\left(a + bx\right)\operatorname{csch}\left(bx + c\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+c)*sinh(b*x+a),x)
```

```
[Out] Integral(sinh(a + b*x)*csch(b*x + c), x)
```

3.153 $\int \operatorname{csch}^2(c+bx)\sinh(a+bx)\,dx$

Optimal. Leaf size=36

$$-\frac{\cosh(a-c)\tanh^{-1}(\cosh(bx+c))}{b}-\frac{\sinh(a-c)\operatorname{csch}(bx+c)}{b}$$

```
[Out] -arctanh(cosh(b*x+c))*cosh(a-c)/b-csch(b*x+c)*sinh(a-c)/b
```

Rubi [A] time = 0.03, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {5626, 2606, 8, 3770}

$$-\frac{\cosh(a-c)\tanh^{-1}(\cosh(bx+c))}{b}-\frac{\sinh(a-c)\operatorname{csch}(bx+c)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + b*x]^2*Sinh[a + b*x],x]
```

```
[Out] -((ArcTanh[Cosh[c + b*x]]*Cosh[a - c])/b) - (Csch[c + b*x]*Sinh[a - c])/b
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 && !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 5626

```
Int[Csch[w_]^(n_.)*Sinh[v_], x_Symbol] :> Dist[Sinh[v - w], Int[Coth[w]*Csc
h[w]^(n - 1), x], x] + Dist[Cosh[v - w], Int[Csch[w]^(n - 1), x], x] /; GtQ
[n, 0] && NeQ[w, v] && FreeQ[v - w, x]
```

Rubi steps

$$\int \operatorname{csch}^2(c+bx)\sinh(a+bx)\,dx = \cosh(a-c)\int \operatorname{csch}(c+bx)\,dx + \sinh(a-c)\int \coth(c+bx)\operatorname{csch}(c+bx)\,dx$$
$$= -\frac{\tanh^{-1}(\cosh(c+bx))\cosh(a-c)}{b} - \frac{(i\sinh(a-c))\,\text{Subst}(\int 1\,dx, x, -i\text{csch}(c}{b}$$
$$= -\frac{\tanh^{-1}(\cosh(c+bx))\cosh(a-c)}{b} - \frac{\operatorname{csch}(c+bx)\sinh(a-c)}{b}$$

Mathematica [C] time = 0.08, size = 90, normalized size = 2.50

$$-\frac{\sinh(a-c)\operatorname{csch}(bx+c)}{b} - \frac{2i\cosh(a-c)\tan^{-1}\left(\frac{(\cosh(c)-\sinh(c))\left(\sinh(c)\sinh\left(\frac{bx}{2}\right)+\cosh(c)\cosh\left(\frac{bx}{2}\right)\right)}{i\cosh(c)\cosh\left(\frac{bx}{2}\right)-i\sinh(c)\cosh\left(\frac{bx}{2}\right)}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + b*x]^2*Sinh[a + b*x],x]
```

```
[Out] ((-2*I)*ArcTan[((Cosh[c] - Sinh[c])*(Cosh[c]*Cosh[(b*x)/2] + Sinh[c]*Sinh[(
b*x)/2]))/(I*Cosh[c]*Cosh[(b*x)/2] - I*Cosh[(b*x)/2]*Sinh[c])]*Cosh[a - c])
/b - (Csch[c + b*x]*Sinh[a - c])/b
```

fricas [B] time = 0.46, size = 617, normalized size = 17.14

$$\frac{4\cosh(bx+c)\cosh(-a+c)\sinh(-a+c) - 2\cosh(bx+c)\sinh(-a+c)^2 - 2\left(\cosh(-a+c)^2-1\right)\cosh(bx+c}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+c)^2*sinh(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/2*(4*cosh(b*x + c)*cosh(-a + c)*sinh(-a + c) - 2*cosh(b*x + c)*sinh(-a +
c)^2 - 2*(cosh(-a + c)^2 - 1)*cosh(b*x + c) - ((cosh(-a + c)^2 + 1)*cosh(b*
x + c)^2 + (cosh(-a + c)^2 - 2*cosh(-a + c)*sinh(-a + c) + sinh(-a + c)^2 +
 1)*sinh(b*x + c)^2 + (cosh(b*x + c)^2 - 1)*sinh(-a + c)^2 - cosh(-a + c)^2
 - 2*(2*cosh(b*x + c)*cosh(-a + c)*sinh(-a + c) - cosh(b*x + c)*sinh(-a + c
)^2 - (cosh(-a + c)^2 + 1)*cosh(b*x + c))*sinh(b*x + c) - 2*(cosh(b*x + c)^
2*cosh(-a + c) - cosh(-a + c))*sinh(-a + c) - 1)*log(cosh(b*x + c) + sinh(b
*x + c) + 1) + ((cosh(-a + c)^2 + 1)*cosh(b*x + c)^2 + (cosh(-a + c)^2 - 2*
cosh(-a + c)*sinh(-a + c) + sinh(-a + c)^2 + 1)*sinh(b*x + c)^2 + (cosh(b*x
 + c)^2 - 1)*sinh(-a + c)^2 - cosh(-a + c)^2 - 2*(2*cosh(b*x + c)*cosh(-a +
 c)*sinh(-a + c) - cosh(b*x + c)*sinh(-a + c)^2 - (cosh(-a + c)^2 + 1)*cosh
```

```
(b*x + c))*sinh(b*x + c) - 2*(cosh(b*x + c)^2*cosh(-a + c) - cosh(-a + c))*
sinh(-a + c) - 1)*log(cosh(b*x + c) + sinh(b*x + c) - 1) - 2*(cosh(-a + c)^
2 - 2*cosh(-a + c)*sinh(-a + c) + sinh(-a + c)^2 - 1)*sinh(b*x + c))/(b*cos
h(b*x + c)^2*cosh(-a + c) + (b*cosh(-a + c) - b*sinh(-a + c))*sinh(b*x + c)
^2 - b*cosh(-a + c) + 2*(b*cosh(b*x + c)*cosh(-a + c) - b*cosh(b*x + c)*sin
h(-a + c))*sinh(b*x + c) - (b*cosh(b*x + c)^2 - b)*sinh(-a + c))
```

giac [B] time = 0.14, size = 104, normalized size = 2.89

$$-\frac{\left(e^{(2a+c)}+e^{(3c)}\right)e^{(-a-2c)}\log\left(e^{(bx+c)}+1\right)-\left(e^{(2a+c)}+e^{(3c)}\right)e^{(-a-2c)}\log\left(\left|e^{(bx+c)}-1\right|\right)+\frac{2\left(e^{(bx+2a)}-e^{(bx+2c)}\right)e^{(-a)}}{e^{(2bx+2c)}-1}}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+c)^2*sinh(b*x+a),x, algorithm="giac")
```

```
[Out] -1/2*((e^(2*a + c) + e^(3*c))*e^(-a - 2*c)*log(e^(b*x + c) + 1) - (e^(2*a +
 c) + e^(3*c))*e^(-a - 2*c)*log(abs(e^(b*x + c) - 1)) + 2*(e^(b*x + 2*a) -
e^(b*x + 2*c))*e^(-a)/(e^(2*b*x + 2*c) - 1))/b
```

maple [B] time = 0.15, size = 172, normalized size = 4.78

$$\frac{e^{bx+a}\left(e^{2a}-e^{2c}\right)}{b\left(-e^{2bx+2a+2c}+e^{2a}\right)}-\frac{\ln\left(e^{bx+a}+e^{a-c}\right)e^{-a-c}e^{2a}}{2b}-\frac{\ln\left(e^{bx+a}+e^{a-c}\right)e^{-a-c}e^{2c}}{2b}+\frac{\ln\left(e^{bx+a}-e^{a-c}\right)e^{-a-c}e^{2a}}{2b}+\frac{\ln\left(e^{bx\cdots}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(b*x+c)^2*sinh(b*x+a),x)
```

```
[Out] 1/b*exp(b*x+a)*(exp(2*a)-exp(2*c))/(-exp(2*b*x+2*a+2*c)+exp(2*a))-1/2*ln(ex
p(b*x+a)+exp(a-c))/b*exp(-a-c)*exp(2*a)-1/2*ln(exp(b*x+a)+exp(a-c))/b*exp(-
a-c)*exp(2*c)+1/2*ln(exp(b*x+a)-exp(a-c))/b*exp(-a-c)*exp(2*a)+1/2*ln(exp(b
*x+a)-exp(a-c))/b*exp(-a-c)*exp(2*c)
```

maxima [B] time = 0.32, size = 103, normalized size = 2.86

$$-\frac{\left(e^{(2a)}+e^{(2c)}\right)e^{(-a-c)}\log\left(e^{(-bx)}+e^{c}\right)}{2b}+\frac{\left(e^{(2a)}+e^{(2c)}\right)e^{(-a-c)}\log\left(e^{(-bx)}-e^{c}\right)}{2b}+\frac{\left(e^{(2a)}-e^{(2c)}\right)e^{(-bx-a)}}{b\left(e^{(-2bx)}-e^{(2c)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+c)^2*sinh(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/2*(e^(2*a) + e^(2*c))*e^(-a - c)*log(e^(-b*x) + e^c)/b + 1/2*(e^(2*a) +
e^(2*c))*e^(-a - c)*log(e^(-b*x) - e^c)/b + (e^(2*a) - e^(2*c))*e^(-b*x - a
)/(b*(e^(-2*b*x) - e^(2*c)))
```

mupad [B] time = 0.20, size = 156, normalized size = 4.33

$$\frac{e^{a+b\,x}\,\left(e^{2\,a-2\,c}-1\right)}{b\,\left(e^{2\,a-2\,c}-e^{2\,a+2\,b\,x}\right)}-\frac{\mathrm{atan}\left(\frac{e^{-a}\,e^{2\,c}\,e^{b\,x}\,\left(\sqrt{-b^2}+e^{2\,a}\,e^{-2\,c}\,\sqrt{-b^2}\right)}{b\,\sqrt{e^{-2\,a}\,e^{2\,c}\,\left(2\,e^{2\,a}\,e^{-2\,c}+e^{4\,a}\,e^{-4\,c}+1\right)}}\right)\,\sqrt{e^{2\,c-2\,a}\,\left(2\,e^{2\,a-2\,c}+e^{4\,a-4\,c}+1\right)}}{\sqrt{-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a + b*x)/sinh(c + b*x)^2,x)
```

```
[Out] (exp(a + b*x)*(exp(2*a - 2*c) - 1))/(b*(exp(2*a - 2*c) - exp(2*a + 2*b*x)))
 - (atan((exp(-a)*exp(2*c)*exp(b*x)*((-b^2)^(1/2) + exp(2*a)*exp(-2*c)*(-b^
2)^(1/2)))/(b*(exp(-2*a)*exp(2*c)*(2*exp(2*a)*exp(-2*c) + exp(4*a)*exp(-4*c
) + 1))^(1/2)))*(exp(2*c - 2*a)*(2*exp(2*a - 2*c) + exp(4*a - 4*c) + 1))^(1
/2))/(-b^2)^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sinh\left(a+bx\right)\operatorname{csch}^2\left(bx+c\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+c)**2*sinh(b*x+a),x)
```

```
[Out] Integral(sinh(a + b*x)*csch(b*x + c)**2, x)
```

3.154 $\int \mathrm{csch}^3(c+bx)\sinh(a+bx)\,dx$

Optimal. Leaf size=39

$$-\frac{\cosh(a-c)\coth(bx+c)}{b}-\frac{\sinh(a-c)\mathrm{csch}^2(bx+c)}{2b}$$

```
[Out] -cosh(a-c)*coth(b*x+c)/b-1/2*csch(b*x+c)^2*sinh(a-c)/b
```

Rubi [A] time = 0.04, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5626, 2606, 30, 3767, 8}

$$-\frac{\cosh(a-c)\coth(bx+c)}{b}-\frac{\sinh(a-c)\mathrm{csch}^2(bx+c)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + b*x]^3*Sinh[a + b*x],x]
```

```
[Out] -((Cosh[a - c]*Coth[c + b*x])/b) - (Csch[c + b*x]^2*Sinh[a - c])/(2*b)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 5626

```
Int[Csch[w_]^(n_.)*Sinh[v_], x_Symbol] :> Dist[Sinh[v - w], Int[Coth[w]*Csc
h[w]^(n - 1), x], x] + Dist[Cosh[v - w], Int[Csch[w]^(n - 1), x], x] /; GtQ
[n, 0] && NeQ[w, v] && FreeQ[v - w, x]
```

Rubi steps

$$\int \operatorname{csch}^3(c+bx)\sinh(a+bx)\,dx = \cosh(a-c)\int \operatorname{csch}^2(c+bx)\,dx + \sinh(a-c)\int \coth(c+bx)\operatorname{csch}^2(c+bx)\,dx$$

$$= -\frac{(i\cosh(a-c))\,\text{Subst}(\int 1\,dx, x, -i\coth(c+bx))}{b} + \frac{\sinh(a-c)\,\text{Subst}(\int x\,dx,}{b}$$

$$= -\frac{\cosh(a-c)\coth(c+bx)}{b} - \frac{\operatorname{csch}^2(c+bx)\sinh(a-c)}{2b}$$

Mathematica [A] time = 0.18, size = 35, normalized size = 0.90

$$-\frac{\operatorname{csch}(c)\operatorname{csch}^2(bx+c)(\cosh(a)-\cosh(a-c)\cosh(2bx+c))}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + b*x]^3*Sinh[a + b*x],x]
```

```
[Out] -1/2*((Cosh[a] - Cosh[a - c]*Cosh[c + 2*b*x])*Csch[c]*Csch[c + b*x]^2)/b
```

fricas [B] time = 0.43, size = 246, normalized size = 6.31

$$-\frac{}{b\cosh(bx+c)^3\cosh(-a+c)^2 - b\cosh(bx+c)\cosh(-a+c)^2 + \left(b\cosh(-a+c)^2 - b\sinh(-a+c)^2\right)\sinh(bx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+c)^3*sinh(b*x+a),x, algorithm="fricas")
```

```
[Out] -2*((2*cosh(-a + c) - sinh(-a + c))*sinh(b*x + c) - cosh(b*x + c)*sinh(-a +
 c))/(b*cosh(b*x + c)^3*cosh(-a + c)^2 - b*cosh(b*x + c)*cosh(-a + c)^2 + (
b*cosh(-a + c)^2 - b*sinh(-a + c)^2)*sinh(b*x + c)^3 + 3*(b*cosh(b*x + c)*c
osh(-a + c)^2 - b*cosh(b*x + c)*sinh(-a + c)^2)*sinh(b*x + c)^2 - (b*cosh(b
*x + c)^3 - b*cosh(b*x + c))*sinh(-a + c)^2 + 3*(b*cosh(b*x + c)^2*cosh(-a
+ c)^2 - b*cosh(-a + c)^2 - (b*cosh(b*x + c)^2 - b)*sinh(-a + c)^2)*sinh(b*
x + c))
```

giac [A] time = 0.12, size = 53, normalized size = 1.36

$$-\frac{\left(2\,e^{(2\,bx+2\,a+2\,c)} - e^{(2\,a)} - e^{(2\,c)}\right)e^{(-a-c)}}{b\left(e^{(2\,bx+2\,c)}-1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+c)^3*sinh(b*x+a),x, algorithm="giac")
```

```
[Out] -(2*e^(2*b*x + 2*a + 2*c) - e^(2*a) - e^(2*c))*e^(-a - c)/(b*(e^(2*b*x + 2*
c) - 1)^2)
```

maple [A] time = 0.17, size = 57, normalized size = 1.46

$$\frac{\left(-2\,e^{2bx+2a+2c}+e^{2a}+e^{2c}\right)e^{3a-c}}{\left(-e^{2bx+2a+2c}+e^{2a}\right)^{2}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(b*x+c)^3*sinh(b*x+a),x)
```

```
[Out] 1/(-exp(2*b*x+2*a+2*c)+exp(2*a))^2/b*(-2*exp(2*b*x+2*a+2*c)+exp(2*a)+exp(2*
c))*exp(3*a-c)
```

maxima [B] time = 0.33, size = 131, normalized size = 3.36

$$-\frac{2\,e^{(-2\,bx+3\,c)}}{b\left(2\,e^{(-2\,bx+a+2\,c)}-e^{(-4\,bx+a)}-e^{(a+4\,c)}\right)}+\frac{e^{(2\,a+3\,c)}}{b\left(2\,e^{(-2\,bx+a+2\,c)}-e^{(-4\,bx+a)}-e^{(a+4\,c)}\right)}+\frac{e^{(5\,c)}}{b\left(2\,e^{(-2\,bx+a+2\,c)}-e^{(-4\,bx+a)}-e^{(a+}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+c)^3*sinh(b*x+a),x, algorithm="maxima")
```

```
[Out] -2*e^(-2*b*x + 3*c)/(b*(2*e^(-2*b*x + a + 2*c) - e^(-4*b*x + a) - e^(a + 4*
c))) + e^(2*a + 3*c)/(b*(2*e^(-2*b*x + a + 2*c) - e^(-4*b*x + a) - e^(a + 4
*c))) + e^(5*c)/(b*(2*e^(-2*b*x + a + 2*c) - e^(-4*b*x + a) - e^(a + 4*c)))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\sinh\left(a+b\,x\right)}{{\sinh\left(c+b\,x\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a + b*x)/sinh(c + b*x)^3,x)
```

```
[Out] int(sinh(a + b*x)/sinh(c + b*x)^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sinh\left(a+bx\right)\operatorname{csch}^{3}\left(bx+c\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+c)**3*sinh(b*x+a),x)
```

```
[Out] Integral(sinh(a + b*x)*csch(b*x + c)**3, x)
```

3.155 $\int \cosh(a+bx)\tanh(c+bx)\,dx$

Optimal. Leaf size=29

$$\frac{\cosh(a+bx)}{b}-\frac{\sinh(a-c)\tan^{-1}(\sinh(bx+c))}{b}$$

```
[Out] cosh(b*x+a)/b-arctan(sinh(b*x+c))*sinh(a-c)/b
```

Rubi [A] time = 0.02, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {5623, 2638, 3770}

$$\frac{\cosh(a+bx)}{b}-\frac{\sinh(a-c)\tan^{-1}(\sinh(bx+c))}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[a + b*x]*Tanh[c + b*x],x]
```

```
[Out] Cosh[a + b*x]/b - (ArcTan[Sinh[c + b*x]]*Sinh[a - c])/b
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 5623

```
Int[Cosh[v_]*Tanh[w_]^(n_.), x_Symbol] :> Int[Sinh[v]*Tanh[w]^(n - 1), x] -
 Dist[Sinh[v - w], Int[Sech[w]*Tanh[w]^(n - 1), x], x] /; GtQ[n, 0] && NeQ[
w, v] && FreeQ[v - w, x]
```

Rubi steps

$$\begin{aligned}\int \cosh(a+bx)\tanh(c+bx)\,dx &= -\left(\sinh(a-c)\int \operatorname{sech}(c+bx)\,dx\right)+\int \sinh(a+bx)\,dx\\ &= \frac{\cosh(a+bx)}{b}-\frac{\tan^{-1}(\sinh(c+bx))\sinh(a-c)}{b}\end{aligned}$$

Mathematica [B] time = 0.06, size = 86, normalized size = 2.97

$$-\frac{2\sinh(a-c)\tan^{-1}\left(\frac{(\cosh(c)-\sinh(c))\left(\sinh(c)\cosh\left(\frac{bx}{2}\right)+\cosh(c)\sinh\left(\frac{bx}{2}\right)\right)}{\cosh(c)\cosh\left(\frac{bx}{2}\right)-\sinh(c)\cosh\left(\frac{bx}{2}\right)}\right)}{b}+\frac{\sinh(a)\sinh(bx)}{b}+\frac{\cosh(a)\cosh(bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[a + b*x]*Tanh[c + b*x],x]
```

```
[Out] (Cosh[a]*Cosh[b*x])/b - (2*ArcTan[((Cosh[c] - Sinh[c])*(Cosh[(b*x)/2]*Sinh[
c] + Cosh[c]*Sinh[(b*x)/2]))/(Cosh[c]*Cosh[(b*x)/2] - Cosh[(b*x)/2]*Sinh[c]
)]*Sinh[a - c])/b + (Sinh[a]*Sinh[b*x])/b
```

fricas [B] time = 0.43, size = 327, normalized size = 11.28

$$\frac{\cosh(bx+c)^2\cosh(-a+c)^2-2\cosh(bx+c)^2\cosh(-a+c)\sinh(-a+c)+\cosh(bx+c)^2\sinh(-a+c)^2+(c}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*tanh(b*x+c),x, algorithm="fricas")
```

```
[Out] 1/2*(cosh(b*x + c)^2*cosh(-a + c)^2 - 2*cosh(b*x + c)^2*cosh(-a + c)*sinh(-
a + c) + cosh(b*x + c)^2*sinh(-a + c)^2 + (cosh(-a + c)^2 - 2*cosh(-a + c)*
sinh(-a + c) + sinh(-a + c)^2)*sinh(b*x + c)^2 + 2*(2*cosh(b*x + c)*cosh(-a
 + c)*sinh(-a + c) - cosh(b*x + c)*sinh(-a + c)^2 - (cosh(-a + c)^2 - 1)*co
sh(b*x + c) - (cosh(-a + c)^2 - 2*cosh(-a + c)*sinh(-a + c) + sinh(-a + c)^
2 - 1)*sinh(b*x + c))*arctan(cosh(b*x + c) + sinh(b*x + c)) + 2*(cosh(b*x +
 c)*cosh(-a + c)^2 - 2*cosh(b*x + c)*cosh(-a + c)*sinh(-a + c) + cosh(b*x +
 c)*sinh(-a + c)^2)*sinh(b*x + c) + 1)/(b*cosh(b*x + c)*cosh(-a + c) - b*co
sh(b*x + c)*sinh(-a + c) + (b*cosh(-a + c) - b*sinh(-a + c))*sinh(b*x + c))
```

giac [A] time = 0.12, size = 53, normalized size = 1.83

$$-\frac{2\left(e^{(2a)}-e^{(2c)}\right)\arctan\left(e^{(bx+c)}\right)e^{(-a-c)}-e^{(bx+a)}-e^{(-bx-a)}}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*tanh(b*x+c),x, algorithm="giac")
```

```
[Out] -1/2*(2*(e^(2*a) - e^(2*c))*arctan(e^(b*x + c))*e^(-a - c) - e^(b*x + a) -
e^(-b*x - a))/b
```

maple [C] time = 0.26, size = 167, normalized size = 5.76

$$\frac{e^{bx+a}}{2b}+\frac{e^{-bx-a}}{2b}+\frac{i\ln\left(e^{bx+a}-ie^{a-c}\right)e^{-a-c}e^{2a}}{2b}-\frac{i\ln\left(e^{bx+a}-ie^{a-c}\right)e^{-a-c}e^{2c}}{2b}-\frac{i\ln\left(e^{bx+a}+ie^{a-c}\right)e^{-a-c}e^{2a}}{2b}+\frac{i\ln\left(e^{bx+\right.}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)*tanh(b*x+c),x)
```

```
[Out] 1/2*exp(b*x+a)/b+1/2*exp(-b*x-a)/b+1/2*I*ln(exp(b*x+a)-I*exp(a-c))/b*exp(-a
-c)*exp(a)^2-1/2*I*ln(exp(b*x+a)-I*exp(a-c))/b*exp(-a-c)*exp(c)^2-1/2*I*ln(
exp(b*x+a)+I*exp(a-c))/b*exp(-a-c)*exp(a)^2+1/2*I*ln(exp(b*x+a)+I*exp(a-c))
/b*exp(-a-c)*exp(c)^2
```

maxima [B] time = 0.42, size = 59, normalized size = 2.03

$$\frac{\left(e^{(2a)}-e^{(2c)}\right)\arctan\left(e^{(-bx-c)}\right)e^{(-a-c)}}{b}+\frac{e^{(bx+a)}}{2b}+\frac{e^{(-bx-a)}}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*tanh(b*x+c),x, algorithm="maxima")
```

```
[Out] (e^(2*a) - e^(2*c))*arctan(e^(-b*x - c))*e^(-a - c)/b + 1/2*e^(b*x + a)/b +
 1/2*e^(-b*x - a)/b
```

mupad [B] time = 0.16, size = 133, normalized size = 4.59

$$\frac{e^{a+bx}}{2b}+\frac{e^{-a-bx}}{2b}+\frac{\mathrm{atan}\left(\frac{e^{-a}e^{2c}e^{bx}\left(\sqrt{b^2}-e^{2a}e^{-2c}\sqrt{b^2}\right)}{b\sqrt{e^{-2a}e^{2c}\left(e^{4a}e^{-4c}-2e^{2a}e^{-2c}+1\right)}}\right)\sqrt{e^{2c-2a}\left(e^{4a-4c}-2e^{2a-2c}+1\right)}}{\sqrt{b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a + b*x)*tanh(c + b*x),x)
```

```
[Out] exp(a + b*x)/(2*b) + exp(- a - b*x)/(2*b) + (atan((exp(-a)*exp(2*c)*exp(b*x
)*((b^2)^(1/2) - exp(2*a)*exp(-2*c)*(b^2)^(1/2)))/(b*(exp(-2*a)*exp(2*c)*(e
xp(4*a)*exp(-4*c) - 2*exp(2*a)*exp(-2*c) + 1))^(1/2)))*(exp(2*c - 2*a)*(exp
(4*a - 4*c) - 2*exp(2*a - 2*c) + 1))^(1/2))/(b^2)^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\cosh\left(a+bx\right)\tanh\left(bx+c\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*tanh(b*x+c),x)
[Out] Integral(cosh(a + b*x)*tanh(b*x + c), x)
```

3.156 $\int \cosh(a+bx)\tanh^2(c+bx)\,dx$

Optimal. Leaf size=45

$$\frac{\sinh(a-c)\text{sech}(bx+c)}{b}-\frac{\cosh(a-c)\tan^{-1}(\sinh(bx+c))}{b}+\frac{\sinh(a+bx)}{b}$$

```
[Out] -arctan(sinh(b*x+c))*cosh(a-c)/b+sech(b*x+c)*sinh(a-c)/b+sinh(b*x+a)/b
```

Rubi [A] time = 0.04, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {5623, 5620, 2637, 3770, 2606, 8}

$$\frac{\sinh(a-c)\text{sech}(bx+c)}{b}-\frac{\cosh(a-c)\tan^{-1}(\sinh(bx+c))}{b}+\frac{\sinh(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[a + b*x]*Tanh[c + b*x]^2,x]
```

```
[Out] -((ArcTan[Sinh[c + b*x]]*Cosh[a - c])/b) + (Sech[c + b*x]*Sinh[a - c])/b +
Sinh[a + b*x]/b
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 5620

```
Int[Sinh[v_]*Tanh[w_]^(n_.), x_Symbol] :> Int[Cosh[v]*Tanh[w]^(n - 1), x] -
 Dist[Cosh[v - w], Int[Sech[w]*Tanh[w]^(n - 1), x], x] /; GtQ[n, 0] && NeQ[
w, v] && FreeQ[v - w, x]
```

Rule 5623

```
Int[Cosh[v_]*Tanh[w_]^(n_.), x_Symbol] :> Int[Sinh[v]*Tanh[w]^(n - 1), x] -
 Dist[Sinh[v - w], Int[Sech[w]*Tanh[w]^(n - 1), x], x] /; GtQ[n, 0] && NeQ[
w, v] && FreeQ[v - w, x]
```

Rubi steps

$$\int \cosh(a+bx)\tanh^2(c+bx)\,dx = -(\sinh(a-c)\int \operatorname{sech}(c+bx)\tanh(c+bx)\,dx) + \int \sinh(a+bx)\tanh(c+bx$$

$$= -(\cosh(a-c)\int \operatorname{sech}(c+bx)\,dx) + \frac{\sinh(a-c)\operatorname{Subst}(\int 1\,dx, x, \operatorname{sech}(c+bx)}{b}$$

$$= -\frac{\tan^{-1}(\sinh(c+bx))\cosh(a-c)}{b} + \frac{\operatorname{sech}(c+bx)\sinh(a-c)}{b} + \frac{\sinh(a+bx)}{b}$$

Mathematica [B] time = 0.10, size = 102, normalized size = 2.27

$$\frac{\sinh(a-c)\operatorname{sech}(bx+c)}{b} - \frac{2\cosh(a-c)\tan^{-1}\left(\frac{(\cosh(c)-\sinh(c))\left(\sinh(c)\cosh\left(\frac{bx}{2}\right)+\cosh(c)\sinh\left(\frac{bx}{2}\right)\right)}{\cosh(c)\cosh\left(\frac{bx}{2}\right)-\sinh(c)\cosh\left(\frac{bx}{2}\right)}\right)}{b} + \frac{\sinh(a)\cosh(bx)}{b} + \frac{c}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[a + b*x]*Tanh[c + b*x]^2,x]
```

```
[Out] (-2*ArcTan[((Cosh[c] - Sinh[c])*(Cosh[(b*x)/2]*Sinh[c] + Cosh[c]*Sinh[(b*x)
/2]))/(Cosh[c]*Cosh[(b*x)/2] - Cosh[(b*x)/2]*Sinh[c])]*Cosh[a - c])/b + (Co
sh[b*x]*Sinh[a])/b + (Sech[c + b*x]*Sinh[a - c])/b + (Cosh[a]*Sinh[b*x])/b
```

fricas [B] time = 0.45, size = 902, normalized size = 20.04

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*tanh(b*x+c)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(cosh(b*x + c)^4*cosh(-a + c)^2 + (cosh(-a + c)^2 - 2*cosh(-a + c)*sinh
(-a + c) + sinh(-a + c)^2)*sinh(b*x + c)^4 + 4*(cosh(b*x + c)*cosh(-a + c)^
```

```
2 - 2*cosh(b*x + c)*cosh(-a + c)*sinh(-a + c) + cosh(b*x + c)*sinh(-a + c)^
2)*sinh(b*x + c)^3 + 3*(cosh(-a + c)^2 - 1)*cosh(b*x + c)^2 + 3*(2*cosh(b*x
 + c)^2*cosh(-a + c)^2 + (2*cosh(b*x + c)^2 + 1)*sinh(-a + c)^2 + cosh(-a +
 c)^2 - 2*(2*cosh(b*x + c)^2*cosh(-a + c) + cosh(-a + c))*sinh(-a + c) - 1)
*sinh(b*x + c)^2 + (cosh(b*x + c)^4 + 3*cosh(b*x + c)^2)*sinh(-a + c)^2 - 2
*((cosh(-a + c)^2 + 1)*cosh(b*x + c)^3 + (cosh(-a + c)^2 - 2*cosh(-a + c)*s
inh(-a + c) + sinh(-a + c)^2 + 1)*sinh(b*x + c)^3 - 3*(2*cosh(b*x + c)*cosh
(-a + c)*sinh(-a + c) - cosh(b*x + c)*sinh(-a + c)^2 - (cosh(-a + c)^2 + 1)
*cosh(b*x + c))*sinh(b*x + c)^2 + (cosh(b*x + c)^3 + cosh(b*x + c))*sinh(-a
 + c)^2 + (cosh(-a + c)^2 + 1)*cosh(b*x + c) + (3*(cosh(-a + c)^2 + 1)*cosh
(b*x + c)^2 + (3*cosh(b*x + c)^2 + 1)*sinh(-a + c)^2 + cosh(-a + c)^2 - 2*(
3*cosh(b*x + c)^2*cosh(-a + c) + cosh(-a + c))*sinh(-a + c) + 1)*sinh(b*x +
 c) - 2*(cosh(b*x + c)^3*cosh(-a + c) + cosh(b*x + c)*cosh(-a + c))*sinh(-a
 + c))*arctan(cosh(b*x + c) + sinh(b*x + c)) + 2*(2*cosh(b*x + c)^3*cosh(-a
 + c)^2 + (2*cosh(b*x + c)^3 + 3*cosh(b*x + c))*sinh(-a + c)^2 + 3*(cosh(-a
 + c)^2 - 1)*cosh(b*x + c) - 2*(2*cosh(b*x + c)^3*cosh(-a + c) + 3*cosh(b*x
 + c)*cosh(-a + c))*sinh(-a + c))*sinh(b*x + c) - 2*(cosh(b*x + c)^4*cosh(-
a + c) + 3*cosh(b*x + c)^2*cosh(-a + c))*sinh(-a + c) - 1)/(b*cosh(b*x + c)
^3*cosh(-a + c) + (b*cosh(-a + c) - b*sinh(-a + c))*sinh(b*x + c)^3 + b*cos
h(b*x + c)*cosh(-a + c) + 3*(b*cosh(b*x + c)*cosh(-a + c) - b*cosh(b*x + c)
*sinh(-a + c))*sinh(b*x + c)^2 + (3*b*cosh(b*x + c)^2*cosh(-a + c) + b*cosh
(-a + c) - (3*b*cosh(b*x + c)^2 + b)*sinh(-a + c))*sinh(b*x + c) - (b*cosh(
b*x + c)^3 + b*cosh(b*x + c))*sinh(-a + c))
```

giac [A] time = 0.13, size = 86, normalized size = 1.91

$$-\frac{2\left(e^{(2a)}+e^{(2c)}\right)\arctan\left(e^{(bx+c)}\right)e^{(-a-c)}-\frac{\left(2e^{(2bx+2a)}-3e^{(2bx+2c)}-1\right)e^{(-a)}}{e^{(3bx+2c)}+e^{(bx)}}-e^{(bx+a)}}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*tanh(b*x+c)^2,x, algorithm="giac")
```

```
[Out] -1/2*(2*(e^(2*a) + e^(2*c))*arctan(e^(b*x + c))*e^(-a - c) - (2*e^(2*b*x +
2*a) - 3*e^(2*b*x + 2*c) - 1)*e^(-a)/(e^(3*b*x + 2*c) + e^(b*x)) - e^(b*x +
 a))/b
```

maple [C] time = 0.35, size = 207, normalized size = 4.60

$$\frac{e^{bx+a}}{2b}-\frac{e^{-bx-a}}{2b}+\frac{e^{bx+a}\left(e^{2a}-e^{2c}\right)}{b\left(e^{2bx+2a+2c}+e^{2a}\right)}+\frac{i\ln\left(e^{bx+a}-ie^{a-c}\right)e^{-a-c}e^{2a}}{2b}+\frac{i\ln\left(e^{bx+a}-ie^{a-c}\right)e^{-a-c}e^{2c}}{2b}-\frac{i\ln\left(e^{bx+a}+ie^{a-}\right.}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)*tanh(b*x+c)^2,x)
```

```
[Out] 1/2*exp(b*x+a)/b-1/2*exp(-b*x-a)/b+1/b*exp(b*x+a)*(exp(2*a)-exp(2*c))/(exp(
2*b*x+2*a+2*c)+exp(2*a))+1/2*I*ln(exp(b*x+a)-I*exp(a-c))/b*exp(-a-c)*exp(2*
a)+1/2*I*ln(exp(b*x+a)-I*exp(a-c))/b*exp(-a-c)*exp(2*c)-1/2*I*ln(exp(b*x+a)
+I*exp(a-c))/b*exp(-a-c)*exp(2*a)-1/2*I*ln(exp(b*x+a)+I*exp(a-c))/b*exp(-a-
c)*exp(2*c)
```

maxima [B] time = 0.41, size = 103, normalized size = 2.29

$$\frac{\left(e^{(2a)} + e^{(2c)}\right)\arctan\left(e^{(-bx-c)}\right)e^{(-a-c)}}{b} - \frac{e^{(-bx-a)}}{2b} + \frac{\left(3e^{(2a)} - 2e^{(2c)}\right)e^{(-2bx-2a)} + e^{(2c)}}{2b\left(e^{(-bx-a+2c)} + e^{(-3bx-a)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*tanh(b*x+c)^2,x, algorithm="maxima")
```

```
[Out] (e^(2*a) + e^(2*c))*arctan(e^(-b*x - c))*e^(-a - c)/b - 1/2*e^(-b*x - a)/b
+ 1/2*((3*e^(2*a) - 2*e^(2*c))*e^(-2*b*x - 2*a) + e^(2*c))/(b*(e^(-b*x - a
+ 2*c) + e^(-3*b*x - a)))
```

mupad [B] time = 0.19, size = 173, normalized size = 3.84

$$\frac{e^{a+bx}}{2b} - \frac{e^{-a-bx}}{2b} - \frac{\operatorname{atan}\left(\frac{e^{-a}\,e^{2c}\,e^{bx}\left(\sqrt{b^2}+e^{2a}\,e^{-2c}\,\sqrt{b^2}\right)}{b\sqrt{e^{-2a}\,e^{2c}\left(2\,e^{2a}\,e^{-2c}+e^{4a}\,e^{-4c}+1\right)}}\right)\sqrt{e^{2c-2a}\left(2\,e^{2a-2c}+e^{4a-4c}+1\right)}}{\sqrt{b^2}} + \frac{e^{a+bx}\left(e^{2a-2c}-1\right)}{b\left(e^{2a-2c}+e^{2a+2bx}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a + b*x)*tanh(c + b*x)^2,x)
```

```
[Out] exp(a + b*x)/(2*b) - exp(- a - b*x)/(2*b) - (atan((exp(-a)*exp(2*c)*exp(b*x
)*((b^2)^(1/2) + exp(2*a)*exp(-2*c)*(b^2)^(1/2)))/(b*(exp(-2*a)*exp(2*c)*(2
*exp(2*a)*exp(-2*c) + exp(4*a)*exp(-4*c) + 1))^(1/2)))*(exp(2*c - 2*a)*(2*e
xp(2*a - 2*c) + exp(4*a - 4*c) + 1))^(1/2))/(b^2)^(1/2) + (exp(a + b*x)*(ex
p(2*a - 2*c) - 1))/(b*(exp(2*a - 2*c) + exp(2*a + 2*b*x)))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \cosh(a + bx)\tanh^2(bx + c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*tanh(b*x+c)**2,x)
```

```
[Out] Integral(cosh(a + b*x)*tanh(b*x + c)**2, x)
```

3.157 $\int \cosh(a+bx)\tanh^3(c+bx)\,dx$

Optimal. Leaf size=72

$$-\frac{3\sinh(a-c)\tan^{-1}(\sinh(bx+c))}{2b}+\frac{\cosh(a-c)\text{sech}(bx+c)}{b}+\frac{\sinh(a-c)\tanh(bx+c)\text{sech}(bx+c)}{2b}+\frac{\cosh(a+$$

[Out] cosh(b*x+a)/b+cosh(a-c)*sech(b*x+c)/b-3/2*arctan(sinh(b*x+c))*sinh(a-c)/b+1/2*sech(b*x+c)*sinh(a-c)*tanh(b*x+c)/b

Rubi [A] time = 0.08, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.467, Rules used = {5623, 5620, 2638, 3770, 2606, 8, 2611}

$$-\frac{3\sinh(a-c)\tan^{-1}(\sinh(bx+c))}{2b}+\frac{\cosh(a-c)\text{sech}(bx+c)}{b}+\frac{\sinh(a-c)\tanh(bx+c)\text{sech}(bx+c)}{2b}+\frac{\cosh(a+$$

Antiderivative was successfully verified.

[In] Int[Cosh[a + b*x]*Tanh[c + b*x]^3,x]

[Out] Cosh[a + b*x]/b + (Cosh[a - c]*Sech[c + b*x])/b - (3*ArcTan[Sinh[c + b*x]]*Sinh[a - c])/(2*b) + (Sech[c + b*x]*Sinh[a - c]*Tanh[c + b*x])/(2*b)

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 5620

```
Int[Sinh[v_]*Tanh[w_]^(n_.), x_Symbol] :> Int[Cosh[v]*Tanh[w]^(n - 1), x] -
 Dist[Cosh[v - w], Int[Sech[w]*Tanh[w]^(n - 1), x], x] /; GtQ[n, 0] && NeQ[
w, v] && FreeQ[v - w, x]
```

Rule 5623

```
Int[Cosh[v_]*Tanh[w_]^(n_.), x_Symbol] :> Int[Sinh[v]*Tanh[w]^(n - 1), x] -
 Dist[Sinh[v - w], Int[Sech[w]*Tanh[w]^(n - 1), x], x] /; GtQ[n, 0] && NeQ[
w, v] && FreeQ[v - w, x]
```

Rubi steps

$$\begin{aligned}\int \cosh(a+bx)\tanh^3(c+bx)\,dx &= -\left(\sinh(a-c)\int \operatorname{sech}(c+bx)\tanh^2(c+bx)\,dx\right)+\int \sinh(a+bx)\tanh^2(c+\\ &= \frac{\operatorname{sech}(c+bx)\sinh(a-c)\tanh(c+bx)}{2b}-\cosh(a-c)\int \operatorname{sech}(c+bx)\tanh(c+\\ &= -\frac{\tan^{-1}(\sinh(c+bx))\sinh(a-c)}{2b}+\frac{\operatorname{sech}(c+bx)\sinh(a-c)\tanh(c+bx)}{2b}+\\ &= \frac{\cosh(a+bx)}{b}+\frac{\cosh(a-c)\operatorname{sech}(c+bx)}{b}-\frac{3\tan^{-1}(\sinh(c+bx))\sinh(a-c)}{2b}\end{aligned}$$

Mathematica [A] time = 0.32, size = 115, normalized size = 1.60

$$\frac{\operatorname{sech}(c)\operatorname{sech}^2(bx+c)(-\cosh(a-bx-c))+\operatorname{sech}(c)\operatorname{sech}^2(bx+c)\cosh(a+bx-c)+\operatorname{sech}(c)\cosh(a-2c)\operatorname{sech}(b}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[a + b*x]*Tanh[c + b*x]^3,x]
```

```
[Out] (Cosh[a - 2*c]*Sech[c]*Sech[c + b*x] - Cosh[a - c - b*x]*Sech[c]*Sech[c + b
*x]^2 + Cosh[a - c + b*x]*Sech[c]*Sech[c + b*x]^2 + Cosh[a]*(4*Cosh[b*x] +
```

```
3*Sech[c]*Sech[c + b*x]) - 12*ArcTan[Sinh[c] + Cosh[c]*Tanh[(b*x)/2]]*Sinh[
a - c] + 4*Sinh[a]*Sinh[b*x])/(4*b)
```

fricas [B] time = 0.45, size = 1737, normalized size = 24.12

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*tanh(b*x+c)^3,x, algorithm="fricas")

[Out] 1/2*(cosh(b*x + c)^6*cosh(-a + c)^2 + (cosh(-a + c)^2 - 2*cosh(-a + c)*sinh
(-a + c) + sinh(-a + c)^2)*sinh(b*x + c)^6 + 6*(cosh(b*x + c)*cosh(-a + c)^
2 - 2*cosh(b*x + c)*cosh(-a + c)*sinh(-a + c) + cosh(b*x + c)*sinh(-a + c)^
2)*sinh(b*x + c)^5 + (5*cosh(-a + c)^2 + 2)*cosh(b*x + c)^4 + (15*cosh(b*x
+ c)^2*cosh(-a + c)^2 + 5*(3*cosh(b*x + c)^2 + 1)*sinh(-a + c)^2 + 5*cosh(-
a + c)^2 - 10*(3*cosh(b*x + c)^2*cosh(-a + c) + cosh(-a + c))*sinh(-a + c)
+ 2)*sinh(b*x + c)^4 + 4*(5*cosh(b*x + c)^3*cosh(-a + c)^2 + 5*(cosh(b*x +
c)^3 + cosh(b*x + c))*sinh(-a + c)^2 + (5*cosh(-a + c)^2 + 2)*cosh(b*x + c)
 - 10*(cosh(b*x + c)^3*cosh(-a + c) + cosh(b*x + c)*cosh(-a + c))*sinh(-a +
 c))*sinh(b*x + c)^3 + (2*cosh(-a + c)^2 + 5)*cosh(b*x + c)^2 + (15*cosh(b*
x + c)^4*cosh(-a + c)^2 + 6*(5*cosh(-a + c)^2 + 2)*cosh(b*x + c)^2 + (15*co
sh(b*x + c)^4 + 30*cosh(b*x + c)^2 + 2)*sinh(-a + c)^2 + 2*cosh(-a + c)^2 -
 2*(15*cosh(b*x + c)^4*cosh(-a + c) + 30*cosh(b*x + c)^2*cosh(-a + c) + 2*c
osh(-a + c))*sinh(-a + c) + 5)*sinh(b*x + c)^2 + (cosh(b*x + c)^6 + 5*cosh(
b*x + c)^4 + 2*cosh(b*x + c)^2)*sinh(-a + c)^2 - 3*((cosh(-a + c)^2 - 1)*co
sh(b*x + c)^5 + (cosh(-a + c)^2 - 2*cosh(-a + c)*sinh(-a + c) + sinh(-a + c
)^2 - 1)*sinh(b*x + c)^5 - 5*(2*cosh(b*x + c)*cosh(-a + c)*sinh(-a + c) - c
osh(b*x + c)*sinh(-a + c)^2 - (cosh(-a + c)^2 - 1)*cosh(b*x + c))*sinh(b*x
+ c)^4 + 2*(cosh(-a + c)^2 - 1)*cosh(b*x + c)^3 + 2*(5*(cosh(-a + c)^2 - 1)
*cosh(b*x + c)^2 + (5*cosh(b*x + c)^2 + 1)*sinh(-a + c)^2 + cosh(-a + c)^2
- 2*(5*cosh(b*x + c)^2*cosh(-a + c) + cosh(-a + c))*sinh(-a + c) - 1)*sinh(
b*x + c)^3 + 2*(5*(cosh(-a + c)^2 - 1)*cosh(b*x + c)^3 + (5*cosh(b*x + c)^3
 + 3*cosh(b*x + c))*sinh(-a + c)^2 + 3*(cosh(-a + c)^2 - 1)*cosh(b*x + c) -
 2*(5*cosh(b*x + c)^3*cosh(-a + c) + 3*cosh(b*x + c)*cosh(-a + c))*sinh(-a
+ c))*sinh(b*x + c)^2 + (cosh(b*x + c)^5 + 2*cosh(b*x + c)^3 + cosh(b*x + c
))*sinh(-a + c)^2 + (cosh(-a + c)^2 - 1)*cosh(b*x + c) + (5*(cosh(-a + c)^2
 - 1)*cosh(b*x + c)^4 + 6*(cosh(-a + c)^2 - 1)*cosh(b*x + c)^2 + (5*cosh(b*
x + c)^4 + 6*cosh(b*x + c)^2 + 1)*sinh(-a + c)^2 + cosh(-a + c)^2 - 2*(5*co
sh(b*x + c)^4*cosh(-a + c) + 6*cosh(b*x + c)^2*cosh(-a + c) + cosh(-a + c))
*sinh(-a + c) - 1)*sinh(b*x + c) - 2*(cosh(b*x + c)^5*cosh(-a + c) + 2*cosh
(b*x + c)^3*cosh(-a + c) + cosh(b*x + c)*cosh(-a + c))*sinh(-a + c))*arctan
(cosh(b*x + c) + sinh(b*x + c)) + 2*(3*cosh(b*x + c)^5*cosh(-a + c)^2 + 2*(
5*cosh(-a + c)^2 + 2)*cosh(b*x + c)^3 + (3*cosh(b*x + c)^5 + 10*cosh(b*x +
c)^3 + 2*cosh(b*x + c))*sinh(-a + c)^2 + (2*cosh(-a + c)^2 + 5)*cosh(b*x +
c) - 2*(3*cosh(b*x + c)^5*cosh(-a + c) + 10*cosh(b*x + c)^3*cosh(-a + c) +
```

```
2*cosh(b*x + c)*cosh(-a + c))*sinh(-a + c))*sinh(b*x + c) - 2*(cosh(b*x + c
)^6*cosh(-a + c) + 5*cosh(b*x + c)^4*cosh(-a + c) + 2*cosh(b*x + c)^2*cosh(
-a + c))*sinh(-a + c) + 1)/(b*cosh(b*x + c)^5*cosh(-a + c) + (b*cosh(-a + c
) - b*sinh(-a + c))*sinh(b*x + c)^5 + 2*b*cosh(b*x + c)^3*cosh(-a + c) + 5*
(b*cosh(b*x + c)*cosh(-a + c) - b*cosh(b*x + c)*sinh(-a + c))*sinh(b*x + c)
^4 + 2*(5*b*cosh(b*x + c)^2*cosh(-a + c) + b*cosh(-a + c) - (5*b*cosh(b*x +
 c)^2 + b)*sinh(-a + c))*sinh(b*x + c)^3 + b*cosh(b*x + c)*cosh(-a + c) + 2
*(5*b*cosh(b*x + c)^3*cosh(-a + c) + 3*b*cosh(b*x + c)*cosh(-a + c) - (5*b*
cosh(b*x + c)^3 + 3*b*cosh(b*x + c))*sinh(-a + c))*sinh(b*x + c)^2 + (5*b*c
osh(b*x + c)^4*cosh(-a + c) + 6*b*cosh(b*x + c)^2*cosh(-a + c) + b*cosh(-a
+ c) - (5*b*cosh(b*x + c)^4 + 6*b*cosh(b*x + c)^2 + b)*sinh(-a + c))*sinh(b
*x + c) - (b*cosh(b*x + c)^5 + 2*b*cosh(b*x + c)^3 + b*cosh(b*x + c))*sinh(
-a + c))
```

giac [A] time = 0.14, size = 114, normalized size = 1.58

$$-\frac{3\left(e^{(2a)}-e^{(2c)}\right)\arctan\left(e^{(bx+c)}\right)e^{(-a-c)}-\frac{\left(3e^{(3bx+2a+2c)}+e^{(3bx+4c)}+e^{(bx+2a)}+3e^{(bx+2c)}\right)e^{(-a)}}{\left(e^{(2bx+2c)}+1\right)^2}-e^{(bx+a)}-e^{(-bx-a)}}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*tanh(b*x+c)^3,x, algorithm="giac")
```

```
[Out] -1/2*(3*(e^(2*a) - e^(2*c))*arctan(e^(b*x + c))*e^(-a - c) - (3*e^(3*b*x +
2*a + 2*c) + e^(3*b*x + 4*c) + e^(b*x + 2*a) + 3*e^(b*x + 2*c))*e^(-a)/(e^(
2*b*x + 2*c) + 1)^2 - e^(b*x + a) - e^(-b*x - a))/b
```

maple [C] time = 0.36, size = 238, normalized size = 3.31

$$\frac{e^{bx+a}}{2b}+\frac{e^{-bx-a}}{2b}+\frac{e^{bx+a}\left(3e^{2bx+4a+2c}+e^{2bx+2a+4c}+e^{4a}+3e^{2a+2c}\right)}{2b\left(e^{2bx+2a+2c}+e^{2a}\right)^2}+\frac{3i\ln\left(e^{bx+a}-ie^{a-c}\right)e^{-a-c}e^{2a}}{4b}-\frac{3i\ln\left(e^{bx+a}-ie^{a-}\right.}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)*tanh(b*x+c)^3,x)
```

```
[Out] 1/2*exp(b*x+a)/b+1/2*exp(-b*x-a)/b+1/2*exp(b*x+a)*(3*exp(2*b*x+4*a+2*c)+exp
(2*b*x+2*a+4*c)+exp(4*a)+3*exp(2*a+2*c))/b/(exp(2*b*x+2*a+2*c)+exp(2*a))^2+
3/4*I*ln(exp(b*x+a)-I*exp(a-c))/b*exp(-a-c)*exp(a)^2-3/4*I*ln(exp(b*x+a)-I*
exp(a-c))/b*exp(-a-c)*exp(c)^2-3/4*I*ln(exp(b*x+a)+I*exp(a-c))/b*exp(-a-c)*
exp(a)^2+3/4*I*ln(exp(b*x+a)+I*exp(a-c))/b*exp(-a-c)*exp(c)^2
```

maxima [B] time = 0.43, size = 149, normalized size = 2.07

$$\frac{3\left(e^{(2a)}-e^{(2c)}\right)\arctan\left(e^{(-bx-c)}\right)e^{(-a-c)}}{2b}+\frac{e^{(-bx-a)}}{2b}+\frac{\left(5e^{(2a+2c)}+e^{(4c)}\right)e^{(-2bx-2a)}+\left(2e^{(4a)}+3e^{(2a+2c)}\right)e^{(-4bx-4a)}+\cdots}{2b\left(e^{(-bx-a+4c)}+2e^{(-3bx-a+2c)}+e^{(-5bx-a)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*tanh(b*x+c)^3,x, algorithm="maxima")
```

```
[Out] 3/2*(e^(2*a) - e^(2*c))*arctan(e^(-b*x - c))*e^(-a - c)/b + 1/2*e^(-b*x - a
)/b + 1/2*((5*e^(2*a + 2*c) + e^(4*c))*e^(-2*b*x - 2*a) + (2*e^(4*a) + 3*e^
(2*a + 2*c))*e^(-4*b*x - 4*a) + e^(4*c))/(b*(e^(-b*x - a + 4*c) + 2*e^(-3*b
*x - a + 2*c) + e^(-5*b*x - a)))
```

mupad **[F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \cosh(a+b\,x)\,\tanh(c+b\,x)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a + b*x)*tanh(c + b*x)^3,x)
```

```
[Out] int(cosh(a + b*x)*tanh(c + b*x)^3, x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \cosh(a+bx)\tanh^3(bx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*tanh(b*x+c)**3,x)
```

```
[Out] Integral(cosh(a + b*x)*tanh(b*x + c)**3, x)
```

3.158 $\int \cosh(a+bx)\coth(c+bx)\,dx$

Optimal. Leaf size=29

$$\frac{\cosh(a+bx)}{b}-\frac{\cosh(a-c)\tanh^{-1}(\cosh(bx+c))}{b}$$

```
[Out] -arctanh(cosh(b*x+c))*cosh(a-c)/b+cosh(b*x+a)/b
```

Rubi [A] time = 0.02, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {5621, 2638, 3770}

$$\frac{\cosh(a+bx)}{b}-\frac{\cosh(a-c)\tanh^{-1}(\cosh(bx+c))}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[a + b*x]*Coth[c + b*x],x]
```

```
[Out] -((ArcTanh[Cosh[c + b*x]]*Cosh[a - c])/b) + Cosh[a + b*x]/b
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 5621

```
Int[Cosh[v_]*Coth[w_]^(n_.), x_Symbol] :> Int[Sinh[v]*Coth[w]^(n - 1), x] +
 Dist[Cosh[v - w], Int[Csch[w]*Coth[w]^(n - 1), x], x] /; GtQ[n, 0] && NeQ[
w, v] && FreeQ[v - w, x]
```

Rubi steps

$$\begin{aligned}\int \cosh(a+bx)\coth(c+bx)\,dx &= \cosh(a-c)\int \operatorname{csch}(c+bx)\,dx+\int \sinh(a+bx)\,dx\\ &= -\frac{\tanh^{-1}(\cosh(c+bx))\cosh(a-c)}{b}+\frac{\cosh(a+bx)}{b}\end{aligned}$$

Mathematica [C] time = 0.06, size = 93, normalized size = 3.21

$$-\frac{2i\cosh(a-c)\tan^{-1}\left(\frac{(\cosh(c)-\sinh(c))\left(\sinh(c)\sinh\left(\frac{bx}{2}\right)+\cosh(c)\cosh\left(\frac{bx}{2}\right)\right)}{i\cosh(c)\cosh\left(\frac{bx}{2}\right)-i\sinh(c)\cosh\left(\frac{bx}{2}\right)}\right)}{b}+\frac{\sinh(a)\sinh(bx)}{b}+\frac{\cosh(a)\cosh(bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[a + b*x]*Coth[c + b*x],x]
```

```
[Out] ((-2*I)*ArcTan[((Cosh[c] - Sinh[c])*(Cosh[c]*Cosh[(b*x)/2] + Sinh[c]*Sinh[(
b*x)/2]))/(I*Cosh[c]*Cosh[(b*x)/2] - I*Cosh[(b*x)/2]*Sinh[c])]*Cosh[a - c])
/b + (Cosh[a]*Cosh[b*x])/b + (Sinh[a]*Sinh[b*x])/b
```

fricas [B] time = 0.44, size = 439, normalized size = 15.14

$$\frac{\cosh(bx+c)^2\cosh(-a+c)^2-2\cosh(bx+c)^2\cosh(-a+c)\sinh(-a+c)+\cosh(bx+c)^2\sinh(-a+c)^2+(}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*coth(b*x+c),x, algorithm="fricas")
```

```
[Out] 1/2*(cosh(b*x + c)^2*cosh(-a + c)^2 - 2*cosh(b*x + c)^2*cosh(-a + c)*sinh(-
a + c) + cosh(b*x + c)^2*sinh(-a + c)^2 + (cosh(-a + c)^2 - 2*cosh(-a + c)*
sinh(-a + c) + sinh(-a + c)^2)*sinh(b*x + c)^2 + (2*cosh(b*x + c)*cosh(-a +
 c)*sinh(-a + c) - cosh(b*x + c)*sinh(-a + c)^2 - (cosh(-a + c)^2 + 1)*cosh
(b*x + c) - (cosh(-a + c)^2 - 2*cosh(-a + c)*sinh(-a + c) + sinh(-a + c)^2
+ 1)*sinh(b*x + c))*log(cosh(b*x + c) + sinh(b*x + c) + 1) - (2*cosh(b*x +
c)*cosh(-a + c)*sinh(-a + c) - cosh(b*x + c)*sinh(-a + c)^2 - (cosh(-a + c)
^2 + 1)*cosh(b*x + c) - (cosh(-a + c)^2 - 2*cosh(-a + c)*sinh(-a + c) + sin
h(-a + c)^2 + 1)*sinh(b*x + c))*log(cosh(b*x + c) + sinh(b*x + c) - 1) + 2*
(cosh(b*x + c)*cosh(-a + c)^2 - 2*cosh(b*x + c)*cosh(-a + c)*sinh(-a + c) +
 cosh(b*x + c)*sinh(-a + c)^2)*sinh(b*x + c) + 1)/(b*cosh(b*x + c)*cosh(-a
+ c) - b*cosh(b*x + c)*sinh(-a + c) + (b*cosh(-a + c) - b*sinh(-a + c))*sin
h(b*x + c))
```

giac [B] time = 0.12, size = 85, normalized size = 2.93

$$-\frac{\left(e^{(2a+c)}+e^{(3c)}\right)e^{(-a-2c)}\log\left(e^{(bx+c)}+1\right)-\left(e^{(2a+c)}+e^{(3c)}\right)e^{(-a-2c)}\log\left(\left|e^{(bx+c)}-1\right|\right)-e^{(bx+a)}-e^{(-bx-a)}}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*coth(b*x+c),x, algorithm="giac")
```

```
[Out] -1/2*((e^(2*a + c) + e^(3*c))*e^(-a - 2*c)*log(e^(b*x + c) + 1) - (e^(2*a +
 c) + e^(3*c))*e^(-a - 2*c)*log(abs(e^(b*x + c) - 1)) - e^(b*x + a) - e^(-b
*x - a))/b
```

maple [B] time = 0.22, size = 155, normalized size = 5.34

$$\frac{e^{bx+a}}{2b}+\frac{e^{-bx-a}}{2b}-\frac{\ln\left(e^{bx+a}+e^{a-c}\right)e^{-a-c}e^{2a}}{2b}-\frac{\ln\left(e^{bx+a}+e^{a-c}\right)e^{-a-c}e^{2c}}{2b}+\frac{\ln\left(e^{bx+a}-e^{a-c}\right)e^{-a-c}e^{2a}}{2b}+\frac{\ln\left(e^{bx+a}-e^{a-c}\right.}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)*coth(b*x+c),x)
```

```
[Out] 1/2*exp(b*x+a)/b+1/2*exp(-b*x-a)/b-1/2*ln(exp(b*x+a)+exp(a-c))/b*exp(-a-c)*
exp(2*a)-1/2*ln(exp(b*x+a)+exp(a-c))/b*exp(-a-c)*exp(2*c)+1/2*ln(exp(b*x+a)
-exp(a-c))/b*exp(-a-c)*exp(2*a)+1/2*ln(exp(b*x+a)-exp(a-c))/b*exp(-a-c)*exp
(2*c)
```

maxima [B] time = 0.33, size = 90, normalized size = 3.10

$$-\frac{\left(e^{(2a)}+e^{(2c)}\right)e^{(-a-c)}\log\left(e^{(-bx)}+e^{c}\right)}{2b}+\frac{\left(e^{(2a)}+e^{(2c)}\right)e^{(-a-c)}\log\left(e^{(-bx)}-e^{c}\right)}{2b}+\frac{e^{(bx+a)}}{2b}+\frac{e^{(-bx-a)}}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*coth(b*x+c),x, algorithm="maxima")
```

```
[Out] -1/2*(e^(2*a) + e^(2*c))*e^(-a - c)*log(e^(-b*x) + e^c)/b + 1/2*(e^(2*a) +
e^(2*c))*e^(-a - c)*log(e^(-b*x) - e^c)/b + 1/2*e^(b*x + a)/b + 1/2*e^(-b*x
 - a)/b
```

mupad [B] time = 0.15, size = 139, normalized size = 4.79

$$\frac{e^{a+b\,x}}{2\,b}+\frac{e^{-a-b\,x}}{2\,b}-\frac{\mathrm{atan}\left(\frac{e^{-a}\,e^{2\,c}\,e^{b\,x}\,\left(\sqrt{-b^2}+e^{2\,a}\,e^{-2\,c}\,\sqrt{-b^2}\right)}{b\,\sqrt{e^{-2\,a}\,e^{2\,c}\,\left(2\,e^{2\,a}\,e^{-2\,c}+e^{4\,a}\,e^{-4\,c}+1\right)}}\right)\,\sqrt{e^{2\,c-2\,a}\,\left(2\,e^{2\,a-2\,c}+e^{4\,a-4\,c}+1\right)}}{\sqrt{-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a + b*x)*coth(c + b*x),x)
```

```
[Out] exp(a + b*x)/(2*b) + exp(- a - b*x)/(2*b) - (atan((exp(-a)*exp(2*c)*exp(b*x
)*((-b^2)^(1/2) + exp(2*a)*exp(-2*c)*(-b^2)^(1/2)))/(b*(exp(-2*a)*exp(2*c)*
(2*exp(2*a)*exp(-2*c) + exp(4*a)*exp(-4*c) + 1))^(1/2)))*(exp(2*c - 2*a)*(2
*exp(2*a - 2*c) + exp(4*a - 4*c) + 1))^(1/2))/(-b^2)^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \cosh (a+bx) \coth (bx+c) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*coth(b*x+c),x)
```

```
[Out] Integral(cosh(a + b*x)*coth(b*x + c), x)
```

3.159 $\int \cosh(a+bx)\coth^2(c+bx)\,dx$

Optimal. Leaf size=46

$$-\frac{\cosh(a-c)\operatorname{csch}(bx+c)}{b}-\frac{\sinh(a-c)\tanh^{-1}(\cosh(bx+c))}{b}+\frac{\sinh(a+bx)}{b}$$

```
[Out] -cosh(a-c)*csch(b*x+c)/b-arctanh(cosh(b*x+c))*sinh(a-c)/b+sinh(b*x+a)/b
```

Rubi [A] time = 0.04, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {5621, 5622, 2637, 3770, 2606, 8}

$$-\frac{\cosh(a-c)\operatorname{csch}(bx+c)}{b}-\frac{\sinh(a-c)\tanh^{-1}(\cosh(bx+c))}{b}+\frac{\sinh(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[a + b*x]*Coth[c + b*x]^2,x]
```

```
[Out] -((Cosh[a - c]*Csch[c + b*x])/b) - (ArcTanh[Cosh[c + b*x]]*Sinh[a - c])/b +
 Sinh[a + b*x]/b
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 5621

```
Int[Cosh[v_]*Coth[w_]^(n_.), x_Symbol] :> Int[Sinh[v]*Coth[w]^(n - 1), x] +
 Dist[Cosh[v - w], Int[Csch[w]*Coth[w]^(n - 1), x], x] /; GtQ[n, 0] && NeQ[
w, v] && FreeQ[v - w, x]
```

Rule 5622

```
Int[Coth[w_]^(n_.)*Sinh[v_], x_Symbol] :> Int[Cosh[v]*Coth[w]^(n - 1), x] +
 Dist[Sinh[v - w], Int[Csch[w]*Coth[w]^(n - 1), x], x] /; GtQ[n, 0] && NeQ[
w, v] && FreeQ[v - w, x]
```

Rubi steps

$$\int \cosh(a+bx)\coth^2(c+bx)\,dx = \cosh(a-c)\int \coth(c+bx)\text{csch}(c+bx)\,dx + \int \coth(c+bx)\sinh(a+bx)\,d$$
$$= -\frac{(i\cosh(a-c))\,\text{Subst}(\int 1\,dx, x, -i\text{csch}(c+bx))}{b} + \sinh(a-c)\int \text{csch}(c+$$
$$= -\frac{\cosh(a-c)\text{csch}(c+bx)}{b} - \frac{\tanh^{-1}(\cosh(c+bx))\sinh(a-c)}{b} + \frac{\sinh(a+b}{b}$$

Mathematica [C] time = 0.10, size = 110, normalized size = 2.39

$$-\frac{\cosh(a-c)\text{csch}(bx+c)}{b} - \frac{2i\sinh(a-c)\tan^{-1}\left(\frac{(\cosh(c)-\sinh(c))\left(\sinh(c)\sinh\left(\frac{bx}{2}\right)+\cosh(c)\cosh\left(\frac{bx}{2}\right)\right)}{i\cosh(c)\cosh\left(\frac{bx}{2}\right)-i\sinh(c)\cosh\left(\frac{bx}{2}\right)}\right)}{b} + \frac{\sinh(a)\cosh(bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[a + b*x]*Coth[c + b*x]^2,x]
```

```
[Out] -((Cosh[a - c]*Csch[c + b*x])/b) + (Cosh[b*x]*Sinh[a])/b - ((2*I)*ArcTan[((
Cosh[c] - Sinh[c])*(Cosh[c]*Cosh[(b*x)/2] + Sinh[c]*Sinh[(b*x)/2]))/(I*Cosh
[c]*Cosh[(b*x)/2] - I*Cosh[(b*x)/2]*Sinh[c])]*Sinh[a - c])/b + (Cosh[a]*Sin
h[b*x])/b
```

fricas [B] time = 0.51, size = 1237, normalized size = 26.89

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*coth(b*x+c)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(cosh(b*x + c)^4*cosh(-a + c)^2 + (cosh(-a + c)^2 - 2*cosh(-a + c)*sinh
(-a + c) + sinh(-a + c)^2)*sinh(b*x + c)^4 + 4*(cosh(b*x + c)*cosh(-a + c)^
2 - 2*cosh(b*x + c)*cosh(-a + c)*sinh(-a + c) + cosh(b*x + c)*sinh(-a + c)^
2)*sinh(b*x + c)^3 - 3*(cosh(-a + c)^2 + 1)*cosh(b*x + c)^2 + 3*(2*cosh(b*x
 + c)^2*cosh(-a + c)^2 + (2*cosh(b*x + c)^2 - 1)*sinh(-a + c)^2 - cosh(-a +
 c)^2 - 2*(2*cosh(b*x + c)^2*cosh(-a + c) - cosh(-a + c))*sinh(-a + c) - 1)
*sinh(b*x + c)^2 + (cosh(b*x + c)^4 - 3*cosh(b*x + c)^2)*sinh(-a + c)^2 - (
(cosh(-a + c)^2 - 1)*cosh(b*x + c)^3 + (cosh(-a + c)^2 - 2*cosh(-a + c)*sin
h(-a + c) + sinh(-a + c)^2 - 1)*sinh(b*x + c)^3 - 3*(2*cosh(b*x + c)*cosh(-
a + c)*sinh(-a + c) - cosh(b*x + c)*sinh(-a + c)^2 - (cosh(-a + c)^2 - 1)*c
osh(b*x + c))*sinh(b*x + c)^2 + (cosh(b*x + c)^3 - cosh(b*x + c))*sinh(-a +
 c)^2 - (cosh(-a + c)^2 - 1)*cosh(b*x + c) + (3*(cosh(-a + c)^2 - 1)*cosh(b
*x + c)^2 + (3*cosh(b*x + c)^2 - 1)*sinh(-a + c)^2 - cosh(-a + c)^2 - 2*(3*
cosh(b*x + c)^2*cosh(-a + c) - cosh(-a + c))*sinh(-a + c) + 1)*sinh(b*x + c
) - 2*(cosh(b*x + c)^3*cosh(-a + c) - cosh(b*x + c)*cosh(-a + c))*sinh(-a +
 c))*log(cosh(b*x + c) + sinh(b*x + c) + 1) + ((cosh(-a + c)^2 - 1)*cosh(b*
x + c)^3 + (cosh(-a + c)^2 - 2*cosh(-a + c)*sinh(-a + c) + sinh(-a + c)^2 -
 1)*sinh(b*x + c)^3 - 3*(2*cosh(b*x + c)*cosh(-a + c)*sinh(-a + c) - cosh(b
*x + c)*sinh(-a + c)^2 - (cosh(-a + c)^2 - 1)*cosh(b*x + c))*sinh(b*x + c)^
2 + (cosh(b*x + c)^3 - cosh(b*x + c))*sinh(-a + c)^2 - (cosh(-a + c)^2 - 1)
*cosh(b*x + c) + (3*(cosh(-a + c)^2 - 1)*cosh(b*x + c)^2 + (3*cosh(b*x + c)
^2 - 1)*sinh(-a + c)^2 - cosh(-a + c)^2 - 2*(3*cosh(b*x + c)^2*cosh(-a + c)
 - cosh(-a + c))*sinh(-a + c) + 1)*sinh(b*x + c) - 2*(cosh(b*x + c)^3*cosh(
-a + c) - cosh(b*x + c)*cosh(-a + c))*sinh(-a + c))*log(cosh(b*x + c) + sin
h(b*x + c) - 1) + 2*(2*cosh(b*x + c)^3*cosh(-a + c)^2 + (2*cosh(b*x + c)^3
- 3*cosh(b*x + c))*sinh(-a + c)^2 - 3*(cosh(-a + c)^2 + 1)*cosh(b*x + c) -
2*(2*cosh(b*x + c)^3*cosh(-a + c) - 3*cosh(b*x + c)*cosh(-a + c))*sinh(-a +
 c))*sinh(b*x + c) - 2*(cosh(b*x + c)^4*cosh(-a + c) - 3*cosh(b*x + c)^2*co
sh(-a + c))*sinh(-a + c) + 1)/(b*cosh(b*x + c)^3*cosh(-a + c) + (b*cosh(-a
+ c) - b*sinh(-a + c))*sinh(b*x + c)^3 - b*cosh(b*x + c)*cosh(-a + c) + 3*(
b*cosh(b*x + c)*cosh(-a + c) - b*cosh(b*x + c)*sinh(-a + c))*sinh(b*x + c)^
2 + (3*b*cosh(b*x + c)^2*cosh(-a + c) - b*cosh(-a + c) - (3*b*cosh(b*x + c)
^2 - b)*sinh(-a + c))*sinh(b*x + c) - (b*cosh(b*x + c)^3 - b*cosh(b*x + c))
*sinh(-a + c))
```

giac [B] time = 0.15, size = 125, normalized size = 2.72

$$-\frac{\left(e^{(2a+c)} - e^{(3c)}\right)e^{(-a-2c)}\log\left(e^{(bx+c)}+1\right) - \left(e^{(2a+c)} - e^{(3c)}\right)e^{(-a-2c)}\log\left(\left|e^{(bx+c)}-1\right|\right) + \frac{\left(2e^{(2bx+2a)}+3e^{(2bx+2c)}-1\right)e^{(-a)}}{e^{(3bx+2c)}-e^{(bx)}} - }{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*coth(b*x+c)^2,x, algorithm="giac")
```

```
[Out] -1/2*((e^(2*a + c) - e^(3*c))*e^(-a - 2*c)*log(e^(b*x + c) + 1) - (e^(2*a +
```

```
c) - e^(3*c))*e^(-a - 2*c)*log(abs(e^(b*x + c) - 1)) + (2*e^(2*b*x + 2*a)
+ 3*e^(2*b*x + 2*c) - 1)*e^(-a)/(e^(3*b*x + 2*c) - e^(b*x)) - e^(b*x + a))/
b
```

maple [B] time = 0.26, size = 195, normalized size = 4.24

$$\frac{e^{bx+a}}{2b}-\frac{e^{-bx-a}}{2b}+\frac{e^{bx+a}\left(e^{2a}+e^{2c}\right)}{b\left(-e^{2bx+2a+2c}+e^{2a}\right)}+\frac{\ln\left(e^{bx+a}-e^{a-c}\right)e^{-a-c}e^{2a}}{2b}-\frac{\ln\left(e^{bx+a}-e^{a-c}\right)e^{-a-c}e^{2c}}{2b}-\frac{\ln\left(e^{bx+a}+e^{a-c}\right)e^{-}}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)*coth(b*x+c)^2,x)
```

```
[Out] 1/2*exp(b*x+a)/b-1/2*exp(-b*x-a)/b+1/b*exp(b*x+a)*(exp(2*a)+exp(2*c))/(-exp
(2*b*x+2*a+2*c)+exp(2*a))+1/2*ln(exp(b*x+a)-exp(a-c))/b*exp(-a-c)*exp(2*a)-
1/2*ln(exp(b*x+a)-exp(a-c))/b*exp(-a-c)*exp(2*c)-1/2*ln(exp(b*x+a)+exp(a-c)
)/b*exp(-a-c)*exp(2*a)+1/2*ln(exp(b*x+a)+exp(a-c))/b*exp(-a-c)*exp(2*c)
```

maxima [B] time = 0.36, size = 144, normalized size = 3.13

$$-\frac{\left(e^{(2a)}-e^{(2c)}\right)e^{(-a-c)}\log\left(e^{(-bx)}+e^{c}\right)}{2b}+\frac{\left(e^{(2a)}-e^{(2c)}\right)e^{(-a-c)}\log\left(e^{(-bx)}-e^{c}\right)}{2b}-\frac{e^{(-bx-a)}}{2b}-\frac{\left(3e^{(2a)}+2e^{(2c)}\right)e^{(-2bx-2a}}{2b\left(e^{(-bx-a+2c)}-e^{(-3bx}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*coth(b*x+c)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(e^(2*a) - e^(2*c))*e^(-a - c)*log(e^(-b*x) + e^c)/b + 1/2*(e^(2*a) -
e^(2*c))*e^(-a - c)*log(e^(-b*x) - e^c)/b - 1/2*e^(-b*x - a)/b - 1/2*((3*e^
(2*a) + 2*e^(2*c))*e^(-2*b*x - 2*a) - e^(2*c))/(b*(e^(-b*x - a + 2*c) - e^(
-3*b*x - a)))
```

mupad [B] time = 1.55, size = 183, normalized size = 3.98

$$\frac{e^{a+b\,x}}{2\,b}-\frac{e^{-a-b\,x}}{2\,b}-\frac{\mathrm{atan}\left(-\frac{e^{-a}\,e^{2\,c}\,e^{b\,x}\,\left(\sqrt{-b^2}-e^{2\,a}\,e^{-2\,c}\,\sqrt{-b^2}\right)}{b\,\sqrt{e^{-2\,a}\,e^{2\,c}\,\left(e^{4\,a}\,e^{-4\,c}-2\,e^{2\,a}\,e^{-2\,c}+1\right)}}\right)\,\sqrt{e^{2\,c-2\,a}\,\left(e^{4\,a-4\,c}-2\,e^{2\,a-2\,c}+1\right)}}{\sqrt{-b^2}}+\frac{e^{a+b\,x}\,\left(e^{2\,a-2\,c}+1\right)}{b\,\left(e^{2\,a-2\,c}-e^{2\,a+2\,b\,x}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a + b*x)*coth(c + b*x)^2,x)
```

```
[Out] exp(a + b*x)/(2*b) - exp(- a - b*x)/(2*b) - (atan(-(exp(-a)*exp(2*c)*exp(b*
x)*((-b^2)^(1/2) - exp(2*a)*exp(-2*c)*(-b^2)^(1/2)))/(b*(exp(-2*a)*exp(2*c)
*(exp(4*a)*exp(-4*c) - 2*exp(2*a)*exp(-2*c) + 1))^(1/2)))*(exp(2*c - 2*a)*(
```

```
exp(4*a - 4*c) - 2*exp(2*a - 2*c) + 1))^(1/2))/(-b^2)^(1/2) + (exp(a + b*x)
*(exp(2*a - 2*c) + 1))/(b*(exp(2*a - 2*c) - exp(2*a + 2*b*x)))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \cosh(a+bx)\coth^2(bx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*coth(b*x+c)**2,x)
```

```
[Out] Integral(cosh(a + b*x)*coth(b*x + c)**2, x)
```

3.160 $\int \cosh(a+bx)\coth^3(c+bx)\,dx$

Optimal. Leaf size=73

$$-\frac{3\cosh(a-c)\tanh^{-1}(\cosh(bx+c))}{2b}-\frac{\sinh(a-c)\text{csch}(bx+c)}{b}-\frac{\cosh(a-c)\coth(bx+c)\text{csch}(bx+c)}{2b}+\frac{\cosh(a}{b}$$

```
[Out] -3/2*arctanh(cosh(b*x+c))*cosh(a-c)/b+cosh(b*x+a)/b-1/2*cosh(a-c)*coth(b*x+
c)*csch(b*x+c)/b-csch(b*x+c)*sinh(a-c)/b
```

Rubi [A] time = 0.09, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.467, Rules used = {5621, 5622, 2638, 3770, 2606, 8, 2611}

$$-\frac{3\cosh(a-c)\tanh^{-1}(\cosh(bx+c))}{2b}-\frac{\sinh(a-c)\text{csch}(bx+c)}{b}-\frac{\cosh(a-c)\coth(bx+c)\text{csch}(bx+c)}{2b}+\frac{\cosh(a}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[a + b*x]*Coth[c + b*x]^3,x]
```

```
[Out] (-3*ArcTanh[Cosh[c + b*x]]*Cosh[a - c])/(2*b) + Cosh[a + b*x]/b - (Cosh[a -
 c]*Coth[c + b*x]*Csch[c + b*x])/(2*b) - (Csch[c + b*x]*Sinh[a - c])/b
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 && !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 5621

```
Int[Cosh[v_]*Coth[w_]^(n_.), x_Symbol] :> Int[Sinh[v]*Coth[w]^(n - 1), x] +
 Dist[Cosh[v - w], Int[Csch[w]*Coth[w]^(n - 1), x], x] /; GtQ[n, 0] && NeQ[
w, v] && FreeQ[v - w, x]
```

Rule 5622

```
Int[Coth[w_]^(n_.)*Sinh[v_], x_Symbol] :> Int[Cosh[v]*Coth[w]^(n - 1), x] +
 Dist[Sinh[v - w], Int[Csch[w]*Coth[w]^(n - 1), x], x] /; GtQ[n, 0] && NeQ[
w, v] && FreeQ[v - w, x]
```

Rubi steps

$$\begin{aligned}\int \cosh(a+bx)\coth^3(c+bx)\,dx &= \cosh(a-c)\int \coth^2(c+bx)\text{csch}(c+bx)\,dx + \int \coth^2(c+bx)\sinh(a+bx)\\ &= -\frac{\cosh(a-c)\coth(c+bx)\text{csch}(c+bx)}{2b} + \frac{1}{2}\cosh(a-c)\int \text{csch}(c+bx)\,dx +\\ &= -\frac{\tanh^{-1}(\cosh(c+bx))\cosh(a-c)}{2b} - \frac{\cosh(a-c)\coth(c+bx)\text{csch}(c+bx)}{2b}\\ &= -\frac{3\tanh^{-1}(\cosh(c+bx))\cosh(a-c)}{2b} + \frac{\cosh(a+bx)}{b} - \frac{\cosh(a-c)\coth(c+}{2b}\end{aligned}$$

Mathematica [A] time = 0.34, size = 70, normalized size = 0.96

$$\frac{\text{csch}^2(bx+c)(2\cosh(a-bx-2c)+\cosh(a+3bx+2c)-5\cosh(a+bx))-12\cosh(a-c)\tanh^{-1}\left(\sinh(c)\tanh\right.}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[a + b*x]*Coth[c + b*x]^3,x]
```

```
[Out] (-12*ArcTanh[Cosh[c] + Sinh[c]*Tanh[(b*x)/2]]*Cosh[a - c] + (2*Cosh[a - 2*c
 - b*x] - 5*Cosh[a + b*x] + Cosh[a + 2*c + 3*b*x])*Csch[c + b*x]^2)/(4*b)
```

fricas [B] time = 0.46, size = 2372, normalized size = 32.49

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*coth(b*x+c)^3,x, algorithm="fricas")
```

```
[Out] 1/4*(2*cosh(b*x + c)^6*cosh(-a + c)^2 + 2*(cosh(-a + c)^2 - 2*cosh(-a + c)*
sinh(-a + c) + sinh(-a + c)^2)*sinh(b*x + c)^6 + 12*(cosh(b*x + c)*cosh(-a
+ c)^2 - 2*cosh(b*x + c)*cosh(-a + c)*sinh(-a + c) + cosh(b*x + c)*sinh(-a
+ c)^2)*sinh(b*x + c)^5 - 2*(5*cosh(-a + c)^2 - 2)*cosh(b*x + c)^4 + 2*(15*
cosh(b*x + c)^2*cosh(-a + c)^2 + 5*(3*cosh(b*x + c)^2 - 1)*sinh(-a + c)^2 -
 5*cosh(-a + c)^2 - 10*(3*cosh(b*x + c)^2*cosh(-a + c) - cosh(-a + c))*sinh
(-a + c) + 2)*sinh(b*x + c)^4 + 8*(5*cosh(b*x + c)^3*cosh(-a + c)^2 + 5*(co
sh(b*x + c)^3 - cosh(b*x + c))*sinh(-a + c)^2 - (5*cosh(-a + c)^2 - 2)*cosh
(b*x + c) - 10*(cosh(b*x + c)^3*cosh(-a + c) - cosh(b*x + c)*cosh(-a + c))*
sinh(-a + c))*sinh(b*x + c)^3 + 2*(2*cosh(-a + c)^2 - 5)*cosh(b*x + c)^2 +
2*(15*cosh(b*x + c)^4*cosh(-a + c)^2 - 6*(5*cosh(-a + c)^2 - 2)*cosh(b*x +
c)^2 + (15*cosh(b*x + c)^4 - 30*cosh(b*x + c)^2 + 2)*sinh(-a + c)^2 + 2*cos
h(-a + c)^2 - 2*(15*cosh(b*x + c)^4*cosh(-a + c) - 30*cosh(b*x + c)^2*cosh(
-a + c) + 2*cosh(-a + c))*sinh(-a + c) - 5)*sinh(b*x + c)^2 + 2*(cosh(b*x +
 c)^6 - 5*cosh(b*x + c)^4 + 2*cosh(b*x + c)^2)*sinh(-a + c)^2 - 3*((cosh(-a
 + c)^2 + 1)*cosh(b*x + c)^5 + (cosh(-a + c)^2 - 2*cosh(-a + c)*sinh(-a + c
) + sinh(-a + c)^2 + 1)*sinh(b*x + c)^5 - 5*(2*cosh(b*x + c)*cosh(-a + c)*s
inh(-a + c) - cosh(b*x + c)*sinh(-a + c)^2 - (cosh(-a + c)^2 + 1)*cosh(b*x
+ c))*sinh(b*x + c)^4 - 2*(cosh(-a + c)^2 + 1)*cosh(b*x + c)^3 + 2*(5*(cosh
(-a + c)^2 + 1)*cosh(b*x + c)^2 + (5*cosh(b*x + c)^2 - 1)*sinh(-a + c)^2 -
cosh(-a + c)^2 - 2*(5*cosh(b*x + c)^2*cosh(-a + c) - cosh(-a + c))*sinh(-a
+ c) - 1)*sinh(b*x + c)^3 + 2*(5*(cosh(-a + c)^2 + 1)*cosh(b*x + c)^3 + (5*
cosh(b*x + c)^3 - 3*cosh(b*x + c))*sinh(-a + c)^2 - 3*(cosh(-a + c)^2 + 1)*
cosh(b*x + c) - 2*(5*cosh(b*x + c)^3*cosh(-a + c) - 3*cosh(b*x + c)*cosh(-a
 + c))*sinh(-a + c))*sinh(b*x + c)^2 + (cosh(b*x + c)^5 - 2*cosh(b*x + c)^3
 + cosh(b*x + c))*sinh(-a + c)^2 + (cosh(-a + c)^2 + 1)*cosh(b*x + c) + (5*
(cosh(-a + c)^2 + 1)*cosh(b*x + c)^4 - 6*(cosh(-a + c)^2 + 1)*cosh(b*x + c)
^2 + (5*cosh(b*x + c)^4 - 6*cosh(b*x + c)^2 + 1)*sinh(-a + c)^2 + cosh(-a +
 c)^2 - 2*(5*cosh(b*x + c)^4*cosh(-a + c) - 6*cosh(b*x + c)^2*cosh(-a + c)
+ cosh(-a + c))*sinh(-a + c) + 1)*sinh(b*x + c) - 2*(cosh(b*x + c)^5*cosh(-
a + c) - 2*cosh(b*x + c)^3*cosh(-a + c) + cosh(b*x + c)*cosh(-a + c))*sinh(
-a + c))*log(cosh(b*x + c) + sinh(b*x + c) + 1) + 3*((cosh(-a + c)^2 + 1)*c
osh(b*x + c)^5 + (cosh(-a + c)^2 - 2*cosh(-a + c)*sinh(-a + c) + sinh(-a +
c)^2 + 1)*sinh(b*x + c)^5 - 5*(2*cosh(b*x + c)*cosh(-a + c)*sinh(-a + c) -
cosh(b*x + c)*sinh(-a + c)^2 - (cosh(-a + c)^2 + 1)*cosh(b*x + c))*sinh(b*x
 + c)^4 - 2*(cosh(-a + c)^2 + 1)*cosh(b*x + c)^3 + 2*(5*(cosh(-a + c)^2 + 1
)*cosh(b*x + c)^2 + (5*cosh(b*x + c)^2 - 1)*sinh(-a + c)^2 - cosh(-a + c)^2
```

```
 - 2*(5*cosh(b*x + c)^2*cosh(-a + c) - cosh(-a + c))*sinh(-a + c) - 1)*sinh
(b*x + c)^3 + 2*(5*(cosh(-a + c)^2 + 1)*cosh(b*x + c)^3 + (5*cosh(b*x + c)^
3 - 3*cosh(b*x + c))*sinh(-a + c)^2 - 3*(cosh(-a + c)^2 + 1)*cosh(b*x + c)
- 2*(5*cosh(b*x + c)^3*cosh(-a + c) - 3*cosh(b*x + c)*cosh(-a + c))*sinh(-a
 + c))*sinh(b*x + c)^2 + (cosh(b*x + c)^5 - 2*cosh(b*x + c)^3 + cosh(b*x +
c))*sinh(-a + c)^2 + (cosh(-a + c)^2 + 1)*cosh(b*x + c) + (5*(cosh(-a + c)^
2 + 1)*cosh(b*x + c)^4 - 6*(cosh(-a + c)^2 + 1)*cosh(b*x + c)^2 + (5*cosh(b
*x + c)^4 - 6*cosh(b*x + c)^2 + 1)*sinh(-a + c)^2 + cosh(-a + c)^2 - 2*(5*c
osh(b*x + c)^4*cosh(-a + c) - 6*cosh(b*x + c)^2*cosh(-a + c) + cosh(-a + c)
)*sinh(-a + c) + 1)*sinh(b*x + c) - 2*(cosh(b*x + c)^5*cosh(-a + c) - 2*cos
h(b*x + c)^3*cosh(-a + c) + cosh(b*x + c)*cosh(-a + c))*sinh(-a + c))*log(c
osh(b*x + c) + sinh(b*x + c) - 1) + 4*(3*cosh(b*x + c)^5*cosh(-a + c)^2 - 2
*(5*cosh(-a + c)^2 - 2)*cosh(b*x + c)^3 + (3*cosh(b*x + c)^5 - 10*cosh(b*x
+ c)^3 + 2*cosh(b*x + c))*sinh(-a + c)^2 + (2*cosh(-a + c)^2 - 5)*cosh(b*x
+ c) - 2*(3*cosh(b*x + c)^5*cosh(-a + c) - 10*cosh(b*x + c)^3*cosh(-a + c)
+ 2*cosh(b*x + c)*cosh(-a + c))*sinh(-a + c))*sinh(b*x + c) - 4*(cosh(b*x +
 c)^6*cosh(-a + c) - 5*cosh(b*x + c)^4*cosh(-a + c) + 2*cosh(b*x + c)^2*cos
h(-a + c))*sinh(-a + c) + 2)/(b*cosh(b*x + c)^5*cosh(-a + c) + (b*cosh(-a +
 c) - b*sinh(-a + c))*sinh(b*x + c)^5 - 2*b*cosh(b*x + c)^3*cosh(-a + c) +
5*(b*cosh(b*x + c)*cosh(-a + c) - b*cosh(b*x + c)*sinh(-a + c))*sinh(b*x +
c)^4 + 2*(5*b*cosh(b*x + c)^2*cosh(-a + c) - b*cosh(-a + c) - (5*b*cosh(b*x
 + c)^2 - b)*sinh(-a + c))*sinh(b*x + c)^3 + b*cosh(b*x + c)*cosh(-a + c) +
 2*(5*b*cosh(b*x + c)^3*cosh(-a + c) - 3*b*cosh(b*x + c)*cosh(-a + c) - (5*
b*cosh(b*x + c)^3 - 3*b*cosh(b*x + c))*sinh(-a + c))*sinh(b*x + c)^2 + (5*b
*cosh(b*x + c)^4*cosh(-a + c) - 6*b*cosh(b*x + c)^2*cosh(-a + c) + b*cosh(-
a + c) - (5*b*cosh(b*x + c)^4 - 6*b*cosh(b*x + c)^2 + b)*sinh(-a + c))*sinh
(b*x + c) - (b*cosh(b*x + c)^5 - 2*b*cosh(b*x + c)^3 + b*cosh(b*x + c))*sin
h(-a + c))
```

giac [B] time = 0.16, size = 151, normalized size = 2.07

$$-\frac{3\left(e^{(2a+c)}+e^{(3c)}\right)e^{(-a-2c)}\log\left(e^{(bx+c)}+1\right)-3\left(e^{(2a+c)}+e^{(3c)}\right)e^{(-a-2c)}\log\left(\left|e^{(bx+c)}-1\right|\right)+\frac{2\left(3e^{(3bx+2a+2c)}-e^{(3bx+4c)}-e^{(b\cdots}\right.}{\left(e^{(2bx+2c)}-\cdots\right.}}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*coth(b*x+c)^3,x, algorithm="giac")
```

```
[Out] -1/4*(3*(e^(2*a + c) + e^(3*c))*e^(-a - 2*c)*log(e^(b*x + c) + 1) - 3*(e^(2
*a + c) + e^(3*c))*e^(-a - 2*c)*log(abs(e^(b*x + c) - 1)) + 2*(3*e^(3*b*x +
 2*a + 2*c) - e^(3*b*x + 4*c) - e^(b*x + 2*a) + 3*e^(b*x + 2*c))*e^(-a)/(e^
(2*b*x + 2*c) - 1)^2 - 2*e^(b*x + a) - 2*e^(-b*x - a))/b
```

maple [B] time = 0.26, size = 228, normalized size = 3.12

$$\frac{e^{bx+a}}{2b}+\frac{e^{-bx-a}}{2b}+\frac{e^{bx+a}\left(-3\,e^{2bx+4a+2c}+e^{2bx+2a+4c}+e^{4a}-3\,e^{2a+2c}\right)}{2b\left(-e^{2bx+2a+2c}+e^{2a}\right)^{2}}-\frac{3\ln\left(e^{bx+a}+e^{a-c}\right)e^{-a-c}e^{2a}}{4b}-\frac{3\ln\left(e^{bx+a}+e^{a\cdot}\right.}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)*coth(b*x+c)^3,x)
```

```
[Out] 1/2*exp(b*x+a)/b+1/2*exp(-b*x-a)/b+1/2*exp(b*x+a)*(-3*exp(2*b*x+4*a+2*c)+ex
p(2*b*x+2*a+4*c)+exp(4*a)-3*exp(2*a+2*c))/b/(-exp(2*b*x+2*a+2*c)+exp(2*a))^
2-3/4*ln(exp(b*x+a)+exp(a-c))/b*exp(-a-c)*exp(2*a)-3/4*ln(exp(b*x+a)+exp(a-
c))/b*exp(-a-c)*exp(2*c)+3/4*ln(exp(b*x+a)-exp(a-c))/b*exp(-a-c)*exp(2*a)+3
/4*ln(exp(b*x+a)-exp(a-c))/b*exp(-a-c)*exp(2*c)
```

maxima [B] time = 0.32, size = 184, normalized size = 2.52

$$-\frac{3\left(e^{(2a)}+e^{(2c)}\right)e^{(-a-c)}\log\left(e^{(-bx)}+e^{c}\right)}{4b}+\frac{3\left(e^{(2a)}+e^{(2c)}\right)e^{(-a-c)}\log\left(e^{(-bx)}-e^{c}\right)}{4b}+\frac{e^{(-bx-a)}}{2b}-\frac{\left(5\,e^{(2a+2c)}-e^{(4c)}\right)e^{(-2}}{2b\left(e^{(-bx-}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*coth(b*x+c)^3,x, algorithm="maxima")
```

```
[Out] -3/4*(e^(2*a) + e^(2*c))*e^(-a - c)*log(e^(-b*x) + e^c)/b + 3/4*(e^(2*a) +
e^(2*c))*e^(-a - c)*log(e^(-b*x) - e^c)/b + 1/2*e^(-b*x - a)/b - 1/2*((5*e^
(2*a + 2*c) - e^(4*c))*e^(-2*b*x - 2*a) - (2*e^(4*a) - 3*e^(2*a + 2*c))*e^(
-4*b*x - 4*a) - e^(4*c))/(b*(e^(-b*x - a + 4*c) - 2*e^(-3*b*x - a + 2*c) +
e^(-5*b*x - a)))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \cosh\left(a+b\,x\right)\,{\mathrm{coth}\left(c+b\,x\right)}^3 \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a + b*x)*coth(c + b*x)^3,x)
```

```
[Out] int(cosh(a + b*x)*coth(c + b*x)^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \cosh\left(a+bx\right)\coth^{3}\left(bx+c\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*coth(b*x+c)**3,x)
[Out] Integral(cosh(a + b*x)*coth(b*x + c)**3, x)
```

3.161 $\int \cosh(a+bx)\,\mathbf{sech}(c+bx)\,dx$

Optimal. Leaf size=26

$$\frac{\sinh(a-c)\log(\cosh(bx+c))}{b}+x\cosh(a-c)$$

```
[Out] x*cosh(a-c)+ln(cosh(b*x+c))*sinh(a-c)/b
```

Rubi [A] time = 0.01, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {5627, 3475, 8}

$$\frac{\sinh(a-c)\log(\cosh(bx+c))}{b}+x\cosh(a-c)$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[a + b*x]*Sech[c + b*x],x]
```

```
[Out] x*Cosh[a - c] + (Log[Cosh[c + b*x]]*Sinh[a - c])/b
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 5627

```
Int[Cosh[v_]*Sech[w_]^(n_.), x_Symbol] :> Dist[Sinh[v - w], Int[Tanh[w]*Sec
h[w]^(n - 1), x], x] + Dist[Cosh[v - w], Int[Sech[w]^(n - 1), x], x] /; GtQ
[n, 0] && NeQ[w, v] && FreeQ[v - w, x]
```

Rubi steps

$$\begin{aligned}\int \cosh(a+bx)\operatorname{sech}(c+bx)\,dx &= \cosh(a-c)\int 1\,dx+\sinh(a-c)\int \tanh(c+bx)\,dx\\ &= x\cosh(a-c)+\frac{\log(\cosh(c+bx))\sinh(a-c)}{b}\end{aligned}$$

Mathematica [A] time = 0.11, size = 26, normalized size = 1.00

$$\frac{\sinh(a-c)\log(\cosh(bx+c))}{b}+x\cosh(a-c)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[a + b*x]*Sech[c + b*x],x]
```

```
[Out] x*Cosh[a - c] + (Log[Cosh[c + b*x]]*Sinh[a - c])/b
```

fricas [B] time = 0.51, size = 86, normalized size = 3.31

$$\frac{2\,bx+\left(\cosh\left(-a+c\right)^{2}-2\,\cosh\left(-a+c\right)\sinh\left(-a+c\right)+\sinh\left(-a+c\right)^{2}-1\right)\log\left(\frac{2\,\cosh(bx+c)}{\cosh(bx+c)-\sinh(bx+c)}\right)}{2\,(b\cosh\left(-a+c\right)-b\sinh\left(-a+c\right))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*sech(b*x+c),x, algorithm="fricas")
```

```
[Out] 1/2*(2*b*x + (cosh(-a + c)^2 - 2*cosh(-a + c)*sinh(-a + c) + sinh(-a + c)^2
 - 1)*log(2*cosh(b*x + c)/(cosh(b*x + c) - sinh(b*x + c))))/(b*cosh(-a + c)
 - b*sinh(-a + c))
```

giac [A] time = 0.12, size = 50, normalized size = 1.92

$$\frac{2\,bxe^{(-a+c)}+\left(e^{(2\,a+c)}-e^{(3\,c)}\right)e^{(-a-2\,c)}\log\left(e^{(2\,bx+2\,c)}+1\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*sech(b*x+c),x, algorithm="giac")
```

```
[Out] 1/2*(2*b*x*e^(-a + c) + (e^(2*a + c) - e^(3*c))*e^(-a - 2*c)*log(e^(2*b*x +
 2*c) + 1))/b
```

maple [B] time = 0.16, size = 146, normalized size = 5.62

$$x\,\mathrm{e}^{a-c}-\mathrm{e}^{-a-c}\mathrm{e}^{2a}x+\mathrm{e}^{-a-c}\mathrm{e}^{2c}x-\frac{\mathrm{e}^{-a-c}\mathrm{e}^{2a}a}{b}+\frac{\mathrm{e}^{-a-c}\mathrm{e}^{2c}a}{b}+\frac{\ln\left(\mathrm{e}^{2bx+2a}+\mathrm{e}^{2a-2c}\right)\mathrm{e}^{-a-c}\mathrm{e}^{2a}}{2b}-\frac{\ln\left(\mathrm{e}^{2bx+2a}+\mathrm{e}^{2a-2c}\right)\mathrm{e}^{-a-c}\mathrm{e}^{2c}}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)*sech(b*x+c),x)
```

```
[Out] x*exp(a-c)-exp(-a-c)*exp(2*a)*x+exp(-a-c)*exp(2*c)*x-1/b*exp(-a-c)*exp(2*a)
*a+1/b*exp(-a-c)*exp(2*c)*a+1/2*ln(exp(2*b*x+2*a)+exp(2*a-2*c))/b*exp(-a-c)
*exp(2*a)-1/2*ln(exp(2*b*x+2*a)+exp(2*a-2*c))/b*exp(-a-c)*exp(2*c)
```

maxima [A] time = 0.32, size = 51, normalized size = 1.96

$$\frac{\left(e^{(2\,a)}-e^{(2\,c)}\right)e^{(-a-c)}\log\left(e^{(-2\,bx)}+e^{(2\,c)}\right)}{2\,b}+\frac{(bx+a)e^{(a-c)}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*sech(b*x+c),x, algorithm="maxima")
```

```
[Out] 1/2*(e^(2*a) - e^(2*c))*e^(-a - c)*log(e^(-2*b*x) + e^(2*c))/b + (b*x + a)*
e^(a - c)/b
```

mupad [B] time = 0.20, size = 64, normalized size = 2.46

$$x\,\mathrm{e}^{c-a}+\frac{\mathrm{e}^{2\,c-2\,a}\,\ln\left(\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}+\mathrm{e}^{2\,a}\,\mathrm{e}^{-2\,c}\right)\,\left(2\,b\,\mathrm{e}^{3\,a-3\,c}-2\,b\,\mathrm{e}^{a-c}\right)}{4\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a + b*x)/cosh(c + b*x),x)
```

```
[Out] x*exp(c - a) + (exp(2*c - 2*a)*log(exp(2*a)*exp(2*b*x) + exp(2*a)*exp(-2*c)
)*(2*b*exp(3*a - 3*c) - 2*b*exp(a - c)))/(4*b^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\cosh\left(a+bx\right)\operatorname{sech}\left(bx+c\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*sech(b*x+c),x)
```

```
[Out] Integral(cosh(a + b*x)*sech(b*x + c), x)
```

3.162 $\int \cosh(a+bx)\mathbf{sech}^2(c+bx)\,dx$

Optimal. Leaf size=35

$$\frac{\cosh(a-c)\tan^{-1}(\sinh(bx+c))}{b}-\frac{\sinh(a-c)\text{sech}(bx+c)}{b}$$

```
[Out] arctan(sinh(b*x+c))*cosh(a-c)/b-sech(b*x+c)*sinh(a-c)/b
```

Rubi [A] time = 0.03, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {5627, 2606, 8, 3770}

$$\frac{\cosh(a-c)\tan^{-1}(\sinh(bx+c))}{b}-\frac{\sinh(a-c)\text{sech}(bx+c)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[a + b*x]*Sech[c + b*x]^2,x]
```

```
[Out] (ArcTan[Sinh[c + b*x]]*Cosh[a - c])/b - (Sech[c + b*x]*Sinh[a - c])/b
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 5627

```
Int[Cosh[v_]*Sech[w_]^(n_.), x_Symbol] :> Dist[Sinh[v - w], Int[Tanh[w]*Sec
h[w]^(n - 1), x], x] + Dist[Cosh[v - w], Int[Sech[w]^(n - 1), x], x] /; GtQ
[n, 0] && NeQ[w, v] && FreeQ[v - w, x]
```

Rubi steps

$$\int \cosh(a+bx)\text{sech}^2(c+bx)\,dx = \cosh(a-c)\int \text{sech}(c+bx)\,dx + \sinh(a-c)\int \text{sech}(c+bx)\tanh(c+bx)\,dx$$
$$= \frac{\tan^{-1}(\sinh(c+bx))\cosh(a-c)}{b} - \frac{\sinh(a-c)\,\text{Subst}(\int 1\,dx, x, \text{sech}(c+bx))}{b}$$
$$= \frac{\tan^{-1}(\sinh(c+bx))\cosh(a-c)}{b} - \frac{\text{sech}(c+bx)\sinh(a-c)}{b}$$

Mathematica [B] time = 0.09, size = 83, normalized size = 2.37

$$\frac{2\cosh(a-c)\tan^{-1}\left(\frac{(\cosh(c)-\sinh(c))\left(\sinh(c)\cosh\left(\frac{bx}{2}\right)+\cosh(c)\sinh\left(\frac{bx}{2}\right)\right)}{\cosh(c)\cosh\left(\frac{bx}{2}\right)-\sinh(c)\cosh\left(\frac{bx}{2}\right)}\right)}{b} - \frac{\sinh(a-c)\text{sech}(bx+c)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[a + b*x]*Sech[c + b*x]^2,x]
```

```
[Out] (2*ArcTan[((Cosh[c] - Sinh[c])*(Cosh[(b*x)/2]*Sinh[c] + Cosh[c]*Sinh[(b*x)/
2]))/(Cosh[c]*Cosh[(b*x)/2] - Cosh[(b*x)/2]*Sinh[c])]*Cosh[a - c])/b - (Sec
h[c + b*x]*Sinh[a - c])/b
```

fricas [B] time = 0.45, size = 405, normalized size = 11.57

$$\frac{2\cosh(bx+c)\cosh(-a+c)\sinh(-a+c) - \cosh(bx+c)\sinh(-a+c)^2 + \left(\left(\cosh(-a+c)^2+1\right)\cosh(bx+c)\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*sech(b*x+c)^2,x, algorithm="fricas")
```

```
[Out] (2*cosh(b*x + c)*cosh(-a + c)*sinh(-a + c) - cosh(b*x + c)*sinh(-a + c)^2 +
 ((cosh(-a + c)^2 + 1)*cosh(b*x + c)^2 + (cosh(-a + c)^2 - 2*cosh(-a + c)*s
inh(-a + c) + sinh(-a + c)^2 + 1)*sinh(b*x + c)^2 + (cosh(b*x + c)^2 + 1)*s
inh(-a + c)^2 + cosh(-a + c)^2 - 2*(2*cosh(b*x + c)*cosh(-a + c)*sinh(-a +
c) - cosh(b*x + c)*sinh(-a + c)^2 - (cosh(-a + c)^2 + 1)*cosh(b*x + c))*sin
h(b*x + c) - 2*(cosh(b*x + c)^2*cosh(-a + c) + cosh(-a + c))*sinh(-a + c) +
 1)*arctan(cosh(b*x + c) + sinh(b*x + c)) - (cosh(-a + c)^2 - 1)*cosh(b*x +
 c) - (cosh(-a + c)^2 - 2*cosh(-a + c)*sinh(-a + c) + sinh(-a + c)^2 - 1)*s
inh(b*x + c))/(b*cosh(b*x + c)^2*cosh(-a + c) + (b*cosh(-a + c) - b*sinh(-a
 + c))*sinh(b*x + c)^2 + b*cosh(-a + c) + 2*(b*cosh(b*x + c)*cosh(-a + c) -
 b*cosh(b*x + c)*sinh(-a + c))*sinh(b*x + c) - (b*cosh(b*x + c)^2 + b)*sinh
(-a + c))
```

giac [A] time = 0.14, size = 68, normalized size = 1.94

$$\frac{\left(e^{(2a)} + e^{(2c)}\right)\arctan\left(e^{(bx+c)}\right)e^{(-a-c)} - \frac{\left(e^{(bx+2a)} - e^{(bx+2c)}\right)e^{(-a)}}{e^{(2bx+2c)}+1}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*sech(b*x+c)^2,x, algorithm="giac")
```

```
[Out] ((e^(2*a) + e^(2*c))*arctan(e^(b*x + c))*e^(-a - c) - (e^(b*x + 2*a) - e^(b
*x + 2*c))*e^(-a)/(e^(2*b*x + 2*c) + 1))/b
```

maple [C] time = 0.28, size = 183, normalized size = 5.23

$$-\frac{e^{bx+a}\left(e^{2a}-e^{2c}\right)}{b\left(e^{2bx+2a+2c}+e^{2a}\right)}+\frac{i\ln\left(e^{bx+a}+ie^{a-c}\right)e^{-a-c}e^{2a}}{2b}+\frac{i\ln\left(e^{bx+a}+ie^{a-c}\right)e^{-a-c}e^{2c}}{2b}-\frac{i\ln\left(e^{bx+a}-ie^{a-c}\right)e^{-a-c}e^{2a}}{2b}-\frac{i\ln}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)*sech(b*x+c)^2,x)
```

```
[Out] -1/b*exp(b*x+a)*(exp(2*a)-exp(2*c))/(exp(2*b*x+2*a+2*c)+exp(2*a))+1/2*I*ln(
exp(b*x+a)+I*exp(a-c))/b*exp(-a-c)*exp(2*a)+1/2*I*ln(exp(b*x+a)+I*exp(a-c))
/b*exp(-a-c)*exp(2*c)-1/2*I*ln(exp(b*x+a)-I*exp(a-c))/b*exp(-a-c)*exp(2*a)-
1/2*I*ln(exp(b*x+a)-I*exp(a-c))/b*exp(-a-c)*exp(2*c)
```

maxima [A] time = 0.42, size = 70, normalized size = 2.00

$$-\frac{\left(e^{(2a)} + e^{(2c)}\right)\arctan\left(e^{(-bx-c)}\right)e^{(-a-c)}}{b} - \frac{\left(e^{(2a)} - e^{(2c)}\right)e^{(-bx-a)}}{b\left(e^{(-2bx)} + e^{(2c)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*sech(b*x+c)^2,x, algorithm="maxima")
```

```
[Out] -(e^(2*a) + e^(2*c))*arctan(e^(-b*x - c))*e^(-a - c)/b - (e^(2*a) - e^(2*c)
)*e^(-b*x - a)/(b*(e^(-2*b*x) + e^(2*c)))
```

mupad [B] time = 1.55, size = 148, normalized size = 4.23

$$\frac{\mathrm{atan}\left(\frac{e^{-a}\,e^{2c}\,e^{bx}\left(\sqrt{b^2}+e^{2a}\,e^{-2c}\,\sqrt{b^2}\right)}{b\sqrt{e^{-2a}\,e^{2c}\left(2\,e^{2a}\,e^{-2c}+e^{4a}\,e^{-4c}+1\right)}}\right)\sqrt{e^{2c-2a}\,\left(2\,e^{2a-2c}+e^{4a-4c}+1\right)}}{\sqrt{b^2}} - \frac{e^{a+bx}\,\left(e^{2a-2c}-1\right)}{b\,\left(e^{2a-2c}+e^{2a+2bx}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a + b*x)/cosh(c + b*x)^2,x)
```

```
[Out] (atan((exp(-a)*exp(2*c)*exp(b*x)*((b^2)^(1/2) + exp(2*a)*exp(-2*c)*(b^2)^(1
/2)))/(b*(exp(-2*a)*exp(2*c)*(2*exp(2*a)*exp(-2*c) + exp(4*a)*exp(-4*c) + 1
))^(1/2)))*(exp(2*c - 2*a)*(2*exp(2*a - 2*c) + exp(4*a - 4*c) + 1))^(1/2))/
(b^2)^(1/2) - (exp(a + b*x)*(exp(2*a - 2*c) - 1))/(b*(exp(2*a - 2*c) + exp(
2*a + 2*b*x)))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \cosh(a+bx)\operatorname{sech}^2(bx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*sech(b*x+c)**2,x)
```

```
[Out] Integral(cosh(a + b*x)*sech(b*x + c)**2, x)
```

3.163 $\int \cosh(a + bx)\mathbf{sech}^3(c + bx)\,dx$

Optimal. Leaf size=38

$$\frac{\cosh(a-c)\tanh(bx+c)}{b} - \frac{\sinh(a-c)\text{sech}^2(bx+c)}{2b}$$

```
[Out] -1/2*sech(b*x+c)^2*sinh(a-c)/b+cosh(a-c)*tanh(b*x+c)/b
```

Rubi [A] time = 0.04, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5627, 2606, 30, 3767, 8}

$$\frac{\cosh(a-c)\tanh(bx+c)}{b} - \frac{\sinh(a-c)\text{sech}^2(bx+c)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[a + b*x]*Sech[c + b*x]^3,x]
```

```
[Out] -(Sech[c + b*x]^2*Sinh[a - c])/(2*b) + (Cosh[a - c]*Tanh[c + b*x])/b
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 5627

```
Int[Cosh[v_]*Sech[w_]^(n_.), x_Symbol] :> Dist[Sinh[v - w], Int[Tanh[w]*Sec
h[w]^(n - 1), x], x] + Dist[Cosh[v - w], Int[Sech[w]^(n - 1), x], x] /; GtQ
[n, 0] && NeQ[w, v] && FreeQ[v - w, x]
```

Rubi steps

$$\int \cosh(a+bx)\mathrm{sech}^3(c+bx)\,dx = \cosh(a-c)\int \mathrm{sech}^2(c+bx)\,dx + \sinh(a-c)\int \mathrm{sech}^2(c+bx)\tanh(c+bx)$$
$$= \frac{(i\cosh(a-c))\,\mathrm{Subst}(\int 1\,dx, x, -i\tanh(c+bx))}{b} - \frac{\sinh(a-c)\,\mathrm{Subst}(\int x\,dx,}{b}$$
$$= -\frac{\mathrm{sech}^2(c+bx)\sinh(a-c)}{2b} + \frac{\cosh(a-c)\tanh(c+bx)}{b}$$

Mathematica [A] time = 0.18, size = 35, normalized size = 0.92

$$-\frac{\mathrm{sech}(c)\mathrm{sech}^2(bx+c)(\sinh(a)-\cosh(a-c)\sinh(2bx+c))}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[a + b*x]*Sech[c + b*x]^3,x]
[Out] -1/2*(Sech[c]*Sech[c + b*x]^2*(Sinh[a] - Cosh[a - c]*Sinh[c + 2*b*x]))/b
```

fricas [B] time = 0.39, size = 248, normalized size = 6.53

$$-\frac{}{b\cosh(bx+c)^3\cosh(-a+c)^2 + 3\,b\cosh(bx+c)\cosh(-a+c)^2 + \left(b\cosh(-a+c)^2 - b\sinh(-a+c)^2\right)\sinh}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*sech(b*x+c)^3,x, algorithm="fricas")
[Out] -2*(2*cosh(b*x + c)*cosh(-a + c) - cosh(b*x + c)*sinh(-a + c) - sinh(b*x +
c)*sinh(-a + c))/(b*cosh(b*x + c)^3*cosh(-a + c)^2 + 3*b*cosh(b*x + c)*cosh
(-a + c)^2 + (b*cosh(-a + c)^2 - b*sinh(-a + c)^2)*sinh(b*x + c)^3 + 3*(b*c
osh(b*x + c)*cosh(-a + c)^2 - b*cosh(b*x + c)*sinh(-a + c)^2)*sinh(b*x + c)
^2 - (b*cosh(b*x + c)^3 + 3*b*cosh(b*x + c))*sinh(-a + c)^2 + (3*b*cosh(b*x
 + c)^2*cosh(-a + c)^2 + b*cosh(-a + c)^2 - (3*b*cosh(b*x + c)^2 + b)*sinh(
-a + c)^2)*sinh(b*x + c))
```

giac [A] time = 0.12, size = 49, normalized size = 1.29

$$-\frac{\left(2\,e^{(2\,bx+2\,a+2\,c)} + e^{(2\,a)} + e^{(2\,c)}\right)e^{(-a-c)}}{b\left(e^{(2\,bx+2\,c)}+1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*sech(b*x+c)^3,x, algorithm="giac")
```

```
[Out] -(2*e^(2*b*x + 2*a + 2*c) + e^(2*a) + e^(2*c))*e^(-a - c)/(b*(e^(2*b*x + 2*
c) + 1)^2)
```

maple [A] time = 0.22, size = 56, normalized size = 1.47

$$-\frac{\left(2\,\mathrm{e}^{2bx+2a+2c}+\mathrm{e}^{2a}+\mathrm{e}^{2c}\right)\mathrm{e}^{3a-c}}{\left(\mathrm{e}^{2bx+2a+2c}+\mathrm{e}^{2a}\right)^{2}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)*sech(b*x+c)^3,x)
```

```
[Out] -1/(exp(2*b*x+2*a+2*c)+exp(2*a))^2/b*(2*exp(2*b*x+2*a+2*c)+exp(2*a)+exp(2*c
))*exp(3*a-c)
```

maxima [B] time = 0.32, size = 119, normalized size = 3.13

$$\frac{2\,e^{(-2\,bx+3\,c)}}{b\left(2\,e^{(-2\,bx+a+2\,c)}+e^{(-4\,bx+a)}+e^{(a+4\,c)}\right)}+\frac{e^{(2\,a+3\,c)}}{b\left(2\,e^{(-2\,bx+a+2\,c)}+e^{(-4\,bx+a)}+e^{(a+4\,c)}\right)}+\frac{e^{(5\,c)}}{b\left(2\,e^{(-2\,bx+a+2\,c)}+e^{(-4\,bx+a)}+e^{(a+4\,c)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*sech(b*x+c)^3,x, algorithm="maxima")
```

```
[Out] 2*e^(-2*b*x + 3*c)/(b*(2*e^(-2*b*x + a + 2*c) + e^(-4*b*x + a) + e^(a + 4*c
))) + e^(2*a + 3*c)/(b*(2*e^(-2*b*x + a + 2*c) + e^(-4*b*x + a) + e^(a + 4*
c))) + e^(5*c)/(b*(2*e^(-2*b*x + a + 2*c) + e^(-4*b*x + a) + e^(a + 4*c)))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\cosh\left(a+b\,x\right)}{{\cosh\left(c+b\,x\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a + b*x)/cosh(c + b*x)^3,x)
```

```
[Out] int(cosh(a + b*x)/cosh(c + b*x)^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \cosh\left(a+bx\right)\operatorname{sech}^{3}\left(bx+c\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*sech(b*x+c)**3,x)
```

```
[Out] Integral(cosh(a + b*x)*sech(b*x + c)**3, x)
```

3.164 $\int \cosh(a+bx)\mathbf{csch}(c+bx)\,dx$

Optimal. Leaf size=26

$$\frac{\cosh(a-c)\log(\sinh(bx+c))}{b}+x\sinh(a-c)$$

```
[Out] cosh(a-c)*ln(sinh(b*x+c))/b+x*sinh(a-c)
```

Rubi [A] time = 0.01, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {5625, 3475, 8}

$$\frac{\cosh(a-c)\log(\sinh(bx+c))}{b}+x\sinh(a-c)$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[a + b*x]*Csch[c + b*x],x]
```

```
[Out] (Cosh[a - c]*Log[Sinh[c + b*x]])/b + x*Sinh[a - c]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 5625

```
Int[Cosh[v_]*Csch[w_]^(n_.), x_Symbol] :> Dist[Cosh[v - w], Int[Coth[w]*Csc
h[w]^(n - 1), x], x] + Dist[Sinh[v - w], Int[Csch[w]^(n - 1), x], x] /; GtQ
[n, 0] && NeQ[w, v] && FreeQ[v - w, x]
```

Rubi steps

$$\begin{aligned}\int \cosh(a+bx)\text{csch}(c+bx)\,dx &= \cosh(a-c)\int \coth(c+bx)\,dx+\sinh(a-c)\int 1\,dx\\ &= \frac{\cosh(a-c)\log(\sinh(c+bx))}{b}+x\sinh(a-c)\end{aligned}$$

Mathematica [A] time = 0.11, size = 26, normalized size = 1.00

$$\frac{\cosh(a-c)\log(\sinh(bx+c))}{b}+x\sinh(a-c)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[a + b*x]*Csch[c + b*x],x]
```

```
[Out] (Cosh[a - c]*Log[Sinh[c + b*x]])/b + x*Sinh[a - c]
```

fricas [B] time = 0.49, size = 87, normalized size = 3.35

$$-\frac{2\,bx-\left(\cosh\left(-a+c\right)^{2}-2\,\cosh\left(-a+c\right)\sinh\left(-a+c\right)+\sinh\left(-a+c\right)^{2}+1\right)\log\left(\frac{2\,\sinh(bx+c)}{\cosh(bx+c)-\sinh(bx+c)}\right)}{2\left(b\cosh\left(-a+c\right)-b\sinh\left(-a+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*csch(b*x+c),x, algorithm="fricas")
```

```
[Out] -1/2*(2*b*x - (cosh(-a + c)^2 - 2*cosh(-a + c)*sinh(-a + c) + sinh(-a + c)^
2 + 1)*log(2*sinh(b*x + c)/(cosh(b*x + c) - sinh(b*x + c))))/(b*cosh(-a + c
) - b*sinh(-a + c))
```

giac [A] time = 0.13, size = 50, normalized size = 1.92

$$-\frac{2\,bxe^{(-a+c)}-\left(e^{(2\,a+c)}+e^{(3\,c)}\right)e^{(-a-2\,c)}\log\left(\left|e^{(2\,bx+2\,c)}-1\right|\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*csch(b*x+c),x, algorithm="giac")
```

```
[Out] -1/2*(2*b*x*e^(-a + c) - (e^(2*a + c) + e^(3*c))*e^(-a - 2*c)*log(abs(e^(2*
b*x + 2*c) - 1)))/b
```

maple [B] time = 0.18, size = 152, normalized size = 5.85

$$x\,\mathrm{e}^{a-c}-\mathrm{e}^{-a-c}\mathrm{e}^{2a}x-\mathrm{e}^{-a-c}\mathrm{e}^{2c}x-\frac{\mathrm{e}^{-a-c}\mathrm{e}^{2a}a}{b}-\frac{\mathrm{e}^{-a-c}\mathrm{e}^{2c}a}{b}+\frac{\ln\left(\mathrm{e}^{2bx+2a}-\mathrm{e}^{2a-2c}\right)\mathrm{e}^{-a-c}\mathrm{e}^{2a}}{2b}+\frac{\ln\left(\mathrm{e}^{2bx+2a}-\mathrm{e}^{2a-2c}\right)\mathrm{e}^{-a-c}\mathrm{e}^{2c}}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)*csch(b*x+c),x)
```

```
[Out] x*exp(a-c)-exp(-a-c)*exp(2*a)*x-exp(-a-c)*exp(2*c)*x-1/b*exp(-a-c)*exp(2*a)
*a-1/b*exp(-a-c)*exp(2*c)*a+1/2*ln(exp(2*b*x+2*a)-exp(2*a-2*c))/b*exp(-a-c)
*exp(2*a)+1/2*ln(exp(2*b*x+2*a)-exp(2*a-2*c))/b*exp(-a-c)*exp(2*c)
```

maxima [B] time = 0.32, size = 80, normalized size = 3.08

$$\frac{\left(e^{(2a)}+e^{(2c)}\right)e^{(-a-c)}\log\left(e^{(-bx)}+e^{c}\right)}{2b}+\frac{\left(e^{(2a)}+e^{(2c)}\right)e^{(-a-c)}\log\left(e^{(-bx)}-e^{c}\right)}{2b}+\frac{(bx+a)e^{(a-c)}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*csch(b*x+c),x, algorithm="maxima")
```

```
[Out] 1/2*(e^(2*a) + e^(2*c))*e^(-a - c)*log(e^(-b*x) + e^c)/b + 1/2*(e^(2*a) + e
^(2*c))*e^(-a - c)*log(e^(-b*x) - e^c)/b + (b*x + a)*e^(a - c)/b
```

mupad [B] time = 1.52, size = 66, normalized size = 2.54

$$\frac{e^{2c-2a}\ln\left(e^{2a}\,e^{2bx}-e^{2a}\,e^{-2c}\right)\left(2\,b\,e^{3a-3c}+2\,b\,e^{a-c}\right)}{4\,b^2}-x\,e^{c-a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a + b*x)/sinh(c + b*x),x)
```

```
[Out] (exp(2*c - 2*a)*log(exp(2*a)*exp(2*b*x) - exp(2*a)*exp(-2*c))*(2*b*exp(3*a
- 3*c) + 2*b*exp(a - c)))/(4*b^2) - x*exp(c - a)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\cosh\left(a+bx\right)\operatorname{csch}\left(bx+c\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*csch(b*x+c),x)
```

```
[Out] Integral(cosh(a + b*x)*csch(b*x + c), x)
```

3.165 $\int \cosh(a+bx)\mathbf{csch}^2(c+bx)\,dx$

Optimal. Leaf size=36

$$-\frac{\cosh(a-c)\text{csch}(bx+c)}{b}-\frac{\sinh(a-c)\tanh^{-1}(\cosh(bx+c))}{b}$$

```
[Out] -cosh(a-c)*csch(b*x+c)/b-arctanh(cosh(b*x+c))*sinh(a-c)/b
```

Rubi [A] time = 0.03, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {5625, 2606, 8, 3770}

$$-\frac{\cosh(a-c)\text{csch}(bx+c)}{b}-\frac{\sinh(a-c)\tanh^{-1}(\cosh(bx+c))}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[a + b*x]*Csch[c + b*x]^2,x]
```

```
[Out] -((Cosh[a - c]*Csch[c + b*x])/b) - (ArcTanh[Cosh[c + b*x]]*Sinh[a - c])/b
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 && !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 5625

```
Int[Cosh[v_]*Csch[w_]^(n_.), x_Symbol] :> Dist[Cosh[v - w], Int[Coth[w]*Csc
h[w]^(n - 1), x], x] + Dist[Sinh[v - w], Int[Csch[w]^(n - 1), x], x] /; GtQ
[n, 0] && NeQ[w, v] && FreeQ[v - w, x]
```

Rubi steps

$$\int \cosh(a+bx)\operatorname{csch}^2(c+bx)\,dx = \cosh(a-c)\int \coth(c+bx)\operatorname{csch}(c+bx)\,dx + \sinh(a-c)\int \operatorname{csch}(c+bx)\,dx$$
$$= -\frac{\tanh^{-1}(\cosh(c+bx))\sinh(a-c)}{b} - \frac{(i\cosh(a-c))\,\text{Subst}(\int 1\,dx, x, -i\operatorname{csch}(c$$
$$= -\frac{\cosh(a-c)\operatorname{csch}(c+bx)}{b} - \frac{\tanh^{-1}(\cosh(c+bx))\sinh(a-c)}{b}$$

Mathematica [C] time = 0.07, size = 90, normalized size = 2.50

$$-\frac{\cosh(a-c)\operatorname{csch}(bx+c)}{b} - \frac{2i\sinh(a-c)\tan^{-1}\left(\frac{(\cosh(c)-\sinh(c))\left(\sinh(c)\sinh\left(\frac{bx}{2}\right)+\cosh(c)\cosh\left(\frac{bx}{2}\right)\right)}{i\cosh(c)\cosh\left(\frac{bx}{2}\right)-i\sinh(c)\cosh\left(\frac{bx}{2}\right)}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[a + b*x]*Csch[c + b*x]^2,x]
```

```
[Out] -((Cosh[a - c]*Csch[c + b*x])/b) - ((2*I)*ArcTan[((Cosh[c] - Sinh[c])*(Cosh
[c]*Cosh[(b*x)/2] + Sinh[c]*Sinh[(b*x)/2]))/(I*Cosh[c]*Cosh[(b*x)/2] - I*Co
sh[(b*x)/2]*Sinh[c])]*Sinh[a - c])/b
```

fricas [B] time = 0.50, size = 617, normalized size = 17.14

$$\frac{4\cosh(bx+c)\cosh(-a+c)\sinh(-a+c) - 2\cosh(bx+c)\sinh(-a+c)^2 - 2\left(\cosh(-a+c)^2+1\right)\cosh(bx+c}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*csch(b*x+c)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(4*cosh(b*x + c)*cosh(-a + c)*sinh(-a + c) - 2*cosh(b*x + c)*sinh(-a +
c)^2 - 2*(cosh(-a + c)^2 + 1)*cosh(b*x + c) - ((cosh(-a + c)^2 - 1)*cosh(b*
x + c)^2 + (cosh(-a + c)^2 - 2*cosh(-a + c)*sinh(-a + c) + sinh(-a + c)^2 -
 1)*sinh(b*x + c)^2 + (cosh(b*x + c)^2 - 1)*sinh(-a + c)^2 - cosh(-a + c)^2
 - 2*(2*cosh(b*x + c)*cosh(-a + c)*sinh(-a + c) - cosh(b*x + c)*sinh(-a + c
)^2 - (cosh(-a + c)^2 - 1)*cosh(b*x + c))*sinh(b*x + c) - 2*(cosh(b*x + c)^
2*cosh(-a + c) - cosh(-a + c))*sinh(-a + c) + 1)*log(cosh(b*x + c) + sinh(b
*x + c) + 1) + ((cosh(-a + c)^2 - 1)*cosh(b*x + c)^2 + (cosh(-a + c)^2 - 2*
cosh(-a + c)*sinh(-a + c) + sinh(-a + c)^2 - 1)*sinh(b*x + c)^2 + (cosh(b*x
 + c)^2 - 1)*sinh(-a + c)^2 - cosh(-a + c)^2 - 2*(2*cosh(b*x + c)*cosh(-a +
 c)*sinh(-a + c) - cosh(b*x + c)*sinh(-a + c)^2 - (cosh(-a + c)^2 - 1)*cosh
```

```
(b*x + c))*sinh(b*x + c) - 2*(cosh(b*x + c)^2*cosh(-a + c) - cosh(-a + c))*
sinh(-a + c) + 1)*log(cosh(b*x + c) + sinh(b*x + c) - 1) - 2*(cosh(-a + c)^
2 - 2*cosh(-a + c)*sinh(-a + c) + sinh(-a + c)^2 + 1)*sinh(b*x + c))/(b*cos
h(b*x + c)^2*cosh(-a + c) + (b*cosh(-a + c) - b*sinh(-a + c))*sinh(b*x + c)
^2 - b*cosh(-a + c) + 2*(b*cosh(b*x + c)*cosh(-a + c) - b*cosh(b*x + c)*sin
h(-a + c))*sinh(b*x + c) - (b*cosh(b*x + c)^2 - b)*sinh(-a + c))
```

giac [B] time = 0.14, size = 106, normalized size = 2.94

$$-\frac{\left(e^{(2a+c)}-e^{(3c)}\right)e^{(-a-2c)}\log\left(e^{(bx+c)}+1\right)-\left(e^{(2a+c)}-e^{(3c)}\right)e^{(-a-2c)}\log\left(\left|e^{(bx+c)}-1\right|\right)+\frac{2\left(e^{(bx+2a)}+e^{(bx+2c)}\right)e^{(-a)}}{e^{(2bx+2c)}-1}}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*csch(b*x+c)^2,x, algorithm="giac")
```

```
[Out] -1/2*((e^(2*a + c) - e^(3*c))*e^(-a - 2*c)*log(e^(b*x + c) + 1) - (e^(2*a +
 c) - e^(3*c))*e^(-a - 2*c)*log(abs(e^(b*x + c) - 1)) + 2*(e^(b*x + 2*a) +
e^(b*x + 2*c))*e^(-a)/(e^(2*b*x + 2*c) - 1))/b
```

maple [B] time = 0.19, size = 170, normalized size = 4.72

$$\frac{e^{bx+a}\left(e^{2a}+e^{2c}\right)}{b\left(-e^{2bx+2a+2c}+e^{2a}\right)}-\frac{\ln\left(e^{bx+a}+e^{a-c}\right)e^{-a-c}e^{2a}}{2b}+\frac{\ln\left(e^{bx+a}+e^{a-c}\right)e^{-a-c}e^{2c}}{2b}+\frac{\ln\left(e^{bx+a}-e^{a-c}\right)e^{-a-c}e^{2a}}{2b}-\frac{\ln\left(e^{bx}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)*csch(b*x+c)^2,x)
```

```
[Out] 1/b*exp(b*x+a)*(exp(2*a)+exp(2*c))/(-exp(2*b*x+2*a+2*c)+exp(2*a))-1/2*ln(ex
p(b*x+a)+exp(a-c))/b*exp(-a-c)*exp(2*a)+1/2*ln(exp(b*x+a)+exp(a-c))/b*exp(-
a-c)*exp(2*c)+1/2*ln(exp(b*x+a)-exp(a-c))/b*exp(-a-c)*exp(2*a)-1/2*ln(exp(b
*x+a)-exp(a-c))/b*exp(-a-c)*exp(2*c)
```

maxima [B] time = 0.32, size = 105, normalized size = 2.92

$$-\frac{\left(e^{(2a)}-e^{(2c)}\right)e^{(-a-c)}\log\left(e^{(-bx)}+e^{c}\right)}{2b}+\frac{\left(e^{(2a)}-e^{(2c)}\right)e^{(-a-c)}\log\left(e^{(-bx)}-e^{c}\right)}{2b}+\frac{\left(e^{(2a)}+e^{(2c)}\right)e^{(-bx-a)}}{b\left(e^{(-2bx)}-e^{(2c)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*csch(b*x+c)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(e^(2*a) - e^(2*c))*e^(-a - c)*log(e^(-b*x) + e^c)/b + 1/2*(e^(2*a) -
e^(2*c))*e^(-a - c)*log(e^(-b*x) - e^c)/b + (e^(2*a) + e^(2*c))*e^(-b*x - a
)/(b*(e^(-2*b*x) - e^(2*c)))
```

mupad [B] time = 0.17, size = 156, normalized size = 4.33

$$\frac{\operatorname{atan}\left(\frac{e^{-a}\,e^{2c}\,e^{bx}\left(\sqrt{-b^2}-e^{2a}\,e^{-2c}\,\sqrt{-b^2}\right)}{b\,\sqrt{e^{-2a}\,e^{2c}\left(e^{4a}\,e^{-4c}-2\,e^{2a}\,e^{-2c}+1\right)}}\right)\sqrt{e^{2c-2a}\,\left(e^{4a-4c}-2\,e^{2a-2c}+1\right)}}{\sqrt{-b^2}}+\frac{e^{a+bx}\,\left(e^{2a-2c}+1\right)}{b\,\left(e^{2a-2c}-e^{2a+2bx}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a + b*x)/sinh(c + b*x)^2,x)
```

```
[Out] (atan((exp(-a)*exp(2*c)*exp(b*x)*((-b^2)^(1/2) - exp(2*a)*exp(-2*c)*(-b^2)^
(1/2)))/(b*(exp(-2*a)*exp(2*c)*(exp(4*a)*exp(-4*c) - 2*exp(2*a)*exp(-2*c) +
 1))^(1/2)))*(exp(2*c - 2*a)*(exp(4*a - 4*c) - 2*exp(2*a - 2*c) + 1))^(1/2)
)/(-b^2)^(1/2) + (exp(a + b*x)*(exp(2*a - 2*c) + 1))/(b*(exp(2*a - 2*c) - e
xp(2*a + 2*b*x)))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \cosh(a+bx)\operatorname{csch}^2(bx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*csch(b*x+c)**2,x)
```

```
[Out] Integral(cosh(a + b*x)*csch(b*x + c)**2, x)
```

3.166 $\int \cosh(a+bx)\mathbf{csch}^3(c+bx)\,dx$

Optimal. Leaf size=39

$$-\frac{\cosh(a-c)\text{csch}^2(bx+c)}{2b}-\frac{\sinh(a-c)\coth(bx+c)}{b}$$

```
[Out] -1/2*cosh(a-c)*csch(b*x+c)^2/b-coth(b*x+c)*sinh(a-c)/b
```

Rubi [A] time = 0.04, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5625, 2606, 30, 3767, 8}

$$-\frac{\cosh(a-c)\text{csch}^2(bx+c)}{2b}-\frac{\sinh(a-c)\coth(bx+c)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[a + b*x]*Csch[c + b*x]^3,x]
```

```
[Out] -(Cosh[a - c]*Csch[c + b*x]^2)/(2*b) - (Coth[c + b*x]*Sinh[a - c])/b
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 5625

```
Int[Cosh[v_]*Csch[w_]^(n_.), x_Symbol] :> Dist[Cosh[v - w], Int[Coth[w]*Csc
h[w]^(n - 1), x], x] + Dist[Sinh[v - w], Int[Csch[w]^(n - 1), x], x] /; GtQ
[n, 0] && NeQ[w, v] && FreeQ[v - w, x]
```

Rubi steps

$$\int \cosh(a+bx)\text{csch}^3(c+bx)\,dx = \cosh(a-c)\int \coth(c+bx)\text{csch}^2(c+bx)\,dx + \sinh(a-c)\int \text{csch}^2(c+bx)\,dx$$

$$= \frac{\cosh(a-c)\,\text{Subst}\left(\int x\,dx, x, -i\text{csch}(c+bx)\right)}{b} - \frac{(i\sinh(a-c))\,\text{Subst}\left(\int 1\,dx, x\right.}{b}$$

$$= -\frac{\cosh(a-c)\text{csch}^2(c+bx)}{2b} - \frac{\coth(c+bx)\sinh(a-c)}{b}$$

Mathematica [A] time = 0.18, size = 35, normalized size = 0.90

$$-\frac{\text{csch}(c)\text{csch}^2(bx+c)(\sinh(a)-\sinh(a-c)\cosh(2bx+c))}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[a + b*x]*Csch[c + b*x]^3,x]
```

```
[Out] -1/2*(Csch[c]*Csch[c + b*x]^2*(Sinh[a] - Cosh[c + 2*b*x]*Sinh[a - c]))/b
```

fricas [B] time = 0.43, size = 243, normalized size = 6.23

$$-\frac{}{b\cosh(bx+c)^3\cosh(-a+c)^2 - b\cosh(bx+c)\cosh(-a+c)^2 + \left(b\cosh(-a+c)^2 - b\sinh(-a+c)^2\right)\sinh(bx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*csch(b*x+c)^3,x, algorithm="fricas")
```

```
[Out] -2*(cosh(b*x + c)*cosh(-a + c) + (cosh(-a + c) - 2*sinh(-a + c))*sinh(b*x +
 c))/(b*cosh(b*x + c)^3*cosh(-a + c)^2 - b*cosh(b*x + c)*cosh(-a + c)^2 + (
b*cosh(-a + c)^2 - b*sinh(-a + c)^2)*sinh(b*x + c)^3 + 3*(b*cosh(b*x + c)*c
osh(-a + c)^2 - b*cosh(b*x + c)*sinh(-a + c)^2)*sinh(b*x + c)^2 - (b*cosh(b
*x + c)^3 - b*cosh(b*x + c))*sinh(-a + c)^2 + 3*(b*cosh(b*x + c)^2*cosh(-a
+ c)^2 - b*cosh(-a + c)^2 - (b*cosh(b*x + c)^2 - b)*sinh(-a + c)^2)*sinh(b*
x + c))
```

giac [A] time = 0.12, size = 51, normalized size = 1.31

$$-\frac{\left(2\,e^{(2\,bx+2\,a+2\,c)} - e^{(2\,a)} + e^{(2\,c)}\right)e^{(-a-c)}}{b\left(e^{(2\,bx+2\,c)}-1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*csch(b*x+c)^3,x, algorithm="giac")
```

```
[Out] -(2*e^(2*b*x + 2*a + 2*c) - e^(2*a) + e^(2*c))*e^(-a - c)/(b*(e^(2*b*x + 2*
c) - 1)^2)
```

maple [A] time = 0.21, size = 59, normalized size = 1.51

$$\frac{\left(-2\,\mathrm{e}^{2bx+2a+2c}+\mathrm{e}^{2a}-\mathrm{e}^{2c}\right)\mathrm{e}^{3a-c}}{\left(-\mathrm{e}^{2bx+2a+2c}+\mathrm{e}^{2a}\right)^{2}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)*csch(b*x+c)^3,x)
```

```
[Out] 1/(-exp(2*b*x+2*a+2*c)+exp(2*a))^2/b*(-2*exp(2*b*x+2*a+2*c)+exp(2*a)-exp(2*
c))*exp(3*a-c)
```

maxima [B] time = 0.32, size = 132, normalized size = 3.38

$$\frac{2\,e^{(-2\,bx+3\,c)}}{b\left(2\,e^{(-2\,bx+a+2\,c)}-e^{(-4\,bx+a)}-e^{(a+4\,c)}\right)}+\frac{e^{(2\,a+3\,c)}}{b\left(2\,e^{(-2\,bx+a+2\,c)}-e^{(-4\,bx+a)}-e^{(a+4\,c)}\right)}-\frac{e^{(5\,c)}}{b\left(2\,e^{(-2\,bx+a+2\,c)}-e^{(-4\,bx+a)}-e^{(a+4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*csch(b*x+c)^3,x, algorithm="maxima")
```

```
[Out] 2*e^(-2*b*x + 3*c)/(b*(2*e^(-2*b*x + a + 2*c) - e^(-4*b*x + a) - e^(a + 4*c
))) + e^(2*a + 3*c)/(b*(2*e^(-2*b*x + a + 2*c) - e^(-4*b*x + a) - e^(a + 4*
c))) - e^(5*c)/(b*(2*e^(-2*b*x + a + 2*c) - e^(-4*b*x + a) - e^(a + 4*c)))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\cosh\left(a+b\,x\right)}{{\sinh\left(c+b\,x\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a + b*x)/sinh(c + b*x)^3,x)
```

```
[Out] int(cosh(a + b*x)/sinh(c + b*x)^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \cosh\left(a+bx\right)\operatorname{csch}^{3}\left(bx+c\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*csch(b*x+c)**3,x)
```

```
[Out] Integral(cosh(a + b*x)*csch(b*x + c)**3, x)
```

3.167 $\int \sinh(a+bx)\sinh(c+dx)\,dx$

Optimal. Leaf size=43

$$\frac{\sinh(a+x(b+d)+c)}{2(b+d)}-\frac{\sinh(a+x(b-d)-c)}{2(b-d)}$$

```
[Out] -1/2*sinh(a-c+(b-d)*x)/(b-d)+1/2*sinh(a+c+(b+d)*x)/(b+d)
```

Rubi [A] time = 0.04, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {5613, 2637}

$$\frac{\sinh(a+x(b+d)+c)}{2(b+d)}-\frac{\sinh(a+x(b-d)-c)}{2(b-d)}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[a + b*x]*Sinh[c + d*x],x]
```

```
[Out] -Sinh[a - c + (b - d)*x]/(2*(b - d)) + Sinh[a + c + (b + d)*x]/(2*(b + d))
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 5613

```
Int[Sinh[v_]^(p_.)*Sinh[w_]^(q_.), x_Symbol] :> Int[ExpandTrigReduce[Sinh[v
]^p*Sinh[w]^q, x], x] /; IGtQ[p, 0] && IGtQ[q, 0] && ((PolynomialQ[v, x] &&
 PolynomialQ[w, x]) || (BinomialQ[{v, w}, x] && IndependentQ[Cancel[v/w], x
]))
```

Rubi steps

$$\begin{aligned}\int \sinh(a+bx)\sinh(c+dx)\,dx &= \int \left(-\frac{1}{2}\cosh(a-c+(b-d)x)+\frac{1}{2}\cosh(a+c+(b+d)x)\right)dx\\ &= -\left(\frac{1}{2}\int \cosh(a-c+(b-d)x)\,dx\right)+\frac{1}{2}\int \cosh(a+c+(b+d)x)\,dx\\ &= -\frac{\sinh(a-c+(b-d)x)}{2(b-d)}+\frac{\sinh(a+c+(b+d)x)}{2(b+d)}\end{aligned}$$

Mathematica [A] time = 0.23, size = 43, normalized size = 1.00

$$\frac{\sinh(a+x(b+d)+c)}{2(b+d)}-\frac{\sinh(a+x(b-d)-c)}{2(b-d)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[a + b*x]*Sinh[c + d*x],x]
```

```
[Out] -1/2*Sinh[a - c + (b - d)*x]/(b - d) + Sinh[a + c + (b + d)*x]/(2*(b + d))
```

fricas [A] time = 0.45, size = 72, normalized size = 1.67

$$-\frac{d\cosh(dx+c)\sinh(bx+a)-b\cosh(bx+a)\sinh(dx+c)}{\left(b^2-d^2\right)\cosh(bx+a)^2-\left(b^2-d^2\right)\sinh(bx+a)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)*sinh(d*x+c),x, algorithm="fricas")
```

```
[Out] -(d*cosh(d*x + c)*sinh(b*x + a) - b*cosh(b*x + a)*sinh(d*x + c))/((b^2 - d^
2)*cosh(b*x + a)^2 - (b^2 - d^2)*sinh(b*x + a)^2)
```

giac [B] time = 0.12, size = 85, normalized size = 1.98

$$\frac{e^{(bx+dx+a+c)}}{4(b+d)}-\frac{e^{(bx-dx+a-c)}}{4(b-d)}+\frac{e^{(-bx+dx-a+c)}}{4(b-d)}-\frac{e^{(-bx-dx-a-c)}}{4(b+d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)*sinh(d*x+c),x, algorithm="giac")
```

```
[Out] 1/4*e^(b*x + d*x + a + c)/(b + d) - 1/4*e^(b*x - d*x + a - c)/(b - d) + 1/4
*e^(-b*x + d*x - a + c)/(b - d) - 1/4*e^(-b*x - d*x - a - c)/(b + d)
```

maple [A] time = 0.16, size = 40, normalized size = 0.93

$$-\frac{\sinh(a-c+(b-d)x)}{2(b-d)}+\frac{\sinh(a+c+(b+d)x)}{2b+2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(b*x+a)*sinh(d*x+c),x)
```

```
[Out] -1/2*sinh(a-c+(b-d)*x)/(b-d)+1/2*sinh(a+c+(b+d)*x)/(b+d)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)*sinh(d*x+c),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(-d/b>0)', see `assume?` for more de
tails)Is -d/b equal to -1?
```

mupad [B] time = 0.15, size = 42, normalized size = 0.98

$$\frac{b\cosh\left(a+b\,x\right)\,\sinh\left(c+d\,x\right)-d\cosh\left(c+d\,x\right)\,\sinh\left(a+b\,x\right)}{b^2-d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a + b*x)*sinh(c + d*x),x)
```

```
[Out] (b*cosh(a + b*x)*sinh(c + d*x) - d*cosh(c + d*x)*sinh(a + b*x))/(b^2 - d^2)
```

sympy [A] time = 1.47, size = 153, normalized size = 3.56

$$\begin{cases} x\sinh\left(a\right)\sinh\left(c\right) & \text{for}\ b=0\wedge d=0 \\ \frac{x\sinh\left(a-dx\right)\sinh\left(c+dx\right)}{2}+\frac{x\cosh\left(a-dx\right)\cosh\left(c+dx\right)}{2}-\frac{\sinh\left(c+dx\right)\cosh\left(a-dx\right)}{2d} & \text{for}\ b=-d \\ \frac{x\sinh\left(a+dx\right)\sinh\left(c+dx\right)}{2}-\frac{x\cosh\left(a+dx\right)\cosh\left(c+dx\right)}{2}+\frac{\sinh\left(c+dx\right)\cosh\left(a+dx\right)}{2d} & \text{for}\ b=d \\ \frac{b\sinh\left(c+dx\right)\cosh\left(a+bx\right)}{b^2-d^2}-\frac{d\sinh\left(a+bx\right)\cosh\left(c+dx\right)}{b^2-d^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)*sinh(d*x+c),x)
```

```
[Out] Piecewise((x*sinh(a)*sinh(c), Eq(b, 0) & Eq(d, 0)), (x*sinh(a - d*x)*sinh(c
 + d*x)/2 + x*cosh(a - d*x)*cosh(c + d*x)/2 - sinh(c + d*x)*cosh(a - d*x)/(
2*d), Eq(b, -d)), (x*sinh(a + d*x)*sinh(c + d*x)/2 - x*cosh(a + d*x)*cosh(c
 + d*x)/2 + sinh(c + d*x)*cosh(a + d*x)/(2*d), Eq(b, d)), (b*sinh(c + d*x)*
cosh(a + b*x)/(b**2 - d**2) - d*sinh(a + b*x)*cosh(c + d*x)/(b**2 - d**2),
True))
```

3.168 $\int \sinh(a+bx)\sinh^2(c+dx)\,dx$

Optimal. Leaf size=62

$$\frac{\cosh(a+x(b-2d)-2c)}{4(b-2d)}+\frac{\cosh(a+x(b+2d)+2c)}{4(b+2d)}-\frac{\cosh(a+bx)}{2b}$$

```
[Out] -1/2*cosh(b*x+a)/b+1/4*cosh(a-2*c+(b-2*d)*x)/(b-2*d)+1/4*cosh(a+2*c+(b+2*d)
*x)/(b+2*d)
```

Rubi [A] time = 0.06, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {5613, 2638}

$$\frac{\cosh(a+x(b-2d)-2c)}{4(b-2d)}+\frac{\cosh(a+x(b+2d)+2c)}{4(b+2d)}-\frac{\cosh(a+bx)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[a + b*x]*Sinh[c + d*x]^2,x]
```

```
[Out] -Cosh[a + b*x]/(2*b) + Cosh[a - 2*c + (b - 2*d)*x]/(4*(b - 2*d)) + Cosh[a +
 2*c + (b + 2*d)*x]/(4*(b + 2*d))
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 5613

```
Int[Sinh[v_]^(p_.)*Sinh[w_]^(q_.), x_Symbol] :> Int[ExpandTrigReduce[Sinh[v
]^p*Sinh[w]^q, x], x] /; IGtQ[p, 0] && IGtQ[q, 0] && ((PolynomialQ[v, x] &&
 PolynomialQ[w, x]) || (BinomialQ[{v, w}, x] && IndependentQ[Cancel[v/w], x
]))
```

Rubi steps

$$\begin{aligned}\int \sinh(a+bx)\sinh^2(c+dx)\,dx &= \int \left(-\frac{1}{2}\sinh(a+bx)+\frac{1}{4}\sinh(a-2c+(b-2d)x)+\frac{1}{4}\sinh(a+2c+(b+2d)\\ &= \frac{1}{4}\int \sinh(a-2c+(b-2d)x)\,dx+\frac{1}{4}\int \sinh(a+2c+(b+2d)x)\,dx-\frac{1}{2}\int \text{si}\\ &= -\frac{\cosh(a+bx)}{2b}+\frac{\cosh(a-2c+(b-2d)x)}{4(b-2d)}+\frac{\cosh(a+2c+(b+2d)x)}{4(b+2d)}\end{aligned}$$

Mathematica [A] time = 0.80, size = 69, normalized size = 1.11

$$\frac{1}{4}\left(\frac{\cosh(a+bx-2c-2dx)}{b-2d}+\frac{\cosh(a+bx+2c+2dx)}{b+2d}-\frac{2\sinh(a)\sinh(bx)}{b}-\frac{2\cosh(a)\cosh(bx)}{b}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[a + b*x]*Sinh[c + d*x]^2,x]
```

```
[Out] ((-2*Cosh[a]*Cosh[b*x])/b + Cosh[a - 2*c + b*x - 2*d*x]/(b - 2*d) + Cosh[a
+ 2*c + b*x + 2*d*x]/(b + 2*d) - (2*Sinh[a]*Sinh[b*x])/b)/4
```

fricas [B] time = 0.45, size = 120, normalized size = 1.94

$$\frac{b^2\cosh\left(bx+a\right)\cosh\left(dx+c\right)^2-4\,bd\cosh\left(dx+c\right)\sinh\left(bx+a\right)\sinh\left(dx+c\right)+b^2\cosh\left(bx+a\right)\sinh\left(dx+c\right)^2}{2\left(\left(b^3-4\,bd^2\right)\cosh\left(bx+a\right)^2-\left(b^3-4\,bd^2\right)\sinh\left(bx+a\right)^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)*sinh(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(b^2*cosh(b*x + a)*cosh(d*x + c)^2 - 4*b*d*cosh(d*x + c)*sinh(b*x + a)*
sinh(d*x + c) + b^2*cosh(b*x + a)*sinh(d*x + c)^2 - (b^2 - 4*d^2)*cosh(b*x
+ a))/((b^3 - 4*b*d^2)*cosh(b*x + a)^2 - (b^3 - 4*b*d^2)*sinh(b*x + a)^2)
```

giac [B] time = 0.15, size = 120, normalized size = 1.94

$$\frac{e^{(bx+2\,dx+a+2\,c)}}{8\,(b+2\,d)}+\frac{e^{(bx-2\,dx+a-2\,c)}}{8\,(b-2\,d)}-\frac{e^{(bx+a)}}{4\,b}+\frac{e^{(-bx+2\,dx-a+2\,c)}}{8\,(b-2\,d)}+\frac{e^{(-bx-2\,dx-a-2\,c)}}{8\,(b+2\,d)}-\frac{e^{(-bx-a)}}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)*sinh(d*x+c)^2,x, algorithm="giac")
```

```
[Out] 1/8*e^(b*x + 2*d*x + a + 2*c)/(b + 2*d) + 1/8*e^(b*x - 2*d*x + a - 2*c)/(b
- 2*d) - 1/4*e^(b*x + a)/b + 1/8*e^(-b*x + 2*d*x - a + 2*c)/(b - 2*d) + 1/8
*e^(-b*x - 2*d*x - a - 2*c)/(b + 2*d) - 1/4*e^(-b*x - a)/b
```

maple [A] time = 0.07, size = 57, normalized size = 0.92

$$-\frac{\cosh\left(bx+a\right)}{2b}+\frac{\cosh\left(a-2c+\left(b-2d\right)x\right)}{4b-8d}+\frac{\cosh\left(a+2c+\left(b+2d\right)x\right)}{4b+8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(b*x+a)*sinh(d*x+c)^2,x)
```

```
[Out] -1/2*cosh(b*x+a)/b+1/4*cosh(a-2*c+(b-2*d)*x)/(b-2*d)+1/4*cosh(a+2*c+(b+2*d)
*x)/(b+2*d)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)*sinh(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(-(2*d)/b>0)', see `assume?` for mor
e details)Is -(2*d)/b equal to -1?
```

mupad [B] time = 0.25, size = 76, normalized size = 1.23

$$\frac{b^2\left(\cosh(a+b\,x)-\cosh(a+b\,x)\,\cosh(c+d\,x)^2\right)-2\,d^2\cosh(a+b\,x)+2\,b\,d\cosh(c+d\,x)\,\sinh(a+b\,x)\,\sinh}{4\,b\,d^2-b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a + b*x)*sinh(c + d*x)^2,x)
```

```
[Out] (b^2*(cosh(a + b*x) - cosh(a + b*x)*cosh(c + d*x)^2) - 2*d^2*cosh(a + b*x)
+ 2*b*d*cosh(c + d*x)*sinh(a + b*x)*sinh(c + d*x))/(4*b*d^2 - b^3)
```

sympy [A] time = 6.35, size = 405, normalized size = 6.53

$$\begin{cases} x\sinh(a)\sinh^2(c) \\ \left(\frac{x\sinh^2(c+dx)}{2}-\frac{x\cosh^2(c+dx)}{2}+\frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right)\sinh(a) \\ \frac{x\sinh(a-2dx)\sinh^2(c+dx)}{4}+\frac{x\sinh(a-2dx)\cosh^2(c+dx)}{4}+\frac{x\sinh(c+dx)\cosh(a-2dx)\cosh(c+dx)}{2}-\frac{\sinh(a-2dx)\sinh(c+dx)\cosh(c+dx)}{4d}- \\ \frac{x\sinh(a+2dx)\sinh^2(c+dx)}{4}+\frac{x\sinh(a+2dx)\cosh^2(c+dx)}{4}-\frac{x\sinh(c+dx)\cosh(a+2dx)\cosh(c+dx)}{2}-\frac{\sinh(a+2dx)\sinh(c+dx)\cosh(c+dx)}{4d}+ \\ \frac{b^2\sinh^2(c+dx)\cosh(a+bx)}{b^3-4bd^2}-\frac{2bd\sinh(a+bx)\sinh(c+dx)\cosh(c+dx)}{b^3-4bd^2}-\frac{2d^2\sinh^2(c+dx)\cosh(a+bx)}{b^3-4bd^2}+\frac{2d^2\cosh(a+bx)\cosh^2(c+dx)}{b^3-4bd^2} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)*sinh(d*x+c)**2,x)
```

```
[Out] Piecewise((x*sinh(a)*sinh(c)**2, Eq(b, 0) & Eq(d, 0)), ((x*sinh(c + d*x)**2
/2 - x*cosh(c + d*x)**2/2 + sinh(c + d*x)*cosh(c + d*x)/(2*d))*sinh(a), Eq(
```

```
b, 0)), (x*sinh(a - 2*d*x)*sinh(c + d*x)**2/4 + x*sinh(a - 2*d*x)*cosh(c +
d*x)**2/4 + x*sinh(c + d*x)*cosh(a - 2*d*x)*cosh(c + d*x)/2 - sinh(a - 2*d*
x)*sinh(c + d*x)*cosh(c + d*x)/(4*d) - sinh(c + d*x)**2*cosh(a - 2*d*x)/(2*
d), Eq(b, -2*d)), (x*sinh(a + 2*d*x)*sinh(c + d*x)**2/4 + x*sinh(a + 2*d*x)
*cosh(c + d*x)**2/4 - x*sinh(c + d*x)*cosh(a + 2*d*x)*cosh(c + d*x)/2 - sin
h(a + 2*d*x)*sinh(c + d*x)*cosh(c + d*x)/(4*d) + sinh(c + d*x)**2*cosh(a +
2*d*x)/(2*d), Eq(b, 2*d)), (b**2*sinh(c + d*x)**2*cosh(a + b*x)/(b**3 - 4*b
*d**2) - 2*b*d*sinh(a + b*x)*sinh(c + d*x)*cosh(c + d*x)/(b**3 - 4*b*d**2)
- 2*d**2*sinh(c + d*x)**2*cosh(a + b*x)/(b**3 - 4*b*d**2) + 2*d**2*cosh(a +
 b*x)*cosh(c + d*x)**2/(b**3 - 4*b*d**2), True))
```

3.169 $\int \sinh(a+bx)\sinh^3(c+dx)\,dx$

Optimal. Leaf size=91

$$-\frac{\sinh(a+x(b-3d)-3c)}{8(b-3d)}+\frac{3\sinh(a+x(b-d)-c)}{8(b-d)}-\frac{3\sinh(a+x(b+d)+c)}{8(b+d)}+\frac{\sinh(a+x(b+3d)+3c)}{8(b+3d)}$$

```
[Out] -1/8*sinh(a-3*c+(b-3*d)*x)/(b-3*d)+3/8*sinh(a-c+(b-d)*x)/(b-d)-3/8*sinh(a+c
+(b+d)*x)/(b+d)+1/8*sinh(a+3*c+(b+3*d)*x)/(b+3*d)
```

Rubi [A] time = 0.07, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {5613, 2637}

$$-\frac{\sinh(a+x(b-3d)-3c)}{8(b-3d)}+\frac{3\sinh(a+x(b-d)-c)}{8(b-d)}-\frac{3\sinh(a+x(b+d)+c)}{8(b+d)}+\frac{\sinh(a+x(b+3d)+3c)}{8(b+3d)}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[a + b*x]*Sinh[c + d*x]^3,x]
```

```
[Out] -Sinh[a - 3*c + (b - 3*d)*x]/(8*(b - 3*d)) + (3*Sinh[a - c + (b - d)*x])/(8
*(b - d)) - (3*Sinh[a + c + (b + d)*x])/(8*(b + d)) + Sinh[a + 3*c + (b + 3
*d)*x]/(8*(b + 3*d))
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 5613

```
Int[Sinh[v_]^(p_.)*Sinh[w_]^(q_.), x_Symbol] :> Int[ExpandTrigReduce[Sinh[v
]^p*Sinh[w]^q, x], x] /; IGtQ[p, 0] && IGtQ[q, 0] && ((PolynomialQ[v, x] &&
 PolynomialQ[w, x]) || (BinomialQ[{v, w}, x] && IndependentQ[Cancel[v/w], x
]))
```

Rubi steps

$$\int \sinh(a+bx)\sinh^3(c+dx)\,dx = \int \left(-\frac{1}{8}\cosh(a-3c+(b-3d)x)+\frac{3}{8}\cosh(a-c+(b-d)x)-\frac{3}{8}\cosh(a+c$$
$$= -\left(\frac{1}{8}\int \cosh(a-3c+(b-3d)x)\,dx\right)+\frac{1}{8}\int \cosh(a+3c+(b+3d)x)\,dx+$$
$$= -\frac{\sinh(a-3c+(b-3d)x)}{8(b-3d)}+\frac{3\sinh(a-c+(b-d)x)}{8(b-d)}-\frac{3\sinh(a+c+(b+}{8(b+d)}$$

Mathematica [A] time = 0.48, size = 86, normalized size = 0.95

$$\frac{1}{8}\left(-\frac{\sinh(a+bx-3c-3dx)}{b-3d}+\frac{3\sinh(a+bx-c-dx)}{b-d}+\frac{\sinh(a+bx+3c+3dx)}{b+3d}-\frac{3\sinh(a+x(b+d)+c)}{b+d}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[a + b*x]*Sinh[c + d*x]^3,x]
```

```
[Out] (-(Sinh[a - 3*c + b*x - 3*d*x]/(b - 3*d)) + (3*Sinh[a - c + b*x - d*x])/(b
- d) + Sinh[a + 3*c + b*x + 3*d*x]/(b + 3*d) - (3*Sinh[a + c + (b + d)*x])/
(b + d))/8
```

fricas [B] time = 0.44, size = 218, normalized size = 2.40

$$-\frac{9\left(b^2d-d^3\right)\cosh\left(dx+c\right)\sinh\left(bx+a\right)\sinh\left(dx+c\right)^2-\left(b^3-bd^2\right)\cosh\left(bx+a\right)\sinh\left(dx+c\right)^3+3\left(\left(b^2d-d^3\right.\right.}{4\left(\left(b^4-10\,b^2d^2+9\,\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)*sinh(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] -1/4*(9*(b^2*d - d^3)*cosh(d*x + c)*sinh(b*x + a)*sinh(d*x + c)^2 - (b^3 -
b*d^2)*cosh(b*x + a)*sinh(d*x + c)^3 + 3*((b^2*d - d^3)*cosh(d*x + c)^3 - (
b^2*d - 9*d^3)*cosh(d*x + c))*sinh(b*x + a) - 3*((b^3 - b*d^2)*cosh(b*x + a
)*cosh(d*x + c)^2 - (b^3 - 9*b*d^2)*cosh(b*x + a))*sinh(d*x + c))/((b^4 - 1
0*b^2*d^2 + 9*d^4)*cosh(b*x + a)^2 - (b^4 - 10*b^2*d^2 + 9*d^4)*sinh(b*x +
a)^2)
```

giac [B] time = 0.15, size = 179, normalized size = 1.97

$$\frac{e^{(bx+3\,dx+a+3\,c)}}{16\,(b+3\,d)}-\frac{3\,e^{(bx+dx+a+c)}}{16\,(b+d)}+\frac{3\,e^{(bx-dx+a-c)}}{16\,(b-d)}-\frac{e^{(bx-3\,dx+a-3\,c)}}{16\,(b-3\,d)}+\frac{e^{(-bx+3\,dx-a+3\,c)}}{16\,(b-3\,d)}-\frac{3\,e^{(-bx+dx-a+c)}}{16\,(b-d)}+\frac{3\,e^{(-bx-dx-a-c)}}{16\,(b+d)}-\frac{e^{(-}}{1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)*sinh(d*x+c)^3,x, algorithm="giac")
```

```
[Out] 1/16*e^(b*x + 3*d*x + a + 3*c)/(b + 3*d) - 3/16*e^(b*x + d*x + a + c)/(b +
d) + 3/16*e^(b*x - d*x + a - c)/(b - d) - 1/16*e^(b*x - 3*d*x + a - 3*c)/(b
 - 3*d) + 1/16*e^(-b*x + 3*d*x - a + 3*c)/(b - 3*d) - 3/16*e^(-b*x + d*x -
a + c)/(b - d) + 3/16*e^(-b*x - d*x - a - c)/(b + d) - 1/16*e^(-b*x - 3*d*x
 - a - 3*c)/(b + 3*d)
```

maple [A] time = 0.30, size = 84, normalized size = 0.92

$$-\frac{\sinh\left(a-3c+(b-3d)\,x\right)}{8\,(b-3d)}+\frac{3\sinh\left(a-c+(b-d)\,x\right)}{8\,(b-d)}-\frac{3\sinh\left(a+c+(b+d)\,x\right)}{8\,(b+d)}+\frac{\sinh\left(a+3c+(b+3d)\,x\right)}{8b+24d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(b*x+a)*sinh(d*x+c)^3,x)
```

```
[Out] -1/8*sinh(a-3*c+(b-3*d)*x)/(b-3*d)+3/8*sinh(a-c+(b-d)*x)/(b-d)-3/8*sinh(a+c
+(b+d)*x)/(b+d)+1/8*sinh(a+3*c+(b+3*d)*x)/(b+3*d)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)*sinh(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(-(3*d)/b>0)', see `assume?` for mor
e details)Is -(3*d)/b equal to -1?
```

mupad [B] time = 0.51, size = 182, normalized size = 2.00

$$\frac{6\,b\,d^2\cosh\left(a+b\,x\right)\,\cosh\left(c+d\,x\right)^2\sinh\left(c+d\,x\right)}{b^4-10\,b^2\,d^2+9\,d^4}-\frac{6\,d^3\cosh\left(c+d\,x\right)^3\sinh\left(a+b\,x\right)}{b^4-10\,b^2\,d^2+9\,d^4}-\frac{3\,d\cosh\left(c+d\,x\right)\,\sinh\left(a-\right.}{b^4-10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a + b*x)*sinh(c + d*x)^3,x)
```

```
[Out] (6*b*d^2*cosh(a + b*x)*cosh(c + d*x)^2*sinh(c + d*x))/(b^4 + 9*d^4 - 10*b^2
*d^2) - (6*d^3*cosh(c + d*x)^3*sinh(a + b*x))/(b^4 + 9*d^4 - 10*b^2*d^2) -
(3*d*cosh(c + d*x)*sinh(a + b*x)*sinh(c + d*x)^2*(b^2 - 3*d^2))/(b^4 + 9*d^
4 - 10*b^2*d^2) - (cosh(a + b*x)*sinh(c + d*x)^3*(7*b*d^2 - b^3))/(b^4 + 9*
d^4 - 10*b^2*d^2)
```

sympy [A] time = 30.31, size = 918, normalized size = 10.09

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)*sinh(d*x+c)**3,x)
```

```
[Out] Piecewise((x*sinh(a)*sinh(c)**3, Eq(b, 0) & Eq(d, 0)), (x*sinh(a - 3*d*x)*s
inh(c + d*x)**3/8 + 3*x*sinh(a - 3*d*x)*sinh(c + d*x)*cosh(c + d*x)**2/8 +
3*x*sinh(c + d*x)**2*cosh(a - 3*d*x)*cosh(c + d*x)/8 + x*cosh(a - 3*d*x)*co
sh(c + d*x)**3/8 - sinh(a - 3*d*x)*sinh(c + d*x)**2*cosh(c + d*x)/(4*d) + s
inh(a - 3*d*x)*cosh(c + d*x)**3/(24*d) - 3*sinh(c + d*x)**3*cosh(a - 3*d*x)
/(8*d), Eq(b, -3*d)), (3*x*sinh(a - d*x)*sinh(c + d*x)**3/8 - 3*x*sinh(a -
d*x)*sinh(c + d*x)*cosh(c + d*x)**2/8 + 3*x*sinh(c + d*x)**2*cosh(a - d*x)*
cosh(c + d*x)/8 - 3*x*cosh(a - d*x)*cosh(c + d*x)**3/8 + 3*sinh(a - d*x)*si
nh(c + d*x)**2*cosh(c + d*x)/(4*d) - 3*sinh(a - d*x)*cosh(c + d*x)**3/(8*d)
 + sinh(c + d*x)**3*cosh(a - d*x)/(8*d), Eq(b, -d)), (3*x*sinh(a + d*x)*sin
h(c + d*x)**3/8 - 3*x*sinh(a + d*x)*sinh(c + d*x)*cosh(c + d*x)**2/8 - 3*x*
sinh(c + d*x)**2*cosh(a + d*x)*cosh(c + d*x)/8 + 3*x*cosh(a + d*x)*cosh(c +
 d*x)**3/8 + 3*sinh(a + d*x)*sinh(c + d*x)**2*cosh(c + d*x)/(4*d) - 3*sinh(
a + d*x)*cosh(c + d*x)**3/(8*d) - sinh(c + d*x)**3*cosh(a + d*x)/(8*d), Eq(
b, d)), (x*sinh(a + 3*d*x)*sinh(c + d*x)**3/8 + 3*x*sinh(a + 3*d*x)*sinh(c
+ d*x)*cosh(c + d*x)**2/8 - 3*x*sinh(c + d*x)**2*cosh(a + 3*d*x)*cosh(c + d
*x)/8 - x*cosh(a + 3*d*x)*cosh(c + d*x)**3/8 - sinh(a + 3*d*x)*sinh(c + d*x
)**2*cosh(c + d*x)/(4*d) + sinh(a + 3*d*x)*cosh(c + d*x)**3/(24*d) + 3*sinh
(c + d*x)**3*cosh(a + 3*d*x)/(8*d), Eq(b, 3*d)), (b**3*sinh(c + d*x)**3*cos
h(a + b*x)/(b**4 - 10*b**2*d**2 + 9*d**4) - 3*b**2*d*sinh(a + b*x)*sinh(c +
 d*x)**2*cosh(c + d*x)/(b**4 - 10*b**2*d**2 + 9*d**4) - 7*b*d**2*sinh(c + d
*x)**3*cosh(a + b*x)/(b**4 - 10*b**2*d**2 + 9*d**4) + 6*b*d**2*sinh(c + d*x
)*cosh(a + b*x)*cosh(c + d*x)**2/(b**4 - 10*b**2*d**2 + 9*d**4) + 9*d**3*si
nh(a + b*x)*sinh(c + d*x)**2*cosh(c + d*x)/(b**4 - 10*b**2*d**2 + 9*d**4) -
 6*d**3*sinh(a + b*x)*cosh(c + d*x)**3/(b**4 - 10*b**2*d**2 + 9*d**4), True
))
```

3.170 $\int \sinh^2(a+bx)\sinh^2(c+dx)\,dx$

Optimal. Leaf size=88

$$\frac{\sinh(2(a-c)+2x(b-d))}{16(b-d)}+\frac{\sinh(2(a+c)+2x(b+d))}{16(b+d)}-\frac{\sinh(2a+2bx)}{8b}-\frac{\sinh(2c+2dx)}{8d}+\frac{x}{4}$$

```
[Out] 1/4*x-1/8*sinh(2*b*x+2*a)/b+1/16*sinh(2*a-2*c+2*(b-d)*x)/(b-d)-1/8*sinh(2*d
*x+2*c)/d+1/16*sinh(2*a+2*c+2*(b+d)*x)/(b+d)
```

Rubi [A] time = 0.07, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {5613, 2637}

$$\frac{\sinh(2(a-c)+2x(b-d))}{16(b-d)}+\frac{\sinh(2(a+c)+2x(b+d))}{16(b+d)}-\frac{\sinh(2a+2bx)}{8b}-\frac{\sinh(2c+2dx)}{8d}+\frac{x}{4}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[a + b*x]^2*Sinh[c + d*x]^2,x]
```

```
[Out] x/4 - Sinh[2*a + 2*b*x]/(8*b) + Sinh[2*(a - c) + 2*(b - d)*x]/(16*(b - d))
- Sinh[2*c + 2*d*x]/(8*d) + Sinh[2*(a + c) + 2*(b + d)*x]/(16*(b + d))
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 5613

```
Int[Sinh[v_]^(p_.)*Sinh[w_]^(q_.), x_Symbol] :> Int[ExpandTrigReduce[Sinh[v
]^p*Sinh[w]^q, x], x] /; IGtQ[p, 0] && IGtQ[q, 0] && ((PolynomialQ[v, x] &&
 PolynomialQ[w, x]) || (BinomialQ[{v, w}, x] && IndependentQ[Cancel[v/w], x
]))
```

Rubi steps

$$\begin{aligned}\int \sinh^2(a+bx)\sinh^2(c+dx)\,dx &= \int \left(\frac{1}{4}-\frac{1}{4}\cosh(2a+2bx)+\frac{1}{8}\cosh(2(a-c)+2(b-d)x)-\frac{1}{4}\cosh(2c+2dx\right.\\ &= \frac{x}{4}+\frac{1}{8}\int \cosh(2(a-c)+2(b-d)x)\,dx+\frac{1}{8}\int \cosh(2(a+c)+2(b+d)x)\,dx\\ &= \frac{x}{4}-\frac{\sinh(2a+2bx)}{8b}+\frac{\sinh(2(a-c)+2(b-d)x)}{16(b-d)}-\frac{\sinh(2c+2dx)}{8d}+\frac{\sinh(\end{aligned}$$

Mathematica [A] time = 0.74, size = 106, normalized size = 1.20

$$\frac{\left(2d^3-2b^2d\right)\sinh(2(a+bx))+bd(b+d)\sinh(2(a+x(b-d)-c))+b(b-d)(d(\sinh(2(a+x(b+d)+c))+4x(b}{16bd(b-d)(b+d)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[a + b*x]^2*Sinh[c + d*x]^2,x]
```

```
[Out] ((-2*b^2*d + 2*d^3)*Sinh[2*(a + b*x)] + b*d*(b + d)*Sinh[2*(a - c + (b - d)
*x)] + b*(b - d)*(-2*(b + d)*Sinh[2*(c + d*x)] + d*(4*(b + d)*x + Sinh[2*(a
 + c + (b + d)*x)])))/(16*b*(b - d)*d*(b + d))
```

fricas [B] time = 0.42, size = 192, normalized size = 2.18

$$\frac{b^2d\cosh(bx+a)\sinh(bx+a)\sinh(dx+c)^2+\left(b^3d-bd^3\right)x+\left(b^2d\cosh(bx+a)\cosh(dx+c)^2-\left(b^2d-d^3\right)\mathrm{c}}{4\left(\left(b^3d-bd^3\right)\cosh(bx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)^2*sinh(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] 1/4*(b^2*d*cosh(b*x + a)*sinh(b*x + a)*sinh(d*x + c)^2 + (b^3*d - b*d^3)*x
+ (b^2*d*cosh(b*x + a)*cosh(d*x + c)^2 - (b^2*d - d^3)*cosh(b*x + a))*sinh(
b*x + a) - (b*d^2*cosh(d*x + c)*sinh(b*x + a)^2 + (b*d^2*cosh(b*x + a)^2 +
b^3 - b*d^2)*cosh(d*x + c))*sinh(d*x + c))/((b^3*d - b*d^3)*cosh(b*x + a)^2
 - (b^3*d - b*d^3)*sinh(b*x + a)^2)
```

giac [A] time = 0.14, size = 156, normalized size = 1.77

$$\frac{1}{4}x+\frac{e^{(2bx+2dx+2a+2c)}}{32(b+d)}+\frac{e^{(2bx-2dx+2a-2c)}}{32(b-d)}-\frac{e^{(2bx+2a)}}{16b}-\frac{e^{(-2bx+2dx-2a+2c)}}{32(b-d)}-\frac{e^{(-2bx-2dx-2a-2c)}}{32(b+d)}+\frac{e^{(-2bx-2a)}}{16b}-\frac{e^{(2dx+2c)}}{16d}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)^2*sinh(d*x+c)^2,x, algorithm="giac")
```

```
[Out] 1/4*x + 1/32*e^(2*b*x + 2*d*x + 2*a + 2*c)/(b + d) + 1/32*e^(2*b*x - 2*d*x
+ 2*a - 2*c)/(b - d) - 1/16*e^(2*b*x + 2*a)/b - 1/32*e^(-2*b*x + 2*d*x - 2*
a + 2*c)/(b - d) - 1/32*e^(-2*b*x - 2*d*x - 2*a - 2*c)/(b + d) + 1/16*e^(-2
*b*x - 2*a)/b - 1/16*e^(2*d*x + 2*c)/d + 1/16*e^(-2*d*x - 2*c)/d
```

maple [A] time = 0.33, size = 83, normalized size = 0.94

$$\frac{x}{4}-\frac{\sinh(2bx+2a)}{8b}-\frac{\sinh(2dx+2c)}{8d}+\frac{\sinh((2b-2d)x+2a-2c)}{16b-16d}+\frac{\sinh((2b+2d)x+2a+2c)}{16b+16d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(b*x+a)^2*sinh(d*x+c)^2,x)
```

```
[Out] 1/4*x-1/8*sinh(2*b*x+2*a)/b-1/8*sinh(2*d*x+2*c)/d+1/16/(b-d)*sinh((2*b-2*d)
*x+2*a-2*c)+1/16/(b+d)*sinh((2*b+2*d)*x+2*a+2*c)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)^2*sinh(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(1-(2*d)/b>0)', see `assume?` for mo
re details)Is 1-(2*d)/b equal to -1?
```

mupad [B] time = 1.92, size = 152, normalized size = 1.73

$$-\frac{d^3\cosh(a+b\,x)\,\sinh(a+b\,x)-b^3\cosh(c+d\,x)\,\sinh(c+d\,x)-b\,d^3\,x+b^3\,d\,x-2\,b^2\,d\cosh(a+b\,x)\,\sinh(a}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a + b*x)^2*sinh(c + d*x)^2,x)
```

```
[Out] -(d^3*cosh(a + b*x)*sinh(a + b*x) - b^3*cosh(c + d*x)*sinh(c + d*x) - b*d^3
*x + b^3*d*x - 2*b^2*d*cosh(a + b*x)*sinh(a + b*x) + 2*b*d^2*cosh(c + d*x)*
sinh(c + d*x) + 2*b^2*d*cosh(a + b*x)*cosh(c + d*x)^2*sinh(a + b*x) - 2*b*d
^2*cosh(a + b*x)^2*cosh(c + d*x)*sinh(c + d*x))/(4*b*d^3 - 4*b^3*d)
```

sympy [A] time = 21.62, size = 1027, normalized size = 11.67

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)**2*sinh(d*x+c)**2,x)
```

```
[Out] Piecewise((x*sinh(a)**2*sinh(c)**2, Eq(b, 0) & Eq(d, 0)), ((x*sinh(c + d*x)
**2/2 - x*cosh(c + d*x)**2/2 + sinh(c + d*x)*cosh(c + d*x)/(2*d))*sinh(a)**
2, Eq(b, 0)), (3*x*sinh(a - d*x)**2*sinh(c + d*x)**2/8 - x*sinh(a - d*x)**2
*cosh(c + d*x)**2/8 + x*sinh(a - d*x)*sinh(c + d*x)*cosh(a - d*x)*cosh(c +
d*x)/2 - x*sinh(c + d*x)**2*cosh(a - d*x)**2/8 + 3*x*cosh(a - d*x)**2*cosh(
c + d*x)**2/8 + sinh(a - d*x)**2*sinh(c + d*x)*cosh(c + d*x)/(8*d) - sinh(a
```

```
 - d*x)*sinh(c + d*x)**2*cosh(a - d*x)/(2*d) - 3*sinh(c + d*x)*cosh(a - d*x
)**2*cosh(c + d*x)/(8*d), Eq(b, -d)), (3*x*sinh(a + d*x)**2*sinh(c + d*x)**
2/8 - x*sinh(a + d*x)**2*cosh(c + d*x)**2/8 - x*sinh(a + d*x)*sinh(c + d*x)
*cosh(a + d*x)*cosh(c + d*x)/2 - x*sinh(c + d*x)**2*cosh(a + d*x)**2/8 + 3*
x*cosh(a + d*x)**2*cosh(c + d*x)**2/8 + 5*sinh(a + d*x)*sinh(c + d*x)**2*co
sh(a + d*x)/(8*d) + sinh(a + d*x)*cosh(a + d*x)*cosh(c + d*x)**2/(8*d) - si
nh(c + d*x)*cosh(a + d*x)**2*cosh(c + d*x)/(2*d), Eq(b, d)), ((x*sinh(a + b
*x)**2/2 - x*cosh(a + b*x)**2/2 + sinh(a + b*x)*cosh(a + b*x)/(2*b))*sinh(c
)**2, Eq(d, 0)), (b**3*d*x*sinh(a + b*x)**2*sinh(c + d*x)**2/(4*b**3*d - 4*
b*d**3) - b**3*d*x*sinh(a + b*x)**2*cosh(c + d*x)**2/(4*b**3*d - 4*b*d**3)
- b**3*d*x*sinh(c + d*x)**2*cosh(a + b*x)**2/(4*b**3*d - 4*b*d**3) + b**3*d
*x*cosh(a + b*x)**2*cosh(c + d*x)**2/(4*b**3*d - 4*b*d**3) + b**3*sinh(a +
b*x)**2*sinh(c + d*x)*cosh(c + d*x)/(4*b**3*d - 4*b*d**3) - b**3*sinh(c + d
*x)*cosh(a + b*x)**2*cosh(c + d*x)/(4*b**3*d - 4*b*d**3) + 2*b**2*d*sinh(a
+ b*x)*sinh(c + d*x)**2*cosh(a + b*x)/(4*b**3*d - 4*b*d**3) - b*d**3*x*sinh
(a + b*x)**2*sinh(c + d*x)**2/(4*b**3*d - 4*b*d**3) + b*d**3*x*sinh(a + b*x
)**2*cosh(c + d*x)**2/(4*b**3*d - 4*b*d**3) + b*d**3*x*sinh(c + d*x)**2*cos
h(a + b*x)**2/(4*b**3*d - 4*b*d**3) - b*d**3*x*cosh(a + b*x)**2*cosh(c + d*
x)**2/(4*b**3*d - 4*b*d**3) - 2*b*d**2*sinh(a + b*x)**2*sinh(c + d*x)*cosh(
c + d*x)/(4*b**3*d - 4*b*d**3) - d**3*sinh(a + b*x)*sinh(c + d*x)**2*cosh(a
 + b*x)/(4*b**3*d - 4*b*d**3) + d**3*sinh(a + b*x)*cosh(a + b*x)*cosh(c + d
*x)**2/(4*b**3*d - 4*b*d**3), True))
```

3.171 $\int \sinh^2(a+bx)\sinh^3(c+dx)\,dx$

Optimal. Leaf size=144

$$-\frac{\cosh(2a+x(2b-3d)-3c)}{16(2b-3d)}+\frac{3\cosh(2a+x(2b-d)-c)}{16(2b-d)}-\frac{3\cosh(2a+x(2b+d)+c)}{16(2b+d)}+\frac{\cosh(2a+x(2b+3d)+3}{16(2b+3d)}$$

```
[Out] -1/16*cosh(2*a-3*c+(2*b-3*d)*x)/(2*b-3*d)+3/16*cosh(2*a-c+(2*b-d)*x)/(2*b-d
)+3/8*cosh(d*x+c)/d-1/24*cosh(3*d*x+3*c)/d-3/16*cosh(2*a+c+(2*b+d)*x)/(2*b+
d)+1/16*cosh(2*a+3*c+(2*b+3*d)*x)/(2*b+3*d)
```

Rubi [A] time = 0.12, antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {5613, 2638}

$$-\frac{\cosh(2a+x(2b-3d)-3c)}{16(2b-3d)}+\frac{3\cosh(2a+x(2b-d)-c)}{16(2b-d)}-\frac{3\cosh(2a+x(2b+d)+c)}{16(2b+d)}+\frac{\cosh(2a+x(2b+3d)+3}{16(2b+3d)}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[a + b*x]^2*Sinh[c + d*x]^3,x]
```

```
[Out] -Cosh[2*a - 3*c + (2*b - 3*d)*x]/(16*(2*b - 3*d)) + (3*Cosh[2*a - c + (2*b
- d)*x])/(16*(2*b - d)) + (3*Cosh[c + d*x])/(8*d) - Cosh[3*c + 3*d*x]/(24*d
) - (3*Cosh[2*a + c + (2*b + d)*x])/(16*(2*b + d)) + Cosh[2*a + 3*c + (2*b
+ 3*d)*x]/(16*(2*b + 3*d))
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 5613

```
Int[Sinh[v_]^(p_.)*Sinh[w_]^(q_.), x_Symbol] :> Int[ExpandTrigReduce[Sinh[v
]^p*Sinh[w]^q, x], x] /; IGtQ[p, 0] && IGtQ[q, 0] && ((PolynomialQ[v, x] &&
 PolynomialQ[w, x]) || (BinomialQ[{v, w}, x] && IndependentQ[Cancel[v/w], x
]))
```

Rubi steps

$$\int \sinh^2(a+bx)\sinh^3(c+dx)\,dx = \int \left(-\frac{1}{16}\sinh(2a-3c+(2b-3d)x) + \frac{3}{16}\sinh(2a-c+(2b-d)x) + \frac{3}{8}\sinh\right.$$
$$= -\left(\frac{1}{16}\int \sinh(2a-3c+(2b-3d)x)\,dx\right) + \frac{1}{16}\int \sinh(2a+3c+(2b+3d)x$$
$$= -\frac{\cosh(2a-3c+(2b-3d)x)}{16(2b-3d)} + \frac{3\cosh(2a-c+(2b-d)x)}{16(2b-d)} + \frac{3\cosh(c+dx}{8d}$$

Mathematica [A] time = 1.60, size = 158, normalized size = 1.10

$$\frac{1}{48}\left(-\frac{3\cosh(2a+2bx-3c-3dx)}{2b-3d} + \frac{9\cosh(2a+2bx-c-dx)}{2b-d} - \frac{9\cosh(2a+2bx+c+dx)}{2b+d} + \frac{3\cosh(2a+2b}{2b+}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[a + b*x]^2*Sinh[c + d*x]^3,x]
```

```
[Out] ((18*Cosh[c]*Cosh[d*x])/d - (2*Cosh[3*c]*Cosh[3*d*x])/d - (3*Cosh[2*a - 3*c
 + 2*b*x - 3*d*x])/(2*b - 3*d) + (9*Cosh[2*a - c + 2*b*x - d*x])/(2*b - d)
- (9*Cosh[2*a + c + 2*b*x + d*x])/(2*b + d) + (3*Cosh[2*a + 3*c + 2*b*x + 3
*d*x])/(2*b + 3*d) + (18*Sinh[c]*Sinh[d*x])/d - (2*Sinh[3*c]*Sinh[3*d*x])/d
)/48
```

fricas [B] time = 0.44, size = 414, normalized size = 2.88

$$\frac{12\left(4b^3d - bd^3\right)\cosh(bx+a)\sinh(bx+a)\sinh(dx+c)^3 - \left(16b^4 - 40b^2d^2 + 9d^4 + 9\left(4b^2d^2 - d^4\right)\cosh(bx+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)^2*sinh(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] 1/24*(12*(4*b^3*d - b*d^3)*cosh(b*x + a)*sinh(b*x + a)*sinh(d*x + c)^3 - (1
6*b^4 - 40*b^2*d^2 + 9*d^4 + 9*(4*b^2*d^2 - d^4)*cosh(b*x + a)^2)*cosh(d*x
+ c)^3 - 9*((4*b^2*d^2 - d^4)*cosh(d*x + c)^3 - (4*b^2*d^2 - 9*d^4)*cosh(d*
x + c))*sinh(b*x + a)^2 + 36*((4*b^3*d - b*d^3)*cosh(b*x + a)*cosh(d*x + c)
^2 - (4*b^3*d - 9*b*d^3)*cosh(b*x + a))*sinh(b*x + a)*sinh(d*x + c) - 3*(9*
(4*b^2*d^2 - d^4)*cosh(d*x + c)*sinh(b*x + a)^2 + (16*b^4 - 40*b^2*d^2 + 9*
d^4 + 9*(4*b^2*d^2 - d^4)*cosh(b*x + a)^2)*cosh(d*x + c))*sinh(d*x + c)^2 +
 9*(16*b^4 - 40*b^2*d^2 + 9*d^4 + (4*b^2*d^2 - 9*d^4)*cosh(b*x + a)^2)*cosh
(d*x + c))/((16*b^4*d - 40*b^2*d^3 + 9*d^5)*cosh(b*x + a)^2 - (16*b^4*d - 4
0*b^2*d^3 + 9*d^5)*sinh(b*x + a)^2)
```

giac [A] time = 0.14, size = 260, normalized size = 1.81

$$\frac{e^{(2\,bx+3\,dx+2\,a+3\,c)}}{32\,(2\,b+3\,d)}-\frac{3\,e^{(2\,bx+dx+2\,a+c)}}{32\,(2\,b+d)}+\frac{3\,e^{(2\,bx-dx+2\,a-c)}}{32\,(2\,b-d)}-\frac{e^{(2\,bx-3\,dx+2\,a-3\,c)}}{32\,(2\,b-3\,d)}-\frac{e^{(-2\,bx+3\,dx-2\,a+3\,c)}}{32\,(2\,b-3\,d)}+\frac{3\,e^{(-2\,bx+dx-2\,a+c)}}{32\,(2\,b-d)}-\frac{3\,e^{(-2}}{32}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)^2*sinh(d*x+c)^3,x, algorithm="giac")
```

```
[Out] 1/32*e^(2*b*x + 3*d*x + 2*a + 3*c)/(2*b + 3*d) - 3/32*e^(2*b*x + d*x + 2*a
+ c)/(2*b + d) + 3/32*e^(2*b*x - d*x + 2*a - c)/(2*b - d) - 1/32*e^(2*b*x -
 3*d*x + 2*a - 3*c)/(2*b - 3*d) - 1/32*e^(-2*b*x + 3*d*x - 2*a + 3*c)/(2*b
- 3*d) + 3/32*e^(-2*b*x + d*x - 2*a + c)/(2*b - d) - 3/32*e^(-2*b*x - d*x -
 2*a - c)/(2*b + d) + 1/32*e^(-2*b*x - 3*d*x - 2*a - 3*c)/(2*b + 3*d) - 1/4
8*e^(3*d*x + 3*c)/d + 3/16*e^(d*x + c)/d + 3/16*e^(-d*x - c)/d - 1/48*e^(-3
*d*x - 3*c)/d
```

maple [A] time = 0.10, size = 133, normalized size = 0.92

$$-\frac{\cosh(2a-3c+(2b-3d)x)}{16(2b-3d)}+\frac{3\cosh(2a-c+(2b-d)x)}{16(2b-d)}+\frac{3\cosh(dx+c)}{8d}-\frac{\cosh(3dx+3c)}{24d}-\frac{3\cosh(2a+c+}{16(2b+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(b*x+a)^2*sinh(d*x+c)^3,x)
```

```
[Out] -1/16*cosh(2*a-3*c+(2*b-3*d)*x)/(2*b-3*d)+3/16*cosh(2*a-c+(2*b-d)*x)/(2*b-d
)+3/8*cosh(d*x+c)/d-1/24*cosh(3*d*x+3*c)/d-3/16*cosh(2*a+c+(2*b+d)*x)/(2*b+
d)+1/16*cosh(2*a+3*c+(2*b+3*d)*x)/(2*b+3*d)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)^2*sinh(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(1-(3*d)/b>0)', see `assume?` for mo
re details)Is 1-(3*d)/b equal to -1?
```

mupad [B] time = 1.94, size = 337, normalized size = 2.34

$$\frac{\cosh(c+d\,x)\,\sinh(a+b\,x)^2\,\sinh(c+d\,x)^2\,\left(8\,b^4-26\,b^2\,d^2+9\,d^4\right)}{d\,\left(16\,b^4-40\,b^2\,d^2+9\,d^4\right)}-\cosh(c+d\,x)^3\,\sinh(a+b\,x)^2\,\left(\frac{3\,a}{16\,b^4-40\,b}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a + b*x)^2*sinh(c + d*x)^3,x)
```

```
[Out] (cosh(c + d*x)*sinh(a + b*x)^2*sinh(c + d*x)^2*(8*b^4 + 9*d^4 - 26*b^2*d^2)
)/(d*(16*b^4 + 9*d^4 - 40*b^2*d^2)) - cosh(c + d*x)^3*sinh(a + b*x)^2*((3*d
^3)/(16*b^4 + 9*d^4 - 40*b^2*d^2) + 1/(3*d)) - (2*cosh(a + b*x)*sinh(a + b*
x)*sinh(c + d*x)^3*(7*b*d^2 - 4*b^3))/(16*b^4 + 9*d^4 - 40*b^2*d^2) - (cosh
(a + b*x)^2*cosh(c + d*x)*sinh(c + d*x)^2*(8*b^4 - 14*b^2*d^2))/(d*(16*b^4
+ 9*d^4 - 40*b^2*d^2)) - cosh(a + b*x)^2*cosh(c + d*x)^3*((3*d^3)/(16*b^4 +
 9*d^4 - 40*b^2*d^2) - 1/(3*d)) + (12*b*d^2*cosh(a + b*x)*cosh(c + d*x)^2*s
inh(a + b*x)*sinh(c + d*x))/(16*b^4 + 9*d^4 - 40*b^2*d^2)
```

sympy [A] time = 109.06, size = 2006, normalized size = 13.93

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)**2*sinh(d*x+c)**3,x)
```

```
[Out] Piecewise((x*sinh(a)**2*sinh(c)**3, Eq(b, 0) & Eq(d, 0)), (x*sinh(a - 3*d*x
/2)**2*sinh(c + d*x)**3/16 + 3*x*sinh(a - 3*d*x/2)**2*sinh(c + d*x)*cosh(c
+ d*x)**2/16 + 3*x*sinh(a - 3*d*x/2)*sinh(c + d*x)**2*cosh(a - 3*d*x/2)*cos
h(c + d*x)/8 + x*sinh(a - 3*d*x/2)*cosh(a - 3*d*x/2)*cosh(c + d*x)**3/8 + x
*sinh(c + d*x)**3*cosh(a - 3*d*x/2)**2/16 + 3*x*sinh(c + d*x)*cosh(a - 3*d*
x/2)**2*cosh(c + d*x)**2/16 + sinh(a - 3*d*x/2)**2*sinh(c + d*x)**2*cosh(c
+ d*x)/d - 5*sinh(a - 3*d*x/2)**2*cosh(c + d*x)**3/(48*d) + sinh(a - 3*d*x/
2)*sinh(c + d*x)**3*cosh(a - 3*d*x/2)/(24*d) + 5*sinh(a - 3*d*x/2)*sinh(c +
 d*x)*cosh(a - 3*d*x/2)*cosh(c + d*x)**2/(4*d) + 9*cosh(a - 3*d*x/2)**2*cos
h(c + d*x)**3/(16*d), Eq(b, -3*d/2)), (3*x*sinh(a - d*x/2)**2*sinh(c + d*x)
**3/16 - 3*x*sinh(a - d*x/2)**2*sinh(c + d*x)*cosh(c + d*x)**2/16 + 3*x*sin
h(a - d*x/2)*sinh(c + d*x)**2*cosh(a - d*x/2)*cosh(c + d*x)/8 - 3*x*sinh(a
- d*x/2)*cosh(a - d*x/2)*cosh(c + d*x)**3/8 + 3*x*sinh(c + d*x)**3*cosh(a -
 d*x/2)**2/16 - 3*x*sinh(c + d*x)*cosh(a - d*x/2)**2*cosh(c + d*x)**2/16 +
17*sinh(a - d*x/2)**2*cosh(c + d*x)**3/(48*d) - 13*sinh(a - d*x/2)*sinh(c +
 d*x)**3*cosh(a - d*x/2)/(8*d) + 7*sinh(a - d*x/2)*sinh(c + d*x)*cosh(a - d
*x/2)*cosh(c + d*x)**2/(4*d) - sinh(c + d*x)**2*cosh(a - d*x/2)**2*cosh(c +
 d*x)/d + 49*cosh(a - d*x/2)**2*cosh(c + d*x)**3/(48*d), Eq(b, -d/2)), (3*x
*sinh(a + d*x/2)**2*sinh(c + d*x)**3/16 - 3*x*sinh(a + d*x/2)**2*sinh(c + d
*x)*cosh(c + d*x)**2/16 - 3*x*sinh(a + d*x/2)*sinh(c + d*x)**2*cosh(a + d*x
/2)*cosh(c + d*x)/8 + 3*x*sinh(a + d*x/2)*cosh(a + d*x/2)*cosh(c + d*x)**3/
8 + 3*x*sinh(c + d*x)**3*cosh(a + d*x/2)**2/16 - 3*x*sinh(c + d*x)*cosh(a +
 d*x/2)**2*cosh(c + d*x)**2/16 + 17*sinh(a + d*x/2)**2*cosh(c + d*x)**3/(48
*d) + 13*sinh(a + d*x/2)*sinh(c + d*x)**3*cosh(a + d*x/2)/(8*d) - 7*sinh(a
+ d*x/2)*sinh(c + d*x)*cosh(a + d*x/2)*cosh(c + d*x)**2/(4*d) - sinh(c + d*
```

```
x)**2*cosh(a + d*x/2)**2*cosh(c + d*x)/d + 49*cosh(a + d*x/2)**2*cosh(c + d
*x)**3/(48*d), Eq(b, d/2)), (x*sinh(a + 3*d*x/2)**2*sinh(c + d*x)**3/16 + 3
*x*sinh(a + 3*d*x/2)**2*sinh(c + d*x)*cosh(c + d*x)**2/16 - 3*x*sinh(a + 3*
d*x/2)*sinh(c + d*x)**2*cosh(a + 3*d*x/2)*cosh(c + d*x)/8 - x*sinh(a + 3*d*
x/2)*cosh(a + 3*d*x/2)*cosh(c + d*x)**3/8 + x*sinh(c + d*x)**3*cosh(a + 3*d
*x/2)**2/16 + 3*x*sinh(c + d*x)*cosh(a + 3*d*x/2)**2*cosh(c + d*x)**2/16 +
sinh(a + 3*d*x/2)**2*sinh(c + d*x)**2*cosh(c + d*x)/d - 5*sinh(a + 3*d*x/2)
**2*cosh(c + d*x)**3/(48*d) - sinh(a + 3*d*x/2)*sinh(c + d*x)**3*cosh(a + 3
*d*x/2)/(24*d) - 5*sinh(a + 3*d*x/2)*sinh(c + d*x)*cosh(a + 3*d*x/2)*cosh(c
 + d*x)**2/(4*d) + 9*cosh(a + 3*d*x/2)**2*cosh(c + d*x)**3/(16*d), Eq(b, 3*
d/2)), ((x*sinh(a + b*x)**2/2 - x*cosh(a + b*x)**2/2 + sinh(a + b*x)*cosh(a
 + b*x)/(2*b))*sinh(c)**3, Eq(d, 0)), (24*b**4*sinh(a + b*x)**2*sinh(c + d*
x)**2*cosh(c + d*x)/(48*b**4*d - 120*b**2*d**3 + 27*d**5) - 16*b**4*sinh(a
+ b*x)**2*cosh(c + d*x)**3/(48*b**4*d - 120*b**2*d**3 + 27*d**5) - 24*b**4*
sinh(c + d*x)**2*cosh(a + b*x)**2*cosh(c + d*x)/(48*b**4*d - 120*b**2*d**3
+ 27*d**5) + 16*b**4*cosh(a + b*x)**2*cosh(c + d*x)**3/(48*b**4*d - 120*b**
2*d**3 + 27*d**5) + 24*b**3*d*sinh(a + b*x)*sinh(c + d*x)**3*cosh(a + b*x)/
(48*b**4*d - 120*b**2*d**3 + 27*d**5) - 78*b**2*d**2*sinh(a + b*x)**2*sinh(
c + d*x)**2*cosh(c + d*x)/(48*b**4*d - 120*b**2*d**3 + 27*d**5) + 40*b**2*d
**2*sinh(a + b*x)**2*cosh(c + d*x)**3/(48*b**4*d - 120*b**2*d**3 + 27*d**5)
 + 42*b**2*d**2*sinh(c + d*x)**2*cosh(a + b*x)**2*cosh(c + d*x)/(48*b**4*d
- 120*b**2*d**3 + 27*d**5) - 40*b**2*d**2*cosh(a + b*x)**2*cosh(c + d*x)**3
/(48*b**4*d - 120*b**2*d**3 + 27*d**5) - 42*b*d**3*sinh(a + b*x)*sinh(c + d
*x)**3*cosh(a + b*x)/(48*b**4*d - 120*b**2*d**3 + 27*d**5) + 36*b*d**3*sinh
(a + b*x)*sinh(c + d*x)*cosh(a + b*x)*cosh(c + d*x)**2/(48*b**4*d - 120*b**
2*d**3 + 27*d**5) + 27*d**4*sinh(a + b*x)**2*sinh(c + d*x)**2*cosh(c + d*x)
/(48*b**4*d - 120*b**2*d**3 + 27*d**5) - 18*d**4*sinh(a + b*x)**2*cosh(c +
d*x)**3/(48*b**4*d - 120*b**2*d**3 + 27*d**5), True))
```

3.172 $\int \sinh^3(a+bx)\sinh^3(c+dx)\,dx$

Optimal. Leaf size=195

$$\frac{3\sinh(a+x(b-3d)-3c)}{32(b-3d)}-\frac{9\sinh(a+x(b-d)-c)}{32(b-d)}-\frac{\sinh(3(a-c)+3x(b-d))}{96(b-d)}+\frac{3\sinh(3a+x(3b-d)-c)}{32(3b-d)}+\frac{9}{}$$

```
[Out] 3/32*sinh(a-3*c+(b-3*d)*x)/(b-3*d)-9/32*sinh(a-c+(b-d)*x)/(b-d)-1/96*sinh(3
*a-3*c+3*(b-d)*x)/(b-d)+3/32*sinh(3*a-c+(3*b-d)*x)/(3*b-d)+9/32*sinh(a+c+(b
+d)*x)/(b+d)+1/96*sinh(3*a+3*c+3*(b+d)*x)/(b+d)-3/32*sinh(3*a+c+(3*b+d)*x)/
(3*b+d)-3/32*sinh(a+3*c+(b+3*d)*x)/(b+3*d)
```

Rubi [A] time = 0.14, antiderivative size = 195, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {5613, 2637}

$$\frac{3\sinh(a+x(b-3d)-3c)}{32(b-3d)}-\frac{9\sinh(a+x(b-d)-c)}{32(b-d)}-\frac{\sinh(3(a-c)+3x(b-d))}{96(b-d)}+\frac{3\sinh(3a+x(3b-d)-c)}{32(3b-d)}+\frac{9}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[a + b*x]^3*Sinh[c + d*x]^3,x]
```

```
[Out] (3*Sinh[a - 3*c + (b - 3*d)*x])/(32*(b - 3*d)) - (9*Sinh[a - c + (b - d)*x]
)/(32*(b - d)) - Sinh[3*(a - c) + 3*(b - d)*x]/(96*(b - d)) + (3*Sinh[3*a -
 c + (3*b - d)*x])/(32*(3*b - d)) + (9*Sinh[a + c + (b + d)*x])/(32*(b + d)
) + Sinh[3*(a + c) + 3*(b + d)*x]/(96*(b + d)) - (3*Sinh[3*a + c + (3*b + d
)*x])/(32*(3*b + d)) - (3*Sinh[a + 3*c + (b + 3*d)*x])/(32*(b + 3*d))
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 5613

```
Int[Sinh[v_]^(p_.)*Sinh[w_]^(q_.), x_Symbol] :> Int[ExpandTrigReduce[Sinh[v
]^p*Sinh[w]^q, x], x] /; IGtQ[p, 0] && IGtQ[q, 0] && ((PolynomialQ[v, x] &&
 PolynomialQ[w, x]) || (BinomialQ[{v, w}, x] && IndependentQ[Cancel[v/w], x
]))
```

Rubi steps

$$\int \sinh^3(a+bx)\sinh^3(c+dx)\,dx = \int \left(\frac{3}{32}\cosh(a-3c+(b-3d)x) - \frac{9}{32}\cosh(a-c+(b-d)x) - \frac{1}{32}\cosh(3(a$$

$$= -\left(\frac{1}{32}\int \cosh(3(a-c)+3(b-d)x)\,dx\right) + \frac{1}{32}\int \cosh(3(a+c)+3(b+d)x)$$

$$= \frac{3\sinh(a-3c+(b-3d)x)}{32(b-3d)} - \frac{9\sinh(a-c+(b-d)x)}{32(b-d)} - \frac{\sinh(3(a-c)+3(b}{96(b-d)}$$

Mathematica [A] time = 1.59, size = 177, normalized size = 0.91

$$\frac{1}{96}\left(\frac{9\sinh(a+bx-3c-3dx)}{b-3d} - \frac{27\sinh(a+bx-c-dx)}{b-d} - \frac{\sinh(3(a+bx-c-dx))}{b-d} + \frac{9\sinh(3a+3bx-c-dx}{3b-d}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[a + b*x]^3*Sinh[c + d*x]^3,x]
```

```
[Out] ((9*Sinh[a - 3*c + b*x - 3*d*x])/(b - 3*d) - (27*Sinh[a - c + b*x - d*x])/(
b - d) - Sinh[3*(a - c + b*x - d*x)]/(b - d) + (9*Sinh[3*a - c + 3*b*x - d*
x])/(3*b - d) - (9*Sinh[3*a + c + 3*b*x + d*x])/(3*b + d) - (9*Sinh[a + 3*c
 + b*x + 3*d*x])/(b + 3*d) + (27*Sinh[a + c + (b + d)*x])/(b + d) + Sinh[3*
(a + c + (b + d)*x)]/(b + d))/96
```

fricas [B] time = 0.46, size = 731, normalized size = 3.75

$$-\frac{\left(\left(9\,b^4d - 82\,b^2d^3 + 9\,d^5\right)\cosh\left(dx+c\right)^3 - 9\left(b^4d - 10\,b^2d^3 + 9\,d^5\right)\cosh\left(dx+c\right)\right)\sinh\left(bx+a\right)^3 - \left(\left(9\,b^5 - 82\,b^3\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)^3*sinh(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] -1/48*(((9*b^4*d - 82*b^2*d^3 + 9*d^5)*cosh(d*x + c)^3 - 9*(b^4*d - 10*b^2*
d^3 + 9*d^5)*cosh(d*x + c))*sinh(b*x + a)^3 - ((9*b^5 - 82*b^3*d^2 + 9*b*d^
4)*cosh(b*x + a)^3 + 3*(9*b^5 - 82*b^3*d^2 + 9*b*d^4)*cosh(b*x + a)*sinh(b*
x + a)^2 - 9*(9*b^5 - 10*b^3*d^2 + b*d^4)*cosh(b*x + a))*sinh(d*x + c)^3 +
3*((9*b^4*d - 82*b^2*d^3 + 9*d^5)*cosh(d*x + c)*sinh(b*x + a)^3 - 3*(81*b^4
*d - 90*b^2*d^3 + 9*d^5 - (9*b^4*d - 82*b^2*d^3 + 9*d^5)*cosh(b*x + a)^2)*c
osh(d*x + c)*sinh(b*x + a))*sinh(d*x + c)^2 - 3*((81*b^4*d - 90*b^2*d^3 + 9
*d^5 - (9*b^4*d - 82*b^2*d^3 + 9*d^5)*cosh(b*x + a)^2)*cosh(d*x + c)^3 - 9*
(9*b^4*d - 82*b^2*d^3 + 9*d^5 - (b^4*d - 10*b^2*d^3 + 9*d^5)*cosh(b*x + a)^
2)*cosh(d*x + c))*sinh(b*x + a) + 3*(9*(b^5 - 10*b^3*d^2 + 9*b*d^4)*cosh(b*
x + a)^3 - ((9*b^5 - 82*b^3*d^2 + 9*b*d^4)*cosh(b*x + a)^3 - 9*(9*b^5 - 10*
```

```
b^3*d^2 + b*d^4)*cosh(b*x + a))*cosh(d*x + c)^2 - 3*((9*b^5 - 82*b^3*d^2 +
9*b*d^4)*cosh(b*x + a)*cosh(d*x + c)^2 - 9*(b^5 - 10*b^3*d^2 + 9*b*d^4)*cos
h(b*x + a))*sinh(b*x + a)^2 - 9*(9*b^5 - 82*b^3*d^2 + 9*b*d^4)*cosh(b*x + a
))*sinh(d*x + c))/((9*b^6 - 91*b^4*d^2 + 91*b^2*d^4 - 9*d^6)*cosh(b*x + a)^
4 - 2*(9*b^6 - 91*b^4*d^2 + 91*b^2*d^4 - 9*d^6)*cosh(b*x + a)^2*sinh(b*x +
a)^2 + (9*b^6 - 91*b^4*d^2 + 91*b^2*d^4 - 9*d^6)*sinh(b*x + a)^4)
```

giac [B] time = 0.15, size = 373, normalized size = 1.91

$$\frac{e^{(3\,bx+3\,dx+3\,a+3\,c)}}{192\,(b+d)}-\frac{3\,e^{(3\,bx+dx+3\,a+c)}}{64\,(3\,b+d)}+\frac{3\,e^{(3\,bx-dx+3\,a-c)}}{64\,(3\,b-d)}-\frac{e^{(3\,bx-3\,dx+3\,a-3\,c)}}{192\,(b-d)}-\frac{3\,e^{(bx+3\,dx+a+3\,c)}}{64\,(b+3\,d)}+\frac{9\,e^{(bx+dx+a+c)}}{64\,(b+d)}-\frac{9\,e^{(bx-dx+}}{64\,(b-}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)^3*sinh(d*x+c)^3,x, algorithm="giac")
```

```
[Out] 1/192*e^(3*b*x + 3*d*x + 3*a + 3*c)/(b + d) - 3/64*e^(3*b*x + d*x + 3*a + c
)/(3*b + d) + 3/64*e^(3*b*x - d*x + 3*a - c)/(3*b - d) - 1/192*e^(3*b*x - 3
*d*x + 3*a - 3*c)/(b - d) - 3/64*e^(b*x + 3*d*x + a + 3*c)/(b + 3*d) + 9/64
*e^(b*x + d*x + a + c)/(b + d) - 9/64*e^(b*x - d*x + a - c)/(b - d) + 3/64*
e^(b*x - 3*d*x + a - 3*c)/(b - 3*d) - 3/64*e^(-b*x + 3*d*x - a + 3*c)/(b -
3*d) + 9/64*e^(-b*x + d*x - a + c)/(b - d) - 9/64*e^(-b*x - d*x - a - c)/(b
 + d) + 3/64*e^(-b*x - 3*d*x - a - 3*c)/(b + 3*d) + 1/192*e^(-3*b*x + 3*d*x
 - 3*a + 3*c)/(b - d) - 3/64*e^(-3*b*x + d*x - 3*a + c)/(3*b - d) + 3/64*e^
(-3*b*x - d*x - 3*a - c)/(3*b + d) - 1/192*e^(-3*b*x - 3*d*x - 3*a - 3*c)/(
b + d)
```

maple [A] time = 0.54, size = 184, normalized size = 0.94

$$\frac{3\sinh\left(a-3c+\left(b-3d\right)x\right)}{32\left(b-3d\right)}-\frac{9\sinh\left(a-c+\left(b-d\right)x\right)}{32\left(b-d\right)}+\frac{9\sinh\left(a+c+\left(b+d\right)x\right)}{32\left(b+d\right)}-\frac{3\sinh\left(a+3c+\left(b+3d\right)x\right)}{32\left(b+3d\right)}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(b*x+a)^3*sinh(d*x+c)^3,x)
```

```
[Out] 3/32*sinh(a-3*c+(b-3*d)*x)/(b-3*d)-9/32*sinh(a-c+(b-d)*x)/(b-d)+9/32*sinh(a
+c+(b+d)*x)/(b+d)-3/32*sinh(a+3*c+(b+3*d)*x)/(b+3*d)-1/96/(b-d)*sinh((3*b-3
*d)*x+3*a-3*c)+3/32*sinh(3*a-c+(3*b-d)*x)/(3*b-d)-3/32*sinh(3*a+c+(3*b+d)*x
)/(3*b+d)+1/96/(b+d)*sinh((3*b+3*d)*x+3*a+3*c)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)^3*sinh(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(-(3*d)/b>0)', see `assume?` for mor
e details)Is -(3*d)/b equal to -1?
```

mupad [B] time = 2.08, size = 906, normalized size = 4.65

$$e^{3a+c+3bx+dx}\left(\frac{-9b^3+3b^2d+9bd^2-3d^3}{576b^4-640b^2d^2+64d^4}+\frac{e^{-6a-6bx}\left(-9b^3-3b^2d+9bd^2+3d^3\right)}{576b^4-640b^2d^2+64d^4}-\frac{e^{-2a-2bx}\left(-81b^3+81\right.}{576b^4-640b}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a + b*x)^3*sinh(c + d*x)^3,x)
```

```
[Out] exp(3*a + c + 3*b*x + d*x)*((9*b*d^2 + 3*b^2*d - 9*b^3 - 3*d^3)/(576*b^4 +
64*d^4 - 640*b^2*d^2) + (exp(- 6*a - 6*b*x)*(9*b*d^2 - 3*b^2*d - 9*b^3 + 3*
d^3))/(576*b^4 + 64*d^4 - 640*b^2*d^2) - (exp(- 2*a - 2*b*x)*(9*b*d^2 + 81*
b^2*d - 81*b^3 - 9*d^3))/(576*b^4 + 64*d^4 - 640*b^2*d^2) - (exp(- 4*a - 4*
b*x)*(9*b*d^2 - 81*b^2*d - 81*b^3 + 9*d^3))/(576*b^4 + 64*d^4 - 640*b^2*d^2
)) - exp(3*a - c + 3*b*x - d*x)*((9*b*d^2 - 3*b^2*d - 9*b^3 + 3*d^3)/(576*b
^4 + 64*d^4 - 640*b^2*d^2) + (exp(- 6*a - 6*b*x)*(9*b*d^2 + 3*b^2*d - 9*b^3
 - 3*d^3))/(576*b^4 + 64*d^4 - 640*b^2*d^2) - (exp(- 2*a - 2*b*x)*(9*b*d^2
- 81*b^2*d - 81*b^3 + 9*d^3))/(576*b^4 + 64*d^4 - 640*b^2*d^2) - (exp(- 4*a
 - 4*b*x)*(9*b*d^2 + 81*b^2*d - 81*b^3 - 9*d^3))/(576*b^4 + 64*d^4 - 640*b^
2*d^2)) + exp(3*a - 3*c + 3*b*x - 3*d*x)*((9*b*d^2 - b^2*d - b^3 + 9*d^3)/(
192*b^4 + 1728*d^4 - 1920*b^2*d^2) + (exp(- 6*a - 6*b*x)*(9*b*d^2 + b^2*d -
 b^3 - 9*d^3))/(192*b^4 + 1728*d^4 - 1920*b^2*d^2) - (exp(- 2*a - 2*b*x)*(9
*b*d^2 - 27*b^2*d - 9*b^3 + 27*d^3))/(192*b^4 + 1728*d^4 - 1920*b^2*d^2) -
(exp(- 4*a - 4*b*x)*(9*b*d^2 + 27*b^2*d - 9*b^3 - 27*d^3))/(192*b^4 + 1728*
d^4 - 1920*b^2*d^2)) - exp(3*a + 3*c + 3*b*x + 3*d*x)*((9*b*d^2 + b^2*d - b
^3 - 9*d^3)/(192*b^4 + 1728*d^4 - 1920*b^2*d^2) + (exp(- 6*a - 6*b*x)*(9*b*
d^2 - b^2*d - b^3 + 9*d^3))/(192*b^4 + 1728*d^4 - 1920*b^2*d^2) - (exp(- 2*
a - 2*b*x)*(9*b*d^2 + 27*b^2*d - 9*b^3 - 27*d^3))/(192*b^4 + 1728*d^4 - 192
0*b^2*d^2) - (exp(- 4*a - 4*b*x)*(9*b*d^2 - 27*b^2*d - 9*b^3 + 27*d^3))/(19
2*b^4 + 1728*d^4 - 1920*b^2*d^2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)**3*sinh(d*x+c)**3,x)
```

```
[Out] Timed out
```

3.173 $\int \cosh(a+bx)\cosh(c+dx)\,dx$

Optimal. Leaf size=43

$$\frac{\sinh(a+x(b-d)-c)}{2(b-d)}+\frac{\sinh(a+x(b+d)+c)}{2(b+d)}$$

```
[Out] 1/2*sinh(a-c+(b-d)*x)/(b-d)+1/2*sinh(a+c+(b+d)*x)/(b+d)
```

Rubi [A] time = 0.04, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {5614, 2637}

$$\frac{\sinh(a+x(b-d)-c)}{2(b-d)}+\frac{\sinh(a+x(b+d)+c)}{2(b+d)}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[a + b*x]*Cosh[c + d*x],x]
```

```
[Out] Sinh[a - c + (b - d)*x]/(2*(b - d)) + Sinh[a + c + (b + d)*x]/(2*(b + d))
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 5614

```
Int[Cosh[v_]^(p_.)*Cosh[w_]^(q_.), x_Symbol] :> Int[ExpandTrigReduce[Cosh[v
]^p*Cosh[w]^q, x], x] /; IGtQ[p, 0] && IGtQ[q, 0] && ((PolynomialQ[v, x] &&
 PolynomialQ[w, x]) || (BinomialQ[{v, w}, x] && IndependentQ[Cancel[v/w], x
]))
```

Rubi steps

$$\begin{aligned}
\int \cosh(a+bx)\cosh(c+dx)\,dx &= \int \left(\frac{1}{2}\cosh(a-c+(b-d)x)+\frac{1}{2}\cosh(a+c+(b+d)x)\right)dx\\
&= \frac{1}{2}\int \cosh(a-c+(b-d)x)\,dx+\frac{1}{2}\int \cosh(a+c+(b+d)x)\,dx\\
&= \frac{\sinh(a-c+(b-d)x)}{2(b-d)}+\frac{\sinh(a+c+(b+d)x)}{2(b+d)}
\end{aligned}$$

Mathematica [A] time = 0.18, size = 43, normalized size = 1.00

$$\frac{\sinh(a+x(b-d)-c)}{2(b-d)}+\frac{\sinh(a+x(b+d)+c)}{2(b+d)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[a + b*x]*Cosh[c + d*x],x]
```

```
[Out] Sinh[a - c + (b - d)*x]/(2*(b - d)) + Sinh[a + c + (b + d)*x]/(2*(b + d))
```

fricas [A] time = 0.43, size = 71, normalized size = 1.65

$$\frac{b\cosh\left(dx+c\right)\sinh\left(bx+a\right)-d\cosh\left(bx+a\right)\sinh\left(dx+c\right)}{\left(b^2-d^2\right)\cosh\left(bx+a\right)^2-\left(b^2-d^2\right)\sinh\left(bx+a\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*cosh(d*x+c),x, algorithm="fricas")
```

```
[Out] (b*cosh(d*x + c)*sinh(b*x + a) - d*cosh(b*x + a)*sinh(d*x + c))/((b^2 - d^2
)*cosh(b*x + a)^2 - (b^2 - d^2)*sinh(b*x + a)^2)
```

giac [B] time = 0.13, size = 85, normalized size = 1.98

$$\frac{e^{(bx+dx+a+c)}}{4\,(b+d)}+\frac{e^{(bx-dx+a-c)}}{4\,(b-d)}-\frac{e^{(-bx+dx-a+c)}}{4\,(b-d)}-\frac{e^{(-bx-dx-a-c)}}{4\,(b+d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*cosh(d*x+c),x, algorithm="giac")
```

```
[Out] 1/4*e^(b*x + d*x + a + c)/(b + d) + 1/4*e^(b*x - d*x + a - c)/(b - d) - 1/4
*e^(-b*x + d*x - a + c)/(b - d) - 1/4*e^(-b*x - d*x - a - c)/(b + d)
```

maple [A] time = 0.21, size = 40, normalized size = 0.93

$$\frac{\sinh\left(a-c+\left(b-d\right)x\right)}{2b-2d}+\frac{\sinh\left(a+c+\left(b+d\right)x\right)}{2b+2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)*cosh(d*x+c),x)
```

```
[Out] 1/2*sinh(a-c+(b-d)*x)/(b-d)+1/2*sinh(a+c+(b+d)*x)/(b+d)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*cosh(d*x+c),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(-d/b>0)', see `assume?` for more de
tails)Is -d/b equal to -1?
```

mupad [B] time = 0.14, size = 42, normalized size = 0.98

$$\frac{b\cosh\left(c+d\,x\right)\,\sinh\left(a+b\,x\right)-d\cosh\left(a+b\,x\right)\,\sinh\left(c+d\,x\right)}{b^2-d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a + b*x)*cosh(c + d*x),x)
```

```
[Out] (b*cosh(c + d*x)*sinh(a + b*x) - d*cosh(a + b*x)*sinh(c + d*x))/(b^2 - d^2)
```

sympy [A] time = 1.47, size = 153, normalized size = 3.56

$$\begin{cases} x\cosh(a)\cosh(c) & \text{for } b=0 \wedge d=0 \\ \frac{x\sinh(a-dx)\sinh(c+dx)}{2}+\frac{x\cosh(a-dx)\cosh(c+dx)}{2}+\frac{\sinh(c+dx)\cosh(a-dx)}{2d} & \text{for } b=-d \\ -\frac{x\sinh(a+dx)\sinh(c+dx)}{2}+\frac{x\cosh(a+dx)\cosh(c+dx)}{2}+\frac{\sinh(c+dx)\cosh(a+dx)}{2d} & \text{for } b=d \\ \frac{b\sinh(a+bx)\cosh(c+dx)}{b^2-d^2}-\frac{d\sinh(c+dx)\cosh(a+bx)}{b^2-d^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*cosh(d*x+c),x)
```

```
[Out] Piecewise((x*cosh(a)*cosh(c), Eq(b, 0) & Eq(d, 0)), (x*sinh(a - d*x)*sinh(c
 + d*x)/2 + x*cosh(a - d*x)*cosh(c + d*x)/2 + sinh(c + d*x)*cosh(a - d*x)/(
2*d), Eq(b, -d)), (-x*sinh(a + d*x)*sinh(c + d*x)/2 + x*cosh(a + d*x)*cosh(
c + d*x)/2 + sinh(c + d*x)*cosh(a + d*x)/(2*d), Eq(b, d)), (b*sinh(a + b*x)
*cosh(c + d*x)/(b**2 - d**2) - d*sinh(c + d*x)*cosh(a + b*x)/(b**2 - d**2),
 True))
```

3.174 $\int \cosh(a+bx)\cosh^2(c+dx)\,dx$

Optimal. Leaf size=62

$$\frac{\sinh(a+x(b-2d)-2c)}{4(b-2d)}+\frac{\sinh(a+x(b+2d)+2c)}{4(b+2d)}+\frac{\sinh(a+bx)}{2b}$$

```
[Out] 1/2*sinh(b*x+a)/b+1/4*sinh(a-2*c+(b-2*d)*x)/(b-2*d)+1/4*sinh(a+2*c+(b+2*d)*
x)/(b+2*d)
```

Rubi [A] time = 0.06, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {5614, 2637}

$$\frac{\sinh(a+x(b-2d)-2c)}{4(b-2d)}+\frac{\sinh(a+x(b+2d)+2c)}{4(b+2d)}+\frac{\sinh(a+bx)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[a + b*x]*Cosh[c + d*x]^2,x]
```

```
[Out] Sinh[a + b*x]/(2*b) + Sinh[a - 2*c + (b - 2*d)*x]/(4*(b - 2*d)) + Sinh[a +
2*c + (b + 2*d)*x]/(4*(b + 2*d))
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 5614

```
Int[Cosh[v_]^(p_.)*Cosh[w_]^(q_.), x_Symbol] :> Int[ExpandTrigReduce[Cosh[v
]^p*Cosh[w]^q, x], x] /; IGtQ[p, 0] && IGtQ[q, 0] && ((PolynomialQ[v, x] &&
 PolynomialQ[w, x]) || (BinomialQ[{v, w}, x] && IndependentQ[Cancel[v/w], x
]))
```

Rubi steps

$$\int \cosh(a+bx)\cosh^2(c+dx)\,dx = \int \left(\frac{1}{2}\cosh(a+bx)+\frac{1}{4}\cosh(a-2c+(b-2d)x)+\frac{1}{4}\cosh(a+2c+(b+2d)\right.$$
$$=\frac{1}{4}\int \cosh(a-2c+(b-2d)x)\,dx+\frac{1}{4}\int \cosh(a+2c+(b+2d)x)\,dx+\frac{1}{2}\int$$
$$=\frac{\sinh(a+bx)}{2b}+\frac{\sinh(a-2c+(b-2d)x)}{4(b-2d)}+\frac{\sinh(a+2c+(b+2d)x)}{4(b+2d)}$$

Mathematica [A] time = 0.72, size = 69, normalized size = 1.11

$$\frac{1}{4}\left(\frac{\sinh(a+bx-2c-2dx)}{b-2d}+\frac{\sinh(a+bx+2c+2dx)}{b+2d}+\frac{2\sinh(a)\cosh(bx)}{b}+\frac{2\cosh(a)\sinh(bx)}{b}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[a + b*x]*Cosh[c + d*x]^2,x]
```

```
[Out] ((2*Cosh[b*x]*Sinh[a])/b + (2*Cosh[a]*Sinh[b*x])/b + Sinh[a - 2*c + b*x - 2
*d*x]/(b - 2*d) + Sinh[a + 2*c + b*x + 2*d*x]/(b + 2*d))/4
```

fricas [B] time = 0.44, size = 115, normalized size = 1.85

$$-\frac{4\,bd\cosh\left(bx+a\right)\cosh\left(dx+c\right)\sinh\left(dx+c\right)-b^{2}\sinh\left(bx+a\right)\sinh\left(dx+c\right)^{2}-\left(b^{2}\cosh\left(dx+c\right)^{2}+b^{2}-4\,d^{2}\right.}{2\left(\left(b^{3}-4\,bd^{2}\right)\cosh\left(bx+a\right)^{2}-\left(b^{3}-4\,bd^{2}\right)\sinh\left(bx+a\right)^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*cosh(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] -1/2*(4*b*d*cosh(b*x + a)*cosh(d*x + c)*sinh(d*x + c) - b^2*sinh(b*x + a)*s
inh(d*x + c)^2 - (b^2*cosh(d*x + c)^2 + b^2 - 4*d^2)*sinh(b*x + a))/((b^3 -
 4*b*d^2)*cosh(b*x + a)^2 - (b^3 - 4*b*d^2)*sinh(b*x + a)^2)
```

giac [B] time = 0.13, size = 120, normalized size = 1.94

$$\frac{e^{\left(bx+2\,dx+a+2\,c\right)}}{8\,\left(b+2\,d\right)}+\frac{e^{\left(bx-2\,dx+a-2\,c\right)}}{8\,\left(b-2\,d\right)}+\frac{e^{\left(bx+a\right)}}{4\,b}-\frac{e^{\left(-bx+2\,dx-a+2\,c\right)}}{8\,\left(b-2\,d\right)}-\frac{e^{\left(-bx-2\,dx-a-2\,c\right)}}{8\,\left(b+2\,d\right)}-\frac{e^{\left(-bx-a\right)}}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*cosh(d*x+c)^2,x, algorithm="giac")
```

```
[Out] 1/8*e^(b*x + 2*d*x + a + 2*c)/(b + 2*d) + 1/8*e^(b*x - 2*d*x + a - 2*c)/(b
- 2*d) + 1/4*e^(b*x + a)/b - 1/8*e^(-b*x + 2*d*x - a + 2*c)/(b - 2*d) - 1/8
*e^(-b*x - 2*d*x - a - 2*c)/(b + 2*d) - 1/4*e^(-b*x - a)/b
```

maple [A] time = 0.21, size = 57, normalized size = 0.92

$$\frac{\sinh\left(bx+a\right)}{2b}+\frac{\sinh\left(a-2c+\left(b-2d\right)x\right)}{4b-8d}+\frac{\sinh\left(a+2c+\left(b+2d\right)x\right)}{4b+8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)*cosh(d*x+c)^2,x)
```

```
[Out] 1/2*sinh(b*x+a)/b+1/4*sinh(a-2*c+(b-2*d)*x)/(b-2*d)+1/4*sinh(a+2*c+(b+2*d)*
x)/(b+2*d)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*cosh(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(-(2*d)/b>0)', see `assume?` for mor
e details)Is -(2*d)/b equal to -1?
```

mupad [B] time = 0.23, size = 68, normalized size = 1.10

$$\frac{2\,d^2\sinh\left(a+b\,x\right)-b^2\cosh\left(c+d\,x\right)^2\sinh\left(a+b\,x\right)+2\,b\,d\cosh\left(a+b\,x\right)\,\cosh\left(c+d\,x\right)\,\sinh\left(c+d\,x\right)}{4\,b\,d^2-b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a + b*x)*cosh(c + d*x)^2,x)
```

```
[Out] (2*d^2*sinh(a + b*x) - b^2*cosh(c + d*x)^2*sinh(a + b*x) + 2*b*d*cosh(a + b
*x)*cosh(c + d*x)*sinh(c + d*x))/(4*b*d^2 - b^3)
```

sympy [A] time = 6.29, size = 408, normalized size = 6.58

$$\begin{cases} x\cosh(a)\cosh^2(c) \\ \left(-\frac{x\sinh^2(c+dx)}{2}+\frac{x\cosh^2(c+dx)}{2}+\frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right)\cosh(a) \\ \frac{x\sinh(a-2dx)\sinh(c+dx)\cosh(c+dx)}{2}+\frac{x\sinh^2(c+dx)\cosh(a-2dx)}{4}+\frac{x\cosh(a-2dx)\cosh^2(c+dx)}{4}+\frac{\sinh(a-2dx)\sinh^2(c+dx)}{2d}+\frac{3\sinh(c+ \\ -\frac{x\sinh(a+2dx)\sinh(c+dx)\cosh(c+dx)}{2}+\frac{x\sinh^2(c+dx)\cosh(a+2dx)}{4}+\frac{x\cosh(a+2dx)\cosh^2(c+dx)}{4}-\frac{\sinh(a+2dx)\sinh^2(c+dx)}{2d}+\frac{3\sinh(\\ \frac{b^2\sinh(a+bx)\cosh^2(c+dx)}{b^3-4bd^2}-\frac{2bd\sinh(c+dx)\cosh(a+bx)\cosh(c+dx)}{b^3-4bd^2}+\frac{2d^2\sinh(a+bx)\sinh^2(c+dx)}{b^3-4bd^2}-\frac{2d^2\sinh(a+bx)\cosh^2(c+dx)}{b^3-4bd^2} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*cosh(d*x+c)**2,x)
```

```
[Out] Piecewise((x*cosh(a)*cosh(c)**2, Eq(b, 0) & Eq(d, 0)), ((-x*sinh(c + d*x)**
2/2 + x*cosh(c + d*x)**2/2 + sinh(c + d*x)*cosh(c + d*x)/(2*d))*cosh(a), Eq
```

```
(b, 0)), (x*sinh(a - 2*d*x)*sinh(c + d*x)*cosh(c + d*x)/2 + x*sinh(c + d*x)
**2*cosh(a - 2*d*x)/4 + x*cosh(a - 2*d*x)*cosh(c + d*x)**2/4 + sinh(a - 2*d
*x)*sinh(c + d*x)**2/(2*d) + 3*sinh(c + d*x)*cosh(a - 2*d*x)*cosh(c + d*x)/
(4*d), Eq(b, -2*d)), (-x*sinh(a + 2*d*x)*sinh(c + d*x)*cosh(c + d*x)/2 + x*
sinh(c + d*x)**2*cosh(a + 2*d*x)/4 + x*cosh(a + 2*d*x)*cosh(c + d*x)**2/4 -
 sinh(a + 2*d*x)*sinh(c + d*x)**2/(2*d) + 3*sinh(c + d*x)*cosh(a + 2*d*x)*c
osh(c + d*x)/(4*d), Eq(b, 2*d)), (b**2*sinh(a + b*x)*cosh(c + d*x)**2/(b**3
 - 4*b*d**2) - 2*b*d*sinh(c + d*x)*cosh(a + b*x)*cosh(c + d*x)/(b**3 - 4*b*
d**2) + 2*d**2*sinh(a + b*x)*sinh(c + d*x)**2/(b**3 - 4*b*d**2) - 2*d**2*si
nh(a + b*x)*cosh(c + d*x)**2/(b**3 - 4*b*d**2), True))
```

3.175 $\int \cosh(a+bx)\cosh^3(c+dx)\,dx$

Optimal. Leaf size=91

$$\frac{\sinh(a+x(b-3d)-3c)}{8(b-3d)}+\frac{3\sinh(a+x(b-d)-c)}{8(b-d)}+\frac{3\sinh(a+x(b+d)+c)}{8(b+d)}+\frac{\sinh(a+x(b+3d)+3c)}{8(b+3d)}$$

```
[Out] 1/8*sinh(a-3*c+(b-3*d)*x)/(b-3*d)+3/8*sinh(a-c+(b-d)*x)/(b-d)+3/8*sinh(a+c+
(b+d)*x)/(b+d)+1/8*sinh(a+3*c+(b+3*d)*x)/(b+3*d)
```

Rubi [A] time = 0.07, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {5614, 2637}

$$\frac{\sinh(a+x(b-3d)-3c)}{8(b-3d)}+\frac{3\sinh(a+x(b-d)-c)}{8(b-d)}+\frac{3\sinh(a+x(b+d)+c)}{8(b+d)}+\frac{\sinh(a+x(b+3d)+3c)}{8(b+3d)}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[a + b*x]*Cosh[c + d*x]^3,x]
```

```
[Out] Sinh[a - 3*c + (b - 3*d)*x]/(8*(b - 3*d)) + (3*Sinh[a - c + (b - d)*x])/(8*
(b - d)) + (3*Sinh[a + c + (b + d)*x])/(8*(b + d)) + Sinh[a + 3*c + (b + 3*
d)*x]/(8*(b + 3*d))
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 5614

```
Int[Cosh[v_]^(p_.)*Cosh[w_]^(q_.), x_Symbol] :> Int[ExpandTrigReduce[Cosh[v
]^p*Cosh[w]^q, x], x] /; IGtQ[p, 0] && IGtQ[q, 0] && ((PolynomialQ[v, x] &&
 PolynomialQ[w, x]) || (BinomialQ[{v, w}, x] && IndependentQ[Cancel[v/w], x
]))
```

Rubi steps

$$\int \cosh(a+bx)\cosh^3(c+dx)\,dx = \int \left(\frac{1}{8}\cosh(a-3c+(b-3d)x)+\frac{3}{8}\cosh(a-c+(b-d)x)+\frac{3}{8}\cosh(a+c+\right.$$

$$=\frac{1}{8}\int \cosh(a-3c+(b-3d)x)\,dx+\frac{1}{8}\int \cosh(a+3c+(b+3d)x)\,dx+\frac{3}{8}\int$$

$$=\frac{\sinh(a-3c+(b-3d)x)}{8(b-3d)}+\frac{3\sinh(a-c+(b-d)x)}{8(b-d)}+\frac{3\sinh(a+c+(b+d)}{8(b+d)}$$

Mathematica [A] time = 0.46, size = 85, normalized size = 0.93

$$\frac{1}{8}\left(\frac{\sinh(a+bx-3c-3dx)}{b-3d}+\frac{3\sinh(a+bx-c-dx)}{b-d}+\frac{\sinh(a+bx+3c+3dx)}{b+3d}+\frac{3\sinh(a+x(b+d)+c)}{b+d}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[a + b*x]*Cosh[c + d*x]^3,x]
```

```
[Out] (Sinh[a - 3*c + b*x - 3*d*x]/(b - 3*d) + (3*Sinh[a - c + b*x - d*x])/(b - d
) + Sinh[a + 3*c + b*x + 3*d*x]/(b + 3*d) + (3*Sinh[a + c + (b + d)*x])/(b
+ d))/8
```

fricas [B] time = 0.45, size = 217, normalized size = 2.38

$$\frac{3\left(b^3-bd^2\right)\cosh\left(dx+c\right)\sinh\left(bx+a\right)\sinh\left(dx+c\right)^2-3\left(b^2d-d^3\right)\cosh\left(bx+a\right)\sinh\left(dx+c\right)^3+\left(\left(b^3-bd^2\right)\right.}{4\left(\left(b^4-10\,b^2d^2+9\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*cosh(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] 1/4*(3*(b^3 - b*d^2)*cosh(d*x + c)*sinh(b*x + a)*sinh(d*x + c)^2 - 3*(b^2*d
 - d^3)*cosh(b*x + a)*sinh(d*x + c)^3 + ((b^3 - b*d^2)*cosh(d*x + c)^3 + 3*
(b^3 - 9*b*d^2)*cosh(d*x + c))*sinh(b*x + a) - 3*(3*(b^2*d - d^3)*cosh(b*x
+ a)*cosh(d*x + c)^2 + (b^2*d - 9*d^3)*cosh(b*x + a))*sinh(d*x + c))/((b^4
- 10*b^2*d^2 + 9*d^4)*cosh(b*x + a)^2 - (b^4 - 10*b^2*d^2 + 9*d^4)*sinh(b*x
 + a)^2)
```

giac [B] time = 0.13, size = 179, normalized size = 1.97

$$\frac{e^{(bx+3\,dx+a+3\,c)}}{16\,(b+3\,d)}+\frac{3\,e^{(bx+dx+a+c)}}{16\,(b+d)}+\frac{3\,e^{(bx-dx+a-c)}}{16\,(b-d)}+\frac{e^{(bx-3\,dx+a-3\,c)}}{16\,(b-3\,d)}-\frac{e^{(-bx+3\,dx-a+3\,c)}}{16\,(b-3\,d)}-\frac{3\,e^{(-bx+dx-a+c)}}{16\,(b-d)}-\frac{3\,e^{(-bx-dx-a-c)}}{16\,(b+d)}-\frac{e^{(-}}{1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*cosh(d*x+c)^3,x, algorithm="giac")
```

```
[Out] 1/16*e^(b*x + 3*d*x + a + 3*c)/(b + 3*d) + 3/16*e^(b*x + d*x + a + c)/(b +
d) + 3/16*e^(b*x - d*x + a - c)/(b - d) + 1/16*e^(b*x - 3*d*x + a - 3*c)/(b
 - 3*d) - 1/16*e^(-b*x + 3*d*x - a + 3*c)/(b - 3*d) - 3/16*e^(-b*x + d*x -
a + c)/(b - d) - 3/16*e^(-b*x - d*x - a - c)/(b + d) - 1/16*e^(-b*x - 3*d*x
 - a - 3*c)/(b + 3*d)
```

maple [A] time = 0.33, size = 84, normalized size = 0.92

$$\frac{\sinh\left(a-3c+(b-3d)\,x\right)}{8b-24d}+\frac{3\sinh\left(a-c+(b-d)\,x\right)}{8\,(b-d)}+\frac{3\sinh\left(a+c+(b+d)\,x\right)}{8\,(b+d)}+\frac{\sinh\left(a+3c+(b+3d)\,x\right)}{8b+24d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)*cosh(d*x+c)^3,x)
```

```
[Out] 1/8*sinh(a-3*c+(b-3*d)*x)/(b-3*d)+3/8*sinh(a-c+(b-d)*x)/(b-d)+3/8*sinh(a+c+
(b+d)*x)/(b+d)+1/8*sinh(a+3*c+(b+3*d)*x)/(b+3*d)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*cosh(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(-(3*d)/b>0)', see `assume?` for mor
e details)Is -(3*d)/b equal to -1?
```

mupad [B] time = 1.90, size = 180, normalized size = 1.98

$$\frac{b\cosh\left(c+d\,x\right)^3\,\sinh\left(a+b\,x\right)\,\left(b^2-7\,d^2\right)}{b^4-10\,b^2\,d^2+9\,d^4}-\frac{3\cosh\left(a+b\,x\right)\,\cosh\left(c+d\,x\right)^2\,\sinh\left(c+d\,x\right)\,\left(b^2\,d-3\,d^3\right)}{b^4-10\,b^2\,d^2+9\,d^4}-\frac{6\,d^3\cosh}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a + b*x)*cosh(c + d*x)^3,x)
```

```
[Out] (b*cosh(c + d*x)^3*sinh(a + b*x)*(b^2 - 7*d^2))/(b^4 + 9*d^4 - 10*b^2*d^2)
- (3*cosh(a + b*x)*cosh(c + d*x)^2*sinh(c + d*x)*(b^2*d - 3*d^3))/(b^4 + 9*
d^4 - 10*b^2*d^2) - (6*d^3*cosh(a + b*x)*sinh(c + d*x)^3)/(b^4 + 9*d^4 - 10
*b^2*d^2) + (6*b*d^2*cosh(c + d*x)*sinh(a + b*x)*sinh(c + d*x)^2)/(b^4 + 9*
d^4 - 10*b^2*d^2)
```

sympy [A] time = 30.70, size = 921, normalized size = 10.12

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*cosh(d*x+c)**3,x)
```

```
[Out] Piecewise((x*cosh(a)*cosh(c)**3, Eq(b, 0) & Eq(d, 0)), (x*sinh(a - 3*d*x)*s
inh(c + d*x)**3/8 + 3*x*sinh(a - 3*d*x)*sinh(c + d*x)*cosh(c + d*x)**2/8 +
3*x*sinh(c + d*x)**2*cosh(a - 3*d*x)*cosh(c + d*x)/8 + x*cosh(a - 3*d*x)*co
sh(c + d*x)**3/8 + sinh(a - 3*d*x)*sinh(c + d*x)**2*cosh(c + d*x)/(4*d) - 7
*sinh(a - 3*d*x)*cosh(c + d*x)**3/(24*d) + sinh(c + d*x)**3*cosh(a - 3*d*x)
```

```
/(8*d), Eq(b, -3*d)), (-3*x*sinh(a - d*x)*sinh(c + d*x)**3/8 + 3*x*sinh(a -
 d*x)*sinh(c + d*x)*cosh(c + d*x)**2/8 - 3*x*sinh(c + d*x)**2*cosh(a - d*x)
*cosh(c + d*x)/8 + 3*x*cosh(a - d*x)*cosh(c + d*x)**3/8 + 3*sinh(a - d*x)*s
inh(c + d*x)**2*cosh(c + d*x)/(4*d) - 5*sinh(a - d*x)*cosh(c + d*x)**3/(8*d
) + 3*sinh(c + d*x)**3*cosh(a - d*x)/(8*d), Eq(b, -d)), (3*x*sinh(a + d*x)*
sinh(c + d*x)**3/8 - 3*x*sinh(a + d*x)*sinh(c + d*x)*cosh(c + d*x)**2/8 - 3
*x*sinh(c + d*x)**2*cosh(a + d*x)*cosh(c + d*x)/8 + 3*x*cosh(a + d*x)*cosh(
c + d*x)**3/8 - 3*sinh(a + d*x)*sinh(c + d*x)**2*cosh(c + d*x)/(4*d) + 5*si
nh(a + d*x)*cosh(c + d*x)**3/(8*d) + 3*sinh(c + d*x)**3*cosh(a + d*x)/(8*d)
, Eq(b, d)), (-x*sinh(a + 3*d*x)*sinh(c + d*x)**3/8 - 3*x*sinh(a + 3*d*x)*s
inh(c + d*x)*cosh(c + d*x)**2/8 + 3*x*sinh(c + d*x)**2*cosh(a + 3*d*x)*cosh
(c + d*x)/8 + x*cosh(a + 3*d*x)*cosh(c + d*x)**3/8 - sinh(a + 3*d*x)*sinh(c
 + d*x)**2*cosh(c + d*x)/(4*d) + 7*sinh(a + 3*d*x)*cosh(c + d*x)**3/(24*d)
+ sinh(c + d*x)**3*cosh(a + 3*d*x)/(8*d), Eq(b, 3*d)), (b**3*sinh(a + b*x)*
cosh(c + d*x)**3/(b**4 - 10*b**2*d**2 + 9*d**4) - 3*b**2*d*sinh(c + d*x)*co
sh(a + b*x)*cosh(c + d*x)**2/(b**4 - 10*b**2*d**2 + 9*d**4) + 6*b*d**2*sinh
(a + b*x)*sinh(c + d*x)**2*cosh(c + d*x)/(b**4 - 10*b**2*d**2 + 9*d**4) - 7
*b*d**2*sinh(a + b*x)*cosh(c + d*x)**3/(b**4 - 10*b**2*d**2 + 9*d**4) - 6*d
**3*sinh(c + d*x)**3*cosh(a + b*x)/(b**4 - 10*b**2*d**2 + 9*d**4) + 9*d**3*
sinh(c + d*x)*cosh(a + b*x)*cosh(c + d*x)**2/(b**4 - 10*b**2*d**2 + 9*d**4)
, True))
```

3.176 $\int \cosh^2(a+bx)\cosh^2(c+dx)\,dx$

Optimal. Leaf size=88

$$\frac{\sinh(2(a-c)+2x(b-d))}{16(b-d)}+\frac{\sinh(2(a+c)+2x(b+d))}{16(b+d)}+\frac{\sinh(2a+2bx)}{8b}+\frac{\sinh(2c+2dx)}{8d}+\frac{x}{4}$$

```
[Out] 1/4*x+1/8*sinh(2*b*x+2*a)/b+1/16*sinh(2*a-2*c+2*(b-d)*x)/(b-d)+1/8*sinh(2*d
*x+2*c)/d+1/16*sinh(2*a+2*c+2*(b+d)*x)/(b+d)
```

Rubi [A] time = 0.07, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {5614, 2637}

$$\frac{\sinh(2(a-c)+2x(b-d))}{16(b-d)}+\frac{\sinh(2(a+c)+2x(b+d))}{16(b+d)}+\frac{\sinh(2a+2bx)}{8b}+\frac{\sinh(2c+2dx)}{8d}+\frac{x}{4}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[a + b*x]^2*Cosh[c + d*x]^2,x]
```

```
[Out] x/4 + Sinh[2*a + 2*b*x]/(8*b) + Sinh[2*(a - c) + 2*(b - d)*x]/(16*(b - d))
+ Sinh[2*c + 2*d*x]/(8*d) + Sinh[2*(a + c) + 2*(b + d)*x]/(16*(b + d))
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 5614

```
Int[Cosh[v_]^(p_.)*Cosh[w_]^(q_.), x_Symbol] :> Int[ExpandTrigReduce[Cosh[v
]^p*Cosh[w]^q, x], x] /; IGtQ[p, 0] && IGtQ[q, 0] && ((PolynomialQ[v, x] &&
 PolynomialQ[w, x]) || (BinomialQ[{v, w}, x] && IndependentQ[Cancel[v/w], x
]))
```

Rubi steps

$$\begin{aligned}\int \cosh^2(a+bx)\cosh^2(c+dx)\,dx &= \int \left(\frac{1}{4}+\frac{1}{4}\cosh(2a+2bx)+\frac{1}{8}\cosh(2(a-c)+2(b-d)x)+\frac{1}{4}\cosh(2c+2d\right.\\ &= \frac{x}{4}+\frac{1}{8}\int \cosh(2(a-c)+2(b-d)x)\,dx+\frac{1}{8}\int \cosh(2(a+c)+2(b+d)x)\,dx\\ &= \frac{x}{4}+\frac{\sinh(2a+2bx)}{8b}+\frac{\sinh(2(a-c)+2(b-d)x)}{16(b-d)}+\frac{\sinh(2c+2dx)}{8d}+\text{sinh}\end{aligned}$$

Mathematica [A] time = 0.70, size = 105, normalized size = 1.19

$$\frac{2d\left(b^2-d^2\right)\sinh(2(a+bx))+bd(b+d)\sinh(2(a+x(b-d)-c))+b(b-d)(d(\sinh(2(a+x(b+d)+c))+4x(b}{16bd(b-d)(b+d)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[a + b*x]^2*Cosh[c + d*x]^2,x]
```

```
[Out] (2*d*(b^2 - d^2)*Sinh[2*(a + b*x)] + b*d*(b + d)*Sinh[2*(a - c + (b - d)*x)
] + b*(b - d)*(2*(b + d)*Sinh[2*(c + d*x)] + d*(4*(b + d)*x + Sinh[2*(a + c
 + (b + d)*x)])))/(16*b*(b - d)*d*(b + d))
```

fricas [B] time = 0.44, size = 192, normalized size = 2.18

$$\frac{b^2d\cosh\left(bx+a\right)\sinh\left(bx+a\right)\sinh\left(dx+c\right)^2+\left(b^3d-bd^3\right)x+\left(b^2d\cosh\left(bx+a\right)\cosh\left(dx+c\right)^2+\left(b^2d-d^3\right)c}{4\left(\left(b^3d-bd^3\right)\cosh\left(bx\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^2*cosh(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] 1/4*(b^2*d*cosh(b*x + a)*sinh(b*x + a)*sinh(d*x + c)^2 + (b^3*d - b*d^3)*x
+ (b^2*d*cosh(b*x + a)*cosh(d*x + c)^2 + (b^2*d - d^3)*cosh(b*x + a))*sinh(
b*x + a) - (b*d^2*cosh(d*x + c)*sinh(b*x + a)^2 + (b*d^2*cosh(b*x + a)^2 -
b^3 + b*d^2)*cosh(d*x + c))*sinh(d*x + c))/((b^3*d - b*d^3)*cosh(b*x + a)^2
 - (b^3*d - b*d^3)*sinh(b*x + a)^2)
```

giac [A] time = 0.14, size = 156, normalized size = 1.77

$$\frac{1}{4}x+\frac{e^{(2bx+2dx+2a+2c)}}{32(b+d)}+\frac{e^{(2bx-2dx+2a-2c)}}{32(b-d)}+\frac{e^{(2bx+2a)}}{16b}-\frac{e^{(-2bx+2dx-2a+2c)}}{32(b-d)}-\frac{e^{(-2bx-2dx-2a-2c)}}{32(b+d)}-\frac{e^{(-2bx-2a)}}{16b}+\frac{e^{(2dx+2c)}}{16d}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^2*cosh(d*x+c)^2,x, algorithm="giac")
```

```
[Out] 1/4*x + 1/32*e^(2*b*x + 2*d*x + 2*a + 2*c)/(b + d) + 1/32*e^(2*b*x - 2*d*x
+ 2*a - 2*c)/(b - d) + 1/16*e^(2*b*x + 2*a)/b - 1/32*e^(-2*b*x + 2*d*x - 2*
a + 2*c)/(b - d) - 1/32*e^(-2*b*x - 2*d*x - 2*a - 2*c)/(b + d) - 1/16*e^(-2
*b*x - 2*a)/b + 1/16*e^(2*d*x + 2*c)/d - 1/16*e^(-2*d*x - 2*c)/d
```

maple [A] time = 0.33, size = 83, normalized size = 0.94

$$\frac{x}{4}+\frac{\sinh\left(2bx+2a\right)}{8b}+\frac{\sinh\left(2dx+2c\right)}{8d}+\frac{\sinh\left(\left(2b-2d\right)x+2a-2c\right)}{16b-16d}+\frac{\sinh\left(\left(2b+2d\right)x+2a+2c\right)}{16b+16d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)^2*cosh(d*x+c)^2,x)
```

```
[Out] 1/4*x+1/8*sinh(2*b*x+2*a)/b+1/8*sinh(2*d*x+2*c)/d+1/16/(b-d)*sinh((2*b-2*d)
*x+2*a-2*c)+1/16/(b+d)*sinh((2*b+2*d)*x+2*a+2*c)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^2*cosh(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(1-(2*d)/b>0)', see `assume?` for mo
re details)Is 1-(2*d)/b equal to -1?
```

mupad [B] time = 1.92, size = 115, normalized size = 1.31

$$\frac{d^3\cosh\left(a+b\,x\right)\,\sinh\left(a+b\,x\right)-b^3\cosh\left(c+d\,x\right)\,\sinh\left(c+d\,x\right)+b\,d^3\,x-b^3\,d\,x-2\,b^2\,d\cosh\left(a+b\,x\right)\,\cosh\left(c+\right.}{4\,b\,d^3-4\,b^3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a + b*x)^2*cosh(c + d*x)^2,x)
```

```
[Out] (d^3*cosh(a + b*x)*sinh(a + b*x) - b^3*cosh(c + d*x)*sinh(c + d*x) + b*d^3*
x - b^3*d*x - 2*b^2*d*cosh(a + b*x)*cosh(c + d*x)^2*sinh(a + b*x) + 2*b*d^2
*cosh(a + b*x)^2*cosh(c + d*x)*sinh(c + d*x))/(4*b*d^3 - 4*b^3*d)
```

sympy [A] time = 21.55, size = 1027, normalized size = 11.67

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)**2*cosh(d*x+c)**2,x)
```

```
[Out] Piecewise((x*cosh(a)**2*cosh(c)**2, Eq(b, 0) & Eq(d, 0)), ((-x*sinh(c + d*x
)**2/2 + x*cosh(c + d*x)**2/2 + sinh(c + d*x)*cosh(c + d*x)/(2*d))*cosh(a)*
*2, Eq(b, 0)), (3*x*sinh(a - d*x)**2*sinh(c + d*x)**2/8 - x*sinh(a - d*x)**
2*cosh(c + d*x)**2/8 + x*sinh(a - d*x)*sinh(c + d*x)*cosh(a - d*x)*cosh(c +
 d*x)/2 - x*sinh(c + d*x)**2*cosh(a - d*x)**2/8 + 3*x*cosh(a - d*x)**2*cosh
(c + d*x)**2/8 + sinh(a - d*x)**2*sinh(c + d*x)*cosh(c + d*x)/(8*d) + sinh(
a - d*x)*sinh(c + d*x)**2*cosh(a - d*x)/(2*d) + 5*sinh(c + d*x)*cosh(a - d*
```

```
x)**2*cosh(c + d*x)/(8*d), Eq(b, -d)), (3*x*sinh(a + d*x)**2*sinh(c + d*x)*
*2/8 - x*sinh(a + d*x)**2*cosh(c + d*x)**2/8 - x*sinh(a + d*x)*sinh(c + d*x
)*cosh(a + d*x)*cosh(c + d*x)/2 - x*sinh(c + d*x)**2*cosh(a + d*x)**2/8 + 3
*x*cosh(a + d*x)**2*cosh(c + d*x)**2/8 - 3*sinh(a + d*x)*sinh(c + d*x)**2*c
osh(a + d*x)/(8*d) + sinh(a + d*x)*cosh(a + d*x)*cosh(c + d*x)**2/(8*d) + s
inh(c + d*x)*cosh(a + d*x)**2*cosh(c + d*x)/(2*d), Eq(b, d)), ((-x*sinh(a +
 b*x)**2/2 + x*cosh(a + b*x)**2/2 + sinh(a + b*x)*cosh(a + b*x)/(2*b))*cosh
(c)**2, Eq(d, 0)), (b**3*d*x*sinh(a + b*x)**2*sinh(c + d*x)**2/(4*b**3*d -
4*b*d**3) - b**3*d*x*sinh(a + b*x)**2*cosh(c + d*x)**2/(4*b**3*d - 4*b*d**3
) - b**3*d*x*sinh(c + d*x)**2*cosh(a + b*x)**2/(4*b**3*d - 4*b*d**3) + b**3
*d*x*cosh(a + b*x)**2*cosh(c + d*x)**2/(4*b**3*d - 4*b*d**3) - b**3*sinh(a
+ b*x)**2*sinh(c + d*x)*cosh(c + d*x)/(4*b**3*d - 4*b*d**3) + b**3*sinh(c +
 d*x)*cosh(a + b*x)**2*cosh(c + d*x)/(4*b**3*d - 4*b*d**3) + 2*b**2*d*sinh(
a + b*x)*cosh(a + b*x)*cosh(c + d*x)**2/(4*b**3*d - 4*b*d**3) - b*d**3*x*si
nh(a + b*x)**2*sinh(c + d*x)**2/(4*b**3*d - 4*b*d**3) + b*d**3*x*sinh(a + b
*x)**2*cosh(c + d*x)**2/(4*b**3*d - 4*b*d**3) + b*d**3*x*sinh(c + d*x)**2*c
osh(a + b*x)**2/(4*b**3*d - 4*b*d**3) - b*d**3*x*cosh(a + b*x)**2*cosh(c +
d*x)**2/(4*b**3*d - 4*b*d**3) - 2*b*d**2*sinh(c + d*x)*cosh(a + b*x)**2*cos
h(c + d*x)/(4*b**3*d - 4*b*d**3) + d**3*sinh(a + b*x)*sinh(c + d*x)**2*cosh
(a + b*x)/(4*b**3*d - 4*b*d**3) - d**3*sinh(a + b*x)*cosh(a + b*x)*cosh(c +
 d*x)**2/(4*b**3*d - 4*b*d**3), True))
```

3.177 $\int \cosh^2(a+bx)\cosh^3(c+dx)\,dx$

Optimal. Leaf size=144

$$\frac{\sinh(2a+x(2b-3d)-3c)}{16(2b-3d)}+\frac{3\sinh(2a+x(2b-d)-c)}{16(2b-d)}+\frac{3\sinh(2a+x(2b+d)+c)}{16(2b+d)}+\frac{\sinh(2a+x(2b+3d)+3c)}{16(2b+3d)}$$

```
[Out] 1/16*sinh(2*a-3*c+(2*b-3*d)*x)/(2*b-3*d)+3/16*sinh(2*a-c+(2*b-d)*x)/(2*b-d)
+3/8*sinh(d*x+c)/d+1/24*sinh(3*d*x+3*c)/d+3/16*sinh(2*a+c+(2*b+d)*x)/(2*b+d
)+1/16*sinh(2*a+3*c+(2*b+3*d)*x)/(2*b+3*d)
```

Rubi [A] time = 0.10, antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {5614, 2637}

$$\frac{\sinh(2a+x(2b-3d)-3c)}{16(2b-3d)}+\frac{3\sinh(2a+x(2b-d)-c)}{16(2b-d)}+\frac{3\sinh(2a+x(2b+d)+c)}{16(2b+d)}+\frac{\sinh(2a+x(2b+3d)+3c)}{16(2b+3d)}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[a + b*x]^2*Cosh[c + d*x]^3,x]
```

```
[Out] Sinh[2*a - 3*c + (2*b - 3*d)*x]/(16*(2*b - 3*d)) + (3*Sinh[2*a - c + (2*b -
 d)*x])/(16*(2*b - d)) + (3*Sinh[c + d*x])/(8*d) + Sinh[3*c + 3*d*x]/(24*d)
 + (3*Sinh[2*a + c + (2*b + d)*x])/(16*(2*b + d)) + Sinh[2*a + 3*c + (2*b +
 3*d)*x]/(16*(2*b + 3*d))
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 5614

```
Int[Cosh[v_]^(p_.)*Cosh[w_]^(q_.), x_Symbol] :> Int[ExpandTrigReduce[Cosh[v
]^p*Cosh[w]^q, x], x] /; IGtQ[p, 0] && IGtQ[q, 0] && ((PolynomialQ[v, x] &&
 PolynomialQ[w, x]) || (BinomialQ[{v, w}, x] && IndependentQ[Cancel[v/w], x
]))
```

Rubi steps

$$\int \cosh^2(a+bx)\cosh^3(c+dx)\,dx = \int \left(\frac{1}{16}\cosh(2a-3c+(2b-3d)x) + \frac{3}{16}\cosh(2a-c+(2b-d)x) + \frac{3}{8}\cosh\right.$$
$$= \frac{1}{16}\int \cosh(2a-3c+(2b-3d)x)\,dx + \frac{1}{16}\int \cosh(2a+3c+(2b+3d)x)\,d$$
$$= \frac{\sinh(2a-3c+(2b-3d)x)}{16(2b-3d)} + \frac{3\sinh(2a-c+(2b-d)x)}{16(2b-d)} + \frac{3\sinh(c+dx)}{8d}$$

Mathematica [A] time = 1.60, size = 158, normalized size = 1.10

$$\frac{1}{48}\left(\frac{3\sinh(2a+2bx-3c-3dx)}{2b-3d} + \frac{9\sinh(2a+2bx-c-dx)}{2b-d} + \frac{9\sinh(2a+2bx+c+dx)}{2b+d} + \frac{3\sinh(2a+2bx}{2b+3}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[a + b*x]^2*Cosh[c + d*x]^3,x]
```

```
[Out] ((18*Cosh[d*x]*Sinh[c])/d + (2*Cosh[3*d*x]*Sinh[3*c])/d + (18*Cosh[c]*Sinh[
d*x])/d + (2*Cosh[3*c]*Sinh[3*d*x])/d + (3*Sinh[2*a - 3*c + 2*b*x - 3*d*x])
/(2*b - 3*d) + (9*Sinh[2*a - c + 2*b*x - d*x])/(2*b - d) + (9*Sinh[2*a + c
+ 2*b*x + d*x])/(2*b + d) + (3*Sinh[2*a + 3*c + 2*b*x + 3*d*x])/(2*b + 3*d)
)/48
```

fricas [B] time = 0.43, size = 397, normalized size = 2.76

$$\frac{36\left(4b^3d-bd^3\right)\cosh(bx+a)\cosh(dx+c)\sinh(bx+a)\sinh(dx+c)^2 + \left(16b^4-40b^2d^2+9d^4-9\left(4b^2d^2-\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^2*cosh(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] 1/24*(36*(4*b^3*d - b*d^3)*cosh(b*x + a)*cosh(d*x + c)*sinh(b*x + a)*sinh(d
*x + c)^2 + (16*b^4 - 40*b^2*d^2 + 9*d^4 - 9*(4*b^2*d^2 - d^4)*cosh(b*x + a
)^2 - 9*(4*b^2*d^2 - d^4)*sinh(b*x + a)^2)*sinh(d*x + c)^3 + 12*((4*b^3*d -
 b*d^3)*cosh(b*x + a)*cosh(d*x + c)^3 + 3*(4*b^3*d - 9*b*d^3)*cosh(b*x + a)
*cosh(d*x + c))*sinh(b*x + a) + 3*(48*b^4 - 120*b^2*d^2 + 27*d^4 - 3*(4*b^2
*d^2 - 9*d^4)*cosh(b*x + a)^2 + (16*b^4 - 40*b^2*d^2 + 9*d^4 - 9*(4*b^2*d^2
 - d^4)*cosh(b*x + a)^2)*cosh(d*x + c)^2 - 3*(4*b^2*d^2 - 9*d^4 + 3*(4*b^2*
d^2 - d^4)*cosh(d*x + c)^2)*sinh(b*x + a)^2)*sinh(d*x + c))/((16*b^4*d - 40
*b^2*d^3 + 9*d^5)*cosh(b*x + a)^2 - (16*b^4*d - 40*b^2*d^3 + 9*d^5)*sinh(b*
x + a)^2)
```

giac [A] time = 0.15, size = 260, normalized size = 1.81

$$\frac{e^{(2\,bx+3\,dx+2\,a+3\,c)}}{32\,(2\,b+3\,d)}+\frac{3\,e^{(2\,bx+dx+2\,a+c)}}{32\,(2\,b+d)}+\frac{3\,e^{(2\,bx-dx+2\,a-c)}}{32\,(2\,b-d)}+\frac{e^{(2\,bx-3\,dx+2\,a-3\,c)}}{32\,(2\,b-3\,d)}-\frac{e^{(-2\,bx+3\,dx-2\,a+3\,c)}}{32\,(2\,b-3\,d)}-\frac{3\,e^{(-2\,bx+dx-2\,a+c)}}{32\,(2\,b-d)}-\frac{3\,e^{(-2}}{32}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^2*cosh(d*x+c)^3,x, algorithm="giac")
```

```
[Out] 1/32*e^(2*b*x + 3*d*x + 2*a + 3*c)/(2*b + 3*d) + 3/32*e^(2*b*x + d*x + 2*a
+ c)/(2*b + d) + 3/32*e^(2*b*x - d*x + 2*a - c)/(2*b - d) + 1/32*e^(2*b*x -
 3*d*x + 2*a - 3*c)/(2*b - 3*d) - 1/32*e^(-2*b*x + 3*d*x - 2*a + 3*c)/(2*b
- 3*d) - 3/32*e^(-2*b*x + d*x - 2*a + c)/(2*b - d) - 3/32*e^(-2*b*x - d*x -
 2*a - c)/(2*b + d) - 1/32*e^(-2*b*x - 3*d*x - 2*a - 3*c)/(2*b + 3*d) + 1/4
8*e^(3*d*x + 3*c)/d + 3/16*e^(d*x + c)/d - 3/16*e^(-d*x - c)/d - 1/48*e^(-3
*d*x - 3*c)/d
```

maple [A] time = 0.39, size = 133, normalized size = 0.92

$$\frac{\sinh(2a-3c+(2b-3d)\,x)}{32b-48d}+\frac{3\sinh(2a-c+(2b-d)\,x)}{16\,(2b-d)}+\frac{3\sinh(dx+c)}{8d}+\frac{\sinh(3dx+3c)}{24d}+\frac{3\sinh(2a+c+(2b}{16\,(2b+d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)^2*cosh(d*x+c)^3,x)
```

```
[Out] 1/16*sinh(2*a-3*c+(2*b-3*d)*x)/(2*b-3*d)+3/16*sinh(2*a-c+(2*b-d)*x)/(2*b-d)
+3/8*sinh(d*x+c)/d+1/24*sinh(3*d*x+3*c)/d+3/16*sinh(2*a+c+(2*b+d)*x)/(2*b+d
)+1/16*sinh(2*a+3*c+(2*b+3*d)*x)/(2*b+3*d)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^2*cosh(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(1-(3*d)/b>0)', see `assume?` for mo
re details)Is 1-(3*d)/b equal to -1?
```

mupad [B] time = 1.95, size = 337, normalized size = 2.34

$$\frac{\cosh(a+b\,x)^2\cosh(c+d\,x)^2\sinh(c+d\,x)\,\left(8\,b^4-26\,b^2\,d^2+9\,d^4\right)}{d\,\left(16\,b^4-40\,b^2\,d^2+9\,d^4\right)}-\sinh(a+b\,x)^2\sinh(c+d\,x)^3\,\left(\frac{3\,a}{16\,b^4-40\,b}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a + b*x)^2*cosh(c + d*x)^3,x)
```

```
[Out] (cosh(a + b*x)^2*cosh(c + d*x)^2*sinh(c + d*x)*(8*b^4 + 9*d^4 - 26*b^2*d^2)
)/(d*(16*b^4 + 9*d^4 - 40*b^2*d^2)) - sinh(a + b*x)^2*sinh(c + d*x)^3*((3*d
^3)/(16*b^4 + 9*d^4 - 40*b^2*d^2) - 1/(3*d)) - (2*cosh(a + b*x)*cosh(c + d*
x)^3*sinh(a + b*x)*(7*b*d^2 - 4*b^3))/(16*b^4 + 9*d^4 - 40*b^2*d^2) - cosh(
a + b*x)^2*sinh(c + d*x)^3*((3*d^3)/(16*b^4 + 9*d^4 - 40*b^2*d^2) + 1/(3*d)
) + (12*b*d^2*cosh(a + b*x)*cosh(c + d*x)*sinh(a + b*x)*sinh(c + d*x)^2)/(1
6*b^4 + 9*d^4 - 40*b^2*d^2) - (2*b^2*cosh(c + d*x)^2*sinh(a + b*x)^2*sinh(c
 + d*x)*(4*b^2 - 7*d^2))/(d*(16*b^4 + 9*d^4 - 40*b^2*d^2))
```

sympy [A] time = 109.36, size = 2003, normalized size = 13.91

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)**2*cosh(d*x+c)**3,x)
```

```
[Out] Piecewise((x*cosh(a)**2*cosh(c)**3, Eq(b, 0) & Eq(d, 0)), (3*x*sinh(a - 3*d
*x/2)**2*sinh(c + d*x)**2*cosh(c + d*x)/16 + x*sinh(a - 3*d*x/2)**2*cosh(c
+ d*x)**3/16 + x*sinh(a - 3*d*x/2)*sinh(c + d*x)**3*cosh(a - 3*d*x/2)/8 + 3
*x*sinh(a - 3*d*x/2)*sinh(c + d*x)*cosh(a - 3*d*x/2)*cosh(c + d*x)**2/8 + 3
*x*sinh(c + d*x)**2*cosh(a - 3*d*x/2)**2*cosh(c + d*x)/16 + x*cosh(a - 3*d*
x/2)**2*cosh(c + d*x)**3/16 + 11*sinh(a - 3*d*x/2)**2*sinh(c + d*x)**3/(48*
d) - sinh(a - 3*d*x/2)**2*sinh(c + d*x)*cosh(c + d*x)**2/d - 3*sinh(a - 3*d
*x/2)*sinh(c + d*x)**2*cosh(a - 3*d*x/2)*cosh(c + d*x)/(4*d) - 5*sinh(a - 3
*d*x/2)*cosh(a - 3*d*x/2)*cosh(c + d*x)**3/(8*d) - 7*sinh(c + d*x)**3*cosh(
a - 3*d*x/2)**2/(16*d), Eq(b, -3*d/2)), (-3*x*sinh(a - d*x/2)**2*sinh(c + d
*x)**2*cosh(c + d*x)/16 + 3*x*sinh(a - d*x/2)**2*cosh(c + d*x)**3/16 - 3*x*
sinh(a - d*x/2)*sinh(c + d*x)**3*cosh(a - d*x/2)/8 + 3*x*sinh(a - d*x/2)*si
nh(c + d*x)*cosh(a - d*x/2)*cosh(c + d*x)**2/8 - 3*x*sinh(c + d*x)**2*cosh(
a - d*x/2)**2*cosh(c + d*x)/16 + 3*x*cosh(a - d*x/2)**2*cosh(c + d*x)**3/16
 + sinh(a - d*x/2)**2*sinh(c + d*x)**3/(48*d) - sinh(a - d*x/2)*sinh(c + d*
x)**2*cosh(a - d*x/2)*cosh(c + d*x)/(4*d) + 3*sinh(a - d*x/2)*cosh(a - d*x/
2)*cosh(c + d*x)**3/(8*d) - 31*sinh(c + d*x)**3*cosh(a - d*x/2)**2/(48*d) +
 sinh(c + d*x)*cosh(a - d*x/2)**2*cosh(c + d*x)**2/d, Eq(b, -d/2)), (-3*x*s
inh(a + d*x/2)**2*sinh(c + d*x)**2*cosh(c + d*x)/16 + 3*x*sinh(a + d*x/2)**
2*cosh(c + d*x)**3/16 + 3*x*sinh(a + d*x/2)*sinh(c + d*x)**3*cosh(a + d*x/2
)/8 - 3*x*sinh(a + d*x/2)*sinh(c + d*x)*cosh(a + d*x/2)*cosh(c + d*x)**2/8
- 3*x*sinh(c + d*x)**2*cosh(a + d*x/2)**2*cosh(c + d*x)/16 + 3*x*cosh(a + d
*x/2)**2*cosh(c + d*x)**3/16 + sinh(a + d*x/2)**2*sinh(c + d*x)**3/(48*d) +
 sinh(a + d*x/2)*sinh(c + d*x)**2*cosh(a + d*x/2)*cosh(c + d*x)/(4*d) - 3*s
inh(a + d*x/2)*cosh(a + d*x/2)*cosh(c + d*x)**3/(8*d) - 31*sinh(c + d*x)**3
```

```
*cosh(a + d*x/2)**2/(48*d) + sinh(c + d*x)*cosh(a + d*x/2)**2*cosh(c + d*x)
**2/d, Eq(b, d/2)), (3*x*sinh(a + 3*d*x/2)**2*sinh(c + d*x)**2*cosh(c + d*x
)/16 + x*sinh(a + 3*d*x/2)**2*cosh(c + d*x)**3/16 - x*sinh(a + 3*d*x/2)*sin
h(c + d*x)**3*cosh(a + 3*d*x/2)/8 - 3*x*sinh(a + 3*d*x/2)*sinh(c + d*x)*cos
h(a + 3*d*x/2)*cosh(c + d*x)**2/8 + 3*x*sinh(c + d*x)**2*cosh(a + 3*d*x/2)*
*2*cosh(c + d*x)/16 + x*cosh(a + 3*d*x/2)**2*cosh(c + d*x)**3/16 + 11*sinh(
a + 3*d*x/2)**2*sinh(c + d*x)**3/(48*d) - sinh(a + 3*d*x/2)**2*sinh(c + d*x
)*cosh(c + d*x)**2/d + 3*sinh(a + 3*d*x/2)*sinh(c + d*x)**2*cosh(a + 3*d*x/
2)*cosh(c + d*x)/(4*d) + 5*sinh(a + 3*d*x/2)*cosh(a + 3*d*x/2)*cosh(c + d*x
)**3/(8*d) - 7*sinh(c + d*x)**3*cosh(a + 3*d*x/2)**2/(16*d), Eq(b, 3*d/2)),
 ((-x*sinh(a + b*x)**2/2 + x*cosh(a + b*x)**2/2 + sinh(a + b*x)*cosh(a + b*
x)/(2*b))*cosh(c)**3, Eq(d, 0)), (16*b**4*sinh(a + b*x)**2*sinh(c + d*x)**3
/(48*b**4*d - 120*b**2*d**3 + 27*d**5) - 24*b**4*sinh(a + b*x)**2*sinh(c +
d*x)*cosh(c + d*x)**2/(48*b**4*d - 120*b**2*d**3 + 27*d**5) - 16*b**4*sinh(
c + d*x)**3*cosh(a + b*x)**2/(48*b**4*d - 120*b**2*d**3 + 27*d**5) + 24*b**
4*sinh(c + d*x)*cosh(a + b*x)**2*cosh(c + d*x)**2/(48*b**4*d - 120*b**2*d**
3 + 27*d**5) + 24*b**3*d*sinh(a + b*x)*cosh(a + b*x)*cosh(c + d*x)**3/(48*b
**4*d - 120*b**2*d**3 + 27*d**5) - 40*b**2*d**2*sinh(a + b*x)**2*sinh(c + d
*x)**3/(48*b**4*d - 120*b**2*d**3 + 27*d**5) + 42*b**2*d**2*sinh(a + b*x)**
2*sinh(c + d*x)*cosh(c + d*x)**2/(48*b**4*d - 120*b**2*d**3 + 27*d**5) + 40
*b**2*d**2*sinh(c + d*x)**3*cosh(a + b*x)**2/(48*b**4*d - 120*b**2*d**3 + 2
7*d**5) - 78*b**2*d**2*sinh(c + d*x)*cosh(a + b*x)**2*cosh(c + d*x)**2/(48*
b**4*d - 120*b**2*d**3 + 27*d**5) + 36*b*d**3*sinh(a + b*x)*sinh(c + d*x)**
2*cosh(a + b*x)*cosh(c + d*x)/(48*b**4*d - 120*b**2*d**3 + 27*d**5) - 42*b*
d**3*sinh(a + b*x)*cosh(a + b*x)*cosh(c + d*x)**3/(48*b**4*d - 120*b**2*d**
3 + 27*d**5) - 18*d**4*sinh(c + d*x)**3*cosh(a + b*x)**2/(48*b**4*d - 120*b
**2*d**3 + 27*d**5) + 27*d**4*sinh(c + d*x)*cosh(a + b*x)**2*cosh(c + d*x)*
*2/(48*b**4*d - 120*b**2*d**3 + 27*d**5), True))
```

3.178 $\int \cosh^3(a+bx)\cosh^3(c+dx)\,dx$

Optimal. Leaf size=195

$$\frac{3\sinh(a+x(b-3d)-3c)}{32(b-3d)}+\frac{9\sinh(a+x(b-d)-c)}{32(b-d)}+\frac{\sinh(3(a-c)+3x(b-d))}{96(b-d)}+\frac{3\sinh(3a+x(3b-d)-c)}{32(3b-d)}+$$

```
[Out] 3/32*sinh(a-3*c+(b-3*d)*x)/(b-3*d)+9/32*sinh(a-c+(b-d)*x)/(b-d)+1/96*sinh(3
*a-3*c+3*(b-d)*x)/(b-d)+3/32*sinh(3*a-c+(3*b-d)*x)/(3*b-d)+9/32*sinh(a+c+(b
+d)*x)/(b+d)+1/96*sinh(3*a+3*c+3*(b+d)*x)/(b+d)+3/32*sinh(3*a+c+(3*b+d)*x)/
(3*b+d)+3/32*sinh(a+3*c+(b+3*d)*x)/(b+3*d)
```

Rubi [A] time = 0.13, antiderivative size = 195, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {5614, 2637}

$$\frac{3\sinh(a+x(b-3d)-3c)}{32(b-3d)}+\frac{9\sinh(a+x(b-d)-c)}{32(b-d)}+\frac{\sinh(3(a-c)+3x(b-d))}{96(b-d)}+\frac{3\sinh(3a+x(3b-d)-c)}{32(3b-d)}+$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[a + b*x]^3*Cosh[c + d*x]^3,x]
```

```
[Out] (3*Sinh[a - 3*c + (b - 3*d)*x])/(32*(b - 3*d)) + (9*Sinh[a - c + (b - d)*x]
)/(32*(b - d)) + Sinh[3*(a - c) + 3*(b - d)*x]/(96*(b - d)) + (3*Sinh[3*a -
 c + (3*b - d)*x])/(32*(3*b - d)) + (9*Sinh[a + c + (b + d)*x])/(32*(b + d)
) + Sinh[3*(a + c) + 3*(b + d)*x]/(96*(b + d)) + (3*Sinh[3*a + c + (3*b + d
)*x])/(32*(3*b + d)) + (3*Sinh[a + 3*c + (b + 3*d)*x])/(32*(b + 3*d))
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 5614

```
Int[Cosh[v_]^(p_.)*Cosh[w_]^(q_.), x_Symbol] :> Int[ExpandTrigReduce[Cosh[v
]^p*Cosh[w]^q, x], x] /; IGtQ[p, 0] && IGtQ[q, 0] && ((PolynomialQ[v, x] &&
 PolynomialQ[w, x]) || (BinomialQ[{v, w}, x] && IndependentQ[Cancel[v/w], x
]))
```

Rubi steps

$$\int \cosh^3(a+bx)\cosh^3(c+dx)\,dx = \int \left(\frac{3}{32}\cosh(a-3c+(b-3d)x) + \frac{9}{32}\cosh(a-c+(b-d)x) + \frac{1}{32}\cosh(3(a\right.$$
$$= \frac{1}{32}\int \cosh(3(a-c)+3(b-d)x)\,dx + \frac{1}{32}\int \cosh(3(a+c)+3(b+d)x)\,dx +$$
$$= \frac{3\sinh(a-3c+(b-3d)x)}{32(b-3d)} + \frac{9\sinh(a-c+(b-d)x)}{32(b-d)} + \frac{\sinh(3(a-c)+3(b}{96(b-d)}$$

Mathematica [A] time = 1.50, size = 176, normalized size = 0.90

$$\frac{1}{96}\left(\frac{9\sinh(a+bx-3c-3dx)}{b-3d} + \frac{27\sinh(a+bx-c-dx)}{b-d} + \frac{\sinh(3(a+bx-c-dx))}{b-d} + \frac{9\sinh(3a+3bx-c-d}{3b-d}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[a + b*x]^3*Cosh[c + d*x]^3,x]
```

```
[Out] ((9*Sinh[a - 3*c + b*x - 3*d*x])/(b - 3*d) + (27*Sinh[a - c + b*x - d*x])/(
b - d) + Sinh[3*(a - c + b*x - d*x)]/(b - d) + (9*Sinh[3*a - c + 3*b*x - d*
x])/(3*b - d) + (9*Sinh[3*a + c + 3*b*x + d*x])/(3*b + d) + (9*Sinh[a + 3*c
 + b*x + 3*d*x])/(b + 3*d) + (27*Sinh[a + c + (b + d)*x])/(b + d) + Sinh[3*
(a + c + (b + d)*x)]/(b + d))/96
```

fricas [B] time = 0.55, size = 726, normalized size = 3.72

$$\frac{\left(\left(9\,b^5-82\,b^3d^2+9\,bd^4\right)\cosh\left(dx+c\right)^3+27\left(b^5-10\,b^3d^2+9\,bd^4\right)\cosh\left(dx+c\right)\right)\sinh\left(bx+a\right)^3-\left(\left(9\,b^4d-82\,b\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^3*cosh(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] 1/48*(((9*b^5 - 82*b^3*d^2 + 9*b*d^4)*cosh(d*x + c)^3 + 27*(b^5 - 10*b^3*d^
2 + 9*b*d^4)*cosh(d*x + c))*sinh(b*x + a)^3 - ((9*b^4*d - 82*b^2*d^3 + 9*d^
5)*cosh(b*x + a)^3 + 3*(9*b^4*d - 82*b^2*d^3 + 9*d^5)*cosh(b*x + a)*sinh(b*
x + a)^2 + 27*(9*b^4*d - 10*b^2*d^3 + d^5)*cosh(b*x + a))*sinh(d*x + c)^3 +
 3*((9*b^5 - 82*b^3*d^2 + 9*b*d^4)*cosh(d*x + c)*sinh(b*x + a)^3 + 3*(27*b^
5 - 30*b^3*d^2 + 3*b*d^4 + (9*b^5 - 82*b^3*d^2 + 9*b*d^4)*cosh(b*x + a)^2)*
cosh(d*x + c)*sinh(b*x + a))*sinh(d*x + c)^2 + 3*((27*b^5 - 30*b^3*d^2 + 3*
b*d^4 + (9*b^5 - 82*b^3*d^2 + 9*b*d^4)*cosh(b*x + a)^2)*cosh(d*x + c)^3 + 9
*(9*b^5 - 82*b^3*d^2 + 9*b*d^4 + 3*(b^5 - 10*b^3*d^2 + 9*b*d^4)*cosh(b*x +
a)^2)*cosh(d*x + c))*sinh(b*x + a) - 3*(3*(b^4*d - 10*b^2*d^3 + 9*d^5)*cosh
(b*x + a)^3 + ((9*b^4*d - 82*b^2*d^3 + 9*d^5)*cosh(b*x + a)^3 + 27*(9*b^4*d
```

```
 - 10*b^2*d^3 + d^5)*cosh(b*x + a))*cosh(d*x + c)^2 + 3*((9*b^4*d - 82*b^2*
d^3 + 9*d^5)*cosh(b*x + a)*cosh(d*x + c)^2 + 3*(b^4*d - 10*b^2*d^3 + 9*d^5)
*cosh(b*x + a))*sinh(b*x + a)^2 + 9*(9*b^4*d - 82*b^2*d^3 + 9*d^5)*cosh(b*x
 + a))*sinh(d*x + c))/((9*b^6 - 91*b^4*d^2 + 91*b^2*d^4 - 9*d^6)*cosh(b*x +
 a)^4 - 2*(9*b^6 - 91*b^4*d^2 + 91*b^2*d^4 - 9*d^6)*cosh(b*x + a)^2*sinh(b*
x + a)^2 + (9*b^6 - 91*b^4*d^2 + 91*b^2*d^4 - 9*d^6)*sinh(b*x + a)^4)
```

giac [B] time = 0.15, size = 373, normalized size = 1.91

$$\frac{e^{(3bx+3dx+3a+3c)}}{192(b+d)}+\frac{3e^{(3bx+dx+3a+c)}}{64(3b+d)}+\frac{3e^{(3bx-dx+3a-c)}}{64(3b-d)}+\frac{e^{(3bx-3dx+3a-3c)}}{192(b-d)}+\frac{3e^{(bx+3dx+a+3c)}}{64(b+3d)}+\frac{9e^{(bx+dx+a+c)}}{64(b+d)}+\frac{9e^{(bx-dx}}{64(b-}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^3*cosh(d*x+c)^3,x, algorithm="giac")
```

```
[Out] 1/192*e^(3*b*x + 3*d*x + 3*a + 3*c)/(b + d) + 3/64*e^(3*b*x + d*x + 3*a + c
)/(3*b + d) + 3/64*e^(3*b*x - d*x + 3*a - c)/(3*b - d) + 1/192*e^(3*b*x - 3
*d*x + 3*a - 3*c)/(b - d) + 3/64*e^(b*x + 3*d*x + a + 3*c)/(b + 3*d) + 9/64
*e^(b*x + d*x + a + c)/(b + d) + 9/64*e^(b*x - d*x + a - c)/(b - d) + 3/64*
e^(b*x - 3*d*x + a - 3*c)/(b - 3*d) - 3/64*e^(-b*x + 3*d*x - a + 3*c)/(b -
3*d) - 9/64*e^(-b*x + d*x - a + c)/(b - d) - 9/64*e^(-b*x - d*x - a - c)/(b
 + d) - 3/64*e^(-b*x - 3*d*x - a - 3*c)/(b + 3*d) - 1/192*e^(-3*b*x + 3*d*x
 - 3*a + 3*c)/(b - d) - 3/64*e^(-3*b*x + d*x - 3*a + c)/(3*b - d) - 3/64*e^
(-3*b*x - d*x - 3*a - c)/(3*b + d) - 1/192*e^(-3*b*x - 3*d*x - 3*a - 3*c)/(
b + d)
```

maple [A] time = 0.55, size = 184, normalized size = 0.94

$$\frac{3\sinh(a-3c+(b-3d)x)}{32(b-3d)}+\frac{9\sinh(a-c+(b-d)x)}{32(b-d)}+\frac{9\sinh(a+c+(b+d)x)}{32(b+d)}+\frac{3\sinh(a+3c+(b+3d)x)}{32(b+3d)}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)^3*cosh(d*x+c)^3,x)
```

```
[Out] 3/32*sinh(a-3*c+(b-3*d)*x)/(b-3*d)+9/32*sinh(a-c+(b-d)*x)/(b-d)+9/32*sinh(a
+c+(b+d)*x)/(b+d)+3/32*sinh(a+3*c+(b+3*d)*x)/(b+3*d)+1/96/(b-d)*sinh((3*b-3
*d)*x+3*a-3*c)+3/32*sinh(3*a-c+(3*b-d)*x)/(3*b-d)+3/32*sinh(3*a+c+(3*b+d)*x
)/(3*b+d)+1/96/(b+d)*sinh((3*b+3*d)*x+3*a+3*c)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^3*cosh(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(-(3*d)/b>0)', see `assume?` for mor
e details)Is -(3*d)/b equal to -1?
```

mupad [B] time = 2.03, size = 908, normalized size = 4.66

$$-e^{3a+c+3bx+dx}\left(\frac{-9b^3+3b^2d+9bd^2-3d^3}{576b^4-640b^2d^2+64d^4}-\frac{e^{-6a-6bx}\left(-9b^3-3b^2d+9bd^2+3d^3\right)}{576b^4-640b^2d^2+64d^4}+\frac{e^{-2a-2bx}\left(-81b^3+8\right.}{576b^4-640\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a + b*x)^3*cosh(c + d*x)^3,x)
```

```
[Out] - exp(3*a + c + 3*b*x + d*x)*((9*b*d^2 + 3*b^2*d - 9*b^3 - 3*d^3)/(576*b^4
+ 64*d^4 - 640*b^2*d^2) - (exp(- 6*a - 6*b*x)*(9*b*d^2 - 3*b^2*d - 9*b^3 +
3*d^3))/(576*b^4 + 64*d^4 - 640*b^2*d^2) + (exp(- 2*a - 2*b*x)*(9*b*d^2 + 8
1*b^2*d - 81*b^3 - 9*d^3))/(576*b^4 + 64*d^4 - 640*b^2*d^2) - (exp(- 4*a -
4*b*x)*(9*b*d^2 - 81*b^2*d - 81*b^3 + 9*d^3))/(576*b^4 + 64*d^4 - 640*b^2*d
^2)) - exp(3*a - c + 3*b*x - d*x)*((9*b*d^2 - 3*b^2*d - 9*b^3 + 3*d^3)/(576
*b^4 + 64*d^4 - 640*b^2*d^2) - (exp(- 6*a - 6*b*x)*(9*b*d^2 + 3*b^2*d - 9*b
^3 - 3*d^3))/(576*b^4 + 64*d^4 - 640*b^2*d^2) + (exp(- 2*a - 2*b*x)*(9*b*d^
2 - 81*b^2*d - 81*b^3 + 9*d^3))/(576*b^4 + 64*d^4 - 640*b^2*d^2) - (exp(- 4
*a - 4*b*x)*(9*b*d^2 + 81*b^2*d - 81*b^3 - 9*d^3))/(576*b^4 + 64*d^4 - 640*
b^2*d^2)) - exp(3*a - 3*c + 3*b*x - 3*d*x)*((9*b*d^2 - b^2*d - b^3 + 9*d^3)
/(192*b^4 + 1728*d^4 - 1920*b^2*d^2) - (exp(- 6*a - 6*b*x)*(9*b*d^2 + b^2*d
 - b^3 - 9*d^3))/(192*b^4 + 1728*d^4 - 1920*b^2*d^2) + (exp(- 2*a - 2*b*x)*
(9*b*d^2 - 27*b^2*d - 9*b^3 + 27*d^3))/(192*b^4 + 1728*d^4 - 1920*b^2*d^2)
- (exp(- 4*a - 4*b*x)*(9*b*d^2 + 27*b^2*d - 9*b^3 - 27*d^3))/(192*b^4 + 172
8*d^4 - 1920*b^2*d^2)) - exp(3*a + 3*c + 3*b*x + 3*d*x)*((9*b*d^2 + b^2*d -
 b^3 - 9*d^3)/(192*b^4 + 1728*d^4 - 1920*b^2*d^2) - (exp(- 6*a - 6*b*x)*(9*
b*d^2 - b^2*d - b^3 + 9*d^3))/(192*b^4 + 1728*d^4 - 1920*b^2*d^2) + (exp(-
2*a - 2*b*x)*(9*b*d^2 + 27*b^2*d - 9*b^3 - 27*d^3))/(192*b^4 + 1728*d^4 - 1
920*b^2*d^2) - (exp(- 4*a - 4*b*x)*(9*b*d^2 - 27*b^2*d - 9*b^3 + 27*d^3))/(
192*b^4 + 1728*d^4 - 1920*b^2*d^2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)**3*cosh(d*x+c)**3,x)
```

```
[Out] Timed out
```

3.179 $\int \cosh(c+dx)\sinh(a+bx)\,dx$

Optimal. Leaf size=43

$$\frac{\cosh(a+x(b-d)-c)}{2(b-d)}+\frac{\cosh(a+x(b+d)+c)}{2(b+d)}$$

```
[Out] 1/2*cosh(a-c+(b-d)*x)/(b-d)+1/2*cosh(a+c+(b+d)*x)/(b+d)
```

Rubi [A] time = 0.05, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {5618, 2638}

$$\frac{\cosh(a+x(b-d)-c)}{2(b-d)}+\frac{\cosh(a+x(b+d)+c)}{2(b+d)}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]*Sinh[a + b*x],x]
```

```
[Out] Cosh[a - c + (b - d)*x]/(2*(b - d)) + Cosh[a + c + (b + d)*x]/(2*(b + d))
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 5618

```
Int[Cosh[w_]^(q_.)*Sinh[v_]^(p_.), x_Symbol] :> Int[ExpandTrigReduce[Sinh[v
]^p*Cosh[w]^q, x], x] /; IGtQ[p, 0] && IGtQ[q, 0] && ((PolynomialQ[v, x] &&
 PolynomialQ[w, x]) || (BinomialQ[{v, w}, x] && IndependentQ[Cancel[v/w], x
]))
```

Rubi steps

$$\begin{aligned}\int \cosh(c+dx)\sinh(a+bx)\,dx &= \int \left(\frac{1}{2}\sinh(a-c+(b-d)x)+\frac{1}{2}\sinh(a+c+(b+d)x)\right)dx\\ &= \frac{1}{2}\int \sinh(a-c+(b-d)x)\,dx+\frac{1}{2}\int \sinh(a+c+(b+d)x)\,dx\\ &= \frac{\cosh(a-c+(b-d)x)}{2(b-d)}+\frac{\cosh(a+c+(b+d)x)}{2(b+d)}\end{aligned}$$

Mathematica [**A**] time = 0.21, size = 43, normalized size = 1.00

$$\frac{\cosh(a+x(b-d)-c)}{2(b-d)}+\frac{\cosh(a+x(b+d)+c)}{2(b+d)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]*Sinh[a + b*x],x]
[Out] Cosh[a - c + (b - d)*x]/(2*(b - d)) + Cosh[a + c + (b + d)*x]/(2*(b + d))
```

fricas [**A**] time = 0.52, size = 71, normalized size = 1.65

$$\frac{b\cosh(bx+a)\cosh(dx+c)-d\sinh(bx+a)\sinh(dx+c)}{\left(b^2-d^2\right)\cosh(bx+a)^2-\left(b^2-d^2\right)\sinh(bx+a)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)*sinh(b*x+a),x, algorithm="fricas")
[Out] (b*cosh(b*x + a)*cosh(d*x + c) - d*sinh(b*x + a)*sinh(d*x + c))/((b^2 - d^2
)*cosh(b*x + a)^2 - (b^2 - d^2)*sinh(b*x + a)^2)
```

giac [**B**] time = 0.12, size = 85, normalized size = 1.98

$$\frac{e^{(bx+dx+a+c)}}{4(b+d)}+\frac{e^{(bx-dx+a-c)}}{4(b-d)}+\frac{e^{(-bx+dx-a+c)}}{4(b-d)}+\frac{e^{(-bx-dx-a-c)}}{4(b+d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)*sinh(b*x+a),x, algorithm="giac")
[Out] 1/4*e^(b*x + d*x + a + c)/(b + d) + 1/4*e^(b*x - d*x + a - c)/(b - d) + 1/4
*e^(-b*x + d*x - a + c)/(b - d) + 1/4*e^(-b*x - d*x - a - c)/(b + d)
```

maple [**A**] time = 0.06, size = 40, normalized size = 0.93

$$\frac{\cosh(a-c+(b-d)x)}{2b-2d}+\frac{\cosh(a+c+(b+d)x)}{2b+2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)*sinh(b*x+a),x)
[Out] 1/2*cosh(a-c+(b-d)*x)/(b-d)+1/2*cosh(a+c+(b+d)*x)/(b+d)
```

maxima [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)*sinh(b*x+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(-d/b>0)', see `assume?` for more de
tails)Is -d/b equal to -1?
```

mupad [B] time = 0.14, size = 42, normalized size = 0.98

$$\frac{b\cosh\left(a+b\,x\right)\,\cosh\left(c+d\,x\right)-d\sinh\left(a+b\,x\right)\,\sinh\left(c+d\,x\right)}{b^2-d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(c + d*x)*sinh(a + b*x),x)
```

```
[Out] (b*cosh(a + b*x)*cosh(c + d*x) - d*sinh(a + b*x)*sinh(c + d*x))/(b^2 - d^2)
```

sympy [A] time = 1.48, size = 153, normalized size = 3.56

$$\begin{cases} x\sinh\left(a\right)\cosh\left(c\right) & \text{for}\: b=0\wedge d=0 \\ \frac{x\sinh\left(a-dx\right)\cosh\left(c+dx\right)}{2}+\frac{x\sinh\left(c+dx\right)\cosh\left(a-dx\right)}{2}+\frac{\sinh\left(a-dx\right)\sinh\left(c+dx\right)}{2d} & \text{for}\: b=-d \\ \frac{x\sinh\left(a+dx\right)\cosh\left(c+dx\right)}{2}-\frac{x\sinh\left(c+dx\right)\cosh\left(a+dx\right)}{2}+\frac{\sinh\left(a+dx\right)\sinh\left(c+dx\right)}{2d} & \text{for}\: b=d \\ \frac{b\cosh\left(a+bx\right)\cosh\left(c+dx\right)}{b^2-d^2}-\frac{d\sinh\left(a+bx\right)\sinh\left(c+dx\right)}{b^2-d^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)*sinh(b*x+a),x)
```

```
[Out] Piecewise((x*sinh(a)*cosh(c), Eq(b, 0) & Eq(d, 0)), (x*sinh(a - d*x)*cosh(c
 + d*x)/2 + x*sinh(c + d*x)*cosh(a - d*x)/2 + sinh(a - d*x)*sinh(c + d*x)/(
2*d), Eq(b, -d)), (x*sinh(a + d*x)*cosh(c + d*x)/2 - x*sinh(c + d*x)*cosh(a
 + d*x)/2 + sinh(a + d*x)*sinh(c + d*x)/(2*d), Eq(b, d)), (b*cosh(a + b*x)*
cosh(c + d*x)/(b**2 - d**2) - d*sinh(a + b*x)*sinh(c + d*x)/(b**2 - d**2),
True))
```

3.180 $\int \cosh^2(c+dx)\sinh(a+bx)\,dx$

Optimal. Leaf size=62

$$\frac{\cosh(a+x(b-2d)-2c)}{4(b-2d)}+\frac{\cosh(a+x(b+2d)+2c)}{4(b+2d)}+\frac{\cosh(a+bx)}{2b}$$

```
[Out] 1/2*cosh(b*x+a)/b+1/4*cosh(a-2*c+(b-2*d)*x)/(b-2*d)+1/4*cosh(a+2*c+(b+2*d)*
x)/(b+2*d)
```

Rubi [A] time = 0.06, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {5618, 2638}

$$\frac{\cosh(a+x(b-2d)-2c)}{4(b-2d)}+\frac{\cosh(a+x(b+2d)+2c)}{4(b+2d)}+\frac{\cosh(a+bx)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]^2*Sinh[a + b*x],x]
```

```
[Out] Cosh[a + b*x]/(2*b) + Cosh[a - 2*c + (b - 2*d)*x]/(4*(b - 2*d)) + Cosh[a +
2*c + (b + 2*d)*x]/(4*(b + 2*d))
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 5618

```
Int[Cosh[w_]^(q_.)*Sinh[v_]^(p_.), x_Symbol] :> Int[ExpandTrigReduce[Sinh[v
]^p*Cosh[w]^q, x], x] /; IGtQ[p, 0] && IGtQ[q, 0] && ((PolynomialQ[v, x] &&
 PolynomialQ[w, x]) || (BinomialQ[{v, w}, x] && IndependentQ[Cancel[v/w], x
]))
```

Rubi steps

$$\int \cosh^2(c+dx)\sinh(a+bx)\,dx=\int \left(\frac{1}{2}\sinh(a+bx)+\frac{1}{4}\sinh(a-2c+(b-2d)x)+\frac{1}{4}\sinh(a+2c+(b+2d)x$$

$$=\frac{1}{4}\int \sinh(a-2c+(b-2d)x)\,dx+\frac{1}{4}\int \sinh(a+2c+(b+2d)x)\,dx+\frac{1}{2}\int \mathrm{s}$$

$$=\frac{\cosh(a+bx)}{2b}+\frac{\cosh(a-2c+(b-2d)x)}{4(b-2d)}+\frac{\cosh(a+2c+(b+2d)x)}{4(b+2d)}$$

Mathematica [A]   time = 0.71, size = 69, normalized size = 1.11

$$\frac{1}{4}\left(\frac{\cosh(a+bx-2c-2dx)}{b-2d}+\frac{\cosh(a+bx+2c+2dx)}{b+2d}+\frac{2\sinh(a)\sinh(bx)}{b}+\frac{2\cosh(a)\cosh(bx)}{b}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]^2*Sinh[a + b*x],x]
```

```
[Out] ((2*Cosh[a]*Cosh[b*x])/b + Cosh[a - 2*c + b*x - 2*d*x]/(b - 2*d) + Cosh[a +
 2*c + b*x + 2*d*x]/(b + 2*d) + (2*Sinh[a]*Sinh[b*x])/b)/4
```

fricas [B]   time = 0.47, size = 119, normalized size = 1.92

$$\frac{b^2\cosh(bx+a)\cosh(dx+c)^2-4\,bd\cosh(dx+c)\sinh(bx+a)\sinh(dx+c)+b^2\cosh(bx+a)\sinh(dx+c)^2}{2\left(\left(b^3-4\,bd^2\right)\cosh(bx+a)^2-\left(b^3-4\,bd^2\right)\sinh(bx+a)^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^2*sinh(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/2*(b^2*cosh(b*x + a)*cosh(d*x + c)^2 - 4*b*d*cosh(d*x + c)*sinh(b*x + a)*
sinh(d*x + c) + b^2*cosh(b*x + a)*sinh(d*x + c)^2 + (b^2 - 4*d^2)*cosh(b*x
+ a))/((b^3 - 4*b*d^2)*cosh(b*x + a)^2 - (b^3 - 4*b*d^2)*sinh(b*x + a)^2)
```

giac [B]   time = 0.15, size = 120, normalized size = 1.94

$$\frac{e^{(bx+2\,dx+a+2\,c)}}{8\,(b+2\,d)}+\frac{e^{(bx-2\,dx+a-2\,c)}}{8\,(b-2\,d)}+\frac{e^{(bx+a)}}{4\,b}+\frac{e^{(-bx+2\,dx-a+2\,c)}}{8\,(b-2\,d)}+\frac{e^{(-bx-2\,dx-a-2\,c)}}{8\,(b+2\,d)}+\frac{e^{(-bx-a)}}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^2*sinh(b*x+a),x, algorithm="giac")
```

```
[Out] 1/8*e^(b*x + 2*d*x + a + 2*c)/(b + 2*d) + 1/8*e^(b*x - 2*d*x + a - 2*c)/(b
- 2*d) + 1/4*e^(b*x + a)/b + 1/8*e^(-b*x + 2*d*x - a + 2*c)/(b - 2*d) + 1/8
*e^(-b*x - 2*d*x - a - 2*c)/(b + 2*d) + 1/4*e^(-b*x - a)/b
```

maple [A]   time = 0.09, size = 57, normalized size = 0.92

$$\frac{\cosh(bx+a)}{2b}+\frac{\cosh(a-2c+(b-2d)\,x)}{4b-8d}+\frac{\cosh(a+2c+(b+2d)\,x)}{4b+8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)^2*sinh(b*x+a),x)
```

[Out] 1/2*cosh(b*x+a)/b+1/4*cosh(a-2*c+(b-2*d)*x)/(b-2*d)+1/4*cosh(a+2*c+(b+2*d)*x)/(b+2*d)

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)^2*sinh(b*x+a),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(-(2*d)/b>0)', see `assume?` for more details)Is -(2*d)/b equal to -1?

mupad [B] time = 0.23, size = 68, normalized size = 1.10

$$\frac{2\,d^2\cosh\left(a+b\,x\right)-b^2\cosh\left(a+b\,x\right)\,{\cosh\left(c+d\,x\right)}^2+2\,b\,d\cosh\left(c+d\,x\right)\,\sinh\left(a+b\,x\right)\,\sinh\left(c+d\,x\right)}{4\,b\,d^2-b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cosh(c + d*x)^2*sinh(a + b*x),x)

[Out] (2*d^2*cosh(a + b*x) - b^2*cosh(a + b*x)*cosh(c + d*x)^2 + 2*b*d*cosh(c + d*x)*sinh(a + b*x)*sinh(c + d*x))/(4*b*d^2 - b^3)

sympy [A] time = 6.42, size = 408, normalized size = 6.58

$$\begin{cases} x\sinh\left(a\right)\cosh^2\left(c\right) \\ \left(-\frac{x\sinh^2\left(c+dx\right)}{2}+\frac{x\cosh^2\left(c+dx\right)}{2}+\frac{\sinh\left(c+dx\right)\cosh\left(c+dx\right)}{2d}\right)\sinh\left(a\right) \\ \frac{x\sinh\left(a-2dx\right)\sinh^2\left(c+dx\right)}{4}+\frac{x\sinh\left(a-2dx\right)\cosh^2\left(c+dx\right)}{4}+\frac{x\sinh\left(c+dx\right)\cosh\left(a-2dx\right)\cosh\left(c+dx\right)}{2}+\frac{3\sinh\left(a-2dx\right)\sinh\left(c+dx\right)\cosh\left(c+dx\right)}{4d} \\ \frac{x\sinh\left(a+2dx\right)\sinh^2\left(c+dx\right)}{4}+\frac{x\sinh\left(a+2dx\right)\cosh^2\left(c+dx\right)}{4}-\frac{x\sinh\left(c+dx\right)\cosh\left(a+2dx\right)\cosh\left(c+dx\right)}{2}+\frac{3\sinh\left(a+2dx\right)\sinh\left(c+dx\right)\cosh\left(c+dx\right)}{4d} \\ \frac{b^2\cosh\left(a+bx\right)\cosh^2\left(c+dx\right)}{b^3-4bd^2}-\frac{2bd\sinh\left(a+bx\right)\sinh\left(c+dx\right)\cosh\left(c+dx\right)}{b^3-4bd^2}+\frac{2d^2\sinh^2\left(c+dx\right)\cosh\left(a+bx\right)}{b^3-4bd^2}-\frac{2d^2\cosh\left(a+bx\right)\cosh^2\left(c+dx\right)}{b^3-4bd^2} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)**2*sinh(b*x+a),x)

[Out] Piecewise((x*sinh(a)*cosh(c)**2, Eq(b, 0) & Eq(d, 0)), ((-x*sinh(c + d*x)**2/2 + x*cosh(c + d*x)**2/2 + sinh(c + d*x)*cosh(c + d*x)/(2*d))*sinh(a), Eq

```
(b, 0)), (x*sinh(a - 2*d*x)*sinh(c + d*x)**2/4 + x*sinh(a - 2*d*x)*cosh(c +
 d*x)**2/4 + x*sinh(c + d*x)*cosh(a - 2*d*x)*cosh(c + d*x)/2 + 3*sinh(a - 2
*d*x)*sinh(c + d*x)*cosh(c + d*x)/(4*d) + sinh(c + d*x)**2*cosh(a - 2*d*x)/
(2*d), Eq(b, -2*d)), (x*sinh(a + 2*d*x)*sinh(c + d*x)**2/4 + x*sinh(a + 2*d
*x)*cosh(c + d*x)**2/4 - x*sinh(c + d*x)*cosh(a + 2*d*x)*cosh(c + d*x)/2 +
3*sinh(a + 2*d*x)*sinh(c + d*x)*cosh(c + d*x)/(4*d) - sinh(c + d*x)**2*cosh
(a + 2*d*x)/(2*d), Eq(b, 2*d)), (b**2*cosh(a + b*x)*cosh(c + d*x)**2/(b**3
- 4*b*d**2) - 2*b*d*sinh(a + b*x)*sinh(c + d*x)*cosh(c + d*x)/(b**3 - 4*b*d
**2) + 2*d**2*sinh(c + d*x)**2*cosh(a + b*x)/(b**3 - 4*b*d**2) - 2*d**2*cos
h(a + b*x)*cosh(c + d*x)**2/(b**3 - 4*b*d**2), True))
```

3.181 $\int \cosh^3(c+dx)\sinh(a+bx)\,dx$

Optimal. Leaf size=91

$$\frac{\cosh(a+x(b-3d)-3c)}{8(b-3d)}+\frac{3\cosh(a+x(b-d)-c)}{8(b-d)}+\frac{3\cosh(a+x(b+d)+c)}{8(b+d)}+\frac{\cosh(a+x(b+3d)+3c)}{8(b+3d)}$$

```
[Out] 1/8*cosh(a-3*c+(b-3*d)*x)/(b-3*d)+3/8*cosh(a-c+(b-d)*x)/(b-d)+3/8*cosh(a+c+
(b+d)*x)/(b+d)+1/8*cosh(a+3*c+(b+3*d)*x)/(b+3*d)
```

Rubi [A] time = 0.08, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {5618, 2638}

$$\frac{\cosh(a+x(b-3d)-3c)}{8(b-3d)}+\frac{3\cosh(a+x(b-d)-c)}{8(b-d)}+\frac{3\cosh(a+x(b+d)+c)}{8(b+d)}+\frac{\cosh(a+x(b+3d)+3c)}{8(b+3d)}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]^3*Sinh[a + b*x],x]
```

```
[Out] Cosh[a - 3*c + (b - 3*d)*x]/(8*(b - 3*d)) + (3*Cosh[a - c + (b - d)*x])/(8*
(b - d)) + (3*Cosh[a + c + (b + d)*x])/(8*(b + d)) + Cosh[a + 3*c + (b + 3*
d)*x]/(8*(b + 3*d))
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 5618

```
Int[Cosh[w_]^(q_.)*Sinh[v_]^(p_.), x_Symbol] :> Int[ExpandTrigReduce[Sinh[v
]^p*Cosh[w]^q, x], x] /; IGtQ[p, 0] && IGtQ[q, 0] && ((PolynomialQ[v, x] &&
 PolynomialQ[w, x]) || (BinomialQ[{v, w}, x] && IndependentQ[Cancel[v/w], x
]))
```

Rubi steps

$$\begin{aligned}\int \cosh^3(c+dx)\sinh(a+bx)\,dx &= \int \left(\frac{1}{8}\sinh(a-3c+(b-3d)x)+\frac{3}{8}\sinh(a-c+(b-d)x)+\frac{3}{8}\sinh(a+c+(b\right.\\ &= \frac{1}{8}\int \sinh(a-3c+(b-3d)x)\,dx+\frac{1}{8}\int \sinh(a+3c+(b+3d)x)\,dx+\frac{3}{8}\int \mathrm{si}\\ &= \frac{\cosh(a-3c+(b-3d)x)}{8(b-3d)}+\frac{3\cosh(a-c+(b-d)x)}{8(b-d)}+\frac{3\cosh(a+c+(b+d}{8(b+d)}\end{aligned}$$

Mathematica [A] time = 0.46, size = 85, normalized size = 0.93

$$\frac{1}{8}\left(\frac{\cosh(a+bx-3c-3dx)}{b-3d}+\frac{3\cosh(a+bx-c-dx)}{b-d}+\frac{\cosh(a+bx+3c+3dx)}{b+3d}+\frac{3\cosh(a+x(b+d)+c)}{b+d}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]^3*Sinh[a + b*x],x]
```

```
[Out] (Cosh[a - 3*c + b*x - 3*d*x]/(b - 3*d) + (3*Cosh[a - c + b*x - d*x])/(b - d
) + Cosh[a + 3*c + b*x + 3*d*x]/(b + 3*d) + (3*Cosh[a + c + (b + d)*x])/(b
+ d))/8
```

fricas [B] time = 0.48, size = 213, normalized size = 2.34

$$\frac{\left(b^3-bd^2\right)\cosh\left(bx+a\right)\cosh\left(dx+c\right)^3+3\left(b^3-bd^2\right)\cosh\left(bx+a\right)\cosh\left(dx+c\right)\sinh\left(dx+c\right)^2-3\left(b^2d-d^3\right)s}{4\left(\left(b^4-10\,b^2d^2+9\,d^4\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^3*sinh(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/4*((b^3 - b*d^2)*cosh(b*x + a)*cosh(d*x + c)^3 + 3*(b^3 - b*d^2)*cosh(b*x
 + a)*cosh(d*x + c)*sinh(d*x + c)^2 - 3*(b^2*d - d^3)*sinh(b*x + a)*sinh(d*
x + c)^3 + 3*(b^3 - 9*b*d^2)*cosh(b*x + a)*cosh(d*x + c) - 3*(b^2*d - 9*d^3
 + 3*(b^2*d - d^3)*cosh(d*x + c)^2)*sinh(b*x + a)*sinh(d*x + c))/((b^4 - 10
*b^2*d^2 + 9*d^4)*cosh(b*x + a)^2 - (b^4 - 10*b^2*d^2 + 9*d^4)*sinh(b*x + a
)^2)
```

giac [B] time = 0.13, size = 179, normalized size = 1.97

$$\frac{e^{(bx+3\,dx+a+3\,c)}}{16\,(b+3\,d)}+\frac{3\,e^{(bx+dx+a+c)}}{16\,(b+d)}+\frac{3\,e^{(bx-dx+a-c)}}{16\,(b-d)}+\frac{e^{(bx-3\,dx+a-3\,c)}}{16\,(b-3\,d)}+\frac{e^{(-bx+3\,dx-a+3\,c)}}{16\,(b-3\,d)}+\frac{3\,e^{(-bx+dx-a+c)}}{16\,(b-d)}+\frac{3\,e^{(-bx-dx-a-c)}}{16\,(b+d)}+\frac{e^{(\cdot}}{1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^3*sinh(b*x+a),x, algorithm="giac")
```

```
[Out] 1/16*e^(b*x + 3*d*x + a + 3*c)/(b + 3*d) + 3/16*e^(b*x + d*x + a + c)/(b +
d) + 3/16*e^(b*x - d*x + a - c)/(b - d) + 1/16*e^(b*x - 3*d*x + a - 3*c)/(b
 - 3*d) + 1/16*e^(-b*x + 3*d*x - a + 3*c)/(b - 3*d) + 3/16*e^(-b*x + d*x -
a + c)/(b - d) + 3/16*e^(-b*x - d*x - a - c)/(b + d) + 1/16*e^(-b*x - 3*d*x
 - a - 3*c)/(b + 3*d)
```

maple [A] time = 0.07, size = 84, normalized size = 0.92

$$\frac{\cosh\left(a-3c+(b-3d)\,x\right)}{8b-24d}+\frac{3\cosh\left(a-c+(b-d)\,x\right)}{8\,(b-d)}+\frac{3\cosh\left(a+c+(b+d)\,x\right)}{8\,(b+d)}+\frac{\cosh\left(a+3c+(b+3d)\,x\right)}{8b+24d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)^3*sinh(b*x+a),x)
```

```
[Out] 1/8*cosh(a-3*c+(b-3*d)*x)/(b-3*d)+3/8*cosh(a-c+(b-d)*x)/(b-d)+3/8*cosh(a+c+
(b+d)*x)/(b+d)+1/8*cosh(a+3*c+(b+3*d)*x)/(b+3*d)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^3*sinh(b*x+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(-(3*d)/b>0)', see `assume?` for mor
e details)Is -(3*d)/b equal to -1?
```

mupad [B] time = 1.87, size = 182, normalized size = 2.00

$$\frac{6\,b\,d^2\cosh\left(a+b\,x\right)\,\cosh\left(c+d\,x\right)\,\sinh\left(c+d\,x\right)^2}{b^4-10\,b^2\,d^2+9\,d^4}-\frac{6\,d^3\sinh\left(a+b\,x\right)\,\sinh\left(c+d\,x\right)^3}{b^4-10\,b^2\,d^2+9\,d^4}-\frac{3\,d\cosh\left(c+d\,x\right)^2\sinh\left(a\right.}{b^4-1(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(c + d*x)^3*sinh(a + b*x),x)
```

```
[Out] (6*b*d^2*cosh(a + b*x)*cosh(c + d*x)*sinh(c + d*x)^2)/(b^4 + 9*d^4 - 10*b^2
*d^2) - (6*d^3*sinh(a + b*x)*sinh(c + d*x)^3)/(b^4 + 9*d^4 - 10*b^2*d^2) -
(3*d*cosh(c + d*x)^2*sinh(a + b*x)*sinh(c + d*x)*(b^2 - 3*d^2))/(b^4 + 9*d^
4 - 10*b^2*d^2) - (cosh(a + b*x)*cosh(c + d*x)^3*(7*b*d^2 - b^3))/(b^4 + 9*
d^4 - 10*b^2*d^2)
```

sympy [A] time = 31.09, size = 921, normalized size = 10.12

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)**3*sinh(b*x+a),x)
```

```
[Out] Piecewise((x*sinh(a)*cosh(c)**3, Eq(b, 0) & Eq(d, 0)), (3*x*sinh(a - 3*d*x)
*sinh(c + d*x)**2*cosh(c + d*x)/8 + x*sinh(a - 3*d*x)*cosh(c + d*x)**3/8 +
x*sinh(c + d*x)**3*cosh(a - 3*d*x)/8 + 3*x*sinh(c + d*x)*cosh(a - 3*d*x)*co
sh(c + d*x)**2/8 + sinh(a - 3*d*x)*sinh(c + d*x)**3/(8*d) + sinh(c + d*x)**
2*cosh(a - 3*d*x)*cosh(c + d*x)/(4*d) - 7*cosh(a - 3*d*x)*cosh(c + d*x)**3/
```

```
(24*d), Eq(b, -3*d)), (-3*x*sinh(a - d*x)*sinh(c + d*x)**2*cosh(c + d*x)/8
+ 3*x*sinh(a - d*x)*cosh(c + d*x)**3/8 - 3*x*sinh(c + d*x)**3*cosh(a - d*x)
/8 + 3*x*sinh(c + d*x)*cosh(a - d*x)*cosh(c + d*x)**2/8 + 3*sinh(a - d*x)*s
inh(c + d*x)**3/(8*d) + 3*sinh(c + d*x)**2*cosh(a - d*x)*cosh(c + d*x)/(4*d
) - 5*cosh(a - d*x)*cosh(c + d*x)**3/(8*d), Eq(b, -d)), (-3*x*sinh(a + d*x)
*sinh(c + d*x)**2*cosh(c + d*x)/8 + 3*x*sinh(a + d*x)*cosh(c + d*x)**3/8 +
3*x*sinh(c + d*x)**3*cosh(a + d*x)/8 - 3*x*sinh(c + d*x)*cosh(a + d*x)*cosh
(c + d*x)**2/8 + 3*sinh(a + d*x)*sinh(c + d*x)**3/(8*d) - 3*sinh(c + d*x)**
2*cosh(a + d*x)*cosh(c + d*x)/(4*d) + 5*cosh(a + d*x)*cosh(c + d*x)**3/(8*d
), Eq(b, d)), (3*x*sinh(a + 3*d*x)*sinh(c + d*x)**2*cosh(c + d*x)/8 + x*sin
h(a + 3*d*x)*cosh(c + d*x)**3/8 - x*sinh(c + d*x)**3*cosh(a + 3*d*x)/8 - 3*
x*sinh(c + d*x)*cosh(a + 3*d*x)*cosh(c + d*x)**2/8 + sinh(a + 3*d*x)*sinh(c
 + d*x)**3/(8*d) - sinh(c + d*x)**2*cosh(a + 3*d*x)*cosh(c + d*x)/(4*d) + 7
*cosh(a + 3*d*x)*cosh(c + d*x)**3/(24*d), Eq(b, 3*d)), (b**3*cosh(a + b*x)*
cosh(c + d*x)**3/(b**4 - 10*b**2*d**2 + 9*d**4) - 3*b**2*d*sinh(a + b*x)*si
nh(c + d*x)*cosh(c + d*x)**2/(b**4 - 10*b**2*d**2 + 9*d**4) + 6*b*d**2*sinh
(c + d*x)**2*cosh(a + b*x)*cosh(c + d*x)/(b**4 - 10*b**2*d**2 + 9*d**4) - 7
*b*d**2*cosh(a + b*x)*cosh(c + d*x)**3/(b**4 - 10*b**2*d**2 + 9*d**4) - 6*d
**3*sinh(a + b*x)*sinh(c + d*x)**3/(b**4 - 10*b**2*d**2 + 9*d**4) + 9*d**3*
sinh(a + b*x)*sinh(c + d*x)*cosh(c + d*x)**2/(b**4 - 10*b**2*d**2 + 9*d**4)
, True))
```

3.182 $\int \cosh(c+dx)\sinh^2(a+bx)\,dx$

Optimal. Leaf size=68

$$\frac{\sinh(2a+x(2b-d)-c)}{4(2b-d)}+\frac{\sinh(2a+x(2b+d)+c)}{4(2b+d)}-\frac{\sinh(c+dx)}{2d}$$

```
[Out] 1/4*sinh(2*a-c+(2*b-d)*x)/(2*b-d)-1/2*sinh(d*x+c)/d+1/4*sinh(2*a+c+(2*b+d)*
x)/(2*b+d)
```

Rubi [A] time = 0.05, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {5618, 2637}

$$\frac{\sinh(2a+x(2b-d)-c)}{4(2b-d)}+\frac{\sinh(2a+x(2b+d)+c)}{4(2b+d)}-\frac{\sinh(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]*Sinh[a + b*x]^2,x]
```

```
[Out] Sinh[2*a - c + (2*b - d)*x]/(4*(2*b - d)) - Sinh[c + d*x]/(2*d) + Sinh[2*a
+ c + (2*b + d)*x]/(4*(2*b + d))
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 5618

```
Int[Cosh[w_]^(q_.)*Sinh[v_]^(p_.), x_Symbol] :> Int[ExpandTrigReduce[Sinh[v
]^p*Cosh[w]^q, x], x] /; IGtQ[p, 0] && IGtQ[q, 0] && ((PolynomialQ[v, x] &&
 PolynomialQ[w, x]) || (BinomialQ[{v, w}, x] && IndependentQ[Cancel[v/w], x
]))
```

Rubi steps

$$\begin{aligned}\int \cosh(c+dx)\sinh^2(a+bx)\,dx &= \int \left(\frac{1}{4}\cosh(2a-c+(2b-d)x)-\frac{1}{2}\cosh(c+dx)+\frac{1}{4}\cosh(2a+c+(2b+d)x\right.\\ &= \frac{1}{4}\int \cosh(2a-c+(2b-d)x)\,dx+\frac{1}{4}\int \cosh(2a+c+(2b+d)x)\,dx-\frac{1}{2}\int c\\ &= \frac{\sinh(2a-c+(2b-d)x)}{4(2b-d)}-\frac{\sinh(c+dx)}{2d}+\frac{\sinh(2a+c+(2b+d)x)}{4(2b+d)}\end{aligned}$$

Mathematica [A] time = 0.73, size = 74, normalized size = 1.09

$$\frac{1}{4}\left(\frac{\sinh(2a+2bx-c-dx)}{2b-d}+\frac{\sinh(2a+2bx+c+dx)}{2b+d}-\frac{2\sinh(c)\cosh(dx)}{d}-\frac{2\cosh(c)\sinh(dx)}{d}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]*Sinh[a + b*x]^2,x]
```

```
[Out] ((-2*Cosh[d*x]*Sinh[c])/d - (2*Cosh[c]*Sinh[d*x])/d + Sinh[2*a - c + 2*b*x
- d*x]/(2*b - d) + Sinh[2*a + c + 2*b*x + d*x]/(2*b + d))/4
```

fricas [A] time = 0.43, size = 114, normalized size = 1.68

$$\frac{4\,bd\cosh\left(bx+a\right)\cosh\left(dx+c\right)\sinh\left(bx+a\right)-\left(d^2\cosh\left(bx+a\right)^2+d^2\sinh\left(bx+a\right)^2+4\,b^2-d^2\right)\sinh\left(dx+c\right)}{2\left(\left(4\,b^2d-d^3\right)\cosh\left(bx+a\right)^2-\left(4\,b^2d-d^3\right)\sinh\left(bx+a\right)^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)*sinh(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(4*b*d*cosh(b*x + a)*cosh(d*x + c)*sinh(b*x + a) - (d^2*cosh(b*x + a)^2
 + d^2*sinh(b*x + a)^2 + 4*b^2 - d^2)*sinh(d*x + c))/((4*b^2*d - d^3)*cosh(
b*x + a)^2 - (4*b^2*d - d^3)*sinh(b*x + a)^2)
```

giac [A] time = 0.12, size = 124, normalized size = 1.82

$$\frac{e^{(2\,bx+dx+2\,a+c)}}{8\,(2\,b+d)}+\frac{e^{(2\,bx-dx+2\,a-c)}}{8\,(2\,b-d)}-\frac{e^{(-2\,bx+dx-2\,a+c)}}{8\,(2\,b-d)}-\frac{e^{(-2\,bx-dx-2\,a-c)}}{8\,(2\,b+d)}-\frac{e^{(dx+c)}}{4\,d}+\frac{e^{(-dx-c)}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)*sinh(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/8*e^(2*b*x + d*x + 2*a + c)/(2*b + d) + 1/8*e^(2*b*x - d*x + 2*a - c)/(2*
b - d) - 1/8*e^(-2*b*x + d*x - 2*a + c)/(2*b - d) - 1/8*e^(-2*b*x - d*x - 2
*a - c)/(2*b + d) - 1/4*e^(d*x + c)/d + 1/4*e^(-d*x - c)/d
```

maple [A] time = 0.21, size = 63, normalized size = 0.93

$$\frac{\sinh\left(2a-c+(2b-d)\,x\right)}{8b-4d}-\frac{\sinh\left(dx+c\right)}{2d}+\frac{\sinh\left(2a+c+(2b+d)\,x\right)}{8b+4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)*sinh(b*x+a)^2,x)
```

```
[Out] 1/4*sinh(2*a-c+(2*b-d)*x)/(2*b-d)-1/2*sinh(d*x+c)/d+1/4*sinh(2*a+c+(2*b+d)*
x)/(2*b+d)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)*sinh(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(1-d/b>0)', see `assume?` for more d
etails)Is 1-d/b equal to -1?
```

mupad [B] time = 1.65, size = 76, normalized size = 1.12

$$\frac{d^2\,\left(\sinh\left(c+d\,x\right)-{\cosh\left(a+b\,x\right)}^2\,\sinh\left(c+d\,x\right)\right)-2\,b^2\,\sinh\left(c+d\,x\right)+2\,b\,d\,\cosh\left(a+b\,x\right)\,\cosh\left(c+d\,x\right)\,\sinh(}{4\,b^2\,d-d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(c + d*x)*sinh(a + b*x)^2,x)
```

```
[Out] (d^2*(sinh(c + d*x) - cosh(a + b*x)^2*sinh(c + d*x)) - 2*b^2*sinh(c + d*x)
+ 2*b*d*cosh(a + b*x)*cosh(c + d*x)*sinh(a + b*x))/(4*b^2*d - d^3)
```

sympy [A] time = 6.54, size = 411, normalized size = 6.04

$$\begin{cases} x \sinh^{2}{\left(a \right)} \cosh{\left(c \right)} \\ \frac{x \sinh^{2}{\left(a - \frac{dx}{2} \right)} \cosh{\left(c + dx \right)}}{4} + \frac{x \sinh{\left(a - \frac{dx}{2} \right)} \sinh{\left(c + dx \right)} \cosh{\left(a - \frac{dx}{2} \right)}}{2} + \frac{x \cosh^{2}{\left(a - \frac{dx}{2} \right)} \cosh{\left(c + dx \right)}}{4} - \frac{3 \sinh{\left(a - \frac{dx}{2} \right)} \cosh{\left(a - \frac{dx}{2} \right)} \cosh{\left(c + dx \right)}}{2d} - \\ \frac{x \sinh^{2}{\left(a + \frac{dx}{2} \right)} \cosh{\left(c + dx \right)}}{4} - \frac{x \sinh{\left(a + \frac{dx}{2} \right)} \sinh{\left(c + dx \right)} \cosh{\left(a + \frac{dx}{2} \right)}}{2} + \frac{x \cosh^{2}{\left(a + \frac{dx}{2} \right)} \cosh{\left(c + dx \right)}}{4} + \frac{3 \sinh{\left(a + \frac{dx}{2} \right)} \cosh{\left(a + \frac{dx}{2} \right)} \cosh{\left(c + dx \right)}}{2d} \\ \left(\frac{x \sinh^{2}{\left(a + bx \right)}}{2} - \frac{x \cosh^{2}{\left(a + bx \right)}}{2} + \frac{\sinh{\left(a + bx \right)} \cosh{\left(a + bx \right)}}{2b}\right) \cosh{\left(c \right)} \\ \frac{2b^{2} \sinh^{2}{\left(a + bx \right)} \sinh{\left(c + dx \right)}}{4b^{2}d - d^{3}} - \frac{2b^{2} \sinh{\left(c + dx \right)} \cosh^{2}{\left(a + bx \right)}}{4b^{2}d - d^{3}} + \frac{2bd \sinh{\left(a + bx \right)} \cosh{\left(a + bx \right)} \cosh{\left(c + dx \right)}}{4b^{2}d - d^{3}} - \frac{d^{2} \sinh^{2}{\left(a + bx \right)} \sinh{\left(c + dx \right)}}{4b^{2}d - d^{3}} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)*sinh(b*x+a)**2,x)
```

```
[Out] Piecewise((x*sinh(a)**2*cosh(c), Eq(b, 0) & Eq(d, 0)), (x*sinh(a - d*x/2)**
2*cosh(c + d*x)/4 + x*sinh(a - d*x/2)*sinh(c + d*x)*cosh(a - d*x/2)/2 + x*c
osh(a - d*x/2)**2*cosh(c + d*x)/4 - 3*sinh(a - d*x/2)*cosh(a - d*x/2)*cosh(
c + d*x)/(2*d) - sinh(c + d*x)*cosh(a - d*x/2)**2/d, Eq(b, -d/2)), (x*sinh(
a + d*x/2)**2*cosh(c + d*x)/4 - x*sinh(a + d*x/2)*sinh(c + d*x)*cosh(a + d*
x/2)/2 + x*cosh(a + d*x/2)**2*cosh(c + d*x)/4 + 3*sinh(a + d*x/2)*cosh(a +
d*x/2)*cosh(c + d*x)/(2*d) - sinh(c + d*x)*cosh(a + d*x/2)**2/d, Eq(b, d/2)
), ((x*sinh(a + b*x)**2/2 - x*cosh(a + b*x)**2/2 + sinh(a + b*x)*cosh(a + b
*x)/(2*b))*cosh(c), Eq(d, 0)), (2*b**2*sinh(a + b*x)**2*sinh(c + d*x)/(4*b*
*2*d - d**3) - 2*b**2*sinh(c + d*x)*cosh(a + b*x)**2/(4*b**2*d - d**3) + 2*
b*d*sinh(a + b*x)*cosh(a + b*x)*cosh(c + d*x)/(4*b**2*d - d**3) - d**2*sinh
(a + b*x)**2*sinh(c + d*x)/(4*b**2*d - d**3), True))
```

3.183 $\int \cosh^2(c+dx)\sinh^2(a+bx)\,dx$

Optimal. Leaf size=88

$$\frac{\sinh(2(a-c)+2x(b-d))}{16(b-d)}+\frac{\sinh(2(a+c)+2x(b+d))}{16(b+d)}+\frac{\sinh(2a+2bx)}{8b}-\frac{\sinh(2c+2dx)}{8d}-\frac{x}{4}$$

```
[Out] -1/4*x+1/8*sinh(2*b*x+2*a)/b+1/16*sinh(2*a-2*c+2*(b-d)*x)/(b-d)-1/8*sinh(2*
d*x+2*c)/d+1/16*sinh(2*a+2*c+2*(b+d)*x)/(b+d)
```

Rubi [A] time = 0.07, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {5618, 2637}

$$\frac{\sinh(2(a-c)+2x(b-d))}{16(b-d)}+\frac{\sinh(2(a+c)+2x(b+d))}{16(b+d)}+\frac{\sinh(2a+2bx)}{8b}-\frac{\sinh(2c+2dx)}{8d}-\frac{x}{4}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]^2*Sinh[a + b*x]^2,x]
```

```
[Out] -x/4 + Sinh[2*a + 2*b*x]/(8*b) + Sinh[2*(a - c) + 2*(b - d)*x]/(16*(b - d))
 - Sinh[2*c + 2*d*x]/(8*d) + Sinh[2*(a + c) + 2*(b + d)*x]/(16*(b + d))
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 5618

```
Int[Cosh[w_]^(q_.)*Sinh[v_]^(p_.), x_Symbol] :> Int[ExpandTrigReduce[Sinh[v
]^p*Cosh[w]^q, x], x] /; IGtQ[p, 0] && IGtQ[q, 0] && ((PolynomialQ[v, x] &&
 PolynomialQ[w, x]) || (BinomialQ[{v, w}, x] && IndependentQ[Cancel[v/w], x
]))
```

Rubi steps

$$\int \cosh^2(c+dx)\sinh^2(a+bx)\,dx = \int \left(-\frac{1}{4}+\frac{1}{4}\cosh(2a+2bx)+\frac{1}{8}\cosh(2(a-c)+2(b-d)x)-\frac{1}{4}\cosh(2c+2$$

$$= -\frac{x}{4}+\frac{1}{8}\int \cosh(2(a-c)+2(b-d)x)\,dx+\frac{1}{8}\int \cosh(2(a+c)+2(b+d)x)$$

$$= -\frac{x}{4}+\frac{\sinh(2a+2bx)}{8b}+\frac{\sinh(2(a-c)+2(b-d)x)}{16(b-d)}-\frac{\sinh(2c+2dx)}{8d}+$$

Mathematica [A] time = 0.72, size = 107, normalized size = 1.22

$$\frac{2d\left(b^2-d^2\right)\sinh(2(a+bx))+bd(b+d)\sinh(2(a+x(b-d)-c))-b(b-d)(-d\sinh(2(a+x(b+d)+c))+2(b}{16bd(b-d)(b+d)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]^2*Sinh[a + b*x]^2,x]
```

```
[Out] (2*d*(b^2 - d^2)*Sinh[2*(a + b*x)] + b*d*(b + d)*Sinh[2*(a - c + (b - d)*x)
] - b*(b - d)*(4*d*(b + d)*x + 2*(b + d)*Sinh[2*(c + d*x)] - d*Sinh[2*(a +
c + (b + d)*x)]))/(16*b*(b - d)*d*(b + d))
```

fricas [B] time = 0.47, size = 192, normalized size = 2.18

$$\frac{b^2d\cosh(bx+a)\sinh(bx+a)\sinh(dx+c)^2-\left(b^3d-bd^3\right)x+\left(b^2d\cosh(bx+a)\cosh(dx+c)^2+\left(b^2d-d^3\right)c}{4\left(\left(b^3d-bd^3\right)\cosh(bx\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^2*sinh(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/4*(b^2*d*cosh(b*x + a)*sinh(b*x + a)*sinh(d*x + c)^2 - (b^3*d - b*d^3)*x
+ (b^2*d*cosh(b*x + a)*cosh(d*x + c)^2 + (b^2*d - d^3)*cosh(b*x + a))*sinh(
b*x + a) - (b*d^2*cosh(d*x + c)*sinh(b*x + a)^2 + (b*d^2*cosh(b*x + a)^2 +
b^3 - b*d^2)*cosh(d*x + c))*sinh(d*x + c))/((b^3*d - b*d^3)*cosh(b*x + a)^2
 - (b^3*d - b*d^3)*sinh(b*x + a)^2)
```

giac [A] time = 0.13, size = 156, normalized size = 1.77

$$-\frac{1}{4}x+\frac{e^{(2bx+2dx+2a+2c)}}{32(b+d)}+\frac{e^{(2bx-2dx+2a-2c)}}{32(b-d)}+\frac{e^{(2bx+2a)}}{16b}-\frac{e^{(-2bx+2dx-2a+2c)}}{32(b-d)}-\frac{e^{(-2bx-2dx-2a-2c)}}{32(b+d)}-\frac{e^{(-2bx-2a)}}{16b}-\frac{e^{(2dx+2c)}}{16d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^2*sinh(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -1/4*x + 1/32*e^(2*b*x + 2*d*x + 2*a + 2*c)/(b + d) + 1/32*e^(2*b*x - 2*d*x
 + 2*a - 2*c)/(b - d) + 1/16*e^(2*b*x + 2*a)/b - 1/32*e^(-2*b*x + 2*d*x - 2
*a + 2*c)/(b - d) - 1/32*e^(-2*b*x - 2*d*x - 2*a - 2*c)/(b + d) - 1/16*e^(-
2*b*x - 2*a)/b - 1/16*e^(2*d*x + 2*c)/d + 1/16*e^(-2*d*x - 2*c)/d
```

maple [A] time = 0.34, size = 83, normalized size = 0.94

$$-\frac{x}{4}+\frac{\sinh(2bx+2a)}{8b}-\frac{\sinh(2dx+2c)}{8d}+\frac{\sinh((2b-2d)x+2a-2c)}{16b-16d}+\frac{\sinh((2b+2d)x+2a+2c)}{16b+16d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)^2*sinh(b*x+a)^2,x)
```

```
[Out] -1/4*x+1/8*sinh(2*b*x+2*a)/b-1/8*sinh(2*d*x+2*c)/d+1/16/(b-d)*sinh((2*b-2*d
)*x+2*a-2*c)+1/16/(b+d)*sinh((2*b+2*d)*x+2*a+2*c)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^2*sinh(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(1-(2*d)/b>0)', see `assume?` for mo
re details)Is 1-(2*d)/b equal to -1?
```

mupad [B] time = 1.96, size = 135, normalized size = 1.53

$$-\frac{d^3\cosh\left(a+b\,x\right)\,\sinh\left(a+b\,x\right)+b^3\cosh\left(c+d\,x\right)\,\sinh\left(c+d\,x\right)-b\,d^3\,x+b^3\,d\,x-2\,b\,d^2\cosh\left(c+d\,x\right)\,\sinh\left(c\right.}{4\,b\,d\,\left(b\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(c + d*x)^2*sinh(a + b*x)^2,x)
```

```
[Out] -(d^3*cosh(a + b*x)*sinh(a + b*x) + b^3*cosh(c + d*x)*sinh(c + d*x) - b*d^3
*x + b^3*d*x - 2*b*d^2*cosh(c + d*x)*sinh(c + d*x) - 2*b^2*d*cosh(a + b*x)*
cosh(c + d*x)^2*sinh(a + b*x) + 2*b*d^2*cosh(a + b*x)^2*cosh(c + d*x)*sinh(
c + d*x))/(4*b*d*(b^2 - d^2))
```

sympy [A] time = 21.98, size = 1027, normalized size = 11.67

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)**2*sinh(b*x+a)**2,x)
```

```
[Out] Piecewise((x*sinh(a)**2*cosh(c)**2, Eq(b, 0) & Eq(d, 0)), ((-x*sinh(c + d*x
)**2/2 + x*cosh(c + d*x)**2/2 + sinh(c + d*x)*cosh(c + d*x)/(2*d))*sinh(a)*
*2, Eq(b, 0)), (-x*sinh(a - d*x)**2*sinh(c + d*x)**2/8 + 3*x*sinh(a - d*x)*
*2*cosh(c + d*x)**2/8 + x*sinh(a - d*x)*sinh(c + d*x)*cosh(a - d*x)*cosh(c
+ d*x)/2 + 3*x*sinh(c + d*x)**2*cosh(a - d*x)**2/8 - x*cosh(a - d*x)**2*cos
```

```
h(c + d*x)**2/8 + 5*sinh(a - d*x)**2*sinh(c + d*x)*cosh(c + d*x)/(8*d) + si
nh(a - d*x)*sinh(c + d*x)**2*cosh(a - d*x)/(2*d) + sinh(c + d*x)*cosh(a - d
*x)**2*cosh(c + d*x)/(8*d), Eq(b, -d)), (-x*sinh(a + d*x)**2*sinh(c + d*x)*
*2/8 + 3*x*sinh(a + d*x)**2*cosh(c + d*x)**2/8 - x*sinh(a + d*x)*sinh(c + d
*x)*cosh(a + d*x)*cosh(c + d*x)/2 + 3*x*sinh(c + d*x)**2*cosh(a + d*x)**2/8
 - x*cosh(a + d*x)**2*cosh(c + d*x)**2/8 + sinh(a + d*x)*sinh(c + d*x)**2*c
osh(a + d*x)/(8*d) + 5*sinh(a + d*x)*cosh(a + d*x)*cosh(c + d*x)**2/(8*d) -
 sinh(c + d*x)*cosh(a + d*x)**2*cosh(c + d*x)/(2*d), Eq(b, d)), ((x*sinh(a
+ b*x)**2/2 - x*cosh(a + b*x)**2/2 + sinh(a + b*x)*cosh(a + b*x)/(2*b))*cos
h(c)**2, Eq(d, 0)), (-b**3*d*x*sinh(a + b*x)**2*sinh(c + d*x)**2/(4*b**3*d
- 4*b*d**3) + b**3*d*x*sinh(a + b*x)**2*cosh(c + d*x)**2/(4*b**3*d - 4*b*d*
*3) + b**3*d*x*sinh(c + d*x)**2*cosh(a + b*x)**2/(4*b**3*d - 4*b*d**3) - b*
*3*d*x*cosh(a + b*x)**2*cosh(c + d*x)**2/(4*b**3*d - 4*b*d**3) + b**3*sinh(
a + b*x)**2*sinh(c + d*x)*cosh(c + d*x)/(4*b**3*d - 4*b*d**3) - b**3*sinh(c
 + d*x)*cosh(a + b*x)**2*cosh(c + d*x)/(4*b**3*d - 4*b*d**3) + 2*b**2*d*sin
h(a + b*x)*cosh(a + b*x)*cosh(c + d*x)**2/(4*b**3*d - 4*b*d**3) + b*d**3*x*
sinh(a + b*x)**2*sinh(c + d*x)**2/(4*b**3*d - 4*b*d**3) - b*d**3*x*sinh(a +
 b*x)**2*cosh(c + d*x)**2/(4*b**3*d - 4*b*d**3) - b*d**3*x*sinh(c + d*x)**2
*cosh(a + b*x)**2/(4*b**3*d - 4*b*d**3) + b*d**3*x*cosh(a + b*x)**2*cosh(c
+ d*x)**2/(4*b**3*d - 4*b*d**3) - 2*b*d**2*sinh(a + b*x)**2*sinh(c + d*x)*c
osh(c + d*x)/(4*b**3*d - 4*b*d**3) + d**3*sinh(a + b*x)*sinh(c + d*x)**2*co
sh(a + b*x)/(4*b**3*d - 4*b*d**3) - d**3*sinh(a + b*x)*cosh(a + b*x)*cosh(c
 + d*x)**2/(4*b**3*d - 4*b*d**3), True))
```

3.184 $\int \cosh^3(c+dx)\sinh^2(a+bx)\,dx$

Optimal. Leaf size=144

$$\frac{\sinh(2a+x(2b-3d)-3c)}{16(2b-3d)}+\frac{3\sinh(2a+x(2b-d)-c)}{16(2b-d)}+\frac{3\sinh(2a+x(2b+d)+c)}{16(2b+d)}+\frac{\sinh(2a+x(2b+3d)+3c)}{16(2b+3d)}$$

```
[Out] 1/16*sinh(2*a-3*c+(2*b-3*d)*x)/(2*b-3*d)+3/16*sinh(2*a-c+(2*b-d)*x)/(2*b-d)
-3/8*sinh(d*x+c)/d-1/24*sinh(3*d*x+3*c)/d+3/16*sinh(2*a+c+(2*b+d)*x)/(2*b+d
)+1/16*sinh(2*a+3*c+(2*b+3*d)*x)/(2*b+3*d)
```

Rubi [A] time = 0.09, antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {5618, 2637}

$$\frac{\sinh(2a+x(2b-3d)-3c)}{16(2b-3d)}+\frac{3\sinh(2a+x(2b-d)-c)}{16(2b-d)}+\frac{3\sinh(2a+x(2b+d)+c)}{16(2b+d)}+\frac{\sinh(2a+x(2b+3d)+3c)}{16(2b+3d)}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]^3*Sinh[a + b*x]^2,x]
```

```
[Out] Sinh[2*a - 3*c + (2*b - 3*d)*x]/(16*(2*b - 3*d)) + (3*Sinh[2*a - c + (2*b -
 d)*x])/(16*(2*b - d)) - (3*Sinh[c + d*x])/(8*d) - Sinh[3*c + 3*d*x]/(24*d)
 + (3*Sinh[2*a + c + (2*b + d)*x])/(16*(2*b + d)) + Sinh[2*a + 3*c + (2*b +
 3*d)*x]/(16*(2*b + 3*d))
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 5618

```
Int[Cosh[w_]^(q_.)*Sinh[v_]^(p_.), x_Symbol] :> Int[ExpandTrigReduce[Sinh[v
]^p*Cosh[w]^q, x], x] /; IGtQ[p, 0] && IGtQ[q, 0] && ((PolynomialQ[v, x] &&
 PolynomialQ[w, x]) || (BinomialQ[{v, w}, x] && IndependentQ[Cancel[v/w], x
]))
```

Rubi steps

$$\int \cosh^3(c+dx)\sinh^2(a+bx)\,dx = \int\left(\frac{1}{16}\cosh(2a-3c+(2b-3d)x)+\frac{3}{16}\cosh(2a-c+(2b-d)x)-\frac{3}{8}\cosh\right.$$
$$=\frac{1}{16}\int\cosh(2a-3c+(2b-3d)x)\,dx+\frac{1}{16}\int\cosh(2a+3c+(2b+3d)x)\,d$$
$$=\frac{\sinh(2a-3c+(2b-3d)x)}{16(2b-3d)}+\frac{3\sinh(2a-c+(2b-d)x)}{16(2b-d)}-\frac{3\sinh(c+dx)}{8d}$$

Mathematica [A] time = 1.53, size = 158, normalized size = 1.10

$$\frac{1}{48}\left(\frac{3\sinh(2a+2bx-3c-3dx)}{2b-3d}+\frac{9\sinh(2a+2bx-c-dx)}{2b-d}+\frac{9\sinh(2a+2bx+c+dx)}{2b+d}+\frac{3\sinh(2a+2bx+}{2b+3d}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]^3*Sinh[a + b*x]^2,x]
```

```
[Out] ((-18*Cosh[d*x]*Sinh[c])/d - (2*Cosh[3*d*x]*Sinh[3*c])/d - (18*Cosh[c]*Sinh
[d*x])/d - (2*Cosh[3*c]*Sinh[3*d*x])/d + (3*Sinh[2*a - 3*c + 2*b*x - 3*d*x]
)/(2*b - 3*d) + (9*Sinh[2*a - c + 2*b*x - d*x])/(2*b - d) + (9*Sinh[2*a + c
 + 2*b*x + d*x])/(2*b + d) + (3*Sinh[2*a + 3*c + 2*b*x + 3*d*x])/(2*b + 3*d
))/48
```

fricas [B] time = 0.45, size = 398, normalized size = 2.76

$$\frac{36\left(4\,b^3d-bd^3\right)\cosh\left(bx+a\right)\cosh\left(dx+c\right)\sinh\left(bx+a\right)\sinh\left(dx+c\right)^2-\left(16\,b^4-40\,b^2d^2+9\,d^4+9\left(4\,b^2d^2-\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^3*sinh(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/24*(36*(4*b^3*d - b*d^3)*cosh(b*x + a)*cosh(d*x + c)*sinh(b*x + a)*sinh(d
*x + c)^2 - (16*b^4 - 40*b^2*d^2 + 9*d^4 + 9*(4*b^2*d^2 - d^4)*cosh(b*x + a
)^2 + 9*(4*b^2*d^2 - d^4)*sinh(b*x + a)^2)*sinh(d*x + c)^3 + 12*((4*b^3*d -
 b*d^3)*cosh(b*x + a)*cosh(d*x + c)^3 + 3*(4*b^3*d - 9*b*d^3)*cosh(b*x + a)
*cosh(d*x + c))*sinh(b*x + a) - 3*(48*b^4 - 120*b^2*d^2 + 27*d^4 + 3*(4*b^2
*d^2 - 9*d^4)*cosh(b*x + a)^2 + (16*b^4 - 40*b^2*d^2 + 9*d^4 + 9*(4*b^2*d^2
 - d^4)*cosh(b*x + a)^2)*cosh(d*x + c)^2 + 3*(4*b^2*d^2 - 9*d^4 + 3*(4*b^2*
d^2 - d^4)*cosh(d*x + c)^2)*sinh(b*x + a)^2)*sinh(d*x + c))/((16*b^4*d - 40
*b^2*d^3 + 9*d^5)*cosh(b*x + a)^2 - (16*b^4*d - 40*b^2*d^3 + 9*d^5)*sinh(b*
x + a)^2)
```

giac [A] time = 0.17, size = 260, normalized size = 1.81

$$\frac{e^{(2\,bx+3\,dx+2\,a+3\,c)}}{32\,(2\,b+3\,d)}+\frac{3\,e^{(2\,bx+dx+2\,a+c)}}{32\,(2\,b+d)}+\frac{3\,e^{(2\,bx-dx+2\,a-c)}}{32\,(2\,b-d)}+\frac{e^{(2\,bx-3\,dx+2\,a-3\,c)}}{32\,(2\,b-3\,d)}-\frac{e^{(-2\,bx+3\,dx-2\,a+3\,c)}}{32\,(2\,b-3\,d)}-\frac{3\,e^{(-2\,bx+dx-2\,a+c)}}{32\,(2\,b-d)}-\frac{3\,e^{(-2}}{32}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^3*sinh(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/32*e^(2*b*x + 3*d*x + 2*a + 3*c)/(2*b + 3*d) + 3/32*e^(2*b*x + d*x + 2*a
+ c)/(2*b + d) + 3/32*e^(2*b*x - d*x + 2*a - c)/(2*b - d) + 1/32*e^(2*b*x -
 3*d*x + 2*a - 3*c)/(2*b - 3*d) - 1/32*e^(-2*b*x + 3*d*x - 2*a + 3*c)/(2*b
- 3*d) - 3/32*e^(-2*b*x + d*x - 2*a + c)/(2*b - d) - 3/32*e^(-2*b*x - d*x -
 2*a - c)/(2*b + d) - 1/32*e^(-2*b*x - 3*d*x - 2*a - 3*c)/(2*b + 3*d) - 1/4
8*e^(3*d*x + 3*c)/d - 3/16*e^(d*x + c)/d + 3/16*e^(-d*x - c)/d + 1/48*e^(-3
*d*x - 3*c)/d
```

maple [A] time = 0.39, size = 133, normalized size = 0.92

$$\frac{\sinh(2a-3c+(2b-3d)\,x)}{32b-48d}+\frac{3\sinh(2a-c+(2b-d)\,x)}{16\,(2b-d)}-\frac{3\sinh(dx+c)}{8d}-\frac{\sinh(3dx+3c)}{24d}+\frac{3\sinh(2a+c+(2b}{16\,(2b+d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)^3*sinh(b*x+a)^2,x)
```

```
[Out] 1/16*sinh(2*a-3*c+(2*b-3*d)*x)/(2*b-3*d)+3/16*sinh(2*a-c+(2*b-d)*x)/(2*b-d)
-3/8*sinh(d*x+c)/d-1/24*sinh(3*d*x+3*c)/d+3/16*sinh(2*a+c+(2*b+d)*x)/(2*b+d
)+1/16*sinh(2*a+3*c+(2*b+3*d)*x)/(2*b+3*d)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^3*sinh(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(1-(3*d)/b>0)', see `assume?` for mo
re details)Is 1-(3*d)/b equal to -1?
```

mupad [B] time = 1.98, size = 337, normalized size = 2.34

$$\frac{\cosh(c+d\,x)^2\sinh(a+b\,x)^2\sinh(c+d\,x)\,\left(8\,b^4-26\,b^2\,d^2+9\,d^4\right)}{d\,\left(16\,b^4-40\,b^2\,d^2+9\,d^4\right)}-\sinh(a+b\,x)^2\sinh(c+d\,x)^3\,\left(\frac{3\,d}{16\,b^4-40\,b^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(c + d*x)^3*sinh(a + b*x)^2,x)
```

```
[Out] (cosh(c + d*x)^2*sinh(a + b*x)^2*sinh(c + d*x)*(8*b^4 + 9*d^4 - 26*b^2*d^2)
)/(d*(16*b^4 + 9*d^4 - 40*b^2*d^2)) - sinh(a + b*x)^2*sinh(c + d*x)^3*((3*d
^3)/(16*b^4 + 9*d^4 - 40*b^2*d^2) + 1/(3*d)) - (2*cosh(a + b*x)*cosh(c + d*
x)^3*sinh(a + b*x)*(7*b*d^2 - 4*b^3))/(16*b^4 + 9*d^4 - 40*b^2*d^2) - (2*co
sh(a + b*x)^2*cosh(c + d*x)^2*sinh(c + d*x)*(4*b^4 - 7*b^2*d^2))/(d*(16*b^4
 + 9*d^4 - 40*b^2*d^2)) - cosh(a + b*x)^2*sinh(c + d*x)^3*((3*d^3)/(16*b^4
+ 9*d^4 - 40*b^2*d^2) - 1/(3*d)) + (12*b*d^2*cosh(a + b*x)*cosh(c + d*x)*si
nh(a + b*x)*sinh(c + d*x)^2)/(16*b^4 + 9*d^4 - 40*b^2*d^2)
```

sympy [A] time = 108.48, size = 2006, normalized size = 13.93

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)**3*sinh(b*x+a)**2,x)
```

```
[Out] Piecewise((x*sinh(a)**2*cosh(c)**3, Eq(b, 0) & Eq(d, 0)), (3*x*sinh(a - 3*d
*x/2)**2*sinh(c + d*x)**2*cosh(c + d*x)/16 + x*sinh(a - 3*d*x/2)**2*cosh(c
+ d*x)**3/16 + x*sinh(a - 3*d*x/2)*sinh(c + d*x)**3*cosh(a - 3*d*x/2)/8 + 3
*x*sinh(a - 3*d*x/2)*sinh(c + d*x)*cosh(a - 3*d*x/2)*cosh(c + d*x)**2/8 + 3
*x*sinh(c + d*x)**2*cosh(a - 3*d*x/2)**2*cosh(c + d*x)/16 + x*cosh(a - 3*d*
x/2)**2*cosh(c + d*x)**3/16 - 5*sinh(a - 3*d*x/2)**2*sinh(c + d*x)**3/(48*d
) + sinh(a - 3*d*x/2)**2*sinh(c + d*x)*cosh(c + d*x)**2/d + 5*sinh(a - 3*d*
x/2)*sinh(c + d*x)**2*cosh(a - 3*d*x/2)*cosh(c + d*x)/(4*d) + sinh(a - 3*d*
x/2)*cosh(a - 3*d*x/2)*cosh(c + d*x)**3/(24*d) + 9*sinh(c + d*x)**3*cosh(a
- 3*d*x/2)**2/(16*d), Eq(b, -3*d/2)), (-3*x*sinh(a - d*x/2)**2*sinh(c + d*x
)**2*cosh(c + d*x)/16 + 3*x*sinh(a - d*x/2)**2*cosh(c + d*x)**3/16 - 3*x*si
nh(a - d*x/2)*sinh(c + d*x)**3*cosh(a - d*x/2)/8 + 3*x*sinh(a - d*x/2)*sinh
(c + d*x)*cosh(a - d*x/2)*cosh(c + d*x)**2/8 - 3*x*sinh(c + d*x)**2*cosh(a
- d*x/2)**2*cosh(c + d*x)/16 + 3*x*cosh(a - d*x/2)**2*cosh(c + d*x)**3/16 +
 17*sinh(a - d*x/2)**2*sinh(c + d*x)**3/(48*d) + 7*sinh(a - d*x/2)*sinh(c +
 d*x)**2*cosh(a - d*x/2)*cosh(c + d*x)/(4*d) - 13*sinh(a - d*x/2)*cosh(a -
d*x/2)*cosh(c + d*x)**3/(8*d) + 49*sinh(c + d*x)**3*cosh(a - d*x/2)**2/(48*
d) - sinh(c + d*x)*cosh(a - d*x/2)**2*cosh(c + d*x)**2/d, Eq(b, -d/2)), (-3
*x*sinh(a + d*x/2)**2*sinh(c + d*x)**2*cosh(c + d*x)/16 + 3*x*sinh(a + d*x/
2)**2*cosh(c + d*x)**3/16 + 3*x*sinh(a + d*x/2)*sinh(c + d*x)**3*cosh(a + d
*x/2)/8 - 3*x*sinh(a + d*x/2)*sinh(c + d*x)*cosh(a + d*x/2)*cosh(c + d*x)**
2/8 - 3*x*sinh(c + d*x)**2*cosh(a + d*x/2)**2*cosh(c + d*x)/16 + 3*x*cosh(a
 + d*x/2)**2*cosh(c + d*x)**3/16 + 17*sinh(a + d*x/2)**2*sinh(c + d*x)**3/(
48*d) - 7*sinh(a + d*x/2)*sinh(c + d*x)**2*cosh(a + d*x/2)*cosh(c + d*x)/(4
*d) + 13*sinh(a + d*x/2)*cosh(a + d*x/2)*cosh(c + d*x)**3/(8*d) + 49*sinh(c
```

```
 + d*x)**3*cosh(a + d*x/2)**2/(48*d) - sinh(c + d*x)*cosh(a + d*x/2)**2*cos
h(c + d*x)**2/d, Eq(b, d/2)), (3*x*sinh(a + 3*d*x/2)**2*sinh(c + d*x)**2*co
sh(c + d*x)/16 + x*sinh(a + 3*d*x/2)**2*cosh(c + d*x)**3/16 - x*sinh(a + 3*
d*x/2)*sinh(c + d*x)**3*cosh(a + 3*d*x/2)/8 - 3*x*sinh(a + 3*d*x/2)*sinh(c
+ d*x)*cosh(a + 3*d*x/2)*cosh(c + d*x)**2/8 + 3*x*sinh(c + d*x)**2*cosh(a +
 3*d*x/2)**2*cosh(c + d*x)/16 + x*cosh(a + 3*d*x/2)**2*cosh(c + d*x)**3/16
- 5*sinh(a + 3*d*x/2)**2*sinh(c + d*x)**3/(48*d) + sinh(a + 3*d*x/2)**2*sin
h(c + d*x)*cosh(c + d*x)**2/d - 5*sinh(a + 3*d*x/2)*sinh(c + d*x)**2*cosh(a
 + 3*d*x/2)*cosh(c + d*x)/(4*d) - sinh(a + 3*d*x/2)*cosh(a + 3*d*x/2)*cosh(
c + d*x)**3/(24*d) + 9*sinh(c + d*x)**3*cosh(a + 3*d*x/2)**2/(16*d), Eq(b,
3*d/2)), ((x*sinh(a + b*x)**2/2 - x*cosh(a + b*x)**2/2 + sinh(a + b*x)*cosh
(a + b*x)/(2*b))*cosh(c)**3, Eq(d, 0)), (-16*b**4*sinh(a + b*x)**2*sinh(c +
 d*x)**3/(48*b**4*d - 120*b**2*d**3 + 27*d**5) + 24*b**4*sinh(a + b*x)**2*s
inh(c + d*x)*cosh(c + d*x)**2/(48*b**4*d - 120*b**2*d**3 + 27*d**5) + 16*b*
*4*sinh(c + d*x)**3*cosh(a + b*x)**2/(48*b**4*d - 120*b**2*d**3 + 27*d**5)
- 24*b**4*sinh(c + d*x)*cosh(a + b*x)**2*cosh(c + d*x)**2/(48*b**4*d - 120*
b**2*d**3 + 27*d**5) + 24*b**3*d*sinh(a + b*x)*cosh(a + b*x)*cosh(c + d*x)*
*3/(48*b**4*d - 120*b**2*d**3 + 27*d**5) + 40*b**2*d**2*sinh(a + b*x)**2*si
nh(c + d*x)**3/(48*b**4*d - 120*b**2*d**3 + 27*d**5) - 78*b**2*d**2*sinh(a
+ b*x)**2*sinh(c + d*x)*cosh(c + d*x)**2/(48*b**4*d - 120*b**2*d**3 + 27*d*
*5) - 40*b**2*d**2*sinh(c + d*x)**3*cosh(a + b*x)**2/(48*b**4*d - 120*b**2*
d**3 + 27*d**5) + 42*b**2*d**2*sinh(c + d*x)*cosh(a + b*x)**2*cosh(c + d*x)
**2/(48*b**4*d - 120*b**2*d**3 + 27*d**5) + 36*b*d**3*sinh(a + b*x)*sinh(c
+ d*x)**2*cosh(a + b*x)*cosh(c + d*x)/(48*b**4*d - 120*b**2*d**3 + 27*d**5)
 - 42*b*d**3*sinh(a + b*x)*cosh(a + b*x)*cosh(c + d*x)**3/(48*b**4*d - 120*
b**2*d**3 + 27*d**5) - 18*d**4*sinh(a + b*x)**2*sinh(c + d*x)**3/(48*b**4*d
 - 120*b**2*d**3 + 27*d**5) + 27*d**4*sinh(a + b*x)**2*sinh(c + d*x)*cosh(c
 + d*x)**2/(48*b**4*d - 120*b**2*d**3 + 27*d**5), True))
```

3.185 $\int \cosh(c+dx)\sinh^3(a+bx)\,dx$

Optimal. Leaf size=97

$$-\frac{3\cosh(a+x(b-d)-c)}{8(b-d)}+\frac{\cosh(3a+x(3b-d)-c)}{8(3b-d)}-\frac{3\cosh(a+x(b+d)+c)}{8(b+d)}+\frac{\cosh(3a+x(3b+d)+c)}{8(3b+d)}$$

```
[Out] -3/8*cosh(a-c+(b-d)*x)/(b-d)+1/8*cosh(3*a-c+(3*b-d)*x)/(3*b-d)-3/8*cosh(a+c
+(b+d)*x)/(b+d)+1/8*cosh(3*a+c+(3*b+d)*x)/(3*b+d)
```

Rubi [A] time = 0.08, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {5618, 2638}

$$-\frac{3\cosh(a+x(b-d)-c)}{8(b-d)}+\frac{\cosh(3a+x(3b-d)-c)}{8(3b-d)}-\frac{3\cosh(a+x(b+d)+c)}{8(b+d)}+\frac{\cosh(3a+x(3b+d)+c)}{8(3b+d)}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]*Sinh[a + b*x]^3,x]
```

```
[Out] (-3*Cosh[a - c + (b - d)*x])/(8*(b - d)) + Cosh[3*a - c + (3*b - d)*x]/(8*(
3*b - d)) - (3*Cosh[a + c + (b + d)*x])/(8*(b + d)) + Cosh[3*a + c + (3*b +
 d)*x]/(8*(3*b + d))
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 5618

```
Int[Cosh[w_]^(q_.)*Sinh[v_]^(p_.), x_Symbol] :> Int[ExpandTrigReduce[Sinh[v
]^p*Cosh[w]^q, x], x] /; IGtQ[p, 0] && IGtQ[q, 0] && ((PolynomialQ[v, x] &&
 PolynomialQ[w, x]) || (BinomialQ[{v, w}, x] && IndependentQ[Cancel[v/w], x
]))
```

Rubi steps

$$\begin{aligned}\int \cosh(c+dx)\sinh^3(a+bx)\,dx &= \int \left(-\frac{3}{8}\sinh(a-c+(b-d)x)+\frac{1}{8}\sinh(3a-c+(3b-d)x)-\frac{3}{8}\sinh(a+c\right.\\ &= \frac{1}{8}\int \sinh(3a-c+(3b-d)x)\,dx+\frac{1}{8}\int \sinh(3a+c+(3b+d)x)\,dx-\frac{3}{8}\int\\ &= -\frac{3\cosh(a-c+(b-d)x)}{8(b-d)}+\frac{\cosh(3a-c+(3b-d)x)}{8(3b-d)}-\frac{3\cosh(a+c+(b}{8(b+d)}\end{aligned}$$

Mathematica [A] time = 0.52, size = 90, normalized size = 0.93

$$\frac{1}{8}\left(-\frac{3\cosh(a+bx-c-dx)}{b-d}+\frac{\cosh(3a+3bx-c-dx)}{3b-d}+\frac{\cosh(3a+3bx+c+dx)}{3b+d}-\frac{3\cosh(a+x(b+d)+c)}{b+d}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]*Sinh[a + b*x]^3,x]
```

```
[Out] ((-3*Cosh[a - c + b*x - d*x])/(b - d) + Cosh[3*a - c + 3*b*x - d*x]/(3*b -
d) + Cosh[3*a + c + 3*b*x + d*x]/(3*b + d) - (3*Cosh[a + c + (b + d)*x])/(b
 + d))/8
```

fricas [B] time = 0.52, size = 243, normalized size = 2.51

$$\frac{9\left(b^3-bd^2\right)\cosh(bx+a)\cosh(dx+c)\sinh(bx+a)^2+3\left(\left(b^3-bd^2\right)\cosh(bx+a)^3-\left(9\,b^3-bd^2\right)\cosh(bx+a)\cdots\right.}{4\left(\left(9\,b^4-10\,b^2d^2+d^4\right)\cosh(bx+a)^4-2\left(9\,b^4-10\,b^2d^2+d^4\right)\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)*sinh(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/4*(9*(b^3 - b*d^2)*cosh(b*x + a)*cosh(d*x + c)*sinh(b*x + a)^2 + 3*((b^3
- b*d^2)*cosh(b*x + a)^3 - (9*b^3 - b*d^2)*cosh(b*x + a))*cosh(d*x + c) - (
(b^2*d - d^3)*sinh(b*x + a)^3 - 3*(9*b^2*d - d^3 - (b^2*d - d^3)*cosh(b*x +
 a)^2)*sinh(b*x + a))*sinh(d*x + c))/((9*b^4 - 10*b^2*d^2 + d^4)*cosh(b*x +
 a)^4 - 2*(9*b^4 - 10*b^2*d^2 + d^4)*cosh(b*x + a)^2*sinh(b*x + a)^2 + (9*b
^4 - 10*b^2*d^2 + d^4)*sinh(b*x + a)^4)
```

giac [B] time = 0.14, size = 183, normalized size = 1.89

$$\frac{e^{(3\,bx+dx+3\,a+c)}}{16\,(3\,b+d)}+\frac{e^{(3\,bx-dx+3\,a-c)}}{16\,(3\,b-d)}-\frac{3\,e^{(bx+dx+a+c)}}{16\,(b+d)}-\frac{3\,e^{(bx-dx+a-c)}}{16\,(b-d)}-\frac{3\,e^{(-bx+dx-a+c)}}{16\,(b-d)}-\frac{3\,e^{(-bx-dx-a-c)}}{16\,(b+d)}+\frac{e^{(-3\,bx+dx-3\,a+c)}}{16\,(3\,b-d)}+\frac{e^{(-3\cdots}}{16\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)*sinh(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/16*e^(3*b*x + d*x + 3*a + c)/(3*b + d) + 1/16*e^(3*b*x - d*x + 3*a - c)/(
3*b - d) - 3/16*e^(b*x + d*x + a + c)/(b + d) - 3/16*e^(b*x - d*x + a - c)/
(b - d) - 3/16*e^(-b*x + d*x - a + c)/(b - d) - 3/16*e^(-b*x - d*x - a - c)
/(b + d) + 1/16*e^(-3*b*x + d*x - 3*a + c)/(3*b - d) + 1/16*e^(-3*b*x - d*x
 - 3*a - c)/(3*b + d)
```

maple [A] time = 0.10, size = 90, normalized size = 0.93

$$-\frac{3\cosh\left(a-c+(b-d)\,x\right)}{8\,(b-d)}+\frac{\cosh\left(3a-c+(3b-d)\,x\right)}{24b-8d}-\frac{3\cosh\left(a+c+(b+d)\,x\right)}{8\,(b+d)}+\frac{\cosh\left(3a+c+(3b+d)\,x\right)}{24b+8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)*sinh(b*x+a)^3,x)
```

```
[Out] -3/8*cosh(a-c+(b-d)*x)/(b-d)+1/8*cosh(3*a-c+(3*b-d)*x)/(3*b-d)-3/8*cosh(a+c
+(b+d)*x)/(b+d)+1/8*cosh(3*a+c+(3*b+d)*x)/(3*b+d)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)*sinh(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(-d/b>0)', see `assume?` for more de
tails)Is -d/b equal to -1?
```

mupad [B] time = 0.53, size = 183, normalized size = 1.89

$$\frac{6\,b^2\,d\,\cosh\left(a+b\,x\right)^2\,\sinh\left(a+b\,x\right)\,\sinh\left(c+d\,x\right)}{9\,b^4-10\,b^2\,d^2+d^4}-\frac{d\,\sinh\left(a+b\,x\right)^3\,\sinh\left(c+d\,x\right)\,\left(7\,b^2-d^2\right)}{9\,b^4-10\,b^2\,d^2+d^4}-\frac{3\,\cosh\left(a+b\,x\right)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(c + d*x)*sinh(a + b*x)^3,x)
```

```
[Out] (6*b^2*d*cosh(a + b*x)^2*sinh(a + b*x)*sinh(c + d*x))/(9*b^4 + d^4 - 10*b^2
*d^2) - (d*sinh(a + b*x)^3*sinh(c + d*x)*(7*b^2 - d^2))/(9*b^4 + d^4 - 10*b
^2*d^2) - (3*cosh(a + b*x)*cosh(c + d*x)*sinh(a + b*x)^2*(b*d^2 - 3*b^3))/(
9*b^4 + d^4 - 10*b^2*d^2) - (6*b^3*cosh(a + b*x)^3*cosh(c + d*x))/(9*b^4 +
d^4 - 10*b^2*d^2)
```

sympy [A] time = 30.37, size = 935, normalized size = 9.64

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)*sinh(b*x+a)**3,x)
```

```
[Out] Piecewise((x*sinh(a)**3*cosh(c), Eq(b, 0) & Eq(d, 0)), (3*x*sinh(a - d*x)**
3*cosh(c + d*x)/8 + 3*x*sinh(a - d*x)**2*sinh(c + d*x)*cosh(a - d*x)/8 - 3*
x*sinh(a - d*x)*cosh(a - d*x)**2*cosh(c + d*x)/8 - 3*x*sinh(c + d*x)*cosh(a
 - d*x)**3/8 - sinh(a - d*x)**3*sinh(c + d*x)/(8*d) - 3*sinh(a - d*x)**2*co
sh(a - d*x)*cosh(c + d*x)/(4*d) + 3*cosh(a - d*x)**3*cosh(c + d*x)/(8*d), E
```

```
q(b, -d)), (x*sinh(a - d*x/3)**3*cosh(c + d*x)/8 + 3*x*sinh(a - d*x/3)**2*s
inh(c + d*x)*cosh(a - d*x/3)/8 + 3*x*sinh(a - d*x/3)*cosh(a - d*x/3)**2*cos
h(c + d*x)/8 + x*sinh(c + d*x)*cosh(a - d*x/3)**3/8 + 7*sinh(a - d*x/3)**3*
sinh(c + d*x)/(8*d) - 3*sinh(a - d*x/3)*sinh(c + d*x)*cosh(a - d*x/3)**2/(4
*d) - 3*cosh(a - d*x/3)**3*cosh(c + d*x)/(8*d), Eq(b, -d/3)), (x*sinh(a + d
*x/3)**3*cosh(c + d*x)/8 - 3*x*sinh(a + d*x/3)**2*sinh(c + d*x)*cosh(a + d*
x/3)/8 + 3*x*sinh(a + d*x/3)*cosh(a + d*x/3)**2*cosh(c + d*x)/8 - x*sinh(c
+ d*x)*cosh(a + d*x/3)**3/8 + 9*sinh(a + d*x/3)**3*sinh(c + d*x)/(8*d) - 3*
sinh(a + d*x/3)**2*cosh(a + d*x/3)*cosh(c + d*x)/(4*d) + cosh(a + d*x/3)**3
*cosh(c + d*x)/(8*d), Eq(b, d/3)), (3*x*sinh(a + d*x)**3*cosh(c + d*x)/8 -
3*x*sinh(a + d*x)**2*sinh(c + d*x)*cosh(a + d*x)/8 - 3*x*sinh(a + d*x)*cosh
(a + d*x)**2*cosh(c + d*x)/8 + 3*x*sinh(c + d*x)*cosh(a + d*x)**3/8 + 5*sin
h(a + d*x)**3*sinh(c + d*x)/(8*d) - 3*sinh(a + d*x)*sinh(c + d*x)*cosh(a +
d*x)**2/(4*d) + 3*cosh(a + d*x)**3*cosh(c + d*x)/(8*d), Eq(b, d)), (9*b**3*
sinh(a + b*x)**2*cosh(a + b*x)*cosh(c + d*x)/(9*b**4 - 10*b**2*d**2 + d**4)
 - 6*b**3*cosh(a + b*x)**3*cosh(c + d*x)/(9*b**4 - 10*b**2*d**2 + d**4) - 7
*b**2*d*sinh(a + b*x)**3*sinh(c + d*x)/(9*b**4 - 10*b**2*d**2 + d**4) + 6*b
**2*d*sinh(a + b*x)*sinh(c + d*x)*cosh(a + b*x)**2/(9*b**4 - 10*b**2*d**2 +
 d**4) - 3*b*d**2*sinh(a + b*x)**2*cosh(a + b*x)*cosh(c + d*x)/(9*b**4 - 10
*b**2*d**2 + d**4) + d**3*sinh(a + b*x)**3*sinh(c + d*x)/(9*b**4 - 10*b**2*
d**2 + d**4), True))
```

3.186 $\int \cosh^2(c+dx)\sinh^3(a+bx)\,dx$

Optimal. Leaf size=138

$$-\frac{3\cosh(a+x(b-2d)-2c)}{16(b-2d)}+\frac{\cosh(3a+x(3b-2d)-2c)}{16(3b-2d)}-\frac{3\cosh(a+x(b+2d)+2c)}{16(b+2d)}+\frac{\cosh(3a+x(3b+2d)+}{16(3b+2d)}$$

```
[Out] -3/8*cosh(b*x+a)/b+1/24*cosh(3*b*x+3*a)/b-3/16*cosh(a-2*c+(b-2*d)*x)/(b-2*d
)+1/16*cosh(3*a-2*c+(3*b-2*d)*x)/(3*b-2*d)-3/16*cosh(a+2*c+(b+2*d)*x)/(b+2*
d)+1/16*cosh(3*a+2*c+(3*b+2*d)*x)/(3*b+2*d)
```

Rubi [A] time = 0.11, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {5618, 2638}

$$-\frac{3\cosh(a+x(b-2d)-2c)}{16(b-2d)}+\frac{\cosh(3a+x(3b-2d)-2c)}{16(3b-2d)}-\frac{3\cosh(a+x(b+2d)+2c)}{16(b+2d)}+\frac{\cosh(3a+x(3b+2d)+}{16(3b+2d)}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]^2*Sinh[a + b*x]^3,x]
```

```
[Out] (-3*Cosh[a + b*x])/(8*b) + Cosh[3*a + 3*b*x]/(24*b) - (3*Cosh[a - 2*c + (b
- 2*d)*x])/(16*(b - 2*d)) + Cosh[3*a - 2*c + (3*b - 2*d)*x]/(16*(3*b - 2*d)
) - (3*Cosh[a + 2*c + (b + 2*d)*x])/(16*(b + 2*d)) + Cosh[3*a + 2*c + (3*b
+ 2*d)*x]/(16*(3*b + 2*d))
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 5618

```
Int[Cosh[w_]^(q_.)*Sinh[v_]^(p_.), x_Symbol] :> Int[ExpandTrigReduce[Sinh[v
]^p*Cosh[w]^q, x], x] /; IGtQ[p, 0] && IGtQ[q, 0] && ((PolynomialQ[v, x] &&
 PolynomialQ[w, x]) || (BinomialQ[{v, w}, x] && IndependentQ[Cancel[v/w], x
]))
```

Rubi steps

$$\int \cosh^2(c+dx)\sinh^3(a+bx)\,dx = \int \left(-\frac{3}{8}\sinh(a+bx)+\frac{1}{8}\sinh(3a+3bx)-\frac{3}{16}\sinh(a-2c+(b-2d)x)+\frac{1}{16}\right.$$

$$=\frac{1}{16}\int \sinh(3a-2c+(3b-2d)x)\,dx+\frac{1}{16}\int \sinh(3a+2c+(3b+2d)x)\,dx$$

$$=-\frac{3\cosh(a+bx)}{8b}+\frac{\cosh(3a+3bx)}{24b}-\frac{3\cosh(a-2c+(b-2d)x)}{16(b-2d)}+\frac{\cosh(3a}{}$$

Mathematica [A] time = 1.64, size = 153, normalized size = 1.11

$$\frac{1}{48}\left(-\frac{9\cosh(a+bx-2c-2dx)}{b-2d}+\frac{3\cosh(3a+3bx-2c-2dx)}{3b-2d}-\frac{9\cosh(a+bx+2c+2dx)}{b+2d}+\frac{3\cosh(3a+3bx}{3b+2}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]^2*Sinh[a + b*x]^3,x]
```

```
[Out] ((-18*Cosh[a]*Cosh[b*x])/b + (2*Cosh[3*a]*Cosh[3*b*x])/b - (9*Cosh[a - 2*c
+ b*x - 2*d*x])/(b - 2*d) + (3*Cosh[3*a - 2*c + 3*b*x - 2*d*x])/(3*b - 2*d)
 - (9*Cosh[a + 2*c + b*x + 2*d*x])/(b + 2*d) + (3*Cosh[3*a + 2*c + 3*b*x +
2*d*x])/(3*b + 2*d) - (18*Sinh[a]*Sinh[b*x])/b + (2*Sinh[3*a]*Sinh[3*b*x])/
b)/48
```

fricas [B] time = 0.41, size = 443, normalized size = 3.21

$$\frac{\left(9b^4-40b^2d^2+16d^4\right)\cosh(bx+a)^3+9\left(\left(b^4-4b^2d^2\right)\cosh(bx+a)^3-\left(9b^4-4b^2d^2\right)\cosh(bx+a)\right)\cosh(dx}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^2*sinh(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/24*((9*b^4 - 40*b^2*d^2 + 16*d^4)*cosh(b*x + a)^3 + 9*((b^4 - 4*b^2*d^2)*
cosh(b*x + a)^3 - (9*b^4 - 4*b^2*d^2)*cosh(b*x + a))*cosh(d*x + c)^2 + 3*(9
*(b^4 - 4*b^2*d^2)*cosh(b*x + a)*cosh(d*x + c)^2 + (9*b^4 - 40*b^2*d^2 + 16
*d^4)*cosh(b*x + a))*sinh(b*x + a)^2 + 9*((b^4 - 4*b^2*d^2)*cosh(b*x + a)^3
 + 3*(b^4 - 4*b^2*d^2)*cosh(b*x + a)*sinh(b*x + a)^2 - (9*b^4 - 4*b^2*d^2)*
cosh(b*x + a))*sinh(d*x + c)^2 - 9*(9*b^4 - 40*b^2*d^2 + 16*d^4)*cosh(b*x +
 a) - 12*((b^3*d - 4*b*d^3)*cosh(d*x + c)*sinh(b*x + a)^3 - 3*(9*b^3*d - 4*
b*d^3 - (b^3*d - 4*b*d^3)*cosh(b*x + a)^2)*cosh(d*x + c)*sinh(b*x + a))*sin
h(d*x + c))/((9*b^5 - 40*b^3*d^2 + 16*b*d^4)*cosh(b*x + a)^4 - 2*(9*b^5 - 4
0*b^3*d^2 + 16*b*d^4)*cosh(b*x + a)^2*sinh(b*x + a)^2 + (9*b^5 - 40*b^3*d^2
 + 16*b*d^4)*sinh(b*x + a)^4)
```

giac [B] time = 0.15, size = 256, normalized size = 1.86

$$\frac{e^{(3\,bx+2\,dx+3\,a+2\,c)}}{32\,(3\,b+2\,d)}+\frac{e^{(3\,bx-2\,dx+3\,a-2\,c)}}{32\,(3\,b-2\,d)}+\frac{e^{(3\,bx+3\,a)}}{48\,b}-\frac{3\,e^{(bx+2\,dx+a+2\,c)}}{32\,(b+2\,d)}-\frac{3\,e^{(bx-2\,dx+a-2\,c)}}{32\,(b-2\,d)}-\frac{3\,e^{(bx+a)}}{16\,b}-\frac{3\,e^{(-bx+2\,dx-a+2\,c)}}{32\,(b-2\,d)}-\frac{3\,e}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^2*sinh(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/32*e^(3*b*x + 2*d*x + 3*a + 2*c)/(3*b + 2*d) + 1/32*e^(3*b*x - 2*d*x + 3*
a - 2*c)/(3*b - 2*d) + 1/48*e^(3*b*x + 3*a)/b - 3/32*e^(b*x + 2*d*x + a + 2
*c)/(b + 2*d) - 3/32*e^(b*x - 2*d*x + a - 2*c)/(b - 2*d) - 3/16*e^(b*x + a)
/b - 3/32*e^(-b*x + 2*d*x - a + 2*c)/(b - 2*d) - 3/32*e^(-b*x - 2*d*x - a -
 2*c)/(b + 2*d) - 3/16*e^(-b*x - a)/b + 1/32*e^(-3*b*x + 2*d*x - 3*a + 2*c)
/(3*b - 2*d) + 1/32*e^(-3*b*x - 2*d*x - 3*a - 2*c)/(3*b + 2*d) + 1/48*e^(-3
*b*x - 3*a)/b
```

maple [A] time = 0.10, size = 127, normalized size = 0.92

$$-\frac{3\cosh\left(bx+a\right)}{8b}+\frac{\cosh\left(3bx+3a\right)}{24b}-\frac{3\cosh\left(a-2c+\left(b-2d\right)x\right)}{16\left(b-2d\right)}+\frac{\cosh\left(3a-2c+\left(3b-2d\right)x\right)}{48b-32d}-\frac{3\cosh\left(a+2c\right.}{16\left(b+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)^2*sinh(b*x+a)^3,x)
```

```
[Out] -3/8*cosh(b*x+a)/b+1/24*cosh(3*b*x+3*a)/b-3/16*cosh(a-2*c+(b-2*d)*x)/(b-2*d
)+1/16*cosh(3*a-2*c+(3*b-2*d)*x)/(3*b-2*d)-3/16*cosh(a+2*c+(b+2*d)*x)/(b+2*
d)+1/16*cosh(3*a+2*c+(3*b+2*d)*x)/(3*b+2*d)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^2*sinh(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(-(2*d)/b>0)', see `assume?` for mor
e details)Is -(2*d)/b equal to -1?
```

mupad [B] time = 2.00, size = 337, normalized size = 2.44

$$\frac{\cosh\left(a+b\,x\right)\,{\cosh\left(c+d\,x\right)}^2\,{\sinh\left(a+b\,x\right)}^2\,\left(9\,b^4-26\,b^2\,d^2+8\,d^4\right)}{b\,\left(9\,b^4-40\,b^2\,d^2+16\,d^4\right)}-{\cosh\left(a+b\,x\right)}^3\,{\sinh\left(c+d\,x\right)}^2\,\left(\frac{}{9\,b^4-40}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(c + d*x)^2*sinh(a + b*x)^3,x)
```

```
[Out] (cosh(a + b*x)*cosh(c + d*x)^2*sinh(a + b*x)^2*(9*b^4 + 8*d^4 - 26*b^2*d^2)
)/(b*(9*b^4 + 16*d^4 - 40*b^2*d^2)) - cosh(a + b*x)^3*sinh(c + d*x)^2*((3*b
^3)/(9*b^4 + 16*d^4 - 40*b^2*d^2) - 1/(3*b)) - cosh(a + b*x)^3*cosh(c + d*x
)^2*((3*b^3)/(9*b^4 + 16*d^4 - 40*b^2*d^2) + 1/(3*b)) - (2*d*cosh(c + d*x)*
sinh(a + b*x)^3*sinh(c + d*x)*(7*b^2 - 4*d^2))/(9*b^4 + 16*d^4 - 40*b^2*d^2
) + (12*b^2*d*cosh(a + b*x)^2*cosh(c + d*x)*sinh(a + b*x)*sinh(c + d*x))/(9
*b^4 + 16*d^4 - 40*b^2*d^2) + (2*d^2*cosh(a + b*x)*sinh(a + b*x)^2*sinh(c +
 d*x)^2*(7*b^2 - 4*d^2))/(b*(9*b^4 + 16*d^4 - 40*b^2*d^2))
```

sympy [A] time = 109.99, size = 2030, normalized size = 14.71

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)**2*sinh(b*x+a)**3,x)
```

```
[Out] Piecewise((x*sinh(a)**3*cosh(c)**2, Eq(b, 0) & Eq(d, 0)), ((-x*sinh(c + d*x
)**2/2 + x*cosh(c + d*x)**2/2 + sinh(c + d*x)*cosh(c + d*x)/(2*d))*sinh(a)*
*3, Eq(b, 0)), (3*x*sinh(a - 2*d*x)**3*sinh(c + d*x)**2/16 + 3*x*sinh(a - 2
*d*x)**3*cosh(c + d*x)**2/16 + 3*x*sinh(a - 2*d*x)**2*sinh(c + d*x)*cosh(a
- 2*d*x)*cosh(c + d*x)/8 - 3*x*sinh(a - 2*d*x)*sinh(c + d*x)**2*cosh(a - 2*
d*x)**2/16 - 3*x*sinh(a - 2*d*x)*cosh(a - 2*d*x)**2*cosh(c + d*x)**2/16 - 3
*x*sinh(c + d*x)*cosh(a - 2*d*x)**3*cosh(c + d*x)/8 + 13*sinh(a - 2*d*x)**3
*sinh(c + d*x)*cosh(c + d*x)/(16*d) + sinh(a - 2*d*x)**2*sinh(c + d*x)**2*c
osh(a - 2*d*x)/(2*d) - 7*sinh(a - 2*d*x)*sinh(c + d*x)*cosh(a - 2*d*x)**2*c
osh(c + d*x)/(8*d) - 49*sinh(c + d*x)**2*cosh(a - 2*d*x)**3/(96*d) - 17*cos
h(a - 2*d*x)**3*cosh(c + d*x)**2/(96*d), Eq(b, -2*d)), (x*sinh(a - 2*d*x/3)
**3*sinh(c + d*x)**2/16 + x*sinh(a - 2*d*x/3)**3*cosh(c + d*x)**2/16 + 3*x*
sinh(a - 2*d*x/3)**2*sinh(c + d*x)*cosh(a - 2*d*x/3)*cosh(c + d*x)/8 + 3*x*
sinh(a - 2*d*x/3)*sinh(c + d*x)**2*cosh(a - 2*d*x/3)**2/16 + 3*x*sinh(a - 2
*d*x/3)*cosh(a - 2*d*x/3)**2*cosh(c + d*x)**2/16 + x*sinh(c + d*x)*cosh(a -
 2*d*x/3)**3*cosh(c + d*x)/8 + 15*sinh(a - 2*d*x/3)**3*sinh(c + d*x)*cosh(c
 + d*x)/(16*d) + 3*sinh(a - 2*d*x/3)**2*sinh(c + d*x)**2*cosh(a - 2*d*x/3)/
(2*d) + 9*sinh(a - 2*d*x/3)*sinh(c + d*x)*cosh(a - 2*d*x/3)**2*cosh(c + d*x
)/(8*d) - 11*sinh(c + d*x)**2*cosh(a - 2*d*x/3)**3/(32*d) + 21*cosh(a - 2*d
*x/3)**3*cosh(c + d*x)**2/(32*d), Eq(b, -2*d/3)), (x*sinh(a + 2*d*x/3)**3*s
inh(c + d*x)**2/16 + x*sinh(a + 2*d*x/3)**3*cosh(c + d*x)**2/16 - 3*x*sinh(
a + 2*d*x/3)**2*sinh(c + d*x)*cosh(a + 2*d*x/3)*cosh(c + d*x)/8 + 3*x*sinh(
a + 2*d*x/3)*sinh(c + d*x)**2*cosh(a + 2*d*x/3)**2/16 + 3*x*sinh(a + 2*d*x/
3)*cosh(a + 2*d*x/3)**2*cosh(c + d*x)**2/16 - x*sinh(c + d*x)*cosh(a + 2*d*
x/3)**3*cosh(c + d*x)/8 + 15*sinh(a + 2*d*x/3)**3*sinh(c + d*x)*cosh(c + d*
```

```
x)/(16*d) - 3*sinh(a + 2*d*x/3)**2*sinh(c + d*x)**2*cosh(a + 2*d*x/3)/(2*d)
 + 9*sinh(a + 2*d*x/3)*sinh(c + d*x)*cosh(a + 2*d*x/3)**2*cosh(c + d*x)/(8*
d) + 11*sinh(c + d*x)**2*cosh(a + 2*d*x/3)**3/(32*d) - 21*cosh(a + 2*d*x/3)
**3*cosh(c + d*x)**2/(32*d), Eq(b, 2*d/3)), (3*x*sinh(a + 2*d*x)**3*sinh(c
+ d*x)**2/16 + 3*x*sinh(a + 2*d*x)**3*cosh(c + d*x)**2/16 - 3*x*sinh(a + 2*
d*x)**2*sinh(c + d*x)*cosh(a + 2*d*x)*cosh(c + d*x)/8 - 3*x*sinh(a + 2*d*x)
*sinh(c + d*x)**2*cosh(a + 2*d*x)**2/16 - 3*x*sinh(a + 2*d*x)*cosh(a + 2*d*
x)**2*cosh(c + d*x)**2/16 + 3*x*sinh(c + d*x)*cosh(a + 2*d*x)**3*cosh(c + d
*x)/8 + 13*sinh(a + 2*d*x)**3*sinh(c + d*x)*cosh(c + d*x)/(16*d) - sinh(a +
 2*d*x)**2*sinh(c + d*x)**2*cosh(a + 2*d*x)/(2*d) - 7*sinh(a + 2*d*x)*sinh(
c + d*x)*cosh(a + 2*d*x)**2*cosh(c + d*x)/(8*d) + 49*sinh(c + d*x)**2*cosh(
a + 2*d*x)**3/(96*d) + 17*cosh(a + 2*d*x)**3*cosh(c + d*x)**2/(96*d), Eq(b,
 2*d)), (27*b**4*sinh(a + b*x)**2*cosh(a + b*x)*cosh(c + d*x)**2/(27*b**5 -
 120*b**3*d**2 + 48*b*d**4) - 18*b**4*cosh(a + b*x)**3*cosh(c + d*x)**2/(27
*b**5 - 120*b**3*d**2 + 48*b*d**4) - 42*b**3*d*sinh(a + b*x)**3*sinh(c + d*
x)*cosh(c + d*x)/(27*b**5 - 120*b**3*d**2 + 48*b*d**4) + 36*b**3*d*sinh(a +
 b*x)*sinh(c + d*x)*cosh(a + b*x)**2*cosh(c + d*x)/(27*b**5 - 120*b**3*d**2
 + 48*b*d**4) + 42*b**2*d**2*sinh(a + b*x)**2*sinh(c + d*x)**2*cosh(a + b*x
)/(27*b**5 - 120*b**3*d**2 + 48*b*d**4) - 78*b**2*d**2*sinh(a + b*x)**2*cos
h(a + b*x)*cosh(c + d*x)**2/(27*b**5 - 120*b**3*d**2 + 48*b*d**4) - 40*b**2
*d**2*sinh(c + d*x)**2*cosh(a + b*x)**3/(27*b**5 - 120*b**3*d**2 + 48*b*d**
4) + 40*b**2*d**2*cosh(a + b*x)**3*cosh(c + d*x)**2/(27*b**5 - 120*b**3*d**
2 + 48*b*d**4) + 24*b*d**3*sinh(a + b*x)**3*sinh(c + d*x)*cosh(c + d*x)/(27
*b**5 - 120*b**3*d**2 + 48*b*d**4) - 24*d**4*sinh(a + b*x)**2*sinh(c + d*x)
**2*cosh(a + b*x)/(27*b**5 - 120*b**3*d**2 + 48*b*d**4) + 24*d**4*sinh(a +
b*x)**2*cosh(a + b*x)*cosh(c + d*x)**2/(27*b**5 - 120*b**3*d**2 + 48*b*d**4
) + 16*d**4*sinh(c + d*x)**2*cosh(a + b*x)**3/(27*b**5 - 120*b**3*d**2 + 48
*b*d**4) - 16*d**4*cosh(a + b*x)**3*cosh(c + d*x)**2/(27*b**5 - 120*b**3*d*
*2 + 48*b*d**4), True))
```

3.187 $\int \cosh^3(c+dx)\sinh^3(a+bx)\,dx$

Optimal. Leaf size=195

$$-\frac{3\cosh(a+x(b-3d)-3c)}{32(b-3d)}-\frac{9\cosh(a+x(b-d)-c)}{32(b-d)}+\frac{\cosh(3(a-c)+3x(b-d))}{96(b-d)}+\frac{3\cosh(3a+x(3b-d)-c)}{32(3b-d)}-$$

```
[Out] -3/32*cosh(a-3*c+(b-3*d)*x)/(b-3*d)-9/32*cosh(a-c+(b-d)*x)/(b-d)+1/96*cosh(
3*a-3*c+3*(b-d)*x)/(b-d)+3/32*cosh(3*a-c+(3*b-d)*x)/(3*b-d)-9/32*cosh(a+c+(
b+d)*x)/(b+d)+1/96*cosh(3*a+3*c+3*(b+d)*x)/(b+d)+3/32*cosh(3*a+c+(3*b+d)*x)
/(3*b+d)-3/32*cosh(a+3*c+(b+3*d)*x)/(b+3*d)
```

Rubi [A] time = 0.15, antiderivative size = 195, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {5618, 2638}

$$-\frac{3\cosh(a+x(b-3d)-3c)}{32(b-3d)}-\frac{9\cosh(a+x(b-d)-c)}{32(b-d)}+\frac{\cosh(3(a-c)+3x(b-d))}{96(b-d)}+\frac{3\cosh(3a+x(3b-d)-c)}{32(3b-d)}-$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]^3*Sinh[a + b*x]^3,x]
```

```
[Out] (-3*Cosh[a - 3*c + (b - 3*d)*x])/(32*(b - 3*d)) - (9*Cosh[a - c + (b - d)*x
])/(32*(b - d)) + Cosh[3*(a - c) + 3*(b - d)*x]/(96*(b - d)) + (3*Cosh[3*a
- c + (3*b - d)*x])/(32*(3*b - d)) - (9*Cosh[a + c + (b + d)*x])/(32*(b + d
)) + Cosh[3*(a + c) + 3*(b + d)*x]/(96*(b + d)) + (3*Cosh[3*a + c + (3*b +
d)*x])/(32*(3*b + d)) - (3*Cosh[a + 3*c + (b + 3*d)*x])/(32*(b + 3*d))
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 5618

```
Int[Cosh[w_]^(q_.)*Sinh[v_]^(p_.), x_Symbol] :> Int[ExpandTrigReduce[Sinh[v
]^p*Cosh[w]^q, x], x] /; IGtQ[p, 0] && IGtQ[q, 0] && ((PolynomialQ[v, x] &&
 PolynomialQ[w, x]) || (BinomialQ[{v, w}, x] && IndependentQ[Cancel[v/w], x
]))
```

Rubi steps

$$\int \cosh^3(c+dx)\sinh^3(a+bx)\,dx = \int \left(-\frac{3}{32}\sinh(a-3c+(b-3d)x) - \frac{9}{32}\sinh(a-c+(b-d)x) + \frac{1}{32}\sinh(3$$
$$= \frac{1}{32}\int \sinh(3(a-c)+3(b-d)x)\,dx + \frac{1}{32}\int \sinh(3(a+c)+3(b+d)x)\,dx$$
$$= -\frac{3\cosh(a-3c+(b-3d)x)}{32(b-3d)} - \frac{9\cosh(a-c+(b-d)x)}{32(b-d)} + \frac{\cosh(3(a-c)+}{96(b-}$$

Mathematica [A] time = 1.69, size = 176, normalized size = 0.90

$$\frac{1}{96}\left(-\frac{9\cosh(a+bx-3c-3dx)}{b-3d} - \frac{27\cosh(a+bx-c-dx)}{b-d} + \frac{\cosh(3(a+bx-c-dx))}{b-d} + \frac{9\cosh(3a+3bx-c}{3b-d}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]^3*Sinh[a + b*x]^3,x]
```

```
[Out] ((-9*Cosh[a - 3*c + b*x - 3*d*x])/(b - 3*d) - (27*Cosh[a - c + b*x - d*x])/
(b - d) + Cosh[3*(a - c + b*x - d*x)]/(b - d) + (9*Cosh[3*a - c + 3*b*x - d
*x])/(3*b - d) + (9*Cosh[3*a + c + 3*b*x + d*x])/(3*b + d) - (9*Cosh[a + 3*
c + b*x + 3*d*x])/(b + 3*d) - (27*Cosh[a + c + (b + d)*x])/(b + d) + Cosh[3
*(a + c + (b + d)*x)]/(b + d))/96
```

fricas [B] time = 0.51, size = 729, normalized size = 3.74

$$\frac{\left(\left(9\,b^5 - 82\,b^3d^2 + 9\,bd^4\right)\cosh\left(bx+a\right)^3 - 9\left(9\,b^5 - 10\,b^3d^2 + bd^4\right)\cosh\left(bx+a\right)\right)\cosh\left(dx+c\right)^3 - \left(\left(9\,b^4d - 82\,b\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^3*sinh(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/48*(((9*b^5 - 82*b^3*d^2 + 9*b*d^4)*cosh(b*x + a)^3 - 9*(9*b^5 - 10*b^3*d
^2 + b*d^4)*cosh(b*x + a))*cosh(d*x + c)^3 - ((9*b^4*d - 82*b^2*d^3 + 9*d^5
)*sinh(b*x + a)^3 - 3*(81*b^4*d - 90*b^2*d^3 + 9*d^5 - (9*b^4*d - 82*b^2*d^
3 + 9*d^5)*cosh(b*x + a)^2)*sinh(b*x + a))*sinh(d*x + c)^3 + 3*((9*b^5 - 82
*b^3*d^2 + 9*b*d^4)*cosh(b*x + a)*cosh(d*x + c)^3 + 27*(b^5 - 10*b^3*d^2 +
9*b*d^4)*cosh(b*x + a)*cosh(d*x + c))*sinh(b*x + a)^2 + 3*(3*(9*b^5 - 82*b^
3*d^2 + 9*b*d^4)*cosh(b*x + a)*cosh(d*x + c)*sinh(b*x + a)^2 + ((9*b^5 - 82
*b^3*d^2 + 9*b*d^4)*cosh(b*x + a)^3 - 9*(9*b^5 - 10*b^3*d^2 + b*d^4)*cosh(b
*x + a))*cosh(d*x + c))*sinh(d*x + c)^2 + 27*((b^5 - 10*b^3*d^2 + 9*b*d^4)*
cosh(b*x + a)^3 - (9*b^5 - 82*b^3*d^2 + 9*b*d^4)*cosh(b*x + a))*cosh(d*x +
c) - 3*((3*b^4*d - 30*b^2*d^3 + 27*d^5 + (9*b^4*d - 82*b^2*d^3 + 9*d^5)*cos
```

```
h(d*x + c)^2)*sinh(b*x + a)^3 - 3*(27*b^4*d - 246*b^2*d^3 + 27*d^5 - 3*(b^4
*d - 10*b^2*d^3 + 9*d^5)*cosh(b*x + a)^2 + (81*b^4*d - 90*b^2*d^3 + 9*d^5 -
 (9*b^4*d - 82*b^2*d^3 + 9*d^5)*cosh(b*x + a)^2)*cosh(d*x + c)^2)*sinh(b*x
+ a))*sinh(d*x + c))/((9*b^6 - 91*b^4*d^2 + 91*b^2*d^4 - 9*d^6)*cosh(b*x +
a)^4 - 2*(9*b^6 - 91*b^4*d^2 + 91*b^2*d^4 - 9*d^6)*cosh(b*x + a)^2*sinh(b*x
 + a)^2 + (9*b^6 - 91*b^4*d^2 + 91*b^2*d^4 - 9*d^6)*sinh(b*x + a)^4)
```

giac [B] time = 0.15, size = 373, normalized size = 1.91

$$\frac{e^{(3\,bx+3\,dx+3\,a+3\,c)}}{192\,(b+d)}+\frac{3\,e^{(3\,bx+dx+3\,a+c)}}{64\,(3\,b+d)}+\frac{3\,e^{(3\,bx-dx+3\,a-c)}}{64\,(3\,b-d)}+\frac{e^{(3\,bx-3\,dx+3\,a-3\,c)}}{192\,(b-d)}-\frac{3\,e^{(bx+3\,dx+a+3\,c)}}{64\,(b+3\,d)}-\frac{9\,e^{(bx+dx+a+c)}}{64\,(b+d)}-\frac{9\,e^{(bx-dx+a}}{64\,(b-d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^3*sinh(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/192*e^(3*b*x + 3*d*x + 3*a + 3*c)/(b + d) + 3/64*e^(3*b*x + d*x + 3*a + c
)/(3*b + d) + 3/64*e^(3*b*x - d*x + 3*a - c)/(3*b - d) + 1/192*e^(3*b*x - 3
*d*x + 3*a - 3*c)/(b - d) - 3/64*e^(b*x + 3*d*x + a + 3*c)/(b + 3*d) - 9/64
*e^(b*x + d*x + a + c)/(b + d) - 9/64*e^(b*x - d*x + a - c)/(b - d) - 3/64*
e^(b*x - 3*d*x + a - 3*c)/(b - 3*d) - 3/64*e^(-b*x + 3*d*x - a + 3*c)/(b -
3*d) - 9/64*e^(-b*x + d*x - a + c)/(b - d) - 9/64*e^(-b*x - d*x - a - c)/(b
 + d) - 3/64*e^(-b*x - 3*d*x - a - 3*c)/(b + 3*d) + 1/192*e^(-3*b*x + 3*d*x
 - 3*a + 3*c)/(b - d) + 3/64*e^(-3*b*x + d*x - 3*a + c)/(3*b - d) + 3/64*e^
(-3*b*x - d*x - 3*a - c)/(3*b + d) + 1/192*e^(-3*b*x - 3*d*x - 3*a - 3*c)/(
b + d)
```

maple [A] time = 0.12, size = 184, normalized size = 0.94

$$-\frac{3\cosh\left(a-3c+(b-3d)\,x\right)}{32\,(b-3d)}-\frac{9\cosh\left(a-c+(b-d)\,x\right)}{32\,(b-d)}-\frac{9\cosh\left(a+c+(b+d)\,x\right)}{32\,(b+d)}-\frac{3\cosh\left(a+3c+(b+3d)\,x\right)}{32\,(b+3d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)^3*sinh(b*x+a)^3,x)
```

```
[Out] -3/32*cosh(a-3*c+(b-3*d)*x)/(b-3*d)-9/32*cosh(a-c+(b-d)*x)/(b-d)-9/32*cosh(
a+c+(b+d)*x)/(b+d)-3/32*cosh(a+3*c+(b+3*d)*x)/(b+3*d)+1/96/(b-d)*cosh((3*b-
3*d)*x+3*a-3*c)+3/32*cosh(3*a-c+(3*b-d)*x)/(3*b-d)+3/32*cosh(3*a+c+(3*b+d)*
x)/(3*b+d)+1/96*cosh((3*b+3*d)*x+3*a+3*c)/(b+d)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^3*sinh(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(-(3*d)/b>0)', see `assume?` for mor
e details)Is -(3*d)/b equal to -1?
```

mupad [B] time = 2.09, size = 908, normalized size = 4.66

$$-e^{3a+c+3bx+dx}\left(\frac{-9b^3+3b^2d+9bd^2-3d^3}{576b^4-640b^2d^2+64d^4}+\frac{e^{-6a-6bx}\left(-9b^3-3b^2d+9bd^2+3d^3\right)}{576b^4-640b^2d^2+64d^4}-\frac{e^{-2a-2bx}\left(-81b^3+\right.}{576b^4-64}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(c + d*x)^3*sinh(a + b*x)^3,x)
```

```
[Out] - exp(3*a + c + 3*b*x + d*x)*((9*b*d^2 + 3*b^2*d - 9*b^3 - 3*d^3)/(576*b^4
+ 64*d^4 - 640*b^2*d^2) + (exp(- 6*a - 6*b*x)*(9*b*d^2 - 3*b^2*d - 9*b^3 +
3*d^3))/(576*b^4 + 64*d^4 - 640*b^2*d^2) - (exp(- 2*a - 2*b*x)*(9*b*d^2 + 8
1*b^2*d - 81*b^3 - 9*d^3))/(576*b^4 + 64*d^4 - 640*b^2*d^2) - (exp(- 4*a -
4*b*x)*(9*b*d^2 - 81*b^2*d - 81*b^3 + 9*d^3))/(576*b^4 + 64*d^4 - 640*b^2*d
^2)) - exp(3*a - c + 3*b*x - d*x)*((9*b*d^2 - 3*b^2*d - 9*b^3 + 3*d^3)/(576
*b^4 + 64*d^4 - 640*b^2*d^2) + (exp(- 6*a - 6*b*x)*(9*b*d^2 + 3*b^2*d - 9*b
^3 - 3*d^3))/(576*b^4 + 64*d^4 - 640*b^2*d^2) - (exp(- 2*a - 2*b*x)*(9*b*d^
2 - 81*b^2*d - 81*b^3 + 9*d^3))/(576*b^4 + 64*d^4 - 640*b^2*d^2) - (exp(- 4
*a - 4*b*x)*(9*b*d^2 + 81*b^2*d - 81*b^3 - 9*d^3))/(576*b^4 + 64*d^4 - 640*
b^2*d^2)) - exp(3*a - 3*c + 3*b*x - 3*d*x)*((9*b*d^2 - b^2*d - b^3 + 9*d^3)
/(192*b^4 + 1728*d^4 - 1920*b^2*d^2) + (exp(- 6*a - 6*b*x)*(9*b*d^2 + b^2*d
 - b^3 - 9*d^3))/(192*b^4 + 1728*d^4 - 1920*b^2*d^2) - (exp(- 2*a - 2*b*x)*
(9*b*d^2 - 27*b^2*d - 9*b^3 + 27*d^3))/(192*b^4 + 1728*d^4 - 1920*b^2*d^2)
- (exp(- 4*a - 4*b*x)*(9*b*d^2 + 27*b^2*d - 9*b^3 - 27*d^3))/(192*b^4 + 172
8*d^4 - 1920*b^2*d^2)) - exp(3*a + 3*c + 3*b*x + 3*d*x)*((9*b*d^2 + b^2*d -
 b^3 - 9*d^3)/(192*b^4 + 1728*d^4 - 1920*b^2*d^2) + (exp(- 6*a - 6*b*x)*(9*
b*d^2 - b^2*d - b^3 + 9*d^3))/(192*b^4 + 1728*d^4 - 1920*b^2*d^2) - (exp(-
2*a - 2*b*x)*(9*b*d^2 + 27*b^2*d - 9*b^3 - 27*d^3))/(192*b^4 + 1728*d^4 - 1
920*b^2*d^2) - (exp(- 4*a - 4*b*x)*(9*b*d^2 - 27*b^2*d - 9*b^3 + 27*d^3))/(
192*b^4 + 1728*d^4 - 1920*b^2*d^2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)**3*sinh(b*x+a)**3,x)
```

```
[Out] Timed out
```

3.188 $\int \sinh(a+bx)\tanh(c+dx)\,dx$

Optimal. Leaf size=121

$$-\frac{e^{-a-bx}\,{}_2F_1\left(1,-\frac{b}{2d};1-\frac{b}{2d};-e^{2(c+dx)}\right)}{b}-\frac{e^{a+bx}\,{}_2F_1\left(1,\frac{b}{2d};\frac{b}{2d}+1;-e^{2(c+dx)}\right)}{b}+\frac{e^{-a-bx}}{2b}+\frac{e^{a+bx}}{2b}$$

```
[Out] 1/2*exp(-b*x-a)/b+1/2*exp(b*x+a)/b-exp(-b*x-a)*hypergeom([1, -1/2*b/d],[1-1
/2*b/d],-exp(2*d*x+2*c))/b-exp(b*x+a)*hypergeom([1, 1/2*b/d],[1+1/2*b/d],-e
xp(2*d*x+2*c))/b
```

Rubi [A] time = 0.11, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {5601, 2194, 2251}

$$-\frac{e^{-a-bx}\,{}_2F_1\left(1,-\frac{b}{2d};1-\frac{b}{2d};-e^{2(c+dx)}\right)}{b}-\frac{e^{a+bx}\,{}_2F_1\left(1,\frac{b}{2d};\frac{b}{2d}+1;-e^{2(c+dx)}\right)}{b}+\frac{e^{-a-bx}}{2b}+\frac{e^{a+bx}}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[a + b*x]*Tanh[c + d*x],x]
```

```
[Out] E^(-a - b*x)/(2*b) + E^(a + b*x)/(2*b) - (E^(-a - b*x)*Hypergeometric2F1[1,
 -b/(2*d), 1 - b/(2*d), -E^(2*(c + d*x))])/b - (E^(a + b*x)*Hypergeometric2
F1[1, b/(2*d), 1 + b/(2*d), -E^(2*(c + d*x))])/b
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 2251

```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_)*(G_)^((h_.)*((f_.
) + (g_.)*(x_))), x_Symbol] :> Simp[(a^p*G^(h*(f + g*x))*Hypergeometric2F1[
-p, (g*h*Log[G])/(d*e*Log[F]), (g*h*Log[G])/(d*e*Log[F]) + 1, Simplify[-((b
*F^(e*(c + d*x)))/a)]])/(g*h*Log[G]), x] /; FreeQ[{F, G, a, b, c, d, e, f,
g, h, p}, x] && (ILtQ[p, 0] || GtQ[a, 0])
```

Rule 5601

```
Int[Sinh[(a_.) + (b_.)*(x_)]*Tanh[(c_.) + (d_.)*(x_)], x_Symbol] :> Int[-(1
/(E^(a + b*x)*2)) + E^(a + b*x)/2 + 1/(E^(a + b*x)*(1 + E^(2*(c + d*x)))) -
 E^(a + b*x)/(1 + E^(2*(c + d*x))), x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2
```

```
 - d^2, 0]
```

Rubi steps

$$\int \sinh(a+bx)\tanh(c+dx)\,dx = \int \left(-\frac{1}{2}e^{-a-bx} + \frac{1}{2}e^{a+bx} + \frac{e^{-a-bx}}{1+e^{2(c+dx)}} - \frac{e^{a+bx}}{1+e^{2(c+dx)}}\right)dx$$
$$= -\left(\frac{1}{2}\int e^{-a-bx}\,dx\right) + \frac{1}{2}\int e^{a+bx}\,dx + \int \frac{e^{-a-bx}}{1+e^{2(c+dx)}}\,dx - \int \frac{e^{a+bx}}{1+e^{2(c+dx)}}\,dx$$
$$= \frac{e^{-a-bx}}{2b} + \frac{e^{a+bx}}{2b} - \frac{e^{-a-bx}\,{}_2F_1\left(1,-\frac{b}{2d};1-\frac{b}{2d};-e^{2(c+dx)}\right)}{b} - \frac{e^{a+bx}\,{}_2F_1\left(1,\frac{b}{2d};1+\frac{b}{2d}\dots\right)}{b}$$

Mathematica [B] time = 14.70, size = 278, normalized size = 2.30

$$\frac{e^{-a-bx-c}\left((b-2d)\left(2b\,\mathrm{sech}(c)e^{2(a+x(b+d)+c)}\,{}_2F_1\left(1,\frac{b}{2d}+1;\frac{b}{2d}+2;-e^{2(c+dx)}\right)-(b+2d)\left(\left(e^{2a}+2e^{2c}+1\right)\mathrm{sech}(c)\,{}_2F\dots\right.\right.\right.}{\dots}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[a + b*x]*Tanh[c + d*x],x]
```

```
[Out] (E^(-a - c - b*x)*(-(b*(b + 2*d)*E^(2*(c + d*x))*(-1 + E^(2*a))*Hypergeomet
ric2F1[1, 1 - b/(2*d), 2 - b/(2*d), -E^(2*(c + d*x))]*Sech[c]) + (b - 2*d)*
(2*b*E^(2*(a + c + (b + d)*x))*Hypergeometric2F1[1, 1 + b/(2*d), 2 + b/(2*d
), -E^(2*(c + d*x))]*Sech[c] - (b + 2*d)*(-Sech[c] - E^(2*a)*Sech[c] + (1 +
 E^(2*a) + 2*E^(2*c))*Hypergeometric2F1[1, -1/2*b/d, 1 - b/(2*d), -E^(2*(c
+ d*x))]*Sech[c] + 2*E^(2*(a + c + b*x))*Hypergeometric2F1[1, b/(2*d), 1 +
b/(2*d), -E^(2*(c + d*x))]*Sech[c] - 4*E^(a + c + b*x)*Cosh[a + b*x]*Tanh[c
]))))/(4*(b^3 - 4*b*d^2))
```

fricas [F] time = 0.48, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\sinh\left(bx+a\right)\tanh\left(dx+c\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)*tanh(d*x+c),x, algorithm="fricas")
```

```
[Out] integral(sinh(b*x + a)*tanh(d*x + c), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sinh\left(bx+a\right)\tanh\left(dx+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)*tanh(d*x+c),x, algorithm="giac")
```

```
[Out] integrate(sinh(b*x + a)*tanh(d*x + c), x)
```

maple [F] time = 0.20, size = 0, normalized size = 0.00

$$\int \sinh\left(bx+a\right)\tanh\left(dx+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(b*x+a)*tanh(d*x+c),x)
```

```
[Out] int(sinh(b*x+a)*tanh(d*x+c),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\left(e^{(2\,bx+2\,a)}+1\right)e^{(-bx-a)}}{2\,b}-\frac{1}{2}\int\frac{2\left(e^{(2\,bx+2\,a)}-1\right)}{e^{(bx+2\,dx+a+2\,c)}+e^{(bx+a)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)*tanh(d*x+c),x, algorithm="maxima")
```

```
[Out] 1/2*(e^(2*b*x + 2*a) + 1)*e^(-b*x - a)/b - 1/2*integrate(2*(e^(2*b*x + 2*a)
 - 1)/(e^(b*x + 2*d*x + a + 2*c) + e^(b*x + a)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sinh\left(a+b\,x\right)\,\tanh\left(c+d\,x\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a + b*x)*tanh(c + d*x),x)
```

```
[Out] int(sinh(a + b*x)*tanh(c + d*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sinh\left(a+bx\right)\tanh\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)*tanh(d*x+c),x)
```

```
[Out] Integral(sinh(a + b*x)*tanh(c + d*x), x)
```

3.189 $\int \coth(c+dx)\sinh(a+bx)\,dx$

Optimal. Leaf size=117

$$-\frac{e^{-a-bx}\,{}_2F_1\left(1,-\frac{b}{2d};1-\frac{b}{2d};e^{2(c+dx)}\right)}{b}-\frac{e^{a+bx}\,{}_2F_1\left(1,\frac{b}{2d};\frac{b}{2d}+1;e^{2(c+dx)}\right)}{b}+\frac{e^{-a-bx}}{2b}+\frac{e^{a+bx}}{2b}$$

```
[Out] 1/2*exp(-b*x-a)/b+1/2*exp(b*x+a)/b-exp(-b*x-a)*hypergeom([1, -1/2*b/d],[1-1
/2*b/d],exp(2*d*x+2*c))/b-exp(b*x+a)*hypergeom([1, 1/2*b/d],[1+1/2*b/d],exp
(2*d*x+2*c))/b
```

Rubi [A] time = 0.11, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {5603, 2194, 2251}

$$-\frac{e^{-a-bx}\,{}_2F_1\left(1,-\frac{b}{2d};1-\frac{b}{2d};e^{2(c+dx)}\right)}{b}-\frac{e^{a+bx}\,{}_2F_1\left(1,\frac{b}{2d};\frac{b}{2d}+1;e^{2(c+dx)}\right)}{b}+\frac{e^{-a-bx}}{2b}+\frac{e^{a+bx}}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[c + d*x]*Sinh[a + b*x],x]
```

```
[Out] E^(-a - b*x)/(2*b) + E^(a + b*x)/(2*b) - (E^(-a - b*x)*Hypergeometric2F1[1,
 -b/(2*d), 1 - b/(2*d), E^(2*(c + d*x))])/b - (E^(a + b*x)*Hypergeometric2F
1[1, b/(2*d), 1 + b/(2*d), E^(2*(c + d*x))])/b
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 2251

```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_)*(G_)^((h_.)*((f_.
) + (g_.)*(x_))), x_Symbol] :> Simp[(a^p*G^(h*(f + g*x))*Hypergeometric2F1[
-p, (g*h*Log[G])/(d*e*Log[F]), (g*h*Log[G])/(d*e*Log[F]) + 1, Simplify[-((b
*F^(e*(c + d*x)))/a)]])/(g*h*Log[G]), x] /; FreeQ[{F, G, a, b, c, d, e, f,
g, h, p}, x] && (ILtQ[p, 0] || GtQ[a, 0])
```

Rule 5603

```
Int[Coth[(c_.) + (d_.)*(x_)]*Sinh[(a_.) + (b_.)*(x_)], x_Symbol] :> Int[-(1
/(E^(a + b*x)*2)) + E^(a + b*x)/2 + 1/(E^(a + b*x)*(1 - E^(2*(c + d*x)))) -
 E^(a + b*x)/(1 - E^(2*(c + d*x))), x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2
```

```
- d^2, 0]
```

Rubi steps

$$\int \coth(c+dx)\sinh(a+bx)\,dx = \int \left(-\frac{1}{2}e^{-a-bx}+\frac{1}{2}e^{a+bx}+\frac{e^{-a-bx}}{1-e^{2(c+dx)}}-\frac{e^{a+bx}}{1-e^{2(c+dx)}}\right)dx$$
$$= -\left(\frac{1}{2}\int e^{-a-bx}\,dx\right)+\frac{1}{2}\int e^{a+bx}\,dx+\int \frac{e^{-a-bx}}{1-e^{2(c+dx)}}\,dx-\int \frac{e^{a+bx}}{1-e^{2(c+dx)}}\,dx$$
$$= \frac{e^{-a-bx}}{2b}+\frac{e^{a+bx}}{2b}-\frac{e^{-a-bx}\,{}_2F_1\left(1,-\frac{b}{2d};1-\frac{b}{2d};e^{2(c+dx)}\right)}{b}-\frac{e^{a+bx}\,{}_2F_1\left(1,\frac{b}{2d};1+\frac{b}{2d};e^{2(c+dx)}\right)}{b}$$

Mathematica [B] time = 6.92, size = 240, normalized size = 2.05

$$\frac{e^{-a-bx+2c}\left(be^{2dx}\,{}_2F_1\left(1,1-\frac{b}{2d};2-\frac{b}{2d};e^{2(c+dx)}\right)-(b-2d)\,{}_2F_1\left(1,-\frac{b}{2d};1-\frac{b}{2d};e^{2(c+dx)}\right)\right)}{b\left(e^{2c}-1\right)(b-2d)}-\frac{e^{a+2c}\left(\frac{e^{bx}\,{}_2F_1\left(1,\frac{b}{2d};\frac{b}{2d}+1;e^{2(c+dx)}\right)}{b} \cdots\right)}{e \cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[c + d*x]*Sinh[a + b*x],x]
```

```
[Out] (Cosh[a]*Cosh[b*x]*Coth[c])/b + (E^(-a + 2*c - b*x)*(b*E^(2*d*x)*Hypergeome
tric2F1[1, 1 - b/(2*d), 2 - b/(2*d), E^(2*(c + d*x))] - (b - 2*d)*Hypergeom
etric2F1[1, -1/2*b/d, 1 - b/(2*d), E^(2*(c + d*x))]))/(b*(b - 2*d)*(-1 + E^
(2*c))) - (E^(a + 2*c)*(-((E^((b + 2*d)*x)*Hypergeometric2F1[1, 1 + b/(2*d)
, 2 + b/(2*d), E^(2*(c + d*x))])/(b + 2*d)) + (E^(b*x)*Hypergeometric2F1[1,
 b/(2*d), 1 + b/(2*d), E^(2*(c + d*x))])/b))/(-1 + E^(2*c)) + (Coth[c]*Sinh
[a]*Sinh[b*x])/b
```

fricas [F] time = 0.46, size = 0, normalized size = 0.00

$$\text{integral}\left(\coth\left(dx+c\right)\sinh\left(bx+a\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)*sinh(b*x+a),x, algorithm="fricas")
```

```
[Out] integral(coth(d*x + c)*sinh(b*x + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \coth\left(dx+c\right)\sinh\left(bx+a\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)*sinh(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(coth(d*x + c)*sinh(b*x + a), x)
```

maple [F] time = 0.26, size = 0, normalized size = 0.00

$$\int \coth\left(dx+c\right)\sinh\left(bx+a\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(d*x+c)*sinh(b*x+a),x)
```

```
[Out] int(coth(d*x+c)*sinh(b*x+a),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\left(e^{(2\,bx+2\,a)}+1\right)e^{(-bx-a)}}{2\,b}-\frac{1}{2}\int\frac{e^{(2\,bx+2\,a)}-1}{e^{(bx+dx+a+c)}+e^{(bx+a)}}\,dx+\frac{1}{2}\int\frac{e^{(2\,bx+2\,a)}-1}{e^{(bx+dx+a+c)}-e^{(bx+a)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)*sinh(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/2*(e^(2*b*x + 2*a) + 1)*e^(-b*x - a)/b - 1/2*integrate((e^(2*b*x + 2*a) -
 1)/(e^(b*x + d*x + a + c) + e^(b*x + a)), x) + 1/2*integrate((e^(2*b*x + 2
*a) - 1)/(e^(b*x + d*x + a + c) - e^(b*x + a)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \mathrm{coth}\left(c+d\,x\right)\,\mathrm{sinh}\left(a+b\,x\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(c + d*x)*sinh(a + b*x),x)
```

```
[Out] int(coth(c + d*x)*sinh(a + b*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sinh{\left(a+bx\right)}\coth{\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)*sinh(b*x+a),x)
```

```
[Out] Integral(sinh(a + b*x)*coth(c + d*x), x)
```

3.190 $\int \cosh(a+bx)\coth(c+dx)\,dx$

Optimal. Leaf size=116

$$\frac{e^{-a-bx}\,{}_2F_1\left(1,-\frac{b}{2d};1-\frac{b}{2d};e^{2(c+dx)}\right)}{b}-\frac{e^{a+bx}\,{}_2F_1\left(1,\frac{b}{2d};\frac{b}{2d}+1;e^{2(c+dx)}\right)}{b}-\frac{e^{-a-bx}}{2b}+\frac{e^{a+bx}}{2b}$$

```
[Out] -1/2*exp(-b*x-a)/b+1/2*exp(b*x+a)/b+exp(-b*x-a)*hypergeom([1, -1/2*b/d],[1-
1/2*b/d],exp(2*d*x+2*c))/b-exp(b*x+a)*hypergeom([1, 1/2*b/d],[1+1/2*b/d],ex
p(2*d*x+2*c))/b
```

Rubi [A] time = 0.11, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {5602, 2194, 2251}

$$\frac{e^{-a-bx}\,{}_2F_1\left(1,-\frac{b}{2d};1-\frac{b}{2d};e^{2(c+dx)}\right)}{b}-\frac{e^{a+bx}\,{}_2F_1\left(1,\frac{b}{2d};\frac{b}{2d}+1;e^{2(c+dx)}\right)}{b}-\frac{e^{-a-bx}}{2b}+\frac{e^{a+bx}}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[a + b*x]*Coth[c + d*x],x]
```

```
[Out] -E^(-a - b*x)/(2*b) + E^(a + b*x)/(2*b) + (E^(-a - b*x)*Hypergeometric2F1[1
, -b/(2*d), 1 - b/(2*d), E^(2*(c + d*x))])/b - (E^(a + b*x)*Hypergeometric2
F1[1, b/(2*d), 1 + b/(2*d), E^(2*(c + d*x))])/b
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 2251

```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_)*(G_)^((h_.)*((f_.
) + (g_.)*(x_))), x_Symbol] :> Simp[(a^p*G^(h*(f + g*x))*Hypergeometric2F1[
-p, (g*h*Log[G])/(d*e*Log[F]), (g*h*Log[G])/(d*e*Log[F]) + 1, Simplify[-((b
*F^(e*(c + d*x)))/a)]])/(g*h*Log[G]), x] /; FreeQ[{F, G, a, b, c, d, e, f,
g, h, p}, x] && (ILtQ[p, 0] || GtQ[a, 0])
```

Rule 5602

```
Int[Cosh[(a_.) + (b_.)*(x_)]*Coth[(c_.) + (d_.)*(x_)], x_Symbol] :> Int[1/(
E^(a + b*x)*2) + E^(a + b*x)/2 - 1/(E^(a + b*x)*(1 - E^(2*(c + d*x)))) - E^
(a + b*x)/(1 - E^(2*(c + d*x))), x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 -
```

```
d^2, 0]
```

Rubi steps

$$\int \cosh(a+bx)\coth(c+dx)\,dx = \int \left(\frac{1}{2}e^{-a-bx} + \frac{1}{2}e^{a+bx} - \frac{e^{-a-bx}}{1-e^{2(c+dx)}} - \frac{e^{a+bx}}{1-e^{2(c+dx)}}\right) dx$$
$$= \frac{1}{2}\int e^{-a-bx}\,dx + \frac{1}{2}\int e^{a+bx}\,dx - \int \frac{e^{-a-bx}}{1-e^{2(c+dx)}}\,dx - \int \frac{e^{a+bx}}{1-e^{2(c+dx)}}\,dx$$
$$= -\frac{e^{-a-bx}}{2b} + \frac{e^{a+bx}}{2b} + \frac{e^{-a-bx}\,{}_2F_1\left(1,-\frac{b}{2d};1-\frac{b}{2d};e^{2(c+dx)}\right)}{b} - \frac{e^{a+bx}\,{}_2F_1\left(1,\frac{b}{2d};1+\frac{b}{2d}\ldots\right)}{b}$$

Mathematica [A] time = 9.56, size = 99, normalized size = 0.85

$$\frac{e^{-a-bx}\left(-2e^{2(a+bx)}\,{}_2F_1\left(1,\frac{b}{2d};\frac{b}{2d}+1;e^{2(c+dx)}\right)+e^{2(a+bx)}+2\,{}_2F_1\left(1,-\frac{b}{2d};1-\frac{b}{2d};e^{2(c+dx)}\right)-1\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[a + b*x]*Coth[c + d*x],x]
[Out] (E^(-a - b*x)*(-1 + E^(2*(a + b*x)) + 2*Hypergeometric2F1[1, -1/2*b/d, 1 -
b/(2*d), E^(2*(c + d*x))] - 2*E^(2*(a + b*x))*Hypergeometric2F1[1, b/(2*d),
 1 + b/(2*d), E^(2*(c + d*x))]))/(2*b)
```

fricas [F] time = 0.50, size = 0, normalized size = 0.00

$$\text{integral}\left(\cosh\left(bx+a\right)\coth\left(dx+c\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*coth(d*x+c),x, algorithm="fricas")
[Out] integral(cosh(b*x + a)*coth(d*x + c), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \cosh\left(bx+a\right)\coth\left(dx+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*coth(d*x+c),x, algorithm="giac")
```

```
[Out] integrate(cosh(b*x + a)*coth(d*x + c), x)
```

maple [F] time = 0.29, size = 0, normalized size = 0.00

$$\int \cosh(bx + a)\coth(dx + c)\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)*coth(d*x+c),x)
```

```
[Out] int(cosh(b*x+a)*coth(d*x+c),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\left(e^{(2\,bx+2\,a)}-1\right)e^{(-bx-a)}}{2\,b} - \frac{1}{2}\int \frac{e^{(2\,bx+2\,a)}+1}{e^{(bx+dx+a+c)}+e^{(bx+a)}}\,dx + \frac{1}{2}\int \frac{e^{(2\,bx+2\,a)}+1}{e^{(bx+dx+a+c)}-e^{(bx+a)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*coth(d*x+c),x, algorithm="maxima")
```

```
[Out] 1/2*(e^(2*b*x + 2*a) - 1)*e^(-b*x - a)/b - 1/2*integrate((e^(2*b*x + 2*a) +
 1)/(e^(b*x + d*x + a + c) + e^(b*x + a)), x) + 1/2*integrate((e^(2*b*x + 2
*a) + 1)/(e^(b*x + d*x + a + c) - e^(b*x + a)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \cosh\left(a+b\,x\right)\,\mathrm{coth}\left(c+d\,x\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a + b*x)*coth(c + d*x),x)
```

```
[Out] int(cosh(a + b*x)*coth(c + d*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \cosh(a + bx)\coth(c + dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*coth(d*x+c),x)
```

```
[Out] Integral(cosh(a + b*x)*coth(c + d*x), x)
```

3.191 $\int \cosh(a+bx)\tanh(c+dx)\,dx$

Optimal. Leaf size=120

$$\frac{e^{-a-bx}\,{}_2F_1\left(1,-\frac{b}{2d};1-\frac{b}{2d};-e^{2(c+dx)}\right)}{b}-\frac{e^{a+bx}\,{}_2F_1\left(1,\frac{b}{2d};\frac{b}{2d}+1;-e^{2(c+dx)}\right)}{b}-\frac{e^{-a-bx}}{2b}+\frac{e^{a+bx}}{2b}$$

```
[Out] -1/2*exp(-b*x-a)/b+1/2*exp(b*x+a)/b+exp(-b*x-a)*hypergeom([1, -1/2*b/d],[1-
1/2*b/d],-exp(2*d*x+2*c))/b-exp(b*x+a)*hypergeom([1, 1/2*b/d],[1+1/2*b/d],-
exp(2*d*x+2*c))/b
```

Rubi [A] time = 0.10, antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {5604, 2194, 2251}

$$\frac{e^{-a-bx}\,{}_2F_1\left(1,-\frac{b}{2d};1-\frac{b}{2d};-e^{2(c+dx)}\right)}{b}-\frac{e^{a+bx}\,{}_2F_1\left(1,\frac{b}{2d};\frac{b}{2d}+1;-e^{2(c+dx)}\right)}{b}-\frac{e^{-a-bx}}{2b}+\frac{e^{a+bx}}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[a + b*x]*Tanh[c + d*x],x]
```

```
[Out] -E^(-a - b*x)/(2*b) + E^(a + b*x)/(2*b) + (E^(-a - b*x)*Hypergeometric2F1[1
, -b/(2*d), 1 - b/(2*d), -E^(2*(c + d*x))])/b - (E^(a + b*x)*Hypergeometric
2F1[1, b/(2*d), 1 + b/(2*d), -E^(2*(c + d*x))])/b
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 2251

```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_)*(G_)^((h_.)*((f_.
) + (g_.)*(x_))), x_Symbol] :> Simp[(a^p*G^(h*(f + g*x))*Hypergeometric2F1[
-p, (g*h*Log[G])/(d*e*Log[F]), (g*h*Log[G])/(d*e*Log[F]) + 1, Simplify[-((b
*F^(e*(c + d*x)))/a)]])/(g*h*Log[G]), x] /; FreeQ[{F, G, a, b, c, d, e, f,
g, h, p}, x] && (ILtQ[p, 0] || GtQ[a, 0])
```

Rule 5604

```
Int[Cosh[(a_.) + (b_.)*(x_)]*Tanh[(c_.) + (d_.)*(x_)], x_Symbol] :> Int[1/(
E^(a + b*x)*2) + E^(a + b*x)/2 - 1/(E^(a + b*x)*(1 + E^(2*(c + d*x)))) - E^
(a + b*x)/(1 + E^(2*(c + d*x))), x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 -
```

```
d^2, 0]
```

Rubi steps

$$\int \cosh(a+bx)\tanh(c+dx)\,dx = \int \left(\frac{1}{2}e^{-a-bx}+\frac{1}{2}e^{a+bx}-\frac{e^{-a-bx}}{1+e^{2(c+dx)}}-\frac{e^{a+bx}}{1+e^{2(c+dx)}}\right)dx$$
$$=\frac{1}{2}\int e^{-a-bx}\,dx+\frac{1}{2}\int e^{a+bx}\,dx-\int\frac{e^{-a-bx}}{1+e^{2(c+dx)}}\,dx-\int\frac{e^{a+bx}}{1+e^{2(c+dx)}}\,dx$$
$$=-\frac{e^{-a-bx}}{2b}+\frac{e^{a+bx}}{2b}+\frac{e^{-a-bx}\,{}_2F_1\left(1,-\frac{b}{2d};1-\frac{b}{2d};-e^{2(c+dx)}\right)}{b}-\frac{e^{a+bx}\,{}_2F_1\left(1,\frac{b}{2d};1+\right.}{b}$$

Mathematica [A] time = 11.71, size = 103, normalized size = 0.86

$$\frac{e^{-a-bx}\left(-2e^{2(a+bx)}\,{}_2F_1\left(1,\frac{b}{2d};\frac{b}{2d}+1;-e^{2(c+dx)}\right)+e^{2(a+bx)}+2\,{}_2F_1\left(1,-\frac{b}{2d};1-\frac{b}{2d};-e^{2(c+dx)}\right)-1\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[a + b*x]*Tanh[c + d*x],x]
```

```
[Out] (E^(-a - b*x)*(-1 + E^(2*(a + b*x)) + 2*Hypergeometric2F1[1, -1/2*b/d, 1 -
b/(2*d), -E^(2*(c + d*x))] - 2*E^(2*(a + b*x))*Hypergeometric2F1[1, b/(2*d)
, 1 + b/(2*d), -E^(2*(c + d*x))]))/(2*b)
```

fricas [F] time = 0.42, size = 0, normalized size = 0.00

$$\text{integral}\left(\cosh\left(bx+a\right)\tanh\left(dx+c\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*tanh(d*x+c),x, algorithm="fricas")
```

```
[Out] integral(cosh(b*x + a)*tanh(d*x + c), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \cosh\left(bx+a\right)\tanh\left(dx+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*tanh(d*x+c),x, algorithm="giac")
```

```
[Out] integrate(cosh(b*x + a)*tanh(d*x + c), x)
```

maple [F] time = 0.24, size = 0, normalized size = 0.00

$$\int \cosh (bx+a)\tanh (dx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)*tanh(d*x+c),x)
[Out] int(cosh(b*x+a)*tanh(d*x+c),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\left(e^{(2\,bx+2\,a)}-1\right)e^{(-bx-a)}}{2\,b}-\frac{1}{2}\int \frac{2\left(e^{(2\,bx+2\,a)}+1\right)}{e^{(bx+2\,dx+a+2\,c)}+e^{(bx+a)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*tanh(d*x+c),x, algorithm="maxima")
[Out] 1/2*(e^(2*b*x + 2*a) - 1)*e^(-b*x - a)/b - 1/2*integrate(2*(e^(2*b*x + 2*a)
 + 1)/(e^(b*x + 2*d*x + a + 2*c) + e^(b*x + a)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \cosh (a+b\,x)\,\tanh (c+d\,x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a + b*x)*tanh(c + d*x),x)
[Out] int(cosh(a + b*x)*tanh(c + d*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \cosh (a+bx)\tanh (c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*tanh(d*x+c),x)
[Out] Integral(cosh(a + b*x)*tanh(c + d*x), x)
```

3.192 $\int \sinh(x)\sinh(2x)\,dx$

Optimal. Leaf size=8

$$\frac{2\sinh^3(x)}{3}$$

[Out] 2/3*sinh(x)^3

Rubi [A] time = 0.01, antiderivative size = 15, normalized size of antiderivative = 1.88, number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4282}

$$\frac{1}{6}\sinh(3x) - \frac{\sinh(x)}{2}$$

Antiderivative was successfully verified.

[In] Int[Sinh[x]*Sinh[2*x],x]

[Out] -Sinh[x]/2 + Sinh[3*x]/6

Rule 4282

```
Int[sin[(a_.) + (b_.)*(x_)]*sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[
a - c + (b - d)*x]/(2*(b - d)), x] - Simp[Sin[a + c + (b + d)*x]/(2*(b + d)
), x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - d^2, 0]
```

Rubi steps

$$\int \sinh(x)\sinh(2x)\,dx = -\frac{\sinh(x)}{2} + \frac{1}{6}\sinh(3x)$$

Mathematica [A] time = 0.01, size = 15, normalized size = 1.88

$$\frac{1}{6}\sinh(3x) - \frac{\sinh(x)}{2}$$

Antiderivative was successfully verified.

[In] Integrate[Sinh[x]*Sinh[2*x],x]

[Out] -1/2*Sinh[x] + Sinh[3*x]/6

fricas [B] time = 0.40, size = 17, normalized size = 2.12

$$\frac{1}{6}\sinh(x)^3 + \frac{1}{2}\left(\cosh(x)^2 - 1\right)\sinh(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)*sinh(2*x),x, algorithm="fricas")
```

```
[Out] 1/6*sinh(x)^3 + 1/2*(cosh(x)^2 - 1)*sinh(x)
```

giac [B] time = 0.13, size = 25, normalized size = 3.12

$$\frac{1}{12}\left(3\,e^{(2\,x)}-1\right)e^{(-3\,x)}+\frac{1}{12}\,e^{(3\,x)}-\frac{1}{4}\,e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)*sinh(2*x),x, algorithm="giac")
```

```
[Out] 1/12*(3*e^(2*x) - 1)*e^(-3*x) + 1/12*e^(3*x) - 1/4*e^x
```

maple [A] time = 0.09, size = 7, normalized size = 0.88

$$\frac{2\left(\sinh^{3}(x)\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)*sinh(2*x),x)
```

```
[Out] 2/3*sinh(x)^3
```

maxima [B] time = 0.45, size = 27, normalized size = 3.38

$$-\frac{1}{12}\left(3\,e^{(-2\,x)}-1\right)e^{(3\,x)}+\frac{1}{4}\,e^{(-x)}-\frac{1}{12}\,e^{(-3\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)*sinh(2*x),x, algorithm="maxima")
```

```
[Out] -1/12*(3*e^(-2*x) - 1)*e^(3*x) + 1/4*e^(-x) - 1/12*e^(-3*x)
```

mupad [B] time = 0.07, size = 6, normalized size = 0.75

$$\frac{2\,\sinh(x)^{3}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(2*x)*sinh(x),x)
```

```
[Out] (2*sinh(x)^3)/3
```

sympy [B] time = 0.44, size = 20, normalized size = 2.50

$$\frac{2\sinh(x)\cosh(2x)}{3} - \frac{\sinh(2x)\cosh(x)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)*sinh(2*x),x)
```

```
[Out] 2*sinh(x)*cosh(2*x)/3 - sinh(2*x)*cosh(x)/3
```

3.193 $\int \sinh(x)\sinh(3x)\,dx$

Optimal. Leaf size=17

$$\frac{1}{8}\sinh(4x) - \frac{1}{4}\sinh(2x)$$

```
[Out] -1/4*sinh(2*x)+1/8*sinh(4*x)
```

Rubi [A] time = 0.01, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4282}

$$\frac{1}{8}\sinh(4x) - \frac{1}{4}\sinh(2x)$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[x]*Sinh[3*x],x]
```

```
[Out] -Sinh[2*x]/4 + Sinh[4*x]/8
```

Rule 4282

```
Int[sin[(a_.) + (b_.)*(x_)]*sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[
a - c + (b - d)*x]/(2*(b - d)), x] - Simp[Sin[a + c + (b + d)*x]/(2*(b + d)
), x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - d^2, 0]
```

Rubi steps

$$\int \sinh(x)\sinh(3x)\,dx = -\frac{1}{4}\sinh(2x) + \frac{1}{8}\sinh(4x)$$

Mathematica [A] time = 0.01, size = 17, normalized size = 1.00

$$\frac{1}{8}\sinh(4x) - \frac{1}{4}\sinh(2x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[x]*Sinh[3*x],x]
```

```
[Out] -1/4*Sinh[2*x] + Sinh[4*x]/8
```

fricas [A] time = 0.40, size = 22, normalized size = 1.29

$$\frac{1}{2}\cosh(x)\sinh(x)^3 + \frac{1}{2}\left(\cosh(x)^3 - \cosh(x)\right)\sinh(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)*sinh(3*x),x, algorithm="fricas")
```

```
[Out] 1/2*cosh(x)*sinh(x)^3 + 1/2*(cosh(x)^3 - cosh(x))*sinh(x)
```

giac [B] time = 0.13, size = 27, normalized size = 1.59

$$\frac{1}{16}\left(2e^{(2x)}-1\right)e^{(-4x)}+\frac{1}{16}e^{(4x)}-\frac{1}{8}e^{(2x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)*sinh(3*x),x, algorithm="giac")
```

```
[Out] 1/16*(2*e^(2*x) - 1)*e^(-4*x) + 1/16*e^(4*x) - 1/8*e^(2*x)
```

maple [A] time = 0.19, size = 14, normalized size = 0.82

$$-\frac{\sinh(2x)}{4}+\frac{\sinh(4x)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)*sinh(3*x),x)
```

```
[Out] -1/4*sinh(2*x)+1/8*sinh(4*x)
```

maxima [B] time = 0.30, size = 27, normalized size = 1.59

$$-\frac{1}{16}\left(2e^{(-2x)}-1\right)e^{(4x)}+\frac{1}{8}e^{(-2x)}-\frac{1}{16}e^{(-4x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)*sinh(3*x),x, algorithm="maxima")
```

```
[Out] -1/16*(2*e^(-2*x) - 1)*e^(4*x) + 1/8*e^(-2*x) - 1/16*e^(-4*x)
```

mupad [B] time = 0.06, size = 13, normalized size = 0.76

$$\frac{\sinh(4x)}{8}-\frac{\sinh(2x)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(3*x)*sinh(x),x)
```

```
[Out] sinh(4*x)/8 - sinh(2*x)/4
```

sympy [A] time = 0.41, size = 20, normalized size = 1.18

$$\frac{3\sinh(x)\cosh(3x)}{8} - \frac{\sinh(3x)\cosh(x)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)*sinh(3*x),x)
```

```
[Out] 3*sinh(x)*cosh(3*x)/8 - sinh(3*x)*cosh(x)/8
```

3.194 $\int \sinh(x)\sinh(4x)\,dx$

Optimal. Leaf size=17

$$\frac{1}{10}\sinh(5x)-\frac{1}{6}\sinh(3x)$$

```
[Out] -1/6*sinh(3*x)+1/10*sinh(5*x)
```

Rubi [A] time = 0.01, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4282}

$$\frac{1}{10}\sinh(5x)-\frac{1}{6}\sinh(3x)$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[x]*Sinh[4*x],x]
```

```
[Out] -Sinh[3*x]/6 + Sinh[5*x]/10
```

Rule 4282

```
Int[sin[(a_.) + (b_.)*(x_)]*sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[
a - c + (b - d)*x]/(2*(b - d)), x] - Simp[Sin[a + c + (b + d)*x]/(2*(b + d)
), x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - d^2, 0]
```

Rubi steps

$$\int \sinh(x)\sinh(4x)\,dx=-\frac{1}{6}\sinh(3x)+\frac{1}{10}\sinh(5x)$$

Mathematica [A] time = 0.01, size = 17, normalized size = 1.00

$$\frac{1}{10}\sinh(5x)-\frac{1}{6}\sinh(3x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[x]*Sinh[4*x],x]
```

```
[Out] -1/6*Sinh[3*x] + Sinh[5*x]/10
```

fricas [B] time = 0.41, size = 36, normalized size = 2.12

$$\frac{1}{10}\sinh(x)^5+\frac{1}{6}\left(6\cosh(x)^2-1\right)\sinh(x)^3+\frac{1}{2}\left(\cosh(x)^4-\cosh(x)^2\right)\sinh(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)*sinh(4*x),x, algorithm="fricas")
```

```
[Out] 1/10*sinh(x)^5 + 1/6*(6*cosh(x)^2 - 1)*sinh(x)^3 + 1/2*(cosh(x)^4 - cosh(x)
^2)*sinh(x)
```

giac [B] time = 0.13, size = 27, normalized size = 1.59

$$\frac{1}{60}\left(5\,e^{(2\,x)}-3\right)e^{(-5\,x)}+\frac{1}{20}\,e^{(5\,x)}-\frac{1}{12}\,e^{(3\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)*sinh(4*x),x, algorithm="giac")
```

```
[Out] 1/60*(5*e^(2*x) - 3)*e^(-5*x) + 1/20*e^(5*x) - 1/12*e^(3*x)
```

maple [A] time = 0.16, size = 14, normalized size = 0.82

$$-\frac{\sinh\left(3x\right)}{6}+\frac{\sinh\left(5x\right)}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)*sinh(4*x),x)
```

```
[Out] -1/6*sinh(3*x)+1/10*sinh(5*x)
```

maxima [B] time = 0.48, size = 27, normalized size = 1.59

$$-\frac{1}{60}\left(5\,e^{(-2\,x)}-3\right)e^{(5\,x)}+\frac{1}{12}\,e^{(-3\,x)}-\frac{1}{20}\,e^{(-5\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)*sinh(4*x),x, algorithm="maxima")
```

```
[Out] -1/60*(5*e^(-2*x) - 3)*e^(5*x) + 1/12*e^(-3*x) - 1/20*e^(-5*x)
```

mupad [B] time = 0.06, size = 14, normalized size = 0.82

$$\frac{4\sinh(x)^3\,\left(6\sinh(x)^2+5\right)}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(4*x)*sinh(x),x)
```

```
[Out] (4*sinh(x)^3*(6*sinh(x)^2 + 5))/15
```

sympy **[A]** time = 0.41, size = 20, normalized size = 1.18

$$\frac{4\sinh\left(x\right)\cosh\left(4x\right)}{15}-\frac{\sinh\left(4x\right)\cosh\left(x\right)}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)*sinh(4*x),x)
[Out] 4*sinh(x)*cosh(4*x)/15 - sinh(4*x)*cosh(x)/15
```

3.195 $\int \sinh(x)\sinh(mx)\,dx$

Optimal. Leaf size=35

$$\frac{\sinh((m+1)x)}{2(m+1)}-\frac{\sinh((1-m)x)}{2(1-m)}$$

```
[Out] -1/2*sinh((1-m)*x)/(1-m)+1/2*sinh((1+m)*x)/(1+m)
```

Rubi [A] time = 0.03, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {5613, 2637}

$$\frac{\sinh((m+1)x)}{2(m+1)}-\frac{\sinh((1-m)x)}{2(1-m)}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[x]*Sinh[m*x],x]
```

```
[Out] -Sinh[(1 - m)*x]/(2*(1 - m)) + Sinh[(1 + m)*x]/(2*(1 + m))
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 5613

```
Int[Sinh[v_]^(p_.)*Sinh[w_]^(q_.), x_Symbol] :> Int[ExpandTrigReduce[Sinh[v
]^p*Sinh[w]^q, x], x] /; IGtQ[p, 0] && IGtQ[q, 0] && ((PolynomialQ[v, x] &&
 PolynomialQ[w, x]) || (BinomialQ[{v, w}, x] && IndependentQ[Cancel[v/w], x
]))
```

Rubi steps

$$\begin{aligned}\int \sinh(x)\sinh(mx)\,dx &= \int \left(-\frac{1}{2}\cosh((1-m)x)+\frac{1}{2}\cosh((1+m)x)\right)dx\\ &= -\left(\frac{1}{2}\int \cosh((1-m)x)\,dx\right)+\frac{1}{2}\int \cosh((1+m)x)\,dx\\ &= -\frac{\sinh((1-m)x)}{2(1-m)}+\frac{\sinh((1+m)x)}{2(1+m)}\end{aligned}$$

Mathematica [A] time = 0.04, size = 25, normalized size = 0.71

$$\frac{m\sinh(x)\cosh(mx)-\cosh(x)\sinh(mx)}{m^2-1}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[x]*Sinh[m*x],x]
[Out] (m*Cosh[m*x]*Sinh[x] - Cosh[x]*Sinh[m*x])/(-1 + m^2)
```

fricas [A] time = 0.41, size = 42, normalized size = 1.20

$$\frac{m\cosh(mx)\sinh(x)-\cosh(x)\sinh(mx)}{\left(m^2-1\right)\cosh(x)^2-\left(m^2-1\right)\sinh(x)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)*sinh(m*x),x, algorithm="fricas")
[Out] (m*cosh(m*x)*sinh(x) - cosh(x)*sinh(m*x))/((m^2 - 1)*cosh(x)^2 - (m^2 - 1)*
sinh(x)^2)
```

giac [B] time = 0.12, size = 59, normalized size = 1.69

$$\frac{e^{(mx+x)}}{4(m+1)}-\frac{e^{(mx-x)}}{4(m-1)}+\frac{e^{(-mx+x)}}{4(m-1)}-\frac{e^{(-mx-x)}}{4(m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)*sinh(m*x),x, algorithm="giac")
[Out] 1/4*e^(m*x + x)/(m + 1) - 1/4*e^(m*x - x)/(m - 1) + 1/4*e^(-m*x + x)/(m - 1
) - 1/4*e^(-m*x - x)/(m + 1)
```

maple [A] time = 0.17, size = 28, normalized size = 0.80

$$-\frac{\sinh((-1+m)x)}{2(-1+m)}+\frac{\sinh((1+m)x)}{2+2m}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)*sinh(m*x),x)
[Out] -1/2/(-1+m)*sinh((-1+m)*x)+1/2*sinh((1+m)*x)/(1+m)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)*sinh(m*x),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(m-2>0)', see `assume?` for more det
ails)Is m-2 equal to -1?
```

mupad [B] time = 0.09, size = 26, normalized size = 0.74

$$-\frac{\sinh(m\,x)\,\cosh(x)-m\cosh(m\,x)\,\sinh(x)}{m^2-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(m*x)*sinh(x),x)
```

```
[Out] -(sinh(m*x)*cosh(x) - m*cosh(m*x)*sinh(x))/(m^2 - 1)
```

sympy [A] time = 0.77, size = 78, normalized size = 2.23

$$\begin{cases} -\frac{x\sinh^2(x)}{2}+\frac{x\cosh^2(x)}{2}-\frac{\sinh(x)\cosh(x)}{2} & \text{for } m=-1 \\ \frac{x\sinh^2(x)}{2}-\frac{x\cosh^2(x)}{2}+\frac{\sinh(x)\cosh(x)}{2} & \text{for } m=1 \\ \frac{m\sinh(x)\cosh(mx)}{m^2-1}-\frac{\sinh(mx)\cosh(x)}{m^2-1} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)*sinh(m*x),x)
```

```
[Out] Piecewise((-x*sinh(x)**2/2 + x*cosh(x)**2/2 - sinh(x)*cosh(x)/2, Eq(m, -1))
, (x*sinh(x)**2/2 - x*cosh(x)**2/2 + sinh(x)*cosh(x)/2, Eq(m, 1)), (m*sinh(
x)*cosh(m*x)/(m**2 - 1) - sinh(m*x)*cosh(x)/(m**2 - 1), True))
```

3.196 $\int \cosh(2x)\sinh(x)\,dx$

Optimal. Leaf size=15

$$\frac{1}{6}\cosh(3x) - \frac{\cosh(x)}{2}$$

```
[Out] -1/2*cosh(x)+1/6*cosh(3*x)
```

Rubi [A] time = 0.01, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4284}

$$\frac{1}{6}\cosh(3x) - \frac{\cosh(x)}{2}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[2*x]*Sinh[x],x]
```

```
[Out] -Cosh[x]/2 + Cosh[3*x]/6
```

Rule 4284

```
Int[cos[(c_.) + (d_.)*(x_)]*sin[(a_.) + (b_.)*(x_)], x_Symbol] :> -Simp[Cos
[a - c + (b - d)*x]/(2*(b - d)), x] - Simp[Cos[a + c + (b + d)*x]/(2*(b + d
)), x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - d^2, 0]
```

Rubi steps

$$\int \cosh(2x)\sinh(x)\,dx = -\frac{\cosh(x)}{2} + \frac{1}{6}\cosh(3x)$$

Mathematica [A] time = 0.01, size = 15, normalized size = 1.00

$$\frac{1}{6}\cosh(3x) - \frac{\cosh(x)}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[2*x]*Sinh[x],x]
```

```
[Out] -1/2*Cosh[x] + Cosh[3*x]/6
```

fricas [A] time = 0.40, size = 19, normalized size = 1.27

$$\frac{1}{6}\cosh(x)^3 + \frac{1}{2}\cosh(x)\sinh(x)^2 - \frac{1}{2}\cosh(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(2*x)*sinh(x),x, algorithm="fricas")
[Out] 1/6*cosh(x)^3 + 1/2*cosh(x)*sinh(x)^2 - 1/2*cosh(x)
```

giac [B] time = 0.11, size = 25, normalized size = 1.67

$$-\frac{1}{12}\left(3\,e^{(2\,x)}-1\right)e^{(-3\,x)}+\frac{1}{12}\,e^{(3\,x)}-\frac{1}{4}\,e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(2*x)*sinh(x),x, algorithm="giac")
[Out] -1/12*(3*e^(2*x) - 1)*e^(-3*x) + 1/12*e^(3*x) - 1/4*e^x
```

maple [A] time = 0.13, size = 12, normalized size = 0.80

$$-\frac{\cosh(x)}{2}+\frac{\cosh\left(3x\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(2*x)*sinh(x),x)
[Out] -1/2*cosh(x)+1/6*cosh(3*x)
```

maxima [B] time = 0.30, size = 27, normalized size = 1.80

$$-\frac{1}{12}\left(3\,e^{(-2\,x)}-1\right)e^{(3\,x)}-\frac{1}{4}\,e^{(-x)}+\frac{1}{12}\,e^{(-3\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(2*x)*sinh(x),x, algorithm="maxima")
[Out] -1/12*(3*e^(-2*x) - 1)*e^(3*x) - 1/4*e^(-x) + 1/12*e^(-3*x)
```

mupad [B] time = 1.42, size = 11, normalized size = 0.73

$$\frac{2\cosh(x)^3}{3}-\cosh(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(2*x)*sinh(x),x)
[Out] (2*cosh(x)^3)/3 - cosh(x)
```

sympy [A] time = 0.45, size = 20, normalized size = 1.33

$$\frac{2\sinh(x)\sinh(2x)}{3} - \frac{\cosh(x)\cosh(2x)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(2*x)*sinh(x),x)
```

```
[Out] 2*sinh(x)*sinh(2*x)/3 - cosh(x)*cosh(2*x)/3
```

3.197 $\int \cosh(3x)\sinh(x)\,dx$

Optimal. Leaf size=17

$$\frac{1}{8}\cosh(4x)-\frac{1}{4}\cosh(2x)$$

```
[Out] -1/4*cosh(2*x)+1/8*cosh(4*x)
```

Rubi [A] time = 0.01, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4284}

$$\frac{1}{8}\cosh(4x)-\frac{1}{4}\cosh(2x)$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[3*x]*Sinh[x],x]
[Out] -Cosh[2*x]/4 + Cosh[4*x]/8
```

Rule 4284

```
Int[cos[(c_.) + (d_.)*(x_)]*sin[(a_.) + (b_.)*(x_)], x_Symbol] :> -Simp[Cos
[a - c + (b - d)*x]/(2*(b - d)), x] - Simp[Cos[a + c + (b + d)*x]/(2*(b + d
)), x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - d^2, 0]
```

Rubi steps

$$\int \cosh(3x)\sinh(x)\,dx=-\frac{1}{4}\cosh(2x)+\frac{1}{8}\cosh(4x)$$

Mathematica [A] time = 0.01, size = 17, normalized size = 1.00

$$\frac{1}{8}\cosh(4x)-\frac{\cosh^2(x)}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[3*x]*Sinh[x],x]
[Out] -1/2*Cosh[x]^2 + Cosh[4*x]/8
```

fricas [B] time = 0.46, size = 33, normalized size = 1.94

$$\frac{1}{8}\cosh(x)^4 + \frac{1}{8}\sinh(x)^4 + \frac{1}{4}\left(3\cosh(x)^2 - 1\right)\sinh(x)^2 - \frac{1}{4}\cosh(x)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(3*x)*sinh(x),x, algorithm="fricas")
[Out] 1/8*cosh(x)^4 + 1/8*sinh(x)^4 + 1/4*(3*cosh(x)^2 - 1)*sinh(x)^2 - 1/4*cosh(
x)^2
```

giac [A] time = 0.11, size = 26, normalized size = 1.53

$$\frac{1}{16}\left(e^{(2x)} + e^{(-2x)}\right)^2 - \frac{1}{8}e^{(2x)} - \frac{1}{8}e^{(-2x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(3*x)*sinh(x),x, algorithm="giac")
[Out] 1/16*(e^(2*x) + e^(-2*x))^2 - 1/8*e^(2*x) - 1/8*e^(-2*x)
```

maple [A] time = 0.16, size = 14, normalized size = 0.82

$$-\frac{\cosh(2x)}{4} + \frac{\cosh(4x)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(3*x)*sinh(x),x)
[Out] -1/4*cosh(2*x)+1/8*cosh(4*x)
```

maxima [B] time = 0.30, size = 27, normalized size = 1.59

$$-\frac{1}{16}\left(2e^{(-2x)} - 1\right)e^{(4x)} - \frac{1}{8}e^{(-2x)} + \frac{1}{16}e^{(-4x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(3*x)*sinh(x),x, algorithm="maxima")
[Out] -1/16*(2*e^(-2*x) - 1)*e^(4*x) - 1/8*e^(-2*x) + 1/16*e^(-4*x)
```

mupad [B] time = 0.06, size = 11, normalized size = 0.65

$$\sinh(x)^4 + \frac{\sinh(x)^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(3*x)*sinh(x),x)
```

```
[Out] sinh(x)^2/2 + sinh(x)^4
```

sympy [A] time = 0.41, size = 20, normalized size = 1.18

$$\frac{3\sinh\left(x\right)\sinh\left(3x\right)}{8}-\frac{\cosh\left(x\right)\cosh\left(3x\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(3*x)*sinh(x),x)
```

```
[Out] 3*sinh(x)*sinh(3*x)/8 - cosh(x)*cosh(3*x)/8
```

3.198 $\int \cosh(4x)\sinh(x)\,dx$

Optimal. Leaf size=17

$$\frac{1}{10}\cosh(5x)-\frac{1}{6}\cosh(3x)$$

```
[Out] -1/6*cosh(3*x)+1/10*cosh(5*x)
```

Rubi [A] time = 0.01, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4284}

$$\frac{1}{10}\cosh(5x)-\frac{1}{6}\cosh(3x)$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[4*x]*Sinh[x],x]
```

```
[Out] -Cosh[3*x]/6 + Cosh[5*x]/10
```

Rule 4284

```
Int[cos[(c_.) + (d_.)*(x_)]*sin[(a_.) + (b_.)*(x_)], x_Symbol] :> -Simp[Cos
[a - c + (b - d)*x]/(2*(b - d)), x] - Simp[Cos[a + c + (b + d)*x]/(2*(b + d
)), x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - d^2, 0]
```

Rubi steps

$$\int \cosh(4x)\sinh(x)\,dx=-\frac{1}{6}\cosh(3x)+\frac{1}{10}\cosh(5x)$$

Mathematica [A] time = 0.01, size = 17, normalized size = 1.00

$$\frac{1}{10}\cosh(5x)-\frac{1}{6}\cosh(3x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[4*x]*Sinh[x],x]
```

```
[Out] -1/6*Cosh[3*x] + Cosh[5*x]/10
```

fricas [B] time = 0.41, size = 38, normalized size = 2.24

$$\frac{1}{10}\cosh(x)^5+\frac{1}{2}\cosh(x)\sinh(x)^4-\frac{1}{6}\cosh(x)^3+\frac{1}{2}\left(2\cosh(x)^3-\cosh(x)\right)\sinh(x)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(4*x)*sinh(x),x, algorithm="fricas")
```

```
[Out] 1/10*cosh(x)^5 + 1/2*cosh(x)*sinh(x)^4 - 1/6*cosh(x)^3 + 1/2*(2*cosh(x)^3 -
 cosh(x))*sinh(x)^2
```

giac [B] time = 0.11, size = 27, normalized size = 1.59

$$-\frac{1}{60}\left(5\,e^{(2\,x)}-3\right)e^{(-5\,x)}+\frac{1}{20}\,e^{(5\,x)}-\frac{1}{12}\,e^{(3\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(4*x)*sinh(x),x, algorithm="giac")
```

```
[Out] -1/60*(5*e^(2*x) - 3)*e^(-5*x) + 1/20*e^(5*x) - 1/12*e^(3*x)
```

maple [A] time = 0.20, size = 14, normalized size = 0.82

$$-\frac{\cosh\left(3x\right)}{6}+\frac{\cosh\left(5x\right)}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(4*x)*sinh(x),x)
```

```
[Out] -1/6*cosh(3*x)+1/10*cosh(5*x)
```

maxima [B] time = 0.33, size = 27, normalized size = 1.59

$$-\frac{1}{60}\left(5\,e^{(-2\,x)}-3\right)e^{(5\,x)}-\frac{1}{12}\,e^{(-3\,x)}+\frac{1}{20}\,e^{(-5\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(4*x)*sinh(x),x, algorithm="maxima")
```

```
[Out] -1/60*(5*e^(-2*x) - 3)*e^(5*x) - 1/12*e^(-3*x) + 1/20*e^(-5*x)
```

mupad [B] time = 1.43, size = 15, normalized size = 0.88

$$\frac{8\cosh(x)^5}{5}-\frac{8\cosh(x)^3}{3}+\cosh(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(4*x)*sinh(x),x)
```

```
[Out] cosh(x) - (8*cosh(x)^3)/3 + (8*cosh(x)^5)/5
```

sympy [A] time = 0.41, size = 20, normalized size = 1.18

$$\frac{4\sinh(x)\sinh(4x)}{15}-\frac{\cosh(x)\cosh(4x)}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(4*x)*sinh(x),x)
```

```
[Out] 4*sinh(x)*sinh(4*x)/15 - cosh(x)*cosh(4*x)/15
```

3.199 $\int \cosh(mx)\sinh(x)\,dx$

Optimal. Leaf size=35

$$\frac{\cosh((1-m)x)}{2(1-m)}+\frac{\cosh((m+1)x)}{2(m+1)}$$

```
[Out] 1/2*cosh((1-m)*x)/(1-m)+1/2*cosh((1+m)*x)/(1+m)
```

Rubi [A] time = 0.03, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {5618, 2638}

$$\frac{\cosh((1-m)x)}{2(1-m)}+\frac{\cosh((m+1)x)}{2(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[m*x]*Sinh[x],x]
```

```
[Out] Cosh[(1 - m)*x]/(2*(1 - m)) + Cosh[(1 + m)*x]/(2*(1 + m))
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 5618

```
Int[Cosh[w_]^(q_.)*Sinh[v_]^(p_.), x_Symbol] :> Int[ExpandTrigReduce[Sinh[v
]^p*Cosh[w]^q, x], x] /; IGtQ[p, 0] && IGtQ[q, 0] && ((PolynomialQ[v, x] &&
 PolynomialQ[w, x]) || (BinomialQ[{v, w}, x] && IndependentQ[Cancel[v/w], x
]))
```

Rubi steps

$$\begin{aligned}\int \cosh(mx)\sinh(x)\,dx &= \int \left(\frac{1}{2}\sinh((1-m)x)+\frac{1}{2}\sinh((1+m)x)\right)dx\\ &= \frac{1}{2}\int \sinh((1-m)x)\,dx+\frac{1}{2}\int \sinh((1+m)x)\,dx\\ &= \frac{\cosh((1-m)x)}{2(1-m)}+\frac{\cosh((1+m)x)}{2(1+m)}\end{aligned}$$

Mathematica [A] time = 0.04, size = 25, normalized size = 0.71

$$\frac{m\sinh(x)\sinh(mx)-\cosh(x)\cosh(mx)}{m^2-1}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[m*x]*Sinh[x],x]
```

```
[Out] (-(Cosh[x]*Cosh[m*x]) + m*Sinh[x]*Sinh[m*x])/(-1 + m^2)
```

fricas [A] time = 0.49, size = 42, normalized size = 1.20

$$\frac{m\sinh(mx)\sinh(x)-\cosh(mx)\cosh(x)}{\left(m^2-1\right)\cosh(x)^2-\left(m^2-1\right)\sinh(x)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(m*x)*sinh(x),x, algorithm="fricas")
```

```
[Out] (m*sinh(m*x)*sinh(x) - cosh(m*x)*cosh(x))/((m^2 - 1)*cosh(x)^2 - (m^2 - 1)*
sinh(x)^2)
```

giac [B] time = 0.12, size = 59, normalized size = 1.69

$$\frac{e^{(mx+x)}}{4(m+1)}-\frac{e^{(mx-x)}}{4(m-1)}-\frac{e^{(-mx+x)}}{4(m-1)}+\frac{e^{(-mx-x)}}{4(m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(m*x)*sinh(x),x, algorithm="giac")
```

```
[Out] 1/4*e^(m*x + x)/(m + 1) - 1/4*e^(m*x - x)/(m - 1) - 1/4*e^(-m*x + x)/(m - 1
) + 1/4*e^(-m*x - x)/(m + 1)
```

maple [A] time = 0.07, size = 28, normalized size = 0.80

$$-\frac{\cosh((-1+m)x)}{2(-1+m)}+\frac{\cosh((1+m)x)}{2+2m}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(m*x)*sinh(x),x)
```

```
[Out] -1/2/(-1+m)*cosh((-1+m)*x)+1/2*cosh((1+m)*x)/(1+m)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(m*x)*sinh(x),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(m-2>0)', see `assume?` for more det
ails)Is m-2 equal to -1?
```

mupad [B] time = 1.44, size = 26, normalized size = 0.74

$$-\frac{\cosh(m\,x)\,\cosh(x)-m\sinh(m\,x)\,\sinh(x)}{m^2-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(m*x)*sinh(x),x)
```

```
[Out] -(cosh(m*x)*cosh(x) - m*sinh(m*x)*sinh(x))/(m^2 - 1)
```

sympy [A] time = 0.90, size = 37, normalized size = 1.06

$$\begin{cases} \frac{\cosh^2(x)}{2} & \text{for } m=-1 \vee m=1 \\ \frac{m\sinh(x)\sinh(mx)}{m^2-1}-\frac{\cosh(x)\cosh(mx)}{m^2-1} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(m*x)*sinh(x),x)
```

```
[Out] Piecewise((cosh(x)**2/2, Eq(m, -1) | Eq(m, 1)), (m*sinh(x)*sinh(m*x)/(m**2
- 1) - cosh(x)*cosh(m*x)/(m**2 - 1), True))
```

3.200 $\int \sinh(x)\tanh(2x)\,dx$

Optimal. Leaf size=19

$$\sinh(x)-\frac{\tan^{-1}\left(\sqrt{2}\sinh(x)\right)}{\sqrt{2}}$$

[Out] sinh(x)-1/2*arctan(sinh(x)*2^(1/2))*2^(1/2)

Rubi [A] time = 0.03, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {12, 321, 203}

$$\sinh(x)-\frac{\tan^{-1}\left(\sqrt{2}\sinh(x)\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Int[Sinh[x]*Tanh[2*x],x]

[Out] -(ArcTan[Sqrt[2]*Sinh[x]]/Sqrt[2]) + Sinh[x]

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}\int \sinh(x)\tanh(2x)\,dx &= -\operatorname{Subst}\left(\int -\frac{2x^2}{1+2x^2}\,dx, x, \sinh(x)\right)\\ &= 2\operatorname{Subst}\left(\int \frac{x^2}{1+2x^2}\,dx, x, \sinh(x)\right)\\ &= \sinh(x) - \operatorname{Subst}\left(\int \frac{1}{1+2x^2}\,dx, x, \sinh(x)\right)\\ &= -\frac{\tan^{-1}\left(\sqrt{2}\sinh(x)\right)}{\sqrt{2}} + \sinh(x)\end{aligned}$$

Mathematica [A] time = 0.02, size = 19, normalized size = 1.00

$$\sinh(x) - \frac{\tan^{-1}\left(\sqrt{2}\sinh(x)\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[x]*Tanh[2*x],x]
[Out] -(ArcTan[Sqrt[2]*Sinh[x]]/Sqrt[2]) + Sinh[x]
```

fricas [B] time = 0.43, size = 115, normalized size = 6.05

$$-\frac{\left(\sqrt{2}\cosh(x)+\sqrt{2}\sinh(x)\right)\arctan\left(\frac{1}{2}\sqrt{2}\cosh(x)+\frac{1}{2}\sqrt{2}\sinh(x)\right)-\left(\sqrt{2}\cosh(x)+\sqrt{2}\sinh(x)\right)\arctan\left(-\cdots\right.}{2\left(\cosh(x)+\sinh(x)\right.\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)*tanh(2*x),x, algorithm="fricas")
[Out] -1/2*((sqrt(2)*cosh(x) + sqrt(2)*sinh(x))*arctan(1/2*sqrt(2)*cosh(x) + 1/2*
sqrt(2)*sinh(x)) - (sqrt(2)*cosh(x) + sqrt(2)*sinh(x))*arctan(-1/2*(sqrt(2)
*cosh(x)^2 + 2*sqrt(2)*cosh(x)*sinh(x) + sqrt(2)*sinh(x)^2 + sqrt(2))/(cosh
(x) - sinh(x))) - cosh(x)^2 - 2*cosh(x)*sinh(x) - sinh(x)^2 + 1)/(cosh(x) +
 sinh(x))
```

giac [B] time = 0.13, size = 36, normalized size = 1.89

$$-\frac{1}{4}\sqrt{2}\left(\pi + 2\arctan\left(\frac{1}{2}\sqrt{2}\left(e^{(2x)}-1\right)e^{(-x)}\right)\right) - \frac{1}{2}e^{(-x)} + \frac{1}{2}e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)*tanh(2*x),x, algorithm="giac")
```

```
[Out] -1/4*sqrt(2)*(pi + 2*arctan(1/2*sqrt(2)*(e^(2*x) - 1)*e^(-x))) - 1/2*e^(-x)
 + 1/2*e^x
```

maple [C] time = 0.26, size = 54, normalized size = 2.84

$$\frac{e^{x}}{2}-\frac{e^{-x}}{2}+\frac{i\sqrt{2}\,\ln\left(e^{2x}-i\sqrt{2}\,e^{x}-1\right)}{4}-\frac{i\sqrt{2}\,\ln\left(e^{2x}+i\sqrt{2}\,e^{x}-1\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)*tanh(2*x),x)
```

```
[Out] 1/2*exp(x)-1/2*exp(-x)+1/4*I*2^(1/2)*ln(exp(2*x)-I*2^(1/2)*exp(x)-1)-1/4*I*
2^(1/2)*ln(exp(2*x)+I*2^(1/2)*exp(x)-1)
```

maxima [B] time = 0.40, size = 53, normalized size = 2.79

$$\frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2e^{(-x)}\right)\right)+\frac{1}{2}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2e^{(-x)}\right)\right)-\frac{1}{2}e^{(-x)}+\frac{1}{2}e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)*tanh(2*x),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*e^(-x))) + 1/2*sqrt(2)*arctan(-
1/2*sqrt(2)*(sqrt(2) - 2*e^(-x))) - 1/2*e^(-x) + 1/2*e^x
```

mupad [B] time = 1.47, size = 47, normalized size = 2.47

$$\frac{e^{x}}{2}-\frac{e^{-x}}{2}-\frac{\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,e^{x}}{2}+\frac{\sqrt{2}\,e^{3x}}{2}\right)}{2}-\frac{\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,e^{x}}{2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(2*x)*sinh(x),x)
```

```
[Out] exp(x)/2 - exp(-x)/2 - (2^(1/2)*atan((2^(1/2)*exp(x))/2 + (2^(1/2)*exp(3*x)
)/2))/2 - (2^(1/2)*atan((2^(1/2)*exp(x))/2))/2
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sinh(x)\tanh(2x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)*tanh(2*x),x)
```

```
[Out] Integral(sinh(x)*tanh(2*x), x)
```

3.201 $\int \sinh(x)\tanh(3x)\,dx$

Optimal. Leaf size=19

$$\sinh(x)-\frac{1}{3}\tan^{-1}(\sinh(x))-\frac{1}{3}\tan^{-1}(2\sinh(x))$$

```
[Out] -1/3*arctan(sinh(x))-1/3*arctan(2*sinh(x))+sinh(x)
```

Rubi [A] time = 0.04, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {1279, 1163, 203}

$$\sinh(x)-\frac{1}{3}\tan^{-1}(\sinh(x))-\frac{1}{3}\tan^{-1}(2\sinh(x))$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[x]*Tanh[3*x],x]
```

```
[Out] -ArcTan[Sinh[x]]/3 - ArcTan[2*Sinh[x]]/3 + Sinh[x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1163

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Eq
Q[c*d^2 - a*e^2, 0] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1279

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[(e*f*(f*x)^(m - 1)*(a + b*x^2 + c*x^4)^(p +
1))/(c*(m + 4*p + 3)), x] - Dist[f^2/(c*(m + 4*p + 3)), Int[(f*x)^(m - 2)*(
a + b*x^2 + c*x^4)^p*Simp[a*e*(m - 1) + (b*e*(m + 2*p + 1) - c*d*(m + 4*p +
 3))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c,
 0] && GtQ[m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] ||
 IntegerQ[m])
```

Rubi steps

$$
\begin{aligned}
\int \sinh(x)\tanh(3x)\,dx &= -\operatorname{Subst}\left(\int \frac{x^2\left(-3-4x^2\right)}{1+5x^2+4x^4}\,dx, x, \sinh(x)\right)\\
&= \sinh(x) + \frac{1}{4}\operatorname{Subst}\left(\int \frac{-4-8x^2}{1+5x^2+4x^4}\,dx, x, \sinh(x)\right)\\
&= \sinh(x) - \frac{2}{3}\operatorname{Subst}\left(\int \frac{1}{1+4x^2}\,dx, x, \sinh(x)\right) - \frac{4}{3}\operatorname{Subst}\left(\int \frac{1}{4+4x^2}\,dx, x, \sinh(x)\right)\\
&= -\frac{1}{3}\tan^{-1}(\sinh(x)) - \frac{1}{3}\tan^{-1}(2\sinh(x)) + \sinh(x)
\end{aligned}
$$

Mathematica [A] time = 0.03, size = 19, normalized size = 1.00

$$\sinh(x) - \frac{1}{3}\tan^{-1}(\sinh(x)) - \frac{1}{3}\tan^{-1}(2\sinh(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[x]*Tanh[3*x],x]
```

```
[Out] -1/3*ArcTan[Sinh[x]] - ArcTan[2*Sinh[x]]/3 + Sinh[x]
```

fricas [B] time = 0.52, size = 76, normalized size = 4.00

$$\frac{2\left(\cosh(x)+\sinh(x)\right)\arctan\left(-\frac{\cosh(x)^2+2\cosh(x)\sinh(x)+\sinh(x)^2}{\cosh(x)-\sinh(x)}\right) - 6\left(\cosh(x)+\sinh(x)\right)\arctan\left(\cosh(x)+\sinh(x\right.}{6\left(\cosh(x)+\sinh(x)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)*tanh(3*x),x, algorithm="fricas")
```

```
[Out] 1/6*(2*(cosh(x) + sinh(x))*arctan(-(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)
^2)/(cosh(x) - sinh(x))) - 6*(cosh(x) + sinh(x))*arctan(cosh(x) + sinh(x))
+ 3*cosh(x)^2 + 6*cosh(x)*sinh(x) + 3*sinh(x)^2 - 3)/(cosh(x) + sinh(x))
```

giac [B] time = 0.14, size = 43, normalized size = 2.26

$$-\frac{1}{3}\pi - \frac{1}{3}\arctan\left(\left(e^{(2x)}-1\right)e^{(-x)}\right) - \frac{1}{3}\arctan\left(\frac{1}{2}\left(e^{(2x)}-1\right)e^{(-x)}\right) - \frac{1}{2}e^{(-x)} + \frac{1}{2}e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)*tanh(3*x),x, algorithm="giac")
```

```
[Out] -1/3*pi - 1/3*arctan((e^(2*x) - 1)*e^(-x)) - 1/3*arctan(1/2*(e^(2*x) - 1)*e
^(-x)) - 1/2*e^(-x) + 1/2*e^x
```

maple [C] time = 0.29, size = 60, normalized size = 3.16

$$\frac{e^{x}}{2}-\frac{e^{-x}}{2}+\frac{i\ln\left(e^{x}-i\right)}{3}-\frac{i\ln\left(e^{x}+i\right)}{3}+\frac{i\ln\left(e^{2x}-ie^{x}-1\right)}{6}-\frac{i\ln\left(e^{2x}+ie^{x}-1\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)*tanh(3*x),x)
```

```
[Out] 1/2*exp(x)-1/2*exp(-x)+1/3*I*ln(exp(x)-I)-1/3*I*ln(exp(x)+I)+1/6*I*ln(exp(2
*x)-I*exp(x)-1)-1/6*I*ln(exp(2*x)+I*exp(x)-1)
```

maxima [B] time = 0.44, size = 46, normalized size = 2.42

$$\frac{1}{3}\arctan\left(\sqrt{3}+2e^{(-x)}\right)+\frac{1}{3}\arctan\left(-\sqrt{3}+2e^{(-x)}\right)+\frac{2}{3}\arctan\left(e^{(-x)}\right)-\frac{1}{2}e^{(-x)}+\frac{1}{2}e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)*tanh(3*x),x, algorithm="maxima")
```

```
[Out] 1/3*arctan(sqrt(3) + 2*e^(-x)) + 1/3*arctan(-sqrt(3) + 2*e^(-x)) + 2/3*arct
an(e^(-x)) - 1/2*e^(-x) + 1/2*e^x
```

mupad [B] time = 1.44, size = 23, normalized size = 1.21

$$\frac{e^{x}}{2}-\mathrm{atan}\left(e^{x}\right)-\frac{\mathrm{atan}\left(e^{3x}\right)}{3}-\frac{e^{-x}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(3*x)*sinh(x),x)
```

```
[Out] exp(x)/2 - atan(exp(x)) - atan(exp(3*x))/3 - exp(-x)/2
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sinh\left(x\right)\tanh\left(3x\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)*tanh(3*x),x)
```

```
[Out] Integral(sinh(x)*tanh(3*x), x)
```

3.202 $\int \sinh(x)\tanh(4x)\,dx$

Optimal. Leaf size=69

$$\sinh(x) - \frac{1}{4}\sqrt{2-\sqrt{2}}\ \tan^{-1}\left(\frac{2\sinh(x)}{\sqrt{2-\sqrt{2}}}\right) - \frac{1}{4}\sqrt{2+\sqrt{2}}\ \tan^{-1}\left(\frac{2\sinh(x)}{\sqrt{2+\sqrt{2}}}\right)$$

```
[Out] sinh(x)-1/4*arctan(2*sinh(x)/(2-2^(1/2))^(1/2))*(2-2^(1/2))^(1/2)-1/4*arcta
n(2*sinh(x)/(2+2^(1/2))^(1/2))*(2+2^(1/2))^(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {12, 1279, 1166, 203}

$$\sinh(x) - \frac{1}{4}\sqrt{2-\sqrt{2}}\ \tan^{-1}\left(\frac{2\sinh(x)}{\sqrt{2-\sqrt{2}}}\right) - \frac{1}{4}\sqrt{2+\sqrt{2}}\ \tan^{-1}\left(\frac{2\sinh(x)}{\sqrt{2+\sqrt{2}}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[x]*Tanh[4*x],x]
```

```
[Out] -(Sqrt[2 - Sqrt[2]]*ArcTan[(2*Sinh[x])/Sqrt[2 - Sqrt[2]]])/4 - (Sqrt[2 + Sq
rt[2]]*ArcTan[(2*Sinh[x])/Sqrt[2 + Sqrt[2]]])/4 + Sinh[x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1279

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[(e*f*(f*x)^(m - 1)*(a + b*x^2 + c*x^4)^(p +
1))/(c*(m + 4*p + 3)), x] - Dist[f^2/(c*(m + 4*p + 3)), Int[(f*x)^(m - 2)*(
a + b*x^2 + c*x^4)^p*Simp[a*e*(m - 1) + (b*e*(m + 2*p + 1) - c*d*(m + 4*p +
 3))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c,
 0] && GtQ[m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] ||
 IntegerQ[m])
```

Rubi steps

$$\begin{aligned}\int \sinh(x)\tanh(4x)\,dx &= -\text{Subst}\left(\int \frac{4x^2\left(-1-2x^2\right)}{1+8x^2+8x^4}\,dx,x,\sinh(x)\right)\\ &= -\left(4\,\text{Subst}\left(\int \frac{x^2\left(-1-2x^2\right)}{1+8x^2+8x^4}\,dx,x,\sinh(x)\right)\right)\\ &= \sinh(x)+\frac{1}{2}\text{Subst}\left(\int \frac{-2-8x^2}{1+8x^2+8x^4}\,dx,x,\sinh(x)\right)\\ &= \sinh(x)+\left(-2+\sqrt{2}\right)\text{Subst}\left(\int \frac{1}{4-2\sqrt{2}+8x^2}\,dx,x,\sinh(x)\right)-\left(2+\sqrt{2}\right)\text{Subst}\left(\int\right.\\ &= -\frac{1}{4}\sqrt{2-\sqrt{2}}\,\tan^{-1}\left(\frac{2\sinh(x)}{\sqrt{2-\sqrt{2}}}\right)-\frac{1}{4}\sqrt{2+\sqrt{2}}\,\tan^{-1}\left(\frac{2\sinh(x)}{\sqrt{2+\sqrt{2}}}\right)+\sinh(x)\end{aligned}$$

Mathematica [A] time = 0.15, size = 69, normalized size = 1.00

$$\sinh(x)-\frac{1}{4}\sqrt{2-\sqrt{2}}\,\tan^{-1}\left(\frac{2\sinh(x)}{\sqrt{2-\sqrt{2}}}\right)-\frac{1}{4}\sqrt{2+\sqrt{2}}\,\tan^{-1}\left(\frac{2\sinh(x)}{\sqrt{2+\sqrt{2}}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[x]*Tanh[4*x],x]
```

```
[Out] -1/4*(Sqrt[2 - Sqrt[2]]*ArcTan[(2*Sinh[x])/Sqrt[2 - Sqrt[2]]]) - (Sqrt[2 +
Sqrt[2]]*ArcTan[(2*Sinh[x])/Sqrt[2 + Sqrt[2]]])/4 + Sinh[x]
```

fricas [B] time = 0.49, size = 168, normalized size = 2.43

$$-\frac{1}{2}\left(\sqrt{\sqrt{2}+2}\,\arctan\left(\frac{1}{2}\left(\sqrt{\sqrt{2}\,e^{(2x)}+e^{(4x)}+1}\,\sqrt{\sqrt{2}+2}\left(\sqrt{2}-2\right)-\left(\left(\sqrt{2}-2\right)e^{(2x)}-\sqrt{2}+2\right)\sqrt{\sqrt{2}+2}\right)e^{(-\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)*tanh(4*x),x, algorithm="fricas")
```

```
[Out] -1/2*(sqrt(sqrt(2) + 2)*arctan(1/2*(sqrt(sqrt(2)*e^(2*x) + e^(4*x) + 1)*sqr
t(sqrt(2) + 2)*(sqrt(2) - 2) - ((sqrt(2) - 2)*e^(2*x) - sqrt(2) + 2)*sqrt(s
qrt(2) + 2))*e^(-x))*e^x - sqrt(-sqrt(2) + 2)*arctan(1/2*(sqrt(-sqrt(2)*e^(
2*x) + e^(4*x) + 1)*(sqrt(2) + 2)*sqrt(-sqrt(2) + 2) - ((sqrt(2) + 2)*e^(2*
x) - sqrt(2) - 2)*sqrt(-sqrt(2) + 2))*e^(-x))*e^x - e^(2*x) + 1)*e^(-x)
```

giac [A] time = 0.22, size = 71, normalized size = 1.03

$$-\frac{1}{4}\sqrt{\sqrt{2}+2}\arctan\left(-\frac{e^{(-x)}-e^{x}}{\sqrt{\sqrt{2}+2}}\right)-\frac{1}{4}\sqrt{-\sqrt{2}+2}\arctan\left(-\frac{e^{(-x)}-e^{x}}{\sqrt{-\sqrt{2}+2}}\right)-\frac{1}{2}e^{(-x)}+\frac{1}{2}e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)*tanh(4*x),x, algorithm="giac")
```

```
[Out] -1/4*sqrt(sqrt(2) + 2)*arctan(-(e^(-x) - e^x)/sqrt(sqrt(2) + 2)) - 1/4*sqrt
(-sqrt(2) + 2)*arctan(-(e^(-x) - e^x)/sqrt(-sqrt(2) + 2)) - 1/2*e^(-x) + 1/
2*e^x
```

maple [C] time = 0.32, size = 42, normalized size = 0.61

$$\frac{e^{x}}{2}-\frac{e^{-x}}{2}+\left(\sum_{\_R=\mathrm{RootOf}\left(2048\_Z^{4}+128\_Z^{2}+1\right)}\_R\ln\left(-8\_R\,e^{x}+e^{2x}-1\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)*tanh(4*x),x)
```

```
[Out] 1/2*exp(x)-1/2*exp(-x)+sum(_R*ln(-8*_R*exp(x)+exp(2*x)-1),_R=RootOf(2048*_Z
^4+128*_Z^2+1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{2}\left(e^{(2x)}-1\right)e^{(-x)}-\frac{1}{2}\int\frac{2\left(e^{(7x)}+e^{x}\right)}{e^{(8x)}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)*tanh(4*x),x, algorithm="maxima")
```

```
[Out] 1/2*(e^(2*x) - 1)*e^(-x) - 1/2*integrate(2*(e^(7*x) + e^x)/(e^(8*x) + 1), x
)
```

mupad [B] time = 0.77, size = 71, normalized size = 1.03

$$\frac{e^x}{2} - \frac{e^{-x}}{2} - \frac{\operatorname{atan}\left(\frac{e^{-x}\left(e^{2x}-1\right)}{\sqrt{\sqrt{2}+2}}\right)\sqrt{\sqrt{2}+2}}{4} - \frac{\operatorname{atan}\left(\frac{e^{-x}\left(e^{2x}-1\right)}{\sqrt{2-\sqrt{2}}}\right)\sqrt{2-\sqrt{2}}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(4*x)*sinh(x),x)
```

```
[Out] exp(x)/2 - exp(-x)/2 - (atan((exp(-x)*(exp(2*x) - 1))/(2^(1/2) + 2)^(1/2))*
(2^(1/2) + 2)^(1/2))/4 - (atan((exp(-x)*(exp(2*x) - 1))/(2 - 2^(1/2))^(1/2)
)*(2 - 2^(1/2))^(1/2))/4
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sinh(x)\tanh(4x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)*tanh(4*x),x)
```

```
[Out] Integral(sinh(x)*tanh(4*x), x)
```

3.203 $\int \sinh(x)\tanh(5x)\,dx$

Optimal. Leaf size=87

$$\sinh(x)-\frac{1}{5}\tan^{-1}(\sinh(x))-\frac{1}{5}\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}\tan^{-1}\left(2\sqrt{\frac{2}{3+\sqrt{5}}}\sinh(x)\right)-\frac{1}{5}\sqrt{\frac{1}{2}\left(3-\sqrt{5}\right)}\tan^{-1}\left(\sqrt{2\left(3+\sqrt{5}\right)}\,\mathrm{s}$$

```
[Out] -1/5*arctan(sinh(x))+sinh(x)-1/5*arctan(sinh(x)*(5^(1/2)+1))*(1/2*5^(1/2)-1
/2)-1/5*arctan(2*sinh(x)*2^(1/2)/(3+5^(1/2))^(1/2))*(1/2+1/2*5^(1/2))
```

Rubi [A] time = 0.29, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 4, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {6742, 2073, 203, 1166}

$$\sinh(x)-\frac{1}{5}\tan^{-1}(\sinh(x))-\frac{1}{5}\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}\tan^{-1}\left(2\sqrt{\frac{2}{3+\sqrt{5}}}\sinh(x)\right)-\frac{1}{5}\sqrt{\frac{1}{2}\left(3-\sqrt{5}\right)}\tan^{-1}\left(\sqrt{2\left(3+\sqrt{5}\right)}\,\mathrm{s}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[x]*Tanh[5*x],x]
```

```
[Out] -ArcTan[Sinh[x]]/5 - (Sqrt[(3 + Sqrt[5])/2]*ArcTan[2*Sqrt[2/(3 + Sqrt[5])]*
Sinh[x]])/5 - (Sqrt[(3 - Sqrt[5])/2]*ArcTan[Sqrt[2*(3 + Sqrt[5])]*Sinh[x]])
/5 + Sinh[x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 2073

```
Int[(P_)^(p_)*(Q_)^(q_.), x_Symbol] :> With[{PP = Factor[P /. x -> Sqrt[x]]
}, Int[ExpandIntegrand[(PP /. x -> x^2)^p*Q^q, x], x] /;  !SumQ[NonfreeFact
ors[PP, x]]] /; FreeQ[q, x] && PolyQ[P, x^2] && PolyQ[Q, x] && ILtQ[p, 0]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \sinh(x)\tanh(5x)\,dx = -\text{Subst}\left(\int \frac{x^2\left(-5-20x^2-16x^4\right)}{1+13x^2+28x^4+16x^6}\,dx,x,\sinh(x)\right)$$
$$= -\text{Subst}\left(\int \left(-1+\frac{1+8x^2+8x^4}{1+13x^2+28x^4+16x^6}\right)dx,x,\sinh(x)\right)$$
$$= \sinh(x)-\text{Subst}\left(\int \frac{1+8x^2+8x^4}{1+13x^2+28x^4+16x^6}\,dx,x,\sinh(x)\right)$$
$$= \sinh(x)-\text{Subst}\left(\int \left(\frac{1}{5\left(1+x^2\right)}+\frac{4\left(1+6x^2\right)}{5\left(1+12x^2+16x^4\right)}\right)dx,x,\sinh(x)\right)$$
$$= \sinh(x)-\frac{1}{5}\text{Subst}\left(\int \frac{1}{1+x^2}\,dx,x,\sinh(x)\right)-\frac{4}{5}\text{Subst}\left(\int \frac{1+6x^2}{1+12x^2+16x^4}\,dx,x,\sin\right.$$
$$= -\frac{1}{5}\tan^{-1}(\sinh(x))+\sinh(x)-\frac{1}{5}\left(4\left(3-\sqrt{5}\right)\right)\text{Subst}\left(\int \frac{1}{6-2\sqrt{5}+16x^2}\,dx,x,\sinh\right.$$
$$= -\frac{1}{5}\tan^{-1}(\sinh(x))-\frac{1}{5}\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}\tan^{-1}\left(2\sqrt{\frac{2}{3+\sqrt{5}}}\sinh(x)\right)-\frac{1}{5}\sqrt{\frac{1}{2}\left(3-\sqrt{5}\right)}$$

Mathematica [A] time = 0.20, size = 81, normalized size = 0.93

$$\frac{1}{10}\left(10\sinh(x)-2\tan^{-1}(\sinh(x))-\sqrt{2\left(3+\sqrt{5}\right)}\tan^{-1}\left(2\sqrt{\frac{2}{3+\sqrt{5}}}\sinh(x)\right)-\sqrt{6-2\sqrt{5}}\tan^{-1}\left(\sqrt{2\left(3+\sqrt{\right.}\right.}\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[x]*Tanh[5*x],x]
```

```
[Out] (-2*ArcTan[Sinh[x]] - Sqrt[2*(3 + Sqrt[5])]*ArcTan[2*Sqrt[2/(3 + Sqrt[5])]*
Sinh[x]] - Sqrt[6 - 2*Sqrt[5]]*ArcTan[Sqrt[2*(3 + Sqrt[5])]*Sinh[x]] + 10*S
inh[x])/10
```

fricas [B] time = 0.46, size = 237, normalized size = 2.72

$$-\frac{1}{10}\left(2\sqrt{2}\sqrt{\sqrt{5}+3}\arctan\left(\frac{1}{8}\left(\sqrt{2\left(\sqrt{5}-1\right)e^{(2x)}+4e^{(4x)}+4}\left(\sqrt{5}\sqrt{2}-3\sqrt{2}\right)\sqrt{\sqrt{5}+3}-2\left(\left(\sqrt{5}\sqrt{2}-3\sqrt{\right.}\right.\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)*tanh(5*x),x, algorithm="fricas")
```

```
[Out] -1/10*(2*sqrt(2)*sqrt(sqrt(5) + 3)*arctan(1/8*(sqrt(2*(sqrt(5) - 1)*e^(2*x)
 + 4*e^(4*x) + 4)*(sqrt(5)*sqrt(2) - 3*sqrt(2))*sqrt(sqrt(5) + 3) - 2*((sqr
t(5)*sqrt(2) - 3*sqrt(2))*e^(2*x) - sqrt(5)*sqrt(2) + 3*sqrt(2))*sqrt(sqrt(
5) + 3))*e^(-x))*e^x - 2*sqrt(2)*sqrt(-sqrt(5) + 3)*arctan(1/8*(sqrt(-2*(sq
rt(5) + 1)*e^(2*x) + 4*e^(4*x) + 4)*(sqrt(5)*sqrt(2) + 3*sqrt(2))*sqrt(-sqr
t(5) + 3) - 2*((sqrt(5)*sqrt(2) + 3*sqrt(2))*e^(2*x) - sqrt(5)*sqrt(2) - 3*
sqrt(2))*sqrt(-sqrt(5) + 3))*e^(-x))*e^x + 4*arctan(e^x)*e^x - 5*e^(2*x) +
5)*e^(-x)
```

giac [A] time = 0.15, size = 81, normalized size = 0.93

$$-\frac{1}{10}\,\pi-\frac{1}{10}\left(\sqrt{5}+1\right)\arctan\left(-\frac{2\left(e^{(-x)}-e^{x}\right)}{\sqrt{5}+1}\right)-\frac{1}{10}\left(\sqrt{5}-1\right)\arctan\left(-\frac{2\left(e^{(-x)}-e^{x}\right)}{\sqrt{5}-1}\right)-\frac{1}{5}\arctan\left(\frac{1}{2}\left(e^{(2x)}-1\right)e^{(-x)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)*tanh(5*x),x, algorithm="giac")
```

```
[Out] -1/10*pi - 1/10*(sqrt(5) + 1)*arctan(-2*(e^(-x) - e^x)/(sqrt(5) + 1)) - 1/1
0*(sqrt(5) - 1)*arctan(-2*(e^(-x) - e^x)/(sqrt(5) - 1)) - 1/5*arctan(1/2*(e
^(2*x) - 1)*e^(-x)) - 1/2*e^(-x) + 1/2*e^x
```

maple [C] time = 0.34, size = 60, normalized size = 0.69

$$\frac{e^{x}}{2}-\frac{e^{-x}}{2}+\frac{i\ln\left(e^{x}-i\right)}{5}-\frac{i\ln\left(e^{x}+i\right)}{5}+\left(\sum_{\_R=\mathrm{RootOf}\left(10000\_Z^{4}+300\_Z^{2}+1\right)}\_R\ln\left(-10\_R\,e^{x}+e^{2x}-1\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)*tanh(5*x),x)
```

```
[Out] 1/2*exp(x)-1/2*exp(-x)+1/5*I*ln(exp(x)-I)-1/5*I*ln(exp(x)+I)+sum(_R*ln(-10*
_R*exp(x)+exp(2*x)-1),_R=RootOf(10000*_Z^4+300*_Z^2+1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{2}\left(e^{(2x)}-1\right)e^{(-x)}-\frac{2}{5}\arctan\left(e^{x}\right)-\frac{1}{2}\int\frac{2\left(3\,e^{(7x)}-e^{(5x)}-e^{(3x)}+3\,e^{x}\right)}{5\left(e^{(8x)}-e^{(6x)}+e^{(4x)}-e^{(2x)}+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)*tanh(5*x),x, algorithm="maxima")
```

```
[Out] 1/2*(e^(2*x) - 1)*e^(-x) - 2/5*arctan(e^x) - 1/2*integrate(2/5*(3*e^(7*x) -
 e^(5*x) - e^(3*x) + 3*e^x)/(e^(8*x) - e^(6*x) + e^(4*x) - e^(2*x) + 1), x)
```

mupad [B] time = 2.66, size = 82, normalized size = 0.94

$$\frac{e^x}{2}-\frac{2\,\mathrm{atan}\left(e^x\right)}{5}-\frac{e^{-x}}{2}-2\,\mathrm{atan}\left(\frac{e^{-x}\,\left(e^{2\,x}-1\right)}{10\,\sqrt{\frac{3}{200}-\frac{\sqrt{5}}{200}}}\right)\,\sqrt{\frac{3}{200}-\frac{\sqrt{5}}{200}}-2\,\mathrm{atan}\left(\frac{e^{-x}\,\left(e^{2\,x}-1\right)}{10\,\sqrt{\frac{\sqrt{5}}{200}+\frac{3}{200}}}\right)\,\sqrt{\frac{\sqrt{5}}{200}+\frac{3}{200}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(5*x)*sinh(x),x)
```

```
[Out] exp(x)/2 - (2*atan(exp(x)))/5 - exp(-x)/2 - 2*atan((exp(-x)*(exp(2*x) - 1))
/(10*(3/200 - 5^(1/2)/200)^(1/2)))*(3/200 - 5^(1/2)/200)^(1/2) - 2*atan((ex
p(-x)*(exp(2*x) - 1))/(10*(5^(1/2)/200 + 3/200)^(1/2)))*(5^(1/2)/200 + 3/20
0)^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sinh\left(x\right)\tanh\left(5x\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)*tanh(5*x),x)
```

```
[Out] Integral(sinh(x)*tanh(5*x), x)
```

3.204 $\int \sinh(x)\tanh(6x)\,dx$

Optimal. Leaf size=87

$$\sinh(x)-\frac{\tan^{-1}\left(\sqrt{2}\sinh(x)\right)}{3\sqrt{2}}-\frac{1}{6}\sqrt{2-\sqrt{3}}\tan^{-1}\left(\frac{2\sinh(x)}{\sqrt{2-\sqrt{3}}}\right)-\frac{1}{6}\sqrt{2+\sqrt{3}}\tan^{-1}\left(\frac{2\sinh(x)}{\sqrt{2+\sqrt{3}}}\right)$$

```
[Out] sinh(x)-1/6*arctan(sinh(x)*2^(1/2))*2^(1/2)-1/6*arctan(2*sinh(x)/(1/2*6^(1/
2)-1/2*2^(1/2)))*(1/2*6^(1/2)-1/2*2^(1/2))-1/6*arctan(2*sinh(x)/(1/2*6^(1/2
)+1/2*2^(1/2)))*(1/2*6^(1/2)+1/2*2^(1/2))
```

Rubi [A] time = 0.26, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 5, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.714, Rules used = {12, 6742, 2073, 203, 1166}

$$\sinh(x)-\frac{\tan^{-1}\left(\sqrt{2}\sinh(x)\right)}{3\sqrt{2}}-\frac{1}{6}\sqrt{2-\sqrt{3}}\tan^{-1}\left(\frac{2\sinh(x)}{\sqrt{2-\sqrt{3}}}\right)-\frac{1}{6}\sqrt{2+\sqrt{3}}\tan^{-1}\left(\frac{2\sinh(x)}{\sqrt{2+\sqrt{3}}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[x]*Tanh[6*x],x]
```

```
[Out] -ArcTan[Sqrt[2]*Sinh[x]]/(3*Sqrt[2]) - (Sqrt[2 - Sqrt[3]]*ArcTan[(2*Sinh[x]
)/Sqrt[2 - Sqrt[3]]])/6 - (Sqrt[2 + Sqrt[3]]*ArcTan[(2*Sinh[x])/Sqrt[2 + Sq
rt[3]]])/6 + Sinh[x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
```

```
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 2073

```
Int[(P_)^(p_)*(Q_)^(q_.), x_Symbol] :> With[{PP = Factor[P /. x -> Sqrt[x]]
}, Int[ExpandIntegrand[(PP /. x -> x^2)^p*Q^q, x], x] /;  !SumQ[NonfreeFact
ors[PP, x]]] /; FreeQ[q, x] && PolyQ[P, x^2] && PolyQ[Q, x] && ILtQ[p, 0]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \sinh(x)\tanh(6x)\,dx = -\text{Subst}\left(\int \frac{2x^2\left(-3-16x^2-16x^4\right)}{1+18x^2+48x^4+32x^6}\,dx, x, \sinh(x)\right)$$

$$= -\left(2\,\text{Subst}\left(\int \frac{x^2\left(-3-16x^2-16x^4\right)}{1+18x^2+48x^4+32x^6}\,dx, x, \sinh(x)\right)\right)$$

$$= -\left(2\,\text{Subst}\left(\int \left(-\frac{1}{2}+\frac{1+12x^2+16x^4}{2\left(1+18x^2+48x^4+32x^6\right)}\right)dx, x, \sinh(x)\right)\right)$$

$$= \sinh(x) - \text{Subst}\left(\int \frac{1+12x^2+16x^4}{1+18x^2+48x^4+32x^6}\,dx, x, \sinh(x)\right)$$

$$= \sinh(x) - \text{Subst}\left(\int \left(\frac{1}{3\left(1+2x^2\right)}+\frac{2\left(1+8x^2\right)}{3\left(1+16x^2+16x^4\right)}\right)dx, x, \sinh(x)\right)$$

$$= \sinh(x) - \frac{1}{3}\text{Subst}\left(\int \frac{1}{1+2x^2}\,dx, x, \sinh(x)\right) - \frac{2}{3}\text{Subst}\left(\int \frac{1+8x^2}{1+16x^2+16x^4}\,dx, x, \text{si}\right.$$

$$= -\frac{\tan^{-1}\left(\sqrt{2}\sinh(x)\right)}{3\sqrt{2}} + \sinh(x) - \frac{1}{3}\left(4\left(2-\sqrt{3}\right)\right)\text{Subst}\left(\int \frac{1}{8-4\sqrt{3}+16x^2}\,dx, x, \text{si}\right.$$

$$= -\frac{\tan^{-1}\left(\sqrt{2}\sinh(x)\right)}{3\sqrt{2}} - \frac{1}{6}\sqrt{2-\sqrt{3}}\,\tan^{-1}\left(\frac{2\sinh(x)}{\sqrt{2-\sqrt{3}}}\right) - \frac{1}{6}\sqrt{2+\sqrt{3}}\,\tan^{-1}\left(\frac{2\sinh(}{\sqrt{2+\sqrt{}}}\right.$$

Mathematica [A] time = 0.13, size = 87, normalized size = 1.00

$$\sinh(x)-\frac{\tan^{-1}\left(\sqrt{2}\sinh(x)\right)}{3\sqrt{2}}-\frac{1}{6}\sqrt{2-\sqrt{3}}\tan^{-1}\left(\frac{2\sinh(x)}{\sqrt{2-\sqrt{3}}}\right)-\frac{1}{6}\sqrt{2+\sqrt{3}}\tan^{-1}\left(\frac{2\sinh(x)}{\sqrt{2+\sqrt{3}}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[x]*Tanh[6*x],x]
```

```
[Out] -1/3*ArcTan[Sqrt[2]*Sinh[x]]/Sqrt[2] - (Sqrt[2 - Sqrt[3]]*ArcTan[(2*Sinh[x]
)/Sqrt[2 - Sqrt[3]]])/6 - (Sqrt[2 + Sqrt[3]]*ArcTan[(2*Sinh[x])/Sqrt[2 + Sq
rt[3]]])/6 + Sinh[x]
```

fricas [B] time = 0.58, size = 205, normalized size = 2.36

$$-\frac{1}{6}\left(2\sqrt{\sqrt{3}+2}\arctan\left(\left(\sqrt{\sqrt{3}e^{(2x)}+e^{(4x)}+1}\sqrt{\sqrt{3}+2}\left(\sqrt{3}-2\right)-\left(\left(\sqrt{3}-2\right)e^{(2x)}-\sqrt{3}+2\right)\sqrt{\sqrt{3}+2}\right)e^{(-x)}\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)*tanh(6*x),x, algorithm="fricas")
```

```
[Out] -1/6*(2*sqrt(sqrt(3) + 2)*arctan((sqrt(sqrt(3)*e^(2*x) + e^(4*x) + 1)*sqrt(
sqrt(3) + 2)*(sqrt(3) - 2) - ((sqrt(3) - 2)*e^(2*x) - sqrt(3) + 2)*sqrt(sqr
t(3) + 2))*e^(-x))*e^x - 2*sqrt(-sqrt(3) + 2)*arctan((sqrt(-sqrt(3)*e^(2*x)
 + e^(4*x) + 1)*(sqrt(3) + 2)*sqrt(-sqrt(3) + 2) - ((sqrt(3) + 2)*e^(2*x) -
 sqrt(3) - 2)*sqrt(-sqrt(3) + 2))*e^(-x))*e^x + sqrt(2)*arctan(1/2*sqrt(2)*
e^(3*x) + 1/2*sqrt(2)*e^x)*e^x + sqrt(2)*arctan(1/2*sqrt(2)*e^x)*e^x - 3*e^
(2*x) + 3)*e^(-x)
```

giac [A] time = 0.15, size = 100, normalized size = 1.15

$$-\frac{1}{12}\left(\sqrt{6}+\sqrt{2}\right)\arctan\left(-\frac{2\left(e^{(-x)}-e^{x}\right)}{\sqrt{6}+\sqrt{2}}\right)-\frac{1}{12}\left(\sqrt{6}-\sqrt{2}\right)\arctan\left(-\frac{2\left(e^{(-x)}-e^{x}\right)}{\sqrt{6}-\sqrt{2}}\right)-\frac{1}{12}\sqrt{2}\left(\pi+2\arctan\left(\frac{1}{2}\sqrt{2}\left(\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)*tanh(6*x),x, algorithm="giac")
```

```
[Out] -1/12*(sqrt(6) + sqrt(2))*arctan(-2*(e^(-x) - e^x)/(sqrt(6) + sqrt(2))) - 1
/12*(sqrt(6) - sqrt(2))*arctan(-2*(e^(-x) - e^x)/(sqrt(6) - sqrt(2))) - 1/1
2*sqrt(2)*(pi + 2*arctan(1/2*sqrt(2)*(e^(2*x) - 1)*e^(-x))) - 1/2*e^(-x) +
1/2*e^x
```

maple [C] time = 0.36, size = 84, normalized size = 0.97

$$\frac{e^x}{2}-\frac{e^{-x}}{2}+\left(\sum_{\_R=\mathrm{RootOf}\left(20736\_Z^4+576\_Z^2+1\right)}\_R\ln\left(-12\_R\,e^x+e^{2x}-1\right)\right)+\frac{i\sqrt{2}\,\ln\left(e^{2x}-i\sqrt{2}\,e^x-1\right)}{12}-\frac{i\sqrt{2}\,\ln\left(e^{2x}+i\right.}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)*tanh(6*x),x)
```

```
[Out] 1/2*exp(x)-1/2*exp(-x)+sum(_R*ln(-12*_R*exp(x)+exp(2*x)-1),_R=RootOf(20736*
_Z^4+576*_Z^2+1))+1/12*I*2^(1/2)*ln(exp(2*x)-I*2^(1/2)*exp(x)-1)-1/12*I*2^(
1/2)*ln(exp(2*x)+I*2^(1/2)*exp(x)-1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{2}\left(e^{(2x)}-1\right)e^{(-x)}-\frac{1}{6}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2e^x\right)\right)-\frac{1}{6}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2e^x\right)\right)-\frac{1}{2}\int\frac{2\left(2e^{(7x)}-e^{(5x)}\right.}{3\left(e^{(8x)}-e\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)*tanh(6*x),x, algorithm="maxima")
```

```
[Out] 1/2*(e^(2*x) - 1)*e^(-x) - 1/6*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*e^x)
) - 1/6*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*e^x)) - 1/2*integrate(2/3*
(2*e^(7*x) - e^(5*x) - e^(3*x) + 2*e^x)/(e^(8*x) - e^(4*x) + 1), x)
```

mupad [B] time = 2.67, size = 98, normalized size = 1.13

$$\frac{e^x}{2}-\frac{e^{-x}}{2}-\frac{\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,e^{-x}\left(e^{2x}-1\right)}{2}\right)}{6}-2\,\mathrm{atan}\left(\frac{e^{-x}\left(e^{2x}-1\right)}{12\sqrt{\frac{1}{72}-\frac{\sqrt{3}}{144}}}\right)\sqrt{\frac{1}{72}-\frac{\sqrt{3}}{144}}-2\,\mathrm{atan}\left(\frac{e^{-x}\left(e^{2x}-1\right)}{12\sqrt{\frac{\sqrt{3}}{144}+\frac{1}{72}}}\right)\sqrt{\frac{\sqrt{3}}{144}+\frac{1}{72}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(6*x)*sinh(x),x)
```

```
[Out] exp(x)/2 - exp(-x)/2 - (2^(1/2)*atan((2^(1/2)*exp(-x)*(exp(2*x) - 1))/2))/6
 - 2*atan((exp(-x)*(exp(2*x) - 1))/(12*(1/72 - 3^(1/2)/144)^(1/2)))*(1/72 -
 3^(1/2)/144)^(1/2) - 2*atan((exp(-x)*(exp(2*x) - 1))/(12*(3^(1/2)/144 + 1/
72)^(1/2)))*(3^(1/2)/144 + 1/72)^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sinh(x)\tanh(6x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)*tanh(6*x),x)
```

```
[Out] Integral(sinh(x)*tanh(6*x), x)
```

3.205 $\int \sinh(x)\tanh(nx)\,dx$

Optimal. Leaf size=81

$$-e^{-x}\,{}_2F_1\left(1,-\frac{1}{2n};1-\frac{1}{2n};-e^{2nx}\right)-e^x\,{}_2F_1\left(1,\frac{1}{2n};\frac{1}{2}\left(2+\frac{1}{n}\right);-e^{2nx}\right)+\frac{e^{-x}}{2}+\frac{e^x}{2}$$

```
[Out] 1/2/exp(x)+1/2*exp(x)-hypergeom([1, -1/2/n],[1-1/2/n],-exp(2*n*x))/exp(x)-e
xp(x)*hypergeom([1, 1/2/n],[1+1/2/n],-exp(2*n*x))
```

Rubi [A] time = 0.07, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {5601, 2194, 2251}

$$-e^{-x}\,{}_2F_1\left(1,-\frac{1}{2n};1-\frac{1}{2n};-e^{2nx}\right)-e^x\,{}_2F_1\left(1,\frac{1}{2n};\frac{1}{2}\left(2+\frac{1}{n}\right);-e^{2nx}\right)+\frac{e^{-x}}{2}+\frac{e^x}{2}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[x]*Tanh[n*x],x]
```

```
[Out] 1/(2*E^x) + E^x/2 - Hypergeometric2F1[1, -1/(2*n), 1 - 1/(2*n), -E^(2*n*x)]
/E^x - E^x*Hypergeometric2F1[1, 1/(2*n), (2 + n^(-1))/2, -E^(2*n*x)]
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 2251

```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_)*(G_)^((h_.)*((f_.
) + (g_.)*(x_))), x_Symbol] :> Simp[(a^p*G^(h*(f + g*x))*Hypergeometric2F1[
-p, (g*h*Log[G])/(d*e*Log[F]), (g*h*Log[G])/(d*e*Log[F]) + 1, Simplify[-((b
*F^(e*(c + d*x)))/a)]])/(g*h*Log[G]), x] /; FreeQ[{F, G, a, b, c, d, e, f,
g, h, p}, x] && (ILtQ[p, 0] || GtQ[a, 0])
```

Rule 5601

```
Int[Sinh[(a_.) + (b_.)*(x_)]*Tanh[(c_.) + (d_.)*(x_)], x_Symbol] :> Int[-(1
/(E^(a + b*x)*2)) + E^(a + b*x)/2 + 1/(E^(a + b*x)*(1 + E^(2*(c + d*x)))) -
 E^(a + b*x)/(1 + E^(2*(c + d*x))), x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2
 - d^2, 0]
```

Rubi steps

$$\int \sinh(x)\tanh(nx)\,dx = \int \left(-\frac{e^{-x}}{2} + \frac{e^x}{2} + \frac{e^{-x}}{1+e^{2nx}} - \frac{e^x}{1+e^{2nx}}\right) dx$$
$$= -\left(\frac{1}{2}\int e^{-x}\,dx\right) + \frac{\int e^x\,dx}{2} + \int \frac{e^{-x}}{1+e^{2nx}}\,dx - \int \frac{e^x}{1+e^{2nx}}\,dx$$
$$= \frac{e^{-x}}{2} + \frac{e^x}{2} - e^{-x}\,{}_2F_1\left(1,-\frac{1}{2n};1-\frac{1}{2n};-e^{2nx}\right) - e^x\,{}_2F_1\left(1,\frac{1}{2n};\frac{1}{2}\left(2+\frac{1}{n}\right);-e^{2nx}\right)$$

Mathematica [B] time = 0.18, size = 164, normalized size = 2.02

$$\frac{1}{2}e^{-2x}\left(-\frac{e^{2nx+x}\,{}_2F_1\left(1,1-\frac{1}{2n};2-\frac{1}{2n};-e^{2nx}\right)}{2n-1} + \frac{e^{(2n+3)x}\,{}_2F_1\left(1,1+\frac{1}{2n};2+\frac{1}{2n};-e^{2nx}\right)}{2n+1} - e^x\left({}_2F_1\left(1,-\frac{1}{2n};1-\frac{1}{2n};-e^{2nx}\right)\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[x]*Tanh[n*x],x]
```

```
[Out] (-((E^(x + 2*n*x)*Hypergeometric2F1[1, 1 - 1/(2*n), 2 - 1/(2*n), -E^(2*n*x)
])/(-1 + 2*n)) + (E^((3 + 2*n)*x)*Hypergeometric2F1[1, 1 + 1/(2*n), 2 + 1/(
2*n), -E^(2*n*x)])/(1 + 2*n) - E^x*(Hypergeometric2F1[1, -1/2*1/n, 1 - 1/(2
*n), -E^(2*n*x)] + E^(2*x)*Hypergeometric2F1[1, 1/(2*n), 1 + 1/(2*n), -E^(2
*n*x)]))/(2*E^(2*x))
```

fricas [F] time = 0.50, size = 0, normalized size = 0.00

$$\text{integral}\left(\sinh(x)\tanh\left(nx\right), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)*tanh(n*x),x, algorithm="fricas")
```

```
[Out] integral(sinh(x)*tanh(n*x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sinh(x)\tanh\left(nx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)*tanh(n*x),x, algorithm="giac")
```

```
[Out] integrate(sinh(x)*tanh(n*x), x)
```

maple [F] time = 0.17, size = 0, normalized size = 0.00

$$\int \sinh(x)\tanh(nx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)*tanh(n*x),x)
```

```
[Out] int(sinh(x)*tanh(n*x),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{2}\left(e^{(2x)}+1\right)e^{(-x)} - \frac{1}{2}\int \frac{2\left(e^{(2x)}-1\right)}{e^{(2nx+x)}+e^{x}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)*tanh(n*x),x, algorithm="maxima")
```

```
[Out] 1/2*(e^(2*x) + 1)*e^(-x) - 1/2*integrate(2*(e^(2*x) - 1)/(e^(2*n*x + x) + e
^x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \tanh(n\,x)\,\sinh(x)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(n*x)*sinh(x),x)
```

```
[Out] int(tanh(n*x)*sinh(x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sinh(x)\tanh(nx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)*tanh(n*x),x)
```

```
[Out] Integral(sinh(x)*tanh(n*x), x)
```

3.206 $\int \coth(2x)\sinh(x)\,dx$

Optimal. Leaf size=10

$$\sinh(x) - \frac{1}{2}\tan^{-1}(\sinh(x))$$

```
[Out] -1/2*arctan(sinh(x))+sinh(x)
```

Rubi [A] time = 0.02, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {388, 203}

$$\sinh(x) - \frac{1}{2}\tan^{-1}(\sinh(x))$$

Antiderivative was successfully verified.

```
[In] Int[Coth[2*x]*Sinh[x],x]
```

```
[Out] -ArcTan[Sinh[x]]/2 + Sinh[x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rubi steps

$$\begin{aligned}
\int \coth(2x)\sinh(x)\,dx &= \text{Subst}\left(\int \frac{1+2x^2}{2+2x^2}\,dx, x, \sinh(x)\right)\\
&= \sinh(x) - \text{Subst}\left(\int \frac{1}{2+2x^2}\,dx, x, \sinh(x)\right)\\
&= -\frac{1}{2}\tan^{-1}(\sinh(x)) + \sinh(x)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 10, normalized size = 1.00

$$\sinh(x) - \frac{1}{2}\tan^{-1}(\sinh(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[2*x]*Sinh[x],x]
[Out] -1/2*ArcTan[Sinh[x]] + Sinh[x]
```

fricas [B] time = 0.44, size = 42, normalized size = 4.20

$$-\frac{2\,(\cosh(x) + \sinh(x))\arctan\left(\cosh(x) + \sinh(x)\right) - \cosh(x)^2 - 2\,\cosh(x)\sinh(x) - \sinh(x)^2 + 1}{2\,(\cosh(x) + \sinh(x))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(2*x)*sinh(x),x, algorithm="fricas")
[Out] -1/2*(2*(cosh(x) + sinh(x))*arctan(cosh(x) + sinh(x)) - cosh(x)^2 - 2*cosh(
x)*sinh(x) - sinh(x)^2 + 1)/(cosh(x) + sinh(x))
```

giac [A] time = 0.13, size = 16, normalized size = 1.60

$$-\arctan\left(e^x\right) - \frac{1}{2}\,e^{(-x)} + \frac{1}{2}\,e^x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(2*x)*sinh(x),x, algorithm="giac")
[Out] -arctan(e^x) - 1/2*e^(-x) + 1/2*e^x
```

maple [A] time = 0.14, size = 9, normalized size = 0.90

$$\sinh(x) - \arctan\left(e^x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(2*x)*sinh(x),x)
[Out] sinh(x)-arctan(exp(x))
```

maxima [A] time = 0.44, size = 16, normalized size = 1.60

$$\arctan\left(e^{(-x)}\right) - \frac{1}{2}\,e^{(-x)} + \frac{1}{2}\,e^x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(2*x)*sinh(x),x, algorithm="maxima")
```

```
[Out] arctan(e^(-x)) - 1/2*e^(-x) + 1/2*e^x
```

mupad [B] time = 0.05, size = 16, normalized size = 1.60

$$\frac{e^x}{2} - \operatorname{atan}\left(e^x\right) - \frac{e^{-x}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(2*x)*sinh(x),x)
```

```
[Out] exp(x)/2 - atan(exp(x)) - exp(-x)/2
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sinh\left(x\right)\coth\left(2x\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(2*x)*sinh(x),x)
```

```
[Out] Integral(sinh(x)*coth(2*x), x)
```

3.207 $\int \coth(3x)\sinh(x)\,dx$

Optimal. Leaf size=20

$$\sinh(x)-\frac{\tan^{-1}\left(\frac{2\sinh(x)}{\sqrt{3}}\right)}{\sqrt{3}}$$

[Out] sinh(x)-1/3*arctan(2/3*sinh(x)*3^(1/2))*3^(1/2)

Rubi [A] time = 0.03, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {388, 203}

$$\sinh(x)-\frac{\tan^{-1}\left(\frac{2\sinh(x)}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[Coth[3*x]*Sinh[x],x]

[Out] -(ArcTan[(2*Sinh[x])/Sqrt[3]]/Sqrt[3]) + Sinh[x]

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rubi steps

$$\int \coth(3x)\sinh(x)\,dx = \text{Subst}\left(\int \frac{1+4x^2}{3+4x^2}\,dx, x, \sinh(x)\right)$$
$$= \sinh(x) - 2\,\text{Subst}\left(\int \frac{1}{3+4x^2}\,dx, x, \sinh(x)\right)$$
$$= -\frac{\tan^{-1}\left(\frac{2\sinh(x)}{\sqrt{3}}\right)}{\sqrt{3}} + \sinh(x)$$

Mathematica [**A**] time = 0.02, size = 20, normalized size = 1.00

$$\sinh(x) - \frac{\tan^{-1}\left(\frac{2\sinh(x)}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[3*x]*Sinh[x],x]
```

```
[Out] -(ArcTan[(2*Sinh[x])/Sqrt[3]]/Sqrt[3]) + Sinh[x]
```

fricas [**B**] time = 0.43, size = 118, normalized size = 5.90

$$-\frac{2\left(\sqrt{3}\cosh(x) + \sqrt{3}\sinh(x)\right)\arctan\left(\frac{1}{3}\sqrt{3}\cosh(x) + \frac{1}{3}\sqrt{3}\sinh(x)\right) - 2\left(\sqrt{3}\cosh(x) + \sqrt{3}\sinh(x)\right)\arctan\left(\right.}{6\left(\cosh(x) + \sinh\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(3*x)*sinh(x),x, algorithm="fricas")
```

```
[Out] -1/6*(2*(sqrt(3)*cosh(x) + sqrt(3)*sinh(x))*arctan(1/3*sqrt(3)*cosh(x) + 1/
3*sqrt(3)*sinh(x)) - 2*(sqrt(3)*cosh(x) + sqrt(3)*sinh(x))*arctan(-1/3*(sqr
t(3)*cosh(x)^2 + 2*sqrt(3)*cosh(x)*sinh(x) + sqrt(3)*sinh(x)^2 + 2*sqrt(3))
/(cosh(x) - sinh(x))) - 3*cosh(x)^2 - 6*cosh(x)*sinh(x) - 3*sinh(x)^2 + 3)/
(cosh(x) + sinh(x))
```

giac [**B**] time = 0.13, size = 36, normalized size = 1.80

$$-\frac{1}{6}\sqrt{3}\left(\pi + 2\arctan\left(\frac{1}{3}\sqrt{3}\left(e^{(2x)} - 1\right)e^{(-x)}\right)\right) - \frac{1}{2}e^{(-x)} + \frac{1}{2}e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(3*x)*sinh(x),x, algorithm="giac")
```

```
[Out] -1/6*sqrt(3)*(pi + 2*arctan(1/3*sqrt(3)*(e^(2*x) - 1)*e^(-x))) - 1/2*e^(-x)
 + 1/2*e^x
```

maple [B] time = 0.22, size = 51, normalized size = 2.55

$$-\frac{1}{\tanh\left(\frac{x}{2}\right)+1}-\frac{\sqrt{3}\,\arctan\left(\frac{\tanh\left(\frac{x}{2}\right)\sqrt{3}}{3}\right)}{3}-\frac{\sqrt{3}\,\arctan\left(\tanh\left(\frac{x}{2}\right)\sqrt{3}\right)}{3}-\frac{1}{\tanh\left(\frac{x}{2}\right)-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(3*x)*sinh(x),x)
```

```
[Out] -1/(tanh(1/2*x)+1)-1/3*3^(1/2)*arctan(1/3*tanh(1/2*x)*3^(1/2))-1/3*3^(1/2)*
arctan(tanh(1/2*x)*3^(1/2))-1/(tanh(1/2*x)-1)
```

maxima [B] time = 0.44, size = 49, normalized size = 2.45

$$\frac{1}{3}\,\sqrt{3}\,\arctan\left(\frac{1}{3}\,\sqrt{3}\left(2\,e^{(-x)}+1\right)\right)+\frac{1}{3}\,\sqrt{3}\,\arctan\left(\frac{1}{3}\,\sqrt{3}\left(2\,e^{(-x)}-1\right)\right)-\frac{1}{2}\,e^{(-x)}+\frac{1}{2}\,e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(3*x)*sinh(x),x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*e^(-x) + 1)) + 1/3*sqrt(3)*arctan(1/3*sqr
t(3)*(2*e^(-x) - 1)) - 1/2*e^(-x) + 1/2*e^x
```

mupad [B] time = 0.08, size = 47, normalized size = 2.35

$$\frac{e^{x}}{2}-\frac{e^{-x}}{2}-\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\,\sqrt{3}\,e^{x}}{3}+\frac{\sqrt{3}\,e^{3\,x}}{3}\right)}{3}-\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{\sqrt{3}\,e^{x}}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(3*x)*sinh(x),x)
```

```
[Out] exp(x)/2 - exp(-x)/2 - (3^(1/2)*atan((2*3^(1/2)*exp(x))/3 + (3^(1/2)*exp(3*
x))/3))/3 - (3^(1/2)*atan((3^(1/2)*exp(x))/3))/3
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sinh(x)\coth(3x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(3*x)*sinh(x),x)
[Out] Integral(sinh(x)*coth(3*x), x)
```

3.208 $\int \coth(4x)\sinh(x)\,dx$

Optimal. Leaf size=28

$$\sinh(x) - \frac{1}{4}\tan^{-1}(\sinh(x)) - \frac{\tan^{-1}\left(\sqrt{2}\sinh(x)\right)}{2\sqrt{2}}$$

```
[Out] -1/4*arctan(sinh(x))+sinh(x)-1/4*arctan(sinh(x)*2^(1/2))*2^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {1676, 1166, 203}

$$\sinh(x) - \frac{1}{4}\tan^{-1}(\sinh(x)) - \frac{\tan^{-1}\left(\sqrt{2}\sinh(x)\right)}{2\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[4*x]*Sinh[x],x]
```

```
[Out] -ArcTan[Sinh[x]]/4 - ArcTan[Sqrt[2]*Sinh[x]]/(2*Sqrt[2]) + Sinh[x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1676

```
Int[(Pq_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Int[ExpandInte
grand[Pq/(a + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^
2] && Expon[Pq, x^2] > 1
```

Rubi steps

$$\int \coth(4x)\sinh(x)\,dx = \text{Subst}\left(\int \frac{1+8x^2+8x^4}{4+12x^2+8x^4}\,dx, x, \sinh(x)\right)$$
$$= \text{Subst}\left(\int \left(1-\frac{3+4x^2}{4+12x^2+8x^4}\right)dx, x, \sinh(x)\right)$$
$$= \sinh(x) - \text{Subst}\left(\int \frac{3+4x^2}{4+12x^2+8x^4}\,dx, x, \sinh(x)\right)$$
$$= \sinh(x) - 2\,\text{Subst}\left(\int \frac{1}{4+8x^2}\,dx, x, \sinh(x)\right) - 2\,\text{Subst}\left(\int \frac{1}{8+8x^2}\,dx, x, \sinh(x)\right)$$
$$= -\frac{1}{4}\tan^{-1}(\sinh(x)) - \frac{\tan^{-1}\left(\sqrt{2}\sinh(x)\right)}{2\sqrt{2}} + \sinh(x)$$

Mathematica [A] time = 0.03, size = 28, normalized size = 1.00

$$\sinh(x) - \frac{1}{4}\tan^{-1}(\sinh(x)) - \frac{\tan^{-1}\left(\sqrt{2}\sinh(x)\right)}{2\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[4*x]*Sinh[x],x]
[Out] -1/4*ArcTan[Sinh[x]] - ArcTan[Sqrt[2]*Sinh[x]]/(2*Sqrt[2]) + Sinh[x]
```

fricas [B] time = 0.50, size = 128, normalized size = 4.57

$$-\frac{\left(\sqrt{2}\cosh(x)+\sqrt{2}\sinh(x)\right)\arctan\left(\frac{1}{2}\sqrt{2}\cosh(x)+\frac{1}{2}\sqrt{2}\sinh(x)\right)-\left(\sqrt{2}\cosh(x)+\sqrt{2}\sinh(x)\right)\arctan\left(-\frac{\sqrt{2}}{\cdots}\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(4*x)*sinh(x),x, algorithm="fricas")
[Out] -1/4*((sqrt(2)*cosh(x) + sqrt(2)*sinh(x))*arctan(1/2*sqrt(2)*cosh(x) + 1/2*
sqrt(2)*sinh(x)) - (sqrt(2)*cosh(x) + sqrt(2)*sinh(x))*arctan(-1/2*(sqrt(2)
*cosh(x)^2 + 2*sqrt(2)*cosh(x)*sinh(x) + sqrt(2)*sinh(x)^2 + sqrt(2))/(cosh
(x) - sinh(x))) + 2*(cosh(x) + sinh(x))*arctan(cosh(x) + sinh(x)) - 2*cosh(
x)^2 - 4*cosh(x)*sinh(x) - 2*sinh(x)^2 + 2)/(cosh(x) + sinh(x))
```

giac [B] time = 0.14, size = 54, normalized size = 1.93

$$-\frac{1}{8}\pi - \frac{1}{8}\sqrt{2}\left(\pi + 2\arctan\left(\frac{1}{2}\sqrt{2}\left(e^{(2x)}-1\right)e^{(-x)}\right)\right) - \frac{1}{4}\arctan\left(\frac{1}{2}\left(e^{(2x)}-1\right)e^{(-x)}\right) - \frac{1}{2}e^{(-x)} + \frac{1}{2}e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(4*x)*sinh(x),x, algorithm="giac")
```

```
[Out] -1/8*pi - 1/8*sqrt(2)*(pi + 2*arctan(1/2*sqrt(2)*(e^(2*x) - 1)*e^(-x))) - 1
/4*arctan(1/2*(e^(2*x) - 1)*e^(-x)) - 1/2*e^(-x) + 1/2*e^x
```

maple [B] time = 0.31, size = 143, normalized size = 5.11

$$-\frac{1}{\tanh\left(\frac{x}{2}\right)+1}+\frac{\sqrt{2}\ \arctan\left(\frac{2\tanh\left(\frac{x}{2}\right)}{-2+2\sqrt{2}}\right)}{-4+4\sqrt{2}}-\frac{\arctan\left(\frac{2\tanh\left(\frac{x}{2}\right)}{-2+2\sqrt{2}}\right)}{-2+2\sqrt{2}}-\frac{\sqrt{2}\ \arctan\left(\frac{2\tanh\left(\frac{x}{2}\right)}{2+2\sqrt{2}}\right)}{2\left(2+2\sqrt{2}\right)}-\frac{\arctan\left(\frac{2\tanh\left(\frac{x}{2}\right)}{2+2\sqrt{2}}\right)}{2+2\sqrt{2}}-\frac{\arctan\left(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(4*x)*sinh(x),x)
```

```
[Out] -1/(tanh(1/2*x)+1)+1/2*2^(1/2)/(-2+2*2^(1/2))*arctan(2*tanh(1/2*x)/(-2+2*2^
(1/2)))-1/(-2+2*2^(1/2))*arctan(2*tanh(1/2*x)/(-2+2*2^(1/2)))-1/2*2^(1/2)/(
2+2*2^(1/2))*arctan(2*tanh(1/2*x)/(2+2*2^(1/2)))-1/(2+2*2^(1/2))*arctan(2*t
anh(1/2*x)/(2+2*2^(1/2)))-1/2*arctan(tanh(1/2*x))-1/(tanh(1/2*x)-1)
```

maxima [B] time = 0.46, size = 60, normalized size = 2.14

$$\frac{1}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2e^{(-x)}\right)\right)+\frac{1}{4}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2e^{(-x)}\right)\right)+\frac{1}{2}\arctan\left(e^{(-x)}\right)-\frac{1}{2}e^{(-x)}+\frac{1}{2}e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(4*x)*sinh(x),x, algorithm="maxima")
```

```
[Out] 1/4*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*e^(-x))) + 1/4*sqrt(2)*arctan(-
1/2*sqrt(2)*(sqrt(2) - 2*e^(-x))) + 1/2*arctan(e^(-x)) - 1/2*e^(-x) + 1/2*e
^x
```

mupad [B] time = 1.43, size = 52, normalized size = 1.86

$$\frac{e^x}{2}-\frac{\mathrm{atan}\left(e^x\right)}{2}-\frac{e^{-x}}{2}-\frac{\sqrt{2}\ \mathrm{atan}\left(\frac{\sqrt{2}\ e^x}{2}+\frac{\sqrt{2}\ e^{3\,x}}{2}\right)}{4}-\frac{\sqrt{2}\ \mathrm{atan}\left(\frac{\sqrt{2}\ e^x}{2}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(4*x)*sinh(x),x)
```

```
[Out] exp(x)/2 - atan(exp(x))/2 - exp(-x)/2 - (2^(1/2)*atan((2^(1/2)*exp(x))/2 +
(2^(1/2)*exp(3*x))/2))/4 - (2^(1/2)*atan((2^(1/2)*exp(x))/2))/4
```

<u>**sympy**</u> **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \sinh(x)\coth(4x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(4*x)*sinh(x),x)
```

```
[Out] Integral(sinh(x)*coth(4*x), x)
```

3.209 $\int \coth(5x)\sinh(x)\,dx$

Optimal. Leaf size=82

$$\sinh(x)-\frac{1}{5}\sqrt{\frac{1}{2}\left(5+\sqrt{5}\right)}\tan^{-1}\left(2\sqrt{\frac{2}{5+\sqrt{5}}}\sinh(x)\right)-\frac{1}{5}\sqrt{\frac{1}{2}\left(5-\sqrt{5}\right)}\tan^{-1}\left(\sqrt{\frac{2}{5}\left(5+\sqrt{5}\right)}\sinh(x)\right)$$

```
[Out] sinh(x)-1/10*arctan(1/5*sinh(x)*(50+10*5^(1/2))^(1/2))*(10-2*5^(1/2))^(1/2)
-1/10*arctan(2*sinh(x)*2^(1/2)/(5+5^(1/2))^(1/2))*(10+2*5^(1/2))^(1/2)
```

Rubi [A] time = 0.19, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {1676, 1166, 203}

$$\sinh(x)-\frac{1}{5}\sqrt{\frac{1}{2}\left(5+\sqrt{5}\right)}\tan^{-1}\left(2\sqrt{\frac{2}{5+\sqrt{5}}}\sinh(x)\right)-\frac{1}{5}\sqrt{\frac{1}{2}\left(5-\sqrt{5}\right)}\tan^{-1}\left(\sqrt{\frac{2}{5}\left(5+\sqrt{5}\right)}\sinh(x)\right)$$

Antiderivative was successfully verified.

```
[In] Int[Coth[5*x]*Sinh[x],x]
```

```
[Out] -(Sqrt[(5 + Sqrt[5])/2]*ArcTan[2*Sqrt[2/(5 + Sqrt[5])]*Sinh[x]])/5 - (Sqrt[
(5 - Sqrt[5])/2]*ArcTan[Sqrt[(2*(5 + Sqrt[5]))/5]*Sinh[x]])/5 + Sinh[x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1676

```
Int[(Pq_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Int[ExpandInte
grand[Pq/(a + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^
2] && Expon[Pq, x^2] > 1
```

Rubi steps

$$\int \coth(5x)\sinh(x)\,dx = \text{Subst}\left(\int \frac{1+12x^2+16x^4}{5+20x^2+16x^4}\,dx, x, \sinh(x)\right)$$
$$= \text{Subst}\left(\int \left(1-\frac{4(1+2x^2)}{5+20x^2+16x^4}\right)dx, x, \sinh(x)\right)$$
$$= \sinh(x) - 4\,\text{Subst}\left(\int \frac{1+2x^2}{5+20x^2+16x^4}\,dx, x, \sinh(x)\right)$$
$$= \sinh(x) - \frac{1}{5}\left(4\left(5-\sqrt{5}\right)\right)\text{Subst}\left(\int \frac{1}{10-2\sqrt{5}+16x^2}\,dx, x, \sinh(x)\right) - \frac{1}{5}\left(4\left(5+\sqrt{5}\right)\right)$$
$$= -\frac{1}{5}\sqrt{\frac{1}{2}\left(5+\sqrt{5}\right)}\tan^{-1}\left(2\sqrt{\frac{2}{5+\sqrt{5}}}\sinh(x)\right) - \frac{1}{5}\sqrt{\frac{1}{2}\left(5-\sqrt{5}\right)}\tan^{-1}\left(\sqrt{\frac{2}{5}\left(5+\sqrt{5}\right)}\right.$$

Mathematica [A] time = 0.24, size = 76, normalized size = 0.93

$$\frac{1}{10}\left(10\sinh(x)-\sqrt{10-2\sqrt{5}}\tan^{-1}\left(\sqrt{2+\frac{2}{\sqrt{5}}}\sinh(x)\right)-\sqrt{2\left(5+\sqrt{5}\right)}\tan^{-1}\left(2\sqrt{\frac{2}{5+\sqrt{5}}}\sinh(x)\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[5*x]*Sinh[x],x]
```

```
[Out] (-(Sqrt[10 - 2*Sqrt[5]]*ArcTan[Sqrt[2 + 2/Sqrt[5]]*Sinh[x]]) - Sqrt[2*(5 +
Sqrt[5])]*ArcTan[2*Sqrt[2/(5 + Sqrt[5])]*Sinh[x]] + 10*Sinh[x])/10
```

fricas [B] time = 0.47, size = 230, normalized size = 2.80

$$-\frac{1}{10}\left(2\sqrt{2}\sqrt{\sqrt{5}+5}\arctan\left(\frac{1}{40}\left(\sqrt{2\left(\sqrt{5}+1\right)e^{(2x)}+4e^{(4x)}+4}\left(\sqrt{5}\sqrt{2}-5\sqrt{2}\right)\sqrt{\sqrt{5}+5}-2\left(\left(\sqrt{5}\sqrt{2}-5\sqrt{\right.\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(5*x)*sinh(x),x, algorithm="fricas")
```

```
[Out] -1/10*(2*sqrt(2)*sqrt(sqrt(5) + 5)*arctan(1/40*(sqrt(2*(sqrt(5) + 1)*e^(2*x
) + 4*e^(4*x) + 4)*(sqrt(5)*sqrt(2) - 5*sqrt(2))*sqrt(sqrt(5) + 5) - 2*((sq
rt(5)*sqrt(2) - 5*sqrt(2))*e^(2*x) - sqrt(5)*sqrt(2) + 5*sqrt(2))*sqrt(sqrt
(5) + 5))*e^(-x))*e^x - 2*sqrt(2)*sqrt(-sqrt(5) + 5)*arctan(1/40*(sqrt(-2*(
sqrt(5) - 1)*e^(2*x) + 4*e^(4*x) + 4)*(sqrt(5)*sqrt(2) + 5*sqrt(2))*sqrt(-s
```

```
qrt(5) + 5) - 2*((sqrt(5)*sqrt(2) + 5*sqrt(2))*e^(2*x) - sqrt(5)*sqrt(2) -
5*sqrt(2))*sqrt(-sqrt(5) + 5))*e^(-x))*e^x - 5*e^(2*x) + 5)*e^(-x)
```

giac [A] time = 0.16, size = 75, normalized size = 0.91

$$-\frac{1}{10}\sqrt{2\sqrt{5}+10}\arctan\left(-\frac{e^{(-x)}-e^{x}}{\sqrt{\frac{1}{2}\sqrt{5}+\frac{5}{2}}}\right)-\frac{1}{10}\sqrt{-2\sqrt{5}+10}\arctan\left(-\frac{e^{(-x)}-e^{x}}{\sqrt{-\frac{1}{2}\sqrt{5}+\frac{5}{2}}}\right)-\frac{1}{2}e^{(-x)}+\frac{1}{2}e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(5*x)*sinh(x),x, algorithm="giac")
```

```
[Out] -1/10*sqrt(2*sqrt(5) + 10)*arctan(-(e^(-x) - e^x)/sqrt(1/2*sqrt(5) + 5/2))
- 1/10*sqrt(-2*sqrt(5) + 10)*arctan(-(e^(-x) - e^x)/sqrt(-1/2*sqrt(5) + 5/2
)) - 1/2*e^(-x) + 1/2*e^x
```

maple [B] time = 0.40, size = 246, normalized size = 3.00

$$-\frac{1}{\tanh\left(\frac{x}{2}\right)+1}-\frac{\arctan\left(\frac{\tanh\left(\frac{x}{2}\right)}{\sqrt{5+2\sqrt{5}}}\right)}{2\sqrt{5+2\sqrt{5}}}-\frac{\sqrt{5}\arctan\left(\frac{\tanh\left(\frac{x}{2}\right)}{\sqrt{5+2\sqrt{5}}}\right)}{10\sqrt{5+2\sqrt{5}}}+\frac{\sqrt{5}\arctan\left(\frac{\tanh\left(\frac{x}{2}\right)}{\sqrt{5-2\sqrt{5}}}\right)}{10\sqrt{5-2\sqrt{5}}}-\frac{\arctan\left(\frac{\tanh\left(\frac{x}{2}\right)}{\sqrt{5-2\sqrt{5}}}\right)}{2\sqrt{5-2\sqrt{5}}}+\frac{\sqrt{5}\ \mathrm{arc}}{2\surd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(5*x)*sinh(x),x)
```

```
[Out] -1/(tanh(1/2*x)+1)-1/2/(5+2*5^(1/2))^(1/2)*arctan(tanh(1/2*x)/(5+2*5^(1/2))
^(1/2))-1/10*5^(1/2)/(5+2*5^(1/2))^(1/2)*arctan(tanh(1/2*x)/(5+2*5^(1/2))^(
1/2))+1/10*5^(1/2)/(5-2*5^(1/2))^(1/2)*arctan(tanh(1/2*x)/(5-2*5^(1/2))^(1/
2))-1/2/(5-2*5^(1/2))^(1/2)*arctan(tanh(1/2*x)/(5-2*5^(1/2))^(1/2))+1/2*5^(
1/2)/(25-10*5^(1/2))^(1/2)*arctan(5*tanh(1/2*x)/(25-10*5^(1/2))^(1/2))-3/2/
(25-10*5^(1/2))^(1/2)*arctan(5*tanh(1/2*x)/(25-10*5^(1/2))^(1/2))-1/2*5^(1/
2)/(25+10*5^(1/2))^(1/2)*arctan(5*tanh(1/2*x)/(25+10*5^(1/2))^(1/2))-3/2/(2
5+10*5^(1/2))^(1/2)*arctan(5*tanh(1/2*x)/(25+10*5^(1/2))^(1/2))-1/(tanh(1/2
*x)-1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{2}\left(e^{(2x)}-1\right)e^{(-x)}-\frac{1}{2}\int\frac{e^{(3x)}+e^{(2x)}+e^{x}}{e^{(4x)}+e^{(3x)}+e^{(2x)}+e^{x}+1}\,dx-\frac{1}{2}\int\frac{e^{(3x)}-e^{(2x)}+e^{x}}{e^{(4x)}-e^{(3x)}+e^{(2x)}-e^{x}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(5*x)*sinh(x),x, algorithm="maxima")
```

```
[Out] 1/2*(e^(2*x) - 1)*e^(-x) - 1/2*integrate((e^(3*x) + e^(2*x) + e^x)/(e^(4*x)
 + e^(3*x) + e^(2*x) + e^x + 1), x) - 1/2*integrate((e^(3*x) - e^(2*x) + e^
x)/(e^(4*x) - e^(3*x) + e^(2*x) - e^x + 1), x)
```

mupad [B] time = 3.33, size = 141, normalized size = 1.72

$$\frac{e^x}{2}-\frac{e^{-x}}{2}+\ln\left(40\,e^x\,\sqrt{-\frac{\sqrt{5}}{200}-\frac{1}{40}}-4\,e^{2\,x}+4\right)\,\sqrt{-\frac{\sqrt{5}}{200}-\frac{1}{40}}+\ln\left(40\,e^x\,\sqrt{\frac{\sqrt{5}}{200}-\frac{1}{40}}-4\,e^{2\,x}+4\right)\,\sqrt{\frac{\sqrt{5}}{200}-\frac{1}{40}}-\mathrm{l}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(5*x)*sinh(x),x)
```

```
[Out] exp(x)/2 - exp(-x)/2 + log(40*exp(x)*(- 5^(1/2)/200 - 1/40)^(1/2) - 4*exp(2
*x) + 4)*(- 5^(1/2)/200 - 1/40)^(1/2) + log(40*exp(x)*(5^(1/2)/200 - 1/40)^
(1/2) - 4*exp(2*x) + 4)*(5^(1/2)/200 - 1/40)^(1/2) - log(4*exp(2*x) + 40*ex
p(x)*(- 5^(1/2)/200 - 1/40)^(1/2) - 4)*(- 5^(1/2)/200 - 1/40)^(1/2) - log(4
*exp(2*x) + 40*exp(x)*(5^(1/2)/200 - 1/40)^(1/2) - 4)*(5^(1/2)/200 - 1/40)^
(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sinh(x)\coth(5x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(5*x)*sinh(x),x)
```

```
[Out] Integral(sinh(x)*coth(5*x), x)
```

3.210 $\int \coth(6x)\sinh(x)\,dx$

Optimal. Leaf size=38

$$\sinh(x)-\frac{1}{6}\tan^{-1}(\sinh(x))-\frac{1}{6}\tan^{-1}(2\sinh(x))-\frac{\tan^{-1}\left(\frac{2\sinh(x)}{\sqrt{3}}\right)}{2\sqrt{3}}$$

```
[Out] -1/6*arctan(sinh(x))-1/6*arctan(2*sinh(x))+sinh(x)-1/6*arctan(2/3*sinh(x)*3
^(1/2))*3^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 3, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {12, 2073, 203}

$$\sinh(x)-\frac{1}{6}\tan^{-1}(\sinh(x))-\frac{1}{6}\tan^{-1}(2\sinh(x))-\frac{\tan^{-1}\left(\frac{2\sinh(x)}{\sqrt{3}}\right)}{2\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[6*x]*Sinh[x],x]
```

```
[Out] -ArcTan[Sinh[x]]/6 - ArcTan[2*Sinh[x]]/6 - ArcTan[(2*Sinh[x])/Sqrt[3]]/(2*S
qrt[3]) + Sinh[x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 2073

```
Int[(P_)^(p_)*(Q_)^(q_.), x_Symbol] :> With[{PP = Factor[P /. x -> Sqrt[x]]
}, Int[ExpandIntegrand[(PP /. x -> x^2)^p*Q^q, x], x] /;  !SumQ[NonfreeFact
ors[PP, x]]] /; FreeQ[q, x] && PolyQ[P, x^2] && PolyQ[Q, x] && ILtQ[p, 0]
```

Rubi steps

$$\int \coth(6x)\sinh(x)\,dx = \text{Subst}\left(\int \frac{1+18x^2+48x^4+32x^6}{2\left(3+19x^2+32x^4+16x^6\right)}\,dx, x, \sinh(x)\right)$$
$$= \frac{1}{2}\text{Subst}\left(\int \frac{1+18x^2+48x^4+32x^6}{3+19x^2+32x^4+16x^6}\,dx, x, \sinh(x)\right)$$
$$= \frac{1}{2}\text{Subst}\left(\int \left(2-\frac{1}{3\left(1+x^2\right)}-\frac{2}{3\left(1+4x^2\right)}-\frac{2}{3+4x^2}\right)dx, x, \sinh(x)\right)$$
$$= \sinh(x)-\frac{1}{6}\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sinh(x)\right)-\frac{1}{3}\text{Subst}\left(\int \frac{1}{1+4x^2}\,dx, x, \sinh(x)\right)-\text{Su}$$
$$= -\frac{1}{6}\tan^{-1}(\sinh(x))-\frac{1}{6}\tan^{-1}(2\sinh(x))-\frac{\tan^{-1}\left(\frac{2\sinh(x)}{\sqrt{3}}\right)}{2\sqrt{3}}+\sinh(x)$$

Mathematica [A] time = 0.06, size = 38, normalized size = 1.00

$$\sinh(x)-\frac{1}{6}\tan^{-1}(\sinh(x))-\frac{1}{6}\tan^{-1}(2\sinh(x))-\frac{\tan^{-1}\left(\frac{2\sinh(x)}{\sqrt{3}}\right)}{2\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[6*x]*Sinh[x],x]
```

```
[Out] -1/6*ArcTan[Sinh[x]] - ArcTan[2*Sinh[x]]/6 - ArcTan[(2*Sinh[x])/Sqrt[3]]/(2
*Sqrt[3]) + Sinh[x]
```

fricas [B] time = 0.46, size = 164, normalized size = 4.32

$$-\frac{\left(\sqrt{3}\cosh(x)+\sqrt{3}\sinh(x)\right)\arctan\left(\frac{1}{3}\sqrt{3}\cosh(x)+\frac{1}{3}\sqrt{3}\sinh(x)\right)-\left(\sqrt{3}\cosh(x)+\sqrt{3}\sinh(x)\right)\arctan\left(-\frac{\sqrt{3}}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(6*x)*sinh(x),x, algorithm="fricas")
```

```
[Out] -1/6*((sqrt(3)*cosh(x) + sqrt(3)*sinh(x))*arctan(1/3*sqrt(3)*cosh(x) + 1/3*
sqrt(3)*sinh(x)) - (sqrt(3)*cosh(x) + sqrt(3)*sinh(x))*arctan(-1/3*(sqrt(3)
*cosh(x)^2 + 2*sqrt(3)*cosh(x)*sinh(x) + sqrt(3)*sinh(x)^2 + 2*sqrt(3))/(co
sh(x) - sinh(x))) - (cosh(x) + sinh(x))*arctan(-(cosh(x)^2 + 2*cosh(x)*sinh
(x) + sinh(x)^2)/(cosh(x) - sinh(x))) + 3*(cosh(x) + sinh(x))*arctan(cosh(x
```

```
) + sinh(x)) - 3*cosh(x)^2 - 6*cosh(x)*sinh(x) - 3*sinh(x)^2 + 3)/(cosh(x)
+ sinh(x))
```

giac [B] time = 0.15, size = 68, normalized size = 1.79

$$-\frac{1}{6}\pi-\frac{1}{12}\sqrt{3}\left(\pi+2\arctan\left(\frac{1}{3}\sqrt{3}\left(e^{(2x)}-1\right)e^{(-x)}\right)\right)-\frac{1}{6}\arctan\left(\left(e^{(2x)}-1\right)e^{(-x)}\right)-\frac{1}{6}\arctan\left(\frac{1}{2}\left(e^{(2x)}-1\right)e^{(-x)}\right)-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(6*x)*sinh(x),x, algorithm="giac")
```

```
[Out] -1/6*pi - 1/12*sqrt(3)*(pi + 2*arctan(1/3*sqrt(3)*(e^(2*x) - 1)*e^(-x))) -
1/6*arctan((e^(2*x) - 1)*e^(-x)) - 1/6*arctan(1/2*(e^(2*x) - 1)*e^(-x)) - 1
/2*e^(-x) + 1/2*e^x
```

maple [B] time = 0.34, size = 172, normalized size = 4.53

$$-\frac{1}{\tanh\left(\frac{x}{2}\right)+1}-\frac{\sqrt{3}\arctan\left(\frac{\tanh\left(\frac{x}{2}\right)\sqrt{3}}{3}\right)}{6}-\frac{\sqrt{3}\arctan\left(\tanh\left(\frac{x}{2}\right)\sqrt{3}\right)}{6}-\frac{\arctan\left(\tanh\left(\frac{x}{2}\right)\right)}{3}-\frac{\sqrt{3}\arctan\left(\frac{2\tanh\left(\frac{x}{2}\right)}{4+2\sqrt{3}}\right)}{3\left(4+2\sqrt{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(6*x)*sinh(x),x)
```

```
[Out] -1/(tanh(1/2*x)+1)-1/6*3^(1/2)*arctan(1/3*tanh(1/2*x)*3^(1/2))-1/6*3^(1/2)*
arctan(tanh(1/2*x)*3^(1/2))-1/3*arctan(tanh(1/2*x))-1/3*3^(1/2)/(4+2*3^(1/2
))*arctan(2*tanh(1/2*x)/(4+2*3^(1/2)))-2/3/(4+2*3^(1/2))*arctan(2*tanh(1/2*
x)/(4+2*3^(1/2)))+1/3*3^(1/2)/(4-2*3^(1/2))*arctan(2*tanh(1/2*x)/(4-2*3^(1/
2)))-2/3/(4-2*3^(1/2))*arctan(2*tanh(1/2*x)/(4-2*3^(1/2)))-1/(tanh(1/2*x)-1
)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{2}\left(e^{(2x)}-1\right)e^{(-x)}-\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2e^{x}+1\right)\right)-\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2e^{x}-1\right)\right)-\frac{1}{3}\arctan\left(e^{x}\right)-\frac{1}{2}\int\frac{e^{(3x}}{3\left(e^{(4x)}-\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(6*x)*sinh(x),x, algorithm="maxima")
```

```
[Out] 1/2*(e^(2*x) - 1)*e^(-x) - 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*e^x + 1)) - 1/
6*sqrt(3)*arctan(1/3*sqrt(3)*(2*e^x - 1)) - 1/3*arctan(e^x) - 1/2*integrate
(1/3*(e^(3*x) + e^x)/(e^(4*x) - e^(2*x) + 1), x)
```

mupad [B] time = 1.61, size = 56, normalized size = 1.47

$$\frac{e^x}{2}-\frac{\operatorname{atan}\left(e^x\right)}{3}-\frac{e^{-x}}{2}-\frac{\operatorname{atan}\left(36\,e^{-x}\,\left(\frac{e^{2\,x}}{36}-\frac{1}{36}\right)\right)}{6}-\frac{\sqrt{3}\,\operatorname{atan}\left(4\,\sqrt{3}\,e^{-x}\,\left(\frac{e^{2\,x}}{12}-\frac{1}{12}\right)\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(6*x)*sinh(x),x)
```

```
[Out] exp(x)/2 - atan(exp(x))/3 - exp(-x)/2 - atan(36*exp(-x)*(exp(2*x)/36 - 1/36
))/6 - (3^(1/2)*atan(4*3^(1/2)*exp(-x)*(exp(2*x)/12 - 1/12)))/6
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sinh (x) \coth (6x)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(6*x)*sinh(x),x)
```

```
[Out] Integral(sinh(x)*coth(6*x), x)
```

3.211 $\int \operatorname{sech}(2x)\sinh(x)\,dx$

Optimal. Leaf size=16

$$-\frac{\tanh^{-1}\left(\sqrt{2}\cosh(x)\right)}{\sqrt{2}}$$

```
[Out] -1/2*arctanh(cosh(x)*2^(1/2))*2^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {4357, 207}

$$-\frac{\tanh^{-1}\left(\sqrt{2}\cosh(x)\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[2*x]*Sinh[x],x]
```

```
[Out] -(ArcTanh[Sqrt[2]*Cosh[x]]/Sqrt[2])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 4357

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Cos[c*(a + b*x)], x]}, -Dist[d/(b*c), Subst[Int[SubstFor[1, Cos[c*(a + b
*x)]/d, u, x], x], x, Cos[c*(a + b*x)]/d], x] /; FunctionOfQ[Cos[c*(a + b*x
)]/d, u, x]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Sin] || EqQ[F, sin])
```

Rubi steps

$$\begin{aligned}\int \operatorname{sech}(2x)\sinh(x)\,dx &= \operatorname{Subst}\left(\int \frac{1}{-1+2x^2}\,dx, x, \cosh(x)\right)\\ &= -\frac{\tanh^{-1}\left(\sqrt{2}\cosh(x)\right)}{\sqrt{2}}\end{aligned}$$

Mathematica [C] time = 0.30, size = 155, normalized size = 9.69

$$\frac{-4\tanh^{-1}\left(\sqrt{2}-i\tanh\left(\frac{x}{2}\right)\right)+\log\left(\sqrt{2}-2\cosh(x)\right)-\log\left(2\cosh(x)+\sqrt{2}\right)-2i\tan^{-1}\left(\frac{\sinh\left(\frac{x}{2}\right)+\cosh\left(\frac{x}{2}\right)}{\left(1+\sqrt{2}\right)\cosh\left(\frac{x}{2}\right)-\left(\sqrt{2}-1\right)\sin}\right.}{4\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[2*x]*Sinh[x],x]
```

```
[Out] ((-2*I)*ArcTan[(Cosh[x/2] + Sinh[x/2])/((1 + Sqrt[2])*Cosh[x/2] - (-1 + Sqr
t[2])*Sinh[x/2])] + (2*I)*ArcTan[(Cosh[x/2] + Sinh[x/2])/((-1 + Sqrt[2])*Co
sh[x/2] - (1 + Sqrt[2])*Sinh[x/2])] - 4*ArcTanh[Sqrt[2] - I*Tanh[x/2]] + Lo
g[Sqrt[2] - 2*Cosh[x]] - Log[Sqrt[2] + 2*Cosh[x]])/(4*Sqrt[2])
```

fricas [B] time = 0.49, size = 35, normalized size = 2.19

$$\frac{1}{4}\sqrt{2}\log\left(\frac{\cosh(x)^2+\sinh(x)^2-2\sqrt{2}\cosh(x)+2}{\cosh(x)^2+\sinh(x)^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(2*x)*sinh(x),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(2)*log((cosh(x)^2 + sinh(x)^2 - 2*sqrt(2)*cosh(x) + 2)/(cosh(x)^2
+ sinh(x)^2))
```

giac [B] time = 0.13, size = 38, normalized size = 2.38

$$-\frac{1}{4}\sqrt{2}\log\left(\sqrt{2}e^x+e^{(2x)}+1\right)+\frac{1}{4}\sqrt{2}\log\left(-\sqrt{2}e^x+e^{(2x)}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(2*x)*sinh(x),x, algorithm="giac")
```

```
[Out] -1/4*sqrt(2)*log(sqrt(2)*e^x + e^(2*x) + 1) + 1/4*sqrt(2)*log(-sqrt(2)*e^x
+ e^(2*x) + 1)
```

maple [B] time = 0.23, size = 39, normalized size = 2.44

$$\frac{\ln\left(1+e^{2x}-e^x\sqrt{2}\right)\sqrt{2}}{4}-\frac{\ln\left(1+e^{2x}+e^x\sqrt{2}\right)\sqrt{2}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(2*x)*sinh(x),x)
```

[Out] 1/4*ln(1+exp(2*x)-exp(x)*2^(1/2))*2^(1/2)-1/4*ln(1+exp(2*x)+exp(x)*2^(1/2))*2^(1/2)

maxima [B] time = 0.40, size = 42, normalized size = 2.62

$$-\frac{1}{4}\sqrt{2}\log\left(\sqrt{2}e^{(-x)} + e^{(-2x)} + 1\right) + \frac{1}{4}\sqrt{2}\log\left(-\sqrt{2}e^{(-x)} + e^{(-2x)} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(2*x)*sinh(x),x, algorithm="maxima")

[Out] -1/4*sqrt(2)*log(sqrt(2)*e^(-x) + e^(-2*x) + 1) + 1/4*sqrt(2)*log(-sqrt(2)*e^(-x) + e^(-2*x) + 1)

mupad [B] time = 0.09, size = 35, normalized size = 2.19

$$-\frac{\sqrt{2}\left(\ln\left(e^{2x} + \sqrt{2}\,e^{x} + 1\right) - \ln\left(e^{2x} - \sqrt{2}\,e^{x} + 1\right)\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sinh(x)/cosh(2*x),x)

[Out] -(2^(1/2)*(log(exp(2*x) + 2^(1/2)*exp(x) + 1) - log(exp(2*x) - 2^(1/2)*exp(x) + 1)))/4

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sinh(x)\operatorname{sech}(2x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(2*x)*sinh(x),x)

[Out] Integral(sinh(x)*sech(2*x), x)

3.212 $\int \operatorname{sech}(3x)\sinh(x)\,dx$

Optimal. Leaf size=21

$$\frac{1}{6}\log\left(3-4\cosh^2(x)\right)-\frac{1}{3}\log(\cosh(x))$$

```
[Out] -1/3*ln(cosh(x))+1/6*ln(3-4*cosh(x)^2)
```

Rubi [A] time = 0.03, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.714, Rules used = {4357, 266, 36, 29, 31}

$$\frac{1}{6}\log\left(3-4\cosh^2(x)\right)-\frac{1}{3}\log(\cosh(x))$$

Antiderivative was successfully verified.

```
[In] Int[Sech[3*x]*Sinh[x],x]
```

```
[Out] -Log[Cosh[x]]/3 + Log[3 - 4*Cosh[x]^2]/6
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4357

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Cos[c*(a + b*x)], x]}, -Dist[d/(b*c), Subst[Int[SubstFor[1, Cos[c*(a + b
```

```
*x)]/d, u, x], x], x, Cos[c*(a + b*x)]/d], x] /; FunctionOfQ[Cos[c*(a + b*x
)]/d, u, x]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Sin] || EqQ[F, sin])
```

Rubi steps

$$\begin{aligned}\int \operatorname{sech}(3x)\sinh(x)\,dx &= \operatorname{Subst}\left(\int \frac{1}{x\left(-3+4x^2\right)}\,dx, x, \cosh(x)\right)\\ &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{x(-3+4x)}\,dx, x, \cosh^2(x)\right)\\ &= -\left(\frac{1}{6}\operatorname{Subst}\left(\int \frac{1}{x}\,dx, x, \cosh^2(x)\right)\right)+\frac{2}{3}\operatorname{Subst}\left(\int \frac{1}{-3+4x}\,dx, x, \cosh^2(x)\right)\\ &= -\frac{1}{3}\log(\cosh(x))+\frac{1}{6}\log\left(3-4\cosh^2(x)\right)\end{aligned}$$

Mathematica [A] time = 0.01, size = 17, normalized size = 0.81

$$-\frac{1}{3}\tanh^{-1}\left(\frac{1}{3}\left(8\sinh^2(x)+5\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[3*x]*Sinh[x],x]
[Out] -1/3*ArcTanh[(5 + 8*Sinh[x]^2)/3]
```

fricas [B] time = 0.43, size = 52, normalized size = 2.48

$$\frac{1}{6}\log\left(\frac{2\cosh(x)^2+2\sinh(x)^2-1}{\cosh(x)^2-2\cosh(x)\sinh(x)+\sinh(x)^2}\right)-\frac{1}{3}\log\left(\frac{2\cosh(x)}{\cosh(x)-\sinh(x)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(3*x)*sinh(x),x, algorithm="fricas")
[Out] 1/6*log((2*cosh(x)^2 + 2*sinh(x)^2 - 1)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + si
nh(x)^2)) - 1/3*log(2*cosh(x)/(cosh(x) - sinh(x)))
```

giac [B] time = 0.11, size = 41, normalized size = 1.95

$$\frac{1}{6}\log\left(\sqrt{3}e^x+e^{(2x)}+1\right)+\frac{1}{6}\log\left(-\sqrt{3}e^x+e^{(2x)}+1\right)-\frac{1}{3}\log\left(e^{(2x)}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(3*x)*sinh(x),x, algorithm="giac")
```

```
[Out] 1/6*log(sqrt(3)*e^x + e^(2*x) + 1) + 1/6*log(-sqrt(3)*e^x + e^(2*x) + 1) -
1/3*log(e^(2*x) + 1)
```

maple [A] time = 0.21, size = 26, normalized size = 1.24

$$-\frac{\ln\left(1+e^{2x}\right)}{3}+\frac{\ln\left(1-e^{2x}+e^{4x}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(3*x)*sinh(x),x)
```

```
[Out] -1/3*ln(1+exp(2*x))+1/6*ln(1-exp(2*x)+exp(4*x))
```

maxima [B] time = 0.44, size = 45, normalized size = 2.14

$$\frac{1}{6}\log\left(\sqrt{3}e^{(-x)}+e^{(-2x)}+1\right)+\frac{1}{6}\log\left(-\sqrt{3}e^{(-x)}+e^{(-2x)}+1\right)-\frac{1}{3}\log\left(e^{(-2x)}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(3*x)*sinh(x),x, algorithm="maxima")
```

```
[Out] 1/6*log(sqrt(3)*e^(-x) + e^(-2*x) + 1) + 1/6*log(-sqrt(3)*e^(-x) + e^(-2*x)
 + 1) - 1/3*log(e^(-2*x) + 1)
```

mupad [B] time = 1.46, size = 27, normalized size = 1.29

$$\frac{\ln\left(e^{2x}-e^{4x}-1\right)}{6}-\frac{\ln\left(3e^{2x}+3\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)/cosh(3*x),x)
```

```
[Out] log(exp(2*x) - exp(4*x) - 1)/6 - log(3*exp(2*x) + 3)/3
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sinh(x)\operatorname{sech}(3x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(3*x)*sinh(x),x)
```

```
[Out] Integral(sinh(x)*sech(3*x), x)
```

3.213 $\int \operatorname{sech}(4x)\sinh(x)\,dx$

Optimal. Leaf size=71

$$\frac{\tanh^{-1}\left(\frac{2\cosh(x)}{\sqrt{2-\sqrt{2}}}\right)}{2\sqrt{2\left(2-\sqrt{2}\right)}}-\frac{\tanh^{-1}\left(\frac{2\cosh(x)}{\sqrt{2+\sqrt{2}}}\right)}{2\sqrt{2\left(2+\sqrt{2}\right)}}$$

```
[Out] 1/2*arctanh(2*cosh(x)/(2-2^(1/2))^(1/2))/(4-2*2^(1/2))^(1/2)-1/2*arctanh(2*
cosh(x)/(2+2^(1/2))^(1/2))/(4+2*2^(1/2))^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {4357, 1093, 207}

$$\frac{\tanh^{-1}\left(\frac{2\cosh(x)}{\sqrt{2-\sqrt{2}}}\right)}{2\sqrt{2\left(2-\sqrt{2}\right)}}-\frac{\tanh^{-1}\left(\frac{2\cosh(x)}{\sqrt{2+\sqrt{2}}}\right)}{2\sqrt{2\left(2+\sqrt{2}\right)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[4*x]*Sinh[x],x]
```

```
[Out] ArcTanh[(2*Cosh[x])/Sqrt[2 - Sqrt[2]]]/(2*Sqrt[2*(2 - Sqrt[2])]) - ArcTanh[
(2*Cosh[x])/Sqrt[2 + Sqrt[2]]]/(2*Sqrt[2*(2 + Sqrt[2])])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 1093

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^2), x], x] - Dist[c/q, Int
[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
0] && PosQ[b^2 - 4*a*c]
```

Rule 4357

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Cos[c*(a + b*x)], x]}, -Dist[d/(b*c), Subst[Int[SubstFor[1, Cos[c*(a + b
```

```
*x)]/d, u, x], x], x, Cos[c*(a + b*x)]/d], x] /; FunctionOfQ[Cos[c*(a + b*x
)]/d, u, x]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Sin] || EqQ[F, sin])
```

Rubi steps

$$\int \operatorname{sech}(4x)\sinh(x)\,dx = \text{Subst}\left(\int \frac{1}{1-8x^2+8x^4}\,dx, x, \cosh(x)\right)$$
$$= \sqrt{2}\,\text{Subst}\left(\int \frac{1}{-4-2\sqrt{2}+8x^2}\,dx, x, \cosh(x)\right) - \sqrt{2}\,\text{Subst}\left(\int \frac{1}{-4+2\sqrt{2}+8x^2}\,dx, x,\right.$$
$$= \frac{\tanh^{-1}\left(\frac{2\cosh(x)}{\sqrt{2-\sqrt{2}}}\right)}{2\sqrt{2\left(2-\sqrt{2}\right)}} - \frac{\tanh^{-1}\left(\frac{2\cosh(x)}{\sqrt{2+\sqrt{2}}}\right)}{2\sqrt{2\left(2+\sqrt{2}\right)}}$$

Mathematica [C] time = 0.03, size = 110, normalized size = 1.55

$$\frac{1}{16}\text{RootSum}\left[\#1^8+1\&, \frac{\#1^2x+2\#1^2\log\left(-\#1\sinh\left(\frac{x}{2}\right)+\#1\cosh\left(\frac{x}{2}\right)-\sinh\left(\frac{x}{2}\right)-\cosh\left(\frac{x}{2}\right)\right)-2\log\left(-\#1\sinh\left(\right.\right.}{\#1^5}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[4*x]*Sinh[x],x]
```

```
[Out] RootSum[1 + #1^8 & , (-x - 2*Log[-Cosh[x/2] - Sinh[x/2] + Cosh[x/2]*#1 - Si
nh[x/2]*#1] + x*#1^2 + 2*Log[-Cosh[x/2] - Sinh[x/2] + Cosh[x/2]*#1 - Sinh[x
/2]*#1]*#1^2)/#1^5 & ]/16
```

fricas [B] time = 0.45, size = 215, normalized size = 3.03

$$\frac{1}{8}\sqrt{\sqrt{2}+2}\log\left(\cosh(x)^2+2\cosh(x)\sinh(x)+\sinh(x)^2+\left(\left(\sqrt{2}-1\right)\cosh(x)+\left(\sqrt{2}-1\right)\sinh(x)\right)\sqrt{\sqrt{2}+2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(4*x)*sinh(x),x, algorithm="fricas")
```

```
[Out] 1/8*sqrt(sqrt(2) + 2)*log(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + ((sqr
t(2) - 1)*cosh(x) + (sqrt(2) - 1)*sinh(x))*sqrt(sqrt(2) + 2) + 1) - 1/8*sqr
t(sqrt(2) + 2)*log(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - ((sqrt(2) -
1)*cosh(x) + (sqrt(2) - 1)*sinh(x))*sqrt(sqrt(2) + 2) + 1) - 1/8*sqrt(-sqrt
(2) + 2)*log(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + ((sqrt(2) + 1)*cos
```

```
h(x) + (sqrt(2) + 1)*sinh(x))*sqrt(-sqrt(2) + 2) + 1) + 1/8*sqrt(-sqrt(2) +
 2)*log(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - ((sqrt(2) + 1)*cosh(x)
+ (sqrt(2) + 1)*sinh(x))*sqrt(-sqrt(2) + 2) + 1)
```

giac [B] time = 0.19, size = 115, normalized size = 1.62

$$-\frac{1}{8}\sqrt{-\sqrt{2}+2}\log\left(\sqrt{\sqrt{2}+2}e^{x}+e^{(2x)}+1\right)+\frac{1}{8}\sqrt{-\sqrt{2}+2}\log\left(-\sqrt{\sqrt{2}+2}e^{x}+e^{(2x)}+1\right)+\frac{1}{8}\sqrt{\sqrt{2}+2}\log\left(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(4*x)*sinh(x),x, algorithm="giac")
```

```
[Out] -1/8*sqrt(-sqrt(2) + 2)*log(sqrt(sqrt(2) + 2)*e^x + e^(2*x) + 1) + 1/8*sqrt
(-sqrt(2) + 2)*log(-sqrt(sqrt(2) + 2)*e^x + e^(2*x) + 1) + 1/8*sqrt(sqrt(2)
 + 2)*log(sqrt(-sqrt(2) + 2)*e^x + e^(2*x) + 1) - 1/8*sqrt(sqrt(2) + 2)*log
(-sqrt(-sqrt(2) + 2)*e^x + e^(2*x) + 1)
```

maple [C] time = 0.25, size = 40, normalized size = 0.56

$$2\left(\sum_{\_R=\mathrm{RootOf}\left(32768\_Z^{4}-512\_Z^{2}+1\right)}\_R\ln\left(\mathrm{e}^{2x}+\left(4096\_R^{3}-48\_R\right)\mathrm{e}^{x}+1\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(4*x)*sinh(x),x)
```

```
[Out] 2*sum(_R*ln(exp(2*x)+(4096*_R^3-48*_R)*exp(x)+1),_R=RootOf(32768*_Z^4-512*_
Z^2+1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{sech}(4x)\sinh(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(4*x)*sinh(x),x, algorithm="maxima")
```

```
[Out] integrate(sech(4*x)*sinh(x), x)
```

mupad [B] time = 1.48, size = 251, normalized size = 3.54

$$\ln\left(3\,\mathrm{e}^{2\,x}-2\,\sqrt{2}+8\,\mathrm{e}^{x}\,\sqrt{\frac{1}{32}-\frac{\sqrt{2}}{64}}-2\,\sqrt{2}\,\mathrm{e}^{2\,x}-8\,\sqrt{2}\,\mathrm{e}^{x}\,\sqrt{\frac{1}{32}-\frac{\sqrt{2}}{64}}+3\right)\sqrt{\frac{1}{32}-\frac{\sqrt{2}}{64}}-\ln\left(3\,\mathrm{e}^{2\,x}-2\,\sqrt{2}-8\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)/cosh(4*x),x)
```

```
[Out] log(3*exp(2*x) - 2*2^(1/2) + 8*exp(x)*(1/32 - 2^(1/2)/64)^(1/2) - 2*2^(1/2)
*exp(2*x) - 8*2^(1/2)*exp(x)*(1/32 - 2^(1/2)/64)^(1/2) + 3)*(1/32 - 2^(1/2)
/64)^(1/2) - log(3*exp(2*x) - 2*2^(1/2) - 8*exp(x)*(1/32 - 2^(1/2)/64)^(1/2
) - 2*2^(1/2)*exp(2*x) + 8*2^(1/2)*exp(x)*(1/32 - 2^(1/2)/64)^(1/2) + 3)*(1
/32 - 2^(1/2)/64)^(1/2) - log(3*exp(2*x) + 2*2^(1/2) - 8*exp(x)*(2^(1/2)/64
 + 1/32)^(1/2) + 2*2^(1/2)*exp(2*x) - 8*2^(1/2)*exp(x)*(2^(1/2)/64 + 1/32)^
(1/2) + 3)*(2^(1/2)/64 + 1/32)^(1/2) + log(3*exp(2*x) + 2*2^(1/2) + 8*exp(x
)*(2^(1/2)/64 + 1/32)^(1/2) + 2*2^(1/2)*exp(2*x) + 8*2^(1/2)*exp(x)*(2^(1/2
)/64 + 1/32)^(1/2) + 3)*(2^(1/2)/64 + 1/32)^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sinh(x)\operatorname{sech}(4x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(4*x)*sinh(x),x)
```

```
[Out] Integral(sinh(x)*sech(4*x), x)
```

3.214 $\int \operatorname{sech}(5x)\sinh(x)\,dx$

Optimal. Leaf size=62

$$-\frac{1}{20}\left(1+\sqrt{5}\right)\log\left(-8\cosh^2(x)-\sqrt{5}+5\right)-\frac{1}{20}\left(1-\sqrt{5}\right)\log\left(-8\cosh^2(x)+\sqrt{5}+5\right)+\frac{1}{5}\log(\cosh(x))$$

```
[Out] 1/5*ln(cosh(x))-1/20*ln(5-8*cosh(x)^2+5^(1/2))*(-5^(1/2)+1)-1/20*ln(5-8*cos
h(x)^2-5^(1/2))*(5^(1/2)+1)
```

Rubi [A] time = 0.08, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.857, Rules used = {4357, 1114, 705, 29, 632, 31}

$$-\frac{1}{20}\left(1+\sqrt{5}\right)\log\left(-8\cosh^2(x)-\sqrt{5}+5\right)-\frac{1}{20}\left(1-\sqrt{5}\right)\log\left(-8\cosh^2(x)+\sqrt{5}+5\right)+\frac{1}{5}\log(\cosh(x))$$

Antiderivative was successfully verified.

```
[In] Int[Sech[5*x]*Sinh[x],x]
```

```
[Out] Log[Cosh[x]]/5 - ((1 + Sqrt[5])*Log[5 - Sqrt[5] - 8*Cosh[x]^2])/20 - ((1 -
Sqrt[5])*Log[5 + Sqrt[5] - 8*Cosh[x]^2])/20
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 632

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 705

```
Int[1/(((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)), x_Symbol]
 :> Dist[e^2/(c*d^2 - b*d*e + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d
^2 - b*d*e + a*e^2), Int[(c*d - b*e - c*e*x)/(a + b*x + c*x^2), x], x] /; F
```

```
reeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^
2, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 4357

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Cos[c*(a + b*x)], x]}, -Dist[d/(b*c), Subst[Int[SubstFor[1, Cos[c*(a + b
*x)]/d, u, x], x], x, Cos[c*(a + b*x)]/d], x] /; FunctionOfQ[Cos[c*(a + b*x
)]/d, u, x]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Sin] || EqQ[F, sin])
```

Rubi steps

$$\begin{aligned}\int \operatorname{sech}(5x)\sinh(x)\,dx &= \operatorname{Subst}\left(\int \frac{1}{x\left(5-20x^2+16x^4\right)}\,dx, x, \cosh(x)\right)\\ &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{x\left(5-20x+16x^2\right)}\,dx, x, \cosh^2(x)\right)\\ &= \frac{1}{10}\operatorname{Subst}\left(\int \frac{1}{x}\,dx, x, \cosh^2(x)\right)+\frac{1}{10}\operatorname{Subst}\left(\int \frac{20-16x}{5-20x+16x^2}\,dx, x, \cosh^2(x)\right)\\ &= \frac{1}{5}\log(\cosh(x))-\frac{1}{5}\left(4\left(1-\sqrt{5}\right)\right)\operatorname{Subst}\left(\int \frac{1}{-10-2\sqrt{5}+16x}\,dx, x, \cosh^2(x)\right)-\frac{1}{5}\left(4\left(1\right.\right.\\ &= \frac{1}{5}\log(\cosh(x))-\frac{1}{20}\left(1+\sqrt{5}\right)\log\left(5-\sqrt{5}-8\cosh^2(x)\right)-\frac{1}{20}\left(1-\sqrt{5}\right)\log\left(5+\sqrt{5}\right.\end{aligned}$$

Mathematica [A] time = 0.08, size = 57, normalized size = 0.92

$$\frac{1}{20}\left(\left(\sqrt{5}-1\right)\log\left(8\sinh^2(x)-\sqrt{5}+3\right)-\left(1+\sqrt{5}\right)\log\left(8\sinh^2(x)+\sqrt{5}+3\right)+4\log(\cosh(x))\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[5*x]*Sinh[x],x]
```

```
[Out] (4*Log[Cosh[x]] + (-1 + Sqrt[5])*Log[3 - Sqrt[5] + 8*Sinh[x]^2] - (1 + Sqrt
[5])*Log[3 + Sqrt[5] + 8*Sinh[x]^2])/20
```

fricas [B] time = 0.44, size = 182, normalized size = 2.94

$$\frac{1}{20}\sqrt{5}\log\left(\frac{4\cosh(x)^4+4\sinh(x)^4-4\left(\sqrt{5}+1\right)\cosh(x)^2+4\left(6\cosh(x)^2-\sqrt{5}-1\right)\sinh(x)^2+\sqrt{5}+7}{2\cosh(x)^4+2\sinh(x)^4+2\left(6\cosh(x)^2-1\right)\sinh(x)^2-2\cosh(x)^2+1}\right)-\frac{1}{20}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(5*x)*sinh(x),x, algorithm="fricas")
```

```
[Out] 1/20*sqrt(5)*log((4*cosh(x)^4 + 4*sinh(x)^4 - 4*(sqrt(5) + 1)*cosh(x)^2 + 4
*(6*cosh(x)^2 - sqrt(5) - 1)*sinh(x)^2 + sqrt(5) + 7)/(2*cosh(x)^4 + 2*sinh
(x)^4 + 2*(6*cosh(x)^2 - 1)*sinh(x)^2 - 2*cosh(x)^2 + 1)) - 1/20*log((2*cos
h(x)^4 + 2*sinh(x)^4 + 2*(6*cosh(x)^2 - 1)*sinh(x)^2 - 2*cosh(x)^2 + 1)/(co
sh(x)^4 - 4*cosh(x)^3*sinh(x) + 6*cosh(x)^2*sinh(x)^2 - 4*cosh(x)*sinh(x)^3
 + sinh(x)^4)) + 1/5*log(2*cosh(x)/(cosh(x) - sinh(x)))
```

giac [B] time = 0.16, size = 118, normalized size = 1.90

$$\frac{1}{20}\left(\sqrt{5}-1\right)\log\left(\frac{1}{2}\sqrt{2\sqrt{5}+10}e^{x}+e^{(2x)}+1\right)+\frac{1}{20}\left(\sqrt{5}-1\right)\log\left(-\frac{1}{2}\sqrt{2\sqrt{5}+10}e^{x}+e^{(2x)}+1\right)-\frac{1}{20}\left(\sqrt{5}+1\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(5*x)*sinh(x),x, algorithm="giac")
```

```
[Out] 1/20*(sqrt(5) - 1)*log(1/2*sqrt(2*sqrt(5) + 10)*e^x + e^(2*x) + 1) + 1/20*(
sqrt(5) - 1)*log(-1/2*sqrt(2*sqrt(5) + 10)*e^x + e^(2*x) + 1) - 1/20*(sqrt(
5) + 1)*log(1/2*sqrt(-2*sqrt(5) + 10)*e^x + e^(2*x) + 1) - 1/20*(sqrt(5) +
1)*log(-1/2*sqrt(-2*sqrt(5) + 10)*e^x + e^(2*x) + 1) + 1/5*log(e^(2*x) + 1)
```

maple [B] time = 0.24, size = 101, normalized size = 1.63

$$\frac{\ln\left(1+e^{2x}\right)}{5}-\frac{\ln\left(e^{4x}+\left(-\frac{1}{2}-\frac{\sqrt{5}}{2}\right)e^{2x}+1\right)}{20}+\frac{\ln\left(e^{4x}+\left(-\frac{1}{2}-\frac{\sqrt{5}}{2}\right)e^{2x}+1\right)\sqrt{5}}{20}-\frac{\ln\left(e^{4x}+\left(\frac{\sqrt{5}}{2}-\frac{1}{2}\right)e^{2x}+1\right)}{20}-\frac{\ln}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(5*x)*sinh(x),x)
```

```
[Out] 1/5*ln(1+exp(2*x))-1/20*ln(exp(4*x)+(-1/2-1/2*5^(1/2))*exp(2*x)+1)+1/20*ln(
exp(4*x)+(-1/2-1/2*5^(1/2))*exp(2*x)+1)*5^(1/2)-1/20*ln(exp(4*x)+(1/2*5^(1/
2)-1/2)*exp(2*x)+1)-1/20*ln(exp(4*x)+(1/2*5^(1/2)-1/2)*exp(2*x)+1)*5^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{2}{5}\int\frac{\left(e^{(6x)}-e^{(4x)}+e^{(2x)}-1\right)e^{(2x)}}{e^{(8x)}-e^{(6x)}+e^{(4x)}-e^{(2x)}+1}\,dx+\frac{2}{5}\int\frac{e^{(6x)}}{e^{(8x)}-e^{(6x)}+e^{(4x)}-e^{(2x)}+1}\,dx+\frac{1}{5}\int\frac{e^{(4x)}}{e^{(8x)}-e^{(6x)}+e^{(4x)}-e^{(2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(5*x)*sinh(x),x, algorithm="maxima")
```

```
[Out] -2/5*integrate((e^(6*x) - e^(4*x) + e^(2*x) - 1)*e^(2*x)/(e^(8*x) - e^(6*x)
 + e^(4*x) - e^(2*x) + 1), x) + 2/5*integrate(e^(6*x)/(e^(8*x) - e^(6*x) +
e^(4*x) - e^(2*x) + 1), x) + 1/5*integrate(e^(4*x)/(e^(8*x) - e^(6*x) + e^(
4*x) - e^(2*x) + 1), x) - 4/5*integrate(e^(2*x)/(e^(8*x) - e^(6*x) + e^(4*x
) - e^(2*x) + 1), x) + 1/5*log(e^(2*x) + 1)
```

mupad [B] time = 1.52, size = 100, normalized size = 1.61

$$\frac{\ln\left(5\,e^{2\,x}+5\right)}{5}-\ln\left(e^{2\,x}+2\,e^{4\,x}+\left(\frac{\sqrt{5}}{20}+\frac{1}{20}\right)\,\left(20\,e^{2\,x}+30\,e^{4\,x}+30\right)+2\right)\,\left(\frac{\sqrt{5}}{20}+\frac{1}{20}\right)+\ln\left(e^{2\,x}+2\,e^{4\,x}-\left(\frac{\sqrt{5}}{20}-\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)/cosh(5*x),x)
```

```
[Out] log(5*exp(2*x) + 5)/5 - log(exp(2*x) + 2*exp(4*x) + (5^(1/2)/20 + 1/20)*(20
*exp(2*x) + 30*exp(4*x) + 30) + 2)*(5^(1/2)/20 + 1/20) + log(exp(2*x) + 2*e
xp(4*x) - (5^(1/2)/20 - 1/20)*(20*exp(2*x) + 30*exp(4*x) + 30) + 2)*(5^(1/2
)/20 - 1/20)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sinh(x)\operatorname{sech}(5x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(5*x)*sinh(x),x)
```

```
[Out] Integral(sinh(x)*sech(5*x), x)
```

3.215 $\int \mathrm{sech}(6x)\sinh(x)\,dx$

Optimal. Leaf size=85

$$\frac{\tanh^{-1}\left(\sqrt{2}\cosh(x)\right)}{3\sqrt{2}}-\frac{\tanh^{-1}\left(\frac{2\cosh(x)}{\sqrt{2-\sqrt{3}}}\right)}{6\sqrt{2-\sqrt{3}}}-\frac{\tanh^{-1}\left(\frac{2\cosh(x)}{\sqrt{2+\sqrt{3}}}\right)}{6\sqrt{2+\sqrt{3}}}$$

```
[Out] 1/6*arctanh(cosh(x)*2^(1/2))*2^(1/2)-1/6*arctanh(2*cosh(x)/(1/2*6^(1/2)-1/2
*2^(1/2)))/(1/2*6^(1/2)-1/2*2^(1/2))-1/6*arctanh(2*cosh(x)/(1/2*6^(1/2)+1/2
*2^(1/2)))/(1/2*6^(1/2)+1/2*2^(1/2))
```

Rubi [A] time = 0.08, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {4357, 2057, 207, 1166}

$$\frac{\tanh^{-1}\left(\sqrt{2}\cosh(x)\right)}{3\sqrt{2}}-\frac{\tanh^{-1}\left(\frac{2\cosh(x)}{\sqrt{2-\sqrt{3}}}\right)}{6\sqrt{2-\sqrt{3}}}-\frac{\tanh^{-1}\left(\frac{2\cosh(x)}{\sqrt{2+\sqrt{3}}}\right)}{6\sqrt{2+\sqrt{3}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[6*x]*Sinh[x],x]
```

```
[Out] ArcTanh[Sqrt[2]*Cosh[x]]/(3*Sqrt[2]) - ArcTanh[(2*Cosh[x])/Sqrt[2 - Sqrt[3]
]]/(6*Sqrt[2 - Sqrt[3]]) - ArcTanh[(2*Cosh[x])/Sqrt[2 + Sqrt[3]]]/(6*Sqrt[2
 + Sqrt[3]])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 2057

```
Int[(P_)^(p_), x_Symbol] :> With[{u = Factor[P /. x -> Sqrt[x]]}, Int[Expan
dIntegrand[(u /. x -> x^2)^p, x], x] /;  !SumQ[NonfreeFactors[u, x]]] /; Po
lyQ[P, x^2] && ILtQ[p, 0]
```

Rule 4357

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Cos[c*(a + b*x)], x]}, -Dist[d/(b*c), Subst[Int[SubstFor[1, Cos[c*(a + b
*x)]/d, u, x], x], x, Cos[c*(a + b*x)]/d], x] /; FunctionOfQ[Cos[c*(a + b*x
)]/d, u, x]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Sin] || EqQ[F, sin])
```

Rubi steps

$$\begin{aligned}\int \operatorname{sech}(6x)\sinh(x)\,dx &= \text{Subst}\left(\int \frac{1}{-1+18x^2-48x^4+32x^6}\,dx,x,\cosh(x)\right)\\ &= \text{Subst}\left(\int \left(-\frac{1}{3\left(-1+2x^2\right)}+\frac{4\left(-1+2x^2\right)}{3\left(1-16x^2+16x^4\right)}\right)dx,x,\cosh(x)\right)\\ &= -\left(\frac{1}{3}\text{Subst}\left(\int \frac{1}{-1+2x^2}\,dx,x,\cosh(x)\right)\right)+\frac{4}{3}\text{Subst}\left(\int \frac{-1+2x^2}{1-16x^2+16x^4}\,dx,x,\cosh(x)\right)\\ &= \frac{\tanh^{-1}\left(\sqrt{2}\cosh(x)\right)}{3\sqrt{2}}+\frac{4}{3}\text{Subst}\left(\int \frac{1}{-8-4\sqrt{3}+16x^2}\,dx,x,\cosh(x)\right)+\frac{4}{3}\text{Subst}\left(\int \right.\\ &= \frac{\tanh^{-1}\left(\sqrt{2}\cosh(x)\right)}{3\sqrt{2}}-\frac{\tanh^{-1}\left(\frac{2\cosh(x)}{\sqrt{2-\sqrt{3}}}\right)}{6\sqrt{2-\sqrt{3}}}-\frac{\tanh^{-1}\left(\frac{2\cosh(x)}{\sqrt{2+\sqrt{3}}}\right)}{6\sqrt{2+\sqrt{3}}}\end{aligned}$$

Mathematica [C] time = 0.28, size = 385, normalized size = 4.53

$$\sqrt{2}\,\text{RootSum}\left[\#1^8-\#1^4+1\&,\frac{\#1^6x+2\#1^6\log\left(-\#1\sinh\left(\frac{x}{2}\right)+\#1\cosh\left(\frac{x}{2}\right)-\sinh\left(\frac{x}{2}\right)-\cosh\left(\frac{x}{2}\right)\right)-\#1^4x-2\#1^4\log\left(-\#1\sinh\left(\frac{x}{2}\right)+\#1\cosh\left(\frac{x}{2}\right)-\text{si}\right.}{}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[6*x]*Sinh[x],x]
```

```
[Out] ((4*I)*ArcTan[(Cosh[x/2] + Sinh[x/2])/((1 + Sqrt[2])*Cosh[x/2] - (-1 + Sqrt
[2])*Sinh[x/2])] - (4*I)*ArcTan[(Cosh[x/2] + Sinh[x/2])/((-1 + Sqrt[2])*Cos
h[x/2] - (1 + Sqrt[2])*Sinh[x/2])] + 8*ArcTanh[Sqrt[2] - I*Tanh[x/2]] - 2*L
```

```
og[Sqrt[2] - 2*Cosh[x]] + 2*Log[Sqrt[2] + 2*Cosh[x]] + Sqrt[2]*RootSum[1 -
#1^4 + #1^8 & , (-x - 2*Log[-Cosh[x/2] - Sinh[x/2] + Cosh[x/2]*#1 - Sinh[x/
2]*#1] + x*#1^2 + 2*Log[-Cosh[x/2] - Sinh[x/2] + Cosh[x/2]*#1 - Sinh[x/2]*#
1]*#1^2 - x*#1^4 - 2*Log[-Cosh[x/2] - Sinh[x/2] + Cosh[x/2]*#1 - Sinh[x/2]*
#1]*#1^4 + x*#1^6 + 2*Log[-Cosh[x/2] - Sinh[x/2] + Cosh[x/2]*#1 - Sinh[x/2]
*#1]*#1^6)/(-#1^3 + 2*#1^7) & ])/(24*Sqrt[2])
```

fricas [B] time = 0.46, size = 250, normalized size = 2.94

$$\frac{1}{12}\sqrt{\sqrt{3}+2}\,\log\left(\cosh(x)^2+2\,\cosh(x)\sinh(x)+\sinh(x)^2+\left(\left(\sqrt{3}-2\right)\cosh(x)+\left(\sqrt{3}-2\right)\sinh(x)\right)\sqrt{\sqrt{3}+2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(6*x)*sinh(x),x, algorithm="fricas")
```

```
[Out] 1/12*sqrt(sqrt(3) + 2)*log(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + ((sq
rt(3) - 2)*cosh(x) + (sqrt(3) - 2)*sinh(x))*sqrt(sqrt(3) + 2) + 1) - 1/12*s
qrt(sqrt(3) + 2)*log(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - ((sqrt(3)
- 2)*cosh(x) + (sqrt(3) - 2)*sinh(x))*sqrt(sqrt(3) + 2) + 1) - 1/12*sqrt(-s
qrt(3) + 2)*log(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + ((sqrt(3) + 2)*
cosh(x) + (sqrt(3) + 2)*sinh(x))*sqrt(-sqrt(3) + 2) + 1) + 1/12*sqrt(-sqrt(
3) + 2)*log(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - ((sqrt(3) + 2)*cosh
(x) + (sqrt(3) + 2)*sinh(x))*sqrt(-sqrt(3) + 2) + 1) + 1/12*sqrt(2)*log((co
sh(x)^2 + sinh(x)^2 + 2*sqrt(2)*cosh(x) + 2)/(cosh(x)^2 + sinh(x)^2))
```

giac [B] time = 0.15, size = 154, normalized size = 1.81

$$-\frac{1}{24}\left(\sqrt{6}-\sqrt{2}\right)\log\left(\frac{1}{2}\left(\sqrt{6}+\sqrt{2}\right)e^{x}+e^{(2\,x)}+1\right)+\frac{1}{24}\left(\sqrt{6}-\sqrt{2}\right)\log\left(-\frac{1}{2}\left(\sqrt{6}+\sqrt{2}\right)e^{x}+e^{(2\,x)}+1\right)-\frac{1}{24}\left(\sqrt{6}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(6*x)*sinh(x),x, algorithm="giac")
```

```
[Out] -1/24*(sqrt(6) - sqrt(2))*log(1/2*(sqrt(6) + sqrt(2))*e^x + e^(2*x) + 1) +
1/24*(sqrt(6) - sqrt(2))*log(-1/2*(sqrt(6) + sqrt(2))*e^x + e^(2*x) + 1) -
1/24*(sqrt(6) + sqrt(2))*log(1/2*(sqrt(6) - sqrt(2))*e^x + e^(2*x) + 1) + 1
/24*(sqrt(6) + sqrt(2))*log(-1/2*(sqrt(6) - sqrt(2))*e^x + e^(2*x) + 1) + 1
/12*sqrt(2)*log(sqrt(2)*e^x + e^(2*x) + 1) - 1/12*sqrt(2)*log(-sqrt(2)*e^x
+ e^(2*x) + 1)
```

maple [C] time = 0.29, size = 78, normalized size = 0.92

$$2\left(\sum_{\_R=\mathrm{RootOf}\left(331776\_Z^4-2304\_Z^2+1\right)}\_R\ln\left(\mathrm{e}^{2x}+\left(13824\_R^3-96\_R\right)\mathrm{e}^{x}+1\right)\right)+\frac{\ln\left(1+\mathrm{e}^{2x}+\mathrm{e}^{x}\sqrt{2}\right)\sqrt{2}}{12}-\frac{\ln\left(1+\mathrm{e}^{2x}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(6*x)*sinh(x),x)
```

```
[Out] 2*sum(_R*ln(exp(2*x)+(13824*_R^3-96*_R)*exp(x)+1),_R=RootOf(331776*_Z^4-230
4*_Z^2+1))+1/12*ln(1+exp(2*x)+exp(x)*2^(1/2))*2^(1/2)-1/12*ln(1+exp(2*x)-ex
p(x)*2^(1/2))*2^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{12}\sqrt{2}\log\left(\sqrt{2}e^{x}+e^{(2x)}+1\right)-\frac{1}{12}\sqrt{2}\log\left(-\sqrt{2}e^{x}+e^{(2x)}+1\right)+\int\frac{e^{(7x)}-e^{(5x)}+e^{(3x)}-e^{x}}{3\left(e^{(8x)}-e^{(4x)}+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(6*x)*sinh(x),x, algorithm="maxima")
```

```
[Out] 1/12*sqrt(2)*log(sqrt(2)*e^x + e^(2*x) + 1) - 1/12*sqrt(2)*log(-sqrt(2)*e^x
 + e^(2*x) + 1) + integrate(1/3*(e^(7*x) - e^(5*x) + e^(3*x) - e^x)/(e^(8*x
) - e^(4*x) + 1), x)
```

mupad [B] time = 1.72, size = 288, normalized size = 3.39

$$\frac{\sqrt{2}\ \ln\left(\mathrm{e}^{2x}+\sqrt{2}\ \mathrm{e}^{x}+1\right)}{12}-\frac{\sqrt{2}\ \ln\left(\mathrm{e}^{2x}-\sqrt{2}\ \mathrm{e}^{x}+1\right)}{12}+\ln\left(7\,\mathrm{e}^{2x}-4\sqrt{3}-24\,\mathrm{e}^{x}\sqrt{\frac{1}{72}-\frac{\sqrt{3}}{144}}-4\sqrt{3}\ \mathrm{e}^{2x}+12\sqrt{3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)/cosh(6*x),x)
```

```
[Out] (2^(1/2)*log(exp(2*x) + 2^(1/2)*exp(x) + 1))/12 - (2^(1/2)*log(exp(2*x) - 2
^(1/2)*exp(x) + 1))/12 + log(7*exp(2*x) - 4*3^(1/2) - 24*exp(x)*(1/72 - 3^(
1/2)/144)^(1/2) - 4*3^(1/2)*exp(2*x) + 12*3^(1/2)*exp(x)*(1/72 - 3^(1/2)/14
4)^(1/2) + 7)*(1/72 - 3^(1/2)/144)^(1/2) - log(7*exp(2*x) - 4*3^(1/2) + 24*
exp(x)*(1/72 - 3^(1/2)/144)^(1/2) - 4*3^(1/2)*exp(2*x) - 12*3^(1/2)*exp(x)*
(1/72 - 3^(1/2)/144)^(1/2) + 7)*(1/72 - 3^(1/2)/144)^(1/2) + log(7*exp(2*x)
 + 4*3^(1/2) - 24*exp(x)*(3^(1/2)/144 + 1/72)^(1/2) + 4*3^(1/2)*exp(2*x) -
12*3^(1/2)*exp(x)*(3^(1/2)/144 + 1/72)^(1/2) + 7)*(3^(1/2)/144 + 1/72)^(1/2
) - log(7*exp(2*x) + 4*3^(1/2) + 24*exp(x)*(3^(1/2)/144 + 1/72)^(1/2) + 4*3
^(1/2)*exp(2*x) + 12*3^(1/2)*exp(x)*(3^(1/2)/144 + 1/72)^(1/2) + 7)*(3^(1/2
)/144 + 1/72)^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sinh(x)\operatorname{sech}(6x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(6*x)*sinh(x),x)
```

```
[Out] Integral(sinh(x)*sech(6*x), x)
```

3.216 $\int \operatorname{csch}(2x)\sinh(x)\,dx$

Optimal. Leaf size=7

$$\frac{1}{2}\tan^{-1}(\sinh(x))$$

```
[Out] 1/2*arctan(sinh(x))
```

Rubi [A] time = 0.01, antiderivative size = 7, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {4288, 3770}

$$\frac{1}{2}\tan^{-1}(\sinh(x))$$

Antiderivative was successfully verified.

```
[In] Int[Csch[2*x]*Sinh[x],x]
```

```
[Out] ArcTan[Sinh[x]]/2
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4288

```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/f^p, Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x], x
] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \operatorname{csch}(2x)\sinh(x)\,dx &= \frac{1}{2}\int \operatorname{sech}(x)\,dx\\ &= \frac{1}{2}\tan^{-1}(\sinh(x))\end{aligned}$$

Mathematica [A] time = 0.00, size = 7, normalized size = 1.00

$$\tan^{-1}\left(\tanh\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[2*x]*Sinh[x],x]
```

```
[Out] ArcTan[Tanh[x/2]]
```

fricas [A] time = 0.42, size = 6, normalized size = 0.86

$$\arctan\left(\cosh(x) + \sinh(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(2*x)*sinh(x),x, algorithm="fricas")
```

```
[Out] arctan(cosh(x) + sinh(x))
```

giac [A] time = 0.11, size = 3, normalized size = 0.43

$$\arctan\left(e^{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(2*x)*sinh(x),x, algorithm="giac")
```

```
[Out] arctan(e^x)
```

maple [A] time = 0.16, size = 4, normalized size = 0.57

$$\arctan\left(\mathrm{e}^{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(2*x)*sinh(x),x)
```

```
[Out] arctan(exp(x))
```

maxima [A] time = 0.43, size = 7, normalized size = 1.00

$$-\arctan\left(e^{(-x)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(2*x)*sinh(x),x, algorithm="maxima")
```

```
[Out] -arctan(e^(-x))
```

mupad [B] time = 0.05, size = 3, normalized size = 0.43

$$\mathrm{atan}\left(\mathrm{e}^{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)/sinh(2*x),x)
```

```
[Out] atan(exp(x))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sinh(x)\operatorname{csch}(2x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(2*x)*sinh(x),x)
```

```
[Out] Integral(sinh(x)*csch(2*x), x)
```

3.217 $\int \operatorname{csch}(3x) \sinh(x) \, dx$

Optimal. Leaf size=15

$$\frac{\tan^{-1}\left(\frac{\tanh(x)}{\sqrt{3}}\right)}{\sqrt{3}}$$

```
[Out] 1/3*arctan(1/3*tanh(x)*3^(1/2))*3^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {203}

$$\frac{\tan^{-1}\left(\frac{\tanh(x)}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[3*x]*Sinh[x],x]
```

```
[Out] ArcTan[Tanh[x]/Sqrt[3]]/Sqrt[3]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \operatorname{csch}(3x) \sinh(x) \, dx &= \text{Subst}\left(\int \frac{1}{3+x^2} \, dx, x, \tanh(x)\right)\\ &= \frac{\tan^{-1}\left(\frac{\tanh(x)}{\sqrt{3}}\right)}{\sqrt{3}}\end{aligned}$$

Mathematica [C] time = 0.03, size = 44, normalized size = 2.93

$$-\frac{1}{4} e^{2x} \left(e^{2x} \, _2F_1\left(\frac{2}{3},1;\frac{5}{3};e^{6x}\right) - 2 \, _2F_1\left(\frac{1}{3},1;\frac{4}{3};e^{6x}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[3*x]*Sinh[x],x]
```

```
[Out] -1/4*(E^(2*x)*(-2*Hypergeometric2F1[1/3, 1, 4/3, E^(6*x)] + E^(2*x)*Hyperge
ometric2F1[2/3, 1, 5/3, E^(6*x)]))
```

fricas [B] time = 0.43, size = 31, normalized size = 2.07

$$-\frac{1}{3}\sqrt{3}\arctan\left(-\frac{3\sqrt{3}\cosh(x)+\sqrt{3}\sinh(x)}{3(\cosh(x)-\sinh(x))}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(3*x)*sinh(x),x, algorithm="fricas")
```

```
[Out] -1/3*sqrt(3)*arctan(-1/3*(3*sqrt(3)*cosh(x) + sqrt(3)*sinh(x))/(cosh(x) - s
inh(x)))
```

giac [A] time = 0.11, size = 19, normalized size = 1.27

$$\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2e^{(2x)}+1\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(3*x)*sinh(x),x, algorithm="giac")
```

```
[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*e^(2*x) + 1))
```

maple [C] time = 0.23, size = 40, normalized size = 2.67

$$\frac{i\ln\left(e^{2x}+\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}{6}-\frac{i\ln\left(e^{2x}+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(3*x)*sinh(x),x)
```

```
[Out] 1/6*I*ln(exp(2*x)+1/2+1/2*I*3^(1/2))*3^(1/2)-1/6*I*ln(exp(2*x)+1/2-1/2*I*3^
(1/2))*3^(1/2)
```

maxima [B] time = 0.43, size = 39, normalized size = 2.60

$$\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2e^{(-x)}+1\right)\right)-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2e^{(-x)}-1\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(3*x)*sinh(x),x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*e^(-x) + 1)) - 1/3*sqrt(3)*arctan(1/3*sqr
t(3)*(2*e^(-x) - 1))
```

mupad [B] time = 1.46, size = 19, normalized size = 1.27

$$\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{\sqrt{3}\,\left(2\,\mathrm{e}^{2\,x}+1\right)}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)/sinh(3*x),x)
```

```
[Out] (3^(1/2)*atan((3^(1/2)*(2*exp(2*x) + 1))/3))/3
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sinh(x)\operatorname{csch}(3x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(3*x)*sinh(x),x)
```

```
[Out] Integral(sinh(x)*csch(3*x), x)
```

3.218 $\int \mathrm{csch}(4x)\sinh(x)\,dx$

Optimal. Leaf size=26

$$\frac{\tan^{-1}\left(\sqrt{2}\sinh(x)\right)}{2\sqrt{2}}-\frac{1}{4}\tan^{-1}(\sinh(x))$$

```
[Out] -1/4*arctan(sinh(x))+1/4*arctan(sinh(x)*2^(1/2))*2^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {1093, 203}

$$\frac{\tan^{-1}\left(\sqrt{2}\sinh(x)\right)}{2\sqrt{2}}-\frac{1}{4}\tan^{-1}(\sinh(x))$$

Antiderivative was successfully verified.

```
[In] Int[Csch[4*x]*Sinh[x],x]
```

```
[Out] -ArcTan[Sinh[x]]/4 + ArcTan[Sqrt[2]*Sinh[x]]/(2*Sqrt[2])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1093

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^2), x], x] - Dist[c/q, Int
[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
0] && PosQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}\int \mathrm{csch}(4x)\sinh(x)\,dx &= \mathrm{Subst}\left(\int \frac{1}{4+12x^2+8x^4}\,dx,x,\sinh(x)\right)\\ &= 2\,\mathrm{Subst}\left(\int \frac{1}{4+8x^2}\,dx,x,\sinh(x)\right)-2\,\mathrm{Subst}\left(\int \frac{1}{8+8x^2}\,dx,x,\sinh(x)\right)\\ &= -\frac{1}{4}\tan^{-1}(\sinh(x))+\frac{\tan^{-1}\left(\sqrt{2}\sinh(x)\right)}{2\sqrt{2}}\end{aligned}$$

Mathematica [A] time = 0.02, size = 26, normalized size = 1.00

$$\frac{\tan^{-1}\left(\sqrt{2}\sinh(x)\right)}{2\sqrt{2}}-\frac{1}{4}\tan^{-1}(\sinh(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[4*x]*Sinh[x],x]
```

```
[Out] -1/4*ArcTan[Sinh[x]] + ArcTan[Sqrt[2]*Sinh[x]]/(2*Sqrt[2])
```

fricas [B] time = 0.45, size = 76, normalized size = 2.92

$$\frac{1}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\cosh(x)+\frac{1}{2}\sqrt{2}\sinh(x)\right)-\frac{1}{4}\sqrt{2}\arctan\left(-\frac{\sqrt{2}\cosh(x)^2+2\sqrt{2}\cosh(x)\sinh(x)+\sqrt{2}\sinh}{2(\cosh(x)-\sinh(x))}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(4*x)*sinh(x),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(2)*arctan(1/2*sqrt(2)*cosh(x) + 1/2*sqrt(2)*sinh(x)) - 1/4*sqrt(2)
*arctan(-1/2*(sqrt(2)*cosh(x)^2 + 2*sqrt(2)*cosh(x)*sinh(x) + sqrt(2)*sinh(
x)^2 + sqrt(2))/(cosh(x) - sinh(x))) - 1/2*arctan(cosh(x) + sinh(x))
```

giac [B] time = 0.12, size = 44, normalized size = 1.69

$$-\frac{1}{8}\pi+\frac{1}{8}\sqrt{2}\left(\pi+2\arctan\left(\frac{1}{2}\sqrt{2}\left(e^{(2x)}-1\right)e^{(-x)}\right)\right)-\frac{1}{4}\arctan\left(\frac{1}{2}\left(e^{(2x)}-1\right)e^{(-x)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(4*x)*sinh(x),x, algorithm="giac")
```

```
[Out] -1/8*pi + 1/8*sqrt(2)*(pi + 2*arctan(1/2*sqrt(2)*(e^(2*x) - 1)*e^(-x))) - 1
/4*arctan(1/2*(e^(2*x) - 1)*e^(-x))
```

maple [C] time = 0.22, size = 62, normalized size = 2.38

$$\frac{i\ln(e^x-i)}{4}-\frac{i\ln(e^x+i)}{4}+\frac{i\sqrt{2}\,\ln\left(e^{2x}+i\sqrt{2}\,e^x-1\right)}{8}-\frac{i\sqrt{2}\,\ln\left(e^{2x}-i\sqrt{2}\,e^x-1\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(4*x)*sinh(x),x)
```

```
[Out] 1/4*I*ln(exp(x)-I)-1/4*I*ln(exp(x)+I)+1/8*I*2^(1/2)*ln(exp(2*x)+I*2^(1/2)*e
xp(x)-1)-1/8*I*2^(1/2)*ln(exp(2*x)-I*2^(1/2)*exp(x)-1)
```

maxima [B] time = 0.44, size = 50, normalized size = 1.92

$$-\frac{1}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2e^{(-x)}\right)\right)-\frac{1}{4}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2e^{(-x)}\right)\right)+\frac{1}{2}\arctan\left(e^{(-x)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(4*x)*sinh(x),x, algorithm="maxima")
```

```
[Out] -1/4*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*e^(-x))) - 1/4*sqrt(2)*arctan(
-1/2*sqrt(2)*(sqrt(2) - 2*e^(-x))) + 1/2*arctan(e^(-x))
```

mupad [B] time = 0.07, size = 42, normalized size = 1.62

$$\frac{\sqrt{2}\left(2\,\mathrm{atan}\left(\frac{\sqrt{2}\,e^{x}}{2}+\frac{\sqrt{2}\,e^{3\,x}}{2}\right)+2\,\mathrm{atan}\left(\frac{\sqrt{2}\,e^{x}}{2}\right)\right)}{8}-\frac{\mathrm{atan}\left(e^{x}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)/sinh(4*x),x)
```

```
[Out] (2^(1/2)*(2*atan((2^(1/2)*exp(x))/2 + (2^(1/2)*exp(3*x))/2) + 2*atan((2^(1/
2)*exp(x))/2)))/8 - atan(exp(x))/2
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sinh(x)\,\mathrm{csch}\,(4x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(4*x)*sinh(x),x)
```

```
[Out] Integral(sinh(x)*csch(4*x), x)
```

3.219 $\int \text{csch}(5x)\sinh(x)\,dx$

Optimal. Leaf size=75

$$\frac{1}{5}\sqrt{\frac{1}{2}\left(5-\sqrt{5}\right)}\tan^{-1}\left(\frac{\tanh(x)}{\sqrt{5-2\sqrt{5}}}\right)-\frac{1}{5}\sqrt{\frac{1}{2}\left(5+\sqrt{5}\right)}\tan^{-1}\left(\frac{\tanh(x)}{\sqrt{5+2\sqrt{5}}}\right)$$

```
[Out] 1/10*arctan(tanh(x)/(5-2*5^(1/2))^(1/2))*(10-2*5^(1/2))^(1/2)-1/10*arctan(t
anh(x)/(5+2*5^(1/2))^(1/2))*(10+2*5^(1/2))^(1/2)
```

Rubi [A] time = 0.11, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {1166, 203}

$$\frac{1}{5}\sqrt{\frac{1}{2}\left(5-\sqrt{5}\right)}\tan^{-1}\left(\frac{\tanh(x)}{\sqrt{5-2\sqrt{5}}}\right)-\frac{1}{5}\sqrt{\frac{1}{2}\left(5+\sqrt{5}\right)}\tan^{-1}\left(\frac{\tanh(x)}{\sqrt{5+2\sqrt{5}}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Csch[5*x]*Sinh[x],x]
```

```
[Out] (Sqrt[(5 - Sqrt[5])/2]*ArcTan[Tanh[x]/Sqrt[5 - 2*Sqrt[5]]])/5 - (Sqrt[(5 +
Sqrt[5])/2]*ArcTan[Tanh[x]/Sqrt[5 + 2*Sqrt[5]]])/5
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \operatorname{csch}(5x)\sinh(x)\,dx = \text{Subst}\left(\int \frac{1-x^2}{5+10x^2+x^4}\,dx, x, \tanh(x)\right)$$
$$= \frac{1}{10}\left(-5+3\sqrt{5}\right)\text{Subst}\left(\int \frac{1}{5-2\sqrt{5}+x^2}\,dx, x, \tanh(x)\right) - \frac{1}{10}\left(5+3\sqrt{5}\right)\text{Subst}\left(\int \frac{}{5+}\right.$$
$$= \frac{1}{5}\sqrt{\frac{1}{2}\left(5-\sqrt{5}\right)}\tan^{-1}\left(\frac{\tanh(x)}{\sqrt{5-2\sqrt{5}}}\right) - \frac{1}{5}\sqrt{\frac{1}{2}\left(5+\sqrt{5}\right)}\tan^{-1}\left(\frac{\tanh(x)}{\sqrt{5+2\sqrt{5}}}\right)$$

Mathematica [A] time = 0.11, size = 84, normalized size = 1.12

$$\frac{\sqrt{5+\sqrt{5}}\tan^{-1}\left(\frac{\left(\sqrt{5}-3\right)\tanh(x)}{\sqrt{10-2\sqrt{5}}}\right)+\sqrt{5-\sqrt{5}}\tan^{-1}\left(\frac{\left(3+\sqrt{5}\right)\tanh(x)}{\sqrt{2\left(5+\sqrt{5}\right)}}\right)}{5\sqrt{2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Csch[5*x]*Sinh[x],x]
```

```
[Out] (Sqrt[5 + Sqrt[5]]*ArcTan[((-3 + Sqrt[5])*Tanh[x])/Sqrt[10 - 2*Sqrt[5]]] +
Sqrt[5 - Sqrt[5]]*ArcTan[((3 + Sqrt[5])*Tanh[x])/Sqrt[2*(5 + Sqrt[5])]])/(5
*Sqrt[2])
```

fricas [B] time = 0.48, size = 171, normalized size = 2.28

$$-\frac{1}{5}\sqrt{2}\sqrt{-\sqrt{5}+5}\arctan\left(\frac{1}{40}\sqrt{5}\sqrt{2}\sqrt{-32\left(\sqrt{5}-1\right)e^{(2x)}+64e^{(4x)}+64}\sqrt{-\sqrt{5}+5}-\frac{1}{20}\left(4\sqrt{5}\sqrt{2}e^{(2x)}+\sqrt{5}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(5*x)*sinh(x),x, algorithm="fricas")
```

```
[Out] -1/5*sqrt(2)*sqrt(-sqrt(5) + 5)*arctan(1/40*sqrt(5)*sqrt(2)*sqrt(-32*(sqrt(
5) - 1)*e^(2*x) + 64*e^(4*x) + 64)*sqrt(-sqrt(5) + 5) - 1/20*(4*sqrt(5)*sqr
t(2)*e^(2*x) + sqrt(5)*sqrt(2) - 5*sqrt(2))*sqrt(-sqrt(5) + 5)) + 1/5*sqrt(
2)*sqrt(sqrt(5) + 5)*arctan(-1/20*(4*sqrt(5)*sqrt(2)*e^(2*x) + sqrt(5)*sqrt
(2) + 5*sqrt(2))*sqrt(sqrt(5) + 5) + 1/5*sqrt(5)*sqrt((sqrt(5) + 1)*e^(2*x)
 + 2*e^(4*x) + 2)*sqrt(sqrt(5) + 5))
```

giac [A] time = 0.15, size = 68, normalized size = 0.91

$$\frac{1}{10}\sqrt{-2\sqrt{5}+10}\arctan\left(-\frac{\sqrt{5}-4e^{(2x)}-1}{\sqrt{2\sqrt{5}+10}}\right)-\frac{1}{10}\sqrt{2\sqrt{5}+10}\arctan\left(\frac{\sqrt{5}+4e^{(2x)}+1}{\sqrt{-2\sqrt{5}+10}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(5*x)*sinh(x),x, algorithm="giac")
```

```
[Out] 1/10*sqrt(-2*sqrt(5) + 10)*arctan(-(sqrt(5) - 4*e^(2*x) - 1)/sqrt(2*sqrt(5)
 + 10)) - 1/10*sqrt(2*sqrt(5) + 10)*arctan((sqrt(5) + 4*e^(2*x) + 1)/sqrt(-
2*sqrt(5) + 10))
```

maple [C] time = 0.25, size = 41, normalized size = 0.55

$$2\left(\sum_{\_R=\mathrm{RootOf}\left(32000\_Z^{4}+400\_Z^{2}+1\right)}\_R\ln\left(4000\_R^{3}-200\_R^{2}+e^{2x}+30\_R-1\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(5*x)*sinh(x),x)
```

```
[Out] 2*sum(_R*ln(4000*_R^3-200*_R^2+exp(2*x)+30*_R-1),_R=RootOf(32000*_Z^4+400*_
Z^2+1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{10}(-1)^{\frac{3}{5}}\log\left((-1)^{\frac{1}{5}}+e^{(-2x)}\right)+\frac{\sqrt{5}\,(-1)^{\frac{3}{5}}\log\left(\frac{\sqrt{5}\,(-1)^{\frac{1}{5}}+(-1)^{\frac{1}{5}}\sqrt{2\sqrt{5}-10}+(-1)^{\frac{1}{5}}-4e^{(-2x)}}{\sqrt{5}\,(-1)^{\frac{1}{5}}-(-1)^{\frac{1}{5}}\sqrt{2\sqrt{5}-10}+(-1)^{\frac{1}{5}}-4e^{(-2x)}}\right)}{10\sqrt{2\sqrt{5}-10}}-\frac{\sqrt{5}\,(-1)^{\frac{3}{5}}\log\left(\frac{\sqrt{5}\,(-1)^{\frac{1}{5}}-(-1)}{\sqrt{5}\,(-1)^{\frac{1}{5}}+(-1)}\right)}{10\sqrt{-2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(5*x)*sinh(x),x, algorithm="maxima")
```

```
[Out] 1/10*(-1)^(3/5)*log((-1)^(1/5) + e^(-2*x)) + 1/10*sqrt(5)*(-1)^(3/5)*log((s
qrt(5)*(-1)^(1/5) + (-1)^(1/5)*sqrt(2*sqrt(5) - 10) + (-1)^(1/5) - 4*e^(-2*
x))/(sqrt(5)*(-1)^(1/5) - (-1)^(1/5)*sqrt(2*sqrt(5) - 10) + (-1)^(1/5) - 4*
e^(-2*x)))/sqrt(2*sqrt(5) - 10) - 1/10*sqrt(5)*(-1)^(3/5)*log((sqrt(5)*(-1)
^(1/5) - (-1)^(1/5)*sqrt(-2*sqrt(5) - 10) - (-1)^(1/5) + 4*e^(-2*x))/(sqrt(
5)*(-1)^(1/5) + (-1)^(1/5)*sqrt(-2*sqrt(5) - 10) - (-1)^(1/5) + 4*e^(-2*x))
)/sqrt(-2*sqrt(5) - 10) - 1/10*log(-(sqrt(5)*(-1)^(1/5) + (-1)^(1/5))*e^(-2
*x) + 2*(-1)^(2/5) + 2*e^(-4*x))/(sqrt(5)*(-1)^(2/5) + (-1)^(2/5)) + 1/10*l
og((sqrt(5)*(-1)^(1/5) - (-1)^(1/5))*e^(-2*x) + 2*(-1)^(2/5) + 2*e^(-4*x))/
(sqrt(5)*(-1)^(2/5) - (-1)^(2/5)) - 1/10*integrate((e^(3*x) + 2*e^(2*x) + 3
*e^x + 4)*e^x/(e^(4*x) + e^(3*x) + e^(2*x) + e^x + 1), x) - 1/10*integrate(
(e^(3*x) - 2*e^(2*x) + 3*e^x - 4)*e^x/(e^(4*x) - e^(3*x) + e^(2*x) - e^x +
1), x) + 1/10*log(e^x + 1) + 1/10*log(e^x - 1)
```

mupad [B] time = 4.26, size = 282, normalized size = 3.76

$$2\,\mathrm{atan}\left(\frac{\frac{e^{2x}}{5}+\frac{9\sqrt{5}}{25}+\frac{6\sqrt{5}\,e^{2x}}{25}+\frac{4}{5}}{5\,e^{2x}\sqrt{\frac{\sqrt{5}}{200}+\frac{1}{40}}+\frac{9\sqrt{5}\sqrt{\frac{\sqrt{5}}{200}+\frac{1}{40}}}{5}+\sqrt{\frac{\sqrt{5}}{200}+\frac{1}{40}}+\frac{9\sqrt{5}\,e^{2x}\sqrt{\frac{\sqrt{5}}{200}+\frac{1}{40}}}{5}}\right)\sqrt{\frac{\sqrt{5}}{200}+\frac{1}{40}}+\sqrt{\frac{1}{40}-\frac{\sqrt{5}}{200}}\left(\ln\left(\frac{9\sqrt{5}}{25}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)/sinh(5*x),x)
```

```
[Out] 2*atan((exp(2*x)/5 + (9*5^(1/2))/25 + (6*5^(1/2)*exp(2*x))/25 + 4/5)/(5*exp
(2*x)*(5^(1/2)/200 + 1/40)^(1/2) + (9*5^(1/2)*(5^(1/2)/200 + 1/40)^(1/2))/5
 + (5^(1/2)/200 + 1/40)^(1/2) + (9*5^(1/2)*exp(2*x)*(5^(1/2)/200 + 1/40)^(1
/2))/5))*(5^(1/2)/200 + 1/40)^(1/2) + (1/40 - 5^(1/2)/200)^(1/2)*(log((5^(1
/2)*(1/40 - 5^(1/2)/200)^(1/2)*9i)/5 - exp(2*x)*(1/40 - 5^(1/2)/200)^(1/2)*
5i - exp(2*x)/5 + (9*5^(1/2))/25 - (1/40 - 5^(1/2)/200)^(1/2)*1i + (6*5^(1/
2)*exp(2*x))/25 + (5^(1/2)*exp(2*x)*(1/40 - 5^(1/2)/200)^(1/2)*9i)/5 - 4/5)
*1i - log(exp(2*x)*(1/40 - 5^(1/2)/200)^(1/2)*5i - exp(2*x)/5 - (5^(1/2)*(1
/40 - 5^(1/2)/200)^(1/2)*9i)/5 + (9*5^(1/2))/25 + (1/40 - 5^(1/2)/200)^(1/2
)*1i + (6*5^(1/2)*exp(2*x))/25 - (5^(1/2)*exp(2*x)*(1/40 - 5^(1/2)/200)^(1/
2)*9i)/5 - 4/5)*1i)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sinh(x)\,\mathrm{csch}(5x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(5*x)*sinh(x),x)
```

```
[Out] Integral(sinh(x)*csch(5*x), x)
```

3.220 $\int \operatorname{csch}(6x)\sinh(x)\,dx$

Optimal. Leaf size=36

$$\frac{1}{6}\tan^{-1}(\sinh(x))+\frac{1}{6}\tan^{-1}(2\sinh(x))-\frac{\tan^{-1}\left(\frac{2\sinh(x)}{\sqrt{3}}\right)}{2\sqrt{3}}$$

```
[Out] 1/6*arctan(sinh(x))+1/6*arctan(2*sinh(x))-1/6*arctan(2/3*sinh(x)*3^(1/2))*3
^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 3, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {12, 2057, 203}

$$\frac{1}{6}\tan^{-1}(\sinh(x))+\frac{1}{6}\tan^{-1}(2\sinh(x))-\frac{\tan^{-1}\left(\frac{2\sinh(x)}{\sqrt{3}}\right)}{2\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[6*x]*Sinh[x],x]
```

```
[Out] ArcTan[Sinh[x]]/6 + ArcTan[2*Sinh[x]]/6 - ArcTan[(2*Sinh[x])/Sqrt[3]]/(2*Sq
rt[3])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 2057

```
Int[(P_)^(p_), x_Symbol] :> With[{u = Factor[P /. x -> Sqrt[x]]}, Int[Expan
dIntegrand[(u /. x -> x^2)^p, x], x] /;  !SumQ[NonfreeFactors[u, x]]] /; Po
lyQ[P, x^2] && ILtQ[p, 0]
```

Rubi steps

$$
\begin{aligned}
\int \operatorname{csch}(6x)\sinh(x)\,dx &= \text{Subst}\left(\int \frac{1}{2\left(3+19x^2+32x^4+16x^6\right)}\,dx, x, \sinh(x)\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \frac{1}{3+19x^2+32x^4+16x^6}\,dx, x, \sinh(x)\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(\frac{1}{3\left(1+x^2\right)}+\frac{2}{3\left(1+4x^2\right)}-\frac{2}{3+4x^2}\right)dx, x, \sinh(x)\right)\\
&= \frac{1}{6}\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sinh(x)\right)+\frac{1}{3}\text{Subst}\left(\int \frac{1}{1+4x^2}\,dx, x, \sinh(x)\right)-\text{Subst}\left(\int \frac{}{3}\right.\\
&= \frac{1}{6}\tan^{-1}(\sinh(x))+\frac{1}{6}\tan^{-1}(2\sinh(x))-\frac{\tan^{-1}\left(\frac{2\sinh(x)}{\sqrt{3}}\right)}{2\sqrt{3}}
\end{aligned}
$$

Mathematica [A] time = 0.04, size = 30, normalized size = 0.83

$$
\frac{1}{6}\left(\tan^{-1}(\sinh(x))+\tan^{-1}(2\sinh(x))-\sqrt{3}\tan^{-1}\left(\frac{2\sinh(x)}{\sqrt{3}}\right)\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[6*x]*Sinh[x],x]
```

```
[Out] (ArcTan[Sinh[x]] + ArcTan[2*Sinh[x]] - Sqrt[3]*ArcTan[(2*Sinh[x])/Sqrt[3]])
/6
```

fricas [B] time = 0.44, size = 107, normalized size = 2.97

$$
-\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\cosh(x)+\frac{1}{3}\sqrt{3}\sinh(x)\right)+\frac{1}{6}\sqrt{3}\arctan\left(-\frac{\sqrt{3}\cosh(x)^2+2\sqrt{3}\cosh(x)\sinh(x)+\sqrt{3}\sin}{3(\cosh(x)-\sinh(x))}\right.
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(6*x)*sinh(x),x, algorithm="fricas")
```

```
[Out] -1/6*sqrt(3)*arctan(1/3*sqrt(3)*cosh(x) + 1/3*sqrt(3)*sinh(x)) + 1/6*sqrt(3
)*arctan(-1/3*(sqrt(3)*cosh(x)^2 + 2*sqrt(3)*cosh(x)*sinh(x) + sqrt(3)*sinh
(x)^2 + 2*sqrt(3))/(cosh(x) - sinh(x))) - 1/6*arctan(-(cosh(x)^2 + 2*cosh(x
)*sinh(x) + sinh(x)^2)/(cosh(x) - sinh(x))) + 1/2*arctan(cosh(x) + sinh(x))
```

giac [B] time = 0.14, size = 58, normalized size = 1.61

$$
\frac{1}{6}\pi-\frac{1}{12}\sqrt{3}\left(\pi+2\arctan\left(\frac{1}{3}\sqrt{3}\left(e^{(2x)}-1\right)e^{(-x)}\right)\right)+\frac{1}{6}\arctan\left(\left(e^{(2x)}-1\right)e^{(-x)}\right)+\frac{1}{6}\arctan\left(\frac{1}{2}\left(e^{(2x)}-1\right)e^{(-x)}\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(6*x)*sinh(x),x, algorithm="giac")
```

```
[Out] 1/6*pi - 1/12*sqrt(3)*(pi + 2*arctan(1/3*sqrt(3)*(e^(2*x) - 1)*e^(-x))) + 1
/6*arctan((e^(2*x) - 1)*e^(-x)) + 1/6*arctan(1/2*(e^(2*x) - 1)*e^(-x))
```

maple [C] time = 0.25, size = 92, normalized size = 2.56

$$\frac{i\ln\left(\mathrm{e}^{x}+i\right)}{6}-\frac{i\ln\left(\mathrm{e}^{x}-i\right)}{6}+\frac{i\ln\left(\mathrm{e}^{2x}+i\mathrm{e}^{x}-1\right)}{12}-\frac{i\ln\left(\mathrm{e}^{2x}-i\mathrm{e}^{x}-1\right)}{12}+\frac{i\sqrt{3}\,\ln\left(\mathrm{e}^{2x}-i\sqrt{3}\,\mathrm{e}^{x}-1\right)}{12}-\frac{i\sqrt{3}\,\ln\left(\mathrm{e}^{2x}+i\sqrt{}\right.}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(6*x)*sinh(x),x)
```

```
[Out] 1/6*I*ln(exp(x)+I)-1/6*I*ln(exp(x)-I)+1/12*I*ln(exp(2*x)+I*exp(x)-1)-1/12*I
*ln(exp(2*x)-I*exp(x)-1)+1/12*I*3^(1/2)*ln(exp(2*x)-I*3^(1/2)*exp(x)-1)-1/1
2*I*3^(1/2)*ln(exp(2*x)+I*3^(1/2)*exp(x)-1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2e^{x}+1\right)\right)-\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2e^{x}-1\right)\right)+\frac{1}{3}\arctan\left(e^{x}\right)+\int\frac{e^{(3x)}+e^{x}}{6\left(e^{(4x)}-e^{(2x)}+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(6*x)*sinh(x),x, algorithm="maxima")
```

```
[Out] -1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*e^x + 1)) - 1/6*sqrt(3)*arctan(1/3*sqrt(
3)*(2*e^x - 1)) + 1/3*arctan(e^x) + integrate(1/6*(e^(3*x) + e^x)/(e^(4*x)
- e^(2*x) + 1), x)
```

mupad [B] time = 0.20, size = 41, normalized size = 1.14

$$\frac{\mathrm{atan}\left(\mathrm{e}^{x}\right)}{3}-\frac{\mathrm{atan}\left(\mathrm{e}^{-x}-\mathrm{e}^{x}\right)}{6}-\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{\sqrt{3}\,\mathrm{e}^{x}}{3}-\frac{\sqrt{3}\,\mathrm{e}^{-x}}{3}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)/sinh(6*x),x)
```

```
[Out] atan(exp(x))/3 - atan(exp(-x) - exp(x))/6 - (3^(1/2)*atan((3^(1/2)*exp(x))/
3 - (3^(1/2)*exp(-x))/3))/6
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sinh\left(x\right)\operatorname{csch}\left(6x\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(6*x)*sinh(x),x)
```

```
[Out] Integral(sinh(x)*csch(6*x), x)
```

3.221 $\int \cosh(x)\sinh(2x)\,dx$

Optimal. Leaf size=8

$$\frac{2\cosh^3(x)}{3}$$

```
[Out] 2/3*cosh(x)^3
```

Rubi [A] time = 0.01, antiderivative size = 15, normalized size of antiderivative = 1.88, number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4284}

$$\frac{\cosh(x)}{2}+\frac{1}{6}\cosh(3x)$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]*Sinh[2*x],x]
```

```
[Out] Cosh[x]/2 + Cosh[3*x]/6
```

Rule 4284

```
Int[cos[(c_.) + (d_.)*(x_)]*sin[(a_.) + (b_.)*(x_)], x_Symbol] :> -Simp[Cos
[a - c + (b - d)*x]/(2*(b - d)), x] - Simp[Cos[a + c + (b + d)*x]/(2*(b + d
)), x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - d^2, 0]
```

Rubi steps

$$\int \cosh(x)\sinh(2x)\,dx=\frac{\cosh(x)}{2}+\frac{1}{6}\cosh(3x)$$

Mathematica [A] time = 0.01, size = 15, normalized size = 1.88

$$\frac{\cosh(x)}{2}+\frac{1}{6}\cosh(3x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]*Sinh[2*x],x]
```

```
[Out] Cosh[x]/2 + Cosh[3*x]/6
```

fricas [B] time = 0.45, size = 19, normalized size = 2.38

$$\frac{1}{6}\cosh(x)^3+\frac{1}{2}\cosh(x)\sinh(x)^2+\frac{1}{2}\cosh(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*sinh(2*x),x, algorithm="fricas")
[Out] 1/6*cosh(x)^3 + 1/2*cosh(x)*sinh(x)^2 + 1/2*cosh(x)
```

giac [B] time = 0.13, size = 25, normalized size = 3.12

$$\frac{1}{12}\left(3\,e^{(2\,x)}+1\right)e^{(-3\,x)}+\frac{1}{12}\,e^{(3\,x)}+\frac{1}{4}\,e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*sinh(2*x),x, algorithm="giac")
[Out] 1/12*(3*e^(2*x) + 1)*e^(-3*x) + 1/12*e^(3*x) + 1/4*e^x
```

maple [A] time = 0.10, size = 7, normalized size = 0.88

$$\frac{2\left(\cosh^{3}(x)\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)*sinh(2*x),x)
[Out] 2/3*cosh(x)^3
```

maxima [B] time = 0.34, size = 27, normalized size = 3.38

$$\frac{1}{12}\left(3\,e^{(-2\,x)}+1\right)e^{(3\,x)}+\frac{1}{4}\,e^{(-x)}+\frac{1}{12}\,e^{(-3\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*sinh(2*x),x, algorithm="maxima")
[Out] 1/12*(3*e^(-2*x) + 1)*e^(3*x) + 1/4*e^(-x) + 1/12*e^(-3*x)
```

mupad [B] time = 0.05, size = 6, normalized size = 0.75

$$\frac{2\cosh(x)^{3}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(2*x)*cosh(x),x)
[Out] (2*cosh(x)^3)/3
```

sympy [B] time = 0.45, size = 20, normalized size = 2.50

$$-\frac{\sinh(x)\sinh(2x)}{3}+\frac{2\cosh(x)\cosh(2x)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*sinh(2*x),x)
```

```
[Out] -sinh(x)*sinh(2*x)/3 + 2*cosh(x)*cosh(2*x)/3
```

3.222 $\int \cosh(x)\sinh(3x)\,dx$

Optimal. Leaf size=17

$$\frac{1}{4}\cosh(2x)+\frac{1}{8}\cosh(4x)$$

```
[Out] 1/4*cosh(2*x)+1/8*cosh(4*x)
```

Rubi [A] time = 0.01, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4284}

$$\frac{1}{4}\cosh(2x)+\frac{1}{8}\cosh(4x)$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]*Sinh[3*x],x]
```

```
[Out] Cosh[2*x]/4 + Cosh[4*x]/8
```

Rule 4284

```
Int[cos[(c_.) + (d_.)*(x_)]*sin[(a_.) + (b_.)*(x_)], x_Symbol] :> -Simp[Cos
[a - c + (b - d)*x]/(2*(b - d)), x] - Simp[Cos[a + c + (b + d)*x]/(2*(b + d
)), x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - d^2, 0]
```

Rubi steps

$$\int \cosh(x)\sinh(3x)\,dx=\frac{1}{4}\cosh(2x)+\frac{1}{8}\cosh(4x)$$

Mathematica [A] time = 0.01, size = 17, normalized size = 1.00

$$\frac{\cosh^2(x)}{2}+\frac{1}{8}\cosh(4x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]*Sinh[3*x],x]
```

```
[Out] Cosh[x]^2/2 + Cosh[4*x]/8
```

fricas [B] time = 0.47, size = 33, normalized size = 1.94

$$\frac{1}{8}\cosh(x)^4 + \frac{1}{8}\sinh(x)^4 + \frac{1}{4}\left(3\cosh(x)^2 + 1\right)\sinh(x)^2 + \frac{1}{4}\cosh(x)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*sinh(3*x),x, algorithm="fricas")
[Out] 1/8*cosh(x)^4 + 1/8*sinh(x)^4 + 1/4*(3*cosh(x)^2 + 1)*sinh(x)^2 + 1/4*cosh(
x)^2
```

giac [A] time = 0.11, size = 26, normalized size = 1.53

$$\frac{1}{16}\left(e^{(2x)} + e^{(-2x)}\right)^2 + \frac{1}{8}e^{(2x)} + \frac{1}{8}e^{(-2x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*sinh(3*x),x, algorithm="giac")
[Out] 1/16*(e^(2*x) + e^(-2*x))^2 + 1/8*e^(2*x) + 1/8*e^(-2*x)
```

maple [A] time = 0.11, size = 12, normalized size = 0.71

$$\cosh^4(x) - \frac{\left(\cosh^2(x)\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)*sinh(3*x),x)
[Out] cosh(x)^4-1/2*cosh(x)^2
```

maxima [B] time = 0.33, size = 27, normalized size = 1.59

$$\frac{1}{16}\left(2e^{(-2x)} + 1\right)e^{(4x)} + \frac{1}{8}e^{(-2x)} + \frac{1}{16}e^{(-4x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*sinh(3*x),x, algorithm="maxima")
[Out] 1/16*(2*e^(-2*x) + 1)*e^(4*x) + 1/8*e^(-2*x) + 1/16*e^(-4*x)
```

mupad [B] time = 1.45, size = 11, normalized size = 0.65

$$\cosh(x)^4 - \frac{\cosh(x)^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(3*x)*cosh(x),x)
```

```
[Out] cosh(x)^4 - cosh(x)^2/2
```

sympy [A] time = 0.41, size = 20, normalized size = 1.18

$$-\frac{\sinh(x)\sinh(3x)}{8}+\frac{3\cosh(x)\cosh(3x)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*sinh(3*x),x)
```

```
[Out] -sinh(x)*sinh(3*x)/8 + 3*cosh(x)*cosh(3*x)/8
```

3.223 $\int \cosh(x)\sinh(4x)\,dx$

Optimal. Leaf size=17

$$\frac{1}{6}\cosh(3x)+\frac{1}{10}\cosh(5x)$$

```
[Out] 1/6*cosh(3*x)+1/10*cosh(5*x)
```

Rubi [A] time = 0.01, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4284}

$$\frac{1}{6}\cosh(3x)+\frac{1}{10}\cosh(5x)$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]*Sinh[4*x],x]
```

```
[Out] Cosh[3*x]/6 + Cosh[5*x]/10
```

Rule 4284

```
Int[cos[(c_.) + (d_.)*(x_)]*sin[(a_.) + (b_.)*(x_)], x_Symbol] :> -Simp[Cos
[a - c + (b - d)*x]/(2*(b - d)), x] - Simp[Cos[a + c + (b + d)*x]/(2*(b + d
)), x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - d^2, 0]
```

Rubi steps

$$\int \cosh(x)\sinh(4x)\,dx=\frac{1}{6}\cosh(3x)+\frac{1}{10}\cosh(5x)$$

Mathematica [A] time = 0.01, size = 17, normalized size = 1.00

$$\frac{1}{6}\cosh(3x)+\frac{1}{10}\cosh(5x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]*Sinh[4*x],x]
```

```
[Out] Cosh[3*x]/6 + Cosh[5*x]/10
```

fricas [B] time = 0.42, size = 36, normalized size = 2.12

$$\frac{1}{10}\cosh(x)^5+\frac{1}{2}\cosh(x)\sinh(x)^4+\frac{1}{6}\cosh(x)^3+\frac{1}{2}\left(2\cosh(x)^3+\cosh(x)\right)\sinh(x)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*sinh(4*x),x, algorithm="fricas")
```

```
[Out] 1/10*cosh(x)^5 + 1/2*cosh(x)*sinh(x)^4 + 1/6*cosh(x)^3 + 1/2*(2*cosh(x)^3 +
 cosh(x))*sinh(x)^2
```

giac [B] time = 0.11, size = 27, normalized size = 1.59

$$\frac{1}{60}\left(5\,e^{(2\,x)}+3\right)e^{(-5\,x)}+\frac{1}{20}\,e^{(5\,x)}+\frac{1}{12}\,e^{(3\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*sinh(4*x),x, algorithm="giac")
```

```
[Out] 1/60*(5*e^(2*x) + 3)*e^(-5*x) + 1/20*e^(5*x) + 1/12*e^(3*x)
```

maple [A] time = 0.10, size = 14, normalized size = 0.82

$$\frac{8\left(\cosh^{5}(x)\right)}{5}-\frac{4\left(\cosh^{3}(x)\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)*sinh(4*x),x)
```

```
[Out] 8/5*cosh(x)^5-4/3*cosh(x)^3
```

maxima [B] time = 0.34, size = 27, normalized size = 1.59

$$\frac{1}{60}\left(5\,e^{(-2\,x)}+3\right)e^{(5\,x)}+\frac{1}{12}\,e^{(-3\,x)}+\frac{1}{20}\,e^{(-5\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*sinh(4*x),x, algorithm="maxima")
```

```
[Out] 1/60*(5*e^(-2*x) + 3)*e^(5*x) + 1/12*e^(-3*x) + 1/20*e^(-5*x)
```

mupad [B] time = 1.44, size = 14, normalized size = 0.82

$$\frac{4\cosh(x)^{3}\,\left(6\cosh(x)^{2}-5\right)}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(4*x)*cosh(x),x)
```

```
[Out] (4*cosh(x)^3*(6*cosh(x)^2 - 5))/15
```

sympy [A] time = 0.41, size = 20, normalized size = 1.18

$$-\frac{\sinh(x)\sinh(4x)}{15}+\frac{4\cosh(x)\cosh(4x)}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*sinh(4*x),x)
```

```
[Out] -sinh(x)*sinh(4*x)/15 + 4*cosh(x)*cosh(4*x)/15
```

3.224 $\int \cosh(x)\sinh(mx)\,dx$

Optimal. Leaf size=35

$$\frac{\cosh((m+1)x)}{2(m+1)}-\frac{\cosh((1-m)x)}{2(1-m)}$$

```
[Out] -1/2*cosh((1-m)*x)/(1-m)+1/2*cosh((1+m)*x)/(1+m)
```

Rubi [A] time = 0.03, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {5618, 2638}

$$\frac{\cosh((m+1)x)}{2(m+1)}-\frac{\cosh((1-m)x)}{2(1-m)}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]*Sinh[m*x],x]
```

```
[Out] -Cosh[(1 - m)*x]/(2*(1 - m)) + Cosh[(1 + m)*x]/(2*(1 + m))
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 5618

```
Int[Cosh[w_]^(q_.)*Sinh[v_]^(p_.), x_Symbol] :> Int[ExpandTrigReduce[Sinh[v
]^p*Cosh[w]^q, x], x] /; IGtQ[p, 0] && IGtQ[q, 0] && ((PolynomialQ[v, x] &&
 PolynomialQ[w, x]) || (BinomialQ[{v, w}, x] && IndependentQ[Cancel[v/w], x
]))
```

Rubi steps

$$\begin{aligned}\int \cosh(x)\sinh(mx)\,dx &= \int \left(-\frac{1}{2}\sinh((1-m)x)+\frac{1}{2}\sinh((1+m)x)\right)dx\\ &= -\left(\frac{1}{2}\int \sinh((1-m)x)\,dx\right)+\frac{1}{2}\int \sinh((1+m)x)\,dx\\ &= -\frac{\cosh((1-m)x)}{2(1-m)}+\frac{\cosh((1+m)x)}{2(1+m)}\end{aligned}$$

Mathematica [A] time = 0.04, size = 25, normalized size = 0.71

$$\frac{m\cosh(x)\cosh(mx)-\sinh(x)\sinh(mx)}{m^2-1}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]*Sinh[m*x],x]
```

```
[Out] (m*Cosh[x]*Cosh[m*x] - Sinh[x]*Sinh[m*x])/(-1 + m^2)
```

fricas [A] time = 0.51, size = 42, normalized size = 1.20

$$\frac{m\cosh(mx)\cosh(x)-\sinh(mx)\sinh(x)}{(m^2-1)\cosh(x)^2-(m^2-1)\sinh(x)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*sinh(m*x),x, algorithm="fricas")
```

```
[Out] (m*cosh(m*x)*cosh(x) - sinh(m*x)*sinh(x))/((m^2 - 1)*cosh(x)^2 - (m^2 - 1)*
sinh(x)^2)
```

giac [B] time = 0.14, size = 59, normalized size = 1.69

$$\frac{e^{(mx+x)}}{4(m+1)}+\frac{e^{(mx-x)}}{4(m-1)}+\frac{e^{(-mx+x)}}{4(m-1)}+\frac{e^{(-mx-x)}}{4(m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*sinh(m*x),x, algorithm="giac")
```

```
[Out] 1/4*e^(m*x + x)/(m + 1) + 1/4*e^(m*x - x)/(m - 1) + 1/4*e^(-m*x + x)/(m - 1
) + 1/4*e^(-m*x - x)/(m + 1)
```

maple [A] time = 0.04, size = 28, normalized size = 0.80

$$\frac{\cosh((-1+m)x)}{-2+2m}+\frac{\cosh((1+m)x)}{2+2m}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)*sinh(m*x),x)
```

```
[Out] 1/2/(-1+m)*cosh((-1+m)*x)+1/2*cosh((1+m)*x)/(1+m)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*sinh(m*x),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(m-2>0)', see `assume?` for more det
ails)Is m-2 equal to -1?
```

mupad [B] time = 1.47, size = 26, normalized size = 0.74

$$-\frac{\sinh(m\,x)\,\sinh(x) - m\cosh(m\,x)\,\cosh(x)}{m^2-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(m*x)*cosh(x),x)
```

```
[Out] -(sinh(m*x)*sinh(x) - m*cosh(m*x)*cosh(x))/(m^2 - 1)
```

sympy [A] time = 0.76, size = 42, normalized size = 1.20

$$\begin{cases} -\frac{\cosh^2(x)}{2} & \text{for } m = -1 \\ \frac{\cosh^2(x)}{2} & \text{for } m = 1 \\ \frac{m\cosh(x)\cosh(mx)}{m^2-1} - \frac{\sinh(x)\sinh(mx)}{m^2-1} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*sinh(m*x),x)
```

```
[Out] Piecewise((-cosh(x)**2/2, Eq(m, -1)), (cosh(x)**2/2, Eq(m, 1)), (m*cosh(x)*
cosh(m*x)/(m**2 - 1) - sinh(x)*sinh(m*x)/(m**2 - 1), True))
```

3.225 $\int \cosh(x)\cosh(2x)\,dx$

Optimal. Leaf size=15

$$\frac{\sinh(x)}{2}+\frac{1}{6}\sinh(3x)$$

```
[Out] 1/2*sinh(x)+1/6*sinh(3*x)
```

Rubi [A] time = 0.01, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4283}

$$\frac{\sinh(x)}{2}+\frac{1}{6}\sinh(3x)$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]*Cosh[2*x],x]
```

```
[Out] Sinh[x]/2 + Sinh[3*x]/6
```

Rule 4283

```
Int[cos[(a_.) + (b_.)*(x_)]*cos[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[
a - c + (b - d)*x]/(2*(b - d)), x] + Simp[Sin[a + c + (b + d)*x]/(2*(b + d)
), x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - d^2, 0]
```

Rubi steps

$$\int \cosh(x)\cosh(2x)\,dx=\frac{\sinh(x)}{2}+\frac{1}{6}\sinh(3x)$$

Mathematica [A] time = 0.01, size = 15, normalized size = 1.00

$$\frac{\sinh(x)}{2}+\frac{1}{6}\sinh(3x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]*Cosh[2*x],x]
```

```
[Out] Sinh[x]/2 + Sinh[3*x]/6
```

fricas [A] time = 0.43, size = 17, normalized size = 1.13

$$\frac{1}{6}\sinh(x)^3+\frac{1}{2}\left(\cosh(x)^2+1\right)\sinh(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*cosh(2*x),x, algorithm="fricas")
[Out] 1/6*sinh(x)^3 + 1/2*(cosh(x)^2 + 1)*sinh(x)
```

giac [B] time = 0.13, size = 25, normalized size = 1.67

$$-\frac{1}{12}\left(3e^{(2x)}+1\right)e^{(-3x)}+\frac{1}{12}e^{(3x)}+\frac{1}{4}e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*cosh(2*x),x, algorithm="giac")
[Out] -1/12*(3*e^(2*x) + 1)*e^(-3*x) + 1/12*e^(3*x) + 1/4*e^x
```

maple [A] time = 0.16, size = 12, normalized size = 0.80

$$\frac{\sinh(x)}{2}+\frac{\sinh(3x)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)*cosh(2*x),x)
[Out] 1/2*sinh(x)+1/6*sinh(3*x)
```

maxima [B] time = 0.34, size = 27, normalized size = 1.80

$$\frac{1}{12}\left(3e^{(-2x)}+1\right)e^{(3x)}-\frac{1}{4}e^{(-x)}-\frac{1}{12}e^{(-3x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*cosh(2*x),x, algorithm="maxima")
[Out] 1/12*(3*e^(-2*x) + 1)*e^(3*x) - 1/4*e^(-x) - 1/12*e^(-3*x)
```

mupad [B] time = 0.06, size = 9, normalized size = 0.60

$$\frac{2\sinh(x)^3}{3}+\sinh(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(2*x)*cosh(x),x)
[Out] sinh(x) + (2*sinh(x)^3)/3
```

sympy [A] time = 0.45, size = 20, normalized size = 1.33

$$-\frac{\sinh(x)\cosh(2x)}{3}+\frac{2\sinh(2x)\cosh(x)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*cosh(2*x),x)
```

```
[Out] -sinh(x)*cosh(2*x)/3 + 2*sinh(2*x)*cosh(x)/3
```

3.226 $\int \cosh(x)\cosh(3x)\,dx$

Optimal. Leaf size=17

$$\frac{1}{4}\sinh(2x)+\frac{1}{8}\sinh(4x)$$

```
[Out] 1/4*sinh(2*x)+1/8*sinh(4*x)
```

Rubi [A] time = 0.01, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4283}

$$\frac{1}{4}\sinh(2x)+\frac{1}{8}\sinh(4x)$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]*Cosh[3*x],x]
```

```
[Out] Sinh[2*x]/4 + Sinh[4*x]/8
```

Rule 4283

```
Int[cos[(a_.) + (b_.)*(x_)]*cos[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[
a - c + (b - d)*x]/(2*(b - d)), x] + Simp[Sin[a + c + (b + d)*x]/(2*(b + d)
), x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - d^2, 0]
```

Rubi steps

$$\int \cosh(x)\cosh(3x)\,dx=\frac{1}{4}\sinh(2x)+\frac{1}{8}\sinh(4x)$$

Mathematica [A] time = 0.01, size = 17, normalized size = 1.00

$$\frac{1}{4}\sinh(2x)+\frac{1}{8}\sinh(4x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]*Cosh[3*x],x]
```

```
[Out] Sinh[2*x]/4 + Sinh[4*x]/8
```

fricas [A] time = 0.39, size = 20, normalized size = 1.18

$$\frac{1}{2}\cosh(x)\sinh(x)^3+\frac{1}{2}\left(\cosh(x)^3+\cosh(x)\right)\sinh(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*cosh(3*x),x, algorithm="fricas")
[Out] 1/2*cosh(x)*sinh(x)^3 + 1/2*(cosh(x)^3 + cosh(x))*sinh(x)
```

giac [B] time = 0.11, size = 27, normalized size = 1.59

$$-\frac{1}{16}\left(2e^{(2x)}+1\right)e^{(-4x)}+\frac{1}{16}e^{(4x)}+\frac{1}{8}e^{(2x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*cosh(3*x),x, algorithm="giac")
[Out] -1/16*(2*e^(2*x) + 1)*e^(-4*x) + 1/16*e^(4*x) + 1/8*e^(2*x)
```

maple [A] time = 0.21, size = 14, normalized size = 0.82

$$\frac{\sinh(2x)}{4}+\frac{\sinh(4x)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)*cosh(3*x),x)
[Out] 1/4*sinh(2*x)+1/8*sinh(4*x)
```

maxima [B] time = 0.33, size = 27, normalized size = 1.59

$$\frac{1}{16}\left(2e^{(-2x)}+1\right)e^{(4x)}-\frac{1}{8}e^{(-2x)}-\frac{1}{16}e^{(-4x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*cosh(3*x),x, algorithm="maxima")
[Out] 1/16*(2*e^(-2*x) + 1)*e^(4*x) - 1/8*e^(-2*x) - 1/16*e^(-4*x)
```

mupad [B] time = 1.45, size = 20, normalized size = 1.18

$$\frac{e^{-4x}\left(e^{2x}-1\right)\left(e^{2x}+1\right)^3}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(3*x)*cosh(x),x)
[Out] (exp(-4*x)*(exp(2*x) - 1)*(exp(2*x) + 1)^3)/16
```

sympy [A] time = 0.41, size = 20, normalized size = 1.18

$$-\frac{\sinh(x)\cosh(3x)}{8}+\frac{3\sinh(3x)\cosh(x)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*cosh(3*x),x)
[Out] -sinh(x)*cosh(3*x)/8 + 3*sinh(3*x)*cosh(x)/8
```

3.227 $\int \cosh(x)\cosh(4x)\,dx$

Optimal. Leaf size=17

$$\frac{1}{6}\sinh(3x)+\frac{1}{10}\sinh(5x)$$

```
[Out] 1/6*sinh(3*x)+1/10*sinh(5*x)
```

Rubi [A] time = 0.01, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4283}

$$\frac{1}{6}\sinh(3x)+\frac{1}{10}\sinh(5x)$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]*Cosh[4*x],x]
```

```
[Out] Sinh[3*x]/6 + Sinh[5*x]/10
```

Rule 4283

```
Int[cos[(a_.) + (b_.)*(x_)]*cos[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[
a - c + (b - d)*x]/(2*(b - d)), x] + Simp[Sin[a + c + (b + d)*x]/(2*(b + d)
), x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - d^2, 0]
```

Rubi steps

$$\int \cosh(x)\cosh(4x)\,dx=\frac{1}{6}\sinh(3x)+\frac{1}{10}\sinh(5x)$$

Mathematica [A] time = 0.00, size = 17, normalized size = 1.00

$$\frac{1}{6}\sinh(3x)+\frac{1}{10}\sinh(5x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]*Cosh[4*x],x]
```

```
[Out] Sinh[3*x]/6 + Sinh[5*x]/10
```

fricas [B] time = 0.41, size = 34, normalized size = 2.00

$$\frac{1}{10}\sinh(x)^5+\frac{1}{6}\left(6\cosh(x)^2+1\right)\sinh(x)^3+\frac{1}{2}\left(\cosh(x)^4+\cosh(x)^2\right)\sinh(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*cosh(4*x),x, algorithm="fricas")
```

```
[Out] 1/10*sinh(x)^5 + 1/6*(6*cosh(x)^2 + 1)*sinh(x)^3 + 1/2*(cosh(x)^4 + cosh(x)
^2)*sinh(x)
```

giac [B] time = 0.11, size = 27, normalized size = 1.59

$$-\frac{1}{60}\left(5\,e^{(2\,x)}+3\right)e^{(-5\,x)}+\frac{1}{20}\,e^{(5\,x)}+\frac{1}{12}\,e^{(3\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*cosh(4*x),x, algorithm="giac")
```

```
[Out] -1/60*(5*e^(2*x) + 3)*e^(-5*x) + 1/20*e^(5*x) + 1/12*e^(3*x)
```

maple [A] time = 0.23, size = 14, normalized size = 0.82

$$\frac{\sinh\left(3x\right)}{6}+\frac{\sinh\left(5x\right)}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)*cosh(4*x),x)
```

```
[Out] 1/6*sinh(3*x)+1/10*sinh(5*x)
```

maxima [B] time = 0.33, size = 27, normalized size = 1.59

$$\frac{1}{60}\left(5\,e^{(-2\,x)}+3\right)e^{(5\,x)}-\frac{1}{12}\,e^{(-3\,x)}-\frac{1}{20}\,e^{(-5\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*cosh(4*x),x, algorithm="maxima")
```

```
[Out] 1/60*(5*e^(-2*x) + 3)*e^(5*x) - 1/12*e^(-3*x) - 1/20*e^(-5*x)
```

mupad [B] time = 1.44, size = 15, normalized size = 0.88

$$\frac{8\sinh(x)^5}{5}+\frac{8\sinh(x)^3}{3}+\sinh(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(4*x)*cosh(x),x)
```

```
[Out] sinh(x) + (8*sinh(x)^3)/3 + (8*sinh(x)^5)/5
```

sympy [**A**] time = 0.41, size = 20, normalized size = 1.18

$$-\frac{\sinh(x)\cosh(4x)}{15}+\frac{4\sinh(4x)\cosh(x)}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*cosh(4*x),x)
[Out] -sinh(x)*cosh(4*x)/15 + 4*sinh(4*x)*cosh(x)/15
```

3.228 $\int \cosh(x)\cosh(mx)\,dx$

Optimal. Leaf size=35

$$\frac{\sinh((1-m)x)}{2(1-m)}+\frac{\sinh((m+1)x)}{2(m+1)}$$

```
[Out] 1/2*sinh((1-m)*x)/(1-m)+1/2*sinh((1+m)*x)/(1+m)
```

Rubi [A] time = 0.03, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {5614, 2637}

$$\frac{\sinh((1-m)x)}{2(1-m)}+\frac{\sinh((m+1)x)}{2(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]*Cosh[m*x],x]
```

```
[Out] Sinh[(1 - m)*x]/(2*(1 - m)) + Sinh[(1 + m)*x]/(2*(1 + m))
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 5614

```
Int[Cosh[v_]^(p_.)*Cosh[w_]^(q_.), x_Symbol] :> Int[ExpandTrigReduce[Cosh[v
]^p*Cosh[w]^q, x], x] /; IGtQ[p, 0] && IGtQ[q, 0] && ((PolynomialQ[v, x] &&
 PolynomialQ[w, x]) || (BinomialQ[{v, w}, x] && IndependentQ[Cancel[v/w], x
]))
```

Rubi steps

$$\begin{aligned}
\int \cosh(x)\cosh(mx)\,dx &= \int \left(\frac{1}{2}\cosh((1-m)x)+\frac{1}{2}\cosh((1+m)x)\right)dx\\
&= \frac{1}{2}\int \cosh((1-m)x)\,dx+\frac{1}{2}\int \cosh((1+m)x)\,dx\\
&= \frac{\sinh((1-m)x)}{2(1-m)}+\frac{\sinh((1+m)x)}{2(1+m)}
\end{aligned}$$

Mathematica [A] time = 0.04, size = 25, normalized size = 0.71

$$\frac{m\cosh(x)\sinh(mx)-\sinh(x)\cosh(mx)}{m^2-1}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]*Cosh[m*x],x]
```

```
[Out] (-(Cosh[m*x]*Sinh[x]) + m*Cosh[x]*Sinh[m*x])/(-1 + m^2)
```

fricas [A] time = 0.42, size = 42, normalized size = 1.20

$$\frac{m\cosh(x)\sinh(mx)-\cosh(mx)\sinh(x)}{(m^2-1)\cosh(x)^2-(m^2-1)\sinh(x)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*cosh(m*x),x, algorithm="fricas")
```

```
[Out] (m*cosh(x)*sinh(m*x) - cosh(m*x)*sinh(x))/((m^2 - 1)*cosh(x)^2 - (m^2 - 1)*
sinh(x)^2)
```

giac [B] time = 0.11, size = 59, normalized size = 1.69

$$\frac{e^{(mx+x)}}{4(m+1)}+\frac{e^{(mx-x)}}{4(m-1)}-\frac{e^{(-mx+x)}}{4(m-1)}-\frac{e^{(-mx-x)}}{4(m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*cosh(m*x),x, algorithm="giac")
```

```
[Out] 1/4*e^(m*x + x)/(m + 1) + 1/4*e^(m*x - x)/(m - 1) - 1/4*e^(-m*x + x)/(m - 1
) - 1/4*e^(-m*x - x)/(m + 1)
```

maple [A] time = 0.18, size = 28, normalized size = 0.80

$$\frac{\sinh((-1+m)x)}{-2+2m}+\frac{\sinh((1+m)x)}{2+2m}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)*cosh(m*x),x)
```

```
[Out] 1/2/(-1+m)*sinh((-1+m)*x)+1/2*sinh((1+m)*x)/(1+m)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*cosh(m*x),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(m-2>0)', see `assume?` for more det
ails)Is m-2 equal to -1?
```

mupad [B] time = 0.07, size = 26, normalized size = 0.74

$$-\frac{\cosh\left(m\,x\right)\,\sinh(x)-m\sinh\left(m\,x\right)\,\cosh(x)}{m^2-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(m*x)*cosh(x),x)
```

```
[Out] -(cosh(m*x)*sinh(x) - m*sinh(m*x)*cosh(x))/(m^2 - 1)
```

sympy [A] time = 0.92, size = 56, normalized size = 1.60

$$\begin{cases} -\frac{x\sinh^2\left(x\right)}{2}+\frac{x\cosh^2\left(x\right)}{2}+\frac{\sinh\left(x\right)\cosh\left(x\right)}{2} & \text{for } m=-1 \vee m=1 \\ \frac{m\sinh\left(mx\right)\cosh\left(x\right)}{m^2-1}-\frac{\sinh\left(x\right)\cosh\left(mx\right)}{m^2-1} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*cosh(m*x),x)
```

```
[Out] Piecewise((-x*sinh(x)**2/2 + x*cosh(x)**2/2 + sinh(x)*cosh(x)/2, Eq(m, -1)
| Eq(m, 1)), (m*sinh(m*x)*cosh(x)/(m**2 - 1) - sinh(x)*cosh(m*x)/(m**2 - 1)
, True))
```

3.229 $\int \cosh(x)\tanh(2x)\,dx$

Optimal. Leaf size=19

$$\cosh(x) - \frac{\tanh^{-1}\left(\sqrt{2}\cosh(x)\right)}{\sqrt{2}}$$

```
[Out] cosh(x)-1/2*arctanh(cosh(x)*2^(1/2))*2^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {12, 321, 207}

$$\cosh(x) - \frac{\tanh^{-1}\left(\sqrt{2}\cosh(x)\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]*Tanh[2*x],x]
```

```
[Out] -(ArcTanh[Sqrt[2]*Cosh[x]]/Sqrt[2]) + Cosh[x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}\int \cosh(x)\tanh(2x)\,dx &= \text{Subst}\left(\int \frac{2x^2}{-1+2x^2}\,dx, x, \cosh(x)\right)\\ &= 2\,\text{Subst}\left(\int \frac{x^2}{-1+2x^2}\,dx, x, \cosh(x)\right)\\ &= \cosh(x) + \text{Subst}\left(\int \frac{1}{-1+2x^2}\,dx, x, \cosh(x)\right)\\ &= -\frac{\tanh^{-1}\left(\sqrt{2}\cosh(x)\right)}{\sqrt{2}} + \cosh(x)\end{aligned}$$

Mathematica [C] time = 0.18, size = 164, normalized size = 8.63

$$\frac{4\sqrt{2}\cosh(x) - 4\tanh^{-1}\left(\sqrt{2} - i\tanh\left(\frac{x}{2}\right)\right) + \log\left(\sqrt{2} - 2\cosh(x)\right) - \log\left(2\cosh(x) + \sqrt{2}\right) - 2i\tan^{-1}\left(\frac{\sin}{(1+\sqrt{2})\cos}\right.}{4\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]*Tanh[2*x],x]
```

```
[Out] ((-2*I)*ArcTan[(Cosh[x/2] + Sinh[x/2])/((1 + Sqrt[2])*Cosh[x/2] - (-1 + Sqr
t[2])*Sinh[x/2])] + (2*I)*ArcTan[(Cosh[x/2] + Sinh[x/2])/((-1 + Sqrt[2])*Co
sh[x/2] - (1 + Sqrt[2])*Sinh[x/2])] - 4*ArcTanh[Sqrt[2] - I*Tanh[x/2]] + 4*
Sqrt[2]*Cosh[x] + Log[Sqrt[2] - 2*Cosh[x]] - Log[Sqrt[2] + 2*Cosh[x]])/(4*S
qrt[2])
```

fricas [B] time = 0.45, size = 73, normalized size = 3.84

$$\frac{2\cosh(x)^2 + \left(\sqrt{2}\cosh(x) + \sqrt{2}\sinh(x)\right)\log\left(\frac{\cosh(x)^2+\sinh(x)^2-2\sqrt{2}\cosh(x)+2}{\cosh(x)^2+\sinh(x)^2}\right) + 4\cosh(x)\sinh(x) + 2\sinh(x)^2 +}{4\left(\cosh(x)+\sinh(x)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*tanh(2*x),x, algorithm="fricas")
```

```
[Out] 1/4*(2*cosh(x)^2 + (sqrt(2)*cosh(x) + sqrt(2)*sinh(x))*log((cosh(x)^2 + sin
h(x)^2 - 2*sqrt(2)*cosh(x) + 2)/(cosh(x)^2 + sinh(x)^2)) + 4*cosh(x)*sinh(x
) + 2*sinh(x)^2 + 2)/(cosh(x) + sinh(x))
```

giac [B] time = 0.11, size = 45, normalized size = 2.37

$$\frac{1}{4}\sqrt{2}\log\left(-\frac{\sqrt{2} - e^{(-x)} - e^{x}}{\sqrt{2} + e^{(-x)} + e^{x}}\right) + \frac{1}{2}e^{(-x)} + \frac{1}{2}e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*tanh(2*x),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(2)*log(-(sqrt(2) - e^(-x) - e^x)/(sqrt(2) + e^(-x) + e^x)) + 1/2*e
^(-x) + 1/2*e^x
```

maple [A] time = 0.07, size = 16, normalized size = 0.84

$$\cosh(x) - \frac{\operatorname{arctanh}\left(\cosh(x)\sqrt{2}\right)\sqrt{2}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)*tanh(2*x),x)
```

```
[Out] cosh(x)-1/2*arctanh(cosh(x)*2^(1/2))*2^(1/2)
```

maxima [B] time = 0.44, size = 52, normalized size = 2.74

$$-\frac{1}{4}\sqrt{2}\log\left(\sqrt{2}e^{(-x)} + e^{(-2x)} + 1\right) + \frac{1}{4}\sqrt{2}\log\left(-\sqrt{2}e^{(-x)} + e^{(-2x)} + 1\right) + \frac{1}{2}e^{(-x)} + \frac{1}{2}e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*tanh(2*x),x, algorithm="maxima")
```

```
[Out] -1/4*sqrt(2)*log(sqrt(2)*e^(-x) + e^(-2*x) + 1) + 1/4*sqrt(2)*log(-sqrt(2)*
e^(-x) + e^(-2*x) + 1) + 1/2*e^(-x) + 1/2*e^x
```

mupad [B] time = 1.45, size = 48, normalized size = 2.53

$$\frac{e^{-x}}{2} + \frac{e^{x}}{2} - \frac{\sqrt{2}\ln\left(e^{2x} + \sqrt{2}\,e^{x} + 1\right)}{4} + \frac{\sqrt{2}\ln\left(e^{2x} - \sqrt{2}\,e^{x} + 1\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(2*x)*cosh(x),x)
```

```
[Out] exp(-x)/2 + exp(x)/2 - (2^(1/2)*log(exp(2*x) + 2^(1/2)*exp(x) + 1))/4 + (2^
(1/2)*log(exp(2*x) - 2^(1/2)*exp(x) + 1))/4
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \cosh(x)\tanh(2x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*tanh(2*x),x)
```

```
[Out] Integral(cosh(x)*tanh(2*x), x)
```

3.230 $\int \cosh(x)\tanh(3x)\,dx$

Optimal. Leaf size=20

$$\cosh(x)-\frac{\tanh^{-1}\left(\frac{2\cosh(x)}{\sqrt{3}}\right)}{\sqrt{3}}$$

[Out] cosh(x)-1/3*arctanh(2/3*cosh(x)*3^(1/2))*3^(1/2)

Rubi [A] time = 0.03, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {388, 206}

$$\cosh(x)-\frac{\tanh^{-1}\left(\frac{2\cosh(x)}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[Cosh[x]*Tanh[3*x],x]

[Out] -(ArcTanh[(2*Cosh[x])/Sqrt[3]]/Sqrt[3]) + Cosh[x]

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rubi steps

$$\int \cosh(x)\tanh(3x)\,dx = \text{Subst}\left(\int \frac{1-4x^2}{3-4x^2}\,dx, x, \cosh(x)\right)$$
$$= \cosh(x) - 2\,\text{Subst}\left(\int \frac{1}{3-4x^2}\,dx, x, \cosh(x)\right)$$
$$= -\frac{\tanh^{-1}\left(\frac{2\cosh(x)}{\sqrt{3}}\right)}{\sqrt{3}} + \cosh(x)$$

Mathematica [C] time = 0.06, size = 55, normalized size = 2.75

$$\cosh(x) - \frac{\tanh^{-1}\left(\frac{2-i\tanh\left(\frac{x}{2}\right)}{\sqrt{3}}\right)}{\sqrt{3}} - \frac{\tanh^{-1}\left(\frac{2+i\tanh\left(\frac{x}{2}\right)}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]*Tanh[3*x],x]
```

```
[Out] -(ArcTanh[(2 - I*Tanh[x/2])/Sqrt[3]]/Sqrt[3]) - ArcTanh[(2 + I*Tanh[x/2])/S
qrt[3]]/Sqrt[3] + Cosh[x]
```

fricas [B] time = 0.41, size = 82, normalized size = 4.10

$$\frac{3\cosh(x)^2 + \left(\sqrt{3}\cosh(x) + \sqrt{3}\sinh(x)\right)\log\left(\frac{2\cosh(x)^2+2\sinh(x)^2-4\sqrt{3}\cosh(x)+5}{2\cosh(x)^2+2\sinh(x)^2-1}\right) + 6\cosh(x)\sinh(x) + 3\sinh(x}{6\left(\cosh(x)+\sinh(x)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*tanh(3*x),x, algorithm="fricas")
```

```
[Out] 1/6*(3*cosh(x)^2 + (sqrt(3)*cosh(x) + sqrt(3)*sinh(x))*log((2*cosh(x)^2 + 2
*sinh(x)^2 - 4*sqrt(3)*cosh(x) + 5)/(2*cosh(x)^2 + 2*sinh(x)^2 - 1)) + 6*co
sh(x)*sinh(x) + 3*sinh(x)^2 + 3)/(cosh(x) + sinh(x))
```

giac [B] time = 0.12, size = 45, normalized size = 2.25

$$\frac{1}{6}\sqrt{3}\log\left(-\frac{\sqrt{3} - e^{(-x)} - e^x}{\sqrt{3} + e^{(-x)} + e^x}\right) + \frac{1}{2}e^{(-x)} + \frac{1}{2}e^x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*tanh(3*x),x, algorithm="giac")
```

```
[Out] 1/6*sqrt(3)*log(-(sqrt(3) - e^(-x) - e^x)/(sqrt(3) + e^(-x) + e^x)) + 1/2*e
^(-x) + 1/2*e^x
```

maple [A] time = 0.08, size = 17, normalized size = 0.85

$$\cosh(x) - \frac{\operatorname{arctanh}\left(\frac{2\cosh(x)\sqrt{3}}{3}\right)\sqrt{3}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)*tanh(3*x),x)
```

```
[Out] cosh(x)-1/3*arctanh(2/3*cosh(x)*3^(1/2))*3^(1/2)
```

maxima [B] time = 0.46, size = 153, normalized size = 7.65

$$-\frac{1}{12}\sqrt{3}\log\left(\sqrt{3}e^{(-x)}+e^{(-2x)}+1\right)+\frac{1}{12}\sqrt{3}\log\left(-\sqrt{3}e^{(-x)}+e^{(-2x)}+1\right)-\frac{1}{12}\sqrt{3}\log\left(\sqrt{3}e^{x}+e^{(2x)}+1\right)+\frac{1}{12}\sqrt{3}\,\text{lo}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*tanh(3*x),x, algorithm="maxima")
```

```
[Out] -1/12*sqrt(3)*log(sqrt(3)*e^(-x) + e^(-2*x) + 1) + 1/12*sqrt(3)*log(-sqrt(3
)*e^(-x) + e^(-2*x) + 1) - 1/12*sqrt(3)*log(sqrt(3)*e^x + e^(2*x) + 1) + 1/
12*sqrt(3)*log(-sqrt(3)*e^x + e^(2*x) + 1) + 1/6*arctan(sqrt(3) + 2*e^(-x))
 + 1/6*arctan(sqrt(3) + 2*e^x) + 1/6*arctan(-sqrt(3) + 2*e^(-x)) + 1/6*arct
an(-sqrt(3) + 2*e^x) + 1/3*arctan(e^(-x)) + 1/3*arctan(e^x) + 1/2*e^(-x) +
1/2*e^x
```

mupad [B] time = 1.46, size = 53, normalized size = 2.65

$$\frac{e^{-x}}{2}+\frac{e^{x}}{2}+\frac{\sqrt{3}\,\ln\left(\frac{e^{2x}}{3}-\frac{\sqrt{3}\,e^{x}}{3}+\frac{1}{3}\right)}{6}-\frac{\sqrt{3}\,\ln\left(\frac{e^{2x}}{3}+\frac{\sqrt{3}\,e^{x}}{3}+\frac{1}{3}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(3*x)*cosh(x),x)
```

```
[Out] exp(-x)/2 + exp(x)/2 + (3^(1/2)*log(exp(2*x)/3 - (3^(1/2)*exp(x))/3 + 1/3))
/6 - (3^(1/2)*log(exp(2*x)/3 + (3^(1/2)*exp(x))/3 + 1/3))/6
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \cosh(x)\tanh(3x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*tanh(3*x),x)
```

```
[Out] Integral(cosh(x)*tanh(3*x), x)
```

3.231 $\int \cosh(x)\tanh(4x)\,dx$

Optimal. Leaf size=69

$$\cosh(x)-\frac{1}{4}\sqrt{2-\sqrt{2}}\ \tanh^{-1}\left(\frac{2\cosh(x)}{\sqrt{2-\sqrt{2}}}\right)-\frac{1}{4}\sqrt{2+\sqrt{2}}\ \tanh^{-1}\left(\frac{2\cosh(x)}{\sqrt{2+\sqrt{2}}}\right)$$

```
[Out] cosh(x)-1/4*arctanh(2*cosh(x)/(2-2^(1/2))^(1/2))*(2-2^(1/2))^(1/2)-1/4*arct
anh(2*cosh(x)/(2+2^(1/2))^(1/2))*(2+2^(1/2))^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {12, 1279, 1166, 207}

$$\cosh(x)-\frac{1}{4}\sqrt{2-\sqrt{2}}\ \tanh^{-1}\left(\frac{2\cosh(x)}{\sqrt{2-\sqrt{2}}}\right)-\frac{1}{4}\sqrt{2+\sqrt{2}}\ \tanh^{-1}\left(\frac{2\cosh(x)}{\sqrt{2+\sqrt{2}}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]*Tanh[4*x],x]
```

```
[Out] -(Sqrt[2 - Sqrt[2]]*ArcTanh[(2*Cosh[x])/Sqrt[2 - Sqrt[2]]])/4 - (Sqrt[2 + S
qrt[2]]*ArcTanh[(2*Cosh[x])/Sqrt[2 + Sqrt[2]]])/4 + Cosh[x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1279

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[(e*f*(f*x)^(m - 1)*(a + b*x^2 + c*x^4)^(p +
1))/(c*(m + 4*p + 3)), x] - Dist[f^2/(c*(m + 4*p + 3)), Int[(f*x)^(m - 2)*(
a + b*x^2 + c*x^4)^p*Simp[a*e*(m - 1) + (b*e*(m + 2*p + 1) - c*d*(m + 4*p +
 3))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c,
 0] && GtQ[m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] ||
 IntegerQ[m])
```

Rubi steps

$$\begin{aligned}\int \cosh(x)\tanh(4x)\,dx &= \text{Subst}\left(\int \frac{4x^2\left(-1+2x^2\right)}{1-8x^2+8x^4}\,dx, x, \cosh(x)\right)\\ &= 4\,\text{Subst}\left(\int \frac{x^2\left(-1+2x^2\right)}{1-8x^2+8x^4}\,dx, x, \cosh(x)\right)\\ &= \cosh(x) - \frac{1}{2}\text{Subst}\left(\int \frac{2-8x^2}{1-8x^2+8x^4}\,dx, x, \cosh(x)\right)\\ &= \cosh(x) - \left(-2+\sqrt{2}\right)\text{Subst}\left(\int \frac{1}{-4+2\sqrt{2}+8x^2}\,dx, x, \cosh(x)\right) + \left(2+\sqrt{2}\right)\text{Subst}\Big(\\ &= -\frac{1}{4}\sqrt{2-\sqrt{2}}\,\tanh^{-1}\left(\frac{2\cosh(x)}{\sqrt{2-\sqrt{2}}}\right) - \frac{1}{4}\sqrt{2+\sqrt{2}}\,\tanh^{-1}\left(\frac{2\cosh(x)}{\sqrt{2+\sqrt{2}}}\right) + \cosh(x)\end{aligned}$$

Mathematica [C] time = 0.02, size = 113, normalized size = 1.64

$$\frac{1}{16}\text{RootSum}\left[\#1^8+1\&, \frac{\#1^6 x + 2\#1^6\log\left(-\#1\sinh\left(\frac{x}{2}\right)+\#1\cosh\left(\frac{x}{2}\right)-\sinh\left(\frac{x}{2}\right)-\cosh\left(\frac{x}{2}\right)\right) - 2\log\left(-\#1\sinh\right.}{\#1^7}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]*Tanh[4*x],x]
```

```
[Out] Cosh[x] + RootSum[1 + #1^8 & , (-x - 2*Log[-Cosh[x/2] - Sinh[x/2] + Cosh[x/
2]*#1 - Sinh[x/2]*#1] + x*#1^6 + 2*Log[-Cosh[x/2] - Sinh[x/2] + Cosh[x/2]*#
1 - Sinh[x/2]*#1]*#1^6)/#1^7 & ]/16
```

fricas [B] time = 0.52, size = 213, normalized size = 3.09

$$-\frac{\sqrt{\sqrt{2}+2}\,(\cosh(x)+\sinh(x))\log\left(\cosh(x)^2+2\cosh(x)\sinh(x)+\sinh(x)^2+\sqrt{\sqrt{2}+2}\,(\cosh(x)+\sinh(x))+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*tanh(4*x),x, algorithm="fricas")
```

```
[Out] -1/8*(sqrt(sqrt(2) + 2)*(cosh(x) + sinh(x))*log(cosh(x)^2 + 2*cosh(x)*sinh(
x) + sinh(x)^2 + sqrt(sqrt(2) + 2)*(cosh(x) + sinh(x)) + 1) - sqrt(sqrt(2)
+ 2)*(cosh(x) + sinh(x))*log(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - sq
rt(sqrt(2) + 2)*(cosh(x) + sinh(x)) + 1) + sqrt(-sqrt(2) + 2)*(cosh(x) + si
nh(x))*log(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + sqrt(-sqrt(2) + 2)*(
cosh(x) + sinh(x)) + 1) - sqrt(-sqrt(2) + 2)*(cosh(x) + sinh(x))*log(cosh(x
)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - sqrt(-sqrt(2) + 2)*(cosh(x) + sinh(x)
) + 1) - 4*cosh(x)^2 - 8*cosh(x)*sinh(x) - 4*sinh(x)^2 - 4)/(cosh(x) + sinh
(x))
```

giac [B] time = 0.21, size = 119, normalized size = 1.72

$$-\frac{1}{8}\sqrt{\sqrt{2}+2}\,\log\left(\sqrt{\sqrt{2}+2}+e^{(-x)}+e^{x}\right)+\frac{1}{8}\sqrt{\sqrt{2}+2}\,\log\left(-\sqrt{\sqrt{2}+2}+e^{(-x)}+e^{x}\right)-\frac{1}{8}\sqrt{-\sqrt{2}+2}\,\log\left(\sqrt{-\sqrt{2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*tanh(4*x),x, algorithm="giac")
```

```
[Out] -1/8*sqrt(sqrt(2) + 2)*log(sqrt(sqrt(2) + 2) + e^(-x) + e^x) + 1/8*sqrt(sqr
t(2) + 2)*log(-sqrt(sqrt(2) + 2) + e^(-x) + e^x) - 1/8*sqrt(-sqrt(2) + 2)*l
og(sqrt(-sqrt(2) + 2) + e^(-x) + e^x) + 1/8*sqrt(-sqrt(2) + 2)*log(-sqrt(-s
qrt(2) + 2) + e^(-x) + e^x) + 1/2*e^(-x) + 1/2*e^x
```

maple [A] time = 0.16, size = 66, normalized size = 0.96

$$\cosh(x)-\frac{\left(1+\sqrt{2}\right)\sqrt{2}\,\operatorname{arctanh}\left(\frac{2\cosh(x)}{\sqrt{2+\sqrt{2}}}\right)}{4\sqrt{2+\sqrt{2}}}-\frac{\left(\sqrt{2}-1\right)\sqrt{2}\,\operatorname{arctanh}\left(\frac{2\cosh(x)}{\sqrt{2-\sqrt{2}}}\right)}{4\sqrt{2-\sqrt{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)*tanh(4*x),x)
```

```
[Out] cosh(x)-1/4*(1+2^(1/2))*2^(1/2)/(2+2^(1/2))^(1/2)*arctanh(2*cosh(x)/(2+2^(1
/2))^(1/2))-1/4*(2^(1/2)-1)*2^(1/2)/(2-2^(1/2))^(1/2)*arctanh(2*cosh(x)/(2-
2^(1/2))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{2}\left(e^{(2x)}+1\right)e^{(-x)}+\frac{1}{2}\int\frac{2\left(e^{(7x)}-e^{x}\right)}{e^{(8x)}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*tanh(4*x),x, algorithm="maxima")
```

```
[Out] 1/2*(e^(2*x) + 1)*e^(-x) + 1/2*integrate(2*(e^(7*x) - e^x)/(e^(8*x) + 1), x
)
```

mupad [B] time = 0.08, size = 133, normalized size = 1.93

$$\frac{e^{-x}}{2}+\frac{e^{x}}{2}+\ln\left(e^{2x}-8\,e^{x}\sqrt{\frac{1}{32}-\frac{\sqrt{2}}{64}}+1\right)\sqrt{\frac{1}{32}-\frac{\sqrt{2}}{64}}-\ln\left(e^{2x}+8\,e^{x}\sqrt{\frac{1}{32}-\frac{\sqrt{2}}{64}}+1\right)\sqrt{\frac{1}{32}-\frac{\sqrt{2}}{64}}+\ln\left(e^{2x}-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(4*x)*cosh(x),x)
```

```
[Out] exp(-x)/2 + exp(x)/2 + log(exp(2*x) - 8*exp(x)*(1/32 - 2^(1/2)/64)^(1/2) +
1)*(1/32 - 2^(1/2)/64)^(1/2) - log(exp(2*x) + 8*exp(x)*(1/32 - 2^(1/2)/64)^
(1/2) + 1)*(1/32 - 2^(1/2)/64)^(1/2) + log(exp(2*x) - 8*exp(x)*(2^(1/2)/64
+ 1/32)^(1/2) + 1)*(2^(1/2)/64 + 1/32)^(1/2) - log(exp(2*x) + 8*exp(x)*(2^(
1/2)/64 + 1/32)^(1/2) + 1)*(2^(1/2)/64 + 1/32)^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\cosh(x)\tanh(4x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*tanh(4*x),x)
```

```
[Out] Integral(cosh(x)*tanh(4*x), x)
```

3.232 $\int \cosh(x)\tanh(5x)\,dx$

Optimal. Leaf size=82

$$\cosh(x)-\frac{1}{5}\sqrt{\frac{1}{2}\left(5+\sqrt{5}\right)}\,\tanh^{-1}\left(2\sqrt{\frac{2}{5+\sqrt{5}}}\,\cosh(x)\right)-\frac{1}{5}\sqrt{\frac{1}{2}\left(5-\sqrt{5}\right)}\,\tanh^{-1}\left(\sqrt{\frac{2}{5}\left(5+\sqrt{5}\right)}\,\cosh(x)\right)$$

```
[Out] cosh(x)-1/10*arctanh(1/5*cosh(x)*(50+10*5^(1/2))^(1/2))*(10-2*5^(1/2))^(1/2
)-1/10*arctanh(2*cosh(x)*2^(1/2)/(5+5^(1/2))^(1/2))*(10+2*5^(1/2))^(1/2)
```

Rubi [A] time = 0.12, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {1676, 1166, 207}

$$\cosh(x)-\frac{1}{5}\sqrt{\frac{1}{2}\left(5+\sqrt{5}\right)}\,\tanh^{-1}\left(2\sqrt{\frac{2}{5+\sqrt{5}}}\,\cosh(x)\right)-\frac{1}{5}\sqrt{\frac{1}{2}\left(5-\sqrt{5}\right)}\,\tanh^{-1}\left(\sqrt{\frac{2}{5}\left(5+\sqrt{5}\right)}\,\cosh(x)\right)$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]*Tanh[5*x],x]
```

```
[Out] -(Sqrt[(5 + Sqrt[5])/2]*ArcTanh[2*Sqrt[2/(5 + Sqrt[5])]*Cosh[x]])/5 - (Sqrt
[(5 - Sqrt[5])/2]*ArcTanh[Sqrt[(2*(5 + Sqrt[5]))/5]*Cosh[x]])/5 + Cosh[x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1676

```
Int[(Pq_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Int[ExpandInte
grand[Pq/(a + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^
2] && Expon[Pq, x^2] > 1
```

Rubi steps

$$\int \cosh(x)\tanh(5x)\,dx = \text{Subst}\left(\int \frac{1-12x^2+16x^4}{5-20x^2+16x^4}\,dx, x, \cosh(x)\right)$$
$$= \text{Subst}\left(\int \left(1-\frac{4\left(1-2x^2\right)}{5-20x^2+16x^4}\right)dx, x, \cosh(x)\right)$$
$$= \cosh(x) - 4\,\text{Subst}\left(\int \frac{1-2x^2}{5-20x^2+16x^4}\,dx, x, \cosh(x)\right)$$
$$= \cosh(x) + \frac{1}{5}\left(4\left(5-\sqrt{5}\right)\right)\text{Subst}\left(\int \frac{1}{-10+2\sqrt{5}+16x^2}\,dx, x, \cosh(x)\right) + \frac{1}{5}\left(4\left(5+\sqrt{\right.\right.$$
$$= -\frac{1}{5}\sqrt{\frac{1}{2}\left(5+\sqrt{5}\right)}\tanh^{-1}\left(2\sqrt{\frac{2}{5+\sqrt{5}}}\cosh(x)\right) - \frac{1}{5}\sqrt{\frac{1}{2}\left(5-\sqrt{5}\right)}\tanh^{-1}\left(\sqrt{\frac{2}{5}\left(5+\right.}\right.$$

Mathematica [C] time = 0.03, size = 249, normalized size = 3.04

$$\frac{1}{4}\text{RootSum}\left[\#1^8-\#1^6+\#1^4-\#1^2+1\&, \frac{\#1^6 x+2\#1^6\log\left(-\#1\sinh\left(\frac{x}{2}\right)+\#1\cosh\left(\frac{x}{2}\right)-\sinh\left(\frac{x}{2}\right)-\cosh\left(\frac{x}{2}\right)\right)}{}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]*Tanh[5*x],x]
```

```
[Out] Cosh[x] + RootSum[1 - #1^2 + #1^4 - #1^6 + #1^8 & , (-x - 2*Log[-Cosh[x/2]
- Sinh[x/2] + Cosh[x/2]*#1 - Sinh[x/2]*#1] + x*#1^2 + 2*Log[-Cosh[x/2] - Si
nh[x/2] + Cosh[x/2]*#1 - Sinh[x/2]*#1]*#1^2 - x*#1^4 - 2*Log[-Cosh[x/2] - S
inh[x/2] + Cosh[x/2]*#1 - Sinh[x/2]*#1]*#1^4 + x*#1^6 + 2*Log[-Cosh[x/2] -
Sinh[x/2] + Cosh[x/2]*#1 - Sinh[x/2]*#1]*#1^6)/(-#1 + 2*#1^3 - 3*#1^5 + 4*#
1^7) & ]/4
```

fricas [B] time = 0.42, size = 293, normalized size = 3.57

$$-\frac{\left(\sqrt{2}\cosh(x)+\sqrt{2}\sinh(x)\right)\sqrt{\sqrt{5}+5}\,\log\left(2\cosh(x)^2+4\cosh(x)\sinh(x)+2\sinh(x)^2+\left(\sqrt{2}\cosh(x)+\sqrt{\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*tanh(5*x),x, algorithm="fricas")
```

```
[Out] -1/20*((sqrt(2)*cosh(x) + sqrt(2)*sinh(x))*sqrt(sqrt(5) + 5)*log(2*cosh(x)^
2 + 4*cosh(x)*sinh(x) + 2*sinh(x)^2 + (sqrt(2)*cosh(x) + sqrt(2)*sinh(x))*s
```

```
qrt(sqrt(5) + 5) + 2) - (sqrt(2)*cosh(x) + sqrt(2)*sinh(x))*sqrt(sqrt(5) +
5)*log(2*cosh(x)^2 + 4*cosh(x)*sinh(x) + 2*sinh(x)^2 - (sqrt(2)*cosh(x) + s
qrt(2)*sinh(x))*sqrt(sqrt(5) + 5) + 2) + (sqrt(2)*cosh(x) + sqrt(2)*sinh(x)
)*sqrt(-sqrt(5) + 5)*log(2*cosh(x)^2 + 4*cosh(x)*sinh(x) + 2*sinh(x)^2 + (s
qrt(2)*cosh(x) + sqrt(2)*sinh(x))*sqrt(-sqrt(5) + 5) + 2) - (sqrt(2)*cosh(x
) + sqrt(2)*sinh(x))*sqrt(-sqrt(5) + 5)*log(2*cosh(x)^2 + 4*cosh(x)*sinh(x)
 + 2*sinh(x)^2 - (sqrt(2)*cosh(x) + sqrt(2)*sinh(x))*sqrt(-sqrt(5) + 5) + 2
) - 10*cosh(x)^2 - 20*cosh(x)*sinh(x) - 10*sinh(x)^2 - 10)/(cosh(x) + sinh(
x))
```

giac [B] time = 0.18, size = 127, normalized size = 1.55

$$-\frac{1}{20}\sqrt{2\sqrt{5}+10}\log\left(\sqrt{\frac{1}{2}\sqrt{5}+\frac{5}{2}}+e^{(-x)}+e^{x}\right)+\frac{1}{20}\sqrt{2\sqrt{5}+10}\log\left(-\sqrt{\frac{1}{2}\sqrt{5}+\frac{5}{2}}+e^{(-x)}+e^{x}\right)-\frac{1}{20}\sqrt{-2\sqrt{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*tanh(5*x),x, algorithm="giac")
```

```
[Out] -1/20*sqrt(2*sqrt(5) + 10)*log(sqrt(1/2*sqrt(5) + 5/2) + e^(-x) + e^x) + 1/
20*sqrt(2*sqrt(5) + 10)*log(-sqrt(1/2*sqrt(5) + 5/2) + e^(-x) + e^x) - 1/20
*sqrt(-2*sqrt(5) + 10)*log(sqrt(-1/2*sqrt(5) + 5/2) + e^(-x) + e^x) + 1/20*
sqrt(-2*sqrt(5) + 10)*log(-sqrt(-1/2*sqrt(5) + 5/2) + e^(-x) + e^x) + 1/2*e
^(-x) + 1/2*e^x
```

maple [A] time = 0.17, size = 70, normalized size = 0.85

$$\cosh(x)-\frac{\sqrt{5}\,\left(\sqrt{5}-1\right)\operatorname{arctanh}\left(\frac{4\cosh(x)}{\sqrt{10-2\sqrt{5}}}\right)}{5\sqrt{10-2\sqrt{5}}}-\frac{\left(\sqrt{5}+1\right)\sqrt{5}\,\operatorname{arctanh}\left(\frac{4\cosh(x)}{\sqrt{10+2\sqrt{5}}}\right)}{5\sqrt{10+2\sqrt{5}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)*tanh(5*x),x)
```

```
[Out] cosh(x)-1/5*5^(1/2)*(5^(1/2)-1)/(10-2*5^(1/2))^(1/2)*arctanh(4*cosh(x)/(10-
2*5^(1/2))^(1/2))-1/5*(5^(1/2)+1)*5^(1/2)/(10+2*5^(1/2))^(1/2)*arctanh(4*co
sh(x)/(10+2*5^(1/2))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{2}\left(e^{(2x)}+1\right)e^{(-x)}+\frac{1}{2}\int\frac{2\left(e^{(7x)}-e^{(5x)}+e^{(3x)}-e^{x}\right)}{e^{(8x)}-e^{(6x)}+e^{(4x)}-e^{(2x)}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*tanh(5*x),x, algorithm="maxima")
```

```
[Out] 1/2*(e^(2*x) + 1)*e^(-x) + 1/2*integrate(2*(e^(7*x) - e^(5*x) + e^(3*x) - e
^x)/(e^(8*x) - e^(6*x) + e^(4*x) - e^(2*x) + 1), x)
```

mupad [B] time = 0.09, size = 141, normalized size = 1.72

$$\frac{e^{-x}}{2}+\frac{e^{x}}{2}+\ln\left(4\,e^{2\,x}-40\,e^{x}\,\sqrt{\frac{1}{40}-\frac{\sqrt{5}}{200}}+4\right)\,\sqrt{\frac{1}{40}-\frac{\sqrt{5}}{200}}-\ln\left(4\,e^{2\,x}+40\,e^{x}\,\sqrt{\frac{1}{40}-\frac{\sqrt{5}}{200}}+4\right)\,\sqrt{\frac{1}{40}-\frac{\sqrt{5}}{200}}+\ln$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(5*x)*cosh(x),x)
```

```
[Out] exp(-x)/2 + exp(x)/2 + log(4*exp(2*x) - 40*exp(x)*(1/40 - 5^(1/2)/200)^(1/2
) + 4)*(1/40 - 5^(1/2)/200)^(1/2) - log(4*exp(2*x) + 40*exp(x)*(1/40 - 5^(1
/2)/200)^(1/2) + 4)*(1/40 - 5^(1/2)/200)^(1/2) + log(4*exp(2*x) - 40*exp(x)
*(5^(1/2)/200 + 1/40)^(1/2) + 4)*(5^(1/2)/200 + 1/40)^(1/2) - log(4*exp(2*x
) + 40*exp(x)*(5^(1/2)/200 + 1/40)^(1/2) + 4)*(5^(1/2)/200 + 1/40)^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \cosh{\left(x \right)} \tanh{\left(5x \right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*tanh(5*x),x)
```

```
[Out] Integral(cosh(x)*tanh(5*x), x)
```

3.233 $\int \cosh(x)\tanh(6x)\,dx$

Optimal. Leaf size=87

$$\cosh(x)-\frac{\tanh^{-1}\left(\sqrt{2}\cosh(x)\right)}{3\sqrt{2}}-\frac{1}{6}\sqrt{2-\sqrt{3}}\ \tanh^{-1}\left(\frac{2\cosh(x)}{\sqrt{2-\sqrt{3}}}\right)-\frac{1}{6}\sqrt{2+\sqrt{3}}\ \tanh^{-1}\left(\frac{2\cosh(x)}{\sqrt{2+\sqrt{3}}}\right)$$

```
[Out] cosh(x)-1/6*arctanh(cosh(x)*2^(1/2))*2^(1/2)-1/6*arctanh(2*cosh(x)/(1/2*6^(
1/2)-1/2*2^(1/2)))*(1/2*6^(1/2)-1/2*2^(1/2))-1/6*arctanh(2*cosh(x)/(1/2*6^(
1/2)+1/2*2^(1/2)))*(1/2*6^(1/2)+1/2*2^(1/2))
```

Rubi [A] time = 0.25, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 5, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.714, Rules used = {12, 6742, 2073, 207, 1166}

$$\cosh(x)-\frac{\tanh^{-1}\left(\sqrt{2}\cosh(x)\right)}{3\sqrt{2}}-\frac{1}{6}\sqrt{2-\sqrt{3}}\ \tanh^{-1}\left(\frac{2\cosh(x)}{\sqrt{2-\sqrt{3}}}\right)-\frac{1}{6}\sqrt{2+\sqrt{3}}\ \tanh^{-1}\left(\frac{2\cosh(x)}{\sqrt{2+\sqrt{3}}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]*Tanh[6*x],x]
```

```
[Out] -ArcTanh[Sqrt[2]*Cosh[x]]/(3*Sqrt[2]) - (Sqrt[2 - Sqrt[3]]*ArcTanh[(2*Cosh[
x])/Sqrt[2 - Sqrt[3]]])/6 - (Sqrt[2 + Sqrt[3]]*ArcTanh[(2*Cosh[x])/Sqrt[2 +
 Sqrt[3]]])/6 + Cosh[x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
```

```
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 2073

```
Int[(P_)^(p_)*(Q_)^(q_.), x_Symbol] :> With[{PP = Factor[P /. x -> Sqrt[x]]
}, Int[ExpandIntegrand[(PP /. x -> x^2)^p*Q^q, x], x] /;  !SumQ[NonfreeFact
ors[PP, x]]] /; FreeQ[q, x] && PolyQ[P, x^2] && PolyQ[Q, x] && ILtQ[p, 0]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}
\int \cosh(x)\tanh(6x)\,dx &= \text{Subst}\left(\int \frac{2x^2\left(-3+16x^2-16x^4\right)}{1-18x^2+48x^4-32x^6}\,dx, x, \cosh(x)\right)\\
&= 2\,\text{Subst}\left(\int \frac{x^2\left(-3+16x^2-16x^4\right)}{1-18x^2+48x^4-32x^6}\,dx, x, \cosh(x)\right)\\
&= 2\,\text{Subst}\left(\int\left(\frac{1}{2}-\frac{1-12x^2+16x^4}{2\left(1-18x^2+48x^4-32x^6\right)}\right)dx, x, \cosh(x)\right)\\
&= \cosh(x)-\text{Subst}\left(\int \frac{1-12x^2+16x^4}{1-18x^2+48x^4-32x^6}\,dx, x, \cosh(x)\right)\\
&= \cosh(x)-\text{Subst}\left(\int\left(-\frac{1}{3\left(-1+2x^2\right)}-\frac{2\left(-1+8x^2\right)}{3\left(1-16x^2+16x^4\right)}\right)dx, x, \cosh(x)\right)\\
&= \cosh(x)+\frac{1}{3}\text{Subst}\left(\int \frac{1}{-1+2x^2}\,dx, x, \cosh(x)\right)+\frac{2}{3}\text{Subst}\left(\int \frac{-1+8x^2}{1-16x^2+16x^4}\,dx, x\right.\\
&= -\frac{\tanh^{-1}\left(\sqrt{2}\cosh(x)\right)}{3\sqrt{2}}+\cosh(x)+\frac{1}{3}\left(4\left(2-\sqrt{3}\right)\right)\text{Subst}\left(\int \frac{1}{-8+4\sqrt{3}+16x^2}\,dx,\right.\\
&= -\frac{\tanh^{-1}\left(\sqrt{2}\cosh(x)\right)}{3\sqrt{2}}-\frac{1}{6}\sqrt{2-\sqrt{3}}\,\tanh^{-1}\left(\frac{2\cosh(x)}{\sqrt{2-\sqrt{3}}}\right)-\frac{1}{6}\sqrt{2+\sqrt{3}}\,\tanh^{-1}\left(\frac{2\,\mathrm{c}}{\sqrt{2}}\right.
\end{aligned}$$

Mathematica [C] time = 0.31, size = 395, normalized size = 4.54

$$\sqrt{2}\,\text{RootSum}\left[\#1^8-\#1^4+1\&,\frac{2\#1^6x+4\#1^6\log\left(-\#1\sinh\left(\frac{x}{2}\right)+\#1\cosh\left(\frac{x}{2}\right)-\sinh\left(\frac{x}{2}\right)-\cosh\left(\frac{x}{2}\right)\right)+\#1^4x+2\#1^4\log\left(-\#1\sinh\left(\frac{x}{2}\right)+\#1\cosh\left(\frac{x}{2}\right)-\right.}{}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]*Tanh[6*x],x]
```

```
[Out] ((-4*I)*ArcTan[(Cosh[x/2] + Sinh[x/2])/((1 + Sqrt[2])*Cosh[x/2] - (-1 + Sqr
t[2])*Sinh[x/2])] + (4*I)*ArcTan[(Cosh[x/2] + Sinh[x/2])/((-1 + Sqrt[2])*Co
sh[x/2] - (1 + Sqrt[2])*Sinh[x/2])] - 8*ArcTanh[Sqrt[2] - I*Tanh[x/2]] + 24
*Sqrt[2]*Cosh[x] + 2*Log[Sqrt[2] - 2*Cosh[x]] - 2*Log[Sqrt[2] + 2*Cosh[x]]
+ Sqrt[2]*RootSum[1 - #1^4 + #1^8 & , (-2*x - 4*Log[-Cosh[x/2] - Sinh[x/2]
+ Cosh[x/2]*#1 - Sinh[x/2]*#1] - x*#1^2 - 2*Log[-Cosh[x/2] - Sinh[x/2] + Co
sh[x/2]*#1 - Sinh[x/2]*#1]*#1^2 + x*#1^4 + 2*Log[-Cosh[x/2] - Sinh[x/2] + C
osh[x/2]*#1 - Sinh[x/2]*#1]*#1^4 + 2*x*#1^6 + 4*Log[-Cosh[x/2] - Sinh[x/2]
+ Cosh[x/2]*#1 - Sinh[x/2]*#1]*#1^6)/(-#1^3 + 2*#1^7) & ])/(24*Sqrt[2])
```

fricas [B] time = 0.48, size = 258, normalized size = 2.97

$$-\frac{\sqrt{\sqrt{3}+2}\,(\cosh(x)+\sinh(x))\log\left(\cosh(x)^2+2\,\cosh(x)\sinh(x)+\sinh(x)^2+\sqrt{\sqrt{3}+2}\,(\cosh(x)+\sinh(x))+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*tanh(6*x),x, algorithm="fricas")
```

```
[Out] -1/12*(sqrt(sqrt(3) + 2)*(cosh(x) + sinh(x))*log(cosh(x)^2 + 2*cosh(x)*sinh
(x) + sinh(x)^2 + sqrt(sqrt(3) + 2)*(cosh(x) + sinh(x)) + 1) - sqrt(sqrt(3)
 + 2)*(cosh(x) + sinh(x))*log(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - s
qrt(sqrt(3) + 2)*(cosh(x) + sinh(x)) + 1) + sqrt(-sqrt(3) + 2)*(cosh(x) + s
inh(x))*log(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + sqrt(-sqrt(3) + 2)*
(cosh(x) + sinh(x)) + 1) - sqrt(-sqrt(3) + 2)*(cosh(x) + sinh(x))*log(cosh(
x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - sqrt(-sqrt(3) + 2)*(cosh(x) + sinh(x
)) + 1) - 6*cosh(x)^2 - (sqrt(2)*cosh(x) + sqrt(2)*sinh(x))*log((cosh(x)^2
+ sinh(x)^2 - 2*sqrt(2)*cosh(x) + 2)/(cosh(x)^2 + sinh(x)^2)) - 12*cosh(x)*
sinh(x) - 6*sinh(x)^2 - 6)/(cosh(x) + sinh(x))
```

giac [B] time = 0.16, size = 157, normalized size = 1.80

$$-\frac{1}{24}\left(\sqrt{6}+\sqrt{2}\right)\log\left(\frac{1}{2}\,\sqrt{6}+\frac{1}{2}\,\sqrt{2}+e^{(-x)}+e^{x}\right)-\frac{1}{24}\left(\sqrt{6}-\sqrt{2}\right)\log\left(\frac{1}{2}\,\sqrt{6}-\frac{1}{2}\,\sqrt{2}+e^{(-x)}+e^{x}\right)+\frac{1}{24}\left(\sqrt{6}-\sqrt{\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*tanh(6*x),x, algorithm="giac")
```

```
[Out] -1/24*(sqrt(6) + sqrt(2))*log(1/2*sqrt(6) + 1/2*sqrt(2) + e^(-x) + e^x) - 1
/24*(sqrt(6) - sqrt(2))*log(1/2*sqrt(6) - 1/2*sqrt(2) + e^(-x) + e^x) + 1/2
4*(sqrt(6) - sqrt(2))*log(-1/2*sqrt(6) + 1/2*sqrt(2) + e^(-x) + e^x) + 1/24
*(sqrt(6) + sqrt(2))*log(-1/2*sqrt(6) - 1/2*sqrt(2) + e^(-x) + e^x) + 1/12*
sqrt(2)*log(-(sqrt(2) - e^(-x) - e^x)/(sqrt(2) + e^(-x) + e^x)) + 1/2*e^(-x
) + 1/2*e^x
```

maple [A] time = 0.18, size = 102, normalized size = 1.17

$$\cosh(x)-\frac{2\left(3+2\sqrt{3}\right)\sqrt{3}\ \operatorname{arctanh}\left(\frac{8\cosh(x)}{2\sqrt{6}+2\sqrt{2}}\right)}{9\left(2\sqrt{6}+2\sqrt{2}\right)}-\frac{2\left(-3+2\sqrt{3}\right)\sqrt{3}\ \operatorname{arctanh}\left(\frac{8\cosh(x)}{2\sqrt{6}-2\sqrt{2}}\right)}{9\left(2\sqrt{6}-2\sqrt{2}\right)}-\frac{\operatorname{arctanh}\left(\cosh(x)\sqrt{2}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)*tanh(6*x),x)
```

```
[Out] cosh(x)-2/9*(3+2*3^(1/2))*3^(1/2)/(2*6^(1/2)+2*2^(1/2))*arctanh(8*cosh(x)/(
2*6^(1/2)+2*2^(1/2)))-2/9*(-3+2*3^(1/2))*3^(1/2)/(2*6^(1/2)-2*2^(1/2))*arct
anh(8*cosh(x)/(2*6^(1/2)-2*2^(1/2)))-1/6*arctanh(cosh(x)*2^(1/2))*2^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{2}\left(e^{(2x)}+1\right)e^{(-x)}-\frac{1}{12}\sqrt{2}\log\left(\sqrt{2}e^{x}+e^{(2x)}+1\right)+\frac{1}{12}\sqrt{2}\log\left(-\sqrt{2}e^{x}+e^{(2x)}+1\right)+\frac{1}{2}\int\frac{2\left(2e^{(7x)}+e^{(5x)}-e^{(3x)}\right.}{3\left(e^{(8x)}-e^{(4x)}+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*tanh(6*x),x, algorithm="maxima")
```

```
[Out] 1/2*(e^(2*x) + 1)*e^(-x) - 1/12*sqrt(2)*log(sqrt(2)*e^x + e^(2*x) + 1) + 1/
12*sqrt(2)*log(-sqrt(2)*e^x + e^(2*x) + 1) + 1/2*integrate(2/3*(2*e^(7*x) +
 e^(5*x) - e^(3*x) - 2*e^x)/(e^(8*x) - e^(4*x) + 1), x)
```

mupad [B] time = 0.10, size = 170, normalized size = 1.95

$$\frac{e^{-x}}{2}+\frac{e^{x}}{2}-\frac{\sqrt{2}\ \ln\left(e^{2x}+\sqrt{2}\ e^{x}+1\right)}{12}+\frac{\sqrt{2}\ \ln\left(e^{2x}-\sqrt{2}\ e^{x}+1\right)}{12}+\ln\left(e^{2x}-12\,e^{x}\sqrt{\frac{1}{72}-\frac{\sqrt{3}}{144}}+1\right)\sqrt{\frac{1}{72}-\frac{\sqrt{3}}{144}}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(6*x)*cosh(x),x)
```

```
[Out] exp(-x)/2 + exp(x)/2 - (2^(1/2)*log(exp(2*x) + 2^(1/2)*exp(x) + 1))/12 + (2
^(1/2)*log(exp(2*x) - 2^(1/2)*exp(x) + 1))/12 + log(exp(2*x) - 12*exp(x)*(1
/72 - 3^(1/2)/144)^(1/2) + 1)*(1/72 - 3^(1/2)/144)^(1/2) - log(exp(2*x) + 1
2*exp(x)*(1/72 - 3^(1/2)/144)^(1/2) + 1)*(1/72 - 3^(1/2)/144)^(1/2) + log(e
xp(2*x) - 12*exp(x)*(3^(1/2)/144 + 1/72)^(1/2) + 1)*(3^(1/2)/144 + 1/72)^(1
/2) - log(exp(2*x) + 12*exp(x)*(3^(1/2)/144 + 1/72)^(1/2) + 1)*(3^(1/2)/144
 + 1/72)^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \cosh(x)\tanh(6x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*tanh(6*x),x)
```

```
[Out] Integral(cosh(x)*tanh(6*x), x)
```

3.234 $\int \cosh(x)\coth(2x)\,dx$

Optimal. Leaf size=10

$$\cosh(x) - \frac{1}{2}\tanh^{-1}(\cosh(x))$$

```
[Out] -1/2*arctanh(cosh(x))+cosh(x)
```

Rubi [A] time = 0.03, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {12, 388, 206}

$$\cosh(x) - \frac{1}{2}\tanh^{-1}(\cosh(x))$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]*Coth[2*x],x]
```

```
[Out] -ArcTanh[Cosh[x]]/2 + Cosh[x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rubi steps

$$\int \cosh(x)\coth(2x)\,dx = -\text{Subst}\left(\int \frac{-1+2x^2}{2\left(1-x^2\right)}\,dx, x, \cosh(x)\right)$$
$$= -\left(\frac{1}{2}\,\text{Subst}\left(\int \frac{-1+2x^2}{1-x^2}\,dx, x, \cosh(x)\right)\right)$$
$$= \cosh(x) - \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \cosh(x)\right)$$
$$= -\frac{1}{2}\tanh^{-1}(\cosh(x)) + \cosh(x)$$

Mathematica [A] time = 0.01, size = 14, normalized size = 1.40

$$\cosh(x) + \frac{1}{2}\log\left(\tanh\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]*Coth[2*x],x]
```

```
[Out] Cosh[x] + Log[Tanh[x/2]]/2
```

fricas [B] time = 0.41, size = 52, normalized size = 5.20

$$\frac{\cosh(x)^2 - (\cosh(x)+\sinh(x))\log(\cosh(x)+\sinh(x)+1) + (\cosh(x)+\sinh(x))\log(\cosh(x)+\sinh(x)-1) +}{2(\cosh(x)+\sinh(x))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*coth(2*x),x, algorithm="fricas")
```

```
[Out] 1/2*(cosh(x)^2 - (cosh(x) + sinh(x))*log(cosh(x) + sinh(x) + 1) + (cosh(x)
+ sinh(x))*log(cosh(x) + sinh(x) - 1) + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)/
(cosh(x) + sinh(x))
```

giac [B] time = 0.11, size = 26, normalized size = 2.60

$$\frac{1}{2}\,e^{(-x)} + \frac{1}{2}\,e^{x} - \frac{1}{2}\,\log\left(e^{x}+1\right) + \frac{1}{2}\,\log\left(|e^{x}-1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*coth(2*x),x, algorithm="giac")
```

```
[Out] 1/2*e^(-x) + 1/2*e^x - 1/2*log(e^x + 1) + 1/2*log(abs(e^x - 1))
```

maple [A] time = 0.15, size = 9, normalized size = 0.90

$$\cosh(x) - \operatorname{arctanh}\left(e^{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)*coth(2*x),x)
[Out] cosh(x)-arctanh(exp(x))
```

maxima [B] time = 0.36, size = 29, normalized size = 2.90

$$\frac{1}{2}\,e^{(-x)} + \frac{1}{2}\,e^{x} - \frac{1}{2}\,\log\left(e^{(-x)} + 1\right) + \frac{1}{2}\,\log\left(e^{(-x)} - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*coth(2*x),x, algorithm="maxima")
[Out] 1/2*e^(-x) + 1/2*e^x - 1/2*log(e^(-x) + 1) + 1/2*log(e^(-x) - 1)
```

mupad [B] time = 1.41, size = 29, normalized size = 2.90

$$\frac{\ln\left(1 - e^{x}\right)}{2} - \frac{\ln\left(-e^{x} - 1\right)}{2} + \frac{e^{-x}}{2} + \frac{e^{x}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(2*x)*cosh(x),x)
[Out] log(1 - exp(x))/2 - log(- exp(x) - 1)/2 + exp(-x)/2 + exp(x)/2
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \cosh\left(x\right)\coth\left(2x\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*coth(2*x),x)
[Out] Integral(cosh(x)*coth(2*x), x)
```

3.235 $\int \cosh(x)\coth(3x)\,dx$

Optimal. Leaf size=45

$$\cosh(x)+\frac{1}{6}\log(1-2\cosh(x))+\frac{1}{6}\log(1-\cosh(x))-\frac{1}{6}\log(\cosh(x)+1)-\frac{1}{6}\log(2\cosh(x)+1)$$

```
[Out] cosh(x)+1/6*ln(1-2*cosh(x))+1/6*ln(1-cosh(x))-1/6*ln(1+cosh(x))-1/6*ln(1+2*
cosh(x))
```

Rubi [A] time = 0.06, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 4, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {1279, 1161, 616, 31}

$$\cosh(x)+\frac{1}{6}\log(1-2\cosh(x))+\frac{1}{6}\log(1-\cosh(x))-\frac{1}{6}\log(\cosh(x)+1)-\frac{1}{6}\log(2\cosh(x)+1)$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]*Coth[3*x],x]
```

```
[Out] Cosh[x] + Log[1 - 2*Cosh[x]]/6 + Log[1 - Cosh[x]]/6 - Log[1 + Cosh[x]]/6 -
Log[1 + 2*Cosh[x]]/6
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 616

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = Rt[b^2
 - 4*a*c, 2]}, Dist[c/q, Int[1/Simp[b/2 - q/2 + c*x, x], x], x] - Dist[c/q,
 Int[1/Simp[b/2 + q/2 + c*x, x], x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[b^2 - 4*a*c] && PerfectSquareQ[b^2 - 4*a*c]
```

Rule 1161

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[(2*d)/e - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
GtQ[(2*d)/e - b/c, 0] || ( !LtQ[(2*d)/e - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
0]))
```

Rule 1279

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[(e*f*(f*x)^(m - 1)*(a + b*x^2 + c*x^4)^(p +
1))/(c*(m + 4*p + 3)), x] - Dist[f^2/(c*(m + 4*p + 3)), Int[(f*x)^(m - 2)*(
a + b*x^2 + c*x^4)^p*Simp[a*e*(m - 1) + (b*e*(m + 2*p + 1) - c*d*(m + 4*p +
 3))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c,
 0] && GtQ[m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] ||
 IntegerQ[m])
```

Rubi steps

$$\begin{aligned}\int \cosh(x)\coth(3x)\,dx &= -\text{Subst}\left(\int \frac{x^2\left(3-4x^2\right)}{1-5x^2+4x^4}\,dx, x, \cosh(x)\right)\\ &= \cosh(x)+\frac{1}{4}\text{Subst}\left(\int \frac{-4+8x^2}{1-5x^2+4x^4}\,dx, x, \cosh(x)\right)\\ &= \cosh(x)+\frac{1}{4}\text{Subst}\left(\int \frac{1}{-\frac{1}{2}-\frac{x}{2}+x^2}\,dx, x, \cosh(x)\right)+\frac{1}{4}\text{Subst}\left(\int \frac{1}{-\frac{1}{2}+\frac{x}{2}+x^2}\,dx, x,\right.\\ &= \cosh(x)+\frac{1}{6}\text{Subst}\left(\int \frac{1}{-1+x}\,dx, x, \cosh(x)\right)+\frac{1}{6}\text{Subst}\left(\int \frac{1}{-\frac{1}{2}+x}\,dx, x, \cosh(x)\right)-\\ &= \cosh(x)+\frac{1}{6}\log(1-2\cosh(x))+\frac{1}{6}\log(1-\cosh(x))-\frac{1}{6}\log(1+\cosh(x))-\frac{1}{6}\log(1\end{aligned}$$

Mathematica [A] time = 0.02, size = 47, normalized size = 1.04

$$\cosh(x)+\frac{1}{3}\log\left(\sinh\left(\frac{x}{2}\right)\right)-\frac{1}{3}\log\left(\cosh\left(\frac{x}{2}\right)\right)+\frac{1}{6}\log(1-2\cosh(x))-\frac{1}{6}\log(2\cosh(x)+1)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]*Coth[3*x],x]
```

```
[Out] Cosh[x] - Log[Cosh[x/2]]/3 + Log[1 - 2*Cosh[x]]/6 - Log[1 + 2*Cosh[x]]/6 +
Log[Sinh[x/2]]/3
```

fricas [B] time = 0.44, size = 104, normalized size = 2.31

$$\frac{3\cosh(x)^2-(\cosh(x)+\sinh(x))\log\left(\frac{2\cosh(x)+1}{\cosh(x)-\sinh(x)}\right)+(\cosh(x)+\sinh(x))\log\left(\frac{2\cosh(x)-1}{\cosh(x)-\sinh(x)}\right)-2\,(\cosh(x)+\mathrm{s}}{6\,(\mathrm{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*coth(3*x),x, algorithm="fricas")
```

```
[Out] 1/6*(3*cosh(x)^2 - (cosh(x) + sinh(x))*log((2*cosh(x) + 1)/(cosh(x) - sinh(
x))) + (cosh(x) + sinh(x))*log((2*cosh(x) - 1)/(cosh(x) - sinh(x))) - 2*(co
sh(x) + sinh(x))*log(cosh(x) + sinh(x) + 1) + 2*(cosh(x) + sinh(x))*log(cos
h(x) + sinh(x) - 1) + 6*cosh(x)*sinh(x) + 3*sinh(x)^2 + 3)/(cosh(x) + sinh(
x))
```

giac [**A**] time = 0.12, size = 55, normalized size = 1.22

$$\frac{1}{2}\,e^{(-x)}+\frac{1}{2}\,e^{x}-\frac{1}{6}\,\log\left(e^{(-x)}+e^{x}+2\right)-\frac{1}{6}\,\log\left(e^{(-x)}+e^{x}+1\right)+\frac{1}{6}\,\log\left(e^{(-x)}+e^{x}-1\right)+\frac{1}{6}\,\log\left(e^{(-x)}+e^{x}-2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*coth(3*x),x, algorithm="giac")
```

```
[Out] 1/2*e^(-x) + 1/2*e^x - 1/6*log(e^(-x) + e^x + 2) - 1/6*log(e^(-x) + e^x + 1
) + 1/6*log(e^(-x) + e^x - 1) + 1/6*log(e^(-x) + e^x - 2)
```

maple [**A**] time = 0.24, size = 50, normalized size = 1.11

$$\frac{e^{x}}{2}+\frac{e^{-x}}{2}+\frac{\ln\left(e^{x}-1\right)}{3}-\frac{\ln\left(e^{x}+1\right)}{3}+\frac{\ln\left(1-e^{x}+e^{2x}\right)}{6}-\frac{\ln\left(1+e^{x}+e^{2x}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)*coth(3*x),x)
```

```
[Out] 1/2*exp(x)+1/2*exp(-x)+1/3*ln(exp(x)-1)-1/3*ln(exp(x)+1)+1/6*ln(1-exp(x)+ex
p(2*x))-1/6*ln(1+exp(x)+exp(2*x))
```

maxima [**A**] time = 0.44, size = 57, normalized size = 1.27

$$\frac{1}{2}\,e^{(-x)}+\frac{1}{2}\,e^{x}-\frac{1}{6}\,\log\left(e^{(-x)}+e^{(-2\,x)}+1\right)-\frac{1}{3}\,\log\left(e^{(-x)}+1\right)+\frac{1}{3}\,\log\left(e^{(-x)}-1\right)+\frac{1}{6}\,\log\left(-e^{(-x)}+e^{(-2\,x)}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*coth(3*x),x, algorithm="maxima")
```

```
[Out] 1/2*e^(-x) + 1/2*e^x - 1/6*log(e^(-x) + e^(-2*x) + 1) - 1/3*log(e^(-x) + 1)
 + 1/3*log(e^(-x) - 1) + 1/6*log(-e^(-x) + e^(-2*x) + 1)
```

mupad [**B**] time = 0.06, size = 57, normalized size = 1.27

$$\frac{\ln\left(6-6\,e^{x}\right)}{3}-\frac{\ln\left(-6\,e^{x}-6\right)}{3}+\frac{e^{-x}}{2}+\frac{\ln\left(e^{x}-e^{2\,x}-1\right)}{6}-\frac{\ln\left(-e^{2\,x}-e^{x}-1\right)}{6}+\frac{e^{x}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(3*x)*cosh(x),x)
```

```
[Out] log(6 - 6*exp(x))/3 - log(- 6*exp(x) - 6)/3 + exp(-x)/2 + log(exp(x) - exp(
2*x) - 1)/6 - log(- exp(2*x) - exp(x) - 1)/6 + exp(x)/2
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \cosh(x)\coth(3x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*coth(3*x),x)
```

```
[Out] Integral(cosh(x)*coth(3*x), x)
```

3.236 $\int \cosh(x)\coth(4x)\,dx$

Optimal. Leaf size=28

$$\cosh(x)-\frac{1}{4}\tanh^{-1}(\cosh(x))-\frac{\tanh^{-1}\left(\sqrt{2}\cosh(x)\right)}{2\sqrt{2}}$$

```
[Out] -1/4*arctanh(cosh(x))+cosh(x)-1/4*arctanh(cosh(x)*2^(1/2))*2^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {1676, 1166, 207}

$$\cosh(x)-\frac{1}{4}\tanh^{-1}(\cosh(x))-\frac{\tanh^{-1}\left(\sqrt{2}\cosh(x)\right)}{2\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]*Coth[4*x],x]
```

```
[Out] -ArcTanh[Cosh[x]]/4 - ArcTanh[Sqrt[2]*Cosh[x]]/(2*Sqrt[2]) + Cosh[x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1676

```
Int[(Pq_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Int[ExpandInte
grand[Pq/(a + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^
2] && Expon[Pq, x^2] > 1
```

Rubi steps

$$\int \cosh(x)\coth(4x)\,dx = -\text{Subst}\left(\int \frac{-1+8x^2-8x^4}{4-12x^2+8x^4}\,dx,x,\cosh(x)\right)$$
$$= -\text{Subst}\left(\int \left(-1+\frac{3-4x^2}{4-12x^2+8x^4}\right)dx,x,\cosh(x)\right)$$
$$= \cosh(x) - \text{Subst}\left(\int \frac{3-4x^2}{4-12x^2+8x^4}\,dx,x,\cosh(x)\right)$$
$$= \cosh(x) + 2\,\text{Subst}\left(\int \frac{1}{-8+8x^2}\,dx,x,\cosh(x)\right) + 2\,\text{Subst}\left(\int \frac{1}{-4+8x^2}\,dx,x,\cosh(x\right.$$
$$= -\frac{1}{4}\tanh^{-1}(\cosh(x)) - \frac{\tanh^{-1}\left(\sqrt{2}\cosh(x)\right)}{2\sqrt{2}} + \cosh(x)$$

Mathematica [C] time = 0.24, size = 192, normalized size = 6.86

$$\frac{8\sqrt{2}\cosh(x) - 4\tanh^{-1}\left(\sqrt{2}-i\tanh\left(\frac{x}{2}\right)\right) + 2\sqrt{2}\log\left(\sinh\left(\frac{x}{2}\right)\right) - 2\sqrt{2}\log\left(\cosh\left(\frac{x}{2}\right)\right) + \log\left(\sqrt{2}-2\cosh(x\right.}{8\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]*Coth[4*x],x]
```

```
[Out] ((-2*I)*ArcTan[(Cosh[x/2] + Sinh[x/2])/((1 + Sqrt[2])*Cosh[x/2] - (-1 + Sqr
t[2])*Sinh[x/2])] + (2*I)*ArcTan[(Cosh[x/2] + Sinh[x/2])/((-1 + Sqrt[2])*Co
sh[x/2] - (1 + Sqrt[2])*Sinh[x/2])] - 4*ArcTanh[Sqrt[2] - I*Tanh[x/2]] + 8*
Sqrt[2]*Cosh[x] - 2*Sqrt[2]*Log[Cosh[x/2]] + Log[Sqrt[2] - 2*Cosh[x]] - Log
[Sqrt[2] + 2*Cosh[x]] + 2*Sqrt[2]*Log[Sinh[x/2]])/(8*Sqrt[2])
```

fricas [B] time = 0.49, size = 101, normalized size = 3.61

$$\frac{4\cosh(x)^2 + \left(\sqrt{2}\cosh(x)+\sqrt{2}\sinh(x)\right)\log\left(\frac{\cosh(x)^2+\sinh(x)^2-2\sqrt{2}\cosh(x)+2}{\cosh(x)^2+\sinh(x)^2}\right) - 2\left(\cosh(x)+\sinh(x)\right)\log\left(\cosh(\right.}{8\left(\cosh(x)+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*coth(4*x),x, algorithm="fricas")
```

```
[Out] 1/8*(4*cosh(x)^2 + (sqrt(2)*cosh(x) + sqrt(2)*sinh(x))*log((cosh(x)^2 + sin
h(x)^2 - 2*sqrt(2)*cosh(x) + 2)/(cosh(x)^2 + sinh(x)^2)) - 2*(cosh(x) + sin
h(x))*log(cosh(x) + sinh(x) + 1) + 2*(cosh(x) + sinh(x))*log(cosh(x) + sinh
(x) - 1) + 8*cosh(x)*sinh(x) + 4*sinh(x)^2 + 4)/(cosh(x) + sinh(x))
```

giac [B] time = 0.12, size = 67, normalized size = 2.39

$$\frac{1}{8}\sqrt{2}\log\left(-\frac{\sqrt{2}-e^{(-x)}-e^{x}}{\sqrt{2}+e^{(-x)}+e^{x}}\right)+\frac{1}{2}e^{(-x)}+\frac{1}{2}e^{x}-\frac{1}{8}\log\left(e^{(-x)}+e^{x}+2\right)+\frac{1}{8}\log\left(e^{(-x)}+e^{x}-2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*coth(4*x),x, algorithm="giac")
```

```
[Out] 1/8*sqrt(2)*log(-(sqrt(2) - e^(-x) - e^x)/(sqrt(2) + e^(-x) + e^x)) + 1/2*e
^(-x) + 1/2*e^x - 1/8*log(e^(-x) + e^x + 2) + 1/8*log(e^(-x) + e^x - 2)
```

maple [B] time = 0.26, size = 63, normalized size = 2.25

$$\frac{e^{x}}{2}+\frac{e^{-x}}{2}-\frac{\ln\left(e^{x}+1\right)}{4}+\frac{\ln\left(e^{x}-1\right)}{4}+\frac{\ln\left(1+e^{2x}-e^{x}\sqrt{2}\right)\sqrt{2}}{8}-\frac{\ln\left(1+e^{2x}+e^{x}\sqrt{2}\right)\sqrt{2}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)*coth(4*x),x)
```

```
[Out] 1/2*exp(x)+1/2*exp(-x)-1/4*ln(exp(x)+1)+1/4*ln(exp(x)-1)+1/8*ln(1+exp(2*x)-
exp(x)*2^(1/2))*2^(1/2)-1/8*ln(1+exp(2*x)+exp(x)*2^(1/2))*2^(1/2)
```

maxima [B] time = 0.44, size = 70, normalized size = 2.50

$$-\frac{1}{8}\sqrt{2}\log\left(\sqrt{2}e^{(-x)}+e^{(-2x)}+1\right)+\frac{1}{8}\sqrt{2}\log\left(-\sqrt{2}e^{(-x)}+e^{(-2x)}+1\right)+\frac{1}{2}e^{(-x)}+\frac{1}{2}e^{x}-\frac{1}{4}\log\left(e^{(-x)}+1\right)+\frac{1}{4}\log\left(e^{(-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*coth(4*x),x, algorithm="maxima")
```

```
[Out] -1/8*sqrt(2)*log(sqrt(2)*e^(-x) + e^(-2*x) + 1) + 1/8*sqrt(2)*log(-sqrt(2)*
e^(-x) + e^(-2*x) + 1) + 1/2*e^(-x) + 1/2*e^x - 1/4*log(e^(-x) + 1) + 1/4*l
og(e^(-x) - 1)
```

mupad [B] time = 1.45, size = 71, normalized size = 2.54

$$\frac{\ln\left(\frac{1}{2}-\frac{e^{x}}{2}\right)}{4}-\frac{\ln\left(-\frac{e^{x}}{2}-\frac{1}{2}\right)}{4}+\frac{e^{-x}}{2}+\frac{e^{x}}{2}-\frac{\sqrt{2}\,\ln\left(-\frac{e^{2x}}{8}-\frac{\sqrt{2}\,e^{x}}{8}-\frac{1}{8}\right)}{8}+\frac{\sqrt{2}\,\ln\left(\frac{\sqrt{2}\,e^{x}}{8}-\frac{e^{2x}}{8}-\frac{1}{8}\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(4*x)*cosh(x),x)
```

```
[Out] log(1/2 - exp(x)/2)/4 - log(- exp(x)/2 - 1/2)/4 + exp(-x)/2 + exp(x)/2 - (2
^(1/2)*log(- exp(2*x)/8 - (2^(1/2)*exp(x))/8 - 1/8))/8 + (2^(1/2)*log((2^(1
/2)*exp(x))/8 - exp(2*x)/8 - 1/8))/8
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \cosh(x)\coth(4x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*coth(4*x),x)
```

```
[Out] Integral(cosh(x)*coth(4*x), x)
```

3.237 $\int \cosh(x)\coth(5x)\,dx$

Optimal. Leaf size=110

$$\cosh(x)+\frac{1}{20}\left(1-\sqrt{5}\right)\log\left(-4\cosh(x)-\sqrt{5}+1\right)+\frac{1}{20}\left(1+\sqrt{5}\right)\log\left(-4\cosh(x)+\sqrt{5}+1\right)-\frac{1}{20}\left(1-\sqrt{5}\right)\log\left(4\right.$$

```
[Out] -1/5*arctanh(cosh(x))+cosh(x)+1/20*ln(1-4*cosh(x)-5^(1/2))*(-5^(1/2)+1)-1/2
0*ln(1+4*cosh(x)-5^(1/2))*(-5^(1/2)+1)+1/20*ln(1-4*cosh(x)+5^(1/2))*(5^(1/2
)+1)-1/20*ln(1+4*cosh(x)+5^(1/2))*(5^(1/2)+1)
```

Rubi [A] time = 0.18, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 4, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {2075, 207, 632, 31}

$$\cosh(x)+\frac{1}{20}\left(1-\sqrt{5}\right)\log\left(-4\cosh(x)-\sqrt{5}+1\right)+\frac{1}{20}\left(1+\sqrt{5}\right)\log\left(-4\cosh(x)+\sqrt{5}+1\right)-\frac{1}{20}\left(1-\sqrt{5}\right)\log\left(4\right.$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]*Coth[5*x],x]
```

```
[Out] -ArcTanh[Cosh[x]]/5 + Cosh[x] + ((1 - Sqrt[5])*Log[1 - Sqrt[5] - 4*Cosh[x]]
)/20 + ((1 + Sqrt[5])*Log[1 + Sqrt[5] - 4*Cosh[x]])/20 - ((1 - Sqrt[5])*Log
[1 - Sqrt[5] + 4*Cosh[x]])/20 - ((1 + Sqrt[5])*Log[1 + Sqrt[5] + 4*Cosh[x]]
)/20
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 632

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2075

```
Int[(P_)^(p_)*(Qm_), x_Symbol] :> With[{PP = Factor[P]}, Int[ExpandIntegran
d[PP^p*Qm, x], x] /; QuadraticProductQ[PP, x]] /; PolyQ[Qm, x] && PolyQ[P,
x] && ILtQ[p, 0]
```

Rubi steps

$$\int \cosh(x)\coth(5x)\,dx = -\text{Subst}\left(\int \frac{x^2\left(5-20x^2+16x^4\right)}{1-13x^2+28x^4-16x^6}\,dx, x, \cosh(x)\right)$$
$$= -\text{Subst}\left(\int \left(-1-\frac{1}{5\left(-1+x^2\right)}-\frac{2(1+x)}{5\left(-1-2x+4x^2\right)}+\frac{2(-1+x)}{5\left(-1+2x+4x^2\right)}\right)dx, x, \cosh$$
$$= \cosh(x)+\frac{1}{5}\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \cosh(x)\right)+\frac{2}{5}\text{Subst}\left(\int \frac{1+x}{-1-2x+4x^2}\,dx, x, \text{co}$$
$$= -\frac{1}{5}\tanh^{-1}(\cosh(x))+\cosh(x)-\frac{1}{5}\left(1-\sqrt{5}\right)\text{Subst}\left(\int \frac{1}{1-\sqrt{5}+4x}\,dx, x, \cosh(x)\right)$$
$$= -\frac{1}{5}\tanh^{-1}(\cosh(x))+\cosh(x)+\frac{1}{20}\left(1-\sqrt{5}\right)\log\left(1-\sqrt{5}-4\cosh(x)\right)+\frac{1}{20}\left(1+\sqrt{\right.$$

Mathematica [A] time = 0.12, size = 133, normalized size = 1.21

$$\frac{1}{100}\left(100\cosh(x)+20\log\left(\sinh\left(\frac{x}{2}\right)\right)-20\log\left(\cosh\left(\frac{x}{2}\right)\right)+\sqrt{5}\left(\sqrt{5}-5\right)\log\left(-4\cosh(x)-\sqrt{5}+1\right)+\sqrt{5}\left(5\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]*Coth[5*x],x]
```

```
[Out] (100*Cosh[x] - 20*Log[Cosh[x/2]] + Sqrt[5]*(-5 + Sqrt[5])*Log[1 - Sqrt[5] -
 4*Cosh[x]] + Sqrt[5]*(5 + Sqrt[5])*Log[1 + Sqrt[5] - 4*Cosh[x]] - Sqrt[5]*
(-5 + Sqrt[5])*Log[1 - Sqrt[5] + 4*Cosh[x]] - Sqrt[5]*(5 + Sqrt[5])*Log[1 +
 Sqrt[5] + 4*Cosh[x]] + 20*Log[Sinh[x/2]])/100
```

fricas [B] time = 0.44, size = 272, normalized size = 2.47

$$10\cosh(x)^2+\left(\sqrt{5}\cosh(x)+\sqrt{5}\sinh(x)\right)\log\left(-\frac{4\left(\sqrt{5}-1\right)\cosh(x)-4\cosh(x)^2-4\sinh(x)^2+\sqrt{5}-7}{2\cosh(x)^2+2\sinh(x)^2+2\cosh(x)+1}\right)+\left(\sqrt{5}\cosh(x)+\sqrt{5}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*coth(5*x),x, algorithm="fricas")
```

```
[Out] 1/20*(10*cosh(x)^2 + (sqrt(5)*cosh(x) + sqrt(5)*sinh(x))*log(-(4*(sqrt(5) -
 1)*cosh(x) - 4*cosh(x)^2 - 4*sinh(x)^2 + sqrt(5) - 7)/(2*cosh(x)^2 + 2*sin
h(x)^2 + 2*cosh(x) + 1)) + (sqrt(5)*cosh(x) + sqrt(5)*sinh(x))*log(-(4*(sqr
t(5) + 1)*cosh(x) - 4*cosh(x)^2 - 4*sinh(x)^2 - sqrt(5) - 7)/(2*cosh(x)^2 +
 2*sinh(x)^2 - 2*cosh(x) + 1)) - (cosh(x) + sinh(x))*log((2*cosh(x)^2 + 2*s
inh(x)^2 + 2*cosh(x) + 1)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + (c
osh(x) + sinh(x))*log((2*cosh(x)^2 + 2*sinh(x)^2 - 2*cosh(x) + 1)/(cosh(x)^
2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) - 4*(cosh(x) + sinh(x))*log(cosh(x) + s
inh(x) + 1) + 4*(cosh(x) + sinh(x))*log(cosh(x) + sinh(x) - 1) + 20*cosh(x)
*sinh(x) + 10*sinh(x)^2 + 10)/(cosh(x) + sinh(x))
```

giac [A] time = 0.16, size = 157, normalized size = 1.43

$$\frac{1}{20}\sqrt{5}\log\left(-\frac{\sqrt{5}-2e^{(-x)}-2e^{x}+1}{\sqrt{5}+2e^{(-x)}+2e^{x}-1}\right)+\frac{1}{20}\sqrt{5}\log\left(-\frac{\sqrt{5}-2e^{(-x)}-2e^{x}-1}{\sqrt{5}+2e^{(-x)}+2e^{x}+1}\right)+\frac{1}{2}e^{(-x)}+\frac{1}{2}e^{x}-\frac{1}{20}\log\left(\left(e^{(-x)}+e^{x}\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*coth(5*x),x, algorithm="giac")
```

```
[Out] 1/20*sqrt(5)*log(-(sqrt(5) - 2*e^(-x) - 2*e^x + 1)/(sqrt(5) + 2*e^(-x) + 2*
e^x - 1)) + 1/20*sqrt(5)*log(-(sqrt(5) - 2*e^(-x) - 2*e^x - 1)/(sqrt(5) + 2
*e^(-x) + 2*e^x + 1)) + 1/2*e^(-x) + 1/2*e^x - 1/20*log((e^(-x) + e^x)^2 +
e^(-x) + e^x - 1) + 1/20*log((e^(-x) + e^x)^2 - e^(-x) - e^x - 1) - 1/10*lo
g(e^(-x) + e^x + 2) + 1/10*log(e^(-x) + e^x - 2)
```

maple [B] time = 0.28, size = 190, normalized size = 1.73

$$\frac{e^{x}}{2}+\frac{e^{-x}}{2}+\frac{\ln\left(e^{x}-1\right)}{5}-\frac{\ln\left(e^{x}+1\right)}{5}-\frac{\ln\left(e^{2x}+\left(\frac{1}{2}-\frac{\sqrt{5}}{2}\right)e^{x}+1\right)}{20}+\frac{\ln\left(e^{2x}+\left(\frac{1}{2}-\frac{\sqrt{5}}{2}\right)e^{x}+1\right)\sqrt{5}}{20}-\frac{\ln\left(e^{2x}+\left(\frac{1}{2}+\frac{\sqrt{5}}{2}\right.\right.}{20}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)*coth(5*x),x)
```

```
[Out] 1/2*exp(x)+1/2*exp(-x)+1/5*ln(exp(x)-1)-1/5*ln(exp(x)+1)-1/20*ln(exp(2*x)+(
1/2-1/2*5^(1/2))*exp(x)+1)+1/20*ln(exp(2*x)+(1/2-1/2*5^(1/2))*exp(x)+1)*5^(
1/2)-1/20*ln(exp(2*x)+(1/2+1/2*5^(1/2))*exp(x)+1)-1/20*ln(exp(2*x)+(1/2+1/2
*5^(1/2))*exp(x)+1)*5^(1/2)+1/20*ln(exp(2*x)+(-1/2-1/2*5^(1/2))*exp(x)+1)+1
/20*ln(exp(2*x)+(-1/2-1/2*5^(1/2))*exp(x)+1)*5^(1/2)+1/20*ln(exp(2*x)+(1/2*
5^(1/2)-1/2)*exp(x)+1)-1/20*ln(exp(2*x)+(1/2*5^(1/2)-1/2)*exp(x)+1)*5^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{2}\left(e^{(2x)}+1\right)e^{(-x)}-\frac{1}{5}\int\frac{\left(e^{(3x)}+e^{(2x)}+e^{x}+1\right)e^{x}}{e^{(4x)}+e^{(3x)}+e^{(2x)}+e^{x}+1}\,dx+\frac{1}{5}\int\frac{\left(e^{(3x)}-e^{(2x)}+e^{x}-1\right)e^{x}}{e^{(4x)}-e^{(3x)}+e^{(2x)}-e^{x}+1}\,dx+\frac{3}{10}\int\frac{e}{e^{(4x)}+e^{(3x)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*coth(5*x),x, algorithm="maxima")
```

```
[Out] 1/2*(e^(2*x) + 1)*e^(-x) - 1/5*integrate((e^(3*x) + e^(2*x) + e^x + 1)*e^x/
(e^(4*x) + e^(3*x) + e^(2*x) + e^x + 1), x) + 1/5*integrate((e^(3*x) - e^(2
*x) + e^x - 1)*e^x/(e^(4*x) - e^(3*x) + e^(2*x) - e^x + 1), x) + 3/10*integ
rate(e^(3*x)/(e^(4*x) + e^(3*x) + e^(2*x) + e^x + 1), x) + 3/10*integrate(e
^(3*x)/(e^(4*x) - e^(3*x) + e^(2*x) - e^x + 1), x) + 1/10*integrate(e^(2*x)
/(e^(4*x) + e^(3*x) + e^(2*x) + e^x + 1), x) - 1/10*integrate(e^(2*x)/(e^(4
*x) - e^(3*x) + e^(2*x) - e^x + 1), x) - 1/10*integrate(e^x/(e^(4*x) + e^(3
*x) + e^(2*x) + e^x + 1), x) - 1/10*integrate(e^x/(e^(4*x) - e^(3*x) + e^(2
*x) - e^x + 1), x) - 1/5*log(e^x + 1) + 1/5*log(e^x - 1)
```

mupad [B] time = 0.10, size = 143, normalized size = 1.30

$$\frac{\ln\left(10-10\,e^{x}\right)}{5}-\frac{\ln\left(-10\,e^{x}-10\right)}{5}+\frac{e^{-x}}{2}+\frac{e^{x}}{2}-\ln\left(-e^{2\,x}-10\,e^{x}\,\left(\frac{\sqrt{5}}{20}-\frac{1}{20}\right)-1\right)\,\left(\frac{\sqrt{5}}{20}-\frac{1}{20}\right)+\ln\left(10\,e^{x}\,\left(\frac{\sqrt{5}}{20}-\frac{1}{20}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(5*x)*cosh(x),x)
```

```
[Out] log(10 - 10*exp(x))/5 - log(- 10*exp(x) - 10)/5 + exp(-x)/2 + exp(x)/2 - lo
g(- exp(2*x) - 10*exp(x)*(5^(1/2)/20 - 1/20) - 1)*(5^(1/2)/20 - 1/20) + log
(10*exp(x)*(5^(1/2)/20 - 1/20) - exp(2*x) - 1)*(5^(1/2)/20 - 1/20) - log(-
exp(2*x) - 10*exp(x)*(5^(1/2)/20 + 1/20) - 1)*(5^(1/2)/20 + 1/20) + log(10*
exp(x)*(5^(1/2)/20 + 1/20) - exp(2*x) - 1)*(5^(1/2)/20 + 1/20)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \cosh\left(x\right)\coth\left(5x\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*coth(5*x),x)
```

```
[Out] Integral(cosh(x)*coth(5*x), x)
```

3.238 $\int \cosh(x)\coth(6x)\,dx$

Optimal. Leaf size=38

$$\cosh(x)-\frac{1}{6}\tanh^{-1}(\cosh(x))-\frac{1}{6}\tanh^{-1}(2\cosh(x))-\frac{\tanh^{-1}\left(\frac{2\cosh(x)}{\sqrt{3}}\right)}{2\sqrt{3}}$$

```
[Out] -1/6*arctanh(cosh(x))-1/6*arctanh(2*cosh(x))+cosh(x)-1/6*arctanh(2/3*cosh(x
)*3^(1/2))*3^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 3, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {12, 2073, 207}

$$\cosh(x)-\frac{1}{6}\tanh^{-1}(\cosh(x))-\frac{1}{6}\tanh^{-1}(2\cosh(x))-\frac{\tanh^{-1}\left(\frac{2\cosh(x)}{\sqrt{3}}\right)}{2\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]*Coth[6*x],x]
```

```
[Out] -ArcTanh[Cosh[x]]/6 - ArcTanh[2*Cosh[x]]/6 - ArcTanh[(2*Cosh[x])/Sqrt[3]]/(
2*Sqrt[3]) + Cosh[x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 2073

```
Int[(P_)^(p_)*(Q_)^(q_.), x_Symbol] :> With[{PP = Factor[P /. x -> Sqrt[x]]
}, Int[ExpandIntegrand[(PP /. x -> x^2)^p*Q^q, x], x] /;  !SumQ[NonfreeFact
ors[PP, x]]] /; FreeQ[q, x] && PolyQ[P, x^2] && PolyQ[Q, x] && ILtQ[p, 0]
```

Rubi steps

$$\int \cosh(x)\coth(6x)\,dx = -\text{Subst}\left(\int \frac{-1+18x^2-48x^4+32x^6}{2\left(3-19x^2+32x^4-16x^6\right)}\,dx, x, \cosh(x)\right)$$
$$= -\left(\frac{1}{2}\text{Subst}\left(\int \frac{-1+18x^2-48x^4+32x^6}{3-19x^2+32x^4-16x^6}\,dx, x, \cosh(x)\right)\right)$$
$$= -\left(\frac{1}{2}\text{Subst}\left(\int \left(-2-\frac{1}{3\left(-1+x^2\right)}-\frac{2}{-3+4x^2}-\frac{2}{3\left(-1+4x^2\right)}\right)dx, x, \cosh(x)\right)\right)$$
$$= \cosh(x)+\frac{1}{6}\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \cosh(x)\right)+\frac{1}{3}\text{Subst}\left(\int \frac{1}{-1+4x^2}\,dx, x, \cosh(x$$
$$= -\frac{1}{6}\tanh^{-1}(\cosh(x))-\frac{1}{6}\tanh^{-1}(2\cosh(x))-\frac{\tanh^{-1}\left(\frac{2\cosh(x)}{\sqrt{3}}\right)}{2\sqrt{3}}+\cosh(x)$$

Mathematica [C] time = 0.08, size = 95, normalized size = 2.50

$$\frac{1}{12}\left(12\cosh(x)-2\sqrt{3}\tanh^{-1}\left(\frac{2-i\tanh\left(\frac{x}{2}\right)}{\sqrt{3}}\right)-2\sqrt{3}\tanh^{-1}\left(\frac{2+i\tanh\left(\frac{x}{2}\right)}{\sqrt{3}}\right)+2\log\left(\sinh\left(\frac{x}{2}\right)\right)-2\log\left(\cosh\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]*Coth[6*x],x]
```

```
[Out] (-2*Sqrt[3]*ArcTanh[(2 - I*Tanh[x/2])/Sqrt[3]] - 2*Sqrt[3]*ArcTanh[(2 + I*T
anh[x/2])/Sqrt[3]] + 12*Cosh[x] - 2*Log[Cosh[x/2]] + Log[1 - 2*Cosh[x]] - L
og[1 + 2*Cosh[x]] + 2*Log[Sinh[x/2]])/12
```

fricas [B] time = 0.57, size = 157, normalized size = 4.13

$$\frac{6\cosh(x)^2+\left(\sqrt{3}\cosh(x)+\sqrt{3}\sinh(x)\right)\log\left(\frac{2\cosh(x)^2+2\sinh(x)^2-4\sqrt{3}\cosh(x)+5}{2\cosh(x)^2+2\sinh(x)^2-1}\right)-(\cosh(x)+\sinh(x))\log\left(\frac{2\,c}{\cosh}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*coth(6*x),x, algorithm="fricas")
```

```
[Out] 1/12*(6*cosh(x)^2 + (sqrt(3)*cosh(x) + sqrt(3)*sinh(x))*log((2*cosh(x)^2 +
2*sinh(x)^2 - 4*sqrt(3)*cosh(x) + 5)/(2*cosh(x)^2 + 2*sinh(x)^2 - 1)) - (co
sh(x) + sinh(x))*log((2*cosh(x) + 1)/(cosh(x) - sinh(x))) + (cosh(x) + sinh
(x))*log((2*cosh(x) - 1)/(cosh(x) - sinh(x))) - 2*(cosh(x) + sinh(x))*log(c
osh(x) + sinh(x) + 1) + 2*(cosh(x) + sinh(x))*log(cosh(x) + sinh(x) - 1) +
12*cosh(x)*sinh(x) + 6*sinh(x)^2 + 6)/(cosh(x) + sinh(x))
```

giac [B] time = 0.12, size = 89, normalized size = 2.34

$$\frac{1}{12}\sqrt{3}\log\left(-\frac{\sqrt{3}-e^{(-x)}-e^{x}}{\sqrt{3}+e^{(-x)}+e^{x}}\right)+\frac{1}{2}e^{(-x)}+\frac{1}{2}e^{x}-\frac{1}{12}\log\left(e^{(-x)}+e^{x}+2\right)-\frac{1}{12}\log\left(e^{(-x)}+e^{x}+1\right)+\frac{1}{12}\log\left(e^{(-x)}+e^{x}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*coth(6*x),x, algorithm="giac")
```

```
[Out] 1/12*sqrt(3)*log(-(sqrt(3) - e^(-x) - e^x)/(sqrt(3) + e^(-x) + e^x)) + 1/2*
e^(-x) + 1/2*e^x - 1/12*log(e^(-x) + e^x + 2) - 1/12*log(e^(-x) + e^x + 1)
+ 1/12*log(e^(-x) + e^x - 1) + 1/12*log(e^(-x) + e^x - 2)
```

maple [B] time = 0.29, size = 87, normalized size = 2.29

$$\frac{e^{x}}{2}+\frac{e^{-x}}{2}+\frac{\ln\left(e^{x}-1\right)}{6}-\frac{\ln\left(e^{x}+1\right)}{6}-\frac{\ln\left(1+e^{x}+e^{2x}\right)}{12}+\frac{\ln\left(1+e^{2x}-e^{x}\sqrt{3}\right)\sqrt{3}}{12}-\frac{\ln\left(1+e^{2x}+e^{x}\sqrt{3}\right)\sqrt{3}}{12}+\frac{\ln\left(1-\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)*coth(6*x),x)
```

```
[Out] 1/2*exp(x)+1/2*exp(-x)+1/6*ln(exp(x)-1)-1/6*ln(exp(x)+1)-1/12*ln(1+exp(x)+e
xp(2*x))+1/12*ln(1+exp(2*x)-exp(x)*3^(1/2))*3^(1/2)-1/12*ln(1+exp(2*x)+exp(
x)*3^(1/2))*3^(1/2)+1/12*ln(1-exp(x)+exp(2*x))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{2}\left(e^{(2x)}+1\right)e^{(-x)}+\frac{1}{2}\int\frac{e^{(3x)}-e^{x}}{e^{(4x)}-e^{(2x)}+1}\,dx-\frac{1}{12}\log\left(e^{(2x)}+e^{x}+1\right)+\frac{1}{12}\log\left(e^{(2x)}-e^{x}+1\right)-\frac{1}{6}\log\left(e^{x}+1\right)+\frac{1}{6}\text{lo}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*coth(6*x),x, algorithm="maxima")
```

```
[Out] 1/2*(e^(2*x) + 1)*e^(-x) + 1/2*integrate((e^(3*x) - e^x)/(e^(4*x) - e^(2*x)
 + 1), x) - 1/12*log(e^(2*x) + e^x + 1) + 1/12*log(e^(2*x) - e^x + 1) - 1/6
*log(e^x + 1) + 1/6*log(e^x - 1)
```

mupad [B] time = 0.09, size = 101, normalized size = 2.66

$$\frac{\ln\left(\frac{1}{3}-\frac{e^{x}}{3}\right)}{6}-\frac{\ln\left(-\frac{e^{x}}{3}-\frac{1}{3}\right)}{6}+\frac{e^{-x}}{2}-\frac{\ln\left(-\frac{e^{2x}}{36}-\frac{e^{x}}{36}-\frac{1}{36}\right)}{12}+\frac{\ln\left(\frac{e^{x}}{36}-\frac{e^{2x}}{36}-\frac{1}{36}\right)}{12}+\frac{e^{x}}{2}-\frac{\sqrt{3}\,\ln\left(-\frac{e^{2x}}{12}-\frac{\sqrt{3}\,e^{x}}{12}-\frac{1}{12}\right)}{12}+\frac{\sqrt{3}\,\text{l}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(6*x)*cosh(x),x)
```

```
[Out] log(1/3 - exp(x)/3)/6 - log(- exp(x)/3 - 1/3)/6 + exp(-x)/2 - log(- exp(2*x
)/36 - exp(x)/36 - 1/36)/12 + log(exp(x)/36 - exp(2*x)/36 - 1/36)/12 + exp(
x)/2 - (3^(1/2)*log(- exp(2*x)/12 - (3^(1/2)*exp(x))/12 - 1/12))/12 + (3^(1
/2)*log((3^(1/2)*exp(x))/12 - exp(2*x)/12 - 1/12))/12
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \cosh(x)\coth(6x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*coth(6*x),x)
```

```
[Out] Integral(cosh(x)*coth(6*x), x)
```

3.239 $\int \cosh(x)\coth(nx)\,dx$

Optimal. Leaf size=76

$$e^{-x}\,{}_2F_1\left(1,-\frac{1}{2n};1-\frac{1}{2n};e^{2nx}\right)-e^{x}\,{}_2F_1\left(1,\frac{1}{2n};\frac{1}{2}\left(2+\frac{1}{n}\right);e^{2nx}\right)-\frac{e^{-x}}{2}+\frac{e^{x}}{2}$$

```
[Out] -1/2/exp(x)+1/2*exp(x)+hypergeom([1, -1/2/n],[1-1/2/n],exp(2*n*x))/exp(x)-e
xp(x)*hypergeom([1, 1/2/n],[1+1/2/n],exp(2*n*x))
```

Rubi [A] time = 0.07, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {5602, 2194, 2251}

$$e^{-x}\,{}_2F_1\left(1,-\frac{1}{2n};1-\frac{1}{2n};e^{2nx}\right)-e^{x}\,{}_2F_1\left(1,\frac{1}{2n};\frac{1}{2}\left(2+\frac{1}{n}\right);e^{2nx}\right)-\frac{e^{-x}}{2}+\frac{e^{x}}{2}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]*Coth[n*x],x]
```

```
[Out] -1/(2*E^x) + E^x/2 + Hypergeometric2F1[1, -1/(2*n), 1 - 1/(2*n), E^(2*n*x)]
/E^x - E^x*Hypergeometric2F1[1, 1/(2*n), (2 + n^(-1))/2, E^(2*n*x)]
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 2251

```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_)*(G_)^((h_.)*((f_.
) + (g_.)*(x_))), x_Symbol] :> Simp[(a^p*G^(h*(f + g*x))*Hypergeometric2F1[
-p, (g*h*Log[G])/(d*e*Log[F]), (g*h*Log[G])/(d*e*Log[F]) + 1, Simplify[-((b
*F^(e*(c + d*x)))/a)]])/(g*h*Log[G]), x] /; FreeQ[{F, G, a, b, c, d, e, f,
g, h, p}, x] && (ILtQ[p, 0] || GtQ[a, 0])
```

Rule 5602

```
Int[Cosh[(a_.) + (b_.)*(x_)]*Coth[(c_.) + (d_.)*(x_)], x_Symbol] :> Int[1/(
E^(a + b*x)*2) + E^(a + b*x)/2 - 1/(E^(a + b*x)*(1 - E^(2*(c + d*x)))) - E^
(a + b*x)/(1 - E^(2*(c + d*x))), x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 -
d^2, 0]
```

Rubi steps

$$\int \cosh(x)\coth(nx)\,dx = \int \left(\frac{e^{-x}}{2}+\frac{e^{x}}{2}-\frac{e^{-x}}{1-e^{2nx}}-\frac{e^{x}}{1-e^{2nx}}\right)dx$$
$$=\frac{1}{2}\int e^{-x}\,dx+\frac{\int e^{x}\,dx}{2}-\int \frac{e^{-x}}{1-e^{2nx}}\,dx-\int \frac{e^{x}}{1-e^{2nx}}\,dx$$
$$=-\frac{e^{-x}}{2}+\frac{e^{x}}{2}+e^{-x}\,{}_2F_1\left(1,-\frac{1}{2n};1-\frac{1}{2n};e^{2nx}\right)-e^{x}\,{}_2F_1\left(1,\frac{1}{2n};\frac{1}{2}\left(2+\frac{1}{n}\right);e^{2nx}\right)$$

Mathematica [B] time = 0.18, size = 156, normalized size = 2.05

$$\frac{1}{2}e^{-2x}\left(-\frac{e^{2nx+x}\,{}_2F_1\left(1,1-\frac{1}{2n};2-\frac{1}{2n};e^{2nx}\right)}{2n-1}-\frac{e^{(2n+3)x}\,{}_2F_1\left(1,1+\frac{1}{2n};2+\frac{1}{2n};e^{2nx}\right)}{2n+1}+e^{x}\,{}_2F_1\left(1,-\frac{1}{2n};1-\frac{1}{2n};e^{2nx}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]*Coth[n*x],x]
```

```
[Out] (-((E^(x + 2*n*x)*Hypergeometric2F1[1, 1 - 1/(2*n), 2 - 1/(2*n), E^(2*n*x)]
)/(-1 + 2*n)) - (E^((3 + 2*n)*x)*Hypergeometric2F1[1, 1 + 1/(2*n), 2 + 1/(2
*n), E^(2*n*x)])/(1 + 2*n) + E^x*Hypergeometric2F1[1, -1/2*1/n, 1 - 1/(2*n)
, E^(2*n*x)] - E^(3*x)*Hypergeometric2F1[1, 1/(2*n), 1 + 1/(2*n), E^(2*n*x)
])/(2*E^(2*x))
```

fricas [F] time = 0.43, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\cosh(x)\coth\left(nx\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*coth(n*x),x, algorithm="fricas")
```

```
[Out] integral(cosh(x)*coth(n*x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \cosh(x)\coth\left(nx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*coth(n*x),x, algorithm="giac")
```

```
[Out] integrate(cosh(x)*coth(n*x), x)
```

maple [F] time = 0.22, size = 0, normalized size = 0.00

$$\int \cosh(x) \coth(nx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)*coth(n*x),x)
[Out] int(cosh(x)*coth(n*x),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{2}\left(e^{(2x)} - 1\right)e^{(-x)} - \frac{1}{2}\int \frac{e^{(2x)} + 1}{e^{(nx+x)} + e^{x}}\, dx + \frac{1}{2}\int \frac{e^{(2x)} + 1}{e^{(nx+x)} - e^{x}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*coth(n*x),x, algorithm="maxima")
[Out] 1/2*(e^(2*x) - 1)*e^(-x) - 1/2*integrate((e^(2*x) + 1)/(e^(n*x + x) + e^x),
 x) + 1/2*integrate((e^(2*x) + 1)/(e^(n*x + x) - e^x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \coth(n\,x)\,\cosh(x)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(n*x)*cosh(x),x)
[Out] int(coth(n*x)*cosh(x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \cosh(x) \coth(nx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*coth(n*x),x)
[Out] Integral(cosh(x)*coth(n*x), x)
```

3.240 $\int \cosh(x)\textbf{sech}(2x)\,dx$

Optimal. Leaf size=15

$$\frac{\tan^{-1}\left(\sqrt{2}\sinh(x)\right)}{\sqrt{2}}$$

```
[Out] 1/2*arctan(sinh(x)*2^(1/2))*2^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {4356, 203}

$$\frac{\tan^{-1}\left(\sqrt{2}\sinh(x)\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]*Sech[2*x],x]
```

```
[Out] ArcTan[Sqrt[2]*Sinh[x]]/Sqrt[2]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 4356

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Sin[c*(a + b*x)], x]}, Dist[d/(b*c), Subst[Int[SubstFor[1, Sin[c*(a + b*
x)]/d, u, x], x], x, Sin[c*(a + b*x)]/d], x] /; FunctionOfQ[Sin[c*(a + b*x)
]/d, u, x]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Cos] || EqQ[F, cos])
```

Rubi steps

$$\begin{aligned}\int \cosh(x)\text{sech}(2x)\,dx &= \text{Subst}\left(\int \frac{1}{1+2x^2}\,dx, x, \sinh(x)\right)\\ &= \frac{\tan^{-1}\left(\sqrt{2}\sinh(x)\right)}{\sqrt{2}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 15, normalized size = 1.00

$$\frac{\tan^{-1}\left(\sqrt{2}\sinh(x)\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]*Sech[2*x],x]
```

```
[Out] ArcTan[Sqrt[2]*Sinh[x]]/Sqrt[2]
```

fricas [B] time = 0.41, size = 68, normalized size = 4.53

$$\frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\cosh(x)+\frac{1}{2}\sqrt{2}\sinh(x)\right)-\frac{1}{2}\sqrt{2}\arctan\left(-\frac{\sqrt{2}\cosh(x)^2+2\sqrt{2}\cosh(x)\sinh(x)+\sqrt{2}\sinh(x)^2+\sqrt{2}}{2\left(\cosh(x)-\sinh(x)\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*sech(2*x),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(2)*arctan(1/2*sqrt(2)*cosh(x) + 1/2*sqrt(2)*sinh(x)) - 1/2*sqrt(2)
*arctan(-1/2*(sqrt(2)*cosh(x)^2 + 2*sqrt(2)*cosh(x)*sinh(x) + sqrt(2)*sinh(
x)^2 + sqrt(2))/(cosh(x) - sinh(x)))
```

giac [B] time = 0.13, size = 39, normalized size = 2.60

$$\frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2e^{x}\right)\right)+\frac{1}{2}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2e^{x}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*sech(2*x),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*e^x)) + 1/2*sqrt(2)*arctan(-1/2
*sqrt(2)*(sqrt(2) - 2*e^x))
```

maple [C] time = 0.22, size = 44, normalized size = 2.93

$$\frac{i\sqrt{2}\,\ln\left(e^{2x}+i\sqrt{2}\,e^{x}-1\right)}{4}-\frac{i\sqrt{2}\,\ln\left(e^{2x}-i\sqrt{2}\,e^{x}-1\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)*sech(2*x),x)
```

```
[Out] 1/4*I*2^(1/2)*ln(exp(2*x)+I*2^(1/2)*exp(x)-1)-1/4*I*2^(1/2)*ln(exp(2*x)-I*2
^(1/2)*exp(x)-1)
```

maxima [B] time = 0.44, size = 43, normalized size = 2.87

$$-\frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2e^{(-x)}\right)\right)-\frac{1}{2}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2e^{(-x)}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*sech(2*x),x, algorithm="maxima")
```

```
[Out] -1/2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*e^(-x))) - 1/2*sqrt(2)*arctan(
-1/2*sqrt(2)*(sqrt(2) - 2*e^(-x)))
```

mupad [B] time = 0.09, size = 32, normalized size = 2.13

$$\frac{\sqrt{2}\left(\mathrm{atan}\left(\frac{\sqrt{2}\,\mathrm{e}^x}{2}+\frac{\sqrt{2}\,\mathrm{e}^{3\,x}}{2}\right)+\mathrm{atan}\left(\frac{\sqrt{2}\,\mathrm{e}^x}{2}\right)\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)/cosh(2*x),x)
```

```
[Out] (2^(1/2)*(atan((2^(1/2)*exp(x))/2 + (2^(1/2)*exp(3*x))/2) + atan((2^(1/2)*e
xp(x))/2)))/2
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \cosh(x)\operatorname{sech}(2x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*sech(2*x),x)
```

```
[Out] Integral(cosh(x)*sech(2*x), x)
```

3.241 $\int \cosh(x)\mathbf{sech}(3x)\,dx$

Optimal. Leaf size=15

$$\frac{\tan^{-1}\left(\sqrt{3}\tanh(x)\right)}{\sqrt{3}}$$

```
[Out] 1/3*arctan(tanh(x)*3^(1/2))*3^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {203}

$$\frac{\tan^{-1}\left(\sqrt{3}\tanh(x)\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]*Sech[3*x],x]
```

```
[Out] ArcTan[Sqrt[3]*Tanh[x]]/Sqrt[3]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \cosh(x)\text{sech}(3x)\,dx &= \text{Subst}\left(\int \frac{1}{1+3x^2}\,dx, x, \tanh(x)\right)\\ &= \frac{\tan^{-1}\left(\sqrt{3}\tanh(x)\right)}{\sqrt{3}}\end{aligned}$$

Mathematica [C] time = 0.02, size = 48, normalized size = 3.20

$$\frac{1}{4}e^{2x}\left(2\,{}_2F_1\left(\frac{1}{3},1;\frac{4}{3};-e^{6x}\right)+e^{2x}\,{}_2F_1\left(\frac{2}{3},1;\frac{5}{3};-e^{6x}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]*Sech[3*x],x]
```

```
[Out] (E^(2*x)*(2*Hypergeometric2F1[1/3, 1, 4/3, -E^(6*x)] + E^(2*x)*Hypergeometr
ic2F1[2/3, 1, 5/3, -E^(6*x)]))/4
```

fricas [B] time = 0.47, size = 31, normalized size = 2.07

$$-\frac{1}{3}\sqrt{3}\arctan\left(-\frac{\sqrt{3}\cosh(x)+3\sqrt{3}\sinh(x)}{3(\cosh(x)-\sinh(x))}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*sech(3*x),x, algorithm="fricas")
```

```
[Out] -1/3*sqrt(3)*arctan(-1/3*(sqrt(3)*cosh(x) + 3*sqrt(3)*sinh(x))/(cosh(x) - s
inh(x)))
```

giac [A] time = 0.13, size = 19, normalized size = 1.27

$$\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2e^{(2x)}-1\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*sech(3*x),x, algorithm="giac")
```

```
[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*e^(2*x) - 1))
```

maple [C] time = 0.21, size = 40, normalized size = 2.67

$$\frac{i\sqrt{3}\ln\left(e^{2x}-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)}{6}-\frac{i\sqrt{3}\ln\left(e^{2x}-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)*sech(3*x),x)
```

```
[Out] 1/6*I*3^(1/2)*ln(exp(2*x)-1/2+1/2*I*3^(1/2))-1/6*I*3^(1/2)*ln(exp(2*x)-1/2-
1/2*I*3^(1/2))
```

maxima [B] time = 0.44, size = 114, normalized size = 7.60

$$-\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2e^{(-2x)}-1\right)\right)-\frac{1}{6}\sqrt{3}\arctan\left(\sqrt{3}+2e^{x}\right)+\frac{1}{6}\sqrt{3}\arctan\left(-\sqrt{3}+2e^{x}\right)+\frac{1}{12}\log\left(\sqrt{3}e^{x}+e^{(}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*sech(3*x),x, algorithm="maxima")
```

```
[Out] -1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*e^(-2*x) - 1)) - 1/6*sqrt(3)*arctan(sqrt
(3) + 2*e^x) + 1/6*sqrt(3)*arctan(-sqrt(3) + 2*e^x) + 1/12*log(sqrt(3)*e^x
+ e^(2*x) + 1) + 1/12*log(-sqrt(3)*e^x + e^(2*x) + 1) - 1/6*log(e^(2*x) + 1
) + 1/6*log(e^(-2*x) + 1) - 1/12*log(-e^(-2*x) + e^(-4*x) + 1)
```

mupad [B] time = 1.49, size = 19, normalized size = 1.27

$$\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{\sqrt{3}\,\left(2\,{\mathrm{e}}^{2\,x}-1\right)}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)/cosh(3*x),x)
```

```
[Out] (3^(1/2)*atan((3^(1/2)*(2*exp(2*x) - 1))/3))/3
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \cosh(x)\,\mathrm{sech}\,(3x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*sech(3*x),x)
```

```
[Out] Integral(cosh(x)*sech(3*x), x)
```

3.242 $\int \cosh(x)\,\mathbf{sech}(4x)\,dx$

Optimal. Leaf size=71

$$\frac{\tan^{-1}\left(\frac{2\sinh(x)}{\sqrt{2-\sqrt{2}}}\right)}{2\sqrt{2\left(2-\sqrt{2}\right)}}-\frac{\tan^{-1}\left(\frac{2\sinh(x)}{\sqrt{2+\sqrt{2}}}\right)}{2\sqrt{2\left(2+\sqrt{2}\right)}}$$

```
[Out] 1/2*arctan(2*sinh(x)/(2-2^(1/2))^(1/2))/(4-2*2^(1/2))^(1/2)-1/2*arctan(2*si
nh(x)/(2+2^(1/2))^(1/2))/(4+2*2^(1/2))^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {4356, 1093, 203}

$$\frac{\tan^{-1}\left(\frac{2\sinh(x)}{\sqrt{2-\sqrt{2}}}\right)}{2\sqrt{2\left(2-\sqrt{2}\right)}}-\frac{\tan^{-1}\left(\frac{2\sinh(x)}{\sqrt{2+\sqrt{2}}}\right)}{2\sqrt{2\left(2+\sqrt{2}\right)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]*Sech[4*x],x]
```

```
[Out] ArcTan[(2*Sinh[x])/Sqrt[2 - Sqrt[2]]]/(2*Sqrt[2*(2 - Sqrt[2])]) - ArcTan[(2
*Sinh[x])/Sqrt[2 + Sqrt[2]]]/(2*Sqrt[2*(2 + Sqrt[2])])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1093

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^2), x], x] - Dist[c/q, Int
[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
0] && PosQ[b^2 - 4*a*c]
```

Rule 4356

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Sin[c*(a + b*x)], x]}, Dist[d/(b*c), Subst[Int[SubstFor[1, Sin[c*(a + b*
x)]/d, u, x], x], x, Sin[c*(a + b*x)]/d], x] /; FunctionOfQ[Sin[c*(a + b*x)
```

```
]/d, u, x]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Cos] || EqQ[F, cos])
```

Rubi steps

$$\begin{aligned}\int \cosh(x)\text{sech}(4x)\,dx &= \text{Subst}\left(\int \frac{1}{1+8x^2+8x^4}\,dx,x,\sinh(x)\right)\\ &= \sqrt{2}\,\text{Subst}\left(\int \frac{1}{4-2\sqrt{2}+8x^2}\,dx,x,\sinh(x)\right)-\sqrt{2}\,\text{Subst}\left(\int \frac{1}{4+2\sqrt{2}+8x^2}\,dx,x,\sin\right.\\ &= \frac{\tan^{-1}\left(\frac{2\sinh(x)}{\sqrt{2-\sqrt{2}}}\right)}{2\sqrt{2\left(2-\sqrt{2}\right)}}-\frac{\tan^{-1}\left(\frac{2\sinh(x)}{\sqrt{2+\sqrt{2}}}\right)}{2\sqrt{2\left(2+\sqrt{2}\right)}}\end{aligned}$$

Mathematica [A] time = 0.11, size = 67, normalized size = 0.94

$$\frac{1}{4}\sqrt{2+\sqrt{2}}\,\tan^{-1}\left(\frac{2\sinh(x)}{\sqrt{2-\sqrt{2}}}\right)-\frac{\tan^{-1}\left(\frac{2\sinh(x)}{\sqrt{2+\sqrt{2}}}\right)}{2\sqrt{2\left(2+\sqrt{2}\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]*Sech[4*x],x]
```

```
[Out] (Sqrt[2 + Sqrt[2]]*ArcTan[(2*Sinh[x])/Sqrt[2 - Sqrt[2]]])/4 - ArcTan[(2*Sin
h[x])/Sqrt[2 + Sqrt[2]]]/(2*Sqrt[2*(2 + Sqrt[2])])
```

fricas [B] time = 0.45, size = 142, normalized size = 2.00

$$-\frac{1}{2}\sqrt{\sqrt{2}+2}\,\arctan\left(-\frac{1}{2}\left(\left(\sqrt{2}e^{(2x)}-\sqrt{2}\right)\sqrt{\sqrt{2}+2}-\sqrt{2}\sqrt{-\sqrt{2}e^{(2x)}+e^{(4x)}+1}\sqrt{\sqrt{2}+2}\right)e^{(-x)}\right)+\frac{1}{2}\sqrt{-\sqrt{2}+\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*sech(4*x),x, algorithm="fricas")
```

```
[Out] -1/2*sqrt(sqrt(2) + 2)*arctan(-1/2*((sqrt(2)*e^(2*x) - sqrt(2))*sqrt(sqrt(2
) + 2) - sqrt(2)*sqrt(-sqrt(2)*e^(2*x) + e^(4*x) + 1)*sqrt(sqrt(2) + 2))*e^
(-x)) + 1/2*sqrt(-sqrt(2) + 2)*arctan(-1/2*((sqrt(2)*e^(2*x) - sqrt(2))*sqr
t(-sqrt(2) + 2) - sqrt(2)*sqrt(sqrt(2)*e^(2*x) + e^(4*x) + 1)*sqrt(-sqrt(2)
 + 2))*e^(-x))
```

giac [B] time = 0.22, size = 135, normalized size = 1.90

$$\frac{1}{4}\sqrt{\sqrt{2}+2}\arctan\left(\frac{\sqrt{\sqrt{2}+2}+2e^{x}}{\sqrt{-\sqrt{2}+2}}\right)+\frac{1}{4}\sqrt{\sqrt{2}+2}\arctan\left(-\frac{\sqrt{\sqrt{2}+2}-2e^{x}}{\sqrt{-\sqrt{2}+2}}\right)-\frac{1}{4}\sqrt{-\sqrt{2}+2}\arctan\left(\frac{\sqrt{-\sqrt{2}+2}+2e^{x}}{\sqrt{\sqrt{2}+2}}\right)-\frac{1}{4}\sqrt{-\sqrt{2}+2}\arctan\left(-\frac{\sqrt{-\sqrt{2}+2}-2e^{x}}{\sqrt{\sqrt{2}+2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*sech(4*x),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(sqrt(2) + 2)*arctan((sqrt(sqrt(2) + 2) + 2*e^x)/sqrt(-sqrt(2) + 2)
) + 1/4*sqrt(sqrt(2) + 2)*arctan(-(sqrt(sqrt(2) + 2) - 2*e^x)/sqrt(-sqrt(2)
 + 2)) - 1/4*sqrt(-sqrt(2) + 2)*arctan((sqrt(-sqrt(2) + 2) + 2*e^x)/sqrt(sq
rt(2) + 2)) - 1/4*sqrt(-sqrt(2) + 2)*arctan(-(sqrt(-sqrt(2) + 2) - 2*e^x)/s
qrt(sqrt(2) + 2))
```

maple [C] time = 0.26, size = 40, normalized size = 0.56

$$2\left(\sum_{\_R=\mathrm{RootOf}\left(32768\_Z^{4}+512\_Z^{2}+1\right)}\_R\ln\left(e^{2x}+\left(-4096\_R^{3}-48\_R\right)e^{x}-1\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)*sech(4*x),x)
```

```
[Out] 2*sum(_R*ln(exp(2*x)+(-4096*_R^3-48*_R)*exp(x)-1),_R=RootOf(32768*_Z^4+512*
_Z^2+1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \cosh(x)\,\mathrm{sech}\,(4\,x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*sech(4*x),x, algorithm="maxima")
```

```
[Out] integrate(cosh(x)*sech(4*x), x)
```

mupad [B] time = 2.95, size = 126, normalized size = 1.77

$$\frac{\mathrm{atan}\left(\frac{3\,e^{2\,x}-2\,\sqrt{2}+2\,\sqrt{2}\,e^{2\,x}-3}{e^{x}\,\sqrt{\sqrt{2}+2}+\sqrt{2}\,e^{x}\,\sqrt{\sqrt{2}+2}}\right)\sqrt{\sqrt{2}+2}}{4}+\frac{\mathrm{atan}\left(\frac{3\,e^{2\,x}+2\,\sqrt{2}-2\,\sqrt{2}\,e^{2\,x}-3}{e^{x}\,\sqrt{2-\sqrt{2}}-\sqrt{2}\,e^{x}\,\sqrt{2-\sqrt{2}}}\right)\sqrt{2-\sqrt{2}}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)/cosh(4*x),x)
```

```
[Out] (atan((3*exp(2*x) - 2*2^(1/2) + 2*2^(1/2)*exp(2*x) - 3)/(exp(x)*(2^(1/2) +
2)^(1/2) + 2^(1/2)*exp(x)*(2^(1/2) + 2)^(1/2)))*(2^(1/2) + 2)^(1/2))/4 + (a
tan((3*exp(2*x) + 2*2^(1/2) - 2*2^(1/2)*exp(2*x) - 3)/(exp(x)*(2 - 2^(1/2))
^(1/2) - 2^(1/2)*exp(x)*(2 - 2^(1/2))^(1/2)))*(2 - 2^(1/2))^(1/2))/4
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \cosh(x)\operatorname{sech}(4x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*sech(4*x),x)
```

```
[Out] Integral(cosh(x)*sech(4*x), x)
```

3.243 $\int \cosh(x)\mathbf{sech}(5x)\,dx$

Optimal. Leaf size=75

$$\frac{1}{5}\sqrt{\frac{1}{2}\left(5+\sqrt{5}\right)}\tan^{-1}\left(\sqrt{5+2\sqrt{5}}\tanh(x)\right)-\frac{1}{5}\sqrt{\frac{1}{2}\left(5-\sqrt{5}\right)}\tan^{-1}\left(\sqrt{5-2\sqrt{5}}\tanh(x)\right)$$

```
[Out] -1/10*arctan((5-2*5^(1/2))^(1/2)*tanh(x))*(10-2*5^(1/2))^(1/2)+1/10*arctan(
(5+2*5^(1/2))^(1/2)*tanh(x))*(10+2*5^(1/2))^(1/2)
```

Rubi [A] time = 0.13, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {1166, 203}

$$\frac{1}{5}\sqrt{\frac{1}{2}\left(5+\sqrt{5}\right)}\tan^{-1}\left(\sqrt{5+2\sqrt{5}}\tanh(x)\right)-\frac{1}{5}\sqrt{\frac{1}{2}\left(5-\sqrt{5}\right)}\tan^{-1}\left(\sqrt{5-2\sqrt{5}}\tanh(x)\right)$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]*Sech[5*x],x]
```

```
[Out] -(Sqrt[(5 - Sqrt[5])/2]*ArcTan[Sqrt[5 - 2*Sqrt[5]]*Tanh[x]])/5 + (Sqrt[(5 +
 Sqrt[5])/2]*ArcTan[Sqrt[5 + 2*Sqrt[5]]*Tanh[x]])/5
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \cosh(x)\text{sech}(5x)\,dx = \text{Subst}\left(\int \frac{1-x^2}{1+10x^2+5x^4}\,dx, x, \tanh(x)\right)$$
$$= \frac{1}{2}\left(-1-\sqrt{5}\right)\text{Subst}\left(\int \frac{1}{5+2\sqrt{5}+5x^2}\,dx, x, \tanh(x)\right) + \frac{1}{2}\left(-1+\sqrt{5}\right)\text{Subst}\left(\int \frac{}{5-2}\right.$$
$$= -\frac{1}{5}\sqrt{\frac{1}{2}\left(5-\sqrt{5}\right)}\tan^{-1}\left(\sqrt{5-2\sqrt{5}}\tanh(x)\right) + \frac{1}{5}\sqrt{\frac{1}{2}\left(5+\sqrt{5}\right)}\tan^{-1}\left(\sqrt{5+2\sqrt{5}}\ \tan\right.$$

Mathematica [A] time = 0.11, size = 84, normalized size = 1.12

$$\frac{\sqrt{5+\sqrt{5}}\tan^{-1}\left(\frac{\left(5+\sqrt{5}\right)\tanh(x)}{\sqrt{10-2\sqrt{5}}}\right)+\sqrt{5-\sqrt{5}}\tan^{-1}\left(\frac{\left(\sqrt{5}-5\right)\tanh(x)}{\sqrt{2\left(5+\sqrt{5}\right)}}\right)}{5\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]*Sech[5*x],x]
```

```
[Out] (Sqrt[5 + Sqrt[5]]*ArcTan[((5 + Sqrt[5])*Tanh[x])/Sqrt[10 - 2*Sqrt[5]]] + S
qrt[5 - Sqrt[5]]*ArcTan[((-5 + Sqrt[5])*Tanh[x])/Sqrt[2*(5 + Sqrt[5])]])/(5
*Sqrt[2])
```

fricas [B] time = 0.45, size = 173, normalized size = 2.31

$$-\frac{1}{5}\sqrt{2}\sqrt{\sqrt{5}+5}\arctan\left(\frac{1}{40}\sqrt{5}\sqrt{2}\sqrt{-32\left(\sqrt{5}+1\right)e^{(2x)}+64e^{(4x)}+64}\sqrt{\sqrt{5}+5}-\frac{1}{20}\left(4\sqrt{5}\sqrt{2}e^{(2x)}-\sqrt{5}\sqrt{2}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*sech(5*x),x, algorithm="fricas")
```

```
[Out] -1/5*sqrt(2)*sqrt(sqrt(5) + 5)*arctan(1/40*sqrt(5)*sqrt(2)*sqrt(-32*(sqrt(5
) + 1)*e^(2*x) + 64*e^(4*x) + 64)*sqrt(sqrt(5) + 5) - 1/20*(4*sqrt(5)*sqrt(
2)*e^(2*x) - sqrt(5)*sqrt(2) - 5*sqrt(2))*sqrt(sqrt(5) + 5)) + 1/5*sqrt(2)*
sqrt(-sqrt(5) + 5)*arctan(-1/20*(4*sqrt(5)*sqrt(2)*e^(2*x) - sqrt(5)*sqrt(2
) + 5*sqrt(2))*sqrt(-sqrt(5) + 5) + 1/5*sqrt(5)*sqrt((sqrt(5) - 1)*e^(2*x)
+ 2*e^(4*x) + 2)*sqrt(-sqrt(5) + 5))
```

giac [A] time = 0.16, size = 68, normalized size = 0.91

$$-\frac{1}{10}\sqrt{-2\sqrt{5}+10}\arctan\left(\frac{\sqrt{5}+4e^{(2x)}-1}{\sqrt{2\sqrt{5}+10}}\right)+\frac{1}{10}\sqrt{2\sqrt{5}+10}\arctan\left(-\frac{\sqrt{5}-4e^{(2x)}+1}{\sqrt{-2\sqrt{5}+10}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*sech(5*x),x, algorithm="giac")
```

```
[Out] -1/10*sqrt(-2*sqrt(5) + 10)*arctan((sqrt(5) + 4*e^(2*x) - 1)/sqrt(2*sqrt(5)
 + 10)) + 1/10*sqrt(2*sqrt(5) + 10)*arctan(-(sqrt(5) - 4*e^(2*x) + 1)/sqrt(
-2*sqrt(5) + 10))
```

maple [C] time = 0.28, size = 41, normalized size = 0.55

$$2\left(\sum_{\_R=\mathrm{RootOf}\left(32000\_Z^4+400\_Z^2+1\right)}\_R\ln\left(-4000\_R^3+200\_R^2+e^{2x}-30\_R+1\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)*sech(5*x),x)
```

```
[Out] 2*sum(_R*ln(-4000*_R^3+200*_R^2+exp(2*x)-30*_R+1),_R=RootOf(32000*_Z^4+400*
_Z^2+1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\sqrt{5}\arctan\left(\frac{\sqrt{5}+4e^{(-2x)}-1}{\sqrt{2\sqrt{5}+10}}\right)}{5\sqrt{2\sqrt{5}+10}}-\frac{\sqrt{5}\arctan\left(-\frac{\sqrt{5}-4e^{(-2x)}+1}{\sqrt{-2\sqrt{5}+10}}\right)}{5\sqrt{-2\sqrt{5}+10}}-\frac{\log\left(-\left(\sqrt{5}+1\right)e^{(-2x)}+2e^{(-4x)}+2\right)}{10\left(\sqrt{5}+1\right)}+\frac{\log\left(\left(\sqrt{5}-1\right)e^{(-}\right.}{10\left(\sqrt{\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*sech(5*x),x, algorithm="maxima")
```

```
[Out] 1/5*sqrt(5)*arctan((sqrt(5) + 4*e^(-2*x) - 1)/sqrt(2*sqrt(5) + 10))/sqrt(2*
sqrt(5) + 10) - 1/5*sqrt(5)*arctan(-(sqrt(5) - 4*e^(-2*x) + 1)/sqrt(-2*sqrt
(5) + 10))/sqrt(-2*sqrt(5) + 10) - 1/10*log(-(sqrt(5) + 1)*e^(-2*x) + 2*e^(
-4*x) + 2)/(sqrt(5) + 1) + 1/10*log((sqrt(5) - 1)*e^(-2*x) + 2*e^(-4*x) + 2
)/(sqrt(5) - 1) - 1/5*integrate((e^(7*x) - 2*e^(5*x) - 2*e^(3*x) + e^x)*e^x
/(e^(8*x) - e^(6*x) + e^(4*x) - e^(2*x) + 1), x) + 1/10*log(e^(2*x) + 1) -
1/10*log(e^(-2*x) + 1)
```

mupad [B] time = 4.15, size = 297, normalized size = 3.96

$$\ln\left(1-\sqrt{-\frac{\sqrt{5}}{200}-\frac{1}{40}}\left(4e^{2x}+\sqrt{-\frac{\sqrt{5}}{200}-\frac{1}{40}}\left(48e^{2x}+\sqrt{-\frac{\sqrt{5}}{200}-\frac{1}{40}}\left(360e^{2x}-360\right)-72\right)-8\right)-e^{2x}\right)\sqrt{-\frac{\sqrt{}}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)/cosh(5*x),x)
```

```
[Out] log(1 - (- 5^(1/2)/200 - 1/40)^(1/2)*(4*exp(2*x) + (- 5^(1/2)/200 - 1/40)^(
1/2)*(48*exp(2*x) + (- 5^(1/2)/200 - 1/40)^(1/2)*(360*exp(2*x) - 360) - 72)
 - 8) - exp(2*x))*(- 5^(1/2)/200 - 1/40)^(1/2) - log((5^(1/2)/200 - 1/40)^(
1/2)*(4*exp(2*x) + (5^(1/2)/200 - 1/40)^(1/2)*((5^(1/2)/200 - 1/40)^(1/2)*(
360*exp(2*x) - 360) - 48*exp(2*x) + 72) - 8) - exp(2*x) + 1)*(5^(1/2)/200 -
 1/40)^(1/2) - log((- 5^(1/2)/200 - 1/40)^(1/2)*(4*exp(2*x) + (- 5^(1/2)/20
0 - 1/40)^(1/2)*((- 5^(1/2)/200 - 1/40)^(1/2)*(360*exp(2*x) - 360) - 48*exp
(2*x) + 72) - 8) - exp(2*x) + 1)*(- 5^(1/2)/200 - 1/40)^(1/2) + log(1 - (5^
(1/2)/200 - 1/40)^(1/2)*(4*exp(2*x) + (5^(1/2)/200 - 1/40)^(1/2)*(48*exp(2*
x) + (5^(1/2)/200 - 1/40)^(1/2)*(360*exp(2*x) - 360) - 72) - 8) - exp(2*x))
*(5^(1/2)/200 - 1/40)^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \cosh(x)\operatorname{sech}(5x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*sech(5*x),x)
```

```
[Out] Integral(cosh(x)*sech(5*x), x)
```

3.244 $\int \cosh(x)\mathbf{sech}(6x)\,dx$

Optimal. Leaf size=85

$$-\frac{\tan^{-1}\left(\sqrt{2}\sinh(x)\right)}{3\sqrt{2}}+\frac{\tan^{-1}\left(\frac{2\sinh(x)}{\sqrt{2-\sqrt{3}}}\right)}{6\sqrt{2-\sqrt{3}}}+\frac{\tan^{-1}\left(\frac{2\sinh(x)}{\sqrt{2+\sqrt{3}}}\right)}{6\sqrt{2+\sqrt{3}}}$$

```
[Out] -1/6*arctan(sinh(x)*2^(1/2))*2^(1/2)+1/6*arctan(2*sinh(x)/(1/2*6^(1/2)-1/2*
2^(1/2)))/(1/2*6^(1/2)-1/2*2^(1/2))+1/6*arctan(2*sinh(x)/(1/2*6^(1/2)+1/2*2
^(1/2)))/(1/2*6^(1/2)+1/2*2^(1/2))
```

Rubi [A] time = 0.06, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {4356, 2057, 203, 1166}

$$-\frac{\tan^{-1}\left(\sqrt{2}\sinh(x)\right)}{3\sqrt{2}}+\frac{\tan^{-1}\left(\frac{2\sinh(x)}{\sqrt{2-\sqrt{3}}}\right)}{6\sqrt{2-\sqrt{3}}}+\frac{\tan^{-1}\left(\frac{2\sinh(x)}{\sqrt{2+\sqrt{3}}}\right)}{6\sqrt{2+\sqrt{3}}}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]*Sech[6*x],x]
```

```
[Out] -ArcTan[Sqrt[2]*Sinh[x]]/(3*Sqrt[2]) + ArcTan[(2*Sinh[x])/Sqrt[2 - Sqrt[3]]
]/(6*Sqrt[2 - Sqrt[3]]) + ArcTan[(2*Sinh[x])/Sqrt[2 + Sqrt[3]]]/(6*Sqrt[2 +
 Sqrt[3]])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 2057

```
Int[(P_)^(p_), x_Symbol] :> With[{u = Factor[P /. x -> Sqrt[x]]}, Int[Expan
dIntegrand[(u /. x -> x^2)^p, x], x] /;  !SumQ[NonfreeFactors[u, x]]] /; Po
lyQ[P, x^2] && ILtQ[p, 0]
```

Rule 4356

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Sin[c*(a + b*x)], x]}, Dist[d/(b*c), Subst[Int[SubstFor[1, Sin[c*(a + b*
x)]/d, u, x], x], x, Sin[c*(a + b*x)]/d], x] /; FunctionOfQ[Sin[c*(a + b*x)
]/d, u, x]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Cos] || EqQ[F, cos])
```

Rubi steps

$$
\begin{aligned}
\int \cosh(x)\operatorname{sech}(6x)\,dx &= \operatorname{Subst}\left(\int \frac{1}{1+18x^2+48x^4+32x^6}\,dx, x, \sinh(x)\right)\\
&= \operatorname{Subst}\left(\int \left(-\frac{1}{3\left(1+2x^2\right)}+\frac{4\left(1+2x^2\right)}{3\left(1+16x^2+16x^4\right)}\right)dx, x, \sinh(x)\right)\\
&= -\left(\frac{1}{3}\operatorname{Subst}\left(\int \frac{1}{1+2x^2}\,dx, x, \sinh(x)\right)\right)+\frac{4}{3}\operatorname{Subst}\left(\int \frac{1+2x^2}{1+16x^2+16x^4}\,dx, x, \sinh(x)\right)\\
&= -\frac{\tan^{-1}\left(\sqrt{2}\sinh(x)\right)}{3\sqrt{2}}+\frac{4}{3}\operatorname{Subst}\left(\int \frac{1}{8-4\sqrt{3}+16x^2}\,dx, x, \sinh(x)\right)+\frac{4}{3}\operatorname{Subst}\left(\int \frac{}{8+}\right.\\
&= -\frac{\tan^{-1}\left(\sqrt{2}\sinh(x)\right)}{3\sqrt{2}}+\frac{\tan^{-1}\left(\frac{2\sinh(x)}{\sqrt{2-\sqrt{3}}}\right)}{6\sqrt{2-\sqrt{3}}}+\frac{\tan^{-1}\left(\frac{2\sinh(x)}{\sqrt{2+\sqrt{3}}}\right)}{6\sqrt{2+\sqrt{3}}}
\end{aligned}
$$

Mathematica [A] time = 0.09, size = 81, normalized size = 0.95

$$
\frac{1}{6}\left(-\sqrt{2}\tan^{-1}\left(\sqrt{2}\sinh(x)\right)+\sqrt{2+\sqrt{3}}\tan^{-1}\left(\frac{2\sinh(x)}{\sqrt{2-\sqrt{3}}}\right)+\sqrt{2-\sqrt{3}}\tan^{-1}\left(\frac{2\sinh(x)}{\sqrt{2+\sqrt{3}}}\right)\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]*Sech[6*x],x]

[Out] (-(Sqrt[2]*ArcTan[Sqrt[2]*Sinh[x]]) + Sqrt[2 + Sqrt[3]]*ArcTan[(2*Sinh[x])/
Sqrt[2 - Sqrt[3]]] + Sqrt[2 - Sqrt[3]]*ArcTan[(2*Sinh[x])/Sqrt[2 + Sqrt[3]]
])/6
```

fricas [B] time = 0.46, size = 156, normalized size = 1.84

$$-\frac{1}{3}\sqrt{\sqrt{3}+2}\arctan\left(-\left(\sqrt{\sqrt{3}+2}\left(e^{(2x)}-1\right)-\sqrt{-\sqrt{3}e^{(2x)}+e^{(4x)}+1}\sqrt{\sqrt{3}+2}\right)e^{(-x)}\right)-\frac{1}{3}\sqrt{-\sqrt{3}+2}\arctan\Big($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*sech(6*x),x, algorithm="fricas")
```

```
[Out] -1/3*sqrt(sqrt(3) + 2)*arctan(-(sqrt(sqrt(3) + 2)*(e^(2*x) - 1) - sqrt(-sqr
t(3)*e^(2*x) + e^(4*x) + 1)*sqrt(sqrt(3) + 2))*e^(-x)) - 1/3*sqrt(-sqrt(3)
+ 2)*arctan(-(sqrt(-sqrt(3) + 2)*(e^(2*x) - 1) - sqrt(sqrt(3)*e^(2*x) + e^(
4*x) + 1)*sqrt(-sqrt(3) + 2))*e^(-x)) - 1/6*sqrt(2)*arctan(1/2*sqrt(2)*e^(3
*x) + 1/2*sqrt(2)*e^x) - 1/6*sqrt(2)*arctan(1/2*sqrt(2)*e^x)
```

giac [B] time = 0.13, size = 177, normalized size = 2.08

$$\frac{1}{12}\left(\sqrt{6}-\sqrt{2}\right)\arctan\left(\frac{\sqrt{6}-\sqrt{2}+4e^{x}}{\sqrt{6}+\sqrt{2}}\right)+\frac{1}{12}\left(\sqrt{6}-\sqrt{2}\right)\arctan\left(-\frac{\sqrt{6}-\sqrt{2}-4e^{x}}{\sqrt{6}+\sqrt{2}}\right)+\frac{1}{12}\left(\sqrt{6}+\sqrt{2}\right)\arctan\Big($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*sech(6*x),x, algorithm="giac")
```

```
[Out] 1/12*(sqrt(6) - sqrt(2))*arctan((sqrt(6) - sqrt(2) + 4*e^x)/(sqrt(6) + sqrt
(2))) + 1/12*(sqrt(6) - sqrt(2))*arctan(-(sqrt(6) - sqrt(2) - 4*e^x)/(sqrt(
6) + sqrt(2))) + 1/12*(sqrt(6) + sqrt(2))*arctan((sqrt(6) + sqrt(2) + 4*e^x
)/(sqrt(6) - sqrt(2))) + 1/12*(sqrt(6) + sqrt(2))*arctan(-(sqrt(6) + sqrt(2
) - 4*e^x)/(sqrt(6) - sqrt(2))) - 1/6*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) +
 2*e^x)) - 1/6*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*e^x))
```

maple [C] time = 0.32, size = 83, normalized size = 0.98

$$\frac{i\sqrt{2}\ \ln\left(e^{2x}-i\sqrt{2}\ e^{x}-1\right)}{12}-\frac{i\sqrt{2}\ \ln\left(e^{2x}+i\sqrt{2}\ e^{x}-1\right)}{12}+2\left(\sum_{\_R=\mathrm{RootOf}\left(331776\_Z^{4}+2304\_Z^{2}+1\right)}\_R\ln\left(e^{2x}+\left(13824\_R\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)*sech(6*x),x)
```

```
[Out] 1/12*I*2^(1/2)*ln(exp(2*x)-I*2^(1/2)*exp(x)-1)-1/12*I*2^(1/2)*ln(exp(2*x)+I
*2^(1/2)*exp(x)-1)+2*sum(_R*ln(exp(2*x)+(13824*_R^3+96*_R)*exp(x)-1),_R=Roo
tOf(331776*_Z^4+2304*_Z^2+1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{6}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2e^{x}\right)\right)-\frac{1}{6}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2e^{x}\right)\right)+\int\frac{e^{(7x)}+e^{(5x)}+e^{(3x)}+e^{x}}{3\left(e^{(8x)}-e^{(4x)}+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*sech(6*x),x, algorithm="maxima")
```

```
[Out] -1/6*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*e^x)) - 1/6*sqrt(2)*arctan(-1/
2*sqrt(2)*(sqrt(2) - 2*e^x)) + integrate(1/3*(e^(7*x) + e^(5*x) + e^(3*x) +
 e^x)/(e^(8*x) - e^(4*x) + 1), x)
```

mupad [B] time = 3.53, size = 206, normalized size = 2.42

$$\frac{\sqrt{2}\,\mathrm{atan}\left(\frac{7\,e^{2\,x}+4\,\sqrt{3}-4\,\sqrt{3}\,e^{2\,x}-7}{\frac{5\,\sqrt{2}\,e^{x}}{2}-\frac{3\,\sqrt{6}\,e^{x}}{2}}\right)}{12}+\frac{\sqrt{2}\,\mathrm{atan}\left(\frac{7\,e^{2\,x}-4\,\sqrt{3}+4\,\sqrt{3}\,e^{2\,x}-7}{\frac{5\,\sqrt{2}\,e^{x}}{2}+\frac{3\,\sqrt{6}\,e^{x}}{2}}\right)}{12}-\frac{\sqrt{6}\,\mathrm{atan}\left(\frac{7\,e^{2\,x}+4\,\sqrt{3}-4\,\sqrt{3}\,e^{2\,x}-7}{\frac{5\,\sqrt{2}\,e^{x}}{2}-\frac{3\,\sqrt{6}\,e^{x}}{2}}\right)}{12}+\frac{\sqrt{6}\,\mathrm{atan}\left(\frac{7\,e^{2}\cdots}{\cdots}\right)}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)/cosh(6*x),x)
```

```
[Out] (2^(1/2)*atan((7*exp(2*x) + 4*3^(1/2) - 4*3^(1/2)*exp(2*x) - 7)/((5*2^(1/2)
*exp(x))/2 - (3*6^(1/2)*exp(x))/2)))/12 + (2^(1/2)*atan((7*exp(2*x) - 4*3^(
1/2) + 4*3^(1/2)*exp(2*x) - 7)/((5*2^(1/2)*exp(x))/2 + (3*6^(1/2)*exp(x))/2
)))/12 - (6^(1/2)*atan((7*exp(2*x) + 4*3^(1/2) - 4*3^(1/2)*exp(2*x) - 7)/((
5*2^(1/2)*exp(x))/2 - (3*6^(1/2)*exp(x))/2)))/12 + (6^(1/2)*atan((7*exp(2*x
) - 4*3^(1/2) + 4*3^(1/2)*exp(2*x) - 7)/((5*2^(1/2)*exp(x))/2 + (3*6^(1/2)*
exp(x))/2)))/12 - (2^(1/2)*atan((2^(1/2)*exp(-x)*(exp(2*x) - 1))/2))/6
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\cosh(x)\operatorname{sech}(6x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*sech(6*x),x)
```

```
[Out] Integral(cosh(x)*sech(6*x), x)
```

3.245 $\int \cosh(x)\mathbf{csch}(2x)\,dx$

Optimal. Leaf size=7

$$-\frac{1}{2}\tanh^{-1}(\cosh(x))$$

```
[Out] -1/2*arctanh(cosh(x))
```

Rubi [A] time = 0.02, antiderivative size = 7, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {4287, 3770}

$$-\frac{1}{2}\tanh^{-1}(\cosh(x))$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]*Csch[2*x],x]
```

```
[Out] -ArcTanh[Cosh[x]]/2
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4287

```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/e^p, Int[(e*Cos[a + b*x])^(m + p)*Sin[a + b*x]^p, x], x
] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \cosh(x)\text{csch}(2x)\,dx &= \frac{1}{2}\int \text{csch}(x)\,dx\\ &= -\frac{1}{2}\tanh^{-1}(\cosh(x))\end{aligned}$$

Mathematica [A] time = 0.00, size = 11, normalized size = 1.57

$$\frac{1}{2}\log\left(\tanh\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]*Csch[2*x],x]
```

```
[Out] Log[Tanh[x/2]]/2
```

fricas [B] time = 0.45, size = 19, normalized size = 2.71

$$-\frac{1}{2}\log\left(\cosh(x)+\sinh(x)+1\right)+\frac{1}{2}\log\left(\cosh(x)+\sinh(x)-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*csch(2*x),x, algorithm="fricas")
```

```
[Out] -1/2*log(cosh(x) + sinh(x) + 1) + 1/2*log(cosh(x) + sinh(x) - 1)
```

giac [B] time = 0.13, size = 16, normalized size = 2.29

$$-\frac{1}{2}\log\left(e^{x}+1\right)+\frac{1}{2}\log\left(|e^{x}-1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*csch(2*x),x, algorithm="giac")
```

```
[Out] -1/2*log(e^x + 1) + 1/2*log(abs(e^x - 1))
```

maple [A] time = 0.15, size = 6, normalized size = 0.86

$$-\operatorname{arctanh}\left(e^{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)*csch(2*x),x)
```

```
[Out] -arctanh(exp(x))
```

maxima [B] time = 0.34, size = 19, normalized size = 2.71

$$-\frac{1}{2}\log\left(e^{(-x)}+1\right)+\frac{1}{2}\log\left(e^{(-x)}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*csch(2*x),x, algorithm="maxima")
```

```
[Out] -1/2*log(e^(-x) + 1) + 1/2*log(e^(-x) - 1)
```

mupad [B] time = 0.06, size = 19, normalized size = 2.71

$$\frac{\ln\left(1-\mathrm{e}^x\right)}{2}-\frac{\ln\left(-\mathrm{e}^x-1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)/sinh(2*x),x)
```

```
[Out] log(1 - exp(x))/2 - log(- exp(x) - 1)/2
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \cosh\left(x\right)\operatorname{csch}\left(2x\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*csch(2*x),x)
```

```
[Out] Integral(cosh(x)*csch(2*x), x)
```

3.246 $\int \cosh(x)\mathbf{csch}(3x)\,dx$

Optimal. Leaf size=21

$$\frac{1}{3}\log(\sinh(x)) - \frac{1}{6}\log\left(4\sinh^2(x) + 3\right)$$

```
[Out] 1/3*ln(sinh(x))-1/6*ln(3+4*sinh(x)^2)
```

Rubi [A] time = 0.03, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.714, Rules used = {4356, 266, 36, 29, 31}

$$\frac{1}{3}\log(\sinh(x)) - \frac{1}{6}\log\left(4\sinh^2(x) + 3\right)$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]*Csch[3*x],x]
```

```
[Out] Log[Sinh[x]]/3 - Log[3 + 4*Sinh[x]^2]/6
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4356

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Sin[c*(a + b*x)], x]}, Dist[d/(b*c), Subst[Int[SubstFor[1, Sin[c*(a + b*
```

```
x)]/d, u, x], x], x, Sin[c*(a + b*x)]/d], x] /; FunctionOfQ[Sin[c*(a + b*x)
]/d, u, x]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Cos] || EqQ[F, cos])
```

Rubi steps

$$\begin{aligned}\int \cosh(x)\operatorname{csch}(3x)\,dx &= \operatorname{Subst}\left(\int \frac{1}{x\left(3+4x^2\right)}\,dx, x, \sinh(x)\right)\\ &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{x(3+4x)}\,dx, x, \sinh^2(x)\right)\\ &= \frac{1}{6}\operatorname{Subst}\left(\int \frac{1}{x}\,dx, x, \sinh^2(x)\right)-\frac{2}{3}\operatorname{Subst}\left(\int \frac{1}{3+4x}\,dx, x, \sinh^2(x)\right)\\ &= \frac{1}{3}\log(\sinh(x))-\frac{1}{6}\log\left(3+4\sinh^2(x)\right)\end{aligned}$$

Mathematica [A] time = 0.01, size = 21, normalized size = 1.00

$$\frac{1}{3}\log(\sinh(x))-\frac{1}{6}\log\left(4\sinh^2(x)+3\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]*Csch[3*x],x]
```

```
[Out] Log[Sinh[x]]/3 - Log[3 + 4*Sinh[x]^2]/6
```

fricas [B] time = 0.40, size = 52, normalized size = 2.48

$$-\frac{1}{6}\log\left(\frac{2\cosh(x)^2+2\sinh(x)^2+1}{\cosh(x)^2-2\cosh(x)\sinh(x)+\sinh(x)^2}\right)+\frac{1}{3}\log\left(\frac{2\sinh(x)}{\cosh(x)-\sinh(x)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*csch(3*x),x, algorithm="fricas")
```

```
[Out] -1/6*log((2*cosh(x)^2 + 2*sinh(x)^2 + 1)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + s
inh(x)^2)) + 1/3*log(2*sinh(x)/(cosh(x) - sinh(x)))
```

giac [B] time = 0.11, size = 40, normalized size = 1.90

$$-\frac{1}{6}\log\left(e^{(2x)}+e^x+1\right)-\frac{1}{6}\log\left(e^{(2x)}-e^x+1\right)+\frac{1}{3}\log\left(e^x+1\right)+\frac{1}{3}\log\left(|e^x-1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*csch(3*x),x, algorithm="giac")
```

```
[Out] -1/6*log(e^(2*x) + e^x + 1) - 1/6*log(e^(2*x) - e^x + 1) + 1/3*log(e^x + 1)
 + 1/3*log(abs(e^x - 1))
```

maple [A] time = 0.22, size = 24, normalized size = 1.14

$$\frac{\ln\left(e^{2x}-1\right)}{3}-\frac{\ln\left(1+e^{2x}+e^{4x}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)*csch(3*x),x)
```

```
[Out] 1/3*ln(exp(2*x)-1)-1/6*ln(1+exp(2*x)+exp(4*x))
```

maxima [B] time = 0.44, size = 47, normalized size = 2.24

$$-\frac{1}{6}\log\left(e^{(-x)}+e^{(-2x)}+1\right)+\frac{1}{3}\log\left(e^{(-x)}+1\right)+\frac{1}{3}\log\left(e^{(-x)}-1\right)-\frac{1}{6}\log\left(-e^{(-x)}+e^{(-2x)}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*csch(3*x),x, algorithm="maxima")
```

```
[Out] -1/6*log(e^(-x) + e^(-2*x) + 1) + 1/3*log(e^(-x) + 1) + 1/3*log(e^(-x) - 1)
 - 1/6*log(-e^(-x) + e^(-2*x) + 1)
```

mupad [B] time = 1.49, size = 29, normalized size = 1.38

$$\frac{\ln\left(3\,e^{2\,x}-3\right)}{3}-\frac{\ln\left(-e^{2\,x}-e^{4\,x}-1\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)/sinh(3*x),x)
```

```
[Out] log(3*exp(2*x) - 3)/3 - log(- exp(2*x) - exp(4*x) - 1)/6
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\cosh\left(x\right)\operatorname{csch}\left(3x\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*csch(3*x),x)
```

```
[Out] Integral(cosh(x)*csch(3*x), x)
```

3.247 $\int \cosh(x)\mathbf{csch}(4x)\,dx$

Optimal. Leaf size=26

$$\frac{\tanh^{-1}\left(\sqrt{2}\cosh(x)\right)}{2\sqrt{2}}-\frac{1}{4}\tanh^{-1}(\cosh(x))$$

```
[Out] -1/4*arctanh(cosh(x))+1/4*arctanh(cosh(x)*2^(1/2))*2^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {1093, 206}

$$\frac{\tanh^{-1}\left(\sqrt{2}\cosh(x)\right)}{2\sqrt{2}}-\frac{1}{4}\tanh^{-1}(\cosh(x))$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]*Csch[4*x],x]
```

```
[Out] -ArcTanh[Cosh[x]]/4 + ArcTanh[Sqrt[2]*Cosh[x]]/(2*Sqrt[2])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 1093

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^2), x], x] - Dist[c/q, Int
[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
0] && PosQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}\int \cosh(x)\text{csch}(4x)\,dx &= -\text{Subst}\left(\int \frac{1}{-4+12x^2-8x^4}\,dx,x,\cosh(x)\right)\\ &= 2\,\text{Subst}\left(\int \frac{1}{4-8x^2}\,dx,x,\cosh(x)\right)-2\,\text{Subst}\left(\int \frac{1}{8-8x^2}\,dx,x,\cosh(x)\right)\\ &= -\frac{1}{4}\tanh^{-1}(\cosh(x))+\frac{\tanh^{-1}\left(\sqrt{2}\cosh(x)\right)}{2\sqrt{2}}\end{aligned}$$

Mathematica [C] time = 0.31, size = 183, normalized size = 7.04

$$\frac{4\tanh^{-1}\left(\sqrt{2}-i\tanh\left(\frac{x}{2}\right)\right)+2\sqrt{2}\log\left(\sinh\left(\frac{x}{2}\right)\right)-2\sqrt{2}\log\left(\cosh\left(\frac{x}{2}\right)\right)-\log\left(\sqrt{2}-2\cosh(x)\right)+\log\left(2\cosh(x\right.}{8\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]*Csch[4*x],x]
```

```
[Out] ((2*I)*ArcTan[(Cosh[x/2] + Sinh[x/2])/((1 + Sqrt[2])*Cosh[x/2] - (-1 + Sqrt
[2])*Sinh[x/2])] - (2*I)*ArcTan[(Cosh[x/2] + Sinh[x/2])/((-1 + Sqrt[2])*Cos
h[x/2] - (1 + Sqrt[2])*Sinh[x/2])] + 4*ArcTanh[Sqrt[2] - I*Tanh[x/2]] - 2*S
qrt[2]*Log[Cosh[x/2]] - Log[Sqrt[2] - 2*Cosh[x]] + Log[Sqrt[2] + 2*Cosh[x]]
 + 2*Sqrt[2]*Log[Sinh[x/2]])/(8*Sqrt[2])
```

fricas [B] time = 0.50, size = 54, normalized size = 2.08

$$\frac{1}{8}\sqrt{2}\log\left(\frac{\cosh(x)^2+\sinh(x)^2+2\sqrt{2}\cosh(x)+2}{\cosh(x)^2+\sinh(x)^2}\right)-\frac{1}{4}\log\left(\cosh(x)+\sinh(x)+1\right)+\frac{1}{4}\log\left(\cosh(x)+\sinh(x)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*csch(4*x),x, algorithm="fricas")
```

```
[Out] 1/8*sqrt(2)*log((cosh(x)^2 + sinh(x)^2 + 2*sqrt(2)*cosh(x) + 2)/(cosh(x)^2
+ sinh(x)^2)) - 1/4*log(cosh(x) + sinh(x) + 1) + 1/4*log(cosh(x) + sinh(x)
- 1)
```

giac [B] time = 0.12, size = 57, normalized size = 2.19

$$-\frac{1}{8}\sqrt{2}\log\left(-\frac{\sqrt{2}-e^{(-x)}-e^x}{\sqrt{2}+e^{(-x)}+e^x}\right)-\frac{1}{8}\log\left(e^{(-x)}+e^x+2\right)+\frac{1}{8}\log\left(e^{(-x)}+e^x-2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*csch(4*x),x, algorithm="giac")
```

```
[Out] -1/8*sqrt(2)*log(-(sqrt(2) - e^(-x) - e^x)/(sqrt(2) + e^(-x) + e^x)) - 1/8*
log(e^(-x) + e^x + 2) + 1/8*log(e^(-x) + e^x - 2)
```

maple [B] time = 0.22, size = 53, normalized size = 2.04

$$-\frac{\ln\left(e^x+1\right)}{4}+\frac{\ln\left(e^x-1\right)}{4}+\frac{\ln\left(1+e^{2x}+e^x\sqrt{2}\right)\sqrt{2}}{8}-\frac{\ln\left(1+e^{2x}-e^x\sqrt{2}\right)\sqrt{2}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)*csch(4*x),x)
```

```
[Out] -1/4*ln(exp(x)+1)+1/4*ln(exp(x)-1)+1/8*ln(1+exp(2*x)+exp(x)*2^(1/2))*2^(1/2
)-1/8*ln(1+exp(2*x)-exp(x)*2^(1/2))*2^(1/2)
```

maxima [B] time = 0.44, size = 60, normalized size = 2.31

$$\frac{1}{8}\sqrt{2}\log\left(\sqrt{2}e^{(-x)}+e^{(-2x)}+1\right)-\frac{1}{8}\sqrt{2}\log\left(-\sqrt{2}e^{(-x)}+e^{(-2x)}+1\right)-\frac{1}{4}\log\left(e^{(-x)}+1\right)+\frac{1}{4}\log\left(e^{(-x)}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*csch(4*x),x, algorithm="maxima")
```

```
[Out] 1/8*sqrt(2)*log(sqrt(2)*e^(-x) + e^(-2*x) + 1) - 1/8*sqrt(2)*log(-sqrt(2)*e
^(-x) + e^(-2*x) + 1) - 1/4*log(e^(-x) + 1) + 1/4*log(e^(-x) - 1)
```

mupad [B] time = 0.06, size = 61, normalized size = 2.35

$$\frac{\ln\left(\frac{1}{2}-\frac{e^x}{2}\right)}{4}-\frac{\ln\left(-\frac{e^x}{2}-\frac{1}{2}\right)}{4}+\frac{\sqrt{2}\,\ln\left(-\frac{e^{2x}}{8}-\frac{\sqrt{2}\,e^x}{8}-\frac{1}{8}\right)}{8}-\frac{\sqrt{2}\,\ln\left(\frac{\sqrt{2}\,e^x}{8}-\frac{e^{2x}}{8}-\frac{1}{8}\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)/sinh(4*x),x)
```

```
[Out] log(1/2 - exp(x)/2)/4 - log(- exp(x)/2 - 1/2)/4 + (2^(1/2)*log(- exp(2*x)/8
 - (2^(1/2)*exp(x))/8 - 1/8))/8 - (2^(1/2)*log((2^(1/2)*exp(x))/8 - exp(2*x
)/8 - 1/8))/8
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\cosh(x)\operatorname{csch}(4x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*csch(4*x),x)
```

```
[Out] Integral(cosh(x)*csch(4*x), x)
```

3.248 $\int \cosh(x)\text{csch}(5x)\,dx$

Optimal. Leaf size=62

$$-\frac{1}{20}\left(1+\sqrt{5}\right)\log\left(8\sinh^2(x)-\sqrt{5}+5\right)-\frac{1}{20}\left(1-\sqrt{5}\right)\log\left(8\sinh^2(x)+\sqrt{5}+5\right)+\frac{1}{5}\log(\sinh(x))$$

```
[Out] 1/5*ln(sinh(x))-1/20*ln(5+8*sinh(x)^2+5^(1/2))*(-5^(1/2)+1)-1/20*ln(5+8*sin
h(x)^2-5^(1/2))*(5^(1/2)+1)
```

Rubi [A] time = 0.08, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.857, Rules used = {4356, 1114, 705, 29, 632, 31}

$$-\frac{1}{20}\left(1+\sqrt{5}\right)\log\left(8\sinh^2(x)-\sqrt{5}+5\right)-\frac{1}{20}\left(1-\sqrt{5}\right)\log\left(8\sinh^2(x)+\sqrt{5}+5\right)+\frac{1}{5}\log(\sinh(x))$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]*Csch[5*x],x]
```

```
[Out] Log[Sinh[x]]/5 - ((1 + Sqrt[5])*Log[5 - Sqrt[5] + 8*Sinh[x]^2])/20 - ((1 -
Sqrt[5])*Log[5 + Sqrt[5] + 8*Sinh[x]^2])/20
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 632

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 705

```
Int[1/(((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)), x_Symbol]
 :> Dist[e^2/(c*d^2 - b*d*e + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d
^2 - b*d*e + a*e^2), Int[(c*d - b*e - c*e*x)/(a + b*x + c*x^2), x], x] /; F
```

```
reeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^
2, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 4356

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Sin[c*(a + b*x)], x]}, Dist[d/(b*c), Subst[Int[SubstFor[1, Sin[c*(a + b*
x)]/d, u, x], x], x, Sin[c*(a + b*x)]/d], x] /; FunctionOfQ[Sin[c*(a + b*x)
]/d, u, x]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Cos] || EqQ[F, cos])
```

Rubi steps

$$\begin{aligned}\int \cosh(x)\operatorname{csch}(5x)\,dx &= \text{Subst}\left(\int \frac{1}{x\left(5+20x^2+16x^4\right)}\,dx, x, \sinh(x)\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x\left(5+20x+16x^2\right)}\,dx, x, \sinh^2(x)\right)\\ &= \frac{1}{10}\text{Subst}\left(\int \frac{1}{x}\,dx, x, \sinh^2(x)\right)+\frac{1}{10}\text{Subst}\left(\int \frac{-20-16x}{5+20x+16x^2}\,dx, x, \sinh^2(x)\right)\\ &= \frac{1}{5}\log(\sinh(x))-\frac{1}{5}\left(4\left(1-\sqrt{5}\right)\right)\text{Subst}\left(\int \frac{1}{10+2\sqrt{5}+16x}\,dx, x, \sinh^2(x)\right)-\frac{1}{5}\left(4\left(1\right.\right.\\ &= \frac{1}{5}\log(\sinh(x))-\frac{1}{20}\left(1+\sqrt{5}\right)\log\left(5-\sqrt{5}+8\sinh^2(x)\right)-\frac{1}{20}\left(1-\sqrt{5}\right)\log\left(5+\sqrt{5}\right.\end{aligned}$$

Mathematica [A] time = 0.06, size = 57, normalized size = 0.92

$$\frac{1}{20}\left(4\log(\sinh(x))-\left(\left(1+\sqrt{5}\right)\log\left(4\cosh(2x)-\sqrt{5}+1\right)\right)+\left(\sqrt{5}-1\right)\log\left(4\cosh(2x)+\sqrt{5}+1\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]*Csch[5*x],x]
```

```
[Out] (-((1 + Sqrt[5])*Log[1 - Sqrt[5] + 4*Cosh[2*x]]) + (-1 + Sqrt[5])*Log[1 + S
qrt[5] + 4*Cosh[2*x]] + 4*Log[Sinh[x]])/20
```

fricas [B] time = 0.43, size = 180, normalized size = 2.90

$$\frac{1}{20}\sqrt{5}\log\left(\frac{4\cosh(x)^4+4\sinh(x)^4+4\left(\sqrt{5}+1\right)\cosh(x)^2+4\left(6\cosh(x)^2+\sqrt{5}+1\right)\sinh(x)^2+\sqrt{5}+7}{2\cosh(x)^4+2\sinh(x)^4+2\left(6\cosh(x)^2+1\right)\sinh(x)^2+2\cosh(x)^2+1}\right)-\frac{1}{20}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*csch(5*x),x, algorithm="fricas")
```

```
[Out] 1/20*sqrt(5)*log((4*cosh(x)^4 + 4*sinh(x)^4 + 4*(sqrt(5) + 1)*cosh(x)^2 + 4
*(6*cosh(x)^2 + sqrt(5) + 1)*sinh(x)^2 + sqrt(5) + 7)/(2*cosh(x)^4 + 2*sinh
(x)^4 + 2*(6*cosh(x)^2 + 1)*sinh(x)^2 + 2*cosh(x)^2 + 1)) - 1/20*log((2*cos
h(x)^4 + 2*sinh(x)^4 + 2*(6*cosh(x)^2 + 1)*sinh(x)^2 + 2*cosh(x)^2 + 1)/(co
sh(x)^4 - 4*cosh(x)^3*sinh(x) + 6*cosh(x)^2*sinh(x)^2 - 4*cosh(x)*sinh(x)^3
 + sinh(x)^4)) + 1/5*log(2*sinh(x)/(cosh(x) - sinh(x)))
```

giac [B] time = 0.14, size = 108, normalized size = 1.74

$$-\frac{1}{20}\left(\sqrt{5}+1\right)\log\left(\frac{1}{2}\left(\sqrt{5}+1\right)e^{x}+e^{(2x)}+1\right)-\frac{1}{20}\left(\sqrt{5}+1\right)\log\left(-\frac{1}{2}\left(\sqrt{5}+1\right)e^{x}+e^{(2x)}+1\right)+\frac{1}{20}\left(\sqrt{5}-1\right)\log\left(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*csch(5*x),x, algorithm="giac")
```

```
[Out] -1/20*(sqrt(5) + 1)*log(1/2*(sqrt(5) + 1)*e^x + e^(2*x) + 1) - 1/20*(sqrt(5
) + 1)*log(-1/2*(sqrt(5) + 1)*e^x + e^(2*x) + 1) + 1/20*(sqrt(5) - 1)*log(1
/2*(sqrt(5) - 1)*e^x + e^(2*x) + 1) + 1/20*(sqrt(5) - 1)*log(-1/2*(sqrt(5)
- 1)*e^x + e^(2*x) + 1) + 1/5*log(e^x + 1) + 1/5*log(abs(e^x - 1))
```

maple [B] time = 0.23, size = 101, normalized size = 1.63

$$\frac{\ln\left(e^{2x}-1\right)}{5}-\frac{\ln\left(e^{4x}+\left(\frac{1}{2}+\frac{\sqrt{5}}{2}\right)e^{2x}+1\right)}{20}+\frac{\ln\left(e^{4x}+\left(\frac{1}{2}+\frac{\sqrt{5}}{2}\right)e^{2x}+1\right)\sqrt{5}}{20}-\frac{\ln\left(e^{4x}+\left(\frac{1}{2}-\frac{\sqrt{5}}{2}\right)e^{2x}+1\right)}{20}-\frac{\ln\left(e^{4x}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)*csch(5*x),x)
```

```
[Out] 1/5*ln(exp(2*x)-1)-1/20*ln(exp(4*x)+(1/2+1/2*5^(1/2))*exp(2*x)+1)+1/20*ln(e
xp(4*x)+(1/2+1/2*5^(1/2))*exp(2*x)+1)*5^(1/2)-1/20*ln(exp(4*x)+(1/2-1/2*5^(
1/2))*exp(2*x)+1)-1/20*ln(exp(4*x)+(1/2-1/2*5^(1/2))*exp(2*x)+1)*5^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{5}\int\frac{\left(e^{(3x)}+e^{(2x)}+e^{x}+1\right)e^{x}}{e^{(4x)}+e^{(3x)}+e^{(2x)}+e^{x}+1}\,dx-\frac{1}{5}\int\frac{\left(e^{(3x)}-e^{(2x)}+e^{x}-1\right)e^{x}}{e^{(4x)}-e^{(3x)}+e^{(2x)}-e^{x}+1}\,dx+\frac{3}{10}\int\frac{e^{(3x)}}{e^{(4x)}+e^{(3x)}+e^{(2x)}+e^{x}+1}\,d$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*csch(5*x),x, algorithm="maxima")
```

```
[Out] -1/5*integrate((e^(3*x) + e^(2*x) + e^x + 1)*e^x/(e^(4*x) + e^(3*x) + e^(2*
x) + e^x + 1), x) - 1/5*integrate((e^(3*x) - e^(2*x) + e^x - 1)*e^x/(e^(4*x
) - e^(3*x) + e^(2*x) - e^x + 1), x) + 3/10*integrate(e^(3*x)/(e^(4*x) + e^
(3*x) + e^(2*x) + e^x + 1), x) - 3/10*integrate(e^(3*x)/(e^(4*x) - e^(3*x)
+ e^(2*x) - e^x + 1), x) + 1/10*integrate(e^(2*x)/(e^(4*x) + e^(3*x) + e^(2
*x) + e^x + 1), x) + 1/10*integrate(e^(2*x)/(e^(4*x) - e^(3*x) + e^(2*x) -
e^x + 1), x) - 1/10*integrate(e^x/(e^(4*x) + e^(3*x) + e^(2*x) + e^x + 1),
x) + 1/10*integrate(e^x/(e^(4*x) - e^(3*x) + e^(2*x) - e^x + 1), x) + 1/5*l
og(e^x + 1) + 1/5*log(e^x - 1)
```

mupad [B] time = 1.51, size = 104, normalized size = 1.68

$$\frac{\ln\left(5-5\,\mathrm{e}^{2\,x}\right)}{5}-\ln\left(2\,\mathrm{e}^{4\,x}-\mathrm{e}^{2\,x}+\left(\frac{\sqrt{5}}{20}+\frac{1}{20}\right)\,\left(30\,\mathrm{e}^{4\,x}-20\,\mathrm{e}^{2\,x}+30\right)+2\right)\,\left(\frac{\sqrt{5}}{20}+\frac{1}{20}\right)+\ln\left(2\,\mathrm{e}^{4\,x}-\mathrm{e}^{2\,x}-\left(\frac{\sqrt{5}}{20}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)/sinh(5*x),x)
```

```
[Out] log(5 - 5*exp(2*x))/5 - log(2*exp(4*x) - exp(2*x) + (5^(1/2)/20 + 1/20)*(30
*exp(4*x) - 20*exp(2*x) + 30) + 2)*(5^(1/2)/20 + 1/20) + log(2*exp(4*x) - e
xp(2*x) - (5^(1/2)/20 - 1/20)*(30*exp(4*x) - 20*exp(2*x) + 30) + 2)*(5^(1/2
)/20 - 1/20)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \cosh\left(x\right)\operatorname{csch}\left(5x\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*csch(5*x),x)
```

```
[Out] Integral(cosh(x)*csch(5*x), x)
```

3.249 $\int \cosh(x)\mathbf{csch}(6x)\,dx$

Optimal. Leaf size=36

$$-\frac{1}{6}\tanh^{-1}(\cosh(x))-\frac{1}{6}\tanh^{-1}(2\cosh(x))+\frac{\tanh^{-1}\left(\frac{2\cosh(x)}{\sqrt{3}}\right)}{2\sqrt{3}}$$

```
[Out] -1/6*arctanh(cosh(x))-1/6*arctanh(2*cosh(x))+1/6*arctanh(2/3*cosh(x)*3^(1/2
))*3^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 3, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {12, 2057, 207}

$$-\frac{1}{6}\tanh^{-1}(\cosh(x))-\frac{1}{6}\tanh^{-1}(2\cosh(x))+\frac{\tanh^{-1}\left(\frac{2\cosh(x)}{\sqrt{3}}\right)}{2\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]*Csch[6*x],x]
```

```
[Out] -ArcTanh[Cosh[x]]/6 - ArcTanh[2*Cosh[x]]/6 + ArcTanh[(2*Cosh[x])/Sqrt[3]]/(
2*Sqrt[3])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 2057

```
Int[(P_)^(p_), x_Symbol] :> With[{u = Factor[P /. x -> Sqrt[x]]}, Int[Expan
dIntegrand[(u /. x -> x^2)^p, x], x] /;  !SumQ[NonfreeFactors[u, x]]] /; Po
lyQ[P, x^2] && ILtQ[p, 0]
```

Rubi steps

$$\int \cosh(x)\operatorname{csch}(6x)\,dx = -\operatorname{Subst}\left(\int \frac{1}{2\left(3-19x^2+32x^4-16x^6\right)}\,dx, x, \cosh(x)\right)$$
$$= -\left(\frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{3-19x^2+32x^4-16x^6}\,dx, x, \cosh(x)\right)\right)$$
$$= -\left(\frac{1}{2}\operatorname{Subst}\left(\int \left(-\frac{1}{3\left(-1+x^2\right)}+\frac{2}{-3+4x^2}-\frac{2}{3\left(-1+4x^2\right)}\right)dx, x, \cosh(x)\right)\right)$$
$$= \frac{1}{6}\operatorname{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \cosh(x)\right)+\frac{1}{3}\operatorname{Subst}\left(\int \frac{1}{-1+4x^2}\,dx, x, \cosh(x)\right)-\operatorname{Subst}\left(\right.$$
$$= -\frac{1}{6}\tanh^{-1}(\cosh(x))-\frac{1}{6}\tanh^{-1}(2\cosh(x))+\frac{\tanh^{-1}\left(\frac{2\cosh(x)}{\sqrt{3}}\right)}{2\sqrt{3}}$$

Mathematica [C] time = 0.07, size = 91, normalized size = 2.53

$$\frac{1}{12}\left(2\sqrt{3}\tanh^{-1}\left(\frac{2-i\tanh\left(\frac{x}{2}\right)}{\sqrt{3}}\right)+2\sqrt{3}\tanh^{-1}\left(\frac{2+i\tanh\left(\frac{x}{2}\right)}{\sqrt{3}}\right)+2\log\left(\sinh\left(\frac{x}{2}\right)\right)-2\log\left(\cosh\left(\frac{x}{2}\right)\right)+\log(\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]*Csch[6*x],x]
```

```
[Out] (2*Sqrt[3]*ArcTanh[(2 - I*Tanh[x/2])/Sqrt[3]] + 2*Sqrt[3]*ArcTanh[(2 + I*Ta
nh[x/2])/Sqrt[3]] - 2*Log[Cosh[x/2]] + Log[1 - 2*Cosh[x]] - Log[1 + 2*Cosh[
x]] + 2*Log[Sinh[x/2]])/12
```

fricas [B] time = 0.46, size = 101, normalized size = 2.81

$$\frac{1}{12}\sqrt{3}\log\left(\frac{2\cosh(x)^2+2\sinh(x)^2+4\sqrt{3}\cosh(x)+5}{2\cosh(x)^2+2\sinh(x)^2-1}\right)-\frac{1}{12}\log\left(\frac{2\cosh(x)+1}{\cosh(x)-\sinh(x)}\right)+\frac{1}{12}\log\left(\frac{2\cosh(x)}{\cosh(x)-\mathrm{si}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*csch(6*x),x, algorithm="fricas")
```

```
[Out] 1/12*sqrt(3)*log((2*cosh(x)^2 + 2*sinh(x)^2 + 4*sqrt(3)*cosh(x) + 5)/(2*cos
h(x)^2 + 2*sinh(x)^2 - 1)) - 1/12*log((2*cosh(x) + 1)/(cosh(x) - sinh(x)))
+ 1/12*log((2*cosh(x) - 1)/(cosh(x) - sinh(x))) - 1/6*log(cosh(x) + sinh(x)
 + 1) + 1/6*log(cosh(x) + sinh(x) - 1)
```

giac [B] time = 0.12, size = 79, normalized size = 2.19

$$-\frac{1}{12}\sqrt{3}\log\left(-\frac{\sqrt{3}-e^{(-x)}-e^{x}}{\sqrt{3}+e^{(-x)}+e^{x}}\right)-\frac{1}{12}\log\left(e^{(-x)}+e^{x}+2\right)-\frac{1}{12}\log\left(e^{(-x)}+e^{x}+1\right)+\frac{1}{12}\log\left(e^{(-x)}+e^{x}-1\right)+\frac{1}{12}\log$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*csch(6*x),x, algorithm="giac")
```

```
[Out] -1/12*sqrt(3)*log(-(sqrt(3) - e^(-x) - e^x)/(sqrt(3) + e^(-x) + e^x)) - 1/1
2*log(e^(-x) + e^x + 2) - 1/12*log(e^(-x) + e^x + 1) + 1/12*log(e^(-x) + e^
x - 1) + 1/12*log(e^(-x) + e^x - 2)
```

maple [B] time = 0.23, size = 77, normalized size = 2.14

$$\frac{\ln\left(e^{x}-1\right)}{6}-\frac{\ln\left(e^{x}+1\right)}{6}+\frac{\ln\left(1+e^{2x}+e^{x}\sqrt{3}\right)\sqrt{3}}{12}-\frac{\ln\left(1+e^{2x}-e^{x}\sqrt{3}\right)\sqrt{3}}{12}-\frac{\ln\left(1+e^{x}+e^{2x}\right)}{12}+\frac{\ln\left(1-e^{x}+e^{2x}\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)*csch(6*x),x)
```

```
[Out] 1/6*ln(exp(x)-1)-1/6*ln(exp(x)+1)+1/12*ln(1+exp(2*x)+exp(x)*3^(1/2))*3^(1/2
)-1/12*ln(1+exp(2*x)-exp(x)*3^(1/2))*3^(1/2)-1/12*ln(1+exp(x)+exp(2*x))+1/1
2*ln(1-exp(x)+exp(2*x))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int\frac{e^{(3x)}-e^{x}}{2\left(e^{(4x)}-e^{(2x)}+1\right)}\,dx-\frac{1}{12}\log\left(e^{(2x)}+e^{x}+1\right)+\frac{1}{12}\log\left(e^{(2x)}-e^{x}+1\right)-\frac{1}{6}\log\left(e^{x}+1\right)+\frac{1}{6}\log\left(e^{x}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*csch(6*x),x, algorithm="maxima")
```

```
[Out] -integrate(1/2*(e^(3*x) - e^x)/(e^(4*x) - e^(2*x) + 1), x) - 1/12*log(e^(2*
x) + e^x + 1) + 1/12*log(e^(2*x) - e^x + 1) - 1/6*log(e^x + 1) + 1/6*log(e^
x - 1)
```

mupad [B] time = 0.08, size = 91, normalized size = 2.53

$$\frac{\ln\left(\frac{1}{3}-\frac{e^{x}}{3}\right)}{6}-\frac{\ln\left(-\frac{e^{x}}{3}-\frac{1}{3}\right)}{6}-\frac{\ln\left(-\frac{e^{2x}}{36}-\frac{e^{x}}{36}-\frac{1}{36}\right)}{12}+\frac{\ln\left(\frac{e^{x}}{36}-\frac{e^{2x}}{36}-\frac{1}{36}\right)}{12}+\frac{\sqrt{3}\,\ln\left(-\frac{e^{2x}}{12}-\frac{\sqrt{3}\,e^{x}}{12}-\frac{1}{12}\right)}{12}-\frac{\sqrt{3}\,\ln\left(\frac{\sqrt{3}\,e^{x}}{12}-\right.}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)/sinh(6*x),x)
```

```
[Out] log(1/3 - exp(x)/3)/6 - log(- exp(x)/3 - 1/3)/6 - log(- exp(2*x)/36 - exp(x
)/36 - 1/36)/12 + log(exp(x)/36 - exp(2*x)/36 - 1/36)/12 + (3^(1/2)*log(- e
xp(2*x)/12 - (3^(1/2)*exp(x))/12 - 1/12))/12 - (3^(1/2)*log((3^(1/2)*exp(x)
)/12 - exp(2*x)/12 - 1/12))/12
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \cosh(x) \operatorname{csch}(6x)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*csch(6*x),x)
```

```
[Out] Integral(cosh(x)*csch(6*x), x)
```

3.250 $\int x^m \cosh(a+bx)\sinh(a+bx)\,dx$

Optimal. Leaf size=70

$$\frac{e^{2a}2^{-m-3}x^m(-bx)^{-m}\Gamma(m+1,-2bx)}{b}+\frac{e^{-2a}2^{-m-3}x^m(bx)^{-m}\Gamma(m+1,2bx)}{b}$$

```
[Out] 2^(-3-m)*exp(2*a)*x^m*GAMMA(1+m,-2*b*x)/b/((-b*x)^m)+2^(-3-m)*x^m*GAMMA(1+m
,2*b*x)/b/exp(2*a)/((b*x)^m)
```

Rubi [A] time = 0.12, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {5448, 12, 3308, 2181}

$$\frac{e^{2a}2^{-m-3}x^m(-bx)^{-m}\text{Gamma}(m+1,-2bx)}{b}+\frac{e^{-2a}2^{-m-3}x^m(bx)^{-m}\text{Gamma}(m+1,2bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*Cosh[a + b*x]*Sinh[a + b*x],x]
```

```
[Out] (2^(-3 - m)*E^(2*a)*x^m*Gamma[1 + m, -2*b*x])/(b*(-(b*x))^m) + (2^(-3 - m)*
x^m*Gamma[1 + m, 2*b*x])/(b*E^(2*a)*(b*x)^m)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] && !I
ntegerQ[m]
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
```

```
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rubi steps

$$
\begin{aligned}
\int x^m \cosh(a+bx)\sinh(a+bx)\,dx &= \int \frac{1}{2}x^m \sinh(2a+2bx)\,dx \\
&= \frac{1}{2}\int x^m \sinh(2a+2bx)\,dx \\
&= \frac{1}{4}\int e^{-i(2ia+2ibx)}x^m\,dx - \frac{1}{4}\int e^{i(2ia+2ibx)}x^m\,dx \\
&= \frac{2^{-3-m}e^{2a}x^m(-bx)^{-m}\Gamma(1+m,-2bx)}{b} + \frac{2^{-3-m}e^{-2a}x^m(bx)^{-m}\Gamma(1+m,2bx)}{b}
\end{aligned}
$$

Mathematica [A] time = 0.05, size = 66, normalized size = 0.94

$$\frac{e^{-2a}2^{-m-3}x^m\left(-b^2x^2\right)^{-m}\left(e^{4a}(bx)^m\Gamma(m+1,-2bx)+(-bx)^m\Gamma(m+1,2bx)\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*Cosh[a + b*x]*Sinh[a + b*x],x]
```

```
[Out] (2^(-3 - m)*x^m*(E^(4*a)*(b*x)^m*Gamma[1 + m, -2*b*x] + (-(b*x))^m*Gamma[1
+ m, 2*b*x]))/(b*E^(2*a)*(-(b^2*x^2))^m)
```

fricas [A] time = 0.62, size = 88, normalized size = 1.26

$$\frac{\cosh\left(m\log\left(2\,b\right)+2\,a\right)\Gamma\left(m+1,2\,bx\right)+\cosh\left(m\log\left(-2\,b\right)-2\,a\right)\Gamma\left(m+1,-2\,bx\right)-\Gamma\left(m+1,2\,bx\right)\sinh\left(m\log\right.}{8\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*cosh(b*x+a)*sinh(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/8*(cosh(m*log(2*b) + 2*a)*gamma(m + 1, 2*b*x) + cosh(m*log(-2*b) - 2*a)*g
amma(m + 1, -2*b*x) - gamma(m + 1, 2*b*x)*sinh(m*log(2*b) + 2*a) - gamma(m
+ 1, -2*b*x)*sinh(m*log(-2*b) - 2*a))/b
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^m \cosh\left(bx+a\right)\sinh\left(bx+a\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*cosh(b*x+a)*sinh(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(x^m*cosh(b*x + a)*sinh(b*x + a), x)
```

maple [F] time = 0.13, size = 0, normalized size = 0.00

$$\int x^m \cosh(bx+a)\sinh(bx+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*cosh(b*x+a)*sinh(b*x+a),x)
```

```
[Out] int(x^m*cosh(b*x+a)*sinh(b*x+a),x)
```

maxima [A] time = 0.42, size = 59, normalized size = 0.84

$$\frac{1}{4}\left(2\,bx\right)^{-m-1}x^{m+1}e^{(-2\,a)}\Gamma\left(m+1,2\,bx\right)-\frac{1}{4}\left(-2\,bx\right)^{-m-1}x^{m+1}e^{(2\,a)}\Gamma\left(m+1,-2\,bx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*cosh(b*x+a)*sinh(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/4*(2*b*x)^(-m - 1)*x^(m + 1)*e^(-2*a)*gamma(m + 1, 2*b*x) - 1/4*(-2*b*x)^
(-m - 1)*x^(m + 1)*e^(2*a)*gamma(m + 1, -2*b*x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^m\,\mathrm{cosh}\left(a+b\,x\right)\,\mathrm{sinh}\left(a+b\,x\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*cosh(a + b*x)*sinh(a + b*x),x)
```

```
[Out] int(x^m*cosh(a + b*x)*sinh(a + b*x), x)
```

sympy [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*cosh(b*x+a)*sinh(b*x+a),x)
```

```
[Out] Exception raised: TypeError
```

3.251 $\int x^3 \cosh(a+bx)\sinh(a+bx)\,dx$

Optimal. Leaf size=94

$$-\frac{3\sinh(a+bx)\cosh(a+bx)}{8b^4}+\frac{3x\sinh^2(a+bx)}{4b^3}-\frac{3x^2\sinh(a+bx)\cosh(a+bx)}{4b^2}+\frac{x^3\sinh^2(a+bx)}{2b}+\frac{3x}{8b^3}+\frac{x^3}{4b}$$

```
[Out] 3/8*x/b^3+1/4*x^3/b-3/8*cosh(b*x+a)*sinh(b*x+a)/b^4-3/4*x^2*cosh(b*x+a)*sin
h(b*x+a)/b^2+3/4*x*sinh(b*x+a)^2/b^3+1/2*x^3*sinh(b*x+a)^2/b
```

Rubi [A] time = 0.07, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {5372, 3311, 30, 2635, 8}

$$-\frac{3x^2\sinh(a+bx)\cosh(a+bx)}{4b^2}+\frac{3x\sinh^2(a+bx)}{4b^3}-\frac{3\sinh(a+bx)\cosh(a+bx)}{8b^4}+\frac{x^3\sinh^2(a+bx)}{2b}+\frac{3x}{8b^3}+\frac{x^3}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Cosh[a + b*x]*Sinh[a + b*x],x]
```

```
[Out] (3*x)/(8*b^3) + x^3/(4*b) - (3*Cosh[a + b*x]*Sinh[a + b*x])/(8*b^4) - (3*x^
2*Cosh[a + b*x]*Sinh[a + b*x])/(4*b^2) + (3*x*Sinh[a + b*x]^2)/(4*b^3) + (x
^3*Sinh[a + b*x]^2)/(2*b)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 3311

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(d*m*(c + d*x)^(m - 1)*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist
[(b^2*(n - 1))/n, Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[(
```

```
d^2*m*(m - 1))/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[(b*(c + d*x)^m*Cos[e + f*x]*(b*Sin[e + f*x])^(n - 1))/(f*n), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 5372

```
Int[Cosh[(a_.) + (b_.)*(x_)^(n_.)]*(x_)^(m_.)*Sinh[(a_.) + (b_.)*(x_)^(n_.)
]^(p_.), x_Symbol] :> Simp[(x^(m - n + 1)*Sinh[a + b*x^n]^(p + 1))/(b*n*(p
+ 1)), x] - Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*Sinh[a + b*x^n]^(
p + 1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int x^3\cosh(a+bx)\sinh(a+bx)\,dx &= \frac{x^3\sinh^2(a+bx)}{2b}-\frac{3\int x^2\sinh^2(a+bx)\,dx}{2b}\\ &= -\frac{3x^2\cosh(a+bx)\sinh(a+bx)}{4b^2}+\frac{3x\sinh^2(a+bx)}{4b^3}+\frac{x^3\sinh^2(a+bx)}{2b}-\\ &= \frac{x^3}{4b}-\frac{3\cosh(a+bx)\sinh(a+bx)}{8b^4}-\frac{3x^2\cosh(a+bx)\sinh(a+bx)}{4b^2}+\frac{3x\sin}{}\\ &= \frac{3x}{8b^3}+\frac{x^3}{4b}-\frac{3\cosh(a+bx)\sinh(a+bx)}{8b^4}-\frac{3x^2\cosh(a+bx)\sinh(a+bx)}{4b^2}+\end{aligned}$$

Mathematica [A] time = 0.11, size = 50, normalized size = 0.53

$$\frac{(4b^3x^3+6bx)\cosh(2(a+bx))-3(2b^2x^2+1)\sinh(2(a+bx))}{16b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Cosh[a + b*x]*Sinh[a + b*x],x]
```

```
[Out] ((6*b*x + 4*b^3*x^3)*Cosh[2*(a + b*x)] - 3*(1 + 2*b^2*x^2)*Sinh[2*(a + b*x)
])/(16*b^4)
```

fricas [A] time = 0.39, size = 74, normalized size = 0.79

$$\frac{(2b^3x^3+3bx)\cosh(bx+a)^2-3(2b^2x^2+1)\cosh(bx+a)\sinh(bx+a)+(2b^3x^3+3bx)\sinh(bx+a)^2}{8b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*cosh(b*x+a)*sinh(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/8*((2*b^3*x^3 + 3*b*x)*cosh(b*x + a)^2 - 3*(2*b^2*x^2 + 1)*cosh(b*x + a)*
sinh(b*x + a) + (2*b^3*x^3 + 3*b*x)*sinh(b*x + a)^2)/b^4
```

giac [A] time = 0.13, size = 73, normalized size = 0.78

$$\frac{\left(4\,b^3x^3-6\,b^2x^2+6\,bx-3\right)e^{(2\,bx+2\,a)}}{32\,b^4}+\frac{\left(4\,b^3x^3+6\,b^2x^2+6\,bx+3\right)e^{(-2\,bx-2\,a)}}{32\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*cosh(b*x+a)*sinh(b*x+a),x, algorithm="giac")
```

```
[Out] 1/32*(4*b^3*x^3 - 6*b^2*x^2 + 6*b*x - 3)*e^(2*b*x + 2*a)/b^4 + 1/32*(4*b^3*
x^3 + 6*b^2*x^2 + 6*b*x + 3)*e^(-2*b*x - 2*a)/b^4
```

maple [B] time = 0.04, size = 203, normalized size = 2.16

$$\frac{\frac{(bx+a)^3\left(\cosh^2(bx+a)\right)}{2}-\frac{3(bx+a)^2\cosh(bx+a)\sinh(bx+a)}{4}-\frac{(bx+a)^3}{4}+\frac{3(bx+a)\left(\cosh^2(bx+a)\right)}{4}-\frac{3\cosh(bx+a)\sinh(bx+a)}{8}-\frac{3bx}{8}-\frac{3a}{8}-3a}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*cosh(b*x+a)*sinh(b*x+a),x)
```

```
[Out] 1/b^4*(1/2*(b*x+a)^3*cosh(b*x+a)^2-3/4*(b*x+a)^2*cosh(b*x+a)*sinh(b*x+a)-1/
4*(b*x+a)^3+3/4*(b*x+a)*cosh(b*x+a)^2-3/8*cosh(b*x+a)*sinh(b*x+a)-3/8*b*x-3
/8*a-3*a*(1/2*(b*x+a)^2*cosh(b*x+a)^2-1/2*(b*x+a)*cosh(b*x+a)*sinh(b*x+a)-1
/4*(b*x+a)^2+1/4*cosh(b*x+a)^2)+3*a^2*(1/2*(b*x+a)*cosh(b*x+a)^2-1/4*cosh(b
*x+a)*sinh(b*x+a)-1/4*b*x-1/4*a)-1/2*cosh(b*x+a)^2*a^3)
```

maxima [A] time = 0.35, size = 86, normalized size = 0.91

$$\frac{\left(4\,b^3x^3e^{(2\,a)}-6\,b^2x^2e^{(2\,a)}+6\,bxe^{(2\,a)}-3\,e^{(2\,a)}\right)e^{(2\,bx)}}{32\,b^4}+\frac{\left(4\,b^3x^3+6\,b^2x^2+6\,bx+3\right)e^{(-2\,bx-2\,a)}}{32\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*cosh(b*x+a)*sinh(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/32*(4*b^3*x^3*e^(2*a) - 6*b^2*x^2*e^(2*a) + 6*b*x*e^(2*a) - 3*e^(2*a))*e^
(2*b*x)/b^4 + 1/32*(4*b^3*x^3 + 6*b^2*x^2 + 6*b*x + 3)*e^(-2*b*x - 2*a)/b^4
```

mupad [B] time = 1.50, size = 64, normalized size = 0.68

$$-\frac{\frac{3\sinh(2\,a+2\,b\,x)}{2}-2\,b^3\,x^3\cosh\left(2\,a+2\,b\,x\right)+3\,b^2\,x^2\sinh\left(2\,a+2\,b\,x\right)-3\,b\,x\cosh\left(2\,a+2\,b\,x\right)}{8\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*cosh(a + b*x)*sinh(a + b*x),x)
```

```
[Out] -((3*sinh(2*a + 2*b*x))/2 - 2*b^3*x^3*cosh(2*a + 2*b*x) + 3*b^2*x^2*sinh(2*
a + 2*b*x) - 3*b*x*cosh(2*a + 2*b*x))/(8*b^4)
```

sympy [A] time = 1.73, size = 119, normalized size = 1.27

$$\begin{cases} \frac{x^3 \sinh^2(a+bx)}{4b} + \frac{x^3 \cosh^2(a+bx)}{4b} - \frac{3x^2 \sinh(a+bx)\cosh(a+bx)}{4b^2} + \frac{3x \sinh^2(a+bx)}{8b^3} + \frac{3x \cosh^2(a+bx)}{8b^3} - \frac{3 \sinh(a+bx)\cosh(a+bx)}{8b^4} & \text{for } b \\ \frac{x^4 \sinh(a)\cosh(a)}{4} & \text{othe} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*cosh(b*x+a)*sinh(b*x+a),x)
```

```
[Out] Piecewise((x**3*sinh(a + b*x)**2/(4*b) + x**3*cosh(a + b*x)**2/(4*b) - 3*x*
*2*sinh(a + b*x)*cosh(a + b*x)/(4*b**2) + 3*x*sinh(a + b*x)**2/(8*b**3) + 3
*x*cosh(a + b*x)**2/(8*b**3) - 3*sinh(a + b*x)*cosh(a + b*x)/(8*b**4), Ne(b
, 0)), (x**4*sinh(a)*cosh(a)/4, True))
```

3.252 $\int x^2 \cosh(a + bx) \sinh(a + bx) \, dx$

Optimal. Leaf size=64

$$\frac{\sinh^2(a+bx)}{4b^3} - \frac{x\sinh(a+bx)\cosh(a+bx)}{2b^2} + \frac{x^2\sinh^2(a+bx)}{2b} + \frac{x^2}{4b}$$

```
[Out] 1/4*x^2/b-1/2*x*cosh(b*x+a)*sinh(b*x+a)/b^2+1/4*sinh(b*x+a)^2/b^3+1/2*x^2*s
inh(b*x+a)^2/b
```

Rubi [A] time = 0.04, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} =$ 0.188, Rules used = {5372, 3310, 30}

$$\frac{\sinh^2(a+bx)}{4b^3} - \frac{x\sinh(a+bx)\cosh(a+bx)}{2b^2} + \frac{x^2\sinh^2(a+bx)}{2b} + \frac{x^2}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Cosh[a + b*x]*Sinh[a + b*x],x]
```

```
[Out] x^2/(4*b) - (x*Cosh[a + b*x]*Sinh[a + b*x])/(2*b^2) + Sinh[a + b*x]^2/(4*b^
3) + (x^2*Sinh[a + b*x]^2)/(2*b)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 3310

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[(d*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[(b*(c + d*x)*Cos[e + f*x]*(b
*Sin[e + f*x])^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 5372

```
Int[Cosh[(a_.) + (b_.)*(x_)^(n_.)]*(x_)^(m_.)*Sinh[(a_.) + (b_.)*(x_)^(n_.)
]^(p_.), x_Symbol] :> Simp[(x^(m - n + 1)*Sinh[a + b*x^n]^(p + 1))/(b*n*(p
+ 1)), x] - Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*Sinh[a + b*x^n]^(
p + 1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

Rubi steps

$$\int x^2\cosh(a+bx)\sinh(a+bx)\,dx = \frac{x^2\sinh^2(a+bx)}{2b} - \frac{\int x\sinh^2(a+bx)\,dx}{b}$$
$$= -\frac{x\cosh(a+bx)\sinh(a+bx)}{2b^2} + \frac{\sinh^2(a+bx)}{4b^3} + \frac{x^2\sinh^2(a+bx)}{2b} + \frac{\int x\,dx}{2b}$$
$$= \frac{x^2}{4b} - \frac{x\cosh(a+bx)\sinh(a+bx)}{2b^2} + \frac{\sinh^2(a+bx)}{4b^3} + \frac{x^2\sinh^2(a+bx)}{2b}$$

Mathematica [A] time = 0.07, size = 39, normalized size = 0.61

$$\frac{(2b^2x^2+1)\cosh(2(a+bx)) - 2bx\sinh(2(a+bx))}{8b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Cosh[a + b*x]*Sinh[a + b*x],x]
[Out] ((1 + 2*b^2*x^2)*Cosh[2*(a + b*x)] - 2*b*x*Sinh[2*(a + b*x)])/(8*b^3)
```

fricas [A] time = 0.44, size = 62, normalized size = 0.97

$$-\frac{4\,bx\cosh\left(bx+a\right)\sinh\left(bx+a\right) - \left(2\,b^2x^2+1\right)\cosh\left(bx+a\right)^2 - \left(2\,b^2x^2+1\right)\sinh\left(bx+a\right)^2}{8\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cosh(b*x+a)*sinh(b*x+a),x, algorithm="fricas")
[Out] -1/8*(4*b*x*cosh(b*x + a)*sinh(b*x + a) - (2*b^2*x^2 + 1)*cosh(b*x + a)^2 -
 (2*b^2*x^2 + 1)*sinh(b*x + a)^2)/b^3
```

giac [A] time = 0.11, size = 57, normalized size = 0.89

$$\frac{\left(2\,b^2x^2 - 2\,bx + 1\right)e^{(2\,bx+2\,a)}}{16\,b^3} + \frac{\left(2\,b^2x^2 + 2\,bx + 1\right)e^{(-2\,bx-2\,a)}}{16\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cosh(b*x+a)*sinh(b*x+a),x, algorithm="giac")
[Out] 1/16*(2*b^2*x^2 - 2*b*x + 1)*e^(2*b*x + 2*a)/b^3 + 1/16*(2*b^2*x^2 + 2*b*x
+ 1)*e^(-2*b*x - 2*a)/b^3
```

maple [B] time = 0.05, size = 114, normalized size = 1.78

$$\frac{\frac{(bx+a)^2\left(\cosh^2(bx+a)\right)}{2} - \frac{(bx+a)\cosh(bx+a)\sinh(bx+a)}{2} - \frac{(bx+a)^2}{4} + \frac{\left(\cosh^2(bx+a)\right)}{4} - 2a\left(\frac{(bx+a)\left(\cosh^2(bx+a)\right)}{2} - \frac{\cosh(bx+a)\sinh(bx+a)}{4} \right.}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2*cosh(b*x+a)*sinh(b*x+a),x)

[Out] 1/b^3*(1/2*(b*x+a)^2*cosh(b*x+a)^2-1/2*(b*x+a)*cosh(b*x+a)*sinh(b*x+a)-1/4*(b*x+a)^2+1/4*cosh(b*x+a)^2-2*a*(1/2*(b*x+a)*cosh(b*x+a)^2-1/4*cosh(b*x+a)*sinh(b*x+a)-1/4*b*x-1/4*a)+1/2*cosh(b*x+a)^2*a^2)

maxima [A] time = 0.32, size = 64, normalized size = 1.00

$$\frac{\left(2\,b^2x^2e^{(2\,a)} - 2\,bxe^{(2\,a)} + e^{(2\,a)}\right)e^{(2\,bx)}}{16\,b^3} + \frac{\left(2\,b^2x^2 + 2\,bx + 1\right)e^{(-2\,bx-2\,a)}}{16\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*cosh(b*x+a)*sinh(b*x+a),x, algorithm="maxima")

[Out] 1/16*(2*b^2*x^2*e^(2*a) - 2*b*x*e^(2*a) + e^(2*a))*e^(2*b*x)/b^3 + 1/16*(2*b^2*x^2 + 2*b*x + 1)*e^(-2*b*x - 2*a)/b^3

mupad [B] time = 0.08, size = 46, normalized size = 0.72

$$\frac{\frac{\cosh(2\,a+2\,b\,x)}{2} - b\,x\sinh\left(2\,a+2\,b\,x\right) + b^2\,x^2\cosh\left(2\,a+2\,b\,x\right)}{4\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2*cosh(a + b*x)*sinh(a + b*x),x)

[Out] (cosh(2*a + 2*b*x)/2 - b*x*sinh(2*a + 2*b*x) + b^2*x^2*cosh(2*a + 2*b*x))/(4*b^3)

sympy [A] time = 0.82, size = 75, normalized size = 1.17

$$\begin{cases} \frac{x^2\sinh^2\left(a+bx\right)}{4b} + \frac{x^2\cosh^2\left(a+bx\right)}{4b} - \frac{x\sinh\left(a+bx\right)\cosh\left(a+bx\right)}{2b^2} + \frac{\sinh^2\left(a+bx\right)}{4b^3} & \text{for } b \neq 0 \\ \frac{x^3\sinh\left(a\right)\cosh\left(a\right)}{3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*cosh(b*x+a)*sinh(b*x+a),x)
```

```
[Out] Piecewise((x**2*sinh(a + b*x)**2/(4*b) + x**2*cosh(a + b*x)**2/(4*b) - x*si
nh(a + b*x)*cosh(a + b*x)/(2*b**2) + sinh(a + b*x)**2/(4*b**3), Ne(b, 0)),
(x**3*sinh(a)*cosh(a)/3, True))
```

3.253 $\int x \cosh(a+bx)\sinh(a+bx)\,dx$

Optimal. Leaf size=44

$$-\frac{\sinh(a+bx)\cosh(a+bx)}{4b^2}+\frac{x\sinh^2(a+bx)}{2b}+\frac{x}{4b}$$

```
[Out] 1/4*x/b-1/4*cosh(b*x+a)*sinh(b*x+a)/b^2+1/2*x*sinh(b*x+a)^2/b
```

Rubi [A] time = 0.02, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {5372, 2635, 8}

$$-\frac{\sinh(a+bx)\cosh(a+bx)}{4b^2}+\frac{x\sinh^2(a+bx)}{2b}+\frac{x}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[x*Cosh[a + b*x]*Sinh[a + b*x],x]
```

```
[Out] x/(4*b) - (Cosh[a + b*x]*Sinh[a + b*x])/(4*b^2) + (x*Sinh[a + b*x]^2)/(2*b)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 5372

```
Int[Cosh[(a_.) + (b_.)*(x_)^(n_.)]*(x_)^(m_.)*Sinh[(a_.) + (b_.)*(x_)^(n_.)
]^(p_.), x_Symbol] :> Simp[(x^(m - n + 1)*Sinh[a + b*x^n]^(p + 1))/(b*n*(p
+ 1)), x] - Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*Sinh[a + b*x^n]^(
p + 1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

Rubi steps

$$\int x\cosh(a+bx)\sinh(a+bx)\,dx = \frac{x\sinh^2(a+bx)}{2b} - \frac{\int \sinh^2(a+bx)\,dx}{2b}$$
$$= -\frac{\cosh(a+bx)\sinh(a+bx)}{4b^2} + \frac{x\sinh^2(a+bx)}{2b} + \frac{\int 1\,dx}{4b}$$
$$= \frac{x}{4b} - \frac{\cosh(a+bx)\sinh(a+bx)}{4b^2} + \frac{x\sinh^2(a+bx)}{2b}$$

Mathematica [A] time = 0.06, size = 28, normalized size = 0.64

$$-\frac{\sinh(2(a+bx)) - 2bx\cosh(2(a+bx))}{8b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Cosh[a + b*x]*Sinh[a + b*x],x]
```

```
[Out] -1/8*(-2*b*x*Cosh[2*(a + b*x)] + Sinh[2*(a + b*x)])/b^2
```

fricas [A] time = 0.56, size = 42, normalized size = 0.95

$$\frac{bx\cosh\left(bx+a\right)^2 + bx\sinh\left(bx+a\right)^2 - \cosh\left(bx+a\right)\sinh\left(bx+a\right)}{4\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x+a)*sinh(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/4*(b*x*cosh(b*x + a)^2 + b*x*sinh(b*x + a)^2 - cosh(b*x + a)*sinh(b*x + a
))/b^2
```

giac [A] time = 0.12, size = 41, normalized size = 0.93

$$\frac{(2\,bx-1)e^{(2\,bx+2\,a)}}{16\,b^2} + \frac{(2\,bx+1)e^{(-2\,bx-2\,a)}}{16\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x+a)*sinh(b*x+a),x, algorithm="giac")
```

```
[Out] 1/16*(2*b*x - 1)*e^(2*b*x + 2*a)/b^2 + 1/16*(2*b*x + 1)*e^(-2*b*x - 2*a)/b^
2
```

maple [A] time = 0.04, size = 53, normalized size = 1.20

$$\frac{\frac{(bx+a)\left(\cosh^2(bx+a)\right)}{2}-\frac{\cosh(bx+a)\sinh(bx+a)}{4}-\frac{bx}{4}-\frac{a}{4}-\frac{a\left(\cosh^2(bx+a)\right)}{2}}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cosh(b*x+a)*sinh(b*x+a),x)
```

```
[Out] 1/b^2*(1/2*(b*x+a)*cosh(b*x+a)^2-1/4*cosh(b*x+a)*sinh(b*x+a)-1/4*b*x-1/4*a-
1/2*a*cosh(b*x+a)^2)
```

maxima [A] time = 0.35, size = 46, normalized size = 1.05

$$\frac{\left(2\,bxe^{(2\,a)}-e^{(2\,a)}\right)e^{(2\,bx)}}{16\,b^2}+\frac{(2\,bx+1)e^{(-2\,bx-2\,a)}}{16\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x+a)*sinh(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/16*(2*b*x*e^(2*a) - e^(2*a))*e^(2*b*x)/b^2 + 1/16*(2*b*x + 1)*e^(-2*b*x -
 2*a)/b^2
```

mupad [B] time = 1.43, size = 28, normalized size = 0.64

$$-\frac{\sinh\left(2\,a+2\,b\,x\right)-2\,b\,x\cosh\left(2\,a+2\,b\,x\right)}{8\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cosh(a + b*x)*sinh(a + b*x),x)
```

```
[Out] -(sinh(2*a + 2*b*x) - 2*b*x*cosh(2*a + 2*b*x))/(8*b^2)
```

sympy [A] time = 0.42, size = 56, normalized size = 1.27

$$\begin{cases} \frac{x\sinh^2\left(a+bx\right)}{4b}+\frac{x\cosh^2\left(a+bx\right)}{4b}-\frac{\sinh\left(a+bx\right)\cosh\left(a+bx\right)}{4b^2} & \text{for}\ b\neq 0 \\ \frac{x^2\sinh\left(a\right)\cosh\left(a\right)}{2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x+a)*sinh(b*x+a),x)
```

```
[Out] Piecewise((x*sinh(a + b*x)**2/(4*b) + x*cosh(a + b*x)**2/(4*b) - sinh(a + b
*x)*cosh(a + b*x)/(4*b**2), Ne(b, 0)), (x**2*sinh(a)*cosh(a)/2, True))
```

3.254 $\int \cosh(a+bx)\sinh(a+bx)\,dx$

Optimal. Leaf size=15

$$\frac{\sinh^2(a+bx)}{2b}$$

[Out] 1/2*sinh(b*x+a)^2/b

Rubi [A] time = 0.01, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2564, 30}

$$\frac{\sinh^2(a+bx)}{2b}$$

Antiderivative was successfully verified.

[In] Int[Cosh[a + b*x]*Sinh[a + b*x],x]

[Out] Sinh[a + b*x]^2/(2*b)

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rubi steps

$$\begin{aligned}\int \cosh(a+bx)\sinh(a+bx)\,dx &= -\frac{\text{Subst}\left(\int x\,dx, x, i\sinh(a+bx)\right)}{b}\\ &= \frac{\sinh^2(a+bx)}{2b}\end{aligned}$$

Mathematica [B] time = 0.01, size = 37, normalized size = 2.47

$$\frac{1}{2}\left(\frac{\sinh(2a)\sinh(2bx)}{2b}+\frac{\cosh(2a)\cosh(2bx)}{2b}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[a + b*x]*Sinh[a + b*x],x]
[Out] ((Cosh[2*a]*Cosh[2*b*x])/(2*b) + (Sinh[2*a]*Sinh[2*b*x])/(2*b))/2
```

fricas [A] time = 0.60, size = 22, normalized size = 1.47

$$\frac{\cosh\left(bx+a\right)^{2}+\sinh\left(bx+a\right)^{2}}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*sinh(b*x+a),x, algorithm="fricas")
[Out] 1/4*(cosh(b*x + a)^2 + sinh(b*x + a)^2)/b
```

giac [B] time = 0.13, size = 29, normalized size = 1.93

$$\frac{e^{\left(2\,bx+2\,a\right)}}{8\,b}+\frac{e^{\left(-2\,bx-2\,a\right)}}{8\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*sinh(b*x+a),x, algorithm="giac")
[Out] 1/8*e^(2*b*x + 2*a)/b + 1/8*e^(-2*b*x - 2*a)/b
```

maple [A] time = 0.02, size = 14, normalized size = 0.93

$$\frac{\cosh^{2}\left(bx+a\right)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)*sinh(b*x+a),x)
[Out] 1/2*cosh(b*x+a)^2/b
```

maxima [A] time = 0.38, size = 13, normalized size = 0.87

$$\frac{\cosh\left(bx+a\right)^{2}}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*sinh(b*x+a),x, algorithm="maxima")
[Out] 1/2*cosh(b*x + a)^2/b
```

mupad [B] time = 1.43, size = 13, normalized size = 0.87

$$\frac{\cosh\left(a+b\,x\right)^2}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a + b*x)*sinh(a + b*x),x)
```

```
[Out] cosh(a + b*x)^2/(2*b)
```

sympy [A] time = 0.18, size = 19, normalized size = 1.27

$$\begin{cases} \frac{\sinh^{2}\left(a+bx\right)}{2b} & \text{for}\: b \neq 0 \\ x \sinh\left(a\right) \cosh\left(a\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*sinh(b*x+a),x)
```

```
[Out] Piecewise((sinh(a + b*x)**2/(2*b), Ne(b, 0)), (x*sinh(a)*cosh(a), True))
```

3.255 $\int \frac{\cosh(a+bx)\sinh(a+bx)}{x}\,dx$

Optimal. Leaf size=27

$$\frac{1}{2}\sinh(2a)\mathrm{Chi}(2bx)+\frac{1}{2}\cosh(2a)\mathrm{Shi}(2bx)$$

```
[Out] 1/2*cosh(2*a)*Shi(2*b*x)+1/2*Chi(2*b*x)*sinh(2*a)
```

Rubi [A] time = 0.07, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {5448, 12, 3303, 3298, 3301}

$$\frac{1}{2}\sinh(2a)\mathrm{Chi}(2bx)+\frac{1}{2}\cosh(2a)\mathrm{Shi}(2bx)$$

Antiderivative was successfully verified.

```
[In] Int[(Cosh[a + b*x]*Sinh[a + b*x])/x,x]
```

```
[Out] (CoshIntegral[2*b*x]*Sinh[2*a])/2 + (Cosh[2*a]*SinhIntegral[2*b*x])/2
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cosh(a+bx)\sinh(a+bx)}{x}\,dx &= \int \frac{\sinh(2a+2bx)}{2x}\,dx \\
&= \frac{1}{2}\int \frac{\sinh(2a+2bx)}{x}\,dx \\
&= \frac{1}{2}\cosh(2a)\int \frac{\sinh(2bx)}{x}\,dx + \frac{1}{2}\sinh(2a)\int \frac{\cosh(2bx)}{x}\,dx \\
&= \frac{1}{2}\text{Chi}(2bx)\sinh(2a) + \frac{1}{2}\cosh(2a)\text{Shi}(2bx)
\end{aligned}$$

Mathematica [A] time = 0.03, size = 25, normalized size = 0.93

$$\frac{1}{2}(\sinh(2a)\text{Chi}(2bx) + \cosh(2a)\text{Shi}(2bx))$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cosh[a + b*x]*Sinh[a + b*x])/x,x]
[Out] (CoshIntegral[2*b*x]*Sinh[2*a] + Cosh[2*a]*SinhIntegral[2*b*x])/2
```

fricas [A] time = 0.58, size = 37, normalized size = 1.37

$$\frac{1}{4}\left(\text{Ei}\left(2\,bx\right) - \text{Ei}\left(-2\,bx\right)\right)\cosh\left(2\,a\right) + \frac{1}{4}\left(\text{Ei}\left(2\,bx\right) + \text{Ei}\left(-2\,bx\right)\right)\sinh\left(2\,a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*sinh(b*x+a)/x,x, algorithm="fricas")
[Out] 1/4*(Ei(2*b*x) - Ei(-2*b*x))*cosh(2*a) + 1/4*(Ei(2*b*x) + Ei(-2*b*x))*sinh(
2*a)
```

giac [A] time = 0.11, size = 23, normalized size = 0.85

$$\frac{1}{4}\,\text{Ei}\left(2\,bx\right)e^{(2\,a)} - \frac{1}{4}\,\text{Ei}\left(-2\,bx\right)e^{(-2\,a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*sinh(b*x+a)/x,x, algorithm="giac")
[Out] 1/4*Ei(2*b*x)*e^(2*a) - 1/4*Ei(-2*b*x)*e^(-2*a)
```

maple [A] time = 0.12, size = 26, normalized size = 0.96

$$\frac{e^{-2a}\operatorname{Ei}(1,2bx)}{4}-\frac{e^{2a}\operatorname{Ei}(1,-2bx)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)*sinh(b*x+a)/x,x)
[Out] 1/4*exp(-2*a)*Ei(1,2*b*x)-1/4*exp(2*a)*Ei(1,-2*b*x)
```

maxima [A] time = 0.38, size = 23, normalized size = 0.85

$$\frac{1}{4}\operatorname{Ei}(2\,bx)\,e^{(2\,a)}-\frac{1}{4}\operatorname{Ei}(-2\,bx)\,e^{(-2\,a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*sinh(b*x+a)/x,x, algorithm="maxima")
[Out] 1/4*Ei(2*b*x)*e^(2*a) - 1/4*Ei(-2*b*x)*e^(-2*a)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.04

$$\int\frac{\cosh\left(a+b\,x\right)\,\sinh\left(a+b\,x\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cosh(a + b*x)*sinh(a + b*x))/x,x)
[Out] int((cosh(a + b*x)*sinh(a + b*x))/x, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sinh\left(a+bx\right)\cosh\left(a+bx\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*sinh(b*x+a)/x,x)
[Out] Integral(sinh(a + b*x)*cosh(a + b*x)/x, x)
```

3.256 $\int \frac{\cosh(a+bx)\sinh(a+bx)}{x^2}\,dx$

Optimal. Leaf size=39

$$b\cosh(2a)\mathrm{Chi}(2bx) + b\sinh(2a)\mathrm{Shi}(2bx) - \frac{\sinh(2a+2bx)}{2x}$$

```
[Out] b*Chi(2*b*x)*cosh(2*a)+b*Shi(2*b*x)*sinh(2*a)-1/2*sinh(2*b*x+2*a)/x
```

Rubi [A] time = 0.09, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {5448, 12, 3297, 3303, 3298, 3301}

$$b\cosh(2a)\mathrm{Chi}(2bx) + b\sinh(2a)\mathrm{Shi}(2bx) - \frac{\sinh(2a+2bx)}{2x}$$

Antiderivative was successfully verified.

```
[In] Int[(Cosh[a + b*x]*Sinh[a + b*x])/x^2,x]
```

```
[Out] b*Cosh[2*a]*CoshIntegral[2*b*x] - Sinh[2*a + 2*b*x]/(2*x) + b*Sinh[2*a]*Sin
hIntegral[2*b*x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
```

```
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\cosh(a+bx)\sinh(a+bx)}{x^2}\,dx &= \int \frac{\sinh(2a+2bx)}{2x^2}\,dx\\ &= \frac{1}{2}\int \frac{\sinh(2a+2bx)}{x^2}\,dx\\ &= -\frac{\sinh(2a+2bx)}{2x}+b\int \frac{\cosh(2a+2bx)}{x}\,dx\\ &= -\frac{\sinh(2a+2bx)}{2x}+(b\cosh(2a))\int \frac{\cosh(2bx)}{x}\,dx+(b\sinh(2a))\int \frac{\sinh(2b}{x}\\ &= b\cosh(2a)\text{Chi}(2bx)-\frac{\sinh(2a+2bx)}{2x}+b\sinh(2a)\text{Shi}(2bx)\end{aligned}$$

Mathematica [A] time = 0.07, size = 42, normalized size = 1.08

$$\frac{1}{2}\left(2b\cosh(2a)\text{Chi}(2bx)+2b\sinh(2a)\text{Shi}(2bx)-\frac{\sinh(2(a+bx))}{x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cosh[a + b*x]*Sinh[a + b*x])/x^2,x]
```

```
[Out] (2*b*Cosh[2*a]*CoshIntegral[2*b*x] - Sinh[2*(a + b*x)]/x + 2*b*Sinh[2*a]*Si
nhIntegral[2*b*x])/2
```

fricas [A] time = 0.47, size = 65, normalized size = 1.67

$$\frac{(bx\mathrm{Ei}(2\,bx)+bx\mathrm{Ei}(-2\,bx))\cosh(2\,a)-2\,\cosh(bx+a)\sinh(bx+a)+(bx\mathrm{Ei}(2\,bx)-bx\mathrm{Ei}(-2\,bx))\sinh(2\,a)}{2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*sinh(b*x+a)/x^2,x, algorithm="fricas")
```

```
[Out] 1/2*((b*x*Ei(2*b*x) + b*x*Ei(-2*b*x))*cosh(2*a) - 2*cosh(b*x + a)*sinh(b*x
+ a) + (b*x*Ei(2*b*x) - b*x*Ei(-2*b*x))*sinh(2*a))/x
```

giac [A] time = 0.14, size = 52, normalized size = 1.33

$$\frac{2\,bx\mathrm{Ei}(2\,bx)\,e^{(2\,a)}+2\,bx\mathrm{Ei}(-2\,bx)\,e^{(-2\,a)}-e^{(2\,bx+2\,a)}+e^{(-2\,bx-2\,a)}}{4\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*sinh(b*x+a)/x^2,x, algorithm="giac")
```

```
[Out] 1/4*(2*b*x*Ei(2*b*x)*e^(2*a) + 2*b*x*Ei(-2*b*x)*e^(-2*a) - e^(2*b*x + 2*a)
+ e^(-2*b*x - 2*a))/x
```

maple [A] time = 0.13, size = 56, normalized size = 1.44

$$\frac{\mathrm{e}^{-2bx-2a}}{4x}-\frac{b\,\mathrm{e}^{-2a}\,\mathrm{Ei}(1,2bx)}{2}-\frac{\mathrm{e}^{2bx+2a}}{4x}-\frac{b\,\mathrm{e}^{2a}\,\mathrm{Ei}(1,-2bx)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)*sinh(b*x+a)/x^2,x)
```

```
[Out] 1/4*exp(-2*b*x-2*a)/x-1/2*b*exp(-2*a)*Ei(1,2*b*x)-1/4*exp(2*b*x+2*a)/x-1/2*
b*exp(2*a)*Ei(1,-2*b*x)
```

maxima [A] time = 0.39, size = 27, normalized size = 0.69

$$\frac{1}{2}\,be^{(-2\,a)}\Gamma(-1,2\,bx)+\frac{1}{2}\,be^{(2\,a)}\Gamma(-1,-2\,bx)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*sinh(b*x+a)/x^2,x, algorithm="maxima")
```

```
[Out] 1/2*b*e^(-2*a)*gamma(-1, 2*b*x) + 1/2*b*e^(2*a)*gamma(-1, -2*b*x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\cosh(a+b\,x)\,\sinh(a+b\,x)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cosh(a + b*x)*sinh(a + b*x))/x^2,x)
```

```
[Out] int((cosh(a + b*x)*sinh(a + b*x))/x^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh(a+bx)\cosh(a+bx)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*sinh(b*x+a)/x**2,x)
```

```
[Out] Integral(sinh(a + b*x)*cosh(a + b*x)/x**2, x)
```

3.257 $\int \frac{\cosh(a+bx)\sinh(a+bx)}{x^3}\,dx$

Optimal. Leaf size=60

$$b^2\sinh(2a)\text{Chi}(2bx)+b^2\cosh(2a)\text{Shi}(2bx)-\frac{\sinh(2a+2bx)}{4x^2}-\frac{b\cosh(2a+2bx)}{2x}$$

```
[Out] -1/2*b*cosh(2*b*x+2*a)/x+b^2*cosh(2*a)*Shi(2*b*x)+b^2*Chi(2*b*x)*sinh(2*a)-
1/4*sinh(2*b*x+2*a)/x^2
```

Rubi [A] time = 0.12, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {5448, 12, 3297, 3303, 3298, 3301}

$$b^2\sinh(2a)\text{Chi}(2bx)+b^2\cosh(2a)\text{Shi}(2bx)-\frac{\sinh(2a+2bx)}{4x^2}-\frac{b\cosh(2a+2bx)}{2x}$$

Antiderivative was successfully verified.

```
[In] Int[(Cosh[a + b*x]*Sinh[a + b*x])/x^3,x]
```

```
[Out] -(b*Cosh[2*a + 2*b*x])/(2*x) + b^2*CoshIntegral[2*b*x]*Sinh[2*a] - Sinh[2*a
 + 2*b*x]/(4*x^2) + b^2*Cosh[2*a]*SinhIntegral[2*b*x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
```

```
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\cosh(a+bx)\sinh(a+bx)}{x^3}\,dx &= \int \frac{\sinh(2a+2bx)}{2x^3}\,dx\\ &= \frac{1}{2}\int \frac{\sinh(2a+2bx)}{x^3}\,dx\\ &= -\frac{\sinh(2a+2bx)}{4x^2}+\frac{1}{2}b\int \frac{\cosh(2a+2bx)}{x^2}\,dx\\ &= -\frac{b\cosh(2a+2bx)}{2x}-\frac{\sinh(2a+2bx)}{4x^2}+b^2\int \frac{\sinh(2a+2bx)}{x}\,dx\\ &= -\frac{b\cosh(2a+2bx)}{2x}-\frac{\sinh(2a+2bx)}{4x^2}+\left(b^2\cosh(2a)\right)\int \frac{\sinh(2bx)}{x}\,dx+\left(b^2\right.\\ &= -\frac{b\cosh(2a+2bx)}{2x}+b^2\text{Chi}(2bx)\sinh(2a)-\frac{\sinh(2a+2bx)}{4x^2}+b^2\cosh(2a)\text{S}\end{aligned}$$

Mathematica [A] time = 0.18, size = 61, normalized size = 1.02

$$\frac{1}{2}\left(2b^2\sinh(2a)\text{Chi}(2bx)+2b^2\cosh(2a)\text{Shi}(2bx)-\frac{\sinh(2(a+bx))+2bx\cosh(2(a+bx))}{2x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cosh[a + b*x]*Sinh[a + b*x])/x^3,x]
```

```
[Out] (2*b^2*CoshIntegral[2*b*x]*Sinh[2*a] - (2*b*x*Cosh[2*(a + b*x)] + Sinh[2*(a
 + b*x)])/(2*x^2) + 2*b^2*Cosh[2*a]*SinhIntegral[2*b*x])/2
```

fricas [A] time = 0.52, size = 104, normalized size = 1.73

$$-\frac{bx\cosh\left(bx+a\right)^{2}+bx\sinh\left(bx+a\right)^{2}-\left(b^{2}x^{2}\mathrm{Ei}\left(2\,bx\right)-b^{2}x^{2}\mathrm{Ei}\left(-2\,bx\right)\right)\cosh\left(2\,a\right)+\cosh\left(bx+a\right)\sinh\left(bx+a\right)}{2\,x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*sinh(b*x+a)/x^3,x, algorithm="fricas")
```

```
[Out] -1/2*(b*x*cosh(b*x + a)^2 + b*x*sinh(b*x + a)^2 - (b^2*x^2*Ei(2*b*x) - b^2*
x^2*Ei(-2*b*x))*cosh(2*a) + cosh(b*x + a)*sinh(b*x + a) - (b^2*x^2*Ei(2*b*x
) + b^2*x^2*Ei(-2*b*x))*sinh(2*a))/x^2
```

giac [A] time = 0.11, size = 86, normalized size = 1.43

$$\frac{4\,b^{2}x^{2}\mathrm{Ei}\left(2\,bx\right)e^{\left(2\,a\right)}-4\,b^{2}x^{2}\mathrm{Ei}\left(-2\,bx\right)e^{\left(-2\,a\right)}-2\,bxe^{\left(2\,bx+2\,a\right)}-2\,bxe^{\left(-2\,bx-2\,a\right)}-e^{\left(2\,bx+2\,a\right)}+e^{\left(-2\,bx-2\,a\right)}}{8\,x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*sinh(b*x+a)/x^3,x, algorithm="giac")
```

```
[Out] 1/8*(4*b^2*x^2*Ei(2*b*x)*e^(2*a) - 4*b^2*x^2*Ei(-2*b*x)*e^(-2*a) - 2*b*x*e^
(2*b*x + 2*a) - 2*b*x*e^(-2*b*x - 2*a) - e^(2*b*x + 2*a) + e^(-2*b*x - 2*a)
)/x^2
```

maple [A] time = 0.12, size = 90, normalized size = 1.50

$$-\frac{b\,\mathrm{e}^{-2bx-2a}}{4x}+\frac{\mathrm{e}^{-2bx-2a}}{8x^{2}}+\frac{b^{2}\mathrm{e}^{-2a}\,\mathrm{Ei}\left(1,2bx\right)}{2}-\frac{\mathrm{e}^{2bx+2a}}{8x^{2}}-\frac{b\,\mathrm{e}^{2bx+2a}}{4x}-\frac{b^{2}\mathrm{e}^{2a}\,\mathrm{Ei}\left(1,-2bx\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)*sinh(b*x+a)/x^3,x)
```

```
[Out] -1/4*b*exp(-2*b*x-2*a)/x+1/8*exp(-2*b*x-2*a)/x^2+1/2*b^2*exp(-2*a)*Ei(1,2*b
*x)-1/8*exp(2*b*x+2*a)/x^2-1/4*b*exp(2*b*x+2*a)/x-1/2*b^2*exp(2*a)*Ei(1,-2*
b*x)
```

maxima [A] time = 0.38, size = 30, normalized size = 0.50

$$b^{2}e^{\left(-2\,a\right)}\Gamma\left(-2,2\,bx\right)-b^{2}e^{\left(2\,a\right)}\Gamma\left(-2,-2\,bx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*sinh(b*x+a)/x^3,x, algorithm="maxima")
```

```
[Out] b^2*e^(-2*a)*gamma(-2, 2*b*x) - b^2*e^(2*a)*gamma(-2, -2*b*x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\cosh\left(a+b\,x\right)\,\sinh\left(a+b\,x\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cosh(a + b*x)*sinh(a + b*x))/x^3,x)
[Out] int((cosh(a + b*x)*sinh(a + b*x))/x^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh\left(a+bx\right)\cosh\left(a+bx\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*sinh(b*x+a)/x**3,x)
[Out] Integral(sinh(a + b*x)*cosh(a + b*x)/x**3, x)
```

3.258 $\int \frac{\cosh(a+bx)\sinh(a+bx)}{x^4}\,dx$

Optimal. Leaf size=85

$$\frac{2}{3}b^3\cosh(2a)\text{Chi}(2bx)+\frac{2}{3}b^3\sinh(2a)\text{Shi}(2bx)-\frac{b^2\sinh(2a+2bx)}{3x}-\frac{\sinh(2a+2bx)}{6x^3}-\frac{b\cosh(2a+2bx)}{6x^2}$$

```
[Out] 2/3*b^3*Chi(2*b*x)*cosh(2*a)-1/6*b*cosh(2*b*x+2*a)/x^2+2/3*b^3*Shi(2*b*x)*s
inh(2*a)-1/6*sinh(2*b*x+2*a)/x^3-1/3*b^2*sinh(2*b*x+2*a)/x
```

Rubi [A] time = 0.15, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {5448, 12, 3297, 3303, 3298, 3301}

$$\frac{2}{3}b^3\cosh(2a)\text{Chi}(2bx)+\frac{2}{3}b^3\sinh(2a)\text{Shi}(2bx)-\frac{b^2\sinh(2a+2bx)}{3x}-\frac{\sinh(2a+2bx)}{6x^3}-\frac{b\cosh(2a+2bx)}{6x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Cosh[a + b*x]*Sinh[a + b*x])/x^4,x]
```

```
[Out] -(b*Cosh[2*a + 2*b*x])/(6*x^2) + (2*b^3*Cosh[2*a]*CoshIntegral[2*b*x])/3 -
Sinh[2*a + 2*b*x]/(6*x^3) - (b^2*Sinh[2*a + 2*b*x])/(3*x) + (2*b^3*Sinh[2*a
]*SinhIntegral[2*b*x])/3
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cosh(a+bx)\sinh(a+bx)}{x^4}\,dx &= \int \frac{\sinh(2a+2bx)}{2x^4}\,dx \\
&= \frac{1}{2}\int \frac{\sinh(2a+2bx)}{x^4}\,dx \\
&= -\frac{\sinh(2a+2bx)}{6x^3} + \frac{1}{3}b\int \frac{\cosh(2a+2bx)}{x^3}\,dx \\
&= -\frac{b\cosh(2a+2bx)}{6x^2} - \frac{\sinh(2a+2bx)}{6x^3} + \frac{1}{3}b^2\int \frac{\sinh(2a+2bx)}{x^2}\,dx \\
&= -\frac{b\cosh(2a+2bx)}{6x^2} - \frac{\sinh(2a+2bx)}{6x^3} - \frac{b^2\sinh(2a+2bx)}{3x} + \frac{1}{3}\left(2b^3\right)\int \frac{\cos
\\
&= -\frac{b\cosh(2a+2bx)}{6x^2} - \frac{\sinh(2a+2bx)}{6x^3} - \frac{b^2\sinh(2a+2bx)}{3x} + \frac{1}{3}\left(2b^3\cosh(2a
\\
&= -\frac{b\cosh(2a+2bx)}{6x^2} + \frac{2}{3}b^3\cosh(2a)\text{Chi}(2bx) - \frac{\sinh(2a+2bx)}{6x^3} - \frac{b^2\sinh(2a}{3x}
\end{aligned}$$

Mathematica [A] time = 0.17, size = 77, normalized size = 0.91

$$-\frac{-4b^3x^3\cosh(2a)\text{Chi}(2bx) - 4b^3x^3\sinh(2a)\text{Shi}(2bx) + 2b^2x^2\sinh(2(a+bx)) + \sinh(2(a+bx)) + bx\cosh(2(a}{6x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cosh[a + b*x]*Sinh[a + b*x])/x^4,x]
```

```
[Out] -1/6*(b*x*Cosh[2*(a + b*x)] - 4*b^3*x^3*Cosh[2*a]*CoshIntegral[2*b*x] + Sin
h[2*(a + b*x)] + 2*b^2*x^2*Sinh[2*(a + b*x)] - 4*b^3*x^3*Sinh[2*a]*SinhInte
gral[2*b*x])/x^3
```

fricas [A] time = 0.45, size = 115, normalized size = 1.35

$$-\frac{bx\cosh(bx+a)^2 + bx\sinh(bx+a)^2 + 2\left(2b^2x^2+1\right)\cosh(bx+a)\sinh(bx+a) - 2\left(b^3x^3\mathrm{Ei}(2bx) + b^3x^3\mathrm{Ei}(-\right.}{6x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*sinh(b*x+a)/x^4,x, algorithm="fricas")
```

```
[Out] -1/6*(b*x*cosh(b*x + a)^2 + b*x*sinh(b*x + a)^2 + 2*(2*b^2*x^2 + 1)*cosh(b*
x + a)*sinh(b*x + a) - 2*(b^3*x^3*Ei(2*b*x) + b^3*x^3*Ei(-2*b*x))*cosh(2*a)
 - 2*(b^3*x^3*Ei(2*b*x) - b^3*x^3*Ei(-2*b*x))*sinh(2*a))/x^3
```

giac [A] time = 0.12, size = 120, normalized size = 1.41

$$\frac{4b^3x^3\mathrm{Ei}(2bx)e^{(2a)} + 4b^3x^3\mathrm{Ei}(-2bx)e^{(-2a)} - 2b^2x^2e^{(2bx+2a)} + 2b^2x^2e^{(-2bx-2a)} - bxe^{(2bx+2a)} - bxe^{(-2bx-2a)} - e^{(2}}{12x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*sinh(b*x+a)/x^4,x, algorithm="giac")
```

```
[Out] 1/12*(4*b^3*x^3*Ei(2*b*x)*e^(2*a) + 4*b^3*x^3*Ei(-2*b*x)*e^(-2*a) - 2*b^2*x
^2*e^(2*b*x + 2*a) + 2*b^2*x^2*e^(-2*b*x - 2*a) - b*x*e^(2*b*x + 2*a) - b*x
*e^(-2*b*x - 2*a) - e^(2*b*x + 2*a) + e^(-2*b*x - 2*a))/x^3
```

maple [A] time = 0.12, size = 124, normalized size = 1.46

$$\frac{b^2\mathrm{e}^{-2bx-2a}}{6x} - \frac{b\,\mathrm{e}^{-2bx-2a}}{12x^2} + \frac{\mathrm{e}^{-2bx-2a}}{12x^3} - \frac{b^3\mathrm{e}^{-2a}\,\mathrm{Ei}(1,2bx)}{3} - \frac{\mathrm{e}^{2bx+2a}}{12x^3} - \frac{b\,\mathrm{e}^{2bx+2a}}{12x^2} - \frac{b^2\mathrm{e}^{2bx+2a}}{6x} - \frac{b^3\mathrm{e}^{2a}\,\mathrm{Ei}(1,-2bx)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)*sinh(b*x+a)/x^4,x)
```

```
[Out] 1/6*b^2*exp(-2*b*x-2*a)/x-1/12*b*exp(-2*b*x-2*a)/x^2+1/12*exp(-2*b*x-2*a)/x
^3-1/3*b^3*exp(-2*a)*Ei(1,2*b*x)-1/12*exp(2*b*x+2*a)/x^3-1/12*b*exp(2*b*x+2
*a)/x^2-1/6*b^2*exp(2*b*x+2*a)/x-1/3*b^3*exp(2*a)*Ei(1,-2*b*x)
```

maxima [A] time = 0.39, size = 31, normalized size = 0.36

$$2b^3e^{(-2a)}\Gamma(-3,2bx) + 2b^3e^{(2a)}\Gamma(-3,-2bx)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*sinh(b*x+a)/x^4,x, algorithm="maxima")
[Out] 2*b^3*e^(-2*a)*gamma(-3, 2*b*x) + 2*b^3*e^(2*a)*gamma(-3, -2*b*x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cosh(a+bx)\,\sinh(a+bx)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cosh(a + b*x)*sinh(a + b*x))/x^4,x)
[Out] int((cosh(a + b*x)*sinh(a + b*x))/x^4, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh(a+bx)\cosh(a+bx)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*sinh(b*x+a)/x**4,x)
[Out] Integral(sinh(a + b*x)*cosh(a + b*x)/x**4, x)
```

3.259 $\int x^m \cosh^2(a+bx)\sinh(a+bx)\,dx$

Optimal. Leaf size=134

$$\frac{e^{3a}3^{-m-1}x^m(-bx)^{-m}\Gamma(m+1,-3bx)}{8b}+\frac{e^a x^m(-bx)^{-m}\Gamma(m+1,-bx)}{8b}+\frac{e^{-a}x^m(bx)^{-m}\Gamma(m+1,bx)}{8b}+\frac{e^{-3a}3^{-m-1}x^m(bx)^{-m}\Gamma(m+1,3bx)}{8b}$$

```
[Out] 1/8*3^(-1-m)*exp(3*a)*x^m*GAMMA(1+m,-3*b*x)/b/((-b*x)^m)+1/8*exp(a)*x^m*GAM
MA(1+m,-b*x)/b/((-b*x)^m)+1/8*x^m*GAMMA(1+m,b*x)/b/exp(a)/((b*x)^m)+1/8*3^(
-1-m)*x^m*GAMMA(1+m,3*b*x)/b/exp(3*a)/((b*x)^m)
```

Rubi [A] time = 0.20, antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {5448, 3308, 2181}

$$\frac{e^{3a}3^{-m-1}x^m(-bx)^{-m}\text{Gamma}(m+1,-3bx)}{8b}+\frac{e^a x^m(-bx)^{-m}\text{Gamma}(m+1,-bx)}{8b}+\frac{e^{-a}x^m(bx)^{-m}\text{Gamma}(m+1,bx)}{8b}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*Cosh[a + b*x]^2*Sinh[a + b*x],x]
```

```
[Out] (3^(-1 - m)*E^(3*a)*x^m*Gamma[1 + m, -3*b*x])/(8*b*(-(b*x))^m) + (E^a*x^m*G
amma[1 + m, -(b*x)])/(8*b*(-(b*x))^m) + (x^m*Gamma[1 + m, b*x])/(8*b*E^a*(b
*x)^m) + (3^(-1 - m)*x^m*Gamma[1 + m, 3*b*x])/(8*b*E^(3*a)*(b*x)^m)
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&  !I
ntegerQ[m]
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
```

```
& IGtQ[p, 0]
```

Rubi steps

$$\int x^m \cosh^2(a+bx)\sinh(a+bx)\,dx = \int \left(\frac{1}{4}x^m \sinh(a+bx) + \frac{1}{4}x^m \sinh(3a+3bx)\right)dx$$
$$= \frac{1}{4}\int x^m \sinh(a+bx)\,dx + \frac{1}{4}\int x^m \sinh(3a+3bx)\,dx$$
$$= \frac{1}{8}\int e^{-i(ia+ibx)}x^m\,dx - \frac{1}{8}\int e^{i(ia+ibx)}x^m\,dx + \frac{1}{8}\int e^{-i(3ia+3ibx)}x^m\,dx - \frac{1}{8}\int$$
$$= \frac{3^{-1-m}e^{3a}x^m(-bx)^{-m}\Gamma(1+m,-3bx)}{8b} + \frac{e^a x^m(-bx)^{-m}\Gamma(1+m,-bx)}{8b} + \frac{e^{-a}x}{}$$

Mathematica [A] time = 0.18, size = 114, normalized size = 0.85

$$\frac{e^{-3a}x^m\left(3^{-m}\left(-b^2x^2\right)^{-m}\left(e^{6a}(bx)^m\Gamma(m+1,-3bx)+(-bx)^m\Gamma(m+1,3bx)\right)+3e^{2a}\left(e^{2a}(-bx)^{-m}\Gamma(m+1,-bx)+(bx\right.\right.}{24b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*Cosh[a + b*x]^2*Sinh[a + b*x],x]
```

```
[Out] (x^m*(3*E^(2*a)*((E^(2*a)*Gamma[1 + m, -(b*x)])/(-(b*x))^m + Gamma[1 + m, b
*x]/(b*x)^m) + (E^(6*a)*(b*x)^m*Gamma[1 + m, -3*b*x] + (-(b*x))^m*Gamma[1 +
 m, 3*b*x])/(3^m*(-(b^2*x^2))^m)))/(24*b*E^(3*a))
```

fricas [A] time = 1.20, size = 162, normalized size = 1.21

$$\frac{\cosh\left(m\log\left(3\,b\right)+3\,a\right)\Gamma\left(m+1,3\,bx\right)+3\,\cosh\left(m\log(b)+a\right)\Gamma\left(m+1,bx\right)+3\,\cosh\left(m\log\left(-b\right)-a\right)\Gamma\left(m+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*cosh(b*x+a)^2*sinh(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/24*(cosh(m*log(3*b) + 3*a)*gamma(m + 1, 3*b*x) + 3*cosh(m*log(b) + a)*gam
ma(m + 1, b*x) + 3*cosh(m*log(-b) - a)*gamma(m + 1, -b*x) + cosh(m*log(-3*b
) - 3*a)*gamma(m + 1, -3*b*x) - gamma(m + 1, 3*b*x)*sinh(m*log(3*b) + 3*a)
- 3*gamma(m + 1, -b*x)*sinh(m*log(-b) - a) - gamma(m + 1, -3*b*x)*sinh(m*lo
g(-3*b) - 3*a) - 3*gamma(m + 1, b*x)*sinh(m*log(b) + a))/b
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^m \cosh(bx+a)^2 \sinh(bx+a)\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*cosh(b*x+a)^2*sinh(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(x^m*cosh(b*x + a)^2*sinh(b*x + a), x)
```

maple [F] time = 0.40, size = 0, normalized size = 0.00

$$\int x^m \left(\cosh^2(bx+a)\right) \sinh(bx+a)\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*cosh(b*x+a)^2*sinh(b*x+a),x)
```

```
[Out] int(x^m*cosh(b*x+a)^2*sinh(b*x+a),x)
```

maxima [A] time = 0.43, size = 113, normalized size = 0.84

$$\frac{1}{8}(3\,bx)^{-m-1}x^{m+1}e^{(-3\,a)}\Gamma(m+1,3\,bx)+\frac{1}{8}(bx)^{-m-1}x^{m+1}e^{(-a)}\Gamma(m+1,bx)-\frac{1}{8}(-bx)^{-m-1}x^{m+1}e^{a}\Gamma(m+1,-bx)-\frac{1}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*cosh(b*x+a)^2*sinh(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/8*(3*b*x)^(-m - 1)*x^(m + 1)*e^(-3*a)*gamma(m + 1, 3*b*x) + 1/8*(b*x)^(-m
 - 1)*x^(m + 1)*e^(-a)*gamma(m + 1, b*x) - 1/8*(-b*x)^(-m - 1)*x^(m + 1)*e^
a*gamma(m + 1, -b*x) - 1/8*(-3*b*x)^(-m - 1)*x^(m + 1)*e^(3*a)*gamma(m + 1,
 -3*b*x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^m \cosh(a+b\,x)^2 \sinh(a+b\,x)\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*cosh(a + b*x)^2*sinh(a + b*x),x)
```

```
[Out] int(x^m*cosh(a + b*x)^2*sinh(a + b*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^m \sinh(a+bx)\cosh^2(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*cosh(b*x+a)**2*sinh(b*x+a),x)
```

```
[Out] Integral(x**m*sinh(a + b*x)*cosh(a + b*x)**2, x)
```

3.260 $\int x^3 \cosh^2(a+bx)\sinh(a+bx)\,dx$

Optimal. Leaf size=117

$$-\frac{2\sinh^3(a+bx)}{27b^4}-\frac{14\sinh(a+bx)}{9b^4}+\frac{2x\cosh^3(a+bx)}{9b^3}+\frac{4x\cosh(a+bx)}{3b^3}-\frac{2x^2\sinh(a+bx)}{3b^2}-\frac{x^2\sinh(a+bx)\cosh}{3b^2}$$

```
[Out] 4/3*x*cosh(b*x+a)/b^3+2/9*x*cosh(b*x+a)^3/b^3+1/3*x^3*cosh(b*x+a)^3/b-14/9*
sinh(b*x+a)/b^4-2/3*x^2*sinh(b*x+a)/b^2-1/3*x^2*cosh(b*x+a)^2*sinh(b*x+a)/b
^2-2/27*sinh(b*x+a)^3/b^4
```

Rubi [A] time = 0.12, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {5373, 3311, 3296, 2637, 2633}

$$-\frac{2x^2\sinh(a+bx)}{3b^2}-\frac{x^2\sinh(a+bx)\cosh^2(a+bx)}{3b^2}-\frac{2\sinh^3(a+bx)}{27b^4}-\frac{14\sinh(a+bx)}{9b^4}+\frac{2x\cosh^3(a+bx)}{9b^3}+\frac{4x\cos}{3}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Cosh[a + b*x]^2*Sinh[a + b*x],x]
```

```
[Out] (4*x*Cosh[a + b*x])/(3*b^3) + (2*x*Cosh[a + b*x]^3)/(9*b^3) + (x^3*Cosh[a +
 b*x]^3)/(3*b) - (14*Sinh[a + b*x])/(9*b^4) - (2*x^2*Sinh[a + b*x])/(3*b^2)
 - (x^2*Cosh[a + b*x]^2*Sinh[a + b*x])/(3*b^2) - (2*Sinh[a + b*x]^3)/(27*b^
4)
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3311

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(d*m*(c + d*x)^(m - 1)*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist
[(b^2*(n - 1))/n, Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[(
d^2*m*(m - 1))/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[(b*(c + d*x)^m*Cos[e + f*x]*(b*Sin[e + f*x])^(n - 1))/(f*n), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 5373

```
Int[Cosh[(a_.) + (b_.)*(x_)^(n_.)]^(p_.)*(x_)^(m_.)*Sinh[(a_.) + (b_.)*(x_)
^(n_.)], x_Symbol] :> Simp[(x^(m - n + 1)*Cosh[a + b*x^n]^(p + 1))/(b*n*(p
+ 1)), x] - Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*Cosh[a + b*x^n]^(
p + 1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int x^3\cosh^2(a+bx)\sinh(a+bx)\,dx &= \frac{x^3\cosh^3(a+bx)}{3b}-\frac{\int x^2\cosh^3(a+bx)\,dx}{b}\\ &= \frac{2x\cosh^3(a+bx)}{9b^3}+\frac{x^3\cosh^3(a+bx)}{3b}-\frac{x^2\cosh^2(a+bx)\sinh(a+bx)}{3b^2}-\\ &= \frac{2x\cosh^3(a+bx)}{9b^3}+\frac{x^3\cosh^3(a+bx)}{3b}-\frac{2x^2\sinh(a+bx)}{3b^2}-\frac{x^2\cosh^2(a+\\ &= \frac{4x\cosh(a+bx)}{3b^3}+\frac{2x\cosh^3(a+bx)}{9b^3}+\frac{x^3\cosh^3(a+bx)}{3b}-\frac{2\sinh(a+bx}{9b^4}\\ &= \frac{4x\cosh(a+bx)}{3b^3}+\frac{2x\cosh^3(a+bx)}{9b^3}+\frac{x^3\cosh^3(a+bx)}{3b}-\frac{14\sinh(a+b}{9b^4}\end{aligned}$$

Mathematica [A] time = 0.41, size = 86, normalized size = 0.74

$$\frac{\left(9b^3x^3+6bx\right)\cosh(3(a+bx))+27bx\left(b^2x^2+6\right)\cosh(a+bx)-2\sinh(a+bx)\left(\left(9b^2x^2+2\right)\cosh(2(a+bx))+\right.}{108b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Cosh[a + b*x]^2*Sinh[a + b*x],x]
```

```
[Out] (27*b*x*(6 + b^2*x^2)*Cosh[a + b*x] + (6*b*x + 9*b^3*x^3)*Cosh[3*(a + b*x)]
 - 2*(82 + 45*b^2*x^2 + (2 + 9*b^2*x^2)*Cosh[2*(a + b*x)])*Sinh[a + b*x])/(
108*b^4)
```

fricas [A] time = 0.49, size = 135, normalized size = 1.15

$$\frac{3\left(3\,b^3x^3+2\,bx\right)\cosh\left(bx+a\right)^3+9\left(3\,b^3x^3+2\,bx\right)\cosh\left(bx+a\right)\sinh\left(bx+a\right)^2-\left(9\,b^2x^2+2\right)\sinh\left(bx+a\right)^3+2}{108\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*cosh(b*x+a)^2*sinh(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/108*(3*(3*b^3*x^3 + 2*b*x)*cosh(b*x + a)^3 + 9*(3*b^3*x^3 + 2*b*x)*cosh(b
*x + a)*sinh(b*x + a)^2 - (9*b^2*x^2 + 2)*sinh(b*x + a)^3 + 27*(b^3*x^3 + 6
*b*x)*cosh(b*x + a) - 3*(27*b^2*x^2 + (9*b^2*x^2 + 2)*cosh(b*x + a)^2 + 54)
*sinh(b*x + a))/b^4
```

giac [A] time = 0.13, size = 140, normalized size = 1.20

$$\frac{\left(9\,b^3x^3-9\,b^2x^2+6\,bx-2\right)e^{(3\,bx+3\,a)}}{216\,b^4}+\frac{\left(b^3x^3-3\,b^2x^2+6\,bx-6\right)e^{(bx+a)}}{8\,b^4}+\frac{\left(b^3x^3+3\,b^2x^2+6\,bx+6\right)e^{(-bx-a)}}{8\,b^4}+\frac{(9\,b^3}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*cosh(b*x+a)^2*sinh(b*x+a),x, algorithm="giac")
```

```
[Out] 1/216*(9*b^3*x^3 - 9*b^2*x^2 + 6*b*x - 2)*e^(3*b*x + 3*a)/b^4 + 1/8*(b^3*x^
3 - 3*b^2*x^2 + 6*b*x - 6)*e^(b*x + a)/b^4 + 1/8*(b^3*x^3 + 3*b^2*x^2 + 6*b
*x + 6)*e^(-b*x - a)/b^4 + 1/216*(9*b^3*x^3 + 9*b^2*x^2 + 6*b*x + 2)*e^(-3*
b*x - 3*a)/b^4
```

maple [B] time = 0.32, size = 244, normalized size = 2.09

$$\frac{\frac{(bx+a)^3\left(\cosh^3(bx+a)\right)}{3}-\frac{2(bx+a)^2\sinh(bx+a)}{3}-\frac{(bx+a)^2\sinh(bx+a)\left(\cosh^2(bx+a)\right)}{3}+\frac{4(bx+a)\cosh(bx+a)}{3}-\frac{40\sinh(bx+a)}{27}+\frac{2(bx+a)\left(\cosh^3(b\right.}{9}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*cosh(b*x+a)^2*sinh(b*x+a),x)
```

```
[Out] 1/b^4*(1/3*(b*x+a)^3*cosh(b*x+a)^3-2/3*(b*x+a)^2*sinh(b*x+a)-1/3*(b*x+a)^2*
sinh(b*x+a)*cosh(b*x+a)^2+4/3*(b*x+a)*cosh(b*x+a)-40/27*sinh(b*x+a)+2/9*(b*
x+a)*cosh(b*x+a)^3-2/27*cosh(b*x+a)^2*sinh(b*x+a)-3*a*(1/3*(b*x+a)^2*cosh(b
*x+a)^3-4/9*(b*x+a)*sinh(b*x+a)-2/9*(b*x+a)*sinh(b*x+a)*cosh(b*x+a)^2+4/9*c
osh(b*x+a)+2/27*cosh(b*x+a)^3)+3*a^2*(1/3*(b*x+a)*cosh(b*x+a)^3-2/9*sinh(b*
x+a)-1/9*cosh(b*x+a)^2*sinh(b*x+a))-1/3*a^3*cosh(b*x+a)^3)
```

maxima [A] time = 0.33, size = 160, normalized size = 1.37

$$\frac{\left(9\,b^3x^3e^{(3\,a)}-9\,b^2x^2e^{(3\,a)}+6\,bxe^{(3\,a)}-2\,e^{(3\,a)}\right)e^{(3\,bx)}}{216\,b^4}+\frac{\left(b^3x^3e^a-3\,b^2x^2e^a+6\,bxe^a-6\,e^a\right)e^{(bx)}}{8\,b^4}+\frac{\left(b^3x^3+3\,b^2x^2+6\,b\right.}{8\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*cosh(b*x+a)^2*sinh(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/216*(9*b^3*x^3*e^(3*a) - 9*b^2*x^2*e^(3*a) + 6*b*x*e^(3*a) - 2*e^(3*a))*e
^(3*b*x)/b^4 + 1/8*(b^3*x^3*e^a - 3*b^2*x^2*e^a + 6*b*x*e^a - 6*e^a)*e^(b*x
)/b^4 + 1/8*(b^3*x^3 + 3*b^2*x^2 + 6*b*x + 6)*e^(-b*x - a)/b^4 + 1/216*(9*b
^3*x^3 + 9*b^2*x^2 + 6*b*x + 2)*e^(-3*b*x - 3*a)/b^4
```

mupad [B] time = 1.54, size = 108, normalized size = 0.92

$$\frac{\frac{2\,x\,\cosh(a+b\,x)^3}{9}+\frac{4\,x\,\cosh(a+b\,x)}{3}}{b^3}-\frac{\frac{2\,x^2\,\sinh(a+b\,x)}{3}+\frac{x^2\,\cosh(a+b\,x)^2\,\sinh(a+b\,x)}{3}}{b^2}-\frac{40\sinh\left(a+b\,x\right)}{27\,b^4}-\frac{2\cosh\left(a+b\,x\right)^2\sinh\left(a\right.}{27\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*cosh(a + b*x)^2*sinh(a + b*x),x)
```

```
[Out] ((4*x*cosh(a + b*x))/3 + (2*x*cosh(a + b*x)^3)/9)/b^3 - ((2*x^2*sinh(a + b*
x))/3 + (x^2*cosh(a + b*x)^2*sinh(a + b*x))/3)/b^2 - (40*sinh(a + b*x))/(27
*b^4) - (2*cosh(a + b*x)^2*sinh(a + b*x))/(27*b^4) + (x^3*cosh(a + b*x)^3)/
(3*b)
```

sympy [A] time = 2.90, size = 146, normalized size = 1.25

$$\begin{cases} \frac{x^3\cosh^3(a+bx)}{3b}+\frac{2x^2\sinh^3(a+bx)}{3b^2}-\frac{x^2\sinh(a+bx)\cosh^2(a+bx)}{b^2}-\frac{4x\sinh^2(a+bx)\cosh(a+bx)}{3b^3}+\frac{14x\cosh^3(a+bx)}{9b^3}+\frac{40\sinh^3(a+bx)}{27b^4}- \\ \frac{x^4\sinh(a)\cosh^2(a)}{4} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*cosh(b*x+a)**2*sinh(b*x+a),x)
```

```
[Out] Piecewise((x**3*cosh(a + b*x)**3/(3*b) + 2*x**2*sinh(a + b*x)**3/(3*b**2) -
 x**2*sinh(a + b*x)*cosh(a + b*x)**2/b**2 - 4*x*sinh(a + b*x)**2*cosh(a + b
*x)/(3*b**3) + 14*x*cosh(a + b*x)**3/(9*b**3) + 40*sinh(a + b*x)**3/(27*b**
4) - 14*sinh(a + b*x)*cosh(a + b*x)**2/(9*b**4), Ne(b, 0)), (x**4*sinh(a)*c
osh(a)**2/4, True))
```

3.261 $\int x^2 \cosh^2(a+bx)\sinh(a+bx)\,dx$

Optimal. Leaf size=83

$$\frac{2\cosh^3(a+bx)}{27b^3}+\frac{4\cosh(a+bx)}{9b^3}-\frac{4x\sinh(a+bx)}{9b^2}-\frac{2x\sinh(a+bx)\cosh^2(a+bx)}{9b^2}+\frac{x^2\cosh^3(a+bx)}{3b}$$

```
[Out] 4/9*cosh(b*x+a)/b^3+2/27*cosh(b*x+a)^3/b^3+1/3*x^2*cosh(b*x+a)^3/b-4/9*x*si
nh(b*x+a)/b^2-2/9*x*cosh(b*x+a)^2*sinh(b*x+a)/b^2
```

Rubi [A] time = 0.07, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {5373, 3310, 3296, 2638}

$$-\frac{4x\sinh(a+bx)}{9b^2}+\frac{2\cosh^3(a+bx)}{27b^3}+\frac{4\cosh(a+bx)}{9b^3}-\frac{2x\sinh(a+bx)\cosh^2(a+bx)}{9b^2}+\frac{x^2\cosh^3(a+bx)}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Cosh[a + b*x]^2*Sinh[a + b*x],x]
```

```
[Out] (4*Cosh[a + b*x])/(9*b^3) + (2*Cosh[a + b*x]^3)/(27*b^3) + (x^2*Cosh[a + b*
x]^3)/(3*b) - (4*x*Sinh[a + b*x])/(9*b^2) - (2*x*Cosh[a + b*x]^2*Sinh[a + b
*x])/(9*b^2)
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3310

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[(d*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[(b*(c + d*x)*Cos[e + f*x]*(b
*Sin[e + f*x])^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 5373

```
Int[Cosh[(a_.) + (b_.)*(x_)^(n_.)]^(p_.)*(x_)^(m_.)*Sinh[(a_.) + (b_.)*(x_)
^(n_.)], x_Symbol] :> Simp[(x^(m - n + 1)*Cosh[a + b*x^n]^(p + 1))/(b*n*(p
+ 1)), x] - Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*Cosh[a + b*x^n]^(
p + 1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int x^2\cosh^2(a+bx)\sinh(a+bx)\,dx &= \frac{x^2\cosh^3(a+bx)}{3b}-\frac{2\int x\cosh^3(a+bx)\,dx}{3b}\\ &= \frac{2\cosh^3(a+bx)}{27b^3}+\frac{x^2\cosh^3(a+bx)}{3b}-\frac{2x\cosh^2(a+bx)\sinh(a+bx)}{9b^2}-\\ &= \frac{2\cosh^3(a+bx)}{27b^3}+\frac{x^2\cosh^3(a+bx)}{3b}-\frac{4x\sinh(a+bx)}{9b^2}-\frac{2x\cosh^2(a+b}{9}\\ &= \frac{4\cosh(a+bx)}{9b^3}+\frac{2\cosh^3(a+bx)}{27b^3}+\frac{x^2\cosh^3(a+bx)}{3b}-\frac{4x\sinh(a+bx)}{9b^2}\end{aligned}$$

Mathematica [A] time = 0.21, size = 65, normalized size = 0.78

$$\frac{27\left(b^2x^2+2\right)\cosh(a+bx)+\left(9b^2x^2+2\right)\cosh(3(a+bx))-6bx(9\sinh(a+bx)+\sinh(3(a+bx)))}{108b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Cosh[a + b*x]^2*Sinh[a + b*x],x]
```

```
[Out] (27*(2 + b^2*x^2)*Cosh[a + b*x] + (2 + 9*b^2*x^2)*Cosh[3*(a + b*x)] - 6*b*x
*(9*Sinh[a + b*x] + Sinh[3*(a + b*x)]))/(108*b^3)
```

fricas [A] time = 0.52, size = 105, normalized size = 1.27

$$-\frac{6\,bx\sinh\left(bx+a\right)^3-\left(9\,b^2x^2+2\right)\cosh\left(bx+a\right)^3-3\left(9\,b^2x^2+2\right)\cosh\left(bx+a\right)\sinh\left(bx+a\right)^2-27\left(b^2x^2+2\right)}{108\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cosh(b*x+a)^2*sinh(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/108*(6*b*x*sinh(b*x + a)^3 - (9*b^2*x^2 + 2)*cosh(b*x + a)^3 - 3*(9*b^2*
x^2 + 2)*cosh(b*x + a)*sinh(b*x + a)^2 - 27*(b^2*x^2 + 2)*cosh(b*x + a) + 1
8*(b*x*cosh(b*x + a)^2 + 3*b*x)*sinh(b*x + a))/b^3
```

giac [A] time = 0.12, size = 108, normalized size = 1.30

$$\frac{\left(9\,b^{2}x^{2}-6\,bx+2\right)e^{(3\,bx+3\,a)}}{216\,b^{3}}+\frac{\left(b^{2}x^{2}-2\,bx+2\right)e^{(bx+a)}}{8\,b^{3}}+\frac{\left(b^{2}x^{2}+2\,bx+2\right)e^{(-bx-a)}}{8\,b^{3}}+\frac{\left(9\,b^{2}x^{2}+6\,bx+2\right)e^{(-3\,bx-3\,a)}}{216\,b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cosh(b*x+a)^2*sinh(b*x+a),x, algorithm="giac")
```

```
[Out] 1/216*(9*b^2*x^2 - 6*b*x + 2)*e^(3*b*x + 3*a)/b^3 + 1/8*(b^2*x^2 - 2*b*x +
2)*e^(b*x + a)/b^3 + 1/8*(b^2*x^2 + 2*b*x + 2)*e^(-b*x - a)/b^3 + 1/216*(9*
b^2*x^2 + 6*b*x + 2)*e^(-3*b*x - 3*a)/b^3
```

maple [A] time = 0.32, size = 131, normalized size = 1.58

$$\frac{\frac{(bx+a)^{2}\left(\cosh^{3}(bx+a)\right)}{3}-\frac{4(bx+a)\sinh(bx+a)}{9}-\frac{2(bx+a)\sinh(bx+a)\left(\cosh^{2}(bx+a)\right)}{9}+\frac{4\cosh(bx+a)}{9}+\frac{2\left(\cosh^{3}(bx+a)\right)}{27}-2a\left(\frac{(bx+a)\left(\cosh^{3}(bx+\right.}{3}\right.}{b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*cosh(b*x+a)^2*sinh(b*x+a),x)
```

```
[Out] 1/b^3*(1/3*(b*x+a)^2*cosh(b*x+a)^3-4/9*(b*x+a)*sinh(b*x+a)-2/9*(b*x+a)*sinh
(b*x+a)*cosh(b*x+a)^2+4/9*cosh(b*x+a)+2/27*cosh(b*x+a)^3-2*a*(1/3*(b*x+a)*c
osh(b*x+a)^3-2/9*sinh(b*x+a)-1/9*cosh(b*x+a)^2*sinh(b*x+a))+1/3*a^2*cosh(b*
x+a)^3)
```

maxima [A] time = 0.40, size = 122, normalized size = 1.47

$$\frac{\left(9\,b^{2}x^{2}e^{(3\,a)}-6\,bxe^{(3\,a)}+2\,e^{(3\,a)}\right)e^{(3\,bx)}}{216\,b^{3}}+\frac{\left(b^{2}x^{2}e^{a}-2\,bxe^{a}+2\,e^{a}\right)e^{(bx)}}{8\,b^{3}}+\frac{\left(b^{2}x^{2}+2\,bx+2\right)e^{(-bx-a)}}{8\,b^{3}}+\frac{\left(9\,b^{2}x^{2}+6\,bx+\right.}{216\,b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cosh(b*x+a)^2*sinh(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/216*(9*b^2*x^2*e^(3*a) - 6*b*x*e^(3*a) + 2*e^(3*a))*e^(3*b*x)/b^3 + 1/8*(
b^2*x^2*e^a - 2*b*x*e^a + 2*e^a)*e^(b*x)/b^3 + 1/8*(b^2*x^2 + 2*b*x + 2)*e^
(-b*x - a)/b^3 + 1/216*(9*b^2*x^2 + 6*b*x + 2)*e^(-3*b*x - 3*a)/b^3
```

mupad [B] time = 1.51, size = 69, normalized size = 0.83

$$\frac{\frac{4\cosh(a+b\,x)}{9}-b\,\left(\frac{2\,x\sinh(a+b\,x)\cosh(a+b\,x)^{2}}{9}+\frac{4\,x\sinh(a+b\,x)}{9}\right)+\frac{2\cosh(a+b\,x)^{3}}{27}+\frac{b^{2}\,x^{2}\cosh(a+b\,x)^{3}}{3}}{b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*cosh(a + b*x)^2*sinh(a + b*x),x)
```

```
[Out] ((4*cosh(a + b*x))/9 - b*((4*x*sinh(a + b*x))/9 + (2*x*cosh(a + b*x)^2*sinh
(a + b*x))/9) + (2*cosh(a + b*x)^3)/27 + (b^2*x^2*cosh(a + b*x)^3)/3)/b^3
```

sympy [A] time = 1.71, size = 105, normalized size = 1.27

$$\begin{cases} \frac{x^2\cosh^3(a+bx)}{3b} + \frac{4x\sinh^3(a+bx)}{9b^2} - \frac{2x\sinh(a+bx)\cosh^2(a+bx)}{3b^2} - \frac{4\sinh^2(a+bx)\cosh(a+bx)}{9b^3} + \frac{14\cosh^3(a+bx)}{27b^3} & \text{for } b \neq 0 \\ \frac{x^3\sinh(a)\cosh^2(a)}{3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*cosh(b*x+a)**2*sinh(b*x+a),x)
```

```
[Out] Piecewise((x**2*cosh(a + b*x)**3/(3*b) + 4*x*sinh(a + b*x)**3/(9*b**2) - 2*
x*sinh(a + b*x)*cosh(a + b*x)**2/(3*b**2) - 4*sinh(a + b*x)**2*cosh(a + b*x
)/(9*b**3) + 14*cosh(a + b*x)**3/(27*b**3), Ne(b, 0)), (x**3*sinh(a)*cosh(a
)**2/3, True))
```

3.262 $\int x \cosh^2(a+bx)\sinh(a+bx)\,dx$

Optimal. Leaf size=45

$$-\frac{\sinh^3(a+bx)}{9b^2}-\frac{\sinh(a+bx)}{3b^2}+\frac{x\cosh^3(a+bx)}{3b}$$

```
[Out] 1/3*x*cosh(b*x+a)^3/b-1/3*sinh(b*x+a)/b^2-1/9*sinh(b*x+a)^3/b^2
```

Rubi [A] time = 0.03, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {5373, 2633}

$$-\frac{\sinh^3(a+bx)}{9b^2}-\frac{\sinh(a+bx)}{3b^2}+\frac{x\cosh^3(a+bx)}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[x*Cosh[a + b*x]^2*Sinh[a + b*x],x]
```

```
[Out] (x*Cosh[a + b*x]^3)/(3*b) - Sinh[a + b*x]/(3*b^2) - Sinh[a + b*x]^3/(9*b^2)
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 5373

```
Int[Cosh[(a_.) + (b_.)*(x_)^(n_.)]^(p_.)*(x_)^(m_.)*Sinh[(a_.) + (b_.)*(x_)
^(n_.)], x_Symbol] :> Simp[(x^(m - n + 1)*Cosh[a + b*x^n]^(p + 1))/(b*n*(p
+ 1)), x] - Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*Cosh[a + b*x^n]^(
p + 1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\int x\cosh^2(a+bx)\sinh(a+bx)\,dx &= \frac{x\cosh^3(a+bx)}{3b}-\frac{\int \cosh^3(a+bx)\,dx}{3b}\\
&= \frac{x\cosh^3(a+bx)}{3b}-\frac{i\,\text{Subst}\left(\int \left(1-x^2\right)\,dx,x,-i\sinh(a+bx)\right)}{3b^2}\\
&= \frac{x\cosh^3(a+bx)}{3b}-\frac{\sinh(a+bx)}{3b^2}-\frac{\sinh^3(a+bx)}{9b^2}
\end{aligned}$$

Mathematica [A] time = 0.15, size = 46, normalized size = 1.02

$$-\frac{9\sinh(a+bx)+\sinh(3(a+bx))-9bx\cosh(a+bx)-3bx\cosh(3(a+bx))}{36b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Cosh[a + b*x]^2*Sinh[a + b*x],x]
```

```
[Out] -1/36*(-9*b*x*Cosh[a + b*x] - 3*b*x*Cosh[3*(a + b*x)] + 9*Sinh[a + b*x] + S
inh[3*(a + b*x)])/b^2
```

fricas [A] time = 0.72, size = 74, normalized size = 1.64

$$\frac{3\,bx\cosh\left(bx+a\right)^{3}+9\,bx\cosh\left(bx+a\right)\sinh\left(bx+a\right)^{2}+9\,bx\cosh\left(bx+a\right)-\sinh\left(bx+a\right)^{3}-3\left(\cosh\left(bx+a\right)^{2}\right.}{36\,b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x+a)^2*sinh(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/36*(3*b*x*cosh(b*x + a)^3 + 9*b*x*cosh(b*x + a)*sinh(b*x + a)^2 + 9*b*x*c
osh(b*x + a) - sinh(b*x + a)^3 - 3*(cosh(b*x + a)^2 + 3)*sinh(b*x + a))/b^2
```

giac [A] time = 0.12, size = 76, normalized size = 1.69

$$\frac{(3\,bx-1)e^{(3\,bx+3\,a)}}{72\,b^2}+\frac{(bx-1)e^{(bx+a)}}{8\,b^2}+\frac{(bx+1)e^{(-bx-a)}}{8\,b^2}+\frac{(3\,bx+1)e^{(-3\,bx-3\,a)}}{72\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x+a)^2*sinh(b*x+a),x, algorithm="giac")
```

```
[Out] 1/72*(3*b*x - 1)*e^(3*b*x + 3*a)/b^2 + 1/8*(b*x - 1)*e^(b*x + a)/b^2 + 1/8*
(b*x + 1)*e^(-b*x - a)/b^2 + 1/72*(3*b*x + 1)*e^(-3*b*x - 3*a)/b^2
```

maple [A] time = 0.32, size = 56, normalized size = 1.24

$$\frac{\frac{(bx+a)\left(\cosh^3(bx+a)\right)}{3}-\frac{2\sinh(bx+a)}{9}-\frac{\left(\cosh^2(bx+a)\right)\sinh(bx+a)}{9}-\frac{a\left(\cosh^3(bx+a)\right)}{3}}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cosh(b*x+a)^2*sinh(b*x+a),x)
```

```
[Out] 1/b^2*(1/3*(b*x+a)*cosh(b*x+a)^3-2/9*sinh(b*x+a)-1/9*cosh(b*x+a)^2*sinh(b*x
+a)-1/3*a*cosh(b*x+a)^3)
```

maxima [B] time = 0.32, size = 84, normalized size = 1.87

$$\frac{\left(3\,bxe^{(3\,a)}-e^{(3\,a)}\right)e^{(3\,bx)}}{72\,b^2}+\frac{(bxe^{a}-e^{a})e^{(bx)}}{8\,b^2}+\frac{(bx+1)e^{(-bx-a)}}{8\,b^2}+\frac{(3\,bx+1)e^{(-3\,bx-3\,a)}}{72\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x+a)^2*sinh(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/72*(3*b*x*e^(3*a) - e^(3*a))*e^(3*b*x)/b^2 + 1/8*(b*x*e^a - e^a)*e^(b*x)/
b^2 + 1/8*(b*x + 1)*e^(-b*x - a)/b^2 + 1/72*(3*b*x + 1)*e^(-3*b*x - 3*a)/b^
2
```

mupad [B] time = 0.08, size = 41, normalized size = 0.91

$$-\frac{-3\,b\,x\cosh\left(a+b\,x\right)^3+\sinh\left(a+b\,x\right)\,\cosh\left(a+b\,x\right)^2+2\sinh\left(a+b\,x\right)}{9\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cosh(a + b*x)^2*sinh(a + b*x),x)
```

```
[Out] -(2*sinh(a + b*x) + cosh(a + b*x)^2*sinh(a + b*x) - 3*b*x*cosh(a + b*x)^3)/
(9*b^2)
```

sympy [A] time = 0.81, size = 61, normalized size = 1.36

$$\begin{cases}\frac{x\cosh^3(a+bx)}{3b}+\frac{2\sinh^3(a+bx)}{9b^2}-\frac{\sinh(a+bx)\cosh^2(a+bx)}{3b^2} & \text{for } b\neq 0\\ \frac{x^2\sinh(a)\cosh^2(a)}{2} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x+a)**2*sinh(b*x+a),x)
```

```
[Out] Piecewise((x*cosh(a + b*x)**3/(3*b) + 2*sinh(a + b*x)**3/(9*b**2) - sinh(a
+ b*x)*cosh(a + b*x)**2/(3*b**2), Ne(b, 0)), (x**2*sinh(a)*cosh(a)**2/2, Tr
ue))
```

3.263 $\int \cosh^2(a+bx)\sinh(a+bx)\,dx$

Optimal. Leaf size=15

$$\frac{\cosh^3(a+bx)}{3b}$$

[Out] 1/3*cosh(b*x+a)^3/b

Rubi [A] time = 0.02, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2565, 30}

$$\frac{\cosh^3(a+bx)}{3b}$$

Antiderivative was successfully verified.

[In] Int[Cosh[a + b*x]^2*Sinh[a + b*x],x]

[Out] Cosh[a + b*x]^3/(3*b)

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rubi steps

$$\begin{aligned}\int \cosh^2(a+bx)\sinh(a+bx)\,dx &= \frac{\operatorname{Subst}\left(\int x^2\,dx, x, \cosh(a+bx)\right)}{b}\\ &= \frac{\cosh^3(a+bx)}{3b}\end{aligned}$$

Mathematica [A] time = 0.00, size = 15, normalized size = 1.00

$$\frac{\cosh^3(a+bx)}{3b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[a + b*x]^2*Sinh[a + b*x],x]
```

```
[Out] Cosh[a + b*x]^3/(3*b)
```

fricas [B] time = 0.93, size = 38, normalized size = 2.53

$$\frac{\cosh\left(bx+a\right)^{3}+3\,\cosh\left(bx+a\right)\sinh\left(bx+a\right)^{2}+3\,\cosh\left(bx+a\right)}{12\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^2*sinh(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/12*(cosh(b*x + a)^3 + 3*cosh(b*x + a)*sinh(b*x + a)^2 + 3*cosh(b*x + a))/
b
```

giac [B] time = 0.14, size = 54, normalized size = 3.60

$$\frac{e^{(3\,bx+3\,a)}}{24\,b}+\frac{e^{(bx+a)}}{8\,b}+\frac{e^{(-bx-a)}}{8\,b}+\frac{e^{(-3\,bx-3\,a)}}{24\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^2*sinh(b*x+a),x, algorithm="giac")
```

```
[Out] 1/24*e^(3*b*x + 3*a)/b + 1/8*e^(b*x + a)/b + 1/8*e^(-b*x - a)/b + 1/24*e^(-
3*b*x - 3*a)/b
```

maple [A] time = 0.02, size = 14, normalized size = 0.93

$$\frac{\cosh^{3}\left(bx+a\right)}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)^2*sinh(b*x+a),x)
```

```
[Out] 1/3*cosh(b*x+a)^3/b
```

maxima [A] time = 0.30, size = 13, normalized size = 0.87

$$\frac{\cosh\left(bx+a\right)^{3}}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^2*sinh(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/3*cosh(b*x + a)^3/b
```

mupad [B] time = 1.46, size = 13, normalized size = 0.87

$$\frac{\cosh\left(a+b\,x\right)^3}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a + b*x)^2*sinh(a + b*x),x)
```

```
[Out] cosh(a + b*x)^3/(3*b)
```

sympy [A] time = 0.38, size = 20, normalized size = 1.33

$$\begin{cases} \frac{\cosh^3\left(a+bx\right)}{3b} & \text{for } b \neq 0 \\ x\sinh\left(a\right)\cosh^2\left(a\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)**2*sinh(b*x+a),x)
```

```
[Out] Piecewise((cosh(a + b*x)**3/(3*b), Ne(b, 0)), (x*sinh(a)*cosh(a)**2, True))
```

3.264 $\int \frac{\cosh^2(a+bx)\sinh(a+bx)}{x}\,dx$

Optimal. Leaf size=47

$$\frac{1}{4}\sinh(a)\text{Chi}(bx)+\frac{1}{4}\sinh(3a)\text{Chi}(3bx)+\frac{1}{4}\cosh(a)\text{Shi}(bx)+\frac{1}{4}\cosh(3a)\text{Shi}(3bx)$$

```
[Out] 1/4*cosh(a)*Shi(b*x)+1/4*cosh(3*a)*Shi(3*b*x)+1/4*Chi(b*x)*sinh(a)+1/4*Chi(
3*b*x)*sinh(3*a)
```

Rubi [A] time = 0.14, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {5448, 3303, 3298, 3301}

$$\frac{1}{4}\sinh(a)\text{Chi}(bx)+\frac{1}{4}\sinh(3a)\text{Chi}(3bx)+\frac{1}{4}\cosh(a)\text{Shi}(bx)+\frac{1}{4}\cosh(3a)\text{Shi}(3bx)$$

Antiderivative was successfully verified.

```
[In] Int[(Cosh[a + b*x]^2*Sinh[a + b*x])/x,x]
```

```
[Out] (CoshIntegral[b*x]*Sinh[a])/4 + (CoshIntegral[3*b*x]*Sinh[3*a])/4 + (Cosh[a
]*SinhIntegral[b*x])/4 + (Cosh[3*a]*SinhIntegral[3*b*x])/4
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\cosh^2(a+bx)\sinh(a+bx)}{x}\,dx &= \int \left(\frac{\sinh(a+bx)}{4x}+\frac{\sinh(3a+3bx)}{4x}\right)dx\\ &= \frac{1}{4}\int \frac{\sinh(a+bx)}{x}\,dx+\frac{1}{4}\int \frac{\sinh(3a+3bx)}{x}\,dx\\ &= \frac{1}{4}\cosh(a)\int \frac{\sinh(bx)}{x}\,dx+\frac{1}{4}\cosh(3a)\int \frac{\sinh(3bx)}{x}\,dx+\frac{1}{4}\sinh(a)\int \frac{\text{co}}{}\\ &= \frac{1}{4}\text{Chi}(bx)\sinh(a)+\frac{1}{4}\text{Chi}(3bx)\sinh(3a)+\frac{1}{4}\cosh(a)\text{Shi}(bx)+\frac{1}{4}\cosh(3a)\end{aligned}$$

Mathematica [A] time = 0.07, size = 39, normalized size = 0.83

$$\frac{1}{4}(\sinh(a)\text{Chi}(bx)+\sinh(3a)\text{Chi}(3bx)+\cosh(a)\text{Shi}(bx)+\cosh(3a)\text{Shi}(3bx))$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cosh[a + b*x]^2*Sinh[a + b*x])/x,x]
```

```
[Out] (CoshIntegral[b*x]*Sinh[a] + CoshIntegral[3*b*x]*Sinh[3*a] + Cosh[a]*SinhIn
tegral[b*x] + Cosh[3*a]*SinhIntegral[3*b*x])/4
```

fricas [A] time = 0.63, size = 67, normalized size = 1.43

$$\frac{1}{8}\left(\text{Ei}\left(3\,bx\right)-\text{Ei}\left(-3\,bx\right)\right)\cosh\left(3\,a\right)+\frac{1}{8}\left(\text{Ei}\left(bx\right)-\text{Ei}\left(-bx\right)\right)\cosh(a)+\frac{1}{8}\left(\text{Ei}\left(3\,bx\right)+\text{Ei}\left(-3\,bx\right)\right)\sinh\left(3\,a\right)+\frac{1}{8}\left(\text{Ei}\left(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^2*sinh(b*x+a)/x,x, algorithm="fricas")
```

```
[Out] 1/8*(Ei(3*b*x) - Ei(-3*b*x))*cosh(3*a) + 1/8*(Ei(b*x) - Ei(-b*x))*cosh(a) +
 1/8*(Ei(3*b*x) + Ei(-3*b*x))*sinh(3*a) + 1/8*(Ei(b*x) + Ei(-b*x))*sinh(a)
```

giac [A] time = 0.12, size = 42, normalized size = 0.89

$$\frac{1}{8}\,\text{Ei}\left(3\,bx\right)e^{(3\,a)}-\frac{1}{8}\,\text{Ei}\left(-bx\right)e^{(-a)}-\frac{1}{8}\,\text{Ei}\left(-3\,bx\right)e^{(-3\,a)}+\frac{1}{8}\,\text{Ei}\left(bx\right)e^{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^2*sinh(b*x+a)/x,x, algorithm="giac")
```

```
[Out] 1/8*Ei(3*b*x)*e^(3*a) - 1/8*Ei(-b*x)*e^(-a) - 1/8*Ei(-3*b*x)*e^(-3*a) + 1/8
*Ei(b*x)*e^a
```

maple [A] time = 0.35, size = 47, normalized size = 1.00

$$\frac{e^{-3a} \operatorname{Ei}(1, 3bx)}{8} + \frac{e^{-a} \operatorname{Ei}(1, bx)}{8} - \frac{e^{a} \operatorname{Ei}(1, -bx)}{8} - \frac{e^{3a} \operatorname{Ei}(1, -3bx)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)^2*sinh(b*x+a)/x,x)
```

```
[Out] 1/8*exp(-3*a)*Ei(1,3*b*x)+1/8*exp(-a)*Ei(1,b*x)-1/8*exp(a)*Ei(1,-b*x)-1/8*e
xp(3*a)*Ei(1,-3*b*x)
```

maxima [A] time = 0.53, size = 42, normalized size = 0.89

$$\frac{1}{8} \operatorname{Ei}(3 bx) e^{(3 a)} - \frac{1}{8} \operatorname{Ei}(-bx) e^{(-a)} - \frac{1}{8} \operatorname{Ei}(-3 bx) e^{(-3 a)} + \frac{1}{8} \operatorname{Ei}(bx) e^{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^2*sinh(b*x+a)/x,x, algorithm="maxima")
```

```
[Out] 1/8*Ei(3*b*x)*e^(3*a) - 1/8*Ei(-b*x)*e^(-a) - 1/8*Ei(-3*b*x)*e^(-3*a) + 1/8
*Ei(b*x)*e^a
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\cosh(a + b x)^2 \sinh(a + b x)}{x} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cosh(a + b*x)^2*sinh(a + b*x))/x,x)
```

```
[Out] int((cosh(a + b*x)^2*sinh(a + b*x))/x, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh(a + bx) \cosh^2(a + bx)}{x} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)**2*sinh(b*x+a)/x,x)
```

```
[Out] Integral(sinh(a + b*x)*cosh(a + b*x)**2/x, x)
```

3.265 $\int \frac{\cosh^2(a+bx)\sinh(a+bx)}{x^2}\,dx$

Optimal. Leaf size=80

$$\frac{1}{4}b\cosh(a)\mathrm{Chi}(bx)+\frac{3}{4}b\cosh(3a)\mathrm{Chi}(3bx)+\frac{1}{4}b\sinh(a)\mathrm{Shi}(bx)+\frac{3}{4}b\sinh(3a)\mathrm{Shi}(3bx)-\frac{\sinh(a+bx)}{4x}-\frac{\sinh(3a+}{4x}$$

```
[Out] 1/4*b*Chi(b*x)*cosh(a)+3/4*b*Chi(3*b*x)*cosh(3*a)+1/4*b*Shi(b*x)*sinh(a)+3/
4*b*Shi(3*b*x)*sinh(3*a)-1/4*sinh(b*x+a)/x-1/4*sinh(3*b*x+3*a)/x
```

Rubi [A] time = 0.19, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {5448, 3297, 3303, 3298, 3301}

$$\frac{1}{4}b\cosh(a)\mathrm{Chi}(bx)+\frac{3}{4}b\cosh(3a)\mathrm{Chi}(3bx)+\frac{1}{4}b\sinh(a)\mathrm{Shi}(bx)+\frac{3}{4}b\sinh(3a)\mathrm{Shi}(3bx)-\frac{\sinh(a+bx)}{4x}-\frac{\sinh(3a+}{4x}$$

Antiderivative was successfully verified.

```
[In] Int[(Cosh[a + b*x]^2*Sinh[a + b*x])/x^2,x]
```

```
[Out] (b*Cosh[a]*CoshIntegral[b*x])/4 + (3*b*Cosh[3*a]*CoshIntegral[3*b*x])/4 - S
inh[a + b*x]/(4*x) - Sinh[3*a + 3*b*x]/(4*x) + (b*Sinh[a]*SinhIntegral[b*x]
)/4 + (3*b*Sinh[3*a]*SinhIntegral[3*b*x])/4
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\cosh^2(a+bx)\sinh(a+bx)}{x^2}\,dx &= \int \left(\frac{\sinh(a+bx)}{4x^2}+\frac{\sinh(3a+3bx)}{4x^2}\right)dx\\ &= \frac{1}{4}\int \frac{\sinh(a+bx)}{x^2}\,dx+\frac{1}{4}\int \frac{\sinh(3a+3bx)}{x^2}\,dx\\ &= -\frac{\sinh(a+bx)}{4x}-\frac{\sinh(3a+3bx)}{4x}+\frac{1}{4}b\int \frac{\cosh(a+bx)}{x}\,dx+\frac{1}{4}(3b)\int \frac{\cosh(\\ &= -\frac{\sinh(a+bx)}{4x}-\frac{\sinh(3a+3bx)}{4x}+\frac{1}{4}(b\cosh(a))\int \frac{\cosh(bx)}{x}\,dx+\frac{1}{4}(3b\cos\\ &= \frac{1}{4}b\cosh(a)\text{Chi}(bx)+\frac{3}{4}b\cosh(3a)\text{Chi}(3bx)-\frac{\sinh(a+bx)}{4x}-\frac{\sinh(3a+3bx}{4x}\end{aligned}$$

Mathematica [A] time = 0.16, size = 70, normalized size = 0.88

$$\frac{bx\cosh(a)\text{Chi}(bx)+3bx\cosh(3a)\text{Chi}(3bx)+bx\sinh(a)\text{Shi}(bx)+3bx\sinh(3a)\text{Shi}(3bx)-\sinh(a+bx)-\sinh(}{4x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cosh[a + b*x]^2*Sinh[a + b*x])/x^2,x]
```

```
[Out] (b*x*Cosh[a]*CoshIntegral[b*x] + 3*b*x*Cosh[3*a]*CoshIntegral[3*b*x] - Sinh
[a + b*x] - Sinh[3*(a + b*x)] + b*x*Sinh[a]*SinhIntegral[b*x] + 3*b*x*Sinh[
3*a]*SinhIntegral[3*b*x])/(4*x)
```

fricas [A] time = 0.55, size = 124, normalized size = 1.55

$$-\frac{2\sinh(bx+a)^3-3(bx\text{Ei}(3bx)+bx\text{Ei}(-3bx))\cosh(3a)-(bx\text{Ei}(bx)+bx\text{Ei}(-bx))\cosh(a)+2\Big(3\cosh(bx+}{8x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^2*sinh(b*x+a)/x^2,x, algorithm="fricas")
```

```
[Out] -1/8*(2*sinh(b*x + a)^3 - 3*(b*x*Ei(3*b*x) + b*x*Ei(-3*b*x))*cosh(3*a) - (b
*x*Ei(b*x) + b*x*Ei(-b*x))*cosh(a) + 2*(3*cosh(b*x + a)^2 + 1)*sinh(b*x + a
) - 3*(b*x*Ei(3*b*x) - b*x*Ei(-3*b*x))*sinh(3*a) - (b*x*Ei(b*x) - b*x*Ei(-b
*x))*sinh(a))/x
```

giac [A] time = 0.14, size = 90, normalized size = 1.12

$$\frac{3\,bx\mathrm{Ei}\left(3\,bx\right)e^{\left(3\,a\right)} + bx\mathrm{Ei}\left(-bx\right)e^{\left(-a\right)} + 3\,bx\mathrm{Ei}\left(-3\,bx\right)e^{\left(-3\,a\right)} + bx\mathrm{Ei}\left(bx\right)e^{a} - e^{\left(3\,bx+3\,a\right)} - e^{\left(bx+a\right)} + e^{\left(-bx-a\right)} + e^{\left(-3\,bx-3\,a\right)}}{8\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^2*sinh(b*x+a)/x^2,x, algorithm="giac")
```

```
[Out] 1/8*(3*b*x*Ei(3*b*x)*e^(3*a) + b*x*Ei(-b*x)*e^(-a) + 3*b*x*Ei(-3*b*x)*e^(-3
*a) + b*x*Ei(b*x)*e^a - e^(3*b*x + 3*a) - e^(b*x + a) + e^(-b*x - a) + e^(-
3*b*x - 3*a))/x
```

maple [A] time = 0.36, size = 104, normalized size = 1.30

$$\frac{\mathrm{e}^{-3bx-3a}}{8x}-\frac{3b\,\mathrm{e}^{-3a}\,\mathrm{Ei}\left(1,3bx\right)}{8}+\frac{\mathrm{e}^{-bx-a}}{8x}-\frac{b\,\mathrm{e}^{-a}\,\mathrm{Ei}\left(1,bx\right)}{8}-\frac{\mathrm{e}^{bx+a}}{8x}-\frac{b\,\mathrm{e}^{a}\,\mathrm{Ei}\left(1,-bx\right)}{8}-\frac{\mathrm{e}^{3bx+3a}}{8x}-\frac{3b\,\mathrm{e}^{3a}\,\mathrm{Ei}\left(1,-3bx\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)^2*sinh(b*x+a)/x^2,x)
```

```
[Out] 1/8*exp(-3*b*x-3*a)/x-3/8*b*exp(-3*a)*Ei(1,3*b*x)+1/8*exp(-b*x-a)/x-1/8*b*e
xp(-a)*Ei(1,b*x)-1/8/x*exp(b*x+a)-1/8*b*exp(a)*Ei(1,-b*x)-1/8/x*exp(3*b*x+3
*a)-3/8*b*exp(3*a)*Ei(1,-3*b*x)
```

maxima [A] time = 0.54, size = 50, normalized size = 0.62

$$\frac{3}{8}\,be^{\left(-3\,a\right)}\Gamma\left(-1, 3\,bx\right) + \frac{1}{8}\,be^{\left(-a\right)}\Gamma\left(-1, bx\right) + \frac{1}{8}\,be^{a}\Gamma\left(-1, -bx\right) + \frac{3}{8}\,be^{\left(3\,a\right)}\Gamma\left(-1, -3\,bx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^2*sinh(b*x+a)/x^2,x, algorithm="maxima")
```

```
[Out] 3/8*b*e^(-3*a)*gamma(-1, 3*b*x) + 1/8*b*e^(-a)*gamma(-1, b*x) + 1/8*b*e^a*g
amma(-1, -b*x) + 3/8*b*e^(3*a)*gamma(-1, -3*b*x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cosh (a+b\,x)^2\,\sinh (a+b\,x)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cosh(a + b*x)^2*sinh(a + b*x))/x^2,x)
```

```
[Out] int((cosh(a + b*x)^2*sinh(a + b*x))/x^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh (a+bx)\cosh^2 (a+bx)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)**2*sinh(b*x+a)/x**2,x)
```

```
[Out] Integral(sinh(a + b*x)*cosh(a + b*x)**2/x**2, x)
```

3.266 $\int \frac{\cosh^2(a+bx)\sinh(a+bx)}{x^3}\,dx$

Optimal. Leaf size=119

$$\frac{1}{8}b^2\sinh(a)\text{Chi}(bx)+\frac{9}{8}b^2\sinh(3a)\text{Chi}(3bx)+\frac{1}{8}b^2\cosh(a)\text{Shi}(bx)+\frac{9}{8}b^2\cosh(3a)\text{Shi}(3bx)-\frac{\sinh(a+bx)}{8x^2}-\frac{\sinh(3}{}$$

```
[Out] -1/8*b*cosh(b*x+a)/x-3/8*b*cosh(3*b*x+3*a)/x+1/8*b^2*cosh(a)*Shi(b*x)+9/8*b
^2*cosh(3*a)*Shi(3*b*x)+1/8*b^2*Chi(b*x)*sinh(a)+9/8*b^2*Chi(3*b*x)*sinh(3*
a)-1/8*sinh(b*x+a)/x^2-1/8*sinh(3*b*x+3*a)/x^2
```

Rubi [A] time = 0.25, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {5448, 3297, 3303, 3298, 3301}

$$\frac{1}{8}b^2\sinh(a)\text{Chi}(bx)+\frac{9}{8}b^2\sinh(3a)\text{Chi}(3bx)+\frac{1}{8}b^2\cosh(a)\text{Shi}(bx)+\frac{9}{8}b^2\cosh(3a)\text{Shi}(3bx)-\frac{\sinh(a+bx)}{8x^2}-\frac{\sinh(3}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Cosh[a + b*x]^2*Sinh[a + b*x])/x^3,x]
```

```
[Out] -(b*Cosh[a + b*x])/(8*x) - (3*b*Cosh[3*a + 3*b*x])/(8*x) + (b^2*CoshIntegra
l[b*x]*Sinh[a])/8 + (9*b^2*CoshIntegral[3*b*x]*Sinh[3*a])/8 - Sinh[a + b*x]
/(8*x^2) - Sinh[3*a + 3*b*x]/(8*x^2) + (b^2*Cosh[a]*SinhIntegral[b*x])/8 +
(9*b^2*Cosh[3*a]*SinhIntegral[3*b*x])/8
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\cosh^2(a+bx)\sinh(a+bx)}{x^3}\,dx &= \int \left(\frac{\sinh(a+bx)}{4x^3}+\frac{\sinh(3a+3bx)}{4x^3}\right)dx\\ &= \frac{1}{4}\int \frac{\sinh(a+bx)}{x^3}\,dx+\frac{1}{4}\int \frac{\sinh(3a+3bx)}{x^3}\,dx\\ &= -\frac{\sinh(a+bx)}{8x^2}-\frac{\sinh(3a+3bx)}{8x^2}+\frac{1}{8}b\int \frac{\cosh(a+bx)}{x^2}\,dx+\frac{1}{8}(3b)\int \frac{\cosh(\\ &= -\frac{b\cosh(a+bx)}{8x}-\frac{3b\cosh(3a+3bx)}{8x}-\frac{\sinh(a+bx)}{8x^2}-\frac{\sinh(3a+3bx)}{8x^2}+\frac{1}{8}\\ &= -\frac{b\cosh(a+bx)}{8x}-\frac{3b\cosh(3a+3bx)}{8x}-\frac{\sinh(a+bx)}{8x^2}-\frac{\sinh(3a+3bx)}{8x^2}+\frac{1}{8}\\ &= -\frac{b\cosh(a+bx)}{8x}-\frac{3b\cosh(3a+3bx)}{8x}+\frac{1}{8}b^2\text{Chi}(bx)\sinh(a)+\frac{9}{8}b^2\text{Chi}(3bx)\end{aligned}$$

Mathematica [A] time = 0.29, size = 105, normalized size = 0.88

$$-\frac{-b^2x^2\sinh(a)\text{Chi}(bx)-9b^2x^2\sinh(3a)\text{Chi}(3bx)-b^2x^2\cosh(a)\text{Shi}(bx)-9b^2x^2\cosh(3a)\text{Shi}(3bx)+\sinh(a+b}{8x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cosh[a + b*x]^2*Sinh[a + b*x])/x^3,x]
```

```
[Out] -1/8*(b*x*Cosh[a + b*x] + 3*b*x*Cosh[3*(a + b*x)] - b^2*x^2*CoshIntegral[b*
x]*Sinh[a] - 9*b^2*x^2*CoshIntegral[3*b*x]*Sinh[3*a] + Sinh[a + b*x] + Sinh
[3*(a + b*x)] - b^2*x^2*Cosh[a]*SinhIntegral[b*x] - 9*b^2*x^2*Cosh[3*a]*Sin
hIntegral[3*b*x])/x^2
```

fricas [A] time = 0.61, size = 196, normalized size = 1.65

$$-\frac{6\,bx\cosh\left(bx+a\right)^{3}+18\,bx\cosh\left(bx+a\right)\sinh\left(bx+a\right)^{2}+2\,bx\cosh\left(bx+a\right)+2\,\sinh\left(bx+a\right)^{3}-9\left(b^{2}x^{2}\mathrm{Ei}\left(3\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^2*sinh(b*x+a)/x^3,x, algorithm="fricas")
```

```
[Out] -1/16*(6*b*x*cosh(b*x + a)^3 + 18*b*x*cosh(b*x + a)*sinh(b*x + a)^2 + 2*b*x
*cosh(b*x + a) + 2*sinh(b*x + a)^3 - 9*(b^2*x^2*Ei(3*b*x) - b^2*x^2*Ei(-3*b
*x))*cosh(3*a) - (b^2*x^2*Ei(b*x) - b^2*x^2*Ei(-b*x))*cosh(a) + 2*(3*cosh(b
*x + a)^2 + 1)*sinh(b*x + a) - 9*(b^2*x^2*Ei(3*b*x) + b^2*x^2*Ei(-3*b*x))*s
inh(3*a) - (b^2*x^2*Ei(b*x) + b^2*x^2*Ei(-b*x))*sinh(a))/x^2
```

giac [A] time = 0.14, size = 156, normalized size = 1.31

$$\frac{9\,b^{2}x^{2}\mathrm{Ei}\left(3\,bx\right)e^{(3\,a)}-b^{2}x^{2}\mathrm{Ei}\left(-bx\right)e^{(-a)}-9\,b^{2}x^{2}\mathrm{Ei}\left(-3\,bx\right)e^{(-3\,a)}+b^{2}x^{2}\mathrm{Ei}\left(bx\right)e^{a}-3\,bxe^{(3\,bx+3\,a)}-bxe^{(bx+a)}-bx}{16\,x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^2*sinh(b*x+a)/x^3,x, algorithm="giac")
```

```
[Out] 1/16*(9*b^2*x^2*Ei(3*b*x)*e^(3*a) - b^2*x^2*Ei(-b*x)*e^(-a) - 9*b^2*x^2*Ei(
-3*b*x)*e^(-3*a) + b^2*x^2*Ei(b*x)*e^a - 3*b*x*e^(3*b*x + 3*a) - b*x*e^(b*x
 + a) - b*x*e^(-b*x - a) - 3*b*x*e^(-3*b*x - 3*a) - e^(3*b*x + 3*a) - e^(b*
x + a) + e^(-b*x - a) + e^(-3*b*x - 3*a))/x^2
```

maple [A] time = 0.37, size = 169, normalized size = 1.42

$$-\frac{3b\,\mathrm{e}^{-3bx-3a}}{16x}+\frac{\mathrm{e}^{-3bx-3a}}{16x^{2}}+\frac{9b^{2}\mathrm{e}^{-3a}\,\mathrm{Ei}\left(1,3bx\right)}{16}-\frac{b\,\mathrm{e}^{-bx-a}}{16x}+\frac{\mathrm{e}^{-bx-a}}{16x^{2}}+\frac{b^{2}\mathrm{e}^{-a}\,\mathrm{Ei}\left(1,bx\right)}{16}-\frac{\mathrm{e}^{bx+a}}{16x^{2}}-\frac{b\,\mathrm{e}^{bx+a}}{16x}-\frac{b^{2}\mathrm{e}^{a}\,\mathrm{Ei}\left(1,-bx\right)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)^2*sinh(b*x+a)/x^3,x)
```

```
[Out] -3/16*b*exp(-3*b*x-3*a)/x+1/16*exp(-3*b*x-3*a)/x^2+9/16*b^2*exp(-3*a)*Ei(1,
3*b*x)-1/16*b*exp(-b*x-a)/x+1/16*exp(-b*x-a)/x^2+1/16*b^2*exp(-a)*Ei(1,b*x)
-1/16/x^2*exp(b*x+a)-1/16*b/x*exp(b*x+a)-1/16*b^2*exp(a)*Ei(1,-b*x)-1/16/x^
2*exp(3*b*x+3*a)-3/16*b/x*exp(3*b*x+3*a)-9/16*b^2*exp(3*a)*Ei(1,-3*b*x)
```

maxima [A] time = 0.41, size = 58, normalized size = 0.49

$$\frac{9}{8}\,b^{2}e^{(-3\,a)}\Gamma\left(-2,3\,bx\right)+\frac{1}{8}\,b^{2}e^{(-a)}\Gamma\left(-2,bx\right)-\frac{1}{8}\,b^{2}e^{a}\Gamma\left(-2,-bx\right)-\frac{9}{8}\,b^{2}e^{(3\,a)}\Gamma\left(-2,-3\,bx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^2*sinh(b*x+a)/x^3,x, algorithm="maxima")
```

```
[Out] 9/8*b^2*e^(-3*a)*gamma(-2, 3*b*x) + 1/8*b^2*e^(-a)*gamma(-2, b*x) - 1/8*b^2
*e^a*gamma(-2, -b*x) - 9/8*b^2*e^(3*a)*gamma(-2, -3*b*x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cosh(a+b\,x)^2\,\sinh(a+b\,x)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cosh(a + b*x)^2*sinh(a + b*x))/x^3,x)
```

```
[Out] int((cosh(a + b*x)^2*sinh(a + b*x))/x^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh(a+bx)\cosh^2(a+bx)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)**2*sinh(b*x+a)/x**3,x)
```

```
[Out] Integral(sinh(a + b*x)*cosh(a + b*x)**2/x**3, x)
```

3.267 $\int \frac{\cosh^2(a+bx)\sinh(a+bx)}{x^4}\,dx$

Optimal. Leaf size=154

$$\frac{1}{24}b^3\cosh(a)\text{Chi}(bx)+\frac{9}{8}b^3\cosh(3a)\text{Chi}(3bx)+\frac{1}{24}b^3\sinh(a)\text{Shi}(bx)+\frac{9}{8}b^3\sinh(3a)\text{Shi}(3bx)-\frac{b^2\sinh(a+bx)}{24x}-$$

```
[Out] 1/24*b^3*Chi(b*x)*cosh(a)+9/8*b^3*Chi(3*b*x)*cosh(3*a)-1/24*b*cosh(b*x+a)/x
^2-1/8*b*cosh(3*b*x+3*a)/x^2+1/24*b^3*Shi(b*x)*sinh(a)+9/8*b^3*Shi(3*b*x)*s
inh(3*a)-1/12*sinh(b*x+a)/x^3-1/24*b^2*sinh(b*x+a)/x-1/12*sinh(3*b*x+3*a)/x
^3-3/8*b^2*sinh(3*b*x+3*a)/x
```

Rubi [A] time = 0.29, antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {5448, 3297, 3303, 3298, 3301}

$$\frac{1}{24}b^3\cosh(a)\text{Chi}(bx)+\frac{9}{8}b^3\cosh(3a)\text{Chi}(3bx)+\frac{1}{24}b^3\sinh(a)\text{Shi}(bx)+\frac{9}{8}b^3\sinh(3a)\text{Shi}(3bx)-\frac{b^2\sinh(a+bx)}{24x}-$$

Antiderivative was successfully verified.

```
[In] Int[(Cosh[a + b*x]^2*Sinh[a + b*x])/x^4,x]
```

```
[Out] -(b*Cosh[a + b*x])/(24*x^2) - (b*Cosh[3*a + 3*b*x])/(8*x^2) + (b^3*Cosh[a]*
CoshIntegral[b*x])/24 + (9*b^3*Cosh[3*a]*CoshIntegral[3*b*x])/8 - Sinh[a +
b*x]/(12*x^3) - (b^2*Sinh[a + b*x])/(24*x) - Sinh[3*a + 3*b*x]/(12*x^3) - (
3*b^2*Sinh[3*a + 3*b*x])/(8*x) + (b^3*Sinh[a]*SinhIntegral[b*x])/24 + (9*b^
3*Sinh[3*a]*SinhIntegral[3*b*x])/8
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
```

```
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cosh^2(a+bx)\sinh(a+bx)}{x^4}\,dx &= \int \left(\frac{\sinh(a+bx)}{4x^4}+\frac{\sinh(3a+3bx)}{4x^4}\right)dx\\
&= \frac{1}{4}\int \frac{\sinh(a+bx)}{x^4}\,dx+\frac{1}{4}\int \frac{\sinh(3a+3bx)}{x^4}\,dx\\
&= -\frac{\sinh(a+bx)}{12x^3}-\frac{\sinh(3a+3bx)}{12x^3}+\frac{1}{12}b\int \frac{\cosh(a+bx)}{x^3}\,dx+\frac{1}{4}b\int \frac{\cosh(3a}{x}\\
&= -\frac{b\cosh(a+bx)}{24x^2}-\frac{b\cosh(3a+3bx)}{8x^2}-\frac{\sinh(a+bx)}{12x^3}-\frac{\sinh(3a+3bx)}{12x^3}+\frac{1}{24}\\
&= -\frac{b\cosh(a+bx)}{24x^2}-\frac{b\cosh(3a+3bx)}{8x^2}-\frac{\sinh(a+bx)}{12x^3}-\frac{b^2\sinh(a+bx)}{24x}-\sin\\
&= -\frac{b\cosh(a+bx)}{24x^2}-\frac{b\cosh(3a+3bx)}{8x^2}-\frac{\sinh(a+bx)}{12x^3}-\frac{b^2\sinh(a+bx)}{24x}-\sin\\
&= -\frac{b\cosh(a+bx)}{24x^2}-\frac{b\cosh(3a+3bx)}{8x^2}+\frac{1}{24}b^3\cosh(a)\text{Chi}(bx)+\frac{9}{8}b^3\cosh(3a
\end{aligned}$$

Mathematica [A] time = 0.36, size = 138, normalized size = 0.90

$$-\frac{-b^3x^3\cosh(a)\text{Chi}(bx)-27b^3x^3\cosh(3a)\text{Chi}(3bx)-b^3x^3\sinh(a)\text{Shi}(bx)-27b^3x^3\sinh(3a)\text{Shi}(3bx)+b^2x^2\sin}{24}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cosh[a + b*x]^2*Sinh[a + b*x])/x^4,x]
```

```
[Out] -1/24*(b*x*Cosh[a + b*x] + 3*b*x*Cosh[3*(a + b*x)] - b^3*x^3*Cosh[a]*CoshIn
tegral[b*x] - 27*b^3*x^3*Cosh[3*a]*CoshIntegral[3*b*x] + 2*Sinh[a + b*x] +
b^2*x^2*Sinh[a + b*x] + 2*Sinh[3*(a + b*x)] + 9*b^2*x^2*Sinh[3*(a + b*x)] -
 b^3*x^3*Sinh[a]*SinhIntegral[b*x] - 27*b^3*x^3*Sinh[3*a]*SinhIntegral[3*b*
x])/x^3
```

fricas [A] time = 0.50, size = 223, normalized size = 1.45

$$-\frac{6\,bx\cosh\left(bx+a\right)^{3}+18\,bx\cosh\left(bx+a\right)\sinh\left(bx+a\right)^{2}+2\left(9\,b^{2}x^{2}+2\right)\sinh\left(bx+a\right)^{3}+2\,bx\cosh\left(bx+a\right)-}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^2*sinh(b*x+a)/x^4,x, algorithm="fricas")
```

```
[Out] -1/48*(6*b*x*cosh(b*x + a)^3 + 18*b*x*cosh(b*x + a)*sinh(b*x + a)^2 + 2*(9*
b^2*x^2 + 2)*sinh(b*x + a)^3 + 2*b*x*cosh(b*x + a) - 27*(b^3*x^3*Ei(3*b*x)
+ b^3*x^3*Ei(-3*b*x))*cosh(3*a) - (b^3*x^3*Ei(b*x) + b^3*x^3*Ei(-b*x))*cosh
(a) + 2*(b^2*x^2 + 3*(9*b^2*x^2 + 2)*cosh(b*x + a)^2 + 2)*sinh(b*x + a) - 2
7*(b^3*x^3*Ei(3*b*x) - b^3*x^3*Ei(-3*b*x))*sinh(3*a) - (b^3*x^3*Ei(b*x) - b
^3*x^3*Ei(-b*x))*sinh(a))/x^3
```

giac [A] time = 0.14, size = 223, normalized size = 1.45

$$\frac{27\,b^{3}x^{3}\mathrm{Ei}\left(3\,bx\right)e^{(3\,a)}+b^{3}x^{3}\mathrm{Ei}\left(-bx\right)e^{(-a)}+27\,b^{3}x^{3}\mathrm{Ei}\left(-3\,bx\right)e^{(-3\,a)}+b^{3}x^{3}\mathrm{Ei}\left(bx\right)e^{a}-9\,b^{2}x^{2}e^{(3\,bx+3\,a)}-b^{2}x^{2}e^{(bx+}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^2*sinh(b*x+a)/x^4,x, algorithm="giac")
```

```
[Out] 1/48*(27*b^3*x^3*Ei(3*b*x)*e^(3*a) + b^3*x^3*Ei(-b*x)*e^(-a) + 27*b^3*x^3*E
i(-3*b*x)*e^(-3*a) + b^3*x^3*Ei(b*x)*e^a - 9*b^2*x^2*e^(3*b*x + 3*a) - b^2*
x^2*e^(b*x + a) + b^2*x^2*e^(-b*x - a) + 9*b^2*x^2*e^(-3*b*x - 3*a) - 3*b*x
*e^(3*b*x + 3*a) - b*x*e^(b*x + a) - b*x*e^(-b*x - a) - 3*b*x*e^(-3*b*x - 3
*a) - 2*e^(3*b*x + 3*a) - 2*e^(b*x + a) + 2*e^(-b*x - a) + 2*e^(-3*b*x - 3*
a))/x^3
```

maple [A] time = 0.37, size = 234, normalized size = 1.52

$$\frac{3b^{2}\mathrm{e}^{-3bx-3a}}{16x}-\frac{b\,\mathrm{e}^{-3bx-3a}}{16x^{2}}+\frac{\mathrm{e}^{-3bx-3a}}{24x^{3}}-\frac{9b^{3}\mathrm{e}^{-3a}\,\mathrm{Ei}\left(1,3bx\right)}{16}+\frac{b^{2}\mathrm{e}^{-bx-a}}{48x}-\frac{b\,\mathrm{e}^{-bx-a}}{48x^{2}}+\frac{\mathrm{e}^{-bx-a}}{24x^{3}}-\frac{b^{3}\mathrm{e}^{-a}\,\mathrm{Ei}\left(1,bx\right)}{48}-\frac{\mathrm{e}^{bx+a}}{24x^{3}}-\frac{b\,\mathrm{e}^{b}}{48}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)^2*sinh(b*x+a)/x^4,x)
```

```
[Out] 3/16*b^2*exp(-3*b*x-3*a)/x-1/16*b*exp(-3*b*x-3*a)/x^2+1/24*exp(-3*b*x-3*a)/
x^3-9/16*b^3*exp(-3*a)*Ei(1,3*b*x)+1/48*b^2*exp(-b*x-a)/x-1/48*b*exp(-b*x-a
)/x^2+1/24*exp(-b*x-a)/x^3-1/48*b^3*exp(-a)*Ei(1,b*x)-1/24/x^3*exp(b*x+a)-1
/48*b/x^2*exp(b*x+a)-1/48*b^2/x*exp(b*x+a)-1/48*b^3*exp(a)*Ei(1,-b*x)-1/24/
x^3*exp(3*b*x+3*a)-1/16*b/x^2*exp(3*b*x+3*a)-3/16*b^2/x*exp(3*b*x+3*a)-9/16
*b^3*exp(3*a)*Ei(1,-3*b*x)
```

maxima [A] time = 0.41, size = 58, normalized size = 0.38

$$\frac{27}{8}\,b^3e^{(-3\,a)}\Gamma\left(-3,3\,bx\right)+\frac{1}{8}\,b^3e^{(-a)}\Gamma\left(-3,bx\right)+\frac{1}{8}\,b^3e^{a}\Gamma\left(-3,-bx\right)+\frac{27}{8}\,b^3e^{(3\,a)}\Gamma\left(-3,-3\,bx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^2*sinh(b*x+a)/x^4,x, algorithm="maxima")
```

```
[Out] 27/8*b^3*e^(-3*a)*gamma(-3, 3*b*x) + 1/8*b^3*e^(-a)*gamma(-3, b*x) + 1/8*b^
3*e^a*gamma(-3, -b*x) + 27/8*b^3*e^(3*a)*gamma(-3, -3*b*x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cosh\left(a+b\,x\right)^2\,\sinh\left(a+b\,x\right)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cosh(a + b*x)^2*sinh(a + b*x))/x^4,x)
```

```
[Out] int((cosh(a + b*x)^2*sinh(a + b*x))/x^4, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh\left(a+bx\right)\cosh^{2}\left(a+bx\right)}{x^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)**2*sinh(b*x+a)/x**4,x)
```

```
[Out] Integral(sinh(a + b*x)*cosh(a + b*x)**2/x**4, x)
```

3.268 $\int x^m \cosh^3(a+bx)\sinh(a+bx)\,dx$

Optimal. Leaf size=139

$$\frac{e^{4a}2^{-2(m+3)}x^m(-bx)^{-m}\Gamma(m+1,-4bx)}{b}+\frac{e^{2a}2^{-m-4}x^m(-bx)^{-m}\Gamma(m+1,-2bx)}{b}+\frac{e^{-2a}2^{-m-4}x^m(bx)^{-m}\Gamma(m+1,2bx)}{b}+$$

```
[Out] exp(4*a)*x^m*GAMMA(1+m,-4*b*x)/(2^(6+2*m))/b/((-b*x)^m)+2^(-4-m)*exp(2*a)*x
^m*GAMMA(1+m,-2*b*x)/b/((-b*x)^m)+2^(-4-m)*x^m*GAMMA(1+m,2*b*x)/b/exp(2*a)/
((b*x)^m)+x^m*GAMMA(1+m,4*b*x)/(2^(6+2*m))/b/exp(4*a)/((b*x)^m)
```

Rubi [A] time = 0.24, antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {5448, 3308, 2181}

$$\frac{e^{4a}2^{-2(m+3)}x^m(-bx)^{-m}\text{Gamma}(m+1,-4bx)}{b}+\frac{e^{2a}2^{-m-4}x^m(-bx)^{-m}\text{Gamma}(m+1,-2bx)}{b}+\frac{e^{-2a}2^{-m-4}x^m(bx)^{-m}\text{G}}{b}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*Cosh[a + b*x]^3*Sinh[a + b*x],x]
```

```
[Out] (E^(4*a)*x^m*Gamma[1 + m, -4*b*x])/(2^(2*(3 + m))*b*(-(b*x))^m) + (2^(-4 -
m)*E^(2*a)*x^m*Gamma[1 + m, -2*b*x])/(b*(-(b*x))^m) + (2^(-4 - m)*x^m*Gamma
[1 + m, 2*b*x])/(b*E^(2*a)*(b*x)^m) + (x^m*Gamma[1 + m, 4*b*x])/(2^(2*(3 +
m))*b*E^(4*a)*(b*x)^m)
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&  !I
ntegerQ[m]
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
```

```
& IGtQ[p, 0]
```

Rubi steps

$$\int x^m \cosh^3(a+bx)\sinh(a+bx)\,dx = \int \left(\frac{1}{4}x^m \sinh(2a+2bx)+\frac{1}{8}x^m \sinh(4a+4bx)\right)dx$$
$$= \frac{1}{8}\int x^m \sinh(4a+4bx)\,dx+\frac{1}{4}\int x^m \sinh(2a+2bx)\,dx$$
$$= \frac{1}{16}\int e^{-i(4ia+4ibx)}x^m\,dx-\frac{1}{16}\int e^{i(4ia+4ibx)}x^m\,dx+\frac{1}{8}\int e^{-i(2ia+2ibx)}x^m\,dx-$$
$$= \frac{4^{-3-m}e^{4a}x^m(-bx)^{-m}\Gamma(1+m,-4bx)}{b}+\frac{2^{-4-m}e^{2a}x^m(-bx)^{-m}\Gamma(1+m,-2bx)}{b}$$

Mathematica [A] time = 0.13, size = 110, normalized size = 0.79

$$\frac{e^{-4a}4^{-m-3}x^m\left(-b^2x^2\right)^{-m}\left((-bx)^m\left(e^{2a}2^{m+2}\Gamma(m+1,2bx)+\Gamma(m+1,4bx)\right)+e^{8a}(bx)^m\Gamma(m+1,-4bx)+e^{6a}2^{m+2}(bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*Cosh[a + b*x]^3*Sinh[a + b*x],x]
```

```
[Out] (4^(-3 - m)*x^m*(E^(8*a)*(b*x)^m*Gamma[1 + m, -4*b*x] + 2^(2 + m)*E^(6*a)*(
b*x)^m*Gamma[1 + m, -2*b*x] + (-(b*x))^m*(2^(2 + m)*E^(2*a)*Gamma[1 + m, 2*
b*x] + Gamma[1 + m, 4*b*x])))/(b*E^(4*a)*(-(b^2*x^2))^m)
```

fricas [A] time = 0.61, size = 172, normalized size = 1.24

$$\frac{\cosh\left(m\log\left(4b\right)+4a\right)\Gamma\left(m+1,4bx\right)+4\cosh\left(m\log\left(2b\right)+2a\right)\Gamma\left(m+1,2bx\right)+4\cosh\left(m\log\left(-2b\right)-2a\right)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*cosh(b*x+a)^3*sinh(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/64*(cosh(m*log(4*b) + 4*a)*gamma(m + 1, 4*b*x) + 4*cosh(m*log(2*b) + 2*a)
*gamma(m + 1, 2*b*x) + 4*cosh(m*log(-2*b) - 2*a)*gamma(m + 1, -2*b*x) + cos
h(m*log(-4*b) - 4*a)*gamma(m + 1, -4*b*x) - gamma(m + 1, 4*b*x)*sinh(m*log(
4*b) + 4*a) - 4*gamma(m + 1, 2*b*x)*sinh(m*log(2*b) + 2*a) - 4*gamma(m + 1,
 -2*b*x)*sinh(m*log(-2*b) - 2*a) - gamma(m + 1, -4*b*x)*sinh(m*log(-4*b) -
4*a))/b
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^m \cosh(bx+a)^3 \sinh(bx+a)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*cosh(b*x+a)^3*sinh(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(x^m*cosh(b*x + a)^3*sinh(b*x + a), x)
```

maple [F] time = 0.42, size = 0, normalized size = 0.00

$$\int x^m \left(\cosh^3(bx+a)\right) \sinh(bx+a)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*cosh(b*x+a)^3*sinh(b*x+a),x)
```

```
[Out] int(x^m*cosh(b*x+a)^3*sinh(b*x+a),x)
```

maxima [A] time = 0.42, size = 117, normalized size = 0.84

$$\frac{1}{16} (4\,bx)^{-m-1} x^{m+1} e^{(-4\,a)} \Gamma(m+1, 4\,bx) + \frac{1}{8} (2\,bx)^{-m-1} x^{m+1} e^{(-2\,a)} \Gamma(m+1, 2\,bx) - \frac{1}{8} (-2\,bx)^{-m-1} x^{m+1} e^{(2\,a)} \Gamma(m+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*cosh(b*x+a)^3*sinh(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/16*(4*b*x)^(-m - 1)*x^(m + 1)*e^(-4*a)*gamma(m + 1, 4*b*x) + 1/8*(2*b*x)^
(-m - 1)*x^(m + 1)*e^(-2*a)*gamma(m + 1, 2*b*x) - 1/8*(-2*b*x)^(-m - 1)*x^(
m + 1)*e^(2*a)*gamma(m + 1, -2*b*x) - 1/16*(-4*b*x)^(-m - 1)*x^(m + 1)*e^(4
*a)*gamma(m + 1, -4*b*x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^m \cosh(a+b\,x)^3 \sinh(a+b\,x)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*cosh(a + b*x)^3*sinh(a + b*x),x)
```

```
[Out] int(x^m*cosh(a + b*x)^3*sinh(a + b*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^m \sinh(a+bx) \cosh^3(a+bx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*cosh(b*x+a)**3*sinh(b*x+a),x)
```

```
[Out] Integral(x**m*sinh(a + b*x)*cosh(a + b*x)**3, x)
```

3.269 $\int x^3 \cosh^3(a+bx)\sinh(a+bx)\,dx$

Optimal. Leaf size=155

$$-\frac{3\sinh(a+bx)\cosh^3(a+bx)}{128b^4}-\frac{45\sinh(a+bx)\cosh(a+bx)}{256b^4}+\frac{3x\cosh^4(a+bx)}{32b^3}+\frac{9x\cosh^2(a+bx)}{32b^3}-\frac{3x^2\sinh(}{}$$

```
[Out] -45/256*x/b^3-3/32*x^3/b+9/32*x*cosh(b*x+a)^2/b^3+3/32*x*cosh(b*x+a)^4/b^3+
1/4*x^3*cosh(b*x+a)^4/b-45/256*cosh(b*x+a)*sinh(b*x+a)/b^4-9/32*x^2*cosh(b*
x+a)*sinh(b*x+a)/b^2-3/128*cosh(b*x+a)^3*sinh(b*x+a)/b^4-3/16*x^2*cosh(b*x+
a)^3*sinh(b*x+a)/b^2
```

Rubi [A] time = 0.14, antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {5373, 3311, 30, 2635, 8}

$$-\frac{3x^2\sinh(a+bx)\cosh^3(a+bx)}{16b^2}-\frac{9x^2\sinh(a+bx)\cosh(a+bx)}{32b^2}+\frac{3x\cosh^4(a+bx)}{32b^3}+\frac{9x\cosh^2(a+bx)}{32b^3}-\frac{3\sinh}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Cosh[a + b*x]^3*Sinh[a + b*x],x]
```

```
[Out] (-45*x)/(256*b^3) - (3*x^3)/(32*b) + (9*x*Cosh[a + b*x]^2)/(32*b^3) + (3*x*
Cosh[a + b*x]^4)/(32*b^3) + (x^3*Cosh[a + b*x]^4)/(4*b) - (45*Cosh[a + b*x]
*Sinh[a + b*x])/(256*b^4) - (9*x^2*Cosh[a + b*x]*Sinh[a + b*x])/(32*b^2) -
(3*Cosh[a + b*x]^3*Sinh[a + b*x])/(128*b^4) - (3*x^2*Cosh[a + b*x]^3*Sinh[a
 + b*x])/(16*b^2)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 3311

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(d*m*(c + d*x)^(m - 1)*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist
[(b^2*(n - 1))/n, Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[(
d^2*m*(m - 1))/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[(b*(c + d*x)^m*Cos[e + f*x]*(b*Sin[e + f*x])^(n - 1))/(f*n), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 5373

```
Int[Cosh[(a_.) + (b_.)*(x_)^(n_.)]^(p_.)*(x_)^(m_.)*Sinh[(a_.) + (b_.)*(x_)
^(n_.)], x_Symbol] :> Simp[(x^(m - n + 1)*Cosh[a + b*x^n]^(p + 1))/(b*n*(p
+ 1)), x] - Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*Cosh[a + b*x^n]^(
p + 1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int x^3\cosh^3(a+bx)\sinh(a+bx)\,dx &= \frac{x^3\cosh^4(a+bx)}{4b}-\frac{3\int x^2\cosh^4(a+bx)\,dx}{4b}\\ &= \frac{3x\cosh^4(a+bx)}{32b^3}+\frac{x^3\cosh^4(a+bx)}{4b}-\frac{3x^2\cosh^3(a+bx)\sinh(a+bx)}{16b^2}-\\ &= \frac{9x\cosh^2(a+bx)}{32b^3}+\frac{3x\cosh^4(a+bx)}{32b^3}+\frac{x^3\cosh^4(a+bx)}{4b}-\frac{9x^2\cosh(a+}{3}\\ &= -\frac{3x^3}{32b}+\frac{9x\cosh^2(a+bx)}{32b^3}+\frac{3x\cosh^4(a+bx)}{32b^3}+\frac{x^3\cosh^4(a+bx)}{4b}-\frac{45\,\mathrm{co}}{}\\ &= -\frac{45x}{256b^3}-\frac{3x^3}{32b}+\frac{9x\cosh^2(a+bx)}{32b^3}+\frac{3x\cosh^4(a+bx)}{32b^3}+\frac{x^3\cosh^4(a+bx}{4b}\end{aligned}$$

Mathematica [A] time = 0.64, size = 91, normalized size = 0.59

$$\frac{32bx\left(2b^2x^2+3\right)\cosh(2(a+bx))+2bx\left(8b^2x^2+3\right)\cosh(4(a+bx))-3\sinh(2(a+bx))\left(\left(8b^2x^2+1\right)\cosh(2(a+\right.\right.}{512b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Cosh[a + b*x]^3*Sinh[a + b*x],x]
```

```
[Out] (32*b*x*(3 + 2*b^2*x^2)*Cosh[2*(a + b*x)] + 2*b*x*(3 + 8*b^2*x^2)*Cosh[4*(a
 + b*x)] - 3*(16 + 32*b^2*x^2 + (1 + 8*b^2*x^2)*Cosh[2*(a + b*x)])*Sinh[2*(
a + b*x)])/(512*b^4)
```

fricas [A] time = 0.47, size = 191, normalized size = 1.23

$$\frac{\left(8\,b^3x^3+3\,bx\right)\cosh\left(bx+a\right)^4-3\left(8\,b^2x^2+1\right)\cosh\left(bx+a\right)\sinh\left(bx+a\right)^3+\left(8\,b^3x^3+3\,bx\right)\sinh\left(bx+a\right)^4+16}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*cosh(b*x+a)^3*sinh(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/256*((8*b^3*x^3 + 3*b*x)*cosh(b*x + a)^4 - 3*(8*b^2*x^2 + 1)*cosh(b*x + a
)*sinh(b*x + a)^3 + (8*b^3*x^3 + 3*b*x)*sinh(b*x + a)^4 + 16*(2*b^3*x^3 + 3
*b*x)*cosh(b*x + a)^2 + 2*(16*b^3*x^3 + 3*(8*b^3*x^3 + 3*b*x)*cosh(b*x + a)
^2 + 24*b*x)*sinh(b*x + a)^2 - 3*((8*b^2*x^2 + 1)*cosh(b*x + a)^3 + 16*(2*b
^2*x^2 + 1)*cosh(b*x + a))*sinh(b*x + a))/b^4
```

giac [A] time = 0.14, size = 145, normalized size = 0.94

$$\frac{\left(32\,b^3x^3-24\,b^2x^2+12\,bx-3\right)e^{(4\,bx+4\,a)}}{2048\,b^4}+\frac{\left(4\,b^3x^3-6\,b^2x^2+6\,bx-3\right)e^{(2\,bx+2\,a)}}{64\,b^4}+\frac{\left(4\,b^3x^3+6\,b^2x^2+6\,bx+3\right)e^{(-2}}{64\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*cosh(b*x+a)^3*sinh(b*x+a),x, algorithm="giac")
```

```
[Out] 1/2048*(32*b^3*x^3 - 24*b^2*x^2 + 12*b*x - 3)*e^(4*b*x + 4*a)/b^4 + 1/64*(4
*b^3*x^3 - 6*b^2*x^2 + 6*b*x - 3)*e^(2*b*x + 2*a)/b^4 + 1/64*(4*b^3*x^3 + 6
*b^2*x^2 + 6*b*x + 3)*e^(-2*b*x - 2*a)/b^4 + 1/2048*(32*b^3*x^3 + 24*b^2*x^
2 + 12*b*x + 3)*e^(-4*b*x - 4*a)/b^4
```

maple [B] time = 0.33, size = 304, normalized size = 1.96

$$\frac{\frac{(bx+a)^3\left(\cosh^4(bx+a)\right)}{4}-\frac{3(bx+a)^2\sinh(bx+a)\left(\cosh^3(bx+a)\right)}{16}-\frac{9(bx+a)^2\cosh(bx+a)\sinh(bx+a)}{32}-\frac{3(bx+a)^3}{32}+\frac{3(bx+a)\left(\cosh^4(bx+a)\right)}{32}-\frac{3(\mathrm{co}}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*cosh(b*x+a)^3*sinh(b*x+a),x)
```

```
[Out] 1/b^4*(1/4*(b*x+a)^3*cosh(b*x+a)^4-3/16*(b*x+a)^2*sinh(b*x+a)*cosh(b*x+a)^3
-9/32*(b*x+a)^2*cosh(b*x+a)*sinh(b*x+a)-3/32*(b*x+a)^3+3/32*(b*x+a)*cosh(b*
x+a)^4-3/128*cosh(b*x+a)^3*sinh(b*x+a)-45/256*cosh(b*x+a)*sinh(b*x+a)-45/25
6*b*x-45/256*a+9/32*(b*x+a)*cosh(b*x+a)^2-3*a*(1/4*(b*x+a)^2*cosh(b*x+a)^4-
1/8*(b*x+a)*sinh(b*x+a)*cosh(b*x+a)^3-3/16*(b*x+a)*cosh(b*x+a)*sinh(b*x+a)-
3/32*(b*x+a)^2+1/32*cosh(b*x+a)^4+3/32*cosh(b*x+a)^2)+3*a^2*(1/4*(b*x+a)*co
sh(b*x+a)^4-1/16*cosh(b*x+a)^3*sinh(b*x+a)-3/32*cosh(b*x+a)*sinh(b*x+a)-3/3
2*b*x-3/32*a)-1/4*a^3*cosh(b*x+a)^4)
```

maxima [A] time = 0.33, size = 171, normalized size = 1.10

$$\frac{\left(32\,b^3x^3e^{(4\,a)}-24\,b^2x^2e^{(4\,a)}+12\,bxe^{(4\,a)}-3\,e^{(4\,a)}\right)e^{(4\,bx)}}{2048\,b^4}+\frac{\left(4\,b^3x^3e^{(2\,a)}-6\,b^2x^2e^{(2\,a)}+6\,bxe^{(2\,a)}-3\,e^{(2\,a)}\right)e^{(2\,bx)}}{64\,b^4}+\frac{(4\,b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*cosh(b*x+a)^3*sinh(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/2048*(32*b^3*x^3*e^(4*a) - 24*b^2*x^2*e^(4*a) + 12*b*x*e^(4*a) - 3*e^(4*a
))*e^(4*b*x)/b^4 + 1/64*(4*b^3*x^3*e^(2*a) - 6*b^2*x^2*e^(2*a) + 6*b*x*e^(2
*a) - 3*e^(2*a))*e^(2*b*x)/b^4 + 1/64*(4*b^3*x^3 + 6*b^2*x^2 + 6*b*x + 3)*e
^(-2*b*x - 2*a)/b^4 + 1/2048*(32*b^3*x^3 + 24*b^2*x^2 + 12*b*x + 3)*e^(-4*b
*x - 4*a)/b^4
```

mupad [B] time = 1.69, size = 125, normalized size = 0.81

$$\frac{\frac{x^3\cosh(2\,a+2\,b\,x)}{8}+\frac{x^3\cosh(4\,a+4\,b\,x)}{32}}{b}-\frac{\frac{3\,x^2\sinh(2\,a+2\,b\,x)}{16}+\frac{3\,x^2\sinh(4\,a+4\,b\,x)}{128}}{b^2}+\frac{\frac{3\,x\cosh(2\,a+2\,b\,x)}{16}+\frac{3\,x\cosh(4\,a+4\,b\,x)}{256}}{b^3}-\frac{3\sinh(2\,a}{32\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*cosh(a + b*x)^3*sinh(a + b*x),x)
```

```
[Out] ((x^3*cosh(2*a + 2*b*x))/8 + (x^3*cosh(4*a + 4*b*x))/32)/b - ((3*x^2*sinh(2
*a + 2*b*x))/16 + (3*x^2*sinh(4*a + 4*b*x))/128)/b^2 + ((3*x*cosh(2*a + 2*b
*x))/16 + (3*x*cosh(4*a + 4*b*x))/256)/b^3 - (3*sinh(2*a + 2*b*x))/(32*b^4)
 - (3*sinh(4*a + 4*b*x))/(1024*b^4)
```

sympy [A] time = 5.26, size = 226, normalized size = 1.46

$$\begin{cases} -\frac{3x^3\sinh^4(a+bx)}{32b}+\frac{3x^3\sinh^2(a+bx)\cosh^2(a+bx)}{16b}+\frac{5x^3\cosh^4(a+bx)}{32b}+\frac{9x^2\sinh^3(a+bx)\cosh(a+bx)}{32b^2}-\frac{15x^2\sinh(a+bx)\cosh^3(a+bx)}{32b^2}-\frac{4}{} \\ \frac{x^4\sinh(a)\cosh^3(a)}{4} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*cosh(b*x+a)**3*sinh(b*x+a),x)
```

```
[Out] Piecewise((-3*x**3*sinh(a + b*x)**4/(32*b) + 3*x**3*sinh(a + b*x)**2*cosh(a
 + b*x)**2/(16*b) + 5*x**3*cosh(a + b*x)**4/(32*b) + 9*x**2*sinh(a + b*x)**
3*cosh(a + b*x)/(32*b**2) - 15*x**2*sinh(a + b*x)*cosh(a + b*x)**3/(32*b**2
) - 45*x*sinh(a + b*x)**4/(256*b**3) + 9*x*sinh(a + b*x)**2*cosh(a + b*x)**
2/(128*b**3) + 51*x*cosh(a + b*x)**4/(256*b**3) + 45*sinh(a + b*x)**3*cosh(
a + b*x)/(256*b**4) - 51*sinh(a + b*x)*cosh(a + b*x)**3/(256*b**4), Ne(b, 0
)), (x**4*sinh(a)*cosh(a)**3/4, True))
```

3.270 $\int x^2 \cosh^3(a+bx)\sinh(a+bx)\,dx$

Optimal. Leaf size=101

$$\frac{\cosh^4(a+bx)}{32b^3}+\frac{3\cosh^2(a+bx)}{32b^3}-\frac{x\sinh(a+bx)\cosh^3(a+bx)}{8b^2}-\frac{3x\sinh(a+bx)\cosh(a+bx)}{16b^2}+\frac{x^2\cosh^4(a+bx)}{4b}$$

```
[Out] -3/32*x^2/b+3/32*cosh(b*x+a)^2/b^3+1/32*cosh(b*x+a)^4/b^3+1/4*x^2*cosh(b*x+
a)^4/b-3/16*x*cosh(b*x+a)*sinh(b*x+a)/b^2-1/8*x*cosh(b*x+a)^3*sinh(b*x+a)/b
^2
```

Rubi [A] time = 0.08, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {5373, 3310, 30}

$$\frac{\cosh^4(a+bx)}{32b^3}+\frac{3\cosh^2(a+bx)}{32b^3}-\frac{x\sinh(a+bx)\cosh^3(a+bx)}{8b^2}-\frac{3x\sinh(a+bx)\cosh(a+bx)}{16b^2}+\frac{x^2\cosh^4(a+bx)}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Cosh[a + b*x]^3*Sinh[a + b*x],x]
```

```
[Out] (-3*x^2)/(32*b) + (3*Cosh[a + b*x]^2)/(32*b^3) + Cosh[a + b*x]^4/(32*b^3) +
 (x^2*Cosh[a + b*x]^4)/(4*b) - (3*x*Cosh[a + b*x]*Sinh[a + b*x])/(16*b^2) -
 (x*Cosh[a + b*x]^3*Sinh[a + b*x])/(8*b^2)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 3310

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[(d*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[(b*(c + d*x)*Cos[e + f*x]*(b
*Sin[e + f*x])^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 5373

```
Int[Cosh[(a_.) + (b_.)*(x_)^(n_.)]^(p_.)*(x_)^(m_.)*Sinh[(a_.) + (b_.)*(x_)
^(n_.)], x_Symbol] :> Simp[(x^(m - n + 1)*Cosh[a + b*x^n]^(p + 1))/(b*n*(p
+ 1)), x] - Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*Cosh[a + b*x^n]^(
p + 1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

Rubi steps

$$\int x^2\cosh^3(a+bx)\sinh(a+bx)\,dx=\frac{x^2\cosh^4(a+bx)}{4b}-\frac{\int x\cosh^4(a+bx)\,dx}{2b}$$
$$=\frac{\cosh^4(a+bx)}{32b^3}+\frac{x^2\cosh^4(a+bx)}{4b}-\frac{x\cosh^3(a+bx)\sinh(a+bx)}{8b^2}-\frac{3\int x$$
$$=\frac{3\cosh^2(a+bx)}{32b^3}+\frac{\cosh^4(a+bx)}{32b^3}+\frac{x^2\cosh^4(a+bx)}{4b}-\frac{3x\cosh(a+bx)\,\mathrm{si}}{16b^2}$$
$$=-\frac{3x^2}{32b}+\frac{3\cosh^2(a+bx)}{32b^3}+\frac{\cosh^4(a+bx)}{32b^3}+\frac{x^2\cosh^4(a+bx)}{4b}-\frac{3x\cosh(}{}$$

Mathematica [A] time = 0.24, size = 70, normalized size = 0.69

$$\frac{16\left(2b^2x^2+1\right)\cosh(2(a+bx))+\left(8b^2x^2+1\right)\cosh(4(a+bx))-4bx(8\sinh(2(a+bx))+\sinh(4(a+bx)))}{256b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Cosh[a + b*x]^3*Sinh[a + b*x],x]
```

```
[Out] (16*(1 + 2*b^2*x^2)*Cosh[2*(a + b*x)] + (1 + 8*b^2*x^2)*Cosh[4*(a + b*x)] -
 4*b*x*(8*Sinh[2*(a + b*x)] + Sinh[4*(a + b*x)]))/(256*b^3)
```

fricas [A] time = 0.71, size = 154, normalized size = 1.52

$$-\frac{16\,bx\cosh\left(bx+a\right)\sinh\left(bx+a\right)^3-\left(8\,b^2x^2+1\right)\cosh\left(bx+a\right)^4-\left(8\,b^2x^2+1\right)\sinh\left(bx+a\right)^4-16\left(2\,b^2x^2+1\right)\mathrm{c}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cosh(b*x+a)^3*sinh(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/256*(16*b*x*cosh(b*x + a)*sinh(b*x + a)^3 - (8*b^2*x^2 + 1)*cosh(b*x + a
)^4 - (8*b^2*x^2 + 1)*sinh(b*x + a)^4 - 16*(2*b^2*x^2 + 1)*cosh(b*x + a)^2
- 2*(16*b^2*x^2 + 3*(8*b^2*x^2 + 1)*cosh(b*x + a)^2 + 8)*sinh(b*x + a)^2 +
16*(b*x*cosh(b*x + a)^3 + 4*b*x*cosh(b*x + a))*sinh(b*x + a))/b^3
```

giac [A] time = 0.12, size = 113, normalized size = 1.12

$$\frac{\left(8\,b^2x^2-4\,bx+1\right)e^{(4\,bx+4\,a)}}{512\,b^3}+\frac{\left(2\,b^2x^2-2\,bx+1\right)e^{(2\,bx+2\,a)}}{32\,b^3}+\frac{\left(2\,b^2x^2+2\,bx+1\right)e^{(-2\,bx-2\,a)}}{32\,b^3}+\frac{\left(8\,b^2x^2+4\,bx+1\right)e^{(-4\,b}}{512\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cosh(b*x+a)^3*sinh(b*x+a),x, algorithm="giac")
```

```
[Out] 1/512*(8*b^2*x^2 - 4*b*x + 1)*e^(4*b*x + 4*a)/b^3 + 1/32*(2*b^2*x^2 - 2*b*x
 + 1)*e^(2*b*x + 2*a)/b^3 + 1/32*(2*b^2*x^2 + 2*b*x + 1)*e^(-2*b*x - 2*a)/b
^3 + 1/512*(8*b^2*x^2 + 4*b*x + 1)*e^(-4*b*x - 4*a)/b^3
```

maple [A] time = 0.33, size = 161, normalized size = 1.59

$$\frac{\frac{(bx+a)^2\left(\cosh^4(bx+a)\right)}{4}-\frac{(bx+a)\sinh(bx+a)\left(\cosh^3(bx+a)\right)}{8}-\frac{3(bx+a)\cosh(bx+a)\sinh(bx+a)}{16}-\frac{3(bx+a)^2}{32}+\frac{\left(\cosh^4(bx+a)\right)}{32}+\frac{3\left(\cosh^2(bx+a)\right)}{32}}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*cosh(b*x+a)^3*sinh(b*x+a),x)
```

```
[Out] 1/b^3*(1/4*(b*x+a)^2*cosh(b*x+a)^4-1/8*(b*x+a)*sinh(b*x+a)*cosh(b*x+a)^3-3/
16*(b*x+a)*cosh(b*x+a)*sinh(b*x+a)-3/32*(b*x+a)^2+1/32*cosh(b*x+a)^4+3/32*c
osh(b*x+a)^2-2*a*(1/4*(b*x+a)*cosh(b*x+a)^4-1/16*cosh(b*x+a)^3*sinh(b*x+a)-
3/32*cosh(b*x+a)*sinh(b*x+a)-3/32*b*x-3/32*a)+1/4*a^2*cosh(b*x+a)^4)
```

maxima [A] time = 0.32, size = 127, normalized size = 1.26

$$\frac{\left(8\,b^2x^2e^{(4\,a)}-4\,bxe^{(4\,a)}+e^{(4\,a)}\right)e^{(4\,bx)}}{512\,b^3}+\frac{\left(2\,b^2x^2e^{(2\,a)}-2\,bxe^{(2\,a)}+e^{(2\,a)}\right)e^{(2\,bx)}}{32\,b^3}+\frac{\left(2\,b^2x^2+2\,bx+1\right)e^{(-2\,bx-2\,a)}}{32\,b^3}+\frac{\left(8\,b^2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cosh(b*x+a)^3*sinh(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/512*(8*b^2*x^2*e^(4*a) - 4*b*x*e^(4*a) + e^(4*a))*e^(4*b*x)/b^3 + 1/32*(2
*b^2*x^2*e^(2*a) - 2*b*x*e^(2*a) + e^(2*a))*e^(2*b*x)/b^3 + 1/32*(2*b^2*x^2
 + 2*b*x + 1)*e^(-2*b*x - 2*a)/b^3 + 1/512*(8*b^2*x^2 + 4*b*x + 1)*e^(-4*b*
x - 4*a)/b^3
```

mupad [B] time = 0.16, size = 89, normalized size = 0.88

$$\frac{3\cosh\left(a+b\,x\right)^2}{32\,b^3}-\frac{\frac{3\,x^2}{32}-\frac{x^2\cosh(a+b\,x)^4}{4}}{b}-\frac{\frac{x\sinh(a+b\,x)\cosh(a+b\,x)^3}{8}+\frac{3\,x\sinh(a+b\,x)\cosh(a+b\,x)}{16}}{b^2}+\frac{\cosh\left(a+b\,x\right)^4}{32\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*cosh(a + b*x)^3*sinh(a + b*x),x)
```

```
[Out] (3*cosh(a + b*x)^2)/(32*b^3) - ((3*x^2)/32 - (x^2*cosh(a + b*x)^4)/4)/b - (
(x*cosh(a + b*x)^3*sinh(a + b*x))/8 + (3*x*cosh(a + b*x)*sinh(a + b*x))/16)
/b^2 + cosh(a + b*x)^4/(32*b^3)
```

sympy [A] time = 2.92, size = 150, normalized size = 1.49

$$\begin{cases} -\frac{3x^2\sinh^4(a+bx)}{32b} + \frac{3x^2\sinh^2(a+bx)\cosh^2(a+bx)}{16b} + \frac{5x^2\cosh^4(a+bx)}{32b} + \frac{3x\sinh^3(a+bx)\cosh(a+bx)}{16b^2} - \frac{5x\sinh(a+bx)\cosh^3(a+bx)}{16b^2} - \frac{3\sin}{} \\ \frac{x^3\sinh(a)\cosh^3(a)}{3} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*cosh(b*x+a)**3*sinh(b*x+a),x)
```

```
[Out] Piecewise((-3*x**2*sinh(a + b*x)**4/(32*b) + 3*x**2*sinh(a + b*x)**2*cosh(a
 + b*x)**2/(16*b) + 5*x**2*cosh(a + b*x)**4/(32*b) + 3*x*sinh(a + b*x)**3*c
osh(a + b*x)/(16*b**2) - 5*x*sinh(a + b*x)*cosh(a + b*x)**3/(16*b**2) - 3*s
inh(a + b*x)**4/(64*b**3) + 5*cosh(a + b*x)**4/(64*b**3), Ne(b, 0)), (x**3*
sinh(a)*cosh(a)**3/3, True))
```

3.271 $\int x\cosh^3(a+bx)\sinh(a+bx)\,dx$

Optimal. Leaf size=65

$$-\frac{\sinh(a+bx)\cosh^3(a+bx)}{16b^2}-\frac{3\sinh(a+bx)\cosh(a+bx)}{32b^2}+\frac{x\cosh^4(a+bx)}{4b}-\frac{3x}{32b}$$

```
[Out] -3/32*x/b+1/4*x*cosh(b*x+a)^4/b-3/32*cosh(b*x+a)*sinh(b*x+a)/b^2-1/16*cosh(
b*x+a)^3*sinh(b*x+a)/b^2
```

Rubi [A] time = 0.04, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {5373, 2635, 8}

$$-\frac{\sinh(a+bx)\cosh^3(a+bx)}{16b^2}-\frac{3\sinh(a+bx)\cosh(a+bx)}{32b^2}+\frac{x\cosh^4(a+bx)}{4b}-\frac{3x}{32b}$$

Antiderivative was successfully verified.

```
[In] Int[x*Cosh[a + b*x]^3*Sinh[a + b*x],x]
```

```
[Out] (-3*x)/(32*b) + (x*Cosh[a + b*x]^4)/(4*b) - (3*Cosh[a + b*x]*Sinh[a + b*x])
/(32*b^2) - (Cosh[a + b*x]^3*Sinh[a + b*x])/(16*b^2)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 5373

```
Int[Cosh[(a_.) + (b_.)*(x_)^(n_.)]^(p_.)*(x_)^(m_.)*Sinh[(a_.) + (b_.)*(x_)
^(n_.)], x_Symbol] :> Simp[(x^(m - n + 1)*Cosh[a + b*x^n]^(p + 1))/(b*n*(p
+ 1)), x] - Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*Cosh[a + b*x^n]^(
p + 1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

Rubi steps

$$\int x\cosh^3(a+bx)\sinh(a+bx)\,dx = \frac{x\cosh^4(a+bx)}{4b} - \frac{\int \cosh^4(a+bx)\,dx}{4b}$$
$$= \frac{x\cosh^4(a+bx)}{4b} - \frac{\cosh^3(a+bx)\sinh(a+bx)}{16b^2} - \frac{3\int \cosh^2(a+bx)\,dx}{16b}$$
$$= \frac{x\cosh^4(a+bx)}{4b} - \frac{3\cosh(a+bx)\sinh(a+bx)}{32b^2} - \frac{\cosh^3(a+bx)\sinh(a+b}{16b^2}$$
$$= -\frac{3x}{32b} + \frac{x\cosh^4(a+bx)}{4b} - \frac{3\cosh(a+bx)\sinh(a+bx)}{32b^2} - \frac{\cosh^3(a+bx)\sin}{16b^2}$$

Mathematica [A] time = 0.16, size = 50, normalized size = 0.77

$$-\frac{8\sinh(2(a+bx)) + \sinh(4(a+bx)) - 16bx\cosh(2(a+bx)) - 4bx\cosh(4(a+bx))}{128b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Cosh[a + b*x]^3*Sinh[a + b*x],x]
```

```
[Out] -1/128*(-16*b*x*Cosh[2*(a + b*x)] - 4*b*x*Cosh[4*(a + b*x)] + 8*Sinh[2*(a +
 b*x)] + Sinh[4*(a + b*x)])/b^2
```

fricas [A] time = 0.62, size = 108, normalized size = 1.66

$$\frac{bx\cosh(bx+a)^4 + bx\sinh(bx+a)^4 + 4\,bx\cosh(bx+a)^2 - \cosh(bx+a)\sinh(bx+a)^3 + 2\left(3\,bx\cosh(bx+a)\right.}{32\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x+a)^3*sinh(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/32*(b*x*cosh(b*x + a)^4 + b*x*sinh(b*x + a)^4 + 4*b*x*cosh(b*x + a)^2 - c
osh(b*x + a)*sinh(b*x + a)^3 + 2*(3*b*x*cosh(b*x + a)^2 + 2*b*x)*sinh(b*x +
 a)^2 - (cosh(b*x + a)^3 + 4*cosh(b*x + a))*sinh(b*x + a))/b^2
```

giac [A] time = 0.14, size = 81, normalized size = 1.25

$$\frac{(4\,bx-1)e^{(4\,bx+4\,a)}}{256\,b^2} + \frac{(2\,bx-1)e^{(2\,bx+2\,a)}}{32\,b^2} + \frac{(2\,bx+1)e^{(-2\,bx-2\,a)}}{32\,b^2} + \frac{(4\,bx+1)e^{(-4\,bx-4\,a)}}{256\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x+a)^3*sinh(b*x+a),x, algorithm="giac")
```

```
[Out] 1/256*(4*b*x - 1)*e^(4*b*x + 4*a)/b^2 + 1/32*(2*b*x - 1)*e^(2*b*x + 2*a)/b^
2 + 1/32*(2*b*x + 1)*e^(-2*b*x - 2*a)/b^2 + 1/256*(4*b*x + 1)*e^(-4*b*x - 4
*a)/b^2
```

maple [A] time = 0.30, size = 69, normalized size = 1.06

$$\frac{\frac{(bx+a)\left(\cosh^4(bx+a)\right)}{4}-\frac{\left(\cosh^3(bx+a)\right)\sinh(bx+a)}{16}-\frac{3\cosh(bx+a)\sinh(bx+a)}{32}-\frac{3bx}{32}-\frac{3a}{32}-\frac{a\left(\cosh^4(bx+a)\right)}{4}}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cosh(b*x+a)^3*sinh(b*x+a),x)
```

```
[Out] 1/b^2*(1/4*(b*x+a)*cosh(b*x+a)^4-1/16*cosh(b*x+a)^3*sinh(b*x+a)-3/32*cosh(b
*x+a)*sinh(b*x+a)-3/32*b*x-3/32*a-1/4*a*cosh(b*x+a)^4)
```

maxima [A] time = 0.33, size = 91, normalized size = 1.40

$$\frac{\left(4\,bxe^{(4\,a)}-e^{(4\,a)}\right)e^{(4\,bx)}}{256\,b^2}+\frac{\left(2\,bxe^{(2\,a)}-e^{(2\,a)}\right)e^{(2\,bx)}}{32\,b^2}+\frac{(2\,bx+1)e^{(-2\,bx-2\,a)}}{32\,b^2}+\frac{(4\,bx+1)e^{(-4\,bx-4\,a)}}{256\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x+a)^3*sinh(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/256*(4*b*x*e^(4*a) - e^(4*a))*e^(4*b*x)/b^2 + 1/32*(2*b*x*e^(2*a) - e^(2*
a))*e^(2*b*x)/b^2 + 1/32*(2*b*x + 1)*e^(-2*b*x - 2*a)/b^2 + 1/256*(4*b*x +
1)*e^(-4*b*x - 4*a)/b^2
```

mupad [B] time = 0.12, size = 57, normalized size = 0.88

$$-\frac{\frac{3\,x}{32}-\frac{x\cosh(a+b\,x)^4}{4}}{b}-\frac{\cosh\left(a+b\,x\right)^3\sinh\left(a+b\,x\right)}{16\,b^2}-\frac{3\cosh\left(a+b\,x\right)\,\sinh\left(a+b\,x\right)}{32\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cosh(a + b*x)^3*sinh(a + b*x),x)
```

```
[Out] - ((3*x)/32 - (x*cosh(a + b*x)^4)/4)/b - (cosh(a + b*x)^3*sinh(a + b*x))/(1
6*b^2) - (3*cosh(a + b*x)*sinh(a + b*x))/(32*b^2)
```

sympy [A] time = 1.68, size = 110, normalized size = 1.69

$$\begin{cases} -\frac{3x\sinh^4(a+bx)}{32b}+\frac{3x\sinh^2(a+bx)\cosh^2(a+bx)}{16b}+\frac{5x\cosh^4(a+bx)}{32b}+\frac{3\sinh^3(a+bx)\cosh(a+bx)}{32b^2}-\frac{5\sinh(a+bx)\cosh^3(a+bx)}{32b^2} & \text{for } b \neq \\ \frac{x^2\sinh(a)\cosh^3(a)}{2} & \text{otherw} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x+a)**3*sinh(b*x+a),x)
```

```
[Out] Piecewise((-3*x*sinh(a + b*x)**4/(32*b) + 3*x*sinh(a + b*x)**2*cosh(a + b*x
)**2/(16*b) + 5*x*cosh(a + b*x)**4/(32*b) + 3*sinh(a + b*x)**3*cosh(a + b*x
)/(32*b**2) - 5*sinh(a + b*x)*cosh(a + b*x)**3/(32*b**2), Ne(b, 0)), (x**2*
sinh(a)*cosh(a)**3/2, True))
```

3.272 $\int \cosh^3(a+bx)\sinh(a+bx)\,dx$

Optimal. Leaf size=15

$$\frac{\cosh^4(a+bx)}{4b}$$

```
[Out] 1/4*cosh(b*x+a)^4/b
```

Rubi [A] time = 0.02, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2565, 30}

$$\frac{\cosh^4(a+bx)}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[a + b*x]^3*Sinh[a + b*x],x]
```

```
[Out] Cosh[a + b*x]^4/(4*b)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rubi steps

$$\begin{aligned}\int \cosh^3(a+bx)\sinh(a+bx)\,dx &= \frac{\text{Subst}\left(\int x^3\,dx, x, \cosh(a+bx)\right)}{b}\\ &= \frac{\cosh^4(a+bx)}{4b}\end{aligned}$$

Mathematica [A] time = 0.00, size = 15, normalized size = 1.00

$$\frac{\cosh^4(a+bx)}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[a + b*x]^3*Sinh[a + b*x],x]
```

```
[Out] Cosh[a + b*x]^4/(4*b)
```

fricas [B] time = 1.02, size = 54, normalized size = 3.60

$$\frac{\cosh\left(bx+a\right)^{4}+\sinh\left(bx+a\right)^{4}+2\left(3\cosh\left(bx+a\right)^{2}+2\right)\sinh\left(bx+a\right)^{2}+4\cosh\left(bx+a\right)^{2}}{32\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^3*sinh(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/32*(cosh(b*x + a)^4 + sinh(b*x + a)^4 + 2*(3*cosh(b*x + a)^2 + 2)*sinh(b*
x + a)^2 + 4*cosh(b*x + a)^2)/b
```

giac [B] time = 0.14, size = 57, normalized size = 3.80

$$\frac{e^{\left(4\,bx+4\,a\right)}}{64\,b}+\frac{e^{\left(2\,bx+2\,a\right)}}{16\,b}+\frac{e^{\left(-2\,bx-2\,a\right)}}{16\,b}+\frac{e^{\left(-4\,bx-4\,a\right)}}{64\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^3*sinh(b*x+a),x, algorithm="giac")
```

```
[Out] 1/64*e^(4*b*x + 4*a)/b + 1/16*e^(2*b*x + 2*a)/b + 1/16*e^(-2*b*x - 2*a)/b +
 1/64*e^(-4*b*x - 4*a)/b
```

maple [A] time = 0.02, size = 14, normalized size = 0.93

$$\frac{\cosh^{4}\left(bx+a\right)}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)^3*sinh(b*x+a),x)
```

```
[Out] 1/4*cosh(b*x+a)^4/b
```

maxima [A] time = 0.32, size = 13, normalized size = 0.87

$$\frac{\cosh\left(bx+a\right)^{4}}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^3*sinh(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/4*cosh(b*x + a)^4/b
```

mupad [B] time = 1.49, size = 13, normalized size = 0.87

$$\frac{\cosh\left(a+b\,x\right)^4}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a + b*x)^3*sinh(a + b*x),x)
```

```
[Out] cosh(a + b*x)^4/(4*b)
```

sympy [A] time = 0.74, size = 20, normalized size = 1.33

$$\begin{cases} \frac{\cosh^{4}{\left(a+bx \right)}}{4b} & \text{for}\: b \neq 0 \\ x \sinh{\left(a \right)} \cosh^{3}{\left(a \right)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)**3*sinh(b*x+a),x)
```

```
[Out] Piecewise((cosh(a + b*x)**4/(4*b), Ne(b, 0)), (x*sinh(a)*cosh(a)**3, True))
```

3.273 $\int \frac{\cosh^3(a+bx)\sinh(a+bx)}{x}\,dx$

Optimal. Leaf size=53

$$\frac{1}{4}\sinh(2a)\text{Chi}(2bx)+\frac{1}{8}\sinh(4a)\text{Chi}(4bx)+\frac{1}{4}\cosh(2a)\text{Shi}(2bx)+\frac{1}{8}\cosh(4a)\text{Shi}(4bx)$$

```
[Out] 1/4*cosh(2*a)*Shi(2*b*x)+1/8*cosh(4*a)*Shi(4*b*x)+1/4*Chi(2*b*x)*sinh(2*a)+
1/8*Chi(4*b*x)*sinh(4*a)
```

Rubi [A] time = 0.14, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {5448, 3303, 3298, 3301}

$$\frac{1}{4}\sinh(2a)\text{Chi}(2bx)+\frac{1}{8}\sinh(4a)\text{Chi}(4bx)+\frac{1}{4}\cosh(2a)\text{Shi}(2bx)+\frac{1}{8}\cosh(4a)\text{Shi}(4bx)$$

Antiderivative was successfully verified.

```
[In] Int[(Cosh[a + b*x]^3*Sinh[a + b*x])/x,x]
```

```
[Out] (CoshIntegral[2*b*x]*Sinh[2*a])/4 + (CoshIntegral[4*b*x]*Sinh[4*a])/8 + (Co
sh[2*a]*SinhIntegral[2*b*x])/4 + (Cosh[4*a]*SinhIntegral[4*b*x])/8
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rubi steps

$$\int \frac{\cosh^3(a+bx)\sinh(a+bx)}{x}\,dx = \int \left(\frac{\sinh(2a+2bx)}{4x}+\frac{\sinh(4a+4bx)}{8x}\right)dx$$
$$= \frac{1}{8}\int \frac{\sinh(4a+4bx)}{x}\,dx+\frac{1}{4}\int \frac{\sinh(2a+2bx)}{x}\,dx$$
$$= \frac{1}{4}\cosh(2a)\int \frac{\sinh(2bx)}{x}\,dx+\frac{1}{8}\cosh(4a)\int \frac{\sinh(4bx)}{x}\,dx+\frac{1}{4}\sinh(2a)\int$$
$$= \frac{1}{4}\text{Chi}(2bx)\sinh(2a)+\frac{1}{8}\text{Chi}(4bx)\sinh(4a)+\frac{1}{4}\cosh(2a)\text{Shi}(2bx)+\frac{1}{8}\cos$$

Mathematica [A] time = 0.08, size = 47, normalized size = 0.89

$$\frac{1}{8}(2\sinh(2a)\text{Chi}(2bx)+\sinh(4a)\text{Chi}(4bx)+2\cosh(2a)\text{Shi}(2bx)+\cosh(4a)\text{Shi}(4bx))$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cosh[a + b*x]^3*Sinh[a + b*x])/x,x]
```

```
[Out] (2*CoshIntegral[2*b*x]*Sinh[2*a] + CoshIntegral[4*b*x]*Sinh[4*a] + 2*Cosh[2
*a]*SinhIntegral[2*b*x] + Cosh[4*a]*SinhIntegral[4*b*x])/8
```

fricas [A] time = 1.13, size = 73, normalized size = 1.38

$$\frac{1}{16}\left(\text{Ei}\left(4\,bx\right)-\text{Ei}\left(-4\,bx\right)\right)\cosh\left(4\,a\right)+\frac{1}{8}\left(\text{Ei}\left(2\,bx\right)-\text{Ei}\left(-2\,bx\right)\right)\cosh\left(2\,a\right)+\frac{1}{16}\left(\text{Ei}\left(4\,bx\right)+\text{Ei}\left(-4\,bx\right)\right)\sinh\left(4\,a\right)+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^3*sinh(b*x+a)/x,x, algorithm="fricas")
```

```
[Out] 1/16*(Ei(4*b*x) - Ei(-4*b*x))*cosh(4*a) + 1/8*(Ei(2*b*x) - Ei(-2*b*x))*cosh
(2*a) + 1/16*(Ei(4*b*x) + Ei(-4*b*x))*sinh(4*a) + 1/8*(Ei(2*b*x) + Ei(-2*b*
x))*sinh(2*a)
```

giac [A] time = 0.12, size = 45, normalized size = 0.85

$$\frac{1}{16}\,\text{Ei}\left(4\,bx\right)e^{(4\,a)}+\frac{1}{8}\,\text{Ei}\left(2\,bx\right)e^{(2\,a)}-\frac{1}{8}\,\text{Ei}\left(-2\,bx\right)e^{(-2\,a)}-\frac{1}{16}\,\text{Ei}\left(-4\,bx\right)e^{(-4\,a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^3*sinh(b*x+a)/x,x, algorithm="giac")
```

```
[Out] 1/16*Ei(4*b*x)*e^(4*a) + 1/8*Ei(2*b*x)*e^(2*a) - 1/8*Ei(-2*b*x)*e^(-2*a) -
1/16*Ei(-4*b*x)*e^(-4*a)
```

maple [A] time = 0.47, size = 50, normalized size = 0.94

$$\frac{e^{-4a}\operatorname{Ei}(1,4bx)}{16}+\frac{e^{-2a}\operatorname{Ei}(1,2bx)}{8}-\frac{e^{2a}\operatorname{Ei}(1,-2bx)}{8}-\frac{e^{4a}\operatorname{Ei}(1,-4bx)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)^3*sinh(b*x+a)/x,x)
```

```
[Out] 1/16*exp(-4*a)*Ei(1,4*b*x)+1/8*exp(-2*a)*Ei(1,2*b*x)-1/8*exp(2*a)*Ei(1,-2*b
*x)-1/16*exp(4*a)*Ei(1,-4*b*x)
```

maxima [A] time = 0.42, size = 45, normalized size = 0.85

$$\frac{1}{16}\operatorname{Ei}(4\,bx)\,e^{(4\,a)}+\frac{1}{8}\operatorname{Ei}(2\,bx)\,e^{(2\,a)}-\frac{1}{8}\operatorname{Ei}(-2\,bx)\,e^{(-2\,a)}-\frac{1}{16}\operatorname{Ei}(-4\,bx)\,e^{(-4\,a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^3*sinh(b*x+a)/x,x, algorithm="maxima")
```

```
[Out] 1/16*Ei(4*b*x)*e^(4*a) + 1/8*Ei(2*b*x)*e^(2*a) - 1/8*Ei(-2*b*x)*e^(-2*a) -
1/16*Ei(-4*b*x)*e^(-4*a)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\cosh(a+b\,x)^3\,\sinh(a+b\,x)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cosh(a + b*x)^3*sinh(a + b*x))/x,x)
```

```
[Out] int((cosh(a + b*x)^3*sinh(a + b*x))/x, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh(a+bx)\cosh^3(a+bx)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)**3*sinh(b*x+a)/x,x)
```

```
[Out] Integral(sinh(a + b*x)*cosh(a + b*x)**3/x, x)
```

3.274 $\int \frac{\cosh^3(a+bx)\sinh(a+bx)}{x^2}\,dx$

Optimal. Leaf size=89

$$\frac{1}{2}b\cosh(2a)\text{Chi}(2bx)+\frac{1}{2}b\cosh(4a)\text{Chi}(4bx)+\frac{1}{2}b\sinh(2a)\text{Shi}(2bx)+\frac{1}{2}b\sinh(4a)\text{Shi}(4bx)-\frac{\sinh(2a+2bx)}{4x}-\frac{\text{si}}{}$$

```
[Out] 1/2*b*Chi(2*b*x)*cosh(2*a)+1/2*b*Chi(4*b*x)*cosh(4*a)+1/2*b*Shi(2*b*x)*sinh
(2*a)+1/2*b*Shi(4*b*x)*sinh(4*a)-1/4*sinh(2*b*x+2*a)/x-1/8*sinh(4*b*x+4*a)/
x
```

Rubi [A] time = 0.19, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {5448, 3297, 3303, 3298, 3301}

$$\frac{1}{2}b\cosh(2a)\text{Chi}(2bx)+\frac{1}{2}b\cosh(4a)\text{Chi}(4bx)+\frac{1}{2}b\sinh(2a)\text{Shi}(2bx)+\frac{1}{2}b\sinh(4a)\text{Shi}(4bx)-\frac{\sinh(2a+2bx)}{4x}-\frac{\text{si}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Cosh[a + b*x]^3*Sinh[a + b*x])/x^2,x]
```

```
[Out] (b*Cosh[2*a]*CoshIntegral[2*b*x])/2 + (b*Cosh[4*a]*CoshIntegral[4*b*x])/2 -
 Sinh[2*a + 2*b*x]/(4*x) - Sinh[4*a + 4*b*x]/(8*x) + (b*Sinh[2*a]*SinhInteg
ral[2*b*x])/2 + (b*Sinh[4*a]*SinhIntegral[4*b*x])/2
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rubi steps

$$\int \frac{\cosh^3(a+bx)\sinh(a+bx)}{x^2}\,dx = \int \left(\frac{\sinh(2a+2bx)}{4x^2}+\frac{\sinh(4a+4bx)}{8x^2}\right)dx$$
$$= \frac{1}{8}\int \frac{\sinh(4a+4bx)}{x^2}\,dx+\frac{1}{4}\int \frac{\sinh(2a+2bx)}{x^2}\,dx$$
$$= -\frac{\sinh(2a+2bx)}{4x}-\frac{\sinh(4a+4bx)}{8x}+\frac{1}{2}b\int \frac{\cosh(2a+2bx)}{x}\,dx+\frac{1}{2}b\int \frac{\cos}{}$$
$$= -\frac{\sinh(2a+2bx)}{4x}-\frac{\sinh(4a+4bx)}{8x}+\frac{1}{2}(b\cosh(2a))\int \frac{\cosh(2bx)}{x}\,dx+\frac{1}{2}(b$$
$$= \frac{1}{2}b\cosh(2a)\text{Chi}(2bx)+\frac{1}{2}b\cosh(4a)\text{Chi}(4bx)-\frac{\sinh(2a+2bx)}{4x}-\frac{\sinh(4a}{8x}$$

Mathematica [A] time = 0.21, size = 80, normalized size = 0.90

$$\frac{4bx\cosh(2a)\text{Chi}(2bx)+4bx\cosh(4a)\text{Chi}(4bx)+4bx\sinh(2a)\text{Shi}(2bx)+4bx\sinh(4a)\text{Shi}(4bx)-2\sinh(2(a+}{8x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cosh[a + b*x]^3*Sinh[a + b*x])/x^2,x]
```

```
[Out] (4*b*x*Cosh[2*a]*CoshIntegral[2*b*x] + 4*b*x*Cosh[4*a]*CoshIntegral[4*b*x]
- 2*Sinh[2*(a + b*x)] - Sinh[4*(a + b*x)] + 4*b*x*Sinh[2*a]*SinhIntegral[2*
b*x] + 4*b*x*Sinh[4*a]*SinhIntegral[4*b*x])/(8*x)
```

fricas [A] time = 0.85, size = 139, normalized size = 1.56

$$-\frac{2\cosh(bx+a)\sinh(bx+a)^3-(bx\text{Ei}(4bx)+bx\text{Ei}(-4bx))\cosh(4a)-(bx\text{Ei}(2bx)+bx\text{Ei}(-2bx))\cosh(2a)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^3*sinh(b*x+a)/x^2,x, algorithm="fricas")
```

```
[Out] -1/4*(2*cosh(b*x + a)*sinh(b*x + a)^3 - (b*x*Ei(4*b*x) + b*x*Ei(-4*b*x))*co
sh(4*a) - (b*x*Ei(2*b*x) + b*x*Ei(-2*b*x))*cosh(2*a) + 2*(cosh(b*x + a)^3 +
 cosh(b*x + a))*sinh(b*x + a) - (b*x*Ei(4*b*x) - b*x*Ei(-4*b*x))*sinh(4*a)
- (b*x*Ei(2*b*x) - b*x*Ei(-2*b*x))*sinh(2*a))/x
```

giac [A] time = 0.12, size = 100, normalized size = 1.12

$$\frac{4\,bx\mathrm{Ei}\left(4\,bx\right)e^{\left(4\,a\right)} + 4\,bx\mathrm{Ei}\left(2\,bx\right)e^{\left(2\,a\right)} + 4\,bx\mathrm{Ei}\left(-2\,bx\right)e^{\left(-2\,a\right)} + 4\,bx\mathrm{Ei}\left(-4\,bx\right)e^{\left(-4\,a\right)} - e^{\left(4\,bx+4\,a\right)} - 2\,e^{\left(2\,bx+2\,a\right)} + 2\,\cdots}{16\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^3*sinh(b*x+a)/x^2,x, algorithm="giac")
```

```
[Out] 1/16*(4*b*x*Ei(4*b*x)*e^(4*a) + 4*b*x*Ei(2*b*x)*e^(2*a) + 4*b*x*Ei(-2*b*x)*
e^(-2*a) + 4*b*x*Ei(-4*b*x)*e^(-4*a) - e^(4*b*x + 4*a) - 2*e^(2*b*x + 2*a)
+ 2*e^(-2*b*x - 2*a) + e^(-4*b*x - 4*a))/x
```

maple [A] time = 0.48, size = 110, normalized size = 1.24

$$\frac{e^{-4bx-4a}}{16x}-\frac{b\,e^{-4a}\,\mathrm{Ei}\left(1,4bx\right)}{4}+\frac{e^{-2bx-2a}}{8x}-\frac{b\,e^{-2a}\,\mathrm{Ei}\left(1,2bx\right)}{4}-\frac{e^{2bx+2a}}{8x}-\frac{b\,e^{2a}\,\mathrm{Ei}\left(1,-2bx\right)}{4}-\frac{e^{4bx+4a}}{16x}-\frac{b\,e^{4a}\,\mathrm{Ei}\left(1,-4bx\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)^3*sinh(b*x+a)/x^2,x)
```

```
[Out] 1/16*exp(-4*b*x-4*a)/x-1/4*b*exp(-4*a)*Ei(1,4*b*x)+1/8*exp(-2*b*x-2*a)/x-1/
4*b*exp(-2*a)*Ei(1,2*b*x)-1/8*exp(2*b*x+2*a)/x-1/4*b*exp(2*a)*Ei(1,-2*b*x)-
1/16/x*exp(4*b*x+4*a)-1/4*b*exp(4*a)*Ei(1,-4*b*x)
```

maxima [A] time = 0.44, size = 53, normalized size = 0.60

$$\frac{1}{4}\,be^{\left(-4\,a\right)}\Gamma\left(-1,4\,bx\right) + \frac{1}{4}\,be^{\left(-2\,a\right)}\Gamma\left(-1,2\,bx\right) + \frac{1}{4}\,be^{\left(2\,a\right)}\Gamma\left(-1,-2\,bx\right) + \frac{1}{4}\,be^{\left(4\,a\right)}\Gamma\left(-1,-4\,bx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^3*sinh(b*x+a)/x^2,x, algorithm="maxima")
```

```
[Out] 1/4*b*e^(-4*a)*gamma(-1, 4*b*x) + 1/4*b*e^(-2*a)*gamma(-1, 2*b*x) + 1/4*b*e
^(2*a)*gamma(-1, -2*b*x) + 1/4*b*e^(4*a)*gamma(-1, -4*b*x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cosh(a+b\,x)^3\sinh(a+b\,x)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cosh(a + b*x)^3*sinh(a + b*x))/x^2,x)
[Out] int((cosh(a + b*x)^3*sinh(a + b*x))/x^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh(a+bx)\cosh^3(a+bx)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)**3*sinh(b*x+a)/x**2,x)
[Out] Integral(sinh(a + b*x)*cosh(a + b*x)**3/x**2, x)
```

3.275 $\int \frac{\cosh^3(a+bx)\sinh(a+bx)}{x^3}\,dx$

Optimal. Leaf size=125

$$\frac{1}{2}b^2\sinh(2a)\text{Chi}(2bx)+b^2\sinh(4a)\text{Chi}(4bx)+\frac{1}{2}b^2\cosh(2a)\text{Shi}(2bx)+b^2\cosh(4a)\text{Shi}(4bx)-\frac{\sinh(2a+2bx)}{8x^2}-\frac{\text{si}}{}$$

```
[Out] -1/4*b*cosh(2*b*x+2*a)/x-1/4*b*cosh(4*b*x+4*a)/x+1/2*b^2*cosh(2*a)*Shi(2*b*
x)+b^2*cosh(4*a)*Shi(4*b*x)+1/2*b^2*Chi(2*b*x)*sinh(2*a)+b^2*Chi(4*b*x)*sin
h(4*a)-1/8*sinh(2*b*x+2*a)/x^2-1/16*sinh(4*b*x+4*a)/x^2
```

Rubi [A] time = 0.25, antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {5448, 3297, 3303, 3298, 3301}

$$\frac{1}{2}b^2\sinh(2a)\text{Chi}(2bx)+b^2\sinh(4a)\text{Chi}(4bx)+\frac{1}{2}b^2\cosh(2a)\text{Shi}(2bx)+b^2\cosh(4a)\text{Shi}(4bx)-\frac{\sinh(2a+2bx)}{8x^2}-\frac{\text{si}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Cosh[a + b*x]^3*Sinh[a + b*x])/x^3,x]
```

```
[Out] -(b*Cosh[2*a + 2*b*x])/(4*x) - (b*Cosh[4*a + 4*b*x])/(4*x) + (b^2*CoshInteg
ral[2*b*x]*Sinh[2*a])/2 + b^2*CoshIntegral[4*b*x]*Sinh[4*a] - Sinh[2*a + 2*
b*x]/(8*x^2) - Sinh[4*a + 4*b*x]/(16*x^2) + (b^2*Cosh[2*a]*SinhIntegral[2*b
*x])/2 + b^2*Cosh[4*a]*SinhIntegral[4*b*x]
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\cosh^3(a+bx)\sinh(a+bx)}{x^3}\,dx &= \int \left(\frac{\sinh(2a+2bx)}{4x^3}+\frac{\sinh(4a+4bx)}{8x^3}\right)dx\\ &= \frac{1}{8}\int \frac{\sinh(4a+4bx)}{x^3}\,dx+\frac{1}{4}\int \frac{\sinh(2a+2bx)}{x^3}\,dx\\ &= -\frac{\sinh(2a+2bx)}{8x^2}-\frac{\sinh(4a+4bx)}{16x^2}+\frac{1}{4}b\int \frac{\cosh(2a+2bx)}{x^2}\,dx+\frac{1}{4}b\int \frac{\cos\end{aligned}$$

$$\begin{aligned}&= -\frac{b\cosh(2a+2bx)}{4x}-\frac{b\cosh(4a+4bx)}{4x}-\frac{\sinh(2a+2bx)}{8x^2}-\frac{\sinh(4a+4bx)}{16x^2}\\ &= -\frac{b\cosh(2a+2bx)}{4x}-\frac{b\cosh(4a+4bx)}{4x}-\frac{\sinh(2a+2bx)}{8x^2}-\frac{\sinh(4a+4bx)}{16x^2}\\ &= -\frac{b\cosh(2a+2bx)}{4x}-\frac{b\cosh(4a+4bx)}{4x}+\frac{1}{2}b^2\text{Chi}(2bx)\sinh(2a)+b^2\text{Chi}(4b\end{aligned}$$

Mathematica [A] time = 0.61, size = 112, normalized size = 0.90

$$b^2\sinh(4a)\text{Chi}(4bx)+b^2\sinh(a)\cosh(a)\text{Chi}(2bx)+\frac{1}{2}b^2\cosh(2a)\text{Shi}(2bx)+b^2\cosh(4a)\text{Shi}(4bx)-\frac{\sinh(2(a+bx))}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cosh[a + b*x]^3*Sinh[a + b*x])/x^3,x]
```

```
[Out] b^2*Cosh[a]*CoshIntegral[2*b*x]*Sinh[a] + b^2*CoshIntegral[4*b*x]*Sinh[4*a]
 - (2*b*x*Cosh[2*(a + b*x)] + Sinh[2*(a + b*x)])/(8*x^2) - (4*b*x*Cosh[4*(a
 + b*x)] + Sinh[4*(a + b*x)])/(16*x^2) + (b^2*Cosh[2*a]*SinhIntegral[2*b*x]
)/2 + b^2*Cosh[4*a]*SinhIntegral[4*b*x]
```

fricas [B] time = 0.91, size = 227, normalized size = 1.82

$$-\frac{bx\cosh(bx+a)^4 + bx\sinh(bx+a)^4 + bx\cosh(bx+a)^2 + \cosh(bx+a)\sinh(bx+a)^3 + \left(6\,bx\cosh(bx+a)\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^3*sinh(b*x+a)/x^3,x, algorithm="fricas")
```

```
[Out] -1/4*(b*x*cosh(b*x + a)^4 + b*x*sinh(b*x + a)^4 + b*x*cosh(b*x + a)^2 + cos
h(b*x + a)*sinh(b*x + a)^3 + (6*b*x*cosh(b*x + a)^2 + b*x)*sinh(b*x + a)^2
- 2*(b^2*x^2*Ei(4*b*x) - b^2*x^2*Ei(-4*b*x))*cosh(4*a) - (b^2*x^2*Ei(2*b*x)
 - b^2*x^2*Ei(-2*b*x))*cosh(2*a) + (cosh(b*x + a)^3 + cosh(b*x + a))*sinh(b
*x + a) - 2*(b^2*x^2*Ei(4*b*x) + b^2*x^2*Ei(-4*b*x))*sinh(4*a) - (b^2*x^2*E
i(2*b*x) + b^2*x^2*Ei(-2*b*x))*sinh(2*a))/x^2
```

giac [A] time = 0.15, size = 168, normalized size = 1.34

$$\frac{16\,b^2x^2\mathrm{Ei}(4\,bx)\,e^{(4\,a)} + 8\,b^2x^2\mathrm{Ei}(2\,bx)\,e^{(2\,a)} - 8\,b^2x^2\mathrm{Ei}(-2\,bx)\,e^{(-2\,a)} - 16\,b^2x^2\mathrm{Ei}(-4\,bx)\,e^{(-4\,a)} - 4\,bxe^{(4\,bx+4\,a)} - 4}{32\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^3*sinh(b*x+a)/x^3,x, algorithm="giac")
```

```
[Out] 1/32*(16*b^2*x^2*Ei(4*b*x)*e^(4*a) + 8*b^2*x^2*Ei(2*b*x)*e^(2*a) - 8*b^2*x^
2*Ei(-2*b*x)*e^(-2*a) - 16*b^2*x^2*Ei(-4*b*x)*e^(-4*a) - 4*b*x*e^(4*b*x + 4
*a) - 4*b*x*e^(2*b*x + 2*a) - 4*b*x*e^(-2*b*x - 2*a) - 4*b*x*e^(-4*b*x - 4*
a) - e^(4*b*x + 4*a) - 2*e^(2*b*x + 2*a) + 2*e^(-2*b*x - 2*a) + e^(-4*b*x -
 4*a))/x^2
```

maple [A] time = 0.52, size = 178, normalized size = 1.42

$$-\frac{b\,\mathrm{e}^{-4bx-4a}}{8x}+\frac{\mathrm{e}^{-4bx-4a}}{32x^2}+\frac{b^2\mathrm{e}^{-4a}\,\mathrm{Ei}(1,4bx)}{2}-\frac{b\,\mathrm{e}^{-2bx-2a}}{8x}+\frac{\mathrm{e}^{-2bx-2a}}{16x^2}+\frac{b^2\mathrm{e}^{-2a}\,\mathrm{Ei}(1,2bx)}{4}-\frac{\mathrm{e}^{2bx+2a}}{16x^2}-\frac{b\,\mathrm{e}^{2bx+2a}}{8x}-\frac{b^2\mathrm{e}^{2a}\,\mathrm{Ei}(}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)^3*sinh(b*x+a)/x^3,x)
```

```
[Out] -1/8*b*exp(-4*b*x-4*a)/x+1/32*exp(-4*b*x-4*a)/x^2+1/2*b^2*exp(-4*a)*Ei(1,4*
b*x)-1/8*b*exp(-2*b*x-2*a)/x+1/16*exp(-2*b*x-2*a)/x^2+1/4*b^2*exp(-2*a)*Ei(
1,2*b*x)-1/16*exp(2*b*x+2*a)/x^2-1/8*b*exp(2*b*x+2*a)/x-1/4*b^2*exp(2*a)*Ei
(1,-2*b*x)-1/32/x^2*exp(4*b*x+4*a)-1/8*b/x*exp(4*b*x+4*a)-1/2*b^2*exp(4*a)*
Ei(1,-4*b*x)
```

maxima [A] time = 0.43, size = 60, normalized size = 0.48

$$b^2 e^{(-4a)}\Gamma(-2, 4bx) + \frac{1}{2} b^2 e^{(-2a)}\Gamma(-2, 2bx) - \frac{1}{2} b^2 e^{(2a)}\Gamma(-2, -2bx) - b^2 e^{(4a)}\Gamma(-2, -4bx)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^3*sinh(b*x+a)/x^3,x, algorithm="maxima")
```

```
[Out] b^2*e^(-4*a)*gamma(-2, 4*b*x) + 1/2*b^2*e^(-2*a)*gamma(-2, 2*b*x) - 1/2*b^2
*e^(2*a)*gamma(-2, -2*b*x) - b^2*e^(4*a)*gamma(-2, -4*b*x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cosh(a + b\,x)^3 \sinh(a + b\,x)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cosh(a + b*x)^3*sinh(a + b*x))/x^3,x)
```

```
[Out] int((cosh(a + b*x)^3*sinh(a + b*x))/x^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh(a + bx)\cosh^3(a + bx)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)**3*sinh(b*x+a)/x**3,x)
```

```
[Out] Integral(sinh(a + b*x)*cosh(a + b*x)**3/x**3, x)
```

3.276 $\int \frac{\cosh^3(a+bx)\sinh(a+bx)}{x^4}\,dx$

Optimal. Leaf size=169

$$\frac{1}{3}b^3\cosh(2a)\text{Chi}(2bx)+\frac{4}{3}b^3\cosh(4a)\text{Chi}(4bx)+\frac{1}{3}b^3\sinh(2a)\text{Shi}(2bx)+\frac{4}{3}b^3\sinh(4a)\text{Shi}(4bx)-\frac{b^2\sinh(2a+2b}{6x}$$

```
[Out] 1/3*b^3*Chi(2*b*x)*cosh(2*a)+4/3*b^3*Chi(4*b*x)*cosh(4*a)-1/12*b*cosh(2*b*x
+2*a)/x^2-1/12*b*cosh(4*b*x+4*a)/x^2+1/3*b^3*Shi(2*b*x)*sinh(2*a)+4/3*b^3*S
hi(4*b*x)*sinh(4*a)-1/12*sinh(2*b*x+2*a)/x^3-1/6*b^2*sinh(2*b*x+2*a)/x-1/24
*sinh(4*b*x+4*a)/x^3-1/3*b^2*sinh(4*b*x+4*a)/x
```

Rubi [A] time = 0.30, antiderivative size = 169, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {5448, 3297, 3303, 3298, 3301}

$$\frac{1}{3}b^3\cosh(2a)\text{Chi}(2bx)+\frac{4}{3}b^3\cosh(4a)\text{Chi}(4bx)+\frac{1}{3}b^3\sinh(2a)\text{Shi}(2bx)+\frac{4}{3}b^3\sinh(4a)\text{Shi}(4bx)-\frac{b^2\sinh(2a+2b}{6x}$$

Antiderivative was successfully verified.

```
[In] Int[(Cosh[a + b*x]^3*Sinh[a + b*x])/x^4,x]
```

```
[Out] -(b*Cosh[2*a + 2*b*x])/(12*x^2) - (b*Cosh[4*a + 4*b*x])/(12*x^2) + (b^3*Cos
h[2*a]*CoshIntegral[2*b*x])/3 + (4*b^3*Cosh[4*a]*CoshIntegral[4*b*x])/3 - S
inh[2*a + 2*b*x]/(12*x^3) - (b^2*Sinh[2*a + 2*b*x])/(6*x) - Sinh[4*a + 4*b*
x]/(24*x^3) - (b^2*Sinh[4*a + 4*b*x])/(3*x) + (b^3*Sinh[2*a]*SinhIntegral[2
*b*x])/3 + (4*b^3*Sinh[4*a]*SinhIntegral[4*b*x])/3
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
```

```
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cosh^3(a+bx)\sinh(a+bx)}{x^4}\,dx &= \int \left(\frac{\sinh(2a+2bx)}{4x^4}+\frac{\sinh(4a+4bx)}{8x^4}\right)dx\\
&= \frac{1}{8}\int \frac{\sinh(4a+4bx)}{x^4}\,dx+\frac{1}{4}\int \frac{\sinh(2a+2bx)}{x^4}\,dx\\
&= -\frac{\sinh(2a+2bx)}{12x^3}-\frac{\sinh(4a+4bx)}{24x^3}+\frac{1}{6}b\int \frac{\cosh(2a+2bx)}{x^3}\,dx+\frac{1}{6}b\int \frac{\cos
\\
&= -\frac{b\cosh(2a+2bx)}{12x^2}-\frac{b\cosh(4a+4bx)}{12x^2}-\frac{\sinh(2a+2bx)}{12x^3}-\frac{\sinh(4a+4bx)}{24x^3}\\
&= -\frac{b\cosh(2a+2bx)}{12x^2}-\frac{b\cosh(4a+4bx)}{12x^2}-\frac{\sinh(2a+2bx)}{12x^3}-\frac{b^2\sinh(2a+2b}{6x}\\
&= -\frac{b\cosh(2a+2bx)}{12x^2}-\frac{b\cosh(4a+4bx)}{12x^2}-\frac{\sinh(2a+2bx)}{12x^3}-\frac{b^2\sinh(2a+2b}{6x}\\
&= -\frac{b\cosh(2a+2bx)}{12x^2}-\frac{b\cosh(4a+4bx)}{12x^2}+\frac{1}{3}b^3\cosh(2a)\mathrm{Chi}(2bx)+\frac{4}{3}b^3\cosh
\end{aligned}$$

Mathematica [A] time = 0.52, size = 150, normalized size = 0.89

$$-\frac{-8b^3x^3\cosh(2a)\mathrm{Chi}(2bx)-32b^3x^3\cosh(4a)\mathrm{Chi}(4bx)-8b^3x^3\sinh(2a)\mathrm{Shi}(2bx)-32b^3x^3\sinh(4a)\mathrm{Shi}(4bx)+}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cosh[a + b*x]^3*Sinh[a + b*x])/x^4,x]
```

```
[Out] -1/24*(2*b*x*Cosh[2*(a + b*x)] + 2*b*x*Cosh[4*(a + b*x)] - 8*b^3*x^3*Cosh[2
*a]*CoshIntegral[2*b*x] - 32*b^3*x^3*Cosh[4*a]*CoshIntegral[4*b*x] + 2*Sinh
[2*(a + b*x)] + 4*b^2*x^2*Sinh[2*(a + b*x)] + Sinh[4*(a + b*x)] + 8*b^2*x^2
*Sinh[4*(a + b*x)] - 8*b^3*x^3*Sinh[2*a]*SinhIntegral[2*b*x] - 32*b^3*x^3*S
inh[4*a]*SinhIntegral[4*b*x])/x^3
```

fricas [A] time = 0.88, size = 261, normalized size = 1.54

$$-\frac{bx\cosh\left(bx+a\right)^{4}+bx\sinh\left(bx+a\right)^{4}+2\left(8\,b^{2}x^{2}+1\right)\cosh\left(bx+a\right)\sinh\left(bx+a\right)^{3}+bx\cosh\left(bx+a\right)^{2}+\left(6\,bx\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^3*sinh(b*x+a)/x^4,x, algorithm="fricas")
```

```
[Out] -1/12*(b*x*cosh(b*x + a)^4 + b*x*sinh(b*x + a)^4 + 2*(8*b^2*x^2 + 1)*cosh(b
*x + a)*sinh(b*x + a)^3 + b*x*cosh(b*x + a)^2 + (6*b*x*cosh(b*x + a)^2 + b*
x)*sinh(b*x + a)^2 - 8*(b^3*x^3*Ei(4*b*x) + b^3*x^3*Ei(-4*b*x))*cosh(4*a) -
 2*(b^3*x^3*Ei(2*b*x) + b^3*x^3*Ei(-2*b*x))*cosh(2*a) + 2*((8*b^2*x^2 + 1)*
cosh(b*x + a)^3 + (2*b^2*x^2 + 1)*cosh(b*x + a))*sinh(b*x + a) - 8*(b^3*x^3
*Ei(4*b*x) - b^3*x^3*Ei(-4*b*x))*sinh(4*a) - 2*(b^3*x^3*Ei(2*b*x) - b^3*x^3
*Ei(-2*b*x))*sinh(2*a))/x^3
```

giac [A] time = 0.14, size = 236, normalized size = 1.40

$$\frac{32\,b^{3}x^{3}\mathrm{Ei}\left(4\,bx\right)e^{(4\,a)}+8\,b^{3}x^{3}\mathrm{Ei}\left(2\,bx\right)e^{(2\,a)}+8\,b^{3}x^{3}\mathrm{Ei}\left(-2\,bx\right)e^{(-2\,a)}+32\,b^{3}x^{3}\mathrm{Ei}\left(-4\,bx\right)e^{(-4\,a)}-8\,b^{2}x^{2}e^{(4\,bx+4\,a)}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^3*sinh(b*x+a)/x^4,x, algorithm="giac")
```

```
[Out] 1/48*(32*b^3*x^3*Ei(4*b*x)*e^(4*a) + 8*b^3*x^3*Ei(2*b*x)*e^(2*a) + 8*b^3*x^
3*Ei(-2*b*x)*e^(-2*a) + 32*b^3*x^3*Ei(-4*b*x)*e^(-4*a) - 8*b^2*x^2*e^(4*b*x
 + 4*a) - 4*b^2*x^2*e^(2*b*x + 2*a) + 4*b^2*x^2*e^(-2*b*x - 2*a) + 8*b^2*x^
2*e^(-4*b*x - 4*a) - 2*b*x*e^(4*b*x + 4*a) - 2*b*x*e^(2*b*x + 2*a) - 2*b*x*
e^(-2*b*x - 2*a) - 2*b*x*e^(-4*b*x - 4*a) - e^(4*b*x + 4*a) - 2*e^(2*b*x +
2*a) + 2*e^(-2*b*x - 2*a) + e^(-4*b*x - 4*a))/x^3
```

maple [A] time = 0.48, size = 246, normalized size = 1.46

$$\frac{b^{2}\mathrm{e}^{-4bx-4a}}{6x}-\frac{b\,\mathrm{e}^{-4bx-4a}}{24x^{2}}+\frac{\mathrm{e}^{-4bx-4a}}{48x^{3}}-\frac{2b^{3}\mathrm{e}^{-4a}\,\mathrm{Ei}\left(1,4bx\right)}{3}+\frac{b^{2}\mathrm{e}^{-2bx-2a}}{12x}-\frac{b\,\mathrm{e}^{-2bx-2a}}{24x^{2}}+\frac{\mathrm{e}^{-2bx-2a}}{24x^{3}}-\frac{b^{3}\mathrm{e}^{-2a}\,\mathrm{Ei}\left(1,2bx\right)}{6}-\frac{\mathrm{e}^{2bx}}{24x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)^3*sinh(b*x+a)/x^4,x)
```

```
[Out] 1/6*b^2*exp(-4*b*x-4*a)/x-1/24*b*exp(-4*b*x-4*a)/x^2+1/48*exp(-4*b*x-4*a)/x
^3-2/3*b^3*exp(-4*a)*Ei(1,4*b*x)+1/12*b^2*exp(-2*b*x-2*a)/x-1/24*b*exp(-2*b
*x-2*a)/x^2+1/24*exp(-2*b*x-2*a)/x^3-1/6*b^3*exp(-2*a)*Ei(1,2*b*x)-1/24*exp
(2*b*x+2*a)/x^3-1/24*b*exp(2*b*x+2*a)/x^2-1/12*b^2*exp(2*b*x+2*a)/x-1/6*b^3
*exp(2*a)*Ei(1,-2*b*x)-1/48/x^3*exp(4*b*x+4*a)-1/24*b/x^2*exp(4*b*x+4*a)-1/
6*b^2/x*exp(4*b*x+4*a)-2/3*b^3*exp(4*a)*Ei(1,-4*b*x)
```

maxima [A] time = 0.42, size = 59, normalized size = 0.35

$$4\,b^3e^{(-4\,a)}\Gamma\left(-3,4\,bx\right)+b^3e^{(-2\,a)}\Gamma\left(-3,2\,bx\right)+b^3e^{(2\,a)}\Gamma\left(-3,-2\,bx\right)+4\,b^3e^{(4\,a)}\Gamma\left(-3,-4\,bx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^3*sinh(b*x+a)/x^4,x, algorithm="maxima")
```

```
[Out] 4*b^3*e^(-4*a)*gamma(-3, 4*b*x) + b^3*e^(-2*a)*gamma(-3, 2*b*x) + b^3*e^(2*
a)*gamma(-3, -2*b*x) + 4*b^3*e^(4*a)*gamma(-3, -4*b*x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cosh\left(a+b\,x\right)^3\,\sinh\left(a+b\,x\right)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cosh(a + b*x)^3*sinh(a + b*x))/x^4,x)
```

```
[Out] int((cosh(a + b*x)^3*sinh(a + b*x))/x^4, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh\left(a+bx\right)\cosh^{3}\left(a+bx\right)}{x^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)**3*sinh(b*x+a)/x**4,x)
```

```
[Out] Integral(sinh(a + b*x)*cosh(a + b*x)**3/x**4, x)
```

3.277 $\int \frac{\cosh(x)\sinh(x)}{x}\,dx$

Optimal. Leaf size=8

$$\frac{\text{Shi}(2x)}{2}$$

```
[Out] 1/2*Shi(2*x)
```

Rubi [A] time = 0.03, antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {5448, 12, 3298}

$$\frac{\text{Shi}(2x)}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(Cosh[x]*Sinh[x])/x,x]
```

```
[Out] SinhIntegral[2*x]/2
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rubi steps

$$\int \frac{\cosh(x)\sinh(x)}{x}\,dx = \int \frac{\sinh(2x)}{2x}\,dx$$
$$= \frac{1}{2}\int \frac{\sinh(2x)}{x}\,dx$$
$$= \frac{\operatorname{Shi}(2x)}{2}$$

Mathematica [A] time = 0.01, size = 8, normalized size = 1.00

$$\frac{\operatorname{Shi}(2x)}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cosh[x]*Sinh[x])/x,x]
[Out] SinhIntegral[2*x]/2
```

fricas [B] time = 0.85, size = 13, normalized size = 1.62

$$\frac{1}{4}\operatorname{Ei}(2x) - \frac{1}{4}\operatorname{Ei}(-2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*sinh(x)/x,x, algorithm="fricas")
[Out] 1/4*Ei(2*x) - 1/4*Ei(-2*x)
```

giac [B] time = 0.13, size = 13, normalized size = 1.62

$$\frac{1}{4}\operatorname{Ei}(2x) - \frac{1}{4}\operatorname{Ei}(-2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*sinh(x)/x,x, algorithm="giac")
[Out] 1/4*Ei(2*x) - 1/4*Ei(-2*x)
```

maple [A] time = 0.13, size = 7, normalized size = 0.88

$$\frac{\operatorname{Shi}(2x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)*sinh(x)/x,x)
```

```
[Out] 1/2*Shi(2*x)
```

maxima [B] time = 0.36, size = 13, normalized size = 1.62

$$\frac{1}{4}\,\mathrm{Ei}\left(2\,x\right)-\frac{1}{4}\,\mathrm{Ei}\left(-2\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*sinh(x)/x,x, algorithm="maxima")
```

```
[Out] 1/4*Ei(2*x) - 1/4*Ei(-2*x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.12

$$\int \frac{\cosh(x)\sinh(x)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cosh(x)*sinh(x))/x,x)
```

```
[Out] int((cosh(x)*sinh(x))/x, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh\left(x\right)\cosh\left(x\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*sinh(x)/x,x)
```

```
[Out] Integral(sinh(x)*cosh(x)/x, x)
```

3.278 $\int \frac{\cosh(x)\sinh(x)}{x^2}\,dx$

Optimal. Leaf size=16

$$\mathrm{Chi}(2x) - \frac{\sinh(2x)}{2x}$$

[Out] Chi(2*x)-1/2*sinh(2*x)/x

Rubi [A] time = 0.05, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5448, 12, 3297, 3301}

$$\mathrm{Chi}(2x) - \frac{\sinh(2x)}{2x}$$

Antiderivative was successfully verified.

[In] Int[(Cosh[x]*Sinh[x])/x^2,x]

[Out] CoshIntegral[2*x] - Sinh[2*x]/(2*x)

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cosh(x)\sinh(x)}{x^2}\,dx &= \int \frac{\sinh(2x)}{2x^2}\,dx\\
&= \frac{1}{2}\int \frac{\sinh(2x)}{x^2}\,dx\\
&= -\frac{\sinh(2x)}{2x} + \int \frac{\cosh(2x)}{x}\,dx\\
&= \text{Chi}(2x) - \frac{\sinh(2x)}{2x}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 16, normalized size = 1.00

$$\text{Chi}(2x) - \frac{\sinh(2x)}{2x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cosh[x]*Sinh[x])/x^2,x]
[Out] CoshIntegral[2*x] - Sinh[2*x]/(2*x)
```

fricas [A] time = 0.92, size = 24, normalized size = 1.50

$$\frac{x\text{Ei}\left(2\,x\right) + x\text{Ei}\left(-2\,x\right) - 2\,\cosh(x)\sinh(x)}{2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*sinh(x)/x^2,x, algorithm="fricas")
[Out] 1/2*(x*Ei(2*x) + x*Ei(-2*x) - 2*cosh(x)*sinh(x))/x
```

giac [B] time = 0.12, size = 30, normalized size = 1.88

$$\frac{2\,x\text{Ei}\left(2\,x\right) + 2\,x\text{Ei}\left(-2\,x\right) - e^{(2\,x)} + e^{(-2\,x)}}{4\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*sinh(x)/x^2,x, algorithm="giac")
[Out] 1/4*(2*x*Ei(2*x) + 2*x*Ei(-2*x) - e^(2*x) + e^(-2*x))/x
```

maple [A] time = 0.12, size = 15, normalized size = 0.94

$$\mathrm{X}\left(2x\right)-\frac{\sinh\left(2x\right)}{2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)*sinh(x)/x^2,x)
```

```
[Out] Chi(2*x)-1/2*sinh(2*x)/x
```

maxima [A] time = 0.36, size = 15, normalized size = 0.94

$$\frac{1}{2}\,\Gamma\left(-1,2\,x\right)+\frac{1}{2}\,\Gamma\left(-1,-2\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*sinh(x)/x^2,x, algorithm="maxima")
```

```
[Out] 1/2*gamma(-1, 2*x) + 1/2*gamma(-1, -2*x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.06

$$\int\frac{\cosh(x)\sinh(x)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cosh(x)*sinh(x))/x^2,x)
```

```
[Out] int((cosh(x)*sinh(x))/x^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sinh\left(x\right)\cosh\left(x\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*sinh(x)/x**2,x)
```

```
[Out] Integral(sinh(x)*cosh(x)/x**2, x)
```

3.279 $\int \frac{\cosh(x)\sinh(x)}{x^3}\,dx$

Optimal. Leaf size=27

$$\text{Shi}(2x) - \frac{\sinh(2x)}{4x^2} - \frac{\cosh(2x)}{2x}$$

```
[Out] -1/2*cosh(2*x)/x+Shi(2*x)-1/4*sinh(2*x)/x^2
```

Rubi [A] time = 0.06, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5448, 12, 3297, 3298}

$$\text{Shi}(2x) - \frac{\sinh(2x)}{4x^2} - \frac{\cosh(2x)}{2x}$$

Antiderivative was successfully verified.

```
[In] Int[(Cosh[x]*Sinh[x])/x^3,x]
```

```
[Out] -Cosh[2*x]/(2*x) - Sinh[2*x]/(4*x^2) + SinhIntegral[2*x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\cosh(x)\sinh(x)}{x^3}\,dx &= \int \frac{\sinh(2x)}{2x^3}\,dx\\ &= \frac{1}{2}\int \frac{\sinh(2x)}{x^3}\,dx\\ &= -\frac{\sinh(2x)}{4x^2} + \frac{1}{2}\int \frac{\cosh(2x)}{x^2}\,dx\\ &= -\frac{\cosh(2x)}{2x} - \frac{\sinh(2x)}{4x^2} + \int \frac{\sinh(2x)}{x}\,dx\\ &= -\frac{\cosh(2x)}{2x} - \frac{\sinh(2x)}{4x^2} + \mathrm{Shi}(2x)\end{aligned}$$

Mathematica [A] time = 0.01, size = 27, normalized size = 1.00

$$\mathrm{Shi}(2x) - \frac{\sinh(2x)}{4x^2} - \frac{\cosh(2x)}{2x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cosh[x]*Sinh[x])/x^3,x]
[Out] -1/2*Cosh[2*x]/x - Sinh[2*x]/(4*x^2) + SinhIntegral[2*x]
```

fricas [A] time = 0.58, size = 43, normalized size = 1.59

$$\frac{x^2\mathrm{Ei}\,(2\,x) - x^2\mathrm{Ei}\,(-2\,x) - x\cosh(x)^2 - x\sinh(x)^2 - \cosh(x)\sinh(x)}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*sinh(x)/x^3,x, algorithm="fricas")
[Out] 1/2*(x^2*Ei(2*x) - x^2*Ei(-2*x) - x*cosh(x)^2 - x*sinh(x)^2 - cosh(x)*sinh(
x))/x^2
```

giac [B] time = 0.11, size = 48, normalized size = 1.78

$$\frac{4\,x^2\mathrm{Ei}\,(2\,x) - 4\,x^2\mathrm{Ei}\,(-2\,x) - 2\,xe^{(2\,x)} - 2\,xe^{(-2\,x)} - e^{(2\,x)} + e^{(-2\,x)}}{8\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*sinh(x)/x^3,x, algorithm="giac")
```

```
[Out] 1/8*(4*x^2*Ei(2*x) - 4*x^2*Ei(-2*x) - 2*x*e^(2*x) - 2*x*e^(-2*x) - e^(2*x)
+ e^(-2*x))/x^2
```

maple [A] time = 0.11, size = 24, normalized size = 0.89

$$-\frac{\cosh(2x)}{2x} + \mathrm{Shi}(2x) - \frac{\sinh(2x)}{4x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)*sinh(x)/x^3,x)
```

```
[Out] -1/2*cosh(2*x)/x+Shi(2*x)-1/4*sinh(2*x)/x^2
```

maxima [A] time = 0.34, size = 13, normalized size = 0.48

$$\Gamma(-2, 2x) - \Gamma(-2, -2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*sinh(x)/x^3,x, algorithm="maxima")
```

```
[Out] gamma(-2, 2*x) - gamma(-2, -2*x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\cosh(x)\sinh(x)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cosh(x)*sinh(x))/x^3,x)
```

```
[Out] int((cosh(x)*sinh(x))/x^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh(x)\cosh(x)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*sinh(x)/x**3,x)
```

```
[Out] Integral(sinh(x)*cosh(x)/x**3, x)
```

3.280 $\int x^m \cosh(a+bx)\sinh^2(a+bx)\,dx$

Optimal. Leaf size=134

$$\frac{e^{3a}3^{-m-1}x^m(-bx)^{-m}\Gamma(m+1,-3bx)}{8b}-\frac{e^a x^m(-bx)^{-m}\Gamma(m+1,-bx)}{8b}+\frac{e^{-a}x^m(bx)^{-m}\Gamma(m+1,bx)}{8b}-\frac{e^{-3a}3^{-m-1}x^m(bx)^{-m}\Gamma}{8b}$$

```
[Out] 1/8*3^(-1-m)*exp(3*a)*x^m*GAMMA(1+m,-3*b*x)/b/((-b*x)^m)-1/8*exp(a)*x^m*GAM
MA(1+m,-b*x)/b/((-b*x)^m)+1/8*x^m*GAMMA(1+m,b*x)/b/exp(a)/((b*x)^m)-1/8*3^(
-1-m)*x^m*GAMMA(1+m,3*b*x)/b/exp(3*a)/((b*x)^m)
```

Rubi [A] time = 0.18, antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {5448, 3307, 2181}

$$\frac{e^{3a}3^{-m-1}x^m(-bx)^{-m}\text{Gamma}(m+1,-3bx)}{8b}-\frac{e^a x^m(-bx)^{-m}\text{Gamma}(m+1,-bx)}{8b}+\frac{e^{-a}x^m(bx)^{-m}\text{Gamma}(m+1,bx)}{8b}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*Cosh[a + b*x]*Sinh[a + b*x]^2,x]
```

```
[Out] (3^(-1 - m)*E^(3*a)*x^m*Gamma[1 + m, -3*b*x])/(8*b*(-(b*x))^m) - (E^a*x^m*G
amma[1 + m, -(b*x)])/(8*b*(-(b*x))^m) + (x^m*Gamma[1 + m, b*x])/(8*b*E^a*(b
*x)^m) - (3^(-1 - m)*x^m*Gamma[1 + m, 3*b*x])/(8*b*E^(3*a)*(b*x)^m)
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] && !I
ntegerQ[m]
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
```

```
& IGtQ[p, 0]
```

Rubi steps

$$\int x^m \cosh(a+bx)\sinh^2(a+bx)\,dx = \int \left(-\frac{1}{4}x^m\cosh(a+bx)+\frac{1}{4}x^m\cosh(3a+3bx)\right)dx$$
$$= -\left(\frac{1}{4}\int x^m\cosh(a+bx)\,dx\right)+\frac{1}{4}\int x^m\cosh(3a+3bx)\,dx$$
$$= -\left(\frac{1}{8}\int e^{-i(ia+ibx)}x^m\,dx\right)-\frac{1}{8}\int e^{i(ia+ibx)}x^m\,dx+\frac{1}{8}\int e^{-i(3ia+3ibx)}x^m\,dx+$$
$$= \frac{3^{-1-m}e^{3a}x^m(-bx)^{-m}\Gamma(1+m,-3bx)}{8b}-\frac{e^a x^m(-bx)^{-m}\Gamma(1+m,-bx)}{8b}+\frac{e^{-a}x}{}$$

Mathematica [A] time = 0.23, size = 114, normalized size = 0.85

$$\frac{e^{-3a}x^m\left(3^{-m}\left(-b^2x^2\right)^{-m}\left(e^{6a}(bx)^m\Gamma(m+1,-3bx)-(-bx)^m\Gamma(m+1,3bx)\right)-3e^{4a}(-bx)^{-m}\Gamma(m+1,-bx)+3e^{2a}(bx}{24b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*Cosh[a + b*x]*Sinh[a + b*x]^2,x]
```

```
[Out] (x^m*((-3*E^(4*a)*Gamma[1 + m, -(b*x)])/(-(b*x))^m + (3*E^(2*a)*Gamma[1 + m
, b*x])/(b*x)^m + (E^(6*a)*(b*x)^m*Gamma[1 + m, -3*b*x] - (-(b*x))^m*Gamma[
1 + m, 3*b*x])/(3^m*(-(b^2*x^2))^m)))/(24*b*E^(3*a))
```

fricas [A] time = 0.78, size = 162, normalized size = 1.21

$$-\frac{\cosh\left(m\log\left(3\,b\right)+3\,a\right)\Gamma\left(m+1,3\,bx\right)-3\,\cosh\left(m\log(b)+a\right)\Gamma\left(m+1,bx\right)+3\,\cosh\left(m\log\left(-b\right)-a\right)\Gamma\left(m+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*cosh(b*x+a)*sinh(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/24*(cosh(m*log(3*b) + 3*a)*gamma(m + 1, 3*b*x) - 3*cosh(m*log(b) + a)*ga
mma(m + 1, b*x) + 3*cosh(m*log(-b) - a)*gamma(m + 1, -b*x) - cosh(m*log(-3*
b) - 3*a)*gamma(m + 1, -3*b*x) - gamma(m + 1, 3*b*x)*sinh(m*log(3*b) + 3*a)
 - 3*gamma(m + 1, -b*x)*sinh(m*log(-b) - a) + gamma(m + 1, -3*b*x)*sinh(m*l
og(-3*b) - 3*a) + 3*gamma(m + 1, b*x)*sinh(m*log(b) + a))/b
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^m \cosh(bx+a)\sinh(bx+a)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*cosh(b*x+a)*sinh(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate(x^m*cosh(b*x + a)*sinh(b*x + a)^2, x)
```

maple [F] time = 0.48, size = 0, normalized size = 0.00

$$\int x^m \cosh(bx+a)\left(\sinh^2(bx+a)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*cosh(b*x+a)*sinh(b*x+a)^2,x)
```

```
[Out] int(x^m*cosh(b*x+a)*sinh(b*x+a)^2,x)
```

maxima [A] time = 0.44, size = 113, normalized size = 0.84

$$-\frac{1}{8}(3bx)^{-m-1}x^{m+1}e^{(-3a)}\Gamma(m+1,3bx)+\frac{1}{8}(bx)^{-m-1}x^{m+1}e^{(-a)}\Gamma(m+1,bx)+\frac{1}{8}(-bx)^{-m-1}x^{m+1}e^{a}\Gamma(m+1,-bx)-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*cosh(b*x+a)*sinh(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -1/8*(3*b*x)^(-m - 1)*x^(m + 1)*e^(-3*a)*gamma(m + 1, 3*b*x) + 1/8*(b*x)^(-
m - 1)*x^(m + 1)*e^(-a)*gamma(m + 1, b*x) + 1/8*(-b*x)^(-m - 1)*x^(m + 1)*e
^a*gamma(m + 1, -b*x) - 1/8*(-3*b*x)^(-m - 1)*x^(m + 1)*e^(3*a)*gamma(m + 1
, -3*b*x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^m\,\cosh(a+b\,x)\,\sinh(a+b\,x)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*cosh(a + b*x)*sinh(a + b*x)^2,x)
```

```
[Out] int(x^m*cosh(a + b*x)*sinh(a + b*x)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^m \sinh^2(a+bx)\cosh(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*cosh(b*x+a)*sinh(b*x+a)**2,x)
```

```
[Out] Integral(x**m*sinh(a + b*x)**2*cosh(a + b*x), x)
```

3.281 $\int x^3 \cosh(a+bx)\sinh^2(a+bx)\,dx$

Optimal. Leaf size=117

$$-\frac{2\cosh^3(a+bx)}{27b^4}+\frac{14\cosh(a+bx)}{9b^4}+\frac{2x\sinh^3(a+bx)}{9b^3}-\frac{4x\sinh(a+bx)}{3b^3}+\frac{2x^2\cosh(a+bx)}{3b^2}-\frac{x^2\sinh^2(a+bx)\cos}{3b^2}$$

```
[Out] 14/9*cosh(b*x+a)/b^4+2/3*x^2*cosh(b*x+a)/b^2-2/27*cosh(b*x+a)^3/b^4-4/3*x*s
inh(b*x+a)/b^3-1/3*x^2*cosh(b*x+a)*sinh(b*x+a)^2/b^2+2/9*x*sinh(b*x+a)^3/b^
3+1/3*x^3*sinh(b*x+a)^3/b
```

Rubi [A] time = 0.14, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {5372, 3311, 3296, 2638, 2633}

$$\frac{2x^2\cosh(a+bx)}{3b^2}-\frac{x^2\sinh^2(a+bx)\cosh(a+bx)}{3b^2}+\frac{2x\sinh^3(a+bx)}{9b^3}-\frac{4x\sinh(a+bx)}{3b^3}-\frac{2\cosh^3(a+bx)}{27b^4}+\frac{14\cosh}{9}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Cosh[a + b*x]*Sinh[a + b*x]^2,x]
```

```
[Out] (14*Cosh[a + b*x])/(9*b^4) + (2*x^2*Cosh[a + b*x])/(3*b^2) - (2*Cosh[a + b*
x]^3)/(27*b^4) - (4*x*Sinh[a + b*x])/(3*b^3) - (x^2*Cosh[a + b*x]*Sinh[a +
b*x]^2)/(3*b^2) + (2*x*Sinh[a + b*x]^3)/(9*b^3) + (x^3*Sinh[a + b*x]^3)/(3*
b)
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3311

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(d*m*(c + d*x)^(m - 1)*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist
[(b^2*(n - 1))/n, Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[(
d^2*m*(m - 1))/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[(b*(c + d*x)^m*Cos[e + f*x]*(b*Sin[e + f*x])^(n - 1))/(f*n), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 5372

```
Int[Cosh[(a_.) + (b_.)*(x_)^(n_.)]*(x_)^(m_.)*Sinh[(a_.) + (b_.)*(x_)^(n_.)
]^(p_.), x_Symbol] :> Simp[(x^(m - n + 1)*Sinh[a + b*x^n]^(p + 1))/(b*n*(p
+ 1)), x] - Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*Sinh[a + b*x^n]^(
p + 1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

Rubi steps

$$\int x^3 \cosh(a+bx)\sinh^2(a+bx)\,dx = \frac{x^3\sinh^3(a+bx)}{3b} - \frac{\int x^2\sinh^3(a+bx)\,dx}{b}$$
$$= -\frac{x^2\cosh(a+bx)\sinh^2(a+bx)}{3b^2} + \frac{2x\sinh^3(a+bx)}{9b^3} + \frac{x^3\sinh^3(a+bx)}{3b} -$$
$$= \frac{2x^2\cosh(a+bx)}{3b^2} - \frac{x^2\cosh(a+bx)\sinh^2(a+bx)}{3b^2} + \frac{2x\sinh^3(a+bx)}{9b^3} +$$
$$= \frac{2\cosh(a+bx)}{9b^4} + \frac{2x^2\cosh(a+bx)}{3b^2} - \frac{2\cosh^3(a+bx)}{27b^4} - \frac{4x\sinh(a+bx)}{3b^3}$$
$$= \frac{14\cosh(a+bx)}{9b^4} + \frac{2x^2\cosh(a+bx)}{3b^2} - \frac{2\cosh^3(a+bx)}{27b^4} - \frac{4x\sinh(a+bx)}{3b^3}$$

Mathematica [A] time = 0.40, size = 84, normalized size = 0.72

$$\frac{81\left(b^2x^2+2\right)\cosh(a+bx) - \left(9b^2x^2+2\right)\cosh(3(a+bx)) + 6bx\sinh(a+bx)\left(\left(3b^2x^2+2\right)\cosh(2(a+bx)) - 3b\right.}{108b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Cosh[a + b*x]*Sinh[a + b*x]^2,x]
```

```
[Out] (81*(2 + b^2*x^2)*Cosh[a + b*x] - (2 + 9*b^2*x^2)*Cosh[3*(a + b*x)] + 6*b*x
*(-26 - 3*b^2*x^2 + (2 + 3*b^2*x^2)*Cosh[2*(a + b*x)])*Sinh[a + b*x])/(108*
b^4)
```

fricas [A] time = 0.54, size = 135, normalized size = 1.15

$$-\frac{\left(9\,b^2x^2+2\right)\cosh\left(bx+a\right)^3+3\left(9\,b^2x^2+2\right)\cosh\left(bx+a\right)\sinh\left(bx+a\right)^2-3\left(3\,b^3x^3+2\,bx\right)\sinh\left(bx+a\right)^3-81}{108\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*cosh(b*x+a)*sinh(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/108*((9*b^2*x^2 + 2)*cosh(b*x + a)^3 + 3*(9*b^2*x^2 + 2)*cosh(b*x + a)*s
inh(b*x + a)^2 - 3*(3*b^3*x^3 + 2*b*x)*sinh(b*x + a)^3 - 81*(b^2*x^2 + 2)*c
osh(b*x + a) + 9*(3*b^3*x^3 - (3*b^3*x^3 + 2*b*x)*cosh(b*x + a)^2 + 18*b*x)
*sinh(b*x + a))/b^4
```

giac [A] time = 0.12, size = 140, normalized size = 1.20

$$\frac{\left(9\,b^3x^3-9\,b^2x^2+6\,bx-2\right)e^{(3\,bx+3\,a)}}{216\,b^4}-\frac{\left(b^3x^3-3\,b^2x^2+6\,bx-6\right)e^{(bx+a)}}{8\,b^4}+\frac{\left(b^3x^3+3\,b^2x^2+6\,bx+6\right)e^{(-bx-a)}}{8\,b^4}-\frac{\left(9\,b^3\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*cosh(b*x+a)*sinh(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/216*(9*b^3*x^3 - 9*b^2*x^2 + 6*b*x - 2)*e^(3*b*x + 3*a)/b^4 - 1/8*(b^3*x^
3 - 3*b^2*x^2 + 6*b*x - 6)*e^(b*x + a)/b^4 + 1/8*(b^3*x^3 + 3*b^2*x^2 + 6*b
*x + 6)*e^(-b*x - a)/b^4 - 1/216*(9*b^3*x^3 + 9*b^2*x^2 + 6*b*x + 2)*e^(-3*
b*x - 3*a)/b^4
```

maple [B] time = 0.30, size = 244, normalized size = 2.09

$$\frac{\frac{(bx+a)^3\left(\sinh^3(bx+a)\right)}{3}+\frac{2(bx+a)^2\cosh(bx+a)}{3}-\frac{(bx+a)^2\cosh(bx+a)\left(\sinh^2(bx+a)\right)}{3}-\frac{4(bx+a)\sinh(bx+a)}{3}+\frac{40\cosh(bx+a)}{27}+\frac{2(bx+a)\left(\sinh^3(b\right.}{9}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*cosh(b*x+a)*sinh(b*x+a)^2,x)
```

```
[Out] 1/b^4*(1/3*(b*x+a)^3*sinh(b*x+a)^3+2/3*(b*x+a)^2*cosh(b*x+a)-1/3*(b*x+a)^2*
cosh(b*x+a)*sinh(b*x+a)^2-4/3*(b*x+a)*sinh(b*x+a)+40/27*cosh(b*x+a)+2/9*(b*
x+a)*sinh(b*x+a)^3-2/27*cosh(b*x+a)*sinh(b*x+a)^2-3*a*(1/3*(b*x+a)^2*sinh(b
*x+a)^3+4/9*(b*x+a)*cosh(b*x+a)-2/9*(b*x+a)*cosh(b*x+a)*sinh(b*x+a)^2-4/9*s
inh(b*x+a)+2/27*sinh(b*x+a)^3)+3*a^2*(1/3*(b*x+a)*sinh(b*x+a)^3+2/9*cosh(b*
x+a)-1/9*cosh(b*x+a)*sinh(b*x+a)^2)-1/3*a^3*sinh(b*x+a)^3)
```

maxima [A] time = 0.33, size = 160, normalized size = 1.37

$$\frac{\left(9\,b^3x^3e^{(3\,a)}-9\,b^2x^2e^{(3\,a)}+6\,bxe^{(3\,a)}-2\,e^{(3\,a)}\right)e^{(3\,bx)}}{216\,b^4}-\frac{\left(b^3x^3e^a-3\,b^2x^2e^a+6\,bxe^a-6\,e^a\right)e^{(bx)}}{8\,b^4}+\frac{\left(b^3x^3+3\,b^2x^2+6\,b\right.}{8\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*cosh(b*x+a)*sinh(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/216*(9*b^3*x^3*e^(3*a) - 9*b^2*x^2*e^(3*a) + 6*b*x*e^(3*a) - 2*e^(3*a))*e
^(3*b*x)/b^4 - 1/8*(b^3*x^3*e^a - 3*b^2*x^2*e^a + 6*b*x*e^a - 6*e^a)*e^(b*x
)/b^4 + 1/8*(b^3*x^3 + 3*b^2*x^2 + 6*b*x + 6)*e^(-b*x - a)/b^4 - 1/216*(9*b
^3*x^3 + 9*b^2*x^2 + 6*b*x + 2)*e^(-3*b*x - 3*a)/b^4
```

mupad [B] time = 0.17, size = 119, normalized size = 1.02

$$\frac{\frac{14\,x\,\sinh(a+b\,x)^3}{9}-\frac{4\,x\,\cosh(a+b\,x)^2\,\sinh(a+b\,x)}{3}}{b^3}+\frac{\frac{2\,x^2\,\cosh(a+b\,x)^3}{3}-x^2\cosh\left(a+b\,x\right)\,\sinh\left(a+b\,x\right)^2}{b^2}+\frac{40\cosh\left(a+b\,x\right)^3}{27\,b^4}-1$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*cosh(a + b*x)*sinh(a + b*x)^2,x)
```

```
[Out] ((14*x*sinh(a + b*x)^3)/9 - (4*x*cosh(a + b*x)^2*sinh(a + b*x))/3)/b^3 + ((
2*x^2*cosh(a + b*x)^3)/3 - x^2*cosh(a + b*x)*sinh(a + b*x)^2)/b^2 + (40*cos
h(a + b*x)^3)/(27*b^4) - (14*cosh(a + b*x)*sinh(a + b*x)^2)/(9*b^4) + (x^3*
sinh(a + b*x)^3)/(3*b)
```

sympy [A] time = 2.93, size = 146, normalized size = 1.25

$$\begin{cases} \frac{x^3\sinh^3(a+bx)}{3b}-\frac{x^2\sinh^2(a+bx)\cosh(a+bx)}{b^2}+\frac{2x^2\cosh^3(a+bx)}{3b^2}+\frac{14x\sinh^3(a+bx)}{9b^3}-\frac{4x\sinh(a+bx)\cosh^2(a+bx)}{3b^3}-\frac{14\sinh^2(a+bx)\cos}{9b^4} \\ \frac{x^4\sinh^2(a)\cosh(a)}{4} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*cosh(b*x+a)*sinh(b*x+a)**2,x)
```

```
[Out] Piecewise((x**3*sinh(a + b*x)**3/(3*b) - x**2*sinh(a + b*x)**2*cosh(a + b*x
)/b**2 + 2*x**2*cosh(a + b*x)**3/(3*b**2) + 14*x*sinh(a + b*x)**3/(9*b**3)
- 4*x*sinh(a + b*x)*cosh(a + b*x)**2/(3*b**3) - 14*sinh(a + b*x)**2*cosh(a
+ b*x)/(9*b**4) + 40*cosh(a + b*x)**3/(27*b**4), Ne(b, 0)), (x**4*sinh(a)**
2*cosh(a)/4, True))
```

3.282 $\int x^2 \cosh(a+bx)\sinh^2(a+bx)\,dx$

Optimal. Leaf size=83

$$\frac{2\sinh^3(a+bx)}{27b^3}-\frac{4\sinh(a+bx)}{9b^3}+\frac{4x\cosh(a+bx)}{9b^2}-\frac{2x\sinh^2(a+bx)\cosh(a+bx)}{9b^2}+\frac{x^2\sinh^3(a+bx)}{3b}$$

```
[Out] 4/9*x*cosh(b*x+a)/b^2-4/9*sinh(b*x+a)/b^3-2/9*x*cosh(b*x+a)*sinh(b*x+a)^2/b
^2+2/27*sinh(b*x+a)^3/b^3+1/3*x^2*sinh(b*x+a)^3/b
```

Rubi [A] time = 0.08, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {5372, 3310, 3296, 2637}

$$\frac{2\sinh^3(a+bx)}{27b^3}-\frac{4\sinh(a+bx)}{9b^3}+\frac{4x\cosh(a+bx)}{9b^2}-\frac{2x\sinh^2(a+bx)\cosh(a+bx)}{9b^2}+\frac{x^2\sinh^3(a+bx)}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Cosh[a + b*x]*Sinh[a + b*x]^2,x]
```

```
[Out] (4*x*Cosh[a + b*x])/(9*b^2) - (4*Sinh[a + b*x])/(9*b^3) - (2*x*Cosh[a + b*x
]*Sinh[a + b*x]^2)/(9*b^2) + (2*Sinh[a + b*x]^3)/(27*b^3) + (x^2*Sinh[a + b
*x]^3)/(3*b)
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3310

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[(d*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[(b*(c + d*x)*Cos[e + f*x]*(b
*Sin[e + f*x])^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 5372

```
Int[Cosh[(a_.) + (b_.)*(x_)^(n_.)]*(x_)^(m_.)*Sinh[(a_.) + (b_.)*(x_)^(n_.)
]^(p_.), x_Symbol] :> Simp[(x^(m - n + 1)*Sinh[a + b*x^n]^(p + 1))/(b*n*(p
+ 1)), x] - Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*Sinh[a + b*x^n]^(
p + 1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

Rubi steps

$$\int x^2\cosh(a+bx)\sinh^2(a+bx)\,dx = \frac{x^2\sinh^3(a+bx)}{3b}-\frac{2\int x\sinh^3(a+bx)\,dx}{3b}$$
$$= -\frac{2x\cosh(a+bx)\sinh^2(a+bx)}{9b^2}+\frac{2\sinh^3(a+bx)}{27b^3}+\frac{x^2\sinh^3(a+bx)}{3b}+$$
$$= \frac{4x\cosh(a+bx)}{9b^2}-\frac{2x\cosh(a+bx)\sinh^2(a+bx)}{9b^2}+\frac{2\sinh^3(a+bx)}{27b^3}+\frac{x}{}$$
$$= \frac{4x\cosh(a+bx)}{9b^2}-\frac{4\sinh(a+bx)}{9b^3}-\frac{2x\cosh(a+bx)\sinh^2(a+bx)}{9b^2}+\frac{2s}{}$$

Mathematica [A] time = 0.44, size = 66, normalized size = 0.80

$$\frac{\sinh(a+bx)\left(\left(9b^2x^2+2\right)\cosh(2(a+bx))-9b^2x^2-26\right)+27bx\cosh(a+bx)-3bx\cosh(3(a+bx))}{54b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Cosh[a + b*x]*Sinh[a + b*x]^2,x]
```

```
[Out] (27*b*x*Cosh[a + b*x] - 3*b*x*Cosh[3*(a + b*x)] + (-26 - 9*b^2*x^2 + (2 + 9
*b^2*x^2)*Cosh[2*(a + b*x)])*Sinh[a + b*x])/(54*b^3)
```

fricas [A] time = 0.46, size = 104, normalized size = 1.25

$$-\frac{6\,bx\cosh\left(bx+a\right)^3+18\,bx\cosh\left(bx+a\right)\sinh\left(bx+a\right)^2-\left(9\,b^2x^2+2\right)\sinh\left(bx+a\right)^3-54\,bx\cosh\left(bx+a\right)+3}{108\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cosh(b*x+a)*sinh(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/108*(6*b*x*cosh(b*x + a)^3 + 18*b*x*cosh(b*x + a)*sinh(b*x + a)^2 - (9*b
^2*x^2 + 2)*sinh(b*x + a)^3 - 54*b*x*cosh(b*x + a) + 3*(9*b^2*x^2 - (9*b^2*
x^2 + 2)*cosh(b*x + a)^2 + 18)*sinh(b*x + a))/b^3
```

giac [A] time = 0.12, size = 108, normalized size = 1.30

$$\frac{\left(9\,b^{2}x^{2}-6\,bx+2\right)e^{(3\,bx+3\,a)}}{216\,b^{3}}-\frac{\left(b^{2}x^{2}-2\,bx+2\right)e^{(bx+a)}}{8\,b^{3}}+\frac{\left(b^{2}x^{2}+2\,bx+2\right)e^{(-bx-a)}}{8\,b^{3}}-\frac{\left(9\,b^{2}x^{2}+6\,bx+2\right)e^{(-3\,bx-3\,a)}}{216\,b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cosh(b*x+a)*sinh(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/216*(9*b^2*x^2 - 6*b*x + 2)*e^(3*b*x + 3*a)/b^3 - 1/8*(b^2*x^2 - 2*b*x +
2)*e^(b*x + a)/b^3 + 1/8*(b^2*x^2 + 2*b*x + 2)*e^(-b*x - a)/b^3 - 1/216*(9*
b^2*x^2 + 6*b*x + 2)*e^(-3*b*x - 3*a)/b^3
```

maple [A] time = 0.33, size = 131, normalized size = 1.58

$$\frac{\frac{(bx+a)^{2}\left(\sinh^{3}(bx+a)\right)}{3}+\frac{4(bx+a)\cosh(bx+a)}{9}-\frac{2(bx+a)\cosh(bx+a)\left(\sinh^{2}(bx+a)\right)}{9}-\frac{4\sinh(bx+a)}{9}+\frac{2\left(\sinh^{3}(bx+a)\right)}{27}-2a\left(\frac{(bx+a)\left(\sinh^{3}(bx+\right.}{3}\right.}{b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*cosh(b*x+a)*sinh(b*x+a)^2,x)
```

```
[Out] 1/b^3*(1/3*(b*x+a)^2*sinh(b*x+a)^3+4/9*(b*x+a)*cosh(b*x+a)-2/9*(b*x+a)*cosh
(b*x+a)*sinh(b*x+a)^2-4/9*sinh(b*x+a)+2/27*sinh(b*x+a)^3-2*a*(1/3*(b*x+a)*s
inh(b*x+a)^3+2/9*cosh(b*x+a)-1/9*cosh(b*x+a)*sinh(b*x+a)^2)+1/3*a^2*sinh(b*
x+a)^3)
```

maxima [A] time = 0.35, size = 122, normalized size = 1.47

$$\frac{\left(9\,b^{2}x^{2}e^{(3\,a)}-6\,bxe^{(3\,a)}+2\,e^{(3\,a)}\right)e^{(3\,bx)}}{216\,b^{3}}-\frac{\left(b^{2}x^{2}e^{a}-2\,bxe^{a}+2\,e^{a}\right)e^{(bx)}}{8\,b^{3}}+\frac{\left(b^{2}x^{2}+2\,bx+2\right)e^{(-bx-a)}}{8\,b^{3}}-\frac{\left(9\,b^{2}x^{2}+6\,bx+2\right.}{216\,b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cosh(b*x+a)*sinh(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/216*(9*b^2*x^2*e^(3*a) - 6*b*x*e^(3*a) + 2*e^(3*a))*e^(3*b*x)/b^3 - 1/8*(
b^2*x^2*e^a - 2*b*x*e^a + 2*e^a)*e^(b*x)/b^3 + 1/8*(b^2*x^2 + 2*b*x + 2)*e^
(-b*x - a)/b^3 - 1/216*(9*b^2*x^2 + 6*b*x + 2)*e^(-3*b*x - 3*a)/b^3
```

mupad [B] time = 1.68, size = 82, normalized size = 0.99

$$\frac{\frac{4\,x\,\cosh(a+b\,x)^{3}}{9}-\frac{2\,x\,\cosh(a+b\,x)\,\sinh(a+b\,x)^{2}}{3}}{b^{2}}+\frac{14\,\sinh\left(a+b\,x\right)^{3}}{27\,b^{3}}-\frac{4\,\cosh\left(a+b\,x\right)^{2}\,\sinh\left(a+b\,x\right)}{9\,b^{3}}+\frac{x^{2}\,\sinh\left(a+b\,x\right)^{3}}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*cosh(a + b*x)*sinh(a + b*x)^2,x)
```

```
[Out] ((4*x*cosh(a + b*x)^3)/9 - (2*x*cosh(a + b*x)*sinh(a + b*x)^2)/3)/b^2 + (14
*sinh(a + b*x)^3)/(27*b^3) - (4*cosh(a + b*x)^2*sinh(a + b*x))/(9*b^3) + (x
^2*sinh(a + b*x)^3)/(3*b)
```

sympy [A] time = 1.70, size = 105, normalized size = 1.27

$$\begin{cases} \frac{x^2 \sinh^3(a+bx)}{3b} - \frac{2x \sinh^2(a+bx)\cosh(a+bx)}{3b^2} + \frac{4x\cosh^3(a+bx)}{9b^2} + \frac{14\sinh^3(a+bx)}{27b^3} - \frac{4\sinh(a+bx)\cosh^2(a+bx)}{9b^3} & \text{for } b \neq 0 \\ \frac{x^3 \sinh^2(a)\cosh(a)}{3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*cosh(b*x+a)*sinh(b*x+a)**2,x)
```

```
[Out] Piecewise((x**2*sinh(a + b*x)**3/(3*b) - 2*x*sinh(a + b*x)**2*cosh(a + b*x)
/(3*b**2) + 4*x*cosh(a + b*x)**3/(9*b**2) + 14*sinh(a + b*x)**3/(27*b**3) -
 4*sinh(a + b*x)*cosh(a + b*x)**2/(9*b**3), Ne(b, 0)), (x**3*sinh(a)**2*cos
h(a)/3, True))
```

3.283 $\int x\cosh(a+bx)\sinh^2(a+bx)\,dx$

Optimal. Leaf size=45

$$-\frac{\cosh^3(a+bx)}{9b^2}+\frac{\cosh(a+bx)}{3b^2}+\frac{x\sinh^3(a+bx)}{3b}$$

```
[Out] 1/3*cosh(b*x+a)/b^2-1/9*cosh(b*x+a)^3/b^2+1/3*x*sinh(b*x+a)^3/b
```

Rubi [A] time = 0.03, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {5372, 2633}

$$-\frac{\cosh^3(a+bx)}{9b^2}+\frac{\cosh(a+bx)}{3b^2}+\frac{x\sinh^3(a+bx)}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[x*Cosh[a + b*x]*Sinh[a + b*x]^2,x]
```

```
[Out] Cosh[a + b*x]/(3*b^2) - Cosh[a + b*x]^3/(9*b^2) + (x*Sinh[a + b*x]^3)/(3*b)
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 5372

```
Int[Cosh[(a_.) + (b_.)*(x_)^(n_.)]*(x_)^(m_.)*Sinh[(a_.) + (b_.)*(x_)^(n_.)
]^(p_.), x_Symbol] :> Simp[(x^(m - n + 1)*Sinh[a + b*x^n]^(p + 1))/(b*n*(p
+ 1)), x] - Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*Sinh[a + b*x^n]^(
p + 1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int x\cosh(a+bx)\sinh^2(a+bx)\,dx&=\frac{x\sinh^3(a+bx)}{3b}-\frac{\int\sinh^3(a+bx)\,dx}{3b}\\&=\frac{x\sinh^3(a+bx)}{3b}+\frac{\operatorname{Subst}\left(\int\left(1-x^2\right)\,dx,x,\cosh(a+bx)\right)}{3b^2}\\&=\frac{\cosh(a+bx)}{3b^2}-\frac{\cosh^3(a+bx)}{9b^2}+\frac{x\sinh^3(a+bx)}{3b}\end{aligned}$$

Mathematica [A] time = 0.14, size = 38, normalized size = 0.84

$$\frac{12bx\sinh^3(a+bx)+9\cosh(a+bx)-\cosh(3(a+bx))}{36b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Cosh[a + b*x]*Sinh[a + b*x]^2,x]
```

```
[Out] (9*Cosh[a + b*x] - Cosh[3*(a + b*x)] + 12*b*x*Sinh[a + b*x]^3)/(36*b^2)
```

fricas [A] time = 0.40, size = 76, normalized size = 1.69

$$\frac{3\,bx\sinh\left(bx+a\right)^3-\cosh\left(bx+a\right)^3-3\,\cosh\left(bx+a\right)\sinh\left(bx+a\right)^2+9\left(bx\cosh\left(bx+a\right)^2-bx\right)\sinh\left(bx+a\right)}{36\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x+a)*sinh(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/36*(3*b*x*sinh(b*x + a)^3 - cosh(b*x + a)^3 - 3*cosh(b*x + a)*sinh(b*x +
a)^2 + 9*(b*x*cosh(b*x + a)^2 - b*x)*sinh(b*x + a) + 9*cosh(b*x + a))/b^2
```

giac [A] time = 0.12, size = 76, normalized size = 1.69

$$\frac{(3\,bx-1)e^{(3\,bx+3\,a)}}{72\,b^2}-\frac{(bx-1)e^{(bx+a)}}{8\,b^2}+\frac{(bx+1)e^{(-bx-a)}}{8\,b^2}-\frac{(3\,bx+1)e^{(-3\,bx-3\,a)}}{72\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x+a)*sinh(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/72*(3*b*x - 1)*e^(3*b*x + 3*a)/b^2 - 1/8*(b*x - 1)*e^(b*x + a)/b^2 + 1/8*
(b*x + 1)*e^(-b*x - a)/b^2 - 1/72*(3*b*x + 1)*e^(-3*b*x - 3*a)/b^2
```

maple [A] time = 0.34, size = 56, normalized size = 1.24

$$\frac{\frac{(bx+a)\left(\sinh^3(bx+a)\right)}{3}+\frac{2\cosh(bx+a)}{9}-\frac{\cosh(bx+a)\left(\sinh^2(bx+a)\right)}{9}-\frac{a\left(\sinh^3(bx+a)\right)}{3}}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cosh(b*x+a)*sinh(b*x+a)^2,x)
```

```
[Out] 1/b^2*(1/3*(b*x+a)*sinh(b*x+a)^3+2/9*cosh(b*x+a)-1/9*cosh(b*x+a)*sinh(b*x+a
)^2-1/3*a*sinh(b*x+a)^3)
```

maxima [B] time = 0.34, size = 84, normalized size = 1.87

$$\frac{\left(3\,bxe^{(3\,a)}-e^{(3\,a)}\right)e^{(3\,bx)}}{72\,b^2}-\frac{\left(bxe^{a}-e^{a}\right)e^{(bx)}}{8\,b^2}+\frac{(bx+1)e^{(-bx-a)}}{8\,b^2}-\frac{(3\,bx+1)e^{(-3\,bx-3\,a)}}{72\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x+a)*sinh(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/72*(3*b*x*e^(3*a) - e^(3*a))*e^(3*b*x)/b^2 - 1/8*(b*x*e^a - e^a)*e^(b*x)/
b^2 + 1/8*(b*x + 1)*e^(-b*x - a)/b^2 - 1/72*(3*b*x + 1)*e^(-3*b*x - 3*a)/b^
2
```

mupad [B] time = 1.68, size = 44, normalized size = 0.98

$$\frac{2\cosh\left(a+b\,x\right)^3-3\cosh\left(a+b\,x\right)\,\sinh\left(a+b\,x\right)^2+3\,b\,x\sinh\left(a+b\,x\right)^3}{9\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cosh(a + b*x)*sinh(a + b*x)^2,x)
```

```
[Out] (2*cosh(a + b*x)^3 - 3*cosh(a + b*x)*sinh(a + b*x)^2 + 3*b*x*sinh(a + b*x)^
3)/(9*b^2)
```

sympy [A] time = 0.80, size = 61, normalized size = 1.36

$$\begin{cases} \frac{x\sinh^3(a+bx)}{3b}-\frac{\sinh^2(a+bx)\cosh(a+bx)}{3b^2}+\frac{2\cosh^3(a+bx)}{9b^2} & \text{for } b\neq 0\\ \frac{x^2\sinh^2(a)\cosh(a)}{2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x+a)*sinh(b*x+a)**2,x)
```

```
[Out] Piecewise((x*sinh(a + b*x)**3/(3*b) - sinh(a + b*x)**2*cosh(a + b*x)/(3*b**
2) + 2*cosh(a + b*x)**3/(9*b**2), Ne(b, 0)), (x**2*sinh(a)**2*cosh(a)/2, Tr
ue))
```

3.284 $\int \cosh(a+bx)\sinh^2(a+bx)\,dx$

Optimal. Leaf size=15

$$\frac{\sinh^3(a+bx)}{3b}$$

```
[Out] 1/3*sinh(b*x+a)^3/b
```

Rubi [A] time = 0.02, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2564, 30}

$$\frac{\sinh^3(a+bx)}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[a + b*x]*Sinh[a + b*x]^2,x]
```

```
[Out] Sinh[a + b*x]^3/(3*b)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rubi steps

$$\begin{aligned}\int \cosh(a+bx)\sinh^2(a+bx)\,dx &= \frac{i\,\text{Subst}\left(\int x^2\,dx, x, i\sinh(a+bx)\right)}{b}\\ &= \frac{\sinh^3(a+bx)}{3b}\end{aligned}$$

Mathematica [A] time = 0.00, size = 15, normalized size = 1.00

$$\frac{\sinh^3(a+bx)}{3b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[a + b*x]*Sinh[a + b*x]^2,x]
```

```
[Out] Sinh[a + b*x]^3/(3*b)
```

fricas [B] time = 0.58, size = 32, normalized size = 2.13

$$\frac{\sinh\left(bx+a\right)^{3}+3\left(\cosh\left(bx+a\right)^{2}-1\right)\sinh\left(bx+a\right)}{12\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*sinh(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/12*(sinh(b*x + a)^3 + 3*(cosh(b*x + a)^2 - 1)*sinh(b*x + a))/b
```

giac [B] time = 0.14, size = 54, normalized size = 3.60

$$\frac{e^{\left(3\,bx+3\,a\right)}}{24\,b}-\frac{e^{\left(bx+a\right)}}{8\,b}+\frac{e^{\left(-bx-a\right)}}{8\,b}-\frac{e^{\left(-3\,bx-3\,a\right)}}{24\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*sinh(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/24*e^(3*b*x + 3*a)/b - 1/8*e^(b*x + a)/b + 1/8*e^(-b*x - a)/b - 1/24*e^(-
3*b*x - 3*a)/b
```

maple [A] time = 0.02, size = 14, normalized size = 0.93

$$\frac{\sinh^{3}\left(bx+a\right)}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)*sinh(b*x+a)^2,x)
```

```
[Out] 1/3*sinh(b*x+a)^3/b
```

maxima [A] time = 0.33, size = 13, normalized size = 0.87

$$\frac{\sinh\left(bx+a\right)^{3}}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*sinh(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/3*sinh(b*x + a)^3/b
```

mupad [B] time = 1.64, size = 13, normalized size = 0.87

$$\frac{\sinh(a+bx)^3}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a + b*x)*sinh(a + b*x)^2,x)
[Out] sinh(a + b*x)^3/(3*b)
```

sympy [A] time = 0.37, size = 20, normalized size = 1.33

$$\begin{cases} \frac{\sinh^3(a+bx)}{3b} & \text{for } b \neq 0 \\ x\sinh^2(a)\cosh(a) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*sinh(b*x+a)**2,x)
[Out] Piecewise((sinh(a + b*x)**3/(3*b), Ne(b, 0)), (x*sinh(a)**2*cosh(a), True))
```

3.285 $\int \frac{\cosh(a+bx)\sinh^2(a+bx)}{x}\,dx$

Optimal. Leaf size=47

$$-\frac{1}{4}\cosh(a)\text{Chi}(bx)+\frac{1}{4}\cosh(3a)\text{Chi}(3bx)-\frac{1}{4}\sinh(a)\text{Shi}(bx)+\frac{1}{4}\sinh(3a)\text{Shi}(3bx)$$

```
[Out] -1/4*Chi(b*x)*cosh(a)+1/4*Chi(3*b*x)*cosh(3*a)-1/4*Shi(b*x)*sinh(a)+1/4*Shi
(3*b*x)*sinh(3*a)
```

Rubi [A] time = 0.12, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {5448, 3303, 3298, 3301}

$$-\frac{1}{4}\cosh(a)\text{Chi}(bx)+\frac{1}{4}\cosh(3a)\text{Chi}(3bx)-\frac{1}{4}\sinh(a)\text{Shi}(bx)+\frac{1}{4}\sinh(3a)\text{Shi}(3bx)$$

Antiderivative was successfully verified.

```
[In] Int[(Cosh[a + b*x]*Sinh[a + b*x]^2)/x,x]
```

```
[Out] -(Cosh[a]*CoshIntegral[b*x])/4 + (Cosh[3*a]*CoshIntegral[3*b*x])/4 - (Sinh[
a]*SinhIntegral[b*x])/4 + (Sinh[3*a]*SinhIntegral[3*b*x])/4
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\cosh(a+bx)\sinh^2(a+bx)}{x}\,dx &= \int \left(-\frac{\cosh(a+bx)}{4x}+\frac{\cosh(3a+3bx)}{4x}\right)dx\\ &= -\left(\frac{1}{4}\int \frac{\cosh(a+bx)}{x}\,dx\right)+\frac{1}{4}\int \frac{\cosh(3a+3bx)}{x}\,dx\\ &= -\left(\frac{1}{4}\cosh(a)\int \frac{\cosh(bx)}{x}\,dx\right)+\frac{1}{4}\cosh(3a)\int \frac{\cosh(3bx)}{x}\,dx-\frac{1}{4}\sinh(a)\int\\ &= -\frac{1}{4}\cosh(a)\mathrm{Chi}(bx)+\frac{1}{4}\cosh(3a)\mathrm{Chi}(3bx)-\frac{1}{4}\sinh(a)\mathrm{Shi}(bx)+\frac{1}{4}\sinh(3a\end{aligned}$$

Mathematica [A] time = 0.07, size = 41, normalized size = 0.87

$$\frac{1}{4}(-\cosh(a)\mathrm{Chi}(bx)+\cosh(3a)\mathrm{Chi}(3bx)-\sinh(a)\mathrm{Shi}(bx)+\sinh(3a)\mathrm{Shi}(3bx))$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cosh[a + b*x]*Sinh[a + b*x]^2)/x,x]
```

```
[Out] (-(Cosh[a]*CoshIntegral[b*x]) + Cosh[3*a]*CoshIntegral[3*b*x] - Sinh[a]*Sin
hIntegral[b*x] + Sinh[3*a]*SinhIntegral[3*b*x])/4
```

fricas [A] time = 0.62, size = 67, normalized size = 1.43

$$\frac{1}{8}\left(\mathrm{Ei}\left(3\,bx\right)+\mathrm{Ei}\left(-3\,bx\right)\right)\cosh\left(3\,a\right)-\frac{1}{8}\left(\mathrm{Ei}\left(bx\right)+\mathrm{Ei}\left(-bx\right)\right)\cosh(a)+\frac{1}{8}\left(\mathrm{Ei}\left(3\,bx\right)-\mathrm{Ei}\left(-3\,bx\right)\right)\sinh\left(3\,a\right)-\frac{1}{8}\left(\mathrm{Ei}\left(b\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*sinh(b*x+a)^2/x,x, algorithm="fricas")
```

```
[Out] 1/8*(Ei(3*b*x) + Ei(-3*b*x))*cosh(3*a) - 1/8*(Ei(b*x) + Ei(-b*x))*cosh(a) +
 1/8*(Ei(3*b*x) - Ei(-3*b*x))*sinh(3*a) - 1/8*(Ei(b*x) - Ei(-b*x))*sinh(a)
```

giac [A] time = 0.14, size = 42, normalized size = 0.89

$$\frac{1}{8}\,\mathrm{Ei}\left(3\,bx\right)e^{(3\,a)}-\frac{1}{8}\,\mathrm{Ei}\left(-bx\right)e^{(-a)}+\frac{1}{8}\,\mathrm{Ei}\left(-3\,bx\right)e^{(-3\,a)}-\frac{1}{8}\,\mathrm{Ei}\left(bx\right)e^{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*sinh(b*x+a)^2/x,x, algorithm="giac")
```

```
[Out] 1/8*Ei(3*b*x)*e^(3*a) - 1/8*Ei(-b*x)*e^(-a) + 1/8*Ei(-3*b*x)*e^(-3*a) - 1/8
*Ei(b*x)*e^a
```

maple [A] time = 0.59, size = 47, normalized size = 1.00

$$-\frac{e^{-3a}\,\mathrm{Ei}\,(1,3bx)}{8}+\frac{e^{-a}\,\mathrm{Ei}\,(1,bx)}{8}+\frac{e^{a}\,\mathrm{Ei}\,(1,-bx)}{8}-\frac{e^{3a}\,\mathrm{Ei}\,(1,-3bx)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)*sinh(b*x+a)^2/x,x)
```

```
[Out] -1/8*exp(-3*a)*Ei(1,3*b*x)+1/8*exp(-a)*Ei(1,b*x)+1/8*exp(a)*Ei(1,-b*x)-1/8*
exp(3*a)*Ei(1,-3*b*x)
```

maxima [A] time = 0.41, size = 42, normalized size = 0.89

$$\frac{1}{8}\,\mathrm{Ei}\,(3\,bx)\,e^{(3\,a)}-\frac{1}{8}\,\mathrm{Ei}\,(-bx)\,e^{(-a)}+\frac{1}{8}\,\mathrm{Ei}\,(-3\,bx)\,e^{(-3\,a)}-\frac{1}{8}\,\mathrm{Ei}\,(bx)\,e^{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*sinh(b*x+a)^2/x,x, algorithm="maxima")
```

```
[Out] 1/8*Ei(3*b*x)*e^(3*a) - 1/8*Ei(-b*x)*e^(-a) + 1/8*Ei(-3*b*x)*e^(-3*a) - 1/8
*Ei(b*x)*e^a
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\cosh\left(a+b\,x\right)\,{\sinh\left(a+b\,x\right)}^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cosh(a + b*x)*sinh(a + b*x)^2)/x,x)
```

```
[Out] int((cosh(a + b*x)*sinh(a + b*x)^2)/x, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh^{2}\left(a+bx\right)\cosh\left(a+bx\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*sinh(b*x+a)**2/x,x)
```

```
[Out] Integral(sinh(a + b*x)**2*cosh(a + b*x)/x, x)
```

3.286 $\int \frac{\cosh(a+bx)\sinh^2(a+bx)}{x^2}\,dx$

Optimal. Leaf size=80

$$-\frac{1}{4}b\sinh(a)\text{Chi}(bx)+\frac{3}{4}b\sinh(3a)\text{Chi}(3bx)-\frac{1}{4}b\cosh(a)\text{Shi}(bx)+\frac{3}{4}b\cosh(3a)\text{Shi}(3bx)+\frac{\cosh(a+bx)}{4x}-\frac{\cosh(3a$$

```
[Out] 1/4*cosh(b*x+a)/x-1/4*cosh(3*b*x+3*a)/x-1/4*b*cosh(a)*Shi(b*x)+3/4*b*cosh(3
*a)*Shi(3*b*x)-1/4*b*Chi(b*x)*sinh(a)+3/4*b*Chi(3*b*x)*sinh(3*a)
```

Rubi [A] time = 0.17, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {5448, 3297, 3303, 3298, 3301}

$$-\frac{1}{4}b\sinh(a)\text{Chi}(bx)+\frac{3}{4}b\sinh(3a)\text{Chi}(3bx)-\frac{1}{4}b\cosh(a)\text{Shi}(bx)+\frac{3}{4}b\cosh(3a)\text{Shi}(3bx)+\frac{\cosh(a+bx)}{4x}-\frac{\cosh(3a$$

Antiderivative was successfully verified.

```
[In] Int[(Cosh[a + b*x]*Sinh[a + b*x]^2)/x^2,x]
```

```
[Out] Cosh[a + b*x]/(4*x) - Cosh[3*a + 3*b*x]/(4*x) - (b*CoshIntegral[b*x]*Sinh[a
])/4 + (3*b*CoshIntegral[3*b*x]*Sinh[3*a])/4 - (b*Cosh[a]*SinhIntegral[b*x]
)/4 + (3*b*Cosh[3*a]*SinhIntegral[3*b*x])/4
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\cosh(a+bx)\sinh^2(a+bx)}{x^2}\,dx &= \int \left(-\frac{\cosh(a+bx)}{4x^2}+\frac{\cosh(3a+3bx)}{4x^2}\right)dx\\ &= -\left(\frac{1}{4}\int \frac{\cosh(a+bx)}{x^2}\,dx\right)+\frac{1}{4}\int \frac{\cosh(3a+3bx)}{x^2}\,dx\\ &= \frac{\cosh(a+bx)}{4x}-\frac{\cosh(3a+3bx)}{4x}-\frac{1}{4}b\int \frac{\sinh(a+bx)}{x}\,dx+\frac{1}{4}(3b)\int \frac{\sinh(3a}{}\\ &= \frac{\cosh(a+bx)}{4x}-\frac{\cosh(3a+3bx)}{4x}-\frac{1}{4}(b\cosh(a))\int \frac{\sinh(bx)}{x}\,dx+\frac{1}{4}(3b\cosh\\ &= \frac{\cosh(a+bx)}{4x}-\frac{\cosh(3a+3bx)}{4x}-\frac{1}{4}b\text{Chi}(bx)\sinh(a)+\frac{3}{4}b\text{Chi}(3bx)\sinh(3a\end{aligned}$$

Mathematica [A] time = 0.22, size = 68, normalized size = 0.85

$$-\frac{bx\sinh(a)\text{Chi}(bx)-3bx\sinh(3a)\text{Chi}(3bx)+bx\cosh(a)\text{Shi}(bx)-3bx\cosh(3a)\text{Shi}(3bx)-\cosh(a+bx)+\cosh}{4x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cosh[a + b*x]*Sinh[a + b*x]^2)/x^2,x]
```

```
[Out] -1/4*(-Cosh[a + b*x] + Cosh[3*(a + b*x)] + b*x*CoshIntegral[b*x]*Sinh[a] -
3*b*x*CoshIntegral[3*b*x]*Sinh[3*a] + b*x*Cosh[a]*SinhIntegral[b*x] - 3*b*x
*Cosh[3*a]*SinhIntegral[3*b*x])/x
```

fricas [A] time = 0.57, size = 126, normalized size = 1.58

$$-\frac{2\cosh(bx+a)^3+6\cosh(bx+a)\sinh(bx+a)^2-3(bx\text{Ei}(3bx)-bx\text{Ei}(-3bx))\cosh(3a)+(bx\text{Ei}(bx)-bx\text{Ei}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*sinh(b*x+a)^2/x^2,x, algorithm="fricas")
```

```
[Out] -1/8*(2*cosh(b*x + a)^3 + 6*cosh(b*x + a)*sinh(b*x + a)^2 - 3*(b*x*Ei(3*b*x
) - b*x*Ei(-3*b*x))*cosh(3*a) + (b*x*Ei(b*x) - b*x*Ei(-b*x))*cosh(a) - 3*(b
*x*Ei(3*b*x) + b*x*Ei(-3*b*x))*sinh(3*a) + (b*x*Ei(b*x) + b*x*Ei(-b*x))*sin
h(a) - 2*cosh(b*x + a))/x
```

giac [A] time = 0.14, size = 91, normalized size = 1.14

$$\frac{3\,bx\mathrm{Ei}\left(3\,bx\right)e^{(3\,a)}+bx\mathrm{Ei}\left(-bx\right)e^{(-a)}-3\,bx\mathrm{Ei}\left(-3\,bx\right)e^{(-3\,a)}-bx\mathrm{Ei}\left(bx\right)e^{a}-e^{(3\,bx+3\,a)}+e^{(bx+a)}+e^{(-bx-a)}-e^{(-3\,bx-}}{8\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*sinh(b*x+a)^2/x^2,x, algorithm="giac")
```

```
[Out] 1/8*(3*b*x*Ei(3*b*x)*e^(3*a) + b*x*Ei(-b*x)*e^(-a) - 3*b*x*Ei(-3*b*x)*e^(-3
*a) - b*x*Ei(b*x)*e^a - e^(3*b*x + 3*a) + e^(b*x + a) + e^(-b*x - a) - e^(-
3*b*x - 3*a))/x
```

maple [A] time = 0.57, size = 104, normalized size = 1.30

$$-\frac{\mathrm{e}^{-3bx-3a}}{8x}+\frac{3b\,\mathrm{e}^{-3a}\,\mathrm{Ei}\left(1,3bx\right)}{8}+\frac{\mathrm{e}^{-bx-a}}{8x}-\frac{b\,\mathrm{e}^{-a}\,\mathrm{Ei}\left(1,bx\right)}{8}+\frac{\mathrm{e}^{bx+a}}{8x}+\frac{b\,\mathrm{e}^{a}\,\mathrm{Ei}\left(1,-bx\right)}{8}-\frac{\mathrm{e}^{3bx+3a}}{8x}-\frac{3b\,\mathrm{e}^{3a}\,\mathrm{Ei}\left(1,-3bx\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)*sinh(b*x+a)^2/x^2,x)
```

```
[Out] -1/8*exp(-3*b*x-3*a)/x+3/8*b*exp(-3*a)*Ei(1,3*b*x)+1/8*exp(-b*x-a)/x-1/8*b*
exp(-a)*Ei(1,b*x)+1/8/x*exp(b*x+a)+1/8*b*exp(a)*Ei(1,-b*x)-1/8/x*exp(3*b*x+
3*a)-3/8*b*exp(3*a)*Ei(1,-3*b*x)
```

maxima [A] time = 0.42, size = 50, normalized size = 0.62

$$-\frac{3}{8}\,be^{(-3\,a)}\Gamma\left(-1,3\,bx\right)+\frac{1}{8}\,be^{(-a)}\Gamma\left(-1,bx\right)-\frac{1}{8}\,be^{a}\Gamma\left(-1,-bx\right)+\frac{3}{8}\,be^{(3\,a)}\Gamma\left(-1,-3\,bx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*sinh(b*x+a)^2/x^2,x, algorithm="maxima")
```

```
[Out] -3/8*b*e^(-3*a)*gamma(-1, 3*b*x) + 1/8*b*e^(-a)*gamma(-1, b*x) - 1/8*b*e^a*
gamma(-1, -b*x) + 3/8*b*e^(3*a)*gamma(-1, -3*b*x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cosh \left(a+b\,x\right)\,\sinh \left(a+b\,x\right)^2}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cosh(a + b*x)*sinh(a + b*x)^2)/x^2,x)
[Out] int((cosh(a + b*x)*sinh(a + b*x)^2)/x^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh^2 (a+bx)\cosh (a+bx)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*sinh(b*x+a)**2/x**2,x)
[Out] Integral(sinh(a + b*x)**2*cosh(a + b*x)/x**2, x)
```

3.287 $\int \frac{\cosh(a+bx)\sinh^2(a+bx)}{x^3}\,dx$

Optimal. Leaf size=119

$$-\frac{1}{8}b^2\cosh(a)\text{Chi}(bx)+\frac{9}{8}b^2\cosh(3a)\text{Chi}(3bx)-\frac{1}{8}b^2\sinh(a)\text{Shi}(bx)+\frac{9}{8}b^2\sinh(3a)\text{Shi}(3bx)+\frac{\cosh(a+bx)}{8x^2}-\text{cosh}$$

```
[Out] -1/8*b^2*Chi(b*x)*cosh(a)+9/8*b^2*Chi(3*b*x)*cosh(3*a)+1/8*cosh(b*x+a)/x^2-
1/8*cosh(3*b*x+3*a)/x^2-1/8*b^2*Shi(b*x)*sinh(a)+9/8*b^2*Shi(3*b*x)*sinh(3*
a)+1/8*b*sinh(b*x+a)/x-3/8*b*sinh(3*b*x+3*a)/x
```

Rubi [A] time = 0.22, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {5448, 3297, 3303, 3298, 3301}

$$-\frac{1}{8}b^2\cosh(a)\text{Chi}(bx)+\frac{9}{8}b^2\cosh(3a)\text{Chi}(3bx)-\frac{1}{8}b^2\sinh(a)\text{Shi}(bx)+\frac{9}{8}b^2\sinh(3a)\text{Shi}(3bx)+\frac{\cosh(a+bx)}{8x^2}-\text{cosh}$$

Antiderivative was successfully verified.

```
[In] Int[(Cosh[a + b*x]*Sinh[a + b*x]^2)/x^3,x]
```

```
[Out] Cosh[a + b*x]/(8*x^2) - Cosh[3*a + 3*b*x]/(8*x^2) - (b^2*Cosh[a]*CoshIntegr
al[b*x])/8 + (9*b^2*Cosh[3*a]*CoshIntegral[3*b*x])/8 + (b*Sinh[a + b*x])/(8
*x) - (3*b*Sinh[3*a + 3*b*x])/(8*x) - (b^2*Sinh[a]*SinhIntegral[b*x])/8 + (
9*b^2*Sinh[3*a]*SinhIntegral[3*b*x])/8
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cosh(a+bx)\sinh^2(a+bx)}{x^3}\,dx &= \int \left(-\frac{\cosh(a+bx)}{4x^3}+\frac{\cosh(3a+3bx)}{4x^3}\right)dx\\
&= -\left(\frac{1}{4}\int \frac{\cosh(a+bx)}{x^3}\,dx\right)+\frac{1}{4}\int \frac{\cosh(3a+3bx)}{x^3}\,dx\\
&= \frac{\cosh(a+bx)}{8x^2}-\frac{\cosh(3a+3bx)}{8x^2}-\frac{1}{8}b\int \frac{\sinh(a+bx)}{x^2}\,dx+\frac{1}{8}(3b)\int \frac{\sinh(3a}{x}\\
&= \frac{\cosh(a+bx)}{8x^2}-\frac{\cosh(3a+3bx)}{8x^2}+\frac{b\sinh(a+bx)}{8x}-\frac{3b\sinh(3a+3bx)}{8x}-\frac{1}{8}b\\
&= \frac{\cosh(a+bx)}{8x^2}-\frac{\cosh(3a+3bx)}{8x^2}+\frac{b\sinh(a+bx)}{8x}-\frac{3b\sinh(3a+3bx)}{8x}-\frac{1}{8}(\\
&= \frac{\cosh(a+bx)}{8x^2}-\frac{\cosh(3a+3bx)}{8x^2}-\frac{1}{8}b^2\cosh(a)\mathrm{Chi}(bx)+\frac{9}{8}b^2\cosh(3a)\mathrm{Chi}(3
\end{aligned}$$

Mathematica [A] time = 0.26, size = 107, normalized size = 0.90

$$\frac{-b^2x^2\cosh(a)\mathrm{Chi}(bx)+9b^2x^2\cosh(3a)\mathrm{Chi}(3bx)-b^2x^2\sinh(a)\mathrm{Shi}(bx)+9b^2x^2\sinh(3a)\mathrm{Shi}(3bx)+bx\sinh(a+}{8x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cosh[a + b*x]*Sinh[a + b*x]^2)/x^3,x]
```

```
[Out] (Cosh[a + b*x] - Cosh[3*(a + b*x)] - b^2*x^2*Cosh[a]*CoshIntegral[b*x] + 9*
b^2*x^2*Cosh[3*a]*CoshIntegral[3*b*x] + b*x*Sinh[a + b*x] - 3*b*x*Sinh[3*(a
 + b*x)] - b^2*x^2*Sinh[a]*SinhIntegral[b*x] + 9*b^2*x^2*Sinh[3*a]*SinhInte
gral[3*b*x])/(8*x^2)
```

fricas [A] time = 0.62, size = 195, normalized size = 1.64

$$-\frac{6\,bx\sinh\left(bx+a\right)^{3}+2\,\cosh\left(bx+a\right)^{3}+6\,\cosh\left(bx+a\right)\sinh\left(bx+a\right)^{2}-9\,\left(b^{2}x^{2}\mathrm{Ei}\left(3\,bx\right)+b^{2}x^{2}\mathrm{Ei}\left(-3\,bx\right)\right)\mathrm{co}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*sinh(b*x+a)^2/x^3,x, algorithm="fricas")
```

```
[Out] -1/16*(6*b*x*sinh(b*x + a)^3 + 2*cosh(b*x + a)^3 + 6*cosh(b*x + a)*sinh(b*x
 + a)^2 - 9*(b^2*x^2*Ei(3*b*x) + b^2*x^2*Ei(-3*b*x))*cosh(3*a) + (b^2*x^2*E
i(b*x) + b^2*x^2*Ei(-b*x))*cosh(a) + 2*(9*b*x*cosh(b*x + a)^2 - b*x)*sinh(b
*x + a) - 9*(b^2*x^2*Ei(3*b*x) - b^2*x^2*Ei(-3*b*x))*sinh(3*a) + (b^2*x^2*E
i(b*x) - b^2*x^2*Ei(-b*x))*sinh(a) - 2*cosh(b*x + a))/x^2
```

giac [A] time = 0.14, size = 156, normalized size = 1.31

$$\frac{9\,b^{2}x^{2}\mathrm{Ei}\left(3\,bx\right)e^{(3\,a)}-b^{2}x^{2}\mathrm{Ei}\left(-bx\right)e^{(-a)}+9\,b^{2}x^{2}\mathrm{Ei}\left(-3\,bx\right)e^{(-3\,a)}-b^{2}x^{2}\mathrm{Ei}\left(bx\right)e^{a}-3\,bxe^{(3\,bx+3\,a)}+bxe^{(bx+a)}-bx}{16\,x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*sinh(b*x+a)^2/x^3,x, algorithm="giac")
```

```
[Out] 1/16*(9*b^2*x^2*Ei(3*b*x)*e^(3*a) - b^2*x^2*Ei(-b*x)*e^(-a) + 9*b^2*x^2*Ei(
-3*b*x)*e^(-3*a) - b^2*x^2*Ei(b*x)*e^a - 3*b*x*e^(3*b*x + 3*a) + b*x*e^(b*x
 + a) - b*x*e^(-b*x - a) + 3*b*x*e^(-3*b*x - 3*a) - e^(3*b*x + 3*a) + e^(b*
x + a) + e^(-b*x - a) - e^(-3*b*x - 3*a))/x^2
```

maple [A] time = 0.58, size = 169, normalized size = 1.42

$$\frac{3b\,\mathrm{e}^{-3bx-3a}}{16x}-\frac{\mathrm{e}^{-3bx-3a}}{16x^{2}}-\frac{9b^{2}\mathrm{e}^{-3a}\,\mathrm{Ei}\left(1,3bx\right)}{16}-\frac{b\,\mathrm{e}^{-bx-a}}{16x}+\frac{\mathrm{e}^{-bx-a}}{16x^{2}}+\frac{b^{2}\mathrm{e}^{-a}\,\mathrm{Ei}\left(1,bx\right)}{16}+\frac{\mathrm{e}^{bx+a}}{16x^{2}}+\frac{b\,\mathrm{e}^{bx+a}}{16x}+\frac{b^{2}\mathrm{e}^{a}\,\mathrm{Ei}\left(1,-bx\right)}{16}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)*sinh(b*x+a)^2/x^3,x)
```

```
[Out] 3/16*b*exp(-3*b*x-3*a)/x-1/16*exp(-3*b*x-3*a)/x^2-9/16*b^2*exp(-3*a)*Ei(1,3
*b*x)-1/16*b*exp(-b*x-a)/x+1/16*exp(-b*x-a)/x^2+1/16*b^2*exp(-a)*Ei(1,b*x)+
1/16/x^2*exp(b*x+a)+1/16*b/x*exp(b*x+a)+1/16*b^2*exp(a)*Ei(1,-b*x)-1/16/x^2
*exp(3*b*x+3*a)-3/16*b/x*exp(3*b*x+3*a)-9/16*b^2*exp(3*a)*Ei(1,-3*b*x)
```

maxima [A] time = 0.43, size = 58, normalized size = 0.49

$$-\frac{9}{8}\,b^{2}e^{(-3\,a)}\Gamma\left(-2,3\,bx\right)+\frac{1}{8}\,b^{2}e^{(-a)}\Gamma\left(-2,bx\right)+\frac{1}{8}\,b^{2}e^{a}\Gamma\left(-2,-bx\right)-\frac{9}{8}\,b^{2}e^{(3\,a)}\Gamma\left(-2,-3\,bx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*sinh(b*x+a)^2/x^3,x, algorithm="maxima")
```

```
[Out] -9/8*b^2*e^(-3*a)*gamma(-2, 3*b*x) + 1/8*b^2*e^(-a)*gamma(-2, b*x) + 1/8*b^
2*e^a*gamma(-2, -b*x) - 9/8*b^2*e^(3*a)*gamma(-2, -3*b*x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cosh\left(a+b\,x\right)\,{\sinh\left(a+b\,x\right)}^2}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cosh(a + b*x)*sinh(a + b*x)^2)/x^3,x)
```

```
[Out] int((cosh(a + b*x)*sinh(a + b*x)^2)/x^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh^{2}{\left(a + bx \right)} \cosh{\left(a + bx \right)}}{x^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*sinh(b*x+a)**2/x**3,x)
```

```
[Out] Integral(sinh(a + b*x)**2*cosh(a + b*x)/x**3, x)
```

3.288 $\int \frac{\cosh(a+bx)\sinh^2(a+bx)}{x^4}\,dx$

Optimal. Leaf size=154

$$-\frac{1}{24}b^3\sinh(a)\text{Chi}(bx)+\frac{9}{8}b^3\sinh(3a)\text{Chi}(3bx)-\frac{1}{24}b^3\cosh(a)\text{Shi}(bx)+\frac{9}{8}b^3\cosh(3a)\text{Shi}(3bx)+\frac{b^2\cosh(a+bx)}{24x}$$

```
[Out] 1/12*cosh(b*x+a)/x^3+1/24*b^2*cosh(b*x+a)/x-1/12*cosh(3*b*x+3*a)/x^3-3/8*b^
2*cosh(3*b*x+3*a)/x-1/24*b^3*cosh(a)*Shi(b*x)+9/8*b^3*cosh(3*a)*Shi(3*b*x)-
1/24*b^3*Chi(b*x)*sinh(a)+9/8*b^3*Chi(3*b*x)*sinh(3*a)+1/24*b*sinh(b*x+a)/x
^2-1/8*b*sinh(3*b*x+3*a)/x^2
```

Rubi [A] time = 0.28, antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {5448, 3297, 3303, 3298, 3301}

$$-\frac{1}{24}b^3\sinh(a)\text{Chi}(bx)+\frac{9}{8}b^3\sinh(3a)\text{Chi}(3bx)-\frac{1}{24}b^3\cosh(a)\text{Shi}(bx)+\frac{9}{8}b^3\cosh(3a)\text{Shi}(3bx)+\frac{b^2\cosh(a+bx)}{24x}$$

Antiderivative was successfully verified.

```
[In] Int[(Cosh[a + b*x]*Sinh[a + b*x]^2)/x^4,x]
```

```
[Out] Cosh[a + b*x]/(12*x^3) + (b^2*Cosh[a + b*x])/(24*x) - Cosh[3*a + 3*b*x]/(12
*x^3) - (3*b^2*Cosh[3*a + 3*b*x])/(8*x) - (b^3*CoshIntegral[b*x]*Sinh[a])/2
4 + (9*b^3*CoshIntegral[3*b*x]*Sinh[3*a])/8 + (b*Sinh[a + b*x])/(24*x^2) -
(b*Sinh[3*a + 3*b*x])/(8*x^2) - (b^3*Cosh[a]*SinhIntegral[b*x])/24 + (9*b^3
*Cosh[3*a]*SinhIntegral[3*b*x])/8
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
```

```
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rubi steps

$$\int \frac{\cosh(a+bx)\sinh^2(a+bx)}{x^4}\,dx = \int \left(-\frac{\cosh(a+bx)}{4x^4}+\frac{\cosh(3a+3bx)}{4x^4}\right)dx$$
$$= -\left(\frac{1}{4}\int \frac{\cosh(a+bx)}{x^4}\,dx\right)+\frac{1}{4}\int \frac{\cosh(3a+3bx)}{x^4}\,dx$$
$$= \frac{\cosh(a+bx)}{12x^3}-\frac{\cosh(3a+3bx)}{12x^3}-\frac{1}{12}b\int \frac{\sinh(a+bx)}{x^3}\,dx+\frac{1}{4}b\int \frac{\sinh(3a}{x^3}$$
$$= \frac{\cosh(a+bx)}{12x^3}-\frac{\cosh(3a+3bx)}{12x^3}+\frac{b\sinh(a+bx)}{24x^2}-\frac{b\sinh(3a+3bx)}{8x^2}-\frac{1}{24}b$$
$$= \frac{\cosh(a+bx)}{12x^3}+\frac{b^2\cosh(a+bx)}{24x}-\frac{\cosh(3a+3bx)}{12x^3}-\frac{3b^2\cosh(3a+3bx)}{8x}+$$
$$= \frac{\cosh(a+bx)}{12x^3}+\frac{b^2\cosh(a+bx)}{24x}-\frac{\cosh(3a+3bx)}{12x^3}-\frac{3b^2\cosh(3a+3bx)}{8x}+$$
$$= \frac{\cosh(a+bx)}{12x^3}+\frac{b^2\cosh(a+bx)}{24x}-\frac{\cosh(3a+3bx)}{12x^3}-\frac{3b^2\cosh(3a+3bx)}{8x}-$$

Mathematica [A] time = 0.32, size = 138, normalized size = 0.90

$$\frac{-b^3x^3\sinh(a)\text{Chi}(bx)+27b^3x^3\sinh(3a)\text{Chi}(3bx)-b^3x^3\cosh(a)\text{Shi}(bx)+27b^3x^3\cosh(3a)\text{Shi}(3bx)+b^2x^2\cosh}{24}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cosh[a + b*x]*Sinh[a + b*x]^2)/x^4,x]
```

```
[Out] (2*Cosh[a + b*x] + b^2*x^2*Cosh[a + b*x] - 2*Cosh[3*(a + b*x)] - 9*b^2*x^2*
Cosh[3*(a + b*x)] - b^3*x^3*CoshIntegral[b*x]*Sinh[a] + 27*b^3*x^3*CoshInte
gral[3*b*x]*Sinh[3*a] + b*x*Sinh[a + b*x] - 3*b*x*Sinh[3*(a + b*x)] - b^3*x
^3*Cosh[a]*SinhIntegral[b*x] + 27*b^3*x^3*Cosh[3*a]*SinhIntegral[3*b*x])/(2
4*x^3)
```

fricas [A] time = 0.48, size = 224, normalized size = 1.45

$$-\frac{6\,bx\sinh\left(bx+a\right)^{3}+2\left(9\,b^{2}x^{2}+2\right)\cosh\left(bx+a\right)^{3}+6\left(9\,b^{2}x^{2}+2\right)\cosh\left(bx+a\right)\sinh\left(bx+a\right)^{2}-2\left(b^{2}x^{2}+2\right)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*sinh(b*x+a)^2/x^4,x, algorithm="fricas")
```

```
[Out] -1/48*(6*b*x*sinh(b*x + a)^3 + 2*(9*b^2*x^2 + 2)*cosh(b*x + a)^3 + 6*(9*b^2
*x^2 + 2)*cosh(b*x + a)*sinh(b*x + a)^2 - 2*(b^2*x^2 + 2)*cosh(b*x + a) - 2
7*(b^3*x^3*Ei(3*b*x) - b^3*x^3*Ei(-3*b*x))*cosh(3*a) + (b^3*x^3*Ei(b*x) - b
^3*x^3*Ei(-b*x))*cosh(a) + 2*(9*b*x*cosh(b*x + a)^2 - b*x)*sinh(b*x + a) -
27*(b^3*x^3*Ei(3*b*x) + b^3*x^3*Ei(-3*b*x))*sinh(3*a) + (b^3*x^3*Ei(b*x) +
b^3*x^3*Ei(-b*x))*sinh(a))/x^3
```

giac [A] time = 0.14, size = 222, normalized size = 1.44

$$\frac{27\,b^{3}x^{3}{\rm Ei}\left(3\,bx\right)e^{\left(3\,a\right)}+b^{3}x^{3}{\rm Ei}\left(-bx\right)e^{\left(-a\right)}-27\,b^{3}x^{3}{\rm Ei}\left(-3\,bx\right)e^{\left(-3\,a\right)}-b^{3}x^{3}{\rm Ei}\left(bx\right)e^{a}-9\,b^{2}x^{2}e^{\left(3\,bx+3\,a\right)}+b^{2}x^{2}e^{(bx+}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*sinh(b*x+a)^2/x^4,x, algorithm="giac")
```

```
[Out] 1/48*(27*b^3*x^3*Ei(3*b*x)*e^(3*a) + b^3*x^3*Ei(-b*x)*e^(-a) - 27*b^3*x^3*E
i(-3*b*x)*e^(-3*a) - b^3*x^3*Ei(b*x)*e^a - 9*b^2*x^2*e^(3*b*x + 3*a) + b^2*
x^2*e^(b*x + a) + b^2*x^2*e^(-b*x - a) - 9*b^2*x^2*e^(-3*b*x - 3*a) - 3*b*x
*e^(3*b*x + 3*a) + b*x*e^(b*x + a) - b*x*e^(-b*x - a) + 3*b*x*e^(-3*b*x - 3
*a) - 2*e^(3*b*x + 3*a) + 2*e^(b*x + a) + 2*e^(-b*x - a) - 2*e^(-3*b*x - 3*
a))/x^3
```

maple [A] time = 0.58, size = 234, normalized size = 1.52

$$-\frac{3b^{2}{\rm e}^{-3bx-3a}}{16x}+\frac{b\,{\rm e}^{-3bx-3a}}{16x^{2}}-\frac{{\rm e}^{-3bx-3a}}{24x^{3}}+\frac{9b^{3}{\rm e}^{-3a}\,{\rm Ei}\left(1,3bx\right)}{16}+\frac{b^{2}{\rm e}^{-bx-a}}{48x}-\frac{b\,{\rm e}^{-bx-a}}{48x^{2}}+\frac{{\rm e}^{-bx-a}}{24x^{3}}-\frac{b^{3}{\rm e}^{-a}\,{\rm Ei}\left(1,bx\right)}{48}+\frac{{\rm e}^{bx+a}}{24x^{3}}+\frac{b}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)*sinh(b*x+a)^2/x^4,x)
```

```
[Out] -3/16*b^2*exp(-3*b*x-3*a)/x+1/16*b*exp(-3*b*x-3*a)/x^2-1/24*exp(-3*b*x-3*a)
/x^3+9/16*b^3*exp(-3*a)*Ei(1,3*b*x)+1/48*b^2*exp(-b*x-a)/x-1/48*b*exp(-b*x-
a)/x^2+1/24*exp(-b*x-a)/x^3-1/48*b^3*exp(-a)*Ei(1,b*x)+1/24/x^3*exp(b*x+a)+
1/48*b/x^2*exp(b*x+a)+1/48*b^2/x*exp(b*x+a)+1/48*b^3*exp(a)*Ei(1,-b*x)-1/24
/x^3*exp(3*b*x+3*a)-1/16*b/x^2*exp(3*b*x+3*a)-3/16*b^2/x*exp(3*b*x+3*a)-9/1
6*b^3*exp(3*a)*Ei(1,-3*b*x)
```

maxima [A] time = 0.44, size = 58, normalized size = 0.38

$$-\frac{27}{8}\,b^{3}e^{(-3\,a)}\Gamma\left(-3,3\,bx\right)+\frac{1}{8}\,b^{3}e^{(-a)}\Gamma\left(-3,bx\right)-\frac{1}{8}\,b^{3}e^{a}\Gamma\left(-3,-bx\right)+\frac{27}{8}\,b^{3}e^{(3\,a)}\Gamma\left(-3,-3\,bx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*sinh(b*x+a)^2/x^4,x, algorithm="maxima")
```

```
[Out] -27/8*b^3*e^(-3*a)*gamma(-3, 3*b*x) + 1/8*b^3*e^(-a)*gamma(-3, b*x) - 1/8*b
^3*e^a*gamma(-3, -b*x) + 27/8*b^3*e^(3*a)*gamma(-3, -3*b*x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cosh\left(a+b\,x\right)\,{\sinh\left(a+b\,x\right)}^2}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cosh(a + b*x)*sinh(a + b*x)^2)/x^4,x)
```

```
[Out] int((cosh(a + b*x)*sinh(a + b*x)^2)/x^4, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh^{2}{\left(a+bx\right)}\cosh{\left(a+bx\right)}}{x^{4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*sinh(b*x+a)**2/x**4,x)
```

```
[Out] Integral(sinh(a + b*x)**2*cosh(a + b*x)/x**4, x)
```

3.289 $\int x^m \cosh^2(a+bx)\sinh^2(a+bx)\,dx$

Optimal. Leaf size=85

$$\frac{e^{4a}2^{-2(m+3)}x^m(-bx)^{-m}\Gamma(m+1,-4bx)}{b}-\frac{e^{-4a}2^{-2(m+3)}x^m(bx)^{-m}\Gamma(m+1,4bx)}{b}-\frac{x^{m+1}}{8(m+1)}$$

```
[Out] -1/8*x^(1+m)/(1+m)+exp(4*a)*x^m*GAMMA(1+m,-4*b*x)/(2^(6+2*m))/b/((-b*x)^m)-
x^m*GAMMA(1+m,4*b*x)/(2^(6+2*m))/b/exp(4*a)/((b*x)^m)
```

Rubi [A] time = 0.13, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {5448, 3307, 2181}

$$\frac{e^{4a}2^{-2(m+3)}x^m(-bx)^{-m}\text{Gamma}(m+1,-4bx)}{b}-\frac{e^{-4a}2^{-2(m+3)}x^m(bx)^{-m}\text{Gamma}(m+1,4bx)}{b}-\frac{x^{m+1}}{8(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*Cosh[a + b*x]^2*Sinh[a + b*x]^2,x]
```

```
[Out] -x^(1 + m)/(8*(1 + m)) + (E^(4*a)*x^m*Gamma[1 + m, -4*b*x])/(2^(2*(3 + m))*
b*(-(b*x))^m) - (x^m*Gamma[1 + m, 4*b*x])/(2^(2*(3 + m))*b*E^(4*a)*(b*x)^m)
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] && !I
ntegerQ[m]
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rubi steps

$$\int x^m \cosh^2(a+bx)\sinh^2(a+bx)\,dx = \int \left(-\frac{x^m}{8}+\frac{1}{8}x^m\cosh(4a+4bx)\right)dx$$
$$= -\frac{x^{1+m}}{8(1+m)}+\frac{1}{8}\int x^m\cosh(4a+4bx)\,dx$$
$$= -\frac{x^{1+m}}{8(1+m)}+\frac{1}{16}\int e^{-i(4ia+4ibx)}x^m\,dx+\frac{1}{16}\int e^{i(4ia+4ibx)}x^m\,dx$$
$$= -\frac{x^{1+m}}{8(1+m)}+\frac{4^{-3-m}e^{4a}x^m(-bx)^{-m}\Gamma(1+m,-4bx)}{b}-\frac{4^{-3-m}e^{-4a}x^m(bx)^{-m}\Gamma(}{b}$$

Mathematica [A] time = 0.22, size = 76, normalized size = 0.89

$$\frac{1}{64}x^m\left(\frac{e^{4a}4^{-m}(-bx)^{-m}\Gamma(m+1,-4bx)}{b}-\frac{e^{-4a}4^{-m}(bx)^{-m}\Gamma(m+1,4bx)}{b}-\frac{8x}{m+1}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*Cosh[a + b*x]^2*Sinh[a + b*x]^2,x]
```

```
[Out] (x^m*((-8*x)/(1 + m) + (E^(4*a)*Gamma[1 + m, -4*b*x])/(4^m*b*(-(b*x))^m) -
Gamma[1 + m, 4*b*x]/(4^m*b*E^(4*a)*(b*x)^m)))/64
```

fricas [A] time = 0.93, size = 122, normalized size = 1.44

$$-\frac{8\,bx\cosh\left(m\log(x)\right)+(m+1)\cosh\left(m\log\left(4\,b\right)+4\,a\right)\Gamma\left(m+1,4\,bx\right)-(m+1)\cosh\left(m\log\left(-4\,b\right)-4\,a\right)\Gamma\left(m\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*cosh(b*x+a)^2*sinh(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/64*(8*b*x*cosh(m*log(x)) + (m + 1)*cosh(m*log(4*b) + 4*a)*gamma(m + 1, 4
*b*x) - (m + 1)*cosh(m*log(-4*b) - 4*a)*gamma(m + 1, -4*b*x) - (m + 1)*gamm
a(m + 1, 4*b*x)*sinh(m*log(4*b) + 4*a) + (m + 1)*gamma(m + 1, -4*b*x)*sinh(
m*log(-4*b) - 4*a) + 8*b*x*sinh(m*log(x)))/(b*m + b)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^m\cosh\left(bx+a\right)^2\sinh\left(bx+a\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*cosh(b*x+a)^2*sinh(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate(x^m*cosh(b*x + a)^2*sinh(b*x + a)^2, x)
```

maple [F] time = 0.43, size = 0, normalized size = 0.00

$$\int x^m \left(\cosh^2 (bx+a)\right) \left(\sinh^2 (bx+a)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*cosh(b*x+a)^2*sinh(b*x+a)^2,x)
```

```
[Out] int(x^m*cosh(b*x+a)^2*sinh(b*x+a)^2,x)
```

maxima [A] time = 0.41, size = 71, normalized size = 0.84

$$-\frac{1}{16}\,(4\,bx)^{-m-1}\,x^{m+1}e^{(-4\,a)}\Gamma\,(m+1,4\,bx)-\frac{1}{16}\,(-4\,bx)^{-m-1}\,x^{m+1}e^{(4\,a)}\Gamma\,(m+1,-4\,bx)-\frac{x^{m+1}}{8\,(m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*cosh(b*x+a)^2*sinh(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -1/16*(4*b*x)^(-m - 1)*x^(m + 1)*e^(-4*a)*gamma(m + 1, 4*b*x) - 1/16*(-4*b*
x)^(-m - 1)*x^(m + 1)*e^(4*a)*gamma(m + 1, -4*b*x) - 1/8*x^(m + 1)/(m + 1)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^m \cosh (a+b\,x)^2 \sinh (a+b\,x)^2\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*cosh(a + b*x)^2*sinh(a + b*x)^2,x)
```

```
[Out] int(x^m*cosh(a + b*x)^2*sinh(a + b*x)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^m \sinh^2 (a+bx) \cosh^2 (a+bx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*cosh(b*x+a)**2*sinh(b*x+a)**2,x)
```

```
[Out] Integral(x**m*sinh(a + b*x)**2*cosh(a + b*x)**2, x)
```

3.290 $\int x^3 \cosh^2(a+bx)\sinh^2(a+bx)\,dx$

Optimal. Leaf size=79

$$-\frac{3\cosh(4a+4bx)}{1024b^4}+\frac{3x\sinh(4a+4bx)}{256b^3}-\frac{3x^2\cosh(4a+4bx)}{128b^2}+\frac{x^3\sinh(4a+4bx)}{32b}-\frac{x^4}{32}$$

```
[Out] -1/32*x^4-3/1024*cosh(4*b*x+4*a)/b^4-3/128*x^2*cosh(4*b*x+4*a)/b^2+3/256*x*
sinh(4*b*x+4*a)/b^3+1/32*x^3*sinh(4*b*x+4*a)/b
```

Rubi [A] time = 0.11, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {5448, 3296, 2638}

$$-\frac{3x^2\cosh(4a+4bx)}{128b^2}+\frac{3x\sinh(4a+4bx)}{256b^3}-\frac{3\cosh(4a+4bx)}{1024b^4}+\frac{x^3\sinh(4a+4bx)}{32b}-\frac{x^4}{32}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Cosh[a + b*x]^2*Sinh[a + b*x]^2,x]
```

```
[Out] -x^4/32 - (3*Cosh[4*a + 4*b*x])/(1024*b^4) - (3*x^2*Cosh[4*a + 4*b*x])/(128
*b^2) + (3*x*Sinh[4*a + 4*b*x])/(256*b^3) + (x^3*Sinh[4*a + 4*b*x])/(32*b)
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rubi steps

$$\int x^3\cosh^2(a+bx)\sinh^2(a+bx)\,dx = \int\left(-\frac{x^3}{8}+\frac{1}{8}x^3\cosh(4a+4bx)\right)dx$$
$$= -\frac{x^4}{32}+\frac{1}{8}\int x^3\cosh(4a+4bx)\,dx$$
$$= -\frac{x^4}{32}+\frac{x^3\sinh(4a+4bx)}{32b}-\frac{3\int x^2\sinh(4a+4bx)\,dx}{32b}$$
$$= -\frac{x^4}{32}-\frac{3x^2\cosh(4a+4bx)}{128b^2}+\frac{x^3\sinh(4a+4bx)}{32b}+\frac{3\int x\cosh(4a+4bx)}{64b^2}$$
$$= -\frac{x^4}{32}-\frac{3x^2\cosh(4a+4bx)}{128b^2}+\frac{3x\sinh(4a+4bx)}{256b^3}+\frac{x^3\sinh(4a+4bx)}{32b}-$$
$$= -\frac{x^4}{32}-\frac{3\cosh(4a+4bx)}{1024b^4}-\frac{3x^2\cosh(4a+4bx)}{128b^2}+\frac{3x\sinh(4a+4bx)}{256b^3}+$$

Mathematica [A] time = 0.21, size = 58, normalized size = 0.73

$$\frac{4bx\left(8b^2x^2+3\right)\sinh(4(a+bx))-3\left(8b^2x^2+1\right)\cosh(4(a+bx))-32b^4x^4}{1024b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Cosh[a + b*x]^2*Sinh[a + b*x]^2,x]
```

```
[Out] (-32*b^4*x^4 - 3*(1 + 8*b^2*x^2)*Cosh[4*(a + b*x)] + 4*b*x*(3 + 8*b^2*x^2)*
Sinh[4*(a + b*x)])/(1024*b^4)
```

fricas [B] time = 0.55, size = 140, normalized size = 1.77

$$-\frac{32\,b^4x^4+3\left(8\,b^2x^2+1\right)\cosh\left(bx+a\right)^4-16\left(8\,b^3x^3+3\,bx\right)\cosh\left(bx+a\right)^3\sinh\left(bx+a\right)+18\left(8\,b^2x^2+1\right)\cosh}{1024\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*cosh(b*x+a)^2*sinh(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/1024*(32*b^4*x^4 + 3*(8*b^2*x^2 + 1)*cosh(b*x + a)^4 - 16*(8*b^3*x^3 + 3
*b*x)*cosh(b*x + a)^3*sinh(b*x + a) + 18*(8*b^2*x^2 + 1)*cosh(b*x + a)^2*si
nh(b*x + a)^2 - 16*(8*b^3*x^3 + 3*b*x)*cosh(b*x + a)*sinh(b*x + a)^3 + 3*(8
*b^2*x^2 + 1)*sinh(b*x + a)^4)/b^4
```

giac [A] time = 0.14, size = 78, normalized size = 0.99

$$-\frac{1}{32}\,x^4+\frac{\left(32\,b^3x^3-24\,b^2x^2+12\,bx-3\right)e^{(4\,bx+4\,a)}}{2048\,b^4}-\frac{\left(32\,b^3x^3+24\,b^2x^2+12\,bx+3\right)e^{(-4\,bx-4\,a)}}{2048\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*cosh(b*x+a)^2*sinh(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -1/32*x^4 + 1/2048*(32*b^3*x^3 - 24*b^2*x^2 + 12*b*x - 3)*e^(4*b*x + 4*a)/b
^4 - 1/2048*(32*b^3*x^3 + 24*b^2*x^2 + 12*b*x + 3)*e^(-4*b*x - 4*a)/b^4
```

maple [B] time = 0.34, size = 404, normalized size = 5.11

$$\frac{\frac{(bx+a)^3\sinh(bx+a)\left(\cosh^3(bx+a)\right)}{4}-\frac{(bx+a)^3\cosh(bx+a)\sinh(bx+a)}{8}-\frac{(bx+a)^4}{32}-\frac{3(bx+a)^2\left(\cosh^4(bx+a)\right)}{16}+\frac{3(bx+a)\sinh(bx+a)\left(\cosh^3(bx+a)\right)}{32}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*cosh(b*x+a)^2*sinh(b*x+a)^2,x)
```

```
[Out] 1/b^4*(1/4*(b*x+a)^3*sinh(b*x+a)*cosh(b*x+a)^3-1/8*(b*x+a)^3*cosh(b*x+a)*si
nh(b*x+a)-1/32*(b*x+a)^4-3/16*(b*x+a)^2*cosh(b*x+a)^4+3/32*(b*x+a)*sinh(b*x
+a)*cosh(b*x+a)^3-3/64*(b*x+a)*cosh(b*x+a)*sinh(b*x+a)-3/128*(b*x+a)^2-3/12
8*cosh(b*x+a)^4+3/128*cosh(b*x+a)^2+3/16*(b*x+a)^2*cosh(b*x+a)^2-3*a*(1/4*(
b*x+a)^2*sinh(b*x+a)*cosh(b*x+a)^3-1/8*(b*x+a)^2*cosh(b*x+a)*sinh(b*x+a)-1/
24*(b*x+a)^3-1/8*(b*x+a)*cosh(b*x+a)^4+1/32*cosh(b*x+a)^3*sinh(b*x+a)-1/64*
cosh(b*x+a)*sinh(b*x+a)-1/64*b*x-1/64*a+1/8*(b*x+a)*cosh(b*x+a)^2)+3*a^2*(1
/4*(b*x+a)*sinh(b*x+a)*cosh(b*x+a)^3-1/8*(b*x+a)*cosh(b*x+a)*sinh(b*x+a)-1/
16*(b*x+a)^2-1/16*cosh(b*x+a)^4+1/16*cosh(b*x+a)^2)-a^3*(1/4*cosh(b*x+a)^3*
sinh(b*x+a)-1/8*cosh(b*x+a)*sinh(b*x+a)-1/8*b*x-1/8*a))
```

maxima [A] time = 0.34, size = 91, normalized size = 1.15

$$-\frac{1}{32}x^4+\frac{\left(32\,b^3x^3e^{(4\,a)}-24\,b^2x^2e^{(4\,a)}+12\,bxe^{(4\,a)}-3\,e^{(4\,a)}\right)e^{(4\,bx)}}{2048\,b^4}-\frac{\left(32\,b^3x^3+24\,b^2x^2+12\,bx+3\right)e^{(-4\,bx-4\,a)}}{2048\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*cosh(b*x+a)^2*sinh(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -1/32*x^4 + 1/2048*(32*b^3*x^3*e^(4*a) - 24*b^2*x^2*e^(4*a) + 12*b*x*e^(4*a
) - 3*e^(4*a))*e^(4*b*x)/b^4 - 1/2048*(32*b^3*x^3 + 24*b^2*x^2 + 12*b*x + 3
)*e^(-4*b*x - 4*a)/b^4
```

mupad [B] time = 1.80, size = 70, normalized size = 0.89

$$-\frac{\frac{3\cosh(4\,a+4\,b\,x)}{1024}-\frac{3\,b\,x\sinh(4\,a+4\,b\,x)}{256}+\frac{3\,b^2\,x^2\cosh(4\,a+4\,b\,x)}{128}-\frac{b^3\,x^3\sinh(4\,a+4\,b\,x)}{32}}{b^4}-\frac{x^4}{32}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*cosh(a + b*x)^2*sinh(a + b*x)^2,x)
```

```
[Out] - ((3*cosh(4*a + 4*b*x))/1024 - (3*b*x*sinh(4*a + 4*b*x))/256 + (3*b^2*x^2*
cosh(4*a + 4*b*x))/128 - (b^3*x^3*sinh(4*a + 4*b*x))/32)/b^4 - x^4/32
```

sympy [A] time = 5.36, size = 250, normalized size = 3.16

$$\begin{cases} -\frac{x^4 \sinh^4(a+bx)}{32} + \frac{x^4 \sinh^2(a+bx)\cosh^2(a+bx)}{16} - \frac{x^4 \cosh^4(a+bx)}{32} + \frac{x^3 \sinh^3(a+bx)\cosh(a+bx)}{8b} + \frac{x^3 \sinh(a+bx)\cosh^3(a+bx)}{8b} - \frac{3x^2 \sin\cdots}{1\cdots} \\ \frac{x^4 \sinh^2(a)\cosh^2(a)}{4} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*cosh(b*x+a)**2*sinh(b*x+a)**2,x)
```

```
[Out] Piecewise((-x**4*sinh(a + b*x)**4/32 + x**4*sinh(a + b*x)**2*cosh(a + b*x)*
*2/16 - x**4*cosh(a + b*x)**4/32 + x**3*sinh(a + b*x)**3*cosh(a + b*x)/(8*b
) + x**3*sinh(a + b*x)*cosh(a + b*x)**3/(8*b) - 3*x**2*sinh(a + b*x)**4/(12
8*b**2) - 9*x**2*sinh(a + b*x)**2*cosh(a + b*x)**2/(64*b**2) - 3*x**2*cosh(
a + b*x)**4/(128*b**2) + 3*x*sinh(a + b*x)**3*cosh(a + b*x)/(64*b**3) + 3*x
*sinh(a + b*x)*cosh(a + b*x)**3/(64*b**3) - 3*sinh(a + b*x)**4/(256*b**4) -
 3*cosh(a + b*x)**4/(256*b**4), Ne(b, 0)), (x**4*sinh(a)**2*cosh(a)**2/4, T
rue))
```

3.291 $\int x^2 \cosh^2(a+bx)\sinh^2(a+bx)\,dx$

Optimal. Leaf size=60

$$\frac{\sinh(4a+4bx)}{256b^3}-\frac{x\cosh(4a+4bx)}{64b^2}+\frac{x^2\sinh(4a+4bx)}{32b}-\frac{x^3}{24}$$

```
[Out] -1/24*x^3-1/64*x*cosh(4*b*x+4*a)/b^2+1/256*sinh(4*b*x+4*a)/b^3+1/32*x^2*sin
h(4*b*x+4*a)/b
```

Rubi [A] time = 0.10, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {5448, 3296, 2637}

$$\frac{\sinh(4a+4bx)}{256b^3}-\frac{x\cosh(4a+4bx)}{64b^2}+\frac{x^2\sinh(4a+4bx)}{32b}-\frac{x^3}{24}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Cosh[a + b*x]^2*Sinh[a + b*x]^2,x]
```

```
[Out] -x^3/24 - (x*Cosh[4*a + 4*b*x])/(64*b^2) + Sinh[4*a + 4*b*x]/(256*b^3) + (x
^2*Sinh[4*a + 4*b*x])/(32*b)
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rubi steps

$$\int x^2\cosh^2(a+bx)\sinh^2(a+bx)\,dx = \int\left(-\frac{x^2}{8}+\frac{1}{8}x^2\cosh(4a+4bx)\right)dx$$
$$= -\frac{x^3}{24}+\frac{1}{8}\int x^2\cosh(4a+4bx)\,dx$$
$$= -\frac{x^3}{24}+\frac{x^2\sinh(4a+4bx)}{32b}-\frac{\int x\sinh(4a+4bx)\,dx}{16b}$$
$$= -\frac{x^3}{24}-\frac{x\cosh(4a+4bx)}{64b^2}+\frac{x^2\sinh(4a+4bx)}{32b}+\frac{\int\cosh(4a+4bx)\,dx}{64b^2}$$
$$= -\frac{x^3}{24}-\frac{x\cosh(4a+4bx)}{64b^2}+\frac{\sinh(4a+4bx)}{256b^3}+\frac{x^2\sinh(4a+4bx)}{32b}$$

Mathematica [A] time = 0.17, size = 48, normalized size = 0.80

$$\frac{3\left(8b^2x^2+1\right)\sinh(4(a+bx))-12bx\cosh(4(a+bx))-32b^3x^3}{768b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Cosh[a + b*x]^2*Sinh[a + b*x]^2,x]
```

```
[Out] (-32*b^3*x^3 - 12*b*x*Cosh[4*(a + b*x)] + 3*(1 + 8*b^2*x^2)*Sinh[4*(a + b*x
)])/(768*b^3)
```

fricas [B] time = 0.77, size = 110, normalized size = 1.83

$$-\frac{8\,b^3x^3+3\,bx\cosh\left(bx+a\right)^4+18\,bx\cosh\left(bx+a\right)^2\sinh\left(bx+a\right)^2+3\,bx\sinh\left(bx+a\right)^4-3\left(8\,b^2x^2+1\right)\cosh(}{192\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cosh(b*x+a)^2*sinh(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/192*(8*b^3*x^3 + 3*b*x*cosh(b*x + a)^4 + 18*b*x*cosh(b*x + a)^2*sinh(b*x
 + a)^2 + 3*b*x*sinh(b*x + a)^4 - 3*(8*b^2*x^2 + 1)*cosh(b*x + a)^3*sinh(b*
x + a) - 3*(8*b^2*x^2 + 1)*cosh(b*x + a)*sinh(b*x + a)^3)/b^3
```

giac [A] time = 0.12, size = 62, normalized size = 1.03

$$-\frac{1}{24}\,x^3+\frac{\left(8\,b^2x^2-4\,bx+1\right)e^{(4\,bx+4\,a)}}{512\,b^3}-\frac{\left(8\,b^2x^2+4\,bx+1\right)e^{(-4\,bx-4\,a)}}{512\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cosh(b*x+a)^2*sinh(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -1/24*x^3 + 1/512*(8*b^2*x^2 - 4*b*x + 1)*e^(4*b*x + 4*a)/b^3 - 1/512*(8*b^
2*x^2 + 4*b*x + 1)*e^(-4*b*x - 4*a)/b^3
```

maple [B] time = 0.32, size = 241, normalized size = 4.02

$$\frac{\frac{(bx+a)^2\sinh(bx+a)\left(\cosh^3(bx+a)\right)}{4}-\frac{(bx+a)^2\cosh(bx+a)\sinh(bx+a)}{8}-\frac{(bx+a)^3}{24}-\frac{(bx+a)\left(\cosh^4(bx+a)\right)}{8}+\frac{\left(\cosh^3(bx+a)\right)\sinh(bx+a)}{32}-\cosh(bx}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*cosh(b*x+a)^2*sinh(b*x+a)^2,x)
```

```
[Out] 1/b^3*(1/4*(b*x+a)^2*sinh(b*x+a)*cosh(b*x+a)^3-1/8*(b*x+a)^2*cosh(b*x+a)*si
nh(b*x+a)-1/24*(b*x+a)^3-1/8*(b*x+a)*cosh(b*x+a)^4+1/32*cosh(b*x+a)^3*sinh(
b*x+a)-1/64*cosh(b*x+a)*sinh(b*x+a)-1/64*b*x-1/64*a+1/8*(b*x+a)*cosh(b*x+a)
^2-2*a*(1/4*(b*x+a)*sinh(b*x+a)*cosh(b*x+a)^3-1/8*(b*x+a)*cosh(b*x+a)*sinh(
b*x+a)-1/16*(b*x+a)^2-1/16*cosh(b*x+a)^4+1/16*cosh(b*x+a)^2)+a^2*(1/4*cosh(
b*x+a)^3*sinh(b*x+a)-1/8*cosh(b*x+a)*sinh(b*x+a)-1/8*b*x-1/8*a))
```

maxima [A] time = 0.34, size = 69, normalized size = 1.15

$$-\frac{1}{24}x^3+\frac{\left(8\,b^2x^2e^{(4\,a)}-4\,bxe^{(4\,a)}+e^{(4\,a)}\right)e^{(4\,bx)}}{512\,b^3}-\frac{\left(8\,b^2x^2+4\,bx+1\right)e^{(-4\,bx-4\,a)}}{512\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cosh(b*x+a)^2*sinh(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -1/24*x^3 + 1/512*(8*b^2*x^2*e^(4*a) - 4*b*x*e^(4*a) + e^(4*a))*e^(4*b*x)/b
^3 - 1/512*(8*b^2*x^2 + 4*b*x + 1)*e^(-4*b*x - 4*a)/b^3
```

mupad [B] time = 0.14, size = 52, normalized size = 0.87

$$\frac{\frac{\sinh(4\,a+4\,b\,x)}{256}+\frac{b^2\,x^2\,\sinh(4\,a+4\,b\,x)}{32}-\frac{b\,x\,\cosh(4\,a+4\,b\,x)}{64}}{b^3}-\frac{x^3}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*cosh(a + b*x)^2*sinh(a + b*x)^2,x)
```

```
[Out] (sinh(4*a + 4*b*x)/256 + (b^2*x^2*sinh(4*a + 4*b*x))/32 - (b*x*cosh(4*a + 4
*b*x))/64)/b^3 - x^3/24
```

sympy [A] time = 3.05, size = 204, normalized size = 3.40

$$\begin{cases} -\frac{x^3\sinh^4(a+bx)}{24} + \frac{x^3\sinh^2(a+bx)\cosh^2(a+bx)}{12} - \frac{x^3\cosh^4(a+bx)}{24} + \frac{x^2\sinh^3(a+bx)\cosh(a+bx)}{8b} + \frac{x^2\sinh(a+bx)\cosh^3(a+bx)}{8b} - \frac{x\sinh^4}{64} \\ \frac{x^3\sinh^2(a)\cosh^2(a)}{3} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*cosh(b*x+a)**2*sinh(b*x+a)**2,x)
```

```
[Out] Piecewise((-x**3*sinh(a + b*x)**4/24 + x**3*sinh(a + b*x)**2*cosh(a + b*x)*
*2/12 - x**3*cosh(a + b*x)**4/24 + x**2*sinh(a + b*x)**3*cosh(a + b*x)/(8*b
) + x**2*sinh(a + b*x)*cosh(a + b*x)**3/(8*b) - x*sinh(a + b*x)**4/(64*b**2
) - 3*x*sinh(a + b*x)**2*cosh(a + b*x)**2/(32*b**2) - x*cosh(a + b*x)**4/(6
4*b**2) + sinh(a + b*x)**3*cosh(a + b*x)/(64*b**3) + sinh(a + b*x)*cosh(a +
 b*x)**3/(64*b**3), Ne(b, 0)), (x**3*sinh(a)**2*cosh(a)**2/3, True))
```

3.292 $\int x \cosh^2(a+bx) \sinh^2(a+bx) \, dx$

Optimal. Leaf size=41

$$-\frac{\cosh(4a+4bx)}{128b^2}+\frac{x\sinh(4a+4bx)}{32b}-\frac{x^2}{16}$$

```
[Out] -1/16*x^2-1/128*cosh(4*b*x+4*a)/b^2+1/32*x*sinh(4*b*x+4*a)/b
```

Rubi [A] time = 0.05, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {5448, 3296, 2638}

$$-\frac{\cosh(4a+4bx)}{128b^2}+\frac{x\sinh(4a+4bx)}{32b}-\frac{x^2}{16}$$

Antiderivative was successfully verified.

```
[In] Int[x*Cosh[a + b*x]^2*Sinh[a + b*x]^2,x]
```

```
[Out] -x^2/16 - Cosh[4*a + 4*b*x]/(128*b^2) + (x*Sinh[4*a + 4*b*x])/(32*b)
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int x\cosh^2(a+bx)\sinh^2(a+bx)\,dx &= \int \left(-\frac{x}{8}+\frac{1}{8}x\cosh(4a+4bx)\right)dx\\ &= -\frac{x^2}{16}+\frac{1}{8}\int x\cosh(4a+4bx)\,dx\\ &= -\frac{x^2}{16}+\frac{x\sinh(4a+4bx)}{32b}-\frac{\int \sinh(4a+4bx)\,dx}{32b}\\ &= -\frac{x^2}{16}-\frac{\cosh(4a+4bx)}{128b^2}+\frac{x\sinh(4a+4bx)}{32b}\end{aligned}$$

Mathematica [A] time = 0.14, size = 41, normalized size = 1.00

$$-\frac{-8a^2-4bx\sinh(4(a+bx))+\cosh(4(a+bx))+8b^2x^2}{128b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Cosh[a + b*x]^2*Sinh[a + b*x]^2,x]
```

```
[Out] -1/128*(-8*a^2 + 8*b^2*x^2 + Cosh[4*(a + b*x)] - 4*b*x*Sinh[4*(a + b*x)])/b
^2
```

fricas [B] time = 0.72, size = 88, normalized size = 2.15

$$\frac{16\,bx\cosh\left(bx+a\right)^3\sinh\left(bx+a\right)+16\,bx\cosh\left(bx+a\right)\sinh\left(bx+a\right)^3-8\,b^2x^2-\cosh\left(bx+a\right)^4-6\,\cosh\left(bx+\right.}{128\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x+a)^2*sinh(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/128*(16*b*x*cosh(b*x + a)^3*sinh(b*x + a) + 16*b*x*cosh(b*x + a)*sinh(b*x
 + a)^3 - 8*b^2*x^2 - cosh(b*x + a)^4 - 6*cosh(b*x + a)^2*sinh(b*x + a)^2 -
 sinh(b*x + a)^4)/b^2
```

giac [A] time = 0.14, size = 46, normalized size = 1.12

$$-\frac{1}{16}\,x^2+\frac{(4\,bx-1)e^{(4\,bx+4\,a)}}{256\,b^2}-\frac{(4\,bx+1)e^{(-4\,bx-4\,a)}}{256\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x+a)^2*sinh(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -1/16*x^2 + 1/256*(4*b*x - 1)*e^(4*b*x + 4*a)/b^2 - 1/256*(4*b*x + 1)*e^(-4
*b*x - 4*a)/b^2
```

maple [B] time = 0.07, size = 116, normalized size = 2.83

$$\frac{\frac{(bx+a)\sinh(bx+a)\left(\cosh^3(bx+a)\right)}{4} - \frac{(bx+a)\cosh(bx+a)\sinh(bx+a)}{8} - \frac{(bx+a)^2}{16} - \frac{\left(\cosh^4(bx+a)\right)}{16} + \frac{\left(\cosh^2(bx+a)\right)}{16} - a\left(\frac{\left(\cosh^3(bx+a)\right)\sinh(bx+}{4}\right.}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cosh(b*x+a)^2*sinh(b*x+a)^2,x)
```

```
[Out] 1/b^2*(1/4*(b*x+a)*sinh(b*x+a)*cosh(b*x+a)^3-1/8*(b*x+a)*cosh(b*x+a)*sinh(b
*x+a)-1/16*(b*x+a)^2-1/16*cosh(b*x+a)^4+1/16*cosh(b*x+a)^2-a*(1/4*cosh(b*x+
a)^3*sinh(b*x+a)-1/8*cosh(b*x+a)*sinh(b*x+a)-1/8*b*x-1/8*a))
```

maxima [A] time = 0.38, size = 51, normalized size = 1.24

$$-\frac{1}{16}x^2 + \frac{\left(4\,bxe^{(4\,a)} - e^{(4\,a)}\right)e^{(4\,bx)}}{256\,b^2} - \frac{(4\,bx+1)e^{(-4\,bx-4\,a)}}{256\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x+a)^2*sinh(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -1/16*x^2 + 1/256*(4*b*x*e^(4*a) - e^(4*a))*e^(4*b*x)/b^2 - 1/256*(4*b*x +
1)*e^(-4*b*x - 4*a)/b^2
```

mupad [B] time = 1.76, size = 36, normalized size = 0.88

$$-\frac{\frac{\cosh(4\,a+4\,b\,x)}{128} - \frac{b\,x\sinh(4\,a+4\,b\,x)}{32}}{b^2} - \frac{x^2}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cosh(a + b*x)^2*sinh(a + b*x)^2,x)
```

```
[Out] - (cosh(4*a + 4*b*x)/128 - (b*x*sinh(4*a + 4*b*x))/32)/b^2 - x^2/16
```

sympy [A] time = 1.71, size = 131, normalized size = 3.20

$$\begin{cases} -\frac{x^2\sinh^4(a+bx)}{16} + \frac{x^2\sinh^2(a+bx)\cosh^2(a+bx)}{8} - \frac{x^2\cosh^4(a+bx)}{16} + \frac{x\sinh^3(a+bx)\cosh(a+bx)}{8b} + \frac{x\sinh(a+bx)\cosh^3(a+bx)}{8b} - \frac{\sinh^4(a+b}{32b^2} \\ \frac{x^2\sinh^2(a)\cosh^2(a)}{2} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x+a)**2*sinh(b*x+a)**2,x)
```

```
[Out] Piecewise((-x**2*sinh(a + b*x)**4/16 + x**2*sinh(a + b*x)**2*cosh(a + b*x)*
*2/8 - x**2*cosh(a + b*x)**4/16 + x*sinh(a + b*x)**3*cosh(a + b*x)/(8*b) +
x*sinh(a + b*x)*cosh(a + b*x)**3/(8*b) - sinh(a + b*x)**4/(32*b**2) - cosh(
a + b*x)**4/(32*b**2), Ne(b, 0)), (x**2*sinh(a)**2*cosh(a)**2/2, True))
```

3.293 $\int \cosh^2(a+bx)\sinh^2(a+bx)\,dx$

Optimal. Leaf size=46

$$\frac{\sinh(a+bx)\cosh^3(a+bx)}{4b}-\frac{\sinh(a+bx)\cosh(a+bx)}{8b}-\frac{x}{8}$$

```
[Out] -1/8*x-1/8*cosh(b*x+a)*sinh(b*x+a)/b+1/4*cosh(b*x+a)^3*sinh(b*x+a)/b
```

Rubi [A] time = 0.04, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {2568, 2635, 8}

$$\frac{\sinh(a+bx)\cosh^3(a+bx)}{4b}-\frac{\sinh(a+bx)\cosh(a+bx)}{8b}-\frac{x}{8}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[a + b*x]^2*Sinh[a + b*x]^2,x]
```

```
[Out] -x/8 - (Cosh[a + b*x]*Sinh[a + b*x])/(8*b) + (Cosh[a + b*x]^3*Sinh[a + b*x]
)/(4*b)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2568

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m - 1)
)/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Cos[e + f*x])^n*(a
*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rubi steps

$$\int \cosh^2(a+bx)\sinh^2(a+bx)\,dx = \frac{\cosh^3(a+bx)\sinh(a+bx)}{4b} - \frac{1}{4}\int \cosh^2(a+bx)\,dx$$
$$= -\frac{\cosh(a+bx)\sinh(a+bx)}{8b} + \frac{\cosh^3(a+bx)\sinh(a+bx)}{4b} - \frac{\int 1\,dx}{8}$$
$$= -\frac{x}{8} - \frac{\cosh(a+bx)\sinh(a+bx)}{8b} + \frac{\cosh^3(a+bx)\sinh(a+bx)}{4b}$$

Mathematica [A] time = 0.02, size = 23, normalized size = 0.50

$$\frac{\sinh(4(a+bx)) - 4(a+bx)}{32b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[a + b*x]^2*Sinh[a + b*x]^2,x]
[Out] (-4*(a + b*x) + Sinh[4*(a + b*x)])/(32*b)
```

fricas [A] time = 0.65, size = 40, normalized size = 0.87

$$\frac{\cosh\left(bx+a\right)^3\sinh\left(bx+a\right) + \cosh\left(bx+a\right)\sinh\left(bx+a\right)^3 - bx}{8\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^2*sinh(b*x+a)^2,x, algorithm="fricas")
[Out] 1/8*(cosh(b*x + a)^3*sinh(b*x + a) + cosh(b*x + a)*sinh(b*x + a)^3 - b*x)/b
```

giac [A] time = 0.12, size = 32, normalized size = 0.70

$$-\frac{1}{8}\,x + \frac{e^{(4\,bx+4\,a)}}{64\,b} - \frac{e^{(-4\,bx-4\,a)}}{64\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^2*sinh(b*x+a)^2,x, algorithm="giac")
[Out] -1/8*x + 1/64*e^(4*b*x + 4*a)/b - 1/64*e^(-4*b*x - 4*a)/b
```

maple [A] time = 0.04, size = 43, normalized size = 0.93

$$\frac{\frac{\left(\cosh^3(bx+a)\right)\sinh(bx+a)}{4} - \frac{\cosh(bx+a)\sinh(bx+a)}{8} - \frac{bx}{8} - \frac{a}{8}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)^2*sinh(b*x+a)^2,x)
```

```
[Out] 1/b*(1/4*cosh(b*x+a)^3*sinh(b*x+a)-1/8*cosh(b*x+a)*sinh(b*x+a)-1/8*b*x-1/8*
a)
```

maxima [A] time = 0.66, size = 39, normalized size = 0.85

$$-\frac{bx+a}{8\,b}+\frac{e^{(4\,bx+4\,a)}}{64\,b}-\frac{e^{(-4\,bx-4\,a)}}{64\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^2*sinh(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -1/8*(b*x + a)/b + 1/64*e^(4*b*x + 4*a)/b - 1/64*e^(-4*b*x - 4*a)/b
```

mupad [B] time = 0.07, size = 18, normalized size = 0.39

$$\frac{\sinh\left(4\,a+4\,b\,x\right)}{32\,b}-\frac{x}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a + b*x)^2*sinh(a + b*x)^2,x)
```

```
[Out] sinh(4*a + 4*b*x)/(32*b) - x/8
```

sympy [A] time = 0.82, size = 92, normalized size = 2.00

$$\begin{cases} -\frac{x\sinh^{4}(a+bx)}{8}+\frac{x\sinh^{2}(a+bx)\cosh^{2}(a+bx)}{4}-\frac{x\cosh^{4}(a+bx)}{8}+\frac{\sinh^{3}(a+bx)\cosh(a+bx)}{8b}+\frac{\sinh(a+bx)\cosh^{3}(a+bx)}{8b} & \text{for } b\neq 0 \\ x\sinh^{2}(a)\cosh^{2}(a) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)**2*sinh(b*x+a)**2,x)
```

```
[Out] Piecewise((-x*sinh(a + b*x)**4/8 + x*sinh(a + b*x)**2*cosh(a + b*x)**2/4 -
x*cosh(a + b*x)**4/8 + sinh(a + b*x)**3*cosh(a + b*x)/(8*b) + sinh(a + b*x)
*cosh(a + b*x)**3/(8*b), Ne(b, 0)), (x*sinh(a)**2*cosh(a)**2, True))
```

3.294 $\int \frac{\cosh^2(a+bx)\sinh^2(a+bx)}{x}\,dx$

Optimal. Leaf size=33

$$\frac{1}{8}\cosh(4a)\mathrm{Chi}(4bx)+\frac{1}{8}\sinh(4a)\mathrm{Shi}(4bx)-\frac{\log(x)}{8}$$

```
[Out] 1/8*Chi(4*b*x)*cosh(4*a)-1/8*ln(x)+1/8*Shi(4*b*x)*sinh(4*a)
```

Rubi [A] time = 0.08, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {5448, 3303, 3298, 3301}

$$\frac{1}{8}\cosh(4a)\mathrm{Chi}(4bx)+\frac{1}{8}\sinh(4a)\mathrm{Shi}(4bx)-\frac{\log(x)}{8}$$

Antiderivative was successfully verified.

```
[In] Int[(Cosh[a + b*x]^2*Sinh[a + b*x]^2)/x,x]
```

```
[Out] (Cosh[4*a]*CoshIntegral[4*b*x])/8 - Log[x]/8 + (Sinh[4*a]*SinhIntegral[4*b*
x])/8
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
```

```
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\cosh^2(a+bx)\sinh^2(a+bx)}{x}\,dx &= \int \left(-\frac{1}{8x}+\frac{\cosh(4a+4bx)}{8x}\right)dx\\ &= -\frac{\log(x)}{8}+\frac{1}{8}\int \frac{\cosh(4a+4bx)}{x}\,dx\\ &= -\frac{\log(x)}{8}+\frac{1}{8}\cosh(4a)\int \frac{\cosh(4bx)}{x}\,dx+\frac{1}{8}\sinh(4a)\int \frac{\sinh(4bx)}{x}\,dx\\ &= \frac{1}{8}\cosh(4a)\mathrm{Chi}(4bx)-\frac{\log(x)}{8}+\frac{1}{8}\sinh(4a)\mathrm{Shi}(4bx)\end{aligned}$$

Mathematica [A] time = 0.10, size = 32, normalized size = 0.97

$$\frac{1}{8}(\cosh(4a)\mathrm{Chi}(4bx)+\sinh(4a)\mathrm{Shi}(4bx)-\log(2bx))$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cosh[a + b*x]^2*Sinh[a + b*x]^2)/x,x]
```

```
[Out] (Cosh[4*a]*CoshIntegral[4*b*x] - Log[2*b*x] + Sinh[4*a]*SinhIntegral[4*b*x]
)/8
```

fricas [A] time = 0.73, size = 41, normalized size = 1.24

$$\frac{1}{16}\left(\mathrm{Ei}\left(4\,bx\right)+\mathrm{Ei}\left(-4\,bx\right)\right)\cosh\left(4\,a\right)+\frac{1}{16}\left(\mathrm{Ei}\left(4\,bx\right)-\mathrm{Ei}\left(-4\,bx\right)\right)\sinh\left(4\,a\right)-\frac{1}{8}\,\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^2*sinh(b*x+a)^2/x,x, algorithm="fricas")
```

```
[Out] 1/16*(Ei(4*b*x) + Ei(-4*b*x))*cosh(4*a) + 1/16*(Ei(4*b*x) - Ei(-4*b*x))*sin
h(4*a) - 1/8*log(x)
```

giac [A] time = 0.12, size = 27, normalized size = 0.82

$$\frac{1}{16}\,\mathrm{Ei}\left(4\,bx\right)e^{(4\,a)}+\frac{1}{16}\,\mathrm{Ei}\left(-4\,bx\right)e^{(-4\,a)}-\frac{1}{8}\,\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^2*sinh(b*x+a)^2/x,x, algorithm="giac")
```

```
[Out] 1/16*Ei(4*b*x)*e^(4*a) + 1/16*Ei(-4*b*x)*e^(-4*a) - 1/8*log(x)
```

maple [A] time = 0.34, size = 30, normalized size = 0.91

$$-\frac{\ln(x)}{8}-\frac{e^{-4a}\operatorname{Ei}(1,4bx)}{16}-\frac{e^{4a}\operatorname{Ei}(1,-4bx)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)^2*sinh(b*x+a)^2/x,x)
```

```
[Out] -1/8*ln(x)-1/16*exp(-4*a)*Ei(1,4*b*x)-1/16*exp(4*a)*Ei(1,-4*b*x)
```

maxima [A] time = 0.80, size = 27, normalized size = 0.82

$$\frac{1}{16}\operatorname{Ei}(4\,bx)\,e^{(4\,a)}+\frac{1}{16}\operatorname{Ei}(-4\,bx)\,e^{(-4\,a)}-\frac{1}{8}\,\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^2*sinh(b*x+a)^2/x,x, algorithm="maxima")
```

```
[Out] 1/16*Ei(4*b*x)*e^(4*a) + 1/16*Ei(-4*b*x)*e^(-4*a) - 1/8*log(x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\cosh(a+b\,x)^2\,\sinh(a+b\,x)^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cosh(a + b*x)^2*sinh(a + b*x)^2)/x,x)
```

```
[Out] int((cosh(a + b*x)^2*sinh(a + b*x)^2)/x, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh^2(a+bx)\cosh^2(a+bx)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)**2*sinh(b*x+a)**2/x,x)
```

```
[Out] Integral(sinh(a + b*x)**2*cosh(a + b*x)**2/x, x)
```

3.295 $\int \frac{\cosh^2(a+bx)\sinh^2(a+bx)}{x^2}\,dx$

Optimal. Leaf size=52

$$\frac{1}{2}b\sinh(4a)\mathrm{Chi}(4bx)+\frac{1}{2}b\cosh(4a)\mathrm{Shi}(4bx)-\frac{\cosh(4a+4bx)}{8x}+\frac{1}{8x}$$

```
[Out] 1/8/x-1/8*cosh(4*b*x+4*a)/x+1/2*b*cosh(4*a)*Shi(4*b*x)+1/2*b*Chi(4*b*x)*sin
h(4*a)
```

Rubi [A] time = 0.11, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {5448, 3297, 3303, 3298, 3301}

$$\frac{1}{2}b\sinh(4a)\mathrm{Chi}(4bx)+\frac{1}{2}b\cosh(4a)\mathrm{Shi}(4bx)-\frac{\cosh(4a+4bx)}{8x}+\frac{1}{8x}$$

Antiderivative was successfully verified.

```
[In] Int[(Cosh[a + b*x]^2*Sinh[a + b*x]^2)/x^2,x]
```

```
[Out] 1/(8*x) - Cosh[4*a + 4*b*x]/(8*x) + (b*CoshIntegral[4*b*x]*Sinh[4*a])/2 + (
b*Cosh[4*a]*SinhIntegral[4*b*x])/2
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\cosh^2(a+bx)\sinh^2(a+bx)}{x^2}\,dx &= \int \left(-\frac{1}{8x^2}+\frac{\cosh(4a+4bx)}{8x^2}\right)dx\\ &= \frac{1}{8x}+\frac{1}{8}\int \frac{\cosh(4a+4bx)}{x^2}\,dx\\ &= \frac{1}{8x}-\frac{\cosh(4a+4bx)}{8x}+\frac{1}{2}b\int \frac{\sinh(4a+4bx)}{x}\,dx\\ &= \frac{1}{8x}-\frac{\cosh(4a+4bx)}{8x}+\frac{1}{2}(b\cosh(4a))\int \frac{\sinh(4bx)}{x}\,dx+\frac{1}{2}(b\sinh(4a))\int\\ &= \frac{1}{8x}-\frac{\cosh(4a+4bx)}{8x}+\frac{1}{2}b\text{Chi}(4bx)\sinh(4a)+\frac{1}{2}b\cosh(4a)\text{Shi}(4bx)\end{aligned}$$

Mathematica [A] time = 0.09, size = 45, normalized size = 0.87

$$\frac{4bx\sinh(4a)\text{Chi}(4bx)+4bx\cosh(4a)\text{Shi}(4bx)-\cosh(4(a+bx))+1}{8x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cosh[a + b*x]^2*Sinh[a + b*x]^2)/x^2,x]
```

```
[Out] (1 - Cosh[4*(a + b*x)] + 4*b*x*CoshIntegral[4*b*x]*Sinh[4*a] + 4*b*x*Cosh[4
*a]*SinhIntegral[4*b*x])/(8*x)
```

fricas [A] time = 0.63, size = 88, normalized size = 1.69

$$-\frac{\cosh(bx+a)^4+6\cosh(bx+a)^2\sinh(bx+a)^2+\sinh(bx+a)^4-2(bx\text{Ei}(4bx)-bx\text{Ei}(-4bx))\cosh(4a)-}{8x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^2*sinh(b*x+a)^2/x^2,x, algorithm="fricas")
```

```
[Out] -1/8*(cosh(b*x + a)^4 + 6*cosh(b*x + a)^2*sinh(b*x + a)^2 + sinh(b*x + a)^4
 - 2*(b*x*Ei(4*b*x) - b*x*Ei(-4*b*x))*cosh(4*a) - 2*(b*x*Ei(4*b*x) + b*x*Ei
(-4*b*x))*sinh(4*a) - 1)/x
```

giac [A] time = 0.15, size = 55, normalized size = 1.06

$$\frac{4\,bx\mathrm{Ei}\left(4\,bx\right)e^{\left(4\,a\right)} - 4\,bx\mathrm{Ei}\left(-4\,bx\right)e^{\left(-4\,a\right)} - e^{\left(4\,bx+4\,a\right)} - e^{\left(-4\,bx-4\,a\right)} + 2}{16\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^2*sinh(b*x+a)^2/x^2,x, algorithm="giac")
```

```
[Out] 1/16*(4*b*x*Ei(4*b*x)*e^(4*a) - 4*b*x*Ei(-4*b*x)*e^(-4*a) - e^(4*b*x + 4*a)
 - e^(-4*b*x - 4*a) + 2)/x
```

maple [A] time = 0.35, size = 61, normalized size = 1.17

$$\frac{1}{8x} - \frac{\mathrm{e}^{-4bx-4a}}{16x} + \frac{b\,\mathrm{e}^{-4a}\,\mathrm{Ei}\left(1,4bx\right)}{4} - \frac{\mathrm{e}^{4bx+4a}}{16x} - \frac{b\,\mathrm{e}^{4a}\,\mathrm{Ei}\left(1,-4bx\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)^2*sinh(b*x+a)^2/x^2,x)
```

```
[Out] 1/8/x-1/16*exp(-4*b*x-4*a)/x+1/4*b*exp(-4*a)*Ei(1,4*b*x)-1/16/x*exp(4*b*x+4
*a)-1/4*b*exp(4*a)*Ei(1,-4*b*x)
```

maxima [A] time = 0.60, size = 32, normalized size = 0.62

$$-\frac{1}{4}\,be^{\left(-4\,a\right)}\Gamma\left(-1,4\,bx\right) + \frac{1}{4}\,be^{\left(4\,a\right)}\Gamma\left(-1,-4\,bx\right) + \frac{1}{8\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^2*sinh(b*x+a)^2/x^2,x, algorithm="maxima")
```

```
[Out] -1/4*b*e^(-4*a)*gamma(-1, 4*b*x) + 1/4*b*e^(4*a)*gamma(-1, -4*b*x) + 1/8/x
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\cosh\left(a+b\,x\right)^2\,\sinh\left(a+b\,x\right)^2}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cosh(a + b*x)^2*sinh(a + b*x)^2)/x^2,x)
```

```
[Out] int((cosh(a + b*x)^2*sinh(a + b*x)^2)/x^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh^2(a+bx)\cosh^2(a+bx)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)**2*sinh(b*x+a)**2/x**2,x)
[Out] Integral(sinh(a + b*x)**2*cosh(a + b*x)**2/x**2, x)
```

3.296 $\int \frac{\cosh^2(a+bx)\sinh^2(a+bx)}{x^3}\,dx$

Optimal. Leaf size=67

$$b^2\cosh(4a)\mathrm{Chi}(4bx)+b^2\sinh(4a)\mathrm{Shi}(4bx)-\frac{\cosh(4a+4bx)}{16x^2}-\frac{b\sinh(4a+4bx)}{4x}+\frac{1}{16x^2}$$

```
[Out] 1/16/x^2+b^2*Chi(4*b*x)*cosh(4*a)-1/16*cosh(4*b*x+4*a)/x^2+b^2*Shi(4*b*x)*s
inh(4*a)-1/4*b*sinh(4*b*x+4*a)/x
```

Rubi [A] time = 0.14, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {5448, 3297, 3303, 3298, 3301}

$$b^2\cosh(4a)\mathrm{Chi}(4bx)+b^2\sinh(4a)\mathrm{Shi}(4bx)-\frac{\cosh(4a+4bx)}{16x^2}-\frac{b\sinh(4a+4bx)}{4x}+\frac{1}{16x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Cosh[a + b*x]^2*Sinh[a + b*x]^2)/x^3,x]
```

```
[Out] 1/(16*x^2) - Cosh[4*a + 4*b*x]/(16*x^2) + b^2*Cosh[4*a]*CoshIntegral[4*b*x]
 - (b*Sinh[4*a + 4*b*x])/(4*x) + b^2*Sinh[4*a]*SinhIntegral[4*b*x]
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\cosh^2(a+bx)\sinh^2(a+bx)}{x^3}\,dx &= \int \left(-\frac{1}{8x^3}+\frac{\cosh(4a+4bx)}{8x^3}\right)dx\\ &= \frac{1}{16x^2}+\frac{1}{8}\int \frac{\cosh(4a+4bx)}{x^3}\,dx\\ &= \frac{1}{16x^2}-\frac{\cosh(4a+4bx)}{16x^2}+\frac{1}{4}b\int \frac{\sinh(4a+4bx)}{x^2}\,dx\\ &= \frac{1}{16x^2}-\frac{\cosh(4a+4bx)}{16x^2}-\frac{b\sinh(4a+4bx)}{4x}+b^2\int \frac{\cosh(4a+4bx)}{x}\,dx\\ &= \frac{1}{16x^2}-\frac{\cosh(4a+4bx)}{16x^2}-\frac{b\sinh(4a+4bx)}{4x}+\left(b^2\cosh(4a)\right)\int \frac{\cosh(4bx)}{x}\\ &= \frac{1}{16x^2}-\frac{\cosh(4a+4bx)}{16x^2}+b^2\cosh(4a)\mathrm{Chi}(4bx)-\frac{b\sinh(4a+4bx)}{4x}+b^2\,\mathrm{s}\end{aligned}$$

Mathematica [A] time = 0.11, size = 65, normalized size = 0.97

$$\frac{16b^2x^2\cosh(4a)\mathrm{Chi}(4bx)+16b^2x^2\sinh(4a)\mathrm{Shi}(4bx)-4bx\sinh(4(a+bx))-\cosh(4(a+bx))+1}{16x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cosh[a + b*x]^2*Sinh[a + b*x]^2)/x^3,x]
```

```
[Out] (1 - Cosh[4*(a + b*x)] + 16*b^2*x^2*Cosh[4*a]*CoshIntegral[4*b*x] - 4*b*x*S
inh[4*(a + b*x)] + 16*b^2*x^2*Sinh[4*a]*SinhIntegral[4*b*x])/(16*x^2)
```

fricas [B] time = 0.46, size = 140, normalized size = 2.09

$$-\frac{16\,bx\cosh(bx+a)^3\sinh(bx+a)+16\,bx\cosh(bx+a)\sinh(bx+a)^3+\cosh(bx+a)^4+6\,\cosh(bx+a)^2\,\mathrm{sin}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^2*sinh(b*x+a)^2/x^3,x, algorithm="fricas")
```

```
[Out] -1/16*(16*b*x*cosh(b*x + a)^3*sinh(b*x + a) + 16*b*x*cosh(b*x + a)*sinh(b*x
 + a)^3 + cosh(b*x + a)^4 + 6*cosh(b*x + a)^2*sinh(b*x + a)^2 + sinh(b*x +
a)^4 - 8*(b^2*x^2*Ei(4*b*x) + b^2*x^2*Ei(-4*b*x))*cosh(4*a) - 8*(b^2*x^2*Ei
(4*b*x) - b^2*x^2*Ei(-4*b*x))*sinh(4*a) - 1)/x^2
```

giac [A] time = 0.14, size = 89, normalized size = 1.33

$$\frac{16\,b^{2}x^{2}\mathrm{Ei}\left(4\,bx\right)e^{\left(4\,a\right)}+16\,b^{2}x^{2}\mathrm{Ei}\left(-4\,bx\right)e^{\left(-4\,a\right)}-4\,bxe^{\left(4\,bx+4\,a\right)}+4\,bxe^{\left(-4\,bx-4\,a\right)}-e^{\left(4\,bx+4\,a\right)}-e^{\left(-4\,bx-4\,a\right)}+2}{32\,x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^2*sinh(b*x+a)^2/x^3,x, algorithm="giac")
```

```
[Out] 1/32*(16*b^2*x^2*Ei(4*b*x)*e^(4*a) + 16*b^2*x^2*Ei(-4*b*x)*e^(-4*a) - 4*b*x
*e^(4*b*x + 4*a) + 4*b*x*e^(-4*b*x - 4*a) - e^(4*b*x + 4*a) - e^(-4*b*x - 4
*a) + 2)/x^2
```

maple [A] time = 0.35, size = 95, normalized size = 1.42

$$\frac{1}{16x^{2}}+\frac{b\,\mathrm{e}^{-4bx-4a}}{8x}-\frac{\mathrm{e}^{-4bx-4a}}{32x^{2}}-\frac{b^{2}\mathrm{e}^{-4a}\,\mathrm{Ei}\left(1,4bx\right)}{2}-\frac{\mathrm{e}^{4bx+4a}}{32x^{2}}-\frac{b\,\mathrm{e}^{4bx+4a}}{8x}-\frac{b^{2}\mathrm{e}^{4a}\,\mathrm{Ei}\left(1,-4bx\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)^2*sinh(b*x+a)^2/x^3,x)
```

```
[Out] 1/16/x^2+1/8*b*exp(-4*b*x-4*a)/x-1/32*exp(-4*b*x-4*a)/x^2-1/2*b^2*exp(-4*a)
*Ei(1,4*b*x)-1/32/x^2*exp(4*b*x+4*a)-1/8*b/x*exp(4*b*x+4*a)-1/2*b^2*exp(4*a
)*Ei(1,-4*b*x)
```

maxima [A] time = 0.51, size = 36, normalized size = 0.54

$$-b^{2}e^{\left(-4\,a\right)}\Gamma\left(-2,4\,bx\right)-b^{2}e^{\left(4\,a\right)}\Gamma\left(-2,-4\,bx\right)+\frac{1}{16\,x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^2*sinh(b*x+a)^2/x^3,x, algorithm="maxima")
```

```
[Out] -b^2*e^(-4*a)*gamma(-2, 4*b*x) - b^2*e^(4*a)*gamma(-2, -4*b*x) + 1/16/x^2
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cosh\left(a+b\,x\right)^{2}\,\sinh\left(a+b\,x\right)^{2}}{x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cosh(a + b*x)^2*sinh(a + b*x)^2)/x^3,x)
```

```
[Out] int((cosh(a + b*x)^2*sinh(a + b*x)^2)/x^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh^2(a+bx)\cosh^2(a+bx)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)**2*sinh(b*x+a)**2/x**3,x)
```

```
[Out] Integral(sinh(a + b*x)**2*cosh(a + b*x)**2/x**3, x)
```

3.297 $\int \frac{\cosh^2(a+bx)\sinh^2(a+bx)}{x^4}\,dx$

Optimal. Leaf size=92

$$\frac{4}{3}b^3\sinh(4a)\text{Chi}(4bx)+\frac{4}{3}b^3\cosh(4a)\text{Shi}(4bx)-\frac{b^2\cosh(4a+4bx)}{3x}-\frac{\cosh(4a+4bx)}{24x^3}-\frac{b\sinh(4a+4bx)}{12x^2}+\frac{1}{24x^3}$$

```
[Out] 1/24/x^3-1/24*cosh(4*b*x+4*a)/x^3-1/3*b^2*cosh(4*b*x+4*a)/x+4/3*b^3*cosh(4*
a)*Shi(4*b*x)+4/3*b^3*Chi(4*b*x)*sinh(4*a)-1/12*b*sinh(4*b*x+4*a)/x^2
```

Rubi [A] time = 0.17, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {5448, 3297, 3303, 3298, 3301}

$$\frac{4}{3}b^3\sinh(4a)\text{Chi}(4bx)+\frac{4}{3}b^3\cosh(4a)\text{Shi}(4bx)-\frac{b^2\cosh(4a+4bx)}{3x}-\frac{b\sinh(4a+4bx)}{12x^2}-\frac{\cosh(4a+4bx)}{24x^3}+\frac{1}{24x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(Cosh[a + b*x]^2*Sinh[a + b*x]^2)/x^4,x]
```

```
[Out] 1/(24*x^3) - Cosh[4*a + 4*b*x]/(24*x^3) - (b^2*Cosh[4*a + 4*b*x])/(3*x) + (
4*b^3*CoshIntegral[4*b*x]*Sinh[4*a])/3 - (b*Sinh[4*a + 4*b*x])/(12*x^2) + (
4*b^3*Cosh[4*a]*SinhIntegral[4*b*x])/3
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cosh^2(a+bx)\sinh^2(a+bx)}{x^4}\,dx &= \int \left(-\frac{1}{8x^4}+\frac{\cosh(4a+4bx)}{8x^4}\right)dx \\
&= \frac{1}{24x^3}+\frac{1}{8}\int \frac{\cosh(4a+4bx)}{x^4}\,dx \\
&= \frac{1}{24x^3}-\frac{\cosh(4a+4bx)}{24x^3}+\frac{1}{6}b\int \frac{\sinh(4a+4bx)}{x^3}\,dx \\
&= \frac{1}{24x^3}-\frac{\cosh(4a+4bx)}{24x^3}-\frac{b\sinh(4a+4bx)}{12x^2}+\frac{1}{3}b^2\int \frac{\cosh(4a+4bx)}{x^2}\,dx \\
&= \frac{1}{24x^3}-\frac{\cosh(4a+4bx)}{24x^3}-\frac{b^2\cosh(4a+4bx)}{3x}-\frac{b\sinh(4a+4bx)}{12x^2}+\frac{1}{3}\Big(4b^{\cdots} \\
&= \frac{1}{24x^3}-\frac{\cosh(4a+4bx)}{24x^3}-\frac{b^2\cosh(4a+4bx)}{3x}-\frac{b\sinh(4a+4bx)}{12x^2}+\frac{1}{3}\Big(4b^{\cdots} \\
&= \frac{1}{24x^3}-\frac{\cosh(4a+4bx)}{24x^3}-\frac{b^2\cosh(4a+4bx)}{3x}+\frac{4}{3}b^3\mathrm{Chi}(4bx)\sinh(4a)-\cdots
\end{aligned}$$

Mathematica [A] time = 0.20, size = 79, normalized size = 0.86

$$-\frac{-32b^3x^3\sinh(4a)\mathrm{Chi}(4bx)-32b^3x^3\cosh(4a)\mathrm{Shi}(4bx)+8b^2x^2\cosh(4(a+bx))+2bx\sinh(4(a+bx))+\cosh\cdots}{24x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cosh[a + b*x]^2*Sinh[a + b*x]^2)/x^4,x]
```

```
[Out] -1/24*(-1 + Cosh[4*(a + b*x)] + 8*b^2*x^2*Cosh[4*(a + b*x)] - 32*b^3*x^3*Co
shIntegral[4*b*x]*Sinh[4*a] + 2*b*x*Sinh[4*(a + b*x)] - 32*b^3*x^3*Cosh[4*a
]*SinhIntegral[4*b*x])/x^3
```

fricas [B] time = 1.11, size = 172, normalized size = 1.87

$$-\frac{8\,bx\cosh\left(bx+a\right)^{3}\sinh\left(bx+a\right)+8\,bx\cosh\left(bx+a\right)\sinh\left(bx+a\right)^{3}+\left(8\,b^{2}x^{2}+1\right)\cosh\left(bx+a\right)^{4}+6\left(8\,b^{2}x^{2}+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^2*sinh(b*x+a)^2/x^4,x, algorithm="fricas")
```

```
[Out] -1/24*(8*b*x*cosh(b*x + a)^3*sinh(b*x + a) + 8*b*x*cosh(b*x + a)*sinh(b*x +
 a)^3 + (8*b^2*x^2 + 1)*cosh(b*x + a)^4 + 6*(8*b^2*x^2 + 1)*cosh(b*x + a)^2
*sinh(b*x + a)^2 + (8*b^2*x^2 + 1)*sinh(b*x + a)^4 - 16*(b^3*x^3*Ei(4*b*x)
- b^3*x^3*Ei(-4*b*x))*cosh(4*a) - 16*(b^3*x^3*Ei(4*b*x) + b^3*x^3*Ei(-4*b*x
))*sinh(4*a) - 1)/x^3
```

giac [A] time = 0.12, size = 123, normalized size = 1.34

$$\frac{32\,b^{3}x^{3}\mathrm{Ei}\left(4\,bx\right)e^{(4\,a)}-32\,b^{3}x^{3}\mathrm{Ei}\left(-4\,bx\right)e^{(-4\,a)}-8\,b^{2}x^{2}e^{(4\,bx+4\,a)}-8\,b^{2}x^{2}e^{(-4\,bx-4\,a)}-2\,bxe^{(4\,bx+4\,a)}+2\,bxe^{(-4\,bx-4\,a}}{48\,x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^2*sinh(b*x+a)^2/x^4,x, algorithm="giac")
```

```
[Out] 1/48*(32*b^3*x^3*Ei(4*b*x)*e^(4*a) - 32*b^3*x^3*Ei(-4*b*x)*e^(-4*a) - 8*b^2
*x^2*e^(4*b*x + 4*a) - 8*b^2*x^2*e^(-4*b*x - 4*a) - 2*b*x*e^(4*b*x + 4*a) +
 2*b*x*e^(-4*b*x - 4*a) - e^(4*b*x + 4*a) - e^(-4*b*x - 4*a) + 2)/x^3
```

maple [A] time = 0.36, size = 129, normalized size = 1.40

$$\frac{1}{24x^{3}}-\frac{b^{2}\mathrm{e}^{-4bx-4a}}{6x}+\frac{b\,\mathrm{e}^{-4bx-4a}}{24x^{2}}-\frac{\mathrm{e}^{-4bx-4a}}{48x^{3}}+\frac{2b^{3}\mathrm{e}^{-4a}\,\mathrm{Ei}\left(1,4bx\right)}{3}-\frac{\mathrm{e}^{4bx+4a}}{48x^{3}}-\frac{b\,\mathrm{e}^{4bx+4a}}{24x^{2}}-\frac{b^{2}\mathrm{e}^{4bx+4a}}{6x}-\frac{2b^{3}\mathrm{e}^{4a}\,\mathrm{Ei}\left(1,-4bx\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)^2*sinh(b*x+a)^2/x^4,x)
```

```
[Out] 1/24/x^3-1/6*b^2*exp(-4*b*x-4*a)/x+1/24*b*exp(-4*b*x-4*a)/x^2-1/48*exp(-4*b
*x-4*a)/x^3+2/3*b^3*exp(-4*a)*Ei(1,4*b*x)-1/48/x^3*exp(4*b*x+4*a)-1/24*b/x^
2*exp(4*b*x+4*a)-1/6*b^2/x*exp(4*b*x+4*a)-2/3*b^3*exp(4*a)*Ei(1,-4*b*x)
```

maxima [A] time = 0.63, size = 36, normalized size = 0.39

$$-4\,b^{3}e^{(-4\,a)}\Gamma\left(-3,4\,bx\right)+4\,b^{3}e^{(4\,a)}\Gamma\left(-3,-4\,bx\right)+\frac{1}{24\,x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^2*sinh(b*x+a)^2/x^4,x, algorithm="maxima")
```

```
[Out] -4*b^3*e^(-4*a)*gamma(-3, 4*b*x) + 4*b^3*e^(4*a)*gamma(-3, -4*b*x) + 1/24/x
^3
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cosh (a + b\,x)^2 \sinh (a + b\,x)^2}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cosh(a + b*x)^2*sinh(a + b*x)^2)/x^4,x)
```

```
[Out] int((cosh(a + b*x)^2*sinh(a + b*x)^2)/x^4, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh^2 (a + bx) \cosh^2 (a + bx)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)**2*sinh(b*x+a)**2/x**4,x)
```

```
[Out] Integral(sinh(a + b*x)**2*cosh(a + b*x)**2/x**4, x)
```

3.298 $\int x^m \cosh^3(a+bx)\sinh^2(a+bx)\,dx$

Optimal. Leaf size=209

$$\frac{e^{5a}5^{-m-1}x^m(-bx)^{-m}\Gamma(m+1,-5bx)}{32b}+\frac{e^{3a}3^{-m-1}x^m(-bx)^{-m}\Gamma(m+1,-3bx)}{32b}-\frac{e^a x^m(-bx)^{-m}\Gamma(m+1,-bx)}{16b}+\frac{e^{-a}x^m(bx)}{}$$

```
[Out] 1/32*5^(-1-m)*exp(5*a)*x^m*GAMMA(1+m,-5*b*x)/b/((-b*x)^m)+1/32*3^(-1-m)*exp
(3*a)*x^m*GAMMA(1+m,-3*b*x)/b/((-b*x)^m)-1/16*exp(a)*x^m*GAMMA(1+m,-b*x)/b/
((-b*x)^m)+1/16*x^m*GAMMA(1+m,b*x)/b/exp(a)/((b*x)^m)-1/32*3^(-1-m)*x^m*GAM
MA(1+m,3*b*x)/b/exp(3*a)/((b*x)^m)-1/32*5^(-1-m)*x^m*GAMMA(1+m,5*b*x)/b/exp
(5*a)/((b*x)^m)
```

Rubi [A] time = 0.29, antiderivative size = 209, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {5448, 3307, 2181}

$$\frac{e^{5a}5^{-m-1}x^m(-bx)^{-m}\text{Gamma}(m+1,-5bx)}{32b}+\frac{e^{3a}3^{-m-1}x^m(-bx)^{-m}\text{Gamma}(m+1,-3bx)}{32b}-\frac{e^a x^m(-bx)^{-m}\text{Gamma}(m}{16b}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*Cosh[a + b*x]^3*Sinh[a + b*x]^2,x]
```

```
[Out] (5^(-1 - m)*E^(5*a)*x^m*Gamma[1 + m, -5*b*x])/(32*b*(-(b*x))^m) + (3^(-1 -
m)*E^(3*a)*x^m*Gamma[1 + m, -3*b*x])/(32*b*(-(b*x))^m) - (E^a*x^m*Gamma[1 +
 m, -(b*x)])/(16*b*(-(b*x))^m) + (x^m*Gamma[1 + m, b*x])/(16*b*E^a*(b*x)^m)
 - (3^(-1 - m)*x^m*Gamma[1 + m, 3*b*x])/(32*b*E^(3*a)*(b*x)^m) - (5^(-1 - m
)*x^m*Gamma[1 + m, 5*b*x])/(32*b*E^(5*a)*(b*x)^m)
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&  !I
ntegerQ[m]
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rubi steps

$$\int x^m \cosh^3(a+bx)\sinh^2(a+bx)\,dx = \int \left(-\frac{1}{8}x^m\cosh(a+bx)+\frac{1}{16}x^m\cosh(3a+3bx)+\frac{1}{16}x^m\cosh(5a+5bx$$
$$= \frac{1}{16}\int x^m\cosh(3a+3bx)\,dx+\frac{1}{16}\int x^m\cosh(5a+5bx)\,dx-\frac{1}{8}\int x^m \text{co}$$
$$= \frac{1}{32}\int e^{-i(3ia+3ibx)}x^m\,dx+\frac{1}{32}\int e^{i(3ia+3ibx)}x^m\,dx+\frac{1}{32}\int e^{-i(5ia+5ibx)}x^m\,d$$
$$= \frac{5^{-1-m}e^{5a}x^m(-bx)^{-m}\Gamma(1+m,-5bx)}{32b}+\frac{3^{-1-m}e^{3a}x^m(-bx)^{-m}\Gamma(1+m,-3b}{32b}$$

Mathematica [A] time = 0.31, size = 175, normalized size = 0.84

$$\frac{e^{-5a}x^m\left(5e^{2a}3^{-m}\left(-b^2x^2\right)^{-m}\left(e^{6a}(bx)^m\Gamma(m+1,-3bx)-(-bx)^m\Gamma(m+1,3bx)\right)+3\,5^{-m}\left(-b^2x^2\right)^{-m}\left(e^{10a}(bx)^m\Gamma(m\right.\right.}{480b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*Cosh[a + b*x]^3*Sinh[a + b*x]^2,x]
```

```
[Out] (x^m*((-30*E^(6*a)*Gamma[1 + m, -(b*x)])/(-(b*x))^m + (30*E^(4*a)*Gamma[1 +
 m, b*x])/(b*x)^m + (5*E^(2*a)*(E^(6*a)*(b*x)^m*Gamma[1 + m, -3*b*x] - (-(b
*x))^m*Gamma[1 + m, 3*b*x]))/(3^m*(-(b^2*x^2))^m) + (3*(E^(10*a)*(b*x)^m*Ga
mma[1 + m, -5*b*x] - (-(b*x))^m*Gamma[1 + m, 5*b*x]))/(5^m*(-(b^2*x^2))^m))
)/(480*b*E^(5*a))
```

fricas [A] time = 0.95, size = 248, normalized size = 1.19

$$-\frac{3\cosh\left(m\log(5b)+5a\right)\Gamma(m+1,5bx)+5\cosh\left(m\log(3b)+3a\right)\Gamma(m+1,3bx)-30\cosh\left(m\log(b)+a\right)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*cosh(b*x+a)^3*sinh(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/480*(3*cosh(m*log(5*b) + 5*a)*gamma(m + 1, 5*b*x) + 5*cosh(m*log(3*b) +
3*a)*gamma(m + 1, 3*b*x) - 30*cosh(m*log(b) + a)*gamma(m + 1, b*x) + 30*cos
```

```
h(m*log(-b) - a)*gamma(m + 1, -b*x) - 5*cosh(m*log(-3*b) - 3*a)*gamma(m + 1
, -3*b*x) - 3*cosh(m*log(-5*b) - 5*a)*gamma(m + 1, -5*b*x) - 3*gamma(m + 1,
 5*b*x)*sinh(m*log(5*b) + 5*a) - 5*gamma(m + 1, 3*b*x)*sinh(m*log(3*b) + 3*
a) - 30*gamma(m + 1, -b*x)*sinh(m*log(-b) - a) + 5*gamma(m + 1, -3*b*x)*sin
h(m*log(-3*b) - 3*a) + 3*gamma(m + 1, -5*b*x)*sinh(m*log(-5*b) - 5*a) + 30*
gamma(m + 1, b*x)*sinh(m*log(b) + a))/b
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^m \cosh(bx+a)^3 \sinh(bx+a)^2\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*cosh(b*x+a)^3*sinh(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate(x^m*cosh(b*x + a)^3*sinh(b*x + a)^2, x)
```

maple [F] time = 0.42, size = 0, normalized size = 0.00

$$\int x^m \left(\cosh^3(bx+a)\right)\left(\sinh^2(bx+a)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*cosh(b*x+a)^3*sinh(b*x+a)^2,x)
```

```
[Out] int(x^m*cosh(b*x+a)^3*sinh(b*x+a)^2,x)
```

maxima [A] time = 0.94, size = 171, normalized size = 0.82

$$-\frac{1}{32}\,(5\,bx)^{-m-1}\,x^{m+1}e^{(-5\,a)}\Gamma\left(m+1,5\,bx\right)-\frac{1}{32}\,(3\,bx)^{-m-1}\,x^{m+1}e^{(-3\,a)}\Gamma\left(m+1,3\,bx\right)+\frac{1}{16}\,(bx)^{-m-1}\,x^{m+1}e^{(-a)}\Gamma\left(m+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*cosh(b*x+a)^3*sinh(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -1/32*(5*b*x)^(-m - 1)*x^(m + 1)*e^(-5*a)*gamma(m + 1, 5*b*x) - 1/32*(3*b*x
)^(-m - 1)*x^(m + 1)*e^(-3*a)*gamma(m + 1, 3*b*x) + 1/16*(b*x)^(-m - 1)*x^(
m + 1)*e^(-a)*gamma(m + 1, b*x) + 1/16*(-b*x)^(-m - 1)*x^(m + 1)*e^a*gamma(
m + 1, -b*x) - 1/32*(-3*b*x)^(-m - 1)*x^(m + 1)*e^(3*a)*gamma(m + 1, -3*b*x
) - 1/32*(-5*b*x)^(-m - 1)*x^(m + 1)*e^(5*a)*gamma(m + 1, -5*b*x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^m\,{\mathrm{cosh}\left(a+b\,x\right)}^3\,{\mathrm{sinh}\left(a+b\,x\right)}^2 \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*cosh(a + b*x)^3*sinh(a + b*x)^2,x)
```

```
[Out] int(x^m*cosh(a + b*x)^3*sinh(a + b*x)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^m \sinh^2(a+bx)\cosh^3(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*cosh(b*x+a)**3*sinh(b*x+a)**2,x)
```

```
[Out] Integral(x**m*sinh(a + b*x)**2*cosh(a + b*x)**3, x)
```

3.299 $\int x^3 \cosh^3(a+bx)\sinh^2(a+bx)\,dx$

Optimal. Leaf size=202

$$\frac{3\cosh(a+bx)}{4b^4}-\frac{\cosh(3a+3bx)}{216b^4}-\frac{3\cosh(5a+5bx)}{5000b^4}-\frac{3x\sinh(a+bx)}{4b^3}+\frac{x\sinh(3a+3bx)}{72b^3}+\frac{3x\sinh(5a+5bx)}{1000b^3}+\frac{3x}{}$$

```
[Out] 3/4*cosh(b*x+a)/b^4+3/8*x^2*cosh(b*x+a)/b^2-1/216*cosh(3*b*x+3*a)/b^4-1/48*
x^2*cosh(3*b*x+3*a)/b^2-3/5000*cosh(5*b*x+5*a)/b^4-3/400*x^2*cosh(5*b*x+5*a
)/b^2-3/4*x*sinh(b*x+a)/b^3-1/8*x^3*sinh(b*x+a)/b+1/72*x*sinh(3*b*x+3*a)/b^
3+1/48*x^3*sinh(3*b*x+3*a)/b+3/1000*x*sinh(5*b*x+5*a)/b^3+1/80*x^3*sinh(5*b
*x+5*a)/b
```

Rubi [A] time = 0.27, antiderivative size = 202, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {5448, 3296, 2638}

$$\frac{3x^2\cosh(a+bx)}{8b^2}-\frac{x^2\cosh(3a+3bx)}{48b^2}-\frac{3x^2\cosh(5a+5bx)}{400b^2}-\frac{3x\sinh(a+bx)}{4b^3}+\frac{x\sinh(3a+3bx)}{72b^3}+\frac{3x\sinh(5a+5b}{1000b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Cosh[a + b*x]^3*Sinh[a + b*x]^2,x]
```

```
[Out] (3*Cosh[a + b*x])/(4*b^4) + (3*x^2*Cosh[a + b*x])/(8*b^2) - Cosh[3*a + 3*b*
x]/(216*b^4) - (x^2*Cosh[3*a + 3*b*x])/(48*b^2) - (3*Cosh[5*a + 5*b*x])/(50
00*b^4) - (3*x^2*Cosh[5*a + 5*b*x])/(400*b^2) - (3*x*Sinh[a + b*x])/(4*b^3)
 - (x^3*Sinh[a + b*x])/(8*b) + (x*Sinh[3*a + 3*b*x])/(72*b^3) + (x^3*Sinh[3
*a + 3*b*x])/(48*b) + (3*x*Sinh[5*a + 5*b*x])/(1000*b^3) + (x^3*Sinh[5*a +
5*b*x])/(80*b)
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
```

```
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rubi steps

$$\int x^3\cosh^3(a+bx)\sinh^2(a+bx)\,dx = \int\left(-\frac{1}{8}x^3\cosh(a+bx)+\frac{1}{16}x^3\cosh(3a+3bx)+\frac{1}{16}x^3\cosh(5a+5bx)\right)$$
$$= \frac{1}{16}\int x^3\cosh(3a+3bx)\,dx+\frac{1}{16}\int x^3\cosh(5a+5bx)\,dx-\frac{1}{8}\int x^3\cosh$$
$$= -\frac{x^3\sinh(a+bx)}{8b}+\frac{x^3\sinh(3a+3bx)}{48b}+\frac{x^3\sinh(5a+5bx)}{80b}-\frac{3\int x^2\sin}{}$$
$$= \frac{3x^2\cosh(a+bx)}{8b^2}-\frac{x^2\cosh(3a+3bx)}{48b^2}-\frac{3x^2\cosh(5a+5bx)}{400b^2}-\frac{x^3\sinh}{8}$$
$$= \frac{3x^2\cosh(a+bx)}{8b^2}-\frac{x^2\cosh(3a+3bx)}{48b^2}-\frac{3x^2\cosh(5a+5bx)}{400b^2}-\frac{3x\sinh}{4}$$
$$= \frac{3\cosh(a+bx)}{4b^4}+\frac{3x^2\cosh(a+bx)}{8b^2}-\frac{\cosh(3a+3bx)}{216b^4}-\frac{x^2\cosh(3a+3}{48b^2}$$

Mathematica [A] time = 1.11, size = 125, normalized size = 0.62

$$\frac{101250\left(b^2x^2+2\right)\cosh(a+bx)-625\left(9b^2x^2+2\right)\cosh(3(a+bx))-81\left(25b^2x^2+2\right)\cosh(5(a+bx))+30bx\sin}{270000b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Cosh[a + b*x]^3*Sinh[a + b*x]^2,x]
```

```
[Out] (101250*(2 + b^2*x^2)*Cosh[a + b*x] - 625*(2 + 9*b^2*x^2)*Cosh[3*(a + b*x)]
 - 81*(2 + 25*b^2*x^2)*Cosh[5*(a + b*x)] + 30*b*x*(-6598 - 825*b^2*x^2 + 8*
(38 + 75*b^2*x^2)*Cosh[2*(a + b*x)] + 9*(6 + 25*b^2*x^2)*Cosh[4*(a + b*x)])
*Sinh[a + b*x])/(270000*b^4)
```

fricas [A] time = 0.42, size = 274, normalized size = 1.36

$$-\frac{81\left(25\,b^2x^2+2\right)\cosh\left(bx+a\right)^5+405\left(25\,b^2x^2+2\right)\cosh\left(bx+a\right)\sinh\left(bx+a\right)^4-135\left(25\,b^3x^3+6\,bx\right)\sinh\left(b\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*cosh(b*x+a)^3*sinh(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/270000*(81*(25*b^2*x^2 + 2)*cosh(b*x + a)^5 + 405*(25*b^2*x^2 + 2)*cosh(
b*x + a)*sinh(b*x + a)^4 - 135*(25*b^3*x^3 + 6*b*x)*sinh(b*x + a)^5 + 625*(
9*b^2*x^2 + 2)*cosh(b*x + a)^3 - 75*(75*b^3*x^3 + 18*(25*b^3*x^3 + 6*b*x)*c
osh(b*x + a)^2 + 50*b*x)*sinh(b*x + a)^3 + 15*(54*(25*b^2*x^2 + 2)*cosh(b*x
 + a)^3 + 125*(9*b^2*x^2 + 2)*cosh(b*x + a))*sinh(b*x + a)^2 - 101250*(b^2*
x^2 + 2)*cosh(b*x + a) + 225*(150*b^3*x^3 - 3*(25*b^3*x^3 + 6*b*x)*cosh(b*x
 + a)^4 - 25*(3*b^3*x^3 + 2*b*x)*cosh(b*x + a)^2 + 900*b*x)*sinh(b*x + a))/
b^4
```

giac [A] time = 0.13, size = 212, normalized size = 1.05

$$\frac{\left(125\, b^{3} x^{3} - 75\, b^{2} x^{2} + 30\, b x - 6\right) e^{\left(5\, b x + 5\, a\right)}}{20000\, b^{4}} + \frac{\left(9\, b^{3} x^{3} - 9\, b^{2} x^{2} + 6\, b x - 2\right) e^{\left(3\, b x + 3\, a\right)}}{864\, b^{4}} - \frac{\left(b^{3} x^{3} - 3\, b^{2} x^{2} + 6\, b x - 6\right) e^{\left(b x + a\right)}}{16\, b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*cosh(b*x+a)^3*sinh(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/20000*(125*b^3*x^3 - 75*b^2*x^2 + 30*b*x - 6)*e^(5*b*x + 5*a)/b^4 + 1/864
*(9*b^3*x^3 - 9*b^2*x^2 + 6*b*x - 2)*e^(3*b*x + 3*a)/b^4 - 1/16*(b^3*x^3 -
3*b^2*x^2 + 6*b*x - 6)*e^(b*x + a)/b^4 + 1/16*(b^3*x^3 + 3*b^2*x^2 + 6*b*x
+ 6)*e^(-b*x - a)/b^4 - 1/864*(9*b^3*x^3 + 9*b^2*x^2 + 6*b*x + 2)*e^(-3*b*x
 - 3*a)/b^4 - 1/20000*(125*b^3*x^3 + 75*b^2*x^2 + 30*b*x + 6)*e^(-5*b*x - 5
*a)/b^4
```

maple [B] time = 0.42, size = 478, normalized size = 2.37

$$\frac{\frac{(bx+a)^{3} \sinh(bx+a)\left(\cosh^{4}(bx+a)\right)}{5} - \frac{2(bx+a)^{3} \sinh(bx+a)}{15} - \frac{(bx+a)^{3} \sinh(bx+a)\left(\cosh^{2}(bx+a)\right)}{15} - \frac{3(bx+a)^{2}\left(\cosh^{5}(bx+a)\right)}{25} - \frac{856(bx+a) \sinh(bx+a)}{1125}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*cosh(b*x+a)^3*sinh(b*x+a)^2,x)
```

```
[Out] 1/b^4*(1/5*(b*x+a)^3*sinh(b*x+a)*cosh(b*x+a)^4-2/15*(b*x+a)^3*sinh(b*x+a)-1
/15*(b*x+a)^3*sinh(b*x+a)*cosh(b*x+a)^2-3/25*(b*x+a)^2*cosh(b*x+a)^5-856/11
25*(b*x+a)*sinh(b*x+a)+6/125*(b*x+a)*sinh(b*x+a)*cosh(b*x+a)^4+22/1125*(b*x
+a)*sinh(b*x+a)*cosh(b*x+a)^2+856/1125*cosh(b*x+a)-6/625*cosh(b*x+a)^5-22/3
375*cosh(b*x+a)^3+2/5*(b*x+a)^2*cosh(b*x+a)+1/15*(b*x+a)^2*cosh(b*x+a)^3-3*
a*(1/5*(b*x+a)^2*sinh(b*x+a)*cosh(b*x+a)^4-2/15*(b*x+a)^2*sinh(b*x+a)-1/15*
(b*x+a)^2*sinh(b*x+a)*cosh(b*x+a)^2-2/25*(b*x+a)*cosh(b*x+a)^5-856/3375*sin
h(b*x+a)+2/125*sinh(b*x+a)*cosh(b*x+a)^4+22/3375*cosh(b*x+a)^2*sinh(b*x+a)+
4/15*(b*x+a)*cosh(b*x+a)+2/45*(b*x+a)*cosh(b*x+a)^3)+3*a^2*(1/5*(b*x+a)*sin
```

```
h(b*x+a)*cosh(b*x+a)^4-2/15*(b*x+a)*sinh(b*x+a)-1/15*(b*x+a)*sinh(b*x+a)*co
sh(b*x+a)^2-1/25*cosh(b*x+a)^5+2/15*cosh(b*x+a)+1/45*cosh(b*x+a)^3)-a^3*(1/
5*sinh(b*x+a)*cosh(b*x+a)^4-1/5*(2/3+1/3*cosh(b*x+a)^2)*sinh(b*x+a)))
```

maxima [A] time = 0.45, size = 245, normalized size = 1.21

$$\frac{\left(125\,b^3x^3e^{(5\,a)} - 75\,b^2x^2e^{(5\,a)} + 30\,bxe^{(5\,a)} - 6\,e^{(5\,a)}\right)e^{(5\,bx)}}{20000\,b^4} + \frac{\left(9\,b^3x^3e^{(3\,a)} - 9\,b^2x^2e^{(3\,a)} + 6\,bxe^{(3\,a)} - 2\,e^{(3\,a)}\right)e^{(3\,bx)}}{864\,b^4} - \cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*cosh(b*x+a)^3*sinh(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/20000*(125*b^3*x^3*e^(5*a) - 75*b^2*x^2*e^(5*a) + 30*b*x*e^(5*a) - 6*e^(5
*a))*e^(5*b*x)/b^4 + 1/864*(9*b^3*x^3*e^(3*a) - 9*b^2*x^2*e^(3*a) + 6*b*x*e
^(3*a) - 2*e^(3*a))*e^(3*b*x)/b^4 - 1/16*(b^3*x^3*e^a - 3*b^2*x^2*e^a + 6*b
*x*e^a - 6*e^a)*e^(b*x)/b^4 + 1/16*(b^3*x^3 + 3*b^2*x^2 + 6*b*x + 6)*e^(-b*
x - a)/b^4 - 1/864*(9*b^3*x^3 + 9*b^2*x^2 + 6*b*x + 2)*e^(-3*b*x - 3*a)/b^4
 - 1/20000*(125*b^3*x^3 + 75*b^2*x^2 + 30*b*x + 6)*e^(-5*b*x - 5*a)/b^4
```

mupad [B] time = 0.50, size = 167, normalized size = 0.83

$$\frac{\frac{x\sinh(3\,a+3\,b\,x)}{72} - \frac{3\,x\sinh(a+b\,x)}{4} + \frac{3\,x\sinh(5\,a+5\,b\,x)}{1000}}{b^3} + \frac{\frac{x^3\sinh(3\,a+3\,b\,x)}{48} + \frac{x^3\sinh(5\,a+5\,b\,x)}{80} - \frac{x^3\sinh(a+b\,x)}{8}}{b} + \frac{3\cosh\left(a+b\,x\right)}{4\,b^4} - \cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*cosh(a + b*x)^3*sinh(a + b*x)^2,x)
```

```
[Out] ((x*sinh(3*a + 3*b*x))/72 - (3*x*sinh(a + b*x))/4 + (3*x*sinh(5*a + 5*b*x))
/1000)/b^3 + ((x^3*sinh(3*a + 3*b*x))/48 + (x^3*sinh(5*a + 5*b*x))/80 - (x^
3*sinh(a + b*x))/8)/b + (3*cosh(a + b*x))/(4*b^4) - cosh(3*a + 3*b*x)/(216*
b^4) - (3*cosh(5*a + 5*b*x))/(5000*b^4) - ((x^2*cosh(3*a + 3*b*x))/48 - (3*
x^2*cosh(a + b*x))/8 + (3*x^2*cosh(5*a + 5*b*x))/400)/b^2
```

sympy [A] time = 8.50, size = 253, normalized size = 1.25

$$\begin{cases} -\frac{2x^3\sinh^5(a+bx)}{15b} + \frac{x^3\sinh^3(a+bx)\cosh^2(a+bx)}{3b} + \frac{2x^2\sinh^4(a+bx)\cosh(a+bx)}{5b^2} - \frac{13x^2\sinh^2(a+bx)\cosh^3(a+bx)}{15b^2} + \frac{26x^2\cosh^5(a+bx)}{75b^2} - \cdots \\ \frac{x^4\sinh^2(a)\cosh^3(a)}{4} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*cosh(b*x+a)**3*sinh(b*x+a)**2,x)
```

```
[Out] Piecewise((-2*x**3*sinh(a + b*x)**5/(15*b) + x**3*sinh(a + b*x)**3*cosh(a +
 b*x)**2/(3*b) + 2*x**2*sinh(a + b*x)**4*cosh(a + b*x)/(5*b**2) - 13*x**2*s
inh(a + b*x)**2*cosh(a + b*x)**3/(15*b**2) + 26*x**2*cosh(a + b*x)**5/(75*b
**2) - 856*x*sinh(a + b*x)**5/(1125*b**3) + 338*x*sinh(a + b*x)**3*cosh(a +
 b*x)**2/(225*b**3) - 52*x*sinh(a + b*x)*cosh(a + b*x)**4/(75*b**3) + 856*s
inh(a + b*x)**4*cosh(a + b*x)/(1125*b**4) - 5114*sinh(a + b*x)**2*cosh(a +
b*x)**3/(3375*b**4) + 12568*cosh(a + b*x)**5/(16875*b**4), Ne(b, 0)), (x**4
*sinh(a)**2*cosh(a)**3/4, True))
```

3.300 $\int x^2 \cosh^3(a+bx)\sinh^2(a+bx)\,dx$

Optimal. Leaf size=148

$$-\frac{\sinh(a+bx)}{4b^3}+\frac{\sinh(3a+3bx)}{216b^3}+\frac{\sinh(5a+5bx)}{1000b^3}+\frac{x\cosh(a+bx)}{4b^2}-\frac{x\cosh(3a+3bx)}{72b^2}-\frac{x\cosh(5a+5bx)}{200b^2}-\frac{x^2 \sin}{}$$

```
[Out] 1/4*x*cosh(b*x+a)/b^2-1/72*x*cosh(3*b*x+3*a)/b^2-1/200*x*cosh(5*b*x+5*a)/b^
2-1/4*sinh(b*x+a)/b^3-1/8*x^2*sinh(b*x+a)/b+1/216*sinh(3*b*x+3*a)/b^3+1/48*
x^2*sinh(3*b*x+3*a)/b+1/1000*sinh(5*b*x+5*a)/b^3+1/80*x^2*sinh(5*b*x+5*a)/b
```

Rubi [A] time = 0.18, antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {5448, 3296, 2637}

$$-\frac{\sinh(a+bx)}{4b^3}+\frac{\sinh(3a+3bx)}{216b^3}+\frac{\sinh(5a+5bx)}{1000b^3}+\frac{x\cosh(a+bx)}{4b^2}-\frac{x\cosh(3a+3bx)}{72b^2}-\frac{x\cosh(5a+5bx)}{200b^2}-\frac{x^2 \sin}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Cosh[a + b*x]^3*Sinh[a + b*x]^2,x]
```

```
[Out] (x*Cosh[a + b*x])/(4*b^2) - (x*Cosh[3*a + 3*b*x])/(72*b^2) - (x*Cosh[5*a +
5*b*x])/(200*b^2) - Sinh[a + b*x]/(4*b^3) - (x^2*Sinh[a + b*x])/(8*b) + Sin
h[3*a + 3*b*x]/(216*b^3) + (x^2*Sinh[3*a + 3*b*x])/(48*b) + Sinh[5*a + 5*b*
x]/(1000*b^3) + (x^2*Sinh[5*a + 5*b*x])/(80*b)
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rubi steps

$$\int x^2\cosh^3(a+bx)\sinh^2(a+bx)\,dx = \int\left(-\frac{1}{8}x^2\cosh(a+bx)+\frac{1}{16}x^2\cosh(3a+3bx)+\frac{1}{16}x^2\cosh(5a+5bx)\right)$$
$$= \frac{1}{16}\int x^2\cosh(3a+3bx)\,dx+\frac{1}{16}\int x^2\cosh(5a+5bx)\,dx-\frac{1}{8}\int x^2\cosh($$
$$= -\frac{x^2\sinh(a+bx)}{8b}+\frac{x^2\sinh(3a+3bx)}{48b}+\frac{x^2\sinh(5a+5bx)}{80b}-\frac{\int x\sinh(5}{4}$$
$$= \frac{x\cosh(a+bx)}{4b^2}-\frac{x\cosh(3a+3bx)}{72b^2}-\frac{x\cosh(5a+5bx)}{200b^2}-\frac{x^2\sinh(a+bx}{8b}$$
$$= \frac{x\cosh(a+bx)}{4b^2}-\frac{x\cosh(3a+3bx)}{72b^2}-\frac{x\cosh(5a+5bx)}{200b^2}-\frac{\sinh(a+bx)}{4b^3}-$$

Mathematica [A] time = 0.32, size = 105, normalized size = 0.71

$$\frac{-6750\left(\left(b^2x^2+2\right)\sinh(a+bx)-2bx\cosh(a+bx)\right)+125\left(\left(9b^2x^2+2\right)\sinh(3(a+bx))-6bx\cosh(3(a+bx))\right)}{54000b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Cosh[a + b*x]^3*Sinh[a + b*x]^2,x]
```

```
[Out] (-6750*(-2*b*x*Cosh[a + b*x] + (2 + b^2*x^2)*Sinh[a + b*x]) + 125*(-6*b*x*C
osh[3*(a + b*x)] + (2 + 9*b^2*x^2)*Sinh[3*(a + b*x)]) + 27*(-10*b*x*Cosh[5*
(a + b*x)] + (2 + 25*b^2*x^2)*Sinh[5*(a + b*x)]))/(54000*b^3)
```

fricas [A] time = 0.57, size = 209, normalized size = 1.41

$$-\frac{270\,bx\cosh\left(bx+a\right)^5+1350\,bx\cosh\left(bx+a\right)\sinh\left(bx+a\right)^4-27\left(25\,b^2x^2+2\right)\sinh\left(bx+a\right)^5+750\,bx\cosh\left(b\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cosh(b*x+a)^3*sinh(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/54000*(270*b*x*cosh(b*x + a)^5 + 1350*b*x*cosh(b*x + a)*sinh(b*x + a)^4
- 27*(25*b^2*x^2 + 2)*sinh(b*x + a)^5 + 750*b*x*cosh(b*x + a)^3 - 5*(225*b^
2*x^2 + 54*(25*b^2*x^2 + 2)*cosh(b*x + a)^2 + 50)*sinh(b*x + a)^3 - 13500*b
*x*cosh(b*x + a) + 450*(6*b*x*cosh(b*x + a)^3 + 5*b*x*cosh(b*x + a))*sinh(b
*x + a)^2 - 15*(9*(25*b^2*x^2 + 2)*cosh(b*x + a)^4 - 450*b^2*x^2 + 25*(9*b^
2*x^2 + 2)*cosh(b*x + a)^2 - 900)*sinh(b*x + a))/b^3
```

giac [A] time = 0.13, size = 164, normalized size = 1.11

$$\frac{\left(25\,b^2x^2-10\,bx+2\right)e^{(5\,bx+5\,a)}}{4000\,b^3}+\frac{\left(9\,b^2x^2-6\,bx+2\right)e^{(3\,bx+3\,a)}}{864\,b^3}-\frac{\left(b^2x^2-2\,bx+2\right)e^{(bx+a)}}{16\,b^3}+\frac{\left(b^2x^2+2\,bx+2\right)e^{(-bx-a)}}{16\,b^3}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cosh(b*x+a)^3*sinh(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/4000*(25*b^2*x^2 - 10*b*x + 2)*e^(5*b*x + 5*a)/b^3 + 1/864*(9*b^2*x^2 - 6
*b*x + 2)*e^(3*b*x + 3*a)/b^3 - 1/16*(b^2*x^2 - 2*b*x + 2)*e^(b*x + a)/b^3
+ 1/16*(b^2*x^2 + 2*b*x + 2)*e^(-b*x - a)/b^3 - 1/864*(9*b^2*x^2 + 6*b*x +
2)*e^(-3*b*x - 3*a)/b^3 - 1/4000*(25*b^2*x^2 + 10*b*x + 2)*e^(-5*b*x - 5*a)
/b^3
```

maple [B] time = 0.38, size = 278, normalized size = 1.88

$$\frac{\frac{(bx+a)^2\sinh(bx+a)\left(\cosh^4(bx+a)\right)}{5}-\frac{2(bx+a)^2\sinh(bx+a)}{15}-\frac{(bx+a)^2\sinh(bx+a)\left(\cosh^2(bx+a)\right)}{15}-\frac{2(bx+a)\left(\cosh^5(bx+a)\right)}{25}-\frac{856\sinh(bx+a)}{3375}+}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*cosh(b*x+a)^3*sinh(b*x+a)^2,x)
```

```
[Out] 1/b^3*(1/5*(b*x+a)^2*sinh(b*x+a)*cosh(b*x+a)^4-2/15*(b*x+a)^2*sinh(b*x+a)-1
/15*(b*x+a)^2*sinh(b*x+a)*cosh(b*x+a)^2-2/25*(b*x+a)*cosh(b*x+a)^5-856/3375
*sinh(b*x+a)+2/125*sinh(b*x+a)*cosh(b*x+a)^4+22/3375*cosh(b*x+a)^2*sinh(b*x
+a)+4/15*(b*x+a)*cosh(b*x+a)+2/45*(b*x+a)*cosh(b*x+a)^3-2*a*(1/5*(b*x+a)*si
nh(b*x+a)*cosh(b*x+a)^4-2/15*(b*x+a)*sinh(b*x+a)-1/15*(b*x+a)*sinh(b*x+a)*c
osh(b*x+a)^2-1/25*cosh(b*x+a)^5+2/15*cosh(b*x+a)+1/45*cosh(b*x+a)^3)+a^2*(1
/5*sinh(b*x+a)*cosh(b*x+a)^4-1/5*(2/3+1/3*cosh(b*x+a)^2)*sinh(b*x+a)))
```

maxima [A] time = 0.38, size = 187, normalized size = 1.26

$$\frac{\left(25\,b^2x^2e^{(5\,a)}-10\,bxe^{(5\,a)}+2\,e^{(5\,a)}\right)e^{(5\,bx)}}{4000\,b^3}+\frac{\left(9\,b^2x^2e^{(3\,a)}-6\,bxe^{(3\,a)}+2\,e^{(3\,a)}\right)e^{(3\,bx)}}{864\,b^3}-\frac{\left(b^2x^2e^{a}-2\,bxe^{a}+2\,e^{a}\right)e^{(bx)}}{16\,b^3}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cosh(b*x+a)^3*sinh(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/4000*(25*b^2*x^2*e^(5*a) - 10*b*x*e^(5*a) + 2*e^(5*a))*e^(5*b*x)/b^3 + 1/
864*(9*b^2*x^2*e^(3*a) - 6*b*x*e^(3*a) + 2*e^(3*a))*e^(3*b*x)/b^3 - 1/16*(b
^2*x^2*e^a - 2*b*x*e^a + 2*e^a)*e^(b*x)/b^3 + 1/16*(b^2*x^2 + 2*b*x + 2)*e^
(-b*x - a)/b^3 - 1/864*(9*b^2*x^2 + 6*b*x + 2)*e^(-3*b*x - 3*a)/b^3 - 1/400
0*(25*b^2*x^2 + 10*b*x + 2)*e^(-5*b*x - 5*a)/b^3
```

mupad [B] time = 1.83, size = 123, normalized size = 0.83

$$\frac{\frac{x^2\sinh(3\,a+3\,b\,x)}{48}+\frac{x^2\sinh(5\,a+5\,b\,x)}{80}-\frac{x^2\sinh(a+b\,x)}{8}}{b}-\frac{\sinh\left(a+b\,x\right)}{4\,b^3}-\frac{\frac{x\cosh(3\,a+3\,b\,x)}{72}-\frac{x\cosh(a+b\,x)}{4}+\frac{x\cosh(5\,a+5\,b\,x)}{200}}{b^2}+\frac{\sinh}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*cosh(a + b*x)^3*sinh(a + b*x)^2,x)
```

```
[Out] ((x^2*sinh(3*a + 3*b*x))/48 + (x^2*sinh(5*a + 5*b*x))/80 - (x^2*sinh(a + b*
x))/8)/b - sinh(a + b*x)/(4*b^3) - ((x*cosh(3*a + 3*b*x))/72 - (x*cosh(a +
b*x))/4 + (x*cosh(5*a + 5*b*x))/200)/b^2 + sinh(3*a + 3*b*x)/(216*b^3) + si
nh(5*a + 5*b*x)/(1000*b^3)
```

sympy [A] time = 5.02, size = 182, normalized size = 1.23

$$\begin{cases} -\frac{2x^2\sinh^5(a+bx)}{15b} + \frac{x^2\sinh^3(a+bx)\cosh^2(a+bx)}{3b} + \frac{4x\sinh^4(a+bx)\cosh(a+bx)}{15b^2} - \frac{26x\sinh^2(a+bx)\cosh^3(a+bx)}{45b^2} + \frac{52x\cosh^5(a+bx)}{225b^2} - 856 \\ \frac{x^3\sinh^2(a)\cosh^3(a)}{3} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*cosh(b*x+a)**3*sinh(b*x+a)**2,x)
```

```
[Out] Piecewise((-2*x**2*sinh(a + b*x)**5/(15*b) + x**2*sinh(a + b*x)**3*cosh(a +
 b*x)**2/(3*b) + 4*x*sinh(a + b*x)**4*cosh(a + b*x)/(15*b**2) - 26*x*sinh(a
 + b*x)**2*cosh(a + b*x)**3/(45*b**2) + 52*x*cosh(a + b*x)**5/(225*b**2) -
856*sinh(a + b*x)**5/(3375*b**3) + 338*sinh(a + b*x)**3*cosh(a + b*x)**2/(6
75*b**3) - 52*sinh(a + b*x)*cosh(a + b*x)**4/(225*b**3), Ne(b, 0)), (x**3*s
inh(a)**2*cosh(a)**3/3, True))
```

3.301 $\int x \cosh^3(a+bx)\sinh^2(a+bx)\,dx$

Optimal. Leaf size=94

$$\frac{\cosh(a+bx)}{8b^2}-\frac{\cosh(3a+3bx)}{144b^2}-\frac{\cosh(5a+5bx)}{400b^2}-\frac{x\sinh(a+bx)}{8b}+\frac{x\sinh(3a+3bx)}{48b}+\frac{x\sinh(5a+5bx)}{80b}$$

```
[Out] 1/8*cosh(b*x+a)/b^2-1/144*cosh(3*b*x+3*a)/b^2-1/400*cosh(5*b*x+5*a)/b^2-1/8
*x*sinh(b*x+a)/b+1/48*x*sinh(3*b*x+3*a)/b+1/80*x*sinh(5*b*x+5*a)/b
```

Rubi [A] time = 0.10, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {5448, 3296, 2638}

$$\frac{\cosh(a+bx)}{8b^2}-\frac{\cosh(3a+3bx)}{144b^2}-\frac{\cosh(5a+5bx)}{400b^2}-\frac{x\sinh(a+bx)}{8b}+\frac{x\sinh(3a+3bx)}{48b}+\frac{x\sinh(5a+5bx)}{80b}$$

Antiderivative was successfully verified.

```
[In] Int[x*Cosh[a + b*x]^3*Sinh[a + b*x]^2,x]
```

```
[Out] Cosh[a + b*x]/(8*b^2) - Cosh[3*a + 3*b*x]/(144*b^2) - Cosh[5*a + 5*b*x]/(40
0*b^2) - (x*Sinh[a + b*x])/(8*b) + (x*Sinh[3*a + 3*b*x])/(48*b) + (x*Sinh[5
*a + 5*b*x])/(80*b)
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rubi steps

$$\int x\cosh^3(a+bx)\sinh^2(a+bx)\,dx = \int \left(-\frac{1}{8}x\cosh(a+bx)+\frac{1}{16}x\cosh(3a+3bx)+\frac{1}{16}x\cosh(5a+5bx)\right)dx$$
$$= \frac{1}{16}\int x\cosh(3a+3bx)\,dx+\frac{1}{16}\int x\cosh(5a+5bx)\,dx-\frac{1}{8}\int x\cosh(a+$$
$$= -\frac{x\sinh(a+bx)}{8b}+\frac{x\sinh(3a+3bx)}{48b}+\frac{x\sinh(5a+5bx)}{80b}-\frac{\int\sinh(5a+5b}{80b}$$
$$= \frac{\cosh(a+bx)}{8b^2}-\frac{\cosh(3a+3bx)}{144b^2}-\frac{\cosh(5a+5bx)}{400b^2}-\frac{x\sinh(a+bx)}{8b}+\frac{x\,s}{}$$

Mathematica [A] time = 0.20, size = 70, normalized size = 0.74

$$\frac{-450bx\sinh(a+bx)+75bx\sinh(3(a+bx))+45bx\sinh(5(a+bx))+450\cosh(a+bx)-25\cosh(3(a+bx))-}{3600b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Cosh[a + b*x]^3*Sinh[a + b*x]^2,x]
```

```
[Out] (450*Cosh[a + b*x] - 25*Cosh[3*(a + b*x)] - 9*Cosh[5*(a + b*x)] - 450*b*x*S
inh[a + b*x] + 75*b*x*Sinh[3*(a + b*x)] + 45*b*x*Sinh[5*(a + b*x)])/(3600*b
^2)
```

fricas [A] time = 1.01, size = 152, normalized size = 1.62

$$\frac{45\,bx\sinh(bx+a)^5-9\cosh(bx+a)^5-45\cosh(bx+a)\sinh(bx+a)^4+75\left(6\,bx\cosh(bx+a)^2+bx\right)\sinh(b}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x+a)^3*sinh(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/3600*(45*b*x*sinh(b*x + a)^5 - 9*cosh(b*x + a)^5 - 45*cosh(b*x + a)*sinh(
b*x + a)^4 + 75*(6*b*x*cosh(b*x + a)^2 + b*x)*sinh(b*x + a)^3 - 25*cosh(b*x
 + a)^3 - 15*(6*cosh(b*x + a)^3 + 5*cosh(b*x + a))*sinh(b*x + a)^2 + 225*(b
*x*cosh(b*x + a)^4 + b*x*cosh(b*x + a)^2 - 2*b*x)*sinh(b*x + a) + 450*cosh(
b*x + a))/b^2
```

giac [A] time = 0.15, size = 116, normalized size = 1.23

$$\frac{(5\,bx-1)e^{(5\,bx+5\,a)}}{800\,b^2}+\frac{(3\,bx-1)e^{(3\,bx+3\,a)}}{288\,b^2}-\frac{(bx-1)e^{(bx+a)}}{16\,b^2}+\frac{(bx+1)e^{(-bx-a)}}{16\,b^2}-\frac{(3\,bx+1)e^{(-3\,bx-3\,a)}}{288\,b^2}-\frac{(5\,bx+1)e^{(-5\,bx-5\,a}}{800\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x+a)^3*sinh(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/800*(5*b*x - 1)*e^(5*b*x + 5*a)/b^2 + 1/288*(3*b*x - 1)*e^(3*b*x + 3*a)/b
^2 - 1/16*(b*x - 1)*e^(b*x + a)/b^2 + 1/16*(b*x + 1)*e^(-b*x - a)/b^2 - 1/2
88*(3*b*x + 1)*e^(-3*b*x - 3*a)/b^2 - 1/800*(5*b*x + 1)*e^(-5*b*x - 5*a)/b^
2
```

maple [A] time = 0.33, size = 129, normalized size = 1.37

$$\frac{\frac{(bx+a)\sinh(bx+a)\left(\cosh^4(bx+a)\right)}{5}-\frac{2(bx+a)\sinh(bx+a)}{15}-\frac{(bx+a)\sinh(bx+a)\left(\cosh^2(bx+a)\right)}{15}-\frac{\left(\cosh^5(bx+a)\right)}{25}+\frac{2\cosh(bx+a)}{15}+\frac{\left(\cosh^3(bx+a\right.}{45}}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cosh(b*x+a)^3*sinh(b*x+a)^2,x)
```

```
[Out] 1/b^2*(1/5*(b*x+a)*sinh(b*x+a)*cosh(b*x+a)^4-2/15*(b*x+a)*sinh(b*x+a)-1/15*
(b*x+a)*sinh(b*x+a)*cosh(b*x+a)^2-1/25*cosh(b*x+a)^5+2/15*cosh(b*x+a)+1/45*
cosh(b*x+a)^3-a*(1/5*sinh(b*x+a)*cosh(b*x+a)^4-1/5*(2/3+1/3*cosh(b*x+a)^2)*
sinh(b*x+a)))
```

maxima [A] time = 0.33, size = 129, normalized size = 1.37

$$\frac{\left(5\,bxe^{(5\,a)}-e^{(5\,a)}\right)e^{(5\,bx)}}{800\,b^2}+\frac{\left(3\,bxe^{(3\,a)}-e^{(3\,a)}\right)e^{(3\,bx)}}{288\,b^2}-\frac{\left(bxe^{a}-e^{a}\right)e^{(bx)}}{16\,b^2}+\frac{(bx+1)e^{(-bx-a)}}{16\,b^2}-\frac{(3\,bx+1)e^{(-3\,bx-3\,a)}}{288\,b^2}-\frac{(5\,bx+}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x+a)^3*sinh(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/800*(5*b*x*e^(5*a) - e^(5*a))*e^(5*b*x)/b^2 + 1/288*(3*b*x*e^(3*a) - e^(3
*a))*e^(3*b*x)/b^2 - 1/16*(b*x*e^a - e^a)*e^(b*x)/b^2 + 1/16*(b*x + 1)*e^(-
b*x - a)/b^2 - 1/288*(3*b*x + 1)*e^(-3*b*x - 3*a)/b^2 - 1/800*(5*b*x + 1)*e
^(-5*b*x - 5*a)/b^2
```

mupad [B] time = 1.53, size = 83, normalized size = 0.88

$$-\frac{b\left(\frac{2\,x\,\sinh(a+b\,x)^5}{15}-\frac{x\cosh(a+b\,x)^2\sinh(a+b\,x)^3}{3}\right)-\frac{2\cosh(a+b\,x)\sinh(a+b\,x)^4}{15}-\frac{26\cosh(a+b\,x)^5}{225}+\frac{13\cosh(a+b\,x)^3\sinh(a+b\,x)^2}{45}}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cosh(a + b*x)^3*sinh(a + b*x)^2,x)
```

```
[Out] -(b*((2*x*sinh(a + b*x)^5)/15 - (x*cosh(a + b*x)^2*sinh(a + b*x)^3)/3) - (2
*cosh(a + b*x)*sinh(a + b*x)^4)/15 - (26*cosh(a + b*x)^5)/225 + (13*cosh(a
+ b*x)^3*sinh(a + b*x)^2)/45)/b^2
```

sympy [A] time = 2.82, size = 112, normalized size = 1.19

$$\begin{cases} -\frac{2x\sinh^5(a+bx)}{15b} + \frac{x\sinh^3(a+bx)\cosh^2(a+bx)}{3b} + \frac{2\sinh^4(a+bx)\cosh(a+bx)}{15b^2} - \frac{13\sinh^2(a+bx)\cosh^3(a+bx)}{45b^2} + \frac{26\cosh^5(a+bx)}{225b^2} & \text{for } b \neq \\ \frac{x^2\sinh^2(a)\cosh^3(a)}{2} & \text{otherw} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x+a)**3*sinh(b*x+a)**2,x)
```

```
[Out] Piecewise((-2*x*sinh(a + b*x)**5/(15*b) + x*sinh(a + b*x)**3*cosh(a + b*x)*
*2/(3*b) + 2*sinh(a + b*x)**4*cosh(a + b*x)/(15*b**2) - 13*sinh(a + b*x)**2
*cosh(a + b*x)**3/(45*b**2) + 26*cosh(a + b*x)**5/(225*b**2), Ne(b, 0)), (x
**2*sinh(a)**2*cosh(a)**3/2, True))
```

3.302 $\int \cosh^3(a+bx)\sinh^2(a+bx)\,dx$

Optimal. Leaf size=31

$$\frac{\sinh^5(a+bx)}{5b}+\frac{\sinh^3(a+bx)}{3b}$$

```
[Out] 1/3*sinh(b*x+a)^3/b+1/5*sinh(b*x+a)^5/b
```

Rubi [A] time = 0.03, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2564, 14}

$$\frac{\sinh^5(a+bx)}{5b}+\frac{\sinh^3(a+bx)}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[a + b*x]^3*Sinh[a + b*x]^2,x]
```

```
[Out] Sinh[a + b*x]^3/(3*b) + Sinh[a + b*x]^5/(5*b)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rubi steps

$$\begin{aligned}\int \cosh^3(a+bx)\sinh^2(a+bx)\,dx &= \frac{i\,\text{Subst}\left(\int x^2\left(1-x^2\right)\,dx,x,i\sinh(a+bx)\right)}{b}\\ &= \frac{i\,\text{Subst}\left(\int \left(x^2-x^4\right)\,dx,x,i\sinh(a+bx)\right)}{b}\\ &= \frac{\sinh^3(a+bx)}{3b}+\frac{\sinh^5(a+bx)}{5b}\end{aligned}$$

Mathematica [A] time = 0.06, size = 27, normalized size = 0.87

$$\frac{\sinh^3(a+bx)(3\cosh(2(a+bx))+7)}{30b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[a + b*x]^3*Sinh[a + b*x]^2,x]
```

```
[Out] ((7 + 3*Cosh[2*(a + b*x)])*Sinh[a + b*x]^3)/(30*b)
```

fricas [B] time = 0.84, size = 64, normalized size = 2.06

$$\frac{3\sinh(bx+a)^5+5\left(6\cosh(bx+a)^2+1\right)\sinh(bx+a)^3+15\left(\cosh(bx+a)^4+\cosh(bx+a)^2-2\right)\sinh(bx+a)}{240\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^3*sinh(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/240*(3*sinh(b*x + a)^5 + 5*(6*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^3 + 15*(
cosh(b*x + a)^4 + cosh(b*x + a)^2 - 2)*sinh(b*x + a))/b
```

giac [B] time = 0.13, size = 82, normalized size = 2.65

$$\frac{e^{(5\,bx+5\,a)}}{160\,b}+\frac{e^{(3\,bx+3\,a)}}{96\,b}-\frac{e^{(bx+a)}}{16\,b}+\frac{e^{(-bx-a)}}{16\,b}-\frac{e^{(-3\,bx-3\,a)}}{96\,b}-\frac{e^{(-5\,bx-5\,a)}}{160\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^3*sinh(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/160*e^(5*b*x + 5*a)/b + 1/96*e^(3*b*x + 3*a)/b - 1/16*e^(b*x + a)/b + 1/1
6*e^(-b*x - a)/b - 1/96*e^(-3*b*x - 3*a)/b - 1/160*e^(-5*b*x - 5*a)/b
```

maple [A] time = 0.33, size = 42, normalized size = 1.35

$$\frac{\frac{\sinh(bx+a)\left(\cosh^4(bx+a)\right)}{5}-\frac{\left(\frac{2}{3}+\frac{\left(\cosh^2(bx+a)\right)}{3}\right)\sinh(bx+a)}{5}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)^3*sinh(b*x+a)^2,x)
```

```
[Out] 1/b*(1/5*sinh(b*x+a)*cosh(b*x+a)^4-1/5*(2/3+1/3*cosh(b*x+a)^2)*sinh(b*x+a))
```

maximA [B] time = 0.62, size = 78, normalized size = 2.52

$$\frac{\left(5\,e^{(-2\,bx-2\,a)}-30\,e^{(-4\,bx-4\,a)}+3\right)e^{(5\,bx+5\,a)}}{480\,b}+\frac{30\,e^{(-bx-a)}-5\,e^{(-3\,bx-3\,a)}-3\,e^{(-5\,bx-5\,a)}}{480\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^3*sinh(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/480*(5*e^(-2*b*x - 2*a) - 30*e^(-4*b*x - 4*a) + 3)*e^(5*b*x + 5*a)/b + 1/
480*(30*e^(-b*x - a) - 5*e^(-3*b*x - 3*a) - 3*e^(-5*b*x - 5*a))/b
```

mupad [B] time = 1.45, size = 26, normalized size = 0.84

$$\frac{3\sinh\left(a+b\,x\right)^5+5\sinh\left(a+b\,x\right)^3}{15\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a + b*x)^3*sinh(a + b*x)^2,x)
```

```
[Out] (5*sinh(a + b*x)^3 + 3*sinh(a + b*x)^5)/(15*b)
```

sympy [A] time = 1.46, size = 44, normalized size = 1.42

$$\begin{cases} -\frac{2\sinh^5(a+bx)}{15b}+\frac{\sinh^3(a+bx)\cosh^2(a+bx)}{3b} & \text{for } b\neq 0\\ x\sinh^2(a)\cosh^3(a) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)**3*sinh(b*x+a)**2,x)
```

```
[Out] Piecewise((-2*sinh(a + b*x)**5/(15*b) + sinh(a + b*x)**3*cosh(a + b*x)**2/(
3*b), Ne(b, 0)), (x*sinh(a)**2*cosh(a)**3, True))
```

3.303 $\int \frac{\cosh^3(a+bx)\sinh^2(a+bx)}{x}\,dx$

Optimal. Leaf size=73

$$-\frac{1}{8}\cosh(a)\text{Chi}(bx)+\frac{1}{16}\cosh(3a)\text{Chi}(3bx)+\frac{1}{16}\cosh(5a)\text{Chi}(5bx)-\frac{1}{8}\sinh(a)\text{Shi}(bx)+\frac{1}{16}\sinh(3a)\text{Shi}(3bx)+\frac{1}{16}\text{ s}$$

```
[Out] -1/8*Chi(b*x)*cosh(a)+1/16*Chi(3*b*x)*cosh(3*a)+1/16*Chi(5*b*x)*cosh(5*a)-1
/8*Shi(b*x)*sinh(a)+1/16*Shi(3*b*x)*sinh(3*a)+1/16*Shi(5*b*x)*sinh(5*a)
```

Rubi [A] time = 0.19, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {5448, 3303, 3298, 3301}

$$-\frac{1}{8}\cosh(a)\text{Chi}(bx)+\frac{1}{16}\cosh(3a)\text{Chi}(3bx)+\frac{1}{16}\cosh(5a)\text{Chi}(5bx)-\frac{1}{8}\sinh(a)\text{Shi}(bx)+\frac{1}{16}\sinh(3a)\text{Shi}(3bx)+\frac{1}{16}\text{ s}$$

Antiderivative was successfully verified.

```
[In] Int[(Cosh[a + b*x]^3*Sinh[a + b*x]^2)/x,x]
```

```
[Out] -(Cosh[a]*CoshIntegral[b*x])/8 + (Cosh[3*a]*CoshIntegral[3*b*x])/16 + (Cosh
[5*a]*CoshIntegral[5*b*x])/16 - (Sinh[a]*SinhIntegral[b*x])/8 + (Sinh[3*a]*
SinhIntegral[3*b*x])/16 + (Sinh[5*a]*SinhIntegral[5*b*x])/16
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\cosh^3(a+bx)\sinh^2(a+bx)}{x}\,dx &= \int \left(-\frac{\cosh(a+bx)}{8x}+\frac{\cosh(3a+3bx)}{16x}+\frac{\cosh(5a+5bx)}{16x}\right)dx\\ &= \frac{1}{16}\int \frac{\cosh(3a+3bx)}{x}\,dx+\frac{1}{16}\int \frac{\cosh(5a+5bx)}{x}\,dx-\frac{1}{8}\int \frac{\cosh(a+bx)}{x}\\ &= -\left(\frac{1}{8}\cosh(a)\int \frac{\cosh(bx)}{x}\,dx\right)+\frac{1}{16}\cosh(3a)\int \frac{\cosh(3bx)}{x}\,dx+\frac{1}{16}\cosh(\\ &= -\frac{1}{8}\cosh(a)\mathrm{Chi}(bx)+\frac{1}{16}\cosh(3a)\mathrm{Chi}(3bx)+\frac{1}{16}\cosh(5a)\mathrm{Chi}(5bx)-\frac{1}{8}\mathrm{s}\end{aligned}$$

Mathematica [A] time = 0.11, size = 61, normalized size = 0.84

$$\frac{1}{16}(-2\cosh(a)\mathrm{Chi}(bx)+\cosh(3a)\mathrm{Chi}(3bx)+\cosh(5a)\mathrm{Chi}(5bx)-2\sinh(a)\mathrm{Shi}(bx)+\sinh(3a)\mathrm{Shi}(3bx)+\sinh(5a)\mathrm{S}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cosh[a + b*x]^3*Sinh[a + b*x]^2)/x,x]
```

```
[Out] (-2*Cosh[a]*CoshIntegral[b*x] + Cosh[3*a]*CoshIntegral[3*b*x] + Cosh[5*a]*C
oshIntegral[5*b*x] - 2*Sinh[a]*SinhIntegral[b*x] + Sinh[3*a]*SinhIntegral[3
*b*x] + Sinh[5*a]*SinhIntegral[5*b*x])/16
```

fricas [A] time = 0.50, size = 103, normalized size = 1.41

$$\frac{1}{32}\left(\mathrm{Ei}\left(5\,bx\right)+\mathrm{Ei}\left(-5\,bx\right)\right)\cosh\left(5\,a\right)+\frac{1}{32}\left(\mathrm{Ei}\left(3\,bx\right)+\mathrm{Ei}\left(-3\,bx\right)\right)\cosh\left(3\,a\right)-\frac{1}{16}\left(\mathrm{Ei}\left(bx\right)+\mathrm{Ei}\left(-bx\right)\right)\cosh(a)+\frac{1}{32}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^3*sinh(b*x+a)^2/x,x, algorithm="fricas")
```

```
[Out] 1/32*(Ei(5*b*x) + Ei(-5*b*x))*cosh(5*a) + 1/32*(Ei(3*b*x) + Ei(-3*b*x))*cos
h(3*a) - 1/16*(Ei(b*x) + Ei(-b*x))*cosh(a) + 1/32*(Ei(5*b*x) - Ei(-5*b*x))*
sinh(5*a) + 1/32*(Ei(3*b*x) - Ei(-3*b*x))*sinh(3*a) - 1/16*(Ei(b*x) - Ei(-b
*x))*sinh(a)
```

giac [A] time = 0.13, size = 64, normalized size = 0.88

$$\frac{1}{32}\,\mathrm{Ei}\left(5\,bx\right)e^{(5\,a)}+\frac{1}{32}\,\mathrm{Ei}\left(3\,bx\right)e^{(3\,a)}-\frac{1}{16}\,\mathrm{Ei}\left(-bx\right)e^{(-a)}+\frac{1}{32}\,\mathrm{Ei}\left(-3\,bx\right)e^{(-3\,a)}+\frac{1}{32}\,\mathrm{Ei}\left(-5\,bx\right)e^{(-5\,a)}-\frac{1}{16}\,\mathrm{Ei}\left(bx\right)e^{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^3*sinh(b*x+a)^2/x,x, algorithm="giac")
```

```
[Out] 1/32*Ei(5*b*x)*e^(5*a) + 1/32*Ei(3*b*x)*e^(3*a) - 1/16*Ei(-b*x)*e^(-a) + 1/
32*Ei(-3*b*x)*e^(-3*a) + 1/32*Ei(-5*b*x)*e^(-5*a) - 1/16*Ei(b*x)*e^a
```

maple [A] time = 0.58, size = 71, normalized size = 0.97

$$-\frac{\mathrm{e}^{-5a}\,\mathrm{Ei}\left(1,5bx\right)}{32}-\frac{\mathrm{e}^{-3a}\,\mathrm{Ei}\left(1,3bx\right)}{32}+\frac{\mathrm{e}^{-a}\,\mathrm{Ei}\left(1,bx\right)}{16}+\frac{\mathrm{e}^{a}\,\mathrm{Ei}\left(1,-bx\right)}{16}-\frac{\mathrm{e}^{3a}\,\mathrm{Ei}\left(1,-3bx\right)}{32}-\frac{\mathrm{e}^{5a}\,\mathrm{Ei}\left(1,-5bx\right)}{32}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)^3*sinh(b*x+a)^2/x,x)
```

```
[Out] -1/32*exp(-5*a)*Ei(1,5*b*x)-1/32*exp(-3*a)*Ei(1,3*b*x)+1/16*exp(-a)*Ei(1,b*
x)+1/16*exp(a)*Ei(1,-b*x)-1/32*exp(3*a)*Ei(1,-3*b*x)-1/32*exp(5*a)*Ei(1,-5*
b*x)
```

maxima [A] time = 0.72, size = 64, normalized size = 0.88

$$\frac{1}{32}\,\mathrm{Ei}\left(5\,bx\right)e^{(5\,a)}+\frac{1}{32}\,\mathrm{Ei}\left(3\,bx\right)e^{(3\,a)}-\frac{1}{16}\,\mathrm{Ei}\left(-bx\right)e^{(-a)}+\frac{1}{32}\,\mathrm{Ei}\left(-3\,bx\right)e^{(-3\,a)}+\frac{1}{32}\,\mathrm{Ei}\left(-5\,bx\right)e^{(-5\,a)}-\frac{1}{16}\,\mathrm{Ei}\left(bx\right)e^{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^3*sinh(b*x+a)^2/x,x, algorithm="maxima")
```

```
[Out] 1/32*Ei(5*b*x)*e^(5*a) + 1/32*Ei(3*b*x)*e^(3*a) - 1/16*Ei(-b*x)*e^(-a) + 1/
32*Ei(-3*b*x)*e^(-3*a) + 1/32*Ei(-5*b*x)*e^(-5*a) - 1/16*Ei(b*x)*e^a
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cosh\left(a+b\,x\right)^3\,\sinh\left(a+b\,x\right)^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cosh(a + b*x)^3*sinh(a + b*x)^2)/x,x)
```

```
[Out] int((cosh(a + b*x)^3*sinh(a + b*x)^2)/x, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh^2(a+bx)\cosh^3(a+bx)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)**3*sinh(b*x+a)**2/x,x)
```

```
[Out] Integral(sinh(a + b*x)**2*cosh(a + b*x)**3/x, x)
```

3.304 $\int \frac{\cosh^3(a+bx)\sinh^2(a+bx)}{x^2}\,dx$

Optimal. Leaf size=124

$$-\frac{1}{8}b\sinh(a)\text{Chi}(bx)+\frac{3}{16}b\sinh(3a)\text{Chi}(3bx)+\frac{5}{16}b\sinh(5a)\text{Chi}(5bx)-\frac{1}{8}b\cosh(a)\text{Shi}(bx)+\frac{3}{16}b\cosh(3a)\text{Shi}(3bx)$$

```
[Out] 1/8*cosh(b*x+a)/x-1/16*cosh(3*b*x+3*a)/x-1/16*cosh(5*b*x+5*a)/x-1/8*b*cosh(
a)*Shi(b*x)+3/16*b*cosh(3*a)*Shi(3*b*x)+5/16*b*cosh(5*a)*Shi(5*b*x)-1/8*b*C
hi(b*x)*sinh(a)+3/16*b*Chi(3*b*x)*sinh(3*a)+5/16*b*Chi(5*b*x)*sinh(5*a)
```

Rubi [A] time = 0.26, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {5448, 3297, 3303, 3298, 3301}

$$-\frac{1}{8}b\sinh(a)\text{Chi}(bx)+\frac{3}{16}b\sinh(3a)\text{Chi}(3bx)+\frac{5}{16}b\sinh(5a)\text{Chi}(5bx)-\frac{1}{8}b\cosh(a)\text{Shi}(bx)+\frac{3}{16}b\cosh(3a)\text{Shi}(3bx)$$

Antiderivative was successfully verified.

```
[In] Int[(Cosh[a + b*x]^3*Sinh[a + b*x]^2)/x^2,x]
```

```
[Out] Cosh[a + b*x]/(8*x) - Cosh[3*a + 3*b*x]/(16*x) - Cosh[5*a + 5*b*x]/(16*x) -
 (b*CoshIntegral[b*x]*Sinh[a])/8 + (3*b*CoshIntegral[3*b*x]*Sinh[3*a])/16 +
 (5*b*CoshIntegral[5*b*x]*Sinh[5*a])/16 - (b*Cosh[a]*SinhIntegral[b*x])/8 +
 (3*b*Cosh[3*a]*SinhIntegral[3*b*x])/16 + (5*b*Cosh[5*a]*SinhIntegral[5*b*x
])/16
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
```

```
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\cosh^3(a+bx)\sinh^2(a+bx)}{x^2}\,dx &= \int \left(-\frac{\cosh(a+bx)}{8x^2}+\frac{\cosh(3a+3bx)}{16x^2}+\frac{\cosh(5a+5bx)}{16x^2}\right)dx\\ &= \frac{1}{16}\int \frac{\cosh(3a+3bx)}{x^2}\,dx+\frac{1}{16}\int \frac{\cosh(5a+5bx)}{x^2}\,dx-\frac{1}{8}\int \frac{\cosh(a+bx)}{x^2}\\ &= \frac{\cosh(a+bx)}{8x}-\frac{\cosh(3a+3bx)}{16x}-\frac{\cosh(5a+5bx)}{16x}-\frac{1}{8}b\int \frac{\sinh(a+bx)}{x}\\ &= \frac{\cosh(a+bx)}{8x}-\frac{\cosh(3a+3bx)}{16x}-\frac{\cosh(5a+5bx)}{16x}-\frac{1}{8}(b\cosh(a))\int \frac{\sinh}{}\\ &= \frac{\cosh(a+bx)}{8x}-\frac{\cosh(3a+3bx)}{16x}-\frac{\cosh(5a+5bx)}{16x}-\frac{1}{8}b\text{Chi}(bx)\sinh(a)+\end{aligned}$$

Mathematica [A] time = 0.36, size = 104, normalized size = 0.84

$$-\frac{2bx\sinh(a)\text{Chi}(bx)-3bx\sinh(3a)\text{Chi}(3bx)-5bx\sinh(5a)\text{Chi}(5bx)+2bx\cosh(a)\text{Shi}(bx)-3bx\cosh(3a)\text{Sh}}{16x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cosh[a + b*x]^3*Sinh[a + b*x]^2)/x^2,x]
```

```
[Out] -1/16*(-2*Cosh[a + b*x] + Cosh[3*(a + b*x)] + Cosh[5*(a + b*x)] + 2*b*x*Cos
hIntegral[b*x]*Sinh[a] - 3*b*x*CoshIntegral[3*b*x]*Sinh[3*a] - 5*b*x*CoshIn
tegral[5*b*x]*Sinh[5*a] + 2*b*x*Cosh[a]*SinhIntegral[b*x] - 3*b*x*Cosh[3*a]
*SinhIntegral[3*b*x] - 5*b*x*Cosh[5*a]*SinhIntegral[5*b*x])/x
```

fricas [B] time = 0.67, size = 214, normalized size = 1.73

$$-\frac{2\cosh(bx+a)^5+10\cosh(bx+a)\sinh(bx+a)^4+2\cosh(bx+a)^3+2\left(10\cosh(bx+a)^3+3\cosh(bx+a)\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^3*sinh(b*x+a)^2/x^2,x, algorithm="fricas")
```

```
[Out] -1/32*(2*cosh(b*x + a)^5 + 10*cosh(b*x + a)*sinh(b*x + a)^4 + 2*cosh(b*x +
a)^3 + 2*(10*cosh(b*x + a)^3 + 3*cosh(b*x + a))*sinh(b*x + a)^2 - 5*(b*x*Ei
(5*b*x) - b*x*Ei(-5*b*x))*cosh(5*a) - 3*(b*x*Ei(3*b*x) - b*x*Ei(-3*b*x))*co
sh(3*a) + 2*(b*x*Ei(b*x) - b*x*Ei(-b*x))*cosh(a) - 5*(b*x*Ei(5*b*x) + b*x*E
i(-5*b*x))*sinh(5*a) - 3*(b*x*Ei(3*b*x) + b*x*Ei(-3*b*x))*sinh(3*a) + 2*(b*
x*Ei(b*x) + b*x*Ei(-b*x))*sinh(a) - 4*cosh(b*x + a))/x
```

giac [A] time = 0.13, size = 144, normalized size = 1.16

$$\frac{5\,bx\mathrm{Ei}(5\,bx)\,e^{(5\,a)}+3\,bx\mathrm{Ei}(3\,bx)\,e^{(3\,a)}+2\,bx\mathrm{Ei}(-bx)\,e^{(-a)}-3\,bx\mathrm{Ei}(-3\,bx)\,e^{(-3\,a)}-5\,bx\mathrm{Ei}(-5\,bx)\,e^{(-5\,a)}-2\,bx\mathrm{Ei}(b}{32\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^3*sinh(b*x+a)^2/x^2,x, algorithm="giac")
```

```
[Out] 1/32*(5*b*x*Ei(5*b*x)*e^(5*a) + 3*b*x*Ei(3*b*x)*e^(3*a) + 2*b*x*Ei(-b*x)*e^
(-a) - 3*b*x*Ei(-3*b*x)*e^(-3*a) - 5*b*x*Ei(-5*b*x)*e^(-5*a) - 2*b*x*Ei(b*x
)*e^a - e^(5*b*x + 5*a) - e^(3*b*x + 3*a) + 2*e^(b*x + a) + 2*e^(-b*x - a)
- e^(-3*b*x - 3*a) - e^(-5*b*x - 5*a))/x
```

maple [A] time = 0.54, size = 158, normalized size = 1.27

$$-\frac{e^{-5bx-5a}}{32x}+\frac{5b\,e^{-5a}\,\mathrm{Ei}(1,5bx)}{32}-\frac{e^{-3bx-3a}}{32x}+\frac{3b\,e^{-3a}\,\mathrm{Ei}(1,3bx)}{32}+\frac{e^{-bx-a}}{16x}-\frac{b\,e^{-a}\,\mathrm{Ei}(1,bx)}{16}+\frac{e^{bx+a}}{16x}+\frac{b\,e^{a}\,\mathrm{Ei}(1,-bx)}{16}-\frac{e^{3b}}{32}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)^3*sinh(b*x+a)^2/x^2,x)
```

```
[Out] -1/32*exp(-5*b*x-5*a)/x+5/32*b*exp(-5*a)*Ei(1,5*b*x)-1/32*exp(-3*b*x-3*a)/x
+3/32*b*exp(-3*a)*Ei(1,3*b*x)+1/16*exp(-b*x-a)/x-1/16*b*exp(-a)*Ei(1,b*x)+1
/16/x*exp(b*x+a)+1/16*b*exp(a)*Ei(1,-b*x)-1/32/x*exp(3*b*x+3*a)-3/32*b*exp(
3*a)*Ei(1,-3*b*x)-1/32/x*exp(5*b*x+5*a)-5/32*b*exp(5*a)*Ei(1,-5*b*x)
```

maxima [A] time = 0.69, size = 76, normalized size = 0.61

$$-\frac{5}{32}\,be^{(-5\,a)}\Gamma(-1,5\,bx)-\frac{3}{32}\,be^{(-3\,a)}\Gamma(-1,3\,bx)+\frac{1}{16}\,be^{(-a)}\Gamma(-1,bx)-\frac{1}{16}\,be^{a}\Gamma(-1,-bx)+\frac{3}{32}\,be^{(3\,a)}\Gamma(-1,-3\,bx)+\frac{5}{32}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^3*sinh(b*x+a)^2/x^2,x, algorithm="maxima")
```

```
[Out] -5/32*b*e^(-5*a)*gamma(-1, 5*b*x) - 3/32*b*e^(-3*a)*gamma(-1, 3*b*x) + 1/16
*b*e^(-a)*gamma(-1, b*x) - 1/16*b*e^a*gamma(-1, -b*x) + 3/32*b*e^(3*a)*gamm
a(-1, -3*b*x) + 5/32*b*e^(5*a)*gamma(-1, -5*b*x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cosh\left(a+b\,x\right)^3\,\sinh\left(a+b\,x\right)^2}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cosh(a + b*x)^3*sinh(a + b*x)^2)/x^2,x)
```

```
[Out] int((cosh(a + b*x)^3*sinh(a + b*x)^2)/x^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh^2(a+bx)\cosh^3(a+bx)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)**3*sinh(b*x+a)**2/x**2,x)
```

```
[Out] Integral(sinh(a + b*x)**2*cosh(a + b*x)**3/x**2, x)
```

3.305 $\int \frac{\cosh^3(a+bx)\sinh^2(a+bx)}{x^3}\,dx$

Optimal. Leaf size=184

$$-\frac{1}{16}b^2\cosh(a)\text{Chi}(bx)+\frac{9}{32}b^2\cosh(3a)\text{Chi}(3bx)+\frac{25}{32}b^2\cosh(5a)\text{Chi}(5bx)-\frac{1}{16}b^2\sinh(a)\text{Shi}(bx)+\frac{9}{32}b^2\sinh(3a)\text{S}$$

```
[Out] -1/16*b^2*Chi(b*x)*cosh(a)+9/32*b^2*Chi(3*b*x)*cosh(3*a)+25/32*b^2*Chi(5*b*
x)*cosh(5*a)+1/16*cosh(b*x+a)/x^2-1/32*cosh(3*b*x+3*a)/x^2-1/32*cosh(5*b*x+
5*a)/x^2-1/16*b^2*Shi(b*x)*sinh(a)+9/32*b^2*Shi(3*b*x)*sinh(3*a)+25/32*b^2*
Shi(5*b*x)*sinh(5*a)+1/16*b*sinh(b*x+a)/x-3/32*b*sinh(3*b*x+3*a)/x-5/32*b*s
inh(5*b*x+5*a)/x
```

Rubi [A] time = 0.34, antiderivative size = 184, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {5448, 3297, 3303, 3298, 3301}

$$-\frac{1}{16}b^2\cosh(a)\text{Chi}(bx)+\frac{9}{32}b^2\cosh(3a)\text{Chi}(3bx)+\frac{25}{32}b^2\cosh(5a)\text{Chi}(5bx)-\frac{1}{16}b^2\sinh(a)\text{Shi}(bx)+\frac{9}{32}b^2\sinh(3a)\text{S}$$

Antiderivative was successfully verified.

```
[In] Int[(Cosh[a + b*x]^3*Sinh[a + b*x]^2)/x^3,x]
```

```
[Out] Cosh[a + b*x]/(16*x^2) - Cosh[3*a + 3*b*x]/(32*x^2) - Cosh[5*a + 5*b*x]/(32
*x^2) - (b^2*Cosh[a]*CoshIntegral[b*x])/16 + (9*b^2*Cosh[3*a]*CoshIntegral[
3*b*x])/32 + (25*b^2*Cosh[5*a]*CoshIntegral[5*b*x])/32 + (b*Sinh[a + b*x])/
(16*x) - (3*b*Sinh[3*a + 3*b*x])/(32*x) - (5*b*Sinh[5*a + 5*b*x])/(32*x) -
(b^2*Sinh[a]*SinhIntegral[b*x])/16 + (9*b^2*Sinh[3*a]*SinhIntegral[3*b*x])/
32 + (25*b^2*Sinh[5*a]*SinhIntegral[5*b*x])/32
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cosh^3(a+bx)\sinh^2(a+bx)}{x^3}\,dx &= \int \left(-\frac{\cosh(a+bx)}{8x^3}+\frac{\cosh(3a+3bx)}{16x^3}+\frac{\cosh(5a+5bx)}{16x^3}\right)dx \\
&= \frac{1}{16}\int \frac{\cosh(3a+3bx)}{x^3}\,dx+\frac{1}{16}\int \frac{\cosh(5a+5bx)}{x^3}\,dx-\frac{1}{8}\int \frac{\cosh(a+bx)}{x^3} \\
&= \frac{\cosh(a+bx)}{16x^2}-\frac{\cosh(3a+3bx)}{32x^2}-\frac{\cosh(5a+5bx)}{32x^2}-\frac{1}{16}b\int \frac{\sinh(a+bx)}{x^2} \\
&= \frac{\cosh(a+bx)}{16x^2}-\frac{\cosh(3a+3bx)}{32x^2}-\frac{\cosh(5a+5bx)}{32x^2}+\frac{b\sinh(a+bx)}{16x}-\frac{3b}{} \\
&= \frac{\cosh(a+bx)}{16x^2}-\frac{\cosh(3a+3bx)}{32x^2}-\frac{\cosh(5a+5bx)}{32x^2}+\frac{b\sinh(a+bx)}{16x}-\frac{3b}{} \\
&= \frac{\cosh(a+bx)}{16x^2}-\frac{\cosh(3a+3bx)}{32x^2}-\frac{\cosh(5a+5bx)}{32x^2}-\frac{1}{16}b^2\cosh(a)\mathrm{Chi}(bx
\end{aligned}$$

Mathematica [A] time = 0.56, size = 162, normalized size = 0.88

$$-\frac{2b^2x^2\cosh(a)\mathrm{Chi}(bx)-9b^2x^2\cosh(3a)\mathrm{Chi}(3bx)-25b^2x^2\cosh(5a)\mathrm{Chi}(5bx)+2b^2x^2\sinh(a)\mathrm{Shi}(bx)-9b^2x^2}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cosh[a + b*x]^3*Sinh[a + b*x]^2)/x^3,x]
```

```
[Out] -1/32*(-2*Cosh[a + b*x] + Cosh[3*(a + b*x)] + Cosh[5*(a + b*x)] + 2*b^2*x^2
*Cosh[a]*CoshIntegral[b*x] - 9*b^2*x^2*Cosh[3*a]*CoshIntegral[3*b*x] - 25*b
^2*x^2*Cosh[5*a]*CoshIntegral[5*b*x] - 2*b*x*Sinh[a + b*x] + 3*b*x*Sinh[3*(
a + b*x)] + 5*b*x*Sinh[5*(a + b*x)] + 2*b^2*x^2*Sinh[a]*SinhIntegral[b*x] -
 9*b^2*x^2*Sinh[3*a]*SinhIntegral[3*b*x] - 25*b^2*x^2*Sinh[5*a]*SinhIntegra
l[5*b*x])/x^2
```

fricas [B] time = 0.59, size = 338, normalized size = 1.84

$$-\frac{10\,bx\sinh\left(bx+a\right)^{5}+2\,\cosh\left(bx+a\right)^{5}+10\,\cosh\left(bx+a\right)\sinh\left(bx+a\right)^{4}+2\left(50\,bx\cosh\left(bx+a\right)^{2}+3\,bx\right)\sinh}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^3*sinh(b*x+a)^2/x^3,x, algorithm="fricas")
```

```
[Out] -1/64*(10*b*x*sinh(b*x + a)^5 + 2*cosh(b*x + a)^5 + 10*cosh(b*x + a)*sinh(b
*x + a)^4 + 2*(50*b*x*cosh(b*x + a)^2 + 3*b*x)*sinh(b*x + a)^3 + 2*cosh(b*x
 + a)^3 + 2*(10*cosh(b*x + a)^3 + 3*cosh(b*x + a))*sinh(b*x + a)^2 - 25*(b^
2*x^2*Ei(5*b*x) + b^2*x^2*Ei(-5*b*x))*cosh(5*a) - 9*(b^2*x^2*Ei(3*b*x) + b^
2*x^2*Ei(-3*b*x))*cosh(3*a) + 2*(b^2*x^2*Ei(b*x) + b^2*x^2*Ei(-b*x))*cosh(a
) + 2*(25*b*x*cosh(b*x + a)^4 + 9*b*x*cosh(b*x + a)^2 - 2*b*x)*sinh(b*x + a
) - 25*(b^2*x^2*Ei(5*b*x) - b^2*x^2*Ei(-5*b*x))*sinh(5*a) - 9*(b^2*x^2*Ei(3
*b*x) - b^2*x^2*Ei(-3*b*x))*sinh(3*a) + 2*(b^2*x^2*Ei(b*x) - b^2*x^2*Ei(-b*
x))*sinh(a) - 4*cosh(b*x + a))/x^2
```

giac [A] time = 0.13, size = 243, normalized size = 1.32

$$\frac{25\,b^{2}x^{2}\mathrm{Ei}\left(5\,bx\right)e^{(5\,a)}+9\,b^{2}x^{2}\mathrm{Ei}\left(3\,bx\right)e^{(3\,a)}-2\,b^{2}x^{2}\mathrm{Ei}\left(-bx\right)e^{(-a)}+9\,b^{2}x^{2}\mathrm{Ei}\left(-3\,bx\right)e^{(-3\,a)}+25\,b^{2}x^{2}\mathrm{Ei}\left(-5\,bx\right)e^{(-5}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^3*sinh(b*x+a)^2/x^3,x, algorithm="giac")
```

```
[Out] 1/64*(25*b^2*x^2*Ei(5*b*x)*e^(5*a) + 9*b^2*x^2*Ei(3*b*x)*e^(3*a) - 2*b^2*x^
2*Ei(-b*x)*e^(-a) + 9*b^2*x^2*Ei(-3*b*x)*e^(-3*a) + 25*b^2*x^2*Ei(-5*b*x)*e
^(-5*a) - 2*b^2*x^2*Ei(b*x)*e^a - 5*b*x*e^(5*b*x + 5*a) - 3*b*x*e^(3*b*x +
3*a) + 2*b*x*e^(b*x + a) - 2*b*x*e^(-b*x - a) + 3*b*x*e^(-3*b*x - 3*a) + 5*
b*x*e^(-5*b*x - 5*a) - e^(5*b*x + 5*a) - e^(3*b*x + 3*a) + 2*e^(b*x + a) +
2*e^(-b*x - a) - e^(-3*b*x - 3*a) - e^(-5*b*x - 5*a))/x^2
```

maple [A] time = 0.53, size = 257, normalized size = 1.40

$$\frac{5b\,\mathrm{e}^{-5bx-5a}}{64x}-\frac{\mathrm{e}^{-5bx-5a}}{64x^{2}}-\frac{25b^{2}\mathrm{e}^{-5a}\,\mathrm{Ei}\left(1,5bx\right)}{64}+\frac{3b\,\mathrm{e}^{-3bx-3a}}{64x}-\frac{\mathrm{e}^{-3bx-3a}}{64x^{2}}-\frac{9b^{2}\mathrm{e}^{-3a}\,\mathrm{Ei}\left(1,3bx\right)}{64}-\frac{b\,\mathrm{e}^{-bx-a}}{32x}+\frac{\mathrm{e}^{-bx-a}}{32x^{2}}+\frac{b^{2}\mathrm{e}^{-a}\,\mathrm{E}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)^3*sinh(b*x+a)^2/x^3,x)
```

```
[Out] 5/64*b*exp(-5*b*x-5*a)/x-1/64*exp(-5*b*x-5*a)/x^2-25/64*b^2*exp(-5*a)*Ei(1,
5*b*x)+3/64*b*exp(-3*b*x-3*a)/x-1/64*exp(-3*b*x-3*a)/x^2-9/64*b^2*exp(-3*a)
*Ei(1,3*b*x)-1/32*b*exp(-b*x-a)/x+1/32*exp(-b*x-a)/x^2+1/32*b^2*exp(-a)*Ei(
1,b*x)+1/32/x^2*exp(b*x+a)+1/32*b/x*exp(b*x+a)+1/32*b^2*exp(a)*Ei(1,-b*x)-1
/64/x^2*exp(3*b*x+3*a)-3/64*b/x*exp(3*b*x+3*a)-9/64*b^2*exp(3*a)*Ei(1,-3*b*
x)-1/64/x^2*exp(5*b*x+5*a)-5/64*b/x*exp(5*b*x+5*a)-25/64*b^2*exp(5*a)*Ei(1,
-5*b*x)
```

maxima [A] time = 0.84, size = 88, normalized size = 0.48

$$-\frac{25}{32}\,b^2e^{(-5\,a)}\Gamma\left(-2,5\,bx\right)-\frac{9}{32}\,b^2e^{(-3\,a)}\Gamma\left(-2,3\,bx\right)+\frac{1}{16}\,b^2e^{(-a)}\Gamma\left(-2,bx\right)+\frac{1}{16}\,b^2e^{a}\Gamma\left(-2,-bx\right)-\frac{9}{32}\,b^2e^{(3\,a)}\Gamma\left(-2,-3\,b\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^3*sinh(b*x+a)^2/x^3,x, algorithm="maxima")
```

```
[Out] -25/32*b^2*e^(-5*a)*gamma(-2, 5*b*x) - 9/32*b^2*e^(-3*a)*gamma(-2, 3*b*x) +
 1/16*b^2*e^(-a)*gamma(-2, b*x) + 1/16*b^2*e^a*gamma(-2, -b*x) - 9/32*b^2*e
^(3*a)*gamma(-2, -3*b*x) - 25/32*b^2*e^(5*a)*gamma(-2, -5*b*x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cosh\left(a+b\,x\right)^3\,\sinh\left(a+b\,x\right)^2}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cosh(a + b*x)^3*sinh(a + b*x)^2)/x^3,x)
```

```
[Out] int((cosh(a + b*x)^3*sinh(a + b*x)^2)/x^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh^{2}\left(a+bx\right)\cosh^{3}\left(a+bx\right)}{x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)**3*sinh(b*x+a)**2/x**3,x)
```

```
[Out] Integral(sinh(a + b*x)**2*cosh(a + b*x)**3/x**3, x)
```

3.306 $\int \frac{\cosh^3(a+bx)\sinh^2(a+bx)}{x^4}\,dx$

Optimal. Leaf size=238

$$-\frac{1}{48}b^3\sinh(a)\text{Chi}(bx)+\frac{9}{32}b^3\sinh(3a)\text{Chi}(3bx)+\frac{125}{96}b^3\sinh(5a)\text{Chi}(5bx)-\frac{1}{48}b^3\cosh(a)\text{Shi}(bx)+\frac{9}{32}b^3\cosh(3a)$$

```
[Out] 1/24*cosh(b*x+a)/x^3+1/48*b^2*cosh(b*x+a)/x-1/48*cosh(3*b*x+3*a)/x^3-3/32*b
^2*cosh(3*b*x+3*a)/x-1/48*cosh(5*b*x+5*a)/x^3-25/96*b^2*cosh(5*b*x+5*a)/x-1
/48*b^3*cosh(a)*Shi(b*x)+9/32*b^3*cosh(3*a)*Shi(3*b*x)+125/96*b^3*cosh(5*a)
*Shi(5*b*x)-1/48*b^3*Chi(b*x)*sinh(a)+9/32*b^3*Chi(3*b*x)*sinh(3*a)+125/96*
b^3*Chi(5*b*x)*sinh(5*a)+1/48*b*sinh(b*x+a)/x^2-1/32*b*sinh(3*b*x+3*a)/x^2-
5/96*b*sinh(5*b*x+5*a)/x^2
```

Rubi [A] time = 0.44, antiderivative size = 238, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {5448, 3297, 3303, 3298, 3301}

$$-\frac{1}{48}b^3\sinh(a)\text{Chi}(bx)+\frac{9}{32}b^3\sinh(3a)\text{Chi}(3bx)+\frac{125}{96}b^3\sinh(5a)\text{Chi}(5bx)-\frac{1}{48}b^3\cosh(a)\text{Shi}(bx)+\frac{9}{32}b^3\cosh(3a)$$

Antiderivative was successfully verified.

```
[In] Int[(Cosh[a + b*x]^3*Sinh[a + b*x]^2)/x^4,x]
```

```
[Out] Cosh[a + b*x]/(24*x^3) + (b^2*Cosh[a + b*x])/(48*x) - Cosh[3*a + 3*b*x]/(48
*x^3) - (3*b^2*Cosh[3*a + 3*b*x])/(32*x) - Cosh[5*a + 5*b*x]/(48*x^3) - (25
*b^2*Cosh[5*a + 5*b*x])/(96*x) - (b^3*CoshIntegral[b*x]*Sinh[a])/48 + (9*b^
3*CoshIntegral[3*b*x]*Sinh[3*a])/32 + (125*b^3*CoshIntegral[5*b*x]*Sinh[5*a
])/96 + (b*Sinh[a + b*x])/(48*x^2) - (b*Sinh[3*a + 3*b*x])/(32*x^2) - (5*b*
Sinh[5*a + 5*b*x])/(96*x^2) - (b^3*Cosh[a]*SinhIntegral[b*x])/48 + (9*b^3*C
osh[3*a]*SinhIntegral[3*b*x])/32 + (125*b^3*Cosh[5*a]*SinhIntegral[5*b*x])/
96
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
```

```
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rubi steps

$$\int \frac{\cosh^3(a+bx)\sinh^2(a+bx)}{x^4}\,dx = \int \left(-\frac{\cosh(a+bx)}{8x^4}+\frac{\cosh(3a+3bx)}{16x^4}+\frac{\cosh(5a+5bx)}{16x^4}\right)dx$$

$$= \frac{1}{16}\int \frac{\cosh(3a+3bx)}{x^4}\,dx + \frac{1}{16}\int \frac{\cosh(5a+5bx)}{x^4}\,dx - \frac{1}{8}\int \frac{\cosh(a+bx)}{x^4}$$

$$= \frac{\cosh(a+bx)}{24x^3} - \frac{\cosh(3a+3bx)}{48x^3} - \frac{\cosh(5a+5bx)}{48x^3} - \frac{1}{24}b\int \frac{\sinh(a+bx)}{x^3}$$

$$= \frac{\cosh(a+bx)}{24x^3} - \frac{\cosh(3a+3bx)}{48x^3} - \frac{\cosh(5a+5bx)}{48x^3} + \frac{b\sinh(a+bx)}{48x^2} - \frac{b\,\mathrm{s}}{}$$

$$= \frac{\cosh(a+bx)}{24x^3} + \frac{b^2\cosh(a+bx)}{48x} - \frac{\cosh(3a+3bx)}{48x^3} - \frac{3b^2\cosh(3a+3bx)}{32x}$$

$$= \frac{\cosh(a+bx)}{24x^3} + \frac{b^2\cosh(a+bx)}{48x} - \frac{\cosh(3a+3bx)}{48x^3} - \frac{3b^2\cosh(3a+3bx)}{32x}$$

$$= \frac{\cosh(a+bx)}{24x^3} + \frac{b^2\cosh(a+bx)}{48x} - \frac{\cosh(3a+3bx)}{48x^3} - \frac{3b^2\cosh(3a+3bx)}{32x}$$

Mathematica [A] time = 0.56, size = 212, normalized size = 0.89

$$-2b^3x^3\sinh(a)\text{Chi}(bx)+27b^3x^3\sinh(3a)\text{Chi}(3bx)+125b^3x^3\sinh(5a)\text{Chi}(5bx)-2b^3x^3\cosh(a)\text{Shi}(bx)+27b^3x$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cosh[a + b*x]^3*Sinh[a + b*x]^2)/x^4,x]
```

```
[Out] (4*Cosh[a + b*x] + 2*b^2*x^2*Cosh[a + b*x] - 2*Cosh[3*(a + b*x)] - 9*b^2*x^
2*Cosh[3*(a + b*x)] - 2*Cosh[5*(a + b*x)] - 25*b^2*x^2*Cosh[5*(a + b*x)] -
2*b^3*x^3*CoshIntegral[b*x]*Sinh[a] + 27*b^3*x^3*CoshIntegral[3*b*x]*Sinh[3
*a] + 125*b^3*x^3*CoshIntegral[5*b*x]*Sinh[5*a] + 2*b*x*Sinh[a + b*x] - 3*b
*x*Sinh[3*(a + b*x)] - 5*b*x*Sinh[5*(a + b*x)] - 2*b^3*x^3*Cosh[a]*SinhInte
gral[b*x] + 27*b^3*x^3*Cosh[3*a]*SinhIntegral[3*b*x] + 125*b^3*x^3*Cosh[5*a
]*SinhIntegral[5*b*x])/(96*x^3)
```

fricas [A] time = 0.58, size = 397, normalized size = 1.67

$$-\frac{10\,bx\sinh\left(bx+a\right)^5+2\left(25\,b^2x^2+2\right)\cosh\left(bx+a\right)^5+10\left(25\,b^2x^2+2\right)\cosh\left(bx+a\right)\sinh\left(bx+a\right)^4+2\left(9\,b^2x^2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^3*sinh(b*x+a)^2/x^4,x, algorithm="fricas")
```

```
[Out] -1/192*(10*b*x*sinh(b*x + a)^5 + 2*(25*b^2*x^2 + 2)*cosh(b*x + a)^5 + 10*(2
5*b^2*x^2 + 2)*cosh(b*x + a)*sinh(b*x + a)^4 + 2*(9*b^2*x^2 + 2)*cosh(b*x +
 a)^3 + 2*(50*b*x*cosh(b*x + a)^2 + 3*b*x)*sinh(b*x + a)^3 + 2*(10*(25*b^2*
x^2 + 2)*cosh(b*x + a)^3 + 3*(9*b^2*x^2 + 2)*cosh(b*x + a))*sinh(b*x + a)^2
 - 4*(b^2*x^2 + 2)*cosh(b*x + a) - 125*(b^3*x^3*Ei(5*b*x) - b^3*x^3*Ei(-5*b
*x))*cosh(5*a) - 27*(b^3*x^3*Ei(3*b*x) - b^3*x^3*Ei(-3*b*x))*cosh(3*a) + 2*
(b^3*x^3*Ei(b*x) - b^3*x^3*Ei(-b*x))*cosh(a) + 2*(25*b*x*cosh(b*x + a)^4 +
9*b*x*cosh(b*x + a)^2 - 2*b*x)*sinh(b*x + a) - 125*(b^3*x^3*Ei(5*b*x) + b^3
*x^3*Ei(-5*b*x))*sinh(5*a) - 27*(b^3*x^3*Ei(3*b*x) + b^3*x^3*Ei(-3*b*x))*si
nh(3*a) + 2*(b^3*x^3*Ei(b*x) + b^3*x^3*Ei(-b*x))*sinh(a))/x^3
```

giac [A] time = 0.13, size = 342, normalized size = 1.44

$$\frac{125\,b^3x^3\text{Ei}\left(5\,bx\right)e^{(5\,a)}+27\,b^3x^3\text{Ei}\left(3\,bx\right)e^{(3\,a)}+2\,b^3x^3\text{Ei}\left(-bx\right)e^{(-a)}-27\,b^3x^3\text{Ei}\left(-3\,bx\right)e^{(-3\,a)}-125\,b^3x^3\text{Ei}\left(-5\,bx\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^3*sinh(b*x+a)^2/x^4,x, algorithm="giac")
```

```
[Out] 1/192*(125*b^3*x^3*Ei(5*b*x)*e^(5*a) + 27*b^3*x^3*Ei(3*b*x)*e^(3*a) + 2*b^3
*x^3*Ei(-b*x)*e^(-a) - 27*b^3*x^3*Ei(-3*b*x)*e^(-3*a) - 125*b^3*x^3*Ei(-5*b
```

```
*x)*e^(-5*a) - 2*b^3*x^3*Ei(b*x)*e^a - 25*b^2*x^2*e^(5*b*x + 5*a) - 9*b^2*x
^2*e^(3*b*x + 3*a) + 2*b^2*x^2*e^(b*x + a) + 2*b^2*x^2*e^(-b*x - a) - 9*b^2
*x^2*e^(-3*b*x - 3*a) - 25*b^2*x^2*e^(-5*b*x - 5*a) - 5*b*x*e^(5*b*x + 5*a)
 - 3*b*x*e^(3*b*x + 3*a) + 2*b*x*e^(b*x + a) - 2*b*x*e^(-b*x - a) + 3*b*x*e
^(-3*b*x - 3*a) + 5*b*x*e^(-5*b*x - 5*a) - 2*e^(5*b*x + 5*a) - 2*e^(3*b*x +
 3*a) + 4*e^(b*x + a) + 4*e^(-b*x - a) - 2*e^(-3*b*x - 3*a) - 2*e^(-5*b*x -
 5*a))/x^3
```

maple [A] time = 0.55, size = 356, normalized size = 1.50

$$-\frac{25b^2\mathrm{e}^{-5bx-5a}}{192x}+\frac{5b\,\mathrm{e}^{-5bx-5a}}{192x^2}-\frac{\mathrm{e}^{-5bx-5a}}{96x^3}+\frac{125b^3\mathrm{e}^{-5a}\,\mathrm{Ei}\,(1,5bx)}{192}-\frac{3b^2\mathrm{e}^{-3bx-3a}}{64x}+\frac{b\,\mathrm{e}^{-3bx-3a}}{64x^2}-\frac{\mathrm{e}^{-3bx-3a}}{96x^3}+\frac{9b^3\mathrm{e}^{-3a}\,\mathrm{Ei}\,(1,3}{64}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)^3*sinh(b*x+a)^2/x^4,x)
```

```
[Out] -25/192*b^2*exp(-5*b*x-5*a)/x+5/192*b*exp(-5*b*x-5*a)/x^2-1/96*exp(-5*b*x-5
*a)/x^3+125/192*b^3*exp(-5*a)*Ei(1,5*b*x)-3/64*b^2*exp(-3*b*x-3*a)/x+1/64*b
*exp(-3*b*x-3*a)/x^2-1/96*exp(-3*b*x-3*a)/x^3+9/64*b^3*exp(-3*a)*Ei(1,3*b*x
)+1/96*b^2*exp(-b*x-a)/x-1/96*b*exp(-b*x-a)/x^2+1/48*exp(-b*x-a)/x^3-1/96*b
^3*exp(-a)*Ei(1,b*x)+1/48/x^3*exp(b*x+a)+1/96*b/x^2*exp(b*x+a)+1/96*b^2/x*e
xp(b*x+a)+1/96*b^3*exp(a)*Ei(1,-b*x)-1/96/x^3*exp(3*b*x+3*a)-1/64*b/x^2*exp
(3*b*x+3*a)-3/64*b^2/x*exp(3*b*x+3*a)-9/64*b^3*exp(3*a)*Ei(1,-3*b*x)-1/96/x
^3*exp(5*b*x+5*a)-5/192*b/x^2*exp(5*b*x+5*a)-25/192*b^2/x*exp(5*b*x+5*a)-12
5/192*b^3*exp(5*a)*Ei(1,-5*b*x)
```

maxima [A] time = 0.45, size = 88, normalized size = 0.37

$$-\frac{125}{32}\,b^3e^{(-5\,a)}\Gamma\left(-3,5\,bx\right)-\frac{27}{32}\,b^3e^{(-3\,a)}\Gamma\left(-3,3\,bx\right)+\frac{1}{16}\,b^3e^{(-a)}\Gamma\left(-3,bx\right)-\frac{1}{16}\,b^3e^{a}\Gamma\left(-3,-bx\right)+\frac{27}{32}\,b^3e^{(3\,a)}\Gamma\left(-3,-3\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^3*sinh(b*x+a)^2/x^4,x, algorithm="maxima")
```

```
[Out] -125/32*b^3*e^(-5*a)*gamma(-3, 5*b*x) - 27/32*b^3*e^(-3*a)*gamma(-3, 3*b*x)
 + 1/16*b^3*e^(-a)*gamma(-3, b*x) - 1/16*b^3*e^a*gamma(-3, -b*x) + 27/32*b^
3*e^(3*a)*gamma(-3, -3*b*x) + 125/32*b^3*e^(5*a)*gamma(-3, -5*b*x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cosh\left(a+b\,x\right)^3\,\sinh\left(a+b\,x\right)^2}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cosh(a + b*x)^3*sinh(a + b*x)^2)/x^4,x)
[Out] int((cosh(a + b*x)^3*sinh(a + b*x)^2)/x^4, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh^2(a+bx)\cosh^3(a+bx)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)**3*sinh(b*x+a)**2/x**4,x)
[Out] Integral(sinh(a + b*x)**2*cosh(a + b*x)**3/x**4, x)
```

3.307 $\int x^m \cosh(a+bx)\sinh^3(a+bx)\,dx$

Optimal. Leaf size=141

$$\frac{e^{4a}2^{-2(m+3)}x^m(-bx)^{-m}\Gamma(m+1,-4bx)}{b}-\frac{e^{2a}2^{-m-4}x^m(-bx)^{-m}\Gamma(m+1,-2bx)}{b}-\frac{e^{-2a}2^{-m-4}x^m(bx)^{-m}\Gamma(m+1,2bx)}{b}+\cdots$$

```
[Out] exp(4*a)*x^m*GAMMA(1+m,-4*b*x)/(2^(6+2*m))/b/((-b*x)^m)-2^(-4-m)*exp(2*a)*x
^m*GAMMA(1+m,-2*b*x)/b/((-b*x)^m)-2^(-4-m)*x^m*GAMMA(1+m,2*b*x)/b/exp(2*a)/
((b*x)^m)+x^m*GAMMA(1+m,4*b*x)/(2^(6+2*m))/b/exp(4*a)/((b*x)^m)
```

Rubi [A] time = 0.21, antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {5448, 3308, 2181}

$$\frac{e^{4a}2^{-2(m+3)}x^m(-bx)^{-m}\text{Gamma}(m+1,-4bx)}{b}-\frac{e^{2a}2^{-m-4}x^m(-bx)^{-m}\text{Gamma}(m+1,-2bx)}{b}-\frac{e^{-2a}2^{-m-4}x^m(bx)^{-m}\text{G}\cdots}{b}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*Cosh[a + b*x]*Sinh[a + b*x]^3,x]
```

```
[Out] (E^(4*a)*x^m*Gamma[1 + m, -4*b*x])/(2^(2*(3 + m))*b*(-(b*x))^m) - (2^(-4 -
m)*E^(2*a)*x^m*Gamma[1 + m, -2*b*x])/(b*(-(b*x))^m) - (2^(-4 - m)*x^m*Gamma
[1 + m, 2*b*x])/(b*E^(2*a)*(b*x)^m) + (x^m*Gamma[1 + m, 4*b*x])/(2^(2*(3 +
m))*b*E^(4*a)*(b*x)^m)
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] && !I
ntegerQ[m]
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
```

```
& IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int x^m \cosh(a+bx)\sinh^3(a+bx)\,dx &= \int \left(-\frac{1}{4}x^m \sinh(2a+2bx)+\frac{1}{8}x^m \sinh(4a+4bx)\right)dx\\ &= \frac{1}{8}\int x^m \sinh(4a+4bx)\,dx-\frac{1}{4}\int x^m \sinh(2a+2bx)\,dx\\ &= \frac{1}{16}\int e^{-i(4ia+4ibx)}x^m\,dx-\frac{1}{16}\int e^{i(4ia+4ibx)}x^m\,dx-\frac{1}{8}\int e^{-i(2ia+2ibx)}x^m\,dx+\\ &= \frac{4^{-3-m}e^{4a}x^m(-bx)^{-m}\Gamma(1+m,-4bx)}{b}-\frac{2^{-4-m}e^{2a}x^m(-bx)^{-m}\Gamma(1+m,-2bx)}{b}\end{aligned}$$

Mathematica [A] time = 0.14, size = 112, normalized size = 0.79

$$\frac{e^{-4a}4^{-m-3}x^m\left(-b^2x^2\right)^{-m}\left((-bx)^m\left(\Gamma(m+1,4bx)-e^{2a}2^{m+2}\Gamma(m+1,2bx)\right)+e^{8a}(bx)^m\Gamma(m+1,-4bx)-e^{6a}2^{m+2}(bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*Cosh[a + b*x]*Sinh[a + b*x]^3,x]
```

```
[Out] (4^(-3 - m)*x^m*(E^(8*a)*(b*x)^m*Gamma[1 + m, -4*b*x] - 2^(2 + m)*E^(6*a)*(
b*x)^m*Gamma[1 + m, -2*b*x] + (-(b*x))^m*(-(2^(2 + m)*E^(2*a)*Gamma[1 + m,
2*b*x]) + Gamma[1 + m, 4*b*x])))/(b*E^(4*a)*(-(b^2*x^2))^m)
```

fricas [A] time = 0.77, size = 172, normalized size = 1.22

$$\frac{\cosh\left(m\log\left(4\,b\right)+4\,a\right)\Gamma\left(m+1,4\,bx\right)-4\,\cosh\left(m\log\left(2\,b\right)+2\,a\right)\Gamma\left(m+1,2\,bx\right)-4\,\cosh\left(m\log\left(-2\,b\right)-2\,a\right)\Gamma}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*cosh(b*x+a)*sinh(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/64*(cosh(m*log(4*b) + 4*a)*gamma(m + 1, 4*b*x) - 4*cosh(m*log(2*b) + 2*a)
*gamma(m + 1, 2*b*x) - 4*cosh(m*log(-2*b) - 2*a)*gamma(m + 1, -2*b*x) + cos
h(m*log(-4*b) - 4*a)*gamma(m + 1, -4*b*x) - gamma(m + 1, 4*b*x)*sinh(m*log(
4*b) + 4*a) + 4*gamma(m + 1, 2*b*x)*sinh(m*log(2*b) + 2*a) + 4*gamma(m + 1,
 -2*b*x)*sinh(m*log(-2*b) - 2*a) - gamma(m + 1, -4*b*x)*sinh(m*log(-4*b) -
4*a))/b
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^m \cosh(bx+a)\sinh(bx+a)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*cosh(b*x+a)*sinh(b*x+a)^3,x, algorithm="giac")
```

```
[Out] integrate(x^m*cosh(b*x + a)*sinh(b*x + a)^3, x)
```

maple [F] time = 0.45, size = 0, normalized size = 0.00

$$\int x^m \cosh(bx+a)\left(\sinh^3(bx+a)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*cosh(b*x+a)*sinh(b*x+a)^3,x)
```

```
[Out] int(x^m*cosh(b*x+a)*sinh(b*x+a)^3,x)
```

maxima [A] time = 0.45, size = 117, normalized size = 0.83

$$\frac{1}{16}\,(4\,bx)^{-m-1}\,x^{m+1}e^{(-4\,a)}\Gamma(m+1,4\,bx)-\frac{1}{8}\,(2\,bx)^{-m-1}\,x^{m+1}e^{(-2\,a)}\Gamma(m+1,2\,bx)+\frac{1}{8}\,(-2\,bx)^{-m-1}\,x^{m+1}e^{(2\,a)}\Gamma(m+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*cosh(b*x+a)*sinh(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/16*(4*b*x)^(-m - 1)*x^(m + 1)*e^(-4*a)*gamma(m + 1, 4*b*x) - 1/8*(2*b*x)^
(-m - 1)*x^(m + 1)*e^(-2*a)*gamma(m + 1, 2*b*x) + 1/8*(-2*b*x)^(-m - 1)*x^(
m + 1)*e^(2*a)*gamma(m + 1, -2*b*x) - 1/16*(-4*b*x)^(-m - 1)*x^(m + 1)*e^(4
*a)*gamma(m + 1, -4*b*x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^m\,\cosh(a+b\,x)\,\sinh(a+b\,x)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*cosh(a + b*x)*sinh(a + b*x)^3,x)
```

```
[Out] int(x^m*cosh(a + b*x)*sinh(a + b*x)^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^m \sinh^3(a+bx)\cosh(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*cosh(b*x+a)*sinh(b*x+a)**3,x)
```

```
[Out] Integral(x**m*sinh(a + b*x)**3*cosh(a + b*x), x)
```

3.308 $\int x^3 \cosh(a+bx)\sinh^3(a+bx)\,dx$

Optimal. Leaf size=155

$$-\frac{3\sinh^3(a+bx)\cosh(a+bx)}{128b^4}+\frac{45\sinh(a+bx)\cosh(a+bx)}{256b^4}+\frac{3x\sinh^4(a+bx)}{32b^3}-\frac{9x\sinh^2(a+bx)}{32b^3}-\frac{3x^2\sinh^3(}{}$$

```
[Out] -45/256*x/b^3-3/32*x^3/b+45/256*cosh(b*x+a)*sinh(b*x+a)/b^4+9/32*x^2*cosh(b
*x+a)*sinh(b*x+a)/b^2-9/32*x*sinh(b*x+a)^2/b^3-3/128*cosh(b*x+a)*sinh(b*x+a
)^3/b^4-3/16*x^2*cosh(b*x+a)*sinh(b*x+a)^3/b^2+3/32*x*sinh(b*x+a)^4/b^3+1/4
*x^3*sinh(b*x+a)^4/b
```

Rubi [A] time = 0.14, antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {5372, 3311, 30, 2635, 8}

$$-\frac{3x^2\sinh^3(a+bx)\cosh(a+bx)}{16b^2}+\frac{9x^2\sinh(a+bx)\cosh(a+bx)}{32b^2}+\frac{3x\sinh^4(a+bx)}{32b^3}-\frac{9x\sinh^2(a+bx)}{32b^3}-\frac{3\sinh^3}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Cosh[a + b*x]*Sinh[a + b*x]^3,x]
```

```
[Out] (-45*x)/(256*b^3) - (3*x^3)/(32*b) + (45*Cosh[a + b*x]*Sinh[a + b*x])/(256*
b^4) + (9*x^2*Cosh[a + b*x]*Sinh[a + b*x])/(32*b^2) - (9*x*Sinh[a + b*x]^2)
/(32*b^3) - (3*Cosh[a + b*x]*Sinh[a + b*x]^3)/(128*b^4) - (3*x^2*Cosh[a + b
*x]*Sinh[a + b*x]^3)/(16*b^2) + (3*x*Sinh[a + b*x]^4)/(32*b^3) + (x^3*Sinh[
a + b*x]^4)/(4*b)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 3311

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(d*m*(c + d*x)^(m - 1)*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist
[(b^2*(n - 1))/n, Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[(
d^2*m*(m - 1))/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[(b*(c + d*x)^m*Cos[e + f*x]*(b*Sin[e + f*x])^(n - 1))/(f*n), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 5372

```
Int[Cosh[(a_.) + (b_.)*(x_)^(n_.)]*(x_)^(m_.)*Sinh[(a_.) + (b_.)*(x_)^(n_.)
]^(p_.), x_Symbol] :> Simp[(x^(m - n + 1)*Sinh[a + b*x^n]^(p + 1))/(b*n*(p
+ 1)), x] - Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*Sinh[a + b*x^n]^(
p + 1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

Rubi steps

$$\int x^3\cosh(a+bx)\sinh^3(a+bx)\,dx = \frac{x^3\sinh^4(a+bx)}{4b} - \frac{3\int x^2\sinh^4(a+bx)\,dx}{4b}$$
$$= -\frac{3x^2\cosh(a+bx)\sinh^3(a+bx)}{16b^2} + \frac{3x\sinh^4(a+bx)}{32b^3} + \frac{x^3\sinh^4(a+bx)}{4b} -$$
$$= \frac{9x^2\cosh(a+bx)\sinh(a+bx)}{32b^2} - \frac{9x\sinh^2(a+bx)}{32b^3} - \frac{3\cosh(a+bx)\sinh^3}{128b^4}$$
$$= -\frac{3x^3}{32b} + \frac{45\cosh(a+bx)\sinh(a+bx)}{256b^4} + \frac{9x^2\cosh(a+bx)\sinh(a+bx)}{32b^2} -$$
$$= -\frac{45x}{256b^3} - \frac{3x^3}{32b} + \frac{45\cosh(a+bx)\sinh(a+bx)}{256b^4} + \frac{9x^2\cosh(a+bx)\sinh(a}{32b^2}$$

Mathematica [A] time = 0.66, size = 95, normalized size = 0.61

$$\frac{48\left(2b^2x^2+1\right)\sinh(2(a+bx))+2bx\left(8b^2x^2+3\right)\cosh(4(a+bx))-\left(3\left(8b^2x^2+1\right)\sinh(2(a+bx))+32bx\left(2b^2x^2\right.\right.}{512b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Cosh[a + b*x]*Sinh[a + b*x]^3,x]
```

```
[Out] (2*b*x*(3 + 8*b^2*x^2)*Cosh[4*(a + b*x)] + 48*(1 + 2*b^2*x^2)*Sinh[2*(a + b
*x)] - Cosh[2*(a + b*x)]*(32*b*x*(3 + 2*b^2*x^2) + 3*(1 + 8*b^2*x^2)*Sinh[2
*(a + b*x)]))/(512*b^4)
```

fricas [A] time = 0.77, size = 191, normalized size = 1.23

$$\frac{\left(8\,b^3x^3+3\,bx\right)\cosh\left(bx+a\right)^4-3\left(8\,b^2x^2+1\right)\cosh\left(bx+a\right)\sinh\left(bx+a\right)^3+\left(8\,b^3x^3+3\,bx\right)\sinh\left(bx+a\right)^4-16}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*cosh(b*x+a)*sinh(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/256*((8*b^3*x^3 + 3*b*x)*cosh(b*x + a)^4 - 3*(8*b^2*x^2 + 1)*cosh(b*x + a
)*sinh(b*x + a)^3 + (8*b^3*x^3 + 3*b*x)*sinh(b*x + a)^4 - 16*(2*b^3*x^3 + 3
*b*x)*cosh(b*x + a)^2 - 2*(16*b^3*x^3 - 3*(8*b^3*x^3 + 3*b*x)*cosh(b*x + a)
^2 + 24*b*x)*sinh(b*x + a)^2 - 3*((8*b^2*x^2 + 1)*cosh(b*x + a)^3 - 16*(2*b
^2*x^2 + 1)*cosh(b*x + a))*sinh(b*x + a))/b^4
```

giac [A] time = 0.12, size = 145, normalized size = 0.94

$$\frac{\left(32\,b^3x^3-24\,b^2x^2+12\,bx-3\right)e^{(4\,bx+4\,a)}}{2048\,b^4}-\frac{\left(4\,b^3x^3-6\,b^2x^2+6\,bx-3\right)e^{(2\,bx+2\,a)}}{64\,b^4}-\frac{\left(4\,b^3x^3+6\,b^2x^2+6\,bx+3\right)e^{(-2}}{64\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*cosh(b*x+a)*sinh(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/2048*(32*b^3*x^3 - 24*b^2*x^2 + 12*b*x - 3)*e^(4*b*x + 4*a)/b^4 - 1/64*(4
*b^3*x^3 - 6*b^2*x^2 + 6*b*x - 3)*e^(2*b*x + 2*a)/b^4 - 1/64*(4*b^3*x^3 + 6
*b^2*x^2 + 6*b*x + 3)*e^(-2*b*x - 2*a)/b^4 + 1/2048*(32*b^3*x^3 + 24*b^2*x^
2 + 12*b*x + 3)*e^(-4*b*x - 4*a)/b^4
```

maple [B] time = 0.36, size = 304, normalized size = 1.96

$$\frac{\frac{(bx+a)^3\left(\sinh^4(bx+a)\right)}{4}-\frac{3(bx+a)^2\cosh(bx+a)\left(\sinh^3(bx+a)\right)}{16}+\frac{9(bx+a)^2\cosh(bx+a)\sinh(bx+a)}{32}-\frac{3(bx+a)^3}{32}+\frac{3(bx+a)\left(\sinh^4(bx+a)\right)}{32}-\frac{3\cos}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*cosh(b*x+a)*sinh(b*x+a)^3,x)
```

```
[Out] 1/b^4*(1/4*(b*x+a)^3*sinh(b*x+a)^4-3/16*(b*x+a)^2*cosh(b*x+a)*sinh(b*x+a)^3
+9/32*(b*x+a)^2*cosh(b*x+a)*sinh(b*x+a)-3/32*(b*x+a)^3+3/32*(b*x+a)*sinh(b*
x+a)^4-3/128*cosh(b*x+a)*sinh(b*x+a)^3+45/256*cosh(b*x+a)*sinh(b*x+a)+27/25
6*b*x+27/256*a-9/32*(b*x+a)*cosh(b*x+a)^2-3*a*(1/4*(b*x+a)^2*sinh(b*x+a)^4-
1/8*(b*x+a)*cosh(b*x+a)*sinh(b*x+a)^3+3/16*(b*x+a)*cosh(b*x+a)*sinh(b*x+a)-
3/32*(b*x+a)^2+1/32*sinh(b*x+a)^4-3/32*cosh(b*x+a)^2)+3*a^2*(1/4*(b*x+a)*si
nh(b*x+a)^4-1/16*cosh(b*x+a)*sinh(b*x+a)^3+3/32*cosh(b*x+a)*sinh(b*x+a)-3/3
2*b*x-3/32*a)-1/4*a^3*sinh(b*x+a)^4)
```

maxima [A] time = 0.35, size = 171, normalized size = 1.10

$$\frac{\left(32\,b^{3}x^{3}e^{(4\,a)}-24\,b^{2}x^{2}e^{(4\,a)}+12\,bxe^{(4\,a)}-3\,e^{(4\,a)}\right)e^{(4\,bx)}}{2048\,b^{4}}-\frac{\left(4\,b^{3}x^{3}e^{(2\,a)}-6\,b^{2}x^{2}e^{(2\,a)}+6\,bxe^{(2\,a)}-3\,e^{(2\,a)}\right)e^{(2\,bx)}}{64\,b^{4}}-\frac{(4\,b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*cosh(b*x+a)*sinh(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/2048*(32*b^3*x^3*e^(4*a) - 24*b^2*x^2*e^(4*a) + 12*b*x*e^(4*a) - 3*e^(4*a
))*e^(4*b*x)/b^4 - 1/64*(4*b^3*x^3*e^(2*a) - 6*b^2*x^2*e^(2*a) + 6*b*x*e^(2
*a) - 3*e^(2*a))*e^(2*b*x)/b^4 - 1/64*(4*b^3*x^3 + 6*b^2*x^2 + 6*b*x + 3)*e
^(-2*b*x - 2*a)/b^4 + 1/2048*(32*b^3*x^3 + 24*b^2*x^2 + 12*b*x + 3)*e^(-4*b
*x - 4*a)/b^4
```

mupad [B] time = 0.26, size = 126, normalized size = 0.81

$$\frac{\frac{3\,x^{2}\sinh(2\,a+2\,b\,x)}{16}-\frac{3\,x^{2}\sinh(4\,a+4\,b\,x)}{128}}{b^{2}}-\frac{\frac{x^{3}\cosh(2\,a+2\,b\,x)}{8}-\frac{x^{3}\cosh(4\,a+4\,b\,x)}{32}}{b}-\frac{\frac{3\,x\cosh(2\,a+2\,b\,x)}{16}-\frac{3\,x\cosh(4\,a+4\,b\,x)}{256}}{b^{3}}+\frac{3\sinh\left(2\,a+\right.}{32\,b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*cosh(a + b*x)*sinh(a + b*x)^3,x)
```

```
[Out] ((3*x^2*sinh(2*a + 2*b*x))/16 - (3*x^2*sinh(4*a + 4*b*x))/128)/b^2 - ((x^3*
cosh(2*a + 2*b*x))/8 - (x^3*cosh(4*a + 4*b*x))/32)/b - ((3*x*cosh(2*a + 2*b
*x))/16 - (3*x*cosh(4*a + 4*b*x))/256)/b^3 + (3*sinh(2*a + 2*b*x))/(32*b^4)
 - (3*sinh(4*a + 4*b*x))/(1024*b^4)
```

sympy [A] time = 5.25, size = 226, normalized size = 1.46

$$\begin{cases} \frac{5x^{3}\sinh^{4}(a+bx)}{32b}+\frac{3x^{3}\sinh^{2}(a+bx)\cosh^{2}(a+bx)}{16b}-\frac{3x^{3}\cosh^{4}(a+bx)}{32b}-\frac{15x^{2}\sinh^{3}(a+bx)\cosh(a+bx)}{32b^{2}}+\frac{9x^{2}\sinh(a+bx)\cosh^{3}(a+bx)}{32b^{2}}+\frac{51x \\ \frac{x^{4}\sinh^{3}(a)\cosh(a)}{4} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*cosh(b*x+a)*sinh(b*x+a)**3,x)
```

```
[Out] Piecewise((5*x**3*sinh(a + b*x)**4/(32*b) + 3*x**3*sinh(a + b*x)**2*cosh(a
+ b*x)**2/(16*b) - 3*x**3*cosh(a + b*x)**4/(32*b) - 15*x**2*sinh(a + b*x)**
3*cosh(a + b*x)/(32*b**2) + 9*x**2*sinh(a + b*x)*cosh(a + b*x)**3/(32*b**2)
 + 51*x*sinh(a + b*x)**4/(256*b**3) + 9*x*sinh(a + b*x)**2*cosh(a + b*x)**2
/(128*b**3) - 45*x*cosh(a + b*x)**4/(256*b**3) - 51*sinh(a + b*x)**3*cosh(a
 + b*x)/(256*b**4) + 45*sinh(a + b*x)*cosh(a + b*x)**3/(256*b**4), Ne(b, 0)
), (x**4*sinh(a)**3*cosh(a)/4, True))
```

3.309 $\int x^2 \cosh(a+bx)\sinh^3(a+bx)\,dx$

Optimal. Leaf size=101

$$\frac{\sinh^4(a+bx)}{32b^3}-\frac{3\sinh^2(a+bx)}{32b^3}-\frac{x\sinh^3(a+bx)\cosh(a+bx)}{8b^2}+\frac{3x\sinh(a+bx)\cosh(a+bx)}{16b^2}+\frac{x^2\sinh^4(a+bx)}{4b}$$

```
[Out] -3/32*x^2/b+3/16*x*cosh(b*x+a)*sinh(b*x+a)/b^2-3/32*sinh(b*x+a)^2/b^3-1/8*x
*cosh(b*x+a)*sinh(b*x+a)^3/b^2+1/32*sinh(b*x+a)^4/b^3+1/4*x^2*sinh(b*x+a)^4
/b
```

Rubi [A] time = 0.08, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {5372, 3310, 30}

$$\frac{\sinh^4(a+bx)}{32b^3}-\frac{3\sinh^2(a+bx)}{32b^3}-\frac{x\sinh^3(a+bx)\cosh(a+bx)}{8b^2}+\frac{3x\sinh(a+bx)\cosh(a+bx)}{16b^2}+\frac{x^2\sinh^4(a+bx)}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Cosh[a + b*x]*Sinh[a + b*x]^3,x]
```

```
[Out] (-3*x^2)/(32*b) + (3*x*Cosh[a + b*x]*Sinh[a + b*x])/(16*b^2) - (3*Sinh[a +
b*x]^2)/(32*b^3) - (x*Cosh[a + b*x]*Sinh[a + b*x]^3)/(8*b^2) + Sinh[a + b*x
]^4/(32*b^3) + (x^2*Sinh[a + b*x]^4)/(4*b)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 3310

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[(d*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[(b*(c + d*x)*Cos[e + f*x]*(b
*Sin[e + f*x])^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 5372

```
Int[Cosh[(a_.) + (b_.)*(x_)^(n_.)]*(x_)^(m_.)*Sinh[(a_.) + (b_.)*(x_)^(n_.)
]^(p_.), x_Symbol] :> Simp[(x^(m - n + 1)*Sinh[a + b*x^n]^(p + 1))/(b*n*(p
+ 1)), x] - Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*Sinh[a + b*x^n]^(
p + 1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

Rubi steps

$$\int x^2\cosh(a+bx)\sinh^3(a+bx)\,dx = \frac{x^2\sinh^4(a+bx)}{4b} - \frac{\int x\sinh^4(a+bx)\,dx}{2b}$$
$$= -\frac{x\cosh(a+bx)\sinh^3(a+bx)}{8b^2} + \frac{\sinh^4(a+bx)}{32b^3} + \frac{x^2\sinh^4(a+bx)}{4b} + \frac{3\int$$
$$= \frac{3x\cosh(a+bx)\sinh(a+bx)}{16b^2} - \frac{3\sinh^2(a+bx)}{32b^3} - \frac{x\cosh(a+bx)\sinh^3(a$$
$$= -\frac{3x^2}{32b} + \frac{3x\cosh(a+bx)\sinh(a+bx)}{16b^2} - \frac{3\sinh^2(a+bx)}{32b^3} - \frac{x\cosh(a+bx)}{8b}$$

Mathematica [A] time = 0.23, size = 72, normalized size = 0.71

$$\frac{-16\left(2b^2x^2+1\right)\cosh(2(a+bx))+\left(8b^2x^2+1\right)\cosh(4(a+bx))+4bx(8\sinh(2(a+bx))-\sinh(4(a+bx)))}{256b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Cosh[a + b*x]*Sinh[a + b*x]^3,x]
```

```
[Out] (-16*(1 + 2*b^2*x^2)*Cosh[2*(a + b*x)] + (1 + 8*b^2*x^2)*Cosh[4*(a + b*x)]
+ 4*b*x*(8*Sinh[2*(a + b*x)] - Sinh[4*(a + b*x)]))/(256*b^3)
```

fricas [A] time = 0.48, size = 154, normalized size = 1.52

$$-\frac{16\,bx\cosh\left(bx+a\right)\sinh\left(bx+a\right)^3-\left(8\,b^2x^2+1\right)\cosh\left(bx+a\right)^4-\left(8\,b^2x^2+1\right)\sinh\left(bx+a\right)^4+16\left(2\,b^2x^2+1\right)c}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cosh(b*x+a)*sinh(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -1/256*(16*b*x*cosh(b*x + a)*sinh(b*x + a)^3 - (8*b^2*x^2 + 1)*cosh(b*x + a
)^4 - (8*b^2*x^2 + 1)*sinh(b*x + a)^4 + 16*(2*b^2*x^2 + 1)*cosh(b*x + a)^2
+ 2*(16*b^2*x^2 - 3*(8*b^2*x^2 + 1)*cosh(b*x + a)^2 + 8)*sinh(b*x + a)^2 +
16*(b*x*cosh(b*x + a)^3 - 4*b*x*cosh(b*x + a))*sinh(b*x + a))/b^3
```

giac [A] time = 0.12, size = 113, normalized size = 1.12

$$\frac{\left(8\,b^2x^2-4\,bx+1\right)e^{(4\,bx+4\,a)}}{512\,b^3}-\frac{\left(2\,b^2x^2-2\,bx+1\right)e^{(2\,bx+2\,a)}}{32\,b^3}-\frac{\left(2\,b^2x^2+2\,bx+1\right)e^{(-2\,bx-2\,a)}}{32\,b^3}+\frac{\left(8\,b^2x^2+4\,bx+1\right)e^{(-4\,b}}{512\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cosh(b*x+a)*sinh(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/512*(8*b^2*x^2 - 4*b*x + 1)*e^(4*b*x + 4*a)/b^3 - 1/32*(2*b^2*x^2 - 2*b*x
 + 1)*e^(2*b*x + 2*a)/b^3 - 1/32*(2*b^2*x^2 + 2*b*x + 1)*e^(-2*b*x - 2*a)/b
^3 + 1/512*(8*b^2*x^2 + 4*b*x + 1)*e^(-4*b*x - 4*a)/b^3
```

maple [A] time = 0.33, size = 161, normalized size = 1.59

$$\frac{\frac{(bx+a)^2\left(\sinh^4(bx+a)\right)}{4}-\frac{(bx+a)\cosh(bx+a)\left(\sinh^3(bx+a)\right)}{8}+\frac{3(bx+a)\cosh(bx+a)\sinh(bx+a)}{16}-\frac{3(bx+a)^2}{32}+\frac{\left(\sinh^4(bx+a)\right)}{32}-\frac{3\left(\cosh^2(bx+a)\right)}{32}}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*cosh(b*x+a)*sinh(b*x+a)^3,x)
```

```
[Out] 1/b^3*(1/4*(b*x+a)^2*sinh(b*x+a)^4-1/8*(b*x+a)*cosh(b*x+a)*sinh(b*x+a)^3+3/
16*(b*x+a)*cosh(b*x+a)*sinh(b*x+a)-3/32*(b*x+a)^2+1/32*sinh(b*x+a)^4-3/32*c
osh(b*x+a)^2-2*a*(1/4*(b*x+a)*sinh(b*x+a)^4-1/16*cosh(b*x+a)*sinh(b*x+a)^3+
3/32*cosh(b*x+a)*sinh(b*x+a)-3/32*b*x-3/32*a)+1/4*a^2*sinh(b*x+a)^4)
```

maxima [A] time = 0.35, size = 127, normalized size = 1.26

$$\frac{\left(8\,b^2x^2e^{(4\,a)}-4\,bxe^{(4\,a)}+e^{(4\,a)}\right)e^{(4\,bx)}}{512\,b^3}-\frac{\left(2\,b^2x^2e^{(2\,a)}-2\,bxe^{(2\,a)}+e^{(2\,a)}\right)e^{(2\,bx)}}{32\,b^3}-\frac{\left(2\,b^2x^2+2\,bx+1\right)e^{(-2\,bx-2\,a)}}{32\,b^3}+\frac{\left(8\,b^2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cosh(b*x+a)*sinh(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/512*(8*b^2*x^2*e^(4*a) - 4*b*x*e^(4*a) + e^(4*a))*e^(4*b*x)/b^3 - 1/32*(2
*b^2*x^2*e^(2*a) - 2*b*x*e^(2*a) + e^(2*a))*e^(2*b*x)/b^3 - 1/32*(2*b^2*x^2
 + 2*b*x + 1)*e^(-2*b*x - 2*a)/b^3 + 1/512*(8*b^2*x^2 + 4*b*x + 1)*e^(-4*b*
x - 4*a)/b^3
```

mupad [B] time = 1.61, size = 89, normalized size = 0.88

$$-\frac{\frac{\cosh(2\,a+2\,b\,x)}{16}-\frac{\cosh(4\,a+4\,b\,x)}{256}+b^2\,\left(\frac{x^2\cosh(2\,a+2\,b\,x)}{8}-\frac{x^2\cosh(4\,a+4\,b\,x)}{32}\right)-b\,\left(\frac{x\sinh(2\,a+2\,b\,x)}{8}-\frac{x\sinh(4\,a+4\,b\,x)}{64}\right)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*cosh(a + b*x)*sinh(a + b*x)^3,x)
```

```
[Out] -(cosh(2*a + 2*b*x)/16 - cosh(4*a + 4*b*x)/256 + b^2*((x^2*cosh(2*a + 2*b*x
))/8 - (x^2*cosh(4*a + 4*b*x))/32) - b*((x*sinh(2*a + 2*b*x))/8 - (x*sinh(4
*a + 4*b*x))/64))/b^3
```

sympy [A] time = 2.90, size = 150, normalized size = 1.49

$$\begin{cases} \frac{5x^2\sinh^4(a+bx)}{32b} + \frac{3x^2\sinh^2(a+bx)\cosh^2(a+bx)}{16b} - \frac{3x^2\cosh^4(a+bx)}{32b} - \frac{5x\sinh^3(a+bx)\cosh(a+bx)}{16b^2} + \frac{3x\sinh(a+bx)\cosh^3(a+bx)}{16b^2} + \frac{5\sinh^4}{64} \\ \frac{x^3\sinh^3(a)\cosh(a)}{3} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*cosh(b*x+a)*sinh(b*x+a)**3,x)
```

```
[Out] Piecewise((5*x**2*sinh(a + b*x)**4/(32*b) + 3*x**2*sinh(a + b*x)**2*cosh(a
+ b*x)**2/(16*b) - 3*x**2*cosh(a + b*x)**4/(32*b) - 5*x*sinh(a + b*x)**3*co
sh(a + b*x)/(16*b**2) + 3*x*sinh(a + b*x)*cosh(a + b*x)**3/(16*b**2) + 5*si
nh(a + b*x)**4/(64*b**3) - 3*cosh(a + b*x)**4/(64*b**3), Ne(b, 0)), (x**3*s
inh(a)**3*cosh(a)/3, True))
```

3.310 $\int x\cosh(a+bx)\sinh^3(a+bx)\,dx$

Optimal. Leaf size=65

$$-\frac{\sinh^3(a+bx)\cosh(a+bx)}{16b^2}+\frac{3\sinh(a+bx)\cosh(a+bx)}{32b^2}+\frac{x\sinh^4(a+bx)}{4b}-\frac{3x}{32b}$$

```
[Out] -3/32*x/b+3/32*cosh(b*x+a)*sinh(b*x+a)/b^2-1/16*cosh(b*x+a)*sinh(b*x+a)^3/b
^2+1/4*x*sinh(b*x+a)^4/b
```

Rubi [A] time = 0.04, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {5372, 2635, 8}

$$-\frac{\sinh^3(a+bx)\cosh(a+bx)}{16b^2}+\frac{3\sinh(a+bx)\cosh(a+bx)}{32b^2}+\frac{x\sinh^4(a+bx)}{4b}-\frac{3x}{32b}$$

Antiderivative was successfully verified.

```
[In] Int[x*Cosh[a + b*x]*Sinh[a + b*x]^3,x]
```

```
[Out] (-3*x)/(32*b) + (3*Cosh[a + b*x]*Sinh[a + b*x])/(32*b^2) - (Cosh[a + b*x]*S
inh[a + b*x]^3)/(16*b^2) + (x*Sinh[a + b*x]^4)/(4*b)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 5372

```
Int[Cosh[(a_.) + (b_.)*(x_)^(n_.)]*(x_)^(m_.)*Sinh[(a_.) + (b_.)*(x_)^(n_.)
]^(p_.), x_Symbol] :> Simp[(x^(m - n + 1)*Sinh[a + b*x^n]^(p + 1))/(b*n*(p
+ 1)), x] - Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*Sinh[a + b*x^n]^(
p + 1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

Rubi steps

$$\int x\cosh(a+bx)\sinh^3(a+bx)\,dx = \frac{x\sinh^4(a+bx)}{4b} - \frac{\int \sinh^4(a+bx)\,dx}{4b}$$
$$= -\frac{\cosh(a+bx)\sinh^3(a+bx)}{16b^2} + \frac{x\sinh^4(a+bx)}{4b} + \frac{3\int \sinh^2(a+bx)\,dx}{16b}$$
$$= \frac{3\cosh(a+bx)\sinh(a+bx)}{32b^2} - \frac{\cosh(a+bx)\sinh^3(a+bx)}{16b^2} + \frac{x\sinh^4(a+b}{4b}$$
$$= -\frac{3x}{32b} + \frac{3\cosh(a+bx)\sinh(a+bx)}{32b^2} - \frac{\cosh(a+bx)\sinh^3(a+bx)}{16b^2} + \frac{x\sin}{}$$

Mathematica [A] time = 0.16, size = 50, normalized size = 0.77

$$-\frac{-8\sinh(2(a+bx)) + \sinh(4(a+bx)) + 16bx\cosh(2(a+bx)) - 4bx\cosh(4(a+bx))}{128b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Cosh[a + b*x]*Sinh[a + b*x]^3,x]
```

```
[Out] -1/128*(16*b*x*Cosh[2*(a + b*x)] - 4*b*x*Cosh[4*(a + b*x)] - 8*Sinh[2*(a +
b*x)] + Sinh[4*(a + b*x)])/b^2
```

fricas [A] time = 0.53, size = 108, normalized size = 1.66

$$\frac{bx\cosh(bx+a)^4 + bx\sinh(bx+a)^4 - 4\,bx\cosh(bx+a)^2 - \cosh(bx+a)\sinh(bx+a)^3 + 2\left(3\,bx\cosh(bx+a)\right.}{32\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x+a)*sinh(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/32*(b*x*cosh(b*x + a)^4 + b*x*sinh(b*x + a)^4 - 4*b*x*cosh(b*x + a)^2 - c
osh(b*x + a)*sinh(b*x + a)^3 + 2*(3*b*x*cosh(b*x + a)^2 - 2*b*x)*sinh(b*x +
 a)^2 - (cosh(b*x + a)^3 - 4*cosh(b*x + a))*sinh(b*x + a))/b^2
```

giac [A] time = 0.12, size = 81, normalized size = 1.25

$$\frac{(4\,bx-1)e^{(4\,bx+4\,a)}}{256\,b^2} - \frac{(2\,bx-1)e^{(2\,bx+2\,a)}}{32\,b^2} - \frac{(2\,bx+1)e^{(-2\,bx-2\,a)}}{32\,b^2} + \frac{(4\,bx+1)e^{(-4\,bx-4\,a)}}{256\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x+a)*sinh(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/256*(4*b*x - 1)*e^(4*b*x + 4*a)/b^2 - 1/32*(2*b*x - 1)*e^(2*b*x + 2*a)/b^
2 - 1/32*(2*b*x + 1)*e^(-2*b*x - 2*a)/b^2 + 1/256*(4*b*x + 1)*e^(-4*b*x - 4
*a)/b^2
```

maple [A] time = 0.33, size = 69, normalized size = 1.06

$$\frac{\frac{(bx+a)\left(\sinh^4(bx+a)\right)}{4}-\frac{\cosh(bx+a)\left(\sinh^3(bx+a)\right)}{16}+\frac{3\cosh(bx+a)\sinh(bx+a)}{32}-\frac{3bx}{32}-\frac{3a}{32}-\frac{a\left(\sinh^4(bx+a)\right)}{4}}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cosh(b*x+a)*sinh(b*x+a)^3,x)
```

```
[Out] 1/b^2*(1/4*(b*x+a)*sinh(b*x+a)^4-1/16*cosh(b*x+a)*sinh(b*x+a)^3+3/32*cosh(b
*x+a)*sinh(b*x+a)-3/32*b*x-3/32*a-1/4*a*sinh(b*x+a)^4)
```

maxima [A] time = 0.33, size = 91, normalized size = 1.40

$$\frac{\left(4\,bxe^{(4\,a)}-e^{(4\,a)}\right)e^{(4\,bx)}}{256\,b^2}-\frac{\left(2\,bxe^{(2\,a)}-e^{(2\,a)}\right)e^{(2\,bx)}}{32\,b^2}-\frac{(2\,bx+1)e^{(-2\,bx-2\,a)}}{32\,b^2}+\frac{(4\,bx+1)e^{(-4\,bx-4\,a)}}{256\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x+a)*sinh(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/256*(4*b*x*e^(4*a) - e^(4*a))*e^(4*b*x)/b^2 - 1/32*(2*b*x*e^(2*a) - e^(2*
a))*e^(2*b*x)/b^2 - 1/32*(2*b*x + 1)*e^(-2*b*x - 2*a)/b^2 + 1/256*(4*b*x +
1)*e^(-4*b*x - 4*a)/b^2
```

mupad [B] time = 0.15, size = 55, normalized size = 0.85

$$-\frac{\frac{\sinh(4\,a+4\,b\,x)}{128}-\frac{\sinh(2\,a+2\,b\,x)}{16}+b\,\left(\frac{x\cosh(2\,a+2\,b\,x)}{8}-\frac{x\cosh(4\,a+4\,b\,x)}{32}\right)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cosh(a + b*x)*sinh(a + b*x)^3,x)
```

```
[Out] -(sinh(4*a + 4*b*x)/128 - sinh(2*a + 2*b*x)/16 + b*((x*cosh(2*a + 2*b*x))/8
 - (x*cosh(4*a + 4*b*x))/32))/b^2
```

sympy [A] time = 1.68, size = 110, normalized size = 1.69

$$\begin{cases} \frac{5x\sinh^4(a+bx)}{32b}+\frac{3x\sinh^2(a+bx)\cosh^2(a+bx)}{16b}-\frac{3x\cosh^4(a+bx)}{32b}-\frac{5\sinh^3(a+bx)\cosh(a+bx)}{32b^2}+\frac{3\sinh(a+bx)\cosh^3(a+bx)}{32b^2} & \text{for } b\neq 0 \\ \frac{x^2\sinh^3(a)\cosh(a)}{2} & \text{otherwi} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x+a)*sinh(b*x+a)**3,x)
```

```
[Out] Piecewise((5*x*sinh(a + b*x)**4/(32*b) + 3*x*sinh(a + b*x)**2*cosh(a + b*x)
**2/(16*b) - 3*x*cosh(a + b*x)**4/(32*b) - 5*sinh(a + b*x)**3*cosh(a + b*x)
/(32*b**2) + 3*sinh(a + b*x)*cosh(a + b*x)**3/(32*b**2), Ne(b, 0)), (x**2*s
inh(a)**3*cosh(a)/2, True))
```

3.311 $\int \cosh(a+bx)\sinh^3(a+bx)\,dx$

Optimal. Leaf size=15

$$\frac{\sinh^4(a+bx)}{4b}$$

```
[Out] 1/4*sinh(b*x+a)^4/b
```

Rubi [A] time = 0.02, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2564, 30}

$$\frac{\sinh^4(a+bx)}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[a + b*x]*Sinh[a + b*x]^3,x]
```

```
[Out] Sinh[a + b*x]^4/(4*b)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rubi steps

$$\begin{aligned}\int \cosh(a+bx)\sinh^3(a+bx)\,dx &= \frac{\text{Subst}\left(\int x^3\,dx, x, i\sinh(a+bx)\right)}{b}\\ &= \frac{\sinh^4(a+bx)}{4b}\end{aligned}$$

Mathematica [A] time = 0.00, size = 15, normalized size = 1.00

$$\frac{\sinh^4(a+bx)}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[a + b*x]*Sinh[a + b*x]^3,x]
```

```
[Out] Sinh[a + b*x]^4/(4*b)
```

fricas [B] time = 0.59, size = 54, normalized size = 3.60

$$\frac{\cosh\left(bx+a\right)^{4}+\sinh\left(bx+a\right)^{4}+2\left(3\cosh\left(bx+a\right)^{2}-2\right)\sinh\left(bx+a\right)^{2}-4\cosh\left(bx+a\right)^{2}}{32\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*sinh(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/32*(cosh(b*x + a)^4 + sinh(b*x + a)^4 + 2*(3*cosh(b*x + a)^2 - 2)*sinh(b*
x + a)^2 - 4*cosh(b*x + a)^2)/b
```

giac [B] time = 0.15, size = 57, normalized size = 3.80

$$\frac{e^{\left(4\,bx+4\,a\right)}}{64\,b}-\frac{e^{\left(2\,bx+2\,a\right)}}{16\,b}-\frac{e^{\left(-2\,bx-2\,a\right)}}{16\,b}+\frac{e^{\left(-4\,bx-4\,a\right)}}{64\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*sinh(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/64*e^(4*b*x + 4*a)/b - 1/16*e^(2*b*x + 2*a)/b - 1/16*e^(-2*b*x - 2*a)/b +
 1/64*e^(-4*b*x - 4*a)/b
```

maple [A] time = 0.06, size = 14, normalized size = 0.93

$$\frac{\sinh^{4}\left(bx+a\right)}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)*sinh(b*x+a)^3,x)
```

```
[Out] 1/4*sinh(b*x+a)^4/b
```

maxima [A] time = 0.32, size = 13, normalized size = 0.87

$$\frac{\sinh\left(bx+a\right)^{4}}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*sinh(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/4*sinh(b*x + a)^4/b
```

mupad [B] time = 0.07, size = 13, normalized size = 0.87

$$\frac{\sinh\left(a+b\,x\right)^4}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a + b*x)*sinh(a + b*x)^3,x)
```

```
[Out] sinh(a + b*x)^4/(4*b)
```

sympy [A] time = 0.73, size = 20, normalized size = 1.33

$$\begin{cases} \frac{\sinh^{4}{\left(a+bx \right)}}{4b} & \text{for}\: b \neq 0 \\ x \sinh^{3}{\left(a \right)} \cosh{\left(a \right)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*sinh(b*x+a)**3,x)
```

```
[Out] Piecewise((sinh(a + b*x)**4/(4*b), Ne(b, 0)), (x*sinh(a)**3*cosh(a), True))
```

3.312 $\int \frac{\cosh(a+bx)\sinh^3(a+bx)}{x}\,dx$

Optimal. Leaf size=53

$$-\frac{1}{4}\sinh(2a)\text{Chi}(2bx)+\frac{1}{8}\sinh(4a)\text{Chi}(4bx)-\frac{1}{4}\cosh(2a)\text{Shi}(2bx)+\frac{1}{8}\cosh(4a)\text{Shi}(4bx)$$

```
[Out] -1/4*cosh(2*a)*Shi(2*b*x)+1/8*cosh(4*a)*Shi(4*b*x)-1/4*Chi(2*b*x)*sinh(2*a)
+1/8*Chi(4*b*x)*sinh(4*a)
```

Rubi [A] time = 0.13, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {5448, 3303, 3298, 3301}

$$-\frac{1}{4}\sinh(2a)\text{Chi}(2bx)+\frac{1}{8}\sinh(4a)\text{Chi}(4bx)-\frac{1}{4}\cosh(2a)\text{Shi}(2bx)+\frac{1}{8}\cosh(4a)\text{Shi}(4bx)$$

Antiderivative was successfully verified.

```
[In] Int[(Cosh[a + b*x]*Sinh[a + b*x]^3)/x,x]
```

```
[Out] -(CoshIntegral[2*b*x]*Sinh[2*a])/4 + (CoshIntegral[4*b*x]*Sinh[4*a])/8 - (C
osh[2*a]*SinhIntegral[2*b*x])/4 + (Cosh[4*a]*SinhIntegral[4*b*x])/8
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\cosh(a+bx)\sinh^3(a+bx)}{x}\,dx &= \int \left(-\frac{\sinh(2a+2bx)}{4x}+\frac{\sinh(4a+4bx)}{8x}\right)dx\\ &= \frac{1}{8}\int \frac{\sinh(4a+4bx)}{x}\,dx-\frac{1}{4}\int \frac{\sinh(2a+2bx)}{x}\,dx\\ &= -\left(\frac{1}{4}\cosh(2a)\int \frac{\sinh(2bx)}{x}\,dx\right)+\frac{1}{8}\cosh(4a)\int \frac{\sinh(4bx)}{x}\,dx-\frac{1}{4}\sinh(2a\\ &= -\frac{1}{4}\text{Chi}(2bx)\sinh(2a)+\frac{1}{8}\text{Chi}(4bx)\sinh(4a)-\frac{1}{4}\cosh(2a)\text{Shi}(2bx)+\frac{1}{8}\text{co}\end{aligned}$$

Mathematica [A] time = 0.08, size = 47, normalized size = 0.89

$$\frac{1}{8}(-2\sinh(2a)\text{Chi}(2bx)+\sinh(4a)\text{Chi}(4bx)-2\cosh(2a)\text{Shi}(2bx)+\cosh(4a)\text{Shi}(4bx))$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cosh[a + b*x]*Sinh[a + b*x]^3)/x,x]
```

```
[Out] (-2*CoshIntegral[2*b*x]*Sinh[2*a] + CoshIntegral[4*b*x]*Sinh[4*a] - 2*Cosh[
2*a]*SinhIntegral[2*b*x] + Cosh[4*a]*SinhIntegral[4*b*x])/8
```

fricas [A] time = 0.61, size = 73, normalized size = 1.38

$$\frac{1}{16}\left(\text{Ei}\left(4\,bx\right)-\text{Ei}\left(-4\,bx\right)\right)\cosh\left(4\,a\right)-\frac{1}{8}\left(\text{Ei}\left(2\,bx\right)-\text{Ei}\left(-2\,bx\right)\right)\cosh\left(2\,a\right)+\frac{1}{16}\left(\text{Ei}\left(4\,bx\right)+\text{Ei}\left(-4\,bx\right)\right)\sinh\left(4\,a\right)-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*sinh(b*x+a)^3/x,x, algorithm="fricas")
```

```
[Out] 1/16*(Ei(4*b*x) - Ei(-4*b*x))*cosh(4*a) - 1/8*(Ei(2*b*x) - Ei(-2*b*x))*cosh
(2*a) + 1/16*(Ei(4*b*x) + Ei(-4*b*x))*sinh(4*a) - 1/8*(Ei(2*b*x) + Ei(-2*b*
x))*sinh(2*a)
```

giac [A] time = 0.12, size = 45, normalized size = 0.85

$$\frac{1}{16}\,\text{Ei}\left(4\,bx\right)e^{(4\,a)}-\frac{1}{8}\,\text{Ei}\left(2\,bx\right)e^{(2\,a)}+\frac{1}{8}\,\text{Ei}\left(-2\,bx\right)e^{(-2\,a)}-\frac{1}{16}\,\text{Ei}\left(-4\,bx\right)e^{(-4\,a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*sinh(b*x+a)^3/x,x, algorithm="giac")
```

```
[Out] 1/16*Ei(4*b*x)*e^(4*a) - 1/8*Ei(2*b*x)*e^(2*a) + 1/8*Ei(-2*b*x)*e^(-2*a) -
1/16*Ei(-4*b*x)*e^(-4*a)
```

maple [A] time = 0.61, size = 50, normalized size = 0.94

$$\frac{e^{-4a}\,\mathrm{Ei}\,(1,4bx)}{16}-\frac{e^{-2a}\,\mathrm{Ei}\,(1,2bx)}{8}+\frac{e^{2a}\,\mathrm{Ei}\,(1,-2bx)}{8}-\frac{e^{4a}\,\mathrm{Ei}\,(1,-4bx)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)*sinh(b*x+a)^3/x,x)
```

```
[Out] 1/16*exp(-4*a)*Ei(1,4*b*x)-1/8*exp(-2*a)*Ei(1,2*b*x)+1/8*exp(2*a)*Ei(1,-2*b
*x)-1/16*exp(4*a)*Ei(1,-4*b*x)
```

maxima [A] time = 0.43, size = 45, normalized size = 0.85

$$\frac{1}{16}\,\mathrm{Ei}\,(4\,bx)\,e^{(4\,a)}-\frac{1}{8}\,\mathrm{Ei}\,(2\,bx)\,e^{(2\,a)}+\frac{1}{8}\,\mathrm{Ei}\,(-2\,bx)\,e^{(-2\,a)}-\frac{1}{16}\,\mathrm{Ei}\,(-4\,bx)\,e^{(-4\,a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*sinh(b*x+a)^3/x,x, algorithm="maxima")
```

```
[Out] 1/16*Ei(4*b*x)*e^(4*a) - 1/8*Ei(2*b*x)*e^(2*a) + 1/8*Ei(-2*b*x)*e^(-2*a) -
1/16*Ei(-4*b*x)*e^(-4*a)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\cosh\left(a+b\,x\right)\,{\sinh\left(a+b\,x\right)}^3}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cosh(a + b*x)*sinh(a + b*x)^3)/x,x)
```

```
[Out] int((cosh(a + b*x)*sinh(a + b*x)^3)/x, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh^{3}{\left(a+bx\right)}\cosh{\left(a+bx\right)}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*sinh(b*x+a)**3/x,x)
```

```
[Out] Integral(sinh(a + b*x)**3*cosh(a + b*x)/x, x)
```

3.313 $\int \frac{\cosh(a+bx)\sinh^3(a+bx)}{x^2}\,dx$

Optimal. Leaf size=89

$$-\frac{1}{2}b\cosh(2a)\text{Chi}(2bx)+\frac{1}{2}b\cosh(4a)\text{Chi}(4bx)-\frac{1}{2}b\sinh(2a)\text{Shi}(2bx)+\frac{1}{2}b\sinh(4a)\text{Shi}(4bx)+\frac{\sinh(2a+2bx)}{4x}-\frac{\text{s}}{}$$

```
[Out] -1/2*b*Chi(2*b*x)*cosh(2*a)+1/2*b*Chi(4*b*x)*cosh(4*a)-1/2*b*Shi(2*b*x)*sin
h(2*a)+1/2*b*Shi(4*b*x)*sinh(4*a)+1/4*sinh(2*b*x+2*a)/x-1/8*sinh(4*b*x+4*a)
/x
```

Rubi [A] time = 0.18, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {5448, 3297, 3303, 3298, 3301}

$$-\frac{1}{2}b\cosh(2a)\text{Chi}(2bx)+\frac{1}{2}b\cosh(4a)\text{Chi}(4bx)-\frac{1}{2}b\sinh(2a)\text{Shi}(2bx)+\frac{1}{2}b\sinh(4a)\text{Shi}(4bx)+\frac{\sinh(2a+2bx)}{4x}-\frac{\text{s}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Cosh[a + b*x]*Sinh[a + b*x]^3)/x^2,x]
```

```
[Out] -(b*Cosh[2*a]*CoshIntegral[2*b*x])/2 + (b*Cosh[4*a]*CoshIntegral[4*b*x])/2
+ Sinh[2*a + 2*b*x]/(4*x) - Sinh[4*a + 4*b*x]/(8*x) - (b*Sinh[2*a]*SinhInte
gral[2*b*x])/2 + (b*Sinh[4*a]*SinhIntegral[4*b*x])/2
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\cosh(a+bx)\sinh^3(a+bx)}{x^2}\,dx &= \int \left(-\frac{\sinh(2a+2bx)}{4x^2}+\frac{\sinh(4a+4bx)}{8x^2}\right)dx\\ &= \frac{1}{8}\int \frac{\sinh(4a+4bx)}{x^2}\,dx-\frac{1}{4}\int \frac{\sinh(2a+2bx)}{x^2}\,dx\\ &= \frac{\sinh(2a+2bx)}{4x}-\frac{\sinh(4a+4bx)}{8x}-\frac{1}{2}b\int \frac{\cosh(2a+2bx)}{x}\,dx+\frac{1}{2}b\int \frac{\cosh(\\ &= \frac{\sinh(2a+2bx)}{4x}-\frac{\sinh(4a+4bx)}{8x}-\frac{1}{2}(b\cosh(2a))\int \frac{\cosh(2bx)}{x}\,dx+\frac{1}{2}(b\cosh\\ &= -\frac{1}{2}b\cosh(2a)\text{Chi}(2bx)+\frac{1}{2}b\cosh(4a)\text{Chi}(4bx)+\frac{\sinh(2a+2bx)}{4x}-\frac{\sinh(4a}{8}\end{aligned}$$

Mathematica [A] time = 0.25, size = 78, normalized size = 0.88

$$-\frac{4bx\cosh(2a)\text{Chi}(2bx)-4bx\cosh(4a)\text{Chi}(4bx)+4bx\sinh(2a)\text{Shi}(2bx)-4bx\sinh(4a)\text{Shi}(4bx)-2\sinh(2(a+}{8x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cosh[a + b*x]*Sinh[a + b*x]^3)/x^2,x]
```

```
[Out] -1/8*(4*b*x*Cosh[2*a]*CoshIntegral[2*b*x] - 4*b*x*Cosh[4*a]*CoshIntegral[4*
b*x] - 2*Sinh[2*(a + b*x)] + Sinh[4*(a + b*x)] + 4*b*x*Sinh[2*a]*SinhIntegr
al[2*b*x] - 4*b*x*Sinh[4*a]*SinhIntegral[4*b*x])/x
```

fricas [A] time = 0.65, size = 139, normalized size = 1.56

$$-\frac{2\cosh(bx+a)\sinh(bx+a)^3-(bx\text{Ei}(4bx)+bx\text{Ei}(-4bx))\cosh(4a)+(bx\text{Ei}(2bx)+bx\text{Ei}(-2bx))\cosh(2a)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*sinh(b*x+a)^3/x^2,x, algorithm="fricas")
```

```
[Out] -1/4*(2*cosh(b*x + a)*sinh(b*x + a)^3 - (b*x*Ei(4*b*x) + b*x*Ei(-4*b*x))*co
sh(4*a) + (b*x*Ei(2*b*x) + b*x*Ei(-2*b*x))*cosh(2*a) + 2*(cosh(b*x + a)^3 -
 cosh(b*x + a))*sinh(b*x + a) - (b*x*Ei(4*b*x) - b*x*Ei(-4*b*x))*sinh(4*a)
+ (b*x*Ei(2*b*x) - b*x*Ei(-2*b*x))*sinh(2*a))/x
```

giac [A] time = 0.15, size = 100, normalized size = 1.12

$$\frac{4\,bx\mathrm{Ei}\left(4\,bx\right)e^{(4\,a)} - 4\,bx\mathrm{Ei}\left(2\,bx\right)e^{(2\,a)} - 4\,bx\mathrm{Ei}\left(-2\,bx\right)e^{(-2\,a)} + 4\,bx\mathrm{Ei}\left(-4\,bx\right)e^{(-4\,a)} - e^{(4\,bx+4\,a)} + 2\,e^{(2\,bx+2\,a)} - 2\,e}{16\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*sinh(b*x+a)^3/x^2,x, algorithm="giac")
```

```
[Out] 1/16*(4*b*x*Ei(4*b*x)*e^(4*a) - 4*b*x*Ei(2*b*x)*e^(2*a) - 4*b*x*Ei(-2*b*x)*
e^(-2*a) + 4*b*x*Ei(-4*b*x)*e^(-4*a) - e^(4*b*x + 4*a) + 2*e^(2*b*x + 2*a)
- 2*e^(-2*b*x - 2*a) + e^(-4*b*x - 4*a))/x
```

maple [A] time = 0.62, size = 110, normalized size = 1.24

$$\frac{e^{-4bx-4a}}{16x}-\frac{b\,e^{-4a}\,\mathrm{Ei}\left(1,4bx\right)}{4}-\frac{e^{-2bx-2a}}{8x}+\frac{b\,e^{-2a}\,\mathrm{Ei}\left(1,2bx\right)}{4}+\frac{e^{2bx+2a}}{8x}+\frac{b\,e^{2a}\,\mathrm{Ei}\left(1,-2bx\right)}{4}-\frac{e^{4bx+4a}}{16x}-\frac{b\,e^{4a}\,\mathrm{Ei}\left(1,-4bx\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)*sinh(b*x+a)^3/x^2,x)
```

```
[Out] 1/16*exp(-4*b*x-4*a)/x-1/4*b*exp(-4*a)*Ei(1,4*b*x)-1/8*exp(-2*b*x-2*a)/x+1/
4*b*exp(-2*a)*Ei(1,2*b*x)+1/8*exp(2*b*x+2*a)/x+1/4*b*exp(2*a)*Ei(1,-2*b*x)-
1/16/x*exp(4*b*x+4*a)-1/4*b*exp(4*a)*Ei(1,-4*b*x)
```

maxima [A] time = 0.44, size = 53, normalized size = 0.60

$$\frac{1}{4}\,be^{(-4\,a)}\Gamma\left(-1,4\,bx\right) - \frac{1}{4}\,be^{(-2\,a)}\Gamma\left(-1,2\,bx\right) - \frac{1}{4}\,be^{(2\,a)}\Gamma\left(-1,-2\,bx\right) + \frac{1}{4}\,be^{(4\,a)}\Gamma\left(-1,-4\,bx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*sinh(b*x+a)^3/x^2,x, algorithm="maxima")
```

```
[Out] 1/4*b*e^(-4*a)*gamma(-1, 4*b*x) - 1/4*b*e^(-2*a)*gamma(-1, 2*b*x) - 1/4*b*e
^(2*a)*gamma(-1, -2*b*x) + 1/4*b*e^(4*a)*gamma(-1, -4*b*x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cosh(a + b\,x)\,\sinh(a + b\,x)^3}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cosh(a + b*x)*sinh(a + b*x)^3)/x^2,x)
[Out] int((cosh(a + b*x)*sinh(a + b*x)^3)/x^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh^3(a + bx)\cosh(a + bx)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*sinh(b*x+a)**3/x**2,x)
[Out] Integral(sinh(a + b*x)**3*cosh(a + b*x)/x**2, x)
```

3.314 $\int \frac{\cosh(a+bx)\sinh^3(a+bx)}{x^3}\,dx$

Optimal. Leaf size=125

$$-\frac{1}{2}b^2\sinh(2a)\text{Chi}(2bx)+b^2\sinh(4a)\text{Chi}(4bx)-\frac{1}{2}b^2\cosh(2a)\text{Shi}(2bx)+b^2\cosh(4a)\text{Shi}(4bx)+\frac{\sinh(2a+2bx)}{8x^2}-$$

```
[Out] 1/4*b*cosh(2*b*x+2*a)/x-1/4*b*cosh(4*b*x+4*a)/x-1/2*b^2*cosh(2*a)*Shi(2*b*x
)+b^2*cosh(4*a)*Shi(4*b*x)-1/2*b^2*Chi(2*b*x)*sinh(2*a)+b^2*Chi(4*b*x)*sinh
(4*a)+1/8*sinh(2*b*x+2*a)/x^2-1/16*sinh(4*b*x+4*a)/x^2
```

Rubi [A] time = 0.23, antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {5448, 3297, 3303, 3298, 3301}

$$-\frac{1}{2}b^2\sinh(2a)\text{Chi}(2bx)+b^2\sinh(4a)\text{Chi}(4bx)-\frac{1}{2}b^2\cosh(2a)\text{Shi}(2bx)+b^2\cosh(4a)\text{Shi}(4bx)+\frac{\sinh(2a+2bx)}{8x^2}-$$

Antiderivative was successfully verified.

```
[In] Int[(Cosh[a + b*x]*Sinh[a + b*x]^3)/x^3,x]
```

```
[Out] (b*Cosh[2*a + 2*b*x])/(4*x) - (b*Cosh[4*a + 4*b*x])/(4*x) - (b^2*CoshIntegr
al[2*b*x]*Sinh[2*a])/2 + b^2*CoshIntegral[4*b*x]*Sinh[4*a] + Sinh[2*a + 2*b
*x]/(8*x^2) - Sinh[4*a + 4*b*x]/(16*x^2) - (b^2*Cosh[2*a]*SinhIntegral[2*b*
x])/2 + b^2*Cosh[4*a]*SinhIntegral[4*b*x]
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rubi steps

$$\int \frac{\cosh(a+bx)\sinh^3(a+bx)}{x^3}\,dx = \int \left(-\frac{\sinh(2a+2bx)}{4x^3}+\frac{\sinh(4a+4bx)}{8x^3}\right)dx$$
$$= \frac{1}{8}\int \frac{\sinh(4a+4bx)}{x^3}\,dx - \frac{1}{4}\int \frac{\sinh(2a+2bx)}{x^3}\,dx$$
$$= \frac{\sinh(2a+2bx)}{8x^2} - \frac{\sinh(4a+4bx)}{16x^2} - \frac{1}{4}b\int \frac{\cosh(2a+2bx)}{x^2}\,dx + \frac{1}{4}b\int \frac{\cosh(}{}$$
$$= \frac{b\cosh(2a+2bx)}{4x} - \frac{b\cosh(4a+4bx)}{4x} + \frac{\sinh(2a+2bx)}{8x^2} - \frac{\sinh(4a+4bx)}{16x^2} -$$
$$= \frac{b\cosh(2a+2bx)}{4x} - \frac{b\cosh(4a+4bx)}{4x} + \frac{\sinh(2a+2bx)}{8x^2} - \frac{\sinh(4a+4bx)}{16x^2} -$$
$$= \frac{b\cosh(2a+2bx)}{4x} - \frac{b\cosh(4a+4bx)}{4x} - \frac{1}{2}b^2\text{Chi}(2bx)\sinh(2a) + b^2\text{Chi}(4bx$$

Mathematica [A] time = 0.56, size = 113, normalized size = 0.90

$$b^2\sinh(4a)\text{Chi}(4bx)+b^2\sinh(a)(-\cosh(a))\text{Chi}(2bx)-\frac{1}{2}b^2\cosh(2a)\text{Shi}(2bx)+b^2\cosh(4a)\text{Shi}(4bx)+\frac{\sinh(2(a+b}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cosh[a + b*x]*Sinh[a + b*x]^3)/x^3,x]
```

```
[Out] -(b^2*Cosh[a]*CoshIntegral[2*b*x]*Sinh[a]) + b^2*CoshIntegral[4*b*x]*Sinh[4
*a] + (2*b*x*Cosh[2*(a + b*x)] + Sinh[2*(a + b*x)])/(8*x^2) - (4*b*x*Cosh[4
*(a + b*x)] + Sinh[4*(a + b*x)])/(16*x^2) - (b^2*Cosh[2*a]*SinhIntegral[2*b
*x])/2 + b^2*Cosh[4*a]*SinhIntegral[4*b*x]
```

fricas [B] time = 0.57, size = 229, normalized size = 1.83

$$-\frac{bx\cosh(bx+a)^4 + bx\sinh(bx+a)^4 - bx\cosh(bx+a)^2 + \cosh(bx+a)\sinh(bx+a)^3 + \left(6\,bx\cosh(bx+a)^2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*sinh(b*x+a)^3/x^3,x, algorithm="fricas")
```

```
[Out] -1/4*(b*x*cosh(b*x + a)^4 + b*x*sinh(b*x + a)^4 - b*x*cosh(b*x + a)^2 + cos
h(b*x + a)*sinh(b*x + a)^3 + (6*b*x*cosh(b*x + a)^2 - b*x)*sinh(b*x + a)^2
- 2*(b^2*x^2*Ei(4*b*x) - b^2*x^2*Ei(-4*b*x))*cosh(4*a) + (b^2*x^2*Ei(2*b*x)
 - b^2*x^2*Ei(-2*b*x))*cosh(2*a) + (cosh(b*x + a)^3 - cosh(b*x + a))*sinh(b
*x + a) - 2*(b^2*x^2*Ei(4*b*x) + b^2*x^2*Ei(-4*b*x))*sinh(4*a) + (b^2*x^2*E
i(2*b*x) + b^2*x^2*Ei(-2*b*x))*sinh(2*a))/x^2
```

giac [A] time = 0.12, size = 168, normalized size = 1.34

$$\frac{16\,b^2x^2\mathrm{Ei}(4\,bx)\,e^{(4\,a)} - 8\,b^2x^2\mathrm{Ei}(2\,bx)\,e^{(2\,a)} + 8\,b^2x^2\mathrm{Ei}(-2\,bx)\,e^{(-2\,a)} - 16\,b^2x^2\mathrm{Ei}(-4\,bx)\,e^{(-4\,a)} - 4\,bxe^{(4\,bx+4\,a)} + \cdots}{32\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*sinh(b*x+a)^3/x^3,x, algorithm="giac")
```

```
[Out] 1/32*(16*b^2*x^2*Ei(4*b*x)*e^(4*a) - 8*b^2*x^2*Ei(2*b*x)*e^(2*a) + 8*b^2*x^
2*Ei(-2*b*x)*e^(-2*a) - 16*b^2*x^2*Ei(-4*b*x)*e^(-4*a) - 4*b*x*e^(4*b*x + 4
*a) + 4*b*x*e^(2*b*x + 2*a) + 4*b*x*e^(-2*b*x - 2*a) - 4*b*x*e^(-4*b*x - 4*
a) - e^(4*b*x + 4*a) + 2*e^(2*b*x + 2*a) - 2*e^(-2*b*x - 2*a) + e^(-4*b*x -
 4*a))/x^2
```

maple [A] time = 0.62, size = 178, normalized size = 1.42

$$-\frac{b\,\mathrm{e}^{-4bx-4a}}{8x}+\frac{\mathrm{e}^{-4bx-4a}}{32x^2}+\frac{b^2\mathrm{e}^{-4a}\,\mathrm{Ei}(1,4bx)}{2}+\frac{b\,\mathrm{e}^{-2bx-2a}}{8x}-\frac{\mathrm{e}^{-2bx-2a}}{16x^2}-\frac{b^2\mathrm{e}^{-2a}\,\mathrm{Ei}(1,2bx)}{4}+\frac{\mathrm{e}^{2bx+2a}}{16x^2}+\frac{b\,\mathrm{e}^{2bx+2a}}{8x}+\frac{b^2\mathrm{e}^{2a}\,\mathrm{Ei}\cdots}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)*sinh(b*x+a)^3/x^3,x)
```

```
[Out] -1/8*b*exp(-4*b*x-4*a)/x+1/32*exp(-4*b*x-4*a)/x^2+1/2*b^2*exp(-4*a)*Ei(1,4*
b*x)+1/8*b*exp(-2*b*x-2*a)/x-1/16*exp(-2*b*x-2*a)/x^2-1/4*b^2*exp(-2*a)*Ei(
1,2*b*x)+1/16*exp(2*b*x+2*a)/x^2+1/8*b*exp(2*b*x+2*a)/x+1/4*b^2*exp(2*a)*Ei
(1,-2*b*x)-1/32/x^2*exp(4*b*x+4*a)-1/8*b/x*exp(4*b*x+4*a)-1/2*b^2*exp(4*a)*
Ei(1,-4*b*x)
```

maxima [A] time = 0.45, size = 60, normalized size = 0.48

$$b^2 e^{(-4\,a)}\Gamma\left(-2, 4\,bx\right) - \frac{1}{2}\,b^2 e^{(-2\,a)}\Gamma\left(-2, 2\,bx\right) + \frac{1}{2}\,b^2 e^{(2\,a)}\Gamma\left(-2, -2\,bx\right) - b^2 e^{(4\,a)}\Gamma\left(-2, -4\,bx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*sinh(b*x+a)^3/x^3,x, algorithm="maxima")
```

```
[Out] b^2*e^(-4*a)*gamma(-2, 4*b*x) - 1/2*b^2*e^(-2*a)*gamma(-2, 2*b*x) + 1/2*b^2
*e^(2*a)*gamma(-2, -2*b*x) - b^2*e^(4*a)*gamma(-2, -4*b*x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cosh\left(a+b\,x\right)\,{\sinh\left(a+b\,x\right)}^3}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cosh(a + b*x)*sinh(a + b*x)^3)/x^3,x)
```

```
[Out] int((cosh(a + b*x)*sinh(a + b*x)^3)/x^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh^{3}\left(a + bx\right)\cosh\left(a + bx\right)}{x^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*sinh(b*x+a)**3/x**3,x)
```

```
[Out] Integral(sinh(a + b*x)**3*cosh(a + b*x)/x**3, x)
```

3.315 $\int \frac{\cosh(a+bx)\sinh^3(a+bx)}{x^4}\,dx$

Optimal. Leaf size=169

$$-\frac{1}{3}b^3\cosh(2a)\text{Chi}(2bx)+\frac{4}{3}b^3\cosh(4a)\text{Chi}(4bx)-\frac{1}{3}b^3\sinh(2a)\text{Shi}(2bx)+\frac{4}{3}b^3\sinh(4a)\text{Shi}(4bx)+\frac{b^2\sinh(2a+2}{6x}$$

```
[Out] -1/3*b^3*Chi(2*b*x)*cosh(2*a)+4/3*b^3*Chi(4*b*x)*cosh(4*a)+1/12*b*cosh(2*b*
x+2*a)/x^2-1/12*b*cosh(4*b*x+4*a)/x^2-1/3*b^3*Shi(2*b*x)*sinh(2*a)+4/3*b^3*
Shi(4*b*x)*sinh(4*a)+1/12*sinh(2*b*x+2*a)/x^3+1/6*b^2*sinh(2*b*x+2*a)/x-1/2
4*sinh(4*b*x+4*a)/x^3-1/3*b^2*sinh(4*b*x+4*a)/x
```

Rubi [A] time = 0.29, antiderivative size = 169, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {5448, 3297, 3303, 3298, 3301}

$$-\frac{1}{3}b^3\cosh(2a)\text{Chi}(2bx)+\frac{4}{3}b^3\cosh(4a)\text{Chi}(4bx)-\frac{1}{3}b^3\sinh(2a)\text{Shi}(2bx)+\frac{4}{3}b^3\sinh(4a)\text{Shi}(4bx)+\frac{b^2\sinh(2a+2}{6x}$$

Antiderivative was successfully verified.

```
[In] Int[(Cosh[a + b*x]*Sinh[a + b*x]^3)/x^4,x]
```

```
[Out] (b*Cosh[2*a + 2*b*x])/(12*x^2) - (b*Cosh[4*a + 4*b*x])/(12*x^2) - (b^3*Cosh
[2*a]*CoshIntegral[2*b*x])/3 + (4*b^3*Cosh[4*a]*CoshIntegral[4*b*x])/3 + Si
nh[2*a + 2*b*x]/(12*x^3) + (b^2*Sinh[2*a + 2*b*x])/(6*x) - Sinh[4*a + 4*b*x
]/(24*x^3) - (b^2*Sinh[4*a + 4*b*x])/(3*x) - (b^3*Sinh[2*a]*SinhIntegral[2*
b*x])/3 + (4*b^3*Sinh[4*a]*SinhIntegral[4*b*x])/3
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
```

```
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cosh(a+bx)\sinh^3(a+bx)}{x^4}\,dx &= \int \left(-\frac{\sinh(2a+2bx)}{4x^4}+\frac{\sinh(4a+4bx)}{8x^4}\right)dx \\
&= \frac{1}{8}\int \frac{\sinh(4a+4bx)}{x^4}\,dx-\frac{1}{4}\int \frac{\sinh(2a+2bx)}{x^4}\,dx \\
&= \frac{\sinh(2a+2bx)}{12x^3}-\frac{\sinh(4a+4bx)}{24x^3}-\frac{1}{6}b\int \frac{\cosh(2a+2bx)}{x^3}\,dx+\frac{1}{6}b\int \frac{\cosh
\end{aligned}$$

$$= \frac{b\cosh(2a+2bx)}{12x^2}-\frac{b\cosh(4a+4bx)}{12x^2}+\frac{\sinh(2a+2bx)}{12x^3}-\frac{\sinh(4a+4bx)}{24x^3}-$$

$$= \frac{b\cosh(2a+2bx)}{12x^2}-\frac{b\cosh(4a+4bx)}{12x^2}+\frac{\sinh(2a+2bx)}{12x^3}+\frac{b^2\sinh(2a+2bx}{6x}$$

$$= \frac{b\cosh(2a+2bx)}{12x^2}-\frac{b\cosh(4a+4bx)}{12x^2}+\frac{\sinh(2a+2bx)}{12x^3}+\frac{b^2\sinh(2a+2bx}{6x}$$

$$= \frac{b\cosh(2a+2bx)}{12x^2}-\frac{b\cosh(4a+4bx)}{12x^2}-\frac{1}{3}b^3\cosh(2a)\text{Chi}(2bx)+\frac{4}{3}b^3\cosh($$

Mathematica [A] time = 0.56, size = 150, normalized size = 0.89

$$-\frac{8b^3x^3\cosh(2a)\text{Chi}(2bx)-32b^3x^3\cosh(4a)\text{Chi}(4bx)+8b^3x^3\sinh(2a)\text{Shi}(2bx)-32b^3x^3\sinh(4a)\text{Shi}(4bx)-4b}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cosh[a + b*x]*Sinh[a + b*x]^3)/x^4,x]
```

```
[Out] -1/24*(-2*b*x*Cosh[2*(a + b*x)] + 2*b*x*Cosh[4*(a + b*x)] + 8*b^3*x^3*Cosh[
2*a]*CoshIntegral[2*b*x] - 32*b^3*x^3*Cosh[4*a]*CoshIntegral[4*b*x] - 2*Sin
h[2*(a + b*x)] - 4*b^2*x^2*Sinh[2*(a + b*x)] + Sinh[4*(a + b*x)] + 8*b^2*x^
2*Sinh[4*(a + b*x)] + 8*b^3*x^3*Sinh[2*a]*SinhIntegral[2*b*x] - 32*b^3*x^3*
Sinh[4*a]*SinhIntegral[4*b*x])/x^3
```

fricas [A] time = 0.68, size = 264, normalized size = 1.56

$$-\frac{bx\cosh\left(bx+a\right)^{4}+bx\sinh\left(bx+a\right)^{4}+2\left(8\,b^{2}x^{2}+1\right)\cosh\left(bx+a\right)\sinh\left(bx+a\right)^{3}-bx\cosh\left(bx+a\right)^{2}+\left(6\,bx\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*sinh(b*x+a)^3/x^4,x, algorithm="fricas")
```

```
[Out] -1/12*(b*x*cosh(b*x + a)^4 + b*x*sinh(b*x + a)^4 + 2*(8*b^2*x^2 + 1)*cosh(b
*x + a)*sinh(b*x + a)^3 - b*x*cosh(b*x + a)^2 + (6*b*x*cosh(b*x + a)^2 - b*
x)*sinh(b*x + a)^2 - 8*(b^3*x^3*Ei(4*b*x) + b^3*x^3*Ei(-4*b*x))*cosh(4*a) +
 2*(b^3*x^3*Ei(2*b*x) + b^3*x^3*Ei(-2*b*x))*cosh(2*a) + 2*((8*b^2*x^2 + 1)*
cosh(b*x + a)^3 - (2*b^2*x^2 + 1)*cosh(b*x + a))*sinh(b*x + a) - 8*(b^3*x^3
*Ei(4*b*x) - b^3*x^3*Ei(-4*b*x))*sinh(4*a) + 2*(b^3*x^3*Ei(2*b*x) - b^3*x^3
*Ei(-2*b*x))*sinh(2*a))/x^3
```

giac [A] time = 0.12, size = 236, normalized size = 1.40

$$\frac{32\,b^{3}x^{3}\mathrm{Ei}\left(4\,bx\right)e^{(4\,a)}-8\,b^{3}x^{3}\mathrm{Ei}\left(2\,bx\right)e^{(2\,a)}-8\,b^{3}x^{3}\mathrm{Ei}\left(-2\,bx\right)e^{(-2\,a)}+32\,b^{3}x^{3}\mathrm{Ei}\left(-4\,bx\right)e^{(-4\,a)}-8\,b^{2}x^{2}e^{(4\,bx+4\,a)}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*sinh(b*x+a)^3/x^4,x, algorithm="giac")
```

```
[Out] 1/48*(32*b^3*x^3*Ei(4*b*x)*e^(4*a) - 8*b^3*x^3*Ei(2*b*x)*e^(2*a) - 8*b^3*x^
3*Ei(-2*b*x)*e^(-2*a) + 32*b^3*x^3*Ei(-4*b*x)*e^(-4*a) - 8*b^2*x^2*e^(4*b*x
 + 4*a) + 4*b^2*x^2*e^(2*b*x + 2*a) - 4*b^2*x^2*e^(-2*b*x - 2*a) + 8*b^2*x^
2*e^(-4*b*x - 4*a) - 2*b*x*e^(4*b*x + 4*a) + 2*b*x*e^(2*b*x + 2*a) + 2*b*x*
e^(-2*b*x - 2*a) - 2*b*x*e^(-4*b*x - 4*a) - e^(4*b*x + 4*a) + 2*e^(2*b*x +
2*a) - 2*e^(-2*b*x - 2*a) + e^(-4*b*x - 4*a))/x^3
```

maple [A] time = 0.63, size = 246, normalized size = 1.46

$$\frac{b^{2}\mathrm{e}^{-4bx-4a}}{6x}-\frac{b\,\mathrm{e}^{-4bx-4a}}{24x^{2}}+\frac{\mathrm{e}^{-4bx-4a}}{48x^{3}}-\frac{2b^{3}\mathrm{e}^{-4a}\,\mathrm{Ei}\left(1,4bx\right)}{3}-\frac{b^{2}\mathrm{e}^{-2bx-2a}}{12x}+\frac{b\,\mathrm{e}^{-2bx-2a}}{24x^{2}}-\frac{\mathrm{e}^{-2bx-2a}}{24x^{3}}+\frac{b^{3}\mathrm{e}^{-2a}\,\mathrm{Ei}\left(1,2bx\right)}{6}+\frac{\mathrm{e}^{2bx}}{24x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)*sinh(b*x+a)^3/x^4,x)
```

```
[Out] 1/6*b^2*exp(-4*b*x-4*a)/x-1/24*b*exp(-4*b*x-4*a)/x^2+1/48*exp(-4*b*x-4*a)/x
^3-2/3*b^3*exp(-4*a)*Ei(1,4*b*x)-1/12*b^2*exp(-2*b*x-2*a)/x+1/24*b*exp(-2*b
*x-2*a)/x^2-1/24*exp(-2*b*x-2*a)/x^3+1/6*b^3*exp(-2*a)*Ei(1,2*b*x)+1/24*exp
(2*b*x+2*a)/x^3+1/24*b*exp(2*b*x+2*a)/x^2+1/12*b^2*exp(2*b*x+2*a)/x+1/6*b^3
*exp(2*a)*Ei(1,-2*b*x)-1/48/x^3*exp(4*b*x+4*a)-1/24*b/x^2*exp(4*b*x+4*a)-1/
6*b^2/x*exp(4*b*x+4*a)-2/3*b^3*exp(4*a)*Ei(1,-4*b*x)
```

maxima [A] time = 0.44, size = 61, normalized size = 0.36

$$4\,b^{3}e^{(-4\,a)}\Gamma\left(-3,4\,bx\right)-b^{3}e^{(-2\,a)}\Gamma\left(-3,2\,bx\right)-b^{3}e^{(2\,a)}\Gamma\left(-3,-2\,bx\right)+4\,b^{3}e^{(4\,a)}\Gamma\left(-3,-4\,bx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*sinh(b*x+a)^3/x^4,x, algorithm="maxima")
```

```
[Out] 4*b^3*e^(-4*a)*gamma(-3, 4*b*x) - b^3*e^(-2*a)*gamma(-3, 2*b*x) - b^3*e^(2*
a)*gamma(-3, -2*b*x) + 4*b^3*e^(4*a)*gamma(-3, -4*b*x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cosh\left(a+b\,x\right)\,{\sinh\left(a+b\,x\right)}^3}{x^4} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cosh(a + b*x)*sinh(a + b*x)^3)/x^4,x)
```

```
[Out] int((cosh(a + b*x)*sinh(a + b*x)^3)/x^4, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh^{3}{\left(a + bx \right)} \cosh{\left(a + bx \right)}}{x^{4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*sinh(b*x+a)**3/x**4,x)
```

```
[Out] Integral(sinh(a + b*x)**3*cosh(a + b*x)/x**4, x)
```

3.316 $\int x^m \cosh^2(a+bx)\sinh^3(a+bx)\,dx$

Optimal. Leaf size=209

$$\frac{e^{5a}5^{-m-1}x^m(-bx)^{-m}\Gamma(m+1,-5bx)}{32b}-\frac{e^{3a}3^{-m-1}x^m(-bx)^{-m}\Gamma(m+1,-3bx)}{32b}-\frac{e^{a}x^m(-bx)^{-m}\Gamma(m+1,-bx)}{16b}-\frac{e^{-a}x^m(bx$$

```
[Out] 1/32*5^(-1-m)*exp(5*a)*x^m*GAMMA(1+m,-5*b*x)/b/((-b*x)^m)-1/32*3^(-1-m)*exp
(3*a)*x^m*GAMMA(1+m,-3*b*x)/b/((-b*x)^m)-1/16*exp(a)*x^m*GAMMA(1+m,-b*x)/b/
((-b*x)^m)-1/16*x^m*GAMMA(1+m,b*x)/b/exp(a)/((b*x)^m)-1/32*3^(-1-m)*x^m*GAM
MA(1+m,3*b*x)/b/exp(3*a)/((b*x)^m)+1/32*5^(-1-m)*x^m*GAMMA(1+m,5*b*x)/b/exp
(5*a)/((b*x)^m)
```

Rubi [A] time = 0.28, antiderivative size = 209, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {5448, 3308, 2181}

$$\frac{e^{5a}5^{-m-1}x^m(-bx)^{-m}\text{Gamma}(m+1,-5bx)}{32b}-\frac{e^{3a}3^{-m-1}x^m(-bx)^{-m}\text{Gamma}(m+1,-3bx)}{32b}-\frac{e^{a}x^m(-bx)^{-m}\text{Gamma}(}{16b}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*Cosh[a + b*x]^2*Sinh[a + b*x]^3,x]
```

```
[Out] (5^(-1 - m)*E^(5*a)*x^m*Gamma[1 + m, -5*b*x])/(32*b*(-(b*x))^m) - (3^(-1 -
m)*E^(3*a)*x^m*Gamma[1 + m, -3*b*x])/(32*b*(-(b*x))^m) - (E^a*x^m*Gamma[1 +
 m, -(b*x)])/(16*b*(-(b*x))^m) - (x^m*Gamma[1 + m, b*x])/(16*b*E^a*(b*x)^m)
 - (3^(-1 - m)*x^m*Gamma[1 + m, 3*b*x])/(32*b*E^(3*a)*(b*x)^m) + (5^(-1 - m
)*x^m*Gamma[1 + m, 5*b*x])/(32*b*E^(5*a)*(b*x)^m)
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&  !I
ntegerQ[m]
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rubi steps

$$\int x^m \cosh^2(a+bx)\sinh^3(a+bx)\,dx = \int\left(-\frac{1}{8}x^m\sinh(a+bx)-\frac{1}{16}x^m\sinh(3a+3bx)+\frac{1}{16}x^m\sinh(5a+5bx)\right)$$

$$= -\left(\frac{1}{16}\int x^m\sinh(3a+3bx)\,dx\right)+\frac{1}{16}\int x^m\sinh(5a+5bx)\,dx-\frac{1}{8}\int x^m$$

$$= -\left(\frac{1}{32}\int e^{-i(3ia+3ibx)}x^m\,dx\right)+\frac{1}{32}\int e^{i(3ia+3ibx)}x^m\,dx+\frac{1}{32}\int e^{-i(5ia+5ibx)}x^m$$

$$= \frac{5^{-1-m}e^{5a}x^m(-bx)^{-m}\Gamma(1+m,-5bx)}{32b}-\frac{3^{-1-m}e^{3a}x^m(-bx)^{-m}\Gamma(1+m,-3bx)}{32b}$$

Mathematica [A] time = 0.27, size = 174, normalized size = 0.83

$$\frac{e^{-5a}x^m\left(-5e^{2a}3^{-m}\left(-b^2x^2\right)^{-m}\left(e^{6a}(bx)^m\Gamma(m+1,-3bx)+(-bx)^m\Gamma(m+1,3bx)\right)+3\,5^{-m}\left(-b^2x^2\right)^{-m}\left(e^{10a}(bx)^m\Gamma(m\right.\right.}{480b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*Cosh[a + b*x]^2*Sinh[a + b*x]^3,x]
```

```
[Out] (x^m*(-30*E^(4*a)*((E^(2*a)*Gamma[1 + m, -(b*x)])/(-(b*x))^m + Gamma[1 + m,
 b*x]/(b*x)^m) - (5*E^(2*a)*(E^(6*a)*(b*x)^m*Gamma[1 + m, -3*b*x] + (-(b*x)
)^m*Gamma[1 + m, 3*b*x]))/(3^m*(-(b^2*x^2))^m) + (3*(E^(10*a)*(b*x)^m*Gamma
[1 + m, -5*b*x] + (-(b*x))^m*Gamma[1 + m, 5*b*x]))/(5^m*(-(b^2*x^2))^m)))/(
480*b*E^(5*a))
```

fricas [A] time = 0.79, size = 248, normalized size = 1.19

$$\frac{3\cosh\left(m\log(5b)+5a\right)\Gamma(m+1,5bx)-5\cosh\left(m\log(3b)+3a\right)\Gamma(m+1,3bx)-30\cosh\left(m\log(b)+a\right)\Gamma(}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*cosh(b*x+a)^2*sinh(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/480*(3*cosh(m*log(5*b) + 5*a)*gamma(m + 1, 5*b*x) - 5*cosh(m*log(3*b) + 3
*a)*gamma(m + 1, 3*b*x) - 30*cosh(m*log(b) + a)*gamma(m + 1, b*x) - 30*cosh
```

```
(m*log(-b) - a)*gamma(m + 1, -b*x) - 5*cosh(m*log(-3*b) - 3*a)*gamma(m + 1,
 -3*b*x) + 3*cosh(m*log(-5*b) - 5*a)*gamma(m + 1, -5*b*x) - 3*gamma(m + 1,
5*b*x)*sinh(m*log(5*b) + 5*a) + 5*gamma(m + 1, 3*b*x)*sinh(m*log(3*b) + 3*a
) + 30*gamma(m + 1, -b*x)*sinh(m*log(-b) - a) + 5*gamma(m + 1, -3*b*x)*sinh
(m*log(-3*b) - 3*a) - 3*gamma(m + 1, -5*b*x)*sinh(m*log(-5*b) - 5*a) + 30*g
amma(m + 1, b*x)*sinh(m*log(b) + a))/b
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^m \cosh(bx+a)^2 \sinh(bx+a)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*cosh(b*x+a)^2*sinh(b*x+a)^3,x, algorithm="giac")
```

```
[Out] integrate(x^m*cosh(b*x + a)^2*sinh(b*x + a)^3, x)
```

maple [F] time = 0.29, size = 0, normalized size = 0.00

$$\int x^m \left(\cosh^2(bx+a)\right)\left(\sinh^3(bx+a)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*cosh(b*x+a)^2*sinh(b*x+a)^3,x)
```

```
[Out] int(x^m*cosh(b*x+a)^2*sinh(b*x+a)^3,x)
```

maxima [A] time = 0.50, size = 171, normalized size = 0.82

$$\frac{1}{32}(5bx)^{-m-1}x^{m+1}e^{(-5a)}\Gamma(m+1,5bx)-\frac{1}{32}(3bx)^{-m-1}x^{m+1}e^{(-3a)}\Gamma(m+1,3bx)-\frac{1}{16}(bx)^{-m-1}x^{m+1}e^{(-a)}\Gamma(m+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*cosh(b*x+a)^2*sinh(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/32*(5*b*x)^(-m - 1)*x^(m + 1)*e^(-5*a)*gamma(m + 1, 5*b*x) - 1/32*(3*b*x)
^(-m - 1)*x^(m + 1)*e^(-3*a)*gamma(m + 1, 3*b*x) - 1/16*(b*x)^(-m - 1)*x^(m
 + 1)*e^(-a)*gamma(m + 1, b*x) + 1/16*(-b*x)^(-m - 1)*x^(m + 1)*e^a*gamma(m
 + 1, -b*x) + 1/32*(-3*b*x)^(-m - 1)*x^(m + 1)*e^(3*a)*gamma(m + 1, -3*b*x)
 - 1/32*(-5*b*x)^(-m - 1)*x^(m + 1)*e^(5*a)*gamma(m + 1, -5*b*x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^m \cosh(a+b\,x)^2 \sinh(a+b\,x)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*cosh(a + b*x)^2*sinh(a + b*x)^3,x)
```

```
[Out] int(x^m*cosh(a + b*x)^2*sinh(a + b*x)^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^m \sinh^3(a+bx)\cosh^2(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*cosh(b*x+a)**2*sinh(b*x+a)**3,x)
```

```
[Out] Integral(x**m*sinh(a + b*x)**3*cosh(a + b*x)**2, x)
```

3.317 $\int x^3 \cosh^2(a+bx)\sinh^3(a+bx)\,dx$

Optimal. Leaf size=202

$$\frac{3\sinh(a+bx)}{4b^4}+\frac{\sinh(3a+3bx)}{216b^4}-\frac{3\sinh(5a+5bx)}{5000b^4}-\frac{3x\cosh(a+bx)}{4b^3}-\frac{x\cosh(3a+3bx)}{72b^3}+\frac{3x\cosh(5a+5bx)}{1000b^3}+$$

```
[Out] -3/4*x*cosh(b*x+a)/b^3-1/8*x^3*cosh(b*x+a)/b-1/72*x*cosh(3*b*x+3*a)/b^3-1/4
8*x^3*cosh(3*b*x+3*a)/b+3/1000*x*cosh(5*b*x+5*a)/b^3+1/80*x^3*cosh(5*b*x+5*
a)/b+3/4*sinh(b*x+a)/b^4+3/8*x^2*sinh(b*x+a)/b^2+1/216*sinh(3*b*x+3*a)/b^4+
1/48*x^2*sinh(3*b*x+3*a)/b^2-3/5000*sinh(5*b*x+5*a)/b^4-3/400*x^2*sinh(5*b*
x+5*a)/b^2
```

Rubi [A] time = 0.26, antiderivative size = 202, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {5448, 3296, 2637}

$$\frac{3x^2\sinh(a+bx)}{8b^2}+\frac{x^2\sinh(3a+3bx)}{48b^2}-\frac{3x^2\sinh(5a+5bx)}{400b^2}+\frac{3\sinh(a+bx)}{4b^4}+\frac{\sinh(3a+3bx)}{216b^4}-\frac{3\sinh(5a+5bx)}{5000b^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Cosh[a + b*x]^2*Sinh[a + b*x]^3,x]
```

```
[Out] (-3*x*Cosh[a + b*x])/(4*b^3) - (x^3*Cosh[a + b*x])/(8*b) - (x*Cosh[3*a + 3*
b*x])/(72*b^3) - (x^3*Cosh[3*a + 3*b*x])/(48*b) + (3*x*Cosh[5*a + 5*b*x])/(
1000*b^3) + (x^3*Cosh[5*a + 5*b*x])/(80*b) + (3*Sinh[a + b*x])/(4*b^4) + (3
*x^2*Sinh[a + b*x])/(8*b^2) + Sinh[3*a + 3*b*x]/(216*b^4) + (x^2*Sinh[3*a +
 3*b*x])/(48*b^2) - (3*Sinh[5*a + 5*b*x])/(5000*b^4) - (3*x^2*Sinh[5*a + 5*
b*x])/(400*b^2)
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
```

```
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rubi steps

$$\int x^3\cosh^2(a+bx)\sinh^3(a+bx)\,dx = \int\left(-\frac{1}{8}x^3\sinh(a+bx)-\frac{1}{16}x^3\sinh(3a+3bx)+\frac{1}{16}x^3\sinh(5a+5bx)\right)dx$$
$$= -\left(\frac{1}{16}\int x^3\sinh(3a+3bx)\,dx\right)+\frac{1}{16}\int x^3\sinh(5a+5bx)\,dx-\frac{1}{8}\int x^3\sinh$$
$$= -\frac{x^3\cosh(a+bx)}{8b}-\frac{x^3\cosh(3a+3bx)}{48b}+\frac{x^3\cosh(5a+5bx)}{80b}-\frac{3\int x^2\cos}{}$$
$$= -\frac{x^3\cosh(a+bx)}{8b}-\frac{x^3\cosh(3a+3bx)}{48b}+\frac{x^3\cosh(5a+5bx)}{80b}+\frac{3x^2\sinh(}{8b^2}$$
$$= -\frac{3x\cosh(a+bx)}{4b^3}-\frac{x^3\cosh(a+bx)}{8b}-\frac{x\cosh(3a+3bx)}{72b^3}-\frac{x^3\cosh(3a+}{48b}$$
$$= -\frac{3x\cosh(a+bx)}{4b^3}-\frac{x^3\cosh(a+bx)}{8b}-\frac{x\cosh(3a+3bx)}{72b^3}-\frac{x^3\cosh(3a+}{48b}$$

Mathematica [A] time = 0.51, size = 136, normalized size = 0.67

$$\frac{-33750\left(bx\left(b^2x^2+6\right)\cosh(a+bx)-3\left(b^2x^2+2\right)\sinh(a+bx)\right)+27\left(5bx\left(25b^2x^2+6\right)\cosh(5(a+bx))-3\left(25\right.\right.}{270000b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Cosh[a + b*x]^2*Sinh[a + b*x]^3,x]
```

```
[Out] (-33750*(b*x*(6 + b^2*x^2)*Cosh[a + b*x] - 3*(2 + b^2*x^2)*Sinh[a + b*x]) -
 625*((6*b*x + 9*b^3*x^3)*Cosh[3*(a + b*x)] - (2 + 9*b^2*x^2)*Sinh[3*(a + b
*x)]) + 27*(5*b*x*(6 + 25*b^2*x^2)*Cosh[5*(a + b*x)] - 3*(2 + 25*b^2*x^2)*S
inh[5*(a + b*x)]))/(270000*b^4)
```

fricas [A] time = 1.55, size = 274, normalized size = 1.36

$$\frac{135\left(25\,b^3x^3+6\,bx\right)\cosh\left(bx+a\right)^5+675\left(25\,b^3x^3+6\,bx\right)\cosh\left(bx+a\right)\sinh\left(bx+a\right)^4-81\left(25\,b^2x^2+2\right)\sinh\left(b\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*cosh(b*x+a)^2*sinh(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/270000*(135*(25*b^3*x^3 + 6*b*x)*cosh(b*x + a)^5 + 675*(25*b^3*x^3 + 6*b*
x)*cosh(b*x + a)*sinh(b*x + a)^4 - 81*(25*b^2*x^2 + 2)*sinh(b*x + a)^5 - 18
75*(3*b^3*x^3 + 2*b*x)*cosh(b*x + a)^3 + 5*(1125*b^2*x^2 - 162*(25*b^2*x^2
+ 2)*cosh(b*x + a)^2 + 250)*sinh(b*x + a)^3 + 225*(6*(25*b^3*x^3 + 6*b*x)*c
osh(b*x + a)^3 - 25*(3*b^3*x^3 + 2*b*x)*cosh(b*x + a))*sinh(b*x + a)^2 - 33
750*(b^3*x^3 + 6*b*x)*cosh(b*x + a) - 15*(27*(25*b^2*x^2 + 2)*cosh(b*x + a)
^4 - 6750*b^2*x^2 - 125*(9*b^2*x^2 + 2)*cosh(b*x + a)^2 - 13500)*sinh(b*x +
 a))/b^4
```

giac [A] time = 0.14, size = 212, normalized size = 1.05

$$\frac{\left(125\,b^3x^3-75\,b^2x^2+30\,bx-6\right)e^{(5\,bx+5\,a)}}{20000\,b^4}-\frac{\left(9\,b^3x^3-9\,b^2x^2+6\,bx-2\right)e^{(3\,bx+3\,a)}}{864\,b^4}-\frac{\left(b^3x^3-3\,b^2x^2+6\,bx-6\right)e^{(bx+}}{16\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*cosh(b*x+a)^2*sinh(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/20000*(125*b^3*x^3 - 75*b^2*x^2 + 30*b*x - 6)*e^(5*b*x + 5*a)/b^4 - 1/864
*(9*b^3*x^3 - 9*b^2*x^2 + 6*b*x - 2)*e^(3*b*x + 3*a)/b^4 - 1/16*(b^3*x^3 -
3*b^2*x^2 + 6*b*x - 6)*e^(b*x + a)/b^4 - 1/16*(b^3*x^3 + 3*b^2*x^2 + 6*b*x
+ 6)*e^(-b*x - a)/b^4 - 1/864*(9*b^3*x^3 + 9*b^2*x^2 + 6*b*x + 2)*e^(-3*b*x
 - 3*a)/b^4 + 1/20000*(125*b^3*x^3 + 75*b^2*x^2 + 30*b*x + 6)*e^(-5*b*x - 5
*a)/b^4
```

maple [B] time = 0.41, size = 439, normalized size = 2.17

$$\frac{\frac{(bx+a)^3\left(\sinh^2(bx+a)\right)\left(\cosh^3(bx+a)\right)}{5}-\frac{2(bx+a)^3\left(\cosh^3(bx+a)\right)}{15}-\frac{3(bx+a)^2\sinh(bx+a)\left(\cosh^4(bx+a)\right)}{25}+\frac{26(bx+a)^2\sinh(bx+a)}{75}+\frac{13(bx+a)^2\sin}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*cosh(b*x+a)^2*sinh(b*x+a)^3,x)
```

```
[Out] 1/b^4*(1/5*(b*x+a)^3*sinh(b*x+a)^2*cosh(b*x+a)^3-2/15*(b*x+a)^3*cosh(b*x+a)
^3-3/25*(b*x+a)^2*sinh(b*x+a)*cosh(b*x+a)^4+26/75*(b*x+a)^2*sinh(b*x+a)+13/
75*(b*x+a)^2*sinh(b*x+a)*cosh(b*x+a)^2+6/125*(b*x+a)*cosh(b*x+a)^5+12568/16
875*sinh(b*x+a)-6/625*sinh(b*x+a)*cosh(b*x+a)^4+434/16875*cosh(b*x+a)^2*sin
h(b*x+a)-52/75*(b*x+a)*cosh(b*x+a)-26/225*(b*x+a)*cosh(b*x+a)^3-3*a*(1/5*(b
*x+a)^2*sinh(b*x+a)^2*cosh(b*x+a)^3-2/15*(b*x+a)^2*cosh(b*x+a)^3-2/25*(b*x+
a)*sinh(b*x+a)*cosh(b*x+a)^4+52/225*(b*x+a)*sinh(b*x+a)+26/225*(b*x+a)*sinh
(b*x+a)*cosh(b*x+a)^2+2/125*cosh(b*x+a)^5-52/225*cosh(b*x+a)-26/675*cosh(b*
x+a)^3)+3*a^2*(1/5*(b*x+a)*sinh(b*x+a)^2*cosh(b*x+a)^3-2/15*(b*x+a)*cosh(b*
x+a)^3-1/25*sinh(b*x+a)*cosh(b*x+a)^4+26/225*sinh(b*x+a)+13/225*cosh(b*x+a)
^2*sinh(b*x+a))-a^3*(1/5*cosh(b*x+a)^3*sinh(b*x+a)^2-2/15*cosh(b*x+a)^3))
```

maxima [A] time = 0.34, size = 245, normalized size = 1.21

$$\frac{\left(125\,b^3x^3e^{(5\,a)} - 75\,b^2x^2e^{(5\,a)} + 30\,bxe^{(5\,a)} - 6\,e^{(5\,a)}\right)e^{(5\,bx)}}{20000\,b^4} - \frac{\left(9\,b^3x^3e^{(3\,a)} - 9\,b^2x^2e^{(3\,a)} + 6\,bxe^{(3\,a)} - 2\,e^{(3\,a)}\right)e^{(3\,bx)}}{864\,b^4} - \frac{\left(b^3\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*cosh(b*x+a)^2*sinh(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/20000*(125*b^3*x^3*e^(5*a) - 75*b^2*x^2*e^(5*a) + 30*b*x*e^(5*a) - 6*e^(5
*a))*e^(5*b*x)/b^4 - 1/864*(9*b^3*x^3*e^(3*a) - 9*b^2*x^2*e^(3*a) + 6*b*x*e
^(3*a) - 2*e^(3*a))*e^(3*b*x)/b^4 - 1/16*(b^3*x^3*e^a - 3*b^2*x^2*e^a + 6*b
*x*e^a - 6*e^a)*e^(b*x)/b^4 - 1/16*(b^3*x^3 + 3*b^2*x^2 + 6*b*x + 6)*e^(-b*
x - a)/b^4 - 1/864*(9*b^3*x^3 + 9*b^2*x^2 + 6*b*x + 2)*e^(-3*b*x - 3*a)/b^4
 + 1/20000*(125*b^3*x^3 + 75*b^2*x^2 + 30*b*x + 6)*e^(-5*b*x - 5*a)/b^4
```

mupad [B] time = 0.27, size = 173, normalized size = 0.86

$$\frac{12568\sinh\left(a+b\,x\right)}{16875\,b^4} - \frac{\frac{x^3\cosh(a+b\,x)^3}{3} - \frac{x^3\cosh(a+b\,x)^5}{5}}{b} - \frac{-\frac{6\,x\cosh(a+b\,x)^5}{125} + \frac{26\,x\cosh(a+b\,x)^3}{225} + \frac{52\,x\cosh(a+b\,x)}{75}}{b^3} + \frac{\frac{26\,x^2\sinh(a+b\,x)}{75}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*cosh(a + b*x)^2*sinh(a + b*x)^3,x)
```

```
[Out] (12568*sinh(a + b*x))/(16875*b^4) - ((x^3*cosh(a + b*x)^3)/3 - (x^3*cosh(a
+ b*x)^5)/5)/b - ((52*x*cosh(a + b*x))/75 + (26*x*cosh(a + b*x)^3)/225 - (6
*x*cosh(a + b*x)^5)/125)/b^3 + ((26*x^2*sinh(a + b*x))/75 + (13*x^2*cosh(a
+ b*x)^2*sinh(a + b*x))/75 - (3*x^2*cosh(a + b*x)^4*sinh(a + b*x))/25)/b^2
+ (434*cosh(a + b*x)^2*sinh(a + b*x))/(16875*b^4) - (6*cosh(a + b*x)^4*sinh
(a + b*x))/(625*b^4)
```

sympy [A] time = 8.51, size = 253, normalized size = 1.25

$$\begin{cases} \frac{x^3\sinh^2(a+bx)\cosh^3(a+bx)}{3b} - \frac{2x^3\cosh^5(a+bx)}{15b} + \frac{26x^2\sinh^5(a+bx)}{75b^2} - \frac{13x^2\sinh^3(a+bx)\cosh^2(a+bx)}{15b^2} + \frac{2x^2\sinh(a+bx)\cosh^4(a+bx)}{5b^2} - 52 \\ \frac{x^4\sinh^3(a)\cosh^2(a)}{4} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*cosh(b*x+a)**2*sinh(b*x+a)**3,x)
```

```
[Out] Piecewise((x**3*sinh(a + b*x)**2*cosh(a + b*x)**3/(3*b) - 2*x**3*cosh(a + b
*x)**5/(15*b) + 26*x**2*sinh(a + b*x)**5/(75*b**2) - 13*x**2*sinh(a + b*x)*
```

```
*3*cosh(a + b*x)**2/(15*b**2) + 2*x**2*sinh(a + b*x)*cosh(a + b*x)**4/(5*b*
*2) - 52*x*sinh(a + b*x)**4*cosh(a + b*x)/(75*b**3) + 338*x*sinh(a + b*x)**
2*cosh(a + b*x)**3/(225*b**3) - 856*x*cosh(a + b*x)**5/(1125*b**3) + 12568*
sinh(a + b*x)**5/(16875*b**4) - 5114*sinh(a + b*x)**3*cosh(a + b*x)**2/(337
5*b**4) + 856*sinh(a + b*x)*cosh(a + b*x)**4/(1125*b**4), Ne(b, 0)), (x**4*
sinh(a)**3*cosh(a)**2/4, True))
```

3.318 $\int x^2 \cosh^2(a+bx)\sinh^3(a+bx)\,dx$

Optimal. Leaf size=148

$$-\frac{\cosh(a+bx)}{4b^3}-\frac{\cosh(3a+3bx)}{216b^3}+\frac{\cosh(5a+5bx)}{1000b^3}+\frac{x\sinh(a+bx)}{4b^2}+\frac{x\sinh(3a+3bx)}{72b^2}-\frac{x\sinh(5a+5bx)}{200b^2}-\frac{x^2\cos}{}$$

```
[Out] -1/4*cosh(b*x+a)/b^3-1/8*x^2*cosh(b*x+a)/b-1/216*cosh(3*b*x+3*a)/b^3-1/48*x
^2*cosh(3*b*x+3*a)/b+1/1000*cosh(5*b*x+5*a)/b^3+1/80*x^2*cosh(5*b*x+5*a)/b+
1/4*x*sinh(b*x+a)/b^2+1/72*x*sinh(3*b*x+3*a)/b^2-1/200*x*sinh(5*b*x+5*a)/b^
2
```

Rubi [A] time = 0.18, antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {5448, 3296, 2638}

$$\frac{x\sinh(a+bx)}{4b^2}+\frac{x\sinh(3a+3bx)}{72b^2}-\frac{x\sinh(5a+5bx)}{200b^2}-\frac{\cosh(a+bx)}{4b^3}-\frac{\cosh(3a+3bx)}{216b^3}+\frac{\cosh(5a+5bx)}{1000b^3}-\frac{x^2\cosh(}{8}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Cosh[a + b*x]^2*Sinh[a + b*x]^3,x]
```

```
[Out] -Cosh[a + b*x]/(4*b^3) - (x^2*Cosh[a + b*x])/(8*b) - Cosh[3*a + 3*b*x]/(216
*b^3) - (x^2*Cosh[3*a + 3*b*x])/(48*b) + Cosh[5*a + 5*b*x]/(1000*b^3) + (x^
2*Cosh[5*a + 5*b*x])/(80*b) + (x*Sinh[a + b*x])/(4*b^2) + (x*Sinh[3*a + 3*b
*x])/(72*b^2) - (x*Sinh[5*a + 5*b*x])/(200*b^2)
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rubi steps

$$\int x^2\cosh^2(a+bx)\sinh^3(a+bx)\,dx = \int\left(-\frac{1}{8}x^2\sinh(a+bx)-\frac{1}{16}x^2\sinh(3a+3bx)+\frac{1}{16}x^2\sinh(5a+5bx)\right)$$
$$= -\left(\frac{1}{16}\int x^2\sinh(3a+3bx)\,dx\right)+\frac{1}{16}\int x^2\sinh(5a+5bx)\,dx-\frac{1}{8}\int x^2$$
$$= -\frac{x^2\cosh(a+bx)}{8b}-\frac{x^2\cosh(3a+3bx)}{48b}+\frac{x^2\cosh(5a+5bx)}{80b}-\frac{\int x\cosh}{}$$
$$= -\frac{x^2\cosh(a+bx)}{8b}-\frac{x^2\cosh(3a+3bx)}{48b}+\frac{x^2\cosh(5a+5bx)}{80b}+\frac{x\sinh(a}{4b^2}$$
$$= -\frac{\cosh(a+bx)}{4b^3}-\frac{x^2\cosh(a+bx)}{8b}-\frac{\cosh(3a+3bx)}{216b^3}-\frac{x^2\cosh(3a+3b}{48b}$$

Mathematica [A] time = 0.46, size = 98, normalized size = 0.66

$$\frac{-6750\left(b^2x^2+2\right)\cosh(a+bx)-125\left(9b^2x^2+2\right)\cosh(3(a+bx))+27\left(25b^2x^2+2\right)\cosh(5(a+bx))+30bx(45}{54000b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Cosh[a + b*x]^2*Sinh[a + b*x]^3,x]
```

```
[Out] (-6750*(2 + b^2*x^2)*Cosh[a + b*x] - 125*(2 + 9*b^2*x^2)*Cosh[3*(a + b*x)]
+ 27*(2 + 25*b^2*x^2)*Cosh[5*(a + b*x)] + 30*b*x*(450*Sinh[a + b*x] + 25*Si
nh[3*(a + b*x)] - 9*Sinh[5*(a + b*x)]))/(54000*b^3)
```

fricas [A] time = 0.66, size = 214, normalized size = 1.45

$$-\frac{270\,bx\sinh\left(bx+a\right)^5-27\left(25\,b^2x^2+2\right)\cosh\left(bx+a\right)^5-135\left(25\,b^2x^2+2\right)\cosh\left(bx+a\right)\sinh\left(bx+a\right)^4+125}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cosh(b*x+a)^2*sinh(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -1/54000*(270*b*x*sinh(b*x + a)^5 - 27*(25*b^2*x^2 + 2)*cosh(b*x + a)^5 - 1
35*(25*b^2*x^2 + 2)*cosh(b*x + a)*sinh(b*x + a)^4 + 125*(9*b^2*x^2 + 2)*cos
h(b*x + a)^3 + 150*(18*b*x*cosh(b*x + a)^2 - 5*b*x)*sinh(b*x + a)^3 - 15*(1
8*(25*b^2*x^2 + 2)*cosh(b*x + a)^3 - 25*(9*b^2*x^2 + 2)*cosh(b*x + a))*sinh
(b*x + a)^2 + 6750*(b^2*x^2 + 2)*cosh(b*x + a) + 450*(3*b*x*cosh(b*x + a)^4
 - 5*b*x*cosh(b*x + a)^2 - 30*b*x)*sinh(b*x + a))/b^3
```

giac [A] time = 0.14, size = 164, normalized size = 1.11

$$\frac{\left(25\,b^2x^2-10\,bx+2\right)e^{(5\,bx+5\,a)}}{4000\,b^3}-\frac{\left(9\,b^2x^2-6\,bx+2\right)e^{(3\,bx+3\,a)}}{864\,b^3}-\frac{\left(b^2x^2-2\,bx+2\right)e^{(bx+a)}}{16\,b^3}-\frac{\left(b^2x^2+2\,bx+2\right)e^{(-bx-a)}}{16\,b^3}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cosh(b*x+a)^2*sinh(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/4000*(25*b^2*x^2 - 10*b*x + 2)*e^(5*b*x + 5*a)/b^3 - 1/864*(9*b^2*x^2 - 6
*b*x + 2)*e^(3*b*x + 3*a)/b^3 - 1/16*(b^2*x^2 - 2*b*x + 2)*e^(b*x + a)/b^3
- 1/16*(b^2*x^2 + 2*b*x + 2)*e^(-b*x - a)/b^3 - 1/864*(9*b^2*x^2 + 6*b*x +
2)*e^(-3*b*x - 3*a)/b^3 + 1/4000*(25*b^2*x^2 + 10*b*x + 2)*e^(-5*b*x - 5*a)
/b^3
```

maple [A] time = 0.30, size = 246, normalized size = 1.66

$$\frac{\frac{(bx+a)^2\left(\sinh^2(bx+a)\right)\left(\cosh^3(bx+a)\right)}{5}-\frac{2(bx+a)^2\left(\cosh^3(bx+a)\right)}{15}-\frac{2(bx+a)\sinh(bx+a)\left(\cosh^4(bx+a)\right)}{25}+\frac{52(bx+a)\sinh(bx+a)}{225}+\frac{26(bx+a)\sinh(bx}{2}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*cosh(b*x+a)^2*sinh(b*x+a)^3,x)
```

```
[Out] 1/b^3*(1/5*(b*x+a)^2*sinh(b*x+a)^2*cosh(b*x+a)^3-2/15*(b*x+a)^2*cosh(b*x+a)
^3-2/25*(b*x+a)*sinh(b*x+a)*cosh(b*x+a)^4+52/225*(b*x+a)*sinh(b*x+a)+26/225
*(b*x+a)*sinh(b*x+a)*cosh(b*x+a)^2+2/125*cosh(b*x+a)^5-52/225*cosh(b*x+a)-2
6/675*cosh(b*x+a)^3-2*a*(1/5*(b*x+a)*sinh(b*x+a)^2*cosh(b*x+a)^3-2/15*(b*x+
a)*cosh(b*x+a)^3-1/25*sinh(b*x+a)*cosh(b*x+a)^4+26/225*sinh(b*x+a)+13/225*c
osh(b*x+a)^2*sinh(b*x+a))+a^2*(1/5*cosh(b*x+a)^3*sinh(b*x+a)^2-2/15*cosh(b*
x+a)^3))
```

maxima [A] time = 0.34, size = 187, normalized size = 1.26

$$\frac{\left(25\,b^2x^2e^{(5\,a)}-10\,bxe^{(5\,a)}+2\,e^{(5\,a)}\right)e^{(5\,bx)}}{4000\,b^3}-\frac{\left(9\,b^2x^2e^{(3\,a)}-6\,bxe^{(3\,a)}+2\,e^{(3\,a)}\right)e^{(3\,bx)}}{864\,b^3}-\frac{\left(b^2x^2e^a-2\,bxe^a+2\,e^a\right)e^{(bx)}}{16\,b^3}-\frac{(b}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cosh(b*x+a)^2*sinh(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/4000*(25*b^2*x^2*e^(5*a) - 10*b*x*e^(5*a) + 2*e^(5*a))*e^(5*b*x)/b^3 - 1/
864*(9*b^2*x^2*e^(3*a) - 6*b*x*e^(3*a) + 2*e^(3*a))*e^(3*b*x)/b^3 - 1/16*(b
^2*x^2*e^a - 2*b*x*e^a + 2*e^a)*e^(b*x)/b^3 - 1/16*(b^2*x^2 + 2*b*x + 2)*e^
(-b*x - a)/b^3 - 1/864*(9*b^2*x^2 + 6*b*x + 2)*e^(-3*b*x - 3*a)/b^3 + 1/400
0*(25*b^2*x^2 + 10*b*x + 2)*e^(-5*b*x - 5*a)/b^3
```

mupad [B] time = 0.23, size = 112, normalized size = 0.76

$$-\frac{780\cosh\left(a+b\,x\right)+130\cosh\left(a+b\,x\right)^3-54\cosh\left(a+b\,x\right)^5-780\,b\,x\sinh\left(a+b\,x\right)+1125\,b^2\,x^2\cosh\left(a+b\,x\right.}{3375}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*cosh(a + b*x)^2*sinh(a + b*x)^3,x)
```

```
[Out] -(780*cosh(a + b*x) + 130*cosh(a + b*x)^3 - 54*cosh(a + b*x)^5 - 780*b*x*si
nh(a + b*x) + 1125*b^2*x^2*cosh(a + b*x)^3 - 675*b^2*x^2*cosh(a + b*x)^5 -
390*b*x*cosh(a + b*x)^2*sinh(a + b*x) + 270*b*x*cosh(a + b*x)^4*sinh(a + b*
x))/(3375*b^3)
```

sympy [A] time = 5.06, size = 182, normalized size = 1.23

$$\begin{cases} \frac{x^2\sinh^2(a+bx)\cosh^3(a+bx)}{3b}-\frac{2x^2\cosh^5(a+bx)}{15b}+\frac{52x\sinh^5(a+bx)}{225b^2}-\frac{26x\sinh^3(a+bx)\cosh^2(a+bx)}{45b^2}+\frac{4x\sinh(a+bx)\cosh^4(a+bx)}{15b^2}-\frac{52\,\mathrm{s}}{} \\ \frac{x^3\sinh^3(a)\cosh^2(a)}{3} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*cosh(b*x+a)**2*sinh(b*x+a)**3,x)
```

```
[Out] Piecewise((x**2*sinh(a + b*x)**2*cosh(a + b*x)**3/(3*b) - 2*x**2*cosh(a + b
*x)**5/(15*b) + 52*x*sinh(a + b*x)**5/(225*b**2) - 26*x*sinh(a + b*x)**3*co
sh(a + b*x)**2/(45*b**2) + 4*x*sinh(a + b*x)*cosh(a + b*x)**4/(15*b**2) - 5
2*sinh(a + b*x)**4*cosh(a + b*x)/(225*b**3) + 338*sinh(a + b*x)**2*cosh(a +
 b*x)**3/(675*b**3) - 856*cosh(a + b*x)**5/(3375*b**3), Ne(b, 0)), (x**3*si
nh(a)**3*cosh(a)**2/3, True))
```

3.319 $\int x\cosh^2(a+bx)\sinh^3(a+bx)\,dx$

Optimal. Leaf size=94

$$\frac{\sinh(a+bx)}{8b^2}+\frac{\sinh(3a+3bx)}{144b^2}-\frac{\sinh(5a+5bx)}{400b^2}-\frac{x\cosh(a+bx)}{8b}-\frac{x\cosh(3a+3bx)}{48b}+\frac{x\cosh(5a+5bx)}{80b}$$

```
[Out] -1/8*x*cosh(b*x+a)/b-1/48*x*cosh(3*b*x+3*a)/b+1/80*x*cosh(5*b*x+5*a)/b+1/8*
sinh(b*x+a)/b^2+1/144*sinh(3*b*x+3*a)/b^2-1/400*sinh(5*b*x+5*a)/b^2
```

Rubi [A] time = 0.09, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {5448, 3296, 2637}

$$\frac{\sinh(a+bx)}{8b^2}+\frac{\sinh(3a+3bx)}{144b^2}-\frac{\sinh(5a+5bx)}{400b^2}-\frac{x\cosh(a+bx)}{8b}-\frac{x\cosh(3a+3bx)}{48b}+\frac{x\cosh(5a+5bx)}{80b}$$

Antiderivative was successfully verified.

```
[In] Int[x*Cosh[a + b*x]^2*Sinh[a + b*x]^3,x]
```

```
[Out] -(x*Cosh[a + b*x])/(8*b) - (x*Cosh[3*a + 3*b*x])/(48*b) + (x*Cosh[5*a + 5*b
*x])/(80*b) + Sinh[a + b*x]/(8*b^2) + Sinh[3*a + 3*b*x]/(144*b^2) - Sinh[5*
a + 5*b*x]/(400*b^2)
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rubi steps

$$\int x\cosh^2(a+bx)\sinh^3(a+bx)\,dx = \int \left(-\frac{1}{8}x\sinh(a+bx) - \frac{1}{16}x\sinh(3a+3bx) + \frac{1}{16}x\sinh(5a+5bx)\right)dx$$
$$= -\left(\frac{1}{16}\int x\sinh(3a+3bx)\,dx\right) + \frac{1}{16}\int x\sinh(5a+5bx)\,dx - \frac{1}{8}\int x\sinh$$
$$= -\frac{x\cosh(a+bx)}{8b} - \frac{x\cosh(3a+3bx)}{48b} + \frac{x\cosh(5a+5bx)}{80b} - \frac{\int\cosh(5a+}{80b}$$
$$= -\frac{x\cosh(a+bx)}{8b} - \frac{x\cosh(3a+3bx)}{48b} + \frac{x\cosh(5a+5bx)}{80b} + \frac{\sinh(a+bx)}{8b^2}$$

Mathematica [A] time = 0.19, size = 70, normalized size = 0.74

$$\frac{450\sinh(a+bx) + 25\sinh(3(a+bx)) - 9\sinh(5(a+bx)) - 450bx\cosh(a+bx) - 75bx\cosh(3(a+bx)) + 45bx}{3600b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Cosh[a + b*x]^2*Sinh[a + b*x]^3,x]
```

```
[Out] (-450*b*x*Cosh[a + b*x] - 75*b*x*Cosh[3*(a + b*x)] + 45*b*x*Cosh[5*(a + b*x
)] + 450*Sinh[a + b*x] + 25*Sinh[3*(a + b*x)] - 9*Sinh[5*(a + b*x)])/(3600*
b^2)
```

fricas [A] time = 0.63, size = 153, normalized size = 1.63

$$\frac{45\,bx\cosh(bx+a)^5 + 225\,bx\cosh(bx+a)\sinh(bx+a)^4 - 75\,bx\cosh(bx+a)^3 - 9\,\sinh(bx+a)^5 - 5\big(18\,\cos}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x+a)^2*sinh(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/3600*(45*b*x*cosh(b*x + a)^5 + 225*b*x*cosh(b*x + a)*sinh(b*x + a)^4 - 75
*b*x*cosh(b*x + a)^3 - 9*sinh(b*x + a)^5 - 5*(18*cosh(b*x + a)^2 - 5)*sinh(
b*x + a)^3 - 450*b*x*cosh(b*x + a) + 225*(2*b*x*cosh(b*x + a)^3 - b*x*cosh(
b*x + a))*sinh(b*x + a)^2 - 15*(3*cosh(b*x + a)^4 - 5*cosh(b*x + a)^2 - 30)
*sinh(b*x + a))/b^2
```

giac [A] time = 0.15, size = 116, normalized size = 1.23

$$\frac{(5\,bx-1)e^{(5\,bx+5\,a)}}{800\,b^2} - \frac{(3\,bx-1)e^{(3\,bx+3\,a)}}{288\,b^2} - \frac{(bx-1)e^{(bx+a)}}{16\,b^2} - \frac{(bx+1)e^{(-bx-a)}}{16\,b^2} - \frac{(3\,bx+1)e^{(-3\,bx-3\,a)}}{288\,b^2} + \frac{(5\,bx+1)e^{(-5\,bx-5}}{800\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x+a)^2*sinh(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/800*(5*b*x - 1)*e^(5*b*x + 5*a)/b^2 - 1/288*(3*b*x - 1)*e^(3*b*x + 3*a)/b
^2 - 1/16*(b*x - 1)*e^(b*x + a)/b^2 - 1/16*(b*x + 1)*e^(-b*x - a)/b^2 - 1/2
88*(3*b*x + 1)*e^(-3*b*x - 3*a)/b^2 + 1/800*(5*b*x + 1)*e^(-5*b*x - 5*a)/b^
2
```

maple [A] time = 0.28, size = 116, normalized size = 1.23

$$\frac{\frac{(bx+a)\left(\sinh^2(bx+a)\right)\left(\cosh^3(bx+a)\right)}{5}-\frac{2(bx+a)\left(\cosh^3(bx+a)\right)}{15}-\frac{\sinh(bx+a)\left(\cosh^4(bx+a)\right)}{25}+\frac{26\sinh(bx+a)}{225}+\frac{13\left(\cosh^2(bx+a)\right)\sinh(bx+a)}{225}-a}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cosh(b*x+a)^2*sinh(b*x+a)^3,x)
```

```
[Out] 1/b^2*(1/5*(b*x+a)*sinh(b*x+a)^2*cosh(b*x+a)^3-2/15*(b*x+a)*cosh(b*x+a)^3-1
/25*sinh(b*x+a)*cosh(b*x+a)^4+26/225*sinh(b*x+a)+13/225*cosh(b*x+a)^2*sinh(
b*x+a)-a*(1/5*cosh(b*x+a)^3*sinh(b*x+a)^2-2/15*cosh(b*x+a)^3))
```

maxima [A] time = 0.35, size = 129, normalized size = 1.37

$$\frac{\left(5\,bxe^{(5\,a)}-e^{(5\,a)}\right)e^{(5\,bx)}}{800\,b^2}-\frac{\left(3\,bxe^{(3\,a)}-e^{(3\,a)}\right)e^{(3\,bx)}}{288\,b^2}-\frac{(bxe^{a}-e^{a})e^{(bx)}}{16\,b^2}-\frac{(bx+1)e^{(-bx-a)}}{16\,b^2}-\frac{(3\,bx+1)e^{(-3\,bx-3\,a)}}{288\,b^2}+\frac{(5\,bx+1}{80}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x+a)^2*sinh(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/800*(5*b*x*e^(5*a) - e^(5*a))*e^(5*b*x)/b^2 - 1/288*(3*b*x*e^(3*a) - e^(3
*a))*e^(3*b*x)/b^2 - 1/16*(b*x*e^a - e^a)*e^(b*x)/b^2 - 1/16*(b*x + 1)*e^(-
b*x - a)/b^2 - 1/288*(3*b*x + 1)*e^(-3*b*x - 3*a)/b^2 + 1/800*(5*b*x + 1)*e
^(-5*b*x - 5*a)/b^2
```

mupad [B] time = 0.14, size = 71, normalized size = 0.76

$$\frac{\frac{26\sinh(a+b\,x)}{225}-b\left(\frac{x\cosh(a+b\,x)^3}{3}-\frac{x\cosh(a+b\,x)^5}{5}\right)+\frac{13\cosh(a+b\,x)^2\sinh(a+b\,x)}{225}-\frac{\cosh(a+b\,x)^4\sinh(a+b\,x)}{25}}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cosh(a + b*x)^2*sinh(a + b*x)^3,x)
```

```
[Out] ((26*sinh(a + b*x))/225 - b*((x*cosh(a + b*x)^3)/3 - (x*cosh(a + b*x)^5)/5)
 + (13*cosh(a + b*x)^2*sinh(a + b*x))/225 - (cosh(a + b*x)^4*sinh(a + b*x))
/25)/b^2
```

sympy [A] time = 2.77, size = 112, normalized size = 1.19

$$\begin{cases} \frac{x\sinh^2(a+bx)\cosh^3(a+bx)}{3b} - \frac{2x\cosh^5(a+bx)}{15b} + \frac{26\sinh^5(a+bx)}{225b^2} - \frac{13\sinh^3(a+bx)\cosh^2(a+bx)}{45b^2} + \frac{2\sinh(a+bx)\cosh^4(a+bx)}{15b^2} & \text{for } b \neq \\ \frac{x^2\sinh^3(a)\cosh^2(a)}{2} & \text{otherw} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x+a)**2*sinh(b*x+a)**3,x)
```

```
[Out] Piecewise((x*sinh(a + b*x)**2*cosh(a + b*x)**3/(3*b) - 2*x*cosh(a + b*x)**5
/(15*b) + 26*sinh(a + b*x)**5/(225*b**2) - 13*sinh(a + b*x)**3*cosh(a + b*x
)**2/(45*b**2) + 2*sinh(a + b*x)*cosh(a + b*x)**4/(15*b**2), Ne(b, 0)), (x*
*2*sinh(a)**3*cosh(a)**2/2, True))
```

3.320 $\int \cosh^2(a+bx)\sinh^3(a+bx)\,dx$

Optimal. Leaf size=31

$$\frac{\cosh^5(a+bx)}{5b}-\frac{\cosh^3(a+bx)}{3b}$$

```
[Out] -1/3*cosh(b*x+a)^3/b+1/5*cosh(b*x+a)^5/b
```

Rubi [A] time = 0.03, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2565, 14}

$$\frac{\cosh^5(a+bx)}{5b}-\frac{\cosh^3(a+bx)}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[a + b*x]^2*Sinh[a + b*x]^3,x]
```

```
[Out] -Cosh[a + b*x]^3/(3*b) + Cosh[a + b*x]^5/(5*b)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rubi steps

$$\begin{aligned}
\int \cosh^2(a+bx)\sinh^3(a+bx)\,dx &= -\frac{\operatorname{Subst}\left(\int x^2\left(1-x^2\right)\,dx,x,\cosh(a+bx)\right)}{b}\\
&= -\frac{\operatorname{Subst}\left(\int \left(x^2-x^4\right)\,dx,x,\cosh(a+bx)\right)}{b}\\
&= -\frac{\cosh^3(a+bx)}{3b}+\frac{\cosh^5(a+bx)}{5b}
\end{aligned}$$

Mathematica [A] time = 0.07, size = 27, normalized size = 0.87

$$\frac{\cosh^3(a+bx)(3\cosh(2(a+bx))-7)}{30b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[a + b*x]^2*Sinh[a + b*x]^3,x]
```

```
[Out] (Cosh[a + b*x]^3*(-7 + 3*Cosh[2*(a + b*x)]))/(30*b)
```

fricas [B] time = 0.85, size = 79, normalized size = 2.55

$$\frac{3\cosh\left(bx+a\right)^5+15\cosh\left(bx+a\right)\sinh\left(bx+a\right)^4-5\cosh\left(bx+a\right)^3+15\left(2\cosh\left(bx+a\right)^3-\cosh\left(bx+a\right)\right)\mathrm{s}}{240\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^2*sinh(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/240*(3*cosh(b*x + a)^5 + 15*cosh(b*x + a)*sinh(b*x + a)^4 - 5*cosh(b*x +
a)^3 + 15*(2*cosh(b*x + a)^3 - cosh(b*x + a))*sinh(b*x + a)^2 - 30*cosh(b*x
 + a))/b
```

giac [B] time = 0.15, size = 82, normalized size = 2.65

$$\frac{e^{(5\,bx+5\,a)}}{160\,b}-\frac{e^{(3\,bx+3\,a)}}{96\,b}-\frac{e^{(bx+a)}}{16\,b}-\frac{e^{(-bx-a)}}{16\,b}-\frac{e^{(-3\,bx-3\,a)}}{96\,b}+\frac{e^{(-5\,bx-5\,a)}}{160\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^2*sinh(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/160*e^(5*b*x + 5*a)/b - 1/96*e^(3*b*x + 3*a)/b - 1/16*e^(b*x + a)/b - 1/1
6*e^(-b*x - a)/b - 1/96*e^(-3*b*x - 3*a)/b + 1/160*e^(-5*b*x - 5*a)/b
```

maple [A] time = 0.06, size = 34, normalized size = 1.10

$$\frac{\frac{\left(\cosh^3(bx+a)\right)\left(\sinh^2(bx+a)\right)}{5}-\frac{2\left(\cosh^3(bx+a)\right)}{15}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)^2*sinh(b*x+a)^3,x)
```

```
[Out] 1/b*(1/5*cosh(b*x+a)^3*sinh(b*x+a)^2-2/15*cosh(b*x+a)^3)
```

maxima [B] time = 0.33, size = 78, normalized size = 2.52

$$-\frac{\left(5\,e^{(-2\,bx-2\,a)}+30\,e^{(-4\,bx-4\,a)}-3\right)e^{(5\,bx+5\,a)}}{480\,b}-\frac{30\,e^{(-bx-a)}+5\,e^{(-3\,bx-3\,a)}-3\,e^{(-5\,bx-5\,a)}}{480\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^2*sinh(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -1/480*(5*e^(-2*b*x - 2*a) + 30*e^(-4*b*x - 4*a) - 3)*e^(5*b*x + 5*a)/b - 1
/480*(30*e^(-b*x - a) + 5*e^(-3*b*x - 3*a) - 3*e^(-5*b*x - 5*a))/b
```

mupad [B] time = 1.45, size = 26, normalized size = 0.84

$$-\frac{5\cosh\left(a+b\,x\right)^3-3\cosh\left(a+b\,x\right)^5}{15\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a + b*x)^2*sinh(a + b*x)^3,x)
```

```
[Out] -(5*cosh(a + b*x)^3 - 3*cosh(a + b*x)^5)/(15*b)
```

sympy [A] time = 1.47, size = 44, normalized size = 1.42

$$\begin{cases} \frac{\sinh^2(a+bx)\cosh^3(a+bx)}{3b}-\frac{2\cosh^5(a+bx)}{15b} & \text{for } b\neq 0 \\ x\sinh^3(a)\cosh^2(a) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)**2*sinh(b*x+a)**3,x)
```

```
[Out] Piecewise((sinh(a + b*x)**2*cosh(a + b*x)**3/(3*b) - 2*cosh(a + b*x)**5/(15
*b), Ne(b, 0)), (x*sinh(a)**3*cosh(a)**2, True))
```

3.321 $\int \frac{\cosh^2(a+bx)\sinh^3(a+bx)}{x}\,dx$

Optimal. Leaf size=73

$$-\frac{1}{8}\sinh(a)\text{Chi}(bx)-\frac{1}{16}\sinh(3a)\text{Chi}(3bx)+\frac{1}{16}\sinh(5a)\text{Chi}(5bx)-\frac{1}{8}\cosh(a)\text{Shi}(bx)-\frac{1}{16}\cosh(3a)\text{Shi}(3bx)+\frac{1}{16}$$

```
[Out] -1/8*cosh(a)*Shi(b*x)-1/16*cosh(3*a)*Shi(3*b*x)+1/16*cosh(5*a)*Shi(5*b*x)-1
/8*Chi(b*x)*sinh(a)-1/16*Chi(3*b*x)*sinh(3*a)+1/16*Chi(5*b*x)*sinh(5*a)
```

Rubi [A] time = 0.18, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {5448, 3303, 3298, 3301}

$$-\frac{1}{8}\sinh(a)\text{Chi}(bx)-\frac{1}{16}\sinh(3a)\text{Chi}(3bx)+\frac{1}{16}\sinh(5a)\text{Chi}(5bx)-\frac{1}{8}\cosh(a)\text{Shi}(bx)-\frac{1}{16}\cosh(3a)\text{Shi}(3bx)+\frac{1}{16}$$

Antiderivative was successfully verified.

```
[In] Int[(Cosh[a + b*x]^2*Sinh[a + b*x]^3)/x,x]
```

```
[Out] -(CoshIntegral[b*x]*Sinh[a])/8 - (CoshIntegral[3*b*x]*Sinh[3*a])/16 + (Cosh
Integral[5*b*x]*Sinh[5*a])/16 - (Cosh[a]*SinhIntegral[b*x])/8 - (Cosh[3*a]*
SinhIntegral[3*b*x])/16 + (Cosh[5*a]*SinhIntegral[5*b*x])/16
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\cosh^2(a+bx)\sinh^3(a+bx)}{x}\,dx &= \int \left(-\frac{\sinh(a+bx)}{8x}-\frac{\sinh(3a+3bx)}{16x}+\frac{\sinh(5a+5bx)}{16x}\right)dx\\ &= -\left(\frac{1}{16}\int \frac{\sinh(3a+3bx)}{x}\,dx\right)+\frac{1}{16}\int \frac{\sinh(5a+5bx)}{x}\,dx-\frac{1}{8}\int \frac{\sinh(a+b\\ &= -\left(\frac{1}{8}\cosh(a)\int \frac{\sinh(bx)}{x}\,dx\right)-\frac{1}{16}\cosh(3a)\int \frac{\sinh(3bx)}{x}\,dx+\frac{1}{16}\cosh(5a\\ &= -\frac{1}{8}\text{Chi}(bx)\sinh(a)-\frac{1}{16}\text{Chi}(3bx)\sinh(3a)+\frac{1}{16}\text{Chi}(5bx)\sinh(5a)-\frac{1}{8}\cos\end{aligned}$$

Mathematica [A] time = 0.12, size = 63, normalized size = 0.86

$$\frac{1}{16}(-2\sinh(a)\text{Chi}(bx)-\sinh(3a)\text{Chi}(3bx)+\sinh(5a)\text{Chi}(5bx)-2\cosh(a)\text{Shi}(bx)-\cosh(3a)\text{Shi}(3bx)+\cosh(5a)\text{Sh}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cosh[a + b*x]^2*Sinh[a + b*x]^3)/x,x]
```

```
[Out] (-2*CoshIntegral[b*x]*Sinh[a] - CoshIntegral[3*b*x]*Sinh[3*a] + CoshIntegra
l[5*b*x]*Sinh[5*a] - 2*Cosh[a]*SinhIntegral[b*x] - Cosh[3*a]*SinhIntegral[3
*b*x] + Cosh[5*a]*SinhIntegral[5*b*x])/16
```

fricas [A] time = 0.56, size = 103, normalized size = 1.41

$$\frac{1}{32}\left(\text{Ei}\left(5\,bx\right)-\text{Ei}\left(-5\,bx\right)\right)\cosh\left(5\,a\right)-\frac{1}{32}\left(\text{Ei}\left(3\,bx\right)-\text{Ei}\left(-3\,bx\right)\right)\cosh\left(3\,a\right)-\frac{1}{16}\left(\text{Ei}\left(bx\right)-\text{Ei}\left(-bx\right)\right)\cosh(a)+\frac{1}{32}\,(\text{I}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^2*sinh(b*x+a)^3/x,x, algorithm="fricas")
```

```
[Out] 1/32*(Ei(5*b*x) - Ei(-5*b*x))*cosh(5*a) - 1/32*(Ei(3*b*x) - Ei(-3*b*x))*cos
h(3*a) - 1/16*(Ei(b*x) - Ei(-b*x))*cosh(a) + 1/32*(Ei(5*b*x) + Ei(-5*b*x))*
sinh(5*a) - 1/32*(Ei(3*b*x) + Ei(-3*b*x))*sinh(3*a) - 1/16*(Ei(b*x) + Ei(-b
*x))*sinh(a)
```

giac [A] time = 0.13, size = 64, normalized size = 0.88

$$\frac{1}{32}\,\mathrm{Ei}\left(5\,bx\right)e^{(5\,a)} - \frac{1}{32}\,\mathrm{Ei}\left(3\,bx\right)e^{(3\,a)} + \frac{1}{16}\,\mathrm{Ei}\left(-bx\right)e^{(-a)} + \frac{1}{32}\,\mathrm{Ei}\left(-3\,bx\right)e^{(-3\,a)} - \frac{1}{32}\,\mathrm{Ei}\left(-5\,bx\right)e^{(-5\,a)} - \frac{1}{16}\,\mathrm{Ei}\left(bx\right)e^{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^2*sinh(b*x+a)^3/x,x, algorithm="giac")
```

```
[Out] 1/32*Ei(5*b*x)*e^(5*a) - 1/32*Ei(3*b*x)*e^(3*a) + 1/16*Ei(-b*x)*e^(-a) + 1/
32*Ei(-3*b*x)*e^(-3*a) - 1/32*Ei(-5*b*x)*e^(-5*a) - 1/16*Ei(b*x)*e^a
```

maple [A] time = 0.40, size = 71, normalized size = 0.97

$$\frac{\mathrm{e}^{-5a}\,\mathrm{Ei}\left(1,5bx\right)}{32} - \frac{\mathrm{e}^{-3a}\,\mathrm{Ei}\left(1,3bx\right)}{32} - \frac{\mathrm{e}^{-a}\,\mathrm{Ei}\left(1,bx\right)}{16} + \frac{\mathrm{e}^{a}\,\mathrm{Ei}\left(1,-bx\right)}{16} + \frac{\mathrm{e}^{3a}\,\mathrm{Ei}\left(1,-3bx\right)}{32} - \frac{\mathrm{e}^{5a}\,\mathrm{Ei}\left(1,-5bx\right)}{32}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)^2*sinh(b*x+a)^3/x,x)
```

```
[Out] 1/32*exp(-5*a)*Ei(1,5*b*x)-1/32*exp(-3*a)*Ei(1,3*b*x)-1/16*exp(-a)*Ei(1,b*x
)+1/16*exp(a)*Ei(1,-b*x)+1/32*exp(3*a)*Ei(1,-3*b*x)-1/32*exp(5*a)*Ei(1,-5*b
*x)
```

maxima [A] time = 0.45, size = 64, normalized size = 0.88

$$\frac{1}{32}\,\mathrm{Ei}\left(5\,bx\right)e^{(5\,a)} - \frac{1}{32}\,\mathrm{Ei}\left(3\,bx\right)e^{(3\,a)} + \frac{1}{16}\,\mathrm{Ei}\left(-bx\right)e^{(-a)} + \frac{1}{32}\,\mathrm{Ei}\left(-3\,bx\right)e^{(-3\,a)} - \frac{1}{32}\,\mathrm{Ei}\left(-5\,bx\right)e^{(-5\,a)} - \frac{1}{16}\,\mathrm{Ei}\left(bx\right)e^{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^2*sinh(b*x+a)^3/x,x, algorithm="maxima")
```

```
[Out] 1/32*Ei(5*b*x)*e^(5*a) - 1/32*Ei(3*b*x)*e^(3*a) + 1/16*Ei(-b*x)*e^(-a) + 1/
32*Ei(-3*b*x)*e^(-3*a) - 1/32*Ei(-5*b*x)*e^(-5*a) - 1/16*Ei(b*x)*e^a
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cosh\left(a+b\,x\right)^2\,\sinh\left(a+b\,x\right)^3}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cosh(a + b*x)^2*sinh(a + b*x)^3)/x,x)
```

```
[Out] int((cosh(a + b*x)^2*sinh(a + b*x)^3)/x, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh^3(a+bx)\cosh^2(a+bx)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)**2*sinh(b*x+a)**3/x,x)
```

```
[Out] Integral(sinh(a + b*x)**3*cosh(a + b*x)**2/x, x)
```

3.322 $\int \frac{\cosh^2(a+bx)\sinh^3(a+bx)}{x^2}\,dx$

Optimal. Leaf size=124

$$-\frac{1}{8}b\cosh(a)\mathrm{Chi}(bx)-\frac{3}{16}b\cosh(3a)\mathrm{Chi}(3bx)+\frac{5}{16}b\cosh(5a)\mathrm{Chi}(5bx)-\frac{1}{8}b\sinh(a)\mathrm{Shi}(bx)-\frac{3}{16}b\sinh(3a)\mathrm{Shi}(3bx$$

```
[Out] -1/8*b*Chi(b*x)*cosh(a)-3/16*b*Chi(3*b*x)*cosh(3*a)+5/16*b*Chi(5*b*x)*cosh(
5*a)-1/8*b*Shi(b*x)*sinh(a)-3/16*b*Shi(3*b*x)*sinh(3*a)+5/16*b*Shi(5*b*x)*s
inh(5*a)+1/8*sinh(b*x+a)/x+1/16*sinh(3*b*x+3*a)/x-1/16*sinh(5*b*x+5*a)/x
```

Rubi [A] time = 0.25, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {5448, 3297, 3303, 3298, 3301}

$$-\frac{1}{8}b\cosh(a)\mathrm{Chi}(bx)-\frac{3}{16}b\cosh(3a)\mathrm{Chi}(3bx)+\frac{5}{16}b\cosh(5a)\mathrm{Chi}(5bx)-\frac{1}{8}b\sinh(a)\mathrm{Shi}(bx)-\frac{3}{16}b\sinh(3a)\mathrm{Shi}(3bx$$

Antiderivative was successfully verified.

```
[In] Int[(Cosh[a + b*x]^2*Sinh[a + b*x]^3)/x^2,x]
```

```
[Out] -(b*Cosh[a]*CoshIntegral[b*x])/8 - (3*b*Cosh[3*a]*CoshIntegral[3*b*x])/16 +
 (5*b*Cosh[5*a]*CoshIntegral[5*b*x])/16 + Sinh[a + b*x]/(8*x) + Sinh[3*a +
3*b*x]/(16*x) - Sinh[5*a + 5*b*x]/(16*x) - (b*Sinh[a]*SinhIntegral[b*x])/8
- (3*b*Sinh[3*a]*SinhIntegral[3*b*x])/16 + (5*b*Sinh[5*a]*SinhIntegral[5*b*
x])/16
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
```

```
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\cosh^2(a+bx)\sinh^3(a+bx)}{x^2}\,dx &= \int \left(-\frac{\sinh(a+bx)}{8x^2}-\frac{\sinh(3a+3bx)}{16x^2}+\frac{\sinh(5a+5bx)}{16x^2}\right)dx\\ &= -\left(\frac{1}{16}\int \frac{\sinh(3a+3bx)}{x^2}\,dx\right)+\frac{1}{16}\int \frac{\sinh(5a+5bx)}{x^2}\,dx-\frac{1}{8}\int \frac{\sinh(a+b\\ &= \frac{\sinh(a+bx)}{8x}+\frac{\sinh(3a+3bx)}{16x}-\frac{\sinh(5a+5bx)}{16x}-\frac{1}{8}b\int \frac{\cosh(a+bx)}{x}\,dx\\ &= \frac{\sinh(a+bx)}{8x}+\frac{\sinh(3a+3bx)}{16x}-\frac{\sinh(5a+5bx)}{16x}-\frac{1}{8}(b\cosh(a))\int \frac{\cosh(}{x}\\ &= -\frac{1}{8}b\cosh(a)\mathrm{Chi}(bx)-\frac{3}{16}b\cosh(3a)\mathrm{Chi}(3bx)+\frac{5}{16}b\cosh(5a)\mathrm{Chi}(5bx)+\end{aligned}$$

Mathematica [A] time = 0.27, size = 106, normalized size = 0.85

$$\frac{-2bx\cosh(a)\mathrm{Chi}(bx)-3bx\cosh(3a)\mathrm{Chi}(3bx)+5bx\cosh(5a)\mathrm{Chi}(5bx)-2bx\sinh(a)\mathrm{Shi}(bx)-3bx\sinh(3a)\mathrm{Shi}}{16x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cosh[a + b*x]^2*Sinh[a + b*x]^3)/x^2,x]
```

```
[Out] (-2*b*x*Cosh[a]*CoshIntegral[b*x] - 3*b*x*Cosh[3*a]*CoshIntegral[3*b*x] + 5
*b*x*Cosh[5*a]*CoshIntegral[5*b*x] + 2*Sinh[a + b*x] + Sinh[3*(a + b*x)] -
Sinh[5*(a + b*x)] - 2*b*x*Sinh[a]*SinhIntegral[b*x] - 3*b*x*Sinh[3*a]*SinhI
ntegral[3*b*x] + 5*b*x*Sinh[5*a]*SinhIntegral[5*b*x])/(16*x)
```

fricas [A] time = 1.01, size = 203, normalized size = 1.64

$$-\frac{2\,\sinh\left(bx+a\right)^{5}+2\left(10\,\cosh\left(bx+a\right)^{2}-1\right)\sinh\left(bx+a\right)^{3}-5\left(bx\mathrm{Ei}\left(5\,bx\right)+bx\mathrm{Ei}\left(-5\,bx\right)\right)\cosh\left(5\,a\right)+3\left(bx\mathrm{E}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^2*sinh(b*x+a)^3/x^2,x, algorithm="fricas")
```

```
[Out] -1/32*(2*sinh(b*x + a)^5 + 2*(10*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^3 - 5*(
b*x*Ei(5*b*x) + b*x*Ei(-5*b*x))*cosh(5*a) + 3*(b*x*Ei(3*b*x) + b*x*Ei(-3*b*
x))*cosh(3*a) + 2*(b*x*Ei(b*x) + b*x*Ei(-b*x))*cosh(a) + 2*(5*cosh(b*x + a)
^4 - 3*cosh(b*x + a)^2 - 2)*sinh(b*x + a) - 5*(b*x*Ei(5*b*x) - b*x*Ei(-5*b*
x))*sinh(5*a) + 3*(b*x*Ei(3*b*x) - b*x*Ei(-3*b*x))*sinh(3*a) + 2*(b*x*Ei(b*
x) - b*x*Ei(-b*x))*sinh(a))/x
```

giac [A] time = 0.15, size = 140, normalized size = 1.13

$$\frac{5\,bx\mathrm{Ei}\left(5\,bx\right)e^{(5\,a)}-3\,bx\mathrm{Ei}\left(3\,bx\right)e^{(3\,a)}-2\,bx\mathrm{Ei}\left(-bx\right)e^{(-a)}-3\,bx\mathrm{Ei}\left(-3\,bx\right)e^{(-3\,a)}+5\,bx\mathrm{Ei}\left(-5\,bx\right)e^{(-5\,a)}-2\,bx\mathrm{Ei}}{32\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^2*sinh(b*x+a)^3/x^2,x, algorithm="giac")
```

```
[Out] 1/32*(5*b*x*Ei(5*b*x)*e^(5*a) - 3*b*x*Ei(3*b*x)*e^(3*a) - 2*b*x*Ei(-b*x)*e^
(-a) - 3*b*x*Ei(-3*b*x)*e^(-3*a) + 5*b*x*Ei(-5*b*x)*e^(-5*a) - 2*b*x*Ei(b*x
)*e^a - e^(5*b*x + 5*a) + e^(3*b*x + 3*a) + 2*e^(b*x + a) - 2*e^(-b*x - a)
- e^(-3*b*x - 3*a) + e^(-5*b*x - 5*a))/x
```

maple [A] time = 0.41, size = 158, normalized size = 1.27

$$\frac{e^{-5bx-5a}}{32x}-\frac{5b\,e^{-5a}\,\mathrm{Ei}\left(1,5bx\right)}{32}-\frac{e^{-3bx-3a}}{32x}+\frac{3b\,e^{-3a}\,\mathrm{Ei}\left(1,3bx\right)}{32}-\frac{e^{-bx-a}}{16x}+\frac{b\,e^{-a}\,\mathrm{Ei}\left(1,bx\right)}{16}+\frac{e^{bx+a}}{16x}+\frac{b\,e^{a}\,\mathrm{Ei}\left(1,-bx\right)}{16}+\frac{e^{3b}}{32}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)^2*sinh(b*x+a)^3/x^2,x)
```

```
[Out] 1/32*exp(-5*b*x-5*a)/x-5/32*b*exp(-5*a)*Ei(1,5*b*x)-1/32*exp(-3*b*x-3*a)/x+
3/32*b*exp(-3*a)*Ei(1,3*b*x)-1/16*exp(-b*x-a)/x+1/16*b*exp(-a)*Ei(1,b*x)+1/
16/x*exp(b*x+a)+1/16*b*exp(a)*Ei(1,-b*x)+1/32/x*exp(3*b*x+3*a)+3/32*b*exp(3
*a)*Ei(1,-3*b*x)-1/32/x*exp(5*b*x+5*a)-5/32*b*exp(5*a)*Ei(1,-5*b*x)
```

maxima [A] time = 0.47, size = 76, normalized size = 0.61

$$\frac{5}{32}\,be^{(-5\,a)}\Gamma\left(-1,5\,bx\right)-\frac{3}{32}\,be^{(-3\,a)}\Gamma\left(-1,3\,bx\right)-\frac{1}{16}\,be^{(-a)}\Gamma\left(-1,bx\right)-\frac{1}{16}\,be^{a}\Gamma\left(-1,-bx\right)-\frac{3}{32}\,be^{(3\,a)}\Gamma\left(-1,-3\,bx\right)+\frac{5}{32}\,b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^2*sinh(b*x+a)^3/x^2,x, algorithm="maxima")
```

```
[Out] 5/32*b*e^(-5*a)*gamma(-1, 5*b*x) - 3/32*b*e^(-3*a)*gamma(-1, 3*b*x) - 1/16*
b*e^(-a)*gamma(-1, b*x) - 1/16*b*e^a*gamma(-1, -b*x) - 3/32*b*e^(3*a)*gamma
(-1, -3*b*x) + 5/32*b*e^(5*a)*gamma(-1, -5*b*x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cosh\left(a+b\,x\right)^2\,\sinh\left(a+b\,x\right)^3}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cosh(a + b*x)^2*sinh(a + b*x)^3)/x^2,x)
```

```
[Out] int((cosh(a + b*x)^2*sinh(a + b*x)^3)/x^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh^{3}\left(a+bx\right)\cosh^{2}\left(a+bx\right)}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)**2*sinh(b*x+a)**3/x**2,x)
```

```
[Out] Integral(sinh(a + b*x)**3*cosh(a + b*x)**2/x**2, x)
```

3.323 $\int \frac{\cosh^2(a+bx)\sinh^3(a+bx)}{x^3}\,dx$

Optimal. Leaf size=184

$$-\frac{1}{16}b^2\sinh(a)\text{Chi}(bx)-\frac{9}{32}b^2\sinh(3a)\text{Chi}(3bx)+\frac{25}{32}b^2\sinh(5a)\text{Chi}(5bx)-\frac{1}{16}b^2\cosh(a)\text{Shi}(bx)-\frac{9}{32}b^2\cosh(3a)$$

```
[Out] 1/16*b*cosh(b*x+a)/x+3/32*b*cosh(3*b*x+3*a)/x-5/32*b*cosh(5*b*x+5*a)/x-1/16
*b^2*cosh(a)*Shi(b*x)-9/32*b^2*cosh(3*a)*Shi(3*b*x)+25/32*b^2*cosh(5*a)*Shi
(5*b*x)-1/16*b^2*Chi(b*x)*sinh(a)-9/32*b^2*Chi(3*b*x)*sinh(3*a)+25/32*b^2*C
hi(5*b*x)*sinh(5*a)+1/16*sinh(b*x+a)/x^2+1/32*sinh(3*b*x+3*a)/x^2-1/32*sinh
(5*b*x+5*a)/x^2
```

Rubi [A] time = 0.34, antiderivative size = 184, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {5448, 3297, 3303, 3298, 3301}

$$-\frac{1}{16}b^2\sinh(a)\text{Chi}(bx)-\frac{9}{32}b^2\sinh(3a)\text{Chi}(3bx)+\frac{25}{32}b^2\sinh(5a)\text{Chi}(5bx)-\frac{1}{16}b^2\cosh(a)\text{Shi}(bx)-\frac{9}{32}b^2\cosh(3a)$$

Antiderivative was successfully verified.

```
[In] Int[(Cosh[a + b*x]^2*Sinh[a + b*x]^3)/x^3,x]
```

```
[Out] (b*Cosh[a + b*x])/(16*x) + (3*b*Cosh[3*a + 3*b*x])/(32*x) - (5*b*Cosh[5*a +
 5*b*x])/(32*x) - (b^2*CoshIntegral[b*x]*Sinh[a])/16 - (9*b^2*CoshIntegral[
3*b*x]*Sinh[3*a])/32 + (25*b^2*CoshIntegral[5*b*x]*Sinh[5*a])/32 + Sinh[a +
 b*x]/(16*x^2) + Sinh[3*a + 3*b*x]/(32*x^2) - Sinh[5*a + 5*b*x]/(32*x^2) -
(b^2*Cosh[a]*SinhIntegral[b*x])/16 - (9*b^2*Cosh[3*a]*SinhIntegral[3*b*x])/
32 + (25*b^2*Cosh[5*a]*SinhIntegral[5*b*x])/32
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cosh^2(a+bx)\sinh^3(a+bx)}{x^3}\,dx &= \int \left(-\frac{\sinh(a+bx)}{8x^3} - \frac{\sinh(3a+3bx)}{16x^3} + \frac{\sinh(5a+5bx)}{16x^3}\right) dx \\
&= -\left(\frac{1}{16}\int \frac{\sinh(3a+3bx)}{x^3}\,dx\right) + \frac{1}{16}\int \frac{\sinh(5a+5bx)}{x^3}\,dx - \frac{1}{8}\int \frac{\sinh(a+b\ldots}{x^3} \\
&= \frac{\sinh(a+bx)}{16x^2} + \frac{\sinh(3a+3bx)}{32x^2} - \frac{\sinh(5a+5bx)}{32x^2} - \frac{1}{16}b\int \frac{\cosh(a+bx)}{x^2}\,d\ldots \\
&= \frac{b\cosh(a+bx)}{16x} + \frac{3b\cosh(3a+3bx)}{32x} - \frac{5b\cosh(5a+5bx)}{32x} + \frac{\sinh(a+bx)}{16x^2} \ldots \\
&= \frac{b\cosh(a+bx)}{16x} + \frac{3b\cosh(3a+3bx)}{32x} - \frac{5b\cosh(5a+5bx)}{32x} + \frac{\sinh(a+bx)}{16x^2} \ldots \\
&= \frac{b\cosh(a+bx)}{16x} + \frac{3b\cosh(3a+3bx)}{32x} - \frac{5b\cosh(5a+5bx)}{32x} - \frac{1}{16}b^2\mathrm{Chi}(bx)\,\mathrm{s}\ldots
\end{aligned}$$

Mathematica [A] time = 0.42, size = 164, normalized size = 0.89

$$\frac{-2b^2x^2\sinh(a)\mathrm{Chi}(bx) - 9b^2x^2\sinh(3a)\mathrm{Chi}(3bx) + 25b^2x^2\sinh(5a)\mathrm{Chi}(5bx) - 2b^2x^2\cosh(a)\mathrm{Shi}(bx) - 9b^2x^2\,\mathrm{co}\ldots}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cosh[a + b*x]^2*Sinh[a + b*x]^3)/x^3,x]
```

```
[Out] (2*b*x*Cosh[a + b*x] + 3*b*x*Cosh[3*(a + b*x)] - 5*b*x*Cosh[5*(a + b*x)] -
2*b^2*x^2*CoshIntegral[b*x]*Sinh[a] - 9*b^2*x^2*CoshIntegral[3*b*x]*Sinh[3*
a] + 25*b^2*x^2*CoshIntegral[5*b*x]*Sinh[5*a] + 2*Sinh[a + b*x] + Sinh[3*(a
 + b*x)] - Sinh[5*(a + b*x)] - 2*b^2*x^2*Cosh[a]*SinhIntegral[b*x] - 9*b^2*
x^2*Cosh[3*a]*SinhIntegral[3*b*x] + 25*b^2*x^2*Cosh[5*a]*SinhIntegral[5*b*x
])/(32*x^2)
```

fricas [B] time = 0.82, size = 336, normalized size = 1.83

$$-\frac{10\,bx\cosh\left(bx+a\right)^{5}+50\,bx\cosh\left(bx+a\right)\sinh\left(bx+a\right)^{4}-6\,bx\cosh\left(bx+a\right)^{3}+2\,\sinh\left(bx+a\right)^{5}+2\left(10\,\cos\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^2*sinh(b*x+a)^3/x^3,x, algorithm="fricas")
```

```
[Out] -1/64*(10*b*x*cosh(b*x + a)^5 + 50*b*x*cosh(b*x + a)*sinh(b*x + a)^4 - 6*b*
x*cosh(b*x + a)^3 + 2*sinh(b*x + a)^5 + 2*(10*cosh(b*x + a)^2 - 1)*sinh(b*x
 + a)^3 - 4*b*x*cosh(b*x + a) + 2*(50*b*x*cosh(b*x + a)^3 - 9*b*x*cosh(b*x
+ a))*sinh(b*x + a)^2 - 25*(b^2*x^2*Ei(5*b*x) - b^2*x^2*Ei(-5*b*x))*cosh(5*
a) + 9*(b^2*x^2*Ei(3*b*x) - b^2*x^2*Ei(-3*b*x))*cosh(3*a) + 2*(b^2*x^2*Ei(b
*x) - b^2*x^2*Ei(-b*x))*cosh(a) + 2*(5*cosh(b*x + a)^4 - 3*cosh(b*x + a)^2
- 2)*sinh(b*x + a) - 25*(b^2*x^2*Ei(5*b*x) + b^2*x^2*Ei(-5*b*x))*sinh(5*a)
+ 9*(b^2*x^2*Ei(3*b*x) + b^2*x^2*Ei(-3*b*x))*sinh(3*a) + 2*(b^2*x^2*Ei(b*x)
 + b^2*x^2*Ei(-b*x))*sinh(a))/x^2
```

giac [A] time = 0.15, size = 239, normalized size = 1.30

$$\frac{25\,b^{2}x^{2}\mathrm{Ei}\left(5\,bx\right)e^{\left(5\,a\right)}-9\,b^{2}x^{2}\mathrm{Ei}\left(3\,bx\right)e^{\left(3\,a\right)}+2\,b^{2}x^{2}\mathrm{Ei}\left(-bx\right)e^{\left(-a\right)}+9\,b^{2}x^{2}\mathrm{Ei}\left(-3\,bx\right)e^{\left(-3\,a\right)}-25\,b^{2}x^{2}\mathrm{Ei}\left(-5\,bx\right)e^{\left(\right.}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^2*sinh(b*x+a)^3/x^3,x, algorithm="giac")
```

```
[Out] 1/64*(25*b^2*x^2*Ei(5*b*x)*e^(5*a) - 9*b^2*x^2*Ei(3*b*x)*e^(3*a) + 2*b^2*x^
2*Ei(-b*x)*e^(-a) + 9*b^2*x^2*Ei(-3*b*x)*e^(-3*a) - 25*b^2*x^2*Ei(-5*b*x)*e
^(-5*a) - 2*b^2*x^2*Ei(b*x)*e^a - 5*b*x*e^(5*b*x + 5*a) + 3*b*x*e^(3*b*x +
3*a) + 2*b*x*e^(b*x + a) + 2*b*x*e^(-b*x - a) + 3*b*x*e^(-3*b*x - 3*a) - 5*
b*x*e^(-5*b*x - 5*a) - e^(5*b*x + 5*a) + e^(3*b*x + 3*a) + 2*e^(b*x + a) -
2*e^(-b*x - a) - e^(-3*b*x - 3*a) + e^(-5*b*x - 5*a))/x^2
```

maple [A] time = 0.42, size = 257, normalized size = 1.40

$$-\frac{5b\,\mathrm{e}^{-5bx-5a}}{64x}+\frac{\mathrm{e}^{-5bx-5a}}{64x^{2}}+\frac{25b^{2}\mathrm{e}^{-5a}\,\mathrm{Ei}\left(1,5bx\right)}{64}+\frac{3b\,\mathrm{e}^{-3bx-3a}}{64x}-\frac{\mathrm{e}^{-3bx-3a}}{64x^{2}}-\frac{9b^{2}\mathrm{e}^{-3a}\,\mathrm{Ei}\left(1,3bx\right)}{64}+\frac{b\,\mathrm{e}^{-bx-a}}{32x}-\frac{\mathrm{e}^{-bx-a}}{32x^{2}}-\frac{b^{2}\mathrm{e}^{\cdots}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)^2*sinh(b*x+a)^3/x^3,x)
```

```
[Out] -5/64*b*exp(-5*b*x-5*a)/x+1/64*exp(-5*b*x-5*a)/x^2+25/64*b^2*exp(-5*a)*Ei(1
,5*b*x)+3/64*b*exp(-3*b*x-3*a)/x-1/64*exp(-3*b*x-3*a)/x^2-9/64*b^2*exp(-3*a
)*Ei(1,3*b*x)+1/32*b*exp(-b*x-a)/x-1/32*exp(-b*x-a)/x^2-1/32*b^2*exp(-a)*Ei
(1,b*x)+1/32/x^2*exp(b*x+a)+1/32*b/x*exp(b*x+a)+1/32*b^2*exp(a)*Ei(1,-b*x)+
1/64/x^2*exp(3*b*x+3*a)+3/64*b/x*exp(3*b*x+3*a)+9/64*b^2*exp(3*a)*Ei(1,-3*b
*x)-1/64/x^2*exp(5*b*x+5*a)-5/64*b/x*exp(5*b*x+5*a)-25/64*b^2*exp(5*a)*Ei(1
,-5*b*x)
```

maxima [A] time = 0.45, size = 88, normalized size = 0.48

$$\frac{25}{32}\,b^2 e^{(-5\,a)}\Gamma\left(-2,5\,bx\right)-\frac{9}{32}\,b^2 e^{(-3\,a)}\Gamma\left(-2,3\,bx\right)-\frac{1}{16}\,b^2 e^{(-a)}\Gamma\left(-2,bx\right)+\frac{1}{16}\,b^2 e^{a}\Gamma\left(-2,-bx\right)+\frac{9}{32}\,b^2 e^{(3\,a)}\Gamma\left(-2,-3\,bx\right)-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^2*sinh(b*x+a)^3/x^3,x, algorithm="maxima")
```

```
[Out] 25/32*b^2*e^(-5*a)*gamma(-2, 5*b*x) - 9/32*b^2*e^(-3*a)*gamma(-2, 3*b*x) -
1/16*b^2*e^(-a)*gamma(-2, b*x) + 1/16*b^2*e^a*gamma(-2, -b*x) + 9/32*b^2*e^
(3*a)*gamma(-2, -3*b*x) - 25/32*b^2*e^(5*a)*gamma(-2, -5*b*x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cosh\left(a+b\,x\right)^2\,\sinh\left(a+b\,x\right)^3}{x^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cosh(a + b*x)^2*sinh(a + b*x)^3)/x^3,x)
```

```
[Out] int((cosh(a + b*x)^2*sinh(a + b*x)^3)/x^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh^{3}{\left(a + bx \right)} \cosh^{2}{\left(a + bx \right)}}{x^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)**2*sinh(b*x+a)**3/x**3,x)
```

```
[Out] Integral(sinh(a + b*x)**3*cosh(a + b*x)**2/x**3, x)
```

3.324 $\int \frac{\cosh^2(a+bx)\sinh^3(a+bx)}{x^4}\,dx$

Optimal. Leaf size=238

$$-\frac{1}{48}b^3\cosh(a)\text{Chi}(bx)-\frac{9}{32}b^3\cosh(3a)\text{Chi}(3bx)+\frac{125}{96}b^3\cosh(5a)\text{Chi}(5bx)-\frac{1}{48}b^3\sinh(a)\text{Shi}(bx)-\frac{9}{32}b^3\sinh(3a$$

```
[Out] -1/48*b^3*Chi(b*x)*cosh(a)-9/32*b^3*Chi(3*b*x)*cosh(3*a)+125/96*b^3*Chi(5*b
*x)*cosh(5*a)+1/48*b*cosh(b*x+a)/x^2+1/32*b*cosh(3*b*x+3*a)/x^2-5/96*b*cosh
(5*b*x+5*a)/x^2-1/48*b^3*Shi(b*x)*sinh(a)-9/32*b^3*Shi(3*b*x)*sinh(3*a)+125
/96*b^3*Shi(5*b*x)*sinh(5*a)+1/24*sinh(b*x+a)/x^3+1/48*b^2*sinh(b*x+a)/x+1/
48*sinh(3*b*x+3*a)/x^3+3/32*b^2*sinh(3*b*x+3*a)/x-1/48*sinh(5*b*x+5*a)/x^3-
25/96*b^2*sinh(5*b*x+5*a)/x
```

Rubi [A] time = 0.41, antiderivative size = 238, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {5448, 3297, 3303, 3298, 3301}

$$-\frac{1}{48}b^3\cosh(a)\text{Chi}(bx)-\frac{9}{32}b^3\cosh(3a)\text{Chi}(3bx)+\frac{125}{96}b^3\cosh(5a)\text{Chi}(5bx)-\frac{1}{48}b^3\sinh(a)\text{Shi}(bx)-\frac{9}{32}b^3\sinh(3a$$

Antiderivative was successfully verified.

```
[In] Int[(Cosh[a + b*x]^2*Sinh[a + b*x]^3)/x^4,x]
```

```
[Out] (b*Cosh[a + b*x])/(48*x^2) + (b*Cosh[3*a + 3*b*x])/(32*x^2) - (5*b*Cosh[5*a
 + 5*b*x])/(96*x^2) - (b^3*Cosh[a]*CoshIntegral[b*x])/48 - (9*b^3*Cosh[3*a]
*CoshIntegral[3*b*x])/32 + (125*b^3*Cosh[5*a]*CoshIntegral[5*b*x])/96 + Sin
h[a + b*x]/(24*x^3) + (b^2*Sinh[a + b*x])/(48*x) + Sinh[3*a + 3*b*x]/(48*x^
3) + (3*b^2*Sinh[3*a + 3*b*x])/(32*x) - Sinh[5*a + 5*b*x]/(48*x^3) - (25*b^
2*Sinh[5*a + 5*b*x])/(96*x) - (b^3*Sinh[a]*SinhIntegral[b*x])/48 - (9*b^3*S
inh[3*a]*SinhIntegral[3*b*x])/32 + (125*b^3*Sinh[5*a]*SinhIntegral[5*b*x])/
96
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
```

```
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rubi steps

$$\int \frac{\cosh^2(a+bx)\sinh^3(a+bx)}{x^4}\,dx = \int \left(-\frac{\sinh(a+bx)}{8x^4} - \frac{\sinh(3a+3bx)}{16x^4} + \frac{\sinh(5a+5bx)}{16x^4}\right) dx$$

$$= -\left(\frac{1}{16}\int \frac{\sinh(3a+3bx)}{x^4}\,dx\right) + \frac{1}{16}\int \frac{\sinh(5a+5bx)}{x^4}\,dx - \frac{1}{8}\int \frac{\sinh(a+b}{x^4}$$

$$= \frac{\sinh(a+bx)}{24x^3} + \frac{\sinh(3a+3bx)}{48x^3} - \frac{\sinh(5a+5bx)}{48x^3} - \frac{1}{24}b\int \frac{\cosh(a+bx)}{x^3}\,d$$

$$= \frac{b\cosh(a+bx)}{48x^2} + \frac{b\cosh(3a+3bx)}{32x^2} - \frac{5b\cosh(5a+5bx)}{96x^2} + \frac{\sinh(a+bx)}{24x^3} +$$

$$= \frac{b\cosh(a+bx)}{48x^2} + \frac{b\cosh(3a+3bx)}{32x^2} - \frac{5b\cosh(5a+5bx)}{96x^2} + \frac{\sinh(a+bx)}{24x^3} +$$

$$= \frac{b\cosh(a+bx)}{48x^2} + \frac{b\cosh(3a+3bx)}{32x^2} - \frac{5b\cosh(5a+5bx)}{96x^2} + \frac{\sinh(a+bx)}{24x^3} +$$

$$= \frac{b\cosh(a+bx)}{48x^2} + \frac{b\cosh(3a+3bx)}{32x^2} - \frac{5b\cosh(5a+5bx)}{96x^2} - \frac{1}{48}b^3\cosh(a)\text{Ch}$$

Mathematica [A] time = 0.51, size = 212, normalized size = 0.89

$$\frac{-2b^3x^3\cosh(a)\text{Chi}(bx) - 27b^3x^3\cosh(3a)\text{Chi}(3bx) + 125b^3x^3\cosh(5a)\text{Chi}(5bx) - 2b^3x^3\sinh(a)\text{Shi}(bx) - 27b}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cosh[a + b*x]^2*Sinh[a + b*x]^3)/x^4,x]
```

```
[Out] (2*b*x*Cosh[a + b*x] + 3*b*x*Cosh[3*(a + b*x)] - 5*b*x*Cosh[5*(a + b*x)] -
2*b^3*x^3*Cosh[a]*CoshIntegral[b*x] - 27*b^3*x^3*Cosh[3*a]*CoshIntegral[3*b
*x] + 125*b^3*x^3*Cosh[5*a]*CoshIntegral[5*b*x] + 4*Sinh[a + b*x] + 2*b^2*x
^2*Sinh[a + b*x] + 2*Sinh[3*(a + b*x)] + 9*b^2*x^2*Sinh[3*(a + b*x)] - 2*Si
nh[5*(a + b*x)] - 25*b^2*x^2*Sinh[5*(a + b*x)] - 2*b^3*x^3*Sinh[a]*SinhInte
gral[b*x] - 27*b^3*x^3*Sinh[3*a]*SinhIntegral[3*b*x] + 125*b^3*x^3*Sinh[5*a
]*SinhIntegral[5*b*x])/(96*x^3)
```

fricas [A] time = 0.56, size = 392, normalized size = 1.65

$$-\frac{10\,bx\cosh\left(bx+a\right)^5 + 50\,bx\cosh\left(bx+a\right)\sinh\left(bx+a\right)^4 + 2\left(25\,b^2x^2+2\right)\sinh\left(bx+a\right)^5 - 6\,bx\cosh\left(bx+a\right)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^2*sinh(b*x+a)^3/x^4,x, algorithm="fricas")
```

```
[Out] -1/192*(10*b*x*cosh(b*x + a)^5 + 50*b*x*cosh(b*x + a)*sinh(b*x + a)^4 + 2*(
25*b^2*x^2 + 2)*sinh(b*x + a)^5 - 6*b*x*cosh(b*x + a)^3 - 2*(9*b^2*x^2 - 10
*(25*b^2*x^2 + 2)*cosh(b*x + a)^2 + 2)*sinh(b*x + a)^3 - 4*b*x*cosh(b*x + a
) + 2*(50*b*x*cosh(b*x + a)^3 - 9*b*x*cosh(b*x + a))*sinh(b*x + a)^2 - 125*
(b^3*x^3*Ei(5*b*x) + b^3*x^3*Ei(-5*b*x))*cosh(5*a) + 27*(b^3*x^3*Ei(3*b*x)
+ b^3*x^3*Ei(-3*b*x))*cosh(3*a) + 2*(b^3*x^3*Ei(b*x) + b^3*x^3*Ei(-b*x))*co
sh(a) + 2*(5*(25*b^2*x^2 + 2)*cosh(b*x + a)^4 - 2*b^2*x^2 - 3*(9*b^2*x^2 +
2)*cosh(b*x + a)^2 - 4)*sinh(b*x + a) - 125*(b^3*x^3*Ei(5*b*x) - b^3*x^3*Ei
(-5*b*x))*sinh(5*a) + 27*(b^3*x^3*Ei(3*b*x) - b^3*x^3*Ei(-3*b*x))*sinh(3*a)
 + 2*(b^3*x^3*Ei(b*x) - b^3*x^3*Ei(-b*x))*sinh(a))/x^3
```

giac [A] time = 0.16, size = 342, normalized size = 1.44

$$\frac{125\,b^3x^3\text{Ei}\left(5\,bx\right)e^{(5\,a)} - 27\,b^3x^3\text{Ei}\left(3\,bx\right)e^{(3\,a)} - 2\,b^3x^3\text{Ei}\left(-bx\right)e^{(-a)} - 27\,b^3x^3\text{Ei}\left(-3\,bx\right)e^{(-3\,a)} + 125\,b^3x^3\text{Ei}\left(-5\,b\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^2*sinh(b*x+a)^3/x^4,x, algorithm="giac")
```

```
[Out] 1/192*(125*b^3*x^3*Ei(5*b*x)*e^(5*a) - 27*b^3*x^3*Ei(3*b*x)*e^(3*a) - 2*b^3
*x^3*Ei(-b*x)*e^(-a) - 27*b^3*x^3*Ei(-3*b*x)*e^(-3*a) + 125*b^3*x^3*Ei(-5*b
```

```
*x)*e^(-5*a) - 2*b^3*x^3*Ei(b*x)*e^a - 25*b^2*x^2*e^(5*b*x + 5*a) + 9*b^2*x
^2*e^(3*b*x + 3*a) + 2*b^2*x^2*e^(b*x + a) - 2*b^2*x^2*e^(-b*x - a) - 9*b^2
*x^2*e^(-3*b*x - 3*a) + 25*b^2*x^2*e^(-5*b*x - 5*a) - 5*b*x*e^(5*b*x + 5*a)
 + 3*b*x*e^(3*b*x + 3*a) + 2*b*x*e^(b*x + a) + 2*b*x*e^(-b*x - a) + 3*b*x*e
^(-3*b*x - 3*a) - 5*b*x*e^(-5*b*x - 5*a) - 2*e^(5*b*x + 5*a) + 2*e^(3*b*x +
 3*a) + 4*e^(b*x + a) - 4*e^(-b*x - a) - 2*e^(-3*b*x - 3*a) + 2*e^(-5*b*x -
 5*a))/x^3
```

maple [A] time = 0.42, size = 356, normalized size = 1.50

$$\frac{25b^2 e^{-5bx-5a}}{192x}-\frac{5b\, e^{-5bx-5a}}{192x^2}+\frac{e^{-5bx-5a}}{96x^3}-\frac{125b^3 e^{-5a}\operatorname{Ei}(1,5bx)}{192}-\frac{3b^2 e^{-3bx-3a}}{64x}+\frac{b\, e^{-3bx-3a}}{64x^2}-\frac{e^{-3bx-3a}}{96x^3}+\frac{9b^3 e^{-3a}\operatorname{Ei}(1,3bx}{64}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)^2*sinh(b*x+a)^3/x^4,x)
```

```
[Out] 25/192*b^2*exp(-5*b*x-5*a)/x-5/192*b*exp(-5*b*x-5*a)/x^2+1/96*exp(-5*b*x-5*
a)/x^3-125/192*b^3*exp(-5*a)*Ei(1,5*b*x)-3/64*b^2*exp(-3*b*x-3*a)/x+1/64*b*
exp(-3*b*x-3*a)/x^2-1/96*exp(-3*b*x-3*a)/x^3+9/64*b^3*exp(-3*a)*Ei(1,3*b*x)
-1/96*b^2*exp(-b*x-a)/x+1/96*b*exp(-b*x-a)/x^2-1/48*exp(-b*x-a)/x^3+1/96*b^
3*exp(-a)*Ei(1,b*x)+1/48/x^3*exp(b*x+a)+1/96*b/x^2*exp(b*x+a)+1/96*b^2/x*ex
p(b*x+a)+1/96*b^3*exp(a)*Ei(1,-b*x)+1/96/x^3*exp(3*b*x+3*a)+1/64*b/x^2*exp(
3*b*x+3*a)+3/64*b^2/x*exp(3*b*x+3*a)+9/64*b^3*exp(3*a)*Ei(1,-3*b*x)-1/96/x^
3*exp(5*b*x+5*a)-5/192*b/x^2*exp(5*b*x+5*a)-25/192*b^2/x*exp(5*b*x+5*a)-125
/192*b^3*exp(5*a)*Ei(1,-5*b*x)
```

maxima [A] time = 0.47, size = 88, normalized size = 0.37

$$\frac{125}{32}\,b^3 e^{(-5\,a)}\Gamma\left(-3,5\,bx\right)-\frac{27}{32}\,b^3 e^{(-3\,a)}\Gamma\left(-3,3\,bx\right)-\frac{1}{16}\,b^3 e^{(-a)}\Gamma\left(-3,bx\right)-\frac{1}{16}\,b^3 e^{a}\Gamma\left(-3,-bx\right)-\frac{27}{32}\,b^3 e^{(3\,a)}\Gamma\left(-3,-3\,bx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^2*sinh(b*x+a)^3/x^4,x, algorithm="maxima")
```

```
[Out] 125/32*b^3*e^(-5*a)*gamma(-3, 5*b*x) - 27/32*b^3*e^(-3*a)*gamma(-3, 3*b*x)
- 1/16*b^3*e^(-a)*gamma(-3, b*x) - 1/16*b^3*e^a*gamma(-3, -b*x) - 27/32*b^3
*e^(3*a)*gamma(-3, -3*b*x) + 125/32*b^3*e^(5*a)*gamma(-3, -5*b*x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cosh\left(a+b\,x\right)^2\,\sinh\left(a+b\,x\right)^3}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cosh(a + b*x)^2*sinh(a + b*x)^3)/x^4,x)
```

```
[Out] int((cosh(a + b*x)^2*sinh(a + b*x)^3)/x^4, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh^3(a+bx)\cosh^2(a+bx)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)**2*sinh(b*x+a)**3/x**4,x)
```

```
[Out] Integral(sinh(a + b*x)**3*cosh(a + b*x)**2/x**4, x)
```

3.325 $\int x^m \cosh^3(a+bx)\sinh^3(a+bx)\,dx$

Optimal. Leaf size=155

$$\frac{e^{6a}2^{-m-7}3^{-m-1}x^m(-bx)^{-m}\Gamma(m+1,-6bx)}{b}-\frac{3e^{2a}2^{-m-7}x^m(-bx)^{-m}\Gamma(m+1,-2bx)}{b}-\frac{3e^{-2a}2^{-m-7}x^m(bx)^{-m}\Gamma(m+1,2b}{b}$$

```
[Out] 2^(-7-m)*3^(-1-m)*exp(6*a)*x^m*GAMMA(1+m,-6*b*x)/b/((-b*x)^m)-3*2^(-7-m)*ex
p(2*a)*x^m*GAMMA(1+m,-2*b*x)/b/((-b*x)^m)-3*2^(-7-m)*x^m*GAMMA(1+m,2*b*x)/b
/exp(2*a)/((b*x)^m)+2^(-7-m)*3^(-1-m)*x^m*GAMMA(1+m,6*b*x)/b/exp(6*a)/((b*x
)^m)
```

Rubi [A] time = 0.24, antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {5448, 3308, 2181}

$$\frac{e^{6a}2^{-m-7}3^{-m-1}x^m(-bx)^{-m}\text{Gamma}(m+1,-6bx)}{b}-\frac{3e^{2a}2^{-m-7}x^m(-bx)^{-m}\text{Gamma}(m+1,-2bx)}{b}-\frac{3e^{-2a}2^{-m-7}x^m(bx}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*Cosh[a + b*x]^3*Sinh[a + b*x]^3,x]
```

```
[Out] (2^(-7 - m)*3^(-1 - m)*E^(6*a)*x^m*Gamma[1 + m, -6*b*x])/(b*(-(b*x))^m) - (
3*2^(-7 - m)*E^(2*a)*x^m*Gamma[1 + m, -2*b*x])/(b*(-(b*x))^m) - (3*2^(-7 -
m)*x^m*Gamma[1 + m, 2*b*x])/(b*E^(2*a)*(b*x)^m) + (2^(-7 - m)*3^(-1 - m)*x^
m*Gamma[1 + m, 6*b*x])/(b*E^(6*a)*(b*x)^m)
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&  !I
ntegerQ[m]
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
```

```
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rubi steps

$$\int x^m\cosh^3(a+bx)\sinh^3(a+bx)\,dx=\int\left(-\frac{3}{32}x^m\sinh(2a+2bx)+\frac{1}{32}x^m\sinh(6a+6bx)\right)dx$$
$$=\frac{1}{32}\int x^m\sinh(6a+6bx)\,dx-\frac{3}{32}\int x^m\sinh(2a+2bx)\,dx$$
$$=\frac{1}{64}\int e^{-i(6ia+6ibx)}x^m\,dx-\frac{1}{64}\int e^{i(6ia+6ibx)}x^m\,dx-\frac{3}{64}\int e^{-i(2ia+2ibx)}x^m\,dx$$
$$=\frac{2^{-7-m}3^{-1-m}e^{6a}x^m(-bx)^{-m}\Gamma(1+m,-6bx)}{b}-\frac{3\,2^{-7-m}e^{2a}x^m(-bx)^{-m}\Gamma(1+$$

Mathematica [A] time = 0.16, size = 119, normalized size = 0.77

$$\frac{e^{-6a}2^{-m-7}3^{-m-1}x^m\left(-b^2x^2\right)^{-m}\left((-bx)^m\left(\Gamma(m+1,6bx)-e^{4a}3^{m+2}\Gamma(m+1,2bx)\right)+e^{12a}(bx)^m\Gamma(m+1,-6bx)-e^{8a}3}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*Cosh[a + b*x]^3*Sinh[a + b*x]^3,x]
```

```
[Out] (2^(-7 - m)*3^(-1 - m)*x^m*(E^(12*a)*(b*x)^m*Gamma[1 + m, -6*b*x] - 3^(2 +
m)*E^(8*a)*(b*x)^m*Gamma[1 + m, -2*b*x] + (-(b*x))^m*(-(3^(2 + m)*E^(4*a)*G
amma[1 + m, 2*b*x]) + Gamma[1 + m, 6*b*x])))/(b*E^(6*a)*(-(b^2*x^2))^m)
```

fricas [A] time = 0.65, size = 172, normalized size = 1.11

$$\frac{\cosh\left(m\log\left(6\,b\right)+6\,a\right)\Gamma\left(m+1,6\,bx\right)-9\,\cosh\left(m\log\left(2\,b\right)+2\,a\right)\Gamma\left(m+1,2\,bx\right)-9\,\cosh\left(m\log\left(-2\,b\right)-2\,a\right)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*cosh(b*x+a)^3*sinh(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/384*(cosh(m*log(6*b) + 6*a)*gamma(m + 1, 6*b*x) - 9*cosh(m*log(2*b) + 2*a
)*gamma(m + 1, 2*b*x) - 9*cosh(m*log(-2*b) - 2*a)*gamma(m + 1, -2*b*x) + co
sh(m*log(-6*b) - 6*a)*gamma(m + 1, -6*b*x) - gamma(m + 1, 6*b*x)*sinh(m*log
(6*b) + 6*a) + 9*gamma(m + 1, 2*b*x)*sinh(m*log(2*b) + 2*a) + 9*gamma(m + 1
, -2*b*x)*sinh(m*log(-2*b) - 2*a) - gamma(m + 1, -6*b*x)*sinh(m*log(-6*b) -
 6*a))/b
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^m \cosh(bx+a)^3 \sinh(bx+a)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*cosh(b*x+a)^3*sinh(b*x+a)^3,x, algorithm="giac")
```

```
[Out] integrate(x^m*cosh(b*x + a)^3*sinh(b*x + a)^3, x)
```

maple [F] time = 0.60, size = 0, normalized size = 0.00

$$\int x^m \left(\cosh^3(bx+a)\right)\left(\sinh^3(bx+a)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*cosh(b*x+a)^3*sinh(b*x+a)^3,x)
```

```
[Out] int(x^m*cosh(b*x+a)^3*sinh(b*x+a)^3,x)
```

maxima [A] time = 0.47, size = 117, normalized size = 0.75

$$\frac{1}{64}(6bx)^{-m-1}x^{m+1}e^{(-6a)}\Gamma(m+1,6bx) - \frac{3}{64}(2bx)^{-m-1}x^{m+1}e^{(-2a)}\Gamma(m+1,2bx) + \frac{3}{64}(-2bx)^{-m-1}x^{m+1}e^{(2a)}\Gamma(m$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*cosh(b*x+a)^3*sinh(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/64*(6*b*x)^(-m - 1)*x^(m + 1)*e^(-6*a)*gamma(m + 1, 6*b*x) - 3/64*(2*b*x)
^(-m - 1)*x^(m + 1)*e^(-2*a)*gamma(m + 1, 2*b*x) + 3/64*(-2*b*x)^(-m - 1)*x
^(m + 1)*e^(2*a)*gamma(m + 1, -2*b*x) - 1/64*(-6*b*x)^(-m - 1)*x^(m + 1)*e^
(6*a)*gamma(m + 1, -6*b*x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^m \cosh(a+b\,x)^3 \sinh(a+b\,x)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*cosh(a + b*x)^3*sinh(a + b*x)^3,x)
```

```
[Out] int(x^m*cosh(a + b*x)^3*sinh(a + b*x)^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^m \sinh^3(a+bx)\cosh^3(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*cosh(b*x+a)**3*sinh(b*x+a)**3,x)
```

```
[Out] Integral(x**m*sinh(a + b*x)**3*cosh(a + b*x)**3, x)
```

3.326 $\int x^3 \cosh^3(a+bx)\sinh^3(a+bx)\,dx$

Optimal. Leaf size=143

$$\frac{9\sinh(2a+2bx)}{256b^4}-\frac{\sinh(6a+6bx)}{6912b^4}-\frac{9x\cosh(2a+2bx)}{128b^3}+\frac{x\cosh(6a+6bx)}{1152b^3}+\frac{9x^2\sinh(2a+2bx)}{128b^2}-\frac{x^2\sinh(6a+6b}{384b^2}$$

```
[Out] -9/128*x*cosh(2*b*x+2*a)/b^3-3/64*x^3*cosh(2*b*x+2*a)/b+1/1152*x*cosh(6*b*x
+6*a)/b^3+1/192*x^3*cosh(6*b*x+6*a)/b+9/256*sinh(2*b*x+2*a)/b^4+9/128*x^2*s
inh(2*b*x+2*a)/b^2-1/6912*sinh(6*b*x+6*a)/b^4-1/384*x^2*sinh(6*b*x+6*a)/b^2
```

Rubi [A] time = 0.20, antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {5448, 3296, 2637}

$$\frac{9x^2\sinh(2a+2bx)}{128b^2}-\frac{x^2\sinh(6a+6bx)}{384b^2}+\frac{9\sinh(2a+2bx)}{256b^4}-\frac{\sinh(6a+6bx)}{6912b^4}-\frac{9x\cosh(2a+2bx)}{128b^3}+\frac{x\cosh(6a+6b}{1152b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Cosh[a + b*x]^3*Sinh[a + b*x]^3,x]
```

```
[Out] (-9*x*Cosh[2*a + 2*b*x])/(128*b^3) - (3*x^3*Cosh[2*a + 2*b*x])/(64*b) + (x*
Cosh[6*a + 6*b*x])/(1152*b^3) + (x^3*Cosh[6*a + 6*b*x])/(192*b) + (9*Sinh[2
*a + 2*b*x])/(256*b^4) + (9*x^2*Sinh[2*a + 2*b*x])/(128*b^2) - Sinh[6*a + 6
*b*x]/(6912*b^4) - (x^2*Sinh[6*a + 6*b*x])/(384*b^2)
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rubi steps

$$\int x^3\cosh^3(a+bx)\sinh^3(a+bx)\,dx = \int\left(-\frac{3}{32}x^3\sinh(2a+2bx)+\frac{1}{32}x^3\sinh(6a+6bx)\right)dx$$
$$=\frac{1}{32}\int x^3\sinh(6a+6bx)\,dx-\frac{3}{32}\int x^3\sinh(2a+2bx)\,dx$$
$$=-\frac{3x^3\cosh(2a+2bx)}{64b}+\frac{x^3\cosh(6a+6bx)}{192b}-\frac{\int x^2\cosh(6a+6bx)\,dx}{64b}+$$
$$=-\frac{3x^3\cosh(2a+2bx)}{64b}+\frac{x^3\cosh(6a+6bx)}{192b}+\frac{9x^2\sinh(2a+2bx)}{128b^2}-\frac{x^2\,\mathrm{s}}{}$$
$$=-\frac{9x\cosh(2a+2bx)}{128b^3}-\frac{3x^3\cosh(2a+2bx)}{64b}+\frac{x\cosh(6a+6bx)}{1152b^3}+\frac{x^3\,\mathrm{co}}{}$$
$$=-\frac{9x\cosh(2a+2bx)}{128b^3}-\frac{3x^3\cosh(2a+2bx)}{64b}+\frac{x\cosh(6a+6bx)}{1152b^3}+\frac{x^3\,\mathrm{co}}{}$$

Mathematica [A] time = 0.90, size = 90, normalized size = 0.63

$$-\frac{-3\left(6b^3x^3+bx\right)\cosh(6(a+bx))+81bx\left(2b^2x^2+3\right)\cosh(2(a+bx))+\sinh(2(a+bx))\left(\left(18b^2x^2+1\right)\cosh(4(a\right.\right.}{3456b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Cosh[a + b*x]^3*Sinh[a + b*x]^3,x]
```

```
[Out] -1/3456*(81*b*x*(3 + 2*b^2*x^2)*Cosh[2*(a + b*x)] - 3*(b*x + 6*b^3*x^3)*Cos
h[6*(a + b*x)] + (-121 - 234*b^2*x^2 + (1 + 18*b^2*x^2)*Cosh[4*(a + b*x)])*
Sinh[2*(a + b*x)])/b^4
```

fricas [A] time = 0.81, size = 248, normalized size = 1.73

$$\frac{3\left(6\,b^3x^3+bx\right)\cosh\left(bx+a\right)^6-10\left(18\,b^2x^2+1\right)\cosh\left(bx+a\right)^3\sinh\left(bx+a\right)^3+45\left(6\,b^3x^3+bx\right)\cosh\left(bx+a\right)^2}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*cosh(b*x+a)^3*sinh(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/3456*(3*(6*b^3*x^3 + b*x)*cosh(b*x + a)^6 - 10*(18*b^2*x^2 + 1)*cosh(b*x
+ a)^3*sinh(b*x + a)^3 + 45*(6*b^3*x^3 + b*x)*cosh(b*x + a)^2*sinh(b*x + a)
^4 - 3*(18*b^2*x^2 + 1)*cosh(b*x + a)*sinh(b*x + a)^5 + 3*(6*b^3*x^3 + b*x)
*sinh(b*x + a)^6 - 81*(2*b^3*x^3 + 3*b*x)*cosh(b*x + a)^2 - 9*(18*b^3*x^3 -
 5*(6*b^3*x^3 + b*x)*cosh(b*x + a)^4 + 27*b*x)*sinh(b*x + a)^2 - 3*((18*b^2
*x^2 + 1)*cosh(b*x + a)^5 - 81*(2*b^2*x^2 + 1)*cosh(b*x + a))*sinh(b*x + a)
)/b^4
```

giac [A] time = 0.15, size = 145, normalized size = 1.01

$$\frac{\left(36\,b^3x^3-18\,b^2x^2+6\,bx-1\right)e^{(6\,bx+6\,a)}}{13824\,b^4}-\frac{3\left(4\,b^3x^3-6\,b^2x^2+6\,bx-3\right)e^{(2\,bx+2\,a)}}{512\,b^4}-\frac{3\left(4\,b^3x^3+6\,b^2x^2+6\,bx+3\right)e^{(-2}}{512\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*cosh(b*x+a)^3*sinh(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/13824*(36*b^3*x^3 - 18*b^2*x^2 + 6*b*x - 1)*e^(6*b*x + 6*a)/b^4 - 3/512*(
4*b^3*x^3 - 6*b^2*x^2 + 6*b*x - 3)*e^(2*b*x + 2*a)/b^4 - 3/512*(4*b^3*x^3 +
 6*b^2*x^2 + 6*b*x + 3)*e^(-2*b*x - 2*a)/b^4 + 1/13824*(36*b^3*x^3 + 18*b^2
*x^2 + 6*b*x + 1)*e^(-6*b*x - 6*a)/b^4
```

maple [B] time = 0.43, size = 499, normalized size = 3.49

$$\frac{\frac{(bx+a)^3\left(\sinh^2(bx+a)\right)\left(\cosh^4(bx+a)\right)}{6}-\frac{(bx+a)^3\left(\cosh^4(bx+a)\right)}{12}-\frac{(bx+a)^2\sinh(bx+a)\left(\cosh^5(bx+a)\right)}{12}+\frac{(bx+a)^2\sinh(bx+a)\left(\cosh^3(bx+a)\right)}{12}+\frac{(bx+}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*cosh(b*x+a)^3*sinh(b*x+a)^3,x)
```

```
[Out] 1/b^4*(1/6*(b*x+a)^3*sinh(b*x+a)^2*cosh(b*x+a)^4-1/12*(b*x+a)^3*cosh(b*x+a)
^4-1/12*(b*x+a)^2*sinh(b*x+a)*cosh(b*x+a)^5+1/12*(b*x+a)^2*sinh(b*x+a)*cosh
(b*x+a)^3+1/8*(b*x+a)^2*cosh(b*x+a)*sinh(b*x+a)+1/24*(b*x+a)^3+1/36*(b*x+a)
*cosh(b*x+a)^6-1/216*sinh(b*x+a)*cosh(b*x+a)^5+1/216*cosh(b*x+a)^3*sinh(b*x
+a)+5/72*cosh(b*x+a)*sinh(b*x+a)+5/72*b*x+5/72*a-1/24*(b*x+a)*cosh(b*x+a)^4
-1/8*(b*x+a)*cosh(b*x+a)^2-3*a*(1/6*(b*x+a)^2*sinh(b*x+a)^2*cosh(b*x+a)^4-1
/12*(b*x+a)^2*cosh(b*x+a)^4-1/18*(b*x+a)*sinh(b*x+a)*cosh(b*x+a)^5+1/18*(b*
x+a)*sinh(b*x+a)*cosh(b*x+a)^3+1/12*(b*x+a)*cosh(b*x+a)*sinh(b*x+a)+1/24*(b
*x+a)^2+1/108*cosh(b*x+a)^6-1/72*cosh(b*x+a)^4-1/24*cosh(b*x+a)^2)+3*a^2*(1
/6*(b*x+a)*sinh(b*x+a)^2*cosh(b*x+a)^4-1/12*(b*x+a)*cosh(b*x+a)^4-1/36*sinh
(b*x+a)*cosh(b*x+a)^5+1/36*cosh(b*x+a)^3*sinh(b*x+a)+1/24*cosh(b*x+a)*sinh(
b*x+a)+1/24*b*x+1/24*a)-a^3*(1/6*cosh(b*x+a)^4*sinh(b*x+a)^2-1/12*cosh(b*x+
a)^4))
```

maxima [A] time = 0.35, size = 171, normalized size = 1.20

$$\frac{\left(36\,b^3x^3e^{(6\,a)}-18\,b^2x^2e^{(6\,a)}+6\,bxe^{(6\,a)}-e^{(6\,a)}\right)e^{(6\,bx)}}{13824\,b^4}-\frac{3\left(4\,b^3x^3e^{(2\,a)}-6\,b^2x^2e^{(2\,a)}+6\,bxe^{(2\,a)}-3\,e^{(2\,a)}\right)e^{(2\,bx)}}{512\,b^4}-\frac{3\left(4\,b\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*cosh(b*x+a)^3*sinh(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/13824*(36*b^3*x^3*e^(6*a) - 18*b^2*x^2*e^(6*a) + 6*b*x*e^(6*a) - e^(6*a))
*e^(6*b*x)/b^4 - 3/512*(4*b^3*x^3*e^(2*a) - 6*b^2*x^2*e^(2*a) + 6*b*x*e^(2*
a) - 3*e^(2*a))*e^(2*b*x)/b^4 - 3/512*(4*b^3*x^3 + 6*b^2*x^2 + 6*b*x + 3)*e
^(-2*b*x - 2*a)/b^4 + 1/13824*(36*b^3*x^3 + 18*b^2*x^2 + 6*b*x + 1)*e^(-6*b
*x - 6*a)/b^4
```

mupad [B] time = 1.72, size = 126, normalized size = 0.88

$$\frac{\frac{9\,x^2\sinh(2\,a+2\,b\,x)}{128}-\frac{x^2\sinh(6\,a+6\,b\,x)}{384}}{b^2}-\frac{\frac{3\,x^3\cosh(2\,a+2\,b\,x)}{64}-\frac{x^3\cosh(6\,a+6\,b\,x)}{192}}{b}-\frac{\frac{9\,x\cosh(2\,a+2\,b\,x)}{128}-\frac{x\cosh(6\,a+6\,b\,x)}{1152}}{b^3}+\frac{9\sinh(2\,a\,-}{256\,b^{}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*cosh(a + b*x)^3*sinh(a + b*x)^3,x)
```

```
[Out] ((9*x^2*sinh(2*a + 2*b*x))/128 - (x^2*sinh(6*a + 6*b*x))/384)/b^2 - ((3*x^3
*cosh(2*a + 2*b*x))/64 - (x^3*cosh(6*a + 6*b*x))/192)/b - ((9*x*cosh(2*a +
2*b*x))/128 - (x*cosh(6*a + 6*b*x))/1152)/b^3 + (9*sinh(2*a + 2*b*x))/(256*
b^4) - sinh(6*a + 6*b*x)/(6912*b^4)
```

sympy [A] time = 14.19, size = 314, normalized size = 2.20

$$\begin{cases} -\frac{x^3\sinh^6(a+bx)}{24b}+\frac{x^3\sinh^4(a+bx)\cosh^2(a+bx)}{8b}+\frac{x^3\sinh^2(a+bx)\cosh^4(a+bx)}{8b}-\frac{x^3\cosh^6(a+bx)}{24b}+\frac{x^2\sinh^5(a+bx)\cosh(a+bx)}{8b^2}-\frac{x^2\sin}{} \\ \frac{x^4\sinh^3(a)\cosh^3(a)}{4} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*cosh(b*x+a)**3*sinh(b*x+a)**3,x)
```

```
[Out] Piecewise((-x**3*sinh(a + b*x)**6/(24*b) + x**3*sinh(a + b*x)**4*cosh(a + b
*x)**2/(8*b) + x**3*sinh(a + b*x)**2*cosh(a + b*x)**4/(8*b) - x**3*cosh(a +
 b*x)**6/(24*b) + x**2*sinh(a + b*x)**5*cosh(a + b*x)/(8*b**2) - x**2*sinh(
a + b*x)**3*cosh(a + b*x)**3/(3*b**2) + x**2*sinh(a + b*x)*cosh(a + b*x)**5
/(8*b**2) - 5*x*sinh(a + b*x)**6/(72*b**3) + x*sinh(a + b*x)**4*cosh(a + b*
x)**2/(12*b**3) + x*sinh(a + b*x)**2*cosh(a + b*x)**4/(12*b**3) - 5*x*cosh(
a + b*x)**6/(72*b**3) + 5*sinh(a + b*x)**5*cosh(a + b*x)/(72*b**4) - 31*sin
h(a + b*x)**3*cosh(a + b*x)**3/(216*b**4) + 5*sinh(a + b*x)*cosh(a + b*x)**
5/(72*b**4), Ne(b, 0)), (x**4*sinh(a)**3*cosh(a)**3/4, True))
```

3.327 $\int x^2 \cosh^3(a+bx)\sinh^3(a+bx)\,dx$

Optimal. Leaf size=105

$$-\frac{3\cosh(2a+2bx)}{128b^3}+\frac{\cosh(6a+6bx)}{3456b^3}+\frac{3x\sinh(2a+2bx)}{64b^2}-\frac{x\sinh(6a+6bx)}{576b^2}-\frac{3x^2\cosh(2a+2bx)}{64b}+\frac{x^2\cosh(6a+}{192b}$$

```
[Out] -3/128*cosh(2*b*x+2*a)/b^3-3/64*x^2*cosh(2*b*x+2*a)/b+1/3456*cosh(6*b*x+6*a
)/b^3+1/192*x^2*cosh(6*b*x+6*a)/b+3/64*x*sinh(2*b*x+2*a)/b^2-1/576*x*sinh(6
*b*x+6*a)/b^2
```

Rubi [A] time = 0.14, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {5448, 3296, 2638}

$$\frac{3x\sinh(2a+2bx)}{64b^2}-\frac{x\sinh(6a+6bx)}{576b^2}-\frac{3\cosh(2a+2bx)}{128b^3}+\frac{\cosh(6a+6bx)}{3456b^3}-\frac{3x^2\cosh(2a+2bx)}{64b}+\frac{x^2\cosh(6a+6}{192b}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Cosh[a + b*x]^3*Sinh[a + b*x]^3,x]
```

```
[Out] (-3*Cosh[2*a + 2*b*x])/(128*b^3) - (3*x^2*Cosh[2*a + 2*b*x])/(64*b) + Cosh[
6*a + 6*b*x]/(3456*b^3) + (x^2*Cosh[6*a + 6*b*x])/(192*b) + (3*x*Sinh[2*a +
 2*b*x])/(64*b^2) - (x*Sinh[6*a + 6*b*x])/(576*b^2)
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rubi steps

$$\int x^2\cosh^3(a+bx)\sinh^3(a+bx)\,dx = \int\left(-\frac{3}{32}x^2\sinh(2a+2bx)+\frac{1}{32}x^2\sinh(6a+6bx)\right)dx$$
$$= \frac{1}{32}\int x^2\sinh(6a+6bx)\,dx - \frac{3}{32}\int x^2\sinh(2a+2bx)\,dx$$
$$= -\frac{3x^2\cosh(2a+2bx)}{64b}+\frac{x^2\cosh(6a+6bx)}{192b}-\frac{\int x\cosh(6a+6bx)\,dx}{96b}+$$
$$= -\frac{3x^2\cosh(2a+2bx)}{64b}+\frac{x^2\cosh(6a+6bx)}{192b}+\frac{3x\sinh(2a+2bx)}{64b^2}-\frac{x\sin}{}$$
$$= -\frac{3\cosh(2a+2bx)}{128b^3}-\frac{3x^2\cosh(2a+2bx)}{64b}+\frac{\cosh(6a+6bx)}{3456b^3}+\frac{x^2\cosh(}{1}$$

Mathematica [A] time = 0.23, size = 72, normalized size = 0.69

$$\frac{-81\left(2b^2x^2+1\right)\cosh(2(a+bx))+\left(18b^2x^2+1\right)\cosh(6(a+bx))+6bx(27\sinh(2(a+bx))-\sinh(6(a+bx)))}{3456b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Cosh[a + b*x]^3*Sinh[a + b*x]^3,x]
```

```
[Out] (-81*(1 + 2*b^2*x^2)*Cosh[2*(a + b*x)] + (1 + 18*b^2*x^2)*Cosh[6*(a + b*x)]
 + 6*b*x*(27*Sinh[2*(a + b*x)] - Sinh[6*(a + b*x)]))/(3456*b^3)
```

fricas [B] time = 0.50, size = 202, normalized size = 1.92

$$-\frac{120\,bx\cosh(bx+a)^3\sinh(bx+a)^3+36\,bx\cosh(bx+a)\sinh(bx+a)^5-\left(18\,b^2x^2+1\right)\cosh(bx+a)^6-15\,(}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cosh(b*x+a)^3*sinh(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -1/3456*(120*b*x*cosh(b*x + a)^3*sinh(b*x + a)^3 + 36*b*x*cosh(b*x + a)*sin
h(b*x + a)^5 - (18*b^2*x^2 + 1)*cosh(b*x + a)^6 - 15*(18*b^2*x^2 + 1)*cosh(
b*x + a)^2*sinh(b*x + a)^4 - (18*b^2*x^2 + 1)*sinh(b*x + a)^6 + 81*(2*b^2*x
^2 + 1)*cosh(b*x + a)^2 - 3*(5*(18*b^2*x^2 + 1)*cosh(b*x + a)^4 - 54*b^2*x^
2 - 27)*sinh(b*x + a)^2 + 36*(b*x*cosh(b*x + a)^5 - 9*b*x*cosh(b*x + a))*si
nh(b*x + a))/b^3
```

giac [A] time = 0.14, size = 113, normalized size = 1.08

$$\frac{\left(18\,b^2x^2-6\,bx+1\right)e^{(6\,bx+6\,a)}}{6912\,b^3}-\frac{3\left(2\,b^2x^2-2\,bx+1\right)e^{(2\,bx+2\,a)}}{256\,b^3}-\frac{3\left(2\,b^2x^2+2\,bx+1\right)e^{(-2\,bx-2\,a)}}{256\,b^3}+\frac{\left(18\,b^2x^2+6\,bx+\right.}{6912\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*cosh(b*x+a)^3*sinh(b*x+a)^3,x, algorithm="giac")

[Out] 1/6912*(18*b^2*x^2 - 6*b*x + 1)*e^(6*b*x + 6*a)/b^3 - 3/256*(2*b^2*x^2 - 2*b*x + 1)*e^(2*b*x + 2*a)/b^3 - 3/256*(2*b^2*x^2 + 2*b*x + 1)*e^(-2*b*x - 2*a)/b^3 + 1/6912*(18*b^2*x^2 + 6*b*x + 1)*e^(-6*b*x - 6*a)/b^3

maple [B] time = 0.33, size = 276, normalized size = 2.63

$$\frac{\frac{(bx+a)^2\left(\sinh^2(bx+a)\right)\left(\cosh^4(bx+a)\right)}{6}-\frac{(bx+a)^2\left(\cosh^4(bx+a)\right)}{12}-\frac{(bx+a)\sinh(bx+a)\left(\cosh^5(bx+a)\right)}{18}+\frac{(bx+a)\sinh(bx+a)\left(\cosh^3(bx+a)\right)}{18}+\frac{(bx+a)\cdots}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2*cosh(b*x+a)^3*sinh(b*x+a)^3,x)

[Out] 1/b^3*(1/6*(b*x+a)^2*sinh(b*x+a)^2*cosh(b*x+a)^4-1/12*(b*x+a)^2*cosh(b*x+a)^4-1/18*(b*x+a)*sinh(b*x+a)*cosh(b*x+a)^5+1/18*(b*x+a)*sinh(b*x+a)*cosh(b*x+a)^3+1/12*(b*x+a)*cosh(b*x+a)*sinh(b*x+a)+1/24*(b*x+a)^2+1/108*cosh(b*x+a)^6-1/72*cosh(b*x+a)^4-1/24*cosh(b*x+a)^2-2*a*(1/6*(b*x+a)*sinh(b*x+a)^2*cosh(b*x+a)^4-1/12*(b*x+a)*cosh(b*x+a)^4-1/36*sinh(b*x+a)*cosh(b*x+a)^5+1/36*cosh(b*x+a)^3*sinh(b*x+a)+1/24*cosh(b*x+a)*sinh(b*x+a)+1/24*b*x+1/24*a)+a^2*(1/6*cosh(b*x+a)^4*sinh(b*x+a)^2-1/12*cosh(b*x+a)^4))

maxima [A] time = 0.34, size = 127, normalized size = 1.21

$$\frac{\left(18\,b^2x^2e^{(6\,a)}-6\,bxe^{(6\,a)}+e^{(6\,a)}\right)e^{(6\,bx)}}{6912\,b^3}-\frac{3\left(2\,b^2x^2e^{(2\,a)}-2\,bxe^{(2\,a)}+e^{(2\,a)}\right)e^{(2\,bx)}}{256\,b^3}-\frac{3\left(2\,b^2x^2+2\,bx+1\right)e^{(-2\,bx-2\,a)}}{256\,b^3}+\frac{(1\cdots}{}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*cosh(b*x+a)^3*sinh(b*x+a)^3,x, algorithm="maxima")

[Out] 1/6912*(18*b^2*x^2*e^(6*a) - 6*b*x*e^(6*a) + e^(6*a))*e^(6*b*x)/b^3 - 3/256*(2*b^2*x^2*e^(2*a) - 2*b*x*e^(2*a) + e^(2*a))*e^(2*b*x)/b^3 - 3/256*(2*b^2*x^2 + 2*b*x + 1)*e^(-2*b*x - 2*a)/b^3 + 1/6912*(18*b^2*x^2 + 6*b*x + 1)*e^(-6*b*x - 6*a)/b^3

mupad [B] time = 0.32, size = 89, normalized size = 0.85

$$-\frac{\frac{3\cosh(2\,a+2\,b\,x)}{128}-\frac{\cosh(6\,a+6\,b\,x)}{3456}+b^2\left(\frac{3\,x^2\cosh(2\,a+2\,b\,x)}{64}-\frac{x^2\cosh(6\,a+6\,b\,x)}{192}\right)-b\left(\frac{3\,x\sinh(2\,a+2\,b\,x)}{64}-\frac{x\sinh(6\,a+6\,b\,x)}{576}\right)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*cosh(a + b*x)^3*sinh(a + b*x)^3,x)
```

```
[Out] -((3*cosh(2*a + 2*b*x))/128 - cosh(6*a + 6*b*x)/3456 + b^2*((3*x^2*cosh(2*a
 + 2*b*x))/64 - (x^2*cosh(6*a + 6*b*x))/192) - b*((3*x*sinh(2*a + 2*b*x))/6
4 - (x*sinh(6*a + 6*b*x))/576))/b^3
```

sympy [A] time = 8.37, size = 212, normalized size = 2.02

$$\begin{cases} -\frac{x^2\sinh^6(a+bx)}{24b} + \frac{x^2\sinh^4(a+bx)\cosh^2(a+bx)}{8b} + \frac{x^2\sinh^2(a+bx)\cosh^4(a+bx)}{8b} - \frac{x^2\cosh^6(a+bx)}{24b} + \frac{x\sinh^5(a+bx)\cosh(a+bx)}{12b^2} - \frac{2x\sinh\ldots \\ \frac{x^3\sinh^3(a)\cosh^3(a)}{3} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*cosh(b*x+a)**3*sinh(b*x+a)**3,x)
```

```
[Out] Piecewise((-x**2*sinh(a + b*x)**6/(24*b) + x**2*sinh(a + b*x)**4*cosh(a + b
*x)**2/(8*b) + x**2*sinh(a + b*x)**2*cosh(a + b*x)**4/(8*b) - x**2*cosh(a +
 b*x)**6/(24*b) + x*sinh(a + b*x)**5*cosh(a + b*x)/(12*b**2) - 2*x*sinh(a +
 b*x)**3*cosh(a + b*x)**3/(9*b**2) + x*sinh(a + b*x)*cosh(a + b*x)**5/(12*b
**2) - sinh(a + b*x)**6/(72*b**3) + sinh(a + b*x)**2*cosh(a + b*x)**4/(18*b
**3) - 7*cosh(a + b*x)**6/(216*b**3), Ne(b, 0)), (x**3*sinh(a)**3*cosh(a)**
3/3, True))
```

3.328 $\int x\cosh^3(a+bx)\sinh^3(a+bx)\,dx$

Optimal. Leaf size=67

$$\frac{3\sinh(2a+2bx)}{128b^2}-\frac{\sinh(6a+6bx)}{1152b^2}-\frac{3x\cosh(2a+2bx)}{64b}+\frac{x\cosh(6a+6bx)}{192b}$$

```
[Out] -3/64*x*cosh(2*b*x+2*a)/b+1/192*x*cosh(6*b*x+6*a)/b+3/128*sinh(2*b*x+2*a)/b
^2-1/1152*sinh(6*b*x+6*a)/b^2
```

Rubi [A] time = 0.07, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {5448, 3296, 2637}

$$\frac{3\sinh(2a+2bx)}{128b^2}-\frac{\sinh(6a+6bx)}{1152b^2}-\frac{3x\cosh(2a+2bx)}{64b}+\frac{x\cosh(6a+6bx)}{192b}$$

Antiderivative was successfully verified.

```
[In] Int[x*Cosh[a + b*x]^3*Sinh[a + b*x]^3,x]
```

```
[Out] (-3*x*Cosh[2*a + 2*b*x])/(64*b) + (x*Cosh[6*a + 6*b*x])/(192*b) + (3*Sinh[2
*a + 2*b*x])/(128*b^2) - Sinh[6*a + 6*b*x]/(1152*b^2)
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rubi steps

$$\int x\cosh^3(a+bx)\sinh^3(a+bx)\,dx = \int\left(-\frac{3}{32}x\sinh(2a+2bx)+\frac{1}{32}x\sinh(6a+6bx)\right)dx$$
$$= \frac{1}{32}\int x\sinh(6a+6bx)\,dx - \frac{3}{32}\int x\sinh(2a+2bx)\,dx$$
$$= -\frac{3x\cosh(2a+2bx)}{64b}+\frac{x\cosh(6a+6bx)}{192b}-\frac{\int\cosh(6a+6bx)\,dx}{192b}+\frac{3\int c}{}$$
$$= -\frac{3x\cosh(2a+2bx)}{64b}+\frac{x\cosh(6a+6bx)}{192b}+\frac{3\sinh(2a+2bx)}{128b^2}-\frac{\sinh(6a}{1152}$$

Mathematica [A] time = 0.15, size = 50, normalized size = 0.75

$$-\frac{-27\sinh(2(a+bx))+\sinh(6(a+bx))+54bx\cosh(2(a+bx))-6bx\cosh(6(a+bx))}{1152b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Cosh[a + b*x]^3*Sinh[a + b*x]^3,x]
```

```
[Out] -1/1152*(54*b*x*Cosh[2*(a + b*x)] - 6*b*x*Cosh[6*(a + b*x)] - 27*Sinh[2*(a
+ b*x)] + Sinh[6*(a + b*x)])/b^2
```

fricas [B] time = 0.65, size = 148, normalized size = 2.21

$$\frac{3\,bx\cosh\left(bx+a\right)^{6}+45\,bx\cosh\left(bx+a\right)^{2}\sinh\left(bx+a\right)^{4}+3\,bx\sinh\left(bx+a\right)^{6}-10\,\cosh\left(bx+a\right)^{3}\sinh\left(bx+a\right)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x+a)^3*sinh(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/576*(3*b*x*cosh(b*x + a)^6 + 45*b*x*cosh(b*x + a)^2*sinh(b*x + a)^4 + 3*b
*x*sinh(b*x + a)^6 - 10*cosh(b*x + a)^3*sinh(b*x + a)^3 - 3*cosh(b*x + a)*s
inh(b*x + a)^5 - 27*b*x*cosh(b*x + a)^2 + 9*(5*b*x*cosh(b*x + a)^4 - 3*b*x)
*sinh(b*x + a)^2 - 3*(cosh(b*x + a)^5 - 9*cosh(b*x + a))*sinh(b*x + a))/b^2
```

giac [A] time = 0.15, size = 81, normalized size = 1.21

$$\frac{(6\,bx-1)e^{(6\,bx+6\,a)}}{2304\,b^2}-\frac{3\,(2\,bx-1)e^{(2\,bx+2\,a)}}{256\,b^2}-\frac{3\,(2\,bx+1)e^{(-2\,bx-2\,a)}}{256\,b^2}+\frac{(6\,bx+1)e^{(-6\,bx-6\,a)}}{2304\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x+a)^3*sinh(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/2304*(6*b*x - 1)*e^(6*b*x + 6*a)/b^2 - 3/256*(2*b*x - 1)*e^(2*b*x + 2*a)/
b^2 - 3/256*(2*b*x + 1)*e^(-2*b*x - 2*a)/b^2 + 1/2304*(6*b*x + 1)*e^(-6*b*x
 - 6*a)/b^2
```

maple [B] time = 0.33, size = 129, normalized size = 1.93

$$\frac{\frac{(bx+a)\left(\sinh^2(bx+a)\right)\left(\cosh^4(bx+a)\right)}{6}-\frac{(bx+a)\left(\cosh^4(bx+a)\right)}{12}-\frac{\sinh(bx+a)\left(\cosh^5(bx+a)\right)}{36}+\frac{\left(\cosh^3(bx+a)\right)\sinh(bx+a)}{36}+\frac{\cosh(bx+a)\sinh(bx+a)}{24}}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cosh(b*x+a)^3*sinh(b*x+a)^3,x)
```

```
[Out] 1/b^2*(1/6*(b*x+a)*sinh(b*x+a)^2*cosh(b*x+a)^4-1/12*(b*x+a)*cosh(b*x+a)^4-1
/36*sinh(b*x+a)*cosh(b*x+a)^5+1/36*cosh(b*x+a)^3*sinh(b*x+a)+1/24*cosh(b*x+
a)*sinh(b*x+a)+1/24*b*x+1/24*a-a*(1/6*cosh(b*x+a)^4*sinh(b*x+a)^2-1/12*cosh
(b*x+a)^4))
```

maxima [A] time = 0.44, size = 91, normalized size = 1.36

$$\frac{\left(6\,bxe^{(6\,a)}-e^{(6\,a)}\right)e^{(6\,bx)}}{2304\,b^2}-\frac{3\left(2\,bxe^{(2\,a)}-e^{(2\,a)}\right)e^{(2\,bx)}}{256\,b^2}-\frac{3\,(2\,bx+1)e^{(-2\,bx-2\,a)}}{256\,b^2}+\frac{(6\,bx+1)e^{(-6\,bx-6\,a)}}{2304\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x+a)^3*sinh(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/2304*(6*b*x*e^(6*a) - e^(6*a))*e^(6*b*x)/b^2 - 3/256*(2*b*x*e^(2*a) - e^(
2*a))*e^(2*b*x)/b^2 - 3/256*(2*b*x + 1)*e^(-2*b*x - 2*a)/b^2 + 1/2304*(6*b*
x + 1)*e^(-6*b*x - 6*a)/b^2
```

mupad [B] time = 0.23, size = 55, normalized size = 0.82

$$-\frac{\frac{\sinh(6\,a+6\,b\,x)}{1152}-\frac{3\sinh(2\,a+2\,b\,x)}{128}+b\,\left(\frac{3\,x\cosh(2\,a+2\,b\,x)}{64}-\frac{x\cosh(6\,a+6\,b\,x)}{192}\right)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cosh(a + b*x)^3*sinh(a + b*x)^3,x)
```

```
[Out] -(sinh(6*a + 6*b*x)/1152 - (3*sinh(2*a + 2*b*x))/128 + b*((3*x*cosh(2*a + 2
*b*x))/64 - (x*cosh(6*a + 6*b*x))/192))/b^2
```

sympy [A] time = 4.94, size = 148, normalized size = 2.21

$$\begin{cases} -\frac{x\sinh^6(a+bx)}{24b} + \frac{x\sinh^4(a+bx)\cosh^2(a+bx)}{8b} + \frac{x\sinh^2(a+bx)\cosh^4(a+bx)}{8b} - \frac{x\cosh^6(a+bx)}{24b} + \frac{\sinh^5(a+bx)\cosh(a+bx)}{24b^2} - \frac{\sinh^3(a+bx)}{9} \\ \frac{x^2\sinh^3(a)\cosh^3(a)}{2} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x+a)**3*sinh(b*x+a)**3,x)
```

```
[Out] Piecewise((-x*sinh(a + b*x)**6/(24*b) + x*sinh(a + b*x)**4*cosh(a + b*x)**2
/(8*b) + x*sinh(a + b*x)**2*cosh(a + b*x)**4/(8*b) - x*cosh(a + b*x)**6/(24
*b) + sinh(a + b*x)**5*cosh(a + b*x)/(24*b**2) - sinh(a + b*x)**3*cosh(a +
b*x)**3/(9*b**2) + sinh(a + b*x)*cosh(a + b*x)**5/(24*b**2), Ne(b, 0)), (x*
*2*sinh(a)**3*cosh(a)**3/2, True))
```

3.329 $\int \cosh^3(a+bx)\sinh^3(a+bx)\,dx$

Optimal. Leaf size=31

$$\frac{\sinh^6(a+bx)}{6b}+\frac{\sinh^4(a+bx)}{4b}$$

```
[Out] 1/4*sinh(b*x+a)^4/b+1/6*sinh(b*x+a)^6/b
```

Rubi [A] time = 0.04, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2564, 14}

$$\frac{\sinh^6(a+bx)}{6b}+\frac{\sinh^4(a+bx)}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[a + b*x]^3*Sinh[a + b*x]^3,x]
```

```
[Out] Sinh[a + b*x]^4/(4*b) + Sinh[a + b*x]^6/(6*b)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rubi steps

$$\begin{aligned}\int \cosh^3(a+bx)\sinh^3(a+bx)\,dx &= \frac{\text{Subst}\left(\int x^3\left(1-x^2\right)\,dx,x,i\sinh(a+bx)\right)}{b}\\ &= \frac{\text{Subst}\left(\int \left(x^3-x^5\right)\,dx,x,i\sinh(a+bx)\right)}{b}\\ &= \frac{\sinh^4(a+bx)}{4b}+\frac{\sinh^6(a+bx)}{6b}\end{aligned}$$

Mathematica [A] time = 0.01, size = 35, normalized size = 1.13

$$\frac{1}{8}\left(\frac{\cosh(6(a+bx))}{24b}-\frac{3\cosh(2(a+bx))}{8b}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[a + b*x]^3*Sinh[a + b*x]^3,x]
```

```
[Out] ((-3*Cosh[2*(a + b*x)])/(8*b) + Cosh[6*(a + b*x)]/(24*b))/8
```

fricas [B] time = 0.67, size = 72, normalized size = 2.32

$$\frac{\cosh\left(bx+a\right)^{6}+15\cosh\left(bx+a\right)^{2}\sinh\left(bx+a\right)^{4}+\sinh\left(bx+a\right)^{6}+3\left(5\cosh\left(bx+a\right)^{4}-3\right)\sinh\left(bx+a\right)^{2}-9}{192\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^3*sinh(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/192*(cosh(b*x + a)^6 + 15*cosh(b*x + a)^2*sinh(b*x + a)^4 + sinh(b*x + a)
^6 + 3*(5*cosh(b*x + a)^4 - 3)*sinh(b*x + a)^2 - 9*cosh(b*x + a)^2)/b
```

giac [B] time = 0.15, size = 57, normalized size = 1.84

$$\frac{e^{(6\,bx+6\,a)}}{384\,b}-\frac{3\,e^{(2\,bx+2\,a)}}{128\,b}-\frac{3\,e^{(-2\,bx-2\,a)}}{128\,b}+\frac{e^{(-6\,bx-6\,a)}}{384\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^3*sinh(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/384*e^(6*b*x + 6*a)/b - 3/128*e^(2*b*x + 2*a)/b - 3/128*e^(-2*b*x - 2*a)/
b + 1/384*e^(-6*b*x - 6*a)/b
```

maple [A] time = 0.06, size = 34, normalized size = 1.10

$$\frac{\frac{\left(\cosh^{4}(bx+a)\right)\left(\sinh^{2}(bx+a)\right)}{6}-\frac{\left(\cosh^{4}(bx+a)\right)}{12}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)^3*sinh(b*x+a)^3,x)
```

```
[Out] 1/b*(1/6*cosh(b*x+a)^4*sinh(b*x+a)^2-1/12*cosh(b*x+a)^4)
```

maxima [B] time = 0.34, size = 56, normalized size = 1.81

$$-\frac{\left(9\,e^{(-4\,bx-4\,a)}-1\right)e^{(6\,bx+6\,a)}}{384\,b}-\frac{9\,e^{(-2\,bx-2\,a)}-e^{(-6\,bx-6\,a)}}{384\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^3*sinh(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -1/384*(9*e^(-4*b*x - 4*a) - 1)*e^(6*b*x + 6*a)/b - 1/384*(9*e^(-2*b*x - 2*
a) - e^(-6*b*x - 6*a))/b
```

mupad [B] time = 1.52, size = 26, normalized size = 0.84

$$\frac{2\sinh\left(a+b\,x\right)^6+3\sinh\left(a+b\,x\right)^4}{12\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a + b*x)^3*sinh(a + b*x)^3,x)
```

```
[Out] (3*sinh(a + b*x)^4 + 2*sinh(a + b*x)^6)/(12*b)
```

sympy [A] time = 2.59, size = 42, normalized size = 1.35

$$\begin{cases}\frac{\sinh^2(a+bx)\cosh^4(a+bx)}{4b}-\frac{\cosh^6(a+bx)}{12b} & \text{for } b\neq 0\\ x\sinh^3(a)\cosh^3(a) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)**3*sinh(b*x+a)**3,x)
```

```
[Out] Piecewise((sinh(a + b*x)**2*cosh(a + b*x)**4/(4*b) - cosh(a + b*x)**6/(12*b
), Ne(b, 0)), (x*sinh(a)**3*cosh(a)**3, True))
```

3.330 $\int \frac{\cosh^3(a+bx)\sinh^3(a+bx)}{x}\,dx$

Optimal. Leaf size=53

$$-\frac{3}{32}\sinh(2a)\text{Chi}(2bx)+\frac{1}{32}\sinh(6a)\text{Chi}(6bx)-\frac{3}{32}\cosh(2a)\text{Shi}(2bx)+\frac{1}{32}\cosh(6a)\text{Shi}(6bx)$$

```
[Out] -3/32*cosh(2*a)*Shi(2*b*x)+1/32*cosh(6*a)*Shi(6*b*x)-3/32*Chi(2*b*x)*sinh(2
*a)+1/32*Chi(6*b*x)*sinh(6*a)
```

Rubi [A] time = 0.16, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {5448, 3303, 3298, 3301}

$$-\frac{3}{32}\sinh(2a)\text{Chi}(2bx)+\frac{1}{32}\sinh(6a)\text{Chi}(6bx)-\frac{3}{32}\cosh(2a)\text{Shi}(2bx)+\frac{1}{32}\cosh(6a)\text{Shi}(6bx)$$

Antiderivative was successfully verified.

```
[In] Int[(Cosh[a + b*x]^3*Sinh[a + b*x]^3)/x,x]
```

```
[Out] (-3*CoshIntegral[2*b*x]*Sinh[2*a])/32 + (CoshIntegral[6*b*x]*Sinh[6*a])/32
- (3*Cosh[2*a]*SinhIntegral[2*b*x])/32 + (Cosh[6*a]*SinhIntegral[6*b*x])/32
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\cosh^3(a+bx)\sinh^3(a+bx)}{x}\,dx &= \int \left(-\frac{3\sinh(2a+2bx)}{32x}+\frac{\sinh(6a+6bx)}{32x}\right)dx\\ &= \frac{1}{32}\int \frac{\sinh(6a+6bx)}{x}\,dx-\frac{3}{32}\int \frac{\sinh(2a+2bx)}{x}\,dx\\ &= -\left(\frac{1}{32}(3\cosh(2a))\int \frac{\sinh(2bx)}{x}\,dx\right)+\frac{1}{32}\cosh(6a)\int \frac{\sinh(6bx)}{x}\,dx-\frac{1}{32}(3\\ &= -\frac{3}{32}\mathrm{Chi}(2bx)\sinh(2a)+\frac{1}{32}\mathrm{Chi}(6bx)\sinh(6a)-\frac{3}{32}\cosh(2a)\mathrm{Shi}(2bx)+\frac{1}{3}\end{aligned}$$

Mathematica [A] time = 0.18, size = 47, normalized size = 0.89

$$\frac{1}{32}(\sinh(6a)\mathrm{Chi}(6bx)-6\sinh(a)\cosh(a)\mathrm{Chi}(2bx)-3\cosh(2a)\mathrm{Shi}(2bx)+\cosh(6a)\mathrm{Shi}(6bx))$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cosh[a + b*x]^3*Sinh[a + b*x]^3)/x,x]
```

```
[Out] (-6*Cosh[a]*CoshIntegral[2*b*x]*Sinh[a] + CoshIntegral[6*b*x]*Sinh[6*a] - 3
*Cosh[2*a]*SinhIntegral[2*b*x] + Cosh[6*a]*SinhIntegral[6*b*x])/32
```

fricas [A] time = 0.68, size = 73, normalized size = 1.38

$$\frac{1}{64}\left(\mathrm{Ei}\left(6\,bx\right)-\mathrm{Ei}\left(-6\,bx\right)\right)\cosh\left(6\,a\right)-\frac{3}{64}\left(\mathrm{Ei}\left(2\,bx\right)-\mathrm{Ei}\left(-2\,bx\right)\right)\cosh\left(2\,a\right)+\frac{1}{64}\left(\mathrm{Ei}\left(6\,bx\right)+\mathrm{Ei}\left(-6\,bx\right)\right)\sinh\left(6\,a\right)-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^3*sinh(b*x+a)^3/x,x, algorithm="fricas")
```

```
[Out] 1/64*(Ei(6*b*x) - Ei(-6*b*x))*cosh(6*a) - 3/64*(Ei(2*b*x) - Ei(-2*b*x))*cos
h(2*a) + 1/64*(Ei(6*b*x) + Ei(-6*b*x))*sinh(6*a) - 3/64*(Ei(2*b*x) + Ei(-2*
b*x))*sinh(2*a)
```

giac [A] time = 0.13, size = 45, normalized size = 0.85

$$\frac{1}{64}\,\mathrm{Ei}\left(6\,bx\right)e^{(6\,a)}-\frac{3}{64}\,\mathrm{Ei}\left(2\,bx\right)e^{(2\,a)}+\frac{3}{64}\,\mathrm{Ei}\left(-2\,bx\right)e^{(-2\,a)}-\frac{1}{64}\,\mathrm{Ei}\left(-6\,bx\right)e^{(-6\,a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^3*sinh(b*x+a)^3/x,x, algorithm="giac")
```

```
[Out] 1/64*Ei(6*b*x)*e^(6*a) - 3/64*Ei(2*b*x)*e^(2*a) + 3/64*Ei(-2*b*x)*e^(-2*a)
- 1/64*Ei(-6*b*x)*e^(-6*a)
```

maple [A] time = 0.68, size = 50, normalized size = 0.94

$$\frac{e^{-6a}\,\mathrm{Ei}\,(1,6bx)}{64}-\frac{3\,e^{-2a}\,\mathrm{Ei}\,(1,2bx)}{64}+\frac{3\,e^{2a}\,\mathrm{Ei}\,(1,-2bx)}{64}-\frac{e^{6a}\,\mathrm{Ei}\,(1,-6bx)}{64}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)^3*sinh(b*x+a)^3/x,x)
```

```
[Out] 1/64*exp(-6*a)*Ei(1,6*b*x)-3/64*exp(-2*a)*Ei(1,2*b*x)+3/64*exp(2*a)*Ei(1,-2
*b*x)-1/64*exp(6*a)*Ei(1,-6*b*x)
```

maxima [A] time = 0.53, size = 45, normalized size = 0.85

$$\frac{1}{64}\,\mathrm{Ei}\,(6\,bx)\,e^{(6\,a)}-\frac{3}{64}\,\mathrm{Ei}\,(2\,bx)\,e^{(2\,a)}+\frac{3}{64}\,\mathrm{Ei}\,(-2\,bx)\,e^{(-2\,a)}-\frac{1}{64}\,\mathrm{Ei}\,(-6\,bx)\,e^{(-6\,a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^3*sinh(b*x+a)^3/x,x, algorithm="maxima")
```

```
[Out] 1/64*Ei(6*b*x)*e^(6*a) - 3/64*Ei(2*b*x)*e^(2*a) + 3/64*Ei(-2*b*x)*e^(-2*a)
- 1/64*Ei(-6*b*x)*e^(-6*a)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\cosh\left(a+b\,x\right)^3\,\sinh\left(a+b\,x\right)^3}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cosh(a + b*x)^3*sinh(a + b*x)^3)/x,x)
```

```
[Out] int((cosh(a + b*x)^3*sinh(a + b*x)^3)/x, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh^{3}\left(a+bx\right)\cosh^{3}\left(a+bx\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)**3*sinh(b*x+a)**3/x,x)
```

```
[Out] Integral(sinh(a + b*x)**3*cosh(a + b*x)**3/x, x)
```

3.331 $\int \frac{\cosh^3(a+bx)\sinh^3(a+bx)}{x^2}\,dx$

Optimal. Leaf size=89

$$-\frac{3}{16}b\cosh(2a)\text{Chi}(2bx)+\frac{3}{16}b\cosh(6a)\text{Chi}(6bx)-\frac{3}{16}b\sinh(2a)\text{Shi}(2bx)+\frac{3}{16}b\sinh(6a)\text{Shi}(6bx)+\frac{3\sinh(2a+2b}{32x}$$

```
[Out] -3/16*b*Chi(2*b*x)*cosh(2*a)+3/16*b*Chi(6*b*x)*cosh(6*a)-3/16*b*Shi(2*b*x)*
sinh(2*a)+3/16*b*Shi(6*b*x)*sinh(6*a)+3/32*sinh(2*b*x+2*a)/x-1/32*sinh(6*b*
x+6*a)/x
```

Rubi [A] time = 0.20, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {5448, 3297, 3303, 3298, 3301}

$$-\frac{3}{16}b\cosh(2a)\text{Chi}(2bx)+\frac{3}{16}b\cosh(6a)\text{Chi}(6bx)-\frac{3}{16}b\sinh(2a)\text{Shi}(2bx)+\frac{3}{16}b\sinh(6a)\text{Shi}(6bx)+\frac{3\sinh(2a+2b}{32x}$$

Antiderivative was successfully verified.

```
[In] Int[(Cosh[a + b*x]^3*Sinh[a + b*x]^3)/x^2,x]
```

```
[Out] (-3*b*Cosh[2*a]*CoshIntegral[2*b*x])/16 + (3*b*Cosh[6*a]*CoshIntegral[6*b*x
])/16 + (3*Sinh[2*a + 2*b*x])/(32*x) - Sinh[6*a + 6*b*x]/(32*x) - (3*b*Sinh
[2*a]*SinhIntegral[2*b*x])/16 + (3*b*Sinh[6*a]*SinhIntegral[6*b*x])/16
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\cosh^3(a+bx)\sinh^3(a+bx)}{x^2}\,dx &= \int \left(-\frac{3\sinh(2a+2bx)}{32x^2}+\frac{\sinh(6a+6bx)}{32x^2}\right)dx\\ &= \frac{1}{32}\int \frac{\sinh(6a+6bx)}{x^2}\,dx-\frac{3}{32}\int \frac{\sinh(2a+2bx)}{x^2}\,dx\\ &= \frac{3\sinh(2a+2bx)}{32x}-\frac{\sinh(6a+6bx)}{32x}-\frac{1}{16}(3b)\int \frac{\cosh(2a+2bx)}{x}\,dx+\frac{1}{16}(\\ &= \frac{3\sinh(2a+2bx)}{32x}-\frac{\sinh(6a+6bx)}{32x}-\frac{1}{16}(3b\cosh(2a))\int \frac{\cosh(2bx)}{x}\,dx+\\ &= -\frac{3}{16}b\cosh(2a)\text{Chi}(2bx)+\frac{3}{16}b\cosh(6a)\text{Chi}(6bx)+\frac{3\sinh(2a+2bx)}{32x}-\end{aligned}$$

Mathematica [A] time = 0.24, size = 78, normalized size = 0.88

$$-\frac{6bx\cosh(2a)\text{Chi}(2bx)-6bx\cosh(6a)\text{Chi}(6bx)+6bx\sinh(2a)\text{Shi}(2bx)-6bx\sinh(6a)\text{Shi}(6bx)-3\sinh(2(a}{32x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cosh[a + b*x]^3*Sinh[a + b*x]^3)/x^2,x]
```

```
[Out] -1/32*(6*b*x*Cosh[2*a]*CoshIntegral[2*b*x] - 6*b*x*Cosh[6*a]*CoshIntegral[6
*b*x] - 3*Sinh[2*(a + b*x)] + Sinh[6*(a + b*x)] + 6*b*x*Sinh[2*a]*SinhInteg
ral[2*b*x] - 6*b*x*Sinh[6*a]*SinhIntegral[6*b*x])/x
```

fricas [B] time = 0.50, size = 159, normalized size = 1.79

$$-\frac{20\cosh(bx+a)^3\sinh(bx+a)^3+6\cosh(bx+a)\sinh(bx+a)^5-3\left(bx\text{Ei}(6bx)+bx\text{Ei}(-6bx)\right)\cosh(6a)+}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^3*sinh(b*x+a)^3/x^2,x, algorithm="fricas")
```

```
[Out] -1/32*(20*cosh(b*x + a)^3*sinh(b*x + a)^3 + 6*cosh(b*x + a)*sinh(b*x + a)^5
 - 3*(b*x*Ei(6*b*x) + b*x*Ei(-6*b*x))*cosh(6*a) + 3*(b*x*Ei(2*b*x) + b*x*Ei
(-2*b*x))*cosh(2*a) + 6*(cosh(b*x + a)^5 - cosh(b*x + a))*sinh(b*x + a) - 3
*(b*x*Ei(6*b*x) - b*x*Ei(-6*b*x))*sinh(6*a) + 3*(b*x*Ei(2*b*x) - b*x*Ei(-2*
b*x))*sinh(2*a))/x
```

giac [A] time = 0.13, size = 100, normalized size = 1.12

$$\frac{6\,bx\mathrm{Ei}\left(6\,bx\right)e^{(6\,a)} - 6\,bx\mathrm{Ei}\left(2\,bx\right)e^{(2\,a)} - 6\,bx\mathrm{Ei}\left(-2\,bx\right)e^{(-2\,a)} + 6\,bx\mathrm{Ei}\left(-6\,bx\right)e^{(-6\,a)} - e^{(6\,bx+6\,a)} + 3\,e^{(2\,bx+2\,a)} - 3\,e^{(-}}{64\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^3*sinh(b*x+a)^3/x^2,x, algorithm="giac")
```

```
[Out] 1/64*(6*b*x*Ei(6*b*x)*e^(6*a) - 6*b*x*Ei(2*b*x)*e^(2*a) - 6*b*x*Ei(-2*b*x)*
e^(-2*a) + 6*b*x*Ei(-6*b*x)*e^(-6*a) - e^(6*b*x + 6*a) + 3*e^(2*b*x + 2*a)
- 3*e^(-2*b*x - 2*a) + e^(-6*b*x - 6*a))/x
```

maple [A] time = 0.68, size = 110, normalized size = 1.24

$$\frac{e^{-6bx-6a}}{64x} - \frac{3b\,e^{-6a}\,\mathrm{Ei}\left(1,6bx\right)}{32} - \frac{3\,e^{-2bx-2a}}{64x} + \frac{3b\,e^{-2a}\,\mathrm{Ei}\left(1,2bx\right)}{32} + \frac{3\,e^{2bx+2a}}{64x} + \frac{3b\,e^{2a}\,\mathrm{Ei}\left(1,-2bx\right)}{32} - \frac{e^{6bx+6a}}{64x} - \frac{3b\,e^{6a}\,\mathrm{Ei}\left(1,\right.}{32}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)^3*sinh(b*x+a)^3/x^2,x)
```

```
[Out] 1/64*exp(-6*b*x-6*a)/x-3/32*b*exp(-6*a)*Ei(1,6*b*x)-3/64*exp(-2*b*x-2*a)/x+
3/32*b*exp(-2*a)*Ei(1,2*b*x)+3/64*exp(2*b*x+2*a)/x+3/32*b*exp(2*a)*Ei(1,-2*
b*x)-1/64/x*exp(6*b*x+6*a)-3/32*b*exp(6*a)*Ei(1,-6*b*x)
```

maxima [A] time = 0.48, size = 53, normalized size = 0.60

$$\frac{3}{32}\,be^{(-6\,a)}\Gamma\left(-1, 6\,bx\right) - \frac{3}{32}\,be^{(-2\,a)}\Gamma\left(-1, 2\,bx\right) - \frac{3}{32}\,be^{(2\,a)}\Gamma\left(-1, -2\,bx\right) + \frac{3}{32}\,be^{(6\,a)}\Gamma\left(-1, -6\,bx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^3*sinh(b*x+a)^3/x^2,x, algorithm="maxima")
```

```
[Out] 3/32*b*e^(-6*a)*gamma(-1, 6*b*x) - 3/32*b*e^(-2*a)*gamma(-1, 2*b*x) - 3/32*
b*e^(2*a)*gamma(-1, -2*b*x) + 3/32*b*e^(6*a)*gamma(-1, -6*b*x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cosh (a + b\,x)^3\,\sinh (a + b\,x)^3}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cosh(a + b*x)^3*sinh(a + b*x)^3)/x^2,x)
```

```
[Out] int((cosh(a + b*x)^3*sinh(a + b*x)^3)/x^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh^3 (a + bx) \cosh^3 (a + bx)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)**3*sinh(b*x+a)**3/x**2,x)
```

```
[Out] Integral(sinh(a + b*x)**3*cosh(a + b*x)**3/x**2, x)
```

3.332 $\int \frac{\cosh^3(a+bx)\sinh^3(a+bx)}{x^3}\,dx$

Optimal. Leaf size=131

$$-\frac{3}{16}b^2\sinh(2a)\mathrm{Chi}(2bx)+\frac{9}{16}b^2\sinh(6a)\mathrm{Chi}(6bx)-\frac{3}{16}b^2\cosh(2a)\mathrm{Shi}(2bx)+\frac{9}{16}b^2\cosh(6a)\mathrm{Shi}(6bx)+\frac{3\sinh(2a+}{64x^2}$$

```
[Out] 3/32*b*cosh(2*b*x+2*a)/x-3/32*b*cosh(6*b*x+6*a)/x-3/16*b^2*cosh(2*a)*Shi(2*
b*x)+9/16*b^2*cosh(6*a)*Shi(6*b*x)-3/16*b^2*Chi(2*b*x)*sinh(2*a)+9/16*b^2*C
hi(6*b*x)*sinh(6*a)+3/64*sinh(2*b*x+2*a)/x^2-1/64*sinh(6*b*x+6*a)/x^2
```

Rubi [A] time = 0.26, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {5448, 3297, 3303, 3298, 3301}

$$-\frac{3}{16}b^2\sinh(2a)\mathrm{Chi}(2bx)+\frac{9}{16}b^2\sinh(6a)\mathrm{Chi}(6bx)-\frac{3}{16}b^2\cosh(2a)\mathrm{Shi}(2bx)+\frac{9}{16}b^2\cosh(6a)\mathrm{Shi}(6bx)+\frac{3\sinh(2a+}{64x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Cosh[a + b*x]^3*Sinh[a + b*x]^3)/x^3,x]
```

```
[Out] (3*b*Cosh[2*a + 2*b*x])/(32*x) - (3*b*Cosh[6*a + 6*b*x])/(32*x) - (3*b^2*Co
shIntegral[2*b*x]*Sinh[2*a])/16 + (9*b^2*CoshIntegral[6*b*x]*Sinh[6*a])/16
+ (3*Sinh[2*a + 2*b*x])/(64*x^2) - Sinh[6*a + 6*b*x]/(64*x^2) - (3*b^2*Cosh
[2*a]*SinhIntegral[2*b*x])/16 + (9*b^2*Cosh[6*a]*SinhIntegral[6*b*x])/16
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rubi steps

$$\int \frac{\cosh^3(a+bx)\sinh^3(a+bx)}{x^3}\,dx = \int \left(-\frac{3\sinh(2a+2bx)}{32x^3}+\frac{\sinh(6a+6bx)}{32x^3}\right)dx$$
$$= \frac{1}{32}\int \frac{\sinh(6a+6bx)}{x^3}\,dx - \frac{3}{32}\int \frac{\sinh(2a+2bx)}{x^3}\,dx$$
$$= \frac{3\sinh(2a+2bx)}{64x^2} - \frac{\sinh(6a+6bx)}{64x^2} - \frac{1}{32}(3b)\int \frac{\cosh(2a+2bx)}{x^2}\,dx + \frac{1}{32}($$
$$= \frac{3b\cosh(2a+2bx)}{32x} - \frac{3b\cosh(6a+6bx)}{32x} + \frac{3\sinh(2a+2bx)}{64x^2} - \frac{\sinh(6a+}{64x^2}$$
$$= \frac{3b\cosh(2a+2bx)}{32x} - \frac{3b\cosh(6a+6bx)}{32x} + \frac{3\sinh(2a+2bx)}{64x^2} - \frac{\sinh(6a+}{64x^2}$$
$$= \frac{3b\cosh(2a+2bx)}{32x} - \frac{3b\cosh(6a+6bx)}{32x} - \frac{3}{16}b^2\mathrm{Chi}(2bx)\sinh(2a) + \frac{9}{16}b^2$$

Mathematica [A] time = 0.25, size = 118, normalized size = 0.90

$$-\frac{12b^2x^2\sinh(2a)\mathrm{Chi}(2bx) - 36b^2x^2\sinh(6a)\mathrm{Chi}(6bx) + 12b^2x^2\cosh(2a)\mathrm{Shi}(2bx) - 36b^2x^2\cosh(6a)\mathrm{Shi}(6bx)}{64x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cosh[a + b*x]^3*Sinh[a + b*x]^3)/x^3,x]
```

```
[Out] -1/64*(-6*b*x*Cosh[2*(a + b*x)] + 6*b*x*Cosh[6*(a + b*x)] + 12*b^2*x^2*Cosh
Integral[2*b*x]*Sinh[2*a] - 36*b^2*x^2*CoshIntegral[6*b*x]*Sinh[6*a] - 3*Si
nh[2*(a + b*x)] + Sinh[6*(a + b*x)] + 12*b^2*x^2*Cosh[2*a]*SinhIntegral[2*b
*x] - 36*b^2*x^2*Cosh[6*a]*SinhIntegral[6*b*x])/x^2
```

fricas [B] time = 1.09, size = 274, normalized size = 2.09

$$-\frac{3\,bx\cosh\left(bx+a\right)^{6}+45\,bx\cosh\left(bx+a\right)^{2}\sinh\left(bx+a\right)^{4}+3\,bx\sinh\left(bx+a\right)^{6}+10\,\cosh\left(bx+a\right)^{3}\sinh\left(bx+a\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^3*sinh(b*x+a)^3/x^3,x, algorithm="fricas")
```

```
[Out] -1/32*(3*b*x*cosh(b*x + a)^6 + 45*b*x*cosh(b*x + a)^2*sinh(b*x + a)^4 + 3*b
*x*sinh(b*x + a)^6 + 10*cosh(b*x + a)^3*sinh(b*x + a)^3 + 3*cosh(b*x + a)*s
inh(b*x + a)^5 - 3*b*x*cosh(b*x + a)^2 + 3*(15*b*x*cosh(b*x + a)^4 - b*x)*s
inh(b*x + a)^2 - 9*(b^2*x^2*Ei(6*b*x) - b^2*x^2*Ei(-6*b*x))*cosh(6*a) + 3*(
b^2*x^2*Ei(2*b*x) - b^2*x^2*Ei(-2*b*x))*cosh(2*a) + 3*(cosh(b*x + a)^5 - co
sh(b*x + a))*sinh(b*x + a) - 9*(b^2*x^2*Ei(6*b*x) + b^2*x^2*Ei(-6*b*x))*sin
h(6*a) + 3*(b^2*x^2*Ei(2*b*x) + b^2*x^2*Ei(-2*b*x))*sinh(2*a))/x^2
```

giac [A] time = 0.13, size = 168, normalized size = 1.28

$$\frac{36\,b^{2}x^{2}\mathrm{Ei}\left(6\,bx\right)e^{(6\,a)}-12\,b^{2}x^{2}\mathrm{Ei}\left(2\,bx\right)e^{(2\,a)}+12\,b^{2}x^{2}\mathrm{Ei}\left(-2\,bx\right)e^{(-2\,a)}-36\,b^{2}x^{2}\mathrm{Ei}\left(-6\,bx\right)e^{(-6\,a)}-6\,bxe^{(6\,bx+6\,a)}+}{128\,x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^3*sinh(b*x+a)^3/x^3,x, algorithm="giac")
```

```
[Out] 1/128*(36*b^2*x^2*Ei(6*b*x)*e^(6*a) - 12*b^2*x^2*Ei(2*b*x)*e^(2*a) + 12*b^2
*x^2*Ei(-2*b*x)*e^(-2*a) - 36*b^2*x^2*Ei(-6*b*x)*e^(-6*a) - 6*b*x*e^(6*b*x
+ 6*a) + 6*b*x*e^(2*b*x + 2*a) + 6*b*x*e^(-2*b*x - 2*a) - 6*b*x*e^(-6*b*x -
 6*a) - e^(6*b*x + 6*a) + 3*e^(2*b*x + 2*a) - 3*e^(-2*b*x - 2*a) + e^(-6*b*
x - 6*a))/x^2
```

maple [A] time = 0.69, size = 178, normalized size = 1.36

$$-\frac{3b\,\mathrm{e}^{-6bx-6a}}{64x}+\frac{\mathrm{e}^{-6bx-6a}}{128x^{2}}+\frac{9b^{2}\mathrm{e}^{-6a}\,\mathrm{Ei}\left(1,6bx\right)}{32}+\frac{3b\,\mathrm{e}^{-2bx-2a}}{64x}-\frac{3\,\mathrm{e}^{-2bx-2a}}{128x^{2}}-\frac{3b^{2}\mathrm{e}^{-2a}\,\mathrm{Ei}\left(1,2bx\right)}{32}+\frac{3\,\mathrm{e}^{2bx+2a}}{128x^{2}}+\frac{3b\,\mathrm{e}^{2bx+2a}}{64x}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)^3*sinh(b*x+a)^3/x^3,x)
```

```
[Out] -3/64*b*exp(-6*b*x-6*a)/x+1/128*exp(-6*b*x-6*a)/x^2+9/32*b^2*exp(-6*a)*Ei(1
,6*b*x)+3/64*b*exp(-2*b*x-2*a)/x-3/128*exp(-2*b*x-2*a)/x^2-3/32*b^2*exp(-2*
a)*Ei(1,2*b*x)+3/128*exp(2*b*x+2*a)/x^2+3/64*b*exp(2*b*x+2*a)/x+3/32*b^2*ex
p(2*a)*Ei(1,-2*b*x)-1/128/x^2*exp(6*b*x+6*a)-3/64*b/x*exp(6*b*x+6*a)-9/32*b
^2*exp(6*a)*Ei(1,-6*b*x)
```

maxima [A] time = 0.44, size = 61, normalized size = 0.47

$$\frac{9}{16}\,b^2e^{(-6\,a)}\Gamma\left(-2,6\,bx\right)-\frac{3}{16}\,b^2e^{(-2\,a)}\Gamma\left(-2,2\,bx\right)+\frac{3}{16}\,b^2e^{(2\,a)}\Gamma\left(-2,-2\,bx\right)-\frac{9}{16}\,b^2e^{(6\,a)}\Gamma\left(-2,-6\,bx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^3*sinh(b*x+a)^3/x^3,x, algorithm="maxima")
```

```
[Out] 9/16*b^2*e^(-6*a)*gamma(-2, 6*b*x) - 3/16*b^2*e^(-2*a)*gamma(-2, 2*b*x) + 3
/16*b^2*e^(2*a)*gamma(-2, -2*b*x) - 9/16*b^2*e^(6*a)*gamma(-2, -6*b*x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cosh\left(a+b\,x\right)^3\,\sinh\left(a+b\,x\right)^3}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cosh(a + b*x)^3*sinh(a + b*x)^3)/x^3,x)
```

```
[Out] int((cosh(a + b*x)^3*sinh(a + b*x)^3)/x^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh^{3}\left(a+bx\right)\cosh^{3}\left(a+bx\right)}{x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)**3*sinh(b*x+a)**3/x**3,x)
```

```
[Out] Integral(sinh(a + b*x)**3*cosh(a + b*x)**3/x**3, x)
```

3.333 $\int \frac{\cosh^3(a+bx)\sinh^3(a+bx)}{x^4}\,dx$

Optimal. Leaf size=169

$$-\frac{1}{8}b^3\cosh(2a)\text{Chi}(2bx)+\frac{9}{8}b^3\cosh(6a)\text{Chi}(6bx)-\frac{1}{8}b^3\sinh(2a)\text{Shi}(2bx)+\frac{9}{8}b^3\sinh(6a)\text{Shi}(6bx)+\frac{b^2\sinh(2a+2b}{16x}$$

```
[Out] -1/8*b^3*Chi(2*b*x)*cosh(2*a)+9/8*b^3*Chi(6*b*x)*cosh(6*a)+1/32*b*cosh(2*b*
x+2*a)/x^2-1/32*b*cosh(6*b*x+6*a)/x^2-1/8*b^3*Shi(2*b*x)*sinh(2*a)+9/8*b^3*
Shi(6*b*x)*sinh(6*a)+1/32*sinh(2*b*x+2*a)/x^3+1/16*b^2*sinh(2*b*x+2*a)/x-1/
96*sinh(6*b*x+6*a)/x^3-3/16*b^2*sinh(6*b*x+6*a)/x
```

Rubi [A] time = 0.32, antiderivative size = 169, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {5448, 3297, 3303, 3298, 3301}

$$-\frac{1}{8}b^3\cosh(2a)\text{Chi}(2bx)+\frac{9}{8}b^3\cosh(6a)\text{Chi}(6bx)-\frac{1}{8}b^3\sinh(2a)\text{Shi}(2bx)+\frac{9}{8}b^3\sinh(6a)\text{Shi}(6bx)+\frac{b^2\sinh(2a+2b}{16x}$$

Antiderivative was successfully verified.

```
[In] Int[(Cosh[a + b*x]^3*Sinh[a + b*x]^3)/x^4,x]
```

```
[Out] (b*Cosh[2*a + 2*b*x])/(32*x^2) - (b*Cosh[6*a + 6*b*x])/(32*x^2) - (b^3*Cosh
[2*a]*CoshIntegral[2*b*x])/8 + (9*b^3*Cosh[6*a]*CoshIntegral[6*b*x])/8 + Si
nh[2*a + 2*b*x]/(32*x^3) + (b^2*Sinh[2*a + 2*b*x])/(16*x) - Sinh[6*a + 6*b*
x]/(96*x^3) - (3*b^2*Sinh[6*a + 6*b*x])/(16*x) - (b^3*Sinh[2*a]*SinhIntegra
l[2*b*x])/8 + (9*b^3*Sinh[6*a]*SinhIntegral[6*b*x])/8
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
```

```
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cosh^3(a+bx)\sinh^3(a+bx)}{x^4}\,dx &= \int \left(-\frac{3\sinh(2a+2bx)}{32x^4}+\frac{\sinh(6a+6bx)}{32x^4}\right)dx \\
&= \frac{1}{32}\int \frac{\sinh(6a+6bx)}{x^4}\,dx-\frac{3}{32}\int \frac{\sinh(2a+2bx)}{x^4}\,dx \\
&= \frac{\sinh(2a+2bx)}{32x^3}-\frac{\sinh(6a+6bx)}{96x^3}-\frac{1}{16}b\int \frac{\cosh(2a+2bx)}{x^3}\,dx+\frac{1}{16}b\int \\
&= \frac{b\cosh(2a+2bx)}{32x^2}-\frac{b\cosh(6a+6bx)}{32x^2}+\frac{\sinh(2a+2bx)}{32x^3}-\frac{\sinh(6a+6bx)}{96x^3} \\
&= \frac{b\cosh(2a+2bx)}{32x^2}-\frac{b\cosh(6a+6bx)}{32x^2}+\frac{\sinh(2a+2bx)}{32x^3}+\frac{b^2\sinh(2a+2}{16x} \\
&= \frac{b\cosh(2a+2bx)}{32x^2}-\frac{b\cosh(6a+6bx)}{32x^2}+\frac{\sinh(2a+2bx)}{32x^3}+\frac{b^2\sinh(2a+2}{16x} \\
&= \frac{b\cosh(2a+2bx)}{32x^2}-\frac{b\cosh(6a+6bx)}{32x^2}-\frac{1}{8}b^3\cosh(2a)\mathrm{Chi}(2bx)+\frac{9}{8}b^3\cos
\end{aligned}$$

Mathematica [A] time = 0.34, size = 150, normalized size = 0.89

$$-\frac{12b^3x^3\cosh(2a)\mathrm{Chi}(2bx)-108b^3x^3\cosh(6a)\mathrm{Chi}(6bx)+12b^3x^3\sinh(2a)\mathrm{Shi}(2bx)-108b^3x^3\sinh(6a)\mathrm{Shi}(6bx}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cosh[a + b*x]^3*Sinh[a + b*x]^3)/x^4,x]
```

```
[Out] -1/96*(-3*b*x*Cosh[2*(a + b*x)] + 3*b*x*Cosh[6*(a + b*x)] + 12*b^3*x^3*Cosh
[2*a]*CoshIntegral[2*b*x] - 108*b^3*x^3*Cosh[6*a]*CoshIntegral[6*b*x] - 3*S
inh[2*(a + b*x)] - 6*b^2*x^2*Sinh[2*(a + b*x)] + Sinh[6*(a + b*x)] + 18*b^2
*x^2*Sinh[6*(a + b*x)] + 12*b^3*x^3*Sinh[2*a]*SinhIntegral[2*b*x] - 108*b^3
*x^3*Sinh[6*a]*SinhIntegral[6*b*x])/x^3
```

fricas [B] time = 0.78, size = 315, normalized size = 1.86

$$-\frac{3\,bx\cosh\left(bx+a\right)^{6}+45\,bx\cosh\left(bx+a\right)^{2}\sinh\left(bx+a\right)^{4}+3\,bx\sinh\left(bx+a\right)^{6}+20\left(18\,b^{2}x^{2}+1\right)\cosh\left(bx+a\right)^{3}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^3*sinh(b*x+a)^3/x^4,x, algorithm="fricas")
```

```
[Out] -1/96*(3*b*x*cosh(b*x + a)^6 + 45*b*x*cosh(b*x + a)^2*sinh(b*x + a)^4 + 3*b
*x*sinh(b*x + a)^6 + 20*(18*b^2*x^2 + 1)*cosh(b*x + a)^3*sinh(b*x + a)^3 +
6*(18*b^2*x^2 + 1)*cosh(b*x + a)*sinh(b*x + a)^5 - 3*b*x*cosh(b*x + a)^2 +
3*(15*b*x*cosh(b*x + a)^4 - b*x)*sinh(b*x + a)^2 - 54*(b^3*x^3*Ei(6*b*x) +
b^3*x^3*Ei(-6*b*x))*cosh(6*a) + 6*(b^3*x^3*Ei(2*b*x) + b^3*x^3*Ei(-2*b*x))*
cosh(2*a) + 6*((18*b^2*x^2 + 1)*cosh(b*x + a)^5 - (2*b^2*x^2 + 1)*cosh(b*x
+ a))*sinh(b*x + a) - 54*(b^3*x^3*Ei(6*b*x) - b^3*x^3*Ei(-6*b*x))*sinh(6*a)
 + 6*(b^3*x^3*Ei(2*b*x) - b^3*x^3*Ei(-2*b*x))*sinh(2*a))/x^3
```

giac [A] time = 0.13, size = 236, normalized size = 1.40

$$\frac{108\,b^{3}x^{3}{\rm Ei}\left(6\,bx\right)e^{\left(6\,a\right)}-12\,b^{3}x^{3}{\rm Ei}\left(2\,bx\right)e^{\left(2\,a\right)}-12\,b^{3}x^{3}{\rm Ei}\left(-2\,bx\right)e^{\left(-2\,a\right)}+108\,b^{3}x^{3}{\rm Ei}\left(-6\,bx\right)e^{\left(-6\,a\right)}-18\,b^{2}x^{2}e^{\left(6\,bx+\right.}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^3*sinh(b*x+a)^3/x^4,x, algorithm="giac")
```

```
[Out] 1/192*(108*b^3*x^3*Ei(6*b*x)*e^(6*a) - 12*b^3*x^3*Ei(2*b*x)*e^(2*a) - 12*b^
3*x^3*Ei(-2*b*x)*e^(-2*a) + 108*b^3*x^3*Ei(-6*b*x)*e^(-6*a) - 18*b^2*x^2*e^
(6*b*x + 6*a) + 6*b^2*x^2*e^(2*b*x + 2*a) - 6*b^2*x^2*e^(-2*b*x - 2*a) + 18
*b^2*x^2*e^(-6*b*x - 6*a) - 3*b*x*e^(6*b*x + 6*a) + 3*b*x*e^(2*b*x + 2*a) +
 3*b*x*e^(-2*b*x - 2*a) - 3*b*x*e^(-6*b*x - 6*a) - e^(6*b*x + 6*a) + 3*e^(2
*b*x + 2*a) - 3*e^(-2*b*x - 2*a) + e^(-6*b*x - 6*a))/x^3
```

maple [A] time = 0.69, size = 246, normalized size = 1.46

$$\frac{3b^{2}{\rm e}^{-6bx-6a}}{32x}-\frac{b\,{\rm e}^{-6bx-6a}}{64x^{2}}+\frac{{\rm e}^{-6bx-6a}}{192x^{3}}-\frac{9b^{3}{\rm e}^{-6a}\,{\rm Ei}\left(1,6bx\right)}{16}-\frac{b^{2}{\rm e}^{-2bx-2a}}{32x}+\frac{b\,{\rm e}^{-2bx-2a}}{64x^{2}}-\frac{{\rm e}^{-2bx-2a}}{64x^{3}}+\frac{b^{3}{\rm e}^{-2a}\,{\rm Ei}\left(1,2bx\right)}{16}+\frac{{\rm e}^{2bx+}}{64x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)^3*sinh(b*x+a)^3/x^4,x)
```

```
[Out] 3/32*b^2*exp(-6*b*x-6*a)/x-1/64*b*exp(-6*b*x-6*a)/x^2+1/192*exp(-6*b*x-6*a)
/x^3-9/16*b^3*exp(-6*a)*Ei(1,6*b*x)-1/32*b^2*exp(-2*b*x-2*a)/x+1/64*b*exp(-
2*b*x-2*a)/x^2-1/64*exp(-2*b*x-2*a)/x^3+1/16*b^3*exp(-2*a)*Ei(1,2*b*x)+1/64
*exp(2*b*x+2*a)/x^3+1/64*b*exp(2*b*x+2*a)/x^2+1/32*b^2*exp(2*b*x+2*a)/x+1/1
6*b^3*exp(2*a)*Ei(1,-2*b*x)-1/192/x^3*exp(6*b*x+6*a)-1/64*b/x^2*exp(6*b*x+6
*a)-3/32*b^2/x*exp(6*b*x+6*a)-9/16*b^3*exp(6*a)*Ei(1,-6*b*x)
```

maxima [A] time = 0.43, size = 61, normalized size = 0.36

$$\frac{27}{8}\,b^3e^{(-6\,a)}\Gamma\left(-3,6\,bx\right)-\frac{3}{8}\,b^3e^{(-2\,a)}\Gamma\left(-3,2\,bx\right)-\frac{3}{8}\,b^3e^{(2\,a)}\Gamma\left(-3,-2\,bx\right)+\frac{27}{8}\,b^3e^{(6\,a)}\Gamma\left(-3,-6\,bx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^3*sinh(b*x+a)^3/x^4,x, algorithm="maxima")
```

```
[Out] 27/8*b^3*e^(-6*a)*gamma(-3, 6*b*x) - 3/8*b^3*e^(-2*a)*gamma(-3, 2*b*x) - 3/
8*b^3*e^(2*a)*gamma(-3, -2*b*x) + 27/8*b^3*e^(6*a)*gamma(-3, -6*b*x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cosh\left(a+b\,x\right)^3\,\sinh\left(a+b\,x\right)^3}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cosh(a + b*x)^3*sinh(a + b*x)^3)/x^4,x)
```

```
[Out] int((cosh(a + b*x)^3*sinh(a + b*x)^3)/x^4, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh^{3}\left(a+bx\right)\cosh^{3}\left(a+bx\right)}{x^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)**3*sinh(b*x+a)**3/x**4,x)
```

```
[Out] Integral(sinh(a + b*x)**3*cosh(a + b*x)**3/x**4, x)
```

3.334 $\int x^m \tanh(a+bx)\,dx$

Optimal. Leaf size=13

$$\mathrm{Int}\left(x^m \tanh(a+bx),x\right)$$

```
[Out] Unintegrable(x^m*tanh(b*x+a),x)
```

Rubi [A] time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int x^m \tanh(a+bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x^m*Tanh[a + b*x],x]
[Out] Defer[Int][x^m*Tanh[a + b*x], x]
```

Rubi steps

$$\int x^m \tanh(a+bx)\,dx = \int x^m \tanh(a+bx)\,dx$$

Mathematica [A] time = 0.48, size = 0, normalized size = 0.00

$$\int x^m \tanh(a+bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^m*Tanh[a + b*x],x]
[Out] Integrate[x^m*Tanh[a + b*x], x]
```

fricas [A] time = 0.93, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(x^m \operatorname{sech}\left(bx+a\right)\sinh\left(bx+a\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*sech(b*x+a)*sinh(b*x+a),x, algorithm="fricas")
[Out] integral(x^m*sech(b*x + a)*sinh(b*x + a), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int x^m \operatorname{sech}(bx+a)\sinh(bx+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*sech(b*x+a)*sinh(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(x^m*sech(b*x + a)*sinh(b*x + a), x)
```

maple [A] time = 0.14, size = 0, normalized size = 0.00

$$\int x^m \operatorname{sech}(bx+a)\sinh(bx+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*sech(b*x+a)*sinh(b*x+a),x)
```

```
[Out] int(x^m*sech(b*x+a)*sinh(b*x+a),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\frac{xe^{\left(2\,bx+m\log(x)+2\,a\right)}}{(m+1)e^{(2\,bx+2\,a)}+m+1}-\int\frac{\left(\left(2\,bxe^{(2\,a)}+(m+1)e^{(2\,a)}\right)e^{(2\,bx)}+m+1\right)x^m}{(m+1)e^{(4\,bx+4\,a)}+2\,(m+1)e^{(2\,bx+2\,a)}+m+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*sech(b*x+a)*sinh(b*x+a),x, algorithm="maxima")
```

```
[Out] x*e^(2*b*x + m*log(x) + 2*a)/((m + 1)*e^(2*b*x + 2*a) + m + 1) - integrate(
((2*b*x*e^(2*a) + (m + 1)*e^(2*a))*e^(2*b*x) + m + 1)*x^m/((m + 1)*e^(4*b*x
 + 4*a) + 2*(m + 1)*e^(2*b*x + 2*a) + m + 1), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.08

$$\int\frac{x^m\sinh(a+b\,x)}{\cosh(a+b\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^m*sinh(a + b*x))/cosh(a + b*x),x)
```

```
[Out] int((x^m*sinh(a + b*x))/cosh(a + b*x), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int x^m \sinh(a+bx)\operatorname{sech}(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*sech(b*x+a)*sinh(b*x+a),x)
```

```
[Out] Integral(x**m*sinh(a + b*x)*sech(a + b*x), x)
```

3.335 $\int x^3 \tanh(a+bx)\,dx$

Optimal. Leaf size=91

$$\frac{3\text{Li}_4\left(-e^{2(a+bx)}\right)}{4b^4}-\frac{3x\text{Li}_3\left(-e^{2(a+bx)}\right)}{2b^3}+\frac{3x^2\text{Li}_2\left(-e^{2(a+bx)}\right)}{2b^2}+\frac{x^3\log\left(e^{2(a+bx)}+1\right)}{b}-\frac{x^4}{4}$$

```
[Out] -1/4*x^4+x^3*ln(1+exp(2*b*x+2*a))/b+3/2*x^2*polylog(2,-exp(2*b*x+2*a))/b^2-
3/2*x*polylog(3,-exp(2*b*x+2*a))/b^3+3/4*polylog(4,-exp(2*b*x+2*a))/b^4
```

Rubi [A] time = 0.16, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {3718, 2190, 2531, 6609, 2282, 6589}

$$\frac{3x^2\text{PolyLog}\left(2,-e^{2(a+bx)}\right)}{2b^2}-\frac{3x\text{PolyLog}\left(3,-e^{2(a+bx)}\right)}{2b^3}+\frac{3\text{PolyLog}\left(4,-e^{2(a+bx)}\right)}{4b^4}+\frac{x^3\log\left(e^{2(a+bx)}+1\right)}{b}-\frac{x^4}{4}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Tanh[a + b*x],x]
```

```
[Out] -x^4/4 + (x^3*Log[1 + E^(2*(a + b*x))])/b + (3*x^2*PolyLog[2, -E^(2*(a + b*
x))])/(2*b^2) - (3*x*PolyLog[3, -E^(2*(a + b*x))])/(2*b^3) + (3*PolyLog[4,
-E^(2*(a + b*x))])/(4*b^4)
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
```

```
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 3718

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c
 + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /;
 FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\int x^3 \tanh(a+bx)\,dx &= -\frac{x^4}{4} + 2\int \frac{e^{2(a+bx)}x^3}{1+e^{2(a+bx)}}\,dx \\
&= -\frac{x^4}{4} + \frac{x^3\log\left(1+e^{2(a+bx)}\right)}{b} - \frac{3\int x^2\log\left(1+e^{2(a+bx)}\right)dx}{b} \\
&= -\frac{x^4}{4} + \frac{x^3\log\left(1+e^{2(a+bx)}\right)}{b} + \frac{3x^2\text{Li}_2\left(-e^{2(a+bx)}\right)}{2b^2} - \frac{3\int x\text{Li}_2\left(-e^{2(a+bx)}\right)dx}{b^2} \\
&= -\frac{x^4}{4} + \frac{x^3\log\left(1+e^{2(a+bx)}\right)}{b} + \frac{3x^2\text{Li}_2\left(-e^{2(a+bx)}\right)}{2b^2} - \frac{3x\text{Li}_3\left(-e^{2(a+bx)}\right)}{2b^3} + \frac{3\int \text{Li}_3\left(-e^{2(a+bx)}\right.}{2b^3} \\
&= -\frac{x^4}{4} + \frac{x^3\log\left(1+e^{2(a+bx)}\right)}{b} + \frac{3x^2\text{Li}_2\left(-e^{2(a+bx)}\right)}{2b^2} - \frac{3x\text{Li}_3\left(-e^{2(a+bx)}\right)}{2b^3} + \frac{3\,\text{Subst}\left(\int \frac{\text{Li}_3(-x)}{x}\right.}{4b^4} \\
&= -\frac{x^4}{4} + \frac{x^3\log\left(1+e^{2(a+bx)}\right)}{b} + \frac{3x^2\text{Li}_2\left(-e^{2(a+bx)}\right)}{2b^2} - \frac{3x\text{Li}_3\left(-e^{2(a+bx)}\right)}{2b^3} + \frac{3\text{Li}_4\left(-e^{2(a+bx)}\right)}{4b^4}
\end{aligned}
$$

Mathematica [A] time = 2.31, size = 88, normalized size = 0.97

$$\frac{4b^3x^3\log\left(e^{-2(a+bx)}+1\right)-6b^2x^2\text{Li}_2\left(-e^{-2(a+bx)}\right)-6bx\text{Li}_3\left(-e^{-2(a+bx)}\right)-3\text{Li}_4\left(-e^{-2(a+bx)}\right)+b^4x^4}{4b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Tanh[a + b*x],x]
```

```
[Out] (b^4*x^4 + 4*b^3*x^3*Log[1 + E^(-2*(a + b*x))] - 6*b^2*x^2*PolyLog[2, -E^(-
2*(a + b*x))] - 6*b*x*PolyLog[3, -E^(-2*(a + b*x))] - 3*PolyLog[4, -E^(-2*(
a + b*x))])/(4*b^4)
```

fricas [C] time = 0.64, size = 257, normalized size = 2.82

$$-\frac{b^4x^4-12\,b^2x^2\text{Li}_2\left(i\,\cosh\left(bx+a\right)+i\,\sinh\left(bx+a\right)\right)-12\,b^2x^2\text{Li}_2\left(-i\,\cosh\left(bx+a\right)-i\,\sinh\left(bx+a\right)\right)+4\,a^3\,\text{lo}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*sech(b*x+a)*sinh(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/4*(b^4*x^4 - 12*b^2*x^2*dilog(I*cosh(b*x + a) + I*sinh(b*x + a)) - 12*b^
2*x^2*dilog(-I*cosh(b*x + a) - I*sinh(b*x + a)) + 4*a^3*log(cosh(b*x + a) +
 sinh(b*x + a) + I) + 4*a^3*log(cosh(b*x + a) + sinh(b*x + a) - I) + 24*b*x
*polylog(3, I*cosh(b*x + a) + I*sinh(b*x + a)) + 24*b*x*polylog(3, -I*cosh(
b*x + a) - I*sinh(b*x + a)) - 4*(b^3*x^3 + a^3)*log(I*cosh(b*x + a) + I*sin
h(b*x + a) + 1) - 4*(b^3*x^3 + a^3)*log(-I*cosh(b*x + a) - I*sinh(b*x + a)
+ 1) - 24*polylog(4, I*cosh(b*x + a) + I*sinh(b*x + a)) - 24*polylog(4, -I*
cosh(b*x + a) - I*sinh(b*x + a)))/b^4
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3\,\text{sech}\left(bx+a\right)\sinh\left(bx+a\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*sech(b*x+a)*sinh(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(x^3*sech(b*x + a)*sinh(b*x + a), x)
```

maple [A] time = 0.32, size = 116, normalized size = 1.27

$$-\frac{x^4}{4}-\frac{2a^3x}{b^3}-\frac{3a^4}{2b^4}+\frac{x^3\ln\left(1+e^{2bx+2a}\right)}{b}+\frac{3x^2\,\text{polylog}\left(2,-e^{2bx+2a}\right)}{2b^2}-\frac{3x\,\text{polylog}\left(3,-e^{2bx+2a}\right)}{2b^3}+\frac{3\,\text{polylog}\left(4,-e^{2bx}\right.}{4b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*sech(b*x+a)*sinh(b*x+a),x)
```

```
[Out] -1/4*x^4-2/b^3*a^3*x-3/2/b^4*a^4+x^3*ln(1+exp(2*b*x+2*a))/b+3/2*x^2*polylog
(2,-exp(2*b*x+2*a))/b^2-3/2*x*polylog(3,-exp(2*b*x+2*a))/b^3+3/4*polylog(4,
-exp(2*b*x+2*a))/b^4+2/b^4*a^3*ln(exp(b*x+a))
```

maxima [A] time = 0.48, size = 84, normalized size = 0.92

$$-\frac{1}{4}x^4+\frac{4b^3x^3\log\left(e^{(2bx+2a)}+1\right)+6b^2x^2\mathrm{Li}_2\left(-e^{(2bx+2a)}\right)-6bx\mathrm{Li}_3(-e^{(2bx+2a)})+3\mathrm{Li}_4(-e^{(2bx+2a)})}{3b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*sech(b*x+a)*sinh(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/4*x^4 + 1/3*(4*b^3*x^3*log(e^(2*b*x + 2*a) + 1) + 6*b^2*x^2*dilog(-e^(2*
b*x + 2*a)) - 6*b*x*polylog(3, -e^(2*b*x + 2*a)) + 3*polylog(4, -e^(2*b*x +
 2*a)))/b^4
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{x^3\sinh(a+bx)}{\cosh(a+bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*sinh(a + b*x))/cosh(a + b*x),x)
```

```
[Out] int((x^3*sinh(a + b*x))/cosh(a + b*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3\sinh(a+bx)\operatorname{sech}(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*sech(b*x+a)*sinh(b*x+a),x)
```

```
[Out] Integral(x**3*sinh(a + b*x)*sech(a + b*x), x)
```

3.336 $\int x^2 \tanh(a+bx)\,dx$

Optimal. Leaf size=65

$$-\frac{\text{Li}_3\left(-e^{2(a+bx)}\right)}{2b^3}+\frac{x\text{Li}_2\left(-e^{2(a+bx)}\right)}{b^2}+\frac{x^2\log\left(e^{2(a+bx)}+1\right)}{b}-\frac{x^3}{3}$$

```
[Out] -1/3*x^3+x^2*ln(1+exp(2*b*x+2*a))/b+x*polylog(2,-exp(2*b*x+2*a))/b^2-1/2*po
lylog(3,-exp(2*b*x+2*a))/b^3
```

Rubi [A] time = 0.14, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3718, 2190, 2531, 2282, 6589}

$$\frac{x\text{PolyLog}\left(2,-e^{2(a+bx)}\right)}{b^2}-\frac{\text{PolyLog}\left(3,-e^{2(a+bx)}\right)}{2b^3}+\frac{x^2\log\left(e^{2(a+bx)}+1\right)}{b}-\frac{x^3}{3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Tanh[a + b*x],x]
```

```
[Out] -x^3/3 + (x^2*Log[1 + E^(2*(a + b*x))])/b + (x*PolyLog[2, -E^(2*(a + b*x))]
)/b^2 - PolyLog[3, -E^(2*(a + b*x))]/(2*b^3)
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
```

```
, g, n}, x] && GtQ[m, 0]
```

Rule 3718

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c
 + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /;
 FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$
\begin{aligned}
\int x^2 \tanh(a+bx)\,dx &= -\frac{x^3}{3}+2\int \frac{e^{2(a+bx)}x^2}{1+e^{2(a+bx)}}\,dx\\
&= -\frac{x^3}{3}+\frac{x^2\log\left(1+e^{2(a+bx)}\right)}{b}-\frac{2\int x\log\left(1+e^{2(a+bx)}\right)dx}{b}\\
&= -\frac{x^3}{3}+\frac{x^2\log\left(1+e^{2(a+bx)}\right)}{b}+\frac{x\mathrm{Li}_2\left(-e^{2(a+bx)}\right)}{b^2}-\frac{\int \mathrm{Li}_2\left(-e^{2(a+bx)}\right)dx}{b^2}\\
&= -\frac{x^3}{3}+\frac{x^2\log\left(1+e^{2(a+bx)}\right)}{b}+\frac{x\mathrm{Li}_2\left(-e^{2(a+bx)}\right)}{b^2}-\frac{\mathrm{Subst}\left(\int \frac{\mathrm{Li}_2(-x)}{x}\,dx,x,e^{2(a+bx)}\right)}{2b^3}\\
&= -\frac{x^3}{3}+\frac{x^2\log\left(1+e^{2(a+bx)}\right)}{b}+\frac{x\mathrm{Li}_2\left(-e^{2(a+bx)}\right)}{b^2}-\frac{\mathrm{Li}_3\left(-e^{2(a+bx)}\right)}{2b^3}
\end{aligned}
$$

Mathematica [A] time = 2.15, size = 66, normalized size = 1.02

$$
\frac{2b^2x^2\left(3\log\left(e^{-2(a+bx)}+1\right)+bx\right)-6bx\mathrm{Li}_2\left(-e^{-2(a+bx)}\right)-3\mathrm{Li}_3\left(-e^{-2(a+bx)}\right)}{6b^3}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Tanh[a + b*x],x]
```

```
[Out] (2*b^2*x^2*(b*x + 3*Log[1 + E^(-2*(a + b*x))]) - 6*b*x*PolyLog[2, -E^(-2*(a
 + b*x))] - 3*PolyLog[3, -E^(-2*(a + b*x))])/(6*b^3)
```

fricas [C] time = 1.81, size = 207, normalized size = 3.18

$$-\frac{b^3x^3 - 6\,bx\mathrm{Li}_2\left(i\,\cosh\left(bx+a\right) + i\,\sinh\left(bx+a\right)\right) - 6\,bx\mathrm{Li}_2\left(-i\,\cosh\left(bx+a\right) - i\,\sinh\left(bx+a\right)\right) - 3\,a^2\log\left(\cos\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sech(b*x+a)*sinh(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/3*(b^3*x^3 - 6*b*x*dilog(I*cosh(b*x + a) + I*sinh(b*x + a)) - 6*b*x*dilo
g(-I*cosh(b*x + a) - I*sinh(b*x + a)) - 3*a^2*log(cosh(b*x + a) + sinh(b*x
+ a) + I) - 3*a^2*log(cosh(b*x + a) + sinh(b*x + a) - I) - 3*(b^2*x^2 - a^2
)*log(I*cosh(b*x + a) + I*sinh(b*x + a) + 1) - 3*(b^2*x^2 - a^2)*log(-I*cos
h(b*x + a) - I*sinh(b*x + a) + 1) + 6*polylog(3, I*cosh(b*x + a) + I*sinh(b
*x + a)) + 6*polylog(3, -I*cosh(b*x + a) - I*sinh(b*x + a)))/b^3
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2\,\mathrm{sech}\left(bx+a\right)\sinh\left(bx+a\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sech(b*x+a)*sinh(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(x^2*sech(b*x + a)*sinh(b*x + a), x)
```

maple [A] time = 0.32, size = 94, normalized size = 1.45

$$-\frac{x^3}{3}-\frac{2a^2\ln\left(\mathrm{e}^{bx+a}\right)}{b^3}+\frac{2a^2x}{b^2}+\frac{4a^3}{3b^3}+\frac{x^2\ln\left(1+\mathrm{e}^{2bx+2a}\right)}{b}+\frac{x\,\mathrm{polylog}\left(2,-\mathrm{e}^{2bx+2a}\right)}{b^2}-\frac{\mathrm{polylog}\left(3,-\mathrm{e}^{2bx+2a}\right)}{2b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*sech(b*x+a)*sinh(b*x+a),x)
```

```
[Out] -1/3*x^3-2/b^3*a^2*ln(exp(b*x+a))+2/b^2*a^2*x+4/3/b^3*a^3+x^2*ln(1+exp(2*b*
x+2*a))/b+x*polylog(2,-exp(2*b*x+2*a))/b^2-1/2*polylog(3,-exp(2*b*x+2*a))/b
^3
```

maxima [A] time = 0.40, size = 63, normalized size = 0.97

$$-\frac{1}{3}\,x^3+\frac{2\,b^2x^2\log\left(e^{(2\,bx+2\,a)}+1\right)+2\,bx\mathrm{Li}_2\left(-e^{(2\,bx+2\,a)}\right)-\mathrm{Li}_3(-e^{(2\,bx+2\,a)})}{2\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sech(b*x+a)*sinh(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/3*x^3 + 1/2*(2*b^2*x^2*log(e^(2*b*x + 2*a) + 1) + 2*b*x*dilog(-e^(2*b*x
+ 2*a)) - polylog(3, -e^(2*b*x + 2*a)))/b^3
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^2 \sinh(a + b\,x)}{\cosh(a + b\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*sinh(a + b*x))/cosh(a + b*x),x)
```

```
[Out] int((x^2*sinh(a + b*x))/cosh(a + b*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \sinh(a + bx) \operatorname{sech}(a + bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*sech(b*x+a)*sinh(b*x+a),x)
```

```
[Out] Integral(x**2*sinh(a + b*x)*sech(a + b*x), x)
```

3.337 $\int x \tanh(a+bx)\,dx$

Optimal. Leaf size=45

$$\frac{\text{Li}_2\left(-e^{2(a+bx)}\right)}{2b^2}+\frac{x\log\left(e^{2(a+bx)}+1\right)}{b}-\frac{x^2}{2}$$

```
[Out] -1/2*x^2+x*ln(1+exp(2*b*x+2*a))/b+1/2*polylog(2,-exp(2*b*x+2*a))/b^2
```

Rubi [A] time = 0.08, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3718, 2190, 2279, 2391}

$$\frac{\text{PolyLog}\left(2,-e^{2(a+bx)}\right)}{2b^2}+\frac{x\log\left(e^{2(a+bx)}+1\right)}{b}-\frac{x^2}{2}$$

Antiderivative was successfully verified.

```
[In] Int[x*Tanh[a + b*x],x]
```

```
[Out] -x^2/2 + (x*Log[1 + E^(2*(a + b*x))])/b + PolyLog[2, -E^(2*(a + b*x))]/(2*b
^2)
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3718

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c
```

```
+ d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /;
FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\int x\tanh(a+bx)\,dx &= -\frac{x^2}{2}+2\int\frac{e^{2(a+bx)}x}{1+e^{2(a+bx)}}\,dx\\ &= -\frac{x^2}{2}+\frac{x\log\left(1+e^{2(a+bx)}\right)}{b}-\frac{\int\log\left(1+e^{2(a+bx)}\right)\,dx}{b}\\ &= -\frac{x^2}{2}+\frac{x\log\left(1+e^{2(a+bx)}\right)}{b}-\frac{\text{Subst}\left(\int\frac{\log(1+x)}{x}\,dx,x,e^{2(a+bx)}\right)}{2b^2}\\ &= -\frac{x^2}{2}+\frac{x\log\left(1+e^{2(a+bx)}\right)}{b}+\frac{\text{Li}_2\left(-e^{2(a+bx)}\right)}{2b^2}\end{aligned}$$

Mathematica [C] time = 3.24, size = 149, normalized size = 3.31

$$\frac{1}{2}\left(x^2\tanh(a)+\frac{-b^2x^2\tanh(a)\sqrt{-\text{csch}^2(a)}\,e^{-\tanh^{-1}(\coth(a))}-\text{Li}_2\left(e^{-2\left(bx+\tanh^{-1}(\coth(a))\right)}\right)+2bx\log\left(1-e^{-2\left(\tanh^{-1}(\text{co}\right.}\right.}{}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x*Tanh[a + b*x],x]
```

```
[Out] (x^2*Tanh[a] + (I*b*Pi*x - I*Pi*Log[1 + E^(2*b*x)] + 2*b*x*Log[1 - E^(-2*(b
*x + ArcTanh[Coth[a]]))] + I*Pi*Log[Cosh[b*x]] + 2*ArcTanh[Coth[a]]*(b*x +
Log[1 - E^(-2*(b*x + ArcTanh[Coth[a]]))] - Log[I*Sinh[b*x + ArcTanh[Coth[a]
]]]) - PolyLog[2, E^(-2*(b*x + ArcTanh[Coth[a]]))] - (b^2*x^2*Sqrt[-Csch[a]
^2]*Tanh[a])/E^ArcTanh[Coth[a]])/b^2)/2
```

fricas [C] time = 0.69, size = 141, normalized size = 3.13

$$-\frac{b^2x^2+2\,a\log\left(\cosh\left(bx+a\right)+\sinh\left(bx+a\right)+i\right)+2\,a\log\left(\cosh\left(bx+a\right)+\sinh\left(bx+a\right)-i\right)-2\left(bx+a\right)\log\left(i\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sech(b*x+a)*sinh(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/2*(b^2*x^2 + 2*a*log(cosh(b*x + a) + sinh(b*x + a) + I) + 2*a*log(cosh(b
*x + a) + sinh(b*x + a) - I) - 2*(b*x + a)*log(I*cosh(b*x + a) + I*sinh(b*x
```

```
 + a) + 1) - 2*(b*x + a)*log(-I*cosh(b*x + a) - I*sinh(b*x + a) + 1) - 2*di
log(I*cosh(b*x + a) + I*sinh(b*x + a)) - 2*dilog(-I*cosh(b*x + a) - I*sinh(
b*x + a)))/b^2
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\operatorname{sech}(bx+a)\sinh(bx+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sech(b*x+a)*sinh(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(x*sech(b*x + a)*sinh(b*x + a), x)
```

maple [A] time = 0.31, size = 70, normalized size = 1.56

$$-\frac{x^2}{2}-\frac{2ax}{b}-\frac{a^2}{b^2}+\frac{x\ln\left(1+e^{2bx+2a}\right)}{b}+\frac{\operatorname{polylog}\left(2,-e^{2bx+2a}\right)}{2b^2}+\frac{2a\ln\left(e^{bx+a}\right)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sech(b*x+a)*sinh(b*x+a),x)
```

```
[Out] -1/2*x^2-2/b*a*x-a^2/b^2+x*ln(1+exp(2*b*x+2*a))/b+1/2*polylog(2,-exp(2*b*x+
2*a))/b^2+2/b^2*a*ln(exp(b*x+a))
```

maxima [A] time = 0.50, size = 40, normalized size = 0.89

$$-\frac{1}{2}x^2+\frac{2\,bx\log\left(e^{(2\,bx+2\,a)}+1\right)+\operatorname{Li}_2\left(-e^{(2\,bx+2\,a)}\right)}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sech(b*x+a)*sinh(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/2*x^2 + 1/2*(2*b*x*log(e^(2*b*x + 2*a) + 1) + dilog(-e^(2*b*x + 2*a)))/b
^2
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x\sinh(a+b\,x)}{\cosh(a+b\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*sinh(a + b*x))/cosh(a + b*x),x)
```

```
[Out] int((x*sinh(a + b*x))/cosh(a + b*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\sinh\left(a+bx\right)\operatorname{sech}\left(a+bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sech(b*x+a)*sinh(b*x+a),x)
[Out] Integral(x*sinh(a + b*x)*sech(a + b*x), x)
```

3.338 $\int \tanh(a+bx)\,dx$

Optimal. Leaf size=11

$$\frac{\log(\cosh(a+bx))}{b}$$

[Out] ln(cosh(b*x+a))/b

Rubi [A] time = 0.01, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3475}

$$\frac{\log(\cosh(a+bx))}{b}$$

Antiderivative was successfully verified.

[In] Int[Tanh[a + b*x],x]

[Out] Log[Cosh[a + b*x]]/b

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \tanh(a+bx)\,dx = \frac{\log(\cosh(a+bx))}{b}$$

Mathematica [A] time = 0.01, size = 11, normalized size = 1.00

$$\frac{\log(\cosh(a+bx))}{b}$$

Antiderivative was successfully verified.

[In] Integrate[Tanh[a + b*x],x]

[Out] Log[Cosh[a + b*x]]/b

fricas [B] time = 0.66, size = 37, normalized size = 3.36

$$-\frac{bx-\log\left(\frac{2\cosh(bx+a)}{\cosh(bx+a)-\sinh(bx+a)}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)*sinh(b*x+a),x, algorithm="fricas")
```

```
[Out] -(b*x - log(2*cosh(b*x + a)/(cosh(b*x + a) - sinh(b*x + a))))/b
```

giac [B] time = 0.13, size = 24, normalized size = 2.18

$$-\frac{bx + a - \log\left(e^{(2\,bx+2\,a)} + 1\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)*sinh(b*x+a),x, algorithm="giac")
```

```
[Out] -(b*x + a - log(e^(2*b*x + 2*a) + 1))/b
```

maple [A] time = 0.03, size = 13, normalized size = 1.18

$$-\frac{\ln\left(\mathrm{sech}\left(bx + a\right)\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(b*x+a)*sinh(b*x+a),x)
```

```
[Out] -1/b*ln(sech(b*x+a))
```

maxima [A] time = 0.31, size = 21, normalized size = 1.91

$$\frac{\log\left(e^{(bx+a)} + e^{(-bx-a)}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)*sinh(b*x+a),x, algorithm="maxima")
```

```
[Out] log(e^(b*x + a) + e^(-b*x - a))/b
```

mupad [B] time = 0.06, size = 11, normalized size = 1.00

$$\frac{\ln\left(\cosh\left(a + b\,x\right)\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a + b*x)/cosh(a + b*x),x)
```

```
[Out] log(cosh(a + b*x))/b
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sinh(a+bx)\operatorname{sech}(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)*sinh(b*x+a),x)
```

```
[Out] Integral(sinh(a + b*x)*sech(a + b*x), x)
```

3.339 $\int \frac{\tanh(a+bx)}{x}\,dx$

Optimal. Leaf size=13

$$\text{Int}\left(\frac{\tanh(a+bx)}{x},x\right)$$

```
[Out] Unintegrable(tanh(b*x+a)/x,x)
```

Rubi [A] time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\tanh(a+bx)}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Tanh[a + b*x]/x,x]
```

```
[Out] Defer[Int][Tanh[a + b*x]/x, x]
```

Rubi steps

$$\int \frac{\tanh(a+bx)}{x}\,dx=\int \frac{\tanh(a+bx)}{x}\,dx$$

Mathematica [A] time = 12.31, size = 0, normalized size = 0.00

$$\int \frac{\tanh(a+bx)}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Tanh[a + b*x]/x,x]
```

```
[Out] Integrate[Tanh[a + b*x]/x, x]
```

fricas [A] time = 0.49, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\operatorname{sech}(bx+a)\sinh(bx+a)}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)*sinh(b*x+a)/x,x, algorithm="fricas")
```

```
[Out] integral(sech(b*x + a)*sinh(b*x + a)/x, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}(bx+a)\sinh(bx+a)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)*sinh(b*x+a)/x,x, algorithm="giac")
[Out] integrate(sech(b*x + a)*sinh(b*x + a)/x, x)
```

maple [A] time = 0.33, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}(bx+a)\sinh(bx+a)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(b*x+a)*sinh(b*x+a)/x,x)
[Out] int(sech(b*x+a)*sinh(b*x+a)/x,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$-2\int \frac{1}{xe^{(2\,bx+2\,a)}+x}\,dx+\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)*sinh(b*x+a)/x,x, algorithm="maxima")
[Out] -2*integrate(1/(x*e^(2*b*x + 2*a) + x), x) + log(x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.08

$$\int \frac{\sinh(a+b\,x)}{x\cosh(a+b\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a + b*x)/(x*cosh(a + b*x)),x)
[Out] int(sinh(a + b*x)/(x*cosh(a + b*x)), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh(a+bx)\operatorname{sech}(a+bx)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)*sinh(b*x+a)/x,x)
```

```
[Out] Integral(sinh(a + b*x)*sech(a + b*x)/x, x)
```

3.340 $\int \frac{\tanh(a+bx)}{x^2} \, dx$

Optimal. Leaf size=13

$$\text{Int}\left(\frac{\tanh(a+bx)}{x^2},x\right)$$

```
[Out] Unintegrable(tanh(b*x+a)/x^2,x)
```

Rubi [A] time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\tanh(a+bx)}{x^2} \, dx$$

Verification is Not applicable to the result.

```
[In] Int[Tanh[a + b*x]/x^2,x]
[Out] Defer[Int][Tanh[a + b*x]/x^2, x]
```

Rubi steps

$$\int \frac{\tanh(a+bx)}{x^2} \, dx = \int \frac{\tanh(a+bx)}{x^2} \, dx$$

Mathematica [A] time = 19.07, size = 0, normalized size = 0.00

$$\int \frac{\tanh(a+bx)}{x^2} \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Tanh[a + b*x]/x^2,x]
[Out] Integrate[Tanh[a + b*x]/x^2, x]
```

fricas [A] time = 0.75, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\operatorname{sech}(bx+a)\sinh(bx+a)}{x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)*sinh(b*x+a)/x^2,x, algorithm="fricas")
```

```
[Out] integral(sech(b*x + a)*sinh(b*x + a)/x^2, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}(bx+a)\sinh(bx+a)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)*sinh(b*x+a)/x^2,x, algorithm="giac")
[Out] integrate(sech(b*x + a)*sinh(b*x + a)/x^2, x)
```

maple [A] time = 0.33, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}(bx+a)\sinh(bx+a)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(b*x+a)*sinh(b*x+a)/x^2,x)
[Out] int(sech(b*x+a)*sinh(b*x+a)/x^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{x} - 2\int \frac{1}{x^2 e^{(2\,bx+2\,a)} + x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)*sinh(b*x+a)/x^2,x, algorithm="maxima")
[Out] -1/x - 2*integrate(1/(x^2*e^(2*b*x + 2*a) + x^2), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.08

$$\int \frac{\sinh(a+b\,x)}{x^2\cosh(a+b\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a + b*x)/(x^2*cosh(a + b*x)),x)
[Out] int(sinh(a + b*x)/(x^2*cosh(a + b*x)), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh(a+bx)\operatorname{sech}(a+bx)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)*sinh(b*x+a)/x**2,x)
```

```
[Out] Integral(sinh(a + b*x)*sech(a + b*x)/x**2, x)
```

3.341 $\int x^m \mathbf{sech}(a+bx)\tanh(a+bx)\,dx$

Optimal. Leaf size=19

$$\text{Int}\left(x^m\tanh(a+bx)\text{sech}(a+bx),x\right)$$

```
[Out] CannotIntegrate(x^m*sech(b*x+a)*tanh(b*x+a),x)
```

Rubi [A] time = 0.39, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int x^m\text{sech}(a+bx)\tanh(a+bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x^m*Sech[a + b*x]*Tanh[a + b*x],x]
[Out] Defer[Int][x^m*Sech[a + b*x]*Tanh[a + b*x], x]
```

Rubi steps

$$\int x^m\text{sech}(a+bx)\tanh(a+bx)\,dx=\int x^m\text{sech}(a+bx)\tanh(a+bx)\,dx$$

Mathematica [A] time = 22.29, size = 0, normalized size = 0.00

$$\int x^m\text{sech}(a+bx)\tanh(a+bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^m*Sech[a + b*x]*Tanh[a + b*x],x]
[Out] Integrate[x^m*Sech[a + b*x]*Tanh[a + b*x], x]
```

fricas [A] time = 0.67, size = 0, normalized size = 0.00

$$\text{integral}\left(x^m\,\text{sech}\,(bx+a)^2\sinh\,(bx+a)\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*sech(b*x+a)^2*sinh(b*x+a),x, algorithm="fricas")
[Out] integral(x^m*sech(b*x + a)^2*sinh(b*x + a), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int x^m \operatorname{sech}(bx+a)^2 \sinh(bx+a)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*sech(b*x+a)^2*sinh(b*x+a),x, algorithm="giac")
[Out] integrate(x^m*sech(b*x + a)^2*sinh(b*x + a), x)
```

maple [A] time = 0.10, size = 0, normalized size = 0.00

$$\int x^m \operatorname{sech}(bx+a)^2 \sinh(bx+a)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*sech(b*x+a)^2*sinh(b*x+a),x)
[Out] int(x^m*sech(b*x+a)^2*sinh(b*x+a),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int x^m \operatorname{sech}(bx+a)^2 \sinh(bx+a)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*sech(b*x+a)^2*sinh(b*x+a),x, algorithm="maxima")
[Out] integrate(x^m*sech(b*x + a)^2*sinh(b*x + a), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{x^m \sinh(a+b\,x)}{\cosh(a+b\,x)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^m*sinh(a + b*x))/cosh(a + b*x)^2,x)
[Out] int((x^m*sinh(a + b*x))/cosh(a + b*x)^2, x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int x^m \sinh(a+bx) \operatorname{sech}^2(a+bx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*sech(b*x+a)**2*sinh(b*x+a),x)
[Out] Integral(x**m*sinh(a + b*x)*sech(a + b*x)**2, x)
```

3.342 $\int x^3 \operatorname{sech}(a+bx)\tanh(a+bx)\,dx$

Optimal. Leaf size=113

$$\frac{6i\text{Li}_3\left(-ie^{a+bx}\right)}{b^4}-\frac{6i\text{Li}_3\left(ie^{a+bx}\right)}{b^4}-\frac{6ix\text{Li}_2\left(-ie^{a+bx}\right)}{b^3}+\frac{6ix\text{Li}_2\left(ie^{a+bx}\right)}{b^3}+\frac{6x^2\tan^{-1}\left(e^{a+bx}\right)}{b^2}-\frac{x^3\text{sech}(a+bx)}{b}$$

```
[Out] 6*x^2*arctan(exp(b*x+a))/b^2-6*I*x*polylog(2,-I*exp(b*x+a))/b^3+6*I*x*polyl
og(2,I*exp(b*x+a))/b^3+6*I*polylog(3,-I*exp(b*x+a))/b^4-6*I*polylog(3,I*exp
(b*x+a))/b^4-x^3*sech(b*x+a)/b
```

Rubi [A] time = 0.11, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {5418, 4180, 2531, 2282, 6589}

$$-\frac{6ix\text{PolyLog}\left(2,-ie^{a+bx}\right)}{b^3}+\frac{6ix\text{PolyLog}\left(2,ie^{a+bx}\right)}{b^3}+\frac{6i\text{PolyLog}\left(3,-ie^{a+bx}\right)}{b^4}-\frac{6i\text{PolyLog}\left(3,ie^{a+bx}\right)}{b^4}+\frac{6x^2\tan^{-1}\left(e\right.}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Sech[a + b*x]*Tanh[a + b*x],x]
```

```
[Out] (6*x^2*ArcTan[E^(a + b*x)])/b^2 - ((6*I)*x*PolyLog[2, (-I)*E^(a + b*x)])/b^
3 + ((6*I)*x*PolyLog[2, I*E^(a + b*x)])/b^3 + ((6*I)*PolyLog[3, (-I)*E^(a +
 b*x)])/b^4 - ((6*I)*PolyLog[3, I*E^(a + b*x)])/b^4 - (x^3*Sech[a + b*x])/b
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
```

```
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5418

```
Int[(x_)^(m_.)*Sech[(a_.) + (b_.)*(x_)^(n_.)]^(p_.)*Tanh[(a_.) + (b_.)*(x_)
^(n_.)]^(q_.), x_Symbol] :> -Simp[(x^(m - n + 1)*Sech[a + b*x^n]^p)/(b*n*p)
, x] + Dist[(m - n + 1)/(b*n*p), Int[x^(m - n)*Sech[a + b*x^n]^p, x], x] /;
 FreeQ[{a, b, p}, x] && RationalQ[m] && IntegerQ[n] && GeQ[m - n, 0] && EqQ
[q, 1]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}\int x^3\operatorname{sech}(a+bx)\tanh(a+bx)\,dx &= -\frac{x^3\operatorname{sech}(a+bx)}{b}+\frac{3\int x^2\operatorname{sech}(a+bx)\,dx}{b}\\ &= \frac{6x^2\tan^{-1}\left(e^{a+bx}\right)}{b^2}-\frac{x^3\operatorname{sech}(a+bx)}{b}-\frac{(6i)\int x\log\left(1-ie^{a+bx}\right)dx}{b^2}+\frac{(6i)\int\\ &= \frac{6x^2\tan^{-1}\left(e^{a+bx}\right)}{b^2}-\frac{6ix\operatorname{Li}_2\left(-ie^{a+bx}\right)}{b^3}+\frac{6ix\operatorname{Li}_2\left(ie^{a+bx}\right)}{b^3}-\frac{x^3\operatorname{sech}(a+bx)}{b}+\\ &= \frac{6x^2\tan^{-1}\left(e^{a+bx}\right)}{b^2}-\frac{6ix\operatorname{Li}_2\left(-ie^{a+bx}\right)}{b^3}+\frac{6ix\operatorname{Li}_2\left(ie^{a+bx}\right)}{b^3}-\frac{x^3\operatorname{sech}(a+bx)}{b}+\\ &= \frac{6x^2\tan^{-1}\left(e^{a+bx}\right)}{b^2}-\frac{6ix\operatorname{Li}_2\left(-ie^{a+bx}\right)}{b^3}+\frac{6ix\operatorname{Li}_2\left(ie^{a+bx}\right)}{b^3}+\frac{6i\operatorname{Li}_3\left(-ie^{a+bx}\right)}{b^4}-\end{aligned}$$

Mathematica [A] time = 2.28, size = 130, normalized size = 1.15

$$-\frac{x^3\operatorname{sech}(a+bx)}{b}+\frac{3i\left(b^2x^2\log\left(1-ie^{a+bx}\right)-b^2x^2\log\left(1+ie^{a+bx}\right)-2bx\operatorname{Li}_2\left(-ie^{a+bx}\right)+2bx\operatorname{Li}_2\left(ie^{a+bx}\right)+2\operatorname{Li}_3\left(-\right.\right.}{b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Sech[a + b*x]*Tanh[a + b*x],x]
```

```
[Out] ((3*I)*(b^2*x^2*Log[1 - I*E^(a + b*x)] - b^2*x^2*Log[1 + I*E^(a + b*x)] - 2
*b*x*PolyLog[2, (-I)*E^(a + b*x)] + 2*b*x*PolyLog[2, I*E^(a + b*x)] + 2*Pol
yLog[3, (-I)*E^(a + b*x)] - 2*PolyLog[3, I*E^(a + b*x)]))/b^4 - (x^3*Sech[a
 + b*x])/b
```

fricas [C] time = 0.77, size = 670, normalized size = 5.93

$$-\frac{2\,b^3x^3\cosh\left(bx+a\right)+2\,b^3x^3\sinh\left(bx+a\right)-\left(6i\,bx\cosh\left(bx+a\right)^2+12i\,bx\cosh\left(bx+a\right)\sinh\left(bx+a\right)+6i\,bx\,\mathrm{si}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*sech(b*x+a)^2*sinh(b*x+a),x, algorithm="fricas")
```

```
[Out] -(2*b^3*x^3*cosh(b*x + a) + 2*b^3*x^3*sinh(b*x + a) - (6*I*b*x*cosh(b*x + a
)^2 + 12*I*b*x*cosh(b*x + a)*sinh(b*x + a) + 6*I*b*x*sinh(b*x + a)^2 + 6*I*
b*x)*dilog(I*cosh(b*x + a) + I*sinh(b*x + a)) - (-6*I*b*x*cosh(b*x + a)^2 -
 12*I*b*x*cosh(b*x + a)*sinh(b*x + a) - 6*I*b*x*sinh(b*x + a)^2 - 6*I*b*x)*
dilog(-I*cosh(b*x + a) - I*sinh(b*x + a)) - (3*I*a^2*cosh(b*x + a)^2 + 6*I*
a^2*cosh(b*x + a)*sinh(b*x + a) + 3*I*a^2*sinh(b*x + a)^2 + 3*I*a^2)*log(co
sh(b*x + a) + sinh(b*x + a) + I) - (-3*I*a^2*cosh(b*x + a)^2 - 6*I*a^2*cosh
(b*x + a)*sinh(b*x + a) - 3*I*a^2*sinh(b*x + a)^2 - 3*I*a^2)*log(cosh(b*x +
 a) + sinh(b*x + a) - I) - (-3*I*b^2*x^2 + (-3*I*b^2*x^2 + 3*I*a^2)*cosh(b*
x + a)^2 + (-6*I*b^2*x^2 + 6*I*a^2)*cosh(b*x + a)*sinh(b*x + a) + (-3*I*b^2
*x^2 + 3*I*a^2)*sinh(b*x + a)^2 + 3*I*a^2)*log(I*cosh(b*x + a) + I*sinh(b*x
 + a) + 1) - (3*I*b^2*x^2 + (3*I*b^2*x^2 - 3*I*a^2)*cosh(b*x + a)^2 + (6*I*
b^2*x^2 - 6*I*a^2)*cosh(b*x + a)*sinh(b*x + a) + (3*I*b^2*x^2 - 3*I*a^2)*si
nh(b*x + a)^2 - 3*I*a^2)*log(-I*cosh(b*x + a) - I*sinh(b*x + a) + 1) - (-6*
I*cosh(b*x + a)^2 - 12*I*cosh(b*x + a)*sinh(b*x + a) - 6*I*sinh(b*x + a)^2
- 6*I)*polylog(3, I*cosh(b*x + a) + I*sinh(b*x + a)) - (6*I*cosh(b*x + a)^2
 + 12*I*cosh(b*x + a)*sinh(b*x + a) + 6*I*sinh(b*x + a)^2 + 6*I)*polylog(3,
 -I*cosh(b*x + a) - I*sinh(b*x + a)))/(b^4*cosh(b*x + a)^2 + 2*b^4*cosh(b*x
 + a)*sinh(b*x + a) + b^4*sinh(b*x + a)^2 + b^4)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3\,\mathrm{sech}\left(bx+a\right)^2\sinh\left(bx+a\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*sech(b*x+a)^2*sinh(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(x^3*sech(b*x + a)^2*sinh(b*x + a), x)
```

maple [F] time = 0.56, size = 0, normalized size = 0.00

$$\int x^3\mathrm{sech}\left(bx+a\right)^2\sinh\left(bx+a\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*sech(b*x+a)^2*sinh(b*x+a),x)
```

```
[Out] int(x^3*sech(b*x+a)^2*sinh(b*x+a),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{2\,x^3 e^{(bx+a)}}{be^{(2\,bx+2\,a)}+b}+6\int \frac{x^2 e^{(bx+a)}}{be^{(2\,bx+2\,a)}+b}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*sech(b*x+a)^2*sinh(b*x+a),x, algorithm="maxima")
```

```
[Out] -2*x^3*e^(b*x + a)/(b*e^(2*b*x + 2*a) + b) + 6*integrate(x^2*e^(b*x + a)/(b
*e^(2*b*x + 2*a) + b), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3\sinh\left(a+b\,x\right)}{{\cosh\left(a+b\,x\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*sinh(a + b*x))/cosh(a + b*x)^2,x)
```

```
[Out] int((x^3*sinh(a + b*x))/cosh(a + b*x)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^{3}\sinh\left(a+bx\right)\operatorname{sech}^{2}\left(a+bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*sech(b*x+a)**2*sinh(b*x+a),x)
```

```
[Out] Integral(x**3*sinh(a + b*x)*sech(a + b*x)**2, x)
```

3.343 $\int x^2\text{sech}(a+bx)\tanh(a+bx)\,dx$

Optimal. Leaf size=69

$$-\frac{2i\text{Li}_2\left(-ie^{a+bx}\right)}{b^3}+\frac{2i\text{Li}_2\left(ie^{a+bx}\right)}{b^3}+\frac{4x\tan^{-1}\left(e^{a+bx}\right)}{b^2}-\frac{x^2\text{sech}(a+bx)}{b}$$

```
[Out] 4*x*arctan(exp(b*x+a))/b^2-2*I*polylog(2,-I*exp(b*x+a))/b^3+2*I*polylog(2,I
*exp(b*x+a))/b^3-x^2*sech(b*x+a)/b
```

Rubi [A] time = 0.06, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {5418, 4180, 2279, 2391}

$$-\frac{2i\text{PolyLog}\left(2,-ie^{a+bx}\right)}{b^3}+\frac{2i\text{PolyLog}\left(2,ie^{a+bx}\right)}{b^3}+\frac{4x\tan^{-1}\left(e^{a+bx}\right)}{b^2}-\frac{x^2\text{sech}(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Sech[a + b*x]*Tanh[a + b*x],x]
```

```
[Out] (4*x*ArcTan[E^(a + b*x)])/b^2 - ((2*I)*PolyLog[2, (-I)*E^(a + b*x)])/b^3 +
((2*I)*PolyLog[2, I*E^(a + b*x)])/b^3 - (x^2*Sech[a + b*x])/b
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5418

```
Int[(x_)^(m_.)*Sech[(a_.) + (b_.)*(x_)^(n_.)]^(p_.)*Tanh[(a_.) + (b_.)*(x_)
^(n_.)]^(q_.), x_Symbol] :> -Simp[(x^(m - n + 1)*Sech[a + b*x^n]^p)/(b*n*p)
, x] + Dist[(m - n + 1)/(b*n*p), Int[x^(m - n)*Sech[a + b*x^n]^p, x], x] /;
 FreeQ[{a, b, p}, x] && RationalQ[m] && IntegerQ[n] && GeQ[m - n, 0] && EqQ
[q, 1]
```

Rubi steps

$$\int x^2\text{sech}(a+bx)\tanh(a+bx)\,dx = -\frac{x^2\text{sech}(a+bx)}{b}+\frac{2\int x\text{sech}(a+bx)\,dx}{b}$$
$$=\frac{4x\tan^{-1}\left(e^{a+bx}\right)}{b^2}-\frac{x^2\text{sech}(a+bx)}{b}-\frac{(2i)\int\log\left(1-ie^{a+bx}\right)\,dx}{b^2}+\frac{(2i)\int\text{lo}$$
$$=\frac{4x\tan^{-1}\left(e^{a+bx}\right)}{b^2}-\frac{x^2\text{sech}(a+bx)}{b}-\frac{(2i)\,\text{Subst}\left(\int\frac{\log(1-ix)}{x}\,dx,x,e^{a+bx}\right)}{b^3}+$$
$$=\frac{4x\tan^{-1}\left(e^{a+bx}\right)}{b^2}-\frac{2i\text{Li}_2\left(-ie^{a+bx}\right)}{b^3}+\frac{2i\text{Li}_2\left(ie^{a+bx}\right)}{b^3}-\frac{x^2\text{sech}(a+bx)}{b}$$

Mathematica [A] time = 0.45, size = 125, normalized size = 1.81

$$-\frac{b^2x^2\text{sech}(a+bx)+2i\left(\text{Li}_2\left(-ie^{a+bx}\right)-\text{Li}_2\left(ie^{a+bx}\right)\right)+(-2ia-2ibx+\pi)\left(\log\left(1-ie^{a+bx}\right)-\log\left(1+ie^{a+bx}\right)\right)-}{b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Sech[a + b*x]*Tanh[a + b*x],x]
```

```
[Out] -(((((-2*I)*a + Pi - (2*I)*b*x)*(Log[1 - I*E^(a + b*x)] - Log[1 + I*E^(a + b
*x)]) - ((-2*I)*a + Pi)*Log[Cot[((2*I)*a + Pi + (2*I)*b*x)/4]] + (2*I)*(Pol
yLog[2, (-I)*E^(a + b*x)] - PolyLog[2, I*E^(a + b*x)]) + b^2*x^2*Sech[a + b
*x])/b^3)
```

fricas [B] time = 0.61, size = 474, normalized size = 6.87

$$-\frac{2\,b^2x^2\cosh\left(bx+a\right)+2\,b^2x^2\sinh\left(bx+a\right)-\left(2i\,\cosh\left(bx+a\right)^2+4i\,\cosh\left(bx+a\right)\sinh\left(bx+a\right)+2i\,\sinh\left(bx\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sech(b*x+a)^2*sinh(b*x+a),x, algorithm="fricas")
```

```
[Out] -(2*b^2*x^2*cosh(b*x + a) + 2*b^2*x^2*sinh(b*x + a) - (2*I*cosh(b*x + a)^2
+ 4*I*cosh(b*x + a)*sinh(b*x + a) + 2*I*sinh(b*x + a)^2 + 2*I)*dilog(I*cosh
(b*x + a) + I*sinh(b*x + a)) - (-2*I*cosh(b*x + a)^2 - 4*I*cosh(b*x + a)*si
nh(b*x + a) - 2*I*sinh(b*x + a)^2 - 2*I)*dilog(-I*cosh(b*x + a) - I*sinh(b*
x + a)) - (-2*I*a*cosh(b*x + a)^2 - 4*I*a*cosh(b*x + a)*sinh(b*x + a) - 2*I
*a*sinh(b*x + a)^2 - 2*I*a)*log(cosh(b*x + a) + sinh(b*x + a) + I) - (2*I*a
*cosh(b*x + a)^2 + 4*I*a*cosh(b*x + a)*sinh(b*x + a) + 2*I*a*sinh(b*x + a)^
2 + 2*I*a)*log(cosh(b*x + a) + sinh(b*x + a) - I) - ((-2*I*b*x - 2*I*a)*cos
h(b*x + a)^2 + (-4*I*b*x - 4*I*a)*cosh(b*x + a)*sinh(b*x + a) + (-2*I*b*x -
 2*I*a)*sinh(b*x + a)^2 - 2*I*b*x - 2*I*a)*log(I*cosh(b*x + a) + I*sinh(b*x
 + a) + 1) - ((2*I*b*x + 2*I*a)*cosh(b*x + a)^2 + (4*I*b*x + 4*I*a)*cosh(b*
x + a)*sinh(b*x + a) + (2*I*b*x + 2*I*a)*sinh(b*x + a)^2 + 2*I*b*x + 2*I*a)
*log(-I*cosh(b*x + a) - I*sinh(b*x + a) + 1))/(b^3*cosh(b*x + a)^2 + 2*b^3*
cosh(b*x + a)*sinh(b*x + a) + b^3*sinh(b*x + a)^2 + b^3)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \operatorname{sech}(bx+a)^2 \sinh(bx+a)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sech(b*x+a)^2*sinh(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(x^2*sech(b*x + a)^2*sinh(b*x + a), x)
```

maple [B] time = 0.33, size = 154, normalized size = 2.23

$$-\frac{2x^2 e^{bx+a}}{b\left(1+e^{2bx+2a}\right)}-\frac{2i\ln\left(1+ie^{bx+a}\right)x}{b^2}-\frac{2i\ln\left(1+ie^{bx+a}\right)a}{b^3}+\frac{2i\ln\left(1-ie^{bx+a}\right)x}{b^2}+\frac{2i\ln\left(1-ie^{bx+a}\right)a}{b^3}-\frac{2i\operatorname{dilog}\left(1+\right.}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*sech(b*x+a)^2*sinh(b*x+a),x)
```

```
[Out] -2*x^2*exp(b*x+a)/b/(1+exp(2*b*x+2*a))-2*I/b^2*ln(1+I*exp(b*x+a))*x-2*I/b^3
*ln(1+I*exp(b*x+a))*a+2*I/b^2*ln(1-I*exp(b*x+a))*x+2*I/b^3*ln(1-I*exp(b*x+a
))*a-2*I/b^3*dilog(1+I*exp(b*x+a))+2*I/b^3*dilog(1-I*exp(b*x+a))-4/b^3*a*ar
ctan(exp(b*x+a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{2\,x^2 e^{(bx+a)}}{be^{(2\,bx+2\,a)}+b}+4\int\frac{xe^{(bx+a)}}{be^{(2\,bx+2\,a)}+b}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sech(b*x+a)^2*sinh(b*x+a),x, algorithm="maxima")
```

```
[Out] -2*x^2*e^(b*x + a)/(b*e^(2*b*x + 2*a) + b) + 4*integrate(x*e^(b*x + a)/(b*e
^(2*b*x + 2*a) + b), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2 \sinh(a + b x)}{\cosh(a + b x)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*sinh(a + b*x))/cosh(a + b*x)^2,x)
```

```
[Out] int((x^2*sinh(a + b*x))/cosh(a + b*x)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \sinh(a + bx) \operatorname{sech}^2(a + bx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*sech(b*x+a)**2*sinh(b*x+a),x)
```

```
[Out] Integral(x**2*sinh(a + b*x)*sech(a + b*x)**2, x)
```

3.344 $\int x\mathbf{sech}(a+bx)\tanh(a+bx)\,dx$

Optimal. Leaf size=24

$$\frac{\tan^{-1}(\sinh(a+bx))}{b^2}-\frac{x\text{sech}(a+bx)}{b}$$

```
[Out] arctan(sinh(b*x+a))/b^2-x*sech(b*x+a)/b
```

Rubi [A] time = 0.02, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {5418, 3770}

$$\frac{\tan^{-1}(\sinh(a+bx))}{b^2}-\frac{x\text{sech}(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sech[a + b*x]*Tanh[a + b*x],x]
```

```
[Out] ArcTan[Sinh[a + b*x]]/b^2 - (x*Sech[a + b*x])/b
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 5418

```
Int[(x_)^(m_.)*Sech[(a_.) + (b_.)*(x_)^(n_.)]^(p_.)*Tanh[(a_.) + (b_.)*(x_)
^(n_.)]^(q_.), x_Symbol] :> -Simp[(x^(m - n + 1)*Sech[a + b*x^n]^p)/(b*n*p)
, x] + Dist[(m - n + 1)/(b*n*p), Int[x^(m - n)*Sech[a + b*x^n]^p, x], x] /;
 FreeQ[{a, b, p}, x] && RationalQ[m] && IntegerQ[n] && GeQ[m - n, 0] && EqQ
[q, 1]
```

Rubi steps

$$\begin{aligned}\int x\text{sech}(a+bx)\tanh(a+bx)\,dx&=-\frac{x\text{sech}(a+bx)}{b}+\frac{\int \text{sech}(a+bx)\,dx}{b}\\&=\frac{\tan^{-1}(\sinh(a+bx))}{b^2}-\frac{x\text{sech}(a+bx)}{b}\end{aligned}$$

Mathematica [A] time = 0.05, size = 32, normalized size = 1.33

$$\frac{2\tan^{-1}\left(\tanh\left(\frac{a}{2}+\frac{bx}{2}\right)\right)}{b^2}-\frac{x\text{sech}(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sech[a + b*x]*Tanh[a + b*x],x]
```

```
[Out] (2*ArcTan[Tanh[a/2 + (b*x)/2]])/b^2 - (x*Sech[a + b*x])/b
```

fricas [B] time = 0.48, size = 116, normalized size = 4.83

$$-\frac{2\left(bx\cosh\left(bx+a\right)+bx\sinh\left(bx+a\right)-\left(\cosh\left(bx+a\right)^2+2\cosh\left(bx+a\right)\sinh\left(bx+a\right)+\sinh\left(bx+a\right)^2+1\right)}{b^2\cosh\left(bx+a\right)^2+2\,b^2\cosh\left(bx+a\right)\sinh\left(bx+a\right)+b^2\sinh\left(bx+a\right)^2+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sech(b*x+a)^2*sinh(b*x+a),x, algorithm="fricas")
```

```
[Out] -2*(b*x*cosh(b*x + a) + b*x*sinh(b*x + a) - (cosh(b*x + a)^2 + 2*cosh(b*x +
 a)*sinh(b*x + a) + sinh(b*x + a)^2 + 1)*arctan(cosh(b*x + a) + sinh(b*x +
a)))/(b^2*cosh(b*x + a)^2 + 2*b^2*cosh(b*x + a)*sinh(b*x + a) + b^2*sinh(b*
x + a)^2 + b^2)
```

giac [B] time = 0.16, size = 70, normalized size = 2.92

$$-\frac{2\left(\pi+bxe^{(bx+a)}+\pi e^{(2\,bx+2\,a)}-\arctan\left(e^{(bx+a)}\right)e^{(2\,bx+2\,a)}-\arctan\left(e^{(bx+a)}\right)\right)}{b^2e^{(2\,bx+2\,a)}+b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sech(b*x+a)^2*sinh(b*x+a),x, algorithm="giac")
```

```
[Out] -2*(pi + b*x*e^(b*x + a) + pi*e^(2*b*x + 2*a) - arctan(e^(b*x + a))*e^(2*b*
x + 2*a) - arctan(e^(b*x + a)))/(b^2*e^(2*b*x + 2*a) + b^2)
```

maple [C] time = 0.17, size = 59, normalized size = 2.46

$$-\frac{2x\,\mathrm{e}^{bx+a}}{b\left(1+\mathrm{e}^{2bx+2a}\right)}+\frac{i\ln\left(\mathrm{e}^{bx+a}+i\right)}{b^2}-\frac{i\ln\left(\mathrm{e}^{bx+a}-i\right)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sech(b*x+a)^2*sinh(b*x+a),x)
```

```
[Out] -2*x*exp(b*x+a)/b/(1+exp(2*b*x+2*a))+I/b^2*ln(exp(b*x+a)+I)-I/b^2*ln(exp(b*
x+a)-I)
```

maxima [A] time = 0.50, size = 37, normalized size = 1.54

$$-\frac{2\,xe^{(bx+a)}}{be^{(2\,bx+2\,a)}+b}+\frac{2\,\arctan\left(e^{(bx+a)}\right)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sech(b*x+a)^2*sinh(b*x+a),x, algorithm="maxima")
```

```
[Out] -2*x*e^(b*x + a)/(b*e^(2*b*x + 2*a) + b) + 2*arctan(e^(b*x + a))/b^2
```

mupad [B] time = 0.07, size = 49, normalized size = 2.04

$$\frac{2\,\mathrm{atan}\left(\frac{\mathrm{e}^{b\,x}\,\mathrm{e}^{a}\,\sqrt{b^4}}{b^2}\right)}{\sqrt{b^4}}-\frac{2\,x\,\mathrm{e}^{a+b\,x}}{b\,\left(\mathrm{e}^{2\,a+2\,b\,x}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*sinh(a + b*x))/cosh(a + b*x)^2,x)
```

```
[Out] (2*atan((exp(b*x)*exp(a)*(b^4)^(1/2))/b^2))/(b^4)^(1/2) - (2*x*exp(a + b*x)
)/(b*(exp(2*a + 2*b*x) + 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\sinh\left(a+bx\right)\operatorname{sech}^2\left(a+bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sech(b*x+a)**2*sinh(b*x+a),x)
```

```
[Out] Integral(x*sinh(a + b*x)*sech(a + b*x)**2, x)
```

3.345 $\int \operatorname{sech}(a+bx)\tanh(a+bx)\,dx$

Optimal. Leaf size=11

$$-\frac{\operatorname{sech}(a+bx)}{b}$$

```
[Out] -sech(b*x+a)/b
```

Rubi [A] time = 0.01, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2606, 8}

$$-\frac{\operatorname{sech}(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[a + b*x]*Tanh[a + b*x],x]
```

```
[Out] -(Sech[a + b*x]/b)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rubi steps

$$\begin{aligned}\int \operatorname{sech}(a+bx)\tanh(a+bx)\,dx &= -\frac{\operatorname{Subst}\left(\int 1\,dx, x, \operatorname{sech}(a+bx)\right)}{b}\\ &= -\frac{\operatorname{sech}(a+bx)}{b}\end{aligned}$$

Mathematica [A] time = 0.01, size = 11, normalized size = 1.00

$$-\frac{\operatorname{sech}(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[a + b*x]*Tanh[a + b*x],x]
```

```
[Out] -(Sech[a + b*x]/b)
```

fricas [B] time = 0.38, size = 54, normalized size = 4.91

$$-\frac{2\left(\cosh\left(bx+a\right)+\sinh\left(bx+a\right)\right)}{b\cosh\left(bx+a\right)^{2}+2\,b\cosh\left(bx+a\right)\sinh\left(bx+a\right)+b\sinh\left(bx+a\right)^{2}+b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)^2*sinh(b*x+a),x, algorithm="fricas")
```

```
[Out] -2*(cosh(b*x + a) + sinh(b*x + a))/(b*cosh(b*x + a)^2 + 2*b*cosh(b*x + a)*s
inh(b*x + a) + b*sinh(b*x + a)^2 + b)
```

giac [B] time = 0.12, size = 23, normalized size = 2.09

$$-\frac{2}{b\left(e^{(bx+a)}+e^{(-bx-a)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)^2*sinh(b*x+a),x, algorithm="giac")
```

```
[Out] -2/(b*(e^(b*x + a) + e^(-b*x - a)))
```

maple [A] time = 0.02, size = 12, normalized size = 1.09

$$-\frac{\operatorname{sech}\left(bx+a\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(b*x+a)^2*sinh(b*x+a),x)
```

```
[Out] -sech(b*x+a)/b
```

maxima [B] time = 0.43, size = 23, normalized size = 2.09

$$-\frac{2}{b\left(e^{(bx+a)}+e^{(-bx-a)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)^2*sinh(b*x+a),x, algorithm="maxima")
```

```
[Out] -2/(b*(e^(b*x + a) + e^(-b*x - a)))
```

mupad [B] time = 0.06, size = 13, normalized size = 1.18

$$-\frac{1}{b\cosh\left(a+b\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a + b*x)/cosh(a + b*x)^2,x)
```

```
[Out] -1/(b*cosh(a + b*x))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sinh\left(a+bx\right)\operatorname{sech}^{2}\left(a+bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)**2*sinh(b*x+a),x)
```

```
[Out] Integral(sinh(a + b*x)*sech(a + b*x)**2, x)
```

3.346 $\int \frac{\operatorname{sech}(a+bx)\tanh(a+bx)}{x}\,dx$

Optimal. Leaf size=19

$$\operatorname{Int}\left(\frac{\tanh(a+bx)\operatorname{sech}(a+bx)}{x},x\right)$$

```
[Out] CannotIntegrate(sech(b*x+a)*tanh(b*x+a)/x,x)
```

Rubi [A] time = 0.18, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\operatorname{sech}(a+bx)\tanh(a+bx)}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Sech[a + b*x]*Tanh[a + b*x])/x,x]
```

```
[Out] Defer[Int][(Sech[a + b*x]*Tanh[a + b*x])/x, x]
```

Rubi steps

$$\int \frac{\operatorname{sech}(a+bx)\tanh(a+bx)}{x}\,dx = \int \frac{\operatorname{sech}(a+bx)\tanh(a+bx)}{x}\,dx$$

Mathematica [A] time = 7.02, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}(a+bx)\tanh(a+bx)}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Sech[a + b*x]*Tanh[a + b*x])/x,x]
```

```
[Out] Integrate[(Sech[a + b*x]*Tanh[a + b*x])/x, x]
```

fricas [A] time = 0.48, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{\operatorname{sech}(bx+a)^2\sinh(bx+a)}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)^2*sinh(b*x+a)/x,x, algorithm="fricas")
```

```
[Out] integral(sech(b*x + a)^2*sinh(b*x + a)/x, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}(bx+a)^2 \sinh(bx+a)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)^2*sinh(b*x+a)/x,x, algorithm="giac")
[Out] integrate(sech(b*x + a)^2*sinh(b*x + a)/x, x)
```

maple [A] time = 0.36, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}(bx+a)^2 \sinh(bx+a)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(b*x+a)^2*sinh(b*x+a)/x,x)
[Out] int(sech(b*x+a)^2*sinh(b*x+a)/x,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{2\,e^{(bx+a)}}{bxe^{(2\,bx+2\,a)}+bx}-2\int \frac{e^{(bx+a)}}{bx^2e^{(2\,bx+2\,a)}+bx^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)^2*sinh(b*x+a)/x,x, algorithm="maxima")
[Out] -2*e^(b*x + a)/(b*x*e^(2*b*x + 2*a) + b*x) - 2*integrate(e^(b*x + a)/(b*x^2
*e^(2*b*x + 2*a) + b*x^2), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{\sinh(a+b\,x)}{x\cosh(a+b\,x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a + b*x)/(x*cosh(a + b*x)^2),x)
[Out] int(sinh(a + b*x)/(x*cosh(a + b*x)^2), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh (a+bx)\operatorname{sech}^2(a+bx)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)**2*sinh(b*x+a)/x,x)
```

```
[Out] Integral(sinh(a + b*x)*sech(a + b*x)**2/x, x)
```

3.347 $\int \frac{\mathbf{sech}(a+bx)\tanh(a+bx)}{x^2}\,dx$

Optimal. Leaf size=19

$$\text{Int}\left(\frac{\tanh(a+bx)\text{sech}(a+bx)}{x^2},x\right)$$

[Out] CannotIntegrate(sech(b*x+a)*tanh(b*x+a)/x^2,x)

Rubi [A] time = 0.20, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\text{sech}(a+bx)\tanh(a+bx)}{x^2}\,dx$$

Verification is Not applicable to the result.

[In] Int[(Sech[a + b*x]*Tanh[a + b*x])/x^2,x]

[Out] Defer[Int][(Sech[a + b*x]*Tanh[a + b*x])/x^2, x]

Rubi steps

$$\int \frac{\text{sech}(a+bx)\tanh(a+bx)}{x^2}\,dx=\int \frac{\text{sech}(a+bx)\tanh(a+bx)}{x^2}\,dx$$

Mathematica [A] time = 8.20, size = 0, normalized size = 0.00

$$\int \frac{\text{sech}(a+bx)\tanh(a+bx)}{x^2}\,dx$$

Verification is Not applicable to the result.

[In] Integrate[(Sech[a + b*x]*Tanh[a + b*x])/x^2,x]

[Out] Integrate[(Sech[a + b*x]*Tanh[a + b*x])/x^2, x]

fricas [A] time = 0.43, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\text{sech}\,(bx+a)^2\sinh\,(bx+a)}{x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(b*x+a)^2*sinh(b*x+a)/x^2,x, algorithm="fricas")

```
[Out] integral(sech(b*x + a)^2*sinh(b*x + a)/x^2, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}(bx+a)^2 \sinh(bx+a)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)^2*sinh(b*x+a)/x^2,x, algorithm="giac")
```

```
[Out] integrate(sech(b*x + a)^2*sinh(b*x + a)/x^2, x)
```

maple [A] time = 0.35, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}(bx+a)^2 \sinh(bx+a)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(b*x+a)^2*sinh(b*x+a)/x^2,x)
```

```
[Out] int(sech(b*x+a)^2*sinh(b*x+a)/x^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{2\,e^{(bx+a)}}{bx^2e^{(2\,bx+2\,a)}+bx^2}-4\int \frac{e^{(bx+a)}}{bx^3e^{(2\,bx+2\,a)}+bx^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)^2*sinh(b*x+a)/x^2,x, algorithm="maxima")
```

```
[Out] -2*e^(b*x + a)/(b*x^2*e^(2*b*x + 2*a) + b*x^2) - 4*integrate(e^(b*x + a)/(b
*x^3*e^(2*b*x + 2*a) + b*x^3), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{\sinh(a+b\,x)}{x^2\cosh(a+b\,x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a + b*x)/(x^2*cosh(a + b*x)^2),x)
```

```
[Out] int(sinh(a + b*x)/(x^2*cosh(a + b*x)^2), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh(a+bx)\operatorname{sech}^2(a+bx)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)**2*sinh(b*x+a)/x**2,x)
```

```
[Out] Integral(sinh(a + b*x)*sech(a + b*x)**2/x**2, x)
```

3.348 $\int x^m \mathbf{sech}^2(a+bx)\tanh(a+bx)\,dx$

Optimal. Leaf size=21

$$\mathrm{Int}\left(x^m\tanh(a+bx)\mathrm{sech}^2(a+bx),x\right)$$

```
[Out] CannotIntegrate(x^m*sech(b*x+a)^2*tanh(b*x+a),x)
```

Rubi [A] time = 0.47, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int x^m\mathrm{sech}^2(a+bx)\tanh(a+bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x^m*Sech[a + b*x]^2*Tanh[a + b*x],x]
[Out] Defer[Int][x^m*Sech[a + b*x]^2*Tanh[a + b*x], x]
```

Rubi steps

$$\int x^m\mathrm{sech}^2(a+bx)\tanh(a+bx)\,dx = \int x^m\mathrm{sech}^2(a+bx)\tanh(a+bx)\,dx$$

Mathematica [A] time = 36.88, size = 0, normalized size = 0.00

$$\int x^m\mathrm{sech}^2(a+bx)\tanh(a+bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^m*Sech[a + b*x]^2*Tanh[a + b*x],x]
[Out] Integrate[x^m*Sech[a + b*x]^2*Tanh[a + b*x], x]
```

fricas [A] time = 0.49, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(x^m\,\mathrm{sech}\,(bx+a)^3\sinh\,(bx+a)\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*sech(b*x+a)^3*sinh(b*x+a),x, algorithm="fricas")
[Out] integral(x^m*sech(b*x + a)^3*sinh(b*x + a), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int x^m \operatorname{sech}(bx+a)^3 \sinh(bx+a)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*sech(b*x+a)^3*sinh(b*x+a),x, algorithm="giac")
[Out] integrate(x^m*sech(b*x + a)^3*sinh(b*x + a), x)
```

maple [A] time = 0.11, size = 0, normalized size = 0.00

$$\int x^m \operatorname{sech}(bx+a)^3 \sinh(bx+a)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*sech(b*x+a)^3*sinh(b*x+a),x)
[Out] int(x^m*sech(b*x+a)^3*sinh(b*x+a),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int x^m \operatorname{sech}(bx+a)^3 \sinh(bx+a)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*sech(b*x+a)^3*sinh(b*x+a),x, algorithm="maxima")
[Out] integrate(x^m*sech(b*x + a)^3*sinh(b*x + a), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{x^m \sinh(a+b\,x)}{\cosh(a+b\,x)^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^m*sinh(a + b*x))/cosh(a + b*x)^3,x)
[Out] int((x^m*sinh(a + b*x))/cosh(a + b*x)^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*sech(b*x+a)**3*sinh(b*x+a),x)
[Out] Timed out
```

3.349 $\int x^3 \operatorname{sech}^2(a+bx)\tanh(a+bx)\,dx$

Optimal. Leaf size=83

$$-\frac{3\mathrm{Li}_2\left(-e^{2(a+bx)}\right)}{2b^4}-\frac{3x\log\left(e^{2(a+bx)}+1\right)}{b^3}+\frac{3x^2\tanh(a+bx)}{2b^2}-\frac{x^3\operatorname{sech}^2(a+bx)}{2b}+\frac{3x^2}{2b^2}$$

```
[Out] 3/2*x^2/b^2-3*x*ln(1+exp(2*b*x+2*a))/b^3-3/2*polylog(2,-exp(2*b*x+2*a))/b^4
-1/2*x^3*sech(b*x+a)^2/b+3/2*x^2*tanh(b*x+a)/b^2
```

Rubi [A] time = 0.18, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5418, 4184, 3718, 2190, 2279, 2391}

$$-\frac{3\text{PolyLog}\left(2,-e^{2(a+bx)}\right)}{2b^4}+\frac{3x^2\tanh(a+bx)}{2b^2}-\frac{3x\log\left(e^{2(a+bx)}+1\right)}{b^3}-\frac{x^3\operatorname{sech}^2(a+bx)}{2b}+\frac{3x^2}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Sech[a + b*x]^2*Tanh[a + b*x],x]
```

```
[Out] (3*x^2)/(2*b^2) - (3*x*Log[1 + E^(2*(a + b*x))])/b^3 - (3*PolyLog[2, -E^(2*
(a + b*x))])/(2*b^4) - (x^3*Sech[a + b*x]^2)/(2*b) + (3*x^2*Tanh[a + b*x])/
(2*b^2)
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3718

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c
 + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /;
 FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 4184

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> -Sim
p[((c + d*x)^m*Cot[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Co
t[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 5418

```
Int[(x_)^(m_.)*Sech[(a_.) + (b_.)*(x_)^(n_.)]^(p_.)*Tanh[(a_.) + (b_.)*(x_)
^(n_.)]^(q_.), x_Symbol] :> -Simp[(x^(m - n + 1)*Sech[a + b*x^n]^p)/(b*n*p)
, x] + Dist[(m - n + 1)/(b*n*p), Int[x^(m - n)*Sech[a + b*x^n]^p, x], x] /;
 FreeQ[{a, b, p}, x] && RationalQ[m] && IntegerQ[n] && GeQ[m - n, 0] && EqQ
[q, 1]
```

Rubi steps

$$
\begin{aligned}
\int x^3 \operatorname{sech}^2(a+bx)\tanh(a+bx)\,dx &= -\frac{x^3\operatorname{sech}^2(a+bx)}{2b}+\frac{3\int x^2\operatorname{sech}^2(a+bx)\,dx}{2b}\\
&= -\frac{x^3\operatorname{sech}^2(a+bx)}{2b}+\frac{3x^2\tanh(a+bx)}{2b^2}-\frac{3\int x\tanh(a+bx)\,dx}{b^2}\\
&= \frac{3x^2}{2b^2}-\frac{x^3\operatorname{sech}^2(a+bx)}{2b}+\frac{3x^2\tanh(a+bx)}{2b^2}-\frac{6\int\frac{e^{2(a+bx)}x}{1+e^{2(a+bx)}}\,dx}{b^2}\\
&= \frac{3x^2}{2b^2}-\frac{3x\log\left(1+e^{2(a+bx)}\right)}{b^3}-\frac{x^3\operatorname{sech}^2(a+bx)}{2b}+\frac{3x^2\tanh(a+bx)}{2b^2}+\frac{3\int
\\
&= \frac{3x^2}{2b^2}-\frac{3x\log\left(1+e^{2(a+bx)}\right)}{b^3}-\frac{x^3\operatorname{sech}^2(a+bx)}{2b}+\frac{3x^2\tanh(a+bx)}{2b^2}+\frac{3\,\mathrm{Su}
\\
&= \frac{3x^2}{2b^2}-\frac{3x\log\left(1+e^{2(a+bx)}\right)}{b^3}-\frac{3\mathrm{Li}_2\left(-e^{2(a+bx)}\right)}{2b^4}-\frac{x^3\operatorname{sech}^2(a+bx)}{2b}+\frac{3x^2\,\mathrm{ta}
\end{aligned}
$$

Mathematica [C] time = 6.14, size = 227, normalized size = 2.73

$$\frac{3x^2\text{sech}(a)\sinh(bx)\text{sech}(a+bx)}{2b^2} - \frac{3\text{csch}(a)\text{sech}(a)\left(b^2x^2e^{-\tanh^{-1}(\coth(a))} - \frac{i\coth(a)\left(i\text{Li}_2\left(e^{2i\left(ibx+i\tanh^{-1}(\coth(a))\right)}\right)-bx\left(-\pi+2i\right.\right.}{\cdots}\right.}{\cdots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^3*Sech[a + b*x]^2*Tanh[a + b*x],x]
```

```
[Out] -1/2*(x^3*Sech[a + b*x]^2)/b - (3*Csch[a]*((b^2*x^2)/E^ArcTanh[Coth[a]] - (
I*Coth[a]*(-(b*x*(-Pi + (2*I)*ArcTanh[Coth[a]])) - Pi*Log[1 + E^(2*b*x)] -
2*(I*b*x + I*ArcTanh[Coth[a]])*Log[1 - E^((2*I)*(I*b*x + I*ArcTanh[Coth[a]]
))] + Pi*Log[Cosh[b*x]] + (2*I)*ArcTanh[Coth[a]]*Log[I*Sinh[b*x + ArcTanh[C
oth[a]]]] + I*PolyLog[2, E^((2*I)*(I*b*x + I*ArcTanh[Coth[a]]))]))/Sqrt[1 -
 Coth[a]^2])*Sech[a])/(2*b^4*Sqrt[Csch[a]^2*(-Cosh[a]^2 + Sinh[a]^2)]) + (3
*x^2*Sech[a]*Sech[a + b*x]*Sinh[b*x])/(2*b^2)
```

fricas [C] time = 0.93, size = 1113, normalized size = 13.41

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*sech(b*x+a)^3*sinh(b*x+a),x, algorithm="fricas")
```

```
[Out] (3*(b^2*x^2 - a^2)*cosh(b*x + a)^4 + 12*(b^2*x^2 - a^2)*cosh(b*x + a)*sinh(
b*x + a)^3 + 3*(b^2*x^2 - a^2)*sinh(b*x + a)^4 - (2*b^3*x^3 - 3*b^2*x^2 + 6
*a^2)*cosh(b*x + a)^2 - (2*b^3*x^3 - 3*b^2*x^2 - 18*(b^2*x^2 - a^2)*cosh(b*
x + a)^2 + 6*a^2)*sinh(b*x + a)^2 - 3*a^2 - 3*(cosh(b*x + a)^4 + 4*cosh(b*x
 + a)*sinh(b*x + a)^3 + sinh(b*x + a)^4 + 2*(3*cosh(b*x + a)^2 + 1)*sinh(b*
x + a)^2 + 2*cosh(b*x + a)^2 + 4*(cosh(b*x + a)^3 + cosh(b*x + a))*sinh(b*x
 + a) + 1)*dilog(I*cosh(b*x + a) + I*sinh(b*x + a)) - 3*(cosh(b*x + a)^4 +
4*cosh(b*x + a)*sinh(b*x + a)^3 + sinh(b*x + a)^4 + 2*(3*cosh(b*x + a)^2 +
1)*sinh(b*x + a)^2 + 2*cosh(b*x + a)^2 + 4*(cosh(b*x + a)^3 + cosh(b*x + a)
)*sinh(b*x + a) + 1)*dilog(-I*cosh(b*x + a) - I*sinh(b*x + a)) + 3*(a*cosh(
b*x + a)^4 + 4*a*cosh(b*x + a)*sinh(b*x + a)^3 + a*sinh(b*x + a)^4 + 2*a*co
sh(b*x + a)^2 + 2*(3*a*cosh(b*x + a)^2 + a)*sinh(b*x + a)^2 + 4*(a*cosh(b*x
 + a)^3 + a*cosh(b*x + a))*sinh(b*x + a) + a)*log(cosh(b*x + a) + sinh(b*x
+ a) + I) + 3*(a*cosh(b*x + a)^4 + 4*a*cosh(b*x + a)*sinh(b*x + a)^3 + a*si
nh(b*x + a)^4 + 2*a*cosh(b*x + a)^2 + 2*(3*a*cosh(b*x + a)^2 + a)*sinh(b*x
+ a)^2 + 4*(a*cosh(b*x + a)^3 + a*cosh(b*x + a))*sinh(b*x + a) + a)*log(cos
h(b*x + a) + sinh(b*x + a) - I) - 3*((b*x + a)*cosh(b*x + a)^4 + 4*(b*x + a
)*cosh(b*x + a)*sinh(b*x + a)^3 + (b*x + a)*sinh(b*x + a)^4 + 2*(b*x + a)*c
```

```
osh(b*x + a)^2 + 2*(3*(b*x + a)*cosh(b*x + a)^2 + b*x + a)*sinh(b*x + a)^2
+ b*x + 4*((b*x + a)*cosh(b*x + a)^3 + (b*x + a)*cosh(b*x + a))*sinh(b*x +
a) + a)*log(I*cosh(b*x + a) + I*sinh(b*x + a) + 1) - 3*((b*x + a)*cosh(b*x
+ a)^4 + 4*(b*x + a)*cosh(b*x + a)*sinh(b*x + a)^3 + (b*x + a)*sinh(b*x + a
)^4 + 2*(b*x + a)*cosh(b*x + a)^2 + 2*(3*(b*x + a)*cosh(b*x + a)^2 + b*x +
a)*sinh(b*x + a)^2 + b*x + 4*((b*x + a)*cosh(b*x + a)^3 + (b*x + a)*cosh(b*
x + a))*sinh(b*x + a) + a)*log(-I*cosh(b*x + a) - I*sinh(b*x + a) + 1) + 2*
(6*(b^2*x^2 - a^2)*cosh(b*x + a)^3 - (2*b^3*x^3 - 3*b^2*x^2 + 6*a^2)*cosh(b
*x + a))*sinh(b*x + a))/(b^4*cosh(b*x + a)^4 + 4*b^4*cosh(b*x + a)*sinh(b*x
 + a)^3 + b^4*sinh(b*x + a)^4 + 2*b^4*cosh(b*x + a)^2 + b^4 + 2*(3*b^4*cosh
(b*x + a)^2 + b^4)*sinh(b*x + a)^2 + 4*(b^4*cosh(b*x + a)^3 + b^4*cosh(b*x
+ a))*sinh(b*x + a))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3 \operatorname{sech}(bx+a)^3 \sinh(bx+a)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*sech(b*x+a)^3*sinh(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(x^3*sech(b*x + a)^3*sinh(b*x + a), x)
```

maple [A] time = 0.23, size = 121, normalized size = 1.46

$$-\frac{x^2\left(2bx\,e^{2bx+2a}+3\,e^{2bx+2a}+3\right)}{b^2\left(1+e^{2bx+2a}\right)^2}+\frac{3x^2}{b^2}+\frac{6ax}{b^3}+\frac{3a^2}{b^4}-\frac{3x\ln\left(1+e^{2bx+2a}\right)}{b^3}-\frac{3\operatorname{polylog}\left(2,-e^{2bx+2a}\right)}{2b^4}-\frac{6a\ln\left(e^{bx+a}\right)}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*sech(b*x+a)^3*sinh(b*x+a),x)
```

```
[Out] -x^2*(2*b*x*exp(2*b*x+2*a)+3*exp(2*b*x+2*a)+3)/b^2/(1+exp(2*b*x+2*a))^2+3*x
^2/b^2+6*a*x/b^3+3/b^4*a^2-3*x*ln(1+exp(2*b*x+2*a))/b^3-3/2*polylog(2,-exp(
2*b*x+2*a))/b^4-6/b^4*a*ln(exp(b*x+a))
```

maxima [A] time = 0.43, size = 110, normalized size = 1.33

$$-\frac{3\,x^2+\left(2\,bx^3e^{(2\,a)}+3\,x^2e^{(2\,a)}\right)e^{(2\,bx)}}{b^2e^{(4\,bx+4\,a)}+2\,b^2e^{(2\,bx+2\,a)}+b^2}+\frac{3\,x^2}{b^2}-\frac{3\left(2\,bx\log\left(e^{(2\,bx+2\,a)}+1\right)+\mathrm{Li}_2\left(-e^{(2\,bx+2\,a)}\right)\right)}{2\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*sech(b*x+a)^3*sinh(b*x+a),x, algorithm="maxima")
```

```
[Out] -(3*x^2 + (2*b*x^3*e^(2*a) + 3*x^2*e^(2*a))*e^(2*b*x))/(b^2*e^(4*b*x + 4*a)
 + 2*b^2*e^(2*b*x + 2*a) + b^2) + 3*x^2/b^2 - 3/2*(2*b*x*log(e^(2*b*x + 2*a
) + 1) + dilog(-e^(2*b*x + 2*a)))/b^4
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3 \sinh(a+b\,x)}{\cosh(a+b\,x)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*sinh(a + b*x))/cosh(a + b*x)^3,x)
```

```
[Out] int((x^3*sinh(a + b*x))/cosh(a + b*x)^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3 \sinh(a+bx)\operatorname{sech}^3(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*sech(b*x+a)**3*sinh(b*x+a),x)
```

```
[Out] Integral(x**3*sinh(a + b*x)*sech(a + b*x)**3, x)
```

3.350 $\int x^2 \operatorname{sech}^2(a+bx)\tanh(a+bx)\,dx$

Optimal. Leaf size=42

$$-\frac{\log(\cosh(a+bx))}{b^3}+\frac{x\tanh(a+bx)}{b^2}-\frac{x^2\operatorname{sech}^2(a+bx)}{2b}$$

```
[Out] -ln(cosh(b*x+a))/b^3-1/2*x^2*sech(b*x+a)^2/b+x*tanh(b*x+a)/b^2
```

Rubi [A] time = 0.06, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {5418, 4184, 3475}

$$\frac{x\tanh(a+bx)}{b^2}-\frac{\log(\cosh(a+bx))}{b^3}-\frac{x^2\operatorname{sech}^2(a+bx)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Sech[a + b*x]^2*Tanh[a + b*x],x]
```

```
[Out] -(Log[Cosh[a + b*x]]/b^3) - (x^2*Sech[a + b*x]^2)/(2*b) + (x*Tanh[a + b*x])
/b^2
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 4184

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> -Sim
p[((c + d*x)^m*Cot[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Co
t[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 5418

```
Int[(x_)^(m_.)*Sech[(a_.) + (b_.)*(x_)^(n_.)]^(p_.)*Tanh[(a_.) + (b_.)*(x_)
^(n_.)]^(q_.), x_Symbol] :> -Simp[(x^(m - n + 1)*Sech[a + b*x^n]^p)/(b*n*p)
, x] + Dist[(m - n + 1)/(b*n*p), Int[x^(m - n)*Sech[a + b*x^n]^p, x], x] /;
 FreeQ[{a, b, p}, x] && RationalQ[m] && IntegerQ[n] && GeQ[m - n, 0] && EqQ
[q, 1]
```

Rubi steps

$$\int x^2\text{sech}^2(a+bx)\tanh(a+bx)\,dx = -\frac{x^2\text{sech}^2(a+bx)}{2b} + \frac{\int x\text{sech}^2(a+bx)\,dx}{b}$$
$$= -\frac{x^2\text{sech}^2(a+bx)}{2b} + \frac{x\tanh(a+bx)}{b^2} - \frac{\int \tanh(a+bx)\,dx}{b^2}$$
$$= -\frac{\log(\cosh(a+bx))}{b^3} - \frac{x^2\text{sech}^2(a+bx)}{2b} + \frac{x\tanh(a+bx)}{b^2}$$

Mathematica [**A**] time = 0.13, size = 55, normalized size = 1.31

$$-\frac{\log(\cosh(a+bx))}{b^3} + \frac{x\tanh(a)}{b^2} + \frac{x\text{sech}(a)\sinh(bx)\text{sech}(a+bx)}{b^2} - \frac{x^2\text{sech}^2(a+bx)}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Sech[a + b*x]^2*Tanh[a + b*x],x]
```

```
[Out] -(Log[Cosh[a + b*x]]/b^3) - (x^2*Sech[a + b*x]^2)/(2*b) + (x*Sech[a]*Sech[a
 + b*x]*Sinh[b*x])/b^2 + (x*Tanh[a])/b^2
```

fricas [**B**] time = 0.89, size = 378, normalized size = 9.00

$$\frac{2\,bx\cosh(bx+a)^4 + 8\,bx\cosh(bx+a)\sinh(bx+a)^3 + 2\,bx\sinh(bx+a)^4 - 2\left(b^2x^2 - bx\right)\cosh(bx+a)^2 - 2\left(b\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sech(b*x+a)^3*sinh(b*x+a),x, algorithm="fricas")
```

```
[Out] (2*b*x*cosh(b*x + a)^4 + 8*b*x*cosh(b*x + a)*sinh(b*x + a)^3 + 2*b*x*sinh(b
*x + a)^4 - 2*(b^2*x^2 - b*x)*cosh(b*x + a)^2 - 2*(b^2*x^2 - 6*b*x*cosh(b*x
 + a)^2 - b*x)*sinh(b*x + a)^2 - (cosh(b*x + a)^4 + 4*cosh(b*x + a)*sinh(b*
x + a)^3 + sinh(b*x + a)^4 + 2*(3*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^2 + 2*
cosh(b*x + a)^2 + 4*(cosh(b*x + a)^3 + cosh(b*x + a))*sinh(b*x + a) + 1)*lo
g(2*cosh(b*x + a)/(cosh(b*x + a) - sinh(b*x + a))) + 4*(2*b*x*cosh(b*x + a)
^3 - (b^2*x^2 - b*x)*cosh(b*x + a))*sinh(b*x + a))/(b^3*cosh(b*x + a)^4 + 4
*b^3*cosh(b*x + a)*sinh(b*x + a)^3 + b^3*sinh(b*x + a)^4 + 2*b^3*cosh(b*x +
 a)^2 + b^3 + 2*(3*b^3*cosh(b*x + a)^2 + b^3)*sinh(b*x + a)^2 + 4*(b^3*cosh
(b*x + a)^3 + b^3*cosh(b*x + a))*sinh(b*x + a))
```

giac [**B**] time = 0.13, size = 142, normalized size = 3.38

$$-\frac{2\,b^2x^2e^{(2\,bx+2\,a)} - 2\,bxe^{(4\,bx+4\,a)} - 2\,bxe^{(2\,bx+2\,a)} + e^{(4\,bx+4\,a)}\log\left(-e^{(2\,bx+2\,a)} - 1\right) + 2\,e^{(2\,bx+2\,a)}\log\left(-e^{(2\,bx+2\,a)} - 1\right) +}{b^3e^{(4\,bx+4\,a)} + 2\,b^3e^{(2\,bx+2\,a)} + b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sech(b*x+a)^3*sinh(b*x+a),x, algorithm="giac")
```

```
[Out] -(2*b^2*x^2*e^(2*b*x + 2*a) - 2*b*x*e^(4*b*x + 4*a) - 2*b*x*e^(2*b*x + 2*a)
 + e^(4*b*x + 4*a)*log(-e^(2*b*x + 2*a) - 1) + 2*e^(2*b*x + 2*a)*log(-e^(2*
b*x + 2*a) - 1) + log(-e^(2*b*x + 2*a) - 1))/(b^3*e^(4*b*x + 4*a) + 2*b^3*e
^(2*b*x + 2*a) + b^3)
```

maple [A] time = 0.15, size = 73, normalized size = 1.74

$$\frac{2x}{b^2}+\frac{2a}{b^3}-\frac{2x\left(bx\,\mathrm{e}^{2bx+2a}+\mathrm{e}^{2bx+2a}+1\right)}{b^2\left(1+\mathrm{e}^{2bx+2a}\right)^2}-\frac{\ln\left(1+\mathrm{e}^{2bx+2a}\right)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*sech(b*x+a)^3*sinh(b*x+a),x)
```

```
[Out] 2*x/b^2+2/b^3*a-2*x*(b*x*exp(2*b*x+2*a)+exp(2*b*x+2*a)+1)/b^2/(1+exp(2*b*x+
2*a))^2-1/b^3*ln(1+exp(2*b*x+2*a))
```

maxima [B] time = 0.41, size = 94, normalized size = 2.24

$$-\frac{2\left(\left(bx^2e^{(2a)}-xe^{(2a)}\right)e^{(2bx)}-xe^{(4bx+4a)}\right)}{b^2e^{(4bx+4a)}+2\,b^2e^{(2bx+2a)}+b^2}-\frac{\log\left(\left(e^{(2bx+2a)}+1\right)e^{(-2a)}\right)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sech(b*x+a)^3*sinh(b*x+a),x, algorithm="maxima")
```

```
[Out] -2*((b*x^2*e^(2*a) - x*e^(2*a))*e^(2*b*x) - x*e^(4*b*x + 4*a))/(b^2*e^(4*b*
x + 4*a) + 2*b^2*e^(2*b*x + 2*a) + b^2) - log((e^(2*b*x + 2*a) + 1)*e^(-2*a
))/b^3
```

mupad [B] time = 1.48, size = 102, normalized size = 2.43

$$\frac{\frac{x^2}{b}-\frac{x^2\,\mathrm{e}^{2\,a+2\,b\,x}}{b}}{2\,\mathrm{e}^{2\,a+2\,b\,x}+\mathrm{e}^{4\,a+4\,b\,x}+1}-\frac{\ln\left(\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}+1\right)}{b^3}+\frac{2\,x}{b^2}-\frac{b\,x^2+2\,x}{b^2\,\left(\mathrm{e}^{2\,a+2\,b\,x}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*sinh(a + b*x))/cosh(a + b*x)^3,x)
```

```
[Out] (x^2/b - (x^2*exp(2*a + 2*b*x))/b)/(2*exp(2*a + 2*b*x) + exp(4*a + 4*b*x) +
 1) - log(exp(2*a)*exp(2*b*x) + 1)/b^3 + (2*x)/b^2 - (2*x + b*x^2)/(b^2*(ex
p(2*a + 2*b*x) + 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \sinh (a + bx) \operatorname{sech}^3 (a + bx) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*sech(b*x+a)**3*sinh(b*x+a),x)
```

```
[Out] Integral(x**2*sinh(a + b*x)*sech(a + b*x)**3, x)
```

3.351 $\int x\text{sech}^2(a+bx)\tanh(a+bx)\,dx$

Optimal. Leaf size=30

$$\frac{\tanh(a+bx)}{2b^2}-\frac{x\text{sech}^2(a+bx)}{2b}$$

```
[Out] -1/2*x*sech(b*x+a)^2/b+1/2*tanh(b*x+a)/b^2
```

Rubi [A] time = 0.03, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {5418, 3767, 8}

$$\frac{\tanh(a+bx)}{2b^2}-\frac{x\text{sech}^2(a+bx)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sech[a + b*x]^2*Tanh[a + b*x],x]
```

```
[Out] -(x*Sech[a + b*x]^2)/(2*b) + Tanh[a + b*x]/(2*b^2)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 5418

```
Int[(x_)^(m_.)*Sech[(a_.) + (b_.)*(x_)^(n_.)]^(p_.)*Tanh[(a_.) + (b_.)*(x_)
^(n_.)]^(q_.), x_Symbol] :> -Simp[(x^(m - n + 1)*Sech[a + b*x^n]^p)/(b*n*p)
, x] + Dist[(m - n + 1)/(b*n*p), Int[x^(m - n)*Sech[a + b*x^n]^p, x], x] /;
 FreeQ[{a, b, p}, x] && RationalQ[m] && IntegerQ[n] && GeQ[m - n, 0] && EqQ
[q, 1]
```

Rubi steps

$$\int x\operatorname{sech}^2(a+bx)\tanh(a+bx)\,dx = -\frac{x\operatorname{sech}^2(a+bx)}{2b} + \frac{\int \operatorname{sech}^2(a+bx)\,dx}{2b}$$
$$= -\frac{x\operatorname{sech}^2(a+bx)}{2b} + \frac{i\,\text{Subst}(\int 1\,dx, x, -i\tanh(a+bx))}{2b^2}$$
$$= -\frac{x\operatorname{sech}^2(a+bx)}{2b} + \frac{\tanh(a+bx)}{2b^2}$$

Mathematica [A] time = 0.08, size = 30, normalized size = 1.00

$$\frac{\tanh(a+bx)}{2b^2} - \frac{x\operatorname{sech}^2(a+bx)}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sech[a + b*x]^2*Tanh[a + b*x],x]
```

```
[Out] -1/2*(x*Sech[a + b*x]^2)/b + Tanh[a + b*x]/(2*b^2)
```

fricas [B] time = 0.43, size = 105, normalized size = 3.50

$$-\frac{2\,(bx\sinh(bx+a) + (bx+1)\cosh(bx+a))}{b^2\cosh(bx+a)^3 + 3\,b^2\cosh(bx+a)\sinh(bx+a)^2 + b^2\sinh(bx+a)^3 + 3\,b^2\cosh(bx+a) + \left(3\,b^2\cosh(bx+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sech(b*x+a)^3*sinh(b*x+a),x, algorithm="fricas")
```

```
[Out] -2*(b*x*sinh(b*x + a) + (b*x + 1)*cosh(b*x + a))/(b^2*cosh(b*x + a)^3 + 3*b
^2*cosh(b*x + a)*sinh(b*x + a)^2 + b^2*sinh(b*x + a)^3 + 3*b^2*cosh(b*x + a
) + (3*b^2*cosh(b*x + a)^2 + b^2)*sinh(b*x + a))
```

giac [B] time = 0.15, size = 184, normalized size = 6.13

$$-\frac{4\,bxe^{(2\,bx+2\,a)} - e^{(4\,bx+4\,a)}\log\left(e^{(2\,bx+2\,a)}+1\right) - 2\,e^{(2\,bx+2\,a)}\log\left(e^{(2\,bx+2\,a)}+1\right) + e^{(4\,bx+4\,a)}\log\left(-e^{(2\,bx+2\,a)}-1\right) + 2\,e}{2\left(b^2e^{(4\,bx+4\,a)} + 2\,b^2e^{(2\,bx+2\,a)}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sech(b*x+a)^3*sinh(b*x+a),x, algorithm="giac")
```

```
[Out] -1/2*(4*b*x*e^(2*b*x + 2*a) - e^(4*b*x + 4*a)*log(e^(2*b*x + 2*a) + 1) - 2*
e^(2*b*x + 2*a)*log(e^(2*b*x + 2*a) + 1) + e^(4*b*x + 4*a)*log(-e^(2*b*x +
```

```
2*a) - 1) + 2*e^(2*b*x + 2*a)*log(-e^(2*b*x + 2*a) - 1) + 2*e^(2*b*x + 2*a)
 - log(e^(2*b*x + 2*a) + 1) + log(-e^(2*b*x + 2*a) - 1) + 2)/(b^2*e^(4*b*x
+ 4*a) + 2*b^2*e^(2*b*x + 2*a) + b^2)
```

maple [A] time = 0.16, size = 43, normalized size = 1.43

$$-\frac{2bx\,\mathrm{e}^{2bx+2a}+\mathrm{e}^{2bx+2a}+1}{b^{2}\left(1+\mathrm{e}^{2bx+2a}\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sech(b*x+a)^3*sinh(b*x+a),x)
[Out] -(2*b*x*exp(2*b*x+2*a)+exp(2*b*x+2*a)+1)/b^2/(1+exp(2*b*x+2*a))^2
```

maxima [B] time = 0.35, size = 131, normalized size = 4.37

$$-\frac{2\,bxe^{(4\,bx+4\,a)}+\left(4\,bxe^{(2\,a)}+e^{(2\,a)}\right)e^{(2\,bx)}+1}{2\left(b^{2}e^{(4\,bx+4\,a)}+2\,b^{2}e^{(2\,bx+2\,a)}+b^{2}\right)}+\frac{2\,bxe^{(4\,bx+4\,a)}-e^{(2\,bx+2\,a)}-1}{2\left(b^{2}e^{(4\,bx+4\,a)}+2\,b^{2}e^{(2\,bx+2\,a)}+b^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sech(b*x+a)^3*sinh(b*x+a),x, algorithm="maxima")
[Out] -1/2*(2*b*x*e^(4*b*x + 4*a) + (4*b*x*e^(2*a) + e^(2*a))*e^(2*b*x) + 1)/(b^2
*e^(4*b*x + 4*a) + 2*b^2*e^(2*b*x + 2*a) + b^2) + 1/2*(2*b*x*e^(4*b*x + 4*a
) - e^(2*b*x + 2*a) - 1)/(b^2*e^(4*b*x + 4*a) + 2*b^2*e^(2*b*x + 2*a) + b^2
)
```

mupad [B] time = 1.46, size = 36, normalized size = 1.20

$$-\frac{\mathrm{e}^{2\,a+2\,b\,x}\,\left(2\,b\,x+1\right)+1}{b^2\,{\left(\mathrm{e}^{2\,a+2\,b\,x}+1\right)}^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*sinh(a + b*x))/cosh(a + b*x)^3,x)
[Out] -(exp(2*a + 2*b*x)*(2*b*x + 1) + 1)/(b^2*(exp(2*a + 2*b*x) + 1)^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\sinh\left(a+bx\right)\operatorname{sech}^{3}\left(a+bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sech(b*x+a)**3*sinh(b*x+a),x)
[Out] Integral(x*sinh(a + b*x)*sech(a + b*x)**3, x)
```

3.352 $\int \operatorname{sech}^2(a+bx)\tanh(a+bx)\,dx$

Optimal. Leaf size=15

$$-\frac{\operatorname{sech}^2(a+bx)}{2b}$$

```
[Out] -1/2*sech(b*x+a)^2/b
```

Rubi [A] time = 0.02, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2606, 30}

$$-\frac{\operatorname{sech}^2(a+bx)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[a + b*x]^2*Tanh[a + b*x],x]
```

```
[Out] -Sech[a + b*x]^2/(2*b)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rubi steps

$$\begin{aligned}\int \operatorname{sech}^2(a+bx)\tanh(a+bx)\,dx &= -\frac{\operatorname{Subst}\left(\int x\,dx, x, \operatorname{sech}(a+bx)\right)}{b}\\ &= -\frac{\operatorname{sech}^2(a+bx)}{2b}\end{aligned}$$

Mathematica [A] time = 0.01, size = 15, normalized size = 1.00

$$-\frac{\operatorname{sech}^2(a+bx)}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[a + b*x]^2*Tanh[a + b*x],x]
```

```
[Out] -1/2*Sech[a + b*x]^2/b
```

fricas [B] time = 0.64, size = 84, normalized size = 5.60

$$-\frac{2\left(\cosh\left(bx+a\right)+\sinh\left(bx+a\right)\right)}{b\cosh\left(bx+a\right)^{3}+3\,b\cosh\left(bx+a\right)\sinh\left(bx+a\right)^{2}+b\sinh\left(bx+a\right)^{3}+3\,b\cosh\left(bx+a\right)+\left(3\,b\cosh\left(bx+a\right)^{2}+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)^3*sinh(b*x+a),x, algorithm="fricas")
```

```
[Out] -2*(cosh(b*x + a) + sinh(b*x + a))/(b*cosh(b*x + a)^3 + 3*b*cosh(b*x + a)*s
inh(b*x + a)^2 + b*sinh(b*x + a)^3 + 3*b*cosh(b*x + a) + (3*b*cosh(b*x + a)
^2 + b)*sinh(b*x + a))
```

giac [B] time = 0.14, size = 27, normalized size = 1.80

$$-\frac{2\,e^{\left(2\,bx+2\,a\right)}}{b\left(e^{\left(2\,bx+2\,a\right)}+1\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)^3*sinh(b*x+a),x, algorithm="giac")
```

```
[Out] -2*e^(2*b*x + 2*a)/(b*(e^(2*b*x + 2*a) + 1)^2)
```

maple [A] time = 0.03, size = 14, normalized size = 0.93

$$-\frac{\mathrm{sech}\left(bx+a\right)^{2}}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(b*x+a)^3*sinh(b*x+a),x)
```

```
[Out] -1/2*sech(b*x+a)^2/b
```

maxima [A] time = 0.32, size = 23, normalized size = 1.53

$$-\frac{2}{b\left(e^{\left(bx+a\right)}+e^{\left(-bx-a\right)}\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)^3*sinh(b*x+a),x, algorithm="maxima")
```

```
[Out] -2/(b*(e^(b*x + a) + e^(-b*x - a))^2)
```

mupad [B] time = 1.45, size = 13, normalized size = 0.87

$$-\frac{1}{2\,b\cosh\left(a+b\,x\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a + b*x)/cosh(a + b*x)^3,x)
```

```
[Out] -1/(2*b*cosh(a + b*x)^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sinh\left(a+bx\right)\operatorname{sech}^3\left(a+bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)**3*sinh(b*x+a),x)
```

```
[Out] Integral(sinh(a + b*x)*sech(a + b*x)**3, x)
```

3.353 $\int \frac{\text{sech}^2(a+bx)\tanh(a+bx)}{x}\,dx$

Optimal. Leaf size=21

$$\text{Int}\left(\frac{\tanh(a+bx)\text{sech}^2(a+bx)}{x},x\right)$$

```
[Out] CannotIntegrate(sech(b*x+a)^2*tanh(b*x+a)/x,x)
```

Rubi [A] time = 0.18, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\text{sech}^2(a+bx)\tanh(a+bx)}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Sech[a + b*x]^2*Tanh[a + b*x])/x,x]
[Out] Defer[Int][(Sech[a + b*x]^2*Tanh[a + b*x])/x, x]
```

Rubi steps

$$\int \frac{\text{sech}^2(a+bx)\tanh(a+bx)}{x}\,dx=\int \frac{\text{sech}^2(a+bx)\tanh(a+bx)}{x}\,dx$$

Mathematica [A] time = 24.69, size = 0, normalized size = 0.00

$$\int \frac{\text{sech}^2(a+bx)\tanh(a+bx)}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Sech[a + b*x]^2*Tanh[a + b*x])/x,x]
[Out] Integrate[(Sech[a + b*x]^2*Tanh[a + b*x])/x, x]
```

fricas [A] time = 0.55, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\text{sech}\left(bx+a\right)^3\sinh\left(bx+a\right)}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)^3*sinh(b*x+a)/x,x, algorithm="fricas")
```

```
[Out] integral(sech(b*x + a)^3*sinh(b*x + a)/x, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}(bx+a)^3 \sinh(bx+a)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)^3*sinh(b*x+a)/x,x, algorithm="giac")
```

```
[Out] integrate(sech(b*x + a)^3*sinh(b*x + a)/x, x)
```

maple [A] time = 0.37, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}(bx+a)^3 \sinh(bx+a)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(b*x+a)^3*sinh(b*x+a)/x,x)
```

```
[Out] int(sech(b*x+a)^3*sinh(b*x+a)/x,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\left(2\,bxe^{(2\,a)} - e^{(2\,a)}\right)e^{(2\,bx)} - 1}{b^2x^2e^{(4\,bx+4\,a)} + 2\,b^2x^2e^{(2\,bx+2\,a)} + b^2x^2} + 4\int \frac{1}{2\left(b^2x^3e^{(2\,bx+2\,a)} + b^2x^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)^3*sinh(b*x+a)/x,x, algorithm="maxima")
```

```
[Out] -((2*b*x*e^(2*a) - e^(2*a))*e^(2*b*x) - 1)/(b^2*x^2*e^(4*b*x + 4*a) + 2*b^2
*x^2*e^(2*b*x + 2*a) + b^2*x^2) + 4*integrate(1/2/(b^2*x^3*e^(2*b*x + 2*a)
+ b^2*x^3), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{\sinh(a+b\,x)}{x\cosh(a+b\,x)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a + b*x)/(x*cosh(a + b*x)^3),x)
```

```
[Out] int(sinh(a + b*x)/(x*cosh(a + b*x)^3), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh(a+bx)\,\mathrm{sech}^3(a+bx)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)**3*sinh(b*x+a)/x,x)
[Out] Integral(sinh(a + b*x)*sech(a + b*x)**3/x, x)
```

3.354 $\int \frac{\text{sech}^2(a+bx)\tanh(a+bx)}{x^2}\,dx$

Optimal. Leaf size=21

$$\text{Int}\left(\frac{\tanh(a+bx)\text{sech}^2(a+bx)}{x^2},x\right)$$

```
[Out] CannotIntegrate(sech(b*x+a)^2*tanh(b*x+a)/x^2,x)
```

Rubi [A] time = 0.21, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\text{sech}^2(a+bx)\tanh(a+bx)}{x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Sech[a + b*x]^2*Tanh[a + b*x])/x^2,x]
[Out] Defer[Int][(Sech[a + b*x]^2*Tanh[a + b*x])/x^2, x]
```

Rubi steps

$$\int \frac{\text{sech}^2(a+bx)\tanh(a+bx)}{x^2}\,dx = \int \frac{\text{sech}^2(a+bx)\tanh(a+bx)}{x^2}\,dx$$

Mathematica [A] time = 21.29, size = 0, normalized size = 0.00

$$\int \frac{\text{sech}^2(a+bx)\tanh(a+bx)}{x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Sech[a + b*x]^2*Tanh[a + b*x])/x^2,x]
[Out] Integrate[(Sech[a + b*x]^2*Tanh[a + b*x])/x^2, x]
```

fricas [A] time = 0.63, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\text{sech}\,(bx+a)^3\sinh\,(bx+a)}{x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)^3*sinh(b*x+a)/x^2,x, algorithm="fricas")
```

```
[Out] integral(sech(b*x + a)^3*sinh(b*x + a)/x^2, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}(bx+a)^3\sinh(bx+a)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)^3*sinh(b*x+a)/x^2,x, algorithm="giac")
```

```
[Out] integrate(sech(b*x + a)^3*sinh(b*x + a)/x^2, x)
```

maple [A] time = 0.37, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}(bx+a)^3\sinh(bx+a)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(b*x+a)^3*sinh(b*x+a)/x^2,x)
```

```
[Out] int(sech(b*x+a)^3*sinh(b*x+a)/x^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{2\left(\left(bxe^{(2a)}-e^{(2a)}\right)e^{(2bx)}-1\right)}{b^2x^3e^{(4bx+4a)}+2b^2x^3e^{(2bx+2a)}+b^2x^3}+12\int \frac{1}{2\left(b^2x^4e^{(2bx+2a)}+b^2x^4\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)^3*sinh(b*x+a)/x^2,x, algorithm="maxima")
```

```
[Out] -2*((b*x*e^(2*a) - e^(2*a))*e^(2*b*x) - 1)/(b^2*x^3*e^(4*b*x + 4*a) + 2*b^2
*x^3*e^(2*b*x + 2*a) + b^2*x^3) + 12*integrate(1/2/(b^2*x^4*e^(2*b*x + 2*a)
 + b^2*x^4), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{\sinh(a+b\,x)}{x^2\cosh(a+b\,x)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a + b*x)/(x^2*cosh(a + b*x)^3),x)
```

```
[Out] int(sinh(a + b*x)/(x^2*cosh(a + b*x)^3), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh(a+bx)\operatorname{sech}^3(a+bx)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)**3*sinh(b*x+a)/x**2,x)
```

```
[Out] Integral(sinh(a + b*x)*sech(a + b*x)**3/x**2, x)
```

3.355 $\int x^m \sinh(a+bx)\tanh(a+bx)\,dx$

Optimal. Leaf size=74

$$-\mathrm{Int}\left(x^m \mathrm{sech}(a+bx),x\right)+\frac{e^a x^m (-bx)^{-m}\Gamma(m+1,-bx)}{2b}-\frac{e^{-a} x^m (bx)^{-m}\Gamma(m+1,bx)}{2b}$$

```
[Out] 1/2*exp(a)*x^m*GAMMA(1+m,-b*x)/b/((-b*x)^m)-1/2*x^m*GAMMA(1+m,b*x)/b/exp(a)
/((b*x)^m)-Unintegrable(x^m*sech(b*x+a),x)
```

Rubi [A] time = 0.11, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int x^m \sinh(a+bx)\tanh(a+bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x^m*Sinh[a + b*x]*Tanh[a + b*x],x]
```

```
[Out] (E^a*x^m*Gamma[1 + m, -(b*x)])/(2*b*(-(b*x))^m) - (x^m*Gamma[1 + m, b*x])/(
2*b*E^a*(b*x)^m) - Defer[Int][x^m*Sech[a + b*x], x]
```

Rubi steps

$$\begin{aligned}\int x^m \sinh(a+bx)\tanh(a+bx)\,dx &= \int x^m \cosh(a+bx)\,dx-\int x^m \mathrm{sech}(a+bx)\,dx\\ &= \frac{1}{2}\int e^{-i(ia+ibx)}x^m\,dx+\frac{1}{2}\int e^{i(ia+ibx)}x^m\,dx-\int x^m\mathrm{sech}(a+bx)\,dx\\ &= \frac{e^a x^m(-bx)^{-m}\Gamma(1+m,-bx)}{2b}-\frac{e^{-a}x^m(bx)^{-m}\Gamma(1+m,bx)}{2b}-\int x^m\mathrm{sech}(a+bx\end{aligned}$$

Mathematica [A] time = 14.08, size = 0, normalized size = 0.00

$$\int x^m \sinh(a+bx)\tanh(a+bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^m*Sinh[a + b*x]*Tanh[a + b*x],x]
```

```
[Out] Integrate[x^m*Sinh[a + b*x]*Tanh[a + b*x], x]
```

fricas [A] time = 0.55, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(x^m\,\mathrm{sech}\left(bx+a\right)\sinh\left(bx+a\right)^2,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*sech(b*x+a)*sinh(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] integral(x^m*sech(b*x + a)*sinh(b*x + a)^2, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int x^m \operatorname{sech}(bx+a)\sinh(bx+a)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*sech(b*x+a)*sinh(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate(x^m*sech(b*x + a)*sinh(b*x + a)^2, x)
```

maple [A] time = 0.32, size = 0, normalized size = 0.00

$$\int x^m \operatorname{sech}(bx+a)\left(\sinh^2(bx+a)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*sech(b*x+a)*sinh(b*x+a)^2,x)
```

```
[Out] int(x^m*sech(b*x+a)*sinh(b*x+a)^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int x^m \operatorname{sech}(bx+a)\sinh(bx+a)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*sech(b*x+a)*sinh(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] integrate(x^m*sech(b*x + a)*sinh(b*x + a)^2, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^m \sinh(a+b\,x)^2}{\cosh(a+b\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^m*sinh(a + b*x)^2)/cosh(a + b*x),x)
```

```
[Out] int((x^m*sinh(a + b*x)^2)/cosh(a + b*x), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int x^m \sinh^2(a+bx)\operatorname{sech}(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*sech(b*x+a)*sinh(b*x+a)**2,x)
```

```
[Out] Integral(x**m*sinh(a + b*x)**2*sech(a + b*x), x)
```

3.356 $\int x^3 \sinh(a+bx)\tanh(a+bx)\,dx$

Optimal. Leaf size=195

$$\frac{6i\text{Li}_4\left(-ie^{a+bx}\right)}{b^4}-\frac{6i\text{Li}_4\left(ie^{a+bx}\right)}{b^4}-\frac{6\cosh(a+bx)}{b^4}-\frac{6ix\text{Li}_3\left(-ie^{a+bx}\right)}{b^3}+\frac{6ix\text{Li}_3\left(ie^{a+bx}\right)}{b^3}+\frac{6x\sinh(a+bx)}{b^3}+\frac{3ix^2\text{Li}_2\left(-\right.}{b^2}$$

```
[Out] -2*x^3*arctan(exp(b*x+a))/b-6*cosh(b*x+a)/b^4-3*x^2*cosh(b*x+a)/b^2+3*I*x^2
*polylog(2,-I*exp(b*x+a))/b^2-3*I*x^2*polylog(2,I*exp(b*x+a))/b^2-6*I*x*pol
ylog(3,-I*exp(b*x+a))/b^3+6*I*x*polylog(3,I*exp(b*x+a))/b^3+6*I*polylog(4,-
I*exp(b*x+a))/b^4-6*I*polylog(4,I*exp(b*x+a))/b^4+6*x*sinh(b*x+a)/b^3+x^3*s
inh(b*x+a)/b
```

Rubi [A] time = 0.20, antiderivative size = 195, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 8, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5449, 3296, 2638, 4180, 2531, 6609, 2282, 6589}

$$\frac{3ix^2\text{PolyLog}\left(2,-ie^{a+bx}\right)}{b^2}-\frac{3ix^2\text{PolyLog}\left(2,ie^{a+bx}\right)}{b^2}-\frac{6ix\text{PolyLog}\left(3,-ie^{a+bx}\right)}{b^3}+\frac{6ix\text{PolyLog}\left(3,ie^{a+bx}\right)}{b^3}+\frac{6i\text{PolyI}}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Sinh[a + b*x]*Tanh[a + b*x],x]
```

```
[Out] (-2*x^3*ArcTan[E^(a + b*x)])/b - (6*Cosh[a + b*x])/b^4 - (3*x^2*Cosh[a + b*
x])/b^2 + ((3*I)*x^2*PolyLog[2, (-I)*E^(a + b*x)])/b^2 - ((3*I)*x^2*PolyLog
[2, I*E^(a + b*x)])/b^2 - ((6*I)*x*PolyLog[3, (-I)*E^(a + b*x)])/b^3 + ((6*
I)*x*PolyLog[3, I*E^(a + b*x)])/b^3 + ((6*I)*PolyLog[4, (-I)*E^(a + b*x)])/
b^4 - ((6*I)*PolyLog[4, I*E^(a + b*x)])/b^4 + (6*x*Sinh[a + b*x])/b^3 + (x^
3*Sinh[a + b*x])/b
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
```

```
, g, n}, x] && GtQ[m, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5449

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) + (b_.)*(x_)]^(n_.)*Tanh[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> Int[(c + d*x)^m*Sinh[a + b*x]^n*Tanh[a + b*
x]^(p - 2), x] - Int[(c + d*x)^m*Sinh[a + b*x]^(n - 2)*Tanh[a + b*x]^p, x]
/; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int x^3\sinh(a+bx)\tanh(a+bx)\,dx = \int x^3\cosh(a+bx)\,dx - \int x^3\text{sech}(a+bx)\,dx$$
$$= -\frac{2x^3\tan^{-1}\left(e^{a+bx}\right)}{b} + \frac{x^3\sinh(a+bx)}{b} + \frac{(3i)\int x^2\log\left(1-ie^{a+bx}\right)dx}{b} - \frac{(3i}{}$$
$$= -\frac{2x^3\tan^{-1}\left(e^{a+bx}\right)}{b} - \frac{3x^2\cosh(a+bx)}{b^2} + \frac{3ix^2\text{Li}_2\left(-ie^{a+bx}\right)}{b^2} - \frac{3ix^2\text{Li}_2\left(ie^{a}\right.}{b^2}$$
$$= -\frac{2x^3\tan^{-1}\left(e^{a+bx}\right)}{b} - \frac{3x^2\cosh(a+bx)}{b^2} + \frac{3ix^2\text{Li}_2\left(-ie^{a+bx}\right)}{b^2} - \frac{3ix^2\text{Li}_2\left(ie^{a}\right.}{b^2}$$
$$= -\frac{2x^3\tan^{-1}\left(e^{a+bx}\right)}{b} - \frac{6\cosh(a+bx)}{b^4} - \frac{3x^2\cosh(a+bx)}{b^2} + \frac{3ix^2\text{Li}_2\left(-ie^{a+}\right.}{b^2}$$
$$= -\frac{2x^3\tan^{-1}\left(e^{a+bx}\right)}{b} - \frac{6\cosh(a+bx)}{b^4} - \frac{3x^2\cosh(a+bx)}{b^2} + \frac{3ix^2\text{Li}_2\left(-ie^{a+}\right.}{b^2}$$

Mathematica [A] time = 1.48, size = 211, normalized size = 1.08

$$-\frac{i\left(b^3x^3\log\left(1-ie^{a+bx}\right) - b^3x^3\log\left(1+ie^{a+bx}\right) + ib^3x^3\sinh(a+bx) - 3b^2x^2\text{Li}_2\left(-ie^{a+bx}\right) + 3b^2x^2\text{Li}_2\left(ie^{a+bx}\right) - \right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Sinh[a + b*x]*Tanh[a + b*x],x]
```

```
[Out] ((-I)*((-6*I)*Cosh[a + b*x] - (3*I)*b^2*x^2*Cosh[a + b*x] + b^3*x^3*Log[1 -
 I*E^(a + b*x)] - b^3*x^3*Log[1 + I*E^(a + b*x)] - 3*b^2*x^2*PolyLog[2, (-I
)*E^(a + b*x)] + 3*b^2*x^2*PolyLog[2, I*E^(a + b*x)] + 6*b*x*PolyLog[3, (-I
)*E^(a + b*x)] - 6*b*x*PolyLog[3, I*E^(a + b*x)] - 6*PolyLog[4, (-I)*E^(a +
 b*x)] + 6*PolyLog[4, I*E^(a + b*x)] + (6*I)*b*x*Sinh[a + b*x] + I*b^3*x^3*
Sinh[a + b*x]))/b^4
```

fricas [C] time = 0.52, size = 609, normalized size = 3.12

$$-\frac{b^3x^3 + 3\,b^2x^2 - \left(b^3x^3 - 3\,b^2x^2 + 6\,bx - 6\right)\cosh\left(bx+a\right)^2 - 2\left(b^3x^3 - 3\,b^2x^2 + 6\,bx - 6\right)\cosh\left(bx+a\right)\sinh\left(bx\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*sech(b*x+a)*sinh(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/2*(b^3*x^3 + 3*b^2*x^2 - (b^3*x^3 - 3*b^2*x^2 + 6*b*x - 6)*cosh(b*x + a)
^2 - 2*(b^3*x^3 - 3*b^2*x^2 + 6*b*x - 6)*cosh(b*x + a)*sinh(b*x + a) - (b^3
```

```
*x^3 - 3*b^2*x^2 + 6*b*x - 6)*sinh(b*x + a)^2 + 6*b*x - (-6*I*b^2*x^2*cosh(
b*x + a) - 6*I*b^2*x^2*sinh(b*x + a))*dilog(I*cosh(b*x + a) + I*sinh(b*x +
a)) - (6*I*b^2*x^2*cosh(b*x + a) + 6*I*b^2*x^2*sinh(b*x + a))*dilog(-I*cosh
(b*x + a) - I*sinh(b*x + a)) - (2*I*a^3*cosh(b*x + a) + 2*I*a^3*sinh(b*x +
a))*log(cosh(b*x + a) + sinh(b*x + a) + I) - (-2*I*a^3*cosh(b*x + a) - 2*I*
a^3*sinh(b*x + a))*log(cosh(b*x + a) + sinh(b*x + a) - I) - ((2*I*b^3*x^3 +
 2*I*a^3)*cosh(b*x + a) + (2*I*b^3*x^3 + 2*I*a^3)*sinh(b*x + a))*log(I*cosh
(b*x + a) + I*sinh(b*x + a) + 1) - ((-2*I*b^3*x^3 - 2*I*a^3)*cosh(b*x + a)
+ (-2*I*b^3*x^3 - 2*I*a^3)*sinh(b*x + a))*log(-I*cosh(b*x + a) - I*sinh(b*x
 + a) + 1) - (-12*I*cosh(b*x + a) - 12*I*sinh(b*x + a))*polylog(4, I*cosh(b
*x + a) + I*sinh(b*x + a)) - (12*I*cosh(b*x + a) + 12*I*sinh(b*x + a))*poly
log(4, -I*cosh(b*x + a) - I*sinh(b*x + a)) - (12*I*b*x*cosh(b*x + a) + 12*I
*b*x*sinh(b*x + a))*polylog(3, I*cosh(b*x + a) + I*sinh(b*x + a)) - (-12*I*
b*x*cosh(b*x + a) - 12*I*b*x*sinh(b*x + a))*polylog(3, -I*cosh(b*x + a) - I
*sinh(b*x + a)) + 6)/(b^4*cosh(b*x + a) + b^4*sinh(b*x + a))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3 \operatorname{sech}(bx+a)\sinh(bx+a)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*sech(b*x+a)*sinh(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate(x^3*sech(b*x + a)*sinh(b*x + a)^2, x)
```

maple [F] time = 0.67, size = 0, normalized size = 0.00

$$\int x^3 \operatorname{sech}(bx+a)\left(\sinh^2(bx+a)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*sech(b*x+a)*sinh(b*x+a)^2,x)
```

```
[Out] int(x^3*sech(b*x+a)*sinh(b*x+a)^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\left(\left(b^3x^3e^{(2a)}-3b^2x^2e^{(2a)}+6bxe^{(2a)}-6e^{(2a)}\right)e^{(bx)}-\left(b^3x^3+3b^2x^2+6bx+6\right)e^{(-bx)}\right)e^{(-a)}}{2b^4}-2\int\frac{x^3e^{(bx+a)}}{e^{(2bx+2a)}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*sech(b*x+a)*sinh(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*((b^3*x^3*e^(2*a) - 3*b^2*x^2*e^(2*a) + 6*b*x*e^(2*a) - 6*e^(2*a))*e^(b
*x) - (b^3*x^3 + 3*b^2*x^2 + 6*b*x + 6)*e^(-b*x))*e^(-a)/b^4 - 2*integrate(
x^3*e^(b*x + a)/(e^(2*b*x + 2*a) + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3 \sinh (a + b x)^2}{\cosh (a + b x)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*sinh(a + b*x)^2)/cosh(a + b*x),x)
```

```
[Out] int((x^3*sinh(a + b*x)^2)/cosh(a + b*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3 \sinh^2 (a + bx) \operatorname{sech} (a + bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*sech(b*x+a)*sinh(b*x+a)**2,x)
```

```
[Out] Integral(x**3*sinh(a + b*x)**2*sech(a + b*x), x)
```

3.357 $\int x^2 \sinh(a+bx)\tanh(a+bx)\,dx$

Optimal. Leaf size=135

$$-\frac{2i\text{Li}_3\left(-ie^{a+bx}\right)}{b^3}+\frac{2i\text{Li}_3\left(ie^{a+bx}\right)}{b^3}+\frac{2\sinh(a+bx)}{b^3}+\frac{2ix\text{Li}_2\left(-ie^{a+bx}\right)}{b^2}-\frac{2ix\text{Li}_2\left(ie^{a+bx}\right)}{b^2}-\frac{2x\cosh(a+bx)}{b^2}-\frac{2x^2\tan^{-1}}{b}$$

```
[Out] -2*x^2*arctan(exp(b*x+a))/b-2*x*cosh(b*x+a)/b^2+2*I*x*polylog(2,-I*exp(b*x+
a))/b^2-2*I*x*polylog(2,I*exp(b*x+a))/b^2-2*I*polylog(3,-I*exp(b*x+a))/b^3+
2*I*polylog(3,I*exp(b*x+a))/b^3+2*sinh(b*x+a)/b^3+x^2*sinh(b*x+a)/b
```

Rubi [A] time = 0.13, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {5449, 3296, 2637, 4180, 2531, 2282, 6589}

$$\frac{2ix\text{PolyLog}\left(2,-ie^{a+bx}\right)}{b^2}-\frac{2ix\text{PolyLog}\left(2,ie^{a+bx}\right)}{b^2}-\frac{2i\text{PolyLog}\left(3,-ie^{a+bx}\right)}{b^3}+\frac{2i\text{PolyLog}\left(3,ie^{a+bx}\right)}{b^3}+\frac{2\sinh(a+bx}{b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Sinh[a + b*x]*Tanh[a + b*x],x]
```

```
[Out] (-2*x^2*ArcTan[E^(a + b*x)])/b - (2*x*Cosh[a + b*x])/b^2 + ((2*I)*x*PolyLog
[2, (-I)*E^(a + b*x)])/b^2 - ((2*I)*x*PolyLog[2, I*E^(a + b*x)])/b^2 - ((2*
I)*PolyLog[3, (-I)*E^(a + b*x)])/b^3 + ((2*I)*PolyLog[3, I*E^(a + b*x)])/b^
3 + (2*Sinh[a + b*x])/b^3 + (x^2*Sinh[a + b*x])/b
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5449

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) + (b_.)*(x_)]^(n_.)*Tanh[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> Int[(c + d*x)^m*Sinh[a + b*x]^n*Tanh[a + b*
x]^(p - 2), x] - Int[(c + d*x)^m*Sinh[a + b*x]^(n - 2)*Tanh[a + b*x]^p, x]
/; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int x^2 \sinh(a+bx)\tanh(a+bx)\,dx = \int x^2\cosh(a+bx)\,dx - \int x^2 \text{sech}(a+bx)\,dx$$
$$= -\frac{2x^2\tan^{-1}\left(e^{a+bx}\right)}{b} + \frac{x^2\sinh(a+bx)}{b} + \frac{(2i)\int x\log\left(1-ie^{a+bx}\right)dx}{b} - \frac{(2i)\int$$
$$= -\frac{2x^2\tan^{-1}\left(e^{a+bx}\right)}{b} - \frac{2x\cosh(a+bx)}{b^2} + \frac{2ix\text{Li}_2\left(-ie^{a+bx}\right)}{b^2} - \frac{2ix\text{Li}_2\left(ie^{a+bx}\right)}{b^2}$$
$$= -\frac{2x^2\tan^{-1}\left(e^{a+bx}\right)}{b} - \frac{2x\cosh(a+bx)}{b^2} + \frac{2ix\text{Li}_2\left(-ie^{a+bx}\right)}{b^2} - \frac{2ix\text{Li}_2\left(ie^{a+bx}\right)}{b^2}$$
$$= -\frac{2x^2\tan^{-1}\left(e^{a+bx}\right)}{b} - \frac{2x\cosh(a+bx)}{b^2} + \frac{2ix\text{Li}_2\left(-ie^{a+bx}\right)}{b^2} - \frac{2ix\text{Li}_2\left(ie^{a+bx}\right)}{b^2}$$

Mathematica [A] time = 1.47, size = 153, normalized size = 1.13

$$-\frac{i\left(b^2x^2\log\left(1-ie^{a+bx}\right) - b^2x^2\log\left(1+ie^{a+bx}\right) + ib^2x^2\sinh(a+bx) - 2bx\text{Li}_2\left(-ie^{a+bx}\right) + 2bx\text{Li}_2\left(ie^{a+bx}\right) + 2\text{Li}_3\left(\right.}{b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Sinh[a + b*x]*Tanh[a + b*x],x]
```

```
[Out] ((-I)*((-2*I)*b*x*Cosh[a + b*x] + b^2*x^2*Log[1 - I*E^(a + b*x)] - b^2*x^2*
Log[1 + I*E^(a + b*x)] - 2*b*x*PolyLog[2, (-I)*E^(a + b*x)] + 2*b*x*PolyLog
[2, I*E^(a + b*x)] + 2*PolyLog[3, (-I)*E^(a + b*x)] - 2*PolyLog[3, I*E^(a +
 b*x)] + (2*I)*Sinh[a + b*x] + I*b^2*x^2*Sinh[a + b*x]))/b^3
```

fricas [C] time = 0.51, size = 477, normalized size = 3.53

$$-\frac{b^2x^2 - \left(b^2x^2 - 2\,bx + 2\right)\cosh\left(bx+a\right)^2 - 2\left(b^2x^2 - 2\,bx + 2\right)\cosh\left(bx+a\right)\sinh\left(bx+a\right) - \left(b^2x^2 - 2\,bx + 2\right)\sin}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sech(b*x+a)*sinh(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/2*(b^2*x^2 - (b^2*x^2 - 2*b*x + 2)*cosh(b*x + a)^2 - 2*(b^2*x^2 - 2*b*x
+ 2)*cosh(b*x + a)*sinh(b*x + a) - (b^2*x^2 - 2*b*x + 2)*sinh(b*x + a)^2 +
2*b*x - (-4*I*b*x*cosh(b*x + a) - 4*I*b*x*sinh(b*x + a))*dilog(I*cosh(b*x +
 a) + I*sinh(b*x + a)) - (4*I*b*x*cosh(b*x + a) + 4*I*b*x*sinh(b*x + a))*di
log(-I*cosh(b*x + a) - I*sinh(b*x + a)) - (-2*I*a^2*cosh(b*x + a) - 2*I*a^2
*sinh(b*x + a))*log(cosh(b*x + a) + sinh(b*x + a) + I) - (2*I*a^2*cosh(b*x
```

```
+ a) + 2*I*a^2*sinh(b*x + a))*log(cosh(b*x + a) + sinh(b*x + a) - I) - ((2*
I*b^2*x^2 - 2*I*a^2)*cosh(b*x + a) + (2*I*b^2*x^2 - 2*I*a^2)*sinh(b*x + a))
*log(I*cosh(b*x + a) + I*sinh(b*x + a) + 1) - ((-2*I*b^2*x^2 + 2*I*a^2)*cos
h(b*x + a) + (-2*I*b^2*x^2 + 2*I*a^2)*sinh(b*x + a))*log(-I*cosh(b*x + a) -
 I*sinh(b*x + a) + 1) - (4*I*cosh(b*x + a) + 4*I*sinh(b*x + a))*polylog(3,
I*cosh(b*x + a) + I*sinh(b*x + a)) - (-4*I*cosh(b*x + a) - 4*I*sinh(b*x + a
))*polylog(3, -I*cosh(b*x + a) - I*sinh(b*x + a)) + 2)/(b^3*cosh(b*x + a) +
 b^3*sinh(b*x + a))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \operatorname{sech}(bx+a) \sinh(bx+a)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sech(b*x+a)*sinh(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate(x^2*sech(b*x + a)*sinh(b*x + a)^2, x)
```

maple [F] time = 0.74, size = 0, normalized size = 0.00

$$\int x^2 \operatorname{sech}(bx+a) \left(\sinh^2(bx+a)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*sech(b*x+a)*sinh(b*x+a)^2,x)
```

```
[Out] int(x^2*sech(b*x+a)*sinh(b*x+a)^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\left(\left(b^2x^2e^{(2a)} - 2bxe^{(2a)} + 2e^{(2a)}\right)e^{(bx)} - \left(b^2x^2 + 2bx + 2\right)e^{(-bx)}\right)e^{(-a)}}{2b^3} - 2\int \frac{x^2e^{(bx+a)}}{e^{(2bx+2a)} + 1} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sech(b*x+a)*sinh(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*((b^2*x^2*e^(2*a) - 2*b*x*e^(2*a) + 2*e^(2*a))*e^(b*x) - (b^2*x^2 + 2*b
*x + 2)*e^(-b*x))*e^(-a)/b^3 - 2*integrate(x^2*e^(b*x + a)/(e^(2*b*x + 2*a)
 + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2 \sinh(a+bx)^2}{\cosh(a+bx)} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*sinh(a + b*x)^2)/cosh(a + b*x),x)
```

```
[Out] int((x^2*sinh(a + b*x)^2)/cosh(a + b*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \sinh^2(a+bx)\operatorname{sech}(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*sech(b*x+a)*sinh(b*x+a)**2,x)
```

```
[Out] Integral(x**2*sinh(a + b*x)**2*sech(a + b*x), x)
```

3.358 $\int x \sinh(a+bx) \tanh(a+bx) \, dx$

Optimal. Leaf size=77

$$\frac{i\text{Li}_2\left(-ie^{a+bx}\right)}{b^2}-\frac{i\text{Li}_2\left(ie^{a+bx}\right)}{b^2}-\frac{\cosh(a+bx)}{b^2}-\frac{2x\tan^{-1}\left(e^{a+bx}\right)}{b}+\frac{x\sinh(a+bx)}{b}$$

```
[Out] -2*x*arctan(exp(b*x+a))/b-cosh(b*x+a)/b^2+I*polylog(2,-I*exp(b*x+a))/b^2-I*
polylog(2,I*exp(b*x+a))/b^2+x*sinh(b*x+a)/b
```

Rubi [A] time = 0.06, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {5449, 3296, 2638, 4180, 2279, 2391}

$$\frac{i\text{PolyLog}\left(2,-ie^{a+bx}\right)}{b^2}-\frac{i\text{PolyLog}\left(2,ie^{a+bx}\right)}{b^2}-\frac{\cosh(a+bx)}{b^2}-\frac{2x\tan^{-1}\left(e^{a+bx}\right)}{b}+\frac{x\sinh(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sinh[a + b*x]*Tanh[a + b*x],x]
```

```
[Out] (-2*x*ArcTan[E^(a + b*x)])/b - Cosh[a + b*x]/b^2 + (I*PolyLog[2, (-I)*E^(a
+ b*x)])/b^2 - (I*PolyLog[2, I*E^(a + b*x)])/b^2 + (x*Sinh[a + b*x])/b
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5449

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) + (b_.)*(x_)]^(n_.)*Tanh[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> Int[(c + d*x)^m*Sinh[a + b*x]^n*Tanh[a + b*
x]^(p - 2), x] - Int[(c + d*x)^m*Sinh[a + b*x]^(n - 2)*Tanh[a + b*x]^p, x]
/; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rubi steps

$$\int x\sinh(a+bx)\tanh(a+bx)\,dx = \int x\cosh(a+bx)\,dx - \int x\text{sech}(a+bx)\,dx$$

$$= -\frac{2x\tan^{-1}\left(e^{a+bx}\right)}{b} + \frac{x\sinh(a+bx)}{b} + \frac{i\int\log\left(1-ie^{a+bx}\right)dx}{b} - \frac{i\int\log\left(1+i\right.}{b}$$

$$= -\frac{2x\tan^{-1}\left(e^{a+bx}\right)}{b} - \frac{\cosh(a+bx)}{b^2} + \frac{x\sinh(a+bx)}{b} + \frac{i\,\text{Subst}\left(\int\frac{\log(1-ix)}{x}dx\right.}{b^2}$$

$$= -\frac{2x\tan^{-1}\left(e^{a+bx}\right)}{b} - \frac{\cosh(a+bx)}{b^2} + \frac{i\text{Li}_2\left(-ie^{a+bx}\right)}{b^2} - \frac{i\text{Li}_2\left(ie^{a+bx}\right)}{b^2} + \frac{x\sinh(}{b}$$

Mathematica [B] time = 0.12, size = 213, normalized size = 2.77

$$-\frac{-i\left(\text{Li}_2\left(-e^{i\left(-ia-ibx+\frac{\pi}{2}\right)}\right)-\text{Li}_2\left(e^{i\left(-ia-ibx+\frac{\pi}{2}\right)}\right)\right)-\left(\left(-ia-ibx+\frac{\pi}{2}\right)\left(\log\left(1-e^{i\left(-ia-ibx+\frac{\pi}{2}\right)}\right)-\log\left(1+e^{i\left(-ia-ibx+\frac{\pi}{2}\right)}\right)\right)\right.}{b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sinh[a + b*x]*Tanh[a + b*x],x]
```

```
[Out] -((-(((-I)*a + Pi/2 - I*b*x)*(Log[1 - E^(I*((-I)*a + Pi/2 - I*b*x))] - Log[
1 + E^(I*((-I)*a + Pi/2 - I*b*x))])) + ((-I)*a + Pi/2)*Log[Tan[((-I)*a + Pi
/2 - I*b*x)/2]] - I*(PolyLog[2, -E^(I*((-I)*a + Pi/2 - I*b*x))] - PolyLog[2
, E^(I*((-I)*a + Pi/2 - I*b*x))]))/b^2) + (Cosh[b*x]*(-Cosh[a] + b*x*Sinh[a
]))/b^2 + ((b*x*Cosh[a] - Sinh[a])*Sinh[b*x])/b^2
```

fricas [B] time = 0.68, size = 322, normalized size = 4.18

$$\frac{(bx-1)\cosh(bx+a)^2 + 2(bx-1)\cosh(bx+a)\sinh(bx+a) + (bx-1)\sinh(bx+a)^2 - bx + (-2i\cosh(bx+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sech(b*x+a)*sinh(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/2*((b*x - 1)*cosh(b*x + a)^2 + 2*(b*x - 1)*cosh(b*x + a)*sinh(b*x + a) +
(b*x - 1)*sinh(b*x + a)^2 - b*x + (-2*I*cosh(b*x + a) - 2*I*sinh(b*x + a))*
dilog(I*cosh(b*x + a) + I*sinh(b*x + a)) + (2*I*cosh(b*x + a) + 2*I*sinh(b*
x + a))*dilog(-I*cosh(b*x + a) - I*sinh(b*x + a)) + (2*I*a*cosh(b*x + a) +
2*I*a*sinh(b*x + a))*log(cosh(b*x + a) + sinh(b*x + a) + I) + (-2*I*a*cosh(
b*x + a) - 2*I*a*sinh(b*x + a))*log(cosh(b*x + a) + sinh(b*x + a) - I) + ((
2*I*b*x + 2*I*a)*cosh(b*x + a) + (2*I*b*x + 2*I*a)*sinh(b*x + a))*log(I*cos
h(b*x + a) + I*sinh(b*x + a) + 1) + ((-2*I*b*x - 2*I*a)*cosh(b*x + a) + (-2
*I*b*x - 2*I*a)*sinh(b*x + a))*log(-I*cosh(b*x + a) - I*sinh(b*x + a) + 1)
- 1)/(b^2*cosh(b*x + a) + b^2*sinh(b*x + a))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\operatorname{sech}(bx+a)\sinh(bx+a)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sech(b*x+a)*sinh(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate(x*sech(b*x + a)*sinh(b*x + a)^2, x)
```

maple [B] time = 0.39, size = 162, normalized size = 2.10

$$\frac{(bx-1)\,\mathrm{e}^{bx+a}}{2b^2} - \frac{(bx+1)\,\mathrm{e}^{-bx-a}}{2b^2} + \frac{i\ln\left(1+i\mathrm{e}^{bx+a}\right)x}{b} + \frac{i\ln\left(1+i\mathrm{e}^{bx+a}\right)a}{b^2} - \frac{i\ln\left(1-i\mathrm{e}^{bx+a}\right)x}{b} - \frac{i\ln\left(1-i\mathrm{e}^{bx+a}\right)a}{b^2} + \frac{i\,\mathrm{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sech(b*x+a)*sinh(b*x+a)^2,x)
```

```
[Out] 1/2*(b*x-1)/b^2*exp(b*x+a)-1/2*(b*x+1)/b^2*exp(-b*x-a)+I/b*ln(1+I*exp(b*x+a
))*x+I/b^2*ln(1+I*exp(b*x+a))*a-I/b*ln(1-I*exp(b*x+a))*x-I/b^2*ln(1-I*exp(b
*x+a))*a+I/b^2*dilog(1+I*exp(b*x+a))-I/b^2*dilog(1-I*exp(b*x+a))+2/b^2*a*ar
ctan(exp(b*x+a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\left(\left(bxe^{(2\,a)} - e^{(2\,a)}\right)e^{(bx)} - (bx+1)e^{(-bx)}\right)e^{(-a)}}{2\,b^2} - 2\int \frac{xe^{(bx+a)}}{e^{(2\,bx+2\,a)}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sech(b*x+a)*sinh(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*((b*x*e^(2*a) - e^(2*a))*e^(b*x) - (b*x + 1)*e^(-b*x))*e^(-a)/b^2 - 2*i
ntegrate(x*e^(b*x + a)/(e^(2*b*x + 2*a) + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x \sinh (a+b x)^2}{\cosh (a+b x)} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*sinh(a + b*x)^2)/cosh(a + b*x),x)
```

```
[Out] int((x*sinh(a + b*x)^2)/cosh(a + b*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x \sinh^2 (a+bx) \operatorname{sech} (a+bx) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sech(b*x+a)*sinh(b*x+a)**2,x)
```

```
[Out] Integral(x*sinh(a + b*x)**2*sech(a + b*x), x)
```

3.359 $\int \sinh(a+bx)\tanh(a+bx)\,dx$

Optimal. Leaf size=23

$$\frac{\sinh(a+bx)}{b}-\frac{\tan^{-1}(\sinh(a+bx))}{b}$$

```
[Out] -arctan(sinh(b*x+a))/b+sinh(b*x+a)/b
```

Rubi [A] time = 0.02, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {2592, 321, 203}

$$\frac{\sinh(a+bx)}{b}-\frac{\tan^{-1}(\sinh(a+bx))}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[a + b*x]*Tanh[a + b*x],x]
```

```
[Out] -(ArcTan[Sinh[a + b*x]]/b) + Sinh[a + b*x]/b
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2592

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, (a*Sin[e + f*x])/ff], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rubi steps

$$\int \sinh(a+bx)\tanh(a+bx)\,dx = \frac{\text{Subst}\left(\int \frac{x^2}{1+x^2}\,dx, x, \sinh(a+bx)\right)}{b}$$
$$= \frac{\sinh(a+bx)}{b} - \frac{\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sinh(a+bx)\right)}{b}$$
$$= -\frac{\tan^{-1}(\sinh(a+bx))}{b} + \frac{\sinh(a+bx)}{b}$$

Mathematica [**A**] time = 0.01, size = 23, normalized size = 1.00

$$\frac{\sinh(a+bx)}{b} - \frac{\tan^{-1}(\sinh(a+bx))}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[a + b*x]*Tanh[a + b*x],x]
```

```
[Out] -(ArcTan[Sinh[a + b*x]]/b) + Sinh[a + b*x]/b
```

fricas [**B**] time = 0.59, size = 86, normalized size = 3.74

$$-\frac{4\left(\cosh\left(bx+a\right)+\sinh\left(bx+a\right)\right)\arctan\left(\cosh\left(bx+a\right)+\sinh\left(bx+a\right)\right)-\cosh\left(bx+a\right)^{2}-2\,\cosh\left(bx+a\right)\sinh}{2\left(b\cosh\left(bx+a\right)+b\sinh\left(bx+a\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)*sinh(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/2*(4*(cosh(b*x + a) + sinh(b*x + a))*arctan(cosh(b*x + a) + sinh(b*x + a
)) - cosh(b*x + a)^2 - 2*cosh(b*x + a)*sinh(b*x + a) - sinh(b*x + a)^2 + 1)
/(b*cosh(b*x + a) + b*sinh(b*x + a))
```

giac [**A**] time = 0.14, size = 32, normalized size = 1.39

$$-\frac{4\,\arctan\left(e^{(bx+a)}\right)-e^{(bx+a)}+e^{(-bx-a)}}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)*sinh(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -1/2*(4*arctan(e^(b*x + a)) - e^(b*x + a) + e^(-b*x - a))/b
```

maple [A] time = 0.08, size = 24, normalized size = 1.04

$$\frac{\sinh(bx+a)}{b}-\frac{2\arctan\left(e^{bx+a}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(b*x+a)*sinh(b*x+a)^2,x)
```

```
[Out] sinh(b*x+a)/b-2*arctan(exp(b*x+a))/b
```

maxima [A] time = 0.45, size = 41, normalized size = 1.78

$$\frac{2\arctan\left(e^{(-bx-a)}\right)}{b}+\frac{e^{(bx+a)}}{2b}-\frac{e^{(-bx-a)}}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)*sinh(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 2*arctan(e^(-b*x - a))/b + 1/2*e^(b*x + a)/b - 1/2*e^(-b*x - a)/b
```

mupad [B] time = 1.43, size = 49, normalized size = 2.13

$$\frac{e^{a+bx}}{2b}-\frac{2\,\mathrm{atan}\left(\frac{e^{bx}\,e^{a}\,\sqrt{b^2}}{b}\right)}{\sqrt{b^2}}-\frac{e^{-a-bx}}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a + b*x)^2/cosh(a + b*x),x)
```

```
[Out] exp(a + b*x)/(2*b) - (2*atan((exp(b*x)*exp(a)*(b^2)^(1/2))/b))/(b^2)^(1/2)
- exp(- a - b*x)/(2*b)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sinh^2(a+bx)\,\mathrm{sech}\,(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)*sinh(b*x+a)**2,x)
```

```
[Out] Integral(sinh(a + b*x)**2*sech(a + b*x), x)
```

3.360 $\int \frac{\sinh(a+bx)\tanh(a+bx)}{x}\,dx$

Optimal. Leaf size=30

$$-\text{Int}\left(\frac{\text{sech}(a+bx)}{x},x\right)+\cosh(a)\text{Chi}(bx)+\sinh(a)\text{Shi}(bx)$$

```
[Out] Chi(b*x)*cosh(a)+Shi(b*x)*sinh(a)-Unintegrable(sech(b*x+a)/x,x)
```

Rubi [A] time = 0.09, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sinh(a+bx)\tanh(a+bx)}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Sinh[a + b*x]*Tanh[a + b*x])/x,x]
```

```
[Out] Cosh[a]*CoshIntegral[b*x] + Sinh[a]*SinhIntegral[b*x] - Defer[Int][Sech[a +
 b*x]/x, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\sinh(a+bx)\tanh(a+bx)}{x}\,dx &= \int \frac{\cosh(a+bx)}{x}\,dx-\int \frac{\text{sech}(a+bx)}{x}\,dx\\ &= \cosh(a)\int \frac{\cosh(bx)}{x}\,dx+\sinh(a)\int \frac{\sinh(bx)}{x}\,dx-\int \frac{\text{sech}(a+bx)}{x}\,dx\\ &= \cosh(a)\text{Chi}(bx)+\sinh(a)\text{Shi}(bx)-\int \frac{\text{sech}(a+bx)}{x}\,dx\end{aligned}$$

Mathematica [A] time = 9.15, size = 0, normalized size = 0.00

$$\int \frac{\sinh(a+bx)\tanh(a+bx)}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Sinh[a + b*x]*Tanh[a + b*x])/x,x]
```

```
[Out] Integrate[(Sinh[a + b*x]*Tanh[a + b*x])/x, x]
```

fricas [A] time = 0.62, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\text{sech}\,(bx+a)\sinh\,(bx+a)^2}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)*sinh(b*x+a)^2/x,x, algorithm="fricas")
[Out] integral(sech(b*x + a)*sinh(b*x + a)^2/x, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}(bx+a)\sinh(bx+a)^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)*sinh(b*x+a)^2/x,x, algorithm="giac")
[Out] integrate(sech(b*x + a)*sinh(b*x + a)^2/x, x)
```

maple [A] time = 0.52, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}(bx+a)\left(\sinh^2(bx+a)\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(b*x+a)*sinh(b*x+a)^2/x,x)
[Out] int(sech(b*x+a)*sinh(b*x+a)^2/x,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}(bx+a)\sinh(bx+a)^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)*sinh(b*x+a)^2/x,x, algorithm="maxima")
[Out] integrate(sech(b*x + a)*sinh(b*x + a)^2/x, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\sinh(a+b\,x)^2}{x\cosh(a+b\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a + b*x)^2/(x*cosh(a + b*x)),x)
```

```
[Out] int(sinh(a + b*x)^2/(x*cosh(a + b*x)), x)
```

sympy [**A**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh^2(a+bx)\,\text{sech}(a+bx)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)*sinh(b*x+a)**2/x,x)
```

```
[Out] Integral(sinh(a + b*x)**2*sech(a + b*x)/x, x)
```

3.361 $\int \frac{\sinh(a+bx)\tanh(a+bx)}{x^2}\,dx$

Optimal. Leaf size=43

$$-\text{Int}\left(\frac{\text{sech}(a+bx)}{x^2},x\right)+b\sinh(a)\text{Chi}(bx)+b\cosh(a)\text{Shi}(bx)-\frac{\cosh(a+bx)}{x}$$

```
[Out] -cosh(b*x+a)/x+b*cosh(a)*Shi(b*x)+b*Chi(b*x)*sinh(a)-Unintegrable(sech(b*x+
a)/x^2,x)
```

Rubi [A] time = 0.11, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sinh(a+bx)\tanh(a+bx)}{x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Sinh[a + b*x]*Tanh[a + b*x])/x^2,x]
```

```
[Out] -(Cosh[a + b*x]/x) + b*CoshIntegral[b*x]*Sinh[a] + b*Cosh[a]*SinhIntegral[b
*x] - Defer[Int][Sech[a + b*x]/x^2, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sinh(a+bx)\tanh(a+bx)}{x^2}\,dx &= \int \frac{\cosh(a+bx)}{x^2}\,dx-\int \frac{\text{sech}(a+bx)}{x^2}\,dx\\
&= -\frac{\cosh(a+bx)}{x}+b\int \frac{\sinh(a+bx)}{x}\,dx-\int \frac{\text{sech}(a+bx)}{x^2}\,dx\\
&= -\frac{\cosh(a+bx)}{x}+(b\cosh(a))\int \frac{\sinh(bx)}{x}\,dx+(b\sinh(a))\int \frac{\cosh(bx)}{x}\,dx-\\
&= -\frac{\cosh(a+bx)}{x}+b\text{Chi}(bx)\sinh(a)+b\cosh(a)\text{Shi}(bx)-\int \frac{\text{sech}(a+bx)}{x^2}\,dx
\end{aligned}$$

Mathematica [A] time = 8.90, size = 0, normalized size = 0.00

$$\int \frac{\sinh(a+bx)\tanh(a+bx)}{x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Sinh[a + b*x]*Tanh[a + b*x])/x^2,x]
```

```
[Out] Integrate[(Sinh[a + b*x]*Tanh[a + b*x])/x^2, x]
```

fricas [A] time = 0.82, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\operatorname{sech}\left(bx+a\right)\sinh\left(bx+a\right)^{2}}{x^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)*sinh(b*x+a)^2/x^2,x, algorithm="fricas")
```

```
[Out] integral(sech(b*x + a)*sinh(b*x + a)^2/x^2, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}\left(bx+a\right)\sinh\left(bx+a\right)^{2}}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)*sinh(b*x+a)^2/x^2,x, algorithm="giac")
```

```
[Out] integrate(sech(b*x + a)*sinh(b*x + a)^2/x^2, x)
```

maple [A] time = 0.52, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}\left(bx+a\right)\left(\sinh^{2}\left(bx+a\right)\right)}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(b*x+a)*sinh(b*x+a)^2/x^2,x)
```

```
[Out] int(sech(b*x+a)*sinh(b*x+a)^2/x^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}\left(bx+a\right)\sinh\left(bx+a\right)^{2}}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)*sinh(b*x+a)^2/x^2,x, algorithm="maxima")
```

```
[Out] integrate(sech(b*x + a)*sinh(b*x + a)^2/x^2, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sinh\left(a+b\,x\right)^{2}}{x^{2}\cosh\left(a+b\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a + b*x)^2/(x^2*cosh(a + b*x)),x)
```

```
[Out] int(sinh(a + b*x)^2/(x^2*cosh(a + b*x)), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh^2{(a+bx)}\operatorname{sech}{(a+bx)}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)*sinh(b*x+a)**2/x**2,x)
```

```
[Out] Integral(sinh(a + b*x)**2*sech(a + b*x)/x**2, x)
```

3.362 $\int x^m \tanh^2(a+bx)\,dx$

Optimal. Leaf size=15

$$\operatorname{Int}\left(x^m \tanh^2(a+bx),x\right)$$

```
[Out] Unintegrable(x^m*tanh(b*x+a)^2,x)
```

Rubi [A] time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int x^m \tanh^2(a+bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x^m*Tanh[a + b*x]^2,x]
[Out] Defer[Int][x^m*Tanh[a + b*x]^2, x]
```

Rubi steps

$$\int x^m \tanh^2(a+bx)\,dx=\int x^m \tanh^2(a+bx)\,dx$$

Mathematica [A] time = 0.62, size = 0, normalized size = 0.00

$$\int x^m \tanh^2(a+bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^m*Tanh[a + b*x]^2,x]
[Out] Integrate[x^m*Tanh[a + b*x]^2, x]
```

fricas [A] time = 0.54, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(x^m \operatorname{sech}(bx+a)^2 \sinh(bx+a)^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*sech(b*x+a)^2*sinh(b*x+a)^2,x, algorithm="fricas")
[Out] integral(x^m*sech(b*x + a)^2*sinh(b*x + a)^2, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int x^m \operatorname{sech}(bx+a)^2 \sinh(bx+a)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*sech(b*x+a)^2*sinh(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate(x^m*sech(b*x + a)^2*sinh(b*x + a)^2, x)
```

maple [A] time = 0.28, size = 0, normalized size = 0.00

$$\int x^m \operatorname{sech}(bx+a)^2 \left(\sinh^2(bx+a)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*sech(b*x+a)^2*sinh(b*x+a)^2,x)
```

```
[Out] int(x^m*sech(b*x+a)^2*sinh(b*x+a)^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\frac{xe^{\left(4\,bx+m\log(x)+4\,a\right)}}{(m+1)e^{(4\,bx+4\,a)}+2\,(m+1)e^{(2\,bx+2\,a)}+m+1}-\int\frac{\left(2\left(2\,bxe^{(4\,a)}+(m+1)e^{(4\,a)}\right)e^{(4\,bx)}+(m+1)e^{(2\,bx+2\,a)}-m-1}{(m+1)e^{(6\,bx+6\,a)}+3\,(m+1)e^{(4\,bx+4\,a)}+3\,(m+1)e^{(2\,bx+2\,a)}+m}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*sech(b*x+a)^2*sinh(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] x*e^(4*b*x + m*log(x) + 4*a)/((m + 1)*e^(4*b*x + 4*a) + 2*(m + 1)*e^(2*b*x
+ 2*a) + m + 1) - integrate((2*(2*b*x*e^(4*a) + (m + 1)*e^(4*a))*e^(4*b*x)
+ (m + 1)*e^(2*b*x + 2*a) - m - 1)*x^m/((m + 1)*e^(6*b*x + 6*a) + 3*(m + 1)
*e^(4*b*x + 4*a) + 3*(m + 1)*e^(2*b*x + 2*a) + m + 1), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.07

$$\int \frac{x^m \sinh(a+b\,x)^2}{\cosh(a+b\,x)^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^m*sinh(a + b*x)^2)/cosh(a + b*x)^2,x)
```

```
[Out] int((x^m*sinh(a + b*x)^2)/cosh(a + b*x)^2, x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*sech(b*x+a)**2*sinh(b*x+a)**2,x)
```

```
[Out] Timed out
```

3.363 $\int x^3 \tanh^2(a+bx)\,dx$

Optimal. Leaf size=89

$$-\frac{3\text{Li}_3\left(-e^{2(a+bx)}\right)}{2b^4}+\frac{3x\text{Li}_2\left(-e^{2(a+bx)}\right)}{b^3}+\frac{3x^2\log\left(e^{2(a+bx)}+1\right)}{b^2}-\frac{x^3\tanh(a+bx)}{b}-\frac{x^3}{b}+\frac{x^4}{4}$$

```
[Out] -x^3/b+1/4*x^4+3*x^2*ln(1+exp(2*b*x+2*a))/b^2+3*x*polylog(2,-exp(2*b*x+2*a)
)/b^3-3/2*polylog(3,-exp(2*b*x+2*a))/b^4-x^3*tanh(b*x+a)/b
```

Rubi [A] time = 0.18, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.583, Rules used = {3720, 3718, 2190, 2531, 2282, 6589, 30}

$$\frac{3x\text{PolyLog}\left(2,-e^{2(a+bx)}\right)}{b^3}-\frac{3\text{PolyLog}\left(3,-e^{2(a+bx)}\right)}{2b^4}+\frac{3x^2\log\left(e^{2(a+bx)}+1\right)}{b^2}-\frac{x^3\tanh(a+bx)}{b}-\frac{x^3}{b}+\frac{x^4}{4}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Tanh[a + b*x]^2,x]
```

```
[Out] -(x^3/b) + x^4/4 + (3*x^2*Log[1 + E^(2*(a + b*x))])/b^2 + (3*x*PolyLog[2, -
E^(2*(a + b*x))])/b^3 - (3*PolyLog[3, -E^(2*(a + b*x))])/(2*b^4) - (x^3*Tan
h[a + b*x])/b
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 3718

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c
 + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /;
 FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 3720

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[(b*(c + d*x)^m*(b*Tan[e + f*x])^(n - 1))/(f*(n - 1)), x] + (-Di
st[(b*d*m)/(f*(n - 1)), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x],
 x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[
{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int x^3 \tanh^2(a+bx)\,dx = -\frac{x^3 \tanh(a+bx)}{b} + \frac{3\int x^2 \tanh(a+bx)\,dx}{b} + \int x^3\,dx$$
$$= -\frac{x^3}{b} + \frac{x^4}{4} - \frac{x^3 \tanh(a+bx)}{b} + \frac{6\int \frac{e^{2(a+bx)}x^2}{1+e^{2(a+bx)}}\,dx}{b}$$
$$= -\frac{x^3}{b} + \frac{x^4}{4} + \frac{3x^2 \log\left(1+e^{2(a+bx)}\right)}{b^2} - \frac{x^3 \tanh(a+bx)}{b} - \frac{6\int x \log\left(1+e^{2(a+bx)}\right)\,dx}{b^2}$$
$$= -\frac{x^3}{b} + \frac{x^4}{4} + \frac{3x^2 \log\left(1+e^{2(a+bx)}\right)}{b^2} + \frac{3x\text{Li}_2\left(-e^{2(a+bx)}\right)}{b^3} - \frac{x^3 \tanh(a+bx)}{b} - \frac{3\int \text{Li}_2\left(-\right.}{b}$$
$$= -\frac{x^3}{b} + \frac{x^4}{4} + \frac{3x^2 \log\left(1+e^{2(a+bx)}\right)}{b^2} + \frac{3x\text{Li}_2\left(-e^{2(a+bx)}\right)}{b^3} - \frac{x^3 \tanh(a+bx)}{b} - \frac{3\,\text{Subst}\left(\int\right.}{}$$
$$= -\frac{x^3}{b} + \frac{x^4}{4} + \frac{3x^2 \log\left(1+e^{2(a+bx)}\right)}{b^2} + \frac{3x\text{Li}_2\left(-e^{2(a+bx)}\right)}{b^3} - \frac{3\text{Li}_3\left(-e^{2(a+bx)}\right)}{2b^4} - \frac{x^3 \tanh(a}{b}$$

Mathematica [A] time = 4.65, size = 104, normalized size = 1.17

$$\frac{2b^2x^2\left(\frac{2bx}{e^{2a}+1} + 3\log\left(e^{-2(a+bx)}+1\right)\right) - 6bx\text{Li}_2\left(-e^{-2(a+bx)}\right) - 3\text{Li}_3\left(-e^{-2(a+bx)}\right)}{2b^4} - \frac{x^3\text{sech}(a)\sinh(bx)\text{sech}(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Tanh[a + b*x]^2,x]
```

```
[Out] x^4/4 + (2*b^2*x^2*((2*b*x)/(1 + E^(2*a)) + 3*Log[1 + E^(-2*(a + b*x))]) -
6*b*x*PolyLog[2, -E^(-2*(a + b*x))] - 3*PolyLog[3, -E^(-2*(a + b*x))])/(2*b
^4) - (x^3*Sech[a]*Sech[a + b*x]*Sinh[b*x])/b
```

fricas [C] time = 0.71, size = 721, normalized size = 8.10

$$\frac{b^4x^4 - 8\,a^3 + \left(b^4x^4 - 8\,b^3x^3 - 8\,a^3\right)\cosh\left(bx+a\right)^2 + 2\left(b^4x^4 - 8\,b^3x^3 - 8\,a^3\right)\cosh\left(bx+a\right)\sinh\left(bx+a\right) + \left(b^4x\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*sech(b*x+a)^2*sinh(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/4*(b^4*x^4 - 8*a^3 + (b^4*x^4 - 8*b^3*x^3 - 8*a^3)*cosh(b*x + a)^2 + 2*(b
^4*x^4 - 8*b^3*x^3 - 8*a^3)*cosh(b*x + a)*sinh(b*x + a) + (b^4*x^4 - 8*b^3*
x^3 - 8*a^3)*sinh(b*x + a)^2 + 24*(b*x*cosh(b*x + a)^2 + 2*b*x*cosh(b*x + a
)*sinh(b*x + a) + b*x*sinh(b*x + a)^2 + b*x)*dilog(I*cosh(b*x + a) + I*sinh
```

```
(b*x + a)) + 24*(b*x*cosh(b*x + a)^2 + 2*b*x*cosh(b*x + a)*sinh(b*x + a) +
b*x*sinh(b*x + a)^2 + b*x)*dilog(-I*cosh(b*x + a) - I*sinh(b*x + a)) + 12*(
a^2*cosh(b*x + a)^2 + 2*a^2*cosh(b*x + a)*sinh(b*x + a) + a^2*sinh(b*x + a)
^2 + a^2)*log(cosh(b*x + a) + sinh(b*x + a) + I) + 12*(a^2*cosh(b*x + a)^2
+ 2*a^2*cosh(b*x + a)*sinh(b*x + a) + a^2*sinh(b*x + a)^2 + a^2)*log(cosh(b
*x + a) + sinh(b*x + a) - I) + 12*(b^2*x^2 + (b^2*x^2 - a^2)*cosh(b*x + a)^
2 + 2*(b^2*x^2 - a^2)*cosh(b*x + a)*sinh(b*x + a) + (b^2*x^2 - a^2)*sinh(b*
x + a)^2 - a^2)*log(I*cosh(b*x + a) + I*sinh(b*x + a) + 1) + 12*(b^2*x^2 +
(b^2*x^2 - a^2)*cosh(b*x + a)^2 + 2*(b^2*x^2 - a^2)*cosh(b*x + a)*sinh(b*x
+ a) + (b^2*x^2 - a^2)*sinh(b*x + a)^2 - a^2)*log(-I*cosh(b*x + a) - I*sinh
(b*x + a) + 1) - 24*(cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh
(b*x + a)^2 + 1)*polylog(3, I*cosh(b*x + a) + I*sinh(b*x + a)) - 24*(cosh(b
*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 + 1)*polylog(3,
 -I*cosh(b*x + a) - I*sinh(b*x + a)))/(b^4*cosh(b*x + a)^2 + 2*b^4*cosh(b*x
 + a)*sinh(b*x + a) + b^4*sinh(b*x + a)^2 + b^4)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3 \operatorname{sech}(bx+a)^2 \sinh(bx+a)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*sech(b*x+a)^2*sinh(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate(x^3*sech(b*x + a)^2*sinh(b*x + a)^2, x)
```

maple [A] time = 0.57, size = 125, normalized size = 1.40

$$\frac{x^4}{4}+\frac{2x^3}{b\left(1+e^{2bx+2a}\right)}-\frac{6a^2\ln\left(e^{bx+a}\right)}{b^4}-\frac{2x^3}{b}+\frac{6a^2x}{b^3}+\frac{4a^3}{b^4}+\frac{3x^2\ln\left(1+e^{2bx+2a}\right)}{b^2}+\frac{3x\,\mathrm{polylog}\left(2,-e^{2bx+2a}\right)}{b^3}-\frac{3\,\mathrm{polylog}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*sech(b*x+a)^2*sinh(b*x+a)^2,x)
```

```
[Out] 1/4*x^4+2*x^3/b/(1+exp(2*b*x+2*a))-6/b^4*a^2*ln(exp(b*x+a))-2*x^3/b+6/b^3*a
^2*x+4/b^4*a^3+3*x^2*ln(1+exp(2*b*x+2*a))/b^2+3*x*polylog(2,-exp(2*b*x+2*a)
)/b^3-3/2*polylog(3,-exp(2*b*x+2*a))/b^4
```

maxima [A] time = 0.42, size = 108, normalized size = 1.21

$$-\frac{2\,x^3}{b}+\frac{bx^4e^{(2\,bx+2\,a)}+bx^4+8\,x^3}{4\left(be^{(2\,bx+2\,a)}+b\right)}+\frac{3\left(2\,b^2x^2\log\left(e^{(2\,bx+2\,a)}+1\right)+2\,bx\mathrm{Li}_2\left(-e^{(2\,bx+2\,a)}\right)-\mathrm{Li}_3(-e^{(2\,bx+2\,a)})\right)}{2\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*sech(b*x+a)^2*sinh(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -2*x^3/b + 1/4*(b*x^4*e^(2*b*x + 2*a) + b*x^4 + 8*x^3)/(b*e^(2*b*x + 2*a) +
 b) + 3/2*(2*b^2*x^2*log(e^(2*b*x + 2*a) + 1) + 2*b*x*dilog(-e^(2*b*x + 2*a
)) - polylog(3, -e^(2*b*x + 2*a)))/b^4
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3 \sinh (a+b\,x)^2}{\cosh (a+b\,x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*sinh(a + b*x)^2)/cosh(a + b*x)^2,x)
```

```
[Out] int((x^3*sinh(a + b*x)^2)/cosh(a + b*x)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3 \sinh^2 (a+bx) \operatorname{sech}^2 (a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*sech(b*x+a)**2*sinh(b*x+a)**2,x)
```

```
[Out] Integral(x**3*sinh(a + b*x)**2*sech(a + b*x)**2, x)
```

3.364 $\int x^2 \tanh^2(a+bx)\,dx$

Optimal. Leaf size=65

$$\frac{\text{Li}_2\left(-e^{2(a+bx)}\right)}{b^3}+\frac{2x\log\left(e^{2(a+bx)}+1\right)}{b^2}-\frac{x^2\tanh(a+bx)}{b}-\frac{x^2}{b}+\frac{x^3}{3}$$

```
[Out] -x^2/b+1/3*x^3+2*x*ln(1+exp(2*b*x+2*a))/b^2+polylog(2,-exp(2*b*x+2*a))/b^3-
x^2*tanh(b*x+a)/b
```

Rubi [A] time = 0.12, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3720, 3718, 2190, 2279, 2391, 30}

$$\frac{\text{PolyLog}\left(2,-e^{2(a+bx)}\right)}{b^3}+\frac{2x\log\left(e^{2(a+bx)}+1\right)}{b^2}-\frac{x^2\tanh(a+bx)}{b}-\frac{x^2}{b}+\frac{x^3}{3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Tanh[a + b*x]^2,x]
```

```
[Out] -(x^2/b) + x^3/3 + (2*x*Log[1 + E^(2*(a + b*x))])/b^2 + PolyLog[2, -E^(2*(a
 + b*x))]/b^3 - (x^2*Tanh[a + b*x])/b
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3718

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c
 + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /;
 FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 3720

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[(b*(c + d*x)^m*(b*Tan[e + f*x])^(n - 1))/(f*(n - 1)), x] + (-Di
st[(b*d*m)/(f*(n - 1)), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x],
 x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[
{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\int x^2 \tanh^2(a+bx)\,dx &= -\frac{x^2\tanh(a+bx)}{b}+\frac{2\int x\tanh(a+bx)\,dx}{b}+\int x^2\,dx\\
&= -\frac{x^2}{b}+\frac{x^3}{3}-\frac{x^2\tanh(a+bx)}{b}+\frac{4\int \frac{e^{2(a+bx)}x}{1+e^{2(a+bx)}}\,dx}{b}\\
&= -\frac{x^2}{b}+\frac{x^3}{3}+\frac{2x\log\left(1+e^{2(a+bx)}\right)}{b^2}-\frac{x^2\tanh(a+bx)}{b}-\frac{2\int \log\left(1+e^{2(a+bx)}\right)\,dx}{b^2}\\
&= -\frac{x^2}{b}+\frac{x^3}{3}+\frac{2x\log\left(1+e^{2(a+bx)}\right)}{b^2}-\frac{x^2\tanh(a+bx)}{b}-\frac{\operatorname{Subst}\left(\int \frac{\log(1+x)}{x}\,dx,x,e^{2(a+bx)}\right)}{b^3}\\
&= -\frac{x^2}{b}+\frac{x^3}{3}+\frac{2x\log\left(1+e^{2(a+bx)}\right)}{b^2}+\frac{\operatorname{Li}_2\left(-e^{2(a+bx)}\right)}{b^3}-\frac{x^2\tanh(a+bx)}{b}
\end{aligned}
$$

Mathematica [C] time = 3.34, size = 168, normalized size = 2.58

$$
\frac{-3b^2x^2\operatorname{sech}(a)\sinh(bx)\operatorname{sech}(a+bx)-3b^2x^2\tanh(a)\sqrt{-\operatorname{csch}^2(a)}\,e^{-\tanh^{-1}(\coth(a))}-3\operatorname{Li}_2\left(e^{-2\left(bx+\tanh^{-1}(\coth(a))\right)}\right)
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2*Tanh[a + b*x]^2,x]
```

```
[Out] ((3*I)*b*Pi*x + b^3*x^3 - (3*I)*Pi*Log[1 + E^(2*b*x)] + 6*b*x*Log[1 - E^(-2
*(b*x + ArcTanh[Coth[a]]))] + (3*I)*Pi*Log[Cosh[b*x]] + 6*ArcTanh[Coth[a]]*
(b*x + Log[1 - E^(-2*(b*x + ArcTanh[Coth[a]]))] - Log[I*Sinh[b*x + ArcTanh[
Coth[a]]]]) - 3*PolyLog[2, E^(-2*(b*x + ArcTanh[Coth[a]]))] - 3*b^2*x^2*Sec
h[a]*Sech[a + b*x]*Sinh[b*x] - (3*b^2*x^2*Sqrt[-Csch[a]^2]*Tanh[a])/E^ArcTa
nh[Coth[a]])/(3*b^3)
```

fricas [C] time = 0.83, size = 515, normalized size = 7.92

$$\frac{b^3x^3 + \left(b^3x^3 - 6\,b^2x^2 + 6\,a^2\right)\cosh\left(bx+a\right)^2 + 2\left(b^3x^3 - 6\,b^2x^2 + 6\,a^2\right)\cosh\left(bx+a\right)\sinh\left(bx+a\right) + \left(b^3x^3 - 6\,b^2x\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sech(b*x+a)^2*sinh(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/3*(b^3*x^3 + (b^3*x^3 - 6*b^2*x^2 + 6*a^2)*cosh(b*x + a)^2 + 2*(b^3*x^3 -
 6*b^2*x^2 + 6*a^2)*cosh(b*x + a)*sinh(b*x + a) + (b^3*x^3 - 6*b^2*x^2 + 6*
a^2)*sinh(b*x + a)^2 + 6*a^2 + 6*(cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*
x + a) + sinh(b*x + a)^2 + 1)*dilog(I*cosh(b*x + a) + I*sinh(b*x + a)) + 6*
(cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 + 1)*dil
og(-I*cosh(b*x + a) - I*sinh(b*x + a)) - 6*(a*cosh(b*x + a)^2 + 2*a*cosh(b*
x + a)*sinh(b*x + a) + a*sinh(b*x + a)^2 + a)*log(cosh(b*x + a) + sinh(b*x
+ a) + I) - 6*(a*cosh(b*x + a)^2 + 2*a*cosh(b*x + a)*sinh(b*x + a) + a*sinh
(b*x + a)^2 + a)*log(cosh(b*x + a) + sinh(b*x + a) - I) + 6*((b*x + a)*cosh
(b*x + a)^2 + 2*(b*x + a)*cosh(b*x + a)*sinh(b*x + a) + (b*x + a)*sinh(b*x
+ a)^2 + b*x + a)*log(I*cosh(b*x + a) + I*sinh(b*x + a) + 1) + 6*((b*x + a)
*cosh(b*x + a)^2 + 2*(b*x + a)*cosh(b*x + a)*sinh(b*x + a) + (b*x + a)*sinh
(b*x + a)^2 + b*x + a)*log(-I*cosh(b*x + a) - I*sinh(b*x + a) + 1))/(b^3*co
sh(b*x + a)^2 + 2*b^3*cosh(b*x + a)*sinh(b*x + a) + b^3*sinh(b*x + a)^2 + b
^3)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2\,\mathrm{sech}\left(bx+a\right)^2\sinh\left(bx+a\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sech(b*x+a)^2*sinh(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate(x^2*sech(b*x + a)^2*sinh(b*x + a)^2, x)
```

maple [A] time = 0.57, size = 99, normalized size = 1.52

$$\frac{x^3}{3}+\frac{2x^2}{b\left(1+\mathrm{e}^{2bx+2a}\right)}-\frac{2x^2}{b}-\frac{4ax}{b^2}-\frac{2a^2}{b^3}+\frac{2x\ln\left(1+\mathrm{e}^{2bx+2a}\right)}{b^2}+\frac{\mathrm{polylog}\left(2,-\mathrm{e}^{2bx+2a}\right)}{b^3}+\frac{4a\ln\left(\mathrm{e}^{bx+a}\right)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*sech(b*x+a)^2*sinh(b*x+a)^2,x)
```

```
[Out] 1/3*x^3+2*x^2/b/(1+exp(2*b*x+2*a))-2*x^2/b-4*a*x/b^2-2/b^3*a^2+2*x*ln(1+exp
(2*b*x+2*a))/b^2+polylog(2,-exp(2*b*x+2*a))/b^3+4/b^3*a*ln(exp(b*x+a))
```

maxima [A] time = 0.42, size = 84, normalized size = 1.29

$$-\frac{2\,x^2}{b}+\frac{bx^3e^{(2\,bx+2\,a)}+bx^3+6\,x^2}{3\left(be^{(2\,bx+2\,a)}+b\right)}+\frac{2\,bx\log\left(e^{(2\,bx+2\,a)}+1\right)+\mathrm{Li}_2\left(-e^{(2\,bx+2\,a)}\right)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sech(b*x+a)^2*sinh(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -2*x^2/b + 1/3*(b*x^3*e^(2*b*x + 2*a) + b*x^3 + 6*x^2)/(b*e^(2*b*x + 2*a) +
 b) + (2*b*x*log(e^(2*b*x + 2*a) + 1) + dilog(-e^(2*b*x + 2*a)))/b^3
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int\frac{x^2\,\sinh\left(a+b\,x\right)^2}{\cosh\left(a+b\,x\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*sinh(a + b*x)^2)/cosh(a + b*x)^2,x)
```

```
[Out] int((x^2*sinh(a + b*x)^2)/cosh(a + b*x)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2\sinh^2\left(a+bx\right)\mathrm{sech}^2\left(a+bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*sech(b*x+a)**2*sinh(b*x+a)**2,x)
```

```
[Out] Integral(x**2*sinh(a + b*x)**2*sech(a + b*x)**2, x)
```

3.365 $\int x \tanh^2(a+bx)\,dx$

Optimal. Leaf size=31

$$\frac{\log(\cosh(a+bx))}{b^2}-\frac{x\tanh(a+bx)}{b}+\frac{x^2}{2}$$

```
[Out] 1/2*x^2+ln(cosh(b*x+a))/b^2-x*tanh(b*x+a)/b
```

Rubi [A] time = 0.03, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {3720, 3475, 30}

$$\frac{\log(\cosh(a+bx))}{b^2}-\frac{x\tanh(a+bx)}{b}+\frac{x^2}{2}$$

Antiderivative was successfully verified.

```
[In] Int[x*Tanh[a + b*x]^2,x]
```

```
[Out] x^2/2 + Log[Cosh[a + b*x]]/b^2 - (x*Tanh[a + b*x])/b
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3720

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[(b*(c + d*x)^m*(b*Tan[e + f*x])^(n - 1))/(f*(n - 1)), x] + (-Di
st[(b*d*m)/(f*(n - 1)), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x],
 x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[
{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rubi steps

$$\int x\tanh^2(a+bx)\,dx = -\frac{x\tanh(a+bx)}{b}+\frac{\int \tanh(a+bx)\,dx}{b}+\int x\,dx$$
$$= \frac{x^2}{2}+\frac{\log(\cosh(a+bx))}{b^2}-\frac{x\tanh(a+bx)}{b}$$

Mathematica [A] time = 0.15, size = 46, normalized size = 1.48

$$\frac{-2bx\tanh(a)+2\log(\cosh(a+bx))-2bx\operatorname{sech}(a)\sinh(bx)\operatorname{sech}(a+bx)+b^2x^2}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Tanh[a + b*x]^2,x]
```

```
[Out] (b^2*x^2 + 2*Log[Cosh[a + b*x]] - 2*b*x*Sech[a]*Sech[a + b*x]*Sinh[b*x] - 2
*b*x*Tanh[a])/(2*b^2)
```

fricas [B] time = 1.51, size = 185, normalized size = 5.97

$$\frac{b^2x^2+\left(b^2x^2-4\,bx\right)\cosh\left(bx+a\right)^2+2\left(b^2x^2-4\,bx\right)\cosh\left(bx+a\right)\sinh\left(bx+a\right)+\left(b^2x^2-4\,bx\right)\sinh\left(bx+a\right)^2}{2\left(b^2\cosh\left(bx+a\right)^2+2\,b^2\cosh\left(bx+a\right)\sinh\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sech(b*x+a)^2*sinh(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(b^2*x^2 + (b^2*x^2 - 4*b*x)*cosh(b*x + a)^2 + 2*(b^2*x^2 - 4*b*x)*cosh
(b*x + a)*sinh(b*x + a) + (b^2*x^2 - 4*b*x)*sinh(b*x + a)^2 + 2*(cosh(b*x +
 a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 + 1)*log(2*cosh(b*x
 + a)/(cosh(b*x + a) - sinh(b*x + a))))/(b^2*cosh(b*x + a)^2 + 2*b^2*cosh(b
*x + a)*sinh(b*x + a) + b^2*sinh(b*x + a)^2 + b^2)
```

giac [B] time = 0.14, size = 95, normalized size = 3.06

$$\frac{b^2x^2e^{(2\,bx+2\,a)}+b^2x^2-4\,bxe^{(2\,bx+2\,a)}+2\,e^{(2\,bx+2\,a)}\log\left(e^{(2\,bx+2\,a)}+1\right)+2\,\log\left(e^{(2\,bx+2\,a)}+1\right)}{2\left(b^2e^{(2\,bx+2\,a)}+b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sech(b*x+a)^2*sinh(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*(b^2*x^2*e^(2*b*x + 2*a) + b^2*x^2 - 4*b*x*e^(2*b*x + 2*a) + 2*e^(2*b*x
 + 2*a)*log(e^(2*b*x + 2*a) + 1) + 2*log(e^(2*b*x + 2*a) + 1))/(b^2*e^(2*b*
x + 2*a) + b^2)
```

maple [A] time = 0.50, size = 54, normalized size = 1.74

$$\frac{x^2}{2} - \frac{2x}{b} - \frac{2a}{b^2} + \frac{2x}{b\left(1 + e^{2bx+2a}\right)} + \frac{\ln\left(1 + e^{2bx+2a}\right)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sech(b*x+a)^2*sinh(b*x+a)^2,x)
```

```
[Out] 1/2*x^2-2*x/b-2*a/b^2+2*x/b/(1+exp(2*b*x+2*a))+1/b^2*ln(1+exp(2*b*x+2*a))
```

maxima [B] time = 0.37, size = 95, normalized size = 3.06

$$-\frac{xe^{(2\,bx+2\,a)}}{be^{(2\,bx+2\,a)} + b} + \frac{bx^2 + \left(bx^2e^{(2\,a)} - 2\,xe^{(2\,a)}\right)e^{(2\,bx)}}{2\left(be^{(2\,bx+2\,a)} + b\right)} + \frac{\log\left(\left(e^{(2\,bx+2\,a)} + 1\right)e^{(-2\,a)}\right)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sech(b*x+a)^2*sinh(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -x*e^(2*b*x + 2*a)/(b*e^(2*b*x + 2*a) + b) + 1/2*(b*x^2 + (b*x^2*e^(2*a) -
2*x*e^(2*a))*e^(2*b*x))/(b*e^(2*b*x + 2*a) + b) + log((e^(2*b*x + 2*a) + 1)
*e^(-2*a))/b^2
```

mupad [B] time = 0.10, size = 45, normalized size = 1.45

$$\frac{\frac{x^2\cosh(a+b\,x)}{2} - \frac{x\sinh(a+b\,x)}{b}}{\cosh\left(a + b\,x\right)} + \frac{\ln\left(\cosh\left(a + b\,x\right)\right)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*sinh(a + b*x)^2)/cosh(a + b*x)^2,x)
```

```
[Out] ((x^2*cosh(a + b*x))/2 - (x*sinh(a + b*x))/b)/cosh(a + b*x) + log(cosh(a +
b*x))/b^2
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\sinh^2\left(a + bx\right)\operatorname{sech}^2\left(a + bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sech(b*x+a)**2*sinh(b*x+a)**2,x)
```

```
[Out] Integral(x*sinh(a + b*x)**2*sech(a + b*x)**2, x)
```

3.366 $\int \tanh^2(a+bx)\,dx$

Optimal. Leaf size=13

$$x-\frac{\tanh(a+bx)}{b}$$

```
[Out] x-tanh(b*x+a)/b
```

Rubi [A] time = 0.01, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3473, 8}

$$x-\frac{\tanh(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[a + b*x]^2,x]
```

```
[Out] x - Tanh[a + b*x]/b
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rubi steps

$$\begin{aligned}\int \tanh^2(a+bx)\,dx &= -\frac{\tanh(a+bx)}{b}+\int 1\,dx\\ &= x-\frac{\tanh(a+bx)}{b}\end{aligned}$$

Mathematica [A] time = 0.01, size = 23, normalized size = 1.77

$$\frac{\tanh^{-1}(\tanh(a+bx))}{b}-\frac{\tanh(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[a + b*x]^2,x]
[Out] ArcTanh[Tanh[a + b*x]]/b - Tanh[a + b*x]/b
```

fricas [B] time = 0.74, size = 33, normalized size = 2.54

$$\frac{(bx+1)\cosh(bx+a)-\sinh(bx+a)}{b\cosh(bx+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)^2*sinh(b*x+a)^2,x, algorithm="fricas")
[Out] ((b*x + 1)*cosh(b*x + a) - sinh(b*x + a))/(b*cosh(b*x + a))
```

giac [A] time = 0.13, size = 24, normalized size = 1.85

$$\frac{bx+a+\frac{2}{e^{(2\,bx+2\,a)}+1}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)^2*sinh(b*x+a)^2,x, algorithm="giac")
[Out] (b*x + a + 2/(e^(2*b*x + 2*a) + 1))/b
```

maple [A] time = 0.11, size = 18, normalized size = 1.38

$$\frac{bx+a-\tanh(bx+a)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(b*x+a)^2*sinh(b*x+a)^2,x)
[Out] 1/b*(b*x+a-tanh(b*x+a))
```

maxima [A] time = 0.32, size = 25, normalized size = 1.92

$$x+\frac{a}{b}-\frac{2}{b\left(e^{(-2\,bx-2\,a)}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)^2*sinh(b*x+a)^2,x, algorithm="maxima")
[Out] x + a/b - 2/(b*(e^(-2*b*x - 2*a) + 1))
```

mupad [B] time = 0.07, size = 20, normalized size = 1.54

$$x+\frac{2}{b\,\left({\mathrm{e}}^{2\,a+2\,b\,x}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a + b*x)^2/cosh(a + b*x)^2,x)
```

```
[Out] x + 2/(b*(exp(2*a + 2*b*x) + 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sinh^{2}{\left(a+bx\right)}\operatorname{sech}^{2}{\left(a+bx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)**2*sinh(b*x+a)**2,x)
```

```
[Out] Integral(sinh(a + b*x)**2*sech(a + b*x)**2, x)
```

3.367 $\int \frac{\tanh^2(a+bx)}{x} \, dx$

Optimal. Leaf size=15

$$\operatorname{Int}\left(\frac{\tanh^2(a+bx)}{x},x\right)$$

```
[Out] Unintegrable(tanh(b*x+a)^2/x,x)
```

Rubi [A] time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\tanh^2(a+bx)}{x} \, dx$$

Verification is Not applicable to the result.

```
[In] Int[Tanh[a + b*x]^2/x,x]
[Out] Defer[Int][Tanh[a + b*x]^2/x, x]
```

Rubi steps

$$\int \frac{\tanh^2(a+bx)}{x} \, dx = \int \frac{\tanh^2(a+bx)}{x} \, dx$$

Mathematica [A] time = 18.88, size = 0, normalized size = 0.00

$$\int \frac{\tanh^2(a+bx)}{x} \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Tanh[a + b*x]^2/x,x]
[Out] Integrate[Tanh[a + b*x]^2/x, x]
```

fricas [A] time = 0.65, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{\operatorname{sech}(bx+a)^2\sinh(bx+a)^2}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)^2*sinh(b*x+a)^2/x,x, algorithm="fricas")
```

```
[Out] integral(sech(b*x + a)^2*sinh(b*x + a)^2/x, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}\left(bx+a\right)^{2}\sinh\left(bx+a\right)^{2}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)^2*sinh(b*x+a)^2/x,x, algorithm="giac")
```

```
[Out] integrate(sech(b*x + a)^2*sinh(b*x + a)^2/x, x)
```

maple [A] time = 0.59, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}\left(bx+a\right)^{2}\left(\sinh^{2}\left(bx+a\right)\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(b*x+a)^2*sinh(b*x+a)^2/x,x)
```

```
[Out] int(sech(b*x+a)^2*sinh(b*x+a)^2/x,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\frac{2}{bxe^{(2\,bx+2\,a)}+bx}+2\int \frac{1}{bx^{2}e^{(2\,bx+2\,a)}+bx^{2}}\,dx+\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)^2*sinh(b*x+a)^2/x,x, algorithm="maxima")
```

```
[Out] 2/(b*x*e^(2*b*x + 2*a) + b*x) + 2*integrate(1/(b*x^2*e^(2*b*x + 2*a) + b*x^
2), x) + log(x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.07

$$\int \frac{\sinh\left(a+b\,x\right)^{2}}{x\cosh\left(a+b\,x\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a + b*x)^2/(x*cosh(a + b*x)^2),x)
```

```
[Out] int(sinh(a + b*x)^2/(x*cosh(a + b*x)^2), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh^2(a+bx)\operatorname{sech}^2(a+bx)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)**2*sinh(b*x+a)**2/x,x)
```

```
[Out] Integral(sinh(a + b*x)**2*sech(a + b*x)**2/x, x)
```

3.368 $\int \frac{\tanh^2(a+bx)}{x^2} dx$

Optimal. Leaf size=15

$$\text{Int}\left(\frac{\tanh^2(a+bx)}{x^2},x\right)$$

```
[Out] Unintegrable(tanh(b*x+a)^2/x^2,x)
```

Rubi [A] time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\tanh^2(a+bx)}{x^2} dx$$

Verification is Not applicable to the result.

```
[In] Int[Tanh[a + b*x]^2/x^2,x]
```

```
[Out] Defer[Int][Tanh[a + b*x]^2/x^2, x]
```

Rubi steps

$$\int \frac{\tanh^2(a+bx)}{x^2} dx = \int \frac{\tanh^2(a+bx)}{x^2} dx$$

Mathematica [A] time = 11.89, size = 0, normalized size = 0.00

$$\int \frac{\tanh^2(a+bx)}{x^2} dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Tanh[a + b*x]^2/x^2,x]
```

```
[Out] Integrate[Tanh[a + b*x]^2/x^2, x]
```

fricas [A] time = 0.68, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\text{sech}\,(bx+a)^2\sinh\,(bx+a)^2}{x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)^2*sinh(b*x+a)^2/x^2,x, algorithm="fricas")
```

```
[Out] integral(sech(b*x + a)^2*sinh(b*x + a)^2/x^2, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}(bx+a)^2 \sinh(bx+a)^2}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)^2*sinh(b*x+a)^2/x^2,x, algorithm="giac")
```

```
[Out] integrate(sech(b*x + a)^2*sinh(b*x + a)^2/x^2, x)
```

maple [A] time = 0.58, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}(bx+a)^2 \left(\sinh^2(bx+a)\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(b*x+a)^2*sinh(b*x+a)^2/x^2,x)
```

```
[Out] int(sech(b*x+a)^2*sinh(b*x+a)^2/x^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{bxe^{(2\,bx+2\,a)} + bx - 2}{bx^2e^{(2\,bx+2\,a)} + bx^2} + 4\int \frac{1}{bx^3e^{(2\,bx+2\,a)} + bx^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)^2*sinh(b*x+a)^2/x^2,x, algorithm="maxima")
```

```
[Out] -(b*x*e^(2*b*x + 2*a) + b*x - 2)/(b*x^2*e^(2*b*x + 2*a) + b*x^2) + 4*integr
ate(1/(b*x^3*e^(2*b*x + 2*a) + b*x^3), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.07

$$\int \frac{\sinh(a+b\,x)^2}{x^2\cosh(a+b\,x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a + b*x)^2/(x^2*cosh(a + b*x)^2),x)
```

```
[Out] int(sinh(a + b*x)^2/(x^2*cosh(a + b*x)^2), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh^2(a+bx)\operatorname{sech}^2(a+bx)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)**2*sinh(b*x+a)**2/x**2,x)
```

```
[Out] Integral(sinh(a + b*x)**2*sech(a + b*x)**2/x**2, x)
```

3.369 $\int x^m \mathbf{sech}(a+bx)\tanh^2(a+bx)\,dx$

Optimal. Leaf size=31

$$\operatorname{Int}\left(x^m\operatorname{sech}(a+bx),x\right)-\operatorname{Int}\left(x^m\operatorname{sech}^3(a+bx),x\right)$$

```
[Out] Unintegrable(x^m*sech(b*x+a),x)-Unintegrable(x^m*sech(b*x+a)^3,x)
```

Rubi [A] time = 0.07, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int x^m\operatorname{sech}(a+bx)\tanh^2(a+bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x^m*Sech[a + b*x]*Tanh[a + b*x]^2,x]
[Out] Defer[Int][x^m*Sech[a + b*x], x] - Defer[Int][x^m*Sech[a + b*x]^3, x]
```

Rubi steps

$$\int x^m\operatorname{sech}(a+bx)\tanh^2(a+bx)\,dx=\int x^m\operatorname{sech}(a+bx)\,dx-\int x^m\operatorname{sech}^3(a+bx)\,dx$$

Mathematica [A] time = 58.51, size = 0, normalized size = 0.00

$$\int x^m\operatorname{sech}(a+bx)\tanh^2(a+bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^m*Sech[a + b*x]*Tanh[a + b*x]^2,x]
[Out] Integrate[x^m*Sech[a + b*x]*Tanh[a + b*x]^2, x]
```

fricas [A] time = 0.72, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(x^m\operatorname{sech}(bx+a)^3\sinh(bx+a)^2,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*sech(b*x+a)^3*sinh(b*x+a)^2,x, algorithm="fricas")
[Out] integral(x^m*sech(b*x + a)^3*sinh(b*x + a)^2, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int x^m \operatorname{sech}(bx+a)^3 \sinh(bx+a)^2\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*sech(b*x+a)^3*sinh(b*x+a)^2,x, algorithm="giac")
[Out] integrate(x^m*sech(b*x + a)^3*sinh(b*x + a)^2, x)
```

maple [A] time = 0.30, size = 0, normalized size = 0.00

$$\int x^m \operatorname{sech}(bx+a)^3 \left(\sinh^2(bx+a)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*sech(b*x+a)^3*sinh(b*x+a)^2,x)
[Out] int(x^m*sech(b*x+a)^3*sinh(b*x+a)^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int x^m \operatorname{sech}(bx+a)^3 \sinh(bx+a)^2\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*sech(b*x+a)^3*sinh(b*x+a)^2,x, algorithm="maxima")
[Out] integrate(x^m*sech(b*x + a)^3*sinh(b*x + a)^2, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{x^m \sinh(a+b\,x)^2}{\cosh(a+b\,x)^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^m*sinh(a + b*x)^2)/cosh(a + b*x)^3,x)
[Out] int((x^m*sinh(a + b*x)^2)/cosh(a + b*x)^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*sech(b*x+a)**3*sinh(b*x+a)**2,x)
[Out] Timed out
```

3.370 $\int x^3 \operatorname{sech}(a+bx)\tanh^2(a+bx)\,dx$

Optimal. Leaf size=240

$$-\frac{3i\text{Li}_2\left(-ie^{a+bx}\right)}{b^4}+\frac{3i\text{Li}_2\left(ie^{a+bx}\right)}{b^4}-\frac{3i\text{Li}_4\left(-ie^{a+bx}\right)}{b^4}+\frac{3i\text{Li}_4\left(ie^{a+bx}\right)}{b^4}+\frac{3ix\text{Li}_3\left(-ie^{a+bx}\right)}{b^3}-\frac{3ix\text{Li}_3\left(ie^{a+bx}\right)}{b^3}+\frac{6x\tan^{-1}\left(e^{a\cdot}\right.}{b^3}$$

```
[Out] 6*x*arctan(exp(b*x+a))/b^3+x^3*arctan(exp(b*x+a))/b-3*I*polylog(2,-I*exp(b*
x+a))/b^4-3/2*I*x^2*polylog(2,-I*exp(b*x+a))/b^2+3*I*polylog(2,I*exp(b*x+a)
)/b^4+3/2*I*x^2*polylog(2,I*exp(b*x+a))/b^2+3*I*x*polylog(3,-I*exp(b*x+a))/
b^3-3*I*x*polylog(3,I*exp(b*x+a))/b^3-3*I*polylog(4,-I*exp(b*x+a))/b^4+3*I*
polylog(4,I*exp(b*x+a))/b^4-3/2*x^2*sech(b*x+a)/b^2-1/2*x^3*sech(b*x+a)*tan
h(b*x+a)/b
```

Rubi [A] time = 0.30, antiderivative size = 240, normalized size of antiderivative = 1.00, number of steps used = 25, number of rules used = 9, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5455, 4180, 2531, 6609, 2282, 6589, 4186, 2279, 2391}

$$-\frac{3ix^2\text{PolyLog}\left(2,-ie^{a+bx}\right)}{2b^2}+\frac{3ix^2\text{PolyLog}\left(2,ie^{a+bx}\right)}{2b^2}+\frac{3ix\text{PolyLog}\left(3,-ie^{a+bx}\right)}{b^3}-\frac{3ix\text{PolyLog}\left(3,ie^{a+bx}\right)}{b^3}-\frac{3i\text{PolyI}}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Sech[a + b*x]*Tanh[a + b*x]^2,x]
```

```
[Out] (6*x*ArcTan[E^(a + b*x)])/b^3 + (x^3*ArcTan[E^(a + b*x)])/b - ((3*I)*PolyLo
g[2, (-I)*E^(a + b*x)])/b^4 - (((3*I)/2)*x^2*PolyLog[2, (-I)*E^(a + b*x)])/
b^2 + ((3*I)*PolyLog[2, I*E^(a + b*x)])/b^4 + (((3*I)/2)*x^2*PolyLog[2, I*E
^(a + b*x)])/b^2 + ((3*I)*x*PolyLog[3, (-I)*E^(a + b*x)])/b^3 - ((3*I)*x*Po
lyLog[3, I*E^(a + b*x)])/b^3 - ((3*I)*PolyLog[4, (-I)*E^(a + b*x)])/b^4 + (
(3*I)*PolyLog[4, I*E^(a + b*x)])/b^4 - (3*x^2*Sech[a + b*x])/(2*b^2) - (x^3
*Sech[a + b*x]*Tanh[a + b*x])/(2*b)
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
```

```
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4186

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> -Simp[(b^2*(c + d*x)^m*Cot[e + f*x]*(b*Csc[e + f*x])^(n - 2))/(f*(n -
 1)), x] + (Dist[(b^2*d^2*m*(m - 1))/(f^2*(n - 1)*(n - 2)), Int[(c + d*x)^(
m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Dist[(b^2*(n - 2))/(n - 1), Int[(
c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x] - Simp[(b^2*d*m*(c + d*x)^(m -
1)*(b*Csc[e + f*x])^(n - 2))/(f^2*(n - 1)*(n - 2)), x]) /; FreeQ[{b, c, d,
e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

Rule 5455

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) + (b_.)*(x_)]*Tanh[(a_.) + (b_.)*
(x_)]^(p_), x_Symbol] :> Int[(c + d*x)^m*Sech[a + b*x]*Tanh[a + b*x]^(p - 2
), x] - Int[(c + d*x)^m*Sech[a + b*x]^3*Tanh[a + b*x]^(p - 2), x] /; FreeQ[
{a, b, c, d, m}, x] && IGtQ[p/2, 0]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
```

```
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int x^3\operatorname{sech}(a+bx)\tanh^2(a+bx)\,dx = \int x^3\operatorname{sech}(a+bx)\,dx - \int x^3\operatorname{sech}^3(a+bx)\,dx$$

$$= \frac{2x^3\tan^{-1}\left(e^{a+bx}\right)}{b} - \frac{3x^2\operatorname{sech}(a+bx)}{2b^2} - \frac{x^3\operatorname{sech}(a+bx)\tanh(a+bx)}{2b} - \frac{1}{2}\int$$

$$= \frac{6x\tan^{-1}\left(e^{a+bx}\right)}{b^3} + \frac{x^3\tan^{-1}\left(e^{a+bx}\right)}{b} - \frac{3ix^2\operatorname{Li}_2\left(-ie^{a+bx}\right)}{b^2} + \frac{3ix^2\operatorname{Li}_2\left(ie^{a+bx}\right)}{b^2}$$

$$= \frac{6x\tan^{-1}\left(e^{a+bx}\right)}{b^3} + \frac{x^3\tan^{-1}\left(e^{a+bx}\right)}{b} - \frac{3ix^2\operatorname{Li}_2\left(-ie^{a+bx}\right)}{2b^2} + \frac{3ix^2\operatorname{Li}_2\left(ie^{a+bx}\right)}{2b^2}$$

$$= \frac{6x\tan^{-1}\left(e^{a+bx}\right)}{b^3} + \frac{x^3\tan^{-1}\left(e^{a+bx}\right)}{b} - \frac{3i\operatorname{Li}_2\left(-ie^{a+bx}\right)}{b^4} - \frac{3ix^2\operatorname{Li}_2\left(-ie^{a+bx}\right)}{2b^2}$$

$$= \frac{6x\tan^{-1}\left(e^{a+bx}\right)}{b^3} + \frac{x^3\tan^{-1}\left(e^{a+bx}\right)}{b} - \frac{3i\operatorname{Li}_2\left(-ie^{a+bx}\right)}{b^4} - \frac{3ix^2\operatorname{Li}_2\left(-ie^{a+bx}\right)}{2b^2}$$

$$= \frac{6x\tan^{-1}\left(e^{a+bx}\right)}{b^3} + \frac{x^3\tan^{-1}\left(e^{a+bx}\right)}{b} - \frac{3i\operatorname{Li}_2\left(-ie^{a+bx}\right)}{b^4} - \frac{3ix^2\operatorname{Li}_2\left(-ie^{a+bx}\right)}{2b^2}$$

Mathematica [A] time = 3.42, size = 245, normalized size = 1.02

$$-\frac{b^3x^3\operatorname{sech}(a)\sinh(bx)\operatorname{sech}^2(a+bx) + b^2x^2(bx\tanh(a)+3)\operatorname{sech}(a+bx) - i\left(b^3x^3\log\left(1-ie^{a+bx}\right) - b^3x^3\log\left(1+\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Sech[a + b*x]*Tanh[a + b*x]^2,x]
```

```
[Out] -1/2*((-I)*(6*b*x*Log[1 - I*E^(a + b*x)] + b^3*x^3*Log[1 - I*E^(a + b*x)] -
 6*b*x*Log[1 + I*E^(a + b*x)] - b^3*x^3*Log[1 + I*E^(a + b*x)] - 3*(2 + b^2
```

```
*x^2)*PolyLog[2, (-I)*E^(a + b*x)] + 3*(2 + b^2*x^2)*PolyLog[2, I*E^(a + b*
x)] + 6*b*x*PolyLog[3, (-I)*E^(a + b*x)] - 6*b*x*PolyLog[3, I*E^(a + b*x)]
- 6*PolyLog[4, (-I)*E^(a + b*x)] + 6*PolyLog[4, I*E^(a + b*x)]) + b^3*x^3*S
ech[a]*Sech[a + b*x]^2*Sinh[b*x] + b^2*x^2*Sech[a + b*x]*(3 + b*x*Tanh[a]))
/b^4
```

fricas [C] time = 0.90, size = 2125, normalized size = 8.85

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*sech(b*x+a)^3*sinh(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/2*(2*(b^3*x^3 + 3*b^2*x^2)*cosh(b*x + a)^3 + 6*(b^3*x^3 + 3*b^2*x^2)*cos
h(b*x + a)*sinh(b*x + a)^2 + 2*(b^3*x^3 + 3*b^2*x^2)*sinh(b*x + a)^3 - 2*(b
^3*x^3 - 3*b^2*x^2)*cosh(b*x + a) - ((3*I*b^2*x^2 + 6*I)*cosh(b*x + a)^4 +
(12*I*b^2*x^2 + 24*I)*cosh(b*x + a)*sinh(b*x + a)^3 + (3*I*b^2*x^2 + 6*I)*s
inh(b*x + a)^4 + 3*I*b^2*x^2 + (6*I*b^2*x^2 + 12*I)*cosh(b*x + a)^2 + (6*I*
b^2*x^2 + (18*I*b^2*x^2 + 36*I)*cosh(b*x + a)^2 + 12*I)*sinh(b*x + a)^2 + (
(12*I*b^2*x^2 + 24*I)*cosh(b*x + a)^3 + (12*I*b^2*x^2 + 24*I)*cosh(b*x + a)
)*sinh(b*x + a) + 6*I)*dilog(I*cosh(b*x + a) + I*sinh(b*x + a)) - ((-3*I*b^
2*x^2 - 6*I)*cosh(b*x + a)^4 + (-12*I*b^2*x^2 - 24*I)*cosh(b*x + a)*sinh(b*
x + a)^3 + (-3*I*b^2*x^2 - 6*I)*sinh(b*x + a)^4 - 3*I*b^2*x^2 + (-6*I*b^2*x
^2 - 12*I)*cosh(b*x + a)^2 + (-6*I*b^2*x^2 + (-18*I*b^2*x^2 - 36*I)*cosh(b*
x + a)^2 - 12*I)*sinh(b*x + a)^2 + ((-12*I*b^2*x^2 - 24*I)*cosh(b*x + a)^3
+ (-12*I*b^2*x^2 - 24*I)*cosh(b*x + a))*sinh(b*x + a) - 6*I)*dilog(-I*cosh(
b*x + a) - I*sinh(b*x + a)) - ((-I*a^3 - 6*I*a)*cosh(b*x + a)^4 + (-4*I*a^3
 - 24*I*a)*cosh(b*x + a)*sinh(b*x + a)^3 + (-I*a^3 - 6*I*a)*sinh(b*x + a)^4
 - I*a^3 + (-2*I*a^3 - 12*I*a)*cosh(b*x + a)^2 + (-2*I*a^3 + (-6*I*a^3 - 36
*I*a)*cosh(b*x + a)^2 - 12*I*a)*sinh(b*x + a)^2 + ((-4*I*a^3 - 24*I*a)*cosh
(b*x + a)^3 + (-4*I*a^3 - 24*I*a)*cosh(b*x + a))*sinh(b*x + a) - 6*I*a)*log
(cosh(b*x + a) + sinh(b*x + a) + I) - ((I*a^3 + 6*I*a)*cosh(b*x + a)^4 + (4
*I*a^3 + 24*I*a)*cosh(b*x + a)*sinh(b*x + a)^3 + (I*a^3 + 6*I*a)*sinh(b*x +
 a)^4 + I*a^3 + (2*I*a^3 + 12*I*a)*cosh(b*x + a)^2 + (2*I*a^3 + (6*I*a^3 +
36*I*a)*cosh(b*x + a)^2 + 12*I*a)*sinh(b*x + a)^2 + ((4*I*a^3 + 24*I*a)*cos
h(b*x + a)^3 + (4*I*a^3 + 24*I*a)*cosh(b*x + a))*sinh(b*x + a) + 6*I*a)*log
(cosh(b*x + a) + sinh(b*x + a) - I) - (-I*b^3*x^3 + (-I*b^3*x^3 - I*a^3 - 6
*I*b*x - 6*I*a)*cosh(b*x + a)^4 + (-4*I*b^3*x^3 - 4*I*a^3 - 24*I*b*x - 24*I
*a)*cosh(b*x + a)*sinh(b*x + a)^3 + (-I*b^3*x^3 - I*a^3 - 6*I*b*x - 6*I*a)*
sinh(b*x + a)^4 - I*a^3 + (-2*I*b^3*x^3 - 2*I*a^3 - 12*I*b*x - 12*I*a)*cosh
(b*x + a)^2 + (-2*I*b^3*x^3 - 2*I*a^3 + (-6*I*b^3*x^3 - 6*I*a^3 - 36*I*b*x
- 36*I*a)*cosh(b*x + a)^2 - 12*I*b*x - 12*I*a)*sinh(b*x + a)^2 - 6*I*b*x +
((-4*I*b^3*x^3 - 4*I*a^3 - 24*I*b*x - 24*I*a)*cosh(b*x + a)^3 + (-4*I*b^3*x
^3 - 4*I*a^3 - 24*I*b*x - 24*I*a)*cosh(b*x + a))*sinh(b*x + a) - 6*I*a)*log
(I*cosh(b*x + a) + I*sinh(b*x + a) + 1) - (I*b^3*x^3 + (I*b^3*x^3 + I*a^3 +
```

```
6*I*b*x + 6*I*a)*cosh(b*x + a)^4 + (4*I*b^3*x^3 + 4*I*a^3 + 24*I*b*x + 24*
I*a)*cosh(b*x + a)*sinh(b*x + a)^3 + (I*b^3*x^3 + I*a^3 + 6*I*b*x + 6*I*a)*
sinh(b*x + a)^4 + I*a^3 + (2*I*b^3*x^3 + 2*I*a^3 + 12*I*b*x + 12*I*a)*cosh(
b*x + a)^2 + (2*I*b^3*x^3 + 2*I*a^3 + (6*I*b^3*x^3 + 6*I*a^3 + 36*I*b*x + 3
6*I*a)*cosh(b*x + a)^2 + 12*I*b*x + 12*I*a)*sinh(b*x + a)^2 + 6*I*b*x + ((4
*I*b^3*x^3 + 4*I*a^3 + 24*I*b*x + 24*I*a)*cosh(b*x + a)^3 + (4*I*b^3*x^3 +
4*I*a^3 + 24*I*b*x + 24*I*a)*cosh(b*x + a))*sinh(b*x + a) + 6*I*a)*log(-I*c
osh(b*x + a) - I*sinh(b*x + a) + 1) - (6*I*cosh(b*x + a)^4 + 24*I*cosh(b*x
+ a)*sinh(b*x + a)^3 + 6*I*sinh(b*x + a)^4 + (36*I*cosh(b*x + a)^2 + 12*I)*
sinh(b*x + a)^2 + 12*I*cosh(b*x + a)^2 + (24*I*cosh(b*x + a)^3 + 24*I*cosh(
b*x + a))*sinh(b*x + a) + 6*I)*polylog(4, I*cosh(b*x + a) + I*sinh(b*x + a)
) - (-6*I*cosh(b*x + a)^4 - 24*I*cosh(b*x + a)*sinh(b*x + a)^3 - 6*I*sinh(b
*x + a)^4 + (-36*I*cosh(b*x + a)^2 - 12*I)*sinh(b*x + a)^2 - 12*I*cosh(b*x
+ a)^2 + (-24*I*cosh(b*x + a)^3 - 24*I*cosh(b*x + a))*sinh(b*x + a) - 6*I)*
polylog(4, -I*cosh(b*x + a) - I*sinh(b*x + a)) - (-6*I*b*x*cosh(b*x + a)^4
- 24*I*b*x*cosh(b*x + a)*sinh(b*x + a)^3 - 6*I*b*x*sinh(b*x + a)^4 - 12*I*b
*x*cosh(b*x + a)^2 + (-36*I*b*x*cosh(b*x + a)^2 - 12*I*b*x)*sinh(b*x + a)^2
 - 6*I*b*x + (-24*I*b*x*cosh(b*x + a)^3 - 24*I*b*x*cosh(b*x + a))*sinh(b*x
+ a))*polylog(3, I*cosh(b*x + a) + I*sinh(b*x + a)) - (6*I*b*x*cosh(b*x + a
)^4 + 24*I*b*x*cosh(b*x + a)*sinh(b*x + a)^3 + 6*I*b*x*sinh(b*x + a)^4 + 12
*I*b*x*cosh(b*x + a)^2 + (36*I*b*x*cosh(b*x + a)^2 + 12*I*b*x)*sinh(b*x + a
)^2 + 6*I*b*x + (24*I*b*x*cosh(b*x + a)^3 + 24*I*b*x*cosh(b*x + a))*sinh(b*
x + a))*polylog(3, -I*cosh(b*x + a) - I*sinh(b*x + a)) - 2*(b^3*x^3 - 3*b^2
*x^2 - 3*(b^3*x^3 + 3*b^2*x^2)*cosh(b*x + a)^2)*sinh(b*x + a))/(b^4*cosh(b*
x + a)^4 + 4*b^4*cosh(b*x + a)*sinh(b*x + a)^3 + b^4*sinh(b*x + a)^4 + 2*b^
4*cosh(b*x + a)^2 + b^4 + 2*(3*b^4*cosh(b*x + a)^2 + b^4)*sinh(b*x + a)^2 +
 4*(b^4*cosh(b*x + a)^3 + b^4*cosh(b*x + a))*sinh(b*x + a))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3 \operatorname{sech}(bx+a)^3 \sinh(bx+a)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*sech(b*x+a)^3*sinh(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate(x^3*sech(b*x + a)^3*sinh(b*x + a)^2, x)
```

maple [F] time = 1.37, size = 0, normalized size = 0.00

$$\int x^3 \operatorname{sech}(bx+a)^3 \left(\sinh^2(bx+a)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*sech(b*x+a)^3*sinh(b*x+a)^2,x)
```

[Out] int(x^3*sech(b*x+a)^3*sinh(b*x+a)^2,x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\left(bx^3e^{(3\,a)} + 3\,x^2e^{(3\,a)}\right)e^{(3\,bx)} - \left(bx^3e^a - 3\,x^2e^a\right)e^{(bx)}}{b^2e^{(4\,bx+4\,a)} + 2\,b^2e^{(2\,bx+2\,a)} + b^2} + 2\int \frac{\left(b^2x^3e^a + 6\,xe^a\right)e^{(bx)}}{2\left(b^2e^{(2\,bx+2\,a)} + b^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*sech(b*x+a)^3*sinh(b*x+a)^2,x, algorithm="maxima")

[Out] -((b*x^3*e^(3*a) + 3*x^2*e^(3*a))*e^(3*b*x) - (b*x^3*e^a - 3*x^2*e^a)*e^(b*x))/(b^2*e^(4*b*x + 4*a) + 2*b^2*e^(2*b*x + 2*a) + b^2) + 2*integrate(1/2*(b^2*x^3*e^a + 6*x*e^a)*e^(b*x)/(b^2*e^(2*b*x + 2*a) + b^2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3\,\sinh\left(a+b\,x\right)^2}{\cosh\left(a+b\,x\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^3*sinh(a + b*x)^2)/cosh(a + b*x)^3,x)

[Out] int((x^3*sinh(a + b*x)^2)/cosh(a + b*x)^3, x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**3*sech(b*x+a)**3*sinh(b*x+a)**2,x)

[Out] Timed out

3.371 $\int x^2\text{sech}(a+bx)\tanh^2(a+bx)\,dx$

Optimal. Leaf size=143

$$\frac{i\text{Li}_3\left(-ie^{a+bx}\right)}{b^3}-\frac{i\text{Li}_3\left(ie^{a+bx}\right)}{b^3}+\frac{\tan^{-1}(\sinh(a+bx))}{b^3}-\frac{ix\text{Li}_2\left(-ie^{a+bx}\right)}{b^2}+\frac{ix\text{Li}_2\left(ie^{a+bx}\right)}{b^2}-\frac{x\text{sech}(a+bx)}{b^2}+\frac{x^2\tan^{-1}\left(e^{a+\cdots}\right)}{b}$$

```
[Out] x^2*arctan(exp(b*x+a))/b+arctan(sinh(b*x+a))/b^3-I*x*polylog(2,-I*exp(b*x+a
))/b^2+I*x*polylog(2,I*exp(b*x+a))/b^2+I*polylog(3,-I*exp(b*x+a))/b^3-I*pol
ylog(3,I*exp(b*x+a))/b^3-x*sech(b*x+a)/b^2-1/2*x^2*sech(b*x+a)*tanh(b*x+a)/
b
```

Rubi [A] time = 0.20, antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {5455, 4180, 2531, 2282, 6589, 4186, 3770}

$$-\frac{ix\text{PolyLog}\left(2,-ie^{a+bx}\right)}{b^2}+\frac{ix\text{PolyLog}\left(2,ie^{a+bx}\right)}{b^2}+\frac{i\text{PolyLog}\left(3,-ie^{a+bx}\right)}{b^3}-\frac{i\text{PolyLog}\left(3,ie^{a+bx}\right)}{b^3}-\frac{x\text{sech}(a+bx)}{b^2}+\cdots$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Sech[a + b*x]*Tanh[a + b*x]^2,x]
```

```
[Out] (x^2*ArcTan[E^(a + b*x)])/b + ArcTan[Sinh[a + b*x]]/b^3 - (I*x*PolyLog[2, (
-I)*E^(a + b*x)])/b^2 + (I*x*PolyLog[2, I*E^(a + b*x)])/b^2 + (I*PolyLog[3,
 (-I)*E^(a + b*x)])/b^3 - (I*PolyLog[3, I*E^(a + b*x)])/b^3 - (x*Sech[a + b
*x])/b^2 - (x^2*Sech[a + b*x]*Tanh[a + b*x])/(2*b)
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4186

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> -Simp[(b^2*(c + d*x)^m*Cot[e + f*x]*(b*Csc[e + f*x])^(n - 2))/(f*(n -
 1)), x] + (Dist[(b^2*d^2*m*(m - 1))/(f^2*(n - 1)*(n - 2)), Int[(c + d*x)^(
m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Dist[(b^2*(n - 2))/(n - 1), Int[(
c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x] - Simp[(b^2*d*m*(c + d*x)^(m -
1)*(b*Csc[e + f*x])^(n - 2))/(f^2*(n - 1)*(n - 2)), x]) /; FreeQ[{b, c, d,
e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

Rule 5455

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) + (b_.)*(x_)]*Tanh[(a_.) + (b_.)*
(x_)]^(p_), x_Symbol] :> Int[(c + d*x)^m*Sech[a + b*x]*Tanh[a + b*x]^(p - 2
), x] - Int[(c + d*x)^m*Sech[a + b*x]^3*Tanh[a + b*x]^(p - 2), x] /; FreeQ[
{a, b, c, d, m}, x] && IGtQ[p/2, 0]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int x^2\mathrm{sech}(a+bx)\tanh^2(a+bx)\,dx = \int x^2\mathrm{sech}(a+bx)\,dx - \int x^2\mathrm{sech}^3(a+bx)\,dx$$
$$= \frac{2x^2\tan^{-1}\left(e^{a+bx}\right)}{b} - \frac{x\mathrm{sech}(a+bx)}{b^2} - \frac{x^2\mathrm{sech}(a+bx)\tanh(a+bx)}{2b} - \frac{1}{2}\int x$$
$$= \frac{x^2\tan^{-1}\left(e^{a+bx}\right)}{b} + \frac{\tan^{-1}(\sinh(a+bx))}{b^3} - \frac{2ix\mathrm{Li}_2\left(-ie^{a+bx}\right)}{b^2} + \frac{2ix\mathrm{Li}_2\left(ie^{a+bx}\right)}{b^2}$$
$$= \frac{x^2\tan^{-1}\left(e^{a+bx}\right)}{b} + \frac{\tan^{-1}(\sinh(a+bx))}{b^3} - \frac{ix\mathrm{Li}_2\left(-ie^{a+bx}\right)}{b^2} + \frac{ix\mathrm{Li}_2\left(ie^{a+bx}\right)}{b^2}$$
$$= \frac{x^2\tan^{-1}\left(e^{a+bx}\right)}{b} + \frac{\tan^{-1}(\sinh(a+bx))}{b^3} - \frac{ix\mathrm{Li}_2\left(-ie^{a+bx}\right)}{b^2} + \frac{ix\mathrm{Li}_2\left(ie^{a+bx}\right)}{b^2}$$
$$= \frac{x^2\tan^{-1}\left(e^{a+bx}\right)}{b} + \frac{\tan^{-1}(\sinh(a+bx))}{b^3} - \frac{ix\mathrm{Li}_2\left(-ie^{a+bx}\right)}{b^2} + \frac{ix\mathrm{Li}_2\left(ie^{a+bx}\right)}{b^2}$$

Mathematica [A] time = 1.65, size = 180, normalized size = 1.26

$$-\frac{x\mathrm{sech}(a)\mathrm{sech}(a+bx)(bx\sinh(a)+2\cosh(a))}{2b^2}+\frac{i\left(b^2x^2\log\left(1-ie^{a+bx}\right)-b^2x^2\log\left(1+ie^{a+bx}\right)-2bx\mathrm{Li}_2\left(-ie^{a+bx}\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Sech[a + b*x]*Tanh[a + b*x]^2,x]
```

```
[Out] ((I/2)*((-4*I)*ArcTan[E^(a + b*x)] + b^2*x^2*Log[1 - I*E^(a + b*x)] - b^2*x
^2*Log[1 + I*E^(a + b*x)] - 2*b*x*PolyLog[2, (-I)*E^(a + b*x)] + 2*b*x*Poly
Log[2, I*E^(a + b*x)] + 2*PolyLog[3, (-I)*E^(a + b*x)] - 2*PolyLog[3, I*E^(
a + b*x)]))/b^3 - (x*Sech[a]*Sech[a + b*x]*(2*Cosh[a] + b*x*Sinh[a]))/(2*b^
2) - (x^2*Sech[a]*Sech[a + b*x]^2*Sinh[b*x])/(2*b)
```

fricas [C] time = 0.73, size = 1549, normalized size = 10.83

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sech(b*x+a)^3*sinh(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/2*(2*(b^2*x^2 + 2*b*x)*cosh(b*x + a)^3 + 6*(b^2*x^2 + 2*b*x)*cosh(b*x +
a)*sinh(b*x + a)^2 + 2*(b^2*x^2 + 2*b*x)*sinh(b*x + a)^3 - 2*(b^2*x^2 - 2*b
*x)*cosh(b*x + a) - (2*I*b*x*cosh(b*x + a)^4 + 8*I*b*x*cosh(b*x + a)*sinh(b
*x + a)^3 + 2*I*b*x*sinh(b*x + a)^4 + 4*I*b*x*cosh(b*x + a)^2 + (12*I*b*x*c
```

```
osh(b*x + a)^2 + 4*I*b*x)*sinh(b*x + a)^2 + 2*I*b*x + (8*I*b*x*cosh(b*x + a
)^3 + 8*I*b*x*cosh(b*x + a))*sinh(b*x + a))*dilog(I*cosh(b*x + a) + I*sinh(
b*x + a)) - (-2*I*b*x*cosh(b*x + a)^4 - 8*I*b*x*cosh(b*x + a)*sinh(b*x + a)
^3 - 2*I*b*x*sinh(b*x + a)^4 - 4*I*b*x*cosh(b*x + a)^2 + (-12*I*b*x*cosh(b*
x + a)^2 - 4*I*b*x)*sinh(b*x + a)^2 - 2*I*b*x + (-8*I*b*x*cosh(b*x + a)^3 -
 8*I*b*x*cosh(b*x + a))*sinh(b*x + a))*dilog(-I*cosh(b*x + a) - I*sinh(b*x
+ a)) - ((I*a^2 + 2*I)*cosh(b*x + a)^4 + (4*I*a^2 + 8*I)*cosh(b*x + a)*sinh
(b*x + a)^3 + (I*a^2 + 2*I)*sinh(b*x + a)^4 + (2*I*a^2 + 4*I)*cosh(b*x + a)
^2 + ((6*I*a^2 + 12*I)*cosh(b*x + a)^2 + 2*I*a^2 + 4*I)*sinh(b*x + a)^2 + I
*a^2 + ((4*I*a^2 + 8*I)*cosh(b*x + a)^3 + (4*I*a^2 + 8*I)*cosh(b*x + a))*si
nh(b*x + a) + 2*I)*log(cosh(b*x + a) + sinh(b*x + a) + I) - ((-I*a^2 - 2*I)
*cosh(b*x + a)^4 + (-4*I*a^2 - 8*I)*cosh(b*x + a)*sinh(b*x + a)^3 + (-I*a^2
 - 2*I)*sinh(b*x + a)^4 + (-2*I*a^2 - 4*I)*cosh(b*x + a)^2 + ((-6*I*a^2 - 1
2*I)*cosh(b*x + a)^2 - 2*I*a^2 - 4*I)*sinh(b*x + a)^2 - I*a^2 + ((-4*I*a^2
- 8*I)*cosh(b*x + a)^3 + (-4*I*a^2 - 8*I)*cosh(b*x + a))*sinh(b*x + a) - 2*
I)*log(cosh(b*x + a) + sinh(b*x + a) - I) - ((-I*b^2*x^2 + I*a^2)*cosh(b*x
+ a)^4 + (-4*I*b^2*x^2 + 4*I*a^2)*cosh(b*x + a)*sinh(b*x + a)^3 + (-I*b^2*x
^2 + I*a^2)*sinh(b*x + a)^4 - I*b^2*x^2 + (-2*I*b^2*x^2 + 2*I*a^2)*cosh(b*x
 + a)^2 + (-2*I*b^2*x^2 + (-6*I*b^2*x^2 + 6*I*a^2)*cosh(b*x + a)^2 + 2*I*a^
2)*sinh(b*x + a)^2 + I*a^2 + ((-4*I*b^2*x^2 + 4*I*a^2)*cosh(b*x + a)^3 + (-
4*I*b^2*x^2 + 4*I*a^2)*cosh(b*x + a))*sinh(b*x + a))*log(I*cosh(b*x + a) +
I*sinh(b*x + a) + 1) - ((I*b^2*x^2 - I*a^2)*cosh(b*x + a)^4 + (4*I*b^2*x^2
- 4*I*a^2)*cosh(b*x + a)*sinh(b*x + a)^3 + (I*b^2*x^2 - I*a^2)*sinh(b*x + a
)^4 + I*b^2*x^2 + (2*I*b^2*x^2 - 2*I*a^2)*cosh(b*x + a)^2 + (2*I*b^2*x^2 +
(6*I*b^2*x^2 - 6*I*a^2)*cosh(b*x + a)^2 - 2*I*a^2)*sinh(b*x + a)^2 - I*a^2
+ ((4*I*b^2*x^2 - 4*I*a^2)*cosh(b*x + a)^3 + (4*I*b^2*x^2 - 4*I*a^2)*cosh(b
*x + a))*sinh(b*x + a))*log(-I*cosh(b*x + a) - I*sinh(b*x + a) + 1) - (-2*I
*cosh(b*x + a)^4 - 8*I*cosh(b*x + a)*sinh(b*x + a)^3 - 2*I*sinh(b*x + a)^4
+ (-12*I*cosh(b*x + a)^2 - 4*I)*sinh(b*x + a)^2 - 4*I*cosh(b*x + a)^2 + (-8
*I*cosh(b*x + a)^3 - 8*I*cosh(b*x + a))*sinh(b*x + a) - 2*I)*polylog(3, I*c
osh(b*x + a) + I*sinh(b*x + a)) - (2*I*cosh(b*x + a)^4 + 8*I*cosh(b*x + a)*
sinh(b*x + a)^3 + 2*I*sinh(b*x + a)^4 + (12*I*cosh(b*x + a)^2 + 4*I)*sinh(b
*x + a)^2 + 4*I*cosh(b*x + a)^2 + (8*I*cosh(b*x + a)^3 + 8*I*cosh(b*x + a))
*sinh(b*x + a) + 2*I)*polylog(3, -I*cosh(b*x + a) - I*sinh(b*x + a)) - 2*(b
^2*x^2 - 3*(b^2*x^2 + 2*b*x)*cosh(b*x + a)^2 - 2*b*x)*sinh(b*x + a))/(b^3*c
osh(b*x + a)^4 + 4*b^3*cosh(b*x + a)*sinh(b*x + a)^3 + b^3*sinh(b*x + a)^4
+ 2*b^3*cosh(b*x + a)^2 + b^3 + 2*(3*b^3*cosh(b*x + a)^2 + b^3)*sinh(b*x +
a)^2 + 4*(b^3*cosh(b*x + a)^3 + b^3*cosh(b*x + a))*sinh(b*x + a))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \operatorname{sech}(bx+a)^3 \sinh(bx+a)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sech(b*x+a)^3*sinh(b*x+a)^2,x, algorithm="giac")
[Out] integrate(x^2*sech(b*x + a)^3*sinh(b*x + a)^2, x)
```

maple [F] time = 1.20, size = 0, normalized size = 0.00

$$\int x^2\text{sech}\left(bx+a\right)^3\left(\sinh^2\left(bx+a\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*sech(b*x+a)^3*sinh(b*x+a)^2,x)
[Out] int(x^2*sech(b*x+a)^3*sinh(b*x+a)^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$2\,b^2\int\frac{x^2e^{(bx+a)}}{2\left(b^2e^{(2\,bx+2\,a)}+b^2\right)}\,dx-\frac{\left(bx^2e^{(3\,a)}+2\,xe^{(3\,a)}\right)e^{(3\,bx)}-\left(bx^2e^a-2\,xe^a\right)e^{(bx)}}{b^2e^{(4\,bx+4\,a)}+2\,b^2e^{(2\,bx+2\,a)}+b^2}+\frac{2\,\arctan\left(e^{(bx+a)}\right)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sech(b*x+a)^3*sinh(b*x+a)^2,x, algorithm="maxima")
[Out] 2*b^2*integrate(1/2*x^2*e^(b*x + a)/(b^2*e^(2*b*x + 2*a) + b^2), x) - ((b*x
^2*e^(3*a) + 2*x*e^(3*a))*e^(3*b*x) - (b*x^2*e^a - 2*x*e^a)*e^(b*x))/(b^2*e
^(4*b*x + 4*a) + 2*b^2*e^(2*b*x + 2*a) + b^2) + 2*arctan(e^(b*x + a))/b^3
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{x^2\,{\sinh\left(a+b\,x\right)}^2}{{\cosh\left(a+b\,x\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*sinh(a + b*x)^2)/cosh(a + b*x)^3,x)
[Out] int((x^2*sinh(a + b*x)^2)/cosh(a + b*x)^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^{2}\sinh^{2}{\left(a+bx\right)}\operatorname{sech}^{3}{\left(a+bx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*sech(b*x+a)**3*sinh(b*x+a)**2,x)
[Out] Integral(x**2*sinh(a + b*x)**2*sech(a + b*x)**3, x)
```

3.372 $\int x\text{sech}(a+bx)\tanh^2(a+bx)\,dx$

Optimal. Leaf size=91

$$-\frac{i\text{Li}_2\left(-ie^{a+bx}\right)}{2b^2}+\frac{i\text{Li}_2\left(ie^{a+bx}\right)}{2b^2}-\frac{\text{sech}(a+bx)}{2b^2}+\frac{x\tan^{-1}\left(e^{a+bx}\right)}{b}-\frac{x\tanh(a+bx)\text{sech}(a+bx)}{2b}$$

```
[Out] x*arctan(exp(b*x+a))/b-1/2*I*polylog(2,-I*exp(b*x+a))/b^2+1/2*I*polylog(2,I
*exp(b*x+a))/b^2-1/2*sech(b*x+a)/b^2-1/2*x*sech(b*x+a)*tanh(b*x+a)/b
```

Rubi [A] time = 0.10, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {5455, 4180, 2279, 2391, 4185}

$$-\frac{i\text{PolyLog}\left(2,-ie^{a+bx}\right)}{2b^2}+\frac{i\text{PolyLog}\left(2,ie^{a+bx}\right)}{2b^2}-\frac{\text{sech}(a+bx)}{2b^2}+\frac{x\tan^{-1}\left(e^{a+bx}\right)}{b}-\frac{x\tanh(a+bx)\text{sech}(a+bx)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sech[a + b*x]*Tanh[a + b*x]^2,x]
```

```
[Out] (x*ArcTan[E^(a + b*x)])/b - ((I/2)*PolyLog[2, (-I)*E^(a + b*x)])/b^2 + ((I/
2)*PolyLog[2, I*E^(a + b*x)])/b^2 - Sech[a + b*x]/(2*b^2) - (x*Sech[a + b*x
]*Tanh[a + b*x])/(2*b)
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4185

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_)), x_Symbol] :>
 -Simp[(b^2*(c + d*x)*Cot[e + f*x]*(b*Csc[e + f*x])^(n - 2))/(f*(n - 1)), x
] + (Dist[(b^2*(n - 2))/(n - 1), Int[(c + d*x)*(b*Csc[e + f*x])^(n - 2), x]
, x] - Simp[(b^2*d*(b*Csc[e + f*x])^(n - 2))/(f^2*(n - 1)*(n - 2)), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2]
```

Rule 5455

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) + (b_.)*(x_)]*Tanh[(a_.) + (b_.)*
(x_)]^(p_), x_Symbol] :> Int[(c + d*x)^m*Sech[a + b*x]*Tanh[a + b*x]^(p - 2
), x] - Int[(c + d*x)^m*Sech[a + b*x]^3*Tanh[a + b*x]^(p - 2), x] /; FreeQ[
{a, b, c, d, m}, x] && IGtQ[p/2, 0]
```

Rubi steps

$$\int x\operatorname{sech}(a+bx)\tanh^2(a+bx)\,dx = \int x\operatorname{sech}(a+bx)\,dx - \int x\operatorname{sech}^3(a+bx)\,dx$$

$$= \frac{2x\tan^{-1}\left(e^{a+bx}\right)}{b} - \frac{\operatorname{sech}(a+bx)}{2b^2} - \frac{x\operatorname{sech}(a+bx)\tanh(a+bx)}{2b} - \frac{1}{2}\int x\operatorname{sec}$$

$$= \frac{x\tan^{-1}\left(e^{a+bx}\right)}{b} - \frac{\operatorname{sech}(a+bx)}{2b^2} - \frac{x\operatorname{sech}(a+bx)\tanh(a+bx)}{2b} - \frac{i\,\mathrm{Subst}\left(\int\right.}{}$$

$$= \frac{x\tan^{-1}\left(e^{a+bx}\right)}{b} - \frac{i\mathrm{Li}_2\left(-ie^{a+bx}\right)}{b^2} + \frac{i\mathrm{Li}_2\left(ie^{a+bx}\right)}{b^2} - \frac{\operatorname{sech}(a+bx)}{2b^2} - \frac{x\operatorname{sech}(a+}{}$$

$$= \frac{x\tan^{-1}\left(e^{a+bx}\right)}{b} - \frac{i\mathrm{Li}_2\left(-ie^{a+bx}\right)}{2b^2} + \frac{i\mathrm{Li}_2\left(ie^{a+bx}\right)}{2b^2} - \frac{\operatorname{sech}(a+bx)}{2b^2} - \frac{x\operatorname{sech}(a+}{}$$

Mathematica [A] time = 0.78, size = 93, normalized size = 1.02

$$-\frac{i\mathrm{Li}_2(-i(\cosh(a+bx)+\sinh(a+bx))) - i\mathrm{Li}_2(i(\cosh(a+bx)+\sinh(a+bx))) + \operatorname{sech}(a+bx) + bx\tanh(a+bx)}{2b^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x*Sech[a + b*x]*Tanh[a + b*x]^2,x]
```

```
[Out] -1/2*(-2*b*x*ArcTan[Cosh[a + b*x] + Sinh[a + b*x]] + I*PolyLog[2, (-I)*(Cos
h[a + b*x] + Sinh[a + b*x])] - I*PolyLog[2, I*(Cosh[a + b*x] + Sinh[a + b*x
])] + Sech[a + b*x] + b*x*Sech[a + b*x]*Tanh[a + b*x])/b^2
```

fricas [B] time = 0.99, size = 1046, normalized size = 11.49

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sech(b*x+a)^3*sinh(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/2*(2*(b*x + 1)*cosh(b*x + a)^3 + 6*(b*x + 1)*cosh(b*x + a)*sinh(b*x + a)
^2 + 2*(b*x + 1)*sinh(b*x + a)^3 - 2*(b*x - 1)*cosh(b*x + a) - (I*cosh(b*x
+ a)^4 + 4*I*cosh(b*x + a)*sinh(b*x + a)^3 + I*sinh(b*x + a)^4 + (6*I*cosh(
b*x + a)^2 + 2*I)*sinh(b*x + a)^2 + 2*I*cosh(b*x + a)^2 + (4*I*cosh(b*x + a
)^3 + 4*I*cosh(b*x + a))*sinh(b*x + a) + I)*dilog(I*cosh(b*x + a) + I*sinh(
b*x + a)) - (-I*cosh(b*x + a)^4 - 4*I*cosh(b*x + a)*sinh(b*x + a)^3 - I*sin
h(b*x + a)^4 + (-6*I*cosh(b*x + a)^2 - 2*I)*sinh(b*x + a)^2 - 2*I*cosh(b*x
+ a)^2 + (-4*I*cosh(b*x + a)^3 - 4*I*cosh(b*x + a))*sinh(b*x + a) - I)*dilo
g(-I*cosh(b*x + a) - I*sinh(b*x + a)) - (-I*a*cosh(b*x + a)^4 - 4*I*a*cosh(
b*x + a)*sinh(b*x + a)^3 - I*a*sinh(b*x + a)^4 - 2*I*a*cosh(b*x + a)^2 + (-
6*I*a*cosh(b*x + a)^2 - 2*I*a)*sinh(b*x + a)^2 + (-4*I*a*cosh(b*x + a)^3 -
4*I*a*cosh(b*x + a))*sinh(b*x + a) - I*a)*log(cosh(b*x + a) + sinh(b*x + a)
 + I) - (I*a*cosh(b*x + a)^4 + 4*I*a*cosh(b*x + a)*sinh(b*x + a)^3 + I*a*si
nh(b*x + a)^4 + 2*I*a*cosh(b*x + a)^2 + (6*I*a*cosh(b*x + a)^2 + 2*I*a)*sin
h(b*x + a)^2 + (4*I*a*cosh(b*x + a)^3 + 4*I*a*cosh(b*x + a))*sinh(b*x + a)
+ I*a)*log(cosh(b*x + a) + sinh(b*x + a) - I) - ((-I*b*x - I*a)*cosh(b*x +
a)^4 + (-4*I*b*x - 4*I*a)*cosh(b*x + a)*sinh(b*x + a)^3 + (-I*b*x - I*a)*si
nh(b*x + a)^4 + (-2*I*b*x - 2*I*a)*cosh(b*x + a)^2 + ((-6*I*b*x - 6*I*a)*co
sh(b*x + a)^2 - 2*I*b*x - 2*I*a)*sinh(b*x + a)^2 - I*b*x + ((-4*I*b*x - 4*I
*a)*cosh(b*x + a)^3 + (-4*I*b*x - 4*I*a)*cosh(b*x + a))*sinh(b*x + a) - I*a
)*log(I*cosh(b*x + a) + I*sinh(b*x + a) + 1) - ((I*b*x + I*a)*cosh(b*x + a)
^4 + (4*I*b*x + 4*I*a)*cosh(b*x + a)*sinh(b*x + a)^3 + (I*b*x + I*a)*sinh(b
*x + a)^4 + (2*I*b*x + 2*I*a)*cosh(b*x + a)^2 + ((6*I*b*x + 6*I*a)*cosh(b*x
 + a)^2 + 2*I*b*x + 2*I*a)*sinh(b*x + a)^2 + I*b*x + ((4*I*b*x + 4*I*a)*cos
h(b*x + a)^3 + (4*I*b*x + 4*I*a)*cosh(b*x + a))*sinh(b*x + a) + I*a)*log(-I
*cosh(b*x + a) - I*sinh(b*x + a) + 1) + 2*(3*(b*x + 1)*cosh(b*x + a)^2 - b*
x + 1)*sinh(b*x + a))/(b^2*cosh(b*x + a)^4 + 4*b^2*cosh(b*x + a)*sinh(b*x +
 a)^3 + b^2*sinh(b*x + a)^4 + 2*b^2*cosh(b*x + a)^2 + 2*(3*b^2*cosh(b*x + a
)^2 + b^2)*sinh(b*x + a)^2 + b^2 + 4*(b^2*cosh(b*x + a)^3 + b^2*cosh(b*x +
a))*sinh(b*x + a))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x \operatorname{sech}(bx+a)^3 \sinh(bx+a)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sech(b*x+a)^3*sinh(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate(x*sech(b*x + a)^3*sinh(b*x + a)^2, x)
```

maple [B] time = 0.68, size = 178, normalized size = 1.96

$$-\frac{e^{bx+a}\left(bx\,e^{2bx+2a}-bx+e^{2bx+2a}+1\right)}{b^{2}\left(1+e^{2bx+2a}\right)^{2}}-\frac{i\ln\left(1+ie^{bx+a}\right)x}{2b}-\frac{i\ln\left(1+ie^{bx+a}\right)a}{2b^{2}}+\frac{i\ln\left(1-ie^{bx+a}\right)x}{2b}+\frac{i\ln\left(1-ie^{bx+a}\right)}{2b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sech(b*x+a)^3*sinh(b*x+a)^2,x)
```

```
[Out] -exp(b*x+a)*(b*x*exp(2*b*x+2*a)-b*x+exp(2*b*x+2*a)+1)/b^2/(1+exp(2*b*x+2*a)
)^2-1/2*I/b*ln(1+I*exp(b*x+a))*x-1/2*I/b^2*ln(1+I*exp(b*x+a))*a+1/2*I/b*ln(
1-I*exp(b*x+a))*x+1/2*I/b^2*ln(1-I*exp(b*x+a))*a-1/2*I/b^2*dilog(1+I*exp(b*
x+a))+1/2*I/b^2*dilog(1-I*exp(b*x+a))-1/b^2*a*arctan(exp(b*x+a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\left(bxe^{(3\,a)}+e^{(3\,a)}\right)e^{(3\,bx)}-\left(bxe^{a}-e^{a}\right)e^{(bx)}}{b^{2}e^{(4\,bx+4\,a)}+2\,b^{2}e^{(2\,bx+2\,a)}+b^{2}}+2\int\frac{xe^{(bx+a)}}{2\left(e^{(2\,bx+2\,a)}+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sech(b*x+a)^3*sinh(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -((b*x*e^(3*a) + e^(3*a))*e^(3*b*x) - (b*x*e^a - e^a)*e^(b*x))/(b^2*e^(4*b*
x + 4*a) + 2*b^2*e^(2*b*x + 2*a) + b^2) + 2*integrate(1/2*x*e^(b*x + a)/(e^
(2*b*x + 2*a) + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{x\,\sinh\left(a+b\,x\right)^{2}}{\cosh\left(a+b\,x\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*sinh(a + b*x)^2)/cosh(a + b*x)^3,x)
```

```
[Out] int((x*sinh(a + b*x)^2)/cosh(a + b*x)^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\sinh^{2}\left(a+bx\right)\operatorname{sech}^{3}\left(a+bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sech(b*x+a)**3*sinh(b*x+a)**2,x)
```

```
[Out] Integral(x*sinh(a + b*x)**2*sech(a + b*x)**3, x)
```

3.373 $\int \operatorname{sech}(a+bx)\tanh^2(a+bx)\,dx$

Optimal. Leaf size=34

$$\frac{\tan^{-1}(\sinh(a+bx))}{2b}-\frac{\tanh(a+bx)\operatorname{sech}(a+bx)}{2b}$$

```
[Out] 1/2*arctan(sinh(b*x+a))/b-1/2*sech(b*x+a)*tanh(b*x+a)/b
```

Rubi [A] time = 0.02, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2611, 3770}

$$\frac{\tan^{-1}(\sinh(a+bx))}{2b}-\frac{\tanh(a+bx)\operatorname{sech}(a+bx)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[a + b*x]*Tanh[a + b*x]^2,x]
```

```
[Out] ArcTan[Sinh[a + b*x]]/(2*b) - (Sech[a + b*x]*Tanh[a + b*x])/(2*b)
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \operatorname{sech}(a+bx)\tanh^2(a+bx)\,dx &= -\frac{\operatorname{sech}(a+bx)\tanh(a+bx)}{2b}+\frac{1}{2}\int \operatorname{sech}(a+bx)\,dx\\ &= \frac{\tan^{-1}(\sinh(a+bx))}{2b}-\frac{\operatorname{sech}(a+bx)\tanh(a+bx)}{2b}\end{aligned}$$

Mathematica [A] time = 0.02, size = 34, normalized size = 1.00

$$\frac{\tan^{-1}(\sinh(a+bx))}{2b}-\frac{\tanh(a+bx)\text{sech}(a+bx)}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[a + b*x]*Tanh[a + b*x]^2,x]
```

```
[Out] ArcTan[Sinh[a + b*x]]/(2*b) - (Sech[a + b*x]*Tanh[a + b*x])/(2*b)
```

fricas [B] time = 0.93, size = 269, normalized size = 7.91

$$-\frac{\cosh\left(bx+a\right)^{3}+3\,\cosh\left(bx+a\right)\sinh\left(bx+a\right)^{2}+\sinh\left(bx+a\right)^{3}-\left(\cosh\left(bx+a\right)^{4}+4\,\cosh\left(bx+a\right)\sinh\left(bx+\right.\right.}{b\cosh\left(bx+a\right)^{4}+4\,b\cosh\left(bx+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)^3*sinh(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -(cosh(b*x + a)^3 + 3*cosh(b*x + a)*sinh(b*x + a)^2 + sinh(b*x + a)^3 - (co
sh(b*x + a)^4 + 4*cosh(b*x + a)*sinh(b*x + a)^3 + sinh(b*x + a)^4 + 2*(3*co
sh(b*x + a)^2 + 1)*sinh(b*x + a)^2 + 2*cosh(b*x + a)^2 + 4*(cosh(b*x + a)^3
 + cosh(b*x + a))*sinh(b*x + a) + 1)*arctan(cosh(b*x + a) + sinh(b*x + a))
+ (3*cosh(b*x + a)^2 - 1)*sinh(b*x + a) - cosh(b*x + a))/(b*cosh(b*x + a)^4
 + 4*b*cosh(b*x + a)*sinh(b*x + a)^3 + b*sinh(b*x + a)^4 + 2*b*cosh(b*x + a
)^2 + 2*(3*b*cosh(b*x + a)^2 + b)*sinh(b*x + a)^2 + 4*(b*cosh(b*x + a)^3 +
b*cosh(b*x + a))*sinh(b*x + a) + b)
```

giac [B] time = 0.15, size = 76, normalized size = 2.24

$$\frac{\pi-\frac{4\left(e^{(bx+a)}-e^{(-bx-a)}\right)}{\left(e^{(bx+a)}-e^{(-bx-a)}\right)^{2}+4}+2\,\arctan\left(\frac{1}{2}\left(e^{(2\,bx+2\,a)}-1\right)e^{(-bx-a)}\right)}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)^3*sinh(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/4*(pi - 4*(e^(b*x + a) - e^(-b*x - a))/((e^(b*x + a) - e^(-b*x - a))^2 +
4) + 2*arctan(1/2*(e^(2*b*x + 2*a) - 1)*e^(-b*x - a)))/b
```

maple [A] time = 0.34, size = 49, normalized size = 1.44

$$-\frac{\sinh\left(bx+a\right)}{b\cosh\left(bx+a\right)^{2}}+\frac{\text{sech}\left(bx+a\right)\tanh\left(bx+a\right)}{2b}+\frac{\arctan\left(e^{bx+a}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(b*x+a)^3*sinh(b*x+a)^2,x)
```

```
[Out] -1/b*sinh(b*x+a)/cosh(b*x+a)^2+1/2*sech(b*x+a)*tanh(b*x+a)/b+arctan(exp(b*x
+a))/b
```

maxima [B] time = 0.42, size = 66, normalized size = 1.94

$$-\frac{\arctan\left(e^{(-bx-a)}\right)}{b} - \frac{e^{(-bx-a)} - e^{(-3\,bx-3\,a)}}{b\left(2\,e^{(-2\,bx-2\,a)} + e^{(-4\,bx-4\,a)} + 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)^3*sinh(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -arctan(e^(-b*x - a))/b - (e^(-b*x - a) - e^(-3*b*x - 3*a))/(b*(2*e^(-2*b*x
 - 2*a) + e^(-4*b*x - 4*a) + 1))
```

mupad [B] time = 1.47, size = 82, normalized size = 2.41

$$\frac{\mathrm{atan}\left(\frac{\mathrm{e}^{b\,x}\,\mathrm{e}^{a}\,\sqrt{b^2}}{b}\right)}{\sqrt{b^2}} + \frac{2\,\mathrm{e}^{a+b\,x}}{b\,\left(2\,\mathrm{e}^{2\,a+2\,b\,x} + \mathrm{e}^{4\,a+4\,b\,x} + 1\right)} - \frac{\mathrm{e}^{a+b\,x}}{b\,\left(\mathrm{e}^{2\,a+2\,b\,x} + 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a + b*x)^2/cosh(a + b*x)^3,x)
```

```
[Out] atan((exp(b*x)*exp(a)*(b^2)^(1/2))/b)/(b^2)^(1/2) + (2*exp(a + b*x))/(b*(2*
exp(2*a + 2*b*x) + exp(4*a + 4*b*x) + 1)) - exp(a + b*x)/(b*(exp(2*a + 2*b*
x) + 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sinh^{2}{\left(a + bx \right)} \operatorname{sech}^{3}{\left(a + bx \right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)**3*sinh(b*x+a)**2,x)
```

```
[Out] Integral(sinh(a + b*x)**2*sech(a + b*x)**3, x)
```

3.374 $\int \frac{\text{sech}(a+bx)\tanh^2(a+bx)}{x}\,dx$

Optimal. Leaf size=31

$$\text{Int}\left(\frac{\text{sech}(a+bx)}{x},x\right)-\text{Int}\left(\frac{\text{sech}^3(a+bx)}{x},x\right)$$

```
[Out] Unintegrable(sech(b*x+a)/x,x)-Unintegrable(sech(b*x+a)^3/x,x)
```

Rubi [A] time = 0.06, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\text{sech}(a+bx)\tanh^2(a+bx)}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Sech[a + b*x]*Tanh[a + b*x]^2)/x,x]
```

```
[Out] Defer[Int][Sech[a + b*x]/x, x] - Defer[Int][Sech[a + b*x]^3/x, x]
```

Rubi steps

$$\int \frac{\text{sech}(a+bx)\tanh^2(a+bx)}{x}\,dx=\int \frac{\text{sech}(a+bx)}{x}\,dx-\int \frac{\text{sech}^3(a+bx)}{x}\,dx$$

Mathematica [A] time = 16.71, size = 0, normalized size = 0.00

$$\int \frac{\text{sech}(a+bx)\tanh^2(a+bx)}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Sech[a + b*x]*Tanh[a + b*x]^2)/x,x]
```

```
[Out] Integrate[(Sech[a + b*x]*Tanh[a + b*x]^2)/x, x]
```

fricas [A] time = 0.65, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\text{sech}\left(bx+a\right)^3\sinh\left(bx+a\right)^2}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)^3*sinh(b*x+a)^2/x,x, algorithm="fricas")
```

```
[Out] integral(sech(b*x + a)^3*sinh(b*x + a)^2/x, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}(bx+a)^3 \sinh(bx+a)^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)^3*sinh(b*x+a)^2/x,x, algorithm="giac")
```

```
[Out] integrate(sech(b*x + a)^3*sinh(b*x + a)^2/x, x)
```

maple [A] time = 0.88, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}(bx+a)^3 \left(\sinh^2(bx+a)\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(b*x+a)^3*sinh(b*x+a)^2/x,x)
```

```
[Out] int(sech(b*x+a)^3*sinh(b*x+a)^2/x,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\left(bxe^{(3a)} - e^{(3a)}\right)e^{(3bx)} - \left(bxe^{a} + e^{a}\right)e^{(bx)}}{b^2x^2e^{(4bx+4a)} + 2b^2x^2e^{(2bx+2a)} + b^2x^2} + 2\int \frac{\left(b^2x^2e^{a} + 2e^{a}\right)e^{(bx)}}{2\left(b^2x^3e^{(2bx+2a)} + b^2x^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)^3*sinh(b*x+a)^2/x,x, algorithm="maxima")
```

```
[Out] -((b*x*e^(3*a) - e^(3*a))*e^(3*b*x) - (b*x*e^a + e^a)*e^(b*x))/(b^2*x^2*e^(
4*b*x + 4*a) + 2*b^2*x^2*e^(2*b*x + 2*a) + b^2*x^2) + 2*integrate(1/2*(b^2*
x^2*e^a + 2*e^a)*e^(b*x)/(b^2*x^3*e^(2*b*x + 2*a) + b^2*x^3), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\sinh(a+b\,x)^2}{x\cosh(a+b\,x)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a + b*x)^2/(x*cosh(a + b*x)^3),x)
```

```
[Out] int(sinh(a + b*x)^2/(x*cosh(a + b*x)^3), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh^2(a+bx)\operatorname{sech}^3(a+bx)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)**3*sinh(b*x+a)**2/x,x)
```

```
[Out] Integral(sinh(a + b*x)**2*sech(a + b*x)**3/x, x)
```

3.375 $\int \frac{\text{sech}(a+bx)\tanh^2(a+bx)}{x^2}\,dx$

Optimal. Leaf size=31

$$\text{Int}\left(\frac{\text{sech}(a+bx)}{x^2},x\right)-\text{Int}\left(\frac{\text{sech}^3(a+bx)}{x^2},x\right)$$

```
[Out] Unintegrable(sech(b*x+a)/x^2,x)-Unintegrable(sech(b*x+a)^3/x^2,x)
```

Rubi [A] time = 0.07, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\text{sech}(a+bx)\tanh^2(a+bx)}{x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Sech[a + b*x]*Tanh[a + b*x]^2)/x^2,x]
```

```
[Out] Defer[Int][Sech[a + b*x]/x^2, x] - Defer[Int][Sech[a + b*x]^3/x^2, x]
```

Rubi steps

$$\int \frac{\text{sech}(a+bx)\tanh^2(a+bx)}{x^2}\,dx=\int \frac{\text{sech}(a+bx)}{x^2}\,dx-\int \frac{\text{sech}^3(a+bx)}{x^2}\,dx$$

Mathematica [A] time = 13.40, size = 0, normalized size = 0.00

$$\int \frac{\text{sech}(a+bx)\tanh^2(a+bx)}{x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Sech[a + b*x]*Tanh[a + b*x]^2)/x^2,x]
```

```
[Out] Integrate[(Sech[a + b*x]*Tanh[a + b*x]^2)/x^2, x]
```

fricas [A] time = 0.59, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\text{sech}\,(bx+a)^3\sinh\,(bx+a)^2}{x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)^3*sinh(b*x+a)^2/x^2,x, algorithm="fricas")
```

```
[Out] integral(sech(b*x + a)^3*sinh(b*x + a)^2/x^2, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\mathrm{sech}\,(bx+a)^3 \sinh\,(bx+a)^2}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)^3*sinh(b*x+a)^2/x^2,x, algorithm="giac")
```

```
[Out] integrate(sech(b*x + a)^3*sinh(b*x + a)^2/x^2, x)
```

maple [A] time = 0.93, size = 0, normalized size = 0.00

$$\int \frac{\mathrm{sech}\,(bx+a)^3 \left(\sinh^2(bx+a)\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(b*x+a)^3*sinh(b*x+a)^2/x^2,x)
```

```
[Out] int(sech(b*x+a)^3*sinh(b*x+a)^2/x^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\left(bxe^{(3a)} - 2e^{(3a)}\right)e^{(3bx)} - \left(bxe^{a} + 2e^{a}\right)e^{(bx)}}{b^2x^3e^{(4bx+4a)} + 2b^2x^3e^{(2bx+2a)} + b^2x^3} + 2\int \frac{\left(b^2x^2e^{a} + 6e^{a}\right)e^{(bx)}}{2\left(b^2x^4e^{(2bx+2a)} + b^2x^4\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)^3*sinh(b*x+a)^2/x^2,x, algorithm="maxima")
```

```
[Out] -((b*x*e^(3*a) - 2*e^(3*a))*e^(3*b*x) - (b*x*e^a + 2*e^a)*e^(b*x))/(b^2*x^3
*e^(4*b*x + 4*a) + 2*b^2*x^3*e^(2*b*x + 2*a) + b^2*x^3) + 2*integrate(1/2*(
b^2*x^2*e^a + 6*e^a)*e^(b*x)/(b^2*x^4*e^(2*b*x + 2*a) + b^2*x^4), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\sinh\,(a+b\,x)^2}{x^2\cosh\,(a+b\,x)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a + b*x)^2/(x^2*cosh(a + b*x)^3),x)
```

```
[Out] int(sinh(a + b*x)^2/(x^2*cosh(a + b*x)^3), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh^2(a+bx)\operatorname{sech}^3(a+bx)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)**3*sinh(b*x+a)**2/x**2,x)
```

```
[Out] Integral(sinh(a + b*x)**2*sech(a + b*x)**3/x**2, x)
```

3.376 $\int x^m \sinh^2(a+bx)\tanh(a+bx)\,dx$

Optimal. Leaf size=85

$$-\mathrm{Int}\left(x^m\tanh(a+bx),x\right)+\frac{e^{2a}2^{-m-3}x^m(-bx)^{-m}\Gamma(m+1,-2bx)}{b}+\frac{e^{-2a}2^{-m-3}x^m(bx)^{-m}\Gamma(m+1,2bx)}{b}$$

```
[Out] 2^(-3-m)*exp(2*a)*x^m*GAMMA(1+m,-2*b*x)/b/((-b*x)^m)+2^(-3-m)*x^m*GAMMA(1+m
,2*b*x)/b/exp(2*a)/((b*x)^m)-Unintegrable(x^m*tanh(b*x+a),x)
```

Rubi [A] time = 0.14, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int x^m\sinh^2(a+bx)\tanh(a+bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x^m*Sinh[a + b*x]^2*Tanh[a + b*x],x]
```

```
[Out] (2^(-3 - m)*E^(2*a)*x^m*Gamma[1 + m, -2*b*x])/(b*(-(b*x))^m) + (2^(-3 - m)*
x^m*Gamma[1 + m, 2*b*x])/(b*E^(2*a)*(b*x)^m) - Defer[Int][x^m*Tanh[a + b*x]
, x]
```

Rubi steps

$$\begin{aligned}\int x^m\sinh^2(a+bx)\tanh(a+bx)\,dx&=\int x^m\cosh(a+bx)\sinh(a+bx)\,dx-\int x^m\tanh(a+bx)\,dx\\&=\int\frac{1}{2}x^m\sinh(2a+2bx)\,dx-\int x^m\tanh(a+bx)\,dx\\&=\frac{1}{2}\int x^m\sinh(2a+2bx)\,dx-\int x^m\tanh(a+bx)\,dx\\&=\frac{1}{4}\int e^{-i(2ia+2ibx)}x^m\,dx-\frac{1}{4}\int e^{i(2ia+2ibx)}x^m\,dx-\int x^m\tanh(a+bx)\,dx\\&=\frac{2^{-3-m}e^{2a}x^m(-bx)^{-m}\Gamma(1+m,-2bx)}{b}+\frac{2^{-3-m}e^{-2a}x^m(bx)^{-m}\Gamma(1+m,2bx)}{b}-\end{aligned}$$

Mathematica [A] time = 22.60, size = 0, normalized size = 0.00

$$\int x^m\sinh^2(a+bx)\tanh(a+bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^m*Sinh[a + b*x]^2*Tanh[a + b*x],x]
[Out] Integrate[x^m*Sinh[a + b*x]^2*Tanh[a + b*x], x]
```

fricas [A] time = 0.40, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(x^{m}\,\mathrm{sech}\left(bx+a\right)\sinh\left(bx+a\right)^{3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*sech(b*x+a)*sinh(b*x+a)^3,x, algorithm="fricas")
[Out] integral(x^m*sech(b*x + a)*sinh(b*x + a)^3, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int x^{m}\,\mathrm{sech}\left(bx+a\right)\sinh\left(bx+a\right)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*sech(b*x+a)*sinh(b*x+a)^3,x, algorithm="giac")
[Out] integrate(x^m*sech(b*x + a)*sinh(b*x + a)^3, x)
```

maple [A] time = 0.43, size = 0, normalized size = 0.00

$$\int x^{m}\mathrm{sech}\left(bx+a\right)\left(\sinh^{3}\left(bx+a\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*sech(b*x+a)*sinh(b*x+a)^3,x)
[Out] int(x^m*sech(b*x+a)*sinh(b*x+a)^3,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int x^{m}\,\mathrm{sech}\left(bx+a\right)\sinh\left(bx+a\right)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*sech(b*x+a)*sinh(b*x+a)^3,x, algorithm="maxima")
[Out] integrate(x^m*sech(b*x + a)*sinh(b*x + a)^3, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{m}\sinh\left(a+b\,x\right)^{3}}{\cosh\left(a+b\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^m*sinh(a + b*x)^3)/cosh(a + b*x),x)
```

```
[Out] int((x^m*sinh(a + b*x)^3)/cosh(a + b*x), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*sech(b*x+a)*sinh(b*x+a)**3,x)
```

```
[Out] Timed out
```

3.377 $\int x^3 \sinh^2(a+bx)\tanh(a+bx)\,dx$

Optimal. Leaf size=185

$$-\frac{3\text{Li}_4\left(-e^{2(a+bx)}\right)}{4b^4}-\frac{3\sinh(a+bx)\cosh(a+bx)}{8b^4}+\frac{3x\text{Li}_3\left(-e^{2(a+bx)}\right)}{2b^3}+\frac{3x\sinh^2(a+bx)}{4b^3}-\frac{3x^2\text{Li}_2\left(-e^{2(a+bx)}\right)}{2b^2}-\frac{3x^2 s}{}$$

```
[Out] 3/8*x/b^3+1/4*x^3/b+1/4*x^4-x^3*ln(1+exp(2*b*x+2*a))/b-3/2*x^2*polylog(2,-e
xp(2*b*x+2*a))/b^2+3/2*x*polylog(3,-exp(2*b*x+2*a))/b^3-3/4*polylog(4,-exp(
2*b*x+2*a))/b^4-3/8*cosh(b*x+a)*sinh(b*x+a)/b^4-3/4*x^2*cosh(b*x+a)*sinh(b*
x+a)/b^2+3/4*x*sinh(b*x+a)^2/b^3+1/2*x^3*sinh(b*x+a)^2/b
```

Rubi [A] time = 0.25, antiderivative size = 185, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 12, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {5449, 5372, 3311, 30, 2635, 8, 3718, 2190, 2531, 6609, 2282, 6589}

$$-\frac{3x^2\text{PolyLog}\left(2,-e^{2(a+bx)}\right)}{2b^2}+\frac{3x\text{PolyLog}\left(3,-e^{2(a+bx)}\right)}{2b^3}-\frac{3\text{PolyLog}\left(4,-e^{2(a+bx)}\right)}{4b^4}-\frac{3x^2\sinh(a+bx)\cosh(a+bx}{4b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Sinh[a + b*x]^2*Tanh[a + b*x],x]
```

```
[Out] (3*x)/(8*b^3) + x^3/(4*b) + x^4/4 - (x^3*Log[1 + E^(2*(a + b*x))])/b - (3*x
^2*PolyLog[2, -E^(2*(a + b*x))])/(2*b^2) + (3*x*PolyLog[3, -E^(2*(a + b*x))
])/(2*b^3) - (3*PolyLog[4, -E^(2*(a + b*x))])/(4*b^4) - (3*Cosh[a + b*x]*Si
nh[a + b*x])/(8*b^4) - (3*x^2*Cosh[a + b*x]*Sinh[a + b*x])/(4*b^2) + (3*x*S
inh[a + b*x]^2)/(4*b^3) + (x^3*Sinh[a + b*x]^2)/(2*b)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
```

```
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 3311

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(d*m*(c + d*x)^(m - 1)*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist
[(b^2*(n - 1))/n, Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[(
d^2*m*(m - 1))/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[(b*(c + d*x)^m*Cos[e + f*x]*(b*Sin[e + f*x])^(n - 1))/(f*n), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 3718

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c
 + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /;
 FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5372

```
Int[Cosh[(a_.) + (b_.)*(x_)^(n_.)]*(x_)^(m_.)*Sinh[(a_.) + (b_.)*(x_)^(n_.)
]^(p_.), x_Symbol] :> Simp[(x^(m - n + 1)*Sinh[a + b*x^n]^(p + 1))/(b*n*(p
```

```
+ 1)), x] - Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*Sinh[a + b*x^n]^(
p + 1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

Rule 5449

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) + (b_.)*(x_)]^(n_.)*Tanh[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> Int[(c + d*x)^m*Sinh[a + b*x]^n*Tanh[a + b*
x]^(p - 2), x] - Int[(c + d*x)^m*Sinh[a + b*x]^(n - 2)*Tanh[a + b*x]^p, x]
/; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int x^3 \sinh^2(a+bx)\tanh(a+bx)\,dx = \int x^3\cosh(a+bx)\sinh(a+bx)\,dx - \int x^3\tanh(a+bx)\,dx$$
$$= \frac{x^4}{4} + \frac{x^3\sinh^2(a+bx)}{2b} - 2\int \frac{e^{2(a+bx)}x^3}{1+e^{2(a+bx)}}\,dx - \frac{3\int x^2\sinh^2(a+bx)\,dx}{2b}$$
$$= \frac{x^4}{4} - \frac{x^3\log\left(1+e^{2(a+bx)}\right)}{b} - \frac{3x^2\cosh(a+bx)\sinh(a+bx)}{4b^2} + \frac{3x\sinh^2(a+$$
$$= \frac{x^3}{4b} + \frac{x^4}{4} - \frac{x^3\log\left(1+e^{2(a+bx)}\right)}{b} - \frac{3x^2\text{Li}_2\left(-e^{2(a+bx)}\right)}{2b^2} - \frac{3\cosh(a+bx)\sinh}{8b^4}$$
$$= \frac{3x}{8b^3} + \frac{x^3}{4b} + \frac{x^4}{4} - \frac{x^3\log\left(1+e^{2(a+bx)}\right)}{b} - \frac{3x^2\text{Li}_2\left(-e^{2(a+bx)}\right)}{2b^2} + \frac{3x\text{Li}_3\left(-e^{2(a-}\right.}{2b^3}$$
$$= \frac{3x}{8b^3} + \frac{x^3}{4b} + \frac{x^4}{4} - \frac{x^3\log\left(1+e^{2(a+bx)}\right)}{b} - \frac{3x^2\text{Li}_2\left(-e^{2(a+bx)}\right)}{2b^2} + \frac{3x\text{Li}_3\left(-e^{2(a-}\right.}{2b^3}$$
$$= \frac{3x}{8b^3} + \frac{x^3}{4b} + \frac{x^4}{4} - \frac{x^3\log\left(1+e^{2(a+bx)}\right)}{b} - \frac{3x^2\text{Li}_2\left(-e^{2(a+bx)}\right)}{2b^2} + \frac{3x\text{Li}_3\left(-e^{2(a-}\right.}{2b^3}$$

Mathematica [A] time = 2.99, size = 191, normalized size = 1.03

$$\frac{1}{16}\left(\frac{12\left(2b^2x^2\text{Li}_2\left(-e^{-2(a+bx)}\right)+2bx\text{Li}_3\left(-e^{-2(a+bx)}\right)+\text{Li}_4\left(-e^{-2(a+bx)}\right)\right)}{b^4} + \frac{\cosh(2bx)\left(2bx\cosh(2a)\left(2b^2x^2+3\right)-\right.}{b^4}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Sinh[a + b*x]^2*Tanh[a + b*x],x]
```

```
[Out] ((-8*x^4)/(1 + E^(2*a)) - (16*x^3*Log[1 + E^(-2*(a + b*x))])/b + (12*(2*b^2
*x^2*PolyLog[2, -E^(-2*(a + b*x))] + 2*b*x*PolyLog[3, -E^(-2*(a + b*x))] +
PolyLog[4, -E^(-2*(a + b*x))]))/b^4 + (Cosh[2*b*x]*(2*b*x*(3 + 2*b^2*x^2)*C
osh[2*a] - 3*(1 + 2*b^2*x^2)*Sinh[2*a]))/b^4 + ((-3*(1 + 2*b^2*x^2)*Cosh[2*
a] + 2*b*x*(3 + 2*b^2*x^2)*Sinh[2*a])*Sinh[2*b*x])/b^4 - 4*x^4*Tanh[a])/16
```

fricas [C] time = 0.60, size = 966, normalized size = 5.22

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*sech(b*x+a)*sinh(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/32*(4*b^3*x^3 + (4*b^3*x^3 - 6*b^2*x^2 + 6*b*x - 3)*cosh(b*x + a)^4 + 4*(
4*b^3*x^3 - 6*b^2*x^2 + 6*b*x - 3)*cosh(b*x + a)*sinh(b*x + a)^3 + (4*b^3*x
```

```
^3 - 6*b^2*x^2 + 6*b*x - 3)*sinh(b*x + a)^4 + 6*b^2*x^2 + 8*(b^4*x^4 - 2*a^
4)*cosh(b*x + a)^2 + 2*(4*b^4*x^4 - 8*a^4 + 3*(4*b^3*x^3 - 6*b^2*x^2 + 6*b*
x - 3)*cosh(b*x + a)^2)*sinh(b*x + a)^2 + 6*b*x - 96*(b^2*x^2*cosh(b*x + a)
^2 + 2*b^2*x^2*cosh(b*x + a)*sinh(b*x + a) + b^2*x^2*sinh(b*x + a)^2)*dilog
(I*cosh(b*x + a) + I*sinh(b*x + a)) - 96*(b^2*x^2*cosh(b*x + a)^2 + 2*b^2*x
^2*cosh(b*x + a)*sinh(b*x + a) + b^2*x^2*sinh(b*x + a)^2)*dilog(-I*cosh(b*x
 + a) - I*sinh(b*x + a)) + 32*(a^3*cosh(b*x + a)^2 + 2*a^3*cosh(b*x + a)*si
nh(b*x + a) + a^3*sinh(b*x + a)^2)*log(cosh(b*x + a) + sinh(b*x + a) + I) +
 32*(a^3*cosh(b*x + a)^2 + 2*a^3*cosh(b*x + a)*sinh(b*x + a) + a^3*sinh(b*x
 + a)^2)*log(cosh(b*x + a) + sinh(b*x + a) - I) - 32*((b^3*x^3 + a^3)*cosh(
b*x + a)^2 + 2*(b^3*x^3 + a^3)*cosh(b*x + a)*sinh(b*x + a) + (b^3*x^3 + a^3
)*sinh(b*x + a)^2)*log(I*cosh(b*x + a) + I*sinh(b*x + a) + 1) - 32*((b^3*x^
3 + a^3)*cosh(b*x + a)^2 + 2*(b^3*x^3 + a^3)*cosh(b*x + a)*sinh(b*x + a) +
(b^3*x^3 + a^3)*sinh(b*x + a)^2)*log(-I*cosh(b*x + a) - I*sinh(b*x + a) + 1
) - 192*(cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2)
*polylog(4, I*cosh(b*x + a) + I*sinh(b*x + a)) - 192*(cosh(b*x + a)^2 + 2*c
osh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2)*polylog(4, -I*cosh(b*x + a) -
 I*sinh(b*x + a)) + 192*(b*x*cosh(b*x + a)^2 + 2*b*x*cosh(b*x + a)*sinh(b*x
 + a) + b*x*sinh(b*x + a)^2)*polylog(3, I*cosh(b*x + a) + I*sinh(b*x + a))
+ 192*(b*x*cosh(b*x + a)^2 + 2*b*x*cosh(b*x + a)*sinh(b*x + a) + b*x*sinh(b
*x + a)^2)*polylog(3, -I*cosh(b*x + a) - I*sinh(b*x + a)) + 4*((4*b^3*x^3 -
 6*b^2*x^2 + 6*b*x - 3)*cosh(b*x + a)^3 + 4*(b^4*x^4 - 2*a^4)*cosh(b*x + a)
)*sinh(b*x + a) + 3)/(b^4*cosh(b*x + a)^2 + 2*b^4*cosh(b*x + a)*sinh(b*x +
a) + b^4*sinh(b*x + a)^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3 \operatorname{sech}(bx+a) \sinh(bx+a)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*sech(b*x+a)*sinh(b*x+a)^3,x, algorithm="giac")
```

```
[Out] integrate(x^3*sech(b*x + a)*sinh(b*x + a)^3, x)
```

maple [A] time = 0.49, size = 189, normalized size = 1.02

$$\frac{x^4}{4}+\frac{\left(4x^3b^3-6x^2b^2+6bx-3\right)e^{2bx+2a}}{32b^4}+\frac{\left(4x^3b^3+6x^2b^2+6bx+3\right)e^{-2bx-2a}}{32b^4}+\frac{2a^3x}{b^3}+\frac{3a^4}{2b^4}-\frac{x^3\ln\left(1+e^{2bx+2a}\right)}{b}-\frac{3}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*sech(b*x+a)*sinh(b*x+a)^3,x)
```

```
[Out] 1/4*x^4+1/32*(4*b^3*x^3-6*b^2*x^2+6*b*x-3)/b^4*exp(2*b*x+2*a)+1/32*(4*b^3*x
^3+6*b^2*x^2+6*b*x+3)/b^4*exp(-2*b*x-2*a)+2/b^3*a^3*x+3/2/b^4*a^4-x^3*ln(1+
```

```
exp(2*b*x+2*a))/b-3/2*x^2*polylog(2,-exp(2*b*x+2*a))/b^2+3/2*x*polylog(3,-e
xp(2*b*x+2*a))/b^3-3/4*polylog(4,-exp(2*b*x+2*a))/b^4-2/b^4*a^3*ln(exp(b*x+
a))
```

maxima [A] time = 0.44, size = 181, normalized size = 0.98

$$\frac{1}{2}x^4 - \frac{\left(8\,b^4x^4e^{(2\,a)} - \left(4\,b^3x^3e^{(4\,a)} - 6\,b^2x^2e^{(4\,a)} + 6\,bxe^{(4\,a)} - 3\,e^{(4\,a)}\right)e^{(2\,bx)} - \left(4\,b^3x^3 + 6\,b^2x^2 + 6\,bx + 3\right)e^{(-2\,bx)}\right)e^{(-2\,a)}}{32\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*sech(b*x+a)*sinh(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/2*x^4 - 1/32*(8*b^4*x^4*e^(2*a) - (4*b^3*x^3*e^(4*a) - 6*b^2*x^2*e^(4*a)
+ 6*b*x*e^(4*a) - 3*e^(4*a))*e^(2*b*x) - (4*b^3*x^3 + 6*b^2*x^2 + 6*b*x + 3
)*e^(-2*b*x))*e^(-2*a)/b^4 - 1/3*(4*b^3*x^3*log(e^(2*b*x + 2*a) + 1) + 6*b^
2*x^2*dilog(-e^(2*b*x + 2*a)) - 6*b*x*polylog(3, -e^(2*b*x + 2*a)) + 3*poly
log(4, -e^(2*b*x + 2*a)))/b^4
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3\,\sinh\left(a+b\,x\right)^3}{\cosh\left(a+b\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*sinh(a + b*x)^3)/cosh(a + b*x),x)
```

```
[Out] int((x^3*sinh(a + b*x)^3)/cosh(a + b*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3\sinh^3\left(a+bx\right)\operatorname{sech}\left(a+bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*sech(b*x+a)*sinh(b*x+a)**3,x)
```

```
[Out] Integral(x**3*sinh(a + b*x)**3*sech(a + b*x), x)
```

3.378 $\int x^2 \sinh^2(a+bx) \tanh(a+bx)\, dx$

Optimal. Leaf size=130

$$\frac{\text{Li}_3\left(-e^{2(a+bx)}\right)}{2b^3}+\frac{\sinh^2(a+bx)}{4b^3}-\frac{x\text{Li}_2\left(-e^{2(a+bx)}\right)}{b^2}-\frac{x\sinh(a+bx)\cosh(a+bx)}{2b^2}-\frac{x^2\log\left(e^{2(a+bx)}+1\right)}{b}+\frac{x^2\sinh^2(a}{2b}$$

```
[Out] 1/4*x^2/b+1/3*x^3-x^2*ln(1+exp(2*b*x+2*a))/b-x*polylog(2,-exp(2*b*x+2*a))/b
^2+1/2*polylog(3,-exp(2*b*x+2*a))/b^3-1/2*x*cosh(b*x+a)*sinh(b*x+a)/b^2+1/4
*sinh(b*x+a)^2/b^3+1/2*x^2*sinh(b*x+a)^2/b
```

Rubi [A] time = 0.19, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5449, 5372, 3310, 30, 3718, 2190, 2531, 2282, 6589}

$$-\frac{x\text{PolyLog}\left(2,-e^{2(a+bx)}\right)}{b^2}+\frac{\text{PolyLog}\left(3,-e^{2(a+bx)}\right)}{2b^3}+\frac{\sinh^2(a+bx)}{4b^3}-\frac{x\sinh(a+bx)\cosh(a+bx)}{2b^2}-\frac{x^2\log\left(e^{2(a+bx}\right.}{b}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Sinh[a + b*x]^2*Tanh[a + b*x],x]
```

```
[Out] x^2/(4*b) + x^3/3 - (x^2*Log[1 + E^(2*(a + b*x))])/b - (x*PolyLog[2, -E^(2*
(a + b*x))])/b^2 + PolyLog[3, -E^(2*(a + b*x))]/(2*b^3) - (x*Cosh[a + b*x]*
Sinh[a + b*x])/(2*b^2) + Sinh[a + b*x]^2/(4*b^3) + (x^2*Sinh[a + b*x]^2)/(2
*b)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
```

```
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 3310

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[(d*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[(b*(c + d*x)*Cos[e + f*x]*(b
*Sin[e + f*x])^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 3718

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c
 + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /;
 FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5372

```
Int[Cosh[(a_.) + (b_.)*(x_)^(n_.)]*(x_)^(m_.)*Sinh[(a_.) + (b_.)*(x_)^(n_.)
]^(p_.), x_Symbol] :> Simp[(x^(m - n + 1)*Sinh[a + b*x^n]^(p + 1))/(b*n*(p
+ 1)), x] - Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*Sinh[a + b*x^n]^(
p + 1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

Rule 5449

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) + (b_.)*(x_)]^(n_.)*Tanh[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> Int[(c + d*x)^m*Sinh[a + b*x]^n*Tanh[a + b*
x]^(p - 2), x] - Int[(c + d*x)^m*Sinh[a + b*x]^(n - 2)*Tanh[a + b*x]^p, x]
/; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int x^2 \sinh^2(a+bx)\tanh(a+bx)\,dx = \int x^2\cosh(a+bx)\sinh(a+bx)\,dx - \int x^2\tanh(a+bx)\,dx$$
$$= \frac{x^3}{3} + \frac{x^2\sinh^2(a+bx)}{2b} - 2\int \frac{e^{2(a+bx)}x^2}{1+e^{2(a+bx)}}\,dx - \frac{\int x\sinh^2(a+bx)\,dx}{b}$$
$$= \frac{x^3}{3} - \frac{x^2\log\left(1+e^{2(a+bx)}\right)}{b} - \frac{x\cosh(a+bx)\sinh(a+bx)}{2b^2} + \frac{\sinh^2(a+bx)}{4b^3}$$
$$= \frac{x^2}{4b} + \frac{x^3}{3} - \frac{x^2\log\left(1+e^{2(a+bx)}\right)}{b} - \frac{x\mathrm{Li}_2\left(-e^{2(a+bx)}\right)}{b^2} - \frac{x\cosh(a+bx)\sinh(}{2b^2}$$
$$= \frac{x^2}{4b} + \frac{x^3}{3} - \frac{x^2\log\left(1+e^{2(a+bx)}\right)}{b} - \frac{x\mathrm{Li}_2\left(-e^{2(a+bx)}\right)}{b^2} - \frac{x\cosh(a+bx)\sinh(}{2b^2}$$
$$= \frac{x^2}{4b} + \frac{x^3}{3} - \frac{x^2\log\left(1+e^{2(a+bx)}\right)}{b} - \frac{x\mathrm{Li}_2\left(-e^{2(a+bx)}\right)}{b^2} + \frac{\mathrm{Li}_3\left(-e^{2(a+bx)}\right)}{2b^3} - \frac{x\,c}{}$$

Mathematica [A] time = 2.73, size = 154, normalized size = 1.18

$$\frac{1}{24}\left(\frac{4\left(2b^2x^2\left(-\frac{2bx}{e^{2a}+1} - 3\log\left(e^{-2(a+bx)}+1\right)\right) + 6bx\mathrm{Li}_2\left(-e^{-2(a+bx)}\right) + 3\mathrm{Li}_3\left(-e^{-2(a+bx)}\right)\right)}{b^3} + \frac{3\cosh(2bx)\left(\cosh(2a\right.}{}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Sinh[a + b*x]^2*Tanh[a + b*x],x]
```

```
[Out] ((4*(2*b^2*x^2*((-2*b*x)/(1 + E^(2*a)) - 3*Log[1 + E^(-2*(a + b*x))]) + 6*b
*x*PolyLog[2, -E^(-2*(a + b*x))] + 3*PolyLog[3, -E^(-2*(a + b*x))]))/b^3 +
(3*Cosh[2*b*x]*((1 + 2*b^2*x^2)*Cosh[2*a] - 2*b*x*Sinh[2*a]))/b^3 + (3*(-2*
b*x*Cosh[2*a] + (1 + 2*b^2*x^2)*Sinh[2*a])*Sinh[2*b*x])/b^3 - 8*x^3*Tanh[a]
)/24
```

fricas [C] time = 0.79, size = 789, normalized size = 6.07

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sech(b*x+a)*sinh(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/48*(3*(2*b^2*x^2 - 2*b*x + 1)*cosh(b*x + a)^4 + 12*(2*b^2*x^2 - 2*b*x + 1
)*cosh(b*x + a)*sinh(b*x + a)^3 + 3*(2*b^2*x^2 - 2*b*x + 1)*sinh(b*x + a)^4
```

```
 + 6*b^2*x^2 + 16*(b^3*x^3 + 2*a^3)*cosh(b*x + a)^2 + 2*(8*b^3*x^3 + 16*a^3
 + 9*(2*b^2*x^2 - 2*b*x + 1)*cosh(b*x + a)^2)*sinh(b*x + a)^2 + 6*b*x - 96*
(b*x*cosh(b*x + a)^2 + 2*b*x*cosh(b*x + a)*sinh(b*x + a) + b*x*sinh(b*x + a
)^2)*dilog(I*cosh(b*x + a) + I*sinh(b*x + a)) - 96*(b*x*cosh(b*x + a)^2 + 2
*b*x*cosh(b*x + a)*sinh(b*x + a) + b*x*sinh(b*x + a)^2)*dilog(-I*cosh(b*x +
 a) - I*sinh(b*x + a)) - 48*(a^2*cosh(b*x + a)^2 + 2*a^2*cosh(b*x + a)*sinh
(b*x + a) + a^2*sinh(b*x + a)^2)*log(cosh(b*x + a) + sinh(b*x + a) + I) - 4
8*(a^2*cosh(b*x + a)^2 + 2*a^2*cosh(b*x + a)*sinh(b*x + a) + a^2*sinh(b*x +
 a)^2)*log(cosh(b*x + a) + sinh(b*x + a) - I) - 48*((b^2*x^2 - a^2)*cosh(b*
x + a)^2 + 2*(b^2*x^2 - a^2)*cosh(b*x + a)*sinh(b*x + a) + (b^2*x^2 - a^2)*
sinh(b*x + a)^2)*log(I*cosh(b*x + a) + I*sinh(b*x + a) + 1) - 48*((b^2*x^2
- a^2)*cosh(b*x + a)^2 + 2*(b^2*x^2 - a^2)*cosh(b*x + a)*sinh(b*x + a) + (b
^2*x^2 - a^2)*sinh(b*x + a)^2)*log(-I*cosh(b*x + a) - I*sinh(b*x + a) + 1)
+ 96*(cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2)*po
lylog(3, I*cosh(b*x + a) + I*sinh(b*x + a)) + 96*(cosh(b*x + a)^2 + 2*cosh(
b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2)*polylog(3, -I*cosh(b*x + a) - I*s
inh(b*x + a)) + 4*(3*(2*b^2*x^2 - 2*b*x + 1)*cosh(b*x + a)^3 + 8*(b^3*x^3 +
 2*a^3)*cosh(b*x + a))*sinh(b*x + a) + 3)/(b^3*cosh(b*x + a)^2 + 2*b^3*cosh
(b*x + a)*sinh(b*x + a) + b^3*sinh(b*x + a)^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \operatorname{sech}(bx+a)\sinh(bx+a)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sech(b*x+a)*sinh(b*x+a)^3,x, algorithm="giac")
```

```
[Out] integrate(x^2*sech(b*x + a)*sinh(b*x + a)^3, x)
```

maple [A] time = 0.48, size = 152, normalized size = 1.17

$$\frac{x^3}{3}+\frac{\left(2x^2b^2-2bx+1\right)e^{2bx+2a}}{16b^3}+\frac{\left(2x^2b^2+2bx+1\right)e^{-2bx-2a}}{16b^3}+\frac{2a^2\ln\left(e^{bx+a}\right)}{b^3}-\frac{2a^2x}{b^2}-\frac{4a^3}{3b^3}-\frac{x^2\ln\left(1+e^{2bx+2a}\right)}{b}-\frac{x\,\mathrm{po}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*sech(b*x+a)*sinh(b*x+a)^3,x)
```

```
[Out] 1/3*x^3+1/16*(2*b^2*x^2-2*b*x+1)/b^3*exp(2*b*x+2*a)+1/16*(2*b^2*x^2+2*b*x+1
)/b^3*exp(-2*b*x-2*a)+2/b^3*a^2*ln(exp(b*x+a))-2/b^2*a^2*x-4/3/b^3*a^3-x^2*
ln(1+exp(2*b*x+2*a))/b-x*polylog(2,-exp(2*b*x+2*a))/b^2+1/2*polylog(3,-exp(
2*b*x+2*a))/b^3
```

maxima [A] time = 0.46, size = 138, normalized size = 1.06

$$\frac{2}{3}x^3-\frac{\left(16\,b^3x^3e^{(2\,a)}-3\left(2\,b^2x^2e^{(4\,a)}-2\,bxe^{(4\,a)}+e^{(4\,a)}\right)e^{(2\,bx)}-3\left(2\,b^2x^2+2\,bx+1\right)e^{(-2\,bx)}\right)e^{(-2\,a)}}{48\,b^3}-\frac{2\,b^2x^2\log\left(e^{(2\,bx}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sech(b*x+a)*sinh(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 2/3*x^3 - 1/48*(16*b^3*x^3*e^(2*a) - 3*(2*b^2*x^2*e^(4*a) - 2*b*x*e^(4*a) +
 e^(4*a))*e^(2*b*x) - 3*(2*b^2*x^2 + 2*b*x + 1)*e^(-2*b*x))*e^(-2*a)/b^3 -
1/2*(2*b^2*x^2*log(e^(2*b*x + 2*a) + 1) + 2*b*x*dilog(-e^(2*b*x + 2*a)) - p
olylog(3, -e^(2*b*x + 2*a)))/b^3
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2 \sinh(a+b\,x)^3}{\cosh(a+b\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*sinh(a + b*x)^3)/cosh(a + b*x),x)
```

```
[Out] int((x^2*sinh(a + b*x)^3)/cosh(a + b*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \sinh^3(a+bx)\,\mathrm{sech}\,(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*sech(b*x+a)*sinh(b*x+a)**3,x)
```

```
[Out] Integral(x**2*sinh(a + b*x)**3*sech(a + b*x), x)
```

3.379 $\int x \sinh^2(a+bx)\tanh(a+bx)\,dx$

Optimal. Leaf size=89

$$-\frac{\text{Li}_2\left(-e^{2(a+bx)}\right)}{2b^2}-\frac{\sinh(a+bx)\cosh(a+bx)}{4b^2}-\frac{x\log\left(e^{2(a+bx)}+1\right)}{b}+\frac{x\sinh^2(a+bx)}{2b}+\frac{x}{4b}+\frac{x^2}{2}$$

```
[Out] 1/4*x/b+1/2*x^2-x*ln(1+exp(2*b*x+2*a))/b-1/2*polylog(2,-exp(2*b*x+2*a))/b^2
-1/4*cosh(b*x+a)*sinh(b*x+a)/b^2+1/2*x*sinh(b*x+a)^2/b
```

Rubi [A] time = 0.12, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5449, 5372, 2635, 8, 3718, 2190, 2279, 2391}

$$-\frac{\text{PolyLog}\left(2,-e^{2(a+bx)}\right)}{2b^2}-\frac{\sinh(a+bx)\cosh(a+bx)}{4b^2}-\frac{x\log\left(e^{2(a+bx)}+1\right)}{b}+\frac{x\sinh^2(a+bx)}{2b}+\frac{x}{4b}+\frac{x^2}{2}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sinh[a + b*x]^2*Tanh[a + b*x],x]
```

```
[Out] x/(4*b) + x^2/2 - (x*Log[1 + E^(2*(a + b*x))])/b - PolyLog[2, -E^(2*(a + b*
x))]/(2*b^2) - (Cosh[a + b*x]*Sinh[a + b*x])/(4*b^2) + (x*Sinh[a + b*x]^2)/
(2*b)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 3718

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c
 + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /;
 FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5372

```
Int[Cosh[(a_.) + (b_.)*(x_)^(n_.)]*(x_)^(m_.)*Sinh[(a_.) + (b_.)*(x_)^(n_.)
]^(p_.), x_Symbol] :> Simp[(x^(m - n + 1)*Sinh[a + b*x^n]^(p + 1))/(b*n*(p
+ 1)), x] - Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*Sinh[a + b*x^n]^(
p + 1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

Rule 5449

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) + (b_.)*(x_)]^(n_.)*Tanh[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> Int[(c + d*x)^m*Sinh[a + b*x]^n*Tanh[a + b*
x]^(p - 2), x] - Int[(c + d*x)^m*Sinh[a + b*x]^(n - 2)*Tanh[a + b*x]^p, x]
/; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rubi steps

$$\int x\sinh^2(a+bx)\tanh(a+bx)\,dx = \int x\cosh(a+bx)\sinh(a+bx)\,dx - \int x\tanh(a+bx)\,dx$$
$$= \frac{x^2}{2} + \frac{x\sinh^2(a+bx)}{2b} - 2\int \frac{e^{2(a+bx)}x}{1+e^{2(a+bx)}}\,dx - \frac{\int \sinh^2(a+bx)\,dx}{2b}$$
$$= \frac{x^2}{2} - \frac{x\log\left(1+e^{2(a+bx)}\right)}{b} - \frac{\cosh(a+bx)\sinh(a+bx)}{4b^2} + \frac{x\sinh^2(a+bx)}{2b} +$$
$$= \frac{x}{4b} + \frac{x^2}{2} - \frac{x\log\left(1+e^{2(a+bx)}\right)}{b} - \frac{\cosh(a+bx)\sinh(a+bx)}{4b^2} + \frac{x\sinh^2(a+b}{2b}$$
$$= \frac{x}{4b} + \frac{x^2}{2} - \frac{x\log\left(1+e^{2(a+bx)}\right)}{b} - \frac{\text{Li}_2\left(-e^{2(a+bx)}\right)}{2b^2} - \frac{\cosh(a+bx)\sinh(a+bx}{4b^2}$$

Mathematica [A] time = 0.23, size = 102, normalized size = 1.15

$$-\frac{4a^2 - 4\text{Li}_2\left(-e^{-2(a+bx)}\right) + 8abx + 8a\log\left(e^{-2(a+bx)}+1\right) + 8bx\log\left(e^{-2(a+bx)}+1\right) + \sinh(2(a+bx)) - 2bx\cosh(2(}{8b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sinh[a + b*x]^2*Tanh[a + b*x],x]
```

```
[Out] -1/8*(4*a^2 + 8*a*b*x + 4*b^2*x^2 - 2*b*x*Cosh[2*(a + b*x)] + 8*a*Log[1 + E
^(-2*(a + b*x))] + 8*b*x*Log[1 + E^(-2*(a + b*x))] - 8*a*Log[Cosh[a + b*x]]
 - 4*PolyLog[2, -E^(-2*(a + b*x))] + Sinh[2*(a + b*x)])/b^2
```

fricas [C] time = 0.73, size = 558, normalized size = 6.27

$$\frac{(2\,bx-1)\cosh\left(bx+a\right)^4 + 4\,(2\,bx-1)\cosh\left(bx+a\right)\sinh\left(bx+a\right)^3 + (2\,bx-1)\sinh\left(bx+a\right)^4 + 8\left(b^2x^2-2\,a^2\right)c}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sech(b*x+a)*sinh(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/16*((2*b*x - 1)*cosh(b*x + a)^4 + 4*(2*b*x - 1)*cosh(b*x + a)*sinh(b*x +
a)^3 + (2*b*x - 1)*sinh(b*x + a)^4 + 8*(b^2*x^2 - 2*a^2)*cosh(b*x + a)^2 +
2*(4*b^2*x^2 + 3*(2*b*x - 1)*cosh(b*x + a)^2 - 8*a^2)*sinh(b*x + a)^2 + 2*b
*x - 16*(cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2)
*dilog(I*cosh(b*x + a) + I*sinh(b*x + a)) - 16*(cosh(b*x + a)^2 + 2*cosh(b*
x + a)*sinh(b*x + a) + sinh(b*x + a)^2)*dilog(-I*cosh(b*x + a) - I*sinh(b*x
 + a)) + 16*(a*cosh(b*x + a)^2 + 2*a*cosh(b*x + a)*sinh(b*x + a) + a*sinh(b
```

```
*x + a)^2)*log(cosh(b*x + a) + sinh(b*x + a) + I) + 16*(a*cosh(b*x + a)^2 +
 2*a*cosh(b*x + a)*sinh(b*x + a) + a*sinh(b*x + a)^2)*log(cosh(b*x + a) + s
inh(b*x + a) - I) - 16*((b*x + a)*cosh(b*x + a)^2 + 2*(b*x + a)*cosh(b*x +
a)*sinh(b*x + a) + (b*x + a)*sinh(b*x + a)^2)*log(I*cosh(b*x + a) + I*sinh(
b*x + a) + 1) - 16*((b*x + a)*cosh(b*x + a)^2 + 2*(b*x + a)*cosh(b*x + a)*s
inh(b*x + a) + (b*x + a)*sinh(b*x + a)^2)*log(-I*cosh(b*x + a) - I*sinh(b*x
 + a) + 1) + 4*((2*b*x - 1)*cosh(b*x + a)^3 + 4*(b^2*x^2 - 2*a^2)*cosh(b*x
+ a))*sinh(b*x + a) + 1)/(b^2*cosh(b*x + a)^2 + 2*b^2*cosh(b*x + a)*sinh(b*
x + a) + b^2*sinh(b*x + a)^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x \operatorname{sech}(bx+a)\sinh(bx+a)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sech(b*x+a)*sinh(b*x+a)^3,x, algorithm="giac")
```

```
[Out] integrate(x*sech(b*x + a)*sinh(b*x + a)^3, x)
```

maple [A] time = 0.47, size = 110, normalized size = 1.24

$$\frac{x^2}{2}+\frac{(2bx-1)\,e^{2bx+2a}}{16b^2}+\frac{(2bx+1)\,e^{-2bx-2a}}{16b^2}+\frac{2ax}{b}+\frac{a^2}{b^2}-\frac{x\ln\left(1+e^{2bx+2a}\right)}{b}-\frac{\operatorname{polylog}\left(2,-e^{2bx+2a}\right)}{2b^2}-\frac{2a\ln\left(e^{bx+a}\right)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sech(b*x+a)*sinh(b*x+a)^3,x)
```

```
[Out] 1/2*x^2+1/16*(2*b*x-1)/b^2*exp(2*b*x+2*a)+1/16*(2*b*x+1)/b^2*exp(-2*b*x-2*a
)+2/b*a*x+a^2/b^2-x*ln(1+exp(2*b*x+2*a))/b-1/2*polylog(2,-exp(2*b*x+2*a))/b
^2-2/b^2*a*ln(exp(b*x+a))
```

maxima [A] time = 0.44, size = 95, normalized size = 1.07

$$x^2-\frac{\left(8\,b^2x^2e^{(2\,a)}-\left(2\,bxe^{(4\,a)}-e^{(4\,a)}\right)e^{(2\,bx)}-(2\,bx+1)e^{(-2\,bx)}\right)e^{(-2\,a)}}{16\,b^2}-\frac{2\,bx\log\left(e^{(2\,bx+2\,a)}+1\right)+\mathrm{Li}_2\left(-e^{(2\,bx+2\,a)}\right)}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sech(b*x+a)*sinh(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] x^2 - 1/16*(8*b^2*x^2*e^(2*a) - (2*b*x*e^(4*a) - e^(4*a))*e^(2*b*x) - (2*b*
x + 1)*e^(-2*b*x))*e^(-2*a)/b^2 - 1/2*(2*b*x*log(e^(2*b*x + 2*a) + 1) + dil
og(-e^(2*b*x + 2*a)))/b^2
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x \sinh (a + b x)^3}{\cosh (a + b x)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*sinh(a + b*x)^3)/cosh(a + b*x),x)
[Out] int((x*sinh(a + b*x)^3)/cosh(a + b*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x \sinh^3 (a + bx) \operatorname{sech} (a + bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sech(b*x+a)*sinh(b*x+a)**3,x)
[Out] Integral(x*sinh(a + b*x)**3*sech(a + b*x), x)
```

3.380 $\int \sinh^2(a+bx)\tanh(a+bx)\,dx$

Optimal. Leaf size=28

$$\frac{\cosh^2(a+bx)}{2b}-\frac{\log(\cosh(a+bx))}{b}$$

```
[Out] 1/2*cosh(b*x+a)^2/b-ln(cosh(b*x+a))/b
```

Rubi [A] time = 0.02, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2590, 14}

$$\frac{\cosh^2(a+bx)}{2b}-\frac{\log(\cosh(a+bx))}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[a + b*x]^2*Tanh[a + b*x],x]
```

```
[Out] Cosh[a + b*x]^2/(2*b) - Log[Cosh[a + b*x]]/b
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2590

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> -Dist[f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rubi steps

$$\begin{aligned}\int \sinh^2(a+bx)\tanh(a+bx)\,dx &= -\frac{\text{Subst}\left(\int \frac{1-x^2}{x}\,dx, x, \cosh(a+bx)\right)}{b}\\ &= -\frac{\text{Subst}\left(\int \left(\frac{1}{x}-x\right)\,dx, x, \cosh(a+bx)\right)}{b}\\ &= \frac{\cosh^2(a+bx)}{2b}-\frac{\log(\cosh(a+bx))}{b}\end{aligned}$$

Mathematica [A] time = 0.02, size = 25, normalized size = 0.89

$$-\frac{\log(\cosh(a+bx))-\frac{1}{2}\cosh^2(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[a + b*x]^2*Tanh[a + b*x],x]
```

```
[Out] -((-1/2*Cosh[a + b*x]^2 + Log[Cosh[a + b*x]])/b)
```

fricas [B] time = 0.59, size = 197, normalized size = 7.04

$$\frac{8\,bx\cosh\left(bx+a\right)^2+\cosh\left(bx+a\right)^4+4\,\cosh\left(bx+a\right)\sinh\left(bx+a\right)^3+\sinh\left(bx+a\right)^4+2\left(4\,bx+3\,\cosh\left(bx+a\right.\right.}{8\left(b\,\mathrm{co}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)*sinh(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/8*(8*b*x*cosh(b*x + a)^2 + cosh(b*x + a)^4 + 4*cosh(b*x + a)*sinh(b*x + a
)^3 + sinh(b*x + a)^4 + 2*(4*b*x + 3*cosh(b*x + a)^2)*sinh(b*x + a)^2 - 8*(
cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2)*log(2*co
sh(b*x + a)/(cosh(b*x + a) - sinh(b*x + a))) + 4*(4*b*x*cosh(b*x + a) + cos
h(b*x + a)^3)*sinh(b*x + a) + 1)/(b*cosh(b*x + a)^2 + 2*b*cosh(b*x + a)*sin
h(b*x + a) + b*sinh(b*x + a)^2)
```

giac [B] time = 0.15, size = 60, normalized size = 2.14

$$\frac{8\,bx-\left(4\,e^{(2\,bx+2\,a)}-1\right)e^{(-2\,bx-2\,a)}+8\,a+e^{(2\,bx+2\,a)}-8\,\log\left(e^{(2\,bx+2\,a)}+1\right)}{8\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)*sinh(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/8*(8*b*x - (4*e^(2*b*x + 2*a) - 1)*e^(-2*b*x - 2*a) + 8*a + e^(2*b*x + 2*
a) - 8*log(e^(2*b*x + 2*a) + 1))/b
```

maple [A] time = 0.12, size = 27, normalized size = 0.96

$$\frac{\sinh^2\left(bx+a\right)}{2b}-\frac{\ln\left(\cosh\left(bx+a\right)\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(b*x+a)*sinh(b*x+a)^3,x)
```

```
[Out] 1/2*sinh(b*x+a)^2/b-ln(cosh(b*x+a))/b
```

maxima [B] time = 0.65, size = 56, normalized size = 2.00

$$-\frac{bx+a}{b}+\frac{e^{(2\,bx+2\,a)}}{8\,b}+\frac{e^{(-2\,bx-2\,a)}}{8\,b}-\frac{\log\left(e^{(-2\,bx-2\,a)}+1\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)*sinh(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -(b*x + a)/b + 1/8*e^(2*b*x + 2*a)/b + 1/8*e^(-2*b*x - 2*a)/b - log(e^(-2*b
*x - 2*a) + 1)/b
```

mupad [B] time = 0.06, size = 48, normalized size = 1.71

$$x-\frac{\ln\left(\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}+1\right)}{b}+\frac{\mathrm{e}^{-2\,a-2\,b\,x}}{8\,b}+\frac{\mathrm{e}^{2\,a+2\,b\,x}}{8\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a + b*x)^3/cosh(a + b*x),x)
```

```
[Out] x - log(exp(2*a)*exp(2*b*x) + 1)/b + exp(- 2*a - 2*b*x)/(8*b) + exp(2*a + 2
*b*x)/(8*b)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sinh^{3}\left(a+bx\right)\operatorname{sech}\left(a+bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)*sinh(b*x+a)**3,x)
```

```
[Out] Integral(sinh(a + b*x)**3*sech(a + b*x), x)
```

3.381 $\int \frac{\sinh^2(a+bx)\tanh(a+bx)}{x}\,dx$

Optimal. Leaf size=42

$$-\text{Int}\left(\frac{\tanh(a+bx)}{x},x\right)+\frac{1}{2}\sinh(2a)\text{Chi}(2bx)+\frac{1}{2}\cosh(2a)\text{Shi}(2bx)$$

```
[Out] 1/2*cosh(2*a)*Shi(2*b*x)+1/2*Chi(2*b*x)*sinh(2*a)-Unintegrable(tanh(b*x+a)/
x,x)
```

Rubi [A] time = 0.10, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sinh^2(a+bx)\tanh(a+bx)}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Sinh[a + b*x]^2*Tanh[a + b*x])/x,x]
```

```
[Out] (CoshIntegral[2*b*x]*Sinh[2*a])/2 + (Cosh[2*a]*SinhIntegral[2*b*x])/2 - Def
er[Int][Tanh[a + b*x]/x, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sinh^2(a+bx)\tanh(a+bx)}{x}\,dx &= \int \frac{\cosh(a+bx)\sinh(a+bx)}{x}\,dx - \int \frac{\tanh(a+bx)}{x}\,dx \\
&= \int \frac{\sinh(2a+2bx)}{2x}\,dx - \int \frac{\tanh(a+bx)}{x}\,dx \\
&= \frac{1}{2}\int \frac{\sinh(2a+2bx)}{x}\,dx - \int \frac{\tanh(a+bx)}{x}\,dx \\
&= \frac{1}{2}\cosh(2a)\int \frac{\sinh(2bx)}{x}\,dx + \frac{1}{2}\sinh(2a)\int \frac{\cosh(2bx)}{x}\,dx - \int \frac{\tanh(a+bx)}{x} \\
&= \frac{1}{2}\text{Chi}(2bx)\sinh(2a) + \frac{1}{2}\cosh(2a)\text{Shi}(2bx) - \int \frac{\tanh(a+bx)}{x}\,dx
\end{aligned}$$

Mathematica [A] time = 19.59, size = 0, normalized size = 0.00

$$\int \frac{\sinh^2(a+bx)\tanh(a+bx)}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Sinh[a + b*x]^2*Tanh[a + b*x])/x,x]
[Out] Integrate[(Sinh[a + b*x]^2*Tanh[a + b*x])/x, x]
```

fricas [A] time = 0.51, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\operatorname{sech}(bx+a)\sinh(bx+a)^3}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)*sinh(b*x+a)^3/x,x, algorithm="fricas")
[Out] integral(sech(b*x + a)*sinh(b*x + a)^3/x, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}(bx+a)\sinh(bx+a)^3}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)*sinh(b*x+a)^3/x,x, algorithm="giac")
[Out] integrate(sech(b*x + a)*sinh(b*x + a)^3/x, x)
```

maple [A] time = 0.60, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}(bx+a)\left(\sinh^3(bx+a)\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(b*x+a)*sinh(b*x+a)^3/x,x)
[Out] int(sech(b*x+a)*sinh(b*x+a)^3/x,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{4}\operatorname{Ei}(2\,bx)\,e^{(2\,a)}-\frac{1}{4}\operatorname{Ei}(-2\,bx)\,e^{(-2\,a)}+2\int \frac{1}{xe^{(2\,bx+2\,a)}+x}\,dx-\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)*sinh(b*x+a)^3/x,x, algorithm="maxima")
[Out] 1/4*Ei(2*b*x)*e^(2*a) - 1/4*Ei(-2*b*x)*e^(-2*a) + 2*integrate(1/(x*e^(2*b*x
 + 2*a) + x), x) - log(x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sinh{(a+b\,x)}^3}{x\cosh{(a+b\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a + b*x)^3/(x*cosh(a + b*x)),x)
```

```
[Out] int(sinh(a + b*x)^3/(x*cosh(a + b*x)), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh^3{(a+bx)}\,\mathrm{sech}\,(a+bx)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)*sinh(b*x+a)**3/x,x)
```

```
[Out] Integral(sinh(a + b*x)**3*sech(a + b*x)/x, x)
```

3.382 $\int \frac{\sinh^2(a+bx)\tanh(a+bx)}{x^2}\,dx$

Optimal. Leaf size=54

$$-\mathrm{Int}\left(\frac{\tanh(a+bx)}{x^2},x\right)+b\cosh(2a)\mathrm{Chi}(2bx)+b\sinh(2a)\mathrm{Shi}(2bx)-\frac{\sinh(2a+2bx)}{2x}$$

```
[Out] b*Chi(2*b*x)*cosh(2*a)+b*Shi(2*b*x)*sinh(2*a)-1/2*sinh(2*b*x+2*a)/x-Uninteg
rable(tanh(b*x+a)/x^2,x)
```

Rubi [A] time = 0.13, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sinh^2(a+bx)\tanh(a+bx)}{x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Sinh[a + b*x]^2*Tanh[a + b*x])/x^2,x]
```

```
[Out] b*Cosh[2*a]*CoshIntegral[2*b*x] - Sinh[2*a + 2*b*x]/(2*x) + b*Sinh[2*a]*Sin
hIntegral[2*b*x] - Defer[Int][Tanh[a + b*x]/x^2, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sinh^2(a+bx)\tanh(a+bx)}{x^2}\,dx &= \int \frac{\cosh(a+bx)\sinh(a+bx)}{x^2}\,dx-\int \frac{\tanh(a+bx)}{x^2}\,dx\\
&= \int \frac{\sinh(2a+2bx)}{2x^2}\,dx-\int \frac{\tanh(a+bx)}{x^2}\,dx\\
&= \frac{1}{2}\int \frac{\sinh(2a+2bx)}{x^2}\,dx-\int \frac{\tanh(a+bx)}{x^2}\,dx\\
&= -\frac{\sinh(2a+2bx)}{2x}+b\int \frac{\cosh(2a+2bx)}{x}\,dx-\int \frac{\tanh(a+bx)}{x^2}\,dx\\
&= -\frac{\sinh(2a+2bx)}{2x}+(b\cosh(2a))\int \frac{\cosh(2bx)}{x}\,dx+(b\sinh(2a))\int \frac{\sinh(2l}{x}\\
&= b\cosh(2a)\mathrm{Chi}(2bx)-\frac{\sinh(2a+2bx)}{2x}+b\sinh(2a)\mathrm{Shi}(2bx)-\int \frac{\tanh(a+}{x^2}
\end{aligned}$$

Mathematica [A] time = 20.20, size = 0, normalized size = 0.00

$$\int \frac{\sinh^2(a+bx)\tanh(a+bx)}{x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Sinh[a + b*x]^2*Tanh[a + b*x])/x^2,x]
[Out] Integrate[(Sinh[a + b*x]^2*Tanh[a + b*x])/x^2, x]
```

fricas [A] time = 0.54, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\operatorname{sech}(bx+a)\sinh(bx+a)^3}{x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)*sinh(b*x+a)^3/x^2,x, algorithm="fricas")
[Out] integral(sech(b*x + a)*sinh(b*x + a)^3/x^2, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}(bx+a)\sinh(bx+a)^3}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)*sinh(b*x+a)^3/x^2,x, algorithm="giac")
[Out] integrate(sech(b*x + a)*sinh(b*x + a)^3/x^2, x)
```

maple [A] time = 0.60, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}(bx+a)\left(\sinh^3(bx+a)\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(b*x+a)*sinh(b*x+a)^3/x^2,x)
[Out] int(sech(b*x+a)*sinh(b*x+a)^3/x^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{2}\,be^{(-2\,a)}\Gamma\left(-1, 2\,bx\right) + \frac{1}{2}\,be^{(2\,a)}\Gamma\left(-1, -2\,bx\right) + \frac{1}{x} + 2\int \frac{1}{x^2e^{(2\,bx+2\,a)} + x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)*sinh(b*x+a)^3/x^2,x, algorithm="maxima")
```

```
[Out] 1/2*b*e^(-2*a)*gamma(-1, 2*b*x) + 1/2*b*e^(2*a)*gamma(-1, -2*b*x) + 1/x + 2
*integrate(1/(x^2*e^(2*b*x + 2*a) + x^2), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sinh(a+b\,x)^3}{x^2\cosh(a+b\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a + b*x)^3/(x^2*cosh(a + b*x)),x)
[Out] int(sinh(a + b*x)^3/(x^2*cosh(a + b*x)), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh^3(a+bx)\operatorname{sech}(a+bx)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)*sinh(b*x+a)**3/x**2,x)
[Out] Integral(sinh(a + b*x)**3*sech(a + b*x)/x**2, x)
```

3.383 $\int x^m \sinh(a+bx)\tanh^2(a+bx)\,dx$

Optimal. Leaf size=80

$$-\text{Int}\left(x^m\tanh(a+bx)\text{sech}(a+bx),x\right)+\frac{e^a x^m(-bx)^{-m}\Gamma(m+1,-bx)}{2b}+\frac{e^{-a}x^m(bx)^{-m}\Gamma(m+1,bx)}{2b}$$

```
[Out] -CannotIntegrate(x^m*sech(b*x+a)*tanh(b*x+a),x)+1/2*exp(a)*x^m*GAMMA(1+m,-b
*x)/b/((-b*x)^m)+1/2*x^m*GAMMA(1+m,b*x)/b/exp(a)/((b*x)^m)
```

Rubi [A] time = 0.17, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int x^m\sinh(a+bx)\tanh^2(a+bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x^m*Sinh[a + b*x]*Tanh[a + b*x]^2,x]
```

```
[Out] (E^a*x^m*Gamma[1 + m, -(b*x)])/(2*b*(-(b*x))^m) + (x^m*Gamma[1 + m, b*x])/(
2*b*E^a*(b*x)^m) - Defer[Int][x^m*Sech[a + b*x]*Tanh[a + b*x], x]
```

Rubi steps

$$\begin{aligned}\int x^m\sinh(a+bx)\tanh^2(a+bx)\,dx &= \int x^m\sinh(a+bx)\,dx-\int x^m\text{sech}(a+bx)\tanh(a+bx)\,dx\\ &= \frac{1}{2}\int e^{-i(ia+ibx)}x^m\,dx-\frac{1}{2}\int e^{i(ia+ibx)}x^m\,dx-\int x^m\text{sech}(a+bx)\tanh(a+bx)\\ &= \frac{e^a x^m(-bx)^{-m}\Gamma(1+m,-bx)}{2b}+\frac{e^{-a}x^m(bx)^{-m}\Gamma(1+m,bx)}{2b}-\int x^m\text{sech}(a+\end{aligned}$$

Mathematica [A] time = 38.12, size = 0, normalized size = 0.00

$$\int x^m\sinh(a+bx)\tanh^2(a+bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^m*Sinh[a + b*x]*Tanh[a + b*x]^2,x]
```

```
[Out] Integrate[x^m*Sinh[a + b*x]*Tanh[a + b*x]^2, x]
```

fricas [A] time = 0.59, size = 0, normalized size = 0.00

$$\text{integral}\left(x^m\,\text{sech}\left(bx+a\right)^2\sinh\left(bx+a\right)^3,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*sech(b*x+a)^2*sinh(b*x+a)^3,x, algorithm="fricas")
[Out] integral(x^m*sech(b*x + a)^2*sinh(b*x + a)^3, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int x^m \operatorname{sech}(bx+a)^2 \sinh(bx+a)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*sech(b*x+a)^2*sinh(b*x+a)^3,x, algorithm="giac")
[Out] integrate(x^m*sech(b*x + a)^2*sinh(b*x + a)^3, x)
```

maple [A] time = 0.42, size = 0, normalized size = 0.00

$$\int x^m \operatorname{sech}(bx+a)^2 \left(\sinh^3(bx+a)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*sech(b*x+a)^2*sinh(b*x+a)^3,x)
[Out] int(x^m*sech(b*x+a)^2*sinh(b*x+a)^3,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int x^m \operatorname{sech}(bx+a)^2 \sinh(bx+a)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*sech(b*x+a)^2*sinh(b*x+a)^3,x, algorithm="maxima")
[Out] integrate(x^m*sech(b*x + a)^2*sinh(b*x + a)^3, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^m \sinh(a+b\,x)^3}{\cosh(a+b\,x)^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^m*sinh(a + b*x)^3)/cosh(a + b*x)^2,x)
[Out] int((x^m*sinh(a + b*x)^3)/cosh(a + b*x)^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*sech(b*x+a)**2*sinh(b*x+a)**3,x)
```

```
[Out] Timed out
```

3.384 $\int x^3 \sinh(a+bx)\tanh^2(a+bx)\,dx$

Optimal. Leaf size=162

$$-\frac{6i\text{Li}_3\left(-ie^{a+bx}\right)}{b^4}+\frac{6i\text{Li}_3\left(ie^{a+bx}\right)}{b^4}-\frac{6\sinh(a+bx)}{b^4}+\frac{6ix\text{Li}_2\left(-ie^{a+bx}\right)}{b^3}-\frac{6ix\text{Li}_2\left(ie^{a+bx}\right)}{b^3}+\frac{6x\cosh(a+bx)}{b^3}-\frac{6x^2\tan^{-1}}{b}$$

```
[Out] -6*x^2*arctan(exp(b*x+a))/b^2+6*x*cosh(b*x+a)/b^3+x^3*cosh(b*x+a)/b+6*I*x*p
olylog(2,-I*exp(b*x+a))/b^3-6*I*x*polylog(2,I*exp(b*x+a))/b^3-6*I*polylog(3
,-I*exp(b*x+a))/b^4+6*I*polylog(3,I*exp(b*x+a))/b^4+x^3*sech(b*x+a)/b-6*sin
h(b*x+a)/b^4-3*x^2*sinh(b*x+a)/b^2
```

Rubi [A] time = 0.18, antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 8, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {5449, 3296, 2637, 5418, 4180, 2531, 2282, 6589}

$$\frac{6ix\text{PolyLog}\left(2,-ie^{a+bx}\right)}{b^3}-\frac{6ix\text{PolyLog}\left(2,ie^{a+bx}\right)}{b^3}-\frac{6i\text{PolyLog}\left(3,-ie^{a+bx}\right)}{b^4}+\frac{6i\text{PolyLog}\left(3,ie^{a+bx}\right)}{b^4}-\frac{6x^2\tan^{-1}\left(e\right.}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Sinh[a + b*x]*Tanh[a + b*x]^2,x]
```

```
[Out] (-6*x^2*ArcTan[E^(a + b*x)])/b^2 + (6*x*Cosh[a + b*x])/b^3 + (x^3*Cosh[a +
b*x])/b + ((6*I)*x*PolyLog[2, (-I)*E^(a + b*x)])/b^3 - ((6*I)*x*PolyLog[2,
I*E^(a + b*x)])/b^3 - ((6*I)*PolyLog[3, (-I)*E^(a + b*x)])/b^4 + ((6*I)*Pol
yLog[3, I*E^(a + b*x)])/b^4 + (x^3*Sech[a + b*x])/b - (6*Sinh[a + b*x])/b^4
 - (3*x^2*Sinh[a + b*x])/b^2
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5418

```
Int[(x_)^(m_.)*Sech[(a_.) + (b_.)*(x_)^(n_.)]^(p_.)*Tanh[(a_.) + (b_.)*(x_)
^(n_.)]^(q_.), x_Symbol] :> -Simp[(x^(m - n + 1)*Sech[a + b*x^n]^p)/(b*n*p)
, x] + Dist[(m - n + 1)/(b*n*p), Int[x^(m - n)*Sech[a + b*x^n]^p, x], x] /;
 FreeQ[{a, b, p}, x] && RationalQ[m] && IntegerQ[n] && GeQ[m - n, 0] && EqQ
[q, 1]
```

Rule 5449

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) + (b_.)*(x_)]^(n_.)*Tanh[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> Int[(c + d*x)^m*Sinh[a + b*x]^n*Tanh[a + b*
x]^(p - 2), x] - Int[(c + d*x)^m*Sinh[a + b*x]^(n - 2)*Tanh[a + b*x]^p, x]
/; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int x^3\sinh(a+bx)\tanh^2(a+bx)\,dx = \int x^3\sinh(a+bx)\,dx - \int x^3\text{sech}(a+bx)\tanh(a+bx)\,dx$$
$$= \frac{x^3\cosh(a+bx)}{b} + \frac{x^3\text{sech}(a+bx)}{b} - \frac{3\int x^2\cosh(a+bx)\,dx}{b} - \frac{3\int x^2\text{sec}}{}$$
$$= -\frac{6x^2\tan^{-1}\left(e^{a+bx}\right)}{b^2} + \frac{x^3\cosh(a+bx)}{b} + \frac{x^3\text{sech}(a+bx)}{b} - \frac{3x^2\sinh(a+}{b^2}$$
$$= -\frac{6x^2\tan^{-1}\left(e^{a+bx}\right)}{b^2} + \frac{6x\cosh(a+bx)}{b^3} + \frac{x^3\cosh(a+bx)}{b} + \frac{6ix\text{Li}_2\left(-ie^{a+}\right.}{b^3}$$
$$= -\frac{6x^2\tan^{-1}\left(e^{a+bx}\right)}{b^2} + \frac{6x\cosh(a+bx)}{b^3} + \frac{x^3\cosh(a+bx)}{b} + \frac{6ix\text{Li}_2\left(-ie^{a+}\right.}{b^3}$$
$$= -\frac{6x^2\tan^{-1}\left(e^{a+bx}\right)}{b^2} + \frac{6x\cosh(a+bx)}{b^3} + \frac{x^3\cosh(a+bx)}{b} + \frac{6ix\text{Li}_2\left(-ie^{a+}\right.}{b^3}$$

Mathematica [A] time = 3.35, size = 196, normalized size = 1.21

$$\frac{b^3x^3\text{sech}(a+bx) - 3i\left(b^2x^2\log\left(1-ie^{a+bx}\right) - b^2x^2\log\left(1+ie^{a+bx}\right) - 2bx\text{Li}_2\left(-ie^{a+bx}\right) + 2bx\text{Li}_2\left(ie^{a+bx}\right) + 2\text{Li}_3\left(\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Sinh[a + b*x]*Tanh[a + b*x]^2,x]
```

```
[Out] ((-3*I)*(b^2*x^2*Log[1 - I*E^(a + b*x)] - b^2*x^2*Log[1 + I*E^(a + b*x)] -
2*b*x*PolyLog[2, (-I)*E^(a + b*x)] + 2*b*x*PolyLog[2, I*E^(a + b*x)] + 2*Po
lyLog[3, (-I)*E^(a + b*x)] - 2*PolyLog[3, I*E^(a + b*x)]) + b^3*x^3*Sech[a
+ b*x] + Cosh[b*x]*(b*x*(6 + b^2*x^2)*Cosh[a] - 3*(2 + b^2*x^2)*Sinh[a]) +
(-3*(2 + b^2*x^2)*Cosh[a] + b*x*(6 + b^2*x^2)*Sinh[a])*Sinh[b*x])/b^4
```

fricas [C] time = 0.51, size = 1213, normalized size = 7.49

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*sech(b*x+a)^2*sinh(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/2*(b^3*x^3 + (b^3*x^3 - 3*b^2*x^2 + 6*b*x - 6)*cosh(b*x + a)^4 + 4*(b^3*x
^3 - 3*b^2*x^2 + 6*b*x - 6)*cosh(b*x + a)*sinh(b*x + a)^3 + (b^3*x^3 - 3*b^
2*x^2 + 6*b*x - 6)*sinh(b*x + a)^4 + 3*b^2*x^2 + 6*(b^3*x^3 + 2*b*x)*cosh(b
*x + a)^2 + 6*(b^3*x^3 + (b^3*x^3 - 3*b^2*x^2 + 6*b*x - 6)*cosh(b*x + a)^2
+ 2*b*x)*sinh(b*x + a)^2 + 6*b*x + (-12*I*b*x*cosh(b*x + a)^3 - 36*I*b*x*co
```

```
sh(b*x + a)*sinh(b*x + a)^2 - 12*I*b*x*sinh(b*x + a)^3 - 12*I*b*x*cosh(b*x
+ a) + (-36*I*b*x*cosh(b*x + a)^2 - 12*I*b*x)*sinh(b*x + a))*dilog(I*cosh(b
*x + a) + I*sinh(b*x + a)) + (12*I*b*x*cosh(b*x + a)^3 + 36*I*b*x*cosh(b*x
+ a)*sinh(b*x + a)^2 + 12*I*b*x*sinh(b*x + a)^3 + 12*I*b*x*cosh(b*x + a) +
(36*I*b*x*cosh(b*x + a)^2 + 12*I*b*x)*sinh(b*x + a))*dilog(-I*cosh(b*x + a)
 - I*sinh(b*x + a)) + (-6*I*a^2*cosh(b*x + a)^3 - 18*I*a^2*cosh(b*x + a)*si
nh(b*x + a)^2 - 6*I*a^2*sinh(b*x + a)^3 - 6*I*a^2*cosh(b*x + a) + (-18*I*a^
2*cosh(b*x + a)^2 - 6*I*a^2)*sinh(b*x + a))*log(cosh(b*x + a) + sinh(b*x +
a) + I) + (6*I*a^2*cosh(b*x + a)^3 + 18*I*a^2*cosh(b*x + a)*sinh(b*x + a)^2
 + 6*I*a^2*sinh(b*x + a)^3 + 6*I*a^2*cosh(b*x + a) + (18*I*a^2*cosh(b*x + a
)^2 + 6*I*a^2)*sinh(b*x + a))*log(cosh(b*x + a) + sinh(b*x + a) - I) + ((6*
I*b^2*x^2 - 6*I*a^2)*cosh(b*x + a)^3 + (18*I*b^2*x^2 - 18*I*a^2)*cosh(b*x +
 a)*sinh(b*x + a)^2 + (6*I*b^2*x^2 - 6*I*a^2)*sinh(b*x + a)^3 + (6*I*b^2*x^
2 - 6*I*a^2)*cosh(b*x + a) + (6*I*b^2*x^2 + (18*I*b^2*x^2 - 18*I*a^2)*cosh(
b*x + a)^2 - 6*I*a^2)*sinh(b*x + a))*log(I*cosh(b*x + a) + I*sinh(b*x + a)
+ 1) + ((-6*I*b^2*x^2 + 6*I*a^2)*cosh(b*x + a)^3 + (-18*I*b^2*x^2 + 18*I*a^
2)*cosh(b*x + a)*sinh(b*x + a)^2 + (-6*I*b^2*x^2 + 6*I*a^2)*sinh(b*x + a)^3
 + (-6*I*b^2*x^2 + 6*I*a^2)*cosh(b*x + a) + (-6*I*b^2*x^2 + (-18*I*b^2*x^2
+ 18*I*a^2)*cosh(b*x + a)^2 + 6*I*a^2)*sinh(b*x + a))*log(-I*cosh(b*x + a)
- I*sinh(b*x + a) + 1) + (12*I*cosh(b*x + a)^3 + 36*I*cosh(b*x + a)*sinh(b*
x + a)^2 + 12*I*sinh(b*x + a)^3 + (36*I*cosh(b*x + a)^2 + 12*I)*sinh(b*x +
a) + 12*I*cosh(b*x + a))*polylog(3, I*cosh(b*x + a) + I*sinh(b*x + a)) + (-
12*I*cosh(b*x + a)^3 - 36*I*cosh(b*x + a)*sinh(b*x + a)^2 - 12*I*sinh(b*x +
 a)^3 + (-36*I*cosh(b*x + a)^2 - 12*I)*sinh(b*x + a) - 12*I*cosh(b*x + a))*
polylog(3, -I*cosh(b*x + a) - I*sinh(b*x + a)) + 4*((b^3*x^3 - 3*b^2*x^2 +
6*b*x - 6)*cosh(b*x + a)^3 + 3*(b^3*x^3 + 2*b*x)*cosh(b*x + a))*sinh(b*x +
a) + 6)/(b^4*cosh(b*x + a)^3 + 3*b^4*cosh(b*x + a)*sinh(b*x + a)^2 + b^4*si
nh(b*x + a)^3 + b^4*cosh(b*x + a) + (3*b^4*cosh(b*x + a)^2 + b^4)*sinh(b*x
+ a))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3 \operatorname{sech}(bx+a)^2 \sinh(bx+a)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*sech(b*x+a)^2*sinh(b*x+a)^3,x, algorithm="giac")
```

```
[Out] integrate(x^3*sech(b*x + a)^2*sinh(b*x + a)^3, x)
```

maple [F] time = 0.75, size = 0, normalized size = 0.00

$$\int x^3 \operatorname{sech}(bx+a)^2 \left(\sinh^3(bx+a)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3*sech(b*x+a)^2*sinh(b*x+a)^3,x)

[Out] int(x^3*sech(b*x+a)^2*sinh(b*x+a)^3,x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\left(b^3x^3e^{(4\,a)} - 3\,b^2x^2e^{(4\,a)} + 6\,bxe^{(4\,a)} - 6\,e^{(4\,a)}\right)e^{(3\,bx)} + 6\left(b^3x^3e^{(2\,a)} + 2\,bxe^{(2\,a)}\right)e^{(bx)} + \left(b^3x^3 + 3\,b^2x^2 + 6\,bx + 6\right)e^{(-}}{2\left(b^4e^{(2\,bx+3\,a)} + b^4e^a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*sech(b*x+a)^2*sinh(b*x+a)^3,x, algorithm="maxima")

[Out] 1/2*((b^3*x^3*e^(4*a) - 3*b^2*x^2*e^(4*a) + 6*b*x*e^(4*a) - 6*e^(4*a))*e^(3*b*x) + 6*(b^3*x^3*e^(2*a) + 2*b*x*e^(2*a))*e^(b*x) + (b^3*x^3 + 3*b^2*x^2 + 6*b*x + 6)*e^(-b*x))/(b^4*e^(2*b*x + 3*a) + b^4*e^a) - 6*integrate(x^2*e^(b*x + a)/(b*e^(2*b*x + 2*a) + b), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3\,\mathrm{sinh}\left(a+b\,x\right)^3}{\mathrm{cosh}\left(a+b\,x\right)^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^3*sinh(a + b*x)^3)/cosh(a + b*x)^2,x)

[Out] int((x^3*sinh(a + b*x)^3)/cosh(a + b*x)^2, x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**3*sech(b*x+a)**2*sinh(b*x+a)**3,x)

[Out] Timed out

3.385 $\int x^2 \sinh(a+bx)\tanh^2(a+bx)\,dx$

Optimal. Leaf size=104

$$\frac{2i\text{Li}_2\left(-ie^{a+bx}\right)}{b^3}-\frac{2i\text{Li}_2\left(ie^{a+bx}\right)}{b^3}+\frac{2\cosh(a+bx)}{b^3}-\frac{4x\tan^{-1}\left(e^{a+bx}\right)}{b^2}-\frac{2x\sinh(a+bx)}{b^2}+\frac{x^2\cosh(a+bx)}{b}+\frac{x^2\text{sech}(a+}{b}$$

```
[Out] -4*x*arctan(exp(b*x+a))/b^2+2*cosh(b*x+a)/b^3+x^2*cosh(b*x+a)/b+2*I*polylog
(2,-I*exp(b*x+a))/b^3-2*I*polylog(2,I*exp(b*x+a))/b^3+x^2*sech(b*x+a)/b-2*x
*sinh(b*x+a)/b^2
```

Rubi [A] time = 0.12, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {5449, 3296, 2638, 5418, 4180, 2279, 2391}

$$\frac{2i\text{PolyLog}\left(2,-ie^{a+bx}\right)}{b^3}-\frac{2i\text{PolyLog}\left(2,ie^{a+bx}\right)}{b^3}-\frac{4x\tan^{-1}\left(e^{a+bx}\right)}{b^2}-\frac{2x\sinh(a+bx)}{b^2}+\frac{2\cosh(a+bx)}{b^3}+\frac{x^2\cosh(a+}{b}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Sinh[a + b*x]*Tanh[a + b*x]^2,x]
```

```
[Out] (-4*x*ArcTan[E^(a + b*x)])/b^2 + (2*Cosh[a + b*x])/b^3 + (x^2*Cosh[a + b*x]
)/b + ((2*I)*PolyLog[2, (-I)*E^(a + b*x)])/b^3 - ((2*I)*PolyLog[2, I*E^(a +
 b*x)])/b^3 + (x^2*Sech[a + b*x])/b - (2*x*Sinh[a + b*x])/b^2
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
```

```
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5418

```
Int[(x_)^(m_.)*Sech[(a_.) + (b_.)*(x_)^(n_.)]^(p_.)*Tanh[(a_.) + (b_.)*(x_)
^(n_.)]^(q_.), x_Symbol] :> -Simp[(x^(m - n + 1)*Sech[a + b*x^n]^p)/(b*n*p)
, x] + Dist[(m - n + 1)/(b*n*p), Int[x^(m - n)*Sech[a + b*x^n]^p, x], x] /;
 FreeQ[{a, b, p}, x] && RationalQ[m] && IntegerQ[n] && GeQ[m - n, 0] && EqQ
[q, 1]
```

Rule 5449

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) + (b_.)*(x_)]^(n_.)*Tanh[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> Int[(c + d*x)^m*Sinh[a + b*x]^n*Tanh[a + b*
x]^(p - 2), x] - Int[(c + d*x)^m*Sinh[a + b*x]^(n - 2)*Tanh[a + b*x]^p, x]
/; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rubi steps

$$\int x^2 \sinh(a+bx)\tanh^2(a+bx)\,dx = \int x^2\sinh(a+bx)\,dx - \int x^2\text{sech}(a+bx)\tanh(a+bx)\,dx$$

$$= \frac{x^2\cosh(a+bx)}{b} + \frac{x^2\text{sech}(a+bx)}{b} - \frac{2\int x\cosh(a+bx)\,dx}{b} - \frac{2\int x\text{sech}(}{b}$$

$$= -\frac{4x\tan^{-1}\left(e^{a+bx}\right)}{b^2} + \frac{x^2\cosh(a+bx)}{b} + \frac{x^2\text{sech}(a+bx)}{b} - \frac{2x\sinh(a+bx}{b^2}$$

$$= -\frac{4x\tan^{-1}\left(e^{a+bx}\right)}{b^2} + \frac{2\cosh(a+bx)}{b^3} + \frac{x^2\cosh(a+bx)}{b} + \frac{x^2\text{sech}(a+bx)}{b}$$

$$= -\frac{4x\tan^{-1}\left(e^{a+bx}\right)}{b^2} + \frac{2\cosh(a+bx)}{b^3} + \frac{x^2\cosh(a+bx)}{b} + \frac{2i\text{Li}_2\left(-ie^{a+bx}\right)}{b^3}$$

Mathematica [A] time = 0.51, size = 172, normalized size = 1.65

$$b^2x^2\operatorname{sech}(a+bx)+\cosh(bx)\left(\cosh(a)\left(b^2x^2+2\right)-2bx\sinh(a)\right)+\sinh(bx)\left(\sinh(a)\left(b^2x^2+2\right)-2bx\cosh(a)\right)-$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Sinh[a + b*x]*Tanh[a + b*x]^2,x]
```

```
[Out] (((-2*I)*a + Pi - (2*I)*b*x)*(Log[1 - I*E^(a + b*x)] - Log[1 + I*E^(a + b*x
)]) - ((-2*I)*a + Pi)*Log[Cot[((2*I)*a + Pi + (2*I)*b*x)/4]] + (2*I)*(PolyL
og[2, (-I)*E^(a + b*x)] - PolyLog[2, I*E^(a + b*x)]) + b^2*x^2*Sech[a + b*x
] + Cosh[b*x]*((2 + b^2*x^2)*Cosh[a] - 2*b*x*Sinh[a]) + (-2*b*x*Cosh[a] + (
2 + b^2*x^2)*Sinh[a])*Sinh[b*x])/b^3
```

fricas [B] time = 0.73, size = 869, normalized size = 8.36

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sech(b*x+a)^2*sinh(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/2*((b^2*x^2 - 2*b*x + 2)*cosh(b*x + a)^4 + 4*(b^2*x^2 - 2*b*x + 2)*cosh(b
*x + a)*sinh(b*x + a)^3 + (b^2*x^2 - 2*b*x + 2)*sinh(b*x + a)^4 + b^2*x^2 +
 2*(3*b^2*x^2 + 2)*cosh(b*x + a)^2 + 2*(3*b^2*x^2 + 3*(b^2*x^2 - 2*b*x + 2)
*cosh(b*x + a)^2 + 2)*sinh(b*x + a)^2 + 2*b*x + (-4*I*cosh(b*x + a)^3 - 12*
I*cosh(b*x + a)*sinh(b*x + a)^2 - 4*I*sinh(b*x + a)^3 + (-12*I*cosh(b*x + a
)^2 - 4*I)*sinh(b*x + a) - 4*I*cosh(b*x + a))*dilog(I*cosh(b*x + a) + I*sin
h(b*x + a)) + (4*I*cosh(b*x + a)^3 + 12*I*cosh(b*x + a)*sinh(b*x + a)^2 + 4
*I*sinh(b*x + a)^3 + (12*I*cosh(b*x + a)^2 + 4*I)*sinh(b*x + a) + 4*I*cosh(
b*x + a))*dilog(-I*cosh(b*x + a) - I*sinh(b*x + a)) + (4*I*a*cosh(b*x + a)^
3 + 12*I*a*cosh(b*x + a)*sinh(b*x + a)^2 + 4*I*a*sinh(b*x + a)^3 + 4*I*a*co
sh(b*x + a) + (12*I*a*cosh(b*x + a)^2 + 4*I*a)*sinh(b*x + a))*log(cosh(b*x
+ a) + sinh(b*x + a) + I) + (-4*I*a*cosh(b*x + a)^3 - 12*I*a*cosh(b*x + a)*
sinh(b*x + a)^2 - 4*I*a*sinh(b*x + a)^3 - 4*I*a*cosh(b*x + a) + (-12*I*a*co
sh(b*x + a)^2 - 4*I*a)*sinh(b*x + a))*log(cosh(b*x + a) + sinh(b*x + a) - I
) + ((4*I*b*x + 4*I*a)*cosh(b*x + a)^3 + (12*I*b*x + 12*I*a)*cosh(b*x + a)*
sinh(b*x + a)^2 + (4*I*b*x + 4*I*a)*sinh(b*x + a)^3 + (4*I*b*x + 4*I*a)*cos
h(b*x + a) + ((12*I*b*x + 12*I*a)*cosh(b*x + a)^2 + 4*I*b*x + 4*I*a)*sinh(b
*x + a))*log(I*cosh(b*x + a) + I*sinh(b*x + a) + 1) + ((-4*I*b*x - 4*I*a)*c
osh(b*x + a)^3 + (-12*I*b*x - 12*I*a)*cosh(b*x + a)*sinh(b*x + a)^2 + (-4*I
*b*x - 4*I*a)*sinh(b*x + a)^3 + (-4*I*b*x - 4*I*a)*cosh(b*x + a) + ((-12*I*
b*x - 12*I*a)*cosh(b*x + a)^2 - 4*I*b*x - 4*I*a)*sinh(b*x + a))*log(-I*cosh
(b*x + a) - I*sinh(b*x + a) + 1) + 4*((b^2*x^2 - 2*b*x + 2)*cosh(b*x + a)^3
 + (3*b^2*x^2 + 2)*cosh(b*x + a))*sinh(b*x + a) + 2)/(b^3*cosh(b*x + a)^3 +
```

```
3*b^3*cosh(b*x + a)*sinh(b*x + a)^2 + b^3*sinh(b*x + a)^3 + b^3*cosh(b*x +
a) + (3*b^3*cosh(b*x + a)^2 + b^3)*sinh(b*x + a))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \operatorname{sech}(bx+a)^2 \sinh(bx+a)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sech(b*x+a)^2*sinh(b*x+a)^3,x, algorithm="giac")
```

```
[Out] integrate(x^2*sech(b*x + a)^2*sinh(b*x + a)^3, x)
```

maple [B] time = 0.48, size = 205, normalized size = 1.97

$$\frac{\left(x^2b^2-2bx+2\right)e^{bx+a}}{2b^3}+\frac{\left(x^2b^2+2bx+2\right)e^{-bx-a}}{2b^3}+\frac{2x^2e^{bx+a}}{b\left(1+e^{2bx+2a}\right)}+\frac{2i\ln\left(1+ie^{bx+a}\right)x}{b^2}+\frac{2i\ln\left(1+ie^{bx+a}\right)a}{b^3}-\frac{2i\ln}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*sech(b*x+a)^2*sinh(b*x+a)^3,x)
```

```
[Out] 1/2*(b^2*x^2-2*b*x+2)/b^3*exp(b*x+a)+1/2*(b^2*x^2+2*b*x+2)/b^3*exp(-b*x-a)+
2*x^2*exp(b*x+a)/b/(1+exp(2*b*x+2*a))+2*I/b^2*ln(1+I*exp(b*x+a))*x+2*I/b^3*
ln(1+I*exp(b*x+a))*a-2*I/b^2*ln(1-I*exp(b*x+a))*x-2*I/b^3*ln(1-I*exp(b*x+a)
)*a+2*I/b^3*dilog(1+I*exp(b*x+a))-2*I/b^3*dilog(1-I*exp(b*x+a))+4/b^3*a*arc
tan(exp(b*x+a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\left(b^2x^2e^{(4a)}-2bxe^{(4a)}+2e^{(4a)}\right)e^{(3bx)}+2\left(3b^2x^2e^{(2a)}+2e^{(2a)}\right)e^{(bx)}+\left(b^2x^2+2bx+2\right)e^{(-bx)}}{2\left(b^3e^{(2bx+3a)}+b^3e^{a}\right)}-4\int\frac{xe^{(bx+a)}}{be^{(2bx+2a)}+b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sech(b*x+a)^2*sinh(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/2*((b^2*x^2*e^(4*a) - 2*b*x*e^(4*a) + 2*e^(4*a))*e^(3*b*x) + 2*(3*b^2*x^2
*e^(2*a) + 2*e^(2*a))*e^(b*x) + (b^2*x^2 + 2*b*x + 2)*e^(-b*x))/(b^3*e^(2*b
*x + 3*a) + b^3*e^a) - 4*integrate(x*e^(b*x + a)/(b*e^(2*b*x + 2*a) + b), x
)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2\sinh(a+b\,x)^3}{\cosh(a+b\,x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*sinh(a + b*x)^3)/cosh(a + b*x)^2,x)
```

```
[Out] int((x^2*sinh(a + b*x)^3)/cosh(a + b*x)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \sinh^3 (a + bx) \operatorname{sech}^2 (a + bx) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*sech(b*x+a)**2*sinh(b*x+a)**3,x)
```

```
[Out] Integral(x**2*sinh(a + b*x)**3*sech(a + b*x)**2, x)
```

3.386 $\int x \sinh(a+bx)\tanh^2(a+bx)\,dx$

Optimal. Leaf size=46

$$-\frac{\sinh(a+bx)}{b^2}-\frac{\tan^{-1}(\sinh(a+bx))}{b^2}+\frac{x\cosh(a+bx)}{b}+\frac{x\text{sech}(a+bx)}{b}$$

```
[Out] -arctan(sinh(b*x+a))/b^2+x*cosh(b*x+a)/b+x*sech(b*x+a)/b-sinh(b*x+a)/b^2
```

Rubi [A] time = 0.05, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {5449, 3296, 2637, 5418, 3770}

$$-\frac{\sinh(a+bx)}{b^2}-\frac{\tan^{-1}(\sinh(a+bx))}{b^2}+\frac{x\cosh(a+bx)}{b}+\frac{x\text{sech}(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sinh[a + b*x]*Tanh[a + b*x]^2,x]
```

```
[Out] -(ArcTan[Sinh[a + b*x]]/b^2) + (x*Cosh[a + b*x])/b + (x*Sech[a + b*x])/b -
Sinh[a + b*x]/b^2
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 5418

```
Int[(x_)^(m_.)*Sech[(a_.) + (b_.)*(x_)^(n_.)]^(p_.)*Tanh[(a_.) + (b_.)*(x_)
^(n_.)]^(q_.), x_Symbol] :> -Simp[(x^(m - n + 1)*Sech[a + b*x^n]^p)/(b*n*p)
, x] + Dist[(m - n + 1)/(b*n*p), Int[x^(m - n)*Sech[a + b*x^n]^p, x], x] /;
 FreeQ[{a, b, p}, x] && RationalQ[m] && IntegerQ[n] && GeQ[m - n, 0] && EqQ
```

```
[q, 1]
```

Rule 5449

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) + (b_.)*(x_)]^(n_.)*Tanh[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> Int[(c + d*x)^m*Sinh[a + b*x]^n*Tanh[a + b*
x]^(p - 2), x] - Int[(c + d*x)^m*Sinh[a + b*x]^(n - 2)*Tanh[a + b*x]^p, x]
/; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int x\sinh(a+bx)\tanh^2(a+bx)\,dx &= \int x\sinh(a+bx)\,dx-\int x\text{sech}(a+bx)\tanh(a+bx)\,dx\\ &= \frac{x\cosh(a+bx)}{b}+\frac{x\text{sech}(a+bx)}{b}-\frac{\int\cosh(a+bx)\,dx}{b}-\frac{\int\text{sech}(a+bx)\,dx}{b}\\ &= -\frac{\tan^{-1}(\sinh(a+bx))}{b^2}+\frac{x\cosh(a+bx)}{b}+\frac{x\text{sech}(a+bx)}{b}-\frac{\sinh(a+bx)}{b^2}\end{aligned}$$

Mathematica [A] time = 0.12, size = 50, normalized size = 1.09

$$-\frac{\sinh(a+bx)}{b^2}-\frac{2\tan^{-1}\left(\tanh\left(\frac{1}{2}(a+bx)\right)\right)}{b^2}+\frac{x\cosh(a+bx)}{b}+\frac{x\text{sech}(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sinh[a + b*x]*Tanh[a + b*x]^2,x]
```

```
[Out] (-2*ArcTan[Tanh[(a + b*x)/2]])/b^2 + (x*Cosh[a + b*x])/b + (x*Sech[a + b*x]
)/b - Sinh[a + b*x]/b^2
```

fricas [B] time = 0.53, size = 283, normalized size = 6.15

$$\frac{(bx-1)\cosh(bx+a)^4+4(bx-1)\cosh(bx+a)\sinh(bx+a)^3+(bx-1)\sinh(bx+a)^4+6bx\cosh(bx+a)^2+}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sech(b*x+a)^2*sinh(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/2*((b*x - 1)*cosh(b*x + a)^4 + 4*(b*x - 1)*cosh(b*x + a)*sinh(b*x + a)^3
+ (b*x - 1)*sinh(b*x + a)^4 + 6*b*x*cosh(b*x + a)^2 + 6*((b*x - 1)*cosh(b*x
```

```
 + a)^2 + b*x)*sinh(b*x + a)^2 + b*x - 4*(cosh(b*x + a)^3 + 3*cosh(b*x + a)
*sinh(b*x + a)^2 + sinh(b*x + a)^3 + (3*cosh(b*x + a)^2 + 1)*sinh(b*x + a)
+ cosh(b*x + a))*arctan(cosh(b*x + a) + sinh(b*x + a)) + 4*((b*x - 1)*cosh(
b*x + a)^3 + 3*b*x*cosh(b*x + a))*sinh(b*x + a) + 1)/(b^2*cosh(b*x + a)^3 +
 3*b^2*cosh(b*x + a)*sinh(b*x + a)^2 + b^2*sinh(b*x + a)^3 + b^2*cosh(b*x +
 a) + (3*b^2*cosh(b*x + a)^2 + b^2)*sinh(b*x + a))
```

giac [B] time = 0.19, size = 102, normalized size = 2.22

$$\frac{bxe^{(4\,bx+4\,a)} + 6\,bxe^{(2\,bx+2\,a)} + bx - 4\,\arctan\left(e^{(bx+a)}\right)e^{(3\,bx+3\,a)} - 4\,\arctan\left(e^{(bx+a)}\right)e^{(bx+a)} - e^{(4\,bx+4\,a)} + 1}{2\left(b^2e^{(3\,bx+3\,a)} + b^2e^{(bx+a)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sech(b*x+a)^2*sinh(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/2*(b*x*e^(4*b*x + 4*a) + 6*b*x*e^(2*b*x + 2*a) + b*x - 4*arctan(e^(b*x +
a))*e^(3*b*x + 3*a) - 4*arctan(e^(b*x + a))*e^(b*x + a) - e^(4*b*x + 4*a) +
 1)/(b^2*e^(3*b*x + 3*a) + b^2*e^(b*x + a))
```

maple [C] time = 0.40, size = 94, normalized size = 2.04

$$\frac{(bx-1)\,e^{bx+a}}{2b^2} + \frac{(bx+1)\,e^{-bx-a}}{2b^2} + \frac{2x\,e^{bx+a}}{b\left(1+e^{2bx+2a}\right)} + \frac{i\ln\left(e^{bx+a}-i\right)}{b^2} - \frac{i\ln\left(e^{bx+a}+i\right)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sech(b*x+a)^2*sinh(b*x+a)^3,x)
```

```
[Out] 1/2*(b*x-1)/b^2*exp(b*x+a)+1/2*(b*x+1)/b^2*exp(-b*x-a)+2*x*exp(b*x+a)/b/(1+
exp(2*b*x+2*a))+I/b^2*ln(exp(b*x+a)-I)-I/b^2*ln(exp(b*x+a)+I)
```

maxima [A] time = 0.80, size = 81, normalized size = 1.76

$$\frac{6\,bxe^{(bx+2\,a)} + \left(bxe^{(4\,a)} - e^{(4\,a)}\right)e^{(3\,bx)} + (bx+1)e^{(-bx)}}{2\left(b^2e^{(2\,bx+3\,a)} + b^2e^a\right)} - \frac{2\,\arctan\left(e^{(bx+a)}\right)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sech(b*x+a)^2*sinh(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/2*(6*b*x*e^(b*x + 2*a) + (b*x*e^(4*a) - e^(4*a))*e^(3*b*x) + (b*x + 1)*e^
(-b*x))/(b^2*e^(2*b*x + 3*a) + b^2*e^a) - 2*arctan(e^(b*x + a))/b^2
```

mupad [B] time = 0.10, size = 90, normalized size = 1.96

$$e^{-a-b\,x}\,\left(\frac{x}{2\,b}+\frac{1}{2\,b^2}\right)-\frac{2\,\mathrm{atan}\left(\frac{e^{b\,x}\,e^a\,\sqrt{b^4}}{b^2}\right)}{\sqrt{b^4}}+e^{a+b\,x}\,\left(\frac{x}{2\,b}-\frac{1}{2\,b^2}\right)+\frac{2\,x\,e^{a+b\,x}}{b\,\left(e^{2\,a+2\,b\,x}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*sinh(a + b*x)^3)/cosh(a + b*x)^2,x)
```

```
[Out] exp(- a - b*x)*(x/(2*b) + 1/(2*b^2)) - (2*atan((exp(b*x)*exp(a)*(b^4)^(1/2)
)/b^2))/(b^4)^(1/2) + exp(a + b*x)*(x/(2*b) - 1/(2*b^2)) + (2*x*exp(a + b*x
))/(b*(exp(2*a + 2*b*x) + 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\sinh^3(a+bx)\,\mathrm{sech}^2(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sech(b*x+a)**2*sinh(b*x+a)**3,x)
```

```
[Out] Integral(x*sinh(a + b*x)**3*sech(a + b*x)**2, x)
```

3.387 $\int \sinh(a+bx)\tanh^2(a+bx)\,dx$

Optimal. Leaf size=21

$$\frac{\cosh(a+bx)}{b}+\frac{\operatorname{sech}(a+bx)}{b}$$

[Out] cosh(b*x+a)/b+sech(b*x+a)/b

Rubi [A] time = 0.02, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2590, 14}

$$\frac{\cosh(a+bx)}{b}+\frac{\operatorname{sech}(a+bx)}{b}$$

Antiderivative was successfully verified.

[In] Int[Sinh[a + b*x]*Tanh[a + b*x]^2,x]

[Out] Cosh[a + b*x]/b + Sech[a + b*x]/b

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2590

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> -Dist[f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rubi steps

$$\begin{aligned}
\int \sinh(a+bx)\tanh^2(a+bx)\,dx &= -\frac{\operatorname{Subst}\left(\int \frac{1-x^2}{x^2}\,dx,x,\cosh(a+bx)\right)}{b}\\
&= -\frac{\operatorname{Subst}\left(\int \left(-1+\frac{1}{x^2}\right)\,dx,x,\cosh(a+bx)\right)}{b}\\
&= \frac{\cosh(a+bx)}{b}+\frac{\operatorname{sech}(a+bx)}{b}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 21, normalized size = 1.00

$$\frac{\cosh(a+bx)}{b}+\frac{\operatorname{sech}(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[a + b*x]*Tanh[a + b*x]^2,x]
[Out] Cosh[a + b*x]/b + Sech[a + b*x]/b
```

fricas [A] time = 0.70, size = 31, normalized size = 1.48

$$\frac{\cosh\left(bx+a\right)^2+\sinh\left(bx+a\right)^2+3}{2\,b\cosh\left(bx+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)^2*sinh(b*x+a)^3,x, algorithm="fricas")
[Out] 1/2*(cosh(b*x + a)^2 + sinh(b*x + a)^2 + 3)/(b*cosh(b*x + a))
```

giac [A] time = 0.15, size = 41, normalized size = 1.95

$$\frac{\frac{4}{e^{(bx+a)}+e^{(-bx-a)}}+e^{(bx+a)}+e^{(-bx-a)}}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)^2*sinh(b*x+a)^3,x, algorithm="giac")
[Out] 1/2*(4/(e^(b*x + a) + e^(-b*x - a)) + e^(b*x + a) + e^(-b*x - a))/b
```

maple [A] time = 0.11, size = 33, normalized size = 1.57

$$\frac{\frac{\sinh^2(bx+a)}{\cosh(bx+a)}+\frac{2}{\cosh(bx+a)}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(b*x+a)^2*sinh(b*x+a)^3,x)
[Out] 1/b*(sinh(b*x+a)^2/cosh(b*x+a)+2/cosh(b*x+a))
```

maxima [B] time = 0.42, size = 54, normalized size = 2.57

$$\frac{e^{(-bx-a)}}{2\,b}+\frac{5\,e^{(-2\,bx-2\,a)}+1}{2\,b\left(e^{(-bx-a)}+e^{(-3\,bx-3\,a)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)^2*sinh(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/2*e^(-b*x - a)/b + 1/2*(5*e^(-2*b*x - 2*a) + 1)/(b*(e^(-b*x - a) + e^(-3*
b*x - 3*a)))
```

mupad [B] time = 0.07, size = 22, normalized size = 1.05

$$\frac{\cosh\left(a+b\,x\right)^2+1}{b\cosh\left(a+b\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a + b*x)^3/cosh(a + b*x)^2,x)
```

```
[Out] (cosh(a + b*x)^2 + 1)/(b*cosh(a + b*x))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sinh^{3}\left(a+bx\right)\operatorname{sech}^{2}\left(a+bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)**2*sinh(b*x+a)**3,x)
```

```
[Out] Integral(sinh(a + b*x)**3*sech(a + b*x)**2, x)
```

3.388 $\int \frac{\sinh(a+bx)\tanh^2(a+bx)}{x}\,dx$

Optimal. Leaf size=36

$$-\text{Int}\left(\frac{\tanh(a+bx)\text{sech}(a+bx)}{x},x\right)+\sinh(a)\text{Chi}(bx)+\cosh(a)\text{Shi}(bx)$$

```
[Out] -CannotIntegrate(sech(b*x+a)*tanh(b*x+a)/x,x)+cosh(a)*Shi(b*x)+Chi(b*x)*sin
h(a)
```

Rubi [A] time = 0.13, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sinh(a+bx)\tanh^2(a+bx)}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Sinh[a + b*x]*Tanh[a + b*x]^2)/x,x]
```

```
[Out] CoshIntegral[b*x]*Sinh[a] + Cosh[a]*SinhIntegral[b*x] - Defer[Int][(Sech[a
+ b*x]*Tanh[a + b*x])/x, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\sinh(a+bx)\tanh^2(a+bx)}{x}\,dx &= \int \frac{\sinh(a+bx)}{x}\,dx - \int \frac{\text{sech}(a+bx)\tanh(a+bx)}{x}\,dx\\ &= \cosh(a)\int \frac{\sinh(bx)}{x}\,dx + \sinh(a)\int \frac{\cosh(bx)}{x}\,dx - \int \frac{\text{sech}(a+bx)\tanh(a+bx)}{x}\,dx\\ &= \text{Chi}(bx)\sinh(a)+\cosh(a)\text{Shi}(bx) - \int \frac{\text{sech}(a+bx)\tanh(a+bx)}{x}\,dx\end{aligned}$$

Mathematica [A] time = 12.09, size = 0, normalized size = 0.00

$$\int \frac{\sinh(a+bx)\tanh^2(a+bx)}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Sinh[a + b*x]*Tanh[a + b*x]^2)/x,x]
```

```
[Out] Integrate[(Sinh[a + b*x]*Tanh[a + b*x]^2)/x, x]
```

fricas [A] time = 0.68, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\operatorname{sech}(bx+a)^2\sinh(bx+a)^3}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)^2*sinh(b*x+a)^3/x,x, algorithm="fricas")
```

```
[Out] integral(sech(b*x + a)^2*sinh(b*x + a)^3/x, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}(bx+a)^2\sinh(bx+a)^3}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)^2*sinh(b*x+a)^3/x,x, algorithm="giac")
```

```
[Out] integrate(sech(b*x + a)^2*sinh(b*x + a)^3/x, x)
```

maple [A] time = 0.67, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}(bx+a)^2\left(\sinh^3(bx+a)\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(b*x+a)^2*sinh(b*x+a)^3/x,x)
```

```
[Out] int(sech(b*x+a)^2*sinh(b*x+a)^3/x,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{2}\operatorname{Ei}(-bx)e^{(-a)}+\frac{1}{2}\operatorname{Ei}(bx)e^{a}+\frac{2e^{(bx+a)}}{bxe^{(2bx+2a)}+bx}+2\int \frac{e^{(bx+a)}}{bx^2e^{(2bx+2a)}+bx^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)^2*sinh(b*x+a)^3/x,x, algorithm="maxima")
```

```
[Out] -1/2*Ei(-b*x)*e^(-a) + 1/2*Ei(b*x)*e^a + 2*e^(b*x + a)/(b*x*e^(2*b*x + 2*a)
 + b*x) + 2*integrate(e^(b*x + a)/(b*x^2*e^(2*b*x + 2*a) + b*x^2), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\sinh(a+b\,x)^3}{x\cosh(a+b\,x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a + b*x)^3/(x*cosh(a + b*x)^2),x)
```

```
[Out] int(sinh(a + b*x)^3/(x*cosh(a + b*x)^2), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh^3(a+bx)\operatorname{sech}^2(a+bx)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)**2*sinh(b*x+a)**3/x,x)
```

```
[Out] Integral(sinh(a + b*x)**3*sech(a + b*x)**2/x, x)
```

3.389 $\int \frac{\sinh(a+bx)\tanh^2(a+bx)}{x^2}\,dx$

Optimal. Leaf size=49

$$-\text{Int}\left(\frac{\tanh(a+bx)\text{sech}(a+bx)}{x^2},x\right)+b\cosh(a)\text{Chi}(bx)+b\sinh(a)\text{Shi}(bx)-\frac{\sinh(a+bx)}{x}$$

```
[Out] -CannotIntegrate(sech(b*x+a)*tanh(b*x+a)/x^2,x)+b*Chi(b*x)*cosh(a)+b*Shi(b*
x)*sinh(a)-sinh(b*x+a)/x
```

Rubi [A] time = 0.13, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sinh(a+bx)\tanh^2(a+bx)}{x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Sinh[a + b*x]*Tanh[a + b*x]^2)/x^2,x]
```

```
[Out] b*Cosh[a]*CoshIntegral[b*x] - Sinh[a + b*x]/x + b*Sinh[a]*SinhIntegral[b*x]
 - Defer[Int][(Sech[a + b*x]*Tanh[a + b*x])/x^2, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\sinh(a+bx)\tanh^2(a+bx)}{x^2}\,dx &= \int \frac{\sinh(a+bx)}{x^2}\,dx-\int \frac{\text{sech}(a+bx)\tanh(a+bx)}{x^2}\,dx\\ &= -\frac{\sinh(a+bx)}{x}+b\int \frac{\cosh(a+bx)}{x}\,dx-\int \frac{\text{sech}(a+bx)\tanh(a+bx)}{x^2}\,dx\\ &= -\frac{\sinh(a+bx)}{x}+(b\cosh(a))\int \frac{\cosh(bx)}{x}\,dx+(b\sinh(a))\int \frac{\sinh(bx)}{x}\,dx-\\ &= b\cosh(a)\text{Chi}(bx)-\frac{\sinh(a+bx)}{x}+b\sinh(a)\text{Shi}(bx)-\int \frac{\text{sech}(a+bx)\tan}{x^2}\end{aligned}$$

Mathematica [A] time = 10.23, size = 0, normalized size = 0.00

$$\int \frac{\sinh(a+bx)\tanh^2(a+bx)}{x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Sinh[a + b*x]*Tanh[a + b*x]^2)/x^2,x]
```

```
[Out] Integrate[(Sinh[a + b*x]*Tanh[a + b*x]^2)/x^2, x]
```

fricas [A] time = 0.87, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\operatorname{sech}(bx+a)^2\sinh(bx+a)^3}{x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)^2*sinh(b*x+a)^3/x^2,x, algorithm="fricas")
```

```
[Out] integral(sech(b*x + a)^2*sinh(b*x + a)^3/x^2, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}(bx+a)^2\sinh(bx+a)^3}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)^2*sinh(b*x+a)^3/x^2,x, algorithm="giac")
```

```
[Out] integrate(sech(b*x + a)^2*sinh(b*x + a)^3/x^2, x)
```

maple [A] time = 0.61, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}(bx+a)^2\left(\sinh^3(bx+a)\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(b*x+a)^2*sinh(b*x+a)^3/x^2,x)
```

```
[Out] int(sech(b*x+a)^2*sinh(b*x+a)^3/x^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{2}\,be^{(-a)}\Gamma\left(-1,bx\right)+\frac{1}{2}\,be^{a}\Gamma\left(-1,-bx\right)+\frac{2\,e^{(bx+a)}}{bx^2e^{(2\,bx+2\,a)}+bx^2}+4\int\frac{e^{(bx+a)}}{bx^3e^{(2\,bx+2\,a)}+bx^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)^2*sinh(b*x+a)^3/x^2,x, algorithm="maxima")
```

```
[Out] 1/2*b*e^(-a)*gamma(-1, b*x) + 1/2*b*e^a*gamma(-1, -b*x) + 2*e^(b*x + a)/(b*
x^2*e^(2*b*x + 2*a) + b*x^2) + 4*integrate(e^(b*x + a)/(b*x^3*e^(2*b*x + 2*
a) + b*x^3), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sinh(a+b\,x)^3}{x^2\cosh(a+b\,x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a + b*x)^3/(x^2*cosh(a + b*x)^2),x)
```

```
[Out] int(sinh(a + b*x)^3/(x^2*cosh(a + b*x)^2), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh^3(a+bx)\,\mathrm{sech}^2(a+bx)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)**2*sinh(b*x+a)**3/x**2,x)
```

```
[Out] Integral(sinh(a + b*x)**3*sech(a + b*x)**2/x**2, x)
```

3.390 $\int x^m \tanh^3(a+bx)\,dx$

Optimal. Leaf size=15

$$\text{Int}\left(x^m \tanh^3(a+bx),x\right)$$

```
[Out] Unintegrable(x^m*tanh(b*x+a)^3,x)
```

Rubi [A] time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} =$ 0.000, Rules used = {}

$$\int x^m \tanh^3(a+bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x^m*Tanh[a + b*x]^3,x]
```

```
[Out] Defer[Int][x^m*Tanh[a + b*x]^3, x]
```

Rubi steps

$$\int x^m \tanh^3(a+bx)\,dx = \int x^m \tanh^3(a+bx)\,dx$$

Mathematica [A] time = 1.53, size = 0, normalized size = 0.00

$$\int x^m \tanh^3(a+bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^m*Tanh[a + b*x]^3,x]
```

```
[Out] Integrate[x^m*Tanh[a + b*x]^3, x]
```

fricas [A] time = 0.74, size = 0, normalized size = 0.00

$$\text{integral}\left(x^m \operatorname{sech}(bx+a)^3 \sinh(bx+a)^3, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*sech(b*x+a)^3*sinh(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] integral(x^m*sech(b*x + a)^3*sinh(b*x + a)^3, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int x^m \operatorname{sech}(bx+a)^3 \sinh(bx+a)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*sech(b*x+a)^3*sinh(b*x+a)^3,x, algorithm="giac")
```

```
[Out] integrate(x^m*sech(b*x + a)^3*sinh(b*x + a)^3, x)
```

maple [A] time = 0.48, size = 0, normalized size = 0.00

$$\int x^m \operatorname{sech}(bx+a)^3 \left(\sinh^3(bx+a)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*sech(b*x+a)^3*sinh(b*x+a)^3,x)
```

```
[Out] int(x^m*sech(b*x+a)^3*sinh(b*x+a)^3,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\frac{xe^{\left(6\,bx+m\log(x)+6\,a\right)}}{(m+1)e^{(6\,bx+6\,a)}+3\,(m+1)e^{(4\,bx+4\,a)}+3\,(m+1)e^{(2\,bx+2\,a)}+m+1}-\int\frac{\left(3\left(2\,bxe^{(6\,a)}+(m+1)e^{(6\,a)}\right)e^{(6}}{(m+1)e^{(8\,bx+8\,a)}+4\,(m+1)e^{(6\,bx+6\,a)}+6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*sech(b*x+a)^3*sinh(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] x*e^(6*b*x + m*log(x) + 6*a)/((m + 1)*e^(6*b*x + 6*a) + 3*(m + 1)*e^(4*b*x
+ 4*a) + 3*(m + 1)*e^(2*b*x + 2*a) + m + 1) - integrate((3*(2*b*x*e^(6*a) +
 (m + 1)*e^(6*a))*e^(6*b*x) - 2*(m + 1)*e^(2*b*x + 2*a) + m + 1)*x^m/((m +
1)*e^(8*b*x + 8*a) + 4*(m + 1)*e^(6*b*x + 6*a) + 6*(m + 1)*e^(4*b*x + 4*a)
+ 4*(m + 1)*e^(2*b*x + 2*a) + m + 1), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.07

$$\int \frac{x^m \sinh(a+b\,x)^3}{\cosh(a+b\,x)^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^m*sinh(a + b*x)^3)/cosh(a + b*x)^3,x)
```

```
[Out] int((x^m*sinh(a + b*x)^3)/cosh(a + b*x)^3, x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*sech(b*x+a)**3*sinh(b*x+a)**3,x)
```

```
[Out] Timed out
```

3.391 $\int x^3 \tanh^3(a+bx)\,dx$

Optimal. Leaf size=183

$$\frac{3\text{Li}_2\left(-e^{2(a+bx)}\right)}{2b^4}+\frac{3\text{Li}_4\left(-e^{2(a+bx)}\right)}{4b^4}-\frac{3x\text{Li}_3\left(-e^{2(a+bx)}\right)}{2b^3}+\frac{3x\log\left(e^{2(a+bx)}+1\right)}{b^3}+\frac{3x^2\text{Li}_2\left(-e^{2(a+bx)}\right)}{2b^2}-\frac{3x^2\tanh(a+bx}{2b^2}$$

[Out] -3/2*x^2/b^2+1/2*x^3/b-1/4*x^4+3*x*ln(1+exp(2*b*x+2*a))/b^3+x^3*ln(1+exp(2*b*x+2*a))/b+3/2*polylog(2,-exp(2*b*x+2*a))/b^4+3/2*x^2*polylog(2,-exp(2*b*x+2*a))/b^2-3/2*x*polylog(3,-exp(2*b*x+2*a))/b^3+3/4*polylog(4,-exp(2*b*x+2*a))/b^4-3/2*x^2*tanh(b*x+a)/b^2-1/2*x^3*tanh(b*x+a)^2/b

Rubi [A] time = 0.33, antiderivative size = 183, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.833, Rules used = {3720, 3718, 2190, 2279, 2391, 30, 2531, 6609, 2282, 6589}

$$\frac{3x^2\text{PolyLog}\left(2,-e^{2(a+bx)}\right)}{2b^2}-\frac{3x\text{PolyLog}\left(3,-e^{2(a+bx)}\right)}{2b^3}+\frac{3\text{PolyLog}\left(2,-e^{2(a+bx)}\right)}{2b^4}+\frac{3\text{PolyLog}\left(4,-e^{2(a+bx)}\right)}{4b^4}-\frac{3x^2 \text{t}}{}$$

Antiderivative was successfully verified.

[In] Int[x^3*Tanh[a + b*x]^3,x]

[Out] (-3*x^2)/(2*b^2) + x^3/(2*b) - x^4/4 + (3*x*Log[1 + E^(2*(a + b*x))])/b^3 + (x^3*Log[1 + E^(2*(a + b*x))])/b + (3*PolyLog[2, -E^(2*(a + b*x))])/(2*b^4) + (3*x^2*PolyLog[2, -E^(2*(a + b*x))])/(2*b^2) - (3*x*PolyLog[3, -E^(2*(a + b*x))])/(2*b^3) + (3*PolyLog[4, -E^(2*(a + b*x))])/(4*b^4) - (3*x^2*Tanh[a + b*x])/(2*b^2) - (x^3*Tanh[a + b*x]^2)/(2*b)

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 3718

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c
 + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /;
 FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 3720

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[(b*(c + d*x)^m*(b*Tan[e + f*x])^(n - 1))/(f*(n - 1)), x] + (-Di
st[(b*d*m)/(f*(n - 1)), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x],
 x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[
{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
```

```
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int x^3\tanh^3(a+bx)\,dx = -\frac{x^3\tanh^2(a+bx)}{2b}+\frac{3\int x^2\tanh^2(a+bx)\,dx}{2b}+\int x^3\tanh(a+bx)\,dx$$

$$= -\frac{x^4}{4}-\frac{3x^2\tanh(a+bx)}{2b^2}-\frac{x^3\tanh^2(a+bx)}{2b}+2\int\frac{e^{2(a+bx)}x^3}{1+e^{2(a+bx)}}\,dx+\frac{3\int x\tanh(a+b}{b^2}$$

$$= -\frac{3x^2}{2b^2}+\frac{x^3}{2b}-\frac{x^4}{4}+\frac{x^3\log\left(1+e^{2(a+bx)}\right)}{b}-\frac{3x^2\tanh(a+bx)}{2b^2}-\frac{x^3\tanh^2(a+bx)}{2b}+\frac{6\int}{}$$

$$= -\frac{3x^2}{2b^2}+\frac{x^3}{2b}-\frac{x^4}{4}+\frac{3x\log\left(1+e^{2(a+bx)}\right)}{b^3}+\frac{x^3\log\left(1+e^{2(a+bx)}\right)}{b}+\frac{3x^2\mathrm{Li}_2\left(-e^{2(a+bx)}\right)}{2b^2}-$$

$$= -\frac{3x^2}{2b^2}+\frac{x^3}{2b}-\frac{x^4}{4}+\frac{3x\log\left(1+e^{2(a+bx)}\right)}{b^3}+\frac{x^3\log\left(1+e^{2(a+bx)}\right)}{b}+\frac{3x^2\mathrm{Li}_2\left(-e^{2(a+bx)}\right)}{2b^2}-$$

$$= -\frac{3x^2}{2b^2}+\frac{x^3}{2b}-\frac{x^4}{4}+\frac{3x\log\left(1+e^{2(a+bx)}\right)}{b^3}+\frac{x^3\log\left(1+e^{2(a+bx)}\right)}{b}+\frac{3\mathrm{Li}_2\left(-e^{2(a+bx)}\right)}{2b^4}+\frac{3}{}$$

$$= -\frac{3x^2}{2b^2}+\frac{x^3}{2b}-\frac{x^4}{4}+\frac{3x\log\left(1+e^{2(a+bx)}\right)}{b^3}+\frac{x^3\log\left(1+e^{2(a+bx)}\right)}{b}+\frac{3\mathrm{Li}_2\left(-e^{2(a+bx)}\right)}{2b^4}+\frac{3}{}$$

Mathematica [A] time = 2.90, size = 230, normalized size = 1.26

$$\frac{1}{4}\left(-\frac{6x^2\mathrm{sech}(a)\sinh(bx)\mathrm{sech}(a+bx)}{b^2}+\frac{e^{2a}\left(2e^{-2a}b^4x^4+4\left(e^{-2a}+1\right)b^3x^3\log\left(e^{-2(a+bx)}+1\right)-3e^{-2a}\left(e^{2a}+1\right)\right(}{}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Tanh[a + b*x]^3,x]
```

```
[Out] ((E^(2*a)*((12*b^2*x^2)/E^(2*a) + (2*b^4*x^4)/E^(2*a) + 12*b*(1 + E^(-2*a))
*x*Log[1 + E^(-2*(a + b*x))] + 4*b^3*(1 + E^(-2*a))*x^3*Log[1 + E^(-2*(a +
```

```
b*x))] - 6*(1 + E^(-2*a))*PolyLog[2, -E^(-2*(a + b*x))] - (3*(1 + E^(2*a))*
(2*b^2*x^2*PolyLog[2, -E^(-2*(a + b*x))] + 2*b*x*PolyLog[3, -E^(-2*(a + b*x
))] + PolyLog[4, -E^(-2*(a + b*x))]))/E^(2*a)))/(b^4*(1 + E^(2*a))) + (2*x^
3*Sech[a + b*x]^2)/b - (6*x^2*Sech[a]*Sech[a + b*x]*Sinh[b*x])/b^2 + x^4*Ta
nh[a])/4
```

fricas [C] time = 0.63, size = 2207, normalized size = 12.06

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*sech(b*x+a)^3*sinh(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -1/4*(b^4*x^4 + (b^4*x^4 - 2*a^4 + 12*b^2*x^2 - 12*a^2)*cosh(b*x + a)^4 + 4
*(b^4*x^4 - 2*a^4 + 12*b^2*x^2 - 12*a^2)*cosh(b*x + a)*sinh(b*x + a)^3 + (b
^4*x^4 - 2*a^4 + 12*b^2*x^2 - 12*a^2)*sinh(b*x + a)^4 - 2*a^4 + 2*(b^4*x^4
- 4*b^3*x^3 - 2*a^4 + 6*b^2*x^2 - 12*a^2)*cosh(b*x + a)^2 + 2*(b^4*x^4 - 4*
b^3*x^3 - 2*a^4 + 6*b^2*x^2 + 3*(b^4*x^4 - 2*a^4 + 12*b^2*x^2 - 12*a^2)*cos
h(b*x + a)^2 - 12*a^2)*sinh(b*x + a)^2 - 12*a^2 - 12*((b^2*x^2 + 1)*cosh(b*
x + a)^4 + 4*(b^2*x^2 + 1)*cosh(b*x + a)*sinh(b*x + a)^3 + (b^2*x^2 + 1)*si
nh(b*x + a)^4 + b^2*x^2 + 2*(b^2*x^2 + 1)*cosh(b*x + a)^2 + 2*(b^2*x^2 + 3*
(b^2*x^2 + 1)*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^2 + 4*((b^2*x^2 + 1)*cosh(
b*x + a)^3 + (b^2*x^2 + 1)*cosh(b*x + a))*sinh(b*x + a) + 1)*dilog(I*cosh(b
*x + a) + I*sinh(b*x + a)) - 12*((b^2*x^2 + 1)*cosh(b*x + a)^4 + 4*(b^2*x^2
 + 1)*cosh(b*x + a)*sinh(b*x + a)^3 + (b^2*x^2 + 1)*sinh(b*x + a)^4 + b^2*x
^2 + 2*(b^2*x^2 + 1)*cosh(b*x + a)^2 + 2*(b^2*x^2 + 3*(b^2*x^2 + 1)*cosh(b*
x + a)^2 + 1)*sinh(b*x + a)^2 + 4*((b^2*x^2 + 1)*cosh(b*x + a)^3 + (b^2*x^2
 + 1)*cosh(b*x + a))*sinh(b*x + a) + 1)*dilog(-I*cosh(b*x + a) - I*sinh(b*x
 + a)) + 4*((a^3 + 3*a)*cosh(b*x + a)^4 + 4*(a^3 + 3*a)*cosh(b*x + a)*sinh(
b*x + a)^3 + (a^3 + 3*a)*sinh(b*x + a)^4 + a^3 + 2*(a^3 + 3*a)*cosh(b*x + a
)^2 + 2*(a^3 + 3*(a^3 + 3*a)*cosh(b*x + a)^2 + 3*a)*sinh(b*x + a)^2 + 4*((a
^3 + 3*a)*cosh(b*x + a)^3 + (a^3 + 3*a)*cosh(b*x + a))*sinh(b*x + a) + 3*a)
*log(cosh(b*x + a) + sinh(b*x + a) + I) + 4*((a^3 + 3*a)*cosh(b*x + a)^4 +
4*(a^3 + 3*a)*cosh(b*x + a)*sinh(b*x + a)^3 + (a^3 + 3*a)*sinh(b*x + a)^4 +
 a^3 + 2*(a^3 + 3*a)*cosh(b*x + a)^2 + 2*(a^3 + 3*(a^3 + 3*a)*cosh(b*x + a)
^2 + 3*a)*sinh(b*x + a)^2 + 4*((a^3 + 3*a)*cosh(b*x + a)^3 + (a^3 + 3*a)*co
sh(b*x + a))*sinh(b*x + a) + 3*a)*log(cosh(b*x + a) + sinh(b*x + a) - I) -
4*(b^3*x^3 + (b^3*x^3 + a^3 + 3*b*x + 3*a)*cosh(b*x + a)^4 + 4*(b^3*x^3 + a
^3 + 3*b*x + 3*a)*cosh(b*x + a)*sinh(b*x + a)^3 + (b^3*x^3 + a^3 + 3*b*x +
3*a)*sinh(b*x + a)^4 + a^3 + 2*(b^3*x^3 + a^3 + 3*b*x + 3*a)*cosh(b*x + a)^
2 + 2*(b^3*x^3 + a^3 + 3*(b^3*x^3 + a^3 + 3*b*x + 3*a)*cosh(b*x + a)^2 + 3*
b*x + 3*a)*sinh(b*x + a)^2 + 3*b*x + 4*((b^3*x^3 + a^3 + 3*b*x + 3*a)*cosh(
b*x + a)^3 + (b^3*x^3 + a^3 + 3*b*x + 3*a)*cosh(b*x + a))*sinh(b*x + a) + 3
*a)*log(I*cosh(b*x + a) + I*sinh(b*x + a) + 1) - 4*(b^3*x^3 + (b^3*x^3 + a^
3 + 3*b*x + 3*a)*cosh(b*x + a)^4 + 4*(b^3*x^3 + a^3 + 3*b*x + 3*a)*cosh(b*x
```

```
 + a)*sinh(b*x + a)^3 + (b^3*x^3 + a^3 + 3*b*x + 3*a)*sinh(b*x + a)^4 + a^3
 + 2*(b^3*x^3 + a^3 + 3*b*x + 3*a)*cosh(b*x + a)^2 + 2*(b^3*x^3 + a^3 + 3*(
b^3*x^3 + a^3 + 3*b*x + 3*a)*cosh(b*x + a)^2 + 3*b*x + 3*a)*sinh(b*x + a)^2
 + 3*b*x + 4*((b^3*x^3 + a^3 + 3*b*x + 3*a)*cosh(b*x + a)^3 + (b^3*x^3 + a^
3 + 3*b*x + 3*a)*cosh(b*x + a))*sinh(b*x + a) + 3*a)*log(-I*cosh(b*x + a) -
 I*sinh(b*x + a) + 1) - 24*(cosh(b*x + a)^4 + 4*cosh(b*x + a)*sinh(b*x + a)
^3 + sinh(b*x + a)^4 + 2*(3*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^2 + 2*cosh(b
*x + a)^2 + 4*(cosh(b*x + a)^3 + cosh(b*x + a))*sinh(b*x + a) + 1)*polylog(
4, I*cosh(b*x + a) + I*sinh(b*x + a)) - 24*(cosh(b*x + a)^4 + 4*cosh(b*x +
a)*sinh(b*x + a)^3 + sinh(b*x + a)^4 + 2*(3*cosh(b*x + a)^2 + 1)*sinh(b*x +
 a)^2 + 2*cosh(b*x + a)^2 + 4*(cosh(b*x + a)^3 + cosh(b*x + a))*sinh(b*x +
a) + 1)*polylog(4, -I*cosh(b*x + a) - I*sinh(b*x + a)) + 24*(b*x*cosh(b*x +
 a)^4 + 4*b*x*cosh(b*x + a)*sinh(b*x + a)^3 + b*x*sinh(b*x + a)^4 + 2*b*x*c
osh(b*x + a)^2 + 2*(3*b*x*cosh(b*x + a)^2 + b*x)*sinh(b*x + a)^2 + b*x + 4*
(b*x*cosh(b*x + a)^3 + b*x*cosh(b*x + a))*sinh(b*x + a))*polylog(3, I*cosh(
b*x + a) + I*sinh(b*x + a)) + 24*(b*x*cosh(b*x + a)^4 + 4*b*x*cosh(b*x + a)
*sinh(b*x + a)^3 + b*x*sinh(b*x + a)^4 + 2*b*x*cosh(b*x + a)^2 + 2*(3*b*x*c
osh(b*x + a)^2 + b*x)*sinh(b*x + a)^2 + b*x + 4*(b*x*cosh(b*x + a)^3 + b*x*
cosh(b*x + a))*sinh(b*x + a))*polylog(3, -I*cosh(b*x + a) - I*sinh(b*x + a)
) + 4*((b^4*x^4 - 2*a^4 + 12*b^2*x^2 - 12*a^2)*cosh(b*x + a)^3 + (b^4*x^4 -
 4*b^3*x^3 - 2*a^4 + 6*b^2*x^2 - 12*a^2)*cosh(b*x + a))*sinh(b*x + a))/(b^4
*cosh(b*x + a)^4 + 4*b^4*cosh(b*x + a)*sinh(b*x + a)^3 + b^4*sinh(b*x + a)^
4 + 2*b^4*cosh(b*x + a)^2 + b^4 + 2*(3*b^4*cosh(b*x + a)^2 + b^4)*sinh(b*x
+ a)^2 + 4*(b^4*cosh(b*x + a)^3 + b^4*cosh(b*x + a))*sinh(b*x + a))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3 \operatorname{sech}(bx+a)^3 \sinh(bx+a)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*sech(b*x+a)^3*sinh(b*x+a)^3,x, algorithm="giac")
```

```
[Out] integrate(x^3*sech(b*x + a)^3*sinh(b*x + a)^3, x)
```

maple [A] time = 0.62, size = 234, normalized size = 1.28

$$-\frac{x^4}{4}+\frac{x^2\left(2bx\,e^{2bx+2a}+3\,e^{2bx+2a}+3\right)}{b^2\left(1+e^{2bx+2a}\right)^2}-\frac{3x^2}{b^2}-\frac{3a^2}{b^4}-\frac{3a^4}{2b^4}+\frac{3x\ln\left(1+e^{2bx+2a}\right)}{b^3}-\frac{3x\,\mathrm{polylog}\left(3,-e^{2bx+2a}\right)}{2b^3}+\frac{x^3\ln\left(1+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*sech(b*x+a)^3*sinh(b*x+a)^3,x)
```

```
[Out] -1/4*x^4+x^2*(2*b*x*exp(2*b*x+2*a)+3*exp(2*b*x+2*a)+3)/b^2/(1+exp(2*b*x+2*a
))^2-3*x^2/b^2-3/b^4*a^2-3/2/b^4*a^4+3*x*ln(1+exp(2*b*x+2*a))/b^3-3/2*x*pol
```

```
ylog(3,-exp(2*b*x+2*a))/b^3+x^3*ln(1+exp(2*b*x+2*a))/b+3/2*x^2*polylog(2,-e
xp(2*b*x+2*a))/b^2-2/b^3*a^3*x-6*a*x/b^3+3/4*polylog(4,-exp(2*b*x+2*a))/b^4
+3/2*polylog(2,-exp(2*b*x+2*a))/b^4+6/b^4*a*ln(exp(b*x+a))+2/b^4*a^3*ln(exp
(b*x+a))
```

maxima [A] time = 1.07, size = 236, normalized size = 1.29

$$\frac{b^2x^4e^{(4\,bx+4\,a)} + b^2x^4 + 12\,x^2 + 2\left(b^2x^4e^{(2\,a)} + 4\,bx^3e^{(2\,a)} + 6\,x^2e^{(2\,a)}\right)e^{(2\,bx)}}{4\left(b^2e^{(4\,bx+4\,a)} + 2\,b^2e^{(2\,bx+2\,a)} + b^2\right)} - \frac{b^4x^4 + 6\,b^2x^2}{2\,b^4} + \frac{4\,b^3x^3\log\left(e^{(2\,bx+2\,a)} + 1\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*sech(b*x+a)^3*sinh(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/4*(b^2*x^4*e^(4*b*x + 4*a) + b^2*x^4 + 12*x^2 + 2*(b^2*x^4*e^(2*a) + 4*b*
x^3*e^(2*a) + 6*x^2*e^(2*a))*e^(2*b*x))/(b^2*e^(4*b*x + 4*a) + 2*b^2*e^(2*b
*x + 2*a) + b^2) - 1/2*(b^4*x^4 + 6*b^2*x^2)/b^4 + 1/3*(4*b^3*x^3*log(e^(2*
b*x + 2*a) + 1) + 6*b^2*x^2*dilog(-e^(2*b*x + 2*a)) - 6*b*x*polylog(3, -e^(
2*b*x + 2*a)) + 3*polylog(4, -e^(2*b*x + 2*a)))/b^4 + 3/2*(2*b*x*log(e^(2*b
*x + 2*a) + 1) + dilog(-e^(2*b*x + 2*a)))/b^4
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3\sinh\left(a+b\,x\right)^3}{\cosh\left(a+b\,x\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*sinh(a + b*x)^3)/cosh(a + b*x)^3,x)
```

```
[Out] int((x^3*sinh(a + b*x)^3)/cosh(a + b*x)^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*sech(b*x+a)**3*sinh(b*x+a)**3,x)
```

```
[Out] Timed out
```

3.392 $\int x^2 \tanh^3(a+bx)\,dx$

Optimal. Leaf size=116

$$-\frac{\text{Li}_3\left(-e^{2(a+bx)}\right)}{2b^3}+\frac{\log(\cosh(a+bx))}{b^3}+\frac{x\text{Li}_2\left(-e^{2(a+bx)}\right)}{b^2}-\frac{x\tanh(a+bx)}{b^2}+\frac{x^2\log\left(e^{2(a+bx)}+1\right)}{b}-\frac{x^2\tanh^2(a+bx)}{2b}$$

```
[Out] 1/2*x^2/b-1/3*x^3+x^2*ln(1+exp(2*b*x+2*a))/b+ln(cosh(b*x+a))/b^3+x*polylog(
2,-exp(2*b*x+2*a))/b^2-1/2*polylog(3,-exp(2*b*x+2*a))/b^3-x*tanh(b*x+a)/b^2
-1/2*x^2*tanh(b*x+a)^2/b
```

Rubi [A] time = 0.20, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {3720, 3475, 30, 3718, 2190, 2531, 2282, 6589}

$$\frac{x\text{PolyLog}\left(2,-e^{2(a+bx)}\right)}{b^2}-\frac{\text{PolyLog}\left(3,-e^{2(a+bx)}\right)}{2b^3}-\frac{x\tanh(a+bx)}{b^2}+\frac{\log(\cosh(a+bx))}{b^3}+\frac{x^2\log\left(e^{2(a+bx)}+1\right)}{b}-\frac{x^2}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Tanh[a + b*x]^3,x]
```

```
[Out] x^2/(2*b) - x^3/3 + (x^2*Log[1 + E^(2*(a + b*x))])/b + Log[Cosh[a + b*x]]/b
^3 + (x*PolyLog[2, -E^(2*(a + b*x))])/b^2 - PolyLog[3, -E^(2*(a + b*x))]/(2
*b^3) - (x*Tanh[a + b*x])/b^2 - (x^2*Tanh[a + b*x]^2)/(2*b)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
```

```
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3718

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c
 + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /;
 FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 3720

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[(b*(c + d*x)^m*(b*Tan[e + f*x])^(n - 1))/(f*(n - 1)), x] + (-Di
st[(b*d*m)/(f*(n - 1)), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x],
 x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[
{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int x^2\tanh^3(a+bx)\,dx = -\frac{x^2\tanh^2(a+bx)}{2b}+\frac{\int x\tanh^2(a+bx)\,dx}{b}+\int x^2\tanh(a+bx)\,dx$$
$$= -\frac{x^3}{3}-\frac{x\tanh(a+bx)}{b^2}-\frac{x^2\tanh^2(a+bx)}{2b}+2\int\frac{e^{2(a+bx)}x^2}{1+e^{2(a+bx)}}\,dx+\frac{\int\tanh(a+bx)\,dx}{b^2}$$
$$= \frac{x^2}{2b}-\frac{x^3}{3}+\frac{x^2\log\left(1+e^{2(a+bx)}\right)}{b}+\frac{\log(\cosh(a+bx))}{b^3}-\frac{x\tanh(a+bx)}{b^2}-\frac{x^2\tanh^2(a+}{2b}$$
$$= \frac{x^2}{2b}-\frac{x^3}{3}+\frac{x^2\log\left(1+e^{2(a+bx)}\right)}{b}+\frac{\log(\cosh(a+bx))}{b^3}+\frac{x\mathrm{Li}_2\left(-e^{2(a+bx)}\right)}{b^2}-\frac{x\tanh(a+}{b^2}$$
$$= \frac{x^2}{2b}-\frac{x^3}{3}+\frac{x^2\log\left(1+e^{2(a+bx)}\right)}{b}+\frac{\log(\cosh(a+bx))}{b^3}+\frac{x\mathrm{Li}_2\left(-e^{2(a+bx)}\right)}{b^2}-\frac{x\tanh(a+}{b^2}$$
$$= \frac{x^2}{2b}-\frac{x^3}{3}+\frac{x^2\log\left(1+e^{2(a+bx)}\right)}{b}+\frac{\log(\cosh(a+bx))}{b^3}+\frac{x\mathrm{Li}_2\left(-e^{2(a+bx)}\right)}{b^2}-\frac{\mathrm{Li}_3\left(-e^{2(a+b}\right.}{2b^3}$$

Mathematica [A] time = 2.38, size = 185, normalized size = 1.59

$$\frac{1}{6}\left(\frac{e^{2a}\left(4e^{-2a}b^2x^3-3\left(e^{-2a}+1\right)\left(2x\mathrm{Li}_2\left(-e^{-2(a+bx)}\right)+\frac{\mathrm{Li}_3\left(-e^{-2(a+bx)}\right)}{b}\right)+6\left(e^{-2a}+1\right)bx^2\log\left(e^{-2(a+bx)}+1\right)-\frac{6\left(e^{-2a}\right.}{}\right.}{\left(e^{2a}+1\right)b^2}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Tanh[a + b*x]^3,x]
```

```
[Out] ((E^(2*a)*((12*x)/E^(2*a) + (4*b^2*x^3)/E^(2*a) + 6*b*(1 + E^(-2*a))*x^2*Lo
g[1 + E^(-2*(a + b*x))] - (6*(1 + E^(-2*a))*(2*b*x - Log[1 + E^(2*(a + b*x)
)]))/b - 3*(1 + E^(-2*a))*(2*x*PolyLog[2, -E^(-2*(a + b*x))] + PolyLog[3, -
E^(-2*(a + b*x))]/b)))/(b^2*(1 + E^(2*a))) + (3*x^2*Sech[a + b*x]^2)/b - (6
*x*Sech[a]*Sech[a + b*x]*Sinh[b*x])/b^2 + 2*x^3*Tanh[a])/6
```

fricas [C] time = 0.94, size = 1649, normalized size = 14.22

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sech(b*x+a)^3*sinh(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -1/3*(b^3*x^3 + (b^3*x^3 + 2*a^3 + 6*b*x + 6*a)*cosh(b*x + a)^4 + 4*(b^3*x^
3 + 2*a^3 + 6*b*x + 6*a)*cosh(b*x + a)*sinh(b*x + a)^3 + (b^3*x^3 + 2*a^3 +
```

```
6*b*x + 6*a)*sinh(b*x + a)^4 + 2*a^3 + 2*(b^3*x^3 - 3*b^2*x^2 + 2*a^3 + 3*
b*x + 6*a)*cosh(b*x + a)^2 + 2*(b^3*x^3 - 3*b^2*x^2 + 2*a^3 + 3*(b^3*x^3 +
2*a^3 + 6*b*x + 6*a)*cosh(b*x + a)^2 + 3*b*x + 6*a)*sinh(b*x + a)^2 - 6*(b*
x*cosh(b*x + a)^4 + 4*b*x*cosh(b*x + a)*sinh(b*x + a)^3 + b*x*sinh(b*x + a)
^4 + 2*b*x*cosh(b*x + a)^2 + 2*(3*b*x*cosh(b*x + a)^2 + b*x)*sinh(b*x + a)^
2 + b*x + 4*(b*x*cosh(b*x + a)^3 + b*x*cosh(b*x + a))*sinh(b*x + a))*dilog(
I*cosh(b*x + a) + I*sinh(b*x + a)) - 6*(b*x*cosh(b*x + a)^4 + 4*b*x*cosh(b*
x + a)*sinh(b*x + a)^3 + b*x*sinh(b*x + a)^4 + 2*b*x*cosh(b*x + a)^2 + 2*(3
*b*x*cosh(b*x + a)^2 + b*x)*sinh(b*x + a)^2 + b*x + 4*(b*x*cosh(b*x + a)^3
+ b*x*cosh(b*x + a))*sinh(b*x + a))*dilog(-I*cosh(b*x + a) - I*sinh(b*x + a
)) - 3*((a^2 + 1)*cosh(b*x + a)^4 + 4*(a^2 + 1)*cosh(b*x + a)*sinh(b*x + a)
^3 + (a^2 + 1)*sinh(b*x + a)^4 + 2*(a^2 + 1)*cosh(b*x + a)^2 + 2*(3*(a^2 +
1)*cosh(b*x + a)^2 + a^2 + 1)*sinh(b*x + a)^2 + a^2 + 4*((a^2 + 1)*cosh(b*x
 + a)^3 + (a^2 + 1)*cosh(b*x + a))*sinh(b*x + a) + 1)*log(cosh(b*x + a) + s
inh(b*x + a) + I) - 3*((a^2 + 1)*cosh(b*x + a)^4 + 4*(a^2 + 1)*cosh(b*x + a
)*sinh(b*x + a)^3 + (a^2 + 1)*sinh(b*x + a)^4 + 2*(a^2 + 1)*cosh(b*x + a)^2
 + 2*(3*(a^2 + 1)*cosh(b*x + a)^2 + a^2 + 1)*sinh(b*x + a)^2 + a^2 + 4*((a^
2 + 1)*cosh(b*x + a)^3 + (a^2 + 1)*cosh(b*x + a))*sinh(b*x + a) + 1)*log(co
sh(b*x + a) + sinh(b*x + a) - I) - 3*((b^2*x^2 - a^2)*cosh(b*x + a)^4 + 4*(
b^2*x^2 - a^2)*cosh(b*x + a)*sinh(b*x + a)^3 + (b^2*x^2 - a^2)*sinh(b*x + a
)^4 + b^2*x^2 + 2*(b^2*x^2 - a^2)*cosh(b*x + a)^2 + 2*(b^2*x^2 + 3*(b^2*x^2
 - a^2)*cosh(b*x + a)^2 - a^2)*sinh(b*x + a)^2 - a^2 + 4*((b^2*x^2 - a^2)*c
osh(b*x + a)^3 + (b^2*x^2 - a^2)*cosh(b*x + a))*sinh(b*x + a))*log(I*cosh(b
*x + a) + I*sinh(b*x + a) + 1) - 3*((b^2*x^2 - a^2)*cosh(b*x + a)^4 + 4*(b^
2*x^2 - a^2)*cosh(b*x + a)*sinh(b*x + a)^3 + (b^2*x^2 - a^2)*sinh(b*x + a)^
4 + b^2*x^2 + 2*(b^2*x^2 - a^2)*cosh(b*x + a)^2 + 2*(b^2*x^2 + 3*(b^2*x^2 -
 a^2)*cosh(b*x + a)^2 - a^2)*sinh(b*x + a)^2 - a^2 + 4*((b^2*x^2 - a^2)*cos
h(b*x + a)^3 + (b^2*x^2 - a^2)*cosh(b*x + a))*sinh(b*x + a))*log(-I*cosh(b*
x + a) - I*sinh(b*x + a) + 1) + 6*(cosh(b*x + a)^4 + 4*cosh(b*x + a)*sinh(b
*x + a)^3 + sinh(b*x + a)^4 + 2*(3*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^2 + 2
*cosh(b*x + a)^2 + 4*(cosh(b*x + a)^3 + cosh(b*x + a))*sinh(b*x + a) + 1)*p
olylog(3, I*cosh(b*x + a) + I*sinh(b*x + a)) + 6*(cosh(b*x + a)^4 + 4*cosh(
b*x + a)*sinh(b*x + a)^3 + sinh(b*x + a)^4 + 2*(3*cosh(b*x + a)^2 + 1)*sinh
(b*x + a)^2 + 2*cosh(b*x + a)^2 + 4*(cosh(b*x + a)^3 + cosh(b*x + a))*sinh(
b*x + a) + 1)*polylog(3, -I*cosh(b*x + a) - I*sinh(b*x + a)) + 4*((b^3*x^3
+ 2*a^3 + 6*b*x + 6*a)*cosh(b*x + a)^3 + (b^3*x^3 - 3*b^2*x^2 + 2*a^3 + 3*b
*x + 6*a)*cosh(b*x + a))*sinh(b*x + a) + 6*a)/(b^3*cosh(b*x + a)^4 + 4*b^3*
cosh(b*x + a)*sinh(b*x + a)^3 + b^3*sinh(b*x + a)^4 + 2*b^3*cosh(b*x + a)^2
 + b^3 + 2*(3*b^3*cosh(b*x + a)^2 + b^3)*sinh(b*x + a)^2 + 4*(b^3*cosh(b*x
+ a)^3 + b^3*cosh(b*x + a))*sinh(b*x + a))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \operatorname{sech}(bx+a)^3 \sinh(bx+a)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sech(b*x+a)^3*sinh(b*x+a)^3,x, algorithm="giac")
```

```
[Out] integrate(x^2*sech(b*x + a)^3*sinh(b*x + a)^3, x)
```

maple [A] time = 0.61, size = 164, normalized size = 1.41

$$-\frac{x^3}{3}+\frac{2x\left(bx\,e^{2bx+2a}+e^{2bx+2a}+1\right)}{b^2\left(1+e^{2bx+2a}\right)^2}-\frac{2\ln\left(e^{bx+a}\right)}{b^3}+\frac{\ln\left(1+e^{2bx+2a}\right)}{b^3}-\frac{2a^2\ln\left(e^{bx+a}\right)}{b^3}+\frac{2a^2x}{b^2}+\frac{4a^3}{3b^3}+\frac{x^2\ln\left(1+e^{2bx+2}\right.}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*sech(b*x+a)^3*sinh(b*x+a)^3,x)
```

```
[Out] -1/3*x^3+2*x*(b*x*exp(2*b*x+2*a)+exp(2*b*x+2*a)+1)/b^2/(1+exp(2*b*x+2*a))^2
-2/b^3*ln(exp(b*x+a))+1/b^3*ln(1+exp(2*b*x+2*a))-2/b^3*a^2*ln(exp(b*x+a))+2
/b^2*a^2*x+4/3/b^3*a^3+x^2*ln(1+exp(2*b*x+2*a))/b+x*polylog(2,-exp(2*b*x+2*
a))/b^2-1/2*polylog(3,-exp(2*b*x+2*a))/b^3
```

maxima [A] time = 0.97, size = 183, normalized size = 1.58

$$-\frac{2}{3}x^3+\frac{b^2x^3e^{(4\,bx+4\,a)}+b^2x^3+2\left(b^2x^3e^{(2\,a)}+3\,bx^2e^{(2\,a)}+3\,xe^{(2\,a)}\right)e^{(2\,bx)}+6\,x}{3\left(b^2e^{(4\,bx+4\,a)}+2\,b^2e^{(2\,bx+2\,a)}+b^2\right)}-\frac{2\,x}{b^2}+\frac{2\,b^2x^2\log\left(e^{(2\,bx+2\,a)}+1\right)+}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sech(b*x+a)^3*sinh(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -2/3*x^3 + 1/3*(b^2*x^3*e^(4*b*x + 4*a) + b^2*x^3 + 2*(b^2*x^3*e^(2*a) + 3*
b*x^2*e^(2*a) + 3*x*e^(2*a))*e^(2*b*x) + 6*x)/(b^2*e^(4*b*x + 4*a) + 2*b^2*
e^(2*b*x + 2*a) + b^2) - 2*x/b^2 + 1/2*(2*b^2*x^2*log(e^(2*b*x + 2*a) + 1)
+ 2*b*x*dilog(-e^(2*b*x + 2*a)) - polylog(3, -e^(2*b*x + 2*a)))/b^3 + log(e
^(2*b*x + 2*a) + 1)/b^3
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2\sinh\left(a+b\,x\right)^3}{\cosh\left(a+b\,x\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*sinh(a + b*x)^3)/cosh(a + b*x)^3,x)
```

```
[Out] int((x^2*sinh(a + b*x)^3)/cosh(a + b*x)^3, x)
```

<u>**sympy**</u> **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*sech(b*x+a)**3*sinh(b*x+a)**3,x)
```

```
[Out] Timed out
```

3.393 $\int x \tanh^3(a+bx)\,dx$

Optimal. Leaf size=82

$$\frac{\text{Li}_2\left(-e^{2(a+bx)}\right)}{2b^2}-\frac{\tanh(a+bx)}{2b^2}+\frac{x\log\left(e^{2(a+bx)}+1\right)}{b}-\frac{x\tanh^2(a+bx)}{2b}+\frac{x}{2b}-\frac{x^2}{2}$$

```
[Out] 1/2*x/b-1/2*x^2+x*ln(1+exp(2*b*x+2*a))/b+1/2*polylog(2,-exp(2*b*x+2*a))/b^2
-1/2*tanh(b*x+a)/b^2-1/2*x*tanh(b*x+a)^2/b
```

Rubi [A] time = 0.12, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.700, Rules used = {3720, 3473, 8, 3718, 2190, 2279, 2391}

$$\frac{\text{PolyLog}\left(2,-e^{2(a+bx)}\right)}{2b^2}-\frac{\tanh(a+bx)}{2b^2}+\frac{x\log\left(e^{2(a+bx)}+1\right)}{b}-\frac{x\tanh^2(a+bx)}{2b}+\frac{x}{2b}-\frac{x^2}{2}$$

Antiderivative was successfully verified.

```
[In] Int[x*Tanh[a + b*x]^3,x]
```

```
[Out] x/(2*b) - x^2/2 + (x*Log[1 + E^(2*(a + b*x))])/b + PolyLog[2, -E^(2*(a + b*
x))]/(2*b^2) - Tanh[a + b*x]/(2*b^2) - (x*Tanh[a + b*x]^2)/(2*b)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3718

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c
 + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /;
 FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 3720

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[(b*(c + d*x)^m*(b*Tan[e + f*x])^(n - 1))/(f*(n - 1)), x] + (-Di
st[(b*d*m)/(f*(n - 1)), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x],
 x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[
{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\int x\tanh^3(a+bx)\,dx &= -\frac{x\tanh^2(a+bx)}{2b}+\frac{\int\tanh^2(a+bx)\,dx}{2b}+\int x\tanh(a+bx)\,dx\\
&= -\frac{x^2}{2}-\frac{\tanh(a+bx)}{2b^2}-\frac{x\tanh^2(a+bx)}{2b}+2\int\frac{e^{2(a+bx)}x}{1+e^{2(a+bx)}}\,dx+\frac{\int 1\,dx}{2b}\\
&= \frac{x}{2b}-\frac{x^2}{2}+\frac{x\log\left(1+e^{2(a+bx)}\right)}{b}-\frac{\tanh(a+bx)}{2b^2}-\frac{x\tanh^2(a+bx)}{2b}-\frac{\int\log\left(1+e^{2(a+bx)}\right)\,d}{b}\\
&= \frac{x}{2b}-\frac{x^2}{2}+\frac{x\log\left(1+e^{2(a+bx)}\right)}{b}-\frac{\tanh(a+bx)}{2b^2}-\frac{x\tanh^2(a+bx)}{2b}-\frac{\text{Subst}\left(\int\frac{\log(1+x)}{x}\,dx,\right.}{2b^2}\\
&= \frac{x}{2b}-\frac{x^2}{2}+\frac{x\log\left(1+e^{2(a+bx)}\right)}{b}+\frac{\text{Li}_2\left(-e^{2(a+bx)}\right)}{2b^2}-\frac{\tanh(a+bx)}{2b^2}-\frac{x\tanh^2(a+bx)}{2b}
\end{aligned}$$

Mathematica [C] time = 6.13, size = 231, normalized size = 2.82

$$\frac{\text{csch}(a)\text{sech}(a)\left(b^2x^2e^{-\tanh^{-1}(\coth(a))}-\frac{i\coth(a)\left(i\text{Li}_2\left(e^{2i\left(ibx+i\tanh^{-1}(\coth(a))\right)}\right)-bx\left(-\pi+2i\tanh^{-1}(\coth(a))\right)-2\left(i\tanh^{-1}(\coth(a))+ibx\right)\log\left(\right.\right.}{}\right.}{2b^2\sqrt{\text{csch}^2(a)\left(\sinh^2(a)-\cosh\right.}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x*Tanh[a + b*x]^3,x]
```

```
[Out] (x*Sech[a + b*x]^2)/(2*b) + (Csch[a]*((b^2*x^2)/E^ArcTanh[Coth[a]] - (I*Cot
h[a]*(-(b*x*(-Pi + (2*I)*ArcTanh[Coth[a]])) - Pi*Log[1 + E^(2*b*x)] - 2*(I*
b*x + I*ArcTanh[Coth[a]])*Log[1 - E^((2*I)*(I*b*x + I*ArcTanh[Coth[a]]))] +
 Pi*Log[Cosh[b*x]] + (2*I)*ArcTanh[Coth[a]]*Log[I*Sinh[b*x + ArcTanh[Coth[a
]]]] + I*PolyLog[2, E^((2*I)*(I*b*x + I*ArcTanh[Coth[a]]))]))/Sqrt[1 - Coth
[a]^2])*Sech[a])/(2*b^2*Sqrt[Csch[a]^2*(-Cosh[a]^2 + Sinh[a]^2)]) - (Sech[a
]*Sech[a + b*x]*Sinh[b*x])/(2*b^2) + (x^2*Tanh[a])/2
```

fricas [C] time = 0.57, size = 1106, normalized size = 13.49

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sech(b*x+a)^3*sinh(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -1/2*((b^2*x^2 - 2*a^2)*cosh(b*x + a)^4 + 4*(b^2*x^2 - 2*a^2)*cosh(b*x + a)
*sinh(b*x + a)^3 + (b^2*x^2 - 2*a^2)*sinh(b*x + a)^4 + b^2*x^2 + 2*(b^2*x^2
 - 2*a^2 - 2*b*x - 1)*cosh(b*x + a)^2 + 2*(b^2*x^2 + 3*(b^2*x^2 - 2*a^2)*co
sh(b*x + a)^2 - 2*a^2 - 2*b*x - 1)*sinh(b*x + a)^2 - 2*a^2 - 2*(cosh(b*x +
a)^4 + 4*cosh(b*x + a)*sinh(b*x + a)^3 + sinh(b*x + a)^4 + 2*(3*cosh(b*x +
a)^2 + 1)*sinh(b*x + a)^2 + 2*cosh(b*x + a)^2 + 4*(cosh(b*x + a)^3 + cosh(b
*x + a))*sinh(b*x + a) + 1)*dilog(I*cosh(b*x + a) + I*sinh(b*x + a)) - 2*(c
osh(b*x + a)^4 + 4*cosh(b*x + a)*sinh(b*x + a)^3 + sinh(b*x + a)^4 + 2*(3*c
osh(b*x + a)^2 + 1)*sinh(b*x + a)^2 + 2*cosh(b*x + a)^2 + 4*(cosh(b*x + a)^
3 + cosh(b*x + a))*sinh(b*x + a) + 1)*dilog(-I*cosh(b*x + a) - I*sinh(b*x +
 a)) + 2*(a*cosh(b*x + a)^4 + 4*a*cosh(b*x + a)*sinh(b*x + a)^3 + a*sinh(b*
x + a)^4 + 2*a*cosh(b*x + a)^2 + 2*(3*a*cosh(b*x + a)^2 + a)*sinh(b*x + a)^
2 + 4*(a*cosh(b*x + a)^3 + a*cosh(b*x + a))*sinh(b*x + a) + a)*log(cosh(b*x
 + a) + sinh(b*x + a) + I) + 2*(a*cosh(b*x + a)^4 + 4*a*cosh(b*x + a)*sinh(
b*x + a)^3 + a*sinh(b*x + a)^4 + 2*a*cosh(b*x + a)^2 + 2*(3*a*cosh(b*x + a)
^2 + a)*sinh(b*x + a)^2 + 4*(a*cosh(b*x + a)^3 + a*cosh(b*x + a))*sinh(b*x
+ a) + a)*log(cosh(b*x + a) + sinh(b*x + a) - I) - 2*((b*x + a)*cosh(b*x +
a)^4 + 4*(b*x + a)*cosh(b*x + a)*sinh(b*x + a)^3 + (b*x + a)*sinh(b*x + a)^
4 + 2*(b*x + a)*cosh(b*x + a)^2 + 2*(3*(b*x + a)*cosh(b*x + a)^2 + b*x + a)
*sinh(b*x + a)^2 + b*x + 4*((b*x + a)*cosh(b*x + a)^3 + (b*x + a)*cosh(b*x
+ a))*sinh(b*x + a) + a)*log(I*cosh(b*x + a) + I*sinh(b*x + a) + 1) - 2*((b
*x + a)*cosh(b*x + a)^4 + 4*(b*x + a)*cosh(b*x + a)*sinh(b*x + a)^3 + (b*x
+ a)*sinh(b*x + a)^4 + 2*(b*x + a)*cosh(b*x + a)^2 + 2*(3*(b*x + a)*cosh(b*
x + a)^2 + b*x + a)*sinh(b*x + a)^2 + b*x + 4*((b*x + a)*cosh(b*x + a)^3 +
(b*x + a)*cosh(b*x + a))*sinh(b*x + a) + a)*log(-I*cosh(b*x + a) - I*sinh(b
*x + a) + 1) + 4*((b^2*x^2 - 2*a^2)*cosh(b*x + a)^3 + (b^2*x^2 - 2*a^2 - 2*
```

```
b*x - 1)*cosh(b*x + a))*sinh(b*x + a) - 2)/(b^2*cosh(b*x + a)^4 + 4*b^2*cos
h(b*x + a)*sinh(b*x + a)^3 + b^2*sinh(b*x + a)^4 + 2*b^2*cosh(b*x + a)^2 +
2*(3*b^2*cosh(b*x + a)^2 + b^2)*sinh(b*x + a)^2 + b^2 + 4*(b^2*cosh(b*x + a
)^3 + b^2*cosh(b*x + a))*sinh(b*x + a))
```

giac **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int x \operatorname{sech}(bx+a)^3 \sinh(bx+a)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sech(b*x+a)^3*sinh(b*x+a)^3,x, algorithm="giac")
```

```
[Out] integrate(x*sech(b*x + a)^3*sinh(b*x + a)^3, x)
```

maple **[A]** time = 0.52, size = 111, normalized size = 1.35

$$-\frac{x^2}{2}+\frac{2bx\,e^{2bx+2a}+e^{2bx+2a}+1}{b^2\left(1+e^{2bx+2a}\right)^2}-\frac{2ax}{b}-\frac{a^2}{b^2}+\frac{x\ln\left(1+e^{2bx+2a}\right)}{b}+\frac{\operatorname{polylog}\left(2,-e^{2bx+2a}\right)}{2b^2}+\frac{2a\ln\left(e^{bx+a}\right)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sech(b*x+a)^3*sinh(b*x+a)^3,x)
```

```
[Out] -1/2*x^2+(2*b*x*exp(2*b*x+2*a)+exp(2*b*x+2*a)+1)/b^2/(1+exp(2*b*x+2*a))^2-2
/b*a*x-a^2/b^2+x*ln(1+exp(2*b*x+2*a))/b+1/2*polylog(2,-exp(2*b*x+2*a))/b^2+
2/b^2*a*ln(exp(b*x+a))
```

maxima **[A]** time = 0.47, size = 131, normalized size = 1.60

$$-x^2+\frac{b^2x^2e^{(4\,bx+4\,a)}+b^2x^2+2\left(b^2x^2e^{(2\,a)}+2\,bxe^{(2\,a)}+e^{(2\,a)}\right)e^{(2\,bx)}+2}{2\left(b^2e^{(4\,bx+4\,a)}+2\,b^2e^{(2\,bx+2\,a)}+b^2\right)}+\frac{2\,bx\log\left(e^{(2\,bx+2\,a)}+1\right)+\operatorname{Li}_2\left(-e^{(2\,bx+2\,a)}\right)}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sech(b*x+a)^3*sinh(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -x^2 + 1/2*(b^2*x^2*e^(4*b*x + 4*a) + b^2*x^2 + 2*(b^2*x^2*e^(2*a) + 2*b*x*
e^(2*a) + e^(2*a))*e^(2*b*x) + 2)/(b^2*e^(4*b*x + 4*a) + 2*b^2*e^(2*b*x + 2
*a) + b^2) + 1/2*(2*b*x*log(e^(2*b*x + 2*a) + 1) + dilog(-e^(2*b*x + 2*a)))
/b^2
```

mupad **[F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x\sinh(a+b\,x)^3}{\cosh(a+b\,x)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*sinh(a + b*x)^3)/cosh(a + b*x)^3,x)
```

```
[Out] int((x*sinh(a + b*x)^3)/cosh(a + b*x)^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\sinh^3(a+bx)\,\mathrm{sech}^3(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sech(b*x+a)**3*sinh(b*x+a)**3,x)
```

```
[Out] Integral(x*sinh(a + b*x)**3*sech(a + b*x)**3, x)
```

3.394 $\int \tanh^3(a+bx)\,dx$

Optimal. Leaf size=27

$$\frac{\log(\cosh(a+bx))}{b}-\frac{\tanh^2(a+bx)}{2b}$$

```
[Out] ln(cosh(b*x+a))/b-1/2*tanh(b*x+a)^2/b
```

Rubi [A] time = 0.02, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3473, 3475}

$$\frac{\log(\cosh(a+bx))}{b}-\frac{\tanh^2(a+bx)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[a + b*x]^3,x]
```

```
[Out] Log[Cosh[a + b*x]]/b - Tanh[a + b*x]^2/(2*b)
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \tanh^3(a+bx)\,dx &= -\frac{\tanh^2(a+bx)}{2b}+\int \tanh(a+bx)\,dx\\ &= \frac{\log(\cosh(a+bx))}{b}-\frac{\tanh^2(a+bx)}{2b}\end{aligned}$$

Mathematica [A] time = 0.01, size = 27, normalized size = 1.00

$$\frac{\log(\cosh(a+bx))}{b}-\frac{\tanh^2(a+bx)}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[a + b*x]^3,x]
```

```
[Out] Log[Cosh[a + b*x]]/b - Tanh[a + b*x]^2/(2*b)
```

fricas [B] time = 0.59, size = 339, normalized size = 12.56

$$-\frac{bx\cosh\left(bx+a\right)^{4}+4\,bx\cosh\left(bx+a\right)\sinh\left(bx+a\right)^{3}+bx\sinh\left(bx+a\right)^{4}+2\left(bx-1\right)\cosh\left(bx+a\right)^{2}+2\left(3\,bx\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)^3*sinh(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -(b*x*cosh(b*x + a)^4 + 4*b*x*cosh(b*x + a)*sinh(b*x + a)^3 + b*x*sinh(b*x
+ a)^4 + 2*(b*x - 1)*cosh(b*x + a)^2 + 2*(3*b*x*cosh(b*x + a)^2 + b*x - 1)*
sinh(b*x + a)^2 + b*x - (cosh(b*x + a)^4 + 4*cosh(b*x + a)*sinh(b*x + a)^3
+ sinh(b*x + a)^4 + 2*(3*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^2 + 2*cosh(b*x
+ a)^2 + 4*(cosh(b*x + a)^3 + cosh(b*x + a))*sinh(b*x + a) + 1)*log(2*cosh(
b*x + a)/(cosh(b*x + a) - sinh(b*x + a))) + 4*(b*x*cosh(b*x + a)^3 + (b*x -
 1)*cosh(b*x + a))*sinh(b*x + a))/(b*cosh(b*x + a)^4 + 4*b*cosh(b*x + a)*si
nh(b*x + a)^3 + b*sinh(b*x + a)^4 + 2*b*cosh(b*x + a)^2 + 2*(3*b*cosh(b*x +
 a)^2 + b)*sinh(b*x + a)^2 + 4*(b*cosh(b*x + a)^3 + b*cosh(b*x + a))*sinh(b
*x + a) + b)
```

giac [B] time = 0.16, size = 65, normalized size = 2.41

$$-\frac{2\,bx+2\,a+\frac{3\,e^{\left(4\,bx+4\,a\right)}+2\,e^{\left(2\,bx+2\,a\right)}+3}{\left(e^{\left(2\,bx+2\,a\right)}+1\right)^{2}}-2\,\log\left(e^{\left(2\,bx+2\,a\right)}+1\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)^3*sinh(b*x+a)^3,x, algorithm="giac")
```

```
[Out] -1/2*(2*b*x + 2*a + (3*e^(4*b*x + 4*a) + 2*e^(2*b*x + 2*a) + 3)/(e^(2*b*x +
 2*a) + 1)^2 - 2*log(e^(2*b*x + 2*a) + 1))/b
```

maple [A] time = 0.13, size = 26, normalized size = 0.96

$$\frac{\ln\left(\cosh\left(bx+a\right)\right)}{b}-\frac{\tanh^{2}\left(bx+a\right)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(b*x+a)^3*sinh(b*x+a)^3,x)
```

```
[Out] ln(cosh(b*x+a))/b-1/2*tanh(b*x+a)^2/b
```

maxima [B] time = 0.79, size = 61, normalized size = 2.26

$$x + \frac{a}{b} + \frac{\log\left(e^{(-2\,bx-2\,a)} + 1\right)}{b} + \frac{2\,e^{(-2\,bx-2\,a)}}{b\left(2\,e^{(-2\,bx-2\,a)} + e^{(-4\,bx-4\,a)} + 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)^3*sinh(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] x + a/b + log(e^(-2*b*x - 2*a) + 1)/b + 2*e^(-2*b*x - 2*a)/(b*(2*e^(-2*b*x
- 2*a) + e^(-4*b*x - 4*a) + 1))
```

mupad [B] time = 1.51, size = 25, normalized size = 0.93

$$\frac{1}{2\,b\cosh\left(a+b\,x\right)^2} + \frac{\ln\left(\cosh\left(a+b\,x\right)\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a + b*x)^3/cosh(a + b*x)^3,x)
```

```
[Out] 1/(2*b*cosh(a + b*x)^2) + log(cosh(a + b*x))/b
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sinh^3\left(a+bx\right)\operatorname{sech}^3\left(a+bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)**3*sinh(b*x+a)**3,x)
```

```
[Out] Integral(sinh(a + b*x)**3*sech(a + b*x)**3, x)
```

3.395 $\int \frac{\tanh^3(a+bx)}{x} \, dx$

Optimal. Leaf size=15

$$\operatorname{Int}\left(\frac{\tanh^3(a+bx)}{x},x\right)$$

```
[Out] Unintegrable(tanh(b*x+a)^3/x,x)
```

Rubi [A] time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\tanh^3(a+bx)}{x} \, dx$$

Verification is Not applicable to the result.

```
[In] Int[Tanh[a + b*x]^3/x,x]
[Out] Defer[Int][Tanh[a + b*x]^3/x, x]
```

Rubi steps

$$\int \frac{\tanh^3(a+bx)}{x} \, dx = \int \frac{\tanh^3(a+bx)}{x} \, dx$$

Mathematica [A] time = 14.11, size = 0, normalized size = 0.00

$$\int \frac{\tanh^3(a+bx)}{x} \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Tanh[a + b*x]^3/x,x]
[Out] Integrate[Tanh[a + b*x]^3/x, x]
```

fricas [A] time = 0.64, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{\operatorname{sech}(bx+a)^3 \sinh(bx+a)^3}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)^3*sinh(b*x+a)^3/x,x, algorithm="fricas")
```

```
[Out] integral(sech(b*x + a)^3*sinh(b*x + a)^3/x, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}(bx+a)^3 \sinh(bx+a)^3}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)^3*sinh(b*x+a)^3/x,x, algorithm="giac")
```

```
[Out] integrate(sech(b*x + a)^3*sinh(b*x + a)^3/x, x)
```

maple [A] time = 1.31, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}(bx+a)^3 \left(\sinh^3(bx+a)\right)}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(b*x+a)^3*sinh(b*x+a)^3/x,x)
```

```
[Out] int(sech(b*x+a)^3*sinh(b*x+a)^3/x,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\left(2\, bxe^{(2\,a)} - e^{(2\,a)}\right)e^{(2\,bx)} - 1}{b^2x^2e^{(4\,bx+4\,a)} + 2\,b^2x^2e^{(2\,bx+2\,a)} + b^2x^2} - \int \frac{2\left(b^2x^2+1\right)}{b^2x^3e^{(2\,bx+2\,a)} + b^2x^3}\, dx + \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)^3*sinh(b*x+a)^3/x,x, algorithm="maxima")
```

```
[Out] ((2*b*x*e^(2*a) - e^(2*a))*e^(2*b*x) - 1)/(b^2*x^2*e^(4*b*x + 4*a) + 2*b^2*
x^2*e^(2*b*x + 2*a) + b^2*x^2) - integrate(2*(b^2*x^2 + 1)/(b^2*x^3*e^(2*b*
x + 2*a) + b^2*x^3), x) + log(x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.07

$$\int \frac{\sinh(a+b\,x)^3}{x\cosh(a+b\,x)^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a + b*x)^3/(x*cosh(a + b*x)^3),x)
```

```
[Out] int(sinh(a + b*x)^3/(x*cosh(a + b*x)^3), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh^3(a+bx)\operatorname{sech}^3(a+bx)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)**3*sinh(b*x+a)**3/x,x)
```

```
[Out] Integral(sinh(a + b*x)**3*sech(a + b*x)**3/x, x)
```

3.396 $\int \frac{\tanh^3(a+bx)}{x^2}\,dx$

Optimal. Leaf size=15

$$\mathrm{Int}\left(\frac{\tanh^3(a+bx)}{x^2},x\right)$$

```
[Out] Unintegrable(tanh(b*x+a)^3/x^2,x)
```

Rubi [A] time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\tanh^3(a+bx)}{x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Tanh[a + b*x]^3/x^2,x]
```

```
[Out] Defer[Int][Tanh[a + b*x]^3/x^2, x]
```

Rubi steps

$$\int \frac{\tanh^3(a+bx)}{x^2}\,dx = \int \frac{\tanh^3(a+bx)}{x^2}\,dx$$

Mathematica [A] time = 8.40, size = 0, normalized size = 0.00

$$\int \frac{\tanh^3(a+bx)}{x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Tanh[a + b*x]^3/x^2,x]
```

```
[Out] Integrate[Tanh[a + b*x]^3/x^2, x]
```

fricas [A] time = 0.69, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\mathrm{sech}\,(bx+a)^3\,\sinh\,(bx+a)^3}{x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)^3*sinh(b*x+a)^3/x^2,x, algorithm="fricas")
```

```
[Out] integral(sech(b*x + a)^3*sinh(b*x + a)^3/x^2, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}(bx+a)^3 \sinh(bx+a)^3}{x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)^3*sinh(b*x+a)^3/x^2,x, algorithm="giac")
```

```
[Out] integrate(sech(b*x + a)^3*sinh(b*x + a)^3/x^2, x)
```

maple [A] time = 1.42, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}(bx+a)^3 \left(\sinh^3(bx+a)\right)}{x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(b*x+a)^3*sinh(b*x+a)^3/x^2,x)
```

```
[Out] int(sech(b*x+a)^3*sinh(b*x+a)^3/x^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{b^2x^2e^{(4\,bx+4\,a)} + b^2x^2 + 2\left(b^2x^2e^{(2\,a)} - bxe^{(2\,a)} + e^{(2\,a)}\right)e^{(2\,bx)} + 2}{b^2x^3e^{(4\,bx+4\,a)} + 2\,b^2x^3e^{(2\,bx+2\,a)} + b^2x^3} - \int \frac{2\left(b^2x^2+3\right)}{b^2x^4e^{(2\,bx+2\,a)} + b^2x^4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)^3*sinh(b*x+a)^3/x^2,x, algorithm="maxima")
```

```
[Out] -(b^2*x^2*e^(4*b*x + 4*a) + b^2*x^2 + 2*(b^2*x^2*e^(2*a) - b*x*e^(2*a) + e^
(2*a))*e^(2*b*x) + 2)/(b^2*x^3*e^(4*b*x + 4*a) + 2*b^2*x^3*e^(2*b*x + 2*a)
+ b^2*x^3) - integrate(2*(b^2*x^2 + 3)/(b^2*x^4*e^(2*b*x + 2*a) + b^2*x^4),
 x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.07

$$\int \frac{\sinh(a+b\,x)^3}{x^2\cosh(a+b\,x)^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a + b*x)^3/(x^2*cosh(a + b*x)^3),x)
```

```
[Out] int(sinh(a + b*x)^3/(x^2*cosh(a + b*x)^3), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh^3(a+bx)\operatorname{sech}^3(a+bx)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)**3*sinh(b*x+a)**3/x**2,x)
```

```
[Out] Integral(sinh(a + b*x)**3*sech(a + b*x)**3/x**2, x)
```

3.397 $\int x^m \coth(a+bx)\,dx$

Optimal. Leaf size=13

$$\text{Int}\left(x^m \coth(a+bx), x\right)$$

```
[Out] Unintegrable(x^m*coth(b*x+a),x)
```

Rubi [A] time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int x^m \coth(a+bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x^m*Coth[a + b*x],x]
```

```
[Out] Defer[Int][x^m*Coth[a + b*x], x]
```

Rubi steps

$$\int x^m \coth(a+bx)\,dx = \int x^m \coth(a+bx)\,dx$$

Mathematica [A] time = 7.58, size = 0, normalized size = 0.00

$$\int x^m \coth(a+bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^m*Coth[a + b*x],x]
```

```
[Out] Integrate[x^m*Coth[a + b*x], x]
```

fricas [A] time = 0.82, size = 0, normalized size = 0.00

$$\text{integral}\left(x^m \cosh\left(bx+a\right) \text{csch}\left(bx+a\right), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*cosh(b*x+a)*csch(b*x+a),x, algorithm="fricas")
```

```
[Out] integral(x^m*cosh(b*x + a)*csch(b*x + a), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int x^m \cosh(bx + a)\operatorname{csch}(bx + a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*cosh(b*x+a)*csch(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(x^m*cosh(b*x + a)*csch(b*x + a), x)
```

maple [A] time = 0.21, size = 0, normalized size = 0.00

$$\int x^m \cosh(bx + a)\operatorname{csch}(bx + a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*cosh(b*x+a)*csch(b*x+a),x)
```

```
[Out] int(x^m*cosh(b*x+a)*csch(b*x+a),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\frac{xe^{\left(2\,bx+m\log(x)+2\,a\right)}}{(m+1)e^{(2\,bx+2\,a)}-m-1}+\int\frac{\left(\left(2\,bxe^{(2\,a)}+(m+1)e^{(2\,a)}\right)e^{(2\,bx)}-m-1\right)x^m}{(m+1)e^{(4\,bx+4\,a)}-2\,(m+1)e^{(2\,bx+2\,a)}+m+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*cosh(b*x+a)*csch(b*x+a),x, algorithm="maxima")
```

```
[Out] x*e^(2*b*x + m*log(x) + 2*a)/((m + 1)*e^(2*b*x + 2*a) - m - 1) + integrate(
((2*b*x*e^(2*a) + (m + 1)*e^(2*a))*e^(2*b*x) - m - 1)*x^m/((m + 1)*e^(4*b*x
 + 4*a) - 2*(m + 1)*e^(2*b*x + 2*a) + m + 1), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.08

$$\int \frac{x^m\cosh(a+b\,x)}{\sinh(a+b\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^m*cosh(a + b*x))/sinh(a + b*x),x)
```

```
[Out] int((x^m*cosh(a + b*x))/sinh(a + b*x), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int x^m \cosh(a + bx)\operatorname{csch}(a + bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*cosh(b*x+a)*csch(b*x+a),x)
```

```
[Out] Integral(x**m*cosh(a + b*x)*csch(a + b*x), x)
```

3.398 $\int x^3 \coth(a+bx)\,dx$

Optimal. Leaf size=87

$$\frac{3\text{Li}_4\left(e^{2(a+bx)}\right)}{4b^4}-\frac{3x\text{Li}_3\left(e^{2(a+bx)}\right)}{2b^3}+\frac{3x^2\text{Li}_2\left(e^{2(a+bx)}\right)}{2b^2}+\frac{x^3\log\left(1-e^{2(a+bx)}\right)}{b}-\frac{x^4}{4}$$

```
[Out] -1/4*x^4+x^3*ln(1-exp(2*b*x+2*a))/b+3/2*x^2*polylog(2,exp(2*b*x+2*a))/b^2-3
/2*x*polylog(3,exp(2*b*x+2*a))/b^3+3/4*polylog(4,exp(2*b*x+2*a))/b^4
```

Rubi [A] time = 0.15, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {3716, 2190, 2531, 6609, 2282, 6589}

$$\frac{3x^2\text{PolyLog}\left(2,e^{2(a+bx)}\right)}{2b^2}-\frac{3x\text{PolyLog}\left(3,e^{2(a+bx)}\right)}{2b^3}+\frac{3\text{PolyLog}\left(4,e^{2(a+bx)}\right)}{4b^4}+\frac{x^3\log\left(1-e^{2(a+bx)}\right)}{b}-\frac{x^4}{4}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Coth[a + b*x],x]
```

```
[Out] -x^4/4 + (x^3*Log[1 - E^(2*(a + b*x))])/b + (3*x^2*PolyLog[2, E^(2*(a + b*x
))])/(2*b^2) - (3*x*PolyLog[3, E^(2*(a + b*x))])/(2*b^3) + (3*PolyLog[4, E^
(2*(a + b*x))])/(4*b^4)
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
```

```
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 3716

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2
*I, Int[((c + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(E^(2*I*k*Pi)*(1 + E^(2*(-(I*
e) + f*fz*x))/E^(2*I*k*Pi))), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Integ
erQ[4*k] && IGtQ[m, 0]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\int x^3\coth(a+bx)\,dx &= -\frac{x^4}{4}-2\int \frac{e^{2(a+bx)}x^3}{1-e^{2(a+bx)}}\,dx\\
&= -\frac{x^4}{4}+\frac{x^3\log\left(1-e^{2(a+bx)}\right)}{b}-\frac{3\int x^2\log\left(1-e^{2(a+bx)}\right)\,dx}{b}\\
&= -\frac{x^4}{4}+\frac{x^3\log\left(1-e^{2(a+bx)}\right)}{b}+\frac{3x^2\text{Li}_2\left(e^{2(a+bx)}\right)}{2b^2}-\frac{3\int x\text{Li}_2\left(e^{2(a+bx)}\right)\,dx}{b^2}\\
&= -\frac{x^4}{4}+\frac{x^3\log\left(1-e^{2(a+bx)}\right)}{b}+\frac{3x^2\text{Li}_2\left(e^{2(a+bx)}\right)}{2b^2}-\frac{3x\text{Li}_3\left(e^{2(a+bx)}\right)}{2b^3}+\frac{3\int \text{Li}_3\left(e^{2(a+bx)}\right)\,dx}{2b^3}\\
&= -\frac{x^4}{4}+\frac{x^3\log\left(1-e^{2(a+bx)}\right)}{b}+\frac{3x^2\text{Li}_2\left(e^{2(a+bx)}\right)}{2b^2}-\frac{3x\text{Li}_3\left(e^{2(a+bx)}\right)}{2b^3}+\frac{3\,\text{Subst}\left(\int\frac{\text{Li}_3(x)}{x}\,dx\right.}{4b^4}\\
&= -\frac{x^4}{4}+\frac{x^3\log\left(1-e^{2(a+bx)}\right)}{b}+\frac{3x^2\text{Li}_2\left(e^{2(a+bx)}\right)}{2b^2}-\frac{3x\text{Li}_3\left(e^{2(a+bx)}\right)}{2b^3}+\frac{3\text{Li}_4\left(e^{2(a+bx)}\right)}{4b^4}
\end{aligned}
$$

Mathematica [A] time = 0.01, size = 91, normalized size = 1.05

$$\frac{3\text{Li}_4\left(e^{2a+2bx}\right)}{4b^4}-\frac{3x\text{Li}_3\left(e^{2a+2bx}\right)}{2b^3}+\frac{3x^2\text{Li}_2\left(e^{2a+2bx}\right)}{2b^2}+\frac{x^3\log\left(1-e^{2a+2bx}\right)}{b}-\frac{x^4}{4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Coth[a + b*x],x]
```

```
[Out] -1/4*x^4 + (x^3*Log[1 - E^(2*a + 2*b*x)])/b + (3*x^2*PolyLog[2, E^(2*a + 2*
b*x)])/(2*b^2) - (3*x*PolyLog[3, E^(2*a + 2*b*x)])/(2*b^3) + (3*PolyLog[4,
E^(2*a + 2*b*x)])/(4*b^4)
```

fricas [C] time = 0.59, size = 216, normalized size = 2.48

$$-\frac{b^4x^4-4\,b^3x^3\log\left(\cosh\left(bx+a\right)+\sinh\left(bx+a\right)+1\right)-12\,b^2x^2\text{Li}_2\left(\cosh\left(bx+a\right)+\sinh\left(bx+a\right)\right)-12\,b^2x^2\text{Li}_2\left(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*cosh(b*x+a)*csch(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/4*(b^4*x^4 - 4*b^3*x^3*log(cosh(b*x + a) + sinh(b*x + a) + 1) - 12*b^2*x
^2*dilog(cosh(b*x + a) + sinh(b*x + a)) - 12*b^2*x^2*dilog(-cosh(b*x + a) -
 sinh(b*x + a)) + 4*a^3*log(cosh(b*x + a) + sinh(b*x + a) - 1) + 24*b*x*pol
ylog(3, cosh(b*x + a) + sinh(b*x + a)) + 24*b*x*polylog(3, -cosh(b*x + a) -
 sinh(b*x + a)) - 4*(b^3*x^3 + a^3)*log(-cosh(b*x + a) - sinh(b*x + a) + 1)
 - 24*polylog(4, cosh(b*x + a) + sinh(b*x + a)) - 24*polylog(4, -cosh(b*x +
 a) - sinh(b*x + a)))/b^4
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3\cosh\left(bx+a\right)\operatorname{csch}\left(bx+a\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*cosh(b*x+a)*csch(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(x^3*cosh(b*x + a)*csch(b*x + a), x)
```

maple [B] time = 0.28, size = 200, normalized size = 2.30

$$-\frac{x^4}{4}-\frac{6x\,\text{polylog}\left(3,\mathrm{e}^{bx+a}\right)}{b^3}+\frac{\ln\left(1+\mathrm{e}^{bx+a}\right)x^3}{b}+\frac{3x^2\,\text{polylog}\left(2,-\mathrm{e}^{bx+a}\right)}{b^2}-\frac{6x\,\text{polylog}\left(3,-\mathrm{e}^{bx+a}\right)}{b^3}+\frac{\ln\left(1-\mathrm{e}^{bx+a}\right)x}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3*cosh(b*x+a)*csch(b*x+a),x)

[Out] -1/4*x^4-6*x*polylog(3,exp(b*x+a))/b^3+1/b*ln(1+exp(b*x+a))*x^3+3*x^2*polylog(2,-exp(b*x+a))/b^2-6*x*polylog(3,-exp(b*x+a))/b^3+1/b*ln(1-exp(b*x+a))*x^3+3*x^2*polylog(2,exp(b*x+a))/b^2-2/b^3*a^3*x+6*polylog(4,-exp(b*x+a))/b^4+6*polylog(4,exp(b*x+a))/b^4-3/2/b^4*a^4+1/b^4*ln(1-exp(b*x+a))*a^3-1/b^4*a^3*ln(exp(b*x+a)-1)+2/b^4*a^3*ln(exp(b*x+a))

maxima [A] time = 0.38, size = 130, normalized size = 1.49

$$-\frac{1}{4}x^4+\frac{b^3x^3\log\left(e^{(bx+a)}+1\right)+3\,b^2x^2\mathrm{Li}_2\left(-e^{(bx+a)}\right)-6\,bx\mathrm{Li}_3(-e^{(bx+a)})+6\,\mathrm{Li}_4(-e^{(bx+a)})}{b^4}+\frac{b^3x^3\log\left(-e^{(bx+a)}+1\right)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*cosh(b*x+a)*csch(b*x+a),x, algorithm="maxima")

[Out] -1/4*x^4 + (b^3*x^3*log(e^(b*x + a) + 1) + 3*b^2*x^2*dilog(-e^(b*x + a)) - 6*b*x*polylog(3, -e^(b*x + a)) + 6*polylog(4, -e^(b*x + a)))/b^4 + (b^3*x^3*log(-e^(b*x + a) + 1) + 3*b^2*x^2*dilog(e^(b*x + a)) - 6*b*x*polylog(3, e^(b*x + a)) + 6*polylog(4, e^(b*x + a)))/b^4

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3\cosh(a+b\,x)}{\sinh(a+b\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^3*cosh(a + b*x))/sinh(a + b*x),x)

[Out] int((x^3*cosh(a + b*x))/sinh(a + b*x), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3\cosh(a+bx)\operatorname{csch}(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**3*cosh(b*x+a)*csch(b*x+a),x)

[Out] Integral(x**3*cosh(a + b*x)*csch(a + b*x), x)

3.399 $\int x^2 \coth(a+bx)\,dx$

Optimal. Leaf size=63

$$-\frac{\text{Li}_3\left(e^{2(a+bx)}\right)}{2b^3}+\frac{x\text{Li}_2\left(e^{2(a+bx)}\right)}{b^2}+\frac{x^2\log\left(1-e^{2(a+bx)}\right)}{b}-\frac{x^3}{3}$$

```
[Out] -1/3*x^3+x^2*ln(1-exp(2*b*x+2*a))/b+x*polylog(2,exp(2*b*x+2*a))/b^2-1/2*pol
ylog(3,exp(2*b*x+2*a))/b^3
```

Rubi [A] time = 0.13, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3716, 2190, 2531, 2282, 6589}

$$\frac{x\text{PolyLog}\left(2,e^{2(a+bx)}\right)}{b^2}-\frac{\text{PolyLog}\left(3,e^{2(a+bx)}\right)}{2b^3}+\frac{x^2\log\left(1-e^{2(a+bx)}\right)}{b}-\frac{x^3}{3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Coth[a + b*x],x]
```

```
[Out] -x^3/3 + (x^2*Log[1 - E^(2*(a + b*x))])/b + (x*PolyLog[2, E^(2*(a + b*x))])
/b^2 - PolyLog[3, E^(2*(a + b*x))]/(2*b^3)
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
```

```
, g, n}, x] && GtQ[m, 0]
```

Rule 3716

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2
*I, Int[((c + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(E^(2*I*k*Pi)*(1 + E^(2*(-(I*
e) + f*fz*x))/E^(2*I*k*Pi))), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Integ
erQ[4*k] && IGtQ[m, 0]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}
\int x^2 \coth(a+bx)\,dx &= -\frac{x^3}{3} - 2\int \frac{e^{2(a+bx)}x^2}{1-e^{2(a+bx)}}\,dx \\
&= -\frac{x^3}{3} + \frac{x^2\log\left(1-e^{2(a+bx)}\right)}{b} - \frac{2\int x\log\left(1-e^{2(a+bx)}\right)dx}{b} \\
&= -\frac{x^3}{3} + \frac{x^2\log\left(1-e^{2(a+bx)}\right)}{b} + \frac{x\text{Li}_2\left(e^{2(a+bx)}\right)}{b^2} - \frac{\int \text{Li}_2\left(e^{2(a+bx)}\right)dx}{b^2} \\
&= -\frac{x^3}{3} + \frac{x^2\log\left(1-e^{2(a+bx)}\right)}{b} + \frac{x\text{Li}_2\left(e^{2(a+bx)}\right)}{b^2} - \frac{\text{Subst}\left(\int \frac{\text{Li}_2(x)}{x}\,dx, x, e^{2(a+bx)}\right)}{2b^3} \\
&= -\frac{x^3}{3} + \frac{x^2\log\left(1-e^{2(a+bx)}\right)}{b} + \frac{x\text{Li}_2\left(e^{2(a+bx)}\right)}{b^2} - \frac{\text{Li}_3\left(e^{2(a+bx)}\right)}{2b^3}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 66, normalized size = 1.05

$$-\frac{\text{Li}_3\left(e^{2a+2bx}\right)}{2b^3} + \frac{x\text{Li}_2\left(e^{2a+2bx}\right)}{b^2} + \frac{x^2\log\left(1-e^{2a+2bx}\right)}{b} - \frac{x^3}{3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Coth[a + b*x],x]
```

```
[Out] -1/3*x^3 + (x^2*Log[1 - E^(2*a + 2*b*x)])/b + (x*PolyLog[2, E^(2*a + 2*b*x)
])/b^2 - PolyLog[3, E^(2*a + 2*b*x)]/(2*b^3)
```

fricas [C] time = 1.00, size = 168, normalized size = 2.67

$$-\frac{b^3x^3 - 3\,b^2x^2\log\left(\cosh\left(bx+a\right)+\sinh\left(bx+a\right)+1\right) - 6\,bx\mathrm{Li}_2\left(\cosh\left(bx+a\right)+\sinh\left(bx+a\right)\right) - 6\,bx\mathrm{Li}_2\left(-\cosh\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cosh(b*x+a)*csch(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/3*(b^3*x^3 - 3*b^2*x^2*log(cosh(b*x + a) + sinh(b*x + a) + 1) - 6*b*x*di
log(cosh(b*x + a) + sinh(b*x + a)) - 6*b*x*dilog(-cosh(b*x + a) - sinh(b*x
+ a)) - 3*a^2*log(cosh(b*x + a) + sinh(b*x + a) - 1) - 3*(b^2*x^2 - a^2)*lo
g(-cosh(b*x + a) - sinh(b*x + a) + 1) + 6*polylog(3, cosh(b*x + a) + sinh(b
*x + a)) + 6*polylog(3, -cosh(b*x + a) - sinh(b*x + a)))/b^3
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2\cosh\left(bx+a\right)\operatorname{csch}\left(bx+a\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cosh(b*x+a)*csch(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(x^2*cosh(b*x + a)*csch(b*x + a), x)
```

maple [B] time = 0.27, size = 166, normalized size = 2.63

$$-\frac{x^3}{3}+\frac{a^2\ln\left(\mathrm{e}^{bx+a}-1\right)}{b^3}-\frac{2a^2\ln\left(\mathrm{e}^{bx+a}\right)}{b^3}+\frac{2a^2x}{b^2}+\frac{4a^3}{3b^3}+\frac{\ln\left(1-\mathrm{e}^{bx+a}\right)x^2}{b}-\frac{\ln\left(1-\mathrm{e}^{bx+a}\right)a^2}{b^3}+\frac{2\,\mathrm{polylog}\left(2,\mathrm{e}^{bx+a}\right)x}{b^2}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*cosh(b*x+a)*csch(b*x+a),x)
```

```
[Out] -1/3*x^3+1/b^3*a^2*ln(exp(b*x+a)-1)-2/b^3*a^2*ln(exp(b*x+a))+2/b^2*a^2*x+4/
3/b^3*a^3+1/b*ln(1-exp(b*x+a))*x^2-1/b^3*ln(1-exp(b*x+a))*a^2+2/b^2*polylog
(2,exp(b*x+a))*x-2/b^3*polylog(3,exp(b*x+a))+1/b*ln(1+exp(b*x+a))*x^2+2/b^2
*polylog(2,-exp(b*x+a))*x-2/b^3*polylog(3,-exp(b*x+a))
```

maxima [A] time = 0.41, size = 96, normalized size = 1.52

$$-\frac{1}{3}\,x^3+\frac{b^2x^2\log\left(e^{(bx+a)}+1\right)+2\,bx\mathrm{Li}_2\left(-e^{(bx+a)}\right)-2\,\mathrm{Li}_3(-e^{(bx+a)})}{b^3}+\frac{b^2x^2\log\left(-e^{(bx+a)}+1\right)+2\,bx\mathrm{Li}_2\left(e^{(bx+a)}\right)-2}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cosh(b*x+a)*csch(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/3*x^3 + (b^2*x^2*log(e^(b*x + a) + 1) + 2*b*x*dilog(-e^(b*x + a)) - 2*po
lylog(3, -e^(b*x + a)))/b^3 + (b^2*x^2*log(-e^(b*x + a) + 1) + 2*b*x*dilog(
e^(b*x + a)) - 2*polylog(3, e^(b*x + a)))/b^3
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^2 \cosh(a+b\,x)}{\sinh(a+b\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*cosh(a + b*x))/sinh(a + b*x),x)
```

```
[Out] int((x^2*cosh(a + b*x))/sinh(a + b*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \cosh(a+bx)\operatorname{csch}(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*cosh(b*x+a)*csch(b*x+a),x)
```

```
[Out] Integral(x**2*cosh(a + b*x)*csch(a + b*x), x)
```

3.400 $\int x \coth(a+bx)\,dx$

Optimal. Leaf size=45

$$\frac{\text{Li}_2\left(e^{2(a+bx)}\right)}{2b^2}+\frac{x\log\left(1-e^{2(a+bx)}\right)}{b}-\frac{x^2}{2}$$

```
[Out] -1/2*x^2+x*ln(1-exp(2*b*x+2*a))/b+1/2*polylog(2,exp(2*b*x+2*a))/b^2
```

Rubi [A] time = 0.08, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3716, 2190, 2279, 2391}

$$\frac{\text{PolyLog}\left(2,e^{2(a+bx)}\right)}{2b^2}+\frac{x\log\left(1-e^{2(a+bx)}\right)}{b}-\frac{x^2}{2}$$

Antiderivative was successfully verified.

```
[In] Int[x*Coth[a + b*x],x]
```

```
[Out] -x^2/2 + (x*Log[1 - E^(2*(a + b*x))])/b + PolyLog[2, E^(2*(a + b*x))]/(2*b^
2)
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3716

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2
```

```
*I, Int[((c + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(E^(2*I*k*Pi)*(1 + E^(2*(-(I*
e) + f*fz*x))/E^(2*I*k*Pi))), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Integ
erQ[4*k] && IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\int x\coth(a+bx)\,dx &= -\frac{x^2}{2}-2\int\frac{e^{2(a+bx)}x}{1-e^{2(a+bx)}}\,dx\\ &= -\frac{x^2}{2}+\frac{x\log\left(1-e^{2(a+bx)}\right)}{b}-\frac{\int\log\left(1-e^{2(a+bx)}\right)\,dx}{b}\\ &= -\frac{x^2}{2}+\frac{x\log\left(1-e^{2(a+bx)}\right)}{b}-\frac{\text{Subst}\left(\int\frac{\log(1-x)}{x}\,dx,x,e^{2(a+bx)}\right)}{2b^2}\\ &= -\frac{x^2}{2}+\frac{x\log\left(1-e^{2(a+bx)}\right)}{b}+\frac{\text{Li}_2\left(e^{2(a+bx)}\right)}{2b^2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 47, normalized size = 1.04

$$\frac{\text{Li}_2\left(e^{2a+2bx}\right)}{2b^2}+\frac{x\log\left(1-e^{2a+2bx}\right)}{b}-\frac{x^2}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Coth[a + b*x],x]
```

```
[Out] -1/2*x^2 + (x*Log[1 - E^(2*a + 2*b*x)])/b + PolyLog[2, E^(2*a + 2*b*x)]/(2*
b^2)
```

fricas [B] time = 0.47, size = 112, normalized size = 2.49

$$-\frac{b^2x^2-2\,bx\log\left(\cosh\left(bx+a\right)+\sinh\left(bx+a\right)+1\right)+2\,a\log\left(\cosh\left(bx+a\right)+\sinh\left(bx+a\right)-1\right)-2\left(bx+a\right)\log\ldots}{\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x+a)*csch(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/2*(b^2*x^2 - 2*b*x*log(cosh(b*x + a) + sinh(b*x + a) + 1) + 2*a*log(cosh
(b*x + a) + sinh(b*x + a) - 1) - 2*(b*x + a)*log(-cosh(b*x + a) - sinh(b*x
+ a) + 1) - 2*dilog(cosh(b*x + a) + sinh(b*x + a)) - 2*dilog(-cosh(b*x + a)
 - sinh(b*x + a)))/b^2
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\cosh(bx+a)\operatorname{csch}(bx+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x+a)*csch(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(x*cosh(b*x + a)*csch(b*x + a), x)
```

maple [B] time = 0.26, size = 122, normalized size = 2.71

$$-\frac{x^2}{2}-\frac{2ax}{b}-\frac{a^2}{b^2}+\frac{\ln\left(1-e^{bx+a}\right)x}{b}+\frac{\ln\left(1-e^{bx+a}\right)a}{b^2}+\frac{\operatorname{polylog}\left(2,e^{bx+a}\right)}{b^2}+\frac{\ln\left(1+e^{bx+a}\right)x}{b}+\frac{\operatorname{polylog}\left(2,-e^{bx+a}\right)}{b^2}+\frac{2a}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cosh(b*x+a)*csch(b*x+a),x)
```

```
[Out] -1/2*x^2-2/b*a*x-a^2/b^2+1/b*ln(1-exp(b*x+a))*x+1/b^2*ln(1-exp(b*x+a))*a+po
lylog(2,exp(b*x+a))/b^2+1/b*ln(1+exp(b*x+a))*x+polylog(2,-exp(b*x+a))/b^2+2
/b^2*a*ln(exp(b*x+a))-1/b^2*a*ln(exp(b*x+a)-1)
```

maxima [A] time = 0.43, size = 58, normalized size = 1.29

$$-\frac{1}{2}x^2+\frac{bx\log\left(e^{(bx+a)}+1\right)+\operatorname{Li}_2\left(-e^{(bx+a)}\right)}{b^2}+\frac{bx\log\left(-e^{(bx+a)}+1\right)+\operatorname{Li}_2\left(e^{(bx+a)}\right)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x+a)*csch(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/2*x^2 + (b*x*log(e^(b*x + a) + 1) + dilog(-e^(b*x + a)))/b^2 + (b*x*log(
-e^(b*x + a) + 1) + dilog(e^(b*x + a)))/b^2
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x\cosh(a+b\,x)}{\sinh(a+b\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*cosh(a + b*x))/sinh(a + b*x),x)
```

```
[Out] int((x*cosh(a + b*x))/sinh(a + b*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\cosh(a+bx)\operatorname{csch}(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x+a)*csch(b*x+a),x)
```

```
[Out] Integral(x*cosh(a + b*x)*csch(a + b*x), x)
```

3.401 $\int \coth(a + bx)\, dx$

Optimal. Leaf size=11

$$\frac{\log(\sinh(a + bx))}{b}$$

```
[Out] ln(sinh(b*x+a))/b
```

Rubi [A] time = 0.01, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3475}

$$\frac{\log(\sinh(a + bx))}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[a + b*x],x]
```

```
[Out] Log[Sinh[a + b*x]]/b
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \coth(a + bx)\, dx = \frac{\log(\sinh(a + bx))}{b}$$

Mathematica [A] time = 0.01, size = 19, normalized size = 1.73

$$\frac{\log(\tanh(a + bx)) + \log(\cosh(a + bx))}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[a + b*x],x]
```

```
[Out] (Log[Cosh[a + b*x]] + Log[Tanh[a + b*x]])/b
```

fricas [B] time = 0.53, size = 37, normalized size = 3.36

$$-\frac{bx - \log\left(\frac{2\,\sinh(bx+a)}{\cosh(bx+a) - \sinh(bx+a)}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*csch(b*x+a),x, algorithm="fricas")
[Out] -(b*x - log(2*sinh(b*x + a)/(cosh(b*x + a) - sinh(b*x + a))))/b
```

giac [B] time = 0.12, size = 25, normalized size = 2.27

$$-\frac{bx + a - \log\left(\left|e^{(2\,bx+2\,a)} - 1\right|\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*csch(b*x+a),x, algorithm="giac")
[Out] -(b*x + a - log(abs(e^(2*b*x + 2*a) - 1)))/b
```

maple [A] time = 0.02, size = 13, normalized size = 1.18

$$-\frac{\ln\left(\operatorname{csch}\left(bx + a\right)\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)*csch(b*x+a),x)
[Out] -1/b*ln(csch(b*x+a))
```

maxima [B] time = 0.34, size = 23, normalized size = 2.09

$$\frac{\log\left(e^{(bx+a)} - e^{(-bx-a)}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*csch(b*x+a),x, algorithm="maxima")
[Out] log(e^(b*x + a) - e^(-b*x - a))/b
```

mupad [B] time = 0.06, size = 11, normalized size = 1.00

$$\frac{\ln\left(\sinh\left(a + b\,x\right)\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a + b*x)/sinh(a + b*x),x)
[Out] log(sinh(a + b*x))/b
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \cosh{(a+bx)}\operatorname{csch}{(a+bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*csch(b*x+a),x)
```

```
[Out] Integral(cosh(a + b*x)*csch(a + b*x), x)
```

3.402 $\int \frac{\coth(a+bx)}{x} dx$

Optimal. Leaf size=13

$$\text{Int}\left(\frac{\coth(a+bx)}{x},x\right)$$

```
[Out] Unintegrable(coth(b*x+a)/x,x)
```

Rubi [A] time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\coth(a+bx)}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Coth[a + b*x]/x,x]
[Out] Defer[Int][Coth[a + b*x]/x, x]
```

Rubi steps

$$\int \frac{\coth(a+bx)}{x}\,dx=\int \frac{\coth(a+bx)}{x}\,dx$$

Mathematica [A] time = 0.46, size = 0, normalized size = 0.00

$$\int \frac{\coth(a+bx)}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Coth[a + b*x]/x,x]
[Out] Integrate[Coth[a + b*x]/x, x]
```

fricas [A] time = 0.56, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\cosh(bx+a)\,\text{csch}(bx+a)}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*csch(b*x+a)/x,x, algorithm="fricas")
```

```
[Out] integral(cosh(b*x + a)*csch(b*x + a)/x, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh(bx+a)\operatorname{csch}(bx+a)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*csch(b*x+a)/x,x, algorithm="giac")
```

```
[Out] integrate(cosh(b*x + a)*csch(b*x + a)/x, x)
```

maple [A] time = 0.40, size = 0, normalized size = 0.00

$$\int \frac{\cosh(bx+a)\operatorname{csch}(bx+a)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)*csch(b*x+a)/x,x)
```

```
[Out] int(cosh(b*x+a)*csch(b*x+a)/x,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{xe^{(bx+a)}+x}\,dx + \int \frac{1}{xe^{(bx+a)}-x}\,dx + \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*csch(b*x+a)/x,x, algorithm="maxima")
```

```
[Out] -integrate(1/(x*e^(b*x + a) + x), x) + integrate(1/(x*e^(b*x + a) - x), x)
+ log(x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.08

$$\int \frac{\cosh(a+b\,x)}{x\sinh(a+b\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a + b*x)/(x*sinh(a + b*x)),x)
```

```
[Out] int(cosh(a + b*x)/(x*sinh(a + b*x)), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh(a+bx)\operatorname{csch}(a+bx)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*csch(b*x+a)/x,x)
```

```
[Out] Integral(cosh(a + b*x)*csch(a + b*x)/x, x)
```

3.403 $\int \frac{\coth(a+bx)}{x^2}\,dx$

Optimal. Leaf size=13

$$\text{Int}\left(\frac{\coth(a+bx)}{x^2},x\right)$$

```
[Out] Unintegrable(coth(b*x+a)/x^2,x)
```

Rubi [A] time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\coth(a+bx)}{x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Coth[a + b*x]/x^2,x]
```

```
[Out] Defer[Int][Coth[a + b*x]/x^2, x]
```

Rubi steps

$$\int \frac{\coth(a+bx)}{x^2}\,dx = \int \frac{\coth(a+bx)}{x^2}\,dx$$

Mathematica [A] time = 0.87, size = 0, normalized size = 0.00

$$\int \frac{\coth(a+bx)}{x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Coth[a + b*x]/x^2,x]
```

```
[Out] Integrate[Coth[a + b*x]/x^2, x]
```

fricas [A] time = 0.45, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\cosh(bx+a)\,\text{csch}(bx+a)}{x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*csch(b*x+a)/x^2,x, algorithm="fricas")
```

```
[Out] integral(cosh(b*x + a)*csch(b*x + a)/x^2, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh(bx+a)\operatorname{csch}(bx+a)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*csch(b*x+a)/x^2,x, algorithm="giac")
```

```
[Out] integrate(cosh(b*x + a)*csch(b*x + a)/x^2, x)
```

maple [A] time = 0.46, size = 0, normalized size = 0.00

$$\int \frac{\cosh(bx+a)\operatorname{csch}(bx+a)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)*csch(b*x+a)/x^2,x)
```

```
[Out] int(cosh(b*x+a)*csch(b*x+a)/x^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{x} - \int \frac{1}{x^2e^{(bx+a)} + x^2}\,dx + \int \frac{1}{x^2e^{(bx+a)} - x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*csch(b*x+a)/x^2,x, algorithm="maxima")
```

```
[Out] -1/x - integrate(1/(x^2*e^(b*x + a) + x^2), x) + integrate(1/(x^2*e^(b*x +
a) - x^2), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.08

$$\int \frac{\cosh(a+b\,x)}{x^2\sinh(a+b\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a + b*x)/(x^2*sinh(a + b*x)),x)
```

```
[Out] int(cosh(a + b*x)/(x^2*sinh(a + b*x)), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh(a+bx)\operatorname{csch}(a+bx)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*csch(b*x+a)/x**2,x)
```

```
[Out] Integral(cosh(a + b*x)*csch(a + b*x)/x**2, x)
```

3.404 $\int x^m \cosh(a+bx)\coth(a+bx)\,dx$

Optimal. Leaf size=72

$$\mathrm{Int}\left(x^m\mathrm{csch}(a+bx),x\right)+\frac{e^a x^m(-bx)^{-m}\Gamma(m+1,-bx)}{2b}+\frac{e^{-a}x^m(bx)^{-m}\Gamma(m+1,bx)}{2b}$$

```
[Out] 1/2*exp(a)*x^m*GAMMA(1+m,-b*x)/b/((-b*x)^m)+1/2*x^m*GAMMA(1+m,b*x)/b/exp(a)
/((b*x)^m)+Unintegrable(x^m*csch(b*x+a),x)
```

Rubi [A] time = 0.09, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int x^m \cosh(a+bx)\coth(a+bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x^m*Cosh[a + b*x]*Coth[a + b*x],x]
```

```
[Out] (E^a*x^m*Gamma[1 + m, -(b*x)])/(2*b*(-(b*x))^m) + (x^m*Gamma[1 + m, b*x])/(
2*b*E^a*(b*x)^m) + Defer[Int][x^m*Csch[a + b*x], x]
```

Rubi steps

$$\begin{aligned}\int x^m \cosh(a+bx)\coth(a+bx)\,dx &= \int x^m\mathrm{csch}(a+bx)\,dx+\int x^m\sinh(a+bx)\,dx\\ &= \frac{1}{2}\int e^{-i(ia+ibx)}x^m\,dx-\frac{1}{2}\int e^{i(ia+ibx)}x^m\,dx+\int x^m\mathrm{csch}(a+bx)\,dx\\ &= \frac{e^a x^m(-bx)^{-m}\Gamma(1+m,-bx)}{2b}+\frac{e^{-a}x^m(bx)^{-m}\Gamma(1+m,bx)}{2b}+\int x^m\mathrm{csch}(a+\end{aligned}$$

Mathematica [A] time = 19.59, size = 0, normalized size = 0.00

$$\int x^m \cosh(a+bx)\coth(a+bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^m*Cosh[a + b*x]*Coth[a + b*x],x]
```

```
[Out] Integrate[x^m*Cosh[a + b*x]*Coth[a + b*x], x]
```

fricas [A] time = 0.46, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(x^m\cosh\left(bx+a\right)^2\mathrm{csch}\left(bx+a\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*cosh(b*x+a)^2*csch(b*x+a),x, algorithm="fricas")
[Out] integral(x^m*cosh(b*x + a)^2*csch(b*x + a), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int x^m \cosh(bx+a)^2 \operatorname{csch}(bx+a)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*cosh(b*x+a)^2*csch(b*x+a),x, algorithm="giac")
[Out] integrate(x^m*cosh(b*x + a)^2*csch(b*x + a), x)
```

maple [A] time = 0.43, size = 0, normalized size = 0.00

$$\int x^m \left(\cosh^2(bx+a)\right) \operatorname{csch}(bx+a)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*cosh(b*x+a)^2*csch(b*x+a),x)
[Out] int(x^m*cosh(b*x+a)^2*csch(b*x+a),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int x^m \cosh(bx+a)^2 \operatorname{csch}(bx+a)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*cosh(b*x+a)^2*csch(b*x+a),x, algorithm="maxima")
[Out] integrate(x^m*cosh(b*x + a)^2*csch(b*x + a), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^m \cosh(a+b\,x)^2}{\sinh(a+b\,x)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^m*cosh(a + b*x)^2)/sinh(a + b*x),x)
[Out] int((x^m*cosh(a + b*x)^2)/sinh(a + b*x), x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*cosh(b*x+a)**2*csch(b*x+a),x)
```

```
[Out] Timed out
```

3.405 $\int x^3 \cosh(a + bx) \coth(a + bx) \, dx$

Optimal. Leaf size=165

$$-\frac{6\text{Li}_4\left(-e^{a+bx}\right)}{b^4}+\frac{6\text{Li}_4\left(e^{a+bx}\right)}{b^4}-\frac{6\sinh(a+bx)}{b^4}+\frac{6x\text{Li}_3\left(-e^{a+bx}\right)}{b^3}-\frac{6x\text{Li}_3\left(e^{a+bx}\right)}{b^3}+\frac{6x\cosh(a+bx)}{b^3}-\frac{3x^2\text{Li}_2\left(-e^{a+bx}\right)}{b^2}$$

```
[Out] -2*x^3*arctanh(exp(b*x+a))/b+6*x*cosh(b*x+a)/b^3+x^3*cosh(b*x+a)/b-3*x^2*po
lylog(2,-exp(b*x+a))/b^2+3*x^2*polylog(2,exp(b*x+a))/b^2+6*x*polylog(3,-exp
(b*x+a))/b^3-6*x*polylog(3,exp(b*x+a))/b^3-6*polylog(4,-exp(b*x+a))/b^4+6*p
olylog(4,exp(b*x+a))/b^4-6*sinh(b*x+a)/b^4-3*x^2*sinh(b*x+a)/b^2
```

Rubi [A] time = 0.18, antiderivative size = 165, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 8, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5450, 3296, 2637, 4182, 2531, 6609, 2282, 6589}

$$-\frac{3x^2\text{PolyLog}\left(2,-e^{a+bx}\right)}{b^2}+\frac{3x^2\text{PolyLog}\left(2,e^{a+bx}\right)}{b^2}+\frac{6x\text{PolyLog}\left(3,-e^{a+bx}\right)}{b^3}-\frac{6x\text{PolyLog}\left(3,e^{a+bx}\right)}{b^3}-\frac{6\text{PolyLog}\left(4\right.}{b^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Cosh[a + b*x]*Coth[a + b*x],x]
```

```
[Out] (-2*x^3*ArcTanh[E^(a + b*x)])/b + (6*x*Cosh[a + b*x])/b^3 + (x^3*Cosh[a + b
*x])/b - (3*x^2*PolyLog[2, -E^(a + b*x)])/b^2 + (3*x^2*PolyLog[2, E^(a + b*
x)])/b^2 + (6*x*PolyLog[3, -E^(a + b*x)])/b^3 - (6*x*PolyLog[3, E^(a + b*x)
])/b^3 - (6*PolyLog[4, -E^(a + b*x)])/b^4 + (6*PolyLog[4, E^(a + b*x)])/b^4
 - (6*Sinh[a + b*x])/b^4 - (3*x^2*Sinh[a + b*x])/b^2
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5450

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(n_.)*Coth[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) +
(d_.)*(x_))^(m_.), x_Symbol] :> Int[(c + d*x)^m*Cosh[a + b*x]^n*Coth[a + b*
x]^(p - 2), x] + Int[(c + d*x)^m*Cosh[a + b*x]^(n - 2)*Coth[a + b*x]^p, x]
/; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int x^3\cosh(a+bx)\coth(a+bx)\,dx = \int x^3\text{csch}(a+bx)\,dx + \int x^3\sinh(a+bx)\,dx$$
$$= -\frac{2x^3\tanh^{-1}\left(e^{a+bx}\right)}{b} + \frac{x^3\cosh(a+bx)}{b} - \frac{3\int x^2\cosh(a+bx)\,dx}{b} - \frac{3\int x^2 \text{l}$$
$$= -\frac{2x^3\tanh^{-1}\left(e^{a+bx}\right)}{b} + \frac{x^3\cosh(a+bx)}{b} - \frac{3x^2\text{Li}_2\left(-e^{a+bx}\right)}{b^2} + \frac{3x^2\text{Li}_2\left(e^{a+bx}\right)}{b^2}$$
$$= -\frac{2x^3\tanh^{-1}\left(e^{a+bx}\right)}{b} + \frac{6x\cosh(a+bx)}{b^3} + \frac{x^3\cosh(a+bx)}{b} - \frac{3x^2\text{Li}_2\left(-e^{a+b}\right.}{b^2}$$
$$= -\frac{2x^3\tanh^{-1}\left(e^{a+bx}\right)}{b} + \frac{6x\cosh(a+bx)}{b^3} + \frac{x^3\cosh(a+bx)}{b} - \frac{3x^2\text{Li}_2\left(-e^{a+b}\right.}{b^2}$$
$$= -\frac{2x^3\tanh^{-1}\left(e^{a+bx}\right)}{b} + \frac{6x\cosh(a+bx)}{b^3} + \frac{x^3\cosh(a+bx)}{b} - \frac{3x^2\text{Li}_2\left(-e^{a+b}\right.}{b^2}$$

Mathematica [A] time = 4.18, size = 202, normalized size = 1.22

$$\frac{b^3x^3\cosh(a+bx) - 2b^3x^3\tanh^{-1}(\sinh(a+bx)+\cosh(a+bx)) - 3b^2x^2\text{Li}_2(-\cosh(a+bx)-\sinh(a+bx)) + 3b}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^3*Cosh[a + b*x]*Coth[a + b*x],x]
```

```
[Out] (-2*b^3*x^3*ArcTanh[Cosh[a + b*x] + Sinh[a + b*x]] + 6*b*x*Cosh[a + b*x] +
b^3*x^3*Cosh[a + b*x] - 3*b^2*x^2*PolyLog[2, -Cosh[a + b*x] - Sinh[a + b*x]
] + 3*b^2*x^2*PolyLog[2, Cosh[a + b*x] + Sinh[a + b*x]] + 6*b*x*PolyLog[3,
-Cosh[a + b*x] - Sinh[a + b*x]] - 6*b*x*PolyLog[3, Cosh[a + b*x] + Sinh[a +
 b*x]] - 6*PolyLog[4, -Cosh[a + b*x] - Sinh[a + b*x]] + 6*PolyLog[4, Cosh[a
 + b*x] + Sinh[a + b*x]] - 6*Sinh[a + b*x] - 3*b^2*x^2*Sinh[a + b*x])/b^4
```

fricas [C] time = 0.80, size = 511, normalized size = 3.10

$$\frac{b^3x^3 + 3b^2x^2 + \left(b^3x^3 - 3b^2x^2 + 6bx - 6\right)\cosh(bx+a)^2 + 2\left(b^3x^3 - 3b^2x^2 + 6bx - 6\right)\cosh(bx+a)\sinh(bx+}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*cosh(b*x+a)^2*csch(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/2*(b^3*x^3 + 3*b^2*x^2 + (b^3*x^3 - 3*b^2*x^2 + 6*b*x - 6)*cosh(b*x + a)^
2 + 2*(b^3*x^3 - 3*b^2*x^2 + 6*b*x - 6)*cosh(b*x + a)*sinh(b*x + a) + (b^3*
```

```
x^3 - 3*b^2*x^2 + 6*b*x - 6)*sinh(b*x + a)^2 + 6*b*x + 6*(b^2*x^2*cosh(b*x
+ a) + b^2*x^2*sinh(b*x + a))*dilog(cosh(b*x + a) + sinh(b*x + a)) - 6*(b^2
*x^2*cosh(b*x + a) + b^2*x^2*sinh(b*x + a))*dilog(-cosh(b*x + a) - sinh(b*x
 + a)) - 2*(b^3*x^3*cosh(b*x + a) + b^3*x^3*sinh(b*x + a))*log(cosh(b*x + a
) + sinh(b*x + a) + 1) - 2*(a^3*cosh(b*x + a) + a^3*sinh(b*x + a))*log(cosh
(b*x + a) + sinh(b*x + a) - 1) + 2*((b^3*x^3 + a^3)*cosh(b*x + a) + (b^3*x^
3 + a^3)*sinh(b*x + a))*log(-cosh(b*x + a) - sinh(b*x + a) + 1) + 12*(cosh(
b*x + a) + sinh(b*x + a))*polylog(4, cosh(b*x + a) + sinh(b*x + a)) - 12*(c
osh(b*x + a) + sinh(b*x + a))*polylog(4, -cosh(b*x + a) - sinh(b*x + a)) -
12*(b*x*cosh(b*x + a) + b*x*sinh(b*x + a))*polylog(3, cosh(b*x + a) + sinh(
b*x + a)) + 12*(b*x*cosh(b*x + a) + b*x*sinh(b*x + a))*polylog(3, -cosh(b*x
 + a) - sinh(b*x + a)) + 6)/(b^4*cosh(b*x + a) + b^4*sinh(b*x + a))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3 \cosh(bx+a)^2 \operatorname{csch}(bx+a)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*cosh(b*x+a)^2*csch(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(x^3*cosh(b*x + a)^2*csch(b*x + a), x)
```

maple [A] time = 0.53, size = 246, normalized size = 1.49

$$\frac{\left(x^3b^3-3x^2b^2+6bx-6\right)e^{bx+a}}{2b^4}+\frac{\left(x^3b^3+3x^2b^2+6bx+6\right)e^{-bx-a}}{2b^4}-\frac{\ln\left(1+e^{bx+a}\right)x^3}{b}-\frac{3x^2\operatorname{polylog}\left(2,-e^{bx+a}\right)}{b^2}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*cosh(b*x+a)^2*csch(b*x+a),x)
```

```
[Out] 1/2*(b^3*x^3-3*b^2*x^2+6*b*x-6)/b^4*exp(b*x+a)+1/2*(b^3*x^3+3*b^2*x^2+6*b*x
+6)/b^4*exp(-b*x-a)-1/b*ln(1+exp(b*x+a))*x^3-3*x^2*polylog(2,-exp(b*x+a))/b
^2+6*x*polylog(3,-exp(b*x+a))/b^3+1/b*ln(1-exp(b*x+a))*x^3+3*x^2*polylog(2,
exp(b*x+a))/b^2-6*x*polylog(3,exp(b*x+a))/b^3-6*polylog(4,-exp(b*x+a))/b^4+
6*polylog(4,exp(b*x+a))/b^4-1/b^4*ln(1+exp(b*x+a))*a^3+1/b^4*ln(1-exp(b*x+a
))*a^3+2/b^4*a^3*arctanh(exp(b*x+a))
```

maxima [A] time = 0.46, size = 206, normalized size = 1.25

$$\frac{\left(\left(b^3x^3e^{(2a)}-3b^2x^2e^{(2a)}+6bxe^{(2a)}-6e^{(2a)}\right)e^{(bx)}+\left(b^3x^3+3b^2x^2+6bx+6\right)e^{(-bx)}\right)e^{(-a)}}{2b^4}-\frac{b^3x^3\log\left(e^{(bx+a)}+1\right)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*cosh(b*x+a)^2*csch(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/2*((b^3*x^3*e^(2*a) - 3*b^2*x^2*e^(2*a) + 6*b*x*e^(2*a) - 6*e^(2*a))*e^(b
*x) + (b^3*x^3 + 3*b^2*x^2 + 6*b*x + 6)*e^(-b*x))*e^(-a)/b^4 - (b^3*x^3*log
(e^(b*x + a) + 1) + 3*b^2*x^2*dilog(-e^(b*x + a)) - 6*b*x*polylog(3, -e^(b*
x + a)) + 6*polylog(4, -e^(b*x + a)))/b^4 + (b^3*x^3*log(-e^(b*x + a) + 1)
+ 3*b^2*x^2*dilog(e^(b*x + a)) - 6*b*x*polylog(3, e^(b*x + a)) + 6*polylog(
4, e^(b*x + a)))/b^4
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3 \cosh \left(a+b\,x\right)^2}{\sinh \left(a+b\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*cosh(a + b*x)^2)/sinh(a + b*x),x)
```

```
[Out] int((x^3*cosh(a + b*x)^2)/sinh(a + b*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^{3} \cosh^{2}{\left(a + bx \right)} \operatorname{csch}{\left(a + bx \right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*cosh(b*x+a)**2*csch(b*x+a),x)
```

```
[Out] Integral(x**3*cosh(a + b*x)**2*csch(a + b*x), x)
```

3.406 $\int x^2 \cosh(a+bx)\coth(a+bx)\,dx$

Optimal. Leaf size=115

$$\frac{2\text{Li}_3\left(-e^{a+bx}\right)}{b^3}-\frac{2\text{Li}_3\left(e^{a+bx}\right)}{b^3}+\frac{2\cosh(a+bx)}{b^3}-\frac{2x\text{Li}_2\left(-e^{a+bx}\right)}{b^2}+\frac{2x\text{Li}_2\left(e^{a+bx}\right)}{b^2}-\frac{2x\sinh(a+bx)}{b^2}+\frac{x^2\cosh(a+bx)}{b}$$

```
[Out] -2*x^2*arctanh(exp(b*x+a))/b+2*cosh(b*x+a)/b^3+x^2*cosh(b*x+a)/b-2*x*polylo
g(2,-exp(b*x+a))/b^2+2*x*polylog(2,exp(b*x+a))/b^2+2*polylog(3,-exp(b*x+a))
/b^3-2*polylog(3,exp(b*x+a))/b^3-2*x*sinh(b*x+a)/b^2
```

Rubi [A] time = 0.12, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {5450, 3296, 2638, 4182, 2531, 2282, 6589}

$$-\frac{2x\text{PolyLog}\left(2,-e^{a+bx}\right)}{b^2}+\frac{2x\text{PolyLog}\left(2,e^{a+bx}\right)}{b^2}+\frac{2\text{PolyLog}\left(3,-e^{a+bx}\right)}{b^3}-\frac{2\text{PolyLog}\left(3,e^{a+bx}\right)}{b^3}-\frac{2x\sinh(a+bx)}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Cosh[a + b*x]*Coth[a + b*x],x]
```

```
[Out] (-2*x^2*ArcTanh[E^(a + b*x)])/b + (2*Cosh[a + b*x])/b^3 + (x^2*Cosh[a + b*x
])/b - (2*x*PolyLog[2, -E^(a + b*x)])/b^2 + (2*x*PolyLog[2, E^(a + b*x)])/b
^2 + (2*PolyLog[3, -E^(a + b*x)])/b^3 - (2*PolyLog[3, E^(a + b*x)])/b^3 - (
2*x*Sinh[a + b*x])/b^2
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5450

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(n_.)*Coth[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) +
(d_.)*(x_))^(m_.), x_Symbol] :> Int[(c + d*x)^m*Cosh[a + b*x]^n*Coth[a + b*
x]^(p - 2), x] + Int[(c + d*x)^m*Cosh[a + b*x]^(n - 2)*Coth[a + b*x]^p, x]
/; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int x^2\cosh(a+bx)\coth(a+bx)\,dx = \int x^2\operatorname{csch}(a+bx)\,dx + \int x^2\sinh(a+bx)\,dx$$
$$= -\frac{2x^2\tanh^{-1}\left(e^{a+bx}\right)}{b} + \frac{x^2\cosh(a+bx)}{b} - \frac{2\int x\cosh(a+bx)\,dx}{b} - \frac{2\int x\operatorname{lo}}{}$$
$$= -\frac{2x^2\tanh^{-1}\left(e^{a+bx}\right)}{b} + \frac{x^2\cosh(a+bx)}{b} - \frac{2x\operatorname{Li}_2\left(-e^{a+bx}\right)}{b^2} + \frac{2x\operatorname{Li}_2\left(e^{a+bx}\right)}{b^2}$$
$$= -\frac{2x^2\tanh^{-1}\left(e^{a+bx}\right)}{b} + \frac{2\cosh(a+bx)}{b^3} + \frac{x^2\cosh(a+bx)}{b} - \frac{2x\operatorname{Li}_2\left(-e^{a+bx}\right.}{b^2}$$
$$= -\frac{2x^2\tanh^{-1}\left(e^{a+bx}\right)}{b} + \frac{2\cosh(a+bx)}{b^3} + \frac{x^2\cosh(a+bx)}{b} - \frac{2x\operatorname{Li}_2\left(-e^{a+bx}\right.}{b^2}$$

Mathematica [A] time = 4.08, size = 138, normalized size = 1.20

$$\frac{b^2x^2\cosh(a+bx) - 2b^2x^2\tanh^{-1}(\sinh(a+bx)+\cosh(a+bx)) - 2bx\operatorname{Li}_2(-\cosh(a+bx)-\sinh(a+bx)) + 2bx}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2*Cosh[a + b*x]*Coth[a + b*x],x]
```

```
[Out] (-2*b^2*x^2*ArcTanh[Cosh[a + b*x] + Sinh[a + b*x]] + 2*Cosh[a + b*x] + b^2*
x^2*Cosh[a + b*x] - 2*b*x*PolyLog[2, -Cosh[a + b*x] - Sinh[a + b*x]] + 2*b*
x*PolyLog[2, Cosh[a + b*x] + Sinh[a + b*x]] + 2*PolyLog[3, -Cosh[a + b*x] -
 Sinh[a + b*x]] - 2*PolyLog[3, Cosh[a + b*x] + Sinh[a + b*x]] - 2*b*x*Sinh[
a + b*x])/b^3
```

fricas [C] time = 0.55, size = 391, normalized size = 3.40

$$\frac{b^2x^2 + \left(b^2x^2 - 2\,bx + 2\right)\cosh\left(bx+a\right)^2 + 2\left(b^2x^2 - 2\,bx + 2\right)\cosh\left(bx+a\right)\sinh\left(bx+a\right) + \left(b^2x^2 - 2\,bx + 2\right)\sin}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cosh(b*x+a)^2*csch(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/2*(b^2*x^2 + (b^2*x^2 - 2*b*x + 2)*cosh(b*x + a)^2 + 2*(b^2*x^2 - 2*b*x +
 2)*cosh(b*x + a)*sinh(b*x + a) + (b^2*x^2 - 2*b*x + 2)*sinh(b*x + a)^2 + 2
*b*x + 4*(b*x*cosh(b*x + a) + b*x*sinh(b*x + a))*dilog(cosh(b*x + a) + sinh
(b*x + a)) - 4*(b*x*cosh(b*x + a) + b*x*sinh(b*x + a))*dilog(-cosh(b*x + a)
 - sinh(b*x + a)) - 2*(b^2*x^2*cosh(b*x + a) + b^2*x^2*sinh(b*x + a))*log(c
```

```
osh(b*x + a) + sinh(b*x + a) + 1) + 2*(a^2*cosh(b*x + a) + a^2*sinh(b*x + a
))*log(cosh(b*x + a) + sinh(b*x + a) - 1) + 2*((b^2*x^2 - a^2)*cosh(b*x + a
) + (b^2*x^2 - a^2)*sinh(b*x + a))*log(-cosh(b*x + a) - sinh(b*x + a) + 1)
- 4*(cosh(b*x + a) + sinh(b*x + a))*polylog(3, cosh(b*x + a) + sinh(b*x + a
)) + 4*(cosh(b*x + a) + sinh(b*x + a))*polylog(3, -cosh(b*x + a) - sinh(b*x
 + a)) + 2)/(b^3*cosh(b*x + a) + b^3*sinh(b*x + a))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \cosh(bx+a)^2 \operatorname{csch}(bx+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cosh(b*x+a)^2*csch(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(x^2*cosh(b*x + a)^2*csch(b*x + a), x)
```

maple [A] time = 0.51, size = 196, normalized size = 1.70

$$\frac{\left(x^2b^2-2bx+2\right)e^{bx+a}}{2b^3}+\frac{\left(x^2b^2+2bx+2\right)e^{-bx-a}}{2b^3}-\frac{2a^2\operatorname{arctanh}\left(e^{bx+a}\right)}{b^3}-\frac{\ln\left(1+e^{bx+a}\right)x^2}{b}+\frac{\ln\left(1+e^{bx+a}\right)a^2}{b^3}-\frac{2\,\mathrm{po}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*cosh(b*x+a)^2*csch(b*x+a),x)
```

```
[Out] 1/2*(b^2*x^2-2*b*x+2)/b^3*exp(b*x+a)+1/2*(b^2*x^2+2*b*x+2)/b^3*exp(-b*x-a)-
2/b^3*a^2*arctanh(exp(b*x+a))-1/b*ln(1+exp(b*x+a))*x^2+1/b^3*ln(1+exp(b*x+a
))*a^2-2/b^2*polylog(2,-exp(b*x+a))*x+2/b^3*polylog(3,-exp(b*x+a))+1/b*ln(1
-exp(b*x+a))*x^2-1/b^3*ln(1-exp(b*x+a))*a^2+2/b^2*polylog(2,exp(b*x+a))*x-2
/b^3*polylog(3,exp(b*x+a))
```

maxima [A] time = 0.43, size = 152, normalized size = 1.32

$$\frac{\left(\left(b^2x^2e^{(2\,a)}-2\,bxe^{(2\,a)}+2\,e^{(2\,a)}\right)e^{(bx)}+\left(b^2x^2+2\,bx+2\right)e^{(-bx)}\right)e^{(-a)}}{2\,b^3}-\frac{b^2x^2\log\left(e^{(bx+a)}+1\right)+2\,bx\mathrm{Li}_2\left(-e^{(bx+a)}\right)-2}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cosh(b*x+a)^2*csch(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/2*((b^2*x^2*e^(2*a) - 2*b*x*e^(2*a) + 2*e^(2*a))*e^(b*x) + (b^2*x^2 + 2*b
*x + 2)*e^(-b*x))*e^(-a)/b^3 - (b^2*x^2*log(e^(b*x + a) + 1) + 2*b*x*dilog(
-e^(b*x + a)) - 2*polylog(3, -e^(b*x + a)))/b^3 + (b^2*x^2*log(-e^(b*x + a)
 + 1) + 2*b*x*dilog(e^(b*x + a)) - 2*polylog(3, e^(b*x + a)))/b^3
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2\cosh\left(a+b\,x\right)^2}{\sinh\left(a+b\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*cosh(a + b*x)^2)/sinh(a + b*x),x)
```

```
[Out] int((x^2*cosh(a + b*x)^2)/sinh(a + b*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2\cosh^2\left(a+bx\right)\operatorname{csch}\left(a+bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*cosh(b*x+a)**2*csch(b*x+a),x)
```

```
[Out] Integral(x**2*cosh(a + b*x)**2*csch(a + b*x), x)
```

3.407 $\int x \cosh(a + bx) \coth(a + bx) \, dx$

Optimal. Leaf size=66

$$-\frac{\text{Li}_2\left(-e^{a+bx}\right)}{b^2}+\frac{\text{Li}_2\left(e^{a+bx}\right)}{b^2}-\frac{\sinh(a+bx)}{b^2}+\frac{x\cosh(a+bx)}{b}-\frac{2x\tanh^{-1}\left(e^{a+bx}\right)}{b}$$

```
[Out] -2*x*arctanh(exp(b*x+a))/b+x*cosh(b*x+a)/b-polylog(2,-exp(b*x+a))/b^2+polyl
og(2,exp(b*x+a))/b^2-sinh(b*x+a)/b^2
```

Rubi [A] time = 0.06, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {5450, 3296, 2637, 4182, 2279, 2391}

$$-\frac{\text{PolyLog}\left(2,-e^{a+bx}\right)}{b^2}+\frac{\text{PolyLog}\left(2,e^{a+bx}\right)}{b^2}-\frac{\sinh(a+bx)}{b^2}+\frac{x\cosh(a+bx)}{b}-\frac{2x\tanh^{-1}\left(e^{a+bx}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[x*Cosh[a + b*x]*Coth[a + b*x],x]
```

```
[Out] (-2*x*ArcTanh[E^(a + b*x)])/b + (x*Cosh[a + b*x])/b - PolyLog[2, -E^(a + b*
x)]/b^2 + PolyLog[2, E^(a + b*x)]/b^2 - Sinh[a + b*x]/b^2
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5450

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(n_.)*Coth[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) +
(d_.)*(x_))^(m_.), x_Symbol] :> Int[(c + d*x)^m*Cosh[a + b*x]^n*Coth[a + b*
x]^(p - 2), x] + Int[(c + d*x)^m*Cosh[a + b*x]^(n - 2)*Coth[a + b*x]^p, x]
/; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int x\cosh(a+bx)\coth(a+bx)\,dx &= \int x\mathrm{csch}(a+bx)\,dx+\int x\sinh(a+bx)\,dx\\ &= -\frac{2x\tanh^{-1}\left(e^{a+bx}\right)}{b}+\frac{x\cosh(a+bx)}{b}-\frac{\int\cosh(a+bx)\,dx}{b}-\frac{\int\log\left(1-e^{a+\cdots}\right.}{b}\\ &= -\frac{2x\tanh^{-1}\left(e^{a+bx}\right)}{b}+\frac{x\cosh(a+bx)}{b}-\frac{\sinh(a+bx)}{b^2}-\frac{\mathrm{Subst}\left(\int\frac{\log(1-x)}{x}\,d\cdots\right.}{b^2}\\ &= -\frac{2x\tanh^{-1}\left(e^{a+bx}\right)}{b}+\frac{x\cosh(a+bx)}{b}-\frac{\mathrm{Li}_2\left(-e^{a+bx}\right)}{b^2}+\frac{\mathrm{Li}_2\left(e^{a+bx}\right)}{b^2}-\frac{\sinh(\cdots}{}\end{aligned}$$

Mathematica [A] time = 0.14, size = 131, normalized size = 1.98

$$-\frac{-\mathrm{Li}_2\left(-e^{-a-bx}\right)+\mathrm{Li}_2\left(e^{-a-bx}\right)-a\log\left(1-e^{-a-bx}\right)-bx\log\left(1-e^{-a-bx}\right)+a\log\left(e^{-a-bx}+1\right)+bx\log\left(e^{-a-bx}+1\cdots\right.}{b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Cosh[a + b*x]*Coth[a + b*x],x]
```

```
[Out] -((-(b*x*Cosh[a + b*x]) - a*Log[1 - E^(-a - b*x)] - b*x*Log[1 - E^(-a - b*x
)] + a*Log[1 + E^(-a - b*x)] + b*x*Log[1 + E^(-a - b*x)] + a*Log[Tanh[(a +
b*x)/2]] - PolyLog[2, -E^(-a - b*x)] + PolyLog[2, E^(-a - b*x)] + Sinh[a +
b*x])/b^2)
```

fricas [B] time = 0.83, size = 255, normalized size = 3.86

$$\frac{(bx-1)\cosh(bx+a)^2 + 2(bx-1)\cosh(bx+a)\sinh(bx+a) + (bx-1)\sinh(bx+a)^2 + bx + 2(\cosh(bx+a) -}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x+a)^2*csch(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/2*((b*x - 1)*cosh(b*x + a)^2 + 2*(b*x - 1)*cosh(b*x + a)*sinh(b*x + a) +
(b*x - 1)*sinh(b*x + a)^2 + b*x + 2*(cosh(b*x + a) + sinh(b*x + a))*dilog(c
osh(b*x + a) + sinh(b*x + a)) - 2*(cosh(b*x + a) + sinh(b*x + a))*dilog(-co
sh(b*x + a) - sinh(b*x + a)) - 2*(b*x*cosh(b*x + a) + b*x*sinh(b*x + a))*lo
g(cosh(b*x + a) + sinh(b*x + a) + 1) - 2*(a*cosh(b*x + a) + a*sinh(b*x + a)
)*log(cosh(b*x + a) + sinh(b*x + a) - 1) + 2*((b*x + a)*cosh(b*x + a) + (b*
x + a)*sinh(b*x + a))*log(-cosh(b*x + a) - sinh(b*x + a) + 1) + 1)/(b^2*cos
h(b*x + a) + b^2*sinh(b*x + a))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\cosh(bx+a)^2\operatorname{csch}(bx+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x+a)^2*csch(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(x*cosh(b*x + a)^2*csch(b*x + a), x)
```

maple [B] time = 0.52, size = 139, normalized size = 2.11

$$\frac{(bx-1)\,e^{bx+a}}{2b^2}+\frac{(bx+1)\,e^{-bx-a}}{2b^2}-\frac{\ln\left(1+e^{bx+a}\right)x}{b}-\frac{\ln\left(1+e^{bx+a}\right)a}{b^2}-\frac{\operatorname{polylog}\left(2,-e^{bx+a}\right)}{b^2}+\frac{\ln\left(1-e^{bx+a}\right)x}{b}+\frac{\ln\left(1-\right.}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cosh(b*x+a)^2*csch(b*x+a),x)
```

```
[Out] 1/2*(b*x-1)/b^2*exp(b*x+a)+1/2*(b*x+1)/b^2*exp(-b*x-a)-1/b*ln(1+exp(b*x+a))
*x-1/b^2*ln(1+exp(b*x+a))*a-polylog(2,-exp(b*x+a))/b^2+1/b*ln(1-exp(b*x+a))
*x+1/b^2*ln(1-exp(b*x+a))*a+polylog(2,exp(b*x+a))/b^2+2/b^2*a*arctanh(exp(b
*x+a))
```

maxima [A] time = 0.43, size = 94, normalized size = 1.42

$$\frac{\left(\left(bxe^{(2a)}-e^{(2a)}\right)e^{(bx)}+(bx+1)e^{(-bx)}\right)e^{(-a)}}{2\,b^2}-\frac{bx\log\left(e^{(bx+a)}+1\right)+\operatorname{Li}_2\left(-e^{(bx+a)}\right)}{b^2}+\frac{bx\log\left(-e^{(bx+a)}+1\right)+\operatorname{Li}_2\left(e^{(bx+\right.}}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x+a)^2*csch(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/2*((b*x*e^(2*a) - e^(2*a))*e^(b*x) + (b*x + 1)*e^(-b*x))*e^(-a)/b^2 - (b*
x*log(e^(b*x + a) + 1) + dilog(-e^(b*x + a)))/b^2 + (b*x*log(-e^(b*x + a) +
 1) + dilog(e^(b*x + a)))/b^2
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x\cosh\left(a+b\,x\right)^2}{\sinh\left(a+b\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*cosh(a + b*x)^2)/sinh(a + b*x),x)
```

```
[Out] int((x*cosh(a + b*x)^2)/sinh(a + b*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\cosh^2\left(a+bx\right)\operatorname{csch}\left(a+bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x+a)**2*csch(b*x+a),x)
```

```
[Out] Integral(x*cosh(a + b*x)**2*csch(a + b*x), x)
```

3.408 $\int \cosh(a + bx) \coth(a + bx) \, dx$

Optimal. Leaf size=23

$$\frac{\cosh(a+bx)}{b} - \frac{\tanh^{-1}(\cosh(a+bx))}{b}$$

```
[Out] -arctanh(cosh(b*x+a))/b+cosh(b*x+a)/b
```

Rubi [A] time = 0.02, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {2592, 321, 206}

$$\frac{\cosh(a+bx)}{b} - \frac{\tanh^{-1}(\cosh(a+bx))}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[a + b*x]*Coth[a + b*x],x]
```

```
[Out] -(ArcTanh[Cosh[a + b*x]]/b) + Cosh[a + b*x]/b
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2592

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, (a*Sin[e + f*x])/ff], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rubi steps

$$\int \cosh(a+bx)\coth(a+bx)\,dx = -\frac{\text{Subst}\left(\int \frac{x^2}{1-x^2}\,dx, x, \cosh(a+bx)\right)}{b}$$
$$= \frac{\cosh(a+bx)}{b} - \frac{\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \cosh(a+bx)\right)}{b}$$
$$= -\frac{\tanh^{-1}(\cosh(a+bx))}{b} + \frac{\cosh(a+bx)}{b}$$

Mathematica [A] time = 0.02, size = 26, normalized size = 1.13

$$\frac{\cosh(a+bx)}{b} + \frac{\log\left(\tanh\left(\frac{1}{2}(a+bx)\right)\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[a + b*x]*Coth[a + b*x],x]
```

```
[Out] Cosh[a + b*x]/b + Log[Tanh[(a + b*x)/2]]/b
```

fricas [B] time = 0.74, size = 113, normalized size = 4.91

$$\frac{\cosh(bx+a)^2 - 2(\cosh(bx+a)+\sinh(bx+a))\log(\cosh(bx+a)+\sinh(bx+a)+1) + 2(\cosh(bx+a)+s}{2(b\cosh(bx+a)+b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^2*csch(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/2*(cosh(b*x + a)^2 - 2*(cosh(b*x + a) + sinh(b*x + a))*log(cosh(b*x + a)
+ sinh(b*x + a) + 1) + 2*(cosh(b*x + a) + sinh(b*x + a))*log(cosh(b*x + a)
+ sinh(b*x + a) - 1) + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 + 1)
/(b*cosh(b*x + a) + b*sinh(b*x + a))
```

giac [A] time = 0.12, size = 44, normalized size = 1.91

$$\frac{e^{(bx+a)} + e^{(-bx-a)} - 2\,\log\left(e^{(bx+a)}+1\right) + 2\,\log\left(\left|e^{(bx+a)}-1\right|\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^2*csch(b*x+a),x, algorithm="giac")
```

```
[Out] 1/2*(e^(b*x + a) + e^(-b*x - a) - 2*log(e^(b*x + a) + 1) + 2*log(abs(e^(b*x
 + a) - 1)))/b
```

maple [A] time = 0.11, size = 21, normalized size = 0.91

$$\frac{\cosh\left(bx+a\right)-2\,\mathrm{arctanh}\left(\mathrm{e}^{bx+a}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)^2*csch(b*x+a),x)
```

```
[Out] 1/b*(cosh(b*x+a)-2*arctanh(exp(b*x+a)))
```

maxima [B] time = 0.35, size = 59, normalized size = 2.57

$$\frac{e^{(bx+a)}}{2\,b}+\frac{e^{(-bx-a)}}{2\,b}-\frac{\log\left(e^{(-bx-a)}+1\right)}{b}+\frac{\log\left(e^{(-bx-a)}-1\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^2*csch(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/2*e^(b*x + a)/b + 1/2*e^(-b*x - a)/b - log(e^(-b*x - a) + 1)/b + log(e^(-
b*x - a) - 1)/b
```

mupad [B] time = 0.09, size = 53, normalized size = 2.30

$$\frac{\mathrm{e}^{a+b\,x}}{2\,b}-\frac{2\,\mathrm{atan}\left(\frac{\mathrm{e}^{b\,x}\,\mathrm{e}^{a}\,\sqrt{-b^2}}{b}\right)}{\sqrt{-b^2}}+\frac{\mathrm{e}^{-a-b\,x}}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a + b*x)^2/sinh(a + b*x),x)
```

```
[Out] exp(a + b*x)/(2*b) - (2*atan((exp(b*x)*exp(a)*(-b^2)^(1/2))/b))/(-b^2)^(1/2
) + exp(- a - b*x)/(2*b)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \cosh^{2}\left(a+bx\right)\operatorname{csch}\left(a+bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)**2*csch(b*x+a),x)
```

```
[Out] Integral(cosh(a + b*x)**2*csch(a + b*x), x)
```

3.409 $\int \frac{\cosh(a+bx)\coth(a+bx)}{x}\,dx$

Optimal. Leaf size=28

$$\operatorname{Int}\left(\frac{\operatorname{csch}(a+bx)}{x},x\right)+\sinh(a)\operatorname{Chi}(bx)+\cosh(a)\operatorname{Shi}(bx)$$

```
[Out] cosh(a)*Shi(b*x)+Chi(b*x)*sinh(a)+Unintegrable(csch(b*x+a)/x,x)
```

Rubi [A] time = 0.08, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\cosh(a+bx)\coth(a+bx)}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Cosh[a + b*x]*Coth[a + b*x])/x,x]
```

```
[Out] CoshIntegral[b*x]*Sinh[a] + Cosh[a]*SinhIntegral[b*x] + Defer[Int][Csch[a +
 b*x]/x, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cosh(a+bx)\coth(a+bx)}{x}\,dx &= \int \frac{\operatorname{csch}(a+bx)}{x}\,dx+\int \frac{\sinh(a+bx)}{x}\,dx\\
&= \cosh(a)\int \frac{\sinh(bx)}{x}\,dx+\sinh(a)\int \frac{\cosh(bx)}{x}\,dx+\int \frac{\operatorname{csch}(a+bx)}{x}\,dx\\
&= \operatorname{Chi}(bx)\sinh(a)+\cosh(a)\operatorname{Shi}(bx)+\int \frac{\operatorname{csch}(a+bx)}{x}\,dx
\end{aligned}$$

Mathematica [A] time = 26.62, size = 0, normalized size = 0.00

$$\int \frac{\cosh(a+bx)\coth(a+bx)}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Cosh[a + b*x]*Coth[a + b*x])/x,x]
```

```
[Out] Integrate[(Cosh[a + b*x]*Coth[a + b*x])/x, x]
```

fricas [A] time = 0.61, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{\cosh\left(bx+a\right)^{2}\operatorname{csch}\left(bx+a\right)}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^2*csch(b*x+a)/x,x, algorithm="fricas")
```

```
[Out] integral(cosh(b*x + a)^2*csch(b*x + a)/x, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh(bx+a)^2 \operatorname{csch}(bx+a)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^2*csch(b*x+a)/x,x, algorithm="giac")
```

```
[Out] integrate(cosh(b*x + a)^2*csch(b*x + a)/x, x)
```

maple [A] time = 0.57, size = 0, normalized size = 0.00

$$\int \frac{\left(\cosh^2(bx+a)\right) \operatorname{csch}(bx+a)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)^2*csch(b*x+a)/x,x)
```

```
[Out] int(cosh(b*x+a)^2*csch(b*x+a)/x,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh(bx+a)^2 \operatorname{csch}(bx+a)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^2*csch(b*x+a)/x,x, algorithm="maxima")
```

```
[Out] integrate(cosh(b*x + a)^2*csch(b*x + a)/x, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\cosh(a+b\,x)^2}{x\sinh(a+b\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a + b*x)^2/(x*sinh(a + b*x)),x)
```

```
[Out] int(cosh(a + b*x)^2/(x*sinh(a + b*x)), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh^2(a+bx)\operatorname{csch}(a+bx)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)**2*csch(b*x+a)/x,x)
```

```
[Out] Integral(cosh(a + b*x)**2*csch(a + b*x)/x, x)
```

3.410 $\int \frac{\cosh(a+bx)\coth(a+bx)}{x^2}\,dx$

Optimal. Leaf size=41

$$\text{Int}\left(\frac{\text{csch}(a+bx)}{x^2},x\right)+b\cosh(a)\text{Chi}(bx)+b\sinh(a)\text{Shi}(bx)-\frac{\sinh(a+bx)}{x}$$

```
[Out] b*Chi(b*x)*cosh(a)+b*Shi(b*x)*sinh(a)-sinh(b*x+a)/x+Unintegrable(csch(b*x+a
)/x^2,x)
```

Rubi [A] time = 0.10, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\cosh(a+bx)\coth(a+bx)}{x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Cosh[a + b*x]*Coth[a + b*x])/x^2,x]
```

```
[Out] b*Cosh[a]*CoshIntegral[b*x] - Sinh[a + b*x]/x + b*Sinh[a]*SinhIntegral[b*x]
 + Defer[Int][Csch[a + b*x]/x^2, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cosh(a+bx)\coth(a+bx)}{x^2}\,dx &= \int \frac{\text{csch}(a+bx)}{x^2}\,dx+\int \frac{\sinh(a+bx)}{x^2}\,dx\\
&= -\frac{\sinh(a+bx)}{x}+b\int \frac{\cosh(a+bx)}{x}\,dx+\int \frac{\text{csch}(a+bx)}{x^2}\,dx\\
&= -\frac{\sinh(a+bx)}{x}+(b\cosh(a))\int \frac{\cosh(bx)}{x}\,dx+(b\sinh(a))\int \frac{\sinh(bx)}{x}\,dx+\int\\
&= b\cosh(a)\text{Chi}(bx)-\frac{\sinh(a+bx)}{x}+b\sinh(a)\text{Shi}(bx)+\int \frac{\text{csch}(a+bx)}{x^2}\,dx
\end{aligned}$$

Mathematica [A] time = 38.16, size = 0, normalized size = 0.00

$$\int \frac{\cosh(a+bx)\coth(a+bx)}{x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Cosh[a + b*x]*Coth[a + b*x])/x^2,x]
```

```
[Out] Integrate[(Cosh[a + b*x]*Coth[a + b*x])/x^2, x]
```

fricas [A] time = 0.39, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\cosh\left(bx+a\right)^{2}\operatorname{csch}\left(bx+a\right)}{x^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^2*csch(b*x+a)/x^2,x, algorithm="fricas")
[Out] integral(cosh(b*x + a)^2*csch(b*x + a)/x^2, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh\left(bx+a\right)^{2}\operatorname{csch}\left(bx+a\right)}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^2*csch(b*x+a)/x^2,x, algorithm="giac")
[Out] integrate(cosh(b*x + a)^2*csch(b*x + a)/x^2, x)
```

maple [A] time = 0.59, size = 0, normalized size = 0.00

$$\int \frac{\left(\cosh^{2}\left(bx+a\right)\right)\operatorname{csch}\left(bx+a\right)}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)^2*csch(b*x+a)/x^2,x)
[Out] int(cosh(b*x+a)^2*csch(b*x+a)/x^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh\left(bx+a\right)^{2}\operatorname{csch}\left(bx+a\right)}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^2*csch(b*x+a)/x^2,x, algorithm="maxima")
[Out] integrate(cosh(b*x + a)^2*csch(b*x + a)/x^2, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{{\cosh\left(a+b\,x\right)}^2}{x^2\,\sinh\left(a+b\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a + b*x)^2/(x^2*sinh(a + b*x)),x)
```

```
[Out] int(cosh(a + b*x)^2/(x^2*sinh(a + b*x)), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh^2(a+bx)\operatorname{csch}(a+bx)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)**2*csch(b*x+a)/x**2,x)
```

```
[Out] Integral(cosh(a + b*x)**2*csch(a + b*x)/x**2, x)
```

3.411 $\int x^m \cosh^2(a+bx)\coth(a+bx)\,dx$

Optimal. Leaf size=83

$$\text{Int}\left(x^m\coth(a+bx),x\right)+\frac{e^{2a}2^{-m-3}x^m(-bx)^{-m}\Gamma(m+1,-2bx)}{b}+\frac{e^{-2a}2^{-m-3}x^m(bx)^{-m}\Gamma(m+1,2bx)}{b}$$

```
[Out] 2^(-3-m)*exp(2*a)*x^m*GAMMA(1+m,-2*b*x)/b/((-b*x)^m)+2^(-3-m)*x^m*GAMMA(1+m
,2*b*x)/b/exp(2*a)/((b*x)^m)+Unintegrable(x^m*coth(b*x+a),x)
```

Rubi [A] time = 0.13, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int x^m\cosh^2(a+bx)\coth(a+bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x^m*Cosh[a + b*x]^2*Coth[a + b*x],x]
```

```
[Out] (2^(-3 - m)*E^(2*a)*x^m*Gamma[1 + m, -2*b*x])/(b*(-(b*x))^m) + (2^(-3 - m)*
x^m*Gamma[1 + m, 2*b*x])/(b*E^(2*a)*(b*x)^m) + Defer[Int][x^m*Coth[a + b*x]
, x]
```

Rubi steps

$$\begin{aligned}
\int x^m\cosh^2(a+bx)\coth(a+bx)\,dx &= \int x^m\coth(a+bx)\,dx+\int x^m\cosh(a+bx)\sinh(a+bx)\,dx\\
&= \int x^m\coth(a+bx)\,dx+\int \frac{1}{2}x^m\sinh(2a+2bx)\,dx\\
&= \frac{1}{2}\int x^m\sinh(2a+2bx)\,dx+\int x^m\coth(a+bx)\,dx\\
&= \frac{1}{4}\int e^{-i(2ia+2ibx)}x^m\,dx-\frac{1}{4}\int e^{i(2ia+2ibx)}x^m\,dx+\int x^m\coth(a+bx)\,dx\\
&= \frac{2^{-3-m}e^{2a}x^m(-bx)^{-m}\Gamma(1+m,-2bx)}{b}+\frac{2^{-3-m}e^{-2a}x^m(bx)^{-m}\Gamma(1+m,2bx)}{b}
\end{aligned}$$

Mathematica [A] time = 23.54, size = 0, normalized size = 0.00

$$\int x^m\cosh^2(a+bx)\coth(a+bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^m*Cosh[a + b*x]^2*Coth[a + b*x],x]
[Out] Integrate[x^m*Cosh[a + b*x]^2*Coth[a + b*x], x]
```

fricas [A] time = 0.54, size = 0, normalized size = 0.00

$$\text{integral}\left(x^m \cosh\left(bx + a\right)^3 \operatorname{csch}\left(bx + a\right), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*cosh(b*x+a)^3*csch(b*x+a),x, algorithm="fricas")
[Out] integral(x^m*cosh(b*x + a)^3*csch(b*x + a), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int x^m \cosh\left(bx + a\right)^3 \operatorname{csch}\left(bx + a\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*cosh(b*x+a)^3*csch(b*x+a),x, algorithm="giac")
[Out] integrate(x^m*cosh(b*x + a)^3*csch(b*x + a), x)
```

maple [A] time = 0.50, size = 0, normalized size = 0.00

$$\int x^m \left(\cosh^3\left(bx + a\right)\right) \operatorname{csch}\left(bx + a\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*cosh(b*x+a)^3*csch(b*x+a),x)
[Out] int(x^m*cosh(b*x+a)^3*csch(b*x+a),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int x^m \cosh\left(bx + a\right)^3 \operatorname{csch}\left(bx + a\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*cosh(b*x+a)^3*csch(b*x+a),x, algorithm="maxima")
[Out] integrate(x^m*cosh(b*x + a)^3*csch(b*x + a), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^m \cosh\left(a + b\,x\right)^3}{\sinh\left(a + b\,x\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^m*cosh(a + b*x)^3)/sinh(a + b*x),x)
```

```
[Out] int((x^m*cosh(a + b*x)^3)/sinh(a + b*x), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*cosh(b*x+a)**3*csch(b*x+a),x)
```

```
[Out] Timed out
```

3.412 $\int x^3 \cosh^2(a + bx) \coth(a + bx) \, dx$

Optimal. Leaf size=180

$$\frac{3\text{Li}_4\left(e^{2(a+bx)}\right)}{4b^4}-\frac{3\sinh(a+bx)\cosh(a+bx)}{8b^4}-\frac{3x\text{Li}_3\left(e^{2(a+bx)}\right)}{2b^3}+\frac{3x\sinh^2(a+bx)}{4b^3}+\frac{3x^2\text{Li}_2\left(e^{2(a+bx)}\right)}{2b^2}-\frac{3x^2\sinh(a+$$

```
[Out] 3/8*x/b^3+1/4*x^3/b-1/4*x^4+x^3*ln(1-exp(2*b*x+2*a))/b+3/2*x^2*polylog(2,ex
p(2*b*x+2*a))/b^2-3/2*x*polylog(3,exp(2*b*x+2*a))/b^3+3/4*polylog(4,exp(2*b
*x+2*a))/b^4-3/8*cosh(b*x+a)*sinh(b*x+a)/b^4-3/4*x^2*cosh(b*x+a)*sinh(b*x+a
)/b^2+3/4*x*sinh(b*x+a)^2/b^3+1/2*x^3*sinh(b*x+a)^2/b
```

Rubi [A] time = 0.24, antiderivative size = 180, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 12, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {5450, 5372, 3311, 30, 2635, 8, 3716, 2190, 2531, 6609, 2282, 6589}

$$\frac{3x^2\text{PolyLog}\left(2,e^{2(a+bx)}\right)}{2b^2}-\frac{3x\text{PolyLog}\left(3,e^{2(a+bx)}\right)}{2b^3}+\frac{3\text{PolyLog}\left(4,e^{2(a+bx)}\right)}{4b^4}-\frac{3x^2\sinh(a+bx)\cosh(a+bx)}{4b^2}+\frac{3x\,\text{si}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Cosh[a + b*x]^2*Coth[a + b*x],x]
```

```
[Out] (3*x)/(8*b^3) + x^3/(4*b) - x^4/4 + (x^3*Log[1 - E^(2*(a + b*x))])/b + (3*x
^2*PolyLog[2, E^(2*(a + b*x))])/(2*b^2) - (3*x*PolyLog[3, E^(2*(a + b*x))])
/(2*b^3) + (3*PolyLog[4, E^(2*(a + b*x))])/(4*b^4) - (3*Cosh[a + b*x]*Sinh[
a + b*x])/(8*b^4) - (3*x^2*Cosh[a + b*x]*Sinh[a + b*x])/(4*b^2) + (3*x*Sinh
[a + b*x]^2)/(4*b^3) + (x^3*Sinh[a + b*x]^2)/(2*b)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
```

```
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 3311

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(d*m*(c + d*x)^(m - 1)*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist
[(b^2*(n - 1))/n, Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[(
d^2*m*(m - 1))/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[(b*(c + d*x)^m*Cos[e + f*x]*(b*Sin[e + f*x])^(n - 1))/(f*n), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 3716

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2
*I, Int[((c + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(E^(2*I*k*Pi)*(1 + E^(2*(-(I*
e) + f*fz*x))/E^(2*I*k*Pi))), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Integ
erQ[4*k] && IGtQ[m, 0]
```

Rule 5372

```
Int[Cosh[(a_.) + (b_.)*(x_)^(n_.)]*(x_)^(m_.)*Sinh[(a_.) + (b_.)*(x_)^(n_.)
]^(p_.), x_Symbol] :> Simp[(x^(m - n + 1)*Sinh[a + b*x^n]^(p + 1))/(b*n*(p
 + 1)), x] - Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*Sinh[a + b*x^n]^(
p + 1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

Rule 5450

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(n_.)*Coth[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) +
(d_.)*(x_))^(m_.), x_Symbol] :> Int[(c + d*x)^m*Cosh[a + b*x]^n*Coth[a + b*
x]^(p - 2), x] + Int[(c + d*x)^m*Cosh[a + b*x]^(n - 2)*Coth[a + b*x]^p, x]
/; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int x^3\cosh^2(a+bx)\coth(a+bx)\,dx = \int x^3\coth(a+bx)\,dx + \int x^3\cosh(a+bx)\sinh(a+bx)\,dx$$
$$= -\frac{x^4}{4} + \frac{x^3\sinh^2(a+bx)}{2b} - 2\int \frac{e^{2(a+bx)}x^3}{1-e^{2(a+bx)}}\,dx - \frac{3\int x^2\sinh^2(a+bx)\,dx}{2b}$$
$$= -\frac{x^4}{4} + \frac{x^3\log\left(1-e^{2(a+bx)}\right)}{b} - \frac{3x^2\cosh(a+bx)\sinh(a+bx)}{4b^2} + \frac{3x\sinh^2(}{4b^3}$$
$$= \frac{x^3}{4b} - \frac{x^4}{4} + \frac{x^3\log\left(1-e^{2(a+bx)}\right)}{b} + \frac{3x^2\text{Li}_2\left(e^{2(a+bx)}\right)}{2b^2} - \frac{3\cosh(a+bx)\sinh}{8b^4}$$
$$= \frac{3x}{8b^3} + \frac{x^3}{4b} - \frac{x^4}{4} + \frac{x^3\log\left(1-e^{2(a+bx)}\right)}{b} + \frac{3x^2\text{Li}_2\left(e^{2(a+bx)}\right)}{2b^2} - \frac{3x\text{Li}_3\left(e^{2(a+b}\right.}{2b^3}$$
$$= \frac{3x}{8b^3} + \frac{x^3}{4b} - \frac{x^4}{4} + \frac{x^3\log\left(1-e^{2(a+bx)}\right)}{b} + \frac{3x^2\text{Li}_2\left(e^{2(a+bx)}\right)}{2b^2} - \frac{3x\text{Li}_3\left(e^{2(a+b}\right.}{2b^3}$$
$$= \frac{3x}{8b^3} + \frac{x^3}{4b} - \frac{x^4}{4} + \frac{x^3\log\left(1-e^{2(a+bx)}\right)}{b} + \frac{3x^2\text{Li}_2\left(e^{2(a+bx)}\right)}{2b^2} - \frac{3x\text{Li}_3\left(e^{2(a+b}\right.}{2b^3}$$

Mathematica [A] time = 2.61, size = 236, normalized size = 1.31

$$\frac{\sinh(a)(\sinh(a)+\cosh(a))\left(16b^3x^3\log\left(1-e^{-a-bx}\right)+16b^3x^3\log\left(e^{-a-bx}+1\right)+4b^3x^3\cosh(2(a+bx))-48b^2x^2}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Cosh[a + b*x]^2*Coth[a + b*x],x]
```

```
[Out] (Sinh[a]*(Cosh[a] + Sinh[a])*(4*b^4*x^4 + 6*b*x*Cosh[2*(a + b*x)] + 4*b^3*x
^3*Cosh[2*(a + b*x)] + 16*b^3*x^3*Log[1 - E^(-a - b*x)] + 16*b^3*x^3*Log[1
+ E^(-a - b*x)] - 48*b^2*x^2*PolyLog[2, -E^(-a - b*x)] - 48*b^2*x^2*PolyLog
[2, E^(-a - b*x)] - 96*b*x*PolyLog[3, -E^(-a - b*x)] - 96*b*x*PolyLog[3, E^
(-a - b*x)] - 96*PolyLog[4, -E^(-a - b*x)] - 96*PolyLog[4, E^(-a - b*x)] -
3*Sinh[2*(a + b*x)] - 6*b^2*x^2*Sinh[2*(a + b*x)]))/(8*b^4*(-1 + E^(2*a)))
```

fricas [C] time = 0.65, size = 876, normalized size = 4.87

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*cosh(b*x+a)^3*csch(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/32*(4*b^3*x^3 + (4*b^3*x^3 - 6*b^2*x^2 + 6*b*x - 3)*cosh(b*x + a)^4 + 4*(
4*b^3*x^3 - 6*b^2*x^2 + 6*b*x - 3)*cosh(b*x + a)*sinh(b*x + a)^3 + (4*b^3*x
```

```
^3 - 6*b^2*x^2 + 6*b*x - 3)*sinh(b*x + a)^4 + 6*b^2*x^2 - 8*(b^4*x^4 - 2*a^
4)*cosh(b*x + a)^2 - 2*(4*b^4*x^4 - 8*a^4 - 3*(4*b^3*x^3 - 6*b^2*x^2 + 6*b*
x - 3)*cosh(b*x + a)^2)*sinh(b*x + a)^2 + 6*b*x + 96*(b^2*x^2*cosh(b*x + a)
^2 + 2*b^2*x^2*cosh(b*x + a)*sinh(b*x + a) + b^2*x^2*sinh(b*x + a)^2)*dilog
(cosh(b*x + a) + sinh(b*x + a)) + 96*(b^2*x^2*cosh(b*x + a)^2 + 2*b^2*x^2*c
osh(b*x + a)*sinh(b*x + a) + b^2*x^2*sinh(b*x + a)^2)*dilog(-cosh(b*x + a)
- sinh(b*x + a)) + 32*(b^3*x^3*cosh(b*x + a)^2 + 2*b^3*x^3*cosh(b*x + a)*si
nh(b*x + a) + b^3*x^3*sinh(b*x + a)^2)*log(cosh(b*x + a) + sinh(b*x + a) +
1) - 32*(a^3*cosh(b*x + a)^2 + 2*a^3*cosh(b*x + a)*sinh(b*x + a) + a^3*sinh
(b*x + a)^2)*log(cosh(b*x + a) + sinh(b*x + a) - 1) + 32*((b^3*x^3 + a^3)*c
osh(b*x + a)^2 + 2*(b^3*x^3 + a^3)*cosh(b*x + a)*sinh(b*x + a) + (b^3*x^3 +
 a^3)*sinh(b*x + a)^2)*log(-cosh(b*x + a) - sinh(b*x + a) + 1) + 192*(cosh(
b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2)*polylog(4, co
sh(b*x + a) + sinh(b*x + a)) + 192*(cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(
b*x + a) + sinh(b*x + a)^2)*polylog(4, -cosh(b*x + a) - sinh(b*x + a)) - 19
2*(b*x*cosh(b*x + a)^2 + 2*b*x*cosh(b*x + a)*sinh(b*x + a) + b*x*sinh(b*x +
 a)^2)*polylog(3, cosh(b*x + a) + sinh(b*x + a)) - 192*(b*x*cosh(b*x + a)^2
 + 2*b*x*cosh(b*x + a)*sinh(b*x + a) + b*x*sinh(b*x + a)^2)*polylog(3, -cos
h(b*x + a) - sinh(b*x + a)) + 4*((4*b^3*x^3 - 6*b^2*x^2 + 6*b*x - 3)*cosh(b
*x + a)^3 - 4*(b^4*x^4 - 2*a^4)*cosh(b*x + a))*sinh(b*x + a) + 3)/(b^4*cosh
(b*x + a)^2 + 2*b^4*cosh(b*x + a)*sinh(b*x + a) + b^4*sinh(b*x + a)^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3 \cosh(bx+a)^3 \operatorname{csch}(bx+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*cosh(b*x+a)^3*csch(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(x^3*cosh(b*x + a)^3*csch(b*x + a), x)
```

maple [A] time = 0.60, size = 272, normalized size = 1.51

$$-\frac{x^4}{4}+\frac{\left(4x^3b^3-6x^2b^2+6bx-3\right)e^{2bx+2a}}{32b^4}+\frac{\left(4x^3b^3+6x^2b^2+6bx+3\right)e^{-2bx-2a}}{32b^4}+\frac{6\,\mathrm{polylog}\left(4,e^{bx+a}\right)}{b^4}-\frac{2a^3x}{b^3}+\frac{\ln\left(1\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*cosh(b*x+a)^3*csch(b*x+a),x)
```

```
[Out] -1/4*x^4+1/32*(4*b^3*x^3-6*b^2*x^2+6*b*x-3)/b^4*exp(2*b*x+2*a)+1/32*(4*b^3*
x^3+6*b^2*x^2+6*b*x+3)/b^4*exp(-2*b*x-2*a)+6*polylog(4,exp(b*x+a))/b^4-2/b^
3*a^3*x+1/b*ln(1-exp(b*x+a))*x^3+3*x^2*polylog(2,exp(b*x+a))/b^2-6*x*polylo
g(3,exp(b*x+a))/b^3+1/b*ln(1+exp(b*x+a))*x^3+3*x^2*polylog(2,-exp(b*x+a))/b
^2-6*x*polylog(3,-exp(b*x+a))/b^3-3/2/b^4*a^4+1/b^4*ln(1-exp(b*x+a))*a^3+6*
```

```
polylog(4,-exp(b*x+a))/b^4-1/b^4*a^3*ln(exp(b*x+a)-1)+2/b^4*a^3*ln(exp(b*x+
a))
```

maxima [A] time = 0.58, size = 225, normalized size = 1.25

$$-\frac{1}{2}x^4+\frac{\left(8\,b^4x^4e^{(2\,a)}+\left(4\,b^3x^3e^{(4\,a)}-6\,b^2x^2e^{(4\,a)}+6\,bxe^{(4\,a)}-3\,e^{(4\,a)}\right)e^{(2\,bx)}+\left(4\,b^3x^3+6\,b^2x^2+6\,bx+3\right)e^{(-2\,bx)}\right)e}{32\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*cosh(b*x+a)^3*csch(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/2*x^4 + 1/32*(8*b^4*x^4*e^(2*a) + (4*b^3*x^3*e^(4*a) - 6*b^2*x^2*e^(4*a)
 + 6*b*x*e^(4*a) - 3*e^(4*a))*e^(2*b*x) + (4*b^3*x^3 + 6*b^2*x^2 + 6*b*x +
3)*e^(-2*b*x))*e^(-2*a)/b^4 + (b^3*x^3*log(e^(b*x + a) + 1) + 3*b^2*x^2*dil
og(-e^(b*x + a)) - 6*b*x*polylog(3, -e^(b*x + a)) + 6*polylog(4, -e^(b*x +
a)))/b^4 + (b^3*x^3*log(-e^(b*x + a) + 1) + 3*b^2*x^2*dilog(e^(b*x + a)) -
6*b*x*polylog(3, e^(b*x + a)) + 6*polylog(4, e^(b*x + a)))/b^4
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3\cosh\left(a+b\,x\right)^3}{\sinh\left(a+b\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*cosh(a + b*x)^3)/sinh(a + b*x),x)
```

```
[Out] int((x^3*cosh(a + b*x)^3)/sinh(a + b*x), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*cosh(b*x+a)**3*csch(b*x+a),x)
```

```
[Out] Timed out
```

3.413 $\int x^2 \cosh^2(a+bx)\coth(a+bx)\,dx$

Optimal. Leaf size=126

$$-\frac{\text{Li}_3\left(e^{2(a+bx)}\right)}{2b^3}+\frac{\sinh^2(a+bx)}{4b^3}+\frac{x\text{Li}_2\left(e^{2(a+bx)}\right)}{b^2}-\frac{x\sinh(a+bx)\cosh(a+bx)}{2b^2}+\frac{x^2\log\left(1-e^{2(a+bx)}\right)}{b}+\frac{x^2\sinh^2(a+}{2b}$$

```
[Out] 1/4*x^2/b-1/3*x^3+x^2*ln(1-exp(2*b*x+2*a))/b+x*polylog(2,exp(2*b*x+2*a))/b^
2-1/2*polylog(3,exp(2*b*x+2*a))/b^3-1/2*x*cosh(b*x+a)*sinh(b*x+a)/b^2+1/4*s
inh(b*x+a)^2/b^3+1/2*x^2*sinh(b*x+a)^2/b
```

Rubi [A] time = 0.20, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5450, 5372, 3310, 30, 3716, 2190, 2531, 2282, 6589}

$$\frac{x\text{PolyLog}\left(2,e^{2(a+bx)}\right)}{b^2}-\frac{\text{PolyLog}\left(3,e^{2(a+bx)}\right)}{2b^3}+\frac{\sinh^2(a+bx)}{4b^3}-\frac{x\sinh(a+bx)\cosh(a+bx)}{2b^2}+\frac{x^2\log\left(1-e^{2(a+bx)}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Cosh[a + b*x]^2*Coth[a + b*x],x]
```

```
[Out] x^2/(4*b) - x^3/3 + (x^2*Log[1 - E^(2*(a + b*x))])/b + (x*PolyLog[2, E^(2*(
a + b*x))])/b^2 - PolyLog[3, E^(2*(a + b*x))]/(2*b^3) - (x*Cosh[a + b*x]*Si
nh[a + b*x])/(2*b^2) + Sinh[a + b*x]^2/(4*b^3) + (x^2*Sinh[a + b*x]^2)/(2*b
)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
```

```
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 3310

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[(d*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[(b*(c + d*x)*Cos[e + f*x]*(b
*Sin[e + f*x])^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 3716

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2
*I, Int[((c + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(E^(2*I*k*Pi)*(1 + E^(2*(-(I*
e) + f*fz*x))/E^(2*I*k*Pi))), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Integ
erQ[4*k] && IGtQ[m, 0]
```

Rule 5372

```
Int[Cosh[(a_.) + (b_.)*(x_)^(n_.)]*(x_)^(m_.)*Sinh[(a_.) + (b_.)*(x_)^(n_.)
]^(p_.), x_Symbol] :> Simp[(x^(m - n + 1)*Sinh[a + b*x^n]^(p + 1))/(b*n*(p
+ 1)), x] - Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*Sinh[a + b*x^n]^(
p + 1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

Rule 5450

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(n_.)*Coth[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) +
(d_.)*(x_))^(m_.), x_Symbol] :> Int[(c + d*x)^m*Cosh[a + b*x]^n*Coth[a + b*
x]^(p - 2), x] + Int[(c + d*x)^m*Cosh[a + b*x]^(n - 2)*Coth[a + b*x]^p, x]
/; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
```

```
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int x^2\cosh^2(a+bx)\coth(a+bx)\,dx=\int x^2\coth(a+bx)\,dx+\int x^2\cosh(a+bx)\sinh(a+bx)\,dx$$
$$=-\frac{x^3}{3}+\frac{x^2\sinh^2(a+bx)}{2b}-2\int\frac{e^{2(a+bx)}x^2}{1-e^{2(a+bx)}}\,dx-\frac{\int x\sinh^2(a+bx)\,dx}{b}$$
$$=-\frac{x^3}{3}+\frac{x^2\log\left(1-e^{2(a+bx)}\right)}{b}-\frac{x\cosh(a+bx)\sinh(a+bx)}{2b^2}+\frac{\sinh^2(a+bx}{4b^3}$$
$$=\frac{x^2}{4b}-\frac{x^3}{3}+\frac{x^2\log\left(1-e^{2(a+bx)}\right)}{b}+\frac{x\mathrm{Li}_2\left(e^{2(a+bx)}\right)}{b^2}-\frac{x\cosh(a+bx)\sinh(a}{2b^2}$$
$$=\frac{x^2}{4b}-\frac{x^3}{3}+\frac{x^2\log\left(1-e^{2(a+bx)}\right)}{b}+\frac{x\mathrm{Li}_2\left(e^{2(a+bx)}\right)}{b^2}-\frac{x\cosh(a+bx)\sinh(a}{2b^2}$$
$$=\frac{x^2}{4b}-\frac{x^3}{3}+\frac{x^2\log\left(1-e^{2(a+bx)}\right)}{b}+\frac{x\mathrm{Li}_2\left(e^{2(a+bx)}\right)}{b^2}-\frac{\mathrm{Li}_3\left(e^{2(a+bx)}\right)}{2b^3}-\frac{x\cosh(}{}$$

Mathematica [A] time = 2.59, size = 178, normalized size = 1.41

$$\frac{\sinh(a)(\sinh(a)+\cosh(a))\left(24b^2x^2\log\left(1-e^{-a-bx}\right)+24b^2x^2\log\left(e^{-a-bx}+1\right)+6b^2x^2\cosh(2(a+bx))-48bx\mathrm{Li}_2}{12\left(e^2\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Cosh[a + b*x]^2*Coth[a + b*x],x]
```

```
[Out] (Sinh[a]*(Cosh[a] + Sinh[a])*(8*b^3*x^3 + 3*Cosh[2*(a + b*x)] + 6*b^2*x^2*C
osh[2*(a + b*x)] + 24*b^2*x^2*Log[1 - E^(-a - b*x)] + 24*b^2*x^2*Log[1 + E^
(-a - b*x)] - 48*b*x*PolyLog[2, -E^(-a - b*x)] - 48*b*x*PolyLog[2, E^(-a -
b*x)] - 48*PolyLog[3, -E^(-a - b*x)] - 48*PolyLog[3, E^(-a - b*x)] - 6*b*x*
Sinh[2*(a + b*x)]))/(12*b^3*(-1 + E^(2*a)))
```

fricas [C] time = 0.92, size = 697, normalized size = 5.53

$$\frac{3\left(2\,b^2x^2-2\,bx+1\right)\cosh\left(bx+a\right)^4+12\left(2\,b^2x^2-2\,bx+1\right)\cosh\left(bx+a\right)\sinh\left(bx+a\right)^3+3\left(2\,b^2x^2-2\,bx+1\right)\mathrm{s}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cosh(b*x+a)^3*csch(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/48*(3*(2*b^2*x^2 - 2*b*x + 1)*cosh(b*x + a)^4 + 12*(2*b^2*x^2 - 2*b*x + 1
)*cosh(b*x + a)*sinh(b*x + a)^3 + 3*(2*b^2*x^2 - 2*b*x + 1)*sinh(b*x + a)^4
 + 6*b^2*x^2 - 16*(b^3*x^3 + 2*a^3)*cosh(b*x + a)^2 - 2*(8*b^3*x^3 + 16*a^3
 - 9*(2*b^2*x^2 - 2*b*x + 1)*cosh(b*x + a)^2)*sinh(b*x + a)^2 + 6*b*x + 96*
(b*x*cosh(b*x + a)^2 + 2*b*x*cosh(b*x + a)*sinh(b*x + a) + b*x*sinh(b*x + a
)^2)*dilog(cosh(b*x + a) + sinh(b*x + a)) + 96*(b*x*cosh(b*x + a)^2 + 2*b*x
*cosh(b*x + a)*sinh(b*x + a) + b*x*sinh(b*x + a)^2)*dilog(-cosh(b*x + a) -
sinh(b*x + a)) + 48*(b^2*x^2*cosh(b*x + a)^2 + 2*b^2*x^2*cosh(b*x + a)*sinh
(b*x + a) + b^2*x^2*sinh(b*x + a)^2)*log(cosh(b*x + a) + sinh(b*x + a) + 1)
 + 48*(a^2*cosh(b*x + a)^2 + 2*a^2*cosh(b*x + a)*sinh(b*x + a) + a^2*sinh(b
*x + a)^2)*log(cosh(b*x + a) + sinh(b*x + a) - 1) + 48*((b^2*x^2 - a^2)*cos
h(b*x + a)^2 + 2*(b^2*x^2 - a^2)*cosh(b*x + a)*sinh(b*x + a) + (b^2*x^2 - a
^2)*sinh(b*x + a)^2)*log(-cosh(b*x + a) - sinh(b*x + a) + 1) - 96*(cosh(b*x
 + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2)*polylog(3, cosh(
b*x + a) + sinh(b*x + a)) - 96*(cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x
+ a) + sinh(b*x + a)^2)*polylog(3, -cosh(b*x + a) - sinh(b*x + a)) + 4*(3*(
2*b^2*x^2 - 2*b*x + 1)*cosh(b*x + a)^3 - 8*(b^3*x^3 + 2*a^3)*cosh(b*x + a))
*sinh(b*x + a) + 3)/(b^3*cosh(b*x + a)^2 + 2*b^3*cosh(b*x + a)*sinh(b*x + a
) + b^3*sinh(b*x + a)^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \cosh(bx+a)^3 \operatorname{csch}(bx+a)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cosh(b*x+a)^3*csch(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(x^2*cosh(b*x + a)^3*csch(b*x + a), x)
```

maple [A] time = 0.62, size = 222, normalized size = 1.76

$$-\frac{x^3}{3}+\frac{\left(2x^2b^2-2bx+1\right)\mathrm{e}^{2bx+2a}}{16b^3}+\frac{\left(2x^2b^2+2bx+1\right)\mathrm{e}^{-2bx-2a}}{16b^3}+\frac{a^2\ln\left(\mathrm{e}^{bx+a}-1\right)}{b^3}-\frac{2a^2\ln\left(\mathrm{e}^{bx+a}\right)}{b^3}+\frac{2a^2x}{b^2}+\frac{4a^3}{3b^3}+\frac{\ln\left(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*cosh(b*x+a)^3*csch(b*x+a),x)
```

```
[Out] -1/3*x^3+1/16*(2*b^2*x^2-2*b*x+1)/b^3*exp(2*b*x+2*a)+1/16*(2*b^2*x^2+2*b*x+
1)/b^3*exp(-2*b*x-2*a)+1/b^3*a^2*ln(exp(b*x+a)-1)-2/b^3*a^2*ln(exp(b*x+a))+
2/b^2*a^2*x+4/3/b^3*a^3+1/b*ln(1-exp(b*x+a))*x^2-1/b^3*ln(1-exp(b*x+a))*a^2
+2/b^2*polylog(2,exp(b*x+a))*x-2/b^3*polylog(3,exp(b*x+a))+1/b*ln(1+exp(b*x
+a))*x^2+2/b^2*polylog(2,-exp(b*x+a))*x-2/b^3*polylog(3,-exp(b*x+a))
```

maxima [A] time = 0.45, size = 171, normalized size = 1.36

$$-\frac{2}{3}x^3+\frac{\left(16\,b^3x^3e^{(2\,a)}+3\left(2\,b^2x^2e^{(4\,a)}-2\,bxe^{(4\,a)}+e^{(4\,a)}\right)e^{(2\,bx)}+3\left(2\,b^2x^2+2\,bx+1\right)e^{(-2\,bx)}\right)e^{(-2\,a)}}{48\,b^3}+\frac{b^2x^2\log\left(e^{(bx+}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cosh(b*x+a)^3*csch(b*x+a),x, algorithm="maxima")
```

```
[Out] -2/3*x^3 + 1/48*(16*b^3*x^3*e^(2*a) + 3*(2*b^2*x^2*e^(4*a) - 2*b*x*e^(4*a)
+ e^(4*a))*e^(2*b*x) + 3*(2*b^2*x^2 + 2*b*x + 1)*e^(-2*b*x))*e^(-2*a)/b^3 +
 (b^2*x^2*log(e^(b*x + a) + 1) + 2*b*x*dilog(-e^(b*x + a)) - 2*polylog(3, -
e^(b*x + a)))/b^3 + (b^2*x^2*log(-e^(b*x + a) + 1) + 2*b*x*dilog(e^(b*x + a
)) - 2*polylog(3, e^(b*x + a)))/b^3
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2\cosh\left(a+b\,x\right)^3}{\sinh\left(a+b\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*cosh(a + b*x)^3)/sinh(a + b*x),x)
```

```
[Out] int((x^2*cosh(a + b*x)^3)/sinh(a + b*x), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*cosh(b*x+a)**3*csch(b*x+a),x)
```

```
[Out] Timed out
```

3.414 $\int x\cosh^2(a+bx)\coth(a+bx)\,dx$

Optimal. Leaf size=88

$$\frac{\text{Li}_2\left(e^{2(a+bx)}\right)}{2b^2}-\frac{\sinh(a+bx)\cosh(a+bx)}{4b^2}+\frac{x\log\left(1-e^{2(a+bx)}\right)}{b}+\frac{x\sinh^2(a+bx)}{2b}+\frac{x}{4b}-\frac{x^2}{2}$$

```
[Out] 1/4*x/b-1/2*x^2+x*ln(1-exp(2*b*x+2*a))/b+1/2*polylog(2,exp(2*b*x+2*a))/b^2-
1/4*cosh(b*x+a)*sinh(b*x+a)/b^2+1/2*x*sinh(b*x+a)^2/b
```

Rubi [A] time = 0.12, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5450, 5372, 2635, 8, 3716, 2190, 2279, 2391}

$$\frac{\text{PolyLog}\left(2,e^{2(a+bx)}\right)}{2b^2}-\frac{\sinh(a+bx)\cosh(a+bx)}{4b^2}+\frac{x\log\left(1-e^{2(a+bx)}\right)}{b}+\frac{x\sinh^2(a+bx)}{2b}+\frac{x}{4b}-\frac{x^2}{2}$$

Antiderivative was successfully verified.

```
[In] Int[x*Cosh[a + b*x]^2*Coth[a + b*x],x]
```

```
[Out] x/(4*b) - x^2/2 + (x*Log[1 - E^(2*(a + b*x))])/b + PolyLog[2, E^(2*(a + b*x
))]/(2*b^2) - (Cosh[a + b*x]*Sinh[a + b*x])/(4*b^2) + (x*Sinh[a + b*x]^2)/(
2*b)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 3716

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2
*I, Int[((c + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(E^(2*I*k*Pi)*(1 + E^(2*(-(I*
e) + f*fz*x))/E^(2*I*k*Pi))), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Integ
erQ[4*k] && IGtQ[m, 0]
```

Rule 5372

```
Int[Cosh[(a_.) + (b_.)*(x_)^(n_.)]*(x_)^(m_.)*Sinh[(a_.) + (b_.)*(x_)^(n_.)
]^(p_.), x_Symbol] :> Simp[(x^(m - n + 1)*Sinh[a + b*x^n]^(p + 1))/(b*n*(p
+ 1)), x] - Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*Sinh[a + b*x^n]^(
p + 1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

Rule 5450

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(n_.)*Coth[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) +
(d_.)*(x_))^(m_.), x_Symbol] :> Int[(c + d*x)^m*Cosh[a + b*x]^n*Coth[a + b*
x]^(p - 2), x] + Int[(c + d*x)^m*Cosh[a + b*x]^(n - 2)*Coth[a + b*x]^p, x]
/; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rubi steps

$$\int x\cosh^2(a+bx)\coth(a+bx)\,dx = \int x\coth(a+bx)\,dx + \int x\cosh(a+bx)\sinh(a+bx)\,dx$$
$$= -\frac{x^2}{2} + \frac{x\sinh^2(a+bx)}{2b} - 2\int \frac{e^{2(a+bx)}x}{1-e^{2(a+bx)}}\,dx - \frac{\int \sinh^2(a+bx)\,dx}{2b}$$
$$= -\frac{x^2}{2} + \frac{x\log\left(1-e^{2(a+bx)}\right)}{b} - \frac{\cosh(a+bx)\sinh(a+bx)}{4b^2} + \frac{x\sinh^2(a+bx)}{2b}$$
$$= \frac{x}{4b} - \frac{x^2}{2} + \frac{x\log\left(1-e^{2(a+bx)}\right)}{b} - \frac{\cosh(a+bx)\sinh(a+bx)}{4b^2} + \frac{x\sinh^2(a+$$
$$= \frac{x}{4b} - \frac{x^2}{2} + \frac{x\log\left(1-e^{2(a+bx)}\right)}{b} + \frac{\text{Li}_2\left(e^{2(a+bx)}\right)}{2b^2} - \frac{\cosh(a+bx)\sinh(a+bx}{4b^2}$$

Mathematica [A] time = 0.25, size = 82, normalized size = 0.93

$$-\frac{4\left(\text{Li}_2\left(e^{-2(a+bx)}\right)-(a+bx)^2\right)-8(a+bx)\log\left(1-e^{-2(a+bx)}\right)+\sinh(2(a+bx))-2bx\cosh(2(a+bx))+8a\log(}{8b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Cosh[a + b*x]^2*Coth[a + b*x],x]
```

```
[Out] -1/8*(-2*b*x*Cosh[2*(a + b*x)] - 8*(a + b*x)*Log[1 - E^(-2*(a + b*x))] + 8*
a*Log[Sinh[a + b*x]] + 4*(-(a + b*x)^2 + PolyLog[2, E^(-2*(a + b*x))]) + Si
nh[2*(a + b*x)])/b^2
```

fricas [B] time = 1.11, size = 488, normalized size = 5.55

$$\frac{(2\,bx-1)\cosh\left(bx+a\right)^4+4\,(2\,bx-1)\cosh\left(bx+a\right)\sinh\left(bx+a\right)^3+(2\,bx-1)\sinh\left(bx+a\right)^4-8\left(b^2x^2-2\,a^2\right)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x+a)^3*csch(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/16*((2*b*x - 1)*cosh(b*x + a)^4 + 4*(2*b*x - 1)*cosh(b*x + a)*sinh(b*x +
a)^3 + (2*b*x - 1)*sinh(b*x + a)^4 - 8*(b^2*x^2 - 2*a^2)*cosh(b*x + a)^2 -
2*(4*b^2*x^2 - 3*(2*b*x - 1)*cosh(b*x + a)^2 - 8*a^2)*sinh(b*x + a)^2 + 2*b
*x + 16*(cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2)
*dilog(cosh(b*x + a) + sinh(b*x + a)) + 16*(cosh(b*x + a)^2 + 2*cosh(b*x +
a)*sinh(b*x + a) + sinh(b*x + a)^2)*dilog(-cosh(b*x + a) - sinh(b*x + a)) +
 16*(b*x*cosh(b*x + a)^2 + 2*b*x*cosh(b*x + a)*sinh(b*x + a) + b*x*sinh(b*x
```

```
 + a)^2)*log(cosh(b*x + a) + sinh(b*x + a) + 1) - 16*(a*cosh(b*x + a)^2 + 2
*a*cosh(b*x + a)*sinh(b*x + a) + a*sinh(b*x + a)^2)*log(cosh(b*x + a) + sin
h(b*x + a) - 1) + 16*((b*x + a)*cosh(b*x + a)^2 + 2*(b*x + a)*cosh(b*x + a)
*sinh(b*x + a) + (b*x + a)*sinh(b*x + a)^2)*log(-cosh(b*x + a) - sinh(b*x +
 a) + 1) + 4*((2*b*x - 1)*cosh(b*x + a)^3 - 4*(b^2*x^2 - 2*a^2)*cosh(b*x +
a))*sinh(b*x + a) + 1)/(b^2*cosh(b*x + a)^2 + 2*b^2*cosh(b*x + a)*sinh(b*x
+ a) + b^2*sinh(b*x + a)^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\cosh(bx+a)^3\operatorname{csch}(bx+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x+a)^3*csch(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(x*cosh(b*x + a)^3*csch(b*x + a), x)
```

maple [B] time = 0.61, size = 162, normalized size = 1.84

$$-\frac{x^2}{2}+\frac{(2bx-1)\,\mathrm{e}^{2bx+2a}}{16b^2}+\frac{(2bx+1)\,\mathrm{e}^{-2bx-2a}}{16b^2}-\frac{2ax}{b}-\frac{a^2}{b^2}+\frac{\ln\left(1-\mathrm{e}^{bx+a}\right)x}{b}+\frac{\ln\left(1-\mathrm{e}^{bx+a}\right)a}{b^2}+\frac{\mathrm{polylog}\left(2,\mathrm{e}^{bx+a}\right)}{b^2}+\frac{\ln\left(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cosh(b*x+a)^3*csch(b*x+a),x)
```

```
[Out] -1/2*x^2+1/16*(2*b*x-1)/b^2*exp(2*b*x+2*a)+1/16*(2*b*x+1)/b^2*exp(-2*b*x-2*
a)-2/b*a*x-a^2/b^2+1/b*ln(1-exp(b*x+a))*x+1/b^2*ln(1-exp(b*x+a))*a+polylog(
2,exp(b*x+a))/b^2+1/b*ln(1+exp(b*x+a))*x+polylog(2,-exp(b*x+a))/b^2+2/b^2*a
*ln(exp(b*x+a))-1/b^2*a*ln(exp(b*x+a)-1)
```

maxima [A] time = 0.41, size = 113, normalized size = 1.28

$$-x^2+\frac{\left(8\,b^2x^2e^{(2\,a)}+\left(2\,bxe^{(4\,a)}-e^{(4\,a)}\right)e^{(2\,bx)}+(2\,bx+1)e^{(-2\,bx)}\right)e^{(-2\,a)}}{16\,b^2}+\frac{bx\log\left(e^{(bx+a)}+1\right)+\mathrm{Li}_2\left(-e^{(bx+a)}\right)}{b^2}+\frac{bx\log}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x+a)^3*csch(b*x+a),x, algorithm="maxima")
```

```
[Out] -x^2 + 1/16*(8*b^2*x^2*e^(2*a) + (2*b*x*e^(4*a) - e^(4*a))*e^(2*b*x) + (2*b
*x + 1)*e^(-2*b*x))*e^(-2*a)/b^2 + (b*x*log(e^(b*x + a) + 1) + dilog(-e^(b*
x + a)))/b^2 + (b*x*log(-e^(b*x + a) + 1) + dilog(e^(b*x + a)))/b^2
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x\cosh(a+b\,x)^3}{\sinh(a+b\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*cosh(a + b*x)^3)/sinh(a + b*x),x)
```

```
[Out] int((x*cosh(a + b*x)^3)/sinh(a + b*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x \cosh^3(a+bx) \operatorname{csch}(a+bx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x+a)**3*csch(b*x+a),x)
```

```
[Out] Integral(x*cosh(a + b*x)**3*csch(a + b*x), x)
```

3.415 $\int \cosh^2(a+bx)\coth(a+bx)\,dx$

Optimal. Leaf size=27

$$\frac{\sinh^2(a+bx)}{2b}+\frac{\log(\sinh(a+bx))}{b}$$

```
[Out] ln(sinh(b*x+a))/b+1/2*sinh(b*x+a)^2/b
```

Rubi [A] time = 0.02, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2590, 14}

$$\frac{\sinh^2(a+bx)}{2b}+\frac{\log(\sinh(a+bx))}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[a + b*x]^2*Coth[a + b*x],x]
```

```
[Out] Log[Sinh[a + b*x]]/b + Sinh[a + b*x]^2/(2*b)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2590

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> -Dist[f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rubi steps

$$\begin{aligned}\int \cosh^2(a+bx)\coth(a+bx)\,dx &= \frac{\text{Subst}\left(\int \frac{1-x^2}{x}\,dx, x, -i\sinh(a+bx)\right)}{b}\\ &= \frac{\text{Subst}\left(\int \left(\frac{1}{x}-x\right)dx, x, -i\sinh(a+bx)\right)}{b}\\ &= \frac{\log(\sinh(a+bx))}{b}+\frac{\sinh^2(a+bx)}{2b}\end{aligned}$$

Mathematica [A] time = 0.02, size = 25, normalized size = 0.93

$$\frac{\sinh^2(a+bx)+2\log(\sinh(a+bx))}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[a + b*x]^2*Coth[a + b*x],x]
```

```
[Out] (2*Log[Sinh[a + b*x]] + Sinh[a + b*x]^2)/(2*b)
```

fricas [B] time = 0.96, size = 203, normalized size = 7.52

$$-\frac{8\,bx\cosh\left(bx+a\right)^2-\cosh\left(bx+a\right)^4-4\,\cosh\left(bx+a\right)\sinh\left(bx+a\right)^3-\sinh\left(bx+a\right)^4+2\left(4\,bx-3\,\cosh\left(bx\right.\right.}{8\left(b\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^3*csch(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/8*(8*b*x*cosh(b*x + a)^2 - cosh(b*x + a)^4 - 4*cosh(b*x + a)*sinh(b*x +
a)^3 - sinh(b*x + a)^4 + 2*(4*b*x - 3*cosh(b*x + a)^2)*sinh(b*x + a)^2 - 8*
(cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2)*log(2*s
inh(b*x + a)/(cosh(b*x + a) - sinh(b*x + a))) + 4*(4*b*x*cosh(b*x + a) - co
sh(b*x + a)^3)*sinh(b*x + a) - 1)/(b*cosh(b*x + a)^2 + 2*b*cosh(b*x + a)*si
nh(b*x + a) + b*sinh(b*x + a)^2)
```

giac [B] time = 0.14, size = 63, normalized size = 2.33

$$-\frac{8\,bx-\left(4\,e^{(2\,bx+2\,a)}+1\right)e^{(-2\,bx-2\,a)}+8\,a-e^{(2\,bx+2\,a)}-8\,\log\left(\left|e^{(2\,bx+2\,a)}-1\right|\right)}{8\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^3*csch(b*x+a),x, algorithm="giac")
```

```
[Out] -1/8*(8*b*x - (4*e^(2*b*x + 2*a) + 1)*e^(-2*b*x - 2*a) + 8*a - e^(2*b*x + 2
*a) - 8*log(abs(e^(2*b*x + 2*a) - 1)))/b
```

maple [A] time = 0.12, size = 26, normalized size = 0.96

$$\frac{\cosh^2\left(bx+a\right)}{2b}+\frac{\ln\left(\sinh\left(bx+a\right)\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)^3*csch(b*x+a),x)
```

```
[Out] 1/2*cosh(b*x+a)^2/b+ln(sinh(b*x+a))/b
```

maxima [B] time = 0.33, size = 70, normalized size = 2.59

$$\frac{bx+a}{b}+\frac{e^{(2\,bx+2\,a)}}{8\,b}+\frac{e^{(-2\,bx-2\,a)}}{8\,b}+\frac{\log\left(e^{(-bx-a)}+1\right)}{b}+\frac{\log\left(e^{(-bx-a)}-1\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^3*csch(b*x+a),x, algorithm="maxima")
```

```
[Out] (b*x + a)/b + 1/8*e^(2*b*x + 2*a)/b + 1/8*e^(-2*b*x - 2*a)/b + log(e^(-b*x
- a) + 1)/b + log(e^(-b*x - a) - 1)/b
```

mupad [B] time = 0.06, size = 49, normalized size = 1.81

$$\frac{\ln\left(e^{2\,a}\,e^{2\,b\,x}-1\right)}{b}-x+\frac{e^{-2\,a-2\,b\,x}}{8\,b}+\frac{e^{2\,a+2\,b\,x}}{8\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a + b*x)^3/sinh(a + b*x),x)
```

```
[Out] log(exp(2*a)*exp(2*b*x) - 1)/b - x + exp(- 2*a - 2*b*x)/(8*b) + exp(2*a + 2
*b*x)/(8*b)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \cosh^3(a+bx)\,\mathrm{csch}\,(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)**3*csch(b*x+a),x)
```

```
[Out] Integral(cosh(a + b*x)**3*csch(a + b*x), x)
```

3.416 $\int \frac{\cosh^2(a+bx)\coth(a+bx)}{x}\,dx$

Optimal. Leaf size=40

$$\text{Int}\left(\frac{\coth(a+bx)}{x},x\right)+\frac{1}{2}\sinh(2a)\text{Chi}(2bx)+\frac{1}{2}\cosh(2a)\text{Shi}(2bx)$$

```
[Out] 1/2*cosh(2*a)*Shi(2*b*x)+1/2*Chi(2*b*x)*sinh(2*a)+Unintegrable(coth(b*x+a)/
x,x)
```

Rubi [A] time = 0.10, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\cosh^2(a+bx)\coth(a+bx)}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Cosh[a + b*x]^2*Coth[a + b*x])/x,x]
```

```
[Out] (CoshIntegral[2*b*x]*Sinh[2*a])/2 + (Cosh[2*a]*SinhIntegral[2*b*x])/2 + Def
er[Int][Coth[a + b*x]/x, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cosh^2(a+bx)\coth(a+bx)}{x}\,dx &= \int \frac{\coth(a+bx)}{x}\,dx+\int \frac{\cosh(a+bx)\sinh(a+bx)}{x}\,dx\\
&= \int \frac{\coth(a+bx)}{x}\,dx+\int \frac{\sinh(2a+2bx)}{2x}\,dx\\
&= \frac{1}{2}\int \frac{\sinh(2a+2bx)}{x}\,dx+\int \frac{\coth(a+bx)}{x}\,dx\\
&= \frac{1}{2}\cosh(2a)\int \frac{\sinh(2bx)}{x}\,dx+\frac{1}{2}\sinh(2a)\int \frac{\cosh(2bx)}{x}\,dx+\int \frac{\coth(a+bx)}{x}\,dx\\
&= \frac{1}{2}\text{Chi}(2bx)\sinh(2a)+\frac{1}{2}\cosh(2a)\text{Shi}(2bx)+\int \frac{\coth(a+bx)}{x}\,dx
\end{aligned}$$

Mathematica [A] time = 11.41, size = 0, normalized size = 0.00

$$\int \frac{\cosh^2(a+bx)\coth(a+bx)}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Cosh[a + b*x]^2*Coth[a + b*x])/x,x]
[Out] Integrate[(Cosh[a + b*x]^2*Coth[a + b*x])/x, x]
```

fricas [A] time = 0.98, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\cosh(bx+a)^3\operatorname{csch}(bx+a)}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^3*csch(b*x+a)/x,x, algorithm="fricas")
[Out] integral(cosh(b*x + a)^3*csch(b*x + a)/x, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh(bx+a)^3\operatorname{csch}(bx+a)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^3*csch(b*x+a)/x,x, algorithm="giac")
[Out] integrate(cosh(b*x + a)^3*csch(b*x + a)/x, x)
```

maple [A] time = 0.74, size = 0, normalized size = 0.00

$$\int \frac{\left(\cosh^3(bx+a)\right)\operatorname{csch}(bx+a)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)^3*csch(b*x+a)/x,x)
[Out] int(cosh(b*x+a)^3*csch(b*x+a)/x,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{4}\operatorname{Ei}(2\,bx)\,e^{(2\,a)}-\frac{1}{4}\operatorname{Ei}(-2\,bx)\,e^{(-2\,a)}-\int \frac{1}{xe^{(bx+a)}+x}\,dx+\int \frac{1}{xe^{(bx+a)}-x}\,dx+\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^3*csch(b*x+a)/x,x, algorithm="maxima")
[Out] 1/4*Ei(2*b*x)*e^(2*a) - 1/4*Ei(-2*b*x)*e^(-2*a) - integrate(1/(x*e^(b*x + a
) + x), x) + integrate(1/(x*e^(b*x + a) - x), x) + log(x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\cosh(a+b\,x)^3}{x\sinh(a+b\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a + b*x)^3/(x*sinh(a + b*x)),x)
[Out] int(cosh(a + b*x)^3/(x*sinh(a + b*x)), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh^3(a+bx)\operatorname{csch}(a+bx)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)**3*csch(b*x+a)/x,x)
[Out] Integral(cosh(a + b*x)**3*csch(a + b*x)/x, x)
```

3.417 $\int \frac{\cosh^2(a+bx)\coth(a+bx)}{x^2}\,dx$

Optimal. Leaf size=52

$$\text{Int}\left(\frac{\coth(a+bx)}{x^2},x\right)+b\cosh(2a)\text{Chi}(2bx)+b\sinh(2a)\text{Shi}(2bx)-\frac{\sinh(2a+2bx)}{2x}$$

```
[Out] b*Chi(2*b*x)*cosh(2*a)+b*Shi(2*b*x)*sinh(2*a)-1/2*sinh(2*b*x+2*a)/x+Uninteg
rable(coth(b*x+a)/x^2,x)
```

Rubi [A] time = 0.13, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\cosh^2(a+bx)\coth(a+bx)}{x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Cosh[a + b*x]^2*Coth[a + b*x])/x^2,x]
```

```
[Out] b*Cosh[2*a]*CoshIntegral[2*b*x] - Sinh[2*a + 2*b*x]/(2*x) + b*Sinh[2*a]*Sin
hIntegral[2*b*x] + Defer[Int][Coth[a + b*x]/x^2, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cosh^2(a+bx)\coth(a+bx)}{x^2}\,dx &= \int \frac{\coth(a+bx)}{x^2}\,dx+\int \frac{\cosh(a+bx)\sinh(a+bx)}{x^2}\,dx\\
&= \int \frac{\coth(a+bx)}{x^2}\,dx+\int \frac{\sinh(2a+2bx)}{2x^2}\,dx\\
&= \frac{1}{2}\int \frac{\sinh(2a+2bx)}{x^2}\,dx+\int \frac{\coth(a+bx)}{x^2}\,dx\\
&= -\frac{\sinh(2a+2bx)}{2x}+b\int \frac{\cosh(2a+2bx)}{x}\,dx+\int \frac{\coth(a+bx)}{x^2}\,dx\\
&= -\frac{\sinh(2a+2bx)}{2x}+(b\cosh(2a))\int \frac{\cosh(2bx)}{x}\,dx+(b\sinh(2a))\int \frac{\sinh(2bx)}{x}\\
&= b\cosh(2a)\text{Chi}(2bx)-\frac{\sinh(2a+2bx)}{2x}+b\sinh(2a)\text{Shi}(2bx)+\int \frac{\coth(a+b}{x^2}
\end{aligned}$$

Mathematica [A] time = 16.85, size = 0, normalized size = 0.00

$$\int \frac{\cosh^2(a+bx)\coth(a+bx)}{x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Cosh[a + b*x]^2*Coth[a + b*x])/x^2,x]
[Out] Integrate[(Cosh[a + b*x]^2*Coth[a + b*x])/x^2, x]
```

fricas [A] time = 0.49, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\cosh\left(bx+a\right)^{3}\operatorname{csch}\left(bx+a\right)}{x^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^3*csch(b*x+a)/x^2,x, algorithm="fricas")
[Out] integral(cosh(b*x + a)^3*csch(b*x + a)/x^2, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh\left(bx+a\right)^{3}\operatorname{csch}\left(bx+a\right)}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^3*csch(b*x+a)/x^2,x, algorithm="giac")
[Out] integrate(cosh(b*x + a)^3*csch(b*x + a)/x^2, x)
```

maple [A] time = 0.72, size = 0, normalized size = 0.00

$$\int \frac{\left(\cosh^{3}\left(bx+a\right)\right)\operatorname{csch}\left(bx+a\right)}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)^3*csch(b*x+a)/x^2,x)
[Out] int(cosh(b*x+a)^3*csch(b*x+a)/x^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{2}\,be^{(-2\,a)}\Gamma\left(-1,2\,bx\right)+\frac{1}{2}\,be^{(2\,a)}\Gamma\left(-1,-2\,bx\right)-\frac{1}{x}-\int \frac{1}{x^{2}e^{(bx+a)}+x^{2}}\,dx+\int \frac{1}{x^{2}e^{(bx+a)}-x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^3*csch(b*x+a)/x^2,x, algorithm="maxima")
```

```
[Out] 1/2*b*e^(-2*a)*gamma(-1, 2*b*x) + 1/2*b*e^(2*a)*gamma(-1, -2*b*x) - 1/x - i
ntegrate(1/(x^2*e^(b*x + a) + x^2), x) + integrate(1/(x^2*e^(b*x + a) - x^2
), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\cosh\left(a+b\,x\right)^3}{x^2\,\sinh\left(a+b\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a + b*x)^3/(x^2*sinh(a + b*x)),x)
```

```
[Out] int(cosh(a + b*x)^3/(x^2*sinh(a + b*x)), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh^{3}{\left(a+bx \right)}\operatorname{csch}{\left(a+bx \right)}}{x^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)**3*csch(b*x+a)/x**2,x)
```

```
[Out] Integral(cosh(a + b*x)**3*csch(a + b*x)/x**2, x)
```

3.418 $\int x \cosh^2(x) \coth^2(x)\, dx$

Optimal. Leaf size=33

$$\frac{3x^2}{4} - \frac{\cosh^2(x)}{4} - x\coth(x) + \log(\sinh(x)) + \frac{1}{2}x\sinh(x)\cosh(x)$$

```
[Out] 3/4*x^2-1/4*cosh(x)^2-x*coth(x)+ln(sinh(x))+1/2*x*cosh(x)*sinh(x)
```

Rubi [A] time = 0.05, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5450, 3310, 30, 3720, 3475}

$$\frac{3x^2}{4} - \frac{\cosh^2(x)}{4} - x\coth(x) + \log(\sinh(x)) + \frac{1}{2}x\sinh(x)\cosh(x)$$

Antiderivative was successfully verified.

```
[In] Int[x*Cosh[x]^2*Coth[x]^2,x]
```

```
[Out] (3*x^2)/4 - Cosh[x]^2/4 - x*Coth[x] + Log[Sinh[x]] + (x*Cosh[x]*Sinh[x])/2
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 3310

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[(d*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[(b*(c + d*x)*Cos[e + f*x]*(b
*Sin[e + f*x])^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3720

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[(b*(c + d*x)^m*(b*Tan[e + f*x])^(n - 1))/(f*(n - 1)), x] + (-Di
st[(b*d*m)/(f*(n - 1)), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x],
 x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[
```

```
{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rule 5450

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(n_.)*Coth[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) +
(d_.)*(x_))^(m_.), x_Symbol] :> Int[(c + d*x)^m*Cosh[a + b*x]^n*Coth[a + b*
x]^(p - 2), x] + Int[(c + d*x)^m*Cosh[a + b*x]^(n - 2)*Coth[a + b*x]^p, x]
/; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int x\cosh^2(x)\coth^2(x)\,dx &= \int x\cosh^2(x)\,dx + \int x\coth^2(x)\,dx\\ &= -\frac{1}{4}\cosh^2(x) - x\coth(x) + \frac{1}{2}x\cosh(x)\sinh(x) + \frac{\int x\,dx}{2} + \int x\,dx + \int \coth(x)\,dx\\ &= \frac{3x^2}{4} - \frac{\cosh^2(x)}{4} - x\coth(x) + \log(\sinh(x)) + \frac{1}{2}x\cosh(x)\sinh(x)\end{aligned}$$

Mathematica [A] time = 0.03, size = 33, normalized size = 1.00

$$\frac{3x^2}{4} + \frac{1}{4}x\sinh(2x) - \frac{1}{8}\cosh(2x) - x\coth(x) + \log(\sinh(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Cosh[x]^2*Coth[x]^2,x]
[Out] (3*x^2)/4 - Cosh[2*x]/8 - x*Coth[x] + Log[Sinh[x]] + (x*Sinh[2*x])/4
```

fricas [B] time = 0.58, size = 336, normalized size = 10.18

$$\frac{(2x-1)\cosh(x)^6 + 6(2x-1)\cosh(x)\sinh(x)^5 + (2x-1)\sinh(x)^6 + \left(12x^2-34x+1\right)\cosh(x)^4 + \left(15(2x-1)\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(x)^2*coth(x)^2,x, algorithm="fricas")
[Out] 1/16*((2*x - 1)*cosh(x)^6 + 6*(2*x - 1)*cosh(x)*sinh(x)^5 + (2*x - 1)*sinh(
x)^6 + (12*x^2 - 34*x + 1)*cosh(x)^4 + (15*(2*x - 1)*cosh(x)^2 + 12*x^2 - 3
4*x + 1)*sinh(x)^4 + 4*(5*(2*x - 1)*cosh(x)^3 + (12*x^2 - 34*x + 1)*cosh(x)
)*sinh(x)^3 - (12*x^2 + 2*x + 1)*cosh(x)^2 + (15*(2*x - 1)*cosh(x)^4 + 6*(1
```

```
2*x^2 - 34*x + 1)*cosh(x)^2 - 12*x^2 - 2*x - 1)*sinh(x)^2 + 16*(cosh(x)^4 +
 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + (6*cosh(x)^2 - 1)*sinh(x)^2 - cosh(x)^2
+ 2*(2*cosh(x)^3 - cosh(x))*sinh(x))*log(2*sinh(x)/(cosh(x) - sinh(x))) + 2
*(3*(2*x - 1)*cosh(x)^5 + 2*(12*x^2 - 34*x + 1)*cosh(x)^3 - (12*x^2 + 2*x +
 1)*cosh(x))*sinh(x) + 2*x + 1)/(cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^
4 + (6*cosh(x)^2 - 1)*sinh(x)^2 - cosh(x)^2 + 2*(2*cosh(x)^3 - cosh(x))*sin
h(x))
```

giac [B] time = 0.12, size = 101, normalized size = 3.06

$$\frac{12\,x^2e^{(4\,x)} - 12\,x^2e^{(2\,x)} + 2\,xe^{(6\,x)} - 34\,xe^{(4\,x)} - 2\,xe^{(2\,x)} + 16\,e^{(4\,x)}\log\left(e^{(2\,x)} - 1\right) - 16\,e^{(2\,x)}\log\left(e^{(2\,x)} - 1\right) + 2\,x - e^{(}}{16\left(e^{(4\,x)} - e^{(2\,x)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(x)^2*coth(x)^2,x, algorithm="giac")
```

```
[Out] 1/16*(12*x^2*e^(4*x) - 12*x^2*e^(2*x) + 2*x*e^(6*x) - 34*x*e^(4*x) - 2*x*e^
(2*x) + 16*e^(4*x)*log(e^(2*x) - 1) - 16*e^(2*x)*log(e^(2*x) - 1) + 2*x - e
^(6*x) + e^(4*x) - e^(2*x) + 1)/(e^(4*x) - e^(2*x))
```

maple [A] time = 0.21, size = 48, normalized size = 1.45

$$\frac{3x^2}{4} + \left(-\frac{1}{16} + \frac{x}{8}\right)e^{2x} + \left(-\frac{1}{16} - \frac{x}{8}\right)e^{-2x} - 2x - \frac{2x}{e^{2x} - 1} + \ln\left(e^{2x} - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cosh(x)^2*coth(x)^2,x)
```

```
[Out] 3/4*x^2+(-1/16+1/8*x)*exp(2*x)+(-1/16-1/8*x)*exp(-2*x)-2*x-2*x/(exp(2*x)-1)
+ln(exp(2*x)-1)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(x)^2*coth(x)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

mupad [B] time = 0.12, size = 48, normalized size = 1.45

$$\ln\left(e^{2\,x} - 1\right) - 2\,x - e^{-2\,x}\left(\frac{x}{8} + \frac{1}{16}\right) + e^{2\,x}\left(\frac{x}{8} - \frac{1}{16}\right) - \frac{2\,x}{e^{2\,x} - 1} + \frac{3\,x^2}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cosh(x)^2*coth(x)^2,x)
```

```
[Out] log(exp(2*x) - 1) - 2*x - exp(-2*x)*(x/8 + 1/16) + exp(2*x)*(x/8 - 1/16) -
(2*x)/(exp(2*x) - 1) + (3*x^2)/4
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\cosh^2(x)\coth^2(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(x)**2*coth(x)**2,x)
```

```
[Out] Integral(x*cosh(x)**2*coth(x)**2, x)
```

3.419 $\int x^2 \cosh^2(x) \coth^2(x) \, dx$

Optimal. Leaf size=73

$$\mathrm{Li}_2\left(e^{2x}\right)+\frac{x^3}{2}-x^2-x^2\coth(x)+\frac{1}{2}x^2\sinh(x)\cosh(x)+\frac{x}{4}+2x\log\left(1-e^{2x}\right)-\frac{1}{2}x\cosh^2(x)+\frac{1}{4}\sinh(x)\cosh(x)$$

```
[Out] 1/4*x-x^2+1/2*x^3-1/2*x*cosh(x)^2-x^2*coth(x)+2*x*ln(1-exp(2*x))+polylog(2,
exp(2*x))+1/4*cosh(x)*sinh(x)+1/2*x^2*cosh(x)*sinh(x)
```

Rubi [A] time = 0.16, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.833, Rules used = {5450, 3311, 30, 2635, 8, 3720, 3716, 2190, 2279, 2391}

$$\mathrm{PolyLog}\left(2,e^{2x}\right)+\frac{x^3}{2}-x^2-x^2\coth(x)+\frac{1}{2}x^2\sinh(x)\cosh(x)+\frac{x}{4}+2x\log\left(1-e^{2x}\right)-\frac{1}{2}x\cosh^2(x)+\frac{1}{4}\sinh(x)\cosh(x)$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Cosh[x]^2*Coth[x]^2,x]
```

```
[Out] x/4 - x^2 + x^3/2 - (x*Cosh[x]^2)/2 - x^2*Coth[x] + 2*x*Log[1 - E^(2*x)] +
PolyLog[2, E^(2*x)] + (Cosh[x]*Sinh[x])/4 + (x^2*Cosh[x]*Sinh[x])/2
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
```

```
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 3311

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(d*m*(c + d*x)^(m - 1)*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist
[(b^2*(n - 1))/n, Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[(
d^2*m*(m - 1))/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[(b*(c + d*x)^m*Cos[e + f*x]*(b*Sin[e + f*x])^(n - 1))/(f*n), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 3716

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2
*I, Int[((c + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(E^(2*I*k*Pi)*(1 + E^(2*(-(I*
e) + f*fz*x))/E^(2*I*k*Pi))), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Integ
erQ[4*k] && IGtQ[m, 0]
```

Rule 3720

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[(b*(c + d*x)^m*(b*Tan[e + f*x])^(n - 1))/(f*(n - 1)), x] + (-Di
st[(b*d*m)/(f*(n - 1)), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x],
 x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[
{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rule 5450

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(n_.)*Coth[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) +
(d_.)*(x_))^(m_.), x_Symbol] :> Int[(c + d*x)^m*Cosh[a + b*x]^n*Coth[a + b*
x]^(p - 2), x] + Int[(c + d*x)^m*Cosh[a + b*x]^(n - 2)*Coth[a + b*x]^p, x]
```

```
/; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rubi steps

$$\int x^2\cosh^2(x)\coth^2(x)\,dx = \int x^2\cosh^2(x)\,dx + \int x^2\coth^2(x)\,dx$$
$$= -\frac{1}{2}x\cosh^2(x) - x^2\coth(x) + \frac{1}{2}x^2\cosh(x)\sinh(x) + \frac{\int x^2\,dx}{2} + \frac{1}{2}\int\cosh^2(x)\,dx +$$
$$= -x^2 + \frac{x^3}{2} - \frac{1}{2}x\cosh^2(x) - x^2\coth(x) + \frac{1}{4}\cosh(x)\sinh(x) + \frac{1}{2}x^2\cosh(x)\sinh(x)$$
$$= \frac{x}{4} - x^2 + \frac{x^3}{2} - \frac{1}{2}x\cosh^2(x) - x^2\coth(x) + 2x\log\left(1-e^{2x}\right) + \frac{1}{4}\cosh(x)\sinh(x) +$$
$$= \frac{x}{4} - x^2 + \frac{x^3}{2} - \frac{1}{2}x\cosh^2(x) - x^2\coth(x) + 2x\log\left(1-e^{2x}\right) + \frac{1}{4}\cosh(x)\sinh(x) +$$
$$= \frac{x}{4} - x^2 + \frac{x^3}{2} - \frac{1}{2}x\cosh^2(x) - x^2\coth(x) + 2x\log\left(1-e^{2x}\right) + \mathrm{Li}_2\left(e^{2x}\right) + \frac{1}{4}\cosh(x$$

Mathematica [A] time = 0.10, size = 64, normalized size = 0.88

$$\frac{1}{8}\left(-8\mathrm{Li}_2\left(e^{-2x}\right) + 4x^3 + 8x^2 + 2x^2\sinh(2x) - 8x^2\coth(x) + 16x\log\left(1-e^{-2x}\right) + \sinh(2x) - 2x\cosh(2x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Cosh[x]^2*Coth[x]^2,x]
```

```
[Out] (8*x^2 + 4*x^3 - 2*x*Cosh[2*x] - 8*x^2*Coth[x] + 16*x*Log[1 - E^(-2*x)] - 8
*PolyLog[2, E^(-2*x)] + Sinh[2*x] + 2*x^2*Sinh[2*x])/8
```

fricas [B] time = 0.47, size = 617, normalized size = 8.45

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cosh(x)^2*coth(x)^2,x, algorithm="fricas")
```

```
[Out] 1/16*((2*x^2 - 2*x + 1)*cosh(x)^6 + 6*(2*x^2 - 2*x + 1)*cosh(x)*sinh(x)^5 +
 (2*x^2 - 2*x + 1)*sinh(x)^6 + (8*x^3 - 34*x^2 + 2*x - 1)*cosh(x)^4 + (8*x^
3 + 15*(2*x^2 - 2*x + 1)*cosh(x)^2 - 34*x^2 + 2*x - 1)*sinh(x)^4 + 4*(5*(2*
x^2 - 2*x + 1)*cosh(x)^3 + (8*x^3 - 34*x^2 + 2*x - 1)*cosh(x))*sinh(x)^3 -
(8*x^3 + 2*x^2 + 2*x + 1)*cosh(x)^2 + (15*(2*x^2 - 2*x + 1)*cosh(x)^4 - 8*x
```

```
^3 + 6*(8*x^3 - 34*x^2 + 2*x - 1)*cosh(x)^2 - 2*x^2 - 2*x - 1)*sinh(x)^2 +
2*x^2 + 32*(cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + (6*cosh(x)^2 - 1)
*sinh(x)^2 - cosh(x)^2 + 2*(2*cosh(x)^3 - cosh(x))*sinh(x))*dilog(cosh(x) +
 sinh(x)) + 32*(cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + (6*cosh(x)^2
- 1)*sinh(x)^2 - cosh(x)^2 + 2*(2*cosh(x)^3 - cosh(x))*sinh(x))*dilog(-cosh
(x) - sinh(x)) + 32*(x*cosh(x)^4 + 4*x*cosh(x)*sinh(x)^3 + x*sinh(x)^4 - x*
cosh(x)^2 + (6*x*cosh(x)^2 - x)*sinh(x)^2 + 2*(2*x*cosh(x)^3 - x*cosh(x))*s
inh(x))*log(cosh(x) + sinh(x) + 1) + 32*(x*cosh(x)^4 + 4*x*cosh(x)*sinh(x)^
3 + x*sinh(x)^4 - x*cosh(x)^2 + (6*x*cosh(x)^2 - x)*sinh(x)^2 + 2*(2*x*cosh
(x)^3 - x*cosh(x))*sinh(x))*log(-cosh(x) - sinh(x) + 1) + 2*(3*(2*x^2 - 2*x
 + 1)*cosh(x)^5 + 2*(8*x^3 - 34*x^2 + 2*x - 1)*cosh(x)^3 - (8*x^3 + 2*x^2 +
 2*x + 1)*cosh(x))*sinh(x) + 2*x + 1)/(cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + si
nh(x)^4 + (6*cosh(x)^2 - 1)*sinh(x)^2 - cosh(x)^2 + 2*(2*cosh(x)^3 - cosh(x
))*sinh(x))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2\cosh(x)^2\coth(x)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cosh(x)^2*coth(x)^2,x, algorithm="giac")
```

```
[Out] integrate(x^2*cosh(x)^2*coth(x)^2, x)
```

maple [A] time = 0.24, size = 87, normalized size = 1.19

$$\frac{x^3}{2}+\left(\frac{1}{16}-\frac{1}{8}x+\frac{1}{8}x^2\right)e^{2x}+\left(-\frac{1}{16}-\frac{1}{8}x-\frac{1}{8}x^2\right)e^{-2x}-\frac{2x^2}{e^{2x}-1}-2x^2+2x\ln\left(e^x+1\right)+2\operatorname{polylog}\left(2,-e^x\right)+2x\ln\left(1-e^x\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*cosh(x)^2*coth(x)^2,x)
```

```
[Out] 1/2*x^3+(1/16-1/8*x+1/8*x^2)*exp(2*x)+(-1/16-1/8*x-1/8*x^2)*exp(-2*x)-2*x^2
/(exp(2*x)-1)-2*x^2+2*x*ln(exp(x)+1)+2*polylog(2,-exp(x))+2*x*ln(1-exp(x))+
2*polylog(2,exp(x))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cosh(x)^2*coth(x)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^2 \cosh(x)^2 \coth(x)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*cosh(x)^2*coth(x)^2,x)
```

```
[Out] int(x^2*cosh(x)^2*coth(x)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \cosh^2(x) \coth^2(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*cosh(x)**2*coth(x)**2,x)
```

```
[Out] Integral(x**2*cosh(x)**2*coth(x)**2, x)
```

3.420 $\int x^3 \cosh^2(x) \coth^2(x)\, dx$

Optimal. Leaf size=102

$$3x\text{Li}_2\left(e^{2x}\right)-\frac{3\text{Li}_3\left(e^{2x}\right)}{2}+\frac{3x^4}{8}-x^3-x^3\coth(x)+\frac{1}{2}x^3\sinh(x)\cosh(x)+\frac{3x^2}{8}+3x^2\log\left(1-e^{2x}\right)-\frac{3}{4}x^2\cosh^2(x)-\frac{3\cosh}{8}$$

```
[Out] 3/8*x^2-x^3+3/8*x^4-3/8*cosh(x)^2-3/4*x^2*cosh(x)^2-x^3*coth(x)+3*x^2*ln(1-
exp(2*x))+3*x*polylog(2,exp(2*x))-3/2*polylog(3,exp(2*x))+3/4*x*cosh(x)*sin
h(x)+1/2*x^3*cosh(x)*sinh(x)
```

Rubi [A] time = 0.18, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 10, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.833, Rules used = {5450, 3311, 30, 3310, 3720, 3716, 2190, 2531, 2282, 6589}

$$3x\text{PolyLog}\left(2,e^{2x}\right)-\frac{3}{2}\text{PolyLog}\left(3,e^{2x}\right)+\frac{3x^4}{8}-x^3+\frac{3x^2}{8}+3x^2\log\left(1-e^{2x}\right)-\frac{3}{4}x^2\cosh^2(x)-x^3\coth(x)+\frac{1}{2}x^3\sinh(x)$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Cosh[x]^2*Coth[x]^2,x]
```

```
[Out] (3*x^2)/8 - x^3 + (3*x^4)/8 - (3*Cosh[x]^2)/8 - (3*x^2*Cosh[x]^2)/4 - x^3*C
oth[x] + 3*x^2*Log[1 - E^(2*x)] + 3*x*PolyLog[2, E^(2*x)] - (3*PolyLog[3, E
^(2*x)])/2 + (3*x*Cosh[x]*Sinh[x])/4 + (x^3*Cosh[x]*Sinh[x])/2
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
```

```
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 3310

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[(d*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[(b*(c + d*x)*Cos[e + f*x]*(b
*Sin[e + f*x])^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 3311

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(d*m*(c + d*x)^(m - 1)*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist
[(b^2*(n - 1))/n, Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[(
d^2*m*(m - 1))/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[(b*(c + d*x)^m*Cos[e + f*x]*(b*Sin[e + f*x])^(n - 1))/(f*n), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 3716

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2
*I, Int[((c + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(E^(2*I*k*Pi)*(1 + E^(2*(-(I*
e) + f*fz*x))/E^(2*I*k*Pi))), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Integ
erQ[4*k] && IGtQ[m, 0]
```

Rule 3720

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[(b*(c + d*x)^m*(b*Tan[e + f*x])^(n - 1))/(f*(n - 1)), x] + (-Di
st[(b*d*m)/(f*(n - 1)), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x],
 x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[
{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rule 5450

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(n_.)*Coth[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) +
(d_.)*(x_))^(m_.), x_Symbol] :> Int[(c + d*x)^m*Cosh[a + b*x]^n*Coth[a + b*
x]^(p - 2), x] + Int[(c + d*x)^m*Cosh[a + b*x]^(n - 2)*Coth[a + b*x]^p, x]
/; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int x^3\cosh^2(x)\coth^2(x)\,dx = \int x^3\cosh^2(x)\,dx + \int x^3\coth^2(x)\,dx$$
$$= -\frac{3}{4}x^2\cosh^2(x) - x^3\coth(x) + \frac{1}{2}x^3\cosh(x)\sinh(x) + \frac{\int x^3\,dx}{2} + \frac{3}{2}\int x\cosh^2(x)\,dx$$
$$= -x^3 + \frac{3x^4}{8} - \frac{3\cosh^2(x)}{8} - \frac{3}{4}x^2\cosh^2(x) - x^3\coth(x) + \frac{3}{4}x\cosh(x)\sinh(x) + \frac{1}{2}x^3 c$$
$$= \frac{3x^2}{8} - x^3 + \frac{3x^4}{8} - \frac{3\cosh^2(x)}{8} - \frac{3}{4}x^2\cosh^2(x) - x^3\coth(x) + 3x^2\log\left(1-e^{2x}\right) + \frac{3}{4}x$$
$$= \frac{3x^2}{8} - x^3 + \frac{3x^4}{8} - \frac{3\cosh^2(x)}{8} - \frac{3}{4}x^2\cosh^2(x) - x^3\coth(x) + 3x^2\log\left(1-e^{2x}\right) + 3x\mathrm{l}$$
$$= \frac{3x^2}{8} - x^3 + \frac{3x^4}{8} - \frac{3\cosh^2(x)}{8} - \frac{3}{4}x^2\cosh^2(x) - x^3\coth(x) + 3x^2\log\left(1-e^{2x}\right) + 3x\mathrm{l}$$
$$= \frac{3x^2}{8} - x^3 + \frac{3x^4}{8} - \frac{3\cosh^2(x)}{8} - \frac{3}{4}x^2\cosh^2(x) - x^3\coth(x) + 3x^2\log\left(1-e^{2x}\right) + 3x\mathrm{l}$$

Mathematica [C] time = 0.15, size = 94, normalized size = 0.92

$$\frac{1}{16}\left(48x\mathrm{Li}_2\left(e^{2x}\right) - 24\mathrm{Li}_3\left(e^{2x}\right) + 6x^4 - 16x^3 + 4x^3\sinh(2x) - 16x^3\coth(x) + 48x^2\log\left(1-e^{2x}\right) - 6x^2\cosh(2x) + \right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Cosh[x]^2*Coth[x]^2,x]
```

```
[Out] ((2*I)*Pi^3 - 16*x^3 + 6*x^4 - 3*Cosh[2*x] - 6*x^2*Cosh[2*x] - 16*x^3*Coth[
x] + 48*x^2*Log[1 - E^(2*x)] + 48*x*PolyLog[2, E^(2*x)] - 24*PolyLog[3, E^(
2*x)] + 6*x*Sinh[2*x] + 4*x^3*Sinh[2*x])/16
```

fricas [C] time = 0.55, size = 875, normalized size = 8.58

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*cosh(x)^2*coth(x)^2,x, algorithm="fricas")
```

```
[Out] 1/32*((4*x^3 - 6*x^2 + 6*x - 3)*cosh(x)^6 + 6*(4*x^3 - 6*x^2 + 6*x - 3)*cos
h(x)*sinh(x)^5 + (4*x^3 - 6*x^2 + 6*x - 3)*sinh(x)^6 + (12*x^4 - 68*x^3 + 6
*x^2 - 6*x + 3)*cosh(x)^4 + (12*x^4 - 68*x^3 + 15*(4*x^3 - 6*x^2 + 6*x - 3)
*cosh(x)^2 + 6*x^2 - 6*x + 3)*sinh(x)^4 + 4*(5*(4*x^3 - 6*x^2 + 6*x - 3)*co
sh(x)^3 + (12*x^4 - 68*x^3 + 6*x^2 - 6*x + 3)*cosh(x))*sinh(x)^3 + 4*x^3 -
(12*x^4 + 4*x^3 + 6*x^2 + 6*x + 3)*cosh(x)^2 + (15*(4*x^3 - 6*x^2 + 6*x - 3
)*cosh(x)^4 - 12*x^4 - 4*x^3 + 6*(12*x^4 - 68*x^3 + 6*x^2 - 6*x + 3)*cosh(x
)^2 - 6*x^2 - 6*x - 3)*sinh(x)^2 + 6*x^2 + 192*(x*cosh(x)^4 + 4*x*cosh(x)*s
inh(x)^3 + x*sinh(x)^4 - x*cosh(x)^2 + (6*x*cosh(x)^2 - x)*sinh(x)^2 + 2*(2
*x*cosh(x)^3 - x*cosh(x))*sinh(x))*dilog(cosh(x) + sinh(x)) + 192*(x*cosh(x
)^4 + 4*x*cosh(x)*sinh(x)^3 + x*sinh(x)^4 - x*cosh(x)^2 + (6*x*cosh(x)^2 -
x)*sinh(x)^2 + 2*(2*x*cosh(x)^3 - x*cosh(x))*sinh(x))*dilog(-cosh(x) - sinh
(x)) + 96*(x^2*cosh(x)^4 + 4*x^2*cosh(x)*sinh(x)^3 + x^2*sinh(x)^4 - x^2*co
sh(x)^2 + (6*x^2*cosh(x)^2 - x^2)*sinh(x)^2 + 2*(2*x^2*cosh(x)^3 - x^2*cosh
(x))*sinh(x))*log(cosh(x) + sinh(x) + 1) + 96*(x^2*cosh(x)^4 + 4*x^2*cosh(x
)*sinh(x)^3 + x^2*sinh(x)^4 - x^2*cosh(x)^2 + (6*x^2*cosh(x)^2 - x^2)*sinh(
x)^2 + 2*(2*x^2*cosh(x)^3 - x^2*cosh(x))*sinh(x))*log(-cosh(x) - sinh(x) +
1) - 192*(cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + (6*cosh(x)^2 - 1)*s
inh(x)^2 - cosh(x)^2 + 2*(2*cosh(x)^3 - cosh(x))*sinh(x))*polylog(3, cosh(x
) + sinh(x)) - 192*(cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + (6*cosh(x
)^2 - 1)*sinh(x)^2 - cosh(x)^2 + 2*(2*cosh(x)^3 - cosh(x))*sinh(x))*polylog
(3, -cosh(x) - sinh(x)) + 2*(3*(4*x^3 - 6*x^2 + 6*x - 3)*cosh(x)^5 + 2*(12*
x^4 - 68*x^3 + 6*x^2 - 6*x + 3)*cosh(x)^3 - (12*x^4 + 4*x^3 + 6*x^2 + 6*x +
 3)*cosh(x))*sinh(x) + 6*x + 3)/(cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^
4 + (6*cosh(x)^2 - 1)*sinh(x)^2 - cosh(x)^2 + 2*(2*cosh(x)^3 - cosh(x))*sin
h(x))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3 \cosh(x)^2 \coth(x)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*cosh(x)^2*coth(x)^2,x, algorithm="giac")
```

```
[Out] integrate(x^3*cosh(x)^2*coth(x)^2, x)
```

maple [A] time = 0.22, size = 117, normalized size = 1.15

$$\frac{3x^4}{8}+\left(-\frac{3}{32}+\frac{3}{16}x-\frac{3}{16}x^2+\frac{1}{8}x^3\right)e^{2x}+\left(-\frac{3}{32}-\frac{3}{16}x-\frac{3}{16}x^2-\frac{1}{8}x^3\right)e^{-2x}-\frac{2x^3}{e^{2x}-1}-2x^3+3x^2\ln\left(e^x+1\right)+6x\,\mathrm{polylo}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*cosh(x)^2*coth(x)^2,x)
```

```
[Out] 3/8*x^4+(-3/32+3/16*x-3/16*x^2+1/8*x^3)*exp(2*x)+(-3/32-3/16*x-3/16*x^2-1/8
*x^3)*exp(-2*x)-2*x^3/(exp(2*x)-1)-2*x^3+3*x^2*ln(exp(x)+1)+6*x*polylog(2,-
exp(x))-6*polylog(3,-exp(x))+3*x^2*ln(1-exp(x))+6*x*polylog(2,exp(x))-6*pol
ylog(3,exp(x))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*cosh(x)^2*coth(x)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^3\cosh(x)^2\coth(x)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*cosh(x)^2*coth(x)^2,x)
```

```
[Out] int(x^3*cosh(x)^2*coth(x)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3\cosh^2(x)\coth^2(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*cosh(x)**2*coth(x)**2,x)
```

```
[Out] Integral(x**3*cosh(x)**2*coth(x)**2, x)
```

3.421 $\int x \cosh^2(x) \coth^3(x) \, dx$

Optimal. Leaf size=63

$$\text{Li}_2\left(e^{2x}\right)-x^2+\frac{3x}{4}+2x\log\left(1-e^{2x}\right)+\frac{1}{2}x\sinh^2(x)-\frac{1}{2}x\coth^2(x)-\frac{\coth(x)}{2}-\frac{1}{4}\sinh(x)\cosh(x)$$

```
[Out] 3/4*x-x^2-1/2*coth(x)-1/2*x*coth(x)^2+2*x*ln(1-exp(2*x))+polylog(2,exp(2*x)
)-1/4*cosh(x)*sinh(x)+1/2*x*sinh(x)^2
```

Rubi [A] time = 0.16, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 10, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {5450, 5372, 2635, 8, 3716, 2190, 2279, 2391, 3720, 3473}

$$\text{PolyLog}\left(2,e^{2x}\right)-x^2+\frac{3x}{4}+2x\log\left(1-e^{2x}\right)+\frac{1}{2}x\sinh^2(x)-\frac{1}{2}x\coth^2(x)-\frac{\coth(x)}{2}-\frac{1}{4}\sinh(x)\cosh(x)$$

Antiderivative was successfully verified.

```
[In] Int[x*Cosh[x]^2*Coth[x]^3,x]
```

```
[Out] (3*x)/4 - x^2 - Coth[x]/2 - (x*Coth[x]^2)/2 + 2*x*Log[1 - E^(2*x)] + PolyLo
g[2, E^(2*x)] - (Cosh[x]*Sinh[x])/4 + (x*Sinh[x]^2)/2
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3716

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2
*I, Int[((c + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(E^(2*I*k*Pi)*(1 + E^(2*(-(I*
e) + f*fz*x))/E^(2*I*k*Pi))), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Integ
erQ[4*k] && IGtQ[m, 0]
```

Rule 3720

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[(b*(c + d*x)^m*(b*Tan[e + f*x])^(n - 1))/(f*(n - 1)), x] + (-Di
st[(b*d*m)/(f*(n - 1)), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x],
 x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[
{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rule 5372

```
Int[Cosh[(a_.) + (b_.)*(x_)^(n_.)]*(x_)^(m_.)*Sinh[(a_.) + (b_.)*(x_)^(n_.)
]^(p_.), x_Symbol] :> Simp[(x^(m - n + 1)*Sinh[a + b*x^n]^(p + 1))/(b*n*(p
+ 1)), x] - Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*Sinh[a + b*x^n]^(
p + 1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

Rule 5450

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(n_.)*Coth[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) +
(d_.)*(x_))^(m_.), x_Symbol] :> Int[(c + d*x)^m*Cosh[a + b*x]^n*Coth[a + b*
x]^(p - 2), x] + Int[(c + d*x)^m*Cosh[a + b*x]^(n - 2)*Coth[a + b*x]^p, x]
/; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rubi steps

$$\int x\cosh^2(x)\coth^3(x)\,dx = \int x\cosh^2(x)\coth(x)\,dx + \int x\coth^3(x)\,dx$$
$$= -\frac{1}{2}x\coth^2(x) + \frac{1}{2}\int \coth^2(x)\,dx + 2\int x\coth(x)\,dx + \int x\cosh(x)\sinh(x)\,dx$$
$$= -\frac{\coth(x)}{2} - \frac{1}{2}x\coth^2(x) + \frac{1}{2}x\sinh^2(x) + \frac{\int 1\,dx}{2} - \frac{1}{2}\int \sinh^2(x)\,dx + 2\left(-\frac{x^2}{2} - 2\right.$$
$$= \frac{x}{2} - \frac{\coth(x)}{2} - \frac{1}{2}x\coth^2(x) - \frac{1}{4}\cosh(x)\sinh(x) + \frac{1}{2}x\sinh^2(x) + \frac{\int 1\,dx}{4} + 2\left(-\frac{x^2}{2}\right.$$
$$= \frac{3x}{4} - \frac{\coth(x)}{2} - \frac{1}{2}x\coth^2(x) - \frac{1}{4}\cosh(x)\sinh(x) + \frac{1}{2}x\sinh^2(x) + 2\left(-\frac{x^2}{2} + x\log\left(\right.\right.$$
$$= \frac{3x}{4} - \frac{\coth(x)}{2} - \frac{1}{2}x\coth^2(x) + 2\left(-\frac{x^2}{2} + x\log\left(1-e^{2x}\right) + \frac{\operatorname{Li}_2\left(e^{2x}\right)}{2}\right) - \frac{1}{4}\cosh(x)\,\mathrm{si}$$

Mathematica [A] time = 0.08, size = 56, normalized size = 0.89

$$\frac{1}{8}\left(-8\operatorname{Li}_2\left(e^{-2x}\right) + 8x^2 + 16x\log\left(1-e^{-2x}\right) - \sinh(2x) + 2x\cosh(2x) - 4\coth(x) - 4x\operatorname{csch}^2(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Cosh[x]^2*Coth[x]^3,x]
```

```
[Out] (8*x^2 + 2*x*Cosh[2*x] - 4*Coth[x] - 4*x*Csch[x]^2 + 16*x*Log[1 - E^(-2*x)]
 - 8*PolyLog[2, E^(-2*x)] - Sinh[2*x])/8
```

fricas [B] time = 0.47, size = 916, normalized size = 14.54

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(x)^2*coth(x)^3,x, algorithm="fricas")
```

```
[Out] 1/16*((2*x - 1)*cosh(x)^8 + 8*(2*x - 1)*cosh(x)*sinh(x)^7 + (2*x - 1)*sinh(
x)^8 - 2*(8*x^2 + 2*x - 1)*cosh(x)^6 + 2*(14*(2*x - 1)*cosh(x)^2 - 8*x^2 -
2*x + 1)*sinh(x)^6 + 4*(14*(2*x - 1)*cosh(x)^3 - 3*(8*x^2 + 2*x - 1)*cosh(x
))*sinh(x)^5 + 4*(8*x^2 - 7*x - 4)*cosh(x)^4 + 2*(35*(2*x - 1)*cosh(x)^4 -
15*(8*x^2 + 2*x - 1)*cosh(x)^2 + 16*x^2 - 14*x - 8)*sinh(x)^4 + 8*(7*(2*x -
 1)*cosh(x)^5 - 5*(8*x^2 + 2*x - 1)*cosh(x)^3 + 2*(8*x^2 - 7*x - 4)*cosh(x)
)*sinh(x)^3 - 2*(8*x^2 + 2*x - 7)*cosh(x)^2 + 2*(14*(2*x - 1)*cosh(x)^6 - 1
5*(8*x^2 + 2*x - 1)*cosh(x)^4 + 12*(8*x^2 - 7*x - 4)*cosh(x)^2 - 8*x^2 - 2*
```

```
x + 7)*sinh(x)^2 + 32*(cosh(x)^6 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6 + (15*co
sh(x)^2 - 2)*sinh(x)^4 - 2*cosh(x)^4 + 4*(5*cosh(x)^3 - 2*cosh(x))*sinh(x)^
3 + (15*cosh(x)^4 - 12*cosh(x)^2 + 1)*sinh(x)^2 + cosh(x)^2 + 2*(3*cosh(x)^
5 - 4*cosh(x)^3 + cosh(x))*sinh(x))*dilog(cosh(x) + sinh(x)) + 32*(cosh(x)^
6 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6 + (15*cosh(x)^2 - 2)*sinh(x)^4 - 2*cosh
(x)^4 + 4*(5*cosh(x)^3 - 2*cosh(x))*sinh(x)^3 + (15*cosh(x)^4 - 12*cosh(x)^
2 + 1)*sinh(x)^2 + cosh(x)^2 + 2*(3*cosh(x)^5 - 4*cosh(x)^3 + cosh(x))*sinh
(x))*dilog(-cosh(x) - sinh(x)) + 32*(x*cosh(x)^6 + 6*x*cosh(x)*sinh(x)^5 +
x*sinh(x)^6 - 2*x*cosh(x)^4 + (15*x*cosh(x)^2 - 2*x)*sinh(x)^4 + 4*(5*x*cos
h(x)^3 - 2*x*cosh(x))*sinh(x)^3 + x*cosh(x)^2 + (15*x*cosh(x)^4 - 12*x*cosh
(x)^2 + x)*sinh(x)^2 + 2*(3*x*cosh(x)^5 - 4*x*cosh(x)^3 + x*cosh(x))*sinh(x
))*log(cosh(x) + sinh(x) + 1) + 32*(x*cosh(x)^6 + 6*x*cosh(x)*sinh(x)^5 + x
*sinh(x)^6 - 2*x*cosh(x)^4 + (15*x*cosh(x)^2 - 2*x)*sinh(x)^4 + 4*(5*x*cosh
(x)^3 - 2*x*cosh(x))*sinh(x)^3 + x*cosh(x)^2 + (15*x*cosh(x)^4 - 12*x*cosh(
x)^2 + x)*sinh(x)^2 + 2*(3*x*cosh(x)^5 - 4*x*cosh(x)^3 + x*cosh(x))*sinh(x)
)*log(-cosh(x) - sinh(x) + 1) + 4*(2*(2*x - 1)*cosh(x)^7 - 3*(8*x^2 + 2*x -
 1)*cosh(x)^5 + 4*(8*x^2 - 7*x - 4)*cosh(x)^3 - (8*x^2 + 2*x - 7)*cosh(x))*
sinh(x) + 2*x + 1)/(cosh(x)^6 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6 + (15*cosh(
x)^2 - 2)*sinh(x)^4 - 2*cosh(x)^4 + 4*(5*cosh(x)^3 - 2*cosh(x))*sinh(x)^3 +
 (15*cosh(x)^4 - 12*cosh(x)^2 + 1)*sinh(x)^2 + cosh(x)^2 + 2*(3*cosh(x)^5 -
 4*cosh(x)^3 + cosh(x))*sinh(x))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\cosh(x)^2\coth(x)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(x)^2*coth(x)^3,x, algorithm="giac")
```

```
[Out] integrate(x*cosh(x)^2*coth(x)^3, x)
```

maple [A] time = 0.30, size = 82, normalized size = 1.30

$$-x^2+\left(-\frac{1}{16}+\frac{x}{8}\right)e^{2x}+\left(\frac{1}{16}+\frac{x}{8}\right)e^{-2x}-\frac{2x\,e^{2x}+e^{2x}-1}{\left(e^{2x}-1\right)^2}+2x\ln\left(e^x+1\right)+2\,\mathrm{polylog}\left(2,-e^x\right)+2x\ln\left(1-e^x\right)+2\,\mathrm{polylo}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cosh(x)^2*coth(x)^3,x)
```

```
[Out] -x^2+(-1/16+1/8*x)*exp(2*x)+(1/16+1/8*x)*exp(-2*x)-(2*x*exp(2*x)+exp(2*x)-1
)/(exp(2*x)-1)^2+2*x*ln(exp(x)+1)+2*polylog(2,-exp(x))+2*x*ln(1-exp(x))+2*p
olylog(2,exp(x))
```

maxima [B] time = 0.66, size = 146, normalized size = 2.32

$$-2\,x^2+2\,x\log\left(e^x+1\right)+2\,x\log\left(-e^x+1\right)+\frac{5}{8}\,x+\frac{16\,x^2+(2\,x-1)e^{(6\,x)}+2\left(8\,x^2-2\,x+1\right)e^{(4\,x)}-\left(32\,x^2+8\,x+11\right)}{16\left(e^{(4\,x)}-2\,e^{(2\,x)}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(x)^2*coth(x)^3,x, algorithm="maxima")
```

```
[Out] -2*x^2 + 2*x*log(e^x + 1) + 2*x*log(-e^x + 1) + 5/8*x + 1/16*(16*x^2 + (2*x
 - 1)*e^(6*x) + 2*(8*x^2 - 2*x + 1)*e^(4*x) - (32*x^2 + 8*x + 11)*e^(2*x) +
 (2*x + 1)*e^(-2*x) - 14*x + 9)/(e^(4*x) - 2*e^(2*x) + 1) - 5/16*(2*x*e^(4*
x) + e^(2*x) - 1)/(e^(4*x) - 2*e^(2*x) + 1) + 2*dilog(-e^x) + 2*dilog(e^x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int x\cosh(x)^2\coth(x)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cosh(x)^2*coth(x)^3,x)
```

```
[Out] int(x*cosh(x)^2*coth(x)^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\cosh^2(x)\coth^3(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(x)**2*coth(x)**3,x)
```

```
[Out] Integral(x*cosh(x)**2*coth(x)**3, x)
```

3.422 $\int x^2 \cosh^2(x) \coth^3(x)\, dx$

Optimal. Leaf size=96

$$2x\mathrm{Li}_2\left(e^{2x}\right)-\mathrm{Li}_3\left(e^{2x}\right)-\frac{2x^3}{3}+\frac{3x^2}{4}+2x^2\log\left(1-e^{2x}\right)+\frac{1}{2}x^2\sinh^2(x)-\frac{1}{2}x^2\coth^2(x)+\frac{\sinh^2(x)}{4}-x\coth(x)+\log(\sinh($$

```
[Out] 3/4*x^2-2/3*x^3-x*coth(x)-1/2*x^2*coth(x)^2+2*x^2*ln(1-exp(2*x))+ln(sinh(x)
)+2*x*polylog(2,exp(2*x))-polylog(3,exp(2*x))-1/2*x*cosh(x)*sinh(x)+1/4*sin
h(x)^2+1/2*x^2*sinh(x)^2
```

Rubi [A] time = 0.28, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 11, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.917, Rules used = {5450, 5372, 3310, 30, 3716, 2190, 2531, 2282, 6589, 3720, 3475}

$$2x\mathrm{PolyLog}\left(2,e^{2x}\right)-\mathrm{PolyLog}\left(3,e^{2x}\right)-\frac{2x^3}{3}+\frac{3x^2}{4}+2x^2\log\left(1-e^{2x}\right)+\frac{1}{2}x^2\sinh^2(x)-\frac{1}{2}x^2\coth^2(x)+\frac{\sinh^2(x)}{4}-x\,\mathrm{cot}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Cosh[x]^2*Coth[x]^3,x]
```

```
[Out] (3*x^2)/4 - (2*x^3)/3 - x*Coth[x] - (x^2*Coth[x]^2)/2 + 2*x^2*Log[1 - E^(2*
x)] + Log[Sinh[x]] + 2*x*PolyLog[2, E^(2*x)] - PolyLog[3, E^(2*x)] - (x*Cos
h[x]*Sinh[x])/2 + Sinh[x]^2/4 + (x^2*Sinh[x]^2)/2
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
```

```
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 3310

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[(d*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[(b*(c + d*x)*Cos[e + f*x]*(b
*Sin[e + f*x])^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3716

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2
*I, Int[((c + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(E^(2*I*k*Pi)*(1 + E^(2*(-(I*
e) + f*fz*x))/E^(2*I*k*Pi))), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Integ
erQ[4*k] && IGtQ[m, 0]
```

Rule 3720

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[(b*(c + d*x)^m*(b*Tan[e + f*x])^(n - 1))/(f*(n - 1)), x] + (-Di
st[(b*d*m)/(f*(n - 1)), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x],
 x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[
{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rule 5372

```
Int[Cosh[(a_.) + (b_.)*(x_)^(n_.)]*(x_)^(m_.)*Sinh[(a_.) + (b_.)*(x_)^(n_.)
]^(p_.), x_Symbol] :> Simp[(x^(m - n + 1)*Sinh[a + b*x^n]^(p + 1))/(b*n*(p
+ 1)), x] - Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*Sinh[a + b*x^n]^(
```

```
p + 1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

Rule 5450

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(n_.)*Coth[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) +
(d_.)*(x_))^(m_.), x_Symbol] :> Int[(c + d*x)^m*Cosh[a + b*x]^n*Coth[a + b*
x]^(p - 2), x] + Int[(c + d*x)^m*Cosh[a + b*x]^(n - 2)*Coth[a + b*x]^p, x]
/; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}\int x^2\cosh^2(x)\coth^3(x)\,dx &= \int x^2\cosh^2(x)\coth(x)\,dx+\int x^2\coth^3(x)\,dx\\ &= -\frac{1}{2}x^2\coth^2(x)+2\int x^2\coth(x)\,dx+\int x\coth^2(x)\,dx+\int x^2\cosh(x)\sinh(x)\,dx\\ &= -x\coth(x)-\frac{1}{2}x^2\coth^2(x)+\frac{1}{2}x^2\sinh^2(x)+2\left(-\frac{x^3}{3}-2\int\frac{e^{2x}x^2}{1-e^{2x}}\,dx\right)+\int x\,dx+\\ &= \frac{x^2}{2}-x\coth(x)-\frac{1}{2}x^2\coth^2(x)+\log(\sinh(x))-\frac{1}{2}x\cosh(x)\sinh(x)+\frac{\sinh^2(x)}{4}+\\ &= \frac{3x^2}{4}-x\coth(x)-\frac{1}{2}x^2\coth^2(x)+\log(\sinh(x))-\frac{1}{2}x\cosh(x)\sinh(x)+\frac{\sinh^2(x)}{4}+\\ &= \frac{3x^2}{4}-x\coth(x)-\frac{1}{2}x^2\coth^2(x)+\log(\sinh(x))-\frac{1}{2}x\cosh(x)\sinh(x)+\frac{\sinh^2(x)}{4}+\\ &= \frac{3x^2}{4}-x\coth(x)-\frac{1}{2}x^2\coth^2(x)+\log(\sinh(x))+2\left(-\frac{x^3}{3}+x^2\log\left(1-e^{2x}\right)+x\mathrm{Li}_2\left(\right.\right.\end{aligned}$$

Mathematica [C] time = 0.28, size = 98, normalized size = 1.02

$$2x\mathrm{Li}_2\left(e^{2x}\right)-\mathrm{Li}_3\left(e^{2x}\right)-\frac{2x^3}{3}+2x^2\log\left(1-e^{2x}\right)+\frac{1}{4}x^2\cosh(2x)-\frac{1}{2}x^2\mathrm{csch}^2(x)-\frac{1}{4}x\sinh(2x)+\frac{1}{8}\cosh(2x)-x\coth(x)+$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Cosh[x]^2*Coth[x]^3,x]
```

```
[Out] (I/12)*Pi^3 - (2*x^3)/3 + Cosh[2*x]/8 + (x^2*Cosh[2*x])/4 - x*Coth[x] - (x^
2*Csch[x]^2)/2 + 2*x^2*Log[1 - E^(2*x)] + Log[Sinh[x]] + 2*x*PolyLog[2, E^(
2*x)] - PolyLog[3, E^(2*x)] - (x*Sinh[2*x])/4
```

fricas [C] time = 0.59, size = 1512, normalized size = 15.75

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cosh(x)^2*coth(x)^3,x, algorithm="fricas")
```

```
[Out] 1/48*(3*(2*x^2 - 2*x + 1)*cosh(x)^8 + 24*(2*x^2 - 2*x + 1)*cosh(x)*sinh(x)^
7 + 3*(2*x^2 - 2*x + 1)*sinh(x)^8 - 2*(16*x^3 + 6*x^2 + 42*x + 3)*cosh(x)^6
 - 2*(16*x^3 - 42*(2*x^2 - 2*x + 1)*cosh(x)^2 + 6*x^2 + 42*x + 3)*sinh(x)^6
 + 12*(14*(2*x^2 - 2*x + 1)*cosh(x)^3 - (16*x^3 + 6*x^2 + 42*x + 3)*cosh(x)
)*sinh(x)^5 + 2*(32*x^3 - 42*x^2 + 48*x + 3)*cosh(x)^4 + 2*(105*(2*x^2 - 2*
x + 1)*cosh(x)^4 + 32*x^3 - 15*(16*x^3 + 6*x^2 + 42*x + 3)*cosh(x)^2 - 42*x
^2 + 48*x + 3)*sinh(x)^4 + 8*(21*(2*x^2 - 2*x + 1)*cosh(x)^5 - 5*(16*x^3 +
6*x^2 + 42*x + 3)*cosh(x)^3 + (32*x^3 - 42*x^2 + 48*x + 3)*cosh(x))*sinh(x)
^3 - 2*(16*x^3 + 6*x^2 + 6*x + 3)*cosh(x)^2 + 2*(42*(2*x^2 - 2*x + 1)*cosh(
x)^6 - 15*(16*x^3 + 6*x^2 + 42*x + 3)*cosh(x)^4 - 16*x^3 + 6*(32*x^3 - 42*x
^2 + 48*x + 3)*cosh(x)^2 - 6*x^2 - 6*x - 3)*sinh(x)^2 + 6*x^2 + 192*(x*cosh
(x)^6 + 6*x*cosh(x)*sinh(x)^5 + x*sinh(x)^6 - 2*x*cosh(x)^4 + (15*x*cosh(x)
^2 - 2*x)*sinh(x)^4 + 4*(5*x*cosh(x)^3 - 2*x*cosh(x))*sinh(x)^3 + x*cosh(x)
^2 + (15*x*cosh(x)^4 - 12*x*cosh(x)^2 + x)*sinh(x)^2 + 2*(3*x*cosh(x)^5 - 4
*x*cosh(x)^3 + x*cosh(x))*sinh(x))*dilog(cosh(x) + sinh(x)) + 192*(x*cosh(x
)^6 + 6*x*cosh(x)*sinh(x)^5 + x*sinh(x)^6 - 2*x*cosh(x)^4 + (15*x*cosh(x)^2
 - 2*x)*sinh(x)^4 + 4*(5*x*cosh(x)^3 - 2*x*cosh(x))*sinh(x)^3 + x*cosh(x)^2
 + (15*x*cosh(x)^4 - 12*x*cosh(x)^2 + x)*sinh(x)^2 + 2*(3*x*cosh(x)^5 - 4*x
*cosh(x)^3 + x*cosh(x))*sinh(x))*dilog(-cosh(x) - sinh(x)) + 48*((2*x^2 + 1
)*cosh(x)^6 + 6*(2*x^2 + 1)*cosh(x)*sinh(x)^5 + (2*x^2 + 1)*sinh(x)^6 - 2*(
2*x^2 + 1)*cosh(x)^4 + (15*(2*x^2 + 1)*cosh(x)^2 - 4*x^2 - 2)*sinh(x)^4 + 4
*(5*(2*x^2 + 1)*cosh(x)^3 - 2*(2*x^2 + 1)*cosh(x))*sinh(x)^3 + (2*x^2 + 1)*
cosh(x)^2 + (15*(2*x^2 + 1)*cosh(x)^4 - 12*(2*x^2 + 1)*cosh(x)^2 + 2*x^2 +
1)*sinh(x)^2 + 2*(3*(2*x^2 + 1)*cosh(x)^5 - 4*(2*x^2 + 1)*cosh(x)^3 + (2*x^
2 + 1)*cosh(x))*sinh(x))*log(cosh(x) + sinh(x) + 1) + 48*(cosh(x)^6 + 6*cos
h(x)*sinh(x)^5 + sinh(x)^6 + (15*cosh(x)^2 - 2)*sinh(x)^4 - 2*cosh(x)^4 + 4
*(5*cosh(x)^3 - 2*cosh(x))*sinh(x)^3 + (15*cosh(x)^4 - 12*cosh(x)^2 + 1)*si
nh(x)^2 + cosh(x)^2 + 2*(3*cosh(x)^5 - 4*cosh(x)^3 + cosh(x))*sinh(x))*log(
cosh(x) + sinh(x) - 1) + 96*(x^2*cosh(x)^6 + 6*x^2*cosh(x)*sinh(x)^5 + x^2*
sinh(x)^6 - 2*x^2*cosh(x)^4 + (15*x^2*cosh(x)^2 - 2*x^2)*sinh(x)^4 + x^2*co
sh(x)^2 + 4*(5*x^2*cosh(x)^3 - 2*x^2*cosh(x))*sinh(x)^3 + (15*x^2*cosh(x)^4
 - 12*x^2*cosh(x)^2 + x^2)*sinh(x)^2 + 2*(3*x^2*cosh(x)^5 - 4*x^2*cosh(x)^3
 + x^2*cosh(x))*sinh(x))*log(-cosh(x) - sinh(x) + 1) - 192*(cosh(x)^6 + 6*c
osh(x)*sinh(x)^5 + sinh(x)^6 + (15*cosh(x)^2 - 2)*sinh(x)^4 - 2*cosh(x)^4 +
```

```
 4*(5*cosh(x)^3 - 2*cosh(x))*sinh(x)^3 + (15*cosh(x)^4 - 12*cosh(x)^2 + 1)*
sinh(x)^2 + cosh(x)^2 + 2*(3*cosh(x)^5 - 4*cosh(x)^3 + cosh(x))*sinh(x))*po
lylog(3, cosh(x) + sinh(x)) - 192*(cosh(x)^6 + 6*cosh(x)*sinh(x)^5 + sinh(x
)^6 + (15*cosh(x)^2 - 2)*sinh(x)^4 - 2*cosh(x)^4 + 4*(5*cosh(x)^3 - 2*cosh(
x))*sinh(x)^3 + (15*cosh(x)^4 - 12*cosh(x)^2 + 1)*sinh(x)^2 + cosh(x)^2 + 2
*(3*cosh(x)^5 - 4*cosh(x)^3 + cosh(x))*sinh(x))*polylog(3, -cosh(x) - sinh(
x)) + 4*(6*(2*x^2 - 2*x + 1)*cosh(x)^7 - 3*(16*x^3 + 6*x^2 + 42*x + 3)*cosh
(x)^5 + 2*(32*x^3 - 42*x^2 + 48*x + 3)*cosh(x)^3 - (16*x^3 + 6*x^2 + 6*x +
3)*cosh(x))*sinh(x) + 6*x + 3)/(cosh(x)^6 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6
 + (15*cosh(x)^2 - 2)*sinh(x)^4 - 2*cosh(x)^4 + 4*(5*cosh(x)^3 - 2*cosh(x))
*sinh(x)^3 + (15*cosh(x)^4 - 12*cosh(x)^2 + 1)*sinh(x)^2 + cosh(x)^2 + 2*(3
*cosh(x)^5 - 4*cosh(x)^3 + cosh(x))*sinh(x))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \cosh(x)^2 \coth(x)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cosh(x)^2*coth(x)^3,x, algorithm="giac")
```

```
[Out] integrate(x^2*cosh(x)^2*coth(x)^3, x)
```

maple [A] time = 0.32, size = 127, normalized size = 1.32

$$-\frac{2x^3}{3}+\left(\frac{1}{16}-\frac{1}{8}x+\frac{1}{8}x^2\right)e^{2x}+\left(\frac{1}{16}+\frac{1}{8}x+\frac{1}{8}x^2\right)e^{-2x}-\frac{2x\left(x\,e^{2x}+e^{2x}-1\right)}{\left(e^{2x}-1\right)^2}+\ln\left(e^x-1\right)+\ln\left(e^x+1\right)-2\ln\left(e^x\right)+2x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*cosh(x)^2*coth(x)^3,x)
```

```
[Out] -2/3*x^3+(1/16-1/8*x+1/8*x^2)*exp(2*x)+(1/16+1/8*x+1/8*x^2)*exp(-2*x)-2*x*(
x*exp(2*x)+exp(2*x)-1)/(exp(2*x)-1)^2+ln(exp(x)-1)+ln(exp(x)+1)-2*ln(exp(x)
)+2*x^2*ln(exp(x)+1)+4*x*polylog(2,-exp(x))-4*polylog(3,-exp(x))+2*x^2*ln(1
-exp(x))+4*x*polylog(2,exp(x))-4*polylog(3,exp(x))
```

maxima [B] time = 0.65, size = 174, normalized size = 1.81

$$-\frac{4}{3}x^3+2x^2\log\left(e^x+1\right)+2x^2\log\left(-e^x+1\right)+4x\mathrm{Li}_2\left(-e^x\right)+4x\mathrm{Li}_2\left(e^x\right)-2x+\frac{32x^3-12x^2+3\left(2x^2-2x+1\right)e^{(6x)}+}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cosh(x)^2*coth(x)^3,x, algorithm="maxima")
```

```
[Out] -4/3*x^3 + 2*x^2*log(e^x + 1) + 2*x^2*log(-e^x + 1) + 4*x*dilog(-e^x) + 4*x
*dilog(e^x) - 2*x + 1/48*(32*x^3 - 12*x^2 + 3*(2*x^2 - 2*x + 1)*e^(6*x) + 2
*(16*x^3 - 6*x^2 + 6*x - 3)*e^(4*x) - 2*(32*x^3 + 42*x^2 + 48*x - 3)*e^(2*x
) + 3*(2*x^2 + 2*x + 1)*e^(-2*x) + 84*x - 6)/(e^(4*x) - 2*e^(2*x) + 1) + lo
g(e^x + 1) + log(e^x - 1) - 4*polylog(3, -e^x) - 4*polylog(3, e^x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^2 \cosh(x)^2 \coth(x)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*cosh(x)^2*coth(x)^3,x)
```

```
[Out] int(x^2*cosh(x)^2*coth(x)^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \cosh^2(x) \coth^3(x) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*cosh(x)**2*coth(x)**3,x)
```

```
[Out] Integral(x**2*cosh(x)**2*coth(x)**3, x)
```

3.423 $\int x^3 \cosh^2(x) \coth^3(x)\, dx$

Optimal. Leaf size=158

$$3x^2\text{Li}_2\left(e^{2x}\right)-3x\text{Li}_3\left(e^{2x}\right)+\frac{3\text{Li}_2\left(e^{2x}\right)}{2}+\frac{3\text{Li}_4\left(e^{2x}\right)}{2}-\frac{x^4}{2}+\frac{3x^3}{4}+2x^3\log\left(1-e^{2x}\right)+\frac{1}{2}x^3\sinh^2(x)-\frac{1}{2}x^3\coth^2(x)-\frac{3x^2}{2}-$$

```
[Out] 3/8*x-3/2*x^2+3/4*x^3-1/2*x^4-3/2*x^2*coth(x)-1/2*x^3*coth(x)^2+3*x*ln(1-ex
p(2*x))+2*x^3*ln(1-exp(2*x))+3/2*polylog(2,exp(2*x))+3*x^2*polylog(2,exp(2*
x))-3*x*polylog(3,exp(2*x))+3/2*polylog(4,exp(2*x))-3/8*cosh(x)*sinh(x)-3/4
*x^2*cosh(x)*sinh(x)+3/4*x*sinh(x)^2+1/2*x^3*sinh(x)^2
```

Rubi [A] time = 0.39, antiderivative size = 158, normalized size of antiderivative = 1.00, number of steps used = 26, number of rules used = 15, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.250, Rules used = {5450, 5372, 3311, 30, 2635, 8, 3716, 2190, 2531, 6609, 2282, 6589, 3720, 2279, 2391}

$$3x^2\text{PolyLog}\left(2,e^{2x}\right)-3x\text{PolyLog}\left(3,e^{2x}\right)+\frac{3}{2}\text{PolyLog}\left(2,e^{2x}\right)+\frac{3}{2}\text{PolyLog}\left(4,e^{2x}\right)-\frac{x^4}{2}+\frac{3x^3}{4}-\frac{3x^2}{2}+2x^3\log\left(1-e^{2x}\right.$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Cosh[x]^2*Coth[x]^3,x]
```

```
[Out] (3*x)/8 - (3*x^2)/2 + (3*x^3)/4 - x^4/2 - (3*x^2*Coth[x])/2 - (x^3*Coth[x]^
2)/2 + 3*x*Log[1 - E^(2*x)] + 2*x^3*Log[1 - E^(2*x)] + (3*PolyLog[2, E^(2*x
)])/2 + 3*x^2*PolyLog[2, E^(2*x)] - 3*x*PolyLog[3, E^(2*x)] + (3*PolyLog[4,
 E^(2*x)])/2 - (3*Cosh[x]*Sinh[x])/8 - (3*x^2*Cosh[x]*Sinh[x])/4 + (3*x*Sin
h[x]^2)/4 + (x^3*Sinh[x]^2)/2
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
```

```
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 3311

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(d*m*(c + d*x)^(m - 1)*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist
[(b^2*(n - 1))/n, Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[(
d^2*m*(m - 1))/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[(b*(c + d*x)^m*Cos[e + f*x]*(b*Sin[e + f*x])^(n - 1))/(f*n), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 3716

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2
*I, Int[((c + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(E^(2*I*k*Pi)*(1 + E^(2*(-(I*
e) + f*fz*x))/E^(2*I*k*Pi))), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Integ
erQ[4*k] && IGtQ[m, 0]
```

Rule 3720

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[(b*(c + d*x)^m*(b*Tan[e + f*x])^(n - 1))/(f*(n - 1)), x] + (-Di
st[(b*d*m)/(f*(n - 1)), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x],
 x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[
{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rule 5372

```
Int[Cosh[(a_.) + (b_.)*(x_)^(n_.)]*(x_)^(m_.)*Sinh[(a_.) + (b_.)*(x_)^(n_.)
]^(p_.), x_Symbol] :> Simp[(x^(m - n + 1)*Sinh[a + b*x^n]^(p + 1))/(b*n*(p
+ 1)), x] - Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*Sinh[a + b*x^n]^(
p + 1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

Rule 5450

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(n_.)*Coth[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) +
(d_.)*(x_))^(m_.), x_Symbol] :> Int[(c + d*x)^m*Cosh[a + b*x]^n*Coth[a + b*
x]^(p - 2), x] + Int[(c + d*x)^m*Cosh[a + b*x]^(n - 2)*Coth[a + b*x]^p, x]
/; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int x^3\cosh^2(x)\coth^3(x)\,dx = \int x^3\cosh^2(x)\coth(x)\,dx + \int x^3\coth^3(x)\,dx$$
$$= -\frac{1}{2}x^3\coth^2(x) + \frac{3}{2}\int x^2\coth^2(x)\,dx + 2\int x^3\coth(x)\,dx + \int x^3\cosh(x)\sinh(x)$$
$$= -\frac{3}{2}x^2\coth(x) - \frac{1}{2}x^3\coth^2(x) + \frac{1}{2}x^3\sinh^2(x) + \frac{3\int x^2\,dx}{2} - \frac{3}{2}\int x^2\sinh^2(x)\,dx +$$
$$= -\frac{3x^2}{2} + \frac{x^3}{2} - \frac{3}{2}x^2\coth(x) - \frac{1}{2}x^3\coth^2(x) - \frac{3}{4}x^2\cosh(x)\sinh(x) + \frac{3}{4}x\sinh^2(x) +$$
$$= -\frac{3x^2}{2} + \frac{3x^3}{4} - \frac{3}{2}x^2\coth(x) - \frac{1}{2}x^3\coth^2(x) + 3x\log\left(1-e^{2x}\right) - \frac{3}{8}\cosh(x)\sinh(x)$$
$$= \frac{3x}{8} - \frac{3x^2}{2} + \frac{3x^3}{4} - \frac{3}{2}x^2\coth(x) - \frac{1}{2}x^3\coth^2(x) + 3x\log\left(1-e^{2x}\right) - \frac{3}{8}\cosh(x)\sinh$$
$$= \frac{3x}{8} - \frac{3x^2}{2} + \frac{3x^3}{4} - \frac{3}{2}x^2\coth(x) - \frac{1}{2}x^3\coth^2(x) + 3x\log\left(1-e^{2x}\right) + \frac{3\text{Li}_2\left(e^{2x}\right)}{2} - \frac{3}{8}$$
$$= \frac{3x}{8} - \frac{3x^2}{2} + \frac{3x^3}{4} - \frac{3}{2}x^2\coth(x) - \frac{1}{2}x^3\coth^2(x) + 3x\log\left(1-e^{2x}\right) + \frac{3\text{Li}_2\left(e^{2x}\right)}{2} + 2$$

Mathematica [A] time = 0.34, size = 133, normalized size = 0.84

$$\frac{1}{32}\left(96x^2\text{Li}_2\left(e^{2x}\right) - 96x\text{Li}_3\left(e^{2x}\right) - 48\text{Li}_2\left(e^{-2x}\right) + 48\text{Li}_4\left(e^{2x}\right) - 16x^4 + 64x^3\log\left(1-e^{2x}\right) + 8x^3\cosh(2x) - 16x^3\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Cosh[x]^2*Coth[x]^3,x]
```

```
[Out] (Pi^4 + 48*x^2 - 16*x^4 + 12*x*Cosh[2*x] + 8*x^3*Cosh[2*x] - 48*x^2*Coth[x]
 - 16*x^3*Csch[x]^2 + 96*x*Log[1 - E^(-2*x)] + 64*x^3*Log[1 - E^(2*x)] - 48
*PolyLog[2, E^(-2*x)] + 96*x^2*PolyLog[2, E^(2*x)] - 96*x*PolyLog[3, E^(2*x
)] + 48*PolyLog[4, E^(2*x)] - 6*Sinh[2*x] - 12*x^2*Sinh[2*x])/32
```

fricas [C] time = 0.54, size = 2067, normalized size = 13.08

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*cosh(x)^2*coth(x)^3,x, algorithm="fricas")
```

```
[Out] 1/32*((4*x^3 - 6*x^2 + 6*x - 3)*cosh(x)^8 + 8*(4*x^3 - 6*x^2 + 6*x - 3)*cos
h(x)*sinh(x)^7 + (4*x^3 - 6*x^2 + 6*x - 3)*sinh(x)^8 - 2*(8*x^4 + 4*x^3 + 4
2*x^2 + 6*x - 3)*cosh(x)^6 - 2*(8*x^4 + 4*x^3 - 14*(4*x^3 - 6*x^2 + 6*x - 3
)*cosh(x)^2 + 42*x^2 + 6*x - 3)*sinh(x)^6 + 4*(14*(4*x^3 - 6*x^2 + 6*x - 3)
*cosh(x)^3 - 3*(8*x^4 + 4*x^3 + 42*x^2 + 6*x - 3)*cosh(x))*sinh(x)^5 + 4*(8
*x^4 - 14*x^3 + 24*x^2 + 3*x)*cosh(x)^4 + 2*(35*(4*x^3 - 6*x^2 + 6*x - 3)*c
osh(x)^4 + 16*x^4 - 28*x^3 - 15*(8*x^4 + 4*x^3 + 42*x^2 + 6*x - 3)*cosh(x)^
2 + 48*x^2 + 6*x)*sinh(x)^4 + 8*(7*(4*x^3 - 6*x^2 + 6*x - 3)*cosh(x)^5 - 5*
(8*x^4 + 4*x^3 + 42*x^2 + 6*x - 3)*cosh(x)^3 + 2*(8*x^4 - 14*x^3 + 24*x^2 +
 3*x)*cosh(x))*sinh(x)^3 + 4*x^3 - 2*(8*x^4 + 4*x^3 + 6*x^2 + 6*x + 3)*cosh
(x)^2 + 2*(14*(4*x^3 - 6*x^2 + 6*x - 3)*cosh(x)^6 - 15*(8*x^4 + 4*x^3 + 42*
x^2 + 6*x - 3)*cosh(x)^4 - 8*x^4 - 4*x^3 + 12*(8*x^4 - 14*x^3 + 24*x^2 + 3*
x)*cosh(x)^2 - 6*x^2 - 6*x - 3)*sinh(x)^2 + 6*x^2 + 96*((2*x^2 + 1)*cosh(x)
^6 + 6*(2*x^2 + 1)*cosh(x)*sinh(x)^5 + (2*x^2 + 1)*sinh(x)^6 - 2*(2*x^2 + 1
)*cosh(x)^4 + (15*(2*x^2 + 1)*cosh(x)^2 - 4*x^2 - 2)*sinh(x)^4 + 4*(5*(2*x^
2 + 1)*cosh(x)^3 - 2*(2*x^2 + 1)*cosh(x))*sinh(x)^3 + (2*x^2 + 1)*cosh(x)^2
 + (15*(2*x^2 + 1)*cosh(x)^4 - 12*(2*x^2 + 1)*cosh(x)^2 + 2*x^2 + 1)*sinh(x
)^2 + 2*(3*(2*x^2 + 1)*cosh(x)^5 - 4*(2*x^2 + 1)*cosh(x)^3 + (2*x^2 + 1)*co
sh(x))*sinh(x))*dilog(cosh(x) + sinh(x)) + 96*((2*x^2 + 1)*cosh(x)^6 + 6*(2
*x^2 + 1)*cosh(x)*sinh(x)^5 + (2*x^2 + 1)*sinh(x)^6 - 2*(2*x^2 + 1)*cosh(x)
^4 + (15*(2*x^2 + 1)*cosh(x)^2 - 4*x^2 - 2)*sinh(x)^4 + 4*(5*(2*x^2 + 1)*co
sh(x)^3 - 2*(2*x^2 + 1)*cosh(x))*sinh(x)^3 + (2*x^2 + 1)*cosh(x)^2 + (15*(2
*x^2 + 1)*cosh(x)^4 - 12*(2*x^2 + 1)*cosh(x)^2 + 2*x^2 + 1)*sinh(x)^2 + 2*(
3*(2*x^2 + 1)*cosh(x)^5 - 4*(2*x^2 + 1)*cosh(x)^3 + (2*x^2 + 1)*cosh(x))*si
nh(x))*dilog(-cosh(x) - sinh(x)) + 32*((2*x^3 + 3*x)*cosh(x)^6 + 6*(2*x^3 +
 3*x)*cosh(x)*sinh(x)^5 + (2*x^3 + 3*x)*sinh(x)^6 - 2*(2*x^3 + 3*x)*cosh(x)
^4 - (4*x^3 - 15*(2*x^3 + 3*x)*cosh(x)^2 + 6*x)*sinh(x)^4 + 4*(5*(2*x^3 + 3
*x)*cosh(x)^3 - 2*(2*x^3 + 3*x)*cosh(x))*sinh(x)^3 + (2*x^3 + 3*x)*cosh(x)^
2 + (15*(2*x^3 + 3*x)*cosh(x)^4 + 2*x^3 - 12*(2*x^3 + 3*x)*cosh(x)^2 + 3*x)
*sinh(x)^2 + 2*(3*(2*x^3 + 3*x)*cosh(x)^5 - 4*(2*x^3 + 3*x)*cosh(x)^3 + (2*
x^3 + 3*x)*cosh(x))*sinh(x))*log(cosh(x) + sinh(x) + 1) + 32*((2*x^3 + 3*x)
*cosh(x)^6 + 6*(2*x^3 + 3*x)*cosh(x)*sinh(x)^5 + (2*x^3 + 3*x)*sinh(x)^6 -
2*(2*x^3 + 3*x)*cosh(x)^4 - (4*x^3 - 15*(2*x^3 + 3*x)*cosh(x)^2 + 6*x)*sinh
(x)^4 + 4*(5*(2*x^3 + 3*x)*cosh(x)^3 - 2*(2*x^3 + 3*x)*cosh(x))*sinh(x)^3 +
 (2*x^3 + 3*x)*cosh(x)^2 + (15*(2*x^3 + 3*x)*cosh(x)^4 + 2*x^3 - 12*(2*x^3
+ 3*x)*cosh(x)^2 + 3*x)*sinh(x)^2 + 2*(3*(2*x^3 + 3*x)*cosh(x)^5 - 4*(2*x^3
 + 3*x)*cosh(x)^3 + (2*x^3 + 3*x)*cosh(x))*sinh(x))*log(-cosh(x) - sinh(x)
+ 1) + 384*(cosh(x)^6 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6 + (15*cosh(x)^2 - 2
)*sinh(x)^4 - 2*cosh(x)^4 + 4*(5*cosh(x)^3 - 2*cosh(x))*sinh(x)^3 + (15*cos
h(x)^4 - 12*cosh(x)^2 + 1)*sinh(x)^2 + cosh(x)^2 + 2*(3*cosh(x)^5 - 4*cosh(
x)^3 + cosh(x))*sinh(x))*polylog(4, cosh(x) + sinh(x)) + 384*(cosh(x)^6 + 6
*cosh(x)*sinh(x)^5 + sinh(x)^6 + (15*cosh(x)^2 - 2)*sinh(x)^4 - 2*cosh(x)^4
 + 4*(5*cosh(x)^3 - 2*cosh(x))*sinh(x)^3 + (15*cosh(x)^4 - 12*cosh(x)^2 + 1
)*sinh(x)^2 + cosh(x)^2 + 2*(3*cosh(x)^5 - 4*cosh(x)^3 + cosh(x))*sinh(x))*
polylog(4, -cosh(x) - sinh(x)) - 384*(x*cosh(x)^6 + 6*x*cosh(x)*sinh(x)^5 +
```

```
 x*sinh(x)^6 - 2*x*cosh(x)^4 + (15*x*cosh(x)^2 - 2*x)*sinh(x)^4 + 4*(5*x*co
sh(x)^3 - 2*x*cosh(x))*sinh(x)^3 + x*cosh(x)^2 + (15*x*cosh(x)^4 - 12*x*cos
h(x)^2 + x)*sinh(x)^2 + 2*(3*x*cosh(x)^5 - 4*x*cosh(x)^3 + x*cosh(x))*sinh(
x))*polylog(3, cosh(x) + sinh(x)) - 384*(x*cosh(x)^6 + 6*x*cosh(x)*sinh(x)^
5 + x*sinh(x)^6 - 2*x*cosh(x)^4 + (15*x*cosh(x)^2 - 2*x)*sinh(x)^4 + 4*(5*x
*cosh(x)^3 - 2*x*cosh(x))*sinh(x)^3 + x*cosh(x)^2 + (15*x*cosh(x)^4 - 12*x*
cosh(x)^2 + x)*sinh(x)^2 + 2*(3*x*cosh(x)^5 - 4*x*cosh(x)^3 + x*cosh(x))*si
nh(x))*polylog(3, -cosh(x) - sinh(x)) + 4*(2*(4*x^3 - 6*x^2 + 6*x - 3)*cosh
(x)^7 - 3*(8*x^4 + 4*x^3 + 42*x^2 + 6*x - 3)*cosh(x)^5 + 4*(8*x^4 - 14*x^3
+ 24*x^2 + 3*x)*cosh(x)^3 - (8*x^4 + 4*x^3 + 6*x^2 + 6*x + 3)*cosh(x))*sinh
(x) + 6*x + 3)/(cosh(x)^6 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6 + (15*cosh(x)^2
 - 2)*sinh(x)^4 - 2*cosh(x)^4 + 4*(5*cosh(x)^3 - 2*cosh(x))*sinh(x)^3 + (15
*cosh(x)^4 - 12*cosh(x)^2 + 1)*sinh(x)^2 + cosh(x)^2 + 2*(3*cosh(x)^5 - 4*c
osh(x)^3 + cosh(x))*sinh(x))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3 \cosh(x)^2 \coth(x)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*cosh(x)^2*coth(x)^3,x, algorithm="giac")
```

```
[Out] integrate(x^3*cosh(x)^2*coth(x)^3, x)
```

maple [A] time = 0.30, size = 184, normalized size = 1.16

$$-\frac{x^4}{2}+\left(-\frac{3}{32}+\frac{3}{16}x-\frac{3}{16}x^2+\frac{1}{8}x^3\right)e^{2x}+\left(\frac{3}{32}+\frac{3}{16}x+\frac{3}{16}x^2+\frac{1}{8}x^3\right)e^{-2x}-\frac{x^2\left(2x\,e^{2x}+3\,e^{2x}-3\right)}{\left(e^{2x}-1\right)^2}-3x^2+3x\ln(e^x+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*cosh(x)^2*coth(x)^3,x)
```

```
[Out] -1/2*x^4+(-3/32+3/16*x-3/16*x^2+1/8*x^3)*exp(2*x)+(3/32+3/16*x+3/16*x^2+1/8
*x^3)*exp(-2*x)-x^2*(2*x*exp(2*x)+3*exp(2*x)-3)/(exp(2*x)-1)^2-3*x^2+3*x*ln
(exp(x)+1)+3*polylog(2,-exp(x))+3*x*ln(1-exp(x))+3*polylog(2,exp(x))+2*x^3*
ln(exp(x)+1)+6*x^2*polylog(2,-exp(x))-12*x*polylog(3,-exp(x))+12*polylog(4,
-exp(x))+2*x^3*ln(1-exp(x))+6*x^2*polylog(2,exp(x))-12*x*polylog(3,exp(x))+
12*polylog(4,exp(x))
```

maxima [A] time = 0.81, size = 238, normalized size = 1.51

$$-x^4+2\,x^3\log\left(e^x+1\right)+2\,x^3\log\left(-e^x+1\right)+6\,x^2\mathrm{Li}_2\left(-e^x\right)+6\,x^2\mathrm{Li}_2\left(e^x\right)-3\,x^2+3\,x\log\left(e^x+1\right)+3\,x\log\left(-e^x+1\right)-12$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*cosh(x)^2*coth(x)^3,x, algorithm="maxima")
```

```
[Out] -x^4 + 2*x^3*log(e^x + 1) + 2*x^3*log(-e^x + 1) + 6*x^2*dilog(-e^x) + 6*x^2
*dilog(e^x) - 3*x^2 + 3*x*log(e^x + 1) + 3*x*log(-e^x + 1) - 12*x*polylog(3
, -e^x) - 12*x*polylog(3, e^x) + 1/32*(16*x^4 - 8*x^3 + 84*x^2 + (4*x^3 - 6
*x^2 + 6*x - 3)*e^(6*x) + 2*(8*x^4 - 4*x^3 + 6*x^2 - 6*x + 3)*e^(4*x) - 4*(
8*x^4 + 14*x^3 + 24*x^2 - 3*x)*e^(2*x) + (4*x^3 + 6*x^2 + 6*x + 3)*e^(-2*x)
 - 12*x - 6)/(e^(4*x) - 2*e^(2*x) + 1) + 3*dilog(-e^x) + 3*dilog(e^x) + 12*
polylog(4, -e^x) + 12*polylog(4, e^x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^3 \cosh(x)^2 \coth(x)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*cosh(x)^2*coth(x)^3,x)
```

```
[Out] int(x^3*cosh(x)^2*coth(x)^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3 \cosh^2(x) \coth^3(x) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*cosh(x)**2*coth(x)**3,x)
```

```
[Out] Integral(x**3*cosh(x)**2*coth(x)**3, x)
```

3.424 $\int x^m \coth(a+bx)\operatorname{csch}(a+bx)\,dx$

Optimal. Leaf size=19

$$\operatorname{Int}\left(x^m \coth(a+bx)\operatorname{csch}(a+bx),x\right)$$

```
[Out] CannotIntegrate(x^m*coth(b*x+a)*csch(b*x+a),x)
```

Rubi [A] time = 0.38, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int x^m \coth(a+bx)\operatorname{csch}(a+bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x^m*Coth[a + b*x]*Csch[a + b*x],x]
[Out] Defer[Int][x^m*Coth[a + b*x]*Csch[a + b*x], x]
```

Rubi steps

$$\int x^m \coth(a+bx)\operatorname{csch}(a+bx)\,dx=\int x^m \coth(a+bx)\operatorname{csch}(a+bx)\,dx$$

Mathematica [A] time = 31.01, size = 0, normalized size = 0.00

$$\int x^m \coth(a+bx)\operatorname{csch}(a+bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^m*Coth[a + b*x]*Csch[a + b*x],x]
[Out] Integrate[x^m*Coth[a + b*x]*Csch[a + b*x], x]
```

fricas [A] time = 0.49, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(x^m \cosh\left(bx+a\right)\operatorname{csch}\left(bx+a\right)^2,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*cosh(b*x+a)*csch(b*x+a)^2,x, algorithm="fricas")
[Out] integral(x^m*cosh(b*x + a)*csch(b*x + a)^2, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int x^m \cosh(bx+a)\operatorname{csch}(bx+a)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*cosh(b*x+a)*csch(b*x+a)^2,x, algorithm="giac")
[Out] integrate(x^m*cosh(b*x + a)*csch(b*x + a)^2, x)
```

maple [A] time = 0.10, size = 0, normalized size = 0.00

$$\int x^m \cosh(bx+a)\operatorname{csch}(bx+a)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*cosh(b*x+a)*csch(b*x+a)^2,x)
[Out] int(x^m*cosh(b*x+a)*csch(b*x+a)^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int x^m \cosh(bx+a)\operatorname{csch}(bx+a)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*cosh(b*x+a)*csch(b*x+a)^2,x, algorithm="maxima")
[Out] integrate(x^m*cosh(b*x + a)*csch(b*x + a)^2, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{x^m \cosh(a+b\,x)}{\sinh(a+b\,x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^m*cosh(a + b*x))/sinh(a + b*x)^2,x)
[Out] int((x^m*cosh(a + b*x))/sinh(a + b*x)^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*cosh(b*x+a)*csch(b*x+a)**2,x)
[Out] Timed out
```

3.425 $\int x^3 \coth(a+bx)\mathbf{csch}(a+bx)\,dx$

Optimal. Leaf size=93

$$\frac{6\text{Li}_3\left(-e^{a+bx}\right)}{b^4}-\frac{6\text{Li}_3\left(e^{a+bx}\right)}{b^4}-\frac{6x\text{Li}_2\left(-e^{a+bx}\right)}{b^3}+\frac{6x\text{Li}_2\left(e^{a+bx}\right)}{b^3}-\frac{6x^2\tanh^{-1}\left(e^{a+bx}\right)}{b^2}-\frac{x^3\text{csch}(a+bx)}{b}$$

```
[Out] -6*x^2*arctanh(exp(b*x+a))/b^2-x^3*csch(b*x+a)/b-6*x*polylog(2,-exp(b*x+a))
/b^3+6*x*polylog(2,exp(b*x+a))/b^3+6*polylog(3,-exp(b*x+a))/b^4-6*polylog(3
,exp(b*x+a))/b^4
```

Rubi [A] time = 0.10, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {5419, 4182, 2531, 2282, 6589}

$$-\frac{6x\text{PolyLog}\left(2,-e^{a+bx}\right)}{b^3}+\frac{6x\text{PolyLog}\left(2,e^{a+bx}\right)}{b^3}+\frac{6\text{PolyLog}\left(3,-e^{a+bx}\right)}{b^4}-\frac{6\text{PolyLog}\left(3,e^{a+bx}\right)}{b^4}-\frac{6x^2\tanh^{-1}\left(e^{a+b}\right.}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Coth[a + b*x]*Csch[a + b*x],x]
```

```
[Out] (-6*x^2*ArcTanh[E^(a + b*x)])/b^2 - (x^3*Csch[a + b*x])/b - (6*x*PolyLog[2,
 -E^(a + b*x)])/b^3 + (6*x*PolyLog[2, E^(a + b*x)])/b^3 + (6*PolyLog[3, -E^
(a + b*x)])/b^4 - (6*PolyLog[3, E^(a + b*x)])/b^4
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
```

```
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5419

```
Int[Coth[(a_.) + (b_.)*(x_)^(n_.)]^(q_.)*Csch[(a_.) + (b_.)*(x_)^(n_.)]^(p_
.)*(x_)^(m_.), x_Symbol] :> -Simp[(x^(m - n + 1)*Csch[a + b*x^n]^p)/(b*n*p)
, x] + Dist[(m - n + 1)/(b*n*p), Int[x^(m - n)*Csch[a + b*x^n]^p, x], x] /;
 FreeQ[{a, b, p}, x] && RationalQ[m] && IntegerQ[n] && GeQ[m - n, 0] && EqQ
[q, 1]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}\int x^3 \coth(a+bx)\operatorname{csch}(a+bx)\,dx &= -\frac{x^3\operatorname{csch}(a+bx)}{b}+\frac{3\int x^2\operatorname{csch}(a+bx)\,dx}{b}\\ &= -\frac{6x^2\tanh^{-1}\left(e^{a+bx}\right)}{b^2}-\frac{x^3\operatorname{csch}(a+bx)}{b}-\frac{6\int x\log\left(1-e^{a+bx}\right)dx}{b^2}+\frac{6\int x\log\ldots\\ &= -\frac{6x^2\tanh^{-1}\left(e^{a+bx}\right)}{b^2}-\frac{x^3\operatorname{csch}(a+bx)}{b}-\frac{6x\operatorname{Li}_2\left(-e^{a+bx}\right)}{b^3}+\frac{6x\operatorname{Li}_2\left(e^{a+bx}\right)}{b^3}+\ldots\\ &= -\frac{6x^2\tanh^{-1}\left(e^{a+bx}\right)}{b^2}-\frac{x^3\operatorname{csch}(a+bx)}{b}-\frac{6x\operatorname{Li}_2\left(-e^{a+bx}\right)}{b^3}+\frac{6x\operatorname{Li}_2\left(e^{a+bx}\right)}{b^3}+\ldots\\ &= -\frac{6x^2\tanh^{-1}\left(e^{a+bx}\right)}{b^2}-\frac{x^3\operatorname{csch}(a+bx)}{b}-\frac{6x\operatorname{Li}_2\left(-e^{a+bx}\right)}{b^3}+\frac{6x\operatorname{Li}_2\left(e^{a+bx}\right)}{b^3}+\ldots\end{aligned}$$

Mathematica [A] time = 7.21, size = 167, normalized size = 1.80

$$-\frac{\operatorname{csch}\left(\frac{1}{2}(a+bx)\right)\operatorname{sech}\left(\frac{1}{2}(a+bx)\right)\left(6b^2x^2\sinh(a+bx)\tanh^{-1}(\sinh(a+bx)+\cosh(a+bx))+6bx\sinh(a+bx)\ldots\right.}{\ldots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^3*Coth[a + b*x]*Csch[a + b*x],x]
```

```
[Out] -1/2*(Csch[(a + b*x)/2]*Sech[(a + b*x)/2]*(b^3*x^3 + 6*b^2*x^2*ArcTanh[Cosh
[a + b*x] + Sinh[a + b*x]]*Sinh[a + b*x] + 6*b*x*PolyLog[2, -Cosh[a + b*x]
- Sinh[a + b*x]]*Sinh[a + b*x] - 6*b*x*PolyLog[2, Cosh[a + b*x] + Sinh[a +
b*x]]*Sinh[a + b*x] - 6*PolyLog[3, -Cosh[a + b*x] - Sinh[a + b*x]]*Sinh[a +
 b*x] + 6*PolyLog[3, Cosh[a + b*x] + Sinh[a + b*x]]*Sinh[a + b*x]))/b^4
```

fricas [C] time = 0.43, size = 551, normalized size = 5.92

$$-\frac{2\,b^3x^3\cosh\left(bx+a\right)+2\,b^3x^3\sinh\left(bx+a\right)-6\left(bx\cosh\left(bx+a\right)^2+2\,bx\cosh\left(bx+a\right)\sinh\left(bx+a\right)+bx\sinh\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*cosh(b*x+a)*csch(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -(2*b^3*x^3*cosh(b*x + a) + 2*b^3*x^3*sinh(b*x + a) - 6*(b*x*cosh(b*x + a)^
2 + 2*b*x*cosh(b*x + a)*sinh(b*x + a) + b*x*sinh(b*x + a)^2 - b*x)*dilog(co
sh(b*x + a) + sinh(b*x + a)) + 6*(b*x*cosh(b*x + a)^2 + 2*b*x*cosh(b*x + a)
*sinh(b*x + a) + b*x*sinh(b*x + a)^2 - b*x)*dilog(-cosh(b*x + a) - sinh(b*x
 + a)) + 3*(b^2*x^2*cosh(b*x + a)^2 + 2*b^2*x^2*cosh(b*x + a)*sinh(b*x + a)
 + b^2*x^2*sinh(b*x + a)^2 - b^2*x^2)*log(cosh(b*x + a) + sinh(b*x + a) + 1
) - 3*(a^2*cosh(b*x + a)^2 + 2*a^2*cosh(b*x + a)*sinh(b*x + a) + a^2*sinh(b
*x + a)^2 - a^2)*log(cosh(b*x + a) + sinh(b*x + a) - 1) + 3*(b^2*x^2 - (b^2
*x^2 - a^2)*cosh(b*x + a)^2 - 2*(b^2*x^2 - a^2)*cosh(b*x + a)*sinh(b*x + a)
 - (b^2*x^2 - a^2)*sinh(b*x + a)^2 - a^2)*log(-cosh(b*x + a) - sinh(b*x + a
) + 1) + 6*(cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)
^2 - 1)*polylog(3, cosh(b*x + a) + sinh(b*x + a)) - 6*(cosh(b*x + a)^2 + 2*
cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 - 1)*polylog(3, -cosh(b*x + a
) - sinh(b*x + a)))/(b^4*cosh(b*x + a)^2 + 2*b^4*cosh(b*x + a)*sinh(b*x + a
) + b^4*sinh(b*x + a)^2 - b^4)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3\cosh\left(bx+a\right)\operatorname{csch}\left(bx+a\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*cosh(b*x+a)*csch(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate(x^3*cosh(b*x + a)*csch(b*x + a)^2, x)
```

maple [A] time = 0.22, size = 174, normalized size = 1.87

$$-\frac{2x^3\mathrm{e}^{bx+a}}{b\left(\mathrm{e}^{2bx+2a}-1\right)}-\frac{6a^2\,\mathrm{arctanh}\left(\mathrm{e}^{bx+a}\right)}{b^4}-\frac{3\ln\left(1+\mathrm{e}^{bx+a}\right)x^2}{b^2}+\frac{3\ln\left(1+\mathrm{e}^{bx+a}\right)a^2}{b^4}-\frac{6x\,\mathrm{polylog}\left(2,-\mathrm{e}^{bx+a}\right)}{b^3}+\frac{6\,\mathrm{poly}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*cosh(b*x+a)*csch(b*x+a)^2,x)
```

```
[Out] -2/b*x^3*exp(b*x+a)/(exp(2*b*x+2*a)-1)-6/b^4*a^2*arctanh(exp(b*x+a))-3/b^2*
ln(1+exp(b*x+a))*x^2+3/b^4*ln(1+exp(b*x+a))*a^2-6*x*polylog(2,-exp(b*x+a))/
b^3+6*polylog(3,-exp(b*x+a))/b^4+3/b^2*ln(1-exp(b*x+a))*x^2-3/b^4*ln(1-exp(
b*x+a))*a^2+6*x*polylog(2,exp(b*x+a))/b^3-6*polylog(3,exp(b*x+a))/b^4
```

maxima [A] time = 0.54, size = 121, normalized size = 1.30

$$-\frac{2\,x^3 e^{(bx+a)}}{be^{(2\,bx+2\,a)}-b}-\frac{3\left(b^2x^2\log\left(e^{(bx+a)}+1\right)+2\,bx\mathrm{Li}_2\left(-e^{(bx+a)}\right)-2\,\mathrm{Li}_3(-e^{(bx+a)})\right)}{b^4}+\frac{3\left(b^2x^2\log\left(-e^{(bx+a)}+1\right)+2\,bx\right.}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*cosh(b*x+a)*csch(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -2*x^3*e^(b*x + a)/(b*e^(2*b*x + 2*a) - b) - 3*(b^2*x^2*log(e^(b*x + a) + 1
) + 2*b*x*dilog(-e^(b*x + a)) - 2*polylog(3, -e^(b*x + a)))/b^4 + 3*(b^2*x^
2*log(-e^(b*x + a) + 1) + 2*b*x*dilog(e^(b*x + a)) - 2*polylog(3, e^(b*x +
a)))/b^4
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3\cosh\left(a+b\,x\right)}{{\mathrm{sinh}\left(a+b\,x\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*cosh(a + b*x))/sinh(a + b*x)^2,x)
```

```
[Out] int((x^3*cosh(a + b*x))/sinh(a + b*x)^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*cosh(b*x+a)*csch(b*x+a)**2,x)
```

```
[Out] Timed out
```

3.426 $\int x^2 \coth(a+bx)\mathbf{csch}(a+bx)\,dx$

Optimal. Leaf size=59

$$-\frac{2\text{Li}_2\left(-e^{a+bx}\right)}{b^3}+\frac{2\text{Li}_2\left(e^{a+bx}\right)}{b^3}-\frac{4x\tanh^{-1}\left(e^{a+bx}\right)}{b^2}-\frac{x^2\text{csch}(a+bx)}{b}$$

```
[Out] -4*x*arctanh(exp(b*x+a))/b^2-x^2*csch(b*x+a)/b-2*polylog(2,-exp(b*x+a))/b^3
+2*polylog(2,exp(b*x+a))/b^3
```

Rubi [A] time = 0.06, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {5419, 4182, 2279, 2391}

$$-\frac{2\text{PolyLog}\left(2,-e^{a+bx}\right)}{b^3}+\frac{2\text{PolyLog}\left(2,e^{a+bx}\right)}{b^3}-\frac{4x\tanh^{-1}\left(e^{a+bx}\right)}{b^2}-\frac{x^2\text{csch}(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Coth[a + b*x]*Csch[a + b*x],x]
```

```
[Out] (-4*x*ArcTanh[E^(a + b*x)])/b^2 - (x^2*Csch[a + b*x])/b - (2*PolyLog[2, -E^
(a + b*x)])/b^3 + (2*PolyLog[2, E^(a + b*x)])/b^3
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5419

```
Int[Coth[(a_.) + (b_.)*(x_)^(n_.)]^(q_.)*Csch[(a_.) + (b_.)*(x_)^(n_.)]^(p_
.)*(x_)^(m_.), x_Symbol] :> -Simp[(x^(m - n + 1)*Csch[a + b*x^n]^p)/(b*n*p)
, x] + Dist[(m - n + 1)/(b*n*p), Int[x^(m - n)*Csch[a + b*x^n]^p, x], x] /;
 FreeQ[{a, b, p}, x] && RationalQ[m] && IntegerQ[n] && GeQ[m - n, 0] && EqQ
[q, 1]
```

Rubi steps

$$\int x^2 \coth(a+bx)\operatorname{csch}(a+bx)\,dx = -\frac{x^2\operatorname{csch}(a+bx)}{b}+\frac{2\int x\operatorname{csch}(a+bx)\,dx}{b}$$
$$= -\frac{4x\tanh^{-1}\left(e^{a+bx}\right)}{b^2}-\frac{x^2\operatorname{csch}(a+bx)}{b}-\frac{2\int \log\left(1-e^{a+bx}\right)\,dx}{b^2}+\frac{2\int \log\left(1\right.}{b}$$
$$= -\frac{4x\tanh^{-1}\left(e^{a+bx}\right)}{b^2}-\frac{x^2\operatorname{csch}(a+bx)}{b}-\frac{2\operatorname{Subst}\left(\int\frac{\log(1-x)}{x}\,dx,x,e^{a+bx}\right)}{b^3}+-\frac{2}{}$$
$$= -\frac{4x\tanh^{-1}\left(e^{a+bx}\right)}{b^2}-\frac{x^2\operatorname{csch}(a+bx)}{b}-\frac{2\operatorname{Li}_2\left(-e^{a+bx}\right)}{b^3}+\frac{2\operatorname{Li}_2\left(e^{a+bx}\right)}{b^3}$$

Mathematica [B] time = 0.79, size = 133, normalized size = 2.25

$$-\frac{b^2x^2\operatorname{csch}(a+bx)-2\operatorname{Li}_2\left(-e^{-a-bx}\right)+2\operatorname{Li}_2\left(e^{-a-bx}\right)-2bx\log\left(1-e^{-a-bx}\right)+2bx\log\left(e^{-a-bx}+1\right)-2a\log\left(1-e^{-a}\right.}{b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Coth[a + b*x]*Csch[a + b*x],x]
```

```
[Out] -((b^2*x^2*Csch[a + b*x] - 2*a*Log[1 - E^(-a - b*x)] - 2*b*x*Log[1 - E^(-a
- b*x)] + 2*a*Log[1 + E^(-a - b*x)] + 2*b*x*Log[1 + E^(-a - b*x)] + 2*a*Log
[Tanh[(a + b*x)/2]] - 2*PolyLog[2, -E^(-a - b*x)] + 2*PolyLog[2, E^(-a - b*
x)])/b^3)
```

fricas [B] time = 0.49, size = 367, normalized size = 6.22

$$-\frac{2\left(b^2x^2\cosh(bx+a)+b^2x^2\sinh(bx+a)-\left(\cosh(bx+a)^2+2\cosh(bx+a)\sinh(bx+a)+\sinh(bx+a)^2-\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cosh(b*x+a)*csch(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -2*(b^2*x^2*cosh(b*x + a) + b^2*x^2*sinh(b*x + a) - (cosh(b*x + a)^2 + 2*co
sh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 - 1)*dilog(cosh(b*x + a) + sinh
(b*x + a)) + (cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x +
a)^2 - 1)*dilog(-cosh(b*x + a) - sinh(b*x + a)) + (b*x*cosh(b*x + a)^2 + 2*
b*x*cosh(b*x + a)*sinh(b*x + a) + b*x*sinh(b*x + a)^2 - b*x)*log(cosh(b*x +
 a) + sinh(b*x + a) + 1) + (a*cosh(b*x + a)^2 + 2*a*cosh(b*x + a)*sinh(b*x
+ a) + a*sinh(b*x + a)^2 - a)*log(cosh(b*x + a) + sinh(b*x + a) - 1) - ((b*
x + a)*cosh(b*x + a)^2 + 2*(b*x + a)*cosh(b*x + a)*sinh(b*x + a) + (b*x + a
)*sinh(b*x + a)^2 - b*x - a)*log(-cosh(b*x + a) - sinh(b*x + a) + 1))/(b^3*
cosh(b*x + a)^2 + 2*b^3*cosh(b*x + a)*sinh(b*x + a) + b^3*sinh(b*x + a)^2 -
 b^3)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \cosh(bx+a)\operatorname{csch}(bx+a)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cosh(b*x+a)*csch(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate(x^2*cosh(b*x + a)*csch(b*x + a)^2, x)
```

maple [B] time = 0.22, size = 134, normalized size = 2.27

$$-\frac{2x^2\mathrm{e}^{bx+a}}{b\left(\mathrm{e}^{2bx+2a}-1\right)}-\frac{2\ln\left(1+\mathrm{e}^{bx+a}\right)x}{b^2}-\frac{2\ln\left(1+\mathrm{e}^{bx+a}\right)a}{b^3}-\frac{2\operatorname{polylog}\left(2,-\mathrm{e}^{bx+a}\right)}{b^3}+\frac{2\ln\left(1-\mathrm{e}^{bx+a}\right)x}{b^2}+\frac{2\ln\left(1-\mathrm{e}^{bx+}\right.}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*cosh(b*x+a)*csch(b*x+a)^2,x)
```

```
[Out] -2*x^2*exp(b*x+a)/b/(exp(2*b*x+2*a)-1)-2/b^2*ln(1+exp(b*x+a))*x-2/b^3*ln(1+
exp(b*x+a))*a-2*polylog(2,-exp(b*x+a))/b^3+2/b^2*ln(1-exp(b*x+a))*x+2/b^3*l
n(1-exp(b*x+a))*a+2*polylog(2,exp(b*x+a))/b^3+4/b^3*a*arctanh(exp(b*x+a))
```

maxima [A] time = 0.90, size = 83, normalized size = 1.41

$$-\frac{2\,x^2e^{(bx+a)}}{be^{(2\,bx+2\,a)}-b}-\frac{2\left(bx\log\left(e^{(bx+a)}+1\right)+\mathrm{Li}_2\left(-e^{(bx+a)}\right)\right)}{b^3}+\frac{2\left(bx\log\left(-e^{(bx+a)}+1\right)+\mathrm{Li}_2\left(e^{(bx+a)}\right)\right)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cosh(b*x+a)*csch(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -2*x^2*e^(b*x + a)/(b*e^(2*b*x + 2*a) - b) - 2*(b*x*log(e^(b*x + a) + 1) +
dilog(-e^(b*x + a)))/b^3 + 2*(b*x*log(-e^(b*x + a) + 1) + dilog(e^(b*x + a)
))/b^3
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^2 \cosh(a + b x)}{\sinh(a + b x)^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*cosh(a + b*x))/sinh(a + b*x)^2,x)
[Out] int((x^2*cosh(a + b*x))/sinh(a + b*x)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \cosh(a + bx) \operatorname{csch}^2(a + bx) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*cosh(b*x+a)*csch(b*x+a)**2,x)
[Out] Integral(x**2*cosh(a + b*x)*csch(a + b*x)**2, x)
```

3.427 $\int x \coth(a+bx)\text{csch}(a+bx)\,dx$

Optimal. Leaf size=25

$$-\frac{\tanh^{-1}(\cosh(a+bx))}{b^2}-\frac{x\text{csch}(a+bx)}{b}$$

```
[Out] -arctanh(cosh(b*x+a))/b^2-x*csch(b*x+a)/b
```

Rubi [A] time = 0.02, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {5419, 3770}

$$-\frac{\tanh^{-1}(\cosh(a+bx))}{b^2}-\frac{x\text{csch}(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[x*Coth[a + b*x]*Csch[a + b*x],x]
```

```
[Out] -(ArcTanh[Cosh[a + b*x]]/b^2) - (x*Csch[a + b*x])/b
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 5419

```
Int[Coth[(a_.) + (b_.)*(x_)^(n_.)]^(q_.)*Csch[(a_.) + (b_.)*(x_)^(n_.)]^(p_
.)*(x_)^(m_.), x_Symbol] :> -Simp[(x^(m - n + 1)*Csch[a + b*x^n]^p)/(b*n*p)
, x] + Dist[(m - n + 1)/(b*n*p), Int[x^(m - n)*Csch[a + b*x^n]^p, x], x] /;
 FreeQ[{a, b, p}, x] && RationalQ[m] && IntegerQ[n] && GeQ[m - n, 0] && EqQ
[q, 1]
```

Rubi steps

$$\begin{aligned}\int x\coth(a+bx)\text{csch}(a+bx)\,dx&=-\frac{x\text{csch}(a+bx)}{b}+\frac{\int \text{csch}(a+bx)\,dx}{b}\\&=-\frac{\tanh^{-1}(\cosh(a+bx))}{b^2}-\frac{x\text{csch}(a+bx)}{b}\end{aligned}$$

Mathematica [B] time = 0.05, size = 114, normalized size = 4.56

$$\frac{\log\left(\sinh\left(\frac{a}{2}+\frac{bx}{2}\right)\right)}{b^2}-\frac{\log\left(\cosh\left(\frac{a}{2}+\frac{bx}{2}\right)\right)}{b^2}-\frac{x\text{csch}(a)}{b}+\frac{x\text{csch}\left(\frac{a}{2}\right)\sinh\left(\frac{bx}{2}\right)\text{csch}\left(\frac{a}{2}+\frac{bx}{2}\right)}{2b}+\frac{x\text{sech}\left(\frac{a}{2}\right)\sinh\left(\frac{bx}{2}\right)\text{se}}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Coth[a + b*x]*Csch[a + b*x],x]
```

```
[Out] -((x*Csch[a])/b) - Log[Cosh[a/2 + (b*x)/2]]/b^2 + Log[Sinh[a/2 + (b*x)/2]]/
b^2 + (x*Csch[a/2]*Csch[a/2 + (b*x)/2]*Sinh[(b*x)/2])/(2*b) + (x*Sech[a/2]*
Sech[a/2 + (b*x)/2]*Sinh[(b*x)/2])/(2*b)
```

fricas [B] time = 0.51, size = 169, normalized size = 6.76

$$-\frac{2\,bx\cosh\left(bx+a\right)+2\,bx\sinh\left(bx+a\right)+\left(\cosh\left(bx+a\right)^2+2\,\cosh\left(bx+a\right)\sinh\left(bx+a\right)+\sinh\left(bx+a\right)^2-1\right)\text{l}}{b^2\cosh\left(bx+a\right)^2+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x+a)*csch(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -(2*b*x*cosh(b*x + a) + 2*b*x*sinh(b*x + a) + (cosh(b*x + a)^2 + 2*cosh(b*x
 + a)*sinh(b*x + a) + sinh(b*x + a)^2 - 1)*log(cosh(b*x + a) + sinh(b*x + a
) + 1) - (cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2
 - 1)*log(cosh(b*x + a) + sinh(b*x + a) - 1))/(b^2*cosh(b*x + a)^2 + 2*b^2*
cosh(b*x + a)*sinh(b*x + a) + b^2*sinh(b*x + a)^2 - b^2)
```

giac [B] time = 0.15, size = 93, normalized size = 3.72

$$-\frac{2\,bxe^{(bx+a)}+e^{(2\,bx+2\,a)}\log\left(e^{(bx+a)}+1\right)-e^{(2\,bx+2\,a)}\log\left(e^{(bx+a)}-1\right)-\log\left(e^{(bx+a)}+1\right)+\log\left(e^{(bx+a)}-1\right)}{b^2e^{(2\,bx+2\,a)}-b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x+a)*csch(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -(2*b*x*e^(b*x + a) + e^(2*b*x + 2*a)*log(e^(b*x + a) + 1) - e^(2*b*x + 2*a
)*log(e^(b*x + a) - 1) - log(e^(b*x + a) + 1) + log(e^(b*x + a) - 1))/(b^2*
e^(2*b*x + 2*a) - b^2)
```

maple [B] time = 0.14, size = 54, normalized size = 2.16

$$-\frac{2\,\mathrm{e}^{bx+a}x}{b\left(\mathrm{e}^{2bx+2a}-1\right)}-\frac{\ln\left(1+\mathrm{e}^{bx+a}\right)}{b^2}+\frac{\ln\left(\mathrm{e}^{bx+a}-1\right)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cosh(b*x+a)*csch(b*x+a)^2,x)
```

```
[Out] -2*exp(b*x+a)*x/b/(exp(2*b*x+2*a)-1)-1/b^2*ln(1+exp(b*x+a))+1/b^2*ln(exp(b*
x+a)-1)
```

maxima [B] time = 0.52, size = 64, normalized size = 2.56

$$-\frac{2\,xe^{(bx+a)}}{be^{(2\,bx+2\,a)}-b}-\frac{\log\left(\left(e^{(bx+a)}+1\right)e^{(-a)}\right)}{b^2}+\frac{\log\left(\left(e^{(bx+a)}-1\right)e^{(-a)}\right)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x+a)*csch(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -2*x*e^(b*x + a)/(b*e^(2*b*x + 2*a) - b) - log((e^(b*x + a) + 1)*e^(-a))/b^
2 + log((e^(b*x + a) - 1)*e^(-a))/b^2
```

mupad [B] time = 0.12, size = 53, normalized size = 2.12

$$-\frac{2\,\mathrm{atan}\left(\frac{\mathrm{e}^{b\,x}\,\mathrm{e}^{a}\,\sqrt{-b^4}}{b^2}\right)}{\sqrt{-b^4}}-\frac{2\,x\,\mathrm{e}^{a+b\,x}}{b\,\left(\mathrm{e}^{2\,a+2\,b\,x}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*cosh(a + b*x))/sinh(a + b*x)^2,x)
```

```
[Out] - (2*atan((exp(b*x)*exp(a)*(-b^4)^(1/2))/b^2))/(-b^4)^(1/2) - (2*x*exp(a +
b*x))/(b*(exp(2*a + 2*b*x) - 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\cosh\left(a+bx\right)\operatorname{csch}^2\left(a+bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x+a)*csch(b*x+a)**2,x)
```

```
[Out] Integral(x*cosh(a + b*x)*csch(a + b*x)**2, x)
```

3.428 $\int \coth(a+bx)\mathbf{csch}(a+bx)\,dx$

Optimal. Leaf size=11

$$-\frac{\operatorname{csch}(a+bx)}{b}$$

```
[Out] -csch(b*x+a)/b
```

Rubi [A] time = 0.01, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2606, 8}

$$-\frac{\operatorname{csch}(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[a + b*x]*Csch[a + b*x],x]
```

```
[Out] -(Csch[a + b*x]/b)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 && !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rubi steps

$$\begin{aligned}\int \coth(a+bx)\operatorname{csch}(a+bx)\,dx &= -\frac{i\,\text{Subst}\left(\int 1\,dx, x, -i\operatorname{csch}(a+bx)\right)}{b}\\ &= -\frac{\operatorname{csch}(a+bx)}{b}\end{aligned}$$

Mathematica [A] time = 0.01, size = 11, normalized size = 1.00

$$-\frac{\operatorname{csch}(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[a + b*x]*Csch[a + b*x],x]
[Out] -(Csch[a + b*x]/b)
```

fricas [B] time = 0.45, size = 56, normalized size = 5.09

$$-\frac{2\,(\cosh(bx+a)+\sinh(bx+a))}{b\cosh(bx+a)^2+2\,b\cosh(bx+a)\sinh(bx+a)+b\sinh(bx+a)^2-b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*csch(b*x+a)^2,x, algorithm="fricas")
[Out] -2*(cosh(b*x + a) + sinh(b*x + a))/(b*cosh(b*x + a)^2 + 2*b*cosh(b*x + a)*s
inh(b*x + a) + b*sinh(b*x + a)^2 - b)
```

giac [B] time = 0.12, size = 25, normalized size = 2.27

$$-\frac{2}{b\left(e^{(bx+a)}-e^{(-bx-a)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*csch(b*x+a)^2,x, algorithm="giac")
[Out] -2/(b*(e^(b*x + a) - e^(-b*x - a)))
```

maple [A] time = 0.03, size = 12, normalized size = 1.09

$$-\frac{\operatorname{csch}(bx+a)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)*csch(b*x+a)^2,x)
[Out] -csch(b*x+a)/b
```

maxima [B] time = 0.52, size = 25, normalized size = 2.27

$$-\frac{2}{b\left(e^{(bx+a)}-e^{(-bx-a)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*csch(b*x+a)^2,x, algorithm="maxima")
[Out] -2/(b*(e^(b*x + a) - e^(-b*x - a)))
```

mupad [B] time = 1.42, size = 13, normalized size = 1.18

$$-\frac{1}{b \sinh(a + b x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a + b*x)/sinh(a + b*x)^2,x)
```

```
[Out] -1/(b*sinh(a + b*x))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \cosh(a + bx) \operatorname{csch}^2(a + bx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*csch(b*x+a)**2,x)
```

```
[Out] Integral(cosh(a + b*x)*csch(a + b*x)**2, x)
```

3.429 $\int \frac{\coth(a+bx)\operatorname{csch}(a+bx)}{x} \, dx$

Optimal. Leaf size=19

$$\operatorname{Int}\left(\frac{\coth(a+bx)\operatorname{csch}(a+bx)}{x},x\right)$$

[Out] CannotIntegrate(coth(b*x+a)*csch(b*x+a)/x,x)

Rubi [A] time = 0.14, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\coth(a+bx)\operatorname{csch}(a+bx)}{x} \, dx$$

Verification is Not applicable to the result.

[In] Int[(Coth[a + b*x]*Csch[a + b*x])/x,x]

[Out] Defer[Int][(Coth[a + b*x]*Csch[a + b*x])/x, x]

Rubi steps

$$\int \frac{\coth(a+bx)\operatorname{csch}(a+bx)}{x} \, dx = \int \frac{\coth(a+bx)\operatorname{csch}(a+bx)}{x} \, dx$$

Mathematica [A] time = 32.05, size = 0, normalized size = 0.00

$$\int \frac{\coth(a+bx)\operatorname{csch}(a+bx)}{x} \, dx$$

Verification is Not applicable to the result.

[In] Integrate[(Coth[a + b*x]*Csch[a + b*x])/x,x]

[Out] Integrate[(Coth[a + b*x]*Csch[a + b*x])/x, x]

fricas [A] time = 0.50, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{\cosh(bx+a)\operatorname{csch}(bx+a)^2}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(b*x+a)*csch(b*x+a)^2/x,x, algorithm="fricas")

```
[Out] integral(cosh(b*x + a)*csch(b*x + a)^2/x, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh(bx+a)\operatorname{csch}(bx+a)^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*csch(b*x+a)^2/x,x, algorithm="giac")
```

```
[Out] integrate(cosh(b*x + a)*csch(b*x + a)^2/x, x)
```

maple [A] time = 0.35, size = 0, normalized size = 0.00

$$\int \frac{\cosh(bx+a)\operatorname{csch}(bx+a)^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)*csch(b*x+a)^2/x,x)
```

```
[Out] int(cosh(b*x+a)*csch(b*x+a)^2/x,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{2\,e^{(bx+a)}}{bxe^{(2\,bx+2\,a)}-bx}-2\int \frac{1}{2\left(bx^2e^{(bx+a)}+bx^2\right)}\,dx-2\int \frac{1}{2\left(bx^2e^{(bx+a)}-bx^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*csch(b*x+a)^2/x,x, algorithm="maxima")
```

```
[Out] -2*e^(b*x + a)/(b*x*e^(2*b*x + 2*a) - b*x) - 2*integrate(1/2/(b*x^2*e^(b*x
+ a) + b*x^2), x) - 2*integrate(1/2/(b*x^2*e^(b*x + a) - b*x^2), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{\cosh(a+b\,x)}{x\sinh(a+b\,x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a + b*x)/(x*sinh(a + b*x)^2),x)
```

```
[Out] int(cosh(a + b*x)/(x*sinh(a + b*x)^2), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh(a+bx)\operatorname{csch}^2(a+bx)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*csch(b*x+a)**2/x,x)
```

```
[Out] Integral(cosh(a + b*x)*csch(a + b*x)**2/x, x)
```

3.430 $\int \frac{\coth(a+bx)\operatorname{csch}(a+bx)}{x^2} dx$

Optimal. Leaf size=19

$$\operatorname{Int}\left(\frac{\coth(a+bx)\operatorname{csch}(a+bx)}{x^2},x\right)$$

```
[Out] CannotIntegrate(coth(b*x+a)*csch(b*x+a)/x^2,x)
```

Rubi [A] time = 0.19, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\coth(a+bx)\operatorname{csch}(a+bx)}{x^2} dx$$

Verification is Not applicable to the result.

```
[In] Int[(Coth[a + b*x]*Csch[a + b*x])/x^2,x]
[Out] Defer[Int][(Coth[a + b*x]*Csch[a + b*x])/x^2, x]
```

Rubi steps

$$\int \frac{\coth(a+bx)\operatorname{csch}(a+bx)}{x^2} dx = \int \frac{\coth(a+bx)\operatorname{csch}(a+bx)}{x^2} dx$$

Mathematica [A] time = 38.19, size = 0, normalized size = 0.00

$$\int \frac{\coth(a+bx)\operatorname{csch}(a+bx)}{x^2} dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Coth[a + b*x]*Csch[a + b*x])/x^2,x]
[Out] Integrate[(Coth[a + b*x]*Csch[a + b*x])/x^2, x]
```

fricas [A] time = 0.51, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{\cosh(bx+a)\operatorname{csch}(bx+a)^2}{x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*csch(b*x+a)^2/x^2,x, algorithm="fricas")
```

```
[Out] integral(cosh(b*x + a)*csch(b*x + a)^2/x^2, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh(bx+a)\operatorname{csch}(bx+a)^2}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*csch(b*x+a)^2/x^2,x, algorithm="giac")
```

```
[Out] integrate(cosh(b*x + a)*csch(b*x + a)^2/x^2, x)
```

maple [A] time = 0.36, size = 0, normalized size = 0.00

$$\int \frac{\cosh(bx+a)\operatorname{csch}(bx+a)^2}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)*csch(b*x+a)^2/x^2,x)
```

```
[Out] int(cosh(b*x+a)*csch(b*x+a)^2/x^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{2\,e^{(bx+a)}}{bx^2e^{(2\,bx+2\,a)}-bx^2}-2\int \frac{1}{bx^3e^{(bx+a)}+bx^3}\,dx-2\int \frac{1}{bx^3e^{(bx+a)}-bx^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*csch(b*x+a)^2/x^2,x, algorithm="maxima")
```

```
[Out] -2*e^(b*x + a)/(b*x^2*e^(2*b*x + 2*a) - b*x^2) - 2*integrate(1/(b*x^3*e^(b*
x + a) + b*x^3), x) - 2*integrate(1/(b*x^3*e^(b*x + a) - b*x^3), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{\cosh(a+b\,x)}{x^2\sinh(a+b\,x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a + b*x)/(x^2*sinh(a + b*x)^2),x)
```

```
[Out] int(cosh(a + b*x)/(x^2*sinh(a + b*x)^2), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh(a+bx)\operatorname{csch}^2(a+bx)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*csch(b*x+a)**2/x**2,x)
```

```
[Out] Integral(cosh(a + b*x)*csch(a + b*x)**2/x**2, x)
```

3.431 $\int x^m \coth^2(a+bx)\,dx$

Optimal. Leaf size=15

$$\text{Int}\left(x^m \coth^2(a+bx),x\right)$$

```
[Out] Unintegrable(x^m*coth(b*x+a)^2,x)
```

Rubi [A] time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int x^m \coth^2(a+bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x^m*Coth[a + b*x]^2,x]
[Out] Defer[Int][x^m*Coth[a + b*x]^2, x]
```

Rubi steps

$$\int x^m \coth^2(a+bx)\,dx=\int x^m \coth^2(a+bx)\,dx$$

Mathematica [A] time = 9.96, size = 0, normalized size = 0.00

$$\int x^m \coth^2(a+bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^m*Coth[a + b*x]^2,x]
[Out] Integrate[x^m*Coth[a + b*x]^2, x]
```

fricas [A] time = 0.46, size = 0, normalized size = 0.00

$$\text{integral}\left(x^m \cosh\left(bx+a\right)^2 \operatorname{csch}\left(bx+a\right)^2,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*cosh(b*x+a)^2*csch(b*x+a)^2,x, algorithm="fricas")
[Out] integral(x^m*cosh(b*x + a)^2*csch(b*x + a)^2, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int x^m \cosh\left(bx+a\right)^2 \operatorname{csch}\left(bx+a\right)^2\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*cosh(b*x+a)^2*csch(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate(x^m*cosh(b*x + a)^2*csch(b*x + a)^2, x)
```

maple [A] time = 0.28, size = 0, normalized size = 0.00

$$\int x^m \left(\cosh^2\left(bx+a\right)\right) \operatorname{csch}\left(bx+a\right)^2\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*cosh(b*x+a)^2*csch(b*x+a)^2,x)
```

```
[Out] int(x^m*cosh(b*x+a)^2*csch(b*x+a)^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\frac{xe^{\left(4\,bx+m\log\left(x\right)+4\,a\right)}}{(m+1)e^{\left(4\,bx+4\,a\right)}-2\,(m+1)e^{\left(2\,bx+2\,a\right)}+m+1}+\int \frac{\left(2\left(2\,bxe^{\left(4\,a\right)}+(m+1)e^{\left(4\,a\right)}\right)e^{\left(4\,bx\right)}-(m+1)e^{\left(2\,bx+2\,a\right)}-m-1\right)x}{(m+1)e^{\left(6\,bx+6\,a\right)}-3\,(m+1)e^{\left(4\,bx+4\,a\right)}+3\,(m+1)e^{\left(2\,bx+2\,a\right)}-m-}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*cosh(b*x+a)^2*csch(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] x*e^(4*b*x + m*log(x) + 4*a)/((m + 1)*e^(4*b*x + 4*a) - 2*(m + 1)*e^(2*b*x
+ 2*a) + m + 1) + integrate((2*(2*b*x*e^(4*a) + (m + 1)*e^(4*a))*e^(4*b*x)
- (m + 1)*e^(2*b*x + 2*a) - m - 1)*x^m/((m + 1)*e^(6*b*x + 6*a) - 3*(m + 1)
*e^(4*b*x + 4*a) + 3*(m + 1)*e^(2*b*x + 2*a) - m - 1), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.07

$$\int \frac{x^m\,\cosh\left(a+b\,x\right)^2}{\sinh\left(a+b\,x\right)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^m*cosh(a + b*x)^2)/sinh(a + b*x)^2,x)
```

```
[Out] int((x^m*cosh(a + b*x)^2)/sinh(a + b*x)^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*cosh(b*x+a)**2*csch(b*x+a)**2,x)
```

```
[Out] Timed out
```

3.432 $\int x^3 \coth^2(a+bx)\,dx$

Optimal. Leaf size=87

$$-\frac{3\text{Li}_3\left(e^{2(a+bx)}\right)}{2b^4}+\frac{3x\text{Li}_2\left(e^{2(a+bx)}\right)}{b^3}+\frac{3x^2\log\left(1-e^{2(a+bx)}\right)}{b^2}-\frac{x^3\coth(a+bx)}{b}-\frac{x^3}{b}+\frac{x^4}{4}$$

```
[Out] -x^3/b+1/4*x^4-x^3*coth(b*x+a)/b+3*x^2*ln(1-exp(2*b*x+2*a))/b^2+3*x*polylog
(2,exp(2*b*x+2*a))/b^3-3/2*polylog(3,exp(2*b*x+2*a))/b^4
```

Rubi [A] time = 0.19, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.583, Rules used = {3720, 3716, 2190, 2531, 2282, 6589, 30}

$$\frac{3x\text{PolyLog}\left(2,e^{2(a+bx)}\right)}{b^3}-\frac{3\text{PolyLog}\left(3,e^{2(a+bx)}\right)}{2b^4}+\frac{3x^2\log\left(1-e^{2(a+bx)}\right)}{b^2}-\frac{x^3\coth(a+bx)}{b}-\frac{x^3}{b}+\frac{x^4}{4}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Coth[a + b*x]^2,x]
```

```
[Out] -(x^3/b) + x^4/4 - (x^3*Coth[a + b*x])/b + (3*x^2*Log[1 - E^(2*(a + b*x))])
/b^2 + (3*x*PolyLog[2, E^(2*(a + b*x))])/b^3 - (3*PolyLog[3, E^(2*(a + b*x)
)])/(2*b^4)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 3716

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2
*I, Int[((c + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(E^(2*I*k*Pi)*(1 + E^(2*(-(I*
e) + f*fz*x))/E^(2*I*k*Pi))), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Integ
erQ[4*k] && IGtQ[m, 0]
```

Rule 3720

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[(b*(c + d*x)^m*(b*Tan[e + f*x])^(n - 1))/(f*(n - 1)), x] + (-Di
st[(b*d*m)/(f*(n - 1)), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x],
 x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[
{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$
\begin{aligned}
\int x^3\coth^2(a+bx)\,dx &= -\frac{x^3\coth(a+bx)}{b}+\frac{3\int x^2\coth(a+bx)\,dx}{b}+\int x^3\,dx\\
&= -\frac{x^3}{b}+\frac{x^4}{4}-\frac{x^3\coth(a+bx)}{b}-\frac{6\int\frac{e^{2(a+bx)}x^2}{1-e^{2(a+bx)}}\,dx}{b}\\
&= -\frac{x^3}{b}+\frac{x^4}{4}-\frac{x^3\coth(a+bx)}{b}+\frac{3x^2\log\left(1-e^{2(a+bx)}\right)}{b^2}-\frac{6\int x\log\left(1-e^{2(a+bx)}\right)\,dx}{b^2}\\
&= -\frac{x^3}{b}+\frac{x^4}{4}-\frac{x^3\coth(a+bx)}{b}+\frac{3x^2\log\left(1-e^{2(a+bx)}\right)}{b^2}+\frac{3x\operatorname{Li}_2\left(e^{2(a+bx)}\right)}{b^3}-\frac{3\int\operatorname{Li}_2\left(e^{2(a+b
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{x^3}{b}+\frac{x^4}{4}-\frac{x^3\coth(a+bx)}{b}+\frac{3x^2\log\left(1-e^{2(a+bx)}\right)}{b^2}+\frac{3x\operatorname{Li}_2\left(e^{2(a+bx)}\right)}{b^3}-\frac{3\,\mathrm{Subst}\left(\int\frac{\operatorname{Li}_2}{x}\right.}{2}\\
&= -\frac{x^3}{b}+\frac{x^4}{4}-\frac{x^3\coth(a+bx)}{b}+\frac{3x^2\log\left(1-e^{2(a+bx)}\right)}{b^2}+\frac{3x\operatorname{Li}_2\left(e^{2(a+bx)}\right)}{b^3}-\frac{3\operatorname{Li}_3\left(e^{2(a+bx)}\right)}{2b^4}
\end{aligned}
$$

Mathematica [B] time = 0.60, size = 204, normalized size = 2.34

$$
-\frac{e^{2a}\left(2e^{-2a}b^3x^3-3\left(1-e^{-2a}\right)b^2x^2\log\left(1-e^{-a-bx}\right)-3\left(1-e^{-2a}\right)b^2x^2\log\left(e^{-a-bx}+1\right)+6\left(1-e^{-2a}\right)\left(bx\operatorname{Li}_2\left(-e^{-a-\right.\right.}{\left(e^{2a}-1\right)b^4}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Coth[a + b*x]^2,x]
```

```
[Out] x^4/4 - (E^(2*a)*((2*b^3*x^3)/E^(2*a) - 3*b^2*(1 - E^(-2*a))*x^2*Log[1 - E^
(-a - b*x)] - 3*b^2*(1 - E^(-2*a))*x^2*Log[1 + E^(-a - b*x)] + 6*(1 - E^(-2
*a))*(b*x*PolyLog[2, -E^(-a - b*x)] + PolyLog[3, -E^(-a - b*x)]) + 6*(1 - E
^(-2*a))*(b*x*PolyLog[2, E^(-a - b*x)] + PolyLog[3, E^(-a - b*x)])))/(b^4*(
-1 + E^(2*a))) + (x^3*Csch[a]*Csch[a + b*x]*Sinh[b*x])/b
```

fricas [C] time = 0.50, size = 632, normalized size = 7.26

$$
-\frac{b^4x^4-8\,a^3-\left(b^4x^4-8\,b^3x^3-8\,a^3\right)\cosh\left(bx+a\right)^2-2\left(b^4x^4-8\,b^3x^3-8\,a^3\right)\cosh\left(bx+a\right)\sinh\left(bx+a\right)-\left(b^4x^4\right.}{}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*cosh(b*x+a)^2*csch(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/4*(b^4*x^4 - 8*a^3 - (b^4*x^4 - 8*b^3*x^3 - 8*a^3)*cosh(b*x + a)^2 - 2*(
b^4*x^4 - 8*b^3*x^3 - 8*a^3)*cosh(b*x + a)*sinh(b*x + a) - (b^4*x^4 - 8*b^3
```

```
*x^3 - 8*a^3)*sinh(b*x + a)^2 - 24*(b*x*cosh(b*x + a)^2 + 2*b*x*cosh(b*x +
a)*sinh(b*x + a) + b*x*sinh(b*x + a)^2 - b*x)*dilog(cosh(b*x + a) + sinh(b*
x + a)) - 24*(b*x*cosh(b*x + a)^2 + 2*b*x*cosh(b*x + a)*sinh(b*x + a) + b*x
*sinh(b*x + a)^2 - b*x)*dilog(-cosh(b*x + a) - sinh(b*x + a)) - 12*(b^2*x^2
*cosh(b*x + a)^2 + 2*b^2*x^2*cosh(b*x + a)*sinh(b*x + a) + b^2*x^2*sinh(b*x
 + a)^2 - b^2*x^2)*log(cosh(b*x + a) + sinh(b*x + a) + 1) - 12*(a^2*cosh(b*
x + a)^2 + 2*a^2*cosh(b*x + a)*sinh(b*x + a) + a^2*sinh(b*x + a)^2 - a^2)*l
og(cosh(b*x + a) + sinh(b*x + a) - 1) + 12*(b^2*x^2 - (b^2*x^2 - a^2)*cosh(
b*x + a)^2 - 2*(b^2*x^2 - a^2)*cosh(b*x + a)*sinh(b*x + a) - (b^2*x^2 - a^2
)*sinh(b*x + a)^2 - a^2)*log(-cosh(b*x + a) - sinh(b*x + a) + 1) + 24*(cosh
(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 - 1)*polylog(
3, cosh(b*x + a) + sinh(b*x + a)) + 24*(cosh(b*x + a)^2 + 2*cosh(b*x + a)*s
inh(b*x + a) + sinh(b*x + a)^2 - 1)*polylog(3, -cosh(b*x + a) - sinh(b*x +
a)))/(b^4*cosh(b*x + a)^2 + 2*b^4*cosh(b*x + a)*sinh(b*x + a) + b^4*sinh(b*
x + a)^2 - b^4)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3 \cosh (bx+a)^2 \operatorname{csch} (bx+a)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*cosh(b*x+a)^2*csch(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate(x^3*cosh(b*x + a)^2*csch(b*x + a)^2, x)
```

maple [B] time = 0.58, size = 198, normalized size = 2.28

$$\frac{x^4}{4}-\frac{2x^3}{b\left(\mathrm{e}^{2bx+2a}-1\right)}+\frac{3a^2\ln\left(\mathrm{e}^{bx+a}-1\right)}{b^4}-\frac{6a^2\ln\left(\mathrm{e}^{bx+a}\right)}{b^4}-\frac{2x^3}{b}+\frac{6a^2x}{b^3}+\frac{4a^3}{b^4}+\frac{3\ln\left(1-\mathrm{e}^{bx+a}\right)x^2}{b^2}-\frac{3\ln\left(1-\mathrm{e}^{bx+a}\right)a^2}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*cosh(b*x+a)^2*csch(b*x+a)^2,x)
```

```
[Out] 1/4*x^4-2*x^3/b/(exp(2*b*x+2*a)-1)+3/b^4*a^2*ln(exp(b*x+a)-1)-6/b^4*a^2*ln(
exp(b*x+a))-2*x^3/b+6/b^3*a^2*x+4/b^4*a^3+3/b^2*ln(1-exp(b*x+a))*x^2-3/b^4*
ln(1-exp(b*x+a))*a^2+6*x*polylog(2,exp(b*x+a))/b^3-6*polylog(3,exp(b*x+a))/
b^4+3/b^2*ln(1+exp(b*x+a))*x^2+6*x*polylog(2,-exp(b*x+a))/b^3-6*polylog(3,-
exp(b*x+a))/b^4
```

maxima [A] time = 0.45, size = 146, normalized size = 1.68

$$-\frac{2\,x^3}{b}+\frac{bx^4e^{(2\,bx+2\,a)}-bx^4-8\,x^3}{4\left(be^{(2\,bx+2\,a)}-b\right)}+\frac{3\left(b^2x^2\log\left(e^{(bx+a)}+1\right)+2\,bx\mathrm{Li}_2\left(-e^{(bx+a)}\right)-2\,\mathrm{Li}_3(-e^{(bx+a)})\right)}{b^4}+\frac{3\left(b^2x^2\log\left(-\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*cosh(b*x+a)^2*csch(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -2*x^3/b + 1/4*(b*x^4*e^(2*b*x + 2*a) - b*x^4 - 8*x^3)/(b*e^(2*b*x + 2*a) -
 b) + 3*(b^2*x^2*log(e^(b*x + a) + 1) + 2*b*x*dilog(-e^(b*x + a)) - 2*polyl
og(3, -e^(b*x + a)))/b^4 + 3*(b^2*x^2*log(-e^(b*x + a) + 1) + 2*b*x*dilog(e
^(b*x + a)) - 2*polylog(3, e^(b*x + a)))/b^4
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3 \cosh\left(a+b\,x\right)^2}{\sinh\left(a+b\,x\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*cosh(a + b*x)^2)/sinh(a + b*x)^2,x)
```

```
[Out] int((x^3*cosh(a + b*x)^2)/sinh(a + b*x)^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*cosh(b*x+a)**2*csch(b*x+a)**2,x)
```

```
[Out] Timed out
```

3.433 $\int x^2 \coth^2(a+bx)\,dx$

Optimal. Leaf size=65

$$\frac{\text{Li}_2\left(e^{2(a+bx)}\right)}{b^3}+\frac{2x\log\left(1-e^{2(a+bx)}\right)}{b^2}-\frac{x^2\coth(a+bx)}{b}-\frac{x^2}{b}+\frac{x^3}{3}$$

```
[Out] -x^2/b+1/3*x^3-x^2*coth(b*x+a)/b+2*x*ln(1-exp(2*b*x+2*a))/b^2+polylog(2,exp
(2*b*x+2*a))/b^3
```

Rubi [A] time = 0.12, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3720, 3716, 2190, 2279, 2391, 30}

$$\frac{\text{PolyLog}\left(2,e^{2(a+bx)}\right)}{b^3}+\frac{2x\log\left(1-e^{2(a+bx)}\right)}{b^2}-\frac{x^2\coth(a+bx)}{b}-\frac{x^2}{b}+\frac{x^3}{3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Coth[a + b*x]^2,x]
```

```
[Out] -(x^2/b) + x^3/3 - (x^2*Coth[a + b*x])/b + (2*x*Log[1 - E^(2*(a + b*x))])/b
^2 + PolyLog[2, E^(2*(a + b*x))]/b^3
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3716

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2
*I, Int[((c + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(E^(2*I*k*Pi)*(1 + E^(2*(-(I*
e) + f*fz*x))/E^(2*I*k*Pi))), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Integ
erQ[4*k] && IGtQ[m, 0]
```

Rule 3720

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[(b*(c + d*x)^m*(b*Tan[e + f*x])^(n - 1))/(f*(n - 1)), x] + (-Di
st[(b*d*m)/(f*(n - 1)), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x],
 x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[
{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\int x^2\coth^2(a+bx)\,dx &= -\frac{x^2\coth(a+bx)}{b}+\frac{2\int x\coth(a+bx)\,dx}{b}+\int x^2\,dx\\
&= -\frac{x^2}{b}+\frac{x^3}{3}-\frac{x^2\coth(a+bx)}{b}-\frac{4\int\frac{e^{2(a+bx)}x}{1-e^{2(a+bx)}}\,dx}{b}\\
&= -\frac{x^2}{b}+\frac{x^3}{3}-\frac{x^2\coth(a+bx)}{b}+\frac{2x\log\left(1-e^{2(a+bx)}\right)}{b^2}-\frac{2\int\log\left(1-e^{2(a+bx)}\right)dx}{b^2}\\
&= -\frac{x^2}{b}+\frac{x^3}{3}-\frac{x^2\coth(a+bx)}{b}+\frac{2x\log\left(1-e^{2(a+bx)}\right)}{b^2}-\frac{\operatorname{Subst}\left(\int\frac{\log(1-x)}{x}\,dx,x,e^{2(a+bx)}\right)}{b^3}\\
&= -\frac{x^2}{b}+\frac{x^3}{3}-\frac{x^2\coth(a+bx)}{b}+\frac{2x\log\left(1-e^{2(a+bx)}\right)}{b^2}+\frac{\operatorname{Li}_2\left(e^{2(a+bx)}\right)}{b^3}
\end{aligned}$$

Mathematica [C] time = 5.06, size = 163, normalized size = 2.51

$$\frac{-b^2x^2e^{-\tanh^{-1}(\tanh(a))}\coth(a)\sqrt{\operatorname{sech}^2(a)}-\operatorname{Li}_2\left(e^{-2\left(bx+\tanh^{-1}(\tanh(a))\right)}\right)+2bx\log\left(1-e^{-2\left(\tanh^{-1}(\tanh(a))+bx\right)}\right)+2\tan}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2*Coth[a + b*x]^2,x]
```

```
[Out] x^3/3 + (I*b*Pi*x - I*Pi*Log[1 + E^(2*b*x)] + 2*b*x*Log[1 - E^(-2*(b*x + Ar
cTanh[Tanh[a]]))] + I*Pi*Log[Cosh[b*x]] + 2*ArcTanh[Tanh[a]]*(b*x + Log[1 -
 E^(-2*(b*x + ArcTanh[Tanh[a]]))] - Log[I*Sinh[b*x + ArcTanh[Tanh[a]]]]) -
PolyLog[2, E^(-2*(b*x + ArcTanh[Tanh[a]]))] - (b^2*x^2*Coth[a]*Sqrt[Sech[a]
^2])/E^ArcTanh[Tanh[a]])/b^3 + (x^2*Csch[a]*Csch[a + b*x]*Sinh[b*x])/b
```

fricas [B] time = 0.54, size = 453, normalized size = 6.97

$$-\frac{b^3x^3 - \left(b^3x^3 - 6\,b^2x^2 + 6\,a^2\right)\cosh\left(bx+a\right)^2 - 2\left(b^3x^3 - 6\,b^2x^2 + 6\,a^2\right)\cosh\left(bx+a\right)\sinh\left(bx+a\right) - \left(b^3x^3 - 6\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cosh(b*x+a)^2*csch(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/3*(b^3*x^3 - (b^3*x^3 - 6*b^2*x^2 + 6*a^2)*cosh(b*x + a)^2 - 2*(b^3*x^3
- 6*b^2*x^2 + 6*a^2)*cosh(b*x + a)*sinh(b*x + a) - (b^3*x^3 - 6*b^2*x^2 + 6
*a^2)*sinh(b*x + a)^2 + 6*a^2 - 6*(cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b
*x + a) + sinh(b*x + a)^2 - 1)*dilog(cosh(b*x + a) + sinh(b*x + a)) - 6*(co
sh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 - 1)*dilog(
-cosh(b*x + a) - sinh(b*x + a)) - 6*(b*x*cosh(b*x + a)^2 + 2*b*x*cosh(b*x +
 a)*sinh(b*x + a) + b*x*sinh(b*x + a)^2 - b*x)*log(cosh(b*x + a) + sinh(b*x
 + a) + 1) + 6*(a*cosh(b*x + a)^2 + 2*a*cosh(b*x + a)*sinh(b*x + a) + a*sin
h(b*x + a)^2 - a)*log(cosh(b*x + a) + sinh(b*x + a) - 1) - 6*((b*x + a)*cos
h(b*x + a)^2 + 2*(b*x + a)*cosh(b*x + a)*sinh(b*x + a) + (b*x + a)*sinh(b*x
 + a)^2 - b*x - a)*log(-cosh(b*x + a) - sinh(b*x + a) + 1))/(b^3*cosh(b*x +
 a)^2 + 2*b^3*cosh(b*x + a)*sinh(b*x + a) + b^3*sinh(b*x + a)^2 - b^3)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2\cosh\left(bx+a\right)^2\operatorname{csch}\left(bx+a\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cosh(b*x+a)^2*csch(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate(x^2*cosh(b*x + a)^2*csch(b*x + a)^2, x)
```

maple [B] time = 0.58, size = 156, normalized size = 2.40

$$\frac{x^3}{3} - \frac{2x^2}{b\left(e^{2bx+2a}-1\right)} - \frac{2x^2}{b} - \frac{4ax}{b^2} - \frac{2a^2}{b^3} + \frac{2\ln\left(1-e^{bx+a}\right)x}{b^2} + \frac{2\ln\left(1-e^{bx+a}\right)a}{b^3} + \frac{2\,\mathrm{polylog}\left(2,e^{bx+a}\right)}{b^3} + \frac{2\ln\left(1+e^{bx+a}\right.}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*cosh(b*x+a)^2*csch(b*x+a)^2,x)
```

```
[Out] 1/3*x^3-2*x^2/b/(exp(2*b*x+2*a)-1)-2*x^2/b-4*a*x/b^2-2/b^3*a^2+2/b^2*ln(1-e
xp(b*x+a))*x+2/b^3*ln(1-exp(b*x+a))*a+2*polylog(2,exp(b*x+a))/b^3+2/b^2*ln(
1+exp(b*x+a))*x+2*polylog(2,-exp(b*x+a))/b^3+4/b^3*a*ln(exp(b*x+a))-2/b^3*a
*ln(exp(b*x+a)-1)
```

maxima [A] time = 0.42, size = 108, normalized size = 1.66

$$-\frac{2\,x^2}{b}+\frac{bx^3e^{(2\,bx+2\,a)}-bx^3-6\,x^2}{3\left(be^{(2\,bx+2\,a)}-b\right)}+\frac{2\left(bx\log\left(e^{(bx+a)}+1\right)+\mathrm{Li}_2\left(-e^{(bx+a)}\right)\right)}{b^3}+\frac{2\left(bx\log\left(-e^{(bx+a)}+1\right)+\mathrm{Li}_2\left(e^{(bx+a)}\right)\right)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cosh(b*x+a)^2*csch(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -2*x^2/b + 1/3*(b*x^3*e^(2*b*x + 2*a) - b*x^3 - 6*x^2)/(b*e^(2*b*x + 2*a) -
 b) + 2*(b*x*log(e^(b*x + a) + 1) + dilog(-e^(b*x + a)))/b^3 + 2*(b*x*log(-
e^(b*x + a) + 1) + dilog(e^(b*x + a)))/b^3
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^2\cosh\left(a+b\,x\right)^2}{\sinh\left(a+b\,x\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*cosh(a + b*x)^2)/sinh(a + b*x)^2,x)
```

```
[Out] int((x^2*cosh(a + b*x)^2)/sinh(a + b*x)^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*cosh(b*x+a)**2*csch(b*x+a)**2,x)
```

```
[Out] Timed out
```

3.434 $\int x \coth^2(a+bx)\,dx$

Optimal. Leaf size=31

$$\frac{\log(\sinh(a+bx))}{b^2} - \frac{x\coth(a+bx)}{b} + \frac{x^2}{2}$$

```
[Out] 1/2*x^2-x*coth(b*x+a)/b+ln(sinh(b*x+a))/b^2
```

Rubi [A] time = 0.03, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {3720, 3475, 30}

$$\frac{\log(\sinh(a+bx))}{b^2} - \frac{x\coth(a+bx)}{b} + \frac{x^2}{2}$$

Antiderivative was successfully verified.

```
[In] Int[x*Coth[a + b*x]^2,x]
```

```
[Out] x^2/2 - (x*Coth[a + b*x])/b + Log[Sinh[a + b*x]]/b^2
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3720

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[(b*(c + d*x)^m*(b*Tan[e + f*x])^(n - 1))/(f*(n - 1)), x] + (-Di
st[(b*d*m)/(f*(n - 1)), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x],
 x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[
{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rubi steps

$$\int x\coth^2(a+bx)\,dx = -\frac{x\coth(a+bx)}{b} + \frac{\int \coth(a+bx)\,dx}{b} + \int x\,dx$$
$$= \frac{x^2}{2} - \frac{x\coth(a+bx)}{b} + \frac{\log(\sinh(a+bx))}{b^2}$$

Mathematica [A] time = 0.16, size = 46, normalized size = 1.48

$$\frac{-2bx\coth(a) + 2\log(\sinh(a+bx)) + 2bx\operatorname{csch}(a)\sinh(bx)\operatorname{csch}(a+bx) + b^2x^2}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Coth[a + b*x]^2,x]
```

```
[Out] (b^2*x^2 - 2*b*x*Coth[a] + 2*Log[Sinh[a + b*x]] + 2*b*x*Csch[a]*Csch[a + b*
x]*Sinh[b*x])/(2*b^2)
```

fricas [B] time = 0.46, size = 189, normalized size = 6.10

$$-\frac{b^2x^2 - \left(b^2x^2 - 4\,bx\right)\cosh\left(bx+a\right)^2 - 2\left(b^2x^2 - 4\,bx\right)\cosh\left(bx+a\right)\sinh\left(bx+a\right) - \left(b^2x^2 - 4\,bx\right)\sinh\left(bx+a\right)^2 \cdots}{2\left(b^2\cosh\left(bx+a\right)^2 + 2\,b^2\cosh\left(bx+a\right)\sinh\left(\cdots\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x+a)^2*csch(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/2*(b^2*x^2 - (b^2*x^2 - 4*b*x)*cosh(b*x + a)^2 - 2*(b^2*x^2 - 4*b*x)*cos
h(b*x + a)*sinh(b*x + a) - (b^2*x^2 - 4*b*x)*sinh(b*x + a)^2 - 2*(cosh(b*x
+ a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 - 1)*log(2*sinh(b*
x + a)/(cosh(b*x + a) - sinh(b*x + a))))/(b^2*cosh(b*x + a)^2 + 2*b^2*cosh(
b*x + a)*sinh(b*x + a) + b^2*sinh(b*x + a)^2 - b^2)
```

giac [B] time = 0.15, size = 98, normalized size = 3.16

$$\frac{b^2x^2e^{(2\,bx+2\,a)} - b^2x^2 - 4\,bxe^{(2\,bx+2\,a)} + 2\,e^{(2\,bx+2\,a)}\log\left(e^{(2\,bx+2\,a)} - 1\right) - 2\,\log\left(e^{(2\,bx+2\,a)} - 1\right)}{2\left(b^2e^{(2\,bx+2\,a)} - b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x+a)^2*csch(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*(b^2*x^2*e^(2*b*x + 2*a) - b^2*x^2 - 4*b*x*e^(2*b*x + 2*a) + 2*e^(2*b*x
 + 2*a)*log(e^(2*b*x + 2*a) - 1) - 2*log(e^(2*b*x + 2*a) - 1))/(b^2*e^(2*b*
x + 2*a) - b^2)
```

maple [A] time = 0.51, size = 54, normalized size = 1.74

$$\frac{x^2}{2}-\frac{2x}{b}-\frac{2a}{b^2}-\frac{2x}{b\left(e^{2bx+2a}-1\right)}+\frac{\ln\left(e^{2bx+2a}-1\right)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cosh(b*x+a)^2*csch(b*x+a)^2,x)
```

```
[Out] 1/2*x^2-2*x/b-2*a/b^2-2*x/b/(exp(2*b*x+2*a)-1)+1/b^2*ln(exp(2*b*x+2*a)-1)
```

maxima [B] time = 0.37, size = 115, normalized size = 3.71

$$-\frac{xe^{(2\,bx+2\,a)}}{be^{(2\,bx+2\,a)}-b}-\frac{bx^2-\left(bx^2e^{(2\,a)}-2\,xe^{(2\,a)}\right)e^{(2\,bx)}}{2\left(be^{(2\,bx+2\,a)}-b\right)}+\frac{\log\left(\left(e^{(bx+a)}+1\right)e^{(-a)}\right)}{b^2}+\frac{\log\left(\left(e^{(bx+a)}-1\right)e^{(-a)}\right)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x+a)^2*csch(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -x*e^(2*b*x + 2*a)/(b*e^(2*b*x + 2*a) - b) - 1/2*(b*x^2 - (b*x^2*e^(2*a) -
2*x*e^(2*a))*e^(2*b*x))/(b*e^(2*b*x + 2*a) - b) + log((e^(b*x + a) + 1)*e^(
-a))/b^2 + log((e^(b*x + a) - 1)*e^(-a))/b^2
```

mupad [B] time = 1.45, size = 45, normalized size = 1.45

$$\frac{\frac{x^2\sinh(a+b\,x)}{2}-\frac{x\cosh(a+b\,x)}{b}}{\sinh\left(a+b\,x\right)}+\frac{\ln\left(\sinh\left(a+b\,x\right)\right)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*cosh(a + b*x)^2)/sinh(a + b*x)^2,x)
```

```
[Out] ((x^2*sinh(a + b*x))/2 - (x*cosh(a + b*x))/b)/sinh(a + b*x) + log(sinh(a +
b*x))/b^2
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\cosh^2\left(a+bx\right)\operatorname{csch}^2\left(a+bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x+a)**2*csch(b*x+a)**2,x)
```

```
[Out] Integral(x*cosh(a + b*x)**2*csch(a + b*x)**2, x)
```

3.435 $\int \coth^2(a+bx)\,dx$

Optimal. Leaf size=13

$$x-\frac{\coth(a+bx)}{b}$$

```
[Out] x-coth(b*x+a)/b
```

Rubi [A] time = 0.01, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3473, 8}

$$x-\frac{\coth(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[a + b*x]^2,x]
```

```
[Out] x - Coth[a + b*x]/b
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rubi steps

$$\begin{aligned}\int \coth^2(a+bx)\,dx &= -\frac{\coth(a+bx)}{b}+\int 1\,dx\\ &= x-\frac{\coth(a+bx)}{b}\end{aligned}$$

Mathematica [C] time = 0.01, size = 27, normalized size = 2.08

$$-\frac{\coth(a+bx)\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\tanh^2(a+bx)\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[a + b*x]^2,x]
```

```
[Out] -((Coth[a + b*x]*Hypergeometric2F1[-1/2, 1, 1/2, Tanh[a + b*x]^2])/b)
```

fricas [B] time = 0.40, size = 33, normalized size = 2.54

$$\frac{(bx+1)\sinh(bx+a)-\cosh(bx+a)}{b\sinh(bx+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^2*csch(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] ((b*x + 1)*sinh(b*x + a) - cosh(b*x + a))/(b*sinh(b*x + a))
```

giac [A] time = 0.14, size = 24, normalized size = 1.85

$$\frac{bx+a-\frac{2}{e^{(2\,bx+2\,a)}-1}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^2*csch(b*x+a)^2,x, algorithm="giac")
```

```
[Out] (b*x + a - 2/(e^(2*b*x + 2*a) - 1))/b
```

maple [A] time = 0.12, size = 18, normalized size = 1.38

$$\frac{bx+a-\coth(bx+a)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)^2*csch(b*x+a)^2,x)
```

```
[Out] 1/b*(b*x+a-coth(b*x+a))
```

maxima [A] time = 0.31, size = 25, normalized size = 1.92

$$x+\frac{a}{b}+\frac{2}{b\left(e^{(-2\,bx-2\,a)}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^2*csch(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] x + a/b + 2/(b*(e^(-2*b*x - 2*a) - 1))
```

mupad [B] time = 0.06, size = 20, normalized size = 1.54

$$x-\frac{2}{b\,\left({\mathrm{e}}^{2\,a+2\,b\,x}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a + b*x)^2/sinh(a + b*x)^2,x)
```

```
[Out] x - 2/(b*(exp(2*a + 2*b*x) - 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \cosh^{2}{\left(a + bx \right)} \operatorname{csch}^{2}{\left(a + bx \right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)**2*csch(b*x+a)**2,x)
```

```
[Out] Integral(cosh(a + b*x)**2*csch(a + b*x)**2, x)
```

3.436 $\int \frac{\coth^2(a+bx)}{x} dx$

Optimal. Leaf size=15

$$\text{Int}\left(\frac{\coth^2(a+bx)}{x},x\right)$$

```
[Out] Unintegrable(coth(b*x+a)^2/x,x)
```

Rubi [A] time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\coth^2(a+bx)}{x} dx$$

Verification is Not applicable to the result.

```
[In] Int[Coth[a + b*x]^2/x,x]
[Out] Defer[Int][Coth[a + b*x]^2/x, x]
```

Rubi steps

$$\int \frac{\coth^2(a+bx)}{x} dx = \int \frac{\coth^2(a+bx)}{x} dx$$

Mathematica [A] time = 0.40, size = 0, normalized size = 0.00

$$\int \frac{\coth^2(a+bx)}{x} dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Coth[a + b*x]^2/x,x]
[Out] Integrate[Coth[a + b*x]^2/x, x]
```

fricas [A] time = 0.41, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\cosh(bx+a)^2 \operatorname{csch}(bx+a)^2}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^2*csch(b*x+a)^2/x,x, algorithm="fricas")
```

```
[Out] integral(cosh(b*x + a)^2*csch(b*x + a)^2/x, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh(bx+a)^2 \operatorname{csch}(bx+a)^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^2*csch(b*x+a)^2/x,x, algorithm="giac")
```

```
[Out] integrate(cosh(b*x + a)^2*csch(b*x + a)^2/x, x)
```

maple [A] time = 0.58, size = 0, normalized size = 0.00

$$\int \frac{\left(\cosh^2(bx+a)\right) \operatorname{csch}(bx+a)^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)^2*csch(b*x+a)^2/x,x)
```

```
[Out] int(cosh(b*x+a)^2*csch(b*x+a)^2/x,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{2}{bxe^{(2\,bx+2\,a)}-bx}+\int \frac{1}{bx^2e^{(bx+a)}+bx^2}\,dx-\int \frac{1}{bx^2e^{(bx+a)}-bx^2}\,dx+\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^2*csch(b*x+a)^2/x,x, algorithm="maxima")
```

```
[Out] -2/(b*x*e^(2*b*x + 2*a) - b*x) + integrate(1/(b*x^2*e^(b*x + a) + b*x^2), x
) - integrate(1/(b*x^2*e^(b*x + a) - b*x^2), x) + log(x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.07

$$\int \frac{\cosh(a+b\,x)^2}{x\sinh(a+b\,x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a + b*x)^2/(x*sinh(a + b*x)^2),x)
```

```
[Out] int(cosh(a + b*x)^2/(x*sinh(a + b*x)^2), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh^2(a+bx)\operatorname{csch}^2(a+bx)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)**2*csch(b*x+a)**2/x,x)
```

```
[Out] Integral(cosh(a + b*x)**2*csch(a + b*x)**2/x, x)
```

3.437 $\int \frac{\coth^2(a+bx)}{x^2} dx$

Optimal. Leaf size=15

$$\text{Int}\left(\frac{\coth^2(a+bx)}{x^2},x\right)$$

```
[Out] Unintegrable(coth(b*x+a)^2/x^2,x)
```

Rubi [A] time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\coth^2(a+bx)}{x^2} dx$$

Verification is Not applicable to the result.

```
[In] Int[Coth[a + b*x]^2/x^2,x]
[Out] Defer[Int][Coth[a + b*x]^2/x^2, x]
```

Rubi steps

$$\int \frac{\coth^2(a+bx)}{x^2} dx = \int \frac{\coth^2(a+bx)}{x^2} dx$$

Mathematica [A] time = 0.92, size = 0, normalized size = 0.00

$$\int \frac{\coth^2(a+bx)}{x^2} dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Coth[a + b*x]^2/x^2,x]
[Out] Integrate[Coth[a + b*x]^2/x^2, x]
```

fricas [A] time = 0.41, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\cosh(bx+a)^2\,\text{csch}(bx+a)^2}{x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^2*csch(b*x+a)^2/x^2,x, algorithm="fricas")
```

```
[Out] integral(cosh(b*x + a)^2*csch(b*x + a)^2/x^2, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh(bx+a)^2 \operatorname{csch}(bx+a)^2}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^2*csch(b*x+a)^2/x^2,x, algorithm="giac")
```

```
[Out] integrate(cosh(b*x + a)^2*csch(b*x + a)^2/x^2, x)
```

maple [A] time = 0.59, size = 0, normalized size = 0.00

$$\int \frac{\left(\cosh^2(bx+a)\right)\operatorname{csch}(bx+a)^2}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)^2*csch(b*x+a)^2/x^2,x)
```

```
[Out] int(cosh(b*x+a)^2*csch(b*x+a)^2/x^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{bxe^{(2\,bx+2\,a)}-bx+2}{bx^2e^{(2\,bx+2\,a)}-bx^2}+2\int \frac{1}{bx^3e^{(bx+a)}+bx^3}\,dx-2\int \frac{1}{bx^3e^{(bx+a)}-bx^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^2*csch(b*x+a)^2/x^2,x, algorithm="maxima")
```

```
[Out] -(b*x*e^(2*b*x + 2*a) - b*x + 2)/(b*x^2*e^(2*b*x + 2*a) - b*x^2) + 2*integr
ate(1/(b*x^3*e^(b*x + a) + b*x^3), x) - 2*integrate(1/(b*x^3*e^(b*x + a) -
b*x^3), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.07

$$\int \frac{\cosh(a+b\,x)^2}{x^2\sinh(a+b\,x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a + b*x)^2/(x^2*sinh(a + b*x)^2),x)
```

```
[Out] int(cosh(a + b*x)^2/(x^2*sinh(a + b*x)^2), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh^2(a+bx)\operatorname{csch}^2(a+bx)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)**2*csch(b*x+a)**2/x**2,x)
```

```
[Out] Integral(cosh(a + b*x)**2*csch(a + b*x)**2/x**2, x)
```

3.438 $\int x^m \cosh(a+bx)\coth^2(a+bx)\,dx$

Optimal. Leaf size=78

$$\text{Int}\left(x^m \coth(a+bx)\text{csch}(a+bx),x\right)+\frac{e^a x^m(-bx)^{-m}\Gamma(m+1,-bx)}{2b}-\frac{e^{-a}x^m(bx)^{-m}\Gamma(m+1,bx)}{2b}$$

```
[Out] CannotIntegrate(x^m*coth(b*x+a)*csch(b*x+a),x)+1/2*exp(a)*x^m*GAMMA(1+m,-b*
x)/b/((-b*x)^m)-1/2*x^m*GAMMA(1+m,b*x)/b/exp(a)/((b*x)^m)
```

Rubi [A] time = 0.15, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int x^m \cosh(a+bx)\coth^2(a+bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x^m*Cosh[a + b*x]*Coth[a + b*x]^2,x]
```

```
[Out] (E^a*x^m*Gamma[1 + m, -(b*x)])/(2*b*(-(b*x))^m) - (x^m*Gamma[1 + m, b*x])/(
2*b*E^a*(b*x)^m) + Defer[Int][x^m*Coth[a + b*x]*Csch[a + b*x], x]
```

Rubi steps

$$\begin{aligned}\int x^m \cosh(a+bx)\coth^2(a+bx)\,dx &= \int x^m \cosh(a+bx)\,dx+\int x^m \coth(a+bx)\text{csch}(a+bx)\,dx\\ &= \frac{1}{2}\int e^{-i(ia+ibx)}x^m\,dx+\frac{1}{2}\int e^{i(ia+ibx)}x^m\,dx+\int x^m \coth(a+bx)\text{csch}(a+\\ &= \frac{e^a x^m(-bx)^{-m}\Gamma(1+m,-bx)}{2b}-\frac{e^{-a}x^m(bx)^{-m}\Gamma(1+m,bx)}{2b}+\int x^m \coth(a-\end{aligned}$$

Mathematica [A] time = 38.88, size = 0, normalized size = 0.00

$$\int x^m \cosh(a+bx)\coth^2(a+bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^m*Cosh[a + b*x]*Coth[a + b*x]^2,x]
```

```
[Out] Integrate[x^m*Cosh[a + b*x]*Coth[a + b*x]^2, x]
```

fricas [A] time = 1.02, size = 0, normalized size = 0.00

$$\text{integral}\left(x^m \cosh(bx+a)^3 \text{csch}(bx+a)^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*cosh(b*x+a)^3*csch(b*x+a)^2,x, algorithm="fricas")
[Out] integral(x^m*cosh(b*x + a)^3*csch(b*x + a)^2, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int x^m \cosh(bx+a)^3 \operatorname{csch}(bx+a)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*cosh(b*x+a)^3*csch(b*x+a)^2,x, algorithm="giac")
[Out] integrate(x^m*cosh(b*x + a)^3*csch(b*x + a)^2, x)
```

maple [A] time = 0.44, size = 0, normalized size = 0.00

$$\int x^m \left(\cosh^3(bx+a)\right) \operatorname{csch}(bx+a)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*cosh(b*x+a)^3*csch(b*x+a)^2,x)
[Out] int(x^m*cosh(b*x+a)^3*csch(b*x+a)^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int x^m \cosh(bx+a)^3 \operatorname{csch}(bx+a)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*cosh(b*x+a)^3*csch(b*x+a)^2,x, algorithm="maxima")
[Out] integrate(x^m*cosh(b*x + a)^3*csch(b*x + a)^2, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^m \cosh(a+b\,x)^3}{\sinh(a+b\,x)^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^m*cosh(a + b*x)^3)/sinh(a + b*x)^2,x)
[Out] int((x^m*cosh(a + b*x)^3)/sinh(a + b*x)^2, x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*cosh(b*x+a)**3*csch(b*x+a)**2,x)
```

```
[Out] Timed out
```

3.439 $\int x^3 \cosh(a+bx)\coth^2(a+bx)\,dx$

Optimal. Leaf size=143

$$\frac{6\text{Li}_3\left(-e^{a+bx}\right)}{b^4}-\frac{6\text{Li}_3\left(e^{a+bx}\right)}{b^4}-\frac{6\cosh(a+bx)}{b^4}-\frac{6x\text{Li}_2\left(-e^{a+bx}\right)}{b^3}+\frac{6x\text{Li}_2\left(e^{a+bx}\right)}{b^3}+\frac{6x\sinh(a+bx)}{b^3}-\frac{3x^2\cosh(a+bx)}{b^2}$$

```
[Out] -6*x^2*arctanh(exp(b*x+a))/b^2-6*cosh(b*x+a)/b^4-3*x^2*cosh(b*x+a)/b^2-x^3*
csch(b*x+a)/b-6*x*polylog(2,-exp(b*x+a))/b^3+6*x*polylog(2,exp(b*x+a))/b^3+
6*polylog(3,-exp(b*x+a))/b^4-6*polylog(3,exp(b*x+a))/b^4+6*x*sinh(b*x+a)/b^
3+x^3*sinh(b*x+a)/b
```

Rubi [A] time = 0.18, antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 8, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {5450, 3296, 2638, 5419, 4182, 2531, 2282, 6589}

$$-\frac{6x\text{PolyLog}\left(2,-e^{a+bx}\right)}{b^3}+\frac{6x\text{PolyLog}\left(2,e^{a+bx}\right)}{b^3}+\frac{6\text{PolyLog}\left(3,-e^{a+bx}\right)}{b^4}-\frac{6\text{PolyLog}\left(3,e^{a+bx}\right)}{b^4}-\frac{3x^2\cosh(a+bx)}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Cosh[a + b*x]*Coth[a + b*x]^2,x]
```

```
[Out] (-6*x^2*ArcTanh[E^(a + b*x)])/b^2 - (6*Cosh[a + b*x])/b^4 - (3*x^2*Cosh[a +
 b*x])/b^2 - (x^3*Csch[a + b*x])/b - (6*x*PolyLog[2, -E^(a + b*x)])/b^3 + (
6*x*PolyLog[2, E^(a + b*x)])/b^3 + (6*PolyLog[3, -E^(a + b*x)])/b^4 - (6*Po
lyLog[3, E^(a + b*x)])/b^4 + (6*x*Sinh[a + b*x])/b^3 + (x^3*Sinh[a + b*x])/
b
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5419

```
Int[Coth[(a_.) + (b_.)*(x_)^(n_.)]^(q_.)*Csch[(a_.) + (b_.)*(x_)^(n_.)]^(p_
.)*(x_)^(m_.), x_Symbol] :> -Simp[(x^(m - n + 1)*Csch[a + b*x^n]^p)/(b*n*p)
, x] + Dist[(m - n + 1)/(b*n*p), Int[x^(m - n)*Csch[a + b*x^n]^p, x], x] /;
 FreeQ[{a, b, p}, x] && RationalQ[m] && IntegerQ[n] && GeQ[m - n, 0] && EqQ
[q, 1]
```

Rule 5450

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(n_.)*Coth[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) +
(d_.)*(x_))^(m_.), x_Symbol] :> Int[(c + d*x)^m*Cosh[a + b*x]^n*Coth[a + b*
x]^(p - 2), x] + Int[(c + d*x)^m*Cosh[a + b*x]^(n - 2)*Coth[a + b*x]^p, x]
/; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int x^3 \cosh(a+bx)\coth^2(a+bx)\,dx = \int x^3 \cosh(a+bx)\,dx + \int x^3 \coth(a+bx)\text{csch}(a+bx)\,dx$$
$$= -\frac{x^3\text{csch}(a+bx)}{b} + \frac{x^3\sinh(a+bx)}{b} + \frac{3\int x^2\text{csch}(a+bx)\,dx}{b} - \frac{3\int x^2 \sinh}{}$$
$$= -\frac{6x^2\tanh^{-1}\left(e^{a+bx}\right)}{b^2} - \frac{3x^2\cosh(a+bx)}{b^2} - \frac{x^3\text{csch}(a+bx)}{b} + \frac{x^3\sinh(a+}{b}$$
$$= -\frac{6x^2\tanh^{-1}\left(e^{a+bx}\right)}{b^2} - \frac{3x^2\cosh(a+bx)}{b^2} - \frac{x^3\text{csch}(a+bx)}{b} - \frac{6x\text{Li}_2\left(-e^{a+b}\right.}{b^3}$$
$$= -\frac{6x^2\tanh^{-1}\left(e^{a+bx}\right)}{b^2} - \frac{6\cosh(a+bx)}{b^4} - \frac{3x^2\cosh(a+bx)}{b^2} - \frac{x^3\text{csch}(a+b}{b}$$
$$= -\frac{6x^2\tanh^{-1}\left(e^{a+bx}\right)}{b^2} - \frac{6\cosh(a+bx)}{b^4} - \frac{3x^2\cosh(a+bx)}{b^2} - \frac{x^3\text{csch}(a+b}{b}$$

Mathematica [A] time = 0.34, size = 225, normalized size = 1.57

$$\frac{\text{csch}\left(\frac{1}{2}(a+bx)\right)\text{sech}\left(\frac{1}{2}(a+bx)\right)\left(b^3x^3\cosh(2(a+bx)) - 3b^2x^2\sinh(2(a+bx)) - 12b^2x^2\sinh(a+bx)\tanh^{-1}(\text{s}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^3*Cosh[a + b*x]*Coth[a + b*x]^2,x]
```

```
[Out] (Csch[(a + b*x)/2]*Sech[(a + b*x)/2]*(-6*b*x - 3*b^3*x^3 + 6*b*x*Cosh[2*(a
+ b*x)] + b^3*x^3*Cosh[2*(a + b*x)] - 12*b^2*x^2*ArcTanh[Cosh[a + b*x] + Si
nh[a + b*x]]*Sinh[a + b*x] - 12*b*x*PolyLog[2, -Cosh[a + b*x] - Sinh[a + b*
x]]*Sinh[a + b*x] + 12*b*x*PolyLog[2, Cosh[a + b*x] + Sinh[a + b*x]]*Sinh[a
 + b*x] + 12*PolyLog[3, -Cosh[a + b*x] - Sinh[a + b*x]]*Sinh[a + b*x] - 12*
PolyLog[3, Cosh[a + b*x] + Sinh[a + b*x]]*Sinh[a + b*x] - 6*Sinh[2*(a + b*x
)] - 3*b^2*x^2*Sinh[2*(a + b*x)]))/(4*b^4)
```

fricas [C] time = 0.43, size = 1055, normalized size = 7.38

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*cosh(b*x+a)^3*csch(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(b^3*x^3 + (b^3*x^3 - 3*b^2*x^2 + 6*b*x - 6)*cosh(b*x + a)^4 + 4*(b^3*x
^3 - 3*b^2*x^2 + 6*b*x - 6)*cosh(b*x + a)*sinh(b*x + a)^3 + (b^3*x^3 - 3*b^
```

```
2*x^2 + 6*b*x - 6)*sinh(b*x + a)^4 + 3*b^2*x^2 - 6*(b^3*x^3 + 2*b*x)*cosh(b
*x + a)^2 - 6*(b^3*x^3 - (b^3*x^3 - 3*b^2*x^2 + 6*b*x - 6)*cosh(b*x + a)^2
+ 2*b*x)*sinh(b*x + a)^2 + 6*b*x + 12*(b*x*cosh(b*x + a)^3 + 3*b*x*cosh(b*x
 + a)*sinh(b*x + a)^2 + b*x*sinh(b*x + a)^3 - b*x*cosh(b*x + a) + (3*b*x*co
sh(b*x + a)^2 - b*x)*sinh(b*x + a))*dilog(cosh(b*x + a) + sinh(b*x + a)) -
12*(b*x*cosh(b*x + a)^3 + 3*b*x*cosh(b*x + a)*sinh(b*x + a)^2 + b*x*sinh(b*
x + a)^3 - b*x*cosh(b*x + a) + (3*b*x*cosh(b*x + a)^2 - b*x)*sinh(b*x + a))
*dilog(-cosh(b*x + a) - sinh(b*x + a)) - 6*(b^2*x^2*cosh(b*x + a)^3 + 3*b^2
*x^2*cosh(b*x + a)*sinh(b*x + a)^2 + b^2*x^2*sinh(b*x + a)^3 - b^2*x^2*cosh
(b*x + a) + (3*b^2*x^2*cosh(b*x + a)^2 - b^2*x^2)*sinh(b*x + a))*log(cosh(b
*x + a) + sinh(b*x + a) + 1) + 6*(a^2*cosh(b*x + a)^3 + 3*a^2*cosh(b*x + a)
*sinh(b*x + a)^2 + a^2*sinh(b*x + a)^3 - a^2*cosh(b*x + a) + (3*a^2*cosh(b*
x + a)^2 - a^2)*sinh(b*x + a))*log(cosh(b*x + a) + sinh(b*x + a) - 1) + 6*(
(b^2*x^2 - a^2)*cosh(b*x + a)^3 + 3*(b^2*x^2 - a^2)*cosh(b*x + a)*sinh(b*x
+ a)^2 + (b^2*x^2 - a^2)*sinh(b*x + a)^3 - (b^2*x^2 - a^2)*cosh(b*x + a) -
(b^2*x^2 - 3*(b^2*x^2 - a^2)*cosh(b*x + a)^2 - a^2)*sinh(b*x + a))*log(-cos
h(b*x + a) - sinh(b*x + a) + 1) - 12*(cosh(b*x + a)^3 + 3*cosh(b*x + a)*sin
h(b*x + a)^2 + sinh(b*x + a)^3 + (3*cosh(b*x + a)^2 - 1)*sinh(b*x + a) - co
sh(b*x + a))*polylog(3, cosh(b*x + a) + sinh(b*x + a)) + 12*(cosh(b*x + a)^
3 + 3*cosh(b*x + a)*sinh(b*x + a)^2 + sinh(b*x + a)^3 + (3*cosh(b*x + a)^2
- 1)*sinh(b*x + a) - cosh(b*x + a))*polylog(3, -cosh(b*x + a) - sinh(b*x +
a)) + 4*((b^3*x^3 - 3*b^2*x^2 + 6*b*x - 6)*cosh(b*x + a)^3 - 3*(b^3*x^3 + 2
*b*x)*cosh(b*x + a))*sinh(b*x + a) + 6)/(b^4*cosh(b*x + a)^3 + 3*b^4*cosh(b
*x + a)*sinh(b*x + a)^2 + b^4*sinh(b*x + a)^3 - b^4*cosh(b*x + a) + (3*b^4*
cosh(b*x + a)^2 - b^4)*sinh(b*x + a))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3 \cosh(bx+a)^3 \operatorname{csch}(bx+a)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*cosh(b*x+a)^3*csch(b*x+a)^2,x, algorithm="giac")
[Out] integrate(x^3*cosh(b*x + a)^3*csch(b*x + a)^2, x)
```

maple [A] time = 0.54, size = 241, normalized size = 1.69

$$\frac{\left(x^3b^3 - 3x^2b^2 + 6bx - 6\right) e^{bx+a}}{2b^4} - \frac{\left(x^3b^3 + 3x^2b^2 + 6bx + 6\right) e^{-bx-a}}{2b^4} - \frac{2x^3 e^{bx+a}}{b\left(e^{2bx+2a} - 1\right)} - \frac{6a^2 \operatorname{arctanh}\left(e^{bx+a}\right)}{b^4} - \frac{3\ln\left(1\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*cosh(b*x+a)^3*csch(b*x+a)^2,x)
```

```
[Out] 1/2*(b^3*x^3-3*b^2*x^2+6*b*x-6)/b^4*exp(b*x+a)-1/2*(b^3*x^3+3*b^2*x^2+6*b*x
+6)/b^4*exp(-b*x-a)-2/b*x^3*exp(b*x+a)/(exp(2*b*x+2*a)-1)-6/b^4*a^2*arctanh
(exp(b*x+a))-3/b^2*ln(1+exp(b*x+a))*x^2+3/b^4*ln(1+exp(b*x+a))*a^2-6*x*poly
log(2,-exp(b*x+a))/b^3+6*polylog(3,-exp(b*x+a))/b^4+3/b^2*ln(1-exp(b*x+a))*
x^2-3/b^4*ln(1-exp(b*x+a))*a^2+6*x*polylog(2,exp(b*x+a))/b^3-6*polylog(3,ex
p(b*x+a))/b^4
```

maxima [A] time = 0.51, size = 216, normalized size = 1.51

$$\frac{\left(b^3x^3e^{(4\,a)} - 3\,b^2x^2e^{(4\,a)} + 6\,bxe^{(4\,a)} - 6\,e^{(4\,a)}\right)e^{(3\,bx)} - 6\left(b^3x^3e^{(2\,a)} + 2\,bxe^{(2\,a)}\right)e^{(bx)} + \left(b^3x^3 + 3\,b^2x^2 + 6\,bx + 6\right)e^{(-bx}}{2\left(b^4e^{(2\,bx+3\,a)} - b^4e^a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*cosh(b*x+a)^3*csch(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*((b^3*x^3*e^(4*a) - 3*b^2*x^2*e^(4*a) + 6*b*x*e^(4*a) - 6*e^(4*a))*e^(3
*b*x) - 6*(b^3*x^3*e^(2*a) + 2*b*x*e^(2*a))*e^(b*x) + (b^3*x^3 + 3*b^2*x^2
+ 6*b*x + 6)*e^(-b*x))/(b^4*e^(2*b*x + 3*a) - b^4*e^a) - 3*(b^2*x^2*log(e^(
b*x + a) + 1) + 2*b*x*dilog(-e^(b*x + a)) - 2*polylog(3, -e^(b*x + a)))/b^4
 + 3*(b^2*x^2*log(-e^(b*x + a) + 1) + 2*b*x*dilog(e^(b*x + a)) - 2*polylog(
3, e^(b*x + a)))/b^4
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3\cosh\left(a+b\,x\right)^3}{\sinh\left(a+b\,x\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*cosh(a + b*x)^3)/sinh(a + b*x)^2,x)
```

```
[Out] int((x^3*cosh(a + b*x)^3)/sinh(a + b*x)^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*cosh(b*x+a)**3*csch(b*x+a)**2,x)
```

```
[Out] Timed out
```

3.440 $\int x^2 \cosh(a+bx)\coth^2(a+bx)\,dx$

Optimal. Leaf size=95

$$-\frac{2\text{Li}_2\left(-e^{a+bx}\right)}{b^3}+\frac{2\text{Li}_2\left(e^{a+bx}\right)}{b^3}+\frac{2\sinh(a+bx)}{b^3}-\frac{2x\cosh(a+bx)}{b^2}-\frac{4x\tanh^{-1}\left(e^{a+bx}\right)}{b^2}+\frac{x^2\sinh(a+bx)}{b}-\frac{x^2\text{csch}(a}{b}$$

```
[Out] -4*x*arctanh(exp(b*x+a))/b^2-2*x*cosh(b*x+a)/b^2-x^2*csch(b*x+a)/b-2*polylo
g(2,-exp(b*x+a))/b^3+2*polylog(2,exp(b*x+a))/b^3+2*sinh(b*x+a)/b^3+x^2*sinh
(b*x+a)/b
```

Rubi [A] time = 0.12, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {5450, 3296, 2637, 5419, 4182, 2279, 2391}

$$-\frac{2\text{PolyLog}\left(2,-e^{a+bx}\right)}{b^3}+\frac{2\text{PolyLog}\left(2,e^{a+bx}\right)}{b^3}+\frac{2\sinh(a+bx)}{b^3}-\frac{2x\cosh(a+bx)}{b^2}-\frac{4x\tanh^{-1}\left(e^{a+bx}\right)}{b^2}+\frac{x^2\sinh(a}{b}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Cosh[a + b*x]*Coth[a + b*x]^2,x]
```

```
[Out] (-4*x*ArcTanh[E^(a + b*x)])/b^2 - (2*x*Cosh[a + b*x])/b^2 - (x^2*Csch[a + b
*x])/b - (2*PolyLog[2, -E^(a + b*x)])/b^3 + (2*PolyLog[2, E^(a + b*x)])/b^3
 + (2*Sinh[a + b*x])/b^3 + (x^2*Sinh[a + b*x])/b
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
```

```
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5419

```
Int[Coth[(a_.) + (b_.)*(x_)^(n_.)]^(q_.)*Csch[(a_.) + (b_.)*(x_)^(n_.)]^(p_
.)*(x_)^(m_.), x_Symbol] :> -Simp[(x^(m - n + 1)*Csch[a + b*x^n]^p)/(b*n*p)
, x] + Dist[(m - n + 1)/(b*n*p), Int[x^(m - n)*Csch[a + b*x^n]^p, x], x] /;
 FreeQ[{a, b, p}, x] && RationalQ[m] && IntegerQ[n] && GeQ[m - n, 0] && EqQ
[q, 1]
```

Rule 5450

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(n_.)*Coth[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) +
(d_.)*(x_))^(m_.), x_Symbol] :> Int[(c + d*x)^m*Cosh[a + b*x]^n*Coth[a + b*
x]^(p - 2), x] + Int[(c + d*x)^m*Cosh[a + b*x]^(n - 2)*Coth[a + b*x]^p, x]
/; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rubi steps

$$\int x^2\cosh(a+bx)\coth^2(a+bx)\,dx = \int x^2\cosh(a+bx)\,dx + \int x^2\coth(a+bx)\text{csch}(a+bx)\,dx$$
$$= -\frac{x^2\text{csch}(a+bx)}{b} + \frac{x^2\sinh(a+bx)}{b} + \frac{2\int x\text{csch}(a+bx)\,dx}{b} - \frac{2\int x\sinh(\ldots}{b}$$
$$= -\frac{4x\tanh^{-1}\left(e^{a+bx}\right)}{b^2} - \frac{2x\cosh(a+bx)}{b^2} - \frac{x^2\text{csch}(a+bx)}{b} + \frac{x^2\sinh(a+bx\ldots}{b}$$
$$= -\frac{4x\tanh^{-1}\left(e^{a+bx}\right)}{b^2} - \frac{2x\cosh(a+bx)}{b^2} - \frac{x^2\text{csch}(a+bx)}{b} + \frac{2\sinh(a+bx)}{b^3}$$
$$= -\frac{4x\tanh^{-1}\left(e^{a+bx}\right)}{b^2} - \frac{2x\cosh(a+bx)}{b^2} - \frac{x^2\text{csch}(a+bx)}{b} - \frac{2\text{Li}_2\left(-e^{a+bx}\right)}{b^3}\ldots$$

Mathematica [B] time = 0.45, size = 230, normalized size = 2.42

$$\frac{\operatorname{csch}\left(\frac{1}{2}(a+bx)\right)\operatorname{sech}\left(\frac{1}{2}(a+bx)\right)\left(b^2x^2\cosh(2(a+bx))+4\text{Li}_2\left(-e^{-a-bx}\right)\sinh(a+bx)-4\text{Li}_2\left(e^{-a-bx}\right)\sinh(a+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Cosh[a + b*x]*Coth[a + b*x]^2,x]
```

```
[Out] (Csch[(a + b*x)/2]*Sech[(a + b*x)/2]*(-2 - 3*b^2*x^2 + 2*Cosh[2*(a + b*x)]
+ b^2*x^2*Cosh[2*(a + b*x)] + 4*a*Log[1 - E^(-a - b*x)]*Sinh[a + b*x] + 4*b
*x*Log[1 - E^(-a - b*x)]*Sinh[a + b*x] - 4*a*Log[1 + E^(-a - b*x)]*Sinh[a +
 b*x] - 4*b*x*Log[1 + E^(-a - b*x)]*Sinh[a + b*x] - 4*a*Log[Tanh[(a + b*x)/
2]]*Sinh[a + b*x] + 4*PolyLog[2, -E^(-a - b*x)]*Sinh[a + b*x] - 4*PolyLog[2
, E^(-a - b*x)]*Sinh[a + b*x] - 2*b*x*Sinh[2*(a + b*x)]))/(4*b^3)
```

fricas [B] time = 0.44, size = 731, normalized size = 7.69

$$\frac{\left(b^2x^2-2\,bx+2\right)\cosh\left(bx+a\right)^4+4\left(b^2x^2-2\,bx+2\right)\cosh\left(bx+a\right)\sinh\left(bx+a\right)^3+\left(b^2x^2-2\,bx+2\right)\sinh\left(bx\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cosh(b*x+a)^3*csch(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/2*((b^2*x^2 - 2*b*x + 2)*cosh(b*x + a)^4 + 4*(b^2*x^2 - 2*b*x + 2)*cosh(b
*x + a)*sinh(b*x + a)^3 + (b^2*x^2 - 2*b*x + 2)*sinh(b*x + a)^4 + b^2*x^2 -
 2*(3*b^2*x^2 + 2)*cosh(b*x + a)^2 - 2*(3*b^2*x^2 - 3*(b^2*x^2 - 2*b*x + 2)
*cosh(b*x + a)^2 + 2)*sinh(b*x + a)^2 + 2*b*x + 4*(cosh(b*x + a)^3 + 3*cosh
(b*x + a)*sinh(b*x + a)^2 + sinh(b*x + a)^3 + (3*cosh(b*x + a)^2 - 1)*sinh(
b*x + a) - cosh(b*x + a))*dilog(cosh(b*x + a) + sinh(b*x + a)) - 4*(cosh(b*
x + a)^3 + 3*cosh(b*x + a)*sinh(b*x + a)^2 + sinh(b*x + a)^3 + (3*cosh(b*x
+ a)^2 - 1)*sinh(b*x + a) - cosh(b*x + a))*dilog(-cosh(b*x + a) - sinh(b*x
+ a)) - 4*(b*x*cosh(b*x + a)^3 + 3*b*x*cosh(b*x + a)*sinh(b*x + a)^2 + b*x*
sinh(b*x + a)^3 - b*x*cosh(b*x + a) + (3*b*x*cosh(b*x + a)^2 - b*x)*sinh(b*
x + a))*log(cosh(b*x + a) + sinh(b*x + a) + 1) - 4*(a*cosh(b*x + a)^3 + 3*a
*cosh(b*x + a)*sinh(b*x + a)^2 + a*sinh(b*x + a)^3 - a*cosh(b*x + a) + (3*a
*cosh(b*x + a)^2 - a)*sinh(b*x + a))*log(cosh(b*x + a) + sinh(b*x + a) - 1)
 + 4*((b*x + a)*cosh(b*x + a)^3 + 3*(b*x + a)*cosh(b*x + a)*sinh(b*x + a)^2
 + (b*x + a)*sinh(b*x + a)^3 - (b*x + a)*cosh(b*x + a) + (3*(b*x + a)*cosh(
b*x + a)^2 - b*x - a)*sinh(b*x + a))*log(-cosh(b*x + a) - sinh(b*x + a) + 1
) + 4*((b^2*x^2 - 2*b*x + 2)*cosh(b*x + a)^3 - (3*b^2*x^2 + 2)*cosh(b*x + a
))*sinh(b*x + a) + 2)/(b^3*cosh(b*x + a)^3 + 3*b^3*cosh(b*x + a)*sinh(b*x +
 a)^2 + b^3*sinh(b*x + a)^3 - b^3*cosh(b*x + a) + (3*b^3*cosh(b*x + a)^2 -
b^3)*sinh(b*x + a))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \cosh(bx+a)^3 \operatorname{csch}(bx+a)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cosh(b*x+a)^3*csch(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate(x^2*cosh(b*x + a)^3*csch(b*x + a)^2, x)
```

maple [A] time = 0.53, size = 185, normalized size = 1.95

$$\frac{(x^2b^2-2bx+2)\,e^{bx+a}}{2b^3}-\frac{(x^2b^2+2bx+2)\,e^{-bx-a}}{2b^3}-\frac{2x^2e^{bx+a}}{b\left(e^{2bx+2a}-1\right)}-\frac{2\ln\left(1+e^{bx+a}\right)x}{b^2}-\frac{2\ln\left(1+e^{bx+a}\right)a}{b^3}-\frac{2\operatorname{polylo}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*cosh(b*x+a)^3*csch(b*x+a)^2,x)
```

```
[Out] 1/2*(b^2*x^2-2*b*x+2)/b^3*exp(b*x+a)-1/2*(b^2*x^2+2*b*x+2)/b^3*exp(-b*x-a)-
2*x^2*exp(b*x+a)/b/(exp(2*b*x+2*a)-1)-2/b^2*ln(1+exp(b*x+a))*x-2/b^3*ln(1+e
xp(b*x+a))*a-2*polylog(2,-exp(b*x+a))/b^3+2/b^2*ln(1-exp(b*x+a))*x+2/b^3*ln
(1-exp(b*x+a))*a+2*polylog(2,exp(b*x+a))/b^3+4/b^3*a*arctanh(exp(b*x+a))
```

maxima [A] time = 0.42, size = 157, normalized size = 1.65

$$\frac{\left(b^2x^2e^{(4a)}-2\,bxe^{(4a)}+2\,e^{(4a)}\right)e^{(3\,bx)}-2\left(3\,b^2x^2e^{(2a)}+2\,e^{(2a)}\right)e^{(bx)}+\left(b^2x^2+2\,bx+2\right)e^{(-bx)}}{2\left(b^3e^{(2\,bx+3\,a)}-b^3e^{a}\right)}-\frac{2\left(bx\log\left(e^{(bx+a)}+1\right.\right.}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cosh(b*x+a)^3*csch(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*((b^2*x^2*e^(4*a) - 2*b*x*e^(4*a) + 2*e^(4*a))*e^(3*b*x) - 2*(3*b^2*x^2
*e^(2*a) + 2*e^(2*a))*e^(b*x) + (b^2*x^2 + 2*b*x + 2)*e^(-b*x))/(b^3*e^(2*b
*x + 3*a) - b^3*e^a) - 2*(b*x*log(e^(b*x + a) + 1) + dilog(-e^(b*x + a)))/b
^3 + 2*(b*x*log(-e^(b*x + a) + 1) + dilog(e^(b*x + a)))/b^3
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2 \cosh(a+b\,x)^3}{\sinh(a+b\,x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*cosh(a + b*x)^3)/sinh(a + b*x)^2,x)
```

```
[Out] int((x^2*cosh(a + b*x)^3)/sinh(a + b*x)^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*cosh(b*x+a)**3*csch(b*x+a)**2,x)
```

```
[Out] Timed out
```

3.441 $\int x\cosh(a+bx)\coth^2(a+bx)\,dx$

Optimal. Leaf size=47

$$-\frac{\cosh(a+bx)}{b^2}-\frac{\tanh^{-1}(\cosh(a+bx))}{b^2}+\frac{x\sinh(a+bx)}{b}-\frac{x\text{csch}(a+bx)}{b}$$

```
[Out] -arctanh(cosh(b*x+a))/b^2-cosh(b*x+a)/b^2-x*csch(b*x+a)/b+x*sinh(b*x+a)/b
```

Rubi [A] time = 0.05, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {5450, 3296, 2638, 5419, 3770}

$$-\frac{\cosh(a+bx)}{b^2}-\frac{\tanh^{-1}(\cosh(a+bx))}{b^2}+\frac{x\sinh(a+bx)}{b}-\frac{x\text{csch}(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[x*Cosh[a + b*x]*Coth[a + b*x]^2,x]
```

```
[Out] -(ArcTanh[Cosh[a + b*x]]/b^2) - Cosh[a + b*x]/b^2 - (x*Csch[a + b*x])/b + (
x*Sinh[a + b*x])/b
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 5419

```
Int[Coth[(a_.) + (b_.)*(x_)^(n_.)]^(q_.)*Csch[(a_.) + (b_.)*(x_)^(n_.)]^(p_
.)*(x_)^(m_.), x_Symbol] :> -Simp[(x^(m - n + 1)*Csch[a + b*x^n]^p)/(b*n*p)
, x] + Dist[(m - n + 1)/(b*n*p), Int[x^(m - n)*Csch[a + b*x^n]^p, x], x] /;
 FreeQ[{a, b, p}, x] && RationalQ[m] && IntegerQ[n] && GeQ[m - n, 0] && EqQ
```

```
[q, 1]
```

Rule 5450

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(n_.)*Coth[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) +
(d_.)*(x_))^(m_.), x_Symbol] :> Int[(c + d*x)^m*Cosh[a + b*x]^n*Coth[a + b*
x]^(p - 2), x] + Int[(c + d*x)^m*Cosh[a + b*x]^(n - 2)*Coth[a + b*x]^p, x]
/; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rubi steps

$$\int x\cosh(a+bx)\coth^2(a+bx)\,dx = \int x\cosh(a+bx)\,dx + \int x\coth(a+bx)\text{csch}(a+bx)\,dx$$
$$= -\frac{x\text{csch}(a+bx)}{b} + \frac{x\sinh(a+bx)}{b} + \frac{\int \text{csch}(a+bx)\,dx}{b} - \frac{\int \sinh(a+bx)\,d}{b}$$
$$= -\frac{\tanh^{-1}(\cosh(a+bx))}{b^2} - \frac{\cosh(a+bx)}{b^2} - \frac{x\text{csch}(a+bx)}{b} + \frac{x\sinh(a+bx}{b}$$

Mathematica [A] time = 0.23, size = 66, normalized size = 1.40

$$\frac{2bx\sinh(a+bx) - 2\cosh(a+bx) + bx\tanh\left(\frac{1}{2}(a+bx)\right) - bx\coth\left(\frac{1}{2}(a+bx)\right) + 2\log\left(\tanh\left(\frac{1}{2}(a+bx)\right)\right)}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Cosh[a + b*x]*Coth[a + b*x]^2,x]
```

```
[Out] (-2*Cosh[a + b*x] - b*x*Coth[(a + b*x)/2] + 2*Log[Tanh[(a + b*x)/2]] + 2*b*
x*Sinh[a + b*x] + b*x*Tanh[(a + b*x)/2])/(2*b^2)
```

fricas [B] time = 0.42, size = 367, normalized size = 7.81

$$\frac{(bx-1)\cosh(bx+a)^4 + 4(bx-1)\cosh(bx+a)\sinh(bx+a)^3 + (bx-1)\sinh(bx+a)^4 - 6bx\cosh(bx+a)^2 -}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x+a)^3*csch(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/2*((b*x - 1)*cosh(b*x + a)^4 + 4*(b*x - 1)*cosh(b*x + a)*sinh(b*x + a)^3
+ (b*x - 1)*sinh(b*x + a)^4 - 6*b*x*cosh(b*x + a)^2 + 6*((b*x - 1)*cosh(b*x
```

```
 + a)^2 - b*x)*sinh(b*x + a)^2 + b*x - 2*(cosh(b*x + a)^3 + 3*cosh(b*x + a)
*sinh(b*x + a)^2 + sinh(b*x + a)^3 + (3*cosh(b*x + a)^2 - 1)*sinh(b*x + a)
- cosh(b*x + a))*log(cosh(b*x + a) + sinh(b*x + a) + 1) + 2*(cosh(b*x + a)^
3 + 3*cosh(b*x + a)*sinh(b*x + a)^2 + sinh(b*x + a)^3 + (3*cosh(b*x + a)^2
- 1)*sinh(b*x + a) - cosh(b*x + a))*log(cosh(b*x + a) + sinh(b*x + a) - 1)
+ 4*((b*x - 1)*cosh(b*x + a)^3 - 3*b*x*cosh(b*x + a))*sinh(b*x + a) + 1)/(b
^2*cosh(b*x + a)^3 + 3*b^2*cosh(b*x + a)*sinh(b*x + a)^2 + b^2*sinh(b*x + a
)^3 - b^2*cosh(b*x + a) + (3*b^2*cosh(b*x + a)^2 - b^2)*sinh(b*x + a))
```

giac [B] time = 0.21, size = 144, normalized size = 3.06

$$\frac{bxe^{(4\,bx+4\,a)} - 6\,bxe^{(2\,bx+2\,a)} + bx - 2\,e^{(3\,bx+3\,a)}\log\left(e^{(bx+a)}+1\right) + 2\,e^{(bx+a)}\log\left(e^{(bx+a)}+1\right) + 2\,e^{(3\,bx+3\,a)}\log\left(e^{(bx+a)}\right.}{2\left(b^2e^{(3\,bx+3\,a)} - b^2e^{(bx+a)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x+a)^3*csch(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*(b*x*e^(4*b*x + 4*a) - 6*b*x*e^(2*b*x + 2*a) + b*x - 2*e^(3*b*x + 3*a)*
log(e^(b*x + a) + 1) + 2*e^(b*x + a)*log(e^(b*x + a) + 1) + 2*e^(3*b*x + 3*
a)*log(e^(b*x + a) - 1) - 2*e^(b*x + a)*log(e^(b*x + a) - 1) - e^(4*b*x + 4
*a) + 1)/(b^2*e^(3*b*x + 3*a) - b^2*e^(b*x + a))
```

maple [A] time = 0.45, size = 89, normalized size = 1.89

$$\frac{(bx-1)\,e^{bx+a}}{2b^2} - \frac{(bx+1)\,e^{-bx-a}}{2b^2} - \frac{2\,e^{bx+a}x}{b\left(e^{2bx+2a}-1\right)} + \frac{\ln\left(e^{bx+a}-1\right)}{b^2} - \frac{\ln\left(1+e^{bx+a}\right)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cosh(b*x+a)^3*csch(b*x+a)^2,x)
```

```
[Out] 1/2*(b*x-1)/b^2*exp(b*x+a)-1/2*(b*x+1)/b^2*exp(-b*x-a)-2*exp(b*x+a)*x/b/(ex
p(2*b*x+2*a)-1)+1/b^2*ln(exp(b*x+a)-1)-1/b^2*ln(1+exp(b*x+a))
```

maxima [B] time = 0.44, size = 109, normalized size = 2.32

$$-\frac{6\,bxe^{(bx+2\,a)} - \left(bxe^{(4\,a)} - e^{(4\,a)}\right)e^{(3\,bx)} - (bx+1)e^{(-bx)}}{2\left(b^2e^{(2\,bx+3\,a)} - b^2e^{a}\right)} - \frac{\log\left(\left(e^{(bx+a)}+1\right)e^{(-a)}\right)}{b^2} + \frac{\log\left(\left(e^{(bx+a)}-1\right)e^{(-a)}\right)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x+a)^3*csch(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(6*b*x*e^(b*x + 2*a) - (b*x*e^(4*a) - e^(4*a))*e^(3*b*x) - (b*x + 1)*e
^(-b*x))/(b^2*e^(2*b*x + 3*a) - b^2*e^a) - log((e^(b*x + a) + 1)*e^(-a))/b^
2 + log((e^(b*x + a) - 1)*e^(-a))/b^2
```

mupad [B] time = 0.09, size = 95, normalized size = 2.02

$$e^{a+b\,x}\,\left(\frac{x}{2\,b}-\frac{1}{2\,b^2}\right)-\frac{2\,\mathrm{atan}\left(\frac{e^{b\,x}\,e^a\,\sqrt{-b^4}}{b^2}\right)}{\sqrt{-b^4}}-e^{-a-b\,x}\,\left(\frac{x}{2\,b}+\frac{1}{2\,b^2}\right)-\frac{2\,x\,e^{a+b\,x}}{b\,\left(e^{2\,a+2\,b\,x}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*cosh(a + b*x)^3)/sinh(a + b*x)^2,x)
```

```
[Out] exp(a + b*x)*(x/(2*b) - 1/(2*b^2)) - (2*atan((exp(b*x)*exp(a)*(-b^4)^(1/2))
/b^2))/(-b^4)^(1/2) - exp(- a - b*x)*(x/(2*b) + 1/(2*b^2)) - (2*x*exp(a + b
*x))/(b*(exp(2*a + 2*b*x) - 1))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x+a)**3*csch(b*x+a)**2,x)
```

```
[Out] Timed out
```

3.442 $\int \cosh(a+bx)\coth^2(a+bx)\,dx$

Optimal. Leaf size=22

$$\frac{\sinh(a+bx)}{b}-\frac{\operatorname{csch}(a+bx)}{b}$$

```
[Out] -csch(b*x+a)/b+sinh(b*x+a)/b
```

Rubi [A] time = 0.02, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2590, 14}

$$\frac{\sinh(a+bx)}{b}-\frac{\operatorname{csch}(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[a + b*x]*Coth[a + b*x]^2,x]
```

```
[Out] -(Csch[a + b*x]/b) + Sinh[a + b*x]/b
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2590

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> -Dist[f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rubi steps

$$\begin{aligned}\int \cosh(a+bx)\coth^2(a+bx)\,dx &= -\frac{i\,\text{Subst}\left(\int \frac{1-x^2}{x^2}\,dx,x,-i\sinh(a+bx)\right)}{b}\\ &= -\frac{i\,\text{Subst}\left(\int \left(-1+\frac{1}{x^2}\right)dx,x,-i\sinh(a+bx)\right)}{b}\\ &= -\frac{\operatorname{csch}(a+bx)}{b}+\frac{\sinh(a+bx)}{b}\end{aligned}$$

Mathematica [A] time = 0.01, size = 22, normalized size = 1.00

$$\frac{\sinh(a+bx)}{b}-\frac{\operatorname{csch}(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[a + b*x]*Coth[a + b*x]^2,x]
[Out] -(Csch[a + b*x]/b) + Sinh[a + b*x]/b
```

fricas [A] time = 0.40, size = 31, normalized size = 1.41

$$\frac{\cosh(bx+a)^2+\sinh(bx+a)^2-3}{2\,b\sinh(bx+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^3*csch(b*x+a)^2,x, algorithm="fricas")
[Out] 1/2*(cosh(b*x + a)^2 + sinh(b*x + a)^2 - 3)/(b*sinh(b*x + a))
```

giac [B] time = 0.14, size = 45, normalized size = 2.05

$$-\frac{\frac{4}{e^{(bx+a)}-e^{(-bx-a)}}-e^{(bx+a)}+e^{(-bx-a)}}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^3*csch(b*x+a)^2,x, algorithm="giac")
[Out] -1/2*(4/(e^(b*x + a) - e^(-b*x - a)) - e^(b*x + a) + e^(-b*x - a))/b
```

maple [A] time = 0.12, size = 33, normalized size = 1.50

$$\frac{\frac{\cosh^2(bx+a)}{\sinh(bx+a)}-\frac{2}{\sinh(bx+a)}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)^3*csch(b*x+a)^2,x)
[Out] 1/b*(1/sinh(b*x+a)*cosh(b*x+a)^2-2/sinh(b*x+a))
```

maxima [B] time = 0.31, size = 56, normalized size = 2.55

$$-\frac{e^{(-bx-a)}}{2\,b}-\frac{5\,e^{(-2\,bx-2\,a)}-1}{2\,b\left(e^{(-bx-a)}-e^{(-3\,bx-3\,a)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^3*csch(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*e^(-b*x - a)/b - 1/2*(5*e^(-2*b*x - 2*a) - 1)/(b*(e^(-b*x - a) - e^(-3
*b*x - 3*a)))
```

mupad [B] time = 1.45, size = 22, normalized size = 1.00

$$\frac{\sinh(a+b\,x)^2-1}{b\sinh(a+b\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a + b*x)^3/sinh(a + b*x)^2,x)
```

```
[Out] (sinh(a + b*x)^2 - 1)/(b*sinh(a + b*x))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \cosh^3(a+bx)\operatorname{csch}^2(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)**3*csch(b*x+a)**2,x)
```

```
[Out] Integral(cosh(a + b*x)**3*csch(a + b*x)**2, x)
```

3.443 $\int \frac{\cosh(a+bx)\coth^2(a+bx)}{x}\,dx$

Optimal. Leaf size=34

$$\text{Int}\left(\frac{\coth(a+bx)\text{csch}(a+bx)}{x},x\right)+\cosh(a)\text{Chi}(bx)+\sinh(a)\text{Shi}(bx)$$

```
[Out] CannotIntegrate(coth(b*x+a)*csch(b*x+a)/x,x)+Chi(b*x)*cosh(a)+Shi(b*x)*sinh
(a)
```

Rubi [A] time = 0.10, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\cosh(a+bx)\coth^2(a+bx)}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Cosh[a + b*x]*Coth[a + b*x]^2)/x,x]
```

```
[Out] Cosh[a]*CoshIntegral[b*x] + Sinh[a]*SinhIntegral[b*x] + Defer[Int][(Coth[a
+ b*x]*Csch[a + b*x])/x, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\cosh(a+bx)\coth^2(a+bx)}{x}\,dx &= \int \frac{\cosh(a+bx)}{x}\,dx+\int \frac{\coth(a+bx)\text{csch}(a+bx)}{x}\,dx\\ &= \cosh(a)\int \frac{\cosh(bx)}{x}\,dx+\sinh(a)\int \frac{\sinh(bx)}{x}\,dx+\int \frac{\coth(a+bx)\text{csch}(a+bx)}{x}\\ &= \cosh(a)\text{Chi}(bx)+\sinh(a)\text{Shi}(bx)+\int \frac{\coth(a+bx)\text{csch}(a+bx)}{x}\,dx\end{aligned}$$

Mathematica [A] time = 24.25, size = 0, normalized size = 0.00

$$\int \frac{\cosh(a+bx)\coth^2(a+bx)}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Cosh[a + b*x]*Coth[a + b*x]^2)/x,x]
```

```
[Out] Integrate[(Cosh[a + b*x]*Coth[a + b*x]^2)/x, x]
```

fricas [A] time = 0.40, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\cosh\left(bx+a\right)^{3}\operatorname{csch}\left(bx+a\right)^{2}}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^3*csch(b*x+a)^2/x,x, algorithm="fricas")
```

```
[Out] integral(cosh(b*x + a)^3*csch(b*x + a)^2/x, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh\left(bx+a\right)^{3}\operatorname{csch}\left(bx+a\right)^{2}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^3*csch(b*x+a)^2/x,x, algorithm="giac")
```

```
[Out] integrate(cosh(b*x + a)^3*csch(b*x + a)^2/x, x)
```

maple [A] time = 0.67, size = 0, normalized size = 0.00

$$\int \frac{\left(\cosh^{3}\left(bx+a\right)\right)\operatorname{csch}\left(bx+a\right)^{2}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)^3*csch(b*x+a)^2/x,x)
```

```
[Out] int(cosh(b*x+a)^3*csch(b*x+a)^2/x,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{2}\,\operatorname{Ei}\left(-bx\right)e^{\left(-a\right)}+\frac{1}{2}\,\operatorname{Ei}\left(bx\right)e^{a}-\frac{2\,e^{\left(bx+a\right)}}{bxe^{\left(2\,bx+2\,a\right)}-bx}-\int \frac{1}{bx^{2}e^{\left(bx+a\right)}+bx^{2}}\,dx-\int \frac{1}{bx^{2}e^{\left(bx+a\right)}-bx^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^3*csch(b*x+a)^2/x,x, algorithm="maxima")
```

```
[Out] 1/2*Ei(-b*x)*e^(-a) + 1/2*Ei(b*x)*e^a - 2*e^(b*x + a)/(b*x*e^(2*b*x + 2*a)
- b*x) - integrate(1/(b*x^2*e^(b*x + a) + b*x^2), x) - integrate(1/(b*x^2*e
^(b*x + a) - b*x^2), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\cosh(a+b\,x)^3}{x\sinh(a+b\,x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a + b*x)^3/(x*sinh(a + b*x)^2),x)
[Out] int(cosh(a + b*x)^3/(x*sinh(a + b*x)^2), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh^3(a+bx)\operatorname{csch}^2(a+bx)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)**3*csch(b*x+a)**2/x,x)
[Out] Integral(cosh(a + b*x)**3*csch(a + b*x)**2/x, x)
```

3.444 $\int \frac{\cosh(a+bx)\coth^2(a+bx)}{x^2}\,dx$

Optimal. Leaf size=47

$$\operatorname{Int}\left(\frac{\coth(a+bx)\operatorname{csch}(a+bx)}{x^2},x\right)+b\sinh(a)\operatorname{Chi}(bx)+b\cosh(a)\operatorname{Shi}(bx)-\frac{\cosh(a+bx)}{x}$$

```
[Out] CannotIntegrate(coth(b*x+a)*csch(b*x+a)/x^2,x)-cosh(b*x+a)/x+b*cosh(a)*Shi(
b*x)+b*Chi(b*x)*sinh(a)
```

Rubi [A] time = 0.14, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\cosh(a+bx)\coth^2(a+bx)}{x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Cosh[a + b*x]*Coth[a + b*x]^2)/x^2,x]
```

```
[Out] -(Cosh[a + b*x]/x) + b*CoshIntegral[b*x]*Sinh[a] + b*Cosh[a]*SinhIntegral[b
*x] + Defer[Int][(Coth[a + b*x]*Csch[a + b*x])/x^2, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cosh(a+bx)\coth^2(a+bx)}{x^2}\,dx &= \int \frac{\cosh(a+bx)}{x^2}\,dx+\int \frac{\coth(a+bx)\operatorname{csch}(a+bx)}{x^2}\,dx\\
&= -\frac{\cosh(a+bx)}{x}+b\int \frac{\sinh(a+bx)}{x}\,dx+\int \frac{\coth(a+bx)\operatorname{csch}(a+bx)}{x^2}\,dx\\
&= -\frac{\cosh(a+bx)}{x}+(b\cosh(a))\int \frac{\sinh(bx)}{x}\,dx+(b\sinh(a))\int \frac{\cosh(bx)}{x}\,dx+\\
&= -\frac{\cosh(a+bx)}{x}+b\operatorname{Chi}(bx)\sinh(a)+b\cosh(a)\operatorname{Shi}(bx)+\int \frac{\coth(a+bx)\operatorname{csc}}{x^2}
\end{aligned}$$

Mathematica [A] time = 21.25, size = 0, normalized size = 0.00

$$\int \frac{\cosh(a+bx)\coth^2(a+bx)}{x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Cosh[a + b*x]*Coth[a + b*x]^2)/x^2,x]
```

```
[Out] Integrate[(Cosh[a + b*x]*Coth[a + b*x]^2)/x^2, x]
```

fricas [A] time = 0.39, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\cosh\left(bx+a\right)^{3}\operatorname{csch}\left(bx+a\right)^{2}}{x^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^3*csch(b*x+a)^2/x^2,x, algorithm="fricas")
```

```
[Out] integral(cosh(b*x + a)^3*csch(b*x + a)^2/x^2, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh\left(bx+a\right)^{3}\operatorname{csch}\left(bx+a\right)^{2}}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^3*csch(b*x+a)^2/x^2,x, algorithm="giac")
```

```
[Out] integrate(cosh(b*x + a)^3*csch(b*x + a)^2/x^2, x)
```

maple [A] time = 0.66, size = 0, normalized size = 0.00

$$\int \frac{\left(\cosh^{3}\left(bx+a\right)\right)\operatorname{csch}\left(bx+a\right)^{2}}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)^3*csch(b*x+a)^2/x^2,x)
```

```
[Out] int(cosh(b*x+a)^3*csch(b*x+a)^2/x^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{2}\,be^{(-a)}\Gamma\left(-1,bx\right)+\frac{1}{2}\,be^{a}\Gamma\left(-1,-bx\right)-\frac{2\,e^{(bx+a)}}{bx^{2}e^{(2\,bx+2\,a)}-bx^{2}}-2\int\frac{1}{bx^{3}e^{(bx+a)}+bx^{3}}\,dx-2\int\frac{1}{bx^{3}e^{(bx+a)}-bx^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^3*csch(b*x+a)^2/x^2,x, algorithm="maxima")
```

```
[Out] -1/2*b*e^(-a)*gamma(-1, b*x) + 1/2*b*e^a*gamma(-1, -b*x) - 2*e^(b*x + a)/(b
*x^2*e^(2*b*x + 2*a) - b*x^2) - 2*integrate(1/(b*x^3*e^(b*x + a) + b*x^3),
x) - 2*integrate(1/(b*x^3*e^(b*x + a) - b*x^3), x)
```

mupad [A]   time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\cosh\left(a+b\,x\right)^3}{x^2\,\sinh\left(a+b\,x\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a + b*x)^3/(x^2*sinh(a + b*x)^2),x)
```

```
[Out] int(cosh(a + b*x)^3/(x^2*sinh(a + b*x)^2), x)
```

sympy [A]   time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh^3\left(a+bx\right)\operatorname{csch}^2\left(a+bx\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)**3*csch(b*x+a)**2/x**2,x)
```

```
[Out] Integral(cosh(a + b*x)**3*csch(a + b*x)**2/x**2, x)
```

3.445 $\int x^m \coth(a+bx)\mathbf{csch}^2(a+bx)\,dx$

Optimal. Leaf size=21

$$\text{Int}\left(x^m \coth(a+bx)\text{csch}^2(a+bx),x\right)$$

```
[Out] CannotIntegrate(x^m*coth(b*x+a)*csch(b*x+a)^2,x)
```

Rubi [A] time = 0.47, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int x^m \coth(a+bx)\text{csch}^2(a+bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x^m*Coth[a + b*x]*Csch[a + b*x]^2,x]
[Out] Defer[Int][x^m*Coth[a + b*x]*Csch[a + b*x]^2, x]
```

Rubi steps

$$\int x^m \coth(a+bx)\text{csch}^2(a+bx)\,dx=\int x^m \coth(a+bx)\text{csch}^2(a+bx)\,dx$$

Mathematica [A] time = 36.78, size = 0, normalized size = 0.00

$$\int x^m \coth(a+bx)\text{csch}^2(a+bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^m*Coth[a + b*x]*Csch[a + b*x]^2,x]
[Out] Integrate[x^m*Coth[a + b*x]*Csch[a + b*x]^2, x]
```

fricas [A] time = 0.41, size = 0, normalized size = 0.00

$$\text{integral}\left(x^m \cosh(bx+a)\,\text{csch}(bx+a)^3\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*cosh(b*x+a)*csch(b*x+a)^3,x, algorithm="fricas")
[Out] integral(x^m*cosh(b*x + a)*csch(b*x + a)^3, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int x^m \cosh(bx+a)\operatorname{csch}(bx+a)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*cosh(b*x+a)*csch(b*x+a)^3,x, algorithm="giac")
[Out] integrate(x^m*cosh(b*x + a)*csch(b*x + a)^3, x)
```

maple [A] time = 0.14, size = 0, normalized size = 0.00

$$\int x^m \cosh(bx+a)\operatorname{csch}(bx+a)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*cosh(b*x+a)*csch(b*x+a)^3,x)
[Out] int(x^m*cosh(b*x+a)*csch(b*x+a)^3,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int x^m \cosh(bx+a)\operatorname{csch}(bx+a)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*cosh(b*x+a)*csch(b*x+a)^3,x, algorithm="maxima")
[Out] integrate(x^m*cosh(b*x + a)*csch(b*x + a)^3, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{x^m \cosh(a+b\,x)}{\sinh(a+b\,x)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^m*cosh(a + b*x))/sinh(a + b*x)^3,x)
[Out] int((x^m*cosh(a + b*x))/sinh(a + b*x)^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*cosh(b*x+a)*csch(b*x+a)**3,x)
[Out] Timed out
```

3.446 $\int x^3 \coth(a+bx)\operatorname{csch}^2(a+bx)\,dx$

Optimal. Leaf size=83

$$\frac{3\text{Li}_2\left(e^{2(a+bx)}\right)}{2b^4}+\frac{3x\log\left(1-e^{2(a+bx)}\right)}{b^3}-\frac{3x^2\coth(a+bx)}{2b^2}-\frac{x^3\text{csch}^2(a+bx)}{2b}-\frac{3x^2}{2b^2}$$

```
[Out] -3/2*x^2/b^2-3/2*x^2*coth(b*x+a)/b^2-1/2*x^3*csch(b*x+a)^2/b+3*x*ln(1-exp(2
*b*x+2*a))/b^3+3/2*polylog(2,exp(2*b*x+2*a))/b^4
```

Rubi [A] time = 0.16, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5419, 4184, 3716, 2190, 2279, 2391}

$$\frac{3\text{PolyLog}\left(2,e^{2(a+bx)}\right)}{2b^4}-\frac{3x^2\coth(a+bx)}{2b^2}+\frac{3x\log\left(1-e^{2(a+bx)}\right)}{b^3}-\frac{x^3\text{csch}^2(a+bx)}{2b}-\frac{3x^2}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Coth[a + b*x]*Csch[a + b*x]^2,x]
```

```
[Out] (-3*x^2)/(2*b^2) - (3*x^2*Coth[a + b*x])/(2*b^2) - (x^3*Csch[a + b*x]^2)/(2
*b) + (3*x*Log[1 - E^(2*(a + b*x))])/b^3 + (3*PolyLog[2, E^(2*(a + b*x))])/
(2*b^4)
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3716

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2
*I, Int[((c + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(E^(2*I*k*Pi)*(1 + E^(2*(-(I*
e) + f*fz*x))/E^(2*I*k*Pi))), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Integ
erQ[4*k] && IGtQ[m, 0]
```

Rule 4184

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> -Sim
p[((c + d*x)^m*Cot[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Co
t[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 5419

```
Int[Coth[(a_.) + (b_.)*(x_)^(n_.)]^(q_.)*Csch[(a_.) + (b_.)*(x_)^(n_.)]^(p_
.)*(x_)^(m_.), x_Symbol] :> -Simp[(x^(m - n + 1)*Csch[a + b*x^n]^p)/(b*n*p)
, x] + Dist[(m - n + 1)/(b*n*p), Int[x^(m - n)*Csch[a + b*x^n]^p, x], x] /;
 FreeQ[{a, b, p}, x] && RationalQ[m] && IntegerQ[n] && GeQ[m - n, 0] && EqQ
[q, 1]
```

Rubi steps

$$\begin{aligned}
\int x^3 \coth(a+bx)\operatorname{csch}^2(a+bx)\,dx &= -\frac{x^3\operatorname{csch}^2(a+bx)}{2b}+\frac{3\int x^2\operatorname{csch}^2(a+bx)\,dx}{2b}\\
&= -\frac{3x^2\coth(a+bx)}{2b^2}-\frac{x^3\operatorname{csch}^2(a+bx)}{2b}+\frac{3\int x\coth(a+bx)\,dx}{b^2}\\
&= -\frac{3x^2}{2b^2}-\frac{3x^2\coth(a+bx)}{2b^2}-\frac{x^3\operatorname{csch}^2(a+bx)}{2b}-\frac{6\int\frac{e^{2(a+bx)}x}{1-e^{2(a+bx)}}\,dx}{b^2}\\
&= -\frac{3x^2}{2b^2}-\frac{3x^2\coth(a+bx)}{2b^2}-\frac{x^3\operatorname{csch}^2(a+bx)}{2b}+\frac{3x\log\left(1-e^{2(a+bx)}\right)}{b^3}-\frac{3\int 1
\end{aligned}$$

$$\begin{aligned}
&= -\frac{3x^2}{2b^2}-\frac{3x^2\coth(a+bx)}{2b^2}-\frac{x^3\operatorname{csch}^2(a+bx)}{2b}+\frac{3x\log\left(1-e^{2(a+bx)}\right)}{b^3}-\frac{3\,\mathrm{Su}
\end{aligned}$$

$$\begin{aligned}
&= -\frac{3x^2}{2b^2}-\frac{3x^2\coth(a+bx)}{2b^2}-\frac{x^3\operatorname{csch}^2(a+bx)}{2b}+\frac{3x\log\left(1-e^{2(a+bx)}\right)}{b^3}+\frac{3\mathrm{Li}_2
\end{aligned}$$

Mathematica [C] time = 6.14, size = 227, normalized size = 2.73

$$\frac{3x^2\operatorname{csch}(a)\sinh(bx)\operatorname{csch}(a+bx)}{2b^2}-\frac{3\operatorname{csch}(a)\operatorname{sech}(a)\left(b^2x^2e^{-\tanh^{-1}(\tanh(a))}-\frac{i\tanh(a)\left(i\operatorname{Li}_2\left(e^{2i\left(ibx+i\tanh^{-1}(\tanh(a))\right)}\right)-bx\left(-\pi+\right.\right.}{}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^3*Coth[a + b*x]*Csch[a + b*x]^2,x]
```

```
[Out] -1/2*(x^3*Csch[a + b*x]^2)/b + (3*x^2*Csch[a]*Csch[a + b*x]*Sinh[b*x])/(2*b
^2) - (3*Csch[a]*Sech[a]*((b^2*x^2)/E^ArcTanh[Tanh[a]] - (I*(-(b*x*(-Pi + (
2*I)*ArcTanh[Tanh[a]])) - Pi*Log[1 + E^(2*b*x)] - 2*(I*b*x + I*ArcTanh[Tanh
[a]])*Log[1 - E^((2*I)*(I*b*x + I*ArcTanh[Tanh[a]]))] + Pi*Log[Cosh[b*x]] +
 (2*I)*ArcTanh[Tanh[a]]*Log[I*Sinh[b*x + ArcTanh[Tanh[a]]]] + I*PolyLog[2,
E^((2*I)*(I*b*x + I*ArcTanh[Tanh[a]]))])*Tanh[a])/Sqrt[1 - Tanh[a]^2]))/(2*
b^4*Sqrt[Sech[a]^2*(Cosh[a]^2 - Sinh[a]^2)])
```

fricas [B] time = 0.43, size = 979, normalized size = 11.80

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*cosh(b*x+a)*csch(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -(3*(b^2*x^2 - a^2)*cosh(b*x + a)^4 + 12*(b^2*x^2 - a^2)*cosh(b*x + a)*sinh
(b*x + a)^3 + 3*(b^2*x^2 - a^2)*sinh(b*x + a)^4 + (2*b^3*x^3 - 3*b^2*x^2 +
6*a^2)*cosh(b*x + a)^2 + (2*b^3*x^3 - 3*b^2*x^2 + 18*(b^2*x^2 - a^2)*cosh(b
*x + a)^2 + 6*a^2)*sinh(b*x + a)^2 - 3*a^2 - 3*(cosh(b*x + a)^4 + 4*cosh(b*
x + a)*sinh(b*x + a)^3 + sinh(b*x + a)^4 + 2*(3*cosh(b*x + a)^2 - 1)*sinh(b
*x + a)^2 - 2*cosh(b*x + a)^2 + 4*(cosh(b*x + a)^3 - cosh(b*x + a))*sinh(b*
x + a) + 1)*dilog(cosh(b*x + a) + sinh(b*x + a)) - 3*(cosh(b*x + a)^4 + 4*c
osh(b*x + a)*sinh(b*x + a)^3 + sinh(b*x + a)^4 + 2*(3*cosh(b*x + a)^2 - 1)*
sinh(b*x + a)^2 - 2*cosh(b*x + a)^2 + 4*(cosh(b*x + a)^3 - cosh(b*x + a))*s
inh(b*x + a) + 1)*dilog(-cosh(b*x + a) - sinh(b*x + a)) - 3*(b*x*cosh(b*x +
 a)^4 + 4*b*x*cosh(b*x + a)*sinh(b*x + a)^3 + b*x*sinh(b*x + a)^4 - 2*b*x*c
osh(b*x + a)^2 + 2*(3*b*x*cosh(b*x + a)^2 - b*x)*sinh(b*x + a)^2 + b*x + 4*
(b*x*cosh(b*x + a)^3 - b*x*cosh(b*x + a))*sinh(b*x + a))*log(cosh(b*x + a)
+ sinh(b*x + a) + 1) + 3*(a*cosh(b*x + a)^4 + 4*a*cosh(b*x + a)*sinh(b*x +
a)^3 + a*sinh(b*x + a)^4 - 2*a*cosh(b*x + a)^2 + 2*(3*a*cosh(b*x + a)^2 - a
)*sinh(b*x + a)^2 + 4*(a*cosh(b*x + a)^3 - a*cosh(b*x + a))*sinh(b*x + a) +
 a)*log(cosh(b*x + a) + sinh(b*x + a) - 1) - 3*((b*x + a)*cosh(b*x + a)^4 +
 4*(b*x + a)*cosh(b*x + a)*sinh(b*x + a)^3 + (b*x + a)*sinh(b*x + a)^4 - 2*
```

```
(b*x + a)*cosh(b*x + a)^2 + 2*(3*(b*x + a)*cosh(b*x + a)^2 - b*x - a)*sinh(
b*x + a)^2 + b*x + 4*((b*x + a)*cosh(b*x + a)^3 - (b*x + a)*cosh(b*x + a))*
sinh(b*x + a) + a)*log(-cosh(b*x + a) - sinh(b*x + a) + 1) + 2*(6*(b^2*x^2
- a^2)*cosh(b*x + a)^3 + (2*b^3*x^3 - 3*b^2*x^2 + 6*a^2)*cosh(b*x + a))*sin
h(b*x + a))/(b^4*cosh(b*x + a)^4 + 4*b^4*cosh(b*x + a)*sinh(b*x + a)^3 + b^
4*sinh(b*x + a)^4 - 2*b^4*cosh(b*x + a)^2 + b^4 + 2*(3*b^4*cosh(b*x + a)^2
- b^4)*sinh(b*x + a)^2 + 4*(b^4*cosh(b*x + a)^3 - b^4*cosh(b*x + a))*sinh(b
*x + a))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3 \cosh(bx+a)\operatorname{csch}(bx+a)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*cosh(b*x+a)*csch(b*x+a)^3,x, algorithm="giac")
```

```
[Out] integrate(x^3*cosh(b*x + a)*csch(b*x + a)^3, x)
```

maple [B] time = 0.26, size = 177, normalized size = 2.13

$$-\frac{x^2\left(2bx\,e^{2bx+2a}+3\,e^{2bx+2a}-3\right)}{b^2\left(e^{2bx+2a}-1\right)^2}-\frac{3x^2}{b^2}-\frac{6ax}{b^3}-\frac{3a^2}{b^4}+\frac{3\ln\left(1-e^{bx+a}\right)x}{b^3}+\frac{3\ln\left(1-e^{bx+a}\right)a}{b^4}+\frac{3\,\mathrm{polylog}\left(2,e^{bx+a}\right)}{b^4}+\frac{3}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*cosh(b*x+a)*csch(b*x+a)^3,x)
```

```
[Out] -x^2*(2*b*x*exp(2*b*x+2*a)+3*exp(2*b*x+2*a)-3)/b^2/(exp(2*b*x+2*a)-1)^2-3*x
^2/b^2-6*a*x/b^3-3/b^4*a^2+3/b^3*ln(1-exp(b*x+a))*x+3/b^4*ln(1-exp(b*x+a))*
a+3*polylog(2,exp(b*x+a))/b^4+3/b^3*ln(1+exp(b*x+a))*x+3*polylog(2,-exp(b*x
+a))/b^4+6/b^4*a*ln(exp(b*x+a))-3/b^4*a*ln(exp(b*x+a)-1)
```

maxima [A] time = 0.48, size = 130, normalized size = 1.57

$$\frac{3\,x^2-\left(2\,bx^3e^{(2\,a)}+3\,x^2e^{(2\,a)}\right)e^{(2\,bx)}}{b^2e^{(4\,bx+4\,a)}-2\,b^2e^{(2\,bx+2\,a)}+b^2}-\frac{3\,x^2}{b^2}+\frac{3\left(bx\log\left(e^{(bx+a)}+1\right)+\mathrm{Li}_2\left(-e^{(bx+a)}\right)\right)}{b^4}+\frac{3\left(bx\log\left(-e^{(bx+a)}+1\right)+\mathrm{Li}_2\right.}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*cosh(b*x+a)*csch(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] (3*x^2 - (2*b*x^3*e^(2*a) + 3*x^2*e^(2*a))*e^(2*b*x))/(b^2*e^(4*b*x + 4*a)
- 2*b^2*e^(2*b*x + 2*a) + b^2) - 3*x^2/b^2 + 3*(b*x*log(e^(b*x + a) + 1) +
dilog(-e^(b*x + a)))/b^4 + 3*(b*x*log(-e^(b*x + a) + 1) + dilog(e^(b*x + a)
))/b^4
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3 \cosh(a + b x)}{\sinh(a + b x)^3} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*cosh(a + b*x))/sinh(a + b*x)^3,x)
[Out] int((x^3*cosh(a + b*x))/sinh(a + b*x)^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*cosh(b*x+a)*csch(b*x+a)**3,x)
[Out] Timed out
```

3.447 $\int x^2 \coth(a+bx)\operatorname{csch}^2(a+bx)\,dx$

Optimal. Leaf size=42

$$\frac{\log(\sinh(a+bx))}{b^3}-\frac{x\coth(a+bx)}{b^2}-\frac{x^2\operatorname{csch}^2(a+bx)}{2b}$$

```
[Out] -x*coth(b*x+a)/b^2-1/2*x^2*csch(b*x+a)^2/b+ln(sinh(b*x+a))/b^3
```

Rubi [A] time = 0.06, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {5419, 4184, 3475}

$$-\frac{x\coth(a+bx)}{b^2}+\frac{\log(\sinh(a+bx))}{b^3}-\frac{x^2\operatorname{csch}^2(a+bx)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Coth[a + b*x]*Csch[a + b*x]^2,x]
```

```
[Out] -((x*Coth[a + b*x])/b^2) - (x^2*Csch[a + b*x]^2)/(2*b) + Log[Sinh[a + b*x]]
/b^3
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 4184

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> -Sim
p[((c + d*x)^m*Cot[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Co
t[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 5419

```
Int[Coth[(a_.) + (b_.)*(x_)^(n_.)]^(q_.)*Csch[(a_.) + (b_.)*(x_)^(n_.)]^(p_
.)*(x_)^(m_.), x_Symbol] :> -Simp[(x^(m - n + 1)*Csch[a + b*x^n]^p)/(b*n*p)
, x] + Dist[(m - n + 1)/(b*n*p), Int[x^(m - n)*Csch[a + b*x^n]^p, x], x] /;
 FreeQ[{a, b, p}, x] && RationalQ[m] && IntegerQ[n] && GeQ[m - n, 0] && EqQ
[q, 1]
```

Rubi steps

$$\int x^2 \coth(a+bx)\operatorname{csch}^2(a+bx)\,dx = -\frac{x^2\operatorname{csch}^2(a+bx)}{2b} + \frac{\int x\operatorname{csch}^2(a+bx)\,dx}{b}$$
$$= -\frac{x\coth(a+bx)}{b^2} - \frac{x^2\operatorname{csch}^2(a+bx)}{2b} + \frac{\int \coth(a+bx)\,dx}{b^2}$$
$$= -\frac{x\coth(a+bx)}{b^2} - \frac{x^2\operatorname{csch}^2(a+bx)}{2b} + \frac{\log(\sinh(a+bx))}{b^3}$$

Mathematica [A] time = 0.12, size = 55, normalized size = 1.31

$$\frac{\log(\sinh(a+bx))}{b^3} - \frac{x\coth(a)}{b^2} + \frac{x\operatorname{csch}(a)\sinh(bx)\operatorname{csch}(a+bx)}{b^2} - \frac{x^2\operatorname{csch}^2(a+bx)}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Coth[a + b*x]*Csch[a + b*x]^2,x]
```

```
[Out] -((x*Coth[a])/b^2) - (x^2*Csch[a + b*x]^2)/(2*b) + Log[Sinh[a + b*x]]/b^3 +
 (x*Csch[a]*Csch[a + b*x]*Sinh[b*x])/b^2
```

fricas [B] time = 0.41, size = 383, normalized size = 9.12

$$-\frac{2\,bx\cosh(bx+a)^4 + 8\,bx\cosh(bx+a)\sinh(bx+a)^3 + 2\,bx\sinh(bx+a)^4 + 2\left(b^2x^2 - bx\right)\cosh(bx+a)^2 + 2}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cosh(b*x+a)*csch(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -(2*b*x*cosh(b*x + a)^4 + 8*b*x*cosh(b*x + a)*sinh(b*x + a)^3 + 2*b*x*sinh(
b*x + a)^4 + 2*(b^2*x^2 - b*x)*cosh(b*x + a)^2 + 2*(b^2*x^2 + 6*b*x*cosh(b*
x + a)^2 - b*x)*sinh(b*x + a)^2 - (cosh(b*x + a)^4 + 4*cosh(b*x + a)*sinh(b
*x + a)^3 + sinh(b*x + a)^4 + 2*(3*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^2 - 2
*cosh(b*x + a)^2 + 4*(cosh(b*x + a)^3 - cosh(b*x + a))*sinh(b*x + a) + 1)*l
og(2*sinh(b*x + a)/(cosh(b*x + a) - sinh(b*x + a))) + 4*(2*b*x*cosh(b*x + a
)^3 + (b^2*x^2 - b*x)*cosh(b*x + a))*sinh(b*x + a))/(b^3*cosh(b*x + a)^4 +
4*b^3*cosh(b*x + a)*sinh(b*x + a)^3 + b^3*sinh(b*x + a)^4 - 2*b^3*cosh(b*x
+ a)^2 + b^3 + 2*(3*b^3*cosh(b*x + a)^2 - b^3)*sinh(b*x + a)^2 + 4*(b^3*cos
h(b*x + a)^3 - b^3*cosh(b*x + a))*sinh(b*x + a))
```

giac [B] time = 0.13, size = 139, normalized size = 3.31

$$-\frac{2\,b^2x^2e^{(2\,bx+2\,a)} + 2\,bxe^{(4\,bx+4\,a)} - 2\,bxe^{(2\,bx+2\,a)} - e^{(4\,bx+4\,a)}\log\left(e^{(2\,bx+2\,a)} - 1\right) + 2\,e^{(2\,bx+2\,a)}\log\left(e^{(2\,bx+2\,a)} - 1\right) - 1}{b^3e^{(4\,bx+4\,a)} - 2\,b^3e^{(2\,bx+2\,a)} + b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cosh(b*x+a)*csch(b*x+a)^3,x, algorithm="giac")
```

```
[Out] -(2*b^2*x^2*e^(2*b*x + 2*a) + 2*b*x*e^(4*b*x + 4*a) - 2*b*x*e^(2*b*x + 2*a)
 - e^(4*b*x + 4*a)*log(e^(2*b*x + 2*a) - 1) + 2*e^(2*b*x + 2*a)*log(e^(2*b*
x + 2*a) - 1) - log(e^(2*b*x + 2*a) - 1))/(b^3*e^(4*b*x + 4*a) - 2*b^3*e^(2
*b*x + 2*a) + b^3)
```

maple [A] time = 0.19, size = 72, normalized size = 1.71

$$-\frac{2x}{b^2}-\frac{2a}{b^3}-\frac{2x\left(bx\,\mathrm{e}^{2bx+2a}+\mathrm{e}^{2bx+2a}-1\right)}{b^2\left(\mathrm{e}^{2bx+2a}-1\right)^2}+\frac{\ln\left(\mathrm{e}^{2bx+2a}-1\right)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*cosh(b*x+a)*csch(b*x+a)^3,x)
```

```
[Out] -2*x/b^2-2/b^3*a-2*x*(b*x*exp(2*b*x+2*a)+exp(2*b*x+2*a)-1)/b^2/(exp(2*b*x+2
*a)-1)^2+1/b^3*ln(exp(2*b*x+2*a)-1)
```

maxima [B] time = 0.60, size = 107, normalized size = 2.55

$$-\frac{2\left(\left(bx^2e^{(2a)}-xe^{(2a)}\right)e^{(2bx)}+xe^{(4bx+4a)}\right)}{b^2e^{(4bx+4a)}-2b^2e^{(2bx+2a)}+b^2}+\frac{\log\left(\left(e^{(bx+a)}+1\right)e^{(-a)}\right)}{b^3}+\frac{\log\left(\left(e^{(bx+a)}-1\right)e^{(-a)}\right)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cosh(b*x+a)*csch(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -2*((b*x^2*e^(2*a) - x*e^(2*a))*e^(2*b*x) + x*e^(4*b*x + 4*a))/(b^2*e^(4*b*
x + 4*a) - 2*b^2*e^(2*b*x + 2*a) + b^2) + log((e^(b*x + a) + 1)*e^(-a))/b^3
 + log((e^(b*x + a) - 1)*e^(-a))/b^3
```

mupad [B] time = 1.47, size = 101, normalized size = 2.40

$$\frac{\ln\left(\mathrm{e}^{2a}\,\mathrm{e}^{2bx}-1\right)}{b^3}-\frac{\frac{x^2}{b}+\frac{x^2\,\mathrm{e}^{2a+2bx}}{b}}{\mathrm{e}^{4a+4bx}-2\,\mathrm{e}^{2a+2bx}+1}-\frac{2x}{b^2}-\frac{b\,x^2+2x}{b^2\,\left(\mathrm{e}^{2a+2bx}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*cosh(a + b*x))/sinh(a + b*x)^3,x)
```

```
[Out] log(exp(2*a)*exp(2*b*x) - 1)/b^3 - (x^2/b + (x^2*exp(2*a + 2*b*x))/b)/(exp(
4*a + 4*b*x) - 2*exp(2*a + 2*b*x) + 1) - (2*x)/b^2 - (2*x + b*x^2)/(b^2*(ex
p(2*a + 2*b*x) - 1))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*cosh(b*x+a)*csch(b*x+a)**3,x)
```

```
[Out] Timed out
```

3.448 $\int x \coth(a+bx)\mathbf{csch}^2(a+bx)\,dx$

Optimal. Leaf size=30

$$-\frac{\coth(a+bx)}{2b^2}-\frac{x\text{csch}^2(a+bx)}{2b}$$

```
[Out] -1/2*coth(b*x+a)/b^2-1/2*x*csch(b*x+a)^2/b
```

Rubi [A] time = 0.03, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {5419, 3767, 8}

$$-\frac{\coth(a+bx)}{2b^2}-\frac{x\text{csch}^2(a+bx)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[x*Coth[a + b*x]*Csch[a + b*x]^2,x]
```

```
[Out] -Coth[a + b*x]/(2*b^2) - (x*Csch[a + b*x]^2)/(2*b)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 5419

```
Int[Coth[(a_.) + (b_.)*(x_)^(n_.)]^(q_.)*Csch[(a_.) + (b_.)*(x_)^(n_.)]^(p_
.)*(x_)^(m_.), x_Symbol] :> -Simp[(x^(m - n + 1)*Csch[a + b*x^n]^p)/(b*n*p)
, x] + Dist[(m - n + 1)/(b*n*p), Int[x^(m - n)*Csch[a + b*x^n]^p, x], x] /;
 FreeQ[{a, b, p}, x] && RationalQ[m] && IntegerQ[n] && GeQ[m - n, 0] && EqQ
[q, 1]
```

Rubi steps

$$\int x\coth(a+bx)\operatorname{csch}^2(a+bx)\,dx = -\frac{x\operatorname{csch}^2(a+bx)}{2b}+\frac{\int \operatorname{csch}^2(a+bx)\,dx}{2b}$$
$$= -\frac{x\operatorname{csch}^2(a+bx)}{2b}-\frac{i\,\mathrm{Subst}(\int 1\,dx,x,-i\coth(a+bx))}{2b^2}$$
$$= -\frac{\coth(a+bx)}{2b^2}-\frac{x\operatorname{csch}^2(a+bx)}{2b}$$

Mathematica [A] time = 0.07, size = 30, normalized size = 1.00

$$-\frac{\coth(a+bx)}{2b^2}-\frac{x\operatorname{csch}^2(a+bx)}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Coth[a + b*x]*Csch[a + b*x]^2,x]
```

```
[Out] -1/2*Coth[a + b*x]/b^2 - (x*Csch[a + b*x]^2)/(2*b)
```

fricas [B] time = 0.39, size = 107, normalized size = 3.57

$$-\frac{2\,(bx\cosh(bx+a)+(bx+1)\sinh(bx+a))}{b^2\cosh(bx+a)^3+3\,b^2\cosh(bx+a)\sinh(bx+a)^2+b^2\sinh(bx+a)^3-b^2\cosh(bx+a)+3\left(b^2\cosh(bx+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x+a)*csch(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -2*(b*x*cosh(b*x + a) + (b*x + 1)*sinh(b*x + a))/(b^2*cosh(b*x + a)^3 + 3*b
^2*cosh(b*x + a)*sinh(b*x + a)^2 + b^2*sinh(b*x + a)^3 - b^2*cosh(b*x + a)
+ 3*(b^2*cosh(b*x + a)^2 - b^2)*sinh(b*x + a))
```

giac [B] time = 0.12, size = 184, normalized size = 6.13

$$-\frac{4\,bxe^{(2\,bx+2\,a)}-e^{(4\,bx+4\,a)}\log\left(e^{(2\,bx+2\,a)}-1\right)+2\,e^{(2\,bx+2\,a)}\log\left(e^{(2\,bx+2\,a)}-1\right)+e^{(4\,bx+4\,a)}\log\left(-e^{(2\,bx+2\,a)}+1\right)-2}{2\left(b^2e^{(4\,bx+4\,a)}-2\,b^2e^{(2\,bx+2\,a}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x+a)*csch(b*x+a)^3,x, algorithm="giac")
```

```
[Out] -1/2*(4*b*x*e^(2*b*x + 2*a) - e^(4*b*x + 4*a)*log(e^(2*b*x + 2*a) - 1) + 2*
e^(2*b*x + 2*a)*log(e^(2*b*x + 2*a) - 1) + e^(4*b*x + 4*a)*log(-e^(2*b*x +
```

```
2*a) + 1) - 2*e^(2*b*x + 2*a)*log(-e^(2*b*x + 2*a) + 1) + 2*e^(2*b*x + 2*a)
 - log(e^(2*b*x + 2*a) - 1) + log(-e^(2*b*x + 2*a) + 1) - 2)/(b^2*e^(4*b*x
+ 4*a) - 2*b^2*e^(2*b*x + 2*a) + b^2)
```

maple [A] time = 0.17, size = 43, normalized size = 1.43

$$-\frac{2bx\,\mathrm{e}^{2bx+2a}+\mathrm{e}^{2bx+2a}-1}{b^{2}\left(\mathrm{e}^{2bx+2a}-1\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cosh(b*x+a)*csch(b*x+a)^3,x)
```

```
[Out] -(2*b*x*exp(2*b*x+2*a)+exp(2*b*x+2*a)-1)/b^2/(exp(2*b*x+2*a)-1)^2
```

maxima [B] time = 0.39, size = 130, normalized size = 4.33

$$\frac{2\,bxe^{(4\,bx+4\,a)}-\left(4\,bxe^{(2\,a)}+e^{(2\,a)}\right)e^{(2\,bx)}+1}{2\left(b^{2}e^{(4\,bx+4\,a)}-2\,b^{2}e^{(2\,bx+2\,a)}+b^{2}\right)}-\frac{2\,bxe^{(4\,bx+4\,a)}+e^{(2\,bx+2\,a)}-1}{2\left(b^{2}e^{(4\,bx+4\,a)}-2\,b^{2}e^{(2\,bx+2\,a)}+b^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x+a)*csch(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/2*(2*b*x*e^(4*b*x + 4*a) - (4*b*x*e^(2*a) + e^(2*a))*e^(2*b*x) + 1)/(b^2*
e^(4*b*x + 4*a) - 2*b^2*e^(2*b*x + 2*a) + b^2) - 1/2*(2*b*x*e^(4*b*x + 4*a)
 + e^(2*b*x + 2*a) - 1)/(b^2*e^(4*b*x + 4*a) - 2*b^2*e^(2*b*x + 2*a) + b^2)
```

mupad [B] time = 1.45, size = 36, normalized size = 1.20

$$-\frac{\mathrm{e}^{2\,a+2\,b\,x}\,\left(2\,b\,x+1\right)-1}{b^2\,{\left(\mathrm{e}^{2\,a+2\,b\,x}-1\right)}^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*cosh(a + b*x))/sinh(a + b*x)^3,x)
```

```
[Out] -(exp(2*a + 2*b*x)*(2*b*x + 1) - 1)/(b^2*(exp(2*a + 2*b*x) - 1)^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\cosh\left(a+bx\right)\operatorname{csch}^{3}\left(a+bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x+a)*csch(b*x+a)**3,x)
```

```
[Out] Integral(x*cosh(a + b*x)*csch(a + b*x)**3, x)
```

3.449 $\int \coth(a+bx)\mathbf{csch}^2(a+bx)\,dx$

Optimal. Leaf size=15

$$-\frac{\text{csch}^2(a+bx)}{2b}$$

```
[Out] -1/2*csch(b*x+a)^2/b
```

Rubi [A] time = 0.02, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2606, 30}

$$-\frac{\text{csch}^2(a+bx)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[a + b*x]*Csch[a + b*x]^2,x]
```

```
[Out] -Csch[a + b*x]^2/(2*b)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rubi steps

$$\begin{aligned}\int \coth(a+bx)\text{csch}^2(a+bx)\,dx &= \frac{\text{Subst}\left(\int x\,dx, x, -i\text{csch}(a+bx)\right)}{b}\\ &= -\frac{\text{csch}^2(a+bx)}{2b}\end{aligned}$$

Mathematica [A] time = 0.01, size = 15, normalized size = 1.00

$$-\frac{\text{csch}^2(a+bx)}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[a + b*x]*Csch[a + b*x]^2,x]
```

```
[Out] -1/2*Csch[a + b*x]^2/b
```

fricas [B] time = 0.39, size = 86, normalized size = 5.73

$$-\frac{2\left(\cosh\left(bx+a\right)+\sinh\left(bx+a\right)\right)}{b\cosh\left(bx+a\right)^{3}+3\,b\cosh\left(bx+a\right)\sinh\left(bx+a\right)^{2}+b\sinh\left(bx+a\right)^{3}-b\cosh\left(bx+a\right)+3\left(b\cosh\left(bx+a\right)^{2}-b\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*csch(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -2*(cosh(b*x + a) + sinh(b*x + a))/(b*cosh(b*x + a)^3 + 3*b*cosh(b*x + a)*s
inh(b*x + a)^2 + b*sinh(b*x + a)^3 - b*cosh(b*x + a) + 3*(b*cosh(b*x + a)^2
 - b)*sinh(b*x + a))
```

giac [B] time = 0.15, size = 27, normalized size = 1.80

$$-\frac{2\,e^{\left(2\,bx+2\,a\right)}}{b{\left(e^{\left(2\,bx+2\,a\right)}-1\right)}^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*csch(b*x+a)^3,x, algorithm="giac")
```

```
[Out] -2*e^(2*b*x + 2*a)/(b*(e^(2*b*x + 2*a) - 1)^2)
```

maple [A] time = 0.07, size = 14, normalized size = 0.93

$$-\frac{\operatorname{csch}\left(bx+a\right)^{2}}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)*csch(b*x+a)^3,x)
```

```
[Out] -1/2*csch(b*x+a)^2/b
```

maxima [A] time = 0.43, size = 25, normalized size = 1.67

$$-\frac{2}{b{\left(e^{\left(bx+a\right)}-e^{\left(-bx-a\right)}\right)}^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*csch(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -2/(b*(e^(b*x + a) - e^(-b*x - a))^2)
```

mupad [B] time = 1.46, size = 13, normalized size = 0.87

$$-\frac{1}{2\,b\sinh\left(a+b\,x\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a + b*x)/sinh(a + b*x)^3,x)
```

```
[Out] -1/(2*b*sinh(a + b*x)^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \cosh\left(a+bx\right)\operatorname{csch}^3\left(a+bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*csch(b*x+a)**3,x)
```

```
[Out] Integral(cosh(a + b*x)*csch(a + b*x)**3, x)
```

3.450 $\int \frac{\coth(a+bx)\mathbf{csch}^2(a+bx)}{x}\,dx$

Optimal. Leaf size=21

$$\text{Int}\left(\frac{\coth(a+bx)\text{csch}^2(a+bx)}{x},x\right)$$

```
[Out] CannotIntegrate(coth(b*x+a)*csch(b*x+a)^2/x,x)
```

Rubi [A] time = 0.16, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\coth(a+bx)\text{csch}^2(a+bx)}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Coth[a + b*x]*Csch[a + b*x]^2)/x,x]
```

```
[Out] Defer[Int][(Coth[a + b*x]*Csch[a + b*x]^2)/x, x]
```

Rubi steps

$$\int \frac{\coth(a+bx)\text{csch}^2(a+bx)}{x}\,dx = \int \frac{\coth(a+bx)\text{csch}^2(a+bx)}{x}\,dx$$

Mathematica [A] time = 15.58, size = 0, normalized size = 0.00

$$\int \frac{\coth(a+bx)\text{csch}^2(a+bx)}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Coth[a + b*x]*Csch[a + b*x]^2)/x,x]
```

```
[Out] Integrate[(Coth[a + b*x]*Csch[a + b*x]^2)/x, x]
```

fricas [A] time = 0.41, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\cosh(bx+a)\,\text{csch}(bx+a)^3}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*csch(b*x+a)^3/x,x, algorithm="fricas")
```

```
[Out] integral(cosh(b*x + a)*csch(b*x + a)^3/x, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh\left(bx+a\right)\operatorname{csch}\left(bx+a\right)^{3}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*csch(b*x+a)^3/x,x, algorithm="giac")
```

```
[Out] integrate(cosh(b*x + a)*csch(b*x + a)^3/x, x)
```

maple [A] time = 0.43, size = 0, normalized size = 0.00

$$\int \frac{\cosh\left(bx+a\right)\operatorname{csch}\left(bx+a\right)^{3}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)*csch(b*x+a)^3/x,x)
```

```
[Out] int(cosh(b*x+a)*csch(b*x+a)^3/x,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\left(2\,bxe^{(2\,a)}-e^{(2\,a)}\right)e^{(2\,bx)}+1}{b^{2}x^{2}e^{(4\,bx+4\,a)}-2\,b^{2}x^{2}e^{(2\,bx+2\,a)}+b^{2}x^{2}}-4\int \frac{1}{4\left(b^{2}x^{3}e^{(bx+a)}+b^{2}x^{3}\right)}\,dx+4\int \frac{1}{4\left(b^{2}x^{3}e^{(bx+a)}-b^{2}x^{3}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*csch(b*x+a)^3/x,x, algorithm="maxima")
```

```
[Out] -((2*b*x*e^(2*a) - e^(2*a))*e^(2*b*x) + 1)/(b^2*x^2*e^(4*b*x + 4*a) - 2*b^2
*x^2*e^(2*b*x + 2*a) + b^2*x^2) - 4*integrate(1/4/(b^2*x^3*e^(b*x + a) + b^
2*x^3), x) + 4*integrate(1/4/(b^2*x^3*e^(b*x + a) - b^2*x^3), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{\cosh\left(a+b\,x\right)}{x\,{\sinh\left(a+b\,x\right)}^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a + b*x)/(x*sinh(a + b*x)^3),x)
```

```
[Out] int(cosh(a + b*x)/(x*sinh(a + b*x)^3), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh(a+bx)\operatorname{csch}^3(a+bx)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*csch(b*x+a)**3/x,x)
```

```
[Out] Integral(cosh(a + b*x)*csch(a + b*x)**3/x, x)
```

3.451 $\int \frac{\coth(a+bx)\operatorname{csch}^2(a+bx)}{x^2}\,dx$

Optimal. Leaf size=21

$$\operatorname{Int}\left(\frac{\coth(a+bx)\operatorname{csch}^2(a+bx)}{x^2},x\right)$$

```
[Out] CannotIntegrate(coth(b*x+a)*csch(b*x+a)^2/x^2,x)
```

Rubi [A] time = 0.24, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\coth(a+bx)\operatorname{csch}^2(a+bx)}{x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Coth[a + b*x]*Csch[a + b*x]^2)/x^2,x]
[Out] Defer[Int][(Coth[a + b*x]*Csch[a + b*x]^2)/x^2, x]
```

Rubi steps

$$\int \frac{\coth(a+bx)\operatorname{csch}^2(a+bx)}{x^2}\,dx=\int \frac{\coth(a+bx)\operatorname{csch}^2(a+bx)}{x^2}\,dx$$

Mathematica [A] time = 20.37, size = 0, normalized size = 0.00

$$\int \frac{\coth(a+bx)\operatorname{csch}^2(a+bx)}{x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Coth[a + b*x]*Csch[a + b*x]^2)/x^2,x]
[Out] Integrate[(Coth[a + b*x]*Csch[a + b*x]^2)/x^2, x]
```

fricas [A] time = 0.41, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{\cosh(bx+a)\operatorname{csch}(bx+a)^3}{x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*csch(b*x+a)^3/x^2,x, algorithm="fricas")
```

```
[Out] integral(cosh(b*x + a)*csch(b*x + a)^3/x^2, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh(bx+a)\operatorname{csch}(bx+a)^3}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*csch(b*x+a)^3/x^2,x, algorithm="giac")
```

```
[Out] integrate(cosh(b*x + a)*csch(b*x + a)^3/x^2, x)
```

maple [A] time = 0.43, size = 0, normalized size = 0.00

$$\int \frac{\cosh(bx+a)\operatorname{csch}(bx+a)^3}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)*csch(b*x+a)^3/x^2,x)
```

```
[Out] int(cosh(b*x+a)*csch(b*x+a)^3/x^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{2\left(\left(bxe^{(2a)}-e^{(2a)}\right)e^{(2bx)}+1\right)}{b^2x^3e^{(4bx+4a)}-2b^2x^3e^{(2bx+2a)}+b^2x^3}-12\int\frac{1}{4\left(b^2x^4e^{(bx+a)}+b^2x^4\right)}\,dx+12\int\frac{1}{4\left(b^2x^4e^{(bx+a)}-b^2x^4\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*csch(b*x+a)^3/x^2,x, algorithm="maxima")
```

```
[Out] -2*((b*x*e^(2*a) - e^(2*a))*e^(2*b*x) + 1)/(b^2*x^3*e^(4*b*x + 4*a) - 2*b^2
*x^3*e^(2*b*x + 2*a) + b^2*x^3) - 12*integrate(1/4/(b^2*x^4*e^(b*x + a) + b
^2*x^4), x) + 12*integrate(1/4/(b^2*x^4*e^(b*x + a) - b^2*x^4), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{\cosh(a+b\,x)}{x^2\,\sinh(a+b\,x)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a + b*x)/(x^2*sinh(a + b*x)^3),x)
```

```
[Out] int(cosh(a + b*x)/(x^2*sinh(a + b*x)^3), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh(a+bx)\operatorname{csch}^3(a+bx)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*csch(b*x+a)**3/x**2,x)
```

```
[Out] Integral(cosh(a + b*x)*csch(a + b*x)**3/x**2, x)
```

3.452 $\int x^m \coth^2(a+bx)\mathbf{csch}(a+bx)\,dx$

Optimal. Leaf size=29

$$\text{Int}\left(x^m\text{csch}^3(a+bx),x\right)+\text{Int}\left(x^m\text{csch}(a+bx),x\right)$$

```
[Out] Unintegrable(x^m*csch(b*x+a),x)+Unintegrable(x^m*csch(b*x+a)^3,x)
```

Rubi [A] time = 0.07, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int x^m \coth^2(a+bx)\text{csch}(a+bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x^m*Coth[a + b*x]^2*Csch[a + b*x],x]
[Out] Defer[Int][x^m*Csch[a + b*x], x] + Defer[Int][x^m*Csch[a + b*x]^3, x]
```

Rubi steps

$$\int x^m \coth^2(a+bx)\text{csch}(a+bx)\,dx=\int x^m\text{csch}(a+bx)\,dx+\int x^m\text{csch}^3(a+bx)\,dx$$

Mathematica [F] time = 180.00, size = 0, normalized size = 0.00

$Aborted

Verification is Not applicable to the result.

```
[In] Integrate[x^m*Coth[a + b*x]^2*Csch[a + b*x],x]
[Out] $Aborted
```

fricas [A] time = 0.41, size = 0, normalized size = 0.00

$$\text{integral}\left(x^m\cosh\left(bx+a\right)^2\text{csch}\left(bx+a\right)^3,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*cosh(b*x+a)^2*csch(b*x+a)^3,x, algorithm="fricas")
[Out] integral(x^m*cosh(b*x + a)^2*csch(b*x + a)^3, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int x^m \cosh(bx+a)^2 \operatorname{csch}(bx+a)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*cosh(b*x+a)^2*csch(b*x+a)^3,x, algorithm="giac")
[Out] integrate(x^m*cosh(b*x + a)^2*csch(b*x + a)^3, x)
```

maple [A] time = 0.30, size = 0, normalized size = 0.00

$$\int x^m \left(\cosh^2(bx+a)\right) \operatorname{csch}(bx+a)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*cosh(b*x+a)^2*csch(b*x+a)^3,x)
[Out] int(x^m*cosh(b*x+a)^2*csch(b*x+a)^3,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int x^m \cosh(bx+a)^2 \operatorname{csch}(bx+a)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*cosh(b*x+a)^2*csch(b*x+a)^3,x, algorithm="maxima")
[Out] integrate(x^m*cosh(b*x + a)^2*csch(b*x + a)^3, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{x^m \cosh(a+b\,x)^2}{\sinh(a+b\,x)^3} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^m*cosh(a + b*x)^2)/sinh(a + b*x)^3,x)
[Out] int((x^m*cosh(a + b*x)^2)/sinh(a + b*x)^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*cosh(b*x+a)**2*csch(b*x+a)**3,x)
[Out] Timed out
```

3.453 $\int x^3 \coth^2(a+bx)\mathbf{csch}(a+bx)\,dx$

Optimal. Leaf size=201

$$-\frac{3\text{Li}_2\left(-e^{a+bx}\right)}{b^4}+\frac{3\text{Li}_2\left(e^{a+bx}\right)}{b^4}-\frac{3\text{Li}_4\left(-e^{a+bx}\right)}{b^4}+\frac{3\text{Li}_4\left(e^{a+bx}\right)}{b^4}+\frac{3x\text{Li}_3\left(-e^{a+bx}\right)}{b^3}-\frac{3x\text{Li}_3\left(e^{a+bx}\right)}{b^3}-\frac{6x\tanh^{-1}\left(e^{a+bx}\right)}{b^3}-\frac{3}{}$$

```
[Out] -6*x*arctanh(exp(b*x+a))/b^3-x^3*arctanh(exp(b*x+a))/b-3/2*x^2*csch(b*x+a)/
b^2-1/2*x^3*coth(b*x+a)*csch(b*x+a)/b-3*polylog(2,-exp(b*x+a))/b^4-3/2*x^2*
polylog(2,-exp(b*x+a))/b^2+3*polylog(2,exp(b*x+a))/b^4+3/2*x^2*polylog(2,ex
p(b*x+a))/b^2+3*x*polylog(3,-exp(b*x+a))/b^3-3*x*polylog(3,exp(b*x+a))/b^3-
3*polylog(4,-exp(b*x+a))/b^4+3*polylog(4,exp(b*x+a))/b^4
```

Rubi [A] time = 0.36, antiderivative size = 201, normalized size of antiderivative = 1.00, number of steps used = 25, number of rules used = 9, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5457, 4182, 2531, 6609, 2282, 6589, 4186, 2279, 2391}

$$-\frac{3x^2\text{PolyLog}\left(2,-e^{a+bx}\right)}{2b^2}+\frac{3x^2\text{PolyLog}\left(2,e^{a+bx}\right)}{2b^2}+\frac{3x\text{PolyLog}\left(3,-e^{a+bx}\right)}{b^3}-\frac{3x\text{PolyLog}\left(3,e^{a+bx}\right)}{b^3}-\frac{3\text{PolyLog}\left(2\right.}{b^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Coth[a + b*x]^2*Csch[a + b*x],x]
```

```
[Out] (-6*x*ArcTanh[E^(a + b*x)])/b^3 - (x^3*ArcTanh[E^(a + b*x)])/b - (3*x^2*Csc
h[a + b*x])/(2*b^2) - (x^3*Coth[a + b*x]*Csch[a + b*x])/(2*b) - (3*PolyLog[
2, -E^(a + b*x)])/b^4 - (3*x^2*PolyLog[2, -E^(a + b*x)])/(2*b^2) + (3*PolyL
og[2, E^(a + b*x)])/b^4 + (3*x^2*PolyLog[2, E^(a + b*x)])/(2*b^2) + (3*x*Po
lyLog[3, -E^(a + b*x)])/b^3 - (3*x*PolyLog[3, E^(a + b*x)])/b^3 - (3*PolyLo
g[4, -E^(a + b*x)])/b^4 + (3*PolyLog[4, E^(a + b*x)])/b^4
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 4186

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> -Simp[(b^2*(c + d*x)^m*Cot[e + f*x]*(b*Csc[e + f*x])^(n - 2))/(f*(n -
 1)), x] + (Dist[(b^2*d^2*m*(m - 1))/(f^2*(n - 1)*(n - 2)), Int[(c + d*x)^(
m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Dist[(b^2*(n - 2))/(n - 1), Int[(
c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x] - Simp[(b^2*d*m*(c + d*x)^(m -
1)*(b*Csc[e + f*x])^(n - 2))/(f^2*(n - 1)*(n - 2)), x]) /; FreeQ[{b, c, d,
e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

Rule 5457

```
Int[Coth[(a_.) + (b_.)*(x_)]^(p_)*Csch[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(
x_))^(m_.), x_Symbol] :> Int[(c + d*x)^m*Csch[a + b*x]*Coth[a + b*x]^(p - 2
), x] + Int[(c + d*x)^m*Csch[a + b*x]^3*Coth[a + b*x]^(p - 2), x] /; FreeQ[
{a, b, c, d, m}, x] && IGtQ[p/2, 0]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int x^3 \coth^2(a+bx)\text{csch}(a+bx)\,dx = \int x^3\text{csch}(a+bx)\,dx + \int x^3\text{csch}^3(a+bx)\,dx$$

$$= -\frac{2x^3\tanh^{-1}\left(e^{a+bx}\right)}{b} - \frac{3x^2\text{csch}(a+bx)}{2b^2} - \frac{x^3\coth(a+bx)\text{csch}(a+bx)}{2b} - \frac{1}{2}$$

$$= -\frac{6x\tanh^{-1}\left(e^{a+bx}\right)}{b^3} - \frac{x^3\tanh^{-1}\left(e^{a+bx}\right)}{b} - \frac{3x^2\text{csch}(a+bx)}{2b^2} - \frac{x^3\coth(a+b}{}$$

$$= -\frac{6x\tanh^{-1}\left(e^{a+bx}\right)}{b^3} - \frac{x^3\tanh^{-1}\left(e^{a+bx}\right)}{b} - \frac{3x^2\text{csch}(a+bx)}{2b^2} - \frac{x^3\coth(a+b}{}$$

$$= -\frac{6x\tanh^{-1}\left(e^{a+bx}\right)}{b^3} - \frac{x^3\tanh^{-1}\left(e^{a+bx}\right)}{b} - \frac{3x^2\text{csch}(a+bx)}{2b^2} - \frac{x^3\coth(a+b}{}$$

$$= -\frac{6x\tanh^{-1}\left(e^{a+bx}\right)}{b^3} - \frac{x^3\tanh^{-1}\left(e^{a+bx}\right)}{b} - \frac{3x^2\text{csch}(a+bx)}{2b^2} - \frac{x^3\coth(a+b}{}$$

$$= -\frac{6x\tanh^{-1}\left(e^{a+bx}\right)}{b^3} - \frac{x^3\tanh^{-1}\left(e^{a+bx}\right)}{b} - \frac{3x^2\text{csch}(a+bx)}{2b^2} - \frac{x^3\coth(a+b}{}$$

Mathematica [A] time = 6.92, size = 280, normalized size = 1.39

$$-\frac{-4b^3x^3\log\left(1-e^{a+bx}\right) + 4b^3x^3\log\left(e^{a+bx}+1\right) + b^3x^3\text{csch}^2\left(\frac{1}{2}(a+bx)\right) + b^3x^3\text{sech}^2\left(\frac{1}{2}(a+bx)\right) + 12\left(b^2x^2+2\right)}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Coth[a + b*x]^2*Csch[a + b*x],x]
```

```
[Out] -1/8*(12*b^2*x^2*Csch[a] + b^3*x^3*Csch[(a + b*x)/2]^2 - 24*b*x*Log[1 - E^(
a + b*x)] - 4*b^3*x^3*Log[1 - E^(a + b*x)] + 24*b*x*Log[1 + E^(a + b*x)] +
4*b^3*x^3*Log[1 + E^(a + b*x)] + 12*(2 + b^2*x^2)*PolyLog[2, -E^(a + b*x)]
- 12*(2 + b^2*x^2)*PolyLog[2, E^(a + b*x)] - 24*b*x*PolyLog[3, -E^(a + b*x)
] + 24*b*x*PolyLog[3, E^(a + b*x)] + 24*PolyLog[4, -E^(a + b*x)] - 24*PolyL
og[4, E^(a + b*x)] + b^3*x^3*Sech[(a + b*x)/2]^2 - 6*b^2*x^2*Csch[a/2]*Csch
```

```
[(a + b*x)/2]*Sinh[(b*x)/2] - 6*b^2*x^2*Sech[a/2]*Sech[(a + b*x)/2]*Sinh[(b
*x)/2])/b^4
```

fricas [C] time = 0.46, size = 1802, normalized size = 8.97

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*cosh(b*x+a)^2*csch(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -1/2*(2*(b^3*x^3 + 3*b^2*x^2)*cosh(b*x + a)^3 + 6*(b^3*x^3 + 3*b^2*x^2)*cos
h(b*x + a)*sinh(b*x + a)^2 + 2*(b^3*x^3 + 3*b^2*x^2)*sinh(b*x + a)^3 + 2*(b
^3*x^3 - 3*b^2*x^2)*cosh(b*x + a) - 3*((b^2*x^2 + 2)*cosh(b*x + a)^4 + 4*(b
^2*x^2 + 2)*cosh(b*x + a)*sinh(b*x + a)^3 + (b^2*x^2 + 2)*sinh(b*x + a)^4 +
 b^2*x^2 - 2*(b^2*x^2 + 2)*cosh(b*x + a)^2 - 2*(b^2*x^2 - 3*(b^2*x^2 + 2)*c
osh(b*x + a)^2 + 2)*sinh(b*x + a)^2 + 4*((b^2*x^2 + 2)*cosh(b*x + a)^3 - (b
^2*x^2 + 2)*cosh(b*x + a))*sinh(b*x + a) + 2)*dilog(cosh(b*x + a) + sinh(b*
x + a)) + 3*((b^2*x^2 + 2)*cosh(b*x + a)^4 + 4*(b^2*x^2 + 2)*cosh(b*x + a)*
sinh(b*x + a)^3 + (b^2*x^2 + 2)*sinh(b*x + a)^4 + b^2*x^2 - 2*(b^2*x^2 + 2)
*cosh(b*x + a)^2 - 2*(b^2*x^2 - 3*(b^2*x^2 + 2)*cosh(b*x + a)^2 + 2)*sinh(b
*x + a)^2 + 4*((b^2*x^2 + 2)*cosh(b*x + a)^3 - (b^2*x^2 + 2)*cosh(b*x + a))
*sinh(b*x + a) + 2)*dilog(-cosh(b*x + a) - sinh(b*x + a)) + (b^3*x^3 + (b^3
*x^3 + 6*b*x)*cosh(b*x + a)^4 + 4*(b^3*x^3 + 6*b*x)*cosh(b*x + a)*sinh(b*x
+ a)^3 + (b^3*x^3 + 6*b*x)*sinh(b*x + a)^4 - 2*(b^3*x^3 + 6*b*x)*cosh(b*x +
 a)^2 - 2*(b^3*x^3 - 3*(b^3*x^3 + 6*b*x)*cosh(b*x + a)^2 + 6*b*x)*sinh(b*x
+ a)^2 + 6*b*x + 4*((b^3*x^3 + 6*b*x)*cosh(b*x + a)^3 - (b^3*x^3 + 6*b*x)*c
osh(b*x + a))*sinh(b*x + a))*log(cosh(b*x + a) + sinh(b*x + a) + 1) + ((a^3
 + 6*a)*cosh(b*x + a)^4 + 4*(a^3 + 6*a)*cosh(b*x + a)*sinh(b*x + a)^3 + (a^
3 + 6*a)*sinh(b*x + a)^4 + a^3 - 2*(a^3 + 6*a)*cosh(b*x + a)^2 - 2*(a^3 - 3
*(a^3 + 6*a)*cosh(b*x + a)^2 + 6*a)*sinh(b*x + a)^2 + 4*((a^3 + 6*a)*cosh(b
*x + a)^3 - (a^3 + 6*a)*cosh(b*x + a))*sinh(b*x + a) + 6*a)*log(cosh(b*x +
a) + sinh(b*x + a) - 1) - (b^3*x^3 + (b^3*x^3 + a^3 + 6*b*x + 6*a)*cosh(b*x
 + a)^4 + 4*(b^3*x^3 + a^3 + 6*b*x + 6*a)*cosh(b*x + a)*sinh(b*x + a)^3 + (
b^3*x^3 + a^3 + 6*b*x + 6*a)*sinh(b*x + a)^4 + a^3 - 2*(b^3*x^3 + a^3 + 6*b
*x + 6*a)*cosh(b*x + a)^2 - 2*(b^3*x^3 + a^3 - 3*(b^3*x^3 + a^3 + 6*b*x + 6
*a)*cosh(b*x + a)^2 + 6*b*x + 6*a)*sinh(b*x + a)^2 + 6*b*x + 4*((b^3*x^3 +
a^3 + 6*b*x + 6*a)*cosh(b*x + a)^3 - (b^3*x^3 + a^3 + 6*b*x + 6*a)*cosh(b*x
 + a))*sinh(b*x + a) + 6*a)*log(-cosh(b*x + a) - sinh(b*x + a) + 1) - 6*(co
sh(b*x + a)^4 + 4*cosh(b*x + a)*sinh(b*x + a)^3 + sinh(b*x + a)^4 + 2*(3*co
sh(b*x + a)^2 - 1)*sinh(b*x + a)^2 - 2*cosh(b*x + a)^2 + 4*(cosh(b*x + a)^3
 - cosh(b*x + a))*sinh(b*x + a) + 1)*polylog(4, cosh(b*x + a) + sinh(b*x +
a)) + 6*(cosh(b*x + a)^4 + 4*cosh(b*x + a)*sinh(b*x + a)^3 + sinh(b*x + a)^
4 + 2*(3*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^2 - 2*cosh(b*x + a)^2 + 4*(cosh
(b*x + a)^3 - cosh(b*x + a))*sinh(b*x + a) + 1)*polylog(4, -cosh(b*x + a) -
 sinh(b*x + a)) + 6*(b*x*cosh(b*x + a)^4 + 4*b*x*cosh(b*x + a)*sinh(b*x + a
```

```
)^3 + b*x*sinh(b*x + a)^4 - 2*b*x*cosh(b*x + a)^2 + 2*(3*b*x*cosh(b*x + a)^
2 - b*x)*sinh(b*x + a)^2 + b*x + 4*(b*x*cosh(b*x + a)^3 - b*x*cosh(b*x + a)
)*sinh(b*x + a))*polylog(3, cosh(b*x + a) + sinh(b*x + a)) - 6*(b*x*cosh(b*
x + a)^4 + 4*b*x*cosh(b*x + a)*sinh(b*x + a)^3 + b*x*sinh(b*x + a)^4 - 2*b*
x*cosh(b*x + a)^2 + 2*(3*b*x*cosh(b*x + a)^2 - b*x)*sinh(b*x + a)^2 + b*x +
 4*(b*x*cosh(b*x + a)^3 - b*x*cosh(b*x + a))*sinh(b*x + a))*polylog(3, -cos
h(b*x + a) - sinh(b*x + a)) + 2*(b^3*x^3 - 3*b^2*x^2 + 3*(b^3*x^3 + 3*b^2*x
^2)*cosh(b*x + a)^2)*sinh(b*x + a))/(b^4*cosh(b*x + a)^4 + 4*b^4*cosh(b*x +
 a)*sinh(b*x + a)^3 + b^4*sinh(b*x + a)^4 - 2*b^4*cosh(b*x + a)^2 + b^4 + 2
*(3*b^4*cosh(b*x + a)^2 - b^4)*sinh(b*x + a)^2 + 4*(b^4*cosh(b*x + a)^3 - b
^4*cosh(b*x + a))*sinh(b*x + a))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3 \cosh(bx+a)^2 \operatorname{csch}(bx+a)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*cosh(b*x+a)^2*csch(b*x+a)^3,x, algorithm="giac")
```

```
[Out] integrate(x^3*cosh(b*x + a)^2*csch(b*x + a)^3, x)
```

maple [A] time = 0.76, size = 340, normalized size = 1.69

$$-\frac{x^2 e^{bx+a}\left(bx\,e^{2bx+2a}+bx+3\,e^{2bx+2a}-3\right)}{b^2\left(e^{2bx+2a}-1\right)^2}-\frac{3a\ln\left(1+e^{bx+a}\right)}{b^4}+\frac{3\ln\left(1-e^{bx+a}\right)a}{b^4}-\frac{\ln\left(1+e^{bx+a}\right)x^3}{2b}-\frac{3x^2\operatorname{polylog}(}{2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*cosh(b*x+a)^2*csch(b*x+a)^3,x)
```

```
[Out] -x^2*exp(b*x+a)*(b*x*exp(2*b*x+2*a)+b*x+3*exp(2*b*x+2*a)-3)/b^2/(exp(2*b*x+
2*a)-1)^2-3/b^4*a*ln(1+exp(b*x+a))+3/b^4*ln(1-exp(b*x+a))*a-1/2/b*ln(1+exp(
b*x+a))*x^3-3/2*x^2*polylog(2,-exp(b*x+a))/b^2+3*x*polylog(3,-exp(b*x+a))/b
^3+1/2/b*ln(1-exp(b*x+a))*x^3+3/2*x^2*polylog(2,exp(b*x+a))/b^2-3*x*polylog
(3,exp(b*x+a))/b^3+3/b^3*ln(1-exp(b*x+a))*x-3/b^3*ln(1+exp(b*x+a))*x-3*poly
log(2,-exp(b*x+a))/b^4-3*polylog(4,-exp(b*x+a))/b^4+3*polylog(2,exp(b*x+a))
/b^4+3*polylog(4,exp(b*x+a))/b^4-1/2/b^4*ln(1+exp(b*x+a))*a^3+1/2/b^4*ln(1-
exp(b*x+a))*a^3+6/b^4*a*arctanh(exp(b*x+a))+1/b^4*a^3*arctanh(exp(b*x+a))
```

maxima [A] time = 0.54, size = 262, normalized size = 1.30

$$-\frac{\left(bx^3e^{(3a)}+3x^2e^{(3a)}\right)e^{(3bx)}+\left(bx^3e^{a}-3x^2e^{a}\right)e^{(bx)}}{b^2e^{(4bx+4a)}-2b^2e^{(2bx+2a)}+b^2}-\frac{b^3x^3\log\left(e^{(bx+a)}+1\right)+3b^2x^2\mathrm{Li}_2\left(-e^{(bx+a)}\right)-6bx\mathrm{Li}_3(-e^{(bx+a}}{2b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*cosh(b*x+a)^2*csch(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -((b*x^3*e^(3*a) + 3*x^2*e^(3*a))*e^(3*b*x) + (b*x^3*e^a - 3*x^2*e^a)*e^(b*
x))/(b^2*e^(4*b*x + 4*a) - 2*b^2*e^(2*b*x + 2*a) + b^2) - 1/2*(b^3*x^3*log(
e^(b*x + a) + 1) + 3*b^2*x^2*dilog(-e^(b*x + a)) - 6*b*x*polylog(3, -e^(b*x
 + a)) + 6*polylog(4, -e^(b*x + a)))/b^4 + 1/2*(b^3*x^3*log(-e^(b*x + a) +
1) + 3*b^2*x^2*dilog(e^(b*x + a)) - 6*b*x*polylog(3, e^(b*x + a)) + 6*polyl
og(4, e^(b*x + a)))/b^4 - 3*(b*x*log(e^(b*x + a) + 1) + dilog(-e^(b*x + a))
)/b^4 + 3*(b*x*log(-e^(b*x + a) + 1) + dilog(e^(b*x + a)))/b^4
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3 \cosh\left(a+b\,x\right)^2}{\sinh\left(a+b\,x\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*cosh(a + b*x)^2)/sinh(a + b*x)^3,x)
```

```
[Out] int((x^3*cosh(a + b*x)^2)/sinh(a + b*x)^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*cosh(b*x+a)**2*csch(b*x+a)**3,x)
```

```
[Out] Timed out
```

3.454 $\int x^2 \coth^2(a+bx)\textbf{csch}(a+bx)\,dx$

Optimal. Leaf size=123

$$\frac{\text{Li}_3\left(-e^{a+bx}\right)}{b^3}-\frac{\text{Li}_3\left(e^{a+bx}\right)}{b^3}-\frac{\tanh^{-1}(\cosh(a+bx))}{b^3}-\frac{x\text{Li}_2\left(-e^{a+bx}\right)}{b^2}+\frac{x\text{Li}_2\left(e^{a+bx}\right)}{b^2}-\frac{x\text{csch}(a+bx)}{b^2}-\frac{x^2\tanh^{-1}\left(e^{a+bx}\right)}{b}$$

```
[Out] -x^2*arctanh(exp(b*x+a))/b-arctanh(cosh(b*x+a))/b^3-x*csch(b*x+a)/b^2-1/2*x
^2*coth(b*x+a)*csch(b*x+a)/b-x*polylog(2,-exp(b*x+a))/b^2+x*polylog(2,exp(b
*x+a))/b^2+polylog(3,-exp(b*x+a))/b^3-polylog(3,exp(b*x+a))/b^3
```

Rubi [A] time = 0.24, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {5457, 4182, 2531, 2282, 6589, 4186, 3770}

$$-\frac{x\text{PolyLog}\left(2,-e^{a+bx}\right)}{b^2}+\frac{x\text{PolyLog}\left(2,e^{a+bx}\right)}{b^2}+\frac{\text{PolyLog}\left(3,-e^{a+bx}\right)}{b^3}-\frac{\text{PolyLog}\left(3,e^{a+bx}\right)}{b^3}-\frac{x\text{csch}(a+bx)}{b^2}-\tanh^{-1}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Coth[a + b*x]^2*Csch[a + b*x],x]
```

```
[Out] -((x^2*ArcTanh[E^(a + b*x)])/b) - ArcTanh[Cosh[a + b*x]]/b^3 - (x*Csch[a +
b*x])/b^2 - (x^2*Coth[a + b*x]*Csch[a + b*x])/(2*b) - (x*PolyLog[2, -E^(a +
 b*x)])/b^2 + (x*PolyLog[2, E^(a + b*x)])/b^2 + PolyLog[3, -E^(a + b*x)]/b^
3 - PolyLog[3, E^(a + b*x)]/b^3
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 4186

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> -Simp[(b^2*(c + d*x)^m*Cot[e + f*x]*(b*Csc[e + f*x])^(n - 2))/(f*(n -
 1)), x] + (Dist[(b^2*d^2*m*(m - 1))/(f^2*(n - 1)*(n - 2)), Int[(c + d*x)^(
m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Dist[(b^2*(n - 2))/(n - 1), Int[(
c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x] - Simp[(b^2*d*m*(c + d*x)^(m -
1)*(b*Csc[e + f*x])^(n - 2))/(f^2*(n - 1)*(n - 2)), x]) /; FreeQ[{b, c, d,
e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

Rule 5457

```
Int[Coth[(a_.) + (b_.)*(x_)]^(p_)*Csch[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(
x_))^(m_.), x_Symbol] :> Int[(c + d*x)^m*Csch[a + b*x]*Coth[a + b*x]^(p - 2
), x] + Int[(c + d*x)^m*Csch[a + b*x]^3*Coth[a + b*x]^(p - 2), x] /; FreeQ[
{a, b, c, d, m}, x] && IGtQ[p/2, 0]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int x^2 \coth^2(a+bx)\operatorname{csch}(a+bx)\,dx = \int x^2\operatorname{csch}(a+bx)\,dx + \int x^2\operatorname{csch}^3(a+bx)\,dx$$
$$= -\frac{2x^2\tanh^{-1}\left(e^{a+bx}\right)}{b} - \frac{x\operatorname{csch}(a+bx)}{b^2} - \frac{x^2\coth(a+bx)\operatorname{csch}(a+bx)}{2b} - \frac{1}{2}\int$$
$$= -\frac{x^2\tanh^{-1}\left(e^{a+bx}\right)}{b} - \frac{\tanh^{-1}(\cosh(a+bx))}{b^3} - \frac{x\operatorname{csch}(a+bx)}{b^2} - \frac{x^2\coth(a}{}$$
$$= -\frac{x^2\tanh^{-1}\left(e^{a+bx}\right)}{b} - \frac{\tanh^{-1}(\cosh(a+bx))}{b^3} - \frac{x\operatorname{csch}(a+bx)}{b^2} - \frac{x^2\coth(a}{}$$
$$= -\frac{x^2\tanh^{-1}\left(e^{a+bx}\right)}{b} - \frac{\tanh^{-1}(\cosh(a+bx))}{b^3} - \frac{x\operatorname{csch}(a+bx)}{b^2} - \frac{x^2\coth(a}{}$$
$$= -\frac{x^2\tanh^{-1}\left(e^{a+bx}\right)}{b} - \frac{\tanh^{-1}(\cosh(a+bx))}{b^3} - \frac{x\operatorname{csch}(a+bx)}{b^2} - \frac{x^2\coth(a}{}$$

Mathematica [A] time = 4.15, size = 222, normalized size = 1.80

$$-\frac{-4b^2x^2\log\left(1-e^{a+bx}\right)+4b^2x^2\log\left(e^{a+bx}+1\right)+b^2x^2\operatorname{csch}^2\left(\frac{1}{2}(a+bx)\right)+b^2x^2\operatorname{sech}^2\left(\frac{1}{2}(a+bx)\right)+8bx\operatorname{Li}_2\left(-e^{a+}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Coth[a + b*x]^2*Csch[a + b*x],x]
```

```
[Out] -1/8*(8*b*x*Csch[a] + b^2*x^2*Csch[(a + b*x)/2]^2 - 8*Log[1 - E^(a + b*x)]
- 4*b^2*x^2*Log[1 - E^(a + b*x)] + 8*Log[1 + E^(a + b*x)] + 4*b^2*x^2*Log[1
 + E^(a + b*x)] + 8*b*x*PolyLog[2, -E^(a + b*x)] - 8*b*x*PolyLog[2, E^(a +
b*x)] - 8*PolyLog[3, -E^(a + b*x)] + 8*PolyLog[3, E^(a + b*x)] + b^2*x^2*Se
ch[(a + b*x)/2]^2 - 4*b*x*Csch[a/2]*Csch[(a + b*x)/2]*Sinh[(b*x)/2] - 4*b*x
*Sech[a/2]*Sech[(a + b*x)/2]*Sinh[(b*x)/2])/b^3
```

fricas [C] time = 0.45, size = 1311, normalized size = 10.66

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cosh(b*x+a)^2*csch(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -1/2*(2*(b^2*x^2 + 2*b*x)*cosh(b*x + a)^3 + 6*(b^2*x^2 + 2*b*x)*cosh(b*x +
a)*sinh(b*x + a)^2 + 2*(b^2*x^2 + 2*b*x)*sinh(b*x + a)^3 + 2*(b^2*x^2 - 2*b
*x)*cosh(b*x + a) - 2*(b*x*cosh(b*x + a)^4 + 4*b*x*cosh(b*x + a)*sinh(b*x +
```

```
a)^3 + b*x*sinh(b*x + a)^4 - 2*b*x*cosh(b*x + a)^2 + 2*(3*b*x*cosh(b*x + a
)^2 - b*x)*sinh(b*x + a)^2 + b*x + 4*(b*x*cosh(b*x + a)^3 - b*x*cosh(b*x +
a))*sinh(b*x + a))*dilog(cosh(b*x + a) + sinh(b*x + a)) + 2*(b*x*cosh(b*x +
 a)^4 + 4*b*x*cosh(b*x + a)*sinh(b*x + a)^3 + b*x*sinh(b*x + a)^4 - 2*b*x*c
osh(b*x + a)^2 + 2*(3*b*x*cosh(b*x + a)^2 - b*x)*sinh(b*x + a)^2 + b*x + 4*
(b*x*cosh(b*x + a)^3 - b*x*cosh(b*x + a))*sinh(b*x + a))*dilog(-cosh(b*x +
a) - sinh(b*x + a)) + ((b^2*x^2 + 2)*cosh(b*x + a)^4 + 4*(b^2*x^2 + 2)*cosh
(b*x + a)*sinh(b*x + a)^3 + (b^2*x^2 + 2)*sinh(b*x + a)^4 + b^2*x^2 - 2*(b^
2*x^2 + 2)*cosh(b*x + a)^2 - 2*(b^2*x^2 - 3*(b^2*x^2 + 2)*cosh(b*x + a)^2 +
 2)*sinh(b*x + a)^2 + 4*((b^2*x^2 + 2)*cosh(b*x + a)^3 - (b^2*x^2 + 2)*cosh
(b*x + a))*sinh(b*x + a) + 2)*log(cosh(b*x + a) + sinh(b*x + a) + 1) - ((a^
2 + 2)*cosh(b*x + a)^4 + 4*(a^2 + 2)*cosh(b*x + a)*sinh(b*x + a)^3 + (a^2 +
 2)*sinh(b*x + a)^4 - 2*(a^2 + 2)*cosh(b*x + a)^2 + 2*(3*(a^2 + 2)*cosh(b*x
 + a)^2 - a^2 - 2)*sinh(b*x + a)^2 + a^2 + 4*((a^2 + 2)*cosh(b*x + a)^3 - (
a^2 + 2)*cosh(b*x + a))*sinh(b*x + a) + 2)*log(cosh(b*x + a) + sinh(b*x + a
) - 1) - ((b^2*x^2 - a^2)*cosh(b*x + a)^4 + 4*(b^2*x^2 - a^2)*cosh(b*x + a)
*sinh(b*x + a)^3 + (b^2*x^2 - a^2)*sinh(b*x + a)^4 + b^2*x^2 - 2*(b^2*x^2 -
 a^2)*cosh(b*x + a)^2 - 2*(b^2*x^2 - 3*(b^2*x^2 - a^2)*cosh(b*x + a)^2 - a^
2)*sinh(b*x + a)^2 - a^2 + 4*((b^2*x^2 - a^2)*cosh(b*x + a)^3 - (b^2*x^2 -
a^2)*cosh(b*x + a))*sinh(b*x + a))*log(-cosh(b*x + a) - sinh(b*x + a) + 1)
+ 2*(cosh(b*x + a)^4 + 4*cosh(b*x + a)*sinh(b*x + a)^3 + sinh(b*x + a)^4 +
2*(3*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^2 - 2*cosh(b*x + a)^2 + 4*(cosh(b*x
 + a)^3 - cosh(b*x + a))*sinh(b*x + a) + 1)*polylog(3, cosh(b*x + a) + sinh
(b*x + a)) - 2*(cosh(b*x + a)^4 + 4*cosh(b*x + a)*sinh(b*x + a)^3 + sinh(b*
x + a)^4 + 2*(3*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^2 - 2*cosh(b*x + a)^2 +
4*(cosh(b*x + a)^3 - cosh(b*x + a))*sinh(b*x + a) + 1)*polylog(3, -cosh(b*x
 + a) - sinh(b*x + a)) + 2*(b^2*x^2 + 3*(b^2*x^2 + 2*b*x)*cosh(b*x + a)^2 -
 2*b*x)*sinh(b*x + a))/(b^3*cosh(b*x + a)^4 + 4*b^3*cosh(b*x + a)*sinh(b*x
+ a)^3 + b^3*sinh(b*x + a)^4 - 2*b^3*cosh(b*x + a)^2 + b^3 + 2*(3*b^3*cosh(
b*x + a)^2 - b^3)*sinh(b*x + a)^2 + 4*(b^3*cosh(b*x + a)^3 - b^3*cosh(b*x +
 a))*sinh(b*x + a))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \cosh(bx+a)^2 \operatorname{csch}(bx+a)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cosh(b*x+a)^2*csch(b*x+a)^3,x, algorithm="giac")
```

```
[Out] integrate(x^2*cosh(b*x + a)^2*csch(b*x + a)^3, x)
```

maple [A] time = 0.72, size = 210, normalized size = 1.71

$$-\frac{x\,e^{bx+a}\left(bx\,e^{2bx+2a}+bx+2\,e^{2bx+2a}-2\right)}{b^2\left(e^{2bx+2a}-1\right)^2}-\frac{a^2\operatorname{arctanh}\left(e^{bx+a}\right)}{b^3}-\frac{\ln\left(1+e^{bx+a}\right)x^2}{2b}+\frac{\ln\left(1+e^{bx+a}\right)a^2}{2b^3}-\frac{\operatorname{polylog}\left(2\right.}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*cosh(b*x+a)^2*csch(b*x+a)^3,x)
```

```
[Out] -x*exp(b*x+a)*(b*x*exp(2*b*x+2*a)+b*x+2*exp(2*b*x+2*a)-2)/b^2/(exp(2*b*x+2*
a)-1)^2-1/b^3*a^2*arctanh(exp(b*x+a))-1/2/b*ln(1+exp(b*x+a))*x^2+1/2/b^3*ln
(1+exp(b*x+a))*a^2-1/b^2*polylog(2,-exp(b*x+a))*x+1/b^3*polylog(3,-exp(b*x+
a))+1/2/b*ln(1-exp(b*x+a))*x^2-1/2/b^3*ln(1-exp(b*x+a))*a^2+1/b^2*polylog(2
,exp(b*x+a))*x-1/b^3*polylog(3,exp(b*x+a))-2/b^3*arctanh(exp(b*x+a))
```

maxima [A] time = 0.49, size = 197, normalized size = 1.60

$$-\frac{\left(bx^2e^{(3\,a)} + 2\,xe^{(3\,a)}\right)e^{(3\,bx)} + \left(bx^2e^{a} - 2\,xe^{a}\right)e^{(bx)}}{b^2e^{(4\,bx+4\,a)} - 2\,b^2e^{(2\,bx+2\,a)} + b^2} - \frac{b^2x^2\log\left(e^{(bx+a)} + 1\right) + 2\,bx\mathrm{Li}_2\left(-e^{(bx+a)}\right) - 2\,\mathrm{Li}_3(-e^{(bx+a)})}{2\,b^3} + \frac{b^2}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cosh(b*x+a)^2*csch(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -((b*x^2*e^(3*a) + 2*x*e^(3*a))*e^(3*b*x) + (b*x^2*e^a - 2*x*e^a)*e^(b*x))/
(b^2*e^(4*b*x + 4*a) - 2*b^2*e^(2*b*x + 2*a) + b^2) - 1/2*(b^2*x^2*log(e^(b
*x + a) + 1) + 2*b*x*dilog(-e^(b*x + a)) - 2*polylog(3, -e^(b*x + a)))/b^3
+ 1/2*(b^2*x^2*log(-e^(b*x + a) + 1) + 2*b*x*dilog(e^(b*x + a)) - 2*polylog
(3, e^(b*x + a)))/b^3 - log(e^(b*x + a) + 1)/b^3 + log(e^(b*x + a) - 1)/b^3
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2\cosh\left(a+b\,x\right)^2}{\sinh\left(a+b\,x\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*cosh(a + b*x)^2)/sinh(a + b*x)^3,x)
```

```
[Out] int((x^2*cosh(a + b*x)^2)/sinh(a + b*x)^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*cosh(b*x+a)**2*csch(b*x+a)**3,x)
```

```
[Out] Timed out
```

3.455 $\int x \coth^2(a+bx)\mathbf{csch}(a+bx)\,dx$

Optimal. Leaf size=82

$$-\frac{\text{Li}_2\left(-e^{a+bx}\right)}{2b^2}+\frac{\text{Li}_2\left(e^{a+bx}\right)}{2b^2}-\frac{\text{csch}(a+bx)}{2b^2}-\frac{x\tanh^{-1}\left(e^{a+bx}\right)}{b}-\frac{x\coth(a+bx)\text{csch}(a+bx)}{2b}$$

```
[Out] -x*arctanh(exp(b*x+a))/b-1/2*csch(b*x+a)/b^2-1/2*x*coth(b*x+a)*csch(b*x+a)/
b-1/2*polylog(2,-exp(b*x+a))/b^2+1/2*polylog(2,exp(b*x+a))/b^2
```

Rubi [A] time = 0.12, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {5457, 4182, 2279, 2391, 4185}

$$-\frac{\text{PolyLog}\left(2,-e^{a+bx}\right)}{2b^2}+\frac{\text{PolyLog}\left(2,e^{a+bx}\right)}{2b^2}-\frac{\text{csch}(a+bx)}{2b^2}-\frac{x\tanh^{-1}\left(e^{a+bx}\right)}{b}-\frac{x\coth(a+bx)\text{csch}(a+bx)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[x*Coth[a + b*x]^2*Csch[a + b*x],x]
```

```
[Out] -((x*ArcTanh[E^(a + b*x)])/b) - Csch[a + b*x]/(2*b^2) - (x*Coth[a + b*x]*Cs
ch[a + b*x])/(2*b) - PolyLog[2, -E^(a + b*x)]/(2*b^2) + PolyLog[2, E^(a + b
*x)]/(2*b^2)
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 4185

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_)), x_Symbol] :>
 -Simp[(b^2*(c + d*x)*Cot[e + f*x]*(b*Csc[e + f*x])^(n - 2))/(f*(n - 1)), x
] + (Dist[(b^2*(n - 2))/(n - 1), Int[(c + d*x)*(b*Csc[e + f*x])^(n - 2), x]
, x] - Simp[(b^2*d*(b*Csc[e + f*x])^(n - 2))/(f^2*(n - 1)*(n - 2)), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2]
```

Rule 5457

```
Int[Coth[(a_.) + (b_.)*(x_)]^(p_)*Csch[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(
x_))^(m_.), x_Symbol] :> Int[(c + d*x)^m*Csch[a + b*x]*Coth[a + b*x]^(p - 2
), x] + Int[(c + d*x)^m*Csch[a + b*x]^3*Coth[a + b*x]^(p - 2), x] /; FreeQ[
{a, b, c, d, m}, x] && IGtQ[p/2, 0]
```

Rubi steps

$$\int x\coth^2(a+bx)\text{csch}(a+bx)\,dx = \int x\text{csch}(a+bx)\,dx + \int x\text{csch}^3(a+bx)\,dx$$
$$= -\frac{2x\tanh^{-1}\left(e^{a+bx}\right)}{b} - \frac{\text{csch}(a+bx)}{2b^2} - \frac{x\coth(a+bx)\text{csch}(a+bx)}{2b} - \frac{1}{2}\int x\text{cs}$$
$$= -\frac{x\tanh^{-1}\left(e^{a+bx}\right)}{b} - \frac{\text{csch}(a+bx)}{2b^2} - \frac{x\coth(a+bx)\text{csch}(a+bx)}{2b} - \frac{\text{Subst}\left(\int\right.}{}$$
$$= -\frac{x\tanh^{-1}\left(e^{a+bx}\right)}{b} - \frac{\text{csch}(a+bx)}{2b^2} - \frac{x\coth(a+bx)\text{csch}(a+bx)}{2b} - \frac{\text{Li}_2\left(-e^{a+}\right.}{b^2}$$
$$= -\frac{x\tanh^{-1}\left(e^{a+bx}\right)}{b} - \frac{\text{csch}(a+bx)}{2b^2} - \frac{x\coth(a+bx)\text{csch}(a+bx)}{2b} - \frac{\text{Li}_2\left(-e^{a+}\right.}{2b^2}$$

Mathematica [A] time = 2.03, size = 144, normalized size = 1.76

$$-\frac{-4\text{Li}_2\left(-e^{-a-bx}\right) + 4\text{Li}_2\left(e^{-a-bx}\right) - 4(a+bx)\left(\log\left(1-e^{-a-bx}\right) - \log\left(e^{-a-bx}+1\right)\right) - 2\tanh\left(\frac{1}{2}(a+bx)\right) + 2\coth\left(\right.}{8b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Coth[a + b*x]^2*Csch[a + b*x],x]
```

```
[Out] -1/8*(2*Coth[(a + b*x)/2] + b*x*Csch[(a + b*x)/2]^2 - 4*(a + b*x)*(Log[1 -
E^(-a - b*x)] - Log[1 + E^(-a - b*x)]) + 4*a*Log[Tanh[(a + b*x)/2]] - 4*Pol
yLog[2, -E^(-a - b*x)] + 4*PolyLog[2, E^(-a - b*x)] + b*x*Sech[(a + b*x)/2]
^2 - 2*Tanh[(a + b*x)/2])/b^2
```

fricas [B] time = 0.43, size = 842, normalized size = 10.27

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x+a)^2*csch(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -1/2*(2*(b*x + 1)*cosh(b*x + a)^3 + 6*(b*x + 1)*cosh(b*x + a)*sinh(b*x + a)
^2 + 2*(b*x + 1)*sinh(b*x + a)^3 + 2*(b*x - 1)*cosh(b*x + a) - (cosh(b*x +
a)^4 + 4*cosh(b*x + a)*sinh(b*x + a)^3 + sinh(b*x + a)^4 + 2*(3*cosh(b*x +
a)^2 - 1)*sinh(b*x + a)^2 - 2*cosh(b*x + a)^2 + 4*(cosh(b*x + a)^3 - cosh(b
*x + a))*sinh(b*x + a) + 1)*dilog(cosh(b*x + a) + sinh(b*x + a)) + (cosh(b*
x + a)^4 + 4*cosh(b*x + a)*sinh(b*x + a)^3 + sinh(b*x + a)^4 + 2*(3*cosh(b*
x + a)^2 - 1)*sinh(b*x + a)^2 - 2*cosh(b*x + a)^2 + 4*(cosh(b*x + a)^3 - co
sh(b*x + a))*sinh(b*x + a) + 1)*dilog(-cosh(b*x + a) - sinh(b*x + a)) + (b*
x*cosh(b*x + a)^4 + 4*b*x*cosh(b*x + a)*sinh(b*x + a)^3 + b*x*sinh(b*x + a)
^4 - 2*b*x*cosh(b*x + a)^2 + 2*(3*b*x*cosh(b*x + a)^2 - b*x)*sinh(b*x + a)^
2 + b*x + 4*(b*x*cosh(b*x + a)^3 - b*x*cosh(b*x + a))*sinh(b*x + a))*log(co
sh(b*x + a) + sinh(b*x + a) + 1) + (a*cosh(b*x + a)^4 + 4*a*cosh(b*x + a)*s
inh(b*x + a)^3 + a*sinh(b*x + a)^4 - 2*a*cosh(b*x + a)^2 + 2*(3*a*cosh(b*x
+ a)^2 - a)*sinh(b*x + a)^2 + 4*(a*cosh(b*x + a)^3 - a*cosh(b*x + a))*sinh(
b*x + a) + a)*log(cosh(b*x + a) + sinh(b*x + a) - 1) - ((b*x + a)*cosh(b*x
+ a)^4 + 4*(b*x + a)*cosh(b*x + a)*sinh(b*x + a)^3 + (b*x + a)*sinh(b*x + a
)^4 - 2*(b*x + a)*cosh(b*x + a)^2 + 2*(3*(b*x + a)*cosh(b*x + a)^2 - b*x -
a)*sinh(b*x + a)^2 + b*x + 4*((b*x + a)*cosh(b*x + a)^3 - (b*x + a)*cosh(b*
x + a))*sinh(b*x + a) + a)*log(-cosh(b*x + a) - sinh(b*x + a) + 1) + 2*(3*(
b*x + 1)*cosh(b*x + a)^2 + b*x - 1)*sinh(b*x + a))/(b^2*cosh(b*x + a)^4 + 4
*b^2*cosh(b*x + a)*sinh(b*x + a)^3 + b^2*sinh(b*x + a)^4 - 2*b^2*cosh(b*x +
 a)^2 + 2*(3*b^2*cosh(b*x + a)^2 - b^2)*sinh(b*x + a)^2 + b^2 + 4*(b^2*cosh
(b*x + a)^3 - b^2*cosh(b*x + a))*sinh(b*x + a))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x \cosh(bx+a)^2 \operatorname{csch}(bx+a)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x+a)^2*csch(b*x+a)^3,x, algorithm="giac")
```

```
[Out] integrate(x*cosh(b*x + a)^2*csch(b*x + a)^3, x)
```

maple [B] time = 0.66, size = 156, normalized size = 1.90

$$-\frac{e^{bx+a}\left(bx\,e^{2bx+2a}+bx+e^{2bx+2a}-1\right)}{b^2\left(e^{2bx+2a}-1\right)^2}-\frac{\ln\left(1+e^{bx+a}\right)x}{2b}-\frac{\ln\left(1+e^{bx+a}\right)a}{2b^2}-\frac{\operatorname{polylog}\left(2,-e^{bx+a}\right)}{2b^2}+\frac{\ln\left(1-e^{bx+a}\right)x}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cosh(b*x+a)^2*csch(b*x+a)^3,x)
```

```
[Out] -exp(b*x+a)*(b*x*exp(2*b*x+2*a)+b*x+exp(2*b*x+2*a)-1)/b^2/(exp(2*b*x+2*a)-1
)^2-1/2/b*ln(1+exp(b*x+a))*x-1/2/b^2*ln(1+exp(b*x+a))*a-1/2*polylog(2,-exp(
b*x+a))/b^2+1/2/b*ln(1-exp(b*x+a))*x+1/2/b^2*ln(1-exp(b*x+a))*a+1/2*polylog
(2,exp(b*x+a))/b^2+1/b^2*a*arctanh(exp(b*x+a))
```

maxima [A] time = 0.45, size = 124, normalized size = 1.51

$$-\frac{\left(bxe^{(3a)} + e^{(3a)}\right)e^{(3bx)} + \left(bxe^{a} - e^{a}\right)e^{(bx)}}{b^2e^{(4bx+4a)} - 2b^2e^{(2bx+2a)} + b^2} - \frac{bx\log\left(e^{(bx+a)} + 1\right) + \mathrm{Li}_2\left(-e^{(bx+a)}\right)}{2b^2} + \frac{bx\log\left(-e^{(bx+a)} + 1\right) + \mathrm{Li}_2\left(e^{(bx+a)}\right)}{2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x+a)^2*csch(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -((b*x*e^(3*a) + e^(3*a))*e^(3*b*x) + (b*x*e^a - e^a)*e^(b*x))/(b^2*e^(4*b*
x + 4*a) - 2*b^2*e^(2*b*x + 2*a) + b^2) - 1/2*(b*x*log(e^(b*x + a) + 1) + d
ilog(-e^(b*x + a)))/b^2 + 1/2*(b*x*log(-e^(b*x + a) + 1) + dilog(e^(b*x + a
)))/b^2
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x\cosh\left(a+b\,x\right)^2}{\sinh\left(a+b\,x\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*cosh(a + b*x)^2)/sinh(a + b*x)^3,x)
```

```
[Out] int((x*cosh(a + b*x)^2)/sinh(a + b*x)^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\cosh^2\left(a + bx\right)\operatorname{csch}^3\left(a + bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x+a)**2*csch(b*x+a)**3,x)
```

```
[Out] Integral(x*cosh(a + b*x)**2*csch(a + b*x)**3, x)
```

3.456 $\int \coth^2(a+bx)\mathbf{csch}(a+bx)\,dx$

Optimal. Leaf size=34

$$-\frac{\tanh^{-1}(\cosh(a+bx))}{2b}-\frac{\coth(a+bx)\text{csch}(a+bx)}{2b}$$

```
[Out] -1/2*arctanh(cosh(b*x+a))/b-1/2*coth(b*x+a)*csch(b*x+a)/b
```

Rubi [A] time = 0.03, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2611, 3770}

$$-\frac{\tanh^{-1}(\cosh(a+bx))}{2b}-\frac{\coth(a+bx)\text{csch}(a+bx)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[a + b*x]^2*Csch[a + b*x],x]
```

```
[Out] -ArcTanh[Cosh[a + b*x]]/(2*b) - (Coth[a + b*x]*Csch[a + b*x])/(2*b)
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \coth^2(a+bx)\text{csch}(a+bx)\,dx &= -\frac{\coth(a+bx)\text{csch}(a+bx)}{2b}+\frac{1}{2}\int \text{csch}(a+bx)\,dx\\ &= -\frac{\tanh^{-1}(\cosh(a+bx))}{2b}-\frac{\coth(a+bx)\text{csch}(a+bx)}{2b}\end{aligned}$$

Mathematica [A] time = 0.03, size = 57, normalized size = 1.68

$$-\frac{\operatorname{csch}^2\left(\frac{1}{2}(a+bx)\right)}{8b}-\frac{\operatorname{sech}^2\left(\frac{1}{2}(a+bx)\right)}{8b}+\frac{\log\left(\tanh\left(\frac{1}{2}(a+bx)\right)\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[a + b*x]^2*Csch[a + b*x],x]
```

```
[Out] -1/8*Csch[(a + b*x)/2]^2/b + Log[Tanh[(a + b*x)/2]]/(2*b) - Sech[(a + b*x)/
2]^2/(8*b)
```

fricas [B] time = 0.40, size = 387, normalized size = 11.38

$$-\frac{2\cosh(bx+a)^3+6\cosh(bx+a)\sinh(bx+a)^2+2\sinh(bx+a)^3+\left(\cosh(bx+a)^4+4\cosh(bx+a)\sinh(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^2*csch(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -1/2*(2*cosh(b*x + a)^3 + 6*cosh(b*x + a)*sinh(b*x + a)^2 + 2*sinh(b*x + a)
^3 + (cosh(b*x + a)^4 + 4*cosh(b*x + a)*sinh(b*x + a)^3 + sinh(b*x + a)^4 +
 2*(3*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^2 - 2*cosh(b*x + a)^2 + 4*(cosh(b*
x + a)^3 - cosh(b*x + a))*sinh(b*x + a) + 1)*log(cosh(b*x + a) + sinh(b*x +
 a) + 1) - (cosh(b*x + a)^4 + 4*cosh(b*x + a)*sinh(b*x + a)^3 + sinh(b*x +
a)^4 + 2*(3*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^2 - 2*cosh(b*x + a)^2 + 4*(c
osh(b*x + a)^3 - cosh(b*x + a))*sinh(b*x + a) + 1)*log(cosh(b*x + a) + sinh
(b*x + a) - 1) + 2*(3*cosh(b*x + a)^2 + 1)*sinh(b*x + a) + 2*cosh(b*x + a))
/(b*cosh(b*x + a)^4 + 4*b*cosh(b*x + a)*sinh(b*x + a)^3 + b*sinh(b*x + a)^4
 - 2*b*cosh(b*x + a)^2 + 2*(3*b*cosh(b*x + a)^2 - b)*sinh(b*x + a)^2 + 4*(b
*cosh(b*x + a)^3 - b*cosh(b*x + a))*sinh(b*x + a) + b)
```

giac [B] time = 0.14, size = 84, normalized size = 2.47

$$-\frac{\frac{4\left(e^{(bx+a)}+e^{(-bx-a)}\right)}{\left(e^{(bx+a)}+e^{(-bx-a)}\right)^2-4}+\log\left(e^{(bx+a)}+e^{(-bx-a)}+2\right)-\log\left(e^{(bx+a)}+e^{(-bx-a)}-2\right)}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^2*csch(b*x+a)^3,x, algorithm="giac")
```

```
[Out] -1/4*(4*(e^(b*x + a) + e^(-b*x - a))/((e^(b*x + a) + e^(-b*x - a))^2 - 4) +
 log(e^(b*x + a) + e^(-b*x - a) + 2) - log(e^(b*x + a) + e^(-b*x - a) - 2))
/b
```

maple [A] time = 0.35, size = 45, normalized size = 1.32

$$\frac{-\frac{\cosh(bx+a)}{\sinh(bx+a)^2}+\frac{\operatorname{csch}(bx+a)\coth(bx+a)}{2}-\operatorname{arctanh}\left(e^{bx+a}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)^2*csch(b*x+a)^3,x)
```

```
[Out] 1/b*(-1/sinh(b*x+a)^2*cosh(b*x+a)+1/2*csch(b*x+a)*coth(b*x+a)-arctanh(exp(b
*x+a)))
```

maxima [B] time = 0.34, size = 84, normalized size = 2.47

$$-\frac{\log\left(e^{(-bx-a)}+1\right)}{2\,b}+\frac{\log\left(e^{(-bx-a)}-1\right)}{2\,b}+\frac{e^{(-bx-a)}+e^{(-3\,bx-3\,a)}}{b\left(2\,e^{(-2\,bx-2\,a)}-e^{(-4\,bx-4\,a)}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^2*csch(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -1/2*log(e^(-b*x - a) + 1)/b + 1/2*log(e^(-b*x - a) - 1)/b + (e^(-b*x - a)
+ e^(-3*b*x - 3*a))/(b*(2*e^(-2*b*x - 2*a) - e^(-4*b*x - 4*a) - 1))
```

mupad [B] time = 1.50, size = 87, normalized size = 2.56

$$-\frac{\operatorname{atan}\left(\frac{e^{b\,x}\,e^{a}\,\sqrt{-b^2}}{b}\right)}{\sqrt{-b^2}}-\frac{2\,e^{a+b\,x}}{b\,\left(e^{4\,a+4\,b\,x}-2\,e^{2\,a+2\,b\,x}+1\right)}-\frac{e^{a+b\,x}}{b\,\left(e^{2\,a+2\,b\,x}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a + b*x)^2/sinh(a + b*x)^3,x)
```

```
[Out] - atan((exp(b*x)*exp(a)*(-b^2)^(1/2))/b)/(-b^2)^(1/2) - (2*exp(a + b*x))/(b
*(exp(4*a + 4*b*x) - 2*exp(2*a + 2*b*x) + 1)) - exp(a + b*x)/(b*(exp(2*a +
2*b*x) - 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \cosh^2(a+bx)\operatorname{csch}^3(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)**2*csch(b*x+a)**3,x)
```

```
[Out] Integral(cosh(a + b*x)**2*csch(a + b*x)**3, x)
```

3.457 $\int \frac{\coth^2(a+bx)\mathbf{csch}(a+bx)}{x}\,dx$

Optimal. Leaf size=29

$$\text{Int}\left(\frac{\text{csch}^3(a+bx)}{x},x\right)+\text{Int}\left(\frac{\text{csch}(a+bx)}{x},x\right)$$

```
[Out] Unintegrable(csch(b*x+a)/x,x)+Unintegrable(csch(b*x+a)^3/x,x)
```

Rubi [A] time = 0.07, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\coth^2(a+bx)\text{csch}(a+bx)}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Coth[a + b*x]^2*Csch[a + b*x])/x,x]
```

```
[Out] Defer[Int][Csch[a + b*x]/x, x] + Defer[Int][Csch[a + b*x]^3/x, x]
```

Rubi steps

$$\int \frac{\coth^2(a+bx)\text{csch}(a+bx)}{x}\,dx=\int \frac{\text{csch}(a+bx)}{x}\,dx+\int \frac{\text{csch}^3(a+bx)}{x}\,dx$$

Mathematica [A] time = 54.01, size = 0, normalized size = 0.00

$$\int \frac{\coth^2(a+bx)\text{csch}(a+bx)}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Coth[a + b*x]^2*Csch[a + b*x])/x,x]
```

```
[Out] Integrate[(Coth[a + b*x]^2*Csch[a + b*x])/x, x]
```

fricas [A] time = 0.42, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\cosh(bx+a)^2\,\text{csch}(bx+a)^3}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^2*csch(b*x+a)^3/x,x, algorithm="fricas")
```

```
[Out] integral(cosh(b*x + a)^2*csch(b*x + a)^3/x, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh\left(bx+a\right)^{2}\operatorname{csch}\left(bx+a\right)^{3}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^2*csch(b*x+a)^3/x,x, algorithm="giac")
```

```
[Out] integrate(cosh(b*x + a)^2*csch(b*x + a)^3/x, x)
```

maple [A] time = 0.92, size = 0, normalized size = 0.00

$$\int \frac{\left(\cosh^{2}\left(bx+a\right)\right)\operatorname{csch}\left(bx+a\right)^{3}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)^2*csch(b*x+a)^3/x,x)
```

```
[Out] int(cosh(b*x+a)^2*csch(b*x+a)^3/x,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\left(bxe^{(3a)}-e^{(3a)}\right)e^{(3bx)}+\left(bxe^{a}+e^{a}\right)e^{(bx)}}{b^{2}x^{2}e^{(4bx+4a)}-2b^{2}x^{2}e^{(2bx+2a)}+b^{2}x^{2}}+2\int \frac{b^{2}x^{2}+2}{4\left(b^{2}x^{3}e^{(bx+a)}+b^{2}x^{3}\right)}\,dx+2\int \frac{b^{2}x^{2}+2}{4\left(b^{2}x^{3}e^{(bx+a)}-b^{2}x^{3}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^2*csch(b*x+a)^3/x,x, algorithm="maxima")
```

```
[Out] -((b*x*e^(3*a) - e^(3*a))*e^(3*b*x) + (b*x*e^a + e^a)*e^(b*x))/(b^2*x^2*e^(
4*b*x + 4*a) - 2*b^2*x^2*e^(2*b*x + 2*a) + b^2*x^2) + 2*integrate(1/4*(b^2*
x^2 + 2)/(b^2*x^3*e^(b*x + a) + b^2*x^3), x) + 2*integrate(1/4*(b^2*x^2 + 2
)/(b^2*x^3*e^(b*x + a) - b^2*x^3), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{{\cosh\left(a+b\,x\right)}^2}{x\,{\sinh\left(a+b\,x\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a + b*x)^2/(x*sinh(a + b*x)^3),x)
```

```
[Out] int(cosh(a + b*x)^2/(x*sinh(a + b*x)^3), x)
```

sympy **[A]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh^2(a+bx)\operatorname{csch}^3(a+bx)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)**2*csch(b*x+a)**3/x,x)
```

```
[Out] Integral(cosh(a + b*x)**2*csch(a + b*x)**3/x, x)
```

3.458 $\int \frac{\coth^2(a+bx)\mathbf{csch}(a+bx)}{x^2}\,dx$

Optimal. Leaf size=29

$$\text{Int}\left(\frac{\text{csch}^3(a+bx)}{x^2},x\right)+\text{Int}\left(\frac{\text{csch}(a+bx)}{x^2},x\right)$$

```
[Out] Unintegrable(csch(b*x+a)/x^2,x)+Unintegrable(csch(b*x+a)^3/x^2,x)
```

Rubi [A] time = 0.08, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\coth^2(a+bx)\text{csch}(a+bx)}{x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Coth[a + b*x]^2*Csch[a + b*x])/x^2,x]
[Out] Defer[Int][Csch[a + b*x]/x^2, x] + Defer[Int][Csch[a + b*x]^3/x^2, x]
```

Rubi steps

$$\int \frac{\coth^2(a+bx)\text{csch}(a+bx)}{x^2}\,dx=\int \frac{\text{csch}(a+bx)}{x^2}\,dx+\int \frac{\text{csch}^3(a+bx)}{x^2}\,dx$$

Mathematica [A] time = 45.88, size = 0, normalized size = 0.00

$$\int \frac{\coth^2(a+bx)\text{csch}(a+bx)}{x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Coth[a + b*x]^2*Csch[a + b*x])/x^2,x]
[Out] Integrate[(Coth[a + b*x]^2*Csch[a + b*x])/x^2, x]
```

fricas [A] time = 0.40, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\cosh(bx+a)^2\,\text{csch}(bx+a)^3}{x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^2*csch(b*x+a)^3/x^2,x, algorithm="fricas")
```

```
[Out] integral(cosh(b*x + a)^2*csch(b*x + a)^3/x^2, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh\left(bx+a\right)^{2} \operatorname{csch}\left(bx+a\right)^{3}}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^2*csch(b*x+a)^3/x^2,x, algorithm="giac")
```

```
[Out] integrate(cosh(b*x + a)^2*csch(b*x + a)^3/x^2, x)
```

maple [A] time = 1.04, size = 0, normalized size = 0.00

$$\int \frac{\left(\cosh^{2}\left(bx+a\right)\right) \operatorname{csch}\left(bx+a\right)^{3}}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)^2*csch(b*x+a)^3/x^2,x)
```

```
[Out] int(cosh(b*x+a)^2*csch(b*x+a)^3/x^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\left(bxe^{(3\,a)}-2\,e^{(3\,a)}\right)e^{(3\,bx)}+\left(bxe^{a}+2\,e^{a}\right)e^{(bx)}}{b^{2}x^{3}e^{(4\,bx+4\,a)}-2\,b^{2}x^{3}e^{(2\,bx+2\,a)}+b^{2}x^{3}}+2\int \frac{b^{2}x^{2}+6}{4\left(b^{2}x^{4}e^{(bx+a)}+b^{2}x^{4}\right)}\,dx+2\int \frac{b^{2}x^{2}+6}{4\left(b^{2}x^{4}e^{(bx+a)}-b^{2}x^{4}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^2*csch(b*x+a)^3/x^2,x, algorithm="maxima")
```

```
[Out] -((b*x*e^(3*a) - 2*e^(3*a))*e^(3*b*x) + (b*x*e^a + 2*e^a)*e^(b*x))/(b^2*x^3
*e^(4*b*x + 4*a) - 2*b^2*x^3*e^(2*b*x + 2*a) + b^2*x^3) + 2*integrate(1/4*(
b^2*x^2 + 6)/(b^2*x^4*e^(b*x + a) + b^2*x^4), x) + 2*integrate(1/4*(b^2*x^2
 + 6)/(b^2*x^4*e^(b*x + a) - b^2*x^4), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\cosh\left(a+b\,x\right)^{2}}{x^{2}\,\sinh\left(a+b\,x\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a + b*x)^2/(x^2*sinh(a + b*x)^3),x)
```

```
[Out] int(cosh(a + b*x)^2/(x^2*sinh(a + b*x)^3), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh^2(a+bx)\operatorname{csch}^3(a+bx)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)**2*csch(b*x+a)**3/x**2,x)
```

```
[Out] Integral(cosh(a + b*x)**2*csch(a + b*x)**3/x**2, x)
```

3.459 $\int x^m \coth^3(a+bx)\,dx$

Optimal. Leaf size=15

$$\text{Int}\left(x^m \coth^3(a+bx),x\right)$$

```
[Out] Unintegrable(x^m*coth(b*x+a)^3,x)
```

Rubi [A] time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int x^m \coth^3(a+bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x^m*Coth[a + b*x]^3,x]
```

```
[Out] Defer[Int][x^m*Coth[a + b*x]^3, x]
```

Rubi steps

$$\int x^m \coth^3(a+bx)\,dx = \int x^m \coth^3(a+bx)\,dx$$

Mathematica [A] time = 122.97, size = 0, normalized size = 0.00

$$\int x^m \coth^3(a+bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^m*Coth[a + b*x]^3,x]
```

```
[Out] Integrate[x^m*Coth[a + b*x]^3, x]
```

fricas [A] time = 0.40, size = 0, normalized size = 0.00

$$\text{integral}\left(x^m \cosh\left(bx+a\right)^3 \operatorname{csch}\left(bx+a\right)^3, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*cosh(b*x+a)^3*csch(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] integral(x^m*cosh(b*x + a)^3*csch(b*x + a)^3, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int x^m \cosh(bx+a)^3 \operatorname{csch}(bx+a)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*cosh(b*x+a)^3*csch(b*x+a)^3,x, algorithm="giac")
```

```
[Out] integrate(x^m*cosh(b*x + a)^3*csch(b*x + a)^3, x)
```

maple [A] time = 0.47, size = 0, normalized size = 0.00

$$\int x^m \left(\cosh^3(bx+a)\right) \operatorname{csch}(bx+a)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*cosh(b*x+a)^3*csch(b*x+a)^3,x)
```

```
[Out] int(x^m*cosh(b*x+a)^3*csch(b*x+a)^3,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\frac{xe^{\left(6\,bx+m\log(x)+6\,a\right)}}{(m+1)e^{(6\,bx+6\,a)} - 3\,(m+1)e^{(4\,bx+4\,a)} + 3\,(m+1)e^{(2\,bx+2\,a)} - m - 1} + \int \frac{\left(3\left(2\,bxe^{(6\,a)} + (m+1)e^{(6\,a)}\right)e^{(6}}{(m+1)e^{(8\,bx+8\,a)} - 4\,(m+1)e^{(6\,bx+6\,a)} + 6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*cosh(b*x+a)^3*csch(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] x*e^(6*b*x + m*log(x) + 6*a)/((m + 1)*e^(6*b*x + 6*a) - 3*(m + 1)*e^(4*b*x
+ 4*a) + 3*(m + 1)*e^(2*b*x + 2*a) - m - 1) + integrate((3*(2*b*x*e^(6*a) +
 (m + 1)*e^(6*a))*e^(6*b*x) - 2*(m + 1)*e^(2*b*x + 2*a) - m - 1)*x^m/((m +
1)*e^(8*b*x + 8*a) - 4*(m + 1)*e^(6*b*x + 6*a) + 6*(m + 1)*e^(4*b*x + 4*a)
- 4*(m + 1)*e^(2*b*x + 2*a) + m + 1), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.07

$$\int \frac{x^m \cosh(a+b\,x)^3}{\sinh(a+b\,x)^3} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^m*cosh(a + b*x)^3)/sinh(a + b*x)^3,x)
```

```
[Out] int((x^m*cosh(a + b*x)^3)/sinh(a + b*x)^3, x)
```

<u>**sympy**</u> **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*cosh(b*x+a)**3*csch(b*x+a)**3,x)
```

```
[Out] Timed out
```

3.460 $\int x^3 \coth^3(a + bx)\,dx$

Optimal. Leaf size=179

$$\frac{3\text{Li}_2\left(e^{2(a+bx)}\right)}{2b^4}+\frac{3\text{Li}_4\left(e^{2(a+bx)}\right)}{4b^4}-\frac{3x\text{Li}_3\left(e^{2(a+bx)}\right)}{2b^3}+\frac{3x\log\left(1-e^{2(a+bx)}\right)}{b^3}+\frac{3x^2\text{Li}_2\left(e^{2(a+bx)}\right)}{2b^2}-\frac{3x^2\coth(a+bx)}{2b^2}+\frac{x^3}{}$$

```
[Out] -3/2*x^2/b^2+1/2*x^3/b-1/4*x^4-3/2*x^2*coth(b*x+a)/b^2-1/2*x^3*coth(b*x+a)^
2/b+3*x*ln(1-exp(2*b*x+2*a))/b^3+x^3*ln(1-exp(2*b*x+2*a))/b+3/2*polylog(2,e
xp(2*b*x+2*a))/b^4+3/2*x^2*polylog(2,exp(2*b*x+2*a))/b^2-3/2*x*polylog(3,ex
p(2*b*x+2*a))/b^3+3/4*polylog(4,exp(2*b*x+2*a))/b^4
```

Rubi [A] time = 0.34, antiderivative size = 179, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.833, Rules used = {3720, 3716, 2190, 2279, 2391, 30, 2531, 6609, 2282, 6589}

$$\frac{3x^2\text{PolyLog}\left(2,e^{2(a+bx)}\right)}{2b^2}-\frac{3x\text{PolyLog}\left(3,e^{2(a+bx)}\right)}{2b^3}+\frac{3\text{PolyLog}\left(2,e^{2(a+bx)}\right)}{2b^4}+\frac{3\text{PolyLog}\left(4,e^{2(a+bx)}\right)}{4b^4}-\frac{3x^2\coth(a}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Coth[a + b*x]^3,x]
```

```
[Out] (-3*x^2)/(2*b^2) + x^3/(2*b) - x^4/4 - (3*x^2*Coth[a + b*x])/(2*b^2) - (x^3
*Coth[a + b*x]^2)/(2*b) + (3*x*Log[1 - E^(2*(a + b*x))])/b^3 + (x^3*Log[1 -
 E^(2*(a + b*x))])/b + (3*PolyLog[2, E^(2*(a + b*x))])/(2*b^4) + (3*x^2*Pol
yLog[2, E^(2*(a + b*x))])/(2*b^2) - (3*x*PolyLog[3, E^(2*(a + b*x))])/(2*b^
3) + (3*PolyLog[4, E^(2*(a + b*x))])/(4*b^4)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 3716

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2
*I, Int[((c + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(E^(2*I*k*Pi)*(1 + E^(2*(-(I*
e) + f*fz*x))/E^(2*I*k*Pi))), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Integ
erQ[4*k] && IGtQ[m, 0]
```

Rule 3720

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[(b*(c + d*x)^m*(b*Tan[e + f*x])^(n - 1))/(f*(n - 1)), x] + (-Di
st[(b*d*m)/(f*(n - 1)), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x],
 x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[
{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
```

```
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int x^3\coth^3(a+bx)\,dx = -\frac{x^3\coth^2(a+bx)}{2b}+\frac{3\int x^2\coth^2(a+bx)\,dx}{2b}+\int x^3\coth(a+bx)\,dx$$

$$= -\frac{x^4}{4}-\frac{3x^2\coth(a+bx)}{2b^2}-\frac{x^3\coth^2(a+bx)}{2b}-2\int\frac{e^{2(a+bx)}x^3}{1-e^{2(a+bx)}}\,dx+\frac{3\int x\coth(a+bx)}{b^2}$$

$$= -\frac{3x^2}{2b^2}+\frac{x^3}{2b}-\frac{x^4}{4}-\frac{3x^2\coth(a+bx)}{2b^2}-\frac{x^3\coth^2(a+bx)}{2b}+\frac{x^3\log\left(1-e^{2(a+bx)}\right)}{b}-\frac{6\int}{}$$

$$= -\frac{3x^2}{2b^2}+\frac{x^3}{2b}-\frac{x^4}{4}-\frac{3x^2\coth(a+bx)}{2b^2}-\frac{x^3\coth^2(a+bx)}{2b}+\frac{3x\log\left(1-e^{2(a+bx)}\right)}{b^3}+\frac{x^3\,\mathrm{lo}}{}$$

$$= -\frac{3x^2}{2b^2}+\frac{x^3}{2b}-\frac{x^4}{4}-\frac{3x^2\coth(a+bx)}{2b^2}-\frac{x^3\coth^2(a+bx)}{2b}+\frac{3x\log\left(1-e^{2(a+bx)}\right)}{b^3}+\frac{x^3\,\mathrm{lo}}{}$$

$$= -\frac{3x^2}{2b^2}+\frac{x^3}{2b}-\frac{x^4}{4}-\frac{3x^2\coth(a+bx)}{2b^2}-\frac{x^3\coth^2(a+bx)}{2b}+\frac{3x\log\left(1-e^{2(a+bx)}\right)}{b^3}+\frac{x^3\,\mathrm{lo}}{}$$

$$= -\frac{3x^2}{2b^2}+\frac{x^3}{2b}-\frac{x^4}{4}-\frac{3x^2\coth(a+bx)}{2b^2}-\frac{x^3\coth^2(a+bx)}{2b}+\frac{3x\log\left(1-e^{2(a+bx)}\right)}{b^3}+\frac{x^3\,\mathrm{lo}}{}$$

Mathematica [B] time = 3.34, size = 390, normalized size = 2.18

$$\frac{1}{4}\left(\frac{6x^2\mathrm{csch}(a)\sinh(bx)\mathrm{csch}(a+bx)}{b^2}-\frac{2e^{2a}\left(e^{-2a}b^4x^4-2e^{-2a}\left(e^{2a}-1\right)b^3x^3\log\left(1-e^{-a-bx}\right)-2e^{-2a}\left(e^{2a}-1\right)b^3x\right.}{}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Coth[a + b*x]^3,x]
```

```
[Out] (x^4*Coth[a] - (2*x^3*Csch[a + b*x]^2)/b - (2*E^(2*a)*((6*b^2*x^2)/E^(2*a)
+ (b^4*x^4)/E^(2*a) - 6*b*(1 - E^(-2*a))*x*Log[1 - E^(-a - b*x)] - (2*b^3*(
```

```
-1 + E^(2*a))*x^3*Log[1 - E^(-a - b*x)])/E^(2*a) - 6*b*(1 - E^(-2*a))*x*Log
[1 + E^(-a - b*x)] - (2*b^3*(-1 + E^(2*a))*x^3*Log[1 + E^(-a - b*x)])/E^(2*
a) + 6*(1 - E^(-2*a))*PolyLog[2, -E^(-a - b*x)] + 6*(1 - E^(-2*a))*PolyLog[
2, E^(-a - b*x)] + 6*(1 - E^(-2*a))*(b^2*x^2*PolyLog[2, -E^(-a - b*x)] + 2*
(b*x*PolyLog[3, -E^(-a - b*x)] + PolyLog[4, -E^(-a - b*x)])) + 6*(1 - E^(-2
*a))*(b^2*x^2*PolyLog[2, E^(-a - b*x)] + 2*(b*x*PolyLog[3, E^(-a - b*x)] +
PolyLog[4, E^(-a - b*x)]))))/(b^4*(-1 + E^(2*a))) + (6*x^2*Csch[a]*Csch[a +
 b*x]*Sinh[b*x])/b^2)/4
```

fricas [C] time = 0.45, size = 1985, normalized size = 11.09

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*cosh(b*x+a)^3*csch(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -1/4*(b^4*x^4 + (b^4*x^4 - 2*a^4 + 12*b^2*x^2 - 12*a^2)*cosh(b*x + a)^4 + 4
*(b^4*x^4 - 2*a^4 + 12*b^2*x^2 - 12*a^2)*cosh(b*x + a)*sinh(b*x + a)^3 + (b
^4*x^4 - 2*a^4 + 12*b^2*x^2 - 12*a^2)*sinh(b*x + a)^4 - 2*a^4 - 2*(b^4*x^4
- 4*b^3*x^3 - 2*a^4 + 6*b^2*x^2 - 12*a^2)*cosh(b*x + a)^2 - 2*(b^4*x^4 - 4*
b^3*x^3 - 2*a^4 + 6*b^2*x^2 - 3*(b^4*x^4 - 2*a^4 + 12*b^2*x^2 - 12*a^2)*cos
h(b*x + a)^2 - 12*a^2)*sinh(b*x + a)^2 - 12*a^2 - 12*((b^2*x^2 + 1)*cosh(b*
x + a)^4 + 4*(b^2*x^2 + 1)*cosh(b*x + a)*sinh(b*x + a)^3 + (b^2*x^2 + 1)*si
nh(b*x + a)^4 + b^2*x^2 - 2*(b^2*x^2 + 1)*cosh(b*x + a)^2 - 2*(b^2*x^2 - 3*
(b^2*x^2 + 1)*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^2 + 4*((b^2*x^2 + 1)*cosh(
b*x + a)^3 - (b^2*x^2 + 1)*cosh(b*x + a))*sinh(b*x + a) + 1)*dilog(cosh(b*x
 + a) + sinh(b*x + a)) - 12*((b^2*x^2 + 1)*cosh(b*x + a)^4 + 4*(b^2*x^2 + 1
)*cosh(b*x + a)*sinh(b*x + a)^3 + (b^2*x^2 + 1)*sinh(b*x + a)^4 + b^2*x^2 -
 2*(b^2*x^2 + 1)*cosh(b*x + a)^2 - 2*(b^2*x^2 - 3*(b^2*x^2 + 1)*cosh(b*x +
a)^2 + 1)*sinh(b*x + a)^2 + 4*((b^2*x^2 + 1)*cosh(b*x + a)^3 - (b^2*x^2 + 1
)*cosh(b*x + a))*sinh(b*x + a) + 1)*dilog(-cosh(b*x + a) - sinh(b*x + a)) -
 4*(b^3*x^3 + (b^3*x^3 + 3*b*x)*cosh(b*x + a)^4 + 4*(b^3*x^3 + 3*b*x)*cosh(
b*x + a)*sinh(b*x + a)^3 + (b^3*x^3 + 3*b*x)*sinh(b*x + a)^4 - 2*(b^3*x^3 +
 3*b*x)*cosh(b*x + a)^2 - 2*(b^3*x^3 - 3*(b^3*x^3 + 3*b*x)*cosh(b*x + a)^2
+ 3*b*x)*sinh(b*x + a)^2 + 3*b*x + 4*((b^3*x^3 + 3*b*x)*cosh(b*x + a)^3 - (
b^3*x^3 + 3*b*x)*cosh(b*x + a))*sinh(b*x + a))*log(cosh(b*x + a) + sinh(b*x
 + a) + 1) + 4*((a^3 + 3*a)*cosh(b*x + a)^4 + 4*(a^3 + 3*a)*cosh(b*x + a)*s
inh(b*x + a)^3 + (a^3 + 3*a)*sinh(b*x + a)^4 + a^3 - 2*(a^3 + 3*a)*cosh(b*x
 + a)^2 - 2*(a^3 - 3*(a^3 + 3*a)*cosh(b*x + a)^2 + 3*a)*sinh(b*x + a)^2 + 4
*((a^3 + 3*a)*cosh(b*x + a)^3 - (a^3 + 3*a)*cosh(b*x + a))*sinh(b*x + a) +
3*a)*log(cosh(b*x + a) + sinh(b*x + a) - 1) - 4*(b^3*x^3 + (b^3*x^3 + a^3 +
 3*b*x + 3*a)*cosh(b*x + a)^4 + 4*(b^3*x^3 + a^3 + 3*b*x + 3*a)*cosh(b*x +
a)*sinh(b*x + a)^3 + (b^3*x^3 + a^3 + 3*b*x + 3*a)*sinh(b*x + a)^4 + a^3 -
2*(b^3*x^3 + a^3 + 3*b*x + 3*a)*cosh(b*x + a)^2 - 2*(b^3*x^3 + a^3 - 3*(b^3
*x^3 + a^3 + 3*b*x + 3*a)*cosh(b*x + a)^2 + 3*b*x + 3*a)*sinh(b*x + a)^2 +
```

```
3*b*x + 4*((b^3*x^3 + a^3 + 3*b*x + 3*a)*cosh(b*x + a)^3 - (b^3*x^3 + a^3 +
 3*b*x + 3*a)*cosh(b*x + a))*sinh(b*x + a) + 3*a)*log(-cosh(b*x + a) - sinh
(b*x + a) + 1) - 24*(cosh(b*x + a)^4 + 4*cosh(b*x + a)*sinh(b*x + a)^3 + si
nh(b*x + a)^4 + 2*(3*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^2 - 2*cosh(b*x + a)
^2 + 4*(cosh(b*x + a)^3 - cosh(b*x + a))*sinh(b*x + a) + 1)*polylog(4, cosh
(b*x + a) + sinh(b*x + a)) - 24*(cosh(b*x + a)^4 + 4*cosh(b*x + a)*sinh(b*x
 + a)^3 + sinh(b*x + a)^4 + 2*(3*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^2 - 2*c
osh(b*x + a)^2 + 4*(cosh(b*x + a)^3 - cosh(b*x + a))*sinh(b*x + a) + 1)*pol
ylog(4, -cosh(b*x + a) - sinh(b*x + a)) + 24*(b*x*cosh(b*x + a)^4 + 4*b*x*c
osh(b*x + a)*sinh(b*x + a)^3 + b*x*sinh(b*x + a)^4 - 2*b*x*cosh(b*x + a)^2
+ 2*(3*b*x*cosh(b*x + a)^2 - b*x)*sinh(b*x + a)^2 + b*x + 4*(b*x*cosh(b*x +
 a)^3 - b*x*cosh(b*x + a))*sinh(b*x + a))*polylog(3, cosh(b*x + a) + sinh(b
*x + a)) + 24*(b*x*cosh(b*x + a)^4 + 4*b*x*cosh(b*x + a)*sinh(b*x + a)^3 +
b*x*sinh(b*x + a)^4 - 2*b*x*cosh(b*x + a)^2 + 2*(3*b*x*cosh(b*x + a)^2 - b*
x)*sinh(b*x + a)^2 + b*x + 4*(b*x*cosh(b*x + a)^3 - b*x*cosh(b*x + a))*sinh
(b*x + a))*polylog(3, -cosh(b*x + a) - sinh(b*x + a)) + 4*((b^4*x^4 - 2*a^4
 + 12*b^2*x^2 - 12*a^2)*cosh(b*x + a)^3 - (b^4*x^4 - 4*b^3*x^3 - 2*a^4 + 6*
b^2*x^2 - 12*a^2)*cosh(b*x + a))*sinh(b*x + a))/(b^4*cosh(b*x + a)^4 + 4*b^
4*cosh(b*x + a)*sinh(b*x + a)^3 + b^4*sinh(b*x + a)^4 - 2*b^4*cosh(b*x + a)
^2 + b^4 + 2*(3*b^4*cosh(b*x + a)^2 - b^4)*sinh(b*x + a)^2 + 4*(b^4*cosh(b*
x + a)^3 - b^4*cosh(b*x + a))*sinh(b*x + a))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3 \cosh(bx+a)^3 \operatorname{csch}(bx+a)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*cosh(b*x+a)^3*csch(b*x+a)^3,x, algorithm="giac")
```

```
[Out] integrate(x^3*cosh(b*x + a)^3*csch(b*x + a)^3, x)
```

maple [B] time = 0.60, size = 375, normalized size = 2.09

$$-\frac{x^2\left(2bx\,e^{2bx+2a}+3\,e^{2bx+2a}-3\right)}{b^2\left(e^{2bx+2a}-1\right)^2}+\frac{\ln\left(1-e^{bx+a}\right)x^3}{b}+\frac{\ln\left(1-e^{bx+a}\right)a^3}{b^4}+\frac{3x^2\operatorname{polylog}\left(2,e^{bx+a}\right)}{b^2}-\frac{2a^3x}{b^3}+\frac{3x^2\operatorname{poly}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*cosh(b*x+a)^3*csch(b*x+a)^3,x)
```

```
[Out] -x^2*(2*b*x*exp(2*b*x+2*a)+3*exp(2*b*x+2*a)-3)/b^2/(exp(2*b*x+2*a)-1)^2+1/b
*ln(1-exp(b*x+a))*x^3+1/b^4*ln(1-exp(b*x+a))*a^3-2/b^3*a^3*x-1/4*x^4+6*poly
log(4,-exp(b*x+a))/b^4+6*polylog(4,exp(b*x+a))/b^4+3*x^2*polylog(2,-exp(b*x
+a))/b^2+3*x^2*polylog(2,exp(b*x+a))/b^2-6*x*polylog(3,-exp(b*x+a))/b^3-6*x
```

```
*polylog(3,exp(b*x+a))/b^3-3*x^2/b^2+3*polylog(2,-exp(b*x+a))/b^4+3*polylog
(2,exp(b*x+a))/b^4-3/2/b^4*a^4+3/b^3*ln(1-exp(b*x+a))*x+3/b^4*ln(1-exp(b*x+
a))*a+3/b^3*ln(1+exp(b*x+a))*x+6/b^4*a*ln(exp(b*x+a))-3/b^4*a*ln(exp(b*x+a)
-1)-3/b^4*a^2+1/b*ln(1+exp(b*x+a))*x^3-6*a*x/b^3-1/b^4*a^3*ln(exp(b*x+a)-1)
+2/b^4*a^3*ln(exp(b*x+a))
```

maxima [A] time = 0.41, size = 302, normalized size = 1.69

$$\frac{b^2x^4e^{(4\,bx+4\,a)} + b^2x^4 + 12\,x^2 - 2\left(b^2x^4e^{(2\,a)} + 4\,bx^3e^{(2\,a)} + 6\,x^2e^{(2\,a)}\right)e^{(2\,bx)}}{4\left(b^2e^{(4\,bx+4\,a)} - 2\,b^2e^{(2\,bx+2\,a)} + b^2\right)} - \frac{b^4x^4 + 6\,b^2x^2}{2\,b^4} + \frac{b^3x^3\log\left(e^{(bx+a)} + 1\right) +}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*cosh(b*x+a)^3*csch(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/4*(b^2*x^4*e^(4*b*x + 4*a) + b^2*x^4 + 12*x^2 - 2*(b^2*x^4*e^(2*a) + 4*b*
x^3*e^(2*a) + 6*x^2*e^(2*a))*e^(2*b*x))/(b^2*e^(4*b*x + 4*a) - 2*b^2*e^(2*b
*x + 2*a) + b^2) - 1/2*(b^4*x^4 + 6*b^2*x^2)/b^4 + (b^3*x^3*log(e^(b*x + a)
 + 1) + 3*b^2*x^2*dilog(-e^(b*x + a)) - 6*b*x*polylog(3, -e^(b*x + a)) + 6*
polylog(4, -e^(b*x + a)))/b^4 + (b^3*x^3*log(-e^(b*x + a) + 1) + 3*b^2*x^2*
dilog(e^(b*x + a)) - 6*b*x*polylog(3, e^(b*x + a)) + 6*polylog(4, e^(b*x +
a)))/b^4 + 3*(b*x*log(e^(b*x + a) + 1) + dilog(-e^(b*x + a)))/b^4 + 3*(b*x*
log(-e^(b*x + a) + 1) + dilog(e^(b*x + a)))/b^4
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3\cosh\left(a+b\,x\right)^3}{\sinh\left(a+b\,x\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*cosh(a + b*x)^3)/sinh(a + b*x)^3,x)
```

```
[Out] int((x^3*cosh(a + b*x)^3)/sinh(a + b*x)^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*cosh(b*x+a)**3*csch(b*x+a)**3,x)
```

```
[Out] Timed out
```

3.461 $\int x^2 \coth^3(a+bx)\,dx$

Optimal. Leaf size=114

$$-\frac{\text{Li}_3\left(e^{2(a+bx)}\right)}{2b^3}+\frac{\log(\sinh(a+bx))}{b^3}+\frac{x\text{Li}_2\left(e^{2(a+bx)}\right)}{b^2}-\frac{x\coth(a+bx)}{b^2}+\frac{x^2\log\left(1-e^{2(a+bx)}\right)}{b}-\frac{x^2\coth^2(a+bx)}{2b}+\frac{x^2}{2b}$$

```
[Out] 1/2*x^2/b-1/3*x^3-x*coth(b*x+a)/b^2-1/2*x^2*coth(b*x+a)^2/b+x^2*ln(1-exp(2*
b*x+2*a))/b+ln(sinh(b*x+a))/b^3+x*polylog(2,exp(2*b*x+2*a))/b^2-1/2*polylog
(3,exp(2*b*x+2*a))/b^3
```

Rubi [A] time = 0.21, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {3720, 3475, 30, 3716, 2190, 2531, 2282, 6589}

$$\frac{x\text{PolyLog}\left(2,e^{2(a+bx)}\right)}{b^2}-\frac{\text{PolyLog}\left(3,e^{2(a+bx)}\right)}{2b^3}-\frac{x\coth(a+bx)}{b^2}+\frac{\log(\sinh(a+bx))}{b^3}+\frac{x^2\log\left(1-e^{2(a+bx)}\right)}{b}-\frac{x^2\coth}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Coth[a + b*x]^3,x]
```

```
[Out] x^2/(2*b) - x^3/3 - (x*Coth[a + b*x])/b^2 - (x^2*Coth[a + b*x]^2)/(2*b) + (
x^2*Log[1 - E^(2*(a + b*x))])/b + Log[Sinh[a + b*x]]/b^3 + (x*PolyLog[2, E^
(2*(a + b*x))])/b^2 - PolyLog[3, E^(2*(a + b*x))]/(2*b^3)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
```

```
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3716

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2
*I, Int[((c + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(E^(2*I*k*Pi)*(1 + E^(2*(-(I*
e) + f*fz*x))/E^(2*I*k*Pi))), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Integ
erQ[4*k] && IGtQ[m, 0]
```

Rule 3720

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[(b*(c + d*x)^m*(b*Tan[e + f*x])^(n - 1))/(f*(n - 1)), x] + (-Di
st[(b*d*m)/(f*(n - 1)), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x],
 x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[
{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int x^2 \coth^3(a+bx)\,dx = -\frac{x^2\coth^2(a+bx)}{2b} + \frac{\int x\coth^2(a+bx)\,dx}{b} + \int x^2\coth(a+bx)\,dx$$
$$= -\frac{x^3}{3} - \frac{x\coth(a+bx)}{b^2} - \frac{x^2\coth^2(a+bx)}{2b} - 2\int \frac{e^{2(a+bx)}x^2}{1-e^{2(a+bx)}}\,dx + \frac{\int \coth(a+bx)\,dx}{b^2} +$$
$$= \frac{x^2}{2b} - \frac{x^3}{3} - \frac{x\coth(a+bx)}{b^2} - \frac{x^2\coth^2(a+bx)}{2b} + \frac{x^2\log\left(1-e^{2(a+bx)}\right)}{b} + \frac{\log(\sinh(a+b}{b^3}$$
$$= \frac{x^2}{2b} - \frac{x^3}{3} - \frac{x\coth(a+bx)}{b^2} - \frac{x^2\coth^2(a+bx)}{2b} + \frac{x^2\log\left(1-e^{2(a+bx)}\right)}{b} + \frac{\log(\sinh(a+b}{b^3}$$
$$= \frac{x^2}{2b} - \frac{x^3}{3} - \frac{x\coth(a+bx)}{b^2} - \frac{x^2\coth^2(a+bx)}{2b} + \frac{x^2\log\left(1-e^{2(a+bx)}\right)}{b} + \frac{\log(\sinh(a+b}{b^3}$$
$$= \frac{x^2}{2b} - \frac{x^3}{3} - \frac{x\coth(a+bx)}{b^2} - \frac{x^2\coth^2(a+bx)}{2b} + \frac{x^2\log\left(1-e^{2(a+bx)}\right)}{b} + \frac{\log(\sinh(a+b}{b^3}$$

Mathematica [B] time = 2.21, size = 295, normalized size = 2.59

$$\frac{x\text{csch}(a)\sinh(bx)\text{csch}(a+bx)}{b^2} - \frac{e^{2a}\left(2e^{-2a}b^3x^3 - 3e^{-2a}\left(e^{2a}-1\right)b^2x^2\log\left(1-e^{-a-bx}\right) - 3e^{-2a}\left(e^{2a}-1\right)b^2x^2\log\left(\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Coth[a + b*x]^3,x]
```

```
[Out] (x^3*Coth[a])/3 - (x^2*Csch[a + b*x]^2)/(2*b) - (E^(2*a)*((6*b*x)/E^(2*a) +
 (2*b^3*x^3)/E^(2*a) - (3*b^2*(-1 + E^(2*a))*x^2*Log[1 - E^(-a - b*x)])/E^(
2*a) - (3*b^2*(-1 + E^(2*a))*x^2*Log[1 + E^(-a - b*x)])/E^(2*a) + 3*(1 - E^
(-2*a))*(b*x - Log[1 - E^(a + b*x)]) + 3*(1 - E^(-2*a))*(b*x - Log[1 + E^(a
 + b*x)]) + 6*(1 - E^(-2*a))*(b*x*PolyLog[2, -E^(-a - b*x)] + PolyLog[3, -E
^(-a - b*x)]) + 6*(1 - E^(-2*a))*(b*x*PolyLog[2, E^(-a - b*x)] + PolyLog[3,
 E^(-a - b*x)])))/(3*b^3*(-1 + E^(2*a))) + (x*Csch[a]*Csch[a + b*x]*Sinh[b*
x])/b^2
```

fricas [C] time = 0.43, size = 1467, normalized size = 12.87

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cosh(b*x+a)^3*csch(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -1/3*(b^3*x^3 + (b^3*x^3 + 2*a^3 + 6*b*x + 6*a)*cosh(b*x + a)^4 + 4*(b^3*x^
3 + 2*a^3 + 6*b*x + 6*a)*cosh(b*x + a)*sinh(b*x + a)^3 + (b^3*x^3 + 2*a^3 +
```

```
 6*b*x + 6*a)*sinh(b*x + a)^4 + 2*a^3 - 2*(b^3*x^3 - 3*b^2*x^2 + 2*a^3 + 3*
b*x + 6*a)*cosh(b*x + a)^2 - 2*(b^3*x^3 - 3*b^2*x^2 + 2*a^3 - 3*(b^3*x^3 +
2*a^3 + 6*b*x + 6*a)*cosh(b*x + a)^2 + 3*b*x + 6*a)*sinh(b*x + a)^2 - 6*(b*
x*cosh(b*x + a)^4 + 4*b*x*cosh(b*x + a)*sinh(b*x + a)^3 + b*x*sinh(b*x + a)
^4 - 2*b*x*cosh(b*x + a)^2 + 2*(3*b*x*cosh(b*x + a)^2 - b*x)*sinh(b*x + a)^
2 + b*x + 4*(b*x*cosh(b*x + a)^3 - b*x*cosh(b*x + a))*sinh(b*x + a))*dilog(
cosh(b*x + a) + sinh(b*x + a)) - 6*(b*x*cosh(b*x + a)^4 + 4*b*x*cosh(b*x +
a)*sinh(b*x + a)^3 + b*x*sinh(b*x + a)^4 - 2*b*x*cosh(b*x + a)^2 + 2*(3*b*x
*cosh(b*x + a)^2 - b*x)*sinh(b*x + a)^2 + b*x + 4*(b*x*cosh(b*x + a)^3 - b*
x*cosh(b*x + a))*sinh(b*x + a))*dilog(-cosh(b*x + a) - sinh(b*x + a)) - 3*(
(b^2*x^2 + 1)*cosh(b*x + a)^4 + 4*(b^2*x^2 + 1)*cosh(b*x + a)*sinh(b*x + a)
^3 + (b^2*x^2 + 1)*sinh(b*x + a)^4 + b^2*x^2 - 2*(b^2*x^2 + 1)*cosh(b*x + a
)^2 - 2*(b^2*x^2 - 3*(b^2*x^2 + 1)*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^2 + 4
*((b^2*x^2 + 1)*cosh(b*x + a)^3 - (b^2*x^2 + 1)*cosh(b*x + a))*sinh(b*x + a
) + 1)*log(cosh(b*x + a) + sinh(b*x + a) + 1) - 3*((a^2 + 1)*cosh(b*x + a)^
4 + 4*(a^2 + 1)*cosh(b*x + a)*sinh(b*x + a)^3 + (a^2 + 1)*sinh(b*x + a)^4 -
 2*(a^2 + 1)*cosh(b*x + a)^2 + 2*(3*(a^2 + 1)*cosh(b*x + a)^2 - a^2 - 1)*si
nh(b*x + a)^2 + a^2 + 4*((a^2 + 1)*cosh(b*x + a)^3 - (a^2 + 1)*cosh(b*x + a
))*sinh(b*x + a) + 1)*log(cosh(b*x + a) + sinh(b*x + a) - 1) - 3*((b^2*x^2
- a^2)*cosh(b*x + a)^4 + 4*(b^2*x^2 - a^2)*cosh(b*x + a)*sinh(b*x + a)^3 +
(b^2*x^2 - a^2)*sinh(b*x + a)^4 + b^2*x^2 - 2*(b^2*x^2 - a^2)*cosh(b*x + a)
^2 - 2*(b^2*x^2 - 3*(b^2*x^2 - a^2)*cosh(b*x + a)^2 - a^2)*sinh(b*x + a)^2
- a^2 + 4*((b^2*x^2 - a^2)*cosh(b*x + a)^3 - (b^2*x^2 - a^2)*cosh(b*x + a))
*sinh(b*x + a))*log(-cosh(b*x + a) - sinh(b*x + a) + 1) + 6*(cosh(b*x + a)^
4 + 4*cosh(b*x + a)*sinh(b*x + a)^3 + sinh(b*x + a)^4 + 2*(3*cosh(b*x + a)^
2 - 1)*sinh(b*x + a)^2 - 2*cosh(b*x + a)^2 + 4*(cosh(b*x + a)^3 - cosh(b*x
+ a))*sinh(b*x + a) + 1)*polylog(3, cosh(b*x + a) + sinh(b*x + a)) + 6*(cos
h(b*x + a)^4 + 4*cosh(b*x + a)*sinh(b*x + a)^3 + sinh(b*x + a)^4 + 2*(3*cos
h(b*x + a)^2 - 1)*sinh(b*x + a)^2 - 2*cosh(b*x + a)^2 + 4*(cosh(b*x + a)^3
- cosh(b*x + a))*sinh(b*x + a) + 1)*polylog(3, -cosh(b*x + a) - sinh(b*x +
a)) + 4*((b^3*x^3 + 2*a^3 + 6*b*x + 6*a)*cosh(b*x + a)^3 - (b^3*x^3 - 3*b^2
*x^2 + 2*a^3 + 3*b*x + 6*a)*cosh(b*x + a))*sinh(b*x + a) + 6*a)/(b^3*cosh(b
*x + a)^4 + 4*b^3*cosh(b*x + a)*sinh(b*x + a)^3 + b^3*sinh(b*x + a)^4 - 2*b
^3*cosh(b*x + a)^2 + b^3 + 2*(3*b^3*cosh(b*x + a)^2 - b^3)*sinh(b*x + a)^2
+ 4*(b^3*cosh(b*x + a)^3 - b^3*cosh(b*x + a))*sinh(b*x + a))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \cosh(bx+a)^3 \operatorname{csch}(bx+a)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cosh(b*x+a)^3*csch(b*x+a)^3,x, algorithm="giac")
```

```
[Out] integrate(x^2*cosh(b*x + a)^3*csch(b*x + a)^3, x)
```

maple [B] time = 0.60, size = 246, normalized size = 2.16

$$-\frac{x^3}{3}-\frac{2x\left(bx\,e^{2bx+2a}+e^{2bx+2a}-1\right)}{b^2\left(e^{2bx+2a}-1\right)^2}+\frac{4a^3}{3b^3}+\frac{a^2\ln\left(e^{bx+a}-1\right)}{b^3}-\frac{2a^2\ln\left(e^{bx+a}\right)}{b^3}+\frac{\ln\left(1-e^{bx+a}\right)x^2}{b}+\frac{2\,\mathrm{polylog}\left(2,e^{bx+}\right.}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*cosh(b*x+a)^3*csch(b*x+a)^3,x)
```

```
[Out] -1/3*x^3-2*x*(b*x*exp(2*b*x+2*a)+exp(2*b*x+2*a)-1)/b^2/(exp(2*b*x+2*a)-1)^2
+4/3/b^3*a^3+1/b^3*a^2*ln(exp(b*x+a)-1)-2/b^3*a^2*ln(exp(b*x+a))+1/b*ln(1-e
xp(b*x+a))*x^2+2/b^2*polylog(2,exp(b*x+a))*x+1/b*ln(1+exp(b*x+a))*x^2+2/b^2
*polylog(2,-exp(b*x+a))*x+2/b^2*a^2*x-2/b^3*ln(exp(b*x+a))+1/b^3*ln(exp(b*x
+a)-1)+1/b^3*ln(1+exp(b*x+a))-2/b^3*polylog(3,exp(b*x+a))-2/b^3*polylog(3,-
exp(b*x+a))-1/b^3*ln(1-exp(b*x+a))*a^2
```

maxima [B] time = 0.72, size = 226, normalized size = 1.98

$$-\frac{2}{3}x^3+\frac{b^2x^3e^{(4\,bx+4\,a)}+b^2x^3-2\left(b^2x^3e^{(2\,a)}+3\,bx^2e^{(2\,a)}+3\,xe^{(2\,a)}\right)e^{(2\,bx)}+6\,x}{3\left(b^2e^{(4\,bx+4\,a)}-2\,b^2e^{(2\,bx+2\,a)}+b^2\right)}-\frac{2\,x}{b^2}+\frac{b^2x^2\log\left(e^{(bx+a)}+1\right)+2\,bx}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cosh(b*x+a)^3*csch(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -2/3*x^3 + 1/3*(b^2*x^3*e^(4*b*x + 4*a) + b^2*x^3 - 2*(b^2*x^3*e^(2*a) + 3*
b*x^2*e^(2*a) + 3*x*e^(2*a))*e^(2*b*x) + 6*x)/(b^2*e^(4*b*x + 4*a) - 2*b^2*
e^(2*b*x + 2*a) + b^2) - 2*x/b^2 + (b^2*x^2*log(e^(b*x + a) + 1) + 2*b*x*di
log(-e^(b*x + a)) - 2*polylog(3, -e^(b*x + a)))/b^3 + (b^2*x^2*log(-e^(b*x
+ a) + 1) + 2*b*x*dilog(e^(b*x + a)) - 2*polylog(3, e^(b*x + a)))/b^3 + log
(e^(b*x + a) + 1)/b^3 + log(e^(b*x + a) - 1)/b^3
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2\cosh\left(a+b\,x\right)^3}{\sinh\left(a+b\,x\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*cosh(a + b*x)^3)/sinh(a + b*x)^3,x)
```

```
[Out] int((x^2*cosh(a + b*x)^3)/sinh(a + b*x)^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*cosh(b*x+a)**3*csch(b*x+a)**3,x)
```

```
[Out] Timed out
```

3.462 $\int x \coth^3(a+bx)\,dx$

Optimal. Leaf size=82

$$\frac{\text{Li}_2\left(e^{2(a+bx)}\right)}{2b^2}-\frac{\coth(a+bx)}{2b^2}+\frac{x\log\left(1-e^{2(a+bx)}\right)}{b}-\frac{x\coth^2(a+bx)}{2b}+\frac{x}{2b}-\frac{x^2}{2}$$

```
[Out] 1/2*x/b-1/2*x^2-1/2*coth(b*x+a)/b^2-1/2*x*coth(b*x+a)^2/b+x*ln(1-exp(2*b*x+
2*a))/b+1/2*polylog(2,exp(2*b*x+2*a))/b^2
```

Rubi [A] time = 0.12, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.700, Rules used = {3720, 3473, 8, 3716, 2190, 2279, 2391}

$$\frac{\text{PolyLog}\left(2,e^{2(a+bx)}\right)}{2b^2}-\frac{\coth(a+bx)}{2b^2}+\frac{x\log\left(1-e^{2(a+bx)}\right)}{b}-\frac{x\coth^2(a+bx)}{2b}+\frac{x}{2b}-\frac{x^2}{2}$$

Antiderivative was successfully verified.

```
[In] Int[x*Coth[a + b*x]^3,x]
```

```
[Out] x/(2*b) - x^2/2 - Coth[a + b*x]/(2*b^2) - (x*Coth[a + b*x]^2)/(2*b) + (x*Lo
g[1 - E^(2*(a + b*x))])/b + PolyLog[2, E^(2*(a + b*x))]/(2*b^2)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3716

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2
*I, Int[((c + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(E^(2*I*k*Pi)*(1 + E^(2*(-(I*
e) + f*fz*x))/E^(2*I*k*Pi))), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Integ
erQ[4*k] && IGtQ[m, 0]
```

Rule 3720

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[(b*(c + d*x)^m*(b*Tan[e + f*x])^(n - 1))/(f*(n - 1)), x] + (-Di
st[(b*d*m)/(f*(n - 1)), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x],
 x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[
{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\int x\coth^3(a+bx)\,dx &= -\frac{x\coth^2(a+bx)}{2b}+\frac{\int \coth^2(a+bx)\,dx}{2b}+\int x\coth(a+bx)\,dx\\
&= -\frac{x^2}{2}-\frac{\coth(a+bx)}{2b^2}-\frac{x\coth^2(a+bx)}{2b}-2\int \frac{e^{2(a+bx)}x}{1-e^{2(a+bx)}}\,dx+\frac{\int 1\,dx}{2b}\\
&= \frac{x}{2b}-\frac{x^2}{2}-\frac{\coth(a+bx)}{2b^2}-\frac{x\coth^2(a+bx)}{2b}+\frac{x\log\left(1-e^{2(a+bx)}\right)}{b}-\frac{\int \log\left(1-e^{2(a+bx)}\right)\,dx}{b}\\
&= \frac{x}{2b}-\frac{x^2}{2}-\frac{\coth(a+bx)}{2b^2}-\frac{x\coth^2(a+bx)}{2b}+\frac{x\log\left(1-e^{2(a+bx)}\right)}{b}-\frac{\text{Subst}\left(\int \frac{\log(1-x)}{x}\,dx,x\right.}{2b^2}\\
&= \frac{x}{2b}-\frac{x^2}{2}-\frac{\coth(a+bx)}{2b^2}-\frac{x\coth^2(a+bx)}{2b}+\frac{x\log\left(1-e^{2(a+bx)}\right)}{b}+\frac{\text{Li}_2\left(e^{2(a+bx)}\right)}{2b^2}
\end{aligned}
$$

Mathematica [C] time = 6.13, size = 231, normalized size = 2.82

$$-\frac{\operatorname{csch}(a)\operatorname{sech}(a)\left(b^2x^2e^{-\tanh^{-1}(\tanh(a))}-\frac{i\tanh(a)\left(i\operatorname{Li}_2\left(e^{2i\left(ibx+i\tanh^{-1}(\tanh(a))\right)}\right)-bx\left(-\pi+2i\tanh^{-1}(\tanh(a))\right)-2\left(i\tanh^{-1}(\tanh(a))+ibx\right)\right.}{}\right.}{2b^2\sqrt{\operatorname{sech}^2(a)\left(\cosh^2(a)-\operatorname{s}\right.}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x*Coth[a + b*x]^3,x]
```

```
[Out] (x^2*Coth[a])/2 - (x*Csch[a + b*x]^2)/(2*b) + (Csch[a]*Csch[a + b*x]*Sinh[b
*x])/(2*b^2) - (Csch[a]*Sech[a]*((b^2*x^2)/E^ArcTanh[Tanh[a]] - (I*(-(b*x*(
-Pi + (2*I)*ArcTanh[Tanh[a]])) - Pi*Log[1 + E^(2*b*x)] - 2*(I*b*x + I*ArcTa
nh[Tanh[a]])*Log[1 - E^((2*I)*(I*b*x + I*ArcTanh[Tanh[a]]))] + Pi*Log[Cosh[
b*x]] + (2*I)*ArcTanh[Tanh[a]]*Log[I*Sinh[b*x + ArcTanh[Tanh[a]]]] + I*Poly
Log[2, E^((2*I)*(I*b*x + I*ArcTanh[Tanh[a]]))])*Tanh[a])/Sqrt[1 - Tanh[a]^2
]))/(2*b^2*Sqrt[Sech[a]^2*(Cosh[a]^2 - Sinh[a]^2)])
```

fricas [B] time = 0.43, size = 975, normalized size = 11.89

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x+a)^3*csch(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -1/2*((b^2*x^2 - 2*a^2)*cosh(b*x + a)^4 + 4*(b^2*x^2 - 2*a^2)*cosh(b*x + a)
*sinh(b*x + a)^3 + (b^2*x^2 - 2*a^2)*sinh(b*x + a)^4 + b^2*x^2 - 2*(b^2*x^2
 - 2*a^2 - 2*b*x - 1)*cosh(b*x + a)^2 - 2*(b^2*x^2 - 3*(b^2*x^2 - 2*a^2)*co
sh(b*x + a)^2 - 2*a^2 - 2*b*x - 1)*sinh(b*x + a)^2 - 2*a^2 - 2*(cosh(b*x +
a)^4 + 4*cosh(b*x + a)*sinh(b*x + a)^3 + sinh(b*x + a)^4 + 2*(3*cosh(b*x +
a)^2 - 1)*sinh(b*x + a)^2 - 2*cosh(b*x + a)^2 + 4*(cosh(b*x + a)^3 - cosh(b
*x + a))*sinh(b*x + a) + 1)*dilog(cosh(b*x + a) + sinh(b*x + a)) - 2*(cosh(
b*x + a)^4 + 4*cosh(b*x + a)*sinh(b*x + a)^3 + sinh(b*x + a)^4 + 2*(3*cosh(
b*x + a)^2 - 1)*sinh(b*x + a)^2 - 2*cosh(b*x + a)^2 + 4*(cosh(b*x + a)^3 -
cosh(b*x + a))*sinh(b*x + a) + 1)*dilog(-cosh(b*x + a) - sinh(b*x + a)) - 2
*(b*x*cosh(b*x + a)^4 + 4*b*x*cosh(b*x + a)*sinh(b*x + a)^3 + b*x*sinh(b*x
+ a)^4 - 2*b*x*cosh(b*x + a)^2 + 2*(3*b*x*cosh(b*x + a)^2 - b*x)*sinh(b*x +
 a)^2 + b*x + 4*(b*x*cosh(b*x + a)^3 - b*x*cosh(b*x + a))*sinh(b*x + a))*lo
g(cosh(b*x + a) + sinh(b*x + a) + 1) + 2*(a*cosh(b*x + a)^4 + 4*a*cosh(b*x
+ a)*sinh(b*x + a)^3 + a*sinh(b*x + a)^4 - 2*a*cosh(b*x + a)^2 + 2*(3*a*cos
h(b*x + a)^2 - a)*sinh(b*x + a)^2 + 4*(a*cosh(b*x + a)^3 - a*cosh(b*x + a))
*sinh(b*x + a) + a)*log(cosh(b*x + a) + sinh(b*x + a) - 1) - 2*((b*x + a)*c
osh(b*x + a)^4 + 4*(b*x + a)*cosh(b*x + a)*sinh(b*x + a)^3 + (b*x + a)*sinh
```

```
(b*x + a)^4 - 2*(b*x + a)*cosh(b*x + a)^2 + 2*(3*(b*x + a)*cosh(b*x + a)^2
- b*x - a)*sinh(b*x + a)^2 + b*x + 4*((b*x + a)*cosh(b*x + a)^3 - (b*x + a)
*cosh(b*x + a))*sinh(b*x + a) + a)*log(-cosh(b*x + a) - sinh(b*x + a) + 1)
+ 4*((b^2*x^2 - 2*a^2)*cosh(b*x + a)^3 - (b^2*x^2 - 2*a^2 - 2*b*x - 1)*cosh
(b*x + a))*sinh(b*x + a) - 2)/(b^2*cosh(b*x + a)^4 + 4*b^2*cosh(b*x + a)*si
nh(b*x + a)^3 + b^2*sinh(b*x + a)^4 - 2*b^2*cosh(b*x + a)^2 + 2*(3*b^2*cosh
(b*x + a)^2 - b^2)*sinh(b*x + a)^2 + b^2 + 4*(b^2*cosh(b*x + a)^3 - b^2*cos
h(b*x + a))*sinh(b*x + a))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x \cosh(bx+a)^3 \operatorname{csch}(bx+a)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x+a)^3*csch(b*x+a)^3,x, algorithm="giac")
```

```
[Out] integrate(x*cosh(b*x + a)^3*csch(b*x + a)^3, x)
```

maple [B] time = 0.59, size = 164, normalized size = 2.00

$$-\frac{x^2}{2}-\frac{2bx\,e^{2bx+2a}+e^{2bx+2a}-1}{b^2\left(e^{2bx+2a}-1\right)^2}-\frac{2ax}{b}-\frac{a^2}{b^2}+\frac{\ln\left(1-e^{bx+a}\right)x}{b}+\frac{\ln\left(1-e^{bx+a}\right)a}{b^2}+\frac{\operatorname{polylog}\left(2,e^{bx+a}\right)}{b^2}+\frac{\ln\left(1+e^{bx+a}\right)x}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cosh(b*x+a)^3*csch(b*x+a)^3,x)
```

```
[Out] -1/2*x^2-(2*b*x*exp(2*b*x+2*a)+exp(2*b*x+2*a)-1)/b^2/(exp(2*b*x+2*a)-1)^2-2
/b*a*x-a^2/b^2+1/b*ln(1-exp(b*x+a))*x+1/b^2*ln(1-exp(b*x+a))*a+polylog(2,ex
p(b*x+a))/b^2+1/b*ln(1+exp(b*x+a))*x+polylog(2,-exp(b*x+a))/b^2+2/b^2*a*ln(
exp(b*x+a))-1/b^2*a*ln(exp(b*x+a)-1)
```

maxima [B] time = 0.38, size = 149, normalized size = 1.82

$$-x^2+\frac{b^2x^2e^{(4\,bx+4\,a)}+b^2x^2-2\left(b^2x^2e^{(2\,a)}+2\,bxe^{(2\,a)}+e^{(2\,a)}\right)e^{(2\,bx)}+2}{2\left(b^2e^{(4\,bx+4\,a)}-2\,b^2e^{(2\,bx+2\,a)}+b^2\right)}+\frac{bx\log\left(e^{(bx+a)}+1\right)+\operatorname{Li}_2\left(-e^{(bx+a)}\right)}{b^2}+\frac{bx\log}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x+a)^3*csch(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -x^2 + 1/2*(b^2*x^2*e^(4*b*x + 4*a) + b^2*x^2 - 2*(b^2*x^2*e^(2*a) + 2*b*x*
e^(2*a) + e^(2*a))*e^(2*b*x) + 2)/(b^2*e^(4*b*x + 4*a) - 2*b^2*e^(2*b*x + 2
```

```
*a) + b^2) + (b*x*log(e^(b*x + a) + 1) + dilog(-e^(b*x + a)))/b^2 + (b*x*lo
g(-e^(b*x + a) + 1) + dilog(e^(b*x + a)))/b^2
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x \cosh (a + b x)^3}{\sinh (a + b x)^3} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*cosh(a + b*x)^3)/sinh(a + b*x)^3,x)
[Out] int((x*cosh(a + b*x)^3)/sinh(a + b*x)^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x+a)**3*csch(b*x+a)**3,x)
[Out] Timed out
```

3.463 $\int \coth^3(a+bx)\,dx$

Optimal. Leaf size=27

$$\frac{\log(\sinh(a+bx))}{b}-\frac{\coth^2(a+bx)}{2b}$$

```
[Out] -1/2*coth(b*x+a)^2/b+ln(sinh(b*x+a))/b
```

Rubi [A] time = 0.02, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3473, 3475}

$$\frac{\log(\sinh(a+bx))}{b}-\frac{\coth^2(a+bx)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[a + b*x]^3,x]
```

```
[Out] -Coth[a + b*x]^2/(2*b) + Log[Sinh[a + b*x]]/b
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \coth^3(a+bx)\,dx &= -\frac{\coth^2(a+bx)}{2b}+\int \coth(a+bx)\,dx\\ &= -\frac{\coth^2(a+bx)}{2b}+\frac{\log(\sinh(a+bx))}{b}\end{aligned}$$

Mathematica [A] time = 0.08, size = 34, normalized size = 1.26

$$-\frac{\coth^2(a+bx)-2\log(\tanh(a+bx))-2\log(\cosh(a+bx))}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[a + b*x]^3,x]
```

```
[Out] -1/2*(Coth[a + b*x]^2 - 2*Log[Cosh[a + b*x]] - 2*Log[Tanh[a + b*x]])/b
```

fricas [B] time = 0.42, size = 346, normalized size = 12.81

$$-\frac{bx\cosh(bx+a)^4 + 4\,bx\cosh(bx+a)\sinh(bx+a)^3 + bx\sinh(bx+a)^4 - 2\,(bx-1)\cosh(bx+a)^2 + 2\left(3\,bx\,\mathrm{c}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^3*csch(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -(b*x*cosh(b*x + a)^4 + 4*b*x*cosh(b*x + a)*sinh(b*x + a)^3 + b*x*sinh(b*x
+ a)^4 - 2*(b*x - 1)*cosh(b*x + a)^2 + 2*(3*b*x*cosh(b*x + a)^2 - b*x + 1)*
sinh(b*x + a)^2 + b*x - (cosh(b*x + a)^4 + 4*cosh(b*x + a)*sinh(b*x + a)^3
+ sinh(b*x + a)^4 + 2*(3*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^2 - 2*cosh(b*x
+ a)^2 + 4*(cosh(b*x + a)^3 - cosh(b*x + a))*sinh(b*x + a) + 1)*log(2*sinh(
b*x + a)/(cosh(b*x + a) - sinh(b*x + a))) + 4*(b*x*cosh(b*x + a)^3 - (b*x -
 1)*cosh(b*x + a))*sinh(b*x + a))/(b*cosh(b*x + a)^4 + 4*b*cosh(b*x + a)*si
nh(b*x + a)^3 + b*sinh(b*x + a)^4 - 2*b*cosh(b*x + a)^2 + 2*(3*b*cosh(b*x +
 a)^2 - b)*sinh(b*x + a)^2 + 4*(b*cosh(b*x + a)^3 - b*cosh(b*x + a))*sinh(b
*x + a) + b)
```

giac [B] time = 0.14, size = 66, normalized size = 2.44

$$-\frac{2\,bx + 2\,a + \frac{3\,e^{(4\,bx+4\,a)} - 2\,e^{(2\,bx+2\,a)} + 3}{\left(e^{(2\,bx+2\,a)} - 1\right)^2} - 2\,\log\left(\left|e^{(2\,bx+2\,a)} - 1\right|\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^3*csch(b*x+a)^3,x, algorithm="giac")
```

```
[Out] -1/2*(2*b*x + 2*a + (3*e^(4*b*x + 4*a) - 2*e^(2*b*x + 2*a) + 3)/(e^(2*b*x +
 2*a) - 1)^2 - 2*log(abs(e^(2*b*x + 2*a) - 1)))/b
```

maple [A] time = 0.17, size = 26, normalized size = 0.96

$$-\frac{\coth^2(bx+a)}{2b} + \frac{\ln(\sinh(bx+a))}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)^3*csch(b*x+a)^3,x)
```

```
[Out] -1/2*coth(b*x+a)^2/b+ln(sinh(b*x+a))/b
```

maxima [B] time = 0.43, size = 79, normalized size = 2.93

$$x + \frac{a}{b} + \frac{\log\left(e^{(-bx-a)} + 1\right)}{b} + \frac{\log\left(e^{(-bx-a)} - 1\right)}{b} + \frac{2\,e^{(-2\,bx-2\,a)}}{b\left(2\,e^{(-2\,bx-2\,a)} - e^{(-4\,bx-4\,a)} - 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^3*csch(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] x + a/b + log(e^(-b*x - a) + 1)/b + log(e^(-b*x - a) - 1)/b + 2*e^(-2*b*x -
 2*a)/(b*(2*e^(-2*b*x - 2*a) - e^(-4*b*x - 4*a) - 1))
```

mupad [B] time = 1.50, size = 25, normalized size = 0.93

$$\frac{\ln\left(\sinh\left(a + b\,x\right)\right)}{b} - \frac{1}{2\,b\,\sinh\left(a + b\,x\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a + b*x)^3/sinh(a + b*x)^3,x)
```

```
[Out] log(sinh(a + b*x))/b - 1/(2*b*sinh(a + b*x)^2)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)**3*csch(b*x+a)**3,x)
```

```
[Out] Timed out
```

3.464 $\int \frac{\coth^3(a+bx)}{x} dx$

Optimal. Leaf size=15

$$\operatorname{Int}\left(\frac{\coth^3(a+bx)}{x},x\right)$$

```
[Out] Unintegrable(coth(b*x+a)^3/x,x)
```

Rubi [A] time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\coth^3(a+bx)}{x} dx$$

Verification is Not applicable to the result.

```
[In] Int[Coth[a + b*x]^3/x,x]
[Out] Defer[Int][Coth[a + b*x]^3/x, x]
```

Rubi steps

$$\int \frac{\coth^3(a+bx)}{x} dx = \int \frac{\coth^3(a+bx)}{x} dx$$

Mathematica [A] time = 0.75, size = 0, normalized size = 0.00

$$\int \frac{\coth^3(a+bx)}{x} dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Coth[a + b*x]^3/x,x]
[Out] Integrate[Coth[a + b*x]^3/x, x]
```

fricas [A] time = 0.40, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{\cosh(bx+a)^3 \operatorname{csch}(bx+a)^3}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^3*csch(b*x+a)^3/x,x, algorithm="fricas")
```

```
[Out] integral(cosh(b*x + a)^3*csch(b*x + a)^3/x, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh\left(bx+a\right)^{3} \operatorname{csch}\left(bx+a\right)^{3}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^3*csch(b*x+a)^3/x,x, algorithm="giac")
```

```
[Out] integrate(cosh(b*x + a)^3*csch(b*x + a)^3/x, x)
```

maple [A] time = 1.35, size = 0, normalized size = 0.00

$$\int \frac{\left(\cosh^{3}\left(bx+a\right)\right) \operatorname{csch}\left(bx+a\right)^{3}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)^3*csch(b*x+a)^3/x,x)
```

```
[Out] int(cosh(b*x+a)^3*csch(b*x+a)^3/x,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\left(2\,bxe^{(2\,a)}-e^{(2\,a)}\right)e^{(2\,bx)}+1}{b^{2}x^{2}e^{(4\,bx+4\,a)}-2\,b^{2}x^{2}e^{(2\,bx+2\,a)}+b^{2}x^{2}}-\int \frac{b^{2}x^{2}+1}{b^{2}x^{3}e^{(bx+a)}+b^{2}x^{3}}\,dx+\int \frac{b^{2}x^{2}+1}{b^{2}x^{3}e^{(bx+a)}-b^{2}x^{3}}\,dx+\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^3*csch(b*x+a)^3/x,x, algorithm="maxima")
```

```
[Out] -((2*b*x*e^(2*a) - e^(2*a))*e^(2*b*x) + 1)/(b^2*x^2*e^(4*b*x + 4*a) - 2*b^2
*x^2*e^(2*b*x + 2*a) + b^2*x^2) - integrate((b^2*x^2 + 1)/(b^2*x^3*e^(b*x +
 a) + b^2*x^3), x) + integrate((b^2*x^2 + 1)/(b^2*x^3*e^(b*x + a) - b^2*x^3
), x) + log(x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.07

$$\int \frac{\cosh\left(a+b\,x\right)^{3}}{x\,\sinh\left(a+b\,x\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a + b*x)^3/(x*sinh(a + b*x)^3),x)
```

```
[Out] int(cosh(a + b*x)^3/(x*sinh(a + b*x)^3), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh^3(a+bx)\operatorname{csch}^3(a+bx)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)**3*csch(b*x+a)**3/x,x)
```

```
[Out] Integral(cosh(a + b*x)**3*csch(a + b*x)**3/x, x)
```

3.465 $\int \frac{\coth^3(a+bx)}{x^2} dx$

Optimal. Leaf size=15

$$\text{Int}\left(\frac{\coth^3(a+bx)}{x^2},x\right)$$

```
[Out] Unintegrable(coth(b*x+a)^3/x^2,x)
```

Rubi [A] time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\coth^3(a+bx)}{x^2} dx$$

Verification is Not applicable to the result.

```
[In] Int[Coth[a + b*x]^3/x^2,x]
[Out] Defer[Int][Coth[a + b*x]^3/x^2, x]
```

Rubi steps

$$\int \frac{\coth^3(a+bx)}{x^2} dx = \int \frac{\coth^3(a+bx)}{x^2} dx$$

Mathematica [A] time = 0.43, size = 0, normalized size = 0.00

$$\int \frac{\coth^3(a+bx)}{x^2} dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Coth[a + b*x]^3/x^2,x]
[Out] Integrate[Coth[a + b*x]^3/x^2, x]
```

fricas [A] time = 0.41, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\cosh(bx+a)^3 \operatorname{csch}(bx+a)^3}{x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^3*csch(b*x+a)^3/x^2,x, algorithm="fricas")
[Out] integral(cosh(b*x + a)^3*csch(b*x + a)^3/x^2, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh(bx+a)^3 \operatorname{csch}(bx+a)^3}{x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^3*csch(b*x+a)^3/x^2,x, algorithm="giac")
[Out] integrate(cosh(b*x + a)^3*csch(b*x + a)^3/x^2, x)
```

maple [A] time = 1.36, size = 0, normalized size = 0.00

$$\int \frac{\left(\cosh^3(bx+a)\right) \operatorname{csch}(bx+a)^3}{x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)^3*csch(b*x+a)^3/x^2,x)
[Out] int(cosh(b*x+a)^3*csch(b*x+a)^3/x^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{b^2x^2e^{(4\,bx+4\,a)} + b^2x^2 - 2\left(b^2x^2e^{(2\,a)} - bxe^{(2\,a)} + e^{(2\,a)}\right)e^{(2\,bx)} + 2}{b^2x^3e^{(4\,bx+4\,a)} - 2\,b^2x^3e^{(2\,bx+2\,a)} + b^2x^3} - \int \frac{b^2x^2+3}{b^2x^4e^{(bx+a)} + b^2x^4}\, dx + \int \frac{b^2x^2+3}{b^2x^4e^{(bx+a)} - b^2x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^3*csch(b*x+a)^3/x^2,x, algorithm="maxima")
[Out] -(b^2*x^2*e^(4*b*x + 4*a) + b^2*x^2 - 2*(b^2*x^2*e^(2*a) - b*x*e^(2*a) + e^
(2*a))*e^(2*b*x) + 2)/(b^2*x^3*e^(4*b*x + 4*a) - 2*b^2*x^3*e^(2*b*x + 2*a)
+ b^2*x^3) - integrate((b^2*x^2 + 3)/(b^2*x^4*e^(b*x + a) + b^2*x^4), x) +
integrate((b^2*x^2 + 3)/(b^2*x^4*e^(b*x + a) - b^2*x^4), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.07

$$\int \frac{\cosh(a+b\,x)^3}{x^2\sinh(a+b\,x)^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a + b*x)^3/(x^2*sinh(a + b*x)^3),x)
```

```
[Out] int(cosh(a + b*x)^3/(x^2*sinh(a + b*x)^3), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)**3*csch(b*x+a)**3/x**2,x)
```

```
[Out] Timed out
```

3.466 $\int x^m \operatorname{csch}(a+bx)\operatorname{sech}(a+bx)\,dx$

Optimal. Leaf size=19

$$\text{Int}\left(x^m\text{csch}(a+bx)\text{sech}(a+bx),x\right)$$

[Out] CannotIntegrate(x^m*csch(b*x+a)*sech(b*x+a),x)

Rubi [A] time = 0.21, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int x^m\text{csch}(a+bx)\text{sech}(a+bx)\,dx$$

Verification is Not applicable to the result.

[In] Int[x^m*Csch[a + b*x]*Sech[a + b*x],x]

[Out] Defer[Int][x^m*Csch[a + b*x]*Sech[a + b*x], x]

Rubi steps

$$\int x^m\text{csch}(a+bx)\text{sech}(a+bx)\,dx=\int x^m\text{csch}(a+bx)\text{sech}(a+bx)\,dx$$

Mathematica [A] time = 9.55, size = 0, normalized size = 0.00

$$\int x^m\text{csch}(a+bx)\text{sech}(a+bx)\,dx$$

Verification is Not applicable to the result.

[In] Integrate[x^m*Csch[a + b*x]*Sech[a + b*x],x]

[Out] Integrate[x^m*Csch[a + b*x]*Sech[a + b*x], x]

fricas [A] time = 0.41, size = 0, normalized size = 0.00

$$\text{integral}\left(x^m\,\text{csch}\left(bx+a\right)\text{sech}\left(bx+a\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^m*csch(b*x+a)*sech(b*x+a),x, algorithm="fricas")

[Out] integral(x^m*csch(b*x + a)*sech(b*x + a), x)

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int x^m \operatorname{csch}(bx+a)\operatorname{sech}(bx+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*csch(b*x+a)*sech(b*x+a),x, algorithm="giac")
[Out] integrate(x^m*csch(b*x + a)*sech(b*x + a), x)
```

maple [A] time = 0.20, size = 0, normalized size = 0.00

$$\int x^m \operatorname{csch}(bx+a)\operatorname{sech}(bx+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*csch(b*x+a)*sech(b*x+a),x)
[Out] int(x^m*csch(b*x+a)*sech(b*x+a),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int x^m \operatorname{csch}(bx+a)\operatorname{sech}(bx+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*csch(b*x+a)*sech(b*x+a),x, algorithm="maxima")
[Out] integrate(x^m*csch(b*x + a)*sech(b*x + a), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{x^m}{\cosh(a+b\,x)\,\sinh(a+b\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(cosh(a + b*x)*sinh(a + b*x)),x)
[Out] int(x^m/(cosh(a + b*x)*sinh(a + b*x)), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int x^m \operatorname{csch}(a+bx)\operatorname{sech}(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*csch(b*x+a)*sech(b*x+a),x)
[Out] Integral(x**m*csch(a + b*x)*sech(a + b*x), x)
```

3.467 $\int x^3 \operatorname{csch}(a+bx) \operatorname{sech}(a+bx)\,dx$

Optimal. Leaf size=148

$$-\frac{3\text{Li}_4\left(-e^{2a+2bx}\right)}{4b^4}+\frac{3\text{Li}_4\left(e^{2a+2bx}\right)}{4b^4}+\frac{3x\text{Li}_3\left(-e^{2a+2bx}\right)}{2b^3}-\frac{3x\text{Li}_3\left(e^{2a+2bx}\right)}{2b^3}-\frac{3x^2\text{Li}_2\left(-e^{2a+2bx}\right)}{2b^2}+\frac{3x^2\text{Li}_2\left(e^{2a+2bx}\right)}{2b^2}-\frac{2x^3}{}$$

```
[Out] -2*x^3*arctanh(exp(2*b*x+2*a))/b-3/2*x^2*polylog(2,-exp(2*b*x+2*a))/b^2+3/2
*x^2*polylog(2,exp(2*b*x+2*a))/b^2+3/2*x*polylog(3,-exp(2*b*x+2*a))/b^3-3/2
*x*polylog(3,exp(2*b*x+2*a))/b^3-3/4*polylog(4,-exp(2*b*x+2*a))/b^4+3/4*pol
ylog(4,exp(2*b*x+2*a))/b^4
```

Rubi [A] time = 0.15, antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {5461, 4182, 2531, 6609, 2282, 6589}

$$-\frac{3x^2\text{PolyLog}\left(2,-e^{2a+2bx}\right)}{2b^2}+\frac{3x^2\text{PolyLog}\left(2,e^{2a+2bx}\right)}{2b^2}+\frac{3x\text{PolyLog}\left(3,-e^{2a+2bx}\right)}{2b^3}-\frac{3x\text{PolyLog}\left(3,e^{2a+2bx}\right)}{2b^3}-\frac{3\text{Po}}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Csch[a + b*x]*Sech[a + b*x],x]
```

```
[Out] (-2*x^3*ArcTanh[E^(2*a + 2*b*x)])/b - (3*x^2*PolyLog[2, -E^(2*a + 2*b*x)])/
(2*b^2) + (3*x^2*PolyLog[2, E^(2*a + 2*b*x)])/(2*b^2) + (3*x*PolyLog[3, -E^
(2*a + 2*b*x)])/(2*b^3) - (3*x*PolyLog[3, E^(2*a + 2*b*x)])/(2*b^3) - (3*Po
lyLog[4, -E^(2*a + 2*b*x)])/(4*b^4) + (3*PolyLog[4, E^(2*a + 2*b*x)])/(4*b^
4)
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5461

```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[2^n, Int[(c + d*x)^m*Csch[2*a + 2*b*x]
^n, x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[m] && IntegerQ[n]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int x^3\text{csch}(a+bx)\text{sech}(a+bx)\,dx = 2\int x^3\text{csch}(2a+2bx)\,dx$$
$$= -\frac{2x^3\tanh^{-1}\left(e^{2a+2bx}\right)}{b} - \frac{3\int x^2\log\left(1-e^{2a+2bx}\right)dx}{b} + \frac{3\int x^2\log\left(1+e^{2a+2\cdots}\right.}{b}$$
$$= -\frac{2x^3\tanh^{-1}\left(e^{2a+2bx}\right)}{b} - \frac{3x^2\text{Li}_2\left(-e^{2a+2bx}\right)}{2b^2} + \frac{3x^2\text{Li}_2\left(e^{2a+2bx}\right)}{2b^2} + \frac{3\int x\text{Li}_2\left(\cdots\right.}{}$$
$$= -\frac{2x^3\tanh^{-1}\left(e^{2a+2bx}\right)}{b} - \frac{3x^2\text{Li}_2\left(-e^{2a+2bx}\right)}{2b^2} + \frac{3x^2\text{Li}_2\left(e^{2a+2bx}\right)}{2b^2} + \frac{3x\text{Li}_3\left(-e\cdots\right.}{2b^3}$$
$$= -\frac{2x^3\tanh^{-1}\left(e^{2a+2bx}\right)}{b} - \frac{3x^2\text{Li}_2\left(-e^{2a+2bx}\right)}{2b^2} + \frac{3x^2\text{Li}_2\left(e^{2a+2bx}\right)}{2b^2} + \frac{3x\text{Li}_3\left(-e\cdots\right.}{2b^3}$$
$$= -\frac{2x^3\tanh^{-1}\left(e^{2a+2bx}\right)}{b} - \frac{3x^2\text{Li}_2\left(-e^{2a+2bx}\right)}{2b^2} + \frac{3x^2\text{Li}_2\left(e^{2a+2bx}\right)}{2b^2} + \frac{3x\text{Li}_3\left(-e\cdots\right.}{2b^3}$$

Mathematica [A] time = 4.35, size = 150, normalized size = 1.01

$$\frac{4b^3x^3\log\left(1-e^{2(a+bx)}\right) - 4b^3x^3\log\left(e^{2(a+bx)}+1\right) - 6b^2x^2\text{Li}_2\left(-e^{2(a+bx)}\right) + 6b^2x^2\text{Li}_2\left(e^{2(a+bx)}\right) + 6bx\text{Li}_3\left(-e^{2(a+bx)}\cdots\right.}{4b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Csch[a + b*x]*Sech[a + b*x],x]
```

```
[Out] (4*b^3*x^3*Log[1 - E^(2*(a + b*x))] - 4*b^3*x^3*Log[1 + E^(2*(a + b*x))] -
6*b^2*x^2*PolyLog[2, -E^(2*(a + b*x))] + 6*b^2*x^2*PolyLog[2, E^(2*(a + b*x
))] + 6*b*x*PolyLog[3, -E^(2*(a + b*x))] - 6*b*x*PolyLog[3, E^(2*(a + b*x))
] - 3*PolyLog[4, -E^(2*(a + b*x))] + 3*PolyLog[4, E^(2*(a + b*x))])/(4*b^4)
```

fricas [C] time = 0.44, size = 448, normalized size = 3.03

$$\frac{b^3x^3\log\left(\cosh\left(bx+a\right)+\sinh\left(bx+a\right)+1\right) + 3\,b^2x^2\text{Li}_2\left(\cosh\left(bx+a\right)+\sinh\left(bx+a\right)\right) - 3\,b^2x^2\text{Li}_2\left(i\,\cosh\left(bx\cdots\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*csch(b*x+a)*sech(b*x+a),x, algorithm="fricas")
```

```
[Out] (b^3*x^3*log(cosh(b*x + a) + sinh(b*x + a) + 1) + 3*b^2*x^2*dilog(cosh(b*x
+ a) + sinh(b*x + a)) - 3*b^2*x^2*dilog(I*cosh(b*x + a) + I*sinh(b*x + a))
- 3*b^2*x^2*dilog(-I*cosh(b*x + a) - I*sinh(b*x + a)) + 3*b^2*x^2*dilog(-co
sh(b*x + a) - sinh(b*x + a)) + a^3*log(cosh(b*x + a) + sinh(b*x + a) + I) +
```

```
 a^3*log(cosh(b*x + a) + sinh(b*x + a) - I) - a^3*log(cosh(b*x + a) + sinh(
b*x + a) - 1) - 6*b*x*polylog(3, cosh(b*x + a) + sinh(b*x + a)) + 6*b*x*pol
ylog(3, I*cosh(b*x + a) + I*sinh(b*x + a)) + 6*b*x*polylog(3, -I*cosh(b*x +
 a) - I*sinh(b*x + a)) - 6*b*x*polylog(3, -cosh(b*x + a) - sinh(b*x + a)) -
 (b^3*x^3 + a^3)*log(I*cosh(b*x + a) + I*sinh(b*x + a) + 1) - (b^3*x^3 + a^
3)*log(-I*cosh(b*x + a) - I*sinh(b*x + a) + 1) + (b^3*x^3 + a^3)*log(-cosh(
b*x + a) - sinh(b*x + a) + 1) + 6*polylog(4, cosh(b*x + a) + sinh(b*x + a))
 - 6*polylog(4, I*cosh(b*x + a) + I*sinh(b*x + a)) - 6*polylog(4, -I*cosh(b
*x + a) - I*sinh(b*x + a)) + 6*polylog(4, -cosh(b*x + a) - sinh(b*x + a)))/
b^4
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3 \operatorname{csch}(bx+a)\operatorname{sech}(bx+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*csch(b*x+a)*sech(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(x^3*csch(b*x + a)*sech(b*x + a), x)
```

maple [A] time = 0.45, size = 241, normalized size = 1.63

$$-\frac{3\,\mathrm{polylog}\left(4,-e^{2bx+2a}\right)}{4b^4}+\frac{6\,\mathrm{polylog}\left(4,e^{bx+a}\right)}{b^4}+\frac{6\,\mathrm{polylog}\left(4,-e^{bx+a}\right)}{b^4}+\frac{\ln\left(1-e^{bx+a}\right)a^3}{b^4}-\frac{3x^2\,\mathrm{polylog}\left(2,-e^{2bx+}\right.}{2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*csch(b*x+a)*sech(b*x+a),x)
```

```
[Out] -3/4*polylog(4,-exp(2*b*x+2*a))/b^4+6*polylog(4,exp(b*x+a))/b^4+6*polylog(4
,-exp(b*x+a))/b^4+1/b^4*ln(1-exp(b*x+a))*a^3-3/2*x^2*polylog(2,-exp(2*b*x+2
*a))/b^2+1/b*ln(1-exp(b*x+a))*x^3+3*x^2*polylog(2,exp(b*x+a))/b^2-6*x*polyl
og(3,exp(b*x+a))/b^3+1/b*ln(1+exp(b*x+a))*x^3+3*x^2*polylog(2,-exp(b*x+a))/
b^2-6*x*polylog(3,-exp(b*x+a))/b^3+3/2*x*polylog(3,-exp(2*b*x+2*a))/b^3-x^3
*ln(1+exp(2*b*x+2*a))/b-1/b^4*a^3*ln(exp(b*x+a)-1)
```

maxima [A] time = 0.55, size = 203, normalized size = 1.37

$$-\frac{4\,b^3x^3\log\left(e^{(2\,bx+2\,a)}+1\right)+6\,b^2x^2\mathrm{Li}_2\left(-e^{(2\,bx+2\,a)}\right)-6\,bx\mathrm{Li}_3(-e^{(2\,bx+2\,a)})+3\,\mathrm{Li}_4(-e^{(2\,bx+2\,a)})}{3\,b^4}+\frac{b^3x^3\log\left(e^{(bx+a)}+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*csch(b*x+a)*sech(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/3*(4*b^3*x^3*log(e^(2*b*x + 2*a) + 1) + 6*b^2*x^2*dilog(-e^(2*b*x + 2*a)
) - 6*b*x*polylog(3, -e^(2*b*x + 2*a)) + 3*polylog(4, -e^(2*b*x + 2*a)))/b^
4 + (b^3*x^3*log(e^(b*x + a) + 1) + 3*b^2*x^2*dilog(-e^(b*x + a)) - 6*b*x*p
olylog(3, -e^(b*x + a)) + 6*polylog(4, -e^(b*x + a)))/b^4 + (b^3*x^3*log(-e
^(b*x + a) + 1) + 3*b^2*x^2*dilog(e^(b*x + a)) - 6*b*x*polylog(3, e^(b*x +
a)) + 6*polylog(4, e^(b*x + a)))/b^4
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3}{\cosh(a+bx)\,\sinh(a+bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(cosh(a + b*x)*sinh(a + b*x)),x)
```

```
[Out] int(x^3/(cosh(a + b*x)*sinh(a + b*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3 \operatorname{csch}(a+bx) \operatorname{sech}(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*csch(b*x+a)*sech(b*x+a),x)
```

```
[Out] Integral(x**3*csch(a + b*x)*sech(a + b*x), x)
```

3.468 $\int x^2\mathrm{csch}(a+bx)\mathrm{sech}(a+bx)\,dx$

Optimal. Leaf size=97

$$\frac{\mathrm{Li}_3\left(-e^{2a+2bx}\right)}{2b^3}-\frac{\mathrm{Li}_3\left(e^{2a+2bx}\right)}{2b^3}-\frac{x\mathrm{Li}_2\left(-e^{2a+2bx}\right)}{b^2}+\frac{x\mathrm{Li}_2\left(e^{2a+2bx}\right)}{b^2}-\frac{2x^2\tanh^{-1}\left(e^{2a+2bx}\right)}{b}$$

```
[Out] -2*x^2*arctanh(exp(2*b*x+2*a))/b-x*polylog(2,-exp(2*b*x+2*a))/b^2+x*polylog
(2,exp(2*b*x+2*a))/b^2+1/2*polylog(3,-exp(2*b*x+2*a))/b^3-1/2*polylog(3,exp
(2*b*x+2*a))/b^3
```

Rubi [A] time = 0.11, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {5461, 4182, 2531, 2282, 6589}

$$-\frac{x\mathrm{PolyLog}\left(2,-e^{2a+2bx}\right)}{b^2}+\frac{x\mathrm{PolyLog}\left(2,e^{2a+2bx}\right)}{b^2}+\frac{\mathrm{PolyLog}\left(3,-e^{2a+2bx}\right)}{2b^3}-\frac{\mathrm{PolyLog}\left(3,e^{2a+2bx}\right)}{2b^3}-\frac{2x^2\tanh^{-1}\left(e^{2a}\right.}{b}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Csch[a + b*x]*Sech[a + b*x],x]
```

```
[Out] (-2*x^2*ArcTanh[E^(2*a + 2*b*x)])/b - (x*PolyLog[2, -E^(2*a + 2*b*x)])/b^2
+ (x*PolyLog[2, E^(2*a + 2*b*x)])/b^2 + PolyLog[3, -E^(2*a + 2*b*x)]/(2*b^3
) - PolyLog[3, E^(2*a + 2*b*x)]/(2*b^3)
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
```

```
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5461

```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[2^n, Int[(c + d*x)^m*Csch[2*a + 2*b*x]
^n, x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[m] && IntegerQ[n]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int x^2\text{csch}(a+bx)\text{sech}(a+bx)\,dx = 2\int x^2\text{csch}(2a+2bx)\,dx$$
$$= -\frac{2x^2\tanh^{-1}\left(e^{2a+2bx}\right)}{b} - \frac{2\int x\log\left(1-e^{2a+2bx}\right)dx}{b} + \frac{2\int x\log\left(1+e^{2a+2bx}\right.}{b}$$
$$= -\frac{2x^2\tanh^{-1}\left(e^{2a+2bx}\right)}{b} - \frac{x\text{Li}_2\left(-e^{2a+2bx}\right)}{b^2} + \frac{x\text{Li}_2\left(e^{2a+2bx}\right)}{b^2} + \frac{\int\text{Li}_2\left(-e^{2a+2b\right.}}{b^2}$$
$$= -\frac{2x^2\tanh^{-1}\left(e^{2a+2bx}\right)}{b} - \frac{x\text{Li}_2\left(-e^{2a+2bx}\right)}{b^2} + \frac{x\text{Li}_2\left(e^{2a+2bx}\right)}{b^2} + \frac{\text{Subst}\left(\int\frac{\text{Li}_2(-}{x}\right.}{2}$$
$$= -\frac{2x^2\tanh^{-1}\left(e^{2a+2bx}\right)}{b} - \frac{x\text{Li}_2\left(-e^{2a+2bx}\right)}{b^2} + \frac{x\text{Li}_2\left(e^{2a+2bx}\right)}{b^2} + \frac{\text{Li}_3\left(-e^{2a+2bx}\right)}{2b^3}$$

Mathematica [A] time = 4.31, size = 108, normalized size = 1.11

$$\frac{2b^2x^2\log\left(1-e^{2(a+bx)}\right)-2b^2x^2\log\left(e^{2(a+bx)}+1\right)-2bx\text{Li}_2\left(-e^{2(a+bx)}\right)+2bx\text{Li}_2\left(e^{2(a+bx)}\right)+\text{Li}_3\left(-e^{2(a+bx)}\right)-\text{Li}_3}{2b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Csch[a + b*x]*Sech[a + b*x],x]
```

```
[Out] (2*b^2*x^2*Log[1 - E^(2*(a + b*x))] - 2*b^2*x^2*Log[1 + E^(2*(a + b*x))] -
2*b*x*PolyLog[2, -E^(2*(a + b*x))] + 2*b*x*PolyLog[2, E^(2*(a + b*x))] + Po
lyLog[3, -E^(2*(a + b*x))] - PolyLog[3, E^(2*(a + b*x))])/(2*b^3)
```

fricas [C] time = 0.44, size = 351, normalized size = 3.62

$$\frac{b^2x^2\log\left(\cosh\left(bx+a\right)+\sinh\left(bx+a\right)+1\right)+2\,bx\text{Li}_2\left(\cosh\left(bx+a\right)+\sinh\left(bx+a\right)\right)-2\,bx\text{Li}_2\left(i\,\cosh\left(bx+a\right)-\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*csch(b*x+a)*sech(b*x+a),x, algorithm="fricas")
```

```
[Out] (b^2*x^2*log(cosh(b*x + a) + sinh(b*x + a) + 1) + 2*b*x*dilog(cosh(b*x + a)
 + sinh(b*x + a)) - 2*b*x*dilog(I*cosh(b*x + a) + I*sinh(b*x + a)) - 2*b*x*
dilog(-I*cosh(b*x + a) - I*sinh(b*x + a)) + 2*b*x*dilog(-cosh(b*x + a) - si
nh(b*x + a)) - a^2*log(cosh(b*x + a) + sinh(b*x + a) + I) - a^2*log(cosh(b*
x + a) + sinh(b*x + a) - I) + a^2*log(cosh(b*x + a) + sinh(b*x + a) - 1) -
(b^2*x^2 - a^2)*log(I*cosh(b*x + a) + I*sinh(b*x + a) + 1) - (b^2*x^2 - a^2
)*log(-I*cosh(b*x + a) - I*sinh(b*x + a) + 1) + (b^2*x^2 - a^2)*log(-cosh(b
*x + a) - sinh(b*x + a) + 1) - 2*polylog(3, cosh(b*x + a) + sinh(b*x + a))
+ 2*polylog(3, I*cosh(b*x + a) + I*sinh(b*x + a)) + 2*polylog(3, -I*cosh(b*
x + a) - I*sinh(b*x + a)) - 2*polylog(3, -cosh(b*x + a) - sinh(b*x + a)))/b
^3
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2\,\mathrm{csch}\left(bx+a\right)\mathrm{sech}\left(bx+a\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*csch(b*x+a)*sech(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(x^2*csch(b*x + a)*sech(b*x + a), x)
```

maple [B] time = 0.45, size = 186, normalized size = 1.92

$$\frac{a^2\ln\left(e^{bx+a}-1\right)}{b^3}-\frac{2\,\text{polylog}\left(3,e^{bx+a}\right)}{b^3}-\frac{2\,\text{polylog}\left(3,-e^{bx+a}\right)}{b^3}+\frac{\text{polylog}\left(3,-e^{2bx+2a}\right)}{2b^3}-\frac{\ln\left(1-e^{bx+a}\right)a^2}{b^3}-\frac{x^2\ln\left(1\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*csch(b*x+a)*sech(b*x+a),x)
```

```
[Out] 1/b^3*a^2*ln(exp(b*x+a)-1)-2/b^3*polylog(3,exp(b*x+a))-2/b^3*polylog(3,-exp
(b*x+a))+1/2*polylog(3,-exp(2*b*x+2*a))/b^3-1/b^3*ln(1-exp(b*x+a))*a^2-x^2*
ln(1+exp(2*b*x+2*a))/b-x*polylog(2,-exp(2*b*x+2*a))/b^2+1/b*ln(1-exp(b*x+a)
)*x^2+2/b^2*polylog(2,exp(b*x+a))*x+1/b*ln(1+exp(b*x+a))*x^2+2/b^2*polylog(
2,-exp(b*x+a))*x
```

maxima [A] time = 0.40, size = 148, normalized size = 1.53

$$-\frac{2\,b^2x^2\log\left(e^{(2\,bx+2\,a)}+1\right)+2\,bx\mathrm{Li}_2\left(-e^{(2\,bx+2\,a)}\right)-\mathrm{Li}_3(-e^{(2\,bx+2\,a)})}{2\,b^3}+\frac{b^2x^2\log\left(e^{(bx+a)}+1\right)+2\,bx\mathrm{Li}_2\left(-e^{(bx+a)}\right)-}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*csch(b*x+a)*sech(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/2*(2*b^2*x^2*log(e^(2*b*x + 2*a) + 1) + 2*b*x*dilog(-e^(2*b*x + 2*a)) -
polylog(3, -e^(2*b*x + 2*a)))/b^3 + (b^2*x^2*log(e^(b*x + a) + 1) + 2*b*x*d
ilog(-e^(b*x + a)) - 2*polylog(3, -e^(b*x + a)))/b^3 + (b^2*x^2*log(-e^(b*x
 + a) + 1) + 2*b*x*dilog(e^(b*x + a)) - 2*polylog(3, e^(b*x + a)))/b^3
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2}{\cosh\left(a+b\,x\right)\,\sinh\left(a+b\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(cosh(a + b*x)*sinh(a + b*x)),x)
```

```
[Out] int(x^2/(cosh(a + b*x)*sinh(a + b*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2\operatorname{csch}\left(a+bx\right)\operatorname{sech}\left(a+bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*csch(b*x+a)*sech(b*x+a),x)
```

```
[Out] Integral(x**2*csch(a + b*x)*sech(a + b*x), x)
```

3.469 $\int x\,\mathrm{csch}(a+bx)\,\mathrm{sech}(a+bx)\,dx$

Optimal. Leaf size=58

$$-\frac{\mathrm{Li}_2\left(-e^{2a+2bx}\right)}{2b^2}+\frac{\mathrm{Li}_2\left(e^{2a+2bx}\right)}{2b^2}-\frac{2x\tanh^{-1}\left(e^{2a+2bx}\right)}{b}$$

```
[Out] -2*x*arctanh(exp(2*b*x+2*a))/b-1/2*polylog(2,-exp(2*b*x+2*a))/b^2+1/2*polyl
og(2,exp(2*b*x+2*a))/b^2
```

Rubi [A] time = 0.06, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {5461, 4182, 2279, 2391}

$$-\frac{\mathrm{PolyLog}\left(2,-e^{2a+2bx}\right)}{2b^2}+\frac{\mathrm{PolyLog}\left(2,e^{2a+2bx}\right)}{2b^2}-\frac{2x\tanh^{-1}\left(e^{2a+2bx}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[x*Csch[a + b*x]*Sech[a + b*x],x]
```

```
[Out] (-2*x*ArcTanh[E^(2*a + 2*b*x)])/b - PolyLog[2, -E^(2*a + 2*b*x)]/(2*b^2) +
PolyLog[2, E^(2*a + 2*b*x)]/(2*b^2)
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5461

```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[2^n, Int[(c + d*x)^m*Csch[2*a + 2*b*x]
^n, x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[m] && IntegerQ[n]
```

Rubi steps

$$\int x\operatorname{csch}(a+bx)\operatorname{sech}(a+bx)\,dx=2\int x\operatorname{csch}(2a+2bx)\,dx$$
$$=-\frac{2x\tanh^{-1}\left(e^{2a+2bx}\right)}{b}-\frac{\int\log\left(1-e^{2a+2bx}\right)dx}{b}+\frac{\int\log\left(1+e^{2a+2bx}\right)dx}{b}$$
$$=-\frac{2x\tanh^{-1}\left(e^{2a+2bx}\right)}{b}-\frac{\operatorname{Subst}\left(\int\frac{\log(1-x)}{x}dx,x,e^{2a+2bx}\right)}{2b^2}+\frac{\operatorname{Subst}\left(\int\frac{\log(1+x)}{x}\right.}{2b^2}$$
$$=-\frac{2x\tanh^{-1}\left(e^{2a+2bx}\right)}{b}-\frac{\operatorname{Li}_2\left(-e^{2a+2bx}\right)}{2b^2}+\frac{\operatorname{Li}_2\left(e^{2a+2bx}\right)}{2b^2}$$

Mathematica [A] time = 0.09, size = 110, normalized size = 1.90

$$\frac{\operatorname{Li}_2\left(-e^{-2(a+bx)}\right)-\operatorname{Li}_2\left(e^{-2(a+bx)}\right)+2a\log\left(1-e^{-2(a+bx)}\right)+2bx\log\left(1-e^{-2(a+bx)}\right)-2a\log\left(e^{-2(a+bx)}+1\right)-2bx\,\mathrm{lo}}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Csch[a + b*x]*Sech[a + b*x],x]
```

```
[Out] (2*a*Log[1 - E^(-2*(a + b*x))] + 2*b*x*Log[1 - E^(-2*(a + b*x))] - 2*a*Log[
1 + E^(-2*(a + b*x))] - 2*b*x*Log[1 + E^(-2*(a + b*x))] - 2*a*Log[Tanh[a +
b*x]] + PolyLog[2, -E^(-2*(a + b*x))] - PolyLog[2, E^(-2*(a + b*x))])/(2*b^
2)
```

fricas [C] time = 0.43, size = 224, normalized size = 3.86

$$\frac{bx\log\left(\cosh\left(bx+a\right)+\sinh\left(bx+a\right)+1\right)+a\log\left(\cosh\left(bx+a\right)+\sinh\left(bx+a\right)+i\right)+a\log\left(\cosh\left(bx+a\right)+\mathrm{si}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*csch(b*x+a)*sech(b*x+a),x, algorithm="fricas")
```

```
[Out] (b*x*log(cosh(b*x + a) + sinh(b*x + a) + 1) + a*log(cosh(b*x + a) + sinh(b*
x + a) + I) + a*log(cosh(b*x + a) + sinh(b*x + a) - I) - a*log(cosh(b*x + a
) + sinh(b*x + a) - 1) - (b*x + a)*log(I*cosh(b*x + a) + I*sinh(b*x + a) +
1) - (b*x + a)*log(-I*cosh(b*x + a) - I*sinh(b*x + a) + 1) + (b*x + a)*log(
```

```
-cosh(b*x + a) - sinh(b*x + a) + 1) + dilog(cosh(b*x + a) + sinh(b*x + a))
- dilog(I*cosh(b*x + a) + I*sinh(b*x + a)) - dilog(-I*cosh(b*x + a) - I*sin
h(b*x + a)) + dilog(-cosh(b*x + a) - sinh(b*x + a)))/b^2
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\,\mathrm{csch}\,(bx+a)\,\mathrm{sech}\,(bx+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*csch(b*x+a)*sech(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(x*csch(b*x + a)*sech(b*x + a), x)
```

maple [B] time = 0.44, size = 125, normalized size = 2.16

$$-\frac{x\ln\left(1+\mathrm{e}^{2bx+2a}\right)}{b}-\frac{\mathrm{polylog}\left(2,-\mathrm{e}^{2bx+2a}\right)}{2b^{2}}+\frac{\ln\left(1-\mathrm{e}^{bx+a}\right)x}{b}+\frac{\ln\left(1-\mathrm{e}^{bx+a}\right)a}{b^{2}}+\frac{\mathrm{polylog}\left(2,\mathrm{e}^{bx+a}\right)}{b^{2}}+\frac{\ln\left(1+\mathrm{e}^{bx+a}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*csch(b*x+a)*sech(b*x+a),x)
```

```
[Out] -x*ln(1+exp(2*b*x+2*a))/b-1/2*polylog(2,-exp(2*b*x+2*a))/b^2+1/b*ln(1-exp(b
*x+a))*x+1/b^2*ln(1-exp(b*x+a))*a+polylog(2,exp(b*x+a))/b^2+1/b*ln(1+exp(b*
x+a))*x+polylog(2,-exp(b*x+a))/b^2-1/b^2*a*ln(exp(b*x+a)-1)
```

maxima [A] time = 0.62, size = 87, normalized size = 1.50

$$-\frac{2\,bx\log\left(e^{(2\,bx+2\,a)}+1\right)+\mathrm{Li}_2\left(-e^{(2\,bx+2\,a)}\right)}{2\,b^{2}}+\frac{bx\log\left(e^{(bx+a)}+1\right)+\mathrm{Li}_2\left(-e^{(bx+a)}\right)}{b^{2}}+\frac{bx\log\left(-e^{(bx+a)}+1\right)+\mathrm{Li}_2\left(e^{(bx}\right.}{b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*csch(b*x+a)*sech(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/2*(2*b*x*log(e^(2*b*x + 2*a) + 1) + dilog(-e^(2*b*x + 2*a)))/b^2 + (b*x*
log(e^(b*x + a) + 1) + dilog(-e^(b*x + a)))/b^2 + (b*x*log(-e^(b*x + a) + 1
) + dilog(e^(b*x + a)))/b^2
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x}{\cosh\left(a+b\,x\right)\,\sinh\left(a+b\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(cosh(a + b*x)*sinh(a + b*x)),x)
```

```
[Out] int(x/(cosh(a + b*x)*sinh(a + b*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x \operatorname{csch}(a+bx) \operatorname{sech}(a+bx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*csch(b*x+a)*sech(b*x+a),x)
```

```
[Out] Integral(x*csch(a + b*x)*sech(a + b*x), x)
```

3.470 $\int \operatorname{csch}(a+bx)\operatorname{sech}(a+bx)\,dx$

Optimal. Leaf size=11

$$\frac{\log(\tanh(a+bx))}{b}$$

```
[Out] ln(tanh(b*x+a))/b
```

Rubi [A] time = 0.01, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2620, 29}

$$\frac{\log(\tanh(a+bx))}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[a + b*x]*Sech[a + b*x],x]
```

```
[Out] Log[Tanh[a + b*x]]/b
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 2620

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rubi steps

$$\begin{aligned}\int \operatorname{csch}(a+bx)\operatorname{sech}(a+bx)\,dx &= \frac{\operatorname{Subst}\left(\int \frac{1}{x}\,dx, x, i\tanh(a+bx)\right)}{b}\\ &= \frac{\log(\tanh(a+bx))}{b}\end{aligned}$$

Mathematica [B] time = 0.01, size = 31, normalized size = 2.82

$$2\left(\frac{\log(\sinh(a+bx))}{2b}-\frac{\log(\cosh(a+bx))}{2b}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[a + b*x]*Sech[a + b*x],x]
```

```
[Out] 2*(-1/2*Log[Cosh[a + b*x]]/b + Log[Sinh[a + b*x]]/(2*b))
```

fricas [B] time = 0.41, size = 60, normalized size = 5.45

$$-\frac{\log\left(\frac{2\cosh(bx+a)}{\cosh(bx+a)-\sinh(bx+a)}\right)-\log\left(\frac{2\sinh(bx+a)}{\cosh(bx+a)-\sinh(bx+a)}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)*sech(b*x+a),x, algorithm="fricas")
```

```
[Out] -(log(2*cosh(b*x + a)/(cosh(b*x + a) - sinh(b*x + a))) - log(2*sinh(b*x + a
)/(cosh(b*x + a) - sinh(b*x + a))))/b
```

giac [B] time = 0.11, size = 41, normalized size = 3.73

$$-\frac{\log\left(e^{(2\,bx+2\,a)}+1\right)-\log\left(e^{(bx+a)}+1\right)-\log\left(\left|e^{(bx+a)}-1\right|\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)*sech(b*x+a),x, algorithm="giac")
```

```
[Out] -(log(e^(2*b*x + 2*a) + 1) - log(e^(b*x + a) + 1) - log(abs(e^(b*x + a) - 1
)))/b
```

maple [A] time = 0.10, size = 12, normalized size = 1.09

$$\frac{\ln(\tanh(bx+a))}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(b*x+a)*sech(b*x+a),x)
```

```
[Out] ln(tanh(b*x+a))/b
```

maxima [B] time = 0.41, size = 50, normalized size = 4.55

$$\frac{\log\left(e^{(-bx-a)}+1\right)}{b}+\frac{\log\left(e^{(-bx-a)}-1\right)}{b}-\frac{\log\left(e^{(-2\,bx-2\,a)}+1\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)*sech(b*x+a),x, algorithm="maxima")
```

```
[Out] log(e^(-b*x - a) + 1)/b + log(e^(-b*x - a) - 1)/b - log(e^(-2*b*x - 2*a) +
1)/b
```

mupad [B] time = 1.45, size = 30, normalized size = 2.73

$$-\frac{2\,\mathrm{atan}\left(\frac{\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}\,\sqrt{-b^2}}{b}\right)}{\sqrt{-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(a + b*x)*sinh(a + b*x)),x)
```

```
[Out] -(2*atan((exp(2*a)*exp(2*b*x)*(-b^2)^(1/2))/b))/(-b^2)^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{csch}\left(a+bx\right)\operatorname{sech}\left(a+bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)*sech(b*x+a),x)
```

```
[Out] Integral(csch(a + b*x)*sech(a + b*x), x)
```

3.471 $\int \frac{\operatorname{csch}(a+bx)\operatorname{sech}(a+bx)}{x}\,dx$

Optimal. Leaf size=18

$$2\mathrm{Int}\left(\frac{\operatorname{csch}(2a+2bx)}{x},x\right)$$

```
[Out] 2*Unintegrable(csch(2*b*x+2*a)/x,x)
```

Rubi [A] time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\operatorname{csch}(a+bx)\operatorname{sech}(a+bx)}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Csch[a + b*x]*Sech[a + b*x])/x,x]
[Out] 2*Defer[Int][Csch[2*a + 2*b*x]/x, x]
```

Rubi steps

$$\int \frac{\operatorname{csch}(a+bx)\operatorname{sech}(a+bx)}{x}\,dx = 2\int \frac{\operatorname{csch}(2a+2bx)}{x}\,dx$$

Mathematica [A] time = 17.61, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}(a+bx)\operatorname{sech}(a+bx)}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Csch[a + b*x]*Sech[a + b*x])/x,x]
[Out] Integrate[(Csch[a + b*x]*Sech[a + b*x])/x, x]
```

fricas [A] time = 0.40, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\operatorname{csch}(bx+a)\operatorname{sech}(bx+a)}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)*sech(b*x+a)/x,x, algorithm="fricas")
```

```
[Out] integral(csch(b*x + a)*sech(b*x + a)/x, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}(bx+a)\operatorname{sech}(bx+a)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)*sech(b*x+a)/x,x, algorithm="giac")
[Out] integrate(csch(b*x + a)*sech(b*x + a)/x, x)
```

maple [A] time = 0.51, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}(bx+a)\operatorname{sech}(bx+a)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(b*x+a)*sech(b*x+a)/x,x)
[Out] int(csch(b*x+a)*sech(b*x+a)/x,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}(bx+a)\operatorname{sech}(bx+a)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)*sech(b*x+a)/x,x, algorithm="maxima")
[Out] integrate(csch(b*x + a)*sech(b*x + a)/x, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.06

$$\int \frac{1}{x\cosh(a+b\,x)\,\sinh(a+b\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*cosh(a + b*x)*sinh(a + b*x)),x)
[Out] int(1/(x*cosh(a + b*x)*sinh(a + b*x)), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}(a+bx)\operatorname{sech}(a+bx)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)*sech(b*x+a)/x,x)
```

```
[Out] Integral(csch(a + b*x)*sech(a + b*x)/x, x)
```

3.472 $\int \frac{\operatorname{csch}(a+bx)\operatorname{sech}(a+bx)}{x^2} dx$

Optimal. Leaf size=18

$$2\text{Int}\left(\frac{\operatorname{csch}(2a+2bx)}{x^2},x\right)$$

```
[Out] 2*Unintegrable(csch(2*b*x+2*a)/x^2,x)
```

Rubi [A] time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\operatorname{csch}(a+bx)\operatorname{sech}(a+bx)}{x^2} dx$$

Verification is Not applicable to the result.

```
[In] Int[(Csch[a + b*x]*Sech[a + b*x])/x^2,x]
[Out] 2*Defer[Int][Csch[2*a + 2*b*x]/x^2, x]
```

Rubi steps

$$\int \frac{\operatorname{csch}(a+bx)\operatorname{sech}(a+bx)}{x^2} dx = 2\int \frac{\operatorname{csch}(2a+2bx)}{x^2} dx$$

Mathematica [A] time = 16.01, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}(a+bx)\operatorname{sech}(a+bx)}{x^2} dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Csch[a + b*x]*Sech[a + b*x])/x^2,x]
[Out] Integrate[(Csch[a + b*x]*Sech[a + b*x])/x^2, x]
```

fricas [A] time = 0.39, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\operatorname{csch}(bx+a)\operatorname{sech}(bx+a)}{x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)*sech(b*x+a)/x^2,x, algorithm="fricas")
```

```
[Out] integral(csch(b*x + a)*sech(b*x + a)/x^2, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}(bx+a)\operatorname{sech}(bx+a)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)*sech(b*x+a)/x^2,x, algorithm="giac")
[Out] integrate(csch(b*x + a)*sech(b*x + a)/x^2, x)
```

maple [A] time = 0.49, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}(bx+a)\operatorname{sech}(bx+a)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(b*x+a)*sech(b*x+a)/x^2,x)
[Out] int(csch(b*x+a)*sech(b*x+a)/x^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}(bx+a)\operatorname{sech}(bx+a)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)*sech(b*x+a)/x^2,x, algorithm="maxima")
[Out] integrate(csch(b*x + a)*sech(b*x + a)/x^2, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.06

$$\int \frac{1}{x^2\cosh(a+b\,x)\,\sinh(a+b\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*cosh(a + b*x)*sinh(a + b*x)),x)
[Out] int(1/(x^2*cosh(a + b*x)*sinh(a + b*x)), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}(a+bx)\operatorname{sech}(a+bx)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)*sech(b*x+a)/x**2,x)
```

```
[Out] Integral(csch(a + b*x)*sech(a + b*x)/x**2, x)
```

3.473 $\int x^m \text{csch}(a+bx)\text{sech}^2(a+bx)\,dx$

Optimal. Leaf size=21

$$\text{Int}\left(x^m\text{csch}(a+bx)\text{sech}^2(a+bx),x\right)$$

```
[Out] CannotIntegrate(x^m*csch(b*x+a)*sech(b*x+a)^2,x)
```

Rubi [A] time = 0.39, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int x^m\text{csch}(a+bx)\text{sech}^2(a+bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x^m*Csch[a + b*x]*Sech[a + b*x]^2,x]
[Out] Defer[Int][x^m*Csch[a + b*x]*Sech[a + b*x]^2, x]
```

Rubi steps

$$\int x^m\text{csch}(a+bx)\text{sech}^2(a+bx)\,dx=\int x^m\text{csch}(a+bx)\text{sech}^2(a+bx)\,dx$$

Mathematica [A] time = 125.60, size = 0, normalized size = 0.00

$$\int x^m\text{csch}(a+bx)\text{sech}^2(a+bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^m*Csch[a + b*x]*Sech[a + b*x]^2,x]
[Out] Integrate[x^m*Csch[a + b*x]*Sech[a + b*x]^2, x]
```

fricas [A] time = 0.40, size = 0, normalized size = 0.00

$$\text{integral}\left(x^m\,\text{csch}\,(bx+a)\,\text{sech}\,(bx+a)^2\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*csch(b*x+a)*sech(b*x+a)^2,x, algorithm="fricas")
[Out] integral(x^m*csch(b*x + a)*sech(b*x + a)^2, x)
```

giac [**A**] time = 0.00, size = 0, normalized size = 0.00

$$\int x^m \operatorname{csch}(bx+a)\operatorname{sech}(bx+a)^2\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*csch(b*x+a)*sech(b*x+a)^2,x, algorithm="giac")
[Out] integrate(x^m*csch(b*x + a)*sech(b*x + a)^2, x)
```

maple [**A**] time = 0.20, size = 0, normalized size = 0.00

$$\int x^m \operatorname{csch}(bx+a)\operatorname{sech}(bx+a)^2\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*csch(b*x+a)*sech(b*x+a)^2,x)
[Out] int(x^m*csch(b*x+a)*sech(b*x+a)^2,x)
```

maxima [**A**] time = 0.00, size = 0, normalized size = 0.00

$$\int x^m \operatorname{csch}(bx+a)\operatorname{sech}(bx+a)^2\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*csch(b*x+a)*sech(b*x+a)^2,x, algorithm="maxima")
[Out] integrate(x^m*csch(b*x + a)*sech(b*x + a)^2, x)
```

mupad [**A**] time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{x^m}{\cosh(a+b\,x)^2\,\sinh(a+b\,x)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(cosh(a + b*x)^2*sinh(a + b*x)),x)
[Out] int(x^m/(cosh(a + b*x)^2*sinh(a + b*x)), x)
```

sympy [**A**] time = 0.00, size = 0, normalized size = 0.00

$$\int x^m \operatorname{csch}(a+bx)\operatorname{sech}^2(a+bx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*csch(b*x+a)*sech(b*x+a)**2,x)
[Out] Integral(x**m*csch(a + b*x)*sech(a + b*x)**2, x)
```

3.474 $\int x^3\mathbf{csch}(a+bx)\mathbf{sech}^2(a+bx)\,dx$

Optimal. Leaf size=226

$$-\frac{6i\text{Li}_3\left(-ie^{a+bx}\right)}{b^4}+\frac{6i\text{Li}_3\left(ie^{a+bx}\right)}{b^4}-\frac{6\text{Li}_4\left(-e^{a+bx}\right)}{b^4}+\frac{6\text{Li}_4\left(e^{a+bx}\right)}{b^4}+\frac{6ix\text{Li}_2\left(-ie^{a+bx}\right)}{b^3}-\frac{6ix\text{Li}_2\left(ie^{a+bx}\right)}{b^3}+\frac{6x\text{Li}_3\left(-e^{a+bx}\right.}{b^3}$$

```
[Out] -6*x^2*arctan(exp(b*x+a))/b^2-2*x^3*arctanh(exp(b*x+a))/b-3*x^2*polylog(2,-
exp(b*x+a))/b^2+6*I*x*polylog(2,-I*exp(b*x+a))/b^3-6*I*x*polylog(2,I*exp(b*
x+a))/b^3+3*x^2*polylog(2,exp(b*x+a))/b^2+6*x*polylog(3,-exp(b*x+a))/b^3-6*
I*polylog(3,-I*exp(b*x+a))/b^4+6*I*polylog(3,I*exp(b*x+a))/b^4-6*x*polylog(
3,exp(b*x+a))/b^3-6*polylog(4,-exp(b*x+a))/b^4+6*polylog(4,exp(b*x+a))/b^4+
x^3*sech(b*x+a)/b
```

Rubi [A] time = 0.34, antiderivative size = 226, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 13, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.722, Rules used = {2622, 321, 207, 5462, 14, 6273, 12, 4182, 2531, 6609, 2282, 6589, 4180}

$$-\frac{3x^2\text{PolyLog}\left(2,-e^{a+bx}\right)}{b^2}+\frac{3x^2\text{PolyLog}\left(2,e^{a+bx}\right)}{b^2}+\frac{6ix\text{PolyLog}\left(2,-ie^{a+bx}\right)}{b^3}-\frac{6ix\text{PolyLog}\left(2,ie^{a+bx}\right)}{b^3}+\frac{6x\text{PolyL}}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Csch[a + b*x]*Sech[a + b*x]^2,x]
```

```
[Out] (-6*x^2*ArcTan[E^(a + b*x)])/b^2 - (2*x^3*ArcTanh[E^(a + b*x)])/b - (3*x^2*
PolyLog[2, -E^(a + b*x)])/b^2 + ((6*I)*x*PolyLog[2, (-I)*E^(a + b*x)])/b^3
- ((6*I)*x*PolyLog[2, I*E^(a + b*x)])/b^3 + (3*x^2*PolyLog[2, E^(a + b*x)])
/b^2 + (6*x*PolyLog[3, -E^(a + b*x)])/b^3 - ((6*I)*PolyLog[3, (-I)*E^(a + b
*x)])/b^4 + ((6*I)*PolyLog[3, I*E^(a + b*x)])/b^4 - (6*x*PolyLog[3, E^(a +
b*x)])/b^3 - (6*PolyLog[4, -E^(a + b*x)])/b^4 + (6*PolyLog[4, E^(a + b*x)])
/b^4 + (x^3*Sech[a + b*x])/b
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
```

```
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5462

```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> With[{u = IntHide[Csch[a + b*x]^n*Sech[a +
b*x]^p, x]}, Dist[(c + d*x)^m, u, x] - Dist[d*m, Int[(c + d*x)^(m - 1)*u, x
], x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n,
p]
```

Rule 6273

```
Int[((a_.) + ArcTanh[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[((c + d*x)^(m + 1)*(a + b*ArcTanh[u]))/(d*(m + 1)), x] - Dist[b/(d*(m +
1)), Int[SimplifyIntegrand[((c + d*x)^(m + 1)*D[u, x])/(1 - u^2), x], x], x
] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && InverseFunctionFreeQ[u, x]
&&  !FunctionOfQ[(c + d*x)^(m + 1), u, x] && FalseQ[PowerVariableExpn[u, m
+ 1, x]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int x^3\text{csch}(a+bx)\text{sech}^2(a+bx)\,dx = -\frac{x^3\tanh^{-1}(\cosh(a+bx))}{b} + \frac{x^3\text{sech}(a+bx)}{b} - 3\int x^2\left(-\frac{\tanh^{-1}(\cosh(a+\right.$$

$$= -\frac{x^3\tanh^{-1}(\cosh(a+bx))}{b} + \frac{x^3\text{sech}(a+bx)}{b} - 3\int\left(-\frac{x^2\tanh^{-1}(\cosh(a+\right.$$

$$= -\frac{x^3\tanh^{-1}(\cosh(a+bx))}{b} + \frac{x^3\text{sech}(a+bx)}{b} + \frac{3\int x^2\tanh^{-1}(\cosh(a+bx)}{b}$$

$$= -\frac{6x^2\tan^{-1}\left(e^{a+bx}\right)}{b^2} + \frac{x^3\text{sech}(a+bx)}{b} + \frac{(6i)\int x\log\left(1-ie^{a+bx}\right)dx}{b^2} - \frac{(6i)\int}{}$$

$$= -\frac{6x^2\tan^{-1}\left(e^{a+bx}\right)}{b^2} + \frac{6ix\text{Li}_2\left(-ie^{a+bx}\right)}{b^3} - \frac{6ix\text{Li}_2\left(ie^{a+bx}\right)}{b^3} + \frac{x^3\text{sech}(a+bx)}{b}$$

$$= -\frac{6x^2\tan^{-1}\left(e^{a+bx}\right)}{b^2} - \frac{2x^3\tanh^{-1}\left(e^{a+bx}\right)}{b} + \frac{6ix\text{Li}_2\left(-ie^{a+bx}\right)}{b^3} - \frac{6ix\text{Li}_2\left(ie^{a+b}\right.}{b^3}$$

$$= -\frac{6x^2\tan^{-1}\left(e^{a+bx}\right)}{b^2} - \frac{2x^3\tanh^{-1}\left(e^{a+bx}\right)}{b} - \frac{3x^2\text{Li}_2\left(-e^{a+bx}\right)}{b^2} + \frac{6ix\text{Li}_2\left(-ie^{a+}\right.}{b^3}$$

$$= -\frac{6x^2\tan^{-1}\left(e^{a+bx}\right)}{b^2} - \frac{2x^3\tanh^{-1}\left(e^{a+bx}\right)}{b} - \frac{3x^2\text{Li}_2\left(-e^{a+bx}\right)}{b^2} + \frac{6ix\text{Li}_2\left(-ie^{a+}\right.}{b^3}$$

$$= -\frac{6x^2\tan^{-1}\left(e^{a+bx}\right)}{b^2} - \frac{2x^3\tanh^{-1}\left(e^{a+bx}\right)}{b} - \frac{3x^2\text{Li}_2\left(-e^{a+bx}\right)}{b^2} + \frac{6ix\text{Li}_2\left(-ie^{a+}\right.}{b^3}$$

$$= -\frac{6x^2\tan^{-1}\left(e^{a+bx}\right)}{b^2} - \frac{2x^3\tanh^{-1}\left(e^{a+bx}\right)}{b} - \frac{3x^2\text{Li}_2\left(-e^{a+bx}\right)}{b^2} + \frac{6ix\text{Li}_2\left(-ie^{a+}\right.}{b^3}$$

Mathematica [A] time = 0.40, size = 282, normalized size = 1.25

$$\frac{b^3x^3\text{sech}(a+bx) - 2b^3x^3\tanh^{-1}(\sinh(a+bx)+\cosh(a+bx)) - 3i\left(b^2x^2\log\left(1-ie^{a+bx}\right) - b^2x^2\log\left(1+ie^{a+bx}\right)\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^3*Csch[a + b*x]*Sech[a + b*x]^2,x]
```

```
[Out] (-2*b^3*x^3*ArcTanh[Cosh[a + b*x] + Sinh[a + b*x]] - 3*b^2*x^2*PolyLog[2, -
Cosh[a + b*x] - Sinh[a + b*x]] + 3*b^2*x^2*PolyLog[2, Cosh[a + b*x] + Sinh[
a + b*x]] - (3*I)*(b^2*x^2*Log[1 - I*E^(a + b*x)] - b^2*x^2*Log[1 + I*E^(a
+ b*x)] - 2*b*x*PolyLog[2, (-I)*E^(a + b*x)] + 2*b*x*PolyLog[2, I*E^(a + b*
x)] + 2*PolyLog[3, (-I)*E^(a + b*x)] - 2*PolyLog[3, I*E^(a + b*x)]) + 6*b*x
*PolyLog[3, -Cosh[a + b*x] - Sinh[a + b*x]] - 6*b*x*PolyLog[3, Cosh[a + b*x
```

```
] + Sinh[a + b*x]] - 6*PolyLog[4, -Cosh[a + b*x] - Sinh[a + b*x]] + 6*PolyL
og[4, Cosh[a + b*x] + Sinh[a + b*x]] + b^3*x^3*Sech[a + b*x])/b^4
```

fricas [C] time = 0.46, size = 1270, normalized size = 5.62

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*csch(b*x+a)*sech(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] (2*b^3*x^3*cosh(b*x + a) + 2*b^3*x^3*sinh(b*x + a) + 3*(b^2*x^2*cosh(b*x +
a)^2 + 2*b^2*x^2*cosh(b*x + a)*sinh(b*x + a) + b^2*x^2*sinh(b*x + a)^2 + b^
2*x^2)*dilog(cosh(b*x + a) + sinh(b*x + a)) + (-6*I*b*x*cosh(b*x + a)^2 - 1
2*I*b*x*cosh(b*x + a)*sinh(b*x + a) - 6*I*b*x*sinh(b*x + a)^2 - 6*I*b*x)*di
log(I*cosh(b*x + a) + I*sinh(b*x + a)) + (6*I*b*x*cosh(b*x + a)^2 + 12*I*b*
x*cosh(b*x + a)*sinh(b*x + a) + 6*I*b*x*sinh(b*x + a)^2 + 6*I*b*x)*dilog(-I
*cosh(b*x + a) - I*sinh(b*x + a)) - 3*(b^2*x^2*cosh(b*x + a)^2 + 2*b^2*x^2*
cosh(b*x + a)*sinh(b*x + a) + b^2*x^2*sinh(b*x + a)^2 + b^2*x^2)*dilog(-cos
h(b*x + a) - sinh(b*x + a)) - (b^3*x^3*cosh(b*x + a)^2 + 2*b^3*x^3*cosh(b*x
 + a)*sinh(b*x + a) + b^3*x^3*sinh(b*x + a)^2 + b^3*x^3)*log(cosh(b*x + a)
+ sinh(b*x + a) + 1) + (-3*I*a^2*cosh(b*x + a)^2 - 6*I*a^2*cosh(b*x + a)*si
nh(b*x + a) - 3*I*a^2*sinh(b*x + a)^2 - 3*I*a^2)*log(cosh(b*x + a) + sinh(b
*x + a) + I) + (3*I*a^2*cosh(b*x + a)^2 + 6*I*a^2*cosh(b*x + a)*sinh(b*x +
a) + 3*I*a^2*sinh(b*x + a)^2 + 3*I*a^2)*log(cosh(b*x + a) + sinh(b*x + a) -
 I) - (a^3*cosh(b*x + a)^2 + 2*a^3*cosh(b*x + a)*sinh(b*x + a) + a^3*sinh(b
*x + a)^2 + a^3)*log(cosh(b*x + a) + sinh(b*x + a) - 1) + (3*I*b^2*x^2 + (3
*I*b^2*x^2 - 3*I*a^2)*cosh(b*x + a)^2 + (6*I*b^2*x^2 - 6*I*a^2)*cosh(b*x +
a)*sinh(b*x + a) + (3*I*b^2*x^2 - 3*I*a^2)*sinh(b*x + a)^2 - 3*I*a^2)*log(I
*cosh(b*x + a) + I*sinh(b*x + a) + 1) + (-3*I*b^2*x^2 + (-3*I*b^2*x^2 + 3*I
*a^2)*cosh(b*x + a)^2 + (-6*I*b^2*x^2 + 6*I*a^2)*cosh(b*x + a)*sinh(b*x + a
) + (-3*I*b^2*x^2 + 3*I*a^2)*sinh(b*x + a)^2 + 3*I*a^2)*log(-I*cosh(b*x + a
) - I*sinh(b*x + a) + 1) + (b^3*x^3 + a^3 + (b^3*x^3 + a^3)*cosh(b*x + a)^2
 + 2*(b^3*x^3 + a^3)*cosh(b*x + a)*sinh(b*x + a) + (b^3*x^3 + a^3)*sinh(b*x
 + a)^2)*log(-cosh(b*x + a) - sinh(b*x + a) + 1) + 6*(cosh(b*x + a)^2 + 2*c
osh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 + 1)*polylog(4, cosh(b*x + a)
+ sinh(b*x + a)) - 6*(cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sin
h(b*x + a)^2 + 1)*polylog(4, -cosh(b*x + a) - sinh(b*x + a)) - 6*(b*x*cosh(
b*x + a)^2 + 2*b*x*cosh(b*x + a)*sinh(b*x + a) + b*x*sinh(b*x + a)^2 + b*x)
*polylog(3, cosh(b*x + a) + sinh(b*x + a)) + (6*I*cosh(b*x + a)^2 + 12*I*co
sh(b*x + a)*sinh(b*x + a) + 6*I*sinh(b*x + a)^2 + 6*I)*polylog(3, I*cosh(b*
x + a) + I*sinh(b*x + a)) + (-6*I*cosh(b*x + a)^2 - 12*I*cosh(b*x + a)*sinh
(b*x + a) - 6*I*sinh(b*x + a)^2 - 6*I)*polylog(3, -I*cosh(b*x + a) - I*sinh
(b*x + a)) + 6*(b*x*cosh(b*x + a)^2 + 2*b*x*cosh(b*x + a)*sinh(b*x + a) + b
*x*sinh(b*x + a)^2 + b*x)*polylog(3, -cosh(b*x + a) - sinh(b*x + a)))/(b^4*
```

```
cosh(b*x + a)^2 + 2*b^4*cosh(b*x + a)*sinh(b*x + a) + b^4*sinh(b*x + a)^2 +
 b^4)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*csch(b*x+a)*sech(b*x+a)^2,x, algorithm="giac")
[Out] Timed out
```

maple [F] time = 1.58, size = 0, normalized size = 0.00

$$\int x^3 \operatorname{csch}(bx+a)\operatorname{sech}(bx+a)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*csch(b*x+a)*sech(b*x+a)^2,x)
[Out] int(x^3*csch(b*x+a)*sech(b*x+a)^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{2\,x^3 e^{(bx+a)}}{be^{(2\,bx+2\,a)}+b} - \frac{b^3x^3\log\left(e^{(bx+a)}+1\right)+3\,b^2x^2\mathrm{Li}_2\left(-e^{(bx+a)}\right)-6\,bx\mathrm{Li}_3(-e^{(bx+a)})+6\,\mathrm{Li}_4(-e^{(bx+a)})}{b^4} + \frac{b^3x^3\log\left(-e^{(bx+\cdots}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*csch(b*x+a)*sech(b*x+a)^2,x, algorithm="maxima")
[Out] 2*x^3*e^(b*x + a)/(b*e^(2*b*x + 2*a) + b) - (b^3*x^3*log(e^(b*x + a) + 1) +
 3*b^2*x^2*dilog(-e^(b*x + a)) - 6*b*x*polylog(3, -e^(b*x + a)) + 6*polylog
(4, -e^(b*x + a)))/b^4 + (b^3*x^3*log(-e^(b*x + a) + 1) + 3*b^2*x^2*dilog(e
^(b*x + a)) - 6*b*x*polylog(3, e^(b*x + a)) + 6*polylog(4, e^(b*x + a)))/b^
4 - 24*integrate(1/4*x^2*e^(b*x + a)/(b*e^(2*b*x + 2*a) + b), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3}{\cosh(a+b\,x)^2\,\sinh(a+b\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(cosh(a + b*x)^2*sinh(a + b*x)),x)
```

```
[Out] int(x^3/(cosh(a + b*x)^2*sinh(a + b*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3 \operatorname{csch}(a + bx) \operatorname{sech}^2(a + bx) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*csch(b*x+a)*sech(b*x+a)**2,x)
```

```
[Out] Integral(x**3*csch(a + b*x)*sech(a + b*x)**2, x)
```

3.475 $\int x^2 \operatorname{csch}(a+bx)\operatorname{sech}^2(a+bx)\,dx$

Optimal. Leaf size=146

$$\frac{2i\text{Li}_2\left(-ie^{a+bx}\right)}{b^3}-\frac{2i\text{Li}_2\left(ie^{a+bx}\right)}{b^3}+\frac{2\text{Li}_3\left(-e^{a+bx}\right)}{b^3}-\frac{2\text{Li}_3\left(e^{a+bx}\right)}{b^3}-\frac{2x\text{Li}_2\left(-e^{a+bx}\right)}{b^2}+\frac{2x\text{Li}_2\left(e^{a+bx}\right)}{b^2}-\frac{4x\tan^{-1}\left(e^{a+bx}\right)}{b^2}-\frac{2\cdots}{\cdots}$$

```
[Out] -4*x*arctan(exp(b*x+a))/b^2-2*x^2*arctanh(exp(b*x+a))/b-2*x*polylog(2,-exp(
b*x+a))/b^2+2*I*polylog(2,-I*exp(b*x+a))/b^3-2*I*polylog(2,I*exp(b*x+a))/b^
3+2*x*polylog(2,exp(b*x+a))/b^2+2*polylog(3,-exp(b*x+a))/b^3-2*polylog(3,ex
p(b*x+a))/b^3+x^2*sech(b*x+a)/b
```

Rubi [A] time = 0.23, antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 14, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.778, Rules used = {2622, 321, 207, 5462, 14, 6273, 12, 4182, 2531, 2282, 6589, 4180, 2279, 2391}

$$-\frac{2x\text{PolyLog}\left(2,-e^{a+bx}\right)}{b^2}+\frac{2x\text{PolyLog}\left(2,e^{a+bx}\right)}{b^2}+\frac{2i\text{PolyLog}\left(2,-ie^{a+bx}\right)}{b^3}-\frac{2i\text{PolyLog}\left(2,ie^{a+bx}\right)}{b^3}+\frac{2\text{PolyLog}\left(3,\cdots\right)}{b^3}\cdots$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Csch[a + b*x]*Sech[a + b*x]^2,x]
```

```
[Out] (-4*x*ArcTan[E^(a + b*x)])/b^2 - (2*x^2*ArcTanh[E^(a + b*x)])/b - (2*x*Poly
Log[2, -E^(a + b*x)])/b^2 + ((2*I)*PolyLog[2, (-I)*E^(a + b*x)])/b^3 - ((2*
I)*PolyLog[2, I*E^(a + b*x)])/b^3 + (2*x*PolyLog[2, E^(a + b*x)])/b^2 + (2*
PolyLog[3, -E^(a + b*x)])/b^3 - (2*PolyLog[3, E^(a + b*x)])/b^3 + (x^2*Sech
[a + b*x])/b
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5462

```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> With[{u = IntHide[Csch[a + b*x]^n*Sech[a +
b*x]^p, x]}, Dist[(c + d*x)^m, u, x] - Dist[d*m, Int[(c + d*x)^(m - 1)*u, x
], x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n,
p]
```

Rule 6273

```
Int[((a_.) + ArcTanh[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[((c + d*x)^(m + 1)*(a + b*ArcTanh[u]))/(d*(m + 1)), x] - Dist[b/(d*(m +
1)), Int[SimplifyIntegrand[((c + d*x)^(m + 1)*D[u, x])/(1 - u^2), x], x], x
] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && InverseFunctionFreeQ[u, x]
&&  !FunctionOfQ[(c + d*x)^(m + 1), u, x] && FalseQ[PowerVariableExpn[u, m
+ 1, x]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int x^2\text{csch}(a+bx)\text{sech}^2(a+bx)\,dx = -\frac{x^2\tanh^{-1}(\cosh(a+bx))}{b} + \frac{x^2\text{sech}(a+bx)}{b} - 2\int x\left(-\frac{\tanh^{-1}(\cosh(a+\right.}{b}$$

$$= -\frac{x^2\tanh^{-1}(\cosh(a+bx))}{b} + \frac{x^2\text{sech}(a+bx)}{b} - 2\int\left(-\frac{x\tanh^{-1}(\cosh(a+\right.}{b}$$

$$= -\frac{x^2\tanh^{-1}(\cosh(a+bx))}{b} + \frac{x^2\text{sech}(a+bx)}{b} + \frac{2\int x\tanh^{-1}(\cosh(a+bx}{b}$$

$$= -\frac{4x\tan^{-1}\left(e^{a+bx}\right)}{b^2} + \frac{x^2\text{sech}(a+bx)}{b} + \frac{(2i)\int\log\left(1-ie^{a+bx}\right)dx}{b^2} - \frac{(2i)\int}{}$$

$$= -\frac{4x\tan^{-1}\left(e^{a+bx}\right)}{b^2} + \frac{x^2\text{sech}(a+bx)}{b} + \frac{(2i)\,\text{Subst}\left(\int\frac{\log(1-ix)}{x}\,dx, x, e^{a+bx}\right)}{b^3}$$

$$= -\frac{4x\tan^{-1}\left(e^{a+bx}\right)}{b^2} - \frac{2x^2\tanh^{-1}\left(e^{a+bx}\right)}{b} + \frac{2i\text{Li}_2\left(-ie^{a+bx}\right)}{b^3} - \frac{2i\text{Li}_2\left(ie^{a+bx}\right)}{b^3}$$

$$= -\frac{4x\tan^{-1}\left(e^{a+bx}\right)}{b^2} - \frac{2x^2\tanh^{-1}\left(e^{a+bx}\right)}{b} - \frac{2x\text{Li}_2\left(-e^{a+bx}\right)}{b^2} + \frac{2i\text{Li}_2\left(-ie^{a+bx}\right.}{b^3}$$

$$= -\frac{4x\tan^{-1}\left(e^{a+bx}\right)}{b^2} - \frac{2x^2\tanh^{-1}\left(e^{a+bx}\right)}{b} - \frac{2x\text{Li}_2\left(-e^{a+bx}\right)}{b^2} + \frac{2i\text{Li}_2\left(-ie^{a+bx}\right.}{b^3}$$

$$= -\frac{4x\tan^{-1}\left(e^{a+bx}\right)}{b^2} - \frac{2x^2\tanh^{-1}\left(e^{a+bx}\right)}{b} - \frac{2x\text{Li}_2\left(-e^{a+bx}\right)}{b^2} + \frac{2i\text{Li}_2\left(-ie^{a+bx}\right.}{b^3}$$

Mathematica [A] time = 0.65, size = 225, normalized size = 1.54

$$\frac{-2\left(b^2x^2\tanh^{-1}(\sinh(a+bx)+\cosh(a+bx)) + bx\text{Li}_2(-\cosh(a+bx)-\sinh(a+bx)) - bx\text{Li}_2(\cosh(a+bx)+\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2*Csch[a + b*x]*Sech[a + b*x]^2,x]
```

```
[Out] (((-2*I)*a + Pi - (2*I)*b*x)*(Log[1 - I*E^(a + b*x)] - Log[1 + I*E^(a + b*x
)]) - ((-2*I)*a + Pi)*Log[Cot[((2*I)*a + Pi + (2*I)*b*x)/4]] + (2*I)*(PolyL
og[2, (-I)*E^(a + b*x)] - PolyLog[2, I*E^(a + b*x)]) - 2*(b^2*x^2*ArcTanh[C
osh[a + b*x] + Sinh[a + b*x]] + b*x*PolyLog[2, -Cosh[a + b*x] - Sinh[a + b*
x]] - b*x*PolyLog[2, Cosh[a + b*x] + Sinh[a + b*x]] - PolyLog[3, -Cosh[a +
b*x] - Sinh[a + b*x]] + PolyLog[3, Cosh[a + b*x] + Sinh[a + b*x]]) + b^2*x^
2*Sech[a + b*x])/b^3
```

fricas [C] time = 0.45, size = 929, normalized size = 6.36

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*csch(b*x+a)*sech(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] (2*b^2*x^2*cosh(b*x + a) + 2*b^2*x^2*sinh(b*x + a) + 2*(b*x*cosh(b*x + a)^2
 + 2*b*x*cosh(b*x + a)*sinh(b*x + a) + b*x*sinh(b*x + a)^2 + b*x)*dilog(cos
h(b*x + a) + sinh(b*x + a)) + (-2*I*cosh(b*x + a)^2 - 4*I*cosh(b*x + a)*sin
h(b*x + a) - 2*I*sinh(b*x + a)^2 - 2*I)*dilog(I*cosh(b*x + a) + I*sinh(b*x
+ a)) + (2*I*cosh(b*x + a)^2 + 4*I*cosh(b*x + a)*sinh(b*x + a) + 2*I*sinh(b
*x + a)^2 + 2*I)*dilog(-I*cosh(b*x + a) - I*sinh(b*x + a)) - 2*(b*x*cosh(b*
x + a)^2 + 2*b*x*cosh(b*x + a)*sinh(b*x + a) + b*x*sinh(b*x + a)^2 + b*x)*d
ilog(-cosh(b*x + a) - sinh(b*x + a)) - (b^2*x^2*cosh(b*x + a)^2 + 2*b^2*x^2
*cosh(b*x + a)*sinh(b*x + a) + b^2*x^2*sinh(b*x + a)^2 + b^2*x^2)*log(cosh(
b*x + a) + sinh(b*x + a) + 1) + (2*I*a*cosh(b*x + a)^2 + 4*I*a*cosh(b*x + a
)*sinh(b*x + a) + 2*I*a*sinh(b*x + a)^2 + 2*I*a)*log(cosh(b*x + a) + sinh(b
*x + a) + I) + (-2*I*a*cosh(b*x + a)^2 - 4*I*a*cosh(b*x + a)*sinh(b*x + a)
- 2*I*a*sinh(b*x + a)^2 - 2*I*a)*log(cosh(b*x + a) + sinh(b*x + a) - I) + (
a^2*cosh(b*x + a)^2 + 2*a^2*cosh(b*x + a)*sinh(b*x + a) + a^2*sinh(b*x + a)
^2 + a^2)*log(cosh(b*x + a) + sinh(b*x + a) - 1) + ((2*I*b*x + 2*I*a)*cosh(
b*x + a)^2 + (4*I*b*x + 4*I*a)*cosh(b*x + a)*sinh(b*x + a) + (2*I*b*x + 2*I
*a)*sinh(b*x + a)^2 + 2*I*b*x + 2*I*a)*log(I*cosh(b*x + a) + I*sinh(b*x + a
) + 1) + ((-2*I*b*x - 2*I*a)*cosh(b*x + a)^2 + (-4*I*b*x - 4*I*a)*cosh(b*x
+ a)*sinh(b*x + a) + (-2*I*b*x - 2*I*a)*sinh(b*x + a)^2 - 2*I*b*x - 2*I*a)*
log(-I*cosh(b*x + a) - I*sinh(b*x + a) + 1) + (b^2*x^2 + (b^2*x^2 - a^2)*co
sh(b*x + a)^2 + 2*(b^2*x^2 - a^2)*cosh(b*x + a)*sinh(b*x + a) + (b^2*x^2 -
a^2)*sinh(b*x + a)^2 - a^2)*log(-cosh(b*x + a) - sinh(b*x + a) + 1) - 2*(co
sh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 + 1)*polylo
g(3, cosh(b*x + a) + sinh(b*x + a)) + 2*(cosh(b*x + a)^2 + 2*cosh(b*x + a)*
sinh(b*x + a) + sinh(b*x + a)^2 + 1)*polylog(3, -cosh(b*x + a) - sinh(b*x +
 a)))/(b^3*cosh(b*x + a)^2 + 2*b^3*cosh(b*x + a)*sinh(b*x + a) + b^3*sinh(b
*x + a)^2 + b^3)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*csch(b*x+a)*sech(b*x+a)^2,x, algorithm="giac")
```

```
[Out] Timed out
```

maple [F] time = 1.75, size = 0, normalized size = 0.00

$$\int x^2 \operatorname{csch}(bx + a) \operatorname{sech}(bx + a)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*csch(b*x+a)*sech(b*x+a)^2,x)
```

```
[Out] int(x^2*csch(b*x+a)*sech(b*x+a)^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{2\,x^2e^{(bx+a)}}{be^{(2\,bx+2\,a)}+b}-\frac{b^2x^2\log\left(e^{(bx+a)}+1\right)+2\,bx\mathrm{Li}_2\left(-e^{(bx+a)}\right)-2\,\mathrm{Li}_3(-e^{(bx+a)})}{b^3}+\frac{b^2x^2\log\left(-e^{(bx+a)}+1\right)+2\,bx\mathrm{Li}_2\left(e^{(b}\right.}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*csch(b*x+a)*sech(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 2*x^2*e^(b*x + a)/(b*e^(2*b*x + 2*a) + b) - (b^2*x^2*log(e^(b*x + a) + 1) +
 2*b*x*dilog(-e^(b*x + a)) - 2*polylog(3, -e^(b*x + a)))/b^3 + (b^2*x^2*log
(-e^(b*x + a) + 1) + 2*b*x*dilog(e^(b*x + a)) - 2*polylog(3, e^(b*x + a)))/
b^3 - 8*integrate(1/2*x*e^(b*x + a)/(b*e^(2*b*x + 2*a) + b), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2}{\cosh\left(a+b\,x\right)^2\,\sinh\left(a+b\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(cosh(a + b*x)^2*sinh(a + b*x)),x)
```

```
[Out] int(x^2/(cosh(a + b*x)^2*sinh(a + b*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2\operatorname{csch}\left(a+bx\right)\operatorname{sech}^2\left(a+bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*csch(b*x+a)*sech(b*x+a)**2,x)
```

```
[Out] Integral(x**2*csch(a + b*x)*sech(a + b*x)**2, x)
```

3.476 $\int x\mathbf{csch}(a+bx)\mathbf{sech}^2(a+bx)\,dx$

Optimal. Leaf size=67

$$-\frac{\text{Li}_2\left(-e^{a+bx}\right)}{b^2}+\frac{\text{Li}_2\left(e^{a+bx}\right)}{b^2}-\frac{\tan^{-1}(\sinh(a+bx))}{b^2}-\frac{2x\tanh^{-1}\left(e^{a+bx}\right)}{b}+\frac{x\text{sech}(a+bx)}{b}$$

```
[Out] -arctan(sinh(b*x+a))/b^2-2*x*arctanh(exp(b*x+a))/b-polylog(2,-exp(b*x+a))/b
^2+polylog(2,exp(b*x+a))/b^2+x*sech(b*x+a)/b
```

Rubi [A] time = 0.11, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {2622, 321, 207, 5462, 6271, 12, 4182, 2279, 2391, 3770}

$$-\frac{\text{PolyLog}\left(2,-e^{a+bx}\right)}{b^2}+\frac{\text{PolyLog}\left(2,e^{a+bx}\right)}{b^2}-\frac{\tan^{-1}(\sinh(a+bx))}{b^2}-\frac{2x\tanh^{-1}\left(e^{a+bx}\right)}{b}+\frac{x\text{sech}(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[x*Csch[a + b*x]*Sech[a + b*x]^2,x]
```

```
[Out] -(ArcTan[Sinh[a + b*x]]/b^2) - (2*x*ArcTanh[E^(a + b*x)])/b - PolyLog[2, -E
^(a + b*x)]/b^2 + PolyLog[2, E^(a + b*x)]/b^2 + (x*Sech[a + b*x])/b
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5462

```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> With[{u = IntHide[Csch[a + b*x]^n*Sech[a +
b*x]^p, x]}, Dist[(c + d*x)^m, u, x] - Dist[d*m, Int[(c + d*x)^(m - 1)*u, x
], x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n,
p]
```

Rule 6271

```
Int[ArcTanh[u_], x_Symbol] :> Simp[x*ArcTanh[u], x] - Int[SimplifyIntegrand
[(x*D[u, x])/(1 - u^2), x], x] /; InverseFunctionFreeQ[u, x]
```

Rubi steps

$$\int x\operatorname{csch}(a+bx)\operatorname{sech}^2(a+bx)\,dx = -\frac{x\tanh^{-1}(\cosh(a+bx))}{b}+\frac{x\operatorname{sech}(a+bx)}{b}-\int\left(-\frac{\tanh^{-1}(\cosh(a+bx))}{b}+\right.$$
$$=-\frac{x\tanh^{-1}(\cosh(a+bx))}{b}+\frac{x\operatorname{sech}(a+bx)}{b}+\frac{\int\tanh^{-1}(\cosh(a+bx))\,dx}{b}-$$
$$=-\frac{\tan^{-1}(\sinh(a+bx))}{b^2}+\frac{x\operatorname{sech}(a+bx)}{b}+\frac{\int bx\operatorname{csch}(a+bx)\,dx}{b}$$
$$=-\frac{\tan^{-1}(\sinh(a+bx))}{b^2}+\frac{x\operatorname{sech}(a+bx)}{b}+\int x\operatorname{csch}(a+bx)\,dx$$
$$=-\frac{\tan^{-1}(\sinh(a+bx))}{b^2}-\frac{2x\tanh^{-1}\left(e^{a+bx}\right)}{b}+\frac{x\operatorname{sech}(a+bx)}{b}-\frac{\int\log\left(1-e^{a+}\right.}{b}$$
$$=-\frac{\tan^{-1}(\sinh(a+bx))}{b^2}-\frac{2x\tanh^{-1}\left(e^{a+bx}\right)}{b}+\frac{x\operatorname{sech}(a+bx)}{b}-\frac{\text{Subst}\left(\int\frac{\log(}{x}\right.}{}$$
$$=-\frac{\tan^{-1}(\sinh(a+bx))}{b^2}-\frac{2x\tanh^{-1}\left(e^{a+bx}\right)}{b}-\frac{\operatorname{Li}_2\left(-e^{a+bx}\right)}{b^2}+\frac{\operatorname{Li}_2\left(e^{a+bx}\right)}{b^2}+\frac{x}{}$$

Mathematica [A] time = 0.29, size = 106, normalized size = 1.58

$$\frac{\operatorname{Li}_2\left(-e^{-a-bx}\right)-\operatorname{Li}_2\left(e^{-a-bx}\right)+(a+bx)\left(\log\left(1-e^{-a-bx}\right)-\log\left(e^{-a-bx}+1\right)\right)+bx\operatorname{sech}(a+bx)-a\log\left(\tanh\left(\frac{1}{2}(a\right.\right.}{b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Csch[a + b*x]*Sech[a + b*x]^2,x]
```

```
[Out] (-2*ArcTan[Tanh[(a + b*x)/2]] + (a + b*x)*(Log[1 - E^(-a - b*x)] - Log[1 +
E^(-a - b*x)]) - a*Log[Tanh[(a + b*x)/2]] + PolyLog[2, -E^(-a - b*x)] - Pol
yLog[2, E^(-a - b*x)] + b*x*Sech[a + b*x])/b^2
```

fricas [B] time = 0.41, size = 401, normalized size = 5.99

$$\frac{2\,bx\cosh(bx+a)+2\,bx\sinh(bx+a)-2\left(\cosh(bx+a)^2+2\,\cosh(bx+a)\sinh(bx+a)+\sinh(bx+a)^2+1\right)a}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*csch(b*x+a)*sech(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] (2*b*x*cosh(b*x + a) + 2*b*x*sinh(b*x + a) - 2*(cosh(b*x + a)^2 + 2*cosh(b*
x + a)*sinh(b*x + a) + sinh(b*x + a)^2 + 1)*arctan(cosh(b*x + a) + sinh(b*x
```

```
 + a)) + (cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2
 + 1)*dilog(cosh(b*x + a) + sinh(b*x + a)) - (cosh(b*x + a)^2 + 2*cosh(b*x
+ a)*sinh(b*x + a) + sinh(b*x + a)^2 + 1)*dilog(-cosh(b*x + a) - sinh(b*x +
 a)) - (b*x*cosh(b*x + a)^2 + 2*b*x*cosh(b*x + a)*sinh(b*x + a) + b*x*sinh(
b*x + a)^2 + b*x)*log(cosh(b*x + a) + sinh(b*x + a) + 1) - (a*cosh(b*x + a)
^2 + 2*a*cosh(b*x + a)*sinh(b*x + a) + a*sinh(b*x + a)^2 + a)*log(cosh(b*x
+ a) + sinh(b*x + a) - 1) + ((b*x + a)*cosh(b*x + a)^2 + 2*(b*x + a)*cosh(b
*x + a)*sinh(b*x + a) + (b*x + a)*sinh(b*x + a)^2 + b*x + a)*log(-cosh(b*x
+ a) - sinh(b*x + a) + 1))/(b^2*cosh(b*x + a)^2 + 2*b^2*cosh(b*x + a)*sinh(
b*x + a) + b^2*sinh(b*x + a)^2 + b^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x \operatorname{csch}(bx+a) \operatorname{sech}(bx+a)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*csch(b*x+a)*sech(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate(x*csch(b*x + a)*sech(b*x + a)^2, x)
```

maple [A] time = 0.52, size = 95, normalized size = 1.42

$$\frac{2x\,e^{bx+a}}{b\left(1+e^{2bx+2a}\right)}-\frac{2\arctan\left(e^{bx+a}\right)}{b^2}-\frac{\operatorname{dilog}\left(1+e^{bx+a}\right)}{b^2}-\frac{\ln\left(1+e^{bx+a}\right)x}{b}-\frac{\operatorname{dilog}\left(e^{bx+a}\right)}{b^2}-\frac{a\ln\left(e^{bx+a}-1\right)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*csch(b*x+a)*sech(b*x+a)^2,x)
```

```
[Out] 2*x*exp(b*x+a)/b/(1+exp(2*b*x+2*a))-2/b^2*arctan(exp(b*x+a))-1/b^2*dilog(1+
exp(b*x+a))-1/b*ln(1+exp(b*x+a))*x-1/b^2*dilog(exp(b*x+a))-1/b^2*a*ln(exp(b
*x+a)-1)
```

maxima [A] time = 0.46, size = 90, normalized size = 1.34

$$\frac{2\,xe^{(bx+a)}}{be^{(2\,bx+2\,a)}+b}-\frac{bx\log\left(e^{(bx+a)}+1\right)+\operatorname{Li}_2\left(-e^{(bx+a)}\right)}{b^2}+\frac{bx\log\left(-e^{(bx+a)}+1\right)+\operatorname{Li}_2\left(e^{(bx+a)}\right)}{b^2}-\frac{2\arctan\left(e^{(bx+a)}\right)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*csch(b*x+a)*sech(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 2*x*e^(b*x + a)/(b*e^(2*b*x + 2*a) + b) - (b*x*log(e^(b*x + a) + 1) + dilog
(-e^(b*x + a)))/b^2 + (b*x*log(-e^(b*x + a) + 1) + dilog(e^(b*x + a)))/b^2
- 2*arctan(e^(b*x + a))/b^2
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x}{\cosh (a+b\,x)^2\,\sinh (a+b\,x)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(cosh(a + b*x)^2*sinh(a + b*x)),x)
```

```
[Out] int(x/(cosh(a + b*x)^2*sinh(a + b*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x \operatorname{csch}(a+bx) \operatorname{sech}^2(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*csch(b*x+a)*sech(b*x+a)**2,x)
```

```
[Out] Integral(x*csch(a + b*x)*sech(a + b*x)**2, x)
```

3.477 $\int \operatorname{csch}(a+bx)\operatorname{sech}^2(a+bx)\,dx$

Optimal. Leaf size=23

$$\frac{\operatorname{sech}(a+bx)}{b}-\frac{\tanh^{-1}(\cosh(a+bx))}{b}$$

```
[Out] -arctanh(cosh(b*x+a))/b+sech(b*x+a)/b
```

Rubi [A] time = 0.03, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2622, 321, 207}

$$\frac{\operatorname{sech}(a+bx)}{b}-\frac{\tanh^{-1}(\cosh(a+bx))}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[a + b*x]*Sech[a + b*x]^2,x]
```

```
[Out] -(ArcTanh[Cosh[a + b*x]]/b) + Sech[a + b*x]/b
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rubi steps

$$\int \operatorname{csch}(a+bx)\operatorname{sech}^2(a+bx)\,dx = \frac{\text{Subst}\left(\int \frac{x^2}{-1+x^2}\,dx, x, \operatorname{sech}(a+bx)\right)}{b}$$
$$= \frac{\operatorname{sech}(a+bx)}{b} + \frac{\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \operatorname{sech}(a+bx)\right)}{b}$$
$$= -\frac{\tanh^{-1}(\cosh(a+bx))}{b} + \frac{\operatorname{sech}(a+bx)}{b}$$

Mathematica [A] time = 0.03, size = 26, normalized size = 1.13

$$\frac{\operatorname{sech}(a+bx)}{b} + \frac{\log\left(\tanh\left(\frac{1}{2}(a+bx)\right)\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[a + b*x]*Sech[a + b*x]^2,x]
```

```
[Out] Log[Tanh[(a + b*x)/2]]/b + Sech[a + b*x]/b
```

fricas [B] time = 0.40, size = 155, normalized size = 6.74

$$-\frac{\left(\cosh(bx+a)^2 + 2\cosh(bx+a)\sinh(bx+a) + \sinh(bx+a)^2 + 1\right)\log(\cosh(bx+a) + \sinh(bx+a) + 1) - }{b\cosh(bx+a)^2 + 2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)*sech(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -((cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 + 1)*l
og(cosh(b*x + a) + sinh(b*x + a) + 1) - (cosh(b*x + a)^2 + 2*cosh(b*x + a)*
sinh(b*x + a) + sinh(b*x + a)^2 + 1)*log(cosh(b*x + a) + sinh(b*x + a) - 1)
 - 2*cosh(b*x + a) - 2*sinh(b*x + a))/(b*cosh(b*x + a)^2 + 2*b*cosh(b*x + a
)*sinh(b*x + a) + b*sinh(b*x + a)^2 + b)
```

giac [B] time = 0.91, size = 64, normalized size = 2.78

$$\frac{\frac{4}{e^{(bx+a)}+e^{(-bx-a)}} - \log\left(e^{(bx+a)} + e^{(-bx-a)} + 2\right) + \log\left(e^{(bx+a)} + e^{(-bx-a)} - 2\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)*sech(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*(4/(e^(b*x + a) + e^(-b*x - a)) - log(e^(b*x + a) + e^(-b*x - a) + 2) +
 log(e^(b*x + a) + e^(-b*x - a) - 2))/b
```

maple [A] time = 0.11, size = 23, normalized size = 1.00

$$\frac{\frac{1}{\cosh(bx+a)} - 2\,\mathrm{arctanh}\left(\mathrm{e}^{bx+a}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(b*x+a)*sech(b*x+a)^2,x)
```

```
[Out] 1/b*(1/cosh(b*x+a)-2*arctanh(exp(b*x+a)))
```

maxima [B] time = 0.34, size = 61, normalized size = 2.65

$$-\frac{\log\left(e^{(-bx-a)}+1\right)}{b} + \frac{\log\left(e^{(-bx-a)}-1\right)}{b} + \frac{2\,e^{(-bx-a)}}{b\left(e^{(-2\,bx-2\,a)}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)*sech(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -log(e^(-b*x - a) + 1)/b + log(e^(-b*x - a) - 1)/b + 2*e^(-b*x - a)/(b*(e^(
-2*b*x - 2*a) + 1))
```

mupad [B] time = 1.47, size = 52, normalized size = 2.26

$$\frac{2\,\mathrm{e}^{a+b\,x}}{b\,\left(\mathrm{e}^{2\,a+2\,b\,x}+1\right)} - \frac{2\,\mathrm{atan}\left(\frac{\mathrm{e}^{b\,x}\,\mathrm{e}^{a}\,\sqrt{-b^2}}{b}\right)}{\sqrt{-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(a + b*x)^2*sinh(a + b*x)),x)
```

```
[Out] (2*exp(a + b*x))/(b*(exp(2*a + 2*b*x) + 1)) - (2*atan((exp(b*x)*exp(a)*(-b^
2)^(1/2))/b))/(-b^2)^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{csch}\left(a+bx\right)\operatorname{sech}^2\left(a+bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)*sech(b*x+a)**2,x)
[Out] Integral(csch(a + b*x)*sech(a + b*x)**2, x)
```

3.478 $\int \frac{\operatorname{csch}(a+bx)\operatorname{sech}^2(a+bx)}{x}\,dx$

Optimal. Leaf size=21

$$\operatorname{Int}\left(\frac{\operatorname{csch}(a+bx)\operatorname{sech}^2(a+bx)}{x},x\right)$$

```
[Out] CannotIntegrate(csch(b*x+a)*sech(b*x+a)^2/x,x)
```

Rubi [A] time = 0.21, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\operatorname{csch}(a+bx)\operatorname{sech}^2(a+bx)}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Csch[a + b*x]*Sech[a + b*x]^2)/x,x]
```

```
[Out] Defer[Int][(Csch[a + b*x]*Sech[a + b*x]^2)/x, x]
```

Rubi steps

$$\int \frac{\operatorname{csch}(a+bx)\operatorname{sech}^2(a+bx)}{x}\,dx = \int \frac{\operatorname{csch}(a+bx)\operatorname{sech}^2(a+bx)}{x}\,dx$$

Mathematica [A] time = 35.91, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}(a+bx)\operatorname{sech}^2(a+bx)}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Csch[a + b*x]*Sech[a + b*x]^2)/x,x]
```

```
[Out] Integrate[(Csch[a + b*x]*Sech[a + b*x]^2)/x, x]
```

fricas [A] time = 0.40, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{\operatorname{csch}(bx+a)\operatorname{sech}(bx+a)^2}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)*sech(b*x+a)^2/x,x, algorithm="fricas")
```

```
[Out] integral(csch(b*x + a)*sech(b*x + a)^2/x, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}(bx+a)\operatorname{sech}(bx+a)^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)*sech(b*x+a)^2/x,x, algorithm="giac")
```

```
[Out] integrate(csch(b*x + a)*sech(b*x + a)^2/x, x)
```

maple [A] time = 0.99, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}(bx+a)\operatorname{sech}(bx+a)^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(b*x+a)*sech(b*x+a)^2/x,x)
```

```
[Out] int(csch(b*x+a)*sech(b*x+a)^2/x,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\frac{2\,e^{(bx+a)}}{bxe^{(2\,bx+2\,a)}+bx}+8\int \frac{e^{(bx+a)}}{4\left(bx^2e^{(2\,bx+2\,a)}+bx^2\right)}\,dx+8\int \frac{1}{8\left(xe^{(bx+a)}+x\right)}\,dx+8\int \frac{1}{8\left(xe^{(bx+a)}-x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)*sech(b*x+a)^2/x,x, algorithm="maxima")
```

```
[Out] 2*e^(b*x + a)/(b*x*e^(2*b*x + 2*a) + b*x) + 8*integrate(1/4*e^(b*x + a)/(b*
x^2*e^(2*b*x + 2*a) + b*x^2), x) + 8*integrate(1/8/(x*e^(b*x + a) + x), x)
+ 8*integrate(1/8/(x*e^(b*x + a) - x), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{1}{x\cosh\left(a+b\,x\right)^2\,\sinh\left(a+b\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*cosh(a + b*x)^2*sinh(a + b*x)),x)
```

```
[Out] int(1/(x*cosh(a + b*x)^2*sinh(a + b*x)), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}(a+bx)\operatorname{sech}^2(a+bx)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)*sech(b*x+a)**2/x,x)
```

```
[Out] Integral(csch(a + b*x)*sech(a + b*x)**2/x, x)
```

3.479 $\int \frac{\operatorname{csch}(a+bx)\operatorname{sech}^2(a+bx)}{x^2} dx$

Optimal. Leaf size=21

$$\operatorname{Int}\left(\frac{\operatorname{csch}(a+bx)\operatorname{sech}^2(a+bx)}{x^2},x\right)$$

```
[Out] CannotIntegrate(csch(b*x+a)*sech(b*x+a)^2/x^2,x)
```

Rubi [A] time = 0.25, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\operatorname{csch}(a+bx)\operatorname{sech}^2(a+bx)}{x^2} dx$$

Verification is Not applicable to the result.

```
[In] Int[(Csch[a + b*x]*Sech[a + b*x]^2)/x^2,x]
```

```
[Out] Defer[Int][(Csch[a + b*x]*Sech[a + b*x]^2)/x^2, x]
```

Rubi steps

$$\int \frac{\operatorname{csch}(a+bx)\operatorname{sech}^2(a+bx)}{x^2} dx = \int \frac{\operatorname{csch}(a+bx)\operatorname{sech}^2(a+bx)}{x^2} dx$$

Mathematica [A] time = 25.90, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}(a+bx)\operatorname{sech}^2(a+bx)}{x^2} dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Csch[a + b*x]*Sech[a + b*x]^2)/x^2,x]
```

```
[Out] Integrate[(Csch[a + b*x]*Sech[a + b*x]^2)/x^2, x]
```

fricas [A] time = 0.41, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{\operatorname{csch}(bx+a)\operatorname{sech}(bx+a)^2}{x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)*sech(b*x+a)^2/x^2,x, algorithm="fricas")
```

```
[Out] integral(csch(b*x + a)*sech(b*x + a)^2/x^2, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}(bx+a)\operatorname{sech}(bx+a)^2}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)*sech(b*x+a)^2/x^2,x, algorithm="giac")
```

```
[Out] integrate(csch(b*x + a)*sech(b*x + a)^2/x^2, x)
```

maple [A] time = 1.09, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}(bx+a)\operatorname{sech}(bx+a)^2}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(b*x+a)*sech(b*x+a)^2/x^2,x)
```

```
[Out] int(csch(b*x+a)*sech(b*x+a)^2/x^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\frac{2\,e^{(bx+a)}}{bx^2e^{(2\,bx+2\,a)}+bx^2}+8\int \frac{e^{(bx+a)}}{2\left(bx^3e^{(2\,bx+2\,a)}+bx^3\right)}\,dx+8\int \frac{1}{8\left(x^2e^{(bx+a)}+x^2\right)}\,dx+8\int \frac{1}{8\left(x^2e^{(bx+a)}-x^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)*sech(b*x+a)^2/x^2,x, algorithm="maxima")
```

```
[Out] 2*e^(b*x + a)/(b*x^2*e^(2*b*x + 2*a) + b*x^2) + 8*integrate(1/2*e^(b*x + a)
/(b*x^3*e^(2*b*x + 2*a) + b*x^3), x) + 8*integrate(1/8/(x^2*e^(b*x + a) + x
^2), x) + 8*integrate(1/8/(x^2*e^(b*x + a) - x^2), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{1}{x^2\cosh\left(a+b\,x\right)^2\sinh\left(a+b\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*cosh(a + b*x)^2*sinh(a + b*x)),x)
```

```
[Out] int(1/(x^2*cosh(a + b*x)^2*sinh(a + b*x)), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}(a+bx)\operatorname{sech}^2(a+bx)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)*sech(b*x+a)**2/x**2,x)
```

```
[Out] Integral(csch(a + b*x)*sech(a + b*x)**2/x**2, x)
```

3.480 $\int x^m \operatorname{csch}(a+bx)\operatorname{sech}^3(a+bx)\,dx$

Optimal. Leaf size=21

$$\operatorname{Int}\left(x^m \operatorname{csch}(a+bx)\operatorname{sech}^3(a+bx),x\right)$$

```
[Out] CannotIntegrate(x^m*csch(b*x+a)*sech(b*x+a)^3,x)
```

Rubi [A] time = 0.48, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int x^m \operatorname{csch}(a+bx)\operatorname{sech}^3(a+bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x^m*Csch[a + b*x]*Sech[a + b*x]^3,x]
[Out] Defer[Int][x^m*Csch[a + b*x]*Sech[a + b*x]^3, x]
```

Rubi steps

$$\int x^m \operatorname{csch}(a+bx)\operatorname{sech}^3(a+bx)\,dx=\int x^m \operatorname{csch}(a+bx)\operatorname{sech}^3(a+bx)\,dx$$

Mathematica [A] time = 82.19, size = 0, normalized size = 0.00

$$\int x^m \operatorname{csch}(a+bx)\operatorname{sech}^3(a+bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^m*Csch[a + b*x]*Sech[a + b*x]^3,x]
[Out] Integrate[x^m*Csch[a + b*x]*Sech[a + b*x]^3, x]
```

fricas [A] time = 0.41, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(x^m \operatorname{csch}(bx+a)\operatorname{sech}(bx+a)^3,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*csch(b*x+a)*sech(b*x+a)^3,x, algorithm="fricas")
[Out] integral(x^m*csch(b*x + a)*sech(b*x + a)^3, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int x^m \operatorname{csch}(bx + a) \operatorname{sech}(bx + a)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*csch(b*x+a)*sech(b*x+a)^3,x, algorithm="giac")
[Out] integrate(x^m*csch(b*x + a)*sech(b*x + a)^3, x)
```

maple [A] time = 0.23, size = 0, normalized size = 0.00

$$\int x^m \operatorname{csch}(bx + a) \operatorname{sech}(bx + a)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*csch(b*x+a)*sech(b*x+a)^3,x)
[Out] int(x^m*csch(b*x+a)*sech(b*x+a)^3,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int x^m \operatorname{csch}(bx + a) \operatorname{sech}(bx + a)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*csch(b*x+a)*sech(b*x+a)^3,x, algorithm="maxima")
[Out] integrate(x^m*csch(b*x + a)*sech(b*x + a)^3, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{x^m}{\cosh(a + b\,x)^3 \sinh(a + b\,x)} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(cosh(a + b*x)^3*sinh(a + b*x)),x)
[Out] int(x^m/(cosh(a + b*x)^3*sinh(a + b*x)), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int x^m \operatorname{csch}(a + bx) \operatorname{sech}^3(a + bx) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*csch(b*x+a)*sech(b*x+a)**3,x)
[Out] Integral(x**m*csch(a + b*x)*sech(a + b*x)**3, x)
```

3.481 $\int x^3\mathbf{csch}(a+bx)\mathbf{sech}^3(a+bx)\,dx$

Optimal. Leaf size=240

$$\frac{3\text{Li}_2\left(-e^{2(a+bx)}\right)}{2b^4}-\frac{3\text{Li}_4\left(-e^{2a+2bx}\right)}{4b^4}+\frac{3\text{Li}_4\left(e^{2a+2bx}\right)}{4b^4}+\frac{3x\text{Li}_3\left(-e^{2a+2bx}\right)}{2b^3}-\frac{3x\text{Li}_3\left(e^{2a+2bx}\right)}{2b^3}+\frac{3x\log\left(e^{2(a+bx)}+1\right)}{b^3}-\frac{3x^2}{}$$

```
[Out] -3/2*x^2/b^2+1/2*x^3/b-2*x^3*arctanh(exp(2*b*x+2*a))/b+3*x*ln(1+exp(2*b*x+2
*a))/b^3+3/2*polylog(2,-exp(2*b*x+2*a))/b^4-3/2*x^2*polylog(2,-exp(2*b*x+2*
a))/b^2+3/2*x^2*polylog(2,exp(2*b*x+2*a))/b^2+3/2*x*polylog(3,-exp(2*b*x+2*
a))/b^3-3/2*x*polylog(3,exp(2*b*x+2*a))/b^3-3/4*polylog(4,-exp(2*b*x+2*a))/
b^4+3/4*polylog(4,exp(2*b*x+2*a))/b^4-3/2*x^2*tanh(b*x+a)/b^2-1/2*x^3*tanh(
b*x+a)^2/b
```

Rubi [A] time = 0.43, antiderivative size = 240, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 16, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.889, Rules used = {2620, 14, 5462, 2551, 12, 4182, 2531, 6609, 2282, 6589, 3720, 3718, 2190, 2279, 2391, 30}

$$-\frac{3x^2\text{PolyLog}\left(2,-e^{2a+2bx}\right)}{2b^2}+\frac{3x^2\text{PolyLog}\left(2,e^{2a+2bx}\right)}{2b^2}+\frac{3x\text{PolyLog}\left(3,-e^{2a+2bx}\right)}{2b^3}-\frac{3x\text{PolyLog}\left(3,e^{2a+2bx}\right)}{2b^3}+\frac{3\text{Po}}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Csch[a + b*x]*Sech[a + b*x]^3,x]
```

```
[Out] (-3*x^2)/(2*b^2) + x^3/(2*b) - (2*x^3*ArcTanh[E^(2*a + 2*b*x)])/b + (3*x*Lo
g[1 + E^(2*(a + b*x))])/b^3 + (3*PolyLog[2, -E^(2*(a + b*x))])/(2*b^4) - (3
*x^2*PolyLog[2, -E^(2*a + 2*b*x)])/(2*b^2) + (3*x^2*PolyLog[2, E^(2*a + 2*b
*x)])/(2*b^2) + (3*x*PolyLog[3, -E^(2*a + 2*b*x)])/(2*b^3) - (3*x*PolyLog[3
, E^(2*a + 2*b*x)])/(2*b^3) - (3*PolyLog[4, -E^(2*a + 2*b*x)])/(4*b^4) + (3
*PolyLog[4, E^(2*a + 2*b*x)])/(4*b^4) - (3*x^2*Tanh[a + b*x])/(2*b^2) - (x^
3*Tanh[a + b*x]^2)/(2*b)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2551

```
Int[Log[u_]*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[((a + b*x)^(m + 1
)*Log[u])/(b*(m + 1)), x] - Dist[1/(b*(m + 1)), Int[SimplifyIntegrand[((a +
 b*x)^(m + 1)*D[u, x])/u, x], x], x] /; FreeQ[{a, b, m}, x] && InverseFunct
```

```
ionFreeQ[u, x] && NeQ[m, -1]
```

Rule 2620

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 3718

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c
 + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /;
 FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 3720

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[(b*(c + d*x)^m*(b*Tan[e + f*x])^(n - 1))/(f*(n - 1)), x] + (-Di
st[(b*d*m)/(f*(n - 1)), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x],
 x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[
{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5462

```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> With[{u = IntHide[Csch[a + b*x]^n*Sech[a +
b*x]^p, x]}, Dist[(c + d*x)^m, u, x] - Dist[d*m, Int[(c + d*x)^(m - 1)*u, x
], x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n,
p]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int x^3\text{csch}(a+bx)\text{sech}^3(a+bx)\,dx = \frac{x^3\log(\tanh(a+bx))}{b} - \frac{x^3\tanh^2(a+bx)}{2b} - 3\int x^2\left(\frac{\log(\tanh(a+bx))}{b} - \text{ta}\right.$$

$$= \frac{x^3\log(\tanh(a+bx))}{b} - \frac{x^3\tanh^2(a+bx)}{2b} - 3\int\left(\frac{x^2\log(\tanh(a+bx))}{b} - x^2\right.$$

$$= \frac{x^3\log(\tanh(a+bx))}{b} - \frac{x^3\tanh^2(a+bx)}{2b} + \frac{3\int x^2\tanh^2(a+bx)\,dx}{2b} - \frac{3\int x}{}$$

$$= -\frac{3x^2\tanh(a+bx)}{2b^2} - \frac{x^3\tanh^2(a+bx)}{2b} + \frac{3\int x\tanh(a+bx)\,dx}{b^2} + \frac{\int 2bx^3\text{cs}}{}$$

$$= -\frac{3x^2}{2b^2} + \frac{x^3}{2b} - \frac{3x^2\tanh(a+bx)}{2b^2} - \frac{x^3\tanh^2(a+bx)}{2b} + 2\int x^3\text{csch}(2a+2bx$$

$$= -\frac{3x^2}{2b^2} + \frac{x^3}{2b} - \frac{2x^3\tanh^{-1}\left(e^{2a+2bx}\right)}{b} + \frac{3x\log\left(1+e^{2(a+bx)}\right)}{b^3} - \frac{3x^2\tanh(a+b}{2b^2}$$

$$= -\frac{3x^2}{2b^2} + \frac{x^3}{2b} - \frac{2x^3\tanh^{-1}\left(e^{2a+2bx}\right)}{b} + \frac{3x\log\left(1+e^{2(a+bx)}\right)}{b^3} - \frac{3x^2\text{Li}_2\left(-e^{2a+2}\right.}{2b^2}$$

$$= -\frac{3x^2}{2b^2} + \frac{x^3}{2b} - \frac{2x^3\tanh^{-1}\left(e^{2a+2bx}\right)}{b} + \frac{3x\log\left(1+e^{2(a+bx)}\right)}{b^3} + \frac{3\text{Li}_2\left(-e^{2(a+bx)}\right.}{2b^4}$$

$$= -\frac{3x^2}{2b^2} + \frac{x^3}{2b} - \frac{2x^3\tanh^{-1}\left(e^{2a+2bx}\right)}{b} + \frac{3x\log\left(1+e^{2(a+bx)}\right)}{b^3} + \frac{3\text{Li}_2\left(-e^{2(a+bx)}\right.}{2b^4}$$

$$= -\frac{3x^2}{2b^2} + \frac{x^3}{2b} - \frac{2x^3\tanh^{-1}\left(e^{2a+2bx}\right)}{b} + \frac{3x\log\left(1+e^{2(a+bx)}\right)}{b^3} + \frac{3\text{Li}_2\left(-e^{2(a+bx)}\right.}{2b^4}$$

Mathematica [A] time = 7.39, size = 462, normalized size = 1.92

$$-\frac{3x^2\text{sech}(a)\sinh(bx)\text{sech}(a+bx)}{2b^2} - \frac{e^{2a}\left(e^{-2a}b^4x^4 - 2\left(1-e^{-2a}\right)b^3x^3\log\left(1-e^{-a-bx}\right) - 2\left(1-e^{-2a}\right)b^3x^3\log\left(e^{-a-}\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Csch[a + b*x]*Sech[a + b*x]^3,x]
```

```
[Out] -1/2*(E^(2*a)*((b^4*x^4)/E^(2*a) - 2*b^3*(1 - E^(-2*a))*x^3*Log[1 - E^(-a -
 b*x)] - 2*b^3*(1 - E^(-2*a))*x^3*Log[1 + E^(-a - b*x)] + 6*(1 - E^(-2*a))*
(b^2*x^2*PolyLog[2, -E^(-a - b*x)] + 2*(b*x*PolyLog[3, -E^(-a - b*x)] + Pol
yLog[4, -E^(-a - b*x)])) + 6*(1 - E^(-2*a))*(b^2*x^2*PolyLog[2, E^(-a - b*x
)] + 2*(b*x*PolyLog[3, E^(-a - b*x)] + PolyLog[4, E^(-a - b*x)]))))/(b^4*(-
1 + E^(2*a))) - (E^(2*a)*((-12*b^2*x^2)/E^(2*a) + (2*b^4*x^4)/E^(2*a) - 12*
b*(1 + E^(-2*a))*x*Log[1 + E^(-2*(a + b*x))] + 4*b^3*(1 + E^(-2*a))*x^3*Log
[1 + E^(-2*(a + b*x))] + 6*(1 + E^(-2*a))*PolyLog[2, -E^(-2*(a + b*x))] - (
3*(1 + E^(2*a))*(2*b^2*x^2*PolyLog[2, -E^(-2*(a + b*x))] + 2*b*x*PolyLog[3,
 -E^(-2*(a + b*x))] + PolyLog[4, -E^(-2*(a + b*x))]))/E^(2*a)))/(4*b^4*(1 +
 E^(2*a))) + (x^4*Csch[a]*Sech[a])/4 + (x^3*Sech[a + b*x]^2)/(2*b) - (3*x^2
*Sech[a]*Sech[a + b*x]*Sinh[b*x])/(2*b^2)
```

fricas [C] time = 0.51, size = 3409, normalized size = 14.20

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*csch(b*x+a)*sech(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -(3*(b^2*x^2 - a^2)*cosh(b*x + a)^4 + 12*(b^2*x^2 - a^2)*cosh(b*x + a)*sinh
(b*x + a)^3 + 3*(b^2*x^2 - a^2)*sinh(b*x + a)^4 - (2*b^3*x^3 - 3*b^2*x^2 +
6*a^2)*cosh(b*x + a)^2 - (2*b^3*x^3 - 3*b^2*x^2 - 18*(b^2*x^2 - a^2)*cosh(b
*x + a)^2 + 6*a^2)*sinh(b*x + a)^2 - 3*a^2 - 3*(b^2*x^2*cosh(b*x + a)^4 + 4
*b^2*x^2*cosh(b*x + a)*sinh(b*x + a)^3 + b^2*x^2*sinh(b*x + a)^4 + 2*b^2*x^
2*cosh(b*x + a)^2 + b^2*x^2 + 2*(3*b^2*x^2*cosh(b*x + a)^2 + b^2*x^2)*sinh(
b*x + a)^2 + 4*(b^2*x^2*cosh(b*x + a)^3 + b^2*x^2*cosh(b*x + a))*sinh(b*x +
 a))*dilog(cosh(b*x + a) + sinh(b*x + a)) + 3*((b^2*x^2 - 1)*cosh(b*x + a)^
4 + 4*(b^2*x^2 - 1)*cosh(b*x + a)*sinh(b*x + a)^3 + (b^2*x^2 - 1)*sinh(b*x
+ a)^4 + b^2*x^2 + 2*(b^2*x^2 - 1)*cosh(b*x + a)^2 + 2*(b^2*x^2 + 3*(b^2*x^
2 - 1)*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^2 + 4*((b^2*x^2 - 1)*cosh(b*x + a
)^3 + (b^2*x^2 - 1)*cosh(b*x + a))*sinh(b*x + a) - 1)*dilog(I*cosh(b*x + a)
 + I*sinh(b*x + a)) + 3*((b^2*x^2 - 1)*cosh(b*x + a)^4 + 4*(b^2*x^2 - 1)*co
sh(b*x + a)*sinh(b*x + a)^3 + (b^2*x^2 - 1)*sinh(b*x + a)^4 + b^2*x^2 + 2*(
b^2*x^2 - 1)*cosh(b*x + a)^2 + 2*(b^2*x^2 + 3*(b^2*x^2 - 1)*cosh(b*x + a)^2
 - 1)*sinh(b*x + a)^2 + 4*((b^2*x^2 - 1)*cosh(b*x + a)^3 + (b^2*x^2 - 1)*co
sh(b*x + a))*sinh(b*x + a) - 1)*dilog(-I*cosh(b*x + a) - I*sinh(b*x + a)) -
 3*(b^2*x^2*cosh(b*x + a)^4 + 4*b^2*x^2*cosh(b*x + a)*sinh(b*x + a)^3 + b^2
*x^2*sinh(b*x + a)^4 + 2*b^2*x^2*cosh(b*x + a)^2 + b^2*x^2 + 2*(3*b^2*x^2*c
osh(b*x + a)^2 + b^2*x^2)*sinh(b*x + a)^2 + 4*(b^2*x^2*cosh(b*x + a)^3 + b^
2*x^2*cosh(b*x + a))*sinh(b*x + a))*dilog(-cosh(b*x + a) - sinh(b*x + a)) -
 (b^3*x^3*cosh(b*x + a)^4 + 4*b^3*x^3*cosh(b*x + a)*sinh(b*x + a)^3 + b^3*x
^3*sinh(b*x + a)^4 + 2*b^3*x^3*cosh(b*x + a)^2 + b^3*x^3 + 2*(3*b^3*x^3*cos
```

```
h(b*x + a)^2 + b^3*x^3)*sinh(b*x + a)^2 + 4*(b^3*x^3*cosh(b*x + a)^3 + b^3*
x^3*cosh(b*x + a))*sinh(b*x + a))*log(cosh(b*x + a) + sinh(b*x + a) + 1) -
((a^3 - 3*a)*cosh(b*x + a)^4 + 4*(a^3 - 3*a)*cosh(b*x + a)*sinh(b*x + a)^3
+ (a^3 - 3*a)*sinh(b*x + a)^4 + a^3 + 2*(a^3 - 3*a)*cosh(b*x + a)^2 + 2*(a^
3 + 3*(a^3 - 3*a)*cosh(b*x + a)^2 - 3*a)*sinh(b*x + a)^2 + 4*((a^3 - 3*a)*c
osh(b*x + a)^3 + (a^3 - 3*a)*cosh(b*x + a))*sinh(b*x + a) - 3*a)*log(cosh(b
*x + a) + sinh(b*x + a) + I) - ((a^3 - 3*a)*cosh(b*x + a)^4 + 4*(a^3 - 3*a)
*cosh(b*x + a)*sinh(b*x + a)^3 + (a^3 - 3*a)*sinh(b*x + a)^4 + a^3 + 2*(a^3
 - 3*a)*cosh(b*x + a)^2 + 2*(a^3 + 3*(a^3 - 3*a)*cosh(b*x + a)^2 - 3*a)*sin
h(b*x + a)^2 + 4*((a^3 - 3*a)*cosh(b*x + a)^3 + (a^3 - 3*a)*cosh(b*x + a))*
sinh(b*x + a) - 3*a)*log(cosh(b*x + a) + sinh(b*x + a) - I) + (a^3*cosh(b*x
 + a)^4 + 4*a^3*cosh(b*x + a)*sinh(b*x + a)^3 + a^3*sinh(b*x + a)^4 + 2*a^3
*cosh(b*x + a)^2 + a^3 + 2*(3*a^3*cosh(b*x + a)^2 + a^3)*sinh(b*x + a)^2 +
4*(a^3*cosh(b*x + a)^3 + a^3*cosh(b*x + a))*sinh(b*x + a))*log(cosh(b*x + a
) + sinh(b*x + a) - 1) + (b^3*x^3 + (b^3*x^3 + a^3 - 3*b*x - 3*a)*cosh(b*x
+ a)^4 + 4*(b^3*x^3 + a^3 - 3*b*x - 3*a)*cosh(b*x + a)*sinh(b*x + a)^3 + (b
^3*x^3 + a^3 - 3*b*x - 3*a)*sinh(b*x + a)^4 + a^3 + 2*(b^3*x^3 + a^3 - 3*b*
x - 3*a)*cosh(b*x + a)^2 + 2*(b^3*x^3 + a^3 + 3*(b^3*x^3 + a^3 - 3*b*x - 3*
a)*cosh(b*x + a)^2 - 3*b*x - 3*a)*sinh(b*x + a)^2 - 3*b*x + 4*((b^3*x^3 + a
^3 - 3*b*x - 3*a)*cosh(b*x + a)^3 + (b^3*x^3 + a^3 - 3*b*x - 3*a)*cosh(b*x
+ a))*sinh(b*x + a) - 3*a)*log(I*cosh(b*x + a) + I*sinh(b*x + a) + 1) + (b^
3*x^3 + (b^3*x^3 + a^3 - 3*b*x - 3*a)*cosh(b*x + a)^4 + 4*(b^3*x^3 + a^3 -
3*b*x - 3*a)*cosh(b*x + a)*sinh(b*x + a)^3 + (b^3*x^3 + a^3 - 3*b*x - 3*a)*
sinh(b*x + a)^4 + a^3 + 2*(b^3*x^3 + a^3 - 3*b*x - 3*a)*cosh(b*x + a)^2 + 2
*(b^3*x^3 + a^3 + 3*(b^3*x^3 + a^3 - 3*b*x - 3*a)*cosh(b*x + a)^2 - 3*b*x -
 3*a)*sinh(b*x + a)^2 - 3*b*x + 4*((b^3*x^3 + a^3 - 3*b*x - 3*a)*cosh(b*x +
 a)^3 + (b^3*x^3 + a^3 - 3*b*x - 3*a)*cosh(b*x + a))*sinh(b*x + a) - 3*a)*l
og(-I*cosh(b*x + a) - I*sinh(b*x + a) + 1) - (b^3*x^3 + (b^3*x^3 + a^3)*cos
h(b*x + a)^4 + 4*(b^3*x^3 + a^3)*cosh(b*x + a)*sinh(b*x + a)^3 + (b^3*x^3 +
 a^3)*sinh(b*x + a)^4 + a^3 + 2*(b^3*x^3 + a^3)*cosh(b*x + a)^2 + 2*(b^3*x^
3 + a^3 + 3*(b^3*x^3 + a^3)*cosh(b*x + a)^2)*sinh(b*x + a)^2 + 4*((b^3*x^3
+ a^3)*cosh(b*x + a)^3 + (b^3*x^3 + a^3)*cosh(b*x + a))*sinh(b*x + a))*log(
-cosh(b*x + a) - sinh(b*x + a) + 1) - 6*(cosh(b*x + a)^4 + 4*cosh(b*x + a)*
sinh(b*x + a)^3 + sinh(b*x + a)^4 + 2*(3*cosh(b*x + a)^2 + 1)*sinh(b*x + a)
^2 + 2*cosh(b*x + a)^2 + 4*(cosh(b*x + a)^3 + cosh(b*x + a))*sinh(b*x + a)
+ 1)*polylog(4, cosh(b*x + a) + sinh(b*x + a)) + 6*(cosh(b*x + a)^4 + 4*cos
h(b*x + a)*sinh(b*x + a)^3 + sinh(b*x + a)^4 + 2*(3*cosh(b*x + a)^2 + 1)*si
nh(b*x + a)^2 + 2*cosh(b*x + a)^2 + 4*(cosh(b*x + a)^3 + cosh(b*x + a))*sin
h(b*x + a) + 1)*polylog(4, I*cosh(b*x + a) + I*sinh(b*x + a)) + 6*(cosh(b*x
 + a)^4 + 4*cosh(b*x + a)*sinh(b*x + a)^3 + sinh(b*x + a)^4 + 2*(3*cosh(b*x
 + a)^2 + 1)*sinh(b*x + a)^2 + 2*cosh(b*x + a)^2 + 4*(cosh(b*x + a)^3 + cos
h(b*x + a))*sinh(b*x + a) + 1)*polylog(4, -I*cosh(b*x + a) - I*sinh(b*x + a
)) - 6*(cosh(b*x + a)^4 + 4*cosh(b*x + a)*sinh(b*x + a)^3 + sinh(b*x + a)^4
 + 2*(3*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^2 + 2*cosh(b*x + a)^2 + 4*(cosh(
b*x + a)^3 + cosh(b*x + a))*sinh(b*x + a) + 1)*polylog(4, -cosh(b*x + a) -
```

```
sinh(b*x + a)) + 6*(b*x*cosh(b*x + a)^4 + 4*b*x*cosh(b*x + a)*sinh(b*x + a)
^3 + b*x*sinh(b*x + a)^4 + 2*b*x*cosh(b*x + a)^2 + 2*(3*b*x*cosh(b*x + a)^2
 + b*x)*sinh(b*x + a)^2 + b*x + 4*(b*x*cosh(b*x + a)^3 + b*x*cosh(b*x + a))
*sinh(b*x + a))*polylog(3, cosh(b*x + a) + sinh(b*x + a)) - 6*(b*x*cosh(b*x
 + a)^4 + 4*b*x*cosh(b*x + a)*sinh(b*x + a)^3 + b*x*sinh(b*x + a)^4 + 2*b*x
*cosh(b*x + a)^2 + 2*(3*b*x*cosh(b*x + a)^2 + b*x)*sinh(b*x + a)^2 + b*x +
4*(b*x*cosh(b*x + a)^3 + b*x*cosh(b*x + a))*sinh(b*x + a))*polylog(3, I*cos
h(b*x + a) + I*sinh(b*x + a)) - 6*(b*x*cosh(b*x + a)^4 + 4*b*x*cosh(b*x + a
)*sinh(b*x + a)^3 + b*x*sinh(b*x + a)^4 + 2*b*x*cosh(b*x + a)^2 + 2*(3*b*x*
cosh(b*x + a)^2 + b*x)*sinh(b*x + a)^2 + b*x + 4*(b*x*cosh(b*x + a)^3 + b*x
*cosh(b*x + a))*sinh(b*x + a))*polylog(3, -I*cosh(b*x + a) - I*sinh(b*x + a
)) + 6*(b*x*cosh(b*x + a)^4 + 4*b*x*cosh(b*x + a)*sinh(b*x + a)^3 + b*x*sin
h(b*x + a)^4 + 2*b*x*cosh(b*x + a)^2 + 2*(3*b*x*cosh(b*x + a)^2 + b*x)*sinh
(b*x + a)^2 + b*x + 4*(b*x*cosh(b*x + a)^3 + b*x*cosh(b*x + a))*sinh(b*x +
a))*polylog(3, -cosh(b*x + a) - sinh(b*x + a)) + 2*(6*(b^2*x^2 - a^2)*cosh(
b*x + a)^3 - (2*b^3*x^3 - 3*b^2*x^2 + 6*a^2)*cosh(b*x + a))*sinh(b*x + a))/
(b^4*cosh(b*x + a)^4 + 4*b^4*cosh(b*x + a)*sinh(b*x + a)^3 + b^4*sinh(b*x +
 a)^4 + 2*b^4*cosh(b*x + a)^2 + b^4 + 2*(3*b^4*cosh(b*x + a)^2 + b^4)*sinh(
b*x + a)^2 + 4*(b^4*cosh(b*x + a)^3 + b^4*cosh(b*x + a))*sinh(b*x + a))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3 \operatorname{csch}(bx+a) \operatorname{sech}(bx+a)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*csch(b*x+a)*sech(b*x+a)^3,x, algorithm="giac")
```

```
[Out] integrate(x^3*csch(b*x + a)*sech(b*x + a)^3, x)
```

maple [A] time = 0.53, size = 359, normalized size = 1.50

$$\frac{x^2\left(2bx\,e^{2bx+2a}+3\,e^{2bx+2a}+3\right)}{b^2\left(1+e^{2bx+2a}\right)^2}-\frac{3x^2}{b^2}-\frac{3a^2}{b^4}+\frac{3x\ln\left(1+e^{2bx+2a}\right)}{b^3}+\frac{3x\,\mathrm{polylog}\left(3,-e^{2bx+2a}\right)}{2b^3}-\frac{x^3\ln\left(1+e^{2bx+2a}\right)}{b}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*csch(b*x+a)*sech(b*x+a)^3,x)
```

```
[Out] x^2*(2*b*x*exp(2*b*x+2*a)+3*exp(2*b*x+2*a)+3)/b^2/(1+exp(2*b*x+2*a))^2-3*x^
2/b^2-3/b^4*a^2+3*x*ln(1+exp(2*b*x+2*a))/b^3+3/2*x*polylog(3,-exp(2*b*x+2*a
))/b^3-x^3*ln(1+exp(2*b*x+2*a))/b-3/2*x^2*polylog(2,-exp(2*b*x+2*a))/b^2+1/
b*ln(1-exp(b*x+a))*x^3+3*x^2*polylog(2,exp(b*x+a))/b^2-6*x*polylog(3,exp(b*
x+a))/b^3+1/b*ln(1+exp(b*x+a))*x^3+3*x^2*polylog(2,-exp(b*x+a))/b^2-6*x*pol
ylog(3,-exp(b*x+a))/b^3+1/b^4*ln(1-exp(b*x+a))*a^3-6*a*x/b^3+6/b^4*a*ln(exp
```

```
(b*x+a))-3/4*polylog(4,-exp(2*b*x+2*a))/b^4+3/2*polylog(2,-exp(2*b*x+2*a))/
b^4+6*polylog(4,exp(b*x+a))/b^4+6*polylog(4,-exp(b*x+a))/b^4-1/b^4*a^3*ln(e
xp(b*x+a)-1)
```

maxima [A] time = 0.36, size = 329, normalized size = 1.37

$$-\frac{1}{2}x^4+\frac{3x^2+\left(2bx^3e^{(2a)}+3x^2e^{(2a)}\right)e^{(2bx)}}{b^2e^{(4bx+4a)}+2b^2e^{(2bx+2a)}+b^2}+\frac{b^4x^4-6b^2x^2}{2b^4}-\frac{4b^3x^3\log\left(e^{(2bx+2a)}+1\right)+6b^2x^2\mathrm{Li}_2\left(-e^{(2bx+2a)}\right)-6}{3b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*csch(b*x+a)*sech(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -1/2*x^4 + (3*x^2 + (2*b*x^3*e^(2*a) + 3*x^2*e^(2*a))*e^(2*b*x))/(b^2*e^(4*
b*x + 4*a) + 2*b^2*e^(2*b*x + 2*a) + b^2) + 1/2*(b^4*x^4 - 6*b^2*x^2)/b^4 -
 1/3*(4*b^3*x^3*log(e^(2*b*x + 2*a) + 1) + 6*b^2*x^2*dilog(-e^(2*b*x + 2*a)
) - 6*b*x*polylog(3, -e^(2*b*x + 2*a)) + 3*polylog(4, -e^(2*b*x + 2*a)))/b^
4 + (b^3*x^3*log(e^(b*x + a) + 1) + 3*b^2*x^2*dilog(-e^(b*x + a)) - 6*b*x*p
olylog(3, -e^(b*x + a)) + 6*polylog(4, -e^(b*x + a)))/b^4 + (b^3*x^3*log(-e
^(b*x + a) + 1) + 3*b^2*x^2*dilog(e^(b*x + a)) - 6*b*x*polylog(3, e^(b*x +
a)) + 6*polylog(4, e^(b*x + a)))/b^4 + 3/2*(2*b*x*log(e^(2*b*x + 2*a) + 1)
+ dilog(-e^(2*b*x + 2*a)))/b^4
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3}{\cosh(a+b\,x)^3\,\sinh(a+b\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(cosh(a + b*x)^3*sinh(a + b*x)),x)
```

```
[Out] int(x^3/(cosh(a + b*x)^3*sinh(a + b*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3\operatorname{csch}(a+bx)\operatorname{sech}^3(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*csch(b*x+a)*sech(b*x+a)**3,x)
```

```
[Out] Integral(x**3*csch(a + b*x)*sech(a + b*x)**3, x)
```

3.482 $\int x^2 \operatorname{csch}(a+bx)\operatorname{sech}^3(a+bx)\,dx$

Optimal. Leaf size=148

$$\frac{\operatorname{Li}_3\left(-e^{2a+2bx}\right)}{2b^3}-\frac{\operatorname{Li}_3\left(e^{2a+2bx}\right)}{2b^3}+\frac{\log(\cosh(a+bx))}{b^3}-\frac{x\operatorname{Li}_2\left(-e^{2a+2bx}\right)}{b^2}+\frac{x\operatorname{Li}_2\left(e^{2a+2bx}\right)}{b^2}-\frac{x\tanh(a+bx)}{b^2}-\frac{2x^2\tanh^{-1}}{b}$$

```
[Out] 1/2*x^2/b-2*x^2*arctanh(exp(2*b*x+2*a))/b+ln(cosh(b*x+a))/b^3-x*polylog(2,-
exp(2*b*x+2*a))/b^2+x*polylog(2,exp(2*b*x+2*a))/b^2+1/2*polylog(3,-exp(2*b*
x+2*a))/b^3-1/2*polylog(3,exp(2*b*x+2*a))/b^3-x*tanh(b*x+a)/b^2-1/2*x^2*tan
h(b*x+a)^2/b
```

Rubi [A] time = 0.25, antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 12, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {2620, 14, 5462, 2551, 12, 4182, 2531, 2282, 6589, 3720, 3475, 30}

$$-\frac{x\text{PolyLog}\left(2,-e^{2a+2bx}\right)}{b^2}+\frac{x\text{PolyLog}\left(2,e^{2a+2bx}\right)}{b^2}+\frac{\text{PolyLog}\left(3,-e^{2a+2bx}\right)}{2b^3}-\frac{\text{PolyLog}\left(3,e^{2a+2bx}\right)}{2b^3}-\frac{x\tanh(a+bx}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Csch[a + b*x]*Sech[a + b*x]^3,x]
```

```
[Out] x^2/(2*b) - (2*x^2*ArcTanh[E^(2*a + 2*b*x)])/b + Log[Cosh[a + b*x]]/b^3 - (
x*PolyLog[2, -E^(2*a + 2*b*x)])/b^2 + (x*PolyLog[2, E^(2*a + 2*b*x)])/b^2 +
 PolyLog[3, -E^(2*a + 2*b*x)]/(2*b^3) - PolyLog[3, E^(2*a + 2*b*x)]/(2*b^3)
 - (x*Tanh[a + b*x])/b^2 - (x^2*Tanh[a + b*x]^2)/(2*b)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2551

```
Int[Log[u_]*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[((a + b*x)^(m + 1
)*Log[u])/(b*(m + 1)), x] - Dist[1/(b*(m + 1)), Int[SimplifyIntegrand[((a +
 b*x)^(m + 1)*D[u, x])/u, x], x], x] /; FreeQ[{a, b, m}, x] && InverseFunct
ionFreeQ[u, x] && NeQ[m, -1]
```

Rule 2620

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3720

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[(b*(c + d*x)^m*(b*Tan[e + f*x])^(n - 1))/(f*(n - 1)), x] + (-Di
st[(b*d*m)/(f*(n - 1)), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x],
 x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[
{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
```

```
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5462

```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> With[{u = IntHide[Csch[a + b*x]^n*Sech[a +
b*x]^p, x]}, Dist[(c + d*x)^m, u, x] - Dist[d*m, Int[(c + d*x)^(m - 1)*u, x
], x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n,
p]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int x^2\text{csch}(a+bx)\text{sech}^3(a+bx)\,dx = \frac{x^2\log(\tanh(a+bx))}{b} - \frac{x^2\tanh^2(a+bx)}{2b} - 2\int x\left(\frac{\log(\tanh(a+bx))}{b} - \text{ta}\right.$$
$$= \frac{x^2\log(\tanh(a+bx))}{b} - \frac{x^2\tanh^2(a+bx)}{2b} - 2\int\left(\frac{x\log(\tanh(a+bx))}{b} - \frac{x\,\text{t}}{}\right.$$
$$= \frac{x^2\log(\tanh(a+bx))}{b} - \frac{x^2\tanh^2(a+bx)}{2b} + \frac{\int x\tanh^2(a+bx)\,dx}{b} - \frac{2\int x\,\text{lo}}{}$$
$$= -\frac{x\tanh(a+bx)}{b^2} - \frac{x^2\tanh^2(a+bx)}{2b} + \frac{\int\tanh(a+bx)\,dx}{b^2} + \frac{\int x\,dx}{b} + \frac{\int 2b}{}$$
$$= \frac{x^2}{2b} + \frac{\log(\cosh(a+bx))}{b^3} - \frac{x\tanh(a+bx)}{b^2} - \frac{x^2\tanh^2(a+bx)}{2b} + 2\int x^2\text{csc}$$
$$= \frac{x^2}{2b} - \frac{2x^2\tanh^{-1}\left(e^{2a+2bx}\right)}{b} + \frac{\log(\cosh(a+bx))}{b^3} - \frac{x\tanh(a+bx)}{b^2} - \frac{x^2\,\text{tanh}}{}$$
$$= \frac{x^2}{2b} - \frac{2x^2\tanh^{-1}\left(e^{2a+2bx}\right)}{b} + \frac{\log(\cosh(a+bx))}{b^3} - \frac{x\text{Li}_2\left(-e^{2a+2bx}\right)}{b^2} + \frac{x\text{Li}_2\left(\right.}{b}$$
$$= \frac{x^2}{2b} - \frac{2x^2\tanh^{-1}\left(e^{2a+2bx}\right)}{b} + \frac{\log(\cosh(a+bx))}{b^3} - \frac{x\text{Li}_2\left(-e^{2a+2bx}\right)}{b^2} + \frac{x\text{Li}_2\left(\right.}{b}$$
$$= \frac{x^2}{2b} - \frac{2x^2\tanh^{-1}\left(e^{2a+2bx}\right)}{b} + \frac{\log(\cosh(a+bx))}{b^3} - \frac{x\text{Li}_2\left(-e^{2a+2bx}\right)}{b^2} + \frac{x\text{Li}_2\left(\right.}{b}$$

Mathematica [B] time = 3.87, size = 362, normalized size = 2.45

$$\frac{1}{6}\left(-\frac{6x\text{sech}(a)\sinh(bx)\text{sech}(a+bx)}{b^2} - \frac{2e^{2a}\left(2e^{-2a}b^3x^3 - 3\left(1-e^{-2a}\right)b^2x^2\log\left(1-e^{-a-bx}\right) - 3\left(1-e^{-2a}\right)b^2x^2\log\right.}{}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Csch[a + b*x]*Sech[a + b*x]^3,x]
```

```
[Out] ((-2*E^(2*a)*((2*b^3*x^3)/E^(2*a) - 3*b^2*(1 - E^(-2*a))*x^2*Log[1 - E^(-a
- b*x)] - 3*b^2*(1 - E^(-2*a))*x^2*Log[1 + E^(-a - b*x)] + 6*(1 - E^(-2*a))
*(b*x*PolyLog[2, -E^(-a - b*x)] + PolyLog[3, -E^(-a - b*x)]) + 6*(1 - E^(-2
*a))*(b*x*PolyLog[2, E^(-a - b*x)] + PolyLog[3, E^(-a - b*x)])))/(b^3*(-1 +
 E^(2*a))) + (-12*b*E^(2*a)*x - 4*b^3*x^3 - 6*b^2*(1 + E^(2*a))*x^2*Log[1 +
 E^(-2*(a + b*x))] + 6*Log[1 + E^(2*(a + b*x))] + 6*E^(2*a)*Log[1 + E^(2*(a
 + b*x))] + 6*b*(1 + E^(2*a))*x*PolyLog[2, -E^(-2*(a + b*x))] + 3*(1 + E^(2
*a))*PolyLog[3, -E^(-2*(a + b*x))])/(b^3*(1 + E^(2*a))) + 2*x^3*Csch[a]*Sec
h[a] + (3*x^2*Sech[a + b*x]^2)/b - (6*x*Sech[a]*Sech[a + b*x]*Sinh[b*x])/b^
2)/6
```

fricas [C] time = 0.48, size = 2523, normalized size = 17.05

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*csch(b*x+a)*sech(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -(2*(b*x + a)*cosh(b*x + a)^4 + 8*(b*x + a)*cosh(b*x + a)*sinh(b*x + a)^3 +
 2*(b*x + a)*sinh(b*x + a)^4 - 2*(b^2*x^2 - b*x - 2*a)*cosh(b*x + a)^2 - 2*
(b^2*x^2 - 6*(b*x + a)*cosh(b*x + a)^2 - b*x - 2*a)*sinh(b*x + a)^2 - 2*(b*
x*cosh(b*x + a)^4 + 4*b*x*cosh(b*x + a)*sinh(b*x + a)^3 + b*x*sinh(b*x + a)
^4 + 2*b*x*cosh(b*x + a)^2 + 2*(3*b*x*cosh(b*x + a)^2 + b*x)*sinh(b*x + a)^
2 + b*x + 4*(b*x*cosh(b*x + a)^3 + b*x*cosh(b*x + a))*sinh(b*x + a))*dilog(
cosh(b*x + a) + sinh(b*x + a)) + 2*(b*x*cosh(b*x + a)^4 + 4*b*x*cosh(b*x +
a)*sinh(b*x + a)^3 + b*x*sinh(b*x + a)^4 + 2*b*x*cosh(b*x + a)^2 + 2*(3*b*x
*cosh(b*x + a)^2 + b*x)*sinh(b*x + a)^2 + b*x + 4*(b*x*cosh(b*x + a)^3 + b*
x*cosh(b*x + a))*sinh(b*x + a))*dilog(I*cosh(b*x + a) + I*sinh(b*x + a)) +
2*(b*x*cosh(b*x + a)^4 + 4*b*x*cosh(b*x + a)*sinh(b*x + a)^3 + b*x*sinh(b*x
 + a)^4 + 2*b*x*cosh(b*x + a)^2 + 2*(3*b*x*cosh(b*x + a)^2 + b*x)*sinh(b*x
+ a)^2 + b*x + 4*(b*x*cosh(b*x + a)^3 + b*x*cosh(b*x + a))*sinh(b*x + a))*d
ilog(-I*cosh(b*x + a) - I*sinh(b*x + a)) - 2*(b*x*cosh(b*x + a)^4 + 4*b*x*c
osh(b*x + a)*sinh(b*x + a)^3 + b*x*sinh(b*x + a)^4 + 2*b*x*cosh(b*x + a)^2
+ 2*(3*b*x*cosh(b*x + a)^2 + b*x)*sinh(b*x + a)^2 + b*x + 4*(b*x*cosh(b*x +
 a)^3 + b*x*cosh(b*x + a))*sinh(b*x + a))*dilog(-cosh(b*x + a) - sinh(b*x +
 a)) - (b^2*x^2*cosh(b*x + a)^4 + 4*b^2*x^2*cosh(b*x + a)*sinh(b*x + a)^3 +
 b^2*x^2*sinh(b*x + a)^4 + 2*b^2*x^2*cosh(b*x + a)^2 + b^2*x^2 + 2*(3*b^2*x
^2*cosh(b*x + a)^2 + b^2*x^2)*sinh(b*x + a)^2 + 4*(b^2*x^2*cosh(b*x + a)^3
+ b^2*x^2*cosh(b*x + a))*sinh(b*x + a))*log(cosh(b*x + a) + sinh(b*x + a) +
 1) + ((a^2 - 1)*cosh(b*x + a)^4 + 4*(a^2 - 1)*cosh(b*x + a)*sinh(b*x + a)^
3 + (a^2 - 1)*sinh(b*x + a)^4 + 2*(a^2 - 1)*cosh(b*x + a)^2 + 2*(3*(a^2 - 1
)*cosh(b*x + a)^2 + a^2 - 1)*sinh(b*x + a)^2 + a^2 + 4*((a^2 - 1)*cosh(b*x
+ a)^3 + (a^2 - 1)*cosh(b*x + a))*sinh(b*x + a) - 1)*log(cosh(b*x + a) + si
nh(b*x + a) + I) + ((a^2 - 1)*cosh(b*x + a)^4 + 4*(a^2 - 1)*cosh(b*x + a)*s
inh(b*x + a)^3 + (a^2 - 1)*sinh(b*x + a)^4 + 2*(a^2 - 1)*cosh(b*x + a)^2 +
2*(3*(a^2 - 1)*cosh(b*x + a)^2 + a^2 - 1)*sinh(b*x + a)^2 + a^2 + 4*((a^2 -
 1)*cosh(b*x + a)^3 + (a^2 - 1)*cosh(b*x + a))*sinh(b*x + a) - 1)*log(cosh(
b*x + a) + sinh(b*x + a) - I) - (a^2*cosh(b*x + a)^4 + 4*a^2*cosh(b*x + a)*
sinh(b*x + a)^3 + a^2*sinh(b*x + a)^4 + 2*a^2*cosh(b*x + a)^2 + 2*(3*a^2*co
sh(b*x + a)^2 + a^2)*sinh(b*x + a)^2 + a^2 + 4*(a^2*cosh(b*x + a)^3 + a^2*c
osh(b*x + a))*sinh(b*x + a))*log(cosh(b*x + a) + sinh(b*x + a) - 1) + ((b^2
*x^2 - a^2)*cosh(b*x + a)^4 + 4*(b^2*x^2 - a^2)*cosh(b*x + a)*sinh(b*x + a)
^3 + (b^2*x^2 - a^2)*sinh(b*x + a)^4 + b^2*x^2 + 2*(b^2*x^2 - a^2)*cosh(b*x
 + a)^2 + 2*(b^2*x^2 + 3*(b^2*x^2 - a^2)*cosh(b*x + a)^2 - a^2)*sinh(b*x +
a)^2 - a^2 + 4*((b^2*x^2 - a^2)*cosh(b*x + a)^3 + (b^2*x^2 - a^2)*cosh(b*x
```

```
+ a))*sinh(b*x + a))*log(I*cosh(b*x + a) + I*sinh(b*x + a) + 1) + ((b^2*x^2
 - a^2)*cosh(b*x + a)^4 + 4*(b^2*x^2 - a^2)*cosh(b*x + a)*sinh(b*x + a)^3 +
 (b^2*x^2 - a^2)*sinh(b*x + a)^4 + b^2*x^2 + 2*(b^2*x^2 - a^2)*cosh(b*x + a
)^2 + 2*(b^2*x^2 + 3*(b^2*x^2 - a^2)*cosh(b*x + a)^2 - a^2)*sinh(b*x + a)^2
 - a^2 + 4*((b^2*x^2 - a^2)*cosh(b*x + a)^3 + (b^2*x^2 - a^2)*cosh(b*x + a)
)*sinh(b*x + a))*log(-I*cosh(b*x + a) - I*sinh(b*x + a) + 1) - ((b^2*x^2 -
a^2)*cosh(b*x + a)^4 + 4*(b^2*x^2 - a^2)*cosh(b*x + a)*sinh(b*x + a)^3 + (b
^2*x^2 - a^2)*sinh(b*x + a)^4 + b^2*x^2 + 2*(b^2*x^2 - a^2)*cosh(b*x + a)^2
 + 2*(b^2*x^2 + 3*(b^2*x^2 - a^2)*cosh(b*x + a)^2 - a^2)*sinh(b*x + a)^2 -
a^2 + 4*((b^2*x^2 - a^2)*cosh(b*x + a)^3 + (b^2*x^2 - a^2)*cosh(b*x + a))*s
inh(b*x + a))*log(-cosh(b*x + a) - sinh(b*x + a) + 1) + 2*(cosh(b*x + a)^4
+ 4*cosh(b*x + a)*sinh(b*x + a)^3 + sinh(b*x + a)^4 + 2*(3*cosh(b*x + a)^2
+ 1)*sinh(b*x + a)^2 + 2*cosh(b*x + a)^2 + 4*(cosh(b*x + a)^3 + cosh(b*x +
a))*sinh(b*x + a) + 1)*polylog(3, cosh(b*x + a) + sinh(b*x + a)) - 2*(cosh(
b*x + a)^4 + 4*cosh(b*x + a)*sinh(b*x + a)^3 + sinh(b*x + a)^4 + 2*(3*cosh(
b*x + a)^2 + 1)*sinh(b*x + a)^2 + 2*cosh(b*x + a)^2 + 4*(cosh(b*x + a)^3 +
cosh(b*x + a))*sinh(b*x + a) + 1)*polylog(3, I*cosh(b*x + a) + I*sinh(b*x +
 a)) - 2*(cosh(b*x + a)^4 + 4*cosh(b*x + a)*sinh(b*x + a)^3 + sinh(b*x + a)
^4 + 2*(3*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^2 + 2*cosh(b*x + a)^2 + 4*(cos
h(b*x + a)^3 + cosh(b*x + a))*sinh(b*x + a) + 1)*polylog(3, -I*cosh(b*x + a
) - I*sinh(b*x + a)) + 2*(cosh(b*x + a)^4 + 4*cosh(b*x + a)*sinh(b*x + a)^3
 + sinh(b*x + a)^4 + 2*(3*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^2 + 2*cosh(b*x
 + a)^2 + 4*(cosh(b*x + a)^3 + cosh(b*x + a))*sinh(b*x + a) + 1)*polylog(3,
 -cosh(b*x + a) - sinh(b*x + a)) + 4*(2*(b*x + a)*cosh(b*x + a)^3 - (b^2*x^
2 - b*x - 2*a)*cosh(b*x + a))*sinh(b*x + a) + 2*a)/(b^3*cosh(b*x + a)^4 + 4
*b^3*cosh(b*x + a)*sinh(b*x + a)^3 + b^3*sinh(b*x + a)^4 + 2*b^3*cosh(b*x +
 a)^2 + b^3 + 2*(3*b^3*cosh(b*x + a)^2 + b^3)*sinh(b*x + a)^2 + 4*(b^3*cosh
(b*x + a)^3 + b^3*cosh(b*x + a))*sinh(b*x + a))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \operatorname{csch}(bx+a)\operatorname{sech}(bx+a)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*csch(b*x+a)*sech(b*x+a)^3,x, algorithm="giac")
```

```
[Out] integrate(x^2*csch(b*x + a)*sech(b*x + a)^3, x)
```

maple [A] time = 0.57, size = 256, normalized size = 1.73

$$\frac{2x\left(bx\,e^{2bx+2a}+e^{2bx+2a}+1\right)}{b^2\left(1+e^{2bx+2a}\right)^2}-\frac{2\,\mathrm{polylog}\left(3,-e^{bx+a}\right)}{b^3}-\frac{2\ln\left(e^{bx+a}\right)}{b^3}+\frac{\ln\left(1+e^{2bx+2a}\right)}{b^3}+\frac{\ln\left(1+e^{bx+a}\right)x^2}{b}+\frac{2\,\mathrm{polylog}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*csch(b*x+a)*sech(b*x+a)^3,x)
```

```
[Out] 2*x*(b*x*exp(2*b*x+2*a)+exp(2*b*x+2*a)+1)/b^2/(1+exp(2*b*x+2*a))^2-2/b^3*po
lylog(3,-exp(b*x+a))-2/b^3*ln(exp(b*x+a))+1/b^3*ln(1+exp(2*b*x+2*a))+1/b*ln
(1+exp(b*x+a))*x^2+2/b^2*polylog(2,-exp(b*x+a))*x-x^2*ln(1+exp(2*b*x+2*a))/
b-x*polylog(2,-exp(2*b*x+2*a))/b^2+1/b*ln(1-exp(b*x+a))*x^2+2/b^2*polylog(2
,exp(b*x+a))*x+1/b^3*a^2*ln(exp(b*x+a)-1)+1/2*polylog(3,-exp(2*b*x+2*a))/b^
3-2/b^3*polylog(3,exp(b*x+a))-1/b^3*ln(1-exp(b*x+a))*a^2
```

maxima [A] time = 0.36, size = 229, normalized size = 1.55

$$\frac{2\left(\left(bx^2e^{(2a)}+xe^{(2a)}\right)e^{(2bx)}+x\right)}{b^2e^{(4bx+4a)}+2b^2e^{(2bx+2a)}+b^2}-\frac{2x}{b^2}-\frac{2b^2x^2\log\left(e^{(2bx+2a)}+1\right)+2bx\mathrm{Li}_2\left(-e^{(2bx+2a)}\right)-\mathrm{Li}_3(-e^{(2bx+2a)})}{2b^3}+\frac{b^2x^2\log}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*csch(b*x+a)*sech(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 2*((b*x^2*e^(2*a) + x*e^(2*a))*e^(2*b*x) + x)/(b^2*e^(4*b*x + 4*a) + 2*b^2*
e^(2*b*x + 2*a) + b^2) - 2*x/b^2 - 1/2*(2*b^2*x^2*log(e^(2*b*x + 2*a) + 1)
+ 2*b*x*dilog(-e^(2*b*x + 2*a)) - polylog(3, -e^(2*b*x + 2*a)))/b^3 + (b^2*
x^2*log(e^(b*x + a) + 1) + 2*b*x*dilog(-e^(b*x + a)) - 2*polylog(3, -e^(b*x
 + a)))/b^3 + (b^2*x^2*log(-e^(b*x + a) + 1) + 2*b*x*dilog(e^(b*x + a)) - 2
*polylog(3, e^(b*x + a)))/b^3 + log(e^(2*b*x + 2*a) + 1)/b^3
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2}{\cosh(a+b\,x)^3\,\sinh(a+b\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(cosh(a + b*x)^3*sinh(a + b*x)),x)
```

```
[Out] int(x^2/(cosh(a + b*x)^3*sinh(a + b*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2\operatorname{csch}(a+bx)\operatorname{sech}^3(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*csch(b*x+a)*sech(b*x+a)**3,x)
```

```
[Out] Integral(x**2*csch(a + b*x)*sech(a + b*x)**3, x)
```

3.483 $\int x\text{csch}(a+bx)\text{sech}^3(a+bx)\,dx$

Optimal. Leaf size=95

$$-\frac{\text{Li}_2\left(-e^{2a+2bx}\right)}{2b^2}+\frac{\text{Li}_2\left(e^{2a+2bx}\right)}{2b^2}-\frac{\tanh(a+bx)}{2b^2}-\frac{2x\tanh^{-1}\left(e^{2a+2bx}\right)}{b}-\frac{x\tanh^2(a+bx)}{2b}+\frac{x}{2b}$$

```
[Out] 1/2*x/b-2*x*arctanh(exp(2*b*x+2*a))/b-1/2*polylog(2,-exp(2*b*x+2*a))/b^2+1/
2*polylog(2,exp(2*b*x+2*a))/b^2-1/2*tanh(b*x+a)/b^2-1/2*x*tanh(b*x+a)^2/b
```

Rubi [A] time = 0.12, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {2620, 14, 5462, 2548, 12, 4182, 2279, 2391, 3473, 8}

$$-\frac{\text{PolyLog}\left(2,-e^{2a+2bx}\right)}{2b^2}+\frac{\text{PolyLog}\left(2,e^{2a+2bx}\right)}{2b^2}-\frac{\tanh(a+bx)}{2b^2}-\frac{x\tanh^2(a+bx)}{2b}-\frac{2x\tanh^{-1}\left(e^{2a+2bx}\right)}{b}+\frac{x}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[x*Csch[a + b*x]*Sech[a + b*x]^3,x]
```

```
[Out] x/(2*b) - (2*x*ArcTanh[E^(2*a + 2*b*x)])/b - PolyLog[2, -E^(2*a + 2*b*x)]/(
2*b^2) + PolyLog[2, E^(2*a + 2*b*x)]/(2*b^2) - Tanh[a + b*x]/(2*b^2) - (x*T
anh[a + b*x]^2)/(2*b)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2548

```
Int[Log[u_], x_Symbol] :> Simp[x*Log[u], x] - Int[SimplifyIntegrand[(x*D[u,
 x])/u, x], x] /; InverseFunctionFreeQ[u, x]
```

Rule 2620

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5462

```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> With[{u = IntHide[Csch[a + b*x]^n*Sech[a +
b*x]^p, x]}, Dist[(c + d*x)^m, u, x] - Dist[d*m, Int[(c + d*x)^(m - 1)*u, x
], x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n,
p]
```

Rubi steps

$$\int x\text{csch}(a+bx)\text{sech}^3(a+bx)\,dx = \frac{x\log(\tanh(a+bx))}{b} - \frac{x\tanh^2(a+bx)}{2b} - \int\left(\frac{\log(\tanh(a+bx))}{b} - \frac{\tanh^2(a\ldots}{2b}\right.$$
$$= \frac{x\log(\tanh(a+bx))}{b} - \frac{x\tanh^2(a+bx)}{2b} + \frac{\int\tanh^2(a+bx)\,dx}{2b} - \frac{\int\log(\tanh\ldots}{b}$$
$$= -\frac{\tanh(a+bx)}{2b^2} - \frac{x\tanh^2(a+bx)}{2b} + \frac{\int 1\,dx}{2b} + \frac{\int 2bx\text{csch}(2a+2bx)\,dx}{b}$$
$$= \frac{x}{2b} - \frac{\tanh(a+bx)}{2b^2} - \frac{x\tanh^2(a+bx)}{2b} + 2\int x\text{csch}(2a+2bx)\,dx$$
$$= \frac{x}{2b} - \frac{2x\tanh^{-1}\left(e^{2a+2bx}\right)}{b} - \frac{\tanh(a+bx)}{2b^2} - \frac{x\tanh^2(a+bx)}{2b} - \frac{\int\log\left(1-e^{2a\ldots}\right.}{b}$$
$$= \frac{x}{2b} - \frac{2x\tanh^{-1}\left(e^{2a+2bx}\right)}{b} - \frac{\tanh(a+bx)}{2b^2} - \frac{x\tanh^2(a+bx)}{2b} - \frac{\text{Subst}\left(\int\frac{\log(\ldots}{\ldots}\right.}{\ldots}$$
$$= \frac{x}{2b} - \frac{2x\tanh^{-1}\left(e^{2a+2bx}\right)}{b} - \frac{\text{Li}_2\left(-e^{2a+2bx}\right)}{2b^2} + \frac{\text{Li}_2\left(e^{2a+2bx}\right)}{2b^2} - \frac{\tanh(a+bx)}{2b^2} - \ldots$$

Mathematica [A] time = 0.43, size = 139, normalized size = 1.46

$$\frac{\text{Li}_2\left(-e^{-2(a+bx)}\right) - \text{Li}_2\left(e^{-2(a+bx)}\right) + 2a\log\left(1-e^{-2(a+bx)}\right) + 2bx\log\left(1-e^{-2(a+bx)}\right) - 2a\log\left(e^{-2(a+bx)}+1\right) - 2bx\log\ldots}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Csch[a + b*x]*Sech[a + b*x]^3,x]
```

```
[Out] (2*a*Log[1 - E^(-2*(a + b*x))] + 2*b*x*Log[1 - E^(-2*(a + b*x))] - 2*a*Log[
1 + E^(-2*(a + b*x))] - 2*b*x*Log[1 + E^(-2*(a + b*x))] + 2*a*Log[Cosh[a +
b*x]] - 2*a*Log[Sinh[a + b*x]] + PolyLog[2, -E^(-2*(a + b*x))] - PolyLog[2,
 E^(-2*(a + b*x))] + b*x*Sech[a + b*x]^2 - Tanh[a + b*x])/(2*b^2)
```

fricas [C] time = 0.47, size = 1543, normalized size = 16.24

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*csch(b*x+a)*sech(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] ((2*b*x + 1)*cosh(b*x + a)^2 + 2*(2*b*x + 1)*cosh(b*x + a)*sinh(b*x + a) +
(2*b*x + 1)*sinh(b*x + a)^2 + (cosh(b*x + a)^4 + 4*cosh(b*x + a)*sinh(b*x +
 a)^3 + sinh(b*x + a)^4 + 2*(3*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^2 + 2*cos
```

```
h(b*x + a)^2 + 4*(cosh(b*x + a)^3 + cosh(b*x + a))*sinh(b*x + a) + 1)*dilog
(cosh(b*x + a) + sinh(b*x + a)) - (cosh(b*x + a)^4 + 4*cosh(b*x + a)*sinh(b
*x + a)^3 + sinh(b*x + a)^4 + 2*(3*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^2 + 2
*cosh(b*x + a)^2 + 4*(cosh(b*x + a)^3 + cosh(b*x + a))*sinh(b*x + a) + 1)*d
ilog(I*cosh(b*x + a) + I*sinh(b*x + a)) - (cosh(b*x + a)^4 + 4*cosh(b*x + a
)*sinh(b*x + a)^3 + sinh(b*x + a)^4 + 2*(3*cosh(b*x + a)^2 + 1)*sinh(b*x +
a)^2 + 2*cosh(b*x + a)^2 + 4*(cosh(b*x + a)^3 + cosh(b*x + a))*sinh(b*x + a
) + 1)*dilog(-I*cosh(b*x + a) - I*sinh(b*x + a)) + (cosh(b*x + a)^4 + 4*cos
h(b*x + a)*sinh(b*x + a)^3 + sinh(b*x + a)^4 + 2*(3*cosh(b*x + a)^2 + 1)*si
nh(b*x + a)^2 + 2*cosh(b*x + a)^2 + 4*(cosh(b*x + a)^3 + cosh(b*x + a))*sin
h(b*x + a) + 1)*dilog(-cosh(b*x + a) - sinh(b*x + a)) + (b*x*cosh(b*x + a)^
4 + 4*b*x*cosh(b*x + a)*sinh(b*x + a)^3 + b*x*sinh(b*x + a)^4 + 2*b*x*cosh(
b*x + a)^2 + 2*(3*b*x*cosh(b*x + a)^2 + b*x)*sinh(b*x + a)^2 + b*x + 4*(b*x
*cosh(b*x + a)^3 + b*x*cosh(b*x + a))*sinh(b*x + a))*log(cosh(b*x + a) + si
nh(b*x + a) + 1) + (a*cosh(b*x + a)^4 + 4*a*cosh(b*x + a)*sinh(b*x + a)^3 +
 a*sinh(b*x + a)^4 + 2*a*cosh(b*x + a)^2 + 2*(3*a*cosh(b*x + a)^2 + a)*sinh
(b*x + a)^2 + 4*(a*cosh(b*x + a)^3 + a*cosh(b*x + a))*sinh(b*x + a) + a)*lo
g(cosh(b*x + a) + sinh(b*x + a) + I) + (a*cosh(b*x + a)^4 + 4*a*cosh(b*x +
a)*sinh(b*x + a)^3 + a*sinh(b*x + a)^4 + 2*a*cosh(b*x + a)^2 + 2*(3*a*cosh(
b*x + a)^2 + a)*sinh(b*x + a)^2 + 4*(a*cosh(b*x + a)^3 + a*cosh(b*x + a))*s
inh(b*x + a) + a)*log(cosh(b*x + a) + sinh(b*x + a) - I) - (a*cosh(b*x + a)
^4 + 4*a*cosh(b*x + a)*sinh(b*x + a)^3 + a*sinh(b*x + a)^4 + 2*a*cosh(b*x +
 a)^2 + 2*(3*a*cosh(b*x + a)^2 + a)*sinh(b*x + a)^2 + 4*(a*cosh(b*x + a)^3
+ a*cosh(b*x + a))*sinh(b*x + a) + a)*log(cosh(b*x + a) + sinh(b*x + a) - 1
) - ((b*x + a)*cosh(b*x + a)^4 + 4*(b*x + a)*cosh(b*x + a)*sinh(b*x + a)^3
+ (b*x + a)*sinh(b*x + a)^4 + 2*(b*x + a)*cosh(b*x + a)^2 + 2*(3*(b*x + a)*
cosh(b*x + a)^2 + b*x + a)*sinh(b*x + a)^2 + b*x + 4*((b*x + a)*cosh(b*x +
a)^3 + (b*x + a)*cosh(b*x + a))*sinh(b*x + a) + a)*log(I*cosh(b*x + a) + I*
sinh(b*x + a) + 1) - ((b*x + a)*cosh(b*x + a)^4 + 4*(b*x + a)*cosh(b*x + a)
*sinh(b*x + a)^3 + (b*x + a)*sinh(b*x + a)^4 + 2*(b*x + a)*cosh(b*x + a)^2
+ 2*(3*(b*x + a)*cosh(b*x + a)^2 + b*x + a)*sinh(b*x + a)^2 + b*x + 4*((b*x
 + a)*cosh(b*x + a)^3 + (b*x + a)*cosh(b*x + a))*sinh(b*x + a) + a)*log(-I*
cosh(b*x + a) - I*sinh(b*x + a) + 1) + ((b*x + a)*cosh(b*x + a)^4 + 4*(b*x
+ a)*cosh(b*x + a)*sinh(b*x + a)^3 + (b*x + a)*sinh(b*x + a)^4 + 2*(b*x + a
)*cosh(b*x + a)^2 + 2*(3*(b*x + a)*cosh(b*x + a)^2 + b*x + a)*sinh(b*x + a)
^2 + b*x + 4*((b*x + a)*cosh(b*x + a)^3 + (b*x + a)*cosh(b*x + a))*sinh(b*x
 + a) + a)*log(-cosh(b*x + a) - sinh(b*x + a) + 1) + 1)/(b^2*cosh(b*x + a)^
4 + 4*b^2*cosh(b*x + a)*sinh(b*x + a)^3 + b^2*sinh(b*x + a)^4 + 2*b^2*cosh(
b*x + a)^2 + 2*(3*b^2*cosh(b*x + a)^2 + b^2)*sinh(b*x + a)^2 + b^2 + 4*(b^2
*cosh(b*x + a)^3 + b^2*cosh(b*x + a))*sinh(b*x + a))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x \operatorname{csch}(bx + a) \operatorname{sech}(bx + a)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*csch(b*x+a)*sech(b*x+a)^3,x, algorithm="giac")
```

```
[Out] integrate(x*csch(b*x + a)*sech(b*x + a)^3, x)
```

maple [B] time = 0.42, size = 166, normalized size = 1.75

$$\frac{2bx\,\mathrm{e}^{2bx+2a}+\mathrm{e}^{2bx+2a}+1}{b^2\left(1+\mathrm{e}^{2bx+2a}\right)^2}-\frac{x\ln\left(1+\mathrm{e}^{2bx+2a}\right)}{b}-\frac{\mathrm{polylog}\left(2,-\mathrm{e}^{2bx+2a}\right)}{2b^2}+\frac{\ln\left(1-\mathrm{e}^{bx+a}\right)x}{b}+\frac{\ln\left(1-\mathrm{e}^{bx+a}\right)a}{b^2}+\frac{\mathrm{polylog}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*csch(b*x+a)*sech(b*x+a)^3,x)
```

```
[Out] (2*b*x*exp(2*b*x+2*a)+exp(2*b*x+2*a)+1)/b^2/(1+exp(2*b*x+2*a))^2-x*ln(1+exp
(2*b*x+2*a))/b-1/2*polylog(2,-exp(2*b*x+2*a))/b^2+1/b*ln(1-exp(b*x+a))*x+1/
b^2*ln(1-exp(b*x+a))*a+polylog(2,exp(b*x+a))/b^2+1/b*ln(1+exp(b*x+a))*x+pol
ylog(2,-exp(b*x+a))/b^2-1/b^2*a*ln(exp(b*x+a)-1)
```

maxima [A] time = 0.36, size = 142, normalized size = 1.49

$$\frac{\left(2\,bxe^{(2\,a)}+e^{(2\,a)}\right)e^{(2\,bx)}+1}{b^2e^{(4\,bx+4\,a)}+2\,b^2e^{(2\,bx+2\,a)}+b^2}-\frac{2\,bx\log\left(e^{(2\,bx+2\,a)}+1\right)+\mathrm{Li}_2\left(-e^{(2\,bx+2\,a)}\right)}{2\,b^2}+\frac{bx\log\left(e^{(bx+a)}+1\right)+\mathrm{Li}_2\left(-e^{(bx+a)}\right)}{b^2}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*csch(b*x+a)*sech(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] ((2*b*x*e^(2*a) + e^(2*a))*e^(2*b*x) + 1)/(b^2*e^(4*b*x + 4*a) + 2*b^2*e^(2
*b*x + 2*a) + b^2) - 1/2*(2*b*x*log(e^(2*b*x + 2*a) + 1) + dilog(-e^(2*b*x
+ 2*a)))/b^2 + (b*x*log(e^(b*x + a) + 1) + dilog(-e^(b*x + a)))/b^2 + (b*x*
log(-e^(b*x + a) + 1) + dilog(e^(b*x + a)))/b^2
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x}{\cosh\left(a+b\,x\right)^3\,\sinh\left(a+b\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(cosh(a + b*x)^3*sinh(a + b*x)),x)
```

```
[Out] int(x/(cosh(a + b*x)^3*sinh(a + b*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\operatorname{csch}\left(a+bx\right)\operatorname{sech}^3\left(a+bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*csch(b*x+a)*sech(b*x+a)**3,x)
```

```
[Out] Integral(x*csch(a + b*x)*sech(a + b*x)**3, x)
```

3.484 $\int \operatorname{csch}(a+bx)\operatorname{sech}^3(a+bx)\,dx$

Optimal. Leaf size=27

$$\frac{\log(\tanh(a+bx))}{b}-\frac{\tanh^2(a+bx)}{2b}$$

```
[Out] ln(tanh(b*x+a))/b-1/2*tanh(b*x+a)^2/b
```

Rubi [A] time = 0.03, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2620, 14}

$$\frac{\log(\tanh(a+bx))}{b}-\frac{\tanh^2(a+bx)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[a + b*x]*Sech[a + b*x]^3,x]
```

```
[Out] Log[Tanh[a + b*x]]/b - Tanh[a + b*x]^2/(2*b)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2620

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rubi steps

$$\begin{aligned}\int \operatorname{csch}(a+bx)\operatorname{sech}^3(a+bx)\,dx &= \frac{\operatorname{Subst}\left(\int \frac{1+x^2}{x}\,dx, x, i\tanh(a+bx)\right)}{b}\\ &= \frac{\operatorname{Subst}\left(\int \left(\frac{1}{x}+x\right)dx, x, i\tanh(a+bx)\right)}{b}\\ &= \frac{\log(\tanh(a+bx))}{b}-\frac{\tanh^2(a+bx)}{2b}\end{aligned}$$

Mathematica [A] time = 0.03, size = 36, normalized size = 1.33

$$-\frac{-\operatorname{sech}^2(a+bx)-2\log(\sinh(a+bx))+2\log(\cosh(a+bx))}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[a + b*x]*Sech[a + b*x]^3,x]
```

```
[Out] -1/2*(2*Log[Cosh[a + b*x]] - 2*Log[Sinh[a + b*x]] - Sech[a + b*x]^2)/b
```

fricas [B] time = 0.43, size = 371, normalized size = 13.74

$$\frac{2\cosh(bx+a)^2-\left(\cosh(bx+a)^4+4\cosh(bx+a)\sinh(bx+a)^3+\sinh(bx+a)^4+2\left(3\cosh(bx+a)^2+1\right)\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)*sech(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] (2*cosh(b*x + a)^2 - (cosh(b*x + a)^4 + 4*cosh(b*x + a)*sinh(b*x + a)^3 + s
inh(b*x + a)^4 + 2*(3*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^2 + 2*cosh(b*x + a
)^2 + 4*(cosh(b*x + a)^3 + cosh(b*x + a))*sinh(b*x + a) + 1)*log(2*cosh(b*x
 + a)/(cosh(b*x + a) - sinh(b*x + a))) + (cosh(b*x + a)^4 + 4*cosh(b*x + a)
*sinh(b*x + a)^3 + sinh(b*x + a)^4 + 2*(3*cosh(b*x + a)^2 + 1)*sinh(b*x + a
)^2 + 2*cosh(b*x + a)^2 + 4*(cosh(b*x + a)^3 + cosh(b*x + a))*sinh(b*x + a)
 + 1)*log(2*sinh(b*x + a)/(cosh(b*x + a) - sinh(b*x + a))) + 4*cosh(b*x + a
)*sinh(b*x + a) + 2*sinh(b*x + a)^2)/(b*cosh(b*x + a)^4 + 4*b*cosh(b*x + a)
*sinh(b*x + a)^3 + b*sinh(b*x + a)^4 + 2*b*cosh(b*x + a)^2 + 2*(3*b*cosh(b*
x + a)^2 + b)*sinh(b*x + a)^2 + 4*(b*cosh(b*x + a)^3 + b*cosh(b*x + a))*sin
h(b*x + a) + b)
```

giac [B] time = 0.14, size = 93, normalized size = 3.44

$$\frac{\frac{e^{(2bx+2a)}+e^{(-2bx-2a)}+6}{e^{(2bx+2a)}+e^{(-2bx-2a)}+2}-\log\left(e^{(2bx+2a)}+e^{(-2bx-2a)}+2\right)+\log\left(e^{(2bx+2a)}+e^{(-2bx-2a)}-2\right)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)*sech(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/2*((e^(2*b*x + 2*a) + e^(-2*b*x - 2*a) + 6)/(e^(2*b*x + 2*a) + e^(-2*b*x
- 2*a) + 2) - log(e^(2*b*x + 2*a) + e^(-2*b*x - 2*a) + 2) + log(e^(2*b*x +
2*a) + e^(-2*b*x - 2*a) - 2))/b
```

maple [A] time = 0.15, size = 26, normalized size = 0.96

$$\frac{1}{2b\cosh\left(bx+a\right)^{2}}+\frac{\ln\left(\tanh\left(bx+a\right)\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(b*x+a)*sech(b*x+a)^3,x)
```

```
[Out] 1/2/b/cosh(b*x+a)^2+ln(tanh(b*x+a))/b
```

maxima [B] time = 0.46, size = 88, normalized size = 3.26

$$\frac{\log\left(e^{(-bx-a)}+1\right)}{b}+\frac{\log\left(e^{(-bx-a)}-1\right)}{b}-\frac{\log\left(e^{(-2\,bx-2\,a)}+1\right)}{b}+\frac{2\,e^{(-2\,bx-2\,a)}}{b\left(2\,e^{(-2\,bx-2\,a)}+e^{(-4\,bx-4\,a)}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)*sech(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] log(e^(-b*x - a) + 1)/b + log(e^(-b*x - a) - 1)/b - log(e^(-2*b*x - 2*a) +
1)/b + 2*e^(-2*b*x - 2*a)/(b*(2*e^(-2*b*x - 2*a) + e^(-4*b*x - 4*a) + 1))
```

mupad [B] time = 0.07, size = 78, normalized size = 2.89

$$\frac{2}{b\,\left({\mathrm{e}}^{2\,a+2\,b\,x}+1\right)}-\frac{2\,\mathrm{atan}\left(\frac{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}\,\sqrt{-b^2}}{b}\right)}{\sqrt{-b^2}}-\frac{2}{b\,\left(2\,{\mathrm{e}}^{2\,a+2\,b\,x}+{\mathrm{e}}^{4\,a+4\,b\,x}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(a + b*x)^3*sinh(a + b*x)),x)
```

```
[Out] 2/(b*(exp(2*a + 2*b*x) + 1)) - (2*atan((exp(2*a)*exp(2*b*x)*(-b^2)^(1/2))/b
))/(-b^2)^(1/2) - 2/(b*(2*exp(2*a + 2*b*x) + exp(4*a + 4*b*x) + 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\operatorname{csch}\left(a+bx\right)\operatorname{sech}^{3}\left(a+bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)*sech(b*x+a)**3,x)
```

```
[Out] Integral(csch(a + b*x)*sech(a + b*x)**3, x)
```

3.485 $\int \frac{\operatorname{csch}(a+bx)\operatorname{sech}^3(a+bx)}{x}\,dx$

Optimal. Leaf size=21

$$\operatorname{Int}\left(\frac{\operatorname{csch}(a+bx)\operatorname{sech}^3(a+bx)}{x},x\right)$$

```
[Out] CannotIntegrate(csch(b*x+a)*sech(b*x+a)^3/x,x)
```

Rubi [A] time = 0.21, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\operatorname{csch}(a+bx)\operatorname{sech}^3(a+bx)}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Csch[a + b*x]*Sech[a + b*x]^3)/x,x]
```

```
[Out] Defer[Int][(Csch[a + b*x]*Sech[a + b*x]^3)/x, x]
```

Rubi steps

$$\int \frac{\operatorname{csch}(a+bx)\operatorname{sech}^3(a+bx)}{x}\,dx = \int \frac{\operatorname{csch}(a+bx)\operatorname{sech}^3(a+bx)}{x}\,dx$$

Mathematica [A] time = 57.73, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}(a+bx)\operatorname{sech}^3(a+bx)}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Csch[a + b*x]*Sech[a + b*x]^3)/x,x]
```

```
[Out] Integrate[(Csch[a + b*x]*Sech[a + b*x]^3)/x, x]
```

fricas [A] time = 0.44, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{\operatorname{csch}(bx+a)\operatorname{sech}(bx+a)^3}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)*sech(b*x+a)^3/x,x, algorithm="fricas")
[Out] integral(csch(b*x + a)*sech(b*x + a)^3/x, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}(bx+a)\operatorname{sech}(bx+a)^3}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)*sech(b*x+a)^3/x,x, algorithm="giac")
[Out] integrate(csch(b*x + a)*sech(b*x + a)^3/x, x)
```

maple [A] time = 1.52, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}(bx+a)\operatorname{sech}(bx+a)^3}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(b*x+a)*sech(b*x+a)^3/x,x)
[Out] int(csch(b*x+a)*sech(b*x+a)^3/x,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\left(2\,bxe^{(2\,a)}-e^{(2\,a)}\right)e^{(2\,bx)}-1}{b^2x^2e^{(4\,bx+4\,a)}+2\,b^2x^2e^{(2\,bx+2\,a)}+b^2x^2}+16\int \frac{b^2x^2-1}{8\left(b^2x^3e^{(2\,bx+2\,a)}+b^2x^3\right)}\,dx-16\int \frac{1}{16\left(xe^{(bx+a)}+x\right)}\,dx+16\int \frac{}{16\,(x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)*sech(b*x+a)^3/x,x, algorithm="maxima")
[Out] ((2*b*x*e^(2*a) - e^(2*a))*e^(2*b*x) - 1)/(b^2*x^2*e^(4*b*x + 4*a) + 2*b^2*
x^2*e^(2*b*x + 2*a) + b^2*x^2) + 16*integrate(1/8*(b^2*x^2 - 1)/(b^2*x^3*e^
(2*b*x + 2*a) + b^2*x^3), x) - 16*integrate(1/16/(x*e^(b*x + a) + x), x) +
16*integrate(1/16/(x*e^(b*x + a) - x), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{1}{x\cosh(a+b\,x)^3\sinh(a+b\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*cosh(a + b*x)^3*sinh(a + b*x)),x)
```

```
[Out] int(1/(x*cosh(a + b*x)^3*sinh(a + b*x)), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}(a+bx)\operatorname{sech}^3(a+bx)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)*sech(b*x+a)**3/x,x)
```

```
[Out] Integral(csch(a + b*x)*sech(a + b*x)**3/x, x)
```

3.486 $\int \frac{\operatorname{csch}(a+bx)\operatorname{sech}^3(a+bx)}{x^2} dx$

Optimal. Leaf size=21

$$\operatorname{Int}\left(\frac{\operatorname{csch}(a+bx)\operatorname{sech}^3(a+bx)}{x^2},x\right)$$

```
[Out] CannotIntegrate(csch(b*x+a)*sech(b*x+a)^3/x^2,x)
```

Rubi [A] time = 0.25, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\operatorname{csch}(a+bx)\operatorname{sech}^3(a+bx)}{x^2} dx$$

Verification is Not applicable to the result.

```
[In] Int[(Csch[a + b*x]*Sech[a + b*x]^3)/x^2,x]
```

```
[Out] Defer[Int][(Csch[a + b*x]*Sech[a + b*x]^3)/x^2, x]
```

Rubi steps

$$\int \frac{\operatorname{csch}(a+bx)\operatorname{sech}^3(a+bx)}{x^2} dx = \int \frac{\operatorname{csch}(a+bx)\operatorname{sech}^3(a+bx)}{x^2} dx$$

Mathematica [A] time = 29.65, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}(a+bx)\operatorname{sech}^3(a+bx)}{x^2} dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Csch[a + b*x]*Sech[a + b*x]^3)/x^2,x]
```

```
[Out] Integrate[(Csch[a + b*x]*Sech[a + b*x]^3)/x^2, x]
```

fricas [A] time = 0.50, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{\operatorname{csch}(bx+a)\operatorname{sech}(bx+a)^3}{x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)*sech(b*x+a)^3/x^2,x, algorithm="fricas")
```

```
[Out] integral(csch(b*x + a)*sech(b*x + a)^3/x^2, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}(bx+a)\operatorname{sech}(bx+a)^3}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)*sech(b*x+a)^3/x^2,x, algorithm="giac")
```

```
[Out] integrate(csch(b*x + a)*sech(b*x + a)^3/x^2, x)
```

maple [A] time = 1.67, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}(bx+a)\operatorname{sech}(bx+a)^3}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(b*x+a)*sech(b*x+a)^3/x^2,x)
```

```
[Out] int(csch(b*x+a)*sech(b*x+a)^3/x^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\frac{2\left(\left(bxe^{(2a)}-e^{(2a)}\right)e^{(2bx)}-1\right)}{b^2x^3e^{(4bx+4a)}+2b^2x^3e^{(2bx+2a)}+b^2x^3}+16\int\frac{b^2x^2-3}{8\left(b^2x^4e^{(2bx+2a)}+b^2x^4\right)}\,dx-16\int\frac{1}{16\left(x^2e^{(bx+a)}+x^2\right)}\,dx+16\int$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)*sech(b*x+a)^3/x^2,x, algorithm="maxima")
```

```
[Out] 2*((b*x*e^(2*a) - e^(2*a))*e^(2*b*x) - 1)/(b^2*x^3*e^(4*b*x + 4*a) + 2*b^2*
x^3*e^(2*b*x + 2*a) + b^2*x^3) + 16*integrate(1/8*(b^2*x^2 - 3)/(b^2*x^4*e^
(2*b*x + 2*a) + b^2*x^4), x) - 16*integrate(1/16/(x^2*e^(b*x + a) + x^2), x
) + 16*integrate(1/16/(x^2*e^(b*x + a) - x^2), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{1}{x^2\cosh(a+b\,x)^3\sinh(a+b\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*cosh(a + b*x)^3*sinh(a + b*x)),x)
```

```
[Out] int(1/(x^2*cosh(a + b*x)^3*sinh(a + b*x)), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}(a+bx)\operatorname{sech}^3(a+bx)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)*sech(b*x+a)**3/x**2,x)
```

```
[Out] Integral(csch(a + b*x)*sech(a + b*x)**3/x**2, x)
```

3.487 $\int x^m \operatorname{csch}^2(a+bx)\operatorname{sech}(a+bx)\,dx$

Optimal. Leaf size=21

$$\text{Int}\left(x^m\text{csch}^2(a+bx)\text{sech}(a+bx),x\right)$$

[Out] CannotIntegrate(x^m*csch(b*x+a)^2*sech(b*x+a),x)

Rubi [A] time = 0.40, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int x^m\text{csch}^2(a+bx)\text{sech}(a+bx)\,dx$$

Verification is Not applicable to the result.

[In] Int[x^m*Csch[a + b*x]^2*Sech[a + b*x],x]

[Out] Defer[Int][x^m*Csch[a + b*x]^2*Sech[a + b*x], x]

Rubi steps

$$\int x^m\text{csch}^2(a+bx)\text{sech}(a+bx)\,dx=\int x^m\text{csch}^2(a+bx)\text{sech}(a+bx)\,dx$$

Mathematica [A] time = 44.68, size = 0, normalized size = 0.00

$$\int x^m\text{csch}^2(a+bx)\text{sech}(a+bx)\,dx$$

Verification is Not applicable to the result.

[In] Integrate[x^m*Csch[a + b*x]^2*Sech[a + b*x],x]

[Out] Integrate[x^m*Csch[a + b*x]^2*Sech[a + b*x], x]

fricas [A] time = 0.43, size = 0, normalized size = 0.00

$$\text{integral}\left(x^m\,\text{csch}\left(bx+a\right)^2\,\text{sech}\left(bx+a\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^m*csch(b*x+a)^2*sech(b*x+a),x, algorithm="fricas")

[Out] integral(x^m*csch(b*x + a)^2*sech(b*x + a), x)

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int x^m \operatorname{csch}(bx+a)^2 \operatorname{sech}(bx+a)\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*csch(b*x+a)^2*sech(b*x+a),x, algorithm="giac")
[Out] integrate(x^m*csch(b*x + a)^2*sech(b*x + a), x)
```

maple [A] time = 0.16, size = 0, normalized size = 0.00

$$\int x^m \operatorname{csch}(bx+a)^2 \operatorname{sech}(bx+a)\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*csch(b*x+a)^2*sech(b*x+a),x)
[Out] int(x^m*csch(b*x+a)^2*sech(b*x+a),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int x^m \operatorname{csch}(bx+a)^2 \operatorname{sech}(bx+a)\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*csch(b*x+a)^2*sech(b*x+a),x, algorithm="maxima")
[Out] integrate(x^m*csch(b*x + a)^2*sech(b*x + a), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{x^m}{\cosh(a+b\,x)\ \sinh(a+b\,x)^2}\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(cosh(a + b*x)*sinh(a + b*x)^2),x)
[Out] int(x^m/(cosh(a + b*x)*sinh(a + b*x)^2), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int x^m \operatorname{csch}^2(a+bx) \operatorname{sech}(a+bx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*csch(b*x+a)**2*sech(b*x+a),x)
[Out] Integral(x**m*csch(a + b*x)**2*sech(a + b*x), x)
```

3.488 $\int x^3 \operatorname{csch}^2(a+bx)\operatorname{sech}(a+bx)\,dx$

Optimal. Leaf size=237

$$\frac{6\text{Li}_3\left(-e^{a+bx}\right)}{b^4}-\frac{6\text{Li}_3\left(e^{a+bx}\right)}{b^4}+\frac{6i\text{Li}_4\left(-ie^{a+bx}\right)}{b^4}-\frac{6i\text{Li}_4\left(ie^{a+bx}\right)}{b^4}-\frac{6x\text{Li}_2\left(-e^{a+bx}\right)}{b^3}+\frac{6x\text{Li}_2\left(e^{a+bx}\right)}{b^3}-\frac{6ix\text{Li}_3\left(-ie^{a+bx}\right)}{b^3}+$$

```
[Out] -2*x^3*arctan(exp(b*x+a))/b-6*x^2*arctanh(exp(b*x+a))/b^2-x^3*csch(b*x+a)/b
-6*x*polylog(2,-exp(b*x+a))/b^3+3*I*x^2*polylog(2,-I*exp(b*x+a))/b^2-3*I*x^
2*polylog(2,I*exp(b*x+a))/b^2+6*x*polylog(2,exp(b*x+a))/b^3+6*polylog(3,-ex
p(b*x+a))/b^4-6*I*x*polylog(3,-I*exp(b*x+a))/b^3+6*I*x*polylog(3,I*exp(b*x+
a))/b^3-6*polylog(3,exp(b*x+a))/b^4+6*I*polylog(4,-I*exp(b*x+a))/b^4-6*I*po
lylog(4,I*exp(b*x+a))/b^4
```

Rubi [A] time = 0.34, antiderivative size = 237, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 13, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.722, Rules used = {2621, 321, 207, 5462, 14, 5205, 12, 4180, 2531, 6609, 2282, 6589, 4182}

$$\frac{3ix^2\text{PolyLog}\left(2,-ie^{a+bx}\right)}{b^2}-\frac{3ix^2\text{PolyLog}\left(2,ie^{a+bx}\right)}{b^2}-\frac{6x\text{PolyLog}\left(2,-e^{a+bx}\right)}{b^3}+\frac{6x\text{PolyLog}\left(2,e^{a+bx}\right)}{b^3}-\frac{6ix\text{PolyLo}}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Csch[a + b*x]^2*Sech[a + b*x],x]
```

```
[Out] (-2*x^3*ArcTan[E^(a + b*x)])/b - (6*x^2*ArcTanh[E^(a + b*x)])/b^2 - (x^3*Cs
ch[a + b*x])/b - (6*x*PolyLog[2, -E^(a + b*x)])/b^3 + ((3*I)*x^2*PolyLog[2,
 (-I)*E^(a + b*x)])/b^2 - ((3*I)*x^2*PolyLog[2, I*E^(a + b*x)])/b^2 + (6*x*
PolyLog[2, E^(a + b*x)])/b^3 + (6*PolyLog[3, -E^(a + b*x)])/b^4 - ((6*I)*x*
PolyLog[3, (-I)*E^(a + b*x)])/b^3 + ((6*I)*x*PolyLog[3, I*E^(a + b*x)])/b^3
 - (6*PolyLog[3, E^(a + b*x)])/b^4 + ((6*I)*PolyLog[4, (-I)*E^(a + b*x)])/b
^4 - ((6*I)*PolyLog[4, I*E^(a + b*x)])/b^4
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2621

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> -Dist[(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
```

```
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5205

```
Int[((a_.) + ArcTan[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Sim
p[((c + d*x)^(m + 1)*(a + b*ArcTan[u]))/(d*(m + 1)), x] - Dist[b/(d*(m + 1)
), Int[SimplifyIntegrand[((c + d*x)^(m + 1)*D[u, x])/(1 + u^2), x], x], x]
/; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && InverseFunctionFreeQ[u, x] &&
  !FunctionOfQ[(c + d*x)^(m + 1), u, x] && FalseQ[PowerVariableExpn[u, m +
1, x]]
```

Rule 5462

```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> With[{u = IntHide[Csch[a + b*x]^n*Sech[a +
b*x]^p, x]}, Dist[(c + d*x)^m, u, x] - Dist[d*m, Int[(c + d*x)^(m - 1)*u, x
], x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n,
p]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int x^3\text{csch}^2(a+bx)\text{sech}(a+bx)\,dx = -\frac{x^3\tan^{-1}(\sinh(a+bx))}{b} - \frac{x^3\text{csch}(a+bx)}{b} - 3\int x^2\left(-\frac{\tan^{-1}(\sinh(a+bx))}{b}\right.$$

$$= -\frac{x^3\tan^{-1}(\sinh(a+bx))}{b} - \frac{x^3\text{csch}(a+bx)}{b} - 3\int\left(-\frac{x^2\tan^{-1}(\sinh(a+bx))}{b}\right.$$

$$= -\frac{x^3\tan^{-1}(\sinh(a+bx))}{b} - \frac{x^3\text{csch}(a+bx)}{b} + \frac{3\int x^2\tan^{-1}(\sinh(a+bx))\,dx}{b}$$

$$= -\frac{6x^2\tanh^{-1}\left(e^{a+bx}\right)}{b^2} - \frac{x^3\text{csch}(a+bx)}{b} - \frac{6\int x\log\left(1-e^{a+bx}\right)dx}{b^2} + \frac{6\int x\,\text{lo}}{}$$

$$= -\frac{6x^2\tanh^{-1}\left(e^{a+bx}\right)}{b^2} - \frac{x^3\text{csch}(a+bx)}{b} - \frac{6x\text{Li}_2\left(-e^{a+bx}\right)}{b^3} + \frac{6x\text{Li}_2\left(e^{a+bx}\right)}{b^3} +$$

$$= -\frac{2x^3\tan^{-1}\left(e^{a+bx}\right)}{b} - \frac{6x^2\tanh^{-1}\left(e^{a+bx}\right)}{b^2} - \frac{x^3\text{csch}(a+bx)}{b} - \frac{6x\text{Li}_2\left(-e^{a+bx}\right)}{b^3}$$

$$= -\frac{2x^3\tan^{-1}\left(e^{a+bx}\right)}{b} - \frac{6x^2\tanh^{-1}\left(e^{a+bx}\right)}{b^2} - \frac{x^3\text{csch}(a+bx)}{b} - \frac{6x\text{Li}_2\left(-e^{a+bx}\right)}{b^3}$$

$$= -\frac{2x^3\tan^{-1}\left(e^{a+bx}\right)}{b} - \frac{6x^2\tanh^{-1}\left(e^{a+bx}\right)}{b^2} - \frac{x^3\text{csch}(a+bx)}{b} - \frac{6x\text{Li}_2\left(-e^{a+bx}\right)}{b^3}$$

$$= -\frac{2x^3\tan^{-1}\left(e^{a+bx}\right)}{b} - \frac{6x^2\tanh^{-1}\left(e^{a+bx}\right)}{b^2} - \frac{x^3\text{csch}(a+bx)}{b} - \frac{6x\text{Li}_2\left(-e^{a+bx}\right)}{b^3}$$

$$= -\frac{2x^3\tan^{-1}\left(e^{a+bx}\right)}{b} - \frac{6x^2\tanh^{-1}\left(e^{a+bx}\right)}{b^2} - \frac{x^3\text{csch}(a+bx)}{b} - \frac{6x\text{Li}_2\left(-e^{a+bx}\right)}{b^3}$$

Mathematica [A] time = 1.81, size = 333, normalized size = 1.41

$$\frac{-2b^3x^3\text{csch}(a) + b^3x^3\text{csch}\left(\frac{a}{2}\right)\sinh\left(\frac{bx}{2}\right)\text{csch}\left(\frac{1}{2}(a+bx)\right) + b^3x^3\text{sech}\left(\frac{a}{2}\right)\sinh\left(\frac{bx}{2}\right)\text{sech}\left(\frac{1}{2}(a+bx)\right) - 12\left(b^2x^2\,\text{t}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^3*Csch[a + b*x]^2*Sech[a + b*x],x]
```

```
[Out] (-2*b^3*x^3*Csch[a] - 12*(b^2*x^2*ArcTanh[Cosh[a + b*x] + Sinh[a + b*x]] +
b*x*PolyLog[2, -Cosh[a + b*x] - Sinh[a + b*x]] - b*x*PolyLog[2, Cosh[a + b*
x] + Sinh[a + b*x]] - PolyLog[3, -Cosh[a + b*x] - Sinh[a + b*x]] + PolyLog[
3, Cosh[a + b*x] + Sinh[a + b*x]]) - (2*I)*(b^3*x^3*Log[1 - I*E^(a + b*x)]
- b^3*x^3*Log[1 + I*E^(a + b*x)] - 3*b^2*x^2*PolyLog[2, (-I)*E^(a + b*x)] +
 3*b^2*x^2*PolyLog[2, I*E^(a + b*x)] + 6*b*x*PolyLog[3, (-I)*E^(a + b*x)] -
 6*b*x*PolyLog[3, I*E^(a + b*x)] - 6*PolyLog[4, (-I)*E^(a + b*x)] + 6*PolyL
```

```
og[4, I*E^(a + b*x)]) + b^3*x^3*Csch[a/2]*Csch[(a + b*x)/2]*Sinh[(b*x)/2] +
 b^3*x^3*Sech[a/2]*Sech[(a + b*x)/2]*Sinh[(b*x)/2])/(2*b^4)
```

fricas [C] time = 0.48, size = 1307, normalized size = 5.51

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*csch(b*x+a)^2*sech(b*x+a),x, algorithm="fricas")
```

```
[Out] -(2*b^3*x^3*cosh(b*x + a) + 2*b^3*x^3*sinh(b*x + a) - 6*(b*x*cosh(b*x + a)^
2 + 2*b*x*cosh(b*x + a)*sinh(b*x + a) + b*x*sinh(b*x + a)^2 - b*x)*dilog(co
sh(b*x + a) + sinh(b*x + a)) - (-3*I*b^2*x^2*cosh(b*x + a)^2 - 6*I*b^2*x^2*
cosh(b*x + a)*sinh(b*x + a) - 3*I*b^2*x^2*sinh(b*x + a)^2 + 3*I*b^2*x^2)*di
log(I*cosh(b*x + a) + I*sinh(b*x + a)) - (3*I*b^2*x^2*cosh(b*x + a)^2 + 6*I
*b^2*x^2*cosh(b*x + a)*sinh(b*x + a) + 3*I*b^2*x^2*sinh(b*x + a)^2 - 3*I*b^
2*x^2)*dilog(-I*cosh(b*x + a) - I*sinh(b*x + a)) + 6*(b*x*cosh(b*x + a)^2 +
 2*b*x*cosh(b*x + a)*sinh(b*x + a) + b*x*sinh(b*x + a)^2 - b*x)*dilog(-cosh
(b*x + a) - sinh(b*x + a)) + 3*(b^2*x^2*cosh(b*x + a)^2 + 2*b^2*x^2*cosh(b*
x + a)*sinh(b*x + a) + b^2*x^2*sinh(b*x + a)^2 - b^2*x^2)*log(cosh(b*x + a)
 + sinh(b*x + a) + 1) - (I*a^3*cosh(b*x + a)^2 + 2*I*a^3*cosh(b*x + a)*sinh
(b*x + a) + I*a^3*sinh(b*x + a)^2 - I*a^3)*log(cosh(b*x + a) + sinh(b*x + a
) + I) - (-I*a^3*cosh(b*x + a)^2 - 2*I*a^3*cosh(b*x + a)*sinh(b*x + a) - I*
a^3*sinh(b*x + a)^2 + I*a^3)*log(cosh(b*x + a) + sinh(b*x + a) - I) - 3*(a^
2*cosh(b*x + a)^2 + 2*a^2*cosh(b*x + a)*sinh(b*x + a) + a^2*sinh(b*x + a)^2
 - a^2)*log(cosh(b*x + a) + sinh(b*x + a) - 1) - (-I*b^3*x^3 - I*a^3 + (I*b
^3*x^3 + I*a^3)*cosh(b*x + a)^2 + (2*I*b^3*x^3 + 2*I*a^3)*cosh(b*x + a)*sin
h(b*x + a) + (I*b^3*x^3 + I*a^3)*sinh(b*x + a)^2)*log(I*cosh(b*x + a) + I*s
inh(b*x + a) + 1) - (I*b^3*x^3 + I*a^3 + (-I*b^3*x^3 - I*a^3)*cosh(b*x + a)
^2 + (-2*I*b^3*x^3 - 2*I*a^3)*cosh(b*x + a)*sinh(b*x + a) + (-I*b^3*x^3 - I
*a^3)*sinh(b*x + a)^2)*log(-I*cosh(b*x + a) - I*sinh(b*x + a) + 1) + 3*(b^2
*x^2 - (b^2*x^2 - a^2)*cosh(b*x + a)^2 - 2*(b^2*x^2 - a^2)*cosh(b*x + a)*si
nh(b*x + a) - (b^2*x^2 - a^2)*sinh(b*x + a)^2 - a^2)*log(-cosh(b*x + a) - s
inh(b*x + a) + 1) - (-6*I*cosh(b*x + a)^2 - 12*I*cosh(b*x + a)*sinh(b*x + a
) - 6*I*sinh(b*x + a)^2 + 6*I)*polylog(4, I*cosh(b*x + a) + I*sinh(b*x + a)
) - (6*I*cosh(b*x + a)^2 + 12*I*cosh(b*x + a)*sinh(b*x + a) + 6*I*sinh(b*x
+ a)^2 - 6*I)*polylog(4, -I*cosh(b*x + a) - I*sinh(b*x + a)) + 6*(cosh(b*x
+ a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 - 1)*polylog(3, co
sh(b*x + a) + sinh(b*x + a)) - (6*I*b*x*cosh(b*x + a)^2 + 12*I*b*x*cosh(b*x
 + a)*sinh(b*x + a) + 6*I*b*x*sinh(b*x + a)^2 - 6*I*b*x)*polylog(3, I*cosh(
b*x + a) + I*sinh(b*x + a)) - (-6*I*b*x*cosh(b*x + a)^2 - 12*I*b*x*cosh(b*x
 + a)*sinh(b*x + a) - 6*I*b*x*sinh(b*x + a)^2 + 6*I*b*x)*polylog(3, -I*cosh
(b*x + a) - I*sinh(b*x + a)) - 6*(cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*
x + a) + sinh(b*x + a)^2 - 1)*polylog(3, -cosh(b*x + a) - sinh(b*x + a)))/(
```

```
b^4*cosh(b*x + a)^2 + 2*b^4*cosh(b*x + a)*sinh(b*x + a) + b^4*sinh(b*x + a)
^2 - b^4)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3 \operatorname{csch}(bx+a)^2 \operatorname{sech}(bx+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*csch(b*x+a)^2*sech(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(x^3*csch(b*x + a)^2*sech(b*x + a), x)
```

maple [F] time = 1.55, size = 0, normalized size = 0.00

$$\int x^3 \operatorname{csch}(bx+a)^2 \operatorname{sech}(bx+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*csch(b*x+a)^2*sech(b*x+a),x)
```

```
[Out] int(x^3*csch(b*x+a)^2*sech(b*x+a),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{2\,x^3e^{(bx+a)}}{be^{(2\,bx+2\,a)}-b}-\frac{3\left(b^2x^2\log\left(e^{(bx+a)}+1\right)+2\,bx\mathrm{Li}_2\left(-e^{(bx+a)}\right)-2\,\mathrm{Li}_3(-e^{(bx+a)})\right)}{b^4}+\frac{3\left(b^2x^2\log\left(-e^{(bx+a)}+1\right)+2\,bx\right.}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*csch(b*x+a)^2*sech(b*x+a),x, algorithm="maxima")
```

```
[Out] -2*x^3*e^(b*x + a)/(b*e^(2*b*x + 2*a) - b) - 3*(b^2*x^2*log(e^(b*x + a) + 1
) + 2*b*x*dilog(-e^(b*x + a)) - 2*polylog(3, -e^(b*x + a)))/b^4 + 3*(b^2*x^
2*log(-e^(b*x + a) + 1) + 2*b*x*dilog(e^(b*x + a)) - 2*polylog(3, e^(b*x +
a)))/b^4 - 8*integrate(1/4*x^3*e^(b*x + a)/(e^(2*b*x + 2*a) + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3}{\cosh(a+b\,x)\,\sinh(a+b\,x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(cosh(a + b*x)*sinh(a + b*x)^2),x)
```

```
[Out] int(x^3/(cosh(a + b*x)*sinh(a + b*x)^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3 \operatorname{csch}^2(a+bx)\operatorname{sech}(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*csch(b*x+a)**2*sech(b*x+a),x)
```

```
[Out] Integral(x**3*csch(a + b*x)**2*sech(a + b*x), x)
```

3.489 $\int x^2 \mathbf{csch}^2(a+bx)\mathbf{sech}(a+bx)\,dx$

Optimal. Leaf size=157

$$-\frac{2\mathrm{Li}_2\left(-e^{a+bx}\right)}{b^3}+\frac{2\mathrm{Li}_2\left(e^{a+bx}\right)}{b^3}-\frac{2i\mathrm{Li}_3\left(-ie^{a+bx}\right)}{b^3}+\frac{2i\mathrm{Li}_3\left(ie^{a+bx}\right)}{b^3}+\frac{2ix\mathrm{Li}_2\left(-ie^{a+bx}\right)}{b^2}-\frac{2ix\mathrm{Li}_2\left(ie^{a+bx}\right)}{b^2}-\frac{4x\tanh^{-1}\left(e^{a+b\right.}}{b^2}$$

```
[Out] -2*x^2*arctan(exp(b*x+a))/b-4*x*arctanh(exp(b*x+a))/b^2-x^2*csch(b*x+a)/b-2
*polylog(2,-exp(b*x+a))/b^3+2*I*x*polylog(2,-I*exp(b*x+a))/b^2-2*I*x*polylo
g(2,I*exp(b*x+a))/b^2+2*polylog(2,exp(b*x+a))/b^3-2*I*polylog(3,-I*exp(b*x+
a))/b^3+2*I*polylog(3,I*exp(b*x+a))/b^3
```

Rubi [A] time = 0.23, antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 14, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.778, Rules used = {2621, 321, 207, 5462, 14, 5205, 12, 4180, 2531, 2282, 6589, 4182, 2279, 2391}

$$\frac{2ix\text{PolyLog}\left(2,-ie^{a+bx}\right)}{b^2}-\frac{2ix\text{PolyLog}\left(2,ie^{a+bx}\right)}{b^2}-\frac{2\text{PolyLog}\left(2,-e^{a+bx}\right)}{b^3}+\frac{2\text{PolyLog}\left(2,e^{a+bx}\right)}{b^3}-\frac{2i\text{PolyLog}\left(3,-\right.}{b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Csch[a + b*x]^2*Sech[a + b*x],x]
```

```
[Out] (-2*x^2*ArcTan[E^(a + b*x)])/b - (4*x*ArcTanh[E^(a + b*x)])/b^2 - (x^2*Csch
[a + b*x])/b - (2*PolyLog[2, -E^(a + b*x)])/b^3 + ((2*I)*x*PolyLog[2, (-I)*
E^(a + b*x)])/b^2 - ((2*I)*x*PolyLog[2, I*E^(a + b*x)])/b^2 + (2*PolyLog[2,
 E^(a + b*x)])/b^3 - ((2*I)*PolyLog[3, (-I)*E^(a + b*x)])/b^3 + ((2*I)*Poly
Log[3, I*E^(a + b*x)])/b^3
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2621

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> -Dist[(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5205

```
Int[((a_.) + ArcTan[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Sim
p[((c + d*x)^(m + 1)*(a + b*ArcTan[u]))/(d*(m + 1)), x] - Dist[b/(d*(m + 1)
), Int[SimplifyIntegrand[((c + d*x)^(m + 1)*D[u, x])/(1 + u^2), x], x], x]
/; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && InverseFunctionFreeQ[u, x] &&
  !FunctionOfQ[(c + d*x)^(m + 1), u, x] && FalseQ[PowerVariableExpn[u, m +
1, x]]
```

Rule 5462

```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> With[{u = IntHide[Csch[a + b*x]^n*Sech[a +
b*x]^p, x]}, Dist[(c + d*x)^m, u, x] - Dist[d*m, Int[(c + d*x)^(m - 1)*u, x
], x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n,
p]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int x^2\text{csch}^2(a+bx)\text{sech}(a+bx)\,dx = -\frac{x^2\tan^{-1}(\sinh(a+bx))}{b} - \frac{x^2\text{csch}(a+bx)}{b} - 2\int x\left(-\frac{\tan^{-1}(\sinh(a+bx)}{b}\right.$$

$$= -\frac{x^2\tan^{-1}(\sinh(a+bx))}{b} - \frac{x^2\text{csch}(a+bx)}{b} - 2\int\left(-\frac{x\tan^{-1}(\sinh(a+bx)}{b}\right.$$

$$= -\frac{x^2\tan^{-1}(\sinh(a+bx))}{b} - \frac{x^2\text{csch}(a+bx)}{b} + \frac{2\int x\tan^{-1}(\sinh(a+bx))\,dx}{b}$$

$$= -\frac{4x\tanh^{-1}\left(e^{a+bx}\right)}{b^2} - \frac{x^2\text{csch}(a+bx)}{b} - \frac{2\int\log\left(1-e^{a+bx}\right)dx}{b^2} + \frac{2\int\log\left(\right.}{}$$

$$= -\frac{4x\tanh^{-1}\left(e^{a+bx}\right)}{b^2} - \frac{x^2\text{csch}(a+bx)}{b} - \frac{2\,\text{Subst}\left(\int\frac{\log(1-x)}{x}\,dx, x, e^{a+bx}\right)}{b^3} +$$

$$= -\frac{2x^2\tan^{-1}\left(e^{a+bx}\right)}{b} - \frac{4x\tanh^{-1}\left(e^{a+bx}\right)}{b^2} - \frac{x^2\text{csch}(a+bx)}{b} - \frac{2\text{Li}_2\left(-e^{a+bx}\right)}{b^3}$$

$$= -\frac{2x^2\tan^{-1}\left(e^{a+bx}\right)}{b} - \frac{4x\tanh^{-1}\left(e^{a+bx}\right)}{b^2} - \frac{x^2\text{csch}(a+bx)}{b} - \frac{2\text{Li}_2\left(-e^{a+bx}\right)}{b^3}$$

$$= -\frac{2x^2\tan^{-1}\left(e^{a+bx}\right)}{b} - \frac{4x\tanh^{-1}\left(e^{a+bx}\right)}{b^2} - \frac{x^2\text{csch}(a+bx)}{b} - \frac{2\text{Li}_2\left(-e^{a+bx}\right)}{b^3}$$

$$= -\frac{2x^2\tan^{-1}\left(e^{a+bx}\right)}{b} - \frac{4x\tanh^{-1}\left(e^{a+bx}\right)}{b^2} - \frac{x^2\text{csch}(a+bx)}{b} - \frac{2\text{Li}_2\left(-e^{a+bx}\right)}{b^3}$$

Mathematica [A] time = 1.93, size = 312, normalized size = 1.99

$$\frac{-2ib^2x^2\log\left(1-ie^{a+bx}\right)+2ib^2x^2\log\left(1+ie^{a+bx}\right)-2b^2x^2\text{csch}(a)+b^2x^2\text{csch}\left(\frac{a}{2}\right)\sinh\left(\frac{bx}{2}\right)\text{csch}\left(\frac{1}{2}(a+bx)\right)+b}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Csch[a + b*x]^2*Sech[a + b*x],x]
```

```
[Out] (-2*b^2*x^2*Csch[a] + 4*a*Log[1 - E^(-a - b*x)] + 4*b*x*Log[1 - E^(-a - b*x
)] - 4*a*Log[1 + E^(-a - b*x)] - 4*b*x*Log[1 + E^(-a - b*x)] - (2*I)*b^2*x^
2*Log[1 - I*E^(a + b*x)] + (2*I)*b^2*x^2*Log[1 + I*E^(a + b*x)] - 4*a*Log[T
anh[(a + b*x)/2]] + 4*PolyLog[2, -E^(-a - b*x)] - 4*PolyLog[2, E^(-a - b*x)
] + (4*I)*b*x*PolyLog[2, (-I)*E^(a + b*x)] - (4*I)*b*x*PolyLog[2, I*E^(a +
b*x)] - (4*I)*PolyLog[3, (-I)*E^(a + b*x)] + (4*I)*PolyLog[3, I*E^(a + b*x)
] + b^2*x^2*Csch[a/2]*Csch[(a + b*x)/2]*Sinh[(b*x)/2] + b^2*x^2*Sech[a/2]*S
ech[(a + b*x)/2]*Sinh[(b*x)/2])/(2*b^3)
```

fricas [C] time = 0.47, size = 964, normalized size = 6.14

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*csch(b*x+a)^2*sech(b*x+a),x, algorithm="fricas")
```

```
[Out] -(2*b^2*x^2*cosh(b*x + a) + 2*b^2*x^2*sinh(b*x + a) - 2*(cosh(b*x + a)^2 +
2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 - 1)*dilog(cosh(b*x + a) +
sinh(b*x + a)) - (-2*I*b*x*cosh(b*x + a)^2 - 4*I*b*x*cosh(b*x + a)*sinh(b*x
 + a) - 2*I*b*x*sinh(b*x + a)^2 + 2*I*b*x)*dilog(I*cosh(b*x + a) + I*sinh(b
*x + a)) - (2*I*b*x*cosh(b*x + a)^2 + 4*I*b*x*cosh(b*x + a)*sinh(b*x + a) +
 2*I*b*x*sinh(b*x + a)^2 - 2*I*b*x)*dilog(-I*cosh(b*x + a) - I*sinh(b*x + a
)) + 2*(cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 -
 1)*dilog(-cosh(b*x + a) - sinh(b*x + a)) + 2*(b*x*cosh(b*x + a)^2 + 2*b*x*
cosh(b*x + a)*sinh(b*x + a) + b*x*sinh(b*x + a)^2 - b*x)*log(cosh(b*x + a)
+ sinh(b*x + a) + 1) - (-I*a^2*cosh(b*x + a)^2 - 2*I*a^2*cosh(b*x + a)*sinh
(b*x + a) - I*a^2*sinh(b*x + a)^2 + I*a^2)*log(cosh(b*x + a) + sinh(b*x + a
) + I) - (I*a^2*cosh(b*x + a)^2 + 2*I*a^2*cosh(b*x + a)*sinh(b*x + a) + I*a
^2*sinh(b*x + a)^2 - I*a^2)*log(cosh(b*x + a) + sinh(b*x + a) - I) + 2*(a*c
osh(b*x + a)^2 + 2*a*cosh(b*x + a)*sinh(b*x + a) + a*sinh(b*x + a)^2 - a)*l
og(cosh(b*x + a) + sinh(b*x + a) - 1) - (-I*b^2*x^2 + (I*b^2*x^2 - I*a^2)*c
osh(b*x + a)^2 + (2*I*b^2*x^2 - 2*I*a^2)*cosh(b*x + a)*sinh(b*x + a) + (I*b
^2*x^2 - I*a^2)*sinh(b*x + a)^2 + I*a^2)*log(I*cosh(b*x + a) + I*sinh(b*x +
 a) + 1) - (I*b^2*x^2 + (-I*b^2*x^2 + I*a^2)*cosh(b*x + a)^2 + (-2*I*b^2*x^
2 + 2*I*a^2)*cosh(b*x + a)*sinh(b*x + a) + (-I*b^2*x^2 + I*a^2)*sinh(b*x +
a)^2 - I*a^2)*log(-I*cosh(b*x + a) - I*sinh(b*x + a) + 1) - 2*((b*x + a)*co
sh(b*x + a)^2 + 2*(b*x + a)*cosh(b*x + a)*sinh(b*x + a) + (b*x + a)*sinh(b*
x + a)^2 - b*x - a)*log(-cosh(b*x + a) - sinh(b*x + a) + 1) - (2*I*cosh(b*x
 + a)^2 + 4*I*cosh(b*x + a)*sinh(b*x + a) + 2*I*sinh(b*x + a)^2 - 2*I)*poly
log(3, I*cosh(b*x + a) + I*sinh(b*x + a)) - (-2*I*cosh(b*x + a)^2 - 4*I*cos
h(b*x + a)*sinh(b*x + a) - 2*I*sinh(b*x + a)^2 + 2*I)*polylog(3, -I*cosh(b*
x + a) - I*sinh(b*x + a)))/(b^3*cosh(b*x + a)^2 + 2*b^3*cosh(b*x + a)*sinh(
b*x + a) + b^3*sinh(b*x + a)^2 - b^3)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*csch(b*x+a)^2*sech(b*x+a),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [F] time = 1.77, size = 0, normalized size = 0.00

$$\int x^2 \operatorname{csch}(bx+a)^2 \operatorname{sech}(bx+a)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*csch(b*x+a)^2*sech(b*x+a),x)
```

```
[Out] int(x^2*csch(b*x+a)^2*sech(b*x+a),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{2\,x^2 e^{(bx+a)}}{b e^{(2\,bx+2\,a)} - b} - \frac{2\left(bx\log\left(e^{(bx+a)}+1\right) + \mathrm{Li}_2\left(-e^{(bx+a)}\right)\right)}{b^3} + \frac{2\left(bx\log\left(-e^{(bx+a)}+1\right) + \mathrm{Li}_2\left(e^{(bx+a)}\right)\right)}{b^3} - 8\int \frac{x^2 e^{(b}}{4\left(e^{(2\,bx+}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*csch(b*x+a)^2*sech(b*x+a),x, algorithm="maxima")
```

```
[Out] -2*x^2*e^(b*x + a)/(b*e^(2*b*x + 2*a) - b) - 2*(b*x*log(e^(b*x + a) + 1) +
dilog(-e^(b*x + a)))/b^3 + 2*(b*x*log(-e^(b*x + a) + 1) + dilog(e^(b*x + a)
))/b^3 - 8*integrate(1/4*x^2*e^(b*x + a)/(e^(2*b*x + 2*a) + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2}{\cosh(a+b\,x)\,\sinh(a+b\,x)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(cosh(a + b*x)*sinh(a + b*x)^2),x)
```

```
[Out] int(x^2/(cosh(a + b*x)*sinh(a + b*x)^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \operatorname{csch}^2(a+bx) \operatorname{sech}(a+bx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*csch(b*x+a)**2*sech(b*x+a),x)
```

```
[Out] Integral(x**2*csch(a + b*x)**2*sech(a + b*x), x)
```

3.490 $\int x\text{csch}^2(a+bx)\text{sech}(a+bx)\,dx$

Optimal. Leaf size=79

$$\frac{i\text{Li}_2\left(-ie^{a+bx}\right)}{b^2}-\frac{i\text{Li}_2\left(ie^{a+bx}\right)}{b^2}-\frac{\tanh^{-1}(\cosh(a+bx))}{b^2}-\frac{2x\tan^{-1}\left(e^{a+bx}\right)}{b}-\frac{x\text{csch}(a+bx)}{b}$$

```
[Out] -2*x*arctan(exp(b*x+a))/b-arctanh(cosh(b*x+a))/b^2-x*csch(b*x+a)/b+I*polylo
g(2,-I*exp(b*x+a))/b^2-I*polylog(2,I*exp(b*x+a))/b^2
```

Rubi [A] time = 0.11, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {2621, 321, 207, 5462, 5203, 12, 4180, 2279, 2391, 3770}

$$\frac{i\text{PolyLog}\left(2,-ie^{a+bx}\right)}{b^2}-\frac{i\text{PolyLog}\left(2,ie^{a+bx}\right)}{b^2}-\frac{\tanh^{-1}(\cosh(a+bx))}{b^2}-\frac{2x\tan^{-1}\left(e^{a+bx}\right)}{b}-\frac{x\text{csch}(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[x*Csch[a + b*x]^2*Sech[a + b*x],x]
```

```
[Out] (-2*x*ArcTan[E^(a + b*x)])/b - ArcTanh[Cosh[a + b*x]]/b^2 - (x*Csch[a + b*x
])/b + (I*PolyLog[2, (-I)*E^(a + b*x)])/b^2 - (I*PolyLog[2, I*E^(a + b*x)])
/b^2
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2621

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> -Dist[(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5203

```
Int[ArcTan[u_], x_Symbol] :> Simp[x*ArcTan[u], x] - Int[SimplifyIntegrand[(
x*D[u, x])/(1 + u^2), x], x] /; InverseFunctionFreeQ[u, x]
```

Rule 5462

```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> With[{u = IntHide[Csch[a + b*x]^n*Sech[a +
b*x]^p, x]}, Dist[(c + d*x)^m, u, x] - Dist[d*m, Int[(c + d*x)^(m - 1)*u, x
], x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n,
p]
```

Rubi steps

$$\int x\text{csch}^2(a+bx)\text{sech}(a+bx)\,dx = -\frac{x\tan^{-1}(\sinh(a+bx))}{b} - \frac{x\text{csch}(a+bx)}{b} - \int \left(-\frac{\tan^{-1}(\sinh(a+bx))}{b} - \text{csc}\right.$$

$$= -\frac{x\tan^{-1}(\sinh(a+bx))}{b} - \frac{x\text{csch}(a+bx)}{b} + \frac{\int \tan^{-1}(\sinh(a+bx))\,dx}{b} + \frac{\int \text{cs}}{}$$

$$= -\frac{\tanh^{-1}(\cosh(a+bx))}{b^2} - \frac{x\text{csch}(a+bx)}{b} - \frac{\int bx\text{sech}(a+bx)\,dx}{b}$$

$$= -\frac{\tanh^{-1}(\cosh(a+bx))}{b^2} - \frac{x\text{csch}(a+bx)}{b} - \int x\text{sech}(a+bx)\,dx$$

$$= -\frac{2x\tan^{-1}\left(e^{a+bx}\right)}{b} - \frac{\tanh^{-1}(\cosh(a+bx))}{b^2} - \frac{x\text{csch}(a+bx)}{b} + \frac{i\int \log\left(1-ie\right.}{b}$$

$$= -\frac{2x\tan^{-1}\left(e^{a+bx}\right)}{b} - \frac{\tanh^{-1}(\cosh(a+bx))}{b^2} - \frac{x\text{csch}(a+bx)}{b} + \frac{i\,\text{Subst}\left(\int \frac{\log}{}\right.}{}$$

$$= -\frac{2x\tan^{-1}\left(e^{a+bx}\right)}{b} - \frac{\tanh^{-1}(\cosh(a+bx))}{b^2} - \frac{x\text{csch}(a+bx)}{b} + \frac{i\text{Li}_2\left(-ie^{a+bx}\right)}{b^2}$$

Mathematica [A] time = 0.83, size = 112, normalized size = 1.42

$$\frac{2i\text{Li}_2(-i(\cosh(a+bx)+\sinh(a+bx))) - 2i\text{Li}_2(i(\cosh(a+bx)+\sinh(a+bx))) + bx\tanh\left(\frac{1}{2}(a+bx)\right) - bx\coth}{2b^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x*Csch[a + b*x]^2*Sech[a + b*x],x]
```

```
[Out] (-4*b*x*ArcTan[Cosh[a + b*x] + Sinh[a + b*x]] - b*x*Coth[(a + b*x)/2] + 2*L
og[Tanh[(a + b*x)/2]] + (2*I)*PolyLog[2, (-I)*(Cosh[a + b*x] + Sinh[a + b*x
])] - (2*I)*PolyLog[2, I*(Cosh[a + b*x] + Sinh[a + b*x])] + b*x*Tanh[(a + b
*x)/2])/(2*b^2)
```

fricas [B] time = 0.44, size = 565, normalized size = 7.15

$$-\frac{2\,bx\cosh(bx+a) + 2\,bx\sinh(bx+a) - \left(-i\,\cosh(bx+a)^2 - 2i\,\cosh(bx+a)\sinh(bx+a) - i\,\sinh(bx+a)^2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*csch(b*x+a)^2*sech(b*x+a),x, algorithm="fricas")
```

```
[Out] -(2*b*x*cosh(b*x + a) + 2*b*x*sinh(b*x + a) - (-I*cosh(b*x + a)^2 - 2*I*cos
h(b*x + a)*sinh(b*x + a) - I*sinh(b*x + a)^2 + I)*dilog(I*cosh(b*x + a) + I
```

```
*sinh(b*x + a)) - (I*cosh(b*x + a)^2 + 2*I*cosh(b*x + a)*sinh(b*x + a) + I*
sinh(b*x + a)^2 - I)*dilog(-I*cosh(b*x + a) - I*sinh(b*x + a)) + (cosh(b*x
+ a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 - 1)*log(cosh(b*x
+ a) + sinh(b*x + a) + 1) - (I*a*cosh(b*x + a)^2 + 2*I*a*cosh(b*x + a)*sinh
(b*x + a) + I*a*sinh(b*x + a)^2 - I*a)*log(cosh(b*x + a) + sinh(b*x + a) +
I) - (-I*a*cosh(b*x + a)^2 - 2*I*a*cosh(b*x + a)*sinh(b*x + a) - I*a*sinh(b
*x + a)^2 + I*a)*log(cosh(b*x + a) + sinh(b*x + a) - I) - (cosh(b*x + a)^2
+ 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 - 1)*log(cosh(b*x + a) +
sinh(b*x + a) - 1) - ((I*b*x + I*a)*cosh(b*x + a)^2 + (2*I*b*x + 2*I*a)*cos
h(b*x + a)*sinh(b*x + a) + (I*b*x + I*a)*sinh(b*x + a)^2 - I*b*x - I*a)*log
(I*cosh(b*x + a) + I*sinh(b*x + a) + 1) - ((-I*b*x - I*a)*cosh(b*x + a)^2 +
 (-2*I*b*x - 2*I*a)*cosh(b*x + a)*sinh(b*x + a) + (-I*b*x - I*a)*sinh(b*x +
 a)^2 + I*b*x + I*a)*log(-I*cosh(b*x + a) - I*sinh(b*x + a) + 1))/(b^2*cosh
(b*x + a)^2 + 2*b^2*cosh(b*x + a)*sinh(b*x + a) + b^2*sinh(b*x + a)^2 - b^2
)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x \operatorname{csch}(bx+a)^2 \operatorname{sech}(bx+a)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*csch(b*x+a)^2*sech(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(x*csch(b*x + a)^2*sech(b*x + a), x)
```

maple [B] time = 0.50, size = 179, normalized size = 2.27

$$-\frac{2\,e^{bx+a}x}{b\left(e^{2bx+2a}-1\right)}+\frac{2a\arctan\left(e^{bx+a}\right)}{b^2}+\frac{i\ln\left(1+ie^{bx+a}\right)x}{b}+\frac{i\ln\left(1+ie^{bx+a}\right)a}{b^2}+\frac{i\,\mathrm{dilog}\left(1+ie^{bx+a}\right)}{b^2}-\frac{i\ln\left(1-ie^{bx+a}\right.}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*csch(b*x+a)^2*sech(b*x+a),x)
```

```
[Out] -2*exp(b*x+a)*x/b/(exp(2*b*x+2*a)-1)+2/b^2*a*arctan(exp(b*x+a))+I/b*ln(1+I*
exp(b*x+a))*x+I/b^2*ln(1+I*exp(b*x+a))*a+I/b^2*dilog(1+I*exp(b*x+a))-I/b*ln
(1-I*exp(b*x+a))*x-I/b^2*ln(1-I*exp(b*x+a))*a-I/b^2*dilog(1-I*exp(b*x+a))-1
/b^2*ln(1+exp(b*x+a))+1/b^2*ln(exp(b*x+a)-1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{2\,xe^{(bx+a)}}{be^{(2\,bx+2\,a)}-b}-\frac{\log\left(\left(e^{(bx+a)}+1\right)e^{(-a)}\right)}{b^2}+\frac{\log\left(\left(e^{(bx+a)}-1\right)e^{(-a)}\right)}{b^2}-8\int\frac{xe^{(bx+a)}}{4\left(e^{(2\,bx+2\,a)}+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*csch(b*x+a)^2*sech(b*x+a),x, algorithm="maxima")
```

```
[Out] -2*x*e^(b*x + a)/(b*e^(2*b*x + 2*a) - b) - log((e^(b*x + a) + 1)*e^(-a))/b^
2 + log((e^(b*x + a) - 1)*e^(-a))/b^2 - 8*integrate(1/4*x*e^(b*x + a)/(e^(2
*b*x + 2*a) + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x}{\cosh(a+b\,x)\,\sinh(a+b\,x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(cosh(a + b*x)*sinh(a + b*x)^2),x)
```

```
[Out] int(x/(cosh(a + b*x)*sinh(a + b*x)^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\operatorname{csch}^2(a+bx)\operatorname{sech}(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*csch(b*x+a)**2*sech(b*x+a),x)
```

```
[Out] Integral(x*csch(a + b*x)**2*sech(a + b*x), x)
```

3.491 $\int \text{csch}^2(a+bx)\text{sech}(a+bx)\,dx$

Optimal. Leaf size=24

$$-\frac{\text{csch}(a+bx)}{b}-\frac{\tan^{-1}(\sinh(a+bx))}{b}$$

```
[Out] -arctan(sinh(b*x+a))/b-csch(b*x+a)/b
```

Rubi [A] time = 0.02, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2621, 321, 207}

$$-\frac{\text{csch}(a+bx)}{b}-\frac{\tan^{-1}(\sinh(a+bx))}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[a + b*x]^2*Sech[a + b*x],x]
```

```
[Out] -(ArcTan[Sinh[a + b*x]]/b) - Csch[a + b*x]/b
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2621

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> -Dist[(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rubi steps

$$\int \operatorname{csch}^2(a+bx)\operatorname{sech}(a+bx)\,dx = -\frac{i\,\mathrm{Subst}\left(\int \frac{x^2}{-1+x^2}\,dx, x, -i\operatorname{csch}(a+bx)\right)}{b}$$
$$= -\frac{\operatorname{csch}(a+bx)}{b} - \frac{i\,\mathrm{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, -i\operatorname{csch}(a+bx)\right)}{b}$$
$$= -\frac{\tan^{-1}(\sinh(a+bx))}{b} - \frac{\operatorname{csch}(a+bx)}{b}$$

Mathematica [C] time = 0.02, size = 29, normalized size = 1.21

$$-\frac{\operatorname{csch}(a+bx)\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};-\sinh^2(a+bx)\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[a + b*x]^2*Sech[a + b*x],x]
[Out] -((Csch[a + b*x]*Hypergeometric2F1[-1/2, 1, 1/2, -Sinh[a + b*x]^2])/b)
```

fricas [B] time = 0.42, size = 103, normalized size = 4.29

$$-\frac{2\left(\left(\cosh(bx+a)^2 + 2\cosh(bx+a)\sinh(bx+a) + \sinh(bx+a)^2 - 1\right)\arctan\left(\cosh(bx+a) + \sinh(bx+a)\right) \cdots}{b\cosh(bx+a)^2 + 2\,b\cosh(bx+a)\sinh(bx+a) + b\sinh(bx+a)^2 - b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^2*sech(b*x+a),x, algorithm="fricas")
[Out] -2*((cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 - 1)
*arctan(cosh(b*x + a) + sinh(b*x + a)) + cosh(b*x + a) + sinh(b*x + a))/(b*
cosh(b*x + a)^2 + 2*b*cosh(b*x + a)*sinh(b*x + a) + b*sinh(b*x + a)^2 - b)
```

giac [B] time = 0.17, size = 54, normalized size = 2.25

$$-\frac{\pi + \frac{4}{e^{(bx+a)} - e^{(-bx-a)}} + 2\arctan\left(\frac{1}{2}\left(e^{(2\,bx+2\,a)} - 1\right)e^{(-bx-a)}\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^2*sech(b*x+a),x, algorithm="giac")
```

```
[Out] -1/2*(pi + 4/(e^(b*x + a) - e^(-b*x - a)) + 2*arctan(1/2*(e^(2*b*x + 2*a) -
 1)*e^(-b*x - a)))/b
```

maple [A] time = 0.10, size = 27, normalized size = 1.12

$$-\frac{1}{b\sinh\left(bx+a\right)}-\frac{2\arctan\left(\mathrm{e}^{bx+a}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(b*x+a)^2*sech(b*x+a),x)
```

```
[Out] -1/b/sinh(b*x+a)-2*arctan(exp(b*x+a))/b
```

maxima [A] time = 0.51, size = 43, normalized size = 1.79

$$\frac{2\arctan\left(e^{(-bx-a)}\right)}{b}+\frac{2\,e^{(-bx-a)}}{b\left(e^{(-2\,bx-2\,a)}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^2*sech(b*x+a),x, algorithm="maxima")
```

```
[Out] 2*arctan(e^(-b*x - a))/b + 2*e^(-b*x - a)/(b*(e^(-2*b*x - 2*a) - 1))
```

mupad [B] time = 1.47, size = 48, normalized size = 2.00

$$-\frac{2\,\mathrm{atan}\left(\frac{\mathrm{e}^{b\,x}\,\mathrm{e}^{a}\,\sqrt{b^2}}{b}\right)}{\sqrt{b^2}}-\frac{2\,\mathrm{e}^{a+b\,x}}{b\,\left(\mathrm{e}^{2\,a+2\,b\,x}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(a + b*x)*sinh(a + b*x)^2),x)
```

```
[Out] - (2*atan((exp(b*x)*exp(a)*(b^2)^(1/2))/b))/(b^2)^(1/2) - (2*exp(a + b*x))/
(b*(exp(2*a + 2*b*x) - 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\operatorname{csch}^{2}\left(a+bx\right)\operatorname{sech}\left(a+bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)**2*sech(b*x+a),x)
```

```
[Out] Integral(csch(a + b*x)**2*sech(a + b*x), x)
```

3.492 $\int \frac{\operatorname{csch}^2(a+bx)\operatorname{sech}(a+bx)}{x} dx$

Optimal. Leaf size=21

$$\operatorname{Int}\left(\frac{\operatorname{csch}^2(a+bx)\operatorname{sech}(a+bx)}{x},x\right)$$

```
[Out] CannotIntegrate(csch(b*x+a)^2*sech(b*x+a)/x,x)
```

Rubi [A] time = 0.16, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\operatorname{csch}^2(a+bx)\operatorname{sech}(a+bx)}{x} dx$$

Verification is Not applicable to the result.

```
[In] Int[(Csch[a + b*x]^2*Sech[a + b*x])/x,x]
```

```
[Out] Defer[Int][(Csch[a + b*x]^2*Sech[a + b*x])/x, x]
```

Rubi steps

$$\int \frac{\operatorname{csch}^2(a+bx)\operatorname{sech}(a+bx)}{x} dx = \int \frac{\operatorname{csch}^2(a+bx)\operatorname{sech}(a+bx)}{x} dx$$

Mathematica [A] time = 23.45, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}^2(a+bx)\operatorname{sech}(a+bx)}{x} dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Csch[a + b*x]^2*Sech[a + b*x])/x,x]
```

```
[Out] Integrate[(Csch[a + b*x]^2*Sech[a + b*x])/x, x]
```

fricas [A] time = 0.39, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{\operatorname{csch}(bx+a)^2\operatorname{sech}(bx+a)}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^2*sech(b*x+a)/x,x, algorithm="fricas")
```

```
[Out] integral(csch(b*x + a)^2*sech(b*x + a)/x, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}(bx+a)^2 \operatorname{sech}(bx+a)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^2*sech(b*x+a)/x,x, algorithm="giac")
```

```
[Out] integrate(csch(b*x + a)^2*sech(b*x + a)/x, x)
```

maple [A] time = 1.01, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}(bx+a)^2 \operatorname{sech}(bx+a)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(b*x+a)^2*sech(b*x+a)/x,x)
```

```
[Out] int(csch(b*x+a)^2*sech(b*x+a)/x,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{2\,e^{(bx+a)}}{bxe^{(2\,bx+2\,a)}-bx}-8\int \frac{e^{(bx+a)}}{4\left(xe^{(2\,bx+2\,a)}+x\right)}\,dx-8\int \frac{1}{8\left(bx^2e^{(bx+a)}+bx^2\right)}\,dx-8\int \frac{1}{8\left(bx^2e^{(bx+a)}-bx^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^2*sech(b*x+a)/x,x, algorithm="maxima")
```

```
[Out] -2*e^(b*x + a)/(b*x*e^(2*b*x + 2*a) - b*x) - 8*integrate(1/4*e^(b*x + a)/(x
*e^(2*b*x + 2*a) + x), x) - 8*integrate(1/8/(b*x^2*e^(b*x + a) + b*x^2), x)
 - 8*integrate(1/8/(b*x^2*e^(b*x + a) - b*x^2), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{1}{x\cosh(a+b\,x)\,\sinh(a+b\,x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*cosh(a + b*x)*sinh(a + b*x)^2),x)
```

```
[Out] int(1/(x*cosh(a + b*x)*sinh(a + b*x)^2), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}^2(a+bx)\operatorname{sech}(a+bx)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)**2*sech(b*x+a)/x,x)
```

```
[Out] Integral(csch(a + b*x)**2*sech(a + b*x)/x, x)
```

3.493 $\int \frac{\operatorname{csch}^2(a+bx)\operatorname{sech}(a+bx)}{x^2}\,dx$

Optimal. Leaf size=21

$$\operatorname{Int}\left(\frac{\operatorname{csch}^2(a+bx)\operatorname{sech}(a+bx)}{x^2},x\right)$$

```
[Out] CannotIntegrate(csch(b*x+a)^2*sech(b*x+a)/x^2,x)
```

Rubi [A] time = 0.21, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\operatorname{csch}^2(a+bx)\operatorname{sech}(a+bx)}{x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Csch[a + b*x]^2*Sech[a + b*x])/x^2,x]
```

```
[Out] Defer[Int][(Csch[a + b*x]^2*Sech[a + b*x])/x^2, x]
```

Rubi steps

$$\int \frac{\operatorname{csch}^2(a+bx)\operatorname{sech}(a+bx)}{x^2}\,dx=\int \frac{\operatorname{csch}^2(a+bx)\operatorname{sech}(a+bx)}{x^2}\,dx$$

Mathematica [A] time = 26.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}^2(a+bx)\operatorname{sech}(a+bx)}{x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Csch[a + b*x]^2*Sech[a + b*x])/x^2,x]
```

```
[Out] Integrate[(Csch[a + b*x]^2*Sech[a + b*x])/x^2, x]
```

fricas [A] time = 0.45, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{\operatorname{csch}(bx+a)^2\operatorname{sech}(bx+a)}{x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^2*sech(b*x+a)/x^2,x, algorithm="fricas")
[Out] integral(csch(b*x + a)^2*sech(b*x + a)/x^2, x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^2*sech(b*x+a)/x^2,x, algorithm="giac")
[Out] Exception raised: AttributeError >> type
```

maple [A] time = 1.10, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}(bx+a)^2 \operatorname{sech}(bx+a)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(b*x+a)^2*sech(b*x+a)/x^2,x)
[Out] int(csch(b*x+a)^2*sech(b*x+a)/x^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{2\,e^{(bx+a)}}{bx^2e^{(2\,bx+2\,a)}-bx^2}-8\int \frac{e^{(bx+a)}}{4\left(x^2e^{(2\,bx+2\,a)}+x^2\right)}\,dx-8\int \frac{1}{4\left(bx^3e^{(bx+a)}+bx^3\right)}\,dx-8\int \frac{1}{4\left(bx^3e^{(bx+a)}-bx^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^2*sech(b*x+a)/x^2,x, algorithm="maxima")
[Out] -2*e^(b*x + a)/(b*x^2*e^(2*b*x + 2*a) - b*x^2) - 8*integrate(1/4*e^(b*x + a
)/(x^2*e^(2*b*x + 2*a) + x^2), x) - 8*integrate(1/4/(b*x^3*e^(b*x + a) + b*
x^3), x) - 8*integrate(1/4/(b*x^3*e^(b*x + a) - b*x^3), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{1}{x^2\cosh(a+b\,x)\,\sinh(a+b\,x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*cosh(a + b*x)*sinh(a + b*x)^2),x)
[Out] int(1/(x^2*cosh(a + b*x)*sinh(a + b*x)^2), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}^2(a+bx)\operatorname{sech}(a+bx)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)**2*sech(b*x+a)/x**2,x)
```

```
[Out] Integral(csch(a + b*x)**2*sech(a + b*x)/x**2, x)
```

3.494 $\int x^m \mathrm{csch}^2(a+bx)\mathrm{sech}^2(a+bx)\,dx$

Optimal. Leaf size=23

$$\mathrm{Int}\left(x^m \mathrm{csch}^2(a+bx)\mathrm{sech}^2(a+bx),x\right)$$

```
[Out] CannotIntegrate(x^m*csch(b*x+a)^2*sech(b*x+a)^2,x)
```

Rubi [A] time = 0.49, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int x^m \mathrm{csch}^2(a+bx)\mathrm{sech}^2(a+bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x^m*Csch[a + b*x]^2*Sech[a + b*x]^2,x]
[Out] Defer[Int][x^m*Csch[a + b*x]^2*Sech[a + b*x]^2, x]
```

Rubi steps

$$\int x^m \mathrm{csch}^2(a+bx)\mathrm{sech}^2(a+bx)\,dx = \int x^m \mathrm{csch}^2(a+bx)\mathrm{sech}^2(a+bx)\,dx$$

Mathematica [A] time = 7.38, size = 0, normalized size = 0.00

$$\int x^m \mathrm{csch}^2(a+bx)\mathrm{sech}^2(a+bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^m*Csch[a + b*x]^2*Sech[a + b*x]^2,x]
[Out] Integrate[x^m*Csch[a + b*x]^2*Sech[a + b*x]^2, x]
```

fricas [A] time = 0.43, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(x^m \operatorname{csch}(bx+a)^2 \operatorname{sech}(bx+a)^2,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*csch(b*x+a)^2*sech(b*x+a)^2,x, algorithm="fricas")
[Out] integral(x^m*csch(b*x + a)^2*sech(b*x + a)^2, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int x^m \operatorname{csch}(bx+a)^2 \operatorname{sech}(bx+a)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*csch(b*x+a)^2*sech(b*x+a)^2,x, algorithm="giac")
[Out] integrate(x^m*csch(b*x + a)^2*sech(b*x + a)^2, x)
```

maple [A] time = 0.17, size = 0, normalized size = 0.00

$$\int x^m \operatorname{csch}(bx+a)^2 \operatorname{sech}(bx+a)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*csch(b*x+a)^2*sech(b*x+a)^2,x)
[Out] int(x^m*csch(b*x+a)^2*sech(b*x+a)^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int x^m \operatorname{csch}(bx+a)^2 \operatorname{sech}(bx+a)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*csch(b*x+a)^2*sech(b*x+a)^2,x, algorithm="maxima")
[Out] integrate(x^m*csch(b*x + a)^2*sech(b*x + a)^2, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{x^m}{\cosh(a+b\,x)^2 \sinh(a+b\,x)^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(cosh(a + b*x)^2*sinh(a + b*x)^2),x)
[Out] int(x^m/(cosh(a + b*x)^2*sinh(a + b*x)^2), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int x^m \operatorname{csch}^2(a+bx) \operatorname{sech}^2(a+bx) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*csch(b*x+a)**2*sech(b*x+a)**2,x)
[Out] Integral(x**m*csch(a + b*x)**2*sech(a + b*x)**2, x)
```

3.495 $\int x^3\text{csch}^2(a+bx)\text{sech}^2(a+bx)\,dx$

Optimal. Leaf size=85

$$-\frac{3\text{Li}_3\left(e^{4(a+bx)}\right)}{8b^4}+\frac{3x\text{Li}_2\left(e^{4(a+bx)}\right)}{2b^3}+\frac{3x^2\log\left(1-e^{4(a+bx)}\right)}{b^2}-\frac{2x^3\coth(2a+2bx)}{b}-\frac{2x^3}{b}$$

```
[Out] -2*x^3/b-2*x^3*coth(2*b*x+2*a)/b+3*x^2*ln(1-exp(4*b*x+4*a))/b^2+3/2*x*polyl
og(2,exp(4*b*x+4*a))/b^3-3/8*polylog(3,exp(4*b*x+4*a))/b^4
```

Rubi [A] time = 0.24, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {5461, 4184, 3716, 2190, 2531, 2282, 6589}

$$\frac{3x\text{PolyLog}\left(2,e^{4(a+bx)}\right)}{2b^3}-\frac{3\text{PolyLog}\left(3,e^{4(a+bx)}\right)}{8b^4}+\frac{3x^2\log\left(1-e^{4(a+bx)}\right)}{b^2}-\frac{2x^3\coth(2a+2bx)}{b}-\frac{2x^3}{b}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Csch[a + b*x]^2*Sech[a + b*x]^2,x]
```

```
[Out] (-2*x^3)/b - (2*x^3*Coth[2*a + 2*b*x])/b + (3*x^2*Log[1 - E^(4*(a + b*x))])
/b^2 + (3*x*PolyLog[2, E^(4*(a + b*x))])/(2*b^3) - (3*PolyLog[3, E^(4*(a +
b*x))])/(8*b^4)
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
```

```
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 3716

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2
*I, Int[((c + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(E^(2*I*k*Pi)*(1 + E^(2*(-(I*
e) + f*fz*x))/E^(2*I*k*Pi))), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Integ
erQ[4*k] && IGtQ[m, 0]
```

Rule 4184

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> -Sim
p[((c + d*x)^m*Cot[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Co
t[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 5461

```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[2^n, Int[(c + d*x)^m*Csch[2*a + 2*b*x]
^n, x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[m] && IntegerQ[n]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int x^3\text{csch}^2(a+bx)\text{sech}^2(a+bx)\,dx = 4\int x^3\text{csch}^2(2a+2bx)\,dx$$
$$= -\frac{2x^3\coth(2a+2bx)}{b} + \frac{6\int x^2\coth(2a+2bx)\,dx}{b}$$
$$= -\frac{2x^3}{b} - \frac{2x^3\coth(2a+2bx)}{b} - \frac{12\int \frac{e^{2(2a+2bx)}x^2}{1-e^{2(2a+2bx)}}\,dx}{b}$$
$$= -\frac{2x^3}{b} - \frac{2x^3\coth(2a+2bx)}{b} + \frac{3x^2\log\left(1-e^{4(a+bx)}\right)}{b^2} - \frac{6\int x\log\left(1-e^{2(2a+\cdots}\right.}{b^2}$$
$$= -\frac{2x^3}{b} - \frac{2x^3\coth(2a+2bx)}{b} + \frac{3x^2\log\left(1-e^{4(a+bx)}\right)}{b^2} + \frac{3x\text{Li}_2\left(e^{4(a+bx)}\right)}{2b^3} - \frac{3\cdots}{\cdots}$$
$$= -\frac{2x^3}{b} - \frac{2x^3\coth(2a+2bx)}{b} + \frac{3x^2\log\left(1-e^{4(a+bx)}\right)}{b^2} + \frac{3x\text{Li}_2\left(e^{4(a+bx)}\right)}{2b^3} - \frac{3\cdots}{\cdots}$$
$$= -\frac{2x^3}{b} - \frac{2x^3\coth(2a+2bx)}{b} + \frac{3x^2\log\left(1-e^{4(a+bx)}\right)}{b^2} + \frac{3x\text{Li}_2\left(e^{4(a+bx)}\right)}{2b^3} - \frac{3\cdots}{\cdots}$$

Mathematica [B] time = 5.76, size = 284, normalized size = 3.34

$$4\left(\frac{x^3\text{csch}(2a)\sinh(2bx)\text{csch}(2a+2bx)}{2b} - \frac{e^{4a}\left(8e^{-4a}b^3x^3 - 6\left(1-e^{-4a}\right)b^2x^2\log\left(1-e^{-a-bx}\right) - 6\left(1-e^{-4a}\right)b^2x^2\,\text{lo}\cdots\right.}{\cdots}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Csch[a + b*x]^2*Sech[a + b*x]^2,x]
```

```
[Out] 4*(-1/8*(E^(4*a)*((8*b^3*x^3)/E^(4*a) - 6*b^2*(1 - E^(-4*a))*x^2*Log[1 - E^
(-a - b*x)] - 6*b^2*(1 - E^(-4*a))*x^2*Log[1 + E^(-a - b*x)] - 6*b^2*(1 - E
^(-4*a))*x^2*Log[1 + E^(-2*(a + b*x))] + 12*(1 - E^(-4*a))*(b*x*PolyLog[2,
-E^(-a - b*x)] + PolyLog[3, -E^(-a - b*x)]) + 12*(1 - E^(-4*a))*(b*x*PolyLo
g[2, E^(-a - b*x)] + PolyLog[3, E^(-a - b*x)]) + (3*(-1 + E^(4*a))*(2*b*x*P
olyLog[2, -E^(-2*(a + b*x))] + PolyLog[3, -E^(-2*(a + b*x))]))/E^(4*a)))/(b
^4*(-1 + E^(4*a))) + (x^3*Csch[2*a]*Csch[2*a + 2*b*x]*Sinh[2*b*x])/(2*b))
```

fricas [C] time = 0.47, size = 1924, normalized size = 22.64

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*csch(b*x+a)^2*sech(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -(4*(b^3*x^3 + a^3)*cosh(b*x + a)^4 + 16*(b^3*x^3 + a^3)*cosh(b*x + a)^3*si
nh(b*x + a) + 24*(b^3*x^3 + a^3)*cosh(b*x + a)^2*sinh(b*x + a)^2 + 16*(b^3*
x^3 + a^3)*cosh(b*x + a)*sinh(b*x + a)^3 + 4*(b^3*x^3 + a^3)*sinh(b*x + a)^
4 - 4*a^3 - 6*(b*x*cosh(b*x + a)^4 + 4*b*x*cosh(b*x + a)^3*sinh(b*x + a) +
6*b*x*cosh(b*x + a)^2*sinh(b*x + a)^2 + 4*b*x*cosh(b*x + a)*sinh(b*x + a)^3
 + b*x*sinh(b*x + a)^4 - b*x)*dilog(cosh(b*x + a) + sinh(b*x + a)) - 6*(b*x
*cosh(b*x + a)^4 + 4*b*x*cosh(b*x + a)^3*sinh(b*x + a) + 6*b*x*cosh(b*x + a
)^2*sinh(b*x + a)^2 + 4*b*x*cosh(b*x + a)*sinh(b*x + a)^3 + b*x*sinh(b*x +
a)^4 - b*x)*dilog(I*cosh(b*x + a) + I*sinh(b*x + a)) - 6*(b*x*cosh(b*x + a)
^4 + 4*b*x*cosh(b*x + a)^3*sinh(b*x + a) + 6*b*x*cosh(b*x + a)^2*sinh(b*x +
 a)^2 + 4*b*x*cosh(b*x + a)*sinh(b*x + a)^3 + b*x*sinh(b*x + a)^4 - b*x)*di
log(-I*cosh(b*x + a) - I*sinh(b*x + a)) - 6*(b*x*cosh(b*x + a)^4 + 4*b*x*co
sh(b*x + a)^3*sinh(b*x + a) + 6*b*x*cosh(b*x + a)^2*sinh(b*x + a)^2 + 4*b*x
*cosh(b*x + a)*sinh(b*x + a)^3 + b*x*sinh(b*x + a)^4 - b*x)*dilog(-cosh(b*x
 + a) - sinh(b*x + a)) - 3*(b^2*x^2*cosh(b*x + a)^4 + 4*b^2*x^2*cosh(b*x +
a)^3*sinh(b*x + a) + 6*b^2*x^2*cosh(b*x + a)^2*sinh(b*x + a)^2 + 4*b^2*x^2*
cosh(b*x + a)*sinh(b*x + a)^3 + b^2*x^2*sinh(b*x + a)^4 - b^2*x^2)*log(cosh
(b*x + a) + sinh(b*x + a) + 1) - 3*(a^2*cosh(b*x + a)^4 + 4*a^2*cosh(b*x +
a)^3*sinh(b*x + a) + 6*a^2*cosh(b*x + a)^2*sinh(b*x + a)^2 + 4*a^2*cosh(b*x
 + a)*sinh(b*x + a)^3 + a^2*sinh(b*x + a)^4 - a^2)*log(cosh(b*x + a) + sinh
(b*x + a) + I) - 3*(a^2*cosh(b*x + a)^4 + 4*a^2*cosh(b*x + a)^3*sinh(b*x +
a) + 6*a^2*cosh(b*x + a)^2*sinh(b*x + a)^2 + 4*a^2*cosh(b*x + a)*sinh(b*x +
 a)^3 + a^2*sinh(b*x + a)^4 - a^2)*log(cosh(b*x + a) + sinh(b*x + a) - I) -
 3*(a^2*cosh(b*x + a)^4 + 4*a^2*cosh(b*x + a)^3*sinh(b*x + a) + 6*a^2*cosh(
b*x + a)^2*sinh(b*x + a)^2 + 4*a^2*cosh(b*x + a)*sinh(b*x + a)^3 + a^2*sinh
(b*x + a)^4 - a^2)*log(cosh(b*x + a) + sinh(b*x + a) - 1) - 3*((b^2*x^2 - a
^2)*cosh(b*x + a)^4 + 4*(b^2*x^2 - a^2)*cosh(b*x + a)^3*sinh(b*x + a) + 6*(
b^2*x^2 - a^2)*cosh(b*x + a)^2*sinh(b*x + a)^2 + 4*(b^2*x^2 - a^2)*cosh(b*x
 + a)*sinh(b*x + a)^3 + (b^2*x^2 - a^2)*sinh(b*x + a)^4 - b^2*x^2 + a^2)*lo
g(I*cosh(b*x + a) + I*sinh(b*x + a) + 1) - 3*((b^2*x^2 - a^2)*cosh(b*x + a)
^4 + 4*(b^2*x^2 - a^2)*cosh(b*x + a)^3*sinh(b*x + a) + 6*(b^2*x^2 - a^2)*co
sh(b*x + a)^2*sinh(b*x + a)^2 + 4*(b^2*x^2 - a^2)*cosh(b*x + a)*sinh(b*x +
a)^3 + (b^2*x^2 - a^2)*sinh(b*x + a)^4 - b^2*x^2 + a^2)*log(-I*cosh(b*x + a
) - I*sinh(b*x + a) + 1) - 3*((b^2*x^2 - a^2)*cosh(b*x + a)^4 + 4*(b^2*x^2
- a^2)*cosh(b*x + a)^3*sinh(b*x + a) + 6*(b^2*x^2 - a^2)*cosh(b*x + a)^2*si
nh(b*x + a)^2 + 4*(b^2*x^2 - a^2)*cosh(b*x + a)*sinh(b*x + a)^3 + (b^2*x^2
- a^2)*sinh(b*x + a)^4 - b^2*x^2 + a^2)*log(-cosh(b*x + a) - sinh(b*x + a)
+ 1) + 6*(cosh(b*x + a)^4 + 4*cosh(b*x + a)^3*sinh(b*x + a) + 6*cosh(b*x +
a)^2*sinh(b*x + a)^2 + 4*cosh(b*x + a)*sinh(b*x + a)^3 + sinh(b*x + a)^4 -
1)*polylog(3, cosh(b*x + a) + sinh(b*x + a)) + 6*(cosh(b*x + a)^4 + 4*cosh(
b*x + a)^3*sinh(b*x + a) + 6*cosh(b*x + a)^2*sinh(b*x + a)^2 + 4*cosh(b*x +
 a)*sinh(b*x + a)^3 + sinh(b*x + a)^4 - 1)*polylog(3, I*cosh(b*x + a) + I*s
inh(b*x + a)) + 6*(cosh(b*x + a)^4 + 4*cosh(b*x + a)^3*sinh(b*x + a) + 6*co
sh(b*x + a)^2*sinh(b*x + a)^2 + 4*cosh(b*x + a)*sinh(b*x + a)^3 + sinh(b*x
+ a)^4 - 1)*polylog(3, -I*cosh(b*x + a) - I*sinh(b*x + a)) + 6*(cosh(b*x +
```

```
a)^4 + 4*cosh(b*x + a)^3*sinh(b*x + a) + 6*cosh(b*x + a)^2*sinh(b*x + a)^2
+ 4*cosh(b*x + a)*sinh(b*x + a)^3 + sinh(b*x + a)^4 - 1)*polylog(3, -cosh(b
*x + a) - sinh(b*x + a)))/(b^4*cosh(b*x + a)^4 + 4*b^4*cosh(b*x + a)^3*sinh
(b*x + a) + 6*b^4*cosh(b*x + a)^2*sinh(b*x + a)^2 + 4*b^4*cosh(b*x + a)*sin
h(b*x + a)^3 + b^4*sinh(b*x + a)^4 - b^4)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3 \operatorname{csch}(bx+a)^2 \operatorname{sech}(bx+a)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*csch(b*x+a)^2*sech(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate(x^3*csch(b*x + a)^2*sech(b*x + a)^2, x)
```

maple [B] time = 0.65, size = 263, normalized size = 3.09

$$-\frac{4x^3}{b\left(e^{2bx+2a}-1\right)\left(1+e^{2bx+2a}\right)}-\frac{12a^2\ln\left(e^{bx+a}\right)}{b^4}+\frac{3a^2\ln\left(e^{bx+a}-1\right)}{b^4}+\frac{8a^3}{b^4}-\frac{3\,\mathrm{polylog}\left(3,-e^{2bx+2a}\right)}{2b^4}-\frac{6\,\mathrm{polylog}\left(3,e\right.}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*csch(b*x+a)^2*sech(b*x+a)^2,x)
```

```
[Out] -4*x^3/b/(exp(2*b*x+2*a)-1)/(1+exp(2*b*x+2*a))-12/b^4*a^2*ln(exp(b*x+a))+3/
b^4*a^2*ln(exp(b*x+a)-1)+8/b^4*a^3-3/2*polylog(3,-exp(2*b*x+2*a))/b^4-6*pol
ylog(3,exp(b*x+a))/b^4-6*polylog(3,-exp(b*x+a))/b^4-4*x^3/b-3/b^4*ln(1-exp(
b*x+a))*a^2+3/b^2*ln(1-exp(b*x+a))*x^2+6*x*polylog(2,exp(b*x+a))/b^3+3/b^2*
ln(1+exp(b*x+a))*x^2+6*x*polylog(2,-exp(b*x+a))/b^3+3*x^2*ln(1+exp(2*b*x+2*
a))/b^2+3*x*polylog(2,-exp(2*b*x+2*a))/b^3+12/b^3*a^2*x
```

maxima [B] time = 0.34, size = 180, normalized size = 2.12

$$-\frac{4\,x^3}{be^{(4\,bx+4\,a)}-b}-\frac{4\,x^3}{b}+\frac{3\left(2\,b^2x^2\log\left(e^{(2\,bx+2\,a)}+1\right)+2\,bx\mathrm{Li}_2\left(-e^{(2\,bx+2\,a)}\right)-\mathrm{Li}_3(-e^{(2\,bx+2\,a)})\right)}{2\,b^4}+\frac{3\left(b^2x^2\log\left(e^{(bx+a)}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*csch(b*x+a)^2*sech(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -4*x^3/(b*e^(4*b*x + 4*a) - b) - 4*x^3/b + 3/2*(2*b^2*x^2*log(e^(2*b*x + 2*
a) + 1) + 2*b*x*dilog(-e^(2*b*x + 2*a)) - polylog(3, -e^(2*b*x + 2*a)))/b^4
 + 3*(b^2*x^2*log(e^(b*x + a) + 1) + 2*b*x*dilog(-e^(b*x + a)) - 2*polylog(
3, -e^(b*x + a)))/b^4 + 3*(b^2*x^2*log(-e^(b*x + a) + 1) + 2*b*x*dilog(e^(b
*x + a)) - 2*polylog(3, e^(b*x + a)))/b^4
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3}{\cosh\left(a+b\,x\right)^2\,\sinh\left(a+b\,x\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(cosh(a + b*x)^2*sinh(a + b*x)^2),x)
```

```
[Out] int(x^3/(cosh(a + b*x)^2*sinh(a + b*x)^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^{3}\operatorname{csch}^{2}\left(a+bx\right)\operatorname{sech}^{2}\left(a+bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*csch(b*x+a)**2*sech(b*x+a)**2,x)
```

```
[Out] Integral(x**3*csch(a + b*x)**2*sech(a + b*x)**2, x)
```

3.496 $\int x^2\text{csch}^2(a+bx)\text{sech}^2(a+bx)\,dx$

Optimal. Leaf size=64

$$\frac{\text{Li}_2\left(e^{4(a+bx)}\right)}{2b^3}+\frac{2x\log\left(1-e^{4(a+bx)}\right)}{b^2}-\frac{2x^2\coth(2a+2bx)}{b}-\frac{2x^2}{b}$$

```
[Out] -2*x^2/b-2*x^2*coth(2*b*x+2*a)/b+2*x*ln(1-exp(4*b*x+4*a))/b^2+1/2*polylog(2
,exp(4*b*x+4*a))/b^3
```

Rubi [A] time = 0.17, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {5461, 4184, 3716, 2190, 2279, 2391}

$$\frac{\text{PolyLog}\left(2,e^{4(a+bx)}\right)}{2b^3}+\frac{2x\log\left(1-e^{4(a+bx)}\right)}{b^2}-\frac{2x^2\coth(2a+2bx)}{b}-\frac{2x^2}{b}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Csch[a + b*x]^2*Sech[a + b*x]^2,x]
```

```
[Out] (-2*x^2)/b - (2*x^2*Coth[2*a + 2*b*x])/b + (2*x*Log[1 - E^(4*(a + b*x))])/b
^2 + PolyLog[2, E^(4*(a + b*x))]/(2*b^3)
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3716

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2
*I, Int[((c + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(E^(2*I*k*Pi)*(1 + E^(2*(-(I*
e) + f*fz*x))/E^(2*I*k*Pi))), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Integ
erQ[4*k] && IGtQ[m, 0]
```

Rule 4184

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> -Sim
p[((c + d*x)^m*Cot[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Co
t[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 5461

```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[2^n, Int[(c + d*x)^m*Csch[2*a + 2*b*x]
^n, x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[m] && IntegerQ[n]
```

Rubi steps

$$
\begin{aligned}
\int x^2\operatorname{csch}^2(a+bx)\operatorname{sech}^2(a+bx)\,dx &= 4\int x^2\operatorname{csch}^2(2a+2bx)\,dx\\
&= -\frac{2x^2\coth(2a+2bx)}{b}+\frac{4\int x\coth(2a+2bx)\,dx}{b}\\
&= -\frac{2x^2}{b}-\frac{2x^2\coth(2a+2bx)}{b}-\frac{8\int\frac{e^{2(2a+2bx)}x}{1-e^{2(2a+2bx)}}\,dx}{b}\\
&= -\frac{2x^2}{b}-\frac{2x^2\coth(2a+2bx)}{b}+\frac{2x\log\left(1-e^{4(a+bx)}\right)}{b^2}-\frac{2\int\log\left(1-e^{2(2a+2bx)}\right)\ldots}{b^2}\\
&= -\frac{2x^2}{b}-\frac{2x^2\coth(2a+2bx)}{b}+\frac{2x\log\left(1-e^{4(a+bx)}\right)}{b^2}-\frac{\operatorname{Subst}\left(\int\frac{\log(1-x)}{x}\,dx\ldots\right)}{2b^3}\\
&= -\frac{2x^2}{b}-\frac{2x^2\coth(2a+2bx)}{b}+\frac{2x\log\left(1-e^{4(a+bx)}\right)}{b^2}+\frac{\operatorname{Li}_2\left(e^{4(a+bx)}\right)}{2b^3}
\end{aligned}
$$

Mathematica [B] time = 4.19, size = 216, normalized size = 3.38

$$
4\left(\frac{x^2\operatorname{csch}(2a)\sinh(2bx)\operatorname{csch}(2a+2bx)}{2b}-\frac{e^{4a}\left(4e^{-4a}b^2x^2+2\left(1-e^{-4a}\right)\operatorname{Li}_2\left(-e^{-a-bx}\right)+2\left(1-e^{-4a}\right)\operatorname{Li}_2\left(e^{-a-bx}\right)\ldots\right.}{\ldots}\right.
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Csch[a + b*x]^2*Sech[a + b*x]^2,x]
```

```
[Out] 4*(-1/4*(E^(4*a)*((4*b^2*x^2)/E^(4*a) - 2*b*(1 - E^(-4*a))*x*Log[1 - E^(-a
- b*x)] - 2*b*(1 - E^(-4*a))*x*Log[1 + E^(-a - b*x)] - 2*b*(1 - E^(-4*a))*x
*Log[1 + E^(-2*(a + b*x))] + 2*(1 - E^(-4*a))*PolyLog[2, -E^(-a - b*x)] + 2
*(1 - E^(-4*a))*PolyLog[2, E^(-a - b*x)] + (1 - E^(-4*a))*PolyLog[2, -E^(-2
*(a + b*x))]))/(b^3*(-1 + E^(4*a))) + (x^2*Csch[2*a]*Csch[2*a + 2*b*x]*Sinh
[2*b*x])/(2*b))
```

fricas [C] time = 0.48, size = 1327, normalized size = 20.73

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*csch(b*x+a)^2*sech(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -2*(2*(b^2*x^2 - a^2)*cosh(b*x + a)^4 + 8*(b^2*x^2 - a^2)*cosh(b*x + a)^3*s
inh(b*x + a) + 12*(b^2*x^2 - a^2)*cosh(b*x + a)^2*sinh(b*x + a)^2 + 8*(b^2*
x^2 - a^2)*cosh(b*x + a)*sinh(b*x + a)^3 + 2*(b^2*x^2 - a^2)*sinh(b*x + a)^
4 + 2*a^2 - (cosh(b*x + a)^4 + 4*cosh(b*x + a)^3*sinh(b*x + a) + 6*cosh(b*x
 + a)^2*sinh(b*x + a)^2 + 4*cosh(b*x + a)*sinh(b*x + a)^3 + sinh(b*x + a)^4
 - 1)*dilog(cosh(b*x + a) + sinh(b*x + a)) - (cosh(b*x + a)^4 + 4*cosh(b*x
+ a)^3*sinh(b*x + a) + 6*cosh(b*x + a)^2*sinh(b*x + a)^2 + 4*cosh(b*x + a)*
sinh(b*x + a)^3 + sinh(b*x + a)^4 - 1)*dilog(I*cosh(b*x + a) + I*sinh(b*x +
 a)) - (cosh(b*x + a)^4 + 4*cosh(b*x + a)^3*sinh(b*x + a) + 6*cosh(b*x + a)
^2*sinh(b*x + a)^2 + 4*cosh(b*x + a)*sinh(b*x + a)^3 + sinh(b*x + a)^4 - 1)
*dilog(-I*cosh(b*x + a) - I*sinh(b*x + a)) - (cosh(b*x + a)^4 + 4*cosh(b*x
+ a)^3*sinh(b*x + a) + 6*cosh(b*x + a)^2*sinh(b*x + a)^2 + 4*cosh(b*x + a)*
sinh(b*x + a)^3 + sinh(b*x + a)^4 - 1)*dilog(-cosh(b*x + a) - sinh(b*x + a)
) - (b*x*cosh(b*x + a)^4 + 4*b*x*cosh(b*x + a)^3*sinh(b*x + a) + 6*b*x*cosh
(b*x + a)^2*sinh(b*x + a)^2 + 4*b*x*cosh(b*x + a)*sinh(b*x + a)^3 + b*x*sin
h(b*x + a)^4 - b*x)*log(cosh(b*x + a) + sinh(b*x + a) + 1) + (a*cosh(b*x +
a)^4 + 4*a*cosh(b*x + a)^3*sinh(b*x + a) + 6*a*cosh(b*x + a)^2*sinh(b*x + a
)^2 + 4*a*cosh(b*x + a)*sinh(b*x + a)^3 + a*sinh(b*x + a)^4 - a)*log(cosh(b
*x + a) + sinh(b*x + a) + I) + (a*cosh(b*x + a)^4 + 4*a*cosh(b*x + a)^3*sin
h(b*x + a) + 6*a*cosh(b*x + a)^2*sinh(b*x + a)^2 + 4*a*cosh(b*x + a)*sinh(b
*x + a)^3 + a*sinh(b*x + a)^4 - a)*log(cosh(b*x + a) + sinh(b*x + a) - I) +
 (a*cosh(b*x + a)^4 + 4*a*cosh(b*x + a)^3*sinh(b*x + a) + 6*a*cosh(b*x + a)
^2*sinh(b*x + a)^2 + 4*a*cosh(b*x + a)*sinh(b*x + a)^3 + a*sinh(b*x + a)^4
- a)*log(cosh(b*x + a) + sinh(b*x + a) - 1) - ((b*x + a)*cosh(b*x + a)^4 +
4*(b*x + a)*cosh(b*x + a)^3*sinh(b*x + a) + 6*(b*x + a)*cosh(b*x + a)^2*sin
h(b*x + a)^2 + 4*(b*x + a)*cosh(b*x + a)*sinh(b*x + a)^3 + (b*x + a)*sinh(b
*x + a)^4 - b*x - a)*log(I*cosh(b*x + a) + I*sinh(b*x + a) + 1) - ((b*x + a
)*cosh(b*x + a)^4 + 4*(b*x + a)*cosh(b*x + a)^3*sinh(b*x + a) + 6*(b*x + a)
*cosh(b*x + a)^2*sinh(b*x + a)^2 + 4*(b*x + a)*cosh(b*x + a)*sinh(b*x + a)^
```

```
3 + (b*x + a)*sinh(b*x + a)^4 - b*x - a)*log(-I*cosh(b*x + a) - I*sinh(b*x
+ a) + 1) - ((b*x + a)*cosh(b*x + a)^4 + 4*(b*x + a)*cosh(b*x + a)^3*sinh(b
*x + a) + 6*(b*x + a)*cosh(b*x + a)^2*sinh(b*x + a)^2 + 4*(b*x + a)*cosh(b*
x + a)*sinh(b*x + a)^3 + (b*x + a)*sinh(b*x + a)^4 - b*x - a)*log(-cosh(b*x
 + a) - sinh(b*x + a) + 1))/(b^3*cosh(b*x + a)^4 + 4*b^3*cosh(b*x + a)^3*si
nh(b*x + a) + 6*b^3*cosh(b*x + a)^2*sinh(b*x + a)^2 + 4*b^3*cosh(b*x + a)*s
inh(b*x + a)^3 + b^3*sinh(b*x + a)^4 - b^3)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \operatorname{csch}(bx+a)^2 \operatorname{sech}(bx+a)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*csch(b*x+a)^2*sech(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate(x^2*csch(b*x + a)^2*sech(b*x + a)^2, x)
```

maple [B] time = 0.66, size = 199, normalized size = 3.11

$$-\frac{4x^2}{b\left(e^{2bx+2a}-1\right)\left(1+e^{2bx+2a}\right)}-\frac{4x^2}{b}-\frac{8ax}{b^2}-\frac{4a^2}{b^3}+\frac{2x\ln\left(1+e^{2bx+2a}\right)}{b^2}+\frac{\operatorname{polylog}\left(2,-e^{2bx+2a}\right)}{b^3}+\frac{2\ln\left(1-e^{bx+a}\right)x}{b^2}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*csch(b*x+a)^2*sech(b*x+a)^2,x)
```

```
[Out] -4*x^2/b/(exp(2*b*x+2*a)-1)/(1+exp(2*b*x+2*a))-4*x^2/b-8*a*x/b^2-4/b^3*a^2+
2*x*ln(1+exp(2*b*x+2*a))/b^2+polylog(2,-exp(2*b*x+2*a))/b^3+2/b^2*ln(1-exp(
b*x+a))*x+2/b^3*ln(1-exp(b*x+a))*a+2*polylog(2,exp(b*x+a))/b^3+2/b^2*ln(1+e
xp(b*x+a))*x+2*polylog(2,-exp(b*x+a))/b^3+8/b^3*a*ln(exp(b*x+a))-2/b^3*a*ln
(exp(b*x+a)-1)
```

maxima [A] time = 0.45, size = 118, normalized size = 1.84

$$-\frac{4x^2}{be^{(4bx+4a)}-b}-\frac{4x^2}{b}+\frac{2bx\log\left(e^{(2bx+2a)}+1\right)+\operatorname{Li}_2\left(-e^{(2bx+2a)}\right)}{b^3}+\frac{2\left(bx\log\left(e^{(bx+a)}+1\right)+\operatorname{Li}_2\left(-e^{(bx+a)}\right)\right)}{b^3}+\frac{2\left(b\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*csch(b*x+a)^2*sech(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -4*x^2/(b*e^(4*b*x + 4*a) - b) - 4*x^2/b + (2*b*x*log(e^(2*b*x + 2*a) + 1)
+ dilog(-e^(2*b*x + 2*a)))/b^3 + 2*(b*x*log(e^(b*x + a) + 1) + dilog(-e^(b*
x + a)))/b^3 + 2*(b*x*log(-e^(b*x + a) + 1) + dilog(e^(b*x + a)))/b^3
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^2}{\cosh\left(a+b\,x\right)^2\,\sinh\left(a+b\,x\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(cosh(a + b*x)^2*sinh(a + b*x)^2),x)
```

```
[Out] int(x^2/(cosh(a + b*x)^2*sinh(a + b*x)^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2\operatorname{csch}^2\left(a+bx\right)\operatorname{sech}^2\left(a+bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*csch(b*x+a)**2*sech(b*x+a)**2,x)
```

```
[Out] Integral(x**2*csch(a + b*x)**2*sech(a + b*x)**2, x)
```

3.497 $\int x\operatorname{csch}^2(a+bx)\operatorname{sech}^2(a+bx)\,dx$

Optimal. Leaf size=30

$$\frac{\log(\sinh(2a+2bx))}{b^2}-\frac{2x\coth(2a+2bx)}{b}$$

```
[Out] -2*x*coth(2*b*x+2*a)/b+ln(sinh(2*b*x+2*a))/b^2
```

Rubi [A] time = 0.06, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {5461, 4184, 3475}

$$\frac{\log(\sinh(2a+2bx))}{b^2}-\frac{2x\coth(2a+2bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[x*Csch[a + b*x]^2*Sech[a + b*x]^2,x]
```

```
[Out] (-2*x*Coth[2*a + 2*b*x])/b + Log[Sinh[2*a + 2*b*x]]/b^2
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 4184

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> -Sim
p[((c + d*x)^m*Cot[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Co
t[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 5461

```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[2^n, Int[(c + d*x)^m*Csch[2*a + 2*b*x]
^n, x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[m] && IntegerQ[n]
```

Rubi steps

$$\int x\text{csch}^2(a+bx)\text{sech}^2(a+bx)\,dx = 4\int x\text{csch}^2(2a+2bx)\,dx$$
$$= -\frac{2x\coth(2a+2bx)}{b} + \frac{2\int\coth(2a+2bx)\,dx}{b}$$
$$= -\frac{2x\coth(2a+2bx)}{b} + \frac{\log(\sinh(2a+2bx))}{b^2}$$

Mathematica [A] time = 0.16, size = 26, normalized size = 0.87

$$\frac{\log(\sinh(2(a+bx))) - 2bx\coth(2(a+bx))}{b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Csch[a + b*x]^2*Sech[a + b*x]^2,x]
```

```
[Out] (-2*b*x*Coth[2*(a + b*x)] + Log[Sinh[2*(a + b*x)]])/b^2
```

fricas [B] time = 0.43, size = 292, normalized size = 9.73

$$-\frac{4\,bx\cosh\left(bx+a\right)^4 + 16\,bx\cosh\left(bx+a\right)^3\sinh\left(bx+a\right) + 24\,bx\cosh\left(bx+a\right)^2\sinh\left(bx+a\right)^2 + 16\,bx\cosh\left(bx-\right.}{b^2\cosh\left(bx+a\right)^{}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*csch(b*x+a)^2*sech(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -(4*b*x*cosh(b*x + a)^4 + 16*b*x*cosh(b*x + a)^3*sinh(b*x + a) + 24*b*x*cos
h(b*x + a)^2*sinh(b*x + a)^2 + 16*b*x*cosh(b*x + a)*sinh(b*x + a)^3 + 4*b*x
*sinh(b*x + a)^4 - (cosh(b*x + a)^4 + 4*cosh(b*x + a)^3*sinh(b*x + a) + 6*c
osh(b*x + a)^2*sinh(b*x + a)^2 + 4*cosh(b*x + a)*sinh(b*x + a)^3 + sinh(b*x
 + a)^4 - 1)*log(4*cosh(b*x + a)*sinh(b*x + a)/(cosh(b*x + a)^2 - 2*cosh(b*
x + a)*sinh(b*x + a) + sinh(b*x + a)^2)))/(b^2*cosh(b*x + a)^4 + 4*b^2*cosh
(b*x + a)^3*sinh(b*x + a) + 6*b^2*cosh(b*x + a)^2*sinh(b*x + a)^2 + 4*b^2*c
osh(b*x + a)*sinh(b*x + a)^3 + b^2*sinh(b*x + a)^4 - b^2)
```

giac [B] time = 0.14, size = 72, normalized size = 2.40

$$-\frac{4\,bxe^{(4\,bx+4\,a)} - e^{(4\,bx+4\,a)}\log\left(e^{(4\,bx+4\,a)}-1\right) + \log\left(e^{(4\,bx+4\,a)}-1\right)}{b^2e^{(4\,bx+4\,a)} - b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*csch(b*x+a)^2*sech(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -(4*b*x*e^(4*b*x + 4*a) - e^(4*b*x + 4*a)*log(e^(4*b*x + 4*a) - 1) + log(e^
(4*b*x + 4*a) - 1))/(b^2*e^(4*b*x + 4*a) - b^2)
```

maple [B] time = 0.53, size = 62, normalized size = 2.07

$$-\frac{4x}{b}-\frac{4a}{b^2}-\frac{4x}{b\left(e^{2bx+2a}-1\right)\left(1+e^{2bx+2a}\right)}+\frac{\ln\left(e^{4bx+4a}-1\right)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*csch(b*x+a)^2*sech(b*x+a)^2,x)
```

```
[Out] -4*x/b-4*a/b^2-4*x/b/(exp(2*b*x+2*a)-1)/(1+exp(2*b*x+2*a))+1/b^2*ln(exp(4*b
*x+4*a)-1)
```

maxima [B] time = 0.32, size = 87, normalized size = 2.90

$$-\frac{4\,xe^{(4\,bx+4\,a)}}{be^{(4\,bx+4\,a)}-b}+\frac{\log\left(\left(e^{(bx+a)}+1\right)e^{(-a)}\right)}{b^2}+\frac{\log\left(\left(e^{(bx+a)}-1\right)e^{(-a)}\right)}{b^2}+\frac{\log\left(\left(e^{(2\,bx+2\,a)}+1\right)e^{(-2\,a)}\right)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*csch(b*x+a)^2*sech(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -4*x*e^(4*b*x + 4*a)/(b*e^(4*b*x + 4*a) - b) + log((e^(b*x + a) + 1)*e^(-a)
)/b^2 + log((e^(b*x + a) - 1)*e^(-a))/b^2 + log((e^(2*b*x + 2*a) + 1)*e^(-2
*a))/b^2
```

mupad [B] time = 0.07, size = 43, normalized size = 1.43

$$\frac{\ln\left(e^{4\,a}\,e^{4\,b\,x}-1\right)}{b^2}-\frac{4\,x}{b}-\frac{4\,x}{b\,\left(e^{4\,a+4\,b\,x}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(cosh(a + b*x)^2*sinh(a + b*x)^2),x)
```

```
[Out] log(exp(4*a)*exp(4*b*x) - 1)/b^2 - (4*x)/b - (4*x)/(b*(exp(4*a + 4*b*x) - 1
))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\operatorname{csch}^2\left(a+bx\right)\operatorname{sech}^2\left(a+bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*csch(b*x+a)**2*sech(b*x+a)**2,x)
```

```
[Out] Integral(x*csch(a + b*x)**2*sech(a + b*x)**2, x)
```

3.498 $\int \operatorname{csch}^2(a+bx)\operatorname{sech}^2(a+bx)\,dx$

Optimal. Leaf size=23

$$-\frac{\tanh(a+bx)}{b}-\frac{\coth(a+bx)}{b}$$

```
[Out] -coth(b*x+a)/b-tanh(b*x+a)/b
```

Rubi [A] time = 0.03, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2620, 14}

$$-\frac{\tanh(a+bx)}{b}-\frac{\coth(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[a + b*x]^2*Sech[a + b*x]^2,x]
```

```
[Out] -(Coth[a + b*x]/b) - Tanh[a + b*x]/b
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2620

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rubi steps

$$\begin{aligned}
\int \operatorname{csch}^2(a+bx)\operatorname{sech}^2(a+bx)\,dx &= \frac{i\,\text{Subst}\left(\int \frac{1+x^2}{x^2}\,dx, x, i\tanh(a+bx)\right)}{b}\\
&= \frac{i\,\text{Subst}\left(\int \left(1+\frac{1}{x^2}\right)dx, x, i\tanh(a+bx)\right)}{b}\\
&= -\frac{\coth(a+bx)}{b}-\frac{\tanh(a+bx)}{b}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 13, normalized size = 0.57

$$-\frac{2\coth(2(a+bx))}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[a + b*x]^2*Sech[a + b*x]^2,x]
```

```
[Out] (-2*Coth[2*(a + b*x)])/b
```

fricas [B] time = 0.40, size = 81, normalized size = 3.52

$$-\frac{4}{b\cosh\left(bx+a\right)^{4}+4\,b\cosh\left(bx+a\right)^{3}\sinh\left(bx+a\right)+6\,b\cosh\left(bx+a\right)^{2}\sinh\left(bx+a\right)^{2}+4\,b\cosh\left(bx+a\right)\sinh\left(bx+a\right)^{3}+b\sinh\left(bx+a\right)^{4}-b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^2*sech(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -4/(b*cosh(b*x + a)^4 + 4*b*cosh(b*x + a)^3*sinh(b*x + a) + 6*b*cosh(b*x +
a)^2*sinh(b*x + a)^2 + 4*b*cosh(b*x + a)*sinh(b*x + a)^3 + b*sinh(b*x + a)^
4 - b)
```

giac [A] time = 0.14, size = 18, normalized size = 0.78

$$-\frac{4}{b\left(e^{(4\,bx+4\,a)}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^2*sech(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -4/(b*(e^(4*b*x + 4*a) - 1))
```

maple [A] time = 0.34, size = 32, normalized size = 1.39

$$\frac{-\frac{1}{\sinh(bx+a)\cosh(bx+a)}-2\tanh\left(bx+a\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(b*x+a)^2*sech(b*x+a)^2,x)
```

```
[Out] 1/b*(-1/sinh(b*x+a)/cosh(b*x+a)-2*tanh(b*x+a))
```

maxima [A] time = 0.31, size = 18, normalized size = 0.78

$$\frac{4}{b\left(e^{(-4\,bx-4\,a)}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^2*sech(b*x+a)^2,x, algorithm="maxima")
[Out] 4/(b*(e^(-4*b*x - 4*a) - 1))
```

mupad [B] time = 0.07, size = 18, normalized size = 0.78

$$-\frac{4}{b\,\left(e^{4\,a+4\,b\,x}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(a + b*x)^2*sinh(a + b*x)^2),x)
[Out] -4/(b*(exp(4*a + 4*b*x) - 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{csch}^2(a+bx)\operatorname{sech}^2(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)**2*sech(b*x+a)**2,x)
[Out] Integral(csch(a + b*x)**2*sech(a + b*x)**2, x)
```

3.499 $\int \frac{\operatorname{csch}^2(a+bx)\operatorname{sech}^2(a+bx)}{x}\,dx$

Optimal. Leaf size=20

$$4\text{Int}\left(\frac{\operatorname{csch}^2(2a+2bx)}{x},x\right)$$

```
[Out] 4*Unintegrable(csch(2*b*x+2*a)^2/x,x)
```

Rubi [A] time = 0.07, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} =$ 0.000, Rules used = {}

$$\int \frac{\operatorname{csch}^2(a+bx)\operatorname{sech}^2(a+bx)}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Csch[a + b*x]^2*Sech[a + b*x]^2)/x,x]
[Out] 4*Defer[Int][Csch[2*a + 2*b*x]^2/x, x]
```

Rubi steps

$$\int \frac{\operatorname{csch}^2(a+bx)\operatorname{sech}^2(a+bx)}{x}\,dx = 4\int \frac{\operatorname{csch}^2(2a+2bx)}{x}\,dx$$

Mathematica [A] time = 23.96, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}^2(a+bx)\operatorname{sech}^2(a+bx)}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Csch[a + b*x]^2*Sech[a + b*x]^2)/x,x]
[Out] Integrate[(Csch[a + b*x]^2*Sech[a + b*x]^2)/x, x]
```

fricas [A] time = 0.42, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\operatorname{csch}(bx+a)^2\operatorname{sech}(bx+a)^2}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^2*sech(b*x+a)^2/x,x, algorithm="fricas")
```

```
[Out] integral(csch(b*x + a)^2*sech(b*x + a)^2/x, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}(bx+a)^2 \operatorname{sech}(bx+a)^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^2*sech(b*x+a)^2/x,x, algorithm="giac")
```

```
[Out] integrate(csch(b*x + a)^2*sech(b*x + a)^2/x, x)
```

maple [A] time = 0.53, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}(bx+a)^2 \operatorname{sech}(bx+a)^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(b*x+a)^2*sech(b*x+a)^2/x,x)
```

```
[Out] int(csch(b*x+a)^2*sech(b*x+a)^2/x,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{4}{bxe^{(4\,bx+4\,a)}-bx}+16\int \frac{1}{8\left(bx^2e^{(2\,bx+2\,a)}+bx^2\right)}\,dx+16\int \frac{1}{16\left(bx^2e^{(bx+a)}+bx^2\right)}\,dx-16\int \frac{1}{16\left(bx^2e^{(bx+a)}-bx^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^2*sech(b*x+a)^2/x,x, algorithm="maxima")
```

```
[Out] -4/(b*x*e^(4*b*x + 4*a) - b*x) + 16*integrate(1/8/(b*x^2*e^(2*b*x + 2*a) +
b*x^2), x) + 16*integrate(1/16/(b*x^2*e^(b*x + a) + b*x^2), x) - 16*integra
te(1/16/(b*x^2*e^(b*x + a) - b*x^2), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{1}{x\cosh\left(a+b\,x\right)^2\sinh\left(a+b\,x\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*cosh(a + b*x)^2*sinh(a + b*x)^2),x)
```

```
[Out] int(1/(x*cosh(a + b*x)^2*sinh(a + b*x)^2), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}^2(a+bx)\operatorname{sech}^2(a+bx)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)**2*sech(b*x+a)**2/x,x)
```

```
[Out] Integral(csch(a + b*x)**2*sech(a + b*x)**2/x, x)
```

3.500 $\int \frac{\operatorname{csch}^2(a+bx)\operatorname{sech}^2(a+bx)}{x^2}\,dx$

Optimal. Leaf size=20

$$4\mathrm{Int}\left(\frac{\operatorname{csch}^2(2a+2bx)}{x^2},x\right)$$

```
[Out] 4*Unintegrable(csch(2*b*x+2*a)^2/x^2,x)
```

Rubi [A] time = 0.07, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\operatorname{csch}^2(a+bx)\operatorname{sech}^2(a+bx)}{x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Csch[a + b*x]^2*Sech[a + b*x]^2)/x^2,x]
[Out] 4*Defer[Int][Csch[2*a + 2*b*x]^2/x^2, x]
```

Rubi steps

$$\int \frac{\operatorname{csch}^2(a+bx)\operatorname{sech}^2(a+bx)}{x^2}\,dx = 4\int \frac{\operatorname{csch}^2(2a+2bx)}{x^2}\,dx$$

Mathematica [A] time = 21.12, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}^2(a+bx)\operatorname{sech}^2(a+bx)}{x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Csch[a + b*x]^2*Sech[a + b*x]^2)/x^2,x]
[Out] Integrate[(Csch[a + b*x]^2*Sech[a + b*x]^2)/x^2, x]
```

fricas [A] time = 0.41, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\operatorname{csch}(bx+a)^2\operatorname{sech}(bx+a)^2}{x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^2*sech(b*x+a)^2/x^2,x, algorithm="fricas")
```

```
[Out] integral(csch(b*x + a)^2*sech(b*x + a)^2/x^2, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}(bx+a)^2 \operatorname{sech}(bx+a)^2}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^2*sech(b*x+a)^2/x^2,x, algorithm="giac")
```

```
[Out] integrate(csch(b*x + a)^2*sech(b*x + a)^2/x^2, x)
```

maple [A] time = 0.53, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}(bx+a)^2 \operatorname{sech}(bx+a)^2}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(b*x+a)^2*sech(b*x+a)^2/x^2,x)
```

```
[Out] int(csch(b*x+a)^2*sech(b*x+a)^2/x^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{4}{bx^2e^{(4\,bx+4\,a)} - bx^2} + 16\int \frac{1}{4\left(bx^3e^{(2\,bx+2\,a)} + bx^3\right)}\,dx + 16\int \frac{1}{8\left(bx^3e^{(bx+a)} + bx^3\right)}\,dx - 16\int \frac{1}{8\left(bx^3e^{(bx+a)} - bx^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^2*sech(b*x+a)^2/x^2,x, algorithm="maxima")
```

```
[Out] -4/(b*x^2*e^(4*b*x + 4*a) - b*x^2) + 16*integrate(1/4/(b*x^3*e^(2*b*x + 2*a
) + b*x^3), x) + 16*integrate(1/8/(b*x^3*e^(b*x + a) + b*x^3), x) - 16*inte
grate(1/8/(b*x^3*e^(b*x + a) - b*x^3), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{1}{x^2\cosh(a+b\,x)^2\sinh(a+b\,x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*cosh(a + b*x)^2*sinh(a + b*x)^2),x)
```

```
[Out] int(1/(x^2*cosh(a + b*x)^2*sinh(a + b*x)^2), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}^2(a+bx)\operatorname{sech}^2(a+bx)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)**2*sech(b*x+a)**2/x**2,x)
```

```
[Out] Integral(csch(a + b*x)**2*sech(a + b*x)**2/x**2, x)
```

3.501 $\int x^m \operatorname{csch}^2(a+bx)\operatorname{sech}^3(a+bx)\,dx$

Optimal. Leaf size=23

$$\operatorname{Int}\left(x^m \operatorname{csch}^2(a+bx)\operatorname{sech}^3(a+bx),x\right)$$

```
[Out] CannotIntegrate(x^m*csch(b*x+a)^2*sech(b*x+a)^3,x)
```

Rubi [A] time = 0.58, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int x^m \operatorname{csch}^2(a+bx)\operatorname{sech}^3(a+bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x^m*Csch[a + b*x]^2*Sech[a + b*x]^3,x]
[Out] Defer[Int][x^m*Csch[a + b*x]^2*Sech[a + b*x]^3, x]
```

Rubi steps

$$\int x^m \operatorname{csch}^2(a+bx)\operatorname{sech}^3(a+bx)\,dx = \int x^m \operatorname{csch}^2(a+bx)\operatorname{sech}^3(a+bx)\,dx$$

Mathematica [A] time = 56.03, size = 0, normalized size = 0.00

$$\int x^m \operatorname{csch}^2(a+bx)\operatorname{sech}^3(a+bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^m*Csch[a + b*x]^2*Sech[a + b*x]^3,x]
[Out] Integrate[x^m*Csch[a + b*x]^2*Sech[a + b*x]^3, x]
```

fricas [A] time = 0.42, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(x^m \operatorname{csch}(bx+a)^2 \operatorname{sech}(bx+a)^3, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*csch(b*x+a)^2*sech(b*x+a)^3,x, algorithm="fricas")
[Out] integral(x^m*csch(b*x + a)^2*sech(b*x + a)^3, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int x^m \operatorname{csch}(bx+a)^2 \operatorname{sech}(bx+a)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*csch(b*x+a)^2*sech(b*x+a)^3,x, algorithm="giac")
[Out] integrate(x^m*csch(b*x + a)^2*sech(b*x + a)^3, x)
```

maple [A] time = 0.21, size = 0, normalized size = 0.00

$$\int x^m \operatorname{csch}(bx+a)^2 \operatorname{sech}(bx+a)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*csch(b*x+a)^2*sech(b*x+a)^3,x)
[Out] int(x^m*csch(b*x+a)^2*sech(b*x+a)^3,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int x^m \operatorname{csch}(bx+a)^2 \operatorname{sech}(bx+a)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*csch(b*x+a)^2*sech(b*x+a)^3,x, algorithm="maxima")
[Out] integrate(x^m*csch(b*x + a)^2*sech(b*x + a)^3, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{x^m}{\cosh(a+b\,x)^3 \sinh(a+b\,x)^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(cosh(a + b*x)^3*sinh(a + b*x)^2),x)
[Out] int(x^m/(cosh(a + b*x)^3*sinh(a + b*x)^2), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int x^m \operatorname{csch}^2(a+bx) \operatorname{sech}^3(a+bx) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*csch(b*x+a)**2*sech(b*x+a)**3,x)
[Out] Integral(x**m*csch(a + b*x)**2*sech(a + b*x)**3, x)
```

3.502 $\int x^2 \operatorname{csch}^2(a+bx)\operatorname{sech}^3(a+bx)\,dx$

Optimal. Leaf size=206

$$-\frac{2\text{Li}_2\left(-e^{a+bx}\right)}{b^3}+\frac{2\text{Li}_2\left(e^{a+bx}\right)}{b^3}-\frac{3i\text{Li}_3\left(-ie^{a+bx}\right)}{b^3}+\frac{3i\text{Li}_3\left(ie^{a+bx}\right)}{b^3}+\frac{\tan^{-1}(\sinh(a+bx))}{b^3}+\frac{3ix\text{Li}_2\left(-ie^{a+bx}\right)}{b^2}-\frac{3ix\text{Li}_2\left(ie^{a+bx}\right)}{b^2}$$

```
[Out] -3*x^2*arctan(exp(b*x+a))/b+arctan(sinh(b*x+a))/b^3-4*x*arctanh(exp(b*x+a))
/b^2-3/2*x^2*csch(b*x+a)/b-2*polylog(2,-exp(b*x+a))/b^3+3*I*x*polylog(2,-I*
exp(b*x+a))/b^2-3*I*x*polylog(2,I*exp(b*x+a))/b^2+2*polylog(2,exp(b*x+a))/b
^3-3*I*polylog(3,-I*exp(b*x+a))/b^3+3*I*polylog(3,I*exp(b*x+a))/b^3-x*sech(
b*x+a)/b^2+1/2*x^2*csch(b*x+a)*sech(b*x+a)^2/b
```

Rubi [A] time = 0.43, antiderivative size = 206, normalized size of antiderivative = 1.00, number of steps used = 29, number of rules used = 18, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.900, Rules used = {2621, 288, 321, 207, 5462, 14, 5205, 12, 4180, 2531, 2282, 6589, 4182, 2279, 2391, 2622, 6271, 3770}

$$\frac{3ix\text{PolyLog}\left(2,-ie^{a+bx}\right)}{b^2}-\frac{3ix\text{PolyLog}\left(2,ie^{a+bx}\right)}{b^2}-\frac{2\text{PolyLog}\left(2,-e^{a+bx}\right)}{b^3}+\frac{2\text{PolyLog}\left(2,e^{a+bx}\right)}{b^3}-\frac{3i\text{PolyLog}\left(3,-ie^{a+bx}\right)}{b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Csch[a + b*x]^2*Sech[a + b*x]^3,x]
```

```
[Out] (-3*x^2*ArcTan[E^(a + b*x)])/b + ArcTan[Sinh[a + b*x]]/b^3 - (4*x*ArcTanh[E
^(a + b*x)])/b^2 - (3*x^2*Csch[a + b*x])/(2*b) - (2*PolyLog[2, -E^(a + b*x)
])/b^3 + ((3*I)*x*PolyLog[2, (-I)*E^(a + b*x)])/b^2 - ((3*I)*x*PolyLog[2, I
*E^(a + b*x)])/b^2 + (2*PolyLog[2, E^(a + b*x)])/b^3 - ((3*I)*PolyLog[3, (-
I)*E^(a + b*x)])/b^3 + ((3*I)*PolyLog[3, I*E^(a + b*x)])/b^3 - (x*Sech[a +
b*x])/b^2 + (x^2*Csch[a + b*x]*Sech[a + b*x]^2)/(2*b)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
```

```
, g, n}, x] && GtQ[m, 0]
```

Rule 2621

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> -Dist[(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5205

```
Int[((a_.) + ArcTan[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Sim
p[((c + d*x)^(m + 1)*(a + b*ArcTan[u]))/(d*(m + 1)), x] - Dist[b/(d*(m + 1)
), Int[SimplifyIntegrand[((c + d*x)^(m + 1)*D[u, x])/(1 + u^2), x], x], x]
/; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && InverseFunctionFreeQ[u, x] &&
  !FunctionOfQ[(c + d*x)^(m + 1), u, x] && FalseQ[PowerVariableExpn[u, m +
```

```
1, x]]
```

Rule 5462

```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> With[{u = IntHide[Csch[a + b*x]^n*Sech[a +
b*x]^p, x]}, Dist[(c + d*x)^m, u, x] - Dist[d*m, Int[(c + d*x)^(m - 1)*u, x
], x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n,
p]
```

Rule 6271

```
Int[ArcTanh[u_], x_Symbol] :> Simp[x*ArcTanh[u], x] - Int[SimplifyIntegrand
[(x*D[u, x])/(1 - u^2), x], x] /; InverseFunctionFreeQ[u, x]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int x^2\text{csch}^2(a+bx)\text{sech}^3(a+bx)\,dx = -\frac{3x^2\tan^{-1}(\sinh(a+bx))}{2b} - \frac{3x^2\text{csch}(a+bx)}{2b} + \frac{x^2\text{csch}(a+bx)\text{sech}^2(a+$$

$$= -\frac{3x^2\tan^{-1}(\sinh(a+bx))}{2b} - \frac{3x^2\text{csch}(a+bx)}{2b} + \frac{x^2\text{csch}(a+bx)\text{sech}^2(a+$$

$$= -\frac{3x^2\tan^{-1}(\sinh(a+bx))}{2b} - \frac{3x^2\text{csch}(a+bx)}{2b} + \frac{x^2\text{csch}(a+bx)\text{sech}^2(a+$$

$$= -\frac{3x^2\tan^{-1}(\sinh(a+bx))}{2b} + \frac{x\tanh^{-1}(\cosh(a+bx))}{b^2} - \frac{3x^2\text{csch}(a+bx)}{2b} -$$

$$= -\frac{3x^2\tan^{-1}(\sinh(a+bx))}{2b} + \frac{x\tanh^{-1}(\cosh(a+bx))}{b^2} - \frac{3x^2\text{csch}(a+bx)}{2b} -$$

$$= \frac{\tan^{-1}(\sinh(a+bx))}{b^3} - \frac{6x\tanh^{-1}\left(e^{a+bx}\right)}{b^2} - \frac{3x^2\text{csch}(a+bx)}{2b} - \frac{x\text{sech}(a+}{b^2}$$

$$= \frac{\tan^{-1}(\sinh(a+bx))}{b^3} - \frac{6x\tanh^{-1}\left(e^{a+bx}\right)}{b^2} - \frac{3x^2\text{csch}(a+bx)}{2b} - \frac{x\text{sech}(a+}{b^2}$$

$$= -\frac{3x^2\tan^{-1}\left(e^{a+bx}\right)}{b} + \frac{\tan^{-1}(\sinh(a+bx))}{b^3} - \frac{4x\tanh^{-1}\left(e^{a+bx}\right)}{b^2} - \frac{3x^2\text{csch}(}{2l}$$

$$= -\frac{3x^2\tan^{-1}\left(e^{a+bx}\right)}{b} + \frac{\tan^{-1}(\sinh(a+bx))}{b^3} - \frac{4x\tanh^{-1}\left(e^{a+bx}\right)}{b^2} - \frac{3x^2\text{csch}(}{2l}$$

$$= -\frac{3x^2\tan^{-1}\left(e^{a+bx}\right)}{b} + \frac{\tan^{-1}(\sinh(a+bx))}{b^3} - \frac{4x\tanh^{-1}\left(e^{a+bx}\right)}{b^2} - \frac{3x^2\text{csch}(}{2l}$$

$$= -\frac{3x^2\tan^{-1}\left(e^{a+bx}\right)}{b} + \frac{\tan^{-1}(\sinh(a+bx))}{b^3} - \frac{4x\tanh^{-1}\left(e^{a+bx}\right)}{b^2} - \frac{3x^2\text{csch}(}{2l}$$

Mathematica [A] time = 7.42, size = 397, normalized size = 1.93

$$\frac{2\left(-a\log\left(\tanh\left(\frac{1}{2}(a+bx)\right)\right) - i\left(i\left(\text{Li}_2\left(-e^{i(ia+ibx)}\right) - \text{Li}_2\left(e^{i(ia+ibx)}\right)\right) + (ia+ibx)\left(\log\left(1-e^{i(ia+ibx)}\right) - \log\left(1+e^{i(ia}\right.\right.\right.\right.}{b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Csch[a + b*x]^2*Sech[a + b*x]^3,x]
```

```
[Out] -((x^2*Csch[a])/b) + (2*(-(a*Log[Tanh[(a + b*x)/2]]) - I*((I*a + I*b*x)*(Lo
g[1 - E^(I*(I*a + I*b*x))] - Log[1 + E^(I*(I*a + I*b*x))]) + I*(PolyLog[2,
-E^(I*(I*a + I*b*x))] - PolyLog[2, E^(I*(I*a + I*b*x))]))))/b^3 - ((I/2)*((
```

```
4*I)*ArcTan[E^(a + b*x)] + 3*b^2*x^2*Log[1 - I*E^(a + b*x)] - 3*b^2*x^2*Log
[1 + I*E^(a + b*x)] - 6*b*x*PolyLog[2, (-I)*E^(a + b*x)] + 6*b*x*PolyLog[2,
 I*E^(a + b*x)] + 6*PolyLog[3, (-I)*E^(a + b*x)] - 6*PolyLog[3, I*E^(a + b*
x)]))/b^3 - (x*Sech[a]*Sech[a + b*x]*(2*Cosh[a] + b*x*Sinh[a]))/(2*b^2) + (
x^2*Csch[a/2]*Csch[a/2 + (b*x)/2]*Sinh[(b*x)/2])/(2*b) + (x^2*Sech[a/2]*Sec
h[a/2 + (b*x)/2]*Sinh[(b*x)/2])/(2*b) - (x^2*Sech[a]*Sech[a + b*x]^2*Sinh[b
*x])/(2*b)
```

fricas [C] time = 0.58, size = 3777, normalized size = 18.33

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*csch(b*x+a)^2*sech(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -1/2*(4*b^2*x^2*cosh(b*x + a)^3 + 2*(3*b^2*x^2 + 2*b*x)*cosh(b*x + a)^5 + 1
0*(3*b^2*x^2 + 2*b*x)*cosh(b*x + a)*sinh(b*x + a)^4 + 2*(3*b^2*x^2 + 2*b*x)
*sinh(b*x + a)^5 + 4*(b^2*x^2 + 5*(3*b^2*x^2 + 2*b*x)*cosh(b*x + a)^2)*sinh
(b*x + a)^3 + 4*(3*b^2*x^2*cosh(b*x + a) + 5*(3*b^2*x^2 + 2*b*x)*cosh(b*x +
 a)^3)*sinh(b*x + a)^2 + 2*(3*b^2*x^2 - 2*b*x)*cosh(b*x + a) - 4*(cosh(b*x
+ a)^6 + 6*cosh(b*x + a)*sinh(b*x + a)^5 + sinh(b*x + a)^6 + (15*cosh(b*x +
 a)^2 + 1)*sinh(b*x + a)^4 + cosh(b*x + a)^4 + 4*(5*cosh(b*x + a)^3 + cosh(
b*x + a))*sinh(b*x + a)^3 + (15*cosh(b*x + a)^4 + 6*cosh(b*x + a)^2 - 1)*si
nh(b*x + a)^2 - cosh(b*x + a)^2 + 2*(3*cosh(b*x + a)^5 + 2*cosh(b*x + a)^3
- cosh(b*x + a))*sinh(b*x + a) - 1)*dilog(cosh(b*x + a) + sinh(b*x + a)) -
(-6*I*b*x*cosh(b*x + a)^6 - 36*I*b*x*cosh(b*x + a)*sinh(b*x + a)^5 - 6*I*b*
x*sinh(b*x + a)^6 - 6*I*b*x*cosh(b*x + a)^4 + (-90*I*b*x*cosh(b*x + a)^2 -
6*I*b*x)*sinh(b*x + a)^4 + 6*I*b*x*cosh(b*x + a)^2 + (-120*I*b*x*cosh(b*x +
 a)^3 - 24*I*b*x*cosh(b*x + a))*sinh(b*x + a)^3 + (-90*I*b*x*cosh(b*x + a)^
4 - 36*I*b*x*cosh(b*x + a)^2 + 6*I*b*x)*sinh(b*x + a)^2 + 6*I*b*x + (-36*I*
b*x*cosh(b*x + a)^5 - 24*I*b*x*cosh(b*x + a)^3 + 12*I*b*x*cosh(b*x + a))*si
nh(b*x + a))*dilog(I*cosh(b*x + a) + I*sinh(b*x + a)) - (6*I*b*x*cosh(b*x +
 a)^6 + 36*I*b*x*cosh(b*x + a)*sinh(b*x + a)^5 + 6*I*b*x*sinh(b*x + a)^6 +
6*I*b*x*cosh(b*x + a)^4 + (90*I*b*x*cosh(b*x + a)^2 + 6*I*b*x)*sinh(b*x + a
)^4 - 6*I*b*x*cosh(b*x + a)^2 + (120*I*b*x*cosh(b*x + a)^3 + 24*I*b*x*cosh(
b*x + a))*sinh(b*x + a)^3 + (90*I*b*x*cosh(b*x + a)^4 + 36*I*b*x*cosh(b*x +
 a)^2 - 6*I*b*x)*sinh(b*x + a)^2 - 6*I*b*x + (36*I*b*x*cosh(b*x + a)^5 + 24
*I*b*x*cosh(b*x + a)^3 - 12*I*b*x*cosh(b*x + a))*sinh(b*x + a))*dilog(-I*co
sh(b*x + a) - I*sinh(b*x + a)) + 4*(cosh(b*x + a)^6 + 6*cosh(b*x + a)*sinh(
b*x + a)^5 + sinh(b*x + a)^6 + (15*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^4 + c
osh(b*x + a)^4 + 4*(5*cosh(b*x + a)^3 + cosh(b*x + a))*sinh(b*x + a)^3 + (1
5*cosh(b*x + a)^4 + 6*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^2 - cosh(b*x + a)^
2 + 2*(3*cosh(b*x + a)^5 + 2*cosh(b*x + a)^3 - cosh(b*x + a))*sinh(b*x + a)
 - 1)*dilog(-cosh(b*x + a) - sinh(b*x + a)) + 4*(b*x*cosh(b*x + a)^6 + 6*b*
x*cosh(b*x + a)*sinh(b*x + a)^5 + b*x*sinh(b*x + a)^6 + b*x*cosh(b*x + a)^4
```

```
 + (15*b*x*cosh(b*x + a)^2 + b*x)*sinh(b*x + a)^4 - b*x*cosh(b*x + a)^2 + 4
*(5*b*x*cosh(b*x + a)^3 + b*x*cosh(b*x + a))*sinh(b*x + a)^3 + (15*b*x*cosh
(b*x + a)^4 + 6*b*x*cosh(b*x + a)^2 - b*x)*sinh(b*x + a)^2 - b*x + 2*(3*b*x
*cosh(b*x + a)^5 + 2*b*x*cosh(b*x + a)^3 - b*x*cosh(b*x + a))*sinh(b*x + a)
)*log(cosh(b*x + a) + sinh(b*x + a) + 1) - ((-3*I*a^2 + 2*I)*cosh(b*x + a)^
6 + (-18*I*a^2 + 12*I)*cosh(b*x + a)*sinh(b*x + a)^5 + (-3*I*a^2 + 2*I)*sin
h(b*x + a)^6 + (-3*I*a^2 + 2*I)*cosh(b*x + a)^4 + ((-45*I*a^2 + 30*I)*cosh(
b*x + a)^2 - 3*I*a^2 + 2*I)*sinh(b*x + a)^4 + ((-60*I*a^2 + 40*I)*cosh(b*x
+ a)^3 + (-12*I*a^2 + 8*I)*cosh(b*x + a))*sinh(b*x + a)^3 + (3*I*a^2 - 2*I)
*cosh(b*x + a)^2 + ((-45*I*a^2 + 30*I)*cosh(b*x + a)^4 + (-18*I*a^2 + 12*I)
*cosh(b*x + a)^2 + 3*I*a^2 - 2*I)*sinh(b*x + a)^2 + 3*I*a^2 + ((-18*I*a^2 +
 12*I)*cosh(b*x + a)^5 + (-12*I*a^2 + 8*I)*cosh(b*x + a)^3 + (6*I*a^2 - 4*I
)*cosh(b*x + a))*sinh(b*x + a) - 2*I)*log(cosh(b*x + a) + sinh(b*x + a) + I
) - ((3*I*a^2 - 2*I)*cosh(b*x + a)^6 + (18*I*a^2 - 12*I)*cosh(b*x + a)*sinh
(b*x + a)^5 + (3*I*a^2 - 2*I)*sinh(b*x + a)^6 + (3*I*a^2 - 2*I)*cosh(b*x +
a)^4 + ((45*I*a^2 - 30*I)*cosh(b*x + a)^2 + 3*I*a^2 - 2*I)*sinh(b*x + a)^4
+ ((60*I*a^2 - 40*I)*cosh(b*x + a)^3 + (12*I*a^2 - 8*I)*cosh(b*x + a))*sinh
(b*x + a)^3 + (-3*I*a^2 + 2*I)*cosh(b*x + a)^2 + ((45*I*a^2 - 30*I)*cosh(b*
x + a)^4 + (18*I*a^2 - 12*I)*cosh(b*x + a)^2 - 3*I*a^2 + 2*I)*sinh(b*x + a)
^2 - 3*I*a^2 + ((18*I*a^2 - 12*I)*cosh(b*x + a)^5 + (12*I*a^2 - 8*I)*cosh(b
*x + a)^3 + (-6*I*a^2 + 4*I)*cosh(b*x + a))*sinh(b*x + a) + 2*I)*log(cosh(b
*x + a) + sinh(b*x + a) - I) + 4*(a*cosh(b*x + a)^6 + 6*a*cosh(b*x + a)*sin
h(b*x + a)^5 + a*sinh(b*x + a)^6 + a*cosh(b*x + a)^4 + (15*a*cosh(b*x + a)^
2 + a)*sinh(b*x + a)^4 + 4*(5*a*cosh(b*x + a)^3 + a*cosh(b*x + a))*sinh(b*x
 + a)^3 - a*cosh(b*x + a)^2 + (15*a*cosh(b*x + a)^4 + 6*a*cosh(b*x + a)^2 -
 a)*sinh(b*x + a)^2 + 2*(3*a*cosh(b*x + a)^5 + 2*a*cosh(b*x + a)^3 - a*cosh
(b*x + a))*sinh(b*x + a) - a)*log(cosh(b*x + a) + sinh(b*x + a) - 1) - ((3*
I*b^2*x^2 - 3*I*a^2)*cosh(b*x + a)^6 + (18*I*b^2*x^2 - 18*I*a^2)*cosh(b*x +
 a)*sinh(b*x + a)^5 + (3*I*b^2*x^2 - 3*I*a^2)*sinh(b*x + a)^6 + (3*I*b^2*x^
2 - 3*I*a^2)*cosh(b*x + a)^4 + (3*I*b^2*x^2 + (45*I*b^2*x^2 - 45*I*a^2)*cos
h(b*x + a)^2 - 3*I*a^2)*sinh(b*x + a)^4 - 3*I*b^2*x^2 + ((60*I*b^2*x^2 - 60
*I*a^2)*cosh(b*x + a)^3 + (12*I*b^2*x^2 - 12*I*a^2)*cosh(b*x + a))*sinh(b*x
 + a)^3 + (-3*I*b^2*x^2 + 3*I*a^2)*cosh(b*x + a)^2 + ((45*I*b^2*x^2 - 45*I*
a^2)*cosh(b*x + a)^4 - 3*I*b^2*x^2 + (18*I*b^2*x^2 - 18*I*a^2)*cosh(b*x + a
)^2 + 3*I*a^2)*sinh(b*x + a)^2 + 3*I*a^2 + ((18*I*b^2*x^2 - 18*I*a^2)*cosh(
b*x + a)^5 + (12*I*b^2*x^2 - 12*I*a^2)*cosh(b*x + a)^3 + (-6*I*b^2*x^2 + 6*
I*a^2)*cosh(b*x + a))*sinh(b*x + a))*log(I*cosh(b*x + a) + I*sinh(b*x + a)
+ 1) - ((-3*I*b^2*x^2 + 3*I*a^2)*cosh(b*x + a)^6 + (-18*I*b^2*x^2 + 18*I*a^
2)*cosh(b*x + a)*sinh(b*x + a)^5 + (-3*I*b^2*x^2 + 3*I*a^2)*sinh(b*x + a)^6
 + (-3*I*b^2*x^2 + 3*I*a^2)*cosh(b*x + a)^4 + (-3*I*b^2*x^2 + (-45*I*b^2*x^
2 + 45*I*a^2)*cosh(b*x + a)^2 + 3*I*a^2)*sinh(b*x + a)^4 + 3*I*b^2*x^2 + ((
-60*I*b^2*x^2 + 60*I*a^2)*cosh(b*x + a)^3 + (-12*I*b^2*x^2 + 12*I*a^2)*cosh
(b*x + a))*sinh(b*x + a)^3 + (3*I*b^2*x^2 - 3*I*a^2)*cosh(b*x + a)^2 + ((-4
5*I*b^2*x^2 + 45*I*a^2)*cosh(b*x + a)^4 + 3*I*b^2*x^2 + (-18*I*b^2*x^2 + 18
*I*a^2)*cosh(b*x + a)^2 - 3*I*a^2)*sinh(b*x + a)^2 - 3*I*a^2 + ((-18*I*b^2*
```

```
x^2 + 18*I*a^2)*cosh(b*x + a)^5 + (-12*I*b^2*x^2 + 12*I*a^2)*cosh(b*x + a)^
3 + (6*I*b^2*x^2 - 6*I*a^2)*cosh(b*x + a))*sinh(b*x + a))*log(-I*cosh(b*x +
 a) - I*sinh(b*x + a) + 1) - 4*((b*x + a)*cosh(b*x + a)^6 + 6*(b*x + a)*cos
h(b*x + a)*sinh(b*x + a)^5 + (b*x + a)*sinh(b*x + a)^6 + (b*x + a)*cosh(b*x
 + a)^4 + (15*(b*x + a)*cosh(b*x + a)^2 + b*x + a)*sinh(b*x + a)^4 + 4*(5*(
b*x + a)*cosh(b*x + a)^3 + (b*x + a)*cosh(b*x + a))*sinh(b*x + a)^3 - (b*x
+ a)*cosh(b*x + a)^2 + (15*(b*x + a)*cosh(b*x + a)^4 + 6*(b*x + a)*cosh(b*x
 + a)^2 - b*x - a)*sinh(b*x + a)^2 - b*x + 2*(3*(b*x + a)*cosh(b*x + a)^5 +
 2*(b*x + a)*cosh(b*x + a)^3 - (b*x + a)*cosh(b*x + a))*sinh(b*x + a) - a)*
log(-cosh(b*x + a) - sinh(b*x + a) + 1) - (6*I*cosh(b*x + a)^6 + 36*I*cosh(
b*x + a)*sinh(b*x + a)^5 + 6*I*sinh(b*x + a)^6 + (90*I*cosh(b*x + a)^2 + 6*
I)*sinh(b*x + a)^4 + 6*I*cosh(b*x + a)^4 + (120*I*cosh(b*x + a)^3 + 24*I*co
sh(b*x + a))*sinh(b*x + a)^3 + (90*I*cosh(b*x + a)^4 + 36*I*cosh(b*x + a)^2
 - 6*I)*sinh(b*x + a)^2 - 6*I*cosh(b*x + a)^2 + (36*I*cosh(b*x + a)^5 + 24*
I*cosh(b*x + a)^3 - 12*I*cosh(b*x + a))*sinh(b*x + a) - 6*I)*polylog(3, I*c
osh(b*x + a) + I*sinh(b*x + a)) - (-6*I*cosh(b*x + a)^6 - 36*I*cosh(b*x + a
)*sinh(b*x + a)^5 - 6*I*sinh(b*x + a)^6 + (-90*I*cosh(b*x + a)^2 - 6*I)*sin
h(b*x + a)^4 - 6*I*cosh(b*x + a)^4 + (-120*I*cosh(b*x + a)^3 - 24*I*cosh(b*
x + a))*sinh(b*x + a)^3 + (-90*I*cosh(b*x + a)^4 - 36*I*cosh(b*x + a)^2 + 6
*I)*sinh(b*x + a)^2 + 6*I*cosh(b*x + a)^2 + (-36*I*cosh(b*x + a)^5 - 24*I*c
osh(b*x + a)^3 + 12*I*cosh(b*x + a))*sinh(b*x + a) + 6*I)*polylog(3, -I*cos
h(b*x + a) - I*sinh(b*x + a)) + 2*(6*b^2*x^2*cosh(b*x + a)^2 + 5*(3*b^2*x^2
 + 2*b*x)*cosh(b*x + a)^4 + 3*b^2*x^2 - 2*b*x)*sinh(b*x + a))/(b^3*cosh(b*x
 + a)^6 + 6*b^3*cosh(b*x + a)*sinh(b*x + a)^5 + b^3*sinh(b*x + a)^6 + b^3*c
osh(b*x + a)^4 - b^3*cosh(b*x + a)^2 + (15*b^3*cosh(b*x + a)^2 + b^3)*sinh(
b*x + a)^4 + 4*(5*b^3*cosh(b*x + a)^3 + b^3*cosh(b*x + a))*sinh(b*x + a)^3
- b^3 + (15*b^3*cosh(b*x + a)^4 + 6*b^3*cosh(b*x + a)^2 - b^3)*sinh(b*x + a
)^2 + 2*(3*b^3*cosh(b*x + a)^5 + 2*b^3*cosh(b*x + a)^3 - b^3*cosh(b*x + a))
*sinh(b*x + a))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \operatorname{csch}(bx+a)^2 \operatorname{sech}(bx+a)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*csch(b*x+a)^2*sech(b*x+a)^3,x, algorithm="giac")
```

```
[Out] integrate(x^2*csch(b*x + a)^2*sech(b*x + a)^3, x)
```

maple [F] time = 2.07, size = 0, normalized size = 0.00

$$\int x^2 \operatorname{csch}(bx+a)^2 \operatorname{sech}(bx+a)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2*csch(b*x+a)^2*sech(b*x+a)^3,x)

[Out] int(x^2*csch(b*x+a)^2*sech(b*x+a)^3,x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-96\,b^2\int\frac{x^2e^{(bx+a)}}{32\left(b^2e^{(2\,bx+2\,a)}+b^2\right)}\,dx-\frac{2\,bx^2e^{(3\,bx+3\,a)}+\left(3\,bx^2e^{(5\,a)}+2\,xe^{(5\,a)}\right)e^{(5\,bx)}+\left(3\,bx^2e^a-2\,xe^a\right)e^{(bx)}}{b^2e^{(6\,bx+6\,a)}+b^2e^{(4\,bx+4\,a)}-b^2e^{(2\,bx+2\,a)}-b^2}-\frac{2\left(bx\log\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*csch(b*x+a)^2*sech(b*x+a)^3,x, algorithm="maxima")

[Out] -96*b^2*integrate(1/32*x^2*e^(b*x + a)/(b^2*e^(2*b*x + 2*a) + b^2), x) - (2*b*x^2*e^(3*b*x + 3*a) + (3*b*x^2*e^(5*a) + 2*x*e^(5*a))*e^(5*b*x) + (3*b*x^2*e^a - 2*x*e^a)*e^(b*x))/(b^2*e^(6*b*x + 6*a) + b^2*e^(4*b*x + 4*a) - b^2*e^(2*b*x + 2*a) - b^2) - 2*(b*x*log(e^(b*x + a) + 1) + dilog(-e^(b*x + a)))/b^3 + 2*(b*x*log(-e^(b*x + a) + 1) + dilog(e^(b*x + a)))/b^3 + 2*arctan(e^(b*x + a))/b^3

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int\frac{x^2}{\cosh\left(a+b\,x\right)^3\,\sinh\left(a+b\,x\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/(cosh(a + b*x)^3*sinh(a + b*x)^2),x)

[Out] int(x^2/(cosh(a + b*x)^3*sinh(a + b*x)^2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2\operatorname{csch}^2\left(a+bx\right)\operatorname{sech}^3\left(a+bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**2*csch(b*x+a)**2*sech(b*x+a)**3,x)

[Out] Integral(x**2*csch(a + b*x)**2*sech(a + b*x)**3, x)

3.503 $\int x\mathrm{csch}^2(a+bx)\mathrm{sech}^3(a+bx)\,dx$

Optimal. Leaf size=120

$$\frac{3i\mathrm{Li}_2\left(-ie^{a+bx}\right)}{2b^2}-\frac{3i\mathrm{Li}_2\left(ie^{a+bx}\right)}{2b^2}-\frac{\mathrm{sech}(a+bx)}{2b^2}-\frac{\tanh^{-1}(\cosh(a+bx))}{b^2}-\frac{3x\tan^{-1}\left(e^{a+bx}\right)}{b}-\frac{3x\mathrm{csch}(a+bx)}{2b}+\frac{x\mathrm{csch}$$

```
[Out] -3*x*arctan(exp(b*x+a))/b-arctanh(cosh(b*x+a))/b^2-3/2*x*csch(b*x+a)/b+3/2*
I*polylog(2,-I*exp(b*x+a))/b^2-3/2*I*polylog(2,I*exp(b*x+a))/b^2-1/2*sech(b
*x+a)/b^2+1/2*x*csch(b*x+a)*sech(b*x+a)^2/b
```

Rubi [A] time = 0.17, antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 12, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {2621, 288, 321, 207, 5462, 5203, 12, 4180, 2279, 2391, 3770, 2622}

$$\frac{3i\mathrm{PolyLog}\left(2,-ie^{a+bx}\right)}{2b^2}-\frac{3i\mathrm{PolyLog}\left(2,ie^{a+bx}\right)}{2b^2}-\frac{\mathrm{sech}(a+bx)}{2b^2}-\frac{\tanh^{-1}(\cosh(a+bx))}{b^2}-\frac{3x\tan^{-1}\left(e^{a+bx}\right)}{b}-\frac{3x\mathrm{csch}$$

Antiderivative was successfully verified.

```
[In] Int[x*Csch[a + b*x]^2*Sech[a + b*x]^3,x]
```

```
[Out] (-3*x*ArcTan[E^(a + b*x)])/b - ArcTanh[Cosh[a + b*x]]/b^2 - (3*x*Csch[a + b
*x])/(2*b) + (((3*I)/2)*PolyLog[2, (-I)*E^(a + b*x)])/b^2 - (((3*I)/2)*Poly
Log[2, I*E^(a + b*x)])/b^2 - Sech[a + b*x]/(2*b^2) + (x*Csch[a + b*x]*Sech[
a + b*x]^2)/(2*b)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
```

```
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2621

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> -Dist[(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
```

```
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5203

```
Int[ArcTan[u_], x_Symbol] :> Simp[x*ArcTan[u], x] - Int[SimplifyIntegrand[(
x*D[u, x])/(1 + u^2), x], x] /; InverseFunctionFreeQ[u, x]
```

Rule 5462

```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> With[{u = IntHide[Csch[a + b*x]^n*Sech[a +
b*x]^p, x]}, Dist[(c + d*x)^m, u, x] - Dist[d*m, Int[(c + d*x)^(m - 1)*u, x
], x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n,
p]
```

Rubi steps

$$\int x\text{csch}^2(a+bx)\text{sech}^3(a+bx)\,dx = -\frac{3x\tan^{-1}(\sinh(a+bx))}{2b} - \frac{3x\text{csch}(a+bx)}{2b} + \frac{x\text{csch}(a+bx)\text{sech}^2(a+bx)}{2b}$$

$$= -\frac{3x\tan^{-1}(\sinh(a+bx))}{2b} - \frac{3x\text{csch}(a+bx)}{2b} + \frac{x\text{csch}(a+bx)\text{sech}^2(a+bx)}{2b}$$

$$= -\frac{3\tanh^{-1}(\cosh(a+bx))}{2b^2} - \frac{3x\text{csch}(a+bx)}{2b} + \frac{x\text{csch}(a+bx)\text{sech}^2(a+bx)}{2b}$$

$$= -\frac{3\tanh^{-1}(\cosh(a+bx))}{2b^2} - \frac{3x\text{csch}(a+bx)}{2b} - \frac{\text{sech}(a+bx)}{2b^2} + \frac{x\text{csch}(a+}{}$$

$$= -\frac{3x\tan^{-1}\left(e^{a+bx}\right)}{b} - \frac{\tanh^{-1}(\cosh(a+bx))}{b^2} - \frac{3x\text{csch}(a+bx)}{2b} - \frac{\text{sech}(a+}{2b^2}$$

$$= -\frac{3x\tan^{-1}\left(e^{a+bx}\right)}{b} - \frac{\tanh^{-1}(\cosh(a+bx))}{b^2} - \frac{3x\text{csch}(a+bx)}{2b} - \frac{\text{sech}(a+}{2b^2}$$

$$= -\frac{3x\tan^{-1}\left(e^{a+bx}\right)}{b} - \frac{\tanh^{-1}(\cosh(a+bx))}{b^2} - \frac{3x\text{csch}(a+bx)}{2b} + \frac{3i\text{Li}_2\left(-ie\right.}{2b^2}$$

Mathematica [A] time = 2.49, size = 209, normalized size = 1.74

$$-\frac{-3i\text{Li}_2\left(-ie^{-a-bx}\right) + 3i\text{Li}_2\left(ie^{-a-bx}\right) - 3ia\log\left(1-ie^{-a-bx}\right) - 3ibx\log\left(1-ie^{-a-bx}\right) + 3ia\log\left(1+ie^{-a-bx}\right) + 3ibx}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Csch[a + b*x]^2*Sech[a + b*x]^3,x]
```

```
[Out] -1/2*(-3*a*ArcTan[Sinh[a + b*x]] + b*x*Coth[(a + b*x)/2] - (3*I)*a*Log[1 -
I*E^(-a - b*x)] - (3*I)*b*x*Log[1 - I*E^(-a - b*x)] + (3*I)*a*Log[1 + I*E^(
-a - b*x)] + (3*I)*b*x*Log[1 + I*E^(-a - b*x)] - 2*Log[Tanh[(a + b*x)/2]] -
 (3*I)*PolyLog[2, (-I)*E^(-a - b*x)] + (3*I)*PolyLog[2, I*E^(-a - b*x)] + S
ech[a + b*x] - b*x*Tanh[(a + b*x)/2] + b*x*Sech[a + b*x]*Tanh[a + b*x])/b^2
```

fricas [B] time = 0.50, size = 2201, normalized size = 18.34

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*csch(b*x+a)^2*sech(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -1/2*(2*(3*b*x + 1)*cosh(b*x + a)^5 + 10*(3*b*x + 1)*cosh(b*x + a)*sinh(b*x
 + a)^4 + 2*(3*b*x + 1)*sinh(b*x + a)^5 + 4*b*x*cosh(b*x + a)^3 + 4*(5*(3*b
*x + 1)*cosh(b*x + a)^2 + b*x)*sinh(b*x + a)^3 + 4*(5*(3*b*x + 1)*cosh(b*x
+ a)^3 + 3*b*x*cosh(b*x + a))*sinh(b*x + a)^2 + 2*(3*b*x - 1)*cosh(b*x + a)
 - (-3*I*cosh(b*x + a)^6 - 18*I*cosh(b*x + a)*sinh(b*x + a)^5 - 3*I*sinh(b*
x + a)^6 + (-45*I*cosh(b*x + a)^2 - 3*I)*sinh(b*x + a)^4 - 3*I*cosh(b*x + a
)^4 + (-60*I*cosh(b*x + a)^3 - 12*I*cosh(b*x + a))*sinh(b*x + a)^3 + (-45*I
*cosh(b*x + a)^4 - 18*I*cosh(b*x + a)^2 + 3*I)*sinh(b*x + a)^2 + 3*I*cosh(b
*x + a)^2 + (-18*I*cosh(b*x + a)^5 - 12*I*cosh(b*x + a)^3 + 6*I*cosh(b*x +
a))*sinh(b*x + a) + 3*I)*dilog(I*cosh(b*x + a) + I*sinh(b*x + a)) - (3*I*co
sh(b*x + a)^6 + 18*I*cosh(b*x + a)*sinh(b*x + a)^5 + 3*I*sinh(b*x + a)^6 +
(45*I*cosh(b*x + a)^2 + 3*I)*sinh(b*x + a)^4 + 3*I*cosh(b*x + a)^4 + (60*I*
cosh(b*x + a)^3 + 12*I*cosh(b*x + a))*sinh(b*x + a)^3 + (45*I*cosh(b*x + a)
^4 + 18*I*cosh(b*x + a)^2 - 3*I)*sinh(b*x + a)^2 - 3*I*cosh(b*x + a)^2 + (1
8*I*cosh(b*x + a)^5 + 12*I*cosh(b*x + a)^3 - 6*I*cosh(b*x + a))*sinh(b*x +
a) - 3*I)*dilog(-I*cosh(b*x + a) - I*sinh(b*x + a)) + 2*(cosh(b*x + a)^6 +
6*cosh(b*x + a)*sinh(b*x + a)^5 + sinh(b*x + a)^6 + (15*cosh(b*x + a)^2 + 1
)*sinh(b*x + a)^4 + cosh(b*x + a)^4 + 4*(5*cosh(b*x + a)^3 + cosh(b*x + a))
*sinh(b*x + a)^3 + (15*cosh(b*x + a)^4 + 6*cosh(b*x + a)^2 - 1)*sinh(b*x +
a)^2 - cosh(b*x + a)^2 + 2*(3*cosh(b*x + a)^5 + 2*cosh(b*x + a)^3 - cosh(b*
x + a))*sinh(b*x + a) - 1)*log(cosh(b*x + a) + sinh(b*x + a) + 1) - (3*I*a*
cosh(b*x + a)^6 + 18*I*a*cosh(b*x + a)*sinh(b*x + a)^5 + 3*I*a*sinh(b*x + a
)^6 + 3*I*a*cosh(b*x + a)^4 + (45*I*a*cosh(b*x + a)^2 + 3*I*a)*sinh(b*x + a
)^4 + (60*I*a*cosh(b*x + a)^3 + 12*I*a*cosh(b*x + a))*sinh(b*x + a)^3 - 3*I
*a*cosh(b*x + a)^2 + (45*I*a*cosh(b*x + a)^4 + 18*I*a*cosh(b*x + a)^2 - 3*I
*a)*sinh(b*x + a)^2 + (18*I*a*cosh(b*x + a)^5 + 12*I*a*cosh(b*x + a)^3 - 6*
I*a*cosh(b*x + a))*sinh(b*x + a) - 3*I*a)*log(cosh(b*x + a) + sinh(b*x + a)
 + I) - (-3*I*a*cosh(b*x + a)^6 - 18*I*a*cosh(b*x + a)*sinh(b*x + a)^5 - 3*
```

```
I*a*sinh(b*x + a)^6 - 3*I*a*cosh(b*x + a)^4 + (-45*I*a*cosh(b*x + a)^2 - 3*
I*a)*sinh(b*x + a)^4 + (-60*I*a*cosh(b*x + a)^3 - 12*I*a*cosh(b*x + a))*sin
h(b*x + a)^3 + 3*I*a*cosh(b*x + a)^2 + (-45*I*a*cosh(b*x + a)^4 - 18*I*a*co
sh(b*x + a)^2 + 3*I*a)*sinh(b*x + a)^2 + (-18*I*a*cosh(b*x + a)^5 - 12*I*a*
cosh(b*x + a)^3 + 6*I*a*cosh(b*x + a))*sinh(b*x + a) + 3*I*a)*log(cosh(b*x
+ a) + sinh(b*x + a) - I) - 2*(cosh(b*x + a)^6 + 6*cosh(b*x + a)*sinh(b*x +
 a)^5 + sinh(b*x + a)^6 + (15*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^4 + cosh(b
*x + a)^4 + 4*(5*cosh(b*x + a)^3 + cosh(b*x + a))*sinh(b*x + a)^3 + (15*cos
h(b*x + a)^4 + 6*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^2 - cosh(b*x + a)^2 + 2
*(3*cosh(b*x + a)^5 + 2*cosh(b*x + a)^3 - cosh(b*x + a))*sinh(b*x + a) - 1)
*log(cosh(b*x + a) + sinh(b*x + a) - 1) - ((3*I*b*x + 3*I*a)*cosh(b*x + a)^
6 + (18*I*b*x + 18*I*a)*cosh(b*x + a)*sinh(b*x + a)^5 + (3*I*b*x + 3*I*a)*s
inh(b*x + a)^6 + (3*I*b*x + 3*I*a)*cosh(b*x + a)^4 + ((45*I*b*x + 45*I*a)*c
osh(b*x + a)^2 + 3*I*b*x + 3*I*a)*sinh(b*x + a)^4 + ((60*I*b*x + 60*I*a)*co
sh(b*x + a)^3 + (12*I*b*x + 12*I*a)*cosh(b*x + a))*sinh(b*x + a)^3 + (-3*I*
b*x - 3*I*a)*cosh(b*x + a)^2 + ((45*I*b*x + 45*I*a)*cosh(b*x + a)^4 + (18*I
*b*x + 18*I*a)*cosh(b*x + a)^2 - 3*I*b*x - 3*I*a)*sinh(b*x + a)^2 - 3*I*b*x
 + ((18*I*b*x + 18*I*a)*cosh(b*x + a)^5 + (12*I*b*x + 12*I*a)*cosh(b*x + a)
^3 + (-6*I*b*x - 6*I*a)*cosh(b*x + a))*sinh(b*x + a) - 3*I*a)*log(I*cosh(b*
x + a) + I*sinh(b*x + a) + 1) - ((-3*I*b*x - 3*I*a)*cosh(b*x + a)^6 + (-18*
I*b*x - 18*I*a)*cosh(b*x + a)*sinh(b*x + a)^5 + (-3*I*b*x - 3*I*a)*sinh(b*x
 + a)^6 + (-3*I*b*x - 3*I*a)*cosh(b*x + a)^4 + ((-45*I*b*x - 45*I*a)*cosh(b
*x + a)^2 - 3*I*b*x - 3*I*a)*sinh(b*x + a)^4 + ((-60*I*b*x - 60*I*a)*cosh(b
*x + a)^3 + (-12*I*b*x - 12*I*a)*cosh(b*x + a))*sinh(b*x + a)^3 + (3*I*b*x
+ 3*I*a)*cosh(b*x + a)^2 + ((-45*I*b*x - 45*I*a)*cosh(b*x + a)^4 + (-18*I*b
*x - 18*I*a)*cosh(b*x + a)^2 + 3*I*b*x + 3*I*a)*sinh(b*x + a)^2 + 3*I*b*x +
 ((-18*I*b*x - 18*I*a)*cosh(b*x + a)^5 + (-12*I*b*x - 12*I*a)*cosh(b*x + a)
^3 + (6*I*b*x + 6*I*a)*cosh(b*x + a))*sinh(b*x + a) + 3*I*a)*log(-I*cosh(b*
x + a) - I*sinh(b*x + a) + 1) + 2*(5*(3*b*x + 1)*cosh(b*x + a)^4 + 6*b*x*co
sh(b*x + a)^2 + 3*b*x - 1)*sinh(b*x + a))/(b^2*cosh(b*x + a)^6 + 6*b^2*cosh
(b*x + a)*sinh(b*x + a)^5 + b^2*sinh(b*x + a)^6 + b^2*cosh(b*x + a)^4 + (15
*b^2*cosh(b*x + a)^2 + b^2)*sinh(b*x + a)^4 - b^2*cosh(b*x + a)^2 + 4*(5*b^
2*cosh(b*x + a)^3 + b^2*cosh(b*x + a))*sinh(b*x + a)^3 + (15*b^2*cosh(b*x +
 a)^4 + 6*b^2*cosh(b*x + a)^2 - b^2)*sinh(b*x + a)^2 - b^2 + 2*(3*b^2*cosh(
b*x + a)^5 + 2*b^2*cosh(b*x + a)^3 - b^2*cosh(b*x + a))*sinh(b*x + a))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x \operatorname{csch}(bx+a)^2 \operatorname{sech}(bx+a)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*csch(b*x+a)^2*sech(b*x+a)^3,x, algorithm="giac")
```

```
[Out] integrate(x*csch(b*x + a)^2*sech(b*x + a)^3, x)
```

maple [B] time = 0.71, size = 232, normalized size = 1.93

$$-\frac{e^{bx+a}\left(3bx\,e^{4bx+4a}+2bx\,e^{2bx+2a}+e^{4bx+4a}+3bx-1\right)}{b^{2}\left(1+e^{2bx+2a}\right)^{2}\left(e^{2bx+2a}-1\right)}+\frac{\ln\left(e^{bx+a}-1\right)}{b^{2}}-\frac{\ln\left(1+e^{bx+a}\right)}{b^{2}}+\frac{3a\arctan\left(e^{bx+a}\right)}{b^{2}}+\frac{3i\ln\left(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*csch(b*x+a)^2*sech(b*x+a)^3,x)
```

```
[Out] -exp(b*x+a)*(3*b*x*exp(4*b*x+4*a)+2*b*x*exp(2*b*x+2*a)+exp(4*b*x+4*a)+3*b*x
-1)/b^2/(1+exp(2*b*x+2*a))^2/(exp(2*b*x+2*a)-1)+1/b^2*ln(exp(b*x+a)-1)-1/b^
2*ln(1+exp(b*x+a))+3/b^2*a*arctan(exp(b*x+a))+3/2*I/b*ln(1+I*exp(b*x+a))*x+
3/2*I/b^2*ln(1+I*exp(b*x+a))*a-3/2*I/b*ln(1-I*exp(b*x+a))*x-3/2*I/b^2*ln(1-
I*exp(b*x+a))*a-3/2*I/b^2*dilog(1-I*exp(b*x+a))+3/2*I/b^2*dilog(1+I*exp(b*x
+a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{2\,bxe^{(3\,bx+3\,a)}+\left(3\,bxe^{(5\,a)}+e^{(5\,a)}\right)e^{(5\,bx)}+\left(3\,bxe^{a}-e^{a}\right)e^{(bx)}}{b^{2}e^{(6\,bx+6\,a)}+b^{2}e^{(4\,bx+4\,a)}-b^{2}e^{(2\,bx+2\,a)}-b^{2}}-\frac{\log\left(\left(e^{(bx+a)}+1\right)e^{(-a)}\right)}{b^{2}}+\frac{\log\left(\left(e^{(bx+a)}-1\right)e^{(-a)}\right)}{b^{2}}-96$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*csch(b*x+a)^2*sech(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -(2*b*x*e^(3*b*x + 3*a) + (3*b*x*e^(5*a) + e^(5*a))*e^(5*b*x) + (3*b*x*e^a
- e^a)*e^(b*x))/(b^2*e^(6*b*x + 6*a) + b^2*e^(4*b*x + 4*a) - b^2*e^(2*b*x +
 2*a) - b^2) - log((e^(b*x + a) + 1)*e^(-a))/b^2 + log((e^(b*x + a) - 1)*e^
(-a))/b^2 - 96*integrate(1/32*x*e^(b*x + a)/(e^(2*b*x + 2*a) + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x}{\cosh\left(a+b\,x\right)^{3}\,\sinh\left(a+b\,x\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(cosh(a + b*x)^3*sinh(a + b*x)^2),x)
```

```
[Out] int(x/(cosh(a + b*x)^3*sinh(a + b*x)^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\operatorname{csch}^{2}\left(a+bx\right)\operatorname{sech}^{3}\left(a+bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*csch(b*x+a)**2*sech(b*x+a)**3,x)
```

```
[Out] Integral(x*csch(a + b*x)**2*sech(a + b*x)**3, x)
```

3.504 $\int \operatorname{csch}^2(a+bx)\operatorname{sech}^3(a+bx)\,dx$

Optimal. Leaf size=49

$$-\frac{3\operatorname{csch}(a+bx)}{2b}-\frac{3\tan^{-1}(\sinh(a+bx))}{2b}+\frac{\operatorname{csch}(a+bx)\operatorname{sech}^2(a+bx)}{2b}$$

```
[Out] -3/2*arctan(sinh(b*x+a))/b-3/2*csch(b*x+a)/b+1/2*csch(b*x+a)*sech(b*x+a)^2/
b
```

Rubi [A] time = 0.04, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {2621, 288, 321, 207}

$$-\frac{3\operatorname{csch}(a+bx)}{2b}-\frac{3\tan^{-1}(\sinh(a+bx))}{2b}+\frac{\operatorname{csch}(a+bx)\operatorname{sech}^2(a+bx)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[a + b*x]^2*Sech[a + b*x]^3,x]
```

```
[Out] (-3*ArcTan[Sinh[a + b*x]])/(2*b) - (3*Csch[a + b*x])/(2*b) + (Csch[a + b*x]
*Sech[a + b*x]^2)/(2*b)
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2621

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> -Dist[(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rubi steps

$$\int \operatorname{csch}^2(a+bx)\operatorname{sech}^3(a+bx)\,dx = -\frac{i\,\text{Subst}\left(\int \frac{x^4}{\left(-1+x^2\right)^2}\,dx, x, -i\operatorname{csch}(a+bx)\right)}{b}$$
$$= \frac{\operatorname{csch}(a+bx)\operatorname{sech}^2(a+bx)}{2b} - \frac{(3i)\,\text{Subst}\left(\int \frac{x^2}{-1+x^2}\,dx, x, -i\operatorname{csch}(a+bx)\right)}{2b}$$
$$= -\frac{3\operatorname{csch}(a+bx)}{2b} + \frac{\operatorname{csch}(a+bx)\operatorname{sech}^2(a+bx)}{2b} - \frac{(3i)\,\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, -\right.}{2b}$$
$$= -\frac{3\tan^{-1}(\sinh(a+bx))}{2b} - \frac{3\operatorname{csch}(a+bx)}{2b} + \frac{\operatorname{csch}(a+bx)\operatorname{sech}^2(a+bx)}{2b}$$

Mathematica [C] time = 0.01, size = 29, normalized size = 0.59

$$-\frac{\operatorname{csch}(a+bx)\,{}_2F_1\left(-\frac{1}{2},2;\frac{1}{2};-\sinh^2(a+bx)\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[a + b*x]^2*Sech[a + b*x]^3,x]
```

```
[Out] -((Csch[a + b*x]*Hypergeometric2F1[-1/2, 2, 1/2, -Sinh[a + b*x]^2])/b)
```

fricas [B] time = 0.41, size = 511, normalized size = 10.43

$$-\frac{3\cosh(bx+a)^5 + 15\cosh(bx+a)\sinh(bx+a)^4 + 3\sinh(bx+a)^5 + 2\left(15\cosh(bx+a)^2+1\right)\sinh(bx+a}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^2*sech(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -(3*cosh(b*x + a)^5 + 15*cosh(b*x + a)*sinh(b*x + a)^4 + 3*sinh(b*x + a)^5
+ 2*(15*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^3 + 2*cosh(b*x + a)^3 + 6*(5*cos
```

```
h(b*x + a)^3 + cosh(b*x + a))*sinh(b*x + a)^2 + 3*(cosh(b*x + a)^6 + 6*cosh
(b*x + a)*sinh(b*x + a)^5 + sinh(b*x + a)^6 + (15*cosh(b*x + a)^2 + 1)*sinh
(b*x + a)^4 + cosh(b*x + a)^4 + 4*(5*cosh(b*x + a)^3 + cosh(b*x + a))*sinh(
b*x + a)^3 + (15*cosh(b*x + a)^4 + 6*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^2 -
 cosh(b*x + a)^2 + 2*(3*cosh(b*x + a)^5 + 2*cosh(b*x + a)^3 - cosh(b*x + a)
)*sinh(b*x + a) - 1)*arctan(cosh(b*x + a) + sinh(b*x + a)) + 3*(5*cosh(b*x
+ a)^4 + 2*cosh(b*x + a)^2 + 1)*sinh(b*x + a) + 3*cosh(b*x + a))/(b*cosh(b*
x + a)^6 + 6*b*cosh(b*x + a)*sinh(b*x + a)^5 + b*sinh(b*x + a)^6 + b*cosh(b
*x + a)^4 + (15*b*cosh(b*x + a)^2 + b)*sinh(b*x + a)^4 + 4*(5*b*cosh(b*x +
a)^3 + b*cosh(b*x + a))*sinh(b*x + a)^3 - b*cosh(b*x + a)^2 + (15*b*cosh(b*
x + a)^4 + 6*b*cosh(b*x + a)^2 - b)*sinh(b*x + a)^2 + 2*(3*b*cosh(b*x + a)^
5 + 2*b*cosh(b*x + a)^3 - b*cosh(b*x + a))*sinh(b*x + a) - b)
```

giac [B] time = 0.15, size = 102, normalized size = 2.08

$$-\frac{3\,\pi + \frac{4\left(3\left(e^{(bx+a)} - e^{(-bx-a)}\right)^{2} + 8\right)}{\left(e^{(bx+a)} - e^{(-bx-a)}\right)^{3} + 4\,e^{(bx+a)} - 4\,e^{(-bx-a)}} + 6\,\arctan\left(\frac{1}{2}\left(e^{(2\,bx+2\,a)} - 1\right)e^{(-bx-a)}\right)}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^2*sech(b*x+a)^3,x, algorithm="giac")
```

```
[Out] -1/4*(3*pi + 4*(3*(e^(b*x + a) - e^(-b*x - a))^2 + 8)/((e^(b*x + a) - e^(-b
*x - a))^3 + 4*e^(b*x + a) - 4*e^(-b*x - a)) + 6*arctan(1/2*(e^(2*b*x + 2*a
) - 1)*e^(-b*x - a)))/b
```

maple [A] time = 0.34, size = 52, normalized size = 1.06

$$-\frac{1}{b\sinh\left(bx+a\right)\cosh\left(bx+a\right)^{2}} - \frac{3\,\mathrm{sech}\left(bx+a\right)\tanh\left(bx+a\right)}{2b} - \frac{3\arctan\left(\mathrm{e}^{bx+a}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(b*x+a)^2*sech(b*x+a)^3,x)
```

```
[Out] -1/b/sinh(b*x+a)/cosh(b*x+a)^2-3/2*sech(b*x+a)*tanh(b*x+a)/b-3*arctan(exp(b
*x+a))/b
```

maxima [B] time = 0.41, size = 90, normalized size = 1.84

$$\frac{3\,\arctan\left(e^{(-bx-a)}\right)}{b} - \frac{3\,e^{(-bx-a)} + 2\,e^{(-3\,bx-3\,a)} + 3\,e^{(-5\,bx-5\,a)}}{b\left(e^{(-2\,bx-2\,a)} - e^{(-4\,bx-4\,a)} - e^{(-6\,bx-6\,a)} + 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^2*sech(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 3*arctan(e^(-b*x - a))/b - (3*e^(-b*x - a) + 2*e^(-3*b*x - 3*a) + 3*e^(-5*b
*x - 5*a))/(b*(e^(-2*b*x - 2*a) - e^(-4*b*x - 4*a) - e^(-6*b*x - 6*a) + 1))
```

mupad [B] time = 0.08, size = 107, normalized size = 2.18

$$\frac{2\,e^{a+b\,x}}{b\,\left(2\,e^{2\,a+2\,b\,x}+e^{4\,a+4\,b\,x}+1\right)}-\frac{3\,\mathrm{atan}\left(\frac{e^{b\,x}\,e^{a}\,\sqrt{b^2}}{b}\right)}{\sqrt{b^2}}-\frac{2\,e^{a+b\,x}}{b\,\left(e^{2\,a+2\,b\,x}-1\right)}-\frac{e^{a+b\,x}}{b\,\left(e^{2\,a+2\,b\,x}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(a + b*x)^3*sinh(a + b*x)^2),x)
```

```
[Out] (2*exp(a + b*x))/(b*(2*exp(2*a + 2*b*x) + exp(4*a + 4*b*x) + 1)) - (3*atan(
(exp(b*x)*exp(a)*(b^2)^(1/2))/b))/(b^2)^(1/2) - (2*exp(a + b*x))/(b*(exp(2*
a + 2*b*x) - 1)) - exp(a + b*x)/(b*(exp(2*a + 2*b*x) + 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{csch}^{2}{\left(a + bx \right)} \operatorname{sech}^{3}{\left(a + bx \right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)**2*sech(b*x+a)**3,x)
```

```
[Out] Integral(csch(a + b*x)**2*sech(a + b*x)**3, x)
```

3.505 $\int \frac{\text{csch}^2(a+bx)\text{sech}^3(a+bx)}{x}\,dx$

Optimal. Leaf size=23

$$\text{Int}\left(\frac{\text{csch}^2(a+bx)\text{sech}^3(a+bx)}{x},x\right)$$

```
[Out] CannotIntegrate(csch(b*x+a)^2*sech(b*x+a)^3/x,x)
```

Rubi [A] time = 0.24, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\text{csch}^2(a+bx)\text{sech}^3(a+bx)}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Csch[a + b*x]^2*Sech[a + b*x]^3)/x,x]
```

```
[Out] Defer[Int][(Csch[a + b*x]^2*Sech[a + b*x]^3)/x, x]
```

Rubi steps

$$\int \frac{\text{csch}^2(a+bx)\text{sech}^3(a+bx)}{x}\,dx = \int \frac{\text{csch}^2(a+bx)\text{sech}^3(a+bx)}{x}\,dx$$

Mathematica [A] time = 43.97, size = 0, normalized size = 0.00

$$\int \frac{\text{csch}^2(a+bx)\text{sech}^3(a+bx)}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Csch[a + b*x]^2*Sech[a + b*x]^3)/x,x]
```

```
[Out] Integrate[(Csch[a + b*x]^2*Sech[a + b*x]^3)/x, x]
```

fricas [A] time = 0.41, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\text{csch}\,(bx+a)^2\,\text{sech}\,(bx+a)^3}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^2*sech(b*x+a)^3/x,x, algorithm="fricas")
```

```
[Out] integral(csch(b*x + a)^2*sech(b*x + a)^3/x, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}(bx+a)^2 \operatorname{sech}(bx+a)^3}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^2*sech(b*x+a)^3/x,x, algorithm="giac")
```

```
[Out] integrate(csch(b*x + a)^2*sech(b*x + a)^3/x, x)
```

maple [A] time = 1.24, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}(bx+a)^2 \operatorname{sech}(bx+a)^3}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(b*x+a)^2*sech(b*x+a)^3/x,x)
```

```
[Out] int(csch(b*x+a)^2*sech(b*x+a)^3/x,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{2\,bxe^{(3\,bx+3\,a)} + \left(3\,bxe^{(5\,a)} - e^{(5\,a)}\right)e^{(5\,bx)} + \left(3\,bxe^{a} + e^{a}\right)e^{(bx)}}{b^2x^2e^{(6\,bx+6\,a)} + b^2x^2e^{(4\,bx+4\,a)} - b^2x^2e^{(2\,bx+2\,a)} - b^2x^2} - 32\int \frac{\left(3\,b^2x^2e^{a} - 2\,e^{a}\right)e^{(bx)}}{32\left(b^2x^3e^{(2\,bx+2\,a)} + b^2x^3\right)}\,dx - 32\int \frac{}{32\left(bx^2e\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^2*sech(b*x+a)^3/x,x, algorithm="maxima")
```

```
[Out] -(2*b*x*e^(3*b*x + 3*a) + (3*b*x*e^(5*a) - e^(5*a))*e^(5*b*x) + (3*b*x*e^a
+ e^a)*e^(b*x))/(b^2*x^2*e^(6*b*x + 6*a) + b^2*x^2*e^(4*b*x + 4*a) - b^2*x^
2*e^(2*b*x + 2*a) - b^2*x^2) - 32*integrate(1/32*(3*b^2*x^2*e^a - 2*e^a)*e^
(b*x)/(b^2*x^3*e^(2*b*x + 2*a) + b^2*x^3), x) - 32*integrate(1/32/(b*x^2*e^
(b*x + a) + b*x^2), x) - 32*integrate(1/32/(b*x^2*e^(b*x + a) - b*x^2), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{x\cosh\left(a+b\,x\right)^3\,\sinh\left(a+b\,x\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*cosh(a + b*x)^3*sinh(a + b*x)^2),x)
```

```
[Out] int(1/(x*cosh(a + b*x)^3*sinh(a + b*x)^2), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}^2(a+bx)\operatorname{sech}^3(a+bx)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)**2*sech(b*x+a)**3/x,x)
```

```
[Out] Integral(csch(a + b*x)**2*sech(a + b*x)**3/x, x)
```

3.506 $\int \frac{\text{csch}^2(a+bx)\text{sech}^3(a+bx)}{x^2}\,dx$

Optimal. Leaf size=23

$$\text{Int}\left(\frac{\text{csch}^2(a+bx)\text{sech}^3(a+bx)}{x^2},x\right)$$

```
[Out] CannotIntegrate(csch(b*x+a)^2*sech(b*x+a)^3/x^2,x)
```

Rubi [A] time = 0.30, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\text{csch}^2(a+bx)\text{sech}^3(a+bx)}{x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Csch[a + b*x]^2*Sech[a + b*x]^3)/x^2,x]
```

```
[Out] Defer[Int][(Csch[a + b*x]^2*Sech[a + b*x]^3)/x^2, x]
```

Rubi steps

$$\int \frac{\text{csch}^2(a+bx)\text{sech}^3(a+bx)}{x^2}\,dx=\int \frac{\text{csch}^2(a+bx)\text{sech}^3(a+bx)}{x^2}\,dx$$

Mathematica [A] time = 35.32, size = 0, normalized size = 0.00

$$\int \frac{\text{csch}^2(a+bx)\text{sech}^3(a+bx)}{x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Csch[a + b*x]^2*Sech[a + b*x]^3)/x^2,x]
```

```
[Out] Integrate[(Csch[a + b*x]^2*Sech[a + b*x]^3)/x^2, x]
```

fricas [A] time = 0.40, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\text{csch}\,(bx+a)^2\,\text{sech}\,(bx+a)^3}{x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^2*sech(b*x+a)^3/x^2,x, algorithm="fricas")
```

```
[Out] integral(csch(b*x + a)^2*sech(b*x + a)^3/x^2, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}(bx+a)^2 \operatorname{sech}(bx+a)^3}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^2*sech(b*x+a)^3/x^2,x, algorithm="giac")
```

```
[Out] integrate(csch(b*x + a)^2*sech(b*x + a)^3/x^2, x)
```

maple [A] time = 1.36, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}(bx+a)^2 \operatorname{sech}(bx+a)^3}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(b*x+a)^2*sech(b*x+a)^3/x^2,x)
```

```
[Out] int(csch(b*x+a)^2*sech(b*x+a)^3/x^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{2\,bxe^{(3\,bx+3\,a)} + \left(3\,bxe^{(5\,a)} - 2\,e^{(5\,a)}\right)e^{(5\,bx)} + (3\,bxe^{a} + 2\,e^{a})e^{(bx)}}{b^2x^3e^{(6\,bx+6\,a)} + b^2x^3e^{(4\,bx+4\,a)} - b^2x^3e^{(2\,bx+2\,a)} - b^2x^3} - 32\int \frac{3\left(b^2x^2e^{a} - 2\,e^{a}\right)e^{(bx)}}{32\left(b^2x^4e^{(2\,bx+2\,a)} + b^2x^4\right)}\,dx - 32\int \frac{}{16\left(bx\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^2*sech(b*x+a)^3/x^2,x, algorithm="maxima")
```

```
[Out] -(2*b*x*e^(3*b*x + 3*a) + (3*b*x*e^(5*a) - 2*e^(5*a))*e^(5*b*x) + (3*b*x*e^
a + 2*e^a)*e^(b*x))/(b^2*x^3*e^(6*b*x + 6*a) + b^2*x^3*e^(4*b*x + 4*a) - b^
2*x^3*e^(2*b*x + 2*a) - b^2*x^3) - 32*integrate(3/32*(b^2*x^2*e^a - 2*e^a)*
e^(b*x)/(b^2*x^4*e^(2*b*x + 2*a) + b^2*x^4), x) - 32*integrate(1/16/(b*x^3*
e^(b*x + a) + b*x^3), x) - 32*integrate(1/16/(b*x^3*e^(b*x + a) - b*x^3), x
)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{x^2\cosh(a+b\,x)^3\sinh(a+b\,x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*cosh(a + b*x)^3*sinh(a + b*x)^2),x)
```

```
[Out] int(1/(x^2*cosh(a + b*x)^3*sinh(a + b*x)^2), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}^2(a+bx)\operatorname{sech}^3(a+bx)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)**2*sech(b*x+a)**3/x**2,x)
```

```
[Out] Integral(csch(a + b*x)**2*sech(a + b*x)**3/x**2, x)
```

3.507 $\int x^m \mathrm{csch}^3(a+bx)\mathrm{sech}(a+bx)\,dx$

Optimal. Leaf size=21

$$\mathrm{Int}\left(x^m\mathrm{csch}^3(a+bx)\mathrm{sech}(a+bx),x\right)$$

```
[Out] CannotIntegrate(x^m*csch(b*x+a)^3*sech(b*x+a),x)
```

Rubi [A] time = 0.45, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int x^m\mathrm{csch}^3(a+bx)\mathrm{sech}(a+bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x^m*Csch[a + b*x]^3*Sech[a + b*x],x]
[Out] Defer[Int][x^m*Csch[a + b*x]^3*Sech[a + b*x], x]
```

Rubi steps

$$\int x^m\mathrm{csch}^3(a+bx)\mathrm{sech}(a+bx)\,dx=\int x^m\mathrm{csch}^3(a+bx)\mathrm{sech}(a+bx)\,dx$$

Mathematica [A] time = 69.58, size = 0, normalized size = 0.00

$$\int x^m\mathrm{csch}^3(a+bx)\mathrm{sech}(a+bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^m*Csch[a + b*x]^3*Sech[a + b*x],x]
[Out] Integrate[x^m*Csch[a + b*x]^3*Sech[a + b*x], x]
```

fricas [A] time = 0.40, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(x^m\,\mathrm{csch}\,(bx+a)^3\,\mathrm{sech}\,(bx+a)\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*csch(b*x+a)^3*sech(b*x+a),x, algorithm="fricas")
[Out] integral(x^m*csch(b*x + a)^3*sech(b*x + a), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int x^m \operatorname{csch}(bx+a)^3 \operatorname{sech}(bx+a)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*csch(b*x+a)^3*sech(b*x+a),x, algorithm="giac")
[Out] integrate(x^m*csch(b*x + a)^3*sech(b*x + a), x)
```

maple [A] time = 0.23, size = 0, normalized size = 0.00

$$\int x^m \operatorname{csch}(bx+a)^3 \operatorname{sech}(bx+a)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*csch(b*x+a)^3*sech(b*x+a),x)
[Out] int(x^m*csch(b*x+a)^3*sech(b*x+a),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int x^m \operatorname{csch}(bx+a)^3 \operatorname{sech}(bx+a)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*csch(b*x+a)^3*sech(b*x+a),x, algorithm="maxima")
[Out] integrate(x^m*csch(b*x + a)^3*sech(b*x + a), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{x^m}{\cosh(a+b\,x)\,\sinh(a+b\,x)^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(cosh(a + b*x)*sinh(a + b*x)^3),x)
[Out] int(x^m/(cosh(a + b*x)*sinh(a + b*x)^3), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int x^m \operatorname{csch}^3(a+bx) \operatorname{sech}(a+bx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*csch(b*x+a)**3*sech(b*x+a),x)
[Out] Integral(x**m*csch(a + b*x)**3*sech(a + b*x), x)
```

3.508 $\int x^3\text{csch}^3(a+bx)\text{sech}(a+bx)\,dx$

Optimal. Leaf size=240

$$\frac{3\text{Li}_2\left(e^{2(a+bx)}\right)}{2b^4}+\frac{3\text{Li}_4\left(-e^{2a+2bx}\right)}{4b^4}-\frac{3\text{Li}_4\left(e^{2a+2bx}\right)}{4b^4}-\frac{3x\text{Li}_3\left(-e^{2a+2bx}\right)}{2b^3}+\frac{3x\text{Li}_3\left(e^{2a+2bx}\right)}{2b^3}+\frac{3x\log\left(1-e^{2(a+bx)}\right)}{b^3}+\frac{3x^2\text{Li}_2}{}$$

```
[Out] -3/2*x^2/b^2+1/2*x^3/b+2*x^3*arctanh(exp(2*b*x+2*a))/b-3/2*x^2*coth(b*x+a)/
b^2-1/2*x^3*coth(b*x+a)^2/b+3*x*ln(1-exp(2*b*x+2*a))/b^3+3/2*polylog(2,exp(
2*b*x+2*a))/b^4+3/2*x^2*polylog(2,-exp(2*b*x+2*a))/b^2-3/2*x^2*polylog(2,ex
p(2*b*x+2*a))/b^2-3/2*x*polylog(3,-exp(2*b*x+2*a))/b^3+3/2*x*polylog(3,exp(
2*b*x+2*a))/b^3+3/4*polylog(4,-exp(2*b*x+2*a))/b^4-3/4*polylog(4,exp(2*b*x+
2*a))/b^4
```

Rubi [A] time = 0.42, antiderivative size = 240, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 16, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.889, Rules used = {2620, 14, 5462, 3720, 3716, 2190, 2279, 2391, 30, 2551, 12, 4182, 2531, 6609, 2282, 6589}

$$\frac{3x^2\text{PolyLog}\left(2,-e^{2a+2bx}\right)}{2b^2}-\frac{3x^2\text{PolyLog}\left(2,e^{2a+2bx}\right)}{2b^2}-\frac{3x\text{PolyLog}\left(3,-e^{2a+2bx}\right)}{2b^3}+\frac{3x\text{PolyLog}\left(3,e^{2a+2bx}\right)}{2b^3}+\frac{3\text{PolyL}}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Csch[a + b*x]^3*Sech[a + b*x],x]
```

```
[Out] (-3*x^2)/(2*b^2) + x^3/(2*b) + (2*x^3*ArcTanh[E^(2*a + 2*b*x)])/b - (3*x^2*
Coth[a + b*x])/(2*b^2) - (x^3*Coth[a + b*x]^2)/(2*b) + (3*x*Log[1 - E^(2*(a
 + b*x))])/b^3 + (3*PolyLog[2, E^(2*(a + b*x))])/(2*b^4) + (3*x^2*PolyLog[2
, -E^(2*a + 2*b*x)])/(2*b^2) - (3*x^2*PolyLog[2, E^(2*a + 2*b*x)])/(2*b^2)
- (3*x*PolyLog[3, -E^(2*a + 2*b*x)])/(2*b^3) + (3*x*PolyLog[3, E^(2*a + 2*b
*x)])/(2*b^3) + (3*PolyLog[4, -E^(2*a + 2*b*x)])/(4*b^4) - (3*PolyLog[4, E^
(2*a + 2*b*x)])/(4*b^4)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2551

```
Int[Log[u_]*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[((a + b*x)^(m + 1
)*Log[u])/(b*(m + 1)), x] - Dist[1/(b*(m + 1)), Int[SimplifyIntegrand[((a +
 b*x)^(m + 1)*D[u, x])/u, x], x], x] /; FreeQ[{a, b, m}, x] && InverseFunct
```

```
ionFreeQ[u, x] && NeQ[m, -1]
```

Rule 2620

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 3716

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2
*I, Int[((c + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(E^(2*I*k*Pi)*(1 + E^(2*(-(I*
e) + f*fz*x))/E^(2*I*k*Pi))), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Integ
erQ[4*k] && IGtQ[m, 0]
```

Rule 3720

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[(b*(c + d*x)^m*(b*Tan[e + f*x])^(n - 1))/(f*(n - 1)), x] + (-Di
st[(b*d*m)/(f*(n - 1)), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x],
 x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[
{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5462

```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> With[{u = IntHide[Csch[a + b*x]^n*Sech[a +
b*x]^p, x]}, Dist[(c + d*x)^m, u, x] - Dist[d*m, Int[(c + d*x)^(m - 1)*u, x
], x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n,
p]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
```

```
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int x^3\text{csch}^3(a+bx)\text{sech}(a+bx)\,dx = -\frac{x^3\coth^2(a+bx)}{2b} - \frac{x^3\log(\tanh(a+bx))}{b} - 3\int x^2\left(-\frac{\coth^2(a+bx)}{2b} - \text{lo}\right.$$

$$= -\frac{x^3\coth^2(a+bx)}{2b} - \frac{x^3\log(\tanh(a+bx))}{b} - 3\int\left(-\frac{x^2\coth^2(a+bx)}{2b} - \frac{x^2}{}\right.$$

$$= -\frac{x^3\coth^2(a+bx)}{2b} - \frac{x^3\log(\tanh(a+bx))}{b} + \frac{3\int x^2\coth^2(a+bx)\,dx}{2b} + \frac{3}{}$$

$$= -\frac{3x^2\coth(a+bx)}{2b^2} - \frac{x^3\coth^2(a+bx)}{2b} + \frac{3\int x\coth(a+bx)\,dx}{b^2} - \frac{\int 2bx^3\text{c}}{}$$

$$= -\frac{3x^2}{2b^2} + \frac{x^3}{2b} - \frac{3x^2\coth(a+bx)}{2b^2} - \frac{x^3\coth^2(a+bx)}{2b} - 2\int x^3\text{csch}(2a+2b$$

$$= -\frac{3x^2}{2b^2} + \frac{x^3}{2b} + \frac{2x^3\tanh^{-1}\left(e^{2a+2bx}\right)}{b} - \frac{3x^2\coth(a+bx)}{2b^2} - \frac{x^3\coth^2(a+bx)}{2b}$$

$$= -\frac{3x^2}{2b^2} + \frac{x^3}{2b} + \frac{2x^3\tanh^{-1}\left(e^{2a+2bx}\right)}{b} - \frac{3x^2\coth(a+bx)}{2b^2} - \frac{x^3\coth^2(a+bx)}{2b}$$

$$= -\frac{3x^2}{2b^2} + \frac{x^3}{2b} + \frac{2x^3\tanh^{-1}\left(e^{2a+2bx}\right)}{b} - \frac{3x^2\coth(a+bx)}{2b^2} - \frac{x^3\coth^2(a+bx)}{2b}$$

$$= -\frac{3x^2}{2b^2} + \frac{x^3}{2b} + \frac{2x^3\tanh^{-1}\left(e^{2a+2bx}\right)}{b} - \frac{3x^2\coth(a+bx)}{2b^2} - \frac{x^3\coth^2(a+bx)}{2b}$$

$$= -\frac{3x^2}{2b^2} + \frac{x^3}{2b} + \frac{2x^3\tanh^{-1}\left(e^{2a+2bx}\right)}{b} - \frac{3x^2\coth(a+bx)}{2b^2} - \frac{x^3\coth^2(a+bx)}{2b}$$

Mathematica [B] time = 7.56, size = 490, normalized size = 2.04

$$\frac{3x^2\text{csch}(a)\sinh(bx)\text{csch}(a+bx)}{2b^2} - \frac{3\left(2b^2x^2\text{Li}_2\left(-e^{-2(a+bx)}\right) + 2bx\text{Li}_3\left(-e^{-2(a+bx)}\right) + \text{Li}_4\left(-e^{-2(a+bx)}\right)\right)}{4b^4} + \frac{e^{2a}\left(e^{-2a}b\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Csch[a + b*x]^3*Sech[a + b*x],x]
```

```
[Out] x^4/(2 + 2*E^(2*a)) - (x^3*Csch[a + b*x]^2)/(2*b) + (x^3*Log[1 + E^(-2*(a +
 b*x))])/b + (E^(2*a)*((-6*b^2*x^2)/E^(2*a) + (b^4*x^4)/E^(2*a) + 6*b*(1 -
E^(-2*a))*x*Log[1 - E^(-a - b*x)] - (2*b^3*(-1 + E^(2*a))*x^3*Log[1 - E^(-a
 - b*x)])/E^(2*a) + 6*b*(1 - E^(-2*a))*x*Log[1 + E^(-a - b*x)] - (2*b^3*(-1
 + E^(2*a))*x^3*Log[1 + E^(-a - b*x)])/E^(2*a) - 6*(1 - E^(-2*a))*PolyLog[2
, -E^(-a - b*x)] - 6*(1 - E^(-2*a))*PolyLog[2, E^(-a - b*x)] + 6*(1 - E^(-2
*a))*(b^2*x^2*PolyLog[2, -E^(-a - b*x)] + 2*(b*x*PolyLog[3, -E^(-a - b*x)]
+ PolyLog[4, -E^(-a - b*x)])) + 6*(1 - E^(-2*a))*(b^2*x^2*PolyLog[2, E^(-a
- b*x)] + 2*(b*x*PolyLog[3, E^(-a - b*x)] + PolyLog[4, E^(-a - b*x)]))))/(2
*b^4*(-1 + E^(2*a))) - (3*(2*b^2*x^2*PolyLog[2, -E^(-2*(a + b*x))] + 2*b*x*
PolyLog[3, -E^(-2*(a + b*x))] + PolyLog[4, -E^(-2*(a + b*x))]))/(4*b^4) - (
x^4*Csch[a]*Sech[a])/4 + (3*x^2*Csch[a]*Csch[a + b*x]*Sinh[b*x])/(2*b^2)
```

fricas [C] time = 0.52, size = 3394, normalized size = 14.14

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*csch(b*x+a)^3*sech(b*x+a),x, algorithm="fricas")
```

```
[Out] -(3*(b^2*x^2 - a^2)*cosh(b*x + a)^4 + 12*(b^2*x^2 - a^2)*cosh(b*x + a)*sinh
(b*x + a)^3 + 3*(b^2*x^2 - a^2)*sinh(b*x + a)^4 + (2*b^3*x^3 - 3*b^2*x^2 +
6*a^2)*cosh(b*x + a)^2 + (2*b^3*x^3 - 3*b^2*x^2 + 18*(b^2*x^2 - a^2)*cosh(b
*x + a)^2 + 6*a^2)*sinh(b*x + a)^2 - 3*a^2 + 3*((b^2*x^2 - 1)*cosh(b*x + a)
^4 + 4*(b^2*x^2 - 1)*cosh(b*x + a)*sinh(b*x + a)^3 + (b^2*x^2 - 1)*sinh(b*x
 + a)^4 + b^2*x^2 - 2*(b^2*x^2 - 1)*cosh(b*x + a)^2 - 2*(b^2*x^2 - 3*(b^2*x
^2 - 1)*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^2 + 4*((b^2*x^2 - 1)*cosh(b*x +
a)^3 - (b^2*x^2 - 1)*cosh(b*x + a))*sinh(b*x + a) - 1)*dilog(cosh(b*x + a)
+ sinh(b*x + a)) - 3*(b^2*x^2*cosh(b*x + a)^4 + 4*b^2*x^2*cosh(b*x + a)*sin
h(b*x + a)^3 + b^2*x^2*sinh(b*x + a)^4 - 2*b^2*x^2*cosh(b*x + a)^2 + b^2*x^
2 + 2*(3*b^2*x^2*cosh(b*x + a)^2 - b^2*x^2)*sinh(b*x + a)^2 + 4*(b^2*x^2*co
sh(b*x + a)^3 - b^2*x^2*cosh(b*x + a))*sinh(b*x + a))*dilog(I*cosh(b*x + a)
 + I*sinh(b*x + a)) - 3*(b^2*x^2*cosh(b*x + a)^4 + 4*b^2*x^2*cosh(b*x + a)*
sinh(b*x + a)^3 + b^2*x^2*sinh(b*x + a)^4 - 2*b^2*x^2*cosh(b*x + a)^2 + b^2
*x^2 + 2*(3*b^2*x^2*cosh(b*x + a)^2 - b^2*x^2)*sinh(b*x + a)^2 + 4*(b^2*x^2
*cosh(b*x + a)^3 - b^2*x^2*cosh(b*x + a))*sinh(b*x + a))*dilog(-I*cosh(b*x
+ a) - I*sinh(b*x + a)) + 3*((b^2*x^2 - 1)*cosh(b*x + a)^4 + 4*(b^2*x^2 - 1
)*cosh(b*x + a)*sinh(b*x + a)^3 + (b^2*x^2 - 1)*sinh(b*x + a)^4 + b^2*x^2 -
 2*(b^2*x^2 - 1)*cosh(b*x + a)^2 - 2*(b^2*x^2 - 3*(b^2*x^2 - 1)*cosh(b*x +
a)^2 - 1)*sinh(b*x + a)^2 + 4*((b^2*x^2 - 1)*cosh(b*x + a)^3 - (b^2*x^2 - 1
)*cosh(b*x + a))*sinh(b*x + a) - 1)*dilog(-cosh(b*x + a) - sinh(b*x + a)) +
```

```
(b^3*x^3 + (b^3*x^3 - 3*b*x)*cosh(b*x + a)^4 + 4*(b^3*x^3 - 3*b*x)*cosh(b*
x + a)*sinh(b*x + a)^3 + (b^3*x^3 - 3*b*x)*sinh(b*x + a)^4 - 2*(b^3*x^3 - 3
*b*x)*cosh(b*x + a)^2 - 2*(b^3*x^3 - 3*(b^3*x^3 - 3*b*x)*cosh(b*x + a)^2 -
3*b*x)*sinh(b*x + a)^2 - 3*b*x + 4*((b^3*x^3 - 3*b*x)*cosh(b*x + a)^3 - (b^
3*x^3 - 3*b*x)*cosh(b*x + a))*sinh(b*x + a))*log(cosh(b*x + a) + sinh(b*x +
 a) + 1) + (a^3*cosh(b*x + a)^4 + 4*a^3*cosh(b*x + a)*sinh(b*x + a)^3 + a^3
*sinh(b*x + a)^4 - 2*a^3*cosh(b*x + a)^2 + a^3 + 2*(3*a^3*cosh(b*x + a)^2 -
 a^3)*sinh(b*x + a)^2 + 4*(a^3*cosh(b*x + a)^3 - a^3*cosh(b*x + a))*sinh(b*
x + a))*log(cosh(b*x + a) + sinh(b*x + a) + I) + (a^3*cosh(b*x + a)^4 + 4*a
^3*cosh(b*x + a)*sinh(b*x + a)^3 + a^3*sinh(b*x + a)^4 - 2*a^3*cosh(b*x + a
)^2 + a^3 + 2*(3*a^3*cosh(b*x + a)^2 - a^3)*sinh(b*x + a)^2 + 4*(a^3*cosh(b
*x + a)^3 - a^3*cosh(b*x + a))*sinh(b*x + a))*log(cosh(b*x + a) + sinh(b*x
+ a) - I) - ((a^3 - 3*a)*cosh(b*x + a)^4 + 4*(a^3 - 3*a)*cosh(b*x + a)*sinh
(b*x + a)^3 + (a^3 - 3*a)*sinh(b*x + a)^4 + a^3 - 2*(a^3 - 3*a)*cosh(b*x +
a)^2 - 2*(a^3 - 3*(a^3 - 3*a)*cosh(b*x + a)^2 - 3*a)*sinh(b*x + a)^2 + 4*((
a^3 - 3*a)*cosh(b*x + a)^3 - (a^3 - 3*a)*cosh(b*x + a))*sinh(b*x + a) - 3*a
)*log(cosh(b*x + a) + sinh(b*x + a) - 1) - (b^3*x^3 + (b^3*x^3 + a^3)*cosh(
b*x + a)^4 + 4*(b^3*x^3 + a^3)*cosh(b*x + a)*sinh(b*x + a)^3 + (b^3*x^3 + a
^3)*sinh(b*x + a)^4 + a^3 - 2*(b^3*x^3 + a^3)*cosh(b*x + a)^2 - 2*(b^3*x^3
+ a^3 - 3*(b^3*x^3 + a^3)*cosh(b*x + a)^2)*sinh(b*x + a)^2 + 4*((b^3*x^3 +
a^3)*cosh(b*x + a)^3 - (b^3*x^3 + a^3)*cosh(b*x + a))*sinh(b*x + a))*log(I*
cosh(b*x + a) + I*sinh(b*x + a) + 1) - (b^3*x^3 + (b^3*x^3 + a^3)*cosh(b*x
+ a)^4 + 4*(b^3*x^3 + a^3)*cosh(b*x + a)*sinh(b*x + a)^3 + (b^3*x^3 + a^3)*
sinh(b*x + a)^4 + a^3 - 2*(b^3*x^3 + a^3)*cosh(b*x + a)^2 - 2*(b^3*x^3 + a^
3 - 3*(b^3*x^3 + a^3)*cosh(b*x + a)^2)*sinh(b*x + a)^2 + 4*((b^3*x^3 + a^3)
*cosh(b*x + a)^3 - (b^3*x^3 + a^3)*cosh(b*x + a))*sinh(b*x + a))*log(-I*cos
h(b*x + a) - I*sinh(b*x + a) + 1) + (b^3*x^3 + (b^3*x^3 + a^3 - 3*b*x - 3*a
)*cosh(b*x + a)^4 + 4*(b^3*x^3 + a^3 - 3*b*x - 3*a)*cosh(b*x + a)*sinh(b*x
+ a)^3 + (b^3*x^3 + a^3 - 3*b*x - 3*a)*sinh(b*x + a)^4 + a^3 - 2*(b^3*x^3 +
 a^3 - 3*b*x - 3*a)*cosh(b*x + a)^2 - 2*(b^3*x^3 + a^3 - 3*(b^3*x^3 + a^3 -
 3*b*x - 3*a)*cosh(b*x + a)^2 - 3*b*x - 3*a)*sinh(b*x + a)^2 - 3*b*x + 4*((
b^3*x^3 + a^3 - 3*b*x - 3*a)*cosh(b*x + a)^3 - (b^3*x^3 + a^3 - 3*b*x - 3*a
)*cosh(b*x + a))*sinh(b*x + a) - 3*a)*log(-cosh(b*x + a) - sinh(b*x + a) +
1) + 6*(cosh(b*x + a)^4 + 4*cosh(b*x + a)*sinh(b*x + a)^3 + sinh(b*x + a)^4
 + 2*(3*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^2 - 2*cosh(b*x + a)^2 + 4*(cosh(
b*x + a)^3 - cosh(b*x + a))*sinh(b*x + a) + 1)*polylog(4, cosh(b*x + a) + s
inh(b*x + a)) - 6*(cosh(b*x + a)^4 + 4*cosh(b*x + a)*sinh(b*x + a)^3 + sinh
(b*x + a)^4 + 2*(3*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^2 - 2*cosh(b*x + a)^2
 + 4*(cosh(b*x + a)^3 - cosh(b*x + a))*sinh(b*x + a) + 1)*polylog(4, I*cosh
(b*x + a) + I*sinh(b*x + a)) - 6*(cosh(b*x + a)^4 + 4*cosh(b*x + a)*sinh(b*
x + a)^3 + sinh(b*x + a)^4 + 2*(3*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^2 - 2*
cosh(b*x + a)^2 + 4*(cosh(b*x + a)^3 - cosh(b*x + a))*sinh(b*x + a) + 1)*po
lylog(4, -I*cosh(b*x + a) - I*sinh(b*x + a)) + 6*(cosh(b*x + a)^4 + 4*cosh(
b*x + a)*sinh(b*x + a)^3 + sinh(b*x + a)^4 + 2*(3*cosh(b*x + a)^2 - 1)*sinh
(b*x + a)^2 - 2*cosh(b*x + a)^2 + 4*(cosh(b*x + a)^3 - cosh(b*x + a))*sinh(
```

```
b*x + a) + 1)*polylog(4, -cosh(b*x + a) - sinh(b*x + a)) - 6*(b*x*cosh(b*x
+ a)^4 + 4*b*x*cosh(b*x + a)*sinh(b*x + a)^3 + b*x*sinh(b*x + a)^4 - 2*b*x*
cosh(b*x + a)^2 + 2*(3*b*x*cosh(b*x + a)^2 - b*x)*sinh(b*x + a)^2 + b*x + 4
*(b*x*cosh(b*x + a)^3 - b*x*cosh(b*x + a))*sinh(b*x + a))*polylog(3, cosh(b
*x + a) + sinh(b*x + a)) + 6*(b*x*cosh(b*x + a)^4 + 4*b*x*cosh(b*x + a)*sin
h(b*x + a)^3 + b*x*sinh(b*x + a)^4 - 2*b*x*cosh(b*x + a)^2 + 2*(3*b*x*cosh(
b*x + a)^2 - b*x)*sinh(b*x + a)^2 + b*x + 4*(b*x*cosh(b*x + a)^3 - b*x*cosh
(b*x + a))*sinh(b*x + a))*polylog(3, I*cosh(b*x + a) + I*sinh(b*x + a)) + 6
*(b*x*cosh(b*x + a)^4 + 4*b*x*cosh(b*x + a)*sinh(b*x + a)^3 + b*x*sinh(b*x
+ a)^4 - 2*b*x*cosh(b*x + a)^2 + 2*(3*b*x*cosh(b*x + a)^2 - b*x)*sinh(b*x +
 a)^2 + b*x + 4*(b*x*cosh(b*x + a)^3 - b*x*cosh(b*x + a))*sinh(b*x + a))*po
lylog(3, -I*cosh(b*x + a) - I*sinh(b*x + a)) - 6*(b*x*cosh(b*x + a)^4 + 4*b
*x*cosh(b*x + a)*sinh(b*x + a)^3 + b*x*sinh(b*x + a)^4 - 2*b*x*cosh(b*x + a
)^2 + 2*(3*b*x*cosh(b*x + a)^2 - b*x)*sinh(b*x + a)^2 + b*x + 4*(b*x*cosh(b
*x + a)^3 - b*x*cosh(b*x + a))*sinh(b*x + a))*polylog(3, -cosh(b*x + a) - s
inh(b*x + a)) + 2*(6*(b^2*x^2 - a^2)*cosh(b*x + a)^3 + (2*b^3*x^3 - 3*b^2*x
^2 + 6*a^2)*cosh(b*x + a))*sinh(b*x + a))/(b^4*cosh(b*x + a)^4 + 4*b^4*cosh
(b*x + a)*sinh(b*x + a)^3 + b^4*sinh(b*x + a)^4 - 2*b^4*cosh(b*x + a)^2 + b
^4 + 2*(3*b^4*cosh(b*x + a)^2 - b^4)*sinh(b*x + a)^2 + 4*(b^4*cosh(b*x + a)
^3 - b^4*cosh(b*x + a))*sinh(b*x + a))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3 \operatorname{csch}(bx+a)^3 \operatorname{sech}(bx+a)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*csch(b*x+a)^3*sech(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(x^3*csch(b*x + a)^3*sech(b*x + a), x)
```

maple [A] time = 0.52, size = 417, normalized size = 1.74

$$-\frac{x^2\left(2bx\,e^{2bx+2a}+3\,e^{2bx+2a}-3\right)}{b^2\left(e^{2bx+2a}-1\right)^2}-\frac{\ln\left(1-e^{bx+a}\right)x^3}{b}-\frac{\ln\left(1-e^{bx+a}\right)a^3}{b^4}-\frac{3x^2\operatorname{polylog}\left(2,e^{bx+a}\right)}{b^2}-\frac{3x^2\operatorname{polylog}\left(2,-e\right.}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*csch(b*x+a)^3*sech(b*x+a),x)
```

```
[Out] -x^2*(2*b*x*exp(2*b*x+2*a)+3*exp(2*b*x+2*a)-3)/b^2/(exp(2*b*x+2*a)-1)^2-1/b
*ln(1-exp(b*x+a))*x^3-1/b^4*ln(1-exp(b*x+a))*a^3+3/2*x^2*polylog(2,-exp(2*b
*x+2*a))/b^2-3/2*x*polylog(3,-exp(2*b*x+2*a))/b^3+3/4*polylog(4,-exp(2*b*x+
2*a))/b^4-6*polylog(4,-exp(b*x+a))/b^4-6*polylog(4,exp(b*x+a))/b^4-3*x^2*po
lylog(2,-exp(b*x+a))/b^2-3*x^2*polylog(2,exp(b*x+a))/b^2+6*x*polylog(3,-exp
```

```
(b*x+a))/b^3+6*x*polylog(3,exp(b*x+a))/b^3-3*x^2/b^2+x^3*ln(1+exp(2*b*x+2*a
))/b+3*polylog(2,-exp(b*x+a))/b^4+3*polylog(2,exp(b*x+a))/b^4+3/b^3*ln(1-ex
p(b*x+a))*x+3/b^4*ln(1-exp(b*x+a))*a+3/b^3*ln(1+exp(b*x+a))*x+6/b^4*a*ln(ex
p(b*x+a))-3/b^4*a*ln(exp(b*x+a)-1)-3/b^4*a^2-1/b*ln(1+exp(b*x+a))*x^3-6*a*x
/b^3+1/b^4*a^3*ln(exp(b*x+a)-1)
```

maxima [A] time = 0.35, size = 352, normalized size = 1.47

$$-\frac{1}{2}x^4+\frac{3x^2-\left(2bx^3e^{(2a)}+3x^2e^{(2a)}\right)e^{(2bx)}}{b^2e^{(4bx+4a)}-2b^2e^{(2bx+2a)}+b^2}+\frac{b^4x^4-6b^2x^2}{2b^4}+\frac{4b^3x^3\log\left(e^{(2bx+2a)}+1\right)+6b^2x^2\mathrm{Li}_2\left(-e^{(2bx+2a)}\right)-}{3b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*csch(b*x+a)^3*sech(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/2*x^4 + (3*x^2 - (2*b*x^3*e^(2*a) + 3*x^2*e^(2*a))*e^(2*b*x))/(b^2*e^(4*
b*x + 4*a) - 2*b^2*e^(2*b*x + 2*a) + b^2) + 1/2*(b^4*x^4 - 6*b^2*x^2)/b^4 +
 1/3*(4*b^3*x^3*log(e^(2*b*x + 2*a) + 1) + 6*b^2*x^2*dilog(-e^(2*b*x + 2*a)
) - 6*b*x*polylog(3, -e^(2*b*x + 2*a)) + 3*polylog(4, -e^(2*b*x + 2*a)))/b^
4 - (b^3*x^3*log(e^(b*x + a) + 1) + 3*b^2*x^2*dilog(-e^(b*x + a)) - 6*b*x*p
olylog(3, -e^(b*x + a)) + 6*polylog(4, -e^(b*x + a)))/b^4 - (b^3*x^3*log(-e
^(b*x + a) + 1) + 3*b^2*x^2*dilog(e^(b*x + a)) - 6*b*x*polylog(3, e^(b*x +
a)) + 6*polylog(4, e^(b*x + a)))/b^4 + 3*(b*x*log(e^(b*x + a) + 1) + dilog(
-e^(b*x + a)))/b^4 + 3*(b*x*log(-e^(b*x + a) + 1) + dilog(e^(b*x + a)))/b^4
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int\frac{x^3}{\cosh\left(a+b\,x\right)\,{\sinh\left(a+b\,x\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(cosh(a + b*x)*sinh(a + b*x)^3),x)
```

```
[Out] int(x^3/(cosh(a + b*x)*sinh(a + b*x)^3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3\operatorname{csch}^3\left(a+bx\right)\operatorname{sech}\left(a+bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*csch(b*x+a)**3*sech(b*x+a),x)
```

```
[Out] Integral(x**3*csch(a + b*x)**3*sech(a + b*x), x)
```

3.509 $\int x^2\mathrm{csch}^3(a+bx)\mathrm{sech}(a+bx)\,dx$

Optimal. Leaf size=148

$$-\frac{\mathrm{Li}_3\left(-e^{2a+2bx}\right)}{2b^3}+\frac{\mathrm{Li}_3\left(e^{2a+2bx}\right)}{2b^3}+\frac{\log(\sinh(a+bx))}{b^3}+\frac{x\mathrm{Li}_2\left(-e^{2a+2bx}\right)}{b^2}-\frac{x\mathrm{Li}_2\left(e^{2a+2bx}\right)}{b^2}-\frac{x\coth(a+bx)}{b^2}+\frac{2x^2\tanh^{-1}}{b}$$

```
[Out] 1/2*x^2/b+2*x^2*arctanh(exp(2*b*x+2*a))/b-x*coth(b*x+a)/b^2-1/2*x^2*coth(b*
x+a)^2/b+ln(sinh(b*x+a))/b^3+x*polylog(2,-exp(2*b*x+2*a))/b^2-x*polylog(2,e
xp(2*b*x+2*a))/b^2-1/2*polylog(3,-exp(2*b*x+2*a))/b^3+1/2*polylog(3,exp(2*b
*x+2*a))/b^3
```

Rubi [A] time = 0.23, antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 12, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {2620, 14, 5462, 3720, 3475, 30, 2551, 12, 4182, 2531, 2282, 6589}

$$\frac{x\mathrm{PolyLog}\left(2,-e^{2a+2bx}\right)}{b^2}-\frac{x\mathrm{PolyLog}\left(2,e^{2a+2bx}\right)}{b^2}-\frac{\mathrm{PolyLog}\left(3,-e^{2a+2bx}\right)}{2b^3}+\frac{\mathrm{PolyLog}\left(3,e^{2a+2bx}\right)}{2b^3}-\frac{x\coth(a+bx)}{b^2}+$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Csch[a + b*x]^3*Sech[a + b*x],x]
```

```
[Out] x^2/(2*b) + (2*x^2*ArcTanh[E^(2*a + 2*b*x)])/b - (x*Coth[a + b*x])/b^2 - (x
^2*Coth[a + b*x]^2)/(2*b) + Log[Sinh[a + b*x]]/b^3 + (x*PolyLog[2, -E^(2*a
+ 2*b*x)])/b^2 - (x*PolyLog[2, E^(2*a + 2*b*x)])/b^2 - PolyLog[3, -E^(2*a +
 2*b*x)]/(2*b^3) + PolyLog[3, E^(2*a + 2*b*x)]/(2*b^3)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2551

```
Int[Log[u_]*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[((a + b*x)^(m + 1
)*Log[u])/(b*(m + 1)), x] - Dist[1/(b*(m + 1)), Int[SimplifyIntegrand[((a +
 b*x)^(m + 1)*D[u, x])/u, x], x], x] /; FreeQ[{a, b, m}, x] && InverseFunct
ionFreeQ[u, x] && NeQ[m, -1]
```

Rule 2620

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3720

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[(b*(c + d*x)^m*(b*Tan[e + f*x])^(n - 1))/(f*(n - 1)), x] + (-Di
st[(b*d*m)/(f*(n - 1)), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x],
 x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[
{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
```

```
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5462

```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> With[{u = IntHide[Csch[a + b*x]^n*Sech[a +
b*x]^p, x]}, Dist[(c + d*x)^m, u, x] - Dist[d*m, Int[(c + d*x)^(m - 1)*u, x
], x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n,
p]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int x^2\text{csch}^3(a+bx)\text{sech}(a+bx)\,dx = -\frac{x^2\coth^2(a+bx)}{2b} - \frac{x^2\log(\tanh(a+bx))}{b} - 2\int x\left(-\frac{\coth^2(a+bx)}{2b} - \text{log}\right.$$

$$= -\frac{x^2\coth^2(a+bx)}{2b} - \frac{x^2\log(\tanh(a+bx))}{b} - 2\int\left(-\frac{x\coth^2(a+bx)}{2b} - \frac{x\,\text{l}}{}\right.$$

$$= -\frac{x^2\coth^2(a+bx)}{2b} - \frac{x^2\log(\tanh(a+bx))}{b} + \frac{\int x\coth^2(a+bx)\,dx}{b} + \frac{2\int x}{}$$

$$= -\frac{x\coth(a+bx)}{b^2} - \frac{x^2\coth^2(a+bx)}{2b} + \frac{\int\coth(a+bx)\,dx}{b^2} + \frac{\int x\,dx}{b} - \frac{\int 2b}{}$$

$$= \frac{x^2}{2b} - \frac{x\coth(a+bx)}{b^2} - \frac{x^2\coth^2(a+bx)}{2b} + \frac{\log(\sinh(a+bx))}{b^3} - 2\int x^2\text{csc}$$

$$= \frac{x^2}{2b} + \frac{2x^2\tanh^{-1}\left(e^{2a+2bx}\right)}{b} - \frac{x\coth(a+bx)}{b^2} - \frac{x^2\coth^2(a+bx)}{2b} + \frac{\log(\text{sin}}{}$$

$$= \frac{x^2}{2b} + \frac{2x^2\tanh^{-1}\left(e^{2a+2bx}\right)}{b} - \frac{x\coth(a+bx)}{b^2} - \frac{x^2\coth^2(a+bx)}{2b} + \frac{\log(\text{sin}}{}$$

$$= \frac{x^2}{2b} + \frac{2x^2\tanh^{-1}\left(e^{2a+2bx}\right)}{b} - \frac{x\coth(a+bx)}{b^2} - \frac{x^2\coth^2(a+bx)}{2b} + \frac{\log(\text{sin}}{}$$

$$= \frac{x^2}{2b} + \frac{2x^2\tanh^{-1}\left(e^{2a+2bx}\right)}{b} - \frac{x\coth(a+bx)}{b^2} - \frac{x^2\coth^2(a+bx)}{2b} + \frac{\log(\text{sin}}{}$$

Mathematica [B] time = 4.47, size = 369, normalized size = 2.49

$$\frac{1}{6}\left(\frac{6x\text{csch}(a)\sinh(bx)\text{csch}(a+bx)}{b^2} + \frac{2b^2x^2\left(\frac{2bx}{e^{2a}+1} + 3\log\left(e^{-2(a+bx)}+1\right)\right) - 6bx\text{Li}_2\left(-e^{-2(a+bx)}\right) - 3\text{Li}_3\left(-e^{-2(a+}\right.}{b^3}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Csch[a + b*x]^3*Sech[a + b*x],x]
```

```
[Out] ((-3*x^2*Csch[a + b*x]^2)/b - (2*E^(2*a)*((6*b*x)/E^(2*a) - (2*b^3*x^3)/E^(
2*a) + (3*b^2*(-1 + E^(2*a))*x^2*Log[1 - E^(-a - b*x)])/E^(2*a) + (3*b^2*(-
1 + E^(2*a))*x^2*Log[1 + E^(-a - b*x)])/E^(2*a) + 3*(1 - E^(-2*a))*(b*x - L
og[1 - E^(a + b*x)]) + 3*(1 - E^(-2*a))*(b*x - Log[1 + E^(a + b*x)]) - 6*(1
 - E^(-2*a))*(b*x*PolyLog[2, -E^(-a - b*x)] + PolyLog[3, -E^(-a - b*x)]) -
6*(1 - E^(-2*a))*(b*x*PolyLog[2, E^(-a - b*x)] + PolyLog[3, E^(-a - b*x)]))
)/(b^3*(-1 + E^(2*a))) + (2*b^2*x^2*((2*b*x)/(1 + E^(2*a)) + 3*Log[1 + E^(-
2*(a + b*x))]) - 6*b*x*PolyLog[2, -E^(-2*(a + b*x))] - 3*PolyLog[3, -E^(-2*
```

```
(a + b*x))])/b^3 - 2*x^3*Csch[a]*Sech[a] + (6*x*Csch[a]*Csch[a + b*x]*Sinh[
b*x])/b^2)/6
```

fricas [C] time = 0.49, size = 2562, normalized size = 17.31

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*csch(b*x+a)^3*sech(b*x+a),x, algorithm="fricas")
```

```
[Out] -(2*(b*x + a)*cosh(b*x + a)^4 + 8*(b*x + a)*cosh(b*x + a)*sinh(b*x + a)^3 +
 2*(b*x + a)*sinh(b*x + a)^4 + 2*(b^2*x^2 - b*x - 2*a)*cosh(b*x + a)^2 + 2*
(b^2*x^2 + 6*(b*x + a)*cosh(b*x + a)^2 - b*x - 2*a)*sinh(b*x + a)^2 + 2*(b*
x*cosh(b*x + a)^4 + 4*b*x*cosh(b*x + a)*sinh(b*x + a)^3 + b*x*sinh(b*x + a)
^4 - 2*b*x*cosh(b*x + a)^2 + 2*(3*b*x*cosh(b*x + a)^2 - b*x)*sinh(b*x + a)^
2 + b*x + 4*(b*x*cosh(b*x + a)^3 - b*x*cosh(b*x + a))*sinh(b*x + a))*dilog(
cosh(b*x + a) + sinh(b*x + a)) - 2*(b*x*cosh(b*x + a)^4 + 4*b*x*cosh(b*x +
a)*sinh(b*x + a)^3 + b*x*sinh(b*x + a)^4 - 2*b*x*cosh(b*x + a)^2 + 2*(3*b*x
*cosh(b*x + a)^2 - b*x)*sinh(b*x + a)^2 + b*x + 4*(b*x*cosh(b*x + a)^3 - b*
x*cosh(b*x + a))*sinh(b*x + a))*dilog(I*cosh(b*x + a) + I*sinh(b*x + a)) -
2*(b*x*cosh(b*x + a)^4 + 4*b*x*cosh(b*x + a)*sinh(b*x + a)^3 + b*x*sinh(b*x
 + a)^4 - 2*b*x*cosh(b*x + a)^2 + 2*(3*b*x*cosh(b*x + a)^2 - b*x)*sinh(b*x
+ a)^2 + b*x + 4*(b*x*cosh(b*x + a)^3 - b*x*cosh(b*x + a))*sinh(b*x + a))*d
ilog(-I*cosh(b*x + a) - I*sinh(b*x + a)) + 2*(b*x*cosh(b*x + a)^4 + 4*b*x*c
osh(b*x + a)*sinh(b*x + a)^3 + b*x*sinh(b*x + a)^4 - 2*b*x*cosh(b*x + a)^2
+ 2*(3*b*x*cosh(b*x + a)^2 - b*x)*sinh(b*x + a)^2 + b*x + 4*(b*x*cosh(b*x +
 a)^3 - b*x*cosh(b*x + a))*sinh(b*x + a))*dilog(-cosh(b*x + a) - sinh(b*x +
 a)) + ((b^2*x^2 - 1)*cosh(b*x + a)^4 + 4*(b^2*x^2 - 1)*cosh(b*x + a)*sinh(
b*x + a)^3 + (b^2*x^2 - 1)*sinh(b*x + a)^4 + b^2*x^2 - 2*(b^2*x^2 - 1)*cosh
(b*x + a)^2 - 2*(b^2*x^2 - 3*(b^2*x^2 - 1)*cosh(b*x + a)^2 - 1)*sinh(b*x +
a)^2 + 4*((b^2*x^2 - 1)*cosh(b*x + a)^3 - (b^2*x^2 - 1)*cosh(b*x + a))*sinh
(b*x + a) - 1)*log(cosh(b*x + a) + sinh(b*x + a) + 1) - (a^2*cosh(b*x + a)^
4 + 4*a^2*cosh(b*x + a)*sinh(b*x + a)^3 + a^2*sinh(b*x + a)^4 - 2*a^2*cosh(
b*x + a)^2 + 2*(3*a^2*cosh(b*x + a)^2 - a^2)*sinh(b*x + a)^2 + a^2 + 4*(a^2
*cosh(b*x + a)^3 - a^2*cosh(b*x + a))*sinh(b*x + a))*log(cosh(b*x + a) + si
nh(b*x + a) + I) - (a^2*cosh(b*x + a)^4 + 4*a^2*cosh(b*x + a)*sinh(b*x + a)
^3 + a^2*sinh(b*x + a)^4 - 2*a^2*cosh(b*x + a)^2 + 2*(3*a^2*cosh(b*x + a)^2
 - a^2)*sinh(b*x + a)^2 + a^2 + 4*(a^2*cosh(b*x + a)^3 - a^2*cosh(b*x + a))
*sinh(b*x + a))*log(cosh(b*x + a) + sinh(b*x + a) - I) + ((a^2 - 1)*cosh(b*
x + a)^4 + 4*(a^2 - 1)*cosh(b*x + a)*sinh(b*x + a)^3 + (a^2 - 1)*sinh(b*x +
 a)^4 - 2*(a^2 - 1)*cosh(b*x + a)^2 + 2*(3*(a^2 - 1)*cosh(b*x + a)^2 - a^2
+ 1)*sinh(b*x + a)^2 + a^2 + 4*((a^2 - 1)*cosh(b*x + a)^3 - (a^2 - 1)*cosh(
b*x + a))*sinh(b*x + a) - 1)*log(cosh(b*x + a) + sinh(b*x + a) - 1) - ((b^2
*x^2 - a^2)*cosh(b*x + a)^4 + 4*(b^2*x^2 - a^2)*cosh(b*x + a)*sinh(b*x + a)
^3 + (b^2*x^2 - a^2)*sinh(b*x + a)^4 + b^2*x^2 - 2*(b^2*x^2 - a^2)*cosh(b*x
```

```
 + a)^2 - 2*(b^2*x^2 - 3*(b^2*x^2 - a^2)*cosh(b*x + a)^2 - a^2)*sinh(b*x +
a)^2 - a^2 + 4*((b^2*x^2 - a^2)*cosh(b*x + a)^3 - (b^2*x^2 - a^2)*cosh(b*x
+ a))*sinh(b*x + a))*log(I*cosh(b*x + a) + I*sinh(b*x + a) + 1) - ((b^2*x^2
 - a^2)*cosh(b*x + a)^4 + 4*(b^2*x^2 - a^2)*cosh(b*x + a)*sinh(b*x + a)^3 +
 (b^2*x^2 - a^2)*sinh(b*x + a)^4 + b^2*x^2 - 2*(b^2*x^2 - a^2)*cosh(b*x + a
)^2 - 2*(b^2*x^2 - 3*(b^2*x^2 - a^2)*cosh(b*x + a)^2 - a^2)*sinh(b*x + a)^2
 - a^2 + 4*((b^2*x^2 - a^2)*cosh(b*x + a)^3 - (b^2*x^2 - a^2)*cosh(b*x + a)
)*sinh(b*x + a))*log(-I*cosh(b*x + a) - I*sinh(b*x + a) + 1) + ((b^2*x^2 -
a^2)*cosh(b*x + a)^4 + 4*(b^2*x^2 - a^2)*cosh(b*x + a)*sinh(b*x + a)^3 + (b
^2*x^2 - a^2)*sinh(b*x + a)^4 + b^2*x^2 - 2*(b^2*x^2 - a^2)*cosh(b*x + a)^2
 - 2*(b^2*x^2 - 3*(b^2*x^2 - a^2)*cosh(b*x + a)^2 - a^2)*sinh(b*x + a)^2 -
a^2 + 4*((b^2*x^2 - a^2)*cosh(b*x + a)^3 - (b^2*x^2 - a^2)*cosh(b*x + a))*s
inh(b*x + a))*log(-cosh(b*x + a) - sinh(b*x + a) + 1) - 2*(cosh(b*x + a)^4
+ 4*cosh(b*x + a)*sinh(b*x + a)^3 + sinh(b*x + a)^4 + 2*(3*cosh(b*x + a)^2
- 1)*sinh(b*x + a)^2 - 2*cosh(b*x + a)^2 + 4*(cosh(b*x + a)^3 - cosh(b*x +
a))*sinh(b*x + a) + 1)*polylog(3, cosh(b*x + a) + sinh(b*x + a)) + 2*(cosh(
b*x + a)^4 + 4*cosh(b*x + a)*sinh(b*x + a)^3 + sinh(b*x + a)^4 + 2*(3*cosh(
b*x + a)^2 - 1)*sinh(b*x + a)^2 - 2*cosh(b*x + a)^2 + 4*(cosh(b*x + a)^3 -
cosh(b*x + a))*sinh(b*x + a) + 1)*polylog(3, I*cosh(b*x + a) + I*sinh(b*x +
 a)) + 2*(cosh(b*x + a)^4 + 4*cosh(b*x + a)*sinh(b*x + a)^3 + sinh(b*x + a)
^4 + 2*(3*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^2 - 2*cosh(b*x + a)^2 + 4*(cos
h(b*x + a)^3 - cosh(b*x + a))*sinh(b*x + a) + 1)*polylog(3, -I*cosh(b*x + a
) - I*sinh(b*x + a)) - 2*(cosh(b*x + a)^4 + 4*cosh(b*x + a)*sinh(b*x + a)^3
 + sinh(b*x + a)^4 + 2*(3*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^2 - 2*cosh(b*x
 + a)^2 + 4*(cosh(b*x + a)^3 - cosh(b*x + a))*sinh(b*x + a) + 1)*polylog(3,
 -cosh(b*x + a) - sinh(b*x + a)) + 4*(2*(b*x + a)*cosh(b*x + a)^3 + (b^2*x^
2 - b*x - 2*a)*cosh(b*x + a))*sinh(b*x + a) + 2*a)/(b^3*cosh(b*x + a)^4 + 4
*b^3*cosh(b*x + a)*sinh(b*x + a)^3 + b^3*sinh(b*x + a)^4 - 2*b^3*cosh(b*x +
 a)^2 + b^3 + 2*(3*b^3*cosh(b*x + a)^2 - b^3)*sinh(b*x + a)^2 + 4*(b^3*cosh
(b*x + a)^3 - b^3*cosh(b*x + a))*sinh(b*x + a))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \operatorname{csch}(bx+a)^3 \operatorname{sech}(bx+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*csch(b*x+a)^3*sech(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(x^2*csch(b*x + a)^3*sech(b*x + a), x)
```

maple [A] time = 0.52, size = 266, normalized size = 1.80

$$-\frac{2x\left(bx\,e^{2bx+2a}+e^{2bx+2a}-1\right)}{b^2\left(e^{2bx+2a}-1\right)^2}-\frac{2\ln\left(e^{bx+a}\right)}{b^3}+\frac{\ln\left(e^{bx+a}-1\right)}{b^3}+\frac{\ln\left(1+e^{bx+a}\right)}{b^3}+\frac{2\,\mathrm{polylog}\left(3,-e^{bx+a}\right)}{b^3}-\frac{\mathrm{polylog}\left(3\right.}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*csch(b*x+a)^3*sech(b*x+a),x)
```

```
[Out] -2*x*(b*x*exp(2*b*x+2*a)+exp(2*b*x+2*a)-1)/b^2/(exp(2*b*x+2*a)-1)^2-2/b^3*l
n(exp(b*x+a))+1/b^3*ln(exp(b*x+a)-1)+1/b^3*ln(1+exp(b*x+a))+2/b^3*polylog(3
,-exp(b*x+a))-1/2*polylog(3,-exp(2*b*x+2*a))/b^3+2/b^3*polylog(3,exp(b*x+a)
)+x^2*ln(1+exp(2*b*x+2*a))/b+x*polylog(2,-exp(2*b*x+2*a))/b^2-1/b*ln(1-exp(
b*x+a))*x^2-2/b^2*polylog(2,exp(b*x+a))*x-1/b*ln(1+exp(b*x+a))*x^2-2/b^2*po
lylog(2,-exp(b*x+a))*x+1/b^3*ln(1-exp(b*x+a))*a^2-1/b^3*a^2*ln(exp(b*x+a)-1
)
```

maxima [A] time = 0.36, size = 243, normalized size = 1.64

$$-\frac{2\left(\left(bx^2e^{(2a)}+xe^{(2a)}\right)e^{(2bx)}-x\right)}{b^2e^{(4bx+4a)}-2b^2e^{(2bx+2a)}+b^2}-\frac{2x}{b^2}+\frac{2b^2x^2\log\left(e^{(2bx+2a)}+1\right)+2bx\mathrm{Li}_2\left(-e^{(2bx+2a)}\right)-\mathrm{Li}_3(-e^{(2bx+2a)})}{2b^3}-\frac{b^2x^2\,\mathrm{l}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*csch(b*x+a)^3*sech(b*x+a),x, algorithm="maxima")
```

```
[Out] -2*((b*x^2*e^(2*a) + x*e^(2*a))*e^(2*b*x) - x)/(b^2*e^(4*b*x + 4*a) - 2*b^2
*e^(2*b*x + 2*a) + b^2) - 2*x/b^2 + 1/2*(2*b^2*x^2*log(e^(2*b*x + 2*a) + 1)
 + 2*b*x*dilog(-e^(2*b*x + 2*a)) - polylog(3, -e^(2*b*x + 2*a)))/b^3 - (b^2
*x^2*log(e^(b*x + a) + 1) + 2*b*x*dilog(-e^(b*x + a)) - 2*polylog(3, -e^(b*
x + a)))/b^3 - (b^2*x^2*log(-e^(b*x + a) + 1) + 2*b*x*dilog(e^(b*x + a)) -
2*polylog(3, e^(b*x + a)))/b^3 + log(e^(b*x + a) + 1)/b^3 + log(e^(b*x + a)
 - 1)/b^3
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2}{\cosh(a+b\,x)\,\sinh(a+b\,x)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(cosh(a + b*x)*sinh(a + b*x)^3),x)
```

```
[Out] int(x^2/(cosh(a + b*x)*sinh(a + b*x)^3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2\operatorname{csch}^3(a+bx)\operatorname{sech}(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*csch(b*x+a)**3*sech(b*x+a),x)
```

```
[Out] Integral(x**2*csch(a + b*x)**3*sech(a + b*x), x)
```

3.510 $\int x\text{csch}^3(a+bx)\text{sech}(a+bx)\,dx$

Optimal. Leaf size=95

$$\frac{\text{Li}_2\left(-e^{2a+2bx}\right)}{2b^2}-\frac{\text{Li}_2\left(e^{2a+2bx}\right)}{2b^2}-\frac{\coth(a+bx)}{2b^2}+\frac{2x\tanh^{-1}\left(e^{2a+2bx}\right)}{b}-\frac{x\coth^2(a+bx)}{2b}+\frac{x}{2b}$$

```
[Out] 1/2*x/b+2*x*arctanh(exp(2*b*x+2*a))/b-1/2*coth(b*x+a)/b^2-1/2*x*coth(b*x+a)
^2/b+1/2*polylog(2,-exp(2*b*x+2*a))/b^2-1/2*polylog(2,exp(2*b*x+2*a))/b^2
```

Rubi [A] time = 0.12, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {2620, 14, 5462, 3473, 8, 2548, 12, 4182, 2279, 2391}

$$\frac{\text{PolyLog}\left(2,-e^{2a+2bx}\right)}{2b^2}-\frac{\text{PolyLog}\left(2,e^{2a+2bx}\right)}{2b^2}-\frac{\coth(a+bx)}{2b^2}+\frac{2x\tanh^{-1}\left(e^{2a+2bx}\right)}{b}-\frac{x\coth^2(a+bx)}{2b}+\frac{x}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[x*Csch[a + b*x]^3*Sech[a + b*x],x]
```

```
[Out] x/(2*b) + (2*x*ArcTanh[E^(2*a + 2*b*x)])/b - Coth[a + b*x]/(2*b^2) - (x*Cot
h[a + b*x]^2)/(2*b) + PolyLog[2, -E^(2*a + 2*b*x)]/(2*b^2) - PolyLog[2, E^(
2*a + 2*b*x)]/(2*b^2)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2548

```
Int[Log[u_], x_Symbol] :> Simp[x*Log[u], x] - Int[SimplifyIntegrand[(x*D[u,
 x])/u, x], x] /; InverseFunctionFreeQ[u, x]
```

Rule 2620

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5462

```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> With[{u = IntHide[Csch[a + b*x]^n*Sech[a +
b*x]^p, x]}, Dist[(c + d*x)^m, u, x] - Dist[d*m, Int[(c + d*x)^(m - 1)*u, x
], x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n,
p]
```

Rubi steps

$$\int x\text{csch}^3(a+bx)\text{sech}(a+bx)\,dx = -\frac{x\coth^2(a+bx)}{2b} - \frac{x\log(\tanh(a+bx))}{b} - \int\left(-\frac{\coth^2(a+bx)}{2b} - \frac{\log(\tanh}{b}\right.$$

$$= -\frac{x\coth^2(a+bx)}{2b} - \frac{x\log(\tanh(a+bx))}{b} + \frac{\int\coth^2(a+bx)\,dx}{2b} + \frac{\int\log(\tan}{}$$

$$= -\frac{\coth(a+bx)}{2b^2} - \frac{x\coth^2(a+bx)}{2b} + \frac{\int 1\,dx}{2b} - \frac{\int 2bx\text{csch}(2a+2bx)\,dx}{b}$$

$$= \frac{x}{2b} - \frac{\coth(a+bx)}{2b^2} - \frac{x\coth^2(a+bx)}{2b} - 2\int x\text{csch}(2a+2bx)\,dx$$

$$= \frac{x}{2b} + \frac{2x\tanh^{-1}\left(e^{2a+2bx}\right)}{b} - \frac{\coth(a+bx)}{2b^2} - \frac{x\coth^2(a+bx)}{2b} + \frac{\int\log\left(1-e\right.}{b}$$

$$= \frac{x}{2b} + \frac{2x\tanh^{-1}\left(e^{2a+2bx}\right)}{b} - \frac{\coth(a+bx)}{2b^2} - \frac{x\coth^2(a+bx)}{2b} + \frac{\text{Subst}\left(\int\frac{\log}{}\right.}{}$$

$$= \frac{x}{2b} + \frac{2x\tanh^{-1}\left(e^{2a+2bx}\right)}{b} - \frac{\coth(a+bx)}{2b^2} - \frac{x\coth^2(a+bx)}{2b} + \frac{\text{Li}_2\left(-e^{2a+2b}\right.}{2b^2}$$

Mathematica [A] time = 0.68, size = 137, normalized size = 1.44

$$-\frac{\text{Li}_2\left(-e^{-2(a+bx)}\right) - \text{Li}_2\left(e^{-2(a+bx)}\right) + 2a\log\left(1-e^{-2(a+bx)}\right) + 2bx\log\left(1-e^{-2(a+bx)}\right) - 2a\log\left(e^{-2(a+bx)}+1\right) - 2bx}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Csch[a + b*x]^3*Sech[a + b*x],x]
```

```
[Out] -1/2*(Coth[a + b*x] + b*x*Csch[a + b*x]^2 + 2*a*Log[1 - E^(-2*(a + b*x))] +
 2*b*x*Log[1 - E^(-2*(a + b*x))] - 2*a*Log[1 + E^(-2*(a + b*x))] - 2*b*x*Lo
g[1 + E^(-2*(a + b*x))] + 2*a*Log[Cosh[a + b*x]] - 2*a*Log[Sinh[a + b*x]] +
 PolyLog[2, -E^(-2*(a + b*x))] - PolyLog[2, E^(-2*(a + b*x))])/b^2
```

fricas [C] time = 0.47, size = 1578, normalized size = 16.61

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*csch(b*x+a)^3*sech(b*x+a),x, algorithm="fricas")
```

```
[Out] -((2*b*x + 1)*cosh(b*x + a)^2 + 2*(2*b*x + 1)*cosh(b*x + a)*sinh(b*x + a) +
 (2*b*x + 1)*sinh(b*x + a)^2 + (cosh(b*x + a)^4 + 4*cosh(b*x + a)*sinh(b*x
+ a)^3 + sinh(b*x + a)^4 + 2*(3*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^2 - 2*co
```

```
sh(b*x + a)^2 + 4*(cosh(b*x + a)^3 - cosh(b*x + a))*sinh(b*x + a) + 1)*dilo
g(cosh(b*x + a) + sinh(b*x + a)) - (cosh(b*x + a)^4 + 4*cosh(b*x + a)*sinh(
b*x + a)^3 + sinh(b*x + a)^4 + 2*(3*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^2 -
2*cosh(b*x + a)^2 + 4*(cosh(b*x + a)^3 - cosh(b*x + a))*sinh(b*x + a) + 1)*
dilog(I*cosh(b*x + a) + I*sinh(b*x + a)) - (cosh(b*x + a)^4 + 4*cosh(b*x +
a)*sinh(b*x + a)^3 + sinh(b*x + a)^4 + 2*(3*cosh(b*x + a)^2 - 1)*sinh(b*x +
 a)^2 - 2*cosh(b*x + a)^2 + 4*(cosh(b*x + a)^3 - cosh(b*x + a))*sinh(b*x +
a) + 1)*dilog(-I*cosh(b*x + a) - I*sinh(b*x + a)) + (cosh(b*x + a)^4 + 4*co
sh(b*x + a)*sinh(b*x + a)^3 + sinh(b*x + a)^4 + 2*(3*cosh(b*x + a)^2 - 1)*s
inh(b*x + a)^2 - 2*cosh(b*x + a)^2 + 4*(cosh(b*x + a)^3 - cosh(b*x + a))*si
nh(b*x + a) + 1)*dilog(-cosh(b*x + a) - sinh(b*x + a)) + (b*x*cosh(b*x + a)
^4 + 4*b*x*cosh(b*x + a)*sinh(b*x + a)^3 + b*x*sinh(b*x + a)^4 - 2*b*x*cosh
(b*x + a)^2 + 2*(3*b*x*cosh(b*x + a)^2 - b*x)*sinh(b*x + a)^2 + b*x + 4*(b*
x*cosh(b*x + a)^3 - b*x*cosh(b*x + a))*sinh(b*x + a))*log(cosh(b*x + a) + s
inh(b*x + a) + 1) + (a*cosh(b*x + a)^4 + 4*a*cosh(b*x + a)*sinh(b*x + a)^3
+ a*sinh(b*x + a)^4 - 2*a*cosh(b*x + a)^2 + 2*(3*a*cosh(b*x + a)^2 - a)*sin
h(b*x + a)^2 + 4*(a*cosh(b*x + a)^3 - a*cosh(b*x + a))*sinh(b*x + a) + a)*l
og(cosh(b*x + a) + sinh(b*x + a) + I) + (a*cosh(b*x + a)^4 + 4*a*cosh(b*x +
 a)*sinh(b*x + a)^3 + a*sinh(b*x + a)^4 - 2*a*cosh(b*x + a)^2 + 2*(3*a*cosh
(b*x + a)^2 - a)*sinh(b*x + a)^2 + 4*(a*cosh(b*x + a)^3 - a*cosh(b*x + a))*
sinh(b*x + a) + a)*log(cosh(b*x + a) + sinh(b*x + a) - I) - (a*cosh(b*x + a
)^4 + 4*a*cosh(b*x + a)*sinh(b*x + a)^3 + a*sinh(b*x + a)^4 - 2*a*cosh(b*x
+ a)^2 + 2*(3*a*cosh(b*x + a)^2 - a)*sinh(b*x + a)^2 + 4*(a*cosh(b*x + a)^3
 - a*cosh(b*x + a))*sinh(b*x + a) + a)*log(cosh(b*x + a) + sinh(b*x + a) -
1) - ((b*x + a)*cosh(b*x + a)^4 + 4*(b*x + a)*cosh(b*x + a)*sinh(b*x + a)^3
 + (b*x + a)*sinh(b*x + a)^4 - 2*(b*x + a)*cosh(b*x + a)^2 + 2*(3*(b*x + a)
*cosh(b*x + a)^2 - b*x - a)*sinh(b*x + a)^2 + b*x + 4*((b*x + a)*cosh(b*x +
 a)^3 - (b*x + a)*cosh(b*x + a))*sinh(b*x + a) + a)*log(I*cosh(b*x + a) + I
*sinh(b*x + a) + 1) - ((b*x + a)*cosh(b*x + a)^4 + 4*(b*x + a)*cosh(b*x + a
)*sinh(b*x + a)^3 + (b*x + a)*sinh(b*x + a)^4 - 2*(b*x + a)*cosh(b*x + a)^2
 + 2*(3*(b*x + a)*cosh(b*x + a)^2 - b*x - a)*sinh(b*x + a)^2 + b*x + 4*((b*
x + a)*cosh(b*x + a)^3 - (b*x + a)*cosh(b*x + a))*sinh(b*x + a) + a)*log(-I
*cosh(b*x + a) - I*sinh(b*x + a) + 1) + ((b*x + a)*cosh(b*x + a)^4 + 4*(b*x
 + a)*cosh(b*x + a)*sinh(b*x + a)^3 + (b*x + a)*sinh(b*x + a)^4 - 2*(b*x +
a)*cosh(b*x + a)^2 + 2*(3*(b*x + a)*cosh(b*x + a)^2 - b*x - a)*sinh(b*x + a
)^2 + b*x + 4*((b*x + a)*cosh(b*x + a)^3 - (b*x + a)*cosh(b*x + a))*sinh(b*
x + a) + a)*log(-cosh(b*x + a) - sinh(b*x + a) + 1) - 1)/(b^2*cosh(b*x + a)
^4 + 4*b^2*cosh(b*x + a)*sinh(b*x + a)^3 + b^2*sinh(b*x + a)^4 - 2*b^2*cosh
(b*x + a)^2 + 2*(3*b^2*cosh(b*x + a)^2 - b^2)*sinh(b*x + a)^2 + b^2 + 4*(b^
2*cosh(b*x + a)^3 - b^2*cosh(b*x + a))*sinh(b*x + a))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x \operatorname{csch}(bx+a)^3 \operatorname{sech}(bx+a)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*csch(b*x+a)^3*sech(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(x*csch(b*x + a)^3*sech(b*x + a), x)
```

maple [B] time = 0.41, size = 170, normalized size = 1.79

$$-\frac{2bx\,\mathrm{e}^{2bx+2a}+\mathrm{e}^{2bx+2a}-1}{b^2\left(\mathrm{e}^{2bx+2a}-1\right)^2}+\frac{x\ln\left(1+\mathrm{e}^{2bx+2a}\right)}{b}+\frac{\mathrm{polylog}\left(2,-\mathrm{e}^{2bx+2a}\right)}{2b^2}-\frac{\ln\left(1-\mathrm{e}^{bx+a}\right)x}{b}-\frac{\ln\left(1-\mathrm{e}^{bx+a}\right)a}{b^2}-\mathrm{polylo}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*csch(b*x+a)^3*sech(b*x+a),x)
```

```
[Out] -(2*b*x*exp(2*b*x+2*a)+exp(2*b*x+2*a)-1)/b^2/(exp(2*b*x+2*a)-1)^2+x*ln(1+ex
p(2*b*x+2*a))/b+1/2*polylog(2,-exp(2*b*x+2*a))/b^2-1/b*ln(1-exp(b*x+a))*x-1
/b^2*ln(1-exp(b*x+a))*a-polylog(2,exp(b*x+a))/b^2-1/b*ln(1+exp(b*x+a))*x-po
lylog(2,-exp(b*x+a))/b^2+1/b^2*a*ln(exp(b*x+a)-1)
```

maxima [A] time = 0.34, size = 145, normalized size = 1.53

$$-\frac{\left(2\,bxe^{(2\,a)}+e^{(2\,a)}\right)e^{(2\,bx)}-1}{b^2e^{(4\,bx+4\,a)}-2\,b^2e^{(2\,bx+2\,a)}+b^2}+\frac{2\,bx\log\left(e^{(2\,bx+2\,a)}+1\right)+\mathrm{Li}_2\left(-e^{(2\,bx+2\,a)}\right)}{2\,b^2}-\frac{bx\log\left(e^{(bx+a)}+1\right)+\mathrm{Li}_2\left(-e^{(bx+a)}\right.}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*csch(b*x+a)^3*sech(b*x+a),x, algorithm="maxima")
```

```
[Out] -((2*b*x*e^(2*a) + e^(2*a))*e^(2*b*x) - 1)/(b^2*e^(4*b*x + 4*a) - 2*b^2*e^(
2*b*x + 2*a) + b^2) + 1/2*(2*b*x*log(e^(2*b*x + 2*a) + 1) + dilog(-e^(2*b*x
 + 2*a)))/b^2 - (b*x*log(e^(b*x + a) + 1) + dilog(-e^(b*x + a)))/b^2 - (b*x
*log(-e^(b*x + a) + 1) + dilog(e^(b*x + a)))/b^2
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x}{\cosh\left(a+b\,x\right)\,{\sinh\left(a+b\,x\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(cosh(a + b*x)*sinh(a + b*x)^3),x)
```

```
[Out] int(x/(cosh(a + b*x)*sinh(a + b*x)^3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\operatorname{csch}^3(a+bx)\operatorname{sech}(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*csch(b*x+a)**3*sech(b*x+a),x)
```

```
[Out] Integral(x*csch(a + b*x)**3*sech(a + b*x), x)
```

3.511 $\int \operatorname{csch}^3(a+bx)\operatorname{sech}(a+bx)\,dx$

Optimal. Leaf size=28

$$-\frac{\coth^2(a+bx)}{2b}-\frac{\log(\tanh(a+bx))}{b}$$

```
[Out] -1/2*coth(b*x+a)^2/b-ln(tanh(b*x+a))/b
```

Rubi [A] time = 0.03, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2620, 14}

$$-\frac{\coth^2(a+bx)}{2b}-\frac{\log(\tanh(a+bx))}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[a + b*x]^3*Sech[a + b*x],x]
```

```
[Out] -Coth[a + b*x]^2/(2*b) - Log[Tanh[a + b*x]]/b
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2620

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rubi steps

$$\begin{aligned}\int \operatorname{csch}^3(a+bx)\operatorname{sech}(a+bx)\,dx &= -\frac{\operatorname{Subst}\left(\int \frac{1+x^2}{x^3}\,dx, x, i\tanh(a+bx)\right)}{b}\\ &= -\frac{\operatorname{Subst}\left(\int \left(\frac{1}{x^3}+\frac{1}{x}\right)dx, x, i\tanh(a+bx)\right)}{b}\\ &= -\frac{\coth^2(a+bx)}{2b}-\frac{\log(\tanh(a+bx))}{b}\end{aligned}$$

Mathematica [A] time = 0.05, size = 34, normalized size = 1.21

$$-\frac{\operatorname{csch}^2(a+bx)+2\log(\sinh(a+bx))-2\log(\cosh(a+bx))}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[a + b*x]^3*Sech[a + b*x],x]
```

```
[Out] -1/2*(Csch[a + b*x]^2 - 2*Log[Cosh[a + b*x]] + 2*Log[Sinh[a + b*x]])/b
```

fricas [B] time = 0.43, size = 379, normalized size = 13.54

$$-\frac{2\cosh(bx+a)^2-\left(\cosh(bx+a)^4+4\cosh(bx+a)\sinh(bx+a)^3+\sinh(bx+a)^4+2\left(3\cosh(bx+a)^2-1\right)\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^3*sech(b*x+a),x, algorithm="fricas")
```

```
[Out] -(2*cosh(b*x + a)^2 - (cosh(b*x + a)^4 + 4*cosh(b*x + a)*sinh(b*x + a)^3 +
sinh(b*x + a)^4 + 2*(3*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^2 - 2*cosh(b*x +
a)^2 + 4*(cosh(b*x + a)^3 - cosh(b*x + a))*sinh(b*x + a) + 1)*log(2*cosh(b*
x + a)/(cosh(b*x + a) - sinh(b*x + a))) + (cosh(b*x + a)^4 + 4*cosh(b*x + a
)*sinh(b*x + a)^3 + sinh(b*x + a)^4 + 2*(3*cosh(b*x + a)^2 - 1)*sinh(b*x +
a)^2 - 2*cosh(b*x + a)^2 + 4*(cosh(b*x + a)^3 - cosh(b*x + a))*sinh(b*x + a
) + 1)*log(2*sinh(b*x + a)/(cosh(b*x + a) - sinh(b*x + a))) + 4*cosh(b*x +
a)*sinh(b*x + a) + 2*sinh(b*x + a)^2)/(b*cosh(b*x + a)^4 + 4*b*cosh(b*x + a
)*sinh(b*x + a)^3 + b*sinh(b*x + a)^4 - 2*b*cosh(b*x + a)^2 + 2*(3*b*cosh(b
*x + a)^2 - b)*sinh(b*x + a)^2 + 4*(b*cosh(b*x + a)^3 - b*cosh(b*x + a))*si
nh(b*x + a) + b)
```

giac [B] time = 0.15, size = 93, normalized size = 3.32

$$\frac{\frac{e^{(2bx+2a)}+e^{(-2bx-2a)}-6}{e^{(2bx+2a)}+e^{(-2bx-2a)}-2}+\log\left(e^{(2bx+2a)}+e^{(-2bx-2a)}+2\right)-\log\left(e^{(2bx+2a)}+e^{(-2bx-2a)}-2\right)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^3*sech(b*x+a),x, algorithm="giac")
```

```
[Out] 1/2*((e^(2*b*x + 2*a) + e^(-2*b*x - 2*a) - 6)/(e^(2*b*x + 2*a) + e^(-2*b*x
- 2*a) - 2) + log(e^(2*b*x + 2*a) + e^(-2*b*x - 2*a) + 2) - log(e^(2*b*x +
2*a) + e^(-2*b*x - 2*a) - 2))/b
```

maple [A] time = 0.18, size = 27, normalized size = 0.96

$$-\frac{1}{2b\sinh\left(bx+a\right)^{2}}-\frac{\ln\left(\tanh\left(bx+a\right)\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(b*x+a)^3*sech(b*x+a),x)
```

```
[Out] -1/2/b/sinh(b*x+a)^2-ln(tanh(b*x+a))/b
```

maxima [B] time = 1.18, size = 91, normalized size = 3.25

$$-\frac{\log\left(e^{(-bx-a)}+1\right)}{b}-\frac{\log\left(e^{(-bx-a)}-1\right)}{b}+\frac{\log\left(e^{(-2\,bx-2\,a)}+1\right)}{b}+\frac{2\,e^{(-2\,bx-2\,a)}}{b\left(2\,e^{(-2\,bx-2\,a)}-e^{(-4\,bx-4\,a)}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^3*sech(b*x+a),x, algorithm="maxima")
```

```
[Out] -log(e^(-b*x - a) + 1)/b - log(e^(-b*x - a) - 1)/b + log(e^(-2*b*x - 2*a) +
 1)/b + 2*e^(-2*b*x - 2*a)/(b*(2*e^(-2*b*x - 2*a) - e^(-4*b*x - 4*a) - 1))
```

mupad [B] time = 1.45, size = 78, normalized size = 2.79

$$\frac{2\,\mathrm{atan}\left(\frac{\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}\,\sqrt{-b^2}}{b}\right)}{\sqrt{-b^2}}-\frac{2}{b\,\left(\mathrm{e}^{2\,a+2\,b\,x}-1\right)}-\frac{2}{b\,\left(\mathrm{e}^{4\,a+4\,b\,x}-2\,\mathrm{e}^{2\,a+2\,b\,x}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(a + b*x)*sinh(a + b*x)^3),x)
```

```
[Out] (2*atan((exp(2*a)*exp(2*b*x)*(-b^2)^(1/2))/b))/(-b^2)^(1/2) - 2/(b*(exp(2*a
 + 2*b*x) - 1)) - 2/(b*(exp(4*a + 4*b*x) - 2*exp(2*a + 2*b*x) + 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{csch}^{3}\left(a+bx\right)\operatorname{sech}\left(a+bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)**3*sech(b*x+a),x)
```

```
[Out] Integral(csch(a + b*x)**3*sech(a + b*x), x)
```

3.512 $\int \frac{\operatorname{csch}^3(a+bx)\operatorname{sech}(a+bx)}{x}\,dx$

Optimal. Leaf size=21

$$\operatorname{Int}\left(\frac{\operatorname{csch}^3(a+bx)\operatorname{sech}(a+bx)}{x},x\right)$$

```
[Out] CannotIntegrate(csch(b*x+a)^3*sech(b*x+a)/x,x)
```

Rubi [A] time = 0.20, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\operatorname{csch}^3(a+bx)\operatorname{sech}(a+bx)}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Csch[a + b*x]^3*Sech[a + b*x])/x,x]
```

```
[Out] Defer[Int][(Csch[a + b*x]^3*Sech[a + b*x])/x, x]
```

Rubi steps

$$\int \frac{\operatorname{csch}^3(a+bx)\operatorname{sech}(a+bx)}{x}\,dx = \int \frac{\operatorname{csch}^3(a+bx)\operatorname{sech}(a+bx)}{x}\,dx$$

Mathematica [A] time = 57.31, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}^3(a+bx)\operatorname{sech}(a+bx)}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Csch[a + b*x]^3*Sech[a + b*x])/x,x]
```

```
[Out] Integrate[(Csch[a + b*x]^3*Sech[a + b*x])/x, x]
```

fricas [A] time = 0.42, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{\operatorname{csch}\left(bx+a\right)^{3}\operatorname{sech}\left(bx+a\right)}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^3*sech(b*x+a)/x,x, algorithm="fricas")
```

```
[Out] integral(csch(b*x + a)^3*sech(b*x + a)/x, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}(bx+a)^3 \operatorname{sech}(bx+a)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^3*sech(b*x+a)/x,x, algorithm="giac")
```

```
[Out] integrate(csch(b*x + a)^3*sech(b*x + a)/x, x)
```

maple [A] time = 1.50, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}(bx+a)^3 \operatorname{sech}(bx+a)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(b*x+a)^3*sech(b*x+a)/x,x)
```

```
[Out] int(csch(b*x+a)^3*sech(b*x+a)/x,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\left(2\,bxe^{(2\,a)}-e^{(2\,a)}\right)e^{(2\,bx)}+1}{b^2x^2e^{(4\,bx+4\,a)}-2\,b^2x^2e^{(2\,bx+2\,a)}+b^2x^2}+16\int\frac{b^2x^2-1}{16\left(b^2x^3e^{(bx+a)}+b^2x^3\right)}\,dx-16\int\frac{b^2x^2-1}{16\left(b^2x^3e^{(bx+a)}-b^2x^3\right)}\,dx-16$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^3*sech(b*x+a)/x,x, algorithm="maxima")
```

```
[Out] -((2*b*x*e^(2*a) - e^(2*a))*e^(2*b*x) + 1)/(b^2*x^2*e^(4*b*x + 4*a) - 2*b^2
*x^2*e^(2*b*x + 2*a) + b^2*x^2) + 16*integrate(1/16*(b^2*x^2 - 1)/(b^2*x^3*
e^(b*x + a) + b^2*x^3), x) - 16*integrate(1/16*(b^2*x^2 - 1)/(b^2*x^3*e^(b*
x + a) - b^2*x^3), x) - 16*integrate(1/8/(x*e^(2*b*x + 2*a) + x), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{1}{x\cosh(a+b\,x)\,\sinh(a+b\,x)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*cosh(a + b*x)*sinh(a + b*x)^3),x)
```

```
[Out] int(1/(x*cosh(a + b*x)*sinh(a + b*x)^3), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}^3(a+bx)\operatorname{sech}(a+bx)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)**3*sech(b*x+a)/x,x)
```

```
[Out] Integral(csch(a + b*x)**3*sech(a + b*x)/x, x)
```

3.513 $\int \frac{\operatorname{csch}^3(a+bx)\operatorname{sech}(a+bx)}{x^2}\,dx$

Optimal. Leaf size=21

$$\operatorname{Int}\left(\frac{\operatorname{csch}^3(a+bx)\operatorname{sech}(a+bx)}{x^2},x\right)$$

```
[Out] CannotIntegrate(csch(b*x+a)^3*sech(b*x+a)/x^2,x)
```

Rubi [A] time = 0.24, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\operatorname{csch}^3(a+bx)\operatorname{sech}(a+bx)}{x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Csch[a + b*x]^3*Sech[a + b*x])/x^2,x]
[Out] Defer[Int][(Csch[a + b*x]^3*Sech[a + b*x])/x^2, x]
```

Rubi steps

$$\int \frac{\operatorname{csch}^3(a+bx)\operatorname{sech}(a+bx)}{x^2}\,dx=\int \frac{\operatorname{csch}^3(a+bx)\operatorname{sech}(a+bx)}{x^2}\,dx$$

Mathematica [A] time = 27.62, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}^3(a+bx)\operatorname{sech}(a+bx)}{x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Csch[a + b*x]^3*Sech[a + b*x])/x^2,x]
[Out] Integrate[(Csch[a + b*x]^3*Sech[a + b*x])/x^2, x]
```

fricas [A] time = 0.45, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{\operatorname{csch}(bx+a)^3\operatorname{sech}(bx+a)}{x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^3*sech(b*x+a)/x^2,x, algorithm="fricas")
```

```
[Out] integral(csch(b*x + a)^3*sech(b*x + a)/x^2, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}(bx+a)^3 \operatorname{sech}(bx+a)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^3*sech(b*x+a)/x^2,x, algorithm="giac")
```

```
[Out] integrate(csch(b*x + a)^3*sech(b*x + a)/x^2, x)
```

maple [A] time = 1.73, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}(bx+a)^3 \operatorname{sech}(bx+a)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(b*x+a)^3*sech(b*x+a)/x^2,x)
```

```
[Out] int(csch(b*x+a)^3*sech(b*x+a)/x^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{2\left(\left(bxe^{(2a)}-e^{(2a)}\right)e^{(2bx)}+1\right)}{b^2x^3e^{(4bx+4a)}-2b^2x^3e^{(2bx+2a)}+b^2x^3}+16\int \frac{b^2x^2-3}{16\left(b^2x^4e^{(bx+a)}+b^2x^4\right)}\,dx-16\int \frac{b^2x^2-3}{16\left(b^2x^4e^{(bx+a)}-b^2x^4\right)}\,dx-16\int$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^3*sech(b*x+a)/x^2,x, algorithm="maxima")
```

```
[Out] -2*((b*x*e^(2*a) - e^(2*a))*e^(2*b*x) + 1)/(b^2*x^3*e^(4*b*x + 4*a) - 2*b^2
*x^3*e^(2*b*x + 2*a) + b^2*x^3) + 16*integrate(1/16*(b^2*x^2 - 3)/(b^2*x^4*
e^(b*x + a) + b^2*x^4), x) - 16*integrate(1/16*(b^2*x^2 - 3)/(b^2*x^4*e^(b*
x + a) - b^2*x^4), x) - 16*integrate(1/8/(x^2*e^(2*b*x + 2*a) + x^2), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{1}{x^2\cosh(a+b\,x)\,\sinh(a+b\,x)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*cosh(a + b*x)*sinh(a + b*x)^3),x)
```

```
[Out] int(1/(x^2*cosh(a + b*x)*sinh(a + b*x)^3), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}^3(a+bx)\operatorname{sech}(a+bx)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)**3*sech(b*x+a)/x**2,x)
```

```
[Out] Integral(csch(a + b*x)**3*sech(a + b*x)/x**2, x)
```

3.514 $\int x^m \mathbf{csch}^3(a+bx)\mathbf{sech}^2(a+bx)\,dx$

Optimal. Leaf size=23

$$\text{Int}\left(x^m\text{csch}^3(a+bx)\text{sech}^2(a+bx),x\right)$$

```
[Out] CannotIntegrate(x^m*csch(b*x+a)^3*sech(b*x+a)^2,x)
```

Rubi [A] time = 0.60, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int x^m\text{csch}^3(a+bx)\text{sech}^2(a+bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x^m*Csch[a + b*x]^3*Sech[a + b*x]^2,x]
[Out] Defer[Int][x^m*Csch[a + b*x]^3*Sech[a + b*x]^2, x]
```

Rubi steps

$$\int x^m\text{csch}^3(a+bx)\text{sech}^2(a+bx)\,dx=\int x^m\text{csch}^3(a+bx)\text{sech}^2(a+bx)\,dx$$

Mathematica [A] time = 69.16, size = 0, normalized size = 0.00

$$\int x^m\text{csch}^3(a+bx)\text{sech}^2(a+bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^m*Csch[a + b*x]^3*Sech[a + b*x]^2,x]
[Out] Integrate[x^m*Csch[a + b*x]^3*Sech[a + b*x]^2, x]
```

fricas [A] time = 0.41, size = 0, normalized size = 0.00

$$\text{integral}\left(x^m\,\text{csch}\,(bx+a)^3\,\text{sech}\,(bx+a)^2\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*csch(b*x+a)^3*sech(b*x+a)^2,x, algorithm="fricas")
[Out] integral(x^m*csch(b*x + a)^3*sech(b*x + a)^2, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int x^m \operatorname{csch}(bx+a)^3 \operatorname{sech}(bx+a)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*csch(b*x+a)^3*sech(b*x+a)^2,x, algorithm="giac")
[Out] integrate(x^m*csch(b*x + a)^3*sech(b*x + a)^2, x)
```

maple [A] time = 0.24, size = 0, normalized size = 0.00

$$\int x^m \operatorname{csch}(bx+a)^3 \operatorname{sech}(bx+a)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*csch(b*x+a)^3*sech(b*x+a)^2,x)
[Out] int(x^m*csch(b*x+a)^3*sech(b*x+a)^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int x^m \operatorname{csch}(bx+a)^3 \operatorname{sech}(bx+a)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*csch(b*x+a)^3*sech(b*x+a)^2,x, algorithm="maxima")
[Out] integrate(x^m*csch(b*x + a)^3*sech(b*x + a)^2, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{x^m}{\cosh(a+b\,x)^2 \sinh(a+b\,x)^3} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(cosh(a + b*x)^2*sinh(a + b*x)^3),x)
[Out] int(x^m/(cosh(a + b*x)^2*sinh(a + b*x)^3), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int x^m \operatorname{csch}^3(a+bx) \operatorname{sech}^2(a+bx) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*csch(b*x+a)**3*sech(b*x+a)**2,x)
[Out] Integral(x**m*csch(a + b*x)**3*sech(a + b*x)**2, x)
```

3.515 $\int x^3 \operatorname{csch}^3(a+bx) \operatorname{sech}^2(a+bx) \, dx$

Optimal. Leaf size=317

$$-\frac{3\text{Li}_2\left(-e^{a+bx}\right)}{b^4}+\frac{3\text{Li}_2\left(e^{a+bx}\right)}{b^4}+\frac{6i\text{Li}_3\left(-ie^{a+bx}\right)}{b^4}-\frac{6i\text{Li}_3\left(ie^{a+bx}\right)}{b^4}+\frac{9\text{Li}_4\left(-e^{a+bx}\right)}{b^4}-\frac{9\text{Li}_4\left(e^{a+bx}\right)}{b^4}-\frac{6ix\text{Li}_2\left(-ie^{a+bx}\right)}{b^3}+\frac{6i\cdots}{\cdots}$$

```
[Out] 6*x^2*arctan(exp(b*x+a))/b^2-6*x*arctanh(exp(b*x+a))/b^3+3*x^3*arctanh(exp(
b*x+a))/b-3/2*x^2*csch(b*x+a)/b^2-3*polylog(2,-exp(b*x+a))/b^4+9/2*x^2*poly
log(2,-exp(b*x+a))/b^2-6*I*polylog(3,I*exp(b*x+a))/b^4-6*I*x*polylog(2,-I*e
xp(b*x+a))/b^3+3*polylog(2,exp(b*x+a))/b^4-9/2*x^2*polylog(2,exp(b*x+a))/b^
2-9*x*polylog(3,-exp(b*x+a))/b^3+6*I*polylog(3,-I*exp(b*x+a))/b^4+6*I*x*pol
ylog(2,I*exp(b*x+a))/b^3+9*x*polylog(3,exp(b*x+a))/b^3+9*polylog(4,-exp(b*x
+a))/b^4-9*polylog(4,exp(b*x+a))/b^4-3/2*x^3*sech(b*x+a)/b-1/2*x^3*csch(b*x
+a)^2*sech(b*x+a)/b
```

Rubi [A] time = 1.18, antiderivative size = 317, normalized size of antiderivative = 1.00, number of steps used = 40, number of rules used = 19, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.950$, Rules used = {2622, 288, 321, 207, 5462, 14, 6273, 12, 4182, 2531, 6609, 2282, 6589, 6742, 4180, 2621, 5205, 2279, 2391}

$$\frac{9x^2\text{PolyLog}\left(2,-e^{a+bx}\right)}{2b^2}-\frac{9x^2\text{PolyLog}\left(2,e^{a+bx}\right)}{2b^2}-\frac{6ix\text{PolyLog}\left(2,-ie^{a+bx}\right)}{b^3}+\frac{6ix\text{PolyLog}\left(2,ie^{a+bx}\right)}{b^3}-\frac{9x\text{PolyLog}\cdots}{b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Csch[a + b*x]^3*Sech[a + b*x]^2,x]
```

```
[Out] (6*x^2*ArcTan[E^(a + b*x)])/b^2 - (6*x*ArcTanh[E^(a + b*x)])/b^3 + (3*x^3*A
rcTanh[E^(a + b*x)])/b - (3*x^2*Csch[a + b*x])/(2*b^2) - (3*PolyLog[2, -E^(
a + b*x)])/b^4 + (9*x^2*PolyLog[2, -E^(a + b*x)])/(2*b^2) - ((6*I)*x*PolyLo
g[2, (-I)*E^(a + b*x)])/b^3 + ((6*I)*x*PolyLog[2, I*E^(a + b*x)])/b^3 + (3*
PolyLog[2, E^(a + b*x)])/b^4 - (9*x^2*PolyLog[2, E^(a + b*x)])/(2*b^2) - (9
*x*PolyLog[3, -E^(a + b*x)])/b^3 + ((6*I)*PolyLog[3, (-I)*E^(a + b*x)])/b^4
 - ((6*I)*PolyLog[3, I*E^(a + b*x)])/b^4 + (9*x*PolyLog[3, E^(a + b*x)])/b^
3 + (9*PolyLog[4, -E^(a + b*x)])/b^4 - (9*PolyLog[4, E^(a + b*x)])/b^4 - (3
*x^3*Sech[a + b*x])/(2*b) - (x^3*Csch[a + b*x]^2*Sech[a + b*x])/(2*b)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2621

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> -Dist[(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5205

```
Int[((a_.) + ArcTan[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Sim
p[((c + d*x)^(m + 1)*(a + b*ArcTan[u]))/(d*(m + 1)), x] - Dist[b/(d*(m + 1)
), Int[SimplifyIntegrand[((c + d*x)^(m + 1)*D[u, x])/(1 + u^2), x], x], x]
/; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && InverseFunctionFreeQ[u, x] &&
```

```
 !FunctionOfQ[(c + d*x)^(m + 1), u, x] && FalseQ[PowerVariableExpn[u, m +
1, x]]
```

Rule 5462

```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> With[{u = IntHide[Csch[a + b*x]^n*Sech[a +
b*x]^p, x]}, Dist[(c + d*x)^m, u, x] - Dist[d*m, Int[(c + d*x)^(m - 1)*u, x
], x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n,
p]
```

Rule 6273

```
Int[((a_.) + ArcTanh[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[((c + d*x)^(m + 1)*(a + b*ArcTanh[u]))/(d*(m + 1)), x] - Dist[b/(d*(m +
1)), Int[SimplifyIntegrand[((c + d*x)^(m + 1)*D[u, x])/(1 - u^2), x], x], x
] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && InverseFunctionFreeQ[u, x]
&&  !FunctionOfQ[(c + d*x)^(m + 1), u, x] && FalseQ[PowerVariableExpn[u, m
+ 1, x]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int x^3\text{csch}^3(a+bx)\text{sech}^2(a+bx)\,dx = \frac{3x^3\tanh^{-1}(\cosh(a+bx))}{2b} - \frac{3x^3\text{sech}(a+bx)}{2b} - \frac{x^3\text{csch}^2(a+bx)\text{sech}(a+}{2b}$$

$$= \frac{3x^3\tanh^{-1}(\cosh(a+bx))}{2b} - \frac{3x^3\text{sech}(a+bx)}{2b} - \frac{x^3\text{csch}^2(a+bx)\text{sech}(a+}{2b}$$

$$= \frac{3x^3\tanh^{-1}(\cosh(a+bx))}{2b} - \frac{3x^3\text{sech}(a+bx)}{2b} - \frac{x^3\text{csch}^2(a+bx)\text{sech}(a+}{2b}$$

$$= -\frac{3x^3\text{sech}(a+bx)}{2b} - \frac{x^3\text{csch}^2(a+bx)\text{sech}(a+bx)}{2b} - \frac{3\int bx^3\text{csch}(a+bx)}{2b}$$

$$= -\frac{3x^3\text{sech}(a+bx)}{2b} - \frac{x^3\text{csch}^2(a+bx)\text{sech}(a+bx)}{2b} - \frac{3}{2}\int x^3\text{csch}(a+bx)$$

$$= \frac{9x^2\tan^{-1}\left(e^{a+bx}\right)}{b^2} - \frac{3x^2\tan^{-1}(\sinh(a+bx))}{2b^2} + \frac{3x^3\tanh^{-1}\left(e^{a+bx}\right)}{b} - 3x^2\text{cs}$$

$$= \frac{9x^2\tan^{-1}\left(e^{a+bx}\right)}{b^2} - \frac{3x^2\tan^{-1}(\sinh(a+bx))}{2b^2} + \frac{3x^3\tanh^{-1}\left(e^{a+bx}\right)}{b} - 3x^2\text{cs}$$

$$= \frac{9x^2\tan^{-1}\left(e^{a+bx}\right)}{b^2} - \frac{3x^2\tan^{-1}(\sinh(a+bx))}{2b^2} + \frac{3x^3\tanh^{-1}\left(e^{a+bx}\right)}{b} - 3x^2\text{cs}$$

$$= \frac{9x^2\tan^{-1}\left(e^{a+bx}\right)}{b^2} - \frac{6x\tanh^{-1}\left(e^{a+bx}\right)}{b^3} + \frac{3x^3\tanh^{-1}\left(e^{a+bx}\right)}{b} - \frac{3x^2\text{csch}(a+}{2b^2}$$

$$= \frac{9x^2\tan^{-1}\left(e^{a+bx}\right)}{b^2} - \frac{6x\tanh^{-1}\left(e^{a+bx}\right)}{b^3} + \frac{3x^3\tanh^{-1}\left(e^{a+bx}\right)}{b} - \frac{3x^2\text{csch}(a+}{2b^2}$$

$$= \frac{6x^2\tan^{-1}\left(e^{a+bx}\right)}{b^2} - \frac{6x\tanh^{-1}\left(e^{a+bx}\right)}{b^3} + \frac{3x^3\tanh^{-1}\left(e^{a+bx}\right)}{b} - \frac{3x^2\text{csch}(a+}{2b^2}$$

$$= \frac{6x^2\tan^{-1}\left(e^{a+bx}\right)}{b^2} - \frac{6x\tanh^{-1}\left(e^{a+bx}\right)}{b^3} + \frac{3x^3\tanh^{-1}\left(e^{a+bx}\right)}{b} - \frac{3x^2\text{csch}(a+}{2b^2}$$

$$= \frac{6x^2\tan^{-1}\left(e^{a+bx}\right)}{b^2} - \frac{6x\tanh^{-1}\left(e^{a+bx}\right)}{b^3} + \frac{3x^3\tanh^{-1}\left(e^{a+bx}\right)}{b} - \frac{3x^2\text{csch}(a+}{2b^2}$$

$$= \frac{6x^2\tan^{-1}\left(e^{a+bx}\right)}{b^2} - \frac{6x\tanh^{-1}\left(e^{a+bx}\right)}{b^3} + \frac{3x^3\tanh^{-1}\left(e^{a+bx}\right)}{b} - \frac{3x^2\text{csch}(a+}{2b^2}$$

Mathematica [A] time = 8.42, size = 433, normalized size = 1.37

$$-\frac{3x^2\text{csch}(a)}{2b^2} + \frac{3x^2\text{csch}\left(\frac{a}{2}\right)\sinh\left(\frac{bx}{2}\right)\text{csch}\left(\frac{a}{2}+\frac{bx}{2}\right)}{4b^2} + \frac{3x^2\text{sech}\left(\frac{a}{2}\right)\sinh\left(\frac{bx}{2}\right)\text{sech}\left(\frac{a}{2}+\frac{bx}{2}\right)}{4b^2} + \frac{3i\left(b^2x^2\log\left(1-ie^{a+b}\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Csch[a + b*x]^3*Sech[a + b*x]^2,x]
```

```
[Out] (-3*x^2*Csch[a])/(2*b^2) - (x^3*Csch[a/2 + (b*x)/2]^2)/(8*b) + ((3*I)*(b^2*
x^2*Log[1 - I*E^(a + b*x)] - b^2*x^2*Log[1 + I*E^(a + b*x)] - 2*b*x*PolyLog
[2, (-I)*E^(a + b*x)] + 2*b*x*PolyLog[2, I*E^(a + b*x)] + 2*PolyLog[3, (-I)
*E^(a + b*x)] - 2*PolyLog[3, I*E^(a + b*x)]))/b^4 - (3*(-2*b*x*Log[1 - E^(a
 + b*x)] + b^3*x^3*Log[1 - E^(a + b*x)] + 2*b*x*Log[1 + E^(a + b*x)] - b^3*
x^3*Log[1 + E^(a + b*x)] + (2 - 3*b^2*x^2)*PolyLog[2, -E^(a + b*x)] + (-2 +
 3*b^2*x^2)*PolyLog[2, E^(a + b*x)] + 6*b*x*PolyLog[3, -E^(a + b*x)] - 6*b*
x*PolyLog[3, E^(a + b*x)] - 6*PolyLog[4, -E^(a + b*x)] + 6*PolyLog[4, E^(a
+ b*x)]))/(2*b^4) - (x^3*Sech[a/2 + (b*x)/2]^2)/(8*b) - (x^3*Sech[a + b*x])
/b + (3*x^2*Csch[a/2]*Csch[a/2 + (b*x)/2]*Sinh[(b*x)/2])/(4*b^2) + (3*x^2*S
ech[a/2]*Sech[a/2 + (b*x)/2]*Sinh[(b*x)/2])/(4*b^2)
```

fricas [C] time = 0.58, size = 5318, normalized size = 16.78

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*csch(b*x+a)^3*sech(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(4*b^3*x^3*cosh(b*x + a)^3 - 6*(b^3*x^3 + b^2*x^2)*cosh(b*x + a)^5 - 30
*(b^3*x^3 + b^2*x^2)*cosh(b*x + a)*sinh(b*x + a)^4 - 6*(b^3*x^3 + b^2*x^2)*
sinh(b*x + a)^5 + 4*(b^3*x^3 - 15*(b^3*x^3 + b^2*x^2)*cosh(b*x + a)^2)*sinh
(b*x + a)^3 + 12*(b^3*x^3*cosh(b*x + a) - 5*(b^3*x^3 + b^2*x^2)*cosh(b*x +
a)^3)*sinh(b*x + a)^2 - 6*(b^3*x^3 - b^2*x^2)*cosh(b*x + a) - 3*((3*b^2*x^2
 - 2)*cosh(b*x + a)^6 + 6*(3*b^2*x^2 - 2)*cosh(b*x + a)*sinh(b*x + a)^5 + (
3*b^2*x^2 - 2)*sinh(b*x + a)^6 - (3*b^2*x^2 - 2)*cosh(b*x + a)^4 - (3*b^2*x
^2 - 15*(3*b^2*x^2 - 2)*cosh(b*x + a)^2 - 2)*sinh(b*x + a)^4 + 3*b^2*x^2 +
4*(5*(3*b^2*x^2 - 2)*cosh(b*x + a)^3 - (3*b^2*x^2 - 2)*cosh(b*x + a))*sinh(
b*x + a)^3 - (3*b^2*x^2 - 2)*cosh(b*x + a)^2 + (15*(3*b^2*x^2 - 2)*cosh(b*x
 + a)^4 - 3*b^2*x^2 - 6*(3*b^2*x^2 - 2)*cosh(b*x + a)^2 + 2)*sinh(b*x + a)^
2 + 2*(3*(3*b^2*x^2 - 2)*cosh(b*x + a)^5 - 2*(3*b^2*x^2 - 2)*cosh(b*x + a)^
3 - (3*b^2*x^2 - 2)*cosh(b*x + a))*sinh(b*x + a) - 2)*dilog(cosh(b*x + a) +
 sinh(b*x + a)) + (12*I*b*x*cosh(b*x + a)^6 + 72*I*b*x*cosh(b*x + a)*sinh(b
*x + a)^5 + 12*I*b*x*sinh(b*x + a)^6 - 12*I*b*x*cosh(b*x + a)^4 + (180*I*b*
x*cosh(b*x + a)^2 - 12*I*b*x)*sinh(b*x + a)^4 - 12*I*b*x*cosh(b*x + a)^2 +
(240*I*b*x*cosh(b*x + a)^3 - 48*I*b*x*cosh(b*x + a))*sinh(b*x + a)^3 + (180
*I*b*x*cosh(b*x + a)^4 - 72*I*b*x*cosh(b*x + a)^2 - 12*I*b*x)*sinh(b*x + a)
^2 + 12*I*b*x + (72*I*b*x*cosh(b*x + a)^5 - 48*I*b*x*cosh(b*x + a)^3 - 24*I
*b*x*cosh(b*x + a))*sinh(b*x + a))*dilog(I*cosh(b*x + a) + I*sinh(b*x + a))
 + (-12*I*b*x*cosh(b*x + a)^6 - 72*I*b*x*cosh(b*x + a)*sinh(b*x + a)^5 - 12
*I*b*x*sinh(b*x + a)^6 + 12*I*b*x*cosh(b*x + a)^4 + (-180*I*b*x*cosh(b*x +
```

```
a)^2 + 12*I*b*x)*sinh(b*x + a)^4 + 12*I*b*x*cosh(b*x + a)^2 + (-240*I*b*x*c
osh(b*x + a)^3 + 48*I*b*x*cosh(b*x + a))*sinh(b*x + a)^3 + (-180*I*b*x*cosh
(b*x + a)^4 + 72*I*b*x*cosh(b*x + a)^2 + 12*I*b*x)*sinh(b*x + a)^2 - 12*I*b
*x + (-72*I*b*x*cosh(b*x + a)^5 + 48*I*b*x*cosh(b*x + a)^3 + 24*I*b*x*cosh(
b*x + a))*sinh(b*x + a))*dilog(-I*cosh(b*x + a) - I*sinh(b*x + a)) + 3*((3*
b^2*x^2 - 2)*cosh(b*x + a)^6 + 6*(3*b^2*x^2 - 2)*cosh(b*x + a)*sinh(b*x + a
)^5 + (3*b^2*x^2 - 2)*sinh(b*x + a)^6 - (3*b^2*x^2 - 2)*cosh(b*x + a)^4 - (
3*b^2*x^2 - 15*(3*b^2*x^2 - 2)*cosh(b*x + a)^2 - 2)*sinh(b*x + a)^4 + 3*b^2
*x^2 + 4*(5*(3*b^2*x^2 - 2)*cosh(b*x + a)^3 - (3*b^2*x^2 - 2)*cosh(b*x + a)
)*sinh(b*x + a)^3 - (3*b^2*x^2 - 2)*cosh(b*x + a)^2 + (15*(3*b^2*x^2 - 2)*c
osh(b*x + a)^4 - 3*b^2*x^2 - 6*(3*b^2*x^2 - 2)*cosh(b*x + a)^2 + 2)*sinh(b*
x + a)^2 + 2*(3*(3*b^2*x^2 - 2)*cosh(b*x + a)^5 - 2*(3*b^2*x^2 - 2)*cosh(b*
x + a)^3 - (3*b^2*x^2 - 2)*cosh(b*x + a))*sinh(b*x + a) - 2)*dilog(-cosh(b*
x + a) - sinh(b*x + a)) + 3*((b^3*x^3 - 2*b*x)*cosh(b*x + a)^6 + 6*(b^3*x^3
 - 2*b*x)*cosh(b*x + a)*sinh(b*x + a)^5 + (b^3*x^3 - 2*b*x)*sinh(b*x + a)^6
 + b^3*x^3 - (b^3*x^3 - 2*b*x)*cosh(b*x + a)^4 - (b^3*x^3 - 15*(b^3*x^3 - 2
*b*x)*cosh(b*x + a)^2 - 2*b*x)*sinh(b*x + a)^4 + 4*(5*(b^3*x^3 - 2*b*x)*cos
h(b*x + a)^3 - (b^3*x^3 - 2*b*x)*cosh(b*x + a))*sinh(b*x + a)^3 - (b^3*x^3
- 2*b*x)*cosh(b*x + a)^2 - (b^3*x^3 - 15*(b^3*x^3 - 2*b*x)*cosh(b*x + a)^4
+ 6*(b^3*x^3 - 2*b*x)*cosh(b*x + a)^2 - 2*b*x)*sinh(b*x + a)^2 - 2*b*x + 2*
(3*(b^3*x^3 - 2*b*x)*cosh(b*x + a)^5 - 2*(b^3*x^3 - 2*b*x)*cosh(b*x + a)^3
- (b^3*x^3 - 2*b*x)*cosh(b*x + a))*sinh(b*x + a))*log(cosh(b*x + a) + sinh(
b*x + a) + 1) + (6*I*a^2*cosh(b*x + a)^6 + 36*I*a^2*cosh(b*x + a)*sinh(b*x
+ a)^5 + 6*I*a^2*sinh(b*x + a)^6 - 6*I*a^2*cosh(b*x + a)^4 + (90*I*a^2*cosh
(b*x + a)^2 - 6*I*a^2)*sinh(b*x + a)^4 - 6*I*a^2*cosh(b*x + a)^2 + (120*I*a
^2*cosh(b*x + a)^3 - 24*I*a^2*cosh(b*x + a))*sinh(b*x + a)^3 + (90*I*a^2*co
sh(b*x + a)^4 - 36*I*a^2*cosh(b*x + a)^2 - 6*I*a^2)*sinh(b*x + a)^2 + 6*I*a
^2 + (36*I*a^2*cosh(b*x + a)^5 - 24*I*a^2*cosh(b*x + a)^3 - 12*I*a^2*cosh(b
*x + a))*sinh(b*x + a))*log(cosh(b*x + a) + sinh(b*x + a) + I) + (-6*I*a^2*
cosh(b*x + a)^6 - 36*I*a^2*cosh(b*x + a)*sinh(b*x + a)^5 - 6*I*a^2*sinh(b*x
 + a)^6 + 6*I*a^2*cosh(b*x + a)^4 + (-90*I*a^2*cosh(b*x + a)^2 + 6*I*a^2)*s
inh(b*x + a)^4 + 6*I*a^2*cosh(b*x + a)^2 + (-120*I*a^2*cosh(b*x + a)^3 + 24
*I*a^2*cosh(b*x + a))*sinh(b*x + a)^3 + (-90*I*a^2*cosh(b*x + a)^4 + 36*I*a
^2*cosh(b*x + a)^2 + 6*I*a^2)*sinh(b*x + a)^2 - 6*I*a^2 + (-36*I*a^2*cosh(b
*x + a)^5 + 24*I*a^2*cosh(b*x + a)^3 + 12*I*a^2*cosh(b*x + a))*sinh(b*x + a
))*log(cosh(b*x + a) + sinh(b*x + a) - I) + 3*((a^3 - 2*a)*cosh(b*x + a)^6
+ 6*(a^3 - 2*a)*cosh(b*x + a)*sinh(b*x + a)^5 + (a^3 - 2*a)*sinh(b*x + a)^6
 - (a^3 - 2*a)*cosh(b*x + a)^4 - (a^3 - 15*(a^3 - 2*a)*cosh(b*x + a)^2 - 2*
a)*sinh(b*x + a)^4 + 4*(5*(a^3 - 2*a)*cosh(b*x + a)^3 - (a^3 - 2*a)*cosh(b*
x + a))*sinh(b*x + a)^3 + a^3 - (a^3 - 2*a)*cosh(b*x + a)^2 + (15*(a^3 - 2*
a)*cosh(b*x + a)^4 - a^3 - 6*(a^3 - 2*a)*cosh(b*x + a)^2 + 2*a)*sinh(b*x +
a)^2 + 2*(3*(a^3 - 2*a)*cosh(b*x + a)^5 - 2*(a^3 - 2*a)*cosh(b*x + a)^3 - (
a^3 - 2*a)*cosh(b*x + a))*sinh(b*x + a) - 2*a)*log(cosh(b*x + a) + sinh(b*x
 + a) - 1) + ((-6*I*b^2*x^2 + 6*I*a^2)*cosh(b*x + a)^6 + (-36*I*b^2*x^2 + 3
6*I*a^2)*cosh(b*x + a)*sinh(b*x + a)^5 + (-6*I*b^2*x^2 + 6*I*a^2)*sinh(b*x
```

```
+ a)^6 + (6*I*b^2*x^2 - 6*I*a^2)*cosh(b*x + a)^4 + (6*I*b^2*x^2 + (-90*I*b^
2*x^2 + 90*I*a^2)*cosh(b*x + a)^2 - 6*I*a^2)*sinh(b*x + a)^4 - 6*I*b^2*x^2
+ ((-120*I*b^2*x^2 + 120*I*a^2)*cosh(b*x + a)^3 + (24*I*b^2*x^2 - 24*I*a^2)
*cosh(b*x + a))*sinh(b*x + a)^3 + (6*I*b^2*x^2 - 6*I*a^2)*cosh(b*x + a)^2 +
 ((-90*I*b^2*x^2 + 90*I*a^2)*cosh(b*x + a)^4 + 6*I*b^2*x^2 + (36*I*b^2*x^2
- 36*I*a^2)*cosh(b*x + a)^2 - 6*I*a^2)*sinh(b*x + a)^2 + 6*I*a^2 + ((-36*I*
b^2*x^2 + 36*I*a^2)*cosh(b*x + a)^5 + (24*I*b^2*x^2 - 24*I*a^2)*cosh(b*x +
a)^3 + (12*I*b^2*x^2 - 12*I*a^2)*cosh(b*x + a))*sinh(b*x + a))*log(I*cosh(b
*x + a) + I*sinh(b*x + a) + 1) + ((6*I*b^2*x^2 - 6*I*a^2)*cosh(b*x + a)^6 +
 (36*I*b^2*x^2 - 36*I*a^2)*cosh(b*x + a)*sinh(b*x + a)^5 + (6*I*b^2*x^2 - 6
*I*a^2)*sinh(b*x + a)^6 + (-6*I*b^2*x^2 + 6*I*a^2)*cosh(b*x + a)^4 + (-6*I*
b^2*x^2 + (90*I*b^2*x^2 - 90*I*a^2)*cosh(b*x + a)^2 + 6*I*a^2)*sinh(b*x + a
)^4 + 6*I*b^2*x^2 + ((120*I*b^2*x^2 - 120*I*a^2)*cosh(b*x + a)^3 + (-24*I*b
^2*x^2 + 24*I*a^2)*cosh(b*x + a))*sinh(b*x + a)^3 + (-6*I*b^2*x^2 + 6*I*a^2
)*cosh(b*x + a)^2 + ((90*I*b^2*x^2 - 90*I*a^2)*cosh(b*x + a)^4 - 6*I*b^2*x^
2 + (-36*I*b^2*x^2 + 36*I*a^2)*cosh(b*x + a)^2 + 6*I*a^2)*sinh(b*x + a)^2 -
 6*I*a^2 + ((36*I*b^2*x^2 - 36*I*a^2)*cosh(b*x + a)^5 + (-24*I*b^2*x^2 + 24
*I*a^2)*cosh(b*x + a)^3 + (-12*I*b^2*x^2 + 12*I*a^2)*cosh(b*x + a))*sinh(b*
x + a))*log(-I*cosh(b*x + a) - I*sinh(b*x + a) + 1) - 3*((b^3*x^3 + a^3 - 2
*b*x - 2*a)*cosh(b*x + a)^6 + 6*(b^3*x^3 + a^3 - 2*b*x - 2*a)*cosh(b*x + a)
*sinh(b*x + a)^5 + (b^3*x^3 + a^3 - 2*b*x - 2*a)*sinh(b*x + a)^6 + b^3*x^3
- (b^3*x^3 + a^3 - 2*b*x - 2*a)*cosh(b*x + a)^4 - (b^3*x^3 + a^3 - 15*(b^3*
x^3 + a^3 - 2*b*x - 2*a)*cosh(b*x + a)^2 - 2*b*x - 2*a)*sinh(b*x + a)^4 + 4
*(5*(b^3*x^3 + a^3 - 2*b*x - 2*a)*cosh(b*x + a)^3 - (b^3*x^3 + a^3 - 2*b*x
- 2*a)*cosh(b*x + a))*sinh(b*x + a)^3 + a^3 - (b^3*x^3 + a^3 - 2*b*x - 2*a)
*cosh(b*x + a)^2 - (b^3*x^3 - 15*(b^3*x^3 + a^3 - 2*b*x - 2*a)*cosh(b*x + a
)^4 + a^3 + 6*(b^3*x^3 + a^3 - 2*b*x - 2*a)*cosh(b*x + a)^2 - 2*b*x - 2*a)*
sinh(b*x + a)^2 - 2*b*x + 2*(3*(b^3*x^3 + a^3 - 2*b*x - 2*a)*cosh(b*x + a)^
5 - 2*(b^3*x^3 + a^3 - 2*b*x - 2*a)*cosh(b*x + a)^3 - (b^3*x^3 + a^3 - 2*b*
x - 2*a)*cosh(b*x + a))*sinh(b*x + a) - 2*a)*log(-cosh(b*x + a) - sinh(b*x
+ a) + 1) - 18*(cosh(b*x + a)^6 + 6*cosh(b*x + a)*sinh(b*x + a)^5 + sinh(b*
x + a)^6 + (15*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^4 - cosh(b*x + a)^4 + 4*(
5*cosh(b*x + a)^3 - cosh(b*x + a))*sinh(b*x + a)^3 + (15*cosh(b*x + a)^4 -
6*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^2 - cosh(b*x + a)^2 + 2*(3*cosh(b*x +
a)^5 - 2*cosh(b*x + a)^3 - cosh(b*x + a))*sinh(b*x + a) + 1)*polylog(4, cos
h(b*x + a) + sinh(b*x + a)) + 18*(cosh(b*x + a)^6 + 6*cosh(b*x + a)*sinh(b*
x + a)^5 + sinh(b*x + a)^6 + (15*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^4 - cos
h(b*x + a)^4 + 4*(5*cosh(b*x + a)^3 - cosh(b*x + a))*sinh(b*x + a)^3 + (15*
cosh(b*x + a)^4 - 6*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^2 - cosh(b*x + a)^2
+ 2*(3*cosh(b*x + a)^5 - 2*cosh(b*x + a)^3 - cosh(b*x + a))*sinh(b*x + a) +
 1)*polylog(4, -cosh(b*x + a) - sinh(b*x + a)) + 18*(b*x*cosh(b*x + a)^6 +
6*b*x*cosh(b*x + a)*sinh(b*x + a)^5 + b*x*sinh(b*x + a)^6 - b*x*cosh(b*x +
a)^4 + (15*b*x*cosh(b*x + a)^2 - b*x)*sinh(b*x + a)^4 - b*x*cosh(b*x + a)^2
 + 4*(5*b*x*cosh(b*x + a)^3 - b*x*cosh(b*x + a))*sinh(b*x + a)^3 + (15*b*x*
cosh(b*x + a)^4 - 6*b*x*cosh(b*x + a)^2 - b*x)*sinh(b*x + a)^2 + b*x + 2*(3
```

```
*b*x*cosh(b*x + a)^5 - 2*b*x*cosh(b*x + a)^3 - b*x*cosh(b*x + a))*sinh(b*x
+ a))*polylog(3, cosh(b*x + a) + sinh(b*x + a)) + (-12*I*cosh(b*x + a)^6 -
72*I*cosh(b*x + a)*sinh(b*x + a)^5 - 12*I*sinh(b*x + a)^6 + (-180*I*cosh(b*
x + a)^2 + 12*I)*sinh(b*x + a)^4 + 12*I*cosh(b*x + a)^4 + (-240*I*cosh(b*x
+ a)^3 + 48*I*cosh(b*x + a))*sinh(b*x + a)^3 + (-180*I*cosh(b*x + a)^4 + 72
*I*cosh(b*x + a)^2 + 12*I)*sinh(b*x + a)^2 + 12*I*cosh(b*x + a)^2 + (-72*I*
cosh(b*x + a)^5 + 48*I*cosh(b*x + a)^3 + 24*I*cosh(b*x + a))*sinh(b*x + a)
- 12*I)*polylog(3, I*cosh(b*x + a) + I*sinh(b*x + a)) + (12*I*cosh(b*x + a)
^6 + 72*I*cosh(b*x + a)*sinh(b*x + a)^5 + 12*I*sinh(b*x + a)^6 + (180*I*cos
h(b*x + a)^2 - 12*I)*sinh(b*x + a)^4 - 12*I*cosh(b*x + a)^4 + (240*I*cosh(b
*x + a)^3 - 48*I*cosh(b*x + a))*sinh(b*x + a)^3 + (180*I*cosh(b*x + a)^4 -
72*I*cosh(b*x + a)^2 - 12*I)*sinh(b*x + a)^2 - 12*I*cosh(b*x + a)^2 + (72*I
*cosh(b*x + a)^5 - 48*I*cosh(b*x + a)^3 - 24*I*cosh(b*x + a))*sinh(b*x + a)
 + 12*I)*polylog(3, -I*cosh(b*x + a) - I*sinh(b*x + a)) - 18*(b*x*cosh(b*x
+ a)^6 + 6*b*x*cosh(b*x + a)*sinh(b*x + a)^5 + b*x*sinh(b*x + a)^6 - b*x*co
sh(b*x + a)^4 + (15*b*x*cosh(b*x + a)^2 - b*x)*sinh(b*x + a)^4 - b*x*cosh(b
*x + a)^2 + 4*(5*b*x*cosh(b*x + a)^3 - b*x*cosh(b*x + a))*sinh(b*x + a)^3 +
 (15*b*x*cosh(b*x + a)^4 - 6*b*x*cosh(b*x + a)^2 - b*x)*sinh(b*x + a)^2 + b
*x + 2*(3*b*x*cosh(b*x + a)^5 - 2*b*x*cosh(b*x + a)^3 - b*x*cosh(b*x + a))*
sinh(b*x + a))*polylog(3, -cosh(b*x + a) - sinh(b*x + a)) + 6*(2*b^3*x^3*co
sh(b*x + a)^2 - b^3*x^3 - 5*(b^3*x^3 + b^2*x^2)*cosh(b*x + a)^4 + b^2*x^2)*
sinh(b*x + a))/(b^4*cosh(b*x + a)^6 + 6*b^4*cosh(b*x + a)*sinh(b*x + a)^5 +
 b^4*sinh(b*x + a)^6 - b^4*cosh(b*x + a)^4 - b^4*cosh(b*x + a)^2 + (15*b^4*
cosh(b*x + a)^2 - b^4)*sinh(b*x + a)^4 + b^4 + 4*(5*b^4*cosh(b*x + a)^3 - b
^4*cosh(b*x + a))*sinh(b*x + a)^3 + (15*b^4*cosh(b*x + a)^4 - 6*b^4*cosh(b*
x + a)^2 - b^4)*sinh(b*x + a)^2 + 2*(3*b^4*cosh(b*x + a)^5 - 2*b^4*cosh(b*x
 + a)^3 - b^4*cosh(b*x + a))*sinh(b*x + a))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3 \operatorname{csch}(bx+a)^3 \operatorname{sech}(bx+a)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*csch(b*x+a)^3*sech(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate(x^3*csch(b*x + a)^3*sech(b*x + a)^2, x)
```

maple [F] time = 2.10, size = 0, normalized size = 0.00

$$\int x^3 \operatorname{csch}(bx+a)^3 \operatorname{sech}(bx+a)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*csch(b*x+a)^3*sech(b*x+a)^2,x)
```

[Out] int(x^3*csch(b*x+a)^3*sech(b*x+a)^2,x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{2\,bx^3e^{(3\,bx+3\,a)} - 3\left(bx^3e^{(5\,a)} + x^2e^{(5\,a)}\right)e^{(5\,bx)} - 3\left(bx^3e^a - x^2e^a\right)e^{(bx)}}{b^2e^{(6\,bx+6\,a)} - b^2e^{(4\,bx+4\,a)} - b^2e^{(2\,bx+2\,a)} + b^2} + \frac{3\left(b^3x^3\log\left(e^{(bx+a)} + 1\right) + 3\,b^2x^2\mathrm{Li}_2\left(-e^{(bx+a)}\right.\right.}{2\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*csch(b*x+a)^3*sech(b*x+a)^2,x, algorithm="maxima")

[Out] (2*b*x^3*e^(3*b*x + 3*a) - 3*(b*x^3*e^(5*a) + x^2*e^(5*a))*e^(5*b*x) - 3*(b*x^3*e^a - x^2*e^a)*e^(b*x))/(b^2*e^(6*b*x + 6*a) - b^2*e^(4*b*x + 4*a) - b^2*e^(2*b*x + 2*a) + b^2) + 3/2*(b^3*x^3*log(e^(b*x + a) + 1) + 3*b^2*x^2*dilog(-e^(b*x + a)) - 6*b*x*polylog(3, -e^(b*x + a)) + 6*polylog(4, -e^(b*x + a)))/b^4 - 3/2*(b^3*x^3*log(-e^(b*x + a) + 1) + 3*b^2*x^2*dilog(e^(b*x + a)) - 6*b*x*polylog(3, e^(b*x + a)) + 6*polylog(4, e^(b*x + a)))/b^4 - 3*(b*x*log(e^(b*x + a) + 1) + dilog(-e^(b*x + a)))/b^4 + 3*(b*x*log(-e^(b*x + a) + 1) + dilog(e^(b*x + a)))/b^4 + 96*integrate(1/16*x^2*e^(b*x + a)/(b*e^(2*b*x + 2*a) + b), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3}{\cosh\left(a+b\,x\right)^2\,\sinh\left(a+b\,x\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3/(cosh(a + b*x)^2*sinh(a + b*x)^3),x)

[Out] int(x^3/(cosh(a + b*x)^2*sinh(a + b*x)^3), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3\operatorname{csch}^3\left(a+bx\right)\operatorname{sech}^2\left(a+bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**3*csch(b*x+a)**3*sech(b*x+a)**2,x)

[Out] Integral(x**3*csch(a + b*x)**3*sech(a + b*x)**2, x)

3.516 $\int x^2 \mathrm{csch}^3(a+bx)\mathrm{sech}^2(a+bx)\,dx$

Optimal. Leaf size=197

$$-\frac{2i\mathrm{Li}_2\left(-ie^{a+bx}\right)}{b^3}+\frac{2i\mathrm{Li}_2\left(ie^{a+bx}\right)}{b^3}-\frac{3\mathrm{Li}_3\left(-e^{a+bx}\right)}{b^3}+\frac{3\mathrm{Li}_3\left(e^{a+bx}\right)}{b^3}-\frac{\tanh^{-1}(\cosh(a+bx))}{b^3}+\frac{3x\mathrm{Li}_2\left(-e^{a+bx}\right)}{b^2}-\frac{3x\mathrm{Li}_2\left(e^{a}\right.}{b^2}$$

```
[Out] 4*x*arctan(exp(b*x+a))/b^2+3*x^2*arctanh(exp(b*x+a))/b-arctanh(cosh(b*x+a))
/b^3-x*csch(b*x+a)/b^2+3*x*polylog(2,-exp(b*x+a))/b^2-2*I*polylog(2,-I*exp(
b*x+a))/b^3+2*I*polylog(2,I*exp(b*x+a))/b^3-3*x*polylog(2,exp(b*x+a))/b^2-3
*polylog(3,-exp(b*x+a))/b^3+3*polylog(3,exp(b*x+a))/b^3-3/2*x^2*sech(b*x+a)
/b-1/2*x^2*csch(b*x+a)^2*sech(b*x+a)/b
```

Rubi [A] time = 0.49, antiderivative size = 197, normalized size of antiderivative = 1.00, number of steps used = 29, number of rules used = 19, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.950, Rules used = {2622, 288, 321, 207, 5462, 14, 6273, 12, 4182, 2531, 2282, 6589, 6742, 4180, 2279, 2391, 2621, 5203, 3770}

$$\frac{3x\mathrm{PolyLog}\left(2,-e^{a+bx}\right)}{b^2}-\frac{3x\mathrm{PolyLog}\left(2,e^{a+bx}\right)}{b^2}-\frac{2i\mathrm{PolyLog}\left(2,-ie^{a+bx}\right)}{b^3}+\frac{2i\mathrm{PolyLog}\left(2,ie^{a+bx}\right)}{b^3}-\frac{3\mathrm{PolyLog}\left(3,-e\right.}{b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Csch[a + b*x]^3*Sech[a + b*x]^2,x]
```

```
[Out] (4*x*ArcTan[E^(a + b*x)])/b^2 + (3*x^2*ArcTanh[E^(a + b*x)])/b - ArcTanh[Co
sh[a + b*x]]/b^3 - (x*Csch[a + b*x])/b^2 + (3*x*PolyLog[2, -E^(a + b*x)])/b
^2 - ((2*I)*PolyLog[2, (-I)*E^(a + b*x)])/b^3 + ((2*I)*PolyLog[2, I*E^(a +
b*x)])/b^3 - (3*x*PolyLog[2, E^(a + b*x)])/b^2 - (3*PolyLog[3, -E^(a + b*x)
])/b^3 + (3*PolyLog[3, E^(a + b*x)])/b^3 - (3*x^2*Sech[a + b*x])/(2*b) - (x
^2*Csch[a + b*x]^2*Sech[a + b*x])/(2*b)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
```

```
, g, n}, x] && GtQ[m, 0]
```

Rule 2621

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> -Dist[(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5203

```
Int[ArcTan[u_], x_Symbol] :> Simp[x*ArcTan[u], x] - Int[SimplifyIntegrand[(
x*D[u, x])/(1 + u^2), x], x] /; InverseFunctionFreeQ[u, x]
```

Rule 5462

```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> With[{u = IntHide[Csch[a + b*x]^n*Sech[a +
b*x]^p, x]}, Dist[(c + d*x)^m, u, x] - Dist[d*m, Int[(c + d*x)^(m - 1)*u, x
], x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n,
p]
```

Rule 6273

```
Int[((a_.) + ArcTanh[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[((c + d*x)^(m + 1)*(a + b*ArcTanh[u]))/(d*(m + 1)), x] - Dist[b/(d*(m +
1)), Int[SimplifyIntegrand[((c + d*x)^(m + 1)*D[u, x])/(1 - u^2), x], x], x
] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && InverseFunctionFreeQ[u, x]
&&  !FunctionOfQ[(c + d*x)^(m + 1), u, x] && FalseQ[PowerVariableExpn[u, m
+ 1, x]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int x^2\text{csch}^3(a+bx)\text{sech}^2(a+bx)\,dx = \frac{3x^2\tanh^{-1}(\cosh(a+bx))}{2b} - \frac{3x^2\text{sech}(a+bx)}{2b} - \frac{x^2\text{csch}^2(a+bx)\text{sech}(a+}{2b}$$

$$= \frac{3x^2\tanh^{-1}(\cosh(a+bx))}{2b} - \frac{3x^2\text{sech}(a+bx)}{2b} - \frac{x^2\text{csch}^2(a+bx)\text{sech}(a+}{2b}$$

$$= \frac{3x^2\tanh^{-1}(\cosh(a+bx))}{2b} - \frac{3x^2\text{sech}(a+bx)}{2b} - \frac{x^2\text{csch}^2(a+bx)\text{sech}(a+}{2b}$$

$$= -\frac{3x^2\text{sech}(a+bx)}{2b} - \frac{x^2\text{csch}^2(a+bx)\text{sech}(a+bx)}{2b} + \frac{\int\left(3x\text{sech}(a+bx)+\right.}{}$$

$$= -\frac{3x^2\text{sech}(a+bx)}{2b} - \frac{x^2\text{csch}^2(a+bx)\text{sech}(a+bx)}{2b} - \frac{3}{2}\int x^2\text{csch}(a+bx)\,d$$

$$= \frac{6x\tan^{-1}\left(e^{a+bx}\right)}{b^2} - \frac{x\tan^{-1}(\sinh(a+bx))}{b^2} + \frac{3x^2\tanh^{-1}\left(e^{a+bx}\right)}{b} - \frac{x\text{csch}(a}{b^2}$$

$$= \frac{6x\tan^{-1}\left(e^{a+bx}\right)}{b^2} - \frac{x\tan^{-1}(\sinh(a+bx))}{b^2} + \frac{3x^2\tanh^{-1}\left(e^{a+bx}\right)}{b} - \frac{x\text{csch}(a}{b^2}$$

$$= \frac{6x\tan^{-1}\left(e^{a+bx}\right)}{b^2} + \frac{3x^2\tanh^{-1}\left(e^{a+bx}\right)}{b} - \frac{\tanh^{-1}(\cosh(a+bx))}{b^3} - \frac{x\text{csch}(a}{b^2}$$

$$= \frac{6x\tan^{-1}\left(e^{a+bx}\right)}{b^2} + \frac{3x^2\tanh^{-1}\left(e^{a+bx}\right)}{b} - \frac{\tanh^{-1}(\cosh(a+bx))}{b^3} - \frac{x\text{csch}(a}{b^2}$$

$$= \frac{4x\tan^{-1}\left(e^{a+bx}\right)}{b^2} + \frac{3x^2\tanh^{-1}\left(e^{a+bx}\right)}{b} - \frac{\tanh^{-1}(\cosh(a+bx))}{b^3} - \frac{x\text{csch}(a}{b^2}$$

$$= \frac{4x\tan^{-1}\left(e^{a+bx}\right)}{b^2} + \frac{3x^2\tanh^{-1}\left(e^{a+bx}\right)}{b} - \frac{\tanh^{-1}(\cosh(a+bx))}{b^3} - \frac{x\text{csch}(a}{b^2}$$

$$= \frac{4x\tan^{-1}\left(e^{a+bx}\right)}{b^2} + \frac{3x^2\tanh^{-1}\left(e^{a+bx}\right)}{b} - \frac{\tanh^{-1}(\cosh(a+bx))}{b^3} - \frac{x\text{csch}(a}{b^2}$$

Mathematica [B] time = 8.07, size = 441, normalized size = 2.24

$$\frac{2\left(-i\left(\text{Li}_2\left(-e^{i\left(-ia-ibx+\frac{\pi}{2}\right)}\right)-\text{Li}_2\left(e^{i\left(-ia-ibx+\frac{\pi}{2}\right)}\right)\right)-\left(\left(-ia-ibx+\frac{\pi}{2}\right)\left(\log\left(1-e^{i\left(-ia-ibx+\frac{\pi}{2}\right)}\right)-\log\left(1+e^{i\left(-ia-ibx+\frac{\pi}{2}\right)}\right)\right.\right.\right.}{b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Csch[a + b*x]^3*Sech[a + b*x]^2,x]
```

```
[Out] -((x*Csch[a])/b^2) - (x^2*Csch[a/2 + (b*x)/2]^2)/(8*b) + (2*(-(((-I)*a + Pi
/2 - I*b*x)*(Log[1 - E^(I*((-I)*a + Pi/2 - I*b*x))] - Log[1 + E^(I*((-I)*a
+ Pi/2 - I*b*x))])) + ((-I)*a + Pi/2)*Log[Tan[((-I)*a + Pi/2 - I*b*x)/2]] -
 I*(PolyLog[2, -E^(I*((-I)*a + Pi/2 - I*b*x))] - PolyLog[2, E^(I*((-I)*a +
Pi/2 - I*b*x))])))/b^3 + (2*Log[1 - E^(a + b*x)] - 3*b^2*x^2*Log[1 - E^(a +
 b*x)] - 2*Log[1 + E^(a + b*x)] + 3*b^2*x^2*Log[1 + E^(a + b*x)] + 6*b*x*Po
lyLog[2, -E^(a + b*x)] - 6*b*x*PolyLog[2, E^(a + b*x)] - 6*PolyLog[3, -E^(a
 + b*x)] + 6*PolyLog[3, E^(a + b*x)])/(2*b^3) - (x^2*Sech[a/2 + (b*x)/2]^2)
/(8*b) - (x^2*Sech[a + b*x])/b + (x*Csch[a/2]*Csch[a/2 + (b*x)/2]*Sinh[(b*x
)/2])/(2*b^2) + (x*Sech[a/2]*Sech[a/2 + (b*x)/2]*Sinh[(b*x)/2])/(2*b^2)
```

fricas [C] time = 0.55, size = 3772, normalized size = 19.15

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*csch(b*x+a)^3*sech(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(4*b^2*x^2*cosh(b*x + a)^3 - 2*(3*b^2*x^2 + 2*b*x)*cosh(b*x + a)^5 - 10
*(3*b^2*x^2 + 2*b*x)*cosh(b*x + a)*sinh(b*x + a)^4 - 2*(3*b^2*x^2 + 2*b*x)*
sinh(b*x + a)^5 + 4*(b^2*x^2 - 5*(3*b^2*x^2 + 2*b*x)*cosh(b*x + a)^2)*sinh(
b*x + a)^3 + 4*(3*b^2*x^2*cosh(b*x + a) - 5*(3*b^2*x^2 + 2*b*x)*cosh(b*x +
a)^3)*sinh(b*x + a)^2 - 2*(3*b^2*x^2 - 2*b*x)*cosh(b*x + a) - 6*(b*x*cosh(b
*x + a)^6 + 6*b*x*cosh(b*x + a)*sinh(b*x + a)^5 + b*x*sinh(b*x + a)^6 - b*x
*cosh(b*x + a)^4 + (15*b*x*cosh(b*x + a)^2 - b*x)*sinh(b*x + a)^4 - b*x*cos
h(b*x + a)^2 + 4*(5*b*x*cosh(b*x + a)^3 - b*x*cosh(b*x + a))*sinh(b*x + a)^
3 + (15*b*x*cosh(b*x + a)^4 - 6*b*x*cosh(b*x + a)^2 - b*x)*sinh(b*x + a)^2
+ b*x + 2*(3*b*x*cosh(b*x + a)^5 - 2*b*x*cosh(b*x + a)^3 - b*x*cosh(b*x + a
))*sinh(b*x + a))*dilog(cosh(b*x + a) + sinh(b*x + a)) + (4*I*cosh(b*x + a)
^6 + 24*I*cosh(b*x + a)*sinh(b*x + a)^5 + 4*I*sinh(b*x + a)^6 + (60*I*cosh(
b*x + a)^2 - 4*I)*sinh(b*x + a)^4 - 4*I*cosh(b*x + a)^4 + (80*I*cosh(b*x +
a)^3 - 16*I*cosh(b*x + a))*sinh(b*x + a)^3 + (60*I*cosh(b*x + a)^4 - 24*I*c
osh(b*x + a)^2 - 4*I)*sinh(b*x + a)^2 - 4*I*cosh(b*x + a)^2 + (24*I*cosh(b*
x + a)^5 - 16*I*cosh(b*x + a)^3 - 8*I*cosh(b*x + a))*sinh(b*x + a) + 4*I)*d
ilog(I*cosh(b*x + a) + I*sinh(b*x + a)) + (-4*I*cosh(b*x + a)^6 - 24*I*cosh
(b*x + a)*sinh(b*x + a)^5 - 4*I*sinh(b*x + a)^6 + (-60*I*cosh(b*x + a)^2 +
4*I)*sinh(b*x + a)^4 + 4*I*cosh(b*x + a)^4 + (-80*I*cosh(b*x + a)^3 + 16*I*
cosh(b*x + a))*sinh(b*x + a)^3 + (-60*I*cosh(b*x + a)^4 + 24*I*cosh(b*x + a
)^2 + 4*I)*sinh(b*x + a)^2 + 4*I*cosh(b*x + a)^2 + (-24*I*cosh(b*x + a)^5 +
 16*I*cosh(b*x + a)^3 + 8*I*cosh(b*x + a))*sinh(b*x + a) - 4*I)*dilog(-I*co
sh(b*x + a) - I*sinh(b*x + a)) + 6*(b*x*cosh(b*x + a)^6 + 6*b*x*cosh(b*x +
a)*sinh(b*x + a)^5 + b*x*sinh(b*x + a)^6 - b*x*cosh(b*x + a)^4 + (15*b*x*co
sh(b*x + a)^2 - b*x)*sinh(b*x + a)^4 - b*x*cosh(b*x + a)^2 + 4*(5*b*x*cosh(
b*x + a)^3 - b*x*cosh(b*x + a))*sinh(b*x + a)^3 + (15*b*x*cosh(b*x + a)^4 -
 6*b*x*cosh(b*x + a)^2 - b*x)*sinh(b*x + a)^2 + b*x + 2*(3*b*x*cosh(b*x + a
```

)^5 - 2*b*x*cosh(b*x + a)^3 - b*x*cosh(b*x + a))*sinh(b*x + a))*dilog(-cosh (b*x + a) - sinh(b*x + a)) + ((3*b^2*x^2 - 2)*cosh(b*x + a)^6 + 6*(3*b^2*x^2 - 2)*cosh(b*x + a)*sinh(b*x + a)^5 + (3*b^2*x^2 - 2)*sinh(b*x + a)^6 - (3*b^2*x^2 - 2)*cosh(b*x + a)^4 - (3*b^2*x^2 - 15*(3*b^2*x^2 - 2)*cosh(b*x + a)^2 - 2)*sinh(b*x + a)^4 + 3*b^2*x^2 + 4*(5*(3*b^2*x^2 - 2)*cosh(b*x + a)^3 - (3*b^2*x^2 - 2)*cosh(b*x + a))*sinh(b*x + a)^3 - (3*b^2*x^2 - 2)*cosh(b*x + a)^2 + (15*(3*b^2*x^2 - 2)*cosh(b*x + a)^4 - 3*b^2*x^2 - 6*(3*b^2*x^2 - 2)*cosh(b*x + a)^2 + 2)*sinh(b*x + a)^2 + 2*(3*(3*b^2*x^2 - 2)*cosh(b*x + a)^5 - 2*(3*b^2*x^2 - 2)*cosh(b*x + a)^3 - (3*b^2*x^2 - 2)*cosh(b*x + a))*sinh(b*x + a) - 2)*log(cosh(b*x + a) + sinh(b*x + a) + 1) + (-4*I*a*cosh(b*x + a)^6 - 24*I*a*cosh(b*x + a)*sinh(b*x + a)^5 - 4*I*a*sinh(b*x + a)^6 + 4*I*a*cosh(b*x + a)^4 + (-60*I*a*cosh(b*x + a)^2 + 4*I*a)*sinh(b*x + a)^4 + (-80*I*a*cosh(b*x + a)^3 + 16*I*a*cosh(b*x + a))*sinh(b*x + a)^3 + 4*I*a*cosh(b*x + a)^2 + (-60*I*a*cosh(b*x + a)^4 + 24*I*a*cosh(b*x + a)^2 + 4*I*a)*sinh(b*x + a)^2 + (-24*I*a*cosh(b*x + a)^5 + 16*I*a*cosh(b*x + a)^3 + 8*I*a*cosh(b*x + a))*sinh(b*x + a) - 4*I*a)*log(cosh(b*x + a) + sinh(b*x + a) + I) + (4*I*a*cosh(b*x + a)^6 + 24*I*a*cosh(b*x + a)*sinh(b*x + a)^5 + 4*I*a*sinh(b*x + a)^6 - 4*I*a*cosh(b*x + a)^4 + (60*I*a*cosh(b*x + a)^2 - 4*I*a)*sinh(b*x + a)^4 + (80*I*a*cosh(b*x + a)^3 - 16*I*a*cosh(b*x + a))*sinh(b*x + a)^3 - 4*I*a*cosh(b*x + a)^2 + (60*I*a*cosh(b*x + a)^4 - 24*I*a*cosh(b*x + a)^2 - 4*I*a)*sinh(b*x + a)^2 + (24*I*a*cosh(b*x + a)^5 - 16*I*a*cosh(b*x + a)^3 - 8*I*a*cosh(b*x + a))*sinh(b*x + a) + 4*I*a)*log(cosh(b*x + a) + sinh(b*x + a) - I) - ((3*a^2 - 2)*cosh(b*x + a)^6 + 6*(3*a^2 - 2)*cosh(b*x + a)*sinh(b*x + a)^5 + (3*a^2 - 2)*sinh(b*x + a)^6 - (3*a^2 - 2)*cosh(b*x + a)^4 + (15*(3*a^2 - 2)*cosh(b*x + a)^2 - 3*a^2 + 2)*sinh(b*x + a)^4 + 4*(5*(3*a^2 - 2)*cosh(b*x + a)^3 - (3*a^2 - 2)*cosh(b*x + a))*sinh(b*x + a)^3 - (3*a^2 - 2)*cosh(b*x + a)^2 + (15*(3*a^2 - 2)*cosh(b*x + a)^4 - 6*(3*a^2 - 2)*cosh(b*x + a)^2 - 3*a^2 + 2)*sinh(b*x + a)^2 + 3*a^2 + 2*(3*(3*a^2 - 2)*cosh(b*x + a)^5 - 2*(3*a^2 - 2)*cosh(b*x + a)^3 - (3*a^2 - 2)*cosh(b*x + a))*sinh(b*x + a) - 2)*log(cosh(b*x + a) + sinh(b*x + a) - 1) + ((-4*I*b*x - 4*I*a)*cosh(b*x + a)^6 + (-24*I*b*x - 24*I*a)*cosh(b*x + a)*sinh(b*x + a)^5 + (-4*I*b*x - 4*I*a)*sinh(b*x + a)^6 + (4*I*b*x + 4*I*a)*cosh(b*x + a)^4 + ((-60*I*b*x - 60*I*a)*cosh(b*x + a)^2 + 4*I*b*x + 4*I*a)*sinh(b*x + a)^4 + ((-80*I*b*x - 80*I*a)*cosh(b*x + a)^3 + (16*I*b*x + 16*I*a)*cosh(b*x + a))*sinh(b*x + a)^3 + (4*I*b*x + 4*I*a)*cosh(b*x + a)^2 + ((-60*I*b*x - 60*I*a)*cosh(b*x + a)^4 + (24*I*b*x + 24*I*a)*cosh(b*x + a)^2 + 4*I*b*x + 4*I*a)*sinh(b*x + a)^2 - 4*I*b*x + ((-24*I*b*x - 24*I*a)*cosh(b*x + a)^5 + (16*I*b*x + 16*I*a)*cosh(b*x + a)^3 + (8*I*b*x + 8*I*a)*cosh(b*x + a))*sinh(b*x + a) - 4*I*a)*log(I*cosh(b*x + a) + I*sinh(b*x + a) + 1) + ((4*I*b*x + 4*I*a)*cosh(b*x + a)^6 + (24*I*b*x + 24*I*a)*cosh(b*x + a)*sinh(b*x + a)^5 + (4*I*b*x + 4*I*a)*sinh(b*x + a)^6 + (-4*I*b*x - 4*I*a)*cosh(b*x + a)^4 + ((60*I*b*x + 60*I*a)*cosh(b*x + a)^2 - 4*I*b*x - 4*I*a)*sinh(b*x + a)^4 + ((80*I*b*x + 80*I*a)*cosh(b*x + a)^3 + (-16*I*b*x - 16*I*a)*cosh(b*x + a))*sinh(b*x + a)^3 + (-4*I*b*x - 4*I*a)*cosh(b*x + a)^2 + ((60*I*b*x + 60*I*a)*cosh(b*x + a)^4 + (-24*I*b*x - 24*I*a)*cosh(b*x + a)^2 - 4*I*b*x - 4*I*a)*sinh(b*

```
x + a)^2 + 4*I*b*x + ((24*I*b*x + 24*I*a)*cosh(b*x + a)^5 + (-16*I*b*x - 16
*I*a)*cosh(b*x + a)^3 + (-8*I*b*x - 8*I*a)*cosh(b*x + a))*sinh(b*x + a) + 4
*I*a)*log(-I*cosh(b*x + a) - I*sinh(b*x + a) + 1) - 3*((b^2*x^2 - a^2)*cosh
(b*x + a)^6 + 6*(b^2*x^2 - a^2)*cosh(b*x + a)*sinh(b*x + a)^5 + (b^2*x^2 -
a^2)*sinh(b*x + a)^6 - (b^2*x^2 - a^2)*cosh(b*x + a)^4 - (b^2*x^2 - 15*(b^2
*x^2 - a^2)*cosh(b*x + a)^2 - a^2)*sinh(b*x + a)^4 + b^2*x^2 + 4*(5*(b^2*x^
2 - a^2)*cosh(b*x + a)^3 - (b^2*x^2 - a^2)*cosh(b*x + a))*sinh(b*x + a)^3 -
 (b^2*x^2 - a^2)*cosh(b*x + a)^2 + (15*(b^2*x^2 - a^2)*cosh(b*x + a)^4 - b^
2*x^2 - 6*(b^2*x^2 - a^2)*cosh(b*x + a)^2 + a^2)*sinh(b*x + a)^2 - a^2 + 2*
(3*(b^2*x^2 - a^2)*cosh(b*x + a)^5 - 2*(b^2*x^2 - a^2)*cosh(b*x + a)^3 - (b
^2*x^2 - a^2)*cosh(b*x + a))*sinh(b*x + a))*log(-cosh(b*x + a) - sinh(b*x +
 a) + 1) + 6*(cosh(b*x + a)^6 + 6*cosh(b*x + a)*sinh(b*x + a)^5 + sinh(b*x
+ a)^6 + (15*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^4 - cosh(b*x + a)^4 + 4*(5*
cosh(b*x + a)^3 - cosh(b*x + a))*sinh(b*x + a)^3 + (15*cosh(b*x + a)^4 - 6*
cosh(b*x + a)^2 - 1)*sinh(b*x + a)^2 - cosh(b*x + a)^2 + 2*(3*cosh(b*x + a)
^5 - 2*cosh(b*x + a)^3 - cosh(b*x + a))*sinh(b*x + a) + 1)*polylog(3, cosh(
b*x + a) + sinh(b*x + a)) - 6*(cosh(b*x + a)^6 + 6*cosh(b*x + a)*sinh(b*x +
 a)^5 + sinh(b*x + a)^6 + (15*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^4 - cosh(b
*x + a)^4 + 4*(5*cosh(b*x + a)^3 - cosh(b*x + a))*sinh(b*x + a)^3 + (15*cos
h(b*x + a)^4 - 6*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^2 - cosh(b*x + a)^2 + 2
*(3*cosh(b*x + a)^5 - 2*cosh(b*x + a)^3 - cosh(b*x + a))*sinh(b*x + a) + 1)
*polylog(3, -cosh(b*x + a) - sinh(b*x + a)) + 2*(6*b^2*x^2*cosh(b*x + a)^2
- 5*(3*b^2*x^2 + 2*b*x)*cosh(b*x + a)^4 - 3*b^2*x^2 + 2*b*x)*sinh(b*x + a))
/(b^3*cosh(b*x + a)^6 + 6*b^3*cosh(b*x + a)*sinh(b*x + a)^5 + b^3*sinh(b*x
+ a)^6 - b^3*cosh(b*x + a)^4 - b^3*cosh(b*x + a)^2 + (15*b^3*cosh(b*x + a)^
2 - b^3)*sinh(b*x + a)^4 + 4*(5*b^3*cosh(b*x + a)^3 - b^3*cosh(b*x + a))*si
nh(b*x + a)^3 + b^3 + (15*b^3*cosh(b*x + a)^4 - 6*b^3*cosh(b*x + a)^2 - b^3
)*sinh(b*x + a)^2 + 2*(3*b^3*cosh(b*x + a)^5 - 2*b^3*cosh(b*x + a)^3 - b^3*
cosh(b*x + a))*sinh(b*x + a))
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*csch(b*x+a)^3*sech(b*x+a)^2,x, algorithm="giac")
```

```
[Out] Exception raised: AttributeError >> type
```

maple [F] time = 1.94, size = 0, normalized size = 0.00

$$\int x^{2} \mathrm{csch}\left(b x +a \right)^{3} \mathrm{sech}\left(b x +a \right)^{2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*csch(b*x+a)^3*sech(b*x+a)^2,x)
```

```
[Out] int(x^2*csch(b*x+a)^3*sech(b*x+a)^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{2\,bx^2e^{(3\,bx+3\,a)} - \left(3\,bx^2e^{(5\,a)} + 2\,xe^{(5\,a)}\right)e^{(5\,bx)} - \left(3\,bx^2e^{a} - 2\,xe^{a}\right)e^{(bx)}}{b^2e^{(6\,bx+6\,a)} - b^2e^{(4\,bx+4\,a)} - b^2e^{(2\,bx+2\,a)} + b^2} + \frac{3\left(b^2x^2\log\left(e^{(bx+a)}+1\right) + 2\,bx\mathrm{Li}_2\left(-e^{(bx+a)}\right)\right.}{2\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*csch(b*x+a)^3*sech(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] (2*b*x^2*e^(3*b*x + 3*a) - (3*b*x^2*e^(5*a) + 2*x*e^(5*a))*e^(5*b*x) - (3*b
*x^2*e^a - 2*x*e^a)*e^(b*x))/(b^2*e^(6*b*x + 6*a) - b^2*e^(4*b*x + 4*a) - b
^2*e^(2*b*x + 2*a) + b^2) + 3/2*(b^2*x^2*log(e^(b*x + a) + 1) + 2*b*x*dilog
(-e^(b*x + a)) - 2*polylog(3, -e^(b*x + a)))/b^3 - 3/2*(b^2*x^2*log(-e^(b*x
 + a) + 1) + 2*b*x*dilog(e^(b*x + a)) - 2*polylog(3, e^(b*x + a)))/b^3 - lo
g(e^(b*x + a) + 1)/b^3 + log(e^(b*x + a) - 1)/b^3 + 32*integrate(1/8*x*e^(b
*x + a)/(b*e^(2*b*x + 2*a) + b), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2}{\cosh\left(a+b\,x\right)^2\,\sinh\left(a+b\,x\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(cosh(a + b*x)^2*sinh(a + b*x)^3),x)
```

```
[Out] int(x^2/(cosh(a + b*x)^2*sinh(a + b*x)^3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2\operatorname{csch}^3\left(a+bx\right)\operatorname{sech}^2\left(a+bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*csch(b*x+a)**3*sech(b*x+a)**2,x)
```

```
[Out] Integral(x**2*csch(a + b*x)**3*sech(a + b*x)**2, x)
```

3.517 $\int x\operatorname{csch}^3(a+bx)\operatorname{sech}^2(a+bx)\,dx$

Optimal. Leaf size=109

$$\frac{3\text{Li}_2\left(-e^{a+bx}\right)}{2b^2}-\frac{3\text{Li}_2\left(e^{a+bx}\right)}{2b^2}-\frac{\text{csch}(a+bx)}{2b^2}+\frac{\tan^{-1}(\sinh(a+bx))}{b^2}+\frac{3x\tanh^{-1}\left(e^{a+bx}\right)}{b}-\frac{3x\text{sech}(a+bx)}{2b}-\frac{x\text{csch}^2(}{}$$

```
[Out] arctan(sinh(b*x+a))/b^2+3*x*arctanh(exp(b*x+a))/b-1/2*csch(b*x+a)/b^2+3/2*p
olylog(2,-exp(b*x+a))/b^2-3/2*polylog(2,exp(b*x+a))/b^2-3/2*x*sech(b*x+a)/b
-1/2*x*csch(b*x+a)^2*sech(b*x+a)/b
```

Rubi [A] time = 0.17, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 12, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {2622, 288, 321, 207, 5462, 6271, 12, 4182, 2279, 2391, 3770, 2621}

$$\frac{3\text{PolyLog}\left(2,-e^{a+bx}\right)}{2b^2}-\frac{3\text{PolyLog}\left(2,e^{a+bx}\right)}{2b^2}-\frac{\text{csch}(a+bx)}{2b^2}+\frac{\tan^{-1}(\sinh(a+bx))}{b^2}+\frac{3x\tanh^{-1}\left(e^{a+bx}\right)}{b}-\frac{3x\text{sech}(a}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[x*Csch[a + b*x]^3*Sech[a + b*x]^2,x]
```

```
[Out] ArcTan[Sinh[a + b*x]]/b^2 + (3*x*ArcTanh[E^(a + b*x)])/b - Csch[a + b*x]/(2
*b^2) + (3*PolyLog[2, -E^(a + b*x)])/(2*b^2) - (3*PolyLog[2, E^(a + b*x)])/
(2*b^2) - (3*x*Sech[a + b*x])/(2*b) - (x*Csch[a + b*x]^2*Sech[a + b*x])/(2*
b)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
```

```
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2621

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> -Dist[(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
```

```
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5462

```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> With[{u = IntHide[Csch[a + b*x]^n*Sech[a +
b*x]^p, x]}, Dist[(c + d*x)^m, u, x] - Dist[d*m, Int[(c + d*x)^(m - 1)*u, x
], x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n,
p]
```

Rule 6271

```
Int[ArcTanh[u_], x_Symbol] :> Simp[x*ArcTanh[u], x] - Int[SimplifyIntegrand
[(x*D[u, x])/(1 - u^2), x], x] /; InverseFunctionFreeQ[u, x]
```

Rubi steps

$$\int x\text{csch}^3(a+bx)\text{sech}^2(a+bx)\,dx = \frac{3x\tanh^{-1}(\cosh(a+bx))}{2b} - \frac{3x\text{sech}(a+bx)}{2b} - \frac{x\text{csch}^2(a+bx)\text{sech}(a+bx}{2b}$$

$$= \frac{3x\tanh^{-1}(\cosh(a+bx))}{2b} - \frac{3x\text{sech}(a+bx)}{2b} - \frac{x\text{csch}^2(a+bx)\text{sech}(a+bx}{2b}$$

$$= \frac{3\tan^{-1}(\sinh(a+bx))}{2b^2} - \frac{3x\text{sech}(a+bx)}{2b} - \frac{x\text{csch}^2(a+bx)\text{sech}(a+bx)}{2b} -$$

$$= \frac{3\tan^{-1}(\sinh(a+bx))}{2b^2} - \frac{\text{csch}(a+bx)}{2b^2} - \frac{3x\text{sech}(a+bx)}{2b} - \frac{x\text{csch}^2(a+bx}{2}$$

$$= \frac{\tan^{-1}(\sinh(a+bx))}{b^2} + \frac{3x\tanh^{-1}\left(e^{a+bx}\right)}{b} - \frac{\text{csch}(a+bx)}{2b^2} - \frac{3x\text{sech}(a+b}{2b}$$

$$= \frac{\tan^{-1}(\sinh(a+bx))}{b^2} + \frac{3x\tanh^{-1}\left(e^{a+bx}\right)}{b} - \frac{\text{csch}(a+bx)}{2b^2} - \frac{3x\text{sech}(a+b}{2b}$$

$$= \frac{\tan^{-1}(\sinh(a+bx))}{b^2} + \frac{3x\tanh^{-1}\left(e^{a+bx}\right)}{b} - \frac{\text{csch}(a+bx)}{2b^2} + \frac{3\text{Li}_2\left(-e^{a+bx}\right)}{2b^2}$$

Mathematica [A] time = 2.99, size = 168, normalized size = 1.54

$$-\frac{12\left(\text{Li}_2\left(-e^{-a-bx}\right)-\text{Li}_2\left(e^{-a-bx}\right)\right)+12(a+bx)\left(\log\left(1-e^{-a-bx}\right)-\log\left(e^{-a-bx}+1\right)\right)-2\tanh\left(\frac{1}{2}(a+bx)\right)+2\,\text{co}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Csch[a + b*x]^3*Sech[a + b*x]^2,x]
```

```
[Out] -1/8*(-16*ArcTan[Tanh[(a + b*x)/2]] + 2*Coth[(a + b*x)/2] + b*x*Csch[(a + b
*x)/2]^2 + 12*(a + b*x)*(Log[1 - E^(-a - b*x)] - Log[1 + E^(-a - b*x)]) - 1
2*a*Log[Tanh[(a + b*x)/2]] + 12*(PolyLog[2, -E^(-a - b*x)] - PolyLog[2, E^(
-a - b*x)]) + b*x*Sech[(a + b*x)/2]^2 + 8*b*x*Sech[a + b*x] - 2*Tanh[(a + b
*x)/2])/b^2
```

fricas [B] time = 0.45, size = 1694, normalized size = 15.54

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*csch(b*x+a)^3*sech(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/2*(2*(3*b*x + 1)*cosh(b*x + a)^5 + 10*(3*b*x + 1)*cosh(b*x + a)*sinh(b*x
 + a)^4 + 2*(3*b*x + 1)*sinh(b*x + a)^5 - 4*b*x*cosh(b*x + a)^3 + 4*(5*(3*b
*x + 1)*cosh(b*x + a)^2 - b*x)*sinh(b*x + a)^3 + 4*(5*(3*b*x + 1)*cosh(b*x
+ a)^3 - 3*b*x*cosh(b*x + a))*sinh(b*x + a)^2 - 4*(cosh(b*x + a)^6 + 6*cosh
(b*x + a)*sinh(b*x + a)^5 + sinh(b*x + a)^6 + (15*cosh(b*x + a)^2 - 1)*sinh
(b*x + a)^4 - cosh(b*x + a)^4 + 4*(5*cosh(b*x + a)^3 - cosh(b*x + a))*sinh(
b*x + a)^3 + (15*cosh(b*x + a)^4 - 6*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^2 -
 cosh(b*x + a)^2 + 2*(3*cosh(b*x + a)^5 - 2*cosh(b*x + a)^3 - cosh(b*x + a)
)*sinh(b*x + a) + 1)*arctan(cosh(b*x + a) + sinh(b*x + a)) + 2*(3*b*x - 1)*
cosh(b*x + a) + 3*(cosh(b*x + a)^6 + 6*cosh(b*x + a)*sinh(b*x + a)^5 + sinh
(b*x + a)^6 + (15*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^4 - cosh(b*x + a)^4 +
4*(5*cosh(b*x + a)^3 - cosh(b*x + a))*sinh(b*x + a)^3 + (15*cosh(b*x + a)^4
 - 6*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^2 - cosh(b*x + a)^2 + 2*(3*cosh(b*x
 + a)^5 - 2*cosh(b*x + a)^3 - cosh(b*x + a))*sinh(b*x + a) + 1)*dilog(cosh(
b*x + a) + sinh(b*x + a)) - 3*(cosh(b*x + a)^6 + 6*cosh(b*x + a)*sinh(b*x +
 a)^5 + sinh(b*x + a)^6 + (15*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^4 - cosh(b
*x + a)^4 + 4*(5*cosh(b*x + a)^3 - cosh(b*x + a))*sinh(b*x + a)^3 + (15*cos
h(b*x + a)^4 - 6*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^2 - cosh(b*x + a)^2 + 2
*(3*cosh(b*x + a)^5 - 2*cosh(b*x + a)^3 - cosh(b*x + a))*sinh(b*x + a) + 1)
*dilog(-cosh(b*x + a) - sinh(b*x + a)) - 3*(b*x*cosh(b*x + a)^6 + 6*b*x*cos
h(b*x + a)*sinh(b*x + a)^5 + b*x*sinh(b*x + a)^6 - b*x*cosh(b*x + a)^4 + (1
5*b*x*cosh(b*x + a)^2 - b*x)*sinh(b*x + a)^4 - b*x*cosh(b*x + a)^2 + 4*(5*b
*x*cosh(b*x + a)^3 - b*x*cosh(b*x + a))*sinh(b*x + a)^3 + (15*b*x*cosh(b*x
+ a)^4 - 6*b*x*cosh(b*x + a)^2 - b*x)*sinh(b*x + a)^2 + b*x + 2*(3*b*x*cosh
(b*x + a)^5 - 2*b*x*cosh(b*x + a)^3 - b*x*cosh(b*x + a))*sinh(b*x + a))*log
(cosh(b*x + a) + sinh(b*x + a) + 1) - 3*(a*cosh(b*x + a)^6 + 6*a*cosh(b*x +
 a)*sinh(b*x + a)^5 + a*sinh(b*x + a)^6 - a*cosh(b*x + a)^4 + (15*a*cosh(b*
x + a)^2 - a)*sinh(b*x + a)^4 + 4*(5*a*cosh(b*x + a)^3 - a*cosh(b*x + a))*s
```

```
inh(b*x + a)^3 - a*cosh(b*x + a)^2 + (15*a*cosh(b*x + a)^4 - 6*a*cosh(b*x +
 a)^2 - a)*sinh(b*x + a)^2 + 2*(3*a*cosh(b*x + a)^5 - 2*a*cosh(b*x + a)^3 -
 a*cosh(b*x + a))*sinh(b*x + a) + a)*log(cosh(b*x + a) + sinh(b*x + a) - 1)
 + 3*((b*x + a)*cosh(b*x + a)^6 + 6*(b*x + a)*cosh(b*x + a)*sinh(b*x + a)^5
 + (b*x + a)*sinh(b*x + a)^6 - (b*x + a)*cosh(b*x + a)^4 + (15*(b*x + a)*co
sh(b*x + a)^2 - b*x - a)*sinh(b*x + a)^4 + 4*(5*(b*x + a)*cosh(b*x + a)^3 -
 (b*x + a)*cosh(b*x + a))*sinh(b*x + a)^3 - (b*x + a)*cosh(b*x + a)^2 + (15
*(b*x + a)*cosh(b*x + a)^4 - 6*(b*x + a)*cosh(b*x + a)^2 - b*x - a)*sinh(b*
x + a)^2 + b*x + 2*(3*(b*x + a)*cosh(b*x + a)^5 - 2*(b*x + a)*cosh(b*x + a)
^3 - (b*x + a)*cosh(b*x + a))*sinh(b*x + a) + a)*log(-cosh(b*x + a) - sinh(
b*x + a) + 1) + 2*(5*(3*b*x + 1)*cosh(b*x + a)^4 - 6*b*x*cosh(b*x + a)^2 +
3*b*x - 1)*sinh(b*x + a))/(b^2*cosh(b*x + a)^6 + 6*b^2*cosh(b*x + a)*sinh(b
*x + a)^5 + b^2*sinh(b*x + a)^6 - b^2*cosh(b*x + a)^4 + (15*b^2*cosh(b*x +
a)^2 - b^2)*sinh(b*x + a)^4 - b^2*cosh(b*x + a)^2 + 4*(5*b^2*cosh(b*x + a)^
3 - b^2*cosh(b*x + a))*sinh(b*x + a)^3 + (15*b^2*cosh(b*x + a)^4 - 6*b^2*co
sh(b*x + a)^2 - b^2)*sinh(b*x + a)^2 + b^2 + 2*(3*b^2*cosh(b*x + a)^5 - 2*b
^2*cosh(b*x + a)^3 - b^2*cosh(b*x + a))*sinh(b*x + a))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x \operatorname{csch}(bx+a)^3 \operatorname{sech}(bx+a)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*csch(b*x+a)^3*sech(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate(x*csch(b*x + a)^3*sech(b*x + a)^2, x)
```

maple [A] time = 0.56, size = 148, normalized size = 1.36

$$-\frac{e^{bx+a}\left(3bx\,e^{4bx+4a}-2bx\,e^{2bx+2a}+e^{4bx+4a}+3bx-1\right)}{b^2\left(1+e^{2bx+2a}\right)\left(e^{2bx+2a}-1\right)^2}+\frac{2\arctan\left(e^{bx+a}\right)}{b^2}+\frac{3a\ln\left(e^{bx+a}-1\right)}{2b^2}+\frac{3\,\mathrm{dilog}\left(e^{bx+a}\right)}{2b^2}+\frac{3}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*csch(b*x+a)^3*sech(b*x+a)^2,x)
```

```
[Out] -exp(b*x+a)*(3*b*x*exp(4*b*x+4*a)-2*b*x*exp(2*b*x+2*a)+exp(4*b*x+4*a)+3*b*x
-1)/b^2/(1+exp(2*b*x+2*a))/(exp(2*b*x+2*a)-1)^2+2/b^2*arctan(exp(b*x+a))+3/
2/b^2*a*ln(exp(b*x+a)-1)+3/2/b^2*dilog(exp(b*x+a))+3/2/b^2*dilog(1+exp(b*x+
a))+3/2/b*ln(1+exp(b*x+a))*x
```

maxima [A] time = 0.46, size = 166, normalized size = 1.52

$$\frac{2\,bxe^{(3\,bx+3\,a)}-\left(3\,bxe^{(5\,a)}+e^{(5\,a)}\right)e^{(5\,bx)}-\left(3\,bxe^{a}-e^{a}\right)e^{(bx)}}{b^2e^{(6\,bx+6\,a)}-b^2e^{(4\,bx+4\,a)}-b^2e^{(2\,bx+2\,a)}+b^2}+\frac{3\left(bx\log\left(e^{(bx+a)}+1\right)+\mathrm{Li}_2\left(-e^{(bx+a)}\right)\right)}{2\,b^2}-\frac{3\left(bx\log\left(-\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*csch(b*x+a)^3*sech(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] (2*b*x*e^(3*b*x + 3*a) - (3*b*x*e^(5*a) + e^(5*a))*e^(5*b*x) - (3*b*x*e^a -
 e^a)*e^(b*x))/(b^2*e^(6*b*x + 6*a) - b^2*e^(4*b*x + 4*a) - b^2*e^(2*b*x +
2*a) + b^2) + 3/2*(b*x*log(e^(b*x + a) + 1) + dilog(-e^(b*x + a)))/b^2 - 3/
2*(b*x*log(-e^(b*x + a) + 1) + dilog(e^(b*x + a)))/b^2 + 2*arctan(e^(b*x +
a))/b^2
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x}{\cosh\left(a+b\,x\right)^2\,\sinh\left(a+b\,x\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(cosh(a + b*x)^2*sinh(a + b*x)^3),x)
```

```
[Out] int(x/(cosh(a + b*x)^2*sinh(a + b*x)^3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x \operatorname{csch}^{3}{\left(a + bx \right)} \operatorname{sech}^{2}{\left(a + bx \right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*csch(b*x+a)**3*sech(b*x+a)**2,x)
```

```
[Out] Integral(x*csch(a + b*x)**3*sech(a + b*x)**2, x)
```

3.518 $\int \operatorname{csch}^3(a+bx)\operatorname{sech}^2(a+bx)\,dx$

Optimal. Leaf size=49

$$-\frac{3\operatorname{sech}(a+bx)}{2b}+\frac{3\tanh^{-1}(\cosh(a+bx))}{2b}-\frac{\operatorname{csch}^2(a+bx)\operatorname{sech}(a+bx)}{2b}$$

```
[Out] 3/2*arctanh(cosh(b*x+a))/b-3/2*sech(b*x+a)/b-1/2*csch(b*x+a)^2*sech(b*x+a)/
b
```

Rubi [A] time = 0.05, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {2622, 288, 321, 207}

$$-\frac{3\operatorname{sech}(a+bx)}{2b}+\frac{3\tanh^{-1}(\cosh(a+bx))}{2b}-\frac{\operatorname{csch}^2(a+bx)\operatorname{sech}(a+bx)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[a + b*x]^3*Sech[a + b*x]^2,x]
```

```
[Out] (3*ArcTanh[Cosh[a + b*x]])/(2*b) - (3*Sech[a + b*x])/(2*b) - (Csch[a + b*x]
^2*Sech[a + b*x])/(2*b)
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rubi steps

$$\int \operatorname{csch}^3(a+bx)\operatorname{sech}^2(a+bx)\,dx = -\frac{\operatorname{Subst}\left(\int \frac{x^4}{\left(-1+x^2\right)^2}\,dx, x, \operatorname{sech}(a+bx)\right)}{b}$$
$$= -\frac{\operatorname{csch}^2(a+bx)\operatorname{sech}(a+bx)}{2b} - \frac{3\operatorname{Subst}\left(\int \frac{x^2}{-1+x^2}\,dx, x, \operatorname{sech}(a+bx)\right)}{2b}$$
$$= -\frac{3\operatorname{sech}(a+bx)}{2b} - \frac{\operatorname{csch}^2(a+bx)\operatorname{sech}(a+bx)}{2b} - \frac{3\operatorname{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \operatorname{sech}\right.}{2b}$$
$$= \frac{3\tanh^{-1}(\cosh(a+bx))}{2b} - \frac{3\operatorname{sech}(a+bx)}{2b} - \frac{\operatorname{csch}^2(a+bx)\operatorname{sech}(a+bx)}{2b}$$

Mathematica [A] time = 0.04, size = 68, normalized size = 1.39

$$-\frac{\operatorname{csch}^2\left(\frac{1}{2}(a+bx)\right)}{8b} - \frac{\operatorname{sech}^2\left(\frac{1}{2}(a+bx)\right)}{8b} - \frac{\operatorname{sech}(a+bx)}{b} - \frac{3\log\left(\tanh\left(\frac{1}{2}(a+bx)\right)\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[a + b*x]^3*Sech[a + b*x]^2,x]
```

```
[Out] -1/8*Csch[(a + b*x)/2]^2/b - (3*Log[Tanh[(a + b*x)/2]])/(2*b) - Sech[(a + b
*x)/2]^2/(8*b) - Sech[a + b*x]/b
```

fricas [B] time = 0.43, size = 709, normalized size = 14.47

$$-\frac{6\cosh(bx+a)^5 + 30\cosh(bx+a)\sinh(bx+a)^4 + 6\sinh(bx+a)^5 + 4\left(15\cosh(bx+a)^2 - 1\right)\sinh(bx+a)^{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^3*sech(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/2*(6*cosh(b*x + a)^5 + 30*cosh(b*x + a)*sinh(b*x + a)^4 + 6*sinh(b*x + a
)^5 + 4*(15*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^3 - 4*cosh(b*x + a)^3 + 12*(
5*cosh(b*x + a)^3 - cosh(b*x + a))*sinh(b*x + a)^2 - 3*(cosh(b*x + a)^6 + 6
*cosh(b*x + a)*sinh(b*x + a)^5 + sinh(b*x + a)^6 + (15*cosh(b*x + a)^2 - 1)
*sinh(b*x + a)^4 - cosh(b*x + a)^4 + 4*(5*cosh(b*x + a)^3 - cosh(b*x + a))*
sinh(b*x + a)^3 + (15*cosh(b*x + a)^4 - 6*cosh(b*x + a)^2 - 1)*sinh(b*x + a
)^2 - cosh(b*x + a)^2 + 2*(3*cosh(b*x + a)^5 - 2*cosh(b*x + a)^3 - cosh(b*x
 + a))*sinh(b*x + a) + 1)*log(cosh(b*x + a) + sinh(b*x + a) + 1) + 3*(cosh(
b*x + a)^6 + 6*cosh(b*x + a)*sinh(b*x + a)^5 + sinh(b*x + a)^6 + (15*cosh(b
*x + a)^2 - 1)*sinh(b*x + a)^4 - cosh(b*x + a)^4 + 4*(5*cosh(b*x + a)^3 - c
osh(b*x + a))*sinh(b*x + a)^3 + (15*cosh(b*x + a)^4 - 6*cosh(b*x + a)^2 - 1
)*sinh(b*x + a)^2 - cosh(b*x + a)^2 + 2*(3*cosh(b*x + a)^5 - 2*cosh(b*x + a
)^3 - cosh(b*x + a))*sinh(b*x + a) + 1)*log(cosh(b*x + a) + sinh(b*x + a) -
 1) + 6*(5*cosh(b*x + a)^4 - 2*cosh(b*x + a)^2 + 1)*sinh(b*x + a) + 6*cosh(
b*x + a))/(b*cosh(b*x + a)^6 + 6*b*cosh(b*x + a)*sinh(b*x + a)^5 + b*sinh(b
*x + a)^6 - b*cosh(b*x + a)^4 + (15*b*cosh(b*x + a)^2 - b)*sinh(b*x + a)^4
+ 4*(5*b*cosh(b*x + a)^3 - b*cosh(b*x + a))*sinh(b*x + a)^3 - b*cosh(b*x +
a)^2 + (15*b*cosh(b*x + a)^4 - 6*b*cosh(b*x + a)^2 - b)*sinh(b*x + a)^2 + 2
*(3*b*cosh(b*x + a)^5 - 2*b*cosh(b*x + a)^3 - b*cosh(b*x + a))*sinh(b*x + a
) + b)
```

giac [B] time = 0.14, size = 110, normalized size = 2.24

$$-\frac{\frac{4\left(3\left(e^{(bx+a)}+e^{(-bx-a)}\right)^2-8\right)}{\left(e^{(bx+a)}+e^{(-bx-a)}\right)^3-4e^{(bx+a)}-4e^{(-bx-a)}}-3\log\left(e^{(bx+a)}+e^{(-bx-a)}+2\right)+3\log\left(e^{(bx+a)}+e^{(-bx-a)}-2\right)}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^3*sech(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -1/4*(4*(3*(e^(b*x + a) + e^(-b*x - a))^2 - 8)/((e^(b*x + a) + e^(-b*x - a)
)^3 - 4*e^(b*x + a) - 4*e^(-b*x - a)) - 3*log(e^(b*x + a) + e^(-b*x - a) +
2) + 3*log(e^(b*x + a) + e^(-b*x - a) - 2))/b
```

maple [A] time = 0.20, size = 43, normalized size = 0.88

$$\frac{-\frac{1}{2\sinh(bx+a)^2\cosh(bx+a)}-\frac{3}{2\cosh(bx+a)}+3\,\mathrm{arctanh}\left(\mathrm{e}^{bx+a}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(b*x+a)^3*sech(b*x+a)^2,x)
```

```
[Out] 1/b*(-1/2/sinh(b*x+a)^2/cosh(b*x+a)-3/2/cosh(b*x+a)+3*arctanh(exp(b*x+a)))
```

maxima [B] time = 0.31, size = 106, normalized size = 2.16

$$\frac{3\log\left(e^{(-bx-a)}+1\right)}{2b}-\frac{3\log\left(e^{(-bx-a)}-1\right)}{2b}+\frac{3e^{(-bx-a)}-2e^{(-3bx-3a)}+3e^{(-5bx-5a)}}{b\left(e^{(-2bx-2a)}+e^{(-4bx-4a)}-e^{(-6bx-6a)}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^3*sech(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 3/2*log(e^(-b*x - a) + 1)/b - 3/2*log(e^(-b*x - a) - 1)/b + (3*e^(-b*x - a)
 - 2*e^(-3*b*x - 3*a) + 3*e^(-5*b*x - 5*a))/(b*(e^(-2*b*x - 2*a) + e^(-4*b*
x - 4*a) - e^(-6*b*x - 6*a) - 1))
```

mupad [B] time = 0.08, size = 111, normalized size = 2.27

$$\frac{3\,\mathrm{atan}\left(\frac{e^{bx}\,e^{a}\,\sqrt{-b^2}}{b}\right)}{\sqrt{-b^2}}-\frac{2\,e^{a+bx}}{b\,\left(e^{4a+4bx}-2\,e^{2a+2bx}+1\right)}-\frac{e^{a+bx}}{b\,\left(e^{2a+2bx}-1\right)}-\frac{2\,e^{a+bx}}{b\,\left(e^{2a+2bx}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(a + b*x)^2*sinh(a + b*x)^3),x)
```

```
[Out] (3*atan((exp(b*x)*exp(a)*(-b^2)^(1/2))/b))/(-b^2)^(1/2) - (2*exp(a + b*x))/
(b*(exp(4*a + 4*b*x) - 2*exp(2*a + 2*b*x) + 1)) - exp(a + b*x)/(b*(exp(2*a
+ 2*b*x) - 1)) - (2*exp(a + b*x))/(b*(exp(2*a + 2*b*x) + 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{csch}^3(a+bx)\operatorname{sech}^2(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)**3*sech(b*x+a)**2,x)
```

```
[Out] Integral(csch(a + b*x)**3*sech(a + b*x)**2, x)
```

3.519 $\int \frac{\operatorname{csch}^3(a+bx)\operatorname{sech}^2(a+bx)}{x}\,dx$

Optimal. Leaf size=23

$$\text{Int}\left(\frac{\operatorname{csch}^3(a+bx)\operatorname{sech}^2(a+bx)}{x},x\right)$$

[Out] CannotIntegrate(csch(b*x+a)^3*sech(b*x+a)^2/x,x)

Rubi [A] time = 0.28, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\operatorname{csch}^3(a+bx)\operatorname{sech}^2(a+bx)}{x}\,dx$$

Verification is Not applicable to the result.

[In] Int[(Csch[a + b*x]^3*Sech[a + b*x]^2)/x,x]

[Out] Defer[Int][(Csch[a + b*x]^3*Sech[a + b*x]^2)/x, x]

Rubi steps

$$\int \frac{\operatorname{csch}^3(a+bx)\operatorname{sech}^2(a+bx)}{x}\,dx=\int \frac{\operatorname{csch}^3(a+bx)\operatorname{sech}^2(a+bx)}{x}\,dx$$

Mathematica [A] time = 67.55, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}^3(a+bx)\operatorname{sech}^2(a+bx)}{x}\,dx$$

Verification is Not applicable to the result.

[In] Integrate[(Csch[a + b*x]^3*Sech[a + b*x]^2)/x,x]

[Out] Integrate[(Csch[a + b*x]^3*Sech[a + b*x]^2)/x, x]

fricas [A] time = 0.41, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\operatorname{csch}(bx+a)^3\operatorname{sech}(bx+a)^2}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^3*sech(b*x+a)^2/x,x, algorithm="fricas")
```

```
[Out] integral(csch(b*x + a)^3*sech(b*x + a)^2/x, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}(bx+a)^3 \operatorname{sech}(bx+a)^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^3*sech(b*x+a)^2/x,x, algorithm="giac")
```

```
[Out] integrate(csch(b*x + a)^3*sech(b*x + a)^2/x, x)
```

maple [A] time = 1.27, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}(bx+a)^3 \operatorname{sech}(bx+a)^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(b*x+a)^3*sech(b*x+a)^2/x,x)
```

```
[Out] int(csch(b*x+a)^3*sech(b*x+a)^2/x,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\frac{2\,bxe^{(3\,bx+3\,a)} - \left(3\,bxe^{(5\,a)} - e^{(5\,a)}\right)e^{(5\,bx)} - \left(3\,bxe^{a} + e^{a}\right)e^{(bx)}}{b^2x^2e^{(6\,bx+6\,a)} - b^2x^2e^{(4\,bx+4\,a)} - b^2x^2e^{(2\,bx+2\,a)} + b^2x^2} - 32\int \frac{3\,b^2x^2 - 2}{64\left(b^2x^3e^{(bx+a)} + b^2x^3\right)}\,dx - 32\int \frac{3\,b^2x^2 -}{64\left(b^2x^3e^{(bx+a}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^3*sech(b*x+a)^2/x,x, algorithm="maxima")
```

```
[Out] (2*b*x*e^(3*b*x + 3*a) - (3*b*x*e^(5*a) - e^(5*a))*e^(5*b*x) - (3*b*x*e^a +
 e^a)*e^(b*x))/(b^2*x^2*e^(6*b*x + 6*a) - b^2*x^2*e^(4*b*x + 4*a) - b^2*x^2
*e^(2*b*x + 2*a) + b^2*x^2) - 32*integrate(1/64*(3*b^2*x^2 - 2)/(b^2*x^3*e^
(b*x + a) + b^2*x^3), x) - 32*integrate(1/64*(3*b^2*x^2 - 2)/(b^2*x^3*e^(b*
x + a) - b^2*x^3), x) - 32*integrate(1/16*e^(b*x + a)/(b*x^2*e^(2*b*x + 2*a
) + b*x^2), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{x\cosh(a+b\,x)^2\sinh(a+b\,x)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*cosh(a + b*x)^2*sinh(a + b*x)^3),x)
[Out] int(1/(x*cosh(a + b*x)^2*sinh(a + b*x)^3), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}^3(a+bx)\operatorname{sech}^2(a+bx)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)**3*sech(b*x+a)**2/x,x)
[Out] Integral(csch(a + b*x)**3*sech(a + b*x)**2/x, x)
```

3.520 $\int \frac{\operatorname{csch}^3(a+bx)\operatorname{sech}^2(a+bx)}{x^2}\,dx$

Optimal. Leaf size=23

$$\operatorname{Int}\left(\frac{\operatorname{csch}^3(a+bx)\operatorname{sech}^2(a+bx)}{x^2},x\right)$$

```
[Out] CannotIntegrate(csch(b*x+a)^3*sech(b*x+a)^2/x^2,x)
```

Rubi [A] time = 0.33, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\operatorname{csch}^3(a+bx)\operatorname{sech}^2(a+bx)}{x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Csch[a + b*x]^3*Sech[a + b*x]^2)/x^2,x]
```

```
[Out] Defer[Int][(Csch[a + b*x]^3*Sech[a + b*x]^2)/x^2, x]
```

Rubi steps

$$\int \frac{\operatorname{csch}^3(a+bx)\operatorname{sech}^2(a+bx)}{x^2}\,dx=\int \frac{\operatorname{csch}^3(a+bx)\operatorname{sech}^2(a+bx)}{x^2}\,dx$$

Mathematica [A] time = 46.12, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}^3(a+bx)\operatorname{sech}^2(a+bx)}{x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Csch[a + b*x]^3*Sech[a + b*x]^2)/x^2,x]
```

```
[Out] Integrate[(Csch[a + b*x]^3*Sech[a + b*x]^2)/x^2, x]
```

fricas [A] time = 0.42, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{\operatorname{csch}(bx+a)^3\operatorname{sech}(bx+a)^2}{x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(b*x+a)^3*sech(b*x+a)^2/x^2,x, algorithm="fricas")

[Out] integral(csch(b*x + a)^3*sech(b*x + a)^2/x^2, x)

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$sage_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(b*x+a)^3*sech(b*x+a)^2/x^2,x, algorithm="giac")

[Out] sage0*x

maple [A] time = 1.39, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}(bx+a)^3 \operatorname{sech}(bx+a)^2}{x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csch(b*x+a)^3*sech(b*x+a)^2/x^2,x)

[Out] int(csch(b*x+a)^3*sech(b*x+a)^2/x^2,x)

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\frac{2\,bxe^{(3\,bx+3\,a)} - \left(3\,bxe^{(5\,a)} - 2\,e^{(5\,a)}\right)e^{(5\,bx)} - \left(3\,bxe^{a} + 2\,e^{a}\right)e^{(bx)}}{b^2x^3e^{(6\,bx+6\,a)} - b^2x^3e^{(4\,bx+4\,a)} - b^2x^3e^{(2\,bx+2\,a)} + b^2x^3} - 32\int \frac{3\left(b^2x^2-2\right)}{64\left(b^2x^4e^{(bx+a)} + b^2x^4\right)}\,dx - 32\int \frac{3\,(b}{64\,(b^2x^4e}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(b*x+a)^3*sech(b*x+a)^2/x^2,x, algorithm="maxima")

[Out] (2*b*x*e^(3*b*x + 3*a) - (3*b*x*e^(5*a) - 2*e^(5*a))*e^(5*b*x) - (3*b*x*e^a + 2*e^a)*e^(b*x))/(b^2*x^3*e^(6*b*x + 6*a) - b^2*x^3*e^(4*b*x + 4*a) - b^2*x^3*e^(2*b*x + 2*a) + b^2*x^3) - 32*integrate(3/64*(b^2*x^2 - 2)/(b^2*x^4*e^(b*x + a) + b^2*x^4), x) - 32*integrate(3/64*(b^2*x^2 - 2)/(b^2*x^4*e^(b*x + a) - b^2*x^4), x) - 32*integrate(1/8*e^(b*x + a)/(b*x^3*e^(2*b*x + 2*a) + b*x^3), x)

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{x^2\cosh(a+b\,x)^2\sinh(a+b\,x)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*cosh(a + b*x)^2*sinh(a + b*x)^3),x)
```

```
[Out] int(1/(x^2*cosh(a + b*x)^2*sinh(a + b*x)^3), x)
```

sympy [A]   time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}^3(a+bx)\operatorname{sech}^2(a+bx)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)**3*sech(b*x+a)**2/x**2,x)
```

```
[Out] Integral(csch(a + b*x)**3*sech(a + b*x)**2/x**2, x)
```

3.521 $\int x^m \mathrm{csch}^3(a+bx)\mathrm{sech}^3(a+bx)\,dx$

Optimal. Leaf size=23

$$\mathrm{Int}\left(x^m \mathrm{csch}^3(a+bx)\mathrm{sech}^3(a+bx),x\right)$$

```
[Out] CannotIntegrate(x^m*csch(b*x+a)^3*sech(b*x+a)^3,x)
```

Rubi [A] time = 0.61, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int x^m \mathrm{csch}^3(a+bx)\mathrm{sech}^3(a+bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x^m*Csch[a + b*x]^3*Sech[a + b*x]^3,x]
[Out] Defer[Int][x^m*Csch[a + b*x]^3*Sech[a + b*x]^3, x]
```

Rubi steps

$$\int x^m \mathrm{csch}^3(a+bx)\mathrm{sech}^3(a+bx)\,dx = \int x^m \mathrm{csch}^3(a+bx)\mathrm{sech}^3(a+bx)\,dx$$

Mathematica [F] time = 180.00, size = 0, normalized size = 0.00

$Aborted

Verification is Not applicable to the result.

```
[In] Integrate[x^m*Csch[a + b*x]^3*Sech[a + b*x]^3,x]
[Out] $Aborted
```

fricas [A] time = 0.41, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(x^m \,\mathrm{csch}\,(bx+a)^3\,\mathrm{sech}\,(bx+a)^3\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*csch(b*x+a)^3*sech(b*x+a)^3,x, algorithm="fricas")
[Out] integral(x^m*csch(b*x + a)^3*sech(b*x + a)^3, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int x^m \operatorname{csch}(bx+a)^3 \operatorname{sech}(bx+a)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*csch(b*x+a)^3*sech(b*x+a)^3,x, algorithm="giac")
[Out] integrate(x^m*csch(b*x + a)^3*sech(b*x + a)^3, x)
```

maple [A] time = 0.27, size = 0, normalized size = 0.00

$$\int x^m \operatorname{csch}(bx+a)^3 \operatorname{sech}(bx+a)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*csch(b*x+a)^3*sech(b*x+a)^3,x)
[Out] int(x^m*csch(b*x+a)^3*sech(b*x+a)^3,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int x^m \operatorname{csch}(bx+a)^3 \operatorname{sech}(bx+a)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*csch(b*x+a)^3*sech(b*x+a)^3,x, algorithm="maxima")
[Out] integrate(x^m*csch(b*x + a)^3*sech(b*x + a)^3, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{x^m}{\cosh(a+b\,x)^3 \sinh(a+b\,x)^3} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(cosh(a + b*x)^3*sinh(a + b*x)^3),x)
[Out] int(x^m/(cosh(a + b*x)^3*sinh(a + b*x)^3), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int x^m \operatorname{csch}^3(a+bx) \operatorname{sech}^3(a+bx) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*csch(b*x+a)**3*sech(b*x+a)**3,x)
[Out] Integral(x**m*csch(a + b*x)**3*sech(a + b*x)**3, x)
```

3.522 $\int x^3 \operatorname{csch}^3(a+bx)\operatorname{sech}^3(a+bx)\,dx$

Optimal. Leaf size=240

$$-\frac{3\text{Li}_2\left(-e^{2a+2bx}\right)}{2b^4}+\frac{3\text{Li}_2\left(e^{2a+2bx}\right)}{2b^4}+\frac{3\text{Li}_4\left(-e^{2a+2bx}\right)}{2b^4}-\frac{3\text{Li}_4\left(e^{2a+2bx}\right)}{2b^4}-\frac{3x\text{Li}_3\left(-e^{2a+2bx}\right)}{b^3}+\frac{3x\text{Li}_3\left(e^{2a+2bx}\right)}{b^3}-\frac{6x\tanh}{}$$

```
[Out] -6*x*arctanh(exp(2*b*x+2*a))/b^3+4*x^3*arctanh(exp(2*b*x+2*a))/b-3*x^2*csch
(2*b*x+2*a)/b^2-2*x^3*coth(2*b*x+2*a)*csch(2*b*x+2*a)/b-3/2*polylog(2,-exp(
2*b*x+2*a))/b^4+3*x^2*polylog(2,-exp(2*b*x+2*a))/b^2+3/2*polylog(2,exp(2*b*
x+2*a))/b^4-3*x^2*polylog(2,exp(2*b*x+2*a))/b^2-3*x*polylog(3,-exp(2*b*x+2*
a))/b^3+3*x*polylog(3,exp(2*b*x+2*a))/b^3+3/2*polylog(4,-exp(2*b*x+2*a))/b^
4-3/2*polylog(4,exp(2*b*x+2*a))/b^4
```

Rubi [A] time = 0.30, antiderivative size = 240, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 9, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.450$, Rules used = {5461, 4186, 4182, 2279, 2391, 2531, 6609, 2282, 6589}

$$\frac{3x^2\text{PolyLog}\left(2,-e^{2a+2bx}\right)}{b^2}-\frac{3x^2\text{PolyLog}\left(2,e^{2a+2bx}\right)}{b^2}-\frac{3x\text{PolyLog}\left(3,-e^{2a+2bx}\right)}{b^3}+\frac{3x\text{PolyLog}\left(3,e^{2a+2bx}\right)}{b^3}-\frac{3\text{Poly}}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Csch[a + b*x]^3*Sech[a + b*x]^3,x]
```

```
[Out] (-6*x*ArcTanh[E^(2*a + 2*b*x)])/b^3 + (4*x^3*ArcTanh[E^(2*a + 2*b*x)])/b -
(3*x^2*Csch[2*a + 2*b*x])/b^2 - (2*x^3*Coth[2*a + 2*b*x]*Csch[2*a + 2*b*x])
/b - (3*PolyLog[2, -E^(2*a + 2*b*x)])/(2*b^4) + (3*x^2*PolyLog[2, -E^(2*a +
 2*b*x)])/b^2 + (3*PolyLog[2, E^(2*a + 2*b*x)])/(2*b^4) - (3*x^2*PolyLog[2,
 E^(2*a + 2*b*x)])/b^2 - (3*x*PolyLog[3, -E^(2*a + 2*b*x)])/b^3 + (3*x*Poly
Log[3, E^(2*a + 2*b*x)])/b^3 + (3*PolyLog[4, -E^(2*a + 2*b*x)])/(2*b^4) - (
3*PolyLog[4, E^(2*a + 2*b*x)])/(2*b^4)
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
```

```
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 4186

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> -Simp[(b^2*(c + d*x)^m*Cot[e + f*x]*(b*Csc[e + f*x])^(n - 2))/(f*(n -
 1)), x] + (Dist[(b^2*d^2*m*(m - 1))/(f^2*(n - 1)*(n - 2)), Int[(c + d*x)^(
m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Dist[(b^2*(n - 2))/(n - 1), Int[(
c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x] - Simp[(b^2*d*m*(c + d*x)^(m -
1)*(b*Csc[e + f*x])^(n - 2))/(f^2*(n - 1)*(n - 2)), x]) /; FreeQ[{b, c, d,
e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

Rule 5461

```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[2^n, Int[(c + d*x)^m*Csch[2*a + 2*b*x]
^n, x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[m] && IntegerQ[n]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int x^3\text{csch}^3(a+bx)\text{sech}^3(a+bx)\,dx = 8\int x^3\text{csch}^3(2a+2bx)\,dx$$

$$= -\frac{3x^2\text{csch}(2a+2bx)}{b^2} - \frac{2x^3\coth(2a+2bx)\text{csch}(2a+2bx)}{b} - 4\int x^3\text{csch}$$

$$= -\frac{6x\tanh^{-1}\left(e^{2a+2bx}\right)}{b^3} + \frac{4x^3\tanh^{-1}\left(e^{2a+2bx}\right)}{b} - \frac{3x^2\text{csch}(2a+2bx)}{b^2} - \frac{2x^3}{}$$

$$= -\frac{6x\tanh^{-1}\left(e^{2a+2bx}\right)}{b^3} + \frac{4x^3\tanh^{-1}\left(e^{2a+2bx}\right)}{b} - \frac{3x^2\text{csch}(2a+2bx)}{b^2} - \frac{2x^3}{}$$

$$= -\frac{6x\tanh^{-1}\left(e^{2a+2bx}\right)}{b^3} + \frac{4x^3\tanh^{-1}\left(e^{2a+2bx}\right)}{b} - \frac{3x^2\text{csch}(2a+2bx)}{b^2} - \frac{2x^3}{}$$

$$= -\frac{6x\tanh^{-1}\left(e^{2a+2bx}\right)}{b^3} + \frac{4x^3\tanh^{-1}\left(e^{2a+2bx}\right)}{b} - \frac{3x^2\text{csch}(2a+2bx)}{b^2} - \frac{2x^3}{}$$

$$= -\frac{6x\tanh^{-1}\left(e^{2a+2bx}\right)}{b^3} + \frac{4x^3\tanh^{-1}\left(e^{2a+2bx}\right)}{b} - \frac{3x^2\text{csch}(2a+2bx)}{b^2} - \frac{2x^3}{}$$

Mathematica [A] time = 7.42, size = 274, normalized size = 1.14

$$\frac{-4b^3x^3\log\left(1-e^{2(a+bx)}\right)+4b^3x^3\log\left(e^{2(a+bx)}+1\right)-b^3x^3\text{csch}^2(a+bx)-b^3x^3\text{sech}^2(a+bx)+\left(6b^2x^2-3\right)\text{Li}_2\left(\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Csch[a + b*x]^3*Sech[a + b*x]^3,x]
```

```
[Out] (-(b^3*x^3*Csch[a + b*x]^2) + 6*b*x*Log[1 - E^(2*(a + b*x))] - 4*b^3*x^3*Lo
g[1 - E^(2*(a + b*x))] - 6*b*x*Log[1 + E^(2*(a + b*x))] + 4*b^3*x^3*Log[1 +
 E^(2*(a + b*x))] + (-3 + 6*b^2*x^2)*PolyLog[2, -E^(2*(a + b*x))] + (3 - 6*
b^2*x^2)*PolyLog[2, E^(2*(a + b*x))] - 6*b*x*PolyLog[3, -E^(2*(a + b*x))] +
 6*b*x*PolyLog[3, E^(2*(a + b*x))] + 3*PolyLog[4, -E^(2*(a + b*x))] - 3*Pol
```

```
yLog[4, E^(2*(a + b*x))] - 3*b^2*x^2*Csch[a]*Sech[a] - b^3*x^3*Sech[a + b*x
]^2 + 3*b^2*x^2*Csch[a]*Csch[a + b*x]*Sinh[b*x] + 3*b^2*x^2*Sech[a]*Sech[a
+ b*x]*Sinh[b*x])/(2*b^4)
```

fricas [C] time = 0.57, size = 6764, normalized size = 28.18

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*csch(b*x+a)^3*sech(b*x+a)^3,x, algorithm="fricas")

[Out] -(2*(2*b^3*x^3 + 3*b^2*x^2)*cosh(b*x + a)^6 + 40*(2*b^3*x^3 + 3*b^2*x^2)*co
sh(b*x + a)^3*sinh(b*x + a)^3 + 30*(2*b^3*x^3 + 3*b^2*x^2)*cosh(b*x + a)^2*
sinh(b*x + a)^4 + 12*(2*b^3*x^3 + 3*b^2*x^2)*cosh(b*x + a)*sinh(b*x + a)^5
+ 2*(2*b^3*x^3 + 3*b^2*x^2)*sinh(b*x + a)^6 + 2*(2*b^3*x^3 - 3*b^2*x^2)*cos
h(b*x + a)^2 + 2*(2*b^3*x^3 + 15*(2*b^3*x^3 + 3*b^2*x^2)*cosh(b*x + a)^4 -
3*b^2*x^2)*sinh(b*x + a)^2 + 3*((2*b^2*x^2 - 1)*cosh(b*x + a)^8 + 56*(2*b^2
*x^2 - 1)*cosh(b*x + a)^3*sinh(b*x + a)^5 + 28*(2*b^2*x^2 - 1)*cosh(b*x + a
)^2*sinh(b*x + a)^6 + 8*(2*b^2*x^2 - 1)*cosh(b*x + a)*sinh(b*x + a)^7 + (2*
b^2*x^2 - 1)*sinh(b*x + a)^8 - 2*(2*b^2*x^2 - 1)*cosh(b*x + a)^4 + 2*(35*(2
*b^2*x^2 - 1)*cosh(b*x + a)^4 - 2*b^2*x^2 + 1)*sinh(b*x + a)^4 + 2*b^2*x^2
+ 8*(7*(2*b^2*x^2 - 1)*cosh(b*x + a)^5 - (2*b^2*x^2 - 1)*cosh(b*x + a))*sin
h(b*x + a)^3 + 4*(7*(2*b^2*x^2 - 1)*cosh(b*x + a)^6 - 3*(2*b^2*x^2 - 1)*cos
h(b*x + a)^2)*sinh(b*x + a)^2 + 8*((2*b^2*x^2 - 1)*cosh(b*x + a)^7 - (2*b^2
*x^2 - 1)*cosh(b*x + a)^3)*sinh(b*x + a) - 1)*dilog(cosh(b*x + a) + sinh(b*
x + a)) - 3*((2*b^2*x^2 - 1)*cosh(b*x + a)^8 + 56*(2*b^2*x^2 - 1)*cosh(b*x
+ a)^3*sinh(b*x + a)^5 + 28*(2*b^2*x^2 - 1)*cosh(b*x + a)^2*sinh(b*x + a)^6
 + 8*(2*b^2*x^2 - 1)*cosh(b*x + a)*sinh(b*x + a)^7 + (2*b^2*x^2 - 1)*sinh(b
*x + a)^8 - 2*(2*b^2*x^2 - 1)*cosh(b*x + a)^4 + 2*(35*(2*b^2*x^2 - 1)*cosh(
b*x + a)^4 - 2*b^2*x^2 + 1)*sinh(b*x + a)^4 + 2*b^2*x^2 + 8*(7*(2*b^2*x^2 -
 1)*cosh(b*x + a)^5 - (2*b^2*x^2 - 1)*cosh(b*x + a))*sinh(b*x + a)^3 + 4*(7
*(2*b^2*x^2 - 1)*cosh(b*x + a)^6 - 3*(2*b^2*x^2 - 1)*cosh(b*x + a)^2)*sinh(
b*x + a)^2 + 8*((2*b^2*x^2 - 1)*cosh(b*x + a)^7 - (2*b^2*x^2 - 1)*cosh(b*x
+ a)^3)*sinh(b*x + a) - 1)*dilog(I*cosh(b*x + a) + I*sinh(b*x + a)) - 3*((2
*b^2*x^2 - 1)*cosh(b*x + a)^8 + 56*(2*b^2*x^2 - 1)*cosh(b*x + a)^3*sinh(b*x
 + a)^5 + 28*(2*b^2*x^2 - 1)*cosh(b*x + a)^2*sinh(b*x + a)^6 + 8*(2*b^2*x^2
 - 1)*cosh(b*x + a)*sinh(b*x + a)^7 + (2*b^2*x^2 - 1)*sinh(b*x + a)^8 - 2*(
2*b^2*x^2 - 1)*cosh(b*x + a)^4 + 2*(35*(2*b^2*x^2 - 1)*cosh(b*x + a)^4 - 2*
b^2*x^2 + 1)*sinh(b*x + a)^4 + 2*b^2*x^2 + 8*(7*(2*b^2*x^2 - 1)*cosh(b*x +
a)^5 - (2*b^2*x^2 - 1)*cosh(b*x + a))*sinh(b*x + a)^3 + 4*(7*(2*b^2*x^2 - 1
)*cosh(b*x + a)^6 - 3*(2*b^2*x^2 - 1)*cosh(b*x + a)^2)*sinh(b*x + a)^2 + 8*
((2*b^2*x^2 - 1)*cosh(b*x + a)^7 - (2*b^2*x^2 - 1)*cosh(b*x + a)^3)*sinh(b*
x + a) - 1)*dilog(-I*cosh(b*x + a) - I*sinh(b*x + a)) + 3*((2*b^2*x^2 - 1)*
cosh(b*x + a)^8 + 56*(2*b^2*x^2 - 1)*cosh(b*x + a)^3*sinh(b*x + a)^5 + 28*(
2*b^2*x^2 - 1)*cosh(b*x + a)^2*sinh(b*x + a)^6 + 8*(2*b^2*x^2 - 1)*cosh(b*x
```

```
 + a)*sinh(b*x + a)^7 + (2*b^2*x^2 - 1)*sinh(b*x + a)^8 - 2*(2*b^2*x^2 - 1)
*cosh(b*x + a)^4 + 2*(35*(2*b^2*x^2 - 1)*cosh(b*x + a)^4 - 2*b^2*x^2 + 1)*s
inh(b*x + a)^4 + 2*b^2*x^2 + 8*(7*(2*b^2*x^2 - 1)*cosh(b*x + a)^5 - (2*b^2*
x^2 - 1)*cosh(b*x + a))*sinh(b*x + a)^3 + 4*(7*(2*b^2*x^2 - 1)*cosh(b*x + a
)^6 - 3*(2*b^2*x^2 - 1)*cosh(b*x + a)^2)*sinh(b*x + a)^2 + 8*((2*b^2*x^2 -
1)*cosh(b*x + a)^7 - (2*b^2*x^2 - 1)*cosh(b*x + a)^3)*sinh(b*x + a) - 1)*di
log(-cosh(b*x + a) - sinh(b*x + a)) + ((2*b^3*x^3 - 3*b*x)*cosh(b*x + a)^8
+ 56*(2*b^3*x^3 - 3*b*x)*cosh(b*x + a)^3*sinh(b*x + a)^5 + 28*(2*b^3*x^3 -
3*b*x)*cosh(b*x + a)^2*sinh(b*x + a)^6 + 8*(2*b^3*x^3 - 3*b*x)*cosh(b*x + a
)*sinh(b*x + a)^7 + (2*b^3*x^3 - 3*b*x)*sinh(b*x + a)^8 + 2*b^3*x^3 - 2*(2*
b^3*x^3 - 3*b*x)*cosh(b*x + a)^4 - 2*(2*b^3*x^3 - 35*(2*b^3*x^3 - 3*b*x)*co
sh(b*x + a)^4 - 3*b*x)*sinh(b*x + a)^4 + 8*(7*(2*b^3*x^3 - 3*b*x)*cosh(b*x
+ a)^5 - (2*b^3*x^3 - 3*b*x)*cosh(b*x + a))*sinh(b*x + a)^3 + 4*(7*(2*b^3*x
^3 - 3*b*x)*cosh(b*x + a)^6 - 3*(2*b^3*x^3 - 3*b*x)*cosh(b*x + a)^2)*sinh(b
*x + a)^2 - 3*b*x + 8*((2*b^3*x^3 - 3*b*x)*cosh(b*x + a)^7 - (2*b^3*x^3 - 3
*b*x)*cosh(b*x + a)^3)*sinh(b*x + a))*log(cosh(b*x + a) + sinh(b*x + a) + 1
) + ((2*a^3 - 3*a)*cosh(b*x + a)^8 + 56*(2*a^3 - 3*a)*cosh(b*x + a)^3*sinh(
b*x + a)^5 + 28*(2*a^3 - 3*a)*cosh(b*x + a)^2*sinh(b*x + a)^6 + 8*(2*a^3 -
3*a)*cosh(b*x + a)*sinh(b*x + a)^7 + (2*a^3 - 3*a)*sinh(b*x + a)^8 - 2*(2*a
^3 - 3*a)*cosh(b*x + a)^4 + 2*(35*(2*a^3 - 3*a)*cosh(b*x + a)^4 - 2*a^3 + 3
*a)*sinh(b*x + a)^4 + 8*(7*(2*a^3 - 3*a)*cosh(b*x + a)^5 - (2*a^3 - 3*a)*co
sh(b*x + a))*sinh(b*x + a)^3 + 2*a^3 + 4*(7*(2*a^3 - 3*a)*cosh(b*x + a)^6 -
 3*(2*a^3 - 3*a)*cosh(b*x + a)^2)*sinh(b*x + a)^2 + 8*((2*a^3 - 3*a)*cosh(b
*x + a)^7 - (2*a^3 - 3*a)*cosh(b*x + a)^3)*sinh(b*x + a) - 3*a)*log(cosh(b*
x + a) + sinh(b*x + a) + I) + ((2*a^3 - 3*a)*cosh(b*x + a)^8 + 56*(2*a^3 -
3*a)*cosh(b*x + a)^3*sinh(b*x + a)^5 + 28*(2*a^3 - 3*a)*cosh(b*x + a)^2*sin
h(b*x + a)^6 + 8*(2*a^3 - 3*a)*cosh(b*x + a)*sinh(b*x + a)^7 + (2*a^3 - 3*a
)*sinh(b*x + a)^8 - 2*(2*a^3 - 3*a)*cosh(b*x + a)^4 + 2*(35*(2*a^3 - 3*a)*c
osh(b*x + a)^4 - 2*a^3 + 3*a)*sinh(b*x + a)^4 + 8*(7*(2*a^3 - 3*a)*cosh(b*x
 + a)^5 - (2*a^3 - 3*a)*cosh(b*x + a))*sinh(b*x + a)^3 + 2*a^3 + 4*(7*(2*a^
3 - 3*a)*cosh(b*x + a)^6 - 3*(2*a^3 - 3*a)*cosh(b*x + a)^2)*sinh(b*x + a)^2
 + 8*((2*a^3 - 3*a)*cosh(b*x + a)^7 - (2*a^3 - 3*a)*cosh(b*x + a)^3)*sinh(b
*x + a) - 3*a)*log(cosh(b*x + a) + sinh(b*x + a) - I) - ((2*a^3 - 3*a)*cosh
(b*x + a)^8 + 56*(2*a^3 - 3*a)*cosh(b*x + a)^3*sinh(b*x + a)^5 + 28*(2*a^3
- 3*a)*cosh(b*x + a)^2*sinh(b*x + a)^6 + 8*(2*a^3 - 3*a)*cosh(b*x + a)*sinh
(b*x + a)^7 + (2*a^3 - 3*a)*sinh(b*x + a)^8 - 2*(2*a^3 - 3*a)*cosh(b*x + a)
^4 + 2*(35*(2*a^3 - 3*a)*cosh(b*x + a)^4 - 2*a^3 + 3*a)*sinh(b*x + a)^4 + 8
*(7*(2*a^3 - 3*a)*cosh(b*x + a)^5 - (2*a^3 - 3*a)*cosh(b*x + a))*sinh(b*x +
 a)^3 + 2*a^3 + 4*(7*(2*a^3 - 3*a)*cosh(b*x + a)^6 - 3*(2*a^3 - 3*a)*cosh(b
*x + a)^2)*sinh(b*x + a)^2 + 8*((2*a^3 - 3*a)*cosh(b*x + a)^7 - (2*a^3 - 3*
a)*cosh(b*x + a)^3)*sinh(b*x + a) - 3*a)*log(cosh(b*x + a) + sinh(b*x + a)
- 1) - ((2*b^3*x^3 + 2*a^3 - 3*b*x - 3*a)*cosh(b*x + a)^8 + 56*(2*b^3*x^3 +
 2*a^3 - 3*b*x - 3*a)*cosh(b*x + a)^3*sinh(b*x + a)^5 + 28*(2*b^3*x^3 + 2*a
^3 - 3*b*x - 3*a)*cosh(b*x + a)^2*sinh(b*x + a)^6 + 8*(2*b^3*x^3 + 2*a^3 -
3*b*x - 3*a)*cosh(b*x + a)*sinh(b*x + a)^7 + (2*b^3*x^3 + 2*a^3 - 3*b*x - 3
```

```
*a)*sinh(b*x + a)^8 + 2*b^3*x^3 - 2*(2*b^3*x^3 + 2*a^3 - 3*b*x - 3*a)*cosh(
b*x + a)^4 - 2*(2*b^3*x^3 - 35*(2*b^3*x^3 + 2*a^3 - 3*b*x - 3*a)*cosh(b*x +
 a)^4 + 2*a^3 - 3*b*x - 3*a)*sinh(b*x + a)^4 + 8*(7*(2*b^3*x^3 + 2*a^3 - 3*
b*x - 3*a)*cosh(b*x + a)^5 - (2*b^3*x^3 + 2*a^3 - 3*b*x - 3*a)*cosh(b*x + a
))*sinh(b*x + a)^3 + 2*a^3 + 4*(7*(2*b^3*x^3 + 2*a^3 - 3*b*x - 3*a)*cosh(b*
x + a)^6 - 3*(2*b^3*x^3 + 2*a^3 - 3*b*x - 3*a)*cosh(b*x + a)^2)*sinh(b*x +
a)^2 - 3*b*x + 8*((2*b^3*x^3 + 2*a^3 - 3*b*x - 3*a)*cosh(b*x + a)^7 - (2*b^
3*x^3 + 2*a^3 - 3*b*x - 3*a)*cosh(b*x + a)^3)*sinh(b*x + a) - 3*a)*log(I*co
sh(b*x + a) + I*sinh(b*x + a) + 1) - ((2*b^3*x^3 + 2*a^3 - 3*b*x - 3*a)*cos
h(b*x + a)^8 + 56*(2*b^3*x^3 + 2*a^3 - 3*b*x - 3*a)*cosh(b*x + a)^3*sinh(b*
x + a)^5 + 28*(2*b^3*x^3 + 2*a^3 - 3*b*x - 3*a)*cosh(b*x + a)^2*sinh(b*x +
a)^6 + 8*(2*b^3*x^3 + 2*a^3 - 3*b*x - 3*a)*cosh(b*x + a)*sinh(b*x + a)^7 +
(2*b^3*x^3 + 2*a^3 - 3*b*x - 3*a)*sinh(b*x + a)^8 + 2*b^3*x^3 - 2*(2*b^3*x^
3 + 2*a^3 - 3*b*x - 3*a)*cosh(b*x + a)^4 - 2*(2*b^3*x^3 - 35*(2*b^3*x^3 + 2
*a^3 - 3*b*x - 3*a)*cosh(b*x + a)^4 + 2*a^3 - 3*b*x - 3*a)*sinh(b*x + a)^4
+ 8*(7*(2*b^3*x^3 + 2*a^3 - 3*b*x - 3*a)*cosh(b*x + a)^5 - (2*b^3*x^3 + 2*a
^3 - 3*b*x - 3*a)*cosh(b*x + a))*sinh(b*x + a)^3 + 2*a^3 + 4*(7*(2*b^3*x^3
+ 2*a^3 - 3*b*x - 3*a)*cosh(b*x + a)^6 - 3*(2*b^3*x^3 + 2*a^3 - 3*b*x - 3*a
)*cosh(b*x + a)^2)*sinh(b*x + a)^2 - 3*b*x + 8*((2*b^3*x^3 + 2*a^3 - 3*b*x
- 3*a)*cosh(b*x + a)^7 - (2*b^3*x^3 + 2*a^3 - 3*b*x - 3*a)*cosh(b*x + a)^3)
*sinh(b*x + a) - 3*a)*log(-I*cosh(b*x + a) - I*sinh(b*x + a) + 1) + ((2*b^3
*x^3 + 2*a^3 - 3*b*x - 3*a)*cosh(b*x + a)^8 + 56*(2*b^3*x^3 + 2*a^3 - 3*b*x
 - 3*a)*cosh(b*x + a)^3*sinh(b*x + a)^5 + 28*(2*b^3*x^3 + 2*a^3 - 3*b*x - 3
*a)*cosh(b*x + a)^2*sinh(b*x + a)^6 + 8*(2*b^3*x^3 + 2*a^3 - 3*b*x - 3*a)*c
osh(b*x + a)*sinh(b*x + a)^7 + (2*b^3*x^3 + 2*a^3 - 3*b*x - 3*a)*sinh(b*x +
 a)^8 + 2*b^3*x^3 - 2*(2*b^3*x^3 + 2*a^3 - 3*b*x - 3*a)*cosh(b*x + a)^4 - 2
*(2*b^3*x^3 - 35*(2*b^3*x^3 + 2*a^3 - 3*b*x - 3*a)*cosh(b*x + a)^4 + 2*a^3
- 3*b*x - 3*a)*sinh(b*x + a)^4 + 8*(7*(2*b^3*x^3 + 2*a^3 - 3*b*x - 3*a)*cos
h(b*x + a)^5 - (2*b^3*x^3 + 2*a^3 - 3*b*x - 3*a)*cosh(b*x + a))*sinh(b*x +
a)^3 + 2*a^3 + 4*(7*(2*b^3*x^3 + 2*a^3 - 3*b*x - 3*a)*cosh(b*x + a)^6 - 3*(
2*b^3*x^3 + 2*a^3 - 3*b*x - 3*a)*cosh(b*x + a)^2)*sinh(b*x + a)^2 - 3*b*x +
 8*((2*b^3*x^3 + 2*a^3 - 3*b*x - 3*a)*cosh(b*x + a)^7 - (2*b^3*x^3 + 2*a^3
- 3*b*x - 3*a)*cosh(b*x + a)^3)*sinh(b*x + a) - 3*a)*log(-cosh(b*x + a) - s
inh(b*x + a) + 1) + 12*(cosh(b*x + a)^8 + 56*cosh(b*x + a)^3*sinh(b*x + a)^
5 + 28*cosh(b*x + a)^2*sinh(b*x + a)^6 + 8*cosh(b*x + a)*sinh(b*x + a)^7 +
sinh(b*x + a)^8 + 2*(35*cosh(b*x + a)^4 - 1)*sinh(b*x + a)^4 - 2*cosh(b*x +
 a)^4 + 8*(7*cosh(b*x + a)^5 - cosh(b*x + a))*sinh(b*x + a)^3 + 4*(7*cosh(b
*x + a)^6 - 3*cosh(b*x + a)^2)*sinh(b*x + a)^2 + 8*(cosh(b*x + a)^7 - cosh(
b*x + a)^3)*sinh(b*x + a) + 1)*polylog(4, cosh(b*x + a) + sinh(b*x + a)) -
12*(cosh(b*x + a)^8 + 56*cosh(b*x + a)^3*sinh(b*x + a)^5 + 28*cosh(b*x + a)
^2*sinh(b*x + a)^6 + 8*cosh(b*x + a)*sinh(b*x + a)^7 + sinh(b*x + a)^8 + 2*
(35*cosh(b*x + a)^4 - 1)*sinh(b*x + a)^4 - 2*cosh(b*x + a)^4 + 8*(7*cosh(b*
x + a)^5 - cosh(b*x + a))*sinh(b*x + a)^3 + 4*(7*cosh(b*x + a)^6 - 3*cosh(b
*x + a)^2)*sinh(b*x + a)^2 + 8*(cosh(b*x + a)^7 - cosh(b*x + a)^3)*sinh(b*x
 + a) + 1)*polylog(4, I*cosh(b*x + a) + I*sinh(b*x + a)) - 12*(cosh(b*x + a
```

```
)^8 + 56*cosh(b*x + a)^3*sinh(b*x + a)^5 + 28*cosh(b*x + a)^2*sinh(b*x + a)
^6 + 8*cosh(b*x + a)*sinh(b*x + a)^7 + sinh(b*x + a)^8 + 2*(35*cosh(b*x + a
)^4 - 1)*sinh(b*x + a)^4 - 2*cosh(b*x + a)^4 + 8*(7*cosh(b*x + a)^5 - cosh(
b*x + a))*sinh(b*x + a)^3 + 4*(7*cosh(b*x + a)^6 - 3*cosh(b*x + a)^2)*sinh(
b*x + a)^2 + 8*(cosh(b*x + a)^7 - cosh(b*x + a)^3)*sinh(b*x + a) + 1)*polyl
og(4, -I*cosh(b*x + a) - I*sinh(b*x + a)) + 12*(cosh(b*x + a)^8 + 56*cosh(b
*x + a)^3*sinh(b*x + a)^5 + 28*cosh(b*x + a)^2*sinh(b*x + a)^6 + 8*cosh(b*x
 + a)*sinh(b*x + a)^7 + sinh(b*x + a)^8 + 2*(35*cosh(b*x + a)^4 - 1)*sinh(b
*x + a)^4 - 2*cosh(b*x + a)^4 + 8*(7*cosh(b*x + a)^5 - cosh(b*x + a))*sinh(
b*x + a)^3 + 4*(7*cosh(b*x + a)^6 - 3*cosh(b*x + a)^2)*sinh(b*x + a)^2 + 8*
(cosh(b*x + a)^7 - cosh(b*x + a)^3)*sinh(b*x + a) + 1)*polylog(4, -cosh(b*x
 + a) - sinh(b*x + a)) - 12*(b*x*cosh(b*x + a)^8 + 56*b*x*cosh(b*x + a)^3*s
inh(b*x + a)^5 + 28*b*x*cosh(b*x + a)^2*sinh(b*x + a)^6 + 8*b*x*cosh(b*x +
a)*sinh(b*x + a)^7 + b*x*sinh(b*x + a)^8 - 2*b*x*cosh(b*x + a)^4 + 2*(35*b*
x*cosh(b*x + a)^4 - b*x)*sinh(b*x + a)^4 + 8*(7*b*x*cosh(b*x + a)^5 - b*x*c
osh(b*x + a))*sinh(b*x + a)^3 + 4*(7*b*x*cosh(b*x + a)^6 - 3*b*x*cosh(b*x +
 a)^2)*sinh(b*x + a)^2 + b*x + 8*(b*x*cosh(b*x + a)^7 - b*x*cosh(b*x + a)^3
)*sinh(b*x + a))*polylog(3, cosh(b*x + a) + sinh(b*x + a)) + 12*(b*x*cosh(b
*x + a)^8 + 56*b*x*cosh(b*x + a)^3*sinh(b*x + a)^5 + 28*b*x*cosh(b*x + a)^2
*sinh(b*x + a)^6 + 8*b*x*cosh(b*x + a)*sinh(b*x + a)^7 + b*x*sinh(b*x + a)^
8 - 2*b*x*cosh(b*x + a)^4 + 2*(35*b*x*cosh(b*x + a)^4 - b*x)*sinh(b*x + a)^
4 + 8*(7*b*x*cosh(b*x + a)^5 - b*x*cosh(b*x + a))*sinh(b*x + a)^3 + 4*(7*b*
x*cosh(b*x + a)^6 - 3*b*x*cosh(b*x + a)^2)*sinh(b*x + a)^2 + b*x + 8*(b*x*c
osh(b*x + a)^7 - b*x*cosh(b*x + a)^3)*sinh(b*x + a))*polylog(3, I*cosh(b*x
+ a) + I*sinh(b*x + a)) + 12*(b*x*cosh(b*x + a)^8 + 56*b*x*cosh(b*x + a)^3*
sinh(b*x + a)^5 + 28*b*x*cosh(b*x + a)^2*sinh(b*x + a)^6 + 8*b*x*cosh(b*x +
 a)*sinh(b*x + a)^7 + b*x*sinh(b*x + a)^8 - 2*b*x*cosh(b*x + a)^4 + 2*(35*b
*x*cosh(b*x + a)^4 - b*x)*sinh(b*x + a)^4 + 8*(7*b*x*cosh(b*x + a)^5 - b*x*
cosh(b*x + a))*sinh(b*x + a)^3 + 4*(7*b*x*cosh(b*x + a)^6 - 3*b*x*cosh(b*x
+ a)^2)*sinh(b*x + a)^2 + b*x + 8*(b*x*cosh(b*x + a)^7 - b*x*cosh(b*x + a)^
3)*sinh(b*x + a))*polylog(3, -I*cosh(b*x + a) - I*sinh(b*x + a)) - 12*(b*x*
cosh(b*x + a)^8 + 56*b*x*cosh(b*x + a)^3*sinh(b*x + a)^5 + 28*b*x*cosh(b*x
+ a)^2*sinh(b*x + a)^6 + 8*b*x*cosh(b*x + a)*sinh(b*x + a)^7 + b*x*sinh(b*x
 + a)^8 - 2*b*x*cosh(b*x + a)^4 + 2*(35*b*x*cosh(b*x + a)^4 - b*x)*sinh(b*x
 + a)^4 + 8*(7*b*x*cosh(b*x + a)^5 - b*x*cosh(b*x + a))*sinh(b*x + a)^3 + 4
*(7*b*x*cosh(b*x + a)^6 - 3*b*x*cosh(b*x + a)^2)*sinh(b*x + a)^2 + b*x + 8*
(b*x*cosh(b*x + a)^7 - b*x*cosh(b*x + a)^3)*sinh(b*x + a))*polylog(3, -cosh
(b*x + a) - sinh(b*x + a)) + 4*(3*(2*b^3*x^3 + 3*b^2*x^2)*cosh(b*x + a)^5 +
 (2*b^3*x^3 - 3*b^2*x^2)*cosh(b*x + a))*sinh(b*x + a))/(b^4*cosh(b*x + a)^8
 + 56*b^4*cosh(b*x + a)^3*sinh(b*x + a)^5 + 28*b^4*cosh(b*x + a)^2*sinh(b*x
 + a)^6 + 8*b^4*cosh(b*x + a)*sinh(b*x + a)^7 + b^4*sinh(b*x + a)^8 - 2*b^4
*cosh(b*x + a)^4 + 2*(35*b^4*cosh(b*x + a)^4 - b^4)*sinh(b*x + a)^4 + b^4 +
 8*(7*b^4*cosh(b*x + a)^5 - b^4*cosh(b*x + a))*sinh(b*x + a)^3 + 4*(7*b^4*c
osh(b*x + a)^6 - 3*b^4*cosh(b*x + a)^2)*sinh(b*x + a)^2 + 8*(b^4*cosh(b*x +
 a)^7 - b^4*cosh(b*x + a)^3)*sinh(b*x + a))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3 \operatorname{csch}(bx+a)^3 \operatorname{sech}(bx+a)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*csch(b*x+a)^3*sech(b*x+a)^3,x, algorithm="giac")
```

```
[Out] integrate(x^3*csch(b*x + a)^3*sech(b*x + a)^3, x)
```

maple [A] time = 0.49, size = 445, normalized size = 1.85

$$-\frac{2x^2 e^{2bx+2a}\left(2bx\, e^{4bx+4a}+3\, e^{4bx+4a}+2bx-3\right)}{b^2\left(e^{2bx+2a}-1\right)^2\left(1+e^{2bx+2a}\right)^2}+\frac{3\operatorname{polylog}\left(2, e^{bx+a}\right)}{b^4}-\frac{12\operatorname{polylog}\left(4, e^{bx+a}\right)}{b^4}+\frac{3\operatorname{polylog}\left(2, -e^{bx}\right.}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*csch(b*x+a)^3*sech(b*x+a)^3,x)
```

```
[Out] -2*x^2*exp(2*b*x+2*a)*(2*b*x*exp(4*b*x+4*a)+3*exp(4*b*x+4*a)+2*b*x-3)/b^2/(
exp(2*b*x+2*a)-1)^2/(1+exp(2*b*x+2*a))^2+3*polylog(2,exp(b*x+a))/b^4-12*pol
ylog(4,exp(b*x+a))/b^4+3*polylog(2,-exp(b*x+a))/b^4-2/b*ln(1-exp(b*x+a))*x^
3-6*x^2*polylog(2,exp(b*x+a))/b^2+12*x*polylog(3,exp(b*x+a))/b^3-2/b*ln(1+e
xp(b*x+a))*x^3-6*x^2*polylog(2,-exp(b*x+a))/b^2+12*x*polylog(3,-exp(b*x+a))
/b^3+3/b^4*ln(1-exp(b*x+a))*a+3/2*polylog(4,-exp(2*b*x+2*a))/b^4-3/2*polylo
g(2,-exp(2*b*x+2*a))/b^4-12*polylog(4,-exp(b*x+a))/b^4-2/b^4*ln(1-exp(b*x+a
))*a^3+2/b^4*a^3*ln(exp(b*x+a)-1)-3/b^4*a*ln(exp(b*x+a)-1)-3*x*ln(1+exp(2*b
*x+2*a))/b^3-3*x*polylog(3,-exp(2*b*x+2*a))/b^3+3/b^3*ln(1-exp(b*x+a))*x+3/
b^3*ln(1+exp(b*x+a))*x+2*x^3*ln(1+exp(2*b*x+2*a))/b+3*x^2*polylog(2,-exp(2*
b*x+2*a))/b^2
```

maxima [A] time = 2.15, size = 381, normalized size = 1.59

$$-\frac{2\left(\left(2\,bx^3e^{(6\,a)}+3\,x^2e^{(6\,a)}\right)e^{(6\,bx)}+\left(2\,bx^3e^{(2\,a)}-3\,x^2e^{(2\,a)}\right)e^{(2\,bx)}\right)}{b^2e^{(8\,bx+8\,a)}-2\,b^2e^{(4\,bx+4\,a)}+b^2}+\frac{2\left(4\,b^3x^3\log\left(e^{(2\,bx+2\,a)}+1\right)+6\,b^2x^2\mathrm{Li}_2\left(-e^{(2\,bx}\right.\right.}{3\,}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*csch(b*x+a)^3*sech(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -2*((2*b*x^3*e^(6*a) + 3*x^2*e^(6*a))*e^(6*b*x) + (2*b*x^3*e^(2*a) - 3*x^2*
e^(2*a))*e^(2*b*x))/(b^2*e^(8*b*x + 8*a) - 2*b^2*e^(4*b*x + 4*a) + b^2) + 2
/3*(4*b^3*x^3*log(e^(2*b*x + 2*a) + 1) + 6*b^2*x^2*dilog(-e^(2*b*x + 2*a))
- 6*b*x*polylog(3, -e^(2*b*x + 2*a)) + 3*polylog(4, -e^(2*b*x + 2*a)))/b^4
- 2*(b^3*x^3*log(e^(b*x + a) + 1) + 3*b^2*x^2*dilog(-e^(b*x + a)) - 6*b*x*p
```

```
olylog(3, -e^(b*x + a)) + 6*polylog(4, -e^(b*x + a)))/b^4 - 2*(b^3*x^3*log(
-e^(b*x + a) + 1) + 3*b^2*x^2*dilog(e^(b*x + a)) - 6*b*x*polylog(3, e^(b*x
+ a)) + 6*polylog(4, e^(b*x + a)))/b^4 - 3/2*(2*b*x*log(e^(2*b*x + 2*a) + 1
) + dilog(-e^(2*b*x + 2*a)))/b^4 + 3*(b*x*log(e^(b*x + a) + 1) + dilog(-e^(
b*x + a)))/b^4 + 3*(b*x*log(-e^(b*x + a) + 1) + dilog(e^(b*x + a)))/b^4
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3}{\cosh\left(a+b\,x\right)^3\,\sinh\left(a+b\,x\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(cosh(a + b*x)^3*sinh(a + b*x)^3),x)
```

```
[Out] int(x^3/(cosh(a + b*x)^3*sinh(a + b*x)^3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^{3}\operatorname{csch}^{3}(a+bx)\operatorname{sech}^{3}(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*csch(b*x+a)**3*sech(b*x+a)**3,x)
```

```
[Out] Integral(x**3*csch(a + b*x)**3*sech(a + b*x)**3, x)
```

3.523 $\int x^2 \operatorname{csch}^3(a+bx)\operatorname{sech}^3(a+bx)\,dx$

Optimal. Leaf size=149

$$-\frac{\operatorname{Li}_3\left(-e^{2a+2bx}\right)}{b^3}+\frac{\operatorname{Li}_3\left(e^{2a+2bx}\right)}{b^3}-\frac{\tanh^{-1}(\cosh(2a+2bx))}{b^3}+\frac{2x\operatorname{Li}_2\left(-e^{2a+2bx}\right)}{b^2}-\frac{2x\operatorname{Li}_2\left(e^{2a+2bx}\right)}{b^2}-\frac{2x\operatorname{csch}(2a+2bx)}{b^2}+$$

```
[Out] 4*x^2*arctanh(exp(2*b*x+2*a))/b-arctanh(cosh(2*b*x+2*a))/b^3-2*x*csch(2*b*x
+2*a)/b^2-2*x^2*coth(2*b*x+2*a)*csch(2*b*x+2*a)/b+2*x*polylog(2,-exp(2*b*x+
2*a))/b^2-2*x*polylog(2,exp(2*b*x+2*a))/b^2-polylog(3,-exp(2*b*x+2*a))/b^3+
polylog(3,exp(2*b*x+2*a))/b^3
```

Rubi [A] time = 0.20, antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {5461, 4186, 3770, 4182, 2531, 2282, 6589}

$$\frac{2x\text{PolyLog}\left(2,-e^{2a+2bx}\right)}{b^2}-\frac{2x\text{PolyLog}\left(2,e^{2a+2bx}\right)}{b^2}-\frac{\text{PolyLog}\left(3,-e^{2a+2bx}\right)}{b^3}+\frac{\text{PolyLog}\left(3,e^{2a+2bx}\right)}{b^3}-\frac{2x\operatorname{csch}(2a+2}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Csch[a + b*x]^3*Sech[a + b*x]^3,x]
```

```
[Out] (4*x^2*ArcTanh[E^(2*a + 2*b*x)])/b - ArcTanh[Cosh[2*a + 2*b*x]]/b^3 - (2*x*
Csch[2*a + 2*b*x])/b^2 - (2*x^2*Coth[2*a + 2*b*x]*Csch[2*a + 2*b*x])/b + (2
*x*PolyLog[2, -E^(2*a + 2*b*x)])/b^2 - (2*x*PolyLog[2, E^(2*a + 2*b*x)])/b^
2 - PolyLog[3, -E^(2*a + 2*b*x)]/b^3 + PolyLog[3, E^(2*a + 2*b*x)]/b^3
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 4186

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> -Simp[(b^2*(c + d*x)^m*Cot[e + f*x]*(b*Csc[e + f*x])^(n - 2))/(f*(n -
 1)), x] + (Dist[(b^2*d^2*m*(m - 1))/(f^2*(n - 1)*(n - 2)), Int[(c + d*x)^(
m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Dist[(b^2*(n - 2))/(n - 1), Int[(
c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x] - Simp[(b^2*d*m*(c + d*x)^(m -
1)*(b*Csc[e + f*x])^(n - 2))/(f^2*(n - 1)*(n - 2)), x]) /; FreeQ[{b, c, d,
e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

Rule 5461

```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[2^n, Int[(c + d*x)^m*Csch[2*a + 2*b*x]
^n, x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[m] && IntegerQ[n]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int x^2\text{csch}^3(a+bx)\text{sech}^3(a+bx)\,dx = 8\int x^2\text{csch}^3(2a+2bx)\,dx$$
$$= -\frac{2x\text{csch}(2a+2bx)}{b^2} - \frac{2x^2\coth(2a+2bx)\text{csch}(2a+2bx)}{b} - 4\int x^2\text{csch}(2a$$
$$= \frac{4x^2\tanh^{-1}\left(e^{2a+2bx}\right)}{b} - \frac{\tanh^{-1}(\cosh(2a+2bx))}{b^3} - \frac{2x\text{csch}(2a+2bx)}{b^2} - \frac{2x}{}$$
$$= \frac{4x^2\tanh^{-1}\left(e^{2a+2bx}\right)}{b} - \frac{\tanh^{-1}(\cosh(2a+2bx))}{b^3} - \frac{2x\text{csch}(2a+2bx)}{b^2} - \frac{2x}{}$$
$$= \frac{4x^2\tanh^{-1}\left(e^{2a+2bx}\right)}{b} - \frac{\tanh^{-1}(\cosh(2a+2bx))}{b^3} - \frac{2x\text{csch}(2a+2bx)}{b^2} - \frac{2x}{}$$
$$= \frac{4x^2\tanh^{-1}\left(e^{2a+2bx}\right)}{b} - \frac{\tanh^{-1}(\cosh(2a+2bx))}{b^3} - \frac{2x\text{csch}(2a+2bx)}{b^2} - \frac{2x}{}$$

Mathematica [A] time = 6.36, size = 192, normalized size = 1.29

$$-\frac{4b^2x^2\log\left(1-e^{2(a+bx)}\right) - 4b^2x^2\log\left(e^{2(a+bx)}+1\right) + b^2x^2\text{csch}^2(a+bx) + b^2x^2\text{sech}^2(a+bx) - 4bx\text{Li}_2\left(-e^{2(a+bx)}\right)}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Csch[a + b*x]^3*Sech[a + b*x]^3,x]
```

```
[Out] -1/2*(4*ArcTanh[E^(2*(a + b*x))] + b^2*x^2*Csch[a + b*x]^2 + 4*b^2*x^2*Log[
1 - E^(2*(a + b*x))] - 4*b^2*x^2*Log[1 + E^(2*(a + b*x))] - 4*b*x*PolyLog[2
, -E^(2*(a + b*x))] + 4*b*x*PolyLog[2, E^(2*(a + b*x))] + 2*PolyLog[3, -E^(
2*(a + b*x))] - 2*PolyLog[3, E^(2*(a + b*x))] + 2*b*x*Csch[a]*Sech[a] + b^2
*x^2*Sech[a + b*x]^2 - 2*b*x*Csch[a]*Csch[a + b*x]*Sinh[b*x] - 2*b*x*Sech[a
]*Sech[a + b*x]*Sinh[b*x])/b^3
```

fricas [C] time = 0.55, size = 4779, normalized size = 32.07

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*csch(b*x+a)^3*sech(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -(4*(b^2*x^2 + b*x)*cosh(b*x + a)^6 + 80*(b^2*x^2 + b*x)*cosh(b*x + a)^3*si
nh(b*x + a)^3 + 60*(b^2*x^2 + b*x)*cosh(b*x + a)^2*sinh(b*x + a)^4 + 24*(b^
2*x^2 + b*x)*cosh(b*x + a)*sinh(b*x + a)^5 + 4*(b^2*x^2 + b*x)*sinh(b*x + a
)^6 + 4*(b^2*x^2 - b*x)*cosh(b*x + a)^2 + 4*(15*(b^2*x^2 + b*x)*cosh(b*x +
```

a)^4 + b^2*x^2 - b*x)*sinh(b*x + a)^2 + 4*(b*x*cosh(b*x + a)^8 + 56*b*x*cos
h(b*x + a)^3*sinh(b*x + a)^5 + 28*b*x*cosh(b*x + a)^2*sinh(b*x + a)^6 + 8*b
*x*cosh(b*x + a)*sinh(b*x + a)^7 + b*x*sinh(b*x + a)^8 - 2*b*x*cosh(b*x + a
)^4 + 2*(35*b*x*cosh(b*x + a)^4 - b*x)*sinh(b*x + a)^4 + 8*(7*b*x*cosh(b*x
+ a)^5 - b*x*cosh(b*x + a))*sinh(b*x + a)^3 + 4*(7*b*x*cosh(b*x + a)^6 - 3*
b*x*cosh(b*x + a)^2)*sinh(b*x + a)^2 + b*x + 8*(b*x*cosh(b*x + a)^7 - b*x*c
osh(b*x + a)^3)*sinh(b*x + a))*dilog(cosh(b*x + a) + sinh(b*x + a)) - 4*(b*
x*cosh(b*x + a)^8 + 56*b*x*cosh(b*x + a)^3*sinh(b*x + a)^5 + 28*b*x*cosh(b*
x + a)^2*sinh(b*x + a)^6 + 8*b*x*cosh(b*x + a)*sinh(b*x + a)^7 + b*x*sinh(b
*x + a)^8 - 2*b*x*cosh(b*x + a)^4 + 2*(35*b*x*cosh(b*x + a)^4 - b*x)*sinh(b
x + a)^4 + 8(7*b*x*cosh(b*x + a)^5 - b*x*cosh(b*x + a))*sinh(b*x + a)^3 +
4*(7*b*x*cosh(b*x + a)^6 - 3*b*x*cosh(b*x + a)^2)*sinh(b*x + a)^2 + b*x +
8*(b*x*cosh(b*x + a)^7 - b*x*cosh(b*x + a)^3)*sinh(b*x + a))*dilog(I*cosh(b
*x + a) + I*sinh(b*x + a)) - 4*(b*x*cosh(b*x + a)^8 + 56*b*x*cosh(b*x + a)^
3*sinh(b*x + a)^5 + 28*b*x*cosh(b*x + a)^2*sinh(b*x + a)^6 + 8*b*x*cosh(b*x
+ a)*sinh(b*x + a)^7 + b*x*sinh(b*x + a)^8 - 2*b*x*cosh(b*x + a)^4 + 2*(35
*b*x*cosh(b*x + a)^4 - b*x)*sinh(b*x + a)^4 + 8*(7*b*x*cosh(b*x + a)^5 - b*
x*cosh(b*x + a))*sinh(b*x + a)^3 + 4*(7*b*x*cosh(b*x + a)^6 - 3*b*x*cosh(b*
x + a)^2)*sinh(b*x + a)^2 + b*x + 8*(b*x*cosh(b*x + a)^7 - b*x*cosh(b*x + a
)^3)*sinh(b*x + a))*dilog(-I*cosh(b*x + a) - I*sinh(b*x + a)) + 4*(b*x*cosh
(b*x + a)^8 + 56*b*x*cosh(b*x + a)^3*sinh(b*x + a)^5 + 28*b*x*cosh(b*x + a)
^2*sinh(b*x + a)^6 + 8*b*x*cosh(b*x + a)*sinh(b*x + a)^7 + b*x*sinh(b*x + a
)^8 - 2*b*x*cosh(b*x + a)^4 + 2*(35*b*x*cosh(b*x + a)^4 - b*x)*sinh(b*x + a
)^4 + 8*(7*b*x*cosh(b*x + a)^5 - b*x*cosh(b*x + a))*sinh(b*x + a)^3 + 4*(7*
b*x*cosh(b*x + a)^6 - 3*b*x*cosh(b*x + a)^2)*sinh(b*x + a)^2 + b*x + 8*(b*x
*cosh(b*x + a)^7 - b*x*cosh(b*x + a)^3)*sinh(b*x + a))*dilog(-cosh(b*x + a)
- sinh(b*x + a)) + ((2*b^2*x^2 - 1)*cosh(b*x + a)^8 + 56*(2*b^2*x^2 - 1)*c
osh(b*x + a)^3*sinh(b*x + a)^5 + 28*(2*b^2*x^2 - 1)*cosh(b*x + a)^2*sinh(b*
x + a)^6 + 8*(2*b^2*x^2 - 1)*cosh(b*x + a)*sinh(b*x + a)^7 + (2*b^2*x^2 - 1
)*sinh(b*x + a)^8 - 2*(2*b^2*x^2 - 1)*cosh(b*x + a)^4 + 2*(35*(2*b^2*x^2 -
1)*cosh(b*x + a)^4 - 2*b^2*x^2 + 1)*sinh(b*x + a)^4 + 2*b^2*x^2 + 8*(7*(2*b
^2*x^2 - 1)*cosh(b*x + a)^5 - (2*b^2*x^2 - 1)*cosh(b*x + a))*sinh(b*x + a)^
3 + 4*(7*(2*b^2*x^2 - 1)*cosh(b*x + a)^6 - 3*(2*b^2*x^2 - 1)*cosh(b*x + a)^
2)*sinh(b*x + a)^2 + 8*((2*b^2*x^2 - 1)*cosh(b*x + a)^7 - (2*b^2*x^2 - 1)*c
osh(b*x + a)^3)*sinh(b*x + a) - 1)*log(cosh(b*x + a) + sinh(b*x + a) + 1) -
((2*a^2 - 1)*cosh(b*x + a)^8 + 56*(2*a^2 - 1)*cosh(b*x + a)^3*sinh(b*x + a
)^5 + 28*(2*a^2 - 1)*cosh(b*x + a)^2*sinh(b*x + a)^6 + 8*(2*a^2 - 1)*cosh(b
*x + a)*sinh(b*x + a)^7 + (2*a^2 - 1)*sinh(b*x + a)^8 - 2*(2*a^2 - 1)*cosh(
b*x + a)^4 + 2*(35*(2*a^2 - 1)*cosh(b*x + a)^4 - 2*a^2 + 1)*sinh(b*x + a)^4
+ 8*(7*(2*a^2 - 1)*cosh(b*x + a)^5 - (2*a^2 - 1)*cosh(b*x + a))*sinh(b*x +
a)^3 + 4*(7*(2*a^2 - 1)*cosh(b*x + a)^6 - 3*(2*a^2 - 1)*cosh(b*x + a)^2)*s
inh(b*x + a)^2 + 2*a^2 + 8*((2*a^2 - 1)*cosh(b*x + a)^7 - (2*a^2 - 1)*cosh(
b*x + a)^3)*sinh(b*x + a) - 1)*log(cosh(b*x + a) + sinh(b*x + a) + I) - ((2
*a^2 - 1)*cosh(b*x + a)^8 + 56*(2*a^2 - 1)*cosh(b*x + a)^3*sinh(b*x + a)^5
+ 28*(2*a^2 - 1)*cosh(b*x + a)^2*sinh(b*x + a)^6 + 8*(2*a^2 - 1)*cosh(b*x +

```
 a)*sinh(b*x + a)^7 + (2*a^2 - 1)*sinh(b*x + a)^8 - 2*(2*a^2 - 1)*cosh(b*x
+ a)^4 + 2*(35*(2*a^2 - 1)*cosh(b*x + a)^4 - 2*a^2 + 1)*sinh(b*x + a)^4 + 8
*(7*(2*a^2 - 1)*cosh(b*x + a)^5 - (2*a^2 - 1)*cosh(b*x + a))*sinh(b*x + a)^
3 + 4*(7*(2*a^2 - 1)*cosh(b*x + a)^6 - 3*(2*a^2 - 1)*cosh(b*x + a)^2)*sinh(
b*x + a)^2 + 2*a^2 + 8*((2*a^2 - 1)*cosh(b*x + a)^7 - (2*a^2 - 1)*cosh(b*x
+ a)^3)*sinh(b*x + a) - 1)*log(cosh(b*x + a) + sinh(b*x + a) - I) + ((2*a^2
 - 1)*cosh(b*x + a)^8 + 56*(2*a^2 - 1)*cosh(b*x + a)^3*sinh(b*x + a)^5 + 28
*(2*a^2 - 1)*cosh(b*x + a)^2*sinh(b*x + a)^6 + 8*(2*a^2 - 1)*cosh(b*x + a)*
sinh(b*x + a)^7 + (2*a^2 - 1)*sinh(b*x + a)^8 - 2*(2*a^2 - 1)*cosh(b*x + a)
^4 + 2*(35*(2*a^2 - 1)*cosh(b*x + a)^4 - 2*a^2 + 1)*sinh(b*x + a)^4 + 8*(7*
(2*a^2 - 1)*cosh(b*x + a)^5 - (2*a^2 - 1)*cosh(b*x + a))*sinh(b*x + a)^3 +
4*(7*(2*a^2 - 1)*cosh(b*x + a)^6 - 3*(2*a^2 - 1)*cosh(b*x + a)^2)*sinh(b*x
+ a)^2 + 2*a^2 + 8*((2*a^2 - 1)*cosh(b*x + a)^7 - (2*a^2 - 1)*cosh(b*x + a)
^3)*sinh(b*x + a) - 1)*log(cosh(b*x + a) + sinh(b*x + a) - 1) - 2*((b^2*x^2
 - a^2)*cosh(b*x + a)^8 + 56*(b^2*x^2 - a^2)*cosh(b*x + a)^3*sinh(b*x + a)^
5 + 28*(b^2*x^2 - a^2)*cosh(b*x + a)^2*sinh(b*x + a)^6 + 8*(b^2*x^2 - a^2)*
cosh(b*x + a)*sinh(b*x + a)^7 + (b^2*x^2 - a^2)*sinh(b*x + a)^8 - 2*(b^2*x^
2 - a^2)*cosh(b*x + a)^4 + 2*(35*(b^2*x^2 - a^2)*cosh(b*x + a)^4 - b^2*x^2
+ a^2)*sinh(b*x + a)^4 + b^2*x^2 + 8*(7*(b^2*x^2 - a^2)*cosh(b*x + a)^5 - (
b^2*x^2 - a^2)*cosh(b*x + a))*sinh(b*x + a)^3 + 4*(7*(b^2*x^2 - a^2)*cosh(b
*x + a)^6 - 3*(b^2*x^2 - a^2)*cosh(b*x + a)^2)*sinh(b*x + a)^2 - a^2 + 8*((
b^2*x^2 - a^2)*cosh(b*x + a)^7 - (b^2*x^2 - a^2)*cosh(b*x + a)^3)*sinh(b*x
+ a))*log(I*cosh(b*x + a) + I*sinh(b*x + a) + 1) - 2*((b^2*x^2 - a^2)*cosh(
b*x + a)^8 + 56*(b^2*x^2 - a^2)*cosh(b*x + a)^3*sinh(b*x + a)^5 + 28*(b^2*x
^2 - a^2)*cosh(b*x + a)^2*sinh(b*x + a)^6 + 8*(b^2*x^2 - a^2)*cosh(b*x + a)
*sinh(b*x + a)^7 + (b^2*x^2 - a^2)*sinh(b*x + a)^8 - 2*(b^2*x^2 - a^2)*cosh
(b*x + a)^4 + 2*(35*(b^2*x^2 - a^2)*cosh(b*x + a)^4 - b^2*x^2 + a^2)*sinh(b
*x + a)^4 + b^2*x^2 + 8*(7*(b^2*x^2 - a^2)*cosh(b*x + a)^5 - (b^2*x^2 - a^2
)*cosh(b*x + a))*sinh(b*x + a)^3 + 4*(7*(b^2*x^2 - a^2)*cosh(b*x + a)^6 - 3
*(b^2*x^2 - a^2)*cosh(b*x + a)^2)*sinh(b*x + a)^2 - a^2 + 8*((b^2*x^2 - a^2
)*cosh(b*x + a)^7 - (b^2*x^2 - a^2)*cosh(b*x + a)^3)*sinh(b*x + a))*log(-I*
cosh(b*x + a) - I*sinh(b*x + a) + 1) + 2*((b^2*x^2 - a^2)*cosh(b*x + a)^8 +
 56*(b^2*x^2 - a^2)*cosh(b*x + a)^3*sinh(b*x + a)^5 + 28*(b^2*x^2 - a^2)*co
sh(b*x + a)^2*sinh(b*x + a)^6 + 8*(b^2*x^2 - a^2)*cosh(b*x + a)*sinh(b*x +
a)^7 + (b^2*x^2 - a^2)*sinh(b*x + a)^8 - 2*(b^2*x^2 - a^2)*cosh(b*x + a)^4
+ 2*(35*(b^2*x^2 - a^2)*cosh(b*x + a)^4 - b^2*x^2 + a^2)*sinh(b*x + a)^4 +
b^2*x^2 + 8*(7*(b^2*x^2 - a^2)*cosh(b*x + a)^5 - (b^2*x^2 - a^2)*cosh(b*x +
 a))*sinh(b*x + a)^3 + 4*(7*(b^2*x^2 - a^2)*cosh(b*x + a)^6 - 3*(b^2*x^2 -
a^2)*cosh(b*x + a)^2)*sinh(b*x + a)^2 - a^2 + 8*((b^2*x^2 - a^2)*cosh(b*x +
 a)^7 - (b^2*x^2 - a^2)*cosh(b*x + a)^3)*sinh(b*x + a))*log(-cosh(b*x + a)
- sinh(b*x + a) + 1) - 4*(cosh(b*x + a)^8 + 56*cosh(b*x + a)^3*sinh(b*x + a
)^5 + 28*cosh(b*x + a)^2*sinh(b*x + a)^6 + 8*cosh(b*x + a)*sinh(b*x + a)^7
+ sinh(b*x + a)^8 + 2*(35*cosh(b*x + a)^4 - 1)*sinh(b*x + a)^4 - 2*cosh(b*x
 + a)^4 + 8*(7*cosh(b*x + a)^5 - cosh(b*x + a))*sinh(b*x + a)^3 + 4*(7*cosh
(b*x + a)^6 - 3*cosh(b*x + a)^2)*sinh(b*x + a)^2 + 8*(cosh(b*x + a)^7 - cos
```

```
h(b*x + a)^3)*sinh(b*x + a) + 1)*polylog(3, cosh(b*x + a) + sinh(b*x + a))
+ 4*(cosh(b*x + a)^8 + 56*cosh(b*x + a)^3*sinh(b*x + a)^5 + 28*cosh(b*x + a
)^2*sinh(b*x + a)^6 + 8*cosh(b*x + a)*sinh(b*x + a)^7 + sinh(b*x + a)^8 + 2
*(35*cosh(b*x + a)^4 - 1)*sinh(b*x + a)^4 - 2*cosh(b*x + a)^4 + 8*(7*cosh(b
*x + a)^5 - cosh(b*x + a))*sinh(b*x + a)^3 + 4*(7*cosh(b*x + a)^6 - 3*cosh(
b*x + a)^2)*sinh(b*x + a)^2 + 8*(cosh(b*x + a)^7 - cosh(b*x + a)^3)*sinh(b*
x + a) + 1)*polylog(3, I*cosh(b*x + a) + I*sinh(b*x + a)) + 4*(cosh(b*x + a
)^8 + 56*cosh(b*x + a)^3*sinh(b*x + a)^5 + 28*cosh(b*x + a)^2*sinh(b*x + a)
^6 + 8*cosh(b*x + a)*sinh(b*x + a)^7 + sinh(b*x + a)^8 + 2*(35*cosh(b*x + a
)^4 - 1)*sinh(b*x + a)^4 - 2*cosh(b*x + a)^4 + 8*(7*cosh(b*x + a)^5 - cosh(
b*x + a))*sinh(b*x + a)^3 + 4*(7*cosh(b*x + a)^6 - 3*cosh(b*x + a)^2)*sinh(
b*x + a)^2 + 8*(cosh(b*x + a)^7 - cosh(b*x + a)^3)*sinh(b*x + a) + 1)*polyl
og(3, -I*cosh(b*x + a) - I*sinh(b*x + a)) - 4*(cosh(b*x + a)^8 + 56*cosh(b*
x + a)^3*sinh(b*x + a)^5 + 28*cosh(b*x + a)^2*sinh(b*x + a)^6 + 8*cosh(b*x
+ a)*sinh(b*x + a)^7 + sinh(b*x + a)^8 + 2*(35*cosh(b*x + a)^4 - 1)*sinh(b*
x + a)^4 - 2*cosh(b*x + a)^4 + 8*(7*cosh(b*x + a)^5 - cosh(b*x + a))*sinh(b
*x + a)^3 + 4*(7*cosh(b*x + a)^6 - 3*cosh(b*x + a)^2)*sinh(b*x + a)^2 + 8*(
cosh(b*x + a)^7 - cosh(b*x + a)^3)*sinh(b*x + a) + 1)*polylog(3, -cosh(b*x
+ a) - sinh(b*x + a)) + 8*(3*(b^2*x^2 + b*x)*cosh(b*x + a)^5 + (b^2*x^2 - b
*x)*cosh(b*x + a))*sinh(b*x + a))/(b^3*cosh(b*x + a)^8 + 56*b^3*cosh(b*x +
a)^3*sinh(b*x + a)^5 + 28*b^3*cosh(b*x + a)^2*sinh(b*x + a)^6 + 8*b^3*cosh(
b*x + a)*sinh(b*x + a)^7 + b^3*sinh(b*x + a)^8 - 2*b^3*cosh(b*x + a)^4 + 2*
(35*b^3*cosh(b*x + a)^4 - b^3)*sinh(b*x + a)^4 + 8*(7*b^3*cosh(b*x + a)^5 -
 b^3*cosh(b*x + a))*sinh(b*x + a)^3 + b^3 + 4*(7*b^3*cosh(b*x + a)^6 - 3*b^
3*cosh(b*x + a)^2)*sinh(b*x + a)^2 + 8*(b^3*cosh(b*x + a)^7 - b^3*cosh(b*x
+ a)^3)*sinh(b*x + a))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \operatorname{csch}(bx+a)^3 \operatorname{sech}(bx+a)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*csch(b*x+a)^3*sech(b*x+a)^3,x, algorithm="giac")
```

```
[Out] integrate(x^2*csch(b*x + a)^3*sech(b*x + a)^3, x)
```

maple [B] time = 0.47, size = 299, normalized size = 2.01

$$-\frac{4x\,\mathrm{e}^{2bx+2a}\left(bx\,\mathrm{e}^{4bx+4a}+\mathrm{e}^{4bx+4a}+bx-1\right)}{b^2\left(\mathrm{e}^{2bx+2a}-1\right)^2\left(1+\mathrm{e}^{2bx+2a}\right)^2}-\frac{\operatorname{polylog}\left(3,-\mathrm{e}^{2bx+2a}\right)}{b^3}+\frac{4\operatorname{polylog}\left(3,\mathrm{e}^{bx+a}\right)}{b^3}+\frac{4\operatorname{polylog}\left(3,-\mathrm{e}^{bx+a}\right)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*csch(b*x+a)^3*sech(b*x+a)^3,x)
```

```
[Out] -4*x*exp(2*b*x+2*a)*(b*x*exp(4*b*x+4*a)+exp(4*b*x+4*a)+b*x-1)/b^2/(exp(2*b*
x+2*a)-1)^2/(1+exp(2*b*x+2*a))^2-polylog(3,-exp(2*b*x+2*a))/b^3+4/b^3*polyl
og(3,exp(b*x+a))+4/b^3*polylog(3,-exp(b*x+a))-2/b^3*a^2*ln(exp(b*x+a)-1)+1/
b^3*ln(exp(b*x+a)-1)-1/b^3*ln(1+exp(2*b*x+2*a))+1/b^3*ln(1+exp(b*x+a))-4/b^
2*polylog(2,exp(b*x+a))*x-2/b*ln(1+exp(b*x+a))*x^2-4/b^2*polylog(2,-exp(b*x
+a))*x+2*x^2*ln(1+exp(2*b*x+2*a))/b+2*x*polylog(2,-exp(2*b*x+2*a))/b^2-2/b*
ln(1-exp(b*x+a))*x^2+2/b^3*ln(1-exp(b*x+a))*a^2
```

maxima [A] time = 2.24, size = 273, normalized size = 1.83

$$-\frac{4\left(\left(bx^2e^{(6a)}+xe^{(6a)}\right)e^{(6bx)}+\left(bx^2e^{(2a)}-xe^{(2a)}\right)e^{(2bx)}\right)}{b^2e^{(8bx+8a)}-2b^2e^{(4bx+4a)}+b^2}+\frac{2b^2x^2\log\left(e^{(2bx+2a)}+1\right)+2bx\mathrm{Li}_2\left(-e^{(2bx+2a)}\right)-\mathrm{Li}_3(-}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*csch(b*x+a)^3*sech(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -4*((b*x^2*e^(6*a) + x*e^(6*a))*e^(6*b*x) + (b*x^2*e^(2*a) - x*e^(2*a))*e^(
2*b*x))/(b^2*e^(8*b*x + 8*a) - 2*b^2*e^(4*b*x + 4*a) + b^2) + (2*b^2*x^2*lo
g(e^(2*b*x + 2*a) + 1) + 2*b*x*dilog(-e^(2*b*x + 2*a)) - polylog(3, -e^(2*b
*x + 2*a)))/b^3 - 2*(b^2*x^2*log(e^(b*x + a) + 1) + 2*b*x*dilog(-e^(b*x + a
)) - 2*polylog(3, -e^(b*x + a)))/b^3 - 2*(b^2*x^2*log(-e^(b*x + a) + 1) + 2
*b*x*dilog(e^(b*x + a)) - 2*polylog(3, e^(b*x + a)))/b^3 - log(e^(2*b*x + 2
*a) + 1)/b^3 + log(e^(b*x + a) + 1)/b^3 + log(e^(b*x + a) - 1)/b^3
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2}{\cosh(a+b\,x)^3\,\sinh(a+b\,x)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(cosh(a + b*x)^3*sinh(a + b*x)^3),x)
```

```
[Out] int(x^2/(cosh(a + b*x)^3*sinh(a + b*x)^3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2\operatorname{csch}^3(a+bx)\operatorname{sech}^3(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*csch(b*x+a)**3*sech(b*x+a)**3,x)
```

```
[Out] Integral(x**2*csch(a + b*x)**3*sech(a + b*x)**3, x)
```

3.524 $\int x\text{csch}^3(a+bx)\text{sech}^3(a+bx)\,dx$

Optimal. Leaf size=91

$$\frac{\text{Li}_2\left(-e^{2a+2bx}\right)}{b^2}-\frac{\text{Li}_2\left(e^{2a+2bx}\right)}{b^2}-\frac{\text{csch}(2a+2bx)}{b^2}+\frac{4x\tanh^{-1}\left(e^{2a+2bx}\right)}{b}-\frac{2x\coth(2a+2bx)\text{csch}(2a+2bx)}{b}$$

```
[Out] 4*x*arctanh(exp(2*b*x+2*a))/b-csch(2*b*x+2*a)/b^2-2*x*coth(2*b*x+2*a)*csch(
2*b*x+2*a)/b+polylog(2,-exp(2*b*x+2*a))/b^2-polylog(2,exp(2*b*x+2*a))/b^2
```

Rubi [A] time = 0.11, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {5461, 4185, 4182, 2279, 2391}

$$\frac{\text{PolyLog}\left(2,-e^{2a+2bx}\right)}{b^2}-\frac{\text{PolyLog}\left(2,e^{2a+2bx}\right)}{b^2}-\frac{\text{csch}(2a+2bx)}{b^2}+\frac{4x\tanh^{-1}\left(e^{2a+2bx}\right)}{b}-\frac{2x\coth(2a+2bx)\text{csch}(2a}{b}$$

Antiderivative was successfully verified.

```
[In] Int[x*Csch[a + b*x]^3*Sech[a + b*x]^3,x]
```

```
[Out] (4*x*ArcTanh[E^(2*a + 2*b*x)])/b - Csch[2*a + 2*b*x]/b^2 - (2*x*Coth[2*a +
2*b*x]*Csch[2*a + 2*b*x])/b + PolyLog[2, -E^(2*a + 2*b*x)]/b^2 - PolyLog[2,
 E^(2*a + 2*b*x)]/b^2
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 4185

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_)), x_Symbol] :>
 -Simp[(b^2*(c + d*x)*Cot[e + f*x]*(b*Csc[e + f*x])^(n - 2))/(f*(n - 1)), x
] + (Dist[(b^2*(n - 2))/(n - 1), Int[(c + d*x)*(b*Csc[e + f*x])^(n - 2), x]
, x] - Simp[(b^2*d*(b*Csc[e + f*x])^(n - 2))/(f^2*(n - 1)*(n - 2)), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2]
```

Rule 5461

```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[2^n, Int[(c + d*x)^m*Csch[2*a + 2*b*x]
^n, x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[m] && IntegerQ[n]
```

Rubi steps

$$\begin{aligned}\int x\text{csch}^3(a+bx)\text{sech}^3(a+bx)\,dx &= 8\int x\text{csch}^3(2a+2bx)\,dx\\ &= -\frac{\text{csch}(2a+2bx)}{b^2}-\frac{2x\coth(2a+2bx)\text{csch}(2a+2bx)}{b}-4\int x\text{csch}(2a+2b\\ &= \frac{4x\tanh^{-1}\left(e^{2a+2bx}\right)}{b}-\frac{\text{csch}(2a+2bx)}{b^2}-\frac{2x\coth(2a+2bx)\text{csch}(2a+2bx)}{b}\\ &= \frac{4x\tanh^{-1}\left(e^{2a+2bx}\right)}{b}-\frac{\text{csch}(2a+2bx)}{b^2}-\frac{2x\coth(2a+2bx)\text{csch}(2a+2bx)}{b}\\ &= \frac{4x\tanh^{-1}\left(e^{2a+2bx}\right)}{b}-\frac{\text{csch}(2a+2bx)}{b^2}-\frac{2x\coth(2a+2bx)\text{csch}(2a+2bx)}{b}\end{aligned}$$

Mathematica [A] time = 1.36, size = 148, normalized size = 1.63

$$-\frac{2\text{Li}_2\left(-e^{-2(a+bx)}\right)-2\text{Li}_2\left(e^{-2(a+bx)}\right)+4a\log\left(1-e^{-2(a+bx)}\right)+4bx\log\left(1-e^{-2(a+bx)}\right)-4a\log\left(e^{-2(a+bx)}+1\right)-4bx}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Csch[a + b*x]^3*Sech[a + b*x]^3,x]
```

```
[Out] -1/2*(Coth[a + b*x] + b*x*Csch[a + b*x]^2 + 4*a*Log[1 - E^(-2*(a + b*x))] +
 4*b*x*Log[1 - E^(-2*(a + b*x))] - 4*a*Log[1 + E^(-2*(a + b*x))] - 4*b*x*Lo
g[1 + E^(-2*(a + b*x))] - 4*a*Log[Tanh[a + b*x]] + 2*PolyLog[2, -E^(-2*(a +
 b*x))] - 2*PolyLog[2, E^(-2*(a + b*x))] + b*x*Sech[a + b*x]^2 - Tanh[a + b
*x])/b^2
```

fricas [C] time = 0.49, size = 3025, normalized size = 33.24

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*csch(b*x+a)^3*sech(b*x+a)^3,x, algorithm="fricas")

[Out] -2*((2*b*x + 1)*cosh(b*x + a)^6 + 20*(2*b*x + 1)*cosh(b*x + a)^3*sinh(b*x + a)^3 + 15*(2*b*x + 1)*cosh(b*x + a)^2*sinh(b*x + a)^4 + 6*(2*b*x + 1)*cosh (b*x + a)*sinh(b*x + a)^5 + (2*b*x + 1)*sinh(b*x + a)^6 + (2*b*x - 1)*cosh(b*x + a)^2 + (15*(2*b*x + 1)*cosh(b*x + a)^4 + 2*b*x - 1)*sinh(b*x + a)^2 + (cosh(b*x + a)^8 + 56*cosh(b*x + a)^3*sinh(b*x + a)^5 + 28*cosh(b*x + a)^2 *sinh(b*x + a)^6 + 8*cosh(b*x + a)*sinh(b*x + a)^7 + sinh(b*x + a)^8 + 2*(3 5*cosh(b*x + a)^4 - 1)*sinh(b*x + a)^4 - 2*cosh(b*x + a)^4 + 8*(7*cosh(b*x + a)^5 - cosh(b*x + a))*sinh(b*x + a)^3 + 4*(7*cosh(b*x + a)^6 - 3*cosh(b*x + a)^2)*sinh(b*x + a)^2 + 8*(cosh(b*x + a)^7 - cosh(b*x + a)^3)*sinh(b*x + a) + 1)*dilog(cosh(b*x + a) + sinh(b*x + a)) - (cosh(b*x + a)^8 + 56*cosh(b*x + a)^3*sinh(b*x + a)^5 + 28*cosh(b*x + a)^2*sinh(b*x + a)^6 + 8*cosh(b* x + a)*sinh(b*x + a)^7 + sinh(b*x + a)^8 + 2*(35*cosh(b*x + a)^4 - 1)*sinh(b*x + a)^4 - 2*cosh(b*x + a)^4 + 8*(7*cosh(b*x + a)^5 - cosh(b*x + a))*sinh (b*x + a)^3 + 4*(7*cosh(b*x + a)^6 - 3*cosh(b*x + a)^2)*sinh(b*x + a)^2 + 8 *(cosh(b*x + a)^7 - cosh(b*x + a)^3)*sinh(b*x + a) + 1)*dilog(I*cosh(b*x + a) + I*sinh(b*x + a)) - (cosh(b*x + a)^8 + 56*cosh(b*x + a)^3*sinh(b*x + a) ^5 + 28*cosh(b*x + a)^2*sinh(b*x + a)^6 + 8*cosh(b*x + a)*sinh(b*x + a)^7 + sinh(b*x + a)^8 + 2*(35*cosh(b*x + a)^4 - 1)*sinh(b*x + a)^4 - 2*cosh(b*x + a)^4 + 8*(7*cosh(b*x + a)^5 - cosh(b*x + a))*sinh(b*x + a)^3 + 4*(7*cosh(b*x + a)^6 - 3*cosh(b*x + a)^2)*sinh(b*x + a)^2 + 8*(cosh(b*x + a)^7 - cosh (b*x + a)^3)*sinh(b*x + a) + 1)*dilog(-I*cosh(b*x + a) - I*sinh(b*x + a)) + (cosh(b*x + a)^8 + 56*cosh(b*x + a)^3*sinh(b*x + a)^5 + 28*cosh(b*x + a)^2 *sinh(b*x + a)^6 + 8*cosh(b*x + a)*sinh(b*x + a)^7 + sinh(b*x + a)^8 + 2*(3 5*cosh(b*x + a)^4 - 1)*sinh(b*x + a)^4 - 2*cosh(b*x + a)^4 + 8*(7*cosh(b*x + a)^5 - cosh(b*x + a))*sinh(b*x + a)^3 + 4*(7*cosh(b*x + a)^6 - 3*cosh(b*x + a)^2)*sinh(b*x + a)^2 + 8*(cosh(b*x + a)^7 - cosh(b*x + a)^3)*sinh(b*x + a) + 1)*dilog(-cosh(b*x + a) - sinh(b*x + a)) + (b*x*cosh(b*x + a)^8 + 56* b*x*cosh(b*x + a)^3*sinh(b*x + a)^5 + 28*b*x*cosh(b*x + a)^2*sinh(b*x + a)^ 6 + 8*b*x*cosh(b*x + a)*sinh(b*x + a)^7 + b*x*sinh(b*x + a)^8 - 2*b*x*cosh(b*x + a)^4 + 2*(35*b*x*cosh(b*x + a)^4 - b*x)*sinh(b*x + a)^4 + 8*(7*b*x*co sh(b*x + a)^5 - b*x*cosh(b*x + a))*sinh(b*x + a)^3 + 4*(7*b*x*cosh(b*x + a) ^6 - 3*b*x*cosh(b*x + a)^2)*sinh(b*x + a)^2 + b*x + 8*(b*x*cosh(b*x + a)^7 - b*x*cosh(b*x + a)^3)*sinh(b*x + a))*log(cosh(b*x + a) + sinh(b*x + a) + 1) + (a*cosh(b*x + a)^8 + 56*a*cosh(b*x + a)^3*sinh(b*x + a)^5 + 28*a*cosh(b *x + a)^2*sinh(b*x + a)^6 + 8*a*cosh(b*x + a)*sinh(b*x + a)^7 + a*sinh(b*x + a)^8 - 2*a*cosh(b*x + a)^4 + 2*(35*a*cosh(b*x + a)^4 - a)*sinh(b*x + a)^4 + 8*(7*a*cosh(b*x + a)^5 - a*cosh(b*x + a))*sinh(b*x + a)^3 + 4*(7*a*cosh(b*x + a)^6 - 3*a*cosh(b*x + a)^2)*sinh(b*x + a)^2 + 8*(a*cosh(b*x + a)^7 - a*cosh(b*x + a)^3)*sinh(b*x + a) + a)*log(cosh(b*x + a) + sinh(b*x + a) + I) + (a*cosh(b*x + a)^8 + 56*a*cosh(b*x + a)^3*sinh(b*x + a)^5 + 28*a*cosh(b *x + a)^2*sinh(b*x + a)^6 + 8*a*cosh(b*x + a)*sinh(b*x + a)^7 + a*sinh(b*x

```
+ a)^8 - 2*a*cosh(b*x + a)^4 + 2*(35*a*cosh(b*x + a)^4 - a)*sinh(b*x + a)^4
 + 8*(7*a*cosh(b*x + a)^5 - a*cosh(b*x + a))*sinh(b*x + a)^3 + 4*(7*a*cosh(
b*x + a)^6 - 3*a*cosh(b*x + a)^2)*sinh(b*x + a)^2 + 8*(a*cosh(b*x + a)^7 -
a*cosh(b*x + a)^3)*sinh(b*x + a) + a)*log(cosh(b*x + a) + sinh(b*x + a) - I
) - (a*cosh(b*x + a)^8 + 56*a*cosh(b*x + a)^3*sinh(b*x + a)^5 + 28*a*cosh(b
*x + a)^2*sinh(b*x + a)^6 + 8*a*cosh(b*x + a)*sinh(b*x + a)^7 + a*sinh(b*x
+ a)^8 - 2*a*cosh(b*x + a)^4 + 2*(35*a*cosh(b*x + a)^4 - a)*sinh(b*x + a)^4
 + 8*(7*a*cosh(b*x + a)^5 - a*cosh(b*x + a))*sinh(b*x + a)^3 + 4*(7*a*cosh(
b*x + a)^6 - 3*a*cosh(b*x + a)^2)*sinh(b*x + a)^2 + 8*(a*cosh(b*x + a)^7 -
a*cosh(b*x + a)^3)*sinh(b*x + a) + a)*log(cosh(b*x + a) + sinh(b*x + a) - 1
) - ((b*x + a)*cosh(b*x + a)^8 + 56*(b*x + a)*cosh(b*x + a)^3*sinh(b*x + a)
^5 + 28*(b*x + a)*cosh(b*x + a)^2*sinh(b*x + a)^6 + 8*(b*x + a)*cosh(b*x +
a)*sinh(b*x + a)^7 + (b*x + a)*sinh(b*x + a)^8 - 2*(b*x + a)*cosh(b*x + a)^
4 + 2*(35*(b*x + a)*cosh(b*x + a)^4 - b*x - a)*sinh(b*x + a)^4 + 8*(7*(b*x
+ a)*cosh(b*x + a)^5 - (b*x + a)*cosh(b*x + a))*sinh(b*x + a)^3 + 4*(7*(b*x
 + a)*cosh(b*x + a)^6 - 3*(b*x + a)*cosh(b*x + a)^2)*sinh(b*x + a)^2 + b*x
+ 8*((b*x + a)*cosh(b*x + a)^7 - (b*x + a)*cosh(b*x + a)^3)*sinh(b*x + a) +
 a)*log(I*cosh(b*x + a) + I*sinh(b*x + a) + 1) - ((b*x + a)*cosh(b*x + a)^8
 + 56*(b*x + a)*cosh(b*x + a)^3*sinh(b*x + a)^5 + 28*(b*x + a)*cosh(b*x + a
)^2*sinh(b*x + a)^6 + 8*(b*x + a)*cosh(b*x + a)*sinh(b*x + a)^7 + (b*x + a)
*sinh(b*x + a)^8 - 2*(b*x + a)*cosh(b*x + a)^4 + 2*(35*(b*x + a)*cosh(b*x +
 a)^4 - b*x - a)*sinh(b*x + a)^4 + 8*(7*(b*x + a)*cosh(b*x + a)^5 - (b*x +
a)*cosh(b*x + a))*sinh(b*x + a)^3 + 4*(7*(b*x + a)*cosh(b*x + a)^6 - 3*(b*x
 + a)*cosh(b*x + a)^2)*sinh(b*x + a)^2 + b*x + 8*((b*x + a)*cosh(b*x + a)^7
 - (b*x + a)*cosh(b*x + a)^3)*sinh(b*x + a) + a)*log(-I*cosh(b*x + a) - I*s
inh(b*x + a) + 1) + ((b*x + a)*cosh(b*x + a)^8 + 56*(b*x + a)*cosh(b*x + a)
^3*sinh(b*x + a)^5 + 28*(b*x + a)*cosh(b*x + a)^2*sinh(b*x + a)^6 + 8*(b*x
+ a)*cosh(b*x + a)*sinh(b*x + a)^7 + (b*x + a)*sinh(b*x + a)^8 - 2*(b*x + a
)*cosh(b*x + a)^4 + 2*(35*(b*x + a)*cosh(b*x + a)^4 - b*x - a)*sinh(b*x + a
)^4 + 8*(7*(b*x + a)*cosh(b*x + a)^5 - (b*x + a)*cosh(b*x + a))*sinh(b*x +
a)^3 + 4*(7*(b*x + a)*cosh(b*x + a)^6 - 3*(b*x + a)*cosh(b*x + a)^2)*sinh(b
*x + a)^2 + b*x + 8*((b*x + a)*cosh(b*x + a)^7 - (b*x + a)*cosh(b*x + a)^3)
*sinh(b*x + a) + a)*log(-cosh(b*x + a) - sinh(b*x + a) + 1) + 2*(3*(2*b*x +
 1)*cosh(b*x + a)^5 + (2*b*x - 1)*cosh(b*x + a))*sinh(b*x + a))/(b^2*cosh(b
*x + a)^8 + 56*b^2*cosh(b*x + a)^3*sinh(b*x + a)^5 + 28*b^2*cosh(b*x + a)^2
*sinh(b*x + a)^6 + 8*b^2*cosh(b*x + a)*sinh(b*x + a)^7 + b^2*sinh(b*x + a)^
8 - 2*b^2*cosh(b*x + a)^4 + 2*(35*b^2*cosh(b*x + a)^4 - b^2)*sinh(b*x + a)^
4 + 8*(7*b^2*cosh(b*x + a)^5 - b^2*cosh(b*x + a))*sinh(b*x + a)^3 + 4*(7*b^
2*cosh(b*x + a)^6 - 3*b^2*cosh(b*x + a)^2)*sinh(b*x + a)^2 + b^2 + 8*(b^2*c
osh(b*x + a)^7 - b^2*cosh(b*x + a)^3)*sinh(b*x + a))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x \operatorname{csch}(bx+a)^3 \operatorname{sech}(bx+a)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*csch(b*x+a)^3*sech(b*x+a)^3,x, algorithm="giac")
```

```
[Out] integrate(x*csch(b*x + a)^3*sech(b*x + a)^3, x)
```

maple [B] time = 0.47, size = 197, normalized size = 2.16

$$-\frac{2\,\mathrm{e}^{2bx+2a}\left(2bx\,\mathrm{e}^{4bx+4a}+\mathrm{e}^{4bx+4a}+2bx-1\right)}{b^{2}\left(1+\mathrm{e}^{2bx+2a}\right)^{2}\left(\mathrm{e}^{2bx+2a}-1\right)^{2}}+\frac{2x\ln\left(1+\mathrm{e}^{2bx+2a}\right)}{b}+\frac{\mathrm{polylog}\left(2,-\mathrm{e}^{2bx+2a}\right)}{b^{2}}-\frac{2\ln\left(1-\mathrm{e}^{bx+a}\right)x}{b}-\frac{2\,\mathrm{l}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*csch(b*x+a)^3*sech(b*x+a)^3,x)
```

```
[Out] -2*exp(2*b*x+2*a)*(2*b*x*exp(4*b*x+4*a)+exp(4*b*x+4*a)+2*b*x-1)/b^2/(1+exp(
2*b*x+2*a))^2/(exp(2*b*x+2*a)-1)^2+2*x*ln(1+exp(2*b*x+2*a))/b+polylog(2,-ex
p(2*b*x+2*a))/b^2-2/b*ln(1-exp(b*x+a))*x-2/b^2*ln(1-exp(b*x+a))*a-2*polylog
(2,exp(b*x+a))/b^2-2/b*ln(1+exp(b*x+a))*x-2*polylog(2,-exp(b*x+a))/b^2+2/b^
2*a*ln(exp(b*x+a)-1)
```

maxima [A] time = 0.49, size = 164, normalized size = 1.80

$$-\frac{2\left(\left(2\,bxe^{(6\,a)}+e^{(6\,a)}\right)e^{(6\,bx)}+\left(2\,bxe^{(2\,a)}-e^{(2\,a)}\right)e^{(2\,bx)}\right)}{b^{2}e^{(8\,bx+8\,a)}-2\,b^{2}e^{(4\,bx+4\,a)}+b^{2}}+\frac{2\,bx\log\left(e^{(2\,bx+2\,a)}+1\right)+\mathrm{Li}_2\left(-e^{(2\,bx+2\,a)}\right)}{b^{2}}-\frac{2\left(bx\log\left(e^{(}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*csch(b*x+a)^3*sech(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -2*((2*b*x*e^(6*a) + e^(6*a))*e^(6*b*x) + (2*b*x*e^(2*a) - e^(2*a))*e^(2*b*
x))/(b^2*e^(8*b*x + 8*a) - 2*b^2*e^(4*b*x + 4*a) + b^2) + (2*b*x*log(e^(2*b
*x + 2*a) + 1) + dilog(-e^(2*b*x + 2*a)))/b^2 - 2*(b*x*log(e^(b*x + a) + 1)
 + dilog(-e^(b*x + a)))/b^2 - 2*(b*x*log(-e^(b*x + a) + 1) + dilog(e^(b*x +
 a)))/b^2
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x}{\cosh\left(a+b\,x\right)^3\,\sinh\left(a+b\,x\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(cosh(a + b*x)^3*sinh(a + b*x)^3),x)
```

```
[Out] int(x/(cosh(a + b*x)^3*sinh(a + b*x)^3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x \operatorname{csch}^3(a+bx) \operatorname{sech}^3(a+bx) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*csch(b*x+a)**3*sech(b*x+a)**3,x)
[Out] Integral(x*csch(a + b*x)**3*sech(a + b*x)**3, x)
```

3.525 $\int \operatorname{csch}^3(a+bx)\operatorname{sech}^3(a+bx)\,dx$

Optimal. Leaf size=43

$$\frac{\tanh^2(a+bx)}{2b}-\frac{\coth^2(a+bx)}{2b}-\frac{2\log(\tanh(a+bx))}{b}$$

```
[Out] -1/2*coth(b*x+a)^2/b-2*ln(tanh(b*x+a))/b+1/2*tanh(b*x+a)^2/b
```

Rubi [A] time = 0.04, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {2620, 266, 43}

$$\frac{\tanh^2(a+bx)}{2b}-\frac{\coth^2(a+bx)}{2b}-\frac{2\log(\tanh(a+bx))}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[a + b*x]^3*Sech[a + b*x]^3,x]
```

```
[Out] -Coth[a + b*x]^2/(2*b) - (2*Log[Tanh[a + b*x]])/b + Tanh[a + b*x]^2/(2*b)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2620

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rubi steps

$$\int \operatorname{csch}^3(a+bx)\operatorname{sech}^3(a+bx)\,dx = -\frac{\operatorname{Subst}\left(\int \frac{(1+x^2)^2}{x^3}\,dx, x, i\tanh(a+bx)\right)}{b}$$
$$= -\frac{\operatorname{Subst}\left(\int \frac{(1+x)^2}{x^2}\,dx, x, -\tanh^2(a+bx)\right)}{2b}$$
$$= -\frac{\operatorname{Subst}\left(\int \left(1+\frac{1}{x^2}+\frac{2}{x}\right)dx, x, -\tanh^2(a+bx)\right)}{2b}$$
$$= -\frac{\coth^2(a+bx)}{2b} - \frac{2\log(\tanh(a+bx))}{b} + \frac{\tanh^2(a+bx)}{2b}$$

Mathematica [A] time = 0.01, size = 47, normalized size = 1.09

$$8\left(-\frac{\operatorname{csch}^2(a+bx)}{16b} - \frac{\operatorname{sech}^2(a+bx)}{16b} - \frac{\log(\tanh(a+bx))}{4b}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[a + b*x]^3*Sech[a + b*x]^3,x]
```

```
[Out] 8*(-1/16*Csch[a + b*x]^2/b - Log[Tanh[a + b*x]]/(4*b) - Sech[a + b*x]^2/(16
*b))
```

fricas [B] time = 0.44, size = 774, normalized size = 18.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^3*sech(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -2*(2*cosh(b*x + a)^6 + 40*cosh(b*x + a)^3*sinh(b*x + a)^3 + 30*cosh(b*x +
a)^2*sinh(b*x + a)^4 + 12*cosh(b*x + a)*sinh(b*x + a)^5 + 2*sinh(b*x + a)^6
 + 2*(15*cosh(b*x + a)^4 + 1)*sinh(b*x + a)^2 + 2*cosh(b*x + a)^2 - (cosh(b
*x + a)^8 + 56*cosh(b*x + a)^3*sinh(b*x + a)^5 + 28*cosh(b*x + a)^2*sinh(b*
x + a)^6 + 8*cosh(b*x + a)*sinh(b*x + a)^7 + sinh(b*x + a)^8 + 2*(35*cosh(b
*x + a)^4 - 1)*sinh(b*x + a)^4 - 2*cosh(b*x + a)^4 + 8*(7*cosh(b*x + a)^5 -
 cosh(b*x + a))*sinh(b*x + a)^3 + 4*(7*cosh(b*x + a)^6 - 3*cosh(b*x + a)^2)
*sinh(b*x + a)^2 + 8*(cosh(b*x + a)^7 - cosh(b*x + a)^3)*sinh(b*x + a) + 1)
*log(2*cosh(b*x + a)/(cosh(b*x + a) - sinh(b*x + a))) + (cosh(b*x + a)^8 +
56*cosh(b*x + a)^3*sinh(b*x + a)^5 + 28*cosh(b*x + a)^2*sinh(b*x + a)^6 + 8
```

```
*cosh(b*x + a)*sinh(b*x + a)^7 + sinh(b*x + a)^8 + 2*(35*cosh(b*x + a)^4 -
1)*sinh(b*x + a)^4 - 2*cosh(b*x + a)^4 + 8*(7*cosh(b*x + a)^5 - cosh(b*x +
a))*sinh(b*x + a)^3 + 4*(7*cosh(b*x + a)^6 - 3*cosh(b*x + a)^2)*sinh(b*x +
a)^2 + 8*(cosh(b*x + a)^7 - cosh(b*x + a)^3)*sinh(b*x + a) + 1)*log(2*sinh(
b*x + a)/(cosh(b*x + a) - sinh(b*x + a))) + 4*(3*cosh(b*x + a)^5 + cosh(b*x
 + a))*sinh(b*x + a))/(b*cosh(b*x + a)^8 + 56*b*cosh(b*x + a)^3*sinh(b*x +
a)^5 + 28*b*cosh(b*x + a)^2*sinh(b*x + a)^6 + 8*b*cosh(b*x + a)*sinh(b*x +
a)^7 + b*sinh(b*x + a)^8 - 2*b*cosh(b*x + a)^4 + 2*(35*b*cosh(b*x + a)^4 -
b)*sinh(b*x + a)^4 + 8*(7*b*cosh(b*x + a)^5 - b*cosh(b*x + a))*sinh(b*x + a
)^3 + 4*(7*b*cosh(b*x + a)^6 - 3*b*cosh(b*x + a)^2)*sinh(b*x + a)^2 + 8*(b*
cosh(b*x + a)^7 - b*cosh(b*x + a)^3)*sinh(b*x + a) + b)
```

giac [B] time = 0.16, size = 96, normalized size = 2.23

$$-\frac{\frac{4\left(e^{(2\,bx+2\,a)}+e^{(-2\,bx-2\,a)}\right)}{\left(e^{(2\,bx+2\,a)}+e^{(-2\,bx-2\,a)}\right)^2-4}-\log\left(e^{(2\,bx+2\,a)}+e^{(-2\,bx-2\,a)}+2\right)+\log\left(e^{(2\,bx+2\,a)}+e^{(-2\,bx-2\,a)}-2\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^3*sech(b*x+a)^3,x, algorithm="giac")
```

```
[Out] -(4*(e^(2*b*x + 2*a) + e^(-2*b*x - 2*a))/((e^(2*b*x + 2*a) + e^(-2*b*x - 2*
a))^2 - 4) - log(e^(2*b*x + 2*a) + e^(-2*b*x - 2*a) + 2) + log(e^(2*b*x + 2
*a) + e^(-2*b*x - 2*a) - 2))/b
```

maple [A] time = 0.23, size = 48, normalized size = 1.12

$$-\frac{1}{2b\sinh\left(bx+a\right)^2\cosh\left(bx+a\right)^2}-\frac{1}{b\cosh\left(bx+a\right)^2}-\frac{2\ln\left(\tanh\left(bx+a\right)\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(b*x+a)^3*sech(b*x+a)^3,x)
```

```
[Out] -1/2/b/sinh(b*x+a)^2/cosh(b*x+a)^2-1/b/cosh(b*x+a)^2-2*ln(tanh(b*x+a))/b
```

maxima [B] time = 0.45, size = 102, normalized size = 2.37

$$-\frac{2\,\log\left(e^{(-bx-a)}+1\right)}{b}-\frac{2\,\log\left(e^{(-bx-a)}-1\right)}{b}+\frac{2\,\log\left(e^{(-2\,bx-2\,a)}+1\right)}{b}+\frac{4\left(e^{(-2\,bx-2\,a)}+e^{(-6\,bx-6\,a)}\right)}{b\left(2\,e^{(-4\,bx-4\,a)}-e^{(-8\,bx-8\,a)}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^3*sech(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -2*log(e^(-b*x - a) + 1)/b - 2*log(e^(-b*x - a) - 1)/b + 2*log(e^(-2*b*x -
2*a) + 1)/b + 4*(e^(-2*b*x - 2*a) + e^(-6*b*x - 6*a))/(b*(2*e^(-4*b*x - 4*a
) - e^(-8*b*x - 8*a) - 1))
```

mupad [B] time = 0.08, size = 96, normalized size = 2.23

$$\frac{4\,\mathrm{atan}\left(\frac{e^{2\,a}\,e^{2\,b\,x}\,\sqrt{-b^2}}{b}\right)}{\sqrt{-b^2}}-\frac{4\,e^{2\,a+2\,b\,x}}{b\,\left(e^{4\,a+4\,b\,x}-1\right)}-\frac{8\,e^{2\,a+2\,b\,x}}{b\,\left(e^{8\,a+8\,b\,x}-2\,e^{4\,a+4\,b\,x}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(a + b*x)^3*sinh(a + b*x)^3),x)
```

```
[Out] (4*atan((exp(2*a)*exp(2*b*x)*(-b^2)^(1/2))/b))/(-b^2)^(1/2) - (4*exp(2*a +
2*b*x))/(b*(exp(4*a + 4*b*x) - 1)) - (8*exp(2*a + 2*b*x))/(b*(exp(8*a + 8*b
*x) - 2*exp(4*a + 4*b*x) + 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{csch}^3(a+bx)\operatorname{sech}^3(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)**3*sech(b*x+a)**3,x)
```

```
[Out] Integral(csch(a + b*x)**3*sech(a + b*x)**3, x)
```

3.526 $\int \frac{\operatorname{csch}^3(a+bx)\operatorname{sech}^3(a+bx)}{x}\,dx$

Optimal. Leaf size=20

$$8\mathrm{Int}\left(\frac{\operatorname{csch}^3(2a+2bx)}{x},x\right)$$

```
[Out] 8*Unintegrable(csch(2*b*x+2*a)^3/x,x)
```

Rubi [A] time = 0.07, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\operatorname{csch}^3(a+bx)\operatorname{sech}^3(a+bx)}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Csch[a + b*x]^3*Sech[a + b*x]^3)/x,x]
```

```
[Out] 8*Defer[Int][Csch[2*a + 2*b*x]^3/x, x]
```

Rubi steps

$$\int \frac{\operatorname{csch}^3(a+bx)\operatorname{sech}^3(a+bx)}{x}\,dx = 8\int \frac{\operatorname{csch}^3(2a+2bx)}{x}\,dx$$

Mathematica [A] time = 60.73, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}^3(a+bx)\operatorname{sech}^3(a+bx)}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Csch[a + b*x]^3*Sech[a + b*x]^3)/x,x]
```

```
[Out] Integrate[(Csch[a + b*x]^3*Sech[a + b*x]^3)/x, x]
```

fricas [A] time = 0.42, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\operatorname{csch}(bx+a)^3\operatorname{sech}(bx+a)^3}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^3*sech(b*x+a)^3/x,x, algorithm="fricas")
```

```
[Out] integral(csch(b*x + a)^3*sech(b*x + a)^3/x, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}(bx+a)^3 \operatorname{sech}(bx+a)^3}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^3*sech(b*x+a)^3/x,x, algorithm="giac")
```

```
[Out] integrate(csch(b*x + a)^3*sech(b*x + a)^3/x, x)
```

maple [A] time = 1.56, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}(bx+a)^3 \operatorname{sech}(bx+a)^3}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(b*x+a)^3*sech(b*x+a)^3/x,x)
```

```
[Out] int(csch(b*x+a)^3*sech(b*x+a)^3/x,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{2\left(\left(2\,bxe^{(6\,a)}-e^{(6\,a)}\right)e^{(6\,bx)}+\left(2\,bxe^{(2\,a)}+e^{(2\,a)}\right)e^{(2\,bx)}\right)}{b^2x^2e^{(8\,bx+8\,a)}-2\,b^2x^2e^{(4\,bx+4\,a)}+b^2x^2}-64\int \frac{2\,b^2x^2-1}{32\left(b^2x^3e^{(2\,bx+2\,a)}+b^2x^3\right)}\,dx+64\int \frac{2\,b^2x^2-}{64\left(b^2x^3e^{(bx+a)}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^3*sech(b*x+a)^3/x,x, algorithm="maxima")
```

```
[Out] -2*((2*b*x*e^(6*a) - e^(6*a))*e^(6*b*x) + (2*b*x*e^(2*a) + e^(2*a))*e^(2*b*
x))/(b^2*x^2*e^(8*b*x + 8*a) - 2*b^2*x^2*e^(4*b*x + 4*a) + b^2*x^2) - 64*in
tegrate(1/32*(2*b^2*x^2 - 1)/(b^2*x^3*e^(2*b*x + 2*a) + b^2*x^3), x) + 64*i
ntegrate(1/64*(2*b^2*x^2 - 1)/(b^2*x^3*e^(b*x + a) + b^2*x^3), x) - 64*inte
grate(1/64*(2*b^2*x^2 - 1)/(b^2*x^3*e^(b*x + a) - b^2*x^3), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{1}{x\cosh(a+b\,x)^3\sinh(a+b\,x)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*cosh(a + b*x)^3*sinh(a + b*x)^3),x)
```

```
[Out] int(1/(x*cosh(a + b*x)^3*sinh(a + b*x)^3), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}^3(a+bx)\operatorname{sech}^3(a+bx)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)**3*sech(b*x+a)**3/x,x)
```

```
[Out] Integral(csch(a + b*x)**3*sech(a + b*x)**3/x, x)
```

3.527 $\int \frac{\operatorname{csch}^3(a+bx)\operatorname{sech}^3(a+bx)}{x^2}\,dx$

Optimal. Leaf size=20

$$8\text{Int}\left(\frac{\operatorname{csch}^3(2a+2bx)}{x^2},x\right)$$

```
[Out] 8*Unintegrable(csch(2*b*x+2*a)^3/x^2,x)
```

Rubi [A] time = 0.07, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\operatorname{csch}^3(a+bx)\operatorname{sech}^3(a+bx)}{x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Csch[a + b*x]^3*Sech[a + b*x]^3)/x^2,x]
[Out] 8*Defer[Int][Csch[2*a + 2*b*x]^3/x^2, x]
```

Rubi steps

$$\int \frac{\operatorname{csch}^3(a+bx)\operatorname{sech}^3(a+bx)}{x^2}\,dx = 8\int \frac{\operatorname{csch}^3(2a+2bx)}{x^2}\,dx$$

Mathematica [A] time = 43.44, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}^3(a+bx)\operatorname{sech}^3(a+bx)}{x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Csch[a + b*x]^3*Sech[a + b*x]^3)/x^2,x]
[Out] Integrate[(Csch[a + b*x]^3*Sech[a + b*x]^3)/x^2, x]
```

fricas [A] time = 0.41, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\operatorname{csch}(bx+a)^3\operatorname{sech}(bx+a)^3}{x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^3*sech(b*x+a)^3/x^2,x, algorithm="fricas")
```

```
[Out] integral(csch(b*x + a)^3*sech(b*x + a)^3/x^2, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}(bx+a)^{3}\operatorname{sech}(bx+a)^{3}}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^3*sech(b*x+a)^3/x^2,x, algorithm="giac")
```

```
[Out] integrate(csch(b*x + a)^3*sech(b*x + a)^3/x^2, x)
```

maple [A] time = 1.70, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}(bx+a)^{3}\operatorname{sech}(bx+a)^{3}}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(b*x+a)^3*sech(b*x+a)^3/x^2,x)
```

```
[Out] int(csch(b*x+a)^3*sech(b*x+a)^3/x^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{4\left(\left(bxe^{(6a)}-e^{(6a)}\right)e^{(6bx)}+\left(bxe^{(2a)}+e^{(2a)}\right)e^{(2bx)}\right)}{b^{2}x^{3}e^{(8bx+8a)}-2b^{2}x^{3}e^{(4bx+4a)}+b^{2}x^{3}}-64\int \frac{2b^{2}x^{2}-3}{32\left(b^{2}x^{4}e^{(2bx+2a)}+b^{2}x^{4}\right)}\,dx+64\int \frac{2b^{2}x^{2}-3}{64\left(b^{2}x^{4}e^{(bx+a)}+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^3*sech(b*x+a)^3/x^2,x, algorithm="maxima")
```

```
[Out] -4*((b*x*e^(6*a) - e^(6*a))*e^(6*b*x) + (b*x*e^(2*a) + e^(2*a))*e^(2*b*x))/
(b^2*x^3*e^(8*b*x + 8*a) - 2*b^2*x^3*e^(4*b*x + 4*a) + b^2*x^3) - 64*integr
ate(1/32*(2*b^2*x^2 - 3)/(b^2*x^4*e^(2*b*x + 2*a) + b^2*x^4), x) + 64*integ
rate(1/64*(2*b^2*x^2 - 3)/(b^2*x^4*e^(b*x + a) + b^2*x^4), x) - 64*integrat
e(1/64*(2*b^2*x^2 - 3)/(b^2*x^4*e^(b*x + a) - b^2*x^4), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{1}{x^{2}\cosh(a+b\,x)^{3}\sinh(a+b\,x)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*cosh(a + b*x)^3*sinh(a + b*x)^3),x)
```

```
[Out] int(1/(x^2*cosh(a + b*x)^3*sinh(a + b*x)^3), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}^3(a+bx)\operatorname{sech}^3(a+bx)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)**3*sech(b*x+a)**3/x**2,x)
```

```
[Out] Integral(csch(a + b*x)**3*sech(a + b*x)**3/x**2, x)
```

3.528 $\int x \cosh^{\frac{5}{2}}(a+bx) \sinh(a+bx) \, dx$

Optimal. Leaf size=87

$$\frac{20iF\left(\frac{1}{2}i(a+bx)\middle|2\right)}{147b^2}-\frac{4\sinh(a+bx)\cosh^{\frac{5}{2}}(a+bx)}{49b^2}-\frac{20\sinh(a+bx)\sqrt{\cosh(a+bx)}}{147b^2}+\frac{2x\cosh^{\frac{7}{2}}(a+bx)}{7b}$$

```
[Out] 2/7*x*cosh(b*x+a)^(7/2)/b+20/147*I*(cosh(1/2*a+1/2*b*x)^2)^(1/2)/cosh(1/2*a
+1/2*b*x)*EllipticF(I*sinh(1/2*a+1/2*b*x),2^(1/2))/b^2-4/49*cosh(b*x+a)^(5/
2)*sinh(b*x+a)/b^2-20/147*sinh(b*x+a)*cosh(b*x+a)^(1/2)/b^2
```

Rubi [A] time = 0.06, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {5373, 2635, 2641}

$$\frac{20iF\left(\frac{1}{2}i(a+bx)\middle|2\right)}{147b^2}-\frac{4\sinh(a+bx)\cosh^{\frac{5}{2}}(a+bx)}{49b^2}-\frac{20\sinh(a+bx)\sqrt{\cosh(a+bx)}}{147b^2}+\frac{2x\cosh^{\frac{7}{2}}(a+bx)}{7b}$$

Antiderivative was successfully verified.

```
[In] Int[x*Cosh[a + b*x]^(5/2)*Sinh[a + b*x],x]
```

```
[Out] (2*x*Cosh[a + b*x]^(7/2))/(7*b) + (((20*I)/147)*EllipticF[(I/2)*(a + b*x),
2])/b^2 - (20*Sqrt[Cosh[a + b*x]]*Sinh[a + b*x])/(147*b^2) - (4*Cosh[a + b*
x]^(5/2)*Sinh[a + b*x])/(49*b^2)
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 5373

```
Int[Cosh[(a_.) + (b_.)*(x_)^(n_.)]^(p_.)*(x_)^(m_.)*Sinh[(a_.) + (b_.)*(x_)
^(n_.)], x_Symbol] :> Simp[(x^(m - n + 1)*Cosh[a + b*x^n]^(p + 1))/(b*n*(p
+ 1)), x] - Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*Cosh[a + b*x^n]^(
```

```
p + 1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

Rubi steps

$$\int x\cosh^{\frac{5}{2}}(a+bx)\sinh(a+bx)\,dx = \frac{2x\cosh^{\frac{7}{2}}(a+bx)}{7b}-\frac{2\int\cosh^{\frac{7}{2}}(a+bx)\,dx}{7b}$$
$$=\frac{2x\cosh^{\frac{7}{2}}(a+bx)}{7b}-\frac{4\cosh^{\frac{5}{2}}(a+bx)\sinh(a+bx)}{49b^2}-\frac{10\int\cosh^{\frac{3}{2}}(a+bx)\,dx}{49b}$$
$$=\frac{2x\cosh^{\frac{7}{2}}(a+bx)}{7b}-\frac{20\sqrt{\cosh(a+bx)}\sinh(a+bx)}{147b^2}-\frac{4\cosh^{\frac{5}{2}}(a+bx)\sinh(a+bx)}{49b^2}$$
$$=\frac{2x\cosh^{\frac{7}{2}}(a+bx)}{7b}+\frac{20iF\left(\frac{1}{2}i(a+bx)\middle|2\right)}{147b^2}-\frac{20\sqrt{\cosh(a+bx)}\sinh(a+bx)}{147b^2}$$

Mathematica [A] time = 0.36, size = 77, normalized size = 0.89

$$\frac{\sqrt{\cosh(a+bx)}\,(-46\sinh(a+bx)-6\sinh(3(a+bx))+63bx\cosh(a+bx)+21bx\cosh(3(a+bx)))+40iF\left(\frac{1}{2}i(a\right.}{294b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Cosh[a + b*x]^(5/2)*Sinh[a + b*x],x]
```

```
[Out] ((40*I)*EllipticF[(I/2)*(a + b*x), 2] + Sqrt[Cosh[a + b*x]]*(63*b*x*Cosh[a
+ b*x] + 21*b*x*Cosh[3*(a + b*x)] - 46*Sinh[a + b*x] - 6*Sinh[3*(a + b*x)])
)/(294*b^2)
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x+a)^(5/2)*sinh(b*x+a),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\cosh\left(bx+a\right)^{\frac{5}{2}}\sinh\left(bx+a\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x+a)^(5/2)*sinh(b*x+a),x, algorithm="giac")
[Out] integrate(x*cosh(b*x + a)^(5/2)*sinh(b*x + a), x)
```

maple [F] time = 0.18, size = 0, normalized size = 0.00

$$\int x\left(\cosh^{\frac{5}{2}}(bx+a)\right)\sinh(bx+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cosh(b*x+a)^(5/2)*sinh(b*x+a),x)
[Out] int(x*cosh(b*x+a)^(5/2)*sinh(b*x+a),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\cosh(bx+a)^{\frac{5}{2}}\sinh(bx+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x+a)^(5/2)*sinh(b*x+a),x, algorithm="maxima")
[Out] integrate(x*cosh(b*x + a)^(5/2)*sinh(b*x + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x\cosh(a+b\,x)^{5/2}\sinh(a+b\,x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cosh(a + b*x)^(5/2)*sinh(a + b*x),x)
[Out] int(x*cosh(a + b*x)^(5/2)*sinh(a + b*x), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x+a)**(5/2)*sinh(b*x+a),x)
[Out] Timed out
```

3.529 $\int x\cosh^{\frac{3}{2}}(a+bx)\sinh(a+bx)\,dx$

Optimal. Leaf size=64

$$\frac{12iE\left(\frac{1}{2}i(a+bx)\middle|2\right)}{25b^2}-\frac{4\sinh(a+bx)\cosh^{\frac{3}{2}}(a+bx)}{25b^2}+\frac{2x\cosh^{\frac{5}{2}}(a+bx)}{5b}$$

```
[Out] 2/5*x*cosh(b*x+a)^(5/2)/b+12/25*I*(cosh(1/2*a+1/2*b*x)^2)^(1/2)/cosh(1/2*a+
1/2*b*x)*EllipticE(I*sinh(1/2*a+1/2*b*x),2^(1/2))/b^2-4/25*cosh(b*x+a)^(3/2
)*sinh(b*x+a)/b^2
```

Rubi [A] time = 0.04, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {5373, 2635, 2639}

$$\frac{12iE\left(\frac{1}{2}i(a+bx)\middle|2\right)}{25b^2}-\frac{4\sinh(a+bx)\cosh^{\frac{3}{2}}(a+bx)}{25b^2}+\frac{2x\cosh^{\frac{5}{2}}(a+bx)}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[x*Cosh[a + b*x]^(3/2)*Sinh[a + b*x],x]
```

```
[Out] (2*x*Cosh[a + b*x]^(5/2))/(5*b) + (((12*I)/25)*EllipticE[(I/2)*(a + b*x), 2
])/b^2 - (4*Cosh[a + b*x]^(3/2)*Sinh[a + b*x])/(25*b^2)
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 5373

```
Int[Cosh[(a_.) + (b_.)*(x_)^(n_.)]^(p_.)*(x_)^(m_.)*Sinh[(a_.) + (b_.)*(x_)
^(n_.)], x_Symbol] :> Simp[(x^(m - n + 1)*Cosh[a + b*x^n]^(p + 1))/(b*n*(p
+ 1)), x] - Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*Cosh[a + b*x^n]^(
p + 1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

Rubi steps

$$\int x\cosh^{\frac{3}{2}}(a+bx)\sinh(a+bx)\,dx=\frac{2x\cosh^{\frac{5}{2}}(a+bx)}{5b}-\frac{2\int\cosh^{\frac{5}{2}}(a+bx)\,dx}{5b}$$
$$=\frac{2x\cosh^{\frac{5}{2}}(a+bx)}{5b}-\frac{4\cosh^{\frac{3}{2}}(a+bx)\sinh(a+bx)}{25b^2}-\frac{6\int\sqrt{\cosh(a+bx)}\ d}{25b}$$
$$=\frac{2x\cosh^{\frac{5}{2}}(a+bx)}{5b}+\frac{12iE\left(\frac{1}{2}i(a+bx)\middle|2\right)}{25b^2}-\frac{4\cosh^{\frac{3}{2}}(a+bx)\sinh(a+bx)}{25b^2}$$

Mathematica [C] time = 2.01, size = 142, normalized size = 2.22

$$\frac{e^{-3(a+bx)}\left(48e^{2(a+bx)}\sqrt{e^{2(a+bx)}+1}\ {}_2F_1\left(-\frac{1}{4},\frac{1}{2};\frac{3}{4};-e^{2(a+bx)}\right)+\left(e^{2(a+bx)}+1\right)\left(2(5bx-12)e^{2(a+bx)}+(5bx-2)e^{4(a+bx)}\right.\right.}{50\sqrt{2}\,b^2\sqrt{e^{-a-bx}+e^{a+bx}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Cosh[a + b*x]^(3/2)*Sinh[a + b*x],x]
```

```
[Out] ((1 + E^(2*(a + b*x)))*(2 + 5*b*x + 2*E^(2*(a + b*x))*(-12 + 5*b*x) + E^(4*
(a + b*x))*(-2 + 5*b*x)) + 48*E^(2*(a + b*x))*Sqrt[1 + E^(2*(a + b*x))]*Hyp
ergeometric2F1[-1/4, 1/2, 3/4, -E^(2*(a + b*x))])/(50*Sqrt[2]*b^2*E^(3*(a +
 b*x))*Sqrt[E^(-a - b*x) + E^(a + b*x)])
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x+a)^(3/2)*sinh(b*x+a),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\cosh(bx+a)^{\frac{3}{2}}\sinh(bx+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x+a)^(3/2)*sinh(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(x*cosh(b*x + a)^(3/2)*sinh(b*x + a), x)
```

maple [F] time = 0.14, size = 0, normalized size = 0.00

$$\int x\left(\cosh^{\frac{3}{2}}\left(bx+a\right)\right)\sinh\left(bx+a\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cosh(b*x+a)^(3/2)*sinh(b*x+a),x)
[Out] int(x*cosh(b*x+a)^(3/2)*sinh(b*x+a),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\cosh\left(bx+a\right)^{\frac{3}{2}}\sinh\left(bx+a\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x+a)^(3/2)*sinh(b*x+a),x, algorithm="maxima")
[Out] integrate(x*cosh(b*x + a)^(3/2)*sinh(b*x + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int x\,\cosh\left(a+b\,x\right)^{3/2}\,\sinh\left(a+b\,x\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cosh(a + b*x)^(3/2)*sinh(a + b*x),x)
[Out] int(x*cosh(a + b*x)^(3/2)*sinh(a + b*x), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x+a)**(3/2)*sinh(b*x+a),x)
[Out] Timed out
```

3.530 $\int x\sqrt{\cosh(a+bx)}\sinh(a+bx)\,dx$

Optimal. Leaf size=64

$$\frac{4iF\left(\frac{1}{2}i(a+bx)\middle|2\right)}{9b^2}-\frac{4\sinh(a+bx)\sqrt{\cosh(a+bx)}}{9b^2}+\frac{2x\cosh^{\frac{3}{2}}(a+bx)}{3b}$$

```
[Out] 2/3*x*cosh(b*x+a)^(3/2)/b+4/9*I*(cosh(1/2*a+1/2*b*x)^2)^(1/2)/cosh(1/2*a+1/
2*b*x)*EllipticF(I*sinh(1/2*a+1/2*b*x),2^(1/2))/b^2-4/9*sinh(b*x+a)*cosh(b*
x+a)^(1/2)/b^2
```

Rubi [A] time = 0.04, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {5373, 2635, 2641}

$$\frac{4iF\left(\frac{1}{2}i(a+bx)\middle|2\right)}{9b^2}-\frac{4\sinh(a+bx)\sqrt{\cosh(a+bx)}}{9b^2}+\frac{2x\cosh^{\frac{3}{2}}(a+bx)}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sqrt[Cosh[a + b*x]]*Sinh[a + b*x],x]
```

```
[Out] (2*x*Cosh[a + b*x]^(3/2))/(3*b) + (((4*I)/9)*EllipticF[(I/2)*(a + b*x), 2])
/b^2 - (4*Sqrt[Cosh[a + b*x]]*Sinh[a + b*x])/(9*b^2)
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 5373

```
Int[Cosh[(a_.) + (b_.)*(x_)^(n_.)]^(p_.)*(x_)^(m_.)*Sinh[(a_.) + (b_.)*(x_)
^(n_.)], x_Symbol] :> Simp[(x^(m - n + 1)*Cosh[a + b*x^n]^(p + 1))/(b*n*(p
+ 1)), x] - Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*Cosh[a + b*x^n]^(
p + 1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

Rubi steps

$$\int x\sqrt{\cosh(a+bx)}\,\sinh(a+bx)\,dx = \frac{2x\cosh^{\frac{3}{2}}(a+bx)}{3b} - \frac{2\int \cosh^{\frac{3}{2}}(a+bx)\,dx}{3b}$$
$$= \frac{2x\cosh^{\frac{3}{2}}(a+bx)}{3b} - \frac{4\sqrt{\cosh(a+bx)}\,\sinh(a+bx)}{9b^2} - \frac{2\int \frac{1}{\sqrt{\cosh(a+bx)}}\,dx}{9b}$$
$$= \frac{2x\cosh^{\frac{3}{2}}(a+bx)}{3b} + \frac{4iF\left(\frac{1}{2}i(a+bx)\middle|2\right)}{9b^2} - \frac{4\sqrt{\cosh(a+bx)}\,\sinh(a+bx)}{9b^2}$$

Mathematica [A] time = 0.16, size = 56, normalized size = 0.88

$$\frac{2\sqrt{\cosh(a+bx)}\,(3bx\cosh(a+bx)-2\sinh(a+bx))+4iF\left(\frac{1}{2}i(a+bx)\middle|2\right)}{9b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sqrt[Cosh[a + b*x]]*Sinh[a + b*x],x]
```

```
[Out] ((4*I)*EllipticF[(I/2)*(a + b*x), 2] + 2*Sqrt[Cosh[a + b*x]]*(3*b*x*Cosh[a
+ b*x] - 2*Sinh[a + b*x]))/(9*b^2)
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sinh(b*x+a)*cosh(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\sqrt{\cosh(bx+a)}\,\sinh(bx+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sinh(b*x+a)*cosh(b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x*sqrt(cosh(b*x + a))*sinh(b*x + a), x)
```

maple [F] time = 0.11, size = 0, normalized size = 0.00

$$\int x \sinh \left(b x+a\right) \left(\sqrt{\cosh}\, \left(b x+a\right)\right)d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sinh(b*x+a)*cosh(b*x+a)^(1/2),x)
[Out] int(x*sinh(b*x+a)*cosh(b*x+a)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x \sqrt{\cosh\left(b x + a\right)} \sinh\left(b x + a\right)\,{d x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sinh(b*x+a)*cosh(b*x+a)^(1/2),x, algorithm="maxima")
[Out] integrate(x*sqrt(cosh(b*x + a))*sinh(b*x + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int x\,\sqrt{\mathrm{cosh}\left(a+b\,x\right)}\,\mathrm{sinh}\left(a+b\,x\right) \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cosh(a + b*x)^(1/2)*sinh(a + b*x),x)
[Out] int(x*cosh(a + b*x)^(1/2)*sinh(a + b*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x \sinh{\left(a + b x \right)} \sqrt{\cosh{\left(a + b x \right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sinh(b*x+a)*cosh(b*x+a)**(1/2),x)
[Out] Integral(x*sinh(a + b*x)*sqrt(cosh(a + b*x)), x)
```

3.531 $\int \frac{x\sinh(a+bx)}{\sqrt{\cosh(a+bx)}}\,dx$

Optimal. Leaf size=37

$$\frac{2x\sqrt{\cosh(a+bx)}}{b}+\frac{4iE\left(\frac{1}{2}i(a+bx)\middle|2\right)}{b^2}$$

```
[Out] 4*I*(cosh(1/2*a+1/2*b*x)^2)^(1/2)/cosh(1/2*a+1/2*b*x)*EllipticE(I*sinh(1/2*
a+1/2*b*x),2^(1/2))/b^2+2*x*cosh(b*x+a)^(1/2)/b
```

Rubi [A] time = 0.03, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {5373, 2639}

$$\frac{2x\sqrt{\cosh(a+bx)}}{b}+\frac{4iE\left(\frac{1}{2}i(a+bx)\middle|2\right)}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Sinh[a + b*x])/Sqrt[Cosh[a + b*x]],x]
```

```
[Out] (2*x*Sqrt[Cosh[a + b*x]])/b + ((4*I)*EllipticE[(I/2)*(a + b*x), 2])/b^2
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 5373

```
Int[Cosh[(a_.) + (b_.)*(x_)^(n_.)]^(p_.)*(x_)^(m_.)*Sinh[(a_.) + (b_.)*(x_)
^(n_.)], x_Symbol] :> Simp[(x^(m - n + 1)*Cosh[a + b*x^n]^(p + 1))/(b*n*(p
+ 1)), x] - Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*Cosh[a + b*x^n]^(
p + 1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{x\sinh(a+bx)}{\sqrt{\cosh(a+bx)}}\,dx&=\frac{2x\sqrt{\cosh(a+bx)}}{b}-\frac{2\int \sqrt{\cosh(a+bx)}\,dx}{b}\\&=\frac{2x\sqrt{\cosh(a+bx)}}{b}+\frac{4iE\left(\frac{1}{2}i(a+bx)\middle|2\right)}{b^2}\end{aligned}$$

Mathematica [C] time = 1.55, size = 190, normalized size = 5.14

$$\frac{\sqrt{2}\,e^{-a-bx}\sqrt{e^{2(a+bx)}+1}\left(18\,{}_3F_2\left(-\frac{1}{4},-\frac{1}{4},\frac{1}{2};\frac{3}{4},\frac{3}{4};-e^{2(a+bx)}\right)-2e^{2(a+bx)}\,{}_3F_2\left(\frac{1}{2},\frac{3}{4},\frac{3}{4};\frac{7}{4},\frac{7}{4};-e^{2(a+bx)}\right)+3bx\left(3\,{}_2F_1\left(-\right.\right.\right.}{9b^2\sqrt{e^{-a-bx}+e^{a+bx}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x*Sinh[a + b*x])/Sqrt[Cosh[a + b*x]],x]
```

```
[Out] (Sqrt[2]*E^(-a - b*x)*Sqrt[1 + E^(2*(a + b*x))]*(3*b*x*(3*Hypergeometric2F1
[-1/4, 1/2, 3/4, -E^(2*(a + b*x))] + E^(2*(a + b*x))*Hypergeometric2F1[1/2,
 3/4, 7/4, -E^(2*(a + b*x))]) + 18*HypergeometricPFQ[{-1/4, -1/4, 1/2}, {3/
4, 3/4}, -E^(2*(a + b*x))] - 2*E^(2*(a + b*x))*HypergeometricPFQ[{1/2, 3/4,
 3/4}, {7/4, 7/4}, -E^(2*(a + b*x))]))/(9*b^2*Sqrt[E^(-a - b*x) + E^(a + b*
x)])
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sinh(b*x+a)/cosh(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\sinh\left(bx+a\right)}{\sqrt{\cosh\left(bx+a\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sinh(b*x+a)/cosh(b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x*sinh(b*x + a)/sqrt(cosh(b*x + a)), x)
```

maple [B] time = 0.15, size = 250, normalized size = 6.76

$$\frac{(bx-2)\left(1+e^{2bx+2a}\right)\sqrt{2}\;e^{-bx-a}}{b^2\sqrt{e^{-bx-a}\left(1+e^{2bx+2a}\right)}}-\frac{2\left(-\frac{2\left(1+e^{2bx+2a}\right)}{\sqrt{\left(1+e^{2bx+2a}\right)e^{bx+a}}}+\frac{i\sqrt{-i\left(e^{bx+a}+i\right)}\;\sqrt{2}\;\sqrt{i\left(e^{bx+a}-i\right)}\;\sqrt{ie^{bx+a}}\;\left(-2i\,\mathrm{EllipticE}\left(\sqrt{-i\left(e^{bx+a}+i\right)},\right.\right.}{\sqrt{e^{3bx+3a}+e^{bx+a}}}\right.}{b^2\sqrt{e^{-bx-a}\left(1+e^{2bx+2a}\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sinh(b*x+a)/cosh(b*x+a)^(1/2),x)
```

```
[Out] (b*x-2)*(exp(b*x+a)^2+1)/b^2*2^(1/2)/((exp(b*x+a)^2+1)/exp(b*x+a))^(1/2)/ex
p(b*x+a)-2/b^2*(-2*(exp(b*x+a)^2+1)/((exp(b*x+a)^2+1)*exp(b*x+a))^(1/2)+I*(
-I*(exp(b*x+a)+I))^(1/2)*2^(1/2)*(I*(exp(b*x+a)-I))^(1/2)*(I*exp(b*x+a))^(1
/2)/(exp(b*x+a)^3+exp(b*x+a))^(1/2)*(-2*I*EllipticE((-I*(exp(b*x+a)+I))^(1/
2),1/2*2^(1/2))+I*EllipticF((-I*(exp(b*x+a)+I))^(1/2),1/2*2^(1/2))))*2^(1/2
)/((exp(b*x+a)^2+1)/exp(b*x+a))^(1/2)*((exp(b*x+a)^2+1)*exp(b*x+a))^(1/2)/e
xp(b*x+a)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x \sinh(bx+a)}{\sqrt{\cosh(bx+a)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sinh(b*x+a)/cosh(b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x*sinh(b*x + a)/sqrt(cosh(b*x + a)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{x \sinh(a+b\,x)}{\sqrt{\cosh(a+b\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*sinh(a + b*x))/cosh(a + b*x)^(1/2),x)
```

```
[Out] int((x*sinh(a + b*x))/cosh(a + b*x)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x \sinh(a+bx)}{\sqrt{\cosh(a+bx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sinh(b*x+a)/cosh(b*x+a)**(1/2),x)
```

```
[Out] Integral(x*sinh(a + b*x)/sqrt(cosh(a + b*x)), x)
```

3.532 $\int \frac{x \sinh(a+bx)}{\cosh^{\frac{3}{2}}(a+bx)}\,dx$

Optimal. Leaf size=37

$$-\frac{2x}{b\sqrt{\cosh(a+bx)}}-\frac{4iF\left(\frac{1}{2}i(a+bx)\middle|2\right)}{b^2}$$

```
[Out] -4*I*(cosh(1/2*a+1/2*b*x)^2)^(1/2)/cosh(1/2*a+1/2*b*x)*EllipticF(I*sinh(1/2
*a+1/2*b*x),2^(1/2))/b^2-2*x/b/cosh(b*x+a)^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {5373, 2641}

$$-\frac{2x}{b\sqrt{\cosh(a+bx)}}-\frac{4iF\left(\frac{1}{2}i(a+bx)\middle|2\right)}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Sinh[a + b*x])/Cosh[a + b*x]^(3/2),x]
```

```
[Out] (-2*x)/(b*Sqrt[Cosh[a + b*x]]) - ((4*I)*EllipticF[(I/2)*(a + b*x), 2])/b^2
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 5373

```
Int[Cosh[(a_.) + (b_.)*(x_)^(n_.)]^(p_.)*(x_)^(m_.)*Sinh[(a_.) + (b_.)*(x_)
^(n_.)], x_Symbol] :> Simp[(x^(m - n + 1)*Cosh[a + b*x^n]^(p + 1))/(b*n*(p
+ 1)), x] - Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*Cosh[a + b*x^n]^(
p + 1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{x\sinh(a+bx)}{\cosh^{\frac{3}{2}}(a+bx)}\,dx = -\frac{2x}{b\sqrt{\cosh(a+bx)}} + \frac{2\int \frac{1}{\sqrt{\cosh(a+bx)}}\,dx}{b}$$
$$= -\frac{2x}{b\sqrt{\cosh(a+bx)}} - \frac{4iF\left(\frac{1}{2}i(a+bx)\middle|2\right)}{b^2}$$

Mathematica [A] time = 0.18, size = 37, normalized size = 1.00

$$-\frac{2x}{b\sqrt{\cosh(a+bx)}} - \frac{4iF\left(\frac{1}{2}i(a+bx)\middle|2\right)}{b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Sinh[a + b*x])/Cosh[a + b*x]^(3/2),x]
```

```
[Out] (-2*x)/(b*Sqrt[Cosh[a + b*x]]) - ((4*I)*EllipticF[(I/2)*(a + b*x), 2])/b^2
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sinh(b*x+a)/cosh(b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\sinh\left(bx+a\right)}{\cosh\left(bx+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sinh(b*x+a)/cosh(b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x*sinh(b*x + a)/cosh(b*x + a)^(3/2), x)
```

maple [F] time = 0.11, size = 0, normalized size = 0.00

$$\int \frac{x\sinh\left(bx+a\right)}{\cosh\left(bx+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sinh(b*x+a)/cosh(b*x+a)^(3/2),x)
[Out] int(x*sinh(b*x+a)/cosh(b*x+a)^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\sinh\left(bx+a\right)}{\cosh\left(bx+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sinh(b*x+a)/cosh(b*x+a)^(3/2),x, algorithm="maxima")
[Out] integrate(x*sinh(b*x + a)/cosh(b*x + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{x\sinh\left(a+b\,x\right)}{{\cosh\left(a+b\,x\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*sinh(a + b*x))/cosh(a + b*x)^(3/2),x)
[Out] int((x*sinh(a + b*x))/cosh(a + b*x)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\sinh\left(a+bx\right)}{\cosh^{\frac{3}{2}}\left(a+bx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sinh(b*x+a)/cosh(b*x+a)**(3/2),x)
[Out] Integral(x*sinh(a + b*x)/cosh(a + b*x)**(3/2), x)
```

3.533 $\int \frac{x\sinh(a+bx)}{\cosh^{\frac{5}{2}}(a+bx)}\,dx$

Optimal. Leaf size=64

$$\frac{4iE\left(\frac{1}{2}i(a+bx)\middle|2\right)}{3b^2}+\frac{4\sinh(a+bx)}{3b^2\sqrt{\cosh(a+bx)}}-\frac{2x}{3b\cosh^{\frac{3}{2}}(a+bx)}$$

```
[Out] -2/3*x/b/cosh(b*x+a)^(3/2)+4/3*I*(cosh(1/2*a+1/2*b*x)^2)^(1/2)/cosh(1/2*a+1
/2*b*x)*EllipticE(I*sinh(1/2*a+1/2*b*x),2^(1/2))/b^2+4/3*sinh(b*x+a)/b^2/co
sh(b*x+a)^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {5373, 2636, 2639}

$$\frac{4iE\left(\frac{1}{2}i(a+bx)\middle|2\right)}{3b^2}+\frac{4\sinh(a+bx)}{3b^2\sqrt{\cosh(a+bx)}}-\frac{2x}{3b\cosh^{\frac{3}{2}}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Sinh[a + b*x])/Cosh[a + b*x]^(5/2),x]
```

```
[Out] (-2*x)/(3*b*Cosh[a + b*x]^(3/2)) + (((4*I)/3)*EllipticE[(I/2)*(a + b*x), 2]
)/b^2 + (4*Sinh[a + b*x])/(3*b^2*Sqrt[Cosh[a + b*x]])
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 5373

```
Int[Cosh[(a_.) + (b_.)*(x_)^(n_.)]^(p_.)*(x_)^(m_.)*Sinh[(a_.) + (b_.)*(x_)
^(n_.)], x_Symbol] :> Simp[(x^(m - n + 1)*Cosh[a + b*x^n]^(p + 1))/(b*n*(p
+ 1)), x] - Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*Cosh[a + b*x^n]^(
```

```
p + 1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{x\sinh(a+bx)}{\cosh^{\frac{5}{2}}(a+bx)}\,dx &= -\frac{2x}{3b\cosh^{\frac{3}{2}}(a+bx)}+\frac{2\int \frac{1}{\cosh^{\frac{3}{2}}(a+bx)}\,dx}{3b}\\ &= -\frac{2x}{3b\cosh^{\frac{3}{2}}(a+bx)}+\frac{4\sinh(a+bx)}{3b^2\sqrt{\cosh(a+bx)}}-\frac{2\int \sqrt{\cosh(a+bx)}\,dx}{3b}\\ &= -\frac{2x}{3b\cosh^{\frac{3}{2}}(a+bx)}+\frac{4iE\left(\frac{1}{2}i(a+bx)\middle|2\right)}{3b^2}+\frac{4\sinh(a+bx)}{3b^2\sqrt{\cosh(a+bx)}}\end{aligned}$$

Mathematica [A] time = 0.20, size = 57, normalized size = 0.89

$$\frac{2\left(\sinh(2(a+bx))+2i\cosh^{\frac{3}{2}}(a+bx)E\left(\frac{1}{2}i(a+bx)\middle|2\right)-bx\right)}{3b^2\cosh^{\frac{3}{2}}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Sinh[a + b*x])/Cosh[a + b*x]^(5/2),x]
```

```
[Out] (2*(-(b*x) + (2*I)*Cosh[a + b*x]^(3/2)*EllipticE[(I/2)*(a + b*x), 2] + Sinh
[2*(a + b*x)]))/(3*b^2*Cosh[a + b*x]^(3/2))
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sinh(b*x+a)/cosh(b*x+a)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\sinh(bx+a)}{\cosh(bx+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sinh(b*x+a)/cosh(b*x+a)^(5/2),x, algorithm="giac")
[Out] integrate(x*sinh(b*x + a)/cosh(b*x + a)^(5/2), x)
```

maple [F] time = 0.11, size = 0, normalized size = 0.00

$$\int \frac{x \sinh (bx+a)}{\cosh (bx+a)^{\frac{5}{2}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sinh(b*x+a)/cosh(b*x+a)^(5/2),x)
[Out] int(x*sinh(b*x+a)/cosh(b*x+a)^(5/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x \sinh (bx+a)}{\cosh (bx+a)^{\frac{5}{2}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sinh(b*x+a)/cosh(b*x+a)^(5/2),x, algorithm="maxima")
[Out] integrate(x*sinh(b*x + a)/cosh(b*x + a)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x \sinh (a+b\,x)}{{\cosh (a+b\,x)}^{5/2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*sinh(a + b*x))/cosh(a + b*x)^(5/2),x)
[Out] int((x*sinh(a + b*x))/cosh(a + b*x)^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sinh(b*x+a)/cosh(b*x+a)**(5/2),x)
[Out] Timed out
```

3.534 $\int \frac{x \sinh(a+bx)}{\cosh^{\frac{7}{2}}(a+bx)} dx$

Optimal. Leaf size=64

$$-\frac{4iF\left(\frac{1}{2}i(a+bx)\middle|2\right)}{15b^2}+\frac{4\sinh(a+bx)}{15b^2\cosh^{\frac{3}{2}}(a+bx)}-\frac{2x}{5b\cosh^{\frac{5}{2}}(a+bx)}$$

```
[Out] -2/5*x/b/cosh(b*x+a)^(5/2)-4/15*I*(cosh(1/2*a+1/2*b*x)^2)^(1/2)/cosh(1/2*a+
1/2*b*x)*EllipticF(I*sinh(1/2*a+1/2*b*x),2^(1/2))/b^2+4/15*sinh(b*x+a)/b^2/
cosh(b*x+a)^(3/2)
```

Rubi [A] time = 0.04, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {5373, 2636, 2641}

$$-\frac{4iF\left(\frac{1}{2}i(a+bx)\middle|2\right)}{15b^2}+\frac{4\sinh(a+bx)}{15b^2\cosh^{\frac{3}{2}}(a+bx)}-\frac{2x}{5b\cosh^{\frac{5}{2}}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Sinh[a + b*x])/Cosh[a + b*x]^(7/2),x]
```

```
[Out] (-2*x)/(5*b*Cosh[a + b*x]^(5/2)) - (((4*I)/15)*EllipticF[(I/2)*(a + b*x), 2
])/b^2 + (4*Sinh[a + b*x])/(15*b^2*Cosh[a + b*x]^(3/2))
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 5373

```
Int[Cosh[(a_.) + (b_.)*(x_)^(n_.)]^(p_.)*(x_)^(m_.)*Sinh[(a_.) + (b_.)*(x_)
^(n_.)], x_Symbol] :> Simp[(x^(m - n + 1)*Cosh[a + b*x^n]^(p + 1))/(b*n*(p
+ 1)), x] - Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*Cosh[a + b*x^n]^(
```

```
p + 1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{x\sinh(a+bx)}{\cosh^{\frac{7}{2}}(a+bx)}\,dx &= -\frac{2x}{5b\cosh^{\frac{5}{2}}(a+bx)}+\frac{2\int \frac{1}{\cosh^{\frac{5}{2}}(a+bx)}\,dx}{5b}\\ &= -\frac{2x}{5b\cosh^{\frac{5}{2}}(a+bx)}+\frac{4\sinh(a+bx)}{15b^2\cosh^{\frac{3}{2}}(a+bx)}+\frac{2\int \frac{1}{\sqrt{\cosh(a+bx)}}\,dx}{15b}\\ &= -\frac{2x}{5b\cosh^{\frac{5}{2}}(a+bx)}-\frac{4iF\left(\frac{1}{2}i(a+bx)\middle|2\right)}{15b^2}+\frac{4\sinh(a+bx)}{15b^2\cosh^{\frac{3}{2}}(a+bx)}\end{aligned}$$

Mathematica [A] time = 0.25, size = 57, normalized size = 0.89

$$\frac{2\left(\sinh(2(a+bx))-2i\cosh^{\frac{5}{2}}(a+bx)F\left(\frac{1}{2}i(a+bx)\middle|2\right)-3bx\right)}{15b^2\cosh^{\frac{5}{2}}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Sinh[a + b*x])/Cosh[a + b*x]^(7/2),x]
[Out] (2*(-3*b*x - (2*I)*Cosh[a + b*x]^(5/2)*EllipticF[(I/2)*(a + b*x), 2] + Sinh
[2*(a + b*x)]))/(15*b^2*Cosh[a + b*x]^(5/2))
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sinh(b*x+a)/cosh(b*x+a)^(7/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\sinh(bx+a)}{\cosh(bx+a)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sinh(b*x+a)/cosh(b*x+a)^(7/2),x, algorithm="giac")
[Out] integrate(x*sinh(b*x + a)/cosh(b*x + a)^(7/2), x)
```

maple [F] time = 0.11, size = 0, normalized size = 0.00

$$\int \frac{x \sinh \left(b x+a\right)}{\cosh \left(b x+a\right)^{\frac{7}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sinh(b*x+a)/cosh(b*x+a)^(7/2),x)
[Out] int(x*sinh(b*x+a)/cosh(b*x+a)^(7/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x \sinh \left(b x+a\right)}{\cosh \left(b x+a\right)^{\frac{7}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sinh(b*x+a)/cosh(b*x+a)^(7/2),x, algorithm="maxima")
[Out] integrate(x*sinh(b*x + a)/cosh(b*x + a)^(7/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x \sinh \left(a+b\,x\right)}{{\cosh \left(a+b\,x\right)}^{7/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*sinh(a + b*x))/cosh(a + b*x)^(7/2),x)
[Out] int((x*sinh(a + b*x))/cosh(a + b*x)^(7/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sinh(b*x+a)/cosh(b*x+a)**(7/2),x)
[Out] Timed out
```

3.535 $\int \frac{x\sinh(a+bx)}{\cosh^{\frac{9}{2}}(a+bx)}\,dx$

Optimal. Leaf size=87

$$\frac{12iE\left(\frac{1}{2}i(a+bx)\middle|2\right)}{35b^2}+\frac{4\sinh(a+bx)}{35b^2\cosh^{\frac{5}{2}}(a+bx)}+\frac{12\sinh(a+bx)}{35b^2\sqrt{\cosh(a+bx)}}-\frac{2x}{7b\cosh^{\frac{7}{2}}(a+bx)}$$

```
[Out] -2/7*x/b/cosh(b*x+a)^(7/2)+12/35*I*(cosh(1/2*a+1/2*b*x)^2)^(1/2)/cosh(1/2*a
+1/2*b*x)*EllipticE(I*sinh(1/2*a+1/2*b*x),2^(1/2))/b^2+4/35*sinh(b*x+a)/b^2
/cosh(b*x+a)^(5/2)+12/35*sinh(b*x+a)/b^2/cosh(b*x+a)^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {5373, 2636, 2639}

$$\frac{12iE\left(\frac{1}{2}i(a+bx)\middle|2\right)}{35b^2}+\frac{4\sinh(a+bx)}{35b^2\cosh^{\frac{5}{2}}(a+bx)}+\frac{12\sinh(a+bx)}{35b^2\sqrt{\cosh(a+bx)}}-\frac{2x}{7b\cosh^{\frac{7}{2}}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Sinh[a + b*x])/Cosh[a + b*x]^(9/2),x]
```

```
[Out] (-2*x)/(7*b*Cosh[a + b*x]^(7/2)) + (((12*I)/35)*EllipticE[(I/2)*(a + b*x),
2])/b^2 + (4*Sinh[a + b*x])/(35*b^2*Cosh[a + b*x]^(5/2)) + (12*Sinh[a + b*x
])/(35*b^2*Sqrt[Cosh[a + b*x]])
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 5373

```
Int[Cosh[(a_.) + (b_.)*(x_)^(n_.)]^(p_.)*(x_)^(m_.)*Sinh[(a_.) + (b_.)*(x_)
^(n_.)], x_Symbol] :> Simp[(x^(m - n + 1)*Cosh[a + b*x^n]^(p + 1))/(b*n*(p
```

```
+ 1)), x] - Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*Cosh[a + b*x^n]^(
p + 1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{x\sinh(a+bx)}{\cosh^{\frac{9}{2}}(a+bx)}\,dx = -\frac{2x}{7b\cosh^{\frac{7}{2}}(a+bx)} + \frac{2\int \frac{1}{\cosh^{\frac{7}{2}}(a+bx)}\,dx}{7b}$$

$$= -\frac{2x}{7b\cosh^{\frac{7}{2}}(a+bx)} + \frac{4\sinh(a+bx)}{35b^2\cosh^{\frac{5}{2}}(a+bx)} + \frac{6\int \frac{1}{\cosh^{\frac{3}{2}}(a+bx)}\,dx}{35b}$$

$$= -\frac{2x}{7b\cosh^{\frac{7}{2}}(a+bx)} + \frac{4\sinh(a+bx)}{35b^2\cosh^{\frac{5}{2}}(a+bx)} + \frac{12\sinh(a+bx)}{35b^2\sqrt{\cosh(a+bx)}} - \frac{6\int \sqrt{\cosh(a+bx)}}{35b}$$

$$= -\frac{2x}{7b\cosh^{\frac{7}{2}}(a+bx)} + \frac{12iE\left(\frac{1}{2}i(a+bx)\middle|2\right)}{35b^2} + \frac{4\sinh(a+bx)}{35b^2\cosh^{\frac{5}{2}}(a+bx)} + \frac{12\sinh(a+bx)}{35b^2\sqrt{\cosh(a+bx)}}$$

Mathematica [A] time = 0.28, size = 69, normalized size = 0.79

$$\frac{10\sinh(2(a+bx)) + 3\sinh(4(a+bx)) + 24i\cosh^{\frac{7}{2}}(a+bx)E\left(\frac{1}{2}i(a+bx)\middle|2\right) - 20bx}{70b^2\cosh^{\frac{7}{2}}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Sinh[a + b*x])/Cosh[a + b*x]^(9/2),x]
```

```
[Out] (-20*b*x + (24*I)*Cosh[a + b*x]^(7/2)*EllipticE[(I/2)*(a + b*x), 2] + 10*Si
nh[2*(a + b*x)] + 3*Sinh[4*(a + b*x)])/(70*b^2*Cosh[a + b*x]^(7/2))
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sinh(b*x+a)/cosh(b*x+a)^(9/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x \sinh (bx + a)}{\cosh (bx + a)^{\frac{9}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sinh(b*x+a)/cosh(b*x+a)^(9/2),x, algorithm="giac")
[Out] integrate(x*sinh(b*x + a)/cosh(b*x + a)^(9/2), x)
```

maple [F] time = 0.11, size = 0, normalized size = 0.00

$$\int \frac{x \sinh (bx + a)}{\cosh (bx + a)^{\frac{9}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sinh(b*x+a)/cosh(b*x+a)^(9/2),x)
[Out] int(x*sinh(b*x+a)/cosh(b*x+a)^(9/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x \sinh (bx + a)}{\cosh (bx + a)^{\frac{9}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sinh(b*x+a)/cosh(b*x+a)^(9/2),x, algorithm="maxima")
[Out] integrate(x*sinh(b*x + a)/cosh(b*x + a)^(9/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x \sinh (a + b\, x)}{{\cosh (a + b\, x)}^{9/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*sinh(a + b*x))/cosh(a + b*x)^(9/2),x)
[Out] int((x*sinh(a + b*x))/cosh(a + b*x)^(9/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sinh(b*x+a)/cosh(b*x+a)**(9/2),x)
```

```
[Out] Timed out
```

3.536 $\int x\mathbf{sech}^{\frac{9}{2}}(a+bx)\sinh(a+bx)\,dx$

Optimal. Leaf size=107

$$\frac{4\sinh(a+bx)\text{sech}^{\frac{5}{2}}(a+bx)}{35b^2}+\frac{12\sinh(a+bx)\sqrt{\text{sech}(a+bx)}}{35b^2}+\frac{12i\sqrt{\cosh(a+bx)}\sqrt{\text{sech}(a+bx)}\,E\left(\frac{1}{2}i(a+bx)\middle|2\right.}{35b^2}$$

```
[Out] -2/7*x*sech(b*x+a)^(7/2)/b+4/35*sech(b*x+a)^(5/2)*sinh(b*x+a)/b^2+12/35*sin
h(b*x+a)*sech(b*x+a)^(1/2)/b^2+12/35*I*(cosh(1/2*a+1/2*b*x)^2)^(1/2)/cosh(1
/2*a+1/2*b*x)*EllipticE(I*sinh(1/2*a+1/2*b*x),2^(1/2))*cosh(b*x+a)^(1/2)*se
ch(b*x+a)^(1/2)/b^2
```

Rubi [A] time = 0.06, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {5444, 3768, 3771, 2639}

$$\frac{4\sinh(a+bx)\text{sech}^{\frac{5}{2}}(a+bx)}{35b^2}+\frac{12\sinh(a+bx)\sqrt{\text{sech}(a+bx)}}{35b^2}+\frac{12i\sqrt{\cosh(a+bx)}\sqrt{\text{sech}(a+bx)}\,E\left(\frac{1}{2}i(a+bx)\middle|2\right.}{35b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sech[a + b*x]^(9/2)*Sinh[a + b*x],x]
```

```
[Out] (((12*I)/35)*Sqrt[Cosh[a + b*x]]*EllipticE[(I/2)*(a + b*x), 2]*Sqrt[Sech[a
+ b*x]])/b^2 - (2*x*Sech[a + b*x]^(7/2))/(7*b) + (12*Sqrt[Sech[a + b*x]]*Si
nh[a + b*x])/(35*b^2) + (4*Sech[a + b*x]^(5/2)*Sinh[a + b*x])/(35*b^2)
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 5444

```
Int[(x_)^(m_.)*Sech[(a_.) + (b_.)*(x_)^(n_.)]^(p_)*Sinh[(a_.) + (b_.)*(x_)^
(n_.)], x_Symbol] :> -Simp[(x^(m - n + 1)*Sech[a + b*x^n]^(p - 1))/(b*n*(p
- 1)), x] + Dist[(m - n + 1)/(b*n*(p - 1)), Int[x^(m - n)*Sech[a + b*x^n]^(
p - 1), x], x] /; FreeQ[{a, b, p}, x] && IntegerQ[n] && GeQ[m - n, 0] && Ne
Q[p, 1]
```

Rubi steps

$$\int x\text{sech}^{\frac{9}{2}}(a+bx)\sinh(a+bx)\,dx = -\frac{2x\text{sech}^{\frac{7}{2}}(a+bx)}{7b}+\frac{2\int \text{sech}^{\frac{7}{2}}(a+bx)\,dx}{7b}$$
$$= -\frac{2x\text{sech}^{\frac{7}{2}}(a+bx)}{7b}+\frac{4\text{sech}^{\frac{5}{2}}(a+bx)\sinh(a+bx)}{35b^2}+\frac{6\int \text{sech}^{\frac{3}{2}}(a+bx)\,dx}{35b}$$
$$= -\frac{2x\text{sech}^{\frac{7}{2}}(a+bx)}{7b}+\frac{12\sqrt{\text{sech}(a+bx)}\sinh(a+bx)}{35b^2}+\frac{4\text{sech}^{\frac{5}{2}}(a+bx)\sin}{35b^2}$$
$$= -\frac{2x\text{sech}^{\frac{7}{2}}(a+bx)}{7b}+\frac{12\sqrt{\text{sech}(a+bx)}\sinh(a+bx)}{35b^2}+\frac{4\text{sech}^{\frac{5}{2}}(a+bx)\sin}{35b^2}$$
$$= \frac{12i\sqrt{\cosh(a+bx)}\,E\left(\frac{1}{2}i(a+bx)\middle|2\right)\sqrt{\text{sech}(a+bx)}}{35b^2}-\frac{2x\text{sech}^{\frac{7}{2}}(a+bx)}{7b}+$$

Mathematica [A] time = 0.33, size = 69, normalized size = 0.64

$$\frac{\text{sech}^{\frac{7}{2}}(a+bx)\left(10\sinh(2(a+bx))+3\sinh(4(a+bx))+24i\cosh^{\frac{7}{2}}(a+bx)E\left(\frac{1}{2}i(a+bx)\middle|2\right)-20bx\right)}{70b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sech[a + b*x]^(9/2)*Sinh[a + b*x],x]
```

```
[Out] (Sech[a + b*x]^(7/2)*(-20*b*x + (24*I)*Cosh[a + b*x]^(7/2)*EllipticE[(I/2)*
(a + b*x), 2] + 10*Sinh[2*(a + b*x)] + 3*Sinh[4*(a + b*x)]))/(70*b^2)
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sech(b*x+a)^(9/2)*sinh(b*x+a),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x \operatorname{sech}(bx+a)^{\frac{9}{2}} \sinh(bx+a)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sech(b*x+a)^(9/2)*sinh(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(x*sech(b*x + a)^(9/2)*sinh(b*x + a), x)
```

maple [F] time = 0.17, size = 0, normalized size = 0.00

$$\int x \operatorname{sech}(bx+a)^{\frac{9}{2}} \sinh(bx+a)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sech(b*x+a)^(9/2)*sinh(b*x+a),x)
```

```
[Out] int(x*sech(b*x+a)^(9/2)*sinh(b*x+a),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x \operatorname{sech}(bx+a)^{\frac{9}{2}} \sinh(bx+a)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sech(b*x+a)^(9/2)*sinh(b*x+a),x, algorithm="maxima")
```

```
[Out] integrate(x*sech(b*x + a)^(9/2)*sinh(b*x + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x \sinh(a+b\,x) \left(\frac{1}{\cosh(a+b\,x)}\right)^{9/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sinh(a + b*x)*(1/cosh(a + b*x))^(9/2),x)
```

```
[Out] int(x*sinh(a + b*x)*(1/cosh(a + b*x))^(9/2), x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sech(b*x+a)**(9/2)*sinh(b*x+a),x)
```

```
[Out] Timed out
```

3.537 $\int x\mathbf{sech}^{\frac{7}{2}}(a+bx)\sinh(a+bx)\,dx$

Optimal. Leaf size=84

$$\frac{4\sinh(a+bx)\text{sech}^{\frac{3}{2}}(a+bx)}{15b^2}-\frac{4i\sqrt{\cosh(a+bx)}\,\sqrt{\text{sech}(a+bx)}\,F\left(\frac{1}{2}i(a+bx)\middle|2\right)}{15b^2}-\frac{2x\text{sech}^{\frac{5}{2}}(a+bx)}{5b}$$

```
[Out] -2/5*x*sech(b*x+a)^(5/2)/b+4/15*sech(b*x+a)^(3/2)*sinh(b*x+a)/b^2-4/15*I*(c
osh(1/2*a+1/2*b*x)^2)^(1/2)/cosh(1/2*a+1/2*b*x)*EllipticF(I*sinh(1/2*a+1/2*
b*x),2^(1/2))*cosh(b*x+a)^(1/2)*sech(b*x+a)^(1/2)/b^2
```

Rubi [A] time = 0.05, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {5444, 3768, 3771, 2641}

$$\frac{4\sinh(a+bx)\text{sech}^{\frac{3}{2}}(a+bx)}{15b^2}-\frac{4i\sqrt{\cosh(a+bx)}\,\sqrt{\text{sech}(a+bx)}\,F\left(\frac{1}{2}i(a+bx)\middle|2\right)}{15b^2}-\frac{2x\text{sech}^{\frac{5}{2}}(a+bx)}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sech[a + b*x]^(7/2)*Sinh[a + b*x],x]
```

```
[Out] (((-4*I)/15)*Sqrt[Cosh[a + b*x]]*EllipticF[(I/2)*(a + b*x), 2]*Sqrt[Sech[a
+ b*x]])/b^2 - (2*x*Sech[a + b*x]^(5/2))/(5*b) + (4*Sech[a + b*x]^(3/2)*Sin
h[a + b*x])/(15*b^2)
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 5444

```
Int[(x_)^(m_.)*Sech[(a_.) + (b_.)*(x_)^(n_.)]^(p_)*Sinh[(a_.) + (b_.)*(x_)^
(n_.)], x_Symbol] :> -Simp[(x^(m - n + 1)*Sech[a + b*x^n]^(p - 1))/(b*n*(p
- 1)), x] + Dist[(m - n + 1)/(b*n*(p - 1)), Int[x^(m - n)*Sech[a + b*x^n]^(
p - 1), x], x] /; FreeQ[{a, b, p}, x] && IntegerQ[n] && GeQ[m - n, 0] && Ne
Q[p, 1]
```

Rubi steps

$$\int x\text{sech}^{\frac{7}{2}}(a+bx)\sinh(a+bx)\,dx = -\frac{2x\text{sech}^{\frac{5}{2}}(a+bx)}{5b}+\frac{2\int \text{sech}^{\frac{5}{2}}(a+bx)\,dx}{5b}$$
$$= -\frac{2x\text{sech}^{\frac{5}{2}}(a+bx)}{5b}+\frac{4\text{sech}^{\frac{3}{2}}(a+bx)\sinh(a+bx)}{15b^2}+\frac{2\int \sqrt{\text{sech}(a+bx)}\ d}{15b}$$
$$= -\frac{2x\text{sech}^{\frac{5}{2}}(a+bx)}{5b}+\frac{4\text{sech}^{\frac{3}{2}}(a+bx)\sinh(a+bx)}{15b^2}+\frac{\left(2\sqrt{\cosh(a+bx)}\sqrt{s}\right.}{}$$
$$= -\frac{4i\sqrt{\cosh(a+bx)}\,F\left(\frac{1}{2}i(a+bx)\middle|2\right)\sqrt{\text{sech}(a+bx)}}{15b^2}-\frac{2x\text{sech}^{\frac{5}{2}}(a+bx)}{5b}+$$

Mathematica [A] time = 0.25, size = 65, normalized size = 0.77

$$-\frac{2\sqrt{\text{sech}(a+bx)}\left(-2\tanh(a+bx)+3bx\text{sech}^2(a+bx)+2i\sqrt{\cosh(a+bx)}\,F\left(\frac{1}{2}i(a+bx)\middle|2\right)\right)}{15b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sech[a + b*x]^(7/2)*Sinh[a + b*x],x]
```

```
[Out] (-2*Sqrt[Sech[a + b*x]]*((2*I)*Sqrt[Cosh[a + b*x]]*EllipticF[(I/2)*(a + b*x
), 2] + 3*b*x*Sech[a + b*x]^2 - 2*Tanh[a + b*x]))/(15*b^2)
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sech(b*x+a)^(7/2)*sinh(b*x+a),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\,\mathrm{sech}\,(bx+a)^{\frac{7}{2}}\sinh\,(bx+a)\,\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sech(b*x+a)^(7/2)*sinh(b*x+a),x, algorithm="giac")
[Out] integrate(x*sech(b*x + a)^(7/2)*sinh(b*x + a), x)
```

maple [F] time = 0.15, size = 0, normalized size = 0.00

$$\int x\mathrm{sech}\,(bx+a)^{\frac{7}{2}}\sinh\,(bx+a)\,\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sech(b*x+a)^(7/2)*sinh(b*x+a),x)
[Out] int(x*sech(b*x+a)^(7/2)*sinh(b*x+a),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\,\mathrm{sech}\,(bx+a)^{\frac{7}{2}}\sinh\,(bx+a)\,\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sech(b*x+a)^(7/2)*sinh(b*x+a),x, algorithm="maxima")
[Out] integrate(x*sech(b*x + a)^(7/2)*sinh(b*x + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x\sinh\left(a+b\,x\right)\,{\left(\frac{1}{\cosh\left(a+b\,x\right)}\right)}^{7/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sinh(a + b*x)*(1/cosh(a + b*x))^(7/2),x)
[Out] int(x*sinh(a + b*x)*(1/cosh(a + b*x))^(7/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sech(b*x+a)**(7/2)*sinh(b*x+a),x)
[Out] Timed out
```

3.538 $\int x\text{sech}^{\frac{5}{2}}(a+bx)\sinh(a+bx)\,dx$

Optimal. Leaf size=84

$$\frac{4\sinh(a+bx)\sqrt{\text{sech}(a+bx)}}{3b^2}+\frac{4i\sqrt{\cosh(a+bx)}\,\sqrt{\text{sech}(a+bx)}\,E\left(\frac{1}{2}i(a+bx)\middle|2\right)}{3b^2}-\frac{2x\text{sech}^{\frac{3}{2}}(a+bx)}{3b}$$

```
[Out] -2/3*x*sech(b*x+a)^(3/2)/b+4/3*sinh(b*x+a)*sech(b*x+a)^(1/2)/b^2+4/3*I*(cos
h(1/2*a+1/2*b*x)^2)^(1/2)/cosh(1/2*a+1/2*b*x)*EllipticE(I*sinh(1/2*a+1/2*b*
x),2^(1/2))*cosh(b*x+a)^(1/2)*sech(b*x+a)^(1/2)/b^2
```

Rubi [A] time = 0.05, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {5444, 3768, 3771, 2639}

$$\frac{4\sinh(a+bx)\sqrt{\text{sech}(a+bx)}}{3b^2}+\frac{4i\sqrt{\cosh(a+bx)}\,\sqrt{\text{sech}(a+bx)}\,E\left(\frac{1}{2}i(a+bx)\middle|2\right)}{3b^2}-\frac{2x\text{sech}^{\frac{3}{2}}(a+bx)}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sech[a + b*x]^(5/2)*Sinh[a + b*x],x]
```

```
[Out] (((4*I)/3)*Sqrt[Cosh[a + b*x]]*EllipticE[(I/2)*(a + b*x), 2]*Sqrt[Sech[a +
b*x]])/b^2 - (2*x*Sech[a + b*x]^(3/2))/(3*b) + (4*Sqrt[Sech[a + b*x]]*Sinh[
a + b*x])/(3*b^2)
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 5444

```
Int[(x_)^(m_.)*Sech[(a_.) + (b_.)*(x_)^(n_.)]^(p_)*Sinh[(a_.) + (b_.)*(x_)^
(n_.)], x_Symbol] :> -Simp[(x^(m - n + 1)*Sech[a + b*x^n]^(p - 1))/(b*n*(p
- 1)), x] + Dist[(m - n + 1)/(b*n*(p - 1)), Int[x^(m - n)*Sech[a + b*x^n]^(
p - 1), x], x] /; FreeQ[{a, b, p}, x] && IntegerQ[n] && GeQ[m - n, 0] && Ne
Q[p, 1]
```

Rubi steps

$$\begin{aligned}\int x\operatorname{sech}^{\frac{5}{2}}(a+bx)\sinh(a+bx)\,dx &= -\frac{2x\operatorname{sech}^{\frac{3}{2}}(a+bx)}{3b}+\frac{2\int \operatorname{sech}^{\frac{3}{2}}(a+bx)\,dx}{3b}\\ &= -\frac{2x\operatorname{sech}^{\frac{3}{2}}(a+bx)}{3b}+\frac{4\sqrt{\operatorname{sech}(a+bx)}\sinh(a+bx)}{3b^2}-\frac{2\int \frac{1}{\sqrt{\operatorname{sech}(a+bx)}}\,dx}{3b}\\ &= -\frac{2x\operatorname{sech}^{\frac{3}{2}}(a+bx)}{3b}+\frac{4\sqrt{\operatorname{sech}(a+bx)}\sinh(a+bx)}{3b^2}-\frac{\left(2\sqrt{\cosh(a+bx)}\sqrt{\operatorname{se}\right.}{}\\ &= \frac{4i\sqrt{\cosh(a+bx)}\,E\left(\frac{1}{2}i(a+bx)\middle|2\right)\sqrt{\operatorname{sech}(a+bx)}}{3b^2}-\frac{2x\operatorname{sech}^{\frac{3}{2}}(a+bx)}{3b}+\frac{4\sqrt{}}{}\end{aligned}$$

Mathematica [A] time = 0.20, size = 57, normalized size = 0.68

$$\frac{2\operatorname{sech}^{\frac{3}{2}}(a+bx)\left(\sinh(2(a+bx))+2i\cosh^{\frac{3}{2}}(a+bx)E\left(\frac{1}{2}i(a+bx)\middle|2\right)-bx\right)}{3b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sech[a + b*x]^(5/2)*Sinh[a + b*x],x]
```

```
[Out] (2*Sech[a + b*x]^(3/2)*(-(b*x) + (2*I)*Cosh[a + b*x]^(3/2)*EllipticE[(I/2)*
(a + b*x), 2] + Sinh[2*(a + b*x)]))/(3*b^2)
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sech(b*x+a)^(5/2)*sinh(b*x+a),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x \operatorname{sech}(bx+a)^{\frac{5}{2}} \sinh(bx+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sech(b*x+a)^(5/2)*sinh(b*x+a),x, algorithm="giac")
[Out] integrate(x*sech(b*x + a)^(5/2)*sinh(b*x + a), x)
```

maple [F] time = 0.15, size = 0, normalized size = 0.00

$$\int x\operatorname{sech}(bx+a)^{\frac{5}{2}} \sinh(bx+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sech(b*x+a)^(5/2)*sinh(b*x+a),x)
[Out] int(x*sech(b*x+a)^(5/2)*sinh(b*x+a),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x \operatorname{sech}(bx+a)^{\frac{5}{2}} \sinh(bx+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sech(b*x+a)^(5/2)*sinh(b*x+a),x, algorithm="maxima")
[Out] integrate(x*sech(b*x + a)^(5/2)*sinh(b*x + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x \sinh(a+b\,x)\left(\frac{1}{\cosh(a+b\,x)}\right)^{5/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sinh(a + b*x)*(1/cosh(a + b*x))^(5/2),x)
[Out] int(x*sinh(a + b*x)*(1/cosh(a + b*x))^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sech(b*x+a)**(5/2)*sinh(b*x+a),x)
[Out] Timed out
```

3.539 $\int x\mathbf{sech}^{\frac{3}{2}}(a+bx)\sinh(a+bx)\,dx$

Optimal. Leaf size=57

$$-\frac{2x\sqrt{\operatorname{sech}(a+bx)}}{b}-\frac{4i\sqrt{\cosh(a+bx)}\sqrt{\operatorname{sech}(a+bx)}\,F\left(\frac{1}{2}i(a+bx)\middle|2\right)}{b^2}$$

```
[Out] -2*x*sech(b*x+a)^(1/2)/b-4*I*(cosh(1/2*a+1/2*b*x)^2)^(1/2)/cosh(1/2*a+1/2*b
*x)*EllipticF(I*sinh(1/2*a+1/2*b*x),2^(1/2))*cosh(b*x+a)^(1/2)*sech(b*x+a)^
(1/2)/b^2
```

Rubi [A] time = 0.04, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {5444, 3771, 2641}

$$-\frac{2x\sqrt{\operatorname{sech}(a+bx)}}{b}-\frac{4i\sqrt{\cosh(a+bx)}\sqrt{\operatorname{sech}(a+bx)}\,F\left(\frac{1}{2}i(a+bx)\middle|2\right)}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sech[a + b*x]^(3/2)*Sinh[a + b*x],x]
```

```
[Out] (-2*x*Sqrt[Sech[a + b*x]])/b - ((4*I)*Sqrt[Cosh[a + b*x]]*EllipticF[(I/2)*(
a + b*x), 2]*Sqrt[Sech[a + b*x]])/b^2
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 5444

```
Int[(x_)^(m_.)*Sech[(a_.) + (b_.)*(x_)^(n_.)]^(p_)*Sinh[(a_.) + (b_.)*(x_)^
(n_.)], x_Symbol] :> -Simp[(x^(m - n + 1)*Sech[a + b*x^n]^(p - 1))/(b*n*(p
- 1)), x] + Dist[(m - n + 1)/(b*n*(p - 1)), Int[x^(m - n)*Sech[a + b*x^n]^(
p - 1), x], x] /; FreeQ[{a, b, p}, x] && IntegerQ[n] && GeQ[m - n, 0] && Ne
Q[p, 1]
```

Rubi steps

$$\int x\text{sech}^{\frac{3}{2}}(a+bx)\sinh(a+bx)\,dx=-\frac{2x\sqrt{\text{sech}(a+bx)}}{b}+\frac{2\int\sqrt{\text{sech}(a+bx)}\,dx}{b}$$
$$=-\frac{2x\sqrt{\text{sech}(a+bx)}}{b}+\frac{\left(2\sqrt{\cosh(a+bx)}\sqrt{\text{sech}(a+bx)}\right)\int\frac{1}{\sqrt{\cosh(a+bx)}}\,dx}{b}$$
$$=-\frac{2x\sqrt{\text{sech}(a+bx)}}{b}-\frac{4i\sqrt{\cosh(a+bx)}\,F\left(\frac{1}{2}i(a+bx)\middle|2\right)\sqrt{\text{sech}(a+bx)}}{b^2}$$

Mathematica [A] time = 0.14, size = 46, normalized size = 0.81

$$-\frac{2\sqrt{\text{sech}(a+bx)}\left(bx+2i\sqrt{\cosh(a+bx)}\,F\left(\frac{1}{2}i(a+bx)\middle|2\right)\right)}{b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sech[a + b*x]^(3/2)*Sinh[a + b*x],x]
```

```
[Out] (-2*(b*x + (2*I)*Sqrt[Cosh[a + b*x]]*EllipticF[(I/2)*(a + b*x), 2])*Sqrt[Se
ch[a + b*x]])/b^2
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sech(b*x+a)^(3/2)*sinh(b*x+a),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\,\text{sech}\left(bx+a\right)^{\frac{3}{2}}\sinh\left(bx+a\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sech(b*x+a)^(3/2)*sinh(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(x*sech(b*x + a)^(3/2)*sinh(b*x + a), x)
```

maple [F] time = 0.15, size = 0, normalized size = 0.00

$$\int x\mathrm{sech}\left(bx+a\right)^{\frac{3}{2}}\sinh\left(bx+a\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sech(b*x+a)^(3/2)*sinh(b*x+a),x)
[Out] int(x*sech(b*x+a)^(3/2)*sinh(b*x+a),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\,\mathrm{sech}\left(bx+a\right)^{\frac{3}{2}}\sinh\left(bx+a\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sech(b*x+a)^(3/2)*sinh(b*x+a),x, algorithm="maxima")
[Out] integrate(x*sech(b*x + a)^(3/2)*sinh(b*x + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int x\sinh\left(a+b\,x\right)\,{\left(\frac{1}{\cosh\left(a+b\,x\right)}\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sinh(a + b*x)*(1/cosh(a + b*x))^(3/2),x)
[Out] int(x*sinh(a + b*x)*(1/cosh(a + b*x))^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sech(b*x+a)**(3/2)*sinh(b*x+a),x)
[Out] Timed out
```

3.540 $\int x\sqrt{\operatorname{sech}(a+bx)}\,\sinh(a+bx)\,dx$

Optimal. Leaf size=57

$$\frac{2x}{b\sqrt{\operatorname{sech}(a+bx)}}+\frac{4i\sqrt{\cosh(a+bx)}\,\sqrt{\operatorname{sech}(a+bx)}\,E\left(\frac{1}{2}i(a+bx)\middle|2\right)}{b^2}$$

```
[Out] 2*x/b/sech(b*x+a)^(1/2)+4*I*(cosh(1/2*a+1/2*b*x)^2)^(1/2)/cosh(1/2*a+1/2*b*
x)*EllipticE(I*sinh(1/2*a+1/2*b*x),2^(1/2))*cosh(b*x+a)^(1/2)*sech(b*x+a)^(
1/2)/b^2
```

Rubi [A] time = 0.04, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {5444, 3771, 2639}

$$\frac{2x}{b\sqrt{\operatorname{sech}(a+bx)}}+\frac{4i\sqrt{\cosh(a+bx)}\,\sqrt{\operatorname{sech}(a+bx)}\,E\left(\frac{1}{2}i(a+bx)\middle|2\right)}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sqrt[Sech[a + b*x]]*Sinh[a + b*x],x]
```

```
[Out] (2*x)/(b*Sqrt[Sech[a + b*x]]) + ((4*I)*Sqrt[Cosh[a + b*x]]*EllipticE[(I/2)*
(a + b*x), 2]*Sqrt[Sech[a + b*x]])/b^2
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 5444

```
Int[(x_)^(m_.)*Sech[(a_.) + (b_.)*(x_)^(n_.)]^(p_)*Sinh[(a_.) + (b_.)*(x_)^
(n_.)], x_Symbol] :> -Simp[(x^(m - n + 1)*Sech[a + b*x^n]^(p - 1))/(b*n*(p
- 1)), x] + Dist[(m - n + 1)/(b*n*(p - 1)), Int[x^(m - n)*Sech[a + b*x^n]^(
p - 1), x], x] /; FreeQ[{a, b, p}, x] && IntegerQ[n] && GeQ[m - n, 0] && Ne
Q[p, 1]
```

Rubi steps

$$\int x\sqrt{\operatorname{sech}(a+bx)}\,\sinh(a+bx)\,dx = \frac{2x}{b\sqrt{\operatorname{sech}(a+bx)}} - \frac{2\int \frac{1}{\sqrt{\operatorname{sech}(a+bx)}}\,dx}{b}$$
$$= \frac{2x}{b\sqrt{\operatorname{sech}(a+bx)}} - \frac{\left(2\sqrt{\cosh(a+bx)}\,\sqrt{\operatorname{sech}(a+bx)}\right)\int \sqrt{\cosh(a+bx)}\,dx}{b}$$
$$= \frac{2x}{b\sqrt{\operatorname{sech}(a+bx)}} + \frac{4i\sqrt{\cosh(a+bx)}\,E\left(\frac{1}{2}i(a+bx)\middle|2\right)\sqrt{\operatorname{sech}(a+bx)}}{b^2}$$

Mathematica [**C**] time = 1.15, size = 100, normalized size = 1.75

$$\frac{\sqrt{2}\,e^{-a-bx}\sqrt{\frac{e^{a+bx}}{e^{2(a+bx)}+1}}\left(4\sqrt{e^{2(a+bx)}+1}\;{}_2F_1\left(-\frac{1}{4},\frac{1}{2};\frac{3}{4};-e^{2(a+bx)}\right)+(bx-2)\left(e^{2(a+bx)}+1\right)\right)}{b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sqrt[Sech[a + b*x]]*Sinh[a + b*x],x]
```

```
[Out] (Sqrt[2]*E^(-a - b*x)*Sqrt[E^(a + b*x)/(1 + E^(2*(a + b*x)))]*((1 + E^(2*(a
 + b*x)))*(-2 + b*x) + 4*Sqrt[1 + E^(2*(a + b*x))]*Hypergeometric2F1[-1/4,
1/2, 3/4, -E^(2*(a + b*x))]))/b^2
```

fricas [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sech(b*x+a)^(1/2)*sinh(b*x+a),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

giac [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int x\sqrt{\operatorname{sech}(bx+a)}\,\sinh(bx+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sech(b*x+a)^(1/2)*sinh(b*x+a),x, algorithm="giac")
```

[Out] integrate(x*sqrt(sech(b*x + a))*sinh(b*x + a), x)

maple [B] time = 0.17, size = 250, normalized size = 4.39

$$\frac{(bx-2)\left(1+e^{2bx+2a}\right)\sqrt{2}\,\sqrt{\frac{e^{bx+a}}{1+e^{2bx+2a}}}\,e^{-bx-a}}{b^2}-\frac{2\left(-\frac{2\left(1+e^{2bx+2a}\right)}{\sqrt{\left(1+e^{2bx+2a}\right)e^{bx+a}}}+\frac{i\sqrt{-i\left(e^{bx+a}+i\right)}\,\sqrt{2}\,\sqrt{i\left(e^{bx+a}-i\right)}\,\sqrt{ie^{bx+a}}\,\left(-2i\,\mathrm{EllipticE}\left(\right.\right.}{\sqrt{e^{3bx+3a}\ldots}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*sech(b*x+a)^(1/2)*sinh(b*x+a),x)

[Out] (b*x-2)*(exp(b*x+a)^2+1)/b^2*2^(1/2)*(exp(b*x+a)/(exp(b*x+a)^2+1))^(1/2)/ex
p(b*x+a)-2/b^2*(-2*(exp(b*x+a)^2+1)/((exp(b*x+a)^2+1)*exp(b*x+a))^(1/2)+I*(
-I*(exp(b*x+a)+I))^(1/2)*2^(1/2)*(I*(exp(b*x+a)-I))^(1/2)*(I*exp(b*x+a))^(1
/2)/(exp(b*x+a)^3+exp(b*x+a))^(1/2)*(-2*I*EllipticE((-I*(exp(b*x+a)+I))^(1/
2),1/2*2^(1/2))+I*EllipticF((-I*(exp(b*x+a)+I))^(1/2),1/2*2^(1/2))))*2^(1/2
)*(exp(b*x+a)/(exp(b*x+a)^2+1))^(1/2)*((exp(b*x+a)^2+1)*exp(b*x+a))^(1/2)/e
xp(b*x+a)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\sqrt{\operatorname{sech}(bx+a)}\,\sinh(bx+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*sech(b*x+a)^(1/2)*sinh(b*x+a),x, algorithm="maxima")

[Out] integrate(x*sqrt(sech(b*x + a))*sinh(b*x + a), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int x\sinh(a+b\,x)\,\sqrt{\frac{1}{\cosh(a+b\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*sinh(a + b*x)*(1/cosh(a + b*x))^(1/2),x)

[Out] int(x*sinh(a + b*x)*(1/cosh(a + b*x))^(1/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\sinh(a+bx)\sqrt{\operatorname{sech}(a+bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sech(b*x+a)**(1/2)*sinh(b*x+a),x)
```

```
[Out] Integral(x*sinh(a + b*x)*sqrt(sech(a + b*x)), x)
```

3.541 $\int \frac{x\sinh(a+bx)}{\sqrt{\operatorname{sech}(a+bx)}}\,dx$

Optimal. Leaf size=84

$$-\frac{4\sinh(a+bx)}{9b^2\sqrt{\operatorname{sech}(a+bx)}}+\frac{4i\sqrt{\cosh(a+bx)}\sqrt{\operatorname{sech}(a+bx)}\,F\left(\frac{1}{2}i(a+bx)\middle|2\right)}{9b^2}+\frac{2x}{3b\operatorname{sech}^{\frac{3}{2}}(a+bx)}$$

```
[Out] 2/3*x/b/sech(b*x+a)^(3/2)-4/9*sinh(b*x+a)/b^2/sech(b*x+a)^(1/2)+4/9*I*(cosh
(1/2*a+1/2*b*x)^2)^(1/2)/cosh(1/2*a+1/2*b*x)*EllipticF(I*sinh(1/2*a+1/2*b*x
),2^(1/2))*cosh(b*x+a)^(1/2)*sech(b*x+a)^(1/2)/b^2
```

Rubi [A] time = 0.05, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {5444, 3769, 3771, 2641}

$$-\frac{4\sinh(a+bx)}{9b^2\sqrt{\operatorname{sech}(a+bx)}}+\frac{4i\sqrt{\cosh(a+bx)}\sqrt{\operatorname{sech}(a+bx)}\,F\left(\frac{1}{2}i(a+bx)\middle|2\right)}{9b^2}+\frac{2x}{3b\operatorname{sech}^{\frac{3}{2}}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Sinh[a + b*x])/Sqrt[Sech[a + b*x]],x]
```

```
[Out] (2*x)/(3*b*Sech[a + b*x]^(3/2)) + (((4*I)/9)*Sqrt[Cosh[a + b*x]]*EllipticF[
(I/2)*(a + b*x), 2]*Sqrt[Sech[a + b*x]])/b^2 - (4*Sinh[a + b*x])/(9*b^2*Sqr
t[Sech[a + b*x]])
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
```

```
EqQ[n^2, 1/4]
```

Rule 5444

```
Int[(x_)^(m_.)*Sech[(a_.) + (b_.)*(x_)^(n_.)]^(p_)*Sinh[(a_.) + (b_.)*(x_)^
(n_.)], x_Symbol] :> -Simp[(x^(m - n + 1)*Sech[a + b*x^n]^(p - 1))/(b*n*(p
- 1)), x] + Dist[(m - n + 1)/(b*n*(p - 1)), Int[x^(m - n)*Sech[a + b*x^n]^(
p - 1), x], x] /; FreeQ[{a, b, p}, x] && IntegerQ[n] && GeQ[m - n, 0] && Ne
Q[p, 1]
```

Rubi steps

$$\int \frac{x\sinh(a+bx)}{\sqrt{\operatorname{sech}(a+bx)}}\,dx = \frac{2x}{3b\operatorname{sech}^{\frac{3}{2}}(a+bx)} - \frac{2\int \frac{1}{\operatorname{sech}^{\frac{3}{2}}(a+bx)}\,dx}{3b}$$

$$= \frac{2x}{3b\operatorname{sech}^{\frac{3}{2}}(a+bx)} - \frac{4\sinh(a+bx)}{9b^2\sqrt{\operatorname{sech}(a+bx)}} - \frac{2\int \sqrt{\operatorname{sech}(a+bx)}\,dx}{9b}$$

$$= \frac{2x}{3b\operatorname{sech}^{\frac{3}{2}}(a+bx)} - \frac{4\sinh(a+bx)}{9b^2\sqrt{\operatorname{sech}(a+bx)}} - \frac{\left(2\sqrt{\cosh(a+bx)}\sqrt{\operatorname{sech}(a+bx)}\right)\int \frac{1}{\sqrt{\cosh(a+bx)}}}{9b}$$

$$= \frac{2x}{3b\operatorname{sech}^{\frac{3}{2}}(a+bx)} + \frac{4i\sqrt{\cosh(a+bx)}\,F\left(\frac{1}{2}i(a+bx)\middle|2\right)\sqrt{\operatorname{sech}(a+bx)}}{9b^2} - \frac{4\sinh(a+bx}{9b^2\sqrt{\operatorname{sech}(a+b}}$$

Mathematica [A] time = 0.18, size = 71, normalized size = 0.85

$$\frac{\sqrt{\operatorname{sech}(a+bx)}\left(-2\sinh(2(a+bx))+3bx\cosh(2(a+bx))+4i\sqrt{\cosh(a+bx)}\,F\left(\frac{1}{2}i(a+bx)\middle|2\right)+3bx\right)}{9b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Sinh[a + b*x])/Sqrt[Sech[a + b*x]],x]
```

```
[Out] (Sqrt[Sech[a + b*x]]*(3*b*x + 3*b*x*Cosh[2*(a + b*x)] + (4*I)*Sqrt[Cosh[a +
 b*x]]*EllipticF[(I/2)*(a + b*x), 2] - 2*Sinh[2*(a + b*x)]))/(9*b^2)
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sinh(b*x+a)/sech(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x \sinh(bx+a)}{\sqrt{\operatorname{sech}(bx+a)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sinh(b*x+a)/sech(b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x*sinh(b*x + a)/sqrt(sech(b*x + a)), x)
```

maple [F] time = 0.14, size = 0, normalized size = 0.00

$$\int \frac{x \sinh(bx+a)}{\sqrt{\operatorname{sech}(bx+a)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sinh(b*x+a)/sech(b*x+a)^(1/2),x)
```

```
[Out] int(x*sinh(b*x+a)/sech(b*x+a)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x \sinh(bx+a)}{\sqrt{\operatorname{sech}(bx+a)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sinh(b*x+a)/sech(b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x*sinh(b*x + a)/sqrt(sech(b*x + a)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x \sinh(a+b\,x)}{\sqrt{\frac{1}{\cosh(a+b\,x)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*sinh(a + b*x))/(1/cosh(a + b*x))^(1/2),x)
```

```
[Out] int((x*sinh(a + b*x))/(1/cosh(a + b*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x \sinh(a + bx)}{\sqrt{\operatorname{sech}(a + bx)}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sinh(b*x+a)/sech(b*x+a)**(1/2),x)
```

```
[Out] Integral(x*sinh(a + b*x)/sqrt(sech(a + b*x)), x)
```

3.542 $\int \frac{x \sinh(a+bx)}{\operatorname{sech}^{\frac{3}{2}}(a+bx)}\,dx$

Optimal. Leaf size=84

$$-\frac{4\sinh(a+bx)}{25b^2\operatorname{sech}^{\frac{3}{2}}(a+bx)}+\frac{12i\sqrt{\cosh(a+bx)}\sqrt{\operatorname{sech}(a+bx)}\,E\left(\frac{1}{2}i(a+bx)\middle|2\right)}{25b^2}+\frac{2x}{5b\operatorname{sech}^{\frac{5}{2}}(a+bx)}$$

```
[Out] 2/5*x/b/sech(b*x+a)^(5/2)-4/25*sinh(b*x+a)/b^2/sech(b*x+a)^(3/2)+12/25*I*(c
osh(1/2*a+1/2*b*x)^2)^(1/2)/cosh(1/2*a+1/2*b*x)*EllipticE(I*sinh(1/2*a+1/2*
b*x),2^(1/2))*cosh(b*x+a)^(1/2)*sech(b*x+a)^(1/2)/b^2
```

Rubi [A] time = 0.05, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {5444, 3769, 3771, 2639}

$$-\frac{4\sinh(a+bx)}{25b^2\operatorname{sech}^{\frac{3}{2}}(a+bx)}+\frac{12i\sqrt{\cosh(a+bx)}\sqrt{\operatorname{sech}(a+bx)}\,E\left(\frac{1}{2}i(a+bx)\middle|2\right)}{25b^2}+\frac{2x}{5b\operatorname{sech}^{\frac{5}{2}}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Sinh[a + b*x])/Sech[a + b*x]^(3/2),x]
```

```
[Out] (2*x)/(5*b*Sech[a + b*x]^(5/2)) + (((12*I)/25)*Sqrt[Cosh[a + b*x]]*Elliptic
E[(I/2)*(a + b*x), 2]*Sqrt[Sech[a + b*x]])/b^2 - (4*Sinh[a + b*x])/(25*b^2*
Sech[a + b*x]^(3/2))
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
```

```
EqQ[n^2, 1/4]
```

Rule 5444

```
Int[(x_)^(m_.)*Sech[(a_.) + (b_.)*(x_)^(n_.)]^(p_)*Sinh[(a_.) + (b_.)*(x_)^
(n_.)], x_Symbol] :> -Simp[(x^(m - n + 1)*Sech[a + b*x^n]^(p - 1))/(b*n*(p
- 1)), x] + Dist[(m - n + 1)/(b*n*(p - 1)), Int[x^(m - n)*Sech[a + b*x^n]^(
p - 1), x], x] /; FreeQ[{a, b, p}, x] && IntegerQ[n] && GeQ[m - n, 0] && Ne
Q[p, 1]
```

Rubi steps

$$\int \frac{x\sinh(a+bx)}{\operatorname{sech}^{\frac{3}{2}}(a+bx)}\,dx = \frac{2x}{5b\operatorname{sech}^{\frac{5}{2}}(a+bx)} - \frac{2\int \frac{1}{\operatorname{sech}^{\frac{5}{2}}(a+bx)}\,dx}{5b}$$

$$= \frac{2x}{5b\operatorname{sech}^{\frac{5}{2}}(a+bx)} - \frac{4\sinh(a+bx)}{25b^2\operatorname{sech}^{\frac{3}{2}}(a+bx)} - \frac{6\int \frac{1}{\sqrt{\operatorname{sech}(a+bx)}}\,dx}{25b}$$

$$= \frac{2x}{5b\operatorname{sech}^{\frac{5}{2}}(a+bx)} - \frac{4\sinh(a+bx)}{25b^2\operatorname{sech}^{\frac{3}{2}}(a+bx)} - \frac{\left(6\sqrt{\cosh(a+bx)}\sqrt{\operatorname{sech}(a+bx)}\right)\int \sqrt{\cosh(a+}}{25b}$$

$$= \frac{2x}{5b\operatorname{sech}^{\frac{5}{2}}(a+bx)} + \frac{12i\sqrt{\cosh(a+bx)}\,E\left(\frac{1}{2}i(a+bx)\middle|2\right)\sqrt{\operatorname{sech}(a+bx)}}{25b^2} - \frac{4\sinh(a+bx}{25b^2\operatorname{sech}^{\frac{3}{2}}(a+}$$

Mathematica [C] time = 2.43, size = 125, normalized size = 1.49

$$\frac{e^{-3(a+bx)}\left(48e^{2(a+bx)}\sqrt{e^{2(a+bx)}+1}\,{}_2F_1\left(-\frac{1}{4},\frac{1}{2};\frac{3}{4};-e^{2(a+bx)}\right)+\left(e^{2(a+bx)}+1\right)\left(2(5bx-12)e^{2(a+bx)}+(5bx-2)e^{4(a+bx)}+\right.}{100b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Sinh[a + b*x])/Sech[a + b*x]^(3/2),x]
```

```
[Out] (((1 + E^(2*(a + b*x)))*(2 + 5*b*x + 2*E^(2*(a + b*x))*(-12 + 5*b*x) + E^(4
*(a + b*x))*(-2 + 5*b*x)) + 48*E^(2*(a + b*x))*Sqrt[1 + E^(2*(a + b*x))]*Hy
pergeometric2F1[-1/4, 1/2, 3/4, -E^(2*(a + b*x))])*Sqrt[Sech[a + b*x]])/(10
0*b^2*E^(3*(a + b*x)))
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sinh(b*x+a)/sech(b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x \sinh (bx+a)}{\operatorname{sech}(bx+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sinh(b*x+a)/sech(b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x*sinh(b*x + a)/sech(b*x + a)^(3/2), x)
```

maple [F] time = 0.18, size = 0, normalized size = 0.00

$$\int \frac{x \sinh (bx+a)}{\operatorname{sech}(bx+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sinh(b*x+a)/sech(b*x+a)^(3/2),x)
```

```
[Out] int(x*sinh(b*x+a)/sech(b*x+a)^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x \sinh (bx+a)}{\operatorname{sech}(bx+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sinh(b*x+a)/sech(b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x*sinh(b*x + a)/sech(b*x + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x \sinh (a+b\,x)}{{\left(\frac{1}{\cosh(a+b\,x)}\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*sinh(a + b*x))/(1/cosh(a + b*x))^(3/2),x)
```

```
[Out] int((x*sinh(a + b*x))/(1/cosh(a + b*x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x \sinh(a + bx)}{\operatorname{sech}^{\frac{3}{2}}(a + bx)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sinh(b*x+a)/sech(b*x+a)**(3/2),x)
```

```
[Out] Integral(x*sinh(a + b*x)/sech(a + b*x)**(3/2), x)
```

3.543 $\int \frac{x\sinh(a+bx)}{\operatorname{sech}^{\frac{5}{2}}(a+bx)}\,dx$

Optimal. Leaf size=107

$$-\frac{4\sinh(a+bx)}{49b^2\operatorname{sech}^{\frac{5}{2}}(a+bx)}-\frac{20\sinh(a+bx)}{147b^2\sqrt{\operatorname{sech}(a+bx)}}+\frac{20i\sqrt{\cosh(a+bx)}\,\sqrt{\operatorname{sech}(a+bx)}\,F\left(\frac{1}{2}i(a+bx)\middle|2\right)}{147b^2}+\frac{2x}{7b\operatorname{sech}^{\frac{7}{2}}(a+\cdots}$$

```
[Out] 2/7*x/b/sech(b*x+a)^(7/2)-4/49*sinh(b*x+a)/b^2/sech(b*x+a)^(5/2)-20/147*sin
h(b*x+a)/b^2/sech(b*x+a)^(1/2)+20/147*I*(cosh(1/2*a+1/2*b*x)^2)^(1/2)/cosh(
1/2*a+1/2*b*x)*EllipticF(I*sinh(1/2*a+1/2*b*x),2^(1/2))*cosh(b*x+a)^(1/2)*s
ech(b*x+a)^(1/2)/b^2
```

Rubi [A] time = 0.07, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {5444, 3769, 3771, 2641}

$$-\frac{4\sinh(a+bx)}{49b^2\operatorname{sech}^{\frac{5}{2}}(a+bx)}-\frac{20\sinh(a+bx)}{147b^2\sqrt{\operatorname{sech}(a+bx)}}+\frac{20i\sqrt{\cosh(a+bx)}\,\sqrt{\operatorname{sech}(a+bx)}\,F\left(\frac{1}{2}i(a+bx)\middle|2\right)}{147b^2}+\frac{2x}{7b\operatorname{sech}^{\frac{7}{2}}(a+\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Sinh[a + b*x])/Sech[a + b*x]^(5/2),x]
```

```
[Out] (2*x)/(7*b*Sech[a + b*x]^(7/2)) + (((20*I)/147)*Sqrt[Cosh[a + b*x]]*Ellipti
cF[(I/2)*(a + b*x), 2]*Sqrt[Sech[a + b*x]])/b^2 - (4*Sinh[a + b*x])/(49*b^2
*Sech[a + b*x]^(5/2)) - (20*Sinh[a + b*x])/(147*b^2*Sqrt[Sech[a + b*x]])
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
```

```
EqQ[n^2, 1/4]
```

Rule 5444

```
Int[(x_)^(m_.)*Sech[(a_.) + (b_.)*(x_)^(n_.)]^(p_)*Sinh[(a_.) + (b_.)*(x_)^
(n_.)], x_Symbol] :> -Simp[(x^(m - n + 1)*Sech[a + b*x^n]^(p - 1))/(b*n*(p
- 1)), x] + Dist[(m - n + 1)/(b*n*(p - 1)), Int[x^(m - n)*Sech[a + b*x^n]^(
p - 1), x], x] /; FreeQ[{a, b, p}, x] && IntegerQ[n] && GeQ[m - n, 0] && Ne
Q[p, 1]
```

Rubi steps

$$\int \frac{x\sinh(a+bx)}{\operatorname{sech}^{\frac{5}{2}}(a+bx)}\,dx = \frac{2x}{7b\operatorname{sech}^{\frac{7}{2}}(a+bx)} - \frac{2\int \frac{1}{\operatorname{sech}^{\frac{7}{2}}(a+bx)}\,dx}{7b}$$

$$= \frac{2x}{7b\operatorname{sech}^{\frac{7}{2}}(a+bx)} - \frac{4\sinh(a+bx)}{49b^2\operatorname{sech}^{\frac{5}{2}}(a+bx)} - \frac{10\int \frac{1}{\operatorname{sech}^{\frac{3}{2}}(a+bx)}\,dx}{49b}$$

$$= \frac{2x}{7b\operatorname{sech}^{\frac{7}{2}}(a+bx)} - \frac{4\sinh(a+bx)}{49b^2\operatorname{sech}^{\frac{5}{2}}(a+bx)} - \frac{20\sinh(a+bx)}{147b^2\sqrt{\operatorname{sech}(a+bx)}} - \frac{10\int \sqrt{\operatorname{sech}(a+bx)}\,dx}{147b}$$

$$= \frac{2x}{7b\operatorname{sech}^{\frac{7}{2}}(a+bx)} - \frac{4\sinh(a+bx)}{49b^2\operatorname{sech}^{\frac{5}{2}}(a+bx)} - \frac{20\sinh(a+bx)}{147b^2\sqrt{\operatorname{sech}(a+bx)}} - \frac{\left(10\sqrt{\cosh(a+bx)}\sqrt{\operatorname{se}\ldots}\right.}{}$$

$$= \frac{2x}{7b\operatorname{sech}^{\frac{7}{2}}(a+bx)} + \frac{20i\sqrt{\cosh(a+bx)}\,F\left(\frac{1}{2}i(a+bx)\middle|2\right)\sqrt{\operatorname{sech}(a+bx)}}{147b^2} - \frac{4\sinh(a+b\ldots}{49b^2\operatorname{sech}^{\frac{5}{2}}(a+\ldots}$$

Mathematica [A] time = 0.29, size = 93, normalized size = 0.87

$$\frac{\sqrt{\operatorname{sech}(a+bx)}\left(-52\sinh(2(a+bx)) - 6\sinh(4(a+bx)) + 84bx\cosh(2(a+bx)) + 21bx\cosh(4(a+bx)) + 80i\sqrt{\ldots}\right.}{588b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Sinh[a + b*x])/Sech[a + b*x]^(5/2),x]
```

```
[Out] (Sqrt[Sech[a + b*x]]*(63*b*x + 84*b*x*Cosh[2*(a + b*x)] + 21*b*x*Cosh[4*(a
+ b*x)] + (80*I)*Sqrt[Cosh[a + b*x]]*EllipticF[(I/2)*(a + b*x), 2] - 52*Sin
h[2*(a + b*x)] - 6*Sinh[4*(a + b*x)]))/(588*b^2)
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sinh(b*x+a)/sech(b*x+a)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x \sinh(bx+a)}{\operatorname{sech}(bx+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sinh(b*x+a)/sech(b*x+a)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(x*sinh(b*x + a)/sech(b*x + a)^(5/2), x)
```

maple [F] time = 0.17, size = 0, normalized size = 0.00

$$\int \frac{x \sinh(bx+a)}{\operatorname{sech}(bx+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sinh(b*x+a)/sech(b*x+a)^(5/2),x)
```

```
[Out] int(x*sinh(b*x+a)/sech(b*x+a)^(5/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x \sinh(bx+a)}{\operatorname{sech}(bx+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sinh(b*x+a)/sech(b*x+a)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(x*sinh(b*x + a)/sech(b*x + a)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x \sinh(a+b\,x)}{\left(\frac{1}{\cosh(a+b\,x)}\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x*sinh(a + b*x))/(1/cosh(a + b*x))^(5/2),x)

[Out] int((x*sinh(a + b*x))/(1/cosh(a + b*x))^(5/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\sinh(a+bx)}{\operatorname{sech}^{\frac{5}{2}}(a+bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*sinh(b*x+a)/sech(b*x+a)**(5/2),x)

[Out] Integral(x*sinh(a + b*x)/sech(a + b*x)**(5/2), x)

3.544 $\int x\cosh(a+bx)\sinh^{\frac{5}{2}}(a+bx)\,dx$

Optimal. Leaf size=121

$$-\frac{4\sinh^{\frac{5}{2}}(a+bx)\cosh(a+bx)}{49b^2}+\frac{20\sqrt{\sinh(a+bx)}\,\cosh(a+bx)}{147b^2}+\frac{20i\sqrt{i\sinh(a+bx)}\,F\left(\frac{1}{2}\left(ia+ibx-\frac{\pi}{2}\right)\middle|2\right)}{147b^2\sqrt{\sinh(a+bx)}}+\frac{2}{}$$

```
[Out] -4/49*cosh(b*x+a)*sinh(b*x+a)^(5/2)/b^2+2/7*x*sinh(b*x+a)^(7/2)/b-20/147*I*
(sin(1/2*I*a+1/4*Pi+1/2*I*b*x)^2)^(1/2)/sin(1/2*I*a+1/4*Pi+1/2*I*b*x)*Ellip
ticF(cos(1/2*I*a+1/4*Pi+1/2*I*b*x),2^(1/2))*(I*sinh(b*x+a))^(1/2)/b^2/sinh(
b*x+a)^(1/2)+20/147*cosh(b*x+a)*sinh(b*x+a)^(1/2)/b^2
```

Rubi [A] time = 0.07, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {5372, 2635, 2642, 2641}

$$-\frac{4\sinh^{\frac{5}{2}}(a+bx)\cosh(a+bx)}{49b^2}+\frac{20\sqrt{\sinh(a+bx)}\,\cosh(a+bx)}{147b^2}+\frac{20i\sqrt{i\sinh(a+bx)}\,F\left(\frac{1}{2}\left(ia+ibx-\frac{\pi}{2}\right)\middle|2\right)}{147b^2\sqrt{\sinh(a+bx)}}+\frac{2}{}$$

Antiderivative was successfully verified.

```
[In] Int[x*Cosh[a + b*x]*Sinh[a + b*x]^(5/2),x]
```

```
[Out] (((20*I)/147)*EllipticF[(I*a - Pi/2 + I*b*x)/2, 2]*Sqrt[I*Sinh[a + b*x]])/(
b^2*Sqrt[Sinh[a + b*x]]) + (20*Cosh[a + b*x]*Sqrt[Sinh[a + b*x]])/(147*b^2)
 - (4*Cosh[a + b*x]*Sinh[a + b*x]^(5/2))/(49*b^2) + (2*x*Sinh[a + b*x]^(7/2
))/(7*b)
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
```

```
d}, x]
```

Rule 5372

```
Int[Cosh[(a_.) + (b_.)*(x_)^(n_.)]*(x_)^(m_.)*Sinh[(a_.) + (b_.)*(x_)^(n_.)
]^(p_.), x_Symbol] :> Simp[(x^(m - n + 1)*Sinh[a + b*x^n]^(p + 1))/(b*n*(p
+ 1)), x] - Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*Sinh[a + b*x^n]^(
p + 1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

Rubi steps

$$\int x\cosh(a+bx)\sinh^{\frac{5}{2}}(a+bx)\,dx = \frac{2x\sinh^{\frac{7}{2}}(a+bx)}{7b} - \frac{2\int \sinh^{\frac{7}{2}}(a+bx)\,dx}{7b}$$
$$= -\frac{4\cosh(a+bx)\sinh^{\frac{5}{2}}(a+bx)}{49b^2} + \frac{2x\sinh^{\frac{7}{2}}(a+bx)}{7b} + \frac{10\int \sinh^{\frac{3}{2}}(a+bx)\,d}{49b}$$
$$= \frac{20\cosh(a+bx)\sqrt{\sinh(a+bx)}}{147b^2} - \frac{4\cosh(a+bx)\sinh^{\frac{5}{2}}(a+bx)}{49b^2} + \frac{2x\sinh^{\frac{7}{2}}}{7}$$
$$= \frac{20\cosh(a+bx)\sqrt{\sinh(a+bx)}}{147b^2} - \frac{4\cosh(a+bx)\sinh^{\frac{5}{2}}(a+bx)}{49b^2} + \frac{2x\sinh^{\frac{7}{2}}}{7}$$
$$= \frac{20iF\left(\frac{1}{2}\left(ia-\frac{\pi}{2}+ibx\right)\middle|2\right)\sqrt{i\sinh(a+bx)}}{147b^2\sqrt{\sinh(a+bx)}} + \frac{20\cosh(a+bx)\sqrt{\sinh(a+bx}}{147b^2}$$

Mathematica [A] time = 0.32, size = 103, normalized size = 0.85

$$\frac{52\sinh(2(a+bx)) - 6\sinh(4(a+bx)) - 84bx\cosh(2(a+bx)) + 21bx\cosh(4(a+bx)) - 80i\sqrt{i\sinh(a+bx)}\,F\left(\frac{1}{4}\right.}{588b^2\sqrt{\sinh(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Cosh[a + b*x]*Sinh[a + b*x]^(5/2),x]
```

```
[Out] (63*b*x - 84*b*x*Cosh[2*(a + b*x)] + 21*b*x*Cosh[4*(a + b*x)] - (80*I)*Elli
pticF[((-2*I)*a + Pi - (2*I)*b*x)/4, 2]*Sqrt[I*Sinh[a + b*x]] + 52*Sinh[2*(
a + b*x)] - 6*Sinh[4*(a + b*x)])/(588*b^2*Sqrt[Sinh[a + b*x]])
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x+a)*sinh(b*x+a)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x+a)*sinh(b*x+a)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:Unable to divide, perhaps due to rounding error%%%{1,[1,1
,0]%%%} / %%%{1,[0,0,2]%%%} Error: Bad Argument Value
```

maple [F] time = 0.14, size = 0, normalized size = 0.00

$$\int x\cosh\left(bx+a\right)\left(\sinh^{\frac{5}{2}}\left(bx+a\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cosh(b*x+a)*sinh(b*x+a)^(5/2),x)
```

```
[Out] int(x*cosh(b*x+a)*sinh(b*x+a)^(5/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\cosh\left(bx+a\right)\sinh\left(bx+a\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x+a)*sinh(b*x+a)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(x*cosh(b*x + a)*sinh(b*x + a)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x\cosh\left(a+b\,x\right)\,{\sinh\left(a+b\,x\right)}^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cosh(a + b*x)*sinh(a + b*x)^(5/2),x)
```

```
[Out] int(x*cosh(a + b*x)*sinh(a + b*x)^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x+a)*sinh(b*x+a)**(5/2),x)
```

```
[Out] Timed out
```

3.545 $\int x \cosh(a+bx) \sinh^{\frac{3}{2}}(a+bx)\,dx$

Optimal. Leaf size=98

$$-\frac{4\sinh^{\frac{3}{2}}(a+bx)\cosh(a+bx)}{25b^2}-\frac{12i\sqrt{\sinh(a+bx)}\,E\left(\frac{1}{2}\left(ia+ibx-\frac{\pi}{2}\right)\middle|2\right)}{25b^2\sqrt{i\sinh(a+bx)}}+\frac{2x\sinh^{\frac{5}{2}}(a+bx)}{5b}$$

```
[Out] -4/25*cosh(b*x+a)*sinh(b*x+a)^(3/2)/b^2+2/5*x*sinh(b*x+a)^(5/2)/b+12/25*I*(
sin(1/2*I*a+1/4*Pi+1/2*I*b*x)^2)^(1/2)/sin(1/2*I*a+1/4*Pi+1/2*I*b*x)*Ellipt
icE(cos(1/2*I*a+1/4*Pi+1/2*I*b*x),2^(1/2))*sinh(b*x+a)^(1/2)/b^2/(I*sinh(b*
x+a))^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {5372, 2635, 2640, 2639}

$$-\frac{4\sinh^{\frac{3}{2}}(a+bx)\cosh(a+bx)}{25b^2}-\frac{12i\sqrt{\sinh(a+bx)}\,E\left(\frac{1}{2}\left(ia+ibx-\frac{\pi}{2}\right)\middle|2\right)}{25b^2\sqrt{i\sinh(a+bx)}}+\frac{2x\sinh^{\frac{5}{2}}(a+bx)}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[x*Cosh[a + b*x]*Sinh[a + b*x]^(3/2),x]
```

```
[Out] (((-12*I)/25)*EllipticE[(I*a - Pi/2 + I*b*x)/2, 2]*Sqrt[Sinh[a + b*x]])/(b^
2*Sqrt[I*Sinh[a + b*x]]) - (4*Cosh[a + b*x]*Sinh[a + b*x]^(3/2))/(25*b^2) +
 (2*x*Sinh[a + b*x]^(5/2))/(5*b)
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
```

```
x]
```

Rule 5372

```
Int[Cosh[(a_.) + (b_.)*(x_)^(n_.)]*(x_)^(m_.)*Sinh[(a_.) + (b_.)*(x_)^(n_.)
]^(p_.), x_Symbol] :> Simp[(x^(m - n + 1)*Sinh[a + b*x^n]^(p + 1))/(b*n*(p
+ 1)), x] - Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*Sinh[a + b*x^n]^(
p + 1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

Rubi steps

$$\int x\cosh(a+bx)\sinh^{\frac{3}{2}}(a+bx)\,dx = \frac{2x\sinh^{\frac{5}{2}}(a+bx)}{5b} - \frac{2\int \sinh^{\frac{5}{2}}(a+bx)\,dx}{5b}$$

$$= -\frac{4\cosh(a+bx)\sinh^{\frac{3}{2}}(a+bx)}{25b^2} + \frac{2x\sinh^{\frac{5}{2}}(a+bx)}{5b} + \frac{6\int \sqrt{\sinh(a+bx)}\,d}{25b}$$

$$= -\frac{4\cosh(a+bx)\sinh^{\frac{3}{2}}(a+bx)}{25b^2} + \frac{2x\sinh^{\frac{5}{2}}(a+bx)}{5b} + \frac{\left(6\sqrt{\sinh(a+bx)}\right)\int}{25b\sqrt{i\sinh}}$$

$$= -\frac{12iE\left(\frac{1}{2}\left(ia-\frac{\pi}{2}+ibx\right)\middle|2\right)\sqrt{\sinh(a+bx)}}{25b^2\sqrt{i\sinh(a+bx)}} - \frac{4\cosh(a+bx)\sinh^{\frac{3}{2}}(a+bx)}{25b^2}$$

Mathematica [C] time = 2.36, size = 143, normalized size = 1.46

$$\frac{e^{-3(a+bx)}\left(48e^{2(a+bx)}\sqrt{1-e^{2(a+bx)}}\ {}_2F_1\left(-\frac{1}{4},\frac{1}{2};\frac{3}{4};e^{2(a+bx)}\right)+\left(e^{2(a+bx)}-1\right)\left((24-10bx)e^{2(a+bx)}+(5bx-2)e^{4(a+bx)}+5\right.\right.}{50\sqrt{2}\,b^2\sqrt{e^{a+bx}-e^{-a-bx}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Cosh[a + b*x]*Sinh[a + b*x]^(3/2),x]
```

```
[Out] ((-1 + E^(2*(a + b*x)))*(2 + 5*b*x + E^(2*(a + b*x))*(24 - 10*b*x) + E^(4*(
a + b*x))*(-2 + 5*b*x)) + 48*E^(2*(a + b*x))*Sqrt[1 - E^(2*(a + b*x))]*Hype
rgeometric2F1[-1/4, 1/2, 3/4, E^(2*(a + b*x))])/(50*Sqrt[2]*b^2*E^(3*(a + b
*x))*Sqrt[-E^(-a - b*x) + E^(a + b*x)])
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x+a)*sinh(b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x \cosh(bx+a) \sinh(bx+a)^{\frac{3}{2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x+a)*sinh(b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x*cosh(b*x + a)*sinh(b*x + a)^(3/2), x)
```

maple [F] time = 0.13, size = 0, normalized size = 0.00

$$\int x \cosh(bx+a) \left(\sinh^{\frac{3}{2}}(bx+a)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cosh(b*x+a)*sinh(b*x+a)^(3/2),x)
```

```
[Out] int(x*cosh(b*x+a)*sinh(b*x+a)^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x \cosh(bx+a) \sinh(bx+a)^{\frac{3}{2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x+a)*sinh(b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x*cosh(b*x + a)*sinh(b*x + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x \cosh(a+b\,x)\,\sinh(a+b\,x)^{3/2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cosh(a + b*x)*sinh(a + b*x)^(3/2),x)
```

```
[Out] int(x*cosh(a + b*x)*sinh(a + b*x)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\sinh^{\frac{3}{2}}(a+bx)\cosh(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x+a)*sinh(b*x+a)**(3/2),x)
```

```
[Out] Integral(x*sinh(a + b*x)**(3/2)*cosh(a + b*x), x)
```

3.546 $\int x\cosh(a+bx)\sqrt{\sinh(a+bx)}\,dx$

Optimal. Leaf size=98

$$-\frac{4\sqrt{\sinh(a+bx)}\cosh(a+bx)}{9b^2}-\frac{4i\sqrt{i\sinh(a+bx)}\,F\left(\frac{1}{2}\left(ia+ibx-\frac{\pi}{2}\right)\middle|2\right)}{9b^2\sqrt{\sinh(a+bx)}}+\frac{2x\sinh^{\frac{3}{2}}(a+bx)}{3b}$$

```
[Out] 2/3*x*sinh(b*x+a)^(3/2)/b+4/9*I*(sin(1/2*I*a+1/4*Pi+1/2*I*b*x)^2)^(1/2)/sin
(1/2*I*a+1/4*Pi+1/2*I*b*x)*EllipticF(cos(1/2*I*a+1/4*Pi+1/2*I*b*x),2^(1/2))
*(I*sinh(b*x+a))^(1/2)/b^2/sinh(b*x+a)^(1/2)-4/9*cosh(b*x+a)*sinh(b*x+a)^(1
/2)/b^2
```

Rubi [A] time = 0.05, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {5372, 2635, 2642, 2641}

$$-\frac{4\sqrt{\sinh(a+bx)}\cosh(a+bx)}{9b^2}-\frac{4i\sqrt{i\sinh(a+bx)}\,F\left(\frac{1}{2}\left(ia+ibx-\frac{\pi}{2}\right)\middle|2\right)}{9b^2\sqrt{\sinh(a+bx)}}+\frac{2x\sinh^{\frac{3}{2}}(a+bx)}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[x*Cosh[a + b*x]*Sqrt[Sinh[a + b*x]],x]
```

```
[Out] (((-4*I)/9)*EllipticF[(I*a - Pi/2 + I*b*x)/2, 2]*Sqrt[I*Sinh[a + b*x]])/(b^
2*Sqrt[Sinh[a + b*x]]) - (4*Cosh[a + b*x]*Sqrt[Sinh[a + b*x]])/(9*b^2) + (2
*x*Sinh[a + b*x]^(3/2))/(3*b)
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
```

```
d}, x]
```

Rule 5372

```
Int[Cosh[(a_.) + (b_.)*(x_)^(n_.)]*(x_)^(m_.)*Sinh[(a_.) + (b_.)*(x_)^(n_.)
]^(p_.), x_Symbol] :> Simp[(x^(m - n + 1)*Sinh[a + b*x^n]^(p + 1))/(b*n*(p
+ 1)), x] - Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*Sinh[a + b*x^n]^(
p + 1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

Rubi steps

$$\int x\cosh(a+bx)\sqrt{\sinh(a+bx)}\,dx = \frac{2x\sinh^{\frac{3}{2}}(a+bx)}{3b} - \frac{2\int\sinh^{\frac{3}{2}}(a+bx)\,dx}{3b}$$
$$= -\frac{4\cosh(a+bx)\sqrt{\sinh(a+bx)}}{9b^2} + \frac{2x\sinh^{\frac{3}{2}}(a+bx)}{3b} + \frac{2\int\frac{1}{\sqrt{\sinh(a+bx)}}\,dx}{9b}$$
$$= -\frac{4\cosh(a+bx)\sqrt{\sinh(a+bx)}}{9b^2} + \frac{2x\sinh^{\frac{3}{2}}(a+bx)}{3b} + \frac{\left(2\sqrt{i\sinh(a+bx)}\right)}{9b\sqrt{\sinh(}}$$
$$= -\frac{4iF\left(\frac{1}{2}\left(ia-\frac{\pi}{2}+ibx\right)\middle|2\right)\sqrt{i\sinh(a+bx)}}{9b^2\sqrt{\sinh(a+bx)}} - \frac{4\cosh(a+bx)\sqrt{\sinh(a+bx}}{9b^2}$$

Mathematica [A] time = 0.21, size = 77, normalized size = 0.79

$$\frac{2\left(3bx\sinh^2(a+bx)-\sinh(2(a+bx))+2i\sqrt{i\sinh(a+bx)}\,F\left(\frac{1}{4}(-2ia-2ibx+\pi)\middle|2\right)\right)}{9b^2\sqrt{\sinh(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Cosh[a + b*x]*Sqrt[Sinh[a + b*x]],x]
```

```
[Out] (2*((2*I)*EllipticF[((-2*I)*a + Pi - (2*I)*b*x)/4, 2]*Sqrt[I*Sinh[a + b*x]]
 + 3*b*x*Sinh[a + b*x]^2 - Sinh[2*(a + b*x)]))/(9*b^2*Sqrt[Sinh[a + b*x]])
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x+a)*sinh(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\cosh\left(bx+a\right)\sqrt{\sinh\left(bx+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x+a)*sinh(b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x*cosh(b*x + a)*sqrt(sinh(b*x + a)), x)
```

maple [F] time = 0.10, size = 0, normalized size = 0.00

$$\int x\cosh\left(bx+a\right)\left(\sqrt{\sinh}\,\left(bx+a\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cosh(b*x+a)*sinh(b*x+a)^(1/2),x)
```

```
[Out] int(x*cosh(b*x+a)*sinh(b*x+a)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\cosh\left(bx+a\right)\sqrt{\sinh\left(bx+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x+a)*sinh(b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x*cosh(b*x + a)*sqrt(sinh(b*x + a)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x\cosh\left(a+b\,x\right)\,\sqrt{\sinh\left(a+b\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cosh(a + b*x)*sinh(a + b*x)^(1/2),x)
```

```
[Out] int(x*cosh(a + b*x)*sinh(a + b*x)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\sqrt{\sinh\left(a+bx\right)}\,\cosh\left(a+bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x+a)*sinh(b*x+a)**(1/2),x)
```

```
[Out] Integral(x*sqrt(sinh(a + b*x))*cosh(a + b*x), x)
```

3.547 $\int \frac{x\cosh(a+bx)}{\sqrt{\sinh(a+bx)}}\,dx$

Optimal. Leaf size=71

$$\frac{2x\sqrt{\sinh(a+bx)}}{b}+\frac{4i\sqrt{\sinh(a+bx)}\,E\left(\frac{1}{2}\left(ia+ibx-\frac{\pi}{2}\right)\middle|2\right)}{b^2\sqrt{i\sinh(a+bx)}}$$

```
[Out] 2*x*sinh(b*x+a)^(1/2)/b-4*I*(sin(1/2*I*a+1/4*Pi+1/2*I*b*x)^2)^(1/2)/sin(1/2
*I*a+1/4*Pi+1/2*I*b*x)*EllipticE(cos(1/2*I*a+1/4*Pi+1/2*I*b*x),2^(1/2))*sin
h(b*x+a)^(1/2)/b^2/(I*sinh(b*x+a))^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {5372, 2640, 2639}

$$\frac{2x\sqrt{\sinh(a+bx)}}{b}+\frac{4i\sqrt{\sinh(a+bx)}\,E\left(\frac{1}{2}\left(ia+ibx-\frac{\pi}{2}\right)\middle|2\right)}{b^2\sqrt{i\sinh(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Cosh[a + b*x])/Sqrt[Sinh[a + b*x]],x]
```

```
[Out] (2*x*Sqrt[Sinh[a + b*x]])/b + ((4*I)*EllipticE[(I*a - Pi/2 + I*b*x)/2, 2]*S
qrt[Sinh[a + b*x]])/(b^2*Sqrt[I*Sinh[a + b*x]])
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 5372

```
Int[Cosh[(a_.) + (b_.)*(x_)^(n_.)]*(x_)^(m_.)*Sinh[(a_.) + (b_.)*(x_)^(n_.)
]^(p_.), x_Symbol] :> Simp[(x^(m - n + 1)*Sinh[a + b*x^n]^(p + 1))/(b*n*(p
+ 1)), x] - Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*Sinh[a + b*x^n]^(
p + 1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{x\cosh(a+bx)}{\sqrt{\sinh(a+bx)}}\,dx = \frac{2x\sqrt{\sinh(a+bx)}}{b} - \frac{2\int \sqrt{\sinh(a+bx)}\,dx}{b}$$
$$= \frac{2x\sqrt{\sinh(a+bx)}}{b} - \frac{\left(2\sqrt{\sinh(a+bx)}\right)\int \sqrt{i\sinh(a+bx)}\,dx}{b\sqrt{i\sinh(a+bx)}}$$
$$= \frac{2x\sqrt{\sinh(a+bx)}}{b} + \frac{4iE\left(\frac{1}{2}\left(ia-\frac{\pi}{2}+ibx\right)\middle|2\right)\sqrt{\sinh(a+bx)}}{b^2\sqrt{i\sinh(a+bx)}}$$

Mathematica [C] time = 1.77, size = 182, normalized size = 2.56

$$\frac{e^{-a-bx}\sqrt{2-2e^{2(a+bx)}}\left(-18\,{}_3F_2\left(-\frac{1}{4},-\frac{1}{4},\frac{1}{2};\frac{3}{4},\frac{3}{4};e^{2(a+bx)}\right)-2e^{2(a+bx)}\,{}_3F_2\left(\frac{1}{2},\frac{3}{4},\frac{3}{4};\frac{7}{4},\frac{7}{4};e^{2(a+bx)}\right)-3bx\left(3\,{}_2F_1\left(-\frac{1}{4},\frac{1}{2};\frac{3}{4}\right.\right.\right.}{9b^2\sqrt{e^{a+bx}-e^{-a-bx}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x*Cosh[a + b*x])/Sqrt[Sinh[a + b*x]],x]
```

```
[Out] (E^(-a - b*x)*Sqrt[2 - 2*E^(2*(a + b*x))]*(-3*b*x*(3*Hypergeometric2F1[-1/4
, 1/2, 3/4, E^(2*(a + b*x))] - E^(2*(a + b*x))*Hypergeometric2F1[1/2, 3/4,
7/4, E^(2*(a + b*x))]) - 18*HypergeometricPFQ[{-1/4, -1/4, 1/2}, {3/4, 3/4}
, E^(2*(a + b*x))] - 2*E^(2*(a + b*x))*HypergeometricPFQ[{1/2, 3/4, 3/4}, {
7/4, 7/4}, E^(2*(a + b*x))]))/(9*b^2*Sqrt[-E^(-a - b*x) + E^(a + b*x)])
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x+a)/sinh(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\cosh(bx+a)}{\sqrt{\sinh(bx+a)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x+a)/sinh(b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x*cosh(b*x + a)/sqrt(sinh(b*x + a)), x)
```

maple [B] time = 0.16, size = 229, normalized size = 3.23

$$\frac{(bx-2)\left(e^{2bx+2a}-1\right)\sqrt{2}\,e^{-bx-a}}{b^{2}\sqrt{\left(e^{2bx+2a}-1\right)e^{-bx-a}}}+\frac{2\left(\frac{2\,e^{2bx+2a}-2}{\sqrt{\left(e^{2bx+2a}-1\right)e^{bx+a}}}-\frac{\sqrt{1+e^{bx+a}}\,\sqrt{2-2\,e^{bx+a}}\,\sqrt{-e^{bx+a}}\,\left(-2\,\mathrm{EllipticE}\left(\sqrt{1+e^{bx+a}},\frac{\sqrt{2}}{2}\right)+\mathrm{EllipticF}\left(\right.\right.}{\sqrt{e^{3bx+3a}-e^{bx+a}}}\right.}{b^{2}\sqrt{\left(e^{2bx+2a}-1\right)e^{-bx-a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cosh(b*x+a)/sinh(b*x+a)^(1/2),x)
```

```
[Out] (b*x-2)*(exp(b*x+a)^2-1)/b^2*2^(1/2)/((exp(b*x+a)^2-1)/exp(b*x+a))^(1/2)/ex
p(b*x+a)+2/b^2*(2*(exp(b*x+a)^2-1)/((exp(b*x+a)^2-1)*exp(b*x+a))^(1/2)-(1+e
xp(b*x+a))^(1/2)*(2-2*exp(b*x+a))^(1/2)*(-exp(b*x+a))^(1/2)/(exp(b*x+a)^3-e
xp(b*x+a))^(1/2)*(-2*EllipticE((1+exp(b*x+a))^(1/2),1/2*2^(1/2))+EllipticF(
(1+exp(b*x+a))^(1/2),1/2*2^(1/2))))*2^(1/2)/((exp(b*x+a)^2-1)/exp(b*x+a))^(
1/2)*((exp(b*x+a)^2-1)*exp(b*x+a))^(1/2)/exp(b*x+a)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\cosh\left(bx+a\right)}{\sqrt{\sinh\left(bx+a\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x+a)/sinh(b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x*cosh(b*x + a)/sqrt(sinh(b*x + a)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x\cosh\left(a+b\,x\right)}{\sqrt{\sinh\left(a+b\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*cosh(a + b*x))/sinh(a + b*x)^(1/2),x)
```

```
[Out] int((x*cosh(a + b*x))/sinh(a + b*x)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\cosh\left(a+bx\right)}{\sqrt{\sinh\left(a+bx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x+a)/sinh(b*x+a)**(1/2),x)
```

```
[Out] Integral(x*cosh(a + b*x)/sqrt(sinh(a + b*x)), x)
```

3.548 $\int \frac{x\cosh(a+bx)}{\sinh^{\frac{3}{2}}(a+bx)}\,dx$

Optimal. Leaf size=71

$$-\frac{2x}{b\sqrt{\sinh(a+bx)}}-\frac{4i\sqrt{i\sinh(a+bx)}\,F\left(\frac{1}{2}\left(ia+ibx-\frac{\pi}{2}\right)\middle|2\right)}{b^2\sqrt{\sinh(a+bx)}}$$

```
[Out] -2*x/b/sinh(b*x+a)^(1/2)+4*I*(sin(1/2*I*a+1/4*Pi+1/2*I*b*x)^2)^(1/2)/sin(1/
2*I*a+1/4*Pi+1/2*I*b*x)*EllipticF(cos(1/2*I*a+1/4*Pi+1/2*I*b*x),2^(1/2))*(I
*sinh(b*x+a))^(1/2)/b^2/sinh(b*x+a)^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {5372, 2642, 2641}

$$-\frac{2x}{b\sqrt{\sinh(a+bx)}}-\frac{4i\sqrt{i\sinh(a+bx)}\,F\left(\frac{1}{2}\left(ia+ibx-\frac{\pi}{2}\right)\middle|2\right)}{b^2\sqrt{\sinh(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Cosh[a + b*x])/Sinh[a + b*x]^(3/2),x]
```

```
[Out] (-2*x)/(b*Sqrt[Sinh[a + b*x]]) - ((4*I)*EllipticF[(I*a - Pi/2 + I*b*x)/2, 2
]*Sqrt[I*Sinh[a + b*x]])/(b^2*Sqrt[Sinh[a + b*x]])
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 5372

```
Int[Cosh[(a_.) + (b_.)*(x_)^(n_.)]*(x_)^(m_.)*Sinh[(a_.) + (b_.)*(x_)^(n_.)
]^(p_.), x_Symbol] :> Simp[(x^(m - n + 1)*Sinh[a + b*x^n]^(p + 1))/(b*n*(p
+ 1)), x] - Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*Sinh[a + b*x^n]^(
p + 1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{x\cosh(a+bx)}{\sinh^{\frac{3}{2}}(a+bx)}\,dx = -\frac{2x}{b\sqrt{\sinh(a+bx)}}+\frac{2\int \frac{1}{\sqrt{\sinh(a+bx)}}\,dx}{b}$$
$$= -\frac{2x}{b\sqrt{\sinh(a+bx)}}+\frac{\left(2\sqrt{i\sinh(a+bx)}\right)\int \frac{1}{\sqrt{i\sinh(a+bx)}}\,dx}{b\sqrt{\sinh(a+bx)}}$$
$$= -\frac{2x}{b\sqrt{\sinh(a+bx)}}-\frac{4iF\left(\frac{1}{2}\left(ia-\frac{\pi}{2}+ibx\right)\middle|2\right)\sqrt{i\sinh(a+bx)}}{b^2\sqrt{\sinh(a+bx)}}$$

Mathematica [A] time = 0.17, size = 56, normalized size = 0.79

$$\frac{-2bx+4i\sqrt{i\sinh(a+bx)}\,F\left(\frac{1}{4}(-2ia-2ibx+\pi)\middle|2\right)}{b^2\sqrt{\sinh(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Cosh[a + b*x])/Sinh[a + b*x]^(3/2),x]
```

```
[Out] (-2*b*x + (4*I)*EllipticF[((-2*I)*a + Pi - (2*I)*b*x)/4, 2]*Sqrt[I*Sinh[a +
 b*x]])/(b^2*Sqrt[Sinh[a + b*x]])
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x+a)/sinh(b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\cosh(bx+a)}{\sinh(bx+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x+a)/sinh(b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x*cosh(b*x + a)/sinh(b*x + a)^(3/2), x)
```

maple [F] time = 0.10, size = 0, normalized size = 0.00

$$\int \frac{x\cosh\left(bx+a\right)}{\sinh\left(bx+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cosh(b*x+a)/sinh(b*x+a)^(3/2),x)
[Out] int(x*cosh(b*x+a)/sinh(b*x+a)^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\cosh\left(bx+a\right)}{\sinh\left(bx+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x+a)/sinh(b*x+a)^(3/2),x, algorithm="maxima")
[Out] integrate(x*cosh(b*x + a)/sinh(b*x + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x\cosh\left(a+b\,x\right)}{{\sinh\left(a+b\,x\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*cosh(a + b*x))/sinh(a + b*x)^(3/2),x)
[Out] int((x*cosh(a + b*x))/sinh(a + b*x)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\cosh\left(a+bx\right)}{\sinh^{\frac{3}{2}}\left(a+bx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x+a)/sinh(b*x+a)**(3/2),x)
[Out] Integral(x*cosh(a + b*x)/sinh(a + b*x)**(3/2), x)
```

3.549 $\int \frac{x\cosh(a+bx)}{\sinh^{\frac{5}{2}}(a+bx)}\,dx$

Optimal. Leaf size=98

$$-\frac{4\cosh(a+bx)}{3b^2\sqrt{\sinh(a+bx)}}-\frac{4i\sqrt{\sinh(a+bx)}\,E\left(\frac{1}{2}\left(ia+ibx-\frac{\pi}{2}\right)\middle|2\right)}{3b^2\sqrt{i\sinh(a+bx)}}-\frac{2x}{3b\sinh^{\frac{3}{2}}(a+bx)}$$

```
[Out] -2/3*x/b/sinh(b*x+a)^(3/2)-4/3*cosh(b*x+a)/b^2/sinh(b*x+a)^(1/2)+4/3*I*(sin
(1/2*I*a+1/4*Pi+1/2*I*b*x)^2)^(1/2)/sin(1/2*I*a+1/4*Pi+1/2*I*b*x)*EllipticE
(cos(1/2*I*a+1/4*Pi+1/2*I*b*x),2^(1/2))*sinh(b*x+a)^(1/2)/b^2/(I*sinh(b*x+a
))^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {5372, 2636, 2640, 2639}

$$-\frac{4\cosh(a+bx)}{3b^2\sqrt{\sinh(a+bx)}}-\frac{4i\sqrt{\sinh(a+bx)}\,E\left(\frac{1}{2}\left(ia+ibx-\frac{\pi}{2}\right)\middle|2\right)}{3b^2\sqrt{i\sinh(a+bx)}}-\frac{2x}{3b\sinh^{\frac{3}{2}}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Cosh[a + b*x])/Sinh[a + b*x]^(5/2),x]
```

```
[Out] (-2*x)/(3*b*Sinh[a + b*x]^(3/2)) - (4*Cosh[a + b*x])/(3*b^2*Sqrt[Sinh[a + b
*x]]) - (((4*I)/3)*EllipticE[(I*a - Pi/2 + I*b*x)/2, 2]*Sqrt[Sinh[a + b*x]]
)/(b^2*Sqrt[I*Sinh[a + b*x]])
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
```

```
x]
```

Rule 5372

```
Int[Cosh[(a_.) + (b_.)*(x_)^(n_.)]*(x_)^(m_.)*Sinh[(a_.) + (b_.)*(x_)^(n_.)
]^(p_.), x_Symbol] :> Simp[(x^(m - n + 1)*Sinh[a + b*x^n]^(p + 1))/(b*n*(p
+ 1)), x] - Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*Sinh[a + b*x^n]^(
p + 1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{x\cosh(a+bx)}{\sinh^{\frac{5}{2}}(a+bx)}\,dx &= -\frac{2x}{3b\sinh^{\frac{3}{2}}(a+bx)}+\frac{2\int \frac{1}{\sinh^{\frac{3}{2}}(a+bx)}\,dx}{3b}\\
&= -\frac{2x}{3b\sinh^{\frac{3}{2}}(a+bx)}-\frac{4\cosh(a+bx)}{3b^2\sqrt{\sinh(a+bx)}}+\frac{2\int \sqrt{\sinh(a+bx)}\,dx}{3b}\\
&= -\frac{2x}{3b\sinh^{\frac{3}{2}}(a+bx)}-\frac{4\cosh(a+bx)}{3b^2\sqrt{\sinh(a+bx)}}+\frac{\left(2\sqrt{\sinh(a+bx)}\right)\int \sqrt{i\sinh(a+bx)}\,dx}{3b\sqrt{i\sinh(a+bx)}}\\
&= -\frac{2x}{3b\sinh^{\frac{3}{2}}(a+bx)}-\frac{4\cosh(a+bx)}{3b^2\sqrt{\sinh(a+bx)}}-\frac{4iE\left(\frac{1}{2}\left(ia-\frac{\pi}{2}+ibx\right)\middle|2\right)\sqrt{\sinh(a+bx)}}{3b^2\sqrt{i\sinh(a+bx)}}
\end{aligned}$$

Mathematica [A] time = 0.21, size = 66, normalized size = 0.67

$$-\frac{2\left(\sinh(2(a+bx))+2i(i\sinh(a+bx))^{3/2}E\left(\frac{1}{4}(-2ia-2ibx+\pi)\middle|2\right)+bx\right)}{3b^2\sinh^{\frac{3}{2}}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Cosh[a + b*x])/Sinh[a + b*x]^(5/2),x]
```

```
[Out] (-2*(b*x + (2*I)*EllipticE[((-2*I)*a + Pi - (2*I)*b*x)/4, 2]*(I*Sinh[a + b*
x])^(3/2) + Sinh[2*(a + b*x)]))/(3*b^2*Sinh[a + b*x]^(3/2))
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x+a)/sinh(b*x+a)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\cosh(bx+a)}{\sinh(bx+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x+a)/sinh(b*x+a)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(x*cosh(b*x + a)/sinh(b*x + a)^(5/2), x)
```

maple [F] time = 0.11, size = 0, normalized size = 0.00

$$\int \frac{x\cosh(bx+a)}{\sinh(bx+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cosh(b*x+a)/sinh(b*x+a)^(5/2),x)
```

```
[Out] int(x*cosh(b*x+a)/sinh(b*x+a)^(5/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\cosh(bx+a)}{\sinh(bx+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x+a)/sinh(b*x+a)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(x*cosh(b*x + a)/sinh(b*x + a)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x\cosh(a+b\,x)}{{\sinh(a+b\,x)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*cosh(a + b*x))/sinh(a + b*x)^(5/2),x)
```

```
[Out] int((x*cosh(a + b*x))/sinh(a + b*x)^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\cosh(a+bx)}{\sinh^{\frac{5}{2}}(a+bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x+a)/sinh(b*x+a)**(5/2),x)
```

```
[Out] Integral(x*cosh(a + b*x)/sinh(a + b*x)**(5/2), x)
```

3.550 $\int \frac{x\cosh(a+bx)}{\sinh^{\frac{7}{2}}(a+bx)}\,dx$

Optimal. Leaf size=98

$$-\frac{4\cosh(a+bx)}{15b^2\sinh^{\frac{3}{2}}(a+bx)}+\frac{4i\sqrt{i\sinh(a+bx)}\,F\left(\frac{1}{2}\left(ia+ibx-\frac{\pi}{2}\right)\middle|2\right)}{15b^2\sqrt{\sinh(a+bx)}}-\frac{2x}{5b\sinh^{\frac{5}{2}}(a+bx)}$$

```
[Out] -2/5*x/b/sinh(b*x+a)^(5/2)-4/15*cosh(b*x+a)/b^2/sinh(b*x+a)^(3/2)-4/15*I*(s
in(1/2*I*a+1/4*Pi+1/2*I*b*x)^2)^(1/2)/sin(1/2*I*a+1/4*Pi+1/2*I*b*x)*Ellipti
cF(cos(1/2*I*a+1/4*Pi+1/2*I*b*x),2^(1/2))*(I*sinh(b*x+a))^(1/2)/b^2/sinh(b*
x+a)^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {5372, 2636, 2642, 2641}

$$-\frac{4\cosh(a+bx)}{15b^2\sinh^{\frac{3}{2}}(a+bx)}+\frac{4i\sqrt{i\sinh(a+bx)}\,F\left(\frac{1}{2}\left(ia+ibx-\frac{\pi}{2}\right)\middle|2\right)}{15b^2\sqrt{\sinh(a+bx)}}-\frac{2x}{5b\sinh^{\frac{5}{2}}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Cosh[a + b*x])/Sinh[a + b*x]^(7/2),x]
```

```
[Out] (-2*x)/(5*b*Sinh[a + b*x]^(5/2)) - (4*Cosh[a + b*x])/(15*b^2*Sinh[a + b*x]^
(3/2)) + (((4*I)/15)*EllipticF[(I*a - Pi/2 + I*b*x)/2, 2]*Sqrt[I*Sinh[a + b
*x]])/(b^2*Sqrt[Sinh[a + b*x]])
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
```

```
d}, x]
```

Rule 5372

```
Int[Cosh[(a_.) + (b_.)*(x_)^(n_.)]*(x_)^(m_.)*Sinh[(a_.) + (b_.)*(x_)^(n_.)
]^(p_.), x_Symbol] :> Simp[(x^(m - n + 1)*Sinh[a + b*x^n]^(p + 1))/(b*n*(p
+ 1)), x] - Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*Sinh[a + b*x^n]^(
p + 1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{x\cosh(a+bx)}{\sinh^{\frac{7}{2}}(a+bx)}\,dx &= -\frac{2x}{5b\sinh^{\frac{5}{2}}(a+bx)} + \frac{2\int \frac{1}{\sinh^{\frac{5}{2}}(a+bx)}\,dx}{5b} \\
&= -\frac{2x}{5b\sinh^{\frac{5}{2}}(a+bx)} - \frac{4\cosh(a+bx)}{15b^2\sinh^{\frac{3}{2}}(a+bx)} - \frac{2\int \frac{1}{\sqrt{\sinh(a+bx)}}\,dx}{15b} \\
&= -\frac{2x}{5b\sinh^{\frac{5}{2}}(a+bx)} - \frac{4\cosh(a+bx)}{15b^2\sinh^{\frac{3}{2}}(a+bx)} - \frac{\left(2\sqrt{i\sinh(a+bx)}\right)\int \frac{1}{\sqrt{i\sinh(a+bx)}}\,dx}{15b\sqrt{\sinh(a+bx)}} \\
&= -\frac{2x}{5b\sinh^{\frac{5}{2}}(a+bx)} - \frac{4\cosh(a+bx)}{15b^2\sinh^{\frac{3}{2}}(a+bx)} + \frac{4iF\left(\frac{1}{2}\left(ia-\frac{\pi}{2}+ibx\right)\middle|2\right)\sqrt{i\sinh(a+bx)}}{15b^2\sqrt{\sinh(a+bx)}}
\end{aligned}$$

Mathematica [A] time = 0.28, size = 67, normalized size = 0.68

$$-\frac{2\left(\sinh(2(a+bx)) - 2i(i\sinh(a+bx))^{5/2}F\left(\frac{1}{4}(-2ia-2ibx+\pi)\middle|2\right)+3bx\right)}{15b^2\sinh^{\frac{5}{2}}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Cosh[a + b*x])/Sinh[a + b*x]^(7/2),x]
```

```
[Out] (-2*(3*b*x - (2*I)*EllipticF[((-2*I)*a + Pi - (2*I)*b*x)/4, 2]*(I*Sinh[a +
b*x])^(5/2) + Sinh[2*(a + b*x)]))/(15*b^2*Sinh[a + b*x]^(5/2))
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x+a)/sinh(b*x+a)^(7/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\cosh(bx+a)}{\sinh(bx+a)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x+a)/sinh(b*x+a)^(7/2),x, algorithm="giac")
```

```
[Out] integrate(x*cosh(b*x + a)/sinh(b*x + a)^(7/2), x)
```

maple [F] time = 0.12, size = 0, normalized size = 0.00

$$\int \frac{x\cosh(bx+a)}{\sinh(bx+a)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cosh(b*x+a)/sinh(b*x+a)^(7/2),x)
```

```
[Out] int(x*cosh(b*x+a)/sinh(b*x+a)^(7/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\cosh(bx+a)}{\sinh(bx+a)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x+a)/sinh(b*x+a)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate(x*cosh(b*x + a)/sinh(b*x + a)^(7/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x\cosh(a+b\,x)}{{\sinh(a+b\,x)}^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*cosh(a + b*x))/sinh(a + b*x)^(7/2),x)
```

```
[Out] int((x*cosh(a + b*x))/sinh(a + b*x)^(7/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x+a)/sinh(b*x+a)**(7/2),x)
```

```
[Out] Timed out
```

3.551 $\int \frac{x \cosh(a+bx)}{\sinh^{\frac{9}{2}}(a+bx)} \, dx$

Optimal. Leaf size=121

$$-\frac{4\cosh(a+bx)}{35b^2\sinh^{\frac{5}{2}}(a+bx)}+\frac{12\cosh(a+bx)}{35b^2\sqrt{\sinh(a+bx)}}+\frac{12i\sqrt{\sinh(a+bx)}\,E\left(\frac{1}{2}\left(ia+ibx-\frac{\pi}{2}\right)\middle|2\right)}{35b^2\sqrt{i\sinh(a+bx)}}-\frac{2x}{7b\sinh^{\frac{7}{2}}(a+bx)}$$

```
[Out] -2/7*x/b/sinh(b*x+a)^(7/2)-4/35*cosh(b*x+a)/b^2/sinh(b*x+a)^(5/2)+12/35*cos
h(b*x+a)/b^2/sinh(b*x+a)^(1/2)-12/35*I*(sin(1/2*I*a+1/4*Pi+1/2*I*b*x)^2)^(1
/2)/sin(1/2*I*a+1/4*Pi+1/2*I*b*x)*EllipticE(cos(1/2*I*a+1/4*Pi+1/2*I*b*x),2
^(1/2))*sinh(b*x+a)^(1/2)/b^2/(I*sinh(b*x+a))^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {5372, 2636, 2640, 2639}

$$-\frac{4\cosh(a+bx)}{35b^2\sinh^{\frac{5}{2}}(a+bx)}+\frac{12\cosh(a+bx)}{35b^2\sqrt{\sinh(a+bx)}}+\frac{12i\sqrt{\sinh(a+bx)}\,E\left(\frac{1}{2}\left(ia+ibx-\frac{\pi}{2}\right)\middle|2\right)}{35b^2\sqrt{i\sinh(a+bx)}}-\frac{2x}{7b\sinh^{\frac{7}{2}}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Cosh[a + b*x])/Sinh[a + b*x]^(9/2),x]
```

```
[Out] (-2*x)/(7*b*Sinh[a + b*x]^(7/2)) - (4*Cosh[a + b*x])/(35*b^2*Sinh[a + b*x]^
(5/2)) + (12*Cosh[a + b*x])/(35*b^2*Sqrt[Sinh[a + b*x]]) + (((12*I)/35)*Ell
ipticE[(I*a - Pi/2 + I*b*x)/2, 2]*Sqrt[Sinh[a + b*x]])/(b^2*Sqrt[I*Sinh[a +
 b*x]])
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 5372

```
Int[Cosh[(a_.) + (b_.)*(x_)^(n_.)]*(x_)^(m_.)*Sinh[(a_.) + (b_.)*(x_)^(n_.)
]^(p_.), x_Symbol] :> Simp[(x^(m - n + 1)*Sinh[a + b*x^n]^(p + 1))/(b*n*(p
+ 1)), x] - Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*Sinh[a + b*x^n]^(
p + 1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{x\cosh(a+bx)}{\sinh^{\frac{9}{2}}(a+bx)}\,dx = -\frac{2x}{7b\sinh^{\frac{7}{2}}(a+bx)} + \frac{2\int \frac{1}{\sinh^{\frac{7}{2}}(a+bx)}\,dx}{7b}$$

$$= -\frac{2x}{7b\sinh^{\frac{7}{2}}(a+bx)} - \frac{4\cosh(a+bx)}{35b^2\sinh^{\frac{5}{2}}(a+bx)} - \frac{6\int \frac{1}{\sinh^{\frac{3}{2}}(a+bx)}\,dx}{35b}$$

$$= -\frac{2x}{7b\sinh^{\frac{7}{2}}(a+bx)} - \frac{4\cosh(a+bx)}{35b^2\sinh^{\frac{5}{2}}(a+bx)} + \frac{12\cosh(a+bx)}{35b^2\sqrt{\sinh(a+bx)}} - \frac{6\int \sqrt{\sinh(a+bx)}\,d}{35b}$$

$$= -\frac{2x}{7b\sinh^{\frac{7}{2}}(a+bx)} - \frac{4\cosh(a+bx)}{35b^2\sinh^{\frac{5}{2}}(a+bx)} + \frac{12\cosh(a+bx)}{35b^2\sqrt{\sinh(a+bx)}} - \frac{\left(6\sqrt{\sinh(a+bx)}\right)\int}{35b\sqrt{i\sinh}}$$

$$= -\frac{2x}{7b\sinh^{\frac{7}{2}}(a+bx)} - \frac{4\cosh(a+bx)}{35b^2\sinh^{\frac{5}{2}}(a+bx)} + \frac{12\cosh(a+bx)}{35b^2\sqrt{\sinh(a+bx)}} + \frac{12iE\left(\frac{1}{2}\left(ia-\frac{\pi}{2}+ib\right.\right.}{35b^2\sqrt{i\sin}}$$

Mathematica [A] time = 0.28, size = 89, normalized size = 0.74

$$-\frac{2\left(\sinh(2(a+bx)) - 6\sinh^3(a+bx)\cosh(a+bx) + 6\sqrt{i\sinh(a+bx)}\,\sinh^3(a+bx)E\left(\frac{1}{4}(-2ia-2ibx+\pi)\middle|2\right)\right.}{35b^2\sinh^{\frac{7}{2}}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Cosh[a + b*x])/Sinh[a + b*x]^(9/2),x]
```

```
[Out] (-2*(5*b*x - 6*Cosh[a + b*x]*Sinh[a + b*x]^3 + 6*EllipticE[((-2*I)*a + Pi -
 (2*I)*b*x)/4, 2]*Sqrt[I*Sinh[a + b*x]]*Sinh[a + b*x]^3 + Sinh[2*(a + b*x)]
))/(35*b^2*Sinh[a + b*x]^(7/2))
```

fricas [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x+a)/sinh(b*x+a)^(9/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\cosh\left(bx+a\right)}{\sinh\left(bx+a\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x+a)/sinh(b*x+a)^(9/2),x, algorithm="giac")
```

```
[Out] integrate(x*cosh(b*x + a)/sinh(b*x + a)^(9/2), x)
```

maple [**F**] time = 0.11, size = 0, normalized size = 0.00

$$\int \frac{x\cosh\left(bx+a\right)}{\sinh\left(bx+a\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cosh(b*x+a)/sinh(b*x+a)^(9/2),x)
```

```
[Out] int(x*cosh(b*x+a)/sinh(b*x+a)^(9/2),x)
```

maxima [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\cosh\left(bx+a\right)}{\sinh\left(bx+a\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x+a)/sinh(b*x+a)^(9/2),x, algorithm="maxima")
```

```
[Out] integrate(x*cosh(b*x + a)/sinh(b*x + a)^(9/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x \cosh(a + b x)}{\sinh(a + b x)^{9/2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*cosh(a + b*x))/sinh(a + b*x)^(9/2),x)
[Out] int((x*cosh(a + b*x))/sinh(a + b*x)^(9/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x+a)/sinh(b*x+a)**(9/2),x)
[Out] Timed out
```

3.552 $\int x\cosh(a+bx)\operatorname{csch}^{\frac{9}{2}}(a+bx)\,dx$

Optimal. Leaf size=121

$$-\frac{4\cosh(a+bx)\operatorname{csch}^{\frac{5}{2}}(a+bx)}{35b^2}+\frac{12\cosh(a+bx)\sqrt{\operatorname{csch}(a+bx)}}{35b^2}+\frac{12iE\left(\frac{1}{2}\left(ia+ibx-\frac{\pi}{2}\right)\middle|2\right)}{35b^2\sqrt{i\sinh(a+bx)}\sqrt{\operatorname{csch}(a+bx)}}-\frac{2x\operatorname{csch}^{\frac{7}{2}}(}{7b}$$

```
[Out] -4/35*cosh(b*x+a)*csch(b*x+a)^(5/2)/b^2-2/7*x*csch(b*x+a)^(7/2)/b+12/35*cos
h(b*x+a)*csch(b*x+a)^(1/2)/b^2-12/35*I*(sin(1/2*I*a+1/4*Pi+1/2*I*b*x)^2)^(1
/2)/sin(1/2*I*a+1/4*Pi+1/2*I*b*x)*EllipticE(cos(1/2*I*a+1/4*Pi+1/2*I*b*x),2
^(1/2))/b^2/csch(b*x+a)^(1/2)/(I*sinh(b*x+a))^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {5445, 3768, 3771, 2639}

$$-\frac{4\cosh(a+bx)\operatorname{csch}^{\frac{5}{2}}(a+bx)}{35b^2}+\frac{12\cosh(a+bx)\sqrt{\operatorname{csch}(a+bx)}}{35b^2}+\frac{12iE\left(\frac{1}{2}\left(ia+ibx-\frac{\pi}{2}\right)\middle|2\right)}{35b^2\sqrt{i\sinh(a+bx)}\sqrt{\operatorname{csch}(a+bx)}}-\frac{2x\operatorname{csch}^{\frac{7}{2}}(}{7b}$$

Antiderivative was successfully verified.

```
[In] Int[x*Cosh[a + b*x]*Csch[a + b*x]^(9/2),x]
```

```
[Out] (12*Cosh[a + b*x]*Sqrt[Csch[a + b*x]])/(35*b^2) - (4*Cosh[a + b*x]*Csch[a +
 b*x]^(5/2))/(35*b^2) - (2*x*Csch[a + b*x]^(7/2))/(7*b) + (((12*I)/35)*Elli
pticE[(I*a - Pi/2 + I*b*x)/2, 2])/(b^2*Sqrt[Csch[a + b*x]]*Sqrt[I*Sinh[a +
b*x]])
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
```

```
EqQ[n^2, 1/4]
```

Rule 5445

```
Int[Cosh[(a_.) + (b_.)*(x_)^(n_.)]*Csch[(a_.) + (b_.)*(x_)^(n_.)]^(p_)*(x_)
^(m_.), x_Symbol] :> -Simp[(x^(m - n + 1)*Csch[a + b*x^n]^(p - 1))/(b*n*(p
- 1)), x] + Dist[(m - n + 1)/(b*n*(p - 1)), Int[x^(m - n)*Csch[a + b*x^n]^(
p - 1), x], x] /; FreeQ[{a, b, p}, x] && IntegerQ[n] && GeQ[m - n, 0] && Ne
Q[p, 1]
```

Rubi steps

$$\int x\cosh(a+bx)\operatorname{csch}^{\frac{9}{2}}(a+bx)\,dx = -\frac{2x\operatorname{csch}^{\frac{7}{2}}(a+bx)}{7b} + \frac{2\int\operatorname{csch}^{\frac{7}{2}}(a+bx)\,dx}{7b}$$
$$= -\frac{4\cosh(a+bx)\operatorname{csch}^{\frac{5}{2}}(a+bx)}{35b^2} - \frac{2x\operatorname{csch}^{\frac{7}{2}}(a+bx)}{7b} - \frac{6\int\operatorname{csch}^{\frac{3}{2}}(a+bx)\,dx}{35b}$$
$$= \frac{12\cosh(a+bx)\sqrt{\operatorname{csch}(a+bx)}}{35b^2} - \frac{4\cosh(a+bx)\operatorname{csch}^{\frac{5}{2}}(a+bx)}{35b^2} - \frac{2x\operatorname{csch}}{7}$$
$$= \frac{12\cosh(a+bx)\sqrt{\operatorname{csch}(a+bx)}}{35b^2} - \frac{4\cosh(a+bx)\operatorname{csch}^{\frac{5}{2}}(a+bx)}{35b^2} - \frac{2x\operatorname{csch}}{7}$$
$$= \frac{12\cosh(a+bx)\sqrt{\operatorname{csch}(a+bx)}}{35b^2} - \frac{4\cosh(a+bx)\operatorname{csch}^{\frac{5}{2}}(a+bx)}{35b^2} - \frac{2x\operatorname{csch}}{7}$$

Mathematica [A] time = 0.53, size = 83, normalized size = 0.69

$$-\frac{2\sqrt{\operatorname{csch}(a+bx)}\left(-6\cosh(a+bx) + (\sinh(2(a+bx)) + 5bx)\operatorname{csch}^3(a+bx) + 6\sqrt{i\sinh(a+bx)}\,E\left(\frac{1}{4}(-2ia-2ib\right.\right.}{35b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Cosh[a + b*x]*Csch[a + b*x]^(9/2),x]
```

```
[Out] (-2*Sqrt[Csch[a + b*x]]*(-6*Cosh[a + b*x] + 6*EllipticE[((-2*I)*a + Pi - (2
*I)*b*x)/4, 2]*Sqrt[I*Sinh[a + b*x]] + Csch[a + b*x]^3*(5*b*x + Sinh[2*(a +
 b*x)])))/(35*b^2)
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x+a)*csch(b*x+a)^(9/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x \cosh(bx+a) \operatorname{csch}(bx+a)^{\frac{9}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x+a)*csch(b*x+a)^(9/2),x, algorithm="giac")
```

```
[Out] integrate(x*cosh(b*x + a)*csch(b*x + a)^(9/2), x)
```

maple [F] time = 0.24, size = 0, normalized size = 0.00

$$\int x \cosh(bx+a) \operatorname{csch}(bx+a)^{\frac{9}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cosh(b*x+a)*csch(b*x+a)^(9/2),x)
```

```
[Out] int(x*cosh(b*x+a)*csch(b*x+a)^(9/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x \cosh(bx+a) \operatorname{csch}(bx+a)^{\frac{9}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x+a)*csch(b*x+a)^(9/2),x, algorithm="maxima")
```

```
[Out] integrate(x*cosh(b*x + a)*csch(b*x + a)^(9/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x \cosh(a+b\,x)\,\left(\frac{1}{\sinh(a+b\,x)}\right)^{9/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cosh(a + b*x)*(1/sinh(a + b*x))^(9/2),x)
```

```
[Out] int(x*cosh(a + b*x)*(1/sinh(a + b*x))^(9/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x+a)*csch(b*x+a)**(9/2),x)
```

```
[Out] Timed out
```

3.553 $\int x\cosh(a+bx)\mathbf{csch}^{\frac{7}{2}}(a+bx)\,dx$

Optimal. Leaf size=98

$$-\frac{4\cosh(a+bx)\text{csch}^{\frac{3}{2}}(a+bx)}{15b^2}+\frac{4i\sqrt{i\sinh(a+bx)}\,\sqrt{\text{csch}(a+bx)}\,F\left(\frac{1}{2}\left(ia+ibx-\frac{\pi}{2}\right)\middle|2\right)}{15b^2}-\frac{2x\text{csch}^{\frac{5}{2}}(a+bx)}{5b}$$

```
[Out] -4/15*cosh(b*x+a)*csch(b*x+a)^(3/2)/b^2-2/5*x*csch(b*x+a)^(5/2)/b-4/15*I*(s
in(1/2*I*a+1/4*Pi+1/2*I*b*x)^2)^(1/2)/sin(1/2*I*a+1/4*Pi+1/2*I*b*x)*Ellipti
cF(cos(1/2*I*a+1/4*Pi+1/2*I*b*x),2^(1/2))*csch(b*x+a)^(1/2)*(I*sinh(b*x+a))
^(1/2)/b^2
```

Rubi [A] time = 0.05, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {5445, 3768, 3771, 2641}

$$-\frac{4\cosh(a+bx)\text{csch}^{\frac{3}{2}}(a+bx)}{15b^2}+\frac{4i\sqrt{i\sinh(a+bx)}\,\sqrt{\text{csch}(a+bx)}\,F\left(\frac{1}{2}\left(ia+ibx-\frac{\pi}{2}\right)\middle|2\right)}{15b^2}-\frac{2x\text{csch}^{\frac{5}{2}}(a+bx)}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[x*Cosh[a + b*x]*Csch[a + b*x]^(7/2),x]
```

```
[Out] (-4*Cosh[a + b*x]*Csch[a + b*x]^(3/2))/(15*b^2) - (2*x*Csch[a + b*x]^(5/2))
/(5*b) + (((4*I)/15)*Sqrt[Csch[a + b*x]]*EllipticF[(I*a - Pi/2 + I*b*x)/2,
2]*Sqrt[I*Sinh[a + b*x]])/b^2
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 5445

```
Int[Cosh[(a_.) + (b_.)*(x_)^(n_.)]*Csch[(a_.) + (b_.)*(x_)^(n_.)]^(p_)*(x_)
^(m_.), x_Symbol] :> -Simp[(x^(m - n + 1)*Csch[a + b*x^n]^(p - 1))/(b*n*(p
- 1)), x] + Dist[(m - n + 1)/(b*n*(p - 1)), Int[x^(m - n)*Csch[a + b*x^n]^(
p - 1), x], x] /; FreeQ[{a, b, p}, x] && IntegerQ[n] && GeQ[m - n, 0] && Ne
Q[p, 1]
```

Rubi steps

$$\int x\cosh(a+bx)\operatorname{csch}^{\frac{7}{2}}(a+bx)\,dx = -\frac{2x\operatorname{csch}^{\frac{5}{2}}(a+bx)}{5b}+\frac{2\int\operatorname{csch}^{\frac{5}{2}}(a+bx)\,dx}{5b}$$

$$= -\frac{4\cosh(a+bx)\operatorname{csch}^{\frac{3}{2}}(a+bx)}{15b^2}-\frac{2x\operatorname{csch}^{\frac{5}{2}}(a+bx)}{5b}-\frac{2\int\sqrt{\operatorname{csch}(a+bx)}\,d}{15b}$$

$$= -\frac{4\cosh(a+bx)\operatorname{csch}^{\frac{3}{2}}(a+bx)}{15b^2}-\frac{2x\operatorname{csch}^{\frac{5}{2}}(a+bx)}{5b}-\frac{\left(2\sqrt{\operatorname{csch}(a+bx)}\sqrt{i}\right.}{}$$

$$= -\frac{4\cosh(a+bx)\operatorname{csch}^{\frac{3}{2}}(a+bx)}{15b^2}-\frac{2x\operatorname{csch}^{\frac{5}{2}}(a+bx)}{5b}+\frac{4i\sqrt{\operatorname{csch}(a+bx)}\,F\left(\right.}{}$$

Mathematica [A] time = 0.30, size = 75, normalized size = 0.77

$$-\frac{2\sqrt{\operatorname{csch}(a+bx)}\left(2\coth(a+bx)+3bx\operatorname{csch}^2(a+bx)+2i\sqrt{i\sinh(a+bx)}\,F\left(\frac{1}{4}(-2ia-2ibx+\pi)\middle|2\right)\right)}{15b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Cosh[a + b*x]*Csch[a + b*x]^(7/2),x]
```

```
[Out] (-2*Sqrt[Csch[a + b*x]]*(2*Coth[a + b*x] + 3*b*x*Csch[a + b*x]^2 + (2*I)*El
lipticF[((-2*I)*a + Pi - (2*I)*b*x)/4, 2]*Sqrt[I*Sinh[a + b*x]]))/(15*b^2)
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x+a)*csch(b*x+a)^(7/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\cosh(bx+a)\operatorname{csch}(bx+a)^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x+a)*csch(b*x+a)^(7/2),x, algorithm="giac")
[Out] integrate(x*cosh(b*x + a)*csch(b*x + a)^(7/2), x)
```

maple [F] time = 0.21, size = 0, normalized size = 0.00

$$\int x\cosh(bx+a)\operatorname{csch}(bx+a)^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cosh(b*x+a)*csch(b*x+a)^(7/2),x)
[Out] int(x*cosh(b*x+a)*csch(b*x+a)^(7/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\cosh(bx+a)\operatorname{csch}(bx+a)^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x+a)*csch(b*x+a)^(7/2),x, algorithm="maxima")
[Out] integrate(x*cosh(b*x + a)*csch(b*x + a)^(7/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x\cosh(a+b\,x)\left(\frac{1}{\sinh(a+b\,x)}\right)^{7/2}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cosh(a + b*x)*(1/sinh(a + b*x))^(7/2),x)
[Out] int(x*cosh(a + b*x)*(1/sinh(a + b*x))^(7/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x+a)*csch(b*x+a)**(7/2),x)
[Out] Timed out
```

3.554 $\int x \cosh(a+bx)\mathbf{csch}^{\frac{5}{2}}(a+bx)\,dx$

Optimal. Leaf size=98

$$-\frac{4\cosh(a+bx)\sqrt{\operatorname{csch}(a+bx)}}{3b^2}-\frac{4iE\left(\frac{1}{2}\left(ia+ibx-\frac{\pi}{2}\right)\middle|2\right)}{3b^2\sqrt{i\sinh(a+bx)}\sqrt{\operatorname{csch}(a+bx)}}-\frac{2x\operatorname{csch}^{\frac{3}{2}}(a+bx)}{3b}$$

```
[Out] -2/3*x*csch(b*x+a)^(3/2)/b-4/3*cosh(b*x+a)*csch(b*x+a)^(1/2)/b^2+4/3*I*(sin
(1/2*I*a+1/4*Pi+1/2*I*b*x)^2)^(1/2)/sin(1/2*I*a+1/4*Pi+1/2*I*b*x)*EllipticE
(cos(1/2*I*a+1/4*Pi+1/2*I*b*x),2^(1/2))/b^2/csch(b*x+a)^(1/2)/(I*sinh(b*x+a
))^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {5445, 3768, 3771, 2639}

$$-\frac{4\cosh(a+bx)\sqrt{\operatorname{csch}(a+bx)}}{3b^2}-\frac{4iE\left(\frac{1}{2}\left(ia+ibx-\frac{\pi}{2}\right)\middle|2\right)}{3b^2\sqrt{i\sinh(a+bx)}\sqrt{\operatorname{csch}(a+bx)}}-\frac{2x\operatorname{csch}^{\frac{3}{2}}(a+bx)}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[x*Cosh[a + b*x]*Csch[a + b*x]^(5/2),x]
```

```
[Out] (-4*Cosh[a + b*x]*Sqrt[Csch[a + b*x]])/(3*b^2) - (2*x*Csch[a + b*x]^(3/2))/
(3*b) - (((4*I)/3)*EllipticE[(I*a - Pi/2 + I*b*x)/2, 2])/(b^2*Sqrt[Csch[a +
 b*x]]*Sqrt[I*Sinh[a + b*x]])
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
```

```
EqQ[n^2, 1/4]
```

Rule 5445

```
Int[Cosh[(a_.) + (b_.)*(x_)^(n_.)]*Csch[(a_.) + (b_.)*(x_)^(n_.)]^(p_)*(x_)
^(m_.), x_Symbol] :> -Simp[(x^(m - n + 1)*Csch[a + b*x^n]^(p - 1))/(b*n*(p
 - 1)), x] + Dist[(m - n + 1)/(b*n*(p - 1)), Int[x^(m - n)*Csch[a + b*x^n]^(
p - 1), x], x] /; FreeQ[{a, b, p}, x] && IntegerQ[n] && GeQ[m - n, 0] && Ne
Q[p, 1]
```

Rubi steps

$$\int x\cosh(a+bx)\text{csch}^{\frac{5}{2}}(a+bx)\,dx = -\frac{2x\text{csch}^{\frac{3}{2}}(a+bx)}{3b}+\frac{2\int \text{csch}^{\frac{3}{2}}(a+bx)\,dx}{3b}$$
$$= -\frac{4\cosh(a+bx)\sqrt{\text{csch}(a+bx)}}{3b^2}-\frac{2x\text{csch}^{\frac{3}{2}}(a+bx)}{3b}+\frac{2\int \frac{1}{\sqrt{\text{csch}(a+bx)}}\,dx}{3b}$$
$$= -\frac{4\cosh(a+bx)\sqrt{\text{csch}(a+bx)}}{3b^2}-\frac{2x\text{csch}^{\frac{3}{2}}(a+bx)}{3b}+\frac{2\int \sqrt{i\sinh(a+}}{3b\sqrt{\text{csch}(a+bx)}\sqrt{i}}$$
$$= -\frac{4\cosh(a+bx)\sqrt{\text{csch}(a+bx)}}{3b^2}-\frac{2x\text{csch}^{\frac{3}{2}}(a+bx)}{3b}-\frac{4iE\left(\frac{1}{2}\left(ia-\frac{\pi}{2}+\right.\right.}{3b^2\sqrt{\text{csch}(a+bx)}\sqrt{i}}$$

Mathematica [A] time = 0.22, size = 70, normalized size = 0.71

$$-\frac{2\sqrt{\text{csch}(a+bx)}\left(2\cosh(a+bx)+bx\text{csch}(a+bx)-2\sqrt{i\sinh(a+bx)}E\left(\frac{1}{4}(-2ia-2ibx+\pi)\middle|2\right)\right)}{3b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Cosh[a + b*x]*Csch[a + b*x]^(5/2),x]
```

```
[Out] (-2*Sqrt[Csch[a + b*x]]*(2*Cosh[a + b*x] + b*x*Csch[a + b*x] - 2*EllipticE[
((-2*I)*a + Pi - (2*I)*b*x)/4, 2]*Sqrt[I*Sinh[a + b*x]]))/(3*b^2)
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x+a)*csch(b*x+a)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x \cosh{(bx+a)} \operatorname{csch}{(bx+a)}^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x+a)*csch(b*x+a)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(x*cosh(b*x + a)*csch(b*x + a)^(5/2), x)
```

maple [F] time = 0.21, size = 0, normalized size = 0.00

$$\int x \cosh{(bx+a)} \operatorname{csch}{(bx+a)}^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cosh(b*x+a)*csch(b*x+a)^(5/2),x)
```

```
[Out] int(x*cosh(b*x+a)*csch(b*x+a)^(5/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x \cosh{(bx+a)} \operatorname{csch}{(bx+a)}^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x+a)*csch(b*x+a)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(x*cosh(b*x + a)*csch(b*x + a)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x \cosh{(a+b\,x)} \left(\frac{1}{\sinh{(a+b\,x)}}\right)^{5/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cosh(a + b*x)*(1/sinh(a + b*x))^(5/2),x)
```

```
[Out] int(x*cosh(a + b*x)*(1/sinh(a + b*x))^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x+a)*csch(b*x+a)**(5/2),x)
```

```
[Out] Timed out
```

3.555 $\int x\cosh(a+bx)\mathbf{csch}^{\frac{3}{2}}(a+bx)\,dx$

Optimal. Leaf size=71

$$-\frac{2x\sqrt{\operatorname{csch}(a+bx)}}{b}-\frac{4i\sqrt{i\sinh(a+bx)}\,\sqrt{\operatorname{csch}(a+bx)}\,F\left(\frac{1}{2}\left(ia+ibx-\frac{\pi}{2}\right)\middle|2\right)}{b^2}$$

```
[Out] -2*x*csch(b*x+a)^(1/2)/b+4*I*(sin(1/2*I*a+1/4*Pi+1/2*I*b*x)^2)^(1/2)/sin(1/
2*I*a+1/4*Pi+1/2*I*b*x)*EllipticF(cos(1/2*I*a+1/4*Pi+1/2*I*b*x),2^(1/2))*cs
ch(b*x+a)^(1/2)*(I*sinh(b*x+a))^(1/2)/b^2
```

Rubi [A] time = 0.04, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {5445, 3771, 2641}

$$-\frac{2x\sqrt{\operatorname{csch}(a+bx)}}{b}-\frac{4i\sqrt{i\sinh(a+bx)}\,\sqrt{\operatorname{csch}(a+bx)}\,F\left(\frac{1}{2}\left(ia+ibx-\frac{\pi}{2}\right)\middle|2\right)}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x*Cosh[a + b*x]*Csch[a + b*x]^(3/2),x]
```

```
[Out] (-2*x*Sqrt[Csch[a + b*x]])/b - ((4*I)*Sqrt[Csch[a + b*x]]*EllipticF[(I*a -
Pi/2 + I*b*x)/2, 2]*Sqrt[I*Sinh[a + b*x]])/b^2
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 5445

```
Int[Cosh[(a_.) + (b_.)*(x_)^(n_.)]*Csch[(a_.) + (b_.)*(x_)^(n_.)]^(p_)*(x_)
^(m_.), x_Symbol] :> -Simp[(x^(m - n + 1)*Csch[a + b*x^n]^(p - 1))/(b*n*(p
- 1)), x] + Dist[(m - n + 1)/(b*n*(p - 1)), Int[x^(m - n)*Csch[a + b*x^n]^(
p - 1), x], x] /; FreeQ[{a, b, p}, x] && IntegerQ[n] && GeQ[m - n, 0] && Ne
Q[p, 1]
```

Rubi steps

$$\int x\cosh(a+bx)\text{csch}^{\frac{3}{2}}(a+bx)\,dx = -\frac{2x\sqrt{\text{csch}(a+bx)}}{b}+\frac{2\int\sqrt{\text{csch}(a+bx)}\,dx}{b}$$
$$= -\frac{2x\sqrt{\text{csch}(a+bx)}}{b}+\frac{\left(2\sqrt{\text{csch}(a+bx)}\sqrt{i\sinh(a+bx)}\right)\int\frac{1}{\sqrt{i\sinh(a+bx)}}\,dx}{b}$$
$$= -\frac{2x\sqrt{\text{csch}(a+bx)}}{b}-\frac{4i\sqrt{\text{csch}(a+bx)}\,F\left(\frac{1}{2}\left(ia-\frac{\pi}{2}+ibx\right)\middle|2\right)\sqrt{i\sinh(a+b}}{b^2}$$

Mathematica [A] time = 0.15, size = 56, normalized size = 0.79

$$-\frac{2\sqrt{\text{csch}(a+bx)}\left(bx-2i\sqrt{i\sinh(a+bx)}\,F\left(\frac{1}{4}(-2ia-2ibx+\pi)\middle|2\right)\right)}{b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Cosh[a + b*x]*Csch[a + b*x]^(3/2),x]
```

```
[Out] (-2*Sqrt[Csch[a + b*x]]*(b*x - (2*I)*EllipticF[((-2*I)*a + Pi - (2*I)*b*x)/
4, 2]*Sqrt[I*Sinh[a + b*x]]))/b^2
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x+a)*csch(b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\cosh(bx+a)\,\text{csch}(bx+a)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x+a)*csch(b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x*cosh(b*x + a)*csch(b*x + a)^(3/2), x)
```

maple [F] time = 0.21, size = 0, normalized size = 0.00

$$\int x \cosh(bx+a) \operatorname{csch}(bx+a)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cosh(b*x+a)*csch(b*x+a)^(3/2),x)
[Out] int(x*cosh(b*x+a)*csch(b*x+a)^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x \cosh(bx+a) \operatorname{csch}(bx+a)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x+a)*csch(b*x+a)^(3/2),x, algorithm="maxima")
[Out] integrate(x*cosh(b*x + a)*csch(b*x + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x \cosh(a+b\,x) \left(\frac{1}{\sinh(a+b\,x)}\right)^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cosh(a + b*x)*(1/sinh(a + b*x))^(3/2),x)
[Out] int(x*cosh(a + b*x)*(1/sinh(a + b*x))^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x+a)*csch(b*x+a)**(3/2),x)
[Out] Timed out
```

3.556 $\int x \cosh(a+bx)\sqrt{\operatorname{csch}(a+bx)}\,dx$

Optimal. Leaf size=71

$$\frac{2x}{b\sqrt{\operatorname{csch}(a+bx)}}+\frac{4iE\left(\frac{1}{2}\left(ia+ibx-\frac{\pi}{2}\right)\middle|2\right)}{b^2\sqrt{i\sinh(a+bx)}\sqrt{\operatorname{csch}(a+bx)}}$$

```
[Out] 2*x/b/csch(b*x+a)^(1/2)-4*I*(sin(1/2*I*a+1/4*Pi+1/2*I*b*x)^2)^(1/2)/sin(1/2
*I*a+1/4*Pi+1/2*I*b*x)*EllipticE(cos(1/2*I*a+1/4*Pi+1/2*I*b*x),2^(1/2))/b^2
/csch(b*x+a)^(1/2)/(I*sinh(b*x+a))^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {5445, 3771, 2639}

$$\frac{2x}{b\sqrt{\operatorname{csch}(a+bx)}}+\frac{4iE\left(\frac{1}{2}\left(ia+ibx-\frac{\pi}{2}\right)\middle|2\right)}{b^2\sqrt{i\sinh(a+bx)}\sqrt{\operatorname{csch}(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[x*Cosh[a + b*x]*Sqrt[Csch[a + b*x]],x]
```

```
[Out] (2*x)/(b*Sqrt[Csch[a + b*x]]) + ((4*I)*EllipticE[(I*a - Pi/2 + I*b*x)/2, 2]
)/(b^2*Sqrt[Csch[a + b*x]]*Sqrt[I*Sinh[a + b*x]])
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 5445

```
Int[Cosh[(a_.) + (b_.)*(x_)^(n_.)]*Csch[(a_.) + (b_.)*(x_)^(n_.)]^(p_)*(x_)
^(m_.), x_Symbol] :> -Simp[(x^(m - n + 1)*Csch[a + b*x^n]^(p - 1))/(b*n*(p
- 1)), x] + Dist[(m - n + 1)/(b*n*(p - 1)), Int[x^(m - n)*Csch[a + b*x^n]^(
p - 1), x], x] /; FreeQ[{a, b, p}, x] && IntegerQ[n] && GeQ[m - n, 0] && Ne
Q[p, 1]
```

Rubi steps

$$\int x\cosh(a+bx)\sqrt{\operatorname{csch}(a+bx)}\,dx=\frac{2x}{b\sqrt{\operatorname{csch}(a+bx)}}-\frac{2\int\frac{1}{\sqrt{\operatorname{csch}(a+bx)}}\,dx}{b}$$
$$=\frac{2x}{b\sqrt{\operatorname{csch}(a+bx)}}-\frac{2\int\sqrt{i\sinh(a+bx)}\,dx}{b\sqrt{\operatorname{csch}(a+bx)}\sqrt{i\sinh(a+bx)}}$$
$$=\frac{2x}{b\sqrt{\operatorname{csch}(a+bx)}}+\frac{4iE\left(\frac{1}{2}\left(ia-\frac{\pi}{2}+ibx\right)\middle|2\right)}{b^2\sqrt{\operatorname{csch}(a+bx)}\sqrt{i\sinh(a+bx)}}$$

Mathematica [C] time = 1.10, size = 183, normalized size = 2.58

$$\frac{e^{-a-bx}\sqrt{2-2e^{2(a+bx)}}\sqrt{\frac{e^{a+bx}}{e^{2(a+bx)}-1}}\left(-18\,{}_3F_2\left(-\frac{1}{4},-\frac{1}{4},\frac{1}{2};\frac{3}{4},\frac{3}{4};e^{2(a+bx)}\right)-2e^{2(a+bx)}\,{}_3F_2\left(\frac{1}{2},\frac{3}{4},\frac{3}{4};\frac{7}{4},\frac{7}{4};e^{2(a+bx)}\right)-3bx\left(3\right.\right.}{9b^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x*Cosh[a + b*x]*Sqrt[Csch[a + b*x]],x]
```

```
[Out] (E^(-a - b*x)*Sqrt[2 - 2*E^(2*(a + b*x))]*Sqrt[E^(a + b*x)/(-1 + E^(2*(a +
b*x)))]*(-3*b*x*(3*Hypergeometric2F1[-1/4, 1/2, 3/4, E^(2*(a + b*x))] - E^(
2*(a + b*x))*Hypergeometric2F1[1/2, 3/4, 7/4, E^(2*(a + b*x))]) - 18*Hyperg
eometricPFQ[{-1/4, -1/4, 1/2}, {3/4, 3/4}, E^(2*(a + b*x))] - 2*E^(2*(a + b
*x))*HypergeometricPFQ[{1/2, 3/4, 3/4}, {7/4, 7/4}, E^(2*(a + b*x))]))/(9*b
^2)
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x+a)*csch(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\cosh\left(bx+a\right)\sqrt{\operatorname{csch}\left(bx+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x+a)*csch(b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x*cosh(b*x + a)*sqrt(csch(b*x + a)), x)
```

maple [B] time = 0.24, size = 229, normalized size = 3.23

$$\frac{(bx-2)\left(e^{2bx+2a}-1\right)\sqrt{2}\,\sqrt{\frac{e^{bx+a}}{e^{2bx+2a}-1}}\;e^{-bx-a}}{b^{2}}+\frac{2\left(\frac{2\,e^{2bx+2a}-2}{\sqrt{\left(e^{2bx+2a}-1\right)e^{bx+a}}}-\frac{\sqrt{1+e^{bx+a}}\;\sqrt{2-2\,e^{bx+a}}\;\sqrt{-e^{bx+a}}\;\left(-2\,\mathrm{EllipticE}\left(\sqrt{1+e^{bx+a}},\frac{\sqrt{2}}{2}\right.\right.}{\sqrt{e^{3bx+3a}-e^{bx+a}}}\right.}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cosh(b*x+a)*csch(b*x+a)^(1/2),x)
```

```
[Out] (b*x-2)*(exp(b*x+a)^2-1)/b^2*2^(1/2)*(exp(b*x+a)/(exp(b*x+a)^2-1))^(1/2)/ex
p(b*x+a)+2/b^2*(2*(exp(b*x+a)^2-1)/((exp(b*x+a)^2-1)*exp(b*x+a))^(1/2)-(1+e
xp(b*x+a))^(1/2)*(2-2*exp(b*x+a))^(1/2)*(-exp(b*x+a))^(1/2)/(exp(b*x+a)^3-e
xp(b*x+a))^(1/2)*(-2*EllipticE((1+exp(b*x+a))^(1/2),1/2*2^(1/2))+EllipticF(
(1+exp(b*x+a))^(1/2),1/2*2^(1/2))))*2^(1/2)*(exp(b*x+a)/(exp(b*x+a)^2-1))^(
1/2)*((exp(b*x+a)^2-1)*exp(b*x+a))^(1/2)/exp(b*x+a)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\cosh\left(bx+a\right)\sqrt{\operatorname{csch}\left(bx+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x+a)*csch(b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x*cosh(b*x + a)*sqrt(csch(b*x + a)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x\cosh\left(a+b\,x\right)\,\sqrt{\frac{1}{\sinh\left(a+b\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cosh(a + b*x)*(1/sinh(a + b*x))^(1/2),x)
```

```
[Out] int(x*cosh(a + b*x)*(1/sinh(a + b*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\cosh\left(a+bx\right)\sqrt{\operatorname{csch}\left(a+bx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x+a)*csch(b*x+a)**(1/2),x)
[Out] Integral(x*cosh(a + b*x)*sqrt(csch(a + b*x)), x)
```

3.557 $\int \frac{x\cosh(a+bx)}{\sqrt{\operatorname{csch}(a+bx)}}\,dx$

Optimal. Leaf size=98

$$-\frac{4\cosh(a+bx)}{9b^2\sqrt{\operatorname{csch}(a+bx)}}-\frac{4i\sqrt{i\sinh(a+bx)}\,\sqrt{\operatorname{csch}(a+bx)}\,F\left(\frac{1}{2}\left(ia+ibx-\frac{\pi}{2}\right)\middle|2\right)}{9b^2}+\frac{2x}{3b\operatorname{csch}^{\frac{3}{2}}(a+bx)}$$

```
[Out] 2/3*x/b/csch(b*x+a)^(3/2)-4/9*cosh(b*x+a)/b^2/csch(b*x+a)^(1/2)+4/9*I*(sin(
1/2*I*a+1/4*Pi+1/2*I*b*x)^2)^(1/2)/sin(1/2*I*a+1/4*Pi+1/2*I*b*x)*EllipticF(
cos(1/2*I*a+1/4*Pi+1/2*I*b*x),2^(1/2))*csch(b*x+a)^(1/2)*(I*sinh(b*x+a))^(1
/2)/b^2
```

Rubi [A] time = 0.05, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {5445, 3769, 3771, 2641}

$$-\frac{4\cosh(a+bx)}{9b^2\sqrt{\operatorname{csch}(a+bx)}}-\frac{4i\sqrt{i\sinh(a+bx)}\,\sqrt{\operatorname{csch}(a+bx)}\,F\left(\frac{1}{2}\left(ia+ibx-\frac{\pi}{2}\right)\middle|2\right)}{9b^2}+\frac{2x}{3b\operatorname{csch}^{\frac{3}{2}}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Cosh[a + b*x])/Sqrt[Csch[a + b*x]],x]
```

```
[Out] (2*x)/(3*b*Csch[a + b*x]^(3/2)) - (4*Cosh[a + b*x])/(9*b^2*Sqrt[Csch[a + b*
x]]) - (((4*I)/9)*Sqrt[Csch[a + b*x]]*EllipticF[(I*a - Pi/2 + I*b*x)/2, 2]*
Sqrt[I*Sinh[a + b*x]])/b^2
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
```

```
EqQ[n^2, 1/4]
```

Rule 5445

```
Int[Cosh[(a_.) + (b_.)*(x_)^(n_.)]*Csch[(a_.) + (b_.)*(x_)^(n_.)]^(p_)*(x_)
^(m_.), x_Symbol] :> -Simp[(x^(m - n + 1)*Csch[a + b*x^n]^(p - 1))/(b*n*(p
- 1)), x] + Dist[(m - n + 1)/(b*n*(p - 1)), Int[x^(m - n)*Csch[a + b*x^n]^(
p - 1), x], x] /; FreeQ[{a, b, p}, x] && IntegerQ[n] && GeQ[m - n, 0] && Ne
Q[p, 1]
```

Rubi steps

$$\int \frac{x\cosh(a+bx)}{\sqrt{\operatorname{csch}(a+bx)}}\,dx = \frac{2x}{3b\operatorname{csch}^{\frac{3}{2}}(a+bx)} - \frac{2\int \frac{1}{\operatorname{csch}^{\frac{3}{2}}(a+bx)}\,dx}{3b}$$

$$= \frac{2x}{3b\operatorname{csch}^{\frac{3}{2}}(a+bx)} - \frac{4\cosh(a+bx)}{9b^2\sqrt{\operatorname{csch}(a+bx)}} + \frac{2\int \sqrt{\operatorname{csch}(a+bx)}\,dx}{9b}$$

$$= \frac{2x}{3b\operatorname{csch}^{\frac{3}{2}}(a+bx)} - \frac{4\cosh(a+bx)}{9b^2\sqrt{\operatorname{csch}(a+bx)}} + \frac{\left(2\sqrt{\operatorname{csch}(a+bx)}\sqrt{i\sinh(a+bx)}\right)\int \frac{1}{\sqrt{i\sinh(a+}}}{9b}$$

$$= \frac{2x}{3b\operatorname{csch}^{\frac{3}{2}}(a+bx)} - \frac{4\cosh(a+bx)}{9b^2\sqrt{\operatorname{csch}(a+bx)}} - \frac{4i\sqrt{\operatorname{csch}(a+bx)}\,F\left(\frac{1}{2}\left(ia-\frac{\pi}{2}+ibx\right)\middle|2\right)\sqrt{i\sin}}{9b^2}$$

Mathematica [A] time = 0.43, size = 67, normalized size = 0.68

$$\frac{-4\coth(a+bx) - \frac{4iF\left(\frac{1}{4}(-2ia-2ibx+\pi)\middle|2\right)}{(i\sinh(a+bx))^{3/2}} + 6bx}{9b^2\operatorname{csch}^{\frac{3}{2}}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Cosh[a + b*x])/Sqrt[Csch[a + b*x]],x]
```

```
[Out] (6*b*x - 4*Coth[a + b*x] - ((4*I)*EllipticF[((-2*I)*a + Pi - (2*I)*b*x)/4,
2])/(I*Sinh[a + b*x])^(3/2))/(9*b^2*Csch[a + b*x]^(3/2))
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x+a)/csch(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\cosh(bx+a)}{\sqrt{\operatorname{csch}(bx+a)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x+a)/csch(b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x*cosh(b*x + a)/sqrt(csch(b*x + a)), x)
```

maple [F] time = 0.14, size = 0, normalized size = 0.00

$$\int \frac{x\cosh(bx+a)}{\sqrt{\operatorname{csch}(bx+a)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cosh(b*x+a)/csch(b*x+a)^(1/2),x)
```

```
[Out] int(x*cosh(b*x+a)/csch(b*x+a)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\cosh(bx+a)}{\sqrt{\operatorname{csch}(bx+a)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x+a)/csch(b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x*cosh(b*x + a)/sqrt(csch(b*x + a)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x\cosh(a+b\,x)}{\sqrt{\frac{1}{\sinh(a+b\,x)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*cosh(a + b*x))/(1/sinh(a + b*x))^(1/2),x)
```

```
[Out] int((x*cosh(a + b*x))/(1/sinh(a + b*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\cosh(a+bx)}{\sqrt{\operatorname{csch}(a+bx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x+a)/csch(b*x+a)**(1/2),x)
```

```
[Out] Integral(x*cosh(a + b*x)/sqrt(csch(a + b*x)), x)
```

3.558 $\int \frac{x \cosh(a+bx)}{\operatorname{csch}^{\frac{3}{2}}(a+bx)}\,dx$

Optimal. Leaf size=98

$$-\frac{4\cosh(a+bx)}{25b^2\operatorname{csch}^{\frac{3}{2}}(a+bx)}-\frac{12iE\left(\frac{1}{2}\left(ia+ibx-\frac{\pi}{2}\right)\middle|2\right)}{25b^2\sqrt{i\sinh(a+bx)}\sqrt{\operatorname{csch}(a+bx)}}+\frac{2x}{5b\operatorname{csch}^{\frac{5}{2}}(a+bx)}$$

```
[Out] 2/5*x/b/csch(b*x+a)^(5/2)-4/25*cosh(b*x+a)/b^2/csch(b*x+a)^(3/2)+12/25*I*(s
in(1/2*I*a+1/4*Pi+1/2*I*b*x)^2)^(1/2)/sin(1/2*I*a+1/4*Pi+1/2*I*b*x)*Ellipti
cE(cos(1/2*I*a+1/4*Pi+1/2*I*b*x),2^(1/2))/b^2/csch(b*x+a)^(1/2)/(I*sinh(b*x
+a))^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {5445, 3769, 3771, 2639}

$$-\frac{4\cosh(a+bx)}{25b^2\operatorname{csch}^{\frac{3}{2}}(a+bx)}-\frac{12iE\left(\frac{1}{2}\left(ia+ibx-\frac{\pi}{2}\right)\middle|2\right)}{25b^2\sqrt{i\sinh(a+bx)}\sqrt{\operatorname{csch}(a+bx)}}+\frac{2x}{5b\operatorname{csch}^{\frac{5}{2}}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Cosh[a + b*x])/Csch[a + b*x]^(3/2),x]
```

```
[Out] (2*x)/(5*b*Csch[a + b*x]^(5/2)) - (4*Cosh[a + b*x])/(25*b^2*Csch[a + b*x]^(
3/2)) - (((12*I)/25)*EllipticE[(I*a - Pi/2 + I*b*x)/2, 2])/(b^2*Sqrt[Csch[a
 + b*x]]*Sqrt[I*Sinh[a + b*x]])
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
```

```
EqQ[n^2, 1/4]
```

Rule 5445

```
Int[Cosh[(a_.) + (b_.)*(x_)^(n_.)]*Csch[(a_.) + (b_.)*(x_)^(n_.)]^(p_)*(x_)
^(m_.), x_Symbol] :> -Simp[(x^(m - n + 1)*Csch[a + b*x^n]^(p - 1))/(b*n*(p
- 1)), x] + Dist[(m - n + 1)/(b*n*(p - 1)), Int[x^(m - n)*Csch[a + b*x^n]^(
p - 1), x], x] /; FreeQ[{a, b, p}, x] && IntegerQ[n] && GeQ[m - n, 0] && Ne
Q[p, 1]
```

Rubi steps

$$\begin{aligned}\int \frac{x\cosh(a+bx)}{\operatorname{csch}^{\frac{3}{2}}(a+bx)}\,dx &= \frac{2x}{5b\operatorname{csch}^{\frac{5}{2}}(a+bx)} - \frac{2\int \frac{1}{\operatorname{csch}^{\frac{5}{2}}(a+bx)}\,dx}{5b}\\ &= \frac{2x}{5b\operatorname{csch}^{\frac{5}{2}}(a+bx)} - \frac{4\cosh(a+bx)}{25b^2\operatorname{csch}^{\frac{3}{2}}(a+bx)} + \frac{6\int \frac{1}{\sqrt{\operatorname{csch}(a+bx)}}\,dx}{25b}\\ &= \frac{2x}{5b\operatorname{csch}^{\frac{5}{2}}(a+bx)} - \frac{4\cosh(a+bx)}{25b^2\operatorname{csch}^{\frac{3}{2}}(a+bx)} + \frac{6\int \sqrt{i\sinh(a+bx)}\,dx}{25b\sqrt{\operatorname{csch}(a+bx)}\sqrt{i\sinh(a+bx)}}\\ &= \frac{2x}{5b\operatorname{csch}^{\frac{5}{2}}(a+bx)} - \frac{4\cosh(a+bx)}{25b^2\operatorname{csch}^{\frac{3}{2}}(a+bx)} - \frac{12iE\left(\frac{1}{2}\left(ia-\frac{\pi}{2}+ibx\right)\middle|2\right)}{25b^2\sqrt{\operatorname{csch}(a+bx)}\sqrt{i\sinh(a+bx)}}\end{aligned}$$

Mathematica [C] time = 2.05, size = 111, normalized size = 1.13

$$\frac{e^{-2(a+bx)}\left(-\frac{48e^{2(a+bx)}\,{}_2F_1\left(-\frac{1}{4},\frac{1}{2};\frac{3}{4};e^{2(a+bx)}\right)}{\sqrt{1-e^{2(a+bx)}}}+(24-10bx)e^{2(a+bx)}+(5bx-2)e^{4(a+bx)}+5bx+2\right)}{50b^2\sqrt{\operatorname{csch}(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Cosh[a + b*x])/Csch[a + b*x]^(3/2),x]
[Out] (2 + 5*b*x + E^(2*(a + b*x))*(24 - 10*b*x) + E^(4*(a + b*x))*(-2 + 5*b*x) -
 (48*E^(2*(a + b*x))*Hypergeometric2F1[-1/4, 1/2, 3/4, E^(2*(a + b*x))])/Sq
rt[1 - E^(2*(a + b*x))])/(50*b^2*E^(2*(a + b*x))*Sqrt[Csch[a + b*x]])
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x+a)/csch(b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\cosh(bx+a)}{\operatorname{csch}(bx+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x+a)/csch(b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x*cosh(b*x + a)/csch(b*x + a)^(3/2), x)
```

maple [F] time = 0.18, size = 0, normalized size = 0.00

$$\int \frac{x\cosh(bx+a)}{\operatorname{csch}(bx+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cosh(b*x+a)/csch(b*x+a)^(3/2),x)
```

```
[Out] int(x*cosh(b*x+a)/csch(b*x+a)^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\cosh(bx+a)}{\operatorname{csch}(bx+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x+a)/csch(b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x*cosh(b*x + a)/csch(b*x + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x\cosh(a+b\,x)}{{\left(\frac{1}{\sinh(a+b\,x)}\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*cosh(a + b*x))/(1/sinh(a + b*x))^(3/2),x)
[Out] int((x*cosh(a + b*x))/(1/sinh(a + b*x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x \cosh(a+bx)}{\operatorname{csch}^{\frac{3}{2}}(a+bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x+a)/csch(b*x+a)**(3/2),x)
[Out] Integral(x*cosh(a + b*x)/csch(a + b*x)**(3/2), x)
```

3.559 $\int \frac{x\cosh(a+bx)}{\operatorname{csch}^{\frac{5}{2}}(a+bx)}\,dx$

Optimal. Leaf size=121

$$-\frac{4\cosh(a+bx)}{49b^2\operatorname{csch}^{\frac{5}{2}}(a+bx)}+\frac{20\cosh(a+bx)}{147b^2\sqrt{\operatorname{csch}(a+bx)}}+\frac{20i\sqrt{i\sinh(a+bx)}\,\sqrt{\operatorname{csch}(a+bx)}\,F\left(\frac{1}{2}\left(ia+ibx-\frac{\pi}{2}\right)\middle|2\right)}{147b^2}+\frac{2}{7b\operatorname{csch}^{\frac{7}{2}}}$$

```
[Out] 2/7*x/b/csch(b*x+a)^(7/2)-4/49*cosh(b*x+a)/b^2/csch(b*x+a)^(5/2)+20/147*cos
h(b*x+a)/b^2/csch(b*x+a)^(1/2)-20/147*I*(sin(1/2*I*a+1/4*Pi+1/2*I*b*x)^2)^(
1/2)/sin(1/2*I*a+1/4*Pi+1/2*I*b*x)*EllipticF(cos(1/2*I*a+1/4*Pi+1/2*I*b*x),
2^(1/2))*csch(b*x+a)^(1/2)*(I*sinh(b*x+a))^(1/2)/b^2
```

Rubi [A] time = 0.07, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {5445, 3769, 3771, 2641}

$$-\frac{4\cosh(a+bx)}{49b^2\operatorname{csch}^{\frac{5}{2}}(a+bx)}+\frac{20\cosh(a+bx)}{147b^2\sqrt{\operatorname{csch}(a+bx)}}+\frac{20i\sqrt{i\sinh(a+bx)}\,\sqrt{\operatorname{csch}(a+bx)}\,F\left(\frac{1}{2}\left(ia+ibx-\frac{\pi}{2}\right)\middle|2\right)}{147b^2}+\frac{2}{7b\operatorname{csch}^{\frac{7}{2}}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Cosh[a + b*x])/Csch[a + b*x]^(5/2),x]
```

```
[Out] (2*x)/(7*b*Csch[a + b*x]^(7/2)) - (4*Cosh[a + b*x])/(49*b^2*Csch[a + b*x]^(
5/2)) + (20*Cosh[a + b*x])/(147*b^2*Sqrt[Csch[a + b*x]]) + (((20*I)/147)*Sq
rt[Csch[a + b*x]]*EllipticF[(I*a - Pi/2 + I*b*x)/2, 2]*Sqrt[I*Sinh[a + b*x]
])/b^2
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 5445

```
Int[Cosh[(a_.) + (b_.)*(x_)^(n_.)]*Csch[(a_.) + (b_.)*(x_)^(n_.)]^(p_)*(x_)
^(m_.), x_Symbol] :> -Simp[(x^(m - n + 1)*Csch[a + b*x^n]^(p - 1))/(b*n*(p
- 1)), x] + Dist[(m - n + 1)/(b*n*(p - 1)), Int[x^(m - n)*Csch[a + b*x^n]^(
p - 1), x], x] /; FreeQ[{a, b, p}, x] && IntegerQ[n] && GeQ[m - n, 0] && Ne
Q[p, 1]
```

Rubi steps

$$\int \frac{x\cosh(a+bx)}{\operatorname{csch}^{\frac{5}{2}}(a+bx)}\,dx = \frac{2x}{7b\operatorname{csch}^{\frac{7}{2}}(a+bx)} - \frac{2\int \frac{1}{\operatorname{csch}^{\frac{7}{2}}(a+bx)}\,dx}{7b}$$

$$= \frac{2x}{7b\operatorname{csch}^{\frac{7}{2}}(a+bx)} - \frac{4\cosh(a+bx)}{49b^2\operatorname{csch}^{\frac{5}{2}}(a+bx)} + \frac{10\int \frac{1}{\operatorname{csch}^{\frac{3}{2}}(a+bx)}\,dx}{49b}$$

$$= \frac{2x}{7b\operatorname{csch}^{\frac{7}{2}}(a+bx)} - \frac{4\cosh(a+bx)}{49b^2\operatorname{csch}^{\frac{5}{2}}(a+bx)} + \frac{20\cosh(a+bx)}{147b^2\sqrt{\operatorname{csch}(a+bx)}} - \frac{10\int \sqrt{\operatorname{csch}(a+bx)}\,d}{147b}$$

$$= \frac{2x}{7b\operatorname{csch}^{\frac{7}{2}}(a+bx)} - \frac{4\cosh(a+bx)}{49b^2\operatorname{csch}^{\frac{5}{2}}(a+bx)} + \frac{20\cosh(a+bx)}{147b^2\sqrt{\operatorname{csch}(a+bx)}} - \frac{\left(10\sqrt{\operatorname{csch}(a+bx)}\sqrt{i}\right.}{}$$

$$= \frac{2x}{7b\operatorname{csch}^{\frac{7}{2}}(a+bx)} - \frac{4\cosh(a+bx)}{49b^2\operatorname{csch}^{\frac{5}{2}}(a+bx)} + \frac{20\cosh(a+bx)}{147b^2\sqrt{\operatorname{csch}(a+bx)}} + \frac{20i\sqrt{\operatorname{csch}(a+bx)}\,F\left(\right.}{}$$

Mathematica [A] time = 0.41, size = 103, normalized size = 0.85

$$\frac{\sqrt{\operatorname{csch}(a+bx)}\left(52\sinh(2(a+bx)) - 6\sinh(4(a+bx)) - 84bx\cosh(2(a+bx)) + 21bx\cosh(4(a+bx)) - 80i\sqrt{i}\right.}{588b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Cosh[a + b*x])/Csch[a + b*x]^(5/2),x]
```

```
[Out] (Sqrt[Csch[a + b*x]]*(63*b*x - 84*b*x*Cosh[2*(a + b*x)] + 21*b*x*Cosh[4*(a
+ b*x)] - (80*I)*EllipticF[((-2*I)*a + Pi - (2*I)*b*x)/4, 2]*Sqrt[I*Sinh[a
+ b*x]] + 52*Sinh[2*(a + b*x)] - 6*Sinh[4*(a + b*x)]))/(588*b^2)
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x+a)/csch(b*x+a)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x \cosh(bx + a)}{\operatorname{csch}(bx + a)^{\frac{5}{2}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x+a)/csch(b*x+a)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(x*cosh(b*x + a)/csch(b*x + a)^(5/2), x)
```

maple [F] time = 0.16, size = 0, normalized size = 0.00

$$\int \frac{x \cosh(bx + a)}{\operatorname{csch}(bx + a)^{\frac{5}{2}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cosh(b*x+a)/csch(b*x+a)^(5/2),x)
```

```
[Out] int(x*cosh(b*x+a)/csch(b*x+a)^(5/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x \cosh(bx + a)}{\operatorname{csch}(bx + a)^{\frac{5}{2}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x+a)/csch(b*x+a)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(x*cosh(b*x + a)/csch(b*x + a)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x\cosh\left(a+b\,x\right)}{{\left(\frac{1}{\mathrm{sinh}\left(a+b\,x\right)}\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*cosh(a + b*x))/(1/sinh(a + b*x))^(5/2),x)
[Out] int((x*cosh(a + b*x))/(1/sinh(a + b*x))^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x+a)/csch(b*x+a)**(5/2),x)
[Out] Timed out
```

3.560 $\int \sqrt{\sinh(x)\tanh(x)}\, dx$

Optimal. Leaf size=13

$$2\coth(x)\sqrt{\sinh(x)\tanh(x)}$$

```
[Out] 2*coth(x)*(sinh(x)*tanh(x))^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {4398, 4400, 2589}

$$2\coth(x)\sqrt{\sinh(x)\tanh(x)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Sinh[x]*Tanh[x]],x]
```

```
[Out] 2*Coth[x]*Sqrt[Sinh[x]*Tanh[x]]
```

Rule 2589

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> -Simp[(b*(a*Sin[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*m
), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n - 1, 0]
```

Rule 4398

```
Int[(u_.)*((a_)*(v_))^(p_), x_Symbol] :> With[{uu = ActivateTrig[u], vv = A
ctivateTrig[v]}, Dist[(a^IntPart[p]*(a*vv)^FracPart[p])/vv^FracPart[p], Int
[uu*vv^p, x], x]] /; FreeQ[{a, p}, x] &&  !IntegerQ[p] &&  !InertTrigFreeQ[
v]
```

Rule 4400

```
Int[(u_.)*((v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> With[{uu = ActivateTri
g[u], vv = ActivateTrig[v], ww = ActivateTrig[w]}, Dist[(vv^m*ww^n)^FracPar
t[p]/(vv^(m*FracPart[p])*ww^(n*FracPart[p])), Int[uu*vv^(m*p)*ww^(n*p), x],
 x]] /; FreeQ[{m, n, p}, x] &&  !IntegerQ[p] && ( !InertTrigFreeQ[v] ||  !I
nertTrigFreeQ[w])
```

Rubi steps

$$\int \sqrt{\sinh(x)\tanh(x)}\,dx = \frac{\sqrt{\sinh(x)\tanh(x)}\int \sqrt{-\sinh(x)\tanh(x)}\,dx}{\sqrt{-\sinh(x)\tanh(x)}}$$
$$= \frac{\sqrt{\sinh(x)\tanh(x)}\int \sqrt{i\sinh(x)}\sqrt{i\tanh(x)}\,dx}{\sqrt{i\sinh(x)}\sqrt{i\tanh(x)}}$$
$$= 2\coth(x)\sqrt{\sinh(x)\tanh(x)}$$

Mathematica [A] time = 0.07, size = 13, normalized size = 1.00

$$2\coth(x)\sqrt{\sinh(x)\tanh(x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Sinh[x]*Tanh[x]],x]
```

```
[Out] 2*Coth[x]*Sqrt[Sinh[x]*Tanh[x]]
```

fricas [B] time = 0.44, size = 53, normalized size = 4.08

$$\frac{2\sqrt{\frac{1}{2}}\left(\cosh(x)^2 + 2\cosh(x)\sinh(x) + \sinh(x)^2 + 1\right)}{\sqrt{\cosh(x)^3 + 3\cosh(x)\sinh(x)^2 + \sinh(x)^3 + \left(3\cosh(x)^2 + 1\right)\sinh(x) + \cosh(x)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sinh(x)*tanh(x))^(1/2),x, algorithm="fricas")
```

```
[Out] 2*sqrt(1/2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)/sqrt(cosh(x)^3
+ 3*cosh(x)*sinh(x)^2 + sinh(x)^3 + (3*cosh(x)^2 + 1)*sinh(x) + cosh(x))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\sinh(x)\tanh(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sinh(x)*tanh(x))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(sinh(x)*tanh(x)), x)
```

maple [B] time = 0.52, size = 42, normalized size = 3.23

$$\frac{\sqrt{2}\sqrt{\frac{\left(e^{2x}-1\right)^2 e^{-x}}{1+e^{2x}}}\left(1+e^{2x}\right)}{e^{2x}-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sinh(x)*tanh(x))^(1/2),x)
```

```
[Out] 2^(1/2)*((exp(2*x)-1)^2*exp(-x)/(1+exp(2*x)))^(1/2)/(exp(2*x)-1)*(1+exp(2*x
))
```

maxima [B] time = 0.77, size = 35, normalized size = 2.69

$$-\frac{\sqrt{2}e^{\left(\frac{1}{2}x\right)}}{\sqrt{e^{(-2x)}+1}}-\frac{\sqrt{2}e^{\left(-\frac{3}{2}x\right)}}{\sqrt{e^{(-2x)}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sinh(x)*tanh(x))^(1/2),x, algorithm="maxima")
```

```
[Out] -sqrt(2)*e^(1/2*x)/sqrt(e^(-2*x) + 1) - sqrt(2)*e^(-3/2*x)/sqrt(e^(-2*x) +
1)
```

mupad [B] time = 1.62, size = 35, normalized size = 2.69

$$2\coth(x)\sqrt{-\left(\frac{e^{-x}}{2}-\frac{e^{x}}{2}\right)\left(e^{2x}-1\right)}\sqrt{\frac{1}{e^{2x}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sinh(x)*tanh(x))^(1/2),x)
```

```
[Out] 2*coth(x)*(-(exp(-x)/2 - exp(x)/2)*(exp(2*x) - 1))^(1/2)*(1/(exp(2*x) + 1))
^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{\sinh(x)\tanh(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sinh(x)*tanh(x))**(1/2),x)
```

```
[Out] Integral(sqrt(sinh(x)*tanh(x)), x)
```

3.561 $\int (\sinh(x)\tanh(x))^{3/2}\,dx$

Optimal. Leaf size=31

$$\frac{2}{3}\sinh(x)\sqrt{\sinh(x)\tanh(x)} + \frac{8}{3}\operatorname{csch}(x)\sqrt{\sinh(x)\tanh(x)}$$

```
[Out] 8/3*csch(x)*(sinh(x)*tanh(x))^(1/2)+2/3*sinh(x)*(sinh(x)*tanh(x))^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {4398, 4400, 2598, 2589}

$$\frac{2}{3}\sinh(x)\sqrt{\sinh(x)\tanh(x)} + \frac{8}{3}\operatorname{csch}(x)\sqrt{\sinh(x)\tanh(x)}$$

Antiderivative was successfully verified.

```
[In] Int[(Sinh[x]*Tanh[x])^(3/2),x]
```

```
[Out] (8*Csch[x]*Sqrt[Sinh[x]*Tanh[x]])/3 + (2*Sinh[x]*Sqrt[Sinh[x]*Tanh[x]])/3
```

Rule 2589

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> -Simp[(b*(a*Sin[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*m
), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n - 1, 0]
```

Rule 2598

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> -Simp[(b*(a*Sin[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f
*m), x] + Dist[(a^2*(m + n - 1))/m, Int[(a*Sin[e + f*x])^(m - 2)*(b*Tan[e +
 f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (GtQ[m, 1] || (EqQ[m, 1] &
& EqQ[n, 1/2])) && IntegersQ[2*m, 2*n]
```

Rule 4398

```
Int[(u_.)*((a_)*(v_))^(p_), x_Symbol] :> With[{uu = ActivateTrig[u], vv = A
ctivateTrig[v]}, Dist[(a^IntPart[p]*(a*vv)^FracPart[p])/vv^FracPart[p], Int
[uu*vv^p, x], x]] /; FreeQ[{a, p}, x] && !IntegerQ[p] && !InertTrigFreeQ[
v]
```

Rule 4400

```
Int[(u_.)*((v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> With[{uu = ActivateTri
g[u], vv = ActivateTrig[v], ww = ActivateTrig[w]}, Dist[(vv^m*ww^n)^FracPar
```

```
t[p]/(vv^(m*FracPart[p])*ww^(n*FracPart[p])), Int[uu*vv^(m*p)*ww^(n*p), x],
 x]] /; FreeQ[{m, n, p}, x] &&  !IntegerQ[p] && ( !InertTrigFreeQ[v] ||  !I
nertTrigFreeQ[w])
```

Rubi steps

$$\begin{aligned}\int (\sinh(x)\tanh(x))^{3/2}\,dx &= -\frac{\sqrt{\sinh(x)\tanh(x)}\int(-\sinh(x)\tanh(x))^{3/2}\,dx}{\sqrt{-\sinh(x)\tanh(x)}}\\ &= -\frac{\sqrt{\sinh(x)\tanh(x)}\int(i\sinh(x))^{3/2}(i\tanh(x))^{3/2}\,dx}{\sqrt{i\sinh(x)}\sqrt{i\tanh(x)}}\\ &= \frac{2}{3}\sinh(x)\sqrt{\sinh(x)\tanh(x)} - \frac{\left(4\sqrt{\sinh(x)\tanh(x)}\right)\int\frac{(i\tanh(x))^{3/2}}{\sqrt{i\sinh(x)}}\,dx}{3\sqrt{i\sinh(x)}\sqrt{i\tanh(x)}}\\ &= \frac{8}{3}\text{csch}(x)\sqrt{\sinh(x)\tanh(x)} + \frac{2}{3}\sinh(x)\sqrt{\sinh(x)\tanh(x)}\end{aligned}$$

Mathematica [A] time = 0.07, size = 23, normalized size = 0.74

$$\frac{2}{3}\sinh(x)\left(4\text{csch}^2(x)+1\right)\sqrt{\sinh(x)\tanh(x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sinh[x]*Tanh[x])^(3/2),x]
```

```
[Out] (2*(1 + 4*Csch[x]^2)*Sinh[x]*Sqrt[Sinh[x]*Tanh[x]])/3
```

fricas [B] time = 0.41, size = 95, normalized size = 3.06

$$\frac{\sqrt{\frac{1}{2}}\left(\cosh(x)^4 + 4\cosh(x)\sinh(x)^3 + \sinh(x)^4 + 2\left(3\cosh(x)^2+7\right)\sinh(x)^2 + 14\cosh(x)^2 + 4\left(\cosh(x)^3 + 7\,\text{c}\right.}{3\sqrt{\cosh(x)^3 + 3\cosh(x)\sinh(x)^2 + \sinh(x)^3 + \left(3\cosh(x)^2+1\right)\sinh(x) + \cosh(x)}\,(\cosh(x) + \text{si}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sinh(x)*tanh(x))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/3*sqrt(1/2)*(cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + 2*(3*cosh(x)^2
 + 7)*sinh(x)^2 + 14*cosh(x)^2 + 4*(cosh(x)^3 + 7*cosh(x))*sinh(x) + 1)/(sq
rt(cosh(x)^3 + 3*cosh(x)*sinh(x)^2 + sinh(x)^3 + (3*cosh(x)^2 + 1)*sinh(x)
+ cosh(x))*(cosh(x) + sinh(x)))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (\sinh(x)\tanh(x))^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sinh(x)*tanh(x))^(3/2),x, algorithm="giac")
[Out] integrate((sinh(x)*tanh(x))^(3/2), x)
```

maple [F] time = 0.44, size = 0, normalized size = 0.00

$$\int (\sinh(x)\tanh(x))^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sinh(x)*tanh(x))^(3/2),x)
[Out] int((sinh(x)*tanh(x))^(3/2),x)
```

maxima [B] time = 0.58, size = 69, normalized size = 2.23

$$-\frac{\sqrt{2}e^{\left(\frac{3}{2}x\right)}}{6\left(e^{(-2x)}+1\right)^{\frac{3}{2}}}-\frac{5\sqrt{2}e^{\left(-\frac{1}{2}x\right)}}{2\left(e^{(-2x)}+1\right)^{\frac{3}{2}}}-\frac{5\sqrt{2}e^{\left(-\frac{5}{2}x\right)}}{2\left(e^{(-2x)}+1\right)^{\frac{3}{2}}}-\frac{\sqrt{2}e^{\left(-\frac{9}{2}x\right)}}{6\left(e^{(-2x)}+1\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sinh(x)*tanh(x))^(3/2),x, algorithm="maxima")
[Out] -1/6*sqrt(2)*e^(3/2*x)/(e^(-2*x) + 1)^(3/2) - 5/2*sqrt(2)*e^(-1/2*x)/(e^(-2
*x) + 1)^(3/2) - 5/2*sqrt(2)*e^(-5/2*x)/(e^(-2*x) + 1)^(3/2) - 1/6*sqrt(2)*
e^(-9/2*x)/(e^(-2*x) + 1)^(3/2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int (\sinh(x)\tanh(x))^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sinh(x)*tanh(x))^(3/2),x)
[Out] int((sinh(x)*tanh(x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (\sinh(x)\tanh(x))^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sinh(x)*tanh(x))**(3/2),x)
```

```
[Out] Integral((sinh(x)*tanh(x))**(3/2), x)
```

3.562 $\int (\sinh(x)\tanh(x))^{5/2}\,dx$

Optimal. Leaf size=50

$$\frac{2}{5}\sinh^2(x)\tanh(x)\sqrt{\sinh(x)\tanh(x)}+\frac{16}{15}\tanh(x)\sqrt{\sinh(x)\tanh(x)}-\frac{64}{15}\coth(x)\sqrt{\sinh(x)\tanh(x)}$$

```
[Out] -64/15*coth(x)*(sinh(x)*tanh(x))^(1/2)+16/15*(sinh(x)*tanh(x))^(1/2)*tanh(x
)+2/5*sinh(x)^2*(sinh(x)*tanh(x))^(1/2)*tanh(x)
```

Rubi [A] time = 0.12, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.556, Rules used = {4398, 4400, 2598, 2594, 2589}

$$\frac{2}{5}\sinh^2(x)\tanh(x)\sqrt{\sinh(x)\tanh(x)}+\frac{16}{15}\tanh(x)\sqrt{\sinh(x)\tanh(x)}-\frac{64}{15}\coth(x)\sqrt{\sinh(x)\tanh(x)}$$

Antiderivative was successfully verified.

```
[In] Int[(Sinh[x]*Tanh[x])^(5/2),x]
```

```
[Out] (-64*Coth[x]*Sqrt[Sinh[x]*Tanh[x]])/15 + (16*Tanh[x]*Sqrt[Sinh[x]*Tanh[x]])
/15 + (2*Sinh[x]^2*Tanh[x]*Sqrt[Sinh[x]*Tanh[x]])/5
```

Rule 2589

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> -Simp[(b*(a*Sin[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*m
), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n - 1, 0]
```

Rule 2594

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sin[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
n - 1)), x] - Dist[(b^2*(m + n - 1))/(n - 1), Int[(a*Sin[e + f*x])^m*(b*Tan
[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] && Int
egersQ[2*m, 2*n] &&  !(GtQ[m, 1] &&  !IntegerQ[(m - 1)/2])
```

Rule 2598

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> -Simp[(b*(a*Sin[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f
*m), x] + Dist[(a^2*(m + n - 1))/m, Int[(a*Sin[e + f*x])^(m - 2)*(b*Tan[e +
 f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (GtQ[m, 1] || (EqQ[m, 1] &
& EqQ[n, 1/2])) && IntegersQ[2*m, 2*n]
```

Rule 4398

```
Int[(u_.)*((a_)*(v_))^(p_), x_Symbol] :> With[{uu = ActivateTrig[u], vv = A
ctivateTrig[v]}, Dist[(a^IntPart[p]*(a*vv)^FracPart[p])/vv^FracPart[p], Int
[uu*vv^p, x], x]] /; FreeQ[{a, p}, x] &&  !IntegerQ[p] &&  !InertTrigFreeQ[
v]
```

Rule 4400

```
Int[(u_.)*((v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> With[{uu = ActivateTri
g[u], vv = ActivateTrig[v], ww = ActivateTrig[w]}, Dist[(vv^m*ww^n)^FracPar
t[p]/(vv^(m*FracPart[p])*ww^(n*FracPart[p])), Int[uu*vv^(m*p)*ww^(n*p), x],
 x]] /; FreeQ[{m, n, p}, x] &&  !IntegerQ[p] && ( !InertTrigFreeQ[v] ||  !I
nertTrigFreeQ[w])
```

Rubi steps

$$\int (\sinh(x)\tanh(x))^{5/2}\,dx = \frac{\sqrt{\sinh(x)\tanh(x)}\int(-\sinh(x)\tanh(x))^{5/2}\,dx}{\sqrt{-\sinh(x)\tanh(x)}}$$
$$= \frac{\sqrt{\sinh(x)\tanh(x)}\int(i\sinh(x))^{5/2}(i\tanh(x))^{5/2}\,dx}{\sqrt{i\sinh(x)}\sqrt{i\tanh(x)}}$$
$$= \frac{2}{5}\sinh^2(x)\tanh(x)\sqrt{\sinh(x)\tanh(x)} + \frac{\left(8\sqrt{\sinh(x)\tanh(x)}\right)\int\sqrt{i\sinh(x)}\,(i\tanh(x$$
$$= \frac{16}{15}\tanh(x)\sqrt{\sinh(x)\tanh(x)} + \frac{2}{5}\sinh^2(x)\tanh(x)\sqrt{\sinh(x)\tanh(x)} - \frac{\left(32\sqrt{\sinh(x$$
$$= -\frac{64}{15}\coth(x)\sqrt{\sinh(x)\tanh(x)} + \frac{16}{15}\tanh(x)\sqrt{\sinh(x)\tanh(x)} + \frac{2}{5}\sinh^2(x)\tanh(x$$

Mathematica [A] time = 0.12, size = 29, normalized size = 0.58

$$-\frac{2}{15}\tanh(x)\sqrt{\sinh(x)\tanh(x)}\left(-3\cosh^2(x)+32\coth^2(x)-5\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sinh[x]*Tanh[x])^(5/2),x]
```

```
[Out] (-2*(-5 - 3*Cosh[x]^2 + 32*Coth[x]^2)*Tanh[x]*Sqrt[Sinh[x]*Tanh[x]])/15
```

fricas [B] time = 0.42, size = 253, normalized size = 5.06

$$\frac{\sqrt{\frac{1}{2}}\left(3\cosh(x)^8 + 24\cosh(x)\sinh(x)^7 + 3\sinh(x)^8 + 12\left(7\cosh(x)^2 - 9\right)\sinh(x)^6 - 108\cosh(x)^6 + 24\left(7\cos\right.\right.}{30\left(\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sinh(x)*tanh(x))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/30*sqrt(1/2)*(3*cosh(x)^8 + 24*cosh(x)*sinh(x)^7 + 3*sinh(x)^8 + 12*(7*co
sh(x)^2 - 9)*sinh(x)^6 - 108*cosh(x)^6 + 24*(7*cosh(x)^3 - 27*cosh(x))*sinh
(x)^5 + 2*(105*cosh(x)^4 - 810*cosh(x)^2 - 151)*sinh(x)^4 - 302*cosh(x)^4 +
 8*(21*cosh(x)^5 - 270*cosh(x)^3 - 151*cosh(x))*sinh(x)^3 + 12*(7*cosh(x)^6
 - 135*cosh(x)^4 - 151*cosh(x)^2 - 9)*sinh(x)^2 - 108*cosh(x)^2 + 8*(3*cosh
(x)^7 - 81*cosh(x)^5 - 151*cosh(x)^3 - 27*cosh(x))*sinh(x) + 3)/((cosh(x)^4
 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + (6*cosh(x)^2 + 1)*sinh(x)^2 + cosh(x)^
2 + 2*(2*cosh(x)^3 + cosh(x))*sinh(x))*sqrt(cosh(x)^3 + 3*cosh(x)*sinh(x)^2
 + sinh(x)^3 + (3*cosh(x)^2 + 1)*sinh(x) + cosh(x)))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (\sinh(x)\tanh(x))^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sinh(x)*tanh(x))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((sinh(x)*tanh(x))^(5/2), x)
```

maple [F] time = 0.44, size = 0, normalized size = 0.00

$$\int (\sinh(x)\tanh(x))^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sinh(x)*tanh(x))^(5/2),x)
```

```
[Out] int((sinh(x)*tanh(x))^(5/2),x)
```

maxima [B] time = 0.69, size = 103, normalized size = 2.06

$$-\frac{\sqrt{2}e^{\left(\frac{5}{2}x\right)}}{20\left(e^{(-2x)}+1\right)^{\frac{5}{2}}}+\frac{7\sqrt{2}e^{\left(\frac{1}{2}x\right)}}{4\left(e^{(-2x)}+1\right)^{\frac{5}{2}}}+\frac{41\sqrt{2}e^{\left(-\frac{3}{2}x\right)}}{6\left(e^{(-2x)}+1\right)^{\frac{5}{2}}}+\frac{41\sqrt{2}e^{\left(-\frac{7}{2}x\right)}}{6\left(e^{(-2x)}+1\right)^{\frac{5}{2}}}+\frac{7\sqrt{2}e^{\left(-\frac{11}{2}x\right)}}{4\left(e^{(-2x)}+1\right)^{\frac{5}{2}}}-\frac{\sqrt{2}e^{\left(-\frac{15}{2}x\right)}}{20\left(e^{(-2x)}+1\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sinh(x)*tanh(x))^(5/2),x, algorithm="maxima")
```

```
[Out] -1/20*sqrt(2)*e^(5/2*x)/(e^(-2*x) + 1)^(5/2) + 7/4*sqrt(2)*e^(1/2*x)/(e^(-2
*x) + 1)^(5/2) + 41/6*sqrt(2)*e^(-3/2*x)/(e^(-2*x) + 1)^(5/2) + 41/6*sqrt(2
)*e^(-7/2*x)/(e^(-2*x) + 1)^(5/2) + 7/4*sqrt(2)*e^(-11/2*x)/(e^(-2*x) + 1)^
(5/2) - 1/20*sqrt(2)*e^(-15/2*x)/(e^(-2*x) + 1)^(5/2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int (\sinh(x)\tanh(x))^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sinh(x)*tanh(x))^(5/2),x)
```

```
[Out] int((sinh(x)*tanh(x))^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sinh(x)*tanh(x))**(5/2),x)
```

```
[Out] Timed out
```

3.563 $\int \sqrt{\cosh(x)\coth(x)}\,dx$

Optimal. Leaf size=13

$$2\tanh(x)\sqrt{\cosh(x)\coth(x)}$$

```
[Out] 2*(cosh(x)*coth(x))^(1/2)*tanh(x)
```

Rubi [A] time = 0.05, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {4398, 4400, 2589}

$$2\tanh(x)\sqrt{\cosh(x)\coth(x)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cosh[x]*Coth[x]],x]
```

```
[Out] 2*Sqrt[Cosh[x]*Coth[x]]*Tanh[x]
```

Rule 2589

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> -Simp[(b*(a*Sin[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*m
), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n - 1, 0]
```

Rule 4398

```
Int[(u_.)*((a_)*(v_))^(p_), x_Symbol] :> With[{uu = ActivateTrig[u], vv = A
ctivateTrig[v]}, Dist[(a^IntPart[p]*(a*vv)^FracPart[p])/vv^FracPart[p], Int
[uu*vv^p, x], x]] /; FreeQ[{a, p}, x] && !IntegerQ[p] && !InertTrigFreeQ[
v]
```

Rule 4400

```
Int[(u_.)*((v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> With[{uu = ActivateTri
g[u], vv = ActivateTrig[v], ww = ActivateTrig[w]}, Dist[(vv^m*ww^n)^FracPar
t[p]/(vv^(m*FracPart[p])*ww^(n*FracPart[p])), Int[uu*vv^(m*p)*ww^(n*p), x],
 x]] /; FreeQ[{m, n, p}, x] && !IntegerQ[p] && ( !InertTrigFreeQ[v] || !I
nertTrigFreeQ[w])
```

Rubi steps

$$\int \sqrt{\cosh(x)\coth(x)}\,dx = \frac{\sqrt{\cosh(x)\coth(x)}\int \sqrt{-i\cosh(x)\coth(x)}\,dx}{\sqrt{-i\cosh(x)\coth(x)}}$$
$$= \frac{\sqrt{\cosh(x)\coth(x)}\int \sqrt{\cosh(x)}\sqrt{-i\coth(x)}\,dx}{\sqrt{\cosh(x)}\sqrt{-i\coth(x)}}$$
$$= 2\sqrt{\cosh(x)\coth(x)}\,\tanh(x)$$

Mathematica [B] time = 0.08, size = 35, normalized size = 2.69

$$\frac{2\left(\sqrt[4]{-\sinh^2(x)}-1\right)\tanh(x)\sqrt{\cosh(x)\coth(x)}}{\sqrt[4]{-\sinh^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Cosh[x]*Coth[x]],x]
```

```
[Out] (2*Sqrt[Cosh[x]*Coth[x]]*(-1 + (-Sinh[x]^2)^(1/4))*Tanh[x])/(-Sinh[x]^2)^(1
/4)
```

fricas [B] time = 0.42, size = 55, normalized size = 4.23

$$\frac{2\sqrt{\frac{1}{2}}\left(\cosh(x)^2+2\cosh(x)\sinh(x)+\sinh(x)^2-1\right)}{\sqrt{\cosh(x)^3+3\cosh(x)\sinh(x)^2+\sinh(x)^3+\left(3\cosh(x)^2-1\right)\sinh(x)-\cosh(x)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cosh(x)*coth(x))^(1/2),x, algorithm="fricas")
```

```
[Out] 2*sqrt(1/2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)/sqrt(cosh(x)^3
+ 3*cosh(x)*sinh(x)^2 + sinh(x)^3 + (3*cosh(x)^2 - 1)*sinh(x) - cosh(x))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\cosh(x)\coth(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cosh(x)*coth(x))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(cosh(x)*coth(x)), x)
```

maple [B] time = 0.52, size = 42, normalized size = 3.23

$$\frac{\sqrt{2}\,\sqrt{\frac{\left(1+\mathrm{e}^{2x}\right)^{2}\mathrm{e}^{-x}}{\mathrm{e}^{2x}-1}}\,\left(\mathrm{e}^{2x}-1\right)}{1+\mathrm{e}^{2x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cosh(x)*coth(x))^(1/2),x)
```

```
[Out] 2^(1/2)*((1+exp(2*x))^2*exp(-x)/(exp(2*x)-1))^(1/2)/(1+exp(2*x))*(exp(2*x)-
1)
```

maxima [B] time = 0.63, size = 54, normalized size = 4.15

$$\frac{\sqrt{2}\,e^{\left(\frac{1}{2}\,x\right)}}{\sqrt{e^{(-x)}+1}\,\sqrt{-e^{(-x)}+1}}-\frac{\sqrt{2}\,e^{\left(-\frac{3}{2}\,x\right)}}{\sqrt{e^{(-x)}+1}\,\sqrt{-e^{(-x)}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cosh(x)*coth(x))^(1/2),x, algorithm="maxima")
```

```
[Out] sqrt(2)*e^(1/2*x)/(sqrt(e^(-x) + 1)*sqrt(-e^(-x) + 1)) - sqrt(2)*e^(-3/2*x)
/(sqrt(e^(-x) + 1)*sqrt(-e^(-x) + 1))
```

mupad [B] time = 1.53, size = 23, normalized size = 1.77

$$4\,\mathrm{e}^{x}\,\sinh(x)\,\sqrt{\frac{\mathrm{e}^{-x}}{2\,\left(\mathrm{e}^{2\,x}-1\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cosh(x)*coth(x))^(1/2),x)
```

```
[Out] 4*exp(x)*sinh(x)*(exp(-x)/(2*(exp(2*x) - 1)))^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{\cosh\left(x\right)\coth\left(x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cosh(x)*coth(x))**(1/2),x)
```

```
[Out] Integral(sqrt(cosh(x)*coth(x)), x)
```

3.564 $\int (\cosh(x)\coth(x))^{3/2}\,dx$

Optimal. Leaf size=31

$$\frac{2}{3}\cosh(x)\sqrt{\cosh(x)\coth(x)}-\frac{8}{3}\text{sech}(x)\sqrt{\cosh(x)\coth(x)}$$

```
[Out] 2/3*cosh(x)*(cosh(x)*coth(x))^(1/2)-8/3*sech(x)*(cosh(x)*coth(x))^(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {4398, 4400, 2598, 2589}

$$\frac{2}{3}\cosh(x)\sqrt{\cosh(x)\coth(x)}-\frac{8}{3}\text{sech}(x)\sqrt{\cosh(x)\coth(x)}$$

Antiderivative was successfully verified.

```
[In] Int[(Cosh[x]*Coth[x])^(3/2),x]
```

```
[Out] (2*Cosh[x]*Sqrt[Cosh[x]*Coth[x]])/3 - (8*Sqrt[Cosh[x]*Coth[x]]*Sech[x])/3
```

Rule 2589

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> -Simp[(b*(a*Sin[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*m
), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n - 1, 0]
```

Rule 2598

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> -Simp[(b*(a*Sin[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f
*m), x] + Dist[(a^2*(m + n - 1))/m, Int[(a*Sin[e + f*x])^(m - 2)*(b*Tan[e +
 f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (GtQ[m, 1] || (EqQ[m, 1] &
& EqQ[n, 1/2])) && IntegersQ[2*m, 2*n]
```

Rule 4398

```
Int[(u_.)*((a_)*(v_))^(p_), x_Symbol] :> With[{uu = ActivateTrig[u], vv = A
ctivateTrig[v]}, Dist[(a^IntPart[p]*(a*vv)^FracPart[p])/vv^FracPart[p], Int
[uu*vv^p, x], x]] /; FreeQ[{a, p}, x] && !IntegerQ[p] && !InertTrigFreeQ[
v]
```

Rule 4400

```
Int[(u_.)*((v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> With[{uu = ActivateTri
g[u], vv = ActivateTrig[v], ww = ActivateTrig[w]}, Dist[(vv^m*ww^n)^FracPar
```

```
t[p]/(vv^(m*FracPart[p])*ww^(n*FracPart[p])), Int[uu*vv^(m*p)*ww^(n*p), x],
 x]] /; FreeQ[{m, n, p}, x] &&  !IntegerQ[p] && ( !InertTrigFreeQ[v] ||  !I
nertTrigFreeQ[w])
```

Rubi steps

$$
\begin{aligned}
\int (\cosh(x)\coth(x))^{3/2}\,dx &= \frac{\left(i\sqrt{\cosh(x)\coth(x)}\right)\int(-i\cosh(x)\coth(x))^{3/2}\,dx}{\sqrt{-i\cosh(x)\coth(x)}}\\
&= \frac{\left(i\sqrt{\cosh(x)\coth(x)}\right)\int\cosh^{\frac{3}{2}}(x)(-i\coth(x))^{3/2}\,dx}{\sqrt{\cosh(x)}\sqrt{-i\coth(x)}}\\
&= \frac{2}{3}\cosh(x)\sqrt{\cosh(x)\coth(x)} + \frac{\left(4i\sqrt{\cosh(x)\coth(x)}\right)\int\frac{(-i\coth(x))^{3/2}}{\sqrt{\cosh(x)}}\,dx}{3\sqrt{\cosh(x)}\sqrt{-i\coth(x)}}\\
&= \frac{2}{3}\cosh(x)\sqrt{\cosh(x)\coth(x)} - \frac{8}{3}\sqrt{\cosh(x)\coth(x)}\,\mathrm{sech}(x)
\end{aligned}
$$

Mathematica [A] time = 0.05, size = 21, normalized size = 0.68

$$\frac{2}{3}\left(\cosh^2(x)-4\right)\mathrm{sech}(x)\sqrt{\cosh(x)\coth(x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cosh[x]*Coth[x])^(3/2),x]
[Out] (2*(-4 + Cosh[x]^2)*Sqrt[Cosh[x]*Coth[x]]*Sech[x])/3
```

fricas [B] time = 0.42, size = 97, normalized size = 3.13

$$\frac{\sqrt{\frac{1}{2}}\left(\cosh(x)^4 + 4\cosh(x)\sinh(x)^3 + \sinh(x)^4 + 2\left(3\cosh(x)^2-7\right)\sinh(x)^2 - 14\cosh(x)^2 + 4\left(\cosh(x)^3-7\right.\right.}{3\sqrt{\cosh(x)^3 + 3\cosh(x)\sinh(x)^2 + \sinh(x)^3 + \left(3\cosh(x)^2-1\right)\sinh(x) - \cosh(x)}\,(\cosh(x)+\mathrm{s}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cosh(x)*coth(x))^(3/2),x, algorithm="fricas")
[Out] 1/3*sqrt(1/2)*(cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + 2*(3*cosh(x)^2
 - 7)*sinh(x)^2 - 14*cosh(x)^2 + 4*(cosh(x)^3 - 7*cosh(x))*sinh(x) + 1)/(sq
rt(cosh(x)^3 + 3*cosh(x)*sinh(x)^2 + sinh(x)^3 + (3*cosh(x)^2 - 1)*sinh(x)
- cosh(x))*(cosh(x) + sinh(x)))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (\cosh(x)\coth(x))^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cosh(x)*coth(x))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((cosh(x)*coth(x))^(3/2), x)
```

maple [F] time = 0.44, size = 0, normalized size = 0.00

$$\int (\cosh(x)\coth(x))^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cosh(x)*coth(x))^(3/2),x)
```

```
[Out] int((cosh(x)*coth(x))^(3/2),x)
```

maxima [B] time = 0.62, size = 109, normalized size = 3.52

$$\frac{\sqrt{2}\,e^{\left(\frac{3}{2}\,x\right)}}{6\left(e^{(-x)}+1\right)^{\frac{3}{2}}\left(-e^{(-x)}+1\right)^{\frac{3}{2}}}-\frac{5\sqrt{2}\,e^{\left(-\frac{1}{2}\,x\right)}}{2\left(e^{(-x)}+1\right)^{\frac{3}{2}}\left(-e^{(-x)}+1\right)^{\frac{3}{2}}}+\frac{5\sqrt{2}\,e^{\left(-\frac{5}{2}\,x\right)}}{2\left(e^{(-x)}+1\right)^{\frac{3}{2}}\left(-e^{(-x)}+1\right)^{\frac{3}{2}}}-\frac{\sqrt{2}\,e^{\left(-\frac{9}{2}\,x\right)}}{6\left(e^{(-x)}+1\right)^{\frac{3}{2}}\left(-e^{(-x)}+1\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cosh(x)*coth(x))^(3/2),x, algorithm="maxima")
```

```
[Out] 1/6*sqrt(2)*e^(3/2*x)/((e^(-x) + 1)^(3/2)*(-e^(-x) + 1)^(3/2)) - 5/2*sqrt(2
)*e^(-1/2*x)/((e^(-x) + 1)^(3/2)*(-e^(-x) + 1)^(3/2)) + 5/2*sqrt(2)*e^(-5/2
*x)/((e^(-x) + 1)^(3/2)*(-e^(-x) + 1)^(3/2)) - 1/6*sqrt(2)*e^(-9/2*x)/((e^(
-x) + 1)^(3/2)*(-e^(-x) + 1)^(3/2))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int (\cosh(x)\coth(x))^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cosh(x)*coth(x))^(3/2),x)
```

```
[Out] int((cosh(x)*coth(x))^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cosh(x)*coth(x))**(3/2),x)
```

```
[Out] Timed out
```

3.565 $\int (\cosh(x)\coth(x))^{5/2}\,dx$

Optimal. Leaf size=50

$$\frac{2}{5}\cosh^2(x)\coth(x)\sqrt{\cosh(x)\coth(x)}-\frac{16}{15}\coth(x)\sqrt{\cosh(x)\coth(x)}+\frac{64}{15}\tanh(x)\sqrt{\cosh(x)\coth(x)}$$

```
[Out] -16/15*coth(x)*(cosh(x)*coth(x))^(1/2)+2/5*cosh(x)^2*coth(x)*(cosh(x)*coth(
x))^(1/2)+64/15*(cosh(x)*coth(x))^(1/2)*tanh(x)
```

Rubi [A] time = 0.13, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.556, Rules used = {4398, 4400, 2598, 2594, 2589}

$$\frac{2}{5}\cosh^2(x)\coth(x)\sqrt{\cosh(x)\coth(x)}-\frac{16}{15}\coth(x)\sqrt{\cosh(x)\coth(x)}+\frac{64}{15}\tanh(x)\sqrt{\cosh(x)\coth(x)}$$

Antiderivative was successfully verified.

```
[In] Int[(Cosh[x]*Coth[x])^(5/2),x]
```

```
[Out] (-16*Coth[x]*Sqrt[Cosh[x]*Coth[x]])/15 + (2*Cosh[x]^2*Coth[x]*Sqrt[Cosh[x]*
Coth[x]])/5 + (64*Sqrt[Cosh[x]*Coth[x]]*Tanh[x])/15
```

Rule 2589

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> -Simp[(b*(a*Sin[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*m
), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n - 1, 0]
```

Rule 2594

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sin[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
n - 1)), x] - Dist[(b^2*(m + n - 1))/(n - 1), Int[(a*Sin[e + f*x])^m*(b*Tan
[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] && Int
egersQ[2*m, 2*n] &&  !(GtQ[m, 1] &&  !IntegerQ[(m - 1)/2])
```

Rule 2598

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> -Simp[(b*(a*Sin[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f
*m), x] + Dist[(a^2*(m + n - 1))/m, Int[(a*Sin[e + f*x])^(m - 2)*(b*Tan[e +
 f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (GtQ[m, 1] || (EqQ[m, 1] &
& EqQ[n, 1/2])) && IntegersQ[2*m, 2*n]
```

Rule 4398

```
Int[(u_.)*((a_)*(v_))^(p_), x_Symbol] :> With[{uu = ActivateTrig[u], vv = A
ctivateTrig[v]}, Dist[(a^IntPart[p]*(a*vv)^FracPart[p])/vv^FracPart[p], Int
[uu*vv^p, x], x]] /; FreeQ[{a, p}, x] &&  !IntegerQ[p] &&  !InertTrigFreeQ[
v]
```

Rule 4400

```
Int[(u_.)*((v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> With[{uu = ActivateTri
g[u], vv = ActivateTrig[v], ww = ActivateTrig[w]}, Dist[(vv^m*ww^n)^FracPar
t[p]/(vv^(m*FracPart[p])*ww^(n*FracPart[p])), Int[uu*vv^(m*p)*ww^(n*p), x],
 x]] /; FreeQ[{m, n, p}, x] &&  !IntegerQ[p] && ( !InertTrigFreeQ[v] ||  !I
nertTrigFreeQ[w])
```

Rubi steps

$$\begin{aligned}\int (\cosh(x)\coth(x))^{5/2}\,dx &= -\frac{\sqrt{\cosh(x)\coth(x)}\int(-i\cosh(x)\coth(x))^{5/2}\,dx}{\sqrt{-i\cosh(x)\coth(x)}}\\ &= -\frac{\sqrt{\cosh(x)\coth(x)}\int\cosh^{\frac{5}{2}}(x)(-i\coth(x))^{5/2}\,dx}{\sqrt{\cosh(x)}\sqrt{-i\coth(x)}}\\ &= \frac{2}{5}\cosh^2(x)\coth(x)\sqrt{\cosh(x)\coth(x)} - \frac{\left(8\sqrt{\cosh(x)\coth(x)}\right)\int\sqrt{\cosh(x)}(-i\coth\\ }{5\sqrt{\cosh(x)}\sqrt{-i\coth(x)}}\\ &= -\frac{16}{15}\coth(x)\sqrt{\cosh(x)\coth(x)} + \frac{2}{5}\cosh^2(x)\coth(x)\sqrt{\cosh(x)\coth(x)} + \frac{\left(32\sqrt{\cos\\}{}\\ &= -\frac{16}{15}\coth(x)\sqrt{\cosh(x)\coth(x)} + \frac{2}{5}\cosh^2(x)\coth(x)\sqrt{\cosh(x)\coth(x)} + \frac{64}{15}\sqrt{\cosh\\ \end{aligned}$$

Mathematica [A] time = 0.32, size = 44, normalized size = 0.88

$$\frac{1}{15}\sqrt{\cosh(x)\coth(x)}\left(64\tanh(x)-10\coth(x)+6\sinh(x)\cosh(x)+57\left(-\sinh^2(x)\right)^{3/4}\operatorname{csch}(x)\operatorname{sech}(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cosh[x]*Coth[x])^(5/2),x]
```

```
[Out] (Sqrt[Cosh[x]*Coth[x]]*(-10*Coth[x] + 6*Cosh[x]*Sinh[x] + 57*Csch[x]*Sech[x
]*(-Sinh[x]^2)^(3/4) + 64*Tanh[x]))/15
```

fricas [B] time = 0.42, size = 259, normalized size = 5.18

$$\frac{\sqrt{\frac{1}{2}}\left(3\,\cosh(x)^{8}+24\,\cosh(x)\sinh(x)^{7}+3\,\sinh(x)^{8}+12\left(7\,\cosh(x)^{2}+9\right)\sinh(x)^{6}+108\,\cosh(x)^{6}+24\left(7\,\cos\right.\right.}{30\,(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cosh(x)*coth(x))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/30*sqrt(1/2)*(3*cosh(x)^8 + 24*cosh(x)*sinh(x)^7 + 3*sinh(x)^8 + 12*(7*co
sh(x)^2 + 9)*sinh(x)^6 + 108*cosh(x)^6 + 24*(7*cosh(x)^3 + 27*cosh(x))*sinh
(x)^5 + 2*(105*cosh(x)^4 + 810*cosh(x)^2 - 151)*sinh(x)^4 - 302*cosh(x)^4 +
 8*(21*cosh(x)^5 + 270*cosh(x)^3 - 151*cosh(x))*sinh(x)^3 + 12*(7*cosh(x)^6
 + 135*cosh(x)^4 - 151*cosh(x)^2 + 9)*sinh(x)^2 + 108*cosh(x)^2 + 8*(3*cosh
(x)^7 + 81*cosh(x)^5 - 151*cosh(x)^3 + 27*cosh(x))*sinh(x) + 3)/((cosh(x)^4
 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + (6*cosh(x)^2 - 1)*sinh(x)^2 - cosh(x)^
2 + 2*(2*cosh(x)^3 - cosh(x))*sinh(x))*sqrt(cosh(x)^3 + 3*cosh(x)*sinh(x)^2
 + sinh(x)^3 + (3*cosh(x)^2 - 1)*sinh(x) - cosh(x)))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int(\cosh(x)\coth(x))^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cosh(x)*coth(x))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((cosh(x)*coth(x))^(5/2), x)
```

maple [F] time = 0.44, size = 0, normalized size = 0.00

$$\int(\cosh(x)\coth(x))^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cosh(x)*coth(x))^(5/2),x)
```

```
[Out] int((cosh(x)*coth(x))^(5/2),x)
```

maxima [B] time = 0.72, size = 163, normalized size = 3.26

$$\frac{\sqrt{2}e^{\left(\frac{5}{2}x\right)}}{20\left(e^{(-x)}+1\right)^{\frac{5}{2}}\left(-e^{(-x)}+1\right)^{\frac{5}{2}}}+\frac{7\sqrt{2}e^{\left(\frac{1}{2}x\right)}}{4\left(e^{(-x)}+1\right)^{\frac{5}{2}}\left(-e^{(-x)}+1\right)^{\frac{5}{2}}}-\frac{41\sqrt{2}e^{\left(-\frac{3}{2}x\right)}}{6\left(e^{(-x)}+1\right)^{\frac{5}{2}}\left(-e^{(-x)}+1\right)^{\frac{5}{2}}}+\frac{41\sqrt{2}e^{\left(-\frac{7}{2}x\right)}}{6\left(e^{(-x)}+1\right)^{\frac{5}{2}}\left(-e^{(-x)}+1\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cosh(x)*coth(x))^(5/2),x, algorithm="maxima")
```

```
[Out] 1/20*sqrt(2)*e^(5/2*x)/((e^(-x) + 1)^(5/2)*(-e^(-x) + 1)^(5/2)) + 7/4*sqrt(
2)*e^(1/2*x)/((e^(-x) + 1)^(5/2)*(-e^(-x) + 1)^(5/2)) - 41/6*sqrt(2)*e^(-3/
2*x)/((e^(-x) + 1)^(5/2)*(-e^(-x) + 1)^(5/2)) + 41/6*sqrt(2)*e^(-7/2*x)/((e
^(-x) + 1)^(5/2)*(-e^(-x) + 1)^(5/2)) - 7/4*sqrt(2)*e^(-11/2*x)/((e^(-x) +
1)^(5/2)*(-e^(-x) + 1)^(5/2)) - 1/20*sqrt(2)*e^(-15/2*x)/((e^(-x) + 1)^(5/2
)*(-e^(-x) + 1)^(5/2))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int (\cosh(x)\coth(x))^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cosh(x)*coth(x))^(5/2),x)
```

```
[Out] int((cosh(x)*coth(x))^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cosh(x)*coth(x))**(5/2),x)
```

```
[Out] Timed out
```

3.566 $\int \frac{b+c+\cosh(x)}{a+b\sinh(x)}\,dx$

Optimal. Leaf size=52

$$\frac{\log(a+b\sinh(x))}{b}-\frac{2(b+c)\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}}$$

```
[Out] ln(a+b*sinh(x))/b-2*(b+c)*arctanh((b-a*tanh(1/2*x))/(a^2+b^2)^(1/2))/(a^2+b
^2)^(1/2)
```

Rubi [A] time = 0.13, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {4401, 2660, 618, 206, 2668, 31}

$$\frac{\log(a+b\sinh(x))}{b}-\frac{2(b+c)\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(b + c + Cosh[x])/(a + b*Sinh[x]),x]
```

```
[Out] (-2*(b + c)*ArcTanh[(b - a*Tanh[x/2])/Sqrt[a^2 + b^2]])/Sqrt[a^2 + b^2] + L
og[a + b*Sinh[x]]/b
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 4401

```
Int[u_, x_Symbol] :> With[{v = ExpandTrig[u, x]}, Int[v, x] /; SumQ[v]] /;
 !InertTrigFreeQ[u]
```

Rubi steps

$$
\begin{aligned}
\int \frac{b+c+\cosh(x)}{a+b\sinh(x)}\,dx &= \int \left(\frac{\left(1+\frac{b}{c}\right)c}{a+b\sinh(x)}+\frac{\cosh(x)}{a+b\sinh(x)}\right)dx\\
&= (b+c)\int \frac{1}{a+b\sinh(x)}\,dx+\int \frac{\cosh(x)}{a+b\sinh(x)}\,dx\\
&= \frac{\operatorname{Subst}\left(\int \frac{1}{a+x}\,dx,x,b\sinh(x)\right)}{b}+(2(b+c))\operatorname{Subst}\left(\int \frac{1}{a+2bx-ax^2}\,dx,x,\tanh\left(\frac{x}{2}\right)\right)\\
&= \frac{\log(a+b\sinh(x))}{b}-(4(b+c))\operatorname{Subst}\left(\int \frac{1}{4\left(a^2+b^2\right)-x^2}\,dx,x,2b-2a\tanh\left(\frac{x}{2}\right)\right)\\
&= -\frac{2(b+c)\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}}+\frac{\log(a+b\sinh(x))}{b}
\end{aligned}
$$

Mathematica [A] time = 0.12, size = 60, normalized size = 1.15

$$
\frac{2(b+c)\tan^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-a^2-b^2}}+\frac{\log(a+b\sinh(x))}{b}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(b + c + Cosh[x])/(a + b*Sinh[x]),x]
```

```
[Out] (2*(b + c)*ArcTan[(b - a*Tanh[x/2])/Sqrt[-a^2 - b^2]])/Sqrt[-a^2 - b^2] + L
og[a + b*Sinh[x]]/b
```

fricas [B] time = 0.43, size = 167, normalized size = 3.21

$$\frac{\sqrt{a^2+b^2}\left(b^2+bc\right)\log\left(\frac{b^2\cosh(x)^2+b^2\sinh(x)^2+2\,ab\cosh(x)+2\,a^2+b^2+2\left(b^2\cosh(x)+ab\right)\sinh(x)-2\,\sqrt{a^2+b^2}\,(b\cosh(x)+b\sinh(x)+a)}{b\cosh(x)^2+b\sinh(x)^2+2\,a\cosh(x)+2\,(b\cosh(x)+a)\sinh(x)-b}\right)-\left(a\right.}{a^2b+b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b+c+cosh(x))/(a+b*sinh(x)),x, algorithm="fricas")
```

```
[Out] (sqrt(a^2 + b^2)*(b^2 + b*c)*log((b^2*cosh(x)^2 + b^2*sinh(x)^2 + 2*a*b*cos
h(x) + 2*a^2 + b^2 + 2*(b^2*cosh(x) + a*b)*sinh(x) - 2*sqrt(a^2 + b^2)*(b*c
osh(x) + b*sinh(x) + a))/(b*cosh(x)^2 + b*sinh(x)^2 + 2*a*cosh(x) + 2*(b*co
sh(x) + a)*sinh(x) - b)) - (a^2 + b^2)*x + (a^2 + b^2)*log(2*(b*sinh(x) + a
)/(cosh(x) - sinh(x))))/(a^2*b + b^3)
```

giac [A] time = 0.14, size = 87, normalized size = 1.67

$$\frac{(b+c)\log\left(\frac{\left|2\,be^x+2\,a-2\,\sqrt{a^2+b^2}\right|}{\left|2\,be^x+2\,a+2\,\sqrt{a^2+b^2}\right|}\right)}{\sqrt{a^2+b^2}}-\frac{x}{b}+\frac{\log\left(\left|be^{(2\,x)}+2\,ae^x-b\right|\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b+c+cosh(x))/(a+b*sinh(x)),x, algorithm="giac")
```

```
[Out] (b + c)*log(abs(2*b*e^x + 2*a - 2*sqrt(a^2 + b^2))/abs(2*b*e^x + 2*a + 2*sq
rt(a^2 + b^2)))/sqrt(a^2 + b^2) - x/b + log(abs(b*e^(2*x) + 2*a*e^x - b))/b
```

maple [B] time = 0.14, size = 120, normalized size = 2.31

$$-\frac{\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{b}-\frac{\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)}{b}+\frac{\ln\left(a\left(\tanh^2\left(\frac{x}{2}\right)\right)-2\tanh\left(\frac{x}{2}\right)b-a\right)}{b}+\frac{2b\,\mathrm{arctanh}\left(\frac{2a\tanh\left(\frac{x}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}}+\frac{2\,\mathrm{arcta}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b+c+cosh(x))/(a+b*sinh(x)),x)
```

```
[Out] -1/b*ln(tanh(1/2*x)-1)-1/b*ln(tanh(1/2*x)+1)+1/b*ln(a*tanh(1/2*x)^2-2*tanh(
1/2*x)*b-a)+2*b/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tanh(1/2*x)-2*b)/(a^2+b^2)
```

```
^(1/2))+2/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tanh(1/2*x)-2*b)/(a^2+b^2)^(1/2)
)*c
```

maxima [B] time = 0.53, size = 122, normalized size = 2.35

$$\frac{b\log\left(\frac{be^{(-x)}-a-\sqrt{a^2+b^2}}{be^{(-x)}-a+\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}}+\frac{c\log\left(\frac{be^{(-x)}-a-\sqrt{a^2+b^2}}{be^{(-x)}-a+\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}}+\frac{\log\left(b\sinh(x)+a\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b+c+cosh(x))/(a+b*sinh(x)),x, algorithm="maxima")
```

```
[Out] b*log((b*e^(-x) - a - sqrt(a^2 + b^2))/(b*e^(-x) - a + sqrt(a^2 + b^2)))/sq
rt(a^2 + b^2) + c*log((b*e^(-x) - a - sqrt(a^2 + b^2))/(b*e^(-x) - a + sqrt
(a^2 + b^2)))/sqrt(a^2 + b^2) + log(b*sinh(x) + a)/b
```

mupad [B] time = 1.83, size = 178, normalized size = 3.42

$$\frac{\ln\left(a^2\,e^x-b\,\sqrt{a^2+b^2}+b^2\,e^x+a\,e^x\,\sqrt{a^2+b^2}\right)\,\left(b^2\,\sqrt{a^2+b^2}+a^2+b^2+b\,c\,\sqrt{a^2+b^2}\right)}{a^2\,b+b^3}-\frac{\ln\left(b\,\sqrt{a^2+b^2}+a^2\,e^{\cdots}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b + c + cosh(x))/(a + b*sinh(x)),x)
```

```
[Out] (log(a^2*exp(x) - b*(a^2 + b^2)^(1/2) + b^2*exp(x) + a*exp(x)*(a^2 + b^2)^(
1/2))*(b^2*(a^2 + b^2)^(1/2) + a^2 + b^2 + b*c*(a^2 + b^2)^(1/2)))/(a^2*b +
 b^3) - (log(b*(a^2 + b^2)^(1/2) + a^2*exp(x) + b^2*exp(x) - a*exp(x)*(a^2
+ b^2)^(1/2))*(b^2*(a^2 + b^2)^(1/2) - a^2 - b^2 + b*c*(a^2 + b^2)^(1/2)))/
(a^2*b + b^3) - x/b
```

sympy [A] time = 88.92, size = 843, normalized size = 16.21

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b+c+cosh(x))/(a+b*sinh(x)),x)
```

```
[Out] Piecewise((zoo*(c*log(tanh(x/2)) + x - 2*log(tanh(x/2) + 1) + log(tanh(x/2)
)), Eq(a, 0) & Eq(b, 0)), ((b*log(tanh(x/2)) + c*log(tanh(x/2)) + x - 2*log
(tanh(x/2) + 1) + log(tanh(x/2)))/b, Eq(a, 0)), (b*x*tanh(x/2)/(b**2*tanh(x
/2) - I*b*sqrt(b**2)) + 2*I*b*sqrt(b**2)/(b**2*tanh(x/2) - I*b*sqrt(b**2))
+ 2*b*log(-I*b/sqrt(b**2) + tanh(x/2))*tanh(x/2)/(b**2*tanh(x/2) - I*b*sqrt
(b**2)) - 2*b*log(tanh(x/2) + 1)*tanh(x/2)/(b**2*tanh(x/2) - I*b*sqrt(b**2)
```

```
) + 2*I*c*sqrt(b**2)/(b**2*tanh(x/2) - I*b*sqrt(b**2)) - I*x*sqrt(b**2)/(b*
*2*tanh(x/2) - I*b*sqrt(b**2)) - 2*I*sqrt(b**2)*log(-I*b/sqrt(b**2) + tanh(
x/2))/(b**2*tanh(x/2) - I*b*sqrt(b**2)) + 2*I*sqrt(b**2)*log(tanh(x/2) + 1)
/(b**2*tanh(x/2) - I*b*sqrt(b**2)), Eq(a, -sqrt(-b**2))), (b*x*tanh(x/2)/(b
**2*tanh(x/2) + I*b*sqrt(b**2)) - 2*I*b*sqrt(b**2)/(b**2*tanh(x/2) + I*b*sq
rt(b**2)) + 2*b*log(I*b/sqrt(b**2) + tanh(x/2))*tanh(x/2)/(b**2*tanh(x/2) +
 I*b*sqrt(b**2)) - 2*b*log(tanh(x/2) + 1)*tanh(x/2)/(b**2*tanh(x/2) + I*b*s
qrt(b**2)) - 2*I*c*sqrt(b**2)/(b**2*tanh(x/2) + I*b*sqrt(b**2)) + I*x*sqrt(
b**2)/(b**2*tanh(x/2) + I*b*sqrt(b**2)) + 2*I*sqrt(b**2)*log(I*b/sqrt(b**2)
 + tanh(x/2))/(b**2*tanh(x/2) + I*b*sqrt(b**2)) - 2*I*sqrt(b**2)*log(tanh(x
/2) + 1)/(b**2*tanh(x/2) + I*b*sqrt(b**2)), Eq(a, sqrt(-b**2))), ((c*x + si
nh(x))/a, Eq(b, 0)), (-b*log(tanh(x/2) - b/a - sqrt(a**2 + b**2)/a)/sqrt(a*
*2 + b**2) + b*log(tanh(x/2) - b/a + sqrt(a**2 + b**2)/a)/sqrt(a**2 + b**2)
 - c*log(tanh(x/2) - b/a - sqrt(a**2 + b**2)/a)/sqrt(a**2 + b**2) + c*log(t
anh(x/2) - b/a + sqrt(a**2 + b**2)/a)/sqrt(a**2 + b**2) + x/b - 2*log(tanh(
x/2) + 1)/b + log(tanh(x/2) - b/a - sqrt(a**2 + b**2)/a)/b + log(tanh(x/2)
- b/a + sqrt(a**2 + b**2)/a)/b, True))
```

3.567 $\int \frac{b+c+\cosh(x)}{a-b\sinh(x)}\,dx$

Optimal. Leaf size=53

$$\frac{2(b+c)\tanh^{-1}\left(\frac{a\tanh\left(\frac{x}{2}\right)+b}{\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}}-\frac{\log(a-b\sinh(x))}{b}$$

```
[Out] -ln(a-b*sinh(x))/b+2*(b+c)*arctanh((b+a*tanh(1/2*x))/(a^2+b^2)^(1/2))/(a^2+
b^2)^(1/2)
```

Rubi [A] time = 0.13, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {4401, 2660, 618, 206, 2668, 31}

$$\frac{2(b+c)\tanh^{-1}\left(\frac{a\tanh\left(\frac{x}{2}\right)+b}{\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}}-\frac{\log(a-b\sinh(x))}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(b + c + Cosh[x])/(a - b*Sinh[x]),x]
```

```
[Out] (2*(b + c)*ArcTanh[(b + a*Tanh[x/2])/Sqrt[a^2 + b^2]])/Sqrt[a^2 + b^2] - Lo
g[a - b*Sinh[x]]/b
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 4401

```
Int[u_, x_Symbol] :> With[{v = ExpandTrig[u, x]}, Int[v, x] /; SumQ[v]] /;
 !InertTrigFreeQ[u]
```

Rubi steps

$$
\begin{aligned}
\int \frac{b+c+\cosh(x)}{a-b\sinh(x)}\,dx &= \int \left(\frac{\left(1+\frac{b}{c}\right)c}{a-b\sinh(x)}+\frac{\cosh(x)}{a-b\sinh(x)}\right)dx \\
&= (b+c)\int \frac{1}{a-b\sinh(x)}\,dx+\int \frac{\cosh(x)}{a-b\sinh(x)}\,dx \\
&= -\frac{\text{Subst}\left(\int \frac{1}{a+x}\,dx,x,-b\sinh(x)\right)}{b}+(2(b+c))\,\text{Subst}\left(\int \frac{1}{a-2bx-ax^2}\,dx,x,\tanh\left(\frac{x}{2}\right)\right) \\
&= -\frac{\log(a-b\sinh(x))}{b}-(4(b+c))\,\text{Subst}\left(\int \frac{1}{4\left(a^2+b^2\right)-x^2}\,dx,x,-2b-2a\tanh\left(\frac{x}{2}\right)\right) \\
&= \frac{2(b+c)\tanh^{-1}\left(\frac{b+a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}}-\frac{\log(a-b\sinh(x))}{b}
\end{aligned}
$$

Mathematica [A] time = 0.11, size = 62, normalized size = 1.17

$$
-\frac{2(b+c)\tan^{-1}\left(\frac{a\tanh\left(\frac{x}{2}\right)+b}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-a^2-b^2}}-\frac{\log(b\sinh(x)-a)}{b}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(b + c + Cosh[x])/(a - b*Sinh[x]),x]
```

```
[Out] (-2*(b + c)*ArcTan[(b + a*Tanh[x/2])/Sqrt[-a^2 - b^2]])/Sqrt[-a^2 - b^2] -
Log[-a + b*Sinh[x]]/b
```

fricas [B] time = 0.42, size = 174, normalized size = 3.28

$$\frac{\sqrt{a^2+b^2}\left(b^2+bc\right)\log\left(\frac{b^2\cosh(x)^2+b^2\sinh(x)^2-2\,ab\cosh(x)+2\,a^2+b^2+2\left(b^2\cosh(x)-ab\right)\sinh(x)+2\,\sqrt{a^2+b^2}\,(b\cosh(x)+b\sinh(x)-a)}{b\cosh(x)^2+b\sinh(x)^2-2\,a\cosh(x)+2\,(b\cosh(x)-a)\sinh(x)-b}\right)+(}{a^2b+b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b+c+cosh(x))/(a-b*sinh(x)),x, algorithm="fricas")
```

```
[Out] (sqrt(a^2 + b^2)*(b^2 + b*c)*log((b^2*cosh(x)^2 + b^2*sinh(x)^2 - 2*a*b*cos
h(x) + 2*a^2 + b^2 + 2*(b^2*cosh(x) - a*b)*sinh(x) + 2*sqrt(a^2 + b^2)*(b*c
osh(x) + b*sinh(x) - a))/(b*cosh(x)^2 + b*sinh(x)^2 - 2*a*cosh(x) + 2*(b*co
sh(x) - a)*sinh(x) - b)) + (a^2 + b^2)*x - (a^2 + b^2)*log(2*(b*sinh(x) - a
)/(cosh(x) - sinh(x))))/(a^2*b + b^3)
```

giac [A] time = 0.17, size = 88, normalized size = 1.66

$$-\frac{(b+c)\log\left(\frac{\left|2\,be^x-2\,a-2\,\sqrt{a^2+b^2}\right|}{\left|2\,be^x-2\,a+2\,\sqrt{a^2+b^2}\right|}\right)}{\sqrt{a^2+b^2}}+\frac{x}{b}-\frac{\log\left(\left|be^{(2\,x)}-2\,ae^x-b\right|\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b+c+cosh(x))/(a-b*sinh(x)),x, algorithm="giac")
```

```
[Out] -(b + c)*log(abs(2*b*e^x - 2*a - 2*sqrt(a^2 + b^2))/abs(2*b*e^x - 2*a + 2*s
qrt(a^2 + b^2)))/sqrt(a^2 + b^2) + x/b - log(abs(b*e^(2*x) - 2*a*e^x - b))/
b
```

maple [B] time = 0.14, size = 119, normalized size = 2.25

$$-\frac{\ln\left(a\left(\tanh^2\left(\frac{x}{2}\right)\right)+2\tanh\left(\frac{x}{2}\right)b-a\right)}{b}+\frac{2b\,\mathrm{arctanh}\left(\frac{2a\tanh\left(\frac{x}{2}\right)+2b}{2\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}}+\frac{2\,\mathrm{arctanh}\left(\frac{2a\tanh\left(\frac{x}{2}\right)+2b}{2\sqrt{a^2+b^2}}\right)c}{\sqrt{a^2+b^2}}+\frac{\ln\left(\tanh\left(\frac{x}{2}\right)-\right.}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b+c+cosh(x))/(a-b*sinh(x)),x)
```

```
[Out] -1/b*ln(a*tanh(1/2*x)^2+2*tanh(1/2*x)*b-a)+2*b/(a^2+b^2)^(1/2)*arctanh(1/2*
(2*a*tanh(1/2*x)+2*b)/(a^2+b^2)^(1/2))+2/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*t
```

```
anh(1/2*x)+2*b)/(a^2+b^2)^(1/2))*c+1/b*ln(tanh(1/2*x)-1)+1/b*ln(tanh(1/2*x)
+1)
```

maxima [B] time = 0.41, size = 119, normalized size = 2.25

$$-\frac{b\log\left(\frac{be^{(-x)}+a-\sqrt{a^2+b^2}}{be^{(-x)}+a+\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}}-\frac{c\log\left(\frac{be^{(-x)}+a-\sqrt{a^2+b^2}}{be^{(-x)}+a+\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}}-\frac{\log\left(b\sinh(x)-a\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b+c+cosh(x))/(a-b*sinh(x)),x, algorithm="maxima")
```

```
[Out] -b*log((b*e^(-x) + a - sqrt(a^2 + b^2))/(b*e^(-x) + a + sqrt(a^2 + b^2)))/s
qrt(a^2 + b^2) - c*log((b*e^(-x) + a - sqrt(a^2 + b^2))/(b*e^(-x) + a + sqr
t(a^2 + b^2)))/sqrt(a^2 + b^2) - log(b*sinh(x) - a)/b
```

mupad [B] time = 0.32, size = 177, normalized size = 3.34

$$\frac{x}{b}+\frac{\ln\left(b\sqrt{a^2+b^2}+a^2\,\mathrm{e}^x+b^2\,\mathrm{e}^x+a\,\mathrm{e}^x\sqrt{a^2+b^2}\right)\left(b^2\sqrt{a^2+b^2}-a^2-b^2+b\,c\sqrt{a^2+b^2}\right)}{a^2\,b+b^3}-\frac{\ln\left(b\sqrt{a^2+b^2}-a^2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b + c + cosh(x))/(a - b*sinh(x)),x)
```

```
[Out] x/b + (log(b*(a^2 + b^2)^(1/2) + a^2*exp(x) + b^2*exp(x) + a*exp(x)*(a^2 +
b^2)^(1/2))*(b^2*(a^2 + b^2)^(1/2) - a^2 - b^2 + b*c*(a^2 + b^2)^(1/2)))/(a
^2*b + b^3) - (log(b*(a^2 + b^2)^(1/2) - a^2*exp(x) - b^2*exp(x) + a*exp(x)
*(a^2 + b^2)^(1/2))*(b^2*(a^2 + b^2)^(1/2) + a^2 + b^2 + b*c*(a^2 + b^2)^(1
/2)))/(a^2*b + b^3)
```

sympy [A] time = 89.17, size = 845, normalized size = 15.94

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b+c+cosh(x))/(a-b*sinh(x)),x)
```

```
[Out] Piecewise((zoo*(c*log(tanh(x/2)) + x - 2*log(tanh(x/2) + 1) + log(tanh(x/2)
)), Eq(a, 0) & Eq(b, 0)), (-(b*log(tanh(x/2)) + c*log(tanh(x/2)) + x - 2*lo
g(tanh(x/2) + 1) + log(tanh(x/2)))/b, Eq(a, 0)), (-b*x*tanh(x/2)/(b**2*tanh
(x/2) + I*b*sqrt(b**2)) + 2*I*b*sqrt(b**2)/(b**2*tanh(x/2) + I*b*sqrt(b**2)
) - 2*b*log(I*b/sqrt(b**2) + tanh(x/2))*tanh(x/2)/(b**2*tanh(x/2) + I*b*sqr
t(b**2)) + 2*b*log(tanh(x/2) + 1)*tanh(x/2)/(b**2*tanh(x/2) + I*b*sqrt(b**2
```

```
)) + 2*I*c*sqrt(b**2)/(b**2*tanh(x/2) + I*b*sqrt(b**2)) - I*x*sqrt(b**2)/(b
**2*tanh(x/2) + I*b*sqrt(b**2)) - 2*I*sqrt(b**2)*log(I*b/sqrt(b**2) + tanh(
x/2))/(b**2*tanh(x/2) + I*b*sqrt(b**2)) + 2*I*sqrt(b**2)*log(tanh(x/2) + 1)
/(b**2*tanh(x/2) + I*b*sqrt(b**2)), Eq(a, -sqrt(-b**2))), (-b*x*tanh(x/2)/(
b**2*tanh(x/2) - I*b*sqrt(b**2)) - 2*I*b*sqrt(b**2)/(b**2*tanh(x/2) - I*b*s
qrt(b**2)) - 2*b*log(-I*b/sqrt(b**2) + tanh(x/2))*tanh(x/2)/(b**2*tanh(x/2)
 - I*b*sqrt(b**2)) + 2*b*log(tanh(x/2) + 1)*tanh(x/2)/(b**2*tanh(x/2) - I*b
*sqrt(b**2)) - 2*I*c*sqrt(b**2)/(b**2*tanh(x/2) - I*b*sqrt(b**2)) + I*x*sqr
t(b**2)/(b**2*tanh(x/2) - I*b*sqrt(b**2)) + 2*I*sqrt(b**2)*log(-I*b/sqrt(b*
*2) + tanh(x/2))/(b**2*tanh(x/2) - I*b*sqrt(b**2)) - 2*I*sqrt(b**2)*log(tan
h(x/2) + 1)/(b**2*tanh(x/2) - I*b*sqrt(b**2)), Eq(a, sqrt(-b**2))), ((c*x +
 sinh(x))/a, Eq(b, 0)), (-b*log(tanh(x/2) + b/a - sqrt(a**2 + b**2)/a)/sqrt
(a**2 + b**2) + b*log(tanh(x/2) + b/a + sqrt(a**2 + b**2)/a)/sqrt(a**2 + b*
*2) - c*log(tanh(x/2) + b/a - sqrt(a**2 + b**2)/a)/sqrt(a**2 + b**2) + c*lo
g(tanh(x/2) + b/a + sqrt(a**2 + b**2)/a)/sqrt(a**2 + b**2) - x/b + 2*log(ta
nh(x/2) + 1)/b - log(tanh(x/2) + b/a - sqrt(a**2 + b**2)/a)/b - log(tanh(x/
2) + b/a + sqrt(a**2 + b**2)/a)/b, True))
```

3.568 $\int \frac{b+c+\sinh(x)}{a+b\cosh(x)}\,dx$

Optimal. Leaf size=57

$$\frac{2(b+c)\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}\,\sqrt{a+b}}+\frac{\log(a+b\cosh(x))}{b}$$

```
[Out] ln(a+b*cosh(x))/b+2*(b+c)*arctanh((a-b)^(1/2)*tanh(1/2*x)/(a+b)^(1/2))/(a-b
)^(1/2)/(a+b)^(1/2)
```

Rubi [A] time = 0.13, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {4401, 2659, 208, 2668, 31}

$$\frac{2(b+c)\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}\,\sqrt{a+b}}+\frac{\log(a+b\cosh(x))}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(b + c + Sinh[x])/(a + b*Cosh[x]),x]
```

```
[Out] (2*(b + c)*ArcTanh[(Sqrt[a - b]*Tanh[x/2])/Sqrt[a + b]])/(Sqrt[a - b]*Sqrt[
a + b]) + Log[a + b*Cosh[x]]/b
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 4401

```
Int[u_, x_Symbol] :> With[{v = ExpandTrig[u, x]}, Int[v, x] /; SumQ[v]] /;
 !InertTrigFreeQ[u]
```

Rubi steps

$$\begin{aligned}\int \frac{b+c+\sinh(x)}{a+b\cosh(x)}\,dx &= \int \left(\frac{b+c}{a+b\cosh(x)}+\frac{\sinh(x)}{a+b\cosh(x)}\right)dx\\ &= (b+c)\int \frac{1}{a+b\cosh(x)}\,dx+\int \frac{\sinh(x)}{a+b\cosh(x)}\,dx\\ &= \frac{\text{Subst}\left(\int \frac{1}{a+x}\,dx, x, b\cosh(x)\right)}{b}+(2(b+c))\,\text{Subst}\left(\int \frac{1}{a+b-(a-b)x^2}\,dx, x, \tanh\left(\frac{x}{2}\right)\right)\\ &= \frac{2(b+c)\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}\,\sqrt{a+b}}+\frac{\log(a+b\cosh(x))}{b}\end{aligned}$$

Mathematica [A] time = 0.10, size = 56, normalized size = 0.98

$$\frac{\log(a+b\cosh(x))}{b}-\frac{2(b+c)\tan^{-1}\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{b^2-a^2}}\right)}{\sqrt{b^2-a^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b + c + Sinh[x])/(a + b*Cosh[x]),x]
```

```
[Out] (-2*(b + c)*ArcTan[((a - b)*Tanh[x/2])/Sqrt[-a^2 + b^2]])/Sqrt[-a^2 + b^2]
+ Log[a + b*Cosh[x]]/b
```

fricas [B] time = 0.44, size = 289, normalized size = 5.07

$$\left[\frac{\sqrt{a^2-b^2}\left(b^2+bc\right)\log\left(\frac{b^2\cosh(x)^2+b^2\sinh(x)^2+2\,ab\cosh(x)+2\,a^2-b^2+2\left(b^2\cosh(x)+ab\right)\sinh(x)-2\,\sqrt{a^2-b^2}\,(b\cosh(x)+b\sinh(x)+a)}{b\cosh(x)^2+b\sinh(x)^2+2\,a\cosh(x)+2\,(b\cosh(x)+a)\sinh(x)+b}\right)-}{a^2b-b^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b+c+sinh(x))/(a+b*cosh(x)),x, algorithm="fricas")
```

```
[Out] [(sqrt(a^2 - b^2)*(b^2 + b*c)*log((b^2*cosh(x)^2 + b^2*sinh(x)^2 + 2*a*b*co
sh(x) + 2*a^2 - b^2 + 2*(b^2*cosh(x) + a*b)*sinh(x) - 2*sqrt(a^2 - b^2)*(b*
cosh(x) + b*sinh(x) + a))/(b*cosh(x)^2 + b*sinh(x)^2 + 2*a*cosh(x) + 2*(b*c
osh(x) + a)*sinh(x) + b)) - (a^2 - b^2)*x + (a^2 - b^2)*log(2*(b*cosh(x) +
a)/(cosh(x) - sinh(x))))/(a^2*b - b^3), -(2*sqrt(-a^2 + b^2)*(b^2 + b*c)*ar
ctan(-sqrt(-a^2 + b^2)*(b*cosh(x) + b*sinh(x) + a)/(a^2 - b^2)) + (a^2 - b^
2)*x - (a^2 - b^2)*log(2*(b*cosh(x) + a)/(cosh(x) - sinh(x))))/(a^2*b - b^3
)]
```

giac [A] time = 0.12, size = 60, normalized size = 1.05

$$\frac{2\,(b+c)\arctan\left(\frac{be^{x}+a}{\sqrt{-a^{2}+b^{2}}}\right)}{\sqrt{-a^{2}+b^{2}}}-\frac{x}{b}+\frac{\log\left(be^{(2\,x)}+2\,ae^{x}+b\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b+c+sinh(x))/(a+b*cosh(x)),x, algorithm="giac")
```

```
[Out] 2*(b + c)*arctan((b*e^x + a)/sqrt(-a^2 + b^2))/sqrt(-a^2 + b^2) - x/b + log
(b*e^(2*x) + 2*a*e^x + b)/b
```

maple [B] time = 0.13, size = 127, normalized size = 2.23

$$-\frac{\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{b}-\frac{\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)}{b}+\frac{\ln\left(a\left(\tanh^{2}\left(\frac{x}{2}\right)\right)-\left(\tanh^{2}\left(\frac{x}{2}\right)\right)b-a-b\right)}{b}+\frac{2b\,\operatorname{arctanh}\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{\sqrt{(a+b)\,(a-b)}}+\frac{2}{-}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b+c+sinh(x))/(a+b*cosh(x)),x)
```

```
[Out] -1/b*ln(tanh(1/2*x)-1)-1/b*ln(tanh(1/2*x)+1)+1/b*ln(a*tanh(1/2*x)^2-tanh(1/
2*x)^2*b-a-b)+2*b/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tanh(1/2*x)/((a+b)*(a-b
))^(1/2))+2/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tanh(1/2*x)/((a+b)*(a-b))^(1/
2))*c
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b+c+sinh(x))/(a+b*cosh(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more details)Is 4*a^2-4*b^2 positive or negative?
```

mupad [B] time = 1.81, size = 198, normalized size = 3.47

$$\frac{\ln\left(b\sqrt{(a+b)\,(a-b)}+a^2\,\mathrm{e}^x-b^2\,\mathrm{e}^x+a\,\mathrm{e}^x\,\sqrt{(a+b)\,(a-b)}\right)\left(b^2\,\sqrt{(a+b)\,(a-b)}+a^2-b^2+b\,c\,\sqrt{(a+b)\,(a-}\right.}{a^2\,b-b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b + c + sinh(x))/(a + b*cosh(x)),x)
```

```
[Out] (log(b*((a + b)*(a - b))^(1/2) + a^2*exp(x) - b^2*exp(x) + a*exp(x)*((a + b
)*(a - b))^(1/2))*(b^2*((a + b)*(a - b))^(1/2) + a^2 - b^2 + b*c*((a + b)*(
a - b))^(1/2)))/(a^2*b - b^3) - x/b - (log(b*((a + b)*(a - b))^(1/2) - a^2*
exp(x) + b^2*exp(x) + a*exp(x)*((a + b)*(a - b))^(1/2))*(b^2*((a + b)*(a -
b))^(1/2) - a^2 + b^2 + b*c*((a + b)*(a - b))^(1/2)))/(a^2*b - b^3)
```

sympy [A] time = 30.82, size = 840, normalized size = 14.74

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b+c+sinh(x))/(a+b*cosh(x)),x)
```

```
[Out] Piecewise((zoo*(2*c*atan(tanh(x/2)) + x - 2*log(tanh(x/2) + 1) + log(tanh(x
/2)**2 + 1)), Eq(a, 0) & Eq(b, 0)), (-1/tanh(x/2) - c/(b*tanh(x/2)) + x/b -
 2*log(tanh(x/2) + 1)/b + 2*log(tanh(x/2))/b, Eq(a, -b)), ((c*x + cosh(x))/
a, Eq(b, 0)), (tanh(x/2) + c*tanh(x/2)/b + x/b - 2*log(tanh(x/2) + 1)/b, Eq
(a, b)), (a*x*sqrt(a/(a - b) + b/(a - b))/(a*b*sqrt(a/(a - b) + b/(a - b))
- b**2*sqrt(a/(a - b) + b/(a - b))) + a*sqrt(a/(a - b) + b/(a - b))*log(-sq
rt(a/(a - b) + b/(a - b)) + tanh(x/2))/(a*b*sqrt(a/(a - b) + b/(a - b)) - b
**2*sqrt(a/(a - b) + b/(a - b))) + a*sqrt(a/(a - b) + b/(a - b))*log(sqrt(a
/(a - b) + b/(a - b)) + tanh(x/2))/(a*b*sqrt(a/(a - b) + b/(a - b)) - b**2*
sqrt(a/(a - b) + b/(a - b))) - 2*a*sqrt(a/(a - b) + b/(a - b))*log(tanh(x/2
) + 1)/(a*b*sqrt(a/(a - b) + b/(a - b)) - b**2*sqrt(a/(a - b) + b/(a - b)))
 - b**2*log(-sqrt(a/(a - b) + b/(a - b)) + tanh(x/2))/(a*b*sqrt(a/(a - b) +
 b/(a - b)) - b**2*sqrt(a/(a - b) + b/(a - b))) + b**2*log(sqrt(a/(a - b) +
 b/(a - b)) + tanh(x/2))/(a*b*sqrt(a/(a - b) + b/(a - b)) - b**2*sqrt(a/(a
- b) + b/(a - b))) - b*c*log(-sqrt(a/(a - b) + b/(a - b)) + tanh(x/2))/(a*b
*sqrt(a/(a - b) + b/(a - b)) - b**2*sqrt(a/(a - b) + b/(a - b))) + b*c*log(
sqrt(a/(a - b) + b/(a - b)) + tanh(x/2))/(a*b*sqrt(a/(a - b) + b/(a - b)) -
```

```
 b**2*sqrt(a/(a - b) + b/(a - b))) - b*x*sqrt(a/(a - b) + b/(a - b))/(a*b*s
qrt(a/(a - b) + b/(a - b)) - b**2*sqrt(a/(a - b) + b/(a - b))) - b*sqrt(a/(
a - b) + b/(a - b))*log(-sqrt(a/(a - b) + b/(a - b)) + tanh(x/2))/(a*b*sqrt
(a/(a - b) + b/(a - b)) - b**2*sqrt(a/(a - b) + b/(a - b))) - b*sqrt(a/(a -
 b) + b/(a - b))*log(sqrt(a/(a - b) + b/(a - b)) + tanh(x/2))/(a*b*sqrt(a/(
a - b) + b/(a - b)) - b**2*sqrt(a/(a - b) + b/(a - b))) + 2*b*sqrt(a/(a - b
) + b/(a - b))*log(tanh(x/2) + 1)/(a*b*sqrt(a/(a - b) + b/(a - b)) - b**2*s
qrt(a/(a - b) + b/(a - b))), True))
```

3.569 $\int \frac{b+c+\sinh(x)}{a-b\cosh(x)}\,dx$

Optimal. Leaf size=59

$$\frac{2(b+c)\tanh^{-1}\left(\frac{\sqrt{a+b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a-b}}\right)}{\sqrt{a-b}\sqrt{a+b}}-\frac{\log(a-b\cosh(x))}{b}$$

```
[Out] -ln(a-b*cosh(x))/b+2*(b+c)*arctanh((a+b)^(1/2)*tanh(1/2*x)/(a-b)^(1/2))/(a-
b)^(1/2)/(a+b)^(1/2)
```

Rubi [A] time = 0.14, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {4401, 2659, 208, 2668, 31}

$$\frac{2(b+c)\tanh^{-1}\left(\frac{\sqrt{a+b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a-b}}\right)}{\sqrt{a-b}\sqrt{a+b}}-\frac{\log(a-b\cosh(x))}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(b + c + Sinh[x])/(a - b*Cosh[x]),x]
```

```
[Out] (2*(b + c)*ArcTanh[(Sqrt[a + b]*Tanh[x/2])/Sqrt[a - b]])/(Sqrt[a - b]*Sqrt[
a + b]) - Log[a - b*Cosh[x]]/b
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 4401

```
Int[u_, x_Symbol] :> With[{v = ExpandTrig[u, x]}, Int[v, x] /; SumQ[v]] /;
 !InertTrigFreeQ[u]
```

Rubi steps

$$\begin{aligned}\int \frac{b+c+\sinh(x)}{a-b\cosh(x)}\,dx &= \int \left(\frac{-b-c}{-a+b\cosh(x)}+\frac{\sinh(x)}{a-b\cosh(x)}\right)dx\\ &= (-b-c)\int \frac{1}{-a+b\cosh(x)}\,dx+\int \frac{\sinh(x)}{a-b\cosh(x)}\,dx\\ &= -\frac{\mathrm{Subst}\left(\int \frac{1}{a+x}\,dx,x,-b\cosh(x)\right)}{b}-(2(b+c))\,\mathrm{Subst}\left(\int \frac{1}{-a+b-(-a-b)x^2}\,dx,x,\tanh\right.\\ &= \frac{2(b+c)\tanh^{-1}\left(\frac{\sqrt{a+b}\,\tanh\left(\frac{x}{2}\right)}{\sqrt{a-b}}\right)}{\sqrt{a-b}\,\sqrt{a+b}}-\frac{\log(a-b\cosh(x))}{b}\end{aligned}$$

Mathematica [A] time = 0.10, size = 56, normalized size = 0.95

$$-\frac{2(b+c)\tan^{-1}\left(\frac{(a+b)\tanh\left(\frac{x}{2}\right)}{\sqrt{b^2-a^2}}\right)}{\sqrt{b^2-a^2}}-\frac{\log(a-b\cosh(x))}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b + c + Sinh[x])/(a - b*Cosh[x]),x]
```

```
[Out] (-2*(b + c)*ArcTan[((a + b)*Tanh[x/2])/Sqrt[-a^2 + b^2]])/Sqrt[-a^2 + b^2]
- Log[a - b*Cosh[x]]/b
```

fricas [B] time = 0.45, size = 299, normalized size = 5.07

$$\left[\frac{\sqrt{a^2-b^2}\left(b^2+bc\right)\log\left(\frac{b^2\cosh(x)^2+b^2\sinh(x)^2-2ab\cosh(x)+2a^2-b^2+2\left(b^2\cosh(x)-ab\right)\sinh(x)+2\sqrt{a^2-b^2}\left(b\cosh(x)+b\sinh(x)-a\right)}{b\cosh(x)^2+b\sinh(x)^2-2a\cosh(x)+2\left(b\cosh(x)-a\right)\sinh(x)+b}\right)+\left(\right.}{a^2b-b^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b+c+sinh(x))/(a-b*cosh(x)),x, algorithm="fricas")
```

```
[Out] [(sqrt(a^2 - b^2)*(b^2 + b*c)*log((b^2*cosh(x)^2 + b^2*sinh(x)^2 - 2*a*b*co
sh(x) + 2*a^2 - b^2 + 2*(b^2*cosh(x) - a*b)*sinh(x) + 2*sqrt(a^2 - b^2)*(b*
cosh(x) + b*sinh(x) - a))/(b*cosh(x)^2 + b*sinh(x)^2 - 2*a*cosh(x) + 2*(b*c
osh(x) - a)*sinh(x) + b)) + (a^2 - b^2)*x - (a^2 - b^2)*log(2*(b*cosh(x) -
a)/(cosh(x) - sinh(x))))/(a^2*b - b^3), (2*sqrt(-a^2 + b^2)*(b^2 + b*c)*arc
tan(-sqrt(-a^2 + b^2)*(b*cosh(x) + b*sinh(x) - a)/(a^2 - b^2)) + (a^2 - b^2
)*x - (a^2 - b^2)*log(2*(b*cosh(x) - a)/(cosh(x) - sinh(x))))/(a^2*b - b^3)
]
```

giac [A] time = 0.12, size = 62, normalized size = 1.05

$$-\frac{2\,(b+c)\arctan\left(\frac{be^{x}-a}{\sqrt{-a^{2}+b^{2}}}\right)}{\sqrt{-a^{2}+b^{2}}}+\frac{x}{b}-\frac{\log\left(be^{(2\,x)}-2\,ae^{x}+b\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b+c+sinh(x))/(a-b*cosh(x)),x, algorithm="giac")
```

```
[Out] -2*(b + c)*arctan((b*e^x - a)/sqrt(-a^2 + b^2))/sqrt(-a^2 + b^2) + x/b - lo
g(b*e^(2*x) - 2*a*e^x + b)/b
```

maple [B] time = 0.13, size = 154, normalized size = 2.61

$$-\frac{\ln\left(a\left(\tanh^{2}\left(\frac{x}{2}\right)\right)+\left(\tanh^{2}\left(\frac{x}{2}\right)\right)b-a+b\right)a}{b\,(a+b)}-\frac{\ln\left(a\left(\tanh^{2}\left(\frac{x}{2}\right)\right)+\left(\tanh^{2}\left(\frac{x}{2}\right)\right)b-a+b\right)}{a+b}+\frac{2b\operatorname{arctanh}\left(\frac{(a+b)\tan}{\sqrt{(a+b)(}}\right.}{\sqrt{(a+b)\,(a-b)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b+c+sinh(x))/(a-b*cosh(x)),x)
```

```
[Out] -1/b/(a+b)*ln(a*tanh(1/2*x)^2+tanh(1/2*x)^2*b-a+b)*a-1/(a+b)*ln(a*tanh(1/2*
x)^2+tanh(1/2*x)^2*b-a+b)+2*b/((a+b)*(a-b))^(1/2)*arctanh((a+b)*tanh(1/2*x)
/((a+b)*(a-b))^(1/2))+2/((a+b)*(a-b))^(1/2)*arctanh((a+b)*tanh(1/2*x)/((a+b
)*(a-b))^(1/2))*c+1/b*ln(tanh(1/2*x)-1)+1/b*ln(tanh(1/2*x)+1)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b+c+sinh(x))/(a-b*cosh(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more details)Is 4*a^2-4*b^2 positive or negative?
```

mupad [B] time = 1.73, size = 199, normalized size = 3.37

$$\frac{x}{b}-\frac{\ln\left(b\sqrt{(a+b)\,(a-b)}+a^2\,e^x-b^2\,e^x-a\,e^x\,\sqrt{(a+b)\,(a-b)}\right)\,\left(b^2\,\sqrt{(a+b)\,(a-b)}+a^2-b^2+b\,c\,\sqrt{(a+b)\,(a}{a^2\,b-b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b + c + sinh(x))/(a - b*cosh(x)),x)
```

```
[Out] x/b - (log(b*((a + b)*(a - b))^(1/2) + a^2*exp(x) - b^2*exp(x) - a*exp(x)*(
(a + b)*(a - b))^(1/2))*(b^2*((a + b)*(a - b))^(1/2) + a^2 - b^2 + b*c*((a
+ b)*(a - b))^(1/2)))/(a^2*b - b^3) + (log(b*((a + b)*(a - b))^(1/2) - a^2*
exp(x) + b^2*exp(x) - a*exp(x)*((a + b)*(a - b))^(1/2))*(b^2*((a + b)*(a -
b))^(1/2) - a^2 + b^2 + b*c*((a + b)*(a - b))^(1/2)))/(a^2*b - b^3)
```

sympy [A] time = 29.67, size = 840, normalized size = 14.24

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b+c+sinh(x))/(a-b*cosh(x)),x)
```

```
[Out] Piecewise((zoo*(2*c*atan(tanh(x/2)) + x - 2*log(tanh(x/2) + 1) + log(tanh(x
/2)**2 + 1)), Eq(a, 0) & Eq(b, 0)), (-tanh(x/2) - c*tanh(x/2)/b - x/b + 2*l
og(tanh(x/2) + 1)/b, Eq(a, -b)), (1/tanh(x/2) + c/(b*tanh(x/2)) - x/b + 2*l
og(tanh(x/2) + 1)/b - 2*log(tanh(x/2))/b, Eq(a, b)), ((c*x + cosh(x))/a, Eq
(b, 0)), (-a*x*sqrt(a/(a + b) - b/(a + b))/(a*b*sqrt(a/(a + b) - b/(a + b))
 + b**2*sqrt(a/(a + b) - b/(a + b))) - a*sqrt(a/(a + b) - b/(a + b))*log(-s
qrt(a/(a + b) - b/(a + b)) + tanh(x/2))/(a*b*sqrt(a/(a + b) - b/(a + b)) +
b**2*sqrt(a/(a + b) - b/(a + b))) - a*sqrt(a/(a + b) - b/(a + b))*log(sqrt(
a/(a + b) - b/(a + b)) + tanh(x/2))/(a*b*sqrt(a/(a + b) - b/(a + b)) + b**2
*sqrt(a/(a + b) - b/(a + b))) + 2*a*sqrt(a/(a + b) - b/(a + b))*log(tanh(x/
2) + 1)/(a*b*sqrt(a/(a + b) - b/(a + b)) + b**2*sqrt(a/(a + b) - b/(a + b))
) - b**2*log(-sqrt(a/(a + b) - b/(a + b)) + tanh(x/2))/(a*b*sqrt(a/(a + b)
- b/(a + b)) + b**2*sqrt(a/(a + b) - b/(a + b))) + b**2*log(sqrt(a/(a + b)
- b/(a + b)) + tanh(x/2))/(a*b*sqrt(a/(a + b) - b/(a + b)) + b**2*sqrt(a/(a
 + b) - b/(a + b))) - b*c*log(-sqrt(a/(a + b) - b/(a + b)) + tanh(x/2))/(a*
b*sqrt(a/(a + b) - b/(a + b)) + b**2*sqrt(a/(a + b) - b/(a + b))) + b*c*log
(sqrt(a/(a + b) - b/(a + b)) + tanh(x/2))/(a*b*sqrt(a/(a + b) - b/(a + b))
```

```
+ b**2*sqrt(a/(a + b) - b/(a + b))) - b*x*sqrt(a/(a + b) - b/(a + b))/(a*b*
sqrt(a/(a + b) - b/(a + b)) + b**2*sqrt(a/(a + b) - b/(a + b))) - b*sqrt(a/
(a + b) - b/(a + b))*log(-sqrt(a/(a + b) - b/(a + b)) + tanh(x/2))/(a*b*sqr
t(a/(a + b) - b/(a + b)) + b**2*sqrt(a/(a + b) - b/(a + b))) - b*sqrt(a/(a
+ b) - b/(a + b))*log(sqrt(a/(a + b) - b/(a + b)) + tanh(x/2))/(a*b*sqrt(a/
(a + b) - b/(a + b)) + b**2*sqrt(a/(a + b) - b/(a + b))) + 2*b*sqrt(a/(a +
b) - b/(a + b))*log(tanh(x/2) + 1)/(a*b*sqrt(a/(a + b) - b/(a + b)) + b**2*
sqrt(a/(a + b) - b/(a + b))), True))
```

3.570 $\int \frac{x(b-a\sinh(x))}{(a+b\sinh(x))^2}\,dx$

Optimal. Leaf size=25

$$\frac{\log(a+b\sinh(x))}{b}-\frac{x\cosh(x)}{a+b\sinh(x)}$$

```
[Out] ln(a+b*sinh(x))/b-x*cosh(x)/(a+b*sinh(x))
```

Rubi [A] time = 0.06, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {5636, 2668, 31}

$$\frac{\log(a+b\sinh(x))}{b}-\frac{x\cosh(x)}{a+b\sinh(x)}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(b - a*Sinh[x]))/(a + b*Sinh[x])^2,x]
```

```
[Out] Log[a + b*Sinh[x]]/b - (x*Cosh[x])/(a + b*Sinh[x])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 5636

```
Int[(((e_.) + (f_.)*(x_))*((A_) + (B_.)*Sinh[(c_.) + (d_.)*(x_)]))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)])^2, x_Symbol] :> Simp[(B*(e + f*x)*Cosh[c + d*x])/(a*d*(a + b*Sinh[c + d*x])), x] - Dist[(B*f)/(a*d), Int[Cosh[c + d*x]/(a + b*Sinh[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && EqQ[a*A + b*B, 0]
```

Rubi steps

$$\int \frac{x(b-a\sinh(x))}{(a+b\sinh(x))^2}\,dx = -\frac{x\cosh(x)}{a+b\sinh(x)} + \int \frac{\cosh(x)}{a+b\sinh(x)}\,dx$$
$$= -\frac{x\cosh(x)}{a+b\sinh(x)} + \frac{\text{Subst}\left(\int \frac{1}{a+x}\,dx, x, b\sinh(x)\right)}{b}$$
$$= \frac{\log(a+b\sinh(x))}{b} - \frac{x\cosh(x)}{a+b\sinh(x)}$$

Mathematica [A] time = 0.18, size = 25, normalized size = 1.00

$$\frac{\log(a+b\sinh(x))}{b} - \frac{x\cosh(x)}{a+b\sinh(x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(b - a*Sinh[x]))/(a + b*Sinh[x])^2,x]
```

```
[Out] Log[a + b*Sinh[x]]/b - (x*Cosh[x])/(a + b*Sinh[x])
```

fricas [B] time = 0.42, size = 134, normalized size = 5.36

$$-\frac{2\,bx\cosh(x)^2 + 2\,bx\sinh(x)^2 + 2\,ax\cosh(x) - \left(b\cosh(x)^2 + b\sinh(x)^2 + 2\,a\cosh(x) + 2\,(b\cosh(x) + a)\,\mathrm{sinh}}{b^2\cosh(x)^2 + b^2\sinh(x)^2 + 2\,ab\cosh(x) - b^2 + 2\left(b^2\cosh(x) +\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b-a*sinh(x))/(a+b*sinh(x))^2,x, algorithm="fricas")
```

```
[Out] -(2*b*x*cosh(x)^2 + 2*b*x*sinh(x)^2 + 2*a*x*cosh(x) - (b*cosh(x)^2 + b*sinh
(x)^2 + 2*a*cosh(x) + 2*(b*cosh(x) + a)*sinh(x) - b)*log(2*(b*sinh(x) + a)/
(cosh(x) - sinh(x))) + 2*(2*b*x*cosh(x) + a*x)*sinh(x))/(b^2*cosh(x)^2 + b^
2*sinh(x)^2 + 2*a*b*cosh(x) - b^2 + 2*(b^2*cosh(x) + a*b)*sinh(x))
```

giac [B] time = 0.16, size = 96, normalized size = 3.84

$$-\frac{2\,bxe^{(2\,x)} - be^{(2\,x)}\log\left(-be^{(2\,x)} - 2\,ae^{x} + b\right) - 2\,ae^{x}\log\left(-be^{(2\,x)} - 2\,ae^{x} + b\right) + 2\,bx + b\log\left(-be^{(2\,x)} - 2\,ae^{x} + b\right)}{b^2e^{(2\,x)} + 2\,abe^{x} - b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b-a*sinh(x))/(a+b*sinh(x))^2,x, algorithm="giac")
```

```
[Out] -(2*b*x*e^(2*x) - b*e^(2*x)*log(-b*e^(2*x) - 2*a*e^x + b) - 2*a*e^x*log(-b*
e^(2*x) - 2*a*e^x + b) + 2*b*x + b*log(-b*e^(2*x) - 2*a*e^x + b))/(b^2*e^(2
*x) + 2*a*b*e^x - b^2)
```

maple [B] time = 0.91, size = 58, normalized size = 2.32

$$-\frac{2x}{b}+\frac{2x\left(a\,e^{x}-b\right)}{b\left(b\,e^{2x}+2a\,e^{x}-b\right)}+\frac{\ln\left(e^{2x}+\frac{2a\,e^{x}}{b}-1\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b-a*sinh(x))/(a+b*sinh(x))^2,x)
```

```
[Out] -2*x/b+2*x*(a*exp(x)-b)/b/(b*exp(2*x)+2*a*exp(x)-b)+1/b*ln(exp(2*x)+2/b*a*e
xp(x)-1)
```

maxima [B] time = 0.71, size = 62, normalized size = 2.48

$$-\frac{2\left(bxe^{(2\,x)}+axe^{x}\right)}{b^{2}e^{(2\,x)}+2\,abe^{x}-b^{2}}+\frac{\log\left(\frac{be^{(2\,x)}+2\,ae^{x}-b}{b}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b-a*sinh(x))/(a+b*sinh(x))^2,x, algorithm="maxima")
```

```
[Out] -2*(b*x*e^(2*x) + a*x*e^x)/(b^2*e^(2*x) + 2*a*b*e^x - b^2) + log((b*e^(2*x)
 + 2*a*e^x - b)/b)/b
```

mupad [B] time = 1.54, size = 103, normalized size = 4.12

$$\frac{\ln\left(2\,a\,e^{x}-b+b\,e^{2\,x}\right)}{b}-\frac{\frac{2\left(x\,a^{2}\,b+x\,b^{3}\right)}{a^{2}\,b+b^{3}}-\frac{2\,e^{x}\left(x\,a^{3}\,b+x\,a\,b^{3}\right)}{b\left(a^{2}\,b+b^{3}\right)}}{2\,a\,e^{x}-b+b\,e^{2\,x}}-\frac{2\,x}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(b - a*sinh(x)))/(a + b*sinh(x))^2,x)
```

```
[Out] log(2*a*exp(x) - b + b*exp(2*x))/b - ((2*(b^3*x + a^2*b*x))/(a^2*b + b^3) -
 (2*exp(x)*(a*b^3*x + a^3*b*x))/(b*(a^2*b + b^3)))/(2*a*exp(x) - b + b*exp(
2*x)) - (2*x)/b
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b-a*sinh(x))/(a+b*sinh(x))**2,x)
```

```
[Out] Timed out
```

3.571 $\int \frac{x(b+a\cosh(x))}{(a+b\cosh(x))^2}\,dx$

Optimal. Leaf size=25

$$\frac{x\sinh(x)}{a+b\cosh(x)}-\frac{\log(a+b\cosh(x))}{b}$$

```
[Out] -ln(a+b*cosh(x))/b+x*sinh(x)/(a+b*cosh(x))
```

Rubi [A] time = 0.06, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {5637, 2668, 31}

$$\frac{x\sinh(x)}{a+b\cosh(x)}-\frac{\log(a+b\cosh(x))}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(b + a*Cosh[x]))/(a + b*Cosh[x])^2,x]
```

```
[Out] -(Log[a + b*Cosh[x]]/b) + (x*Sinh[x])/(a + b*Cosh[x])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 5637

```
Int[((Cosh[(c_.) + (d_.)*(x_)]*(B_.) + (A_))*((e_.) + (f_.)*(x_)))/(Cosh[(c
_.) + (d_.)*(x_)]*(b_.) + (a_))^2, x_Symbol] :> Simp[(B*(e + f*x)*Sinh[c +
d*x])/(a*d*(a + b*Cosh[c + d*x])), x] - Dist[(B*f)/(a*d), Int[Sinh[c + d*x]
/(a + b*Cosh[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && EqQ
[a*A - b*B, 0]
```

Rubi steps

$$\int \frac{x(b+a\cosh(x))}{(a+b\cosh(x))^2}\,dx = \frac{x\sinh(x)}{a+b\cosh(x)} - \int \frac{\sinh(x)}{a+b\cosh(x)}\,dx$$
$$= \frac{x\sinh(x)}{a+b\cosh(x)} - \frac{\text{Subst}\left(\int \frac{1}{a+x}\,dx, x, b\cosh(x)\right)}{b}$$
$$= -\frac{\log(a+b\cosh(x))}{b} + \frac{x\sinh(x)}{a+b\cosh(x)}$$

Mathematica [A] time = 0.13, size = 25, normalized size = 1.00

$$\frac{x\sinh(x)}{a+b\cosh(x)} - \frac{\log(a+b\cosh(x))}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(b + a*Cosh[x]))/(a + b*Cosh[x])^2,x]
```

```
[Out] -(Log[a + b*Cosh[x]]/b) + (x*Sinh[x])/(a + b*Cosh[x])
```

fricas [B] time = 0.44, size = 129, normalized size = 5.16

$$\frac{2\,bx\cosh(x)^2 + 2\,bx\sinh(x)^2 + 2\,ax\cosh(x) - \left(b\cosh(x)^2 + b\sinh(x)^2 + 2\,a\cosh(x) + 2\,(b\cosh(x) + a)\sinh(\ldots}{b^2\cosh(x)^2 + b^2\sinh(x)^2 + 2\,ab\cosh(x) + b^2 + 2\left(b^2\cosh(x) + \ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b+a*cosh(x))/(a+b*cosh(x))^2,x, algorithm="fricas")
```

```
[Out] (2*b*x*cosh(x)^2 + 2*b*x*sinh(x)^2 + 2*a*x*cosh(x) - (b*cosh(x)^2 + b*sinh(
x)^2 + 2*a*cosh(x) + 2*(b*cosh(x) + a)*sinh(x) + b)*log(2*(b*cosh(x) + a)/(
cosh(x) - sinh(x))) + 2*(2*b*x*cosh(x) + a*x)*sinh(x))/(b^2*cosh(x)^2 + b^2
*sinh(x)^2 + 2*a*b*cosh(x) + b^2 + 2*(b^2*cosh(x) + a*b)*sinh(x))
```

giac [B] time = 0.14, size = 100, normalized size = 4.00

$$\frac{2\,bxe^{(2\,x)} - be^{(2\,x)}\log\left(-be^{(2\,x)} - 2\,ae^{x} - b\right) - 2\,ae^{x}\log\left(-be^{(2\,x)} - 2\,ae^{x} - b\right) - 2\,bx - b\log\left(-be^{(2\,x)} - 2\,ae^{x} - b\right)}{b^2e^{(2\,x)} + 2\,abe^{x} + b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b+a*cosh(x))/(a+b*cosh(x))^2,x, algorithm="giac")
```

```
[Out] (2*b*x*e^(2*x) - b*e^(2*x)*log(-b*e^(2*x) - 2*a*e^x - b) - 2*a*e^x*log(-b*e
^(2*x) - 2*a*e^x - b) - 2*b*x - b*log(-b*e^(2*x) - 2*a*e^x - b))/(b^2*e^(2*
x) + 2*a*b*e^x + b^2)
```

maple [B] time = 0.48, size = 55, normalized size = 2.20

$$\frac{2x}{b}-\frac{2x\,(a\,e^{x}+b)}{b\left(b\,e^{2x}+2a\,e^{x}+b\right)}-\frac{\ln\left(e^{2x}+\frac{2a\,e^{x}}{b}+1\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b+a*cosh(x))/(a+b*cosh(x))^2,x)
```

```
[Out] 2*x/b-2*x*(a*exp(x)+b)/b/(b*exp(2*x)+2*a*exp(x)+b)-1/b*ln(exp(2*x)+2/b*a*ex
p(x)+1)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b+a*cosh(x))/(a+b*cosh(x))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more details)Is 4*a^2-4*b^2 positive or negative?
```

mupad [B] time = 1.53, size = 105, normalized size = 4.20

$$\frac{2\,x}{b}+\frac{\frac{2\left(b^3\,x-a^2\,b\,x\right)}{a^2\,b-b^3}+\frac{2\,e^{x}\left(a\,b^3\,x-a^3\,b\,x\right)}{b\left(a^2\,b-b^3\right)}}{b+2\,a\,e^{x}+b\,e^{2\,x}}-\frac{\ln\left(b+2\,a\,e^{x}+b\,e^{2\,x}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(b + a*cosh(x)))/(a + b*cosh(x))^2,x)
```

```
[Out] (2*x)/b + ((2*(b^3*x - a^2*b*x))/(a^2*b - b^3) + (2*exp(x)*(a*b^3*x - a^3*b
*x))/(b*(a^2*b - b^3)))/(b + 2*a*exp(x) + b*exp(2*x)) - log(b + 2*a*exp(x)
+ b*exp(2*x))/b
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b+a*cosh(x))/(a+b*cosh(x))**2,x)
```

```
[Out] Timed out
```

3.572 $\int \frac{a+b\,\mathbf{sech}(x)}{c+d\cosh(x)}\,dx$

Optimal. Leaf size=62

$$\frac{2(ac-bd)\tanh^{-1}\left(\frac{\sqrt{c-d}\tanh\left(\frac{x}{2}\right)}{\sqrt{c+d}}\right)}{c\sqrt{c-d}\,\sqrt{c+d}}+\frac{b\tan^{-1}(\sinh(x))}{c}$$

```
[Out] b*arctan(sinh(x))/c+2*(a*c-b*d)*arctanh((c-d)^(1/2)*tanh(1/2*x)/(c+d)^(1/2)
)/c/(c-d)^(1/2)/(c+d)^(1/2)
```

Rubi [A] time = 0.16, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2828, 3001, 3770, 2659, 208}

$$\frac{2(ac-bd)\tanh^{-1}\left(\frac{\sqrt{c-d}\tanh\left(\frac{x}{2}\right)}{\sqrt{c+d}}\right)}{c\sqrt{c-d}\,\sqrt{c+d}}+\frac{b\tan^{-1}(\sinh(x))}{c}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sech[x])/(c + d*Cosh[x]),x]
```

```
[Out] (b*ArcTan[Sinh[x]])/c + (2*(a*c - b*d)*ArcTanh[(Sqrt[c - d]*Tanh[x/2])/Sqrt
[c + d]])/(c*Sqrt[c - d]*Sqrt[c + d])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2828

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.)*((a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)])^(m_.), x_Symbol] :> Int[((a + b*Sin[e + f*x])^m*(d + c*Sin[e +
 f*x])^n)/Sin[e + f*x]^n, x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IntegerQ
[n]
```

Rule 3001

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{a+b\,\mathrm{sech}(x)}{c+d\cosh(x)}\,dx &= \int \frac{(b+a\cosh(x))\mathrm{sech}(x)}{c+d\cosh(x)}\,dx\\
&= \frac{b\int \mathrm{sech}(x)\,dx}{c} + \frac{(ac-bd)\int \frac{1}{c+d\cosh(x)}\,dx}{c}\\
&= \frac{b\tan^{-1}(\sinh(x))}{c} + \frac{(2(ac-bd))\,\mathrm{Subst}\left(\int \frac{1}{c+d-(c-d)x^2}\,dx, x, \tanh\left(\frac{x}{2}\right)\right)}{c}\\
&= \frac{b\tan^{-1}(\sinh(x))}{c} + \frac{2(ac-bd)\tanh^{-1}\left(\frac{\sqrt{c-d}\,\tanh\left(\frac{x}{2}\right)}{\sqrt{c+d}}\right)}{c\sqrt{c-d}\,\sqrt{c+d}}
\end{aligned}
$$

Mathematica [A] time = 0.13, size = 63, normalized size = 1.02

$$
\frac{2\left(\frac{(bd-ac)\tan^{-1}\left(\frac{(c-d)\tanh\left(\frac{x}{2}\right)}{\sqrt{d^2-c^2}}\right)}{\sqrt{d^2-c^2}} + b\tan^{-1}\left(\tanh\left(\frac{x}{2}\right)\right)\right)}{c}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sech[x])/(c + d*Cosh[x]),x]
```

```
[Out] (2*(b*ArcTan[Tanh[x/2]] + ((-(a*c) + b*d)*ArcTan[((c - d)*Tanh[x/2])/Sqrt[-
c^2 + d^2]])/Sqrt[-c^2 + d^2]))/c
```

fricas [A] time = 0.77, size = 249, normalized size = 4.02

$$\left[-\frac{(ac-bd)\sqrt{c^2-d^2}\,\log\left(\frac{d^2\cosh(x)^2+d^2\sinh(x)^2+2\,cd\cosh(x)+2\,c^2-d^2+2\,\left(d^2\cosh(x)+cd\right)\sinh(x)+2\,\sqrt{c^2-d^2}\,(d\cosh(x)+d\sinh(x)+c)}{d\cosh(x)^2+d\sinh(x)^2+2\,c\cosh(x)+2\,(d\cosh(x)+c)\sinh(x)+d}\right)-}{c^3-cd^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(x))/(c+d*cosh(x)),x, algorithm="fricas")
```

```
[Out] [-((a*c - b*d)*sqrt(c^2 - d^2)*log((d^2*cosh(x)^2 + d^2*sinh(x)^2 + 2*c*d*c
osh(x) + 2*c^2 - d^2 + 2*(d^2*cosh(x) + c*d)*sinh(x) + 2*sqrt(c^2 - d^2)*(d
*cosh(x) + d*sinh(x) + c))/(d*cosh(x)^2 + d*sinh(x)^2 + 2*c*cosh(x) + 2*(d*
cosh(x) + c)*sinh(x) + d)) - 2*(b*c^2 - b*d^2)*arctan(cosh(x) + sinh(x)))/(
c^3 - c*d^2), -2*((a*c - b*d)*sqrt(-c^2 + d^2)*arctan(-sqrt(-c^2 + d^2)*(d*
cosh(x) + d*sinh(x) + c)/(c^2 - d^2)) - (b*c^2 - b*d^2)*arctan(cosh(x) + si
nh(x)))/(c^3 - c*d^2)]
```

giac [A] time = 0.14, size = 53, normalized size = 0.85

$$\frac{2\,b\arctan\left(e^x\right)}{c}+\frac{2\,(ac-bd)\arctan\left(\frac{de^x+c}{\sqrt{-c^2+d^2}}\right)}{\sqrt{-c^2+d^2}\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(x))/(c+d*cosh(x)),x, algorithm="giac")
```

```
[Out] 2*b*arctan(e^x)/c + 2*(a*c - b*d)*arctan((d*e^x + c)/sqrt(-c^2 + d^2))/(sqr
t(-c^2 + d^2)*c)
```

maple [A] time = 0.19, size = 89, normalized size = 1.44

$$\frac{2\,\mathrm{arctanh}\left(\frac{(c-d)\tanh\left(\frac{x}{2}\right)}{\sqrt{(c+d)(c-d)}}\right)a}{\sqrt{(c+d)\,(c-d)}}-\frac{2\,\mathrm{arctanh}\left(\frac{(c-d)\tanh\left(\frac{x}{2}\right)}{\sqrt{(c+d)(c-d)}}\right)bd}{c\sqrt{(c+d)\,(c-d)}}+\frac{2b\arctan\left(\tanh\left(\frac{x}{2}\right)\right)}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sech(x))/(c+d*cosh(x)),x)
```

```
[Out] 2/((c+d)*(c-d))^(1/2)*arctanh((c-d)*tanh(1/2*x)/((c+d)*(c-d))^(1/2))*a-2/c/
((c+d)*(c-d))^(1/2)*arctanh((c-d)*tanh(1/2*x)/((c+d)*(c-d))^(1/2))*b*d+2*b/
c*arctan(tanh(1/2*x))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(x))/(c+d*cosh(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*c^2-4*d^2>0)', see `assume?` for
more details)Is 4*c^2-4*d^2 positive or negative?
```

mupad [B] time = 6.35, size = 636, normalized size = 10.26

$$\frac{\ln\left(\frac{\sqrt{(c+d)\,(c-d)}\,(a\,c-b\,d)\left(\frac{32\left(a^2\,c^2\,d-2\,a\,b\,c\,d^2-4\,\mathrm{e}^x\,b^2\,c^3-2\,b^2\,c^2\,d+3\,\mathrm{e}^x\,b^2\,c\,d^2+2\,b^2\,d^3\right)}{d^5}+\frac{\sqrt{(c+d)\,(c-d)}\left(\frac{32\,c^2\left(2\,b\,d^2-4\,a\,c^2\,\mathrm{e}^x+a\,d^2\,\mathrm{e}^x-2\,a\,c\,d+3\,b\,c\,d\,\mathrm{e}^x\right)}{d^5}-\frac{32\,c^2\,\sqrt{(c+ \cdots}{\cdots}\right.}{c\,d^2-c^3}\right.}{c\,d^2-c^3}\right.}{c\,d^2-c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cosh(x))/(c + d*cosh(x)),x)
```

```
[Out] (b*log(exp(x) + 1i)*1i)/c - (b*log(exp(x) - 1i)*1i)/c + (log((((c + d)*(c -
 d))^(1/2)*(a*c - b*d)*((32*(2*b^2*d^3 + a^2*c^2*d - 2*b^2*c^2*d - 4*b^2*c^
3*exp(x) + 3*b^2*c*d^2*exp(x) - 2*a*b*c*d^2))/d^5 + (((c + d)*(c - d))^(1/2
)*((32*c^2*(2*b*d^2 - 4*a*c^2*exp(x) + a*d^2*exp(x) - 2*a*c*d + 3*b*c*d*exp
(x)))/d^5 - (32*c^2*((c + d)*(c - d))^(1/2)*(a*c - b*d)*(3*c^2*d - 2*d^3 +
4*c^3*exp(x) - 3*c*d^2*exp(x)))/(d^5*(c*d^2 - c^3)))*(a*c - b*d))/(c*d^2 -
c^3)))/(c*d^2 - c^3) - (32*b*(a*c - b*d)*(2*b*d - a*d*exp(x) + 4*b*c*exp(x)
))/d^5)*((c + d)*(c - d))^(1/2)*(a*c - b*d))/(c*d^2 - c^3) - (log(- (32*b*(
a*c - b*d)*(2*b*d - a*d*exp(x) + 4*b*c*exp(x)))/d^5 - (((c + d)*(c - d))^(1
/2)*(a*c - b*d)*((32*(2*b^2*d^3 + a^2*c^2*d - 2*b^2*c^2*d - 4*b^2*c^3*exp(x
) + 3*b^2*c*d^2*exp(x) - 2*a*b*c*d^2))/d^5 - (((c + d)*(c - d))^(1/2)*((32*
c^2*(2*b*d^2 - 4*a*c^2*exp(x) + a*d^2*exp(x) - 2*a*c*d + 3*b*c*d*exp(x)))/d
^5 + (32*c^2*((c + d)*(c - d))^(1/2)*(a*c - b*d)*(3*c^2*d - 2*d^3 + 4*c^3*e
xp(x) - 3*c*d^2*exp(x)))/(d^5*(c*d^2 - c^3)))*(a*c - b*d))/(c*d^2 - c^3)))/
(c*d^2 - c^3))*((c + d)*(c - d))^(1/2)*(a*c - b*d))/(c*d^2 - c^3)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b\,\mathrm{sech}\,(x)}{c + d\cosh\,(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(x))/(c+d*cosh(x)),x)
```

```
[Out] Integral((a + b*sech(x))/(c + d*cosh(x)), x)
```

3.573 $\int \frac{a+b\text{csch}(x)}{c+d\sinh(x)}\,dx$

Optimal. Leaf size=58

$$-\frac{2(ac-bd)\tanh^{-1}\left(\frac{d-c\tanh\left(\frac{x}{2}\right)}{\sqrt{c^2+d^2}}\right)}{c\sqrt{c^2+d^2}}-\frac{b\tanh^{-1}(\cosh(x))}{c}$$

```
[Out] -b*arctanh(cosh(x))/c-2*(a*c-b*d)*arctanh((d-c*tanh(1/2*x))/(c^2+d^2)^(1/2)
)/c/(c^2+d^2)^(1/2)
```

Rubi [A] time = 0.17, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {2828, 3001, 3770, 2660, 618, 206}

$$-\frac{2(ac-bd)\tanh^{-1}\left(\frac{d-c\tanh\left(\frac{x}{2}\right)}{\sqrt{c^2+d^2}}\right)}{c\sqrt{c^2+d^2}}-\frac{b\tanh^{-1}(\cosh(x))}{c}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Csch[x])/(c + d*Sinh[x]),x]
```

```
[Out] -((b*ArcTanh[Cosh[x]])/c) - (2*(a*c - b*d)*ArcTanh[(d - c*Tanh[x/2])/Sqrt[c
^2 + d^2]])/(c*Sqrt[c^2 + d^2])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2828

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.)*((a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)])^(m_.), x_Symbol] :> Int[((a + b*Sin[e + f*x])^m*(d + c*Sin[e +
 f*x])^n)/Sin[e + f*x]^n, x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IntegerQ
[n]
```

Rule 3001

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{a+b\operatorname{csch}(x)}{c+d\sinh(x)}\,dx &= -\left(i\int \frac{\operatorname{csch}(x)(ib+ia\sinh(x))}{c+d\sinh(x)}\,dx\right)\\
&= \frac{b\int \operatorname{csch}(x)\,dx}{c}+\frac{(ac-bd)\int \frac{1}{c+d\sinh(x)}\,dx}{c}\\
&= -\frac{b\tanh^{-1}(\cosh(x))}{c}+\frac{(2(ac-bd))\operatorname{Subst}\left(\int \frac{1}{c+2dx-cx^2}\,dx,x,\tanh\left(\frac{x}{2}\right)\right)}{c}\\
&= -\frac{b\tanh^{-1}(\cosh(x))}{c}-\frac{(4(ac-bd))\operatorname{Subst}\left(\int \frac{1}{4\left(c^2+d^2\right)-x^2}\,dx,x,2d-2c\tanh\left(\frac{x}{2}\right)\right)}{c}\\
&= -\frac{b\tanh^{-1}(\cosh(x))}{c}-\frac{2(ac-bd)\tanh^{-1}\left(\frac{d-c\tanh\left(\frac{x}{2}\right)}{\sqrt{c^2+d^2}}\right)}{c\sqrt{c^2+d^2}}
\end{aligned}$$

Mathematica [A] time = 0.14, size = 67, normalized size = 1.16

$$\frac{\frac{2(ac-bd)\tan^{-1}\left(\frac{d-c\tanh\left(\frac{x}{2}\right)}{\sqrt{-c^2-d^2}}\right)}{\sqrt{-c^2-d^2}}+b\log\left(\tanh\left(\frac{x}{2}\right)\right)}{c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Csch[x])/(c + d*Sinh[x]),x]
```

```
[Out] ((2*(a*c - b*d)*ArcTan[(d - c*Tanh[x/2])/Sqrt[-c^2 - d^2]])/Sqrt[-c^2 - d^2
] + b*Log[Tanh[x/2]])/c
```

fricas [B] time = 0.76, size = 172, normalized size = 2.97

$$-\frac{(ac-bd)\sqrt{c^2+d^2}\,\log\left(\frac{d^2\cosh(x)^2+d^2\sinh(x)^2+2\,cd\cosh(x)+2\,c^2+d^2+2\left(d^2\cosh(x)+cd\right)\sinh(x)+2\,\sqrt{c^2+d^2}\,(d\cosh(x)+d\sinh(x)+c)}{d\cosh(x)^2+d\sinh(x)^2+2\,c\cosh(x)+2\,(d\cosh(x)+c)\sinh(x)-d}\right)+}{c^3+cd^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*csch(x))/(c+d*sinh(x)),x, algorithm="fricas")
```

```
[Out] -((a*c - b*d)*sqrt(c^2 + d^2)*log((d^2*cosh(x)^2 + d^2*sinh(x)^2 + 2*c*d*co
sh(x) + 2*c^2 + d^2 + 2*(d^2*cosh(x) + c*d)*sinh(x) + 2*sqrt(c^2 + d^2)*(d*
cosh(x) + d*sinh(x) + c))/(d*cosh(x)^2 + d*sinh(x)^2 + 2*c*cosh(x) + 2*(d*c
osh(x) + c)*sinh(x) - d)) + (b*c^2 + b*d^2)*log(cosh(x) + sinh(x) + 1) - (b
*c^2 + b*d^2)*log(cosh(x) + sinh(x) - 1))/(c^3 + c*d^2)
```

giac [A] time = 0.16, size = 90, normalized size = 1.55

$$-\frac{b\log\left(e^x+1\right)}{c}+\frac{b\log\left(|e^x-1|\right)}{c}+\frac{(ac-bd)\log\left(\frac{\left|2\,de^x+2\,c-2\,\sqrt{c^2+d^2}\right|}{\left|2\,de^x+2\,c+2\,\sqrt{c^2+d^2}\right|}\right)}{\sqrt{c^2+d^2}\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*csch(x))/(c+d*sinh(x)),x, algorithm="giac")
```

```
[Out] -b*log(e^x + 1)/c + b*log(abs(e^x - 1))/c + (a*c - b*d)*log(abs(2*d*e^x + 2
*c - 2*sqrt(c^2 + d^2))/abs(2*d*e^x + 2*c + 2*sqrt(c^2 + d^2)))/(sqrt(c^2 +
 d^2)*c)
```

maple [A] time = 0.18, size = 86, normalized size = 1.48

$$\frac{b\ln\left(\tanh\left(\frac{x}{2}\right)\right)}{c}+\frac{2\,\mathrm{arctanh}\left(\frac{2c\tanh\left(\frac{x}{2}\right)-2d}{2\sqrt{c^2+d^2}}\right)a}{\sqrt{c^2+d^2}}-\frac{2\,\mathrm{arctanh}\left(\frac{2c\tanh\left(\frac{x}{2}\right)-2d}{2\sqrt{c^2+d^2}}\right)bd}{c\sqrt{c^2+d^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*csch(x))/(c+d*sinh(x)),x)
```

```
[Out] b/c*ln(tanh(1/2*x))+2/(c^2+d^2)^(1/2)*arctanh(1/2*(2*c*tanh(1/2*x)-2*d)/(c^
2+d^2)^(1/2))*a-2/c/(c^2+d^2)^(1/2)*arctanh(1/2*(2*c*tanh(1/2*x)-2*d)/(c^2+
d^2)^(1/2))*b*d
```

maxima [B] time = 0.53, size = 141, normalized size = 2.43

$$-b\left(\frac{d\log\left(\frac{de^{(-x)}-c-\sqrt{c^2+d^2}}{de^{(-x)}-c+\sqrt{c^2+d^2}}\right)}{\sqrt{c^2+d^2}\,c}+\frac{\log\left(e^{(-x)}+1\right)}{c}-\frac{\log\left(e^{(-x)}-1\right)}{c}\right)+\frac{a\log\left(\frac{de^{(-x)}-c-\sqrt{c^2+d^2}}{de^{(-x)}-c+\sqrt{c^2+d^2}}\right)}{\sqrt{c^2+d^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*csch(x))/(c+d*sinh(x)),x, algorithm="maxima")
```

```
[Out] -b*(d*log((d*e^(-x) - c - sqrt(c^2 + d^2))/(d*e^(-x) - c + sqrt(c^2 + d^2))
)/(sqrt(c^2 + d^2)*c) + log(e^(-x) + 1)/c - log(e^(-x) - 1)/c) + a*log((d*e
^(-x) - c - sqrt(c^2 + d^2))/(d*e^(-x) - c + sqrt(c^2 + d^2)))/sqrt(c^2 + d
^2)
```

mupad [B] time = 3.37, size = 539, normalized size = 9.29

$$\frac{b\,\ln\left(e^x-1\right)}{c}-\frac{b\,\ln\left(e^x+1\right)}{c}-\frac{\ln\left(\frac{\left(\frac{32\left(a^2\,c^2\,d-2\,a\,b\,c\,d^2-4\,e^x\,b^2\,c^3+2\,b^2\,c^2\,d-3\,e^x\,b^2\,c\,d^2+2\,b^2\,d^3\right)}{d^5}-\frac{(a\,c-b\,d)\left(\frac{32\,c^2\left(2\,b\,d^2+4\,a\,c^2\,e^x+a\,d^2\,e^x-2\,a\,c\,d-3\,b\,c\,d\,e^x\right)}{d^5}+\cdots}{c\,\sqrt{c^2+d^2}}\right.}{c\,\sqrt{c^2+d^2}}\right.}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/sinh(x))/(c + d*sinh(x)),x)
```

```
[Out] (b*log(exp(x) - 1))/c - (b*log(exp(x) + 1))/c - (log((((32*(2*b^2*d^3 + a^2
*c^2*d + 2*b^2*c^2*d - 4*b^2*c^3*exp(x) - 3*b^2*c*d^2*exp(x) - 2*a*b*c*d^2)
)/d^5 - ((a*c - b*d)*((32*c^2*(2*b*d^2 + 4*a*c^2*exp(x) + a*d^2*exp(x) - 2*
a*c*d - 3*b*c*d*exp(x)))/d^5 + (32*c*(a*c - b*d)*(3*c^2*d + 2*d^3 - 4*c^3*e
xp(x) - 3*c*d^2*exp(x)))/(d^5*(c^2 + d^2)^(1/2))))/(c*(c^2 + d^2)^(1/2)))*(
a*c - b*d))/(c*(c^2 + d^2)^(1/2)) + (32*b*(a*c - b*d)*(a*d*exp(x) - 2*b*d +
 4*b*c*exp(x)))/d^5)*(a*c - b*d)*(c^2 + d^2)^(1/2))/(c*d^2 + c^3) + (log((3
2*b*(a*c - b*d)*(a*d*exp(x) - 2*b*d + 4*b*c*exp(x)))/d^5 - (((32*(2*b^2*d^3
 + a^2*c^2*d + 2*b^2*c^2*d - 4*b^2*c^3*exp(x) - 3*b^2*c*d^2*exp(x) - 2*a*b*
c*d^2))/d^5 + ((a*c - b*d)*((32*c^2*(2*b*d^2 + 4*a*c^2*exp(x) + a*d^2*exp(x
) - 2*a*c*d - 3*b*c*d*exp(x)))/d^5 - (32*c*(a*c - b*d)*(3*c^2*d + 2*d^3 - 4
*c^3*exp(x) - 3*c*d^2*exp(x)))/(d^5*(c^2 + d^2)^(1/2))))/(c*(c^2 + d^2)^(1/
```

```
2)))*(a*c - b*d))/(c*(c^2 + d^2)^(1/2)))*(a*c - b*d)*(c^2 + d^2)^(1/2))/(c*
d^2 + c^3)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a+b\operatorname{csch}(x)}{c+d\sinh(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*csch(x))/(c+d*sinh(x)),x)
```

```
[Out] Integral((a + b*csch(x))/(c + d*sinh(x)), x)
```

3.574 $\int \frac{1+\sinh^2(x)}{1-\sinh^2(x)} \, dx$

Optimal. Leaf size=19

$$\sqrt{2} \tanh^{-1}\left(\sqrt{2} \tanh(x)\right) - x$$

[Out] -x+arctanh(2^(1/2)*tanh(x))*2^(1/2)

Rubi [A] time = 0.04, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {3171, 3181, 206}

$$\sqrt{2} \tanh^{-1}\left(\sqrt{2} \tanh(x)\right) - x$$

Antiderivative was successfully verified.

[In] Int[(1 + Sinh[x]^2)/(1 - Sinh[x]^2),x]

[Out] -x + Sqrt[2]*ArcTanh[Sqrt[2]*Tanh[x]]

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3171

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]^2)/((a_) + (b_.)*sin[(e_.) + (f_
.)*(x_)]^2), x_Symbol] :> Simp[(B*x)/b, x] + Dist[(A*b - a*B)/b, Int[1/(a +
 b*Sin[e + f*x]^2), x], x] /; FreeQ[{a, b, e, f, A, B}, x]
```

Rule 3181

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[1/(a + (a + b)*ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

Rubi steps

$$\int \frac{1+\sinh^2(x)}{1-\sinh^2(x)}\,dx = -x+2\int \frac{1}{1-\sinh^2(x)}\,dx$$
$$= -x+2\,\text{Subst}\left(\int \frac{1}{1-2x^2}\,dx, x, \tanh(x)\right)$$
$$= -x+\sqrt{2}\,\tanh^{-1}\left(\sqrt{2}\,\tanh(x)\right)$$

Mathematica [A] time = 0.03, size = 24, normalized size = 1.26

$$-2\left(\frac{x}{2}-\frac{\tanh^{-1}\left(\sqrt{2}\,\tanh(x)\right)}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Sinh[x]^2)/(1 - Sinh[x]^2),x]
```

```
[Out] -2*(x/2 - ArcTanh[Sqrt[2]*Tanh[x]]/Sqrt[2])
```

fricas [B] time = 0.42, size = 70, normalized size = 3.68

$$\frac{1}{2}\sqrt{2}\log\left(-\frac{3\left(2\sqrt{2}-3\right)\cosh(x)^2-4\left(3\sqrt{2}-4\right)\cosh(x)\sinh(x)+3\left(2\sqrt{2}-3\right)\sinh(x)^2-2\sqrt{2}+3}{\cosh(x)^2+\sinh(x)^2-3}\right)-x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+sinh(x)^2)/(1-sinh(x)^2),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(2)*log(-(3*(2*sqrt(2) - 3)*cosh(x)^2 - 4*(3*sqrt(2) - 4)*cosh(x)*s
inh(x) + 3*(2*sqrt(2) - 3)*sinh(x)^2 - 2*sqrt(2) + 3)/(cosh(x)^2 + sinh(x)^
2 - 3)) - x
```

giac [B] time = 0.14, size = 41, normalized size = 2.16

$$-\frac{1}{2}\sqrt{2}\log\left(\frac{\left|-4\sqrt{2}+2\,e^{(2\,x)}-6\right|}{\left|4\sqrt{2}+2\,e^{(2\,x)}-6\right|}\right)-x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+sinh(x)^2)/(1-sinh(x)^2),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(2)*log(abs(-4*sqrt(2) + 2*e^(2*x) - 6)/abs(4*sqrt(2) + 2*e^(2*x)
- 6)) - x
```

maple [B] time = 0.16, size = 54, normalized size = 2.84

$$\sqrt{2}\ \operatorname{arctanh}\left(\frac{\left(2\tanh\left(\frac{x}{2}\right)-2\right)\sqrt{2}}{4}\right)+\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)+\sqrt{2}\ \operatorname{arctanh}\left(\frac{\left(2\tanh\left(\frac{x}{2}\right)+2\right)\sqrt{2}}{4}\right)-\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+sinh(x)^2)/(1-sinh(x)^2),x)
```

```
[Out] 2^(1/2)*arctanh(1/4*(2*tanh(1/2*x)-2)*2^(1/2))+ln(tanh(1/2*x)-1)+2^(1/2)*ar
ctanh(1/4*(2*tanh(1/2*x)+2)*2^(1/2))-ln(tanh(1/2*x)+1)
```

maxima [B] time = 0.46, size = 64, normalized size = 3.37

$$\frac{1}{2}\sqrt{2}\log\left(-\frac{\sqrt{2}-e^{(-x)}+1}{\sqrt{2}+e^{(-x)}-1}\right)-\frac{1}{2}\sqrt{2}\log\left(-\frac{\sqrt{2}-e^{(-x)}-1}{\sqrt{2}+e^{(-x)}+1}\right)-x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+sinh(x)^2)/(1-sinh(x)^2),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(2)*log(-(sqrt(2) - e^(-x) + 1)/(sqrt(2) + e^(-x) - 1)) - 1/2*sqrt(
2)*log(-(sqrt(2) - e^(-x) - 1)/(sqrt(2) + e^(-x) + 1)) - x
```

mupad [B] time = 0.13, size = 56, normalized size = 2.95

$$\frac{\sqrt{2}\ \ln\left(8\,e^{2\,x}+\frac{\sqrt{2}\,\left(12\,e^{2\,x}-4\right)}{2}\right)}{2}-\frac{\sqrt{2}\ \ln\left(8\,e^{2\,x}-\frac{\sqrt{2}\,\left(12\,e^{2\,x}-4\right)}{2}\right)}{2}-x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(sinh(x)^2 + 1)/(sinh(x)^2 - 1),x)
```

```
[Out] (2^(1/2)*log(8*exp(2*x) + (2^(1/2)*(12*exp(2*x) - 4))/2))/2 - (2^(1/2)*log(
8*exp(2*x) - (2^(1/2)*(12*exp(2*x) - 4))/2))/2 - x
```

sympy [B] time = 6.54, size = 238, normalized size = 12.53

$$-\frac{1331714x}{941664\sqrt{2}+1331714}-\frac{941664\sqrt{2}\,x}{941664\sqrt{2}+1331714}+\frac{941664\log\left(\tanh\left(\frac{x}{2}\right)-1+\sqrt{2}\right)}{941664\sqrt{2}+1331714}+\frac{665857\sqrt{2}\log\left(\tanh\left(\frac{x}{2}\right)-1\right.}{941664\sqrt{2}+133171}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+sinh(x)**2)/(1-sinh(x)**2),x)
```

```
[Out] -1331714*x/(941664*sqrt(2) + 1331714) - 941664*sqrt(2)*x/(941664*sqrt(2) +
1331714) + 941664*log(tanh(x/2) - 1 + sqrt(2))/(941664*sqrt(2) + 1331714) +
 665857*sqrt(2)*log(tanh(x/2) - 1 + sqrt(2))/(941664*sqrt(2) + 1331714) + 9
41664*log(tanh(x/2) + 1 + sqrt(2))/(941664*sqrt(2) + 1331714) + 665857*sqrt
(2)*log(tanh(x/2) + 1 + sqrt(2))/(941664*sqrt(2) + 1331714) - 665857*sqrt(2
)*log(tanh(x/2) - sqrt(2) - 1)/(941664*sqrt(2) + 1331714) - 941664*log(tanh
(x/2) - sqrt(2) - 1)/(941664*sqrt(2) + 1331714) - 665857*sqrt(2)*log(tanh(x
/2) - sqrt(2) + 1)/(941664*sqrt(2) + 1331714) - 941664*log(tanh(x/2) - sqrt
(2) + 1)/(941664*sqrt(2) + 1331714)
```

3.575 $\int \frac{1-\sinh^2(x)}{1+\sinh^2(x)}\,dx$

Optimal. Leaf size=8

$$2\tanh(x) - x$$

```
[Out] -x+2*tanh(x)
```

Rubi [A] time = 0.04, antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {3171, 3175, 3767, 8}

$$2\tanh(x) - x$$

Antiderivative was successfully verified.

```
[In] Int[(1 - Sinh[x]^2)/(1 + Sinh[x]^2),x]
```

```
[Out] -x + 2*Tanh[x]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3171

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]^2)/((a_) + (b_.)*sin[(e_.) + (f_
.)*(x_)]^2), x_Symbol] :> Simp[(B*x)/b, x] + Dist[(A*b - a*B)/b, Int[1/(a +
 b*Sin[e + f*x]^2), x], x] /; FreeQ[{a, b, e, f, A, B}, x]
```

Rule 3175

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
a^p, Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1-\sinh^2(x)}{1+\sinh^2(x)}\,dx &= -x+2\int \frac{1}{1+\sinh^2(x)}\,dx\\ &= -x+2\int \operatorname{sech}^2(x)\,dx\\ &= -x+2i\,\text{Subst}(\int 1\,dx, x, -i\tanh(x))\\ &= -x+2\tanh(x)\end{aligned}$$

Mathematica [A] time = 0.02, size = 8, normalized size = 1.00

$$2\tanh(x)-x$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - Sinh[x]^2)/(1 + Sinh[x]^2),x]
```

```
[Out] -x + 2*Tanh[x]
```

fricas [B] time = 0.40, size = 17, normalized size = 2.12

$$-\frac{(x+2)\cosh(x)-2\,\sinh(x)}{\cosh(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-sinh(x)^2)/(1+sinh(x)^2),x, algorithm="fricas")
```

```
[Out] -((x + 2)*cosh(x) - 2*sinh(x))/cosh(x)
```

giac [A] time = 0.13, size = 14, normalized size = 1.75

$$-x-\frac{4}{e^{(2x)}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-sinh(x)^2)/(1+sinh(x)^2),x, algorithm="giac")
```

```
[Out] -x - 4/(e^(2*x) + 1)
```

maple [B] time = 0.15, size = 34, normalized size = 4.25

$$\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)-\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)+\frac{4\tanh\left(\frac{x}{2}\right)}{\tanh^2\left(\frac{x}{2}\right)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-sinh(x)^2)/(1+sinh(x)^2),x)
```

```
[Out] ln(tanh(1/2*x)-1)-ln(tanh(1/2*x)+1)+4*tanh(1/2*x)/(tanh(1/2*x)^2+1)
```

maxima [A] time = 0.34, size = 14, normalized size = 1.75

$$-x+\frac{4}{e^{(-2x)}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-sinh(x)^2)/(1+sinh(x)^2),x, algorithm="maxima")
```

```
[Out] -x + 4/(e^(-2*x) + 1)
```

mupad [B] time = 0.04, size = 14, normalized size = 1.75

$$-x-\frac{4}{\mathrm{e}^{2x}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(sinh(x)^2 - 1)/(sinh(x)^2 + 1),x)
```

```
[Out] - x - 4/(exp(2*x) + 1)
```

sympy [B] time = 1.03, size = 41, normalized size = 5.12

$$-\frac{x\tanh^2\left(\frac{x}{2}\right)}{\tanh^2\left(\frac{x}{2}\right)+1}-\frac{x}{\tanh^2\left(\frac{x}{2}\right)+1}+\frac{4\tanh\left(\frac{x}{2}\right)}{\tanh^2\left(\frac{x}{2}\right)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-sinh(x)**2)/(1+sinh(x)**2),x)
```

```
[Out] -x*tanh(x/2)**2/(tanh(x/2)**2 + 1) - x/(tanh(x/2)**2 + 1) + 4*tanh(x/2)/(ta
nh(x/2)**2 + 1)
```

3.576 $\int \frac{1+\cosh^2(x)}{1-\cosh^2(x)}\,dx$

Optimal. Leaf size=8

$$2\coth(x)-x$$

```
[Out] -x+2*coth(x)
```

Rubi [A] time = 0.04, antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {3171, 3175, 3767, 8}

$$2\coth(x)-x$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Cosh[x]^2)/(1 - Cosh[x]^2),x]
```

```
[Out] -x + 2*Coth[x]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3171

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]^2)/((a_) + (b_.)*sin[(e_.) + (f_
.)*(x_)]^2), x_Symbol] :> Simp[(B*x)/b, x] + Dist[(A*b - a*B)/b, Int[1/(a +
 b*Sin[e + f*x]^2), x], x] /; FreeQ[{a, b, e, f, A, B}, x]
```

Rule 3175

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
a^p, Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1+\cosh^2(x)}{1-\cosh^2(x)}\,dx &= -x+2\int \frac{1}{1-\cosh^2(x)}\,dx \\
&= -x-2\int \operatorname{csch}^2(x)\,dx \\
&= -x+2i\,\text{Subst}\Big(\int 1\,dx, x, -i\coth(x)\Big) \\
&= -x+2\coth(x)
\end{aligned}
$$

Mathematica [A] time = 0.01, size = 8, normalized size = 1.00

$$2\coth(x)-x$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Cosh[x]^2)/(1 - Cosh[x]^2),x]
[Out] -x + 2*Coth[x]
```

fricas [B] time = 0.40, size = 17, normalized size = 2.12

$$-\frac{(x+2)\sinh(x)-2\cosh(x)}{\sinh(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+cosh(x)^2)/(1-cosh(x)^2),x, algorithm="fricas")
[Out] -((x + 2)*sinh(x) - 2*cosh(x))/sinh(x)
```

giac [A] time = 0.11, size = 14, normalized size = 1.75

$$-x+\frac{4}{e^{(2x)}-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+cosh(x)^2)/(1-cosh(x)^2),x, algorithm="giac")
[Out] -x + 4/(e^(2*x) - 1)
```

maple [B] time = 0.14, size = 28, normalized size = 3.50

$$\tanh\left(\frac{x}{2}\right)+\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)-\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)+\frac{1}{\tanh\left(\frac{x}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+cosh(x)^2)/(1-cosh(x)^2),x)
```

```
[Out] tanh(1/2*x)+ln(tanh(1/2*x)-1)-ln(tanh(1/2*x)+1)+1/tanh(1/2*x)
```

maxima [A] time = 0.36, size = 14, normalized size = 1.75

$$-x - \frac{4}{e^{(-2x)} - 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+cosh(x)^2)/(1-cosh(x)^2),x, algorithm="maxima")
```

```
[Out] -x - 4/(e^(-2*x) - 1)
```

mupad [B] time = 0.05, size = 14, normalized size = 1.75

$$\frac{4}{e^{2x} - 1} - x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(cosh(x)^2 + 1)/(cosh(x)^2 - 1),x)
```

```
[Out] 4/(exp(2*x) - 1) - x
```

sympy [B] time = 0.91, size = 12, normalized size = 1.50

$$-x + \tanh\left(\frac{x}{2}\right) + \frac{1}{\tanh\left(\frac{x}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+cosh(x)**2)/(1-cosh(x)**2),x)
```

```
[Out] -x + tanh(x/2) + 1/tanh(x/2)
```

3.577 $\int \frac{1-\cosh^2(x)}{1+\cosh^2(x)} dx$

Optimal. Leaf size=19

$$\sqrt{2} \tanh^{-1}\left(\frac{\tanh(x)}{\sqrt{2}}\right) - x$$

```
[Out] -x+arctanh(1/2*2^(1/2)*tanh(x))*2^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {3171, 3181, 206}

$$\sqrt{2} \tanh^{-1}\left(\frac{\tanh(x)}{\sqrt{2}}\right) - x$$

Antiderivative was successfully verified.

```
[In] Int[(1 - Cosh[x]^2)/(1 + Cosh[x]^2),x]
```

```
[Out] -x + Sqrt[2]*ArcTanh[Tanh[x]/Sqrt[2]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3171

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]^2)/((a_) + (b_.)*sin[(e_.) + (f_
.)*(x_)]^2), x_Symbol] :> Simp[(B*x)/b, x] + Dist[(A*b - a*B)/b, Int[1/(a +
 b*Sin[e + f*x]^2), x], x] /; FreeQ[{a, b, e, f, A, B}, x]
```

Rule 3181

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[1/(a + (a + b)*ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

Rubi steps

$$\int \frac{1-\cosh^2(x)}{1+\cosh^2(x)}\,dx = -x + 2\int \frac{1}{1+\cosh^2(x)}\,dx$$
$$= -x + 2\,\text{Subst}\left(\int \frac{1}{1-2x^2}\,dx, x, \coth(x)\right)$$
$$= -x + \sqrt{2}\tanh^{-1}\left(\frac{\tanh(x)}{\sqrt{2}}\right)$$

Mathematica [A] time = 0.03, size = 24, normalized size = 1.26

$$-2\left(\frac{x}{2} - \frac{\tanh^{-1}\left(\frac{\tanh(x)}{\sqrt{2}}\right)}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - Cosh[x]^2)/(1 + Cosh[x]^2),x]
```

```
[Out] -2*(x/2 - ArcTanh[Tanh[x]/Sqrt[2]]/Sqrt[2])
```

fricas [B] time = 0.44, size = 70, normalized size = 3.68

$$\frac{1}{2}\sqrt{2}\log\left(-\frac{3\left(2\sqrt{2}-3\right)\cosh(x)^2 - 4\left(3\sqrt{2}-4\right)\cosh(x)\sinh(x) + 3\left(2\sqrt{2}-3\right)\sinh(x)^2 + 2\sqrt{2}-3}{\cosh(x)^2+\sinh(x)^2+3}\right) - x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-cosh(x)^2)/(1+cosh(x)^2),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(2)*log(-(3*(2*sqrt(2) - 3)*cosh(x)^2 - 4*(3*sqrt(2) - 4)*cosh(x)*s
inh(x) + 3*(2*sqrt(2) - 3)*sinh(x)^2 + 2*sqrt(2) - 3)/(cosh(x)^2 + sinh(x)^
2 + 3)) - x
```

giac [B] time = 0.14, size = 38, normalized size = 2.00

$$\frac{1}{2}\sqrt{2}\log\left(-\frac{2\sqrt{2}-e^{(2x)}-3}{2\sqrt{2}+e^{(2x)}+3}\right) - x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-cosh(x)^2)/(1+cosh(x)^2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(2)*log(-(2*sqrt(2) - e^(2*x) - 3)/(2*sqrt(2) + e^(2*x) + 3)) - x
```

maple [B] time = 0.12, size = 102, normalized size = 5.37

$$\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)+\frac{\sqrt{2}\ \ln\left(\frac{\tanh^{2}\left(\frac{x}{2}\right)+\sqrt{2}\ \tanh\left(\frac{x}{2}\right)+1}{\tanh^{2}\left(\frac{x}{2}\right)-\sqrt{2}\ \tanh\left(\frac{x}{2}\right)+1}\right)}{4}-\frac{\sqrt{2}\ \ln\left(\frac{\tanh^{2}\left(\frac{x}{2}\right)-\sqrt{2}\ \tanh\left(\frac{x}{2}\right)+1}{\tanh^{2}\left(\frac{x}{2}\right)+\sqrt{2}\ \tanh\left(\frac{x}{2}\right)+1}\right)}{4}-\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-cosh(x)^2)/(1+cosh(x)^2),x)
```

```
[Out] ln(tanh(1/2*x)-1)+1/4*2^(1/2)*ln((tanh(1/2*x)^2+2^(1/2)*tanh(1/2*x)+1)/(tan
h(1/2*x)^2-2^(1/2)*tanh(1/2*x)+1))-1/4*2^(1/2)*ln((tanh(1/2*x)^2-2^(1/2)*ta
nh(1/2*x)+1)/(tanh(1/2*x)^2+2^(1/2)*tanh(1/2*x)+1))-ln(tanh(1/2*x)+1)
```

maxima [B] time = 0.57, size = 102, normalized size = 5.37

$$\frac{3}{16}\sqrt{2}\log\left(-\frac{2\sqrt{2}-e^{(2x)}-3}{2\sqrt{2}+e^{(2x)}+3}\right)-\frac{5}{16}\sqrt{2}\log\left(-\frac{2\sqrt{2}-e^{(-2x)}-3}{2\sqrt{2}+e^{(-2x)}+3}\right)-2x+\frac{1}{4}\log\left(e^{(4x)}+6e^{(2x)}+1\right)-\frac{1}{4}\log\left(6e^{(-2x}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-cosh(x)^2)/(1+cosh(x)^2),x, algorithm="maxima")
```

```
[Out] 3/16*sqrt(2)*log(-(2*sqrt(2) - e^(2*x) - 3)/(2*sqrt(2) + e^(2*x) + 3)) - 5/
16*sqrt(2)*log(-(2*sqrt(2) - e^(-2*x) - 3)/(2*sqrt(2) + e^(-2*x) + 3)) - 2*
x + 1/4*log(e^(4*x) + 6*e^(2*x) + 1) - 1/4*log(6*e^(-2*x) + e^(-4*x) + 1)
```

mupad [B] time = 0.11, size = 56, normalized size = 2.95

$$\frac{\sqrt{2}\ \ln\left(-8\,\mathrm{e}^{2\,x}-\frac{\sqrt{2}\,\left(12\,\mathrm{e}^{2\,x}+4\right)}{2}\right)}{2}-x-\frac{\sqrt{2}\ \ln\left(\frac{\sqrt{2}\,\left(12\,\mathrm{e}^{2\,x}+4\right)}{2}-8\,\mathrm{e}^{2\,x}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(cosh(x)^2 - 1)/(cosh(x)^2 + 1),x)
```

```
[Out] (2^(1/2)*log(- 8*exp(2*x) - (2^(1/2)*(12*exp(2*x) + 4))/2))/2 - x - (2^(1/2
)*log((2^(1/2)*(12*exp(2*x) + 4))/2 - 8*exp(2*x)))/2
```

sympy [B] time = 3.02, size = 61, normalized size = 3.21

$$-x-\frac{\sqrt{2}\log\left(4\tanh^{2}\left(\frac{x}{2}\right)-4\sqrt{2}\tanh\left(\frac{x}{2}\right)+4\right)}{2}+\frac{\sqrt{2}\log\left(4\tanh^{2}\left(\frac{x}{2}\right)+4\sqrt{2}\tanh\left(\frac{x}{2}\right)+4\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-cosh(x)**2)/(1+cosh(x)**2),x)
```

```
[Out] -x - sqrt(2)*log(4*tanh(x/2)**2 - 4*sqrt(2)*tanh(x/2) + 4)/2 + sqrt(2)*log(
4*tanh(x/2)**2 + 4*sqrt(2)*tanh(x/2) + 4)/2
```

3.578 $\int \frac{a+b\,\mathrm{sech}^2(x)}{c+d\cosh(x)}\,dx$

Optimal. Leaf size=74

$$\frac{2\left(ac^2+bd^2\right)\tanh^{-1}\left(\frac{\sqrt{c-d}\,\tanh\left(\frac{x}{2}\right)}{\sqrt{c+d}}\right)}{c^2\sqrt{c-d}\,\sqrt{c+d}}-\frac{bd\tan^{-1}(\sinh(x))}{c^2}+\frac{b\tanh(x)}{c}$$

```
[Out] -b*d*arctan(sinh(x))/c^2+2*(a*c^2+b*d^2)*arctanh((c-d)^(1/2)*tanh(1/2*x)/(c
+d)^(1/2))/c^2/(c-d)^(1/2)/(c+d)^(1/2)+b*tanh(x)/c
```

Rubi [A] time = 0.25, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {4234, 3056, 3001, 3770, 2659, 208}

$$\frac{2\left(ac^2+bd^2\right)\tanh^{-1}\left(\frac{\sqrt{c-d}\,\tanh\left(\frac{x}{2}\right)}{\sqrt{c+d}}\right)}{c^2\sqrt{c-d}\,\sqrt{c+d}}-\frac{bd\tan^{-1}(\sinh(x))}{c^2}+\frac{b\tanh(x)}{c}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sech[x]^2)/(c + d*Cosh[x]),x]
```

```
[Out] -((b*d*ArcTan[Sinh[x]])/c^2) + (2*(a*c^2 + b*d^2)*ArcTanh[(Sqrt[c - d]*Tanh
[x/2])/Sqrt[c + d]])/(c^2*Sqrt[c - d]*Sqrt[c + d]) + (b*Tanh[x])/c
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 3001

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
```

```
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3056

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((A*b^2 + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin
[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)), x] + Dist[1/((m +
1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
+ f*x])^n*Simp[a*(m + 1)*(b*c - a*d)*(A + C) + d*(A*b^2 + a^2*C)*(m + n + 2
) - (c*(A*b^2 + a^2*C) + b*(m + 1)*(b*c - a*d)*(A + C))*Sin[e + f*x] - d*(A
*b^2 + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d,
e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d
^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] && !IntegerQ[n]) || !(
IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] && !IntegerQ[m]) || EqQ[a, 0]
)))
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4234

```
Int[(u_)*((A_) + (C_.)*sec[(a_.) + (b_.)*(x_)]^2), x_Symbol] :> Int[(Activa
teTrig[u]*(C + A*Cos[a + b*x]^2))/Cos[a + b*x]^2, x] /; FreeQ[{a, b, A, C},
 x] && KnownSineIntegrandQ[u, x]
```

Rubi steps

$$\int \frac{a+b\operatorname{sech}^2(x)}{c+d\cosh(x)}\,dx = \int \frac{\left(b+a\cosh^2(x)\right)\operatorname{sech}^2(x)}{c+d\cosh(x)}\,dx$$

$$= \frac{b\tanh(x)}{c} + \frac{\int \frac{(-bd+ac\cosh(x))\operatorname{sech}(x)}{c+d\cosh(x)}\,dx}{c}$$

$$= \frac{b\tanh(x)}{c} - \frac{(bd)\int \operatorname{sech}(x)\,dx}{c^2} + \left(a+\frac{bd^2}{c^2}\right)\int \frac{1}{c+d\cosh(x)}\,dx$$

$$= -\frac{bd\tan^{-1}(\sinh(x))}{c^2} + \frac{b\tanh(x)}{c} + \left(2\left(a+\frac{bd^2}{c^2}\right)\right)\operatorname{Subst}\left(\int \frac{1}{c+d-(c-d)x^2}\,dx, x, \tanh\left(\right.\right.$$

$$= -\frac{bd\tan^{-1}(\sinh(x))}{c^2} + \frac{2\left(a+\frac{bd^2}{c^2}\right)\tanh^{-1}\left(\frac{\sqrt{c-d}\,\tanh\left(\frac{x}{2}\right)}{\sqrt{c+d}}\right)}{\sqrt{c-d}\,\sqrt{c+d}} + \frac{b\tanh(x)}{c}$$

Mathematica [A] time = 0.22, size = 127, normalized size = 1.72

$$-\frac{2\operatorname{sech}(x)\left(a\cosh^2(x)+b\right)\left(2\cosh(x)\left(\left(ac^2+bd^2\right)\tan^{-1}\left(\frac{(c-d)\tanh\left(\frac{x}{2}\right)}{\sqrt{d^2-c^2}}\right)+bd\sqrt{d^2-c^2}\,\tan^{-1}\left(\tanh\left(\frac{x}{2}\right)\right)\right)-bc\sqrt{d^2\right.}{c^2\sqrt{d^2-c^2}\,(a\cosh(2x)+a+2b)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sech[x]^2)/(c + d*Cosh[x]),x]
```

```
[Out] (-2*(b + a*Cosh[x]^2)*Sech[x]*(2*(b*d*Sqrt[-c^2 + d^2]*ArcTan[Tanh[x/2]] +
(a*c^2 + b*d^2)*ArcTan[((c - d)*Tanh[x/2])/Sqrt[-c^2 + d^2]])*Cosh[x] - b*c
*Sqrt[-c^2 + d^2]*Sinh[x]))/(c^2*Sqrt[-c^2 + d^2]*(a + 2*b + a*Cosh[2*x]))
```

fricas [B] time = 0.79, size = 598, normalized size = 8.08

$$\left[-\frac{2\,bc^3-2\,bcd^2-\left(ac^2+bd^2+\left(ac^2+bd^2\right)\cosh(x)^2+2\left(ac^2+bd^2\right)\cosh(x)\sinh(x)+\left(ac^2+bd^2\right)\sinh(x)^2\right)\sqrt{c^2}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(x)^2)/(c+d*cosh(x)),x, algorithm="fricas")
```

```
[Out] [-(2*b*c^3 - 2*b*c*d^2 - (a*c^2 + b*d^2 + (a*c^2 + b*d^2)*cosh(x)^2 + 2*(a*
c^2 + b*d^2)*cosh(x)*sinh(x) + (a*c^2 + b*d^2)*sinh(x)^2)*sqrt(c^2 - d^2)*l
og((d^2*cosh(x)^2 + d^2*sinh(x)^2 + 2*c*d*cosh(x) + 2*c^2 - d^2 + 2*(d^2*co
```

```
sh(x) + c*d)*sinh(x) - 2*sqrt(c^2 - d^2)*(d*cosh(x) + d*sinh(x) + c))/(d*co
sh(x)^2 + d*sinh(x)^2 + 2*c*cosh(x) + 2*(d*cosh(x) + c)*sinh(x) + d)) + 2*(
b*c^2*d - b*d^3 + (b*c^2*d - b*d^3)*cosh(x)^2 + 2*(b*c^2*d - b*d^3)*cosh(x)
*sinh(x) + (b*c^2*d - b*d^3)*sinh(x)^2)*arctan(cosh(x) + sinh(x)))/(c^4 - c
^2*d^2 + (c^4 - c^2*d^2)*cosh(x)^2 + 2*(c^4 - c^2*d^2)*cosh(x)*sinh(x) + (c
^4 - c^2*d^2)*sinh(x)^2), -2*(b*c^3 - b*c*d^2 + (a*c^2 + b*d^2 + (a*c^2 + b
*d^2)*cosh(x)^2 + 2*(a*c^2 + b*d^2)*cosh(x)*sinh(x) + (a*c^2 + b*d^2)*sinh(
x)^2)*sqrt(-c^2 + d^2)*arctan(-sqrt(-c^2 + d^2)*(d*cosh(x) + d*sinh(x) + c)
/(c^2 - d^2)) + (b*c^2*d - b*d^3 + (b*c^2*d - b*d^3)*cosh(x)^2 + 2*(b*c^2*d
 - b*d^3)*cosh(x)*sinh(x) + (b*c^2*d - b*d^3)*sinh(x)^2)*arctan(cosh(x) + s
inh(x)))/(c^4 - c^2*d^2 + (c^4 - c^2*d^2)*cosh(x)^2 + 2*(c^4 - c^2*d^2)*cos
h(x)*sinh(x) + (c^4 - c^2*d^2)*sinh(x)^2)]
```

giac [A] time = 0.14, size = 71, normalized size = 0.96

$$-\frac{2\,bd\arctan\left(e^{x}\right)}{c^{2}}+\frac{2\left(ac^{2}+bd^{2}\right)\arctan\left(\frac{de^{x}+c}{\sqrt{-c^{2}+d^{2}}}\right)}{\sqrt{-c^{2}+d^{2}}\,c^{2}}-\frac{2\,b}{c\left(e^{(2\,x)}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(x)^2)/(c+d*cosh(x)),x, algorithm="giac")
```

```
[Out] -2*b*d*arctan(e^x)/c^2 + 2*(a*c^2 + b*d^2)*arctan((d*e^x + c)/sqrt(-c^2 + d
^2))/(sqrt(-c^2 + d^2)*c^2) - 2*b/(c*(e^(2*x) + 1))
```

maple [A] time = 0.18, size = 112, normalized size = 1.51

$$\frac{2\operatorname{arctanh}\left(\frac{(c-d)\tanh\left(\frac{x}{2}\right)}{\sqrt{(c+d)(c-d)}}\right)a}{\sqrt{(c+d)\,(c-d)}}+\frac{2\operatorname{arctanh}\left(\frac{(c-d)\tanh\left(\frac{x}{2}\right)}{\sqrt{(c+d)(c-d)}}\right)b\,d^{2}}{c^{2}\sqrt{(c+d)\,(c-d)}}+\frac{2b\tanh\left(\frac{x}{2}\right)}{c\left(\tanh^{2}\left(\frac{x}{2}\right)+1\right)}-\frac{2bd\arctan\left(\tanh\left(\frac{x}{2}\right)\right)}{c^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sech(x)^2)/(c+d*cosh(x)),x)
```

```
[Out] 2/((c+d)*(c-d))^(1/2)*arctanh((c-d)*tanh(1/2*x)/((c+d)*(c-d))^(1/2))*a+2/c^
2/((c+d)*(c-d))^(1/2)*arctanh((c-d)*tanh(1/2*x)/((c+d)*(c-d))^(1/2))*b*d^2+
2*b/c*tanh(1/2*x)/(tanh(1/2*x)^2+1)-2*b/c^2*d*arctan(tanh(1/2*x))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(x)^2)/(c+d*cosh(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*c^2-4*d^2>0)', see `assume?` for
more details)Is 4*c^2-4*d^2 positive or negative?
```

mupad [B] time = 6.90, size = 704, normalized size = 9.51

$$\frac{\ln\left(\frac{\sqrt{(c+d)\,(c-d)}\left(\frac{32\left(a^2\,c^4+2\,a\,b\,c^2\,d^2-4\,e^x\,b^2\,c^3\,d-2\,b^2\,c^2\,d^2+3\,e^x\,b^2\,c\,d^3+2\,b^2\,d^4\right)}{c^2\,d^4}-\frac{\sqrt{(c+d)\,(c-d)}\,\left(a\,c^2+b\,d^2\right)\left(\frac{32\,c\left(2\,b\,d^3+4\,a\,c^3\,e^x+2\,a\,c^2\,d-a\,c\,d^2\,e^x+3\,b\,c\,d^2\,e^x\right)}{d^5}+\frac{32\,\sqrt{(c}}{}\right.}{c^2\left(c^2-d^2\right)}\right.}{c^2\left(c^2-d^2\right)}\right.}{c^4-c^2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cosh(x)^2)/(c + d*cosh(x)),x)
```

```
[Out] (log((((c + d)*(c - d))^(1/2)*((32*(a^2*c^4 + 2*b^2*d^4 - 2*b^2*c^2*d^2 + 3
*b^2*c*d^3*exp(x) - 4*b^2*c^3*d*exp(x) + 2*a*b*c^2*d^2))/(c^2*d^4) - (((c +
 d)*(c - d))^(1/2)*(a*c^2 + b*d^2)*((32*c*(2*b*d^3 + 4*a*c^3*exp(x) + 2*a*c
^2*d - a*c*d^2*exp(x) + 3*b*c*d^2*exp(x)))/d^5 + (32*((c + d)*(c - d))^(1/2
)*(a*c^2 + b*d^2)*(3*c^2*d - 2*d^3 + 4*c^3*exp(x) - 3*c*d^2*exp(x)))/(d^5*(
c^2 - d^2))))/(c^2*(c^2 - d^2)))*(a*c^2 + b*d^2))/(c^2*(c^2 - d^2)) - (32*b
*(a*c^2 + b*d^2)*(2*b*d + a*c*exp(x) + 4*b*c*exp(x)))/(c^3*d^3))*((c + d)*(
c - d))^(1/2)*(a*c^2 + b*d^2))/(c^4 - c^2*d^2) - (2*b)/(c*(exp(2*x) + 1)) -
 (log(- (32*b*(a*c^2 + b*d^2)*(2*b*d + a*c*exp(x) + 4*b*c*exp(x)))/(c^3*d^3
) - (((c + d)*(c - d))^(1/2)*((32*(a^2*c^4 + 2*b^2*d^4 - 2*b^2*c^2*d^2 + 3*
b^2*c*d^3*exp(x) - 4*b^2*c^3*d*exp(x) + 2*a*b*c^2*d^2))/(c^2*d^4) + (((c +
d)*(c - d))^(1/2)*(a*c^2 + b*d^2)*((32*c*(2*b*d^3 + 4*a*c^3*exp(x) + 2*a*c^
2*d - a*c*d^2*exp(x) + 3*b*c*d^2*exp(x)))/d^5 - (32*((c + d)*(c - d))^(1/2)
*(a*c^2 + b*d^2)*(3*c^2*d - 2*d^3 + 4*c^3*exp(x) - 3*c*d^2*exp(x)))/(d^5*(c
^2 - d^2))))/(c^2*(c^2 - d^2)))*(a*c^2 + b*d^2))/(c^2*(c^2 - d^2)))*((c + d
)*(c - d))^(1/2)*(a*c^2 + b*d^2))/(c^4 - c^2*d^2) + (b*d*log(exp(x) - 1i)*1
i)/c^2 - (b*d*log(exp(x) + 1i)*1i)/c^2
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b\,\mathrm{sech}^2\,(x)}{c + d\cosh\,(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(x)**2)/(c+d*cosh(x)),x)
```

```
[Out] Integral((a + b*sech(x)**2)/(c + d*cosh(x)), x)
```

3.579 $\int \frac{a+b\operatorname{csch}^2(x)}{c+d\sinh(x)}\,dx$

Optimal. Leaf size=69

$$-\frac{2\left(ac^2+bd^2\right)\tanh^{-1}\left(\frac{d-c\tanh\left(\frac{x}{2}\right)}{\sqrt{c^2+d^2}}\right)}{c^2\sqrt{c^2+d^2}}+\frac{bd\tanh^{-1}(\cosh(x))}{c^2}-\frac{b\coth(x)}{c}$$

```
[Out] b*d*arctanh(cosh(x))/c^2-b*coth(x)/c-2*(a*c^2+b*d^2)*arctanh((d-c*tanh(1/2*
x))/(c^2+d^2)^(1/2))/c^2/(c^2+d^2)^(1/2)
```

Rubi [A] time = 0.26, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.412, Rules used = {4233, 3056, 3001, 3770, 2660, 618, 206}

$$-\frac{2\left(ac^2+bd^2\right)\tanh^{-1}\left(\frac{d-c\tanh\left(\frac{x}{2}\right)}{\sqrt{c^2+d^2}}\right)}{c^2\sqrt{c^2+d^2}}+\frac{bd\tanh^{-1}(\cosh(x))}{c^2}-\frac{b\coth(x)}{c}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Csch[x]^2)/(c + d*Sinh[x]),x]
```

```
[Out] (b*d*ArcTanh[Cosh[x]])/c^2 - (2*(a*c^2 + b*d^2)*ArcTanh[(d - c*Tanh[x/2])/S
qrt[c^2 + d^2]])/(c^2*Sqrt[c^2 + d^2]) - (b*Coth[x])/c
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 3001

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3056

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((A*b^2 + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin
[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)), x] + Dist[1/((m +
1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
+ f*x])^n*Simp[a*(m + 1)*(b*c - a*d)*(A + C) + d*(A*b^2 + a^2*C)*(m + n + 2
) - (c*(A*b^2 + a^2*C) + b*(m + 1)*(b*c - a*d)*(A + C))*Sin[e + f*x] - d*(A
*b^2 + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d,
e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d
^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(
IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0]
)))
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4233

```
Int[(csc[(a_.) + (b_.)*(x_)]^2*(C_.) + (A_))*(u_), x_Symbol] :> Int[(Activa
teTrig[u]*(C + A*Sin[a + b*x]^2))/Sin[a + b*x]^2, x] /; FreeQ[{a, b, A, C},
 x] && KnownSineIntegrandQ[u, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{a+b\,\text{csch}^2(x)}{c+d\sinh(x)}\,dx &= -\int \frac{\text{csch}^2(x)\left(-b-a\sinh^2(x)\right)}{c+d\sinh(x)}\,dx \\
&= -\frac{b\coth(x)}{c} - \frac{i\int \frac{\text{csch}(x)(-ibd+iac\sinh(x))}{c+d\sinh(x)}\,dx}{c} \\
&= -\frac{b\coth(x)}{c} - \frac{(bd)\int \text{csch}(x)\,dx}{c^2} + \left(a+\frac{bd^2}{c^2}\right)\int \frac{1}{c+d\sinh(x)}\,dx \\
&= \frac{bd\tanh^{-1}(\cosh(x))}{c^2} - \frac{b\coth(x)}{c} + \left(2\left(a+\frac{bd^2}{c^2}\right)\right)\text{Subst}\left(\int \frac{1}{c+2dx-cx^2}\,dx, x, \tanh\left(\frac{x}{2}\right)\right. \\
&= \frac{bd\tanh^{-1}(\cosh(x))}{c^2} - \frac{b\coth(x)}{c} - \left(4\left(a+\frac{bd^2}{c^2}\right)\right)\text{Subst}\left(\int \frac{1}{4\left(c^2+d^2\right)-x^2}\,dx, x, 2d-2c\right. \\
&= \frac{bd\tanh^{-1}(\cosh(x))}{c^2} - \frac{2\left(a+\frac{bd^2}{c^2}\right)\tanh^{-1}\left(\frac{d-c\tanh\left(\frac{x}{2}\right)}{\sqrt{c^2+d^2}}\right)}{\sqrt{c^2+d^2}} - \frac{b\coth(x)}{c}
\end{aligned}
$$

Mathematica [A] time = 0.42, size = 125, normalized size = 1.81

$$
-\frac{\text{csch}\left(\frac{x}{2}\right)\text{sech}\left(\frac{x}{2}\right)\left(\sinh(x)\left(bd\sqrt{-c^2-d^2}\,\log\left(\tanh\left(\frac{x}{2}\right)\right)-2\left(ac^2+bd^2\right)\tan^{-1}\left(\frac{d-c\tanh\left(\frac{x}{2}\right)}{\sqrt{-c^2-d^2}}\right)\right)+bc\sqrt{-c^2-d^2}\,\cos\right.}{2c^2\sqrt{-c^2-d^2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Csch[x]^2)/(c + d*Sinh[x]),x]
```

```
[Out] -1/2*(Csch[x/2]*Sech[x/2]*(b*c*Sqrt[-c^2 - d^2]*Cosh[x] + (-2*(a*c^2 + b*d^
2)*ArcTan[(d - c*Tanh[x/2])/Sqrt[-c^2 - d^2]] + b*d*Sqrt[-c^2 - d^2]*Log[Ta
nh[x/2]])*Sinh[x]))/(c^2*Sqrt[-c^2 - d^2])
```

fricas [B] time = 0.79, size = 401, normalized size = 5.81

$$
\frac{2\,bc^3+2\,bcd^2+\left(ac^2+bd^2-\left(ac^2+bd^2\right)\cosh(x)^2-2\left(ac^2+bd^2\right)\cosh(x)\sinh(x)-\left(ac^2+bd^2\right)\sinh(x)^2\right)\sqrt{c^2+}}{}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*csch(x)^2)/(c+d*sinh(x)),x, algorithm="fricas")
```

```
[Out] (2*b*c^3 + 2*b*c*d^2 + (a*c^2 + b*d^2 - (a*c^2 + b*d^2)*cosh(x)^2 - 2*(a*c^
2 + b*d^2)*cosh(x)*sinh(x) - (a*c^2 + b*d^2)*sinh(x)^2)*sqrt(c^2 + d^2)*log
((d^2*cosh(x)^2 + d^2*sinh(x)^2 + 2*c*d*cosh(x) + 2*c^2 + d^2 + 2*(d^2*cosh
(x) + c*d)*sinh(x) - 2*sqrt(c^2 + d^2)*(d*cosh(x) + d*sinh(x) + c))/(d*cosh
(x)^2 + d*sinh(x)^2 + 2*c*cosh(x) + 2*(d*cosh(x) + c)*sinh(x) - d)) + (b*c^
2*d + b*d^3 - (b*c^2*d + b*d^3)*cosh(x)^2 - 2*(b*c^2*d + b*d^3)*cosh(x)*sin
h(x) - (b*c^2*d + b*d^3)*sinh(x)^2)*log(cosh(x) + sinh(x) + 1) - (b*c^2*d +
 b*d^3 - (b*c^2*d + b*d^3)*cosh(x)^2 - 2*(b*c^2*d + b*d^3)*cosh(x)*sinh(x)
- (b*c^2*d + b*d^3)*sinh(x)^2)*log(cosh(x) + sinh(x) - 1))/(c^4 + c^2*d^2 -
 (c^4 + c^2*d^2)*cosh(x)^2 - 2*(c^4 + c^2*d^2)*cosh(x)*sinh(x) - (c^4 + c^2
*d^2)*sinh(x)^2)
```

giac [A] time = 0.16, size = 109, normalized size = 1.58

$$\frac{bd\log\left(e^{x}+1\right)}{c^{2}}-\frac{bd\log\left(|e^{x}-1|\right)}{c^{2}}+\frac{\left(ac^{2}+bd^{2}\right)\log\left(\frac{|2\,de^{x}+2\,c-2\,\sqrt{c^{2}+d^{2}}|}{|2\,de^{x}+2\,c+2\,\sqrt{c^{2}+d^{2}}|}\right)}{\sqrt{c^{2}+d^{2}}\,c^{2}}-\frac{2\,b}{c\left(e^{(2\,x)}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*csch(x)^2)/(c+d*sinh(x)),x, algorithm="giac")
```

```
[Out] b*d*log(e^x + 1)/c^2 - b*d*log(abs(e^x - 1))/c^2 + (a*c^2 + b*d^2)*log(abs(
2*d*e^x + 2*c - 2*sqrt(c^2 + d^2))/abs(2*d*e^x + 2*c + 2*sqrt(c^2 + d^2)))/
(sqrt(c^2 + d^2)*c^2) - 2*b/(c*(e^(2*x) - 1))
```

maple [A] time = 0.20, size = 112, normalized size = 1.62

$$-\frac{b\tanh\left(\frac{x}{2}\right)}{2c}-\frac{b}{2c\tanh\left(\frac{x}{2}\right)}-\frac{bd\ln\left(\tanh\left(\frac{x}{2}\right)\right)}{c^{2}}+\frac{2\,\mathrm{arctanh}\left(\frac{2c\tanh\left(\frac{x}{2}\right)-2d}{2\sqrt{c^{2}+d^{2}}}\right)a}{\sqrt{c^{2}+d^{2}}}+\frac{2\,\mathrm{arctanh}\left(\frac{2c\tanh\left(\frac{x}{2}\right)-2d}{2\sqrt{c^{2}+d^{2}}}\right)b\,d^{2}}{c^{2}\sqrt{c^{2}+d^{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*csch(x)^2)/(c+d*sinh(x)),x)
```

```
[Out] -1/2*b/c*tanh(1/2*x)-1/2*b/c/tanh(1/2*x)-1/c^2*b*d*ln(tanh(1/2*x))+2/(c^2+d
^2)^(1/2)*arctanh(1/2*(2*c*tanh(1/2*x)-2*d)/(c^2+d^2)^(1/2))*a+2/c^2/(c^2+d
^2)^(1/2)*arctanh(1/2*(2*c*tanh(1/2*x)-2*d)/(c^2+d^2)^(1/2))*b*d^2
```

maxima [B] time = 0.53, size = 158, normalized size = 2.29

$$b\left(\frac{d^{2}\log\left(\frac{de^{(-x)}-c-\sqrt{c^{2}+d^{2}}}{de^{(-x)}-c+\sqrt{c^{2}+d^{2}}}\right)}{\sqrt{c^{2}+d^{2}}\,c^{2}}+\frac{d\log\left(e^{(-x)}+1\right)}{c^{2}}-\frac{d\log\left(e^{(-x)}-1\right)}{c^{2}}+\frac{2}{ce^{(-2\,x)}-c}\right)+\frac{a\log\left(\frac{de^{(-x)}-c-\sqrt{c^{2}+d^{2}}}{de^{(-x)}-c+\sqrt{c^{2}+d^{2}}}\right)}{\sqrt{c^{2}+d^{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*csch(x)^2)/(c+d*sinh(x)),x, algorithm="maxima")
```

```
[Out] b*(d^2*log((d*e^(-x) - c - sqrt(c^2 + d^2))/(d*e^(-x) - c + sqrt(c^2 + d^2)
))/(sqrt(c^2 + d^2)*c^2) + d*log(e^(-x) + 1)/c^2 - d*log(e^(-x) - 1)/c^2 +
2/(c*e^(-2*x) - c)) + a*log((d*e^(-x) - c - sqrt(c^2 + d^2))/(d*e^(-x) - c
+ sqrt(c^2 + d^2)))/sqrt(c^2 + d^2)
```

mupad [B] time = 3.77, size = 613, normalized size = 8.88

$$\frac{b\,d\,\ln\left(\mathrm{e}^x+1\right)}{c^2}-\frac{b\,d\,\ln\left(\mathrm{e}^x-1\right)}{c^2}-\frac{2\,b}{c\,\left(\mathrm{e}^{2\,x}-1\right)}-\frac{\ln\left(\frac{\left(a\,c^2+b\,d^2\right)\left(\frac{32\,\left(a^2\,c^4+2\,a\,b\,c^2\,d^2-4\,\mathrm{e}^x\,b^2\,c^3\,d+2\,b^2\,c^2\,d^2-3\,\mathrm{e}^x\,b^2\,c\,d^3+2\,b^2\,d^4\right)}{c^2\,d^4}-\frac{\left(a\,c^2+b\,d^2\right)\left(\frac{32\,c\,(\cdots}{\cdots}\right.}{\cdots}\right.}{c^2\,\sqrt{c^2\cdots}}\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/sinh(x)^2)/(c + d*sinh(x)),x)
```

```
[Out] (b*d*log(exp(x) + 1))/c^2 - (b*d*log(exp(x) - 1))/c^2 - (2*b)/(c*(exp(2*x)
- 1)) - (log(((a*c^2 + b*d^2)*((32*(a^2*c^4 + 2*b^2*d^4 + 2*b^2*c^2*d^2 - 3
*b^2*c*d^3*exp(x) - 4*b^2*c^3*d*exp(x) + 2*a*b*c^2*d^2))/(c^2*d^4) - ((a*c^
2 + b*d^2)*((32*c*(4*a*c^3*exp(x) - 2*b*d^3 - 2*a*c^2*d + a*c*d^2*exp(x) +
3*b*c*d^2*exp(x)))/d^5 + (32*(a*c^2 + b*d^2)*(3*c^2*d + 2*d^3 - 4*c^3*exp(x
) - 3*c*d^2*exp(x)))/(d^5*(c^2 + d^2)^(1/2))))/(c^2*(c^2 + d^2)^(1/2))))/(c
^2*(c^2 + d^2)^(1/2)) - (32*b*(a*c^2 + b*d^2)*(2*b*d + a*c*exp(x) - 4*b*c*e
xp(x)))/(c^3*d^3))*(a*c^2 + b*d^2)*(c^2 + d^2)^(1/2))/(c^4 + c^2*d^2) + (lo
g(- ((a*c^2 + b*d^2)*((32*(a^2*c^4 + 2*b^2*d^4 + 2*b^2*c^2*d^2 - 3*b^2*c*d^
3*exp(x) - 4*b^2*c^3*d*exp(x) + 2*a*b*c^2*d^2))/(c^2*d^4) + ((a*c^2 + b*d^2
)*((32*c*(4*a*c^3*exp(x) - 2*b*d^3 - 2*a*c^2*d + a*c*d^2*exp(x) + 3*b*c*d^2
*exp(x)))/d^5 - (32*(a*c^2 + b*d^2)*(3*c^2*d + 2*d^3 - 4*c^3*exp(x) - 3*c*d
^2*exp(x)))/(d^5*(c^2 + d^2)^(1/2))))/(c^2*(c^2 + d^2)^(1/2))))/(c^2*(c^2 +
 d^2)^(1/2)) - (32*b*(a*c^2 + b*d^2)*(2*b*d + a*c*exp(x) - 4*b*c*exp(x)))/(
c^3*d^3))*(a*c^2 + b*d^2)*(c^2 + d^2)^(1/2))/(c^4 + c^2*d^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b\operatorname{csch}^{2}{\left(x \right)}}{c + d\sinh{\left(x \right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*csch(x)**2)/(c+d*sinh(x)),x)
[Out] Integral((a + b*csch(x)**2)/(c + d*sinh(x)), x)
```

3.580 $\int (a\cosh(x) + b\sinh(x))\,dx$

Optimal. Leaf size=9

$$a\sinh(x) + b\cosh(x)$$

```
[Out] b*cosh(x)+a*sinh(x)
```

Rubi [A] time = 0.01, antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2637, 2638}

$$a\sinh(x) + b\cosh(x)$$

Antiderivative was successfully verified.

```
[In] Int[a*Cosh[x] + b*Sinh[x],x]
```

```
[Out] b*Cosh[x] + a*Sinh[x]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int (a\cosh(x) + b\sinh(x))\,dx &= a\int \cosh(x)\,dx + b\int \sinh(x)\,dx\\ &= b\cosh(x) + a\sinh(x)\end{aligned}$$

Mathematica [A] time = 0.00, size = 9, normalized size = 1.00

$$a\sinh(x) + b\cosh(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[a*Cosh[x] + b*Sinh[x],x]
```

```
[Out] b*Cosh[x] + a*Sinh[x]
```

fricas [A] time = 0.40, size = 9, normalized size = 1.00

$$b \cosh(x) + a \sinh(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a*cosh(x)+b*sinh(x),x, algorithm="fricas")
```

```
[Out] b*cosh(x) + a*sinh(x)
```

giac [B] time = 0.13, size = 23, normalized size = 2.56

$$\frac{1}{2} b\left(e^{(-x)} + e^{x}\right) - \frac{1}{2} a\left(e^{(-x)} - e^{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a*cosh(x)+b*sinh(x),x, algorithm="giac")
```

```
[Out] 1/2*b*(e^(-x) + e^x) - 1/2*a*(e^(-x) - e^x)
```

maple [A] time = 0.02, size = 10, normalized size = 1.11

$$b \cosh(x) + a \sinh(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a*cosh(x)+b*sinh(x),x)
```

```
[Out] b*cosh(x)+a*sinh(x)
```

maxima [A] time = 0.35, size = 9, normalized size = 1.00

$$b \cosh(x) + a \sinh(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a*cosh(x)+b*sinh(x),x, algorithm="maxima")
```

```
[Out] b*cosh(x) + a*sinh(x)
```

mupad [B] time = 1.50, size = 9, normalized size = 1.00

$$b \cosh(x) + a \sinh(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a*cosh(x) + b*sinh(x),x)
```

```
[Out] b*cosh(x) + a*sinh(x)
```

sympy **[A]** time = 0.11, size = 8, normalized size = 0.89

$$a\sinh(x) + b\cosh(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a*cosh(x)+b*sinh(x),x)
```

```
[Out] a*sinh(x) + b*cosh(x)
```

3.581 $\int (a\cosh(x)+b\sinh(x))^2\,dx$

Optimal. Leaf size=37

$$\frac{1}{2}x\left(a^2-b^2\right)+\frac{1}{2}(a\sinh(x)+b\cosh(x))(a\cosh(x)+b\sinh(x))$$

```
[Out] 1/2*(a^2-b^2)*x+1/2*(b*cosh(x)+a*sinh(x))*(a*cosh(x)+b*sinh(x))
```

Rubi [A] time = 0.02, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {3073, 8}

$$\frac{1}{2}x\left(a^2-b^2\right)+\frac{1}{2}(a\sinh(x)+b\cosh(x))(a\cosh(x)+b\sinh(x))$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cosh[x] + b*Sinh[x])^2,x]
```

```
[Out] ((a^2 - b^2)*x)/2 + ((b*Cosh[x] + a*Sinh[x])*(a*Cosh[x] + b*Sinh[x]))/2
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3073

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> -Simp[((b*Cos[c + d*x] - a*Sin[c + d*x])*(a*Cos[c + d*x] + b*Si
n[c + d*x])^(n - 1))/(d*n), x] + Dist[((n - 1)*(a^2 + b^2))/n, Int[(a*Cos[c
 + d*x] + b*Sin[c + d*x])^(n - 2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 + b^2, 0] &&  !IntegerQ[(n - 1)/2] && GtQ[n, 1]
```

Rubi steps

$$\begin{aligned}\int (a\cosh(x)+b\sinh(x))^2\,dx&=\frac{1}{2}(b\cosh(x)+a\sinh(x))(a\cosh(x)+b\sinh(x))+\frac{1}{2}\left(a^2-b^2\right)\int 1\,dx\\&=\frac{1}{2}\left(a^2-b^2\right)x+\frac{1}{2}(b\cosh(x)+a\sinh(x))(a\cosh(x)+b\sinh(x))\end{aligned}$$

Mathematica [A] time = 0.06, size = 36, normalized size = 0.97

$$\frac{1}{4}\left(\left(a^2+b^2\right)\sinh(2x)+2x(a-b)(a+b)+2ab\cosh(2x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cosh[x] + b*Sinh[x])^2,x]
```

```
[Out] (2*(a - b)*(a + b)*x + 2*a*b*Cosh[2*x] + (a^2 + b^2)*Sinh[2*x])/4
```

fricas [A] time = 0.41, size = 42, normalized size = 1.14

$$\frac{1}{2}\,ab\cosh(x)^2+\frac{1}{2}\,ab\sinh(x)^2+\frac{1}{2}\left(a^2+b^2\right)\cosh(x)\sinh(x)+\frac{1}{2}\left(a^2-b^2\right)x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cosh(x)+b*sinh(x))^2,x, algorithm="fricas")
```

```
[Out] 1/2*a*b*cosh(x)^2 + 1/2*a*b*sinh(x)^2 + 1/2*(a^2 + b^2)*cosh(x)*sinh(x) + 1
/2*(a^2 - b^2)*x
```

giac [B] time = 0.13, size = 74, normalized size = 2.00

$$\frac{1}{8}\,a^2e^{(2\,x)}+\frac{1}{4}\,abe^{(2\,x)}+\frac{1}{8}\,b^2e^{(2\,x)}+\frac{1}{2}\left(a^2-b^2\right)x-\frac{1}{8}\left(2\,a^2e^{(2\,x)}-2\,b^2e^{(2\,x)}+a^2-2\,ab+b^2\right)e^{(-2\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cosh(x)+b*sinh(x))^2,x, algorithm="giac")
```

```
[Out] 1/8*a^2*e^(2*x) + 1/4*a*b*e^(2*x) + 1/8*b^2*e^(2*x) + 1/2*(a^2 - b^2)*x - 1
/8*(2*a^2*e^(2*x) - 2*b^2*e^(2*x) + a^2 - 2*a*b + b^2)*e^(-2*x)
```

maple [A] time = 0.15, size = 37, normalized size = 1.00

$$b^2\left(\frac{\cosh(x)\sinh(x)}{2}-\frac{x}{2}\right)+ab\left(\cosh^2(x)\right)+a^2\left(\frac{\cosh(x)\sinh(x)}{2}+\frac{x}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cosh(x)+b*sinh(x))^2,x)
```

```
[Out] b^2*(1/2*cosh(x)*sinh(x)-1/2*x)+a*b*cosh(x)^2+a^2*(1/2*cosh(x)*sinh(x)+1/2*
x)
```

maxima [A] time = 0.55, size = 46, normalized size = 1.24

$$ab\cosh(x)^2+\frac{1}{8}\,a^2\left(4\,x+e^{(2\,x)}-e^{(-2\,x)}\right)-\frac{1}{8}\,b^2\left(4\,x-e^{(2\,x)}+e^{(-2\,x)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cosh(x)+b*sinh(x))^2,x, algorithm="maxima")
```

```
[Out] a*b*cosh(x)^2 + 1/8*a^2*(4*x + e^(2*x) - e^(-2*x)) - 1/8*b^2*(4*x - e^(2*x)
 + e^(-2*x))
```

mupad [B] time = 1.54, size = 39, normalized size = 1.05

$$\frac{a^2\sinh(2x)}{4}+\frac{b^2\sinh(2x)}{4}+\frac{a^2x}{2}-\frac{b^2x}{2}+\frac{a\,b\cosh(2x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cosh(x) + b*sinh(x))^2,x)
```

```
[Out] (a^2*sinh(2*x))/4 + (b^2*sinh(2*x))/4 + (a^2*x)/2 - (b^2*x)/2 + (a*b*cosh(2
*x))/2
```

sympy [B] time = 0.19, size = 78, normalized size = 2.11

$$-\frac{a^2x\sinh^2(x)}{2}+\frac{a^2x\cosh^2(x)}{2}+\frac{a^2\sinh(x)\cosh(x)}{2}+ab\cosh^2(x)+\frac{b^2x\sinh^2(x)}{2}-\frac{b^2x\cosh^2(x)}{2}+\frac{b^2\sinh(x)\cos}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cosh(x)+b*sinh(x))**2,x)
```

```
[Out] -a**2*x*sinh(x)**2/2 + a**2*x*cosh(x)**2/2 + a**2*sinh(x)*cosh(x)/2 + a*b*c
osh(x)**2 + b**2*x*sinh(x)**2/2 - b**2*x*cosh(x)**2/2 + b**2*sinh(x)*cosh(x
)/2
```

3.582 $\int (a\cosh(x)+b\sinh(x))^3\,dx$

Optimal. Leaf size=35

$$\left(a^2-b^2\right)(a\sinh(x)+b\cosh(x))+\frac{1}{3}(a\sinh(x)+b\cosh(x))^3$$

```
[Out] (a^2-b^2)*(b*cosh(x)+a*sinh(x))+1/3*(b*cosh(x)+a*sinh(x))^3
```

Rubi [A] time = 0.02, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {3072}

$$\left(a^2-b^2\right)(a\sinh(x)+b\cosh(x))+\frac{1}{3}(a\sinh(x)+b\cosh(x))^3$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cosh[x] + b*Sinh[x])^3,x]
```

```
[Out] (a^2 - b^2)*(b*Cosh[x] + a*Sinh[x]) + (b*Cosh[x] + a*Sinh[x])^3/3
```

Rule 3072

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> -Dist[d^(-1), Subst[Int[(a^2 + b^2 - x^2)^((n - 1)/2), x], x, b
*Cos[c + d*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 +
b^2, 0] && IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\begin{aligned}\int (a\cosh(x)+b\sinh(x))^3\,dx &= i\,\text{Subst}\left(\int \left(a^2-b^2-x^2\right)\,dx, x, -ib\cosh(x)-ia\sinh(x)\right)\\ &= \left(a^2-b^2\right)(b\cosh(x)+a\sinh(x))+\frac{1}{3}(b\cosh(x)+a\sinh(x))^3\end{aligned}$$

Mathematica [A] time = 0.14, size = 63, normalized size = 1.80

$$\frac{1}{12}\left(9a\left(a^2-b^2\right)\sinh(x)+a\left(a^2+3b^2\right)\sinh(3x)+9b\left(a^2-b^2\right)\cosh(x)+b\left(3a^2+b^2\right)\cosh(3x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cosh[x] + b*Sinh[x])^3,x]
```

```
[Out] (9*b*(a^2 - b^2)*Cosh[x] + b*(3*a^2 + b^2)*Cosh[3*x] + 9*a*(a^2 - b^2)*Sinh
[x] + a*(a^2 + 3*b^2)*Sinh[3*x])/12
```

fricas [B] time = 0.41, size = 97, normalized size = 2.77

$$\frac{1}{12}\left(3\,a^2b+b^3\right)\cosh(x)^3+\frac{1}{4}\left(3\,a^2b+b^3\right)\cosh(x)\sinh(x)^2+\frac{1}{12}\left(a^3+3\,ab^2\right)\sinh(x)^3+\frac{3}{4}\left(a^2b-b^3\right)\cosh(x)+\frac{1}{4}\left(3\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cosh(x)+b*sinh(x))^3,x, algorithm="fricas")
```

```
[Out] 1/12*(3*a^2*b + b^3)*cosh(x)^3 + 1/4*(3*a^2*b + b^3)*cosh(x)*sinh(x)^2 + 1/
12*(a^3 + 3*a*b^2)*sinh(x)^3 + 3/4*(a^2*b - b^3)*cosh(x) + 1/4*(3*a^3 - 3*a
*b^2 + (a^3 + 3*a*b^2)*cosh(x)^2)*sinh(x)
```

giac [B] time = 0.14, size = 134, normalized size = 3.83

$$\frac{1}{24}\,a^3e^{(3\,x)}+\frac{1}{8}\,a^2be^{(3\,x)}+\frac{1}{8}\,ab^2e^{(3\,x)}+\frac{1}{24}\,b^3e^{(3\,x)}+\frac{3}{8}\,a^3e^x+\frac{3}{8}\,a^2be^x-\frac{3}{8}\,ab^2e^x-\frac{3}{8}\,b^3e^x-\frac{1}{24}\left(9\,a^3e^{(2\,x)}-9\,a^2be^{(2\,x)}-9\,ab\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cosh(x)+b*sinh(x))^3,x, algorithm="giac")
```

```
[Out] 1/24*a^3*e^(3*x) + 1/8*a^2*b*e^(3*x) + 1/8*a*b^2*e^(3*x) + 1/24*b^3*e^(3*x)
 + 3/8*a^3*e^x + 3/8*a^2*b*e^x - 3/8*a*b^2*e^x - 3/8*b^3*e^x - 1/24*(9*a^3*
e^(2*x) - 9*a^2*b*e^(2*x) - 9*a*b^2*e^(2*x) + 9*b^3*e^(2*x) + a^3 - 3*a^2*b
 + 3*a*b^2 - b^3)*e^(-3*x)
```

maple [A] time = 0.46, size = 48, normalized size = 1.37

$$b^3\left(-\frac{2}{3}+\frac{\left(\sinh^2(x)\right)}{3}\right)\cosh(x)+a\,b^2\left(\sinh^3(x)\right)+a^2b\left(\cosh^3(x)\right)+a^3\left(\frac{2}{3}+\frac{\left(\cosh^2(x)\right)}{3}\right)\sinh(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cosh(x)+b*sinh(x))^3,x)
```

```
[Out] b^3*(-2/3+1/3*sinh(x)^2)*cosh(x)+a*b^2*sinh(x)^3+a^2*b*cosh(x)^3+a^3*(2/3+1
/3*cosh(x)^2)*sinh(x)
```

maxima [B] time = 0.39, size = 69, normalized size = 1.97

$$a^2b\cosh(x)^3+ab^2\sinh(x)^3+\frac{1}{24}\,b^3\left(e^{(3\,x)}-9\,e^{(-x)}+e^{(-3\,x)}-9\,e^x\right)+\frac{1}{24}\,a^3\left(e^{(3\,x)}-9\,e^{(-x)}-e^{(-3\,x)}+9\,e^x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cosh(x)+b*sinh(x))^3,x, algorithm="maxima")
```

```
[Out] a^2*b*cosh(x)^3 + a*b^2*sinh(x)^3 + 1/24*b^3*(e^(3*x) - 9*e^(-x) + e^(-3*x)
 - 9*e^x) + 1/24*a^3*(e^(3*x) - 9*e^(-x) - e^(-3*x) + 9*e^x)
```

mupad [B] time = 0.10, size = 53, normalized size = 1.51

$$\cosh(x)^3 \left(a^2 b - \frac{2 b^3}{3}\right) + \sinh(x)^3 \left(a b^2 - \frac{2 a^3}{3}\right) + a^3 \cosh(x)^2 \sinh(x) + b^3 \cosh(x) \sinh(x)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cosh(x) + b*sinh(x))^3,x)
```

```
[Out] cosh(x)^3*(a^2*b - (2*b^3)/3) + sinh(x)^3*(a*b^2 - (2*a^3)/3) + a^3*cosh(x)
^2*sinh(x) + b^3*cosh(x)*sinh(x)^2
```

sympy [B] time = 0.33, size = 66, normalized size = 1.89

$$-\frac{2a^3 \sinh^3(x)}{3} + a^3 \sinh(x) \cosh^2(x) + a^2 b \cosh^3(x) + ab^2 \sinh^3(x) + b^3 \sinh^2(x) \cosh(x) - \frac{2b^3 \cosh^3(x)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cosh(x)+b*sinh(x))**3,x)
```

```
[Out] -2*a**3*sinh(x)**3/3 + a**3*sinh(x)*cosh(x)**2 + a**2*b*cosh(x)**3 + a*b**2
*sinh(x)**3 + b**3*sinh(x)**2*cosh(x) - 2*b**3*cosh(x)**3/3
```

3.583 $\int (a\cosh(x)+b\sinh(x))^4\,dx$

Optimal. Leaf size=72

$$\frac{3}{8}x\left(a^2-b^2\right)^2+\frac{3}{8}\left(a^2-b^2\right)(a\sinh(x)+b\cosh(x))(a\cosh(x)+b\sinh(x))+\frac{1}{4}(a\sinh(x)+b\cosh(x))(a\cosh(x)+b\,\mathrm{si}$$

```
[Out] 3/8*(a^2-b^2)^2*x+3/8*(a^2-b^2)*(b*cosh(x)+a*sinh(x))*(a*cosh(x)+b*sinh(x))
+1/4*(b*cosh(x)+a*sinh(x))*(a*cosh(x)+b*sinh(x))^3
```

Rubi [A] time = 0.04, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {3073, 8}

$$\frac{3}{8}x\left(a^2-b^2\right)^2+\frac{3}{8}\left(a^2-b^2\right)(a\sinh(x)+b\cosh(x))(a\cosh(x)+b\sinh(x))+\frac{1}{4}(a\sinh(x)+b\cosh(x))(a\cosh(x)+b\,\mathrm{si}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cosh[x] + b*Sinh[x])^4,x]
```

```
[Out] (3*(a^2 - b^2)^2*x)/8 + (3*(a^2 - b^2)*(b*Cosh[x] + a*Sinh[x])*(a*Cosh[x] +
 b*Sinh[x]))/8 + ((b*Cosh[x] + a*Sinh[x])*(a*Cosh[x] + b*Sinh[x])^3)/4
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3073

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> -Simp[((b*Cos[c + d*x] - a*Sin[c + d*x])*(a*Cos[c + d*x] + b*Si
n[c + d*x])^(n - 1))/(d*n), x] + Dist[((n - 1)*(a^2 + b^2))/n, Int[(a*Cos[c
 + d*x] + b*Sin[c + d*x])^(n - 2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 + b^2, 0] &&  !IntegerQ[(n - 1)/2] && GtQ[n, 1]
```

Rubi steps

$$\begin{aligned}\int (a\cosh(x)+b\sinh(x))^4\,dx &= \frac{1}{4}(b\cosh(x)+a\sinh(x))(a\cosh(x)+b\sinh(x))^3+\frac{1}{4}\left(3\left(a^2-b^2\right)\right)\int (a\cosh(x\\ &= \frac{3}{8}\left(a^2-b^2\right)(b\cosh(x)+a\sinh(x))(a\cosh(x)+b\sinh(x))+\frac{1}{4}(b\cosh(x)+a\,\mathrm{si}\\ &= \frac{3}{8}\left(a^2-b^2\right)^2x+\frac{3}{8}\left(a^2-b^2\right)(b\cosh(x)+a\sinh(x))(a\cosh(x)+b\sinh(x))+\frac{1}{4}\end{aligned}$$

Mathematica [A] time = 0.15, size = 87, normalized size = 1.21

$$\frac{1}{32}\left(8\left(a^4-b^4\right)\sinh(2x)+16ab\left(a^2-b^2\right)\cosh(2x)+4ab\left(a^2+b^2\right)\cosh(4x)+\left(a^4+6a^2b^2+b^4\right)\sinh(4x)+12x($$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cosh[x] + b*Sinh[x])^4,x]
```

```
[Out] (12*(a - b)^2*(a + b)^2*x + 16*a*b*(a^2 - b^2)*Cosh[2*x] + 4*a*b*(a^2 + b^2
)*Cosh[4*x] + 8*(a^4 - b^4)*Sinh[2*x] + (a^4 + 6*a^2*b^2 + b^4)*Sinh[4*x])/
32
```

fricas [B] time = 0.43, size = 168, normalized size = 2.33

$$\frac{1}{8}\left(a^3b+ab^3\right)\cosh(x)^4+\frac{1}{8}\left(a^4+6a^2b^2+b^4\right)\cosh(x)\sinh(x)^3+\frac{1}{8}\left(a^3b+ab^3\right)\sinh(x)^4+\frac{1}{2}\left(a^3b-ab^3\right)\cosh(x)^2+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cosh(x)+b*sinh(x))^4,x, algorithm="fricas")
```

```
[Out] 1/8*(a^3*b + a*b^3)*cosh(x)^4 + 1/8*(a^4 + 6*a^2*b^2 + b^4)*cosh(x)*sinh(x)
^3 + 1/8*(a^3*b + a*b^3)*sinh(x)^4 + 1/2*(a^3*b - a*b^3)*cosh(x)^2 + 1/4*(2
*a^3*b - 2*a*b^3 + 3*(a^3*b + a*b^3)*cosh(x)^2)*sinh(x)^2 + 3/8*(a^4 - 2*a^
2*b^2 + b^4)*x + 1/8*((a^4 + 6*a^2*b^2 + b^4)*cosh(x)^3 + 4*(a^4 - b^4)*cos
h(x))*sinh(x)
```

giac [B] time = 0.12, size = 208, normalized size = 2.89

$$\frac{1}{64}a^4e^{(4x)}+\frac{1}{16}a^3be^{(4x)}+\frac{3}{32}a^2b^2e^{(4x)}+\frac{1}{16}ab^3e^{(4x)}+\frac{1}{64}b^4e^{(4x)}+\frac{1}{8}a^4e^{(2x)}+\frac{1}{4}a^3be^{(2x)}-\frac{1}{4}ab^3e^{(2x)}-\frac{1}{8}b^4e^{(2x)}+\frac{3}{8}\left(a^4\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cosh(x)+b*sinh(x))^4,x, algorithm="giac")
```

```
[Out] 1/64*a^4*e^(4*x) + 1/16*a^3*b*e^(4*x) + 3/32*a^2*b^2*e^(4*x) + 1/16*a*b^3*e
^(4*x) + 1/64*b^4*e^(4*x) + 1/8*a^4*e^(2*x) + 1/4*a^3*b*e^(2*x) - 1/4*a*b^3
*e^(2*x) - 1/8*b^4*e^(2*x) + 3/8*(a^4 - 2*a^2*b^2 + b^4)*x - 1/64*(18*a^4*e
^(4*x) - 36*a^2*b^2*e^(4*x) + 18*b^4*e^(4*x) + 8*a^4*e^(2*x) - 16*a^3*b*e^(
2*x) + 16*a*b^3*e^(2*x) - 8*b^4*e^(2*x) + a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b
^3 + b^4)*e^(-4*x)
```

maple [A] time = 0.46, size = 90, normalized size = 1.25

$$b^4\left(\left(\frac{\left(\sinh^3(x)\right)}{4}-\frac{3\sinh(x)}{8}\right)\cosh(x)+\frac{3x}{8}\right)+a\,b^3\left(\sinh^4(x)\right)+6a^2b^2\left(\frac{\sinh(x)\left(\cosh^3(x)\right)}{4}-\frac{\cosh(x)\sinh(x)}{8}-\frac{x}{8}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cosh(x)+b*sinh(x))^4,x)
```

```
[Out] b^4*((1/4*sinh(x)^3-3/8*sinh(x))*cosh(x)+3/8*x)+a*b^3*sinh(x)^4+6*a^2*b^2*(
1/4*sinh(x)*cosh(x)^3-1/8*cosh(x)*sinh(x)-1/8*x)+a^3*b*cosh(x)^4+a^4*((1/4*
cosh(x)^3+3/8*cosh(x))*sinh(x)+3/8*x)
```

maxima [A] time = 0.57, size = 103, normalized size = 1.43

$$a^3b\cosh(x)^4+ab^3\sinh(x)^4+\frac{1}{64}a^4\left(24x+e^{(4x)}+8e^{(2x)}-8e^{(-2x)}-e^{(-4x)}\right)+\frac{1}{64}b^4\left(24x+e^{(4x)}-8e^{(2x)}+8e^{(-2x}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cosh(x)+b*sinh(x))^4,x, algorithm="maxima")
```

```
[Out] a^3*b*cosh(x)^4 + a*b^3*sinh(x)^4 + 1/64*a^4*(24*x + e^(4*x) + 8*e^(2*x) -
8*e^(-2*x) - e^(-4*x)) + 1/64*b^4*(24*x + e^(4*x) - 8*e^(2*x) + 8*e^(-2*x)
- e^(-4*x)) - 3/32*a^2*b^2*(8*x - e^(4*x) + e^(-4*x))
```

mupad [B] time = 1.56, size = 143, normalized size = 1.99

$$\cosh(x)\sinh(x)^3\left(-\frac{3a^4}{8}+\frac{3a^2b^2}{4}+\frac{5b^4}{8}\right)-\cosh(x)^4\left(ab^3-a^3b\right)+\cosh(x)^3\sinh(x)\left(\frac{5a^4}{8}+\frac{3a^2b^2}{4}-\frac{3b^4}{8}\right)+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cosh(x) + b*sinh(x))^4,x)
```

```
[Out] cosh(x)*sinh(x)^3*((5*b^4)/8 - (3*a^4)/8 + (3*a^2*b^2)/4) - cosh(x)^4*(a*b^
3 - a^3*b) + cosh(x)^3*sinh(x)*((5*a^4)/8 - (3*b^4)/8 + (3*a^2*b^2)/4) + (3
*x*cosh(x)^4*(a^2 - b^2)^2)/8 + (3*x*sinh(x)^4*(a^2 - b^2)^2)/8 + 2*a*b^3*c
osh(x)^2*sinh(x)^2 - (3*x*cosh(x)^2*sinh(x)^2*(a^2 - b^2)^2)/4
```

sympy [B] time = 0.69, size = 265, normalized size = 3.68

$$\frac{3a^4x\sinh^4(x)}{8}-\frac{3a^4x\sinh^2(x)\cosh^2(x)}{4}+\frac{3a^4x\cosh^4(x)}{8}-\frac{3a^4\sinh^3(x)\cosh(x)}{8}+\frac{5a^4\sinh(x)\cosh^3(x)}{8}+a^3b\,\mathrm{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cosh(x)+b*sinh(x))**4,x)
```

```
[Out] 3*a**4*x*sinh(x)**4/8 - 3*a**4*x*sinh(x)**2*cosh(x)**2/4 + 3*a**4*x*cosh(x)
**4/8 - 3*a**4*sinh(x)**3*cosh(x)/8 + 5*a**4*sinh(x)*cosh(x)**3/8 + a**3*b*
cosh(x)**4 - 3*a**2*b**2*x*sinh(x)**4/4 + 3*a**2*b**2*x*sinh(x)**2*cosh(x)*
```

```
*2/2 - 3*a**2*b**2*x*cosh(x)**4/4 + 3*a**2*b**2*sinh(x)**3*cosh(x)/4 + 3*a*
*2*b**2*sinh(x)*cosh(x)**3/4 + a*b**3*sinh(x)**4 + 3*b**4*x*sinh(x)**4/8 -
3*b**4*x*sinh(x)**2*cosh(x)**2/4 + 3*b**4*x*cosh(x)**4/8 + 5*b**4*sinh(x)**
3*cosh(x)/8 - 3*b**4*sinh(x)*cosh(x)**3/8
```

3.584 $\int (a\cosh(x)+b\sinh(x))^5\,dx$

Optimal. Leaf size=61

$$\frac{2}{3}\left(a^2-b^2\right)(a\sinh(x)+b\cosh(x))^3+\left(a^2-b^2\right)^2(a\sinh(x)+b\cosh(x))+\frac{1}{5}(a\sinh(x)+b\cosh(x))^5$$

```
[Out] (a^2-b^2)^2*(b*cosh(x)+a*sinh(x))+2/3*(a^2-b^2)*(b*cosh(x)+a*sinh(x))^3+1/5
*(b*cosh(x)+a*sinh(x))^5
```

Rubi [A] time = 0.05, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {3072, 194}

$$\frac{2}{3}\left(a^2-b^2\right)(a\sinh(x)+b\cosh(x))^3+\left(a^2-b^2\right)^2(a\sinh(x)+b\cosh(x))+\frac{1}{5}(a\sinh(x)+b\cosh(x))^5$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cosh[x] + b*Sinh[x])^5,x]
```

```
[Out] (a^2 - b^2)^2*(b*Cosh[x] + a*Sinh[x]) + (2*(a^2 - b^2)*(b*Cosh[x] + a*Sinh[
x])^3)/3 + (b*Cosh[x] + a*Sinh[x])^5/5
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 3072

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> -Dist[d^(-1), Subst[Int[(a^2 + b^2 - x^2)^((n - 1)/2), x], x, b
*Cos[c + d*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 +
b^2, 0] && IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\begin{aligned}\int (a\cosh(x)+b\sinh(x))^5\,dx &= i\,\text{Subst}\left(\int \left(a^2-b^2-x^2\right)^2\,dx, x, -ib\cosh(x)-ia\sinh(x)\right)\\ &= i\,\text{Subst}\left(\int \left(a^4\left(1+\frac{-2a^2b^2+b^4}{a^4}\right)-2a^2\left(1-\frac{b^2}{a^2}\right)x^2+x^4\right)dx, x, -ib\cosh(x)-\right.\\ &= \left(a^2-b^2\right)^2(b\cosh(x)+a\sinh(x))+\frac{2}{3}\left(a^2-b^2\right)(b\cosh(x)+a\sinh(x))^3+\frac{1}{5}(b\end{aligned}$$

Mathematica [B] time = 0.23, size = 133, normalized size = 2.18

$$\frac{1}{240}\left(150a\left(a^2-b^2\right)^2\sinh(x)+150b\left(a^2-b^2\right)^2\cosh(x)+25a\left(a^4+2a^2b^2-3b^4\right)\sinh(3x)+3a\left(a^4+10a^2b^2+5b\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cosh[x] + b*Sinh[x])^5,x]
```

```
[Out] (150*b*(a^2 - b^2)^2*Cosh[x] - 25*b*(-3*a^4 + 2*a^2*b^2 + b^4)*Cosh[3*x] +
3*b*(5*a^4 + 10*a^2*b^2 + b^4)*Cosh[5*x] + 150*a*(a^2 - b^2)^2*Sinh[x] + 25
*a*(a^4 + 2*a^2*b^2 - 3*b^4)*Sinh[3*x] + 3*a*(a^4 + 10*a^2*b^2 + 5*b^4)*Sin
h[5*x])/240
```

fricas [B] time = 0.41, size = 298, normalized size = 4.89

$$\frac{1}{80}\left(5\,a^4b+10\,a^2b^3+b^5\right)\cosh(x)^5+\frac{1}{16}\left(5\,a^4b+10\,a^2b^3+b^5\right)\cosh(x)\sinh(x)^4+\frac{1}{80}\left(a^5+10\,a^3b^2+5\,ab^4\right)\sinh(x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cosh(x)+b*sinh(x))^5,x, algorithm="fricas")
```

```
[Out] 1/80*(5*a^4*b + 10*a^2*b^3 + b^5)*cosh(x)^5 + 1/16*(5*a^4*b + 10*a^2*b^3 +
b^5)*cosh(x)*sinh(x)^4 + 1/80*(a^5 + 10*a^3*b^2 + 5*a*b^4)*sinh(x)^5 + 5/48
*(3*a^4*b - 2*a^2*b^3 - b^5)*cosh(x)^3 + 1/48*(5*a^5 + 10*a^3*b^2 - 15*a*b^
4 + 6*(a^5 + 10*a^3*b^2 + 5*a*b^4)*cosh(x)^2)*sinh(x)^3 + 1/16*(2*(5*a^4*b
+ 10*a^2*b^3 + b^5)*cosh(x)^3 + 5*(3*a^4*b - 2*a^2*b^3 - b^5)*cosh(x))*sinh
(x)^2 + 5/8*(a^4*b - 2*a^2*b^3 + b^5)*cosh(x) + 1/16*(10*a^5 - 20*a^3*b^2 +
 10*a*b^4 + (a^5 + 10*a^3*b^2 + 5*a*b^4)*cosh(x)^4 + 5*(a^5 + 2*a^3*b^2 - 3
*a*b^4)*cosh(x)^2)*sinh(x)
```

giac [B] time = 0.14, size = 344, normalized size = 5.64

$$\frac{1}{160}\,a^5e^{(5\,x)}+\frac{1}{32}\,a^4be^{(5\,x)}+\frac{1}{16}\,a^3b^2e^{(5\,x)}+\frac{1}{16}\,a^2b^3e^{(5\,x)}+\frac{1}{32}\,ab^4e^{(5\,x)}+\frac{1}{160}\,b^5e^{(5\,x)}+\frac{5}{96}\,a^5e^{(3\,x)}+\frac{5}{32}\,a^4be^{(3\,x)}+\frac{5}{48}\,a^3b^2e^{(3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cosh(x)+b*sinh(x))^5,x, algorithm="giac")
```

```
[Out] 1/160*a^5*e^(5*x) + 1/32*a^4*b*e^(5*x) + 1/16*a^3*b^2*e^(5*x) + 1/16*a^2*b^
3*e^(5*x) + 1/32*a*b^4*e^(5*x) + 1/160*b^5*e^(5*x) + 5/96*a^5*e^(3*x) + 5/3
2*a^4*b*e^(3*x) + 5/48*a^3*b^2*e^(3*x) - 5/48*a^2*b^3*e^(3*x) - 5/32*a*b^4*
e^(3*x) - 5/96*b^5*e^(3*x) + 5/16*a^5*e^x + 5/16*a^4*b*e^x - 5/8*a^3*b^2*e^
x - 5/8*a^2*b^3*e^x + 5/16*a*b^4*e^x + 5/16*b^5*e^x - 1/480*(150*a^5*e^(4*x
) - 150*a^4*b*e^(4*x) - 300*a^3*b^2*e^(4*x) + 300*a^2*b^3*e^(4*x) + 150*a*b
```

```
^4*e^(4*x) - 150*b^5*e^(4*x) + 25*a^5*e^(2*x) - 75*a^4*b*e^(2*x) + 50*a^3*b
^2*e^(2*x) + 50*a^2*b^3*e^(2*x) - 75*a*b^4*e^(2*x) + 25*b^5*e^(2*x) + 3*a^5
 - 15*a^4*b + 30*a^3*b^2 - 30*a^2*b^3 + 15*a*b^4 - 3*b^5)*e^(-5*x)
```

maple [A] time = 0.42, size = 114, normalized size = 1.87

$$b^5\left(\frac{8}{15}+\frac{\left(\sinh^4(x)\right)}{5}-\frac{4\left(\sinh^2(x)\right)}{15}\right)\cosh(x)+a\,b^4\left(\sinh^5(x)\right)+10a^2b^3\left(\frac{\left(\sinh^2(x)\right)\left(\cosh^3(x)\right)}{5}-\frac{2\left(\cosh^3(x)\right)}{15}\right)+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cosh(x)+b*sinh(x))^5,x)
```

```
[Out] b^5*(8/15+1/5*sinh(x)^4-4/15*sinh(x)^2)*cosh(x)+a*b^4*sinh(x)^5+10*a^2*b^3*
(1/5*sinh(x)^2*cosh(x)^3-2/15*cosh(x)^3)+10*a^3*b^2*(1/5*sinh(x)*cosh(x)^4-
1/5*(2/3+1/3*cosh(x)^2)*sinh(x))+a^4*b*cosh(x)^5+a^5*(8/15+1/5*cosh(x)^4+4/
15*cosh(x)^2)*sinh(x)
```

maxima [B] time = 0.38, size = 191, normalized size = 3.13

$$a^4b\cosh(x)^5+ab^4\sinh(x)^5+\frac{1}{48}\left(\left(5\,e^{(-2\,x)}-30\,e^{(-4\,x)}+3\right)e^{(5\,x)}+30\,e^{(-x)}-5\,e^{(-3\,x)}-3\,e^{(-5\,x)}\right)a^3b^2-\frac{1}{48}\left(\left(5\,e^{(-2\,x)}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cosh(x)+b*sinh(x))^5,x, algorithm="maxima")
```

```
[Out] a^4*b*cosh(x)^5 + a*b^4*sinh(x)^5 + 1/48*((5*e^(-2*x) - 30*e^(-4*x) + 3)*e^
(5*x) + 30*e^(-x) - 5*e^(-3*x) - 3*e^(-5*x))*a^3*b^2 - 1/48*((5*e^(-2*x) +
30*e^(-4*x) - 3)*e^(5*x) + 30*e^(-x) + 5*e^(-3*x) - 3*e^(-5*x))*a^2*b^3 + 1
/480*a^5*(3*e^(5*x) + 25*e^(3*x) - 150*e^(-x) - 25*e^(-3*x) - 3*e^(-5*x) +
150*e^x) + 1/480*b^5*(3*e^(5*x) - 25*e^(3*x) + 150*e^(-x) - 25*e^(-3*x) + 3
*e^(-5*x) + 150*e^x)
```

mupad [B] time = 0.12, size = 117, normalized size = 1.92

$$\cosh(x)^5\left(a^4\,b-\frac{4\,a^2\,b^3}{3}+\frac{8\,b^5}{15}\right)+\sinh(x)^5\left(\frac{8\,a^5}{15}-\frac{4\,a^3\,b^2}{3}+a\,b^4\right)-\cosh(x)^2\sinh(x)^3\left(\frac{4\,a^5}{3}-\frac{10\,a^3\,b^2}{3}\right)+a^5\cos$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cosh(x) + b*sinh(x))^5,x)
```

```
[Out] cosh(x)^5*(a^4*b + (8*b^5)/15 - (4*a^2*b^3)/3) + sinh(x)^5*(a*b^4 + (8*a^5)
/15 - (4*a^3*b^2)/3) - cosh(x)^2*sinh(x)^3*((4*a^5)/3 - (10*a^3*b^2)/3) + a
```

```
^5*cosh(x)^4*sinh(x) - cosh(x)^3*sinh(x)^2*((4*b^5)/3 - (10*a^2*b^3)/3) + b
^5*cosh(x)*sinh(x)^4
```

sympy [B] time = 1.18, size = 172, normalized size = 2.82

$$\frac{8a^5\sinh^5(x)}{15} - \frac{4a^5\sinh^3(x)\cosh^2(x)}{3} + a^5\sinh(x)\cosh^4(x) + a^4b\cosh^5(x) - \frac{4a^3b^2\sinh^5(x)}{3} + \frac{10a^3b^2\sinh^3(x)\text{co}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cosh(x)+b*sinh(x))**5,x)
```

```
[Out] 8*a**5*sinh(x)**5/15 - 4*a**5*sinh(x)**3*cosh(x)**2/3 + a**5*sinh(x)*cosh(x
)**4 + a**4*b*cosh(x)**5 - 4*a**3*b**2*sinh(x)**5/3 + 10*a**3*b**2*sinh(x)*
*3*cosh(x)**2/3 + 10*a**2*b**3*sinh(x)**2*cosh(x)**3/3 - 4*a**2*b**3*cosh(x
)**5/3 + a*b**4*sinh(x)**5 + b**5*sinh(x)**4*cosh(x) - 4*b**5*sinh(x)**2*co
sh(x)**3/3 + 8*b**5*cosh(x)**5/15
```

3.585 $\int \frac{1}{a\cosh(x)+b\sinh(x)}\,dx$

Optimal. Leaf size=38

$$\frac{\tan^{-1}\left(\frac{a\sinh(x)+b\cosh(x)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}$$

```
[Out] arctan((b*cosh(x)+a*sinh(x))/(a^2-b^2)^(1/2))/(a^2-b^2)^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {3074, 206}

$$\frac{\tan^{-1}\left(\frac{a\sinh(x)+b\cosh(x)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cosh[x] + b*Sinh[x])^(-1),x]
```

```
[Out] ArcTan[(b*Cosh[x] + a*Sinh[x])/Sqrt[a^2 - b^2]]/Sqrt[a^2 - b^2]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3074

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> -Dist[d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d
*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{a\cosh(x)+b\sinh(x)}\,dx &= i\,\text{Subst}\left(\int \frac{1}{a^2-b^2-x^2}\,dx, x, -ib\cosh(x)-ia\sinh(x)\right)\\
&= \frac{\tan^{-1}\left(\frac{b\cosh(x)+a\sinh(x)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}
\end{aligned}$$

Mathematica [A] time = 0.05, size = 46, normalized size = 1.21

$$\frac{2\tan^{-1}\left(\frac{a\tanh\left(\frac{x}{2}\right)+b}{\sqrt{a-b}\sqrt{a+b}}\right)}{\sqrt{a-b}\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cosh[x] + b*Sinh[x])^(-1),x]
```

```
[Out] (2*ArcTan[(b + a*Tanh[x/2])/(Sqrt[a - b]*Sqrt[a + b])])/(Sqrt[a - b]*Sqrt[a
 + b])
```

fricas [A] time = 0.42, size = 148, normalized size = 3.89

$$\left[-\frac{\sqrt{-a^2+b^2}\log\left(\frac{(a+b)\cosh(x)^2+2\,(a+b)\cosh(x)\sinh(x)+(a+b)\sinh(x)^2-2\,\sqrt{-a^2+b^2}\,(\cosh(x)+\sinh(x))-a+b}{(a+b)\cosh(x)^2+2\,(a+b)\cosh(x)\sinh(x)+(a+b)\sinh(x)^2+a-b}\right)}{a^2-b^2}, -\frac{2\arctan\left(\frac{\sqrt{a}}{(a+b)\cosh(x}\right)}{\sqrt{a^2-b}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cosh(x)+b*sinh(x)),x, algorithm="fricas")
```

```
[Out] [-sqrt(-a^2 + b^2)*log(((a + b)*cosh(x)^2 + 2*(a + b)*cosh(x)*sinh(x) + (a
+ b)*sinh(x)^2 - 2*sqrt(-a^2 + b^2)*(cosh(x) + sinh(x)) - a + b)/((a + b)*c
osh(x)^2 + 2*(a + b)*cosh(x)*sinh(x) + (a + b)*sinh(x)^2 + a - b))/(a^2 - b
^2), -2*arctan(sqrt(a^2 - b^2)/((a + b)*cosh(x) + (a + b)*sinh(x)))/sqrt(a^
2 - b^2)]
```

giac [A] time = 0.13, size = 35, normalized size = 0.92

$$\frac{2\arctan\left(\frac{ae^x+be^x}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cosh(x)+b*sinh(x)),x, algorithm="giac")
```

```
[Out] 2*arctan((a*e^x + b*e^x)/sqrt(a^2 - b^2))/sqrt(a^2 - b^2)
```

maple [A] time = 0.20, size = 39, normalized size = 1.03

$$\frac{2\arctan\left(\frac{2a\tanh\left(\frac{x}{2}\right)+2b}{2\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*cosh(x)+b*sinh(x)),x)
```

```
[Out] 2/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tanh(1/2*x)+2*b)/(a^2-b^2)^(1/2))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cosh(x)+b*sinh(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [B] time = 0.10, size = 35, normalized size = 0.92

$$\frac{2\,\mathrm{atan}\left(\frac{\mathrm{e}^x\,\sqrt{a^2-b^2}}{a-b}\right)}{\sqrt{a^2-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*cosh(x) + b*sinh(x)),x)
```

```
[Out] (2*atan((exp(x)*(a^2 - b^2)^(1/2))/(a - b)))/(a^2 - b^2)^(1/2)
```

sympy [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cosh(x)+b*sinh(x)),x)
```

```
[Out] Exception raised: AttributeError
```

3.586 $\int \frac{1}{(a\cosh(x)+b\sinh(x))^2} \, dx$

Optimal. Leaf size=17

$$\frac{\sinh(x)}{a(a\cosh(x)+b\sinh(x))}$$

[Out] sinh(x)/a/(a*cosh(x)+b*sinh(x))

Rubi [A] time = 0.01, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {3075}

$$\frac{\sinh(x)}{a(a\cosh(x)+b\sinh(x))}$$

Antiderivative was successfully verified.

[In] Int[(a*Cosh[x] + b*Sinh[x])^(-2),x]

[Out] Sinh[x]/(a*(a*Cosh[x] + b*Sinh[x]))

Rule 3075

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-2), x
_Symbol] :> Simp[Sin[c + d*x]/(a*d*(a*Cos[c + d*x] + b*Sin[c + d*x])), x] /
; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rubi steps

$$\int \frac{1}{(a\cosh(x)+b\sinh(x))^2} \, dx = \frac{\sinh(x)}{a(a\cosh(x)+b\sinh(x))}$$

Mathematica [A] time = 0.03, size = 17, normalized size = 1.00

$$\frac{\sinh(x)}{a(a\cosh(x)+b\sinh(x))}$$

Antiderivative was successfully verified.

[In] Integrate[(a*Cosh[x] + b*Sinh[x])^(-2),x]

[Out] Sinh[x]/(a*(a*Cosh[x] + b*Sinh[x]))

fricas [B] time = 0.40, size = 62, normalized size = 3.65

$$-\frac{2}{\left(a^2+2\,ab+b^2\right)\cosh(x)^2+2\left(a^2+2\,ab+b^2\right)\cosh(x)\sinh(x)+\left(a^2+2\,ab+b^2\right)\sinh(x)^2+a^2-b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cosh(x)+b*sinh(x))^2,x, algorithm="fricas")
```

```
[Out] -2/((a^2 + 2*a*b + b^2)*cosh(x)^2 + 2*(a^2 + 2*a*b + b^2)*cosh(x)*sinh(x) +
 (a^2 + 2*a*b + b^2)*sinh(x)^2 + a^2 - b^2)
```

giac [A] time = 0.12, size = 26, normalized size = 1.53

$$-\frac{2}{\left(ae^{(2\,x)}+be^{(2\,x)}+a-b\right)(a+b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cosh(x)+b*sinh(x))^2,x, algorithm="giac")
```

```
[Out] -2/((a*e^(2*x) + b*e^(2*x) + a - b)*(a + b))
```

maple [A] time = 0.24, size = 29, normalized size = 1.71

$$\frac{2\tanh\left(\frac{x}{2}\right)}{a\left(a+2\tanh\left(\frac{x}{2}\right)b+a\left(\tanh^2\left(\frac{x}{2}\right)\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*cosh(x)+b*sinh(x))^2,x)
```

```
[Out] 2/a*tanh(1/2*x)/(a+2*tanh(1/2*x)*b+a*tanh(1/2*x)^2)
```

maxima [A] time = 0.36, size = 29, normalized size = 1.71

$$\frac{2}{a^2-b^2+\left(a^2-2\,ab+b^2\right)e^{(-2\,x)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cosh(x)+b*sinh(x))^2,x, algorithm="maxima")
```

```
[Out] 2/(a^2 - b^2 + (a^2 - 2*a*b + b^2)*e^(-2*x))
```

mupad [B] time = 1.54, size = 22, normalized size = 1.29

$$-\frac{2}{(a+b)\left(a-b+\mathrm{e}^{2x}\,(a+b)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*cosh(x) + b*sinh(x))^2,x)
[Out] -2/((a + b)*(a - b + exp(2*x)*(a + b)))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cosh(x)+b*sinh(x))**2,x)
[Out] Timed out
```

3.587 $\int \frac{1}{(a\cosh(x)+b\sinh(x))^3}\,dx$

Optimal. Leaf size=77

$$\frac{a\sinh(x)+b\cosh(x)}{2\left(a^2-b^2\right)(a\cosh(x)+b\sinh(x))^2}+\frac{\tan^{-1}\left(\frac{a\sinh(x)+b\cosh(x)}{\sqrt{a^2-b^2}}\right)}{2\left(a^2-b^2\right)^{3/2}}$$

```
[Out] 1/2*arctan((b*cosh(x)+a*sinh(x))/(a^2-b^2)^(1/2))/(a^2-b^2)^(3/2)+1/2*(b*co
sh(x)+a*sinh(x))/(a^2-b^2)/(a*cosh(x)+b*sinh(x))^2
```

Rubi [A] time = 0.05, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {3076, 3074, 206}

$$\frac{a\sinh(x)+b\cosh(x)}{2\left(a^2-b^2\right)(a\cosh(x)+b\sinh(x))^2}+\frac{\tan^{-1}\left(\frac{a\sinh(x)+b\cosh(x)}{\sqrt{a^2-b^2}}\right)}{2\left(a^2-b^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cosh[x] + b*Sinh[x])^(-3),x]
```

```
[Out] ArcTan[(b*Cosh[x] + a*Sinh[x])/Sqrt[a^2 - b^2]]/(2*(a^2 - b^2)^(3/2)) + (b*
Cosh[x] + a*Sinh[x])/(2*(a^2 - b^2)*(a*Cosh[x] + b*Sinh[x])^2)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3074

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> -Dist[d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d
*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3076

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Simp[((b*Cos[c + d*x] - a*Sin[c + d*x])*(a*Cos[c + d*x] + b*Sin
[c + d*x])^(n + 1))/(d*(n + 1)*(a^2 + b^2)), x] + Dist[(n + 2)/((n + 1)*(a^
2 + b^2)), Int[(a*Cos[c + d*x] + b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{
```

```
a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && LtQ[n, -1] && NeQ[n, -2]
```

Rubi steps

$$\int \frac{1}{(a\cosh(x)+b\sinh(x))^3}\,dx = \frac{b\cosh(x)+a\sinh(x)}{2\left(a^2-b^2\right)(a\cosh(x)+b\sinh(x))^2}+\frac{\int \frac{1}{a\cosh(x)+b\sinh(x)}\,dx}{2\left(a^2-b^2\right)}$$

$$= \frac{b\cosh(x)+a\sinh(x)}{2\left(a^2-b^2\right)(a\cosh(x)+b\sinh(x))^2}+\frac{i\,\mathrm{Subst}\left(\int \frac{1}{a^2-b^2-x^2}\,dx,x,-ib\cosh(x)-ia\right.}{2\left(a^2-b^2\right)}$$

$$= \frac{\tan^{-1}\left(\frac{b\cosh(x)+a\sinh(x)}{\sqrt{a^2-b^2}}\right)}{2\left(a^2-b^2\right)^{3/2}}+\frac{b\cosh(x)+a\sinh(x)}{2\left(a^2-b^2\right)(a\cosh(x)+b\sinh(x))^2}$$

Mathematica [A] time = 0.49, size = 96, normalized size = 1.25

$$\frac{1}{2}\left(\frac{2\tan^{-1}\left(\frac{a\tanh\left(\frac{x}{2}\right)+b}{\sqrt{a-b}\sqrt{a+b}}\right)}{(a-b)^{3/2}(a+b)^{3/2}}+\frac{b}{a(a-b)(a+b)(a\cosh(x)+b\sinh(x))}+\frac{\sinh(x)}{a(a\cosh(x)+b\sinh(x))^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cosh[x] + b*Sinh[x])^(-3),x]
```

```
[Out] ((2*ArcTan[(b + a*Tanh[x/2])/(Sqrt[a - b]*Sqrt[a + b])])/((a - b)^(3/2)*(a
+ b)^(3/2)) + Sinh[x]/(a*(a*Cosh[x] + b*Sinh[x])^2) + b/(a*(a - b)*(a + b)*
(a*Cosh[x] + b*Sinh[x])))/2
```

fricas [B] time = 0.45, size = 1495, normalized size = 19.42

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cosh(x)+b*sinh(x))^3,x, algorithm="fricas")
```

```
[Out] [1/2*(2*(a^3 + a^2*b - a*b^2 - b^3)*cosh(x)^3 + 6*(a^3 + a^2*b - a*b^2 - b^
3)*cosh(x)*sinh(x)^2 + 2*(a^3 + a^2*b - a*b^2 - b^3)*sinh(x)^3 + ((a^2 + 2*
a*b + b^2)*cosh(x)^4 + 4*(a^2 + 2*a*b + b^2)*cosh(x)*sinh(x)^3 + (a^2 + 2*a
*b + b^2)*sinh(x)^4 + 2*(a^2 - b^2)*cosh(x)^2 + 2*(3*(a^2 + 2*a*b + b^2)*co
sh(x)^2 + a^2 - b^2)*sinh(x)^2 + a^2 - 2*a*b + b^2 + 4*((a^2 + 2*a*b + b^2)
```

```
*cosh(x)^3 + (a^2 - b^2)*cosh(x))*sinh(x))*sqrt(-a^2 + b^2)*log(((a + b)*co
sh(x)^2 + 2*(a + b)*cosh(x)*sinh(x) + (a + b)*sinh(x)^2 + 2*sqrt(-a^2 + b^2
)*(cosh(x) + sinh(x)) - a + b)/((a + b)*cosh(x)^2 + 2*(a + b)*cosh(x)*sinh(
x) + (a + b)*sinh(x)^2 + a - b)) - 2*(a^3 - a^2*b - a*b^2 + b^3)*cosh(x) -
2*(a^3 - a^2*b - a*b^2 + b^3 - 3*(a^3 + a^2*b - a*b^2 - b^3)*cosh(x)^2)*sin
h(x))/(a^6 - 2*a^5*b - a^4*b^2 + 4*a^3*b^3 - a^2*b^4 - 2*a*b^5 + b^6 + (a^6
 + 2*a^5*b - a^4*b^2 - 4*a^3*b^3 - a^2*b^4 + 2*a*b^5 + b^6)*cosh(x)^4 + 4*(
a^6 + 2*a^5*b - a^4*b^2 - 4*a^3*b^3 - a^2*b^4 + 2*a*b^5 + b^6)*cosh(x)*sinh
(x)^3 + (a^6 + 2*a^5*b - a^4*b^2 - 4*a^3*b^3 - a^2*b^4 + 2*a*b^5 + b^6)*sin
h(x)^4 + 2*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^2 + 2*(a^6 - 3*a^4*b
^2 + 3*a^2*b^4 - b^6 + 3*(a^6 + 2*a^5*b - a^4*b^2 - 4*a^3*b^3 - a^2*b^4 + 2
*a*b^5 + b^6)*cosh(x)^2)*sinh(x)^2 + 4*((a^6 + 2*a^5*b - a^4*b^2 - 4*a^3*b^
3 - a^2*b^4 + 2*a*b^5 + b^6)*cosh(x)^3 + (a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6
)*cosh(x))*sinh(x)), ((a^3 + a^2*b - a*b^2 - b^3)*cosh(x)^3 + 3*(a^3 + a^2*
b - a*b^2 - b^3)*cosh(x)*sinh(x)^2 + (a^3 + a^2*b - a*b^2 - b^3)*sinh(x)^3
- ((a^2 + 2*a*b + b^2)*cosh(x)^4 + 4*(a^2 + 2*a*b + b^2)*cosh(x)*sinh(x)^3
+ (a^2 + 2*a*b + b^2)*sinh(x)^4 + 2*(a^2 - b^2)*cosh(x)^2 + 2*(3*(a^2 + 2*a
*b + b^2)*cosh(x)^2 + a^2 - b^2)*sinh(x)^2 + a^2 - 2*a*b + b^2 + 4*((a^2 +
2*a*b + b^2)*cosh(x)^3 + (a^2 - b^2)*cosh(x))*sinh(x))*sqrt(a^2 - b^2)*arct
an(sqrt(a^2 - b^2)/((a + b)*cosh(x) + (a + b)*sinh(x))) - (a^3 - a^2*b - a*
b^2 + b^3)*cosh(x) - (a^3 - a^2*b - a*b^2 + b^3 - 3*(a^3 + a^2*b - a*b^2 -
b^3)*cosh(x)^2)*sinh(x))/(a^6 - 2*a^5*b - a^4*b^2 + 4*a^3*b^3 - a^2*b^4 - 2
*a*b^5 + b^6 + (a^6 + 2*a^5*b - a^4*b^2 - 4*a^3*b^3 - a^2*b^4 + 2*a*b^5 + b
^6)*cosh(x)^4 + 4*(a^6 + 2*a^5*b - a^4*b^2 - 4*a^3*b^3 - a^2*b^4 + 2*a*b^5
+ b^6)*cosh(x)*sinh(x)^3 + (a^6 + 2*a^5*b - a^4*b^2 - 4*a^3*b^3 - a^2*b^4 +
 2*a*b^5 + b^6)*sinh(x)^4 + 2*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^2
 + 2*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6 + 3*(a^6 + 2*a^5*b - a^4*b^2 - 4*a^
3*b^3 - a^2*b^4 + 2*a*b^5 + b^6)*cosh(x)^2)*sinh(x)^2 + 4*((a^6 + 2*a^5*b -
 a^4*b^2 - 4*a^3*b^3 - a^2*b^4 + 2*a*b^5 + b^6)*cosh(x)^3 + (a^6 - 3*a^4*b^
2 + 3*a^2*b^4 - b^6)*cosh(x))*sinh(x))]
```

giac [A] time = 0.14, size = 88, normalized size = 1.14

$$\frac{\arctan\left(\frac{ae^{x}+be^{x}}{\sqrt{a^{2}-b^{2}}}\right)}{\left(a^{2}-b^{2}\right)^{\frac{3}{2}}}+\frac{ae^{(3\,x)}+be^{(3\,x)}-ae^{x}+be^{x}}{\left(a^{2}-b^{2}\right)\left(ae^{(2\,x)}+be^{(2\,x)}+a-b\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cosh(x)+b*sinh(x))^3,x, algorithm="giac")
```

```
[Out] arctan((a*e^x + b*e^x)/sqrt(a^2 - b^2))/(a^2 - b^2)^(3/2) + (a*e^(3*x) + b*
e^(3*x) - a*e^x + b*e^x)/((a^2 - b^2)*(a*e^(2*x) + b*e^(2*x) + a - b)^2)
```

maple [B] time = 0.26, size = 167, normalized size = 2.17

$$\frac{-\frac{\left(a^2-2b^2\right)\left(\tanh^3\left(\frac{x}{2}\right)\right)}{\left(a^2-b^2\right)a} + \frac{b\left(a^2+2b^2\right)\left(\tanh^2\left(\frac{x}{2}\right)\right)}{\left(a^2-b^2\right)a^2} + \frac{\left(a^2+2b^2\right)\tanh\left(\frac{x}{2}\right)}{\left(a^2-b^2\right)a} + \frac{2b}{2a^2-2b^2}}{\left(a+2\tanh\left(\frac{x}{2}\right)b+a\left(\tanh^2\left(\frac{x}{2}\right)\right)\right)^2} + \frac{\arctan\left(\frac{2a\tanh\left(\frac{x}{2}\right)+2b}{2\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*cosh(x)+b*sinh(x))^3,x)
```

```
[Out] 2*(-1/2*(a^2-2*b^2)/(a^2-b^2)/a*tanh(1/2*x)^3+1/2*b*(a^2+2*b^2)/(a^2-b^2)/a
^2*tanh(1/2*x)^2+1/2*(a^2+2*b^2)/(a^2-b^2)/a*tanh(1/2*x)+1/2*b/(a^2-b^2))/(
a+2*tanh(1/2*x)*b+a*tanh(1/2*x)^2)^2+1/(a^2-b^2)^(3/2)*arctan(1/2*(2*a*tanh
(1/2*x)+2*b)/(a^2-b^2)^(1/2))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cosh(x)+b*sinh(x))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [B] time = 1.61, size = 157, normalized size = 2.04

$$\frac{e^x}{(a+b)\,(a-b)\,\left(a-b+e^{2x}\,(a+b)\right)} - \frac{2\,e^x}{(a+b)\,\left(e^{4x}\,(a+b)^2+(a-b)^2+2\,e^{2x}\,(a+b)\,(a-b)\right)} - \frac{\mathrm{atan}\left(\frac{e^x\,\sqrt{a^6-3\,a^4\,b^2\cdots}}{-a^3+a^2\,b+\cdots}\right)}{\sqrt{a^6-3\,a^4\,b^2+3\cdots}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*cosh(x) + b*sinh(x))^3,x)
```

```
[Out] exp(x)/((a + b)*(a - b)*(a - b + exp(2*x)*(a + b))) - (2*exp(x))/((a + b)*(
exp(4*x)*(a + b)^2 + (a - b)^2 + 2*exp(2*x)*(a + b)*(a - b))) - atan((exp(x
)*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)^(1/2))/(a*b^2 + a^2*b - a^3 - b^3))/(
a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)^(1/2)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cosh(x)+b*sinh(x))**3,x)
```

```
[Out] Timed out
```

3.588 $\int \frac{1}{(a\cosh(x)+b\sinh(x))^4}\,dx$

Optimal. Leaf size=67

$$\frac{2\sinh(x)}{3a(a^2-b^2)(a\cosh(x)+b\sinh(x))}+\frac{a\sinh(x)+b\cosh(x)}{3(a^2-b^2)(a\cosh(x)+b\sinh(x))^3}$$

```
[Out] 1/3*(b*cosh(x)+a*sinh(x))/(a^2-b^2)/(a*cosh(x)+b*sinh(x))^3+2/3*sinh(x)/a/(
a^2-b^2)/(a*cosh(x)+b*sinh(x))
```

Rubi [A] time = 0.03, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {3076, 3075}

$$\frac{2\sinh(x)}{3a(a^2-b^2)(a\cosh(x)+b\sinh(x))}+\frac{a\sinh(x)+b\cosh(x)}{3(a^2-b^2)(a\cosh(x)+b\sinh(x))^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cosh[x] + b*Sinh[x])^(-4),x]
```

```
[Out] (b*Cosh[x] + a*Sinh[x])/(3*(a^2 - b^2)*(a*Cosh[x] + b*Sinh[x])^3) + (2*Sinh
[x])/(3*a*(a^2 - b^2)*(a*Cosh[x] + b*Sinh[x]))
```

Rule 3075

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-2), x
_Symbol] :> Simp[Sin[c + d*x]/(a*d*(a*Cos[c + d*x] + b*Sin[c + d*x])), x] /
; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3076

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Simp[((b*Cos[c + d*x] - a*Sin[c + d*x])*(a*Cos[c + d*x] + b*Sin
[c + d*x])^(n + 1))/(d*(n + 1)*(a^2 + b^2)), x] + Dist[(n + 2)/((n + 1)*(a^
2 + b^2)), Int[(a*Cos[c + d*x] + b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{
a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && LtQ[n, -1] && NeQ[n, -2]
```

Rubi steps

$$\int \frac{1}{(a\cosh(x)+b\sinh(x))^4}\,dx = \frac{b\cosh(x)+a\sinh(x)}{3\left(a^2-b^2\right)(a\cosh(x)+b\sinh(x))^3} + \frac{2\int \frac{1}{(a\cosh(x)+b\sinh(x))^2}\,dx}{3\left(a^2-b^2\right)}$$
$$= \frac{b\cosh(x)+a\sinh(x)}{3\left(a^2-b^2\right)(a\cosh(x)+b\sinh(x))^3} + \frac{2\sinh(x)}{3a\left(a^2-b^2\right)(a\cosh(x)+b\sinh(x))}$$

Mathematica [A] time = 0.14, size = 64, normalized size = 0.96

$$\frac{\sinh(x)\left(\left(a^2+b^2\right)\cosh(2x)+2a^2-b^2\right)+ab\cosh(3x)}{3a(a-b)(a+b)(a\cosh(x)+b\sinh(x))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cosh[x] + b*Sinh[x])^(-4),x]
```

```
[Out] (a*b*Cosh[3*x] + (2*a^2 - b^2 + (a^2 + b^2)*Cosh[2*x])*Sinh[x])/(3*a*(a - b
)*(a + b)*(a*Cosh[x] + b*Sinh[x])^3)
```

fricas [B] time = 0.42, size = 527, normalized size = 7.87

$$-\frac{}{3\left(\left(a^5+5a^4b+10a^3b^2+10a^2b^3+5ab^4+b^5\right)\cosh(x)^5+5\left(a^5+5a^4b+10a^3b^2+10a^2b^3+5ab^4+b^5\right)\cos\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cosh(x)+b*sinh(x))^4,x, algorithm="fricas")
```

```
[Out] -8/3*((2*a + b)*cosh(x) + (a + 2*b)*sinh(x))/((a^5 + 5*a^4*b + 10*a^3*b^2 +
 10*a^2*b^3 + 5*a*b^4 + b^5)*cosh(x)^5 + 5*(a^5 + 5*a^4*b + 10*a^3*b^2 + 10
*a^2*b^3 + 5*a*b^4 + b^5)*cosh(x)*sinh(x)^4 + (a^5 + 5*a^4*b + 10*a^3*b^2 +
 10*a^2*b^3 + 5*a*b^4 + b^5)*sinh(x)^5 + 3*(a^5 + 3*a^4*b + 2*a^3*b^2 - 2*a
^2*b^3 - 3*a*b^4 - b^5)*cosh(x)^3 + (3*a^5 + 9*a^4*b + 6*a^3*b^2 - 6*a^2*b^
3 - 9*a*b^4 - 3*b^5 + 10*(a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4
 + b^5)*cosh(x)^2)*sinh(x)^3 + (10*(a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3
 + 5*a*b^4 + b^5)*cosh(x)^3 + 9*(a^5 + 3*a^4*b + 2*a^3*b^2 - 2*a^2*b^3 - 3*
a*b^4 - b^5)*cosh(x))*sinh(x)^2 + 2*(2*a^5 + a^4*b - 4*a^3*b^2 - 2*a^2*b^3
+ 2*a*b^4 + b^5)*cosh(x) + (2*a^5 + 4*a^4*b - 4*a^3*b^2 - 8*a^2*b^3 + 2*a*b
^4 + 4*b^5 + 5*(a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*co
sh(x)^4 + 9*(a^5 + 3*a^4*b + 2*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^5)*cosh(x)
^2)*sinh(x))
```

giac [A] time = 0.14, size = 53, normalized size = 0.79

$$-\frac{4\left(3\,ae^{(2\,x)}+3\,be^{(2\,x)}+a-b\right)}{3\left(a^2+2\,ab+b^2\right)\left(ae^{(2\,x)}+be^{(2\,x)}+a-b\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cosh(x)+b*sinh(x))^4,x, algorithm="giac")
```

```
[Out] -4/3*(3*a*e^(2*x) + 3*b*e^(2*x) + a - b)/((a^2 + 2*a*b + b^2)*(a*e^(2*x) +
b*e^(2*x) + a - b)^3)
```

maple [A] time = 0.28, size = 87, normalized size = 1.30

$$-\frac{2\left(-\frac{\tanh^5\left(\frac{x}{2}\right)}{a}-\frac{2b\left(\tanh^4\left(\frac{x}{2}\right)\right)}{a^2}-\frac{2\left(a^2+2b^2\right)\left(\tanh^3\left(\frac{x}{2}\right)\right)}{3a^3}-\frac{2b\left(\tanh^2\left(\frac{x}{2}\right)\right)}{a^2}-\frac{\tanh\left(\frac{x}{2}\right)}{a}\right)}{\left(a+2\tanh\left(\frac{x}{2}\right)b+a\left(\tanh^2\left(\frac{x}{2}\right)\right)\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*cosh(x)+b*sinh(x))^4,x)
```

```
[Out] -2*(-1/a*tanh(1/2*x)^5-2/a^2*b*tanh(1/2*x)^4-2/3/a^3*(a^2+2*b^2)*tanh(1/2*x
)^3-2/a^2*b*tanh(1/2*x)^2-1/a*tanh(1/2*x))/(a+2*tanh(1/2*x)*b+a*tanh(1/2*x)
^2)^3
```

maxima [B] time = 0.62, size = 498, normalized size = 7.43

$$\frac{4\,(a-b)e^{(-2\,x)}}{a^5+a^4b-2\,a^3b^2-2\,a^2b^3+ab^4+b^5+3\left(a^5-a^4b-2\,a^3b^2+2\,a^2b^3+ab^4-b^5\right)e^{(-2\,x)}+3\left(a^5-3\,a^4b+2\,a^3b^2+2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cosh(x)+b*sinh(x))^4,x, algorithm="maxima")
```

```
[Out] 4*(a - b)*e^(-2*x)/(a^5 + a^4*b - 2*a^3*b^2 - 2*a^2*b^3 + a*b^4 + b^5 + 3*(
a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*e^(-2*x) + 3*(a^5 - 3*a^
4*b + 2*a^3*b^2 + 2*a^2*b^3 - 3*a*b^4 + b^5)*e^(-4*x) + (a^5 - 5*a^4*b + 10
*a^3*b^2 - 10*a^2*b^3 + 5*a*b^4 - b^5)*e^(-6*x)) + 4/3*a/(a^5 + a^4*b - 2*a
^3*b^2 - 2*a^2*b^3 + a*b^4 + b^5 + 3*(a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 +
 a*b^4 - b^5)*e^(-2*x) + 3*(a^5 - 3*a^4*b + 2*a^3*b^2 + 2*a^2*b^3 - 3*a*b^4
 + b^5)*e^(-4*x) + (a^5 - 5*a^4*b + 10*a^3*b^2 - 10*a^2*b^3 + 5*a*b^4 - b^5
)*e^(-6*x)) + 4/3*b/(a^5 + a^4*b - 2*a^3*b^2 - 2*a^2*b^3 + a*b^4 + b^5 + 3*
(a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*e^(-2*x) + 3*(a^5 - 3*a
```

```
^4*b + 2*a^3*b^2 + 2*a^2*b^3 - 3*a*b^4 + b^5)*e^(-4*x) + (a^5 - 5*a^4*b + 1
0*a^3*b^2 - 10*a^2*b^3 + 5*a*b^4 - b^5)*e^(-6*x))
```

mupad [B] time = 1.70, size = 47, normalized size = 0.70

$$-\frac{a\left(4\,{\mathrm{e}}^{2\,x}+\frac{4}{3}\right)+b\left(4\,{\mathrm{e}}^{2\,x}-\frac{4}{3}\right)}{{\left(a+b\right)}^2\,{\left(a-b+a\,{\mathrm{e}}^{2\,x}+b\,{\mathrm{e}}^{2\,x}\right)}^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*cosh(x) + b*sinh(x))^4,x)
```

```
[Out] -(a*(4*exp(2*x) + 4/3) + b*(4*exp(2*x) - 4/3))/((a + b)^2*(a - b + a*exp(2*
x) + b*exp(2*x))^3)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cosh(x)+b*sinh(x))**4,x)
```

```
[Out] Timed out
```

3.589 $\int \frac{1}{(a\cosh(x)+b\sinh(x))^5}\,dx$

Optimal. Leaf size=112

$$\frac{3(a\sinh(x)+b\cosh(x))}{8\left(a^2-b^2\right)^2(a\cosh(x)+b\sinh(x))^2}+\frac{a\sinh(x)+b\cosh(x)}{4\left(a^2-b^2\right)(a\cosh(x)+b\sinh(x))^4}+\frac{3\tan^{-1}\left(\frac{a\sinh(x)+b\cosh(x)}{\sqrt{a^2-b^2}}\right)}{8\left(a^2-b^2\right)^{5/2}}$$

```
[Out] 3/8*arctan((b*cosh(x)+a*sinh(x))/(a^2-b^2)^(1/2))/(a^2-b^2)^(5/2)+1/4*(b*co
sh(x)+a*sinh(x))/(a^2-b^2)/(a*cosh(x)+b*sinh(x))^4+3/8*(b*cosh(x)+a*sinh(x)
)/(a^2-b^2)^2/(a*cosh(x)+b*sinh(x))^2
```

Rubi [A] time = 0.07, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {3076, 3074, 206}

$$\frac{3(a\sinh(x)+b\cosh(x))}{8\left(a^2-b^2\right)^2(a\cosh(x)+b\sinh(x))^2}+\frac{a\sinh(x)+b\cosh(x)}{4\left(a^2-b^2\right)(a\cosh(x)+b\sinh(x))^4}+\frac{3\tan^{-1}\left(\frac{a\sinh(x)+b\cosh(x)}{\sqrt{a^2-b^2}}\right)}{8\left(a^2-b^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cosh[x] + b*Sinh[x])^(-5),x]
```

```
[Out] (3*ArcTan[(b*Cosh[x] + a*Sinh[x])/Sqrt[a^2 - b^2]])/(8*(a^2 - b^2)^(5/2)) +
 (b*Cosh[x] + a*Sinh[x])/(4*(a^2 - b^2)*(a*Cosh[x] + b*Sinh[x])^4) + (3*(b*
Cosh[x] + a*Sinh[x]))/(8*(a^2 - b^2)^2*(a*Cosh[x] + b*Sinh[x])^2)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3074

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> -Dist[d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d
*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3076

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Simp[((b*Cos[c + d*x] - a*Sin[c + d*x])*(a*Cos[c + d*x] + b*Sin
[c + d*x])^(n + 1))/(d*(n + 1)*(a^2 + b^2)), x] + Dist[(n + 2)/((n + 1)*(a^
```

```
2 + b^2)), Int[(a*Cos[c + d*x] + b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{
a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && LtQ[n, -1] && NeQ[n, -2]
```

Rubi steps

$$\int \frac{1}{(a\cosh(x)+b\sinh(x))^5}\,dx = \frac{b\cosh(x)+a\sinh(x)}{4\left(a^2-b^2\right)(a\cosh(x)+b\sinh(x))^4}+\frac{3\int \frac{1}{(a\cosh(x)+b\sinh(x))^3}\,dx}{4\left(a^2-b^2\right)}$$

$$= \frac{b\cosh(x)+a\sinh(x)}{4\left(a^2-b^2\right)(a\cosh(x)+b\sinh(x))^4}+\frac{3(b\cosh(x)+a\sinh(x))}{8\left(a^2-b^2\right)^2(a\cosh(x)+b\sinh(x))^2}+$$

$$= \frac{b\cosh(x)+a\sinh(x)}{4\left(a^2-b^2\right)(a\cosh(x)+b\sinh(x))^4}+\frac{3(b\cosh(x)+a\sinh(x))}{8\left(a^2-b^2\right)^2(a\cosh(x)+b\sinh(x))^2}+$$

$$= \frac{3\tan^{-1}\left(\frac{b\cosh(x)+a\sinh(x)}{\sqrt{a^2-b^2}}\right)}{8\left(a^2-b^2\right)^{5/2}}+\frac{b\cosh(x)+a\sinh(x)}{4\left(a^2-b^2\right)(a\cosh(x)+b\sinh(x))^4}+\frac{3(b\,\mathrm{c}}{8\left(a^2-b^2\right)}$$

Mathematica [A] time = 1.06, size = 147, normalized size = 1.31

$$\frac{1}{8}\left(\frac{6\tan^{-1}\left(\frac{a\tanh\left(\frac{x}{2}\right)+b}{\sqrt{a-b}\sqrt{a+b}}\right)}{(a-b)^{5/2}(a+b)^{5/2}}+\frac{b\left(3(a\cosh(x)+b\sinh(x))^2+2(a-b)(a+b)\right)}{a(a-b)^2(a+b)^2(a\cosh(x)+b\sinh(x))^3}+\frac{\sinh(x)\left(\frac{3(a\cosh(x)+b\sinh(x))^2}{(a-b)(a+b)}+2\right)}{a(a\cosh(x)+b\sinh(x))^4}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cosh[x] + b*Sinh[x])^(-5),x]
```

```
[Out] ((6*ArcTan[(b + a*Tanh[x/2])/(Sqrt[a - b]*Sqrt[a + b])])/((a - b)^(5/2)*(a
+ b)^(5/2)) + (b*(2*(a - b)*(a + b) + 3*(a*Cosh[x] + b*Sinh[x])^2))/(a*(a -
 b)^2*(a + b)^2*(a*Cosh[x] + b*Sinh[x])^3) + (Sinh[x]*(2 + (3*(a*Cosh[x] +
b*Sinh[x])^2)/((a - b)*(a + b))))/(a*(a*Cosh[x] + b*Sinh[x])^4))/8
```

fricas [B] time = 0.56, size = 6874, normalized size = 61.38

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cosh(x)+b*sinh(x))^5,x, algorithm="fricas")
```

[Out] [1/8*(6*(a^5 + 3*a^4*b + 2*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^5)*cosh(x)^7 + 42*(a^5 + 3*a^4*b + 2*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^5)*cosh(x)*sinh(x)^6 + 6*(a^5 + 3*a^4*b + 2*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^5)*sinh(x)^7 + 22*(a^5 + a^4*b - 2*a^3*b^2 - 2*a^2*b^3 + a*b^4 + b^5)*cosh(x)^5 + 2*(11*a^5 + 11*a^4*b - 22*a^3*b^2 - 22*a^2*b^3 + 11*a*b^4 + 11*b^5 + 63*(a^5 + 3*a^4*b + 2*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^5)*cosh(x)^2)*sinh(x)^5 + 10*(21*(a^5 + 3*a^4*b + 2*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^5)*cosh(x)^3 + 11*(a^5 + a^4*b - 2*a^3*b^2 - 2*a^2*b^3 + a*b^4 + b^5)*cosh(x))*sinh(x)^4 - 22*(a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^3 - 2*(11*a^5 - 11*a^4*b - 22*a^3*b^2 + 22*a^2*b^3 + 11*a*b^4 - 11*b^5 - 105*(a^5 + 3*a^4*b + 2*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^5)*cosh(x)^4 - 110*(a^5 + a^4*b - 2*a^3*b^2 - 2*a^2*b^3 + a*b^4 + b^5)*cosh(x)^2)*sinh(x)^3 + 2*(63*(a^5 + 3*a^4*b + 2*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^5)*cosh(x)^5 + 110*(a^5 + a^4*b - 2*a^3*b^2 - 2*a^2*b^3 + a*b^4 + b^5)*cosh(x)^3 - 33*(a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x))*sinh(x)^2 - 3*((a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*cosh(x)^8 + 8*(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*cosh(x)*sinh(x)^7 + (a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*sinh(x)^8 + 4*(a^4 + 2*a^3*b - 2*a*b^3 - b^4)*cosh(x)^6 + 4*(a^4 + 2*a^3*b - 2*a*b^3 - b^4 + 7*(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*cosh(x)^2)*sinh(x)^6 + 8*(7*(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*cosh(x)^3 + 3*(a^4 + 2*a^3*b - 2*a*b^3 - b^4)*cosh(x))*sinh(x)^5 + 6*(a^4 - 2*a^2*b^2 + b^4)*cosh(x)^4 + 2*(35*(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*cosh(x)^4 + 3*a^4 - 6*a^2*b^2 + 3*b^4 + 30*(a^4 + 2*a^3*b - 2*a*b^3 - b^4)*cosh(x)^2)*sinh(x)^4 + a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4 + 8*(7*(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*cosh(x)^5 + 10*(a^4 + 2*a^3*b - 2*a*b^3 - b^4)*cosh(x)^3 + 3*(a^4 - 2*a^2*b^2 + b^4)*cosh(x))*sinh(x)^3 + 4*(a^4 - 2*a^3*b + 2*a*b^3 - b^4)*cosh(x)^2 + 4*(7*(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*cosh(x)^6 + 15*(a^4 + 2*a^3*b - 2*a*b^3 - b^4)*cosh(x)^4 + a^4 - 2*a^3*b + 2*a*b^3 - b^4 + 9*(a^4 - 2*a^2*b^2 + b^4)*cosh(x)^2)*sinh(x)^2 + 8*((a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*cosh(x)^7 + 3*(a^4 + 2*a^3*b - 2*a*b^3 - b^4)*cosh(x)^5 + 3*(a^4 - 2*a^2*b^2 + b^4)*cosh(x)^3 + (a^4 - 2*a^3*b + 2*a*b^3 - b^4)*cosh(x))*sinh(x))*sqrt(-a^2 + b^2)*log(((a + b)*cosh(x)^2 + 2*(a + b)*cosh(x)*sinh(x) + (a + b)*sinh(x)^2 - 2*sqrt(-a^2 + b^2)*(cosh(x) + sinh(x)) - a + b)/((a + b)*cosh(x)^2 + 2*(a + b)*cosh(x)*sinh(x) + (a + b)*sinh(x)^2 + a - b)) - 6*(a^5 - 3*a^4*b + 2*a^3*b^2 + 2*a^2*b^3 - 3*a*b^4 + b^5)*cosh(x) + 2*(21*(a^5 + 3*a^4*b + 2*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^5)*cosh(x)^6 - 3*a^5 + 9*a^4*b - 6*a^3*b^2 - 6*a^2*b^3 + 9*a*b^4 - 3*b^5 + 55*(a^5 + a^4*b - 2*a^3*b^2 - 2*a^2*b^3 + a*b^4 + b^5)*cosh(x)^4 - 33*(a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^2)*sinh(x))/(a^10 - 4*a^9*b + 3*a^8*b^2 + 8*a^7*b^3 - 14*a^6*b^4 + 14*a^4*b^6 - 8*a^3*b^7 - 3*a^2*b^8 + 4*a*b^9 - b^10 + (a^10 + 4*a^9*b + 3*a^8*b^2 - 8*a^7*b^3 - 14*a^6*b^4 + 14*a^4*b^6 + 8*a^3*b^7 - 3*a^2*b^8 - 4*a*b^9 - b^10)*cosh(x)^8 + 8*(a^10 + 4*a^9*b + 3*a^8*b^2 - 8*a^7*b^3 - 14*a^6*b^4 + 14*a^4*b^6 + 8*a^3*b^7 - 3*a^2*b^8 - 4*a*b^9 - b^10)*cosh(x)*sinh(x)^7 + (a^10 + 4*a^9*b + 3*a^8*b^2 - 8*a^7*b^3 - 14*a^6*b^4 + 14*a^4*b^6 + 8*a^3*b^7 - 3*a^2*b^8 - 4*a*

```
b^9 - b^10)*sinh(x)^8 + 4*(a^10 + 2*a^9*b - 3*a^8*b^2 - 8*a^7*b^3 + 2*a^6*b
^4 + 12*a^5*b^5 + 2*a^4*b^6 - 8*a^3*b^7 - 3*a^2*b^8 + 2*a*b^9 + b^10)*cosh(
x)^6 + 4*(a^10 + 2*a^9*b - 3*a^8*b^2 - 8*a^7*b^3 + 2*a^6*b^4 + 12*a^5*b^5 +
 2*a^4*b^6 - 8*a^3*b^7 - 3*a^2*b^8 + 2*a*b^9 + b^10 + 7*(a^10 + 4*a^9*b + 3
*a^8*b^2 - 8*a^7*b^3 - 14*a^6*b^4 + 14*a^4*b^6 + 8*a^3*b^7 - 3*a^2*b^8 - 4*
a*b^9 - b^10)*cosh(x)^2)*sinh(x)^6 + 8*(7*(a^10 + 4*a^9*b + 3*a^8*b^2 - 8*a
^7*b^3 - 14*a^6*b^4 + 14*a^4*b^6 + 8*a^3*b^7 - 3*a^2*b^8 - 4*a*b^9 - b^10)*
cosh(x)^3 + 3*(a^10 + 2*a^9*b - 3*a^8*b^2 - 8*a^7*b^3 + 2*a^6*b^4 + 12*a^5*
b^5 + 2*a^4*b^6 - 8*a^3*b^7 - 3*a^2*b^8 + 2*a*b^9 + b^10)*cosh(x))*sinh(x)^
5 + 6*(a^10 - 5*a^8*b^2 + 10*a^6*b^4 - 10*a^4*b^6 + 5*a^2*b^8 - b^10)*cosh(
x)^4 + 2*(3*a^10 - 15*a^8*b^2 + 30*a^6*b^4 - 30*a^4*b^6 + 15*a^2*b^8 - 3*b^
10 + 35*(a^10 + 4*a^9*b + 3*a^8*b^2 - 8*a^7*b^3 - 14*a^6*b^4 + 14*a^4*b^6 +
 8*a^3*b^7 - 3*a^2*b^8 - 4*a*b^9 - b^10)*cosh(x)^4 + 30*(a^10 + 2*a^9*b - 3
*a^8*b^2 - 8*a^7*b^3 + 2*a^6*b^4 + 12*a^5*b^5 + 2*a^4*b^6 - 8*a^3*b^7 - 3*a
^2*b^8 + 2*a*b^9 + b^10)*cosh(x)^2)*sinh(x)^4 + 8*(7*(a^10 + 4*a^9*b + 3*a^
8*b^2 - 8*a^7*b^3 - 14*a^6*b^4 + 14*a^4*b^6 + 8*a^3*b^7 - 3*a^2*b^8 - 4*a*b
^9 - b^10)*cosh(x)^5 + 10*(a^10 + 2*a^9*b - 3*a^8*b^2 - 8*a^7*b^3 + 2*a^6*b
^4 + 12*a^5*b^5 + 2*a^4*b^6 - 8*a^3*b^7 - 3*a^2*b^8 + 2*a*b^9 + b^10)*cosh(
x)^3 + 3*(a^10 - 5*a^8*b^2 + 10*a^6*b^4 - 10*a^4*b^6 + 5*a^2*b^8 - b^10)*co
sh(x))*sinh(x)^3 + 4*(a^10 - 2*a^9*b - 3*a^8*b^2 + 8*a^7*b^3 + 2*a^6*b^4 -
12*a^5*b^5 + 2*a^4*b^6 + 8*a^3*b^7 - 3*a^2*b^8 - 2*a*b^9 + b^10)*cosh(x)^2
+ 4*(a^10 - 2*a^9*b - 3*a^8*b^2 + 8*a^7*b^3 + 2*a^6*b^4 - 12*a^5*b^5 + 2*a^
4*b^6 + 8*a^3*b^7 - 3*a^2*b^8 - 2*a*b^9 + b^10 + 7*(a^10 + 4*a^9*b + 3*a^8*
b^2 - 8*a^7*b^3 - 14*a^6*b^4 + 14*a^4*b^6 + 8*a^3*b^7 - 3*a^2*b^8 - 4*a*b^9
 - b^10)*cosh(x)^6 + 15*(a^10 + 2*a^9*b - 3*a^8*b^2 - 8*a^7*b^3 + 2*a^6*b^4
 + 12*a^5*b^5 + 2*a^4*b^6 - 8*a^3*b^7 - 3*a^2*b^8 + 2*a*b^9 + b^10)*cosh(x)
^4 + 9*(a^10 - 5*a^8*b^2 + 10*a^6*b^4 - 10*a^4*b^6 + 5*a^2*b^8 - b^10)*cosh
(x)^2)*sinh(x)^2 + 8*((a^10 + 4*a^9*b + 3*a^8*b^2 - 8*a^7*b^3 - 14*a^6*b^4
+ 14*a^4*b^6 + 8*a^3*b^7 - 3*a^2*b^8 - 4*a*b^9 - b^10)*cosh(x)^7 + 3*(a^10
+ 2*a^9*b - 3*a^8*b^2 - 8*a^7*b^3 + 2*a^6*b^4 + 12*a^5*b^5 + 2*a^4*b^6 - 8*
a^3*b^7 - 3*a^2*b^8 + 2*a*b^9 + b^10)*cosh(x)^5 + 3*(a^10 - 5*a^8*b^2 + 10*
a^6*b^4 - 10*a^4*b^6 + 5*a^2*b^8 - b^10)*cosh(x)^3 + (a^10 - 2*a^9*b - 3*a^
8*b^2 + 8*a^7*b^3 + 2*a^6*b^4 - 12*a^5*b^5 + 2*a^4*b^6 + 8*a^3*b^7 - 3*a^2*
b^8 - 2*a*b^9 + b^10)*cosh(x))*sinh(x)), 1/4*(3*(a^5 + 3*a^4*b + 2*a^3*b^2
 - 2*a^2*b^3 - 3*a*b^4 - b^5)*cosh(x)^7 + 21*(a^5 + 3*a^4*b + 2*a^3*b^2 - 2*
a^2*b^3 - 3*a*b^4 - b^5)*cosh(x)*sinh(x)^6 + 3*(a^5 + 3*a^4*b + 2*a^3*b^2 -
 2*a^2*b^3 - 3*a*b^4 - b^5)*sinh(x)^7 + 11*(a^5 + a^4*b - 2*a^3*b^2 - 2*a^2
*b^3 + a*b^4 + b^5)*cosh(x)^5 + (11*a^5 + 11*a^4*b - 22*a^3*b^2 - 22*a^2*b^
3 + 11*a*b^4 + 11*b^5 + 63*(a^5 + 3*a^4*b + 2*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4
 - b^5)*cosh(x)^2)*sinh(x)^5 + 5*(21*(a^5 + 3*a^4*b + 2*a^3*b^2 - 2*a^2*b^3
 - 3*a*b^4 - b^5)*cosh(x)^3 + 11*(a^5 + a^4*b - 2*a^3*b^2 - 2*a^2*b^3 + a*b
^4 + b^5)*cosh(x))*sinh(x)^4 - 11*(a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*
b^4 - b^5)*cosh(x)^3 - (11*a^5 - 11*a^4*b - 22*a^3*b^2 + 22*a^2*b^3 + 11*a*
b^4 - 11*b^5 - 105*(a^5 + 3*a^4*b + 2*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^5)*
cosh(x)^4 - 110*(a^5 + a^4*b - 2*a^3*b^2 - 2*a^2*b^3 + a*b^4 + b^5)*cosh(x)
```

```
^2)*sinh(x)^3 + (63*(a^5 + 3*a^4*b + 2*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^5)
*cosh(x)^5 + 110*(a^5 + a^4*b - 2*a^3*b^2 - 2*a^2*b^3 + a*b^4 + b^5)*cosh(x
)^3 - 33*(a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x))*sinh(
x)^2 - 3*((a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*cosh(x)^8 + 8*(a^4 +
4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*cosh(x)*sinh(x)^7 + (a^4 + 4*a^3*b + 6
*a^2*b^2 + 4*a*b^3 + b^4)*sinh(x)^8 + 4*(a^4 + 2*a^3*b - 2*a*b^3 - b^4)*cos
h(x)^6 + 4*(a^4 + 2*a^3*b - 2*a*b^3 - b^4 + 7*(a^4 + 4*a^3*b + 6*a^2*b^2 +
4*a*b^3 + b^4)*cosh(x)^2)*sinh(x)^6 + 8*(7*(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a
*b^3 + b^4)*cosh(x)^3 + 3*(a^4 + 2*a^3*b - 2*a*b^3 - b^4)*cosh(x))*sinh(x)^
5 + 6*(a^4 - 2*a^2*b^2 + b^4)*cosh(x)^4 + 2*(35*(a^4 + 4*a^3*b + 6*a^2*b^2
+ 4*a*b^3 + b^4)*cosh(x)^4 + 3*a^4 - 6*a^2*b^2 + 3*b^4 + 30*(a^4 + 2*a^3*b
- 2*a*b^3 - b^4)*cosh(x)^2)*sinh(x)^4 + a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3
 + b^4 + 8*(7*(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*cosh(x)^5 + 10*(a
^4 + 2*a^3*b - 2*a*b^3 - b^4)*cosh(x)^3 + 3*(a^4 - 2*a^2*b^2 + b^4)*cosh(x)
)*sinh(x)^3 + 4*(a^4 - 2*a^3*b + 2*a*b^3 - b^4)*cosh(x)^2 + 4*(7*(a^4 + 4*a
^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*cosh(x)^6 + 15*(a^4 + 2*a^3*b - 2*a*b^3 -
 b^4)*cosh(x)^4 + a^4 - 2*a^3*b + 2*a*b^3 - b^4 + 9*(a^4 - 2*a^2*b^2 + b^4)
*cosh(x)^2)*sinh(x)^2 + 8*((a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*cosh
(x)^7 + 3*(a^4 + 2*a^3*b - 2*a*b^3 - b^4)*cosh(x)^5 + 3*(a^4 - 2*a^2*b^2 +
b^4)*cosh(x)^3 + (a^4 - 2*a^3*b + 2*a*b^3 - b^4)*cosh(x))*sinh(x))*sqrt(a^2
 - b^2)*arctan(sqrt(a^2 - b^2)/((a + b)*cosh(x) + (a + b)*sinh(x))) - 3*(a^
5 - 3*a^4*b + 2*a^3*b^2 + 2*a^2*b^3 - 3*a*b^4 + b^5)*cosh(x) + (21*(a^5 + 3
*a^4*b + 2*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^5)*cosh(x)^6 - 3*a^5 + 9*a^4*b
 - 6*a^3*b^2 - 6*a^2*b^3 + 9*a*b^4 - 3*b^5 + 55*(a^5 + a^4*b - 2*a^3*b^2 -
2*a^2*b^3 + a*b^4 + b^5)*cosh(x)^4 - 33*(a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^
3 + a*b^4 - b^5)*cosh(x)^2)*sinh(x))/(a^10 - 4*a^9*b + 3*a^8*b^2 + 8*a^7*b^
3 - 14*a^6*b^4 + 14*a^4*b^6 - 8*a^3*b^7 - 3*a^2*b^8 + 4*a*b^9 - b^10 + (a^1
0 + 4*a^9*b + 3*a^8*b^2 - 8*a^7*b^3 - 14*a^6*b^4 + 14*a^4*b^6 + 8*a^3*b^7 -
 3*a^2*b^8 - 4*a*b^9 - b^10)*cosh(x)^8 + 8*(a^10 + 4*a^9*b + 3*a^8*b^2 - 8*
a^7*b^3 - 14*a^6*b^4 + 14*a^4*b^6 + 8*a^3*b^7 - 3*a^2*b^8 - 4*a*b^9 - b^10)
*cosh(x)*sinh(x)^7 + (a^10 + 4*a^9*b + 3*a^8*b^2 - 8*a^7*b^3 - 14*a^6*b^4 +
 14*a^4*b^6 + 8*a^3*b^7 - 3*a^2*b^8 - 4*a*b^9 - b^10)*sinh(x)^8 + 4*(a^10 +
 2*a^9*b - 3*a^8*b^2 - 8*a^7*b^3 + 2*a^6*b^4 + 12*a^5*b^5 + 2*a^4*b^6 - 8*a
^3*b^7 - 3*a^2*b^8 + 2*a*b^9 + b^10)*cosh(x)^6 + 4*(a^10 + 2*a^9*b - 3*a^8*
b^2 - 8*a^7*b^3 + 2*a^6*b^4 + 12*a^5*b^5 + 2*a^4*b^6 - 8*a^3*b^7 - 3*a^2*b^
8 + 2*a*b^9 + b^10 + 7*(a^10 + 4*a^9*b + 3*a^8*b^2 - 8*a^7*b^3 - 14*a^6*b^4
 + 14*a^4*b^6 + 8*a^3*b^7 - 3*a^2*b^8 - 4*a*b^9 - b^10)*cosh(x)^2)*sinh(x)^
6 + 8*(7*(a^10 + 4*a^9*b + 3*a^8*b^2 - 8*a^7*b^3 - 14*a^6*b^4 + 14*a^4*b^6
+ 8*a^3*b^7 - 3*a^2*b^8 - 4*a*b^9 - b^10)*cosh(x)^3 + 3*(a^10 + 2*a^9*b - 3
*a^8*b^2 - 8*a^7*b^3 + 2*a^6*b^4 + 12*a^5*b^5 + 2*a^4*b^6 - 8*a^3*b^7 - 3*a
^2*b^8 + 2*a*b^9 + b^10)*cosh(x))*sinh(x)^5 + 6*(a^10 - 5*a^8*b^2 + 10*a^6*
b^4 - 10*a^4*b^6 + 5*a^2*b^8 - b^10)*cosh(x)^4 + 2*(3*a^10 - 15*a^8*b^2 + 3
0*a^6*b^4 - 30*a^4*b^6 + 15*a^2*b^8 - 3*b^10 + 35*(a^10 + 4*a^9*b + 3*a^8*b
^2 - 8*a^7*b^3 - 14*a^6*b^4 + 14*a^4*b^6 + 8*a^3*b^7 - 3*a^2*b^8 - 4*a*b^9
- b^10)*cosh(x)^4 + 30*(a^10 + 2*a^9*b - 3*a^8*b^2 - 8*a^7*b^3 + 2*a^6*b^4
```

```
+ 12*a^5*b^5 + 2*a^4*b^6 - 8*a^3*b^7 - 3*a^2*b^8 + 2*a*b^9 + b^10)*cosh(x)^
2)*sinh(x)^4 + 8*(7*(a^10 + 4*a^9*b + 3*a^8*b^2 - 8*a^7*b^3 - 14*a^6*b^4 +
14*a^4*b^6 + 8*a^3*b^7 - 3*a^2*b^8 - 4*a*b^9 - b^10)*cosh(x)^5 + 10*(a^10 +
 2*a^9*b - 3*a^8*b^2 - 8*a^7*b^3 + 2*a^6*b^4 + 12*a^5*b^5 + 2*a^4*b^6 - 8*a
^3*b^7 - 3*a^2*b^8 + 2*a*b^9 + b^10)*cosh(x)^3 + 3*(a^10 - 5*a^8*b^2 + 10*a
^6*b^4 - 10*a^4*b^6 + 5*a^2*b^8 - b^10)*cosh(x))*sinh(x)^3 + 4*(a^10 - 2*a^
9*b - 3*a^8*b^2 + 8*a^7*b^3 + 2*a^6*b^4 - 12*a^5*b^5 + 2*a^4*b^6 + 8*a^3*b^
7 - 3*a^2*b^8 - 2*a*b^9 + b^10)*cosh(x)^2 + 4*(a^10 - 2*a^9*b - 3*a^8*b^2 +
 8*a^7*b^3 + 2*a^6*b^4 - 12*a^5*b^5 + 2*a^4*b^6 + 8*a^3*b^7 - 3*a^2*b^8 - 2
*a*b^9 + b^10 + 7*(a^10 + 4*a^9*b + 3*a^8*b^2 - 8*a^7*b^3 - 14*a^6*b^4 + 14
*a^4*b^6 + 8*a^3*b^7 - 3*a^2*b^8 - 4*a*b^9 - b^10)*cosh(x)^6 + 15*(a^10 + 2
*a^9*b - 3*a^8*b^2 - 8*a^7*b^3 + 2*a^6*b^4 + 12*a^5*b^5 + 2*a^4*b^6 - 8*a^3
*b^7 - 3*a^2*b^8 + 2*a*b^9 + b^10)*cosh(x)^4 + 9*(a^10 - 5*a^8*b^2 + 10*a^6
*b^4 - 10*a^4*b^6 + 5*a^2*b^8 - b^10)*cosh(x)^2)*sinh(x)^2 + 8*((a^10 + 4*a
^9*b + 3*a^8*b^2 - 8*a^7*b^3 - 14*a^6*b^4 + 14*a^4*b^6 + 8*a^3*b^7 - 3*a^2*
b^8 - 4*a*b^9 - b^10)*cosh(x)^7 + 3*(a^10 + 2*a^9*b - 3*a^8*b^2 - 8*a^7*b^3
 + 2*a^6*b^4 + 12*a^5*b^5 + 2*a^4*b^6 - 8*a^3*b^7 - 3*a^2*b^8 + 2*a*b^9 + b
^10)*cosh(x)^5 + 3*(a^10 - 5*a^8*b^2 + 10*a^6*b^4 - 10*a^4*b^6 + 5*a^2*b^8
- b^10)*cosh(x)^3 + (a^10 - 2*a^9*b - 3*a^8*b^2 + 8*a^7*b^3 + 2*a^6*b^4 - 1
2*a^5*b^5 + 2*a^4*b^6 + 8*a^3*b^7 - 3*a^2*b^8 - 2*a*b^9 + b^10)*cosh(x))*si
nh(x))]
```

giac [B] time = 0.16, size = 236, normalized size = 2.11

$$\frac{3\,\arctan\left(\frac{ae^{x}+be^{x}}{\sqrt{a^2-b^2}}\right)}{4\left(a^4-2\,a^2b^2+b^4\right)\sqrt{a^2-b^2}}+\frac{3\,a^3e^{(7\,x)}+9\,a^2be^{(7\,x)}+9\,ab^2e^{(7\,x)}+3\,b^3e^{(7\,x)}+11\,a^3e^{(5\,x)}+11\,a^2be^{(5\,x)}-11\,ab^2e^{(5}}{4\left(a^4-2\,a^2b\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cosh(x)+b*sinh(x))^5,x, algorithm="giac")
```

```
[Out] 3/4*arctan((a*e^x + b*e^x)/sqrt(a^2 - b^2))/((a^4 - 2*a^2*b^2 + b^4)*sqrt(a
^2 - b^2)) + 1/4*(3*a^3*e^(7*x) + 9*a^2*b*e^(7*x) + 9*a*b^2*e^(7*x) + 3*b^3
*e^(7*x) + 11*a^3*e^(5*x) + 11*a^2*b*e^(5*x) - 11*a*b^2*e^(5*x) - 11*b^3*e^
(5*x) - 11*a^3*e^(3*x) + 11*a^2*b*e^(3*x) + 11*a*b^2*e^(3*x) - 11*b^3*e^(3*
x) - 3*a^3*e^x + 9*a^2*b*e^x - 9*a*b^2*e^x + 3*b^3*e^x)/((a^4 - 2*a^2*b^2 +
 b^4)*(a*e^(2*x) + b*e^(2*x) + a - b)^4)
```

maple [B] time = 0.30, size = 462, normalized size = 4.12

$$\frac{-\frac{\left(5a^4-16a^2b^2+8b^4\right)\left(\tanh^7\left(\frac{x}{2}\right)\right)}{4a\left(a^4-2a^2b^2+b^4\right)}-\frac{3b\left(a^4-16a^2b^2+8b^4\right)\left(\tanh^6\left(\frac{x}{2}\right)\right)}{4\left(a^4-2a^2b^2+b^4\right)a^2}+\frac{\left(3a^6+36a^4b^2+56a^2b^4-32b^6\right)\left(\tanh^5\left(\frac{x}{2}\right)\right)}{4a^3\left(a^4-2a^2b^2+b^4\right)}+\frac{b\left(15a^6+114a^4b^2-8a^2b^4-16b^6\right)}{4a^4\left(a^4-2a^2b^2+b^4\right)}}{\left(a+2\tanh\left(\frac{x}{2}\right)b\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*cosh(x)+b*sinh(x))^5,x)
```

```
[Out] 2*(-1/8*(5*a^4-16*a^2*b^2+8*b^4)/a/(a^4-2*a^2*b^2+b^4)*tanh(1/2*x)^7-3/8*b*
(a^4-16*a^2*b^2+8*b^4)/(a^4-2*a^2*b^2+b^4)/a^2*tanh(1/2*x)^6+1/8/a^3*(3*a^6
+36*a^4*b^2+56*a^2*b^4-32*b^6)/(a^4-2*a^2*b^2+b^4)*tanh(1/2*x)^5+1/8/a^4*b*
(15*a^6+114*a^4*b^2-8*a^2*b^4-16*b^6)/(a^4-2*a^2*b^2+b^4)*tanh(1/2*x)^4-1/8
/a^3*(3*a^6-84*a^4*b^2-56*a^2*b^4+32*b^6)/(a^4-2*a^2*b^2+b^4)*tanh(1/2*x)^3
+1/8*b*(23*a^4+64*a^2*b^2-24*b^4)/(a^4-2*a^2*b^2+b^4)/a^2*tanh(1/2*x)^2+1/8
*(5*a^4+24*a^2*b^2-8*b^4)/a/(a^4-2*a^2*b^2+b^4)*tanh(1/2*x)+1/8*(5*a^2-2*b^
2)*b/(a^4-2*a^2*b^2+b^4))/(a+2*tanh(1/2*x)*b+a*tanh(1/2*x)^2)^4+3/4/(a^4-2*
a^2*b^2+b^4)/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tanh(1/2*x)+2*b)/(a^2-b^2)^(1/
2))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cosh(x)+b*sinh(x))^5,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [B] time = 1.64, size = 354, normalized size = 3.16

$$\frac{3\,\mathrm{atan}\left(\frac{e^x\,\sqrt{a^{10}-5\,a^8\,b^2+10\,a^6\,b^4-10\,a^4\,b^6+5\,a^2\,b^8-b^{10}}}{a^5-a^4\,b-2\,a^3\,b^2+2\,a^2\,b^3+a\,b^4-b^5}\right)}{4\,\sqrt{a^{10}-5\,a^8\,b^2+10\,a^6\,b^4-10\,a^4\,b^6+5\,a^2\,b^8-b^{10}}}-\frac{4\,e^{3\,x}}{(a+b)\,\left(e^{8\,x}\,(a+b)^4+(a-b)^4+4\,e^{2\,x}\,(a+b)\,(a-b)^3+4\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*cosh(x) + b*sinh(x))^5,x)
```

```
[Out] (3*atan((exp(x)*(a^10 - b^10 + 5*a^2*b^8 - 10*a^4*b^6 + 10*a^6*b^4 - 5*a^8*
b^2)^(1/2))/(a*b^4 - a^4*b + a^5 - b^5 + 2*a^2*b^3 - 2*a^3*b^2)))/(4*(a^10
- b^10 + 5*a^2*b^8 - 10*a^4*b^6 + 10*a^6*b^4 - 5*a^8*b^2)^(1/2)) - (4*exp(3
*x))/((a + b)*(exp(8*x)*(a + b)^4 + (a - b)^4 + 4*exp(2*x)*(a + b)*(a - b)^
3 + 4*exp(6*x)*(a + b)^3*(a - b) + 6*exp(4*x)*(a + b)^2*(a - b)^2)) - (2*ex
p(x))/((a + b)^2*(exp(6*x)*(a + b)^3 + (a - b)^3 + 3*exp(2*x)*(a + b)*(a -
b)^2 + 3*exp(4*x)*(a + b)^2*(a - b))) + (3*exp(x))/(4*(a + b)^2*(a - b)^2*(
a - b + exp(2*x)*(a + b))) + exp(x)/(2*(a + b)^2*(a - b)*(exp(4*x)*(a + b)^
2 + (a - b)^2 + 2*exp(2*x)*(a + b)*(a - b)))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cosh(x)+b*sinh(x))**5,x)
```

```
[Out] Timed out
```

3.590 $\int \sqrt{a\cosh(x)+b\sinh(x)}\,dx$

Optimal. Leaf size=65

$$-\frac{2i\sqrt{a\cosh(x)+b\sinh(x)}\,E\left(\frac{1}{2}\left(ix-\tan^{-1}(a,-ib)\right)\middle|2\right)}{\sqrt{\frac{a\cosh(x)+b\sinh(x)}{\sqrt{a^2-b^2}}}}$$

```
[Out] -2*I*(cos(1/2*I*x-1/2*arctan(a,-I*b))^2)^(1/2)/cos(1/2*I*x-1/2*arctan(a,-I*
b))*EllipticE(sin(1/2*I*x-1/2*arctan(a,-I*b)),2^(1/2))*(a*cosh(x)+b*sinh(x)
)^(1/2)/((a*cosh(x)+b*sinh(x))/(a^2-b^2)^(1/2))^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3078, 2639}

$$-\frac{2i\sqrt{a\cosh(x)+b\sinh(x)}\,E\left(\frac{1}{2}\left(ix-\tan^{-1}(a,-ib)\right)\middle|2\right)}{\sqrt{\frac{a\cosh(x)+b\sinh(x)}{\sqrt{a^2-b^2}}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*Cosh[x] + b*Sinh[x]],x]
```

```
[Out] ((-2*I)*EllipticE[(I*x - ArcTan[a, (-I)*b])/2, 2]*Sqrt[a*Cosh[x] + b*Sinh[x
]])/Sqrt[(a*Cosh[x] + b*Sinh[x])/Sqrt[a^2 - b^2]]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3078

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Dist[(a*Cos[c + d*x] + b*Sin[c + d*x])^n/((a*Cos[c + d*x] + b*S
in[c + d*x])/Sqrt[a^2 + b^2])^n, Int[Cos[c + d*x - ArcTan[a, b]]^n, x], x]
/; FreeQ[{a, b, c, d, n}, x] && !(GeQ[n, 1] || LeQ[n, -1]) && !(GtQ[a^2 +
 b^2, 0] || EqQ[a^2 + b^2, 0])
```

Rubi steps

$$\int \sqrt{a\cosh(x)+b\sinh(x)}\,dx = \frac{\sqrt{a\cosh(x)+b\sinh(x)}\int\sqrt{\cosh\left(x+i\tan^{-1}(a,-ib)\right)}\,dx}{\sqrt{\frac{a\cosh(x)+b\sinh(x)}{\sqrt{a^2-b^2}}}}$$
$$= -\frac{2iE\left(\frac{1}{2}\left(ix-\tan^{-1}(a,-ib)\right)\middle|2\right)\sqrt{a\cosh(x)+b\sinh(x)}}{\sqrt{\frac{a\cosh(x)+b\sinh(x)}{\sqrt{a^2-b^2}}}}$$

Mathematica [C] time = 0.76, size = 206, normalized size = 3.17

$$\frac{b\left(b^2-a^2\right)\sinh\left(\tanh^{-1}\left(\frac{b}{a}\right)+x\right)\,{}_2F_1\left(-\frac{1}{2},-\frac{1}{4};\frac{3}{4};\cosh^2\left(x+\tanh^{-1}\left(\frac{b}{a}\right)\right)\right)+\sqrt{-\sinh^2\left(\tanh^{-1}\left(\frac{b}{a}\right)+x\right)}\left(2a^2b\right.}{ab\sqrt{1-\frac{b^2}{a^2}}\sqrt{-\sinh}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[a*Cosh[x] + b*Sinh[x]],x]
```

```
[Out] (b*(-a^2 + b^2)*HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cosh[x + ArcTanh[b/a
]]^2]*Sinh[x + ArcTanh[b/a]] + Sqrt[-Sinh[x + ArcTanh[b/a]]^2]*(2*a^3*Sqrt[
1 - b^2/a^2]*Cosh[x] - 2*a*(a^2 - b^2)*Cosh[x + ArcTanh[b/a]] + 2*a^2*b*Sqr
t[1 - b^2/a^2]*Sinh[x] + a^2*b*Sinh[x + ArcTanh[b/a]] - b^3*Sinh[x + ArcTan
h[b/a]]))/(a*b*Sqrt[1 - b^2/a^2]*Sqrt[a*Cosh[x] + b*Sinh[x]]*Sqrt[-Sinh[x +
 ArcTanh[b/a]]^2])
```

fricas [F] time = 0.42, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{a\cosh(x)+b\sinh(x)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cosh(x)+b*sinh(x))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a*cosh(x) + b*sinh(x)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{a\cosh(x)+b\sinh(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cosh(x)+b*sinh(x))^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(a*cosh(x) + b*sinh(x)), x)
```

maple [A] time = 0.40, size = 33, normalized size = 0.51

$$-\frac{\sqrt{a^2-b^2}\ \cosh(x)}{\sqrt{-\sinh(x)\sqrt{a^2-b^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cosh(x)+b*sinh(x))^(1/2),x)
[Out] -(a^2-b^2)^(1/2)/(-sinh(x)*(a^2-b^2)^(1/2))^(1/2)*cosh(x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a\cosh(x) + b\sinh(x)}\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cosh(x)+b*sinh(x))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(a*cosh(x) + b*sinh(x)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \sqrt{a\cosh(x) + b\sinh(x)}\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cosh(x) + b*sinh(x))^(1/2),x)
[Out] int((a*cosh(x) + b*sinh(x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a\cosh\left(x\right) + b\sinh\left(x\right)}\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cosh(x)+b*sinh(x))**(1/2),x)
[Out] Integral(sqrt(a*cosh(x) + b*sinh(x)), x)
```

3.591 $\int (a\cosh(x) + b\sinh(x))^{3/2}\,dx$

Optimal. Leaf size=103

$$\frac{2}{3}(a\sinh(x)+b\cosh(x))\sqrt{a\cosh(x)+b\sinh(x)}-\frac{2i\left(a^2-b^2\right)\sqrt{\frac{a\cosh(x)+b\sinh(x)}{\sqrt{a^2-b^2}}}\,F\left(\frac{1}{2}\left(ix-\tan^{-1}(a,-ib)\right)\middle|2\right)}{3\sqrt{a\cosh(x)+b\sinh(x)}}$$

```
[Out] 2/3*(b*cosh(x)+a*sinh(x))*(a*cosh(x)+b*sinh(x))^(1/2)-2/3*I*(a^2-b^2)*(cos(
1/2*I*x-1/2*arctan(a,-I*b))^2)^(1/2)/cos(1/2*I*x-1/2*arctan(a,-I*b))*Ellipt
icF(sin(1/2*I*x-1/2*arctan(a,-I*b)),2^(1/2))*((a*cosh(x)+b*sinh(x))/(a^2-b^
2)^(1/2))^(1/2)/(a*cosh(x)+b*sinh(x))^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {3073, 3078, 2641}

$$\frac{2}{3}(a\sinh(x)+b\cosh(x))\sqrt{a\cosh(x)+b\sinh(x)}-\frac{2i\left(a^2-b^2\right)\sqrt{\frac{a\cosh(x)+b\sinh(x)}{\sqrt{a^2-b^2}}}\,F\left(\frac{1}{2}\left(ix-\tan^{-1}(a,-ib)\right)\middle|2\right)}{3\sqrt{a\cosh(x)+b\sinh(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cosh[x] + b*Sinh[x])^(3/2),x]
```

```
[Out] (2*(b*Cosh[x] + a*Sinh[x])*Sqrt[a*Cosh[x] + b*Sinh[x]])/3 - (((2*I)/3)*(a^2
 - b^2)*EllipticF[(I*x - ArcTan[a, (-I)*b])/2, 2]*Sqrt[(a*Cosh[x] + b*Sinh[
x])/Sqrt[a^2 - b^2]])/Sqrt[a*Cosh[x] + b*Sinh[x]]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3073

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> -Simp[((b*Cos[c + d*x] - a*Sin[c + d*x])*(a*Cos[c + d*x] + b*Si
n[c + d*x])^(n - 1))/(d*n), x] + Dist[((n - 1)*(a^2 + b^2))/n, Int[(a*Cos[c
 + d*x] + b*Sin[c + d*x])^(n - 2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 + b^2, 0] &&  !IntegerQ[(n - 1)/2] && GtQ[n, 1]
```

Rule 3078

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Dist[(a*Cos[c + d*x] + b*Sin[c + d*x])^n/((a*Cos[c + d*x] + b*S
```

```
in[c + d*x])/Sqrt[a^2 + b^2])^n, Int[Cos[c + d*x - ArcTan[a, b]]^n, x], x]
/; FreeQ[{a, b, c, d, n}, x] &&  !(GeQ[n, 1] || LeQ[n, -1]) &&  !(GtQ[a^2 +
 b^2, 0] || EqQ[a^2 + b^2, 0])
```

Rubi steps

$$\int (a\cosh(x)+b\sinh(x))^{3/2}\,dx = \frac{2}{3}(b\cosh(x)+a\sinh(x))\sqrt{a\cosh(x)+b\sinh(x)} + \frac{1}{3}\left(a^2-b^2\right)\int \frac{1}{\sqrt{a\cosh(x)}}$$

$$= \frac{2}{3}(b\cosh(x)+a\sinh(x))\sqrt{a\cosh(x)+b\sinh(x)} + \frac{\left(\left(a^2-b^2\right)\sqrt{\frac{a\cosh(x)+b\sinh(x)}{\sqrt{a^2-b^2}}}\right.}{3\sqrt{a\cosh(x)}}$$

$$= \frac{2}{3}(b\cosh(x)+a\sinh(x))\sqrt{a\cosh(x)+b\sinh(x)} - \frac{2i\left(a^2-b^2\right)F\left(\frac{1}{2}\left(ix-\tan^{-1}\right.\right.}{3\sqrt{a\cosh(}}$$

Mathematica [C] time = 0.60, size = 92, normalized size = 0.89

$$\frac{2}{3}\sqrt{a\cosh(x)+b\sinh(x)}\left(-b\sqrt{1-\frac{a^2}{b^2}}\sqrt{\cosh^2\left(\tanh^{-1}\left(\frac{a}{b}\right)+x\right)}\operatorname{sech}\left(\tanh^{-1}\left(\frac{a}{b}\right)+x\right){}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\sinh^2\left(x\right.\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cosh[x] + b*Sinh[x])^(3/2),x]
```

```
[Out] (2*(b*Cosh[x] - Sqrt[1 - a^2/b^2]*b*Sqrt[Cosh[x + ArcTanh[a/b]]^2]*Hypergeo
metricPFQ[{1/4, 1/2}, {5/4}, -Sinh[x + ArcTanh[a/b]]^2]*Sech[x + ArcTanh[a/
b]] + a*Sinh[x])*Sqrt[a*Cosh[x] + b*Sinh[x]])/3
```

fricas [F] time = 0.41, size = 0, normalized size = 0.00

$$\text{integral}\left((a\cosh(x)+b\sinh(x))^{\frac{3}{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cosh(x)+b*sinh(x))^(3/2),x, algorithm="fricas")
```

```
[Out] integral((a*cosh(x) + b*sinh(x))^(3/2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\cosh(x)+b\sinh(x))^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cosh(x)+b*sinh(x))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a*cosh(x) + b*sinh(x))^(3/2), x)
```

maple [A] time = 0.60, size = 171, normalized size = 1.66

$$-\frac{\sqrt{-\sqrt{a^2-b^2}\,\left(\sinh^3(x)\right)}\,\left(\cosh(x)\sqrt{-\sqrt{a^2-b^2}\,\left(\sinh^3(x)\right)}\,\sqrt{\sinh(x)\sqrt{a^2-b^2}}\,\left(a^2-b^2\right)+\sinh(x)\left(a^2-b^2\right)^{\frac{3}{2}}\cdots\right.}{2\sinh(x)^2\sqrt{a^2-b^2}\,\sqrt{\sinh(x)\sqrt{a^2-b^2}}\,\sqrt{-\sinh(x)\sqrt{a^2-b^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cosh(x)+b*sinh(x))^(3/2),x)
```

```
[Out] -1/2*(-(a^2-b^2)^(1/2)*sinh(x)^3)^(1/2)*(cosh(x)*(-(a^2-b^2)^(1/2)*sinh(x)^
3)^(1/2)*(sinh(x)*(a^2-b^2)^(1/2))^(1/2)*(a^2-b^2)+sinh(x)*(a^2-b^2)^(3/2)*
arctan((sinh(x)*(a^2-b^2)^(1/2))^(1/2)*cosh(x)/(-(a^2-b^2)^(1/2)*sinh(x)^3)
^(1/2)))/sinh(x)^2/(a^2-b^2)^(1/2)/(sinh(x)*(a^2-b^2)^(1/2))^(1/2)/(-sinh(x
)*(a^2-b^2)^(1/2))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\cosh(x)+b\sinh(x))^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cosh(x)+b*sinh(x))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a*cosh(x) + b*sinh(x))^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (a\cosh(x)+b\sinh(x))^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cosh(x) + b*sinh(x))^(3/2),x)
```

```
[Out] int((a*cosh(x) + b*sinh(x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\cosh(x)+b\sinh(x))^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cosh(x)+b*sinh(x))**(3/2),x)
```

```
[Out] Integral((a*cosh(x) + b*sinh(x))**(3/2), x)
```

3.592 $\int (a\cosh(x)+b\sinh(x))^{5/2}\,dx$

Optimal. Leaf size=103

$$\frac{2}{5}(a\sinh(x)+b\cosh(x))(a\cosh(x)+b\sinh(x))^{3/2}-\frac{6i\left(a^2-b^2\right)\sqrt{a\cosh(x)+b\sinh(x)}\,E\left(\frac{1}{2}\left(ix-\tan^{-1}(a,-ib)\right)\middle|2\right)}{5\sqrt{\frac{a\cosh(x)+b\sinh(x)}{\sqrt{a^2-b^2}}}}$$

```
[Out] 2/5*(b*cosh(x)+a*sinh(x))*(a*cosh(x)+b*sinh(x))^(3/2)-6/5*I*(a^2-b^2)*(cos(
1/2*I*x-1/2*arctan(a,-I*b))^2)^(1/2)/cos(1/2*I*x-1/2*arctan(a,-I*b))*Ellipt
icE(sin(1/2*I*x-1/2*arctan(a,-I*b)),2^(1/2))*(a*cosh(x)+b*sinh(x))^(1/2)/((
a*cosh(x)+b*sinh(x))/(a^2-b^2)^(1/2))^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {3073, 3078, 2639}

$$\frac{2}{5}(a\sinh(x)+b\cosh(x))(a\cosh(x)+b\sinh(x))^{3/2}-\frac{6i\left(a^2-b^2\right)\sqrt{a\cosh(x)+b\sinh(x)}\,E\left(\frac{1}{2}\left(ix-\tan^{-1}(a,-ib)\right)\middle|2\right)}{5\sqrt{\frac{a\cosh(x)+b\sinh(x)}{\sqrt{a^2-b^2}}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cosh[x] + b*Sinh[x])^(5/2),x]
```

```
[Out] (2*(b*Cosh[x] + a*Sinh[x])*(a*Cosh[x] + b*Sinh[x])^(3/2))/5 - (((6*I)/5)*(a
^2 - b^2)*EllipticE[(I*x - ArcTan[a, (-I)*b])/2, 2]*Sqrt[a*Cosh[x] + b*Sinh
[x]])/Sqrt[(a*Cosh[x] + b*Sinh[x])/Sqrt[a^2 - b^2]]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3073

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> -Simp[((b*Cos[c + d*x] - a*Sin[c + d*x])*(a*Cos[c + d*x] + b*Si
n[c + d*x])^(n - 1))/(d*n), x] + Dist[((n - 1)*(a^2 + b^2))/n, Int[(a*Cos[c
 + d*x] + b*Sin[c + d*x])^(n - 2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 + b^2, 0] &&  !IntegerQ[(n - 1)/2] && GtQ[n, 1]
```

Rule 3078

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Dist[(a*Cos[c + d*x] + b*Sin[c + d*x])^n/((a*Cos[c + d*x] + b*S
in[c + d*x])/Sqrt[a^2 + b^2])^n, Int[Cos[c + d*x - ArcTan[a, b]]^n, x], x]
/; FreeQ[{a, b, c, d, n}, x] &&  !(GeQ[n, 1] || LeQ[n, -1]) &&  !(GtQ[a^2 +
 b^2, 0] || EqQ[a^2 + b^2, 0])
```

Rubi steps

$$\int (a\cosh(x)+b\sinh(x))^{5/2}\,dx = \frac{2}{5}(b\cosh(x)+a\sinh(x))(a\cosh(x)+b\sinh(x))^{3/2}+\frac{1}{5}\left(3\left(a^2-b^2\right)\right)\int \sqrt{a\cosh}$$

$$= \frac{2}{5}(b\cosh(x)+a\sinh(x))(a\cosh(x)+b\sinh(x))^{3/2}+\frac{\left(3\left(a^2-b^2\right)\sqrt{a\cosh(x)+}\right.}{}$$

$$= \frac{2}{5}(b\cosh(x)+a\sinh(x))(a\cosh(x)+b\sinh(x))^{3/2}-\frac{6i\left(a^2-b^2\right)E\left(\frac{1}{2}\left(ix-\tan\right.\right.}{5\sqrt{}}$$

Mathematica [C] time = 0.86, size = 193, normalized size = 1.87

$$\frac{(a\cosh(x)+b\sinh(x))\left(b\left(a^2+b^2\right)\sinh(2x)+6a\left(a^2-b^2\right)+2ab^2\cosh(2x)\right)-\frac{3(a-b)^2(a+b)^2\left(b\sinh\left(\tanh^{-1}\left(\frac{b}{a}\right)+x\right){}_2F_1\left(-\right.\right.}{}}{5b\sqrt{a\cosh(x)+b\sinh(x)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a*Cosh[x] + b*Sinh[x])^(5/2),x]
```

```
[Out] ((a*Cosh[x] + b*Sinh[x])*(6*a*(a^2 - b^2) + 2*a*b^2*Cosh[2*x] + b*(a^2 + b^
2)*Sinh[2*x]) - (3*(a - b)^2*(a + b)^2*(b*HypergeometricPFQ[{-1/2, -1/4}, {
3/4}, Cosh[x + ArcTanh[b/a]]^2]*Sinh[x + ArcTanh[b/a]] + Sqrt[-Sinh[x + Arc
Tanh[b/a]]^2]*(2*a*Cosh[x + ArcTanh[b/a]] - b*Sinh[x + ArcTanh[b/a]])))/(a*
Sqrt[1 - b^2/a^2]*Sqrt[-Sinh[x + ArcTanh[b/a]]^2]))/(5*b*Sqrt[a*Cosh[x] + b
*Sinh[x]])
```

fricas [F] time = 0.43, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(a^2\cosh(x)^2+2\,ab\cosh(x)\sinh(x)+b^2\sinh(x)^2\right)\sqrt{a\cosh(x)+b\sinh(x)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*cosh(x)+b*sinh(x))^(5/2),x, algorithm="fricas")

[Out] integral((a^2*cosh(x)^2 + 2*a*b*cosh(x)*sinh(x) + b^2*sinh(x)^2)*sqrt(a*cos
h(x) + b*sinh(x)), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\cosh(x) + b\sinh(x))^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*cosh(x)+b*sinh(x))^(5/2),x, algorithm="giac")

[Out] integrate((a*cosh(x) + b*sinh(x))^(5/2), x)

maple [A] time = 0.41, size = 51, normalized size = 0.50

$$\frac{-\frac{(a^2-b^2)^{\frac{3}{2}}(\cosh^3(x))}{3} + (a^2-b^2)^{\frac{3}{2}}\cosh(x)}{\sqrt{-\sinh(x)\sqrt{a^2-b^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a*cosh(x)+b*sinh(x))^(5/2),x)

[Out] 1/(-sinh(x)*(a^2-b^2)^(1/2))^(1/2)*(-1/3*(a^2-b^2)^(3/2)*cosh(x)^3+(a^2-b^2
)^(3/2)*cosh(x))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\cosh(x) + b\sinh(x))^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*cosh(x)+b*sinh(x))^(5/2),x, algorithm="maxima")

[Out] integrate((a*cosh(x) + b*sinh(x))^(5/2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (a\cosh(x) + b\sinh(x))^{5/2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a*cosh(x) + b*sinh(x))^(5/2),x)

```
[Out] int((a*cosh(x) + b*sinh(x))^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cosh(x)+b*sinh(x))**(5/2),x)
```

```
[Out] Timed out
```

3.593 $\int \frac{1}{\sqrt{a\cosh(x)+b\sinh(x)}}\,dx$

Optimal. Leaf size=65

$$-\frac{2i\sqrt{\frac{a\cosh(x)+b\sinh(x)}{\sqrt{a^2-b^2}}}\,F\left(\frac{1}{2}\left(ix-\tan^{-1}(a,-ib)\right)\middle|2\right)}{\sqrt{a\cosh(x)+b\sinh(x)}}$$

```
[Out] -2*I*(cos(1/2*I*x-1/2*arctan(a,-I*b))^2)^(1/2)/cos(1/2*I*x-1/2*arctan(a,-I*
b))*EllipticF(sin(1/2*I*x-1/2*arctan(a,-I*b)),2^(1/2))*((a*cosh(x)+b*sinh(x
))/(a^2-b^2)^(1/2))^(1/2)/(a*cosh(x)+b*sinh(x))^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3078, 2641}

$$-\frac{2i\sqrt{\frac{a\cosh(x)+b\sinh(x)}{\sqrt{a^2-b^2}}}\,F\left(\frac{1}{2}\left(ix-\tan^{-1}(a,-ib)\right)\middle|2\right)}{\sqrt{a\cosh(x)+b\sinh(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a*Cosh[x] + b*Sinh[x]],x]
```

```
[Out] ((-2*I)*EllipticF[(I*x - ArcTan[a, (-I)*b])/2, 2]*Sqrt[(a*Cosh[x] + b*Sinh[
x])/Sqrt[a^2 - b^2]])/Sqrt[a*Cosh[x] + b*Sinh[x]]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3078

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Dist[(a*Cos[c + d*x] + b*Sin[c + d*x])^n/((a*Cos[c + d*x] + b*S
in[c + d*x])/Sqrt[a^2 + b^2])^n, Int[Cos[c + d*x - ArcTan[a, b]]^n, x], x]
/; FreeQ[{a, b, c, d, n}, x] &&  !(GeQ[n, 1] || LeQ[n, -1]) &&  !(GtQ[a^2 +
 b^2, 0] || EqQ[a^2 + b^2, 0])
```

Rubi steps

$$\int \frac{1}{\sqrt{a\cosh(x)+b\sinh(x)}}\,dx = \frac{\sqrt{\frac{a\cosh(x)+b\sinh(x)}{\sqrt{a^2-b^2}}}\int \frac{1}{\sqrt{\cosh\left(x+i\tan^{-1}(a,-ib)\right)}}\,dx}{\sqrt{a\cosh(x)+b\sinh(x)}}$$
$$= -\frac{2iF\left(\frac{1}{2}\left(ix-\tan^{-1}(a,-ib)\right)\middle|2\right)\sqrt{\frac{a\cosh(x)+b\sinh(x)}{\sqrt{a^2-b^2}}}}{\sqrt{a\cosh(x)+b\sinh(x)}}$$

Mathematica [C] time = 0.10, size = 81, normalized size = 1.25

$$\frac{2\sqrt{a\cosh(x)+b\sinh(x)}\sqrt{\cosh^2\left(\tanh^{-1}\left(\frac{a}{b}\right)+x\right)}\operatorname{sech}\left(\tanh^{-1}\left(\frac{a}{b}\right)+x\right){}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\sinh^2\left(x+\tanh^{-1}\left(\frac{a}{b}\right)\right)\right)}{b\sqrt{1-\frac{a^2}{b^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a*Cosh[x] + b*Sinh[x]],x]
```

```
[Out] (2*Sqrt[Cosh[x + ArcTanh[a/b]]^2]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, -Sin
h[x + ArcTanh[a/b]]^2]*Sech[x + ArcTanh[a/b]]*Sqrt[a*Cosh[x] + b*Sinh[x]])/
(Sqrt[1 - a^2/b^2]*b)
```

fricas [F] time = 0.42, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{\sqrt{a\cosh(x)+b\sinh(x)}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cosh(x)+b*sinh(x))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(1/sqrt(a*cosh(x) + b*sinh(x)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a\cosh(x)+b\sinh(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cosh(x)+b*sinh(x))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(a*cosh(x) + b*sinh(x)), x)
```

maple [A] time = 0.41, size = 97, normalized size = 1.49

$$\frac{\sqrt{-\sqrt{a^2-b^2}\,\left(\sinh^3(x)\right)}\,\arctan\left(\frac{\sqrt{\sinh(x)\sqrt{a^2-b^2}}\,\cosh(x)}{\sqrt{-\sqrt{a^2-b^2}\,\left(\sinh^3(x)\right)}}\right)}{\sqrt{\sinh(x)\sqrt{a^2-b^2}}\,\sinh(x)\sqrt{-\sinh(x)\sqrt{a^2-b^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*cosh(x)+b*sinh(x))^(1/2),x)
```

```
[Out] (-(a^2-b^2)^(1/2)*sinh(x)^3)^(1/2)/(sinh(x)*(a^2-b^2)^(1/2))^(1/2)*arctan((
sinh(x)*(a^2-b^2)^(1/2))^(1/2)*cosh(x)/(-(a^2-b^2)^(1/2)*sinh(x)^3)^(1/2))/
sinh(x)/(-sinh(x)*(a^2-b^2)^(1/2))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a\cosh(x)+b\sinh(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cosh(x)+b*sinh(x))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(a*cosh(x) + b*sinh(x)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sqrt{a\cosh(x)+b\sinh(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*cosh(x) + b*sinh(x))^(1/2),x)
```

```
[Out] int(1/(a*cosh(x) + b*sinh(x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a\cosh{(x)}+b\sinh{(x)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cosh(x)+b*sinh(x))**(1/2),x)
```

```
[Out] Integral(1/sqrt(a*cosh(x) + b*sinh(x)), x)
```

3.594 $\int \frac{1}{(a\cosh(x)+b\sinh(x))^{3/2}}\,dx$

Optimal. Leaf size=112

$$\frac{2(a\sinh(x)+b\cosh(x))}{(a^2-b^2)\sqrt{a\cosh(x)+b\sinh(x)}}+\frac{2i\sqrt{a\cosh(x)+b\sinh(x)}\,E\left(\frac{1}{2}\left(ix-\tan^{-1}(a,-ib)\right)\middle|2\right)}{(a^2-b^2)\sqrt{\frac{a\cosh(x)+b\sinh(x)}{\sqrt{a^2-b^2}}}}$$

```
[Out] 2*(b*cosh(x)+a*sinh(x))/(a^2-b^2)/(a*cosh(x)+b*sinh(x))^(1/2)+2*I*(cos(1/2*
I*x-1/2*arctan(a,-I*b))^2)^(1/2)/cos(1/2*I*x-1/2*arctan(a,-I*b))*EllipticE(
sin(1/2*I*x-1/2*arctan(a,-I*b)),2^(1/2))*(a*cosh(x)+b*sinh(x))^(1/2)/(a^2-b
^2)/((a*cosh(x)+b*sinh(x))/(a^2-b^2)^(1/2))^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {3076, 3078, 2639}

$$\frac{2(a\sinh(x)+b\cosh(x))}{(a^2-b^2)\sqrt{a\cosh(x)+b\sinh(x)}}+\frac{2i\sqrt{a\cosh(x)+b\sinh(x)}\,E\left(\frac{1}{2}\left(ix-\tan^{-1}(a,-ib)\right)\middle|2\right)}{(a^2-b^2)\sqrt{\frac{a\cosh(x)+b\sinh(x)}{\sqrt{a^2-b^2}}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cosh[x] + b*Sinh[x])^(-3/2),x]
```

```
[Out] (2*(b*Cosh[x] + a*Sinh[x]))/((a^2 - b^2)*Sqrt[a*Cosh[x] + b*Sinh[x]]) + ((2
*I)*EllipticE[(I*x - ArcTan[a, (-I)*b])/2, 2]*Sqrt[a*Cosh[x] + b*Sinh[x]])/
((a^2 - b^2)*Sqrt[(a*Cosh[x] + b*Sinh[x])/Sqrt[a^2 - b^2]])
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3076

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Simp[((b*Cos[c + d*x] - a*Sin[c + d*x])*(a*Cos[c + d*x] + b*Sin
[c + d*x])^(n + 1))/(d*(n + 1)*(a^2 + b^2)), x] + Dist[(n + 2)/((n + 1)*(a^
2 + b^2)), Int[(a*Cos[c + d*x] + b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{
a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && LtQ[n, -1] && NeQ[n, -2]
```

Rule 3078

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Dist[(a*Cos[c + d*x] + b*Sin[c + d*x])^n/((a*Cos[c + d*x] + b*S
in[c + d*x])/Sqrt[a^2 + b^2])^n, Int[Cos[c + d*x - ArcTan[a, b]]^n, x], x]
/; FreeQ[{a, b, c, d, n}, x] && !(GeQ[n, 1] || LeQ[n, -1]) && !(GtQ[a^2 +
 b^2, 0] || EqQ[a^2 + b^2, 0])
```

Rubi steps

$$\int \frac{1}{(a\cosh(x)+b\sinh(x))^{3/2}}\,dx = \frac{2(b\cosh(x)+a\sinh(x))}{\left(a^2-b^2\right)\sqrt{a\cosh(x)+b\sinh(x)}}+\frac{\int \sqrt{a\cosh(x)+b\sinh(x)}\,dx}{-a^2+b^2}$$

$$= \frac{2(b\cosh(x)+a\sinh(x))}{\left(a^2-b^2\right)\sqrt{a\cosh(x)+b\sinh(x)}}+\frac{\sqrt{a\cosh(x)+b\sinh(x)}\int \sqrt{\cosh\left(x+i\right.}}{\left(-a^2+b^2\right)\sqrt{\frac{a\cosh(x)+b\sinh}{\sqrt{a^2-b^2}}}}$$

$$= \frac{2(b\cosh(x)+a\sinh(x))}{\left(a^2-b^2\right)\sqrt{a\cosh(x)+b\sinh(x)}}+\frac{2iE\left(\frac{1}{2}\left(ix-\tan^{-1}(a,-ib)\right)\middle|2\right)\sqrt{a\cosh(}}{\left(a^2-b^2\right)\sqrt{\frac{a\cosh(x)+b\sinh(x)}{\sqrt{a^2-b^2}}}}$$

Mathematica [C] time = 0.46, size = 148, normalized size = 1.32

$$\frac{b\sinh\left(\tanh^{-1}\left(\frac{b}{a}\right)+x\right){}_2F_1\left(-\frac{1}{2},-\frac{1}{4};\frac{3}{4};\cosh^2\left(x+\tanh^{-1}\left(\frac{b}{a}\right)\right)\right)-\sqrt{-\sinh^2\left(\tanh^{-1}\left(\frac{b}{a}\right)+x\right)}\left(2a\sqrt{1-\frac{b^2}{a^2}}\cos\right.}{ab\sqrt{1-\frac{b^2}{a^2}}\sqrt{-\sinh^2\left(\tanh^{-1}\left(\frac{b}{a}\right)+x\right)}\sqrt{a\cosh(x)+b\sin}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a*Cosh[x] + b*Sinh[x])^(-3/2),x]
```

```
[Out] (b*HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cosh[x + ArcTanh[b/a]]^2]*Sinh[x
+ ArcTanh[b/a]] - Sqrt[-Sinh[x + ArcTanh[b/a]]^2]*(2*a*Sqrt[1 - b^2/a^2]*Co
sh[x] - 2*a*Cosh[x + ArcTanh[b/a]] + b*Sinh[x + ArcTanh[b/a]]))/(a*b*Sqrt[1
 - b^2/a^2]*Sqrt[a*Cosh[x] + b*Sinh[x]]*Sqrt[-Sinh[x + ArcTanh[b/a]]^2])
```

fricas [F] time = 0.43, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\sqrt{a\cosh(x)+b\sinh(x)}}{a^2\cosh(x)^2+2\,ab\cosh(x)\sinh(x)+b^2\sinh(x)^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cosh(x)+b*sinh(x))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a*cosh(x) + b*sinh(x))/(a^2*cosh(x)^2 + 2*a*b*cosh(x)*sinh(x)
 + b^2*sinh(x)^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a\cosh(x)+b\sinh(x))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cosh(x)+b*sinh(x))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a*cosh(x) + b*sinh(x))^(-3/2), x)
```

maple [A] time = 0.34, size = 33, normalized size = 0.29

$$\frac{\operatorname{arctanh}(\cosh(x))}{\sqrt{a^2-b^2}\,\sqrt{-\sinh(x)\sqrt{a^2-b^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*cosh(x)+b*sinh(x))^(3/2),x)
```

```
[Out] 1/(a^2-b^2)^(1/2)/(-sinh(x)*(a^2-b^2)^(1/2))^(1/2)*arctanh(cosh(x))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a\cosh(x)+b\sinh(x))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cosh(x)+b*sinh(x))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a*cosh(x) + b*sinh(x))^(-3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(a\cosh(x)+b\sinh(x))^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*cosh(x) + b*sinh(x))^(3/2),x)
```

```
[Out] int(1/(a*cosh(x) + b*sinh(x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a\cosh(x) + b\sinh(x))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cosh(x)+b*sinh(x))**(3/2),x)
```

```
[Out] Integral((a*cosh(x) + b*sinh(x))**(-3/2), x)
```

3.595 $\int \frac{1}{(a\cosh(x)+b\sinh(x))^{5/2}}\,dx$

Optimal. Leaf size=116

$$\frac{2(a\sinh(x)+b\cosh(x))}{3\left(a^2-b^2\right)(a\cosh(x)+b\sinh(x))^{3/2}}-\frac{2i\sqrt{\frac{a\cosh(x)+b\sinh(x)}{\sqrt{a^2-b^2}}}\,F\left(\frac{1}{2}\left(ix-\tan^{-1}(a,-ib)\right)\middle|2\right)}{3\left(a^2-b^2\right)\sqrt{a\cosh(x)+b\sinh(x)}}$$

```
[Out] 2/3*(b*cosh(x)+a*sinh(x))/(a^2-b^2)/(a*cosh(x)+b*sinh(x))^(3/2)-2/3*I*(cos(
1/2*I*x-1/2*arctan(a,-I*b))^2)^(1/2)/cos(1/2*I*x-1/2*arctan(a,-I*b))*Ellipt
icF(sin(1/2*I*x-1/2*arctan(a,-I*b)),2^(1/2))*((a*cosh(x)+b*sinh(x))/(a^2-b^
2)^(1/2))^(1/2)/(a^2-b^2)/(a*cosh(x)+b*sinh(x))^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {3076, 3078, 2641}

$$\frac{2(a\sinh(x)+b\cosh(x))}{3\left(a^2-b^2\right)(a\cosh(x)+b\sinh(x))^{3/2}}-\frac{2i\sqrt{\frac{a\cosh(x)+b\sinh(x)}{\sqrt{a^2-b^2}}}\,F\left(\frac{1}{2}\left(ix-\tan^{-1}(a,-ib)\right)\middle|2\right)}{3\left(a^2-b^2\right)\sqrt{a\cosh(x)+b\sinh(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cosh[x] + b*Sinh[x])^(-5/2),x]
```

```
[Out] (2*(b*Cosh[x] + a*Sinh[x]))/(3*(a^2 - b^2)*(a*Cosh[x] + b*Sinh[x])^(3/2)) -
 (((2*I)/3)*EllipticF[(I*x - ArcTan[a, (-I)*b])/2, 2]*Sqrt[(a*Cosh[x] + b*S
inh[x])/Sqrt[a^2 - b^2]])/((a^2 - b^2)*Sqrt[a*Cosh[x] + b*Sinh[x]])
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3076

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Simp[((b*Cos[c + d*x] - a*Sin[c + d*x])*(a*Cos[c + d*x] + b*Sin
[c + d*x])^(n + 1))/(d*(n + 1)*(a^2 + b^2)), x] + Dist[(n + 2)/((n + 1)*(a^
2 + b^2)), Int[(a*Cos[c + d*x] + b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{
a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && LtQ[n, -1] && NeQ[n, -2]
```

Rule 3078

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Dist[(a*Cos[c + d*x] + b*Sin[c + d*x])^n/((a*Cos[c + d*x] + b*S
in[c + d*x])/Sqrt[a^2 + b^2])^n, Int[Cos[c + d*x - ArcTan[a, b]]^n, x], x]
/; FreeQ[{a, b, c, d, n}, x] && !(GeQ[n, 1] || LeQ[n, -1]) && !(GtQ[a^2 +
 b^2, 0] || EqQ[a^2 + b^2, 0])
```

Rubi steps

$$\int \frac{1}{(a\cosh(x)+b\sinh(x))^{5/2}}\,dx = \frac{2(b\cosh(x)+a\sinh(x))}{3\left(a^2-b^2\right)(a\cosh(x)+b\sinh(x))^{3/2}}+\frac{\int \frac{1}{\sqrt{a\cosh(x)+b\sinh(x)}}\,dx}{3\left(a^2-b^2\right)}$$

$$= \frac{2(b\cosh(x)+a\sinh(x))}{3\left(a^2-b^2\right)(a\cosh(x)+b\sinh(x))^{3/2}}+\frac{\sqrt{\frac{a\cosh(x)+b\sinh(x)}{\sqrt{a^2-b^2}}}\int \frac{1}{\sqrt{\cosh\left(x+i\tan^{-1}(a,\right.}}}{3\left(a^2-b^2\right)\sqrt{a\cosh(x)+b\sinh(}}$$

$$= \frac{2(b\cosh(x)+a\sinh(x))}{3\left(a^2-b^2\right)(a\cosh(x)+b\sinh(x))^{3/2}}-\frac{2iF\left(\frac{1}{2}\left(ix-\tan^{-1}(a,-ib)\right)\middle|2\right)\sqrt{\frac{a\cos}{}}}{3\left(a^2-b^2\right)\sqrt{a\cosh(x)+b\sin}}$$

Mathematica [C] time = 0.59, size = 133, normalized size = 1.15

$$-\frac{2\left((a\cosh(x)+b\sinh(x))^2\sqrt{\cosh^2\left(\tanh^{-1}\left(\frac{a}{b}\right)+x\right)}\operatorname{sech}\left(\tanh^{-1}\left(\frac{a}{b}\right)+x\right){}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\sinh^2\left(x+\tanh^{-1}\left(\right.\right.\right.\right.}{3b\sqrt{1-\frac{a^2}{b^2}}(b-a)(a+b)(a\cosh(x)+b\sinh(x))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cosh[x] + b*Sinh[x])^(-5/2),x]
```

```
[Out] (-2*(Sqrt[1 - a^2/b^2]*b*(b*Cosh[x] + a*Sinh[x]) + Sqrt[Cosh[x + ArcTanh[a/
b]]^2]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, -Sinh[x + ArcTanh[a/b]]^2]*Sech
[x + ArcTanh[a/b]]*(a*Cosh[x] + b*Sinh[x])^2))/(3*Sqrt[1 - a^2/b^2]*b*(-a +
 b)*(a + b)*(a*Cosh[x] + b*Sinh[x])^(3/2))
```

fricas [F] time = 0.42, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{a\cosh(x)+b\sinh(x)}}{a^3\cosh(x)^3+3\,a^2b\cosh(x)^2\sinh(x)+3\,ab^2\cosh(x)\sinh(x)^2+b^3\sinh(x)^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cosh(x)+b*sinh(x))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a*cosh(x) + b*sinh(x))/(a^3*cosh(x)^3 + 3*a^2*b*cosh(x)^2*sin
h(x) + 3*a*b^2*cosh(x)*sinh(x)^2 + b^3*sinh(x)^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a\cosh(x) + b\sinh(x))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cosh(x)+b*sinh(x))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((a*cosh(x) + b*sinh(x))^(-5/2), x)
```

maple [A] time = 0.46, size = 37, normalized size = 0.32

$$-\frac{\cosh(x)}{(a^2 - b^2)\sinh(x)\sqrt{-\sinh(x)\sqrt{a^2 - b^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*cosh(x)+b*sinh(x))^(5/2),x)
```

```
[Out] -cosh(x)/(a^2-b^2)/sinh(x)/(-sinh(x)*(a^2-b^2)^(1/2))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a\cosh(x) + b\sinh(x))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cosh(x)+b*sinh(x))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((a*cosh(x) + b*sinh(x))^(-5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(a\cosh(x) + b\sinh(x))^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*cosh(x) + b*sinh(x))^(5/2),x)
```

```
[Out] int(1/(a*cosh(x) + b*sinh(x))^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cosh(x)+b*sinh(x))**(5/2),x)
```

```
[Out] Timed out
```

3.596 $\int (a\cosh(c+dx)+a\sinh(c+dx))\,dx$

Optimal. Leaf size=23

$$\frac{a\sinh(c+dx)}{d}+\frac{a\cosh(c+dx)}{d}$$

```
[Out] a*cosh(d*x+c)/d+a*sinh(d*x+c)/d
```

Rubi [A] time = 0.02, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2637, 2638}

$$\frac{a\sinh(c+dx)}{d}+\frac{a\cosh(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[a*Cosh[c + d*x] + a*Sinh[c + d*x],x]
```

```
[Out] (a*Cosh[c + d*x])/d + (a*Sinh[c + d*x])/d
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int (a\cosh(c+dx)+a\sinh(c+dx))\,dx &= a\int \cosh(c+dx)\,dx+a\int \sinh(c+dx)\,dx\\ &= \frac{a\cosh(c+dx)}{d}+\frac{a\sinh(c+dx)}{d}\end{aligned}$$

Mathematica [A] time = 0.02, size = 45, normalized size = 1.96

$$\frac{a\sinh(c)\sinh(dx)}{d}+\frac{a\cosh(c)\cosh(dx)}{d}+\frac{a\sinh(c)\cosh(dx)}{d}+\frac{a\cosh(c)\sinh(dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[a*Cosh[c + d*x] + a*Sinh[c + d*x],x]
```

```
[Out] (a*Cosh[c]*Cosh[d*x])/d + (a*Cosh[d*x]*Sinh[c])/d + (a*Cosh[c]*Sinh[d*x])/d
 + (a*Sinh[c]*Sinh[d*x])/d
```

fricas [A] time = 0.41, size = 21, normalized size = 0.91

$$\frac{a\cosh(dx+c)+a\sinh(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a*cosh(d*x+c)+a*sinh(d*x+c),x, algorithm="fricas")
```

```
[Out] (a*cosh(d*x + c) + a*sinh(d*x + c))/d
```

giac [B] time = 0.13, size = 56, normalized size = 2.43

$$\frac{1}{2}a\left(\frac{e^{(dx+c)}}{d}+\frac{e^{(-dx-c)}}{d}\right)+\frac{1}{2}a\left(\frac{e^{(dx+c)}}{d}-\frac{e^{(-dx-c)}}{d}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a*cosh(d*x+c)+a*sinh(d*x+c),x, algorithm="giac")
```

```
[Out] 1/2*a*(e^(d*x + c)/d + e^(-d*x - c)/d) + 1/2*a*(e^(d*x + c)/d - e^(-d*x - c
)/d)
```

maple [A] time = 0.02, size = 19, normalized size = 0.83

$$\frac{a(\cosh(dx+c)+\sinh(dx+c))}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a*cosh(d*x+c)+a*sinh(d*x+c),x)
```

```
[Out] a*(cosh(d*x+c)+sinh(d*x+c))/d
```

maxima [A] time = 0.31, size = 23, normalized size = 1.00

$$\frac{a\cosh(dx+c)}{d}+\frac{a\sinh(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a*cosh(d*x+c)+a*sinh(d*x+c),x, algorithm="maxima")
```

```
[Out] a*cosh(d*x + c)/d + a*sinh(d*x + c)/d
```

mupad [B] time = 0.06, size = 11, normalized size = 0.48

$$\frac{a\,\mathrm{e}^{c+d\,x}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a*cosh(c + d*x) + a*sinh(c + d*x),x)
```

```
[Out] (a*exp(c + d*x))/d
```

sympy [A] time = 0.16, size = 29, normalized size = 1.26

$$a\left(\begin{cases}\frac{\sinh(c+dx)}{d} & \text{for}\ d\neq 0\\ x\cosh(c) & \text{otherwise}\end{cases}\right)+a\left(\begin{cases}\frac{\cosh(c+dx)}{d} & \text{for}\ d\neq 0\\ x\sinh(c) & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a*cosh(d*x+c)+a*sinh(d*x+c),x)
```

```
[Out] a*Piecewise((sinh(c + d*x)/d, Ne(d, 0)), (x*cosh(c), True)) + a*Piecewise((
cosh(c + d*x)/d, Ne(d, 0)), (x*sinh(c), True))
```

3.597 $\int (a\cosh(c+dx)+a\sinh(c+dx))^2\,dx$

Optimal. Leaf size=26

$$\frac{(a\sinh(c+dx)+a\cosh(c+dx))^2}{2d}$$

```
[Out] 1/2*(a*cosh(d*x+c)+a*sinh(d*x+c))^2/d
```

Rubi [A] time = 0.02, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {3071}

$$\frac{(a\sinh(c+dx)+a\cosh(c+dx))^2}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cosh[c + d*x] + a*Sinh[c + d*x])^2,x]
```

```
[Out] (a*Cosh[c + d*x] + a*Sinh[c + d*x])^2/(2*d)
```

Rule 3071

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Simp[(a*(a*Cos[c + d*x] + b*Sin[c + d*x])^n)/(b*d*n), x] /; Fre
eQ[{a, b, c, d, n}, x] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\int (a\cosh(c+dx)+a\sinh(c+dx))^2\,dx=\frac{(a\cosh(c+dx)+a\sinh(c+dx))^2}{2d}$$

Mathematica [A] time = 0.05, size = 25, normalized size = 0.96

$$\frac{a^2(\sinh(c+dx)+\cosh(c+dx))^2}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cosh[c + d*x] + a*Sinh[c + d*x])^2,x]
```

```
[Out] (a^2*(Cosh[c + d*x] + Sinh[c + d*x])^2)/(2*d)
```

fricas [A] time = 0.41, size = 43, normalized size = 1.65

$$\frac{a^2\cosh(dx+c)+a^2\sinh(dx+c)}{2\,(d\cosh(dx+c)-d\sinh(dx+c))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cosh(d*x+c)+a*sinh(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/2*(a^2*cosh(d*x + c) + a^2*sinh(d*x + c))/(d*cosh(d*x + c) - d*sinh(d*x +
 c))
```

giac [A] time = 0.12, size = 17, normalized size = 0.65

$$\frac{a^2e^{(2\,dx+2\,c)}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cosh(d*x+c)+a*sinh(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/2*a^2*e^(2*d*x + 2*c)/d
```

maple [A] time = 0.02, size = 24, normalized size = 0.92

$$\frac{a^2\,(\cosh(dx+c)+\sinh(dx+c))^2}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cosh(d*x+c)+a*sinh(d*x+c))^2,x)
```

```
[Out] 1/2*a^2*(cosh(d*x+c)+sinh(d*x+c))^2/d
```

maxima [B] time = 0.30, size = 88, normalized size = 3.38

$$\frac{1}{8}\,a^2\left(4\,x+\frac{e^{(2\,dx+2\,c)}}{d}-\frac{e^{(-2\,dx-2\,c)}}{d}\right)-\frac{1}{8}\,a^2\left(4\,x-\frac{e^{(2\,dx+2\,c)}}{d}+\frac{e^{(-2\,dx-2\,c)}}{d}\right)+\frac{a^2\cosh(dx+c)^2}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cosh(d*x+c)+a*sinh(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/8*a^2*(4*x + e^(2*d*x + 2*c)/d - e^(-2*d*x - 2*c)/d) - 1/8*a^2*(4*x - e^(
2*d*x + 2*c)/d + e^(-2*d*x - 2*c)/d) + a^2*cosh(d*x + c)^2/d
```

mupad [B] time = 0.07, size = 17, normalized size = 0.65

$$\frac{a^2\,\mathrm{e}^{2\,c+2\,d\,x}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cosh(c + d*x) + a*sinh(c + d*x))^2,x)
```

```
[Out] (a^2*exp(2*c + 2*d*x))/(2*d)
```

sympy [**A**] time = 0.21, size = 44, normalized size = 1.69

$$\begin{cases} \frac{a^2\sinh^2(c+dx)}{d} + \frac{a^2\sinh(c+dx)\cosh(c+dx)}{d} & \text{for } d \neq 0 \\ x\,(a\sinh(c) + a\cosh(c))^2 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cosh(d*x+c)+a*sinh(d*x+c))**2,x)
```

```
[Out] Piecewise((a**2*sinh(c + d*x)**2/d + a**2*sinh(c + d*x)*cosh(c + d*x)/d, Ne
(d, 0)), (x*(a*sinh(c) + a*cosh(c))**2, True))
```

3.598 $\int (a\cosh(c+dx)+a\sinh(c+dx))^3\,dx$

Optimal. Leaf size=26

$$\frac{(a\sinh(c+dx)+a\cosh(c+dx))^3}{3d}$$

```
[Out] 1/3*(a*cosh(d*x+c)+a*sinh(d*x+c))^3/d
```

Rubi [A] time = 0.02, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {3071}

$$\frac{(a\sinh(c+dx)+a\cosh(c+dx))^3}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cosh[c + d*x] + a*Sinh[c + d*x])^3,x]
```

```
[Out] (a*Cosh[c + d*x] + a*Sinh[c + d*x])^3/(3*d)
```

Rule 3071

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Simp[(a*(a*Cos[c + d*x] + b*Sin[c + d*x])^n)/(b*d*n), x] /; Fre
eQ[{a, b, c, d, n}, x] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\int (a\cosh(c+dx)+a\sinh(c+dx))^3\,dx=\frac{(a\cosh(c+dx)+a\sinh(c+dx))^3}{3d}$$

Mathematica [A] time = 0.08, size = 25, normalized size = 0.96

$$\frac{a^3(\sinh(c+dx)+\cosh(c+dx))^3}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cosh[c + d*x] + a*Sinh[c + d*x])^3,x]
```

```
[Out] (a^3*(Cosh[c + d*x] + Sinh[c + d*x])^3)/(3*d)
```

fricas [B] time = 0.40, size = 64, normalized size = 2.46

$$\frac{a^3\cosh(dx+c)^2 + 2\,a^3\cosh(dx+c)\sinh(dx+c) + a^3\sinh(dx+c)^2}{3\,(d\cosh(dx+c) - d\sinh(dx+c))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cosh(d*x+c)+a*sinh(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/3*(a^3*cosh(d*x + c)^2 + 2*a^3*cosh(d*x + c)*sinh(d*x + c) + a^3*sinh(d*x
 + c)^2)/(d*cosh(d*x + c) - d*sinh(d*x + c))
```

giac [A] time = 0.14, size = 17, normalized size = 0.65

$$\frac{a^3e^{(3\,dx+3\,c)}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cosh(d*x+c)+a*sinh(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/3*a^3*e^(3*d*x + 3*c)/d
```

maple [A] time = 0.02, size = 24, normalized size = 0.92

$$\frac{a^3\,(\cosh(dx+c) + \sinh(dx+c))^3}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cosh(d*x+c)+a*sinh(d*x+c))^3,x)
```

```
[Out] 1/3*a^3*(cosh(d*x+c)+sinh(d*x+c))^3/d
```

maxima [B] time = 0.31, size = 146, normalized size = 5.62

$$\frac{a^3\cosh(dx+c)^3}{d} + \frac{a^3\sinh(dx+c)^3}{d} + \frac{1}{24}\,a^3\left(\frac{e^{(3\,dx+3\,c)}}{d} + \frac{9\,e^{(dx+c)}}{d} - \frac{9\,e^{(-dx-c)}}{d} - \frac{e^{(-3\,dx-3\,c)}}{d}\right) + \frac{1}{24}\,a^3\left(\frac{e^{(3\,dx+3\,c)}}{d} - \frac{9}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cosh(d*x+c)+a*sinh(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] a^3*cosh(d*x + c)^3/d + a^3*sinh(d*x + c)^3/d + 1/24*a^3*(e^(3*d*x + 3*c)/d
 + 9*e^(d*x + c)/d - 9*e^(-d*x - c)/d - e^(-3*d*x - 3*c)/d) + 1/24*a^3*(e^(
3*d*x + 3*c)/d - 9*e^(d*x + c)/d - 9*e^(-d*x - c)/d + e^(-3*d*x - 3*c)/d)
```

mupad [B] time = 0.07, size = 17, normalized size = 0.65

$$\frac{a^3\,\mathrm{e}^{3\,c+3\,d\,x}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cosh(c + d*x) + a*sinh(c + d*x))^3,x)
```

```
[Out] (a^3*exp(3*c + 3*d*x))/(3*d)
```

sympy [A] time = 0.46, size = 83, normalized size = 3.19

$$\begin{cases} \frac{a^3\sinh^3(c+dx)}{3d} + \frac{a^3\sinh^2(c+dx)\cosh(c+dx)}{d} + \frac{a^3\sinh(c+dx)\cosh^2(c+dx)}{d} + \frac{a^3\cosh^3(c+dx)}{3d} & \text{for}\ d \neq 0 \\ x\left(a\sinh(c) + a\cosh(c)\right)^3 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cosh(d*x+c)+a*sinh(d*x+c))**3,x)
```

```
[Out] Piecewise((a**3*sinh(c + d*x)**3/(3*d) + a**3*sinh(c + d*x)**2*cosh(c + d*x
)/d + a**3*sinh(c + d*x)*cosh(c + d*x)**2/d + a**3*cosh(c + d*x)**3/(3*d),
Ne(d, 0)), (x*(a*sinh(c) + a*cosh(c))**3, True))
```

3.599 $\int (a\cosh(c+dx)+a\sinh(c+dx))^n\,dx$

Optimal. Leaf size=26

$$\frac{(a\sinh(c+dx)+a\cosh(c+dx))^n}{dn}$$

```
[Out] (a*cosh(d*x+c)+a*sinh(d*x+c))^n/d/n
```

Rubi [A] time = 0.02, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {3071}

$$\frac{(a\sinh(c+dx)+a\cosh(c+dx))^n}{dn}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cosh[c + d*x] + a*Sinh[c + d*x])^n,x]
```

```
[Out] (a*Cosh[c + d*x] + a*Sinh[c + d*x])^n/(d*n)
```

Rule 3071

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Simp[(a*(a*Cos[c + d*x] + b*Sin[c + d*x])^n)/(b*d*n), x] /; Fre
eQ[{a, b, c, d, n}, x] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\int (a\cosh(c+dx)+a\sinh(c+dx))^n\,dx=\frac{(a\cosh(c+dx)+a\sinh(c+dx))^n}{dn}$$

Mathematica [A] time = 0.08, size = 24, normalized size = 0.92

$$\frac{(a(\sinh(c+dx)+\cosh(c+dx)))^n}{dn}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cosh[c + d*x] + a*Sinh[c + d*x])^n,x]
```

```
[Out] (a*(Cosh[c + d*x] + Sinh[c + d*x]))^n/(d*n)
```

fricas [A] time = 0.42, size = 34, normalized size = 1.31

$$\frac{\cosh\left(dnx + cn + n\log(a)\right) + \sinh\left(dnx + cn + n\log(a)\right)}{dn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cosh(d*x+c)+a*sinh(d*x+c))^n,x, algorithm="fricas")
[Out] (cosh(d*n*x + c*n + n*log(a)) + sinh(d*n*x + c*n + n*log(a)))/(d*n)
```

giac [A] time = 0.12, size = 20, normalized size = 0.77

$$\frac{e^{\left(dnx+cn+n\log(a)\right)}}{dn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cosh(d*x+c)+a*sinh(d*x+c))^n,x, algorithm="giac")
[Out] e^(d*n*x + c*n + n*log(a))/(d*n)
```

maple [A] time = 0.02, size = 27, normalized size = 1.04

$$\frac{(a\cosh(dx+c) + a\sinh(dx+c))^n}{dn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cosh(d*x+c)+a*sinh(d*x+c))^n,x)
[Out] (a*cosh(d*x+c)+a*sinh(d*x+c))^n/d/n
```

maxima [A] time = 0.57, size = 18, normalized size = 0.69

$$\frac{a^n e^{((dx+c)n)}}{dn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cosh(d*x+c)+a*sinh(d*x+c))^n,x, algorithm="maxima")
[Out] a^n*e^((d*x + c)*n)/(d*n)
```

mupad [B] time = 1.63, size = 17, normalized size = 0.65

$$\frac{\left(a\,\mathrm{e}^{c+d\,x}\right)^n}{d\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cosh(c + d*x) + a*sinh(c + d*x))^n,x)
```

```
[Out] (a*exp(c + d*x))^n/(d*n)
```

sympy [A] time = 0.19, size = 36, normalized size = 1.38

$$\begin{cases} x & \text{for } d = 0 \wedge n = 0 \\ x \left(a \sinh{\left(c \right)} + a \cosh{\left(c \right)}\right)^{n} & \text{for } d = 0 \\ x & \text{for } n = 0 \\ \frac{\left(a \sinh{\left(c + d x \right)} + a \cosh{\left(c + d x \right)}\right)^{n}}{d n} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cosh(d*x+c)+a*sinh(d*x+c))**n,x)
```

```
[Out] Piecewise((x, Eq(d, 0) & Eq(n, 0)), (x*(a*sinh(c) + a*cosh(c))**n, Eq(d, 0)
), (x, Eq(n, 0)), ((a*sinh(c + d*x) + a*cosh(c + d*x))**n/(d*n), True))
```

3.600 $\int \frac{1}{a\cosh(c+dx)+a\sinh(c+dx)}\,dx$

Optimal. Leaf size=24

$$-\frac{1}{d(a\sinh(c+dx)+a\cosh(c+dx))}$$

[Out] -1/d/(a*cosh(d*x+c)+a*sinh(d*x+c))

Rubi [A] time = 0.02, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {3071}

$$-\frac{1}{d(a\sinh(c+dx)+a\cosh(c+dx))}$$

Antiderivative was successfully verified.

[In] Int[(a*Cosh[c + d*x] + a*Sinh[c + d*x])^(-1),x]

[Out] -(1/(d*(a*Cosh[c + d*x] + a*Sinh[c + d*x])))

Rule 3071

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Simp[(a*(a*Cos[c + d*x] + b*Sin[c + d*x])^n)/(b*d*n), x] /; Fre
eQ[{a, b, c, d, n}, x] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\int \frac{1}{a\cosh(c+dx)+a\sinh(c+dx)}\,dx = -\frac{1}{d(a\cosh(c+dx)+a\sinh(c+dx))}$$

Mathematica [A] time = 0.04, size = 24, normalized size = 1.00

$$-\frac{1}{d(a\sinh(c+dx)+a\cosh(c+dx))}$$

Antiderivative was successfully verified.

[In] Integrate[(a*Cosh[c + d*x] + a*Sinh[c + d*x])^(-1),x]

[Out] -(1/(d*(a*Cosh[c + d*x] + a*Sinh[c + d*x])))

fricas [A] time = 0.40, size = 23, normalized size = 0.96

$$-\frac{1}{ad\cosh\left(dx+c\right)+ad\sinh\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cosh(d*x+c)+a*sinh(d*x+c)),x, algorithm="fricas")
[Out] -1/(a*d*cosh(d*x + c) + a*d*sinh(d*x + c))
```

giac [A] time = 0.13, size = 17, normalized size = 0.71

$$-\frac{e^{(-dx-c)}}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cosh(d*x+c)+a*sinh(d*x+c)),x, algorithm="giac")
[Out] -e^(-d*x - c)/(a*d)
```

maple [A] time = 0.02, size = 24, normalized size = 1.00

$$-\frac{1}{da\left(\cosh\left(dx+c\right)+\sinh\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*cosh(d*x+c)+a*sinh(d*x+c)),x)
[Out] -1/d/a/(cosh(d*x+c)+sinh(d*x+c))
```

maxima [A] time = 0.31, size = 17, normalized size = 0.71

$$-\frac{e^{(-dx-c)}}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cosh(d*x+c)+a*sinh(d*x+c)),x, algorithm="maxima")
[Out] -e^(-d*x - c)/(a*d)
```

mupad [B] time = 0.07, size = 17, normalized size = 0.71

$$-\frac{\mathrm{e}^{-c-d\,x}}{a\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*cosh(c + d*x) + a*sinh(c + d*x)),x)
```

```
[Out] -exp(- c - d*x)/(a*d)
```

sympy [A] time = 0.45, size = 34, normalized size = 1.42

$$\begin{cases} -\frac{1}{ad\sinh(c+dx)+ad\cosh(c+dx)} & \text{for } d \neq 0 \\ \frac{x}{a\sinh(c)+a\cosh(c)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cosh(d*x+c)+a*sinh(d*x+c)),x)
```

```
[Out] Piecewise((-1/(a*d*sinh(c + d*x) + a*d*cosh(c + d*x)), Ne(d, 0)), (x/(a*sin
h(c) + a*cosh(c)), True))
```

3.601 $\int \frac{1}{(a\cosh(c+dx)+a\sinh(c+dx))^2}\,dx$

Optimal. Leaf size=26

$$-\frac{1}{2d(a\sinh(c+dx)+a\cosh(c+dx))^2}$$

```
[Out] -1/2/d/(a*cosh(d*x+c)+a*sinh(d*x+c))^2
```

Rubi [A] time = 0.02, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {3071}

$$-\frac{1}{2d(a\sinh(c+dx)+a\cosh(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cosh[c + d*x] + a*Sinh[c + d*x])^(-2),x]
```

```
[Out] -1/(2*d*(a*Cosh[c + d*x] + a*Sinh[c + d*x])^2)
```

Rule 3071

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Simp[(a*(a*Cos[c + d*x] + b*Sin[c + d*x])^n)/(b*d*n), x] /; Fre
eQ[{a, b, c, d, n}, x] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\int \frac{1}{(a\cosh(c+dx)+a\sinh(c+dx))^2}\,dx = -\frac{1}{2d(a\cosh(c+dx)+a\sinh(c+dx))^2}$$

Mathematica [A] time = 0.04, size = 26, normalized size = 1.00

$$-\frac{1}{2d(a\sinh(c+dx)+a\cosh(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cosh[c + d*x] + a*Sinh[c + d*x])^(-2),x]
```

```
[Out] -1/2*1/(d*(a*Cosh[c + d*x] + a*Sinh[c + d*x])^2)
```

fricas [B] time = 0.40, size = 49, normalized size = 1.88

$$-\frac{1}{2\left(a^2d\cosh\left(dx+c\right)^2+2\,a^2d\cosh\left(dx+c\right)\sinh\left(dx+c\right)+a^2d\sinh\left(dx+c\right)^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cosh(d*x+c)+a*sinh(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/2/(a^2*d*cosh(d*x + c)^2 + 2*a^2*d*cosh(d*x + c)*sinh(d*x + c) + a^2*d*s
inh(d*x + c)^2)
```

giac [A] time = 0.12, size = 17, normalized size = 0.65

$$-\frac{e^{(-2\,dx-2\,c)}}{2\,a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cosh(d*x+c)+a*sinh(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/2*e^(-2*d*x - 2*c)/(a^2*d)
```

maple [A] time = 0.03, size = 24, normalized size = 0.92

$$-\frac{1}{2d\,a^2\left(\cosh\left(dx+c\right)+\sinh\left(dx+c\right)\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*cosh(d*x+c)+a*sinh(d*x+c))^2,x)
```

```
[Out] -1/2/d/a^2/(cosh(d*x+c)+sinh(d*x+c))^2
```

maxima [A] time = 0.33, size = 17, normalized size = 0.65

$$-\frac{e^{(-2\,dx-2\,c)}}{2\,a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cosh(d*x+c)+a*sinh(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/2*e^(-2*d*x - 2*c)/(a^2*d)
```

mupad [B] time = 0.10, size = 17, normalized size = 0.65

$$-\frac{\mathrm{e}^{-2\,c-2\,d\,x}}{2\,a^2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*cosh(c + d*x) + a*sinh(c + d*x))^2,x)
```

```
[Out] -exp(- 2*c - 2*d*x)/(2*a^2*d)
```

sympy [A] time = 0.82, size = 66, normalized size = 2.54

$$\begin{cases} -\frac{1}{2a^2 d \sinh^2(c+dx)+4a^2 d \sinh(c+dx)\cosh(c+dx)+2a^2 d \cosh^2(c+dx)} & \text{for } d \neq 0 \\ \frac{x}{(a\sinh(c)+a\cosh(c))^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cosh(d*x+c)+a*sinh(d*x+c))**2,x)
```

```
[Out] Piecewise((-1/(2*a**2*d*sinh(c + d*x)**2 + 4*a**2*d*sinh(c + d*x)*cosh(c +
d*x) + 2*a**2*d*cosh(c + d*x)**2), Ne(d, 0)), (x/(a*sinh(c) + a*cosh(c))**2
, True))
```

3.602 $\int \frac{1}{(a\cosh(c+dx)+a\sinh(c+dx))^3}\,dx$

Optimal. Leaf size=26

$$-\frac{1}{3d(a\sinh(c+dx)+a\cosh(c+dx))^3}$$

```
[Out] -1/3/d/(a*cosh(d*x+c)+a*sinh(d*x+c))^3
```

Rubi [A] time = 0.02, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {3071}

$$-\frac{1}{3d(a\sinh(c+dx)+a\cosh(c+dx))^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cosh[c + d*x] + a*Sinh[c + d*x])^(-3),x]
```

```
[Out] -1/(3*d*(a*Cosh[c + d*x] + a*Sinh[c + d*x])^3)
```

Rule 3071

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Simp[(a*(a*Cos[c + d*x] + b*Sin[c + d*x])^n)/(b*d*n), x] /; Fre
eQ[{a, b, c, d, n}, x] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\int \frac{1}{(a\cosh(c+dx)+a\sinh(c+dx))^3}\,dx = -\frac{1}{3d(a\cosh(c+dx)+a\sinh(c+dx))^3}$$

Mathematica [A] time = 0.05, size = 26, normalized size = 1.00

$$-\frac{1}{3d(a\sinh(c+dx)+a\cosh(c+dx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cosh[c + d*x] + a*Sinh[c + d*x])^(-3),x]
```

```
[Out] -1/3*1/(d*(a*Cosh[c + d*x] + a*Sinh[c + d*x])^3)
```

fricas [B] time = 0.40, size = 71, normalized size = 2.73

$$-\frac{1}{3\left(a^3d\cosh\left(dx+c\right)^3+3\,a^3d\cosh\left(dx+c\right)^2\sinh\left(dx+c\right)+3\,a^3d\cosh\left(dx+c\right)\sinh\left(dx+c\right)^2+a^3d\sinh\left(dx+c\right)^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cosh(d*x+c)+a*sinh(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/3/(a^3*d*cosh(d*x + c)^3 + 3*a^3*d*cosh(d*x + c)^2*sinh(d*x + c) + 3*a^3
*d*cosh(d*x + c)*sinh(d*x + c)^2 + a^3*d*sinh(d*x + c)^3)
```

giac [A] time = 0.13, size = 17, normalized size = 0.65

$$-\frac{e^{(-3\,dx-3\,c)}}{3\,a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cosh(d*x+c)+a*sinh(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/3*e^(-3*d*x - 3*c)/(a^3*d)
```

maple [A] time = 0.02, size = 24, normalized size = 0.92

$$-\frac{1}{3d\,a^3\left(\cosh\left(dx+c\right)+\sinh\left(dx+c\right)\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*cosh(d*x+c)+a*sinh(d*x+c))^3,x)
```

```
[Out] -1/3/d/a^3/(cosh(d*x+c)+sinh(d*x+c))^3
```

maxima [A] time = 0.32, size = 17, normalized size = 0.65

$$-\frac{e^{(-3\,dx-3\,c)}}{3\,a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cosh(d*x+c)+a*sinh(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/3*e^(-3*d*x - 3*c)/(a^3*d)
```

mupad [B] time = 1.47, size = 17, normalized size = 0.65

$$-\frac{\mathrm{e}^{-3\,c-3\,d\,x}}{3\,a^3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*cosh(c + d*x) + a*sinh(c + d*x))^3,x)
```

```
[Out] -exp(- 3*c - 3*d*x)/(3*a^3*d)
```

sympy [A] time = 1.42, size = 90, normalized size = 3.46

$$\begin{cases} -\frac{1}{3a^3d\sinh^3(c+dx)+9a^3d\sinh^2(c+dx)\cosh(c+dx)+9a^3d\sinh(c+dx)\cosh^2(c+dx)+3a^3d\cosh^3(c+dx)} & \text{for } d \neq 0 \\ \frac{x}{(a\sinh(c)+a\cosh(c))^3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cosh(d*x+c)+a*sinh(d*x+c))**3,x)
```

```
[Out] Piecewise((-1/(3*a**3*d*sinh(c + d*x)**3 + 9*a**3*d*sinh(c + d*x)**2*cosh(c
 + d*x) + 9*a**3*d*sinh(c + d*x)*cosh(c + d*x)**2 + 3*a**3*d*cosh(c + d*x)*
*3), Ne(d, 0)), (x/(a*sinh(c) + a*cosh(c))**3, True))
```

3.603 $\int \sqrt{a\cosh(c+dx)+a\sinh(c+dx)}\,dx$

Optimal. Leaf size=26

$$\frac{2\sqrt{a\sinh(c+dx)+a\cosh(c+dx)}}{d}$$

```
[Out] 2*(a*cosh(d*x+c)+a*sinh(d*x+c))^(1/2)/d
```

Rubi [A] time = 0.02, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {3071}

$$\frac{2\sqrt{a\sinh(c+dx)+a\cosh(c+dx)}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*Cosh[c + d*x] + a*Sinh[c + d*x]],x]
```

```
[Out] (2*Sqrt[a*Cosh[c + d*x] + a*Sinh[c + d*x]])/d
```

Rule 3071

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Simp[(a*(a*Cos[c + d*x] + b*Sin[c + d*x])^n)/(b*d*n), x] /; Fre
eQ[{a, b, c, d, n}, x] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\int \sqrt{a\cosh(c+dx)+a\sinh(c+dx)}\,dx = \frac{2\sqrt{a\cosh(c+dx)+a\sinh(c+dx)}}{d}$$

Mathematica [A] time = 0.02, size = 24, normalized size = 0.92

$$\frac{2\sqrt{a(\sinh(c+dx)+\cosh(c+dx))}}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*Cosh[c + d*x] + a*Sinh[c + d*x]],x]
```

```
[Out] (2*Sqrt[a*(Cosh[c + d*x] + Sinh[c + d*x])])/d
```

fricas [A] time = 0.38, size = 24, normalized size = 0.92

$$\frac{2\,\sqrt{a\cosh\left(dx+c\right)+a\sinh\left(dx+c\right)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cosh(d*x+c)+a*sinh(d*x+c))^(1/2),x, algorithm="fricas")
[Out] 2*sqrt(a*cosh(d*x + c) + a*sinh(d*x + c))/d
```

giac [A] time = 0.12, size = 17, normalized size = 0.65

$$\frac{2\,\sqrt{a}e^{\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cosh(d*x+c)+a*sinh(d*x+c))^(1/2),x, algorithm="giac")
[Out] 2*sqrt(a)*e^(1/2*d*x + 1/2*c)/d
```

maple [A] time = 0.02, size = 25, normalized size = 0.96

$$\frac{2\sqrt{a\cosh\left(dx+c\right)+a\sinh\left(dx+c\right)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cosh(d*x+c)+a*sinh(d*x+c))^(1/2),x)
[Out] 2*(a*cosh(d*x+c)+a*sinh(d*x+c))^(1/2)/d
```

maxima [A] time = 0.36, size = 17, normalized size = 0.65

$$\frac{2\,\sqrt{a}e^{\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cosh(d*x+c)+a*sinh(d*x+c))^(1/2),x, algorithm="maxima")
[Out] 2*sqrt(a)*e^(1/2*d*x + 1/2*c)/d
```

mupad [B] time = 1.58, size = 15, normalized size = 0.58

$$\frac{2\,\sqrt{a\,\mathrm{e}^{c+d\,x}}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cosh(c + d*x) + a*sinh(c + d*x))^(1/2),x)
```

```
[Out] (2*(a*exp(c + d*x))^(1/2))/d
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a \sinh (c + dx) + a \cosh (c + dx)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cosh(d*x+c)+a*sinh(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(a*sinh(c + d*x) + a*cosh(c + d*x)), x)
```

3.604 $\int \frac{1}{\sqrt{a\cosh(c+dx)+a\sinh(c+dx)}}\,dx$

Optimal. Leaf size=26

$$-\frac{2}{d\sqrt{a\sinh(c+dx)+a\cosh(c+dx)}}$$

```
[Out] -2/d/(a*cosh(d*x+c)+a*sinh(d*x+c))^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {3071}

$$-\frac{2}{d\sqrt{a\sinh(c+dx)+a\cosh(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a*Cosh[c + d*x] + a*Sinh[c + d*x]],x]
[Out] -2/(d*Sqrt[a*Cosh[c + d*x] + a*Sinh[c + d*x]])
```

Rule 3071

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Simp[(a*(a*Cos[c + d*x] + b*Sin[c + d*x])^n)/(b*d*n), x] /; Fre
eQ[{a, b, c, d, n}, x] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{a\cosh(c+dx)+a\sinh(c+dx)}}\,dx = -\frac{2}{d\sqrt{a\cosh(c+dx)+a\sinh(c+dx)}}$$

Mathematica [A] time = 0.04, size = 24, normalized size = 0.92

$$-\frac{2}{d\sqrt{a(\sinh(c+dx)+\cosh(c+dx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a*Cosh[c + d*x] + a*Sinh[c + d*x]],x]
[Out] -2/(d*Sqrt[a*(Cosh[c + d*x] + Sinh[c + d*x])])
```

fricas [A] time = 0.40, size = 42, normalized size = 1.62

$$-\frac{2\sqrt{a\cosh\left(dx+c\right)+a\sinh\left(dx+c\right)}}{ad\cosh\left(dx+c\right)+ad\sinh\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cosh(d*x+c)+a*sinh(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] -2*sqrt(a*cosh(d*x + c) + a*sinh(d*x + c))/(a*d*cosh(d*x + c) + a*d*sinh(d*
x + c))
```

giac [A] time = 0.13, size = 17, normalized size = 0.65

$$-\frac{2\,e^{\left(-\frac{1}{2}\,dx-\frac{1}{2}\,c\right)}}{\sqrt{a}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cosh(d*x+c)+a*sinh(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] -2*e^(-1/2*d*x - 1/2*c)/(sqrt(a)*d)
```

maple [A] time = 0.02, size = 25, normalized size = 0.96

$$-\frac{2}{d\sqrt{a\cosh\left(dx+c\right)+a\sinh\left(dx+c\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*cosh(d*x+c)+a*sinh(d*x+c))^(1/2),x)
```

```
[Out] -2/d/(a*cosh(d*x+c)+a*sinh(d*x+c))^(1/2)
```

maxima [A] time = 0.36, size = 17, normalized size = 0.65

$$-\frac{2\,e^{\left(-\frac{1}{2}\,dx-\frac{1}{2}\,c\right)}}{\sqrt{a}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cosh(d*x+c)+a*sinh(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] -2*e^(-1/2*d*x - 1/2*c)/(sqrt(a)*d)
```

mupad [B] time = 1.57, size = 15, normalized size = 0.58

$$-\frac{2}{d\sqrt{a\,\mathrm{e}^{c+d\,x}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*cosh(c + d*x) + a*sinh(c + d*x))^(1/2),x)
[Out] -2/(d*(a*exp(c + d*x))^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a\sinh\left(c+dx\right)+a\cosh\left(c+dx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cosh(d*x+c)+a*sinh(d*x+c))**(1/2),x)
[Out] Integral(1/sqrt(a*sinh(c + d*x) + a*cosh(c + d*x)), x)
```

3.605 $\int (a\cosh(c+dx)-a\sinh(c+dx))\,dx$

Optimal. Leaf size=24

$$\frac{a\sinh(c+dx)}{d}-\frac{a\cosh(c+dx)}{d}$$

```
[Out] -a*cosh(d*x+c)/d+a*sinh(d*x+c)/d
```

Rubi [A] time = 0.01, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2637, 2638}

$$\frac{a\sinh(c+dx)}{d}-\frac{a\cosh(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[a*Cosh[c + d*x] - a*Sinh[c + d*x],x]
```

```
[Out] -((a*Cosh[c + d*x])/d) + (a*Sinh[c + d*x])/d
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int (a\cosh(c+dx)-a\sinh(c+dx))\,dx &= a\int \cosh(c+dx)\,dx-a\int \sinh(c+dx)\,dx\\ &= -\frac{a\cosh(c+dx)}{d}+\frac{a\sinh(c+dx)}{d}\end{aligned}$$

Mathematica [A] time = 0.02, size = 47, normalized size = 1.96

$$-\frac{a\sinh(c)\sinh(dx)}{d}-\frac{a\cosh(c)\cosh(dx)}{d}+\frac{a\sinh(c)\cosh(dx)}{d}+\frac{a\cosh(c)\sinh(dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[a*Cosh[c + d*x] - a*Sinh[c + d*x],x]
```

```
[Out] -((a*Cosh[c]*Cosh[d*x])/d) + (a*Cosh[d*x]*Sinh[c])/d + (a*Cosh[c]*Sinh[d*x]
)/d - (a*Sinh[c]*Sinh[d*x])/d
```

fricas [A] time = 0.38, size = 22, normalized size = 0.92

$$-\frac{a}{d\cosh(dx+c)+d\sinh(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a*cosh(d*x+c)-a*sinh(d*x+c),x, algorithm="fricas")
```

```
[Out] -a/(d*cosh(d*x + c) + d*sinh(d*x + c))
```

giac [B] time = 0.11, size = 56, normalized size = 2.33

$$-\frac{1}{2}a\left(\frac{e^{(dx+c)}}{d}+\frac{e^{(-dx-c)}}{d}\right)+\frac{1}{2}a\left(\frac{e^{(dx+c)}}{d}-\frac{e^{(-dx-c)}}{d}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a*cosh(d*x+c)-a*sinh(d*x+c),x, algorithm="giac")
```

```
[Out] -1/2*a*(e^(d*x + c)/d + e^(-d*x - c)/d) + 1/2*a*(e^(d*x + c)/d - e^(-d*x -
c)/d)
```

maple [A] time = 0.02, size = 21, normalized size = 0.88

$$\frac{a(\sinh(dx+c)-\cosh(dx+c))}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a*cosh(d*x+c)-a*sinh(d*x+c),x)
```

```
[Out] a*(sinh(d*x+c)-cosh(d*x+c))/d
```

maxima [A] time = 0.36, size = 24, normalized size = 1.00

$$-\frac{a\cosh(dx+c)}{d}+\frac{a\sinh(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a*cosh(d*x+c)-a*sinh(d*x+c),x, algorithm="maxima")
```

```
[Out] -a*cosh(d*x + c)/d + a*sinh(d*x + c)/d
```

mupad [B] time = 0.05, size = 15, normalized size = 0.62

$$-\frac{a\,\mathrm{e}^{-c-d\,x}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a*cosh(c + d*x) - a*sinh(c + d*x),x)
```

```
[Out] -(a*exp(- c - d*x))/d
```

sympy [A] time = 0.15, size = 29, normalized size = 1.21

$$a\left(\begin{cases} \frac{\sinh{\left(c+dx \right)}}{d} & \text{for}\: d \neq 0 \\ x\cosh{\left(c \right)} & \text{otherwise} \end{cases}\right) - a\left(\begin{cases} \frac{\cosh{\left(c+dx \right)}}{d} & \text{for}\: d \neq 0 \\ x\sinh{\left(c \right)} & \text{otherwise} \end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a*cosh(d*x+c)-a*sinh(d*x+c),x)
```

```
[Out] a*Piecewise((sinh(c + d*x)/d, Ne(d, 0)), (x*cosh(c), True)) - a*Piecewise((
cosh(c + d*x)/d, Ne(d, 0)), (x*sinh(c), True))
```

3.606 $\int (a\cosh(c+dx) - a\sinh(c+dx))^2\,dx$

Optimal. Leaf size=27

$$-\frac{(a\cosh(c+dx) - a\sinh(c+dx))^2}{2d}$$

```
[Out] -1/2*(a*cosh(d*x+c)-a*sinh(d*x+c))^2/d
```

Rubi [A] time = 0.02, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {3071}

$$-\frac{(a\cosh(c+dx) - a\sinh(c+dx))^2}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cosh[c + d*x] - a*Sinh[c + d*x])^2,x]
```

```
[Out] -(a*Cosh[c + d*x] - a*Sinh[c + d*x])^2/(2*d)
```

Rule 3071

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Simp[(a*(a*Cos[c + d*x] + b*Sin[c + d*x])^n)/(b*d*n), x] /; Fre
eQ[{a, b, c, d, n}, x] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\int (a\cosh(c+dx) - a\sinh(c+dx))^2\,dx = -\frac{(a\cosh(c+dx) - a\sinh(c+dx))^2}{2d}$$

Mathematica [A] time = 0.03, size = 27, normalized size = 1.00

$$-\frac{(a\cosh(c+dx) - a\sinh(c+dx))^2}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cosh[c + d*x] - a*Sinh[c + d*x])^2,x]
```

```
[Out] -1/2*(a*Cosh[c + d*x] - a*Sinh[c + d*x])^2/d
```

fricas [A] time = 0.38, size = 43, normalized size = 1.59

$$-\frac{a^2}{2\left(d\cosh\left(dx+c\right)^2+2\,d\cosh\left(dx+c\right)\sinh\left(dx+c\right)+d\sinh\left(dx+c\right)^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cosh(d*x+c)-a*sinh(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/2*a^2/(d*cosh(d*x + c)^2 + 2*d*cosh(d*x + c)*sinh(d*x + c) + d*sinh(d*x
+ c)^2)
```

giac [A] time = 0.14, size = 17, normalized size = 0.63

$$-\frac{a^2e^{(-2\,dx-2\,c)}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cosh(d*x+c)-a*sinh(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/2*a^2*e^(-2*d*x - 2*c)/d
```

maple [A] time = 0.02, size = 26, normalized size = 0.96

$$-\frac{a^2\left(\cosh\left(dx+c\right)-\sinh\left(dx+c\right)\right)^2}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cosh(d*x+c)-a*sinh(d*x+c))^2,x)
```

```
[Out] -1/2*a^2*(cosh(d*x+c)-sinh(d*x+c))^2/d
```

maxima [B] time = 0.30, size = 89, normalized size = 3.30

$$\frac{1}{8}\,a^2\left(4\,x+\frac{e^{(2\,dx+2\,c)}}{d}-\frac{e^{(-2\,dx-2\,c)}}{d}\right)-\frac{1}{8}\,a^2\left(4\,x-\frac{e^{(2\,dx+2\,c)}}{d}+\frac{e^{(-2\,dx-2\,c)}}{d}\right)-\frac{a^2\cosh\left(dx+c\right)^2}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cosh(d*x+c)-a*sinh(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/8*a^2*(4*x + e^(2*d*x + 2*c)/d - e^(-2*d*x - 2*c)/d) - 1/8*a^2*(4*x - e^(
2*d*x + 2*c)/d + e^(-2*d*x - 2*c)/d) - a^2*cosh(d*x + c)^2/d
```

mupad [B] time = 0.06, size = 17, normalized size = 0.63

$$-\frac{a^2\,\mathrm{e}^{-2\,c-2\,d\,x}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cosh(c + d*x) - a*sinh(c + d*x))^2,x)
[Out] -(a^2*exp(- 2*c - 2*d*x))/(2*d)
```

sympy [A] time = 0.21, size = 44, normalized size = 1.63

$$\begin{cases} -\frac{a^2\sinh^2(c+dx)}{d} + \frac{a^2\sinh(c+dx)\cosh(c+dx)}{d} & \text{for } d \neq 0 \\ x\left(-a\sinh(c) + a\cosh(c)\right)^2 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cosh(d*x+c)-a*sinh(d*x+c))**2,x)
[Out] Piecewise((-a**2*sinh(c + d*x)**2/d + a**2*sinh(c + d*x)*cosh(c + d*x)/d, N
e(d, 0)), (x*(-a*sinh(c) + a*cosh(c))**2, True))
```

3.607 $\int (a\cosh(c+dx)-a\sinh(c+dx))^3\,dx$

Optimal. Leaf size=27

$$-\frac{(a\cosh(c+dx)-a\sinh(c+dx))^3}{3d}$$

```
[Out] -1/3*(a*cosh(d*x+c)-a*sinh(d*x+c))^3/d
```

Rubi [A] time = 0.01, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {3071}

$$-\frac{(a\cosh(c+dx)-a\sinh(c+dx))^3}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cosh[c + d*x] - a*Sinh[c + d*x])^3,x]
```

```
[Out] -(a*Cosh[c + d*x] - a*Sinh[c + d*x])^3/(3*d)
```

Rule 3071

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Simp[(a*(a*Cos[c + d*x] + b*Sin[c + d*x])^n)/(b*d*n), x] /; Fre
eQ[{a, b, c, d, n}, x] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\int (a\cosh(c+dx)-a\sinh(c+dx))^3\,dx=-\frac{(a\cosh(c+dx)-a\sinh(c+dx))^3}{3d}$$

Mathematica [A] time = 0.02, size = 27, normalized size = 1.00

$$-\frac{(a\cosh(c+dx)-a\sinh(c+dx))^3}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cosh[c + d*x] - a*Sinh[c + d*x])^3,x]
```

```
[Out] -1/3*(a*Cosh[c + d*x] - a*Sinh[c + d*x])^3/d
```

fricas [B] time = 0.40, size = 62, normalized size = 2.30

$$-\frac{a^3}{3\left(d\cosh\left(dx+c\right)^3+3\,d\cosh\left(dx+c\right)^2\sinh\left(dx+c\right)+3\,d\cosh\left(dx+c\right)\sinh\left(dx+c\right)^2+d\sinh\left(dx+c\right)^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cosh(d*x+c)-a*sinh(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/3*a^3/(d*cosh(d*x + c)^3 + 3*d*cosh(d*x + c)^2*sinh(d*x + c) + 3*d*cosh(
d*x + c)*sinh(d*x + c)^2 + d*sinh(d*x + c)^3)
```

giac [A] time = 0.11, size = 17, normalized size = 0.63

$$-\frac{a^3e^{(-3\,dx-3\,c)}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cosh(d*x+c)-a*sinh(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/3*a^3*e^(-3*d*x - 3*c)/d
```

maple [A] time = 0.02, size = 26, normalized size = 0.96

$$-\frac{a^3\left(\cosh\left(dx+c\right)-\sinh\left(dx+c\right)\right)^3}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cosh(d*x+c)-a*sinh(d*x+c))^3,x)
```

```
[Out] -1/3*a^3*(cosh(d*x+c)-sinh(d*x+c))^3/d
```

maxima [B] time = 0.31, size = 147, normalized size = 5.44

$$-\frac{a^3\cosh\left(dx+c\right)^3}{d}+\frac{a^3\sinh\left(dx+c\right)^3}{d}+\frac{1}{24}\,a^3\left(\frac{e^{(3\,dx+3\,c)}}{d}+\frac{9\,e^{(dx+c)}}{d}-\frac{9\,e^{(-dx-c)}}{d}-\frac{e^{(-3\,dx-3\,c)}}{d}\right)-\frac{1}{24}\,a^3\left(\frac{e^{(3\,dx+3\,c)}}{d}-\frac{9}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cosh(d*x+c)-a*sinh(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -a^3*cosh(d*x + c)^3/d + a^3*sinh(d*x + c)^3/d + 1/24*a^3*(e^(3*d*x + 3*c)/
d + 9*e^(d*x + c)/d - 9*e^(-d*x - c)/d - e^(-3*d*x - 3*c)/d) - 1/24*a^3*(e^
(3*d*x + 3*c)/d - 9*e^(d*x + c)/d - 9*e^(-d*x - c)/d + e^(-3*d*x - 3*c)/d)
```

mupad [B] time = 1.85, size = 17, normalized size = 0.63

$$-\frac{a^3\,\mathrm{e}^{-3\,c-3\,d\,x}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cosh(c + d*x) - a*sinh(c + d*x))^3,x)
```

```
[Out] -(a^3*exp(- 3*c - 3*d*x))/(3*d)
```

sympy [A] time = 0.47, size = 83, normalized size = 3.07

$$\begin{cases} \frac{a^3\sinh^3(c+dx)}{3d} - \frac{a^3\sinh^2(c+dx)\cosh(c+dx)}{d} + \frac{a^3\sinh(c+dx)\cosh^2(c+dx)}{d} - \frac{a^3\cosh^3(c+dx)}{3d} & \text{for } d \neq 0 \\ x\left(-a\sinh(c) + a\cosh(c)\right)^3 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cosh(d*x+c)-a*sinh(d*x+c))**3,x)
```

```
[Out] Piecewise((a**3*sinh(c + d*x)**3/(3*d) - a**3*sinh(c + d*x)**2*cosh(c + d*x
)/d + a**3*sinh(c + d*x)*cosh(c + d*x)**2/d - a**3*cosh(c + d*x)**3/(3*d),
Ne(d, 0)), (x*(-a*sinh(c) + a*cosh(c))**3, True))
```

3.608 $\int (a\cosh(c+dx)-a\sinh(c+dx))^n\,dx$

Optimal. Leaf size=28

$$-\frac{(a\cosh(c+dx)-a\sinh(c+dx))^n}{dn}$$

```
[Out] -(a*cosh(d*x+c)-a*sinh(d*x+c))^n/d/n
```

Rubi [A] time = 0.02, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {3071}

$$-\frac{(a\cosh(c+dx)-a\sinh(c+dx))^n}{dn}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cosh[c + d*x] - a*Sinh[c + d*x])^n,x]
```

```
[Out] -((a*Cosh[c + d*x] - a*Sinh[c + d*x])^n/(d*n))
```

Rule 3071

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Simp[(a*(a*Cos[c + d*x] + b*Sin[c + d*x])^n)/(b*d*n), x] /; Fre
eQ[{a, b, c, d, n}, x] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\int (a\cosh(c+dx)-a\sinh(c+dx))^n\,dx=-\frac{(a\cosh(c+dx)-a\sinh(c+dx))^n}{dn}$$

Mathematica [A] time = 0.05, size = 27, normalized size = 0.96

$$-\frac{(a(\cosh(c+dx)-\sinh(c+dx)))^n}{dn}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cosh[c + d*x] - a*Sinh[c + d*x])^n,x]
```

```
[Out] -((a*(Cosh[c + d*x] - Sinh[c + d*x]))^n/(d*n))
```

fricas [A] time = 0.40, size = 39, normalized size = 1.39

$$-\frac{\cosh\left(-dnx - cn + n\log(a)\right) + \sinh\left(-dnx - cn + n\log(a)\right)}{dn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cosh(d*x+c)-a*sinh(d*x+c))^n,x, algorithm="fricas")
[Out] -(cosh(-d*n*x - c*n + n*log(a)) + sinh(-d*n*x - c*n + n*log(a)))/(d*n)
```

giac [A] time = 0.14, size = 23, normalized size = 0.82

$$-\frac{e^{\left(-dnx - cn + n\log(a)\right)}}{dn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cosh(d*x+c)-a*sinh(d*x+c))^n,x, algorithm="giac")
[Out] -e^(-d*n*x - c*n + n*log(a))/(d*n)
```

maple [A] time = 0.02, size = 29, normalized size = 1.04

$$-\frac{(a\cosh(dx+c) - a\sinh(dx+c))^n}{dn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cosh(d*x+c)-a*sinh(d*x+c))^n,x)
[Out] -(a*cosh(d*x+c)-a*sinh(d*x+c))^n/d/n
```

maxima [A] time = 0.49, size = 20, normalized size = 0.71

$$-\frac{a^n e^{(-(dx+c)n)}}{dn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cosh(d*x+c)-a*sinh(d*x+c))^n,x, algorithm="maxima")
[Out] -a^n*e^(-(d*x + c)*n)/(d*n)
```

mupad [B] time = 1.66, size = 21, normalized size = 0.75

$$-\frac{\left(a\,\mathrm{e}^{-c-d\,x}\right)^n}{d\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cosh(c + d*x) - a*sinh(c + d*x))^n,x)
```

```
[Out] -(a*exp(- c - d*x))^n/(d*n)
```

sympy [A] time = 0.19, size = 37, normalized size = 1.32

$$\begin{cases} x & \text{for}\: d = 0 \wedge n = 0 \\ x \left(- a \sinh{\left(c \right)} + a \cosh{\left(c \right)}\right)^{n} & \text{for}\: d = 0 \\ x & \text{for}\: n = 0 \\ - \frac{\left(- a \sinh{\left(c + d x \right)} + a \cosh{\left(c + d x \right)}\right)^{n}}{d n} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cosh(d*x+c)-a*sinh(d*x+c))**n,x)
```

```
[Out] Piecewise((x, Eq(d, 0) & Eq(n, 0)), (x*(-a*sinh(c) + a*cosh(c))**n, Eq(d, 0
)), (x, Eq(n, 0)), (-(-a*sinh(c + d*x) + a*cosh(c + d*x))**n/(d*n), True))
```

3.609 $\int \frac{1}{a\cosh(c+dx)-a\sinh(c+dx)}\,dx$

Optimal. Leaf size=24

$$\frac{1}{d(a\cosh(c+dx)-a\sinh(c+dx))}$$

[Out] 1/d/(a*cosh(d*x+c)-a*sinh(d*x+c))

Rubi [A] time = 0.02, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {3071}

$$\frac{1}{d(a\cosh(c+dx)-a\sinh(c+dx))}$$

Antiderivative was successfully verified.

[In] Int[(a*Cosh[c + d*x] - a*Sinh[c + d*x])^(-1),x]

[Out] 1/(d*(a*Cosh[c + d*x] - a*Sinh[c + d*x]))

Rule 3071

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Simp[(a*(a*Cos[c + d*x] + b*Sin[c + d*x])^n)/(b*d*n), x] /; Fre
eQ[{a, b, c, d, n}, x] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\int \frac{1}{a\cosh(c+dx)-a\sinh(c+dx)}\,dx=\frac{1}{d(a\cosh(c+dx)-a\sinh(c+dx))}$$

Mathematica [A] time = 0.01, size = 22, normalized size = 0.92

$$\frac{1}{ad\cosh(c+dx)-ad\sinh(c+dx)}$$

Antiderivative was successfully verified.

[In] Integrate[(a*Cosh[c + d*x] - a*Sinh[c + d*x])^(-1),x]

[Out] (a*d*Cosh[c + d*x] - a*d*Sinh[c + d*x])^(-1)

fricas [A] time = 0.41, size = 20, normalized size = 0.83

$$\frac{\cosh(dx+c)+\sinh(dx+c)}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cosh(d*x+c)-a*sinh(d*x+c)),x, algorithm="fricas")
[Out] (cosh(d*x + c) + sinh(d*x + c))/(a*d)
```

giac [A] time = 0.14, size = 13, normalized size = 0.54

$$\frac{e^{(dx+c)}}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cosh(d*x+c)-a*sinh(d*x+c)),x, algorithm="giac")
[Out] e^(d*x + c)/(a*d)
```

maple [A] time = 0.02, size = 25, normalized size = 1.04

$$\frac{1}{da(\cosh(dx+c)-\sinh(dx+c))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*cosh(d*x+c)-a*sinh(d*x+c)),x)
[Out] 1/d/a/(cosh(d*x+c)-sinh(d*x+c))
```

maxima [A] time = 0.51, size = 13, normalized size = 0.54

$$\frac{e^{(dx+c)}}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cosh(d*x+c)-a*sinh(d*x+c)),x, algorithm="maxima")
[Out] e^(d*x + c)/(a*d)
```

mupad [B] time = 1.53, size = 13, normalized size = 0.54

$$\frac{\mathrm{e}^{c+d\,x}}{a\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*cosh(c + d*x) - a*sinh(c + d*x)),x)
```

```
[Out] exp(c + d*x)/(a*d)
```

sympy [A] time = 0.35, size = 32, normalized size = 1.33

$$\begin{cases} \frac{1}{-ad\sinh(c+dx)+ad\cosh(c+dx)} & \text{for } d \neq 0 \\ \frac{x}{-a\sinh(c)+a\cosh(c)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cosh(d*x+c)-a*sinh(d*x+c)),x)
```

```
[Out] Piecewise((1/(-a*d*sinh(c + d*x) + a*d*cosh(c + d*x)), Ne(d, 0)), (x/(-a*si
nh(c) + a*cosh(c)), True))
```

3.610 $\int \frac{1}{(a\cosh(c+dx)-a\sinh(c+dx))^2}\,dx$

Optimal. Leaf size=27

$$\frac{1}{2d(a\cosh(c+dx)-a\sinh(c+dx))^2}$$

```
[Out] 1/2/d/(a*cosh(d*x+c)-a*sinh(d*x+c))^2
```

Rubi [A] time = 0.02, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {3071}

$$\frac{1}{2d(a\cosh(c+dx)-a\sinh(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cosh[c + d*x] - a*Sinh[c + d*x])^(-2),x]
```

```
[Out] 1/(2*d*(a*Cosh[c + d*x] - a*Sinh[c + d*x])^2)
```

Rule 3071

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Simp[(a*(a*Cos[c + d*x] + b*Sin[c + d*x])^n)/(b*d*n), x] /; Fre
eQ[{a, b, c, d, n}, x] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\int \frac{1}{(a\cosh(c+dx)-a\sinh(c+dx))^2}\,dx = \frac{1}{2d(a\cosh(c+dx)-a\sinh(c+dx))^2}$$

Mathematica [A] time = 0.04, size = 27, normalized size = 1.00

$$\frac{1}{2d(a\cosh(c+dx)-a\sinh(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cosh[c + d*x] - a*Sinh[c + d*x])^(-2),x]
```

```
[Out] 1/(2*d*(a*Cosh[c + d*x] - a*Sinh[c + d*x])^2)
```

fricas [A] time = 0.83, size = 41, normalized size = 1.52

$$\frac{\cosh(dx+c)+\sinh(dx+c)}{2\left(a^2d\cosh(dx+c)-a^2d\sinh(dx+c)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cosh(d*x+c)-a*sinh(d*x+c))^2,x, algorithm="fricas")
[Out] 1/2*(cosh(d*x + c) + sinh(d*x + c))/(a^2*d*cosh(d*x + c) - a^2*d*sinh(d*x +
 c))
```

giac [A] time = 0.12, size = 17, normalized size = 0.63

$$\frac{e^{(2\,dx+2\,c)}}{2\,a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cosh(d*x+c)-a*sinh(d*x+c))^2,x, algorithm="giac")
[Out] 1/2*e^(2*d*x + 2*c)/(a^2*d)
```

maple [A] time = 0.02, size = 26, normalized size = 0.96

$$\frac{1}{2d\,a^2\left(\cosh(dx+c)-\sinh(dx+c)\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*cosh(d*x+c)-a*sinh(d*x+c))^2,x)
[Out] 1/2/d/a^2/(cosh(d*x+c)-sinh(d*x+c))^2
```

maxima [A] time = 0.35, size = 17, normalized size = 0.63

$$\frac{e^{(2\,dx+2\,c)}}{2\,a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cosh(d*x+c)-a*sinh(d*x+c))^2,x, algorithm="maxima")
[Out] 1/2*e^(2*d*x + 2*c)/(a^2*d)
```

mupad [B] time = 0.10, size = 17, normalized size = 0.63

$$\frac{e^{2\,c+2\,d\,x}}{2\,a^2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*cosh(c + d*x) - a*sinh(c + d*x))^2,x)
```

```
[Out] exp(2*c + 2*d*x)/(2*a^2*d)
```

sympy [A] time = 0.62, size = 65, normalized size = 2.41

$$\begin{cases} \frac{1}{2a^2 d \sinh^2(c+dx) - 4a^2 d \sinh(c+dx)\cosh(c+dx) + 2a^2 d \cosh^2(c+dx)} & \text{for } d \neq 0 \\ \frac{x}{(-a\sinh(c) + a\cosh(c))^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cosh(d*x+c)-a*sinh(d*x+c))**2,x)
```

```
[Out] Piecewise((1/(2*a**2*d*sinh(c + d*x)**2 - 4*a**2*d*sinh(c + d*x)*cosh(c + d
*x) + 2*a**2*d*cosh(c + d*x)**2), Ne(d, 0)), (x/(-a*sinh(c) + a*cosh(c))**2
, True))
```

3.611 $\int \frac{1}{(a\cosh(c+dx)-a\sinh(c+dx))^3}\,dx$

Optimal. Leaf size=27

$$\frac{1}{3d(a\cosh(c+dx)-a\sinh(c+dx))^3}$$

```
[Out] 1/3/d/(a*cosh(d*x+c)-a*sinh(d*x+c))^3
```

Rubi [A] time = 0.02, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {3071}

$$\frac{1}{3d(a\cosh(c+dx)-a\sinh(c+dx))^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cosh[c + d*x] - a*Sinh[c + d*x])^(-3),x]
```

```
[Out] 1/(3*d*(a*Cosh[c + d*x] - a*Sinh[c + d*x])^3)
```

Rule 3071

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Simp[(a*(a*Cos[c + d*x] + b*Sin[c + d*x])^n)/(b*d*n), x] /; Fre
eQ[{a, b, c, d, n}, x] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\int \frac{1}{(a\cosh(c+dx)-a\sinh(c+dx))^3}\,dx = \frac{1}{3d(a\cosh(c+dx)-a\sinh(c+dx))^3}$$

Mathematica [A] time = 0.07, size = 27, normalized size = 1.00

$$\frac{1}{3d(a\cosh(c+dx)-a\sinh(c+dx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cosh[c + d*x] - a*Sinh[c + d*x])^(-3),x]
```

```
[Out] 1/(3*d*(a*Cosh[c + d*x] - a*Sinh[c + d*x])^3)
```

fricas [B] time = 0.42, size = 59, normalized size = 2.19

$$\frac{\cosh\left(dx+c\right)^{2}+2\,\cosh\left(dx+c\right)\sinh\left(dx+c\right)+\sinh\left(dx+c\right)^{2}}{3\left(a^{3}d\cosh\left(dx+c\right)-a^{3}d\sinh\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cosh(d*x+c)-a*sinh(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/3*(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2)/(a^
3*d*cosh(d*x + c) - a^3*d*sinh(d*x + c))
```

giac [A] time = 0.14, size = 17, normalized size = 0.63

$$\frac{e^{\left(3\,dx+3\,c\right)}}{3\,a^{3}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cosh(d*x+c)-a*sinh(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/3*e^(3*d*x + 3*c)/(a^3*d)
```

maple [A] time = 0.02, size = 26, normalized size = 0.96

$$\frac{1}{3d\,a^{3}\left(\cosh\left(dx+c\right)-\sinh\left(dx+c\right)\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*cosh(d*x+c)-a*sinh(d*x+c))^3,x)
```

```
[Out] 1/3/d/a^3/(cosh(d*x+c)-sinh(d*x+c))^3
```

maxima [A] time = 0.52, size = 17, normalized size = 0.63

$$\frac{e^{\left(3\,dx+3\,c\right)}}{3\,a^{3}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cosh(d*x+c)-a*sinh(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/3*e^(3*d*x + 3*c)/(a^3*d)
```

mupad [B] time = 0.06, size = 17, normalized size = 0.63

$$\frac{\mathrm{e}^{3\,c+3\,d\,x}}{3\,a^{3}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*cosh(c + d*x) - a*sinh(c + d*x))^3,x)
```

```
[Out] exp(3*c + 3*d*x)/(3*a^3*d)
```

sympy [A] time = 1.12, size = 88, normalized size = 3.26

$$\begin{cases} \frac{1}{-3a^3d\sinh^3(c+dx)+9a^3d\sinh^2(c+dx)\cosh(c+dx)-9a^3d\sinh(c+dx)\cosh^2(c+dx)+3a^3d\cosh^3(c+dx)} & \text{for } d \neq 0 \\ \frac{x}{(-a\sinh(c)+a\cosh(c))^3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cosh(d*x+c)-a*sinh(d*x+c))**3,x)
```

```
[Out] Piecewise((1/(-3*a**3*d*sinh(c + d*x)**3 + 9*a**3*d*sinh(c + d*x)**2*cosh(c
 + d*x) - 9*a**3*d*sinh(c + d*x)*cosh(c + d*x)**2 + 3*a**3*d*cosh(c + d*x)*
*3), Ne(d, 0)), (x/(-a*sinh(c) + a*cosh(c))**3, True))
```

3.612 $\int \sqrt{a\cosh(c+dx)-a\sinh(c+dx)}\,dx$

Optimal. Leaf size=27

$$-\frac{2\sqrt{a\cosh(c+dx)-a\sinh(c+dx)}}{d}$$

```
[Out] -2*(a*cosh(d*x+c)-a*sinh(d*x+c))^(1/2)/d
```

Rubi [A] time = 0.02, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {3071}

$$-\frac{2\sqrt{a\cosh(c+dx)-a\sinh(c+dx)}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*Cosh[c + d*x] - a*Sinh[c + d*x]],x]
```

```
[Out] (-2*Sqrt[a*Cosh[c + d*x] - a*Sinh[c + d*x]])/d
```

Rule 3071

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Simp[(a*(a*Cos[c + d*x] + b*Sin[c + d*x])^n)/(b*d*n), x] /; Fre
eQ[{a, b, c, d, n}, x] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\int \sqrt{a\cosh(c+dx)-a\sinh(c+dx)}\,dx=-\frac{2\sqrt{a\cosh(c+dx)-a\sinh(c+dx)}}{d}$$

Mathematica [A] time = 0.02, size = 26, normalized size = 0.96

$$-\frac{2\sqrt{a(\cosh(c+dx)-\sinh(c+dx))}}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*Cosh[c + d*x] - a*Sinh[c + d*x]],x]
```

```
[Out] (-2*Sqrt[a*(Cosh[c + d*x] - Sinh[c + d*x])])/d
```

fricas [A] time = 0.43, size = 24, normalized size = 0.89

$$-\frac{2\sqrt{\frac{a}{\cosh(dx+c)+\sinh(dx+c)}}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cosh(d*x+c)-a*sinh(d*x+c))^(1/2),x, algorithm="fricas")
[Out] -2*sqrt(a/(cosh(d*x + c) + sinh(d*x + c)))/d
```

giac [A] time = 0.14, size = 17, normalized size = 0.63

$$-\frac{2\sqrt{a}e^{\left(-\frac{1}{2}dx-\frac{1}{2}c\right)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cosh(d*x+c)-a*sinh(d*x+c))^(1/2),x, algorithm="giac")
[Out] -2*sqrt(a)*e^(-1/2*d*x - 1/2*c)/d
```

maple [A] time = 0.02, size = 26, normalized size = 0.96

$$-\frac{2\sqrt{a\cosh(dx+c)-a\sinh(dx+c)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cosh(d*x+c)-a*sinh(d*x+c))^(1/2),x)
[Out] -2*(a*cosh(d*x+c)-a*sinh(d*x+c))^(1/2)/d
```

maxima [A] time = 0.31, size = 17, normalized size = 0.63

$$-\frac{2\sqrt{a}e^{\left(-\frac{1}{2}dx-\frac{1}{2}c\right)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cosh(d*x+c)-a*sinh(d*x+c))^(1/2),x, algorithm="maxima")
[Out] -2*sqrt(a)*e^(-1/2*d*x - 1/2*c)/d
```

mupad [B] time = 1.55, size = 18, normalized size = 0.67

$$-\frac{2\sqrt{a\,\mathrm{e}^{-c-d\,x}}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cosh(c + d*x) - a*sinh(c + d*x))^(1/2),x)
```

```
[Out] -(2*(a*exp(- c - d*x))^(1/2))/d
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{-a \sinh\left(c+dx\right)+a \cosh\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cosh(d*x+c)-a*sinh(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(-a*sinh(c + d*x) + a*cosh(c + d*x)), x)
```

3.613 $\int \frac{1}{\sqrt{a\cosh(c+dx)-a\sinh(c+dx)}}\,dx$

Optimal. Leaf size=27

$$\frac{2}{d\sqrt{a\cosh(c+dx)-a\sinh(c+dx)}}$$

```
[Out] 2/d/(a*cosh(d*x+c)-a*sinh(d*x+c))^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {3071}

$$\frac{2}{d\sqrt{a\cosh(c+dx)-a\sinh(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a*Cosh[c + d*x] - a*Sinh[c + d*x]],x]
[Out] 2/(d*Sqrt[a*Cosh[c + d*x] - a*Sinh[c + d*x]])
```

Rule 3071

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Simp[(a*(a*Cos[c + d*x] + b*Sin[c + d*x])^n)/(b*d*n), x] /; Fre
eQ[{a, b, c, d, n}, x] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{a\cosh(c+dx)-a\sinh(c+dx)}}\,dx = \frac{2}{d\sqrt{a\cosh(c+dx)-a\sinh(c+dx)}}$$

Mathematica [A] time = 0.03, size = 26, normalized size = 0.96

$$\frac{2}{d\sqrt{a(\cosh(c+dx)-\sinh(c+dx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a*Cosh[c + d*x] - a*Sinh[c + d*x]],x]
[Out] 2/(d*Sqrt[a*(Cosh[c + d*x] - Sinh[c + d*x])])
```

fricas [A] time = 0.42, size = 40, normalized size = 1.48

$$\frac{2\sqrt{\frac{a}{\cosh(dx+c)+\sinh(dx+c)}}\left(\cosh\left(dx+c\right)+\sinh\left(dx+c\right)\right)}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cosh(d*x+c)-a*sinh(d*x+c))^(1/2),x, algorithm="fricas")
[Out] 2*sqrt(a/(cosh(d*x + c) + sinh(d*x + c)))*(cosh(d*x + c) + sinh(d*x + c))/(
a*d)
```

giac [A] time = 0.12, size = 17, normalized size = 0.63

$$\frac{2\,e^{\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}}{\sqrt{a}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cosh(d*x+c)-a*sinh(d*x+c))^(1/2),x, algorithm="giac")
[Out] 2*e^(1/2*d*x + 1/2*c)/(sqrt(a)*d)
```

maple [A] time = 0.02, size = 26, normalized size = 0.96

$$\frac{2}{d\sqrt{a\cosh\left(dx+c\right)-a\sinh\left(dx+c\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*cosh(d*x+c)-a*sinh(d*x+c))^(1/2),x)
[Out] 2/d/(a*cosh(d*x+c)-a*sinh(d*x+c))^(1/2)
```

maxima [A] time = 0.35, size = 17, normalized size = 0.63

$$\frac{2\,e^{\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}}{\sqrt{a}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cosh(d*x+c)-a*sinh(d*x+c))^(1/2),x, algorithm="maxima")
[Out] 2*e^(1/2*d*x + 1/2*c)/(sqrt(a)*d)
```

mupad [**B**] time = 0.17, size = 27, normalized size = 1.00

$$\frac{2\,\mathrm{e}^{c+d\,x}\,\sqrt{a\,\mathrm{e}^{-c-d\,x}}}{a\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*cosh(c + d*x) - a*sinh(c + d*x))^(1/2),x)
[Out] (2*exp(c + d*x)*(a*exp(- c - d*x))^(1/2))/(a*d)
```

sympy [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-a\sinh\left(c+dx\right)+a\cosh\left(c+dx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cosh(d*x+c)-a*sinh(d*x+c))**(1/2),x)
[Out] Integral(1/sqrt(-a*sinh(c + d*x) + a*cosh(c + d*x)), x)
```

3.614 $\int (a\,\mathrm{sech}(x) + b\tanh(x))^5\,dx$

Optimal. Leaf size=124

$$-\frac{1}{8}ab^2\left(3a^2+7b^2\right)\sinh(x)-\frac{1}{8}\mathrm{sech}^2(x)(a+b\sinh(x))^2\left(2b\left(a^2+2b^2\right)-a\left(3a^2+5b^2\right)\sinh(x)\right)+\frac{1}{8}a\left(3a^4+10a^2b^2\right.$$

```
[Out] 1/8*a*(3*a^4+10*a^2*b^2+15*b^4)*arctan(sinh(x))+b^5*ln(cosh(x))-1/8*a*b^2*(
3*a^2+7*b^2)*sinh(x)-1/4*sech(x)^4*(b-a*sinh(x))*(a+b*sinh(x))^4-1/8*sech(x
)^2*(a+b*sinh(x))^2*(2*b*(a^2+2*b^2)-a*(3*a^2+5*b^2)*sinh(x))
```

Rubi [A] time = 0.19, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.727, Rules used = {4391, 2668, 739, 819, 774, 635, 204, 260}

$$-\frac{1}{8}ab^2\left(3a^2+7b^2\right)\sinh(x)+\frac{1}{8}a\left(10a^2b^2+3a^4+15b^4\right)\tan^{-1}(\sinh(x))-\frac{1}{8}\mathrm{sech}^2(x)(a+b\sinh(x))^2\left(2b\left(a^2+2b^2\right)-\right.$$

Antiderivative was successfully verified.

```
[In] Int[(a*Sech[x] + b*Tanh[x])^5,x]
```

```
[Out] (a*(3*a^4 + 10*a^2*b^2 + 15*b^4)*ArcTan[Sinh[x]])/8 + b^5*Log[Cosh[x]] - (a
*b^2*(3*a^2 + 7*b^2)*Sinh[x])/8 - (Sech[x]^4*(b - a*Sinh[x])*(a + b*Sinh[x]
)^4)/4 - (Sech[x]^2*(a + b*Sinh[x])^2*(2*b*(a^2 + 2*b^2) - a*(3*a^2 + 5*b^2
)*Sinh[x]))/8
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 739

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m - 1)*(a*e - c*d*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)), x] +
 Dist[1/((p + 1)*(-2*a*c)), Int[(d + e*x)^(m - 2)*Simp[a*e^2*(m - 1) - c*d^
2*(2*p + 3) - d*c*e*(m + 2*p + 2)*x, x]*(a + c*x^2)^(p + 1), x], x] /; Free
Q[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && I
ntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 774

```
Int[(((d_.) + (e_.)*(x_))*((f_) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2), x_Sym
bol] :> Simp[(e*g*x)/c, x] + Dist[1/c, Int[(c*d*f - a*e*g + c*(e*f + d*g)*x
)/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e, f, g}, x]
```

Rule 819

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m - 1)*(a + c*x^2)^(p + 1)*(a*(e*f + d*g
) - (c*d*f - a*e*g)*x))/(2*a*c*(p + 1)), x] - Dist[1/(2*a*c*(p + 1)), Int[(
d + e*x)^(m - 2)*(a + c*x^2)^(p + 1)*Simp[a*e*(e*f*(m - 1) + d*g*m) - c*d^2
*f*(2*p + 3) + e*(a*e*g*m - c*d*f*(m + 2*p + 2))*x, x], x], x] /; FreeQ[{a,
 c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] &&
(EqQ[d, 0] || (EqQ[m, 2] && EqQ[p, -3] && RationalQ[a, c, d, e, f, g]) ||
!ILtQ[m + 2*p + 3, 0])
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 4391

```
Int[(u_.)*((b_.)*sec[(c_.) + (d_.)*(x_)]^(n_.) + (a_.)*tan[(c_.) + (d_.)*(x
_)]^(n_.))^(p_), x_Symbol] :> Int[ActivateTrig[u]*Sec[c + d*x]^(n*p)*(b + a
*Sin[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

Rubi steps

$$\int (a\text{sech}(x) + b\tanh(x))^5\,dx = \int \text{sech}^5(x)(a + b\sinh(x))^5\,dx$$

$$= -\left(b^5\,\text{Subst}\left(\int \frac{(a+x)^5}{\left(-b^2-x^2\right)^3}\,dx, x, b\sinh(x)\right)\right)$$

$$= -\frac{1}{4}\text{sech}^4(x)(b - a\sinh(x))(a + b\sinh(x))^4 - \frac{1}{4}b^3\,\text{Subst}\left(\int \frac{(a+x)^3\left(-3a^2 - 4b^2\right.}{\left(-b^2-x^2\right)^2}\right.$$

$$= -\frac{1}{4}\text{sech}^4(x)(b - a\sinh(x))(a + b\sinh(x))^4 - \frac{1}{8}\text{sech}^2(x)(a + b\sinh(x))^2\left(2b\left(a^2 +\right.\right.$$

$$= -\frac{1}{8}ab^2\left(3a^2 + 7b^2\right)\sinh(x) - \frac{1}{4}\text{sech}^4(x)(b - a\sinh(x))(a + b\sinh(x))^4 - \frac{1}{8}\text{sech}^2$$

$$= -\frac{1}{8}ab^2\left(3a^2 + 7b^2\right)\sinh(x) - \frac{1}{4}\text{sech}^4(x)(b - a\sinh(x))(a + b\sinh(x))^4 - \frac{1}{8}\text{sech}^2$$

$$= \frac{1}{8}a\left(3a^4 + 10a^2b^2 + 15b^4\right)\tan^{-1}(\sinh(x)) + b^5\log(\cosh(x)) - \frac{1}{8}ab^2\left(3a^2 + 7b^2\right)\text{s}$$

Mathematica [B] time = 1.96, size = 355, normalized size = 2.86

$$\frac{\frac{2\text{sech}^2(x)\left(a\left(3a^2-5b^2\right)\sinh(x)+6a^2b-2b^3\right)(a+b\sinh(x))^6}{a^2+b^2} + \frac{b\left(2ab^5\left(5b^2-3a^2\right)\sinh^5(x)+4b^4\left(-9a^4+12a^2b^2+b^4\right)\sinh^4(x)+10ab^3\left(-9a^4+8a^2b^2+b^4\right)\sinh\right.}{}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Sech[x] + b*Tanh[x])^5,x]
```

```
[Out] (4*Sech[x]^4*(b + a*Sinh[x])*(a + b*Sinh[x])^6 + (2*Sech[x]^2*(a + b*Sinh[x
])^6*(6*a^2*b - 2*b^3 + a*(3*a^2 - 5*b^2)*Sinh[x]))/(a^2 + b^2) + (b*(((a^2
 + b^2)^2*((3*a^5 + 10*a^3*b^2 + 15*a*b^4 + 8*b^4*Sqrt[-b^2])*Log[Sqrt[-b^2
] - b*Sinh[x]] + (-3*a^5 - 10*a^3*b^2 - 15*a*b^4 + 8*(-b^2)^(5/2))*Log[Sqrt
[-b^2] + b*Sinh[x]]))/Sqrt[-b^2] - 10*a*b*(9*a^6 + 6*a^4*b^2 + 8*a^2*b^4 +
3*b^6)*Sinh[x] - 8*b^2*(15*a^6 - 4*a^4*b^2 + 2*a^2*b^4 + b^6)*Sinh[x]^2 + 1
0*a*b^3*(-9*a^4 + 8*a^2*b^2 + b^4)*Sinh[x]^3 + 4*b^4*(-9*a^4 + 12*a^2*b^2 +
 b^4)*Sinh[x]^4 + 2*a*b^5*(-3*a^2 + 5*b^2)*Sinh[x]^5))/(a^2 + b^2))/(16*(a^
2 + b^2))
```

fricas [B] time = 0.46, size = 2040, normalized size = 16.45

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*sech(x)+b*tanh(x))^5,x, algorithm="fricas")

[Out] -1/4*(4*b^5*x*cosh(x)^8 + 4*b^5*x*sinh(x)^8 - (3*a^5 + 10*a^3*b^2 - 25*a*b^4)*cosh(x)^7 + (32*b^5*x*cosh(x) - 3*a^5 - 10*a^3*b^2 + 25*a*b^4)*sinh(x)^7 + 16*(b^5*x + 5*a^2*b^3 - b^5)*cosh(x)^6 + (112*b^5*x*cosh(x)^2 + 16*b^5*x + 80*a^2*b^3 - 16*b^5 - 7*(3*a^5 + 10*a^3*b^2 - 25*a*b^4)*cosh(x))*sinh(x)^6 + 4*b^5*x - (11*a^5 - 70*a^3*b^2 + 15*a*b^4)*cosh(x)^5 + (224*b^5*x*cosh(x)^3 - 11*a^5 + 70*a^3*b^2 - 15*a*b^4 - 21*(3*a^5 + 10*a^3*b^2 - 25*a*b^4)*cosh(x)^2 + 96*(b^5*x + 5*a^2*b^3 - b^5)*cosh(x))*sinh(x)^5 + 8*(3*b^5*x + 10*a^4*b - 2*b^5)*cosh(x)^4 + (280*b^5*x*cosh(x)^4 + 24*b^5*x + 80*a^4*b - 16*b^5 - 35*(3*a^5 + 10*a^3*b^2 - 25*a*b^4)*cosh(x)^3 + 240*(b^5*x + 5*a^2*b^3 - b^5)*cosh(x)^2 - 5*(11*a^5 - 70*a^3*b^2 + 15*a*b^4)*cosh(x))*sinh(x)^4 + (11*a^5 - 70*a^3*b^2 + 15*a*b^4)*cosh(x)^3 + (224*b^5*x*cosh(x)^5 + 11*a^5 - 70*a^3*b^2 + 15*a*b^4 - 35*(3*a^5 + 10*a^3*b^2 - 25*a*b^4)*cosh(x)^4 + 320*(b^5*x + 5*a^2*b^3 - b^5)*cosh(x)^3 - 10*(11*a^5 - 70*a^3*b^2 + 15*a*b^4)*cosh(x)^2 + 32*(3*b^5*x + 10*a^4*b - 2*b^5)*cosh(x))*sinh(x)^3 + 16*(b^5*x + 5*a^2*b^3 - b^5)*cosh(x)^2 + (112*b^5*x*cosh(x)^6 + 16*b^5*x - 21*(3*a^5 + 10*a^3*b^2 - 25*a*b^4)*cosh(x)^5 + 80*a^2*b^3 - 16*b^5 + 240*(b^5*x + 5*a^2*b^3 - b^5)*cosh(x)^4 - 10*(11*a^5 - 70*a^3*b^2 + 15*a*b^4)*cosh(x)^3 + 48*(3*b^5*x + 10*a^4*b - 2*b^5)*cosh(x)^2 + 3*(11*a^5 - 70*a^3*b^2 + 15*a*b^4)*cosh(x))*sinh(x)^2 - ((3*a^5 + 10*a^3*b^2 + 15*a*b^4)*cosh(x)^8 + 8*(3*a^5 + 10*a^3*b^2 + 15*a*b^4)*cosh(x)*sinh(x)^7 + (3*a^5 + 10*a^3*b^2 + 15*a*b^4)*sinh(x)^8 + 4*(3*a^5 + 10*a^3*b^2 + 15*a*b^4)*cosh(x)^6 + 4*(3*a^5 + 10*a^3*b^2 + 15*a*b^4 + 7*(3*a^5 + 10*a^3*b^2 + 15*a*b^4)*cosh(x)^2)*sinh(x)^6 + 8*(7*(3*a^5 + 10*a^3*b^2 + 15*a*b^4)*cosh(x)^3 + 3*(3*a^5 + 10*a^3*b^2 + 15*a*b^4)*cosh(x))*sinh(x)^5 + 3*a^5 + 10*a^3*b^2 + 15*a*b^4 + 6*(3*a^5 + 10*a^3*b^2 + 15*a*b^4)*cosh(x)^4 + 2*(9*a^5 + 30*a^3*b^2 + 45*a*b^4 + 35*(3*a^5 + 10*a^3*b^2 + 15*a*b^4)*cosh(x)^4 + 30*(3*a^5 + 10*a^3*b^2 + 15*a*b^4)*cosh(x)^2)*sinh(x)^4 + 8*(7*(3*a^5 + 10*a^3*b^2 + 15*a*b^4)*cosh(x)^5 + 10*(3*a^5 + 10*a^3*b^2 + 15*a*b^4)*cosh(x)^3 + 3*(3*a^5 + 10*a^3*b^2 + 15*a*b^4)*cosh(x))*sinh(x)^3 + 4*(3*a^5 + 10*a^3*b^2 + 15*a*b^4)*cosh(x)^2 + 4*(7*(3*a^5 + 10*a^3*b^2 + 15*a*b^4)*cosh(x)^6 + 3*a^5 + 10*a^3*b^2 + 15*a*b^4 + 15*(3*a^5 + 10*a^3*b^2 + 15*a*b^4)*cosh(x)^4 + 9*(3*a^5 + 10*a^3*b^2 + 15*a*b^4)*cosh(x)^2)*sinh(x)^2 + 8*((3*a^5 + 10*a^3*b^2 + 15*a*b^4)*cosh(x)^7 + 3*(3*a^5 + 10*a^3*b^2 + 15*a*b^4)*cosh(x)^5 + 3*(3*a^5 + 10*a^3*b^2 + 15*a*b^4)*cosh(x)^3 + (3*a^5 + 10*a^3*b^2 + 15*a*b^4)*cosh(x))*sinh(x))*arctan(cosh(x) + sinh(x)) + (3*a^5 + 10*a^3*b^2 - 25*a*b^4)*cosh(x) - 4*(b^5*cosh(x)^8 + 8*b^5*cosh(x)*sinh(x)^7 + b^5*sinh(x)^8 + 4*b^5*cosh(x)^6 + 6*b^5*cosh(x)^4 + 4*b^5*cosh(x)^2 + 4*(7*b^5*cosh(x)^2 + b^5)*sinh(x)^6 + 8*(7*b^5*cosh(x)^3 + 3*b^5*cosh(x))*sinh(x)^5 + b^5 + 2*(35*b^5*cosh(x)^4 + 30*b^5*cosh(x)^2 + 3*b^5)*sinh(x)^4 + 8*(7*b^5*cosh(x)^5 + 10*b^5*cosh(x)^3 + 3*b^5*cosh(x))*sinh(x)^3 + 4*(7*b^5*cosh(x)^6 + 15*b^5*cosh(x)^4 + 9*b^5*cosh(x)^2 + b^5)*sinh(x)^2 + 8*(b^5*cosh(x)^7 + 3*b^5*cosh(x)^5 + 3*b^5

```
*cosh(x)^3 + b^5*cosh(x))*sinh(x))*log(2*cosh(x)/(cosh(x) - sinh(x))) + (32
*b^5*x*cosh(x)^7 - 7*(3*a^5 + 10*a^3*b^2 - 25*a*b^4)*cosh(x)^6 + 96*(b^5*x
+ 5*a^2*b^3 - b^5)*cosh(x)^5 + 3*a^5 + 10*a^3*b^2 - 25*a*b^4 - 5*(11*a^5 -
70*a^3*b^2 + 15*a*b^4)*cosh(x)^4 + 32*(3*b^5*x + 10*a^4*b - 2*b^5)*cosh(x)^
3 + 3*(11*a^5 - 70*a^3*b^2 + 15*a*b^4)*cosh(x)^2 + 32*(b^5*x + 5*a^2*b^3 -
b^5)*cosh(x))*sinh(x))/(cosh(x)^8 + 8*cosh(x)*sinh(x)^7 + sinh(x)^8 + 4*(7*
cosh(x)^2 + 1)*sinh(x)^6 + 4*cosh(x)^6 + 8*(7*cosh(x)^3 + 3*cosh(x))*sinh(x
)^5 + 2*(35*cosh(x)^4 + 30*cosh(x)^2 + 3)*sinh(x)^4 + 6*cosh(x)^4 + 8*(7*co
sh(x)^5 + 10*cosh(x)^3 + 3*cosh(x))*sinh(x)^3 + 4*(7*cosh(x)^6 + 15*cosh(x)
^4 + 9*cosh(x)^2 + 1)*sinh(x)^2 + 4*cosh(x)^2 + 8*(cosh(x)^7 + 3*cosh(x)^5
+ 3*cosh(x)^3 + cosh(x))*sinh(x) + 1)
```

giac [B] time = 0.14, size = 240, normalized size = 1.94

$$\frac{1}{2}\, b^5 \log\left(\left(e^{(-x)} - e^x\right)^2 + 4\right) + \frac{1}{16}\left(\pi + 2\,\arctan\left(\frac{1}{2}\left(e^{(2\,x)} - 1\right)e^{(-x)}\right)\right)\left(3\,a^5 + 10\,a^3b^2 + 15\,ab^4\right) - \frac{3\,b^5\left(e^{(-x)} - e^x\right)^4 + 3}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sech(x)+b*tanh(x))^5,x, algorithm="giac")
```

```
[Out] 1/2*b^5*log((e^(-x) - e^x)^2 + 4) + 1/16*(pi + 2*arctan(1/2*(e^(2*x) - 1)*e
^(-x)))*(3*a^5 + 10*a^3*b^2 + 15*a*b^4) - 1/4*(3*b^5*(e^(-x) - e^x)^4 + 3*a
^5*(e^(-x) - e^x)^3 + 10*a^3*b^2*(e^(-x) - e^x)^3 - 25*a*b^4*(e^(-x) - e^x)
^3 + 80*a^2*b^3*(e^(-x) - e^x)^2 + 8*b^5*(e^(-x) - e^x)^2 + 20*a^5*(e^(-x)
- e^x) - 40*a^3*b^2*(e^(-x) - e^x) - 60*a*b^4*(e^(-x) - e^x) + 80*a^4*b + 1
60*a^2*b^3)/((e^(-x) - e^x)^2 + 4)^2
```

maple [A] time = 0.34, size = 201, normalized size = 1.62

$$\frac{a^5 \tanh(x)\mathrm{sech}(x)^3}{4} + \frac{3a^5\mathrm{sech}(x)\tanh(x)}{8} + \frac{3a^5 \arctan\left(\mathrm{e}^x\right)}{4} - \frac{5a^4b}{4\cosh(x)^4} - \frac{10a^3b^2\sinh(x)}{3\cosh(x)^4} + \frac{5a^3b^2\tanh(x)\mathrm{sech}(x)^3}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*sech(x)+b*tanh(x))^5,x)
```

```
[Out] 1/4*a^5*tanh(x)*sech(x)^3+3/8*a^5*sech(x)*tanh(x)+3/4*a^5*arctan(exp(x))-5/
4*a^4*b/cosh(x)^4-10/3*a^3*b^2*sinh(x)/cosh(x)^4+5/6*a^3*b^2*tanh(x)*sech(x
)^3+5/4*a^3*b^2*sech(x)*tanh(x)+5/2*a^3*b^2*arctan(exp(x))-5*a^2*b^3*sinh(x
)^2/cosh(x)^4-5/2*a^2*b^3/cosh(x)^4-5*a*b^4*sinh(x)^3/cosh(x)^4-5*a*b^4*sin
h(x)/cosh(x)^4+5/4*a*b^4*tanh(x)*sech(x)^3+15/8*a*b^4*sech(x)*tanh(x)+15/4*
a*b^4*arctan(exp(x))+b^5*ln(cosh(x))-1/2*b^5*tanh(x)^2-1/4*b^5*tanh(x)^4
```

maxima [B] time = 0.54, size = 279, normalized size = 2.25

$$\frac{5}{2}\,a^2b^3\tanh(x)^4+b^5\left(x+\frac{4\left(e^{(-2\,x)}+e^{(-4\,x)}+e^{(-6\,x)}\right)}{4\,e^{(-2\,x)}+6\,e^{(-4\,x)}+4\,e^{(-6\,x)}+e^{(-8\,x)}+1}+\log\left(e^{(-2\,x)}+1\right)\right)-\frac{5}{4}\,ab^4\left(\frac{5\,e^{(-x)}-3\,e^{(-3\,x)}+}{4\,e^{(-2\,x)}+6\,e^{(-4\,x)}+}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sech(x)+b*tanh(x))^5,x, algorithm="maxima")
```

```
[Out] 5/2*a^2*b^3*tanh(x)^4 + b^5*(x + 4*(e^(-2*x) + e^(-4*x) + e^(-6*x))/(4*e^(-
2*x) + 6*e^(-4*x) + 4*e^(-6*x) + e^(-8*x) + 1) + log(e^(-2*x) + 1)) - 5/4*a
*b^4*((5*e^(-x) - 3*e^(-3*x) + 3*e^(-5*x) - 5*e^(-7*x))/(4*e^(-2*x) + 6*e^(
-4*x) + 4*e^(-6*x) + e^(-8*x) + 1) + 3*arctan(e^(-x))) + 1/4*a^5*((3*e^(-x)
 + 11*e^(-3*x) - 11*e^(-5*x) - 3*e^(-7*x))/(4*e^(-2*x) + 6*e^(-4*x) + 4*e^(
-6*x) + e^(-8*x) + 1) - 3*arctan(e^(-x))) + 5/2*a^3*b^2*((e^(-x) - 7*e^(-3*
x) + 7*e^(-5*x) - e^(-7*x))/(4*e^(-2*x) + 6*e^(-4*x) + 4*e^(-6*x) + e^(-8*x
) + 1) - arctan(e^(-x))) - 20*a^4*b/(e^(-x) + e^x)^4
```

mupad [B] time = 1.92, size = 495, normalized size = 3.99

$$\frac{e^x\,\left(4\,a^5-40\,a^3\,b^2+20\,a\,b^4\right)-20\,a^4\,b-4\,b^5+40\,a^2\,b^3}{4\,e^{2\,x}+6\,e^{4\,x}+4\,e^{6\,x}+e^{8\,x}+1}+b^5\,\ln\left(\left(\frac{3\,a^5\,e^x}{4}-2\,\sqrt{-\frac{9\,a^{10}}{64}-\frac{15\,a^8\,b^2}{16}-\frac{95\,a^6\,b^4}{32}-\frac{75}{}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*tanh(x) + a/cosh(x))^5,x)
```

```
[Out] (exp(x)*(20*a*b^4 + 4*a^5 - 40*a^3*b^2) - 20*a^4*b - 4*b^5 + 40*a^2*b^3)/(4
*exp(2*x) + 6*exp(4*x) + 4*exp(6*x) + exp(8*x) + 1) + b^5*log(((3*a^5*exp(x
))/4 - 2*(- (9*a^10)/64 - (225*a^2*b^8)/64 - (75*a^4*b^6)/16 - (95*a^6*b^4)
/32 - (15*a^8*b^2)/16)^(1/2) + (15*a*b^4*exp(x))/4 + (5*a^3*b^2*exp(x))/2)*
(2*(- (9*a^10)/64 - (225*a^2*b^8)/64 - (75*a^4*b^6)/16 - (95*a^6*b^4)/32 -
(15*a^8*b^2)/16)^(1/2) + (3*a^5*exp(x))/4 + (15*a*b^4*exp(x))/4 + (5*a^3*b^
2*exp(x))/2)) - (exp(x)*(30*a*b^4 + 6*a^5 - 60*a^3*b^2) - 40*a^4*b - 8*b^5
+ 80*a^2*b^3)/(3*exp(2*x) + 3*exp(4*x) + exp(6*x) + 1) - b^5*x + (atan((4*e
xp(x)*((15*a*b^4)/4 + (3*a^5)/4 + (5*a^3*b^2)/2))/(9*a^10 + 225*a^2*b^8 + 3
00*a^4*b^6 + 190*a^6*b^4 + 60*a^8*b^2)^(1/2))*(9*a^10 + 225*a^2*b^8 + 300*a
^4*b^6 + 190*a^6*b^4 + 60*a^8*b^2)^(1/2))/4 + (exp(x)*((3*a^5)/4 - (25*a*b^
4)/4 + (5*a^3*b^2)/2) + 4*b^5 - 20*a^2*b^3)/(exp(2*x) + 1) + (exp(x)*((45*a
*b^4)/2 + a^5/2 - 25*a^3*b^2) - 20*a^4*b - 8*b^5 + 60*a^2*b^3)/(2*exp(2*x)
+ exp(4*x) + 1)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\,\text{sech}\,(x) + b\tanh\,(x))^5\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sech(x)+b*tanh(x))**5,x)
```

```
[Out] Integral((a*sech(x) + b*tanh(x))**5, x)
```

3.615 $\int (a\,\mathrm{sech}(x) + b\tanh(x))^4\,dx$

Optimal. Leaf size=100

$$-\frac{4}{3}ab\left(a^2+2b^2\right)\cosh(x)-\frac{1}{3}b^2\left(2a^2+3b^2\right)\sinh(x)\cosh(x)+\frac{1}{3}\mathrm{sech}(x)(a+b\sinh(x))^2\left(\left(2a^2+3b^2\right)\sinh(x)+ab\right)$$

```
[Out] b^4*x-4/3*a*b*(a^2+2*b^2)*cosh(x)-1/3*b^2*(2*a^2+3*b^2)*cosh(x)*sinh(x)-1/3
*sech(x)^3*(b-a*sinh(x))*(a+b*sinh(x))^3+1/3*sech(x)*(a+b*sinh(x))^2*(a*b+(
2*a^2+3*b^2)*sinh(x))
```

Rubi [A] time = 0.21, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {4391, 2691, 2861, 2734}

$$-\frac{4}{3}ab\left(a^2+2b^2\right)\cosh(x)-\frac{1}{3}b^2\left(2a^2+3b^2\right)\sinh(x)\cosh(x)+\frac{1}{3}\mathrm{sech}(x)(a+b\sinh(x))^2\left(\left(2a^2+3b^2\right)\sinh(x)+ab\right)$$

Antiderivative was successfully verified.

```
[In] Int[(a*Sech[x] + b*Tanh[x])^4,x]
```

```
[Out] b^4*x - (4*a*b*(a^2 + 2*b^2)*Cosh[x])/3 - (b^2*(2*a^2 + 3*b^2)*Cosh[x]*Sinh
[x])/3 - (Sech[x]^3*(b - a*Sinh[x])*(a + b*Sinh[x])^3)/3 + (Sech[x]*(a + b*
Sinh[x])^2*(a*b + (2*a^2 + 3*b^2)*Sinh[x]))/3
```

Rule 2691

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[((g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x]
)^(m - 1)*(b + a*Sin[e + f*x]))/(f*g*(p + 1)), x] + Dist[1/(g^2*(p + 1)), I
nt[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 2)*(b^2*(m - 1) + a^2
*(p + 2) + a*b*(m + p + 1)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g},
x] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] && LtQ[p, -1] && (IntegersQ[2*m, 2*p]
|| IntegerQ[m])
```

Rule 2734

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[((2*a*c + b*d)*x)/2, x] + (-Simp[((b*c + a*d)*Co
s[e + f*x])/f, x] - Simp[(b*d*Cos[e + f*x]*Sin[e + f*x])/(2*f), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2861

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[((g*
```

```
Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m*(d + c*Sin[e + f*x]))/(f*g*(p
+ 1)), x] + Dist[1/(g^2*(p + 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e
 + f*x])^(m - 1)*Simp[a*c*(p + 2) + b*d*m + b*c*(m + p + 2)*Sin[e + f*x], x
], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b^2, 0] && GtQ[m,
 0] && LtQ[p, -1] && IntegerQ[2*m] &&  !(EqQ[m, 1] && NeQ[c^2 - d^2, 0] &&
SimplerQ[c + d*x, a + b*x])
```

Rule 4391

```
Int[(u_.)*((b_.)*sec[(c_.) + (d_.)*(x_)]^(n_.) + (a_.)*tan[(c_.) + (d_.)*(x
_)]^(n_.))^(p_), x_Symbol] :> Int[ActivateTrig[u]*Sec[c + d*x]^(n*p)*(b + a
*Sin[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

Rubi steps

$$\int (a\text{sech}(x)+b\tanh(x))^4\,dx = \int \text{sech}^4(x)(a+b\sinh(x))^4\,dx$$
$$= -\frac{1}{3}\text{sech}^3(x)(b-a\sinh(x))(a+b\sinh(x))^3 - \frac{1}{3}\int \text{sech}^2(x)(a+b\sinh(x))^2\left(-2a^2\right.$$
$$= -\frac{1}{3}\text{sech}^3(x)(b-a\sinh(x))(a+b\sinh(x))^3 + \frac{1}{3}\text{sech}(x)(a+b\sinh(x))^2\left(ab+\left(2a\right.\right.$$
$$= b^4x - \frac{4}{3}ab\left(a^2+2b^2\right)\cosh(x) - \frac{1}{3}b^2\left(2a^2+3b^2\right)\cosh(x)\sinh(x) - \frac{1}{3}\text{sech}^3(x)(b-$$

Mathematica [A] time = 0.19, size = 79, normalized size = 0.79

$$\frac{1}{3}\left(-4ab\left(a^2-b^2\right)\text{sech}^3(x)+2\left(a^4+3a^2b^2-2b^4\right)\tanh(x)+\left(a^4-6a^2b^2+b^4\right)\tanh(x)\text{sech}^2(x)-12ab^3\text{sech}(x)+\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Sech[x] + b*Tanh[x])^4,x]
```

```
[Out] (3*b^4*x - 12*a*b^3*Sech[x] - 4*a*b*(a^2 - b^2)*Sech[x]^3 + 2*(a^4 + 3*a^2*
b^2 - 2*b^4)*Tanh[x] + (a^4 - 6*a^2*b^2 + b^4)*Sech[x]^2*Tanh[x])/3
```

fricas [B] time = 0.40, size = 207, normalized size = 2.07

$$-\frac{24\,ab^3\cosh(x)^2+16\,a^3b+8\,ab^3-\left(3\,b^4x-2\,a^4-6\,a^2b^2+4\,b^4\right)\cosh(x)^3-2\left(a^4+3\,a^2b^2-2\,b^4\right)\sinh(x)^3+3\left(\right.}{3\left(\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sech(x)+b*tanh(x))^4,x, algorithm="fricas")
```

```
[Out] -1/3*(24*a*b^3*cosh(x)^2 + 16*a^3*b + 8*a*b^3 - (3*b^4*x - 2*a^4 - 6*a^2*b^
2 + 4*b^4)*cosh(x)^3 - 2*(a^4 + 3*a^2*b^2 - 2*b^4)*sinh(x)^3 + 3*(8*a*b^3 -
 (3*b^4*x - 2*a^4 - 6*a^2*b^2 + 4*b^4)*cosh(x))*sinh(x)^2 - 3*(3*b^4*x - 2*
a^4 - 6*a^2*b^2 + 4*b^4)*cosh(x) - 6*(a^4 - 3*a^2*b^2 + (a^4 + 3*a^2*b^2 -
2*b^4)*cosh(x)^2)*sinh(x))/(cosh(x)^3 + 3*cosh(x)*sinh(x)^2 + 3*cosh(x))
```

giac [A] time = 0.13, size = 110, normalized size = 1.10

$$b^4x - \frac{4\left(6\,ab^3e^{(5\,x)} + 9\,a^2b^2e^{(4\,x)} - 3\,b^4e^{(4\,x)} + 8\,a^3be^{(3\,x)} + 4\,ab^3e^{(3\,x)} + 3\,a^4e^{(2\,x)} - 3\,b^4e^{(2\,x)} + 6\,ab^3e^{x} + a^4 + 3\,a^2b^2}{3\left(e^{(2\,x)}+1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sech(x)+b*tanh(x))^4,x, algorithm="giac")
```

```
[Out] b^4*x - 4/3*(6*a*b^3*e^(5*x) + 9*a^2*b^2*e^(4*x) - 3*b^4*e^(4*x) + 8*a^3*b*
e^(3*x) + 4*a*b^3*e^(3*x) + 3*a^4*e^(2*x) - 3*b^4*e^(2*x) + 6*a*b^3*e^x + a
^4 + 3*a^2*b^2 - 2*b^4)/(e^(2*x) + 1)^3
```

maple [A] time = 0.39, size = 94, normalized size = 0.94

$$a^4\left(\frac{2}{3}+\frac{\mathrm{sech}(x)^2}{3}\right)\tanh(x) - \frac{4a^3b}{3\cosh(x)^3} + 6a^2b^2\left(-\frac{\sinh(x)}{2\cosh(x)^3} + \frac{\left(\frac{2}{3}+\frac{\mathrm{sech}(x)^2}{3}\right)\tanh(x)}{2}\right) + 4a\,b^3\left(-\frac{\sinh^2(x)}{\cosh(x)^3} - \frac{}{3\,\mathrm{c}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*sech(x)+b*tanh(x))^4,x)
```

```
[Out] a^4*(2/3+1/3*sech(x)^2)*tanh(x)-4/3*a^3*b/cosh(x)^3+6*a^2*b^2*(-1/2*sinh(x)
/cosh(x)^3+1/2*(2/3+1/3*sech(x)^2)*tanh(x))+4*a*b^3*(-sinh(x)^2/cosh(x)^3-2
/3/cosh(x)^3)+b^4*(x-tanh(x)-1/3*tanh(x)^3)
```

maxima [B] time = 0.38, size = 210, normalized size = 2.10

$$2\,a^2b^2\tanh(x)^3 + \frac{1}{3}\,b^4\left(3\,x - \frac{4\left(3\,e^{(-2\,x)} + 3\,e^{(-4\,x)} + 2\right)}{3\,e^{(-2\,x)} + 3\,e^{(-4\,x)} + e^{(-6\,x)} + 1}\right) - \frac{8}{3}\,ab^3\left(\frac{3\,e^{(-x)}}{3\,e^{(-2\,x)} + 3\,e^{(-4\,x)} + e^{(-6\,x)} + 1} + \frac{2}{3\,e^{(-2\,x)} + 3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sech(x)+b*tanh(x))^4,x, algorithm="maxima")
```

```
[Out] 2*a^2*b^2*tanh(x)^3 + 1/3*b^4*(3*x - 4*(3*e^(-2*x) + 3*e^(-4*x) + 2)/(3*e^(
-2*x) + 3*e^(-4*x) + e^(-6*x) + 1)) - 8/3*a*b^3*(3*e^(-x)/(3*e^(-2*x) + 3*e
```

^(-4*x) + e^(-6*x) + 1) + 2*e^(-3*x)/(3*e^(-2*x) + 3*e^(-4*x) + e^(-6*x) + 1) + 3*e^(-5*x)/(3*e^(-2*x) + 3*e^(-4*x) + e^(-6*x) + 1)) + 4/3*a^4*(3*e^(-2*x)/(3*e^(-2*x) + 3*e^(-4*x) + e^(-6*x) + 1) + 1/(3*e^(-2*x) + 3*e^(-4*x) + e^(-6*x) + 1)) - 32/3*a^3*b/(e^(-x) + e^x)^3

mupad [B] time = 1.52, size = 145, normalized size = 1.45

$$\frac{e^x\left(\frac{32\,a\,b^3}{3}-\frac{32\,a^3\,b}{3}\right)-4\,a^4-4\,b^4+24\,a^2\,b^2}{2\,e^{2\,x}+e^{4\,x}+1}-\frac{12\,a^2\,b^2+8\,e^x\,a\,b^3-4\,b^4}{e^{2\,x}+1}+b^4\,x-\frac{e^x\left(\frac{32\,a\,b^3}{3}-\frac{32\,a^3\,b}{3}\right)-\frac{8\,a^4}{3}-\frac{8\,b^4}{3}+16}{3\,e^{2\,x}+3\,e^{4\,x}+e^{6\,x}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*tanh(x) + a/cosh(x))^4,x)
```

```
[Out] (exp(x)*((32*a*b^3)/3 - (32*a^3*b)/3) - 4*a^4 - 4*b^4 + 24*a^2*b^2)/(2*exp(
2*x) + exp(4*x) + 1) - (12*a^2*b^2 - 4*b^4 + 8*a*b^3*exp(x))/(exp(2*x) + 1)
 + b^4*x - (exp(x)*((32*a*b^3)/3 - (32*a^3*b)/3) - (8*a^4)/3 - (8*b^4)/3 +
16*a^2*b^2)/(3*exp(2*x) + 3*exp(4*x) + exp(6*x) + 1)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\,\mathrm{sech}\,(x)+b\tanh(x))^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sech(x)+b*tanh(x))**4,x)
```

```
[Out] Integral((a*sech(x) + b*tanh(x))**4, x)
```

3.616 $\int (a\,\mathrm{sech}(x) + b\tanh(x))^3\,dx$

Optimal. Leaf size=58

$$\frac{1}{2}a\left(a^2+3b^2\right)\tan^{-1}(\sinh(x))-\frac{1}{2}ab^2\sinh(x)-\frac{1}{2}\mathrm{sech}^2(x)(b-a\sinh(x))(a+b\sinh(x))^2+b^3\log(\cosh(x))$$

```
[Out] 1/2*a*(a^2+3*b^2)*arctan(sinh(x))+b^3*ln(cosh(x))-1/2*a*b^2*sinh(x)-1/2*sec
h(x)^2*(b-a*sinh(x))*(a+b*sinh(x))^2
```

Rubi [A] time = 0.11, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.636, Rules used = {4391, 2668, 739, 774, 635, 204, 260}

$$\frac{1}{2}a\left(a^2+3b^2\right)\tan^{-1}(\sinh(x))-\frac{1}{2}ab^2\sinh(x)-\frac{1}{2}\mathrm{sech}^2(x)(b-a\sinh(x))(a+b\sinh(x))^2+b^3\log(\cosh(x))$$

Antiderivative was successfully verified.

```
[In] Int[(a*Sech[x] + b*Tanh[x])^3,x]
```

```
[Out] (a*(a^2 + 3*b^2)*ArcTan[Sinh[x]])/2 + b^3*Log[Cosh[x]] - (a*b^2*Sinh[x])/2
- (Sech[x]^2*(b - a*Sinh[x])*(a + b*Sinh[x])^2)/2
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] && !NiceSqrtQ[-(a*c)]
```

Rule 739

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m - 1)*(a*e - c*d*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)), x] +
 Dist[1/((p + 1)*(-2*a*c)), Int[(d + e*x)^(m - 2)*Simp[a*e^2*(m - 1) - c*d^
```

```
2*(2*p + 3) - d*c*e*(m + 2*p + 2)*x, x]*(a + c*x^2)^(p + 1), x], x] /; Free
Q[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && I
ntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 774

```
Int[(((d_.) + (e_.)*(x_))*((f_) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2), x_Sym
bol] :> Simp[(e*g*x)/c, x] + Dist[1/c, Int[(c*d*f - a*e*g + c*(e*f + d*g)*x
)/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e, f, g}, x]
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 4391

```
Int[(u_.)*((b_.)*sec[(c_.) + (d_.)*(x_)]^(n_.) + (a_.)*tan[(c_.) + (d_.)*(x
_)]^(n_.))^(p_), x_Symbol] :> Int[ActivateTrig[u]*Sec[c + d*x]^(n*p)*(b + a
*Sin[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

Rubi steps

$$\int (a\text{sech}(x) + b\tanh(x))^3\,dx = \int \text{sech}^3(x)(a + b\sinh(x))^3\,dx$$

$$= b^3\,\text{Subst}\left(\int \frac{(a+x)^3}{\left(-b^2-x^2\right)^2}\,dx, x, b\sinh(x)\right)$$

$$= -\frac{1}{2}\text{sech}^2(x)(b - a\sinh(x))(a + b\sinh(x))^2 + \frac{1}{2}b\,\text{Subst}\left(\int \frac{(a+x)\left(-a^2-2b^2+a\right.}{-b^2-x^2}\right.$$

$$= -\frac{1}{2}ab^2\sinh(x) - \frac{1}{2}\text{sech}^2(x)(b - a\sinh(x))(a + b\sinh(x))^2 - \frac{1}{2}b\,\text{Subst}\left(\int \frac{ab^2 -}{}\right.$$

$$= -\frac{1}{2}ab^2\sinh(x) - \frac{1}{2}\text{sech}^2(x)(b - a\sinh(x))(a + b\sinh(x))^2 - b^3\,\text{Subst}\left(\int \frac{x}{-b^2-}\right.$$

$$= \frac{1}{2}a\left(a^2+3b^2\right)\tan^{-1}(\sinh(x)) + b^3\log(\cosh(x)) - \frac{1}{2}ab^2\sinh(x) - \frac{1}{2}\text{sech}^2(x)(b -$$

Mathematica [B] time = 1.99, size = 194, normalized size = 3.34

$$\frac{1}{4}\left(\frac{b\left(\left(a^3+3ab^2-2\left(-b^2\right)^{3/2}\right)\log\left(\sqrt{-b^2}-b\sinh(x)\right)-\left(a^3+3ab^2+2\left(-b^2\right)^{3/2}\right)\log\left(\sqrt{-b^2}+b\sinh(x)\right)\right)}{\sqrt{-b^2}}+\frac{2\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Sech[x] + b*Tanh[x])^3,x]
```

```
[Out] ((b*((a^3 + 3*a*b^2 - 2*(-b^2)^(3/2))*Log[Sqrt[-b^2] - b*Sinh[x]] - (a^3 +
3*a*b^2 + 2*(-b^2)^(3/2))*Log[Sqrt[-b^2] + b*Sinh[x]]))/Sqrt[-b^2] + (2*a^4
*b*Sech[x]^2)/(a^2 + b^2) + (a*(2*a^4 - 4*a^2*b^2 - 7*b^4 + b^4*Cosh[2*x])*
Sech[x]*Tanh[x])/(a^2 + b^2) - (2*b*(-4*a^4 - 2*a^2*b^2 + b^4 + a*b^3*Sinh[
x])*Tanh[x]^2)/(a^2 + b^2))/4
```

fricas [B] time = 0.41, size = 502, normalized size = 8.66

$$-\frac{b^3x\cosh(x)^4+b^3x\sinh(x)^4+b^3x-\left(a^3-3\,ab^2\right)\cosh(x)^3+\left(4\,b^3x\cosh(x)-a^3+3\,ab^2\right)\sinh(x)^3+2\left(b^3x+3\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sech(x)+b*tanh(x))^3,x, algorithm="fricas")
```

```
[Out] -(b^3*x*cosh(x)^4 + b^3*x*sinh(x)^4 + b^3*x - (a^3 - 3*a*b^2)*cosh(x)^3 + (
4*b^3*x*cosh(x) - a^3 + 3*a*b^2)*sinh(x)^3 + 2*(b^3*x + 3*a^2*b - b^3)*cosh
(x)^2 + (6*b^3*x*cosh(x)^2 + 2*b^3*x + 6*a^2*b - 2*b^3 - 3*(a^3 - 3*a*b^2)*
cosh(x))*sinh(x)^2 - ((a^3 + 3*a*b^2)*cosh(x)^4 + 4*(a^3 + 3*a*b^2)*cosh(x)
*sinh(x)^3 + (a^3 + 3*a*b^2)*sinh(x)^4 + a^3 + 3*a*b^2 + 2*(a^3 + 3*a*b^2)*
cosh(x)^2 + 2*(a^3 + 3*a*b^2 + 3*(a^3 + 3*a*b^2)*cosh(x)^2)*sinh(x)^2 + 4*(
(a^3 + 3*a*b^2)*cosh(x)^3 + (a^3 + 3*a*b^2)*cosh(x))*sinh(x))*arctan(cosh(x
) + sinh(x)) + (a^3 - 3*a*b^2)*cosh(x) - (b^3*cosh(x)^4 + 4*b^3*cosh(x)*sin
h(x)^3 + b^3*sinh(x)^4 + 2*b^3*cosh(x)^2 + b^3 + 2*(3*b^3*cosh(x)^2 + b^3)*
sinh(x)^2 + 4*(b^3*cosh(x)^3 + b^3*cosh(x))*sinh(x))*log(2*cosh(x)/(cosh(x)
 - sinh(x))) + (4*b^3*x*cosh(x)^3 + a^3 - 3*a*b^2 - 3*(a^3 - 3*a*b^2)*cosh(
x)^2 + 4*(b^3*x + 3*a^2*b - b^3)*cosh(x))*sinh(x))/(cosh(x)^4 + 4*cosh(x)*s
inh(x)^3 + sinh(x)^4 + 2*(3*cosh(x)^2 + 1)*sinh(x)^2 + 2*cosh(x)^2 + 4*(cos
h(x)^3 + cosh(x))*sinh(x) + 1)
```

giac [B] time = 0.12, size = 117, normalized size = 2.02

$$\frac{1}{2}\,b^3\log\left(\left(e^{(-x)}-e^x\right)^2+4\right)+\frac{1}{4}\left(\pi+2\,\arctan\left(\frac{1}{2}\left(e^{(2\,x)}-1\right)e^{(-x)}\right)\right)\left(a^3+3\,ab^2\right)-\frac{b^3\left(e^{(-x)}-e^x\right)^2+2\,a^3\left(e^{(-x)}-e^x\right)}{2\left(\left(e^{(-x)}-e^{x}\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sech(x)+b*tanh(x))^3,x, algorithm="giac")
```

```
[Out] 1/2*b^3*log((e^(-x) - e^x)^2 + 4) + 1/4*(pi + 2*arctan(1/2*(e^(2*x) - 1)*e^
(-x)))*(a^3 + 3*a*b^2) - 1/2*(b^3*(e^(-x) - e^x)^2 + 2*a^3*(e^(-x) - e^x) -
 6*a*b^2*(e^(-x) - e^x) + 12*a^2*b)/((e^(-x) - e^x)^2 + 4)
```

maple [A] time = 0.37, size = 75, normalized size = 1.29

$$\frac{a^3\mathrm{sech}(x)\tanh(x)}{2}+a^3\arctan\left(\mathrm{e}^x\right)-\frac{3a^2b}{2\cosh(x)^2}-\frac{3a\,b^2\sinh(x)}{\cosh(x)^2}+\frac{3a\,b^2\mathrm{sech}(x)\tanh(x)}{2}+3a\,b^2\arctan\left(\mathrm{e}^x\right)+b^3\ln\left(\mathrm{co}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*sech(x)+b*tanh(x))^3,x)
```

```
[Out] 1/2*a^3*sech(x)*tanh(x)+a^3*arctan(exp(x))-3/2*a^2*b/cosh(x)^2-3*a*b^2/cosh
(x)^2*sinh(x)+3/2*a*b^2*sech(x)*tanh(x)+3*a*b^2*arctan(exp(x))+b^3*ln(cosh(
x))-1/2*b^3*tanh(x)^2
```

maxima [B] time = 0.56, size = 120, normalized size = 2.07

$$\frac{3}{2}a^2b\tanh(x)^2+b^3\left(x+\frac{2\,e^{(-2\,x)}}{2\,e^{(-2\,x)}+e^{(-4\,x)}+1}+\log\left(e^{(-2\,x)}+1\right)\right)-3\,ab^2\left(\frac{e^{(-x)}-e^{(-3\,x)}}{2\,e^{(-2\,x)}+e^{(-4\,x)}+1}+\arctan\left(e^{(-x)}\right)\right)+a^3\left(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sech(x)+b*tanh(x))^3,x, algorithm="maxima")
```

```
[Out] 3/2*a^2*b*tanh(x)^2 + b^3*(x + 2*e^(-2*x)/(2*e^(-2*x) + e^(-4*x) + 1) + log
(e^(-2*x) + 1)) - 3*a*b^2*((e^(-x) - e^(-3*x))/(2*e^(-2*x) + e^(-4*x) + 1)
+ arctan(e^(-x))) + a^3*((e^(-x) - e^(-3*x))/(2*e^(-2*x) + e^(-4*x) + 1) -
arctan(e^(-x)))
```

mupad [B] time = 1.61, size = 233, normalized size = 4.02

$$\mathrm{atan}\left(\frac{\mathrm{e}^x\,\left(a^3+3\,a\,b^2\right)}{\sqrt{a^6+6\,a^4\,b^2+9\,a^2\,b^4}}\right)\,\sqrt{a^6+6\,a^4\,b^2+9\,a^2\,b^4}+\frac{\mathrm{e}^x\,\left(6\,a\,b^2-2\,a^3\right)+6\,a^2\,b-2\,b^3}{2\,\mathrm{e}^{2\,x}+\mathrm{e}^{4\,x}+1}+b^3\,\ln\left(\left(a^3\,\mathrm{e}^x-2\,\sqrt{-\frac{a^6}{4}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*tanh(x) + a/cosh(x))^3,x)
```

```
[Out] atan((exp(x)*(3*a*b^2 + a^3))/(a^6 + 9*a^2*b^4 + 6*a^4*b^2)^(1/2))*(a^6 + 9
*a^2*b^4 + 6*a^4*b^2)^(1/2) + (exp(x)*(6*a*b^2 - 2*a^3) + 6*a^2*b - 2*b^3)/
(2*exp(2*x) + exp(4*x) + 1) + b^3*log((a^3*exp(x) - 2*(- a^6/4 - (9*a^2*b^4
```

```
)/4 - (3*a^4*b^2)/2)^(1/2) + 3*a*b^2*exp(x))*(2*(- a^6/4 - (9*a^2*b^4)/4 -
(3*a^4*b^2)/2)^(1/2) + a^3*exp(x) + 3*a*b^2*exp(x))) - b^3*x - (exp(x)*(3*a
*b^2 - a^3) + 6*a^2*b - 2*b^3)/(exp(2*x) + 1)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a \operatorname{sech}(x) + b \tanh(x))^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sech(x)+b*tanh(x))**3,x)
```

```
[Out] Integral((a*sech(x) + b*tanh(x))**3, x)
```

3.617 $\int (a\,\mathbf{sech}(x) + b\tanh(x))^2\,dx$

Optimal. Leaf size=29

$$-ab\cosh(x) - \operatorname{sech}(x)(b - a\sinh(x))(a + b\sinh(x)) + b^2x$$

```
[Out] b^2*x-a*b*cosh(x)-sech(x)*(b-a*sinh(x))*(a+b*sinh(x))
```

Rubi [A] time = 0.06, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {4391, 2691, 2638}

$$-ab\cosh(x) - \operatorname{sech}(x)(b - a\sinh(x))(a + b\sinh(x)) + b^2x$$

Antiderivative was successfully verified.

```
[In] Int[(a*Sech[x] + b*Tanh[x])^2,x]
```

```
[Out] b^2*x - a*b*Cosh[x] - Sech[x]*(b - a*Sinh[x])*(a + b*Sinh[x])
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2691

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[((g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x]
)^(m - 1)*(b + a*Sin[e + f*x]))/(f*g*(p + 1)), x] + Dist[1/(g^2*(p + 1)), I
nt[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 2)*(b^2*(m - 1) + a^2
*(p + 2) + a*b*(m + p + 1)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g},
x] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] && LtQ[p, -1] && (IntegersQ[2*m, 2*p]
|| IntegerQ[m])
```

Rule 4391

```
Int[(u_.)*((b_.)*sec[(c_.) + (d_.)*(x_)]^(n_.) + (a_.)*tan[(c_.) + (d_.)*(x
_)]^(n_.))^(p_), x_Symbol] :> Int[ActivateTrig[u]*Sec[c + d*x]^(n*p)*(b + a
*Sin[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

Rubi steps

$$\int (a\text{sech}(x) + b\tanh(x))^2\,dx = \int \text{sech}^2(x)(a + b\sinh(x))^2\,dx$$
$$= -\text{sech}(x)(b - a\sinh(x))(a + b\sinh(x)) - \int \left(-b^2 + ab\sinh(x)\right)\,dx$$
$$= b^2x - \text{sech}(x)(b - a\sinh(x))(a + b\sinh(x)) - (ab)\int \sinh(x)\,dx$$
$$= b^2x - ab\cosh(x) - \text{sech}(x)(b - a\sinh(x))(a + b\sinh(x))$$

Mathematica [A] time = 0.05, size = 26, normalized size = 0.90

$$\left(a^2 - b^2\right)\tanh(x) - 2ab\text{sech}(x) + b^2\tanh^{-1}(\tanh(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Sech[x] + b*Tanh[x])^2,x]
[Out] b^2*ArcTanh[Tanh[x]] - 2*a*b*Sech[x] + (a^2 - b^2)*Tanh[x]
```

fricas [A] time = 0.39, size = 42, normalized size = 1.45

$$-\frac{2\,ab - \left(b^2x - a^2 + b^2\right)\cosh(x) - \left(a^2 - b^2\right)\sinh(x)}{\cosh(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sech(x)+b*tanh(x))^2,x, algorithm="fricas")
[Out] -(2*a*b - (b^2*x - a^2 + b^2)*cosh(x) - (a^2 - b^2)*sinh(x))/cosh(x)
```

giac [A] time = 0.12, size = 31, normalized size = 1.07

$$b^2x - \frac{2\left(2\,abe^x + a^2 - b^2\right)}{e^{(2\,x)} + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sech(x)+b*tanh(x))^2,x, algorithm="giac")
[Out] b^2*x - 2*(2*a*b*e^x + a^2 - b^2)/(e^(2*x) + 1)
```

maple [A] time = 0.35, size = 26, normalized size = 0.90

$$a^2\tanh(x) - \frac{2ab}{\cosh(x)} + b^2(x - \tanh(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*sech(x)+b*tanh(x))^2,x)
```

```
[Out] a^2*tanh(x)-2*a*b/cosh(x)+b^2*(x-tanh(x))
```

maxima [A] time = 0.40, size = 43, normalized size = 1.48

$$b^2\left(x - \frac{2}{e^{(-2x)} + 1}\right) - \frac{4\,ab}{e^{(-x)} + e^x} + \frac{2\,a^2}{e^{(-2x)} + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sech(x)+b*tanh(x))^2,x, algorithm="maxima")
```

```
[Out] b^2*(x - 2/(e^(-2*x) + 1)) - 4*a*b/(e^(-x) + e^x) + 2*a^2/(e^(-2*x) + 1)
```

mupad [B] time = 1.57, size = 33, normalized size = 1.14

$$b^2\,x - \frac{2\,a^2 + 4\,e^x\,a\,b - 2\,b^2}{e^{2\,x} + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*tanh(x) + a/cosh(x))^2,x)
```

```
[Out] b^2*x - (2*a^2 - 2*b^2 + 4*a*b*exp(x))/(exp(2*x) + 1)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\,\mathrm{sech}\,(x) + b\tanh\,(x)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sech(x)+b*tanh(x))**2,x)
```

```
[Out] Integral((a*sech(x) + b*tanh(x))**2, x)
```

3.618 $\int (a\,\mathrm{sech}(x) + b\tanh(x))\,dx$

Optimal. Leaf size=11

$$a\tan^{-1}(\sinh(x)) + b\log(\cosh(x))$$

```
[Out] a*arctan(sinh(x))+b*ln(cosh(x))
```

Rubi [A] time = 0.01, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {3770, 3475}

$$a\tan^{-1}(\sinh(x)) + b\log(\cosh(x))$$

Antiderivative was successfully verified.

```
[In] Int[a*Sech[x] + b*Tanh[x],x]
```

```
[Out] a*ArcTan[Sinh[x]] + b*Log[Cosh[x]]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int (a\,\mathrm{sech}(x) + b\tanh(x))\,dx &= a\int \mathrm{sech}(x)\,dx + b\int \tanh(x)\,dx\\ &= a\tan^{-1}(\sinh(x)) + b\log(\cosh(x))\end{aligned}$$

Mathematica [A] time = 0.00, size = 16, normalized size = 1.45

$$2a\tan^{-1}\left(\tanh\left(\frac{x}{2}\right)\right) + b\log(\cosh(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[a*Sech[x] + b*Tanh[x],x]
```

[Out] 2*a*ArcTan[Tanh[x/2]] + b*Log[Cosh[x]]

fricas [B] time = 0.44, size = 30, normalized size = 2.73

$$-bx + 2\,a\arctan\left(\cosh(x) + \sinh(x)\right) + b\log\left(\frac{2\,\cosh(x)}{\cosh(x) - \sinh(x)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(a*sech(x)+b*tanh(x),x, algorithm="fricas")

[Out] -b*x + 2*a*arctan(cosh(x) + sinh(x)) + b*log(2*cosh(x)/(cosh(x) - sinh(x)))

giac [A] time = 0.11, size = 21, normalized size = 1.91

$$-b\left(x - \log\left(e^{(2\,x)} + 1\right)\right) + 2\,a\arctan\left(e^x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(a*sech(x)+b*tanh(x),x, algorithm="giac")

[Out] -b*(x - log(e^(2*x) + 1)) + 2*a*arctan(e^x)

maple [A] time = 0.02, size = 12, normalized size = 1.09

$$a\arctan\left(\sinh(x)\right) + b\ln\left(\cosh(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(a*sech(x)+b*tanh(x),x)

[Out] a*arctan(sinh(x))+b*ln(cosh(x))

maxima [A] time = 0.32, size = 11, normalized size = 1.00

$$a\arctan\left(\sinh(x)\right) + b\log\left(\cosh(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(a*sech(x)+b*tanh(x),x, algorithm="maxima")

[Out] a*arctan(sinh(x)) + b*log(cosh(x))

mupad [B] time = 0.07, size = 40, normalized size = 3.64

$$b\,\ln\left(4\,a^2\,\mathrm{e}^{2\,x} + 4\,a^2\right) - b\,x + 2\,\mathrm{atan}\left(\frac{a\,\mathrm{e}^x}{\sqrt{a^2}}\right)\,\sqrt{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(b*tanh(x) + a/cosh(x),x)
```

```
[Out] b*log(4*a^2*exp(2*x) + 4*a^2) - b*x + 2*atan((a*exp(x))/(a^2)^(1/2))*(a^2)^
(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\operatorname{sech}(x) + b\tanh(x))\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a*sech(x)+b*tanh(x),x)
```

```
[Out] Integral(a*sech(x) + b*tanh(x), x)
```

3.619 $\int \frac{1}{a\mathbf{sech}(x)+b\tanh(x)}\,dx$

Optimal. Leaf size=11

$$\frac{\log(a+b\sinh(x))}{b}$$

[Out] ln(a+b*sinh(x))/b

Rubi [A] time = 0.04, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {3159, 2668, 31}

$$\frac{\log(a+b\sinh(x))}{b}$$

Antiderivative was successfully verified.

[In] Int[(a*Sech[x] + b*Tanh[x])^(-1),x]

[Out] Log[a + b*Sinh[x]]/b

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 3159

```
Int[((a_.) + (b_.)*sec[(d_.) + (e_.)*(x_)] + (c_.)*tan[(d_.) + (e_.)*(x_)])
^(-1), x_Symbol] :> Int[Cos[d + e*x]/(b + a*Cos[d + e*x] + c*Sin[d + e*x]),
 x] /; FreeQ[{a, b, c, d, e}, x]
```

Rubi steps

$$\int \frac{1}{a\text{sech}(x) + b\tanh(x)}\,dx = \int \frac{\cosh(x)}{a + b\sinh(x)}\,dx$$
$$= \frac{\text{Subst}\left(\int \frac{1}{a+x}\,dx, x, b\sinh(x)\right)}{b}$$
$$= \frac{\log(a + b\sinh(x))}{b}$$

Mathematica [A] time = 0.01, size = 11, normalized size = 1.00

$$\frac{\log(a + b\sinh(x))}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Sech[x] + b*Tanh[x])^(-1),x]
[Out] Log[a + b*Sinh[x]]/b
```

fricas [B] time = 0.41, size = 27, normalized size = 2.45

$$-\frac{x - \log\left(\frac{2\,(b\sinh(x)+a)}{\cosh(x)-\sinh(x)}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sech(x)+b*tanh(x)),x, algorithm="fricas")
[Out] -(x - log(2*(b*sinh(x) + a)/(cosh(x) - sinh(x))))/b
```

giac [A] time = 0.12, size = 22, normalized size = 2.00

$$\frac{\log\left(\left|-b\left(e^{(-x)} - e^{x}\right) + 2\,a\right|\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sech(x)+b*tanh(x)),x, algorithm="giac")
[Out] log(abs(-b*(e^(-x) - e^x) + 2*a))/b
```

maple [B] time = 0.21, size = 50, normalized size = 4.55

$$-\frac{\ln\left(\tanh\left(\frac{x}{2}\right) - 1\right)}{b} - \frac{\ln\left(\tanh\left(\frac{x}{2}\right) + 1\right)}{b} + \frac{\ln\left(a\left(\tanh^{2}\left(\frac{x}{2}\right)\right) - 2\tanh\left(\frac{x}{2}\right)b - a\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*sech(x)+b*tanh(x)),x)
```

```
[Out] -1/b*ln(tanh(1/2*x)-1)-1/b*ln(tanh(1/2*x)+1)+1/b*ln(a*tanh(1/2*x)^2-2*tanh(
1/2*x)*b-a)
```

maxima [B] time = 0.32, size = 28, normalized size = 2.55

$$\frac{x}{b} + \frac{\log\left(-2\, ae^{(-x)} + be^{(-2\,x)} - b\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sech(x)+b*tanh(x)),x, algorithm="maxima")
```

```
[Out] x/b + log(-2*a*e^(-x) + b*e^(-2*x) - b)/b
```

mupad [B] time = 0.08, size = 25, normalized size = 2.27

$$-\frac{x - \ln\left(2\,a\,e^{x} - b + b\,e^{2\,x}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*tanh(x) + a/cosh(x)),x)
```

```
[Out] -(x - log(2*a*exp(x) - b + b*exp(2*x)))/b
```

sympy [A] time = 0.44, size = 32, normalized size = 2.91

$$\begin{cases} \frac{x}{b} + \frac{\log\left(\frac{a\operatorname{sech}(x)}{b} + \tanh(x)\right)}{b} - \frac{\log(\tanh(x)+1)}{b} & \text{for}\ b \neq 0 \\ \frac{\tanh(x)}{a\operatorname{sech}(x)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sech(x)+b*tanh(x)),x)
```

```
[Out] Piecewise((x/b + log(a*sech(x)/b + tanh(x))/b - log(tanh(x) + 1)/b, Ne(b, 0
)), (tanh(x)/(a*sech(x)), True))
```

3.620 $\int \frac{1}{(a\,\mathbf{sech}(x)+b\tanh(x))^2}\,dx$

Optimal. Leaf size=62

$$\frac{2a\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{b^2\sqrt{a^2+b^2}}-\frac{\cosh(x)}{b(a+b\sinh(x))}+\frac{x}{b^2}$$

```
[Out] x/b^2-cosh(x)/b/(a+b*sinh(x))+2*a*arctanh((b-a*tanh(1/2*x))/(a^2+b^2)^(1/2)
)/b^2/(a^2+b^2)^(1/2)
```

Rubi [A] time = 0.13, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.546, Rules used = {4391, 2693, 2735, 2660, 618, 206}

$$\frac{2a\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{b^2\sqrt{a^2+b^2}}-\frac{\cosh(x)}{b(a+b\sinh(x))}+\frac{x}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Sech[x] + b*Tanh[x])^(-2),x]
```

```
[Out] x/b^2 + (2*a*ArcTanh[(b - a*Tanh[x/2])/Sqrt[a^2 + b^2]])/(b^2*Sqrt[a^2 + b^
2]) - Cosh[x]/(b*(a + b*Sinh[x]))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2693

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x
])^(m + 1))/(b*f*(m + 1)), x] + Dist[(g^2*(p - 1))/(b*(m + 1)), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1)*Sin[e + f*x], x], x] /; Free
Q[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && In
tegersQ[2*m, 2*p]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 4391

```
Int[(u_.)*((b_.)*sec[(c_.) + (d_.)*(x_)]^(n_.) + (a_.)*tan[(c_.) + (d_.)*(x
_)]^(n_.))^(p_), x_Symbol] :> Int[ActivateTrig[u]*Sec[c + d*x]^(n*p)*(b + a
*Sin[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{(a\operatorname{sech}(x)+b\tanh(x))^2}\,dx &= \int \frac{\cosh^2(x)}{(a+b\sinh(x))^2}\,dx\\
&= -\frac{\cosh(x)}{b(a+b\sinh(x))}+\frac{\int \frac{\sinh(x)}{a+b\sinh(x)}\,dx}{b}\\
&= \frac{x}{b^2}-\frac{\cosh(x)}{b(a+b\sinh(x))}-\frac{a\int \frac{1}{a+b\sinh(x)}\,dx}{b^2}\\
&= \frac{x}{b^2}-\frac{\cosh(x)}{b(a+b\sinh(x))}-\frac{(2a)\operatorname{Subst}\left(\int \frac{1}{a+2bx-ax^2}\,dx,x,\tanh\left(\frac{x}{2}\right)\right)}{b^2}\\
&= \frac{x}{b^2}-\frac{\cosh(x)}{b(a+b\sinh(x))}+\frac{(4a)\operatorname{Subst}\left(\int \frac{1}{4\left(a^2+b^2\right)-x^2}\,dx,x,2b-2a\tanh\left(\frac{x}{2}\right)\right)}{b^2}\\
&= \frac{x}{b^2}+\frac{2a\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{b^2\sqrt{a^2+b^2}}-\frac{\cosh(x)}{b(a+b\sinh(x))}
\end{aligned}
$$

Mathematica [C] time = 2.08, size = 502, normalized size = 8.10

$$-\frac{\cosh(x)\left(\sqrt{a+ib}\sqrt{-\frac{b(\sinh(x)-i)}{a+ib}}\left(\sqrt{a-ib}\left(a^2+b^2\right)\sqrt{1+i\sinh(x)}\sqrt{-\frac{b(\sinh(x)+i)}{a-ib}}-2\sqrt[4]{-1}\,b^{3/2}(b+ia)\sinh(x)\sin\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Sech[x] + b*Tanh[x])^(-2),x]
```

```
[Out] -((Cosh[x]*(-2*a*Sqrt[a - I*b]*Sqrt[a + I*b]*ArcTanh[Sqrt[-((b*(I + Sinh[x]
))/(a - I*b))]/Sqrt[-((b*(-I + Sinh[x]))/(a + I*b))]]*Sqrt[1 + I*Sinh[x]]*(
a + b*Sinh[x]) + 2*a*(a - I*b)*ArcTanh[(Sqrt[a - I*b]*Sqrt[-((b*(I + Sinh[x
]))/(a - I*b))])/(Sqrt[a + I*b]*Sqrt[-((b*(-I + Sinh[x]))/(a + I*b))])]*Sqr
t[1 + I*Sinh[x]]*(a + b*Sinh[x]) + Sqrt[a + I*b]*Sqrt[-((b*(-I + Sinh[x]))/
(a + I*b))]*(-2*(-1)^(1/4)*a*Sqrt[b]*(I*a + b)*ArcSin[((1/2 + I/2)*Sqrt[a -
 I*b]*Sqrt[-((b*(I + Sinh[x]))/(a - I*b))])/Sqrt[b]] - 2*(-1)^(1/4)*b^(3/2)
*(I*a + b)*ArcSin[((1/2 + I/2)*Sqrt[a - I*b]*Sqrt[-((b*(I + Sinh[x]))/(a -
I*b))])/Sqrt[b]]*Sinh[x] + Sqrt[a - I*b]*(a^2 + b^2)*Sqrt[1 + I*Sinh[x]]*Sq
rt[-((b*(I + Sinh[x]))/(a - I*b))])))/((a - I*b)^(3/2)*(a + I*b)^(3/2)*b*Sq
rt[1 + I*Sinh[x]]*Sqrt[-((b*(-I + Sinh[x]))/(a + I*b))]*Sqrt[-((b*(I + Sinh
[x]))/(a - I*b))]*(a + b*Sinh[x])))
```

fricas [B] time = 0.42, size = 362, normalized size = 5.84

$$-\frac{\left(a^2b+b^3\right)x\cosh(x)^2+\left(a^2b+b^3\right)x\sinh(x)^2-2\,a^2b-2\,b^3+\left(ab\cosh(x)^2+ab\sinh(x)^2+2\,a^2\cosh(x)-ab+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sech(x)+b*tanh(x))^2,x, algorithm="fricas")
```

```
[Out] -((a^2*b + b^3)*x*cosh(x)^2 + (a^2*b + b^3)*x*sinh(x)^2 - 2*a^2*b - 2*b^3 +
 (a*b*cosh(x)^2 + a*b*sinh(x)^2 + 2*a^2*cosh(x) - a*b + 2*(a*b*cosh(x) + a^
2)*sinh(x))*sqrt(a^2 + b^2)*log((b^2*cosh(x)^2 + b^2*sinh(x)^2 + 2*a*b*cosh
(x) + 2*a^2 + b^2 + 2*(b^2*cosh(x) + a*b)*sinh(x) + 2*sqrt(a^2 + b^2)*(b*co
sh(x) + b*sinh(x) + a))/(b*cosh(x)^2 + b*sinh(x)^2 + 2*a*cosh(x) + 2*(b*cos
h(x) + a)*sinh(x) - b)) - (a^2*b + b^3)*x + 2*(a^3 + a*b^2 + (a^3 + a*b^2)*
x)*cosh(x) + 2*(a^3 + a*b^2 + (a^2*b + b^3)*x*cosh(x) + (a^3 + a*b^2)*x)*si
nh(x))/(a^2*b^3 + b^5 - (a^2*b^3 + b^5)*cosh(x)^2 - (a^2*b^3 + b^5)*sinh(x)
^2 - 2*(a^3*b^2 + a*b^4)*cosh(x) - 2*(a^3*b^2 + a*b^4 + (a^2*b^3 + b^5)*cos
h(x))*sinh(x))
```

giac [A] time = 0.14, size = 97, normalized size = 1.56

$$-\frac{a\log\left(\frac{\left|2\,be^{x}+2\,a-2\,\sqrt{a^{2}+b^{2}}\right|}{\left|2\,be^{x}+2\,a+2\,\sqrt{a^{2}+b^{2}}\right|}\right)}{\sqrt{a^{2}+b^{2}}\,b^{2}}+\frac{x}{b^{2}}+\frac{2\,(ae^{x}-b)}{\left(be^{(2\,x)}+2\,ae^{x}-b\right)b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sech(x)+b*tanh(x))^2,x, algorithm="giac")
```

```
[Out] -a*log(abs(2*b*e^x + 2*a - 2*sqrt(a^2 + b^2))/abs(2*b*e^x + 2*a + 2*sqrt(a^
2 + b^2)))/(sqrt(a^2 + b^2)*b^2) + x/b^2 + 2*(a*e^x - b)/((b*e^(2*x) + 2*a*
e^x - b)*b^2)
```

maple [B] time = 0.28, size = 119, normalized size = 1.92

$$-\frac{\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{b^{2}}+\frac{\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)}{b^{2}}+\frac{2\tanh\left(\frac{x}{2}\right)}{\left(a\left(\tanh^{2}\left(\frac{x}{2}\right)\right)-2\tanh\left(\frac{x}{2}\right)b-a\right)a}+\frac{2}{b\left(a\left(\tanh^{2}\left(\frac{x}{2}\right)\right)-2\tanh\left(\frac{x}{2}\right)b-a\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*sech(x)+b*tanh(x))^2,x)
```

```
[Out] -1/b^2*ln(tanh(1/2*x)-1)+1/b^2*ln(tanh(1/2*x)+1)+2/(a*tanh(1/2*x)^2-2*tanh(
1/2*x)*b-a)/a*tanh(1/2*x)+2/b/(a*tanh(1/2*x)^2-2*tanh(1/2*x)*b-a)-2/b^2*a/(
a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tanh(1/2*x)-2*b)/(a^2+b^2)^(1/2))
```

maxima [A] time = 0.43, size = 100, normalized size = 1.61

$$-\frac{2\left(ae^{(-x)}+b\right)}{2\,ab^{2}e^{(-x)}-b^{3}e^{(-2\,x)}+b^{3}}-\frac{a\log\left(\frac{be^{(-x)}-a-\sqrt{a^{2}+b^{2}}}{be^{(-x)}-a+\sqrt{a^{2}+b^{2}}}\right)}{\sqrt{a^{2}+b^{2}}\,b^{2}}+\frac{x}{b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sech(x)+b*tanh(x))^2,x, algorithm="maxima")
```

```
[Out] -2*(a*e^(-x) + b)/(2*a*b^2*e^(-x) - b^3*e^(-2*x) + b^3) - a*log((b*e^(-x) -
 a - sqrt(a^2 + b^2))/(b*e^(-x) - a + sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*b^
2) + x/b^2
```

mupad [B] time = 1.74, size = 132, normalized size = 2.13

$$\frac{x}{b^{2}}-\frac{\frac{2}{b}-\frac{2\,a\,\mathrm{e}^{x}}{b^{2}}}{2\,a\,\mathrm{e}^{x}-b+b\,\mathrm{e}^{2\,x}}-\frac{a\,\ln\left(\frac{2\,a\,\mathrm{e}^{x}}{b^{3}}-\frac{2\,a\,(b-a\,\mathrm{e}^{x})}{b^{3}\,\sqrt{a^{2}+b^{2}}}\right)}{b^{2}\,\sqrt{a^{2}+b^{2}}}+\frac{a\,\ln\left(\frac{2\,a\,\mathrm{e}^{x}}{b^{3}}+\frac{2\,a\,(b-a\,\mathrm{e}^{x})}{b^{3}\,\sqrt{a^{2}+b^{2}}}\right)}{b^{2}\,\sqrt{a^{2}+b^{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*tanh(x) + a/cosh(x))^2,x)
```

```
[Out] x/b^2 - (2/b - (2*a*exp(x))/b^2)/(2*a*exp(x) - b + b*exp(2*x)) - (a*log((2*
a*exp(x))/b^3 - (2*a*(b - a*exp(x)))/(b^3*(a^2 + b^2)^(1/2))))/(b^2*(a^2 +
b^2)^(1/2)) + (a*log((2*a*exp(x))/b^3 + (2*a*(b - a*exp(x)))/(b^3*(a^2 + b^
2)^(1/2))))/(b^2*(a^2 + b^2)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a\,\mathrm{sech}\,(x) + b\tanh\,(x))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sech(x)+b*tanh(x))**2,x)
```

```
[Out] Integral((a*sech(x) + b*tanh(x))**(-2), x)
```

3.621 $\int \frac{1}{(a\,\mathbf{sech}(x)+b\tanh(x))^3}\,dx$

Optimal. Leaf size=48

$$-\frac{a^2+b^2}{2b^3(a+b\sinh(x))^2}+\frac{2a}{b^3(a+b\sinh(x))}+\frac{\log(a+b\sinh(x))}{b^3}$$

```
[Out] ln(a+b*sinh(x))/b^3+1/2*(-a^2-b^2)/b^3/(a+b*sinh(x))^2+2*a/b^3/(a+b*sinh(x)
)
```

Rubi [A] time = 0.08, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {4391, 2668, 697}

$$-\frac{a^2+b^2}{2b^3(a+b\sinh(x))^2}+\frac{2a}{b^3(a+b\sinh(x))}+\frac{\log(a+b\sinh(x))}{b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Sech[x] + b*Tanh[x])^(-3),x]
```

```
[Out] Log[a + b*Sinh[x]]/b^3 - (a^2 + b^2)/(2*b^3*(a + b*Sinh[x])^2) + (2*a)/(b^3
*(a + b*Sinh[x]))
```

Rule 697

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 4391

```
Int[(u_.)*((b_.)*sec[(c_.) + (d_.)*(x_)]^(n_.) + (a_.)*tan[(c_.) + (d_.)*(x
_)]^(n_.))^(p_), x_Symbol] :> Int[ActivateTrig[u]*Sec[c + d*x]^(n*p)*(b + a
*Sin[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

Rubi steps

$$\int \frac{1}{(a\text{sech}(x) + b\tanh(x))^3}\,dx = \int \frac{\cosh^3(x)}{(a + b\sinh(x))^3}\,dx$$
$$= -\frac{\text{Subst}\left(\int \frac{-b^2-x^2}{(a+x)^3}\,dx, x, b\sinh(x)\right)}{b^3}$$
$$= -\frac{\text{Subst}\left(\int \left(\frac{1}{-a-x} + \frac{-a^2-b^2}{(a+x)^3} + \frac{2a}{(a+x)^2}\right)dx, x, b\sinh(x)\right)}{b^3}$$
$$= \frac{\log(a + b\sinh(x))}{b^3} - \frac{a^2 + b^2}{2b^3(a + b\sinh(x))^2} + \frac{2a}{b^3(a + b\sinh(x))}$$

Mathematica [A] time = 0.14, size = 42, normalized size = 0.88

$$-\frac{\frac{-3a^2-4ab\sinh(x)+b^2}{2(a+b\sinh(x))^2} - \log(a + b\sinh(x))}{b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Sech[x] + b*Tanh[x])^(-3),x]
```

```
[Out] -((-Log[a + b*Sinh[x]] + (-3*a^2 + b^2 - 4*a*b*Sinh[x])/(2*(a + b*Sinh[x])^
2))/b^3)
```

fricas [B] time = 0.42, size = 543, normalized size = 11.31

$$-\frac{b^2x\cosh(x)^4 + b^2x\sinh(x)^4 + 4(abx - ab)\cosh(x)^3 + 4\left(b^2x\cosh(x) + abx - ab\right)\sinh(x)^3 + b^2x - 2\left(3a^2 - b\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sech(x)+b*tanh(x))^3,x, algorithm="fricas")
```

```
[Out] -(b^2*x*cosh(x)^4 + b^2*x*sinh(x)^4 + 4*(a*b*x - a*b)*cosh(x)^3 + 4*(b^2*x*
cosh(x) + a*b*x - a*b)*sinh(x)^3 + b^2*x - 2*(3*a^2 - b^2 - (2*a^2 - b^2)*x
)*cosh(x)^2 + 2*(3*b^2*x*cosh(x)^2 - 3*a^2 + b^2 + (2*a^2 - b^2)*x + 6*(a*b
*x - a*b)*cosh(x))*sinh(x)^2 - 4*(a*b*x - a*b)*cosh(x) - (b^2*cosh(x)^4 + b
^2*sinh(x)^4 + 4*a*b*cosh(x)^3 + 4*(b^2*cosh(x) + a*b)*sinh(x)^3 - 4*a*b*co
sh(x) + 2*(2*a^2 - b^2)*cosh(x)^2 + 2*(3*b^2*cosh(x)^2 + 6*a*b*cosh(x) + 2*
a^2 - b^2)*sinh(x)^2 + b^2 + 4*(b^2*cosh(x)^3 + 3*a*b*cosh(x)^2 - a*b + (2*
a^2 - b^2)*cosh(x))*sinh(x))*log(2*(b*sinh(x) + a)/(cosh(x) - sinh(x))) + 4
*(b^2*x*cosh(x)^3 - a*b*x + 3*(a*b*x - a*b)*cosh(x)^2 + a*b - (3*a^2 - b^2
```

```
- (2*a^2 - b^2)*x)*cosh(x))*sinh(x))/(b^5*cosh(x)^4 + b^5*sinh(x)^4 + 4*a*b
^4*cosh(x)^3 - 4*a*b^4*cosh(x) + b^5 + 4*(b^5*cosh(x) + a*b^4)*sinh(x)^3 +
2*(2*a^2*b^3 - b^5)*cosh(x)^2 + 2*(3*b^5*cosh(x)^2 + 6*a*b^4*cosh(x) + 2*a^
2*b^3 - b^5)*sinh(x)^2 + 4*(b^5*cosh(x)^3 + 3*a*b^4*cosh(x)^2 - a*b^4 + (2*
a^2*b^3 - b^5)*cosh(x))*sinh(x))
```

giac [A] time = 0.13, size = 75, normalized size = 1.56

$$\frac{\log\left(\left|-b\left(e^{(-x)}-e^{x}\right)+2\,a\right|\right)}{b^{3}}-\frac{3\,b\left(e^{(-x)}-e^{x}\right)^{2}-4\,a\left(e^{(-x)}-e^{x}\right)+4\,b}{2\left(b\left(e^{(-x)}-e^{x}\right)-2\,a\right)^{2}b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sech(x)+b*tanh(x))^3,x, algorithm="giac")
```

```
[Out] log(abs(-b*(e^(-x) - e^x) + 2*a))/b^3 - 1/2*(3*b*(e^(-x) - e^x)^2 - 4*a*(e^
(-x) - e^x) + 4*b)/((b*(e^(-x) - e^x) - 2*a)^2*b^2)
```

maple [B] time = 0.27, size = 241, normalized size = 5.02

$$-\frac{\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{b^{3}}-\frac{\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)}{b^{3}}+\frac{2a\left(\tanh^{3}\left(\frac{x}{2}\right)\right)}{b^{2}\left(a\left(\tanh^{2}\left(\frac{x}{2}\right)\right)-2\tanh\left(\frac{x}{2}\right)b-a\right)^{2}}-\frac{2\left(\tanh^{3}\left(\frac{x}{2}\right)\right)}{\left(a\left(\tanh^{2}\left(\frac{x}{2}\right)\right)-2\tanh\left(\frac{x}{2}\right)b-a\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*sech(x)+b*tanh(x))^3,x)
```

```
[Out] -1/b^3*ln(tanh(1/2*x)-1)-1/b^3*ln(tanh(1/2*x)+1)+2/b^2/(a*tanh(1/2*x)^2-2*t
anh(1/2*x)*b-a)^2*a*tanh(1/2*x)^3-2/(a*tanh(1/2*x)^2-2*tanh(1/2*x)*b-a)^2/a
*tanh(1/2*x)^3-6/b/(a*tanh(1/2*x)^2-2*tanh(1/2*x)*b-a)^2*tanh(1/2*x)^2+2*b/
(a*tanh(1/2*x)^2-2*tanh(1/2*x)*b-a)^2/a^2*tanh(1/2*x)^2-2/b^2/(a*tanh(1/2*x
)^2-2*tanh(1/2*x)*b-a)^2*a*tanh(1/2*x)+2/(a*tanh(1/2*x)^2-2*tanh(1/2*x)*b-a
)^2/a*tanh(1/2*x)+1/b^3*ln(a*tanh(1/2*x)^2-2*tanh(1/2*x)*b-a)
```

maxima [B] time = 0.48, size = 117, normalized size = 2.44

$$\frac{2\left(2\,abe^{(-x)}-2\,abe^{(-3\,x)}+\left(3\,a^{2}-b^{2}\right)e^{(-2\,x)}\right)}{4\,ab^{4}e^{(-x)}-4\,ab^{4}e^{(-3\,x)}+b^{5}e^{(-4\,x)}+b^{5}+2\left(2\,a^{2}b^{3}-b^{5}\right)e^{(-2\,x)}}+\frac{x}{b^{3}}+\frac{\log\left(-2\,ae^{(-x)}+be^{(-2\,x)}-b\right)}{b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sech(x)+b*tanh(x))^3,x, algorithm="maxima")
```

```
[Out] 2*(2*a*b*e^(-x) - 2*a*b*e^(-3*x) + (3*a^2 - b^2)*e^(-2*x))/(4*a*b^4*e^(-x)
- 4*a*b^4*e^(-3*x) + b^5*e^(-4*x) + b^5 + 2*(2*a^2*b^3 - b^5)*e^(-2*x)) + x
/b^3 + log(-2*a*e^(-x) + b*e^(-2*x) - b)/b^3
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\left(b\tanh(x) + \frac{a}{\cosh(x)}\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*tanh(x) + a/cosh(x))^3,x)
```

```
[Out] int(1/(b*tanh(x) + a/cosh(x))^3, x)
```

sympy [A] time = 2.89, size = 651, normalized size = 13.56

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sech(x)+b*tanh(x))**3,x)
```

```
[Out] Piecewise((2*a**2*x*sech(x)**2/(2*a**2*b**3*sech(x)**2 + 4*a*b**4*tanh(x)*s
ech(x) + 2*b**5*tanh(x)**2) + 2*a**2*log(a*sech(x)/b + tanh(x))*sech(x)**2/
(2*a**2*b**3*sech(x)**2 + 4*a*b**4*tanh(x)*sech(x) + 2*b**5*tanh(x)**2) - 2
*a**2*log(tanh(x) + 1)*sech(x)**2/(2*a**2*b**3*sech(x)**2 + 4*a*b**4*tanh(x
)*sech(x) + 2*b**5*tanh(x)**2) + a**2*sech(x)**2/(2*a**2*b**3*sech(x)**2 +
4*a*b**4*tanh(x)*sech(x) + 2*b**5*tanh(x)**2) + 4*a*b*x*tanh(x)*sech(x)/(2*
a**2*b**3*sech(x)**2 + 4*a*b**4*tanh(x)*sech(x) + 2*b**5*tanh(x)**2) + 4*a*
b*log(a*sech(x)/b + tanh(x))*tanh(x)*sech(x)/(2*a**2*b**3*sech(x)**2 + 4*a*
b**4*tanh(x)*sech(x) + 2*b**5*tanh(x)**2) - 4*a*b*log(tanh(x) + 1)*tanh(x)*
sech(x)/(2*a**2*b**3*sech(x)**2 + 4*a*b**4*tanh(x)*sech(x) + 2*b**5*tanh(x)
**2) + 2*b**2*x*tanh(x)**2/(2*a**2*b**3*sech(x)**2 + 4*a*b**4*tanh(x)*sech(
x) + 2*b**5*tanh(x)**2) + 2*b**2*log(a*sech(x)/b + tanh(x))*tanh(x)**2/(2*a
**2*b**3*sech(x)**2 + 4*a*b**4*tanh(x)*sech(x) + 2*b**5*tanh(x)**2) - 2*b**
2*log(tanh(x) + 1)*tanh(x)**2/(2*a**2*b**3*sech(x)**2 + 4*a*b**4*tanh(x)*se
ch(x) + 2*b**5*tanh(x)**2) - b**2*tanh(x)**2/(2*a**2*b**3*sech(x)**2 + 4*a*
b**4*tanh(x)*sech(x) + 2*b**5*tanh(x)**2) - b**2/(2*a**2*b**3*sech(x)**2 +
4*a*b**4*tanh(x)*sech(x) + 2*b**5*tanh(x)**2), Ne(b, 0)), ((-2*tanh(x)**3/(
3*sech(x)**3) + tanh(x)/sech(x)**3)/a**3, True))
```

3.622 $\int \frac{1}{(a\,\mathbf{sech}(x)+b\tanh(x))^4}\,dx$

Optimal. Leaf size=146

$$\frac{a\cosh^3(x)}{2b\left(a^2+b^2\right)(a+b\sinh(x))^2}+\frac{a\left(2a^2+3b^2\right)\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{b^4\left(a^2+b^2\right)^{3/2}}-\frac{\cosh(x)\left(2\left(a^2+b^2\right)+ab\sinh(x)\right)}{2b^3\left(a^2+b^2\right)(a+b\sinh(x))}-\frac{\cosh^3}{3b(a+b\,\mathrm{si}}$$

```
[Out] x/b^4+a*(2*a^2+3*b^2)*arctanh((b-a*tanh(1/2*x))/(a^2+b^2)^(1/2))/b^4/(a^2+b
^2)^(3/2)-1/3*cosh(x)^3/b/(a+b*sinh(x))^3+1/2*a*cosh(x)^3/b/(a^2+b^2)/(a+b*
sinh(x))^2-1/2*cosh(x)*(2*a^2+2*b^2+a*b*sinh(x))/b^3/(a^2+b^2)/(a+b*sinh(x)
)
```

Rubi [A] time = 0.37, antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.727, Rules used = {4391, 2693, 2864, 2863, 2735, 2660, 618, 206}

$$\frac{a\left(2a^2+3b^2\right)\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{b^4\left(a^2+b^2\right)^{3/2}}+\frac{a\cosh^3(x)}{2b\left(a^2+b^2\right)(a+b\sinh(x))^2}-\frac{\cosh(x)\left(2\left(a^2+b^2\right)+ab\sinh(x)\right)}{2b^3\left(a^2+b^2\right)(a+b\sinh(x))}-\frac{\cosh^3}{3b(a+b\,\mathrm{si}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Sech[x] + b*Tanh[x])^(-4),x]
```

```
[Out] x/b^4 + (a*(2*a^2 + 3*b^2)*ArcTanh[(b - a*Tanh[x/2])/Sqrt[a^2 + b^2]])/(b^4
*(a^2 + b^2)^(3/2)) - Cosh[x]^3/(3*b*(a + b*Sinh[x])^3) + (a*Cosh[x]^3)/(2*
b*(a^2 + b^2)*(a + b*Sinh[x])^2) - (Cosh[x]*(2*(a^2 + b^2) + a*b*Sinh[x]))/
(2*b^3*(a^2 + b^2)*(a + b*Sinh[x]))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2693

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x
])^(m + 1))/(b*f*(m + 1)), x] + Dist[(g^2*(p - 1))/(b*(m + 1)), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1)*Sin[e + f*x], x], x] /; Free
Q[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && In
tegersQ[2*m, 2*p]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2863

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(g*(g*
Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*(b*c*(m + p + 1) - a*d*p
 + b*d*(m + 1)*Sin[e + f*x]))/(b^2*f*(m + 1)*(m + p + 1)), x] + Dist[(g^2*(
p - 1))/(b^2*(m + 1)*(m + p + 1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[
e + f*x])^(m + 1)*Simp[b*d*(m + 1) + (b*c*(m + p + 1) - a*d*p)*Sin[e + f*x]
, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ
[m, -1] && GtQ[p, 1] && NeQ[m + p + 1, 0] && IntegerQ[2*m]
```

Rule 2864

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[((b*c
 - a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m + 1))/(f*g*(a^2 -
b^2)*(m + 1)), x] + Dist[1/((a^2 - b^2)*(m + 1)), Int[(g*Cos[e + f*x])^p*(a
 + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m + p +
2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[a^
2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 4391

```
Int[(u_.)*((b_.)*sec[(c_.) + (d_.)*(x_)]^(n_.) + (a_.)*tan[(c_.) + (d_.)*(x
_)]^(n_.))^(p_), x_Symbol] :> Int[ActivateTrig[u]*Sec[c + d*x]^(n*p)*(b + a
```

```
*Sin[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

Rubi steps

$$\int \frac{1}{(a\text{sech}(x)+b\tanh(x))^4}\,dx = \int \frac{\cosh^4(x)}{(a+b\sinh(x))^4}\,dx$$

$$= -\frac{\cosh^3(x)}{3b(a+b\sinh(x))^3} + \frac{\int \frac{\cosh^2(x)\sinh(x)}{(a+b\sinh(x))^3}\,dx}{b}$$

$$= -\frac{\cosh^3(x)}{3b(a+b\sinh(x))^3} + \frac{a\cosh^3(x)}{2b\left(a^2+b^2\right)(a+b\sinh(x))^2} + \frac{i\int \frac{\cosh^2(x)(-2ib+ia\sinh(x))}{(a+b\sinh(x))^2}\,d$$

$$= -\frac{\cosh^3(x)}{3b(a+b\sinh(x))^3} + \frac{a\cosh^3(x)}{2b\left(a^2+b^2\right)(a+b\sinh(x))^2} - \frac{\cosh(x)\left(2\left(a^2+b^2\right)+ab}{2b^3\left(a^2+b^2\right)(a+b\sin}$$

$$= \frac{x}{b^4} - \frac{\cosh^3(x)}{3b(a+b\sinh(x))^3} + \frac{a\cosh^3(x)}{2b\left(a^2+b^2\right)(a+b\sinh(x))^2} - \frac{\cosh(x)\left(2\left(a^2+b^2\right)\right.}{2b^3\left(a^2+b^2\right)(a+}$$

$$= \frac{x}{b^4} - \frac{\cosh^3(x)}{3b(a+b\sinh(x))^3} + \frac{a\cosh^3(x)}{2b\left(a^2+b^2\right)(a+b\sinh(x))^2} - \frac{\cosh(x)\left(2\left(a^2+b^2\right)\right.}{2b^3\left(a^2+b^2\right)(a+}$$

$$= \frac{x}{b^4} - \frac{\cosh^3(x)}{3b(a+b\sinh(x))^3} + \frac{a\cosh^3(x)}{2b\left(a^2+b^2\right)(a+b\sinh(x))^2} - \frac{\cosh(x)\left(2\left(a^2+b^2\right)\right.}{2b^3\left(a^2+b^2\right)(a+}$$

$$= \frac{x}{b^4} + \frac{a\left(2a^2+3b^2\right)\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{b^4\left(a^2+b^2\right)^{3/2}} - \frac{\cosh^3(x)}{3b(a+b\sinh(x))^3} + \frac{a\cosh^3}{2b\left(a^2+b^2\right)(a+}$$

Mathematica [C] time = 6.46, size = 3430, normalized size = 23.49

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a*Sech[x] + b*Tanh[x])^(-4),x]
```

```
[Out] ((-I)*Sech[x]*(a + b*Sinh[x])^4*(((I/3)*b*(((-I)*b)/(a - I*b) - (b*Sinh[x])
/(a - I*b))^(5/2)*((I*b)/(a + I*b) - (b*Sinh[x])/(a + I*b))^(5/2))/((((-I)*
```

```
a*b)/(a - I*b) - b^2/(a - I*b))*(((-I)*a*b)/(a + I*b) + b^2/(a + I*b))*(a +
 b*Sinh[x])^3) - (((I/2)*a*b^3*(((-I)*b)/(a - I*b) - (b*Sinh[x])/(a - I*b))
^(5/2)*((I*b)/(a + I*b) - (b*Sinh[x])/(a + I*b))^(5/2))/((a^2 + b^2)*(((-I)
*a*b)/(a - I*b) - b^2/(a - I*b))*(((-I)*a*b)/(a + I*b) + b^2/(a + I*b))*(a
+ b*Sinh[x])^2) - (-(((((3*I)*a^2*b^5)/(a^2 + b^2)^2 - ((2*I)*b^5*(3*a^2 +
2*b^2))/(a^2 + b^2)^2)*(((-I)*b)/(a - I*b) - (b*Sinh[x])/(a - I*b))^(5/2)*(
(I*b)/(a + I*b) - (b*Sinh[x])/(a + I*b))^(5/2))/((((-I)*a*b)/(a - I*b) - b^
2/(a - I*b))*(((-I)*a*b)/(a + I*b) + b^2/(a + I*b))*(a + b*Sinh[x]))) - ((1
6*Sqrt[2]*(a - I*b)*b^6*(3*a^2 + 4*b^2)*(((-I)*b)/(a - I*b) - (b*Sinh[x])/(
a - I*b))^(5/2)*Sqrt[(I*b)/(a + I*b) - (b*Sinh[x])/(a + I*b)]*(1 - ((I/2)*(
a - I*b)*(((-I)*b)/(a - I*b) - (b*Sinh[x])/(a - I*b)))/b)^(5/2)*((5*(1/(2*(
1 - ((I/2)*(a - I*b)*(((-I)*b)/(a - I*b) - (b*Sinh[x])/(a - I*b)))/b)^2) +
(1 - ((I/2)*(a - I*b)*(((-I)*b)/(a - I*b) - (b*Sinh[x])/(a - I*b)))/b)^(-1)
))/8 + (((15*I)/32)*b^3*(((-I)*(a - I*b)*(((-I)*b)/(a - I*b) - (b*Sinh[x])/
(a - I*b)))/b + ((a - I*b)^2*(((-I)*b)/(a - I*b) - (b*Sinh[x])/(a - I*b))^2
)/(3*b^2) + ((-1)^(1/4)*Sqrt[2]*Sqrt[a - I*b]*ArcSin[((-1)^(1/4)*Sqrt[a - I
*b]*Sqrt[((-I)*b)/(a - I*b) - (b*Sinh[x])/(a - I*b)])/(Sqrt[2]*Sqrt[b])]*Sq
rt[((-I)*b)/(a - I*b) - (b*Sinh[x])/(a - I*b)])/(Sqrt[b]*Sqrt[1 - ((I/2)*(a
 - I*b)*(((-I)*b)/(a - I*b) - (b*Sinh[x])/(a - I*b)))/b])))/((a - I*b)^3*((
(-I)*b)/(a - I*b) - (b*Sinh[x])/(a - I*b))^3*(1 - ((I/2)*(a - I*b)*(((-I)*b
)/(a - I*b) - (b*Sinh[x])/(a - I*b)))/b)^2)))/(5*(a + I*b)*(a^2 + b^2)^3*Sq
rt[((-I)*(a + I*b)*((I*b)/(a + I*b) - (b*Sinh[x])/(a + I*b)))/b]) + (I*(((4
*I)*a*b^7*(3*a^2 + 4*b^2))/(a^2 + b^2)^3 - (I*a*b^7*(6*a^2 + 7*b^2))/(a^2 +
 b^2)^3)*((-4*Sqrt[2]*(((-I)*b)/(a - I*b) - (b*Sinh[x])/(a - I*b))^(3/2)*Sq
rt[(I*b)/(a + I*b) - (b*Sinh[x])/(a + I*b)]*(1 - ((I/2)*(a - I*b)*(((-I)*b)
/(a - I*b) - (b*Sinh[x])/(a - I*b)))/b)^(5/2)*((3/(4*(1 - ((I/2)*(a - I*b)*
(((-I)*b)/(a - I*b) - (b*Sinh[x])/(a - I*b)))/b)^2) + (1 - ((I/2)*(a - I*b)
*(((-I)*b)/(a - I*b) - (b*Sinh[x])/(a - I*b)))/b)^(-1))/2 - (3*b^2*(((-I)*(
a - I*b)*(((-I)*b)/(a - I*b) - (b*Sinh[x])/(a - I*b)))/b + ((-1)^(1/4)*Sqrt
[2]*Sqrt[a - I*b]*ArcSin[((-1)^(1/4)*Sqrt[a - I*b]*Sqrt[((-I)*b)/(a - I*b)
- (b*Sinh[x])/(a - I*b)])/(Sqrt[2]*Sqrt[b])]*Sqrt[((-I)*b)/(a - I*b) - (b*S
inh[x])/(a - I*b)])/(Sqrt[b]*Sqrt[1 - ((I/2)*(a - I*b)*(((-I)*b)/(a - I*b)
- (b*Sinh[x])/(a - I*b)))/b])))/(8*(a - I*b)^2*(((-I)*b)/(a - I*b) - (b*Sin
h[x])/(a - I*b))^2*(1 - ((I/2)*(a - I*b)*(((-I)*b)/(a - I*b) - (b*Sinh[x])/
(a - I*b)))/b)^2)))/(3*(a + I*b)*Sqrt[((-I)*(a + I*b)*((I*b)/(a + I*b) - (b
*Sinh[x])/(a + I*b)))/b]) - (I*((I*a*b)/(a - I*b) + b^2/(a - I*b))*(((-I)*(
(I*a*b)/(a - I*b) + b^2/(a - I*b))*(((-I)*((I*a*b)/(a + I*b) - b^2/(a + I*b
))*(((2*I)*Sqrt[a - I*b]*ArcTanh[(Sqrt[a - I*b]*Sqrt[((-I)*b)/(a - I*b) - (
b*Sinh[x])/(a - I*b)])/(Sqrt[a + I*b]*Sqrt[(I*b)/(a + I*b) - (b*Sinh[x])/(a
 + I*b)])])/(Sqrt[a + I*b]*b) - ((2*I)*Sqrt[(I*a*b)/(a + I*b) - b^2/(a + I*
b)]*ArcTanh[(Sqrt[(I*a*b)/(a + I*b) - b^2/(a + I*b)]*Sqrt[((-I)*b)/(a - I*b
) - (b*Sinh[x])/(a - I*b)])/(Sqrt[(I*a*b)/(a - I*b) + b^2/(a - I*b)]*Sqrt[(
I*b)/(a + I*b) - (b*Sinh[x])/(a + I*b)])])/(b*Sqrt[(I*a*b)/(a - I*b) + b^2/
(a - I*b)])))/b + ((2*I)*Sqrt[2]*(a - I*b)*Sqrt[((-I)*b)/(a - I*b) - (b*Sin
h[x])/(a - I*b)]*Sqrt[(I*b)/(a + I*b) - (b*Sinh[x])/(a + I*b)]*(1 - ((I/2)*
```

```
(a - I*b)*(((-I)*b)/(a - I*b) - (b*Sinh[x])/(a - I*b)))/b)^(3/2)*(-(((-1)^(
3/4)*Sqrt[b]*ArcSin[((-1)^(1/4)*Sqrt[a - I*b]*Sqrt[((-I)*b)/(a - I*b) - (b*
Sinh[x])/(a - I*b)])/(Sqrt[2]*Sqrt[b])])/(Sqrt[2]*Sqrt[a - I*b]*Sqrt[((-I)*
b)/(a - I*b) - (b*Sinh[x])/(a - I*b)]*(1 - ((I/2)*(a - I*b)*(((-I)*b)/(a -
I*b) - (b*Sinh[x])/(a - I*b)))/b)^(3/2))) + 1/(2*(1 - ((I/2)*(a - I*b)*(((-
I)*b)/(a - I*b) - (b*Sinh[x])/(a - I*b)))/b))))/((a + I*b)*b*Sqrt[((-I)*(a
+ I*b)*((I*b)/(a + I*b) - (b*Sinh[x])/(a + I*b)))/b])))/b - (4*Sqrt[2]*Sqrt
[((-I)*b)/(a - I*b) - (b*Sinh[x])/(a - I*b)]*Sqrt[(I*b)/(a + I*b) - (b*Sinh
[x])/(a + I*b)]*(1 - ((I/2)*(a - I*b)*(((-I)*b)/(a - I*b) - (b*Sinh[x])/(a
- I*b)))/b)^(5/2)*((-3*(-1)^(3/4)*Sqrt[b]*ArcSin[((-1)^(1/4)*Sqrt[a - I*b]*
Sqrt[((-I)*b)/(a - I*b) - (b*Sinh[x])/(a - I*b)])/(Sqrt[2]*Sqrt[b])])/(4*Sq
rt[2]*Sqrt[a - I*b]*Sqrt[((-I)*b)/(a - I*b) - (b*Sinh[x])/(a - I*b)]*(1 - (
(I/2)*(a - I*b)*(((-I)*b)/(a - I*b) - (b*Sinh[x])/(a - I*b)))/b)^(5/2)) + (
3/(2*(1 - ((I/2)*(a - I*b)*(((-I)*b)/(a - I*b) - (b*Sinh[x])/(a - I*b)))/b)
^2) + (1 - ((I/2)*(a - I*b)*(((-I)*b)/(a - I*b) - (b*Sinh[x])/(a - I*b)))/b
)^(-1))/4))/((a + I*b)*Sqrt[((-I)*(a + I*b)*((I*b)/(a + I*b) - (b*Sinh[x])/
(a + I*b)))/b]))))/b))/b)/((((-I)*a*b)/(a - I*b) - b^2/(a - I*b))*(((-I)*a*b
)/(a + I*b) + b^2/(a + I*b))))/(2*(((-I)*a*b)/(a - I*b) - b^2/(a - I*b))*((
(-I)*a*b)/(a + I*b) + b^2/(a + I*b))))/(3*(((-I)*a*b)/(a - I*b) - b^2/(a -
I*b))*(((-I)*a*b)/(a + I*b) + b^2/(a + I*b)))))/((1 - (a + b*Sinh[x])/(a -
I*b))^(3/2)*(1 - (a + b*Sinh[x])/(a + I*b))^(3/2)*(a*Sech[x] + b*Tanh[x])^4
)
```

fricas [B] time = 0.49, size = 2978, normalized size = 20.40

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sech(x)+b*tanh(x))^4,x, algorithm="fricas")
```

```
[Out] -1/6*(6*(a^4*b^3 + 2*a^2*b^5 + b^7)*x*cosh(x)^6 + 6*(a^4*b^3 + 2*a^2*b^5 +
b^7)*x*sinh(x)^6 - 22*a^4*b^3 - 38*a^2*b^5 - 16*b^7 + 6*(6*a^5*b^2 + 11*a^3
*b^4 + 5*a*b^6 + 6*(a^5*b^2 + 2*a^3*b^4 + a*b^6)*x)*cosh(x)^5 + 6*(6*a^5*b^
2 + 11*a^3*b^4 + 5*a*b^6 + 6*(a^4*b^3 + 2*a^2*b^5 + b^7)*x*cosh(x) + 6*(a^5
*b^2 + 2*a^3*b^4 + a*b^6)*x)*sinh(x)^5 + 6*(18*a^6*b + 27*a^4*b^3 + 5*a^2*b
^5 - 4*b^7 + 3*(4*a^6*b + 7*a^4*b^3 + 2*a^2*b^5 - b^7)*x)*cosh(x)^4 + 6*(18
*a^6*b + 27*a^4*b^3 + 5*a^2*b^5 - 4*b^7 + 15*(a^4*b^3 + 2*a^2*b^5 + b^7)*x*
cosh(x)^2 + 3*(4*a^6*b + 7*a^4*b^3 + 2*a^2*b^5 - b^7)*x + 5*(6*a^5*b^2 + 11
*a^3*b^4 + 5*a*b^6 + 6*(a^5*b^2 + 2*a^3*b^4 + a*b^6)*x)*cosh(x))*sinh(x)^4
+ 4*(22*a^7 + 5*a^5*b^2 - 41*a^3*b^4 - 24*a*b^6 + 6*(2*a^7 + a^5*b^2 - 4*a^
3*b^4 - 3*a*b^6)*x)*cosh(x)^3 + 4*(22*a^7 + 5*a^5*b^2 - 41*a^3*b^4 - 24*a*b
^6 + 30*(a^4*b^3 + 2*a^2*b^5 + b^7)*x*cosh(x)^3 + 15*(6*a^5*b^2 + 11*a^3*b^
4 + 5*a*b^6 + 6*(a^5*b^2 + 2*a^3*b^4 + a*b^6)*x)*cosh(x)^2 + 6*(2*a^7 + a^5
*b^2 - 4*a^3*b^4 - 3*a*b^6)*x + 6*(18*a^6*b + 27*a^4*b^3 + 5*a^2*b^5 - 4*b^
7 + 3*(4*a^6*b + 7*a^4*b^3 + 2*a^2*b^5 - b^7)*x)*cosh(x))*sinh(x)^3 - 6*(26
```

```
*a^6*b + 38*a^4*b^3 + 8*a^2*b^5 - 4*b^7 + 3*(4*a^6*b + 7*a^4*b^3 + 2*a^2*b^
5 - b^7)*x)*cosh(x)^2 - 6*(26*a^6*b + 38*a^4*b^3 + 8*a^2*b^5 - 4*b^7 - 15*(
a^4*b^3 + 2*a^2*b^5 + b^7)*x*cosh(x)^4 - 10*(6*a^5*b^2 + 11*a^3*b^4 + 5*a*b
^6 + 6*(a^5*b^2 + 2*a^3*b^4 + a*b^6)*x)*cosh(x)^3 - 6*(18*a^6*b + 27*a^4*b^
3 + 5*a^2*b^5 - 4*b^7 + 3*(4*a^6*b + 7*a^4*b^3 + 2*a^2*b^5 - b^7)*x)*cosh(x
)^2 + 3*(4*a^6*b + 7*a^4*b^3 + 2*a^2*b^5 - b^7)*x - 2*(22*a^7 + 5*a^5*b^2 -
 41*a^3*b^4 - 24*a*b^6 + 6*(2*a^7 + a^5*b^2 - 4*a^3*b^4 - 3*a*b^6)*x)*cosh(
x))*sinh(x)^2 + 3*((2*a^3*b^3 + 3*a*b^5)*cosh(x)^6 + (2*a^3*b^3 + 3*a*b^5)*
sinh(x)^6 - 2*a^3*b^3 - 3*a*b^5 + 6*(2*a^4*b^2 + 3*a^2*b^4)*cosh(x)^5 + 6*(
2*a^4*b^2 + 3*a^2*b^4 + (2*a^3*b^3 + 3*a*b^5)*cosh(x))*sinh(x)^5 + 3*(8*a^5
*b + 10*a^3*b^3 - 3*a*b^5)*cosh(x)^4 + 3*(8*a^5*b + 10*a^3*b^3 - 3*a*b^5 +
5*(2*a^3*b^3 + 3*a*b^5)*cosh(x)^2 + 10*(2*a^4*b^2 + 3*a^2*b^4)*cosh(x))*sin
h(x)^4 + 4*(4*a^6 - 9*a^2*b^4)*cosh(x)^3 + 4*(4*a^6 - 9*a^2*b^4 + 5*(2*a^3*
b^3 + 3*a*b^5)*cosh(x)^3 + 15*(2*a^4*b^2 + 3*a^2*b^4)*cosh(x)^2 + 3*(8*a^5*
b + 10*a^3*b^3 - 3*a*b^5)*cosh(x))*sinh(x)^3 - 3*(8*a^5*b + 10*a^3*b^3 - 3*
a*b^5)*cosh(x)^2 - 3*(8*a^5*b + 10*a^3*b^3 - 3*a*b^5 - 5*(2*a^3*b^3 + 3*a*b
^5)*cosh(x)^4 - 20*(2*a^4*b^2 + 3*a^2*b^4)*cosh(x)^3 - 6*(8*a^5*b + 10*a^3*
b^3 - 3*a*b^5)*cosh(x)^2 - 4*(4*a^6 - 9*a^2*b^4)*cosh(x))*sinh(x)^2 + 6*(2*
a^4*b^2 + 3*a^2*b^4)*cosh(x) + 6*(2*a^4*b^2 + 3*a^2*b^4 + (2*a^3*b^3 + 3*a*
b^5)*cosh(x)^5 + 5*(2*a^4*b^2 + 3*a^2*b^4)*cosh(x)^4 + 2*(8*a^5*b + 10*a^3*
b^3 - 3*a*b^5)*cosh(x)^3 + 2*(4*a^6 - 9*a^2*b^4)*cosh(x)^2 - (8*a^5*b + 10*
a^3*b^3 - 3*a*b^5)*cosh(x))*sinh(x))*sqrt(a^2 + b^2)*log((b^2*cosh(x)^2 + b
^2*sinh(x)^2 + 2*a*b*cosh(x) + 2*a^2 + b^2 + 2*(b^2*cosh(x) + a*b)*sinh(x)
+ 2*sqrt(a^2 + b^2)*(b*cosh(x) + b*sinh(x) + a))/(b*cosh(x)^2 + b*sinh(x)^2
 + 2*a*cosh(x) + 2*(b*cosh(x) + a)*sinh(x) - b)) - 6*(a^4*b^3 + 2*a^2*b^5 +
 b^7)*x + 6*(16*a^5*b^2 + 27*a^3*b^4 + 11*a*b^6 + 6*(a^5*b^2 + 2*a^3*b^4 +
a*b^6)*x)*cosh(x) + 6*(16*a^5*b^2 + 27*a^3*b^4 + 11*a*b^6 + 6*(a^4*b^3 + 2*
a^2*b^5 + b^7)*x*cosh(x)^5 + 5*(6*a^5*b^2 + 11*a^3*b^4 + 5*a*b^6 + 6*(a^5*b
^2 + 2*a^3*b^4 + a*b^6)*x)*cosh(x)^4 + 4*(18*a^6*b + 27*a^4*b^3 + 5*a^2*b^5
 - 4*b^7 + 3*(4*a^6*b + 7*a^4*b^3 + 2*a^2*b^5 - b^7)*x)*cosh(x)^3 + 2*(22*a
^7 + 5*a^5*b^2 - 41*a^3*b^4 - 24*a*b^6 + 6*(2*a^7 + a^5*b^2 - 4*a^3*b^4 - 3
*a*b^6)*x)*cosh(x)^2 + 6*(a^5*b^2 + 2*a^3*b^4 + a*b^6)*x - 2*(26*a^6*b + 38
*a^4*b^3 + 8*a^2*b^5 - 4*b^7 + 3*(4*a^6*b + 7*a^4*b^3 + 2*a^2*b^5 - b^7)*x)
*cosh(x))*sinh(x))/(a^4*b^7 + 2*a^2*b^9 + b^11 - (a^4*b^7 + 2*a^2*b^9 + b^1
1)*cosh(x)^6 - (a^4*b^7 + 2*a^2*b^9 + b^11)*sinh(x)^6 - 6*(a^5*b^6 + 2*a^3*
b^8 + a*b^10)*cosh(x)^5 - 6*(a^5*b^6 + 2*a^3*b^8 + a*b^10 + (a^4*b^7 + 2*a^
2*b^9 + b^11)*cosh(x))*sinh(x)^5 - 3*(4*a^6*b^5 + 7*a^4*b^7 + 2*a^2*b^9 - b
^11)*cosh(x)^4 - 3*(4*a^6*b^5 + 7*a^4*b^7 + 2*a^2*b^9 - b^11 + 5*(a^4*b^7 +
 2*a^2*b^9 + b^11)*cosh(x)^2 + 10*(a^5*b^6 + 2*a^3*b^8 + a*b^10)*cosh(x))*s
inh(x)^4 - 4*(2*a^7*b^4 + a^5*b^6 - 4*a^3*b^8 - 3*a*b^10)*cosh(x)^3 - 4*(2*
a^7*b^4 + a^5*b^6 - 4*a^3*b^8 - 3*a*b^10 + 5*(a^4*b^7 + 2*a^2*b^9 + b^11)*c
osh(x)^3 + 15*(a^5*b^6 + 2*a^3*b^8 + a*b^10)*cosh(x)^2 + 3*(4*a^6*b^5 + 7*a
^4*b^7 + 2*a^2*b^9 - b^11)*cosh(x))*sinh(x)^3 + 3*(4*a^6*b^5 + 7*a^4*b^7 +
2*a^2*b^9 - b^11)*cosh(x)^2 + 3*(4*a^6*b^5 + 7*a^4*b^7 + 2*a^2*b^9 - b^11 -
 5*(a^4*b^7 + 2*a^2*b^9 + b^11)*cosh(x)^4 - 20*(a^5*b^6 + 2*a^3*b^8 + a*b^1
```

```
0)*cosh(x)^3 - 6*(4*a^6*b^5 + 7*a^4*b^7 + 2*a^2*b^9 - b^11)*cosh(x)^2 - 4*(
2*a^7*b^4 + a^5*b^6 - 4*a^3*b^8 - 3*a*b^10)*cosh(x))*sinh(x)^2 - 6*(a^5*b^6
 + 2*a^3*b^8 + a*b^10)*cosh(x) - 6*(a^5*b^6 + 2*a^3*b^8 + a*b^10 + (a^4*b^7
 + 2*a^2*b^9 + b^11)*cosh(x)^5 + 5*(a^5*b^6 + 2*a^3*b^8 + a*b^10)*cosh(x)^4
 + 2*(4*a^6*b^5 + 7*a^4*b^7 + 2*a^2*b^9 - b^11)*cosh(x)^3 + 2*(2*a^7*b^4 +
a^5*b^6 - 4*a^3*b^8 - 3*a*b^10)*cosh(x)^2 - (4*a^6*b^5 + 7*a^4*b^7 + 2*a^2*
b^9 - b^11)*cosh(x))*sinh(x))
```

giac [A] time = 0.17, size = 267, normalized size = 1.83

$$-\frac{\left(2\,a^{3}+3\,ab^{2}\right)\log\left(\frac{\left|2\,be^{x}+2\,a-2\,\sqrt{a^{2}+b^{2}}\right|}{\left|2\,be^{x}+2\,a+2\,\sqrt{a^{2}+b^{2}}\right|}\right)}{2\left(a^{2}b^{4}+b^{6}\right)\sqrt{a^{2}+b^{2}}}+\frac{18\,a^{3}b^{2}e^{(5\,x)}+15\,ab^{4}e^{(5\,x)}+54\,a^{4}be^{(4\,x)}+27\,a^{2}b^{3}e^{(4\,x)}-12\,b^{5}e^{(4\,x)}+44\,a}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sech(x)+b*tanh(x))^4,x, algorithm="giac")
```

```
[Out] -1/2*(2*a^3 + 3*a*b^2)*log(abs(2*b*e^x + 2*a - 2*sqrt(a^2 + b^2))/abs(2*b*e
^x + 2*a + 2*sqrt(a^2 + b^2)))/((a^2*b^4 + b^6)*sqrt(a^2 + b^2)) + 1/3*(18*
a^3*b^2*e^(5*x) + 15*a*b^4*e^(5*x) + 54*a^4*b*e^(4*x) + 27*a^2*b^3*e^(4*x)
- 12*b^5*e^(4*x) + 44*a^5*e^(3*x) - 34*a^3*b^2*e^(3*x) - 48*a*b^4*e^(3*x) -
 78*a^4*b*e^(2*x) - 36*a^2*b^3*e^(2*x) + 12*b^5*e^(2*x) + 48*a^3*b^2*e^x +
33*a*b^4*e^x - 11*a^2*b^3 - 8*b^5)/((a^2*b^4 + b^6)*(b*e^(2*x) + 2*a*e^x -
b)^3) + x/b^4
```

maple [B] time = 0.31, size = 972, normalized size = 6.66

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*sech(x)+b*tanh(x))^4,x)
```

```
[Out] -1/b^4*ln(tanh(1/2*x)-1)+1/b^4*ln(tanh(1/2*x)+1)+2/3*b/(a*tanh(1/2*x)^2-2*t
anh(1/2*x)*b-a)^3/(a^2+b^2)+1/b^2/(a*tanh(1/2*x)^2-2*tanh(1/2*x)*b-a)^3*a^3
/(a^2+b^2)*tanh(1/2*x)^5+2*b^2/(a*tanh(1/2*x)^2-2*tanh(1/2*x)*b-a)^3/a/(a^2
+b^2)*tanh(1/2*x)^5+2/b^3/(a*tanh(1/2*x)^2-2*tanh(1/2*x)*b-a)^3/(a^2+b^2)*a
^4*tanh(1/2*x)^4-3/b/(a*tanh(1/2*x)^2-2*tanh(1/2*x)*b-a)^3/(a^2+b^2)*a^2*ta
nh(1/2*x)^4-4*b^3/(a*tanh(1/2*x)^2-2*tanh(1/2*x)*b-a)^3/(a^2+b^2)/a^2*tanh(
1/2*x)^4-12/b^2/(a*tanh(1/2*x)^2-2*tanh(1/2*x)*b-a)^3*a^3/(a^2+b^2)*tanh(1/
2*x)^3+8/3*b^2/(a*tanh(1/2*x)^2-2*tanh(1/2*x)*b-a)^3/a/(a^2+b^2)*tanh(1/2*x
)^3+8/3*b^4/(a*tanh(1/2*x)^2-2*tanh(1/2*x)*b-a)^3/a^3/(a^2+b^2)*tanh(1/2*x)
^3-4/b^3/(a*tanh(1/2*x)^2-2*tanh(1/2*x)*b-a)^3*a^4/(a^2+b^2)*tanh(1/2*x)^2+
16/b/(a*tanh(1/2*x)^2-2*tanh(1/2*x)*b-a)^3*a^2/(a^2+b^2)*tanh(1/2*x)^2+4*b^
```

```
3/(a*tanh(1/2*x)^2-2*tanh(1/2*x)*b-a)^3/a^2/(a^2+b^2)*tanh(1/2*x)^2+11/b^2/
(a*tanh(1/2*x)^2-2*tanh(1/2*x)*b-a)^3*a^3/(a^2+b^2)*tanh(1/2*x)+2*b^2/(a*ta
nh(1/2*x)^2-2*tanh(1/2*x)*b-a)^3/a/(a^2+b^2)*tanh(1/2*x)+2/(a*tanh(1/2*x)^2
-2*tanh(1/2*x)*b-a)^3*a/(a^2+b^2)*tanh(1/2*x)^5-4*b/(a*tanh(1/2*x)^2-2*tanh
(1/2*x)*b-a)^3/(a^2+b^2)*tanh(1/2*x)^4-2/(a*tanh(1/2*x)^2-2*tanh(1/2*x)*b-a
)^3*a/(a^2+b^2)*tanh(1/2*x)^3+14*b/(a*tanh(1/2*x)^2-2*tanh(1/2*x)*b-a)^3/(a
^2+b^2)*tanh(1/2*x)^2+8/(a*tanh(1/2*x)^2-2*tanh(1/2*x)*b-a)^3*a/(a^2+b^2)*t
anh(1/2*x)+2/b^3/(a*tanh(1/2*x)^2-2*tanh(1/2*x)*b-a)^3/(a^2+b^2)*a^4+5/3/b/
(a*tanh(1/2*x)^2-2*tanh(1/2*x)*b-a)^3/(a^2+b^2)*a^2-2/b^4*a^3/(a^2+b^2)^(3/
2)*arctanh(1/2*(2*a*tanh(1/2*x)-2*b)/(a^2+b^2)^(1/2))-3/b^2*a/(a^2+b^2)^(3/
2)*arctanh(1/2*(2*a*tanh(1/2*x)-2*b)/(a^2+b^2)^(1/2))
```

maxima [B] time = 0.78, size = 375, normalized size = 2.57

$$-\frac{\left(2\,a^{2}+3\,b^{2}\right)a\log\left(\frac{be^{(-x)}-a-\sqrt{a^{2}+b^{2}}}{be^{(-x)}-a+\sqrt{a^{2}+b^{2}}}\right)}{2\left(a^{2}b^{4}+b^{6}\right)\sqrt{a^{2}+b^{2}}}-\frac{11\,a^{2}b^{3}+8\,b^{5}+3\left(16\,a^{3}b^{2}+11\,ab^{4}\right)e^{(-x)}+6\left(13\,a^{4}b+6\,a^{2}b^{3}-2\,b^{5}\right)e^{(-2}}{3\left(a^{2}b^{7}+b^{9}+6\left(a^{3}b^{6}+ab^{8}\right)e^{(-x)}+3\left(4\,a^{4}b^{5}+3\,a^{2}b^{7}-b^{9}\right)e^{(-2\,x)}+4\left(2\,a\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sech(x)+b*tanh(x))^4,x, algorithm="maxima")
```

```
[Out] -1/2*(2*a^2 + 3*b^2)*a*log((b*e^(-x) - a - sqrt(a^2 + b^2))/(b*e^(-x) - a +
 sqrt(a^2 + b^2)))/((a^2*b^4 + b^6)*sqrt(a^2 + b^2)) - 1/3*(11*a^2*b^3 + 8*
b^5 + 3*(16*a^3*b^2 + 11*a*b^4)*e^(-x) + 6*(13*a^4*b + 6*a^2*b^3 - 2*b^5)*e
^(-2*x) + 2*(22*a^5 - 17*a^3*b^2 - 24*a*b^4)*e^(-3*x) - 3*(18*a^4*b + 9*a^2
*b^3 - 4*b^5)*e^(-4*x) + 3*(6*a^3*b^2 + 5*a*b^4)*e^(-5*x))/(a^2*b^7 + b^9 +
 6*(a^3*b^6 + a*b^8)*e^(-x) + 3*(4*a^4*b^5 + 3*a^2*b^7 - b^9)*e^(-2*x) + 4*
(2*a^5*b^4 - a^3*b^6 - 3*a*b^8)*e^(-3*x) - 3*(4*a^4*b^5 + 3*a^2*b^7 - b^9)*
e^(-4*x) + 6*(a^3*b^6 + a*b^8)*e^(-5*x) - (a^2*b^7 + b^9)*e^(-6*x)) + x/b^4
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(b\tanh(x)+\frac{a}{\cosh(x)}\right)^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*tanh(x) + a/cosh(x))^4,x)
```

```
[Out] int(1/(b*tanh(x) + a/cosh(x))^4, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a\operatorname{sech}(x)+b\tanh(x)\right)^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sech(x)+b*tanh(x))**4,x)
```

```
[Out] Integral((a*sech(x) + b*tanh(x))**(-4), x)
```

3.623 $\int \frac{1}{(a\,\mathbf{sech}(x)+b\tanh(x))^5}\,dx$

Optimal. Leaf size=95

$$-\frac{\left(a^2+b^2\right)^2}{4b^5(a+b\sinh(x))^4}+\frac{4a\left(a^2+b^2\right)}{3b^5(a+b\sinh(x))^3}-\frac{3a^2+b^2}{b^5(a+b\sinh(x))^2}+\frac{4a}{b^5(a+b\sinh(x))}+\frac{\log(a+b\sinh(x))}{b^5}$$

```
[Out] ln(a+b*sinh(x))/b^5-1/4*(a^2+b^2)^2/b^5/(a+b*sinh(x))^4+4/3*a*(a^2+b^2)/b^5
/(a+b*sinh(x))^3+(-3*a^2-b^2)/b^5/(a+b*sinh(x))^2+4*a/b^5/(a+b*sinh(x))
```

Rubi [A] time = 0.12, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {4391, 2668, 697}

$$-\frac{\left(a^2+b^2\right)^2}{4b^5(a+b\sinh(x))^4}+\frac{4a\left(a^2+b^2\right)}{3b^5(a+b\sinh(x))^3}-\frac{3a^2+b^2}{b^5(a+b\sinh(x))^2}+\frac{4a}{b^5(a+b\sinh(x))}+\frac{\log(a+b\sinh(x))}{b^5}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Sech[x] + b*Tanh[x])^(-5),x]
```

```
[Out] Log[a + b*Sinh[x]]/b^5 - (a^2 + b^2)^2/(4*b^5*(a + b*Sinh[x])^4) + (4*a*(a^
2 + b^2))/(3*b^5*(a + b*Sinh[x])^3) - (3*a^2 + b^2)/(b^5*(a + b*Sinh[x])^2)
 + (4*a)/(b^5*(a + b*Sinh[x]))
```

Rule 697

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 4391

```
Int[(u_.)*((b_.)*sec[(c_.) + (d_.)*(x_)]^(n_.) + (a_.)*tan[(c_.) + (d_.)*(x
_)]^(n_.))^(p_), x_Symbol] :> Int[ActivateTrig[u]*Sec[c + d*x]^(n*p)*(b + a
*Sin[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

Rubi steps

$$\int \frac{1}{(a\text{sech}(x) + b\tanh(x))^5}\,dx = \int \frac{\cosh^5(x)}{(a + b\sinh(x))^5}\,dx$$
$$= \frac{\text{Subst}\left(\int \frac{\left(-b^2-x^2\right)^2}{(a+x)^5}\,dx, x, b\sinh(x)\right)}{b^5}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{\left(a^2+b^2\right)^2}{(a+x)^5} - \frac{4a\left(a^2+b^2\right)}{(a+x)^4} + \frac{2\left(3a^2+b^2\right)}{(a+x)^3} - \frac{4a}{(a+x)^2} + \frac{1}{a+x}\right)dx, x, b\sinh(x)\right)}{b^5}$$
$$= \frac{\log(a + b\sinh(x))}{b^5} - \frac{\left(a^2 + b^2\right)^2}{4b^5(a + b\sinh(x))^4} + \frac{4a\left(a^2 + b^2\right)}{3b^5(a + b\sinh(x))^3} - \frac{3a^2 + b^2}{b^5(a + b\sinh}$$

Mathematica [A] time = 0.25, size = 83, normalized size = 0.87

$$\frac{-\frac{\left(a^2+b^2\right)^2}{4(a+b\sinh(x))^4} + \frac{4a\left(a^2+b^2\right)}{3(a+b\sinh(x))^3} - \frac{3a^2+b^2}{(a+b\sinh(x))^2} + \frac{4a}{a+b\sinh(x)} + \log(a + b\sinh(x))}{b^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Sech[x] + b*Tanh[x])^(-5),x]
```

```
[Out] (Log[a + b*Sinh[x]] - (a^2 + b^2)^2/(4*(a + b*Sinh[x])^4) + (4*a*(a^2 + b^2
))/(3*(a + b*Sinh[x])^3) - (3*a^2 + b^2)/(a + b*Sinh[x])^2 + (4*a)/(a + b*S
inh[x]))/b^5
```

fricas [B] time = 0.46, size = 2640, normalized size = 27.79

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sech(x)+b*tanh(x))^5,x, algorithm="fricas")
```

```
[Out] -1/3*(3*b^4*x*cosh(x)^8 + 3*b^4*x*sinh(x)^8 + 24*(a*b^3*x - a*b^3)*cosh(x)^
7 + 24*(b^4*x*cosh(x) + a*b^3*x - a*b^3)*sinh(x)^7 - 12*(9*a^2*b^2 - b^4 -
(6*a^2*b^2 - b^4)*x)*cosh(x)^6 + 12*(7*b^4*x*cosh(x)^2 - 9*a^2*b^2 + b^4 +
(6*a^2*b^2 - b^4)*x + 14*(a*b^3*x - a*b^3)*cosh(x))*sinh(x)^6 - 8*(22*a^3*b
 - 11*a*b^3 - 3*(4*a^3*b - 3*a*b^3)*x)*cosh(x)^5 + 8*(21*b^4*x*cosh(x)^3 -
22*a^3*b + 11*a*b^3 + 63*(a*b^3*x - a*b^3)*cosh(x)^2 + 3*(4*a^3*b - 3*a*b^3
)*x - 9*(9*a^2*b^2 - b^4 - (6*a^2*b^2 - b^4)*x)*cosh(x))*sinh(x)^5 + 3*b^4*
```

```
x - 2*(50*a^4 - 112*a^2*b^2 + 6*b^4 - 3*(8*a^4 - 24*a^2*b^2 + 3*b^4)*x)*cos
h(x)^4 + 2*(105*b^4*x*cosh(x)^4 - 50*a^4 + 112*a^2*b^2 - 6*b^4 + 420*(a*b^3
*x - a*b^3)*cosh(x)^3 - 90*(9*a^2*b^2 - b^4 - (6*a^2*b^2 - b^4)*x)*cosh(x)^
2 + 3*(8*a^4 - 24*a^2*b^2 + 3*b^4)*x - 20*(22*a^3*b - 11*a*b^3 - 3*(4*a^3*b
 - 3*a*b^3)*x)*cosh(x))*sinh(x)^4 + 8*(22*a^3*b - 11*a*b^3 - 3*(4*a^3*b - 3
*a*b^3)*x)*cosh(x)^3 + 8*(21*b^4*x*cosh(x)^5 + 105*(a*b^3*x - a*b^3)*cosh(x
)^4 + 22*a^3*b - 11*a*b^3 - 30*(9*a^2*b^2 - b^4 - (6*a^2*b^2 - b^4)*x)*cosh
(x)^3 - 10*(22*a^3*b - 11*a*b^3 - 3*(4*a^3*b - 3*a*b^3)*x)*cosh(x)^2 - 3*(4
*a^3*b - 3*a*b^3)*x - (50*a^4 - 112*a^2*b^2 + 6*b^4 - 3*(8*a^4 - 24*a^2*b^2
 + 3*b^4)*x)*cosh(x))*sinh(x)^3 - 12*(9*a^2*b^2 - b^4 - (6*a^2*b^2 - b^4)*x
)*cosh(x)^2 + 4*(21*b^4*x*cosh(x)^6 + 126*(a*b^3*x - a*b^3)*cosh(x)^5 - 45*
(9*a^2*b^2 - b^4 - (6*a^2*b^2 - b^4)*x)*cosh(x)^4 - 27*a^2*b^2 + 3*b^4 - 20
*(22*a^3*b - 11*a*b^3 - 3*(4*a^3*b - 3*a*b^3)*x)*cosh(x)^3 - 3*(50*a^4 - 11
2*a^2*b^2 + 6*b^4 - 3*(8*a^4 - 24*a^2*b^2 + 3*b^4)*x)*cosh(x)^2 + 3*(6*a^2*
b^2 - b^4)*x + 6*(22*a^3*b - 11*a*b^3 - 3*(4*a^3*b - 3*a*b^3)*x)*cosh(x))*s
inh(x)^2 - 24*(a*b^3*x - a*b^3)*cosh(x) - 3*(b^4*cosh(x)^8 + b^4*sinh(x)^8
+ 8*a*b^3*cosh(x)^7 + 8*(b^4*cosh(x) + a*b^3)*sinh(x)^7 + 4*(6*a^2*b^2 - b^
4)*cosh(x)^6 + 4*(7*b^4*cosh(x)^2 + 14*a*b^3*cosh(x) + 6*a^2*b^2 - b^4)*sin
h(x)^6 + 8*(4*a^3*b - 3*a*b^3)*cosh(x)^5 + 8*(7*b^4*cosh(x)^3 + 21*a*b^3*co
sh(x)^2 + 4*a^3*b - 3*a*b^3 + 3*(6*a^2*b^2 - b^4)*cosh(x))*sinh(x)^5 - 8*a*
b^3*cosh(x) + 2*(8*a^4 - 24*a^2*b^2 + 3*b^4)*cosh(x)^4 + 2*(35*b^4*cosh(x)^
4 + 140*a*b^3*cosh(x)^3 + 8*a^4 - 24*a^2*b^2 + 3*b^4 + 30*(6*a^2*b^2 - b^4)
*cosh(x)^2 + 20*(4*a^3*b - 3*a*b^3)*cosh(x))*sinh(x)^4 + b^4 - 8*(4*a^3*b -
 3*a*b^3)*cosh(x)^3 + 8*(7*b^4*cosh(x)^5 + 35*a*b^3*cosh(x)^4 - 4*a^3*b + 3
*a*b^3 + 10*(6*a^2*b^2 - b^4)*cosh(x)^3 + 10*(4*a^3*b - 3*a*b^3)*cosh(x)^2
+ (8*a^4 - 24*a^2*b^2 + 3*b^4)*cosh(x))*sinh(x)^3 + 4*(6*a^2*b^2 - b^4)*cos
h(x)^2 + 4*(7*b^4*cosh(x)^6 + 42*a*b^3*cosh(x)^5 + 15*(6*a^2*b^2 - b^4)*cos
h(x)^4 + 6*a^2*b^2 - b^4 + 20*(4*a^3*b - 3*a*b^3)*cosh(x)^3 + 3*(8*a^4 - 24
*a^2*b^2 + 3*b^4)*cosh(x)^2 - 6*(4*a^3*b - 3*a*b^3)*cosh(x))*sinh(x)^2 + 8*
(b^4*cosh(x)^7 + 7*a*b^3*cosh(x)^6 + 3*(6*a^2*b^2 - b^4)*cosh(x)^5 + 5*(4*a
^3*b - 3*a*b^3)*cosh(x)^4 - a*b^3 + (8*a^4 - 24*a^2*b^2 + 3*b^4)*cosh(x)^3
- 3*(4*a^3*b - 3*a*b^3)*cosh(x)^2 + (6*a^2*b^2 - b^4)*cosh(x))*sinh(x))*log
(2*(b*sinh(x) + a)/(cosh(x) - sinh(x))) + 8*(3*b^4*x*cosh(x)^7 + 21*(a*b^3*
x - a*b^3)*cosh(x)^6 - 9*(9*a^2*b^2 - b^4 - (6*a^2*b^2 - b^4)*x)*cosh(x)^5
- 3*a*b^3*x - 5*(22*a^3*b - 11*a*b^3 - 3*(4*a^3*b - 3*a*b^3)*x)*cosh(x)^4 +
 3*a*b^3 - (50*a^4 - 112*a^2*b^2 + 6*b^4 - 3*(8*a^4 - 24*a^2*b^2 + 3*b^4)*x
)*cosh(x)^3 + 3*(22*a^3*b - 11*a*b^3 - 3*(4*a^3*b - 3*a*b^3)*x)*cosh(x)^2 -
 3*(9*a^2*b^2 - b^4 - (6*a^2*b^2 - b^4)*x)*cosh(x))*sinh(x))/(b^9*cosh(x)^8
 + b^9*sinh(x)^8 + 8*a*b^8*cosh(x)^7 - 8*a*b^8*cosh(x) + b^9 + 8*(b^9*cosh(
x) + a*b^8)*sinh(x)^7 + 4*(6*a^2*b^7 - b^9)*cosh(x)^6 + 4*(7*b^9*cosh(x)^2
+ 14*a*b^8*cosh(x) + 6*a^2*b^7 - b^9)*sinh(x)^6 + 8*(4*a^3*b^6 - 3*a*b^8)*c
osh(x)^5 + 8*(7*b^9*cosh(x)^3 + 21*a*b^8*cosh(x)^2 + 4*a^3*b^6 - 3*a*b^8 +
3*(6*a^2*b^7 - b^9)*cosh(x))*sinh(x)^5 + 2*(8*a^4*b^5 - 24*a^2*b^7 + 3*b^9)
*cosh(x)^4 + 2*(35*b^9*cosh(x)^4 + 140*a*b^8*cosh(x)^3 + 8*a^4*b^5 - 24*a^2
*b^7 + 3*b^9 + 30*(6*a^2*b^7 - b^9)*cosh(x)^2 + 20*(4*a^3*b^6 - 3*a*b^8)*co
```

```
sh(x))*sinh(x)^4 - 8*(4*a^3*b^6 - 3*a*b^8)*cosh(x)^3 + 8*(7*b^9*cosh(x)^5 +
 35*a*b^8*cosh(x)^4 - 4*a^3*b^6 + 3*a*b^8 + 10*(6*a^2*b^7 - b^9)*cosh(x)^3
+ 10*(4*a^3*b^6 - 3*a*b^8)*cosh(x)^2 + (8*a^4*b^5 - 24*a^2*b^7 + 3*b^9)*cos
h(x))*sinh(x)^3 + 4*(6*a^2*b^7 - b^9)*cosh(x)^2 + 4*(7*b^9*cosh(x)^6 + 42*a
*b^8*cosh(x)^5 + 6*a^2*b^7 - b^9 + 15*(6*a^2*b^7 - b^9)*cosh(x)^4 + 20*(4*a
^3*b^6 - 3*a*b^8)*cosh(x)^3 + 3*(8*a^4*b^5 - 24*a^2*b^7 + 3*b^9)*cosh(x)^2
- 6*(4*a^3*b^6 - 3*a*b^8)*cosh(x))*sinh(x)^2 + 8*(b^9*cosh(x)^7 + 7*a*b^8*c
osh(x)^6 - a*b^8 + 3*(6*a^2*b^7 - b^9)*cosh(x)^5 + 5*(4*a^3*b^6 - 3*a*b^8)*
cosh(x)^4 + (8*a^4*b^5 - 24*a^2*b^7 + 3*b^9)*cosh(x)^3 - 3*(4*a^3*b^6 - 3*a
*b^8)*cosh(x)^2 + (6*a^2*b^7 - b^9)*cosh(x))*sinh(x))
```

giac [A] time = 0.17, size = 152, normalized size = 1.60

$$\frac{\log\left(\left|-b\left(e^{(-x)}-e^{x}\right)+2\,a\right|\right)}{b^{5}}-\frac{25\,b^{3}\left(e^{(-x)}-e^{x}\right)^{4}-104\,ab^{2}\left(e^{(-x)}-e^{x}\right)^{3}+168\,a^{2}b\left(e^{(-x)}-e^{x}\right)^{2}+48\,b^{3}\left(e^{(-x)}-e^{x}\right)^{2}-9}{12\,\left(b\left(e^{(-x)}-e^{x}\right)-2\,a\right)^{4}b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sech(x)+b*tanh(x))^5,x, algorithm="giac")
```

```
[Out] log(abs(-b*(e^(-x) - e^x) + 2*a))/b^5 - 1/12*(25*b^3*(e^(-x) - e^x)^4 - 104
*a*b^2*(e^(-x) - e^x)^3 + 168*a^2*b*(e^(-x) - e^x)^2 + 48*b^3*(e^(-x) - e^x
)^2 - 96*a^3*(e^(-x) - e^x) - 64*a*b^2*(e^(-x) - e^x) + 32*a^2*b + 48*b^3)/
((b*(e^(-x) - e^x) - 2*a)^4*b^4)
```

maple [B] time = 0.34, size = 721, normalized size = 7.59

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*sech(x)+b*tanh(x))^5,x)
```

```
[Out] -1/b^5*ln(tanh(1/2*x)-1)-1/b^5*ln(tanh(1/2*x)+1)+2/b^4/(a*tanh(1/2*x)^2-2*t
anh(1/2*x)*b-a)^4*a^3*tanh(1/2*x)^7-2/(a*tanh(1/2*x)^2-2*tanh(1/2*x)*b-a)^4
/a*tanh(1/2*x)^7-14/b^3/(a*tanh(1/2*x)^2-2*tanh(1/2*x)*b-a)^4*a^2*tanh(1/2*
x)^6+6*b/(a*tanh(1/2*x)^2-2*tanh(1/2*x)*b-a)^4/a^2*tanh(1/2*x)^6-6/b^4/(a*t
anh(1/2*x)^2-2*tanh(1/2*x)*b-a)^4*a^3*tanh(1/2*x)^5+104/3/b^2/(a*tanh(1/2*x
)^2-2*tanh(1/2*x)*b-a)^4*a*tanh(1/2*x)^5+2/3/(a*tanh(1/2*x)^2-2*tanh(1/2*x)
*b-a)^4/a*tanh(1/2*x)^5-8*b^2/(a*tanh(1/2*x)^2-2*tanh(1/2*x)*b-a)^4/a^3*tan
h(1/2*x)^5+28/b^3/(a*tanh(1/2*x)^2-2*tanh(1/2*x)*b-a)^4*a^2*tanh(1/2*x)^4-1
00/3/b/(a*tanh(1/2*x)^2-2*tanh(1/2*x)*b-a)^4*tanh(1/2*x)^4-28/3*b/(a*tanh(1
/2*x)^2-2*tanh(1/2*x)*b-a)^4/a^2*tanh(1/2*x)^4+4*b^3/(a*tanh(1/2*x)^2-2*tan
h(1/2*x)*b-a)^4/a^4*tanh(1/2*x)^4+6/b^4/(a*tanh(1/2*x)^2-2*tanh(1/2*x)*b-a)
^4*a^3*tanh(1/2*x)^3-104/3/b^2/(a*tanh(1/2*x)^2-2*tanh(1/2*x)*b-a)^4*a*tanh
```

```
(1/2*x)^3-2/3/(a*tanh(1/2*x)^2-2*tanh(1/2*x)*b-a)^4/a*tanh(1/2*x)^3+8*b^2/(
a*tanh(1/2*x)^2-2*tanh(1/2*x)*b-a)^4/a^3*tanh(1/2*x)^3-14/b^3/(a*tanh(1/2*x
)^2-2*tanh(1/2*x)*b-a)^4*a^2*tanh(1/2*x)^2+6*b/(a*tanh(1/2*x)^2-2*tanh(1/2*
x)*b-a)^4/a^2*tanh(1/2*x)^2-2/b^4/(a*tanh(1/2*x)^2-2*tanh(1/2*x)*b-a)^4*a^3
*tanh(1/2*x)+2/(a*tanh(1/2*x)^2-2*tanh(1/2*x)*b-a)^4/a*tanh(1/2*x)+1/b^5*ln
(a*tanh(1/2*x)^2-2*tanh(1/2*x)*b-a)
```

maxima [B] time = 0.36, size = 297, normalized size = 3.13

$$\frac{4\left(6\,ab^{3}e^{(-x)}-6\,ab^{3}e^{(-7\,x)}+3\left(9\,a^{2}b^{2}-b^{4}\right)e^{(-2\,x)}+22\left(2\,a^{3}b-ab^{3}\right)e^{(-3\,x)}+\left(25\,a^{4}-56\,a^{2}b^{2}+3\,b^{4}\right)e^{(-}}{3\left(8\,ab^{8}e^{(-x)}-8\,ab^{8}e^{(-7\,x)}+b^{9}e^{(-8\,x)}+b^{9}+4\left(6\,a^{2}b^{7}-b^{9}\right)e^{(-2\,x)}+8\left(4\,a^{3}b^{6}-3\,ab^{8}\right)e^{(-3\,x)}+2\left(8\,a^{4}b^{5}-24\,a^{2}b^{7}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sech(x)+b*tanh(x))^5,x, algorithm="maxima")
```

```
[Out] 4/3*(6*a*b^3*e^(-x) - 6*a*b^3*e^(-7*x) + 3*(9*a^2*b^2 - b^4)*e^(-2*x) + 22*
(2*a^3*b - a*b^3)*e^(-3*x) + (25*a^4 - 56*a^2*b^2 + 3*b^4)*e^(-4*x) - 22*(2
*a^3*b - a*b^3)*e^(-5*x) + 3*(9*a^2*b^2 - b^4)*e^(-6*x))/(8*a*b^8*e^(-x) -
8*a*b^8*e^(-7*x) + b^9*e^(-8*x) + b^9 + 4*(6*a^2*b^7 - b^9)*e^(-2*x) + 8*(4
*a^3*b^6 - 3*a*b^8)*e^(-3*x) + 2*(8*a^4*b^5 - 24*a^2*b^7 + 3*b^9)*e^(-4*x)
- 8*(4*a^3*b^6 - 3*a*b^8)*e^(-5*x) + 4*(6*a^2*b^7 - b^9)*e^(-6*x)) + x/b^5
+ log(-2*a*e^(-x) + b*e^(-2*x) - b)/b^5
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(b\tanh(x)+\frac{a}{\cosh(x)}\right)^{5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*tanh(x) + a/cosh(x))^5,x)
```

```
[Out] int(1/(b*tanh(x) + a/cosh(x))^5, x)
```

sympy [A] time = 16.64, size = 2162, normalized size = 22.76

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sech(x)+b*tanh(x))**5,x)
```

```
[Out] Piecewise((36*a**4*x*sech(x)**4/(36*a**4*b**5*sech(x)**4 + 144*a**3*b**6*ta
nh(x)*sech(x)**3 + 216*a**2*b**7*tanh(x)**2*sech(x)**2 + 144*a*b**8*tanh(x)
**3*sech(x) + 36*b**9*tanh(x)**4) + 36*a**4*log(a*sech(x)/b + tanh(x))*sech
```

```
(x)**4/(36*a**4*b**5*sech(x)**4 + 144*a**3*b**6*tanh(x)*sech(x)**3 + 216*a*
*2*b**7*tanh(x)**2*sech(x)**2 + 144*a*b**8*tanh(x)**3*sech(x) + 36*b**9*tan
h(x)**4) - 36*a**4*log(tanh(x) + 1)*sech(x)**4/(36*a**4*b**5*sech(x)**4 + 1
44*a**3*b**6*tanh(x)*sech(x)**3 + 216*a**2*b**7*tanh(x)**2*sech(x)**2 + 144
*a*b**8*tanh(x)**3*sech(x) + 36*b**9*tanh(x)**4) + 20*a**4*sech(x)**4/(36*a
**4*b**5*sech(x)**4 + 144*a**3*b**6*tanh(x)*sech(x)**3 + 216*a**2*b**7*tanh
(x)**2*sech(x)**2 + 144*a*b**8*tanh(x)**3*sech(x) + 36*b**9*tanh(x)**4) + 1
44*a**3*b*x*tanh(x)*sech(x)**3/(36*a**4*b**5*sech(x)**4 + 144*a**3*b**6*tan
h(x)*sech(x)**3 + 216*a**2*b**7*tanh(x)**2*sech(x)**2 + 144*a*b**8*tanh(x)*
*3*sech(x) + 36*b**9*tanh(x)**4) + 144*a**3*b*log(a*sech(x)/b + tanh(x))*ta
nh(x)*sech(x)**3/(36*a**4*b**5*sech(x)**4 + 144*a**3*b**6*tanh(x)*sech(x)**
3 + 216*a**2*b**7*tanh(x)**2*sech(x)**2 + 144*a*b**8*tanh(x)**3*sech(x) + 3
6*b**9*tanh(x)**4) - 144*a**3*b*log(tanh(x) + 1)*tanh(x)*sech(x)**3/(36*a**
4*b**5*sech(x)**4 + 144*a**3*b**6*tanh(x)*sech(x)**3 + 216*a**2*b**7*tanh(x
)**2*sech(x)**2 + 144*a*b**8*tanh(x)**3*sech(x) + 36*b**9*tanh(x)**4) + 44*
a**3*b*tanh(x)*sech(x)**3/(36*a**4*b**5*sech(x)**4 + 144*a**3*b**6*tanh(x)*
sech(x)**3 + 216*a**2*b**7*tanh(x)**2*sech(x)**2 + 144*a*b**8*tanh(x)**3*se
ch(x) + 36*b**9*tanh(x)**4) + 216*a**2*b**2*x*tanh(x)**2*sech(x)**2/(36*a**
4*b**5*sech(x)**4 + 144*a**3*b**6*tanh(x)*sech(x)**3 + 216*a**2*b**7*tanh(x
)**2*sech(x)**2 + 144*a*b**8*tanh(x)**3*sech(x) + 36*b**9*tanh(x)**4) + 216
*a**2*b**2*log(a*sech(x)/b + tanh(x))*tanh(x)**2*sech(x)**2/(36*a**4*b**5*s
ech(x)**4 + 144*a**3*b**6*tanh(x)*sech(x)**3 + 216*a**2*b**7*tanh(x)**2*sec
h(x)**2 + 144*a*b**8*tanh(x)**3*sech(x) + 36*b**9*tanh(x)**4) - 216*a**2*b*
*2*log(tanh(x) + 1)*tanh(x)**2*sech(x)**2/(36*a**4*b**5*sech(x)**4 + 144*a*
*3*b**6*tanh(x)*sech(x)**3 + 216*a**2*b**7*tanh(x)**2*sech(x)**2 + 144*a*b*
*8*tanh(x)**3*sech(x) + 36*b**9*tanh(x)**4) - 6*a**2*b**2*sech(x)**2/(36*a*
*4*b**5*sech(x)**4 + 144*a**3*b**6*tanh(x)*sech(x)**3 + 216*a**2*b**7*tanh(
x)**2*sech(x)**2 + 144*a*b**8*tanh(x)**3*sech(x) + 36*b**9*tanh(x)**4) + 14
4*a*b**3*x*tanh(x)**3*sech(x)/(36*a**4*b**5*sech(x)**4 + 144*a**3*b**6*tanh
(x)*sech(x)**3 + 216*a**2*b**7*tanh(x)**2*sech(x)**2 + 144*a*b**8*tanh(x)**
3*sech(x) + 36*b**9*tanh(x)**4) + 144*a*b**3*log(a*sech(x)/b + tanh(x))*tan
h(x)**3*sech(x)/(36*a**4*b**5*sech(x)**4 + 144*a**3*b**6*tanh(x)*sech(x)**3
 + 216*a**2*b**7*tanh(x)**2*sech(x)**2 + 144*a*b**8*tanh(x)**3*sech(x) + 36
*b**9*tanh(x)**4) - 144*a*b**3*log(tanh(x) + 1)*tanh(x)**3*sech(x)/(36*a**4
*b**5*sech(x)**4 + 144*a**3*b**6*tanh(x)*sech(x)**3 + 216*a**2*b**7*tanh(x)
**2*sech(x)**2 + 144*a*b**8*tanh(x)**3*sech(x) + 36*b**9*tanh(x)**4) - 52*a
*b**3*tanh(x)**3*sech(x)/(36*a**4*b**5*sech(x)**4 + 144*a**3*b**6*tanh(x)*s
ech(x)**3 + 216*a**2*b**7*tanh(x)**2*sech(x)**2 + 144*a*b**8*tanh(x)**3*sec
h(x) + 36*b**9*tanh(x)**4) - 24*a*b**3*tanh(x)*sech(x)/(36*a**4*b**5*sech(x
)**4 + 144*a**3*b**6*tanh(x)*sech(x)**3 + 216*a**2*b**7*tanh(x)**2*sech(x)*
*2 + 144*a*b**8*tanh(x)**3*sech(x) + 36*b**9*tanh(x)**4) + 36*b**4*x*tanh(x
)**4/(36*a**4*b**5*sech(x)**4 + 144*a**3*b**6*tanh(x)*sech(x)**3 + 216*a**2
*b**7*tanh(x)**2*sech(x)**2 + 144*a*b**8*tanh(x)**3*sech(x) + 36*b**9*tanh(
x)**4) + 36*b**4*log(a*sech(x)/b + tanh(x))*tanh(x)**4/(36*a**4*b**5*sech(x
)**4 + 144*a**3*b**6*tanh(x)*sech(x)**3 + 216*a**2*b**7*tanh(x)**2*sech(x)*
```

```
*2 + 144*a*b**8*tanh(x)**3*sech(x) + 36*b**9*tanh(x)**4) - 36*b**4*log(tanh
(x) + 1)*tanh(x)**4/(36*a**4*b**5*sech(x)**4 + 144*a**3*b**6*tanh(x)*sech(x
)**3 + 216*a**2*b**7*tanh(x)**2*sech(x)**2 + 144*a*b**8*tanh(x)**3*sech(x)
+ 36*b**9*tanh(x)**4) - 28*b**4*tanh(x)**4/(36*a**4*b**5*sech(x)**4 + 144*a
**3*b**6*tanh(x)*sech(x)**3 + 216*a**2*b**7*tanh(x)**2*sech(x)**2 + 144*a*b
**8*tanh(x)**3*sech(x) + 36*b**9*tanh(x)**4) - 18*b**4*tanh(x)**2/(36*a**4*
b**5*sech(x)**4 + 144*a**3*b**6*tanh(x)*sech(x)**3 + 216*a**2*b**7*tanh(x)*
*2*sech(x)**2 + 144*a*b**8*tanh(x)**3*sech(x) + 36*b**9*tanh(x)**4) - 9*b**
4/(36*a**4*b**5*sech(x)**4 + 144*a**3*b**6*tanh(x)*sech(x)**3 + 216*a**2*b*
*7*tanh(x)**2*sech(x)**2 + 144*a*b**8*tanh(x)**3*sech(x) + 36*b**9*tanh(x)*
*4), Ne(b, 0)), ((8*tanh(x)**5/(15*sech(x)**5) - 4*tanh(x)**3/(3*sech(x)**5
) + tanh(x)/sech(x)**5)/a**5, True))
```

3.624 $\int (\text{sech}(x) + i\tanh(x))^5 \, dx$

Optimal. Leaf size=40

$$\frac{4i}{1 - i\sinh(x)} - \frac{2i}{(1 - i\sinh(x))^2} + i\log(\sinh(x) + i)$$

```
[Out] I*ln(I+sinh(x))-2*I/(1-I*sinh(x))^2+4*I/(1-I*sinh(x))
```

Rubi [A] time = 0.06, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {4391, 2667, 43}

$$\frac{4i}{1 - i\sinh(x)} - \frac{2i}{(1 - i\sinh(x))^2} + i\log(\sinh(x) + i)$$

Antiderivative was successfully verified.

```
[In] Int[(Sech[x] + I*Tanh[x])^5,x]
```

```
[Out] I*Log[I + Sinh[x]] - (2*I)/(1 - I*Sinh[x])^2 + (4*I)/(1 - I*Sinh[x])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 4391

```
Int[(u_.)*((b_.)*sec[(c_.) + (d_.)*(x_)]^(n_.) + (a_.)*tan[(c_.) + (d_.)*(x
_)]^(n_.))^(p_), x_Symbol] :> Int[ActivateTrig[u]*Sec[c + d*x]^(n*p)*(b + a
*Sin[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

Rubi steps

$$\int (\operatorname{sech}(x)+i\tanh(x))^5\,dx=\int \operatorname{sech}^5(x)(1+i\sinh(x))^5\,dx$$
$$=-\left(i\,\text{Subst}\left(\int \frac{(1+x)^2}{(1-x)^3}\,dx,x,i\sinh(x)\right)\right)$$
$$=-\left(i\,\text{Subst}\left(\int \left(\frac{1}{1-x}-\frac{4}{(-1+x)^3}-\frac{4}{(-1+x)^2}\right)dx,x,i\sinh(x)\right)\right)$$
$$=i\log(i+\sinh(x))-\frac{2i}{(1-i\sinh(x))^2}+\frac{4i}{1-i\sinh(x)}$$

Mathematica [A] time = 0.11, size = 62, normalized size = 1.55

$$-\frac{11}{4}i\tanh^4(x)-\frac{1}{2}i\tanh^2(x)-\frac{5}{4}i\operatorname{sech}^4(x)+\tan^{-1}(\sinh(x))+i\log(\cosh(x))-\tanh(x)\operatorname{sech}^3(x)-5\tanh^3(x)\operatorname{sech}(x)+$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sech[x] + I*Tanh[x])^5,x]
```

```
[Out] ArcTan[Sinh[x]] + I*Log[Cosh[x]] - ((5*I)/4)*Sech[x]^4 + Sech[x]*Tanh[x] -
Sech[x]^3*Tanh[x] - (I/2)*Tanh[x]^2 - 5*Sech[x]*Tanh[x]^3 - ((11*I)/4)*Tanh
[x]^4
```

fricas [B] time = 0.42, size = 92, normalized size = 2.30

$$\frac{-i\,xe^{(4\,x)}+4\,(x-2)e^{(3\,x)}+(6i\,x-8i)e^{(2\,x)}-4\,(x-2)e^{x}+\left(2i\,e^{(4\,x)}-8\,e^{(3\,x)}-12i\,e^{(2\,x)}+8\,e^{x}+2i\right)\log\left(e^{x}+i\right)-i}{e^{(4\,x)}+4i\,e^{(3\,x)}-6\,e^{(2\,x)}-4i\,e^{x}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sech(x)+I*tanh(x))^5,x, algorithm="fricas")
```

```
[Out] (-I*x*e^(4*x) + 4*(x - 2)*e^(3*x) + (6*I*x - 8*I)*e^(2*x) - 4*(x - 2)*e^x +
 (2*I*e^(4*x) - 8*e^(3*x) - 12*I*e^(2*x) + 8*e^x + 2*I)*log(e^x + I) - I*x)
/(e^(4*x) + 4*I*e^(3*x) - 6*e^(2*x) - 4*I*e^x + 1)
```

giac [A] time = 0.15, size = 34, normalized size = 0.85

$$-i\,x-\frac{8\left(e^{(3\,x)}+i\,e^{(2\,x)}-e^{x}\right)}{\left(e^{x}+i\right)^4}+2i\,\log\left(e^{x}+i\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sech(x)+I*tanh(x))^5,x, algorithm="giac")
```

```
[Out] -I*x - 8*(e^(3*x) + I*e^(2*x) - e^x)/(e^x + I)^4 + 2*I*log(e^x + I)
```

maple [B] time = 0.43, size = 78, normalized size = 1.95

$$\frac{8\left(\frac{\operatorname{sech}(x)^3}{4}+\frac{3\operatorname{sech}(x)}{8}\right)\tanh(x)}{3}+2\arctan\left(e^x\right)+\frac{5i}{4\cosh(x)^4}-\frac{5\sinh(x)}{3\cosh(x)^4}+\frac{5i\left(\sinh^2(x)\right)}{\cosh(x)^4}-\frac{5\left(\sinh^3(x)\right)}{\cosh(x)^4}+i\ln\left(\cosh(x\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sech(x)+I*tanh(x))^5,x)
```

```
[Out] 8/3*(1/4*sech(x)^3+3/8*sech(x))*tanh(x)+2*arctan(exp(x))+5/4*I/cosh(x)^4-5/
3*sinh(x)/cosh(x)^4+5*I*sinh(x)^2/cosh(x)^4-5*sinh(x)^3/cosh(x)^4+I*ln(cosh
(x))-1/2*I*tanh(x)^2-1/4*I*tanh(x)^4
```

maxima [B] time = 0.50, size = 235, normalized size = 5.88

$$-\frac{5}{2}i\,\tanh(x)^4+i\,x-\frac{5\left(5\,e^{(-x)}-3\,e^{(-3\,x)}+3\,e^{(-5\,x)}-5\,e^{(-7\,x)}\right)}{4\left(4\,e^{(-2\,x)}+6\,e^{(-4\,x)}+4\,e^{(-6\,x)}+e^{(-8\,x)}+1\right)}+\frac{3\,e^{(-x)}+11\,e^{(-3\,x)}-11\,e^{(-5\,x)}-3\,e^{(-7\,x)}}{4\left(4\,e^{(-2\,x)}+6\,e^{(-4\,x)}+4\,e^{(-6\,x)}+e^{(-8\,x)}+1\right)}-\frac{}{2\left(4\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sech(x)+I*tanh(x))^5,x, algorithm="maxima")
```

```
[Out] -5/2*I*tanh(x)^4 + I*x - 5/4*(5*e^(-x) - 3*e^(-3*x) + 3*e^(-5*x) - 5*e^(-7*
x))/(4*e^(-2*x) + 6*e^(-4*x) + 4*e^(-6*x) + e^(-8*x) + 1) + 1/4*(3*e^(-x) +
 11*e^(-3*x) - 11*e^(-5*x) - 3*e^(-7*x))/(4*e^(-2*x) + 6*e^(-4*x) + 4*e^(-6
*x) + e^(-8*x) + 1) - 5/2*(e^(-x) - 7*e^(-3*x) + 7*e^(-5*x) - e^(-7*x))/(4*
e^(-2*x) + 6*e^(-4*x) + 4*e^(-6*x) + e^(-8*x) + 1) + 4*I*(e^(-2*x) + e^(-4*
x) + e^(-6*x))/(4*e^(-2*x) + 6*e^(-4*x) + 4*e^(-6*x) + e^(-8*x) + 1) - 20*I
/(e^(-x) + e^x)^4 - 2*arctan(e^(-x)) + I*log(e^(-2*x) + 1)
```

mupad [B] time = 0.28, size = 90, normalized size = 2.25

$$-x\,1\mathrm{i}+\ln\left(\mathrm{e}^x+1\mathrm{i}\right)\,2\mathrm{i}+\frac{16\mathrm{i}}{\mathrm{e}^{2\,x}-1+\mathrm{e}^x\,2\mathrm{i}}-\frac{8\mathrm{i}}{\mathrm{e}^{4\,x}-6\,\mathrm{e}^{2\,x}+1+\mathrm{e}^{3\,x}\,4\mathrm{i}-\mathrm{e}^x\,4\mathrm{i}}-\frac{8}{\mathrm{e}^x+1\mathrm{i}}+\frac{16}{\mathrm{e}^{2\,x}\,3\mathrm{i}+\mathrm{e}^{3\,x}-3\,\mathrm{e}^x-\mathrm{i}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((tanh(x)*1i + 1/cosh(x))^5,x)
```

```
[Out] log(exp(x) + 1i)*2i - x*1i + 16i/(exp(2*x) + exp(x)*2i - 1) - 8i/(exp(3*x)*
4i - 6*exp(2*x) + exp(4*x) - exp(x)*4i + 1) - 8/(exp(x) + 1i) + 16/(exp(2*x
)*3i + exp(3*x) - 3*exp(x) - 1i)
```

<u>**sympy**</u> **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (i \tanh (x) + \operatorname{sech} (x))^5 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sech(x)+I*tanh(x))**5,x)
```

```
[Out] Integral((I*tanh(x) + sech(x))**5, x)
```

3.625 $\int (\text{sech}(x) + i\tanh(x))^4\, dx$

Optimal. Leaf size=38

$$x - \frac{2i\cosh^3(x)}{3(1 - i\sinh(x))^3} + \frac{2i\cosh(x)}{1 - i\sinh(x)}$$

```
[Out] x-2/3*I*cosh(x)^3/(1-I*sinh(x))^3+2*I*cosh(x)/(1-I*sinh(x))
```

Rubi [A] time = 0.11, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {4391, 2670, 2680, 8}

$$x - \frac{2i\cosh^3(x)}{3(1 - i\sinh(x))^3} + \frac{2i\cosh(x)}{1 - i\sinh(x)}$$

Antiderivative was successfully verified.

```
[In] Int[(Sech[x] + I*Tanh[x])^4,x]
```

```
[Out] x - (((2*I)/3)*Cosh[x]^3)/(1 - I*Sinh[x])^3 + ((2*I)*Cosh[x])/(1 - I*Sinh[x
])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2670

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*m), Int[(g*Cos[e + f*x])^(2*m + p)/(a
 - b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2,
 0] && IntegerQ[m] && LtQ[p, -1] && GeQ[2*m + p, 0]
```

Rule 2680

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(2*g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f
*x])^(m + 1))/(b*f*(2*m + p + 1)), x] + Dist[(g^2*(p - 1))/(b^2*(2*m + p +
1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 4391

```
Int[(u_.)*((b_.)*sec[(c_.) + (d_.)*(x_)]^(n_.) + (a_.)*tan[(c_.) + (d_.)*(x
_)]^(n_.))^(p_), x_Symbol] :> Int[ActivateTrig[u]*Sec[c + d*x]^(n*p)*(b + a
```

```
*Sin[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

Rubi steps

$$\begin{aligned}
\int (\operatorname{sech}(x)+i\tanh(x))^4\,dx &= \int \operatorname{sech}^4(x)(1+i\sinh(x))^4\,dx\\
&= \int \frac{\cosh^4(x)}{(1-i\sinh(x))^4}\,dx\\
&= -\frac{2i\cosh^3(x)}{3(1-i\sinh(x))^3}-\int \frac{\cosh^2(x)}{(1-i\sinh(x))^2}\,dx\\
&= -\frac{2i\cosh^3(x)}{3(1-i\sinh(x))^3}+\frac{2i\cosh(x)}{1-i\sinh(x)}+\int 1\,dx\\
&= x-\frac{2i\cosh^3(x)}{3(1-i\sinh(x))^3}+\frac{2i\cosh(x)}{1-i\sinh(x)}
\end{aligned}$$

Mathematica [A] time = 0.14, size = 74, normalized size = 1.95

$$\frac{3(3x-8i)\cosh\left(\frac{x}{2}\right)+(-3x+16i)\cosh\left(\frac{3x}{2}\right)-6i\sinh\left(\frac{x}{2}\right)(2x+x\cosh(x)-4i)}{6\left(\cosh\left(\frac{x}{2}\right)-i\sinh\left(\frac{x}{2}\right)\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sech[x] + I*Tanh[x])^4,x]
```

```
[Out] (3*(-8*I + 3*x)*Cosh[x/2] + (16*I - 3*x)*Cosh[(3*x)/2] - (6*I)*(-4*I + 2*x
+ x*Cosh[x])*Sinh[x/2])/(6*(Cosh[x/2] - I*Sinh[x/2])^3)
```

fricas [A] time = 0.40, size = 52, normalized size = 1.37

$$\frac{3\,xe^{(3\,x)}+(9i\,x+24i)e^{(2\,x)}-3\,(3\,x+8)e^{x}-3i\,x-16i}{3\,e^{(3\,x)}+9i\,e^{(2\,x)}-9\,e^{x}-3i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sech(x)+I*tanh(x))^4,x, algorithm="fricas")
```

```
[Out] (3*x*e^(3*x) + (9*I*x + 24*I)*e^(2*x) - 3*(3*x + 8)*e^x - 3*I*x - 16*I)/(3*
e^(3*x) + 9*I*e^(2*x) - 9*e^x - 3*I)
```

giac [A] time = 0.12, size = 22, normalized size = 0.58

$$x-\frac{-24i\,e^{(2\,x)}+24\,e^{x}+16i}{3\,(e^{x}+i)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sech(x)+I*tanh(x))^4,x, algorithm="giac")
```

```
[Out] x - 1/3*(-24*I*e^(2*x) + 24*e^x + 16*I)/(e^x + I)^3
```

maple [A] time = 0.34, size = 60, normalized size = 1.58

$$-2\left(\frac{2}{3}+\frac{\mathrm{sech}(x)^{2}}{3}\right)\tanh(x)-\frac{4i}{3\cosh(x)^{3}}+\frac{3\sinh(x)}{\cosh(x)^{3}}-4i\left(-\frac{\sinh^{2}(x)}{\cosh(x)^{3}}-\frac{2}{3\cosh(x)^{3}}\right)+x-\tanh(x)-\frac{\left(\tanh^{3}(x)\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sech(x)+I*tanh(x))^4,x)
```

```
[Out] -2*(2/3+1/3*sech(x)^2)*tanh(x)-4/3*I/cosh(x)^3+3*sinh(x)/cosh(x)^3-4*I*(-si
nh(x)^2/cosh(x)^3-2/3/cosh(x)^3)+x-tanh(x)-1/3*tanh(x)^3
```

maxima [B] time = 0.49, size = 181, normalized size = 4.76

$$-2\tanh(x)^{3}+x-\frac{4\left(3e^{(-2x)}+3e^{(-4x)}+2\right)}{3\left(3e^{(-2x)}+3e^{(-4x)}+e^{(-6x)}+1\right)}+\frac{8ie^{(-x)}}{3e^{(-2x)}+3e^{(-4x)}+e^{(-6x)}+1}+\frac{4e^{(-2x)}}{3e^{(-2x)}+3e^{(-4x)}+e^{(-6x)}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sech(x)+I*tanh(x))^4,x, algorithm="maxima")
```

```
[Out] -2*tanh(x)^3 + x - 4/3*(3*e^(-2*x) + 3*e^(-4*x) + 2)/(3*e^(-2*x) + 3*e^(-4*
x) + e^(-6*x) + 1) + 8*I*e^(-x)/(3*e^(-2*x) + 3*e^(-4*x) + e^(-6*x) + 1) +
4*e^(-2*x)/(3*e^(-2*x) + 3*e^(-4*x) + e^(-6*x) + 1) + 16/3*I*e^(-3*x)/(3*e^
(-2*x) + 3*e^(-4*x) + e^(-6*x) + 1) + 8*I*e^(-5*x)/(3*e^(-2*x) + 3*e^(-4*x)
 + e^(-6*x) + 1) + 4/3/(3*e^(-2*x) + 3*e^(-4*x) + e^(-6*x) + 1) - 32/3*I/(e
^(-x) + e^x)^3
```

mupad [B] time = 1.63, size = 65, normalized size = 1.71

$$x+\frac{\frac{e^{2x}\,8\mathrm{i}}{3}-\frac{8}{3}\mathrm{i}}{e^{2x}\,3\mathrm{i}+e^{3x}-3\,e^{x}-\mathrm{i}}+\frac{e^{x}\,8\mathrm{i}}{3\,\left(e^{2x}-1+e^{x}\,2\mathrm{i}\right)}+\frac{8\mathrm{i}}{3\,\left(e^{x}+1\mathrm{i}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((tanh(x)*1i + 1/cosh(x))^4,x)
```

```
[Out] x + ((exp(2*x)*8i)/3 - 8i/3)/(exp(2*x)*3i + exp(3*x) - 3*exp(x) - 1i) + (ex
p(x)*8i)/(3*(exp(2*x) + exp(x)*2i - 1)) + 8i/(3*(exp(x) + 1i))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (i\tanh(x) + \operatorname{sech}(x))^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sech(x)+I*tanh(x))**4,x)
```

```
[Out] Integral((I*tanh(x) + sech(x))**4, x)
```

3.626 $\int (\text{sech}(x) + i\tanh(x))^3\, dx$

Optimal. Leaf size=26

$$-\frac{2i}{1-i\sinh(x)} - i\log(\sinh(x)+i)$$

[Out] -I*ln(I+sinh(x))-2*I/(1-I*sinh(x))

Rubi [A] time = 0.05, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {4391, 2667, 43}

$$-\frac{2i}{1-i\sinh(x)} - i\log(\sinh(x)+i)$$

Antiderivative was successfully verified.

[In] Int[(Sech[x] + I*Tanh[x])^3,x]

[Out] (-I)*Log[I + Sinh[x]] - (2*I)/(1 - I*Sinh[x])

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 4391

```
Int[(u_.)*((b_.)*sec[(c_.) + (d_.)*(x_)]^(n_.) + (a_.)*tan[(c_.) + (d_.)*(x
_)]^(n_.))^(p_), x_Symbol] :> Int[ActivateTrig[u]*Sec[c + d*x]^(n*p)*(b + a
*Sin[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

Rubi steps

$$\begin{aligned}\int (\operatorname{sech}(x)+i\tanh(x))^3\,dx &= \int \operatorname{sech}^3(x)(1+i\sinh(x))^3\,dx\\ &= -\left(i\,\mathrm{Subst}\left(\int \frac{1+x}{(1-x)^2}\,dx,x,i\sinh(x)\right)\right)\\ &= -\left(i\,\mathrm{Subst}\left(\int \left(\frac{2}{(-1+x)^2}+\frac{1}{-1+x}\right)dx,x,i\sinh(x)\right)\right)\\ &= -i\log(i+\sinh(x))-\frac{2i}{1-i\sinh(x)}\end{aligned}$$

Mathematica [A] time = 0.03, size = 39, normalized size = 1.50

$$\frac{1}{2}i\tanh^2(x)-\frac{3}{2}i\operatorname{sech}^2(x)-\tan^{-1}(\sinh(x))-i\log(\cosh(x))+2\tanh(x)\operatorname{sech}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sech[x] + I*Tanh[x])^3,x]
```

```
[Out] -ArcTan[Sinh[x]] - I*Log[Cosh[x]] - ((3*I)/2)*Sech[x]^2 + 2*Sech[x]*Tanh[x]
 + (I/2)*Tanh[x]^2
```

fricas [B] time = 0.43, size = 49, normalized size = 1.88

$$\frac{i\,xe^{(2\,x)}-2\,(x-2)e^x+\left(-2i\,e^{(2\,x)}+4\,e^x+2i\right)\log\left(e^x+i\right)-i\,x}{e^{(2\,x)}+2i\,e^x-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sech(x)+I*tanh(x))^3,x, algorithm="fricas")
```

```
[Out] (I*x*e^(2*x) - 2*(x - 2)*e^x + (-2*I*e^(2*x) + 4*e^x + 2*I)*log(e^x + I) -
I*x)/(e^(2*x) + 2*I*e^x - 1)
```

giac [A] time = 0.12, size = 21, normalized size = 0.81

$$i\,x+\frac{4\,e^x}{(e^x+i)^2}-2i\,\log\left(e^x+i\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sech(x)+I*tanh(x))^3,x, algorithm="giac")
```

```
[Out] I*x + 4*e^x/(e^x + I)^2 - 2*I*log(e^x + I)
```

maple [A] time = 0.34, size = 41, normalized size = 1.58

$$-\mathrm{sech}(x)\tanh(x)-2\arctan\left(\mathrm{e}^{x}\right)-\frac{3i}{2\cosh(x)^{2}}+\frac{3\sinh(x)}{\cosh(x)^{2}}-i\ln\left(\cosh(x)\right)+\frac{i\left(\tanh^{2}(x)\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sech(x)+I*tanh(x))^3,x)
```

```
[Out] -sech(x)*tanh(x)-2*arctan(exp(x))-3/2*I/cosh(x)^2+3/cosh(x)^2*sinh(x)-I*ln(
cosh(x))+1/2*I*tanh(x)^2
```

maxima [B] time = 0.71, size = 73, normalized size = 2.81

$$\frac{3}{2}i\tanh(x)^{2}-ix+\frac{4\left(e^{(-x)}-e^{(-3x)}\right)}{2e^{(-2x)}+e^{(-4x)}+1}-\frac{2ie^{(-2x)}}{2e^{(-2x)}+e^{(-4x)}+1}+2\arctan\left(e^{(-x)}\right)-i\log\left(e^{(-2x)}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sech(x)+I*tanh(x))^3,x, algorithm="maxima")
```

```
[Out] 3/2*I*tanh(x)^2 - I*x + 4*(e^(-x) - e^(-3*x))/(2*e^(-2*x) + e^(-4*x) + 1) -
 2*I*e^(-2*x)/(2*e^(-2*x) + e^(-4*x) + 1) + 2*arctan(e^(-x)) - I*log(e^(-2*
x) + 1)
```

mupad [B] time = 1.65, size = 39, normalized size = 1.50

$$x\,1\mathrm{i}-\ln\left(\mathrm{e}^{x}+1\mathrm{i}\right)\,2\mathrm{i}-\frac{4\mathrm{i}}{\mathrm{e}^{2x}-1+\mathrm{e}^{x}\,2\mathrm{i}}+\frac{4}{\mathrm{e}^{x}+1\mathrm{i}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((tanh(x)*1i + 1/cosh(x))^3,x)
```

```
[Out] x*1i - log(exp(x) + 1i)*2i - 4i/(exp(2*x) + exp(x)*2i - 1) + 4/(exp(x) + 1i
)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(i\tanh\left(x\right)+\mathrm{sech}\left(x\right)\right)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sech(x)+I*tanh(x))**3,x)
```

```
[Out] Integral((I*tanh(x) + sech(x))**3, x)
```

3.627 $\int (\text{sech}(x) + i \tanh(x))^2 \, dx$

Optimal. Leaf size=20

$$-x - \frac{2i\cosh(x)}{1 - i\sinh(x)}$$

```
[Out] -x-2*I*cosh(x)/(1-I*sinh(x))
```

Rubi [A] time = 0.08, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {4391, 2670, 2680, 8}

$$-x - \frac{2i\cosh(x)}{1 - i\sinh(x)}$$

Antiderivative was successfully verified.

```
[In] Int[(Sech[x] + I*Tanh[x])^2,x]
```

```
[Out] -x - ((2*I)*Cosh[x])/(1 - I*Sinh[x])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2670

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*m), Int[(g*Cos[e + f*x])^(2*m + p)/(a
 - b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2,
 0] && IntegerQ[m] && LtQ[p, -1] && GeQ[2*m + p, 0]
```

Rule 2680

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(2*g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f
*x])^(m + 1))/(b*f*(2*m + p + 1)), x] + Dist[(g^2*(p - 1))/(b^2*(2*m + p +
1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 4391

```
Int[(u_.)*((b_.)*sec[(c_.) + (d_.)*(x_)]^(n_.) + (a_.)*tan[(c_.) + (d_.)*(x
_)]^(n_.))^(p_), x_Symbol] :> Int[ActivateTrig[u]*Sec[c + d*x]^(n*p)*(b + a
```

```
*Sin[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

Rubi steps

$$\begin{aligned}\int (\operatorname{sech}(x)+i\tanh(x))^2\,dx &= \int \operatorname{sech}^2(x)(1+i\sinh(x))^2\,dx\\ &= \int \frac{\cosh^2(x)}{(1-i\sinh(x))^2}\,dx\\ &= -\frac{2i\cosh(x)}{1-i\sinh(x)}-\int 1\,dx\\ &= -x-\frac{2i\cosh(x)}{1-i\sinh(x)}\end{aligned}$$

Mathematica [A] time = 0.01, size = 14, normalized size = 0.70

$$-x+2\tanh(x)-2i\operatorname{sech}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sech[x] + I*Tanh[x])^2,x]
[Out] -x - (2*I)*Sech[x] + 2*Tanh[x]
```

fricas [A] time = 0.41, size = 17, normalized size = 0.85

$$-\frac{xe^x+ix+4i}{e^x+i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sech(x)+I*tanh(x))^2,x, algorithm="fricas")
[Out] -(x*e^x + I*x + 4*I)/(e^x + I)
```

giac [A] time = 0.14, size = 12, normalized size = 0.60

$$-x-\frac{4i}{e^x+i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sech(x)+I*tanh(x))^2,x, algorithm="giac")
[Out] -x - 4*I/(e^x + I)
```

maple [A] time = 0.36, size = 16, normalized size = 0.80

$$2\tanh(x) - \frac{2i}{\cosh(x)} - x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sech(x)+I*tanh(x))^2,x)
[Out] 2*tanh(x)-2*I/cosh(x)-x
```

maxima [A] time = 0.32, size = 25, normalized size = 1.25

$$-x - \frac{4i}{e^{(-x)} + e^{x}} + \frac{4}{e^{(-2x)} + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sech(x)+I*tanh(x))^2,x, algorithm="maxima")
[Out] -x - 4*I/(e^(-x) + e^x) + 4/(e^(-2*x) + 1)
```

mupad [B] time = 1.64, size = 14, normalized size = 0.70

$$-x - \frac{4\mathrm{i}}{\mathrm{e}^{x} + 1\mathrm{i}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((tanh(x)*1i + 1/cosh(x))^2,x)
[Out] - x - 4i/(exp(x) + 1i)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (i\tanh(x) + \operatorname{sech}(x))^2\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sech(x)+I*tanh(x))**2,x)
[Out] Integral((I*tanh(x) + sech(x))**2, x)
```

3.628 $\int (\text{sech}(x) + i \tanh(x)) \, dx$

Optimal. Leaf size=11

$$\tan^{-1}(\sinh(x)) + i \log(\cosh(x))$$

```
[Out] arctan(sinh(x))+I*ln(cosh(x))
```

Rubi [A] time = 0.01, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {3770, 3475}

$$\tan^{-1}(\sinh(x)) + i \log(\cosh(x))$$

Antiderivative was successfully verified.

```
[In] Int[Sech[x] + I*Tanh[x],x]
```

```
[Out] ArcTan[Sinh[x]] + I*Log[Cosh[x]]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned} \int (\text{sech}(x) + i \tanh(x)) \, dx &= i \int \tanh(x) \, dx + \int \text{sech}(x) \, dx \\ &= \tan^{-1}(\sinh(x)) + i \log(\cosh(x)) \end{aligned}$$

Mathematica [A] time = 0.00, size = 17, normalized size = 1.55

$$2 \tan^{-1}\left(\tanh\left(\frac{x}{2}\right)\right) + i \log(\cosh(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[x] + I*Tanh[x],x]
```

```
[Out] 2*ArcTan[Tanh[x/2]] + I*Log[Cosh[x]]
```

fricas [A] time = 0.42, size = 11, normalized size = 1.00

$$-ix + 2i \log(e^x + i)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)+I*tanh(x),x, algorithm="fricas")
[Out] -I*x + 2*I*log(e^x + I)
```

giac [A] time = 0.14, size = 18, normalized size = 1.64

$$-ix + 2\arctan(e^x) + i \log\left(e^{(2x)} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)+I*tanh(x),x, algorithm="giac")
[Out] -I*x + 2*arctan(e^x) + I*log(e^(2*x) + 1)
```

maple [A] time = 0.02, size = 11, normalized size = 1.00

$$\arctan(\sinh(x)) + i \ln(\cosh(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(x)+I*tanh(x),x)
[Out] arctan(sinh(x))+I*ln(cosh(x))
```

maxima [A] time = 0.31, size = 9, normalized size = 0.82

$$\arctan(\sinh(x)) + i \log(\cosh(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)+I*tanh(x),x, algorithm="maxima")
[Out] arctan(sinh(x)) + I*log(cosh(x))
```

mupad [B] time = 1.59, size = 14, normalized size = 1.27

$$-x\,1\mathrm{i} + \ln(\mathrm{e}^x + 1\mathrm{i})\,2\mathrm{i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)*1i + 1/cosh(x),x)
```

```
[Out] log(exp(x) + 1i)*2i - x*1i
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (i\tanh(x) + \operatorname{sech}(x))\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)+I*tanh(x),x)
```

```
[Out] Integral(I*tanh(x) + sech(x), x)
```

3.629 $\int \frac{1}{\operatorname{sech}(x)+i\tanh(x)}\,dx$

Optimal. Leaf size=13

$$-i\log(-\sinh(x)+i)$$

[Out] -I*ln(I-sinh(x))

Rubi [A] time = 0.03, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {3159, 2667, 31}

$$-i\log(-\sinh(x)+i)$$

Antiderivative was successfully verified.

[In] Int[(Sech[x] + I*Tanh[x])^(-1),x]

[Out] (-I)*Log[I - Sinh[x]]

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 3159

```
Int[((a_.) + (b_.)*sec[(d_.) + (e_.)*(x_)] + (c_.)*tan[(d_.) + (e_.)*(x_)])
^(-1), x_Symbol] :> Int[Cos[d + e*x]/(b + a*Cos[d + e*x] + c*Sin[d + e*x]),
 x] /; FreeQ[{a, b, c, d, e}, x]
```

Rubi steps

$$\int \frac{1}{\operatorname{sech}(x) + i\tanh(x)}\,dx = \int \frac{\cosh(x)}{1 + i\sinh(x)}\,dx$$
$$= -\left(i\,\text{Subst}\left(\int \frac{1}{1+x}\,dx, x, i\sinh(x)\right)\right)$$
$$= -i\log(i - \sinh(x))$$

Mathematica [A] time = 0.02, size = 17, normalized size = 1.31

$$2\tan^{-1}\left(\tanh\left(\frac{x}{2}\right)\right) - i\log(\cosh(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sech[x] + I*Tanh[x])^(-1),x]
[Out] 2*ArcTan[Tanh[x/2]] - I*Log[Cosh[x]]
```

fricas [A] time = 0.43, size = 11, normalized size = 0.85

$$i\,x - 2i\,\log\left(e^{x} - i\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sech(x)+I*tanh(x)),x, algorithm="fricas")
[Out] I*x - 2*I*log(e^x - I)
```

giac [A] time = 0.14, size = 13, normalized size = 1.00

$$i\,x - 2i\,\log\left(i\,e^{x} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sech(x)+I*tanh(x)),x, algorithm="giac")
[Out] I*x - 2*I*log(I*e^x + 1)
```

maple [B] time = 0.24, size = 33, normalized size = 2.54

$$i\ln\left(\tanh\left(\frac{x}{2}\right)-1\right) + i\ln\left(\tanh\left(\frac{x}{2}\right)+1\right) - 2i\ln\left(\tanh\left(\frac{x}{2}\right)-i\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sech(x)+I*tanh(x)),x)
```

[Out] I*ln(tanh(1/2*x)-1)+I*ln(tanh(1/2*x)+1)-2*I*ln(tanh(1/2*x)-I)

maxima [A] time = 0.34, size = 15, normalized size = 1.15

$$-i\,x - 2i\,\log\left(i\,e^{(-x)} - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(sech(x)+I*tanh(x)),x, algorithm="maxima")

[Out] -I*x - 2*I*log(I*e^(-x) - 1)

mupad [B] time = 1.56, size = 14, normalized size = 1.08

$$x\,1\mathrm{i} - \ln\left(\mathrm{e}^{x} - \mathrm{i}\right)\,2\mathrm{i}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(tanh(x)*1i + 1/cosh(x)),x)

[Out] x*1i - log(exp(x) - 1i)*2i

sympy [B] time = 0.21, size = 22, normalized size = 1.69

$$-ix - i\log\left(i\tanh\left(x\right) + \operatorname{sech}\left(x\right)\right) + i\log\left(\tanh\left(x\right) + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(sech(x)+I*tanh(x)),x)

[Out] -I*x - I*log(I*tanh(x) + sech(x)) + I*log(tanh(x) + 1)

3.630 $\int \frac{1}{(\mathbf{sech}(x)+i\tanh(x))^2}\,dx$

Optimal. Leaf size=20

$$-x+\frac{2i\cosh(x)}{1+i\sinh(x)}$$

[Out] -x+2*I*cosh(x)/(1+I*sinh(x))

Rubi [A] time = 0.04, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {4391, 2680, 8}

$$-x+\frac{2i\cosh(x)}{1+i\sinh(x)}$$

Antiderivative was successfully verified.

```
[In] Int[(Sech[x] + I*Tanh[x])^(-2),x]
```

```
[Out] -x + ((2*I)*Cosh[x])/(1 + I*Sinh[x])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2680

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(2*g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f
*x])^(m + 1))/(b*f*(2*m + p + 1)), x] + Dist[(g^2*(p - 1))/(b^2*(2*m + p +
1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 4391

```
Int[(u_.)*((b_.)*sec[(c_.) + (d_.)*(x_)]^(n_.) + (a_.)*tan[(c_.) + (d_.)*(x
_)]^(n_.))^(p_), x_Symbol] :> Int[ActivateTrig[u]*Sec[c + d*x]^(n*p)*(b + a
*Sin[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

Rubi steps

$$\int \frac{1}{(\operatorname{sech}(x) + i\tanh(x))^2}\,dx = \int \frac{\cosh^2(x)}{(1+i\sinh(x))^2}\,dx$$
$$= \frac{2i\cosh(x)}{1+i\sinh(x)} - \int 1\,dx$$
$$= -x + \frac{2i\cosh(x)}{1+i\sinh(x)}$$

Mathematica [A] time = 0.03, size = 31, normalized size = 1.55

$$-x + \frac{4\sinh\left(\frac{x}{2}\right)}{\cosh\left(\frac{x}{2}\right) + i\sinh\left(\frac{x}{2}\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sech[x] + I*Tanh[x])^(-2),x]
[Out] -x + (4*Sinh[x/2])/(Cosh[x/2] + I*Sinh[x/2])
```

fricas [A] time = 0.41, size = 17, normalized size = 0.85

$$-\frac{xe^x - i\,x - 4i}{e^x - i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sech(x)+I*tanh(x))^2,x, algorithm="fricas")
[Out] -(x*e^x - I*x - 4*I)/(e^x - I)
```

giac [A] time = 0.14, size = 12, normalized size = 0.60

$$-x + \frac{4i}{e^x - i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sech(x)+I*tanh(x))^2,x, algorithm="giac")
[Out] -x + 4*I/(e^x - I)
```

maple [A] time = 0.28, size = 29, normalized size = 1.45

$$\frac{4}{\tanh\left(\frac{x}{2}\right) - i} + \ln\left(\tanh\left(\frac{x}{2}\right) - 1\right) - \ln\left(\tanh\left(\frac{x}{2}\right) + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sech(x)+I*tanh(x))^2,x)
```

```
[Out] 4/(tanh(1/2*x)-I)+ln(tanh(1/2*x)-1)-ln(tanh(1/2*x)+1)
```

maxima [A] time = 0.35, size = 14, normalized size = 0.70

$$-x+\frac{4i}{e^{(-x)}+i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sech(x)+I*tanh(x))^2,x, algorithm="maxima")
```

```
[Out] -x + 4*I/(e^(-x) + I)
```

mupad [B] time = 1.50, size = 14, normalized size = 0.70

$$-x+\frac{4\mathrm{i}}{\mathrm{e}^{x}-\mathrm{i}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(tanh(x)*1i + 1/cosh(x))^2,x)
```

```
[Out] 4i/(exp(x) - 1i) - x
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\left(i\tanh\left(x\right)+\operatorname{sech}\left(x\right)\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sech(x)+I*tanh(x))**2,x)
```

```
[Out] Integral((I*tanh(x) + sech(x))**(-2), x)
```

3.631 $\int \frac{1}{(\mathbf{sech}(x)+i\tanh(x))^3}\,dx$

Optimal. Leaf size=28

$$\frac{2i}{1+i\sinh(x)}+i\log(-\sinh(x)+i)$$

```
[Out] I*ln(I-sinh(x))+2*I/(1+I*sinh(x))
```

Rubi [A] time = 0.05, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {4391, 2667, 43}

$$\frac{2i}{1+i\sinh(x)}+i\log(-\sinh(x)+i)$$

Antiderivative was successfully verified.

```
[In] Int[(Sech[x] + I*Tanh[x])^(-3),x]
```

```
[Out] I*Log[I - Sinh[x]] + (2*I)/(1 + I*Sinh[x])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 4391

```
Int[(u_.)*((b_.)*sec[(c_.) + (d_.)*(x_)]^(n_.) + (a_.)*tan[(c_.) + (d_.)*(x
_)]^(n_.))^(p_), x_Symbol] :> Int[ActivateTrig[u]*Sec[c + d*x]^(n*p)*(b + a
*Sin[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

Rubi steps

$$\int \frac{1}{(\operatorname{sech}(x)+i\tanh(x))^3}\,dx = \int \frac{\cosh^3(x)}{(1+i\sinh(x))^3}\,dx$$
$$= -\left(i\,\mathrm{Subst}\left(\int \frac{1-x}{(1+x)^2}\,dx, x, i\sinh(x)\right)\right)$$
$$= -\left(i\,\mathrm{Subst}\left(\int \left(\frac{1}{-1-x}+\frac{2}{(1+x)^2}\right)dx, x, i\sinh(x)\right)\right)$$
$$= i\log(i-\sinh(x)) + \frac{2i}{1+i\sinh(x)}$$

Mathematica [**A**] time = 0.04, size = 40, normalized size = 1.43

$$-2\tan^{-1}\left(\tanh\left(\frac{x}{2}\right)\right) + i\log(\cosh(x)) + \frac{2i}{\left(\cosh\left(\frac{x}{2}\right)+i\sinh\left(\frac{x}{2}\right)\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sech[x] + I*Tanh[x])^(-3),x]
[Out] -2*ArcTan[Tanh[x/2]] + I*Log[Cosh[x]] + (2*I)/(Cosh[x/2] + I*Sinh[x/2])^2
```

fricas [**B**] time = 0.42, size = 49, normalized size = 1.75

$$\frac{-i\,xe^{(2\,x)} - 2\,(x-2)e^x + \left(2i\,e^{(2\,x)} + 4\,e^x - 2i\right)\log\left(e^x - i\right) + i\,x}{e^{(2\,x)} - 2i\,e^x - 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sech(x)+I*tanh(x))^3,x, algorithm="fricas")
[Out] (-I*x*e^(2*x) - 2*(x - 2)*e^x + (2*I*e^(2*x) + 4*e^x - 2*I)*log(e^x - I) +
I*x)/(e^(2*x) - 2*I*e^x - 1)
```

giac [**A**] time = 0.13, size = 27, normalized size = 0.96

$$\frac{4\,e^x}{\left(e^x - i\right)^2} - i\,\log\left(i\,e^x\right) + 2i\,\log\left(-i\,e^x - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sech(x)+I*tanh(x))^3,x, algorithm="giac")
[Out] 4*e^x/(e^x - I)^2 - I*log(I*e^x) + 2*I*log(-I*e^x - 1)
```

maple [B] time = 0.34, size = 56, normalized size = 2.00

$$-i\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)-i\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)+2i\ln\left(\tanh\left(\frac{x}{2}\right)-i\right)-\frac{4i}{\left(\tanh\left(\frac{x}{2}\right)-i\right)^{2}}-\frac{4}{\tanh\left(\frac{x}{2}\right)-i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sech(x)+I*tanh(x))^3,x)
```

```
[Out] -I*ln(tanh(1/2*x)-1)-I*ln(tanh(1/2*x)+1)+2*I*ln(tanh(1/2*x)-I)-4*I/(tanh(1/
2*x)-I)^2-4/(tanh(1/2*x)-I)
```

maxima [A] time = 0.51, size = 33, normalized size = 1.18

$$i\,x-\frac{4\,e^{(-x)}}{2i\,e^{(-x)}+e^{(-2\,x)}-1}+2i\,\log\left(e^{(-x)}+i\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sech(x)+I*tanh(x))^3,x, algorithm="maxima")
```

```
[Out] I*x - 4*e^(-x)/(2*I*e^(-x) + e^(-2*x) - 1) + 2*I*log(e^(-x) + I)
```

mupad [B] time = 0.19, size = 41, normalized size = 1.46

$$-x\,1\mathrm{i}+\ln\left(\mathrm{e}^{x}-\mathrm{i}\right)\,2\mathrm{i}-\frac{4\mathrm{i}}{1-\mathrm{e}^{2\,x}+\mathrm{e}^{x}\,2\mathrm{i}}+\frac{4}{\mathrm{e}^{x}-\mathrm{i}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(tanh(x)*1i + 1/cosh(x))^3,x)
```

```
[Out] log(exp(x) - 1i)*2i - x*1i - 4i/(exp(x)*2i - exp(2*x) + 1) + 4/(exp(x) - 1i
)
```

sympy [B] time = 1.80, size = 432, normalized size = 15.43

$$-\frac{2ix\tanh^{2}(x)}{-2\tanh^{2}(x)+4i\tanh(x)\operatorname{sech}(x)+2\operatorname{sech}^{2}(x)}-\frac{4x\tanh(x)\operatorname{sech}(x)}{-2\tanh^{2}(x)+4i\tanh(x)\operatorname{sech}(x)+2\operatorname{sech}^{2}(x)}+\frac{}{-2\tanh^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sech(x)+I*tanh(x))**3,x)
```

```
[Out] -2*I*x*tanh(x)**2/(-2*tanh(x)**2 + 4*I*tanh(x)*sech(x) + 2*sech(x)**2) - 4*
x*tanh(x)*sech(x)/(-2*tanh(x)**2 + 4*I*tanh(x)*sech(x) + 2*sech(x)**2) + 2*
I*x*sech(x)**2/(-2*tanh(x)**2 + 4*I*tanh(x)*sech(x) + 2*sech(x)**2) - 2*I*l
```

```
og(I*tanh(x) + sech(x))*tanh(x)**2/(-2*tanh(x)**2 + 4*I*tanh(x)*sech(x) + 2
*sech(x)**2) - 4*log(I*tanh(x) + sech(x))*tanh(x)*sech(x)/(-2*tanh(x)**2 +
4*I*tanh(x)*sech(x) + 2*sech(x)**2) + 2*I*log(I*tanh(x) + sech(x))*sech(x)*
*2/(-2*tanh(x)**2 + 4*I*tanh(x)*sech(x) + 2*sech(x)**2) + 2*I*log(tanh(x) +
 1)*tanh(x)**2/(-2*tanh(x)**2 + 4*I*tanh(x)*sech(x) + 2*sech(x)**2) + 4*log
(tanh(x) + 1)*tanh(x)*sech(x)/(-2*tanh(x)**2 + 4*I*tanh(x)*sech(x) + 2*sech
(x)**2) - 2*I*log(tanh(x) + 1)*sech(x)**2/(-2*tanh(x)**2 + 4*I*tanh(x)*sech
(x) + 2*sech(x)**2) + I*tanh(x)**2/(-2*tanh(x)**2 + 4*I*tanh(x)*sech(x) + 2
*sech(x)**2) + I*sech(x)**2/(-2*tanh(x)**2 + 4*I*tanh(x)*sech(x) + 2*sech(x
)**2) + I/(-2*tanh(x)**2 + 4*I*tanh(x)*sech(x) + 2*sech(x)**2)
```

3.632 $\int \frac{1}{(\operatorname{sech}(x)+i\tanh(x))^4}\,dx$

Optimal. Leaf size=38

$$x+\frac{2i\cosh^3(x)}{3(1+i\sinh(x))^3}-\frac{2i\cosh(x)}{1+i\sinh(x)}$$

```
[Out] x+2/3*I*cosh(x)^3/(1+I*sinh(x))^3-2*I*cosh(x)/(1+I*sinh(x))
```

Rubi [A] time = 0.08, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {4391, 2680, 8}

$$x+\frac{2i\cosh^3(x)}{3(1+i\sinh(x))^3}-\frac{2i\cosh(x)}{1+i\sinh(x)}$$

Antiderivative was successfully verified.

```
[In] Int[(Sech[x] + I*Tanh[x])^(-4),x]
```

```
[Out] x + (((2*I)/3)*Cosh[x]^3)/(1 + I*Sinh[x])^3 - ((2*I)*Cosh[x])/(1 + I*Sinh[x
])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2680

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(2*g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f
*x])^(m + 1))/(b*f*(2*m + p + 1)), x] + Dist[(g^2*(p - 1))/(b^2*(2*m + p +
1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 4391

```
Int[(u_.)*((b_.)*sec[(c_.) + (d_.)*(x_)]^(n_.) + (a_.)*tan[(c_.) + (d_.)*(x
_)]^(n_.))^(p_), x_Symbol] :> Int[ActivateTrig[u]*Sec[c + d*x]^(n*p)*(b + a
*Sin[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

Rubi steps

$$\int \frac{1}{(\operatorname{sech}(x)+i\tanh(x))^4}\,dx = \int \frac{\cosh^4(x)}{(1+i\sinh(x))^4}\,dx$$
$$= \frac{2i\cosh^3(x)}{3(1+i\sinh(x))^3} - \int \frac{\cosh^2(x)}{(1+i\sinh(x))^2}\,dx$$
$$= \frac{2i\cosh^3(x)}{3(1+i\sinh(x))^3} - \frac{2i\cosh(x)}{1+i\sinh(x)} + \int 1\,dx$$
$$= x + \frac{2i\cosh^3(x)}{3(1+i\sinh(x))^3} - \frac{2i\cosh(x)}{1+i\sinh(x)}$$

Mathematica [A] time = 0.08, size = 75, normalized size = 1.97

$$\frac{3(3x+8i)\cosh\left(\frac{x}{2}\right) - (3x+16i)\cosh\left(\frac{3x}{2}\right) + 6i\sinh\left(\frac{x}{2}\right)(2x+x\cosh(x)+4i)}{6\left(\cosh\left(\frac{x}{2}\right)+i\sinh\left(\frac{x}{2}\right)\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sech[x] + I*Tanh[x])^(-4),x]
```

```
[Out] (3*(8*I + 3*x)*Cosh[x/2] - (16*I + 3*x)*Cosh[(3*x)/2] + (6*I)*(4*I + 2*x +
x*Cosh[x])*Sinh[x/2])/(6*(Cosh[x/2] + I*Sinh[x/2])^3)
```

fricas [A] time = 0.42, size = 52, normalized size = 1.37

$$\frac{3\,xe^{(3\,x)} + (-9i\,x - 24i)e^{(2\,x)} - 3\,(3\,x+8)e^{x} + 3i\,x + 16i}{3\,e^{(3\,x)} - 9i\,e^{(2\,x)} - 9\,e^{x} + 3i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sech(x)+I*tanh(x))^4,x, algorithm="fricas")
```

```
[Out] (3*x*e^(3*x) + (-9*I*x - 24*I)*e^(2*x) - 3*(3*x + 8)*e^x + 3*I*x + 16*I)/(3
*e^(3*x) - 9*I*e^(2*x) - 9*e^x + 3*I)
```

giac [A] time = 0.14, size = 22, normalized size = 0.58

$$x - \frac{24i\,e^{(2\,x)} + 24\,e^{x} - 16i}{3\,(e^{x}-i)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sech(x)+I*tanh(x))^4,x, algorithm="giac")
```

```
[Out] x - 1/3*(24*I*e^(2*x) + 24*e^x - 16*I)/(e^x - I)^3
```

maple [A] time = 0.36, size = 41, normalized size = 1.08

$$-\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)+\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)+\frac{8i}{\left(\tanh\left(\frac{x}{2}\right)-i\right)^{2}}-\frac{16}{3\left(\tanh\left(\frac{x}{2}\right)-i\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sech(x)+I*tanh(x))^4,x)
```

```
[Out] -ln(tanh(1/2*x)-1)+ln(tanh(1/2*x)+1)+8*I/(tanh(1/2*x)-I)^2-16/3/(tanh(1/2*x
)-I)^3
```

maxima [A] time = 0.42, size = 40, normalized size = 1.05

$$x-\frac{24\,e^{(-x)}-24i\,e^{(-2\,x)}+16i}{9\,e^{(-x)}-9i\,e^{(-2\,x)}-3\,e^{(-3\,x)}+3i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sech(x)+I*tanh(x))^4,x, algorithm="maxima")
```

```
[Out] x - (24*e^(-x) - 24*I*e^(-2*x) + 16*I)/(9*e^(-x) - 9*I*e^(-2*x) - 3*e^(-3*x
) + 3*I)
```

mupad [B] time = 0.17, size = 67, normalized size = 1.76

$$x+\frac{\frac{e^{2\,x}\,8i}{3}-\frac{8}{3}i}{e^{2\,x}\,3i-e^{3\,x}+3\,e^{x}-i}-\frac{8i}{3\,\left(e^{x}-i\right)}+\frac{e^{x}\,8i}{3\,\left(1-e^{2\,x}+e^{x}\,2i\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(tanh(x)*1i + 1/cosh(x))^4,x)
```

```
[Out] x + ((exp(2*x)*8i)/3 - 8i/3)/(exp(2*x)*3i - exp(3*x) + 3*exp(x) - 1i) - 8i/
(3*(exp(x) - 1i)) + (exp(x)*8i)/(3*(exp(x)*2i - exp(2*x) + 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\left(i\tanh\left(x\right)+\operatorname{sech}\left(x\right)\right)^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sech(x)+I*tanh(x))**4,x)
```

```
[Out] Integral((I*tanh(x) + sech(x))**(-4), x)
```

3.633 $\int \frac{1}{(\mathbf{sech}(x)+i\tanh(x))^5}\,dx$

Optimal. Leaf size=42

$$-\frac{4i}{1+i\sinh(x)}+\frac{2i}{(1+i\sinh(x))^2}-i\log(-\sinh(x)+i)$$

```
[Out] -I*ln(I-sinh(x))+2*I/(1+I*sinh(x))^2-4*I/(1+I*sinh(x))
```

Rubi [A] time = 0.06, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {4391, 2667, 43}

$$-\frac{4i}{1+i\sinh(x)}+\frac{2i}{(1+i\sinh(x))^2}-i\log(-\sinh(x)+i)$$

Antiderivative was successfully verified.

```
[In] Int[(Sech[x] + I*Tanh[x])^(-5),x]
```

```
[Out] (-I)*Log[I - Sinh[x]] + (2*I)/(1 + I*Sinh[x])^2 - (4*I)/(1 + I*Sinh[x])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 4391

```
Int[(u_.)*((b_.)*sec[(c_.) + (d_.)*(x_)]^(n_.) + (a_.)*tan[(c_.) + (d_.)*(x
_)]^(n_.))^(p_), x_Symbol] :> Int[ActivateTrig[u]*Sec[c + d*x]^(n*p)*(b + a
*Sin[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

Rubi steps

$$\int \frac{1}{(\text{sech}(x)+i\tanh(x))^5}\,dx = \int \frac{\cosh^5(x)}{(1+i\sinh(x))^5}\,dx$$
$$= -\left(i\,\text{Subst}\left(\int \frac{(1-x)^2}{(1+x)^3}\,dx, x, i\sinh(x)\right)\right)$$
$$= -\left(i\,\text{Subst}\left(\int \left(\frac{4}{(1+x)^3}-\frac{4}{(1+x)^2}+\frac{1}{1+x}\right)dx, x, i\sinh(x)\right)\right)$$
$$= -i\log(i-\sinh(x)) + \frac{2i}{(1+i\sinh(x))^2} - \frac{4i}{1+i\sinh(x)}$$

Mathematica [A] time = 0.11, size = 45, normalized size = 1.07

$$2\tan^{-1}\left(\tanh\left(\frac{x}{2}\right)\right) - i\log(\cosh(x)) + \frac{4\sinh(x)-2i}{\left(\cosh\left(\frac{x}{2}\right)+i\sinh\left(\frac{x}{2}\right)\right)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sech[x] + I*Tanh[x])^(-5),x]
```

```
[Out] 2*ArcTan[Tanh[x/2]] - I*Log[Cosh[x]] + (-2*I + 4*Sinh[x])/(Cosh[x/2] + I*Si
nh[x/2])^4
```

fricas [B] time = 0.42, size = 92, normalized size = 2.19

$$\frac{i\,xe^{(4\,x)} + 4\,(x-2)e^{(3\,x)} + (-6i\,x+8i)e^{(2\,x)} - 4\,(x-2)e^{x} + \left(-2i\,e^{(4\,x)} - 8\,e^{(3\,x)} + 12i\,e^{(2\,x)} + 8\,e^{x} - 2i\right)\log\left(e^{x}-i\right) + }{e^{(4\,x)} - 4i\,e^{(3\,x)} - 6\,e^{(2\,x)} + 4i\,e^{x} + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sech(x)+I*tanh(x))^5,x, algorithm="fricas")
```

```
[Out] (I*x*e^(4*x) + 4*(x - 2)*e^(3*x) + (-6*I*x + 8*I)*e^(2*x) - 4*(x - 2)*e^x +
 (-2*I*e^(4*x) - 8*e^(3*x) + 12*I*e^(2*x) + 8*e^x - 2*I)*log(e^x - I) + I*x
)/(e^(4*x) - 4*I*e^(3*x) - 6*e^(2*x) + 4*I*e^x + 1)
```

giac [A] time = 0.13, size = 38, normalized size = 0.90

$$-\frac{8\left(e^{(3\,x)} - i\,e^{(2\,x)} - e^{x}\right)}{\left(e^{x}-i\right)^4} + i\,\log\left(i\,e^{x}\right) - 2i\,\log\left(e^{x}-i\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sech(x)+I*tanh(x))^5,x, algorithm="giac")
```

```
[Out] -8*(e^(3*x) - I*e^(2*x) - e^x)/(e^x - I)^4 + I*log(I*e^x) - 2*I*log(e^x - I
)
```

maple [A] time = 0.42, size = 68, normalized size = 1.62

$$i\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)+i\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)+\frac{8i}{\left(\tanh\left(\frac{x}{2}\right)-i\right)^{4}}-2i\ln\left(\tanh\left(\frac{x}{2}\right)-i\right)-\frac{8i}{\left(\tanh\left(\frac{x}{2}\right)-i\right)^{2}}+\frac{16}{\left(\tanh\left(\frac{x}{2}\right)-i\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sech(x)+I*tanh(x))^5,x)
```

```
[Out] I*ln(tanh(1/2*x)-1)+I*ln(tanh(1/2*x)+1)+8*I/(tanh(1/2*x)-I)^4-2*I*ln(tanh(1
/2*x)-I)-8*I/(tanh(1/2*x)-I)^2+16/(tanh(1/2*x)-I)^3
```

maxima [A] time = 0.39, size = 60, normalized size = 1.43

$$-i\,x-\frac{8\,e^{(-x)}-8i\,e^{(-2\,x)}-8\,e^{(-3\,x)}}{-4i\,e^{(-x)}-6\,e^{(-2\,x)}+4i\,e^{(-3\,x)}+e^{(-4\,x)}+1}-2i\,\log\left(e^{(-x)}+i\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sech(x)+I*tanh(x))^5,x, algorithm="maxima")
```

```
[Out] -I*x - (8*e^(-x) - 8*I*e^(-2*x) - 8*e^(-3*x))/(-4*I*e^(-x) - 6*e^(-2*x) + 4
*I*e^(-3*x) + e^(-4*x) + 1) - 2*I*log(e^(-x) + I)
```

mupad [B] time = 1.68, size = 94, normalized size = 2.24

$$x\,1\mathrm{i}-\ln\left(\mathrm{e}^{x}-\mathrm{i}\right)\,2\mathrm{i}-\frac{16}{\mathrm{e}^{2\,x}\,3\mathrm{i}-\mathrm{e}^{3\,x}+3\,\mathrm{e}^{x}-\mathrm{i}}+\frac{8\mathrm{i}}{\mathrm{e}^{4\,x}-6\,\mathrm{e}^{2\,x}+1-\mathrm{e}^{3\,x}\,4\mathrm{i}+\mathrm{e}^{x}\,4\mathrm{i}}+\frac{16\mathrm{i}}{1-\mathrm{e}^{2\,x}+\mathrm{e}^{x}\,2\mathrm{i}}-\frac{8}{\mathrm{e}^{x}-\mathrm{i}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(tanh(x)*1i + 1/cosh(x))^5,x)
```

```
[Out] x*1i - log(exp(x) - 1i)*2i - 16/(exp(2*x)*3i - exp(3*x) + 3*exp(x) - 1i) +
8i/(exp(4*x) - exp(3*x)*4i - 6*exp(2*x) + exp(x)*4i + 1) + 16i/(exp(x)*2i -
 exp(2*x) + 1) - 8/(exp(x) - 1i)
```

sympy [B] time = 8.42, size = 1445, normalized size = 34.40

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sech(x)+I*tanh(x))**5,x)
```

```
[Out] -36*I*x*tanh(x)**4/(36*tanh(x)**4 - 144*I*tanh(x)**3*sech(x) - 216*tanh(x)*
*2*sech(x)**2 + 144*I*tanh(x)*sech(x)**3 + 36*sech(x)**4) - 144*x*tanh(x)**
3*sech(x)/(36*tanh(x)**4 - 144*I*tanh(x)**3*sech(x) - 216*tanh(x)**2*sech(x
)**2 + 144*I*tanh(x)*sech(x)**3 + 36*sech(x)**4) + 216*I*x*tanh(x)**2*sech(
x)**2/(36*tanh(x)**4 - 144*I*tanh(x)**3*sech(x) - 216*tanh(x)**2*sech(x)**2
 + 144*I*tanh(x)*sech(x)**3 + 36*sech(x)**4) + 144*x*tanh(x)*sech(x)**3/(36
*tanh(x)**4 - 144*I*tanh(x)**3*sech(x) - 216*tanh(x)**2*sech(x)**2 + 144*I*
tanh(x)*sech(x)**3 + 36*sech(x)**4) - 36*I*x*sech(x)**4/(36*tanh(x)**4 - 14
4*I*tanh(x)**3*sech(x) - 216*tanh(x)**2*sech(x)**2 + 144*I*tanh(x)*sech(x)*
*3 + 36*sech(x)**4) - 36*I*log(I*tanh(x) + sech(x))*tanh(x)**4/(36*tanh(x)*
*4 - 144*I*tanh(x)**3*sech(x) - 216*tanh(x)**2*sech(x)**2 + 144*I*tanh(x)*s
ech(x)**3 + 36*sech(x)**4) - 144*log(I*tanh(x) + sech(x))*tanh(x)**3*sech(x
)/(36*tanh(x)**4 - 144*I*tanh(x)**3*sech(x) - 216*tanh(x)**2*sech(x)**2 + 1
44*I*tanh(x)*sech(x)**3 + 36*sech(x)**4) + 216*I*log(I*tanh(x) + sech(x))*t
anh(x)**2*sech(x)**2/(36*tanh(x)**4 - 144*I*tanh(x)**3*sech(x) - 216*tanh(x
)**2*sech(x)**2 + 144*I*tanh(x)*sech(x)**3 + 36*sech(x)**4) + 144*log(I*tan
h(x) + sech(x))*tanh(x)*sech(x)**3/(36*tanh(x)**4 - 144*I*tanh(x)**3*sech(x
) - 216*tanh(x)**2*sech(x)**2 + 144*I*tanh(x)*sech(x)**3 + 36*sech(x)**4) -
 36*I*log(I*tanh(x) + sech(x))*sech(x)**4/(36*tanh(x)**4 - 144*I*tanh(x)**3
*sech(x) - 216*tanh(x)**2*sech(x)**2 + 144*I*tanh(x)*sech(x)**3 + 36*sech(x
)**4) + 36*I*log(tanh(x) + 1)*tanh(x)**4/(36*tanh(x)**4 - 144*I*tanh(x)**3*
sech(x) - 216*tanh(x)**2*sech(x)**2 + 144*I*tanh(x)*sech(x)**3 + 36*sech(x)
**4) + 144*log(tanh(x) + 1)*tanh(x)**3*sech(x)/(36*tanh(x)**4 - 144*I*tanh(
x)**3*sech(x) - 216*tanh(x)**2*sech(x)**2 + 144*I*tanh(x)*sech(x)**3 + 36*s
ech(x)**4) - 216*I*log(tanh(x) + 1)*tanh(x)**2*sech(x)**2/(36*tanh(x)**4 -
144*I*tanh(x)**3*sech(x) - 216*tanh(x)**2*sech(x)**2 + 144*I*tanh(x)*sech(x
)**3 + 36*sech(x)**4) - 144*log(tanh(x) + 1)*tanh(x)*sech(x)**3/(36*tanh(x)
**4 - 144*I*tanh(x)**3*sech(x) - 216*tanh(x)**2*sech(x)**2 + 144*I*tanh(x)*
sech(x)**3 + 36*sech(x)**4) + 36*I*log(tanh(x) + 1)*sech(x)**4/(36*tanh(x)*
*4 - 144*I*tanh(x)**3*sech(x) - 216*tanh(x)**2*sech(x)**2 + 144*I*tanh(x)*s
ech(x)**3 + 36*sech(x)**4) + 28*I*tanh(x)**4/(36*tanh(x)**4 - 144*I*tanh(x)
**3*sech(x) - 216*tanh(x)**2*sech(x)**2 + 144*I*tanh(x)*sech(x)**3 + 36*sec
h(x)**4) + 52*tanh(x)**3*sech(x)/(36*tanh(x)**4 - 144*I*tanh(x)**3*sech(x)
- 216*tanh(x)**2*sech(x)**2 + 144*I*tanh(x)*sech(x)**3 + 36*sech(x)**4) + 1
8*I*tanh(x)**2/(36*tanh(x)**4 - 144*I*tanh(x)**3*sech(x) - 216*tanh(x)**2*s
ech(x)**2 + 144*I*tanh(x)*sech(x)**3 + 36*sech(x)**4) + 44*tanh(x)*sech(x)*
*3/(36*tanh(x)**4 - 144*I*tanh(x)**3*sech(x) - 216*tanh(x)**2*sech(x)**2 +
144*I*tanh(x)*sech(x)**3 + 36*sech(x)**4) + 24*tanh(x)*sech(x)/(36*tanh(x)*
*4 - 144*I*tanh(x)**3*sech(x) - 216*tanh(x)**2*sech(x)**2 + 144*I*tanh(x)*s
ech(x)**3 + 36*sech(x)**4) - 20*I*sech(x)**4/(36*tanh(x)**4 - 144*I*tanh(x)
**3*sech(x) - 216*tanh(x)**2*sech(x)**2 + 144*I*tanh(x)*sech(x)**3 + 36*sec
h(x)**4) - 6*I*sech(x)**2/(36*tanh(x)**4 - 144*I*tanh(x)**3*sech(x) - 216*t
anh(x)**2*sech(x)**2 + 144*I*tanh(x)*sech(x)**3 + 36*sech(x)**4) + 9*I/(36*
```

```
tanh(x)**4 - 144*I*tanh(x)**3*sech(x) - 216*tanh(x)**2*sech(x)**2 + 144*I*t
anh(x)*sech(x)**3 + 36*sech(x)**4)
```

3.634 $\int (\text{sech}(x) - i\tanh(x))^5 \, dx$

Optimal. Leaf size=42

$$-\frac{4i}{1+i\sinh(x)}+\frac{2i}{(1+i\sinh(x))^2}-i\log(-\sinh(x)+i)$$

```
[Out] -I*ln(I-sinh(x))+2*I/(1+I*sinh(x))^2-4*I/(1+I*sinh(x))
```

Rubi [A] time = 0.06, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {4391, 2667, 43}

$$-\frac{4i}{1+i\sinh(x)}+\frac{2i}{(1+i\sinh(x))^2}-i\log(-\sinh(x)+i)$$

Antiderivative was successfully verified.

```
[In] Int[(Sech[x] - I*Tanh[x])^5,x]
```

```
[Out] (-I)*Log[I - Sinh[x]] + (2*I)/(1 + I*Sinh[x])^2 - (4*I)/(1 + I*Sinh[x])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 4391

```
Int[(u_.)*((b_.)*sec[(c_.) + (d_.)*(x_)]^(n_.) + (a_.)*tan[(c_.) + (d_.)*(x
_)]^(n_.))^(p_), x_Symbol] :> Int[ActivateTrig[u]*Sec[c + d*x]^(n*p)*(b + a
*Sin[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

Rubi steps

$$\int (\operatorname{sech}(x) - i\tanh(x))^5\,dx = \int \operatorname{sech}^5(x)(1 - i\sinh(x))^5\,dx$$
$$= i\,\text{Subst}\left(\int \frac{(1+x)^2}{(1-x)^3}\,dx, x, -i\sinh(x)\right)$$
$$= i\,\text{Subst}\left(\int \left(\frac{1}{1-x} - \frac{4}{(-1+x)^3} - \frac{4}{(-1+x)^2}\right)dx, x, -i\sinh(x)\right)$$
$$= -i\log(i - \sinh(x)) + \frac{2i}{(1+i\sinh(x))^2} - \frac{4i}{1+i\sinh(x)}$$

Mathematica [A] time = 0.06, size = 62, normalized size = 1.48

$$\frac{11}{4}i\tanh^4(x)+\frac{1}{2}i\tanh^2(x)+\frac{5}{4}i\operatorname{sech}^4(x)+\tan^{-1}(\sinh(x))-i\log(\cosh(x))-\tanh(x)\operatorname{sech}^3(x)-5\tanh^3(x)\operatorname{sech}(x)+\text{ta}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sech[x] - I*Tanh[x])^5,x]
```

```
[Out] ArcTan[Sinh[x]] - I*Log[Cosh[x]] + ((5*I)/4)*Sech[x]^4 + Sech[x]*Tanh[x] -
Sech[x]^3*Tanh[x] + (I/2)*Tanh[x]^2 - 5*Sech[x]*Tanh[x]^3 + ((11*I)/4)*Tanh
[x]^4
```

fricas [B] time = 0.42, size = 92, normalized size = 2.19

$$\frac{i\,xe^{(4\,x)} + 4\,(x-2)e^{(3\,x)} + (-6i\,x + 8i)e^{(2\,x)} - 4\,(x-2)e^{x} + \left(-2i\,e^{(4\,x)} - 8\,e^{(3\,x)} + 12i\,e^{(2\,x)} + 8\,e^{x} - 2i\right)\log\left(e^{x} - i\right) + i}{e^{(4\,x)} - 4i\,e^{(3\,x)} - 6\,e^{(2\,x)} + 4i\,e^{x} + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sech(x)-I*tanh(x))^5,x, algorithm="fricas")
```

```
[Out] (I*x*e^(4*x) + 4*(x - 2)*e^(3*x) + (-6*I*x + 8*I)*e^(2*x) - 4*(x - 2)*e^x +
 (-2*I*e^(4*x) - 8*e^(3*x) + 12*I*e^(2*x) + 8*e^x - 2*I)*log(e^x - I) + I*x
)/(e^(4*x) - 4*I*e^(3*x) - 6*e^(2*x) + 4*I*e^x + 1)
```

giac [A] time = 0.12, size = 34, normalized size = 0.81

$$i\,x - \frac{8\left(e^{(3\,x)} - i\,e^{(2\,x)} - e^{x}\right)}{\left(e^{x} - i\right)^{4}} - 2i\,\log\left(e^{x} - i\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sech(x)-I*tanh(x))^5,x, algorithm="giac")
```

```
[Out] I*x - 8*(e^(3*x) - I*e^(2*x) - e^x)/(e^x - I)^4 - 2*I*log(e^x - I)
```

maple [B] time = 0.39, size = 78, normalized size = 1.86

$$\frac{8\left(\frac{\operatorname{sech}(x)^{3}}{4}+\frac{3\operatorname{sech}(x)}{8}\right)\tanh(x)}{3}+2\arctan\left(e^{x}\right)-\frac{5i}{4\cosh(x)^{4}}-\frac{5\sinh(x)}{3\cosh(x)^{4}}-\frac{5i\left(\sinh^{2}(x)\right)}{\cosh(x)^{4}}-\frac{5\left(\sinh^{3}(x)\right)}{\cosh(x)^{4}}-i\ln\left(\cosh(x\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sech(x)-I*tanh(x))^5,x)
```

```
[Out] 8/3*(1/4*sech(x)^3+3/8*sech(x))*tanh(x)+2*arctan(exp(x))-5/4*I/cosh(x)^4-5/
3*sinh(x)/cosh(x)^4-5*I*sinh(x)^2/cosh(x)^4-5*sinh(x)^3/cosh(x)^4-I*ln(cosh
(x))+1/2*I*tanh(x)^2+1/4*I*tanh(x)^4
```

maxima [B] time = 0.61, size = 235, normalized size = 5.60

$$\frac{5}{2}i\tanh(x)^{4}-ix-\frac{5\left(5e^{(-x)}-3e^{(-3x)}+3e^{(-5x)}-5e^{(-7x)}\right)}{4\left(4e^{(-2x)}+6e^{(-4x)}+4e^{(-6x)}+e^{(-8x)}+1\right)}+\frac{3e^{(-x)}+11e^{(-3x)}-11e^{(-5x)}-3e^{(-7x)}}{4\left(4e^{(-2x)}+6e^{(-4x)}+4e^{(-6x)}+e^{(-8x)}+1\right)}-\frac{}{2(4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sech(x)-I*tanh(x))^5,x, algorithm="maxima")
```

```
[Out] 5/2*I*tanh(x)^4 - I*x - 5/4*(5*e^(-x) - 3*e^(-3*x) + 3*e^(-5*x) - 5*e^(-7*x
))/(4*e^(-2*x) + 6*e^(-4*x) + 4*e^(-6*x) + e^(-8*x) + 1) + 1/4*(3*e^(-x) +
11*e^(-3*x) - 11*e^(-5*x) - 3*e^(-7*x))/(4*e^(-2*x) + 6*e^(-4*x) + 4*e^(-6*
x) + e^(-8*x) + 1) - 5/2*(e^(-x) - 7*e^(-3*x) + 7*e^(-5*x) - e^(-7*x))/(4*e
^(-2*x) + 6*e^(-4*x) + 4*e^(-6*x) + e^(-8*x) + 1) - 4*I*(e^(-2*x) + e^(-4*x
) + e^(-6*x))/(4*e^(-2*x) + 6*e^(-4*x) + 4*e^(-6*x) + e^(-8*x) + 1) + 20*I/
(e^(-x) + e^x)^4 - 2*arctan(e^(-x)) - I*log(e^(-2*x) + 1)
```

mupad [B] time = 1.60, size = 94, normalized size = 2.24

$$x\,1\mathrm{i}-\ln\left(e^{x}-\mathrm{i}\right)\,2\mathrm{i}-\frac{16}{e^{2x}\,3\mathrm{i}-e^{3x}+3\,e^{x}-\mathrm{i}}+\frac{8\mathrm{i}}{e^{4x}-6\,e^{2x}+1-e^{3x}\,4\mathrm{i}+e^{x}\,4\mathrm{i}}+\frac{16\mathrm{i}}{1-e^{2x}+e^{x}\,2\mathrm{i}}-\frac{8}{e^{x}-\mathrm{i}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(tanh(x)*1i - 1/cosh(x))^5,x)
```

```
[Out] x*1i - log(exp(x) - 1i)*2i - 16/(exp(2*x)*3i - exp(3*x) + 3*exp(x) - 1i) +
8i/(exp(4*x) - exp(3*x)*4i - 6*exp(2*x) + exp(x)*4i + 1) + 16i/(exp(x)*2i -
 exp(2*x) + 1) - 8/(exp(x) - 1i)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (-i\tanh(x) + \mathrm{sech}(x))^5 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sech(x)-I*tanh(x))**5,x)
```

```
[Out] Integral((-I*tanh(x) + sech(x))**5, x)
```

3.635 $\int (\mathbf{sech}(x) - i\tanh(x))^4\,dx$

Optimal. Leaf size=38

$$x + \frac{2i\cosh^3(x)}{3(1+i\sinh(x))^3} - \frac{2i\cosh(x)}{1+i\sinh(x)}$$

```
[Out] x+2/3*I*cosh(x)^3/(1+I*sinh(x))^3-2*I*cosh(x)/(1+I*sinh(x))
```

Rubi [A] time = 0.11, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {4391, 2670, 2680, 8}

$$x + \frac{2i\cosh^3(x)}{3(1+i\sinh(x))^3} - \frac{2i\cosh(x)}{1+i\sinh(x)}$$

Antiderivative was successfully verified.

```
[In] Int[(Sech[x] - I*Tanh[x])^4,x]
```

```
[Out] x + (((2*I)/3)*Cosh[x]^3)/(1 + I*Sinh[x])^3 - ((2*I)*Cosh[x])/(1 + I*Sinh[x
])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2670

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*m), Int[(g*Cos[e + f*x])^(2*m + p)/(a
 - b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2,
 0] && IntegerQ[m] && LtQ[p, -1] && GeQ[2*m + p, 0]
```

Rule 2680

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(2*g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f
*x])^(m + 1))/(b*f*(2*m + p + 1)), x] + Dist[(g^2*(p - 1))/(b^2*(2*m + p +
1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 4391

```
Int[(u_.)*((b_.)*sec[(c_.) + (d_.)*(x_)]^(n_.) + (a_.)*tan[(c_.) + (d_.)*(x
_)]^(n_.))^(p_), x_Symbol] :> Int[ActivateTrig[u]*Sec[c + d*x]^(n*p)*(b + a
*Sin[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

Rubi steps

$$\begin{aligned}
\int (\operatorname{sech}(x)-i\tanh(x))^4\,dx &= \int \operatorname{sech}^4(x)(1-i\sinh(x))^4\,dx\\
&= \int \frac{\cosh^4(x)}{(1+i\sinh(x))^4}\,dx\\
&= \frac{2i\cosh^3(x)}{3(1+i\sinh(x))^3}-\int \frac{\cosh^2(x)}{(1+i\sinh(x))^2}\,dx\\
&= \frac{2i\cosh^3(x)}{3(1+i\sinh(x))^3}-\frac{2i\cosh(x)}{1+i\sinh(x)}+\int 1\,dx\\
&= x+\frac{2i\cosh^3(x)}{3(1+i\sinh(x))^3}-\frac{2i\cosh(x)}{1+i\sinh(x)}
\end{aligned}$$

Mathematica [A] time = 0.05, size = 75, normalized size = 1.97

$$\frac{3(3x+8i)\cosh\left(\frac{x}{2}\right)-(3x+16i)\cosh\left(\frac{3x}{2}\right)+6i\sinh\left(\frac{x}{2}\right)(2x+x\cosh(x)+4i)}{6\left(\cosh\left(\frac{x}{2}\right)+i\sinh\left(\frac{x}{2}\right)\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sech[x] - I*Tanh[x])^4,x]
```

```
[Out] (3*(8*I + 3*x)*Cosh[x/2] - (16*I + 3*x)*Cosh[(3*x)/2] + (6*I)*(4*I + 2*x +
x*Cosh[x])*Sinh[x/2])/(6*(Cosh[x/2] + I*Sinh[x/2])^3)
```

fricas [A] time = 0.44, size = 52, normalized size = 1.37

$$\frac{3\,xe^{(3\,x)}+(-9i\,x-24i)e^{(2\,x)}-3\,(3\,x+8)e^{x}+3i\,x+16i}{3\,e^{(3\,x)}-9i\,e^{(2\,x)}-9\,e^{x}+3i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sech(x)-I*tanh(x))^4,x, algorithm="fricas")
```

```
[Out] (3*x*e^(3*x) + (-9*I*x - 24*I)*e^(2*x) - 3*(3*x + 8)*e^x + 3*I*x + 16*I)/(3
*e^(3*x) - 9*I*e^(2*x) - 9*e^x + 3*I)
```

giac [A] time = 0.14, size = 22, normalized size = 0.58

$$x - \frac{24i\,e^{(2\,x)} + 24\,e^{x} - 16i}{3\,(e^{x} - i)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sech(x)-I*tanh(x))^4,x, algorithm="giac")
```

```
[Out] x - 1/3*(24*I*e^(2*x) + 24*e^x - 16*I)/(e^x - I)^3
```

maple [A] time = 0.36, size = 60, normalized size = 1.58

$$-2\left(\frac{2}{3}+\frac{\mathrm{sech}(x)^{2}}{3}\right)\tanh(x)+\frac{4i}{3\cosh(x)^{3}}+\frac{3\sinh(x)}{\cosh(x)^{3}}+4i\left(-\frac{\sinh^{2}(x)}{\cosh(x)^{3}}-\frac{2}{3\cosh(x)^{3}}\right)+x-\tanh(x)-\frac{\left(\tanh^{3}(x)\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sech(x)-I*tanh(x))^4,x)
```

```
[Out] -2*(2/3+1/3*sech(x)^2)*tanh(x)+4/3*I/cosh(x)^3+3*sinh(x)/cosh(x)^3+4*I*(-si
nh(x)^2/cosh(x)^3-2/3/cosh(x)^3)+x-tanh(x)-1/3*tanh(x)^3
```

maxima [B] time = 0.37, size = 181, normalized size = 4.76

$$-2\,\tanh(x)^{3}+x-\frac{4\left(3\,e^{(-2\,x)}+3\,e^{(-4\,x)}+2\right)}{3\left(3\,e^{(-2\,x)}+3\,e^{(-4\,x)}+e^{(-6\,x)}+1\right)}-\frac{8i\,e^{(-x)}}{3\,e^{(-2\,x)}+3\,e^{(-4\,x)}+e^{(-6\,x)}+1}+\frac{4\,e^{(-2\,x)}}{3\,e^{(-2\,x)}+3\,e^{(-4\,x)}+e^{(-6\,x)}+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sech(x)-I*tanh(x))^4,x, algorithm="maxima")
```

```
[Out] -2*tanh(x)^3 + x - 4/3*(3*e^(-2*x) + 3*e^(-4*x) + 2)/(3*e^(-2*x) + 3*e^(-4*
x) + e^(-6*x) + 1) - 8*I*e^(-x)/(3*e^(-2*x) + 3*e^(-4*x) + e^(-6*x) + 1) +
4*e^(-2*x)/(3*e^(-2*x) + 3*e^(-4*x) + e^(-6*x) + 1) - 16/3*I*e^(-3*x)/(3*e^
(-2*x) + 3*e^(-4*x) + e^(-6*x) + 1) - 8*I*e^(-5*x)/(3*e^(-2*x) + 3*e^(-4*x)
 + e^(-6*x) + 1) + 4/3/(3*e^(-2*x) + 3*e^(-4*x) + e^(-6*x) + 1) + 32/3*I/(e
^(-x) + e^x)^3
```

mupad [B] time = 0.13, size = 67, normalized size = 1.76

$$x+\frac{\frac{\mathrm{e}^{2\,x}\,8\mathrm{i}}{3}-\frac{8}{3}\mathrm{i}}{\mathrm{e}^{2\,x}\,3\mathrm{i}-\mathrm{e}^{3\,x}+3\,\mathrm{e}^{x}-\mathrm{i}}-\frac{8\mathrm{i}}{3\,\left(\mathrm{e}^{x}-\mathrm{i}\right)}+\frac{\mathrm{e}^{x}\,8\mathrm{i}}{3\,\left(1-\mathrm{e}^{2\,x}+\mathrm{e}^{x}\,2\mathrm{i}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((tanh(x)*1i - 1/cosh(x))^4,x)
```

```
[Out] x + ((exp(2*x)*8i)/3 - 8i/3)/(exp(2*x)*3i - exp(3*x) + 3*exp(x) - 1i) - 8i/
(3*(exp(x) - 1i)) + (exp(x)*8i)/(3*(exp(x)*2i - exp(2*x) + 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (-i\tanh(x) + \operatorname{sech}(x))^4 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sech(x)-I*tanh(x))**4,x)
```

```
[Out] Integral((-I*tanh(x) + sech(x))**4, x)
```

3.636 $\int (\text{sech}(x) - i\tanh(x))^3\,dx$

Optimal. Leaf size=28

$$\frac{2i}{1+i\sinh(x)}+i\log(-\sinh(x)+i)$$

```
[Out] I*ln(I-sinh(x))+2*I/(1+I*sinh(x))
```

Rubi [A] time = 0.06, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {4391, 2667, 43}

$$\frac{2i}{1+i\sinh(x)}+i\log(-\sinh(x)+i)$$

Antiderivative was successfully verified.

```
[In] Int[(Sech[x] - I*Tanh[x])^3,x]
```

```
[Out] I*Log[I - Sinh[x]] + (2*I)/(1 + I*Sinh[x])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 4391

```
Int[(u_.)*((b_.)*sec[(c_.) + (d_.)*(x_)]^(n_.) + (a_.)*tan[(c_.) + (d_.)*(x
_)]^(n_.))^(p_), x_Symbol] :> Int[ActivateTrig[u]*Sec[c + d*x]^(n*p)*(b + a
*Sin[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

Rubi steps

$$
\begin{aligned}
\int (\operatorname{sech}(x)-i\tanh(x))^3\,dx &= \int \operatorname{sech}^3(x)(1-i\sinh(x))^3\,dx \\
&= i\,\text{Subst}\left(\int \frac{1+x}{(1-x)^2}\,dx, x, -i\sinh(x)\right) \\
&= i\,\text{Subst}\left(\int \left(\frac{2}{(-1+x)^2}+\frac{1}{-1+x}\right)dx, x, -i\sinh(x)\right) \\
&= i\log(i-\sinh(x))+\frac{2i}{1+i\sinh(x)}
\end{aligned}
$$

Mathematica [A] time = 0.03, size = 39, normalized size = 1.39

$$
-\frac{1}{2}i\tanh^2(x)+\frac{3}{2}i\operatorname{sech}^2(x)-\tan^{-1}(\sinh(x))+i\log(\cosh(x))+2\tanh(x)\operatorname{sech}(x)
$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sech[x] - I*Tanh[x])^3,x]
```

```
[Out] -ArcTan[Sinh[x]] + I*Log[Cosh[x]] + ((3*I)/2)*Sech[x]^2 + 2*Sech[x]*Tanh[x]
 - (I/2)*Tanh[x]^2
```

fricas [B] time = 0.41, size = 49, normalized size = 1.75

$$
\frac{-i\,xe^{(2\,x)}-2\,(x-2)e^{x}+\left(2i\,e^{(2\,x)}+4\,e^{x}-2i\right)\log\left(e^{x}-i\right)+i\,x}{e^{(2\,x)}-2i\,e^{x}-1}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sech(x)-I*tanh(x))^3,x, algorithm="fricas")
```

```
[Out] (-I*x*e^(2*x) - 2*(x - 2)*e^x + (2*I*e^(2*x) + 4*e^x - 2*I)*log(e^x - I) +
I*x)/(e^(2*x) - 2*I*e^x - 1)
```

giac [A] time = 0.14, size = 21, normalized size = 0.75

$$
-i\,x+\frac{4\,e^{x}}{\left(e^{x}-i\right)^2}+2i\,\log\left(e^{x}-i\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sech(x)-I*tanh(x))^3,x, algorithm="giac")
```

```
[Out] -I*x + 4*e^x/(e^x - I)^2 + 2*I*log(e^x - I)
```

maple [A] time = 0.37, size = 41, normalized size = 1.46

$$-\operatorname{sech}(x)\tanh(x)-2\arctan\left(\mathrm{e}^{x}\right)+\frac{3i}{2\cosh(x)^{2}}+\frac{3\sinh(x)}{\cosh(x)^{2}}+i\ln\left(\cosh(x)\right)-\frac{i\left(\tanh^{2}(x)\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sech(x)-I*tanh(x))^3,x)
```

```
[Out] -sech(x)*tanh(x)-2*arctan(exp(x))+3/2*I/cosh(x)^2+3/cosh(x)^2*sinh(x)+I*ln(
cosh(x))-1/2*I*tanh(x)^2
```

maxima [B] time = 0.86, size = 73, normalized size = 2.61

$$-\frac{3}{2}i\tanh(x)^{2}+ix+\frac{4\left(e^{(-x)}-e^{(-3x)}\right)}{2e^{(-2x)}+e^{(-4x)}+1}+\frac{2ie^{(-2x)}}{2e^{(-2x)}+e^{(-4x)}+1}+2\arctan\left(e^{(-x)}\right)+i\log\left(e^{(-2x)}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sech(x)-I*tanh(x))^3,x, algorithm="maxima")
```

```
[Out] -3/2*I*tanh(x)^2 + I*x + 4*(e^(-x) - e^(-3*x))/(2*e^(-2*x) + e^(-4*x) + 1)
+ 2*I*e^(-2*x)/(2*e^(-2*x) + e^(-4*x) + 1) + 2*arctan(e^(-x)) + I*log(e^(-2
*x) + 1)
```

mupad [B] time = 0.16, size = 41, normalized size = 1.46

$$-x\,1\mathrm{i}+\ln\left(\mathrm{e}^{x}-\mathrm{i}\right)\,2\mathrm{i}-\frac{4\mathrm{i}}{1-\mathrm{e}^{2x}+\mathrm{e}^{x}\,2\mathrm{i}}+\frac{4}{\mathrm{e}^{x}-\mathrm{i}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(tanh(x)*1i - 1/cosh(x))^3,x)
```

```
[Out] log(exp(x) - 1i)*2i - x*1i - 4i/(exp(x)*2i - exp(2*x) + 1) + 4/(exp(x) - 1i
)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(-i\tanh\left(x\right)+\operatorname{sech}\left(x\right)\right)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sech(x)-I*tanh(x))**3,x)
```

```
[Out] Integral((-I*tanh(x) + sech(x))**3, x)
```

3.637 $\int (\mathrm{sech}(x) - i\tanh(x))^2\,dx$

Optimal. Leaf size=20

$$-x+\frac{2i\cosh(x)}{1+i\sinh(x)}$$

[Out] -x+2*I*cosh(x)/(1+I*sinh(x))

Rubi [A] time = 0.07, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {4391, 2670, 2680, 8}

$$-x+\frac{2i\cosh(x)}{1+i\sinh(x)}$$

Antiderivative was successfully verified.

[In] Int[(Sech[x] - I*Tanh[x])^2,x]

[Out] -x + ((2*I)*Cosh[x])/(1 + I*Sinh[x])

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2670

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*m), Int[(g*Cos[e + f*x])^(2*m + p)/(a
 - b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2,
 0] && IntegerQ[m] && LtQ[p, -1] && GeQ[2*m + p, 0]
```

Rule 2680

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(2*g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f
*x])^(m + 1))/(b*f*(2*m + p + 1)), x] + Dist[(g^2*(p - 1))/(b^2*(2*m + p +
1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 4391

```
Int[(u_.)*((b_.)*sec[(c_.) + (d_.)*(x_)]^(n_.) + (a_.)*tan[(c_.) + (d_.)*(x
_)]^(n_.))^(p_), x_Symbol] :> Int[ActivateTrig[u]*Sec[c + d*x]^(n*p)*(b + a
```

```
*Sin[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

Rubi steps

$$\begin{aligned}\int (\operatorname{sech}(x)-i\tanh(x))^2\,dx &= \int \operatorname{sech}^2(x)(1-i\sinh(x))^2\,dx\\ &= \int \frac{\cosh^2(x)}{(1+i\sinh(x))^2}\,dx\\ &= \frac{2i\cosh(x)}{1+i\sinh(x)}-\int 1\,dx\\ &= -x+\frac{2i\cosh(x)}{1+i\sinh(x)}\end{aligned}$$

Mathematica [A] time = 0.01, size = 14, normalized size = 0.70

$$-x+2\tanh(x)+2i\operatorname{sech}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sech[x] - I*Tanh[x])^2,x]
[Out] -x + (2*I)*Sech[x] + 2*Tanh[x]
```

fricas [A] time = 0.42, size = 17, normalized size = 0.85

$$-\frac{xe^x-ix-4i}{e^x-i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sech(x)-I*tanh(x))^2,x, algorithm="fricas")
[Out] -(x*e^x - I*x - 4*I)/(e^x - I)
```

giac [A] time = 0.12, size = 12, normalized size = 0.60

$$-x+\frac{4i}{e^x-i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sech(x)-I*tanh(x))^2,x, algorithm="giac")
[Out] -x + 4*I/(e^x - I)
```

maple [A] time = 0.35, size = 16, normalized size = 0.80

$$2\tanh(x) + \frac{2i}{\cosh(x)} - x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sech(x)-I*tanh(x))^2,x)
```

```
[Out] 2*tanh(x)+2*I/cosh(x)-x
```

maxima [A] time = 0.41, size = 25, normalized size = 1.25

$$-x + \frac{4i}{e^{(-x)} + e^{x}} + \frac{4}{e^{(-2x)} + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sech(x)-I*tanh(x))^2,x, algorithm="maxima")
```

```
[Out] -x + 4*I/(e^(-x) + e^x) + 4/(e^(-2*x) + 1)
```

mupad [B] time = 1.56, size = 14, normalized size = 0.70

$$-x + \frac{4\mathrm{i}}{\mathrm{e}^{x} - \mathrm{i}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((tanh(x)*1i - 1/cosh(x))^2,x)
```

```
[Out] 4i/(exp(x) - 1i) - x
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (-i\tanh(x) + \operatorname{sech}(x))^2\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sech(x)-I*tanh(x))**2,x)
```

```
[Out] Integral((-I*tanh(x) + sech(x))**2, x)
```

3.638 $\int (\mathbf{sech}(x) - i\tanh(x))\,dx$

Optimal. Leaf size=11

$$\tan^{-1}(\sinh(x)) - i\log(\cosh(x))$$

```
[Out] arctan(sinh(x))-I*ln(cosh(x))
```

Rubi [A] time = 0.01, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {3770, 3475}

$$\tan^{-1}(\sinh(x)) - i\log(\cosh(x))$$

Antiderivative was successfully verified.

```
[In] Int[Sech[x] - I*Tanh[x],x]
```

```
[Out] ArcTan[Sinh[x]] - I*Log[Cosh[x]]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int (\operatorname{sech}(x) - i\tanh(x))\,dx &= -(i\int \tanh(x)\,dx) + \int \operatorname{sech}(x)\,dx\\ &= \tan^{-1}(\sinh(x)) - i\log(\cosh(x))\end{aligned}$$

Mathematica [A] time = 0.00, size = 17, normalized size = 1.55

$$2\tan^{-1}\left(\tanh\left(\frac{x}{2}\right)\right) - i\log(\cosh(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[x] - I*Tanh[x],x]
```

```
[Out] 2*ArcTan[Tanh[x/2]] - I*Log[Cosh[x]]
```

fricas [A] time = 0.42, size = 11, normalized size = 1.00

$$ix - 2i\log(e^x - i)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)-I*tanh(x),x, algorithm="fricas")
[Out] I*x - 2*I*log(e^x - I)
```

giac [A] time = 0.13, size = 18, normalized size = 1.64

$$ix + 2\arctan(e^x) - i\log\left(e^{(2x)} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)-I*tanh(x),x, algorithm="giac")
[Out] I*x + 2*arctan(e^x) - I*log(e^(2*x) + 1)
```

maple [A] time = 0.02, size = 11, normalized size = 1.00

$$\arctan(\sinh(x)) - i\ln(\cosh(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(x)-I*tanh(x),x)
[Out] arctan(sinh(x))-I*ln(cosh(x))
```

maxima [A] time = 0.31, size = 9, normalized size = 0.82

$$\arctan(\sinh(x)) - i\log(\cosh(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)-I*tanh(x),x, algorithm="maxima")
[Out] arctan(sinh(x)) - I*log(cosh(x))
```

mupad [B] time = 1.47, size = 14, normalized size = 1.27

$$x\,1\mathrm{i} - \ln(\mathrm{e}^x - \mathrm{i})\,2\mathrm{i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cosh(x) - tanh(x)*1i,x)
```

```
[Out] x*1i - log(exp(x) - 1i)*2i
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (-i\tanh (x) + \operatorname{sech} (x))\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)-I*tanh(x),x)
```

```
[Out] Integral(-I*tanh(x) + sech(x), x)
```

3.639 $\int \frac{1}{\mathbf{sech}(x)-i\tanh(x)}\,dx$

Optimal. Leaf size=11

$$i\log(\sinh(x)+i)$$

[Out] I*ln(I+sinh(x))

Rubi [A] time = 0.03, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {3159, 2667, 31}

$$i\log(\sinh(x)+i)$$

Antiderivative was successfully verified.

[In] Int[(Sech[x] - I*Tanh[x])^(-1),x]

[Out] I*Log[I + Sinh[x]]

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 3159

```
Int[((a_.) + (b_.)*sec[(d_.) + (e_.)*(x_)] + (c_.)*tan[(d_.) + (e_.)*(x_)])
^(-1), x_Symbol] :> Int[Cos[d + e*x]/(b + a*Cos[d + e*x] + c*Sin[d + e*x]),
 x] /; FreeQ[{a, b, c, d, e}, x]
```

Rubi steps

$$\int \frac{1}{\operatorname{sech}(x) - i\tanh(x)}\,dx = \int \frac{\cosh(x)}{1 - i\sinh(x)}\,dx$$
$$= i\,\text{Subst}\left(\int \frac{1}{1+x}\,dx, x, -i\sinh(x)\right)$$
$$= i\log(i + \sinh(x))$$

Mathematica [A] time = 0.02, size = 17, normalized size = 1.55

$$2\tan^{-1}\left(\tanh\left(\frac{x}{2}\right)\right) + i\log(\cosh(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sech[x] - I*Tanh[x])^(-1),x]
[Out] 2*ArcTan[Tanh[x/2]] + I*Log[Cosh[x]]
```

fricas [A] time = 0.44, size = 11, normalized size = 1.00

$$-i\,x + 2i\,\log\left(e^{x} + i\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sech(x)-I*tanh(x)),x, algorithm="fricas")
[Out] -I*x + 2*I*log(e^x + I)
```

giac [A] time = 0.12, size = 13, normalized size = 1.18

$$-i\,x + 2i\,\log\left(-i\,e^{x} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sech(x)-I*tanh(x)),x, algorithm="giac")
[Out] -I*x + 2*I*log(-I*e^x + 1)
```

maple [B] time = 0.23, size = 33, normalized size = 3.00

$$-i\ln\left(\tanh\left(\frac{x}{2}\right)-1\right) - i\ln\left(\tanh\left(\frac{x}{2}\right)+1\right) + 2i\ln\left(\tanh\left(\frac{x}{2}\right)+i\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sech(x)-I*tanh(x)),x)
```

```
[Out] -I*ln(tanh(1/2*x)-1)-I*ln(tanh(1/2*x)+1)+2*I*ln(tanh(1/2*x)+I)
```

maxima [B] time = 0.32, size = 15, normalized size = 1.36

$$ix + 2i \log\left(ie^{(-x)} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sech(x)-I*tanh(x)),x, algorithm="maxima")
[Out] I*x + 2*I*log(I*e^(-x) + 1)
```

mupad [B] time = 0.13, size = 14, normalized size = 1.27

$$-x\,1\mathrm{i} + \ln\left(\mathrm{e}^x + 1\mathrm{i}\right)\,2\mathrm{i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/(tanh(x)*1i - 1/cosh(x)),x)
[Out] log(exp(x) + 1i)*2i - x*1i
```

sympy [B] time = 0.32, size = 22, normalized size = 2.00

$$ix + i\log\left(-i\tanh\left(x\right) + \operatorname{sech}\left(x\right)\right) - i\log\left(\tanh\left(x\right) + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sech(x)-I*tanh(x)),x)
[Out] I*x + I*log(-I*tanh(x) + sech(x)) - I*log(tanh(x) + 1)
```

3.640 $\int \frac{1}{(\textbf{sech}(x)-i\tanh(x))^2}\,dx$

Optimal. Leaf size=20

$$-x-\frac{2i\cosh(x)}{1-i\sinh(x)}$$

```
[Out] -x-2*I*cosh(x)/(1-I*sinh(x))
```

Rubi [A] time = 0.05, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {4391, 2680, 8}

$$-x-\frac{2i\cosh(x)}{1-i\sinh(x)}$$

Antiderivative was successfully verified.

```
[In] Int[(Sech[x] - I*Tanh[x])^(-2),x]
```

```
[Out] -x - ((2*I)*Cosh[x])/(1 - I*Sinh[x])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2680

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(2*g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f
*x])^(m + 1))/(b*f*(2*m + p + 1)), x] + Dist[(g^2*(p - 1))/(b^2*(2*m + p +
1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 4391

```
Int[(u_.)*((b_.)*sec[(c_.) + (d_.)*(x_)]^(n_.) + (a_.)*tan[(c_.) + (d_.)*(x
_)]^(n_.))^(p_), x_Symbol] :> Int[ActivateTrig[u]*Sec[c + d*x]^(n*p)*(b + a
*Sin[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

Rubi steps

$$\int \frac{1}{(\operatorname{sech}(x) - i\tanh(x))^2}\,dx = \int \frac{\cosh^2(x)}{(1 - i\sinh(x))^2}\,dx$$
$$= -\frac{2i\cosh(x)}{1 - i\sinh(x)} - \int 1\,dx$$
$$= -x - \frac{2i\cosh(x)}{1 - i\sinh(x)}$$

Mathematica [A] time = 0.03, size = 31, normalized size = 1.55

$$-x + \frac{4\sinh\left(\frac{x}{2}\right)}{\cosh\left(\frac{x}{2}\right) - i\sinh\left(\frac{x}{2}\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sech[x] - I*Tanh[x])^(-2),x]
[Out] -x + (4*Sinh[x/2])/(Cosh[x/2] - I*Sinh[x/2])
```

fricas [A] time = 0.42, size = 17, normalized size = 0.85

$$-\frac{xe^x + ix + 4i}{e^x + i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sech(x)-I*tanh(x))^2,x, algorithm="fricas")
[Out] -(x*e^x + I*x + 4*I)/(e^x + I)
```

giac [A] time = 0.12, size = 12, normalized size = 0.60

$$-x - \frac{4i}{e^x + i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sech(x)-I*tanh(x))^2,x, algorithm="giac")
[Out] -x - 4*I/(e^x + I)
```

maple [A] time = 0.29, size = 29, normalized size = 1.45

$$\frac{4}{\tanh\left(\frac{x}{2}\right) + i} + \ln\left(\tanh\left(\frac{x}{2}\right) - 1\right) - \ln\left(\tanh\left(\frac{x}{2}\right) + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sech(x)-I*tanh(x))^2,x)
[Out] 4/(tanh(1/2*x)+I)+ln(tanh(1/2*x)-1)-ln(tanh(1/2*x)+1)
```

maxima [A] time = 0.50, size = 14, normalized size = 0.70

$$-x - \frac{4i}{e^{(-x)} - i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sech(x)-I*tanh(x))^2,x, algorithm="maxima")
[Out] -x - 4*I/(e^(-x) - I)
```

mupad [B] time = 1.51, size = 14, normalized size = 0.70

$$-x - \frac{4\mathrm{i}}{\mathrm{e}^{x} + 1\mathrm{i}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(tanh(x)*1i - 1/cosh(x))^2,x)
[Out] - x - 4i/(exp(x) + 1i)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-i \tanh{\left(x \right)} + \operatorname{sech}{\left(x \right)}\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sech(x)-I*tanh(x))**2,x)
[Out] Integral((-I*tanh(x) + sech(x))**(-2), x)
```

3.641 $\int \frac{1}{(\mathbf{sech}(x)-i\tanh(x))^3}\,dx$

Optimal. Leaf size=26

$$-\frac{2i}{1-i\sinh(x)}-i\log(\sinh(x)+i)$$

[Out] -I*ln(I+sinh(x))-2*I/(1-I*sinh(x))

Rubi [A] time = 0.05, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {4391, 2667, 43}

$$-\frac{2i}{1-i\sinh(x)}-i\log(\sinh(x)+i)$$

Antiderivative was successfully verified.

[In] Int[(Sech[x] - I*Tanh[x])^(-3),x]

[Out] (-I)*Log[I + Sinh[x]] - (2*I)/(1 - I*Sinh[x])

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 4391

```
Int[(u_.)*((b_.)*sec[(c_.) + (d_.)*(x_)]^(n_.) + (a_.)*tan[(c_.) + (d_.)*(x
_)]^(n_.))^(p_), x_Symbol] :> Int[ActivateTrig[u]*Sec[c + d*x]^(n*p)*(b + a
*Sin[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

Rubi steps

$$\int \frac{1}{(\operatorname{sech}(x)-i\tanh(x))^3}\,dx=\int \frac{\cosh^3(x)}{(1-i\sinh(x))^3}\,dx$$
$$=i\,\text{Subst}\left(\int \frac{1-x}{(1+x)^2}\,dx,x,-i\sinh(x)\right)$$
$$=i\,\text{Subst}\left(\int \left(\frac{1}{-1-x}+\frac{2}{(1+x)^2}\right)dx,x,-i\sinh(x)\right)$$
$$=-i\log(i+\sinh(x))-\frac{2i}{1-i\sinh(x)}$$

Mathematica [A] time = 0.04, size = 27, normalized size = 1.04

$$\frac{2}{\sinh(x)+i}-2\tan^{-1}\left(\tanh\left(\frac{x}{2}\right)\right)-i\log(\cosh(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sech[x] - I*Tanh[x])^(-3),x]
[Out] -2*ArcTan[Tanh[x/2]] - I*Log[Cosh[x]] + 2/(I + Sinh[x])
```

fricas [B] time = 0.45, size = 49, normalized size = 1.88

$$\frac{i\,xe^{(2\,x)}-2\,(x-2)e^{x}+\left(-2i\,e^{(2\,x)}+4\,e^{x}+2i\right)\log\left(e^{x}+i\right)-i\,x}{e^{(2\,x)}+2i\,e^{x}-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sech(x)-I*tanh(x))^3,x, algorithm="fricas")
[Out] (I*x*e^(2*x) - 2*(x - 2)*e^x + (-2*I*e^(2*x) + 4*e^x + 2*I)*log(e^x + I) -
I*x)/(e^(2*x) + 2*I*e^x - 1)
```

giac [A] time = 0.14, size = 27, normalized size = 1.04

$$\frac{4\,e^{x}}{\left(e^{x}+i\right)^{2}}+i\,\log\left(-i\,e^{x}\right)-2i\,\log\left(i\,e^{x}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sech(x)-I*tanh(x))^3,x, algorithm="giac")
[Out] 4*e^x/(e^x + I)^2 + I*log(-I*e^x) - 2*I*log(I*e^x - 1)
```

maple [B] time = 0.41, size = 56, normalized size = 2.15

$$i\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)+i\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)+\frac{4i}{\left(\tanh\left(\frac{x}{2}\right)+i\right)^{2}}-2i\ln\left(\tanh\left(\frac{x}{2}\right)+i\right)-\frac{4}{\tanh\left(\frac{x}{2}\right)+i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sech(x)-I*tanh(x))^3,x)
```

```
[Out] I*ln(tanh(1/2*x)-1)+I*ln(tanh(1/2*x)+1)+4*I/(tanh(1/2*x)+I)^2-2*I*ln(tanh(1
/2*x)+I)-4/(tanh(1/2*x)+I)
```

maxima [A] time = 0.56, size = 33, normalized size = 1.27

$$-i\,x-\frac{4\,e^{(-x)}}{-2i\,e^{(-x)}+e^{(-2\,x)}-1}-2i\,\log\left(e^{(-x)}-i\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sech(x)-I*tanh(x))^3,x, algorithm="maxima")
```

```
[Out] -I*x - 4*e^(-x)/(-2*I*e^(-x) + e^(-2*x) - 1) - 2*I*log(e^(-x) - I)
```

mupad [B] time = 0.17, size = 39, normalized size = 1.50

$$x\,1\mathrm{i}-\ln\left(\mathrm{e}^{x}+1\mathrm{i}\right)\,2\mathrm{i}-\frac{4\mathrm{i}}{\mathrm{e}^{2\,x}-1+\mathrm{e}^{x}\,2\mathrm{i}}+\frac{4}{\mathrm{e}^{x}+1\mathrm{i}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/(tanh(x)*1i - 1/cosh(x))^3,x)
```

```
[Out] x*1i - log(exp(x) + 1i)*2i - 4i/(exp(2*x) + exp(x)*2i - 1) + 4/(exp(x) + 1i
)
```

sympy [B] time = 1.79, size = 432, normalized size = 16.62

$$\frac{2ix\tanh^{2}(x)}{-2\tanh^{2}(x)-4i\tanh(x)\operatorname{sech}(x)+2\operatorname{sech}^{2}(x)}-\frac{4x\tanh(x)\operatorname{sech}(x)}{-2\tanh^{2}(x)-4i\tanh(x)\operatorname{sech}(x)+2\operatorname{sech}^{2}(x)}-\frac{}{-2\tanh^{2}(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sech(x)-I*tanh(x))**3,x)
```

```
[Out] 2*I*x*tanh(x)**2/(-2*tanh(x)**2 - 4*I*tanh(x)*sech(x) + 2*sech(x)**2) - 4*x
*tanh(x)*sech(x)/(-2*tanh(x)**2 - 4*I*tanh(x)*sech(x) + 2*sech(x)**2) - 2*I
*x*sech(x)**2/(-2*tanh(x)**2 - 4*I*tanh(x)*sech(x) + 2*sech(x)**2) + 2*I*lo
```

```
g(-I*tanh(x) + sech(x))*tanh(x)**2/(-2*tanh(x)**2 - 4*I*tanh(x)*sech(x) + 2
*sech(x)**2) - 4*log(-I*tanh(x) + sech(x))*tanh(x)*sech(x)/(-2*tanh(x)**2 -
 4*I*tanh(x)*sech(x) + 2*sech(x)**2) - 2*I*log(-I*tanh(x) + sech(x))*sech(x
)**2/(-2*tanh(x)**2 - 4*I*tanh(x)*sech(x) + 2*sech(x)**2) - 2*I*log(tanh(x)
 + 1)*tanh(x)**2/(-2*tanh(x)**2 - 4*I*tanh(x)*sech(x) + 2*sech(x)**2) + 4*l
og(tanh(x) + 1)*tanh(x)*sech(x)/(-2*tanh(x)**2 - 4*I*tanh(x)*sech(x) + 2*se
ch(x)**2) + 2*I*log(tanh(x) + 1)*sech(x)**2/(-2*tanh(x)**2 - 4*I*tanh(x)*se
ch(x) + 2*sech(x)**2) - I*tanh(x)**2/(-2*tanh(x)**2 - 4*I*tanh(x)*sech(x) +
 2*sech(x)**2) - I*sech(x)**2/(-2*tanh(x)**2 - 4*I*tanh(x)*sech(x) + 2*sech
(x)**2) - I/(-2*tanh(x)**2 - 4*I*tanh(x)*sech(x) + 2*sech(x)**2)
```

3.642 $\int \frac{1}{(\mathbf{sech}(x) - i\tanh(x))^4}\,dx$

Optimal. Leaf size=38

$$x - \frac{2i\cosh^3(x)}{3(1 - i\sinh(x))^3} + \frac{2i\cosh(x)}{1 - i\sinh(x)}$$

```
[Out] x-2/3*I*cosh(x)^3/(1-I*sinh(x))^3+2*I*cosh(x)/(1-I*sinh(x))
```

Rubi [A] time = 0.08, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {4391, 2680, 8}

$$x - \frac{2i\cosh^3(x)}{3(1 - i\sinh(x))^3} + \frac{2i\cosh(x)}{1 - i\sinh(x)}$$

Antiderivative was successfully verified.

```
[In] Int[(Sech[x] - I*Tanh[x])^(-4),x]
```

```
[Out] x - (((2*I)/3)*Cosh[x]^3)/(1 - I*Sinh[x])^3 + ((2*I)*Cosh[x])/(1 - I*Sinh[x
])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2680

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(2*g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f
*x])^(m + 1))/(b*f*(2*m + p + 1)), x] + Dist[(g^2*(p - 1))/(b^2*(2*m + p +
1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 4391

```
Int[(u_.)*((b_.)*sec[(c_.) + (d_.)*(x_)]^(n_.) + (a_.)*tan[(c_.) + (d_.)*(x
_)]^(n_.))^(p_), x_Symbol] :> Int[ActivateTrig[u]*Sec[c + d*x]^(n*p)*(b + a
*Sin[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

Rubi steps

$$\int \frac{1}{(\operatorname{sech}(x) - i\tanh(x))^4}\,dx = \int \frac{\cosh^4(x)}{(1 - i\sinh(x))^4}\,dx$$
$$= -\frac{2i\cosh^3(x)}{3(1 - i\sinh(x))^3} - \int \frac{\cosh^2(x)}{(1 - i\sinh(x))^2}\,dx$$
$$= -\frac{2i\cosh^3(x)}{3(1 - i\sinh(x))^3} + \frac{2i\cosh(x)}{1 - i\sinh(x)} + \int 1\,dx$$
$$= x - \frac{2i\cosh^3(x)}{3(1 - i\sinh(x))^3} + \frac{2i\cosh(x)}{1 - i\sinh(x)}$$

Mathematica [A] time = 0.07, size = 74, normalized size = 1.95

$$\frac{3(3x - 8i)\cosh\left(\frac{x}{2}\right) + (-3x + 16i)\cosh\left(\frac{3x}{2}\right) - 6i\sinh\left(\frac{x}{2}\right)(2x + x\cosh(x) - 4i)}{6\left(\cosh\left(\frac{x}{2}\right) - i\sinh\left(\frac{x}{2}\right)\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sech[x] - I*Tanh[x])^(-4),x]
```

```
[Out] (3*(-8*I + 3*x)*Cosh[x/2] + (16*I - 3*x)*Cosh[(3*x)/2] - (6*I)*(-4*I + 2*x
+ x*Cosh[x])*Sinh[x/2])/(6*(Cosh[x/2] - I*Sinh[x/2])^3)
```

fricas [A] time = 0.42, size = 52, normalized size = 1.37

$$\frac{3\,xe^{(3\,x)} + (9i\,x + 24i)e^{(2\,x)} - 3\,(3\,x + 8)e^{x} - 3i\,x - 16i}{3\,e^{(3\,x)} + 9i\,e^{(2\,x)} - 9\,e^{x} - 3i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sech(x)-I*tanh(x))^4,x, algorithm="fricas")
```

```
[Out] (3*x*e^(3*x) + (9*I*x + 24*I)*e^(2*x) - 3*(3*x + 8)*e^x - 3*I*x - 16*I)/(3*
e^(3*x) + 9*I*e^(2*x) - 9*e^x - 3*I)
```

giac [A] time = 0.14, size = 22, normalized size = 0.58

$$x - \frac{-24i\,e^{(2\,x)} + 24\,e^{x} + 16i}{3\,(e^{x} + i)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sech(x)-I*tanh(x))^4,x, algorithm="giac")
```

```
[Out] x - 1/3*(-24*I*e^(2*x) + 24*e^x + 16*I)/(e^x + I)^3
```

maple [A] time = 0.40, size = 41, normalized size = 1.08

$$-\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)+\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)-\frac{8i}{\left(\tanh\left(\frac{x}{2}\right)+i\right)^{2}}-\frac{16}{3\left(\tanh\left(\frac{x}{2}\right)+i\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sech(x)-I*tanh(x))^4,x)
```

```
[Out] -ln(tanh(1/2*x)-1)+ln(tanh(1/2*x)+1)-8*I/(tanh(1/2*x)+I)^2-16/3/(tanh(1/2*x
)+I)^3
```

maxima [A] time = 0.33, size = 40, normalized size = 1.05

$$x-\frac{24\,e^{(-x)}+24i\,e^{(-2\,x)}-16i}{9\,e^{(-x)}+9i\,e^{(-2\,x)}-3\,e^{(-3\,x)}-3i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sech(x)-I*tanh(x))^4,x, algorithm="maxima")
```

```
[Out] x - (24*e^(-x) + 24*I*e^(-2*x) - 16*I)/(9*e^(-x) + 9*I*e^(-2*x) - 3*e^(-3*x
) - 3*I)
```

mupad [B] time = 1.61, size = 65, normalized size = 1.71

$$x+\frac{\frac{\mathrm{e}^{2\,x}\,8\mathrm{i}}{3}-\frac{8}{3}\mathrm{i}}{\mathrm{e}^{2\,x}\,3\mathrm{i}+\mathrm{e}^{3\,x}-3\,\mathrm{e}^{x}-\mathrm{i}}+\frac{\mathrm{e}^{x}\,8\mathrm{i}}{3\,\left(\mathrm{e}^{2\,x}-1+\mathrm{e}^{x}\,2\mathrm{i}\right)}+\frac{8\mathrm{i}}{3\,\left(\mathrm{e}^{x}+1\mathrm{i}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(tanh(x)*1i - 1/cosh(x))^4,x)
```

```
[Out] x + ((exp(2*x)*8i)/3 - 8i/3)/(exp(2*x)*3i + exp(3*x) - 3*exp(x) - 1i) + (ex
p(x)*8i)/(3*(exp(2*x) + exp(x)*2i - 1)) + 8i/(3*(exp(x) + 1i))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\left(-i\tanh\left(x\right)+\operatorname{sech}\left(x\right)\right)^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sech(x)-I*tanh(x))**4,x)
```

```
[Out] Integral((-I*tanh(x) + sech(x))**(-4), x)
```

3.643 $\int \frac{1}{(\mathbf{sech}(x) - i\tanh(x))^5}\,dx$

Optimal. Leaf size=40

$$\frac{4i}{1 - i\sinh(x)} - \frac{2i}{(1 - i\sinh(x))^2} + i\log(\sinh(x) + i)$$

```
[Out] I*ln(I+sinh(x))-2*I/(1-I*sinh(x))^2+4*I/(1-I*sinh(x))
```

Rubi [A] time = 0.06, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {4391, 2667, 43}

$$\frac{4i}{1 - i\sinh(x)} - \frac{2i}{(1 - i\sinh(x))^2} + i\log(\sinh(x) + i)$$

Antiderivative was successfully verified.

```
[In] Int[(Sech[x] - I*Tanh[x])^(-5),x]
[Out] I*Log[I + Sinh[x]] - (2*I)/(1 - I*Sinh[x])^2 + (4*I)/(1 - I*Sinh[x])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 4391

```
Int[(u_.)*((b_.)*sec[(c_.) + (d_.)*(x_)]^(n_.) + (a_.)*tan[(c_.) + (d_.)*(x
_)]^(n_.))^(p_), x_Symbol] :> Int[ActivateTrig[u]*Sec[c + d*x]^(n*p)*(b + a
*Sin[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

Rubi steps

$$\int \frac{1}{(\mathrm{sech}(x)-i\tanh(x))^5}\,dx = \int \frac{\cosh^5(x)}{(1-i\sinh(x))^5}\,dx$$
$$= i\,\mathrm{Subst}\left(\int \frac{(1-x)^2}{(1+x)^3}\,dx, x, -i\sinh(x)\right)$$
$$= i\,\mathrm{Subst}\left(\int \left(\frac{4}{(1+x)^3}-\frac{4}{(1+x)^2}+\frac{1}{1+x}\right)dx, x, -i\sinh(x)\right)$$
$$= i\log(i+\sinh(x)) - \frac{2i}{(1-i\sinh(x))^2} + \frac{4i}{1-i\sinh(x)}$$

Mathematica [A] time = 0.10, size = 45, normalized size = 1.12

$$2\tan^{-1}\left(\tanh\left(\frac{x}{2}\right)\right) + i\log(\cosh(x)) + \frac{4\sinh(x)+2i}{\left(\cosh\left(\frac{x}{2}\right)-i\sinh\left(\frac{x}{2}\right)\right)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sech[x] - I*Tanh[x])^(-5),x]
```

```
[Out] 2*ArcTan[Tanh[x/2]] + I*Log[Cosh[x]] + (2*I + 4*Sinh[x])/(Cosh[x/2] - I*Sin
h[x/2])^4
```

fricas [B] time = 0.43, size = 92, normalized size = 2.30

$$\frac{-i\,xe^{(4\,x)} + 4\,(x-2)e^{(3\,x)} + (6i\,x-8i)e^{(2\,x)} - 4\,(x-2)e^{x} + \left(2i\,e^{(4\,x)} - 8\,e^{(3\,x)} - 12i\,e^{(2\,x)} + 8\,e^{x} + 2i\right)\log\left(e^{x}+i\right) - i\,x}{e^{(4\,x)} + 4i\,e^{(3\,x)} - 6\,e^{(2\,x)} - 4i\,e^{x} + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sech(x)-I*tanh(x))^5,x, algorithm="fricas")
```

```
[Out] (-I*x*e^(4*x) + 4*(x - 2)*e^(3*x) + (6*I*x - 8*I)*e^(2*x) - 4*(x - 2)*e^x +
 (2*I*e^(4*x) - 8*e^(3*x) - 12*I*e^(2*x) + 8*e^x + 2*I)*log(e^x + I) - I*x)
/(e^(4*x) + 4*I*e^(3*x) - 6*e^(2*x) - 4*I*e^x + 1)
```

giac [A] time = 0.12, size = 38, normalized size = 0.95

$$-\frac{8\left(e^{(3\,x)} + i\,e^{(2\,x)} - e^{x}\right)}{\left(e^{x}+i\right)^4} - i\,\log\left(-i\,e^{x}\right) + 2i\,\log\left(e^{x}+i\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sech(x)-I*tanh(x))^5,x, algorithm="giac")
```

```
[Out] -8*(e^(3*x) + I*e^(2*x) - e^x)/(e^x + I)^4 - I*log(-I*e^x) + 2*I*log(e^x +
I)
```

maple [A] time = 0.40, size = 68, normalized size = 1.70

$$-i\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)-i\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)+\frac{8i}{\left(\tanh\left(\frac{x}{2}\right)+i\right)^{2}}+2i\ln\left(\tanh\left(\frac{x}{2}\right)+i\right)-\frac{8i}{\left(\tanh\left(\frac{x}{2}\right)+i\right)^{4}}+\frac{16}{\left(\tanh\left(\frac{x}{2}\right)+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sech(x)-I*tanh(x))^5,x)
```

```
[Out] -I*ln(tanh(1/2*x)-1)-I*ln(tanh(1/2*x)+1)+8*I/(tanh(1/2*x)+I)^2+2*I*ln(tanh(
1/2*x)+I)-8*I/(tanh(1/2*x)+I)^4+16/(tanh(1/2*x)+I)^3
```

maxima [B] time = 0.37, size = 60, normalized size = 1.50

$$i\,x-\frac{8\,e^{(-x)}+8i\,e^{(-2\,x)}-8\,e^{(-3\,x)}}{4i\,e^{(-x)}-6\,e^{(-2\,x)}-4i\,e^{(-3\,x)}+e^{(-4\,x)}+1}+2i\,\log\left(e^{(-x)}-i\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sech(x)-I*tanh(x))^5,x, algorithm="maxima")
```

```
[Out] I*x - (8*e^(-x) + 8*I*e^(-2*x) - 8*e^(-3*x))/(4*I*e^(-x) - 6*e^(-2*x) - 4*I
*e^(-3*x) + e^(-4*x) + 1) + 2*I*log(e^(-x) - I)
```

mupad [B] time = 1.69, size = 90, normalized size = 2.25

$$-x\,1\mathrm{i}+\ln\left(\mathrm{e}^{x}+1\mathrm{i}\right)\,2\mathrm{i}+\frac{16\mathrm{i}}{\mathrm{e}^{2\,x}-1+\mathrm{e}^{x}\,2\mathrm{i}}-\frac{8\mathrm{i}}{\mathrm{e}^{4\,x}-6\,\mathrm{e}^{2\,x}+1+\mathrm{e}^{3\,x}\,4\mathrm{i}-\mathrm{e}^{x}\,4\mathrm{i}}-\frac{8}{\mathrm{e}^{x}+1\mathrm{i}}+\frac{16}{\mathrm{e}^{2\,x}\,3\mathrm{i}+\mathrm{e}^{3\,x}-3\,\mathrm{e}^{x}-\mathrm{i}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/(tanh(x)*1i - 1/cosh(x))^5,x)
```

```
[Out] log(exp(x) + 1i)*2i - x*1i + 16i/(exp(2*x) + exp(x)*2i - 1) - 8i/(exp(3*x)*
4i - 6*exp(2*x) + exp(4*x) - exp(x)*4i + 1) - 8/(exp(x) + 1i) + 16/(exp(2*x
)*3i + exp(3*x) - 3*exp(x) - 1i)
```

sympy [B] time = 8.39, size = 1445, normalized size = 36.12

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sech(x)-I*tanh(x))**5,x)

[Out] 36*I*x*tanh(x)**4/(36*tanh(x)**4 + 144*I*tanh(x)**3*sech(x) - 216*tanh(x)**
2*sech(x)**2 - 144*I*tanh(x)*sech(x)**3 + 36*sech(x)**4) - 144*x*tanh(x)**3
*sech(x)/(36*tanh(x)**4 + 144*I*tanh(x)**3*sech(x) - 216*tanh(x)**2*sech(x)
**2 - 144*I*tanh(x)*sech(x)**3 + 36*sech(x)**4) - 216*I*x*tanh(x)**2*sech(x
)**2/(36*tanh(x)**4 + 144*I*tanh(x)**3*sech(x) - 216*tanh(x)**2*sech(x)**2
- 144*I*tanh(x)*sech(x)**3 + 36*sech(x)**4) + 144*x*tanh(x)*sech(x)**3/(36*
tanh(x)**4 + 144*I*tanh(x)**3*sech(x) - 216*tanh(x)**2*sech(x)**2 - 144*I*t
anh(x)*sech(x)**3 + 36*sech(x)**4) + 36*I*x*sech(x)**4/(36*tanh(x)**4 + 144
*I*tanh(x)**3*sech(x) - 216*tanh(x)**2*sech(x)**2 - 144*I*tanh(x)*sech(x)**
3 + 36*sech(x)**4) + 36*I*log(-I*tanh(x) + sech(x))*tanh(x)**4/(36*tanh(x)*
*4 + 144*I*tanh(x)**3*sech(x) - 216*tanh(x)**2*sech(x)**2 - 144*I*tanh(x)*s
ech(x)**3 + 36*sech(x)**4) - 144*log(-I*tanh(x) + sech(x))*tanh(x)**3*sech(
x)/(36*tanh(x)**4 + 144*I*tanh(x)**3*sech(x) - 216*tanh(x)**2*sech(x)**2 -
144*I*tanh(x)*sech(x)**3 + 36*sech(x)**4) - 216*I*log(-I*tanh(x) + sech(x))
*tanh(x)**2*sech(x)**2/(36*tanh(x)**4 + 144*I*tanh(x)**3*sech(x) - 216*tanh
(x)**2*sech(x)**2 - 144*I*tanh(x)*sech(x)**3 + 36*sech(x)**4) + 144*log(-I*
tanh(x) + sech(x))*tanh(x)*sech(x)**3/(36*tanh(x)**4 + 144*I*tanh(x)**3*sec
h(x) - 216*tanh(x)**2*sech(x)**2 - 144*I*tanh(x)*sech(x)**3 + 36*sech(x)**4
) + 36*I*log(-I*tanh(x) + sech(x))*sech(x)**4/(36*tanh(x)**4 + 144*I*tanh(x
)**3*sech(x) - 216*tanh(x)**2*sech(x)**2 - 144*I*tanh(x)*sech(x)**3 + 36*se
ch(x)**4) - 36*I*log(tanh(x) + 1)*tanh(x)**4/(36*tanh(x)**4 + 144*I*tanh(x)
**3*sech(x) - 216*tanh(x)**2*sech(x)**2 - 144*I*tanh(x)*sech(x)**3 + 36*sec
h(x)**4) + 144*log(tanh(x) + 1)*tanh(x)**3*sech(x)/(36*tanh(x)**4 + 144*I*t
anh(x)**3*sech(x) - 216*tanh(x)**2*sech(x)**2 - 144*I*tanh(x)*sech(x)**3 +
36*sech(x)**4) + 216*I*log(tanh(x) + 1)*tanh(x)**2*sech(x)**2/(36*tanh(x)**
4 + 144*I*tanh(x)**3*sech(x) - 216*tanh(x)**2*sech(x)**2 - 144*I*tanh(x)*se
ch(x)**3 + 36*sech(x)**4) - 144*log(tanh(x) + 1)*tanh(x)*sech(x)**3/(36*tan
h(x)**4 + 144*I*tanh(x)**3*sech(x) - 216*tanh(x)**2*sech(x)**2 - 144*I*tanh
(x)*sech(x)**3 + 36*sech(x)**4) - 36*I*log(tanh(x) + 1)*sech(x)**4/(36*tanh
(x)**4 + 144*I*tanh(x)**3*sech(x) - 216*tanh(x)**2*sech(x)**2 - 144*I*tanh(
x)*sech(x)**3 + 36*sech(x)**4) - 28*I*tanh(x)**4/(36*tanh(x)**4 + 144*I*tan
h(x)**3*sech(x) - 216*tanh(x)**2*sech(x)**2 - 144*I*tanh(x)*sech(x)**3 + 36
*sech(x)**4) + 52*tanh(x)**3*sech(x)/(36*tanh(x)**4 + 144*I*tanh(x)**3*sech
(x) - 216*tanh(x)**2*sech(x)**2 - 144*I*tanh(x)*sech(x)**3 + 36*sech(x)**4)
 - 18*I*tanh(x)**2/(36*tanh(x)**4 + 144*I*tanh(x)**3*sech(x) - 216*tanh(x)*
*2*sech(x)**2 - 144*I*tanh(x)*sech(x)**3 + 36*sech(x)**4) + 44*tanh(x)*sech
(x)**3/(36*tanh(x)**4 + 144*I*tanh(x)**3*sech(x) - 216*tanh(x)**2*sech(x)**
2 - 144*I*tanh(x)*sech(x)**3 + 36*sech(x)**4) + 24*tanh(x)*sech(x)/(36*tanh
(x)**4 + 144*I*tanh(x)**3*sech(x) - 216*tanh(x)**2*sech(x)**2 - 144*I*tanh(
x)*sech(x)**3 + 36*sech(x)**4) + 20*I*sech(x)**4/(36*tanh(x)**4 + 144*I*tan
h(x)**3*sech(x) - 216*tanh(x)**2*sech(x)**2 - 144*I*tanh(x)*sech(x)**3 + 36
*sech(x)**4) + 6*I*sech(x)**2/(36*tanh(x)**4 + 144*I*tanh(x)**3*sech(x) - 2
16*tanh(x)**2*sech(x)**2 - 144*I*tanh(x)*sech(x)**3 + 36*sech(x)**4) - 9*I/
```

```
(36*tanh(x)**4 + 144*I*tanh(x)**3*sech(x) - 216*tanh(x)**2*sech(x)**2 - 144
*I*tanh(x)*sech(x)**3 + 36*sech(x)**4)
```

3.644 $\int (a\coth(x) + b\,\mathbf{csch}(x))^5\,dx$

Optimal. Leaf size=124

$$a^5\log(\sinh(x))+\frac{1}{8}a^2b\left(7a^2-3b^2\right)\cosh(x)-\frac{1}{8}\text{csch}^2(x)(a\cosh(x)+b)^2\left(b\left(5a^2-3b^2\right)\cosh(x)+2a\left(2a^2-b^2\right)\right)-\frac{1}{8}b$$

```
[Out] -1/8*b*(15*a^4-10*a^2*b^2+3*b^4)*arctanh(cosh(x))+1/8*a^2*b*(7*a^2-3*b^2)*c
osh(x)-1/8*(b+a*cosh(x))^2*(2*a*(2*a^2-b^2)+b*(5*a^2-3*b^2)*cosh(x))*csch(x
)^2-1/4*(b+a*cosh(x))^4*(a+b*cosh(x))*csch(x)^4+a^5*ln(sinh(x))
```

Rubi [A] time = 0.24, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.727, Rules used = {4392, 2668, 739, 819, 774, 635, 204, 260}

$$\frac{1}{8}a^2b\left(7a^2-3b^2\right)\cosh(x)-\frac{1}{8}b\left(-10a^2b^2+15a^4+3b^4\right)\tanh^{-1}(\cosh(x))-\frac{1}{8}\text{csch}^2(x)(a\cosh(x)+b)^2\left(b\left(5a^2-3b^2\right)\right.$$

Antiderivative was successfully verified.

```
[In] Int[(a*Coth[x] + b*Csch[x])^5,x]
```

```
[Out] -(b*(15*a^4 - 10*a^2*b^2 + 3*b^4)*ArcTanh[Cosh[x]])/8 + (a^2*b*(7*a^2 - 3*b
^2)*Cosh[x])/8 - ((b + a*Cosh[x])^2*(2*a*(2*a^2 - b^2) + b*(5*a^2 - 3*b^2)*
Cosh[x])*Csch[x]^2)/8 - ((b + a*Cosh[x])^4*(a + b*Cosh[x])*Csch[x]^4)/4 + a
^5*Log[Sinh[x]]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] && !NiceSqrtQ[-(a*c)]
```

Rule 739

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m - 1)*(a*e - c*d*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)), x] +
 Dist[1/((p + 1)*(-2*a*c)), Int[(d + e*x)^(m - 2)*Simp[a*e^2*(m - 1) - c*d^
2*(2*p + 3) - d*c*e*(m + 2*p + 2)*x, x]*(a + c*x^2)^(p + 1), x], x] /; Free
Q[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && I
ntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 774

```
Int[(((d_.) + (e_.)*(x_))*((f_) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2), x_Sym
bol] :> Simp[(e*g*x)/c, x] + Dist[1/c, Int[(c*d*f - a*e*g + c*(e*f + d*g)*x
)/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e, f, g}, x]
```

Rule 819

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m - 1)*(a + c*x^2)^(p + 1)*(a*(e*f + d*g
) - (c*d*f - a*e*g)*x))/(2*a*c*(p + 1)), x] - Dist[1/(2*a*c*(p + 1)), Int[(
d + e*x)^(m - 2)*(a + c*x^2)^(p + 1)*Simp[a*e*(e*f*(m - 1) + d*g*m) - c*d^2
*f*(2*p + 3) + e*(a*e*g*m - c*d*f*(m + 2*p + 2))*x, x], x], x] /; FreeQ[{a,
 c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] &&
(EqQ[d, 0] || (EqQ[m, 2] && EqQ[p, -3] && RationalQ[a, c, d, e, f, g]) ||
!ILtQ[m + 2*p + 3, 0])
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 4392

```
Int[(cot[(c_.) + (d_.)*(x_)]^(n_.)*(a_.) + csc[(c_.) + (d_.)*(x_)]^(n_.)*(b
_.))^(p_)*(u_.), x_Symbol] :> Int[ActivateTrig[u]*Csc[c + d*x]^(n*p)*(b + a
*Cos[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

Rubi steps

$$\int (a\coth(x)+b\text{csch}(x))^5\,dx = -\left(i\int (ib+ia\cosh(x))^5\text{csch}^5(x)\,dx\right)$$
$$= -\left(a^5\,\text{Subst}\left(\int \frac{(ib+x)^5}{\left(-a^2-x^2\right)^3}\,dx, x, ia\cosh(x)\right)\right)$$
$$= -\frac{1}{4}(b+a\cosh(x))^4(a+b\cosh(x))\text{csch}^4(x) - \frac{1}{4}a^3\,\text{Subst}\left(\int \frac{(ib+x)^3\left(-4a^2+3b\right.}{\left(-a^2-x^2\right)^2}\right.$$
$$= -\frac{1}{8}(b+a\cosh(x))^2\left(2a\left(2a^2-b^2\right)+b\left(5a^2-3b^2\right)\cosh(x)\right)\text{csch}^2(x) - \frac{1}{4}(b+a\,\text{co}$$
$$= \frac{1}{8}a^2b\left(7a^2-3b^2\right)\cosh(x) - \frac{1}{8}(b+a\cosh(x))^2\left(2a\left(2a^2-b^2\right)+b\left(5a^2-3b^2\right)\text{cosh}\right.$$
$$= \frac{1}{8}a^2b\left(7a^2-3b^2\right)\cosh(x) - \frac{1}{8}(b+a\cosh(x))^2\left(2a\left(2a^2-b^2\right)+b\left(5a^2-3b^2\right)\text{cosh}\right.$$
$$= -\frac{1}{8}b\left(15a^4-10a^2b^2+3b^4\right)\tanh^{-1}(\cosh(x)) + \frac{1}{8}a^2b\left(7a^2-3b^2\right)\cosh(x) - \frac{1}{8}(b+$$

Mathematica [A] time = 0.47, size = 244, normalized size = 1.97

$$-\frac{1}{64}\text{csch}^4(x)\left(-24a^5\log(\sinh(x)) - 8a^5\cosh(4x)\log(\sinh(x)) - 16a^5 + 50a^4b\cosh(3x) - 45a^4b\log\left(\tanh\left(\frac{x}{2}\right)\right)\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Coth[x] + b*Csch[x])^5,x]
```

```
[Out] -1/64*(Csch[x]^4*(-16*a^5 + 80*a*b^4 + 2*b*(15*a^4 + 70*a^2*b^2 + 11*b^4)*C
osh[x] + 50*a^4*b*Cosh[3*x] + 20*a^2*b^3*Cosh[3*x] - 6*b^5*Cosh[3*x] - 24*a
^5*Log[Sinh[x]] - 8*a^5*Cosh[4*x]*Log[Sinh[x]] - 45*a^4*b*Log[Tanh[x/2]] +
30*a^2*b^3*Log[Tanh[x/2]] - 9*b^5*Log[Tanh[x/2]] - 15*a^4*b*Cosh[4*x]*Log[T
anh[x/2]] + 10*a^2*b^3*Cosh[4*x]*Log[Tanh[x/2]] - 3*b^5*Cosh[4*x]*Log[Tanh[
x/2]] + 4*Cosh[2*x]*(8*(a^5 + 5*a^3*b^2) + 8*a^5*Log[Sinh[x]] + b*(15*a^4 -
 10*a^2*b^2 + 3*b^4)*Log[Tanh[x/2]])))
```

fricas [B] time = 0.47, size = 2716, normalized size = 21.90

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*coth(x)+b*csch(x))^5,x, algorithm="fricas")
```

```
[Out] -1/8*(8*a^5*x*cosh(x)^8 + 8*a^5*x*sinh(x)^8 + 2*(25*a^4*b + 10*a^2*b^3 - 3*
b^5)*cosh(x)^7 + 2*(32*a^5*x*cosh(x) + 25*a^4*b + 10*a^2*b^3 - 3*b^5)*sinh(
x)^7 - 32*(a^5*x - a^5 - 5*a^3*b^2)*cosh(x)^6 + 2*(112*a^5*x*cosh(x)^2 - 16
*a^5*x + 16*a^5 + 80*a^3*b^2 + 7*(25*a^4*b + 10*a^2*b^3 - 3*b^5)*cosh(x))*s
inh(x)^6 + 8*a^5*x + 2*(15*a^4*b + 70*a^2*b^3 + 11*b^5)*cosh(x)^5 + 2*(224*
a^5*x*cosh(x)^3 + 15*a^4*b + 70*a^2*b^3 + 11*b^5 + 21*(25*a^4*b + 10*a^2*b^
3 - 3*b^5)*cosh(x)^2 - 96*(a^5*x - a^5 - 5*a^3*b^2)*cosh(x))*sinh(x)^5 + 16
*(3*a^5*x - 2*a^5 + 10*a*b^4)*cosh(x)^4 + 2*(280*a^5*x*cosh(x)^4 + 24*a^5*x
 - 16*a^5 + 80*a*b^4 + 35*(25*a^4*b + 10*a^2*b^3 - 3*b^5)*cosh(x)^3 - 240*(
a^5*x - a^5 - 5*a^3*b^2)*cosh(x)^2 + 5*(15*a^4*b + 70*a^2*b^3 + 11*b^5)*cos
h(x))*sinh(x)^4 + 2*(15*a^4*b + 70*a^2*b^3 + 11*b^5)*cosh(x)^3 + 2*(224*a^5
*x*cosh(x)^5 + 15*a^4*b + 70*a^2*b^3 + 11*b^5 + 35*(25*a^4*b + 10*a^2*b^3 -
 3*b^5)*cosh(x)^4 - 320*(a^5*x - a^5 - 5*a^3*b^2)*cosh(x)^3 + 10*(15*a^4*b
+ 70*a^2*b^3 + 11*b^5)*cosh(x)^2 + 32*(3*a^5*x - 2*a^5 + 10*a*b^4)*cosh(x))
*sinh(x)^3 - 32*(a^5*x - a^5 - 5*a^3*b^2)*cosh(x)^2 + 2*(112*a^5*x*cosh(x)^
6 - 16*a^5*x + 21*(25*a^4*b + 10*a^2*b^3 - 3*b^5)*cosh(x)^5 + 16*a^5 + 80*a
^3*b^2 - 240*(a^5*x - a^5 - 5*a^3*b^2)*cosh(x)^4 + 10*(15*a^4*b + 70*a^2*b^
3 + 11*b^5)*cosh(x)^3 + 48*(3*a^5*x - 2*a^5 + 10*a*b^4)*cosh(x)^2 + 3*(15*a
^4*b + 70*a^2*b^3 + 11*b^5)*cosh(x))*sinh(x)^2 + 2*(25*a^4*b + 10*a^2*b^3 -
 3*b^5)*cosh(x) - ((8*a^5 - 15*a^4*b + 10*a^2*b^3 - 3*b^5)*cosh(x)^8 + 8*(8
*a^5 - 15*a^4*b + 10*a^2*b^3 - 3*b^5)*cosh(x)*sinh(x)^7 + (8*a^5 - 15*a^4*b
 + 10*a^2*b^3 - 3*b^5)*sinh(x)^8 - 4*(8*a^5 - 15*a^4*b + 10*a^2*b^3 - 3*b^5
)*cosh(x)^6 - 4*(8*a^5 - 15*a^4*b + 10*a^2*b^3 - 3*b^5 - 7*(8*a^5 - 15*a^4*
b + 10*a^2*b^3 - 3*b^5)*cosh(x)^2)*sinh(x)^6 + 8*(7*(8*a^5 - 15*a^4*b + 10*
a^2*b^3 - 3*b^5)*cosh(x)^3 - 3*(8*a^5 - 15*a^4*b + 10*a^2*b^3 - 3*b^5)*cosh
(x))*sinh(x)^5 + 8*a^5 - 15*a^4*b + 10*a^2*b^3 - 3*b^5 + 6*(8*a^5 - 15*a^4*
b + 10*a^2*b^3 - 3*b^5)*cosh(x)^4 + 2*(24*a^5 - 45*a^4*b + 30*a^2*b^3 - 9*b
^5 + 35*(8*a^5 - 15*a^4*b + 10*a^2*b^3 - 3*b^5)*cosh(x)^4 - 30*(8*a^5 - 15*
a^4*b + 10*a^2*b^3 - 3*b^5)*cosh(x)^2)*sinh(x)^4 + 8*(7*(8*a^5 - 15*a^4*b +
 10*a^2*b^3 - 3*b^5)*cosh(x)^5 - 10*(8*a^5 - 15*a^4*b + 10*a^2*b^3 - 3*b^5)
*cosh(x)^3 + 3*(8*a^5 - 15*a^4*b + 10*a^2*b^3 - 3*b^5)*cosh(x))*sinh(x)^3 -
 4*(8*a^5 - 15*a^4*b + 10*a^2*b^3 - 3*b^5)*cosh(x)^2 + 4*(7*(8*a^5 - 15*a^4
*b + 10*a^2*b^3 - 3*b^5)*cosh(x)^6 - 8*a^5 + 15*a^4*b - 10*a^2*b^3 + 3*b^5
- 15*(8*a^5 - 15*a^4*b + 10*a^2*b^3 - 3*b^5)*cosh(x)^4 + 9*(8*a^5 - 15*a^4*
b + 10*a^2*b^3 - 3*b^5)*cosh(x)^2)*sinh(x)^2 + 8*((8*a^5 - 15*a^4*b + 10*a^
2*b^3 - 3*b^5)*cosh(x)^7 - 3*(8*a^5 - 15*a^4*b + 10*a^2*b^3 - 3*b^5)*cosh(x
)^5 + 3*(8*a^5 - 15*a^4*b + 10*a^2*b^3 - 3*b^5)*cosh(x)^3 - (8*a^5 - 15*a^4
*b + 10*a^2*b^3 - 3*b^5)*cosh(x))*sinh(x))*log(cosh(x) + sinh(x) + 1) - ((8
*a^5 + 15*a^4*b - 10*a^2*b^3 + 3*b^5)*cosh(x)^8 + 8*(8*a^5 + 15*a^4*b - 10*
a^2*b^3 + 3*b^5)*cosh(x)*sinh(x)^7 + (8*a^5 + 15*a^4*b - 10*a^2*b^3 + 3*b^5
)*sinh(x)^8 - 4*(8*a^5 + 15*a^4*b - 10*a^2*b^3 + 3*b^5)*cosh(x)^6 - 4*(8*a^
5 + 15*a^4*b - 10*a^2*b^3 + 3*b^5 - 7*(8*a^5 + 15*a^4*b - 10*a^2*b^3 + 3*b^
5)*cosh(x)^2)*sinh(x)^6 + 8*(7*(8*a^5 + 15*a^4*b - 10*a^2*b^3 + 3*b^5)*cosh
(x)^3 - 3*(8*a^5 + 15*a^4*b - 10*a^2*b^3 + 3*b^5)*cosh(x))*sinh(x)^5 + 8*a^
5 + 15*a^4*b - 10*a^2*b^3 + 3*b^5 + 6*(8*a^5 + 15*a^4*b - 10*a^2*b^3 + 3*b^
```

```
5)*cosh(x)^4 + 2*(24*a^5 + 45*a^4*b - 30*a^2*b^3 + 9*b^5 + 35*(8*a^5 + 15*a
^4*b - 10*a^2*b^3 + 3*b^5)*cosh(x)^4 - 30*(8*a^5 + 15*a^4*b - 10*a^2*b^3 +
3*b^5)*cosh(x)^2)*sinh(x)^4 + 8*(7*(8*a^5 + 15*a^4*b - 10*a^2*b^3 + 3*b^5)*
cosh(x)^5 - 10*(8*a^5 + 15*a^4*b - 10*a^2*b^3 + 3*b^5)*cosh(x)^3 + 3*(8*a^5
 + 15*a^4*b - 10*a^2*b^3 + 3*b^5)*cosh(x))*sinh(x)^3 - 4*(8*a^5 + 15*a^4*b
- 10*a^2*b^3 + 3*b^5)*cosh(x)^2 + 4*(7*(8*a^5 + 15*a^4*b - 10*a^2*b^3 + 3*b
^5)*cosh(x)^6 - 8*a^5 - 15*a^4*b + 10*a^2*b^3 - 3*b^5 - 15*(8*a^5 + 15*a^4*
b - 10*a^2*b^3 + 3*b^5)*cosh(x)^4 + 9*(8*a^5 + 15*a^4*b - 10*a^2*b^3 + 3*b^
5)*cosh(x)^2)*sinh(x)^2 + 8*((8*a^5 + 15*a^4*b - 10*a^2*b^3 + 3*b^5)*cosh(x
)^7 - 3*(8*a^5 + 15*a^4*b - 10*a^2*b^3 + 3*b^5)*cosh(x)^5 + 3*(8*a^5 + 15*a
^4*b - 10*a^2*b^3 + 3*b^5)*cosh(x)^3 - (8*a^5 + 15*a^4*b - 10*a^2*b^3 + 3*b
^5)*cosh(x))*sinh(x))*log(cosh(x) + sinh(x) - 1) + 2*(32*a^5*x*cosh(x)^7 +
7*(25*a^4*b + 10*a^2*b^3 - 3*b^5)*cosh(x)^6 - 96*(a^5*x - a^5 - 5*a^3*b^2)*
cosh(x)^5 + 25*a^4*b + 10*a^2*b^3 - 3*b^5 + 5*(15*a^4*b + 70*a^2*b^3 + 11*b
^5)*cosh(x)^4 + 32*(3*a^5*x - 2*a^5 + 10*a*b^4)*cosh(x)^3 + 3*(15*a^4*b + 7
0*a^2*b^3 + 11*b^5)*cosh(x)^2 - 32*(a^5*x - a^5 - 5*a^3*b^2)*cosh(x))*sinh(
x))/(cosh(x)^8 + 8*cosh(x)*sinh(x)^7 + sinh(x)^8 + 4*(7*cosh(x)^2 - 1)*sinh
(x)^6 - 4*cosh(x)^6 + 8*(7*cosh(x)^3 - 3*cosh(x))*sinh(x)^5 + 2*(35*cosh(x)
^4 - 30*cosh(x)^2 + 3)*sinh(x)^4 + 6*cosh(x)^4 + 8*(7*cosh(x)^5 - 10*cosh(x
)^3 + 3*cosh(x))*sinh(x)^3 + 4*(7*cosh(x)^6 - 15*cosh(x)^4 + 9*cosh(x)^2 -
1)*sinh(x)^2 - 4*cosh(x)^2 + 8*(cosh(x)^7 - 3*cosh(x)^5 + 3*cosh(x)^3 - cos
h(x))*sinh(x) + 1)
```

giac [B] time = 0.14, size = 234, normalized size = 1.89

$$\frac{1}{16}\left(8\,a^{5}-15\,a^{4}b+10\,a^{2}b^{3}-3\,b^{5}\right)\log\left(e^{(-x)}+e^{x}+2\right)+\frac{1}{16}\left(8\,a^{5}+15\,a^{4}b-10\,a^{2}b^{3}+3\,b^{5}\right)\log\left(e^{(-x)}+e^{x}-2\right)-\frac{3\,a}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*coth(x)+b*csch(x))^5,x, algorithm="giac")
```

```
[Out] 1/16*(8*a^5 - 15*a^4*b + 10*a^2*b^3 - 3*b^5)*log(e^(-x) + e^x + 2) + 1/16*(
8*a^5 + 15*a^4*b - 10*a^2*b^3 + 3*b^5)*log(e^(-x) + e^x - 2) - 1/4*(3*a^5*(
e^(-x) + e^x)^4 + 25*a^4*b*(e^(-x) + e^x)^3 + 10*a^2*b^3*(e^(-x) + e^x)^3 -
 3*b^5*(e^(-x) + e^x)^3 - 8*a^5*(e^(-x) + e^x)^2 + 80*a^3*b^2*(e^(-x) + e^x
)^2 - 60*a^4*b*(e^(-x) + e^x) + 40*a^2*b^3*(e^(-x) + e^x) + 20*b^5*(e^(-x)
+ e^x) - 160*a^3*b^2 + 80*a*b^4)/((e^(-x) + e^x)^2 - 4)^2
```

maple [A] time = 0.38, size = 201, normalized size = 1.62

$$a^{5}\ln\left(\sinh(x)\right)-\frac{a^{5}\left(\coth^{2}(x)\right)}{2}-\frac{a^{5}\left(\coth^{4}(x)\right)}{4}-\frac{5a^{4}b\left(\cosh^{3}(x)\right)}{\sinh(x)^{4}}+\frac{5a^{4}b\cosh(x)}{\sinh(x)^{4}}-\frac{5a^{4}b\coth(x)\mathrm{csch}(x)^{3}}{4}+\frac{15a^{4}b\,\mathrm{csc}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*coth(x)+b*csch(x))^5,x)
```

```
[Out] a^5*ln(sinh(x))-1/2*a^5*coth(x)^2-1/4*a^5*coth(x)^4-5*a^4*b/sinh(x)^4*cosh(
x)^3+5*a^4*b/sinh(x)^4*cosh(x)-5/4*a^4*b*coth(x)*csch(x)^3+15/8*a^4*b*csch(
x)*coth(x)-15/4*a^4*b*arctanh(exp(x))-5*a^3*b^2/sinh(x)^4*cosh(x)^2+5/2*a^3
*b^2/sinh(x)^4-10/3*a^2*b^3/sinh(x)^4*cosh(x)+5/6*a^2*b^3*coth(x)*csch(x)^3
-5/4*a^2*b^3*csch(x)*coth(x)+5/2*a^2*b^3*arctanh(exp(x))-5/4*a*b^4/sinh(x)^
4-1/4*b^5*coth(x)*csch(x)^3+3/8*b^5*csch(x)*coth(x)-3/4*b^5*arctanh(exp(x))
```

maxima [B] time = 0.39, size = 330, normalized size = 2.66

$$-\frac{5}{2}\,a^3b^2\coth(x)^4+a^5\left(x+\frac{4\left(e^{(-2\,x)}-e^{(-4\,x)}+e^{(-6\,x)}\right)}{4\,e^{(-2\,x)}-6\,e^{(-4\,x)}+4\,e^{(-6\,x)}-e^{(-8\,x)}-1}+\log\left(e^{(-x)}+1\right)+\log\left(e^{(-x)}-1\right)\right)+\frac{5}{8}\,a^4b\left(\frac{2\,(\cdots}{4\,e\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*coth(x)+b*csch(x))^5,x, algorithm="maxima")
```

```
[Out] -5/2*a^3*b^2*coth(x)^4 + a^5*(x + 4*(e^(-2*x) - e^(-4*x) + e^(-6*x))/(4*e^(
-2*x) - 6*e^(-4*x) + 4*e^(-6*x) - e^(-8*x) - 1) + log(e^(-x) + 1) + log(e^(
-x) - 1)) + 5/8*a^4*b*(2*(5*e^(-x) + 3*e^(-3*x) + 3*e^(-5*x) + 5*e^(-7*x))/
(4*e^(-2*x) - 6*e^(-4*x) + 4*e^(-6*x) - e^(-8*x) - 1) - 3*log(e^(-x) + 1) +
 3*log(e^(-x) - 1)) - 1/8*b^5*(2*(3*e^(-x) - 11*e^(-3*x) - 11*e^(-5*x) + 3*
e^(-7*x))/(4*e^(-2*x) - 6*e^(-4*x) + 4*e^(-6*x) - e^(-8*x) - 1) + 3*log(e^(
-x) + 1) - 3*log(e^(-x) - 1)) + 5/4*a^2*b^3*(2*(e^(-x) + 7*e^(-3*x) + 7*e^(
-5*x) + e^(-7*x))/(4*e^(-2*x) - 6*e^(-4*x) + 4*e^(-6*x) - e^(-8*x) - 1) + l
og(e^(-x) + 1) - log(e^(-x) - 1)) - 20*a*b^4/(e^(-x) - e^x)^4
```

mupad [B] time = 0.20, size = 392, normalized size = 3.16

$$\ln\left(\frac{15\,a^4\,b}{4}+\frac{3\,b^5}{4}-\frac{5\,a^2\,b^3}{2}-\frac{3\,b^5\,\mathrm{e}^x}{4}-\frac{15\,a^4\,b\,\mathrm{e}^x}{4}+\frac{5\,a^2\,b^3\,\mathrm{e}^x}{2}\right)\left(a^5+\frac{15\,a^4\,b}{8}-\frac{5\,a^2\,b^3}{4}+\frac{3\,b^5}{8}\right)-\frac{\mathrm{e}^x\,\left(20\,a^4\,b+4\cdots\right.}{6\,\mathrm{e}\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b/sinh(x) + a*coth(x))^5,x)
```

```
[Out] log((15*a^4*b)/4 + (3*b^5)/4 - (5*a^2*b^3)/2 - (3*b^5*exp(x))/4 - (15*a^4*b
*exp(x))/4 + (5*a^2*b^3*exp(x))/2)*((15*a^4*b)/8 + a^5 + (3*b^5)/8 - (5*a^2
*b^3)/4) - (exp(x)*(20*a^4*b + 4*b^5 + 40*a^2*b^3) + 20*a*b^4 + 4*a^5 + 40*
a^3*b^2)/(6*exp(4*x) - 4*exp(2*x) - 4*exp(6*x) + exp(8*x) + 1) - (exp(x)*(3
0*a^4*b + 6*b^5 + 60*a^2*b^3) + 40*a*b^4 + 8*a^5 + 80*a^3*b^2)/(3*exp(2*x)
- 3*exp(4*x) + exp(6*x) - 1) - a^5*x - log((5*a^2*b^3)/2 - (3*b^5)/4 - (15*
a^4*b)/4 - (3*b^5*exp(x))/4 - (15*a^4*b*exp(x))/4 + (5*a^2*b^3*exp(x))/2)*(
(15*a^4*b)/8 - a^5 + (3*b^5)/8 - (5*a^2*b^3)/4) - (exp(x)*((25*a^4*b)/4 - (
```

```
3*b^5)/4 + (5*a^2*b^3)/2) + 4*a^5 + 20*a^3*b^2)/(exp(2*x) - 1) - (exp(x)*((
45*a^4*b)/2 + b^5/2 + 25*a^2*b^3) + 20*a*b^4 + 8*a^5 + 60*a^3*b^2)/(exp(4*x
) - 2*exp(2*x) + 1)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\coth(x) + b\operatorname{csch}(x))^5 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*coth(x)+b*csch(x))**5,x)
```

```
[Out] Integral((a*coth(x) + b*csch(x))**5, x)
```

3.645 $\int (a\coth(x) + b\,\mathbf{csch}(x))^4\,dx$

Optimal. Leaf size=101

$$a^4x+\frac{4}{3}ab\left(2a^2-b^2\right)\sinh(x)+\frac{1}{3}a^2\left(3a^2-2b^2\right)\sinh(x)\cosh(x)-\frac{1}{3}\operatorname{csch}(x)(a\cosh(x)+b)^2\left(\left(3a^2-2b^2\right)\cosh(x)+\right.$$

```
[Out] a^4*x-1/3*(b+a*cosh(x))^2*(a*b+(3*a^2-2*b^2)*cosh(x))*csch(x)-1/3*(b+a*cosh
(x))^3*(a+b*cosh(x))*csch(x)^3+4/3*a*b*(2*a^2-b^2)*sinh(x)+1/3*a^2*(3*a^2-2
*b^2)*cosh(x)*sinh(x)
```

Rubi [A] time = 0.24, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {4392, 2691, 2861, 2734}

$$\frac{4}{3}ab\left(2a^2-b^2\right)\sinh(x)+\frac{1}{3}a^2\left(3a^2-2b^2\right)\sinh(x)\cosh(x)-\frac{1}{3}\operatorname{csch}(x)(a\cosh(x)+b)^2\left(\left(3a^2-2b^2\right)\cosh(x)+ab\right)+$$

Antiderivative was successfully verified.

```
[In] Int[(a*Coth[x] + b*Csch[x])^4,x]
```

```
[Out] a^4*x - ((b + a*Cosh[x])^2*(a*b + (3*a^2 - 2*b^2)*Cosh[x])*Csch[x])/3 - ((b
 + a*Cosh[x])^3*(a + b*Cosh[x])*Csch[x]^3)/3 + (4*a*b*(2*a^2 - b^2)*Sinh[x]
)/3 + (a^2*(3*a^2 - 2*b^2)*Cosh[x]*Sinh[x])/3
```

Rule 2691

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[((g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x]
)^(m - 1)*(b + a*Sin[e + f*x]))/(f*g*(p + 1)), x] + Dist[1/(g^2*(p + 1)), I
nt[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 2)*(b^2*(m - 1) + a^2
*(p + 2) + a*b*(m + p + 1)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g},
x] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] && LtQ[p, -1] && (IntegersQ[2*m, 2*p]
|| IntegerQ[m])
```

Rule 2734

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[((2*a*c + b*d)*x)/2, x] + (-Simp[((b*c + a*d)*Co
s[e + f*x])/f, x] - Simp[(b*d*Cos[e + f*x]*Sin[e + f*x])/(2*f), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2861

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[((g*
```

```
Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m*(d + c*Sin[e + f*x]))/(f*g*(p
+ 1)), x] + Dist[1/(g^2*(p + 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e
 + f*x])^(m - 1)*Simp[a*c*(p + 2) + b*d*m + b*c*(m + p + 2)*Sin[e + f*x], x
], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b^2, 0] && GtQ[m,
 0] && LtQ[p, -1] && IntegerQ[2*m] &&  !(EqQ[m, 1] && NeQ[c^2 - d^2, 0] &&
SimplerQ[c + d*x, a + b*x])
```

Rule 4392

```
Int[(cot[(c_.) + (d_.)*(x_)]^(n_.)*(a_.) + csc[(c_.) + (d_.)*(x_)]^(n_.)*(b
_.))^(p_)*(u_.), x_Symbol] :> Int[ActivateTrig[u]*Csc[c + d*x]^(n*p)*(b + a
*Cos[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

Rubi steps

$$\begin{aligned}\int (a\coth(x)+b\text{csch}(x))^4\,dx &= \int (ib+ia\cosh(x))^4\text{csch}^4(x)\,dx\\ &= -\frac{1}{3}(b+a\cosh(x))^3(a+b\cosh(x))\text{csch}^3(x)+\frac{1}{3}\int (ib+ia\cosh(x))^2\left(-3a^2+2b^2\right.\\ &= -\frac{1}{3}(b+a\cosh(x))^2\left(ab+\left(3a^2-2b^2\right)\cosh(x)\right)\text{csch}(x)-\frac{1}{3}(b+a\cosh(x))^3(a+b\\ &= a^4x-\frac{1}{3}(b+a\cosh(x))^2\left(ab+\left(3a^2-2b^2\right)\cosh(x)\right)\text{csch}(x)-\frac{1}{3}(b+a\cosh(x))^3(\end{aligned}$$

Mathematica [A] time = 0.28, size = 95, normalized size = 0.94

$$-\frac{1}{12}\text{csch}^3(x)\left(9a^4x\sinh(x)-3a^4x\sinh(3x)+4a^4\cosh(3x)+24a^3b\cosh(2x)-8a^3b+6a^2b^2\cosh(3x)+6b^2\left(3a\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Coth[x] + b*Csch[x])^4,x]
```

```
[Out] -1/12*(Csch[x]^3*(-8*a^3*b + 16*a*b^3 + 6*b^2*(3*a^2 + b^2)*Cosh[x] + 24*a^
3*b*Cosh[2*x] + 4*a^4*Cosh[3*x] + 6*a^2*b^2*Cosh[3*x] - 2*b^4*Cosh[3*x] + 9
*a^4*x*Sinh[x] - 3*a^4*x*Sinh[3*x]))
```

fricas [B] time = 0.42, size = 209, normalized size = 2.07

$$-\frac{24\,a^3b\cosh(x)^2-8\,a^3b+16\,ab^3+2\left(2\,a^4+3\,a^2b^2-b^4\right)\cosh(x)^3-\left(3\,a^4x+4\,a^4+6\,a^2b^2-2\,b^4\right)\sinh(x)^3+6\left(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*coth(x)+b*csch(x))^4,x, algorithm="fricas")
```

```
[Out] -1/3*(24*a^3*b*cosh(x)^2 - 8*a^3*b + 16*a*b^3 + 2*(2*a^4 + 3*a^2*b^2 - b^4)
*cosh(x)^3 - (3*a^4*x + 4*a^4 + 6*a^2*b^2 - 2*b^4)*sinh(x)^3 + 6*(4*a^3*b +
 (2*a^4 + 3*a^2*b^2 - b^4)*cosh(x))*sinh(x)^2 + 6*(3*a^2*b^2 + b^4)*cosh(x)
 + 3*(3*a^4*x + 4*a^4 + 6*a^2*b^2 - 2*b^4 - (3*a^4*x + 4*a^4 + 6*a^2*b^2 -
2*b^4)*cosh(x)^2)*sinh(x))/(sinh(x)^3 + 3*(cosh(x)^2 - 1)*sinh(x))
```

giac [A] time = 0.12, size = 112, normalized size = 1.11

$$a^4x - \frac{4\left(6\,a^3be^{(5\,x)} + 3\,a^4e^{(4\,x)} + 9\,a^2b^2e^{(4\,x)} - 4\,a^3be^{(3\,x)} + 8\,ab^3e^{(3\,x)} - 3\,a^4e^{(2\,x)} + 3\,b^4e^{(2\,x)} + 6\,a^3be^{x} + 2\,a^4 + 3\,a^2b\right.}{3\left(e^{(2\,x)} - 1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*coth(x)+b*csch(x))^4,x, algorithm="giac")
```

```
[Out] a^4*x - 4/3*(6*a^3*b*e^(5*x) + 3*a^4*e^(4*x) + 9*a^2*b^2*e^(4*x) - 4*a^3*b*
e^(3*x) + 8*a*b^3*e^(3*x) - 3*a^4*e^(2*x) + 3*b^4*e^(2*x) + 6*a^3*b*e^x + 2
*a^4 + 3*a^2*b^2 - b^4)/(e^(2*x) - 1)^3
```

maple [A] time = 0.35, size = 94, normalized size = 0.93

$$a^4\left(x - \coth(x) - \frac{\left(\coth^3(x)\right)}{3}\right) + 4a^3b\left(-\frac{\cosh^2(x)}{\sinh(x)^3} + \frac{2}{3\sinh(x)^3}\right) + 6a^2b^2\left(-\frac{\cosh(x)}{2\sinh(x)^3} - \frac{\left(\frac{2}{3} - \frac{\operatorname{csch}(x)^2}{3}\right)\coth(x)}{2}\right) -$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*coth(x)+b*csch(x))^4,x)
```

```
[Out] a^4*(x-coth(x)-1/3*coth(x)^3)+4*a^3*b*(-1/sinh(x)^3*cosh(x)^2+2/3/sinh(x)^3
)+6*a^2*b^2*(-1/2/sinh(x)^3*cosh(x)-1/2*(2/3-1/3*csch(x)^2)*coth(x))-4/3*a*
b^3/sinh(x)^3+b^4*(2/3-1/3*csch(x)^2)*coth(x)
```

maxima [B] time = 0.69, size = 214, normalized size = 2.12

$$-2\,a^2b^2\coth(x)^3 + \frac{1}{3}\,a^4\left(3\,x - \frac{4\left(3\,e^{(-2\,x)} - 3\,e^{(-4\,x)} - 2\right)}{3\,e^{(-2\,x)} - 3\,e^{(-4\,x)} + e^{(-6\,x)} - 1}\right) + \frac{8}{3}\,a^3b\left(\frac{3\,e^{(-x)}}{3\,e^{(-2\,x)} - 3\,e^{(-4\,x)} + e^{(-6\,x)} - 1} - \frac{}{3\,e^{(-2\,x)} - 3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*coth(x)+b*csch(x))^4,x, algorithm="maxima")
```

```
[Out] -2*a^2*b^2*coth(x)^3 + 1/3*a^4*(3*x - 4*(3*e^(-2*x) - 3*e^(-4*x) - 2)/(3*e^
(-2*x) - 3*e^(-4*x) + e^(-6*x) - 1)) + 8/3*a^3*b*(3*e^(-x)/(3*e^(-2*x) - 3*
```

```
e^(-4*x) + e^(-6*x) - 1) - 2*e^(-3*x)/(3*e^(-2*x) - 3*e^(-4*x) + e^(-6*x) -
 1) + 3*e^(-5*x)/(3*e^(-2*x) - 3*e^(-4*x) + e^(-6*x) - 1)) + 4/3*b^4*(3*e^(
-2*x)/(3*e^(-2*x) - 3*e^(-4*x) + e^(-6*x) - 1) - 1/(3*e^(-2*x) - 3*e^(-4*x)
 + e^(-6*x) - 1)) + 32/3*a*b^3/(e^(-x) - e^x)^3
```

mupad [B] time = 1.55, size = 146, normalized size = 1.45

$$a^4\,x-\frac{4\,a^4+8\,{\mathrm{e}}^x\,a^3\,b+12\,a^2\,b^2}{{\mathrm{e}}^{2\,x}-1}-\frac{{\mathrm{e}}^x\,\left(\frac{32\,a^3\,b}{3}+\frac{32\,a\,b^3}{3}\right)+4\,a^4+4\,b^4+24\,a^2\,b^2}{{\mathrm{e}}^{4\,x}-2\,{\mathrm{e}}^{2\,x}+1}-\frac{{\mathrm{e}}^x\,\left(\frac{32\,a^3\,b}{3}+\frac{32\,a\,b^3}{3}\right)+\frac{8\,a^4}{3}+\frac{8\,b^4}{3}+16\ldots}{3\,{\mathrm{e}}^{2\,x}-3\,{\mathrm{e}}^{4\,x}+{\mathrm{e}}^{6\,x}-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b/sinh(x) + a*coth(x))^4,x)
```

```
[Out] a^4*x - (4*a^4 + 12*a^2*b^2 + 8*a^3*b*exp(x))/(exp(2*x) - 1) - (exp(x)*((32
*a*b^3)/3 + (32*a^3*b)/3) + 4*a^4 + 4*b^4 + 24*a^2*b^2)/(exp(4*x) - 2*exp(2
*x) + 1) - (exp(x)*((32*a*b^3)/3 + (32*a^3*b)/3) + (8*a^4)/3 + (8*b^4)/3 +
16*a^2*b^2)/(3*exp(2*x) - 3*exp(4*x) + exp(6*x) - 1)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\coth(x) + b\,\mathrm{csch}(x))^4\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*coth(x)+b*csch(x))**4,x)
```

```
[Out] Integral((a*coth(x) + b*csch(x))**4, x)
```

3.646 $\int (a\coth(x)+b\mathbf{csch}(x))^3\,dx$

Optimal. Leaf size=59

$$a^3\log(\sinh(x))-\frac{1}{2}b\left(3a^2-b^2\right)\tanh^{-1}(\cosh(x))+\frac{1}{2}a^2b\cosh(x)-\frac{1}{2}\text{csch}^2(x)(a\cosh(x)+b)^2(a+b\cosh(x))$$

```
[Out] -1/2*b*(3*a^2-b^2)*arctanh(cosh(x))+1/2*a^2*b*cosh(x)-1/2*(b+a*cosh(x))^2*(
a+b*cosh(x))*csch(x)^2+a^3*ln(sinh(x))
```

Rubi [A] time = 0.12, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.636, Rules used = {4392, 2668, 739, 774, 635, 204, 260}

$$-\frac{1}{2}b\left(3a^2-b^2\right)\tanh^{-1}(\cosh(x))+\frac{1}{2}a^2b\cosh(x)+a^3\log(\sinh(x))-\frac{1}{2}\text{csch}^2(x)(a\cosh(x)+b)^2(a+b\cosh(x))$$

Antiderivative was successfully verified.

```
[In] Int[(a*Coth[x] + b*Csch[x])^3,x]
```

```
[Out] -(b*(3*a^2 - b^2)*ArcTanh[Cosh[x]])/2 + (a^2*b*Cosh[x])/2 - ((b + a*Cosh[x]
)^2*(a + b*Cosh[x])*Csch[x]^2)/2 + a^3*Log[Sinh[x]]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 739

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m - 1)*(a*e - c*d*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)), x] +
 Dist[1/((p + 1)*(-2*a*c)), Int[(d + e*x)^(m - 2)*Simp[a*e^2*(m - 1) - c*d^
```

```
2*(2*p + 3) - d*c*e*(m + 2*p + 2)*x, x]*(a + c*x^2)^(p + 1), x], x] /; Free
Q[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && I
ntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 774

```
Int[(((d_.) + (e_.)*(x_))*((f_) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2), x_Sym
bol] :> Simp[(e*g*x)/c, x] + Dist[1/c, Int[(c*d*f - a*e*g + c*(e*f + d*g)*x
)/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e, f, g}, x]
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 4392

```
Int[(cot[(c_.) + (d_.)*(x_)]^(n_.)*(a_.) + csc[(c_.) + (d_.)*(x_)]^(n_.)*(b
_.))^(p_)*(u_.), x_Symbol] :> Int[ActivateTrig[u]*Csc[c + d*x]^(n*p)*(b + a
*Cos[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

Rubi steps

$$\int (a\coth(x)+b\text{csch}(x))^3\,dx = i\int (ib+ia\cosh(x))^3\text{csch}^3(x)\,dx$$

$$= a^3\,\text{Subst}\left(\int \frac{(ib+x)^3}{\left(-a^2-x^2\right)^2}\,dx, x, ia\cosh(x)\right)$$

$$= -\frac{1}{2}(b+a\cosh(x))^2(a+b\cosh(x))\text{csch}^2(x)+\frac{1}{2}a\,\text{Subst}\left(\int \frac{(ib+x)\left(-2a^2+b^2+\right.}{-a^2-x^2}\right.$$

$$= \frac{1}{2}a^2b\cosh(x)-\frac{1}{2}(b+a\cosh(x))^2(a+b\cosh(x))\text{csch}^2(x)-\frac{1}{2}a\,\text{Subst}\left(\int \frac{ia^2b-}{}\right.$$

$$= \frac{1}{2}a^2b\cosh(x)-\frac{1}{2}(b+a\cosh(x))^2(a+b\cosh(x))\text{csch}^2(x)-a^3\,\text{Subst}\left(\int \frac{x}{-a^2-x}\right.$$

$$= -\frac{1}{2}b\left(3a^2-b^2\right)\tanh^{-1}(\cosh(x))+\frac{1}{2}a^2b\cosh(x)-\frac{1}{2}(b+a\cosh(x))^2(a+b\cosh($$

Mathematica [A] time = 0.25, size = 99, normalized size = 1.68

$$-\frac{1}{4}\text{csch}^2(x)\left(2a^3\log(\sinh(x))+2a^3+2b\left(3a^2+b^2\right)\cosh(x)+3a^2b\log\left(\tanh\left(\frac{x}{2}\right)\right)+\cosh(2x)\left(b\left(b^2-3a^2\right)\text{lo}\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Coth[x] + b*Csch[x])^3,x]
```

```
[Out] -1/4*(Csch[x]^2*(2*a^3 + 6*a*b^2 + 2*b*(3*a^2 + b^2)*Cosh[x] + 2*a^3*Log[Si
nh[x]] + 3*a^2*b*Log[Tanh[x/2]] - b^3*Log[Tanh[x/2]] + Cosh[2*x]*(-2*a^3*Lo
g[Sinh[x]] + b*(-3*a^2 + b^2)*Log[Tanh[x/2]])))
```

fricas [B] time = 0.42, size = 674, normalized size = 11.42

$$-\frac{2a^3x\cosh(x)^4+2a^3x\sinh(x)^4+2a^3x+2\left(3a^2b+b^3\right)\cosh(x)^3+2\left(4a^3x\cosh(x)+3a^2b+b^3\right)\sinh(x)^3-4}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*coth(x)+b*csch(x))^3,x, algorithm="fricas")
```

```
[Out] -1/2*(2*a^3*x*cosh(x)^4 + 2*a^3*x*sinh(x)^4 + 2*a^3*x + 2*(3*a^2*b + b^3)*c
osh(x)^3 + 2*(4*a^3*x*cosh(x) + 3*a^2*b + b^3)*sinh(x)^3 - 4*(a^3*x - a^3 -
 3*a*b^2)*cosh(x)^2 + 2*(6*a^3*x*cosh(x)^2 - 2*a^3*x + 2*a^3 + 6*a*b^2 + 3*
(3*a^2*b + b^3)*cosh(x))*sinh(x)^2 + 2*(3*a^2*b + b^3)*cosh(x) - ((2*a^3 -
3*a^2*b + b^3)*cosh(x)^4 + 4*(2*a^3 - 3*a^2*b + b^3)*cosh(x)*sinh(x)^3 + (2
*a^3 - 3*a^2*b + b^3)*sinh(x)^4 + 2*a^3 - 3*a^2*b + b^3 - 2*(2*a^3 - 3*a^2*
b + b^3)*cosh(x)^2 - 2*(2*a^3 - 3*a^2*b + b^3 - 3*(2*a^3 - 3*a^2*b + b^3)*c
osh(x)^2)*sinh(x)^2 + 4*((2*a^3 - 3*a^2*b + b^3)*cosh(x)^3 - (2*a^3 - 3*a^2
*b + b^3)*cosh(x))*sinh(x))*log(cosh(x) + sinh(x) + 1) - ((2*a^3 + 3*a^2*b
- b^3)*cosh(x)^4 + 4*(2*a^3 + 3*a^2*b - b^3)*cosh(x)*sinh(x)^3 + (2*a^3 + 3
*a^2*b - b^3)*sinh(x)^4 + 2*a^3 + 3*a^2*b - b^3 - 2*(2*a^3 + 3*a^2*b - b^3)
*cosh(x)^2 - 2*(2*a^3 + 3*a^2*b - b^3 - 3*(2*a^3 + 3*a^2*b - b^3)*cosh(x)^2
)*sinh(x)^2 + 4*((2*a^3 + 3*a^2*b - b^3)*cosh(x)^3 - (2*a^3 + 3*a^2*b - b^3
)*cosh(x))*sinh(x))*log(cosh(x) + sinh(x) - 1) + 2*(4*a^3*x*cosh(x)^3 + 3*a
^2*b + b^3 + 3*(3*a^2*b + b^3)*cosh(x)^2 - 4*(a^3*x - a^3 - 3*a*b^2)*cosh(x
))*sinh(x))/(cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + 2*(3*cosh(x)^2 -
 1)*sinh(x)^2 - 2*cosh(x)^2 + 4*(cosh(x)^3 - cosh(x))*sinh(x) + 1)
```

giac [B] time = 0.14, size = 115, normalized size = 1.95

$$\frac{1}{4}\left(2a^3-3a^2b+b^3\right)\log\left(e^{(-x)}+e^x+2\right)+\frac{1}{4}\left(2a^3+3a^2b-b^3\right)\log\left(e^{(-x)}+e^x-2\right)-\frac{a^3\left(e^{(-x)}+e^x\right)^2+6a^2b\left(e^{(-x)}\right.}{2\left(\left(e^{(-x)}\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*coth(x)+b*csch(x))^3,x, algorithm="giac")
```

```
[Out] 1/4*(2*a^3 - 3*a^2*b + b^3)*log(e^(-x) + e^x + 2) + 1/4*(2*a^3 + 3*a^2*b -
b^3)*log(e^(-x) + e^x - 2) - 1/2*(a^3*(e^(-x) + e^x)^2 + 6*a^2*b*(e^(-x) +
e^x) + 2*b^3*(e^(-x) + e^x) + 12*a*b^2)/((e^(-x) + e^x)^2 - 4)
```

maple [A] time = 0.36, size = 75, normalized size = 1.27

$$a^3 \ln(\sinh(x)) - \frac{a^3\left(\coth^2(x)\right)}{2} - \frac{3a^2 b\cosh(x)}{\sinh(x)^2} + \frac{3a^2 b\,\mathrm{csch}(x)\coth(x)}{2} - 3a^2 b\,\mathrm{arctanh}\left(e^x\right) - \frac{3a\,b^2}{2\sinh(x)^2} - \frac{b^3\mathrm{csch}(x)\,\mathrm{co}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*coth(x)+b*csch(x))^3,x)
```

```
[Out] a^3*ln(sinh(x))-1/2*a^3*coth(x)^2-3*a^2*b/sinh(x)^2*cosh(x)+3/2*a^2*b*csch(
x)*coth(x)-3*a^2*b*arctanh(exp(x))-3/2*a*b^2/sinh(x)^2-1/2*b^3*csch(x)*coth
(x)+b^3*arctanh(exp(x))
```

maxima [B] time = 0.44, size = 152, normalized size = 2.58

$$-\frac{3}{2}ab^2\coth(x)^2 + a^3\left(x + \frac{2e^{(-2x)}}{2e^{(-2x)} - e^{(-4x)} - 1} + \log\left(e^{(-x)} + 1\right) + \log\left(e^{(-x)} - 1\right)\right) + \frac{1}{2}b^3\left(\frac{2\left(e^{(-x)} + e^{(-3x)}\right)}{2e^{(-2x)} - e^{(-4x)} - 1} + \log\left(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*coth(x)+b*csch(x))^3,x, algorithm="maxima")
```

```
[Out] -3/2*a*b^2*coth(x)^2 + a^3*(x + 2*e^(-2*x)/(2*e^(-2*x) - e^(-4*x) - 1) + lo
g(e^(-x) + 1) + log(e^(-x) - 1)) + 1/2*b^3*(2*(e^(-x) + e^(-3*x))/(2*e^(-2*
x) - e^(-4*x) - 1) + log(e^(-x) + 1) - log(e^(-x) - 1)) + 3/2*a^2*b*(2*(e^(
-x) + e^(-3*x))/(2*e^(-2*x) - e^(-4*x) - 1) - log(e^(-x) + 1) + log(e^(-x)
- 1))
```

mupad [B] time = 1.54, size = 169, normalized size = 2.86

$$\ln\left(b^3 - 3\,a^2\,b + b^3\,e^x - 3\,a^2\,b\,e^x\right)\left(a^3 - \frac{3\,a^2\,b}{2} + \frac{b^3}{2}\right) - \frac{6\,a\,b^2 + 2\,a^3 + e^x\,\left(3\,a^2\,b + b^3\right)}{e^{2\,x} - 1} - \frac{e^x\,\left(6\,a^2\,b + 2\,b^3\right) + 6\,a\,b^2}{e^{4\,x} - 2\,e^{2\,x} + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b/sinh(x) + a*coth(x))^3,x)
```

```
[Out] log(b^3 - 3*a^2*b + b^3*exp(x) - 3*a^2*b*exp(x))*(a^3 - (3*a^2*b)/2 + b^3/2
) - (6*a*b^2 + 2*a^3 + exp(x)*(3*a^2*b + b^3))/(exp(2*x) - 1) - (exp(x)*(6*
```

```
a^2*b + 2*b^3) + 6*a*b^2 + 2*a^3)/(exp(4*x) - 2*exp(2*x) + 1) - a^3*x + log
(3*a^2*b - b^3 + b^3*exp(x) - 3*a^2*b*exp(x))*((3*a^2*b)/2 + a^3 - b^3/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\coth(x) + b\operatorname{csch}(x))^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*coth(x)+b*csch(x))**3,x)
```

```
[Out] Integral((a*coth(x) + b*csch(x))**3, x)
```

3.647 $\int (a\coth(x) + b\mathbf{csch}(x))^2\,dx$

Optimal. Leaf size=27

$$a^2x + ab\sinh(x) - \operatorname{csch}(x)(a\cosh(x) + b)(a + b\cosh(x))$$

[Out] a^2*x-(b+a*cosh(x))*(a+b*cosh(x))*csch(x)+a*b*sinh(x)

Rubi [A] time = 0.07, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} =$ 0.273, Rules used = {4392, 2691, 2637}

$$a^2x + ab\sinh(x) - \operatorname{csch}(x)(a\cosh(x) + b)(a + b\cosh(x))$$

Antiderivative was successfully verified.

[In] Int[(a*Coth[x] + b*Csch[x])^2,x]

[Out] a^2*x - (b + a*Cosh[x])*(a + b*Cosh[x])*Csch[x] + a*b*Sinh[x]

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2691

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[((g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x]
)^(m - 1)*(b + a*Sin[e + f*x]))/(f*g*(p + 1)), x] + Dist[1/(g^2*(p + 1)), I
nt[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 2)*(b^2*(m - 1) + a^2
*(p + 2) + a*b*(m + p + 1)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g},
x] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] && LtQ[p, -1] && (IntegersQ[2*m, 2*p]
|| IntegerQ[m])
```

Rule 4392

```
Int[(cot[(c_.) + (d_.)*(x_)]^(n_.)*(a_.) + csc[(c_.) + (d_.)*(x_)]^(n_.)*(b
_.))^(p_)*(u_.), x_Symbol] :> Int[ActivateTrig[u]*Csc[c + d*x]^(n*p)*(b + a
*Cos[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

Rubi steps

$$\begin{aligned}\int (a\coth(x)+b\text{csch}(x))^2\,dx &= -\int (ib+ia\cosh(x))^2\text{csch}^2(x)\,dx\\ &= -(b+a\cosh(x))(a+b\cosh(x))\text{csch}(x)-\int \left(-a^2-ab\cosh(x)\right)\,dx\\ &= a^2x-(b+a\cosh(x))(a+b\cosh(x))\text{csch}(x)+(ab)\int \cosh(x)\,dx\\ &= a^2x-(b+a\cosh(x))(a+b\cosh(x))\text{csch}(x)+ab\sinh(x)\end{aligned}$$

Mathematica [A] time = 0.13, size = 23, normalized size = 0.85

$$a(ax-2b\text{csch}(x))-\left(a^2+b^2\right)\coth(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Coth[x] + b*Csch[x])^2,x]
[Out] -((a^2 + b^2)*Coth[x]) + a*(a*x - 2*b*Csch[x])
```

fricas [A] time = 0.40, size = 37, normalized size = 1.37

$$-\frac{2\,ab+\left(a^2+b^2\right)\cosh(x)-\left(a^2x+a^2+b^2\right)\sinh(x)}{\sinh(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*coth(x)+b*csch(x))^2,x, algorithm="fricas")
[Out] -(2*a*b + (a^2 + b^2)*cosh(x) - (a^2*x + a^2 + b^2)*sinh(x))/sinh(x)
```

giac [A] time = 0.13, size = 29, normalized size = 1.07

$$a^2x-\frac{2\left(2\,abe^x+a^2+b^2\right)}{e^{(2\,x)}-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*coth(x)+b*csch(x))^2,x, algorithm="giac")
[Out] a^2*x - 2*(2*a*b*e^x + a^2 + b^2)/(e^(2*x) - 1)
```

maple [A] time = 0.34, size = 27, normalized size = 1.00

$$a^2\,(x-\coth(x))-\frac{2ab}{\sinh(x)}-b^2\coth(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*coth(x)+b*csch(x))^2,x)
```

```
[Out] a^2*(x-coth(x))-2*a*b/sinh(x)-b^2*coth(x)
```

maxima [A] time = 0.45, size = 45, normalized size = 1.67

$$a^2\left(x+\frac{2}{e^{(-2x)}-1}\right)+\frac{4\,ab}{e^{(-x)}-e^x}+\frac{2\,b^2}{e^{(-2x)}-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*coth(x)+b*csch(x))^2,x, algorithm="maxima")
```

```
[Out] a^2*(x + 2/(e^(-2*x) - 1)) + 4*a*b/(e^(-x) - e^x) + 2*b^2/(e^(-2*x) - 1)
```

mupad [B] time = 1.61, size = 33, normalized size = 1.22

$$a^2\,x-\frac{2\,a^2+4\,\mathrm{e}^x\,a\,b+2\,b^2}{\mathrm{e}^{2\,x}-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b/sinh(x) + a*coth(x))^2,x)
```

```
[Out] a^2*x - (2*a^2 + 2*b^2 + 4*a*b*exp(x))/(exp(2*x) - 1)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a\coth\left(x\right)+b\operatorname{csch}\left(x\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*coth(x)+b*csch(x))**2,x)
```

```
[Out] Integral((a*coth(x) + b*csch(x))**2, x)
```

3.648 $\int (a\coth(x) + b\textbf{csch}(x))\,dx$

Optimal. Leaf size=12

$$a\log(\sinh(x)) - b\tanh^{-1}(\cosh(x))$$

```
[Out] -b*arctanh(cosh(x))+a*ln(sinh(x))
```

Rubi [A] time = 0.01, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {3475, 3770}

$$a\log(\sinh(x)) - b\tanh^{-1}(\cosh(x))$$

Antiderivative was successfully verified.

```
[In] Int[a*Coth[x] + b*Csch[x],x]
```

```
[Out] -(b*ArcTanh[Cosh[x]]) + a*Log[Sinh[x]]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int (a\coth(x) + b\text{csch}(x))\,dx &= a\int \coth(x)\,dx + b\int \text{csch}(x)\,dx\\ &= -b\tanh^{-1}(\cosh(x)) + a\log(\sinh(x))\end{aligned}$$

Mathematica [A] time = 0.00, size = 15, normalized size = 1.25

$$a\log(\sinh(x)) + b\log\left(\tanh\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[a*Coth[x] + b*Csch[x],x]
```

[Out] a*Log[Sinh[x]] + b*Log[Tanh[x/2]]

fricas [B] time = 0.43, size = 29, normalized size = 2.42

$$-ax + (a - b) \log\left(\cosh(x) + \sinh(x) + 1\right) + (a + b) \log\left(\cosh(x) + \sinh(x) - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(a*coth(x)+b*csch(x),x, algorithm="fricas")

[Out] -a*x + (a - b)*log(cosh(x) + sinh(x) + 1) + (a + b)*log(cosh(x) + sinh(x) - 1)

giac [B] time = 0.13, size = 33, normalized size = 2.75

$$-a\left(x - \log\left(\left|e^{(2x)} - 1\right|\right)\right) - b\left(\log\left(e^{x} + 1\right) - \log\left(|e^{x} - 1|\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(a*coth(x)+b*csch(x),x, algorithm="giac")

[Out] -a*(x - log(abs(e^(2*x) - 1))) - b*(log(e^x + 1) - log(abs(e^x - 1)))

maple [A] time = 0.02, size = 14, normalized size = 1.17

$$a \ln\left(\sinh(x)\right) + b \ln\left(\tanh\left(\frac{x}{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(a*coth(x)+b*csch(x),x)

[Out] a*ln(sinh(x))+b*ln(tanh(1/2*x))

maxima [A] time = 0.39, size = 13, normalized size = 1.08

$$a \log\left(\sinh(x)\right) + b \log\left(\tanh\left(\frac{1}{2}\,x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(a*coth(x)+b*csch(x),x, algorithm="maxima")

[Out] a*log(sinh(x)) + b*log(tanh(1/2*x))

mupad [B] time = 0.06, size = 35, normalized size = 2.92

$$\ln\left(-2\,b - 2\,b\,e^{x}\right)\,\left(a - b\right) - a\,x + \ln\left(2\,b - 2\,b\,e^{x}\right)\,\left(a + b\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(b/sinh(x) + a*coth(x),x)
```

```
[Out] log(- 2*b - 2*b*exp(x))*(a - b) - a*x + log(2*b - 2*b*exp(x))*(a + b)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\coth(x) + b\operatorname{csch}(x))\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a*coth(x)+b*csch(x),x)
```

```
[Out] Integral(a*coth(x) + b*csch(x), x)
```

3.649 $\int \frac{1}{a\coth(x)+b\,\mathbf{csch}(x)}\,dx$

Optimal. Leaf size=11

$$\frac{\log(a\cosh(x)+b)}{a}$$

[Out] ln(b+a*cosh(x))/a

Rubi [A] time = 0.05, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {3160, 2668, 31}

$$\frac{\log(a\cosh(x)+b)}{a}$$

Antiderivative was successfully verified.

[In] Int[(a*Coth[x] + b*Csch[x])^(-1),x]

[Out] Log[b + a*Cosh[x]]/a

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 3160

```
Int[((a_.) + csc[(d_.) + (e_.)*(x_)]*(b_.) + cot[(d_.) + (e_.)*(x_)]*(c_.))
^(-1), x_Symbol] :> Int[Sin[d + e*x]/(b + a*Sin[d + e*x] + c*Cos[d + e*x]),
 x] /; FreeQ[{a, b, c, d, e}, x]
```

Rubi steps

$$\int \frac{1}{a\coth(x)+b\operatorname{csch}(x)}\,dx = i\int \frac{\sinh(x)}{ib+ia\cosh(x)}\,dx$$
$$= \frac{\text{Subst}\left(\int \frac{1}{ib+x}\,dx, x, ia\cosh(x)\right)}{a}$$
$$= \frac{\log(b+a\cosh(x))}{a}$$

Mathematica [A] time = 0.02, size = 11, normalized size = 1.00

$$\frac{\log(a\cosh(x)+b)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Coth[x] + b*Csch[x])^(-1),x]
[Out] Log[b + a*Cosh[x]]/a
```

fricas [B] time = 0.42, size = 27, normalized size = 2.45

$$-\frac{x-\log\left(\frac{2(a\cosh(x)+b)}{\cosh(x)-\sinh(x)}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*coth(x)+b*csch(x)),x, algorithm="fricas")
[Out] -(x - log(2*(a*cosh(x) + b)/(cosh(x) - sinh(x))))/a
```

giac [A] time = 0.15, size = 19, normalized size = 1.73

$$\frac{\log\left(\left|a\left(e^{(-x)}+e^{x}\right)+2b\right|\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*coth(x)+b*csch(x)),x, algorithm="giac")
[Out] log(abs(a*(e^(-x) + e^x) + 2*b))/a
```

maple [B] time = 0.21, size = 51, normalized size = 4.64

$$-\frac{\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{a}-\frac{\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)}{a}+\frac{\ln\left(a\left(\tanh^{2}\left(\frac{x}{2}\right)\right)-\left(\tanh^{2}\left(\frac{x}{2}\right)\right)b+a+b\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*coth(x)+b*csch(x)),x)
```

```
[Out] -1/a*ln(tanh(1/2*x)-1)-1/a*ln(tanh(1/2*x)+1)+1/a*ln(a*tanh(1/2*x)^2-tanh(1/
2*x)^2*b+a+b)
```

maxima [B] time = 0.31, size = 26, normalized size = 2.36

$$\frac{x}{a} + \frac{\log\left(2\,be^{(-x)} + ae^{(-2\,x)} + a\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*coth(x)+b*csch(x)),x, algorithm="maxima")
```

```
[Out] x/a + log(2*b*e^(-x) + a*e^(-2*x) + a)/a
```

mupad [B] time = 0.09, size = 23, normalized size = 2.09

$$-\frac{x - \ln\left(a + 2\,b\,\mathrm{e}^{x} + a\,\mathrm{e}^{2\,x}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b/sinh(x) + a*coth(x)),x)
```

```
[Out] -(x - log(a + 2*b*exp(x) + a*exp(2*x)))/a
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{a\coth(x) + b\,\mathrm{csch}\,(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*coth(x)+b*csch(x)),x)
```

```
[Out] Integral(1/(a*coth(x) + b*csch(x)), x)
```

3.650 $\int \frac{1}{(a\coth(x)+b\textbf{csch}(x))^2}\,dx$

Optimal. Leaf size=67

$$-\frac{2b\tan^{-1}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{a^2\sqrt{a-b}\sqrt{a+b}}+\frac{x}{a^2}-\frac{\sinh(x)}{a(a\cosh(x)+b)}$$

```
[Out] x/a^2-sinh(x)/a/(b+a*cosh(x))-2*b*arctan((a-b)^(1/2)*tanh(1/2*x)/(a+b)^(1/2
))/a^2/(a-b)^(1/2)/(a+b)^(1/2)
```

Rubi [A] time = 0.13, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.454, Rules used = {4392, 2693, 2735, 2659, 205}

$$-\frac{2b\tan^{-1}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{a^2\sqrt{a-b}\sqrt{a+b}}+\frac{x}{a^2}-\frac{\sinh(x)}{a(a\cosh(x)+b)}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Coth[x] + b*Csch[x])^(-2),x]
```

```
[Out] x/a^2 - (2*b*ArcTan[(Sqrt[a - b]*Tanh[x/2])/Sqrt[a + b]])/(a^2*Sqrt[a - b]*
Sqrt[a + b]) - Sinh[x]/(a*(b + a*Cosh[x]))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2693

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x
])^(m + 1))/(b*f*(m + 1)), x] + Dist[(g^2*(p - 1))/(b*(m + 1)), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1)*Sin[e + f*x], x], x] /; Free
Q[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && In
```

```
tegersQ[2*m, 2*p]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 4392

```
Int[(cot[(c_.) + (d_.)*(x_)]^(n_.)*(a_.) + csc[(c_.) + (d_.)*(x_)]^(n_.)*(b
_.))^(p_)*(u_.), x_Symbol] :> Int[ActivateTrig[u]*Csc[c + d*x]^(n*p)*(b + a
*Cos[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(a\coth(x)+b\operatorname{csch}(x))^2}\,dx &= -\int \frac{\sinh^2(x)}{(ib+ia\cosh(x))^2}\,dx\\
&= -\frac{\sinh(x)}{a(b+a\cosh(x))}+\frac{i\int \frac{\cosh(x)}{ib+ia\cosh(x)}\,dx}{a}\\
&= \frac{x}{a^2}-\frac{\sinh(x)}{a(b+a\cosh(x))}-\frac{(ib)\int \frac{1}{ib+ia\cosh(x)}\,dx}{a^2}\\
&= \frac{x}{a^2}-\frac{\sinh(x)}{a(b+a\cosh(x))}-\frac{(2ib)\operatorname{Subst}\left(\int \frac{1}{ia+ib-(-ia+ib)x^2}\,dx,x,\tanh\left(\frac{x}{2}\right)\right)}{a^2}\\
&= \frac{x}{a^2}-\frac{2b\tan^{-1}\left(\frac{\sqrt{a-b}\,\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{a^2\sqrt{a-b}\,\sqrt{a+b}}-\frac{\sinh(x)}{a(b+a\cosh(x))}
\end{aligned}$$

Mathematica [A] time = 0.35, size = 61, normalized size = 0.91

$$\frac{\frac{2b\tan^{-1}\left(\frac{(b-a)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}-\frac{a\sinh(x)}{a\cosh(x)+b}+x}{a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Coth[x] + b*Csch[x])^(-2),x]
```

```
[Out] (x + (2*b*ArcTan[((-a + b)*Tanh[x/2])/Sqrt[a^2 - b^2]])/Sqrt[a^2 - b^2] - (
a*Sinh[x])/(b + a*Cosh[x]))/a^2
```

fricas [B] time = 0.45, size = 682, normalized size = 10.18

$$\left[\frac{\left(a^3 - ab^2\right)x\cosh(x)^2 + \left(a^3 - ab^2\right)x\sinh(x)^2 + 2\,a^3 - 2\,ab^2 - \left(ab\cosh(x)^2 + ab\sinh(x)^2 + 2\,b^2\cosh(x) + ab + \right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*coth(x)+b*csch(x))^2,x, algorithm="fricas")
```

```
[Out] [((a^3 - a*b^2)*x*cosh(x)^2 + (a^3 - a*b^2)*x*sinh(x)^2 + 2*a^3 - 2*a*b^2 -
 (a*b*cosh(x)^2 + a*b*sinh(x)^2 + 2*b^2*cosh(x) + a*b + 2*(a*b*cosh(x) + b^
2)*sinh(x))*sqrt(-a^2 + b^2)*log((a^2*cosh(x)^2 + a^2*sinh(x)^2 + 2*a*b*cos
h(x) - a^2 + 2*b^2 + 2*(a^2*cosh(x) + a*b)*sinh(x) + 2*sqrt(-a^2 + b^2)*(a*
cosh(x) + a*sinh(x) + b))/(a*cosh(x)^2 + a*sinh(x)^2 + 2*b*cosh(x) + 2*(a*c
osh(x) + b)*sinh(x) + a)) + (a^3 - a*b^2)*x + 2*(a^2*b - b^3 + (a^2*b - b^3
)*x)*cosh(x) + 2*(a^2*b - b^3 + (a^3 - a*b^2)*x*cosh(x) + (a^2*b - b^3)*x)*
sinh(x))/(a^5 - a^3*b^2 + (a^5 - a^3*b^2)*cosh(x)^2 + (a^5 - a^3*b^2)*sinh(
x)^2 + 2*(a^4*b - a^2*b^3)*cosh(x) + 2*(a^4*b - a^2*b^3 + (a^5 - a^3*b^2)*c
osh(x))*sinh(x)), ((a^3 - a*b^2)*x*cosh(x)^2 + (a^3 - a*b^2)*x*sinh(x)^2 +
2*a^3 - 2*a*b^2 + 2*(a*b*cosh(x)^2 + a*b*sinh(x)^2 + 2*b^2*cosh(x) + a*b +
2*(a*b*cosh(x) + b^2)*sinh(x))*sqrt(a^2 - b^2)*arctan(-(a*cosh(x) + a*sinh(
x) + b)/sqrt(a^2 - b^2)) + (a^3 - a*b^2)*x + 2*(a^2*b - b^3 + (a^2*b - b^3)
*x)*cosh(x) + 2*(a^2*b - b^3 + (a^3 - a*b^2)*x*cosh(x) + (a^2*b - b^3)*x)*s
inh(x))/(a^5 - a^3*b^2 + (a^5 - a^3*b^2)*cosh(x)^2 + (a^5 - a^3*b^2)*sinh(x
)^2 + 2*(a^4*b - a^2*b^3)*cosh(x) + 2*(a^4*b - a^2*b^3 + (a^5 - a^3*b^2)*co
sh(x))*sinh(x))]
```

giac [A] time = 0.12, size = 68, normalized size = 1.01

$$-\frac{2\,b\arctan\left(\frac{ae^x+b}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}\,a^2} + \frac{x}{a^2} + \frac{2\,(be^x + a)}{\left(ae^{(2\,x)} + 2\,be^x + a\right)a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*coth(x)+b*csch(x))^2,x, algorithm="giac")
```

```
[Out] -2*b*arctan((a*e^x + b)/sqrt(a^2 - b^2))/(sqrt(a^2 - b^2)*a^2) + x/a^2 + 2*
(b*e^x + a)/((a*e^(2*x) + 2*b*e^x + a)*a^2)
```

maple [A] time = 0.22, size = 95, normalized size = 1.42

$$-\frac{\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{a^{2}}+\frac{\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)}{a^{2}}-\frac{2\tanh\left(\frac{x}{2}\right)}{a\left(a\left(\tanh^{2}\left(\frac{x}{2}\right)\right)-\left(\tanh^{2}\left(\frac{x}{2}\right)\right)b+a+b\right)}-\frac{2b\arctan\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{a^{2}\sqrt{(a+b)(a-b)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*coth(x)+b*csch(x))^2,x)
```

```
[Out] -1/a^2*ln(tanh(1/2*x)-1)+1/a^2*ln(tanh(1/2*x)+1)-2/a*tanh(1/2*x)/(a*tanh(1/
2*x)^2-tanh(1/2*x)^2*b+a+b)-2/a^2*b/((a+b)*(a-b))^(1/2)*arctan((a-b)*tanh(1
/2*x)/((a+b)*(a-b))^(1/2))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*coth(x)+b*csch(x))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [B] time = 1.79, size = 139, normalized size = 2.07

$$\frac{x}{a^{2}}+\frac{\frac{2}{a}+\frac{2\,b\,e^{x}}{a^{2}}}{a+2\,b\,e^{x}+a\,e^{2\,x}}+\frac{b\,\ln\left(\frac{2\,b\,e^{x}}{a^{3}}-\frac{2\,b\,(a+b\,e^{x})}{a^{3}\,\sqrt{a+b}\,\sqrt{b-a}}\right)}{a^{2}\,\sqrt{a+b}\,\sqrt{b-a}}-\frac{b\,\ln\left(\frac{2\,b\,e^{x}}{a^{3}}+\frac{2\,b\,(a+b\,e^{x})}{a^{3}\,\sqrt{a+b}\,\sqrt{b-a}}\right)}{a^{2}\,\sqrt{a+b}\,\sqrt{b-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b/sinh(x) + a*coth(x))^2,x)
```

```
[Out] x/a^2 + (2/a + (2*b*exp(x))/a^2)/(a + 2*b*exp(x) + a*exp(2*x)) + (b*log((2*
b*exp(x))/a^3 - (2*b*(a + b*exp(x)))/(a^3*(a + b)^(1/2)*(b - a)^(1/2))))/(a
^2*(a + b)^(1/2)*(b - a)^(1/2)) - (b*log((2*b*exp(x))/a^3 + (2*b*(a + b*exp
(x)))/(a^3*(a + b)^(1/2)*(b - a)^(1/2))))/(a^2*(a + b)^(1/2)*(b - a)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a\coth(x)+b\operatorname{csch}(x)\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*coth(x)+b*csch(x))**2,x)
```

```
[Out] Integral((a*coth(x) + b*csch(x))**(-2), x)
```

3.651 $\int \frac{1}{(a\coth(x)+b\mathbf{csch}(x))^3}\,dx$

Optimal. Leaf size=50

$$\frac{2b}{a^3(a\cosh(x)+b)}+\frac{\log(a\cosh(x)+b)}{a^3}+\frac{a^2-b^2}{2a^3(a\cosh(x)+b)^2}$$

```
[Out] 1/2*(a^2-b^2)/a^3/(b+a*cosh(x))^2+2*b/a^3/(b+a*cosh(x))+ln(b+a*cosh(x))/a^3
```

Rubi [A] time = 0.11, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {4392, 2668, 697}

$$\frac{a^2-b^2}{2a^3(a\cosh(x)+b)^2}+\frac{2b}{a^3(a\cosh(x)+b)}+\frac{\log(a\cosh(x)+b)}{a^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Coth[x] + b*Csch[x])^(-3),x]
```

```
[Out] (a^2 - b^2)/(2*a^3*(b + a*Cosh[x])^2) + (2*b)/(a^3*(b + a*Cosh[x])) + Log[b
 + a*Cosh[x]]/a^3
```

Rule 697

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 4392

```
Int[(cot[(c_.) + (d_.)*(x_)]^(n_.)*(a_.) + csc[(c_.) + (d_.)*(x_)]^(n_.)*(b
_.))^(p_)*(u_.), x_Symbol] :> Int[ActivateTrig[u]*Csc[c + d*x]^(n*p)*(b + a
*Cos[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

Rubi steps

$$\int \frac{1}{(a\coth(x)+b\text{csch}(x))^3}\,dx = -\left(i\int \frac{\sinh^3(x)}{(ib+ia\cosh(x))^3}\,dx\right)$$
$$= -\frac{\text{Subst}\left(\int \frac{-a^2-x^2}{(ib+x)^3}\,dx, x, ia\cosh(x)\right)}{a^3}$$
$$= -\frac{\text{Subst}\left(\int \left(\frac{-a^2+b^2}{(ib+x)^3}+\frac{2ib}{(ib+x)^2}-\frac{1}{ib+x}\right)dx, x, ia\cosh(x)\right)}{a^3}$$
$$= \frac{a^2-b^2}{2a^3(b+a\cosh(x))^2}+\frac{2b}{a^3(b+a\cosh(x))}+\frac{\log(b+a\cosh(x))}{a^3}$$

Mathematica [A] time = 0.11, size = 77, normalized size = 1.54

$$\frac{a^2\cosh(2x)\log(a\cosh(x)+b)+a^2\log(a\cosh(x)+b)+a^2+2b^2\log(a\cosh(x)+b)+4ab\cosh(x)(\log(a\cosh}{2a^3(a\cosh(x)+b)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Coth[x] + b*Csch[x])^(-3),x]
```

```
[Out] (a^2 + 3*b^2 + a^2*Log[b + a*Cosh[x]] + 2*b^2*Log[b + a*Cosh[x]] + a^2*Cosh
[2*x]*Log[b + a*Cosh[x]] + 4*a*b*Cosh[x]*(1 + Log[b + a*Cosh[x]]))/(2*a^3*(
b + a*Cosh[x])^2)
```

fricas [B] time = 0.78, size = 521, normalized size = 10.42

$$-\frac{a^2x\cosh(x)^4+a^2x\sinh(x)^4+4(abx-ab)\cosh(x)^3+4\left(a^2x\cosh(x)+abx-ab\right)\sinh(x)^3+a^2x-2\left(a^2+3b\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*coth(x)+b*csch(x))^3,x, algorithm="fricas")
```

```
[Out] -(a^2*x*cosh(x)^4 + a^2*x*sinh(x)^4 + 4*(a*b*x - a*b)*cosh(x)^3 + 4*(a^2*x*
cosh(x) + a*b*x - a*b)*sinh(x)^3 + a^2*x - 2*(a^2 + 3*b^2 - (a^2 + 2*b^2)*x
)*cosh(x)^2 + 2*(3*a^2*x*cosh(x)^2 - a^2 - 3*b^2 + (a^2 + 2*b^2)*x + 6*(a*b
*x - a*b)*cosh(x))*sinh(x)^2 + 4*(a*b*x - a*b)*cosh(x) - (a^2*cosh(x)^4 + a
^2*sinh(x)^4 + 4*a*b*cosh(x)^3 + 4*(a^2*cosh(x) + a*b)*sinh(x)^3 + 4*a*b*co
sh(x) + 2*(a^2 + 2*b^2)*cosh(x)^2 + 2*(3*a^2*cosh(x)^2 + 6*a*b*cosh(x) + a^
2 + 2*b^2)*sinh(x)^2 + a^2 + 4*(a^2*cosh(x)^3 + 3*a*b*cosh(x)^2 + a*b + (a^
2 + 2*b^2)*cosh(x))*sinh(x))*log(2*(a*cosh(x) + b)/(cosh(x) - sinh(x))) + 4
```

```
*(a^2*x*cosh(x)^3 + a*b*x + 3*(a*b*x - a*b)*cosh(x)^2 - a*b - (a^2 + 3*b^2
- (a^2 + 2*b^2)*x)*cosh(x))*sinh(x))/(a^5*cosh(x)^4 + a^5*sinh(x)^4 + 4*a^4
*b*cosh(x)^3 + 4*a^4*b*cosh(x) + a^5 + 4*(a^5*cosh(x) + a^4*b)*sinh(x)^3 +
2*(a^5 + 2*a^3*b^2)*cosh(x)^2 + 2*(3*a^5*cosh(x)^2 + 6*a^4*b*cosh(x) + a^5
+ 2*a^3*b^2)*sinh(x)^2 + 4*(a^5*cosh(x)^3 + 3*a^4*b*cosh(x)^2 + a^4*b + (a^
5 + 2*a^3*b^2)*cosh(x))*sinh(x))
```

giac [A] time = 0.15, size = 66, normalized size = 1.32

$$\frac{\log\left(\left|a\left(e^{(-x)}+e^{x}\right)+2\,b\right|\right)}{a^{3}}-\frac{3\,a\left(e^{(-x)}+e^{x}\right)^{2}+4\,b\left(e^{(-x)}+e^{x}\right)-4\,a}{2\left(a\left(e^{(-x)}+e^{x}\right)+2\,b\right)^{2}a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*coth(x)+b*csch(x))^3,x, algorithm="giac")
```

```
[Out] log(abs(a*(e^(-x) + e^x) + 2*b))/a^3 - 1/2*(3*a*(e^(-x) + e^x)^2 + 4*b*(e^(
-x) + e^x) - 4*a)/((a*(e^(-x) + e^x) + 2*b)^2*a^2)
```

maple [B] time = 0.22, size = 144, normalized size = 2.88

$$-\frac{\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{a^{3}}-\frac{\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)}{a^{3}}-\frac{2}{a^{2}\left(a\left(\tanh^{2}\left(\frac{x}{2}\right)\right)-\left(\tanh^{2}\left(\frac{x}{2}\right)\right)b+a+b\right)}+\frac{\ln\left(a\left(\tanh^{2}\left(\frac{x}{2}\right)\right)-\left(\tanh^{2}\left(\frac{x}{2}\right.\right.\right.}{a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*coth(x)+b*csch(x))^3,x)
```

```
[Out] -1/a^3*ln(tanh(1/2*x)-1)-1/a^3*ln(tanh(1/2*x)+1)-2/a^2/(a*tanh(1/2*x)^2-tan
h(1/2*x)^2*b+a+b)+1/a^3*ln(a*tanh(1/2*x)^2-tanh(1/2*x)^2*b+a+b)+2/(a-b)/(a*
tanh(1/2*x)^2-tanh(1/2*x)^2*b+a+b)^2+2/a/(a-b)/(a*tanh(1/2*x)^2-tanh(1/2*x)
^2*b+a+b)^2*b
```

maxima [B] time = 0.68, size = 111, normalized size = 2.22

$$\frac{2\left(2\,abe^{(-x)}+2\,abe^{(-3\,x)}+\left(a^{2}+3\,b^{2}\right)e^{(-2\,x)}\right)}{4\,a^{4}be^{(-x)}+4\,a^{4}be^{(-3\,x)}+a^{5}e^{(-4\,x)}+a^{5}+2\left(a^{5}+2\,a^{3}b^{2}\right)e^{(-2\,x)}}+\frac{x}{a^{3}}+\frac{\log\left(2\,be^{(-x)}+ae^{(-2\,x)}+a\right)}{a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*coth(x)+b*csch(x))^3,x, algorithm="maxima")
```

```
[Out] 2*(2*a*b*e^(-x) + 2*a*b*e^(-3*x) + (a^2 + 3*b^2)*e^(-2*x))/(4*a^4*b*e^(-x)
+ 4*a^4*b*e^(-3*x) + a^5*e^(-4*x) + a^5 + 2*(a^5 + 2*a^3*b^2)*e^(-2*x)) + x
/a^3 + log(2*b*e^(-x) + a*e^(-2*x) + a)/a^3
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\left(\frac{b}{\sinh(x)} + a\coth(x)\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b/sinh(x) + a*coth(x))^3,x)
```

```
[Out] int(1/(b/sinh(x) + a*coth(x))^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a\coth(x) + b\operatorname{csch}(x))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*coth(x)+b*csch(x))**3,x)
```

```
[Out] Integral((a*coth(x) + b*csch(x))**(-3), x)
```

3.652 $\int \frac{1}{(a\coth(x)+b\,\mathbf{csch}(x))^4}\,dx$

Optimal. Leaf size=159

$$\frac{x}{a^4}-\frac{b\sinh^3(x)}{2a\left(a^2-b^2\right)(a\cosh(x)+b)^2}-\frac{b\left(3a^2-2b^2\right)\tan^{-1}\left(\frac{\sqrt{a-b}\,\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{a^4(a-b)^{3/2}(a+b)^{3/2}}-\frac{\sinh(x)\left(2\left(a^2-b^2\right)-ab\cosh(x)\right)}{2a^3\left(a^2-b^2\right)(a\cosh(x)+b)}-\frac{\text{si}}{3a(a\,\text{co}}$$

```
[Out] x/a^4-b*(3*a^2-2*b^2)*arctan((a-b)^(1/2)*tanh(1/2*x)/(a+b)^(1/2))/a^4/(a-b)
^(3/2)/(a+b)^(3/2)-1/2*(2*a^2-2*b^2-a*b*cosh(x))*sinh(x)/a^3/(a^2-b^2)/(b+a
*cosh(x))-1/3*sinh(x)^3/a/(b+a*cosh(x))^3-1/2*b*sinh(x)^3/a/(a^2-b^2)/(b+a*
cosh(x))^2
```

Rubi [A] time = 0.37, antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.636, Rules used = {4392, 2693, 2864, 2863, 2735, 2659, 205}

$$-\frac{b\left(3a^2-2b^2\right)\tan^{-1}\left(\frac{\sqrt{a-b}\,\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{a^4(a-b)^{3/2}(a+b)^{3/2}}-\frac{b\sinh^3(x)}{2a\left(a^2-b^2\right)(a\cosh(x)+b)^2}-\frac{\sinh(x)\left(2\left(a^2-b^2\right)-ab\cosh(x)\right)}{2a^3\left(a^2-b^2\right)(a\cosh(x)+b)}+\frac{x}{a^4}-\frac{}{3a(a}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Coth[x] + b*Csch[x])^(-4),x]
```

```
[Out] x/a^4 - (b*(3*a^2 - 2*b^2)*ArcTan[(Sqrt[a - b]*Tanh[x/2])/Sqrt[a + b]])/(a^
4*(a - b)^(3/2)*(a + b)^(3/2)) - ((2*(a^2 - b^2) - a*b*Cosh[x])*Sinh[x])/(2
*a^3*(a^2 - b^2)*(b + a*Cosh[x])) - Sinh[x]^3/(3*a*(b + a*Cosh[x])^3) - (b*
Sinh[x]^3)/(2*a*(a^2 - b^2)*(b + a*Cosh[x])^2)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2693

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x
])^(m + 1))/(b*f*(m + 1)), x] + Dist[(g^2*(p - 1))/(b*(m + 1)), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1)*Sin[e + f*x], x], x] /; Free
Q[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && In
tegersQ[2*m, 2*p]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2863

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(g*(g*
Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*(b*c*(m + p + 1) - a*d*p
 + b*d*(m + 1)*Sin[e + f*x]))/(b^2*f*(m + 1)*(m + p + 1)), x] + Dist[(g^2*(
p - 1))/(b^2*(m + 1)*(m + p + 1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[
e + f*x])^(m + 1)*Simp[b*d*(m + 1) + (b*c*(m + p + 1) - a*d*p)*Sin[e + f*x]
, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ
[m, -1] && GtQ[p, 1] && NeQ[m + p + 1, 0] && IntegerQ[2*m]
```

Rule 2864

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[((b*c
 - a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m + 1))/(f*g*(a^2 -
b^2)*(m + 1)), x] + Dist[1/((a^2 - b^2)*(m + 1)), Int[(g*Cos[e + f*x])^p*(a
 + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m + p +
2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[a^
2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 4392

```
Int[(cot[(c_.) + (d_.)*(x_)]^(n_.)*(a_.) + csc[(c_.) + (d_.)*(x_)]^(n_.)*(b
_.))^(p_)*(u_.), x_Symbol] :> Int[ActivateTrig[u]*Csc[c + d*x]^(n*p)*(b + a
*Cos[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

Rubi steps

$$\int \frac{1}{(a\coth(x)+b\text{csch}(x))^4}\,dx = \int \frac{\sinh^4(x)}{(ib+ia\cosh(x))^4}\,dx$$
$$= -\frac{\sinh^3(x)}{3a(b+a\cosh(x))^3} - \frac{i\int \frac{\cosh(x)\sinh^2(x)}{(ib+ia\cosh(x))^3}\,dx}{a}$$
$$= -\frac{\sinh^3(x)}{3a(b+a\cosh(x))^3} - \frac{b\sinh^3(x)}{2a\left(a^2-b^2\right)(b+a\cosh(x))^2} + \frac{i\int \frac{(2ia+ib\cosh(x))\sinh^2(x)}{(ib+ia\cosh(x))^2}\,dx}{2a\left(a^2-b^2\right)}$$
$$= -\frac{\left(2\left(a^2-b^2\right)-ab\cosh(x)\right)\sinh(x)}{2a^3\left(a^2-b^2\right)(b+a\cosh(x))} - \frac{\sinh^3(x)}{3a(b+a\cosh(x))^3} - \frac{b\sinh^3(x)}{2a\left(a^2-b^2\right)(b+a\cosh(x))^2}$$
$$= \frac{x}{a^4} - \frac{\left(2\left(a^2-b^2\right)-ab\cosh(x)\right)\sinh(x)}{2a^3\left(a^2-b^2\right)(b+a\cosh(x))} - \frac{\sinh^3(x)}{3a(b+a\cosh(x))^3} - \frac{b\sinh^3(x)}{2a\left(a^2-b^2\right)(b+\ldots}$$
$$= \frac{x}{a^4} - \frac{\left(2\left(a^2-b^2\right)-ab\cosh(x)\right)\sinh(x)}{2a^3\left(a^2-b^2\right)(b+a\cosh(x))} - \frac{\sinh^3(x)}{3a(b+a\cosh(x))^3} - \frac{b\sinh^3\ldots}{2a\left(a^2-b^2\right)(b+\ldots}$$
$$= \frac{x}{a^4} - \frac{b\left(3a^2-2b^2\right)\tan^{-1}\left(\frac{\sqrt{a-b}\,\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{a^4(a-b)^{3/2}(a+b)^{3/2}} - \frac{\left(2\left(a^2-b^2\right)-ab\cosh(x)\right)\sinh(x)}{2a^3\left(a^2-b^2\right)(b+a\cosh(x))} - \ldots$$

Mathematica [A] time = 0.47, size = 150, normalized size = 0.94

$$\frac{\sinh(x)\left(-\frac{a\left(8a^2-11b^2\right)(a\cosh(x)+b)^2}{(a-b)(a+b)} - \frac{6b\left(2b^2-3a^2\right)\text{csch}(x)(a\cosh(x)+b)^3\tan^{-1}\left(\frac{(b-a)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{3/2}} + 2a\left(a^2-b^2\right) + 7ab(a\cosh(x)+b)\ldots\right.}{6a^4(a\cosh(x)+b)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Coth[x] + b*Csch[x])^(-4),x]
```

```
[Out] ((2*a*(a^2 - b^2) + 7*a*b*(b + a*Cosh[x]) - (a*(8*a^2 - 11*b^2)*(b + a*Cosh
[x])^2)/((a - b)*(a + b)) + 6*x*(b + a*Cosh[x])^3*Csch[x] - (6*b*(-3*a^2 +
2*b^2)*ArcTan[((-a + b)*Tanh[x/2])/Sqrt[a^2 - b^2]]*(b + a*Cosh[x])^3*Csch[
x])/(a^2 - b^2)^(3/2))*Sinh[x])/(6*a^4*(b + a*Cosh[x])^3)
```

fricas [B] time = 0.55, size = 5830, normalized size = 36.67

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*coth(x)+b*csch(x))^4,x, algorithm="fricas")
```

```
[Out] [1/6*(6*(a^7 - 2*a^5*b^2 + a^3*b^4)*x*cosh(x)^6 + 6*(a^7 - 2*a^5*b^2 + a^3*
b^4)*x*sinh(x)^6 + 16*a^7 - 38*a^5*b^2 + 22*a^3*b^4 + 6*(5*a^6*b - 11*a^4*b
^3 + 6*a^2*b^5 + 6*(a^6*b - 2*a^4*b^3 + a^2*b^5)*x)*cosh(x)^5 + 6*(5*a^6*b
- 11*a^4*b^3 + 6*a^2*b^5 + 6*(a^7 - 2*a^5*b^2 + a^3*b^4)*x*cosh(x) + 6*(a^6
*b - 2*a^4*b^3 + a^2*b^5)*x)*sinh(x)^5 + 6*(4*a^7 + 5*a^5*b^2 - 27*a^3*b^4
+ 18*a*b^6 + 3*(a^7 + 2*a^5*b^2 - 7*a^3*b^4 + 4*a*b^6)*x)*cosh(x)^4 + 6*(4*
a^7 + 5*a^5*b^2 - 27*a^3*b^4 + 18*a*b^6 + 15*(a^7 - 2*a^5*b^2 + a^3*b^4)*x*
cosh(x)^2 + 3*(a^7 + 2*a^5*b^2 - 7*a^3*b^4 + 4*a*b^6)*x + 5*(5*a^6*b - 11*a
^4*b^3 + 6*a^2*b^5 + 6*(a^6*b - 2*a^4*b^3 + a^2*b^5)*x)*cosh(x))*sinh(x)^4
+ 4*(24*a^6*b - 41*a^4*b^3 - 5*a^2*b^5 + 22*b^7 + 6*(3*a^6*b - 4*a^4*b^3 -
a^2*b^5 + 2*b^7)*x)*cosh(x)^3 + 4*(24*a^6*b - 41*a^4*b^3 - 5*a^2*b^5 + 22*b
^7 + 30*(a^7 - 2*a^5*b^2 + a^3*b^4)*x*cosh(x)^3 + 15*(5*a^6*b - 11*a^4*b^3
+ 6*a^2*b^5 + 6*(a^6*b - 2*a^4*b^3 + a^2*b^5)*x)*cosh(x)^2 + 6*(3*a^6*b - 4
*a^4*b^3 - a^2*b^5 + 2*b^7)*x + 6*(4*a^7 + 5*a^5*b^2 - 27*a^3*b^4 + 18*a*b^
6 + 3*(a^7 + 2*a^5*b^2 - 7*a^3*b^4 + 4*a*b^6)*x)*cosh(x))*sinh(x)^3 + 6*(4*
a^7 + 8*a^5*b^2 - 38*a^3*b^4 + 26*a*b^6 + 3*(a^7 + 2*a^5*b^2 - 7*a^3*b^4 +
4*a*b^6)*x)*cosh(x)^2 + 6*(4*a^7 + 8*a^5*b^2 - 38*a^3*b^4 + 26*a*b^6 + 15*(
a^7 - 2*a^5*b^2 + a^3*b^4)*x*cosh(x)^4 + 10*(5*a^6*b - 11*a^4*b^3 + 6*a^2*b
^5 + 6*(a^6*b - 2*a^4*b^3 + a^2*b^5)*x)*cosh(x)^3 + 6*(4*a^7 + 5*a^5*b^2 -
27*a^3*b^4 + 18*a*b^6 + 3*(a^7 + 2*a^5*b^2 - 7*a^3*b^4 + 4*a*b^6)*x)*cosh(x
)^2 + 3*(a^7 + 2*a^5*b^2 - 7*a^3*b^4 + 4*a*b^6)*x + 2*(24*a^6*b - 41*a^4*b^
3 - 5*a^2*b^5 + 22*b^7 + 6*(3*a^6*b - 4*a^4*b^3 - a^2*b^5 + 2*b^7)*x)*cosh(
x))*sinh(x)^2 - 3*((3*a^5*b - 2*a^3*b^3)*cosh(x)^6 + (3*a^5*b - 2*a^3*b^3)*
sinh(x)^6 + 3*a^5*b - 2*a^3*b^3 + 6*(3*a^4*b^2 - 2*a^2*b^4)*cosh(x)^5 + 6*(
3*a^4*b^2 - 2*a^2*b^4 + (3*a^5*b - 2*a^3*b^3)*cosh(x))*sinh(x)^5 + 3*(3*a^5
*b + 10*a^3*b^3 - 8*a*b^5)*cosh(x)^4 + 3*(3*a^5*b + 10*a^3*b^3 - 8*a*b^5 +
5*(3*a^5*b - 2*a^3*b^3)*cosh(x)^2 + 10*(3*a^4*b^2 - 2*a^2*b^4)*cosh(x))*sin
h(x)^4 + 4*(9*a^4*b^2 - 4*b^6)*cosh(x)^3 + 4*(9*a^4*b^2 - 4*b^6 + 5*(3*a^5*
b - 2*a^3*b^3)*cosh(x)^3 + 15*(3*a^4*b^2 - 2*a^2*b^4)*cosh(x)^2 + 3*(3*a^5*
b + 10*a^3*b^3 - 8*a*b^5)*cosh(x))*sinh(x)^3 + 3*(3*a^5*b + 10*a^3*b^3 - 8*
a*b^5)*cosh(x)^2 + 3*(3*a^5*b + 10*a^3*b^3 - 8*a*b^5 + 5*(3*a^5*b - 2*a^3*b
^3)*cosh(x)^4 + 20*(3*a^4*b^2 - 2*a^2*b^4)*cosh(x)^3 + 6*(3*a^5*b + 10*a^3*
b^3 - 8*a*b^5)*cosh(x)^2 + 4*(9*a^4*b^2 - 4*b^6)*cosh(x))*sinh(x)^2 + 6*(3*
a^4*b^2 - 2*a^2*b^4)*cosh(x) + 6*(3*a^4*b^2 - 2*a^2*b^4 + (3*a^5*b - 2*a^3*
b^3)*cosh(x)^5 + 5*(3*a^4*b^2 - 2*a^2*b^4)*cosh(x)^4 + 2*(3*a^5*b + 10*a^3*
b^3 - 8*a*b^5)*cosh(x)^3 + 2*(9*a^4*b^2 - 4*b^6)*cosh(x)^2 + (3*a^5*b + 10*
a^3*b^3 - 8*a*b^5)*cosh(x))*sinh(x))*sqrt(-a^2 + b^2)*log((a^2*cosh(x)^2 +
a^2*sinh(x)^2 + 2*a*b*cosh(x) - a^2 + 2*b^2 + 2*(a^2*cosh(x) + a*b)*sinh(x)
 + 2*sqrt(-a^2 + b^2)*(a*cosh(x) + a*sinh(x) + b))/(a*cosh(x)^2 + a*sinh(x)
^2 + 2*b*cosh(x) + 2*(a*cosh(x) + b)*sinh(x) + a)) + 6*(a^7 - 2*a^5*b^2 + a
^3*b^4)*x + 6*(11*a^6*b - 27*a^4*b^3 + 16*a^2*b^5 + 6*(a^6*b - 2*a^4*b^3 +
```

```
a^2*b^5)*x)*cosh(x) + 6*(11*a^6*b - 27*a^4*b^3 + 16*a^2*b^5 + 6*(a^7 - 2*a^
5*b^2 + a^3*b^4)*x*cosh(x)^5 + 5*(5*a^6*b - 11*a^4*b^3 + 6*a^2*b^5 + 6*(a^6
*b - 2*a^4*b^3 + a^2*b^5)*x)*cosh(x)^4 + 4*(4*a^7 + 5*a^5*b^2 - 27*a^3*b^4
+ 18*a*b^6 + 3*(a^7 + 2*a^5*b^2 - 7*a^3*b^4 + 4*a*b^6)*x)*cosh(x)^3 + 2*(24
*a^6*b - 41*a^4*b^3 - 5*a^2*b^5 + 22*b^7 + 6*(3*a^6*b - 4*a^4*b^3 - a^2*b^5
 + 2*b^7)*x)*cosh(x)^2 + 6*(a^6*b - 2*a^4*b^3 + a^2*b^5)*x + 2*(4*a^7 + 8*a
^5*b^2 - 38*a^3*b^4 + 26*a*b^6 + 3*(a^7 + 2*a^5*b^2 - 7*a^3*b^4 + 4*a*b^6)*
x)*cosh(x))*sinh(x))/(a^11 - 2*a^9*b^2 + a^7*b^4 + (a^11 - 2*a^9*b^2 + a^7*
b^4)*cosh(x)^6 + (a^11 - 2*a^9*b^2 + a^7*b^4)*sinh(x)^6 + 6*(a^10*b - 2*a^8
*b^3 + a^6*b^5)*cosh(x)^5 + 6*(a^10*b - 2*a^8*b^3 + a^6*b^5 + (a^11 - 2*a^9
*b^2 + a^7*b^4)*cosh(x))*sinh(x)^5 + 3*(a^11 + 2*a^9*b^2 - 7*a^7*b^4 + 4*a^
5*b^6)*cosh(x)^4 + 3*(a^11 + 2*a^9*b^2 - 7*a^7*b^4 + 4*a^5*b^6 + 5*(a^11 -
2*a^9*b^2 + a^7*b^4)*cosh(x)^2 + 10*(a^10*b - 2*a^8*b^3 + a^6*b^5)*cosh(x))
*sinh(x)^4 + 4*(3*a^10*b - 4*a^8*b^3 - a^6*b^5 + 2*a^4*b^7)*cosh(x)^3 + 4*(
3*a^10*b - 4*a^8*b^3 - a^6*b^5 + 2*a^4*b^7 + 5*(a^11 - 2*a^9*b^2 + a^7*b^4)
*cosh(x)^3 + 15*(a^10*b - 2*a^8*b^3 + a^6*b^5)*cosh(x)^2 + 3*(a^11 + 2*a^9*
b^2 - 7*a^7*b^4 + 4*a^5*b^6)*cosh(x))*sinh(x)^3 + 3*(a^11 + 2*a^9*b^2 - 7*a
^7*b^4 + 4*a^5*b^6)*cosh(x)^2 + 3*(a^11 + 2*a^9*b^2 - 7*a^7*b^4 + 4*a^5*b^6
 + 5*(a^11 - 2*a^9*b^2 + a^7*b^4)*cosh(x)^4 + 20*(a^10*b - 2*a^8*b^3 + a^6*
b^5)*cosh(x)^3 + 6*(a^11 + 2*a^9*b^2 - 7*a^7*b^4 + 4*a^5*b^6)*cosh(x)^2 + 4
*(3*a^10*b - 4*a^8*b^3 - a^6*b^5 + 2*a^4*b^7)*cosh(x))*sinh(x)^2 + 6*(a^10*
b - 2*a^8*b^3 + a^6*b^5)*cosh(x) + 6*(a^10*b - 2*a^8*b^3 + a^6*b^5 + (a^11
- 2*a^9*b^2 + a^7*b^4)*cosh(x)^5 + 5*(a^10*b - 2*a^8*b^3 + a^6*b^5)*cosh(x)
^4 + 2*(a^11 + 2*a^9*b^2 - 7*a^7*b^4 + 4*a^5*b^6)*cosh(x)^3 + 2*(3*a^10*b -
 4*a^8*b^3 - a^6*b^5 + 2*a^4*b^7)*cosh(x)^2 + (a^11 + 2*a^9*b^2 - 7*a^7*b^4
 + 4*a^5*b^6)*cosh(x))*sinh(x)), 1/3*(3*(a^7 - 2*a^5*b^2 + a^3*b^4)*x*cosh(
x)^6 + 3*(a^7 - 2*a^5*b^2 + a^3*b^4)*x*sinh(x)^6 + 8*a^7 - 19*a^5*b^2 + 11*
a^3*b^4 + 3*(5*a^6*b - 11*a^4*b^3 + 6*a^2*b^5 + 6*(a^6*b - 2*a^4*b^3 + a^2*
b^5)*x)*cosh(x)^5 + 3*(5*a^6*b - 11*a^4*b^3 + 6*a^2*b^5 + 6*(a^7 - 2*a^5*b^
2 + a^3*b^4)*x*cosh(x) + 6*(a^6*b - 2*a^4*b^3 + a^2*b^5)*x)*sinh(x)^5 + 3*(
4*a^7 + 5*a^5*b^2 - 27*a^3*b^4 + 18*a*b^6 + 3*(a^7 + 2*a^5*b^2 - 7*a^3*b^4
+ 4*a*b^6)*x)*cosh(x)^4 + 3*(4*a^7 + 5*a^5*b^2 - 27*a^3*b^4 + 18*a*b^6 + 15
*(a^7 - 2*a^5*b^2 + a^3*b^4)*x*cosh(x)^2 + 3*(a^7 + 2*a^5*b^2 - 7*a^3*b^4 +
 4*a*b^6)*x + 5*(5*a^6*b - 11*a^4*b^3 + 6*a^2*b^5 + 6*(a^6*b - 2*a^4*b^3 +
a^2*b^5)*x)*cosh(x))*sinh(x)^4 + 2*(24*a^6*b - 41*a^4*b^3 - 5*a^2*b^5 + 22*
b^7 + 6*(3*a^6*b - 4*a^4*b^3 - a^2*b^5 + 2*b^7)*x)*cosh(x)^3 + 2*(24*a^6*b
- 41*a^4*b^3 - 5*a^2*b^5 + 22*b^7 + 30*(a^7 - 2*a^5*b^2 + a^3*b^4)*x*cosh(x
)^3 + 15*(5*a^6*b - 11*a^4*b^3 + 6*a^2*b^5 + 6*(a^6*b - 2*a^4*b^3 + a^2*b^5
)*x)*cosh(x)^2 + 6*(3*a^6*b - 4*a^4*b^3 - a^2*b^5 + 2*b^7)*x + 6*(4*a^7 + 5
*a^5*b^2 - 27*a^3*b^4 + 18*a*b^6 + 3*(a^7 + 2*a^5*b^2 - 7*a^3*b^4 + 4*a*b^6
)*x)*cosh(x))*sinh(x)^3 + 3*(4*a^7 + 8*a^5*b^2 - 38*a^3*b^4 + 26*a*b^6 + 3*
(a^7 + 2*a^5*b^2 - 7*a^3*b^4 + 4*a*b^6)*x)*cosh(x)^2 + 3*(4*a^7 + 8*a^5*b^2
 - 38*a^3*b^4 + 26*a*b^6 + 15*(a^7 - 2*a^5*b^2 + a^3*b^4)*x*cosh(x)^4 + 10*
(5*a^6*b - 11*a^4*b^3 + 6*a^2*b^5 + 6*(a^6*b - 2*a^4*b^3 + a^2*b^5)*x)*cosh
(x)^3 + 6*(4*a^7 + 5*a^5*b^2 - 27*a^3*b^4 + 18*a*b^6 + 3*(a^7 + 2*a^5*b^2 -
```

```
 7*a^3*b^4 + 4*a*b^6)*x)*cosh(x)^2 + 3*(a^7 + 2*a^5*b^2 - 7*a^3*b^4 + 4*a*b
^6)*x + 2*(24*a^6*b - 41*a^4*b^3 - 5*a^2*b^5 + 22*b^7 + 6*(3*a^6*b - 4*a^4*
b^3 - a^2*b^5 + 2*b^7)*x)*cosh(x))*sinh(x)^2 + 3*((3*a^5*b - 2*a^3*b^3)*cos
h(x)^6 + (3*a^5*b - 2*a^3*b^3)*sinh(x)^6 + 3*a^5*b - 2*a^3*b^3 + 6*(3*a^4*b
^2 - 2*a^2*b^4)*cosh(x)^5 + 6*(3*a^4*b^2 - 2*a^2*b^4 + (3*a^5*b - 2*a^3*b^3
)*cosh(x))*sinh(x)^5 + 3*(3*a^5*b + 10*a^3*b^3 - 8*a*b^5)*cosh(x)^4 + 3*(3*
a^5*b + 10*a^3*b^3 - 8*a*b^5 + 5*(3*a^5*b - 2*a^3*b^3)*cosh(x)^2 + 10*(3*a^
4*b^2 - 2*a^2*b^4)*cosh(x))*sinh(x)^4 + 4*(9*a^4*b^2 - 4*b^6)*cosh(x)^3 + 4
*(9*a^4*b^2 - 4*b^6 + 5*(3*a^5*b - 2*a^3*b^3)*cosh(x)^3 + 15*(3*a^4*b^2 - 2
*a^2*b^4)*cosh(x)^2 + 3*(3*a^5*b + 10*a^3*b^3 - 8*a*b^5)*cosh(x))*sinh(x)^3
 + 3*(3*a^5*b + 10*a^3*b^3 - 8*a*b^5)*cosh(x)^2 + 3*(3*a^5*b + 10*a^3*b^3 -
 8*a*b^5 + 5*(3*a^5*b - 2*a^3*b^3)*cosh(x)^4 + 20*(3*a^4*b^2 - 2*a^2*b^4)*c
osh(x)^3 + 6*(3*a^5*b + 10*a^3*b^3 - 8*a*b^5)*cosh(x)^2 + 4*(9*a^4*b^2 - 4*
b^6)*cosh(x))*sinh(x)^2 + 6*(3*a^4*b^2 - 2*a^2*b^4)*cosh(x) + 6*(3*a^4*b^2
- 2*a^2*b^4 + (3*a^5*b - 2*a^3*b^3)*cosh(x)^5 + 5*(3*a^4*b^2 - 2*a^2*b^4)*c
osh(x)^4 + 2*(3*a^5*b + 10*a^3*b^3 - 8*a*b^5)*cosh(x)^3 + 2*(9*a^4*b^2 - 4*
b^6)*cosh(x)^2 + (3*a^5*b + 10*a^3*b^3 - 8*a*b^5)*cosh(x))*sinh(x))*sqrt(a^
2 - b^2)*arctan(-(a*cosh(x) + a*sinh(x) + b)/sqrt(a^2 - b^2)) + 3*(a^7 - 2*
a^5*b^2 + a^3*b^4)*x + 3*(11*a^6*b - 27*a^4*b^3 + 16*a^2*b^5 + 6*(a^6*b - 2
*a^4*b^3 + a^2*b^5)*x)*cosh(x) + 3*(11*a^6*b - 27*a^4*b^3 + 16*a^2*b^5 + 6*
(a^7 - 2*a^5*b^2 + a^3*b^4)*x*cosh(x)^5 + 5*(5*a^6*b - 11*a^4*b^3 + 6*a^2*b
^5 + 6*(a^6*b - 2*a^4*b^3 + a^2*b^5)*x)*cosh(x)^4 + 4*(4*a^7 + 5*a^5*b^2 -
27*a^3*b^4 + 18*a*b^6 + 3*(a^7 + 2*a^5*b^2 - 7*a^3*b^4 + 4*a*b^6)*x)*cosh(x
)^3 + 2*(24*a^6*b - 41*a^4*b^3 - 5*a^2*b^5 + 22*b^7 + 6*(3*a^6*b - 4*a^4*b^
3 - a^2*b^5 + 2*b^7)*x)*cosh(x)^2 + 6*(a^6*b - 2*a^4*b^3 + a^2*b^5)*x + 2*(
4*a^7 + 8*a^5*b^2 - 38*a^3*b^4 + 26*a*b^6 + 3*(a^7 + 2*a^5*b^2 - 7*a^3*b^4
+ 4*a*b^6)*x)*cosh(x))*sinh(x))/(a^11 - 2*a^9*b^2 + a^7*b^4 + (a^11 - 2*a^9
*b^2 + a^7*b^4)*cosh(x)^6 + (a^11 - 2*a^9*b^2 + a^7*b^4)*sinh(x)^6 + 6*(a^1
0*b - 2*a^8*b^3 + a^6*b^5)*cosh(x)^5 + 6*(a^10*b - 2*a^8*b^3 + a^6*b^5 + (a
^11 - 2*a^9*b^2 + a^7*b^4)*cosh(x))*sinh(x)^5 + 3*(a^11 + 2*a^9*b^2 - 7*a^7
*b^4 + 4*a^5*b^6)*cosh(x)^4 + 3*(a^11 + 2*a^9*b^2 - 7*a^7*b^4 + 4*a^5*b^6 +
 5*(a^11 - 2*a^9*b^2 + a^7*b^4)*cosh(x)^2 + 10*(a^10*b - 2*a^8*b^3 + a^6*b^
5)*cosh(x))*sinh(x)^4 + 4*(3*a^10*b - 4*a^8*b^3 - a^6*b^5 + 2*a^4*b^7)*cosh
(x)^3 + 4*(3*a^10*b - 4*a^8*b^3 - a^6*b^5 + 2*a^4*b^7 + 5*(a^11 - 2*a^9*b^2
 + a^7*b^4)*cosh(x)^3 + 15*(a^10*b - 2*a^8*b^3 + a^6*b^5)*cosh(x)^2 + 3*(a^
11 + 2*a^9*b^2 - 7*a^7*b^4 + 4*a^5*b^6)*cosh(x))*sinh(x)^3 + 3*(a^11 + 2*a^
9*b^2 - 7*a^7*b^4 + 4*a^5*b^6)*cosh(x)^2 + 3*(a^11 + 2*a^9*b^2 - 7*a^7*b^4
+ 4*a^5*b^6 + 5*(a^11 - 2*a^9*b^2 + a^7*b^4)*cosh(x)^4 + 20*(a^10*b - 2*a^8
*b^3 + a^6*b^5)*cosh(x)^3 + 6*(a^11 + 2*a^9*b^2 - 7*a^7*b^4 + 4*a^5*b^6)*co
sh(x)^2 + 4*(3*a^10*b - 4*a^8*b^3 - a^6*b^5 + 2*a^4*b^7)*cosh(x))*sinh(x)^2
 + 6*(a^10*b - 2*a^8*b^3 + a^6*b^5)*cosh(x) + 6*(a^10*b - 2*a^8*b^3 + a^6*b
^5 + (a^11 - 2*a^9*b^2 + a^7*b^4)*cosh(x)^5 + 5*(a^10*b - 2*a^8*b^3 + a^6*b
^5)*cosh(x)^4 + 2*(a^11 + 2*a^9*b^2 - 7*a^7*b^4 + 4*a^5*b^6)*cosh(x)^3 + 2*
(3*a^10*b - 4*a^8*b^3 - a^6*b^5 + 2*a^4*b^7)*cosh(x)^2 + (a^11 + 2*a^9*b^2
- 7*a^7*b^4 + 4*a^5*b^6)*cosh(x))*sinh(x))]
```

giac [A] time = 0.18, size = 242, normalized size = 1.52

$$-\frac{\left(3\,a^{2}b-2\,b^{3}\right)\arctan\left(\frac{ae^{x}+b}{\sqrt{a^{2}-b^{2}}}\right)}{\left(a^{6}-a^{4}b^{2}\right)\sqrt{a^{2}-b^{2}}}+\frac{15\,a^{4}be^{(5\,x)}-18\,a^{2}b^{3}e^{(5\,x)}+12\,a^{5}e^{(4\,x)}+27\,a^{3}b^{2}e^{(4\,x)}-54\,ab^{4}e^{(4\,x)}+48\,a^{4}be^{(3\,x)}}{3\,(a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*coth(x)+b*csch(x))^4,x, algorithm="giac")
```

```
[Out] -(3*a^2*b - 2*b^3)*arctan((a*e^x + b)/sqrt(a^2 - b^2))/((a^6 - a^4*b^2)*sqr
t(a^2 - b^2)) + 1/3*(15*a^4*b*e^(5*x) - 18*a^2*b^3*e^(5*x) + 12*a^5*e^(4*x)
 + 27*a^3*b^2*e^(4*x) - 54*a*b^4*e^(4*x) + 48*a^4*b*e^(3*x) - 34*a^2*b^3*e^
(3*x) - 44*b^5*e^(3*x) + 12*a^5*e^(2*x) + 36*a^3*b^2*e^(2*x) - 78*a*b^4*e^(
2*x) + 33*a^4*b*e^x - 48*a^2*b^3*e^x + 8*a^5 - 11*a^3*b^2)/((a^6 - a^4*b^2)
*(a*e^(2*x) + 2*b*e^x + a)^3) + x/a^4
```

maple [B] time = 0.23, size = 507, normalized size = 3.19

$$-\frac{\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{a^{4}}+\frac{\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)}{a^{4}}-\frac{2\left(\tanh^{5}\left(\frac{x}{2}\right)\right)}{\left(a\left(\tanh^{2}\left(\frac{x}{2}\right)\right)-\left(\tanh^{2}\left(\frac{x}{2}\right)\right)b+a+b\right)^{3}(a+b)}+\frac{(\tanh}{a\left(a\left(\tanh^{2}\left(\frac{x}{2}\right)\right)-\left(\tanh\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*coth(x)+b*csch(x))^4,x)
```

```
[Out] -1/a^4*ln(tanh(1/2*x)-1)+1/a^4*ln(tanh(1/2*x)+1)-2/(a*tanh(1/2*x)^2-tanh(1/
2*x)^2*b+a+b)^3/(a+b)*tanh(1/2*x)^5+1/a/(a*tanh(1/2*x)^2-tanh(1/2*x)^2*b+a+
b)^3/(a+b)*tanh(1/2*x)^5*b+3/a^2/(a*tanh(1/2*x)^2-tanh(1/2*x)^2*b+a+b)^3/(a
+b)*tanh(1/2*x)^5*b^2-2/a^3/(a*tanh(1/2*x)^2-tanh(1/2*x)^2*b+a+b)^3/(a+b)*t
anh(1/2*x)^5*b^3-20/3/a/(a*tanh(1/2*x)^2-tanh(1/2*x)^2*b+a+b)^3*tanh(1/2*x)
^3+4/a^3/(a*tanh(1/2*x)^2-tanh(1/2*x)^2*b+a+b)^3*tanh(1/2*x)^3*b^2-2/(a*tan
h(1/2*x)^2-tanh(1/2*x)^2*b+a+b)^3/(a-b)*tanh(1/2*x)-1/a/(a*tanh(1/2*x)^2-ta
nh(1/2*x)^2*b+a+b)^3/(a-b)*tanh(1/2*x)*b+3/a^2/(a*tanh(1/2*x)^2-tanh(1/2*x)
^2*b+a+b)^3/(a-b)*tanh(1/2*x)*b^2+2/a^3/(a*tanh(1/2*x)^2-tanh(1/2*x)^2*b+a+
b)^3/(a-b)*tanh(1/2*x)*b^3-3/a^2*b/(a^2-b^2)/((a+b)*(a-b))^(1/2)*arctan((a-
b)*tanh(1/2*x)/((a+b)*(a-b))^(1/2))+2/a^4*b^3/(a^2-b^2)/((a+b)*(a-b))^(1/2)
*arctan((a-b)*tanh(1/2*x)/((a+b)*(a-b))^(1/2))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*coth(x)+b*csch(x))^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(\frac{b}{\sinh(x)} + a\coth(x)\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b/sinh(x) + a*coth(x))^4,x)
```

```
[Out] int(1/(b/sinh(x) + a*coth(x))^4, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a\coth\left(x\right) + b\operatorname{csch}\left(x\right)\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*coth(x)+b*csch(x))**4,x)
```

```
[Out] Integral((a*coth(x) + b*csch(x))**(-4), x)
```

3.653 $\int \frac{1}{(a\coth(x)+b\,\mathbf{csch}(x))^5}\,dx$

Optimal. Leaf size=98

$$\frac{4b}{a^5(a\cosh(x)+b)}+\frac{\log(a\cosh(x)+b)}{a^5}-\frac{\left(a^2-b^2\right)^2}{4a^5(a\cosh(x)+b)^4}-\frac{4b\left(a^2-b^2\right)}{3a^5(a\cosh(x)+b)^3}+\frac{a^2-3b^2}{a^5(a\cosh(x)+b)^2}$$

```
[Out] -1/4*(a^2-b^2)^2/a^5/(b+a*cosh(x))^4-4/3*b*(a^2-b^2)/a^5/(b+a*cosh(x))^3+(a
^2-3*b^2)/a^5/(b+a*cosh(x))^2+4*b/a^5/(b+a*cosh(x))+ln(b+a*cosh(x))/a^5
```

Rubi [A] time = 0.16, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {4392, 2668, 697}

$$-\frac{\left(a^2-b^2\right)^2}{4a^5(a\cosh(x)+b)^4}-\frac{4b\left(a^2-b^2\right)}{3a^5(a\cosh(x)+b)^3}+\frac{a^2-3b^2}{a^5(a\cosh(x)+b)^2}+\frac{4b}{a^5(a\cosh(x)+b)}+\frac{\log(a\cosh(x)+b)}{a^5}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Coth[x] + b*Csch[x])^(-5),x]
```

```
[Out] -(a^2 - b^2)^2/(4*a^5*(b + a*Cosh[x])^4) - (4*b*(a^2 - b^2))/(3*a^5*(b + a*
Cosh[x])^3) + (a^2 - 3*b^2)/(a^5*(b + a*Cosh[x])^2) + (4*b)/(a^5*(b + a*Cos
h[x])) + Log[b + a*Cosh[x]]/a^5
```

Rule 697

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 4392

```
Int[(cot[(c_.) + (d_.)*(x_)]^(n_.)*(a_.) + csc[(c_.) + (d_.)*(x_)]^(n_.)*(b
_.))^(p_)*(u_.), x_Symbol] :> Int[ActivateTrig[u]*Csc[c + d*x]^(n*p)*(b + a
*Cos[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

Rubi steps

$$\int \frac{1}{(a\coth(x)+b\text{csch}(x))^5}\,dx = i\int \frac{\sinh^5(x)}{(ib+ia\cosh(x))^5}\,dx$$
$$= \frac{\text{Subst}\left(\int \frac{\left(-a^2-x^2\right)^2}{(ib+x)^5}\,dx, x, ia\cosh(x)\right)}{a^5}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{\left(a^2-b^2\right)^2}{(ib+x)^5}+\frac{4ib\left(-a^2+b^2\right)}{(ib+x)^4}+\frac{2\left(a^2-3b^2\right)}{(ib+x)^3}-\frac{4ib}{(ib+x)^2}+\frac{1}{ib+x}\right)dx, x, ia\cosh(x)\right)}{a^5}$$
$$= -\frac{\left(a^2-b^2\right)^2}{4a^5(b+a\cosh(x))^4}-\frac{4b\left(a^2-b^2\right)}{3a^5(b+a\cosh(x))^3}+\frac{a^2-3b^2}{a^5(b+a\cosh(x))^2}+\frac{4b}{a^5(b+a\,\text{c}}$$

Mathematica [A] time = 0.32, size = 138, normalized size = 1.41

$$\frac{12a^4\cosh^4(x)\log(a\cosh(x)+b)-3a^4+48a^3b\cosh^3(x)(\log(a\cosh(x)+b)+1)+12a^2\cosh^2(x)\left(a^2+6b^2\log\right.}{12a^5(a\,\text{c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Coth[x] + b*Csch[x])^(-5),x]
```

```
[Out] (-3*a^4 + 2*a^2*b^2 + 25*b^4 + 12*b^4*Log[b + a*Cosh[x]] + 12*a^4*Cosh[x]^4
*Log[b + a*Cosh[x]] + 48*a^3*b*Cosh[x]^3*(1 + Log[b + a*Cosh[x]]) + 12*a^2*
Cosh[x]^2*(a^2 + 9*b^2 + 6*b^2*Log[b + a*Cosh[x]]) + 8*a*b*Cosh[x]*(a^2 + 1
1*b^2 + 6*b^2*Log[b + a*Cosh[x]]))/(12*a^5*(b + a*Cosh[x])^4)
```

fricas [B] time = 0.49, size = 2564, normalized size = 26.16

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*coth(x)+b*csch(x))^5,x, algorithm="fricas")
```

```
[Out] -1/3*(3*a^4*x*cosh(x)^8 + 3*a^4*x*sinh(x)^8 + 24*(a^3*b*x - a^3*b)*cosh(x)^
7 + 24*(a^4*x*cosh(x) + a^3*b*x - a^3*b)*sinh(x)^7 - 12*(a^4 + 9*a^2*b^2 -
(a^4 + 6*a^2*b^2)*x)*cosh(x)^6 + 12*(7*a^4*x*cosh(x)^2 - a^4 - 9*a^2*b^2 +
(a^4 + 6*a^2*b^2)*x + 14*(a^3*b*x - a^3*b)*cosh(x))*sinh(x)^6 - 8*(11*a^3*b
 + 22*a*b^3 - 3*(3*a^3*b + 4*a*b^3)*x)*cosh(x)^5 + 8*(21*a^4*x*cosh(x)^3 -
11*a^3*b - 22*a*b^3 + 63*(a^3*b*x - a^3*b)*cosh(x)^2 + 3*(3*a^3*b + 4*a*b^3
```

)*x - 9*(a^4 + 9*a^2*b^2 - (a^4 + 6*a^2*b^2)*x)*cosh(x))*sinh(x)^5 + 3*a^4*
x - 2*(6*a^4 + 112*a^2*b^2 + 50*b^4 - 3*(3*a^4 + 24*a^2*b^2 + 8*b^4)*x)*cos
h(x)^4 + 2*(105*a^4*x*cosh(x)^4 - 6*a^4 - 112*a^2*b^2 - 50*b^4 + 420*(a^3*b
*x - a^3*b)*cosh(x)^3 - 90*(a^4 + 9*a^2*b^2 - (a^4 + 6*a^2*b^2)*x)*cosh(x)^
2 + 3*(3*a^4 + 24*a^2*b^2 + 8*b^4)*x - 20*(11*a^3*b + 22*a*b^3 - 3*(3*a^3*b
 + 4*a*b^3)*x)*cosh(x))*sinh(x)^4 - 8*(11*a^3*b + 22*a*b^3 - 3*(3*a^3*b + 4
*a*b^3)*x)*cosh(x)^3 + 8*(21*a^4*x*cosh(x)^5 + 105*(a^3*b*x - a^3*b)*cosh(x
)^4 - 11*a^3*b - 22*a*b^3 - 30*(a^4 + 9*a^2*b^2 - (a^4 + 6*a^2*b^2)*x)*cosh
(x)^3 - 10*(11*a^3*b + 22*a*b^3 - 3*(3*a^3*b + 4*a*b^3)*x)*cosh(x)^2 + 3*(3
*a^3*b + 4*a*b^3)*x - (6*a^4 + 112*a^2*b^2 + 50*b^4 - 3*(3*a^4 + 24*a^2*b^2
 + 8*b^4)*x)*cosh(x))*sinh(x)^3 - 12*(a^4 + 9*a^2*b^2 - (a^4 + 6*a^2*b^2)*x
)*cosh(x)^2 + 4*(21*a^4*x*cosh(x)^6 + 126*(a^3*b*x - a^3*b)*cosh(x)^5 - 45*
(a^4 + 9*a^2*b^2 - (a^4 + 6*a^2*b^2)*x)*cosh(x)^4 - 3*a^4 - 27*a^2*b^2 - 20
*(11*a^3*b + 22*a*b^3 - 3*(3*a^3*b + 4*a*b^3)*x)*cosh(x)^3 - 3*(6*a^4 + 112
*a^2*b^2 + 50*b^4 - 3*(3*a^4 + 24*a^2*b^2 + 8*b^4)*x)*cosh(x)^2 + 3*(a^4 +
6*a^2*b^2)*x - 6*(11*a^3*b + 22*a*b^3 - 3*(3*a^3*b + 4*a*b^3)*x)*cosh(x))*s
inh(x)^2 + 24*(a^3*b*x - a^3*b)*cosh(x) - 3*(a^4*cosh(x)^8 + a^4*sinh(x)^8
+ 8*a^3*b*cosh(x)^7 + 8*(a^4*cosh(x) + a^3*b)*sinh(x)^7 + 4*(a^4 + 6*a^2*b^
2)*cosh(x)^6 + 4*(7*a^4*cosh(x)^2 + 14*a^3*b*cosh(x) + a^4 + 6*a^2*b^2)*sin
h(x)^6 + 8*(3*a^3*b + 4*a*b^3)*cosh(x)^5 + 8*(7*a^4*cosh(x)^3 + 21*a^3*b*co
sh(x)^2 + 3*a^3*b + 4*a*b^3 + 3*(a^4 + 6*a^2*b^2)*cosh(x))*sinh(x)^5 + 8*a^
3*b*cosh(x) + 2*(3*a^4 + 24*a^2*b^2 + 8*b^4)*cosh(x)^4 + 2*(35*a^4*cosh(x)^
4 + 140*a^3*b*cosh(x)^3 + 3*a^4 + 24*a^2*b^2 + 8*b^4 + 30*(a^4 + 6*a^2*b^2)
cosh(x)^2 + 20(3*a^3*b + 4*a*b^3)*cosh(x))*sinh(x)^4 + a^4 + 8*(3*a^3*b +
 4*a*b^3)*cosh(x)^3 + 8*(7*a^4*cosh(x)^5 + 35*a^3*b*cosh(x)^4 + 3*a^3*b + 4
*a*b^3 + 10*(a^4 + 6*a^2*b^2)*cosh(x)^3 + 10*(3*a^3*b + 4*a*b^3)*cosh(x)^2
+ (3*a^4 + 24*a^2*b^2 + 8*b^4)*cosh(x))*sinh(x)^3 + 4*(a^4 + 6*a^2*b^2)*cos
h(x)^2 + 4*(7*a^4*cosh(x)^6 + 42*a^3*b*cosh(x)^5 + 15*(a^4 + 6*a^2*b^2)*cos
h(x)^4 + a^4 + 6*a^2*b^2 + 20*(3*a^3*b + 4*a*b^3)*cosh(x)^3 + 3*(3*a^4 + 24
*a^2*b^2 + 8*b^4)*cosh(x)^2 + 6*(3*a^3*b + 4*a*b^3)*cosh(x))*sinh(x)^2 + 8*
(a^4*cosh(x)^7 + 7*a^3*b*cosh(x)^6 + 3*(a^4 + 6*a^2*b^2)*cosh(x)^5 + 5*(3*a
^3*b + 4*a*b^3)*cosh(x)^4 + a^3*b + (3*a^4 + 24*a^2*b^2 + 8*b^4)*cosh(x)^3
+ 3*(3*a^3*b + 4*a*b^3)*cosh(x)^2 + (a^4 + 6*a^2*b^2)*cosh(x))*sinh(x))*log
(2*(a*cosh(x) + b)/(cosh(x) - sinh(x))) + 8*(3*a^4*x*cosh(x)^7 + 21*(a^3*b*
x - a^3*b)*cosh(x)^6 - 9*(a^4 + 9*a^2*b^2 - (a^4 + 6*a^2*b^2)*x)*cosh(x)^5
+ 3*a^3*b*x - 5*(11*a^3*b + 22*a*b^3 - 3*(3*a^3*b + 4*a*b^3)*x)*cosh(x)^4 -
 3*a^3*b - (6*a^4 + 112*a^2*b^2 + 50*b^4 - 3*(3*a^4 + 24*a^2*b^2 + 8*b^4)*x
)*cosh(x)^3 - 3*(11*a^3*b + 22*a*b^3 - 3*(3*a^3*b + 4*a*b^3)*x)*cosh(x)^2 -
 3*(a^4 + 9*a^2*b^2 - (a^4 + 6*a^2*b^2)*x)*cosh(x))*sinh(x))/(a^9*cosh(x)^8
 + a^9*sinh(x)^8 + 8*a^8*b*cosh(x)^7 + 8*a^8*b*cosh(x) + a^9 + 8*(a^9*cosh(
x) + a^8*b)*sinh(x)^7 + 4*(a^9 + 6*a^7*b^2)*cosh(x)^6 + 4*(7*a^9*cosh(x)^2
+ 14*a^8*b*cosh(x) + a^9 + 6*a^7*b^2)*sinh(x)^6 + 8*(3*a^8*b + 4*a^6*b^3)*c
osh(x)^5 + 8*(7*a^9*cosh(x)^3 + 21*a^8*b*cosh(x)^2 + 3*a^8*b + 4*a^6*b^3 +
3*(a^9 + 6*a^7*b^2)*cosh(x))*sinh(x)^5 + 2*(3*a^9 + 24*a^7*b^2 + 8*a^5*b^4)
cosh(x)^4 + 2(35*a^9*cosh(x)^4 + 140*a^8*b*cosh(x)^3 + 3*a^9 + 24*a^7*b^2

```
 + 8*a^5*b^4 + 30*(a^9 + 6*a^7*b^2)*cosh(x)^2 + 20*(3*a^8*b + 4*a^6*b^3)*co
sh(x))*sinh(x)^4 + 8*(3*a^8*b + 4*a^6*b^3)*cosh(x)^3 + 8*(7*a^9*cosh(x)^5 +
 35*a^8*b*cosh(x)^4 + 3*a^8*b + 4*a^6*b^3 + 10*(a^9 + 6*a^7*b^2)*cosh(x)^3
+ 10*(3*a^8*b + 4*a^6*b^3)*cosh(x)^2 + (3*a^9 + 24*a^7*b^2 + 8*a^5*b^4)*cos
h(x))*sinh(x)^3 + 4*(a^9 + 6*a^7*b^2)*cosh(x)^2 + 4*(7*a^9*cosh(x)^6 + 42*a
^8*b*cosh(x)^5 + a^9 + 6*a^7*b^2 + 15*(a^9 + 6*a^7*b^2)*cosh(x)^4 + 20*(3*a
^8*b + 4*a^6*b^3)*cosh(x)^3 + 3*(3*a^9 + 24*a^7*b^2 + 8*a^5*b^4)*cosh(x)^2
+ 6*(3*a^8*b + 4*a^6*b^3)*cosh(x))*sinh(x)^2 + 8*(a^9*cosh(x)^7 + 7*a^8*b*c
osh(x)^6 + a^8*b + 3*(a^9 + 6*a^7*b^2)*cosh(x)^5 + 5*(3*a^8*b + 4*a^6*b^3)*
cosh(x)^4 + (3*a^9 + 24*a^7*b^2 + 8*a^5*b^4)*cosh(x)^3 + 3*(3*a^8*b + 4*a^6
*b^3)*cosh(x)^2 + (a^9 + 6*a^7*b^2)*cosh(x))*sinh(x))
```

giac [A] time = 0.18, size = 135, normalized size = 1.38

$$\frac{\log\left(\left|a\left(e^{(-x)}+e^{x}\right)+2\,b\right|\right)}{a^5}-\frac{25\,a^3\left(e^{(-x)}+e^{x}\right)^4+104\,a^2b\left(e^{(-x)}+e^{x}\right)^3-48\,a^3\left(e^{(-x)}+e^{x}\right)^2+168\,ab^2\left(e^{(-x)}+e^{x}\right)^2-}{12\left(a\left(e^{(-x)}+e^{x}\right)+2\,b\right)^4a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*coth(x)+b*csch(x))^5,x, algorithm="giac")
```

```
[Out] log(abs(a*(e^(-x) + e^x) + 2*b))/a^5 - 1/12*(25*a^3*(e^(-x) + e^x)^4 + 104*
a^2*b*(e^(-x) + e^x)^3 - 48*a^3*(e^(-x) + e^x)^2 + 168*a*b^2*(e^(-x) + e^x)
^2 - 64*a^2*b*(e^(-x) + e^x) + 96*b^3*(e^(-x) + e^x) + 48*a^3 - 32*a*b^2)/(
(a*(e^(-x) + e^x) + 2*b)^4*a^4)
```

maple [B] time = 0.24, size = 309, normalized size = 3.15

$$-\frac{\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{a^5}-\frac{\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)}{a^5}+\frac{8}{(a-b)^2\left(a\left(\tanh^2\left(\frac{x}{2}\right)\right)-\left(\tanh^2\left(\frac{x}{2}\right)\right)b+a+b\right)^3}+\frac{}{3a\,(a-b)^2\left(a\left(\tanh^2\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*coth(x)+b*csch(x))^5,x)
```

```
[Out] -1/a^5*ln(tanh(1/2*x)-1)-1/a^5*ln(tanh(1/2*x)+1)+8/(a-b)^2/(a*tanh(1/2*x)^2
-tanh(1/2*x)^2*b+a+b)^3+16/3/a/(a-b)^2/(a*tanh(1/2*x)^2-tanh(1/2*x)^2*b+a+b
)^3*b-8/3/a^2/(a-b)^2/(a*tanh(1/2*x)^2-tanh(1/2*x)^2*b+a+b)^3*b^2+1/a^5*ln(
a*tanh(1/2*x)^2-tanh(1/2*x)^2*b+a+b)-2/a^3/(a*tanh(1/2*x)^2-tanh(1/2*x)^2*b
+a+b)^2-4*a/(a-b)^2/(a*tanh(1/2*x)^2-tanh(1/2*x)^2*b+a+b)^4-8/(a-b)^2/(a*ta
nh(1/2*x)^2-tanh(1/2*x)^2*b+a+b)^4*b-4/a/(a-b)^2/(a*tanh(1/2*x)^2-tanh(1/2*
x)^2*b+a+b)^4*b^2-2/a^4/(a*tanh(1/2*x)^2-tanh(1/2*x)^2*b+a+b)
```

maxima [B] time = 0.60, size = 285, normalized size = 2.91

$$\frac{4\left(6\,a^3be^{(-x)} + 6\,a^3be^{(-7\,x)} + 3\left(a^4 + 9\,a^2b^2\right)e^{(-2\,x)} + 22\left(a^3b + 2\,ab^3\right)e^{(-3\,x)} + \left(3\,a^4 + 56\,a^2b^2 + 25\,b^4\right)e^{(-}}{3\left(8\,a^8be^{(-x)} + 8\,a^8be^{(-7\,x)} + a^9e^{(-8\,x)} + a^9 + 4\left(a^9 + 6\,a^7b^2\right)e^{(-2\,x)} + 8\left(3\,a^8b + 4\,a^6b^3\right)e^{(-3\,x)} + 2\left(3\,a^9 + 24\,a^7b^2 + 8\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*coth(x)+b*csch(x))^5,x, algorithm="maxima")
```

```
[Out] 4/3*(6*a^3*b*e^(-x) + 6*a^3*b*e^(-7*x) + 3*(a^4 + 9*a^2*b^2)*e^(-2*x) + 22*
(a^3*b + 2*a*b^3)*e^(-3*x) + (3*a^4 + 56*a^2*b^2 + 25*b^4)*e^(-4*x) + 22*(a
^3*b + 2*a*b^3)*e^(-5*x) + 3*(a^4 + 9*a^2*b^2)*e^(-6*x))/(8*a^8*b*e^(-x) +
8*a^8*b*e^(-7*x) + a^9*e^(-8*x) + a^9 + 4*(a^9 + 6*a^7*b^2)*e^(-2*x) + 8*(3
*a^8*b + 4*a^6*b^3)*e^(-3*x) + 2*(3*a^9 + 24*a^7*b^2 + 8*a^5*b^4)*e^(-4*x)
+ 8*(3*a^8*b + 4*a^6*b^3)*e^(-5*x) + 4*(a^9 + 6*a^7*b^2)*e^(-6*x)) + x/a^5
+ log(2*b*e^(-x) + a*e^(-2*x) + a)/a^5
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(\frac{b}{\sinh(x)} + a\coth(x)\right)^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b/sinh(x) + a*coth(x))^5,x)
```

```
[Out] int(1/(b/sinh(x) + a*coth(x))^5, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a\coth\left(x\right) + b\operatorname{csch}\left(x\right)\right)^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*coth(x)+b*csch(x))**5,x)
```

```
[Out] Integral((a*coth(x) + b*csch(x))**(-5), x)
```

3.654 $\int (\coth(x) + \operatorname{csch}(x))^5 \, dx$

Optimal. Leaf size=28

$$\frac{4}{1-\cosh(x)} - \frac{2}{(1-\cosh(x))^2} + \log(1-\cosh(x))$$

```
[Out] -2/(1-cosh(x))^2+4/(1-cosh(x))+ln(1-cosh(x))
```

Rubi [A] time = 0.07, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {4392, 2667, 43}

$$\frac{4}{1-\cosh(x)} - \frac{2}{(1-\cosh(x))^2} + \log(1-\cosh(x))$$

Antiderivative was successfully verified.

```
[In] Int[(Coth[x] + Csch[x])^5,x]
```

```
[Out] -2/(1 - Cosh[x])^2 + 4/(1 - Cosh[x]) + Log[1 - Cosh[x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 4392

```
Int[(cot[(c_.) + (d_.)*(x_)]^(n_.)*(a_.) + csc[(c_.) + (d_.)*(x_)]^(n_.)*(b
_.))^(p_)*(u_.), x_Symbol] :> Int[ActivateTrig[u]*Csc[c + d*x]^(n*p)*(b + a
*Cos[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

Rubi steps

$$\int (\coth(x)+\operatorname{csch}(x))^5\,dx = -\left(i\int (i+i\cosh(x))^5\operatorname{csch}^5(x)\,dx\right)$$
$$= -\operatorname{Subst}\left(\int \frac{(i+x)^2}{(i-x)^3}\,dx, x, i\cosh(x)\right)$$
$$= -\operatorname{Subst}\left(\int \left(\frac{1}{i-x}+\frac{4}{(-i+x)^3}-\frac{4i}{(-i+x)^2}\right)dx, x, i\cosh(x)\right)$$
$$= \frac{2}{(i-i\cosh(x))^2}+\frac{4i}{i-i\cosh(x)}+\log(1-\cosh(x))$$

Mathematica [A] time = 0.09, size = 53, normalized size = 1.89

$$-\frac{1}{2}\operatorname{csch}^4\left(\frac{x}{2}\right)-2\operatorname{csch}^2\left(\frac{x}{2}\right)+6\log\left(\sinh\left(\frac{x}{2}\right)\right)+\log(\sinh(x))-5\log\left(\tanh\left(\frac{x}{2}\right)\right)-6\log\left(\cosh\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Coth[x] + Csch[x])^5,x]
```

```
[Out] -2*Csch[x/2]^2 - Csch[x/2]^4/2 - 6*Log[Cosh[x/2]] + 6*Log[Sinh[x/2]] + Log[
Sinh[x]] - 5*Log[Tanh[x/2]]
```

fricas [B] time = 0.43, size = 270, normalized size = 9.64

$$-\frac{x\cosh(x)^4+x\sinh(x)^4-4\,(x-2)\cosh(x)^3+4\,(x\cosh(x)-x+2)\sinh(x)^3+2\,(3\,x-4)\cosh(x)^2+2\left(3\,x\cos\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((coth(x)+csch(x))^5,x, algorithm="fricas")
```

```
[Out] -(x*cosh(x)^4 + x*sinh(x)^4 - 4*(x - 2)*cosh(x)^3 + 4*(x*cosh(x) - x + 2)*s
inh(x)^3 + 2*(3*x - 4)*cosh(x)^2 + 2*(3*x*cosh(x)^2 - 6*(x - 2)*cosh(x) + 3
*x - 4)*sinh(x)^2 - 4*(x - 2)*cosh(x) - 2*(cosh(x)^4 + 4*(cosh(x) - 1)*sinh
(x)^3 + sinh(x)^4 - 4*cosh(x)^3 + 6*(cosh(x)^2 - 2*cosh(x) + 1)*sinh(x)^2 +
 6*cosh(x)^2 + 4*(cosh(x)^3 - 3*cosh(x)^2 + 3*cosh(x) - 1)*sinh(x) - 4*cosh
(x) + 1)*log(cosh(x) + sinh(x) - 1) + 4*(x*cosh(x)^3 - 3*(x - 2)*cosh(x)^2
+ (3*x - 4)*cosh(x) - x + 2)*sinh(x) + x)/(cosh(x)^4 + 4*(cosh(x) - 1)*sinh
(x)^3 + sinh(x)^4 - 4*cosh(x)^3 + 6*(cosh(x)^2 - 2*cosh(x) + 1)*sinh(x)^2 +
 6*cosh(x)^2 + 4*(cosh(x)^3 - 3*cosh(x)^2 + 3*cosh(x) - 1)*sinh(x) - 4*cosh
(x) + 1)
```

giac [A] time = 0.12, size = 33, normalized size = 1.18

$$-x - \frac{8\left(e^{(3x)} - e^{(2x)} + e^{x}\right)}{(e^{x} - 1)^{4}} + 2\log\left(|e^{x} - 1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((coth(x)+csch(x))^5,x, algorithm="giac")
```

```
[Out] -x - 8*(e^(3*x) - e^(2*x) + e^x)/(e^x - 1)^4 + 2*log(abs(e^x - 1))
```

maple [B] time = 0.32, size = 71, normalized size = 2.54

$$\ln(\sinh(x)) - \frac{\left(\coth^{2}(x)\right)}{2} - \frac{\left(\coth^{4}(x)\right)}{4} - \frac{5\left(\cosh^{3}(x)\right)}{\sinh(x)^{4}} + \frac{5\cosh(x)}{3\sinh(x)^{4}} + \frac{8\left(-\frac{\operatorname{csch}(x)^{3}}{4} + \frac{3\operatorname{csch}(x)}{8}\right)\coth(x)}{3} - 2\operatorname{arctanh}(e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((coth(x)+csch(x))^5,x)
```

```
[Out] ln(sinh(x))-1/2*coth(x)^2-1/4*coth(x)^4-5/sinh(x)^4*cosh(x)^3+5/3/sinh(x)^4
*cosh(x)+8/3*(-1/4*csch(x)^3+3/8*csch(x))*coth(x)-2*arctanh(exp(x))-5/sinh(
x)^4*cosh(x)^2+5/4/sinh(x)^4
```

maxima [B] time = 0.58, size = 236, normalized size = 8.43

$$-\frac{5}{2}\coth(x)^{4} + x + \frac{5\left(5e^{(-x)} + 3e^{(-3x)} + 3e^{(-5x)} + 5e^{(-7x)}\right)}{4\left(4e^{(-2x)} - 6e^{(-4x)} + 4e^{(-6x)} - e^{(-8x)} - 1\right)} - \frac{3e^{(-x)} - 11e^{(-3x)} - 11e^{(-5x)} + 3e^{(-7x)}}{4\left(4e^{(-2x)} - 6e^{(-4x)} + 4e^{(-6x)} - e^{(-8x)} - 1\right)} + \frac{5}{2\left(4e\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((coth(x)+csch(x))^5,x, algorithm="maxima")
```

```
[Out] -5/2*coth(x)^4 + x + 5/4*(5*e^(-x) + 3*e^(-3*x) + 3*e^(-5*x) + 5*e^(-7*x))/
(4*e^(-2*x) - 6*e^(-4*x) + 4*e^(-6*x) - e^(-8*x) - 1) - 1/4*(3*e^(-x) - 11*
e^(-3*x) - 11*e^(-5*x) + 3*e^(-7*x))/(4*e^(-2*x) - 6*e^(-4*x) + 4*e^(-6*x)
- e^(-8*x) - 1) + 5/2*(e^(-x) + 7*e^(-3*x) + 7*e^(-5*x) + e^(-7*x))/(4*e^(-
2*x) - 6*e^(-4*x) + 4*e^(-6*x) - e^(-8*x) - 1) + 4*(e^(-2*x) - e^(-4*x) + e
^(-6*x))/(4*e^(-2*x) - 6*e^(-4*x) + 4*e^(-6*x) - e^(-8*x) - 1) - 20/(e^(-x)
 - e^x)^4 + 2*log(e^(-x) - 1)
```

mupad [B] time = 1.52, size = 81, normalized size = 2.89

$$2\ln\left(e^{x} - 1\right) - x + \frac{16}{3e^{2x} - e^{3x} - 3e^{x} + 1} - \frac{16}{e^{2x} - 2e^{x} + 1} - \frac{8}{6e^{2x} - 4e^{3x} + e^{4x} - 4e^{x} + 1} - \frac{8}{e^{x} - 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((coth(x) + 1/sinh(x))^5,x)
```

```
[Out] 2*log(exp(x) - 1) - x + 16/(3*exp(2*x) - exp(3*x) - 3*exp(x) + 1) - 16/(exp
(2*x) - 2*exp(x) + 1) - 8/(6*exp(2*x) - 4*exp(3*x) + exp(4*x) - 4*exp(x) +
1) - 8/(exp(x) - 1)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (\coth(x) + \operatorname{csch}(x))^5 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((coth(x)+csch(x))**5,x)
```

```
[Out] Integral((coth(x) + csch(x))**5, x)
```

3.655 $\int (\coth(x)+\mathrm{csch}(x))^4\,dx$

Optimal. Leaf size=30

$$x+\frac{2\sinh^3(x)}{3(1-\cosh(x))^3}+\frac{2\sinh(x)}{1-\cosh(x)}$$

```
[Out] x+2*sinh(x)/(1-cosh(x))+2/3*sinh(x)^3/(1-cosh(x))^3
```

Rubi [A] time = 0.11, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {4392, 2670, 2680, 8}

$$x+\frac{2\sinh^3(x)}{3(1-\cosh(x))^3}+\frac{2\sinh(x)}{1-\cosh(x)}$$

Antiderivative was successfully verified.

```
[In] Int[(Coth[x] + Csch[x])^4,x]
```

```
[Out] x + (2*Sinh[x])/(1 - Cosh[x]) + (2*Sinh[x]^3)/(3*(1 - Cosh[x])^3)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2670

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*m), Int[(g*Cos[e + f*x])^(2*m + p)/(a
 - b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2,
 0] && IntegerQ[m] && LtQ[p, -1] && GeQ[2*m + p, 0]
```

Rule 2680

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(2*g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f
*x])^(m + 1))/(b*f*(2*m + p + 1)), x] + Dist[(g^2*(p - 1))/(b^2*(2*m + p +
1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 4392

```
Int[(cot[(c_.) + (d_.)*(x_)]^(n_.)*(a_.) + csc[(c_.) + (d_.)*(x_)]^(n_.)*(b
_.))^(p_)*(u_.), x_Symbol] :> Int[ActivateTrig[u]*Csc[c + d*x]^(n*p)*(b + a
```

```
*Cos[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

Rubi steps

$$\begin{aligned}\int (\coth(x)+\operatorname{csch}(x))^4\,dx &= \int (i+i\cosh(x))^4\operatorname{csch}^4(x)\,dx\\ &= \int \frac{\sinh^4(x)}{(i-i\cosh(x))^4}\,dx\\ &= \frac{2\sinh^3(x)}{3(1-\cosh(x))^3}-\int \frac{\sinh^2(x)}{(i-i\cosh(x))^2}\,dx\\ &= \frac{2\sinh(x)}{1-\cosh(x)}+\frac{2\sinh^3(x)}{3(1-\cosh(x))^3}+\int 1\,dx\\ &= x+\frac{2\sinh(x)}{1-\cosh(x)}+\frac{2\sinh^3(x)}{3(1-\cosh(x))^3}\end{aligned}$$

Mathematica [A] time = 0.05, size = 30, normalized size = 1.00

$$x-\frac{8}{3}\coth\left(\frac{x}{2}\right)-\frac{2}{3}\coth\left(\frac{x}{2}\right)\operatorname{csch}^2\left(\frac{x}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Coth[x] + Csch[x])^4,x]
```

```
[Out] x - (8*Coth[x/2])/3 - (2*Coth[x/2]*Csch[x/2]^2)/3
```

fricas [B] time = 0.44, size = 68, normalized size = 2.27

$$\frac{3\,x\cosh(x)^2+3\,x\sinh(x)^2-4\,(3\,x+10)\cosh(x)+2\,(3\,x\cosh(x)-3\,x-4)\sinh(x)+9\,x+24}{3\left(\cosh(x)^2+2\,(\cosh(x)-1)\sinh(x)+\sinh(x)^2-4\,\cosh(x)+3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((coth(x)+csch(x))^4,x, algorithm="fricas")
```

```
[Out] 1/3*(3*x*cosh(x)^2 + 3*x*sinh(x)^2 - 4*(3*x + 10)*cosh(x) + 2*(3*x*cosh(x)
- 3*x - 4)*sinh(x) + 9*x + 24)/(cosh(x)^2 + 2*(cosh(x) - 1)*sinh(x) + sinh(
x)^2 - 4*cosh(x) + 3)
```

giac [A] time = 0.11, size = 22, normalized size = 0.73

$$x-\frac{8\left(3\,e^{(2\,x)}-3\,e^{x}+2\right)}{3\,(e^{x}-1)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((coth(x)+csch(x))^4,x, algorithm="giac")
```

```
[Out] x - 8/3*(3*e^(2*x) - 3*e^x + 2)/(e^x - 1)^3
```

maple [A] time = 0.37, size = 49, normalized size = 1.63

$$x-\coth(x)-\frac{\left(\coth^{3}(x)\right)}{3}-\frac{4\left(\cosh^{2}(x)\right)}{\sinh(x)^{3}}+\frac{4}{3\sinh(x)^{3}}-\frac{3\cosh(x)}{\sinh(x)^{3}}-2\left(\frac{2}{3}-\frac{\operatorname{csch}(x)^{2}}{3}\right)\coth(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((coth(x)+csch(x))^4,x)
```

```
[Out] x-coth(x)-1/3*coth(x)^3-4/sinh(x)^3*cosh(x)^2+4/3/sinh(x)^3-3/sinh(x)^3*cos
h(x)-2*(2/3-1/3*csch(x)^2)*coth(x)
```

maxima [B] time = 0.52, size = 183, normalized size = 6.10

$$-2\coth(x)^{3}+x-\frac{4\left(3e^{(-2x)}-3e^{(-4x)}-2\right)}{3\left(3e^{(-2x)}-3e^{(-4x)}+e^{(-6x)}-1\right)}+\frac{8e^{(-x)}}{3e^{(-2x)}-3e^{(-4x)}+e^{(-6x)}-1}+\frac{4e^{(-2x)}}{3e^{(-2x)}-3e^{(-4x)}+e^{(-6x)}-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((coth(x)+csch(x))^4,x, algorithm="maxima")
```

```
[Out] -2*coth(x)^3 + x - 4/3*(3*e^(-2*x) - 3*e^(-4*x) - 2)/(3*e^(-2*x) - 3*e^(-4*
x) + e^(-6*x) - 1) + 8*e^(-x)/(3*e^(-2*x) - 3*e^(-4*x) + e^(-6*x) - 1) + 4*
e^(-2*x)/(3*e^(-2*x) - 3*e^(-4*x) + e^(-6*x) - 1) - 16/3*e^(-3*x)/(3*e^(-2*
x) - 3*e^(-4*x) + e^(-6*x) - 1) + 8*e^(-5*x)/(3*e^(-2*x) - 3*e^(-4*x) + e^(
-6*x) - 1) - 4/3/(3*e^(-2*x) - 3*e^(-4*x) + e^(-6*x) - 1) + 32/3/(e^(-x) -
e^x)^3
```

mupad [B] time = 0.05, size = 59, normalized size = 1.97

$$x-\frac{8\,\mathrm{e}^{x}}{3\,\left(\mathrm{e}^{2\,x}-2\,\mathrm{e}^{x}+1\right)}+\frac{\frac{8\,\mathrm{e}^{2\,x}}{3}+\frac{8}{3}}{3\,\mathrm{e}^{2\,x}-\mathrm{e}^{3\,x}-3\,\mathrm{e}^{x}+1}-\frac{8}{3\,\left(\mathrm{e}^{x}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((coth(x) + 1/sinh(x))^4,x)
```

```
[Out] x - (8*exp(x))/(3*(exp(2*x) - 2*exp(x) + 1)) + ((8*exp(2*x))/3 + 8/3)/(3*ex
p(2*x) - exp(3*x) - 3*exp(x) + 1) - 8/(3*(exp(x) - 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (\coth(x) + \operatorname{csch}(x))^4 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((coth(x)+csch(x))**4,x)
```

```
[Out] Integral((coth(x) + csch(x))**4, x)
```

3.656 $\int (\coth(x) + \textbf{csch}(x))^3 \, dx$

Optimal. Leaf size=18

$$\frac{2}{1-\cosh(x)} + \log(1-\cosh(x))$$

```
[Out] 2/(1-cosh(x))+ln(1-cosh(x))
```

Rubi [A] time = 0.06, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {4392, 2667, 43}

$$\frac{2}{1-\cosh(x)} + \log(1-\cosh(x))$$

Antiderivative was successfully verified.

```
[In] Int[(Coth[x] + Csch[x])^3,x]
```

```
[Out] 2/(1 - Cosh[x]) + Log[1 - Cosh[x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 4392

```
Int[(cot[(c_.) + (d_.)*(x_)]^(n_.)*(a_.) + csc[(c_.) + (d_.)*(x_)]^(n_.)*(b
_.))^(p_)*(u_.), x_Symbol] :> Int[ActivateTrig[u]*Csc[c + d*x]^(n*p)*(b + a
*Cos[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

Rubi steps

$$
\begin{aligned}
\int (\coth(x)+\operatorname{csch}(x))^3\,dx &= i\int (i+i\cosh(x))^3\operatorname{csch}^3(x)\,dx\\
&= \operatorname{Subst}\left(\int \frac{i+x}{(i-x)^2}\,dx, x, i\cosh(x)\right)\\
&= \operatorname{Subst}\left(\int \left(\frac{2i}{(-i+x)^2}+\frac{1}{-i+x}\right)dx, x, i\cosh(x)\right)\\
&= \frac{2i}{i-i\cosh(x)}+\log(1-\cosh(x))
\end{aligned}
$$

Mathematica [B] time = 0.05, size = 41, normalized size = 2.28

$$
-\operatorname{csch}^2\left(\frac{x}{2}\right)-2\log\left(\sinh\left(\frac{x}{2}\right)\right)+\log(\sinh(x))+3\log\left(\tanh\left(\frac{x}{2}\right)\right)+2\log\left(\cosh\left(\frac{x}{2}\right)\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[(Coth[x] + Csch[x])^3,x]
```

```
[Out] -Csch[x/2]^2 + 2*Log[Cosh[x/2]] - 2*Log[Sinh[x/2]] + Log[Sinh[x]] + 3*Log[T
anh[x/2]]
```

fricas [B] time = 0.43, size = 91, normalized size = 5.06

$$
-\frac{x\cosh(x)^2+x\sinh(x)^2-2\,(x-2)\cosh(x)-2\left(\cosh(x)^2+2\,(\cosh(x)-1)\sinh(x)+\sinh(x)^2-2\,\cosh(x)+1\right.}{\cosh(x)^2+2\,(\cosh(x)-1)\sinh(x)+\sinh(x)^2-2\,\cosh}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((coth(x)+csch(x))^3,x, algorithm="fricas")
```

```
[Out] -(x*cosh(x)^2 + x*sinh(x)^2 - 2*(x - 2)*cosh(x) - 2*(cosh(x)^2 + 2*(cosh(x)
 - 1)*sinh(x) + sinh(x)^2 - 2*cosh(x) + 1)*log(cosh(x) + sinh(x) - 1) + 2*(
x*cosh(x) - x + 2)*sinh(x) + x)/(cosh(x)^2 + 2*(cosh(x) - 1)*sinh(x) + sinh
(x)^2 - 2*cosh(x) + 1)
```

giac [A] time = 0.13, size = 22, normalized size = 1.22

$$
-x-\frac{4\,e^x}{(e^x-1)^2}+2\,\log\left(|e^x-1|\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((coth(x)+csch(x))^3,x, algorithm="giac")
```

```
[Out] -x - 4*e^x/(e^x - 1)^2 + 2*log(abs(e^x - 1))
```

maple [A] time = 0.37, size = 35, normalized size = 1.94

$$\ln(\sinh(x)) - \frac{(\coth^2(x))}{2} - \frac{3\cosh(x)}{\sinh(x)^2} + \coth(x)\operatorname{csch}(x) - 2\operatorname{arctanh}(e^x) - \frac{3}{2\sinh(x)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((coth(x)+csch(x))^3,x)
[Out] ln(sinh(x))-1/2*coth(x)^2-3/sinh(x)^2*cosh(x)+coth(x)*csch(x)-2*arctanh(exp
(x))-3/2/sinh(x)^2
```

maxima [B] time = 0.36, size = 66, normalized size = 3.67

$$-\frac{3}{2}\coth(x)^2 + x + \frac{4\left(e^{(-x)} + e^{(-3x)}\right)}{2e^{(-2x)} - e^{(-4x)} - 1} + \frac{2e^{(-2x)}}{2e^{(-2x)} - e^{(-4x)} - 1} + 2\log\left(e^{(-x)} - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((coth(x)+csch(x))^3,x, algorithm="maxima")
[Out] -3/2*coth(x)^2 + x + 4*(e^(-x) + e^(-3*x))/(2*e^(-2*x) - e^(-4*x) - 1) + 2*
e^(-2*x)/(2*e^(-2*x) - e^(-4*x) - 1) + 2*log(e^(-x) - 1)
```

mupad [B] time = 0.05, size = 33, normalized size = 1.83

$$2\ln(e^x - 1) - x - \frac{4}{e^{2x} - 2e^x + 1} - \frac{4}{e^x - 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((coth(x) + 1/sinh(x))^3,x)
[Out] 2*log(exp(x) - 1) - x - 4/(exp(2*x) - 2*exp(x) + 1) - 4/(exp(x) - 1)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (\coth(x) + \operatorname{csch}(x))^3\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((coth(x)+csch(x))**3,x)
[Out] Integral((coth(x) + csch(x))**3, x)
```

3.657 $\int (\coth(x) + \mathbf{csch}(x))^2 \, dx$

Optimal. Leaf size=14

$$x + \frac{2\sinh(x)}{1 - \cosh(x)}$$

[Out] x+2*sinh(x)/(1-cosh(x))

Rubi [A] time = 0.08, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {4392, 2670, 2680, 8}

$$x + \frac{2\sinh(x)}{1 - \cosh(x)}$$

Antiderivative was successfully verified.

[In] Int[(Coth[x] + Csch[x])^2,x]

[Out] x + (2*Sinh[x])/(1 - Cosh[x])

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2670

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*m), Int[(g*Cos[e + f*x])^(2*m + p)/(a
 - b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2,
 0] && IntegerQ[m] && LtQ[p, -1] && GeQ[2*m + p, 0]
```

Rule 2680

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(2*g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f
*x])^(m + 1))/(b*f*(2*m + p + 1)), x] + Dist[(g^2*(p - 1))/(b^2*(2*m + p +
1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 4392

```
Int[(cot[(c_.) + (d_.)*(x_)]^(n_.)*(a_.) + csc[(c_.) + (d_.)*(x_)]^(n_.)*(b
_.))^(p_)*(u_.), x_Symbol] :> Int[ActivateTrig[u]*Csc[c + d*x]^(n*p)*(b + a
```

```
*Cos[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

Rubi steps

$$\begin{aligned}\int (\coth(x)+\operatorname{csch}(x))^2\,dx &= -\int (i+i\cosh(x))^2\operatorname{csch}^2(x)\,dx\\ &= -\int \frac{\sinh^2(x)}{(i-i\cosh(x))^2}\,dx\\ &= \frac{2\sinh(x)}{1-\cosh(x)}+\int 1\,dx\\ &= x+\frac{2\sinh(x)}{1-\cosh(x)}\end{aligned}$$

Mathematica [A] time = 0.03, size = 10, normalized size = 0.71

$$x-2\coth\left(\frac{x}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Coth[x] + Csch[x])^2,x]
[Out] x - 2*Coth[x/2]
```

fricas [A] time = 0.40, size = 22, normalized size = 1.57

$$\frac{x\cosh(x)+x\sinh(x)-x-4}{\cosh(x)+\sinh(x)-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((coth(x)+csch(x))^2,x, algorithm="fricas")
[Out] (x*cosh(x) + x*sinh(x) - x - 4)/(cosh(x) + sinh(x) - 1)
```

giac [A] time = 0.13, size = 10, normalized size = 0.71

$$x-\frac{4}{e^x-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((coth(x)+csch(x))^2,x, algorithm="giac")
```

```
[Out] x - 4/(e^x - 1)
```

maple [A] time = 0.37, size = 13, normalized size = 0.93

$$x - 2\coth(x) - \frac{2}{\sinh(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((coth(x)+csch(x))^2,x)
```

```
[Out] x-2*coth(x)-2/sinh(x)
```

maxima [B] time = 0.31, size = 25, normalized size = 1.79

$$x + \frac{4}{e^{(-x)} - e^{x}} + \frac{4}{e^{(-2x)} - 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((coth(x)+csch(x))^2,x, algorithm="maxima")
```

```
[Out] x + 4/(e^(-x) - e^x) + 4/(e^(-2*x) - 1)
```

mupad [B] time = 1.66, size = 10, normalized size = 0.71

$$x - \frac{4}{e^{x} - 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((coth(x) + 1/sinh(x))^2,x)
```

```
[Out] x - 4/(exp(x) - 1)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (\coth(x) + \operatorname{csch}(x))^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((coth(x)+csch(x))**2,x)
```

```
[Out] Integral((coth(x) + csch(x))**2, x)
```

3.658 $\int (\coth(x) + \operatorname{csch}(x))\,dx$

Optimal. Leaf size=9

$$\log(\sinh(x)) - \tanh^{-1}(\cosh(x))$$

```
[Out] -arctanh(cosh(x))+ln(sinh(x))
```

Rubi [A] time = 0.01, antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {3475, 3770}

$$\log(\sinh(x)) - \tanh^{-1}(\cosh(x))$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x] + Csch[x],x]
[Out] -ArcTanh[Cosh[x]] + Log[Sinh[x]]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int (\coth(x) + \operatorname{csch}(x))\,dx &= \int \coth(x)\,dx + \int \operatorname{csch}(x)\,dx\\ &= -\tanh^{-1}(\cosh(x)) + \log(\sinh(x))\end{aligned}$$

Mathematica [A] time = 0.00, size = 11, normalized size = 1.22

$$\log(\sinh(x)) + \log\left(\tanh\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x] + Csch[x],x]
```

```
[Out] Log[Sinh[x]] + Log[Tanh[x/2]]
```

fricas [A] time = 0.40, size = 13, normalized size = 1.44

$$-x + 2 \log(\cosh(x) + \sinh(x) - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)+csch(x),x, algorithm="fricas")
[Out] -x + 2*log(cosh(x) + sinh(x) - 1)
```

giac [B] time = 0.11, size = 25, normalized size = 2.78

$$-x - \log(e^x + 1) + \log\left(\left|e^{(2x)} - 1\right|\right) + \log(|e^x - 1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)+csch(x),x, algorithm="giac")
[Out] -x - log(e^x + 1) + log(abs(e^(2*x) - 1)) + log(abs(e^x - 1))
```

maple [A] time = 0.03, size = 10, normalized size = 1.11

$$\ln(\sinh(x)) + \ln\left(\tanh\left(\frac{x}{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)+csch(x),x)
[Out] ln(sinh(x))+ln(tanh(1/2*x))
```

maxima [A] time = 0.55, size = 9, normalized size = 1.00

$$\log(\sinh(x)) + \log\left(\tanh\left(\frac{1}{2}x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)+csch(x),x, algorithm="maxima")
[Out] log(sinh(x)) + log(tanh(1/2*x))
```

mupad [B] time = 0.04, size = 11, normalized size = 1.22

$$2\ln(\mathrm{e}^x - 1) - x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x) + 1/sinh(x),x)
```

```
[Out] 2*log(exp(x) - 1) - x
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (\coth(x) + \operatorname{csch}(x))\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)+csch(x),x)
```

```
[Out] Integral(coth(x) + csch(x), x)
```

3.659 $\int \frac{1}{\coth(x)+\mathbf{csch}(x)}\,dx$

Optimal. Leaf size=5

$$\log(\cosh(x)+1)$$

[Out] ln(1+cosh(x))

Rubi [A] time = 0.03, antiderivative size = 5, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {3160, 2667, 31}

$$\log(\cosh(x)+1)$$

Antiderivative was successfully verified.

[In] Int[(Coth[x] + Csch[x])^(-1),x]

[Out] Log[1 + Cosh[x]]

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 3160

```
Int[((a_.) + csc[(d_.) + (e_.)*(x_)]*(b_.) + cot[(d_.) + (e_.)*(x_)]*(c_.))
^(-1), x_Symbol] :> Int[Sin[d + e*x]/(b + a*Sin[d + e*x] + c*Cos[d + e*x]),
 x] /; FreeQ[{a, b, c, d, e}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{\coth(x)+\operatorname{csch}(x)}\,dx &= i\int \frac{\sinh(x)}{i+i\cosh(x)}\,dx \\
&= \operatorname{Subst}\left(\int \frac{1}{i+x}\,dx, x, i\cosh(x)\right) \\
&= \log(1+\cosh(x))
\end{aligned}
$$

Mathematica [A] time = 0.02, size = 9, normalized size = 1.80

$$2\log\left(\cosh\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Coth[x] + Csch[x])^(-1),x]
```

```
[Out] 2*Log[Cosh[x/2]]
```

fricas [B] time = 0.41, size = 13, normalized size = 2.60

$$-x+2\,\log\left(\cosh(x)+\sinh(x)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(coth(x)+csch(x)),x, algorithm="fricas")
```

```
[Out] -x + 2*log(cosh(x) + sinh(x) + 1)
```

giac [B] time = 0.13, size = 11, normalized size = 2.20

$$-x+2\,\log\left(e^{x}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(coth(x)+csch(x)),x, algorithm="giac")
```

```
[Out] -x + 2*log(e^x + 1)
```

maple [B] time = 0.21, size = 20, normalized size = 4.00

$$-\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)-\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(coth(x)+csch(x)),x)
```

```
[Out] -ln(tanh(1/2*x)-1)-ln(tanh(1/2*x)+1)
```

maxima [B] time = 0.33, size = 11, normalized size = 2.20

$$x + 2 \log\left(e^{(-x)} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(coth(x)+csch(x)),x, algorithm="maxima")
[Out] x + 2*log(e^(-x) + 1)
```

mupad [B] time = 0.04, size = 11, normalized size = 2.20

$$2 \ln(\mathrm{e}^x + 1) - x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(coth(x) + 1/sinh(x)),x)
[Out] 2*log(exp(x) + 1) - x
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\coth(x) + \operatorname{csch}(x)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(coth(x)+csch(x)),x)
[Out] Integral(1/(coth(x) + csch(x)), x)
```

3.660 $\int \frac{1}{(\coth(x)+\operatorname{csch}(x))^2} \, dx$

Optimal. Leaf size=12

$$x - \frac{2\sinh(x)}{\cosh(x)+1}$$

```
[Out] x-2*sinh(x)/(1+cosh(x))
```

Rubi [A] time = 0.05, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {4392, 2680, 8}

$$x - \frac{2\sinh(x)}{\cosh(x)+1}$$

Antiderivative was successfully verified.

```
[In] Int[(Coth[x] + Csch[x])^(-2),x]
```

```
[Out] x - (2*Sinh[x])/(1 + Cosh[x])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2680

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(2*g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f
*x])^(m + 1))/(b*f*(2*m + p + 1)), x] + Dist[(g^2*(p - 1))/(b^2*(2*m + p +
1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 4392

```
Int[(cot[(c_.) + (d_.)*(x_)]^(n_.)*(a_.) + csc[(c_.) + (d_.)*(x_)]^(n_.)*(b
_.))^(p_)*(u_.), x_Symbol] :> Int[ActivateTrig[u]*Csc[c + d*x]^(n*p)*(b + a
*Cos[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

Rubi steps

$$\int \frac{1}{(\coth(x)+\operatorname{csch}(x))^2}\,dx = -\int \frac{\sinh^2(x)}{(i+i\cosh(x))^2}\,dx$$
$$= -\frac{2\sinh(x)}{1+\cosh(x)} + \int 1\,dx$$
$$= x - \frac{2\sinh(x)}{1+\cosh(x)}$$

Mathematica [A] time = 0.02, size = 10, normalized size = 0.83

$$x - 2\tanh\left(\frac{x}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Coth[x] + Csch[x])^(-2),x]
[Out] x - 2*Tanh[x/2]
```

fricas [A] time = 0.40, size = 20, normalized size = 1.67

$$\frac{x\cosh(x)+x\sinh(x)+x+4}{\cosh(x)+\sinh(x)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(coth(x)+csch(x))^2,x, algorithm="fricas")
[Out] (x*cosh(x) + x*sinh(x) + x + 4)/(cosh(x) + sinh(x) + 1)
```

giac [A] time = 0.13, size = 10, normalized size = 0.83

$$x + \frac{4}{e^x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(coth(x)+csch(x))^2,x, algorithm="giac")
[Out] x + 4/(e^x + 1)
```

maple [A] time = 0.19, size = 24, normalized size = 2.00

$$-2\tanh\left(\frac{x}{2}\right) - \ln\left(\tanh\left(\frac{x}{2}\right)-1\right) + \ln\left(\tanh\left(\frac{x}{2}\right)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(coth(x)+csch(x))^2,x)
```

```
[Out] -2*tanh(1/2*x)-ln(tanh(1/2*x)-1)+ln(tanh(1/2*x)+1)
```

maxima [A] time = 0.47, size = 12, normalized size = 1.00

$$x - \frac{4}{e^{(-x)} + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(coth(x)+csch(x))^2,x, algorithm="maxima")
```

```
[Out] x - 4/(e^(-x) + 1)
```

mupad [B] time = 0.18, size = 10, normalized size = 0.83

$$x + \frac{4}{e^x + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(coth(x) + 1/sinh(x))^2,x)
```

```
[Out] x + 4/(exp(x) + 1)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(\coth{(x)} + \operatorname{csch}{(x)})^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(coth(x)+csch(x))**2,x)
```

```
[Out] Integral((coth(x) + csch(x))**(-2), x)
```

3.661 $\int \frac{1}{(\coth(x)+\mathbf{csch}(x))^3} \, dx$

Optimal. Leaf size=14

$$\frac{2}{\cosh(x)+1}+\log(\cosh(x)+1)$$

[Out] 2/(1+cosh(x))+ln(1+cosh(x))

Rubi [A] time = 0.06, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {4392, 2667, 43}

$$\frac{2}{\cosh(x)+1}+\log(\cosh(x)+1)$$

Antiderivative was successfully verified.

[In] Int[(Coth[x] + Csch[x])^(-3),x]

[Out] 2/(1 + Cosh[x]) + Log[1 + Cosh[x]]

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 4392

```
Int[(cot[(c_.) + (d_.)*(x_)]^(n_.)*(a_.) + csc[(c_.) + (d_.)*(x_)]^(n_.)*(b
_.))^(p_)*(u_.), x_Symbol] :> Int[ActivateTrig[u]*Csc[c + d*x]^(n*p)*(b + a
*Cos[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

Rubi steps

$$\int \frac{1}{(\coth(x)+\operatorname{csch}(x))^3}\,dx = -\left(i\int \frac{\sinh^3(x)}{(i+i\cosh(x))^3}\,dx\right)$$
$$= -\operatorname{Subst}\left(\int \frac{i-x}{(i+x)^2}\,dx, x, i\cosh(x)\right)$$
$$= -\operatorname{Subst}\left(\int \left(\frac{1}{-i-x}+\frac{2i}{(i+x)^2}\right)dx, x, i\cosh(x)\right)$$
$$= \frac{2i}{i+i\cosh(x)} + \log(1+\cosh(x))$$

Mathematica [A] time = 0.02, size = 18, normalized size = 1.29

$$\operatorname{sech}^2\left(\frac{x}{2}\right) + 2\log\left(\cosh\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Coth[x] + Csch[x])^(-3),x]
```

```
[Out] 2*Log[Cosh[x/2]] + Sech[x/2]^2
```

fricas [B] time = 0.41, size = 89, normalized size = 6.36

$$-\frac{x\cosh(x)^2 + x\sinh(x)^2 + 2\,(x-2)\cosh(x) - 2\left(\cosh(x)^2 + 2\,(\cosh(x)+1)\sinh(x) + \sinh(x)^2 + 2\,\cosh(x) + \right.}{\cosh(x)^2 + 2\,(\cosh(x)+1)\sinh(x) + \sinh(x)^2 + 2\,\mathrm{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(coth(x)+csch(x))^3,x, algorithm="fricas")
```

```
[Out] -(x*cosh(x)^2 + x*sinh(x)^2 + 2*(x - 2)*cosh(x) - 2*(cosh(x)^2 + 2*(cosh(x)
 + 1)*sinh(x) + sinh(x)^2 + 2*cosh(x) + 1)*log(cosh(x) + sinh(x) + 1) + 2*(
x*cosh(x) + x - 2)*sinh(x) + x)/(cosh(x)^2 + 2*(cosh(x) + 1)*sinh(x) + sinh
(x)^2 + 2*cosh(x) + 1)
```

giac [A] time = 0.12, size = 21, normalized size = 1.50

$$-x + \frac{4\,e^x}{(e^x+1)^2} + 2\,\log\left(e^x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(coth(x)+csch(x))^3,x, algorithm="giac")
```

```
[Out] -x + 4*e^x/(e^x + 1)^2 + 2*log(e^x + 1)
```

maple [A] time = 0.19, size = 28, normalized size = 2.00

$$-\left(\tanh^2\left(\frac{x}{2}\right)\right)-\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)-\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(coth(x)+csch(x))^3,x)
[Out] -tanh(1/2*x)^2-ln(tanh(1/2*x)-1)-ln(tanh(1/2*x)+1)
```

maxima [B] time = 0.44, size = 31, normalized size = 2.21

$$x+\frac{4\,e^{(-x)}}{2\,e^{(-x)}+e^{(-2\,x)}+1}+2\,\log\left(e^{(-x)}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(coth(x)+csch(x))^3,x, algorithm="maxima")
[Out] x + 4*e^(-x)/(2*e^(-x) + e^(-2*x) + 1) + 2*log(e^(-x) + 1)
```

mupad [B] time = 1.55, size = 33, normalized size = 2.36

$$2\,\ln\left(\mathrm{e}^x+1\right)-x-\frac{4}{\mathrm{e}^{2\,x}+2\,\mathrm{e}^x+1}+\frac{4}{\mathrm{e}^x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(coth(x) + 1/sinh(x))^3,x)
[Out] 2*log(exp(x) + 1) - x - 4/(exp(2*x) + 2*exp(x) + 1) + 4/(exp(x) + 1)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\left(\coth\left(x\right)+\operatorname{csch}\left(x\right)\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(coth(x)+csch(x))**3,x)
[Out] Integral((coth(x) + csch(x))**(-3), x)
```

3.662 $\int \frac{1}{(\coth(x)+\mathbf{csch}(x))^4} dx$

Optimal. Leaf size=26

$$x-\frac{2\sinh^3(x)}{3(\cosh(x)+1)^3}-\frac{2\sinh(x)}{\cosh(x)+1}$$

```
[Out] x-2*sinh(x)/(1+cosh(x))-2/3*sinh(x)^3/(1+cosh(x))^3
```

Rubi [A] time = 0.08, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {4392, 2680, 8}

$$x-\frac{2\sinh^3(x)}{3(\cosh(x)+1)^3}-\frac{2\sinh(x)}{\cosh(x)+1}$$

Antiderivative was successfully verified.

```
[In] Int[(Coth[x] + Csch[x])^(-4),x]
```

```
[Out] x - (2*Sinh[x])/(1 + Cosh[x]) - (2*Sinh[x]^3)/(3*(1 + Cosh[x])^3)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2680

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(2*g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f
*x])^(m + 1))/(b*f*(2*m + p + 1)), x] + Dist[(g^2*(p - 1))/(b^2*(2*m + p +
1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 4392

```
Int[(cot[(c_.) + (d_.)*(x_)]^(n_.)*(a_.) + csc[(c_.) + (d_.)*(x_)]^(n_.)*(b
_.))^(p_)*(u_.), x_Symbol] :> Int[ActivateTrig[u]*Csc[c + d*x]^(n*p)*(b + a
*Cos[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

Rubi steps

$$\int \frac{1}{(\coth(x)+\operatorname{csch}(x))^4}\,dx = \int \frac{\sinh^4(x)}{(i+i\cosh(x))^4}\,dx$$
$$= -\frac{2\sinh^3(x)}{3(1+\cosh(x))^3} - \int \frac{\sinh^2(x)}{(i+i\cosh(x))^2}\,dx$$
$$= -\frac{2\sinh(x)}{1+\cosh(x)} - \frac{2\sinh^3(x)}{3(1+\cosh(x))^3} + \int 1\,dx$$
$$= x - \frac{2\sinh(x)}{1+\cosh(x)} - \frac{2\sinh^3(x)}{3(1+\cosh(x))^3}$$

Mathematica [A] time = 0.02, size = 30, normalized size = 1.15

$$x - \frac{8}{3}\tanh\left(\frac{x}{2}\right) + \frac{2}{3}\tanh\left(\frac{x}{2}\right)\operatorname{sech}^2\left(\frac{x}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Coth[x] + Csch[x])^(-4),x]
```

```
[Out] x - (8*Tanh[x/2])/3 + (2*Sech[x/2]^2*Tanh[x/2])/3
```

fricas [B] time = 0.41, size = 68, normalized size = 2.62

$$\frac{3\,x\cosh(x)^2 + 3\,x\sinh(x)^2 + 4\,(3\,x+10)\cosh(x) + 2\,(3\,x\cosh(x) + 3\,x + 4)\sinh(x) + 9\,x + 24}{3\left(\cosh(x)^2 + 2\,(\cosh(x)+1)\sinh(x) + \sinh(x)^2 + 4\,\cosh(x) + 3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(coth(x)+csch(x))^4,x, algorithm="fricas")
```

```
[Out] 1/3*(3*x*cosh(x)^2 + 3*x*sinh(x)^2 + 4*(3*x + 10)*cosh(x) + 2*(3*x*cosh(x)
+ 3*x + 4)*sinh(x) + 9*x + 24)/(cosh(x)^2 + 2*(cosh(x) + 1)*sinh(x) + sinh(
x)^2 + 4*cosh(x) + 3)
```

giac [A] time = 0.12, size = 22, normalized size = 0.85

$$x + \frac{8\left(3\,e^{(2\,x)} + 3\,e^x + 2\right)}{3\,(e^x+1)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(coth(x)+csch(x))^4,x, algorithm="giac")
```

```
[Out] x + 8/3*(3*e^(2*x) + 3*e^x + 2)/(e^x + 1)^3
```

maple [A] time = 0.21, size = 32, normalized size = 1.23

$$-\frac{2\left(\tanh^{3}\left(\frac{x}{2}\right)\right)}{3}-2\tanh\left(\frac{x}{2}\right)-\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)+\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(coth(x)+csch(x))^4,x)
[Out] -2/3*tanh(1/2*x)^3-2*tanh(1/2*x)-ln(tanh(1/2*x)-1)+ln(tanh(1/2*x)+1)
```

maxima [A] time = 0.66, size = 38, normalized size = 1.46

$$x-\frac{8\left(3\,e^{(-x)}+3\,e^{(-2\,x)}+2\right)}{3\left(3\,e^{(-x)}+3\,e^{(-2\,x)}+e^{(-3\,x)}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(coth(x)+csch(x))^4,x, algorithm="maxima")
[Out] x - 8/3*(3*e^(-x) + 3*e^(-2*x) + 2)/(3*e^(-x) + 3*e^(-2*x) + e^(-3*x) + 1)
```

mupad [B] time = 1.55, size = 57, normalized size = 2.19

$$x+\frac{\frac{8\,e^{2\,x}}{3}+\frac{8}{3}}{3\,e^{2\,x}+e^{3\,x}+3\,e^{x}+1}+\frac{8\,e^{x}}{3\,\left(e^{2\,x}+2\,e^{x}+1\right)}+\frac{8}{3\,\left(e^{x}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(coth(x) + 1/sinh(x))^4,x)
[Out] x + ((8*exp(2*x))/3 + 8/3)/(3*exp(2*x) + exp(3*x) + 3*exp(x) + 1) + (8*exp(
x))/(3*(exp(2*x) + 2*exp(x) + 1)) + 8/(3*(exp(x) + 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\left(\coth\left(x\right)+\operatorname{csch}\left(x\right)\right)^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(coth(x)+csch(x))**4,x)
[Out] Integral((coth(x) + csch(x))**(-4), x)
```

3.663 $\int \frac{1}{(\coth(x)+\mathbf{csch}(x))^5}\,dx$

Optimal. Leaf size=22

$$\frac{4}{\cosh(x)+1}-\frac{2}{(\cosh(x)+1)^2}+\log(\cosh(x)+1)$$

```
[Out] -2/(1+cosh(x))^2+4/(1+cosh(x))+ln(1+cosh(x))
```

Rubi [A] time = 0.06, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {4392, 2667, 43}

$$\frac{4}{\cosh(x)+1}-\frac{2}{(\cosh(x)+1)^2}+\log(\cosh(x)+1)$$

Antiderivative was successfully verified.

```
[In] Int[(Coth[x] + Csch[x])^(-5),x]
```

```
[Out] -2/(1 + Cosh[x])^2 + 4/(1 + Cosh[x]) + Log[1 + Cosh[x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 4392

```
Int[(cot[(c_.) + (d_.)*(x_)]^(n_.)*(a_.) + csc[(c_.) + (d_.)*(x_)]^(n_.)*(b
_.))^(p_)*(u_.), x_Symbol] :> Int[ActivateTrig[u]*Csc[c + d*x]^(n*p)*(b + a
*Cos[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(\coth(x)+\operatorname{csch}(x))^5}\,dx &= i\int \frac{\sinh^5(x)}{(i+i\cosh(x))^5}\,dx\\ &= \operatorname{Subst}\left(\int \frac{(i-x)^2}{(i+x)^3}\,dx, x, i\cosh(x)\right)\\ &= \operatorname{Subst}\left(\int \left(-\frac{4}{(i+x)^3}-\frac{4i}{(i+x)^2}+\frac{1}{i+x}\right)dx, x, i\cosh(x)\right)\\ &= \frac{2}{(i+i\cosh(x))^2}+\frac{4i}{i+i\cosh(x)}+\log(1+\cosh(x))\end{aligned}$$

Mathematica [A] time = 0.02, size = 32, normalized size = 1.45

$$-\frac{1}{2}\operatorname{sech}^4\left(\frac{x}{2}\right)+2\operatorname{sech}^2\left(\frac{x}{2}\right)+2\log\left(\cosh\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Coth[x] + Csch[x])^(-5),x]
```

```
[Out] 2*Log[Cosh[x/2]] + 2*Sech[x/2]^2 - Sech[x/2]^4/2
```

fricas [B] time = 0.41, size = 266, normalized size = 12.09

$$-\frac{x\cosh(x)^4+x\sinh(x)^4+4\,(x-2)\cosh(x)^3+4\,(x\cosh(x)+x-2)\sinh(x)^3+2\,(3\,x-4)\cosh(x)^2+2\,\Big(3\,x\,\mathrm{co}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(coth(x)+csch(x))^5,x, algorithm="fricas")
```

```
[Out] -(x*cosh(x)^4 + x*sinh(x)^4 + 4*(x - 2)*cosh(x)^3 + 4*(x*cosh(x) + x - 2)*s
inh(x)^3 + 2*(3*x - 4)*cosh(x)^2 + 2*(3*x*cosh(x)^2 + 6*(x - 2)*cosh(x) + 3
*x - 4)*sinh(x)^2 + 4*(x - 2)*cosh(x) - 2*(cosh(x)^4 + 4*(cosh(x) + 1)*sinh
(x)^3 + sinh(x)^4 + 4*cosh(x)^3 + 6*(cosh(x)^2 + 2*cosh(x) + 1)*sinh(x)^2 +
 6*cosh(x)^2 + 4*(cosh(x)^3 + 3*cosh(x)^2 + 3*cosh(x) + 1)*sinh(x) + 4*cosh
(x) + 1)*log(cosh(x) + sinh(x) + 1) + 4*(x*cosh(x)^3 + 3*(x - 2)*cosh(x)^2
+ (3*x - 4)*cosh(x) + x - 2)*sinh(x) + x)/(cosh(x)^4 + 4*(cosh(x) + 1)*sinh
(x)^3 + sinh(x)^4 + 4*cosh(x)^3 + 6*(cosh(x)^2 + 2*cosh(x) + 1)*sinh(x)^2 +
 6*cosh(x)^2 + 4*(cosh(x)^3 + 3*cosh(x)^2 + 3*cosh(x) + 1)*sinh(x) + 4*cosh
(x) + 1)
```

giac [A] time = 0.12, size = 30, normalized size = 1.36

$$-x+\frac{8\left(e^{(3\,x)}+e^{(2\,x)}+e^{x}\right)}{\left(e^{x}+1\right)^4}+2\,\log\left(e^{x}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(coth(x)+csch(x))^5,x, algorithm="giac")
```

```
[Out] -x + 8*(e^(3*x) + e^(2*x) + e^x)/(e^x + 1)^4 + 2*log(e^x + 1)
```

maple [A] time = 0.20, size = 36, normalized size = 1.64

$$-\frac{\left(\tanh^{4}\left(\frac{x}{2}\right)\right)}{2}-\left(\tanh^{2}\left(\frac{x}{2}\right)\right)-\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)-\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(coth(x)+csch(x))^5,x)
```

```
[Out] -1/2*tanh(1/2*x)^4-tanh(1/2*x)^2-ln(tanh(1/2*x)-1)-ln(tanh(1/2*x)+1)
```

maxima [B] time = 0.68, size = 52, normalized size = 2.36

$$x+\frac{8\left(e^{(-x)}+e^{(-2x)}+e^{(-3x)}\right)}{4e^{(-x)}+6e^{(-2x)}+4e^{(-3x)}+e^{(-4x)}+1}+2\log\left(e^{(-x)}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(coth(x)+csch(x))^5,x, algorithm="maxima")
```

```
[Out] x + 8*(e^(-x) + e^(-2*x) + e^(-3*x))/(4*e^(-x) + 6*e^(-2*x) + 4*e^(-3*x) +
e^(-4*x) + 1) + 2*log(e^(-x) + 1)
```

mupad [B] time = 1.61, size = 79, normalized size = 3.59

$$2\ln\left(e^{x}+1\right)-x-\frac{16}{e^{2x}+2e^{x}+1}-\frac{8}{6e^{2x}+4e^{3x}+e^{4x}+4e^{x}+1}+\frac{8}{e^{x}+1}+\frac{16}{3e^{2x}+e^{3x}+3e^{x}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(coth(x) + 1/sinh(x))^5,x)
```

```
[Out] 2*log(exp(x) + 1) - x - 16/(exp(2*x) + 2*exp(x) + 1) - 8/(6*exp(2*x) + 4*ex
p(3*x) + exp(4*x) + 4*exp(x) + 1) + 8/(exp(x) + 1) + 16/(3*exp(2*x) + exp(3
*x) + 3*exp(x) + 1)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\left(\coth\left(x\right)+\operatorname{csch}\left(x\right)\right)^{5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(coth(x)+csch(x))**5,x)
[Out] Integral((coth(x) + csch(x))**(-5), x)
```

3.664 $\int (-\coth(x)+\mathbf{csch}(x))^5\,dx$

Optimal. Leaf size=24

$$-\frac{4}{\cosh(x)+1}+\frac{2}{(\cosh(x)+1)^2}-\log(\cosh(x)+1)$$

```
[Out] 2/(1+cosh(x))^2-4/(1+cosh(x))-ln(1+cosh(x))
```

Rubi [A] time = 0.06, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {4392, 2667, 43}

$$-\frac{4}{\cosh(x)+1}+\frac{2}{(\cosh(x)+1)^2}-\log(\cosh(x)+1)$$

Antiderivative was successfully verified.

```
[In] Int[(-Coth[x] + Csch[x])^5,x]
```

```
[Out] 2/(1 + Cosh[x])^2 - 4/(1 + Cosh[x]) - Log[1 + Cosh[x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 4392

```
Int[(cot[(c_.) + (d_.)*(x_)]^(n_.)*(a_.) + csc[(c_.) + (d_.)*(x_)]^(n_.)*(b
_.))^(p_)*(u_.), x_Symbol] :> Int[ActivateTrig[u]*Csc[c + d*x]^(n*p)*(b + a
*Cos[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

Rubi steps

$$\int (-\coth(x)+\operatorname{csch}(x))^5\,dx = -\left(i\int (i-i\cosh(x))^5\operatorname{csch}^5(x)\,dx\right)$$
$$= \operatorname{Subst}\left(\int \frac{(i+x)^2}{(i-x)^3}\,dx, x, -i\cosh(x)\right)$$
$$= \operatorname{Subst}\left(\int \left(\frac{1}{i-x}+\frac{4}{(-i+x)^3}-\frac{4i}{(-i+x)^2}\right)dx, x, -i\cosh(x)\right)$$
$$= -\frac{2}{(i+i\cosh(x))^2}-\frac{4i}{i+i\cosh(x)}-\log(1+\cosh(x))$$

Mathematica [B] time = 0.09, size = 55, normalized size = 2.29

$$\frac{1}{2}\operatorname{sech}^4\left(\frac{x}{2}\right)-2\operatorname{sech}^2\left(\frac{x}{2}\right)+6\log\left(\sinh\left(\frac{x}{2}\right)\right)-\log(\sinh(x))-5\log\left(\tanh\left(\frac{x}{2}\right)\right)-6\log\left(\cosh\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-Coth[x] + Csch[x])^5,x]
```

```
[Out] -6*Log[Cosh[x/2]] + 6*Log[Sinh[x/2]] - Log[Sinh[x]] - 5*Log[Tanh[x/2]] - 2*
Sech[x/2]^2 + Sech[x/2]^4/2
```

fricas [B] time = 0.40, size = 265, normalized size = 11.04

$$\frac{x\cosh(x)^4+x\sinh(x)^4+4\,(x-2)\cosh(x)^3+4\,(x\cosh(x)+x-2)\sinh(x)^3+2\,(3\,x-4)\cosh(x)^2+2\,\Big(3\,x\cos}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-coth(x)+csch(x))^5,x, algorithm="fricas")
```

```
[Out] (x*cosh(x)^4 + x*sinh(x)^4 + 4*(x - 2)*cosh(x)^3 + 4*(x*cosh(x) + x - 2)*si
nh(x)^3 + 2*(3*x - 4)*cosh(x)^2 + 2*(3*x*cosh(x)^2 + 6*(x - 2)*cosh(x) + 3*
x - 4)*sinh(x)^2 + 4*(x - 2)*cosh(x) - 2*(cosh(x)^4 + 4*(cosh(x) + 1)*sinh(
x)^3 + sinh(x)^4 + 4*cosh(x)^3 + 6*(cosh(x)^2 + 2*cosh(x) + 1)*sinh(x)^2 +
6*cosh(x)^2 + 4*(cosh(x)^3 + 3*cosh(x)^2 + 3*cosh(x) + 1)*sinh(x) + 4*cosh(
x) + 1)*log(cosh(x) + sinh(x) + 1) + 4*(x*cosh(x)^3 + 3*(x - 2)*cosh(x)^2 +
 (3*x - 4)*cosh(x) + x - 2)*sinh(x) + x)/(cosh(x)^4 + 4*(cosh(x) + 1)*sinh(
x)^3 + sinh(x)^4 + 4*cosh(x)^3 + 6*(cosh(x)^2 + 2*cosh(x) + 1)*sinh(x)^2 +
6*cosh(x)^2 + 4*(cosh(x)^3 + 3*cosh(x)^2 + 3*cosh(x) + 1)*sinh(x) + 4*cosh(
x) + 1)
```

giac [A] time = 0.13, size = 28, normalized size = 1.17

$$x - \frac{8\left(e^{(3x)} + e^{(2x)} + e^{x}\right)}{\left(e^{x}+1\right)^{4}} - 2\log\left(e^{x}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-coth(x)+csch(x))^5,x, algorithm="giac")
```

```
[Out] x - 8*(e^(3*x) + e^(2*x) + e^x)/(e^x + 1)^4 - 2*log(e^x + 1)
```

maple [B] time = 0.35, size = 73, normalized size = 3.04

$$-\ln\left(\sinh(x)\right)+\frac{\left(\coth^{2}(x)\right)}{2}+\frac{\left(\coth^{4}(x)\right)}{4}-\frac{5\left(\cosh^{3}(x)\right)}{\sinh(x)^{4}}+\frac{5\cosh(x)}{3\sinh(x)^{4}}+\frac{8\left(-\frac{\operatorname{csch}(x)^{3}}{4}+\frac{3\operatorname{csch}(x)}{8}\right)\coth(x)}{3}-2\operatorname{arctanh}(e$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-coth(x)+csch(x))^5,x)
```

```
[Out] -ln(sinh(x))+1/2*coth(x)^2+1/4*coth(x)^4-5/sinh(x)^4*cosh(x)^3+5/3/sinh(x)^
4*cosh(x)+8/3*(-1/4*csch(x)^3+3/8*csch(x))*coth(x)-2*arctanh(exp(x))+5/sinh
(x)^4*cosh(x)^2-5/4/sinh(x)^4
```

maxima [B] time = 0.35, size = 238, normalized size = 9.92

$$\frac{5}{2}\coth(x)^{4}-x+\frac{5\left(5e^{(-x)}+3e^{(-3x)}+3e^{(-5x)}+5e^{(-7x)}\right)}{4\left(4e^{(-2x)}-6e^{(-4x)}+4e^{(-6x)}-e^{(-8x)}-1\right)}-\frac{3e^{(-x)}-11e^{(-3x)}-11e^{(-5x)}+3e^{(-7x)}}{4\left(4e^{(-2x)}-6e^{(-4x)}+4e^{(-6x)}-e^{(-8x)}-1\right)}+\frac{5\left(e^{(-\right.}}{2\left(4e^{(-2\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-coth(x)+csch(x))^5,x, algorithm="maxima")
```

```
[Out] 5/2*coth(x)^4 - x + 5/4*(5*e^(-x) + 3*e^(-3*x) + 3*e^(-5*x) + 5*e^(-7*x))/(
4*e^(-2*x) - 6*e^(-4*x) + 4*e^(-6*x) - e^(-8*x) - 1) - 1/4*(3*e^(-x) - 11*e
^(-3*x) - 11*e^(-5*x) + 3*e^(-7*x))/(4*e^(-2*x) - 6*e^(-4*x) + 4*e^(-6*x) -
 e^(-8*x) - 1) + 5/2*(e^(-x) + 7*e^(-3*x) + 7*e^(-5*x) + e^(-7*x))/(4*e^(-2
*x) - 6*e^(-4*x) + 4*e^(-6*x) - e^(-8*x) - 1) - 4*(e^(-2*x) - e^(-4*x) + e^
(-6*x))/(4*e^(-2*x) - 6*e^(-4*x) + 4*e^(-6*x) - e^(-8*x) - 1) + 20/(e^(-x)
- e^x)^4 - 2*log(e^(-x) + 1)
```

mupad [B] time = 1.50, size = 77, normalized size = 3.21

$$x-2\ln\left(e^{x}+1\right)+\frac{16}{e^{2x}+2e^{x}+1}+\frac{8}{6e^{2x}+4e^{3x}+e^{4x}+4e^{x}+1}-\frac{8}{e^{x}+1}-\frac{16}{3e^{2x}+e^{3x}+3e^{x}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(coth(x) - 1/sinh(x))^5,x)
```

```
[Out] x - 2*log(exp(x) + 1) + 16/(exp(2*x) + 2*exp(x) + 1) + 8/(6*exp(2*x) + 4*ex
p(3*x) + exp(4*x) + 4*exp(x) + 1) - 8/(exp(x) + 1) - 16/(3*exp(2*x) + exp(3
*x) + 3*exp(x) + 1)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int 5\coth(x)\,\mathrm{csch}^4(x)\,dx-\int\left(-10\coth^2(x)\,\mathrm{csch}^3(x)\right)dx-\int 10\coth^3(x)\,\mathrm{csch}^2(x)\,dx-\int\left(-5\coth^4(x)\,\mathrm{csch}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-coth(x)+csch(x))**5,x)
```

```
[Out] -Integral(5*coth(x)*csch(x)**4, x) - Integral(-10*coth(x)**2*csch(x)**3, x)
 - Integral(10*coth(x)**3*csch(x)**2, x) - Integral(-5*coth(x)**4*csch(x),
x) - Integral(coth(x)**5, x) - Integral(-csch(x)**5, x)
```

3.665 $\int (-\coth(x) + \mathbf{csch}(x))^4 \, dx$

Optimal. Leaf size=26

$$x - \frac{2\sinh^3(x)}{3(\cosh(x)+1)^3} - \frac{2\sinh(x)}{\cosh(x)+1}$$

```
[Out] x-2*sinh(x)/(1+cosh(x))-2/3*sinh(x)^3/(1+cosh(x))^3
```

Rubi [A] time = 0.12, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {4392, 2670, 2680, 8}

$$x - \frac{2\sinh^3(x)}{3(\cosh(x)+1)^3} - \frac{2\sinh(x)}{\cosh(x)+1}$$

Antiderivative was successfully verified.

```
[In] Int[(-Coth[x] + Csch[x])^4,x]
```

```
[Out] x - (2*Sinh[x])/(1 + Cosh[x]) - (2*Sinh[x]^3)/(3*(1 + Cosh[x])^3)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2670

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*m), Int[(g*Cos[e + f*x])^(2*m + p)/(a
 - b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2,
 0] && IntegerQ[m] && LtQ[p, -1] && GeQ[2*m + p, 0]
```

Rule 2680

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(2*g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f
*x])^(m + 1))/(b*f*(2*m + p + 1)), x] + Dist[(g^2*(p - 1))/(b^2*(2*m + p +
1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 4392

```
Int[(cot[(c_.) + (d_.)*(x_)]^(n_.)*(a_.) + csc[(c_.) + (d_.)*(x_)]^(n_.)*(b
_.))^(p_)*(u_.), x_Symbol] :> Int[ActivateTrig[u]*Csc[c + d*x]^(n*p)*(b + a
```

```
*Cos[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

Rubi steps

$$\begin{aligned}\int(-\coth(x)+\operatorname{csch}(x))^4\,dx &= \int(i-i\cosh(x))^4\operatorname{csch}^4(x)\,dx\\ &= \int\frac{\sinh^4(x)}{(i+i\cosh(x))^4}\,dx\\ &= -\frac{2\sinh^3(x)}{3(1+\cosh(x))^3}-\int\frac{\sinh^2(x)}{(i+i\cosh(x))^2}\,dx\\ &= -\frac{2\sinh(x)}{1+\cosh(x)}-\frac{2\sinh^3(x)}{3(1+\cosh(x))^3}+\int 1\,dx\\ &= x-\frac{2\sinh(x)}{1+\cosh(x)}-\frac{2\sinh^3(x)}{3(1+\cosh(x))^3}\end{aligned}$$

Mathematica [A] time = 0.01, size = 30, normalized size = 1.15

$$2\tanh^{-1}\left(\tanh\left(\frac{x}{2}\right)\right)-\frac{2}{3}\tanh^3\left(\frac{x}{2}\right)-2\tanh\left(\frac{x}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-Coth[x] + Csch[x])^4,x]
[Out] 2*ArcTanh[Tanh[x/2]] - 2*Tanh[x/2] - (2*Tanh[x/2]^3)/3
```

fricas [B] time = 0.44, size = 68, normalized size = 2.62

$$\frac{3\,x\cosh(x)^2+3\,x\sinh(x)^2+4\,(3\,x+10)\cosh(x)+2\,(3\,x\cosh(x)+3\,x+4)\sinh(x)+9\,x+24}{3\left(\cosh(x)^2+2\,(\cosh(x)+1)\sinh(x)+\sinh(x)^2+4\,\cosh(x)+3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-coth(x)+csch(x))^4,x, algorithm="fricas")
[Out] 1/3*(3*x*cosh(x)^2 + 3*x*sinh(x)^2 + 4*(3*x + 10)*cosh(x) + 2*(3*x*cosh(x)
+ 3*x + 4)*sinh(x) + 9*x + 24)/(cosh(x)^2 + 2*(cosh(x) + 1)*sinh(x) + sinh(
x)^2 + 4*cosh(x) + 3)
```

giac [A] time = 0.13, size = 22, normalized size = 0.85

$$x+\frac{8\left(3\,e^{(2\,x)}+3\,e^x+2\right)}{3\,(e^x+1)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-coth(x)+csch(x))^4,x, algorithm="giac")
```

```
[Out] x + 8/3*(3*e^(2*x) + 3*e^x + 2)/(e^x + 1)^3
```

maple [A] time = 0.37, size = 49, normalized size = 1.88

$$x-\coth(x)-\frac{\left(\coth^3(x)\right)}{3}+\frac{4\left(\cosh^2(x)\right)}{\sinh(x)^3}-\frac{4}{3\sinh(x)^3}-\frac{3\cosh(x)}{\sinh(x)^3}-2\left(\frac{2}{3}-\frac{\operatorname{csch}(x)^2}{3}\right)\coth(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-coth(x)+csch(x))^4,x)
```

```
[Out] x-coth(x)-1/3*coth(x)^3+4/sinh(x)^3*cosh(x)^2-4/3/sinh(x)^3-3/sinh(x)^3*cos
h(x)-2*(2/3-1/3*csch(x)^2)*coth(x)
```

maxima [B] time = 0.45, size = 183, normalized size = 7.04

$$-2\coth(x)^3+x-\frac{4\left(3e^{(-2x)}-3e^{(-4x)}-2\right)}{3\left(3e^{(-2x)}-3e^{(-4x)}+e^{(-6x)}-1\right)}-\frac{8e^{(-x)}}{3e^{(-2x)}-3e^{(-4x)}+e^{(-6x)}-1}+\frac{4e^{(-2x)}}{3e^{(-2x)}-3e^{(-4x)}+e^{(-6x)}-1}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-coth(x)+csch(x))^4,x, algorithm="maxima")
```

```
[Out] -2*coth(x)^3 + x - 4/3*(3*e^(-2*x) - 3*e^(-4*x) - 2)/(3*e^(-2*x) - 3*e^(-4*
x) + e^(-6*x) - 1) - 8*e^(-x)/(3*e^(-2*x) - 3*e^(-4*x) + e^(-6*x) - 1) + 4*
e^(-2*x)/(3*e^(-2*x) - 3*e^(-4*x) + e^(-6*x) - 1) + 16/3*e^(-3*x)/(3*e^(-2*
x) - 3*e^(-4*x) + e^(-6*x) - 1) - 8*e^(-5*x)/(3*e^(-2*x) - 3*e^(-4*x) + e^(
-6*x) - 1) - 4/3/(3*e^(-2*x) - 3*e^(-4*x) + e^(-6*x) - 1) - 32/3/(e^(-x) -
e^x)^3
```

mupad [B] time = 1.51, size = 57, normalized size = 2.19

$$x+\frac{\frac{8\,e^{2x}}{3}+\frac{8}{3}}{3\,e^{2x}+e^{3x}+3\,e^{x}+1}+\frac{8\,e^{x}}{3\left(e^{2x}+2\,e^{x}+1\right)}+\frac{8}{3\,\left(e^{x}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((coth(x) - 1/sinh(x))^4,x)
```

```
[Out] x + ((8*exp(2*x))/3 + 8/3)/(3*exp(2*x) + exp(3*x) + 3*exp(x) + 1) + (8*exp(
x))/(3*(exp(2*x) + 2*exp(x) + 1)) + 8/(3*(exp(x) + 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (-\coth(x) + \operatorname{csch}(x))^4 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-coth(x)+csch(x))**4,x)
```

```
[Out] Integral((-coth(x) + csch(x))**4, x)
```

3.666 $\int (-\coth(x) + \mathbf{csch}(x))^3 \, dx$

Optimal. Leaf size=16

$$-\frac{2}{\cosh(x)+1} - \log(\cosh(x)+1)$$

```
[Out] -2/(1+cosh(x))-ln(1+cosh(x))
```

Rubi [A] time = 0.06, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {4392, 2667, 43}

$$-\frac{2}{\cosh(x)+1} - \log(\cosh(x)+1)$$

Antiderivative was successfully verified.

```
[In] Int[(-Coth[x] + Csch[x])^3,x]
```

```
[Out] -2/(1 + Cosh[x]) - Log[1 + Cosh[x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 4392

```
Int[(cot[(c_.) + (d_.)*(x_)]^(n_.)*(a_.) + csc[(c_.) + (d_.)*(x_)]^(n_.)*(b
_.))^(p_)*(u_.), x_Symbol] :> Int[ActivateTrig[u]*Csc[c + d*x]^(n*p)*(b + a
*Cos[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

Rubi steps

$$\int (-\coth(x)+\operatorname{csch}(x))^3\,dx = i\int (i-i\cosh(x))^3\operatorname{csch}^3(x)\,dx$$
$$= -\operatorname{Subst}\left(\int \frac{i+x}{(i-x)^2}\,dx, x, -i\cosh(x)\right)$$
$$= -\operatorname{Subst}\left(\int \left(\frac{2i}{(-i+x)^2}+\frac{1}{-i+x}\right)dx, x, -i\cosh(x)\right)$$
$$= -\frac{2i}{i+i\cosh(x)} - \log(1+\cosh(x))$$

Mathematica [B] time = 0.05, size = 43, normalized size = 2.69

$$-\operatorname{sech}^2\left(\frac{x}{2}\right) - 2\log\left(\sinh\left(\frac{x}{2}\right)\right) - \log(\sinh(x)) + 3\log\left(\tanh\left(\frac{x}{2}\right)\right) + 2\log\left(\cosh\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-Coth[x] + Csch[x])^3,x]
```

```
[Out] 2*Log[Cosh[x/2]] - 2*Log[Sinh[x/2]] - Log[Sinh[x]] + 3*Log[Tanh[x/2]] - Sec
h[x/2]^2
```

fricas [B] time = 0.40, size = 88, normalized size = 5.50

$$\frac{x\cosh(x)^2 + x\sinh(x)^2 + 2(x-2)\cosh(x) - 2\left(\cosh(x)^2 + 2(\cosh(x)+1)\sinh(x) + \sinh(x)^2 + 2\cosh(x) + 1\right.}{\cosh(x)^2 + 2(\cosh(x)+1)\sinh(x) + \sinh(x)^2 + 2\cos}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-coth(x)+csch(x))^3,x, algorithm="fricas")
```

```
[Out] (x*cosh(x)^2 + x*sinh(x)^2 + 2*(x - 2)*cosh(x) - 2*(cosh(x)^2 + 2*(cosh(x)
+ 1)*sinh(x) + sinh(x)^2 + 2*cosh(x) + 1)*log(cosh(x) + sinh(x) + 1) + 2*(x
*cosh(x) + x - 2)*sinh(x) + x)/(cosh(x)^2 + 2*(cosh(x) + 1)*sinh(x) + sinh(
x)^2 + 2*cosh(x) + 1)
```

giac [A] time = 0.11, size = 19, normalized size = 1.19

$$x - \frac{4e^x}{(e^x+1)^2} - 2\log(e^x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-coth(x)+csch(x))^3,x, algorithm="giac")
```

```
[Out] x - 4*e^x/(e^x + 1)^2 - 2*log(e^x + 1)
```

maple [B] time = 0.37, size = 37, normalized size = 2.31

$$-\ln(\sinh(x)) + \frac{\left(\coth^2(x)\right)}{2} - \frac{3\cosh(x)}{\sinh(x)^2} + \coth(x)\operatorname{csch}(x) - 2\operatorname{arctanh}(\mathrm{e}^x) + \frac{3}{2\sinh(x)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-coth(x)+csch(x))^3,x)
```

```
[Out] -ln(sinh(x))+1/2*coth(x)^2-3/sinh(x)^2*cosh(x)+coth(x)*csch(x)-2*arctanh(ex
p(x))+3/2/sinh(x)^2
```

maxima [B] time = 0.51, size = 68, normalized size = 4.25

$$\frac{3}{2}\coth(x)^2 - x + \frac{4\left(e^{(-x)} + e^{(-3x)}\right)}{2e^{(-2x)} - e^{(-4x)} - 1} - \frac{2e^{(-2x)}}{2e^{(-2x)} - e^{(-4x)} - 1} - 2\log\left(e^{(-x)} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-coth(x)+csch(x))^3,x, algorithm="maxima")
```

```
[Out] 3/2*coth(x)^2 - x + 4*(e^(-x) + e^(-3*x))/(2*e^(-2*x) - e^(-4*x) - 1) - 2*e
^(-2*x)/(2*e^(-2*x) - e^(-4*x) - 1) - 2*log(e^(-x) + 1)
```

mupad [B] time = 1.54, size = 31, normalized size = 1.94

$$x - 2\ln(\mathrm{e}^x + 1) + \frac{4}{\mathrm{e}^{2x} + 2\,\mathrm{e}^x + 1} - \frac{4}{\mathrm{e}^x + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(coth(x) - 1/sinh(x))^3,x)
```

```
[Out] x - 2*log(exp(x) + 1) + 4/(exp(2*x) + 2*exp(x) + 1) - 4/(exp(x) + 1)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int 3\coth(x)\operatorname{csch}^2(x)\,dx - \int\left(-3\coth^2(x)\operatorname{csch}(x)\right)dx - \int\coth^3(x)\,dx - \int\left(-\operatorname{csch}^3(x)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-coth(x)+csch(x))**3,x)
```

```
[Out] -Integral(3*coth(x)*csch(x)**2, x) - Integral(-3*coth(x)**2*csch(x), x) - I
ntegral(coth(x)**3, x) - Integral(-csch(x)**3, x)
```

3.667 $\int (-\coth(x) + \textbf{csch}(x))^2 \, dx$

Optimal. Leaf size=12

$$x - \frac{2\sinh(x)}{\cosh(x) + 1}$$

```
[Out] x-2*sinh(x)/(1+cosh(x))
```

Rubi [A] time = 0.08, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {4392, 2670, 2680, 8}

$$x - \frac{2\sinh(x)}{\cosh(x) + 1}$$

Antiderivative was successfully verified.

```
[In] Int[(-Coth[x] + Csch[x])^2,x]
```

```
[Out] x - (2*Sinh[x])/(1 + Cosh[x])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2670

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*m), Int[(g*Cos[e + f*x])^(2*m + p)/(a
 - b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2,
 0] && IntegerQ[m] && LtQ[p, -1] && GeQ[2*m + p, 0]
```

Rule 2680

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(2*g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f
*x])^(m + 1))/(b*f*(2*m + p + 1)), x] + Dist[(g^2*(p - 1))/(b^2*(2*m + p +
1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 4392

```
Int[(cot[(c_.) + (d_.)*(x_)]^(n_.)*(a_.) + csc[(c_.) + (d_.)*(x_)]^(n_.)*(b
_.))^(p_)*(u_.), x_Symbol] :> Int[ActivateTrig[u]*Csc[c + d*x]^(n*p)*(b + a
```

```
*Cos[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

Rubi steps

$$\begin{aligned}\int (-\coth(x)+\operatorname{csch}(x))^2\,dx &= -\int (i-i\cosh(x))^2\operatorname{csch}^2(x)\,dx\\ &= -\int \frac{\sinh^2(x)}{(i+i\cosh(x))^2}\,dx\\ &= -\frac{2\sinh(x)}{1+\cosh(x)}+\int 1\,dx\\ &= x-\frac{2\sinh(x)}{1+\cosh(x)}\end{aligned}$$

Mathematica [A] time = 0.01, size = 18, normalized size = 1.50

$$2\tanh^{-1}\left(\tanh\left(\frac{x}{2}\right)\right)-2\tanh\left(\frac{x}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-Coth[x] + Csch[x])^2,x]
[Out] 2*ArcTanh[Tanh[x/2]] - 2*Tanh[x/2]
```

fricas [A] time = 0.39, size = 20, normalized size = 1.67

$$\frac{x\cosh(x)+x\sinh(x)+x+4}{\cosh(x)+\sinh(x)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-coth(x)+csch(x))^2,x, algorithm="fricas")
[Out] (x*cosh(x) + x*sinh(x) + x + 4)/(cosh(x) + sinh(x) + 1)
```

giac [A] time = 0.11, size = 10, normalized size = 0.83

$$x+\frac{4}{e^x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-coth(x)+csch(x))^2,x, algorithm="giac")
```

```
[Out] x + 4/(e^x + 1)
```

maple [A] time = 0.37, size = 13, normalized size = 1.08

$$x - 2\coth(x) + \frac{2}{\sinh(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-coth(x)+csch(x))^2,x)
[Out] x-2*coth(x)+2/sinh(x)
```

maxima [B] time = 0.40, size = 25, normalized size = 2.08

$$x - \frac{4}{e^{(-x)} - e^{x}} + \frac{4}{e^{(-2x)} - 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-coth(x)+csch(x))^2,x, algorithm="maxima")
[Out] x - 4/(e^(-x) - e^x) + 4/(e^(-2*x) - 1)
```

mupad [B] time = 0.06, size = 10, normalized size = 0.83

$$x + \frac{4}{e^{x} + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((coth(x) - 1/sinh(x))^2,x)
[Out] x + 4/(exp(x) + 1)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(-\coth(x) + \operatorname{csch}(x)\right)^{2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-coth(x)+csch(x))**2,x)
[Out] Integral((-coth(x) + csch(x))**2, x)
```

3.668 $\int (-\coth(x) + \mathbf{csch}(x))\, dx$

Optimal. Leaf size=11

$$-\log(\sinh(x)) - \tanh^{-1}(\cosh(x))$$

```
[Out] -arctanh(cosh(x))-ln(sinh(x))
```

Rubi [A] time = 0.01, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3475, 3770}

$$-\log(\sinh(x)) - \tanh^{-1}(\cosh(x))$$

Antiderivative was successfully verified.

```
[In] Int[-Coth[x] + Csch[x],x]
```

```
[Out] -ArcTanh[Cosh[x]] - Log[Sinh[x]]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int (-\coth(x)+\operatorname{csch}(x))\, dx &= -\int \coth(x)\, dx + \int \operatorname{csch}(x)\, dx\\ &= -\tanh^{-1}(\cosh(x)) - \log(\sinh(x))\end{aligned}$$

Mathematica [A] time = 0.01, size = 13, normalized size = 1.18

$$\log\left(\tanh\left(\frac{x}{2}\right)\right) - \log(\sinh(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[-Coth[x] + Csch[x],x]
```

```
[Out] -Log[Sinh[x]] + Log[Tanh[x/2]]
```

fricas [A] time = 0.40, size = 11, normalized size = 1.00

$$x - 2 \log(\cosh(x) + \sinh(x) + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-coth(x)+csch(x),x, algorithm="fricas")
[Out] x - 2*log(cosh(x) + sinh(x) + 1)
```

giac [B] time = 0.12, size = 25, normalized size = 2.27

$$x - \log(e^x + 1) - \log\left(\left|e^{(2x)} - 1\right|\right) + \log(|e^x - 1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-coth(x)+csch(x),x, algorithm="giac")
[Out] x - log(e^x + 1) - log(abs(e^(2*x) - 1)) + log(abs(e^x - 1))
```

maple [A] time = 0.02, size = 12, normalized size = 1.09

$$-\ln(\sinh(x)) + \ln\left(\tanh\left(\frac{x}{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-coth(x)+csch(x),x)
[Out] -ln(sinh(x))+ln(tanh(1/2*x))
```

maxima [A] time = 0.61, size = 11, normalized size = 1.00

$$-\log(\sinh(x)) + \log\left(\tanh\left(\frac{1}{2}x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-coth(x)+csch(x),x, algorithm="maxima")
[Out] -log(sinh(x)) + log(tanh(1/2*x))
```

mupad [B] time = 1.54, size = 9, normalized size = 0.82

$$x - 2\ln(e^x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sinh(x) - coth(x),x)
```

```
[Out] x - 2*log(exp(x) + 1)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (-\coth(x) + \operatorname{csch}(x))\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-coth(x)+csch(x),x)
```

```
[Out] Integral(-coth(x) + csch(x), x)
```

3.669 $\int \frac{1}{-\coth(x)+\mathbf{csch}(x)}\,dx$

Optimal. Leaf size=9

$$-\log(1-\cosh(x))$$

```
[Out] -ln(1-cosh(x))
```

Rubi [A] time = 0.03, antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3160, 2667, 31}

$$-\log(1-\cosh(x))$$

Antiderivative was successfully verified.

```
[In] Int[(-Coth[x] + Csch[x])^(-1),x]
```

```
[Out] -Log[1 - Cosh[x]]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 3160

```
Int[((a_.) + csc[(d_.) + (e_.)*(x_)]*(b_.) + cot[(d_.) + (e_.)*(x_)]*(c_.))
^(-1), x_Symbol] :> Int[Sin[d + e*x]/(b + a*Sin[d + e*x] + c*Cos[d + e*x]),
 x] /; FreeQ[{a, b, c, d, e}, x]
```

Rubi steps

$$\int \frac{1}{-\coth(x)+\operatorname{csch}(x)}\,dx = i\int \frac{\sinh(x)}{i-i\cosh(x)}\,dx$$
$$= -\operatorname{Subst}\left(\int \frac{1}{i+x}\,dx, x, -i\cosh(x)\right)$$
$$= -\log(1-\cosh(x))$$

Mathematica [A] time = 0.02, size = 9, normalized size = 1.00

$$-2\log\left(\sinh\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-Coth[x] + Csch[x])^(-1),x]
[Out] -2*Log[Sinh[x/2]]
```

fricas [A] time = 0.40, size = 11, normalized size = 1.22

$$x-2\,\log\left(\cosh(x)+\sinh(x)-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-coth(x)+csch(x)),x, algorithm="fricas")
[Out] x - 2*log(cosh(x) + sinh(x) - 1)
```

giac [A] time = 0.13, size = 10, normalized size = 1.11

$$x-2\,\log\left(|e^{x}-1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-coth(x)+csch(x)),x, algorithm="giac")
[Out] x - 2*log(abs(e^x - 1))
```

maple [B] time = 0.19, size = 23, normalized size = 2.56

$$\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)+\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)-2\ln\left(\tanh\left(\frac{x}{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-coth(x)+csch(x)),x)
```

```
[Out] ln(tanh(1/2*x)-1)+ln(tanh(1/2*x)+1)-2*ln(tanh(1/2*x))
```

maxima [A] time = 0.37, size = 13, normalized size = 1.44

$$-x - 2\log\left(e^{(-x)} - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-coth(x)+csch(x)),x, algorithm="maxima")
[Out] -x - 2*log(e^(-x) - 1)
```

mupad [B] time = 0.04, size = 9, normalized size = 1.00

$$x - 2\ln\left(e^x - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/(coth(x) - 1/sinh(x)),x)
[Out] x - 2*log(exp(x) - 1)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{\coth(x) - \operatorname{csch}(x)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-coth(x)+csch(x)),x)
[Out] -Integral(1/(coth(x) - csch(x)), x)
```

3.670 $\int \frac{1}{(-\coth(x)+\mathbf{csch}(x))^2}\,dx$

Optimal. Leaf size=14

$$x+\frac{2\sinh(x)}{1-\cosh(x)}$$

[Out] x+2*sinh(x)/(1-cosh(x))

Rubi [A] time = 0.05, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {4392, 2680, 8}

$$x+\frac{2\sinh(x)}{1-\cosh(x)}$$

Antiderivative was successfully verified.

[In] Int[(-Coth[x] + Csch[x])^(-2),x]

[Out] x + (2*Sinh[x])/(1 - Cosh[x])

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2680

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(2*g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f
*x])^(m + 1))/(b*f*(2*m + p + 1)), x] + Dist[(g^2*(p - 1))/(b^2*(2*m + p +
1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 4392

```
Int[(cot[(c_.) + (d_.)*(x_)]^(n_.)*(a_.) + csc[(c_.) + (d_.)*(x_)]^(n_.)*(b
_.))^(p_)*(u_.), x_Symbol] :> Int[ActivateTrig[u]*Csc[c + d*x]^(n*p)*(b + a
*Cos[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

Rubi steps

$$\int \frac{1}{(-\coth(x)+\operatorname{csch}(x))^2}\,dx = -\int \frac{\sinh^2(x)}{(i-i\cosh(x))^2}\,dx$$
$$= \frac{2\sinh(x)}{1-\cosh(x)} + \int 1\,dx$$
$$= x + \frac{2\sinh(x)}{1-\cosh(x)}$$

Mathematica [C] time = 0.01, size = 24, normalized size = 1.71

$$-2\coth\left(\frac{x}{2}\right)\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\tanh^2\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-Coth[x] + Csch[x])^(-2),x]
[Out] -2*Coth[x/2]*Hypergeometric2F1[-1/2, 1, 1/2, Tanh[x/2]^2]
```

fricas [A] time = 0.39, size = 22, normalized size = 1.57

$$\frac{x\cosh(x)+x\sinh(x)-x-4}{\cosh(x)+\sinh(x)-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-coth(x)+csch(x))^2,x, algorithm="fricas")
[Out] (x*cosh(x) + x*sinh(x) - x - 4)/(cosh(x) + sinh(x) - 1)
```

giac [A] time = 0.13, size = 10, normalized size = 0.71

$$x - \frac{4}{e^x-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-coth(x)+csch(x))^2,x, algorithm="giac")
[Out] x - 4/(e^x - 1)
```

maple [A] time = 0.20, size = 26, normalized size = 1.86

$$-\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)+\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)-\frac{2}{\tanh\left(\frac{x}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-coth(x)+csch(x))^2,x)
```

```
[Out] -ln(tanh(1/2*x)-1)+ln(tanh(1/2*x)+1)-2/tanh(1/2*x)
```

maxima [A] time = 0.37, size = 12, normalized size = 0.86

$$x + \frac{4}{e^{(-x)} - 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-coth(x)+csch(x))^2,x, algorithm="maxima")
```

```
[Out] x + 4/(e^(-x) - 1)
```

mupad [B] time = 1.67, size = 10, normalized size = 0.71

$$x - \frac{4}{e^x - 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(coth(x) - 1/sinh(x))^2,x)
```

```
[Out] x - 4/(exp(x) - 1)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(-\coth(x) + \operatorname{csch}(x))^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-coth(x)+csch(x))**2,x)
```

```
[Out] Integral((-coth(x) + csch(x))**(-2), x)
```

3.671 $\int \frac{1}{(-\coth(x)+\mathbf{csch}(x))^3} dx$

Optimal. Leaf size=20

$$-\frac{2}{1-\cosh(x)} - \log(1-\cosh(x))$$

```
[Out] -2/(1-cosh(x))-ln(1-cosh(x))
```

Rubi [A] time = 0.06, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {4392, 2667, 43}

$$-\frac{2}{1-\cosh(x)} - \log(1-\cosh(x))$$

Antiderivative was successfully verified.

```
[In] Int[(-Coth[x] + Csch[x])^(-3),x]
```

```
[Out] -2/(1 - Cosh[x]) - Log[1 - Cosh[x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 4392

```
Int[(cot[(c_.) + (d_.)*(x_)]^(n_.)*(a_.) + csc[(c_.) + (d_.)*(x_)]^(n_.)*(b
_.))^(p_)*(u_.), x_Symbol] :> Int[ActivateTrig[u]*Csc[c + d*x]^(n*p)*(b + a
*Cos[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

Rubi steps

$$\int \frac{1}{(-\coth(x)+\operatorname{csch}(x))^3}\,dx = -\left(i\int \frac{\sinh^3(x)}{(i-i\cosh(x))^3}\,dx\right)$$
$$= \text{Subst}\left(\int \frac{i-x}{(i+x)^2}\,dx, x, -i\cosh(x)\right)$$
$$= \text{Subst}\left(\int \left(\frac{1}{-i-x}+\frac{2i}{(i+x)^2}\right)dx, x, -i\cosh(x)\right)$$
$$= -\frac{2i}{i-i\cosh(x)} - \log(1-\cosh(x))$$

Mathematica [A] time = 0.03, size = 18, normalized size = 0.90

$$\operatorname{csch}^2\left(\frac{x}{2}\right) - 2\log\left(\sinh\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-Coth[x] + Csch[x])^(-3),x]
```

```
[Out] Csch[x/2]^2 - 2*Log[Sinh[x/2]]
```

fricas [B] time = 0.44, size = 90, normalized size = 4.50

$$\frac{x\cosh(x)^2 + x\sinh(x)^2 - 2(x-2)\cosh(x) - 2\left(\cosh(x)^2 + 2(\cosh(x)-1)\sinh(x) + \sinh(x)^2 - 2\cosh(x) + 1\right)\mathrm{l}}{\cosh(x)^2 + 2(\cosh(x)-1)\sinh(x) + \sinh(x)^2 - 2\cosh(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-coth(x)+csch(x))^3,x, algorithm="fricas")
```

```
[Out] (x*cosh(x)^2 + x*sinh(x)^2 - 2*(x - 2)*cosh(x) - 2*(cosh(x)^2 + 2*(cosh(x)
- 1)*sinh(x) + sinh(x)^2 - 2*cosh(x) + 1)*log(cosh(x) + sinh(x) - 1) + 2*(x
*cosh(x) - x + 2)*sinh(x) + x)/(cosh(x)^2 + 2*(cosh(x) - 1)*sinh(x) + sinh(
x)^2 - 2*cosh(x) + 1)
```

giac [A] time = 0.13, size = 20, normalized size = 1.00

$$x + \frac{4\,e^x}{(e^x-1)^2} - 2\,\log\left(|e^x - 1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-coth(x)+csch(x))^3,x, algorithm="giac")
```

```
[Out] x + 4*e^x/(e^x - 1)^2 - 2*log(abs(e^x - 1))
```

maple [A] time = 0.20, size = 29, normalized size = 1.45

$$\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)+\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)+\frac{1}{\tanh\left(\frac{x}{2}\right)^{2}}-2\ln\left(\tanh\left(\frac{x}{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-coth(x)+csch(x))^3,x)
[Out] ln(tanh(1/2*x)-1)+ln(tanh(1/2*x)+1)+1/tanh(1/2*x)^2-2*ln(tanh(1/2*x))
```

maxima [A] time = 0.40, size = 35, normalized size = 1.75

$$-x-\frac{4\,e^{(-x)}}{2\,e^{(-x)}-e^{(-2\,x)}-1}-2\,\log\left(e^{(-x)}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-coth(x)+csch(x))^3,x, algorithm="maxima")
[Out] -x - 4*e^(-x)/(2*e^(-x) - e^(-2*x) - 1) - 2*log(e^(-x) - 1)
```

mupad [B] time = 1.51, size = 31, normalized size = 1.55

$$x-2\,\ln\left(\mathrm{e}^{x}-1\right)+\frac{4}{\mathrm{e}^{2\,x}-2\,\mathrm{e}^{x}+1}+\frac{4}{\mathrm{e}^{x}-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/(coth(x) - 1/sinh(x))^3,x)
[Out] x - 2*log(exp(x) - 1) + 4/(exp(2*x) - 2*exp(x) + 1) + 4/(exp(x) - 1)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int\frac{1}{\coth^{3}(x)-3\coth^{2}(x)\operatorname{csch}(x)+3\coth(x)\operatorname{csch}^{2}(x)-\operatorname{csch}^{3}(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-coth(x)+csch(x))**3,x)
[Out] -Integral(1/(coth(x)**3 - 3*coth(x)**2*csch(x) + 3*coth(x)*csch(x)**2 - csc
h(x)**3), x)
```

3.672 $\int \frac{1}{(-\coth(x)+\mathbf{csch}(x))^4}\,dx$

Optimal. Leaf size=30

$$x+\frac{2\sinh^3(x)}{3(1-\cosh(x))^3}+\frac{2\sinh(x)}{1-\cosh(x)}$$

```
[Out] x+2*sinh(x)/(1-cosh(x))+2/3*sinh(x)^3/(1-cosh(x))^3
```

Rubi [A] time = 0.08, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {4392, 2680, 8}

$$x+\frac{2\sinh^3(x)}{3(1-\cosh(x))^3}+\frac{2\sinh(x)}{1-\cosh(x)}$$

Antiderivative was successfully verified.

```
[In] Int[(-Coth[x] + Csch[x])^(-4),x]
```

```
[Out] x + (2*Sinh[x])/(1 - Cosh[x]) + (2*Sinh[x]^3)/(3*(1 - Cosh[x])^3)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2680

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(2*g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f
*x])^(m + 1))/(b*f*(2*m + p + 1)), x] + Dist[(g^2*(p - 1))/(b^2*(2*m + p +
1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 4392

```
Int[(cot[(c_.) + (d_.)*(x_)]^(n_.)*(a_.) + csc[(c_.) + (d_.)*(x_)]^(n_.)*(b
_.))^(p_)*(u_.), x_Symbol] :> Int[ActivateTrig[u]*Csc[c + d*x]^(n*p)*(b + a
*Cos[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

Rubi steps

$$\int \frac{1}{(-\coth(x)+\operatorname{csch}(x))^4}\,dx = \int \frac{\sinh^4(x)}{(i-i\cosh(x))^4}\,dx$$
$$= \frac{2\sinh^3(x)}{3(1-\cosh(x))^3} - \int \frac{\sinh^2(x)}{(i-i\cosh(x))^2}\,dx$$
$$= \frac{2\sinh(x)}{1-\cosh(x)} + \frac{2\sinh^3(x)}{3(1-\cosh(x))^3} + \int 1\,dx$$
$$= x + \frac{2\sinh(x)}{1-\cosh(x)} + \frac{2\sinh^3(x)}{3(1-\cosh(x))^3}$$

Mathematica [C] time = 0.01, size = 28, normalized size = 0.93

$$-\frac{2}{3}\coth^3\left(\frac{x}{2}\right) {}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};\tanh^2\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-Coth[x] + Csch[x])^(-4),x]
```

```
[Out] (-2*Coth[x/2]^3*Hypergeometric2F1[-3/2, 1, -1/2, Tanh[x/2]^2])/3
```

fricas [B] time = 0.41, size = 68, normalized size = 2.27

$$\frac{3\,x\cosh(x)^2 + 3\,x\sinh(x)^2 - 4\,(3\,x+10)\cosh(x) + 2\,(3\,x\cosh(x) - 3\,x - 4)\sinh(x) + 9\,x + 24}{3\left(\cosh(x)^2 + 2\,(\cosh(x)-1)\sinh(x) + \sinh(x)^2 - 4\,\cosh(x) + 3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-coth(x)+csch(x))^4,x, algorithm="fricas")
```

```
[Out] 1/3*(3*x*cosh(x)^2 + 3*x*sinh(x)^2 - 4*(3*x + 10)*cosh(x) + 2*(3*x*cosh(x)
- 3*x - 4)*sinh(x) + 9*x + 24)/(cosh(x)^2 + 2*(cosh(x) - 1)*sinh(x) + sinh(
x)^2 - 4*cosh(x) + 3)
```

giac [A] time = 0.14, size = 22, normalized size = 0.73

$$x - \frac{8\left(3\,e^{(2\,x)} - 3\,e^{x} + 2\right)}{3\,(e^{x}-1)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-coth(x)+csch(x))^4,x, algorithm="giac")
```

```
[Out] x - 8/3*(3*e^(2*x) - 3*e^x + 2)/(e^x - 1)^3
```

maple [A] time = 0.22, size = 34, normalized size = 1.13

$$-\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)+\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)-\frac{2}{3\tanh\left(\frac{x}{2}\right)^{3}}-\frac{2}{\tanh\left(\frac{x}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-coth(x)+csch(x))^4,x)
[Out] -ln(tanh(1/2*x)-1)+ln(tanh(1/2*x)+1)-2/3/tanh(1/2*x)^3-2/tanh(1/2*x)
```

maxima [A] time = 0.37, size = 38, normalized size = 1.27

$$x-\frac{8\left(3e^{(-x)}-3e^{(-2x)}-2\right)}{3\left(3e^{(-x)}-3e^{(-2x)}+e^{(-3x)}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-coth(x)+csch(x))^4,x, algorithm="maxima")
[Out] x - 8/3*(3*e^(-x) - 3*e^(-2*x) - 2)/(3*e^(-x) - 3*e^(-2*x) + e^(-3*x) - 1)
```

mupad [B] time = 1.53, size = 59, normalized size = 1.97

$$x-\frac{8\,e^{x}}{3\left(e^{2x}-2\,e^{x}+1\right)}+\frac{\frac{8\,e^{2x}}{3}+\frac{8}{3}}{3\,e^{2x}-e^{3x}-3\,e^{x}+1}-\frac{8}{3\,\left(e^{x}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(coth(x) - 1/sinh(x))^4,x)
[Out] x - (8*exp(x))/(3*(exp(2*x) - 2*exp(x) + 1)) + ((8*exp(2*x))/3 + 8/3)/(3*ex
p(2*x) - exp(3*x) - 3*exp(x) + 1) - 8/(3*(exp(x) - 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\left(-\coth\left(x\right)+\operatorname{csch}\left(x\right)\right)^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-coth(x)+csch(x))**4,x)
[Out] Integral((-coth(x) + csch(x))**(-4), x)
```

3.673 $\int \frac{1}{(-\coth(x)+\mathbf{csch}(x))^5}\,dx$

Optimal. Leaf size=30

$$-\frac{4}{1-\cosh(x)}+\frac{2}{(1-\cosh(x))^2}-\log(1-\cosh(x))$$

```
[Out] 2/(1-cosh(x))^2-4/(1-cosh(x))-ln(1-cosh(x))
```

Rubi [A] time = 0.06, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {4392, 2667, 43}

$$-\frac{4}{1-\cosh(x)}+\frac{2}{(1-\cosh(x))^2}-\log(1-\cosh(x))$$

Antiderivative was successfully verified.

```
[In] Int[(-Coth[x] + Csch[x])^(-5),x]
```

```
[Out] 2/(1 - Cosh[x])^2 - 4/(1 - Cosh[x]) - Log[1 - Cosh[x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 4392

```
Int[(cot[(c_.) + (d_.)*(x_)]^(n_.)*(a_.) + csc[(c_.) + (d_.)*(x_)]^(n_.)*(b
_.))^(p_)*(u_.), x_Symbol] :> Int[ActivateTrig[u]*Csc[c + d*x]^(n*p)*(b + a
*Cos[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

Rubi steps

$$\int \frac{1}{(-\coth(x)+\operatorname{csch}(x))^5}\,dx = i\int \frac{\sinh^5(x)}{(i-i\cosh(x))^5}\,dx$$
$$= -\text{Subst}\left(\int \frac{(i-x)^2}{(i+x)^3}\,dx, x, -i\cosh(x)\right)$$
$$= -\text{Subst}\left(\int \left(-\frac{4}{(i+x)^3}-\frac{4i}{(i+x)^2}+\frac{1}{i+x}\right)dx, x, -i\cosh(x)\right)$$
$$= -\frac{2}{(i-i\cosh(x))^2}-\frac{4i}{i-i\cosh(x)}-\log(1-\cosh(x))$$

Mathematica [A] time = 0.02, size = 32, normalized size = 1.07

$$\frac{1}{2}\operatorname{csch}^4\left(\frac{x}{2}\right)+2\operatorname{csch}^2\left(\frac{x}{2}\right)-2\log\left(\sinh\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-Coth[x] + Csch[x])^(-5),x]
```

```
[Out] 2*Csch[x/2]^2 + Csch[x/2]^4/2 - 2*Log[Sinh[x/2]]
```

fricas [B] time = 0.43, size = 269, normalized size = 8.97

$$\frac{x\cosh(x)^4 + x\sinh(x)^4 - 4\,(x-2)\cosh(x)^3 + 4\,(x\cosh(x) - x + 2)\sinh(x)^3 + 2\,(3\,x-4)\cosh(x)^2 + 2\left(3\,x\cosh\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-coth(x)+csch(x))^5,x, algorithm="fricas")
```

```
[Out] (x*cosh(x)^4 + x*sinh(x)^4 - 4*(x - 2)*cosh(x)^3 + 4*(x*cosh(x) - x + 2)*si
nh(x)^3 + 2*(3*x - 4)*cosh(x)^2 + 2*(3*x*cosh(x)^2 - 6*(x - 2)*cosh(x) + 3*
x - 4)*sinh(x)^2 - 4*(x - 2)*cosh(x) - 2*(cosh(x)^4 + 4*(cosh(x) - 1)*sinh(
x)^3 + sinh(x)^4 - 4*cosh(x)^3 + 6*(cosh(x)^2 - 2*cosh(x) + 1)*sinh(x)^2 +
6*cosh(x)^2 + 4*(cosh(x)^3 - 3*cosh(x)^2 + 3*cosh(x) - 1)*sinh(x) - 4*cosh(
x) + 1)*log(cosh(x) + sinh(x) - 1) + 4*(x*cosh(x)^3 - 3*(x - 2)*cosh(x)^2 +
 (3*x - 4)*cosh(x) - x + 2)*sinh(x) + x)/(cosh(x)^4 + 4*(cosh(x) - 1)*sinh(
x)^3 + sinh(x)^4 - 4*cosh(x)^3 + 6*(cosh(x)^2 - 2*cosh(x) + 1)*sinh(x)^2 +
6*cosh(x)^2 + 4*(cosh(x)^3 - 3*cosh(x)^2 + 3*cosh(x) - 1)*sinh(x) - 4*cosh(
x) + 1)
```

giac [A] time = 0.11, size = 31, normalized size = 1.03

$$x+\frac{8\left(e^{(3\,x)}-e^{(2\,x)}+e^{x}\right)}{\left(e^{x}-1\right)^4}-2\,\log\left(\left|e^{x}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-coth(x)+csch(x))^5,x, algorithm="giac")
```

```
[Out] x + 8*(e^(3*x) - e^(2*x) + e^x)/(e^x - 1)^4 - 2*log(abs(e^x - 1))
```

maple [A] time = 0.21, size = 37, normalized size = 1.23

$$\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)+\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)+\frac{1}{2\tanh\left(\frac{x}{2}\right)^{4}}+\frac{1}{\tanh\left(\frac{x}{2}\right)^{2}}-2\ln\left(\tanh\left(\frac{x}{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-coth(x)+csch(x))^5,x)
```

```
[Out] ln(tanh(1/2*x)-1)+ln(tanh(1/2*x)+1)+1/2/tanh(1/2*x)^4+1/tanh(1/2*x)^2-2*ln(
tanh(1/2*x))
```

maxima [B] time = 0.42, size = 58, normalized size = 1.93

$$-x-\frac{8\left(e^{(-x)}-e^{(-2\,x)}+e^{(-3\,x)}\right)}{4\,e^{(-x)}-6\,e^{(-2\,x)}+4\,e^{(-3\,x)}-e^{(-4\,x)}-1}-2\,\log\left(e^{(-x)}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-coth(x)+csch(x))^5,x, algorithm="maxima")
```

```
[Out] -x - 8*(e^(-x) - e^(-2*x) + e^(-3*x))/(4*e^(-x) - 6*e^(-2*x) + 4*e^(-3*x) -
 e^(-4*x) - 1) - 2*log(e^(-x) - 1)
```

mupad [B] time = 1.52, size = 79, normalized size = 2.63

$$x-2\,\ln\left(\mathrm{e}^{x}-1\right)-\frac{16}{3\,\mathrm{e}^{2\,x}-\mathrm{e}^{3\,x}-3\,\mathrm{e}^{x}+1}+\frac{16}{\mathrm{e}^{2\,x}-2\,\mathrm{e}^{x}+1}+\frac{8}{6\,\mathrm{e}^{2\,x}-4\,\mathrm{e}^{3\,x}+\mathrm{e}^{4\,x}-4\,\mathrm{e}^{x}+1}+\frac{8}{\mathrm{e}^{x}-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/(coth(x) - 1/sinh(x))^5,x)
```

```
[Out] x - 2*log(exp(x) - 1) - 16/(3*exp(2*x) - exp(3*x) - 3*exp(x) + 1) + 16/(exp
(2*x) - 2*exp(x) + 1) + 8/(6*exp(2*x) - 4*exp(3*x) + exp(4*x) - 4*exp(x) +
1) + 8/(exp(x) - 1)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int\frac{1}{\coth^{5}(x)-5\coth^{4}(x)\operatorname{csch}(x)+10\coth^{3}(x)\operatorname{csch}^{2}(x)-10\coth^{2}(x)\operatorname{csch}^{3}(x)+5\coth(x)\operatorname{csch}^{4}(x)-\operatorname{csc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-coth(x)+csch(x))**5,x)
```

```
[Out] -Integral(1/(coth(x)**5 - 5*coth(x)**4*csch(x) + 10*coth(x)**3*csch(x)**2 -
 10*coth(x)**2*csch(x)**3 + 5*coth(x)*csch(x)**4 - csch(x)**5), x)
```

3.674 $\int (\operatorname{csch}(x) + \sinh(x))\,dx$

Optimal. Leaf size=8

$$\cosh(x) - \tanh^{-1}(\cosh(x))$$

```
[Out] -arctanh(cosh(x))+cosh(x)
```

Rubi [A] time = 0.01, antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {3770, 2638}

$$\cosh(x) - \tanh^{-1}(\cosh(x))$$

Antiderivative was successfully verified.

```
[In] Int[Csch[x] + Sinh[x],x]
```

```
[Out] -ArcTanh[Cosh[x]] + Cosh[x]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int (\operatorname{csch}(x)+\sinh(x))\,dx &= \int \operatorname{csch}(x)\,dx+\int \sinh(x)\,dx\\ &= -\tanh^{-1}(\cosh(x))+\cosh(x)\end{aligned}$$

Mathematica [A] time = 0.00, size = 10, normalized size = 1.25

$$\cosh(x) + \log\left(\tanh\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[x] + Sinh[x],x]
```

[Out] Cosh[x] + Log[Tanh[x/2]]

fricas [B] time = 0.40, size = 53, normalized size = 6.62

$$\frac{\cosh(x)^2 - 2(\cosh(x) + \sinh(x))\log(\cosh(x) + \sinh(x) + 1) + 2(\cosh(x) + \sinh(x))\log(\cosh(x) + \sinh(x) - 1}{2(\cosh(x) + \sinh(x))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)+sinh(x),x, algorithm="fricas")
```

```
[Out] 1/2*(cosh(x)^2 - 2*(cosh(x) + sinh(x))*log(cosh(x) + sinh(x) + 1) + 2*(cosh
(x) + sinh(x))*log(cosh(x) + sinh(x) - 1) + 2*cosh(x)*sinh(x) + sinh(x)^2 +
 1)/(cosh(x) + sinh(x))
```

giac [B] time = 0.11, size = 24, normalized size = 3.00

$$\frac{1}{2}e^{(-x)} + \frac{1}{2}e^x - \log(e^x + 1) + \log(|e^x - 1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)+sinh(x),x, algorithm="giac")
```

```
[Out] 1/2*e^(-x) + 1/2*e^x - log(e^x + 1) + log(abs(e^x - 1))
```

maple [A] time = 0.02, size = 9, normalized size = 1.12

$$\ln\left(\tanh\left(\frac{x}{2}\right)\right) + \cosh(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(x)+sinh(x),x)
```

```
[Out] ln(tanh(1/2*x))+cosh(x)
```

maxima [A] time = 0.32, size = 8, normalized size = 1.00

$$\cosh(x) + \log\left(\tanh\left(\frac{1}{2}x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)+sinh(x),x, algorithm="maxima")
```

```
[Out] cosh(x) + log(tanh(1/2*x))
```

mupad [B] time = 0.04, size = 27, normalized size = 3.38

$$\ln\left(2-2\,\mathrm{e}^{x}\right)-\ln\left(-2\,\mathrm{e}^{x}-2\right)+\frac{\mathrm{e}^{-x}}{2}+\frac{\mathrm{e}^{x}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x) + 1/sinh(x),x)
[Out] log(2 - 2*exp(x)) - log(- 2*exp(x) - 2) + exp(-x)/2 + exp(x)/2
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(\sinh\left(x\right)+\operatorname{csch}\left(x\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)+sinh(x),x)
[Out] Integral(sinh(x) + csch(x), x)
```

3.675 $\int (\operatorname{csch}(x) + \sinh(x))^2 \, dx$

Optimal. Leaf size=22

$$\frac{3x}{2} - \frac{3\coth(x)}{2} + \frac{1}{2}\cosh^2(x)\coth(x)$$

```
[Out] 3/2*x-3/2*coth(x)+1/2*cosh(x)^2*coth(x)
```

Rubi [A] time = 0.02, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {290, 325, 206}

$$\frac{3x}{2} - \frac{3\coth(x)}{2} + \frac{1}{2}\cosh^2(x)\coth(x)$$

Antiderivative was successfully verified.

```
[In] Int[(Csch[x] + Sinh[x])^2,x]
```

```
[Out] (3*x)/2 - (3*Coth[x])/2 + (Cosh[x]^2*Coth[x])/2
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rubi steps

$$
\begin{aligned}
\int (\operatorname{csch}(x)+\sinh(x))^2\,dx &= \text{Subst}\left(\int \frac{1}{x^2\left(1-x^2\right)^2}\,dx,x,\tanh(x)\right)\\
&= \frac{1}{2}\cosh^2(x)\coth(x)+\frac{3}{2}\text{Subst}\left(\int \frac{1}{x^2\left(1-x^2\right)}\,dx,x,\tanh(x)\right)\\
&= -\frac{3\coth(x)}{2}+\frac{1}{2}\cosh^2(x)\coth(x)+\frac{3}{2}\text{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,\tanh(x)\right)\\
&= \frac{3x}{2}-\frac{3\coth(x)}{2}+\frac{1}{2}\cosh^2(x)\coth(x)
\end{aligned}
$$

Mathematica [A] time = 0.00, size = 18, normalized size = 0.82

$$
\frac{3x}{2}+\frac{1}{4}\sinh(2x)-\coth(x)
$$

Antiderivative was successfully verified.

```
[In] Integrate[(Csch[x] + Sinh[x])^2,x]
```

```
[Out] (3*x)/2 - Coth[x] + Sinh[2*x]/4
```

fricas [A] time = 0.41, size = 32, normalized size = 1.45

$$
\frac{\cosh(x)^3+3\,\cosh(x)\sinh(x)^2+4\,(3\,x+2)\sinh(x)-9\,\cosh(x)}{8\,\sinh(x)}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((csch(x)+sinh(x))^2,x, algorithm="fricas")
```

```
[Out] 1/8*(cosh(x)^3 + 3*cosh(x)*sinh(x)^2 + 4*(3*x + 2)*sinh(x) - 9*cosh(x))/sin
h(x)
```

giac [B] time = 0.11, size = 39, normalized size = 1.77

$$
\frac{3}{2}\,x-\frac{3\,e^{(4\,x)}+14\,e^{(2\,x)}-1}{8\,\left(e^{(4\,x)}-e^{(2\,x)}\right)}+\frac{1}{8}\,e^{(2\,x)}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((csch(x)+sinh(x))^2,x, algorithm="giac")
```

```
[Out] 3/2*x - 1/8*(3*e^(4*x) + 14*e^(2*x) - 1)/(e^(4*x) - e^(2*x)) + 1/8*e^(2*x)
```

maple [A] time = 0.40, size = 15, normalized size = 0.68

$$-\coth(x)+\frac{3x}{2}+\frac{\cosh(x)\sinh(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((csch(x)+sinh(x))^2,x)
[Out] -coth(x)+3/2*x+1/2*cosh(x)*sinh(x)
```

maxima [A] time = 0.36, size = 26, normalized size = 1.18

$$\frac{3}{2}\,x+\frac{2}{e^{(-2\,x)}-1}+\frac{1}{8}\,e^{(2\,x)}-\frac{1}{8}\,e^{(-2\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((csch(x)+sinh(x))^2,x, algorithm="maxima")
[Out] 3/2*x + 2/(e^(-2*x) - 1) + 1/8*e^(2*x) - 1/8*e^(-2*x)
```

mupad [B] time = 1.58, size = 26, normalized size = 1.18

$$\frac{3\,x}{2}-\frac{\mathrm{e}^{-2\,x}}{8}+\frac{\mathrm{e}^{2\,x}}{8}-\frac{2}{\mathrm{e}^{2\,x}-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sinh(x) + 1/sinh(x))^2,x)
[Out] (3*x)/2 - exp(-2*x)/8 + exp(2*x)/8 - 2/(exp(2*x) - 1)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(\sinh{\left(x\right)}+\operatorname{csch}{\left(x\right)}\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((csch(x)+sinh(x))**2,x)
[Out] Integral((sinh(x) + csch(x))**2, x)
```

3.676 $\int (\operatorname{csch}(x) + \sinh(x))^3 \, dx$

Optimal. Leaf size=34

$$\frac{5\cosh^3(x)}{6} + \frac{5\cosh(x)}{2} - \frac{1}{2}\cosh^3(x)\coth^2(x) - \frac{5}{2}\tanh^{-1}(\cosh(x))$$

```
[Out] -5/2*arctanh(cosh(x))+5/2*cosh(x)+5/6*cosh(x)^3-1/2*cosh(x)^3*coth(x)^2
```

Rubi [A] time = 0.05, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.714, Rules used = {4397, 2592, 288, 302, 206}

$$\frac{5\cosh^3(x)}{6} + \frac{5\cosh(x)}{2} - \frac{1}{2}\cosh^3(x)\coth^2(x) - \frac{5}{2}\tanh^{-1}(\cosh(x))$$

Antiderivative was successfully verified.

```
[In] Int[(Csch[x] + Sinh[x])^3,x]
```

```
[Out] (-5*ArcTanh[Cosh[x]])/2 + (5*Cosh[x])/2 + (5*Cosh[x]^3)/6 - (Cosh[x]^3*Coth
[x]^2)/2
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 2592

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, (a*Sin[e + f*x])/ff], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 4397

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rubi steps

$$\begin{aligned}
\int (\operatorname{csch}(x)+\sinh(x))^3\,dx &= \int \cosh^3(x)\coth^3(x)\,dx\\
&= \text{Subst}\left(\int \frac{x^6}{\left(1-x^2\right)^2}\,dx,x,\cosh(x)\right)\\
&= -\frac{1}{2}\cosh^3(x)\coth^2(x)-\frac{5}{2}\text{Subst}\left(\int \frac{x^4}{1-x^2}\,dx,x,\cosh(x)\right)\\
&= -\frac{1}{2}\cosh^3(x)\coth^2(x)-\frac{5}{2}\text{Subst}\left(\int \left(-1-x^2+\frac{1}{1-x^2}\right)dx,x,\cosh(x)\right)\\
&= \frac{5\cosh(x)}{2}+\frac{5\cosh^3(x)}{6}-\frac{1}{2}\cosh^3(x)\coth^2(x)-\frac{5}{2}\text{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,\cosh(x)\right)\\
&= -\frac{5}{2}\tanh^{-1}(\cosh(x))+\frac{5\cosh(x)}{2}+\frac{5\cosh^3(x)}{6}-\frac{1}{2}\cosh^3(x)\coth^2(x)
\end{aligned}$$

Mathematica [A] time = 0.07, size = 45, normalized size = 1.32

$$\frac{1}{48}\operatorname{csch}^2(x)\left(-50\cosh(x)+25\cosh(3x)+\cosh(5x)-60\log\left(\tanh\left(\frac{x}{2}\right)\right)+60\cosh(2x)\log\left(\tanh\left(\frac{x}{2}\right)\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Csch[x] + Sinh[x])^3,x]
```

```
[Out] (Csch[x]^2*(-50*Cosh[x] + 25*Cosh[3*x] + Cosh[5*x] - 60*Log[Tanh[x/2]] + 60
*Cosh[2*x]*Log[Tanh[x/2]]))/48
```

fricas [B] time = 0.43, size = 616, normalized size = 18.12

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((csch(x)+sinh(x))^3,x, algorithm="fricas")
```

```
[Out] 1/24*(cosh(x)^10 + 10*cosh(x)*sinh(x)^9 + sinh(x)^10 + 5*(9*cosh(x)^2 + 5)*
sinh(x)^8 + 25*cosh(x)^8 + 40*(3*cosh(x)^3 + 5*cosh(x))*sinh(x)^7 + 10*(21*
cosh(x)^4 + 70*cosh(x)^2 - 5)*sinh(x)^6 - 50*cosh(x)^6 + 4*(63*cosh(x)^5 +
350*cosh(x)^3 - 75*cosh(x))*sinh(x)^5 + 10*(21*cosh(x)^6 + 175*cosh(x)^4 -
75*cosh(x)^2 - 5)*sinh(x)^4 - 50*cosh(x)^4 + 40*(3*cosh(x)^7 + 35*cosh(x)^5
 - 25*cosh(x)^3 - 5*cosh(x))*sinh(x)^3 + 5*(9*cosh(x)^8 + 140*cosh(x)^6 - 1
50*cosh(x)^4 - 60*cosh(x)^2 + 5)*sinh(x)^2 + 25*cosh(x)^2 - 60*(cosh(x)^7 +
 7*cosh(x)*sinh(x)^6 + sinh(x)^7 + (21*cosh(x)^2 - 2)*sinh(x)^5 - 2*cosh(x)
^5 + 5*(7*cosh(x)^3 - 2*cosh(x))*sinh(x)^4 + (35*cosh(x)^4 - 20*cosh(x)^2 +
 1)*sinh(x)^3 + cosh(x)^3 + (21*cosh(x)^5 - 20*cosh(x)^3 + 3*cosh(x))*sinh(
x)^2 + (7*cosh(x)^6 - 10*cosh(x)^4 + 3*cosh(x)^2)*sinh(x))*log(cosh(x) + si
nh(x) + 1) + 60*(cosh(x)^7 + 7*cosh(x)*sinh(x)^6 + sinh(x)^7 + (21*cosh(x)^
2 - 2)*sinh(x)^5 - 2*cosh(x)^5 + 5*(7*cosh(x)^3 - 2*cosh(x))*sinh(x)^4 + (3
5*cosh(x)^4 - 20*cosh(x)^2 + 1)*sinh(x)^3 + cosh(x)^3 + (21*cosh(x)^5 - 20*
cosh(x)^3 + 3*cosh(x))*sinh(x)^2 + (7*cosh(x)^6 - 10*cosh(x)^4 + 3*cosh(x)^
2)*sinh(x))*log(cosh(x) + sinh(x) - 1) + 10*(cosh(x)^9 + 20*cosh(x)^7 - 30*
cosh(x)^5 - 20*cosh(x)^3 + 5*cosh(x))*sinh(x) + 1)/(cosh(x)^7 + 7*cosh(x)*s
inh(x)^6 + sinh(x)^7 + (21*cosh(x)^2 - 2)*sinh(x)^5 - 2*cosh(x)^5 + 5*(7*co
sh(x)^3 - 2*cosh(x))*sinh(x)^4 + (35*cosh(x)^4 - 20*cosh(x)^2 + 1)*sinh(x)^
3 + cosh(x)^3 + (21*cosh(x)^5 - 20*cosh(x)^3 + 3*cosh(x))*sinh(x)^2 + (7*co
sh(x)^6 - 10*cosh(x)^4 + 3*cosh(x)^2)*sinh(x))
```

giac [B] time = 0.11, size = 62, normalized size = 1.82

$$\frac{1}{24}\left(e^{(-x)}+e^{x}\right)^{3}-\frac{e^{(-x)}+e^{x}}{\left(e^{(-x)}+e^{x}\right)^{2}-4}+e^{(-x)}+e^{x}-\frac{5}{4}\,\log\left(e^{(-x)}+e^{x}+2\right)+\frac{5}{4}\,\log\left(e^{(-x)}+e^{x}-2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((csch(x)+sinh(x))^3,x, algorithm="giac")
```

```
[Out] 1/24*(e^(-x) + e^x)^3 - (e^(-x) + e^x)/((e^(-x) + e^x)^2 - 4) + e^(-x) + e^
x - 5/4*log(e^(-x) + e^x + 2) + 5/4*log(e^(-x) + e^x - 2)
```

maple [A] time = 0.47, size = 28, normalized size = 0.82

$$-\frac{\coth(x)\mathrm{csch}(x)}{2}-5\,\mathrm{arctanh}\left(\mathrm{e}^{x}\right)+3\cosh(x)+\left(-\frac{2}{3}+\frac{\left(\sinh^{2}(x)\right)}{3}\right)\cosh(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((csch(x)+sinh(x))^3,x)
```

```
[Out] -1/2*coth(x)*csch(x)-5*arctanh(exp(x))+3*cosh(x)+(-2/3+1/3*sinh(x)^2)*cosh(
x)
```

maxima [B] time = 0.37, size = 67, normalized size = 1.97

$$\frac{e^{(-x)} + e^{(-3x)}}{2e^{(-2x)} - e^{(-4x)} - 1} + \frac{1}{24}e^{(3x)} + \frac{9}{8}e^{(-x)} + \frac{1}{24}e^{(-3x)} + \frac{9}{8}e^{x} - \frac{5}{2}\log\left(e^{(-x)} + 1\right) + \frac{5}{2}\log\left(e^{(-x)} - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((csch(x)+sinh(x))^3,x, algorithm="maxima")
```

```
[Out] (e^(-x) + e^(-3*x))/(2*e^(-2*x) - e^(-4*x) - 1) + 1/24*e^(3*x) + 9/8*e^(-x)
 + 1/24*e^(-3*x) + 9/8*e^x - 5/2*log(e^(-x) + 1) + 5/2*log(e^(-x) - 1)
```

mupad [B] time = 0.06, size = 71, normalized size = 2.09

$$\frac{5\ln\left(5 - 5e^{x}\right)}{2} - \frac{5\ln\left(-5e^{x} - 5\right)}{2} + \frac{9e^{-x}}{8} + \frac{e^{-3x}}{24} + \frac{e^{3x}}{24} + \frac{9e^{x}}{8} - \frac{e^{x}}{e^{2x} - 1} - \frac{2e^{x}}{e^{4x} - 2e^{2x} + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sinh(x) + 1/sinh(x))^3,x)
```

```
[Out] (5*log(5 - 5*exp(x)))/2 - (5*log(- 5*exp(x) - 5))/2 + (9*exp(-x))/8 + exp(-
3*x)/24 + exp(3*x)/24 + (9*exp(x))/8 - exp(x)/(exp(2*x) - 1) - (2*exp(x))/(
exp(4*x) - 2*exp(2*x) + 1)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (\sinh(x) + \operatorname{csch}(x))^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((csch(x)+sinh(x))**3,x)
```

```
[Out] Integral((sinh(x) + csch(x))**3, x)
```

3.677 $\int \sqrt{\operatorname{csch}(x)+\sinh(x)}\,dx$

Optimal. Leaf size=13

$$2\tanh(x)\sqrt{\cosh(x)\coth(x)}$$

[Out] 2*(cosh(x)*coth(x))^(1/2)*tanh(x)

Rubi [A] time = 0.07, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {4397, 4398, 4400, 2589}

$$2\tanh(x)\sqrt{\cosh(x)\coth(x)}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[Csch[x] + Sinh[x]],x]

[Out] 2*Sqrt[Cosh[x]*Coth[x]]*Tanh[x]

Rule 2589

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> -Simp[(b*(a*Sin[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*m
), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n - 1, 0]
```

Rule 4397

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rule 4398

```
Int[(u_.)*((a_)*(v_))^(p_), x_Symbol] :> With[{uu = ActivateTrig[u], vv = A
ctivateTrig[v]}, Dist[(a^IntPart[p]*(a*vv)^FracPart[p])/vv^FracPart[p], Int
[uu*vv^p, x], x]] /; FreeQ[{a, p}, x] &&  !IntegerQ[p] &&  !InertTrigFreeQ[
v]
```

Rule 4400

```
Int[(u_.)*((v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> With[{uu = ActivateTri
g[u], vv = ActivateTrig[v], ww = ActivateTrig[w]}, Dist[(vv^m*ww^n)^FracPar
t[p]/(vv^(m*FracPart[p])*ww^(n*FracPart[p])), Int[uu*vv^(m*p)*ww^(n*p), x],
 x]] /; FreeQ[{m, n, p}, x] &&  !IntegerQ[p] && ( !InertTrigFreeQ[v] ||  !I
nertTrigFreeQ[w])
```

Rubi steps

$$\int \sqrt{\operatorname{csch}(x)+\sinh(x)}\,dx = \int \sqrt{\cosh(x)\coth(x)}\,dx$$
$$= \frac{\sqrt{\cosh(x)\coth(x)}\int \sqrt{-i\cosh(x)\coth(x)}\,dx}{\sqrt{-i\cosh(x)\coth(x)}}$$
$$= \frac{\sqrt{\cosh(x)\coth(x)}\int \sqrt{\cosh(x)}\sqrt{-i\coth(x)}\,dx}{\sqrt{\cosh(x)}\sqrt{-i\coth(x)}}$$
$$= 2\sqrt{\cosh(x)\coth(x)}\,\tanh(x)$$

Mathematica [B] time = 0.07, size = 35, normalized size = 2.69

$$\frac{2\left(\sqrt[4]{-\sinh^2(x)}-1\right)\tanh(x)\sqrt{\cosh(x)\coth(x)}}{\sqrt[4]{-\sinh^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Csch[x] + Sinh[x]],x]
```

```
[Out] (2*Sqrt[Cosh[x]*Coth[x]]*(-1 + (-Sinh[x]^2)^(1/4))*Tanh[x])/(-Sinh[x]^2)^(1
/4)
```

fricas [B] time = 0.41, size = 55, normalized size = 4.23

$$\frac{2\sqrt{\frac{1}{2}}\left(\cosh(x)^2+2\cosh(x)\sinh(x)+\sinh(x)^2-1\right)}{\sqrt{\cosh(x)^3+3\cosh(x)\sinh(x)^2+\sinh(x)^3+\left(3\cosh(x)^2-1\right)\sinh(x)-\cosh(x)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((csch(x)+sinh(x))^(1/2),x, algorithm="fricas")
```

```
[Out] 2*sqrt(1/2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)/sqrt(cosh(x)^3
+ 3*cosh(x)*sinh(x)^2 + sinh(x)^3 + (3*cosh(x)^2 - 1)*sinh(x) - cosh(x))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\operatorname{csch}(x)+\sinh(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((csch(x)+sinh(x))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(csch(x) + sinh(x)), x)
```

maple [B] time = 0.58, size = 42, normalized size = 3.23

$$\frac{\sqrt{2}\,\sqrt{\frac{\left(1+e^{2x}\right)^2 e^{-x}}{e^{2x}-1}}\,\left(e^{2x}-1\right)}{1+e^{2x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((csch(x)+sinh(x))^(1/2),x)
```

```
[Out] 2^(1/2)*((1+exp(2*x))^2*exp(-x)/(exp(2*x)-1))^(1/2)/(1+exp(2*x))*(exp(2*x)-
1)
```

maxima [B] time = 0.52, size = 54, normalized size = 4.15

$$\frac{\sqrt{2}\,e^{\left(\frac{1}{2}\,x\right)}}{\sqrt{e^{(-x)}+1}\,\sqrt{-e^{(-x)}+1}} - \frac{\sqrt{2}\,e^{\left(-\frac{3}{2}\,x\right)}}{\sqrt{e^{(-x)}+1}\,\sqrt{-e^{(-x)}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((csch(x)+sinh(x))^(1/2),x, algorithm="maxima")
```

```
[Out] sqrt(2)*e^(1/2*x)/(sqrt(e^(-x) + 1)*sqrt(-e^(-x) + 1)) - sqrt(2)*e^(-3/2*x)
/(sqrt(e^(-x) + 1)*sqrt(-e^(-x) + 1))
```

mupad [B] time = 1.54, size = 13, normalized size = 1.00

$$2\tanh(x)\sqrt{\sinh(x)+\frac{1}{\sinh(x)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sinh(x) + 1/sinh(x))^(1/2),x)
```

```
[Out] 2*tanh(x)*(sinh(x) + 1/sinh(x))^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\sinh(x)+\operatorname{csch}(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((csch(x)+sinh(x))**(1/2),x)
```

```
[Out] Integral(sqrt(sinh(x) + csch(x)), x)
```

3.678 $\int (\text{csch}(x) + \sinh(x))^{3/2} \, dx$

Optimal. Leaf size=31

$$\frac{2}{3}\cosh(x)\sqrt{\cosh(x)\coth(x)} - \frac{8}{3}\text{sech}(x)\sqrt{\cosh(x)\coth(x)}$$

```
[Out] 2/3*cosh(x)*(cosh(x)*coth(x))^(1/2)-8/3*sech(x)*(cosh(x)*coth(x))^(1/2)
```

Rubi [A] time = 0.12, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.556, Rules used = {4397, 4398, 4400, 2598, 2589}

$$\frac{2}{3}\cosh(x)\sqrt{\cosh(x)\coth(x)} - \frac{8}{3}\text{sech}(x)\sqrt{\cosh(x)\coth(x)}$$

Antiderivative was successfully verified.

```
[In] Int[(Csch[x] + Sinh[x])^(3/2),x]
```

```
[Out] (2*Cosh[x]*Sqrt[Cosh[x]*Coth[x]])/3 - (8*Sqrt[Cosh[x]*Coth[x]]*Sech[x])/3
```

Rule 2589

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> -Simp[(b*(a*Sin[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*m
), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n - 1, 0]
```

Rule 2598

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> -Simp[(b*(a*Sin[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f
*m), x] + Dist[(a^2*(m + n - 1))/m, Int[(a*Sin[e + f*x])^(m - 2)*(b*Tan[e +
 f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (GtQ[m, 1] || (EqQ[m, 1] &
& EqQ[n, 1/2])) && IntegersQ[2*m, 2*n]
```

Rule 4397

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rule 4398

```
Int[(u_.)*((a_)*(v_))^(p_), x_Symbol] :> With[{uu = ActivateTrig[u], vv = A
ctivateTrig[v]}, Dist[(a^IntPart[p]*(a*vv)^FracPart[p])/vv^FracPart[p], Int
[uu*vv^p, x], x]] /; FreeQ[{a, p}, x] && !IntegerQ[p] && !InertTrigFreeQ[
v]
```

Rule 4400

```
Int[(u_.)*((v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> With[{uu = ActivateTri
g[u], vv = ActivateTrig[v], ww = ActivateTrig[w]}, Dist[(vv^m*ww^n)^FracPar
t[p]/(vv^(m*FracPart[p])*ww^(n*FracPart[p])), Int[uu*vv^(m*p)*ww^(n*p), x],
 x]] /; FreeQ[{m, n, p}, x] &&  !IntegerQ[p] && ( !InertTrigFreeQ[v] ||  !I
nertTrigFreeQ[w])
```

Rubi steps

$$\begin{aligned}
\int (\operatorname{csch}(x)+\sinh(x))^{3/2}\,dx &= \int (\cosh(x)\coth(x))^{3/2}\,dx \\
&= \frac{\left(i\sqrt{\cosh(x)\coth(x)}\right)\int (-i\cosh(x)\coth(x))^{3/2}\,dx}{\sqrt{-i\cosh(x)\coth(x)}} \\
&= \frac{\left(i\sqrt{\cosh(x)\coth(x)}\right)\int \cosh^{\frac{3}{2}}(x)(-i\coth(x))^{3/2}\,dx}{\sqrt{\cosh(x)}\sqrt{-i\coth(x)}} \\
&= \frac{2}{3}\cosh(x)\sqrt{\cosh(x)\coth(x)}+\frac{\left(4i\sqrt{\cosh(x)\coth(x)}\right)\int \frac{(-i\coth(x))^{3/2}}{\sqrt{\cosh(x)}}\,dx}{3\sqrt{\cosh(x)}\sqrt{-i\coth(x)}} \\
&= \frac{2}{3}\cosh(x)\sqrt{\cosh(x)\coth(x)}-\frac{8}{3}\sqrt{\cosh(x)\coth(x)}\operatorname{sech}(x)
\end{aligned}$$

Mathematica [A] time = 0.05, size = 21, normalized size = 0.68

$$\frac{2}{3}\left(\cosh^2(x)-4\right)\operatorname{sech}(x)\sqrt{\cosh(x)\coth(x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Csch[x] + Sinh[x])^(3/2),x]
```

```
[Out] (2*(-4 + Cosh[x]^2)*Sqrt[Cosh[x]*Coth[x]]*Sech[x])/3
```

fricas [B] time = 0.43, size = 97, normalized size = 3.13

$$\frac{\sqrt{\frac{1}{2}}\left(\cosh(x)^4+4\cosh(x)\sinh(x)^3+\sinh(x)^4+2\left(3\cosh(x)^2-7\right)\sinh(x)^2-14\cosh(x)^2+4\left(\cosh(x)^3-7\right.\right.}{3\sqrt{\cosh(x)^3+3\cosh(x)\sinh(x)^2+\sinh(x)^3+\left(3\cosh(x)^2-1\right)\sinh(x)-\cosh(x)}\,(\cosh(x)+\mathrm{s}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((csch(x)+sinh(x))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/3*sqrt(1/2)*(cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + 2*(3*cosh(x)^2
 - 7)*sinh(x)^2 - 14*cosh(x)^2 + 4*(cosh(x)^3 - 7*cosh(x))*sinh(x) + 1)/(sq
rt(cosh(x)^3 + 3*cosh(x)*sinh(x)^2 + sinh(x)^3 + (3*cosh(x)^2 - 1)*sinh(x)
- cosh(x))*(cosh(x) + sinh(x)))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (\operatorname{csch}(x) + \sinh(x))^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((csch(x)+sinh(x))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((csch(x) + sinh(x))^(3/2), x)
```

maple [F] time = 0.49, size = 0, normalized size = 0.00

$$\int (\operatorname{csch}(x) + \sinh(x))^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((csch(x)+sinh(x))^(3/2),x)
```

```
[Out] int((csch(x)+sinh(x))^(3/2),x)
```

maxima [B] time = 0.54, size = 109, normalized size = 3.52

$$\frac{\sqrt{2}\, e^{\left(\frac{3}{2}\, x\right)}}{6 \left(e^{(-x)} + 1\right)^{\frac{3}{2}} \left(-e^{(-x)} + 1\right)^{\frac{3}{2}}} - \frac{5 \sqrt{2}\, e^{\left(-\frac{1}{2}\, x\right)}}{2 \left(e^{(-x)} + 1\right)^{\frac{3}{2}} \left(-e^{(-x)} + 1\right)^{\frac{3}{2}}} + \frac{5 \sqrt{2}\, e^{\left(-\frac{5}{2}\, x\right)}}{2 \left(e^{(-x)} + 1\right)^{\frac{3}{2}} \left(-e^{(-x)} + 1\right)^{\frac{3}{2}}} - \frac{\sqrt{2}\, e^{\left(-\frac{9}{2}\, x\right)}}{6 \left(e^{(-x)} + 1\right)^{\frac{3}{2}} \left(-e^{(-x)} + 1\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((csch(x)+sinh(x))^(3/2),x, algorithm="maxima")
```

```
[Out] 1/6*sqrt(2)*e^(3/2*x)/((e^(-x) + 1)^(3/2)*(-e^(-x) + 1)^(3/2)) - 5/2*sqrt(2
)*e^(-1/2*x)/((e^(-x) + 1)^(3/2)*(-e^(-x) + 1)^(3/2)) + 5/2*sqrt(2)*e^(-5/2
*x)/((e^(-x) + 1)^(3/2)*(-e^(-x) + 1)^(3/2)) - 1/6*sqrt(2)*e^(-9/2*x)/((e^(
-x) + 1)^(3/2)*(-e^(-x) + 1)^(3/2))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int {\left(\sinh(x) + \frac{1}{\sinh(x)}\right)}^{3/2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sinh(x) + 1/sinh(x))^(3/2),x)
[Out] int((sinh(x) + 1/sinh(x))^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((csch(x)+sinh(x))**(3/2),x)
[Out] Timed out
```

3.679 $\int (\mathrm{csch}(x)+\sinh(x))^{5/2}\,dx$

Optimal. Leaf size=50

$$\frac{2}{5}\cosh^2(x)\coth(x)\sqrt{\cosh(x)\coth(x)}-\frac{16}{15}\coth(x)\sqrt{\cosh(x)\coth(x)}+\frac{64}{15}\tanh(x)\sqrt{\cosh(x)\coth(x)}$$

```
[Out] -16/15*coth(x)*(cosh(x)*coth(x))^(1/2)+2/5*cosh(x)^2*coth(x)*(cosh(x)*coth(
x))^(1/2)+64/15*(cosh(x)*coth(x))^(1/2)*tanh(x)
```

Rubi [A] time = 0.16, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {4397, 4398, 4400, 2598, 2594, 2589}

$$\frac{2}{5}\cosh^2(x)\coth(x)\sqrt{\cosh(x)\coth(x)}-\frac{16}{15}\coth(x)\sqrt{\cosh(x)\coth(x)}+\frac{64}{15}\tanh(x)\sqrt{\cosh(x)\coth(x)}$$

Antiderivative was successfully verified.

```
[In] Int[(Csch[x] + Sinh[x])^(5/2),x]
```

```
[Out] (-16*Coth[x]*Sqrt[Cosh[x]*Coth[x]])/15 + (2*Cosh[x]^2*Coth[x]*Sqrt[Cosh[x]*
Coth[x]])/5 + (64*Sqrt[Cosh[x]*Coth[x]]*Tanh[x])/15
```

Rule 2589

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> -Simp[(b*(a*Sin[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*m
), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n - 1, 0]
```

Rule 2594

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sin[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
n - 1)), x] - Dist[(b^2*(m + n - 1))/(n - 1), Int[(a*Sin[e + f*x])^m*(b*Tan
[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] && Int
egersQ[2*m, 2*n] &&  !(GtQ[m, 1] &&  !IntegerQ[(m - 1)/2])
```

Rule 2598

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> -Simp[(b*(a*Sin[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f
*m), x] + Dist[(a^2*(m + n - 1))/m, Int[(a*Sin[e + f*x])^(m - 2)*(b*Tan[e +
 f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (GtQ[m, 1] || (EqQ[m, 1] &
& EqQ[n, 1/2])) && IntegersQ[2*m, 2*n]
```

Rule 4397

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rule 4398

```
Int[(u_.)*((a_)*(v_))^(p_), x_Symbol] :> With[{uu = ActivateTrig[u], vv = A
ctivateTrig[v]}, Dist[(a^IntPart[p]*(a*vv)^FracPart[p])/vv^FracPart[p], Int
[uu*vv^p, x], x]] /; FreeQ[{a, p}, x] &&  !IntegerQ[p] &&  !InertTrigFreeQ[
v]
```

Rule 4400

```
Int[(u_.)*((v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> With[{uu = ActivateTri
g[u], vv = ActivateTrig[v], ww = ActivateTrig[w]}, Dist[(vv^m*ww^n)^FracPar
t[p]/(vv^(m*FracPart[p])*ww^(n*FracPart[p])), Int[uu*vv^(m*p)*ww^(n*p), x],
 x]] /; FreeQ[{m, n, p}, x] &&  !IntegerQ[p] && ( !InertTrigFreeQ[v] ||  !I
nertTrigFreeQ[w])
```

Rubi steps

$$
\begin{aligned}
\int (\operatorname{csch}(x)+\sinh(x))^{5/2}\,dx &= \int (\cosh(x)\coth(x))^{5/2}\,dx \\
&= -\frac{\sqrt{\cosh(x)\coth(x)}\int (-i\cosh(x)\coth(x))^{5/2}\,dx}{\sqrt{-i\cosh(x)\coth(x)}} \\
&= -\frac{\sqrt{\cosh(x)\coth(x)}\int \cosh^{\frac{5}{2}}(x)(-i\coth(x))^{5/2}\,dx}{\sqrt{\cosh(x)}\sqrt{-i\coth(x)}} \\
&= \frac{2}{5}\cosh^2(x)\coth(x)\sqrt{\cosh(x)\coth(x)} - \frac{\left(8\sqrt{\cosh(x)\coth(x)}\right)\int \sqrt{\cosh(x)}\,(-i\operatorname{co}}{5\sqrt{\cosh(x)}\sqrt{-i\coth(x)}} \\
&= -\frac{16}{15}\coth(x)\sqrt{\cosh(x)\coth(x)} + \frac{2}{5}\cosh^2(x)\coth(x)\sqrt{\cosh(x)\coth(x)} + \frac{\left(32\sqrt{\operatorname{c}}\right.}{} \\
&= -\frac{16}{15}\coth(x)\sqrt{\cosh(x)\coth(x)} + \frac{2}{5}\cosh^2(x)\coth(x)\sqrt{\cosh(x)\coth(x)} + \frac{64}{15}\sqrt{\operatorname{co}}
\end{aligned}
$$

Mathematica [A] time = 0.32, size = 44, normalized size = 0.88

$$
\frac{1}{15}\sqrt{\cosh(x)\coth(x)}\left(64\tanh(x)-10\coth(x)+6\sinh(x)\cosh(x)+57\left(-\sinh^2(x)\right)^{3/4}\operatorname{csch}(x)\operatorname{sech}(x)\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[(Csch[x] + Sinh[x])^(5/2),x]
```

```
[Out] (Sqrt[Cosh[x]*Coth[x]]*(-10*Coth[x] + 6*Cosh[x]*Sinh[x] + 57*Csch[x]*Sech[x
]*(-Sinh[x]^2)^(3/4) + 64*Tanh[x]))/15
```

fricas [B] time = 0.44, size = 259, normalized size = 5.18

$$\frac{\sqrt{\frac{1}{2}}\left(3\,\cosh(x)^{8}+24\,\cosh(x)\sinh(x)^{7}+3\,\sinh(x)^{8}+12\left(7\,\cosh(x)^{2}+9\right)\sinh(x)^{6}+108\,\cosh(x)^{6}+24\left(7\,\cos\right.\right.}{30\left(\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((csch(x)+sinh(x))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/30*sqrt(1/2)*(3*cosh(x)^8 + 24*cosh(x)*sinh(x)^7 + 3*sinh(x)^8 + 12*(7*co
sh(x)^2 + 9)*sinh(x)^6 + 108*cosh(x)^6 + 24*(7*cosh(x)^3 + 27*cosh(x))*sinh
(x)^5 + 2*(105*cosh(x)^4 + 810*cosh(x)^2 - 151)*sinh(x)^4 - 302*cosh(x)^4 +
 8*(21*cosh(x)^5 + 270*cosh(x)^3 - 151*cosh(x))*sinh(x)^3 + 12*(7*cosh(x)^6
 + 135*cosh(x)^4 - 151*cosh(x)^2 + 9)*sinh(x)^2 + 108*cosh(x)^2 + 8*(3*cosh
(x)^7 + 81*cosh(x)^5 - 151*cosh(x)^3 + 27*cosh(x))*sinh(x) + 3)/((cosh(x)^4
 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + (6*cosh(x)^2 - 1)*sinh(x)^2 - cosh(x)^
2 + 2*(2*cosh(x)^3 - cosh(x))*sinh(x))*sqrt(cosh(x)^3 + 3*cosh(x)*sinh(x)^2
 + sinh(x)^3 + (3*cosh(x)^2 - 1)*sinh(x) - cosh(x)))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int(\operatorname{csch}(x)+\sinh(x))^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((csch(x)+sinh(x))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((csch(x) + sinh(x))^(5/2), x)
```

maple [F] time = 0.48, size = 0, normalized size = 0.00

$$\int(\operatorname{csch}(x)+\sinh(x))^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((csch(x)+sinh(x))^(5/2),x)
```

```
[Out] int((csch(x)+sinh(x))^(5/2),x)
```

maxima [B] time = 0.86, size = 163, normalized size = 3.26

$$\frac{\sqrt{2}e^{\left(\frac{5}{2}x\right)}}{20\left(e^{(-x)}+1\right)^{\frac{5}{2}}\left(-e^{(-x)}+1\right)^{\frac{5}{2}}}+\frac{7\sqrt{2}e^{\left(\frac{1}{2}x\right)}}{4\left(e^{(-x)}+1\right)^{\frac{5}{2}}\left(-e^{(-x)}+1\right)^{\frac{5}{2}}}-\frac{41\sqrt{2}e^{\left(-\frac{3}{2}x\right)}}{6\left(e^{(-x)}+1\right)^{\frac{5}{2}}\left(-e^{(-x)}+1\right)^{\frac{5}{2}}}+\frac{41\sqrt{2}e^{\left(-\frac{7}{2}x\right)}}{6\left(e^{(-x)}+1\right)^{\frac{5}{2}}\left(-e^{(-x)}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((csch(x)+sinh(x))^(5/2),x, algorithm="maxima")
```

```
[Out] 1/20*sqrt(2)*e^(5/2*x)/((e^(-x) + 1)^(5/2)*(-e^(-x) + 1)^(5/2)) + 7/4*sqrt(
2)*e^(1/2*x)/((e^(-x) + 1)^(5/2)*(-e^(-x) + 1)^(5/2)) - 41/6*sqrt(2)*e^(-3/
2*x)/((e^(-x) + 1)^(5/2)*(-e^(-x) + 1)^(5/2)) + 41/6*sqrt(2)*e^(-7/2*x)/((e
^(-x) + 1)^(5/2)*(-e^(-x) + 1)^(5/2)) - 7/4*sqrt(2)*e^(-11/2*x)/((e^(-x) +
1)^(5/2)*(-e^(-x) + 1)^(5/2)) - 1/20*sqrt(2)*e^(-15/2*x)/((e^(-x) + 1)^(5/2
)*(-e^(-x) + 1)^(5/2))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \left(\sinh(x)+\frac{1}{\sinh(x)}\right)^{5/2}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sinh(x) + 1/sinh(x))^(5/2),x)
```

```
[Out] int((sinh(x) + 1/sinh(x))^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((csch(x)+sinh(x))**(5/2),x)
```

```
[Out] Timed out
```

3.680 $\int (-\cosh(x) + \mathbf{sech}(x))\, dx$

Optimal. Leaf size=8

$$\tan^{-1}(\sinh(x)) - \sinh(x)$$

```
[Out] arctan(sinh(x))-sinh(x)
```

Rubi [A] time = 0.01, antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2637, 3770}

$$\tan^{-1}(\sinh(x)) - \sinh(x)$$

Antiderivative was successfully verified.

```
[In] Int[-Cosh[x] + Sech[x],x]
```

```
[Out] ArcTan[Sinh[x]] - Sinh[x]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned} \int (-\cosh(x) + \operatorname{sech}(x))\, dx &= -\int \cosh(x)\, dx + \int \operatorname{sech}(x)\, dx \\ &= \tan^{-1}(\sinh(x)) - \sinh(x) \end{aligned}$$

Mathematica [A] time = 0.00, size = 14, normalized size = 1.75

$$2\tan^{-1}\left(\tanh\left(\frac{x}{2}\right)\right) - \sinh(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[-Cosh[x] + Sech[x],x]
```

```
[Out] 2*ArcTan[Tanh[x/2]] - Sinh[x]
```

fricas [B] time = 0.47, size = 42, normalized size = 5.25

$$\frac{4\left(\cosh(x)+\sinh(x)\right)\arctan\left(\cosh(x)+\sinh(x)\right)-\cosh(x)^2-2\,\cosh(x)\sinh(x)-\sinh(x)^2+1}{2\left(\cosh(x)+\sinh(x)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-cosh(x)+sech(x),x, algorithm="fricas")
```

```
[Out] 1/2*(4*(cosh(x) + sinh(x))*arctan(cosh(x) + sinh(x)) - cosh(x)^2 - 2*cosh(x
)*sinh(x) - sinh(x)^2 + 1)/(cosh(x) + sinh(x))
```

giac [A] time = 0.13, size = 16, normalized size = 2.00

$$2\,\arctan\left(e^x\right)+\frac{1}{2}\,e^{(-x)}-\frac{1}{2}\,e^x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-cosh(x)+sech(x),x, algorithm="giac")
```

```
[Out] 2*arctan(e^x) + 1/2*e^(-x) - 1/2*e^x
```

maple [A] time = 0.02, size = 9, normalized size = 1.12

$$\arctan\left(\sinh(x)\right)-\sinh(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-cosh(x)+sech(x),x)
```

```
[Out] arctan(sinh(x))-sinh(x)
```

maxima [A] time = 0.52, size = 8, normalized size = 1.00

$$\arctan\left(\sinh(x)\right)-\sinh(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-cosh(x)+sech(x),x, algorithm="maxima")
```

```
[Out] arctan(sinh(x)) - sinh(x)
```

mupad [B] time = 1.49, size = 16, normalized size = 2.00

$$\frac{\mathrm{e}^{-x}}{2}+2\,\mathrm{atan}\left(\mathrm{e}^x\right)-\frac{\mathrm{e}^x}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cosh(x) - cosh(x),x)
```

```
[Out] exp(-x)/2 + 2*atan(exp(x)) - exp(x)/2
```

sympy [F]   time = 0.00, size = 0, normalized size = 0.00

$$\int (-\cosh(x) + \operatorname{sech}(x))\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-cosh(x)+sech(x),x)
```

```
[Out] Integral(-cosh(x) + sech(x), x)
```

3.681 $\int (-\cosh(x) + \mathbf{sech}(x))^2 \, dx$

Optimal. Leaf size=22

$$-\frac{3x}{2} + \frac{3\tanh(x)}{2} + \frac{1}{2}\sinh^2(x)\tanh(x)$$

```
[Out] -3/2*x+3/2*tanh(x)+1/2*sinh(x)^2*tanh(x)
```

Rubi [A] time = 0.02, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {288, 321, 206}

$$-\frac{3x}{2} + \frac{3\tanh(x)}{2} + \frac{1}{2}\sinh^2(x)\tanh(x)$$

Antiderivative was successfully verified.

```
[In] Int[(-Cosh[x] + Sech[x])^2,x]
```

```
[Out] (-3*x)/2 + (3*Tanh[x])/2 + (Sinh[x]^2*Tanh[x])/2
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int (-\cosh(x)+\operatorname{sech}(x))^2\,dx = \text{Subst}\left(\int \frac{x^4}{(1-x^2)^2}\,dx, x, \tanh(x)\right)$$
$$= \frac{1}{2}\sinh^2(x)\tanh(x) - \frac{3}{2}\text{Subst}\left(\int \frac{x^2}{1-x^2}\,dx, x, \tanh(x)\right)$$
$$= \frac{3\tanh(x)}{2} + \frac{1}{2}\sinh^2(x)\tanh(x) - \frac{3}{2}\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \tanh(x)\right)$$
$$= -\frac{3x}{2} + \frac{3\tanh(x)}{2} + \frac{1}{2}\sinh^2(x)\tanh(x)$$

Mathematica [A] time = 0.03, size = 16, normalized size = 0.73

$$-\frac{3x}{2} + \frac{1}{4}\sinh(2x) + \tanh(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-Cosh[x] + Sech[x])^2,x]
[Out] (-3*x)/2 + Sinh[2*x]/4 + Tanh[x]
```

fricas [A] time = 0.48, size = 30, normalized size = 1.36

$$\frac{\sinh(x)^3 - 4\,(3\,x+2)\cosh(x) + 3\left(\cosh(x)^2+3\right)\sinh(x)}{8\,\cosh(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-cosh(x)+sech(x))^2,x, algorithm="fricas")
[Out] 1/8*(sinh(x)^3 - 4*(3*x + 2)*cosh(x) + 3*(cosh(x)^2 + 3)*sinh(x))/cosh(x)
```

giac [B] time = 0.15, size = 37, normalized size = 1.68

$$-\frac{3}{2}\,x + \frac{3\,e^{(4\,x)} - 14\,e^{(2\,x)} - 1}{8\left(e^{(4\,x)} + e^{(2\,x)}\right)} + \frac{1}{8}\,e^{(2\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-cosh(x)+sech(x))^2,x, algorithm="giac")
[Out] -3/2*x + 1/8*(3*e^(4*x) - 14*e^(2*x) - 1)/(e^(4*x) + e^(2*x)) + 1/8*e^(2*x)
```

maple [A] time = 0.35, size = 13, normalized size = 0.59

$$\frac{\cosh(x)\sinh(x)}{2} - \frac{3x}{2} + \tanh(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-cosh(x)+sech(x))^2,x)
[Out] 1/2*cosh(x)*sinh(x)-3/2*x+tanh(x)
```

maxima [A] time = 0.34, size = 26, normalized size = 1.18

$$-\frac{3}{2}x + \frac{2}{e^{(-2x)} + 1} + \frac{1}{8}e^{(2x)} - \frac{1}{8}e^{(-2x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-cosh(x)+sech(x))^2,x, algorithm="maxima")
[Out] -3/2*x + 2/(e^(-2*x) + 1) + 1/8*e^(2*x) - 1/8*e^(-2*x)
```

mupad [B] time = 1.54, size = 26, normalized size = 1.18

$$\frac{e^{2x}}{8} - \frac{e^{-2x}}{8} - \frac{3x}{2} - \frac{2}{e^{2x} + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cosh(x) - 1/cosh(x))^2,x)
[Out] exp(2*x)/8 - exp(-2*x)/8 - (3*x)/2 - 2/(exp(2*x) + 1)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (-\cosh(x) + \operatorname{sech}(x))^2\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-cosh(x)+sech(x))**2,x)
[Out] Integral((-cosh(x) + sech(x))**2, x)
```

3.682 $\int (-\cosh(x) + \mathbf{sech}(x))^3 \, dx$

Optimal. Leaf size=34

$$-\frac{5\sinh^3(x)}{6} + \frac{5\sinh(x)}{2} + \frac{1}{2}\sinh^3(x)\tanh^2(x) - \frac{5}{2}\tan^{-1}(\sinh(x))$$

```
[Out] -5/2*arctan(sinh(x))+5/2*sinh(x)-5/6*sinh(x)^3+1/2*sinh(x)^3*tanh(x)^2
```

Rubi [A] time = 0.05, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.556, Rules used = {4397, 2592, 288, 302, 203}

$$-\frac{5\sinh^3(x)}{6} + \frac{5\sinh(x)}{2} + \frac{1}{2}\sinh^3(x)\tanh^2(x) - \frac{5}{2}\tan^{-1}(\sinh(x))$$

Antiderivative was successfully verified.

```
[In] Int[(-Cosh[x] + Sech[x])^3,x]
```

```
[Out] (-5*ArcTan[Sinh[x]])/2 + (5*Sinh[x])/2 - (5*Sinh[x]^3)/6 + (Sinh[x]^3*Tanh[
x]^2)/2
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 2592

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, (a*Sin[e + f*x])/ff], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 4397

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rubi steps

$$\begin{aligned}\int (-\cosh(x)+\operatorname{sech}(x))^3\,dx &= -\int \sinh^3(x)\tanh^3(x)\,dx\\
&= -\operatorname{Subst}\left(\int \frac{x^6}{\left(1+x^2\right)^2}\,dx,x,\sinh(x)\right)\\
&= \frac{1}{2}\sinh^3(x)\tanh^2(x)-\frac{5}{2}\operatorname{Subst}\left(\int \frac{x^4}{1+x^2}\,dx,x,\sinh(x)\right)\\
&= \frac{1}{2}\sinh^3(x)\tanh^2(x)-\frac{5}{2}\operatorname{Subst}\left(\int \left(-1+x^2+\frac{1}{1+x^2}\right)dx,x,\sinh(x)\right)\\
&= \frac{5\sinh(x)}{2}-\frac{5\sinh^3(x)}{6}+\frac{1}{2}\sinh^3(x)\tanh^2(x)-\frac{5}{2}\operatorname{Subst}\left(\int \frac{1}{1+x^2}\,dx,x,\sinh(x)\right)\\
&= -\frac{5}{2}\tan^{-1}(\sinh(x))+\frac{5\sinh(x)}{2}-\frac{5\sinh^3(x)}{6}+\frac{1}{2}\sinh^3(x)\tanh^2(x)\end{aligned}$$

Mathematica [A] time = 0.03, size = 37, normalized size = 1.09

$$-\frac{1}{48}\operatorname{sech}^2(x)\left(-50\sinh(x)-25\sinh(3x)+\sinh(5x)+60\tan^{-1}(\sinh(x))+60\cosh(2x)\tan^{-1}(\sinh(x))\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-Cosh[x] + Sech[x])^3,x]
```

```
[Out] -1/48*(Sech[x]^2*(60*ArcTan[Sinh[x]] + 60*ArcTan[Sinh[x]]*Cosh[2*x] - 50*Si
nh[x] - 25*Sinh[3*x] + Sinh[5*x]))
```

fricas [B] time = 0.45, size = 486, normalized size = 14.29

$$-\frac{\cosh(x)^{10}+10\cosh(x)\sinh(x)^9+\sinh(x)^{10}+5\left(9\cosh(x)^2-5\right)\sinh(x)^8-25\cosh(x)^8+40\left(3\cosh(x)^3-\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-cosh(x)+sech(x))^3,x, algorithm="fricas")
```

```
[Out] -1/24*(cosh(x)^10 + 10*cosh(x)*sinh(x)^9 + sinh(x)^10 + 5*(9*cosh(x)^2 - 5)
*sinh(x)^8 - 25*cosh(x)^8 + 40*(3*cosh(x)^3 - 5*cosh(x))*sinh(x)^7 + 10*(21
*cosh(x)^4 - 70*cosh(x)^2 - 5)*sinh(x)^6 - 50*cosh(x)^6 + 4*(63*cosh(x)^5 -
 350*cosh(x)^3 - 75*cosh(x))*sinh(x)^5 + 10*(21*cosh(x)^6 - 175*cosh(x)^4 -
 75*cosh(x)^2 + 5)*sinh(x)^4 + 50*cosh(x)^4 + 40*(3*cosh(x)^7 - 35*cosh(x)^
5 - 25*cosh(x)^3 + 5*cosh(x))*sinh(x)^3 + 5*(9*cosh(x)^8 - 140*cosh(x)^6 -
150*cosh(x)^4 + 60*cosh(x)^2 + 5)*sinh(x)^2 + 120*(cosh(x)^7 + 7*cosh(x)*si
nh(x)^6 + sinh(x)^7 + (21*cosh(x)^2 + 2)*sinh(x)^5 + 2*cosh(x)^5 + 5*(7*cos
h(x)^3 + 2*cosh(x))*sinh(x)^4 + (35*cosh(x)^4 + 20*cosh(x)^2 + 1)*sinh(x)^3
 + cosh(x)^3 + (21*cosh(x)^5 + 20*cosh(x)^3 + 3*cosh(x))*sinh(x)^2 + (7*cos
h(x)^6 + 10*cosh(x)^4 + 3*cosh(x)^2)*sinh(x))*arctan(cosh(x) + sinh(x)) + 2
5*cosh(x)^2 + 10*(cosh(x)^9 - 20*cosh(x)^7 - 30*cosh(x)^5 + 20*cosh(x)^3 +
5*cosh(x))*sinh(x) - 1)/(cosh(x)^7 + 7*cosh(x)*sinh(x)^6 + sinh(x)^7 + (21*
cosh(x)^2 + 2)*sinh(x)^5 + 2*cosh(x)^5 + 5*(7*cosh(x)^3 + 2*cosh(x))*sinh(x
)^4 + (35*cosh(x)^4 + 20*cosh(x)^2 + 1)*sinh(x)^3 + cosh(x)^3 + (21*cosh(x)
^5 + 20*cosh(x)^3 + 3*cosh(x))*sinh(x)^2 + (7*cosh(x)^6 + 10*cosh(x)^4 + 3*
cosh(x)^2)*sinh(x))
```

giac [B] time = 0.12, size = 66, normalized size = 1.94

$$-\frac{5}{4}\,\pi + \frac{1}{24}\,{\left(e^{(-x)} - e^{x}\right)}^{3} - \frac{e^{(-x)} - e^{x}}{{\left(e^{(-x)} - e^{x}\right)}^{2} + 4} - \frac{5}{2}\,\arctan\left(\frac{1}{2}\,{\left(e^{(2\,x)} - 1\right)}e^{(-x)}\right) - e^{(-x)} + e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-cosh(x)+sech(x))^3,x, algorithm="giac")
```

```
[Out] -5/4*pi + 1/24*(e^(-x) - e^x)^3 - (e^(-x) - e^x)/((e^(-x) - e^x)^2 + 4) - 5
/2*arctan(1/2*(e^(2*x) - 1)*e^(-x)) - e^(-x) + e^x
```

maple [A] time = 0.46, size = 29, normalized size = 0.85

$$-\left(\frac{2}{3}+\frac{\left(\cosh^{2}(x)\right)}{3}\right)\sinh(x)+3\sinh(x)-5\arctan\left(\mathrm{e}^{x}\right)+\frac{\mathrm{sech}(x)\tanh(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-cosh(x)+sech(x))^3,x)
```

```
[Out] -(2/3+1/3*cosh(x)^2)*sinh(x)+3*sinh(x)-5*arctan(exp(x))+1/2*sech(x)*tanh(x)
```

maxima [B] time = 0.99, size = 56, normalized size = 1.65

$$\frac{e^{(-x)} - e^{(-3x)}}{2e^{(-2x)} + e^{(-4x)} + 1} + 5 \arctan\left(e^{(-x)}\right) - \frac{1}{24}e^{(3x)} - \frac{9}{8}e^{(-x)} + \frac{1}{24}e^{(-3x)} + \frac{9}{8}e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-cosh(x)+sech(x))^3,x, algorithm="maxima")
```

```
[Out] (e^(-x) - e^(-3*x))/(2*e^(-2*x) + e^(-4*x) + 1) + 5*arctan(e^(-x)) - 1/24*e
^(3*x) - 9/8*e^(-x) + 1/24*e^(-3*x) + 9/8*e^x
```

mupad [B] time = 0.06, size = 57, normalized size = 1.68

$$\frac{e^{-3x}}{24} - \frac{9e^{-x}}{8} - \frac{e^{3x}}{24} - 5\,\mathrm{atan}\left(e^{x}\right) + \frac{9e^{x}}{8} + \frac{e^{x}}{e^{2x}+1} - \frac{2e^{x}}{2e^{2x}+e^{4x}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(cosh(x) - 1/cosh(x))^3,x)
```

```
[Out] exp(-3*x)/24 - (9*exp(-x))/8 - exp(3*x)/24 - 5*atan(exp(x)) + (9*exp(x))/8
+ exp(x)/(exp(2*x) + 1) - (2*exp(x))/(2*exp(2*x) + exp(4*x) + 1)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int 3\cosh(x)\,\mathrm{sech}^2(x)\,dx - \int \left(-3\cosh^2(x)\,\mathrm{sech}(x)\right)dx - \int \cosh^3(x)\,dx - \int \left(-\mathrm{sech}^3(x)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-cosh(x)+sech(x))**3,x)
```

```
[Out] -Integral(3*cosh(x)*sech(x)**2, x) - Integral(-3*cosh(x)**2*sech(x), x) - I
ntegral(cosh(x)**3, x) - Integral(-sech(x)**3, x)
```

3.683 $\int \sqrt{-\cosh(x)+\mathbf{sech}(x)}\,dx$

Optimal. Leaf size=14

$$2\coth(x)\sqrt{-\sinh(x)\tanh(x)}$$

[Out] 2*coth(x)*(-sinh(x)*tanh(x))^(1/2)

Rubi [A] time = 0.06, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {4397, 4400, 2589}

$$2\coth(x)\sqrt{-\sinh(x)\tanh(x)}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[-Cosh[x] + Sech[x]],x]

[Out] 2*Coth[x]*Sqrt[-(Sinh[x]*Tanh[x])]

Rule 2589

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> -Simp[(b*(a*Sin[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*m
), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n - 1, 0]
```

Rule 4397

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rule 4400

```
Int[(u_.)*((v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> With[{uu = ActivateTri
g[u], vv = ActivateTrig[v], ww = ActivateTrig[w]}, Dist[(vv^m*ww^n)^FracPar
t[p]/(vv^(m*FracPart[p])*ww^(n*FracPart[p])), Int[uu*vv^(m*p)*ww^(n*p), x],
 x]] /; FreeQ[{m, n, p}, x] &&  !IntegerQ[p] && ( !InertTrigFreeQ[v] ||  !I
nertTrigFreeQ[w])
```

Rubi steps

$$\int \sqrt{-\cosh(x)+\operatorname{sech}(x)}\,dx = \int \sqrt{-\sinh(x)\tanh(x)}\,dx$$
$$= \frac{\sqrt{-\sinh(x)\tanh(x)}\int \sqrt{i\sinh(x)}\sqrt{i\tanh(x)}\,dx}{\sqrt{i\sinh(x)}\sqrt{i\tanh(x)}}$$
$$= 2\coth(x)\sqrt{-\sinh(x)\tanh(x)}$$

Mathematica [A] time = 0.06, size = 14, normalized size = 1.00

$$2\coth(x)\sqrt{-\sinh(x)\tanh(x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-Cosh[x] + Sech[x]],x]
```

```
[Out] 2*Coth[x]*Sqrt[-(Sinh[x]*Tanh[x])]
```

fricas [B] time = 0.42, size = 57, normalized size = 4.07

$$2\sqrt{\frac{1}{2}}\left(\cosh(x)^2+2\cosh(x)\sinh(x)+\sinh(x)^2+1\right)\sqrt{-\frac{1}{\cosh(x)^3+3\cosh(x)\sinh(x)^2+\sinh(x)^3+\left(3\cosh(\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-cosh(x)+sech(x))^(1/2),x, algorithm="fricas")
```

```
[Out] 2*sqrt(1/2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)*sqrt(-1/(cosh(x
)^3 + 3*cosh(x)*sinh(x)^2 + sinh(x)^3 + (3*cosh(x)^2 + 1)*sinh(x) + cosh(x)
))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{-\cosh(x)+\operatorname{sech}(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-cosh(x)+sech(x))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-cosh(x) + sech(x)), x)
```

maple [B] time = 0.62, size = 43, normalized size = 3.07

$$\frac{\sqrt{2}\sqrt{-\frac{\left(e^{2x}-1\right)^2 e^{-x}}{1+e^{2x}}}\left(1+e^{2x}\right)}{e^{2x}-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-cosh(x)+sech(x))^(1/2),x)
```

```
[Out] 2^(1/2)*(-(exp(2*x)-1)^2*exp(-x)/(1+exp(2*x)))^(1/2)/(exp(2*x)-1)*(1+exp(2*
x))
```

maxima [B] time = 0.88, size = 39, normalized size = 2.79

$$-\frac{\sqrt{2}e^{\left(\frac{1}{2}x\right)}}{\sqrt{-e^{(-2x)}-1}}-\frac{\sqrt{2}e^{\left(-\frac{3}{2}x\right)}}{\sqrt{-e^{(-2x)}-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-cosh(x)+sech(x))^(1/2),x, algorithm="maxima")
```

```
[Out] -sqrt(2)*e^(1/2*x)/sqrt(-e^(-2*x) - 1) - sqrt(2)*e^(-3/2*x)/sqrt(-e^(-2*x)
- 1)
```

mupad [B] time = 1.60, size = 15, normalized size = 1.07

$$2\coth(x)\sqrt{\frac{1}{\cosh(x)}-\cosh(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cosh(x) - cosh(x))^(1/2),x)
```

```
[Out] 2*coth(x)*(1/cosh(x) - cosh(x))^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{-\cosh(x)+\operatorname{sech}(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-cosh(x)+sech(x))**(1/2),x)
```

```
[Out] Integral(sqrt(-cosh(x) + sech(x)), x)
```

3.684 $\int (-\cosh(x) + \mathbf{sech}(x))^{3/2}\,dx$

Optimal. Leaf size=33

$$-\frac{2}{3}\sinh(x)\sqrt{-\sinh(x)\tanh(x)} - \frac{8}{3}\operatorname{csch}(x)\sqrt{-\sinh(x)\tanh(x)}$$

```
[Out] -8/3*csch(x)*(-sinh(x)*tanh(x))^(1/2)-2/3*sinh(x)*(-sinh(x)*tanh(x))^(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {4397, 4400, 2598, 2589}

$$-\frac{2}{3}\sinh(x)\sqrt{-\sinh(x)\tanh(x)} - \frac{8}{3}\operatorname{csch}(x)\sqrt{-\sinh(x)\tanh(x)}$$

Antiderivative was successfully verified.

```
[In] Int[(-Cosh[x] + Sech[x])^(3/2),x]
```

```
[Out] (-8*Csch[x]*Sqrt[-(Sinh[x]*Tanh[x])])/3 - (2*Sinh[x]*Sqrt[-(Sinh[x]*Tanh[x]
)])/3
```

Rule 2589

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> -Simp[(b*(a*Sin[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*m
), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n - 1, 0]
```

Rule 2598

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> -Simp[(b*(a*Sin[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f
*m), x] + Dist[(a^2*(m + n - 1))/m, Int[(a*Sin[e + f*x])^(m - 2)*(b*Tan[e +
 f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (GtQ[m, 1] || (EqQ[m, 1] &
& EqQ[n, 1/2])) && IntegersQ[2*m, 2*n]
```

Rule 4397

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rule 4400

```
Int[(u_.)*((v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> With[{uu = ActivateTri
g[u], vv = ActivateTrig[v], ww = ActivateTrig[w]}, Dist[(vv^m*ww^n)^FracPar
t[p]/(vv^(m*FracPart[p])*ww^(n*FracPart[p])), Int[uu*vv^(m*p)*ww^(n*p), x],
```

```
 x]] /; FreeQ[{m, n, p}, x] &&  !IntegerQ[p] && ( !InertTrigFreeQ[v] ||  !I
nertTrigFreeQ[w])
```

Rubi steps

$$\begin{aligned}\int(-\cosh(x)+\operatorname{sech}(x))^{3/2}\,dx &= \int(-\sinh(x)\tanh(x))^{3/2}\,dx\\ &= \frac{\sqrt{-\sinh(x)\tanh(x)}\,\int(i\sinh(x))^{3/2}(i\tanh(x))^{3/2}\,dx}{\sqrt{i\sinh(x)}\,\sqrt{i\tanh(x)}}\\ &= -\frac{2}{3}\sinh(x)\sqrt{-\sinh(x)\tanh(x)}+\frac{\left(4\sqrt{-\sinh(x)\tanh(x)}\right)\int\frac{(i\tanh(x))^{3/2}}{\sqrt{i\sinh(x)}}\,dx}{3\sqrt{i\sinh(x)}\,\sqrt{i\tanh(x)}}\\ &= -\frac{8}{3}\operatorname{csch}(x)\sqrt{-\sinh(x)\tanh(x)}-\frac{2}{3}\sinh(x)\sqrt{-\sinh(x)\tanh(x)}\end{aligned}$$

Mathematica [A] time = 0.09, size = 24, normalized size = 0.73

$$\frac{2}{3}\coth(x)\left(4\operatorname{csch}^2(x)+1\right)(-\sinh(x)\tanh(x))^{3/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-Cosh[x] + Sech[x])^(3/2),x]
```

```
[Out] (2*Coth[x]*(1 + 4*Csch[x]^2)*(-(Sinh[x]*Tanh[x]))^(3/2))/3
```

fricas [B] time = 0.43, size = 99, normalized size = 3.00

$$-\frac{\sqrt{\frac{1}{2}}\left(\cosh(x)^4+4\cosh(x)\sinh(x)^3+\sinh(x)^4+2\left(3\cosh(x)^2+7\right)\sinh(x)^2+14\cosh(x)^2+4\left(\cosh(x)^3+7\right.\right.}{3\left(\cosh(x)+\sinh(x)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-cosh(x)+sech(x))^(3/2),x, algorithm="fricas")
```

```
[Out] -1/3*sqrt(1/2)*(cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + 2*(3*cosh(x)^
2 + 7)*sinh(x)^2 + 14*cosh(x)^2 + 4*(cosh(x)^3 + 7*cosh(x))*sinh(x) + 1)*sq
rt(-1/(cosh(x)^3 + 3*cosh(x)*sinh(x)^2 + sinh(x)^3 + (3*cosh(x)^2 + 1)*sinh
(x) + cosh(x)))/(cosh(x) + sinh(x))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int(-\cosh(x)+\operatorname{sech}(x))^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-cosh(x)+sech(x))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((-cosh(x) + sech(x))^(3/2), x)
```

maple [F] time = 0.52, size = 0, normalized size = 0.00

$$\int(-\cosh(x)+\operatorname{sech}(x))^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-cosh(x)+sech(x))^(3/2),x)
```

```
[Out] int((-cosh(x)+sech(x))^(3/2),x)
```

maxima [B] time = 0.80, size = 77, normalized size = 2.33

$$-\frac{\sqrt{2}e^{\left(\frac{3}{2}x\right)}}{6\left(-e^{(-2x)}-1\right)^{\frac{3}{2}}}-\frac{5\sqrt{2}e^{\left(-\frac{1}{2}x\right)}}{2\left(-e^{(-2x)}-1\right)^{\frac{3}{2}}}-\frac{5\sqrt{2}e^{\left(-\frac{5}{2}x\right)}}{2\left(-e^{(-2x)}-1\right)^{\frac{3}{2}}}-\frac{\sqrt{2}e^{\left(-\frac{9}{2}x\right)}}{6\left(-e^{(-2x)}-1\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-cosh(x)+sech(x))^(3/2),x, algorithm="maxima")
```

```
[Out] -1/6*sqrt(2)*e^(3/2*x)/(-e^(-2*x) - 1)^(3/2) - 5/2*sqrt(2)*e^(-1/2*x)/(-e^(
-2*x) - 1)^(3/2) - 5/2*sqrt(2)*e^(-5/2*x)/(-e^(-2*x) - 1)^(3/2) - 1/6*sqrt(
2)*e^(-9/2*x)/(-e^(-2*x) - 1)^(3/2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int\left(\frac{1}{\cosh(x)}-\cosh(x)\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cosh(x) - cosh(x))^(3/2),x)
```

```
[Out] int((1/cosh(x) - cosh(x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int(-\cosh(x)+\operatorname{sech}(x))^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-cosh(x)+sech(x))**(3/2),x)
[Out] Integral((-cosh(x) + sech(x))**(3/2), x)
```

3.685 $\int (-\cosh(x) + \mathbf{sech}(x))^{5/2}\, dx$

Optimal. Leaf size=53

$$\frac{2}{5}\sinh^2(x)\tanh(x)\sqrt{-\sinh(x)\tanh(x)}+\frac{16}{15}\tanh(x)\sqrt{-\sinh(x)\tanh(x)}-\frac{64}{15}\coth(x)\sqrt{-\sinh(x)\tanh(x)}$$

```
[Out] -64/15*coth(x)*(-sinh(x)*tanh(x))^(1/2)+16/15*(-sinh(x)*tanh(x))^(1/2)*tanh
(x)+2/5*sinh(x)^2*(-sinh(x)*tanh(x))^(1/2)*tanh(x)
```

Rubi [A] time = 0.13, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.454, Rules used = {4397, 4400, 2598, 2594, 2589}

$$\frac{2}{5}\sinh^2(x)\tanh(x)\sqrt{-\sinh(x)\tanh(x)}+\frac{16}{15}\tanh(x)\sqrt{-\sinh(x)\tanh(x)}-\frac{64}{15}\coth(x)\sqrt{-\sinh(x)\tanh(x)}$$

Antiderivative was successfully verified.

```
[In] Int[(-Cosh[x] + Sech[x])^(5/2),x]
```

```
[Out] (-64*Coth[x]*Sqrt[-(Sinh[x]*Tanh[x])])/15 + (16*Tanh[x]*Sqrt[-(Sinh[x]*Tanh
[x])])/15 + (2*Sinh[x]^2*Tanh[x]*Sqrt[-(Sinh[x]*Tanh[x])])/5
```

Rule 2589

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> -Simp[(b*(a*Sin[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*m
), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n - 1, 0]
```

Rule 2594

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sin[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
n - 1)), x] - Dist[(b^2*(m + n - 1))/(n - 1), Int[(a*Sin[e + f*x])^m*(b*Tan
[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] && Int
egersQ[2*m, 2*n] &&  !(GtQ[m, 1] &&  !IntegerQ[(m - 1)/2])
```

Rule 2598

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> -Simp[(b*(a*Sin[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f
*m), x] + Dist[(a^2*(m + n - 1))/m, Int[(a*Sin[e + f*x])^(m - 2)*(b*Tan[e +
 f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (GtQ[m, 1] || (EqQ[m, 1] &
& EqQ[n, 1/2])) && IntegersQ[2*m, 2*n]
```

Rule 4397

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rule 4400

```
Int[(u_.)*((v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> With[{uu = ActivateTri
g[u], vv = ActivateTrig[v], ww = ActivateTrig[w]}, Dist[(vv^m*ww^n)^FracPar
t[p]/(vv^(m*FracPart[p])*ww^(n*FracPart[p])), Int[uu*vv^(m*p)*ww^(n*p), x],
 x]] /; FreeQ[{m, n, p}, x] &&  !IntegerQ[p] && ( !InertTrigFreeQ[v] ||  !I
nertTrigFreeQ[w])
```

Rubi steps

$$\int(-\cosh(x)+\text{sech}(x))^{5/2}\,dx=\int(-\sinh(x)\tanh(x))^{5/2}\,dx$$

$$=\frac{\sqrt{-\sinh(x)\tanh(x)}\int(i\sinh(x))^{5/2}(i\tanh(x))^{5/2}\,dx}{\sqrt{i\sinh(x)}\sqrt{i\tanh(x)}}$$

$$=\frac{2}{5}\sinh^2(x)\tanh(x)\sqrt{-\sinh(x)\tanh(x)}+\frac{\left(8\sqrt{-\sinh(x)\tanh(x)}\right)\int\sqrt{i\sinh(x)}\,(}{5\sqrt{i\sinh(x)}\sqrt{i\tanh(x)}}$$

$$=\frac{16}{15}\tanh(x)\sqrt{-\sinh(x)\tanh(x)}+\frac{2}{5}\sinh^2(x)\tanh(x)\sqrt{-\sinh(x)\tanh(x)}-\frac{(32}{}$$

$$=-\frac{64}{15}\coth(x)\sqrt{-\sinh(x)\tanh(x)}+\frac{16}{15}\tanh(x)\sqrt{-\sinh(x)\tanh(x)}+\frac{2}{5}\sinh^2(x$$

Mathematica [A] time = 0.14, size = 30, normalized size = 0.57

$$\frac{2}{15}\text{csch}(x)(-\sinh(x)\tanh(x))^{3/2}\left(-3\cosh^2(x)+32\coth^2(x)-5\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-Cosh[x] + Sech[x])^(5/2),x]
```

```
[Out] (2*(-5 - 3*Cosh[x]^2 + 32*Coth[x]^2)*Csch[x]*(-(Sinh[x]*Tanh[x]))^(3/2))/15
```

fricas [B] time = 0.47, size = 257, normalized size = 4.85

$$\sqrt{\frac{1}{2}}\left(3\cosh(x)^8+24\cosh(x)\sinh(x)^7+3\sinh(x)^8+12\left(7\cosh(x)^2-9\right)\sinh(x)^6-108\cosh(x)^6+24\left(7\cosh\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-cosh(x)+sech(x))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/30*sqrt(1/2)*(3*cosh(x)^8 + 24*cosh(x)*sinh(x)^7 + 3*sinh(x)^8 + 12*(7*co
sh(x)^2 - 9)*sinh(x)^6 - 108*cosh(x)^6 + 24*(7*cosh(x)^3 - 27*cosh(x))*sinh
(x)^5 + 2*(105*cosh(x)^4 - 810*cosh(x)^2 - 151)*sinh(x)^4 - 302*cosh(x)^4 +
 8*(21*cosh(x)^5 - 270*cosh(x)^3 - 151*cosh(x))*sinh(x)^3 + 12*(7*cosh(x)^6
 - 135*cosh(x)^4 - 151*cosh(x)^2 - 9)*sinh(x)^2 - 108*cosh(x)^2 + 8*(3*cosh
(x)^7 - 81*cosh(x)^5 - 151*cosh(x)^3 - 27*cosh(x))*sinh(x) + 3)*sqrt(-1/(co
sh(x)^3 + 3*cosh(x)*sinh(x)^2 + sinh(x)^3 + (3*cosh(x)^2 + 1)*sinh(x) + cos
h(x)))/(cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + (6*cosh(x)^2 + 1)*sin
h(x)^2 + cosh(x)^2 + 2*(2*cosh(x)^3 + cosh(x))*sinh(x))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (-\cosh(x) + \operatorname{sech}(x))^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-cosh(x)+sech(x))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((-cosh(x) + sech(x))^(5/2), x)
```

maple [F] time = 0.54, size = 0, normalized size = 0.00

$$\int (-\cosh(x) + \operatorname{sech}(x))^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-cosh(x)+sech(x))^(5/2),x)
```

```
[Out] int((-cosh(x)+sech(x))^(5/2),x)
```

maxima [B] time = 0.57, size = 115, normalized size = 2.17

$$-\frac{\sqrt{2}e^{\left(\frac{5}{2}x\right)}}{20\left(-e^{(-2x)}-1\right)^{\frac{5}{2}}}+\frac{7\sqrt{2}e^{\left(\frac{1}{2}x\right)}}{4\left(-e^{(-2x)}-1\right)^{\frac{5}{2}}}+\frac{41\sqrt{2}e^{\left(-\frac{3}{2}x\right)}}{6\left(-e^{(-2x)}-1\right)^{\frac{5}{2}}}+\frac{41\sqrt{2}e^{\left(-\frac{7}{2}x\right)}}{6\left(-e^{(-2x)}-1\right)^{\frac{5}{2}}}+\frac{7\sqrt{2}e^{\left(-\frac{11}{2}x\right)}}{4\left(-e^{(-2x)}-1\right)^{\frac{5}{2}}}-\frac{\sqrt{2}e^{\left(-\frac{15}{2}x\right)}}{20\left(-e^{(-2x)}-1\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-cosh(x)+sech(x))^(5/2),x, algorithm="maxima")
```

```
[Out] -1/20*sqrt(2)*e^(5/2*x)/(-e^(-2*x) - 1)^(5/2) + 7/4*sqrt(2)*e^(1/2*x)/(-e^(
-2*x) - 1)^(5/2) + 41/6*sqrt(2)*e^(-3/2*x)/(-e^(-2*x) - 1)^(5/2) + 41/6*sqr
```

```
t(2)*e^(-7/2*x)/(-e^(-2*x) - 1)^(5/2) + 7/4*sqrt(2)*e^(-11/2*x)/(-e^(-2*x)
- 1)^(5/2) - 1/20*sqrt(2)*e^(-15/2*x)/(-e^(-2*x) - 1)^(5/2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \left(\frac{1}{\cosh(x)} - \cosh(x)\right)^{5/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cosh(x) - cosh(x))^(5/2),x)
```

```
[Out] int((1/cosh(x) - cosh(x))^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-cosh(x)+sech(x))**(5/2),x)
```

```
[Out] Timed out
```

3.686 $\int \frac{1}{\sinh(x)+\tanh(x)}\,dx$

Optimal. Leaf size=18

$$-\frac{1}{2(\cosh(x)+1)}-\frac{1}{2}\tanh^{-1}(\cosh(x))$$

```
[Out] -1/2*arctanh(cosh(x))-1/2/(1+cosh(x))
```

Rubi [A] time = 0.08, antiderivative size = 24, normalized size of antiderivative = 1.33, number of steps used = 6, number of rules used = 6, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.857, Rules used = {4397, 2706, 2606, 30, 2611, 3770}

$$\frac{\text{csch}^2(x)}{2}-\frac{1}{2}\tanh^{-1}(\cosh(x))-\frac{1}{2}\coth(x)\text{csch}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(Sinh[x] + Tanh[x])^(-1),x]
```

```
[Out] -ArcTanh[Cosh[x]]/2 - (Coth[x]*Csch[x])/2 + Csch[x]^2/2
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2706

```
Int[((g_.)*tan[(e_.) + (f_.)*(x_)])^(p_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Dist[1/a, Int[Sec[e + f*x]^2*(g*Tan[e + f*x])^p, x], x]
```

```
 - Dist[1/(b*g), Int[Sec[e + f*x]*(g*Tan[e + f*x])^(p + 1), x], x] /; FreeQ
[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && NeQ[p, -1]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4397

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sinh(x)+\tanh(x)}\,dx &= -\left(i\int \frac{\coth(x)}{-i-i\cosh(x)}\,dx\right)\\
&= \int \coth^2(x)\text{csch}(x)\,dx - \int \coth(x)\text{csch}^2(x)\,dx\\
&= -\frac{1}{2}\coth(x)\text{csch}(x) + \frac{1}{2}\int \text{csch}(x)\,dx - \text{Subst}(\int x\,dx, x, -i\text{csch}(x))\\
&= -\frac{1}{2}\tanh^{-1}(\cosh(x)) - \frac{1}{2}\coth(x)\text{csch}(x) + \frac{\text{csch}^2(x)}{2}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 35, normalized size = 1.94

$$-\frac{1}{4}\text{sech}^2\left(\frac{x}{2}\right)+\frac{1}{2}\log\left(\sinh\left(\frac{x}{2}\right)\right)-\frac{1}{2}\log\left(\cosh\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sinh[x] + Tanh[x])^(-1),x]
[Out] -1/2*Log[Cosh[x/2]] + Log[Sinh[x/2]]/2 - Sech[x/2]^2/4
```

fricas [B] time = 0.45, size = 96, normalized size = 5.33

$$-\frac{\left(\cosh(x)^2+2\left(\cosh(x)+1\right)\sinh(x)+\sinh(x)^2+2\,\cosh(x)+1\right)\log\left(\cosh(x)+\sinh(x)+1\right)-\left(\cosh(x)^2+2\,(c\right.}{2\left(\cosh(x)^2+2\left(\cosh(x)+1\right)\sinh(x)\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sinh(x)+tanh(x)),x, algorithm="fricas")
```

```
[Out] -1/2*((cosh(x)^2 + 2*(cosh(x) + 1)*sinh(x) + sinh(x)^2 + 2*cosh(x) + 1)*log
(cosh(x) + sinh(x) + 1) - (cosh(x)^2 + 2*(cosh(x) + 1)*sinh(x) + sinh(x)^2
+ 2*cosh(x) + 1)*log(cosh(x) + sinh(x) - 1) + 2*cosh(x) + 2*sinh(x))/(cosh(
x)^2 + 2*(cosh(x) + 1)*sinh(x) + sinh(x)^2 + 2*cosh(x) + 1)
```

giac [B] time = 0.13, size = 43, normalized size = 2.39

$$\frac{e^{(-x)} + e^{x} - 2}{4\left(e^{(-x)} + e^{x} + 2\right)} - \frac{1}{4}\log\left(e^{(-x)} + e^{x} + 2\right) + \frac{1}{4}\log\left(e^{(-x)} + e^{x} - 2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sinh(x)+tanh(x)),x, algorithm="giac")
```

```
[Out] 1/4*(e^(-x) + e^x - 2)/(e^(-x) + e^x + 2) - 1/4*log(e^(-x) + e^x + 2) + 1/4
*log(e^(-x) + e^x - 2)
```

maple [A] time = 0.20, size = 17, normalized size = 0.94

$$\frac{\left(\tanh^{2}\left(\frac{x}{2}\right)\right)}{4} + \frac{\ln\left(\tanh\left(\frac{x}{2}\right)\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sinh(x)+tanh(x)),x)
```

```
[Out] 1/4*tanh(1/2*x)^2+1/2*ln(tanh(1/2*x))
```

maxima [B] time = 0.35, size = 39, normalized size = 2.17

$$-\frac{e^{(-x)}}{2\,e^{(-x)} + e^{(-2\,x)} + 1} - \frac{1}{2}\log\left(e^{(-x)} + 1\right) + \frac{1}{2}\log\left(e^{(-x)} - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sinh(x)+tanh(x)),x, algorithm="maxima")
```

```
[Out] -e^(-x)/(2*e^(-x) + e^(-2*x) + 1) - 1/2*log(e^(-x) + 1) + 1/2*log(e^(-x) -
1)
```

mupad [B] time = 0.04, size = 39, normalized size = 2.17

$$\frac{\ln\left(1 - \mathrm{e}^{x}\right)}{2} - \frac{\ln\left(-\mathrm{e}^{x} - 1\right)}{2} + \frac{1}{\mathrm{e}^{2\,x} + 2\,\mathrm{e}^{x} + 1} - \frac{1}{\mathrm{e}^{x} + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sinh(x) + tanh(x)),x)
```

```
[Out] log(1 - exp(x))/2 - log(- exp(x) - 1)/2 + 1/(exp(2*x) + 2*exp(x) + 1) - 1/(
exp(x) + 1)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sinh(x) + \tanh(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sinh(x)+tanh(x)),x)
```

```
[Out] Integral(1/(sinh(x) + tanh(x)), x)
```

3.687 $\int \frac{1}{\sinh(x)-\tanh(x)}\,dx$

Optimal. Leaf size=20

$$\frac{1}{2(1-\cosh(x))}-\frac{1}{2}\tanh^{-1}(\cosh(x))$$

```
[Out] -1/2*arctanh(cosh(x))+1/2/(1-cosh(x))
```

Rubi [A] time = 0.07, antiderivative size = 24, normalized size of antiderivative = 1.20, number of steps used = 6, number of rules used = 6, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {4397, 2706, 2606, 30, 2611, 3770}

$$-\frac{1}{2}\text{csch}^2(x)-\frac{1}{2}\tanh^{-1}(\cosh(x))-\frac{1}{2}\coth(x)\text{csch}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(Sinh[x] - Tanh[x])^(-1),x]
```

```
[Out] -ArcTanh[Cosh[x]]/2 - (Coth[x]*Csch[x])/2 - Csch[x]^2/2
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 && !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2706

```
Int[((g_.)*tan[(e_.) + (f_.)*(x_)])^(p_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Dist[1/a, Int[Sec[e + f*x]^2*(g*Tan[e + f*x])^p, x], x]
 - Dist[1/(b*g), Int[Sec[e + f*x]*(g*Tan[e + f*x])^(p + 1), x], x] /; FreeQ
```

```
[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && NeQ[p, -1]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4397

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sinh(x)-\tanh(x)}\,dx &= -\left(i\int \frac{\coth(x)}{i-i\cosh(x)}\,dx\right)\\ &= \int \coth^2(x)\text{csch}(x)\,dx+\int \coth(x)\text{csch}^2(x)\,dx\\ &= -\frac{1}{2}\coth(x)\text{csch}(x)+\frac{1}{2}\int \text{csch}(x)\,dx+\text{Subst}\left(\int x\,dx,x,-i\text{csch}(x)\right)\\ &= -\frac{1}{2}\tanh^{-1}(\cosh(x))-\frac{1}{2}\coth(x)\text{csch}(x)-\frac{\text{csch}^2(x)}{2}\end{aligned}$$

Mathematica [B] time = 0.06, size = 50, normalized size = 2.50

$$-\frac{1}{4}\text{csch}^2\left(\frac{x}{2}\right)\left(\log\left(\sinh\left(\frac{x}{2}\right)\right)-\log\left(\cosh\left(\frac{x}{2}\right)\right)+\cosh(x)\left(\log\left(\cosh\left(\frac{x}{2}\right)\right)-\log\left(\sinh\left(\frac{x}{2}\right)\right)\right)+1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sinh[x] - Tanh[x])^(-1),x]
```

```
[Out] -1/4*(Csch[x/2]^2*(1 - Log[Cosh[x/2]] + Cosh[x]*(Log[Cosh[x/2]] - Log[Sinh[
x/2]]) + Log[Sinh[x/2]]))
```

fricas [B] time = 0.44, size = 96, normalized size = 4.80

$$-\frac{\left(\cosh(x)^2+2\left(\cosh(x)-1\right)\sinh(x)+\sinh(x)^2-2\cosh(x)+1\right)\log\left(\cosh(x)+\sinh(x)+1\right)-\left(\cosh(x)^2+2\left(c\right.\right.}{2\left(\cosh(x)^2+2\left(\cosh(x)-1\right)\sinh(x)\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sinh(x)-tanh(x)),x, algorithm="fricas")
```

```
[Out] -1/2*((cosh(x)^2 + 2*(cosh(x) - 1)*sinh(x) + sinh(x)^2 - 2*cosh(x) + 1)*log
(cosh(x) + sinh(x) + 1) - (cosh(x)^2 + 2*(cosh(x) - 1)*sinh(x) + sinh(x)^2
- 2*cosh(x) + 1)*log(cosh(x) + sinh(x) - 1) + 2*cosh(x) + 2*sinh(x))/(cosh(
x)^2 + 2*(cosh(x) - 1)*sinh(x) + sinh(x)^2 - 2*cosh(x) + 1)
```

giac [B] time = 0.12, size = 43, normalized size = 2.15

$$-\frac{e^{(-x)} + e^{x} + 2}{4\left(e^{(-x)} + e^{x} - 2\right)} - \frac{1}{4}\log\left(e^{(-x)} + e^{x} + 2\right) + \frac{1}{4}\log\left(e^{(-x)} + e^{x} - 2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sinh(x)-tanh(x)),x, algorithm="giac")
```

```
[Out] -1/4*(e^(-x) + e^x + 2)/(e^(-x) + e^x - 2) - 1/4*log(e^(-x) + e^x + 2) + 1/
4*log(e^(-x) + e^x - 2)
```

maple [A] time = 0.20, size = 17, normalized size = 0.85

$$-\frac{1}{4\tanh\left(\frac{x}{2}\right)^{2}} + \frac{\ln\left(\tanh\left(\frac{x}{2}\right)\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sinh(x)-tanh(x)),x)
```

```
[Out] -1/4/tanh(1/2*x)^2+1/2*ln(tanh(1/2*x))
```

maxima [B] time = 0.34, size = 40, normalized size = 2.00

$$\frac{e^{(-x)}}{2\,e^{(-x)} - e^{(-2\,x)} - 1} - \frac{1}{2}\log\left(e^{(-x)} + 1\right) + \frac{1}{2}\log\left(e^{(-x)} - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sinh(x)-tanh(x)),x, algorithm="maxima")
```

```
[Out] e^(-x)/(2*e^(-x) - e^(-2*x) - 1) - 1/2*log(e^(-x) + 1) + 1/2*log(e^(-x) - 1
)
```

mupad [B] time = 0.04, size = 41, normalized size = 2.05

$$\frac{\ln\left(1-\mathrm{e}^{x}\right)}{2} - \frac{\ln\left(-\mathrm{e}^{x}-1\right)}{2} - \frac{1}{\mathrm{e}^{2\,x}-2\,\mathrm{e}^{x}+1} - \frac{1}{\mathrm{e}^{x}-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sinh(x) - tanh(x)),x)
```

```
[Out] log(1 - exp(x))/2 - log(- exp(x) - 1)/2 - 1/(exp(2*x) - 2*exp(x) + 1) - 1/(
exp(x) - 1)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sinh(x) - \tanh(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sinh(x)-tanh(x)),x)
```

```
[Out] Integral(1/(sinh(x) - tanh(x)), x)
```

3.688 $\int \frac{\sinh(x)}{a\cosh(x)+b\sinh(x)}\,dx$

Optimal. Leaf size=39

$$\frac{a\log(a\cosh(x)+b\sinh(x))}{a^2-b^2}-\frac{bx}{a^2-b^2}$$

```
[Out] -b*x/(a^2-b^2)+a*ln(a*cosh(x)+b*sinh(x))/(a^2-b^2)
```

Rubi [A] time = 0.07, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3097, 3133}

$$\frac{a\log(a\cosh(x)+b\sinh(x))}{a^2-b^2}-\frac{bx}{a^2-b^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[x]/(a*Cosh[x] + b*Sinh[x]),x]
```

```
[Out] -((b*x)/(a^2 - b^2)) + (a*Log[a*Cosh[x] + b*Sinh[x]])/(a^2 - b^2)
```

Rule 3097

```
Int[sin[(c_.) + (d_.)*(x_)]/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.
) + (d_.)*(x_)]), x_Symbol] :> Simp[(b*x)/(a^2 + b^2), x] - Dist[a/(a^2 + b
^2), Int[(b*Cos[c + d*x] - a*Sin[c + d*x])/(a*Cos[c + d*x] + b*Sin[c + d*x]
), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3133

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)]), x
_Symbol] :> Simp[((b*B + c*C)*x)/(b^2 + c^2), x] + Simp[((c*B - b*C)*Log[a
+ b*Cos[d + e*x] + c*Sin[d + e*x]])/(e*(b^2 + c^2)), x] /; FreeQ[{a, b, c,
d, e, A, B, C}, x] && NeQ[b^2 + c^2, 0] && EqQ[A*(b^2 + c^2) - a*(b*B + c*C
), 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sinh(x)}{a\cosh(x)+b\sinh(x)}\,dx &= -\frac{bx}{a^2-b^2}+\frac{(ia)\int \frac{-ib\cosh(x)-ia\sinh(x)}{a\cosh(x)+b\sinh(x)}\,dx}{a^2-b^2}\\ &= -\frac{bx}{a^2-b^2}+\frac{a\log(a\cosh(x)+b\sinh(x))}{a^2-b^2}\end{aligned}$$

Mathematica [A] time = 0.07, size = 29, normalized size = 0.74

$$\frac{a\log(a\cosh(x)+b\sinh(x))-bx}{a^2-b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[x]/(a*Cosh[x] + b*Sinh[x]),x]
```

```
[Out] (-(b*x) + a*Log[a*Cosh[x] + b*Sinh[x]])/(a^2 - b^2)
```

fricas [A] time = 0.44, size = 43, normalized size = 1.10

$$-\frac{(a+b)x-a\log\left(\frac{2\,(a\cosh(x)+b\sinh(x))}{\cosh(x)-\sinh(x)}\right)}{a^2-b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)/(a*cosh(x)+b*sinh(x)),x, algorithm="fricas")
```

```
[Out] -((a + b)*x - a*log(2*(a*cosh(x) + b*sinh(x))/(cosh(x) - sinh(x))))/(a^2 -
b^2)
```

giac [A] time = 0.13, size = 43, normalized size = 1.10

$$\frac{a\log\left(\left|ae^{(2\,x)}+be^{(2\,x)}+a-b\right|\right)}{a^2-b^2}-\frac{x}{a-b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)/(a*cosh(x)+b*sinh(x)),x, algorithm="giac")
```

```
[Out] a*log(abs(a*e^(2*x) + b*e^(2*x) + a - b))/(a^2 - b^2) - x/(a - b)
```

maple [A] time = 0.20, size = 70, normalized size = 1.79

$$-\frac{4\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{4a+4b}+\frac{a\ln\left(a+2\tanh\left(\frac{x}{2}\right)b+a\left(\tanh^2\left(\frac{x}{2}\right)\right)\right)}{(a+b)\,(a-b)}-\frac{4\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)}{4a-4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)/(a*cosh(x)+b*sinh(x)),x)
```

```
[Out] -4/(4*a+4*b)*ln(tanh(1/2*x)-1)+a/(a+b)/(a-b)*ln(a+2*tanh(1/2*x)*b+a*tanh(1/
2*x)^2)-4/(4*a-4*b)*ln(tanh(1/2*x)+1)
```

maxima [A] time = 0.31, size = 40, normalized size = 1.03

$$\frac{a\log\left(-(a-b)e^{(-2x)}-a-b\right)}{a^2-b^2}+\frac{x}{a+b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)/(a*cosh(x)+b*sinh(x)),x, algorithm="maxima")
```

```
[Out] a*log(-(a - b)*e^(-2*x) - a - b)/(a^2 - b^2) + x/(a + b)
```

mupad [B] time = 0.09, size = 30, normalized size = 0.77

$$-\frac{b\,x-a\,\ln\left(a\cosh(x)+b\sinh(x)\right)}{a^2-b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)/(a*cosh(x) + b*sinh(x)),x)
```

```
[Out] -(b*x - a*log(a*cosh(x) + b*sinh(x)))/(a^2 - b^2)
```

sympy [A] time = 0.55, size = 143, normalized size = 3.67

$$\begin{cases} \tilde{\infty}x & \text{for } a=0\wedge b=0 \\ \frac{x}{b} & \text{for } a=0 \\ -\frac{x\sinh(x)}{-2b\sinh(x)+2b\cosh(x)}+\frac{x\cosh(x)}{-2b\sinh(x)+2b\cosh(x)}-\frac{\cosh(x)}{-2b\sinh(x)+2b\cosh(x)} & \text{for } a=-b \\ \frac{x\sinh(x)}{2b\sinh(x)+2b\cosh(x)}+\frac{x\cosh(x)}{2b\sinh(x)+2b\cosh(x)}+\frac{\cosh(x)}{2b\sinh(x)+2b\cosh(x)} & \text{for } a=b \\ \frac{a\log\left(\cosh(x)+\frac{b\sinh(x)}{a}\right)}{a^2-b^2}-\frac{bx}{a^2-b^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)/(a*cosh(x)+b*sinh(x)),x)
```

```
[Out] Piecewise((zoo*x, Eq(a, 0) & Eq(b, 0)), (x/b, Eq(a, 0)), (-x*sinh(x)/(-2*b*
sinh(x) + 2*b*cosh(x)) + x*cosh(x)/(-2*b*sinh(x) + 2*b*cosh(x)) - cosh(x)/(
-2*b*sinh(x) + 2*b*cosh(x)), Eq(a, -b)), (x*sinh(x)/(2*b*sinh(x) + 2*b*cosh
(x)) + x*cosh(x)/(2*b*sinh(x) + 2*b*cosh(x)) + cosh(x)/(2*b*sinh(x) + 2*b*c
osh(x)), Eq(a, b)), (a*log(cosh(x) + b*sinh(x)/a)/(a**2 - b**2) - b*x/(a**2
 - b**2), True))
```

3.689 $\int \frac{\sinh^2(x)}{a\cosh(x)+b\sinh(x)}\,dx$

Optimal. Leaf size=74

$$\frac{a\sinh(x)}{a^2-b^2}-\frac{b\cosh(x)}{a^2-b^2}-\frac{a^2\tan^{-1}\left(\frac{a\sinh(x)+b\cosh(x)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{3/2}}$$

```
[Out] -a^2*arctan((b*cosh(x)+a*sinh(x))/(a^2-b^2)^(1/2))/(a^2-b^2)^(3/2)-b*cosh(x
)/(a^2-b^2)+a*sinh(x)/(a^2-b^2)
```

Rubi [A] time = 0.08, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3099, 3074, 206, 2638}

$$\frac{a\sinh(x)}{a^2-b^2}-\frac{b\cosh(x)}{a^2-b^2}-\frac{a^2\tan^{-1}\left(\frac{a\sinh(x)+b\cosh(x)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[x]^2/(a*Cosh[x] + b*Sinh[x]),x]
```

```
[Out] -((a^2*ArcTan[(b*Cosh[x] + a*Sinh[x])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(3/2))
- (b*Cosh[x])/(a^2 - b^2) + (a*Sinh[x])/(a^2 - b^2)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3074

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> -Dist[d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d
*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3099

```
Int[sin[(c_.) + (d_.)*(x_)]^(m_)/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin
[(c_.) + (d_.)*(x_)]), x_Symbol] :> -Simp[(a*Sin[c + d*x]^(m - 1))/(d*(a^2
+ b^2)*(m - 1)), x] + (Dist[a^2/(a^2 + b^2), Int[Sin[c + d*x]^(m - 2)/(a*Co
s[c + d*x] + b*Sin[c + d*x]), x], x] + Dist[b/(a^2 + b^2), Int[Sin[c + d*x]
^(m - 1), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && GtQ[m,
1]
```

Rubi steps

$$\begin{aligned}\int \frac{\sinh^2(x)}{a\cosh(x)+b\sinh(x)}\,dx &= \frac{a\sinh(x)}{a^2-b^2}-\frac{a^2\int \frac{1}{a\cosh(x)+b\sinh(x)}\,dx}{a^2-b^2}-\frac{b\int \sinh(x)\,dx}{a^2-b^2}\\ &= -\frac{b\cosh(x)}{a^2-b^2}+\frac{a\sinh(x)}{a^2-b^2}-\frac{\left(ia^2\right)\text{Subst}\left(\int \frac{1}{a^2-b^2-x^2}\,dx,x,-ib\cosh(x)-ia\sinh(x)\right)}{a^2-b^2}\\ &= -\frac{a^2\tan^{-1}\left(\frac{b\cosh(x)+a\sinh(x)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{3/2}}-\frac{b\cosh(x)}{a^2-b^2}+\frac{a\sinh(x)}{a^2-b^2}\end{aligned}$$

Mathematica [A] time = 0.20, size = 89, normalized size = 1.20

$$\frac{a\left(\sqrt{a-b}\,(a+b)\sinh(x)-2a\sqrt{a+b}\,\tan^{-1}\left(\frac{a\tanh\left(\frac{x}{2}\right)+b}{\sqrt{a-b}\,\sqrt{a+b}}\right)\right)-b\sqrt{a-b}\,(a+b)\cosh(x)}{(a-b)^{3/2}(a+b)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[x]^2/(a*Cosh[x] + b*Sinh[x]),x]
```

```
[Out] (-(Sqrt[a - b]*b*(a + b)*Cosh[x]) + a*(-2*a*Sqrt[a + b]*ArcTan[(b + a*Tanh[
x/2])/(Sqrt[a - b]*Sqrt[a + b])] + Sqrt[a - b]*(a + b)*Sinh[x]))/((a - b)^(
3/2)*(a + b)^2)
```

fricas [B] time = 0.45, size = 435, normalized size = 5.88

$$\left[-\frac{a^3+a^2b-ab^2-b^3-\left(a^3-a^2b-ab^2+b^3\right)\cosh(x)^2-2\left(a^3-a^2b-ab^2+b^3\right)\cosh(x)\sinh(x)-\left(a^3-a^2b-\right.}{2\left(\left(a^4-2a^2b^2+\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^2/(a*cosh(x)+b*sinh(x)),x, algorithm="fricas")
```

```
[Out] [-1/2*(a^3 + a^2*b - a*b^2 - b^3 - (a^3 - a^2*b - a*b^2 + b^3)*cosh(x)^2 -
2*(a^3 - a^2*b - a*b^2 + b^3)*cosh(x)*sinh(x) - (a^3 - a^2*b - a*b^2 + b^3)
*sinh(x)^2 - 2*(a^2*cosh(x) + a^2*sinh(x))*sqrt(-a^2 + b^2)*log(((a + b)*co
sh(x)^2 + 2*(a + b)*cosh(x)*sinh(x) + (a + b)*sinh(x)^2 - 2*sqrt(-a^2 + b^2
)*(cosh(x) + sinh(x)) - a + b)/((a + b)*cosh(x)^2 + 2*(a + b)*cosh(x)*sinh(
x) + (a + b)*sinh(x)^2 + a - b)))/((a^4 - 2*a^2*b^2 + b^4)*cosh(x) + (a^4 -
 2*a^2*b^2 + b^4)*sinh(x)), -1/2*(a^3 + a^2*b - a*b^2 - b^3 - (a^3 - a^2*b
- a*b^2 + b^3)*cosh(x)^2 - 2*(a^3 - a^2*b - a*b^2 + b^3)*cosh(x)*sinh(x) -
(a^3 - a^2*b - a*b^2 + b^3)*sinh(x)^2 - 4*(a^2*cosh(x) + a^2*sinh(x))*sqrt(
a^2 - b^2)*arctan(sqrt(a^2 - b^2)/((a + b)*cosh(x) + (a + b)*sinh(x))))/((a
^4 - 2*a^2*b^2 + b^4)*cosh(x) + (a^4 - 2*a^2*b^2 + b^4)*sinh(x))]
```

giac [A] time = 0.12, size = 61, normalized size = 0.82

$$-\frac{2\,a^2\arctan\left(\frac{ae^x+be^x}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{\frac{3}{2}}}-\frac{e^{(-x)}}{2\,(a-b)}+\frac{e^x}{2\,(a+b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^2/(a*cosh(x)+b*sinh(x)),x, algorithm="giac")
```

```
[Out] -2*a^2*arctan((a*e^x + b*e^x)/sqrt(a^2 - b^2))/(a^2 - b^2)^(3/2) - 1/2*e^(-
x)/(a - b) + 1/2*e^x/(a + b)
```

maple [A] time = 0.20, size = 93, normalized size = 1.26

$$-\frac{8}{(8a-8b)\left(\tanh\left(\frac{x}{2}\right)+1\right)}-\frac{2a^2\arctan\left(\frac{2a\tanh\left(\frac{x}{2}\right)+2b}{2\sqrt{a^2-b^2}}\right)}{(a+b)\,(a-b)\,\sqrt{a^2-b^2}}-\frac{8}{(8a+8b)\left(\tanh\left(\frac{x}{2}\right)-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)^2/(a*cosh(x)+b*sinh(x)),x)
```

```
[Out] -8/(8*a-8*b)/(tanh(1/2*x)+1)-2*a^2/(a+b)/(a-b)/(a^2-b^2)^(1/2)*arctan(1/2*(
2*a*tanh(1/2*x)+2*b)/(a^2-b^2)^(1/2))-8/(8*a+8*b)/(tanh(1/2*x)-1)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^2/(a*cosh(x)+b*sinh(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [B] time = 1.72, size = 157, normalized size = 2.12

$$\frac{e^x}{2\,a+2\,b}-\frac{e^{-x}}{2\,a-2\,b}-\frac{a^2\,\ln\left(-\frac{2\,a^2}{{(a+b)}^{5/2}\,\sqrt{b-a}}-\frac{2\,a^2\,e^x}{-a^3-a^2\,b+a\,b^2+b^3}\right)}{{(a+b)}^{3/2}\,{(b-a)}^{3/2}}+\frac{a^2\,\ln\left(\frac{2\,a^2}{{(a+b)}^{5/2}\,\sqrt{b-a}}-\frac{2\,a^2\,e^x}{-a^3-a^2\,b+a\,b^2+b^3}\right)}{{(a+b)}^{3/2}\,{(b-a)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)^2/(a*cosh(x) + b*sinh(x)),x)
```

```
[Out] exp(x)/(2*a + 2*b) - exp(-x)/(2*a - 2*b) - (a^2*log(- (2*a^2)/((a + b)^(5/2
)*(b - a)^(1/2)) - (2*a^2*exp(x))/(a*b^2 - a^2*b - a^3 + b^3)))/((a + b)^(3
/2)*(b - a)^(3/2)) + (a^2*log((2*a^2)/((a + b)^(5/2)*(b - a)^(1/2)) - (2*a^
2*exp(x))/(a*b^2 - a^2*b - a^3 + b^3)))/((a + b)^(3/2)*(b - a)^(3/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)**2/(a*cosh(x)+b*sinh(x)),x)
```

```
[Out] Timed out
```

3.690 $\int \frac{\sinh^3(x)}{a\cosh(x)+b\sinh(x)}\,dx$

Optimal. Leaf size=101

$$\frac{a^2bx}{(a^2-b^2)^2}+\frac{bx}{2(a^2-b^2)}+\frac{a\sinh^2(x)}{2(a^2-b^2)}-\frac{b\sinh(x)\cosh(x)}{2(a^2-b^2)}-\frac{a^3\log(a\cosh(x)+b\sinh(x))}{(a^2-b^2)^2}$$

```
[Out] a^2*b*x/(a^2-b^2)^2+1/2*b*x/(a^2-b^2)-a^3*ln(a*cosh(x)+b*sinh(x))/(a^2-b^2)
^2-1/2*b*cosh(x)*sinh(x)/(a^2-b^2)+1/2*a*sinh(x)^2/(a^2-b^2)
```

Rubi [A] time = 0.13, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {3099, 3097, 3133, 2635, 8}

$$\frac{a^2bx}{(a^2-b^2)^2}+\frac{bx}{2(a^2-b^2)}+\frac{a\sinh^2(x)}{2(a^2-b^2)}-\frac{b\sinh(x)\cosh(x)}{2(a^2-b^2)}-\frac{a^3\log(a\cosh(x)+b\sinh(x))}{(a^2-b^2)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[x]^3/(a*Cosh[x] + b*Sinh[x]),x]
```

```
[Out] (a^2*b*x)/(a^2 - b^2)^2 + (b*x)/(2*(a^2 - b^2)) - (a^3*Log[a*Cosh[x] + b*Si
nh[x]])/(a^2 - b^2)^2 - (b*Cosh[x]*Sinh[x])/(2*(a^2 - b^2)) + (a*Sinh[x]^2)
/(2*(a^2 - b^2))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 3097

```
Int[sin[(c_.) + (d_.)*(x_)]/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.
) + (d_.)*(x_)]), x_Symbol] :> Simp[(b*x)/(a^2 + b^2), x] - Dist[a/(a^2 + b
^2), Int[(b*Cos[c + d*x] - a*Sin[c + d*x])/(a*Cos[c + d*x] + b*Sin[c + d*x]
), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3099

```
Int[sin[(c_.) + (d_.)*(x_)]^(m_)/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin
[(c_.) + (d_.)*(x_)]), x_Symbol] :> -Simp[(a*Sin[c + d*x]^(m - 1))/(d*(a^2
+ b^2)*(m - 1)), x] + (Dist[a^2/(a^2 + b^2), Int[Sin[c + d*x]^(m - 2)/(a*Co
s[c + d*x] + b*Sin[c + d*x]), x], x] + Dist[b/(a^2 + b^2), Int[Sin[c + d*x]
^(m - 1), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && GtQ[m,
1]
```

Rule 3133

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)]), x
_Symbol] :> Simp[((b*B + c*C)*x)/(b^2 + c^2), x] + Simp[((c*B - b*C)*Log[a
+ b*Cos[d + e*x] + c*Sin[d + e*x]])/(e*(b^2 + c^2)), x] /; FreeQ[{a, b, c,
d, e, A, B, C}, x] && NeQ[b^2 + c^2, 0] && EqQ[A*(b^2 + c^2) - a*(b*B + c*C
), 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sinh^3(x)}{a\cosh(x)+b\sinh(x)}\,dx &= \frac{a\sinh^2(x)}{2\left(a^2-b^2\right)}-\frac{a^2\int \frac{\sinh(x)}{a\cosh(x)+b\sinh(x)}\,dx}{a^2-b^2}-\frac{b\int \sinh^2(x)\,dx}{a^2-b^2}\\ &= \frac{a^2bx}{\left(a^2-b^2\right)^2}-\frac{b\cosh(x)\sinh(x)}{2\left(a^2-b^2\right)}+\frac{a\sinh^2(x)}{2\left(a^2-b^2\right)}-\frac{\left(ia^3\right)\int \frac{-ib\cosh(x)-ia\sinh(x)}{a\cosh(x)+b\sinh(x)}\,dx}{\left(a^2-b^2\right)^2}+\frac{}{2}\\ &= \frac{a^2bx}{\left(a^2-b^2\right)^2}+\frac{bx}{2\left(a^2-b^2\right)}-\frac{a^3\log(a\cosh(x)+b\sinh(x))}{\left(a^2-b^2\right)^2}-\frac{b\cosh(x)\sinh(x)}{2\left(a^2-b^2\right)}+\end{aligned}$$

Mathematica [A] time = 0.16, size = 75, normalized size = 0.74

$$\frac{-4a^3\log(a\cosh(x)+b\sinh(x))+\left(b^3-a^2b\right)\sinh(2x)+a\left(a^2-b^2\right)\cosh(2x)+6a^2bx-2b^3x}{4(a-b)^2(a+b)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[x]^3/(a*Cosh[x] + b*Sinh[x]),x]
```

```
[Out] (6*a^2*b*x - 2*b^3*x + a*(a^2 - b^2)*Cosh[2*x] - 4*a^3*Log[a*Cosh[x] + b*Si
nh[x]] + (-(a^2*b) + b^3)*Sinh[2*x])/(4*(a - b)^2*(a + b)^2)
```

fricas [B] time = 0.43, size = 337, normalized size = 3.34

$$\frac{\left(a^3 - a^2b - ab^2 + b^3\right)\cosh(x)^4 + 4\left(a^3 - a^2b - ab^2 + b^3\right)\cosh(x)\sinh(x)^3 + \left(a^3 - a^2b - ab^2 + b^3\right)\sinh(x)^4 + 4\left(2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^3/(a*cosh(x)+b*sinh(x)),x, algorithm="fricas")
```

```
[Out] 1/8*((a^3 - a^2*b - a*b^2 + b^3)*cosh(x)^4 + 4*(a^3 - a^2*b - a*b^2 + b^3)*
cosh(x)*sinh(x)^3 + (a^3 - a^2*b - a*b^2 + b^3)*sinh(x)^4 + 4*(2*a^3 + 3*a^
2*b - b^3)*x*cosh(x)^2 + a^3 + a^2*b - a*b^2 - b^3 + 2*(3*(a^3 - a^2*b - a*
b^2 + b^3)*cosh(x)^2 + 2*(2*a^3 + 3*a^2*b - b^3)*x)*sinh(x)^2 - 8*(a^3*cosh
(x)^2 + 2*a^3*cosh(x)*sinh(x) + a^3*sinh(x)^2)*log(2*(a*cosh(x) + b*sinh(x)
)/(cosh(x) - sinh(x))) + 4*((a^3 - a^2*b - a*b^2 + b^3)*cosh(x)^3 + 2*(2*a^
3 + 3*a^2*b - b^3)*x*cosh(x))*sinh(x))/((a^4 - 2*a^2*b^2 + b^4)*cosh(x)^2 +
 2*(a^4 - 2*a^2*b^2 + b^4)*cosh(x)*sinh(x) + (a^4 - 2*a^2*b^2 + b^4)*sinh(x
)^2)
```

giac [A] time = 0.14, size = 114, normalized size = 1.13

$$-\frac{a^3\log\left(\left|ae^{(2x)} + be^{(2x)} + a - b\right|\right)}{a^4 - 2a^2b^2 + b^4} + \frac{(2a-b)x}{2\left(a^2 - 2ab + b^2\right)} - \frac{\left(4ae^{(2x)} - 2be^{(2x)} - a + b\right)e^{(-2x)}}{8\left(a^2 - 2ab + b^2\right)} + \frac{e^{(2x)}}{8(a+b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^3/(a*cosh(x)+b*sinh(x)),x, algorithm="giac")
```

```
[Out] -a^3*log(abs(a*e^(2*x) + b*e^(2*x) + a - b))/(a^4 - 2*a^2*b^2 + b^4) + 1/2*
(2*a - b)*x/(a^2 - 2*a*b + b^2) - 1/8*(4*a*e^(2*x) - 2*b*e^(2*x) - a + b)*e
^(-2*x)/(a^2 - 2*a*b + b^2) + 1/8*e^(2*x)/(a + b)
```

maple [A] time = 0.22, size = 175, normalized size = 1.73

$$\frac{8}{(16a+16b)\left(\tanh\left(\frac{x}{2}\right)-1\right)^2} + \frac{16}{(32a+32b)\left(\tanh\left(\frac{x}{2}\right)-1\right)} + \frac{\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)a}{(a+b)^2} + \frac{\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)b}{2(a+b)^2} - \frac{a^3\ln\left(a+2\right.}{(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)^3/(a*cosh(x)+b*sinh(x)),x)
```

```
[Out] 8/(16*a+16*b)/(tanh(1/2*x)-1)^2+16/(32*a+32*b)/(tanh(1/2*x)-1)+1/(a+b)^2*ln
(tanh(1/2*x)-1)*a+1/2/(a+b)^2*ln(tanh(1/2*x)-1)*b-a^3/(a-b)^2/(a+b)^2*ln(a+
2*tanh(1/2*x)*b+a*tanh(1/2*x)^2)-16/(32*a-32*b)/(tanh(1/2*x)+1)+8/(16*a-16*
```

```
b)/(tanh(1/2*x)+1)^2+1/(a-b)^2*ln(tanh(1/2*x)+1)*a-1/2/(a-b)^2*ln(tanh(1/2*
x)+1)*b
```

maxima [A] time = 0.56, size = 87, normalized size = 0.86

$$-\frac{a^3\log\left(-(a-b)e^{(-2x)}-a-b\right)}{a^4-2a^2b^2+b^4}-\frac{(2a+b)x}{2\left(a^2+2ab+b^2\right)}+\frac{e^{(2x)}}{8(a+b)}+\frac{e^{(-2x)}}{8(a-b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^3/(a*cosh(x)+b*sinh(x)),x, algorithm="maxima")
```

```
[Out] -a^3*log(-(a - b)*e^(-2*x) - a - b)/(a^4 - 2*a^2*b^2 + b^4) - 1/2*(2*a + b)
*x/(a^2 + 2*a*b + b^2) + 1/8*e^(2*x)/(a + b) + 1/8*e^(-2*x)/(a - b)
```

mupad [B] time = 1.96, size = 86, normalized size = 0.85

$$\frac{e^{-2x}}{8a-8b}+\frac{e^{2x}}{8a+8b}-\frac{a^3\ln\left(a-b+a\,e^{2x}+b\,e^{2x}\right)}{a^4-2a^2b^2+b^4}+\frac{x\,(2a-b)}{2(a-b)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)^3/(a*cosh(x) + b*sinh(x)),x)
```

```
[Out] exp(-2*x)/(8*a - 8*b) + exp(2*x)/(8*a + 8*b) - (a^3*log(a - b + a*exp(2*x)
+ b*exp(2*x)))/(a^4 + b^4 - 2*a^2*b^2) + (x*(2*a - b))/(2*(a - b)^2)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)**3/(a*cosh(x)+b*sinh(x)),x)
```

```
[Out] Timed out
```

3.691 $\int \frac{\cosh(x)}{a\cosh(x)+b\sinh(x)}\,dx$

Optimal. Leaf size=39

$$\frac{ax}{a^2-b^2}-\frac{b\log(a\cosh(x)+b\sinh(x))}{a^2-b^2}$$

```
[Out] a*x/(a^2-b^2)-b*ln(a*cosh(x)+b*sinh(x))/(a^2-b^2)
```

Rubi [A] time = 0.06, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3098, 3133}

$$\frac{ax}{a^2-b^2}-\frac{b\log(a\cosh(x)+b\sinh(x))}{a^2-b^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]/(a*Cosh[x] + b*Sinh[x]),x]
```

```
[Out] (a*x)/(a^2 - b^2) - (b*Log[a*Cosh[x] + b*Sinh[x]])/(a^2 - b^2)
```

Rule 3098

```
Int[cos[(c_.) + (d_.)*(x_)]/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.
) + (d_.)*(x_)]), x_Symbol] :> Simp[(a*x)/(a^2 + b^2), x] + Dist[b/(a^2 + b
^2), Int[(b*Cos[c + d*x] - a*Sin[c + d*x])/(a*Cos[c + d*x] + b*Sin[c + d*x]
), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3133

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)]), x
_Symbol] :> Simp[((b*B + c*C)*x)/(b^2 + c^2), x] + Simp[((c*B - b*C)*Log[a
+ b*Cos[d + e*x] + c*Sin[d + e*x]])/(e*(b^2 + c^2)), x] /; FreeQ[{a, b, c,
d, e, A, B, C}, x] && NeQ[b^2 + c^2, 0] && EqQ[A*(b^2 + c^2) - a*(b*B + c*C
), 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\cosh(x)}{a\cosh(x)+b\sinh(x)}\,dx &= \frac{ax}{a^2-b^2}-\frac{(ib)\int \frac{-ib\cosh(x)-ia\sinh(x)}{a\cosh(x)+b\sinh(x)}\,dx}{a^2-b^2}\\ &= \frac{ax}{a^2-b^2}-\frac{b\log(a\cosh(x)+b\sinh(x))}{a^2-b^2}\end{aligned}$$

Mathematica [A] time = 0.05, size = 29, normalized size = 0.74

$$\frac{ax - b\log(a\cosh(x) + b\sinh(x))}{a^2 - b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]/(a*Cosh[x] + b*Sinh[x]),x]
[Out] (a*x - b*Log[a*Cosh[x] + b*Sinh[x]])/(a^2 - b^2)
```

fricas [A] time = 0.43, size = 42, normalized size = 1.08

$$\frac{(a+b)x - b\log\left(\frac{2(a\cosh(x)+b\sinh(x))}{\cosh(x)-\sinh(x)}\right)}{a^2 - b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)/(a*cosh(x)+b*sinh(x)),x, algorithm="fricas")
[Out] ((a + b)*x - b*log(2*(a*cosh(x) + b*sinh(x))/(cosh(x) - sinh(x))))/(a^2 - b
^2)
```

giac [A] time = 0.12, size = 43, normalized size = 1.10

$$-\frac{b\log\left(\left|ae^{(2x)} + be^{(2x)} + a - b\right|\right)}{a^2 - b^2} + \frac{x}{a - b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)/(a*cosh(x)+b*sinh(x)),x, algorithm="giac")
[Out] -b*log(abs(a*e^(2*x) + b*e^(2*x) + a - b))/(a^2 - b^2) + x/(a - b)
```

maple [A] time = 0.19, size = 71, normalized size = 1.82

$$-\frac{2\ln\left(\tanh\left(\frac{x}{2}\right) - 1\right)}{2b + 2a} - \frac{b\ln\left(a + 2\tanh\left(\frac{x}{2}\right)b + a\left(\tanh^2\left(\frac{x}{2}\right)\right)\right)}{(a-b)(a+b)} + \frac{2\ln\left(\tanh\left(\frac{x}{2}\right) + 1\right)}{2a - 2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)/(a*cosh(x)+b*sinh(x)),x)
[Out] -2/(2*b+2*a)*ln(tanh(1/2*x)-1)-b/(a-b)/(a+b)*ln(a+2*tanh(1/2*x)*b+a*tanh(1/
2*x)^2)+2/(2*a-2*b)*ln(tanh(1/2*x)+1)
```

maxima [A] time = 0.31, size = 41, normalized size = 1.05

$$-\frac{b\log\left(-(a-b)e^{(-2x)}-a-b\right)}{a^2-b^2}+\frac{x}{a+b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)/(a*cosh(x)+b*sinh(x)),x, algorithm="maxima")
```

```
[Out] -b*log(-(a - b)*e^(-2*x) - a - b)/(a^2 - b^2) + x/(a + b)
```

mupad [B] time = 1.52, size = 29, normalized size = 0.74

$$\frac{a\,x-b\,\ln\left(a\cosh(x)+b\sinh(x)\right)}{a^2-b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)/(a*cosh(x) + b*sinh(x)),x)
```

```
[Out] (a*x - b*log(a*cosh(x) + b*sinh(x)))/(a^2 - b^2)
```

sympy [A] time = 0.58, size = 150, normalized size = 3.85

$$\begin{cases} \tilde{\infty}\log\left(\sinh(x)\right) & \text{for } a=0 \wedge b=0 \\ \frac{\log\left(\sinh(x)\right)}{b} & \text{for } a=0 \\ \frac{x\sinh(x)}{-2b\sinh(x)+2b\cosh(x)}-\frac{x\cosh(x)}{-2b\sinh(x)+2b\cosh(x)}-\frac{\cosh(x)}{-2b\sinh(x)+2b\cosh(x)} & \text{for } a=-b \\ \frac{x\sinh(x)}{2b\sinh(x)+2b\cosh(x)}+\frac{x\cosh(x)}{2b\sinh(x)+2b\cosh(x)}-\frac{\cosh(x)}{2b\sinh(x)+2b\cosh(x)} & \text{for } a=b \\ \frac{ax}{a^2-b^2}-\frac{b\log\left(\cosh(x)+\frac{b\sinh(x)}{a}\right)}{a^2-b^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)/(a*cosh(x)+b*sinh(x)),x)
```

```
[Out] Piecewise((zoo*log(sinh(x)), Eq(a, 0) & Eq(b, 0)), (log(sinh(x))/b, Eq(a, 0
)), (x*sinh(x)/(-2*b*sinh(x) + 2*b*cosh(x)) - x*cosh(x)/(-2*b*sinh(x) + 2*b
*cosh(x)) - cosh(x)/(-2*b*sinh(x) + 2*b*cosh(x)), Eq(a, -b)), (x*sinh(x)/(2
*b*sinh(x) + 2*b*cosh(x)) + x*cosh(x)/(2*b*sinh(x) + 2*b*cosh(x)) - cosh(x)
/(2*b*sinh(x) + 2*b*cosh(x)), Eq(a, b)), (a*x/(a**2 - b**2) - b*log(cosh(x)
 + b*sinh(x)/a)/(a**2 - b**2), True))
```

3.692 $\int \frac{\cosh^2(x)}{a\cosh(x)+b\sinh(x)}\,dx$

Optimal. Leaf size=74

$$\frac{a\sinh(x)}{a^2-b^2}-\frac{b\cosh(x)}{a^2-b^2}-\frac{b^2\tan^{-1}\left(\frac{a\sinh(x)+b\cosh(x)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{3/2}}$$

```
[Out] -b^2*arctan((b*cosh(x)+a*sinh(x))/(a^2-b^2)^(1/2))/(a^2-b^2)^(3/2)-b*cosh(x
)/(a^2-b^2)+a*sinh(x)/(a^2-b^2)
```

Rubi [A] time = 0.08, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3100, 2637, 3074, 206}

$$\frac{a\sinh(x)}{a^2-b^2}-\frac{b\cosh(x)}{a^2-b^2}-\frac{b^2\tan^{-1}\left(\frac{a\sinh(x)+b\cosh(x)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]^2/(a*Cosh[x] + b*Sinh[x]),x]
```

```
[Out] -((b^2*ArcTan[(b*Cosh[x] + a*Sinh[x])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(3/2))
- (b*Cosh[x])/(a^2 - b^2) + (a*Sinh[x])/(a^2 - b^2)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3074

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> -Dist[d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d
*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3100

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_)/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin
[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[(b*Cos[c + d*x]^(m - 1))/(d*(a^2 +
 b^2)*(m - 1)), x] + (Dist[a/(a^2 + b^2), Int[Cos[c + d*x]^(m - 1), x], x]
+ Dist[b^2/(a^2 + b^2), Int[Cos[c + d*x]^(m - 2)/(a*Cos[c + d*x] + b*Sin[c
+ d*x]), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && GtQ[m, 1
]
```

Rubi steps

$$\begin{aligned}\int \frac{\cosh^2(x)}{a\cosh(x)+b\sinh(x)}\,dx &= -\frac{b\cosh(x)}{a^2-b^2}+\frac{a\int \cosh(x)\,dx}{a^2-b^2}-\frac{b^2\int \frac{1}{a\cosh(x)+b\sinh(x)}\,dx}{a^2-b^2}\\ &= -\frac{b\cosh(x)}{a^2-b^2}+\frac{a\sinh(x)}{a^2-b^2}-\frac{\left(ib^2\right)\text{Subst}\left(\int \frac{1}{a^2-b^2-x^2}\,dx,x,-ib\cosh(x)-ia\sinh(x)\right)}{a^2-b^2}\\ &= -\frac{b^2\tan^{-1}\left(\frac{b\cosh(x)+a\sinh(x)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{3/2}}-\frac{b\cosh(x)}{a^2-b^2}+\frac{a\sinh(x)}{a^2-b^2}\end{aligned}$$

Mathematica [A] time = 0.17, size = 80, normalized size = 1.08

$$\frac{a\sinh(x)}{a^2-b^2}+\frac{b\cosh(x)}{b^2-a^2}-\frac{2b^2\tan^{-1}\left(\frac{a\tanh\left(\frac{x}{2}\right)+b}{\sqrt{a-b}\,\sqrt{a+b}}\right)}{(a-b)^{3/2}(a+b)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]^2/(a*Cosh[x] + b*Sinh[x]),x]
```

```
[Out] (-2*b^2*ArcTan[(b + a*Tanh[x/2])/(Sqrt[a - b]*Sqrt[a + b])])/((a - b)^(3/2)
*(a + b)^(3/2)) + (b*Cosh[x])/(-a^2 + b^2) + (a*Sinh[x])/(a^2 - b^2)
```

fricas [B] time = 0.47, size = 435, normalized size = 5.88

$$\left[-\frac{a^3+a^2b-ab^2-b^3-\left(a^3-a^2b-ab^2+b^3\right)\cosh(x)^2-2\left(a^3-a^2b-ab^2+b^3\right)\cosh(x)\sinh(x)-\left(a^3-a^2b-ab\right.}{2\left(\left(a^4-2a^2b^2+\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^2/(a*cosh(x)+b*sinh(x)),x, algorithm="fricas")
```

```
[Out] [-1/2*(a^3 + a^2*b - a*b^2 - b^3 - (a^3 - a^2*b - a*b^2 + b^3)*cosh(x)^2 -
2*(a^3 - a^2*b - a*b^2 + b^3)*cosh(x)*sinh(x) - (a^3 - a^2*b - a*b^2 + b^3)
*sinh(x)^2 - 2*(b^2*cosh(x) + b^2*sinh(x))*sqrt(-a^2 + b^2)*log(((a + b)*co
sh(x)^2 + 2*(a + b)*cosh(x)*sinh(x) + (a + b)*sinh(x)^2 - 2*sqrt(-a^2 + b^2
)*(cosh(x) + sinh(x)) - a + b)/((a + b)*cosh(x)^2 + 2*(a + b)*cosh(x)*sinh(
x) + (a + b)*sinh(x)^2 + a - b)))/((a^4 - 2*a^2*b^2 + b^4)*cosh(x) + (a^4 -
 2*a^2*b^2 + b^4)*sinh(x)), -1/2*(a^3 + a^2*b - a*b^2 - b^3 - (a^3 - a^2*b
- a*b^2 + b^3)*cosh(x)^2 - 2*(a^3 - a^2*b - a*b^2 + b^3)*cosh(x)*sinh(x) -
(a^3 - a^2*b - a*b^2 + b^3)*sinh(x)^2 - 4*(b^2*cosh(x) + b^2*sinh(x))*sqrt(
a^2 - b^2)*arctan(sqrt(a^2 - b^2)/((a + b)*cosh(x) + (a + b)*sinh(x))))/((a
^4 - 2*a^2*b^2 + b^4)*cosh(x) + (a^4 - 2*a^2*b^2 + b^4)*sinh(x))]
```

giac [A] time = 0.13, size = 61, normalized size = 0.82

$$-\frac{2\,b^2\arctan\left(\frac{ae^x+be^x}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{\frac{3}{2}}}-\frac{e^{(-x)}}{2\,(a-b)}+\frac{e^x}{2\,(a+b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^2/(a*cosh(x)+b*sinh(x)),x, algorithm="giac")
```

```
[Out] -2*b^2*arctan((a*e^x + b*e^x)/sqrt(a^2 - b^2))/(a^2 - b^2)^(3/2) - 1/2*e^(-
x)/(a - b) + 1/2*e^x/(a + b)
```

maple [A] time = 0.23, size = 93, normalized size = 1.26

$$-\frac{2}{(2a-2b)\left(\tanh\left(\frac{x}{2}\right)+1\right)}-\frac{2b^2\arctan\left(\frac{2a\tanh\left(\frac{x}{2}\right)+2b}{2\sqrt{a^2-b^2}}\right)}{(a-b)\,(a+b)\sqrt{a^2-b^2}}-\frac{2}{(2b+2a)\left(\tanh\left(\frac{x}{2}\right)-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^2/(a*cosh(x)+b*sinh(x)),x)
```

```
[Out] -2/(2*a-2*b)/(tanh(1/2*x)+1)-2*b^2/(a-b)/(a+b)/(a^2-b^2)^(1/2)*arctan(1/2*(
2*a*tanh(1/2*x)+2*b)/(a^2-b^2)^(1/2))-2/(2*b+2*a)/(tanh(1/2*x)-1)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^2/(a*cosh(x)+b*sinh(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [B] time = 1.72, size = 157, normalized size = 2.12

$$\frac{e^x}{2a+2b}-\frac{e^{-x}}{2a-2b}-\frac{b^2\ln\left(-\frac{2b^2}{(a+b)^{5/2}\sqrt{b-a}}-\frac{2b^2e^x}{-a^3-a^2b+ab^2+b^3}\right)}{(a+b)^{3/2}(b-a)^{3/2}}+\frac{b^2\ln\left(\frac{2b^2}{(a+b)^{5/2}\sqrt{b-a}}-\frac{2b^2e^x}{-a^3-a^2b+ab^2+b^3}\right)}{(a+b)^{3/2}(b-a)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^2/(a*cosh(x) + b*sinh(x)),x)
```

```
[Out] exp(x)/(2*a + 2*b) - exp(-x)/(2*a - 2*b) - (b^2*log(- (2*b^2)/((a + b)^(5/2
)*(b - a)^(1/2)) - (2*b^2*exp(x))/(a*b^2 - a^2*b - a^3 + b^3)))/((a + b)^(3
/2)*(b - a)^(3/2)) + (b^2*log((2*b^2)/((a + b)^(5/2)*(b - a)^(1/2)) - (2*b^
2*exp(x))/(a*b^2 - a^2*b - a^3 + b^3)))/((a + b)^(3/2)*(b - a)^(3/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)**2/(a*cosh(x)+b*sinh(x)),x)
```

```
[Out] Timed out
```

3.693 $\int \frac{\cosh^3(x)}{a\cosh(x)+b\sinh(x)}\,dx$

Optimal. Leaf size=101

$$-\frac{ab^2x}{\left(a^2-b^2\right)^2}+\frac{ax}{2\left(a^2-b^2\right)}-\frac{b\cosh^2(x)}{2\left(a^2-b^2\right)}+\frac{a\sinh(x)\cosh(x)}{2\left(a^2-b^2\right)}+\frac{b^3\log(a\cosh(x)+b\sinh(x))}{\left(a^2-b^2\right)^2}$$

```
[Out] -a*b^2*x/(a^2-b^2)^2+1/2*a*x/(a^2-b^2)-1/2*b*cosh(x)^2/(a^2-b^2)+b^3*ln(a*c
osh(x)+b*sinh(x))/(a^2-b^2)^2+1/2*a*cosh(x)*sinh(x)/(a^2-b^2)
```

Rubi [A] time = 0.12, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {3100, 2635, 8, 3098, 3133}

$$-\frac{ab^2x}{\left(a^2-b^2\right)^2}+\frac{ax}{2\left(a^2-b^2\right)}-\frac{b\cosh^2(x)}{2\left(a^2-b^2\right)}+\frac{a\sinh(x)\cosh(x)}{2\left(a^2-b^2\right)}+\frac{b^3\log(a\cosh(x)+b\sinh(x))}{\left(a^2-b^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]^3/(a*Cosh[x] + b*Sinh[x]),x]
```

```
[Out] -((a*b^2*x)/(a^2 - b^2)^2) + (a*x)/(2*(a^2 - b^2)) - (b*Cosh[x]^2)/(2*(a^2
- b^2)) + (b^3*Log[a*Cosh[x] + b*Sinh[x]])/(a^2 - b^2)^2 + (a*Cosh[x]*Sinh[
x])/(2*(a^2 - b^2))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 3098

```
Int[cos[(c_.) + (d_.)*(x_)]/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.
) + (d_.)*(x_)]), x_Symbol] :> Simp[(a*x)/(a^2 + b^2), x] + Dist[b/(a^2 + b
^2), Int[(b*Cos[c + d*x] - a*Sin[c + d*x])/(a*Cos[c + d*x] + b*Sin[c + d*x]
), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3100

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_)/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin
[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[(b*Cos[c + d*x]^(m - 1))/(d*(a^2 +
 b^2)*(m - 1)), x] + (Dist[a/(a^2 + b^2), Int[Cos[c + d*x]^(m - 1), x], x]
+ Dist[b^2/(a^2 + b^2), Int[Cos[c + d*x]^(m - 2)/(a*Cos[c + d*x] + b*Sin[c
+ d*x]), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && GtQ[m, 1
]
```

Rule 3133

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)]), x
_Symbol] :> Simp[((b*B + c*C)*x)/(b^2 + c^2), x] + Simp[((c*B - b*C)*Log[a
+ b*Cos[d + e*x] + c*Sin[d + e*x]])/(e*(b^2 + c^2)), x] /; FreeQ[{a, b, c,
d, e, A, B, C}, x] && NeQ[b^2 + c^2, 0] && EqQ[A*(b^2 + c^2) - a*(b*B + c*C
), 0]
```

Rubi steps

$$\int \frac{\cosh^3(x)}{a\cosh(x)+b\sinh(x)}\,dx = -\frac{b\cosh^2(x)}{2\left(a^2-b^2\right)}+\frac{a\int \cosh^2(x)\,dx}{a^2-b^2}-\frac{b^2\int \frac{\cosh(x)}{a\cosh(x)+b\sinh(x)}\,dx}{a^2-b^2}$$

$$= -\frac{ab^2x}{\left(a^2-b^2\right)^2}-\frac{b\cosh^2(x)}{2\left(a^2-b^2\right)}+\frac{a\cosh(x)\sinh(x)}{2\left(a^2-b^2\right)}+\frac{\left(ib^3\right)\int \frac{-ib\cosh(x)-ia\sinh(x)}{a\cosh(x)+b\sinh(x)}\,dx}{\left(a^2-b^2\right)^2}+ -\frac{}{2}$$

$$= -\frac{ab^2x}{\left(a^2-b^2\right)^2}+\frac{ax}{2\left(a^2-b^2\right)}-\frac{b\cosh^2(x)}{2\left(a^2-b^2\right)}+\frac{b^3\log(a\cosh(x)+b\sinh(x))}{\left(a^2-b^2\right)^2}+\frac{a\cosh}{2(a}$$

Mathematica [A] time = 0.12, size = 75, normalized size = 0.74

$$\frac{2a^3x+\left(b^3-a^2b\right)\cosh(2x)+a\left(a^2-b^2\right)\sinh(2x)+4b^3\log(a\cosh(x)+b\sinh(x))-6ab^2x}{4(a-b)^2(a+b)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]^3/(a*Cosh[x] + b*Sinh[x]),x]
```

```
[Out] (2*a^3*x - 6*a*b^2*x + (-(a^2*b) + b^3)*Cosh[2*x] + 4*b^3*Log[a*Cosh[x] + b
*Sinh[x]] + a*(a^2 - b^2)*Sinh[2*x])/(4*(a - b)^2*(a + b)^2)
```

fricas [B] time = 0.44, size = 331, normalized size = 3.28

$$\frac{(a^3 - a^2b - ab^2 + b^3)\cosh(x)^4 + 4(a^3 - a^2b - ab^2 + b^3)\cosh(x)\sinh(x)^3 + (a^3 - a^2b - ab^2 + b^3)\sinh(x)^4 + 4(}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^3/(a*cosh(x)+b*sinh(x)),x, algorithm="fricas")
```

```
[Out] 1/8*((a^3 - a^2*b - a*b^2 + b^3)*cosh(x)^4 + 4*(a^3 - a^2*b - a*b^2 + b^3)*
cosh(x)*sinh(x)^3 + (a^3 - a^2*b - a*b^2 + b^3)*sinh(x)^4 + 4*(a^3 - 3*a*b^
2 - 2*b^3)*x*cosh(x)^2 - a^3 - a^2*b + a*b^2 + b^3 + 2*(3*(a^3 - a^2*b - a*
b^2 + b^3)*cosh(x)^2 + 2*(a^3 - 3*a*b^2 - 2*b^3)*x)*sinh(x)^2 + 8*(b^3*cosh
(x)^2 + 2*b^3*cosh(x)*sinh(x) + b^3*sinh(x)^2)*log(2*(a*cosh(x) + b*sinh(x)
)/(cosh(x) - sinh(x))) + 4*((a^3 - a^2*b - a*b^2 + b^3)*cosh(x)^3 + 2*(a^3
- 3*a*b^2 - 2*b^3)*x*cosh(x))*sinh(x))/((a^4 - 2*a^2*b^2 + b^4)*cosh(x)^2 +
 2*(a^4 - 2*a^2*b^2 + b^4)*cosh(x)*sinh(x) + (a^4 - 2*a^2*b^2 + b^4)*sinh(x
)^2)
```

giac [A] time = 0.13, size = 111, normalized size = 1.10

$$\frac{b^3\log\left(\left|ae^{(2x)} + be^{(2x)} + a - b\right|\right)}{a^4 - 2a^2b^2 + b^4} + \frac{(a-2b)x}{2(a^2 - 2ab + b^2)} - \frac{\left(2ae^{(2x)} - 4be^{(2x)} + a - b\right)e^{(-2x)}}{8(a^2 - 2ab + b^2)} + \frac{e^{(2x)}}{8(a+b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^3/(a*cosh(x)+b*sinh(x)),x, algorithm="giac")
```

```
[Out] b^3*log(abs(a*e^(2*x) + b*e^(2*x) + a - b))/(a^4 - 2*a^2*b^2 + b^4) + 1/2*(
a - 2*b)*x/(a^2 - 2*a*b + b^2) - 1/8*(2*a*e^(2*x) - 4*b*e^(2*x) + a - b)*e^
(-2*x)/(a^2 - 2*a*b + b^2) + 1/8*e^(2*x)/(a + b)
```

maple [A] time = 0.23, size = 175, normalized size = 1.73

$$\frac{1}{(2b+2a)\left(\tanh\left(\frac{x}{2}\right)-1\right)^2} + \frac{2}{(4a+4b)\left(\tanh\left(\frac{x}{2}\right)-1\right)} - \frac{\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)a}{2(a+b)^2} - \frac{\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)b}{(a+b)^2} + \frac{b^3\ln\left(a + 2\tanh\right.}{(a-}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^3/(a*cosh(x)+b*sinh(x)),x)
```

```
[Out] 1/(2*b+2*a)/(tanh(1/2*x)-1)^2+2/(4*a+4*b)/(tanh(1/2*x)-1)-1/2/(a+b)^2*ln(ta
nh(1/2*x)-1)*a-1/(a+b)^2*ln(tanh(1/2*x)-1)*b+b^3/(a-b)^2/(a+b)^2*ln(a+2*tan
```

```
h(1/2*x)*b+a*tanh(1/2*x)^2)-1/(2*a-2*b)/(tanh(1/2*x)+1)^2+2/(4*a-4*b)/(tanh
(1/2*x)+1)+1/2/(a-b)^2*ln(tanh(1/2*x)+1)*a-1/(a-b)^2*ln(tanh(1/2*x)+1)*b
```

maxima [A] time = 0.51, size = 86, normalized size = 0.85

$$\frac{b^3 \log\left(-(a-b)e^{(-2x)} - a - b\right)}{a^4 - 2a^2b^2 + b^4} + \frac{(a+2b)x}{2\left(a^2 + 2ab + b^2\right)} + \frac{e^{(2x)}}{8(a+b)} - \frac{e^{(-2x)}}{8(a-b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^3/(a*cosh(x)+b*sinh(x)),x, algorithm="maxima")
```

```
[Out] b^3*log(-(a - b)*e^(-2*x) - a - b)/(a^4 - 2*a^2*b^2 + b^4) + 1/2*(a + 2*b)*
x/(a^2 + 2*a*b + b^2) + 1/8*e^(2*x)/(a + b) - 1/8*e^(-2*x)/(a - b)
```

mupad [B] time = 1.67, size = 84, normalized size = 0.83

$$\frac{e^{2x}}{8a+8b} - \frac{e^{-2x}}{8a-8b} + \frac{b^3 \ln\left(a - b + a e^{2x} + b e^{2x}\right)}{a^4 - 2a^2b^2 + b^4} + \frac{x(a-2b)}{2(a-b)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^3/(a*cosh(x) + b*sinh(x)),x)
```

```
[Out] exp(2*x)/(8*a + 8*b) - exp(-2*x)/(8*a - 8*b) + (b^3*log(a - b + a*exp(2*x)
+ b*exp(2*x)))/(a^4 + b^4 - 2*a^2*b^2) + (x*(a - 2*b))/(2*(a - b)^2)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)**3/(a*cosh(x)+b*sinh(x)),x)
```

```
[Out] Timed out
```

3.694 $\int \frac{\tanh(x)}{b\cosh(x)+a\sinh(x)}\,dx$

Optimal. Leaf size=50

$$\frac{b\tanh^{-1}\left(\frac{a\cosh(x)+b\sinh(x)}{\sqrt{a^2-b^2}}\right)}{a\sqrt{a^2-b^2}}+\frac{\tan^{-1}(\sinh(x))}{a}$$

```
[Out] arctan(sinh(x))/a+b*arctanh((a*cosh(x)+b*sinh(x))/(a^2-b^2)^(1/2))/a/(a^2-b
^2)^(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3110, 3770, 3074, 204}

$$\frac{b\tanh^{-1}\left(\frac{a\cosh(x)+b\sinh(x)}{\sqrt{a^2-b^2}}\right)}{a\sqrt{a^2-b^2}}+\frac{\tan^{-1}(\sinh(x))}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[x]/(b*Cosh[x] + a*Sinh[x]),x]
```

```
[Out] ArcTan[Sinh[x]]/a + (b*ArcTanh[(a*Cosh[x] + b*Sinh[x])/Sqrt[a^2 - b^2]])/(a
*Sqrt[a^2 - b^2])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 3074

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> -Dist[d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d
*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3110

```
Int[(cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.))/(cos[(c_.
) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Int[Ex
pandTrig[(cos[c + d*x]^m*sin[c + d*x]^n)/(a*cos[c + d*x] + b*sin[c + d*x]),
 x], x] /; FreeQ[{a, b, c, d, m, n}, x] && IntegersQ[m, n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\tanh(x)}{b\cosh(x)+a\sinh(x)}\,dx = -\left(i\int\left(\frac{i\operatorname{sech}(x)}{a}-\frac{ib}{a(b\cosh(x)+a\sinh(x))}\right)dx\right)$$
$$=\frac{\int \operatorname{sech}(x)\,dx}{a}-\frac{b\int\frac{1}{b\cosh(x)+a\sinh(x)}\,dx}{a}$$
$$=\frac{\tan^{-1}(\sinh(x))}{a}-\frac{(ib)\operatorname{Subst}\left(\int\frac{1}{-a^2+b^2-x^2}\,dx,x,-ia\cosh(x)-ib\sinh(x)\right)}{a}$$
$$=\frac{\tan^{-1}(\sinh(x))}{a}+\frac{b\tanh^{-1}\left(\frac{a\cosh(x)+b\sinh(x)}{\sqrt{a^2-b^2}}\right)}{a\sqrt{a^2-b^2}}$$

Mathematica [A] time = 0.13, size = 60, normalized size = 1.20

$$\frac{2\left(\tan^{-1}\left(\tanh\left(\frac{x}{2}\right)\right)-\frac{b\tan^{-1}\left(\frac{a+b\tanh\left(\frac{x}{2}\right)}{\sqrt{b-a}\sqrt{a+b}}\right)}{\sqrt{b-a}\sqrt{a+b}}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]/(b*Cosh[x] + a*Sinh[x]),x]
```

```
[Out] (2*(ArcTan[Tanh[x/2]] - (b*ArcTan[(a + b*Tanh[x/2])/(Sqrt[-a + b]*Sqrt[a +
b])])/(Sqrt[-a + b]*Sqrt[a + b])))/a
```

fricas [A] time = 0.47, size = 200, normalized size = 4.00

$$\left[\frac{\sqrt{a^2-b^2}\,b\log\left(\frac{(a+b)\cosh(x)^2+2\,(a+b)\cosh(x)\sinh(x)+(a+b)\sinh(x)^2+2\,\sqrt{a^2-b^2}\,(\cosh(x)+\sinh(x))+a-b}{(a+b)\cosh(x)^2+2\,(a+b)\cosh(x)\sinh(x)+(a+b)\sinh(x)^2-a+b}\right)+2\left(a^2-b^2\right)\arctan(\cosh}{a^3-ab^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/(b*cosh(x)+a*sinh(x)),x, algorithm="fricas")
```

```
[Out] [(sqrt(a^2 - b^2)*b*log(((a + b)*cosh(x)^2 + 2*(a + b)*cosh(x)*sinh(x) + (a
 + b)*sinh(x)^2 + 2*sqrt(a^2 - b^2)*(cosh(x) + sinh(x)) + a - b)/((a + b)*c
osh(x)^2 + 2*(a + b)*cosh(x)*sinh(x) + (a + b)*sinh(x)^2 - a + b)) + 2*(a^2
 - b^2)*arctan(cosh(x) + sinh(x)))/(a^3 - a*b^2), -2*(sqrt(-a^2 + b^2)*b*ar
ctan(sqrt(-a^2 + b^2)/((a + b)*cosh(x) + (a + b)*sinh(x))) - (a^2 - b^2)*ar
ctan(cosh(x) + sinh(x)))/(a^3 - a*b^2)]
```

giac [A] time = 0.12, size = 48, normalized size = 0.96

$$-\frac{2\,b\arctan\left(\frac{ae^{x}+be^{x}}{\sqrt{-a^{2}+b^{2}}}\right)}{\sqrt{-a^{2}+b^{2}}\,a}+\frac{2\,\arctan\left(e^{x}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/(b*cosh(x)+a*sinh(x)),x, algorithm="giac")
```

```
[Out] -2*b*arctan((a*e^x + b*e^x)/sqrt(-a^2 + b^2))/(sqrt(-a^2 + b^2)*a) + 2*arct
an(e^x)/a
```

maple [A] time = 0.30, size = 54, normalized size = 1.08

$$-\frac{2b\arctan\left(\frac{2\tanh\left(\frac{x}{2}\right)b+2a}{2\sqrt{-a^{2}+b^{2}}}\right)}{a\sqrt{-a^{2}+b^{2}}}+\frac{2\arctan\left(\tanh\left(\frac{x}{2}\right)\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)/(b*cosh(x)+a*sinh(x)),x)
```

```
[Out] -2/a*b/(-a^2+b^2)^(1/2)*arctan(1/2*(2*tanh(1/2*x)*b+2*a)/(-a^2+b^2)^(1/2))+
2/a*arctan(tanh(1/2*x))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/(b*cosh(x)+a*sinh(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more details)Is 4*a^2-4*b^2 positive or negative?
```

mupad [B] time = 3.53, size = 164, normalized size = 3.28

$$\frac{b\ \ln\left(32\,a\,b^2\,\mathrm{e}^x+32\,a^2\,b\,\mathrm{e}^x+32\,a\,b\,\sqrt{a^2-b^2}\right)}{a\,\sqrt{a^2-b^2}}-\frac{b\ \ln\left(32\,a\,b^2\,\mathrm{e}^x+32\,a^2\,b\,\mathrm{e}^x-32\,a\,b\,\sqrt{a^2-b^2}\right)}{a\,\sqrt{a^2-b^2}}+\frac{\ln\left(32\,a\,b\,\mathrm{e}^x-3\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)/(b*cosh(x) + a*sinh(x)),x)
```

```
[Out] (log(a*b*32i - a^2*32i - 32*a^2*exp(x) + 32*a*b*exp(x))*1i)/a - (log(a*b*32
i - a^2*32i + 32*a^2*exp(x) - 32*a*b*exp(x))*1i)/a - (b*log(32*a*b^2*exp(x)
 + 32*a^2*b*exp(x) - 32*a*b*(a^2 - b^2)^(1/2)))/(a*(a^2 - b^2)^(1/2)) + (b*
log(32*a*b^2*exp(x) + 32*a^2*b*exp(x) + 32*a*b*(a^2 - b^2)^(1/2)))/(a*(a^2
- b^2)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh(x)}{a\sinh(x)+b\cosh(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/(b*cosh(x)+a*sinh(x)),x)
```

```
[Out] Integral(tanh(x)/(a*sinh(x) + b*cosh(x)), x)
```

3.695 $\int \frac{\coth(x)}{b\cosh(x)+a\sinh(x)}\,dx$

Optimal. Leaf size=51

$$\frac{a\tanh^{-1}\left(\frac{a\cosh(x)+b\sinh(x)}{\sqrt{a^2-b^2}}\right)}{b\sqrt{a^2-b^2}}-\frac{\tanh^{-1}(\cosh(x))}{b}$$

```
[Out] -arctanh(cosh(x))/b+a*arctanh((a*cosh(x)+b*sinh(x))/(a^2-b^2)^(1/2))/b/(a^2
-b^2)^(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3110, 3770, 3074, 206}

$$\frac{a\tanh^{-1}\left(\frac{a\cosh(x)+b\sinh(x)}{\sqrt{a^2-b^2}}\right)}{b\sqrt{a^2-b^2}}-\frac{\tanh^{-1}(\cosh(x))}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]/(b*Cosh[x] + a*Sinh[x]),x]
```

```
[Out] -(ArcTanh[Cosh[x]]/b) + (a*ArcTanh[(a*Cosh[x] + b*Sinh[x])/Sqrt[a^2 - b^2]]
)/(b*Sqrt[a^2 - b^2])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3074

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> -Dist[d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d
*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3110

```
Int[(cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.))/(cos[(c_.
) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Int[Ex
pandTrig[(cos[c + d*x]^m*sin[c + d*x]^n)/(a*cos[c + d*x] + b*sin[c + d*x]),
 x], x] /; FreeQ[{a, b, c, d, m, n}, x] && IntegersQ[m, n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\coth(x)}{b\cosh(x)+a\sinh(x)}\,dx = i\int\left(-\frac{i\operatorname{csch}(x)}{b}-\frac{a}{b(ib\cosh(x)+ia\sinh(x))}\right)dx$$
$$=\frac{\int \operatorname{csch}(x)\,dx}{b}-\frac{(ia)\int \frac{1}{ib\cosh(x)+ia\sinh(x)}\,dx}{b}$$
$$=-\frac{\tanh^{-1}(\cosh(x))}{b}+\frac{a\,\mathrm{Subst}\left(\int \frac{1}{a^2-b^2-x^2}\,dx,x,a\cosh(x)+b\sinh(x)\right)}{b}$$
$$=-\frac{\tanh^{-1}(\cosh(x))}{b}+\frac{a\tanh^{-1}\left(\frac{a\cosh(x)+b\sinh(x)}{\sqrt{a^2-b^2}}\right)}{b\sqrt{a^2-b^2}}$$

Mathematica [A] time = 0.08, size = 59, normalized size = 1.16

$$\frac{\log\left(\tanh\left(\frac{x}{2}\right)\right)-\frac{2a\tan^{-1}\left(\frac{a+b\tanh\left(\frac{x}{2}\right)}{\sqrt{b-a}\sqrt{a+b}}\right)}{\sqrt{b-a}\sqrt{a+b}}}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]/(b*Cosh[x] + a*Sinh[x]),x]
```

```
[Out] ((-2*a*ArcTan[(a + b*Tanh[x/2])/(Sqrt[-a + b]*Sqrt[a + b])])/(Sqrt[-a + b]*
Sqrt[a + b]) + Log[Tanh[x/2]])/b
```

fricas [A] time = 0.46, size = 239, normalized size = 4.69

$$\left[\frac{\sqrt{a^2-b^2}\,a\log\left(\frac{(a+b)\cosh(x)^2+2\,(a+b)\cosh(x)\sinh(x)+(a+b)\sinh(x)^2+2\,\sqrt{a^2-b^2}\,(\cosh(x)+\sinh(x))+a-b}{(a+b)\cosh(x)^2+2\,(a+b)\cosh(x)\sinh(x)+(a+b)\sinh(x)^2-a+b}\right)-\left(a^2-b^2\right)\log\left(\cosh(x)+\right.}{a^2b-b^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)/(b*cosh(x)+a*sinh(x)),x, algorithm="fricas")
```

```
[Out] [(sqrt(a^2 - b^2)*a*log(((a + b)*cosh(x)^2 + 2*(a + b)*cosh(x)*sinh(x) + (a
 + b)*sinh(x)^2 + 2*sqrt(a^2 - b^2)*(cosh(x) + sinh(x)) + a - b)/((a + b)*c
osh(x)^2 + 2*(a + b)*cosh(x)*sinh(x) + (a + b)*sinh(x)^2 - a + b)) - (a^2 -
 b^2)*log(cosh(x) + sinh(x) + 1) + (a^2 - b^2)*log(cosh(x) + sinh(x) - 1))/
(a^2*b - b^3), -(2*sqrt(-a^2 + b^2)*a*arctan(sqrt(-a^2 + b^2)/((a + b)*cosh
(x) + (a + b)*sinh(x))) + (a^2 - b^2)*log(cosh(x) + sinh(x) + 1) - (a^2 - b
^2)*log(cosh(x) + sinh(x) - 1))/(a^2*b - b^3)]
```

giac [A] time = 0.12, size = 60, normalized size = 1.18

$$-\frac{2\,a\arctan\left(\frac{ae^{x}+be^{x}}{\sqrt{-a^{2}+b^{2}}}\right)}{\sqrt{-a^{2}+b^{2}}\,b}-\frac{\log\left(e^{x}+1\right)}{b}+\frac{\log\left(\left|e^{x}-1\right|\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)/(b*cosh(x)+a*sinh(x)),x, algorithm="giac")
```

```
[Out] -2*a*arctan((a*e^x + b*e^x)/sqrt(-a^2 + b^2))/(sqrt(-a^2 + b^2)*b) - log(e^
x + 1)/b + log(abs(e^x - 1))/b
```

maple [A] time = 0.30, size = 53, normalized size = 1.04

$$-\frac{2a\arctan\left(\frac{2\tanh\left(\frac{x}{2}\right)b+2a}{2\sqrt{-a^{2}+b^{2}}}\right)}{b\sqrt{-a^{2}+b^{2}}}+\frac{\ln\left(\tanh\left(\frac{x}{2}\right)\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)/(b*cosh(x)+a*sinh(x)),x)
```

```
[Out] -2*a/b/(-a^2+b^2)^(1/2)*arctan(1/2*(2*tanh(1/2*x)*b+2*a)/(-a^2+b^2)^(1/2))+
1/b*ln(tanh(1/2*x))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)/(b*cosh(x)+a*sinh(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more details)Is 4*a^2-4*b^2 positive or negative?
```

mupad [B] time = 1.78, size = 177, normalized size = 3.47

$$\frac{\ln\left(32\,a\,b-32\,b^2+32\,b^2\,\mathrm{e}^x-32\,a\,b\,\mathrm{e}^x\right)}{b}-\frac{\ln\left(32\,a\,b-32\,b^2-32\,b^2\,\mathrm{e}^x+32\,a\,b\,\mathrm{e}^x\right)}{b}-\frac{a\,\ln\left(32\,a\,b^2\,\mathrm{e}^x+32\,a^2\,b\,\mathrm{e}^x-\right.}{a^2\,b-}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)/(b*cosh(x) + a*sinh(x)),x)
```

```
[Out] log(32*a*b - 32*b^2 + 32*b^2*exp(x) - 32*a*b*exp(x))/b - log(32*a*b - 32*b^
2 - 32*b^2*exp(x) + 32*a*b*exp(x))/b - (a*log(32*a*b^2*exp(x) + 32*a^2*b*ex
p(x) - 32*a*b*(a^2 - b^2)^(1/2))*(a^2 - b^2)^(1/2))/(a^2*b - b^3) + (a*log(
32*a*b^2*exp(x) + 32*a^2*b*exp(x) + 32*a*b*(a^2 - b^2)^(1/2))*(a^2 - b^2)^(
1/2))/(a^2*b - b^3)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\coth(x)}{a\sinh(x)+b\cosh(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)/(b*cosh(x)+a*sinh(x)),x)
```

```
[Out] Integral(coth(x)/(a*sinh(x) + b*cosh(x)), x)
```

3.696 $\int \frac{\sinh(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx$

Optimal. Leaf size=66

$$-\frac{a}{\left(a^2-b^2\right)\left(a\cosh(x)+b\sinh(x)\right)}-\frac{b\tan^{-1}\left(\frac{a\sinh(x)+b\cosh(x)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{3/2}}$$

```
[Out] -b*arctan((b*cosh(x)+a*sinh(x))/(a^2-b^2)^(1/2))/(a^2-b^2)^(3/2)-a/(a^2-b^2
)/(a*cosh(x)+b*sinh(x))
```

Rubi [A] time = 0.06, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3154, 3074, 206}

$$-\frac{a}{\left(a^2-b^2\right)\left(a\cosh(x)+b\sinh(x)\right)}-\frac{b\tan^{-1}\left(\frac{a\sinh(x)+b\cosh(x)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[x]/(a*Cosh[x] + b*Sinh[x])^2,x]
```

```
[Out] -((b*ArcTan[(b*Cosh[x] + a*Sinh[x])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(3/2)) -
a/((a^2 - b^2)*(a*Cosh[x] + b*Sinh[x]))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3074

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> -Dist[d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d
*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3154

```
Int[((A_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])/((a_.) + cos[(d_.) + (e_.)*(x_)
]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])^2, x_Symbol] :> -Simp[(b*C + (a*C
- c*A)*Cos[d + e*x] + b*A*Sin[d + e*x])/(e*(a^2 - b^2 - c^2)*(a + b*Cos[d +
 e*x] + c*Sin[d + e*x])), x] + Dist[(a*A - c*C)/(a^2 - b^2 - c^2), Int[1/(a
 + b*Cos[d + e*x] + c*Sin[d + e*x]), x], x] /; FreeQ[{a, b, c, d, e, A, C},
```

```
x] && NeQ[a^2 - b^2 - c^2, 0] && NeQ[a*A - c*C, 0]
```

Rubi steps

$$\int \frac{\sinh(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx = -\frac{a}{\left(a^2-b^2\right)(a\cosh(x)+b\sinh(x))} - \frac{b\int \frac{1}{a\cosh(x)+b\sinh(x)}\,dx}{a^2-b^2}$$

$$= -\frac{a}{\left(a^2-b^2\right)(a\cosh(x)+b\sinh(x))} - \frac{(ib)\,\mathrm{Subst}\left(\int \frac{1}{a^2-b^2-x^2}\,dx, x, -ib\cosh(x) -$$

$$= -\frac{b\tan^{-1}\left(\frac{b\cosh(x)+a\sinh(x)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{3/2}} - \frac{a}{\left(a^2-b^2\right)(a\cosh(x)+b\sinh(x))}$$

Mathematica [A] time = 0.19, size = 125, normalized size = 1.89

$$-\frac{2b^2\sqrt{a+b}\,\sinh(x)\tan^{-1}\left(\frac{a\tanh\left(\frac{x}{2}\right)+b}{\sqrt{a-b}\,\sqrt{a+b}}\right)+2ab\sqrt{a+b}\,\cosh(x)\tan^{-1}\left(\frac{a\tanh\left(\frac{x}{2}\right)+b}{\sqrt{a-b}\,\sqrt{a+b}}\right)+a\sqrt{a-b}\,(a+b)}{(a-b)^{3/2}(a+b)^2(a\cosh(x)+b\sinh(x))}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[x]/(a*Cosh[x] + b*Sinh[x])^2,x]
```

```
[Out] -((a*Sqrt[a - b]*(a + b) + 2*a*b*Sqrt[a + b]*ArcTan[(b + a*Tanh[x/2])/(Sqrt
[a - b]*Sqrt[a + b])]*Cosh[x] + 2*b^2*Sqrt[a + b]*ArcTan[(b + a*Tanh[x/2])/
(Sqrt[a - b]*Sqrt[a + b])]*Sinh[x])/((a - b)^(3/2)*(a + b)^2*(a*Cosh[x] + b
*Sinh[x])))
```

fricas [B] time = 0.45, size = 594, normalized size = 9.00

$$\left[\frac{\left(\left(ab+b^2\right)\cosh(x)^2+2\left(ab+b^2\right)\cosh(x)\sinh(x)+\left(ab+b^2\right)\sinh(x)^2+ab-b^2\right)\sqrt{-a^2+b^2}\,\log\left(\frac{(a+b)\cosh(x)^2+2}{(a}\right.}{a^5-a^4b-2\,a^3b^2+2\,a^2b^3+ab^4-b^5+\left(a^5+a^4b-2\,a^3b^2-2\,a^2b^3+ab^4+b^5\right)\cosh(x)^2+2\left(a^5+a^4b-\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)/(a*cosh(x)+b*sinh(x))^2,x, algorithm="fricas")
```

```
[Out] [(((a*b + b^2)*cosh(x)^2 + 2*(a*b + b^2)*cosh(x)*sinh(x) + (a*b + b^2)*sinh
(x)^2 + a*b - b^2)*sqrt(-a^2 + b^2)*log(((a + b)*cosh(x)^2 + 2*(a + b)*cosh
```

```
(x)*sinh(x) + (a + b)*sinh(x)^2 - 2*sqrt(-a^2 + b^2)*(cosh(x) + sinh(x)) -
a + b)/((a + b)*cosh(x)^2 + 2*(a + b)*cosh(x)*sinh(x) + (a + b)*sinh(x)^2 +
 a - b)) - 2*(a^3 - a*b^2)*cosh(x) - 2*(a^3 - a*b^2)*sinh(x))/(a^5 - a^4*b
- 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5 + (a^5 + a^4*b - 2*a^3*b^2 - 2*a^2*b^
3 + a*b^4 + b^5)*cosh(x)^2 + 2*(a^5 + a^4*b - 2*a^3*b^2 - 2*a^2*b^3 + a*b^4
 + b^5)*cosh(x)*sinh(x) + (a^5 + a^4*b - 2*a^3*b^2 - 2*a^2*b^3 + a*b^4 + b^
5)*sinh(x)^2), 2*(((a*b + b^2)*cosh(x)^2 + 2*(a*b + b^2)*cosh(x)*sinh(x) +
(a*b + b^2)*sinh(x)^2 + a*b - b^2)*sqrt(a^2 - b^2)*arctan(sqrt(a^2 - b^2)/(
(a + b)*cosh(x) + (a + b)*sinh(x))) - (a^3 - a*b^2)*cosh(x) - (a^3 - a*b^2)
*sinh(x))/(a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5 + (a^5 + a^4*b
 - 2*a^3*b^2 - 2*a^2*b^3 + a*b^4 + b^5)*cosh(x)^2 + 2*(a^5 + a^4*b - 2*a^3*
b^2 - 2*a^2*b^3 + a*b^4 + b^5)*cosh(x)*sinh(x) + (a^5 + a^4*b - 2*a^3*b^2 -
 2*a^2*b^3 + a*b^4 + b^5)*sinh(x)^2)]
```

giac [A] time = 0.13, size = 72, normalized size = 1.09

$$-\frac{2\,b\arctan\left(\frac{ae^{x}+be^{x}}{\sqrt{a^{2}-b^{2}}}\right)}{\left(a^{2}-b^{2}\right)^{\frac{3}{2}}}-\frac{2\,ae^{x}}{\left(a^{2}-b^{2}\right)\left(ae^{(2\,x)}+be^{(2\,x)}+a-b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)/(a*cosh(x)+b*sinh(x))^2,x, algorithm="giac")
```

```
[Out] -2*b*arctan((a*e^x + b*e^x)/sqrt(a^2 - b^2))/(a^2 - b^2)^(3/2) - 2*a*e^x/((
a^2 - b^2)*(a*e^(2*x) + b*e^(2*x) + a - b))
```

maple [A] time = 0.26, size = 99, normalized size = 1.50

$$\frac{-8\tanh\left(\frac{x}{2}\right)b-8a}{\left(4a^{2}-4b^{2}\right)\left(a+2\tanh\left(\frac{x}{2}\right)b+a\left(\tanh^{2}\left(\frac{x}{2}\right)\right)\right)}-\frac{8b\arctan\left(\frac{2a\tanh\left(\frac{x}{2}\right)+2b}{2\sqrt{a^{2}-b^{2}}}\right)}{\left(4a^{2}-4b^{2}\right)\sqrt{a^{2}-b^{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)/(a*cosh(x)+b*sinh(x))^2,x)
```

```
[Out] 4*(-2*tanh(1/2*x)*b-2*a)/(4*a^2-4*b^2)/(a+2*tanh(1/2*x)*b+a*tanh(1/2*x)^2)-
8*b/(4*a^2-4*b^2)/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tanh(1/2*x)+2*b)/(a^2-b^2
)^(1/2))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)/(a*cosh(x)+b*sinh(x))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [B] time = 1.65, size = 183, normalized size = 2.77

$$-\frac{2\,\mathrm{atan}\left(\frac{\mathrm{e}^x\,\left(b^2\,\sqrt{a^6-3\,a^4\,b^2+3\,a^2\,b^4-b^6}+a\,b\,\sqrt{a^6-3\,a^4\,b^2+3\,a^2\,b^4-b^6}\right)}{a^4\,\sqrt{b^2}-2\,a^2\,{\left(b^2\right)}^{3/2}+b^4\,\sqrt{b^2}+a\,b\,{\left(b^2\right)}^{3/2}-a\,b^3\,\sqrt{b^2}}\right)\,\sqrt{b^2}}{\sqrt{a^6-3\,a^4\,b^2+3\,a^2\,b^4-b^6}}-\frac{2\,a\,\mathrm{e}^x}{\left(a+b\right)\,\left(a-b\right)\,\left(a-b+\mathrm{e}^{2\,x}\,\left(a+b\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)/(a*cosh(x) + b*sinh(x))^2,x)
```

```
[Out] - (2*atan((exp(x)*(b^2*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)^(1/2) + a*b*(a^6
 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)^(1/2)))/(a^4*(b^2)^(1/2) - 2*a^2*(b^2)^(3/2
) + b^4*(b^2)^(1/2) + a*b*(b^2)^(3/2) - a*b^3*(b^2)^(1/2)))*(b^2)^(1/2))/(a
^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)^(1/2) - (2*a*exp(x))/((a + b)*(a - b)*(a
- b + exp(2*x)*(a + b)))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)/(a*cosh(x)+b*sinh(x))**2,x)
```

```
[Out] Timed out
```

3.697 $\int \frac{\sinh^2(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx$

Optimal. Leaf size=68

$$\frac{x(a^2+b^2)}{(a^2-b^2)^2}-\frac{a}{(a^2-b^2)(a\coth(x)+b)}-\frac{2ab\log(a\cosh(x)+b\sinh(x))}{(a^2-b^2)^2}$$

```
[Out] (a^2+b^2)*x/(a^2-b^2)^2-a/(a^2-b^2)/(b+a*coth(x))-2*a*b*ln(a*cosh(x)+b*sinh
(x))/(a^2-b^2)^2
```

Rubi [A] time = 0.14, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3085, 3483, 3531, 3530}

$$\frac{x(a^2+b^2)}{(a^2-b^2)^2}-\frac{a}{(a^2-b^2)(a\coth(x)+b)}-\frac{2ab\log(a\cosh(x)+b\sinh(x))}{(a^2-b^2)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[x]^2/(a*Cosh[x] + b*Sinh[x])^2,x]
```

```
[Out] ((a^2 + b^2)*x)/(a^2 - b^2)^2 - a/((a^2 - b^2)*(b + a*Coth[x])) - (2*a*b*Lo
g[a*Cosh[x] + b*Sinh[x]])/(a^2 - b^2)^2
```

Rule 3085

```
Int[sin[(c_.) + (d_.)*(x_)]^(m_)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin
[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[(b + a*Cot[c + d*x])^n, x] /;
 FreeQ[{a, b, c, d}, x] && EqQ[m + n, 0] && IntegerQ[n] && NeQ[a^2 + b^2, 0
]
```

Rule 3483

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(a +
b*Tan[c + d*x])^(n + 1))/(d*(n + 1)*(a^2 + b^2)), x] + Dist[1/(a^2 + b^2),
Int[(a - b*Tan[c + d*x])*(a + b*Tan[c + d*x])^(n + 1), x], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 + b^2, 0] && LtQ[n, -1]
```

Rule 3530

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c*Log[RemoveContent[a*Cos[e + f*x] + b*Sin[e + f
*x], x]])/(b*f), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
```

```
NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

Rule 3531

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[((a*c + b*d)*x)/(a^2 + b^2), x] + Dist[(b*c - a
*d)/(a^2 + b^2), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && Ne
Q[a*c + b*d, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sinh^2(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx &= -\int \frac{1}{(-ib-ia\coth(x))^2}\,dx\\ &= -\frac{a}{(a^2-b^2)(b+a\coth(x))}-\frac{\int \frac{-ib+ia\coth(x)}{-ib-ia\coth(x)}\,dx}{a^2-b^2}\\ &= \frac{(a^2+b^2)x}{(a^2-b^2)^2}-\frac{a}{(a^2-b^2)(b+a\coth(x))}-\frac{(2iab)\int \frac{-a-b\coth(x)}{-ib-ia\coth(x)}\,dx}{(a^2-b^2)^2}\\ &= \frac{(a^2+b^2)x}{(a^2-b^2)^2}-\frac{a}{(a^2-b^2)(b+a\coth(x))}-\frac{2ab\log(a\cosh(x)+b\sinh(x))}{(a^2-b^2)^2}\end{aligned}$$

Mathematica [A] time = 0.28, size = 61, normalized size = 0.90

$$\frac{x\left(a^2+b^2\right)-\frac{a(a-b)(a+b)\sinh(x)}{a\cosh(x)+b\sinh(x)}-2ab\log(a\cosh(x)+b\sinh(x))}{(a-b)^2(a+b)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[x]^2/(a*Cosh[x] + b*Sinh[x])^2,x]
```

```
[Out] ((a^2 + b^2)*x - 2*a*b*Log[a*Cosh[x] + b*Sinh[x]] - (a*(a - b)*(a + b)*Sinh
[x])/(a*Cosh[x] + b*Sinh[x]))/((a - b)^2*(a + b)^2)
```

fricas [B] time = 0.44, size = 348, normalized size = 5.12

$$\frac{\left(a^3+3a^2b+3ab^2+b^3\right)x\cosh(x)^2+2\left(a^3+3a^2b+3ab^2+b^3\right)x\cosh(x)\sinh(x)+\left(a^3+3a^2b+3ab^2+b^3\right)x\,\mathrm{si}}{a^5-a^4b-2a^3b^2+2a^2b^3+ab^4-b^5+\left(a^5+a^4b-2a^3b^2-2a^2b^3+ab^4\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^2/(a*cosh(x)+b*sinh(x))^2,x, algorithm="fricas")
```

```
[Out] ((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*x*cosh(x)^2 + 2*(a^3 + 3*a^2*b + 3*a*b^2 +
 b^3)*x*cosh(x)*sinh(x) + (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*x*sinh(x)^2 + 2*a
^3 - 2*a^2*b + (a^3 + a^2*b - a*b^2 - b^3)*x - 2*(a^2*b - a*b^2 + (a^2*b +
a*b^2)*cosh(x)^2 + 2*(a^2*b + a*b^2)*cosh(x)*sinh(x) + (a^2*b + a*b^2)*sinh
(x)^2)*log(2*(a*cosh(x) + b*sinh(x))/(cosh(x) - sinh(x))))/(a^5 - a^4*b - 2
*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5 + (a^5 + a^4*b - 2*a^3*b^2 - 2*a^2*b^3 +
 a*b^4 + b^5)*cosh(x)^2 + 2*(a^5 + a^4*b - 2*a^3*b^2 - 2*a^2*b^3 + a*b^4 +
b^5)*cosh(x)*sinh(x) + (a^5 + a^4*b - 2*a^3*b^2 - 2*a^2*b^3 + a*b^4 + b^5)*
sinh(x)^2)
```

giac [A] time = 0.14, size = 113, normalized size = 1.66

$$-\frac{2\,ab\log\left(\left|ae^{(2\,x)}+be^{(2\,x)}+a-b\right|\right)}{a^4-2\,a^2b^2+b^4}+\frac{x}{a^2-2\,ab+b^2}+\frac{2\left(abe^{(2\,x)}+a^2-ab\right)}{\left(a^3-a^2b-ab^2+b^3\right)\left(ae^{(2\,x)}+be^{(2\,x)}+a-b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^2/(a*cosh(x)+b*sinh(x))^2,x, algorithm="giac")
```

```
[Out] -2*a*b*log(abs(a*e^(2*x) + b*e^(2*x) + a - b))/(a^4 - 2*a^2*b^2 + b^4) + x/
(a^2 - 2*a*b + b^2) + 2*(a*b*e^(2*x) + a^2 - a*b)/((a^3 - a^2*b - a*b^2 + b
^3)*(a*e^(2*x) + b*e^(2*x) + a - b))
```

maple [B] time = 0.26, size = 146, normalized size = 2.15

$$-\frac{\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{(a+b)^2}-\frac{2a^3\tanh\left(\frac{x}{2}\right)}{(a-b)^2(a+b)^2\left(a+2\tanh\left(\frac{x}{2}\right)b+a\left(\tanh^2\left(\frac{x}{2}\right)\right)\right)}+\frac{2a\tanh\left(\frac{x}{2}\right)b^2}{(a-b)^2(a+b)^2\left(a+2\tanh\left(\frac{x}{2}\right)b+a\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)^2/(a*cosh(x)+b*sinh(x))^2,x)
```

```
[Out] -1/(a+b)^2*ln(tanh(1/2*x)-1)-2*a^3/(a-b)^2/(a+b)^2*tanh(1/2*x)/(a+2*tanh(1/
2*x)*b+a*tanh(1/2*x)^2)+2*a/(a-b)^2/(a+b)^2*tanh(1/2*x)/(a+2*tanh(1/2*x)*b+
a*tanh(1/2*x)^2)*b^2-2*a/(a-b)^2/(a+b)^2*b*ln(a+2*tanh(1/2*x)*b+a*tanh(1/2*
x)^2)+1/(a-b)^2*ln(tanh(1/2*x)+1)
```

maxima [A] time = 0.50, size = 104, normalized size = 1.53

$$-\frac{2\,ab\log\left(-(a-b)e^{(-2\,x)}-a-b\right)}{a^4-2\,a^2b^2+b^4}-\frac{2\,a^2}{a^4-2\,a^2b^2+b^4+\left(a^4-2\,a^3b+2\,ab^3-b^4\right)e^{(-2\,x)}}+\frac{x}{a^2+2\,ab+b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^2/(a*cosh(x)+b*sinh(x))^2,x, algorithm="maxima")
```

```
[Out] -2*a*b*log(-(a - b)*e^(-2*x) - a - b)/(a^4 - 2*a^2*b^2 + b^4) - 2*a^2/(a^4
- 2*a^2*b^2 + b^4 + (a^4 - 2*a^3*b + 2*a*b^3 - b^4)*e^(-2*x)) + x/(a^2 + 2*
a*b + b^2)
```

mupad [B] time = 0.47, size = 108, normalized size = 1.59

$$\frac{\frac{a^2\cosh(x)}{b\left(a^2-b^2\right)}+\frac{a\,x\cosh(x)\left(a^2+b^2\right)}{\left(a^2-b^2\right)^2}+\frac{b\,x\sinh(x)\left(a^2+b^2\right)}{\left(a^2-b^2\right)^2}}{a\cosh(x)+b\sinh(x)}+\ln\left(a\cosh(x)+b\sinh(x)\right)\left(\frac{1}{2\,(a+b)^2}-\frac{1}{2\,(a-b)^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)^2/(a*cosh(x) + b*sinh(x))^2,x)
```

```
[Out] ((a^2*cosh(x))/(b*(a^2 - b^2)) + (a*x*cosh(x)*(a^2 + b^2))/(a^2 - b^2)^2 +
(b*x*sinh(x)*(a^2 + b^2))/(a^2 - b^2)^2)/(a*cosh(x) + b*sinh(x)) + log(a*co
sh(x) + b*sinh(x))*(1/(2*(a + b)^2) - 1/(2*(a - b)^2))
```

sympy [A] time = 1.42, size = 983, normalized size = 14.46

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)**2/(a*cosh(x)+b*sinh(x))**2,x)
```

```
[Out] Piecewise((zoo*x, Eq(a, 0) & Eq(b, 0)), (2*x*sinh(x)**2/(8*b**2*sinh(x)**2
- 16*b**2*sinh(x)*cosh(x) + 8*b**2*cosh(x)**2) - 4*x*sinh(x)*cosh(x)/(8*b**
2*sinh(x)**2 - 16*b**2*sinh(x)*cosh(x) + 8*b**2*cosh(x)**2) + 2*x*cosh(x)**
2/(8*b**2*sinh(x)**2 - 16*b**2*sinh(x)*cosh(x) + 8*b**2*cosh(x)**2) + 3*sin
h(x)**2/(8*b**2*sinh(x)**2 - 16*b**2*sinh(x)*cosh(x) + 8*b**2*cosh(x)**2) -
 cosh(x)**2/(8*b**2*sinh(x)**2 - 16*b**2*sinh(x)*cosh(x) + 8*b**2*cosh(x)**
2), Eq(a, -b)), (2*x*sinh(x)**2/(8*b**2*sinh(x)**2 + 16*b**2*sinh(x)*cosh(x
) + 8*b**2*cosh(x)**2) + 4*x*sinh(x)*cosh(x)/(8*b**2*sinh(x)**2 + 16*b**2*s
inh(x)*cosh(x) + 8*b**2*cosh(x)**2) + 2*x*cosh(x)**2/(8*b**2*sinh(x)**2 + 1
6*b**2*sinh(x)*cosh(x) + 8*b**2*cosh(x)**2) - 3*sinh(x)**2/(8*b**2*sinh(x)*
*2 + 16*b**2*sinh(x)*cosh(x) + 8*b**2*cosh(x)**2) + cosh(x)**2/(8*b**2*sinh
(x)**2 + 16*b**2*sinh(x)*cosh(x) + 8*b**2*cosh(x)**2), Eq(a, b)), ((x - sin
h(x)/cosh(x))/a**2, Eq(b, 0)), (x/b**2, Eq(a, 0)), (a**4*cosh(x)/(a**5*b*co
sh(x) + a**4*b**2*sinh(x) - 2*a**3*b**3*cosh(x) - 2*a**2*b**4*sinh(x) + a*b
**5*cosh(x) + b**6*sinh(x)) + a**3*b*x*cosh(x)/(a**5*b*cosh(x) + a**4*b**2*
sinh(x) - 2*a**3*b**3*cosh(x) - 2*a**2*b**4*sinh(x) + a*b**5*cosh(x) + b**6
```

```
*sinh(x)) + a**2*b**2*x*sinh(x)/(a**5*b*cosh(x) + a**4*b**2*sinh(x) - 2*a**
3*b**3*cosh(x) - 2*a**2*b**4*sinh(x) + a*b**5*cosh(x) + b**6*sinh(x)) - 2*a
**2*b**2*log(cosh(x) + b*sinh(x)/a)*cosh(x)/(a**5*b*cosh(x) + a**4*b**2*sin
h(x) - 2*a**3*b**3*cosh(x) - 2*a**2*b**4*sinh(x) + a*b**5*cosh(x) + b**6*si
nh(x)) - a**2*b**2*cosh(x)/(a**5*b*cosh(x) + a**4*b**2*sinh(x) - 2*a**3*b**
3*cosh(x) - 2*a**2*b**4*sinh(x) + a*b**5*cosh(x) + b**6*sinh(x)) + a*b**3*x
*cosh(x)/(a**5*b*cosh(x) + a**4*b**2*sinh(x) - 2*a**3*b**3*cosh(x) - 2*a**2
*b**4*sinh(x) + a*b**5*cosh(x) + b**6*sinh(x)) - 2*a*b**3*log(cosh(x) + b*s
inh(x)/a)*sinh(x)/(a**5*b*cosh(x) + a**4*b**2*sinh(x) - 2*a**3*b**3*cosh(x)
 - 2*a**2*b**4*sinh(x) + a*b**5*cosh(x) + b**6*sinh(x)) + b**4*x*sinh(x)/(a
**5*b*cosh(x) + a**4*b**2*sinh(x) - 2*a**3*b**3*cosh(x) - 2*a**2*b**4*sinh(
x) + a*b**5*cosh(x) + b**6*sinh(x)), True))
```

3.698 $\int \frac{\sinh^3(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx$

Optimal. Leaf size=195

$$-\frac{a^3}{b^3(a+b)^2\left(1-\tanh\left(\frac{x}{2}\right)\right)}+\frac{a^3}{b^3(a-b)^2\left(\tanh\left(\frac{x}{2}\right)+1\right)}+\frac{2a^2\left(a+b\tanh\left(\frac{x}{2}\right)\right)}{\left(a^2-b^2\right)^2\left(a\tanh^2\left(\frac{x}{2}\right)+a+2b\tanh\left(\frac{x}{2}\right)\right)}+\frac{3a^2b\tan^{-1}\left(\cdots\right.}{\left(a^2\cdots\right.}$$

```
[Out] 3*a^2*b*arctan((b*cosh(x)+a*sinh(x))/(a^2-b^2)^(1/2))/(a^2-b^2)^(5/2)+(2*a^
2+b^2)*cosh(x)/(-a^2*b^2+b^4)+a*(a^2+2*b^2)*sinh(x)/b^3/(a^2-b^2)-a^3/b^3/(
a+b)^2/(1-tanh(1/2*x))+a^3/(a-b)^2/b^3/(1+tanh(1/2*x))+2*a^2*(a+b*tanh(1/2*
x))/(a^2-b^2)^2/(a+2*b*tanh(1/2*x)+a*tanh(1/2*x)^2)
```

Rubi [A] time = 1.22, antiderivative size = 301, normalized size of antiderivative = 1.54, number of steps used = 16, number of rules used = 10, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {4401, 2637, 2638, 6742, 638, 618, 204, 3100, 3074, 206}

$$\frac{3a^3\sinh(x)}{b^3\left(a^2-b^2\right)}-\frac{3a^2\cosh(x)}{b^2\left(a^2-b^2\right)}+\frac{2a^2\left(a+b\tanh\left(\frac{x}{2}\right)\right)}{\left(a^2-b^2\right)^2\left(a\tanh^2\left(\frac{x}{2}\right)+a+2b\tanh\left(\frac{x}{2}\right)\right)}-\frac{a^3}{b^3(a+b)^2\left(1-\tanh\left(\frac{x}{2}\right)\right)}+\frac{a^3}{b^3(a-b)^2\left(\tan\cdots\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[x]^3/(a*Cosh[x] + b*Sinh[x])^2,x]
```

```
[Out] (-3*a^2*ArcTan[(b*Cosh[x] + a*Sinh[x])/Sqrt[a^2 - b^2]])/(b*(a^2 - b^2)^(3/
2)) + (2*a^2*b*ArcTan[(b + a*Tanh[x/2])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(5/2)
 + (2*a^2*(3*a^2 - b^2)*ArcTan[(b + a*Tanh[x/2])/Sqrt[a^2 - b^2]])/(b*(a^2
- b^2)^(5/2)) + Cosh[x]/b^2 - (3*a^2*Cosh[x])/(b^2*(a^2 - b^2)) - (2*a*Sinh
[x])/b^3 + (3*a^3*Sinh[x])/(b^3*(a^2 - b^2)) - a^3/(b^3*(a + b)^2*(1 - Tanh
[x/2])) + a^3/((a - b)^2*b^3*(1 + Tanh[x/2])) + (2*a^2*(a + b*Tanh[x/2]))/(
(a^2 - b^2)^2*(a + 2*b*Tanh[x/2] + a*Tanh[x/2]^2))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 638

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)*(a + b*x + c*x^2)^(p + 1))/((p +
 1)*(b^2 - 4*a*c)), x] - Dist[((2*p + 3)*(2*c*d - b*e))/((p + 1)*(b^2 - 4*a
*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
 NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3074

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> -Dist[d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d
*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3100

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_)/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin
[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[(b*Cos[c + d*x]^(m - 1))/(d*(a^2 +
 b^2)*(m - 1)), x] + (Dist[a/(a^2 + b^2), Int[Cos[c + d*x]^(m - 1), x], x]
+ Dist[b^2/(a^2 + b^2), Int[Cos[c + d*x]^(m - 2)/(a*Cos[c + d*x] + b*Sin[c
+ d*x]), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && GtQ[m, 1
]
```

Rule 4401

```
Int[u_, x_Symbol] :> With[{v = ExpandTrig[u, x]}, Int[v, x] /; SumQ[v]] /;
 !InertTrigFreeQ[u]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{\sinh^3(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx = i\int\left(\frac{2ia\cosh(x)}{b^3} - \frac{i\sinh(x)}{b^2} - \frac{ia^3\cosh^3(x)}{b^3(ia\cosh(x)+ib\sinh(x))^2} - \frac{3ia^2\cosh^2(}{b^3(a\cosh(x)+b}\right.$$

$$= -\frac{(2a)\int\cosh(x)\,dx}{b^3} + \frac{\left(3a^2\right)\int\frac{\cosh^2(x)}{a\cosh(x)+b\sinh(x)}\,dx}{b^3} + \frac{a^3\int\frac{\cosh^3(x)}{(ia\cosh(x)+ib\sinh(x))^2}\,dx}{b^3}$$

$$= \frac{\cosh(x)}{b^2} - \frac{3a^2\cosh(x)}{b^2\left(a^2-b^2\right)} - \frac{2a\sinh(x)}{b^3} + \frac{\left(2a^3\right)\text{Subst}\left(\int\frac{\left(-1-x^2\right)^3}{\left(1-x^2\right)^2\left(a+2bx+ax^2\right)^2}\,dx, x, \right.}{b^3}$$

$$= \frac{\cosh(x)}{b^2} - \frac{3a^2\cosh(x)}{b^2\left(a^2-b^2\right)} - \frac{2a\sinh(x)}{b^3} + \frac{3a^3\sinh(x)}{b^3\left(a^2-b^2\right)} + \frac{\left(2a^3\right)\text{Subst}\left(\int\left(-\frac{1}{2(a+b)^2(}\right.\right.}{}$$

$$= -\frac{3a^2\tan^{-1}\left(\frac{b\cosh(x)+a\sinh(x)}{\sqrt{a^2-b^2}}\right)}{b\left(a^2-b^2\right)^{3/2}} + \frac{\cosh(x)}{b^2} - \frac{3a^2\cosh(x)}{b^2\left(a^2-b^2\right)} - \frac{2a\sinh(x)}{b^3} + \frac{3a^3\sin}{b^3\left(a^2-\right.}$$

$$= -\frac{3a^2\tan^{-1}\left(\frac{b\cosh(x)+a\sinh(x)}{\sqrt{a^2-b^2}}\right)}{b\left(a^2-b^2\right)^{3/2}} + \frac{\cosh(x)}{b^2} - \frac{3a^2\cosh(x)}{b^2\left(a^2-b^2\right)} - \frac{2a\sinh(x)}{b^3} + \frac{3a^3\sin}{b^3\left(a^2-\right.}$$

$$= -\frac{3a^2\tan^{-1}\left(\frac{b\cosh(x)+a\sinh(x)}{\sqrt{a^2-b^2}}\right)}{b\left(a^2-b^2\right)^{3/2}} + \frac{2a^2\left(3a^2-b^2\right)\tan^{-1}\left(\frac{b+a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{b\left(a^2-b^2\right)^{5/2}} + \frac{\cosh(x)}{b^2} -$$

$$= -\frac{3a^2\tan^{-1}\left(\frac{b\cosh(x)+a\sinh(x)}{\sqrt{a^2-b^2}}\right)}{b\left(a^2-b^2\right)^{3/2}} + \frac{2a^2b\tan^{-1}\left(\frac{b+a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{5/2}} + \frac{2a^2\left(3a^2-b^2\right)\tan^{-1}}{b\left(a^2-b^2\right)}$$

Mathematica [A] time = 0.46, size = 205, normalized size = 1.05

$$\frac{-b\cosh(x)\left((a-b)^{3/2}(a+b)^2\sinh(x)-6a^3\sqrt{a+b}\,\tan^{-1}\left(\frac{a\tanh\left(\frac{x}{2}\right)+b}{\sqrt{a-b}\,\sqrt{a+b}}\right)\right)+a\left(a^2\sqrt{a-b}\,(a+b)-2b^2\sqrt{a-b}\,(a+b\right.}{(a-b)^{5/2}(a+b)^3(a\cosh(x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[x]^3/(a*Cosh[x] + b*Sinh[x])^2,x]
```

```
[Out] (a*Sqrt[a - b]*(a^3 + a^2*b + a*b^2 + b^3)*Cosh[x]^2 - b*Cosh[x]*(-6*a^3*Sq
rt[a + b]*ArcTan[(b + a*Tanh[x/2])/(Sqrt[a - b]*Sqrt[a + b])] + (a - b)^(3/
2)*(a + b)^2*Sinh[x]) + a*(a^2*Sqrt[a - b]*(a + b) + 6*a*b^2*Sqrt[a + b]*Ar
cTan[(b + a*Tanh[x/2])/(Sqrt[a - b]*Sqrt[a + b])]*Sinh[x] - 2*Sqrt[a - b]*b
^2*(a + b)*Sinh[x]^2))/((a - b)^(5/2)*(a + b)^3*(a*Cosh[x] + b*Sinh[x]))
```

fricas [B] time = 0.49, size = 1633, normalized size = 8.37

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^3/(a*cosh(x)+b*sinh(x))^2,x, algorithm="fricas")
```

```
[Out] [1/2*(a^5 + a^4*b - 2*a^3*b^2 - 2*a^2*b^3 + a*b^4 + b^5 + (a^5 - a^4*b - 2*
a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^4 + 4*(a^5 - a^4*b - 2*a^3*b^2 +
 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)*sinh(x)^3 + (a^5 - a^4*b - 2*a^3*b^2 + 2*
a^2*b^3 + a*b^4 - b^5)*sinh(x)^4 + 6*(a^5 - a*b^4)*cosh(x)^2 + 6*(a^5 - a*b
^4 + (a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^2)*sinh(x)
^2 - 6*((a^3*b + a^2*b^2)*cosh(x)^3 + 3*(a^3*b + a^2*b^2)*cosh(x)*sinh(x)^2
 + (a^3*b + a^2*b^2)*sinh(x)^3 + (a^3*b - a^2*b^2)*cosh(x) + (a^3*b - a^2*b
^2 + 3*(a^3*b + a^2*b^2)*cosh(x)^2)*sinh(x))*sqrt(-a^2 + b^2)*log(((a + b)*
cosh(x)^2 + 2*(a + b)*cosh(x)*sinh(x) + (a + b)*sinh(x)^2 - 2*sqrt(-a^2 + b
^2)*(cosh(x) + sinh(x)) - a + b)/((a + b)*cosh(x)^2 + 2*(a + b)*cosh(x)*sin
h(x) + (a + b)*sinh(x)^2 + a - b)) + 4*((a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^
3 + a*b^4 - b^5)*cosh(x)^3 + 3*(a^5 - a*b^4)*cosh(x))*sinh(x))/((a^7 + a^6*
b - 3*a^5*b^2 - 3*a^4*b^3 + 3*a^3*b^4 + 3*a^2*b^5 - a*b^6 - b^7)*cosh(x)^3
+ 3*(a^7 + a^6*b - 3*a^5*b^2 - 3*a^4*b^3 + 3*a^3*b^4 + 3*a^2*b^5 - a*b^6 -
b^7)*cosh(x)*sinh(x)^2 + (a^7 + a^6*b - 3*a^5*b^2 - 3*a^4*b^3 + 3*a^3*b^4 +
 3*a^2*b^5 - a*b^6 - b^7)*sinh(x)^3 + (a^7 - a^6*b - 3*a^5*b^2 + 3*a^4*b^3
+ 3*a^3*b^4 - 3*a^2*b^5 - a*b^6 + b^7)*cosh(x) + (a^7 - a^6*b - 3*a^5*b^2 +
 3*a^4*b^3 + 3*a^3*b^4 - 3*a^2*b^5 - a*b^6 + b^7 + 3*(a^7 + a^6*b - 3*a^5*b
^2 - 3*a^4*b^3 + 3*a^3*b^4 + 3*a^2*b^5 - a*b^6 - b^7)*cosh(x)^2)*sinh(x)),
1/2*(a^5 + a^4*b - 2*a^3*b^2 - 2*a^2*b^3 + a*b^4 + b^5 + (a^5 - a^4*b - 2*a
^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^4 + 4*(a^5 - a^4*b - 2*a^3*b^2 +
2*a^2*b^3 + a*b^4 - b^5)*cosh(x)*sinh(x)^3 + (a^5 - a^4*b - 2*a^3*b^2 + 2*a
```

```
^2*b^3 + a*b^4 - b^5)*sinh(x)^4 + 6*(a^5 - a*b^4)*cosh(x)^2 + 6*(a^5 - a*b^
4 + (a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^2)*sinh(x)^
2 - 12*((a^3*b + a^2*b^2)*cosh(x)^3 + 3*(a^3*b + a^2*b^2)*cosh(x)*sinh(x)^2
 + (a^3*b + a^2*b^2)*sinh(x)^3 + (a^3*b - a^2*b^2)*cosh(x) + (a^3*b - a^2*b
^2 + 3*(a^3*b + a^2*b^2)*cosh(x)^2)*sinh(x))*sqrt(a^2 - b^2)*arctan(sqrt(a^
2 - b^2)/((a + b)*cosh(x) + (a + b)*sinh(x))) + 4*((a^5 - a^4*b - 2*a^3*b^2
 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^3 + 3*(a^5 - a*b^4)*cosh(x))*sinh(x))/(
(a^7 + a^6*b - 3*a^5*b^2 - 3*a^4*b^3 + 3*a^3*b^4 + 3*a^2*b^5 - a*b^6 - b^7)
*cosh(x)^3 + 3*(a^7 + a^6*b - 3*a^5*b^2 - 3*a^4*b^3 + 3*a^3*b^4 + 3*a^2*b^5
 - a*b^6 - b^7)*cosh(x)*sinh(x)^2 + (a^7 + a^6*b - 3*a^5*b^2 - 3*a^4*b^3 +
3*a^3*b^4 + 3*a^2*b^5 - a*b^6 - b^7)*sinh(x)^3 + (a^7 - a^6*b - 3*a^5*b^2 +
 3*a^4*b^3 + 3*a^3*b^4 - 3*a^2*b^5 - a*b^6 + b^7)*cosh(x) + (a^7 - a^6*b -
3*a^5*b^2 + 3*a^4*b^3 + 3*a^3*b^4 - 3*a^2*b^5 - a*b^6 + b^7 + 3*(a^7 + a^6*
b - 3*a^5*b^2 - 3*a^4*b^3 + 3*a^3*b^4 + 3*a^2*b^5 - a*b^6 - b^7)*cosh(x)^2)
*sinh(x))]
```

giac [A] time = 0.13, size = 174, normalized size = 0.89

$$\frac{6\,a^2b\arctan\left(\frac{ae^x+be^x}{\sqrt{a^2-b^2}}\right)}{\left(a^4-2\,a^2b^2+b^4\right)\sqrt{a^2-b^2}}+\frac{e^x}{2\left(a^2+2\,ab+b^2\right)}+\frac{5\,a^3e^{(2\,x)}+3\,a^2be^{(2\,x)}+3\,ab^2e^{(2\,x)}+b^3e^{(2\,x)}+a^3+a^2b-ab^2-b^3}{2\left(a^4-2\,a^2b^2+b^4\right)\left(ae^{(3\,x)}+be^{(3\,x)}+ae^x-be^x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^3/(a*cosh(x)+b*sinh(x))^2,x, algorithm="giac")
```

```
[Out] 6*a^2*b*arctan((a*e^x + b*e^x)/sqrt(a^2 - b^2))/((a^4 - 2*a^2*b^2 + b^4)*sq
rt(a^2 - b^2)) + 1/2*e^x/(a^2 + 2*a*b + b^2) + 1/2*(5*a^3*e^(2*x) + 3*a^2*b
*e^(2*x) + 3*a*b^2*e^(2*x) + b^3*e^(2*x) + a^3 + a^2*b - a*b^2 - b^3)/((a^4
 - 2*a^2*b^2 + b^4)*(a*e^(3*x) + b*e^(3*x) + a*e^x - b*e^x))
```

maple [A] time = 0.28, size = 164, normalized size = 0.84

$$-\frac{1}{\left(a+b\right)^2\left(\tanh\left(\frac{x}{2}\right)-1\right)}+\frac{2a^2\tanh\left(\frac{x}{2}\right)b}{\left(a-b\right)^2\left(a+b\right)^2\left(a+2\tanh\left(\frac{x}{2}\right)b+a\left(\tanh^2\left(\frac{x}{2}\right)\right)\right)}+\frac{2a^3}{\left(a-b\right)^2\left(a+b\right)^2\left(a+2\tanh\left(\frac{x}{2}\right)b\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)^3/(a*cosh(x)+b*sinh(x))^2,x)
```

```
[Out] -1/(a+b)^2/(tanh(1/2*x)-1)+2*a^2/(a-b)^2/(a+b)^2/(a+2*tanh(1/2*x)*b+a*tanh(
1/2*x)^2)*tanh(1/2*x)*b+2*a^3/(a-b)^2/(a+b)^2/(a+2*tanh(1/2*x)*b+a*tanh(1/2
*x)^2)+6*a^2/(a-b)^2/(a+b)^2*b/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tanh(1/2*x)+
2*b)/(a^2-b^2)^(1/2))+1/(a-b)^2/(tanh(1/2*x)+1)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^3/(a*cosh(x)+b*sinh(x))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [B] time = 1.79, size = 255, normalized size = 1.31

$$\frac{e^{-x}}{2\,(a-b)^2}+\frac{e^{x}}{2\,(a+b)^2}+\frac{6\,\mathrm{atan}\left(\frac{a^2\,b\,e^x\,\sqrt{a^{10}-5\,a^8\,b^2+10\,a^6\,b^4-10\,a^4\,b^6+5\,a^2\,b^8-b^{10}}}{a^5\,\sqrt{a^4\,b^2}-b^5\,\sqrt{a^4\,b^2}+2\,a^2\,b^3\,\sqrt{a^4\,b^2}-2\,a^3\,b^2\,\sqrt{a^4\,b^2}+a\,b^4\,\sqrt{a^4\,b^2}-a^4\,b\,\sqrt{a^4\,b^2}}\right)\,\sqrt{a^4\,b^2}}{\sqrt{a^{10}-5\,a^8\,b^2+10\,a^6\,b^4-10\,a^4\,b^6+5\,a^2\,b^8-b^{10}}}+\frac{}{(a+b)^2\,(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)^3/(a*cosh(x) + b*sinh(x))^2,x)
```

```
[Out] exp(-x)/(2*(a - b)^2) + exp(x)/(2*(a + b)^2) + (6*atan((a^2*b*exp(x)*(a^10
- b^10 + 5*a^2*b^8 - 10*a^4*b^6 + 10*a^6*b^4 - 5*a^8*b^2)^(1/2))/(a^5*(a^4*
b^2)^(1/2) - b^5*(a^4*b^2)^(1/2) + 2*a^2*b^3*(a^4*b^2)^(1/2) - 2*a^3*b^2*(a
^4*b^2)^(1/2) + a*b^4*(a^4*b^2)^(1/2) - a^4*b*(a^4*b^2)^(1/2)))*(a^4*b^2)^(
1/2))/(a^10 - b^10 + 5*a^2*b^8 - 10*a^4*b^6 + 10*a^6*b^4 - 5*a^8*b^2)^(1/2)
 + (2*a^3*exp(x))/((a + b)^2*(a - b)^2*(a - b + exp(2*x)*(a + b)))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)**3/(a*cosh(x)+b*sinh(x))**2,x)
```

```
[Out] Timed out
```

3.699 $\int \frac{\cosh(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx$

Optimal. Leaf size=64

$$\frac{b}{\left(a^2-b^2\right)\left(a\cosh(x)+b\sinh(x)\right)}+\frac{a\tan^{-1}\left(\frac{a\sinh(x)+b\cosh(x)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{3/2}}$$

```
[Out] a*arctan((b*cosh(x)+a*sinh(x))/(a^2-b^2)^(1/2))/(a^2-b^2)^(3/2)+b/(a^2-b^2)
/(a*cosh(x)+b*sinh(x))
```

Rubi [A] time = 0.06, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3155, 3074, 206}

$$\frac{b}{\left(a^2-b^2\right)\left(a\cosh(x)+b\sinh(x)\right)}+\frac{a\tan^{-1}\left(\frac{a\sinh(x)+b\cosh(x)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]/(a*Cosh[x] + b*Sinh[x])^2,x]
```

```
[Out] (a*ArcTan[(b*Cosh[x] + a*Sinh[x])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(3/2) + b/(
(a^2 - b^2)*(a*Cosh[x] + b*Sinh[x]))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3074

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> -Dist[d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d
*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3155

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.))/((a_.) + cos[(d_.) + (e_.)*(x_)
]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])^2, x_Symbol] :> Simp[(c*B + c*A*Co
s[d + e*x] + (a*B - b*A)*Sin[d + e*x])/(e*(a^2 - b^2 - c^2)*(a + b*Cos[d +
e*x] + c*Sin[d + e*x])), x] + Dist[(a*A - b*B)/(a^2 - b^2 - c^2), Int[1/(a
+ b*Cos[d + e*x] + c*Sin[d + e*x]), x], x] /; FreeQ[{a, b, c, d, e, A, B},
```

```
x] && NeQ[a^2 - b^2 - c^2, 0] && NeQ[a*A - b*B, 0]
```

Rubi steps

$$\int \frac{\cosh(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx = \frac{b}{\left(a^2-b^2\right)(a\cosh(x)+b\sinh(x))} + \frac{a\int \frac{1}{a\cosh(x)+b\sinh(x)}\,dx}{a^2-b^2}$$

$$= \frac{b}{\left(a^2-b^2\right)(a\cosh(x)+b\sinh(x))} + \frac{(ia)\,\text{Subst}\left(\int \frac{1}{a^2-b^2-x^2}\,dx, x, -ib\cosh(x) -\right.}{a^2-b^2}$$

$$= \frac{a\tan^{-1}\left(\frac{b\cosh(x)+a\sinh(x)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{3/2}} + \frac{b}{\left(a^2-b^2\right)(a\cosh(x)+b\sinh(x))}$$

Mathematica [A] time = 0.13, size = 124, normalized size = 1.94

$$\frac{2a^2\sqrt{a+b}\,\cosh(x)\tan^{-1}\left(\frac{a\tanh\left(\frac{x}{2}\right)+b}{\sqrt{a-b}\,\sqrt{a+b}}\right)+2ab\sqrt{a+b}\,\sinh(x)\tan^{-1}\left(\frac{a\tanh\left(\frac{x}{2}\right)+b}{\sqrt{a-b}\,\sqrt{a+b}}\right)+b\sqrt{a-b}\,(a+b)}{(a-b)^{3/2}(a+b)^2(a\cosh(x)+b\sinh(x))}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]/(a*Cosh[x] + b*Sinh[x])^2,x]
```

```
[Out] (Sqrt[a - b]*b*(a + b) + 2*a^2*Sqrt[a + b]*ArcTan[(b + a*Tanh[x/2])/(Sqrt[a
 - b]*Sqrt[a + b])]*Cosh[x] + 2*a*b*Sqrt[a + b]*ArcTan[(b + a*Tanh[x/2])/(S
qrt[a - b]*Sqrt[a + b])]*Sinh[x])/((a - b)^(3/2)*(a + b)^2*(a*Cosh[x] + b*S
inh[x]))
```

fricas [B] time = 0.46, size = 596, normalized size = 9.31

$$\left[\frac{\left(\left(a^2+ab\right)\cosh(x)^2+2\left(a^2+ab\right)\cosh(x)\sinh(x)+\left(a^2+ab\right)\sinh(x)^2+a^2-ab\right)\sqrt{-a^2+b^2}\,\log\left(\frac{(a+b)\cosh(x)^2+\cdots}{\cdots}\right.}{a^5-a^4b-2a^3b^2+2a^2b^3+ab^4-b^5+\left(a^5+a^4b-2a^3b^2-2a^2b^3+ab^4+b^5\right)\cosh(x)^2+2\left(a^5+a^4b\cdots\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)/(a*cosh(x)+b*sinh(x))^2,x, algorithm="fricas")
```

```
[Out] [(((a^2 + a*b)*cosh(x)^2 + 2*(a^2 + a*b)*cosh(x)*sinh(x) + (a^2 + a*b)*sinh
(x)^2 + a^2 - a*b)*sqrt(-a^2 + b^2)*log(((a + b)*cosh(x)^2 + 2*(a + b)*cosh
```

```
(x)*sinh(x) + (a + b)*sinh(x)^2 + 2*sqrt(-a^2 + b^2)*(cosh(x) + sinh(x)) -
a + b)/((a + b)*cosh(x)^2 + 2*(a + b)*cosh(x)*sinh(x) + (a + b)*sinh(x)^2 +
 a - b)) + 2*(a^2*b - b^3)*cosh(x) + 2*(a^2*b - b^3)*sinh(x))/(a^5 - a^4*b
- 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5 + (a^5 + a^4*b - 2*a^3*b^2 - 2*a^2*b^
3 + a*b^4 + b^5)*cosh(x)^2 + 2*(a^5 + a^4*b - 2*a^3*b^2 - 2*a^2*b^3 + a*b^4
 + b^5)*cosh(x)*sinh(x) + (a^5 + a^4*b - 2*a^3*b^2 - 2*a^2*b^3 + a*b^4 + b^
5)*sinh(x)^2), -2*(((a^2 + a*b)*cosh(x)^2 + 2*(a^2 + a*b)*cosh(x)*sinh(x) +
 (a^2 + a*b)*sinh(x)^2 + a^2 - a*b)*sqrt(a^2 - b^2)*arctan(sqrt(a^2 - b^2)/
((a + b)*cosh(x) + (a + b)*sinh(x))) - (a^2*b - b^3)*cosh(x) - (a^2*b - b^3
)*sinh(x))/(a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5 + (a^5 + a^4*
b - 2*a^3*b^2 - 2*a^2*b^3 + a*b^4 + b^5)*cosh(x)^2 + 2*(a^5 + a^4*b - 2*a^3
*b^2 - 2*a^2*b^3 + a*b^4 + b^5)*cosh(x)*sinh(x) + (a^5 + a^4*b - 2*a^3*b^2
- 2*a^2*b^3 + a*b^4 + b^5)*sinh(x)^2)]
```

giac [A] time = 0.13, size = 72, normalized size = 1.12

$$\frac{2\,a\arctan\left(\frac{ae^{x}+be^{x}}{\sqrt{a^{2}-b^{2}}}\right)}{\left(a^{2}-b^{2}\right)^{\frac{3}{2}}}+\frac{2\,be^{x}}{\left(a^{2}-b^{2}\right)\left(ae^{(2\,x)}+be^{(2\,x)}+a-b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)/(a*cosh(x)+b*sinh(x))^2,x, algorithm="giac")
```

```
[Out] 2*a*arctan((a*e^x + b*e^x)/sqrt(a^2 - b^2))/(a^2 - b^2)^(3/2) + 2*b*e^x/((a
^2 - b^2)*(a*e^(2*x) + b*e^(2*x) + a - b))
```

maple [A] time = 0.30, size = 98, normalized size = 1.53

$$\frac{\frac{2b^{2}\tanh\left(\frac{x}{2}\right)}{a\left(a^{2}-b^{2}\right)}+\frac{2b}{a^{2}-b^{2}}}{a+2\tanh\left(\frac{x}{2}\right)b+a\left(\tanh^{2}\left(\frac{x}{2}\right)\right)}+\frac{2a\arctan\left(\frac{2a\tanh\left(\frac{x}{2}\right)+2b}{2\sqrt{a^{2}-b^{2}}}\right)}{\left(a^{2}-b^{2}\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)/(a*cosh(x)+b*sinh(x))^2,x)
```

```
[Out] 2*(b^2/a/(a^2-b^2)*tanh(1/2*x)+b/(a^2-b^2))/(a+2*tanh(1/2*x)*b+a*tanh(1/2*x
)^2)+2*a/(a^2-b^2)^(3/2)*arctan(1/2*(2*a*tanh(1/2*x)+2*b)/(a^2-b^2)^(1/2))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)/(a*cosh(x)+b*sinh(x))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [**B**] time = 1.71, size = 183, normalized size = 2.86

$$\frac{2\,\mathrm{atan}\left(\frac{e^x\,\left(a^2\,\sqrt{a^6-3\,a^4\,b^2+3\,a^2\,b^4-b^6}+a\,b\,\sqrt{a^6-3\,a^4\,b^2+3\,a^2\,b^4-b^6}\right)}{a^4\,\sqrt{a^2}-2\,b^2\,{\left(a^2\right)}^{3/2}+b^4\,\sqrt{a^2}+a\,b\,{\left(a^2\right)}^{3/2}-a^3\,b\,\sqrt{a^2}}\right)\,\sqrt{a^2}}{\sqrt{a^6-3\,a^4\,b^2+3\,a^2\,b^4-b^6}}+\frac{2\,b\,e^x}{\left(a+b\right)\,\left(a-b\right)\,\left(a-b+e^{2\,x}\,\left(a+b\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)/(a*cosh(x) + b*sinh(x))^2,x)
```

```
[Out] (2*atan((exp(x)*(a^2*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)^(1/2) + a*b*(a^6 -
 b^6 + 3*a^2*b^4 - 3*a^4*b^2)^(1/2)))/(a^4*(a^2)^(1/2) - 2*b^2*(a^2)^(3/2)
+ b^4*(a^2)^(1/2) + a*b*(a^2)^(3/2) - a^3*b*(a^2)^(1/2)))*(a^2)^(1/2))/(a^6
 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)^(1/2) + (2*b*exp(x))/((a + b)*(a - b)*(a -
b + exp(2*x)*(a + b)))
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)/(a*cosh(x)+b*sinh(x))**2,x)
```

```
[Out] Timed out
```

3.700 $\int \frac{\cosh^2(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx$

Optimal. Leaf size=67

$$\frac{x(a^2+b^2)}{(a^2-b^2)^2}+\frac{b}{(a^2-b^2)(a+b\tanh(x))}-\frac{2ab\log(a\cosh(x)+b\sinh(x))}{(a^2-b^2)^2}$$

```
[Out] (a^2+b^2)*x/(a^2-b^2)^2-2*a*b*ln(a*cosh(x)+b*sinh(x))/(a^2-b^2)^2+b/(a^2-b^
2)/(a+b*tanh(x))
```

Rubi [A] time = 0.13, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3086, 3483, 3531, 3530}

$$\frac{x(a^2+b^2)}{(a^2-b^2)^2}+\frac{b}{(a^2-b^2)(a+b\tanh(x))}-\frac{2ab\log(a\cosh(x)+b\sinh(x))}{(a^2-b^2)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]^2/(a*Cosh[x] + b*Sinh[x])^2,x]
```

```
[Out] ((a^2 + b^2)*x)/(a^2 - b^2)^2 - (2*a*b*Log[a*Cosh[x] + b*Sinh[x]])/(a^2 - b
^2)^2 + b/((a^2 - b^2)*(a + b*Tanh[x]))
```

Rule 3086

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin
[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[(a + b*Tan[c + d*x])^n, x] /;
 FreeQ[{a, b, c, d}, x] && EqQ[m + n, 0] && IntegerQ[n] && NeQ[a^2 + b^2, 0
]
```

Rule 3483

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(a +
b*Tan[c + d*x])^(n + 1))/(d*(n + 1)*(a^2 + b^2)), x] + Dist[1/(a^2 + b^2),
Int[(a - b*Tan[c + d*x])*(a + b*Tan[c + d*x])^(n + 1), x], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 + b^2, 0] && LtQ[n, -1]
```

Rule 3530

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c*Log[RemoveContent[a*Cos[e + f*x] + b*Sin[e + f
*x], x]])/(b*f), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
```

```
NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

Rule 3531

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[((a*c + b*d)*x)/(a^2 + b^2), x] + Dist[(b*c - a
*d)/(a^2 + b^2), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && Ne
Q[a*c + b*d, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\cosh^2(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx &= \int \frac{1}{(a+b\tanh(x))^2}\,dx\\ &= \frac{b}{\left(a^2-b^2\right)(a+b\tanh(x))}+\frac{\int \frac{a-b\tanh(x)}{a+b\tanh(x)}\,dx}{a^2-b^2}\\ &= \frac{\left(a^2+b^2\right)x}{\left(a^2-b^2\right)^2}+\frac{b}{\left(a^2-b^2\right)(a+b\tanh(x))}-\frac{(2iab)\int \frac{-ib-ia\tanh(x)}{a+b\tanh(x)}\,dx}{\left(a^2-b^2\right)^2}\\ &= \frac{\left(a^2+b^2\right)x}{\left(a^2-b^2\right)^2}-\frac{2ab\log(a\cosh(x)+b\sinh(x))}{\left(a^2-b^2\right)^2}+\frac{b}{\left(a^2-b^2\right)(a+b\tanh(x))}\end{aligned}$$

Mathematica [A] time = 0.32, size = 66, normalized size = 0.99

$$\frac{x\left(a^2+b^2\right)+\frac{b^2\left(b^2-a^2\right)\sinh(x)}{a(a\cosh(x)+b\sinh(x))}-2ab\log(a\cosh(x)+b\sinh(x))}{(a-b)^2(a+b)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]^2/(a*Cosh[x] + b*Sinh[x])^2,x]
```

```
[Out] ((a^2 + b^2)*x - 2*a*b*Log[a*Cosh[x] + b*Sinh[x]] + (b^2*(-a^2 + b^2)*Sinh[
x])/(a*(a*Cosh[x] + b*Sinh[x])))/((a - b)^2*(a + b)^2)
```

fricas [B] time = 0.45, size = 348, normalized size = 5.19

$$\frac{\left(a^3+3a^2b+3ab^2+b^3\right)x\cosh(x)^2+2\left(a^3+3a^2b+3ab^2+b^3\right)x\cosh(x)\sinh(x)+\left(a^3+3a^2b+3ab^2+b^3\right)x}{a^5-a^4b-2a^3b^2+2a^2b^3+ab^4-b^5+\left(a^5+a^4b-2a^3b^2-2a^2b^3+ab\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^2/(a*cosh(x)+b*sinh(x))^2,x, algorithm="fricas")
```

```
[Out] ((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*x*cosh(x)^2 + 2*(a^3 + 3*a^2*b + 3*a*b^2 +
 b^3)*x*cosh(x)*sinh(x) + (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*x*sinh(x)^2 + 2*a
*b^2 - 2*b^3 + (a^3 + a^2*b - a*b^2 - b^3)*x - 2*(a^2*b - a*b^2 + (a^2*b +
a*b^2)*cosh(x)^2 + 2*(a^2*b + a*b^2)*cosh(x)*sinh(x) + (a^2*b + a*b^2)*sinh
(x)^2)*log(2*(a*cosh(x) + b*sinh(x))/(cosh(x) - sinh(x))))/(a^5 - a^4*b - 2
*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5 + (a^5 + a^4*b - 2*a^3*b^2 - 2*a^2*b^3 +
 a*b^4 + b^5)*cosh(x)^2 + 2*(a^5 + a^4*b - 2*a^3*b^2 - 2*a^2*b^3 + a*b^4 +
b^5)*cosh(x)*sinh(x) + (a^5 + a^4*b - 2*a^3*b^2 - 2*a^2*b^3 + a*b^4 + b^5)*
sinh(x)^2)
```

giac [A] time = 0.13, size = 114, normalized size = 1.70

$$-\frac{2\,ab\log\left(\left|ae^{(2\,x)}+be^{(2\,x)}+a-b\right|\right)}{a^4-2\,a^2b^2+b^4}+\frac{x}{a^2-2\,ab+b^2}+\frac{2\left(abe^{(2\,x)}+ab-b^2\right)}{\left(a^3-a^2b-ab^2+b^3\right)\left(ae^{(2\,x)}+be^{(2\,x)}+a-b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^2/(a*cosh(x)+b*sinh(x))^2,x, algorithm="giac")
```

```
[Out] -2*a*b*log(abs(a*e^(2*x) + b*e^(2*x) + a - b))/(a^4 - 2*a^2*b^2 + b^4) + x/
(a^2 - 2*a*b + b^2) + 2*(a*b*e^(2*x) + a*b - b^2)/((a^3 - a^2*b - a*b^2 + b
^3)*(a*e^(2*x) + b*e^(2*x) + a - b))
```

maple [B] time = 0.26, size = 149, normalized size = 2.22

$$-\frac{\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{(a+b)^2}-\frac{2a\tanh\left(\frac{x}{2}\right)b^2}{(a-b)^2\,(a+b)^2\left(a+2\tanh\left(\frac{x}{2}\right)b+a\left(\tanh^2\left(\frac{x}{2}\right)\right)\right)}+\frac{2b^4\tanh\left(\frac{x}{2}\right)}{(a-b)^2\,(a+b)^2\,a\left(a+2\tanh\left(\frac{x}{2}\right)b+a\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^2/(a*cosh(x)+b*sinh(x))^2,x)
```

```
[Out] -1/(a+b)^2*ln(tanh(1/2*x)-1)-2*a/(a-b)^2/(a+b)^2*tanh(1/2*x)/(a+2*tanh(1/2*
x)*b+a*tanh(1/2*x)^2)*b^2+2*b^4/(a-b)^2/(a+b)^2/a*tanh(1/2*x)/(a+2*tanh(1/2
*x)*b+a*tanh(1/2*x)^2)-2*a/(a-b)^2/(a+b)^2*b*ln(a+2*tanh(1/2*x)*b+a*tanh(1/
2*x)^2)+1/(a-b)^2*ln(tanh(1/2*x)+1)
```

maxima [A] time = 0.51, size = 104, normalized size = 1.55

$$-\frac{2\,ab\log\left(-(a-b)e^{(-2\,x)}-a-b\right)}{a^4-2\,a^2b^2+b^4}-\frac{2\,b^2}{a^4-2\,a^2b^2+b^4+\left(a^4-2\,a^3b+2\,ab^3-b^4\right)e^{(-2\,x)}}+\frac{x}{a^2+2\,ab+b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^2/(a*cosh(x)+b*sinh(x))^2,x, algorithm="maxima")
```

```
[Out] -2*a*b*log(-(a - b)*e^(-2*x) - a - b)/(a^4 - 2*a^2*b^2 + b^4) - 2*b^2/(a^4
- 2*a^2*b^2 + b^4 + (a^4 - 2*a^3*b + 2*a*b^3 - b^4)*e^(-2*x)) + x/(a^2 + 2*
a*b + b^2)
```

mupad [B] time = 0.42, size = 104, normalized size = 1.55

$$\frac{\frac{b\cosh(x)}{a^2-b^2}+\frac{x\sinh(x)\left(a^2\,b+b^3\right)}{\left(a^2-b^2\right)^2}+\frac{a\,x\cosh(x)\left(a^2+b^2\right)}{\left(a^2-b^2\right)^2}}{a\cosh(x)+b\sinh(x)}+\ln\left(a\cosh(x)+b\sinh(x)\right)\left(\frac{1}{2\,(a+b)^2}-\frac{1}{2\,(a-b)^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^2/(a*cosh(x) + b*sinh(x))^2,x)
```

```
[Out] ((b*cosh(x))/(a^2 - b^2) + (x*sinh(x)*(a^2*b + b^3))/(a^2 - b^2)^2 + (a*x*c
osh(x)*(a^2 + b^2))/(a^2 - b^2)^2)/(a*cosh(x) + b*sinh(x)) + log(a*cosh(x)
+ b*sinh(x))*(1/(2*(a + b)^2) - 1/(2*(a - b)^2))
```

sympy [A] time = 1.49, size = 952, normalized size = 14.21

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)**2/(a*cosh(x)+b*sinh(x))**2,x)
```

```
[Out] Piecewise((zoo*(x - cosh(x)/sinh(x)), Eq(a, 0) & Eq(b, 0)), (2*x*sinh(x)**2
/(8*b**2*sinh(x)**2 - 16*b**2*sinh(x)*cosh(x) + 8*b**2*cosh(x)**2) - 4*x*si
nh(x)*cosh(x)/(8*b**2*sinh(x)**2 - 16*b**2*sinh(x)*cosh(x) + 8*b**2*cosh(x)
**2) + 2*x*cosh(x)**2/(8*b**2*sinh(x)**2 - 16*b**2*sinh(x)*cosh(x) + 8*b**2
*cosh(x)**2) - sinh(x)**2/(8*b**2*sinh(x)**2 - 16*b**2*sinh(x)*cosh(x) + 8*
b**2*cosh(x)**2) + 3*cosh(x)**2/(8*b**2*sinh(x)**2 - 16*b**2*sinh(x)*cosh(x
) + 8*b**2*cosh(x)**2), Eq(a, -b)), (2*x*sinh(x)**2/(8*b**2*sinh(x)**2 + 16
*b**2*sinh(x)*cosh(x) + 8*b**2*cosh(x)**2) + 4*x*sinh(x)*cosh(x)/(8*b**2*si
nh(x)**2 + 16*b**2*sinh(x)*cosh(x) + 8*b**2*cosh(x)**2) + 2*x*cosh(x)**2/(8
*b**2*sinh(x)**2 + 16*b**2*sinh(x)*cosh(x) + 8*b**2*cosh(x)**2) + sinh(x)**
2/(8*b**2*sinh(x)**2 + 16*b**2*sinh(x)*cosh(x) + 8*b**2*cosh(x)**2) - 3*cos
h(x)**2/(8*b**2*sinh(x)**2 + 16*b**2*sinh(x)*cosh(x) + 8*b**2*cosh(x)**2),
Eq(a, b)), ((x - cosh(x)/sinh(x))/b**2, Eq(a, 0)), (a**3*x*cosh(x)/(a**5*co
sh(x) + a**4*b*sinh(x) - 2*a**3*b**2*cosh(x) - 2*a**2*b**3*sinh(x) + a*b**4
*cosh(x) + b**5*sinh(x)) + a**2*b*x*sinh(x)/(a**5*cosh(x) + a**4*b*sinh(x)
- 2*a**3*b**2*cosh(x) - 2*a**2*b**3*sinh(x) + a*b**4*cosh(x) + b**5*sinh(x)
```

```
) - 2*a**2*b*log(cosh(x) + b*sinh(x)/a)*cosh(x)/(a**5*cosh(x) + a**4*b*sinh
(x) - 2*a**3*b**2*cosh(x) - 2*a**2*b**3*sinh(x) + a*b**4*cosh(x) + b**5*sin
h(x)) + a**2*b*cosh(x)/(a**5*cosh(x) + a**4*b*sinh(x) - 2*a**3*b**2*cosh(x)
 - 2*a**2*b**3*sinh(x) + a*b**4*cosh(x) + b**5*sinh(x)) + a*b**2*x*cosh(x)/
(a**5*cosh(x) + a**4*b*sinh(x) - 2*a**3*b**2*cosh(x) - 2*a**2*b**3*sinh(x)
+ a*b**4*cosh(x) + b**5*sinh(x)) - 2*a*b**2*log(cosh(x) + b*sinh(x)/a)*sinh
(x)/(a**5*cosh(x) + a**4*b*sinh(x) - 2*a**3*b**2*cosh(x) - 2*a**2*b**3*sinh
(x) + a*b**4*cosh(x) + b**5*sinh(x)) + b**3*x*sinh(x)/(a**5*cosh(x) + a**4*
b*sinh(x) - 2*a**3*b**2*cosh(x) - 2*a**2*b**3*sinh(x) + a*b**4*cosh(x) + b*
*5*sinh(x)) - b**3*cosh(x)/(a**5*cosh(x) + a**4*b*sinh(x) - 2*a**3*b**2*cos
h(x) - 2*a**2*b**3*sinh(x) + a*b**4*cosh(x) + b**5*sinh(x)), True))
```

3.701 $\int \frac{\cosh^3(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx$

Optimal. Leaf size=133

$$-\frac{3ab^2\tan^{-1}\left(\frac{a\sinh(x)+b\cosh(x)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{5/2}}-\frac{2b^3\left(a+b\tanh\left(\frac{x}{2}\right)\right)}{a\left(a^2-b^2\right)^2\left(a\tanh^2\left(\frac{x}{2}\right)+a+2b\tanh\left(\frac{x}{2}\right)\right)}+\frac{1}{(a+b)^2\left(1-\tanh\left(\frac{x}{2}\right)\right)}-\frac{\cdots}{(a-b)^2\,(\cdots}$$

```
[Out] -3*a*b^2*arctan((b*cosh(x)+a*sinh(x))/(a^2-b^2)^(1/2))/(a^2-b^2)^(5/2)+1/(a
+b)^2/(1-tanh(1/2*x))-1/(a-b)^2/(1+tanh(1/2*x))-2*b^3*(a+b*tanh(1/2*x))/a/(
a^2-b^2)^2/(a+2*b*tanh(1/2*x)+a*tanh(1/2*x)^2)
```

Rubi [A] time = 0.78, antiderivative size = 193, normalized size of antiderivative = 1.45, number of steps used = 8, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6742, 638, 618, 204}

$$-\frac{2b^3\left(a+b\tanh\left(\frac{x}{2}\right)\right)}{a\left(a^2-b^2\right)^2\left(a\tanh^2\left(\frac{x}{2}\right)+a+2b\tanh\left(\frac{x}{2}\right)\right)}-\frac{2b^4\tan^{-1}\left(\frac{a\tanh\left(\frac{x}{2}\right)+b}{\sqrt{a^2-b^2}}\right)}{a\left(a^2-b^2\right)^{5/2}}-\frac{2b^2\left(3a^2-b^2\right)\tan^{-1}\left(\frac{a\tanh\left(\frac{x}{2}\right)+b}{\sqrt{a^2-b^2}}\right)}{a\left(a^2-b^2\right)^{5/2}}+\frac{\cdots}{(a+\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]^3/(a*Cosh[x] + b*Sinh[x])^2,x]
```

```
[Out] (-2*b^4*ArcTan[(b + a*Tanh[x/2])/Sqrt[a^2 - b^2]])/(a*(a^2 - b^2)^(5/2)) -
(2*b^2*(3*a^2 - b^2)*ArcTan[(b + a*Tanh[x/2])/Sqrt[a^2 - b^2]])/(a*(a^2 - b
^2)^(5/2)) + 1/((a + b)^2*(1 - Tanh[x/2])) - 1/((a - b)^2*(1 + Tanh[x/2]))
- (2*b^3*(a + b*Tanh[x/2]))/(a*(a^2 - b^2)^2*(a + 2*b*Tanh[x/2] + a*Tanh[x/
2]^2))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 638

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)*(a + b*x + c*x^2)^(p + 1))/((p +
 1)*(b^2 - 4*a*c)), x] - Dist[((2*p + 3)*(2*c*d - b*e))/((p + 1)*(b^2 - 4*a
*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
 NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{\cosh^3(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx = 2\,\text{Subst}\left(\int \frac{\left(1+x^2\right)^3}{\left(1-x^2\right)^2\left(a+2bx+ax^2\right)^2}\,dx, x, \tanh\left(\frac{x}{2}\right)\right)$$

$$= 2\,\text{Subst}\left(\int\left(\frac{1}{2(a+b)^2(-1+x)^2}+\frac{1}{2(a-b)^2(1+x)^2}-\frac{2b^3x}{a\left(-a^2+b^2\right)\left(a+2bx+a\right.}\right.\right.$$

$$= \frac{1}{(a+b)^2\left(1-\tanh\left(\frac{x}{2}\right)\right)}-\frac{1}{(a-b)^2\left(1+\tanh\left(\frac{x}{2}\right)\right)}+\frac{\left(4b^3\right)\text{Subst}\left(\int \frac{x}{\left(a+2bx+ax^2\right)^2}\right.}{a\left(a^2-b^2\right)}$$

$$= \frac{1}{(a+b)^2\left(1-\tanh\left(\frac{x}{2}\right)\right)}-\frac{1}{(a-b)^2\left(1+\tanh\left(\frac{x}{2}\right)\right)}-\frac{2b^3\left(a+b\tanh\right.}{a\left(a^2-b^2\right)^2\left(a+2b\tanh\left(\frac{x}{2}\right.\right.}$$

$$= -\frac{2b^2\left(3a^2-b^2\right)\tan^{-1}\left(\frac{b+a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{a\left(a^2-b^2\right)^{5/2}}+\frac{1}{(a+b)^2\left(1-\tanh\left(\frac{x}{2}\right)\right)}-\frac{1}{(a-b)^2\left(1+\tan\right.}$$

$$= -\frac{2b^4\tan^{-1}\left(\frac{b+a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{a\left(a^2-b^2\right)^{5/2}}-\frac{2b^2\left(3a^2-b^2\right)\tan^{-1}\left(\frac{b+a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{a\left(a^2-b^2\right)^{5/2}}+\frac{1}{(a+b)^2\left(1-\tan\right.}$$

Mathematica [A] time = 0.37, size = 204, normalized size = 1.53

$$\frac{-2a^2b\sqrt{a-b}\,(a+b)\cosh^2(x)+b\sqrt{a-b}\,\left(a^3+a^2b+ab^2+b^3\right)\sinh^2(x)-6ab^3\sqrt{a+b}\,\sinh(x)\tan^{-1}\left(\frac{a\tanh\left(\frac{x}{2}\right)+b}{\sqrt{a-b}\,\sqrt{a+b}}\right.}{(a-b)^{5/2}(a+b)^3(a\cosh(x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]^3/(a*Cosh[x] + b*Sinh[x])^2,x]
```

```
[Out] (-(Sqrt[a - b]*b^3*(a + b)) - 2*a^2*Sqrt[a - b]*b*(a + b)*Cosh[x]^2 - 6*a*b
^3*Sqrt[a + b]*ArcTan[(b + a*Tanh[x/2])/(Sqrt[a - b]*Sqrt[a + b])]*Sinh[x]
+ Sqrt[a - b]*b*(a^3 + a^2*b + a*b^2 + b^3)*Sinh[x]^2 + a*Cosh[x]*(-6*a*b^2
*Sqrt[a + b]*ArcTan[(b + a*Tanh[x/2])/(Sqrt[a - b]*Sqrt[a + b])] + (a - b)^
(3/2)*(a + b)^2*Sinh[x]))/((a - b)^(5/2)*(a + b)^3*(a*Cosh[x] + b*Sinh[x]))
```

fricas [B] time = 0.48, size = 1645, normalized size = 12.37

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^3/(a*cosh(x)+b*sinh(x))^2,x, algorithm="fricas")
```

```
[Out] [-1/2*(a^5 + a^4*b - 2*a^3*b^2 - 2*a^2*b^3 + a*b^4 + b^5 - (a^5 - a^4*b - 2
*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^4 - 4*(a^5 - a^4*b - 2*a^3*b^2
+ 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)*sinh(x)^3 - (a^5 - a^4*b - 2*a^3*b^2 + 2
*a^2*b^3 + a*b^4 - b^5)*sinh(x)^4 + 6*(a^4*b - b^5)*cosh(x)^2 + 6*(a^4*b -
b^5 - (a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^2)*sinh(x
)^2 + 6*((a^2*b^2 + a*b^3)*cosh(x)^3 + 3*(a^2*b^2 + a*b^3)*cosh(x)*sinh(x)^
2 + (a^2*b^2 + a*b^3)*sinh(x)^3 + (a^2*b^2 - a*b^3)*cosh(x) + (a^2*b^2 - a*
b^3 + 3*(a^2*b^2 + a*b^3)*cosh(x)^2)*sinh(x))*sqrt(-a^2 + b^2)*log(((a + b)
*cosh(x)^2 + 2*(a + b)*cosh(x)*sinh(x) + (a + b)*sinh(x)^2 + 2*sqrt(-a^2 +
b^2)*(cosh(x) + sinh(x)) - a + b)/((a + b)*cosh(x)^2 + 2*(a + b)*cosh(x)*si
nh(x) + (a + b)*sinh(x)^2 + a - b)) - 4*((a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b
^3 + a*b^4 - b^5)*cosh(x)^3 - 3*(a^4*b - b^5)*cosh(x))*sinh(x))/((a^7 + a^6
*b - 3*a^5*b^2 - 3*a^4*b^3 + 3*a^3*b^4 + 3*a^2*b^5 - a*b^6 - b^7)*cosh(x)^3
 + 3*(a^7 + a^6*b - 3*a^5*b^2 - 3*a^4*b^3 + 3*a^3*b^4 + 3*a^2*b^5 - a*b^6 -
 b^7)*cosh(x)*sinh(x)^2 + (a^7 + a^6*b - 3*a^5*b^2 - 3*a^4*b^3 + 3*a^3*b^4
+ 3*a^2*b^5 - a*b^6 - b^7)*sinh(x)^3 + (a^7 - a^6*b - 3*a^5*b^2 + 3*a^4*b^3
 + 3*a^3*b^4 - 3*a^2*b^5 - a*b^6 + b^7)*cosh(x) + (a^7 - a^6*b - 3*a^5*b^2
+ 3*a^4*b^3 + 3*a^3*b^4 - 3*a^2*b^5 - a*b^6 + b^7 + 3*(a^7 + a^6*b - 3*a^5*
b^2 - 3*a^4*b^3 + 3*a^3*b^4 + 3*a^2*b^5 - a*b^6 - b^7)*cosh(x)^2)*sinh(x)),
 -1/2*(a^5 + a^4*b - 2*a^3*b^2 - 2*a^2*b^3 + a*b^4 + b^5 - (a^5 - a^4*b - 2
*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^4 - 4*(a^5 - a^4*b - 2*a^3*b^2
+ 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)*sinh(x)^3 - (a^5 - a^4*b - 2*a^3*b^2 + 2
*a^2*b^3 + a*b^4 - b^5)*sinh(x)^4 + 6*(a^4*b - b^5)*cosh(x)^2 + 6*(a^4*b -
b^5 - (a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^2)*sinh(x
)^2 - 12*((a^2*b^2 + a*b^3)*cosh(x)^3 + 3*(a^2*b^2 + a*b^3)*cosh(x)*sinh(x)
^2 + (a^2*b^2 + a*b^3)*sinh(x)^3 + (a^2*b^2 - a*b^3)*cosh(x) + (a^2*b^2 - a
*b^3 + 3*(a^2*b^2 + a*b^3)*cosh(x)^2)*sinh(x))*sqrt(a^2 - b^2)*arctan(sqrt(
a^2 - b^2)/((a + b)*cosh(x) + (a + b)*sinh(x))) - 4*((a^5 - a^4*b - 2*a^3*b
```

```
^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^3 - 3*(a^4*b - b^5)*cosh(x))*sinh(x))
/((a^7 + a^6*b - 3*a^5*b^2 - 3*a^4*b^3 + 3*a^3*b^4 + 3*a^2*b^5 - a*b^6 - b^
7)*cosh(x)^3 + 3*(a^7 + a^6*b - 3*a^5*b^2 - 3*a^4*b^3 + 3*a^3*b^4 + 3*a^2*b
^5 - a*b^6 - b^7)*cosh(x)*sinh(x)^2 + (a^7 + a^6*b - 3*a^5*b^2 - 3*a^4*b^3
+ 3*a^3*b^4 + 3*a^2*b^5 - a*b^6 - b^7)*sinh(x)^3 + (a^7 - a^6*b - 3*a^5*b^2
 + 3*a^4*b^3 + 3*a^3*b^4 - 3*a^2*b^5 - a*b^6 + b^7)*cosh(x) + (a^7 - a^6*b
- 3*a^5*b^2 + 3*a^4*b^3 + 3*a^3*b^4 - 3*a^2*b^5 - a*b^6 + b^7 + 3*(a^7 + a^
6*b - 3*a^5*b^2 - 3*a^4*b^3 + 3*a^3*b^4 + 3*a^2*b^5 - a*b^6 - b^7)*cosh(x)^
2)*sinh(x))]
```

giac [A] time = 0.14, size = 174, normalized size = 1.31

$$-\frac{6\,ab^2\arctan\left(\frac{ae^x+be^x}{\sqrt{a^2-b^2}}\right)}{\left(a^4-2\,a^2b^2+b^4\right)\sqrt{a^2-b^2}}+\frac{e^x}{2\left(a^2+2\,ab+b^2\right)}-\frac{a^3e^{(2\,x)}+3\,a^2be^{(2\,x)}+3\,ab^2e^{(2\,x)}+5\,b^3e^{(2\,x)}+a^3+a^2b-ab^2-}{2\left(a^4-2\,a^2b^2+b^4\right)\left(ae^{(3\,x)}+be^{(3\,x)}+ae^x-be^x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^3/(a*cosh(x)+b*sinh(x))^2,x, algorithm="giac")
```

```
[Out] -6*a*b^2*arctan((a*e^x + b*e^x)/sqrt(a^2 - b^2))/((a^4 - 2*a^2*b^2 + b^4)*s
qrt(a^2 - b^2)) + 1/2*e^x/(a^2 + 2*a*b + b^2) - 1/2*(a^3*e^(2*x) + 3*a^2*b*
e^(2*x) + 3*a*b^2*e^(2*x) + 5*b^3*e^(2*x) + a^3 + a^2*b - a*b^2 - b^3)/((a^
4 - 2*a^2*b^2 + b^4)*(a*e^(3*x) + b*e^(3*x) + a*e^x - b*e^x))
```

maple [A] time = 0.27, size = 167, normalized size = 1.26

$$-\frac{1}{(a+b)^2\left(\tanh\left(\frac{x}{2}\right)-1\right)}-\frac{2b^4\tanh\left(\frac{x}{2}\right)}{(a-b)^2(a+b)^2a\left(a+2\tanh\left(\frac{x}{2}\right)b+a\left(\tanh^2\left(\frac{x}{2}\right)\right)\right)}-\frac{2b^3}{(a-b)^2(a+b)^2\left(a+2\tanh\left(\frac{x}{2}\right)\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^3/(a*cosh(x)+b*sinh(x))^2,x)
```

```
[Out] -1/(a+b)^2/(tanh(1/2*x)-1)-2*b^4/(a-b)^2/(a+b)^2/a*tanh(1/2*x)/(a+2*tanh(1/
2*x)*b+a*tanh(1/2*x)^2)-2*b^3/(a-b)^2/(a+b)^2/(a+2*tanh(1/2*x)*b+a*tanh(1/2
*x)^2)-6*b^2/(a-b)^2/(a+b)^2*a/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tanh(1/2*x)+
2*b)/(a^2-b^2)^(1/2))-1/(a-b)^2/(tanh(1/2*x)+1)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^3/(a*cosh(x)+b*sinh(x))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [B] time = 1.86, size = 255, normalized size = 1.92

$$\frac{e^x}{2\,(a+b)^2}-\frac{e^{-x}}{2\,(a-b)^2}-\frac{6\,\mathrm{atan}\left(\frac{a\,b^2\,e^x\,\sqrt{a^{10}-5\,a^8\,b^2+10\,a^6\,b^4-10\,a^4\,b^6+5\,a^2\,b^8-b^{10}}}{a^5\,\sqrt{a^2\,b^4}-b^5\,\sqrt{a^2\,b^4}+2\,a^2\,b^3\,\sqrt{a^2\,b^4}-2\,a^3\,b^2\,\sqrt{a^2\,b^4}+a\,b^4\,\sqrt{a^2\,b^4}-a^4\,b\,\sqrt{a^2\,b^4}}\right)\,\sqrt{a^2\,b^4}}{\sqrt{a^{10}-5\,a^8\,b^2+10\,a^6\,b^4-10\,a^4\,b^6+5\,a^2\,b^8-b^{10}}}-\frac{\cdots}{(a+b)^2\,(a \cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^3/(a*cosh(x) + b*sinh(x))^2,x)
```

```
[Out] exp(x)/(2*(a + b)^2) - exp(-x)/(2*(a - b)^2) - (6*atan((a*b^2*exp(x)*(a^10
- b^10 + 5*a^2*b^8 - 10*a^4*b^6 + 10*a^6*b^4 - 5*a^8*b^2)^(1/2))/(a^5*(a^2*
b^4)^(1/2) - b^5*(a^2*b^4)^(1/2) + 2*a^2*b^3*(a^2*b^4)^(1/2) - 2*a^3*b^2*(a
^2*b^4)^(1/2) + a*b^4*(a^2*b^4)^(1/2) - a^4*b*(a^2*b^4)^(1/2)))*(a^2*b^4)^(
1/2))/(a^10 - b^10 + 5*a^2*b^8 - 10*a^4*b^6 + 10*a^6*b^4 - 5*a^8*b^2)^(1/2)
 - (2*b^3*exp(x))/((a + b)^2*(a - b)^2*(a - b + exp(2*x)*(a + b)))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)**3/(a*cosh(x)+b*sinh(x))**2,x)
```

```
[Out] Timed out
```

3.702 $\int \frac{\sinh(x)}{(a\cosh(x)+b\sinh(x))^3}\,dx$

Optimal. Leaf size=19

$$\frac{\tanh^2(x)}{2a(a+b\tanh(x))^2}$$

```
[Out] 1/2*tanh(x)^2/a/(a+b*tanh(x))^2
```

Rubi [A] time = 0.03, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3087, 37}

$$\frac{\tanh^2(x)}{2a(a+b\tanh(x))^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[x]/(a*Cosh[x] + b*Sinh[x])^3,x]
```

```
[Out] Tanh[x]^2/(2*a*(a + b*Tanh[x])^2)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 3087

```
Int[sin[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[1/d, Subst[Int[(x^m*(a + b*x
)^n)/(1 + x^2)^((m + n + 2)/2), x], x, Tan[c + d*x]], x] /; FreeQ[{a, b, c,
 d}, x] && IntegerQ[n] && IntegerQ[(m + n)/2] && NeQ[n, -1] &&  !(GtQ[n, 0]
 && GtQ[m, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{\sinh(x)}{(a\cosh(x)+b\sinh(x))^3}\,dx &= -\operatorname{Subst}\left(\int \frac{x}{(a-ibx)^3}\,dx, x, i\tanh(x)\right)\\ &= \frac{\tanh^2(x)}{2a(a+b\tanh(x))^2}\end{aligned}$$

Mathematica [B] time = 0.12, size = 54, normalized size = 2.84

$$-\frac{a^2 + ab\sinh(2x) + b^2\cosh(2x) - b^2}{2a(a-b)(a+b)(a\cosh(x) + b\sinh(x))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[x]/(a*Cosh[x] + b*Sinh[x])^3,x]
```

```
[Out] -1/2*(a^2 - b^2 + b^2*Cosh[2*x] + a*b*Sinh[2*x])/(a*(a - b)*(a + b)*(a*Cosh
[x] + b*Sinh[x])^2)
```

fricas [B] time = 0.41, size = 216, normalized size = 11.37

$$-\frac{}{(a^4 + 4\,a^3b + 6\,a^2b^2 + 4\,ab^3 + b^4)\cosh(x)^3 + 3\,(a^4 + 4\,a^3b + 6\,a^2b^2 + 4\,ab^3 + b^4)\cosh(x)\sinh(x)^2 + (a^4 + 4\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)/(a*cosh(x)+b*sinh(x))^3,x, algorithm="fricas")
```

```
[Out] -2*(a*cosh(x) + (a + 2*b)*sinh(x))/((a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 +
b^4)*cosh(x)^3 + 3*(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*cosh(x)*sinh
(x)^2 + (a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*sinh(x)^3 + (3*a^4 + 4*
a^3*b - 2*a^2*b^2 - 4*a*b^3 - b^4)*cosh(x) + (a^4 + 4*a^3*b + 2*a^2*b^2 - 4
*a*b^3 - 3*b^4 + 3*(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*cosh(x)^2)*s
inh(x))
```

giac [B] time = 0.13, size = 50, normalized size = 2.63

$$-\frac{2\left(ae^{(2\,x)} + be^{(2\,x)} - b\right)}{\left(a^2 + 2\,ab + b^2\right)\left(ae^{(2\,x)} + be^{(2\,x)} + a - b\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)/(a*cosh(x)+b*sinh(x))^3,x, algorithm="giac")
```

```
[Out] -2*(a*e^(2*x) + b*e^(2*x) - b)/((a^2 + 2*a*b + b^2)*(a*e^(2*x) + b*e^(2*x)
+ a - b)^2)
```

maple [A] time = 0.28, size = 31, normalized size = 1.63

$$\frac{2\left(\tanh^2\left(\frac{x}{2}\right)\right)}{a\left(a + 2\tanh\left(\frac{x}{2}\right)b + a\left(\tanh^2\left(\frac{x}{2}\right)\right)\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sinh(x)/(a*cosh(x)+b*sinh(x))^3,x)

[Out] 2/a*tanh(1/2*x)^2/(a+2*tanh(1/2*x)*b+a*tanh(1/2*x)^2)^2

maxima [B] time = 0.46, size = 167, normalized size = 8.79

$$-\frac{2\,(a-b)e^{(-2\,x)}}{a^4-2\,a^2b^2+b^4+2\left(a^4-2\,a^3b+2\,ab^3-b^4\right)e^{(-2\,x)}+\left(a^4-4\,a^3b+6\,a^2b^2-4\,ab^3+b^4\right)e^{(-4\,x)}}-\frac{}{a^4-2\,a^2b^2+b^4+}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(x)/(a*cosh(x)+b*sinh(x))^3,x, algorithm="maxima")

[Out] -2*(a - b)*e^(-2*x)/(a^4 - 2*a^2*b^2 + b^4 + 2*(a^4 - 2*a^3*b + 2*a*b^3 - b^4)*e^(-2*x) + (a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4)*e^(-4*x)) - 2*b/(a^4 - 2*a^2*b^2 + b^4 + 2*(a^4 - 2*a^3*b + 2*a*b^3 - b^4)*e^(-2*x) + (a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4)*e^(-4*x))

mupad [B] time = 1.60, size = 42, normalized size = 2.21

$$\frac{2\,b-\mathrm{e}^{2\,x}\,\left(2\,a+2\,b\right)}{{\left(a+b\right)}^2\,{\left(a-b+a\,\mathrm{e}^{2\,x}+b\,\mathrm{e}^{2\,x}\right)}^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sinh(x)/(a*cosh(x) + b*sinh(x))^3,x)

[Out] (2*b - exp(2*x)*(2*a + 2*b))/((a + b)^2*(a - b + a*exp(2*x) + b*exp(2*x))^2)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(x)/(a*cosh(x)+b*sinh(x))**3,x)

[Out] Timed out

3.703 $\int \frac{\sinh^3(x)}{(a\cosh(x)+b\sinh(x))^3}\,dx$

Optimal. Leaf size=104

$$-\frac{bx\left(3a^2+b^2\right)}{\left(a^2-b^2\right)^3}+\frac{2ab}{\left(a^2-b^2\right)^2(a\coth(x)+b)}-\frac{a}{2\left(a^2-b^2\right)(a\coth(x)+b)^2}+\frac{a\left(a^2+3b^2\right)\log(a\cosh(x)+b\sinh(x)}{\left(a^2-b^2\right)^3}$$

```
[Out] -b*(3*a^2+b^2)*x/(a^2-b^2)^3-1/2*a/(a^2-b^2)/(b+a*coth(x))^2+2*a*b/(a^2-b^2
)^2/(b+a*coth(x))+a*(a^2+3*b^2)*ln(a*cosh(x)+b*sinh(x))/(a^2-b^2)^3
```

Rubi [A] time = 0.24, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {3085, 3483, 3529, 3531, 3530}

$$-\frac{bx\left(3a^2+b^2\right)}{\left(a^2-b^2\right)^3}+\frac{2ab}{\left(a^2-b^2\right)^2(a\coth(x)+b)}-\frac{a}{2\left(a^2-b^2\right)(a\coth(x)+b)^2}+\frac{a\left(a^2+3b^2\right)\log(a\cosh(x)+b\sinh(x)}{\left(a^2-b^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[x]^3/(a*Cosh[x] + b*Sinh[x])^3,x]
```

```
[Out] -((b*(3*a^2 + b^2)*x)/(a^2 - b^2)^3) - a/(2*(a^2 - b^2)*(b + a*Coth[x])^2)
+ (2*a*b)/((a^2 - b^2)^2*(b + a*Coth[x])) + (a*(a^2 + 3*b^2)*Log[a*Cosh[x]
+ b*Sinh[x]])/(a^2 - b^2)^3
```

Rule 3085

```
Int[sin[(c_.) + (d_.)*(x_)]^(m_)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin
[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[(b + a*Cot[c + d*x])^n, x] /;
 FreeQ[{a, b, c, d}, x] && EqQ[m + n, 0] && IntegerQ[n] && NeQ[a^2 + b^2, 0
]
```

Rule 3483

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(a +
b*Tan[c + d*x])^(n + 1))/(d*(n + 1)*(a^2 + b^2)), x] + Dist[1/(a^2 + b^2),
Int[(a - b*Tan[c + d*x])*(a + b*Tan[c + d*x])^(n + 1), x], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 + b^2, 0] && LtQ[n, -1]
```

Rule 3529

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[((b*c - a*d)*(a + b*Tan[e + f*x])^(m + 1))
```

```
/(f*(m + 1)*(a^2 + b^2)), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])
^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a,
 b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

Rule 3530

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c*Log[RemoveContent[a*Cos[e + f*x] + b*Sin[e + f
*x], x]])/(b*f), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

Rule 3531

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[((a*c + b*d)*x)/(a^2 + b^2), x] + Dist[(b*c - a
*d)/(a^2 + b^2), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && Ne
Q[a*c + b*d, 0]
```

Rubi steps

$$\int \frac{\sinh^3(x)}{(a\cosh(x)+b\sinh(x))^3}\,dx = i\int \frac{1}{(-ib-ia\coth(x))^3}\,dx$$
$$= -\frac{a}{2\left(a^2-b^2\right)(b+a\coth(x))^2} + \frac{i\int \frac{-ib+ia\coth(x)}{(-ib-ia\coth(x))^2}\,dx}{a^2-b^2}$$
$$= -\frac{a}{2\left(a^2-b^2\right)(b+a\coth(x))^2} + \frac{2ab}{\left(a^2-b^2\right)^2(b+a\coth(x))} + \frac{i\int \frac{-a^2-b^2+2ab\coth(x)}{-ib-ia\coth(x)}}{\left(a^2-b^2\right)^2}$$
$$= -\frac{b\left(3a^2+b^2\right)x}{\left(a^2-b^2\right)^3} - \frac{a}{2\left(a^2-b^2\right)(b+a\coth(x))^2} + \frac{2ab}{\left(a^2-b^2\right)^2(b+a\coth(x))} + \frac{(i}{}$$
$$= -\frac{b\left(3a^2+b^2\right)x}{\left(a^2-b^2\right)^3} - \frac{a}{2\left(a^2-b^2\right)(b+a\coth(x))^2} + \frac{2ab}{\left(a^2-b^2\right)^2(b+a\coth(x))} + \frac{a(}{}$$

Mathematica [A] time = 0.78, size = 117, normalized size = 1.12

$$\frac{a^3}{2(a-b)^2(a+b)^2(a\cosh(x)+b\sinh(x))^2} - \frac{bx\left(3a^2+b^2\right)}{(a-b)^3(a+b)^3} + \frac{\left(a^3+3ab^2\right)\log(a\cosh(x)+b\sinh(x))}{\left(a^2-b^2\right)^3} + \frac{}{(a-b)^2(a+}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[x]^3/(a*Cosh[x] + b*Sinh[x])^3,x]
```

```
[Out] -((b*(3*a^2 + b^2)*x)/((a - b)^3*(a + b)^3)) + ((a^3 + 3*a*b^2)*Log[a*Cosh[
x] + b*Sinh[x]])/(a^2 - b^2)^3 + a^3/(2*(a - b)^2*(a + b)^2*(a*Cosh[x] + b*
Sinh[x])^2) + (3*a*b*Sinh[x])/((a - b)^2*(a + b)^2*(a*Cosh[x] + b*Sinh[x]))
```

fricas [B] time = 0.48, size = 1268, normalized size = 12.19

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^3/(a*cosh(x)+b*sinh(x))^3,x, algorithm="fricas")
```

```
[Out] -((a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*x*cosh(x)^4 + 4
*(a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*x*cosh(x)*sinh(x
)^3 + (a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*x*sinh(x)^4
 + 6*a^4*b - 12*a^3*b^2 + 6*a^2*b^3 - 2*(a^5 - 3*a^4*b - a^3*b^2 + 3*a^2*b^
3 - (a^5 + 3*a^4*b + 2*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^5)*x)*cosh(x)^2 -
2*(a^5 - 3*a^4*b - a^3*b^2 + 3*a^2*b^3 - 3*(a^5 + 5*a^4*b + 10*a^3*b^2 + 10
*a^2*b^3 + 5*a*b^4 + b^5)*x*cosh(x)^2 - (a^5 + 3*a^4*b + 2*a^3*b^2 - 2*a^2*
b^3 - 3*a*b^4 - b^5)*x)*sinh(x)^2 + (a^5 + a^4*b - 2*a^3*b^2 - 2*a^2*b^3 +
a*b^4 + b^5)*x - (a^5 - 2*a^4*b + 4*a^3*b^2 - 6*a^2*b^3 + 3*a*b^4 + (a^5 +
2*a^4*b + 4*a^3*b^2 + 6*a^2*b^3 + 3*a*b^4)*cosh(x)^4 + 4*(a^5 + 2*a^4*b + 4
*a^3*b^2 + 6*a^2*b^3 + 3*a*b^4)*cosh(x)*sinh(x)^3 + (a^5 + 2*a^4*b + 4*a^3*
b^2 + 6*a^2*b^3 + 3*a*b^4)*sinh(x)^4 + 2*(a^5 + 2*a^3*b^2 - 3*a*b^4)*cosh(x
)^2 + 2*(a^5 + 2*a^3*b^2 - 3*a*b^4 + 3*(a^5 + 2*a^4*b + 4*a^3*b^2 + 6*a^2*b
^3 + 3*a*b^4)*cosh(x)^2)*sinh(x)^2 + 4*((a^5 + 2*a^4*b + 4*a^3*b^2 + 6*a^2*
b^3 + 3*a*b^4)*cosh(x)^3 + (a^5 + 2*a^3*b^2 - 3*a*b^4)*cosh(x))*sinh(x))*lo
g(2*(a*cosh(x) + b*sinh(x))/(cosh(x) - sinh(x))) + 4*((a^5 + 5*a^4*b + 10*a
^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*x*cosh(x)^3 - (a^5 - 3*a^4*b - a^3*b^2
 + 3*a^2*b^3 - (a^5 + 3*a^4*b + 2*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^5)*x)*c
osh(x))*sinh(x))/(a^8 - 2*a^7*b - 2*a^6*b^2 + 6*a^5*b^3 - 6*a^3*b^5 + 2*a^2
*b^6 + 2*a*b^7 - b^8 + (a^8 + 2*a^7*b - 2*a^6*b^2 - 6*a^5*b^3 + 6*a^3*b^5 +
 2*a^2*b^6 - 2*a*b^7 - b^8)*cosh(x)^4 + 4*(a^8 + 2*a^7*b - 2*a^6*b^2 - 6*a^
5*b^3 + 6*a^3*b^5 + 2*a^2*b^6 - 2*a*b^7 - b^8)*cosh(x)*sinh(x)^3 + (a^8 + 2
*a^7*b - 2*a^6*b^2 - 6*a^5*b^3 + 6*a^3*b^5 + 2*a^2*b^6 - 2*a*b^7 - b^8)*sin
h(x)^4 + 2*(a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8)*cosh(x)^2 + 2*(a
^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8 + 3*(a^8 + 2*a^7*b - 2*a^6*b^2
 - 6*a^5*b^3 + 6*a^3*b^5 + 2*a^2*b^6 - 2*a*b^7 - b^8)*cosh(x)^2)*sinh(x)^2
+ 4*((a^8 + 2*a^7*b - 2*a^6*b^2 - 6*a^5*b^3 + 6*a^3*b^5 + 2*a^2*b^6 - 2*a*b
^7 - b^8)*cosh(x)^3 + (a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8)*cosh(
x))*sinh(x))
```

giac [B] time = 0.14, size = 251, normalized size = 2.41

$$\frac{\left(a^3+3\,ab^2\right)\log\left(\left|ae^{(2\,x)}+be^{(2\,x)}+a-b\right|\right)}{a^6-3\,a^4b^2+3\,a^2b^4-b^6}-\frac{x}{a^3-3\,a^2b+3\,ab^2-b^3}-\frac{3\,a^4e^{(4\,x)}+3\,a^3be^{(4\,x)}+9\,a^2b^2e^{(4\,x)}+9\,ab^3e^{(4\,x)}}{2\left(a^5-a^4b-\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^3/(a*cosh(x)+b*sinh(x))^3,x, algorithm="giac")
```

```
[Out] (a^3 + 3*a*b^2)*log(abs(a*e^(2*x) + b*e^(2*x) + a - b))/(a^6 - 3*a^4*b^2 +
3*a^2*b^4 - b^6) - x/(a^3 - 3*a^2*b + 3*a*b^2 - b^3) - 1/2*(3*a^4*e^(4*x) +
 3*a^3*b*e^(4*x) + 9*a^2*b^2*e^(4*x) + 9*a*b^3*e^(4*x) + 2*a^4*e^(2*x) + 10
*a^3*b*e^(2*x) + 6*a^2*b^2*e^(2*x) - 18*a*b^3*e^(2*x) + 3*a^4 + 3*a^3*b - 1
5*a^2*b^2 + 9*a*b^3)/((a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*(
a*e^(2*x) + b*e^(2*x) + a - b)^2)
```

maple [B] time = 0.28, size = 404, normalized size = 3.88

$$-\frac{\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{\left(a+b\right)^3}+\frac{4a^4\left(\tanh^3\left(\frac{x}{2}\right)\right)b}{\left(a-b\right)^3\left(a+b\right)^3\left(a+2\tanh\left(\frac{x}{2}\right)b+a\left(\tanh^2\left(\frac{x}{2}\right)\right)\right)^2}-\frac{4a^2\left(\tanh^3\left(\frac{x}{2}\right)\right)b^3}{\left(a-b\right)^3\left(a+b\right)^3\left(a+2\tanh\left(\frac{x}{2}\right)b+a\left(\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)^3/(a*cosh(x)+b*sinh(x))^3,x)
```

```
[Out] -1/(a+b)^3*ln(tanh(1/2*x)-1)+4*a^4/(a-b)^3/(a+b)^3/(a+2*tanh(1/2*x)*b+a*tan
h(1/2*x)^2)^2*tanh(1/2*x)^3*b-4*a^2/(a-b)^3/(a+b)^3/(a+2*tanh(1/2*x)*b+a*ta
nh(1/2*x)^2)^2*tanh(1/2*x)^3*b^3-2*a^5/(a-b)^3/(a+b)^3/(a+2*tanh(1/2*x)*b+a
*tanh(1/2*x)^2)^2*tanh(1/2*x)^2+12*a^3/(a-b)^3/(a+b)^3/(a+2*tanh(1/2*x)*b+a
*tanh(1/2*x)^2)^2*tanh(1/2*x)^2*b^2-10*a/(a-b)^3/(a+b)^3/(a+2*tanh(1/2*x)*b
+a*tanh(1/2*x)^2)^2*tanh(1/2*x)^2*b^4+4*a^4/(a-b)^3/(a+b)^3/(a+2*tanh(1/2*x
)*b+a*tanh(1/2*x)^2)^2*tanh(1/2*x)*b-4*a^2/(a-b)^3/(a+b)^3/(a+2*tanh(1/2*x)
*b+a*tanh(1/2*x)^2)^2*tanh(1/2*x)*b^3+a^3/(a-b)^3/(a+b)^3*ln(a+2*tanh(1/2*x
)*b+a*tanh(1/2*x)^2)+3*a/(a-b)^3/(a+b)^3*ln(a+2*tanh(1/2*x)*b+a*tanh(1/2*x)
^2)*b^2-1/(a-b)^3*ln(tanh(1/2*x)+1)
```

maxima [B] time = 0.45, size = 289, normalized size = 2.78

$$\frac{\left(a^3+3\,ab^2\right)\log\left(-(a-b)e^{(-2\,x)}-a-b\right)}{a^6-3\,a^4b^2+3\,a^2b^4-b^6}+\frac{2\,(}{a^7+a^6b-3\,a^5b^2-3\,a^4b^3+3\,a^3b^4+3\,a^2b^5-ab^6-b^7+2\left(a^7-a^6b-3\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^3/(a*cosh(x)+b*sinh(x))^3,x, algorithm="maxima")
```

```
[Out] (a^3 + 3*a*b^2)*log(-(a - b)*e^(-2*x) - a - b)/(a^6 - 3*a^4*b^2 + 3*a^2*b^4
 - b^6) + 2*(3*a^3*b + 3*a^2*b^2 + (a^4 + 2*a^3*b - 3*a^2*b^2)*e^(-2*x))/(a
^7 + a^6*b - 3*a^5*b^2 - 3*a^4*b^3 + 3*a^3*b^4 + 3*a^2*b^5 - a*b^6 - b^7 +
2*(a^7 - a^6*b - 3*a^5*b^2 + 3*a^4*b^3 + 3*a^3*b^4 - 3*a^2*b^5 - a*b^6 + b^
7)*e^(-2*x) + (a^7 - 3*a^6*b + a^5*b^2 + 5*a^4*b^3 - 5*a^3*b^4 - a^2*b^5 +
3*a*b^6 - b^7)*e^(-4*x)) + x/(a^3 + 3*a^2*b + 3*a*b^2 + b^3)
```

mupad [B] time = 1.70, size = 159, normalized size = 1.53

$$\frac{\ln\left(a-b+a\,e^{2\,x}+b\,e^{2\,x}\right)\,\left(a^3+3\,a\,b^2\right)}{a^6-3\,a^4\,b^2+3\,a^2\,b^4-b^6}-\frac{x}{(a-b)^3}-\frac{2\,\left(3\,a^2\,b-a^3\right)}{(a+b)^3\,(a-b)^2\,\left(a-b+e^{2\,x}\,(a+b)\right)}-\frac{}{(a+b)^3\,(a-b)\,\left(e^{4\,x}\,(\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)^3/(a*cosh(x) + b*sinh(x))^3,x)
```

```
[Out] (log(a - b + a*exp(2*x) + b*exp(2*x))*(3*a*b^2 + a^3))/(a^6 - b^6 + 3*a^2*b
^4 - 3*a^4*b^2) - x/(a - b)^3 - (2*(3*a^2*b - a^3))/((a + b)^3*(a - b)^2*(a
 - b + exp(2*x)*(a + b))) - (2*a^3)/((a + b)^3*(a - b)*(exp(4*x)*(a + b)^2
+ (a - b)^2 + 2*exp(2*x)*(a + b)*(a - b)))
```

sympy [A] time = 4.09, size = 3813, normalized size = 36.66

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)**3/(a*cosh(x)+b*sinh(x))**3,x)
```

```
[Out] Piecewise((zoo*x, Eq(a, 0) & Eq(b, 0)), (-3*x*sinh(x)**3/(-24*b**3*sinh(x)*
*3 + 72*b**3*sinh(x)**2*cosh(x) - 72*b**3*sinh(x)*cosh(x)**2 + 24*b**3*cosh
(x)**3) + 9*x*sinh(x)**2*cosh(x)/(-24*b**3*sinh(x)**3 + 72*b**3*sinh(x)**2*
cosh(x) - 72*b**3*sinh(x)*cosh(x)**2 + 24*b**3*cosh(x)**3) - 9*x*sinh(x)*co
sh(x)**2/(-24*b**3*sinh(x)**3 + 72*b**3*sinh(x)**2*cosh(x) - 72*b**3*sinh(x
)*cosh(x)**2 + 24*b**3*cosh(x)**3) + 3*x*cosh(x)**3/(-24*b**3*sinh(x)**3 +
72*b**3*sinh(x)**2*cosh(x) - 72*b**3*sinh(x)*cosh(x)**2 + 24*b**3*cosh(x)**
3) - 7*sinh(x)**3/(-24*b**3*sinh(x)**3 + 72*b**3*sinh(x)**2*cosh(x) - 72*b*
*3*sinh(x)*cosh(x)**2 + 24*b**3*cosh(x)**3) + 6*sinh(x)*cosh(x)**2/(-24*b**
3*sinh(x)**3 + 72*b**3*sinh(x)**2*cosh(x) - 72*b**3*sinh(x)*cosh(x)**2 + 24
*b**3*cosh(x)**3) - 3*cosh(x)**3/(-24*b**3*sinh(x)**3 + 72*b**3*sinh(x)**2*
cosh(x) - 72*b**3*sinh(x)*cosh(x)**2 + 24*b**3*cosh(x)**3), Eq(a, -b)), (3*
x*sinh(x)**3/(24*b**3*sinh(x)**3 + 72*b**3*sinh(x)**2*cosh(x) + 72*b**3*sin
h(x)*cosh(x)**2 + 24*b**3*cosh(x)**3) + 9*x*sinh(x)**2*cosh(x)/(24*b**3*sin
h(x)**3 + 72*b**3*sinh(x)**2*cosh(x) + 72*b**3*sinh(x)*cosh(x)**2 + 24*b**3
*cosh(x)**3) + 9*x*sinh(x)*cosh(x)**2/(24*b**3*sinh(x)**3 + 72*b**3*sinh(x)
**2*cosh(x) + 72*b**3*sinh(x)*cosh(x)**2 + 24*b**3*cosh(x)**3) + 3*x*cosh(x
```

```
)**3/(24*b**3*sinh(x)**3 + 72*b**3*sinh(x)**2*cosh(x) + 72*b**3*sinh(x)*cos
h(x)**2 + 24*b**3*cosh(x)**3) - 7*sinh(x)**3/(24*b**3*sinh(x)**3 + 72*b**3*
sinh(x)**2*cosh(x) + 72*b**3*sinh(x)*cosh(x)**2 + 24*b**3*cosh(x)**3) + 6*s
inh(x)*cosh(x)**2/(24*b**3*sinh(x)**3 + 72*b**3*sinh(x)**2*cosh(x) + 72*b**
3*sinh(x)*cosh(x)**2 + 24*b**3*cosh(x)**3) + 3*cosh(x)**3/(24*b**3*sinh(x)*
*3 + 72*b**3*sinh(x)**2*cosh(x) + 72*b**3*sinh(x)*cosh(x)**2 + 24*b**3*cosh
(x)**3), Eq(a, b)), (x/b**3, Eq(a, 0)), (2*a**5*log(cosh(x) + b*sinh(x)/a)*
cosh(x)**2/(2*a**8*cosh(x)**2 + 4*a**7*b*sinh(x)*cosh(x) + 2*a**6*b**2*sinh
(x)**2 - 6*a**6*b**2*cosh(x)**2 - 12*a**5*b**3*sinh(x)*cosh(x) - 6*a**4*b**
4*sinh(x)**2 + 6*a**4*b**4*cosh(x)**2 + 12*a**3*b**5*sinh(x)*cosh(x) + 6*a*
*2*b**6*sinh(x)**2 - 2*a**2*b**6*cosh(x)**2 - 4*a*b**7*sinh(x)*cosh(x) - 2*
b**8*sinh(x)**2) - a**5*sinh(x)**2/(2*a**8*cosh(x)**2 + 4*a**7*b*sinh(x)*co
sh(x) + 2*a**6*b**2*sinh(x)**2 - 6*a**6*b**2*cosh(x)**2 - 12*a**5*b**3*sinh
(x)*cosh(x) - 6*a**4*b**4*sinh(x)**2 + 6*a**4*b**4*cosh(x)**2 + 12*a**3*b**
5*sinh(x)*cosh(x) + 6*a**2*b**6*sinh(x)**2 - 2*a**2*b**6*cosh(x)**2 - 4*a*b
**7*sinh(x)*cosh(x) - 2*b**8*sinh(x)**2) - 2*a**5*cosh(x)**2/(2*a**8*cosh(x
)**2 + 4*a**7*b*sinh(x)*cosh(x) + 2*a**6*b**2*sinh(x)**2 - 6*a**6*b**2*cosh
(x)**2 - 12*a**5*b**3*sinh(x)*cosh(x) - 6*a**4*b**4*sinh(x)**2 + 6*a**4*b**
4*cosh(x)**2 + 12*a**3*b**5*sinh(x)*cosh(x) + 6*a**2*b**6*sinh(x)**2 - 2*a*
*2*b**6*cosh(x)**2 - 4*a*b**7*sinh(x)*cosh(x) - 2*b**8*sinh(x)**2) - 6*a**4
*b*x*cosh(x)**2/(2*a**8*cosh(x)**2 + 4*a**7*b*sinh(x)*cosh(x) + 2*a**6*b**2
*sinh(x)**2 - 6*a**6*b**2*cosh(x)**2 - 12*a**5*b**3*sinh(x)*cosh(x) - 6*a**
4*b**4*sinh(x)**2 + 6*a**4*b**4*cosh(x)**2 + 12*a**3*b**5*sinh(x)*cosh(x) +
 6*a**2*b**6*sinh(x)**2 - 2*a**2*b**6*cosh(x)**2 - 4*a*b**7*sinh(x)*cosh(x)
 - 2*b**8*sinh(x)**2) + 4*a**4*b*log(cosh(x) + b*sinh(x)/a)*sinh(x)*cosh(x)
/(2*a**8*cosh(x)**2 + 4*a**7*b*sinh(x)*cosh(x) + 2*a**6*b**2*sinh(x)**2 - 6
*a**6*b**2*cosh(x)**2 - 12*a**5*b**3*sinh(x)*cosh(x) - 6*a**4*b**4*sinh(x)*
*2 + 6*a**4*b**4*cosh(x)**2 + 12*a**3*b**5*sinh(x)*cosh(x) + 6*a**2*b**6*si
nh(x)**2 - 2*a**2*b**6*cosh(x)**2 - 4*a*b**7*sinh(x)*cosh(x) - 2*b**8*sinh(
x)**2) - 12*a**3*b**2*x*sinh(x)*cosh(x)/(2*a**8*cosh(x)**2 + 4*a**7*b*sinh(
x)*cosh(x) + 2*a**6*b**2*sinh(x)**2 - 6*a**6*b**2*cosh(x)**2 - 12*a**5*b**3
*sinh(x)*cosh(x) - 6*a**4*b**4*sinh(x)**2 + 6*a**4*b**4*cosh(x)**2 + 12*a**
3*b**5*sinh(x)*cosh(x) + 6*a**2*b**6*sinh(x)**2 - 2*a**2*b**6*cosh(x)**2 -
4*a*b**7*sinh(x)*cosh(x) - 2*b**8*sinh(x)**2) + 2*a**3*b**2*log(cosh(x) + b
*sinh(x)/a)*sinh(x)**2/(2*a**8*cosh(x)**2 + 4*a**7*b*sinh(x)*cosh(x) + 2*a*
*6*b**2*sinh(x)**2 - 6*a**6*b**2*cosh(x)**2 - 12*a**5*b**3*sinh(x)*cosh(x)
- 6*a**4*b**4*sinh(x)**2 + 6*a**4*b**4*cosh(x)**2 + 12*a**3*b**5*sinh(x)*co
sh(x) + 6*a**2*b**6*sinh(x)**2 - 2*a**2*b**6*cosh(x)**2 - 4*a*b**7*sinh(x)*
cosh(x) - 2*b**8*sinh(x)**2) + 6*a**3*b**2*log(cosh(x) + b*sinh(x)/a)*cosh(
x)**2/(2*a**8*cosh(x)**2 + 4*a**7*b*sinh(x)*cosh(x) + 2*a**6*b**2*sinh(x)**
2 - 6*a**6*b**2*cosh(x)**2 - 12*a**5*b**3*sinh(x)*cosh(x) - 6*a**4*b**4*sin
h(x)**2 + 6*a**4*b**4*cosh(x)**2 + 12*a**3*b**5*sinh(x)*cosh(x) + 6*a**2*b*
*6*sinh(x)**2 - 2*a**2*b**6*cosh(x)**2 - 4*a*b**7*sinh(x)*cosh(x) - 2*b**8*
sinh(x)**2) + 4*a**3*b**2*sinh(x)**2/(2*a**8*cosh(x)**2 + 4*a**7*b*sinh(x)*
cosh(x) + 2*a**6*b**2*sinh(x)**2 - 6*a**6*b**2*cosh(x)**2 - 12*a**5*b**3*si
```

```
nh(x)*cosh(x) - 6*a**4*b**4*sinh(x)**2 + 6*a**4*b**4*cosh(x)**2 + 12*a**3*b
**5*sinh(x)*cosh(x) + 6*a**2*b**6*sinh(x)**2 - 2*a**2*b**6*cosh(x)**2 - 4*a
*b**7*sinh(x)*cosh(x) - 2*b**8*sinh(x)**2) + 2*a**3*b**2*cosh(x)**2/(2*a**8
*cosh(x)**2 + 4*a**7*b*sinh(x)*cosh(x) + 2*a**6*b**2*sinh(x)**2 - 6*a**6*b*
*2*cosh(x)**2 - 12*a**5*b**3*sinh(x)*cosh(x) - 6*a**4*b**4*sinh(x)**2 + 6*a
**4*b**4*cosh(x)**2 + 12*a**3*b**5*sinh(x)*cosh(x) + 6*a**2*b**6*sinh(x)**2
 - 2*a**2*b**6*cosh(x)**2 - 4*a*b**7*sinh(x)*cosh(x) - 2*b**8*sinh(x)**2) -
 6*a**2*b**3*x*sinh(x)**2/(2*a**8*cosh(x)**2 + 4*a**7*b*sinh(x)*cosh(x) + 2
*a**6*b**2*sinh(x)**2 - 6*a**6*b**2*cosh(x)**2 - 12*a**5*b**3*sinh(x)*cosh(
x) - 6*a**4*b**4*sinh(x)**2 + 6*a**4*b**4*cosh(x)**2 + 12*a**3*b**5*sinh(x)
*cosh(x) + 6*a**2*b**6*sinh(x)**2 - 2*a**2*b**6*cosh(x)**2 - 4*a*b**7*sinh(
x)*cosh(x) - 2*b**8*sinh(x)**2) - 2*a**2*b**3*x*cosh(x)**2/(2*a**8*cosh(x)*
*2 + 4*a**7*b*sinh(x)*cosh(x) + 2*a**6*b**2*sinh(x)**2 - 6*a**6*b**2*cosh(x
)**2 - 12*a**5*b**3*sinh(x)*cosh(x) - 6*a**4*b**4*sinh(x)**2 + 6*a**4*b**4*
cosh(x)**2 + 12*a**3*b**5*sinh(x)*cosh(x) + 6*a**2*b**6*sinh(x)**2 - 2*a**2
*b**6*cosh(x)**2 - 4*a*b**7*sinh(x)*cosh(x) - 2*b**8*sinh(x)**2) + 12*a**2*
b**3*log(cosh(x) + b*sinh(x)/a)*sinh(x)*cosh(x)/(2*a**8*cosh(x)**2 + 4*a**7
*b*sinh(x)*cosh(x) + 2*a**6*b**2*sinh(x)**2 - 6*a**6*b**2*cosh(x)**2 - 12*a
**5*b**3*sinh(x)*cosh(x) - 6*a**4*b**4*sinh(x)**2 + 6*a**4*b**4*cosh(x)**2
+ 12*a**3*b**5*sinh(x)*cosh(x) + 6*a**2*b**6*sinh(x)**2 - 2*a**2*b**6*cosh(
x)**2 - 4*a*b**7*sinh(x)*cosh(x) - 2*b**8*sinh(x)**2) - 4*a*b**4*x*sinh(x)*
cosh(x)/(2*a**8*cosh(x)**2 + 4*a**7*b*sinh(x)*cosh(x) + 2*a**6*b**2*sinh(x)
**2 - 6*a**6*b**2*cosh(x)**2 - 12*a**5*b**3*sinh(x)*cosh(x) - 6*a**4*b**4*s
inh(x)**2 + 6*a**4*b**4*cosh(x)**2 + 12*a**3*b**5*sinh(x)*cosh(x) + 6*a**2*
b**6*sinh(x)**2 - 2*a**2*b**6*cosh(x)**2 - 4*a*b**7*sinh(x)*cosh(x) - 2*b**
8*sinh(x)**2) + 6*a*b**4*log(cosh(x) + b*sinh(x)/a)*sinh(x)**2/(2*a**8*cosh
(x)**2 + 4*a**7*b*sinh(x)*cosh(x) + 2*a**6*b**2*sinh(x)**2 - 6*a**6*b**2*co
sh(x)**2 - 12*a**5*b**3*sinh(x)*cosh(x) - 6*a**4*b**4*sinh(x)**2 + 6*a**4*b
**4*cosh(x)**2 + 12*a**3*b**5*sinh(x)*cosh(x) + 6*a**2*b**6*sinh(x)**2 - 2*
a**2*b**6*cosh(x)**2 - 4*a*b**7*sinh(x)*cosh(x) - 2*b**8*sinh(x)**2) - 3*a*
b**4*sinh(x)**2/(2*a**8*cosh(x)**2 + 4*a**7*b*sinh(x)*cosh(x) + 2*a**6*b**2
*sinh(x)**2 - 6*a**6*b**2*cosh(x)**2 - 12*a**5*b**3*sinh(x)*cosh(x) - 6*a**
4*b**4*sinh(x)**2 + 6*a**4*b**4*cosh(x)**2 + 12*a**3*b**5*sinh(x)*cosh(x) +
 6*a**2*b**6*sinh(x)**2 - 2*a**2*b**6*cosh(x)**2 - 4*a*b**7*sinh(x)*cosh(x)
 - 2*b**8*sinh(x)**2) - 2*b**5*x*sinh(x)**2/(2*a**8*cosh(x)**2 + 4*a**7*b*s
inh(x)*cosh(x) + 2*a**6*b**2*sinh(x)**2 - 6*a**6*b**2*cosh(x)**2 - 12*a**5*
b**3*sinh(x)*cosh(x) - 6*a**4*b**4*sinh(x)**2 + 6*a**4*b**4*cosh(x)**2 + 12
*a**3*b**5*sinh(x)*cosh(x) + 6*a**2*b**6*sinh(x)**2 - 2*a**2*b**6*cosh(x)**
2 - 4*a*b**7*sinh(x)*cosh(x) - 2*b**8*sinh(x)**2), True))
```

3.704 $\int \frac{\cosh(x)}{(a\cosh(x)+b\sinh(x))^3}\,dx$

Optimal. Leaf size=19

$$-\frac{\coth^2(x)}{2b(a\coth(x)+b)^2}$$

```
[Out] -1/2*coth(x)^2/b/(b+a*coth(x))^2
```

Rubi [A] time = 0.03, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3088, 37}

$$-\frac{\coth^2(x)}{2b(a\coth(x)+b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]/(a*Cosh[x] + b*Sinh[x])^3,x]
```

```
[Out] -Coth[x]^2/(2*b*(b + a*Coth[x])^2)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 3088

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[(x^m*(b +
 a*x)^n)/(1 + x^2)^((m + n + 2)/2), x], x, Cot[c + d*x]], x] /; FreeQ[{a, b
, c, d}, x] && IntegerQ[n] && IntegerQ[(m + n)/2] && NeQ[n, -1] &&  !(GtQ[n
, 0] && GtQ[m, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{\cosh(x)}{(a\cosh(x)+b\sinh(x))^3}\,dx &= i\,\text{Subst}\left(\int \frac{x}{(-ib+ax)^3}\,dx, x, -i\coth(x)\right)\\ &= -\frac{\coth^2(x)}{2b(b+a\coth(x))^2}\end{aligned}$$

Mathematica [B] time = 0.06, size = 40, normalized size = 2.11

$$\frac{a\sinh(2x)+b\cosh(2x)}{2(a-b)(a+b)(a\cosh(x)+b\sinh(x))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]/(a*Cosh[x] + b*Sinh[x])^3,x]
```

```
[Out] (b*Cosh[2*x] + a*Sinh[2*x])/(2*(a - b)*(a + b)*(a*Cosh[x] + b*Sinh[x])^2)
```

fricas [B] time = 0.42, size = 216, normalized size = 11.37

$$-\frac{}{\left(a^4+4\,a^3b+6\,a^2b^2+4\,ab^3+b^4\right)\cosh(x)^3+3\left(a^4+4\,a^3b+6\,a^2b^2+4\,ab^3+b^4\right)\cosh(x)\sinh(x)^2+\left(a^4+4\,a\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)/(a*cosh(x)+b*sinh(x))^3,x, algorithm="fricas")
```

```
[Out] -2*((2*a + b)*cosh(x) + b*sinh(x))/((a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 +
b^4)*cosh(x)^3 + 3*(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*cosh(x)*sinh
(x)^2 + (a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*sinh(x)^3 + (3*a^4 + 4*
a^3*b - 2*a^2*b^2 - 4*a*b^3 - b^4)*cosh(x) + (a^4 + 4*a^3*b + 2*a^2*b^2 - 4
*a*b^3 - 3*b^4 + 3*(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*cosh(x)^2)*s
inh(x))
```

giac [B] time = 0.13, size = 48, normalized size = 2.53

$$-\frac{2\left(ae^{(2x)}+be^{(2x)}+a\right)}{\left(a^2+2\,ab+b^2\right)\left(ae^{(2x)}+be^{(2x)}+a-b\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)/(a*cosh(x)+b*sinh(x))^3,x, algorithm="giac")
```

```
[Out] -2*(a*e^(2*x) + b*e^(2*x) + a)/((a^2 + 2*a*b + b^2)*(a*e^(2*x) + b*e^(2*x)
+ a - b)^2)
```

maple [B] time = 0.25, size = 55, normalized size = 2.89

$$-\frac{2\left(-\frac{\tanh^3\left(\frac{x}{2}\right)}{a}-\frac{b\left(\tanh^2\left(\frac{x}{2}\right)\right)}{a^2}-\frac{\tanh\left(\frac{x}{2}\right)}{a}\right)}{\left(a+2\tanh\left(\frac{x}{2}\right)b+a\left(\tanh^2\left(\frac{x}{2}\right)\right)\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)/(a*cosh(x)+b*sinh(x))^3,x)
```

```
[Out] -2*(-1/a*tanh(1/2*x)^3-1/a^2*b*tanh(1/2*x)^2-1/a*tanh(1/2*x))/(a+2*tanh(1/2
*x)*b+a*tanh(1/2*x)^2)^2
```

maxima [B] time = 0.33, size = 167, normalized size = 8.79

$$\frac{2(a-b)e^{(-2x)}}{a^4-2a^2b^2+b^4+2\left(a^4-2a^3b+2ab^3-b^4\right)e^{(-2x)}+\left(a^4-4a^3b+6a^2b^2-4ab^3+b^4\right)e^{(-4x)}}+\frac{}{a^4-2a^2b^2+b^4+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)/(a*cosh(x)+b*sinh(x))^3,x, algorithm="maxima")
```

```
[Out] 2*(a - b)*e^(-2*x)/(a^4 - 2*a^2*b^2 + b^4 + 2*(a^4 - 2*a^3*b + 2*a*b^3 - b^
4)*e^(-2*x) + (a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4)*e^(-4*x)) + 2*a/(
a^4 - 2*a^2*b^2 + b^4 + 2*(a^4 - 2*a^3*b + 2*a*b^3 - b^4)*e^(-2*x) + (a^4 -
 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4)*e^(-4*x))
```

mupad [B] time = 1.52, size = 42, normalized size = 2.21

$$-\frac{2a+e^{2x}\,(2a+2b)}{(a+b)^2\left(a-b+a\,e^{2x}+b\,e^{2x}\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)/(a*cosh(x) + b*sinh(x))^3,x)
```

```
[Out] -(2*a + exp(2*x)*(2*a + 2*b))/((a + b)^2*(a - b + a*exp(2*x) + b*exp(2*x))^
2)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)/(a*cosh(x)+b*sinh(x))**3,x)
```

```
[Out] Timed out
```

3.705 $\int \frac{\cosh^3(x)}{(a\cosh(x)+b\sinh(x))^3}\,dx$

Optimal. Leaf size=104

$$\frac{ax\left(a^2+3b^2\right)}{\left(a^2-b^2\right)^3}+\frac{2ab}{\left(a^2-b^2\right)^2(a+b\tanh(x))}+\frac{b}{2\left(a^2-b^2\right)(a+b\tanh(x))^2}-\frac{b\left(3a^2+b^2\right)\log(a\cosh(x)+b\sinh(x))}{\left(a^2-b^2\right)^3}$$

```
[Out] a*(a^2+3*b^2)*x/(a^2-b^2)^3-b*(3*a^2+b^2)*ln(a*cosh(x)+b*sinh(x))/(a^2-b^2)
^3+1/2*b/(a^2-b^2)/(a+b*tanh(x))^2+2*a*b/(a^2-b^2)^2/(a+b*tanh(x))
```

Rubi [A] time = 0.20, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {3086, 3483, 3529, 3531, 3530}

$$\frac{ax\left(a^2+3b^2\right)}{\left(a^2-b^2\right)^3}+\frac{2ab}{\left(a^2-b^2\right)^2(a+b\tanh(x))}+\frac{b}{2\left(a^2-b^2\right)(a+b\tanh(x))^2}-\frac{b\left(3a^2+b^2\right)\log(a\cosh(x)+b\sinh(x))}{\left(a^2-b^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]^3/(a*Cosh[x] + b*Sinh[x])^3,x]
```

```
[Out] (a*(a^2 + 3*b^2)*x)/(a^2 - b^2)^3 - (b*(3*a^2 + b^2)*Log[a*Cosh[x] + b*Sinh
[x]])/(a^2 - b^2)^3 + b/(2*(a^2 - b^2)*(a + b*Tanh[x])^2) + (2*a*b)/((a^2 -
 b^2)^2*(a + b*Tanh[x]))
```

Rule 3086

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin
[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[(a + b*Tan[c + d*x])^n, x] /;
 FreeQ[{a, b, c, d}, x] && EqQ[m + n, 0] && IntegerQ[n] && NeQ[a^2 + b^2, 0
]
```

Rule 3483

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(a +
b*Tan[c + d*x])^(n + 1))/(d*(n + 1)*(a^2 + b^2)), x] + Dist[1/(a^2 + b^2),
Int[(a - b*Tan[c + d*x])*(a + b*Tan[c + d*x])^(n + 1), x], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 + b^2, 0] && LtQ[n, -1]
```

Rule 3529

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[((b*c - a*d)*(a + b*Tan[e + f*x])^(m + 1))
```

```
/(f*(m + 1)*(a^2 + b^2)), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])
^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a,
 b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

Rule 3530

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c*Log[RemoveContent[a*Cos[e + f*x] + b*Sin[e + f
*x], x]])/(b*f), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

Rule 3531

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[((a*c + b*d)*x)/(a^2 + b^2), x] + Dist[(b*c - a
*d)/(a^2 + b^2), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && Ne
Q[a*c + b*d, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cosh^3(x)}{(a\cosh(x)+b\sinh(x))^3}\,dx &= \int \frac{1}{(a+b\tanh(x))^3}\,dx \\
&= \frac{b}{2\left(a^2-b^2\right)(a+b\tanh(x))^2}+\frac{\int \frac{a-b\tanh(x)}{(a+b\tanh(x))^2}\,dx}{a^2-b^2} \\
&= \frac{b}{2\left(a^2-b^2\right)(a+b\tanh(x))^2}+\frac{2ab}{\left(a^2-b^2\right)^2(a+b\tanh(x))}+\frac{\int \frac{a^2+b^2-2ab\tanh(x)}{a+b\tanh(x)}\,dx}{\left(a^2-b^2\right)^2} \\
&= \frac{a\left(a^2+3b^2\right)x}{\left(a^2-b^2\right)^3}+\frac{b}{2\left(a^2-b^2\right)(a+b\tanh(x))^2}+\frac{2ab}{\left(a^2-b^2\right)^2(a+b\tanh(x))}-\frac{(ib
\end{aligned}
$$

$$
= \frac{a\left(a^2+3b^2\right)x}{\left(a^2-b^2\right)^3}-\frac{b\left(3a^2+b^2\right)\log(a\cosh(x)+b\sinh(x))}{\left(a^2-b^2\right)^3}+\frac{b}{2\left(a^2-b^2\right)(a+b\tan
$$

Mathematica [A] time = 1.05, size = 119, normalized size = 1.14

$$
\frac{ax\left(a^2+3b^2\right)}{(a-b)^3(a+b)^3}+\frac{\left(-3a^2b-b^3\right)\log(a\cosh(x)+b\sinh(x))}{\left(a^2-b^2\right)^3}-\frac{b^3}{2(a-b)^2(a+b)^2(a\cosh(x)+b\sinh(x))^2}-\frac{}{(a-b)^2(a
$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]^3/(a*Cosh[x] + b*Sinh[x])^3,x]
```

```
[Out] (a*(a^2 + 3*b^2)*x)/((a - b)^3*(a + b)^3) + ((-3*a^2*b - b^3)*Log[a*Cosh[x]
 + b*Sinh[x]])/(a^2 - b^2)^3 - b^3/(2*(a - b)^2*(a + b)^2*(a*Cosh[x] + b*Si
nh[x])^2) - (3*b^2*Sinh[x])/((a - b)^2*(a + b)^2*(a*Cosh[x] + b*Sinh[x]))
```

fricas [B] time = 0.48, size = 1269, normalized size = 12.20

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^3/(a*cosh(x)+b*sinh(x))^3,x, algorithm="fricas")
```

```
[Out] ((a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*x*cosh(x)^4 + 4*
(a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*x*cosh(x)*sinh(x)
^3 + (a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*x*sinh(x)^4
+ 6*a^3*b^2 - 12*a^2*b^3 + 6*a*b^4 + 2*(3*a^3*b^2 - a^2*b^3 - 3*a*b^4 + b^5
 + (a^5 + 3*a^4*b + 2*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^5)*x)*cosh(x)^2 + 2
*(3*a^3*b^2 - a^2*b^3 - 3*a*b^4 + b^5 + 3*(a^5 + 5*a^4*b + 10*a^3*b^2 + 10*
a^2*b^3 + 5*a*b^4 + b^5)*x*cosh(x)^2 + (a^5 + 3*a^4*b + 2*a^3*b^2 - 2*a^2*b
^3 - 3*a*b^4 - b^5)*x)*sinh(x)^2 + (a^5 + a^4*b - 2*a^3*b^2 - 2*a^2*b^3 + a
*b^4 + b^5)*x - (3*a^4*b - 6*a^3*b^2 + 4*a^2*b^3 - 2*a*b^4 + b^5 + (3*a^4*b
 + 6*a^3*b^2 + 4*a^2*b^3 + 2*a*b^4 + b^5)*cosh(x)^4 + 4*(3*a^4*b + 6*a^3*b^
2 + 4*a^2*b^3 + 2*a*b^4 + b^5)*cosh(x)*sinh(x)^3 + (3*a^4*b + 6*a^3*b^2 + 4
*a^2*b^3 + 2*a*b^4 + b^5)*sinh(x)^4 + 2*(3*a^4*b - 2*a^2*b^3 - b^5)*cosh(x)
^2 + 2*(3*a^4*b - 2*a^2*b^3 - b^5 + 3*(3*a^4*b + 6*a^3*b^2 + 4*a^2*b^3 + 2*
a*b^4 + b^5)*cosh(x)^2)*sinh(x)^2 + 4*((3*a^4*b + 6*a^3*b^2 + 4*a^2*b^3 + 2
*a*b^4 + b^5)*cosh(x)^3 + (3*a^4*b - 2*a^2*b^3 - b^5)*cosh(x))*sinh(x))*log
(2*(a*cosh(x) + b*sinh(x))/(cosh(x) - sinh(x))) + 4*((a^5 + 5*a^4*b + 10*a^
3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*x*cosh(x)^3 + (3*a^3*b^2 - a^2*b^3 - 3*
a*b^4 + b^5 + (a^5 + 3*a^4*b + 2*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^5)*x)*co
sh(x))*sinh(x))/(a^8 - 2*a^7*b - 2*a^6*b^2 + 6*a^5*b^3 - 6*a^3*b^5 + 2*a^2*
b^6 + 2*a*b^7 - b^8 + (a^8 + 2*a^7*b - 2*a^6*b^2 - 6*a^5*b^3 + 6*a^3*b^5 +
2*a^2*b^6 - 2*a*b^7 - b^8)*cosh(x)^4 + 4*(a^8 + 2*a^7*b - 2*a^6*b^2 - 6*a^5
*b^3 + 6*a^3*b^5 + 2*a^2*b^6 - 2*a*b^7 - b^8)*cosh(x)*sinh(x)^3 + (a^8 + 2*
a^7*b - 2*a^6*b^2 - 6*a^5*b^3 + 6*a^3*b^5 + 2*a^2*b^6 - 2*a*b^7 - b^8)*sinh
(x)^4 + 2*(a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8)*cosh(x)^2 + 2*(a^
8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8 + 3*(a^8 + 2*a^7*b - 2*a^6*b^2
- 6*a^5*b^3 + 6*a^3*b^5 + 2*a^2*b^6 - 2*a*b^7 - b^8)*cosh(x)^2)*sinh(x)^2 +
 4*((a^8 + 2*a^7*b - 2*a^6*b^2 - 6*a^5*b^3 + 6*a^3*b^5 + 2*a^2*b^6 - 2*a*b^
7 - b^8)*cosh(x)^3 + (a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8)*cosh(x
))*sinh(x))
```

giac [B] time = 0.15, size = 251, normalized size = 2.41

$$-\frac{\left(3\,a^{2}b+b^{3}\right)\log\left(\left|ae^{(2\,x)}+be^{(2\,x)}+a-b\right|\right)}{a^{6}-3\,a^{4}b^{2}+3\,a^{2}b^{4}-b^{6}}+\frac{x}{a^{3}-3\,a^{2}b+3\,ab^{2}-b^{3}}+\frac{9\,a^{3}be^{(4\,x)}+9\,a^{2}b^{2}e^{(4\,x)}+3\,ab^{3}e^{(4\,x)}+3\,b^{4}e^{(\cdots}}{2\left(a^{5}-a^{4}b\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^3/(a*cosh(x)+b*sinh(x))^3,x, algorithm="giac")
```

```
[Out] -(3*a^2*b + b^3)*log(abs(a*e^(2*x) + b*e^(2*x) + a - b))/(a^6 - 3*a^4*b^2 +
 3*a^2*b^4 - b^6) + x/(a^3 - 3*a^2*b + 3*a*b^2 - b^3) + 1/2*(9*a^3*b*e^(4*x
) + 9*a^2*b^2*e^(4*x) + 3*a*b^3*e^(4*x) + 3*b^4*e^(4*x) + 18*a^3*b*e^(2*x)
- 6*a^2*b^2*e^(2*x) - 10*a*b^3*e^(2*x) - 2*b^4*e^(2*x) + 9*a^3*b - 15*a^2*b
^2 + 3*a*b^3 + 3*b^4)/((a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*
(a*e^(2*x) + b*e^(2*x) + a - b)^2)
```

maple [B] time = 0.29, size = 494, normalized size = 4.75

$$-\frac{\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{\left(a+b\right)^{3}}-\frac{6b^{2}a^{3}\left(\tanh^{3}\left(\frac{x}{2}\right)\right)}{\left(a-b\right)^{3}\left(a+b\right)^{3}\left(a+2\tanh\left(\frac{x}{2}\right)b+a\left(\tanh^{2}\left(\frac{x}{2}\right)\right)\right)^{2}}+\frac{8b^{4}a\left(\tanh^{3}\left(\frac{x}{2}\right)\right)}{\left(a-b\right)^{3}\left(a+b\right)^{3}\left(a+2\tanh\left(\frac{x}{2}\right)b+a\left(\cdots\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^3/(a*cosh(x)+b*sinh(x))^3,x)
```

```
[Out] -1/(a+b)^3*ln(tanh(1/2*x)-1)-6*b^2/(a-b)^3/(a+b)^3/(a+2*tanh(1/2*x)*b+a*tan
h(1/2*x)^2)^2*a^3*tanh(1/2*x)^3+8*b^4/(a-b)^3/(a+b)^3/(a+2*tanh(1/2*x)*b+a*
tanh(1/2*x)^2)^2*a*tanh(1/2*x)^3-2*b^6/(a-b)^3/(a+b)^3/(a+2*tanh(1/2*x)*b+a
*tanh(1/2*x)^2)^2/a*tanh(1/2*x)^3-10*b^3/(a-b)^3/(a+b)^3/(a+2*tanh(1/2*x)*b
+a*tanh(1/2*x)^2)^2*a^2*tanh(1/2*x)^2+12*b^5/(a-b)^3/(a+b)^3/(a+2*tanh(1/2*
x)*b+a*tanh(1/2*x)^2)^2*tanh(1/2*x)^2-2*b^7/(a-b)^3/(a+b)^3/(a+2*tanh(1/2*x
)*b+a*tanh(1/2*x)^2)^2/a^2*tanh(1/2*x)^2-6*b^2/(a-b)^3/(a+b)^3/(a+2*tanh(1/
2*x)*b+a*tanh(1/2*x)^2)^2*a^3*tanh(1/2*x)+8*b^4/(a-b)^3/(a+b)^3/(a+2*tanh(1
/2*x)*b+a*tanh(1/2*x)^2)^2*a*tanh(1/2*x)-2*b^6/(a-b)^3/(a+b)^3/(a+2*tanh(1/
2*x)*b+a*tanh(1/2*x)^2)^2/a*tanh(1/2*x)-3*b/(a-b)^3/(a+b)^3*ln(a+2*tanh(1/2
*x)*b+a*tanh(1/2*x)^2)*a^2-b^3/(a-b)^3/(a+b)^3*ln(a+2*tanh(1/2*x)*b+a*tanh(
1/2*x)^2)+1/(a-b)^3*ln(tanh(1/2*x)+1)
```

maxima [B] time = 0.47, size = 292, normalized size = 2.81

$$-\frac{\left(3\,a^{2}b+b^{3}\right)\log\left(-(a-b)e^{(-2\,x)}-a-b\right)}{a^{6}-3\,a^{4}b^{2}+3\,a^{2}b^{4}-b^{6}}-\frac{2\cdots}{a^{7}+a^{6}b-3\,a^{5}b^{2}-3\,a^{4}b^{3}+3\,a^{3}b^{4}+3\,a^{2}b^{5}-ab^{6}-b^{7}+2\left(a^{7}-a^{6}b-\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^3/(a*cosh(x)+b*sinh(x))^3,x, algorithm="maxima")
```

```
[Out] -(3*a^2*b + b^3)*log(-(a - b)*e^(-2*x) - a - b)/(a^6 - 3*a^4*b^2 + 3*a^2*b^
4 - b^6) - 2*(3*a^2*b^2 + 3*a*b^3 + (3*a^2*b^2 - 2*a*b^3 - b^4)*e^(-2*x))/(
a^7 + a^6*b - 3*a^5*b^2 - 3*a^4*b^3 + 3*a^3*b^4 + 3*a^2*b^5 - a*b^6 - b^7 +
 2*(a^7 - a^6*b - 3*a^5*b^2 + 3*a^4*b^3 + 3*a^3*b^4 - 3*a^2*b^5 - a*b^6 + b
^7)*e^(-2*x) + (a^7 - 3*a^6*b + a^5*b^2 + 5*a^4*b^3 - 5*a^3*b^4 - a^2*b^5 +
 3*a*b^6 - b^7)*e^(-4*x)) + x/(a^3 + 3*a^2*b + 3*a*b^2 + b^3)
```

mupad [B] time = 1.63, size = 159, normalized size = 1.53

$$\frac{x}{(a-b)^3}-\frac{\ln\left(a-b+a\,e^{2\,x}+b\,e^{2\,x}\right)\,\left(3\,a^2\,b+b^3\right)}{a^6-3\,a^4\,b^2+3\,a^2\,b^4-b^6}+\frac{2\,\left(3\,a\,b^2-b^3\right)}{(a+b)^3\,(a-b)^2\,\left(a-b+e^{2\,x}\,(a+b)\right)}+\frac{}{(a+b)^3\,(a-b)\,\left(e^{4\,x}\,\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^3/(a*cosh(x) + b*sinh(x))^3,x)
```

```
[Out] x/(a - b)^3 - (log(a - b + a*exp(2*x) + b*exp(2*x))*(3*a^2*b + b^3))/(a^6 -
 b^6 + 3*a^2*b^4 - 3*a^4*b^2) + (2*(3*a*b^2 - b^3))/((a + b)^3*(a - b)^2*(a
 - b + exp(2*x)*(a + b))) + (2*b^3)/((a + b)^3*(a - b)*(exp(4*x)*(a + b)^2
+ (a - b)^2 + 2*exp(2*x)*(a + b)*(a - b)))
```

sympy [A] time = 4.39, size = 3840, normalized size = 36.92

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)**3/(a*cosh(x)+b*sinh(x))**3,x)
```

```
[Out] Piecewise((zoo*(log(sinh(x)) - cosh(x)**2/(2*sinh(x)**2)), Eq(a, 0) & Eq(b,
 0)), (3*x*sinh(x)**3/(-24*b**3*sinh(x)**3 + 72*b**3*sinh(x)**2*cosh(x) - 7
2*b**3*sinh(x)*cosh(x)**2 + 24*b**3*cosh(x)**3) - 9*x*sinh(x)**2*cosh(x)/(-
24*b**3*sinh(x)**3 + 72*b**3*sinh(x)**2*cosh(x) - 72*b**3*sinh(x)*cosh(x)**
2 + 24*b**3*cosh(x)**3) + 9*x*sinh(x)*cosh(x)**2/(-24*b**3*sinh(x)**3 + 72*
b**3*sinh(x)**2*cosh(x) - 72*b**3*sinh(x)*cosh(x)**2 + 24*b**3*cosh(x)**3)
- 3*x*cosh(x)**3/(-24*b**3*sinh(x)**3 + 72*b**3*sinh(x)**2*cosh(x) - 72*b**
3*sinh(x)*cosh(x)**2 + 24*b**3*cosh(x)**3) - sinh(x)**3/(-24*b**3*sinh(x)**
3 + 72*b**3*sinh(x)**2*cosh(x) - 72*b**3*sinh(x)*cosh(x)**2 + 24*b**3*cosh(
x)**3) + 6*sinh(x)*cosh(x)**2/(-24*b**3*sinh(x)**3 + 72*b**3*sinh(x)**2*cos
h(x) - 72*b**3*sinh(x)*cosh(x)**2 + 24*b**3*cosh(x)**3) - 9*cosh(x)**3/(-24
*b**3*sinh(x)**3 + 72*b**3*sinh(x)**2*cosh(x) - 72*b**3*sinh(x)*cosh(x)**2
+ 24*b**3*cosh(x)**3), Eq(a, -b)), (3*x*sinh(x)**3/(24*b**3*sinh(x)**3 + 72
*b**3*sinh(x)**2*cosh(x) + 72*b**3*sinh(x)*cosh(x)**2 + 24*b**3*cosh(x)**3)
 + 9*x*sinh(x)**2*cosh(x)/(24*b**3*sinh(x)**3 + 72*b**3*sinh(x)**2*cosh(x)
```

```
+ 72*b**3*sinh(x)*cosh(x)**2 + 24*b**3*cosh(x)**3) + 9*x*sinh(x)*cosh(x)**2
/(24*b**3*sinh(x)**3 + 72*b**3*sinh(x)**2*cosh(x) + 72*b**3*sinh(x)*cosh(x)
**2 + 24*b**3*cosh(x)**3) + 3*x*cosh(x)**3/(24*b**3*sinh(x)**3 + 72*b**3*si
nh(x)**2*cosh(x) + 72*b**3*sinh(x)*cosh(x)**2 + 24*b**3*cosh(x)**3) + sinh(
x)**3/(24*b**3*sinh(x)**3 + 72*b**3*sinh(x)**2*cosh(x) + 72*b**3*sinh(x)*co
sh(x)**2 + 24*b**3*cosh(x)**3) - 6*sinh(x)*cosh(x)**2/(24*b**3*sinh(x)**3 +
 72*b**3*sinh(x)**2*cosh(x) + 72*b**3*sinh(x)*cosh(x)**2 + 24*b**3*cosh(x)*
*3) - 9*cosh(x)**3/(24*b**3*sinh(x)**3 + 72*b**3*sinh(x)**2*cosh(x) + 72*b*
*3*sinh(x)*cosh(x)**2 + 24*b**3*cosh(x)**3), Eq(a, b)), ((log(sinh(x)) - co
sh(x)**2/(2*sinh(x)**2))/b**3, Eq(a, 0)), (2*a**5*x*cosh(x)**2/(2*a**8*cosh
(x)**2 + 4*a**7*b*sinh(x)*cosh(x) + 2*a**6*b**2*sinh(x)**2 - 6*a**6*b**2*co
sh(x)**2 - 12*a**5*b**3*sinh(x)*cosh(x) - 6*a**4*b**4*sinh(x)**2 + 6*a**4*b
**4*cosh(x)**2 + 12*a**3*b**5*sinh(x)*cosh(x) + 6*a**2*b**6*sinh(x)**2 - 2*
a**2*b**6*cosh(x)**2 - 4*a*b**7*sinh(x)*cosh(x) - 2*b**8*sinh(x)**2) + 4*a*
*4*b*x*sinh(x)*cosh(x)/(2*a**8*cosh(x)**2 + 4*a**7*b*sinh(x)*cosh(x) + 2*a*
*6*b**2*sinh(x)**2 - 6*a**6*b**2*cosh(x)**2 - 12*a**5*b**3*sinh(x)*cosh(x)
- 6*a**4*b**4*sinh(x)**2 + 6*a**4*b**4*cosh(x)**2 + 12*a**3*b**5*sinh(x)*co
sh(x) + 6*a**2*b**6*sinh(x)**2 - 2*a**2*b**6*cosh(x)**2 - 4*a*b**7*sinh(x)*
cosh(x) - 2*b**8*sinh(x)**2) - 6*a**4*b*log(cosh(x) + b*sinh(x)/a)*cosh(x)*
*2/(2*a**8*cosh(x)**2 + 4*a**7*b*sinh(x)*cosh(x) + 2*a**6*b**2*sinh(x)**2 -
 6*a**6*b**2*cosh(x)**2 - 12*a**5*b**3*sinh(x)*cosh(x) - 6*a**4*b**4*sinh(x
)**2 + 6*a**4*b**4*cosh(x)**2 + 12*a**3*b**5*sinh(x)*cosh(x) + 6*a**2*b**6*
sinh(x)**2 - 2*a**2*b**6*cosh(x)**2 - 4*a*b**7*sinh(x)*cosh(x) - 2*b**8*sin
h(x)**2) + 3*a**4*b*cosh(x)**2/(2*a**8*cosh(x)**2 + 4*a**7*b*sinh(x)*cosh(x
) + 2*a**6*b**2*sinh(x)**2 - 6*a**6*b**2*cosh(x)**2 - 12*a**5*b**3*sinh(x)*
cosh(x) - 6*a**4*b**4*sinh(x)**2 + 6*a**4*b**4*cosh(x)**2 + 12*a**3*b**5*si
nh(x)*cosh(x) + 6*a**2*b**6*sinh(x)**2 - 2*a**2*b**6*cosh(x)**2 - 4*a*b**7*
sinh(x)*cosh(x) - 2*b**8*sinh(x)**2) + 2*a**3*b**2*x*sinh(x)**2/(2*a**8*cos
h(x)**2 + 4*a**7*b*sinh(x)*cosh(x) + 2*a**6*b**2*sinh(x)**2 - 6*a**6*b**2*c
osh(x)**2 - 12*a**5*b**3*sinh(x)*cosh(x) - 6*a**4*b**4*sinh(x)**2 + 6*a**4*
b**4*cosh(x)**2 + 12*a**3*b**5*sinh(x)*cosh(x) + 6*a**2*b**6*sinh(x)**2 - 2
*a**2*b**6*cosh(x)**2 - 4*a*b**7*sinh(x)*cosh(x) - 2*b**8*sinh(x)**2) + 6*a
**3*b**2*x*cosh(x)**2/(2*a**8*cosh(x)**2 + 4*a**7*b*sinh(x)*cosh(x) + 2*a**
6*b**2*sinh(x)**2 - 6*a**6*b**2*cosh(x)**2 - 12*a**5*b**3*sinh(x)*cosh(x) -
 6*a**4*b**4*sinh(x)**2 + 6*a**4*b**4*cosh(x)**2 + 12*a**3*b**5*sinh(x)*cos
h(x) + 6*a**2*b**6*sinh(x)**2 - 2*a**2*b**6*cosh(x)**2 - 4*a*b**7*sinh(x)*c
osh(x) - 2*b**8*sinh(x)**2) - 12*a**3*b**2*log(cosh(x) + b*sinh(x)/a)*sinh(
x)*cosh(x)/(2*a**8*cosh(x)**2 + 4*a**7*b*sinh(x)*cosh(x) + 2*a**6*b**2*sinh
(x)**2 - 6*a**6*b**2*cosh(x)**2 - 12*a**5*b**3*sinh(x)*cosh(x) - 6*a**4*b**
4*sinh(x)**2 + 6*a**4*b**4*cosh(x)**2 + 12*a**3*b**5*sinh(x)*cosh(x) + 6*a*
*2*b**6*sinh(x)**2 - 2*a**2*b**6*cosh(x)**2 - 4*a*b**7*sinh(x)*cosh(x) - 2*
b**8*sinh(x)**2) + 12*a**2*b**3*x*sinh(x)*cosh(x)/(2*a**8*cosh(x)**2 + 4*a*
*7*b*sinh(x)*cosh(x) + 2*a**6*b**2*sinh(x)**2 - 6*a**6*b**2*cosh(x)**2 - 12
*a**5*b**3*sinh(x)*cosh(x) - 6*a**4*b**4*sinh(x)**2 + 6*a**4*b**4*cosh(x)**
2 + 12*a**3*b**5*sinh(x)*cosh(x) + 6*a**2*b**6*sinh(x)**2 - 2*a**2*b**6*cos
```

```
h(x)**2 - 4*a*b**7*sinh(x)*cosh(x) - 2*b**8*sinh(x)**2) - 6*a**2*b**3*log(c
osh(x) + b*sinh(x)/a)*sinh(x)**2/(2*a**8*cosh(x)**2 + 4*a**7*b*sinh(x)*cosh
(x) + 2*a**6*b**2*sinh(x)**2 - 6*a**6*b**2*cosh(x)**2 - 12*a**5*b**3*sinh(x
)*cosh(x) - 6*a**4*b**4*sinh(x)**2 + 6*a**4*b**4*cosh(x)**2 + 12*a**3*b**5*
sinh(x)*cosh(x) + 6*a**2*b**6*sinh(x)**2 - 2*a**2*b**6*cosh(x)**2 - 4*a*b**
7*sinh(x)*cosh(x) - 2*b**8*sinh(x)**2) - 2*a**2*b**3*log(cosh(x) + b*sinh(x
)/a)*cosh(x)**2/(2*a**8*cosh(x)**2 + 4*a**7*b*sinh(x)*cosh(x) + 2*a**6*b**2
*sinh(x)**2 - 6*a**6*b**2*cosh(x)**2 - 12*a**5*b**3*sinh(x)*cosh(x) - 6*a**
4*b**4*sinh(x)**2 + 6*a**4*b**4*cosh(x)**2 + 12*a**3*b**5*sinh(x)*cosh(x) +
 6*a**2*b**6*sinh(x)**2 - 2*a**2*b**6*cosh(x)**2 - 4*a*b**7*sinh(x)*cosh(x)
 - 2*b**8*sinh(x)**2) - 2*a**2*b**3*sinh(x)**2/(2*a**8*cosh(x)**2 + 4*a**7*
b*sinh(x)*cosh(x) + 2*a**6*b**2*sinh(x)**2 - 6*a**6*b**2*cosh(x)**2 - 12*a*
*5*b**3*sinh(x)*cosh(x) - 6*a**4*b**4*sinh(x)**2 + 6*a**4*b**4*cosh(x)**2 +
 12*a**3*b**5*sinh(x)*cosh(x) + 6*a**2*b**6*sinh(x)**2 - 2*a**2*b**6*cosh(x
)**2 - 4*a*b**7*sinh(x)*cosh(x) - 2*b**8*sinh(x)**2) - 4*a**2*b**3*cosh(x)*
*2/(2*a**8*cosh(x)**2 + 4*a**7*b*sinh(x)*cosh(x) + 2*a**6*b**2*sinh(x)**2 -
 6*a**6*b**2*cosh(x)**2 - 12*a**5*b**3*sinh(x)*cosh(x) - 6*a**4*b**4*sinh(x
)**2 + 6*a**4*b**4*cosh(x)**2 + 12*a**3*b**5*sinh(x)*cosh(x) + 6*a**2*b**6*
sinh(x)**2 - 2*a**2*b**6*cosh(x)**2 - 4*a*b**7*sinh(x)*cosh(x) - 2*b**8*sin
h(x)**2) + 6*a*b**4*x*sinh(x)**2/(2*a**8*cosh(x)**2 + 4*a**7*b*sinh(x)*cosh
(x) + 2*a**6*b**2*sinh(x)**2 - 6*a**6*b**2*cosh(x)**2 - 12*a**5*b**3*sinh(x
)*cosh(x) - 6*a**4*b**4*sinh(x)**2 + 6*a**4*b**4*cosh(x)**2 + 12*a**3*b**5*
sinh(x)*cosh(x) + 6*a**2*b**6*sinh(x)**2 - 2*a**2*b**6*cosh(x)**2 - 4*a*b**
7*sinh(x)*cosh(x) - 2*b**8*sinh(x)**2) - 4*a*b**4*log(cosh(x) + b*sinh(x)/a
)*sinh(x)*cosh(x)/(2*a**8*cosh(x)**2 + 4*a**7*b*sinh(x)*cosh(x) + 2*a**6*b*
*2*sinh(x)**2 - 6*a**6*b**2*cosh(x)**2 - 12*a**5*b**3*sinh(x)*cosh(x) - 6*a
**4*b**4*sinh(x)**2 + 6*a**4*b**4*cosh(x)**2 + 12*a**3*b**5*sinh(x)*cosh(x)
 + 6*a**2*b**6*sinh(x)**2 - 2*a**2*b**6*cosh(x)**2 - 4*a*b**7*sinh(x)*cosh(
x) - 2*b**8*sinh(x)**2) - 2*b**5*log(cosh(x) + b*sinh(x)/a)*sinh(x)**2/(2*a
**8*cosh(x)**2 + 4*a**7*b*sinh(x)*cosh(x) + 2*a**6*b**2*sinh(x)**2 - 6*a**6
*b**2*cosh(x)**2 - 12*a**5*b**3*sinh(x)*cosh(x) - 6*a**4*b**4*sinh(x)**2 +
6*a**4*b**4*cosh(x)**2 + 12*a**3*b**5*sinh(x)*cosh(x) + 6*a**2*b**6*sinh(x)
**2 - 2*a**2*b**6*cosh(x)**2 - 4*a*b**7*sinh(x)*cosh(x) - 2*b**8*sinh(x)**2
) + 2*b**5*sinh(x)**2/(2*a**8*cosh(x)**2 + 4*a**7*b*sinh(x)*cosh(x) + 2*a**
6*b**2*sinh(x)**2 - 6*a**6*b**2*cosh(x)**2 - 12*a**5*b**3*sinh(x)*cosh(x) -
 6*a**4*b**4*sinh(x)**2 + 6*a**4*b**4*cosh(x)**2 + 12*a**3*b**5*sinh(x)*cos
h(x) + 6*a**2*b**6*sinh(x)**2 - 2*a**2*b**6*cosh(x)**2 - 4*a*b**7*sinh(x)*c
osh(x) - 2*b**8*sinh(x)**2) + b**5*cosh(x)**2/(2*a**8*cosh(x)**2 + 4*a**7*b
*sinh(x)*cosh(x) + 2*a**6*b**2*sinh(x)**2 - 6*a**6*b**2*cosh(x)**2 - 12*a**
5*b**3*sinh(x)*cosh(x) - 6*a**4*b**4*sinh(x)**2 + 6*a**4*b**4*cosh(x)**2 +
12*a**3*b**5*sinh(x)*cosh(x) + 6*a**2*b**6*sinh(x)**2 - 2*a**2*b**6*cosh(x)
**2 - 4*a*b**7*sinh(x)*cosh(x) - 2*b**8*sinh(x)**2), True))
```

3.706 $\int \frac{\cosh(x)\sinh(x)}{a\cosh(x)+b\sinh(x)}\,dx$

Optimal. Leaf size=72

$$-\frac{b\sinh(x)}{a^2-b^2}+\frac{a\cosh(x)}{a^2-b^2}+\frac{ab\tan^{-1}\left(\frac{a\sinh(x)+b\cosh(x)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{3/2}}$$

```
[Out] a*b*arctan((b*cosh(x)+a*sinh(x))/(a^2-b^2)^(1/2))/(a^2-b^2)^(3/2)+a*cosh(x)
/(a^2-b^2)-b*sinh(x)/(a^2-b^2)
```

Rubi [A] time = 0.09, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {3109, 2637, 2638, 3074, 206}

$$-\frac{b\sinh(x)}{a^2-b^2}+\frac{a\cosh(x)}{a^2-b^2}+\frac{ab\tan^{-1}\left(\frac{a\sinh(x)+b\cosh(x)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cosh[x]*Sinh[x])/(a*Cosh[x] + b*Sinh[x]),x]
```

```
[Out] (a*b*ArcTan[(b*Cosh[x] + a*Sinh[x])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(3/2) + (
a*Cosh[x])/(a^2 - b^2) - (b*Sinh[x])/(a^2 - b^2)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3074

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> -Dist[d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d
*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3109

```
Int[(cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.))/(cos[(c_.
) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[b
/(a^2 + b^2), Int[Cos[c + d*x]^m*Sin[c + d*x]^(n - 1), x], x] + (Dist[a/(a^
2 + b^2), Int[Cos[c + d*x]^(m - 1)*Sin[c + d*x]^n, x], x] - Dist[(a*b)/(a^2
 + b^2), Int[(Cos[c + d*x]^(m - 1)*Sin[c + d*x]^(n - 1))/(a*Cos[c + d*x] +
b*Sin[c + d*x]), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] &&
IGtQ[m, 0] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cosh(x)\sinh(x)}{a\cosh(x)+b\sinh(x)}\,dx &= \frac{a\int \sinh(x)\,dx}{a^2-b^2}-\frac{b\int \cosh(x)\,dx}{a^2-b^2}+\frac{(ab)\int \frac{1}{a\cosh(x)+b\sinh(x)}\,dx}{a^2-b^2}\\
&= \frac{a\cosh(x)}{a^2-b^2}-\frac{b\sinh(x)}{a^2-b^2}+\frac{(iab)\,\mathrm{Subst}\left(\int \frac{1}{a^2-b^2-x^2}\,dx, x, -ib\cosh(x)-ia\sinh(x)\right)}{a^2-b^2}\\
&= \frac{ab\tan^{-1}\left(\frac{b\cosh(x)+a\sinh(x)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{3/2}}+\frac{a\cosh(x)}{a^2-b^2}-\frac{b\sinh(x)}{a^2-b^2}
\end{aligned}$$

Mathematica [A] time = 0.21, size = 79, normalized size = 1.10

$$\frac{b\sinh(x)}{b^2-a^2}+\frac{a\cosh(x)}{a^2-b^2}+\frac{2ab\tan^{-1}\left(\frac{a\tanh\left(\frac{x}{2}\right)+b}{\sqrt{a-b}\,\sqrt{a+b}}\right)}{(a-b)^{3/2}(a+b)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cosh[x]*Sinh[x])/(a*Cosh[x] + b*Sinh[x]),x]
```

```
[Out] (2*a*b*ArcTan[(b + a*Tanh[x/2])/(Sqrt[a - b]*Sqrt[a + b])])/((a - b)^(3/2)*
(a + b)^(3/2)) + (a*Cosh[x])/(a^2 - b^2) + (b*Sinh[x])/(-a^2 + b^2)
```

fricas [B] time = 0.49, size = 427, normalized size = 5.93

$$\left[\frac{a^3 + a^2b - ab^2 - b^3 + \left(a^3 - a^2b - ab^2 + b^3\right)\cosh(x)^2 + 2\left(a^3 - a^2b - ab^2 + b^3\right)\cosh(x)\sinh(x) + \left(a^3 - a^2b - ab^2\right.}{2\left(\left(a^4 - 2a^2b^2 + b\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*sinh(x)/(a*cosh(x)+b*sinh(x)),x, algorithm="fricas")
```

```
[Out] [1/2*(a^3 + a^2*b - a*b^2 - b^3 + (a^3 - a^2*b - a*b^2 + b^3)*cosh(x)^2 + 2
*(a^3 - a^2*b - a*b^2 + b^3)*cosh(x)*sinh(x) + (a^3 - a^2*b - a*b^2 + b^3)*
sinh(x)^2 + 2*(a*b*cosh(x) + a*b*sinh(x))*sqrt(-a^2 + b^2)*log(((a + b)*cos
h(x)^2 + 2*(a + b)*cosh(x)*sinh(x) + (a + b)*sinh(x)^2 + 2*sqrt(-a^2 + b^2)
*(cosh(x) + sinh(x)) - a + b)/((a + b)*cosh(x)^2 + 2*(a + b)*cosh(x)*sinh(x
) + (a + b)*sinh(x)^2 + a - b)))/((a^4 - 2*a^2*b^2 + b^4)*cosh(x) + (a^4 -
2*a^2*b^2 + b^4)*sinh(x)), 1/2*(a^3 + a^2*b - a*b^2 - b^3 + (a^3 - a^2*b -
a*b^2 + b^3)*cosh(x)^2 + 2*(a^3 - a^2*b - a*b^2 + b^3)*cosh(x)*sinh(x) + (a
^3 - a^2*b - a*b^2 + b^3)*sinh(x)^2 - 4*(a*b*cosh(x) + a*b*sinh(x))*sqrt(a^
2 - b^2)*arctan(sqrt(a^2 - b^2)/((a + b)*cosh(x) + (a + b)*sinh(x))))/((a^4
 - 2*a^2*b^2 + b^4)*cosh(x) + (a^4 - 2*a^2*b^2 + b^4)*sinh(x))]
```

giac [A] time = 0.12, size = 60, normalized size = 0.83

$$\frac{2\,ab\arctan\left(\frac{ae^x+be^x}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{\frac{3}{2}}} + \frac{e^{(-x)}}{2\,(a-b)} + \frac{e^x}{2\,(a+b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*sinh(x)/(a*cosh(x)+b*sinh(x)),x, algorithm="giac")
```

```
[Out] 2*a*b*arctan((a*e^x + b*e^x)/sqrt(a^2 - b^2))/(a^2 - b^2)^(3/2) + 1/2*e^(-x
)/(a - b) + 1/2*e^x/(a + b)
```

maple [A] time = 0.21, size = 92, normalized size = 1.28

$$\frac{4}{(4a-4b)\left(\tanh\left(\frac{x}{2}\right)+1\right)} + \frac{2ab\arctan\left(\frac{2a\tanh\left(\frac{x}{2}\right)+2b}{2\sqrt{a^2-b^2}}\right)}{(a+b)\,(a-b)\sqrt{a^2-b^2}} - \frac{4}{(4a+4b)\left(\tanh\left(\frac{x}{2}\right)-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)*sinh(x)/(a*cosh(x)+b*sinh(x)),x)
```

```
[Out] 4/(4*a-4*b)/(tanh(1/2*x)+1)+2*a*b/(a+b)/(a-b)/(a^2-b^2)^(1/2)*arctan(1/2*(2
*a*tanh(1/2*x)+2*b)/(a^2-b^2)^(1/2))-4/(4*a+4*b)/(tanh(1/2*x)-1)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*sinh(x)/(a*cosh(x)+b*sinh(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [B] time = 1.69, size = 157, normalized size = 2.18

$$\frac{e^x}{2\,a+2\,b}+\frac{e^{-x}}{2\,a-2\,b}+\frac{2\,\mathrm{atan}\left(\frac{a\,b\,e^x\,\sqrt{a^6-3\,a^4\,b^2+3\,a^2\,b^4-b^6}}{a^3\,\sqrt{a^2\,b^2}+b^3\,\sqrt{a^2\,b^2}-a\,b^2\,\sqrt{a^2\,b^2}-a^2\,b\,\sqrt{a^2\,b^2}}\right)\,\sqrt{a^2\,b^2}}{\sqrt{a^6-3\,a^4\,b^2+3\,a^2\,b^4-b^6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cosh(x)*sinh(x))/(a*cosh(x) + b*sinh(x)),x)
```

```
[Out] exp(x)/(2*a + 2*b) + exp(-x)/(2*a - 2*b) + (2*atan((a*b*exp(x)*(a^6 - b^6 +
 3*a^2*b^4 - 3*a^4*b^2)^(1/2))/(a^3*(a^2*b^2)^(1/2) + b^3*(a^2*b^2)^(1/2) -
 a*b^2*(a^2*b^2)^(1/2) - a^2*b*(a^2*b^2)^(1/2)))*(a^2*b^2)^(1/2))/(a^6 - b^
6 + 3*a^2*b^4 - 3*a^4*b^2)^(1/2)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*sinh(x)/(a*cosh(x)+b*sinh(x)),x)
```

```
[Out] Timed out
```

3.707 $\int \frac{\cosh(x)\sinh^2(x)}{a\cosh(x)+b\sinh(x)}\,dx$

Optimal. Leaf size=102

$$-\frac{ax}{2\left(a^2-b^2\right)}-\frac{ab^2x}{\left(a^2-b^2\right)^2}-\frac{b\sinh^2(x)}{2\left(a^2-b^2\right)}+\frac{a\sinh(x)\cosh(x)}{2\left(a^2-b^2\right)}+\frac{a^2b\log(a\cosh(x)+b\sinh(x))}{\left(a^2-b^2\right)^2}$$

```
[Out] -a*b^2*x/(a^2-b^2)^2-1/2*a*x/(a^2-b^2)+a^2*b*ln(a*cosh(x)+b*sinh(x))/(a^2-b
^2)^2+1/2*a*cosh(x)*sinh(x)/(a^2-b^2)-1/2*b*sinh(x)^2/(a^2-b^2)
```

Rubi [A] time = 0.16, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {3109, 2564, 30, 2635, 8, 3097, 3133}

$$-\frac{ax}{2\left(a^2-b^2\right)}-\frac{ab^2x}{\left(a^2-b^2\right)^2}-\frac{b\sinh^2(x)}{2\left(a^2-b^2\right)}+\frac{a\sinh(x)\cosh(x)}{2\left(a^2-b^2\right)}+\frac{a^2b\log(a\cosh(x)+b\sinh(x))}{\left(a^2-b^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Cosh[x]*Sinh[x]^2)/(a*Cosh[x] + b*Sinh[x]),x]
```

```
[Out] -((a*b^2*x)/(a^2 - b^2)^2) - (a*x)/(2*(a^2 - b^2)) + (a^2*b*Log[a*Cosh[x] +
 b*Sinh[x]])/(a^2 - b^2)^2 + (a*Cosh[x]*Sinh[x])/(2*(a^2 - b^2)) - (b*Sinh[
x]^2)/(2*(a^2 - b^2))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 3097

```
Int[sin[(c_.) + (d_.)*(x_)]/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.
) + (d_.)*(x_)]), x_Symbol] :> Simp[(b*x)/(a^2 + b^2), x] - Dist[a/(a^2 + b
^2), Int[(b*Cos[c + d*x] - a*Sin[c + d*x])/(a*Cos[c + d*x] + b*Sin[c + d*x]
), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3109

```
Int[(cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.))/(cos[(c_.
) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[b
/(a^2 + b^2), Int[Cos[c + d*x]^m*Sin[c + d*x]^(n - 1), x], x] + (Dist[a/(a^
2 + b^2), Int[Cos[c + d*x]^(m - 1)*Sin[c + d*x]^n, x], x] - Dist[(a*b)/(a^2
 + b^2), Int[(Cos[c + d*x]^(m - 1)*Sin[c + d*x]^(n - 1))/(a*Cos[c + d*x] +
b*Sin[c + d*x]), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] &&
IGtQ[m, 0] && IGtQ[n, 0]
```

Rule 3133

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)]), x
_Symbol] :> Simp[((b*B + c*C)*x)/(b^2 + c^2), x] + Simp[((c*B - b*C)*Log[a
+ b*Cos[d + e*x] + c*Sin[d + e*x]])/(e*(b^2 + c^2)), x] /; FreeQ[{a, b, c,
d, e, A, B, C}, x] && NeQ[b^2 + c^2, 0] && EqQ[A*(b^2 + c^2) - a*(b*B + c*C
), 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cosh(x)\sinh^2(x)}{a\cosh(x)+b\sinh(x)}\,dx &= \frac{a\int \sinh^2(x)\,dx}{a^2-b^2}-\frac{b\int \cosh(x)\sinh(x)\,dx}{a^2-b^2}+\frac{(ab)\int \frac{\sinh(x)}{a\cosh(x)+b\sinh(x)}\,dx}{a^2-b^2}\\
&= -\frac{ab^2x}{\left(a^2-b^2\right)^2}+\frac{a\cosh(x)\sinh(x)}{2\left(a^2-b^2\right)}+\frac{\left(ia^2b\right)\int \frac{-ib\cosh(x)-ia\sinh(x)}{a\cosh(x)+b\sinh(x)}\,dx}{\left(a^2-b^2\right)^2}-\frac{a\int 1\,dx}{2\left(a^2-b^2\right)}+\\
&= -\frac{ab^2x}{\left(a^2-b^2\right)^2}-\frac{ax}{2\left(a^2-b^2\right)}+\frac{a^2b\log(a\cosh(x)+b\sinh(x))}{\left(a^2-b^2\right)^2}+\frac{a\cosh(x)\sinh(x)}{2\left(a^2-b^2\right)}
\end{aligned}$$

Mathematica [A] time = 0.25, size = 73, normalized size = 0.72

$$\frac{\left(b^3-a^2b\right)\cosh(2x)+a\left(-2x\left(a^2+b^2\right)+\left(a^2-b^2\right)\sinh(2x)+4ab\log(a\cosh(x)+b\sinh(x))\right)}{4(a-b)^2(a+b)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cosh[x]*Sinh[x]^2)/(a*Cosh[x] + b*Sinh[x]),x]
```

```
[Out] ((-(a^2*b) + b^3)*Cosh[2*x] + a*(-2*(a^2 + b^2)*x + 4*a*b*Log[a*Cosh[x] + b
*Sinh[x]] + (a^2 - b^2)*Sinh[2*x]))/(4*(a - b)^2*(a + b)^2)
```

fricas [B] time = 0.45, size = 334, normalized size = 3.27

$$\frac{\left(a^3-a^2b-ab^2+b^3\right)\cosh(x)^4+4\left(a^3-a^2b-ab^2+b^3\right)\cosh(x)\sinh(x)^3+\left(a^3-a^2b-ab^2+b^3\right)\sinh(x)^4-4\left(a\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*sinh(x)^2/(a*cosh(x)+b*sinh(x)),x, algorithm="fricas")
```

```
[Out] 1/8*((a^3 - a^2*b - a*b^2 + b^3)*cosh(x)^4 + 4*(a^3 - a^2*b - a*b^2 + b^3)*
cosh(x)*sinh(x)^3 + (a^3 - a^2*b - a*b^2 + b^3)*sinh(x)^4 - 4*(a^3 + 2*a^2*
b + a*b^2)*x*cosh(x)^2 - a^3 - a^2*b + a*b^2 + b^3 + 2*(3*(a^3 - a^2*b - a*
b^2 + b^3)*cosh(x)^2 - 2*(a^3 + 2*a^2*b + a*b^2)*x)*sinh(x)^2 + 8*(a^2*b*co
sh(x)^2 + 2*a^2*b*cosh(x)*sinh(x) + a^2*b*sinh(x)^2)*log(2*(a*cosh(x) + b*s
inh(x))/(cosh(x) - sinh(x))) + 4*((a^3 - a^2*b - a*b^2 + b^3)*cosh(x)^3 - 2
*(a^3 + 2*a^2*b + a*b^2)*x*cosh(x))*sinh(x))/((a^4 - 2*a^2*b^2 + b^4)*cosh(
x)^2 + 2*(a^4 - 2*a^2*b^2 + b^4)*cosh(x)*sinh(x) + (a^4 - 2*a^2*b^2 + b^4)*
sinh(x)^2)
```

giac [A] time = 0.11, size = 101, normalized size = 0.99

$$\frac{a^2b\log\left(\left|ae^{(2x)}+be^{(2x)}+a-b\right|\right)}{a^4-2a^2b^2+b^4}-\frac{ax}{2\left(a^2-2ab+b^2\right)}+\frac{\left(2ae^{(2x)}-a+b\right)e^{(-2x)}}{8\left(a^2-2ab+b^2\right)}+\frac{e^{(2x)}}{8(a+b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*sinh(x)^2/(a*cosh(x)+b*sinh(x)),x, algorithm="giac")
```

```
[Out] a^2*b*log(abs(a*e^(2*x) + b*e^(2*x) + a - b))/(a^4 - 2*a^2*b^2 + b^4) - 1/2
*a*x/(a^2 - 2*a*b + b^2) + 1/8*(2*a*e^(2*x) - a + b)*e^(-2*x)/(a^2 - 2*a*b
+ b^2) + 1/8*e^(2*x)/(a + b)
```

maple [A] time = 0.21, size = 145, normalized size = 1.42

$$\frac{4}{(8a+8b)\left(\tanh\left(\frac{x}{2}\right)-1\right)^2}+\frac{8}{(16a+16b)\left(\tanh\left(\frac{x}{2}\right)-1\right)}+\frac{\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)a}{2(a+b)^2}+\frac{a^2b\ln\left(a+2\tanh\left(\frac{x}{2}\right)b+a\left(\tanh^2\right.\right.}{(a-b)^2(a+b)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)*sinh(x)^2/(a*cosh(x)+b*sinh(x)),x)
```

```
[Out] 4/(8*a+8*b)/(tanh(1/2*x)-1)^2+8/(16*a+16*b)/(tanh(1/2*x)-1)+1/2/(a+b)^2*ln(
tanh(1/2*x)-1)*a+a^2*b/(a-b)^2/(a+b)^2*ln(a+2*tanh(1/2*x)*b+a*tanh(1/2*x)^2
)-4/(8*a-8*b)/(tanh(1/2*x)+1)^2+8/(16*a-16*b)/(tanh(1/2*x)+1)-1/2/(a-b)^2*l
n(tanh(1/2*x)+1)*a
```

maxima [A] time = 0.33, size = 83, normalized size = 0.81

$$\frac{a^2b\log\left(-(a-b)e^{(-2x)}-a-b\right)}{a^4-2a^2b^2+b^4}-\frac{ax}{2\left(a^2+2ab+b^2\right)}+\frac{e^{(2x)}}{8(a+b)}-\frac{e^{(-2x)}}{8(a-b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*sinh(x)^2/(a*cosh(x)+b*sinh(x)),x, algorithm="maxima")
```

```
[Out] a^2*b*log(-(a - b)*e^(-2*x) - a - b)/(a^4 - 2*a^2*b^2 + b^4) - 1/2*a*x/(a^2
 + 2*a*b + b^2) + 1/8*e^(2*x)/(a + b) - 1/8*e^(-2*x)/(a - b)
```

mupad [B] time = 1.68, size = 81, normalized size = 0.79

$$\frac{e^{2x}}{8a+8b}-\frac{e^{-2x}}{8a-8b}-\frac{ax}{2(a-b)^2}+\frac{a^2b\ln\left(a-b+ae^{2x}+be^{2x}\right)}{a^4-2a^2b^2+b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cosh(x)*sinh(x)^2)/(a*cosh(x) + b*sinh(x)),x)
```

```
[Out] exp(2*x)/(8*a + 8*b) - exp(-2*x)/(8*a - 8*b) - (a*x)/(2*(a - b)^2) + (a^2*b
*log(a - b + a*exp(2*x) + b*exp(2*x)))/(a^4 + b^4 - 2*a^2*b^2)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*sinh(x)**2/(a*cosh(x)+b*sinh(x)),x)
```

```
[Out] Timed out
```

3.708 $\int \frac{\cosh(x)\sinh^3(x)}{a\cosh(x)+b\sinh(x)}\,dx$

Optimal. Leaf size=137

$$-\frac{b\sinh^3(x)}{3(a^2-b^2)}+\frac{a^2b\sinh(x)}{(a^2-b^2)^2}+\frac{a\cosh^3(x)}{3(a^2-b^2)}-\frac{a\cosh(x)}{a^2-b^2}-\frac{ab^2\cosh(x)}{(a^2-b^2)^2}-\frac{a^3b\tan^{-1}\left(\frac{a\sinh(x)+b\cosh(x)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{5/2}}$$

```
[Out] -a^3*b*arctan((b*cosh(x)+a*sinh(x))/(a^2-b^2)^(1/2))/(a^2-b^2)^(5/2)-a*b^2*
cosh(x)/(a^2-b^2)^2-a*cosh(x)/(a^2-b^2)+1/3*a*cosh(x)^3/(a^2-b^2)+a^2*b*sin
h(x)/(a^2-b^2)^2-1/3*b*sinh(x)^3/(a^2-b^2)
```

Rubi [A] time = 0.20, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {3109, 2564, 30, 2633, 3099, 3074, 206, 2638}

$$-\frac{b\sinh^3(x)}{3(a^2-b^2)}+\frac{a^2b\sinh(x)}{(a^2-b^2)^2}+\frac{a\cosh^3(x)}{3(a^2-b^2)}-\frac{a\cosh(x)}{a^2-b^2}-\frac{ab^2\cosh(x)}{(a^2-b^2)^2}-\frac{a^3b\tan^{-1}\left(\frac{a\sinh(x)+b\cosh(x)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cosh[x]*Sinh[x]^3)/(a*Cosh[x] + b*Sinh[x]),x]
```

```
[Out] -((a^3*b*ArcTan[(b*Cosh[x] + a*Sinh[x])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(5/2)
) - (a*b^2*Cosh[x])/(a^2 - b^2)^2 - (a*Cosh[x])/(a^2 - b^2) + (a*Cosh[x]^3)
/(3*(a^2 - b^2)) + (a^2*b*Sinh[x])/(a^2 - b^2)^2 - (b*Sinh[x]^3)/(3*(a^2 -
b^2))
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
```

```
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3074

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> -Dist[d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d
*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3099

```
Int[sin[(c_.) + (d_.)*(x_)]^(m_)/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin
[(c_.) + (d_.)*(x_)]), x_Symbol] :> -Simp[(a*Sin[c + d*x]^(m - 1))/(d*(a^2
+ b^2)*(m - 1)), x] + (Dist[a^2/(a^2 + b^2), Int[Sin[c + d*x]^(m - 2)/(a*Co
s[c + d*x] + b*Sin[c + d*x]), x], x] + Dist[b/(a^2 + b^2), Int[Sin[c + d*x]
^(m - 1), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && GtQ[m,
1]
```

Rule 3109

```
Int[(cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.))/(cos[(c_.
) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[b
/(a^2 + b^2), Int[Cos[c + d*x]^m*Sin[c + d*x]^(n - 1), x], x] + (Dist[a/(a^
2 + b^2), Int[Cos[c + d*x]^(m - 1)*Sin[c + d*x]^n, x], x] - Dist[(a*b)/(a^2
 + b^2), Int[(Cos[c + d*x]^(m - 1)*Sin[c + d*x]^(n - 1))/(a*Cos[c + d*x] +
b*Sin[c + d*x]), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] &&
IGtQ[m, 0] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{\cosh(x)\sinh^3(x)}{a\cosh(x)+b\sinh(x)}\,dx = \frac{a\int \sinh^3(x)\,dx}{a^2-b^2} - \frac{b\int \cosh(x)\sinh^2(x)\,dx}{a^2-b^2} + \frac{(ab)\int \frac{\sinh^2(x)}{a\cosh(x)+b\sinh(x)}\,dx}{a^2-b^2}$$
$$= \frac{a^2b\sinh(x)}{\left(a^2-b^2\right)^2} - \frac{\left(a^3b\right)\int \frac{1}{a\cosh(x)+b\sinh(x)}\,dx}{\left(a^2-b^2\right)^2} - \frac{\left(ab^2\right)\int \sinh(x)\,dx}{\left(a^2-b^2\right)^2} - \frac{a\,\mathrm{Subst}\left(\int \left(1-x\right.\right.}{a^2}$$
$$= -\frac{ab^2\cosh(x)}{\left(a^2-b^2\right)^2} - \frac{a\cosh(x)}{a^2-b^2} + \frac{a\cosh^3(x)}{3\left(a^2-b^2\right)} + \frac{a^2b\sinh(x)}{\left(a^2-b^2\right)^2} - \frac{b\sinh^3(x)}{3\left(a^2-b^2\right)} - \frac{\left(ia^3b\right)\mathrm{Sub}}{}$$
$$= -\frac{a^3b\tan^{-1}\left(\frac{b\cosh(x)+a\sinh(x)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{5/2}} - \frac{ab^2\cosh(x)}{\left(a^2-b^2\right)^2} - \frac{a\cosh(x)}{a^2-b^2} + \frac{a\cosh^3(x)}{3\left(a^2-b^2\right)} + \frac{a^2b\sinh}{\left(a^2-b^2\right.}$$

Mathematica [A] time = 1.15, size = 180, normalized size = 1.31

$$\frac{-3a\sqrt{a-b}\,\sqrt{a+b}\,\left(3a^2+b^2\right)\cosh(x) + a\sqrt{a-b}\,\sqrt{a+b}\,\left(a^2-b^2\right)\cosh(3x) + b\left(-24a^3\tan^{-1}\left(\frac{a\tanh\left(\frac{x}{2}\right)+b}{\sqrt{a-b}\,\sqrt{a+b}}\right) + 3\sqrt{a}\right.}{12(a-b)^{5/2}(a+b)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cosh[x]*Sinh[x]^3)/(a*Cosh[x] + b*Sinh[x]),x]
```

```
[Out] (-3*a*Sqrt[a - b]*Sqrt[a + b]*(3*a^2 + b^2)*Cosh[x] + a*Sqrt[a - b]*Sqrt[a
+ b]*(a^2 - b^2)*Cosh[3*x] + b*(-24*a^3*ArcTan[(b + a*Tanh[x/2])/(Sqrt[a -
b]*Sqrt[a + b])] + 3*Sqrt[a - b]*Sqrt[a + b]*(5*a^2 - b^2)*Sinh[x] - Sqrt[a
 - b]*Sqrt[a + b]*(a^2 - b^2)*Sinh[3*x]))/(12*(a - b)^(5/2)*(a + b)^(5/2))
```

fricas [B] time = 0.48, size = 1861, normalized size = 13.58

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*sinh(x)^3/(a*cosh(x)+b*sinh(x)),x, algorithm="fricas")
```

```
[Out] [1/24*((a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^6 + 6*(a
^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)*sinh(x)^5 + (a^5
- a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*sinh(x)^6 + a^5 + a^4*b - 2*
a^3*b^2 - 2*a^2*b^3 + a*b^4 + b^5 - 3*(3*a^5 - 5*a^4*b - 2*a^3*b^2 + 6*a^2*
b^3 - a*b^4 - b^5)*cosh(x)^4 - 3*(3*a^5 - 5*a^4*b - 2*a^3*b^2 + 6*a^2*b^3 -
```

```
a*b^4 - b^5 - 5*(a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x
)^2)*sinh(x)^4 + 4*(5*(a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*c
osh(x)^3 - 3*(3*a^5 - 5*a^4*b - 2*a^3*b^2 + 6*a^2*b^3 - a*b^4 - b^5)*cosh(x
))*sinh(x)^3 - 3*(3*a^5 + 5*a^4*b - 2*a^3*b^2 - 6*a^2*b^3 - a*b^4 + b^5)*co
sh(x)^2 - 3*(3*a^5 + 5*a^4*b - 2*a^3*b^2 - 6*a^2*b^3 - a*b^4 + b^5 - 5*(a^5
 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^4 + 6*(3*a^5 - 5*a^
4*b - 2*a^3*b^2 + 6*a^2*b^3 - a*b^4 - b^5)*cosh(x)^2)*sinh(x)^2 - 24*(a^3*b
*cosh(x)^3 + 3*a^3*b*cosh(x)^2*sinh(x) + 3*a^3*b*cosh(x)*sinh(x)^2 + a^3*b*
sinh(x)^3)*sqrt(-a^2 + b^2)*log(((a + b)*cosh(x)^2 + 2*(a + b)*cosh(x)*sinh
(x) + (a + b)*sinh(x)^2 + 2*sqrt(-a^2 + b^2)*(cosh(x) + sinh(x)) - a + b)/(
(a + b)*cosh(x)^2 + 2*(a + b)*cosh(x)*sinh(x) + (a + b)*sinh(x)^2 + a - b))
 + 6*((a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^5 - 2*(3*
a^5 - 5*a^4*b - 2*a^3*b^2 + 6*a^2*b^3 - a*b^4 - b^5)*cosh(x)^3 - (3*a^5 + 5
*a^4*b - 2*a^3*b^2 - 6*a^2*b^3 - a*b^4 + b^5)*cosh(x))*sinh(x))/((a^6 - 3*a
^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^3 + 3*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)
*cosh(x)^2*sinh(x) + 3*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)*sinh(x)^
2 + (a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*sinh(x)^3), 1/24*((a^5 - a^4*b - 2*
a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^6 + 6*(a^5 - a^4*b - 2*a^3*b^2 +
 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)*sinh(x)^5 + (a^5 - a^4*b - 2*a^3*b^2 + 2*
a^2*b^3 + a*b^4 - b^5)*sinh(x)^6 + a^5 + a^4*b - 2*a^3*b^2 - 2*a^2*b^3 + a*
b^4 + b^5 - 3*(3*a^5 - 5*a^4*b - 2*a^3*b^2 + 6*a^2*b^3 - a*b^4 - b^5)*cosh(
x)^4 - 3*(3*a^5 - 5*a^4*b - 2*a^3*b^2 + 6*a^2*b^3 - a*b^4 - b^5 - 5*(a^5 -
a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^2)*sinh(x)^4 + 4*(5*(a
^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^3 - 3*(3*a^5 - 5*
a^4*b - 2*a^3*b^2 + 6*a^2*b^3 - a*b^4 - b^5)*cosh(x))*sinh(x)^3 - 3*(3*a^5
+ 5*a^4*b - 2*a^3*b^2 - 6*a^2*b^3 - a*b^4 + b^5)*cosh(x)^2 - 3*(3*a^5 + 5*a
^4*b - 2*a^3*b^2 - 6*a^2*b^3 - a*b^4 + b^5 - 5*(a^5 - a^4*b - 2*a^3*b^2 + 2
*a^2*b^3 + a*b^4 - b^5)*cosh(x)^4 + 6*(3*a^5 - 5*a^4*b - 2*a^3*b^2 + 6*a^2*
b^3 - a*b^4 - b^5)*cosh(x)^2)*sinh(x)^2 + 48*(a^3*b*cosh(x)^3 + 3*a^3*b*cos
h(x)^2*sinh(x) + 3*a^3*b*cosh(x)*sinh(x)^2 + a^3*b*sinh(x)^3)*sqrt(a^2 - b^
2)*arctan(sqrt(a^2 - b^2)/((a + b)*cosh(x) + (a + b)*sinh(x))) + 6*((a^5 -
a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^5 - 2*(3*a^5 - 5*a^4*b
 - 2*a^3*b^2 + 6*a^2*b^3 - a*b^4 - b^5)*cosh(x)^3 - (3*a^5 + 5*a^4*b - 2*a^
3*b^2 - 6*a^2*b^3 - a*b^4 + b^5)*cosh(x))*sinh(x))/((a^6 - 3*a^4*b^2 + 3*a^
2*b^4 - b^6)*cosh(x)^3 + 3*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^2*si
nh(x) + 3*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)*sinh(x)^2 + (a^6 - 3*
a^4*b^2 + 3*a^2*b^4 - b^6)*sinh(x)^3)]
```

giac [A] time = 0.13, size = 163, normalized size = 1.19

$$-\frac{2\,a^{3}b\arctan\left(\frac{ae^{x}+be^{x}}{\sqrt{a^{2}-b^{2}}}\right)}{\left(a^{4}-2\,a^{2}b^{2}+b^{4}\right)\sqrt{a^{2}-b^{2}}}-\frac{\left(9\,ae^{(2\,x)}-3\,be^{(2\,x)}-a+b\right)e^{(-3\,x)}}{24\,\left(a^{2}-2\,ab+b^{2}\right)}+\frac{a^{2}e^{(3\,x)}+2\,abe^{(3\,x)}+b^{2}e^{(3\,x)}-9\,a^{2}e^{x}-12\,abe^{x}}{24\,\left(a^{3}+3\,a^{2}b+3\,ab^{2}+b^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*sinh(x)^3/(a*cosh(x)+b*sinh(x)),x, algorithm="giac")
```

```
[Out] -2*a^3*b*arctan((a*e^x + b*e^x)/sqrt(a^2 - b^2))/((a^4 - 2*a^2*b^2 + b^4)*s
qrt(a^2 - b^2)) - 1/24*(9*a*e^(2*x) - 3*b*e^(2*x) - a + b)*e^(-3*x)/(a^2 -
2*a*b + b^2) + 1/24*(a^2*e^(3*x) + 2*a*b*e^(3*x) + b^2*e^(3*x) - 9*a^2*e^x
- 12*a*b*e^x - 3*b^2*e^x)/(a^3 + 3*a^2*b + 3*a*b^2 + b^3)
```

maple [A] time = 0.22, size = 166, normalized size = 1.21

$$-\frac{16}{3\left(\tanh\left(\frac{x}{2}\right)-1\right)^{3}(16a+16b)}-\frac{8}{(16a+16b)\left(\tanh\left(\frac{x}{2}\right)-1\right)^{2}}+\frac{a}{2(a+b)^{2}\left(\tanh\left(\frac{x}{2}\right)-1\right)}-\frac{2a^{3}b\arctan\left(\frac{2a\tanh\left(\frac{x}{2}\right)+2}{2\sqrt{a^{2}-b^{2}}}\right)}{(a-b)^{2}(a+b)^{2}\sqrt{a^{2}-b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)*sinh(x)^3/(a*cosh(x)+b*sinh(x)),x)
```

```
[Out] -16/3/(tanh(1/2*x)-1)^3/(16*a+16*b)-8/(16*a+16*b)/(tanh(1/2*x)-1)^2+1/2*a/(
a+b)^2/(tanh(1/2*x)-1)-2*a^3*b/(a-b)^2/(a+b)^2/(a^2-b^2)^(1/2)*arctan(1/2*(
2*a*tanh(1/2*x)+2*b)/(a^2-b^2)^(1/2))-8/(16*a-16*b)/(tanh(1/2*x)+1)^2+16/3/
(tanh(1/2*x)+1)^3/(16*a-16*b)-1/2*a/(a-b)^2/(tanh(1/2*x)+1)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*sinh(x)^3/(a*cosh(x)+b*sinh(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [B] time = 1.78, size = 261, normalized size = 1.91

$$\frac{e^{-3x}}{24a-24b}+\frac{e^{3x}}{24a+24b}-\frac{e^{x}\,(3a+b)}{8(a+b)^{2}}-\frac{e^{-x}\,(3a-b)}{8(a-b)^{2}}-\frac{2\,\mathrm{atan}\left(\frac{a^{3}\,b\,e^{x}\,\sqrt{a^{10}-5a^{8}b^{2}+10a^{6}b^{4}-10a^{4}b^{6}+5a^{2}b^{8}-b^{10}}}{a^{5}\sqrt{a^{6}b^{2}}-b^{5}\sqrt{a^{6}b^{2}}+2a^{2}b^{3}\sqrt{a^{6}b^{2}}-2a^{3}b^{2}\sqrt{a^{6}b^{2}}+a\,b^{4}\sqrt{a^{6}b^{2}}-}\right)}{\sqrt{a^{10}-5a^{8}b^{2}+10a^{6}b^{4}-10a^{4}b^{6}+5a^{2}b^{8}-}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cosh(x)*sinh(x)^3)/(a*cosh(x) + b*sinh(x)),x)
```

```
[Out] exp(-3*x)/(24*a - 24*b) + exp(3*x)/(24*a + 24*b) - (exp(x)*(3*a + b))/(8*(a
 + b)^2) - (exp(-x)*(3*a - b))/(8*(a - b)^2) - (2*atan((a^3*b*exp(x)*(a^10
```

```
- b^10 + 5*a^2*b^8 - 10*a^4*b^6 + 10*a^6*b^4 - 5*a^8*b^2)^(1/2))/(a^5*(a^6*
b^2)^(1/2) - b^5*(a^6*b^2)^(1/2) + 2*a^2*b^3*(a^6*b^2)^(1/2) - 2*a^3*b^2*(a
^6*b^2)^(1/2) + a*b^4*(a^6*b^2)^(1/2) - a^4*b*(a^6*b^2)^(1/2)))*(a^6*b^2)^(
1/2))/(a^10 - b^10 + 5*a^2*b^8 - 10*a^4*b^6 + 10*a^6*b^4 - 5*a^8*b^2)^(1/2)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*sinh(x)**3/(a*cosh(x)+b*sinh(x)),x)
```

```
[Out] Timed out
```

3.709 $\int \frac{\cosh^2(x)\sinh(x)}{a\cosh(x)+b\sinh(x)}\,dx$

Optimal. Leaf size=102

$$\frac{a^2bx}{(a^2-b^2)^2}-\frac{bx}{2(a^2-b^2)}+\frac{a\sinh^2(x)}{2(a^2-b^2)}-\frac{b\sinh(x)\cosh(x)}{2(a^2-b^2)}-\frac{ab^2\log(a\cosh(x)+b\sinh(x))}{(a^2-b^2)^2}$$

```
[Out] a^2*b*x/(a^2-b^2)^2-1/2*b*x/(a^2-b^2)-a*b^2*ln(a*cosh(x)+b*sinh(x))/(a^2-b^
2)^2-1/2*b*cosh(x)*sinh(x)/(a^2-b^2)+1/2*a*sinh(x)^2/(a^2-b^2)
```

Rubi [A] time = 0.16, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {3109, 2635, 8, 2564, 30, 3098, 3133}

$$\frac{a^2bx}{(a^2-b^2)^2}-\frac{bx}{2(a^2-b^2)}+\frac{a\sinh^2(x)}{2(a^2-b^2)}-\frac{b\sinh(x)\cosh(x)}{2(a^2-b^2)}-\frac{ab^2\log(a\cosh(x)+b\sinh(x))}{(a^2-b^2)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Cosh[x]^2*Sinh[x])/(a*Cosh[x] + b*Sinh[x]),x]
```

```
[Out] (a^2*b*x)/(a^2 - b^2)^2 - (b*x)/(2*(a^2 - b^2)) - (a*b^2*Log[a*Cosh[x] + b*
Sinh[x]])/(a^2 - b^2)^2 - (b*Cosh[x]*Sinh[x])/(2*(a^2 - b^2)) + (a*Sinh[x]^
2)/(2*(a^2 - b^2))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 3098

```
Int[cos[(c_.) + (d_.)*(x_)]/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.
) + (d_.)*(x_)]), x_Symbol] :> Simp[(a*x)/(a^2 + b^2), x] + Dist[b/(a^2 + b
^2), Int[(b*Cos[c + d*x] - a*Sin[c + d*x])/(a*Cos[c + d*x] + b*Sin[c + d*x]
), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3109

```
Int[(cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.))/(cos[(c_.
) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[b
/(a^2 + b^2), Int[Cos[c + d*x]^m*Sin[c + d*x]^(n - 1), x], x] + (Dist[a/(a^
2 + b^2), Int[Cos[c + d*x]^(m - 1)*Sin[c + d*x]^n, x], x] - Dist[(a*b)/(a^2
 + b^2), Int[(Cos[c + d*x]^(m - 1)*Sin[c + d*x]^(n - 1))/(a*Cos[c + d*x] +
b*Sin[c + d*x]), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] &&
IGtQ[m, 0] && IGtQ[n, 0]
```

Rule 3133

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)]), x
_Symbol] :> Simp[((b*B + c*C)*x)/(b^2 + c^2), x] + Simp[((c*B - b*C)*Log[a
+ b*Cos[d + e*x] + c*Sin[d + e*x]])/(e*(b^2 + c^2)), x] /; FreeQ[{a, b, c,
d, e, A, B, C}, x] && NeQ[b^2 + c^2, 0] && EqQ[A*(b^2 + c^2) - a*(b*B + c*C
), 0]
```

Rubi steps

$$\int \frac{\cosh^2(x)\sinh(x)}{a\cosh(x)+b\sinh(x)}\,dx = \frac{a\int \cosh(x)\sinh(x)\,dx}{a^2-b^2} - \frac{b\int \cosh^2(x)\,dx}{a^2-b^2} + \frac{(ab)\int \frac{\cosh(x)}{a\cosh(x)+b\sinh(x)}\,dx}{a^2-b^2}$$

$$= \frac{a^2bx}{\left(a^2-b^2\right)^2} - \frac{b\cosh(x)\sinh(x)}{2\left(a^2-b^2\right)} - \frac{\left(iab^2\right)\int \frac{-ib\cosh(x)-ia\sinh(x)}{a\cosh(x)+b\sinh(x)}\,dx}{\left(a^2-b^2\right)^2} - \frac{a\,\mathrm{Subst}(\int x\,dx}{a^2-}$$

$$= \frac{a^2bx}{\left(a^2-b^2\right)^2} - \frac{bx}{2\left(a^2-b^2\right)} - \frac{ab^2\log(a\cosh(x)+b\sinh(x))}{\left(a^2-b^2\right)^2} - \frac{b\cosh(x)\sinh(x)}{2\left(a^2-b^2\right)} +$$

Mathematica [A] time = 0.25, size = 73, normalized size = 0.72

$$\frac{a\left(a^2-b^2\right)\cosh(2x)+b\left(2x\left(a^2+b^2\right)+\left(b^2-a^2\right)\sinh(2x)-4ab\log(a\cosh(x)+b\sinh(x))\right)}{4(a-b)^2(a+b)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cosh[x]^2*Sinh[x])/(a*Cosh[x] + b*Sinh[x]),x]
```

```
[Out] (a*(a^2 - b^2)*Cosh[2*x] + b*(2*(a^2 + b^2)*x - 4*a*b*Log[a*Cosh[x] + b*Sin
h[x]] + (-a^2 + b^2)*Sinh[2*x]))/(4*(a - b)^2*(a + b)^2)
```

fricas [B] time = 0.47, size = 334, normalized size = 3.27

$$\left(a^3-a^2b-ab^2+b^3\right)\cosh(x)^4+4\left(a^3-a^2b-ab^2+b^3\right)\cosh(x)\sinh(x)^3+\left(a^3-a^2b-ab^2+b^3\right)\sinh(x)^4+4\,(a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^2*sinh(x)/(a*cosh(x)+b*sinh(x)),x, algorithm="fricas")
```

```
[Out] 1/8*((a^3 - a^2*b - a*b^2 + b^3)*cosh(x)^4 + 4*(a^3 - a^2*b - a*b^2 + b^3)*
cosh(x)*sinh(x)^3 + (a^3 - a^2*b - a*b^2 + b^3)*sinh(x)^4 + 4*(a^2*b + 2*a*
b^2 + b^3)*x*cosh(x)^2 + a^3 + a^2*b - a*b^2 - b^3 + 2*(3*(a^3 - a^2*b - a*
b^2 + b^3)*cosh(x)^2 + 2*(a^2*b + 2*a*b^2 + b^3)*x)*sinh(x)^2 - 8*(a*b^2*co
sh(x)^2 + 2*a*b^2*cosh(x)*sinh(x) + a*b^2*sinh(x)^2)*log(2*(a*cosh(x) + b*s
inh(x))/(cosh(x) - sinh(x))) + 4*((a^3 - a^2*b - a*b^2 + b^3)*cosh(x)^3 + 2
*(a^2*b + 2*a*b^2 + b^3)*x*cosh(x))*sinh(x))/((a^4 - 2*a^2*b^2 + b^4)*cosh(
x)^2 + 2*(a^4 - 2*a^2*b^2 + b^4)*cosh(x)*sinh(x) + (a^4 - 2*a^2*b^2 + b^4)*
sinh(x)^2)
```

giac [A] time = 0.12, size = 102, normalized size = 1.00

$$-\frac{ab^2\log\left(\left|ae^{(2\,x)}+be^{(2\,x)}+a-b\right|\right)}{a^4-2\,a^2b^2+b^4}+\frac{bx}{2\left(a^2-2\,ab+b^2\right)}-\frac{\left(2\,be^{(2\,x)}-a+b\right)e^{(-2\,x)}}{8\left(a^2-2\,ab+b^2\right)}+\frac{e^{(2\,x)}}{8\,(a+b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^2*sinh(x)/(a*cosh(x)+b*sinh(x)),x, algorithm="giac")
```

```
[Out] -a*b^2*log(abs(a*e^(2*x) + b*e^(2*x) + a - b))/(a^4 - 2*a^2*b^2 + b^4) + 1/
2*b*x/(a^2 - 2*a*b + b^2) - 1/8*(2*b*e^(2*x) - a + b)*e^(-2*x)/(a^2 - 2*a*b
 + b^2) + 1/8*e^(2*x)/(a + b)
```

maple [A] time = 0.21, size = 146, normalized size = 1.43

$$\frac{2}{(4a+4b)\left(\tanh\left(\frac{x}{2}\right)-1\right)^2}+\frac{4}{(8a+8b)\left(\tanh\left(\frac{x}{2}\right)-1\right)}-\frac{\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)b}{2\,(a+b)^2}-\frac{a\,b^2\ln\left(a+2\tanh\left(\frac{x}{2}\right)b+a\left(\tanh^2\left(\right.\right.\right.}{(a-b)^2\,(a+b)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^2*sinh(x)/(a*cosh(x)+b*sinh(x)),x)
```

```
[Out] 2/(4*a+4*b)/(tanh(1/2*x)-1)^2+4/(8*a+8*b)/(tanh(1/2*x)-1)-1/2/(a+b)^2*ln(ta
nh(1/2*x)-1)*b-a*b^2/(a-b)^2/(a+b)^2*ln(a+2*tanh(1/2*x)*b+a*tanh(1/2*x)^2)-
4/(8*a-8*b)/(tanh(1/2*x)+1)+2/(4*a-4*b)/(tanh(1/2*x)+1)^2+1/2/(a-b)^2*ln(ta
nh(1/2*x)+1)*b
```

maxima [A] time = 0.50, size = 84, normalized size = 0.82

$$-\frac{ab^2\log\left(-(a-b)e^{(-2\,x)}-a-b\right)}{a^4-2\,a^2b^2+b^4}+\frac{bx}{2\left(a^2+2\,ab+b^2\right)}+\frac{e^{(2\,x)}}{8\,(a+b)}+\frac{e^{(-2\,x)}}{8\,(a-b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^2*sinh(x)/(a*cosh(x)+b*sinh(x)),x, algorithm="maxima")
```

```
[Out] -a*b^2*log(-(a - b)*e^(-2*x) - a - b)/(a^4 - 2*a^2*b^2 + b^4) + 1/2*b*x/(a^
2 + 2*a*b + b^2) + 1/8*e^(2*x)/(a + b) + 1/8*e^(-2*x)/(a - b)
```

mupad [B] time = 1.91, size = 81, normalized size = 0.79

$$\frac{e^{-2\,x}}{8\,a-8\,b}+\frac{e^{2\,x}}{8\,a+8\,b}+\frac{b\,x}{2\,(a-b)^2}-\frac{a\,b^2\,\ln\left(a-b+a\,e^{2\,x}+b\,e^{2\,x}\right)}{a^4-2\,a^2\,b^2+b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cosh(x)^2*sinh(x))/(a*cosh(x) + b*sinh(x)),x)
```

```
[Out] exp(-2*x)/(8*a - 8*b) + exp(2*x)/(8*a + 8*b) + (b*x)/(2*(a - b)^2) - (a*b^2
*log(a - b + a*exp(2*x) + b*exp(2*x)))/(a^4 + b^4 - 2*a^2*b^2)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)**2*sinh(x)/(a*cosh(x)+b*sinh(x)),x)
```

```
[Out] Timed out
```

3.710 $\int \frac{\cosh^2(x)\sinh^2(x)}{a\cosh(x)+b\sinh(x)}\,dx$

Optimal. Leaf size=122

$$\frac{a\sinh^3(x)}{3\left(a^2-b^2\right)}-\frac{ab^2\sinh(x)}{\left(a^2-b^2\right)^2}-\frac{b\cosh^3(x)}{3\left(a^2-b^2\right)}+\frac{a^2b\cosh(x)}{\left(a^2-b^2\right)^2}+\frac{a^2b^2\tan^{-1}\left(\frac{a\sinh(x)+b\cosh(x)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{5/2}}$$

```
[Out] a^2*b^2*arctan((b*cosh(x)+a*sinh(x))/(a^2-b^2)^(1/2))/(a^2-b^2)^(5/2)+a^2*b
*cosh(x)/(a^2-b^2)^2-1/3*b*cosh(x)^3/(a^2-b^2)-a*b^2*sinh(x)/(a^2-b^2)^2+1/
3*a*sinh(x)^3/(a^2-b^2)
```

Rubi [A] time = 0.23, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {3109, 2565, 30, 2564, 2637, 2638, 3074, 206}

$$\frac{a\sinh^3(x)}{3\left(a^2-b^2\right)}-\frac{ab^2\sinh(x)}{\left(a^2-b^2\right)^2}-\frac{b\cosh^3(x)}{3\left(a^2-b^2\right)}+\frac{a^2b\cosh(x)}{\left(a^2-b^2\right)^2}+\frac{a^2b^2\tan^{-1}\left(\frac{a\sinh(x)+b\cosh(x)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cosh[x]^2*Sinh[x]^2)/(a*Cosh[x] + b*Sinh[x]),x]
```

```
[Out] (a^2*b^2*ArcTan[(b*Cosh[x] + a*Sinh[x])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(5/2)
 + (a^2*b*Cosh[x])/(a^2 - b^2)^2 - (b*Cosh[x]^3)/(3*(a^2 - b^2)) - (a*b^2*S
inh[x])/(a^2 - b^2)^2 + (a*Sinh[x]^3)/(3*(a^2 - b^2))
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] && !(In
```

```
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3074

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> -Dist[d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d
*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3109

```
Int[(cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.))/(cos[(c_.
) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[b
/(a^2 + b^2), Int[Cos[c + d*x]^m*Sin[c + d*x]^(n - 1), x], x] + (Dist[a/(a^
2 + b^2), Int[Cos[c + d*x]^(m - 1)*Sin[c + d*x]^n, x], x] - Dist[(a*b)/(a^2
 + b^2), Int[(Cos[c + d*x]^(m - 1)*Sin[c + d*x]^(n - 1))/(a*Cos[c + d*x] +
b*Sin[c + d*x]), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] &&
IGtQ[m, 0] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{\cosh^2(x)\sinh^2(x)}{a\cosh(x)+b\sinh(x)}\,dx = \frac{a\int \cosh(x)\sinh^2(x)\,dx}{a^2-b^2} - \frac{b\int \cosh^2(x)\sinh(x)\,dx}{a^2-b^2} + \frac{(ab)\int \frac{\cosh(x)\sinh(x)}{a\cosh(x)+b\sinh(x)}\,dx}{a^2-b^2}$$

$$= \frac{\left(a^2b\right)\int \sinh(x)\,dx}{\left(a^2-b^2\right)^2} - \frac{\left(ab^2\right)\int \cosh(x)\,dx}{\left(a^2-b^2\right)^2} + \frac{\left(a^2b^2\right)\int \frac{1}{a\cosh(x)+b\sinh(x)}\,dx}{\left(a^2-b^2\right)^2} + \frac{(ia)\,\mathrm{Sub}}{}$$

$$= \frac{a^2b\cosh(x)}{\left(a^2-b^2\right)^2} - \frac{b\cosh^3(x)}{3\left(a^2-b^2\right)} - \frac{ab^2\sinh(x)}{\left(a^2-b^2\right)^2} + \frac{a\sinh^3(x)}{3\left(a^2-b^2\right)} + \frac{\left(ia^2b^2\right)\mathrm{Subst}\left(\int \frac{1}{a^2-b^2-x^2}\right.}{\left(a\right.}$$

$$= \frac{a^2b^2\tan^{-1}\left(\frac{b\cosh(x)+a\sinh(x)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{5/2}} + \frac{a^2b\cosh(x)}{\left(a^2-b^2\right)^2} - \frac{b\cosh^3(x)}{3\left(a^2-b^2\right)} - \frac{ab^2\sinh(x)}{\left(a^2-b^2\right)^2} + \frac{a\sinh^3}{3\left(a^2-\right.}$$

Mathematica [A] time = 1.01, size = 179, normalized size = 1.47

$$\frac{3b\sqrt{a-b}\sqrt{a+b}\left(3a^2+b^2\right)\cosh(x) - b\sqrt{a-b}\sqrt{a+b}\left(a^2-b^2\right)\cosh(3x) + a\left(-3\sqrt{a-b}\sqrt{a+b}\left(a^2+3b^2\right)\sinh\right.}{12(a-b)^{5/2}(a+b)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cosh[x]^2*Sinh[x]^2)/(a*Cosh[x] + b*Sinh[x]),x]
```

```
[Out] (3*Sqrt[a - b]*b*Sqrt[a + b]*(3*a^2 + b^2)*Cosh[x] - Sqrt[a - b]*b*Sqrt[a +
 b]*(a^2 - b^2)*Cosh[3*x] + a*(24*a*b^2*ArcTan[(b + a*Tanh[x/2])/(Sqrt[a -
b]*Sqrt[a + b])] - 3*Sqrt[a - b]*Sqrt[a + b]*(a^2 + 3*b^2)*Sinh[x] + Sqrt[a
 - b]*Sqrt[a + b]*(a^2 - b^2)*Sinh[3*x]))/(12*(a - b)^(5/2)*(a + b)^(5/2))
```

fricas [B] time = 0.48, size = 1847, normalized size = 15.14

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^2*sinh(x)^2/(a*cosh(x)+b*sinh(x)),x, algorithm="fricas")
```

```
[Out] [1/24*((a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^6 + 6*(a
^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)*sinh(x)^5 + (a^5
- a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*sinh(x)^6 - a^5 - a^4*b + 2*
a^3*b^2 + 2*a^2*b^3 - a*b^4 - b^5 - 3*(a^5 - 3*a^4*b + 2*a^3*b^2 + 2*a^2*b^
3 - 3*a*b^4 + b^5)*cosh(x)^4 - 3*(a^5 - 3*a^4*b + 2*a^3*b^2 + 2*a^2*b^3 - 3
*a*b^4 + b^5 - 5*(a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x
```

```
)^2)*sinh(x)^4 + 4*(5*(a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*c
osh(x)^3 - 3*(a^5 - 3*a^4*b + 2*a^3*b^2 + 2*a^2*b^3 - 3*a*b^4 + b^5)*cosh(x
))*sinh(x)^3 + 3*(a^5 + 3*a^4*b + 2*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^5)*co
sh(x)^2 + 3*(a^5 + 3*a^4*b + 2*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^5 + 5*(a^5
 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^4 - 6*(a^5 - 3*a^4*
b + 2*a^3*b^2 + 2*a^2*b^3 - 3*a*b^4 + b^5)*cosh(x)^2)*sinh(x)^2 - 24*(a^2*b
^2*cosh(x)^3 + 3*a^2*b^2*cosh(x)^2*sinh(x) + 3*a^2*b^2*cosh(x)*sinh(x)^2 +
a^2*b^2*sinh(x)^3)*sqrt(-a^2 + b^2)*log(((a + b)*cosh(x)^2 + 2*(a + b)*cosh
(x)*sinh(x) + (a + b)*sinh(x)^2 - 2*sqrt(-a^2 + b^2)*(cosh(x) + sinh(x)) -
a + b)/((a + b)*cosh(x)^2 + 2*(a + b)*cosh(x)*sinh(x) + (a + b)*sinh(x)^2 +
 a - b)) + 6*((a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^5
 - 2*(a^5 - 3*a^4*b + 2*a^3*b^2 + 2*a^2*b^3 - 3*a*b^4 + b^5)*cosh(x)^3 + (a
^5 + 3*a^4*b + 2*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^5)*cosh(x))*sinh(x))/((a
^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^3 + 3*(a^6 - 3*a^4*b^2 + 3*a^2*b^
4 - b^6)*cosh(x)^2*sinh(x) + 3*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)*
sinh(x)^2 + (a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*sinh(x)^3), 1/24*((a^5 - a^
4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^6 + 6*(a^5 - a^4*b - 2*a
^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)*sinh(x)^5 + (a^5 - a^4*b - 2*a^3*
b^2 + 2*a^2*b^3 + a*b^4 - b^5)*sinh(x)^6 - a^5 - a^4*b + 2*a^3*b^2 + 2*a^2*
b^3 - a*b^4 - b^5 - 3*(a^5 - 3*a^4*b + 2*a^3*b^2 + 2*a^2*b^3 - 3*a*b^4 + b^
5)*cosh(x)^4 - 3*(a^5 - 3*a^4*b + 2*a^3*b^2 + 2*a^2*b^3 - 3*a*b^4 + b^5 - 5
*(a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^2)*sinh(x)^4 +
 4*(5*(a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^3 - 3*(a^
5 - 3*a^4*b + 2*a^3*b^2 + 2*a^2*b^3 - 3*a*b^4 + b^5)*cosh(x))*sinh(x)^3 + 3
*(a^5 + 3*a^4*b + 2*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^5)*cosh(x)^2 + 3*(a^5
 + 3*a^4*b + 2*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^5 + 5*(a^5 - a^4*b - 2*a^3
*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^4 - 6*(a^5 - 3*a^4*b + 2*a^3*b^2 +
2*a^2*b^3 - 3*a*b^4 + b^5)*cosh(x)^2)*sinh(x)^2 - 48*(a^2*b^2*cosh(x)^3 + 3
*a^2*b^2*cosh(x)^2*sinh(x) + 3*a^2*b^2*cosh(x)*sinh(x)^2 + a^2*b^2*sinh(x)^
3)*sqrt(a^2 - b^2)*arctan(sqrt(a^2 - b^2)/((a + b)*cosh(x) + (a + b)*sinh(x
))) + 6*((a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^5 - 2*
(a^5 - 3*a^4*b + 2*a^3*b^2 + 2*a^2*b^3 - 3*a*b^4 + b^5)*cosh(x)^3 + (a^5 +
3*a^4*b + 2*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^5)*cosh(x))*sinh(x))/((a^6 -
3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^3 + 3*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b
^6)*cosh(x)^2*sinh(x) + 3*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)*sinh(
x)^2 + (a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*sinh(x)^3)]
```

giac [A] time = 0.12, size = 159, normalized size = 1.30

$$\frac{2\,a^2b^2\arctan\left(\frac{ae^x+be^x}{\sqrt{a^2-b^2}}\right)}{\left(a^4-2\,a^2b^2+b^4\right)\sqrt{a^2-b^2}}+\frac{\left(3\,ae^{(2\,x)}+3\,be^{(2\,x)}-a+b\right)e^{(-3\,x)}}{24\left(a^2-2\,ab+b^2\right)}+\frac{a^2e^{(3\,x)}+2\,abe^{(3\,x)}+b^2e^{(3\,x)}-3\,a^2e^x+3\,b^2e^x}{24\left(a^3+3\,a^2b+3\,ab^2+b^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^2*sinh(x)^2/(a*cosh(x)+b*sinh(x)),x, algorithm="giac")
```

```
[Out] 2*a^2*b^2*arctan((a*e^x + b*e^x)/sqrt(a^2 - b^2))/((a^4 - 2*a^2*b^2 + b^4)*
sqrt(a^2 - b^2)) + 1/24*(3*a*e^(2*x) + 3*b*e^(2*x) - a + b)*e^(-3*x)/(a^2 -
 2*a*b + b^2) + 1/24*(a^2*e^(3*x) + 2*a*b*e^(3*x) + b^2*e^(3*x) - 3*a^2*e^x
 + 3*b^2*e^x)/(a^3 + 3*a^2*b + 3*a*b^2 + b^3)
```

maple [A] time = 0.25, size = 168, normalized size = 1.38

$$-\frac{8}{3\left(\tanh\left(\frac{x}{2}\right)-1\right)^{3}(8a+8b)}-\frac{4}{(8a+8b)\left(\tanh\left(\frac{x}{2}\right)-1\right)^{2}}-\frac{b}{2\,(a+b)^{2}\left(\tanh\left(\frac{x}{2}\right)-1\right)}+\frac{2a^{2}b^{2}\arctan\left(\frac{2a\tanh\left(\frac{x}{2}\right)+2b}{2\sqrt{a^{2}-b^{2}}}\right)}{(a-b)^{2}(a+b)^{2}\sqrt{a^{2}-b^{2}}}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^2*sinh(x)^2/(a*cosh(x)+b*sinh(x)),x)
```

```
[Out] -8/3/(tanh(1/2*x)-1)^3/(8*a+8*b)-4/(8*a+8*b)/(tanh(1/2*x)-1)^2-1/2*b/(a+b)^
2/(tanh(1/2*x)-1)+2*a^2*b^2/(a-b)^2/(a+b)^2/(a^2-b^2)^(1/2)*arctan(1/2*(2*a
*tanh(1/2*x)+2*b)/(a^2-b^2)^(1/2))-8/3/(tanh(1/2*x)+1)^3/(8*a-8*b)+4/(8*a-8
*b)/(tanh(1/2*x)+1)^2+1/2*b/(a-b)^2/(tanh(1/2*x)+1)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^2*sinh(x)^2/(a*cosh(x)+b*sinh(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [B] time = 1.75, size = 260, normalized size = 2.13

$$\frac{e^{3\,x}}{24\,a+24\,b}-\frac{e^{-3\,x}}{24\,a-24\,b}-\frac{e^{x}\,(a-b)}{8\,(a+b)^{2}}+\frac{2\,\mathrm{atan}\left(\frac{a^{2}\,b^{2}\,e^{x}\,\sqrt{a^{10}-5\,a^{8}\,b^{2}+10\,a^{6}\,b^{4}-10\,a^{4}\,b^{6}+5\,a^{2}\,b^{8}-b^{10}}}{a^{5}\,\sqrt{a^{4}\,b^{4}}-b^{5}\,\sqrt{a^{4}\,b^{4}}+2\,a^{2}\,b^{3}\,\sqrt{a^{4}\,b^{4}}-2\,a^{3}\,b^{2}\,\sqrt{a^{4}\,b^{4}}+a\,b^{4}\,\sqrt{a^{4}\,b^{4}}-a^{4}\,b\,\sqrt{a^{4}\,b^{4}}}\right)\sqrt{a^{4}\,b}}{\sqrt{a^{10}-5\,a^{8}\,b^{2}+10\,a^{6}\,b^{4}-10\,a^{4}\,b^{6}+5\,a^{2}\,b^{8}-b^{10}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cosh(x)^2*sinh(x)^2)/(a*cosh(x) + b*sinh(x)),x)
```

```
[Out] exp(3*x)/(24*a + 24*b) - exp(-3*x)/(24*a - 24*b) - (exp(x)*(a - b))/(8*(a +
 b)^2) + (2*atan((a^2*b^2*exp(x)*(a^10 - b^10 + 5*a^2*b^8 - 10*a^4*b^6 + 10
```

```
*a^6*b^4 - 5*a^8*b^2)^(1/2))/(a^5*(a^4*b^4)^(1/2) - b^5*(a^4*b^4)^(1/2) + 2
*a^2*b^3*(a^4*b^4)^(1/2) - 2*a^3*b^2*(a^4*b^4)^(1/2) + a*b^4*(a^4*b^4)^(1/2
) - a^4*b*(a^4*b^4)^(1/2)))*(a^4*b^4)^(1/2))/(a^10 - b^10 + 5*a^2*b^8 - 10*
a^4*b^6 + 10*a^6*b^4 - 5*a^8*b^2)^(1/2) + (exp(-x)*(a + b))/(8*(a - b)^2)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)**2*sinh(x)**2/(a*cosh(x)+b*sinh(x)),x)
```

```
[Out] Timed out
```

3.711 $\int \frac{\cosh^2(x)\sinh^3(x)}{a\cosh(x)+b\sinh(x)}\,dx$

Optimal. Leaf size=194

$$\frac{bx}{8\left(a^2-b^2\right)}-\frac{a^2bx}{2\left(a^2-b^2\right)^2}+\frac{a\sinh^4(x)}{4\left(a^2-b^2\right)}-\frac{ab^2\sinh^2(x)}{2\left(a^2-b^2\right)^2}-\frac{b\sinh(x)\cosh^3(x)}{4\left(a^2-b^2\right)}+\frac{b\sinh(x)\cosh(x)}{8\left(a^2-b^2\right)}+\frac{a^2b\sinh(x)\cosh(x}{2\left(a^2-b^2\right)^2}$$

```
[Out] -a^2*b^3*x/(a^2-b^2)^3-1/2*a^2*b*x/(a^2-b^2)^2+1/8*b*x/(a^2-b^2)+a^3*b^2*ln
(a*cosh(x)+b*sinh(x))/(a^2-b^2)^3+1/2*a^2*b*cosh(x)*sinh(x)/(a^2-b^2)^2+1/8
*b*cosh(x)*sinh(x)/(a^2-b^2)-1/4*b*cosh(x)^3*sinh(x)/(a^2-b^2)-1/2*a*b^2*si
nh(x)^2/(a^2-b^2)^2+1/4*a*sinh(x)^4/(a^2-b^2)
```

Rubi [A] time = 0.34, antiderivative size = 194, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {3109, 2568, 2635, 8, 2564, 30, 3097, 3133}

$$\frac{bx}{8\left(a^2-b^2\right)}-\frac{a^2bx}{2\left(a^2-b^2\right)^2}-\frac{a^2b^3x}{\left(a^2-b^2\right)^3}+\frac{a\sinh^4(x)}{4\left(a^2-b^2\right)}-\frac{ab^2\sinh^2(x)}{2\left(a^2-b^2\right)^2}-\frac{b\sinh(x)\cosh^3(x)}{4\left(a^2-b^2\right)}+\frac{b\sinh(x)\cosh(x)}{8\left(a^2-b^2\right)}+\frac{a^2b\sin}{2(}$$

Antiderivative was successfully verified.

```
[In] Int[(Cosh[x]^2*Sinh[x]^3)/(a*Cosh[x] + b*Sinh[x]),x]
```

```
[Out] -((a^2*b^3*x)/(a^2 - b^2)^3) - (a^2*b*x)/(2*(a^2 - b^2)^2) + (b*x)/(8*(a^2
- b^2)) + (a^3*b^2*Log[a*Cosh[x] + b*Sinh[x]])/(a^2 - b^2)^3 + (a^2*b*Cosh[
x]*Sinh[x])/(2*(a^2 - b^2)^2) + (b*Cosh[x]*Sinh[x])/(8*(a^2 - b^2)) - (b*Co
sh[x]^3*Sinh[x])/(4*(a^2 - b^2)) - (a*b^2*Sinh[x]^2)/(2*(a^2 - b^2)^2) + (a
*Sinh[x]^4)/(4*(a^2 - b^2))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
```

```
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2568

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m - 1)
)/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Cos[e + f*x])^n*(a
*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 3097

```
Int[sin[(c_.) + (d_.)*(x_)]/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.
) + (d_.)*(x_)]), x_Symbol] :> Simp[(b*x)/(a^2 + b^2), x] - Dist[a/(a^2 + b
^2), Int[(b*Cos[c + d*x] - a*Sin[c + d*x])/(a*Cos[c + d*x] + b*Sin[c + d*x]
), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3109

```
Int[(cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.))/(cos[(c_.
) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[b
/(a^2 + b^2), Int[Cos[c + d*x]^m*Sin[c + d*x]^(n - 1), x], x] + (Dist[a/(a^
2 + b^2), Int[Cos[c + d*x]^(m - 1)*Sin[c + d*x]^n, x], x] - Dist[(a*b)/(a^2
 + b^2), Int[(Cos[c + d*x]^(m - 1)*Sin[c + d*x]^(n - 1))/(a*Cos[c + d*x] +
b*Sin[c + d*x]), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] &&
IGtQ[m, 0] && IGtQ[n, 0]
```

Rule 3133

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)]), x
_Symbol] :> Simp[((b*B + c*C)*x)/(b^2 + c^2), x] + Simp[((c*B - b*C)*Log[a
+ b*Cos[d + e*x] + c*Sin[d + e*x]])/(e*(b^2 + c^2)), x] /; FreeQ[{a, b, c,
d, e, A, B, C}, x] && NeQ[b^2 + c^2, 0] && EqQ[A*(b^2 + c^2) - a*(b*B + c*C
), 0]
```

Rubi steps

$$\int \frac{\cosh^2(x)\sinh^3(x)}{a\cosh(x)+b\sinh(x)}\,dx = \frac{a\int \cosh(x)\sinh^3(x)\,dx}{a^2-b^2} - \frac{b\int \cosh^2(x)\sinh^2(x)\,dx}{a^2-b^2} + \frac{(ab)\int \frac{\cosh(x)\sinh^2(x)}{a\cosh(x)+b\sinh(x)}\,dx}{a^2-b^2}$$

$$= -\frac{b\cosh^3(x)\sinh(x)}{4\left(a^2-b^2\right)} + \frac{\left(a^2b\right)\int \sinh^2(x)\,dx}{\left(a^2-b^2\right)^2} - \frac{\left(ab^2\right)\int \cosh(x)\sinh(x)\,dx}{\left(a^2-b^2\right)^2} + \frac{\left(a^2b^2\right)}{}$$

$$= -\frac{a^2b^3x}{\left(a^2-b^2\right)^3} + \frac{a^2b\cosh(x)\sinh(x)}{2\left(a^2-b^2\right)^2} + \frac{b\cosh(x)\sinh(x)}{8\left(a^2-b^2\right)} - \frac{b\cosh^3(x)\sinh(x)}{4\left(a^2-b^2\right)} + \frac{a}{4}$$

$$= -\frac{a^2b^3x}{\left(a^2-b^2\right)^3} - \frac{a^2bx}{2\left(a^2-b^2\right)^2} + \frac{bx}{8\left(a^2-b^2\right)} + \frac{a^3b^2\log(a\cosh(x)+b\sinh(x))}{\left(a^2-b^2\right)^3} + \frac{a^2b}{}$$

Mathematica [**A**] time = 0.64, size = 128, normalized size = 0.66

$$\frac{-4a\left(a^4-b^4\right)\cosh(2x) + a\left(a^2-b^2\right)^2\cosh(4x) - b\left(-8a^2\left(a^2-b^2\right)\sinh(2x) + \left(a^2-b^2\right)^2\sinh(4x) + 4\left(3a^4x - 8a\right.\right.}{32(a-b)^3(a+b)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cosh[x]^2*Sinh[x]^3)/(a*Cosh[x] + b*Sinh[x]),x]
```

```
[Out] (-4*a*(a^4 - b^4)*Cosh[2*x] + a*(a^2 - b^2)^2*Cosh[4*x] - b*(4*(3*a^4*x + 6
*a^2*b^2*x - b^4*x - 8*a^3*b*Log[a*Cosh[x] + b*Sinh[x]]) - 8*a^2*(a^2 - b^2
)*Sinh[2*x] + (a^2 - b^2)^2*Sinh[4*x]))/(32*(a - b)^3*(a + b)^3)
```

fricas [**B**] time = 0.46, size = 1158, normalized size = 5.97

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^2*sinh(x)^3/(a*cosh(x)+b*sinh(x)),x, algorithm="fricas")
```

```
[Out] 1/64*((a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^8 + 8*(a^
5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)*sinh(x)^7 + (a^5 -
 a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*sinh(x)^8 - 4*(a^5 - 2*a^4*b
+ 2*a^2*b^3 - a*b^4)*cosh(x)^6 - 4*(a^5 - 2*a^4*b + 2*a^2*b^3 - a*b^4 - 7*(
a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^2)*sinh(x)^6 - 8
*(3*a^4*b + 8*a^3*b^2 + 6*a^2*b^3 - b^5)*x*cosh(x)^4 + 8*(7*(a^5 - a^4*b -
2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^3 - 3*(a^5 - 2*a^4*b + 2*a^2*b
^3 - a*b^4)*cosh(x))*sinh(x)^5 + a^5 + a^4*b - 2*a^3*b^2 - 2*a^2*b^3 + a*b^
```

```
4 + b^5 + 2*(35*(a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)
^4 - 30*(a^5 - 2*a^4*b + 2*a^2*b^3 - a*b^4)*cosh(x)^2 - 4*(3*a^4*b + 8*a^3*
b^2 + 6*a^2*b^3 - b^5)*x)*sinh(x)^4 + 8*(7*(a^5 - a^4*b - 2*a^3*b^2 + 2*a^2
*b^3 + a*b^4 - b^5)*cosh(x)^5 - 10*(a^5 - 2*a^4*b + 2*a^2*b^3 - a*b^4)*cosh
(x)^3 - 4*(3*a^4*b + 8*a^3*b^2 + 6*a^2*b^3 - b^5)*x*cosh(x))*sinh(x)^3 - 4*
(a^5 + 2*a^4*b - 2*a^2*b^3 - a*b^4)*cosh(x)^2 + 4*(7*(a^5 - a^4*b - 2*a^3*b
^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^6 - a^5 - 2*a^4*b + 2*a^2*b^3 + a*b^4
 - 15*(a^5 - 2*a^4*b + 2*a^2*b^3 - a*b^4)*cosh(x)^4 - 12*(3*a^4*b + 8*a^3*b
^2 + 6*a^2*b^3 - b^5)*x*cosh(x)^2)*sinh(x)^2 + 64*(a^3*b^2*cosh(x)^4 + 4*a^
3*b^2*cosh(x)^3*sinh(x) + 6*a^3*b^2*cosh(x)^2*sinh(x)^2 + 4*a^3*b^2*cosh(x)
*sinh(x)^3 + a^3*b^2*sinh(x)^4)*log(2*(a*cosh(x) + b*sinh(x))/(cosh(x) - si
nh(x))) + 8*((a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^7
- 3*(a^5 - 2*a^4*b + 2*a^2*b^3 - a*b^4)*cosh(x)^5 - 4*(3*a^4*b + 8*a^3*b^2
+ 6*a^2*b^3 - b^5)*x*cosh(x)^3 - (a^5 + 2*a^4*b - 2*a^2*b^3 - a*b^4)*cosh(x
))*sinh(x))/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^4 + 4*(a^6 - 3*a^4
*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^3*sinh(x) + 6*(a^6 - 3*a^4*b^2 + 3*a^2*b^4
- b^6)*cosh(x)^2*sinh(x)^2 + 4*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)*
sinh(x)^3 + (a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*sinh(x)^4)
```

giac [A] time = 0.12, size = 199, normalized size = 1.03

$$\frac{a^3b^2\log\left(\left|ae^{(2x)}+be^{(2x)}+a-b\right|\right)}{a^6-3a^4b^2+3a^2b^4-b^6}-\frac{\left(3ab-b^2\right)x}{8\left(a^3-3a^2b+3ab^2-b^3\right)}+\frac{\left(18abe^{(4x)}-6b^2e^{(4x)}-4a^2e^{(2x)}+4abe^{(2x)}+a\right.}{64\left(a^3-3a^2b+3ab^2-b^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^2*sinh(x)^3/(a*cosh(x)+b*sinh(x)),x, algorithm="giac")
```

```
[Out] a^3*b^2*log(abs(a*e^(2*x) + b*e^(2*x) + a - b))/(a^6 - 3*a^4*b^2 + 3*a^2*b^
4 - b^6) - 1/8*(3*a*b - b^2)*x/(a^3 - 3*a^2*b + 3*a*b^2 - b^3) + 1/64*(18*a
*b*e^(4*x) - 6*b^2*e^(4*x) - 4*a^2*e^(2*x) + 4*a*b*e^(2*x) + a^2 - 2*a*b +
b^2)*e^(-4*x)/(a^3 - 3*a^2*b + 3*a*b^2 - b^3) + 1/64*(a*e^(4*x) + b*e^(4*x)
 - 4*a*e^(2*x))/(a^2 + 2*a*b + b^2)
```

maple [A] time = 0.23, size = 321, normalized size = 1.65

$$\frac{4}{(16a+16b)\left(\tanh\left(\frac{x}{2}\right)-1\right)^4}+\frac{16}{(32a+32b)\left(\tanh\left(\frac{x}{2}\right)-1\right)^3}+\frac{a}{8\,(a+b)^2\left(\tanh\left(\frac{x}{2}\right)-1\right)^2}+\frac{3b}{8\,(a+b)^2\left(\tanh\left(\frac{x}{2}\right)-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^2*sinh(x)^3/(a*cosh(x)+b*sinh(x)),x)
```

```
[Out] 4/(16*a+16*b)/(tanh(1/2*x)-1)^4+16/(32*a+32*b)/(tanh(1/2*x)-1)^3+1/8/(a+b)^
2/(tanh(1/2*x)-1)^2*a+3/8/(a+b)^2/(tanh(1/2*x)-1)^2*b-1/8*a/(a+b)^2/(tanh(1
```

```
/2*x)-1)+1/8*b/(a+b)^2/(tanh(1/2*x)-1)+3/8*b/(a+b)^3*ln(tanh(1/2*x)-1)*a+1/
8*b^2/(a+b)^3*ln(tanh(1/2*x)-1)+a^3*b^2/(a-b)^3/(a+b)^3*ln(a+2*tanh(1/2*x)*
b+a*tanh(1/2*x)^2)+4/(16*a-16*b)/(tanh(1/2*x)+1)^4-16/(32*a-32*b)/(tanh(1/2
*x)+1)^3+1/8*a/(a-b)^2/(tanh(1/2*x)+1)+1/8*b/(a-b)^2/(tanh(1/2*x)+1)+1/8/(a
-b)^2/(tanh(1/2*x)+1)^2*a-3/8/(a-b)^2/(tanh(1/2*x)+1)^2*b-3/8*b/(a-b)^3*ln(
tanh(1/2*x)+1)*a+1/8*b^2/(a-b)^3*ln(tanh(1/2*x)+1)
```

maxima [A] time = 0.50, size = 153, normalized size = 0.79

$$\frac{a^3b^2\log\left(-(a-b)e^{(-2x)}-a-b\right)}{a^6-3a^4b^2+3a^2b^4-b^6}-\frac{\left(3ab+b^2\right)x}{8\left(a^3+3a^2b+3ab^2+b^3\right)}-\frac{\left(4ae^{(-2x)}-a-b\right)e^{(4x)}}{64\left(a^2+2ab+b^2\right)}-\frac{4ae^{(-2x)}-(a-b)e^{(-4x)}}{64\left(a^2-2ab+b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^2*sinh(x)^3/(a*cosh(x)+b*sinh(x)),x, algorithm="maxima")
```

```
[Out] a^3*b^2*log(-(a - b)*e^(-2*x) - a - b)/(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)
- 1/8*(3*a*b + b^2)*x/(a^3 + 3*a^2*b + 3*a*b^2 + b^3) - 1/64*(4*a*e^(-2*x)
- a - b)*e^(4*x)/(a^2 + 2*a*b + b^2) - 1/64*(4*a*e^(-2*x) - (a - b)*e^(-4*x
))/(a^2 - 2*a*b + b^2)
```

mupad [B] time = 1.87, size = 127, normalized size = 0.65

$$\frac{e^{-4x}}{64a-64b}+\frac{e^{4x}}{64a+64b}-\frac{x\left(3ab-b^2\right)}{8(a-b)^3}-\frac{a\,e^{2x}}{16(a+b)^2}-\frac{a\,e^{-2x}}{16(a-b)^2}+\frac{a^3b^2\ln\left(a-b+a\,e^{2x}+b\,e^{2x}\right)}{a^6-3a^4b^2+3a^2b^4-b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cosh(x)^2*sinh(x)^3)/(a*cosh(x) + b*sinh(x)),x)
```

```
[Out] exp(-4*x)/(64*a - 64*b) + exp(4*x)/(64*a + 64*b) - (x*(3*a*b - b^2))/(8*(a
- b)^3) - (a*exp(2*x))/(16*(a + b)^2) - (a*exp(-2*x))/(16*(a - b)^2) + (a^3
*b^2*log(a - b + a*exp(2*x) + b*exp(2*x)))/(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b
^2)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)**2*sinh(x)**3/(a*cosh(x)+b*sinh(x)),x)
```

```
[Out] Timed out
```

3.712 $\int \frac{\cosh^3(x)\sinh(x)}{a\cosh(x)+b\sinh(x)}\,dx$

Optimal. Leaf size=137

$$-\frac{b\sinh^3(x)}{3\left(a^2-b^2\right)}-\frac{b\sinh(x)}{a^2-b^2}+\frac{a^2b\sinh(x)}{\left(a^2-b^2\right)^2}+\frac{a\cosh^3(x)}{3\left(a^2-b^2\right)}-\frac{ab^2\cosh(x)}{\left(a^2-b^2\right)^2}-\frac{ab^3\tan^{-1}\left(\frac{a\sinh(x)+b\cosh(x)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{5/2}}$$

```
[Out] -a*b^3*arctan((b*cosh(x)+a*sinh(x))/(a^2-b^2)^(1/2))/(a^2-b^2)^(5/2)-a*b^2*
cosh(x)/(a^2-b^2)^2+1/3*a*cosh(x)^3/(a^2-b^2)+a^2*b*sinh(x)/(a^2-b^2)^2-b*s
inh(x)/(a^2-b^2)-1/3*b*sinh(x)^3/(a^2-b^2)
```

Rubi [A] time = 0.20, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {3109, 2633, 2565, 30, 3100, 2637, 3074, 206}

$$-\frac{b\sinh^3(x)}{3\left(a^2-b^2\right)}-\frac{b\sinh(x)}{a^2-b^2}+\frac{a^2b\sinh(x)}{\left(a^2-b^2\right)^2}+\frac{a\cosh^3(x)}{3\left(a^2-b^2\right)}-\frac{ab^2\cosh(x)}{\left(a^2-b^2\right)^2}-\frac{ab^3\tan^{-1}\left(\frac{a\sinh(x)+b\cosh(x)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cosh[x]^3*Sinh[x])/(a*Cosh[x] + b*Sinh[x]),x]
```

```
[Out] -((a*b^3*ArcTan[(b*Cosh[x] + a*Sinh[x])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(5/2)
) - (a*b^2*Cosh[x])/(a^2 - b^2)^2 + (a*Cosh[x]^3)/(3*(a^2 - b^2)) + (a^2*b*
Sinh[x])/(a^2 - b^2)^2 - (b*Sinh[x])/(a^2 - b^2) - (b*Sinh[x]^3)/(3*(a^2 -
b^2))
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
```

```
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3074

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> -Dist[d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d
*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3100

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_)/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin
[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[(b*Cos[c + d*x]^(m - 1))/(d*(a^2 +
 b^2)*(m - 1)), x] + (Dist[a/(a^2 + b^2), Int[Cos[c + d*x]^(m - 1), x], x]
+ Dist[b^2/(a^2 + b^2), Int[Cos[c + d*x]^(m - 2)/(a*Cos[c + d*x] + b*Sin[c
+ d*x]), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && GtQ[m, 1
]
```

Rule 3109

```
Int[(cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.))/(cos[(c_.
) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[b
/(a^2 + b^2), Int[Cos[c + d*x]^m*Sin[c + d*x]^(n - 1), x], x] + (Dist[a/(a^
2 + b^2), Int[Cos[c + d*x]^(m - 1)*Sin[c + d*x]^n, x], x] - Dist[(a*b)/(a^2
 + b^2), Int[(Cos[c + d*x]^(m - 1)*Sin[c + d*x]^(n - 1))/(a*Cos[c + d*x] +
b*Sin[c + d*x]), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] &&
IGtQ[m, 0] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{\cosh^3(x)\sinh(x)}{a\cosh(x)+b\sinh(x)}\,dx = \frac{a\int \cosh^2(x)\sinh(x)\,dx}{a^2-b^2} - \frac{b\int \cosh^3(x)\,dx}{a^2-b^2} + \frac{(ab)\int \frac{\cosh^2(x)}{a\cosh(x)+b\sinh(x)}\,dx}{a^2-b^2}$$

$$= -\frac{ab^2\cosh(x)}{\left(a^2-b^2\right)^2} + \frac{\left(a^2b\right)\int \cosh(x)\,dx}{\left(a^2-b^2\right)^2} - \frac{\left(ab^3\right)\int \frac{1}{a\cosh(x)+b\sinh(x)}\,dx}{\left(a^2-b^2\right)^2} + \frac{a\,\mathrm{Subst}\left(\int x\right.}{a^2}$$

$$= -\frac{ab^2\cosh(x)}{\left(a^2-b^2\right)^2} + \frac{a\cosh^3(x)}{3\left(a^2-b^2\right)} + \frac{a^2b\sinh(x)}{\left(a^2-b^2\right)^2} - \frac{b\sinh(x)}{a^2-b^2} - \frac{b\sinh^3(x)}{3\left(a^2-b^2\right)} - \frac{\left(iab^3\right)\mathrm{Su}}{}$$

$$= -\frac{ab^3\tan^{-1}\left(\frac{b\cosh(x)+a\sinh(x)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{5/2}} - \frac{ab^2\cosh(x)}{\left(a^2-b^2\right)^2} + \frac{a\cosh^3(x)}{3\left(a^2-b^2\right)} + \frac{a^2b\sinh(x)}{\left(a^2-b^2\right)^2} - \frac{b\sinh}{a^2-}$$

Mathematica [A] time = 1.18, size = 167, normalized size = 1.22

$$\frac{1}{12}\left(\frac{3b\left(a^2+3b^2\right)\sinh(x)}{(a-b)^2(a+b)^2} + \frac{3a\left(a^2-5b^2\right)\cosh(x)}{(a-b)^2(a+b)^2} - \frac{a^2b\sinh(3x)}{(a-b)^2(a+b)^2} + \frac{b^3\sinh(3x)}{(a-b)^2(a+b)^2} - \frac{24ab^3\tan^{-1}\left(\frac{a\tanh\left(\frac{x}{2}\right)}{\sqrt{a-b}\sqrt{a}}\right.}{(a-b)^{5/2}(a+b)^{5/}}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cosh[x]^3*Sinh[x])/(a*Cosh[x] + b*Sinh[x]),x]
```

```
[Out] ((-24*a*b^3*ArcTan[(b + a*Tanh[x/2])/(Sqrt[a - b]*Sqrt[a + b])])/((a - b)^(
5/2)*(a + b)^(5/2)) + (3*a*(a^2 - 5*b^2)*Cosh[x])/((a - b)^2*(a + b)^2) + (
a*Cosh[3*x])/((a - b)*(a + b)) + (3*b*(a^2 + 3*b^2)*Sinh[x])/((a - b)^2*(a
+ b)^2) - (a^2*b*Sinh[3*x])/((a - b)^2*(a + b)^2) + (b^3*Sinh[3*x])/((a - b
)^2*(a + b)^2))/12
```

fricas [B] time = 0.47, size = 1829, normalized size = 13.35

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^3*sinh(x)/(a*cosh(x)+b*sinh(x)),x, algorithm="fricas")
```

```
[Out] [1/24*((a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^6 + 6*(a
^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)*sinh(x)^5 + (a^5
- a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*sinh(x)^6 + a^5 + a^4*b - 2*
```

```
a^3*b^2 - 2*a^2*b^3 + a*b^4 + b^5 + 3*(a^5 + a^4*b - 6*a^3*b^2 + 2*a^2*b^3
+ 5*a*b^4 - 3*b^5)*cosh(x)^4 + 3*(a^5 + a^4*b - 6*a^3*b^2 + 2*a^2*b^3 + 5*a
*b^4 - 3*b^5 + 5*(a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x
)^2)*sinh(x)^4 + 4*(5*(a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*c
osh(x)^3 + 3*(a^5 + a^4*b - 6*a^3*b^2 + 2*a^2*b^3 + 5*a*b^4 - 3*b^5)*cosh(x
))*sinh(x)^3 + 3*(a^5 - a^4*b - 6*a^3*b^2 - 2*a^2*b^3 + 5*a*b^4 + 3*b^5)*co
sh(x)^2 + 3*(a^5 - a^4*b - 6*a^3*b^2 - 2*a^2*b^3 + 5*a*b^4 + 3*b^5 + 5*(a^5
 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^4 + 6*(a^5 + a^4*b
- 6*a^3*b^2 + 2*a^2*b^3 + 5*a*b^4 - 3*b^5)*cosh(x)^2)*sinh(x)^2 - 24*(a*b^3
*cosh(x)^3 + 3*a*b^3*cosh(x)^2*sinh(x) + 3*a*b^3*cosh(x)*sinh(x)^2 + a*b^3*
sinh(x)^3)*sqrt(-a^2 + b^2)*log(((a + b)*cosh(x)^2 + 2*(a + b)*cosh(x)*sinh
(x) + (a + b)*sinh(x)^2 + 2*sqrt(-a^2 + b^2)*(cosh(x) + sinh(x)) - a + b)/(
(a + b)*cosh(x)^2 + 2*(a + b)*cosh(x)*sinh(x) + (a + b)*sinh(x)^2 + a - b))
 + 6*((a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^5 + 2*(a^
5 + a^4*b - 6*a^3*b^2 + 2*a^2*b^3 + 5*a*b^4 - 3*b^5)*cosh(x)^3 + (a^5 - a^4
*b - 6*a^3*b^2 - 2*a^2*b^3 + 5*a*b^4 + 3*b^5)*cosh(x))*sinh(x))/((a^6 - 3*a
^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^3 + 3*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)
*cosh(x)^2*sinh(x) + 3*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)*sinh(x)^
2 + (a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*sinh(x)^3), 1/24*((a^5 - a^4*b - 2*
a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^6 + 6*(a^5 - a^4*b - 2*a^3*b^2 +
 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)*sinh(x)^5 + (a^5 - a^4*b - 2*a^3*b^2 + 2*
a^2*b^3 + a*b^4 - b^5)*sinh(x)^6 + a^5 + a^4*b - 2*a^3*b^2 - 2*a^2*b^3 + a*
b^4 + b^5 + 3*(a^5 + a^4*b - 6*a^3*b^2 + 2*a^2*b^3 + 5*a*b^4 - 3*b^5)*cosh(
x)^4 + 3*(a^5 + a^4*b - 6*a^3*b^2 + 2*a^2*b^3 + 5*a*b^4 - 3*b^5 + 5*(a^5 -
a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^2)*sinh(x)^4 + 4*(5*(a
^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^3 + 3*(a^5 + a^4*
b - 6*a^3*b^2 + 2*a^2*b^3 + 5*a*b^4 - 3*b^5)*cosh(x))*sinh(x)^3 + 3*(a^5 -
a^4*b - 6*a^3*b^2 - 2*a^2*b^3 + 5*a*b^4 + 3*b^5)*cosh(x)^2 + 3*(a^5 - a^4*b
 - 6*a^3*b^2 - 2*a^2*b^3 + 5*a*b^4 + 3*b^5 + 5*(a^5 - a^4*b - 2*a^3*b^2 + 2
*a^2*b^3 + a*b^4 - b^5)*cosh(x)^4 + 6*(a^5 + a^4*b - 6*a^3*b^2 + 2*a^2*b^3
+ 5*a*b^4 - 3*b^5)*cosh(x)^2)*sinh(x)^2 + 48*(a*b^3*cosh(x)^3 + 3*a*b^3*cos
h(x)^2*sinh(x) + 3*a*b^3*cosh(x)*sinh(x)^2 + a*b^3*sinh(x)^3)*sqrt(a^2 - b^
2)*arctan(sqrt(a^2 - b^2)/((a + b)*cosh(x) + (a + b)*sinh(x))) + 6*((a^5 -
a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^5 + 2*(a^5 + a^4*b - 6
*a^3*b^2 + 2*a^2*b^3 + 5*a*b^4 - 3*b^5)*cosh(x)^3 + (a^5 - a^4*b - 6*a^3*b^
2 - 2*a^2*b^3 + 5*a*b^4 + 3*b^5)*cosh(x))*sinh(x))/((a^6 - 3*a^4*b^2 + 3*a^
2*b^4 - b^6)*cosh(x)^3 + 3*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^2*si
nh(x) + 3*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)*sinh(x)^2 + (a^6 - 3*
a^4*b^2 + 3*a^2*b^4 - b^6)*sinh(x)^3)]
```

giac [A] time = 0.14, size = 163, normalized size = 1.19

$$-\frac{2\,ab^{3}\arctan\left(\frac{ae^{x}+be^{x}}{\sqrt{a^{2}-b^{2}}}\right)}{\left(a^{4}-2\,a^{2}b^{2}+b^{4}\right)\sqrt{a^{2}-b^{2}}}+\frac{\left(3\,ae^{(2\,x)}-9\,be^{(2\,x)}+a-b\right)e^{(-3\,x)}}{24\,\left(a^{2}-2\,ab+b^{2}\right)}+\frac{a^{2}e^{(3\,x)}+2\,abe^{(3\,x)}+b^{2}e^{(3\,x)}+3\,a^{2}e^{x}+12\,abe^{x}+}{24\,\left(a^{3}+3\,a^{2}b+3\,ab^{2}+b^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^3*sinh(x)/(a*cosh(x)+b*sinh(x)),x, algorithm="giac")
```

```
[Out] -2*a*b^3*arctan((a*e^x + b*e^x)/sqrt(a^2 - b^2))/((a^4 - 2*a^2*b^2 + b^4)*s
qrt(a^2 - b^2)) + 1/24*(3*a*e^(2*x) - 9*b*e^(2*x) + a - b)*e^(-3*x)/(a^2 -
2*a*b + b^2) + 1/24*(a^2*e^(3*x) + 2*a*b*e^(3*x) + b^2*e^(3*x) + 3*a^2*e^x
+ 12*a*b*e^x + 9*b^2*e^x)/(a^3 + 3*a^2*b + 3*a*b^2 + b^3)
```

maple [A] time = 0.21, size = 200, normalized size = 1.46

$$-\frac{4}{3\left(\tanh\left(\frac{x}{2}\right)-1\right)^{3}(4a+4b)}-\frac{2}{(4a+4b)\left(\tanh\left(\frac{x}{2}\right)-1\right)^{2}}-\frac{a}{2(a+b)^{2}\left(\tanh\left(\frac{x}{2}\right)-1\right)}-\frac{b}{(a+b)^{2}\left(\tanh\left(\frac{x}{2}\right)-1\right)}-\frac{2a}{(a\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^3*sinh(x)/(a*cosh(x)+b*sinh(x)),x)
```

```
[Out] -4/3/(tanh(1/2*x)-1)^3/(4*a+4*b)-2/(4*a+4*b)/(tanh(1/2*x)-1)^2-1/2*a/(a+b)^
2/(tanh(1/2*x)-1)-b/(a+b)^2/(tanh(1/2*x)-1)-2*a*b^3/(a-b)^2/(a+b)^2/(a^2-b^
2)^(1/2)*arctan(1/2*(2*a*tanh(1/2*x)+2*b)/(a^2-b^2)^(1/2))-2/(4*a-4*b)/(tan
h(1/2*x)+1)^2+4/3/(tanh(1/2*x)+1)^3/(4*a-4*b)+1/2*a/(a-b)^2/(tanh(1/2*x)+1)
-b/(a-b)^2/(tanh(1/2*x)+1)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^3*sinh(x)/(a*cosh(x)+b*sinh(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [B] time = 1.76, size = 259, normalized size = 1.89

$$\frac{e^{-3x}}{24a-24b}+\frac{e^{3x}}{24a+24b}+\frac{e^{x}\,(a+3b)}{8(a+b)^{2}}-\frac{2\,\mathrm{atan}\left(\frac{a\,b^{3}\,e^{x}\,\sqrt{a^{10}-5a^{8}b^{2}+10a^{6}b^{4}-10a^{4}b^{6}+5a^{2}b^{8}-b^{10}}}{a^{5}\sqrt{a^{2}b^{6}}-b^{5}\sqrt{a^{2}b^{6}}+2a^{2}b^{3}\sqrt{a^{2}b^{6}}-2a^{3}b^{2}\sqrt{a^{2}b^{6}}+a\,b^{4}\sqrt{a^{2}b^{6}}-a^{4}b\sqrt{a^{2}b^{6}}}\right)\sqrt{\cdots}}{\sqrt{a^{10}-5a^{8}b^{2}+10a^{6}b^{4}-10a^{4}b^{6}+5a^{2}b^{8}-b^{10}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cosh(x)^3*sinh(x))/(a*cosh(x) + b*sinh(x)),x)
```

```
[Out] exp(-3*x)/(24*a - 24*b) + exp(3*x)/(24*a + 24*b) + (exp(x)*(a + 3*b))/(8*(a
 + b)^2) - (2*atan((a*b^3*exp(x)*(a^10 - b^10 + 5*a^2*b^8 - 10*a^4*b^6 + 10
*a^6*b^4 - 5*a^8*b^2)^(1/2))/(a^5*(a^2*b^6)^(1/2) - b^5*(a^2*b^6)^(1/2) + 2
*a^2*b^3*(a^2*b^6)^(1/2) - 2*a^3*b^2*(a^2*b^6)^(1/2) + a*b^4*(a^2*b^6)^(1/2
) - a^4*b*(a^2*b^6)^(1/2)))*(a^2*b^6)^(1/2))/(a^10 - b^10 + 5*a^2*b^8 - 10*
a^4*b^6 + 10*a^6*b^4 - 5*a^8*b^2)^(1/2) + (exp(-x)*(a - 3*b))/(8*(a - b)^2)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)**3*sinh(x)/(a*cosh(x)+b*sinh(x)),x)
```

```
[Out] Timed out
```

3.713 $\int \frac{\cosh^3(x)\sinh^2(x)}{a\cosh(x)+b\sinh(x)}\,dx$

Optimal. Leaf size=194

$$-\frac{ax}{8\left(a^2-b^2\right)}-\frac{ab^2x}{2\left(a^2-b^2\right)^2}+\frac{a^2b\sinh^2(x)}{2\left(a^2-b^2\right)^2}-\frac{b\cosh^4(x)}{4\left(a^2-b^2\right)}+\frac{a\sinh(x)\cosh^3(x)}{4\left(a^2-b^2\right)}-\frac{a\sinh(x)\cosh(x)}{8\left(a^2-b^2\right)}-\frac{ab^2\sinh(x)\cosh}{2\left(a^2-b^2\right)^2}$$

```
[Out] a^3*b^2*x/(a^2-b^2)^3-1/2*a*b^2*x/(a^2-b^2)^2-1/8*a*x/(a^2-b^2)-1/4*b*cosh(
x)^4/(a^2-b^2)-a^2*b^3*ln(a*cosh(x)+b*sinh(x))/(a^2-b^2)^3-1/2*a*b^2*cosh(x
)*sinh(x)/(a^2-b^2)^2-1/8*a*cosh(x)*sinh(x)/(a^2-b^2)+1/4*a*cosh(x)^3*sinh(
x)/(a^2-b^2)+1/2*a^2*b*sinh(x)^2/(a^2-b^2)^2
```

Rubi [A] time = 0.34, antiderivative size = 194, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.450, Rules used = {3109, 2565, 30, 2568, 2635, 8, 2564, 3098, 3133}

$$-\frac{ax}{8\left(a^2-b^2\right)}-\frac{ab^2x}{2\left(a^2-b^2\right)^2}+\frac{a^3b^2x}{\left(a^2-b^2\right)^3}+\frac{a^2b\sinh^2(x)}{2\left(a^2-b^2\right)^2}-\frac{b\cosh^4(x)}{4\left(a^2-b^2\right)}+\frac{a\sinh(x)\cosh^3(x)}{4\left(a^2-b^2\right)}-\frac{a\sinh(x)\cosh(x)}{8\left(a^2-b^2\right)}-\frac{ab^2}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Cosh[x]^3*Sinh[x]^2)/(a*Cosh[x] + b*Sinh[x]),x]
```

```
[Out] (a^3*b^2*x)/(a^2 - b^2)^3 - (a*b^2*x)/(2*(a^2 - b^2)^2) - (a*x)/(8*(a^2 - b
^2)) - (b*Cosh[x]^4)/(4*(a^2 - b^2)) - (a^2*b^3*Log[a*Cosh[x] + b*Sinh[x]])
/(a^2 - b^2)^3 - (a*b^2*Cosh[x]*Sinh[x])/(2*(a^2 - b^2)^2) - (a*Cosh[x]*Sin
h[x])/(8*(a^2 - b^2)) + (a*Cosh[x]^3*Sinh[x])/(4*(a^2 - b^2)) + (a^2*b*Sinh
[x]^2)/(2*(a^2 - b^2)^2)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] && !(In
```

```
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2568

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m - 1)
)/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Cos[e + f*x])^n*(a
*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 3098

```
Int[cos[(c_.) + (d_.)*(x_)]/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.
) + (d_.)*(x_)]), x_Symbol] :> Simp[(a*x)/(a^2 + b^2), x] + Dist[b/(a^2 + b
^2), Int[(b*Cos[c + d*x] - a*Sin[c + d*x])/(a*Cos[c + d*x] + b*Sin[c + d*x]
), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3109

```
Int[(cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.))/(cos[(c_.
) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[b
/(a^2 + b^2), Int[Cos[c + d*x]^m*Sin[c + d*x]^(n - 1), x], x] + (Dist[a/(a^
2 + b^2), Int[Cos[c + d*x]^(m - 1)*Sin[c + d*x]^n, x], x] - Dist[(a*b)/(a^2
 + b^2), Int[(Cos[c + d*x]^(m - 1)*Sin[c + d*x]^(n - 1))/(a*Cos[c + d*x] +
b*Sin[c + d*x]), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] &&
IGtQ[m, 0] && IGtQ[n, 0]
```

Rule 3133

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)]), x
```

```
_Symbol] :> Simp[((b*B + c*C)*x)/(b^2 + c^2), x] + Simp[((c*B - b*C)*Log[a
+ b*Cos[d + e*x] + c*Sin[d + e*x]])/(e*(b^2 + c^2)), x] /; FreeQ[{a, b, c,
d, e, A, B, C}, x] && NeQ[b^2 + c^2, 0] && EqQ[A*(b^2 + c^2) - a*(b*B + c*C
), 0]
```

Rubi steps

$$\int \frac{\cosh^3(x)\sinh^2(x)}{a\cosh(x)+b\sinh(x)}\,dx = \frac{a\int \cosh^2(x)\sinh^2(x)\,dx}{a^2-b^2} - \frac{b\int \cosh^3(x)\sinh(x)\,dx}{a^2-b^2} + \frac{(ab)\int \frac{\cosh^2(x)\sinh(x)}{a\cosh(x)+b\sinh(x)}\,dx}{a^2-b^2}$$

$$= \frac{a\cosh^3(x)\sinh(x)}{4\left(a^2-b^2\right)} + \frac{\left(a^2b\right)\int \cosh(x)\sinh(x)\,dx}{\left(a^2-b^2\right)^2} - \frac{\left(ab^2\right)\int \cosh^2(x)\,dx}{\left(a^2-b^2\right)^2} + \frac{\left(a^2b^2\right.}{}$$

$$= \frac{a^3b^2x}{\left(a^2-b^2\right)^3} - \frac{b\cosh^4(x)}{4\left(a^2-b^2\right)} - \frac{ab^2\cosh(x)\sinh(x)}{2\left(a^2-b^2\right)^2} - \frac{a\cosh(x)\sinh(x)}{8\left(a^2-b^2\right)} + \frac{a\cosh^3(x)}{4\left(a^2-\right.}$$

$$= \frac{a^3b^2x}{\left(a^2-b^2\right)^3} - \frac{ab^2x}{2\left(a^2-b^2\right)^2} - \frac{ax}{8\left(a^2-b^2\right)} - \frac{b\cosh^4(x)}{4\left(a^2-b^2\right)} - \frac{a^2b^3\log(a\cosh(x)+b\sin}{\left(a^2-b^2\right)^3}$$

Mathematica [A] time = 0.61, size = 126, normalized size = 0.65

$$\frac{4b\left(a^4-b^4\right)\cosh(2x) - b\left(a^2-b^2\right)^2\cosh(4x) + a\left(8b^2\left(b^2-a^2\right)\sinh(2x) + \left(a^2-b^2\right)^2\sinh(4x) - 4\left(x\left(a^4-6a^2\right.\right.\right.}{32(a-b)^3(a+b)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cosh[x]^3*Sinh[x]^2)/(a*Cosh[x] + b*Sinh[x]),x]
```

```
[Out] (4*b*(a^4 - b^4)*Cosh[2*x] - b*(a^2 - b^2)^2*Cosh[4*x] + a*(-4*((a^4 - 6*a^
2*b^2 - 3*b^4)*x + 8*a*b^3*Log[a*Cosh[x] + b*Sinh[x]]) + 8*b^2*(-a^2 + b^2)
*Sinh[2*x] + (a^2 - b^2)^2*Sinh[4*x]))/(32*(a - b)^3*(a + b)^3)
```

fricas [B] time = 0.61, size = 1162, normalized size = 5.99

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^3*sinh(x)^2/(a*cosh(x)+b*sinh(x)),x, algorithm="fricas")
```

```
[Out] 1/64*((a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^8 + 8*(a^
5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)*sinh(x)^7 + (a^5 -
 a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*sinh(x)^8 + 4*(a^4*b - 2*a^3*
b^2 + 2*a*b^4 - b^5)*cosh(x)^6 + 4*(a^4*b - 2*a^3*b^2 + 2*a*b^4 - b^5 + 7*(
a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^2)*sinh(x)^6 - 8
*(a^5 - 6*a^3*b^2 - 8*a^2*b^3 - 3*a*b^4)*x*cosh(x)^4 + 8*(7*(a^5 - a^4*b -
2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^3 + 3*(a^4*b - 2*a^3*b^2 + 2*a
*b^4 - b^5)*cosh(x))*sinh(x)^5 - a^5 - a^4*b + 2*a^3*b^2 + 2*a^2*b^3 - a*b^
4 - b^5 + 2*(35*(a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)
^4 + 30*(a^4*b - 2*a^3*b^2 + 2*a*b^4 - b^5)*cosh(x)^2 - 4*(a^5 - 6*a^3*b^2
- 8*a^2*b^3 - 3*a*b^4)*x)*sinh(x)^4 + 8*(7*(a^5 - a^4*b - 2*a^3*b^2 + 2*a^2
*b^3 + a*b^4 - b^5)*cosh(x)^5 + 10*(a^4*b - 2*a^3*b^2 + 2*a*b^4 - b^5)*cosh
(x)^3 - 4*(a^5 - 6*a^3*b^2 - 8*a^2*b^3 - 3*a*b^4)*x*cosh(x))*sinh(x)^3 + 4*
(a^4*b + 2*a^3*b^2 - 2*a*b^4 - b^5)*cosh(x)^2 + 4*(7*(a^5 - a^4*b - 2*a^3*b
^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^6 + a^4*b + 2*a^3*b^2 - 2*a*b^4 - b^5
 + 15*(a^4*b - 2*a^3*b^2 + 2*a*b^4 - b^5)*cosh(x)^4 - 12*(a^5 - 6*a^3*b^2 -
 8*a^2*b^3 - 3*a*b^4)*x*cosh(x)^2)*sinh(x)^2 - 64*(a^2*b^3*cosh(x)^4 + 4*a^
2*b^3*cosh(x)^3*sinh(x) + 6*a^2*b^3*cosh(x)^2*sinh(x)^2 + 4*a^2*b^3*cosh(x)
*sinh(x)^3 + a^2*b^3*sinh(x)^4)*log(2*(a*cosh(x) + b*sinh(x))/(cosh(x) - si
nh(x))) + 8*((a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^7
+ 3*(a^4*b - 2*a^3*b^2 + 2*a*b^4 - b^5)*cosh(x)^5 - 4*(a^5 - 6*a^3*b^2 - 8*
a^2*b^3 - 3*a*b^4)*x*cosh(x)^3 + (a^4*b + 2*a^3*b^2 - 2*a*b^4 - b^5)*cosh(x
))*sinh(x))/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^4 + 4*(a^6 - 3*a^4
*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^3*sinh(x) + 6*(a^6 - 3*a^4*b^2 + 3*a^2*b^4
- b^6)*cosh(x)^2*sinh(x)^2 + 4*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)*
sinh(x)^3 + (a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*sinh(x)^4)
```

giac [A] time = 0.12, size = 202, normalized size = 1.04

$$-\frac{a^2b^3\log\left(\left|ae^{(2x)}+be^{(2x)}+a-b\right|\right)}{a^6-3a^4b^2+3a^2b^4-b^6}-\frac{\left(a^2-3ab\right)x}{8\left(a^3-3a^2b+3ab^2-b^3\right)}+\frac{\left(6a^2e^{(4x)}-18abe^{(4x)}+4abe^{(2x)}-4b^2e^{(2x)}-a\right.}{64\left(a^3-3a^2b+3ab^2-b^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^3*sinh(x)^2/(a*cosh(x)+b*sinh(x)),x, algorithm="giac")
```

```
[Out] -a^2*b^3*log(abs(a*e^(2*x) + b*e^(2*x) + a - b))/(a^6 - 3*a^4*b^2 + 3*a^2*b
^4 - b^6) - 1/8*(a^2 - 3*a*b)*x/(a^3 - 3*a^2*b + 3*a*b^2 - b^3) + 1/64*(6*a
^2*e^(4*x) - 18*a*b*e^(4*x) + 4*a*b*e^(2*x) - 4*b^2*e^(2*x) - a^2 + 2*a*b -
 b^2)*e^(-4*x)/(a^3 - 3*a^2*b + 3*a*b^2 - b^3) + 1/64*(a*e^(4*x) + b*e^(4*x
) + 4*b*e^(2*x))/(a^2 + 2*a*b + b^2)
```

maple [A] time = 0.22, size = 322, normalized size = 1.66

$$\frac{2}{(8a+8b)\left(\tanh\left(\frac{x}{2}\right)-1\right)^{4}}+\frac{8}{\left(\tanh\left(\frac{x}{2}\right)-1\right)^{3}(16a+16b)}+\frac{3a}{8(a+b)^{2}\left(\tanh\left(\frac{x}{2}\right)-1\right)^{2}}+\frac{5b}{8(a+b)^{2}\left(\tanh\left(\frac{x}{2}\right)-1\right)^{2}}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^3*sinh(x)^2/(a*cosh(x)+b*sinh(x)),x)
```

```
[Out] 2/(8*a+8*b)/(tanh(1/2*x)-1)^4+8/(tanh(1/2*x)-1)^3/(16*a+16*b)+3/8/(a+b)^2/(
tanh(1/2*x)-1)^2*a+5/8/(a+b)^2/(tanh(1/2*x)-1)^2*b+1/8*a/(a+b)^2/(tanh(1/2*
x)-1)+3/8*b/(a+b)^2/(tanh(1/2*x)-1)+1/8*a^2/(a+b)^3*ln(tanh(1/2*x)-1)+3/8*b
/(a+b)^3*ln(tanh(1/2*x)-1)*a-a^2*b^3/(a-b)^3/(a+b)^3*ln(a+2*tanh(1/2*x)*b+a
*tanh(1/2*x)^2)-2/(8*a-8*b)/(tanh(1/2*x)+1)^4+8/(tanh(1/2*x)+1)^3/(16*a-16*
b)+1/8*a/(a-b)^2/(tanh(1/2*x)+1)-3/8*b/(a-b)^2/(tanh(1/2*x)+1)-3/8/(a-b)^2/
(tanh(1/2*x)+1)^2*a+5/8/(a-b)^2/(tanh(1/2*x)+1)^2*b-1/8*a^2/(a-b)^3*ln(tanh
(1/2*x)+1)+3/8*b/(a-b)^3*ln(tanh(1/2*x)+1)*a
```

maxima [A] time = 0.33, size = 150, normalized size = 0.77

$$-\frac{a^{2}b^{3}\log\left(-(a-b)e^{(-2x)}-a-b\right)}{a^{6}-3a^{4}b^{2}+3a^{2}b^{4}-b^{6}}-\frac{\left(a^{2}+3ab\right)x}{8\left(a^{3}+3a^{2}b+3ab^{2}+b^{3}\right)}+\frac{\left(4be^{(-2x)}+a+b\right)e^{(4x)}}{64\left(a^{2}+2ab+b^{2}\right)}+\frac{4be^{(-2x)}-(a-b)e^{(-4x}}{64\left(a^{2}-2ab+b^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^3*sinh(x)^2/(a*cosh(x)+b*sinh(x)),x, algorithm="maxima")
```

```
[Out] -a^2*b^3*log(-(a - b)*e^(-2*x) - a - b)/(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)
 - 1/8*(a^2 + 3*a*b)*x/(a^3 + 3*a^2*b + 3*a*b^2 + b^3) + 1/64*(4*b*e^(-2*x)
 + a + b)*e^(4*x)/(a^2 + 2*a*b + b^2) + 1/64*(4*b*e^(-2*x) - (a - b)*e^(-4*
x))/(a^2 - 2*a*b + b^2)
```

mupad [B] time = 1.82, size = 129, normalized size = 0.66

$$\frac{e^{4x}}{64a+64b}-\frac{e^{-4x}}{64a-64b}+\frac{x\left(3ab-a^{2}\right)}{8(a-b)^{3}}+\frac{b\,e^{2x}}{16(a+b)^{2}}+\frac{b\,e^{-2x}}{16(a-b)^{2}}-\frac{a^{2}b^{3}\ln\left(a-b+a\,e^{2x}+b\,e^{2x}\right)}{a^{6}-3a^{4}b^{2}+3a^{2}b^{4}-b^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cosh(x)^3*sinh(x)^2)/(a*cosh(x) + b*sinh(x)),x)
```

```
[Out] exp(4*x)/(64*a + 64*b) - exp(-4*x)/(64*a - 64*b) + (x*(3*a*b - a^2))/(8*(a
- b)^3) + (b*exp(2*x))/(16*(a + b)^2) + (b*exp(-2*x))/(16*(a - b)^2) - (a^2
*b^3*log(a - b + a*exp(2*x) + b*exp(2*x)))/(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b
^2)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)**3*sinh(x)**2/(a*cosh(x)+b*sinh(x)),x)
```

```
[Out] Timed out
```

3.714 $\int \frac{\cosh^3(x)\sinh^3(x)}{a\cosh(x)+b\sinh(x)}\,dx$

Optimal. Leaf size=212

$$-\frac{b\sinh^5(x)}{5(a^2-b^2)}-\frac{b\sinh^3(x)}{3(a^2-b^2)}+\frac{a^2b\sinh^3(x)}{3(a^2-b^2)^2}+\frac{a\cosh^5(x)}{5(a^2-b^2)}-\frac{a\cosh^3(x)}{3(a^2-b^2)}-\frac{ab^2\cosh^3(x)}{3(a^2-b^2)^2}-\frac{a^2b^3\sinh(x)}{(a^2-b^2)^3}+\frac{a^3b^2\cosh(x)}{(a^2-b^2)^3}+$$

```
[Out] a^3*b^3*arctan((b*cosh(x)+a*sinh(x))/(a^2-b^2)^(1/2))/(a^2-b^2)^(7/2)+a^3*b
^2*cosh(x)/(a^2-b^2)^3-1/3*a*b^2*cosh(x)^3/(a^2-b^2)^2-1/3*a*cosh(x)^3/(a^2
-b^2)+1/5*a*cosh(x)^5/(a^2-b^2)-a^2*b^3*sinh(x)/(a^2-b^2)^3+1/3*a^2*b*sinh(
x)^3/(a^2-b^2)^2-1/3*b*sinh(x)^3/(a^2-b^2)-1/5*b*sinh(x)^5/(a^2-b^2)
```

Rubi [A] time = 0.43, antiderivative size = 212, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 9, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.450, Rules used = {3109, 2564, 14, 2565, 30, 2637, 2638, 3074, 206}

$$-\frac{b\sinh^5(x)}{5(a^2-b^2)}-\frac{b\sinh^3(x)}{3(a^2-b^2)}+\frac{a^2b\sinh^3(x)}{3(a^2-b^2)^2}-\frac{a^2b^3\sinh(x)}{(a^2-b^2)^3}+\frac{a\cosh^5(x)}{5(a^2-b^2)}-\frac{a\cosh^3(x)}{3(a^2-b^2)}-\frac{ab^2\cosh^3(x)}{3(a^2-b^2)^2}+\frac{a^3b^2\cosh(x)}{(a^2-b^2)^3}+$$

Antiderivative was successfully verified.

```
[In] Int[(Cosh[x]^3*Sinh[x]^3)/(a*Cosh[x] + b*Sinh[x]),x]
```

```
[Out] (a^3*b^3*ArcTan[(b*Cosh[x] + a*Sinh[x])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(7/2)
 + (a^3*b^2*Cosh[x])/(a^2 - b^2)^3 - (a*b^2*Cosh[x]^3)/(3*(a^2 - b^2)^2) -
(a*Cosh[x]^3)/(3*(a^2 - b^2)) + (a*Cosh[x]^5)/(5*(a^2 - b^2)) - (a^2*b^3*Si
nh[x])/(a^2 - b^2)^3 + (a^2*b*Sinh[x]^3)/(3*(a^2 - b^2)^2) - (b*Sinh[x]^3)/
(3*(a^2 - b^2)) - (b*Sinh[x]^5)/(5*(a^2 - b^2))
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3074

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> -Dist[d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d
*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3109

```
Int[(cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.))/(cos[(c_.
) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[b
/(a^2 + b^2), Int[Cos[c + d*x]^m*Sin[c + d*x]^(n - 1), x], x] + (Dist[a/(a^
2 + b^2), Int[Cos[c + d*x]^(m - 1)*Sin[c + d*x]^n, x], x] - Dist[(a*b)/(a^2
 + b^2), Int[(Cos[c + d*x]^(m - 1)*Sin[c + d*x]^(n - 1))/(a*Cos[c + d*x] +
b*Sin[c + d*x]), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] &&
IGtQ[m, 0] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{\cosh^3(x)\sinh^3(x)}{a\cosh(x)+b\sinh(x)}\,dx = \frac{a\int \cosh^2(x)\sinh^3(x)\,dx}{a^2-b^2} - \frac{b\int \cosh^3(x)\sinh^2(x)\,dx}{a^2-b^2} + \frac{(ab)\int \frac{\cosh^2(x)\sinh^2(x)}{a\cosh(x)+b\sinh(x)}\,dx}{a^2-b^2}$$

$$= \frac{(a^2b)\int \cosh(x)\sinh^2(x)\,dx}{(a^2-b^2)^2} - \frac{(ab^2)\int \cosh^2(x)\sinh(x)\,dx}{(a^2-b^2)^2} + \frac{(a^2b^2)\int \frac{\cosh(x)\sin}{a\cosh(x)+b}}{(a^2-b^2)^2}$$

$$= \frac{(a^3b^2)\int \sinh(x)\,dx}{(a^2-b^2)^3} - \frac{(a^2b^3)\int \cosh(x)\,dx}{(a^2-b^2)^3} + \frac{(a^3b^3)\int \frac{1}{a\cosh(x)+b\sinh(x)}\,dx}{(a^2-b^2)^3} + (ia^2$$

$$= \frac{a^3b^2\cosh(x)}{(a^2-b^2)^3} - \frac{ab^2\cosh^3(x)}{3(a^2-b^2)^2} - \frac{a\cosh^3(x)}{3(a^2-b^2)} + \frac{a\cosh^5(x)}{5(a^2-b^2)} - \frac{a^2b^3\sinh(x)}{(a^2-b^2)^3} + \frac{a^2b\,\mathrm{si}}{3(a^2}$$

$$= \frac{a^3b^3\tan^{-1}\left(\frac{b\cosh(x)+a\sinh(x)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{7/2}} + \frac{a^3b^2\cosh(x)}{(a^2-b^2)^3} - \frac{ab^2\cosh^3(x)}{3(a^2-b^2)^2} - \frac{a\cosh^3(x)}{3(a^2-b^2)} + \frac{a\,\mathrm{c}}{5(a}$$

Mathematica [A] time = 2.30, size = 325, normalized size = 1.53

$$\frac{1}{32}\left(\frac{2b(3a^2+b^2)\sinh(3x)}{3(a-b)^2(a+b)^2} - \frac{2a(a^2+3b^2)\cosh(3x)}{3(a-b)^2(a+b)^2} - 3\left(\frac{2b\sinh(x)}{b^2-a^2} + \frac{2a\cosh(x)}{a^2-b^2} + \frac{4ab\tan^{-1}\left(\frac{a\tanh\left(\frac{x}{2}\right)+b}{\sqrt{a-b}\sqrt{a+b}}\right)}{(a-b)^{3/2}(a+b)^{3/2}}\right) -$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cosh[x]^3*Sinh[x]^3)/(a*Cosh[x] + b*Sinh[x]),x]
```

```
[Out] ((4*a*b*(3*a^4 + 10*a^2*b^2 + 3*b^4)*ArcTan[(b + a*Tanh[x/2])/(Sqrt[a - b]*
Sqrt[a + b])])/((a - b)^(7/2)*(a + b)^(7/2)) + (2*a*(a^4 + 10*a^2*b^2 + 5*b
^4)*Cosh[x])/((a - b)^3*(a + b)^3) - (2*a*(a^2 + 3*b^2)*Cosh[3*x])/(3*(a -
b)^2*(a + b)^2) + (2*a*Cosh[5*x])/(5*(a - b)*(a + b)) + (2*b*(5*a^4 + 10*a^
2*b^2 + b^4)*Sinh[x])/((-a + b)^3*(a + b)^3) - 3*((4*a*b*ArcTan[(b + a*Tanh
[x/2])/(Sqrt[a - b]*Sqrt[a + b])])/((a - b)^(3/2)*(a + b)^(3/2)) + (2*a*Cos
h[x])/(a^2 - b^2) + (2*b*Sinh[x])/(-a^2 + b^2)) + (2*b*(3*a^2 + b^2)*Sinh[3
*x])/(3*(a - b)^2*(a + b)^2) - (2*b*Sinh[5*x])/(5*(a - b)*(a + b)))/32
```

fricas [B] time = 0.67, size = 4935, normalized size = 23.28

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(x)^3*sinh(x)^3/(a*cosh(x)+b*sinh(x)),x, algorithm="fricas")

[Out] [1/480*(3*(a^7 - a^6*b - 3*a^5*b^2 + 3*a^4*b^3 + 3*a^3*b^4 - 3*a^2*b^5 - a*b^6 + b^7)*cosh(x)^10 + 30*(a^7 - a^6*b - 3*a^5*b^2 + 3*a^4*b^3 + 3*a^3*b^4 - 3*a^2*b^5 - a*b^6 + b^7)*cosh(x)*sinh(x)^9 + 3*(a^7 - a^6*b - 3*a^5*b^2 + 3*a^4*b^3 + 3*a^3*b^4 - 3*a^2*b^5 - a*b^6 + b^7)*sinh(x)^10 - 5*(a^7 - 3*a^6*b + a^5*b^2 + 5*a^4*b^3 - 5*a^3*b^4 - a^2*b^5 + 3*a*b^6 - b^7)*cosh(x)^8 - 5*(a^7 - 3*a^6*b + a^5*b^2 + 5*a^4*b^3 - 5*a^3*b^4 - a^2*b^5 + 3*a*b^6 - b^7 - 27*(a^7 - a^6*b - 3*a^5*b^2 + 3*a^4*b^3 + 3*a^3*b^4 - 3*a^2*b^5 - a*b^6 + b^7)*cosh(x)^2)*sinh(x)^8 + 40*(9*(a^7 - a^6*b - 3*a^5*b^2 + 3*a^4*b^3 + 3*a^3*b^4 - 3*a^2*b^5 - a*b^6 + b^7)*cosh(x)^3 - (a^7 - 3*a^6*b + a^5*b^2 + 5*a^4*b^3 - 5*a^3*b^4 - a^2*b^5 + 3*a*b^6 - b^7)*cosh(x))*sinh(x)^7 + 3*a^7 + 3*a^6*b - 9*a^5*b^2 - 9*a^4*b^3 + 9*a^3*b^4 + 9*a^2*b^5 - 3*a*b^6 - 3*b^7 - 30*(a^7 + a^6*b - 9*a^5*b^2 + 7*a^4*b^3 + 7*a^3*b^4 - 9*a^2*b^5 + a*b^6 + b^7)*cosh(x)^6 - 10*(3*a^7 + 3*a^6*b - 27*a^5*b^2 + 21*a^4*b^3 + 21*a^3*b^4 - 27*a^2*b^5 + 3*a*b^6 + 3*b^7 - 63*(a^7 - a^6*b - 3*a^5*b^2 + 3*a^4*b^3 + 3*a^3*b^4 - 3*a^2*b^5 - a*b^6 + b^7)*cosh(x)^4 + 14*(a^7 - 3*a^6*b + a^5*b^2 + 5*a^4*b^3 - 5*a^3*b^4 - a^2*b^5 + 3*a*b^6 - b^7)*cosh(x)^2)*sinh(x)^6 + 4*(189*(a^7 - a^6*b - 3*a^5*b^2 + 3*a^4*b^3 + 3*a^3*b^4 - 3*a^2*b^5 - a*b^6 + b^7)*cosh(x)^5 - 70*(a^7 - 3*a^6*b + a^5*b^2 + 5*a^4*b^3 - 5*a^3*b^4 - a^2*b^5 + 3*a*b^6 - b^7)*cosh(x)^3 - 45*(a^7 + a^6*b - 9*a^5*b^2 + 7*a^4*b^3 + 7*a^3*b^4 - 9*a^2*b^5 + a*b^6 + b^7)*cosh(x))*sinh(x)^5 - 30*(a^7 - a^6*b - 9*a^5*b^2 - 7*a^4*b^3 + 7*a^3*b^4 + 9*a^2*b^5 + a*b^6 - b^7)*cosh(x)^4 - 10*(3*a^7 - 3*a^6*b - 27*a^5*b^2 - 21*a^4*b^3 + 21*a^3*b^4 + 27*a^2*b^5 + 3*a*b^6 - 3*b^7 - 63*(a^7 - a^6*b - 3*a^5*b^2 + 3*a^4*b^3 + 3*a^3*b^4 - 3*a^2*b^5 - a*b^6 + b^7)*cosh(x)^6 + 35*(a^7 - 3*a^6*b + a^5*b^2 + 5*a^4*b^3 - 5*a^3*b^4 - a^2*b^5 + 3*a*b^6 - b^7)*cosh(x)^4 + 45*(a^7 + a^6*b - 9*a^5*b^2 + 7*a^4*b^3 + 7*a^3*b^4 - 9*a^2*b^5 + a*b^6 + b^7)*cosh(x)^2)*sinh(x)^4 + 40*(9*(a^7 - a^6*b - 3*a^5*b^2 + 3*a^4*b^3 + 3*a^3*b^4 - 3*a^2*b^5 - a*b^6 + b^7)*cosh(x)^7 - 7*(a^7 - 3*a^6*b + a^5*b^2 + 5*a^4*b^3 - 5*a^3*b^4 - a^2*b^5 + 3*a*b^6 - b^7)*cosh(x)^5 - 15*(a^7 + a^6*b - 9*a^5*b^2 + 7*a^4*b^3 + 7*a^3*b^4 - 9*a^2*b^5 + a*b^6 + b^7)*cosh(x)^3 - 3*(a^7 - a^6*b - 9*a^5*b^2 - 7*a^4*b^3 + 7*a^3*b^4 + 9*a^2*b^5 + a*b^6 - b^7)*cosh(x))*sinh(x)^3 - 5*(a^7 + 3*a^6*b + a^5*b^2 - 5*a^4*b^3 - 5*a^3*b^4 + a^2*b^5 + 3*a*b^6 + b^7)*cosh(x)^2 + 5*(27*(a^7 - a^6*b - 3*a^5*b^2 + 3*a^4*b^3 + 3*a^3*b^4 - 3*a^2*b^5 - a*b^6 + b^7)*cosh(x)^8 - a^7 - 3*a^6*b - a^5*b^2 + 5*a^4*b^3 + 5*a^3*b^4 - a^2*b^5 - 3*a*b^6 - b^7 - 28*(a^7 - 3*a^6*b + a^5*b^2 + 5*a^4*b^3 - 5*a^3*b^4 - a^2*b^5 + 3*a*b^6 - b^7)*cosh(x)^6 - 90*(a^7 + a^6*b - 9*a^5*b^2 + 7*a^4*b^3 + 7*a^3*b^4 - 9*a^2*b^5 + a*b^6 + b^7)*cosh(x)^4 - 36*(a^7 - a^6*b - 9*a^5*b^2 - 7*a^4*b^3 + 7*a^3*b^4 + 9*a^2*b^5 + a*b^6 - b^7)*cosh(x)^2)*sinh(x)^2 + 480*(a^3*b^3*cosh(x)^5 + 5*a^3*b^3*cosh(x)^4*sinh(x) + 10*a^3*b^3*cosh(x)^3*sinh(x)^2 + 10*a^3*b^3*cosh(x)^2*sinh(x)^3 + 5*a^3*b^3*cosh(x)*sinh(x)^4 + a^3*b^3*sinh(x)^5)*sqrt(-a^2 + b^2)*log(((

a + b)*cosh(x)^2 + 2*(a + b)*cosh(x)*sinh(x) + (a + b)*sinh(x)^2 + 2*sqrt(-a^2 + b^2)*(cosh(x) + sinh(x)) - a + b)/((a + b)*cosh(x)^2 + 2*(a + b)*cosh(x)*sinh(x) + (a + b)*sinh(x)^2 + a - b)) + 10*(3*(a^7 - a^6*b - 3*a^5*b^2 + 3*a^4*b^3 + 3*a^3*b^4 - 3*a^2*b^5 - a*b^6 + b^7)*cosh(x)^9 - 4*(a^7 - 3*a^6*b + a^5*b^2 + 5*a^4*b^3 - 5*a^3*b^4 - a^2*b^5 + 3*a*b^6 - b^7)*cosh(x)^7 - 18*(a^7 + a^6*b - 9*a^5*b^2 + 7*a^4*b^3 + 7*a^3*b^4 - 9*a^2*b^5 + a*b^6 + b^7)*cosh(x)^5 - 12*(a^7 - a^6*b - 9*a^5*b^2 - 7*a^4*b^3 + 7*a^3*b^4 + 9*a^2*b^5 + a*b^6 - b^7)*cosh(x)^3 - (a^7 + 3*a^6*b + a^5*b^2 - 5*a^4*b^3 - 5*a^3*b^4 + a^2*b^5 + 3*a*b^6 + b^7)*cosh(x))*sinh(x))/((a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8)*cosh(x)^5 + 5*(a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8)*cosh(x)^4*sinh(x) + 10*(a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8)*cosh(x)^3*sinh(x)^2 + 10*(a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8)*cosh(x)^2*sinh(x)^3 + 5*(a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8)*cosh(x)*sinh(x)^4 + (a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8)*sinh(x)^5), 1/480*(3*(a^7 - a^6*b - 3*a^5*b^2 + 3*a^4*b^3 + 3*a^3*b^4 - 3*a^2*b^5 - a*b^6 + b^7)*cosh(x)^10 + 30*(a^7 - a^6*b - 3*a^5*b^2 + 3*a^4*b^3 + 3*a^3*b^4 - 3*a^2*b^5 - a*b^6 + b^7)*cosh(x)*sinh(x)^9 + 3*(a^7 - a^6*b - 3*a^5*b^2 + 3*a^4*b^3 + 3*a^3*b^4 - 3*a^2*b^5 - a*b^6 + b^7)*sinh(x)^10 - 5*(a^7 - 3*a^6*b + a^5*b^2 + 5*a^4*b^3 - 5*a^3*b^4 - a^2*b^5 + 3*a*b^6 - b^7)*cosh(x)^8 - 5*(a^7 - 3*a^6*b + a^5*b^2 + 5*a^4*b^3 - 5*a^3*b^4 - a^2*b^5 + 3*a*b^6 - b^7 - 27*(a^7 - a^6*b - 3*a^5*b^2 + 3*a^4*b^3 + 3*a^3*b^4 - 3*a^2*b^5 - a*b^6 + b^7)*cosh(x)^2)*sinh(x)^8 + 40*(9*(a^7 - a^6*b - 3*a^5*b^2 + 3*a^4*b^3 + 3*a^3*b^4 - 3*a^2*b^5 - a*b^6 + b^7)*cosh(x)^3 - (a^7 - 3*a^6*b + a^5*b^2 + 5*a^4*b^3 - 5*a^3*b^4 - a^2*b^5 + 3*a*b^6 - b^7)*cosh(x))*sinh(x)^7 + 3*a^7 + 3*a^6*b - 9*a^5*b^2 - 9*a^4*b^3 + 9*a^3*b^4 + 9*a^2*b^5 - 3*a*b^6 - 3*b^7 - 30*(a^7 + a^6*b - 9*a^5*b^2 + 7*a^4*b^3 + 7*a^3*b^4 - 9*a^2*b^5 + a*b^6 + b^7)*cosh(x)^6 - 10*(3*a^7 + 3*a^6*b - 27*a^5*b^2 + 21*a^4*b^3 + 21*a^3*b^4 - 27*a^2*b^5 + 3*a*b^6 + 3*b^7 - 63*(a^7 - a^6*b - 3*a^5*b^2 + 3*a^4*b^3 + 3*a^3*b^4 - 3*a^2*b^5 - a*b^6 + b^7)*cosh(x)^4 + 14*(a^7 - 3*a^6*b + a^5*b^2 + 5*a^4*b^3 - 5*a^3*b^4 - a^2*b^5 + 3*a*b^6 - b^7)*cosh(x)^2)*sinh(x)^6 + 4*(189*(a^7 - a^6*b - 3*a^5*b^2 + 3*a^4*b^3 + 3*a^3*b^4 - 3*a^2*b^5 - a*b^6 + b^7)*cosh(x)^5 - 70*(a^7 - 3*a^6*b + a^5*b^2 + 5*a^4*b^3 - 5*a^3*b^4 - a^2*b^5 + 3*a*b^6 - b^7)*cosh(x)^3 - 45*(a^7 + a^6*b - 9*a^5*b^2 + 7*a^4*b^3 + 7*a^3*b^4 - 9*a^2*b^5 + a*b^6 + b^7)*cosh(x))*sinh(x)^5 - 30*(a^7 - a^6*b - 9*a^5*b^2 - 7*a^4*b^3 + 7*a^3*b^4 + 9*a^2*b^5 + a*b^6 - b^7)*cosh(x)^4 - 10*(3*a^7 - 3*a^6*b - 27*a^5*b^2 - 21*a^4*b^3 + 21*a^3*b^4 + 27*a^2*b^5 + 3*a*b^6 - 3*b^7 - 63*(a^7 - a^6*b - 3*a^5*b^2 + 3*a^4*b^3 + 3*a^3*b^4 - 3*a^2*b^5 - a*b^6 + b^7)*cosh(x)^6 + 35*(a^7 - 3*a^6*b + a^5*b^2 + 5*a^4*b^3 - 5*a^3*b^4 - a^2*b^5 + 3*a*b^6 - b^7)*cosh(x)^4 + 45*(a^7 + a^6*b - 9*a^5*b^2 + 7*a^4*b^3 + 7*a^3*b^4 - 9*a^2*b^5 + a*b^6 + b^7)*cosh(x)^2)*sinh(x)^4 + 40*(9*(a^7 - a^6*b - 3*a^5*b^2 + 3*a^4*b^3 + 3*a^3*b^4 - 3*a^2*b^5 - a*b^6 + b^7)*cosh(x)^7 - 7*(a^7 - 3*a^6*b + a^5*b^2 + 5*a^4*b^3 - 5*a^3*b^4 - a^2*b^5 + 3*a*b^6 - b^7)*cosh(x)^5 - 15*(a^7 + a^6*b - 9*a^5*b^2 + 7*a^4*b^3 + 7*a^3*b^4 - 9*a^2*b^5 + a*b^6 + b^7)*cosh(x)^3 - 3*(a^7 - a^6*b - 9*a^5*b^2 - 7*a^4*b^3 + 7*a^3*b^4 + 9*a^2*b^5 + a*b^6 - b^7

```
)*cosh(x))*sinh(x)^3 - 5*(a^7 + 3*a^6*b + a^5*b^2 - 5*a^4*b^3 - 5*a^3*b^4 +
 a^2*b^5 + 3*a*b^6 + b^7)*cosh(x)^2 + 5*(27*(a^7 - a^6*b - 3*a^5*b^2 + 3*a^
4*b^3 + 3*a^3*b^4 - 3*a^2*b^5 - a*b^6 + b^7)*cosh(x)^8 - a^7 - 3*a^6*b - a^
5*b^2 + 5*a^4*b^3 + 5*a^3*b^4 - a^2*b^5 - 3*a*b^6 - b^7 - 28*(a^7 - 3*a^6*b
 + a^5*b^2 + 5*a^4*b^3 - 5*a^3*b^4 - a^2*b^5 + 3*a*b^6 - b^7)*cosh(x)^6 - 9
0*(a^7 + a^6*b - 9*a^5*b^2 + 7*a^4*b^3 + 7*a^3*b^4 - 9*a^2*b^5 + a*b^6 + b^
7)*cosh(x)^4 - 36*(a^7 - a^6*b - 9*a^5*b^2 - 7*a^4*b^3 + 7*a^3*b^4 + 9*a^2*
b^5 + a*b^6 - b^7)*cosh(x)^2)*sinh(x)^2 - 960*(a^3*b^3*cosh(x)^5 + 5*a^3*b^
3*cosh(x)^4*sinh(x) + 10*a^3*b^3*cosh(x)^3*sinh(x)^2 + 10*a^3*b^3*cosh(x)^2
*sinh(x)^3 + 5*a^3*b^3*cosh(x)*sinh(x)^4 + a^3*b^3*sinh(x)^5)*sqrt(a^2 - b^
2)*arctan(sqrt(a^2 - b^2)/((a + b)*cosh(x) + (a + b)*sinh(x))) + 10*(3*(a^7
 - a^6*b - 3*a^5*b^2 + 3*a^4*b^3 + 3*a^3*b^4 - 3*a^2*b^5 - a*b^6 + b^7)*cos
h(x)^9 - 4*(a^7 - 3*a^6*b + a^5*b^2 + 5*a^4*b^3 - 5*a^3*b^4 - a^2*b^5 + 3*a
*b^6 - b^7)*cosh(x)^7 - 18*(a^7 + a^6*b - 9*a^5*b^2 + 7*a^4*b^3 + 7*a^3*b^4
 - 9*a^2*b^5 + a*b^6 + b^7)*cosh(x)^5 - 12*(a^7 - a^6*b - 9*a^5*b^2 - 7*a^4
*b^3 + 7*a^3*b^4 + 9*a^2*b^5 + a*b^6 - b^7)*cosh(x)^3 - (a^7 + 3*a^6*b + a^
5*b^2 - 5*a^4*b^3 - 5*a^3*b^4 + a^2*b^5 + 3*a*b^6 + b^7)*cosh(x))*sinh(x))/
((a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8)*cosh(x)^5 + 5*(a^8 - 4*a^6
*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8)*cosh(x)^4*sinh(x) + 10*(a^8 - 4*a^6*b^2
 + 6*a^4*b^4 - 4*a^2*b^6 + b^8)*cosh(x)^3*sinh(x)^2 + 10*(a^8 - 4*a^6*b^2 +
 6*a^4*b^4 - 4*a^2*b^6 + b^8)*cosh(x)^2*sinh(x)^3 + 5*(a^8 - 4*a^6*b^2 + 6*
a^4*b^4 - 4*a^2*b^6 + b^8)*cosh(x)*sinh(x)^4 + (a^8 - 4*a^6*b^2 + 6*a^4*b^4
 - 4*a^2*b^6 + b^8)*sinh(x)^5)]
```

giac [A] time = 0.13, size = 325, normalized size = 1.53

$$\frac{2\,a^{3}b^{3}\arctan\left(\frac{ae^{x}+be^{x}}{\sqrt{a^{2}-b^{2}}}\right)}{\left(a^{6}-3\,a^{4}b^{2}+3\,a^{2}b^{4}-b^{6}\right)\sqrt{a^{2}-b^{2}}}-\frac{\left(30\,a^{2}e^{(4\,x)}-120\,abe^{(4\,x)}+30\,b^{2}e^{(4\,x)}+5\,a^{2}e^{(2\,x)}-5\,b^{2}e^{(2\,x)}-3\,a^{2}+6\,ab-3\right.}{480\,\left(a^{3}-3\,a^{2}b+3\,ab^{2}-b^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^3*sinh(x)^3/(a*cosh(x)+b*sinh(x)),x, algorithm="giac")
```

```
[Out] 2*a^3*b^3*arctan((a*e^x + b*e^x)/sqrt(a^2 - b^2))/((a^6 - 3*a^4*b^2 + 3*a^2
*b^4 - b^6)*sqrt(a^2 - b^2)) - 1/480*(30*a^2*e^(4*x) - 120*a*b*e^(4*x) + 30
*b^2*e^(4*x) + 5*a^2*e^(2*x) - 5*b^2*e^(2*x) - 3*a^2 + 6*a*b - 3*b^2)*e^(-5
*x)/(a^3 - 3*a^2*b + 3*a*b^2 - b^3) + 1/480*(3*a^4*e^(5*x) + 12*a^3*b*e^(5*
x) + 18*a^2*b^2*e^(5*x) + 12*a*b^3*e^(5*x) + 3*b^4*e^(5*x) - 5*a^4*e^(3*x)
- 10*a^3*b*e^(3*x) + 10*a*b^3*e^(3*x) + 5*b^4*e^(3*x) - 30*a^4*e^x - 180*a^
3*b*e^x - 300*a^2*b^2*e^x - 180*a*b^3*e^x - 30*b^4*e^x)/(a^5 + 5*a^4*b + 10
*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)
```

maple [A] time = 0.22, size = 344, normalized size = 1.62

$$-\frac{16}{5\left(16a+16b\right)\left(\tanh\left(\frac{x}{2}\right)-1\right)^{5}}-\frac{4}{\left(8a+8b\right)\left(\tanh\left(\frac{x}{2}\right)-1\right)^{4}}-\frac{a}{8\left(a+b\right)^{2}\left(\tanh\left(\frac{x}{2}\right)-1\right)^{2}}-\frac{3b}{8\left(a+b\right)^{2}\left(\tanh\left(\frac{x}{2}\right)-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^3*sinh(x)^3/(a*cosh(x)+b*sinh(x)),x)
```

```
[Out] -16/5/(16*a+16*b)/(tanh(1/2*x)-1)^5-4/(8*a+8*b)/(tanh(1/2*x)-1)^4-1/8/(a+b)
^2/(tanh(1/2*x)-1)^2*a-3/8/(a+b)^2/(tanh(1/2*x)-1)^2*b-5/12/(a+b)^2/(tanh(1
/2*x)-1)^3*a-7/12/(a+b)^2/(tanh(1/2*x)-1)^3*b+1/8*a^2/(a+b)^3/(tanh(1/2*x)-
1)+3/8*a/(a+b)^3/(tanh(1/2*x)-1)*b+2*a^3*b^3/(a-b)^3/(a+b)^3/(a^2-b^2)^(1/2
)*arctan(1/2*(2*a*tanh(1/2*x)+2*b)/(a^2-b^2)^(1/2))-4/(8*a-8*b)/(tanh(1/2*x
)+1)^4+16/5/(16*a-16*b)/(tanh(1/2*x)+1)^5+5/12/(a-b)^2/(tanh(1/2*x)+1)^3*a-
7/12/(a-b)^2/(tanh(1/2*x)+1)^3*b-1/8/(a-b)^2/(tanh(1/2*x)+1)^2*a+3/8/(a-b)^
2/(tanh(1/2*x)+1)^2*b-1/8*a^2/(a-b)^3/(tanh(1/2*x)+1)+3/8*a/(a-b)^3/(tanh(1
/2*x)+1)*b
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^3*sinh(x)^3/(a*cosh(x)+b*sinh(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [B] time = 2.07, size = 371, normalized size = 1.75

$$\frac{e^{-5x}}{160\,a-160\,b}+\frac{e^{5x}}{160\,a+160\,b}+\frac{2\,\mathrm{atan}\left(\frac{a^3\,b^3\,e^x\,\sqrt{a^{14}-7\,a^{12}\,b^2+21\,a^{10}\,b^4-35\,a^8\,b^6+35\,a^6\,b^8-21\,a^4\,b^{10}+7\,a^2\,b^{12}-b^{14}}}{a^7\,\sqrt{a^6\,b^6}+b^7\,\sqrt{a^6\,b^6}-3\,a^2\,b^5\,\sqrt{a^6\,b^6}+3\,a^3\,b^4\,\sqrt{a^6\,b^6}+3\,a^4\,b^3\,\sqrt{a^6\,b^6}-3\,a^5\,b^2\,\sqrt{a^6\,b^6}-a\,b^6\,\sqrt{a^6\,b^6}}\right.}{\sqrt{a^{14}-7\,a^{12}\,b^2+21\,a^{10}\,b^4-35\,a^8\,b^6+35\,a^6\,b^8-21\,a^4\,b^{10}+7\,a^2\,b^{12}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cosh(x)^3*sinh(x)^3)/(a*cosh(x) + b*sinh(x)),x)
```

```
[Out] exp(-5*x)/(160*a - 160*b) + exp(5*x)/(160*a + 160*b) + (2*atan((a^3*b^3*exp
(x)*(a^14 - b^14 + 7*a^2*b^12 - 21*a^4*b^10 + 35*a^6*b^8 - 35*a^8*b^6 + 21*
```

```
a^10*b^4 - 7*a^12*b^2)^(1/2))/(a^7*(a^6*b^6)^(1/2) + b^7*(a^6*b^6)^(1/2) -
3*a^2*b^5*(a^6*b^6)^(1/2) + 3*a^3*b^4*(a^6*b^6)^(1/2) + 3*a^4*b^3*(a^6*b^6)
^(1/2) - 3*a^5*b^2*(a^6*b^6)^(1/2) - a*b^6*(a^6*b^6)^(1/2) - a^6*b*(a^6*b^6
)^(1/2)))*(a^6*b^6)^(1/2))/(a^14 - b^14 + 7*a^2*b^12 - 21*a^4*b^10 + 35*a^6
*b^8 - 35*a^8*b^6 + 21*a^10*b^4 - 7*a^12*b^2)^(1/2) - (exp(-x)*(a^2 - 4*a*b
 + b^2))/(16*(a - b)^3) - (exp(-3*x)*(a + b))/(96*(a - b)^2) - (exp(3*x)*(a
 - b))/(96*(a + b)^2) - (exp(x)*(4*a*b + a^2 + b^2))/(16*(a + b)^3)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)**3*sinh(x)**3/(a*cosh(x)+b*sinh(x)),x)
```

```
[Out] Timed out
```

3.715 $\int \frac{\cosh(x)\sinh(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx$

Optimal. Leaf size=93

$$-\frac{2abx}{(a^2-b^2)^2}+\frac{b\sinh(x)}{(a^2-b^2)(a\cosh(x)+b\sinh(x))}+\frac{a^2\log(a\cosh(x)+b\sinh(x))}{(a^2-b^2)^2}+\frac{b^2\log(a\cosh(x)+b\sinh(x))}{(a^2-b^2)^2}$$

```
[Out] -2*a*b*x/(a^2-b^2)^2+a^2*ln(a*cosh(x)+b*sinh(x))/(a^2-b^2)^2+b^2*ln(a*cosh(
x)+b*sinh(x))/(a^2-b^2)^2+b*sinh(x)/(a^2-b^2)/(a*cosh(x)+b*sinh(x))
```

Rubi [A] time = 0.20, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {3111, 3098, 3133, 3097, 3075}

$$-\frac{2abx}{(a^2-b^2)^2}+\frac{b\sinh(x)}{(a^2-b^2)(a\cosh(x)+b\sinh(x))}+\frac{a^2\log(a\cosh(x)+b\sinh(x))}{(a^2-b^2)^2}+\frac{b^2\log(a\cosh(x)+b\sinh(x))}{(a^2-b^2)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Cosh[x]*Sinh[x])/(a*Cosh[x] + b*Sinh[x])^2,x]
```

```
[Out] (-2*a*b*x)/(a^2 - b^2)^2 + (a^2*Log[a*Cosh[x] + b*Sinh[x]])/(a^2 - b^2)^2 +
 (b^2*Log[a*Cosh[x] + b*Sinh[x]])/(a^2 - b^2)^2 + (b*Sinh[x])/((a^2 - b^2)*
(a*Cosh[x] + b*Sinh[x]))
```

Rule 3075

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-2), x
_Symbol] :> Simp[Sin[c + d*x]/(a*d*(a*Cos[c + d*x] + b*Sin[c + d*x])), x] /
; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3097

```
Int[sin[(c_.) + (d_.)*(x_)]/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.
) + (d_.)*(x_)]), x_Symbol] :> Simp[(b*x)/(a^2 + b^2), x] - Dist[a/(a^2 + b
^2), Int[(b*Cos[c + d*x] - a*Sin[c + d*x])/(a*Cos[c + d*x] + b*Sin[c + d*x]
), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3098

```
Int[cos[(c_.) + (d_.)*(x_)]/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.
) + (d_.)*(x_)]), x_Symbol] :> Simp[(a*x)/(a^2 + b^2), x] + Dist[b/(a^2 + b
^2), Int[(b*Cos[c + d*x] - a*Sin[c + d*x])/(a*Cos[c + d*x] + b*Sin[c + d*x]
```

```
), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3111

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.)*(cos[(c_.)
+ (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol] :> Dis
t[b/(a^2 + b^2), Int[Cos[c + d*x]^m*Sin[c + d*x]^(n - 1)*(a*Cos[c + d*x] +
b*Sin[c + d*x])^(p + 1), x], x] + (Dist[a/(a^2 + b^2), Int[Cos[c + d*x]^(m
- 1)*Sin[c + d*x]^n*(a*Cos[c + d*x] + b*Sin[c + d*x])^(p + 1), x], x] - Dis
t[(a*b)/(a^2 + b^2), Int[Cos[c + d*x]^(m - 1)*Sin[c + d*x]^(n - 1)*(a*Cos[c
 + d*x] + b*Sin[c + d*x])^p, x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 +
 b^2, 0] && IGtQ[m, 0] && IGtQ[n, 0] && ILtQ[p, 0]
```

Rule 3133

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)]), x
_Symbol] :> Simp[((b*B + c*C)*x)/(b^2 + c^2), x] + Simp[((c*B - b*C)*Log[a
+ b*Cos[d + e*x] + c*Sin[d + e*x]])/(e*(b^2 + c^2)), x] /; FreeQ[{a, b, c,
d, e, A, B, C}, x] && NeQ[b^2 + c^2, 0] && EqQ[A*(b^2 + c^2) - a*(b*B + c*C
), 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\cosh(x)\sinh(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx &= \frac{a\int \frac{\sinh(x)}{a\cosh(x)+b\sinh(x)}\,dx}{a^2-b^2}-\frac{b\int \frac{\cosh(x)}{a\cosh(x)+b\sinh(x)}\,dx}{a^2-b^2}+\frac{(ab)\int \frac{1}{(a\cosh(x)+b\sinh(x))^2}\,dx}{a^2-b^2}\\ &= -\frac{2abx}{\left(a^2-b^2\right)^2}+\frac{b\sinh(x)}{\left(a^2-b^2\right)(a\cosh(x)+b\sinh(x))}+\frac{\left(ia^2\right)\int \frac{-ib\cosh(x)-ia\sinh(x)}{a\cosh(x)+b\sinh(x)}\,dx}{\left(a^2-b^2\right)^2}\\ &= -\frac{2abx}{\left(a^2-b^2\right)^2}+\frac{a^2\log(a\cosh(x)+b\sinh(x))}{\left(a^2-b^2\right)^2}+\frac{b^2\log(a\cosh(x)+b\sinh(x))}{\left(a^2-b^2\right)^2}+\end{aligned}$$

Mathematica [A] time = 0.24, size = 60, normalized size = 0.65

$$\frac{\left(a^2+b^2\right)\log(a\cosh(x)+b\sinh(x))-2abx+\frac{b(a-b)(a+b)\sinh(x)}{a\cosh(x)+b\sinh(x)}}{(a-b)^2(a+b)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cosh[x]*Sinh[x])/(a*Cosh[x] + b*Sinh[x])^2,x]
```

```
[Out] (-2*a*b*x + (a^2 + b^2)*Log[a*Cosh[x] + b*Sinh[x]] + ((a - b)*b*(a + b)*Sin
h[x])/(a*Cosh[x] + b*Sinh[x]))/((a - b)^2*(a + b)^2)
```

fricas [B] time = 0.61, size = 376, normalized size = 4.04

$$-\frac{\left(a^3+3\,a^2b+3\,ab^2+b^3\right)x\cosh(x)^2+2\left(a^3+3\,a^2b+3\,ab^2+b^3\right)x\cosh(x)\sinh(x)+\left(a^3+3\,a^2b+3\,ab^2+b^3\right)}{a^5-a^4b-2\,a^3b^2+2\,a^2b^3+ab^4-b^5+\left(a^5+a^4b\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*sinh(x)/(a*cosh(x)+b*sinh(x))^2,x, algorithm="fricas")
```

```
[Out] -((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*x*cosh(x)^2 + 2*(a^3 + 3*a^2*b + 3*a*b^2
+ b^3)*x*cosh(x)*sinh(x) + (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*x*sinh(x)^2 + 2*
a^2*b - 2*a*b^2 + (a^3 + a^2*b - a*b^2 - b^3)*x - (a^3 - a^2*b + a*b^2 - b^
3 + (a^3 + a^2*b + a*b^2 + b^3)*cosh(x)^2 + 2*(a^3 + a^2*b + a*b^2 + b^3)*c
osh(x)*sinh(x) + (a^3 + a^2*b + a*b^2 + b^3)*sinh(x)^2)*log(2*(a*cosh(x) +
b*sinh(x))/(cosh(x) - sinh(x))))/(a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b
^4 - b^5 + (a^5 + a^4*b - 2*a^3*b^2 - 2*a^2*b^3 + a*b^4 + b^5)*cosh(x)^2 +
2*(a^5 + a^4*b - 2*a^3*b^2 - 2*a^2*b^3 + a*b^4 + b^5)*cosh(x)*sinh(x) + (a^
5 + a^4*b - 2*a^3*b^2 - 2*a^2*b^3 + a*b^4 + b^5)*sinh(x)^2)
```

giac [A] time = 0.13, size = 128, normalized size = 1.38

$$\frac{\left(a^2+b^2\right)\log\left(\left|ae^{(2\,x)}+be^{(2\,x)}+a-b\right|\right)}{a^4-2\,a^2b^2+b^4}-\frac{x}{a^2-2\,ab+b^2}-\frac{a^2e^{(2\,x)}+b^2e^{(2\,x)}+a^2-b^2}{\left(a^3-a^2b-ab^2+b^3\right)\left(ae^{(2\,x)}+be^{(2\,x)}+a-b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*sinh(x)/(a*cosh(x)+b*sinh(x))^2,x, algorithm="giac")
```

```
[Out] (a^2 + b^2)*log(abs(a*e^(2*x) + b*e^(2*x) + a - b))/(a^4 - 2*a^2*b^2 + b^4)
 - x/(a^2 - 2*a*b + b^2) - (a^2*e^(2*x) + b^2*e^(2*x) + a^2 - b^2)/((a^3 -
a^2*b - a*b^2 + b^3)*(a*e^(2*x) + b*e^(2*x) + a - b))
```

maple [A] time = 0.25, size = 181, normalized size = 1.95

$$-\frac{\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{(a+b)^2}+\frac{2a^2\tanh\left(\frac{x}{2}\right)b}{(a-b)^2(a+b)^2\left(a+2\tanh\left(\frac{x}{2}\right)b+a\left(\tanh^2\left(\frac{x}{2}\right)\right)\right)}-\frac{2b^3\tanh\left(\frac{x}{2}\right)}{(a-b)^2(a+b)^2\left(a+2\tanh\left(\frac{x}{2}\right)b+a\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)*sinh(x)/(a*cosh(x)+b*sinh(x))^2,x)
```

```
[Out] -1/(a+b)^2*ln(tanh(1/2*x)-1)+2*a^2/(a-b)^2/(a+b)^2/(a+2*tanh(1/2*x)*b+a*tan
h(1/2*x)^2)*tanh(1/2*x)*b-2/(a-b)^2/(a+b)^2*b^3*tanh(1/2*x)/(a+2*tanh(1/2*x
)*b+a*tanh(1/2*x)^2)+1/(a-b)^2/(a+b)^2*ln(a+2*tanh(1/2*x)*b+a*tanh(1/2*x)^2
)*a^2+1/(a-b)^2/(a+b)^2*ln(a+2*tanh(1/2*x)*b+a*tanh(1/2*x)^2)*b^2-1/(a-b)^2
*ln(tanh(1/2*x)+1)
```

maxima [A] time = 0.45, size = 107, normalized size = 1.15

$$\frac{2\,ab}{a^4 - 2\,a^2b^2 + b^4 + \left(a^4 - 2\,a^3b + 2\,ab^3 - b^4\right)e^{(-2\,x)}} + \frac{\left(a^2 + b^2\right)\log\left(-(a-b)e^{(-2\,x)} - a - b\right)}{a^4 - 2\,a^2b^2 + b^4} + \frac{x}{a^2 + 2\,ab + b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*sinh(x)/(a*cosh(x)+b*sinh(x))^2,x, algorithm="maxima")
```

```
[Out] 2*a*b/(a^4 - 2*a^2*b^2 + b^4 + (a^4 - 2*a^3*b + 2*a*b^3 - b^4)*e^(-2*x)) +
(a^2 + b^2)*log(-(a - b)*e^(-2*x) - a - b)/(a^4 - 2*a^2*b^2 + b^4) + x/(a^2
 + 2*a*b + b^2)
```

mupad [B] time = 1.93, size = 98, normalized size = 1.05

$$\ln\left(a\cosh(x) + b\sinh(x)\right)\left(\frac{1}{2\,(a+b)^2} + \frac{1}{2\,(a-b)^2}\right) - \frac{\frac{a\cosh(x)}{a^2-b^2} + \frac{2\,a^2\,b\,x\cosh(x)}{\left(a^2-b^2\right)^2} + \frac{2\,a\,b^2\,x\sinh(x)}{\left(a^2-b^2\right)^2}}{a\cosh(x) + b\sinh(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cosh(x)*sinh(x))/(a*cosh(x) + b*sinh(x))^2,x)
```

```
[Out] log(a*cosh(x) + b*sinh(x))*(1/(2*(a + b)^2) + 1/(2*(a - b)^2)) - ((a*cosh(x
))/(a^2 - b^2) + (2*a^2*b*x*cosh(x))/(a^2 - b^2)^2 + (2*a*b^2*x*sinh(x))/(a
^2 - b^2)^2)/(a*cosh(x) + b*sinh(x))
```

sympy [A] time = 1.39, size = 962, normalized size = 10.34

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*sinh(x)/(a*cosh(x)+b*sinh(x))**2,x)
```

```
[Out] Piecewise((zoo*log(sinh(x)), Eq(a, 0) & Eq(b, 0)), (-2*x*sinh(x)**2/(8*b**2
*sinh(x)**2 - 16*b**2*sinh(x)*cosh(x) + 8*b**2*cosh(x)**2) + 4*x*sinh(x)*co
sh(x)/(8*b**2*sinh(x)**2 - 16*b**2*sinh(x)*cosh(x) + 8*b**2*cosh(x)**2) - 2
*x*cosh(x)**2/(8*b**2*sinh(x)**2 - 16*b**2*sinh(x)*cosh(x) + 8*b**2*cosh(x)
**2) + sinh(x)**2/(8*b**2*sinh(x)**2 - 16*b**2*sinh(x)*cosh(x) + 8*b**2*cos
```

```
h(x)**2) + cosh(x)**2/(8*b**2*sinh(x)**2 - 16*b**2*sinh(x)*cosh(x) + 8*b**2
*cosh(x)**2), Eq(a, -b)), (2*x*sinh(x)**2/(8*b**2*sinh(x)**2 + 16*b**2*sinh
(x)*cosh(x) + 8*b**2*cosh(x)**2) + 4*x*sinh(x)*cosh(x)/(8*b**2*sinh(x)**2 +
 16*b**2*sinh(x)*cosh(x) + 8*b**2*cosh(x)**2) + 2*x*cosh(x)**2/(8*b**2*sinh
(x)**2 + 16*b**2*sinh(x)*cosh(x) + 8*b**2*cosh(x)**2) + sinh(x)**2/(8*b**2*
sinh(x)**2 + 16*b**2*sinh(x)*cosh(x) + 8*b**2*cosh(x)**2) + cosh(x)**2/(8*b
**2*sinh(x)**2 + 16*b**2*sinh(x)*cosh(x) + 8*b**2*cosh(x)**2), Eq(a, b)), (
log(sinh(x))/b**2, Eq(a, 0)), (a**3*log(cosh(x) + b*sinh(x)/a)*cosh(x)/(a**
5*cosh(x) + a**4*b*sinh(x) - 2*a**3*b**2*cosh(x) - 2*a**2*b**3*sinh(x) + a*
b**4*cosh(x) + b**5*sinh(x)) - a**3*cosh(x)/(a**5*cosh(x) + a**4*b*sinh(x)
- 2*a**3*b**2*cosh(x) - 2*a**2*b**3*sinh(x) + a*b**4*cosh(x) + b**5*sinh(x)
) - 2*a**2*b*x*cosh(x)/(a**5*cosh(x) + a**4*b*sinh(x) - 2*a**3*b**2*cosh(x)
 - 2*a**2*b**3*sinh(x) + a*b**4*cosh(x) + b**5*sinh(x)) + a**2*b*log(cosh(x
) + b*sinh(x)/a)*sinh(x)/(a**5*cosh(x) + a**4*b*sinh(x) - 2*a**3*b**2*cosh(
x) - 2*a**2*b**3*sinh(x) + a*b**4*cosh(x) + b**5*sinh(x)) - 2*a*b**2*x*sinh
(x)/(a**5*cosh(x) + a**4*b*sinh(x) - 2*a**3*b**2*cosh(x) - 2*a**2*b**3*sinh
(x) + a*b**4*cosh(x) + b**5*sinh(x)) + a*b**2*log(cosh(x) + b*sinh(x)/a)*co
sh(x)/(a**5*cosh(x) + a**4*b*sinh(x) - 2*a**3*b**2*cosh(x) - 2*a**2*b**3*si
nh(x) + a*b**4*cosh(x) + b**5*sinh(x)) + a*b**2*cosh(x)/(a**5*cosh(x) + a**
4*b*sinh(x) - 2*a**3*b**2*cosh(x) - 2*a**2*b**3*sinh(x) + a*b**4*cosh(x) +
b**5*sinh(x)) + b**3*log(cosh(x) + b*sinh(x)/a)*sinh(x)/(a**5*cosh(x) + a**
4*b*sinh(x) - 2*a**3*b**2*cosh(x) - 2*a**2*b**3*sinh(x) + a*b**4*cosh(x) +
b**5*sinh(x)), True))
```

3.716 $\int \frac{\cosh(x)\sinh^2(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx$

Optimal. Leaf size=165

$$\frac{a^2\sinh(x)}{\left(a^2-b^2\right)^2}+\frac{b^2\sinh(x)}{\left(a^2-b^2\right)^2}-\frac{2ab\cosh(x)}{\left(a^2-b^2\right)^2}-\frac{a^2b}{\left(a^2-b^2\right)^2(a\cosh(x)+b\sinh(x))}-\frac{2ab^2\tan^{-1}\left(\frac{a\sinh(x)+b\cosh(x)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{5/2}}-\frac{a^3\tan^{-1}}{(}$$

```
[Out] -a^3*arctan((b*cosh(x)+a*sinh(x))/(a^2-b^2)^(1/2))/(a^2-b^2)^(5/2)-2*a*b^2*
arctan((b*cosh(x)+a*sinh(x))/(a^2-b^2)^(1/2))/(a^2-b^2)^(5/2)-2*a*b*cosh(x)
/(a^2-b^2)^2+a^2*sinh(x)/(a^2-b^2)^2+b^2*sinh(x)/(a^2-b^2)^2-a^2*b/(a^2-b^2
)^2/(a*cosh(x)+b*sinh(x))
```

Rubi [A] time = 0.31, antiderivative size = 165, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 8, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {3111, 3109, 2637, 2638, 3074, 206, 3099, 3154}

$$\frac{a^2\sinh(x)}{\left(a^2-b^2\right)^2}+\frac{b^2\sinh(x)}{\left(a^2-b^2\right)^2}-\frac{2ab\cosh(x)}{\left(a^2-b^2\right)^2}-\frac{a^2b}{\left(a^2-b^2\right)^2(a\cosh(x)+b\sinh(x))}-\frac{a^3\tan^{-1}\left(\frac{a\sinh(x)+b\cosh(x)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{5/2}}-\frac{2ab^2\tan^{-1}}{(a}$$

Antiderivative was successfully verified.

```
[In] Int[(Cosh[x]*Sinh[x]^2)/(a*Cosh[x] + b*Sinh[x])^2,x]
```

```
[Out] -((a^3*ArcTan[(b*Cosh[x] + a*Sinh[x])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(5/2))
- (2*a*b^2*ArcTan[(b*Cosh[x] + a*Sinh[x])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(5/
2) - (2*a*b*Cosh[x])/(a^2 - b^2)^2 + (a^2*Sinh[x])/(a^2 - b^2)^2 + (b^2*Sin
h[x])/(a^2 - b^2)^2 - (a^2*b)/((a^2 - b^2)^2*(a*Cosh[x] + b*Sinh[x]))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3074

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> -Dist[d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d
*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3099

```
Int[sin[(c_.) + (d_.)*(x_)]^(m_)/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin
[(c_.) + (d_.)*(x_)]), x_Symbol] :> -Simp[(a*Sin[c + d*x]^(m - 1))/(d*(a^2
+ b^2)*(m - 1)), x] + (Dist[a^2/(a^2 + b^2), Int[Sin[c + d*x]^(m - 2)/(a*Co
s[c + d*x] + b*Sin[c + d*x]), x], x] + Dist[b/(a^2 + b^2), Int[Sin[c + d*x]
^(m - 1), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && GtQ[m,
1]
```

Rule 3109

```
Int[(cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.))/(cos[(c_.
) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[b
/(a^2 + b^2), Int[Cos[c + d*x]^m*Sin[c + d*x]^(n - 1), x], x] + (Dist[a/(a^
2 + b^2), Int[Cos[c + d*x]^(m - 1)*Sin[c + d*x]^n, x], x] - Dist[(a*b)/(a^2
 + b^2), Int[(Cos[c + d*x]^(m - 1)*Sin[c + d*x]^(n - 1))/(a*Cos[c + d*x] +
b*Sin[c + d*x]), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] &&
IGtQ[m, 0] && IGtQ[n, 0]
```

Rule 3111

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.)*(cos[(c_.)
+ (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol] :> Dis
t[b/(a^2 + b^2), Int[Cos[c + d*x]^m*Sin[c + d*x]^(n - 1)*(a*Cos[c + d*x] +
b*Sin[c + d*x])^(p + 1), x], x] + (Dist[a/(a^2 + b^2), Int[Cos[c + d*x]^(m
- 1)*Sin[c + d*x]^n*(a*Cos[c + d*x] + b*Sin[c + d*x])^(p + 1), x], x] - Dis
t[(a*b)/(a^2 + b^2), Int[Cos[c + d*x]^(m - 1)*Sin[c + d*x]^(n - 1)*(a*Cos[c
 + d*x] + b*Sin[c + d*x])^p, x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 +
 b^2, 0] && IGtQ[m, 0] && IGtQ[n, 0] && ILtQ[p, 0]
```

Rule 3154

```
Int[((A_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])/((a_.) + cos[(d_.) + (e_.)*(x_)
]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])^2, x_Symbol] :> -Simp[(b*C + (a*C
- c*A)*Cos[d + e*x] + b*A*Sin[d + e*x])/(e*(a^2 - b^2 - c^2)*(a + b*Cos[d +
 e*x] + c*Sin[d + e*x])), x] + Dist[(a*A - c*C)/(a^2 - b^2 - c^2), Int[1/(a
```

```
+ b*Cos[d + e*x] + c*Sin[d + e*x]), x], x] /; FreeQ[{a, b, c, d, e, A, C},
x] && NeQ[a^2 - b^2 - c^2, 0] && NeQ[a*A - c*C, 0]
```

Rubi steps

$$\int \frac{\cosh(x)\sinh^2(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx = \frac{a\int \frac{\sinh^2(x)}{a\cosh(x)+b\sinh(x)}\,dx}{a^2-b^2} - \frac{b\int \frac{\cosh(x)\sinh(x)}{a\cosh(x)+b\sinh(x)}\,dx}{a^2-b^2} + \frac{(ab)\int \frac{\sinh(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx}{a^2-b^2}$$

$$= \frac{a^2\sinh(x)}{\left(a^2-b^2\right)^2} - \frac{a^2 b}{\left(a^2-b^2\right)^2(a\cosh(x)+b\sinh(x))} - \frac{a^3\int \frac{1}{a\cosh(x)+b\sinh(x)}\,dx}{\left(a^2-b^2\right)^2} - 2\frac{(a$$

$$= -\frac{2ab\cosh(x)}{\left(a^2-b^2\right)^2} + \frac{a^2\sinh(x)}{\left(a^2-b^2\right)^2} + \frac{b^2\sinh(x)}{\left(a^2-b^2\right)^2} - \frac{a^2 b}{\left(a^2-b^2\right)^2(a\cosh(x)+b\sinh(x))} -$$

$$= -\frac{a^3\tan^{-1}\left(\frac{b\cosh(x)+a\sinh(x)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{5/2}} - \frac{2ab^2\tan^{-1}\left(\frac{b\cosh(x)+a\sinh(x)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{5/2}} - \frac{2ab\cosh(x)}{\left(a^2-b^2\right)^2} + \frac{a^2}{(a}$$

Mathematica [A] time = 1.10, size = 222, normalized size = 1.35

$$\frac{b\left(\sqrt{a-b}\sqrt{a+b}\left(a^2+b^2\right)\sinh^2(x) - 2a\left(a^2+2b^2\right)\sinh(x)\tan^{-1}\left(\frac{a\tanh\left(\frac{x}{2}\right)+b}{\sqrt{a-b}\sqrt{a+b}}\right) + a^2\left(-\sqrt{a-b}\right)\sqrt{a+b}\right) + a\cosh}{(a-b)^{5/2}(a+b)^{5/2}(a\cosh}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cosh[x]*Sinh[x]^2)/(a*Cosh[x] + b*Sinh[x])^2,x]
```

```
[Out] (-2*a^2*Sqrt[a - b]*b*Sqrt[a + b]*Cosh[x]^2 + a*Cosh[x]*(-2*a*(a^2 + 2*b^2)
*ArcTan[(b + a*Tanh[x/2])/(Sqrt[a - b]*Sqrt[a + b])] + Sqrt[a - b]*Sqrt[a +
 b]*(a^2 - b^2)*Sinh[x]) + b*(-(a^2*Sqrt[a - b]*Sqrt[a + b]) - 2*a*(a^2 + 2
*b^2)*ArcTan[(b + a*Tanh[x/2])/(Sqrt[a - b]*Sqrt[a + b])]*Sinh[x] + Sqrt[a
- b]*Sqrt[a + b]*(a^2 + b^2)*Sinh[x]^2))/((a - b)^(5/2)*(a + b)^(5/2)*(a*Co
sh[x] + b*Sinh[x]))
```

fricas [B] time = 0.55, size = 1819, normalized size = 11.02

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*sinh(x)^2/(a*cosh(x)+b*sinh(x))^2,x, algorithm="fricas")

[Out] [-1/2*(a^5 + a^4*b - 2*a^3*b^2 - 2*a^2*b^3 + a*b^4 + b^5 - (a^5 - a^4*b - 2
*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^4 - 4*(a^5 - a^4*b - 2*a^3*b^2
+ 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)*sinh(x)^3 - (a^5 - a^4*b - 2*a^3*b^2 + 2
*a^2*b^3 + a*b^4 - b^5)*sinh(x)^4 + 2*(5*a^4*b - 4*a^2*b^3 - b^5)*cosh(x)^2
 + 2*(5*a^4*b - 4*a^2*b^3 - b^5 - 3*(a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 +
a*b^4 - b^5)*cosh(x)^2)*sinh(x)^2 + 2*((a^4 + a^3*b + 2*a^2*b^2 + 2*a*b^3)*
cosh(x)^3 + 3*(a^4 + a^3*b + 2*a^2*b^2 + 2*a*b^3)*cosh(x)*sinh(x)^2 + (a^4
+ a^3*b + 2*a^2*b^2 + 2*a*b^3)*sinh(x)^3 + (a^4 - a^3*b + 2*a^2*b^2 - 2*a*b
^3)*cosh(x) + (a^4 - a^3*b + 2*a^2*b^2 - 2*a*b^3 + 3*(a^4 + a^3*b + 2*a^2*b
^2 + 2*a*b^3)*cosh(x)^2)*sinh(x))*sqrt(-a^2 + b^2)*log(((a + b)*cosh(x)^2 +
 2*(a + b)*cosh(x)*sinh(x) + (a + b)*sinh(x)^2 + 2*sqrt(-a^2 + b^2)*(cosh(x
) + sinh(x)) - a + b)/((a + b)*cosh(x)^2 + 2*(a + b)*cosh(x)*sinh(x) + (a +
 b)*sinh(x)^2 + a - b)) - 4*((a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 -
 b^5)*cosh(x)^3 - (5*a^4*b - 4*a^2*b^3 - b^5)*cosh(x))*sinh(x))/((a^7 + a^6
*b - 3*a^5*b^2 - 3*a^4*b^3 + 3*a^3*b^4 + 3*a^2*b^5 - a*b^6 - b^7)*cosh(x)^3
 + 3*(a^7 + a^6*b - 3*a^5*b^2 - 3*a^4*b^3 + 3*a^3*b^4 + 3*a^2*b^5 - a*b^6 -
 b^7)*cosh(x)*sinh(x)^2 + (a^7 + a^6*b - 3*a^5*b^2 - 3*a^4*b^3 + 3*a^3*b^4
+ 3*a^2*b^5 - a*b^6 - b^7)*sinh(x)^3 + (a^7 - a^6*b - 3*a^5*b^2 + 3*a^4*b^3
 + 3*a^3*b^4 - 3*a^2*b^5 - a*b^6 + b^7)*cosh(x) + (a^7 - a^6*b - 3*a^5*b^2
+ 3*a^4*b^3 + 3*a^3*b^4 - 3*a^2*b^5 - a*b^6 + b^7 + 3*(a^7 + a^6*b - 3*a^5*
b^2 - 3*a^4*b^3 + 3*a^3*b^4 + 3*a^2*b^5 - a*b^6 - b^7)*cosh(x)^2)*sinh(x)),
 -1/2*(a^5 + a^4*b - 2*a^3*b^2 - 2*a^2*b^3 + a*b^4 + b^5 - (a^5 - a^4*b - 2
*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^4 - 4*(a^5 - a^4*b - 2*a^3*b^2
+ 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)*sinh(x)^3 - (a^5 - a^4*b - 2*a^3*b^2 + 2
*a^2*b^3 + a*b^4 - b^5)*sinh(x)^4 + 2*(5*a^4*b - 4*a^2*b^3 - b^5)*cosh(x)^2
 + 2*(5*a^4*b - 4*a^2*b^3 - b^5 - 3*(a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 +
a*b^4 - b^5)*cosh(x)^2)*sinh(x)^2 - 4*((a^4 + a^3*b + 2*a^2*b^2 + 2*a*b^3)*
cosh(x)^3 + 3*(a^4 + a^3*b + 2*a^2*b^2 + 2*a*b^3)*cosh(x)*sinh(x)^2 + (a^4
+ a^3*b + 2*a^2*b^2 + 2*a*b^3)*sinh(x)^3 + (a^4 - a^3*b + 2*a^2*b^2 - 2*a*b
^3)*cosh(x) + (a^4 - a^3*b + 2*a^2*b^2 - 2*a*b^3 + 3*(a^4 + a^3*b + 2*a^2*b
^2 + 2*a*b^3)*cosh(x)^2)*sinh(x))*sqrt(a^2 - b^2)*arctan(sqrt(a^2 - b^2)/((
a + b)*cosh(x) + (a + b)*sinh(x))) - 4*((a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^
3 + a*b^4 - b^5)*cosh(x)^3 - (5*a^4*b - 4*a^2*b^3 - b^5)*cosh(x))*sinh(x))/
((a^7 + a^6*b - 3*a^5*b^2 - 3*a^4*b^3 + 3*a^3*b^4 + 3*a^2*b^5 - a*b^6 - b^7
)*cosh(x)^3 + 3*(a^7 + a^6*b - 3*a^5*b^2 - 3*a^4*b^3 + 3*a^3*b^4 + 3*a^2*b^
5 - a*b^6 - b^7)*cosh(x)*sinh(x)^2 + (a^7 + a^6*b - 3*a^5*b^2 - 3*a^4*b^3 +
 3*a^3*b^4 + 3*a^2*b^5 - a*b^6 - b^7)*sinh(x)^3 + (a^7 - a^6*b - 3*a^5*b^2
+ 3*a^4*b^3 + 3*a^3*b^4 - 3*a^2*b^5 - a*b^6 + b^7)*cosh(x) + (a^7 - a^6*b -
 3*a^5*b^2 + 3*a^4*b^3 + 3*a^3*b^4 - 3*a^2*b^5 - a*b^6 + b^7 + 3*(a^7 + a^6
*b - 3*a^5*b^2 - 3*a^4*b^3 + 3*a^3*b^4 + 3*a^2*b^5 - a*b^6 - b^7)*cosh(x)^2
)*sinh(x))]
```

giac [A] time = 0.14, size = 179, normalized size = 1.08

$$-\frac{2\left(a^{3}+2\,ab^{2}\right)\arctan\left(\frac{ae^{x}+be^{x}}{\sqrt{a^{2}-b^{2}}}\right)}{\left(a^{4}-2\,a^{2}b^{2}+b^{4}\right)\sqrt{a^{2}-b^{2}}}+\frac{e^{x}}{2\left(a^{2}+2\,ab+b^{2}\right)}-\frac{a^{3}e^{(2\,x)}+7\,a^{2}be^{(2\,x)}+3\,ab^{2}e^{(2\,x)}+b^{3}e^{(2\,x)}+a^{3}+a^{2}b-ab^{2}-}{2\left(a^{4}-2\,a^{2}b^{2}+b^{4}\right)\left(ae^{(3\,x)}+be^{(3\,x)}+ae^{x}-be^{x}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*sinh(x)^2/(a*cosh(x)+b*sinh(x))^2,x, algorithm="giac")
```

```
[Out] -2*(a^3 + 2*a*b^2)*arctan((a*e^x + b*e^x)/sqrt(a^2 - b^2))/((a^4 - 2*a^2*b^
2 + b^4)*sqrt(a^2 - b^2)) + 1/2*e^x/(a^2 + 2*a*b + b^2) - 1/2*(a^3*e^(2*x)
+ 7*a^2*b*e^(2*x) + 3*a*b^2*e^(2*x) + b^3*e^(2*x) + a^3 + a^2*b - a*b^2 - b
^3)/((a^4 - 2*a^2*b^2 + b^4)*(a*e^(3*x) + b*e^(3*x) + a*e^x - b*e^x))
```

maple [A] time = 0.27, size = 219, normalized size = 1.33

$$-\frac{1}{\left(a+b\right)^{2}\left(\tanh\left(\frac{x}{2}\right)-1\right)}-\frac{2a\tanh\left(\frac{x}{2}\right)b^{2}}{\left(a-b\right)^{2}\left(a+b\right)^{2}\left(a+2\tanh\left(\frac{x}{2}\right)b+a\left(\tanh^{2}\left(\frac{x}{2}\right)\right)\right)}-\frac{2a^{2}b}{\left(a-b\right)^{2}\left(a+b\right)^{2}\left(a+2\tanh\left(\frac{x}{2}\right)b\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)*sinh(x)^2/(a*cosh(x)+b*sinh(x))^2,x)
```

```
[Out] -1/(a+b)^2/(tanh(1/2*x)-1)-2*a/(a-b)^2/(a+b)^2*tanh(1/2*x)/(a+2*tanh(1/2*x)
*b+a*tanh(1/2*x)^2)*b^2-2*a^2/(a-b)^2/(a+b)^2/(a+2*tanh(1/2*x)*b+a*tanh(1/2
*x)^2)*b-2*a^3/(a-b)^2/(a+b)^2/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tanh(1/2*x)+
2*b)/(a^2-b^2)^(1/2))-4*b^2/(a-b)^2/(a+b)^2*a/(a^2-b^2)^(1/2)*arctan(1/2*(2
*a*tanh(1/2*x)+2*b)/(a^2-b^2)^(1/2))-1/(a-b)^2/(tanh(1/2*x)+1)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*sinh(x)^2/(a*cosh(x)+b*sinh(x))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [B] time = 1.82, size = 397, normalized size = 2.41

$$\frac{e^x}{2\,(a+b)^2}-\frac{e^{-x}}{2\,(a-b)^2}-\frac{2\,\mathrm{atan}\left(\frac{e^x\left(a^3\,\sqrt{a^{10}-5\,a^8\,b^2+10\,a^6\,b^4-10\,a^4\,b^6+5\,a^2\,b^8-b^{10}}+2\,a\,b^2\,\sqrt{a^{10}-5\,a^8\,b^2+10\,a^6}\right.}{a^5\,\sqrt{a^6+4\,a^4\,b^2+4\,a^2\,b^4}-b^5\,\sqrt{a^6+4\,a^4\,b^2+4\,a^2\,b^4}+2\,a^2\,b^3\,\sqrt{a^6+4\,a^4\,b^2+4\,a^2\,b^4}-2\,a^3\,b^2\,\sqrt{a^6+4\,a^4\,b^2+4\,a^2\,b^4}}\right.}{\sqrt{a^{10}-5\,a^8\,b^2+10\,a^6\,b^4-10\,a^4\,b^6+}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cosh(x)*sinh(x)^2)/(a*cosh(x) + b*sinh(x))^2,x)
```

```
[Out] exp(x)/(2*(a + b)^2) - exp(-x)/(2*(a - b)^2) - (2*atan((exp(x)*(a^3*(a^10 -
 b^10 + 5*a^2*b^8 - 10*a^4*b^6 + 10*a^6*b^4 - 5*a^8*b^2)^(1/2) + 2*a*b^2*(a
^10 - b^10 + 5*a^2*b^8 - 10*a^4*b^6 + 10*a^6*b^4 - 5*a^8*b^2)^(1/2)))/(a^5*
(a^6 + 4*a^2*b^4 + 4*a^4*b^2)^(1/2) - b^5*(a^6 + 4*a^2*b^4 + 4*a^4*b^2)^(1/
2) + 2*a^2*b^3*(a^6 + 4*a^2*b^4 + 4*a^4*b^2)^(1/2) - 2*a^3*b^2*(a^6 + 4*a^2
*b^4 + 4*a^4*b^2)^(1/2) + a*b^4*(a^6 + 4*a^2*b^4 + 4*a^4*b^2)^(1/2) - a^4*b
*(a^6 + 4*a^2*b^4 + 4*a^4*b^2)^(1/2)))*(a^6 + 4*a^2*b^4 + 4*a^4*b^2)^(1/2))
/(a^10 - b^10 + 5*a^2*b^8 - 10*a^4*b^6 + 10*a^6*b^4 - 5*a^8*b^2)^(1/2) - (2
*a^2*b*exp(x))/((a + b)^2*(a - b)^2*(a - b + exp(2*x)*(a + b)))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*sinh(x)**2/(a*cosh(x)+b*sinh(x))**2,x)
```

```
[Out] Timed out
```

3.717 $\int \frac{\cosh(x)\sinh^3(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx$

Optimal. Leaf size=215

$$\frac{abx\left(a^2+b^2\right)}{\left(a^2-b^2\right)^3}+\frac{abx}{\left(a^2-b^2\right)^2}+\frac{a^2\sinh^2(x)}{2\left(a^2-b^2\right)^2}+\frac{b^2\sinh^2(x)}{2\left(a^2-b^2\right)^2}-\frac{a^2b}{\left(a^2-b^2\right)^2(a\coth(x)+b)}-\frac{ab\sinh(x)\cosh(x)}{\left(a^2-b^2\right)^2}-\frac{3a^2b^2\log}{}$$

```
[Out] a^3*b*x/(a^2-b^2)^3+a*b^3*x/(a^2-b^2)^3+a*b*x/(a^2-b^2)^2+a*b*(a^2+b^2)*x/(
a^2-b^2)^3-a^2*b/(a^2-b^2)^2/(b+a*coth(x))-a^4*ln(a*cosh(x)+b*sinh(x))/(a^2
-b^2)^3-3*a^2*b^2*ln(a*cosh(x)+b*sinh(x))/(a^2-b^2)^3-a*b*cosh(x)*sinh(x)/(
a^2-b^2)^2+1/2*a^2*sinh(x)^2/(a^2-b^2)^2+1/2*b^2*sinh(x)^2/(a^2-b^2)^2
```

Rubi [A] time = 0.54, antiderivative size = 215, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 13, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.722, Rules used = {3111, 3109, 2564, 30, 2635, 8, 3097, 3133, 3099, 3085, 3483, 3531, 3530}

$$\frac{a^3bx}{\left(a^2-b^2\right)^3}+\frac{abx\left(a^2+b^2\right)}{\left(a^2-b^2\right)^3}+\frac{abx}{\left(a^2-b^2\right)^2}+\frac{ab^3x}{\left(a^2-b^2\right)^3}+\frac{a^2\sinh^2(x)}{2\left(a^2-b^2\right)^2}+\frac{b^2\sinh^2(x)}{2\left(a^2-b^2\right)^2}-\frac{a^2b}{\left(a^2-b^2\right)^2(a\coth(x)+b)}-\frac{ab\sin}{(a}$$

Antiderivative was successfully verified.

```
[In] Int[(Cosh[x]*Sinh[x]^3)/(a*Cosh[x] + b*Sinh[x])^2,x]
```

```
[Out] (a^3*b*x)/(a^2 - b^2)^3 + (a*b^3*x)/(a^2 - b^2)^3 + (a*b*x)/(a^2 - b^2)^2 +
 (a*b*(a^2 + b^2)*x)/(a^2 - b^2)^3 - (a^2*b)/((a^2 - b^2)^2*(b + a*Coth[x])
) - (a^4*Log[a*Cosh[x] + b*Sinh[x]])/(a^2 - b^2)^3 - (3*a^2*b^2*Log[a*Cosh[
x] + b*Sinh[x]])/(a^2 - b^2)^3 - (a*b*Cosh[x]*Sinh[x])/(a^2 - b^2)^2 + (a^2
*Sinh[x]^2)/(2*(a^2 - b^2)^2) + (b^2*Sinh[x]^2)/(2*(a^2 - b^2)^2)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
```

```
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 3085

```
Int[sin[(c_.) + (d_.)*(x_)]^(m_)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin
[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[(b + a*Cot[c + d*x])^n, x] /;
 FreeQ[{a, b, c, d}, x] && EqQ[m + n, 0] && IntegerQ[n] && NeQ[a^2 + b^2, 0
]
```

Rule 3097

```
Int[sin[(c_.) + (d_.)*(x_)]/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.
) + (d_.)*(x_)]), x_Symbol] :> Simp[(b*x)/(a^2 + b^2), x] - Dist[a/(a^2 + b
^2), Int[(b*Cos[c + d*x] - a*Sin[c + d*x])/(a*Cos[c + d*x] + b*Sin[c + d*x]
), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3099

```
Int[sin[(c_.) + (d_.)*(x_)]^(m_)/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin
[(c_.) + (d_.)*(x_)]), x_Symbol] :> -Simp[(a*Sin[c + d*x]^(m - 1))/(d*(a^2
+ b^2)*(m - 1)), x] + (Dist[a^2/(a^2 + b^2), Int[Sin[c + d*x]^(m - 2)/(a*Co
s[c + d*x] + b*Sin[c + d*x]), x], x] + Dist[b/(a^2 + b^2), Int[Sin[c + d*x]
^(m - 1), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && GtQ[m,
1]
```

Rule 3109

```
Int[(cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.))/(cos[(c_.
) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[b
/(a^2 + b^2), Int[Cos[c + d*x]^m*Sin[c + d*x]^(n - 1), x], x] + (Dist[a/(a^
2 + b^2), Int[Cos[c + d*x]^(m - 1)*Sin[c + d*x]^n, x], x] - Dist[(a*b)/(a^2
 + b^2), Int[(Cos[c + d*x]^(m - 1)*Sin[c + d*x]^(n - 1))/(a*Cos[c + d*x] +
b*Sin[c + d*x]), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] &&
IGtQ[m, 0] && IGtQ[n, 0]
```

Rule 3111

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.)*(cos[(c_.)
+ (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol] :> Dis
t[b/(a^2 + b^2), Int[Cos[c + d*x]^m*Sin[c + d*x]^(n - 1)*(a*Cos[c + d*x] +
b*Sin[c + d*x])^(p + 1), x], x] + (Dist[a/(a^2 + b^2), Int[Cos[c + d*x]^(m
- 1)*Sin[c + d*x]^n*(a*Cos[c + d*x] + b*Sin[c + d*x])^(p + 1), x], x] - Dis
t[(a*b)/(a^2 + b^2), Int[Cos[c + d*x]^(m - 1)*Sin[c + d*x]^(n - 1)*(a*Cos[c
 + d*x] + b*Sin[c + d*x])^p, x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 +
 b^2, 0] && IGtQ[m, 0] && IGtQ[n, 0] && ILtQ[p, 0]
```

Rule 3133

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)]), x
_Symbol] :> Simp[((b*B + c*C)*x)/(b^2 + c^2), x] + Simp[((c*B - b*C)*Log[a
+ b*Cos[d + e*x] + c*Sin[d + e*x]])/(e*(b^2 + c^2)), x] /; FreeQ[{a, b, c,
d, e, A, B, C}, x] && NeQ[b^2 + c^2, 0] && EqQ[A*(b^2 + c^2) - a*(b*B + c*C
), 0]
```

Rule 3483

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(a +
b*Tan[c + d*x])^(n + 1))/(d*(n + 1)*(a^2 + b^2)), x] + Dist[1/(a^2 + b^2),
Int[(a - b*Tan[c + d*x])*(a + b*Tan[c + d*x])^(n + 1), x], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 + b^2, 0] && LtQ[n, -1]
```

Rule 3530

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c*Log[RemoveContent[a*Cos[e + f*x] + b*Sin[e + f
*x], x]])/(b*f), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

Rule 3531

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[((a*c + b*d)*x)/(a^2 + b^2), x] + Dist[(b*c - a
*d)/(a^2 + b^2), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && Ne
Q[a*c + b*d, 0]
```

Rubi steps

$$\int \frac{\cosh(x)\sinh^3(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx = \frac{a\int \frac{\sinh^3(x)}{a\cosh(x)+b\sinh(x)}\,dx}{a^2-b^2} - \frac{b\int \frac{\cosh(x)\sinh^2(x)}{a\cosh(x)+b\sinh(x)}\,dx}{a^2-b^2} + \frac{(ab)\int \frac{\sinh^2(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx}{a^2-b^2}$$

$$= \frac{a^2\sinh^2(x)}{2\left(a^2-b^2\right)^2} - \frac{a^3\int \frac{\sinh(x)}{a\cosh(x)+b\sinh(x)}\,dx}{\left(a^2-b^2\right)^2} - 2\frac{(ab)\int \sinh^2(x)\,dx}{\left(a^2-b^2\right)^2} + \frac{b^2\int \cosh(x)\,\mathrm{s}}{\left(a^2-b\right.}$$

$$= \frac{a^3bx}{\left(a^2-b^2\right)^3} + \frac{ab^3x}{\left(a^2-b^2\right)^3} - \frac{a^2b}{\left(a^2-b^2\right)^2(b+a\coth(x))} + \frac{a^2\sinh^2(x)}{2\left(a^2-b^2\right)^2} - \frac{\left(ia^4\right)\int}{}$$

$$= \frac{a^3bx}{\left(a^2-b^2\right)^3} + \frac{ab^3x}{\left(a^2-b^2\right)^3} + \frac{ab\left(a^2+b^2\right)x}{\left(a^2-b^2\right)^3} - \frac{a^2b}{\left(a^2-b^2\right)^2(b+a\coth(x))} - \frac{a^4\log}{}$$

$$= \frac{a^3bx}{\left(a^2-b^2\right)^3} + \frac{ab^3x}{\left(a^2-b^2\right)^3} + \frac{ab\left(a^2+b^2\right)x}{\left(a^2-b^2\right)^3} - \frac{a^2b}{\left(a^2-b^2\right)^2(b+a\coth(x))} - \frac{a^4\log}{}$$

Mathematica [A] time = 1.03, size = 176, normalized size = 0.82

$$\frac{a\left(a^2-b^2\right)^2\cosh(3x) - 2b\sinh(x)\left(\left(a^2-b^2\right)^2\cosh(2x) + 2a\left(3a^3+2a\left(a^2+3b^2\right)\log(a\cosh(x)+b\sinh(x)) - \right.\right.}{8(a-b)^3(a+b)^3(}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cosh[x]*Sinh[x]^3)/(a*Cosh[x] + b*Sinh[x])^2,x]
```

```
[Out] (a*(a^2 - b^2)^2*Cosh[3*x] + a*Cosh[x]*(a^4 + 2*a^2*b^2 - 3*b^4 + 24*a^3*b*
x + 8*a*b^3*x - 8*a^2*(a^2 + 3*b^2)*Log[a*Cosh[x] + b*Sinh[x]]) - 2*b*((a^2
 - b^2)^2*Cosh[2*x] + 2*a*(3*a^3 - 3*a*b^2 - 6*a^2*b*x - 2*b^3*x + 2*a*(a^2
 + 3*b^2)*Log[a*Cosh[x] + b*Sinh[x]]))*Sinh[x])/(8*(a - b)^3*(a + b)^3*(a*C
osh[x] + b*Sinh[x]))
```

fricas [B] time = 0.58, size = 1655, normalized size = 7.70

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*sinh(x)^3/(a*cosh(x)+b*sinh(x))^2,x, algorithm="fricas")
```

```
[Out] 1/8*((a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^6 + 6*(a^5
 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)*sinh(x)^5 + (a^5 -
```

```
a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*sinh(x)^6 + a^5 + a^4*b - 2*a^
3*b^2 - 2*a^2*b^3 + a*b^4 + b^5 + (a^5 - 3*a^4*b + 2*a^3*b^2 + 2*a^2*b^3 -
3*a*b^4 + b^5 + 8*(a^5 + 4*a^4*b + 6*a^3*b^2 + 4*a^2*b^3 + a*b^4)*x)*cosh(x
)^4 + (a^5 - 3*a^4*b + 2*a^3*b^2 + 2*a^2*b^3 - 3*a*b^4 + b^5 + 15*(a^5 - a^
4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^2 + 8*(a^5 + 4*a^4*b + 6
*a^3*b^2 + 4*a^2*b^3 + a*b^4)*x)*sinh(x)^4 + 4*(5*(a^5 - a^4*b - 2*a^3*b^2
+ 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^3 + (a^5 - 3*a^4*b + 2*a^3*b^2 + 2*a^2*b
^3 - 3*a*b^4 + b^5 + 8*(a^5 + 4*a^4*b + 6*a^3*b^2 + 4*a^2*b^3 + a*b^4)*x)*c
osh(x))*sinh(x)^3 + (a^5 + 19*a^4*b - 14*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^
5 + 8*(a^5 + 2*a^4*b - 2*a^2*b^3 - a*b^4)*x)*cosh(x)^2 + (a^5 + 19*a^4*b -
14*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^5 + 15*(a^5 - a^4*b - 2*a^3*b^2 + 2*a^
2*b^3 + a*b^4 - b^5)*cosh(x)^4 + 6*(a^5 - 3*a^4*b + 2*a^3*b^2 + 2*a^2*b^3 -
 3*a*b^4 + b^5 + 8*(a^5 + 4*a^4*b + 6*a^3*b^2 + 4*a^2*b^3 + a*b^4)*x)*cosh(
x)^2 + 8*(a^5 + 2*a^4*b - 2*a^2*b^3 - a*b^4)*x)*sinh(x)^2 - 8*((a^5 + a^4*b
 + 3*a^3*b^2 + 3*a^2*b^3)*cosh(x)^4 + 4*(a^5 + a^4*b + 3*a^3*b^2 + 3*a^2*b^
3)*cosh(x)*sinh(x)^3 + (a^5 + a^4*b + 3*a^3*b^2 + 3*a^2*b^3)*sinh(x)^4 + (a
^5 - a^4*b + 3*a^3*b^2 - 3*a^2*b^3)*cosh(x)^2 + (a^5 - a^4*b + 3*a^3*b^2 -
3*a^2*b^3 + 6*(a^5 + a^4*b + 3*a^3*b^2 + 3*a^2*b^3)*cosh(x)^2)*sinh(x)^2 +
2*(2*(a^5 + a^4*b + 3*a^3*b^2 + 3*a^2*b^3)*cosh(x)^3 + (a^5 - a^4*b + 3*a^3
*b^2 - 3*a^2*b^3)*cosh(x))*sinh(x))*log(2*(a*cosh(x) + b*sinh(x))/(cosh(x)
- sinh(x))) + 2*(3*(a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh
(x)^5 + 2*(a^5 - 3*a^4*b + 2*a^3*b^2 + 2*a^2*b^3 - 3*a*b^4 + b^5 + 8*(a^5 +
 4*a^4*b + 6*a^3*b^2 + 4*a^2*b^3 + a*b^4)*x)*cosh(x)^3 + (a^5 + 19*a^4*b -
14*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^5 + 8*(a^5 + 2*a^4*b - 2*a^2*b^3 - a*b
^4)*x)*cosh(x))*sinh(x))/((a^7 + a^6*b - 3*a^5*b^2 - 3*a^4*b^3 + 3*a^3*b^4
+ 3*a^2*b^5 - a*b^6 - b^7)*cosh(x)^4 + 4*(a^7 + a^6*b - 3*a^5*b^2 - 3*a^4*b
^3 + 3*a^3*b^4 + 3*a^2*b^5 - a*b^6 - b^7)*cosh(x)*sinh(x)^3 + (a^7 + a^6*b
- 3*a^5*b^2 - 3*a^4*b^3 + 3*a^3*b^4 + 3*a^2*b^5 - a*b^6 - b^7)*sinh(x)^4 +
(a^7 - a^6*b - 3*a^5*b^2 + 3*a^4*b^3 + 3*a^3*b^4 - 3*a^2*b^5 - a*b^6 + b^7)
*cosh(x)^2 + (a^7 - a^6*b - 3*a^5*b^2 + 3*a^4*b^3 + 3*a^3*b^4 - 3*a^2*b^5 -
 a*b^6 + b^7 + 6*(a^7 + a^6*b - 3*a^5*b^2 - 3*a^4*b^3 + 3*a^3*b^4 + 3*a^2*b
^5 - a*b^6 - b^7)*cosh(x)^2)*sinh(x)^2 + 2*(2*(a^7 + a^6*b - 3*a^5*b^2 - 3*
a^4*b^3 + 3*a^3*b^4 + 3*a^2*b^5 - a*b^6 - b^7)*cosh(x)^3 + (a^7 - a^6*b - 3
*a^5*b^2 + 3*a^4*b^3 + 3*a^3*b^4 - 3*a^2*b^5 - a*b^6 + b^7)*cosh(x))*sinh(x
))
```

giac [A] time = 0.14, size = 238, normalized size = 1.11

$$\frac{ax}{a^3-3a^2b+3ab^2-b^3}-\frac{\left(a^4+3a^2b^2\right)\log\left(\left|ae^{(2x)}+be^{(2x)}+a-b\right|\right)}{a^6-3a^4b^2+3a^2b^4-b^6}+\frac{e^{(2x)}}{8\left(a^2+2ab+b^2\right)}+\frac{2a^3e^{(4x)}-4a^2be^{(4x)}+2}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*sinh(x)^3/(a*cosh(x)+b*sinh(x))^2,x, algorithm="giac")
```

```
[Out] a*x/(a^3 - 3*a^2*b + 3*a*b^2 - b^3) - (a^4 + 3*a^2*b^2)*log(abs(a*e^(2*x) +
 b*e^(2*x) + a - b))/(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6) + 1/8*e^(2*x)/(a^2
 + 2*a*b + b^2) + 1/8*(2*a^3*e^(4*x) - 4*a^2*b*e^(4*x) + 2*a*b^2*e^(4*x) +
3*a^3*e^(2*x) + 11*a^2*b*e^(2*x) + a*b^2*e^(2*x) + b^3*e^(2*x) + a^3 + a^2*
b - a*b^2 - b^3)/((a^4 - 2*a^2*b^2 + b^4)*(a*e^(4*x) + b*e^(4*x) + a*e^(2*x
) - b*e^(2*x)))
```

maple [A] time = 0.27, size = 253, normalized size = 1.18

$$\frac{1}{2\left(a+b\right)^{2}\left(\tanh\left(\frac{x}{2}\right)-1\right)^{2}}+\frac{1}{2\left(a+b\right)^{2}\left(\tanh\left(\frac{x}{2}\right)-1\right)}+\frac{a\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{\left(a+b\right)^{3}}-\frac{2a^{4}b\tanh\left(\frac{x}{2}\right)}{\left(a-b\right)^{3}\left(a+b\right)^{3}\left(a+2\tanh\left(\frac{x}{2}\right)b+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)*sinh(x)^3/(a*cosh(x)+b*sinh(x))^2,x)
```

```
[Out] 1/2/(a+b)^2/(tanh(1/2*x)-1)^2+1/2/(a+b)^2/(tanh(1/2*x)-1)+a/(a+b)^3*ln(tanh
(1/2*x)-1)-2*a^4/(a-b)^3/(a+b)^3*b*tanh(1/2*x)/(a+2*tanh(1/2*x)*b+a*tanh(1/
2*x)^2)+2*a^2/(a-b)^3/(a+b)^3*b^3*tanh(1/2*x)/(a+2*tanh(1/2*x)*b+a*tanh(1/2
*x)^2)-a^4/(a-b)^3/(a+b)^3*ln(a+2*tanh(1/2*x)*b+a*tanh(1/2*x)^2)-3*a^2/(a-b
)^3/(a+b)^3*ln(a+2*tanh(1/2*x)*b+a*tanh(1/2*x)^2)*b^2+1/2/(a-b)^2/(tanh(1/2
*x)+1)^2-1/2/(a-b)^2/(tanh(1/2*x)+1)+a/(a-b)^3*ln(tanh(1/2*x)+1)
```

maxima [A] time = 0.52, size = 241, normalized size = 1.12

$$-\frac{ax}{a^{3}+3\,a^{2}b+3\,ab^{2}+b^{3}}-\frac{\left(a^{4}+3\,a^{2}b^{2}\right)\log\left(-(a-b)e^{(-2\,x)}-a-b\right)}{a^{6}-3\,a^{4}b^{2}+3\,a^{2}b^{4}-b^{6}}+\frac{a^{4}-2\,a^{3}b+2\,ab^{3}-b^{4}+\left(a^{4}\right.}{8\left(\left(a^{6}-3\,a^{4}b^{2}+3\,a^{2}b^{4}-b^{6}\right)e^{(-2\,x)}+\left(a^{6}-\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*sinh(x)^3/(a*cosh(x)+b*sinh(x))^2,x, algorithm="maxima")
```

```
[Out] -a*x/(a^3 + 3*a^2*b + 3*a*b^2 + b^3) - (a^4 + 3*a^2*b^2)*log(-(a - b)*e^(-2
*x) - a - b)/(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6) + 1/8*(a^4 - 2*a^3*b + 2*a
*b^3 - b^4 + (a^4 - 20*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4)*e^(-2*x))/((a^6 -
 3*a^4*b^2 + 3*a^2*b^4 - b^6)*e^(-2*x) + (a^6 - 2*a^5*b - a^4*b^2 + 4*a^3*b
^3 - a^2*b^4 - 2*a*b^5 + b^6)*e^(-4*x)) + 1/8*e^(-2*x)/(a^2 - 2*a*b + b^2)
```

mupad [B] time = 1.82, size = 127, normalized size = 0.59

$$\frac{e^{2\,x}}{8\,\left(a+b\right)^{2}}+\frac{e^{-2\,x}}{8\,\left(a-b\right)^{2}}+\frac{a\,x}{\left(a-b\right)^{3}}-\frac{\ln\left(a-b+a\,e^{2\,x}+b\,e^{2\,x}\right)\,\left(a^{4}+3\,a^{2}\,b^{2}\right)}{a^{6}-3\,a^{4}\,b^{2}+3\,a^{2}\,b^{4}-b^{6}}+\frac{2\,a^{3}\,b}{\left(a+b\right)^{3}\,\left(a-b\right)^{2}\,\left(a-b+e^{2\,x}\,\left(a+b\right)\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cosh(x)*sinh(x)^3)/(a*cosh(x) + b*sinh(x))^2,x)
```

```
[Out] exp(2*x)/(8*(a + b)^2) + exp(-2*x)/(8*(a - b)^2) + (a*x)/(a - b)^3 - (log(a
 - b + a*exp(2*x) + b*exp(2*x))*(a^4 + 3*a^2*b^2))/(a^6 - b^6 + 3*a^2*b^4 -
 3*a^4*b^2) + (2*a^3*b)/((a + b)^3*(a - b)^2*(a - b + exp(2*x)*(a + b)))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*sinh(x)**3/(a*cosh(x)+b*sinh(x))**2,x)
```

```
[Out] Timed out
```

3.718 $\int \frac{\cosh^2(x)\sinh(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx$

Optimal. Leaf size=163

$$-\frac{2ab\sinh(x)}{(a^2-b^2)^2}+\frac{b^2\cosh(x)}{(a^2-b^2)^2}+\frac{a^2\cosh(x)}{(a^2-b^2)^2}+\frac{ab^2}{(a^2-b^2)^2(a\cosh(x)+b\sinh(x))}+\frac{2a^2b\tan^{-1}\left(\frac{a\sinh(x)+b\cosh(x)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{5/2}}+\frac{b^3\tan\ldots}{\ldots}$$

```
[Out] 2*a^2*b*arctan((b*cosh(x)+a*sinh(x))/(a^2-b^2)^(1/2))/(a^2-b^2)^(5/2)+b^3*a
rctan((b*cosh(x)+a*sinh(x))/(a^2-b^2)^(1/2))/(a^2-b^2)^(5/2)+a^2*cosh(x)/(a
^2-b^2)^2+b^2*cosh(x)/(a^2-b^2)^2-2*a*b*sinh(x)/(a^2-b^2)^2+a*b^2/(a^2-b^2)
^2/(a*cosh(x)+b*sinh(x))
```

Rubi [A] time = 0.32, antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 8, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {3111, 3100, 2637, 3074, 206, 3109, 2638, 3155}

$$-\frac{2ab\sinh(x)}{(a^2-b^2)^2}+\frac{b^2\cosh(x)}{(a^2-b^2)^2}+\frac{a^2\cosh(x)}{(a^2-b^2)^2}+\frac{ab^2}{(a^2-b^2)^2(a\cosh(x)+b\sinh(x))}+\frac{b^3\tan^{-1}\left(\frac{a\sinh(x)+b\cosh(x)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{5/2}}+\frac{2a^2b\tan\ldots}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[(Cosh[x]^2*Sinh[x])/(a*Cosh[x] + b*Sinh[x])^2,x]
```

```
[Out] (2*a^2*b*ArcTan[(b*Cosh[x] + a*Sinh[x])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(5/2)
 + (b^3*ArcTan[(b*Cosh[x] + a*Sinh[x])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(5/2)
+ (a^2*Cosh[x])/(a^2 - b^2)^2 + (b^2*Cosh[x])/(a^2 - b^2)^2 - (2*a*b*Sinh[x
])/(a^2 - b^2)^2 + (a*b^2)/((a^2 - b^2)^2*(a*Cosh[x] + b*Sinh[x]))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3074

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> -Dist[d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d
*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3100

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_)/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin
[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[(b*Cos[c + d*x]^(m - 1))/(d*(a^2 +
 b^2)*(m - 1)), x] + (Dist[a/(a^2 + b^2), Int[Cos[c + d*x]^(m - 1), x], x]
+ Dist[b^2/(a^2 + b^2), Int[Cos[c + d*x]^(m - 2)/(a*Cos[c + d*x] + b*Sin[c
+ d*x]), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && GtQ[m, 1
]
```

Rule 3109

```
Int[(cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.))/(cos[(c_.
) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[b
/(a^2 + b^2), Int[Cos[c + d*x]^m*Sin[c + d*x]^(n - 1), x], x] + (Dist[a/(a^
2 + b^2), Int[Cos[c + d*x]^(m - 1)*Sin[c + d*x]^n, x], x] - Dist[(a*b)/(a^2
 + b^2), Int[(Cos[c + d*x]^(m - 1)*Sin[c + d*x]^(n - 1))/(a*Cos[c + d*x] +
b*Sin[c + d*x]), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] &&
IGtQ[m, 0] && IGtQ[n, 0]
```

Rule 3111

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.)*(cos[(c_.)
+ (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol] :> Dis
t[b/(a^2 + b^2), Int[Cos[c + d*x]^m*Sin[c + d*x]^(n - 1)*(a*Cos[c + d*x] +
b*Sin[c + d*x])^(p + 1), x], x] + (Dist[a/(a^2 + b^2), Int[Cos[c + d*x]^(m
- 1)*Sin[c + d*x]^n*(a*Cos[c + d*x] + b*Sin[c + d*x])^(p + 1), x], x] - Dis
t[(a*b)/(a^2 + b^2), Int[Cos[c + d*x]^(m - 1)*Sin[c + d*x]^(n - 1)*(a*Cos[c
 + d*x] + b*Sin[c + d*x])^p, x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 +
 b^2, 0] && IGtQ[m, 0] && IGtQ[n, 0] && ILtQ[p, 0]
```

Rule 3155

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.))/((a_.) + cos[(d_.) + (e_.)*(x_)
]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])^2, x_Symbol] :> Simp[(c*B + c*A*Co
s[d + e*x] + (a*B - b*A)*Sin[d + e*x])/(e*(a^2 - b^2 - c^2)*(a + b*Cos[d +
e*x] + c*Sin[d + e*x])), x] + Dist[(a*A - b*B)/(a^2 - b^2 - c^2), Int[1/(a
```

```
+ b*Cos[d + e*x] + c*Sin[d + e*x]), x], x] /; FreeQ[{a, b, c, d, e, A, B},
x] && NeQ[a^2 - b^2 - c^2, 0] && NeQ[a*A - b*B, 0]
```

Rubi steps

$$\int \frac{\cosh^2(x)\sinh(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx = \frac{a\int \frac{\cosh(x)\sinh(x)}{a\cosh(x)+b\sinh(x)}\,dx}{a^2-b^2} - \frac{b\int \frac{\cosh^2(x)}{a\cosh(x)+b\sinh(x)}\,dx}{a^2-b^2} + \frac{(ab)\int \frac{\cosh(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx}{a^2-b^2}$$

$$= \frac{b^2\cosh(x)}{\left(a^2-b^2\right)^2} + \frac{ab^2}{\left(a^2-b^2\right)^2(a\cosh(x)+b\sinh(x))} + \frac{a^2\int \sinh(x)\,dx}{\left(a^2-b^2\right)^2} - 2\frac{(ab)\int c}{\left(a^2\right.}$$

$$= \frac{a^2\cosh(x)}{\left(a^2-b^2\right)^2} + \frac{b^2\cosh(x)}{\left(a^2-b^2\right)^2} - \frac{2ab\sinh(x)}{\left(a^2-b^2\right)^2} + \frac{ab^2}{\left(a^2-b^2\right)^2(a\cosh(x)+b\sinh(x))} +$$

$$= \frac{2a^2b\tan^{-1}\left(\frac{b\cosh(x)+a\sinh(x)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{5/2}} + \frac{b^3\tan^{-1}\left(\frac{b\cosh(x)+a\sinh(x)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{5/2}} + \frac{a^2\cosh(x)}{\left(a^2-b^2\right)^2} + \frac{b^2}{(a^2}$$

Mathematica [A] time = 0.90, size = 264, normalized size = 1.62

$$\frac{1}{4}\left(\frac{4\left(a^2+b^2\right)\cosh(x)}{(a-b)^2(a+b)^2} + \frac{6b\left(3a^2+b^2\right)\tan^{-1}\left(\frac{a\tanh\left(\frac{x}{2}\right)+b}{\sqrt{a-b}\sqrt{a+b}}\right)}{(a-b)^{5/2}(a+b)^{5/2}} + \frac{a\left(a^2+3b^2\right)}{(a-b)^2(a+b)^2(a\cosh(x)+b\sinh(x))} - \frac{8ab\sin}{(a-b)^2(}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cosh[x]^2*Sinh[x])/(a*Cosh[x] + b*Sinh[x])^2,x]
```

```
[Out] -1/4*(a*Sqrt[a - b]*(a + b) + 2*a*b*Sqrt[a + b]*ArcTan[(b + a*Tanh[x/2])/(S
qrt[a - b]*Sqrt[a + b])]*Cosh[x] + 2*b^2*Sqrt[a + b]*ArcTan[(b + a*Tanh[x/2
])/(Sqrt[a - b]*Sqrt[a + b])]*Sinh[x])/((a - b)^(3/2)*(a + b)^2*(a*Cosh[x]
+ b*Sinh[x])) + ((6*b*(3*a^2 + b^2)*ArcTan[(b + a*Tanh[x/2])/(Sqrt[a - b]*S
qrt[a + b])])/((a - b)^(5/2)*(a + b)^(5/2)) + (4*(a^2 + b^2)*Cosh[x])/((a -
 b)^2*(a + b)^2) - (8*a*b*Sinh[x])/((a - b)^2*(a + b)^2) + (a*(a^2 + 3*b^2)
)/((a - b)^2*(a + b)^2*(a*Cosh[x] + b*Sinh[x])))/4
```

fricas [B] time = 0.47, size = 1805, normalized size = 11.07

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^2*sinh(x)/(a*cosh(x)+b*sinh(x))^2,x, algorithm="fricas")
```

```
[Out] [1/2*(a^5 + a^4*b - 2*a^3*b^2 - 2*a^2*b^3 + a*b^4 + b^5 + (a^5 - a^4*b - 2*
a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^4 + 4*(a^5 - a^4*b - 2*a^3*b^2 +
 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)*sinh(x)^3 + (a^5 - a^4*b - 2*a^3*b^2 + 2*
a^2*b^3 + a*b^4 - b^5)*sinh(x)^4 + 2*(a^5 + 4*a^3*b^2 - 5*a*b^4)*cosh(x)^2
+ 2*(a^5 + 4*a^3*b^2 - 5*a*b^4 + 3*(a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a
*b^4 - b^5)*cosh(x)^2)*sinh(x)^2 - 2*((2*a^3*b + 2*a^2*b^2 + a*b^3 + b^4)*c
osh(x)^3 + 3*(2*a^3*b + 2*a^2*b^2 + a*b^3 + b^4)*cosh(x)*sinh(x)^2 + (2*a^3
*b + 2*a^2*b^2 + a*b^3 + b^4)*sinh(x)^3 + (2*a^3*b - 2*a^2*b^2 + a*b^3 - b^
4)*cosh(x) + (2*a^3*b - 2*a^2*b^2 + a*b^3 - b^4 + 3*(2*a^3*b + 2*a^2*b^2 +
a*b^3 + b^4)*cosh(x)^2)*sinh(x))*sqrt(-a^2 + b^2)*log(((a + b)*cosh(x)^2 +
2*(a + b)*cosh(x)*sinh(x) + (a + b)*sinh(x)^2 - 2*sqrt(-a^2 + b^2)*(cosh(x)
 + sinh(x)) - a + b)/((a + b)*cosh(x)^2 + 2*(a + b)*cosh(x)*sinh(x) + (a +
b)*sinh(x)^2 + a - b)) + 4*((a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 -
b^5)*cosh(x)^3 + (a^5 + 4*a^3*b^2 - 5*a*b^4)*cosh(x))*sinh(x))/((a^7 + a^6*
b - 3*a^5*b^2 - 3*a^4*b^3 + 3*a^3*b^4 + 3*a^2*b^5 - a*b^6 - b^7)*cosh(x)^3
+ 3*(a^7 + a^6*b - 3*a^5*b^2 - 3*a^4*b^3 + 3*a^3*b^4 + 3*a^2*b^5 - a*b^6 -
b^7)*cosh(x)*sinh(x)^2 + (a^7 + a^6*b - 3*a^5*b^2 - 3*a^4*b^3 + 3*a^3*b^4 +
 3*a^2*b^5 - a*b^6 - b^7)*sinh(x)^3 + (a^7 - a^6*b - 3*a^5*b^2 + 3*a^4*b^3
+ 3*a^3*b^4 - 3*a^2*b^5 - a*b^6 + b^7)*cosh(x) + (a^7 - a^6*b - 3*a^5*b^2 +
 3*a^4*b^3 + 3*a^3*b^4 - 3*a^2*b^5 - a*b^6 + b^7 + 3*(a^7 + a^6*b - 3*a^5*b
^2 - 3*a^4*b^3 + 3*a^3*b^4 + 3*a^2*b^5 - a*b^6 - b^7)*cosh(x)^2)*sinh(x)),
1/2*(a^5 + a^4*b - 2*a^3*b^2 - 2*a^2*b^3 + a*b^4 + b^5 + (a^5 - a^4*b - 2*a
^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^4 + 4*(a^5 - a^4*b - 2*a^3*b^2 +
2*a^2*b^3 + a*b^4 - b^5)*cosh(x)*sinh(x)^3 + (a^5 - a^4*b - 2*a^3*b^2 + 2*a
^2*b^3 + a*b^4 - b^5)*sinh(x)^4 + 2*(a^5 + 4*a^3*b^2 - 5*a*b^4)*cosh(x)^2 +
 2*(a^5 + 4*a^3*b^2 - 5*a*b^4 + 3*(a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*
b^4 - b^5)*cosh(x)^2)*sinh(x)^2 - 4*((2*a^3*b + 2*a^2*b^2 + a*b^3 + b^4)*co
sh(x)^3 + 3*(2*a^3*b + 2*a^2*b^2 + a*b^3 + b^4)*cosh(x)*sinh(x)^2 + (2*a^3*
b + 2*a^2*b^2 + a*b^3 + b^4)*sinh(x)^3 + (2*a^3*b - 2*a^2*b^2 + a*b^3 - b^4
)*cosh(x) + (2*a^3*b - 2*a^2*b^2 + a*b^3 - b^4 + 3*(2*a^3*b + 2*a^2*b^2 + a
*b^3 + b^4)*cosh(x)^2)*sinh(x))*sqrt(a^2 - b^2)*arctan(sqrt(a^2 - b^2)/((a
+ b)*cosh(x) + (a + b)*sinh(x))) + 4*((a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3
+ a*b^4 - b^5)*cosh(x)^3 + (a^5 + 4*a^3*b^2 - 5*a*b^4)*cosh(x))*sinh(x))/((
a^7 + a^6*b - 3*a^5*b^2 - 3*a^4*b^3 + 3*a^3*b^4 + 3*a^2*b^5 - a*b^6 - b^7)*
cosh(x)^3 + 3*(a^7 + a^6*b - 3*a^5*b^2 - 3*a^4*b^3 + 3*a^3*b^4 + 3*a^2*b^5
- a*b^6 - b^7)*cosh(x)*sinh(x)^2 + (a^7 + a^6*b - 3*a^5*b^2 - 3*a^4*b^3 + 3
*a^3*b^4 + 3*a^2*b^5 - a*b^6 - b^7)*sinh(x)^3 + (a^7 - a^6*b - 3*a^5*b^2 +
3*a^4*b^3 + 3*a^3*b^4 - 3*a^2*b^5 - a*b^6 + b^7)*cosh(x) + (a^7 - a^6*b - 3
*a^5*b^2 + 3*a^4*b^3 + 3*a^3*b^4 - 3*a^2*b^5 - a*b^6 + b^7 + 3*(a^7 + a^6*b
 - 3*a^5*b^2 - 3*a^4*b^3 + 3*a^3*b^4 + 3*a^2*b^5 - a*b^6 - b^7)*cosh(x)^2)*
sinh(x))]
```

giac [A] time = 0.16, size = 179, normalized size = 1.10

$$\frac{2\left(2\,a^2b+b^3\right)\arctan\left(\frac{ae^x+be^x}{\sqrt{a^2-b^2}}\right)}{\left(a^4-2\,a^2b^2+b^4\right)\sqrt{a^2-b^2}}+\frac{e^x}{2\left(a^2+2\,ab+b^2\right)}+\frac{a^3e^{(2\,x)}+3\,a^2be^{(2\,x)}+7\,ab^2e^{(2\,x)}+b^3e^{(2\,x)}+a^3+a^2b-ab^2-}{2\left(a^4-2\,a^2b^2+b^4\right)\left(ae^{(3\,x)}+be^{(3\,x)}+ae^x-be^x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^2*sinh(x)/(a*cosh(x)+b*sinh(x))^2,x, algorithm="giac")
```

```
[Out] 2*(2*a^2*b + b^3)*arctan((a*e^x + b*e^x)/sqrt(a^2 - b^2))/((a^4 - 2*a^2*b^2
 + b^4)*sqrt(a^2 - b^2)) + 1/2*e^x/(a^2 + 2*a*b + b^2) + 1/2*(a^3*e^(2*x) +
 3*a^2*b*e^(2*x) + 7*a*b^2*e^(2*x) + b^3*e^(2*x) + a^3 + a^2*b - a*b^2 - b^
3)/((a^4 - 2*a^2*b^2 + b^4)*(a*e^(3*x) + b*e^(3*x) + a*e^x - b*e^x))
```

maple [A] time = 0.26, size = 217, normalized size = 1.33

$$-\frac{1}{\left(a+b\right)^2\left(\tanh\left(\frac{x}{2}\right)-1\right)}+\frac{2b^3\tanh\left(\frac{x}{2}\right)}{\left(a-b\right)^2\left(a+b\right)^2\left(a+2\tanh\left(\frac{x}{2}\right)b+a\left(\tanh^2\left(\frac{x}{2}\right)\right)\right)}+\frac{2b^2a}{\left(a-b\right)^2\left(a+b\right)^2\left(a+2\tanh\left(\frac{x}{2}\right)\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^2*sinh(x)/(a*cosh(x)+b*sinh(x))^2,x)
```

```
[Out] -1/(a+b)^2/(tanh(1/2*x)-1)+2/(a-b)^2/(a+b)^2*b^3*tanh(1/2*x)/(a+2*tanh(1/2*
x)*b+a*tanh(1/2*x)^2)+2*b^2/(a-b)^2/(a+b)^2/(a+2*tanh(1/2*x)*b+a*tanh(1/2*x
)^2)*a+4*a^2/(a-b)^2/(a+b)^2*b/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tanh(1/2*x)+
2*b)/(a^2-b^2)^(1/2))+2*b^3/(a-b)^2/(a+b)^2/(a^2-b^2)^(1/2)*arctan(1/2*(2*a
*tanh(1/2*x)+2*b)/(a^2-b^2)^(1/2))+1/(a-b)^2/(tanh(1/2*x)+1)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^2*sinh(x)/(a*cosh(x)+b*sinh(x))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [B] time = 1.82, size = 397, normalized size = 2.44

$$\frac{e^{-x}}{2(a-b)^2}+\frac{e^x}{2(a+b)^2}+\frac{2\,\mathrm{atan}\left(\frac{e^x\left(b^3\sqrt{a^{10}-5a^8b^2+10a^6b^4-10a^4b^6+5a^2b^8-b^{10}}+2a^2b\sqrt{a^{10}-5a^8b^2+10a^6b^4}\right.}{a^5\sqrt{4a^4b^2+4a^2b^4+b^6}-b^5\sqrt{4a^4b^2+4a^2b^4+b^6}+2a^2b^3\sqrt{4a^4b^2+4a^2b^4+b^6}-2a^3b^2\sqrt{4a^4b^2+4a^2b^4+b^6}+}\right.}{\sqrt{a^{10}-5a^8b^2+10a^6b^4-10a^4b^6+}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cosh(x)^2*sinh(x))/(a*cosh(x) + b*sinh(x))^2,x)
```

```
[Out] exp(-x)/(2*(a - b)^2) + exp(x)/(2*(a + b)^2) + (2*atan((exp(x)*(b^3*(a^10 -
 b^10 + 5*a^2*b^8 - 10*a^4*b^6 + 10*a^6*b^4 - 5*a^8*b^2)^(1/2) + 2*a^2*b*(a
^10 - b^10 + 5*a^2*b^8 - 10*a^4*b^6 + 10*a^6*b^4 - 5*a^8*b^2)^(1/2)))/(a^5*
(b^6 + 4*a^2*b^4 + 4*a^4*b^2)^(1/2) - b^5*(b^6 + 4*a^2*b^4 + 4*a^4*b^2)^(1/
2) + 2*a^2*b^3*(b^6 + 4*a^2*b^4 + 4*a^4*b^2)^(1/2) - 2*a^3*b^2*(b^6 + 4*a^2
*b^4 + 4*a^4*b^2)^(1/2) + a*b^4*(b^6 + 4*a^2*b^4 + 4*a^4*b^2)^(1/2) - a^4*b
*(b^6 + 4*a^2*b^4 + 4*a^4*b^2)^(1/2)))*(b^6 + 4*a^2*b^4 + 4*a^4*b^2)^(1/2))
/(a^10 - b^10 + 5*a^2*b^8 - 10*a^4*b^6 + 10*a^6*b^4 - 5*a^8*b^2)^(1/2) + (2
*a*b^2*exp(x))/((a + b)^2*(a - b)^2*(a - b + exp(2*x)*(a + b)))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)**2*sinh(x)/(a*cosh(x)+b*sinh(x))**2,x)
```

```
[Out] Timed out
```

3.719 $\int \frac{\cosh^2(x)\sinh^2(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx$

Optimal. Leaf size=205

$$-\frac{a^2x}{2\left(a^2-b^2\right)^2}-\frac{4a^2b^2x}{\left(a^2-b^2\right)^3}+\frac{b^2x}{2\left(a^2-b^2\right)^2}-\frac{ab\sinh^2(x)}{\left(a^2-b^2\right)^2}+\frac{a^2\sinh(x)\cosh(x)}{2\left(a^2-b^2\right)^2}+\frac{ab^2\sinh(x)}{\left(a^2-b^2\right)^2(a\cosh(x)+b\sinh(x))}+$$

```
[Out] -4*a^2*b^2*x/(a^2-b^2)^3-1/2*a^2*x/(a^2-b^2)^2+1/2*b^2*x/(a^2-b^2)^2+2*a^3*
b*ln(a*cosh(x)+b*sinh(x))/(a^2-b^2)^3+2*a*b^3*ln(a*cosh(x)+b*sinh(x))/(a^2-
b^2)^3+1/2*a^2*cosh(x)*sinh(x)/(a^2-b^2)^2+1/2*b^2*cosh(x)*sinh(x)/(a^2-b^2
)^2-a*b*sinh(x)^2/(a^2-b^2)^2+a*b^2*sinh(x)/(a^2-b^2)^2/(a*cosh(x)+b*sinh(x
))
```

Rubi [A] time = 0.66, antiderivative size = 205, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 10, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3111, 3109, 2635, 8, 2564, 30, 3098, 3133, 3097, 3075}

$$-\frac{a^2x}{2\left(a^2-b^2\right)^2}-\frac{4a^2b^2x}{\left(a^2-b^2\right)^3}+\frac{b^2x}{2\left(a^2-b^2\right)^2}-\frac{ab\sinh^2(x)}{\left(a^2-b^2\right)^2}+\frac{a^2\sinh(x)\cosh(x)}{2\left(a^2-b^2\right)^2}+\frac{ab^2\sinh(x)}{\left(a^2-b^2\right)^2(a\cosh(x)+b\sinh(x))}+$$

Antiderivative was successfully verified.

```
[In] Int[(Cosh[x]^2*Sinh[x]^2)/(a*Cosh[x] + b*Sinh[x])^2,x]
```

```
[Out] (-4*a^2*b^2*x)/(a^2 - b^2)^3 - (a^2*x)/(2*(a^2 - b^2)^2) + (b^2*x)/(2*(a^2
- b^2)^2) + (2*a^3*b*Log[a*Cosh[x] + b*Sinh[x]])/(a^2 - b^2)^3 + (2*a*b^3*L
og[a*Cosh[x] + b*Sinh[x]])/(a^2 - b^2)^3 + (a^2*Cosh[x]*Sinh[x])/(2*(a^2 -
b^2)^2) + (b^2*Cosh[x]*Sinh[x])/(2*(a^2 - b^2)^2) - (a*b*Sinh[x]^2)/(a^2 -
b^2)^2 + (a*b^2*Sinh[x])/((a^2 - b^2)^2*(a*Cosh[x] + b*Sinh[x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
```

```
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 3075

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-2), x
_Symbol] :> Simp[Sin[c + d*x]/(a*d*(a*Cos[c + d*x] + b*Sin[c + d*x])), x] /
; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3097

```
Int[sin[(c_.) + (d_.)*(x_)]/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.
) + (d_.)*(x_)]), x_Symbol] :> Simp[(b*x)/(a^2 + b^2), x] - Dist[a/(a^2 + b
^2), Int[(b*Cos[c + d*x] - a*Sin[c + d*x])/(a*Cos[c + d*x] + b*Sin[c + d*x]
), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3098

```
Int[cos[(c_.) + (d_.)*(x_)]/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.
) + (d_.)*(x_)]), x_Symbol] :> Simp[(a*x)/(a^2 + b^2), x] + Dist[b/(a^2 + b
^2), Int[(b*Cos[c + d*x] - a*Sin[c + d*x])/(a*Cos[c + d*x] + b*Sin[c + d*x]
), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3109

```
Int[(cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.))/(cos[(c_.
) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[b
/(a^2 + b^2), Int[Cos[c + d*x]^m*Sin[c + d*x]^(n - 1), x], x] + (Dist[a/(a^
2 + b^2), Int[Cos[c + d*x]^(m - 1)*Sin[c + d*x]^n, x], x] - Dist[(a*b)/(a^2
 + b^2), Int[(Cos[c + d*x]^(m - 1)*Sin[c + d*x]^(n - 1))/(a*Cos[c + d*x] +
b*Sin[c + d*x]), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] &&
IGtQ[m, 0] && IGtQ[n, 0]
```

Rule 3111

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.)*(cos[(c_.)
+ (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol] :> Dis
t[b/(a^2 + b^2), Int[Cos[c + d*x]^m*Sin[c + d*x]^(n - 1)*(a*Cos[c + d*x] +
```

```
b*Sin[c + d*x])^(p + 1), x], x] + (Dist[a/(a^2 + b^2), Int[Cos[c + d*x]^(m
- 1)*Sin[c + d*x]^n*(a*Cos[c + d*x] + b*Sin[c + d*x])^(p + 1), x], x] - Dis
t[(a*b)/(a^2 + b^2), Int[Cos[c + d*x]^(m - 1)*Sin[c + d*x]^(n - 1)*(a*Cos[c
 + d*x] + b*Sin[c + d*x])^p, x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 +
 b^2, 0] && IGtQ[m, 0] && IGtQ[n, 0] && ILtQ[p, 0]
```

Rule 3133

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)]), x
_Symbol] :> Simp[((b*B + c*C)*x)/(b^2 + c^2), x] + Simp[((c*B - b*C)*Log[a
+ b*Cos[d + e*x] + c*Sin[d + e*x]])/(e*(b^2 + c^2)), x] /; FreeQ[{a, b, c,
d, e, A, B, C}, x] && NeQ[b^2 + c^2, 0] && EqQ[A*(b^2 + c^2) - a*(b*B + c*C
), 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\cosh^2(x)\sinh^2(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx &= \frac{a\int \frac{\cosh(x)\sinh^2(x)}{a\cosh(x)+b\sinh(x)}\,dx}{a^2-b^2} - \frac{b\int \frac{\cosh^2(x)\sinh(x)}{a\cosh(x)+b\sinh(x)}\,dx}{a^2-b^2} + \frac{(ab)\int \frac{\cosh(x)\sinh(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx}{a^2-b^2} \\ &= \frac{a^2\int \sinh^2(x)\,dx}{\left(a^2-b^2\right)^2} - 2\frac{(ab)\int \cosh(x)\sinh(x)\,dx}{\left(a^2-b^2\right)^2} + 2\frac{\left(a^2b\right)\int \frac{\sinh(x)}{a\cosh(x)+b\sinh(x)}\,dx}{\left(a^2-b^2\right)^2} \\ &= \frac{a^2\cosh(x)\sinh(x)}{2\left(a^2-b^2\right)^2} + \frac{b^2\cosh(x)\sinh(x)}{2\left(a^2-b^2\right)^2} + \frac{ab^2\sinh(x)}{\left(a^2-b^2\right)^2(a\cosh(x)+b\sinh(x))} \\ &= -\frac{a^2x}{2\left(a^2-b^2\right)^2} + \frac{b^2x}{2\left(a^2-b^2\right)^2} + 2\left(-\frac{a^2b^2x}{\left(a^2-b^2\right)^3} + \frac{a^3b\log(a\cosh(x)+b\sinh(x}{\left(a^2-b^2\right)^3}\right.\end{aligned}$$

Mathematica [A] time = 2.05, size = 174, normalized size = 0.85

$$\frac{1}{8}\left(\frac{2\left(a^2+b^2\right)\sinh(2x)}{(a-b)^2(a+b)^2} + \frac{16ab\left(a^2+b^2\right)\log(a\cosh(x)+b\sinh(x))}{\left(a^2-b^2\right)^3} - \frac{\sinh(x)}{a^2\cosh(x)+ab\sinh(x)} - \frac{4x\left(a^4+6a^2b}{(a-b)^3(a-}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cosh[x]^2*Sinh[x]^2)/(a*Cosh[x] + b*Sinh[x])^2,x]
```

```
[Out] ((-4*(a^4 + 6*a^2*b^2 + b^4)*x)/((a - b)^3*(a + b)^3) - (4*a*b*Cosh[2*x])/(
(a - b)^2*(a + b)^2) + (16*a*b*(a^2 + b^2)*Log[a*Cosh[x] + b*Sinh[x]])/(a^2
```

```
 - b^2)^3 + ((a^4 + 6*a^2*b^2 + b^4)*Sinh[x])/(a*(a - b)^2*(a + b)^2*(a*Cos
h[x] + b*Sinh[x])) - Sinh[x]/(a^2*Cosh[x] + a*b*Sinh[x]) + (2*(a^2 + b^2)*S
inh[2*x])/((a - b)^2*(a + b)^2))/8
```

fricas [B] time = 0.55, size = 1726, normalized size = 8.42

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^2*sinh(x)^2/(a*cosh(x)+b*sinh(x))^2,x, algorithm="fricas"
)
```

```
[Out] 1/8*((a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^6 + 6*(a^5
 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)*sinh(x)^5 + (a^5 -
a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*sinh(x)^6 - a^5 - a^4*b + 2*a^
3*b^2 + 2*a^2*b^3 - a*b^4 - b^5 + (a^5 - 3*a^4*b + 2*a^3*b^2 + 2*a^2*b^3 -
3*a*b^4 + b^5 - 4*(a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)
*x)*cosh(x)^4 + (a^5 - 3*a^4*b + 2*a^3*b^2 + 2*a^2*b^3 - 3*a*b^4 + b^5 + 15
*(a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^2 - 4*(a^5 + 5
*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*x)*sinh(x)^4 + 4*(5*(a^5
- a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^3 + (a^5 - 3*a^4*b +
 2*a^3*b^2 + 2*a^2*b^3 - 3*a*b^4 + b^5 - 4*(a^5 + 5*a^4*b + 10*a^3*b^2 + 10
*a^2*b^3 + 5*a*b^4 + b^5)*x)*cosh(x))*sinh(x)^3 - (a^5 + 3*a^4*b + 18*a^3*b
^2 - 18*a^2*b^3 - 3*a*b^4 - b^5 + 4*(a^5 + 3*a^4*b + 2*a^3*b^2 - 2*a^2*b^3
- 3*a*b^4 - b^5)*x)*cosh(x)^2 - (a^5 + 3*a^4*b + 18*a^3*b^2 - 18*a^2*b^3 -
3*a*b^4 - b^5 - 15*(a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh
(x)^4 - 6*(a^5 - 3*a^4*b + 2*a^3*b^2 + 2*a^2*b^3 - 3*a*b^4 + b^5 - 4*(a^5 +
 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*x)*cosh(x)^2 + 4*(a^5 +
 3*a^4*b + 2*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^5)*x)*sinh(x)^2 + 16*((a^4*b
 + a^3*b^2 + a^2*b^3 + a*b^4)*cosh(x)^4 + 4*(a^4*b + a^3*b^2 + a^2*b^3 + a*
b^4)*cosh(x)*sinh(x)^3 + (a^4*b + a^3*b^2 + a^2*b^3 + a*b^4)*sinh(x)^4 + (a
^4*b - a^3*b^2 + a^2*b^3 - a*b^4)*cosh(x)^2 + (a^4*b - a^3*b^2 + a^2*b^3 -
a*b^4 + 6*(a^4*b + a^3*b^2 + a^2*b^3 + a*b^4)*cosh(x)^2)*sinh(x)^2 + 2*(2*(
a^4*b + a^3*b^2 + a^2*b^3 + a*b^4)*cosh(x)^3 + (a^4*b - a^3*b^2 + a^2*b^3 -
 a*b^4)*cosh(x))*sinh(x))*log(2*(a*cosh(x) + b*sinh(x))/(cosh(x) - sinh(x))
) + 2*(3*(a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^5 + 2*
(a^5 - 3*a^4*b + 2*a^3*b^2 + 2*a^2*b^3 - 3*a*b^4 + b^5 - 4*(a^5 + 5*a^4*b +
 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*x)*cosh(x)^3 - (a^5 + 3*a^4*b + 1
8*a^3*b^2 - 18*a^2*b^3 - 3*a*b^4 - b^5 + 4*(a^5 + 3*a^4*b + 2*a^3*b^2 - 2*a
^2*b^3 - 3*a*b^4 - b^5)*x)*cosh(x))*sinh(x))/((a^7 + a^6*b - 3*a^5*b^2 - 3*
a^4*b^3 + 3*a^3*b^4 + 3*a^2*b^5 - a*b^6 - b^7)*cosh(x)^4 + 4*(a^7 + a^6*b -
 3*a^5*b^2 - 3*a^4*b^3 + 3*a^3*b^4 + 3*a^2*b^5 - a*b^6 - b^7)*cosh(x)*sinh(
x)^3 + (a^7 + a^6*b - 3*a^5*b^2 - 3*a^4*b^3 + 3*a^3*b^4 + 3*a^2*b^5 - a*b^6
 - b^7)*sinh(x)^4 + (a^7 - a^6*b - 3*a^5*b^2 + 3*a^4*b^3 + 3*a^3*b^4 - 3*a^
2*b^5 - a*b^6 + b^7)*cosh(x)^2 + (a^7 - a^6*b - 3*a^5*b^2 + 3*a^4*b^3 + 3*a
```

```
^3*b^4 - 3*a^2*b^5 - a*b^6 + b^7 + 6*(a^7 + a^6*b - 3*a^5*b^2 - 3*a^4*b^3 +
 3*a^3*b^4 + 3*a^2*b^5 - a*b^6 - b^7)*cosh(x)^2)*sinh(x)^2 + 2*(2*(a^7 + a^
6*b - 3*a^5*b^2 - 3*a^4*b^3 + 3*a^3*b^4 + 3*a^2*b^5 - a*b^6 - b^7)*cosh(x)^
3 + (a^7 - a^6*b - 3*a^5*b^2 + 3*a^4*b^3 + 3*a^3*b^4 - 3*a^2*b^5 - a*b^6 +
b^7)*cosh(x))*sinh(x))
```

giac [A] time = 0.15, size = 232, normalized size = 1.13

$$-\frac{(a+b)x}{2\left(a^3-3\,a^2b+3\,ab^2-b^3\right)}+\frac{2\left(a^3b+ab^3\right)\log\left(\left|ae^{(2\,x)}+be^{(2\,x)}+a-b\right|\right)}{a^6-3\,a^4b^2+3\,a^2b^4-b^6}+\frac{e^{(2\,x)}}{8\left(a^2+2\,ab+b^2\right)}+\frac{a^3e^{(4\,x)}-3\,a^2be^{(4}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^2*sinh(x)^2/(a*cosh(x)+b*sinh(x))^2,x, algorithm="giac")
```

```
[Out] -1/2*(a + b)*x/(a^3 - 3*a^2*b + 3*a*b^2 - b^3) + 2*(a^3*b + a*b^3)*log(abs(
a*e^(2*x) + b*e^(2*x) + a - b))/(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6) + 1/8*e
^(2*x)/(a^2 + 2*a*b + b^2) + 1/8*(a^3*e^(4*x) - 3*a^2*b*e^(4*x) + 3*a*b^2*e
^(4*x) - b^3*e^(4*x) - 8*a^2*b*e^(2*x) - 8*a*b^2*e^(2*x) - a^3 - a^2*b + a*
b^2 + b^3)/((a^4 - 2*a^2*b^2 + b^4)*(a*e^(4*x) + b*e^(4*x) + a*e^(2*x) - b*
e^(2*x)))
```

maple [A] time = 0.26, size = 286, normalized size = 1.40

$$\frac{1}{2\,(a+b)^2\left(\tanh\left(\frac{x}{2}\right)-1\right)^2}+\frac{1}{2\,(a+b)^2\left(\tanh\left(\frac{x}{2}\right)-1\right)}+\frac{a\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{2\,(a+b)^3}-\frac{\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)b}{2\,(a+b)^3}+\frac{}{(a-b)^3\,(a+b)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^2*sinh(x)^2/(a*cosh(x)+b*sinh(x))^2,x)
```

```
[Out] 1/2/(a+b)^2/(tanh(1/2*x)-1)^2+1/2/(a+b)^2/(tanh(1/2*x)-1)+1/2*a/(a+b)^3*ln(
tanh(1/2*x)-1)-1/2/(a+b)^3*ln(tanh(1/2*x)-1)*b+2*a^3*b^2/(a-b)^3/(a+b)^3*ta
nh(1/2*x)/(a+2*tanh(1/2*x)*b+a*tanh(1/2*x)^2)-2*a*b^4/(a-b)^3/(a+b)^3*tanh(
1/2*x)/(a+2*tanh(1/2*x)*b+a*tanh(1/2*x)^2)+2*a^3*b/(a-b)^3/(a+b)^3*ln(a+2*t
anh(1/2*x)*b+a*tanh(1/2*x)^2)+2*a*b^3/(a-b)^3/(a+b)^3*ln(a+2*tanh(1/2*x)*b+
a*tanh(1/2*x)^2)-1/2/(a-b)^2/(tanh(1/2*x)+1)^2+1/2/(a-b)^2/(tanh(1/2*x)+1)-
1/2*a/(a-b)^3*ln(tanh(1/2*x)+1)-1/2/(a-b)^3*ln(tanh(1/2*x)+1)*b
```

maxima [A] time = 0.55, size = 244, normalized size = 1.19

$$-\frac{(a-b)x}{2\left(a^3+3\,a^2b+3\,ab^2+b^3\right)}+\frac{2\left(a^3b+ab^3\right)\log\left(-(a-b)e^{(-2\,x)}-a-b\right)}{a^6-3\,a^4b^2+3\,a^2b^4-b^6}+\frac{a^4-2\,a^3b+2\,ab^3-b^4+}{8\left(\left(a^6-3\,a^4b^2+3\,a^2b^4-b^6\right)e^{(-2\,x)}+\left(\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^2*sinh(x)^2/(a*cosh(x)+b*sinh(x))^2,x, algorithm="maxima"
)
```

```
[Out] -1/2*(a - b)*x/(a^3 + 3*a^2*b + 3*a*b^2 + b^3) + 2*(a^3*b + a*b^3)*log(-(a
- b)*e^(-2*x) - a - b)/(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6) + 1/8*(a^4 - 2*a
^3*b + 2*a*b^3 - b^4 + (a^4 - 4*a^3*b + 22*a^2*b^2 - 4*a*b^3 + b^4)*e^(-2*x
))/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*e^(-2*x) + (a^6 - 2*a^5*b - a^4*b^2
 + 4*a^3*b^3 - a^2*b^4 - 2*a*b^5 + b^6)*e^(-4*x)) - 1/8*e^(-2*x)/(a^2 - 2*a
*b + b^2)
```

mupad [B] time = 1.85, size = 132, normalized size = 0.64

$$\frac{e^{2x}}{8(a+b)^2}-\frac{e^{-2x}}{8(a-b)^2}+\frac{\ln\left(a-b+a\,e^{2x}+b\,e^{2x}\right)\left(2\,a^3\,b+2\,a\,b^3\right)}{a^6-3\,a^4\,b^2+3\,a^2\,b^4-b^6}-\frac{x\,(a+b)}{2(a-b)^3}-\frac{2\,a^2\,b^2}{(a+b)^3\,(a-b)^2\,\left(a-b+e^{2x}\,(a+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cosh(x)^2*sinh(x)^2)/(a*cosh(x) + b*sinh(x))^2,x)
```

```
[Out] exp(2*x)/(8*(a + b)^2) - exp(-2*x)/(8*(a - b)^2) + (log(a - b + a*exp(2*x)
+ b*exp(2*x))*(2*a*b^3 + 2*a^3*b))/(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2) - (x
*(a + b))/(2*(a - b)^3) - (2*a^2*b^2)/((a + b)^3*(a - b)^2*(a - b + exp(2*x
)*(a + b)))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)**2*sinh(x)**2/(a*cosh(x)+b*sinh(x))**2,x)
```

```
[Out] Timed out
```

3.720 $\int \frac{\cosh^2(x)\sinh^3(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx$

Optimal. Leaf size=261

$$-\frac{2ab\sinh^3(x)}{3\left(a^2-b^2\right)^2}+\frac{a^2\cosh^3(x)}{3\left(a^2-b^2\right)^2}+\frac{b^2\cosh^3(x)}{3\left(a^2-b^2\right)^2}-\frac{a^2\cosh(x)}{\left(a^2-b^2\right)^2}-\frac{4a^2b^2\cosh(x)}{\left(a^2-b^2\right)^3}+\frac{2ab^3\sinh(x)}{\left(a^2-b^2\right)^3}-\frac{3a^2b^3\tan^{-1}\left(\frac{a\sinh(x)+b\,\mathrm{co}}{\sqrt{a^2-b^2}}\right.}{\left(a^2-b^2\right)^{7/2}}$$

```
[Out] -2*a^4*b*arctan((b*cosh(x)+a*sinh(x))/(a^2-b^2)^(1/2))/(a^2-b^2)^(7/2)-3*a^
2*b^3*arctan((b*cosh(x)+a*sinh(x))/(a^2-b^2)^(1/2))/(a^2-b^2)^(7/2)-4*a^2*b
^2*cosh(x)/(a^2-b^2)^3-a^2*cosh(x)/(a^2-b^2)^2+1/3*a^2*cosh(x)^3/(a^2-b^2)^
2+1/3*b^2*cosh(x)^3/(a^2-b^2)^2+2*a^3*b*sinh(x)/(a^2-b^2)^3+2*a*b^3*sinh(x)
/(a^2-b^2)^3-2/3*a*b*sinh(x)^3/(a^2-b^2)^2-a^3*b^2/(a^2-b^2)^3/(a*cosh(x)+b
*sinh(x))
```

Rubi [A] time = 1.03, antiderivative size = 261, normalized size of antiderivative = 1.00, number of steps used = 33, number of rules used = 12, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {3111, 3109, 2565, 30, 2564, 2637, 2638, 3074, 206, 2633, 3099, 3154}

$$-\frac{2ab\sinh^3(x)}{3\left(a^2-b^2\right)^2}+\frac{2a^3b\sinh(x)}{\left(a^2-b^2\right)^3}+\frac{2ab^3\sinh(x)}{\left(a^2-b^2\right)^3}+\frac{a^2\cosh^3(x)}{3\left(a^2-b^2\right)^2}+\frac{b^2\cosh^3(x)}{3\left(a^2-b^2\right)^2}-\frac{a^2\cosh(x)}{\left(a^2-b^2\right)^2}-\frac{4a^2b^2\cosh(x)}{\left(a^2-b^2\right)^3}-\frac{}{\left(a^2-b^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(Cosh[x]^2*Sinh[x]^3)/(a*Cosh[x] + b*Sinh[x])^2,x]
```

```
[Out] (-2*a^4*b*ArcTan[(b*Cosh[x] + a*Sinh[x])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(7/2
) - (3*a^2*b^3*ArcTan[(b*Cosh[x] + a*Sinh[x])/Sqrt[a^2 - b^2]])/(a^2 - b^2)
^(7/2) - (4*a^2*b^2*Cosh[x])/(a^2 - b^2)^3 - (a^2*Cosh[x])/(a^2 - b^2)^2 +
(a^2*Cosh[x]^3)/(3*(a^2 - b^2)^2) + (b^2*Cosh[x]^3)/(3*(a^2 - b^2)^2) + (2*
a^3*b*Sinh[x])/(a^2 - b^2)^3 + (2*a*b^3*Sinh[x])/(a^2 - b^2)^3 - (2*a*b*Sin
h[x]^3)/(3*(a^2 - b^2)^2) - (a^3*b^2)/((a^2 - b^2)^3*(a*Cosh[x] + b*Sinh[x]
))
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3074

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> -Dist[d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d
*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3099

```
Int[sin[(c_.) + (d_.)*(x_)]^(m_)/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin
[(c_.) + (d_.)*(x_)]), x_Symbol] :> -Simp[(a*Sin[c + d*x]^(m - 1))/(d*(a^2
+ b^2)*(m - 1)), x] + (Dist[a^2/(a^2 + b^2), Int[Sin[c + d*x]^(m - 2)/(a*Co
s[c + d*x] + b*Sin[c + d*x]), x], x] + Dist[b/(a^2 + b^2), Int[Sin[c + d*x]
^(m - 1), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && GtQ[m,
```

```
1]
```

Rule 3109

```
Int[(cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.))/(cos[(c_.
) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[b
/(a^2 + b^2), Int[Cos[c + d*x]^m*Sin[c + d*x]^(n - 1), x], x] + (Dist[a/(a^
2 + b^2), Int[Cos[c + d*x]^(m - 1)*Sin[c + d*x]^n, x], x] - Dist[(a*b)/(a^2
 + b^2), Int[(Cos[c + d*x]^(m - 1)*Sin[c + d*x]^(n - 1))/(a*Cos[c + d*x] +
b*Sin[c + d*x]), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] &&
IGtQ[m, 0] && IGtQ[n, 0]
```

Rule 3111

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.)*(cos[(c_.)
+ (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol] :> Dis
t[b/(a^2 + b^2), Int[Cos[c + d*x]^m*Sin[c + d*x]^(n - 1)*(a*Cos[c + d*x] +
b*Sin[c + d*x])^(p + 1), x], x] + (Dist[a/(a^2 + b^2), Int[Cos[c + d*x]^(m
- 1)*Sin[c + d*x]^n*(a*Cos[c + d*x] + b*Sin[c + d*x])^(p + 1), x], x] - Dis
t[(a*b)/(a^2 + b^2), Int[Cos[c + d*x]^(m - 1)*Sin[c + d*x]^(n - 1)*(a*Cos[c
 + d*x] + b*Sin[c + d*x])^p, x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 +
 b^2, 0] && IGtQ[m, 0] && IGtQ[n, 0] && ILtQ[p, 0]
```

Rule 3154

```
Int[((A_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])/((a_.) + cos[(d_.) + (e_.)*(x_)
]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])^2, x_Symbol] :> -Simp[(b*C + (a*C
- c*A)*Cos[d + e*x] + b*A*Sin[d + e*x])/(e*(a^2 - b^2 - c^2)*(a + b*Cos[d +
 e*x] + c*Sin[d + e*x])), x] + Dist[(a*A - c*C)/(a^2 - b^2 - c^2), Int[1/(a
 + b*Cos[d + e*x] + c*Sin[d + e*x]), x], x] /; FreeQ[{a, b, c, d, e, A, C},
 x] && NeQ[a^2 - b^2 - c^2, 0] && NeQ[a*A - c*C, 0]
```

Rubi steps

$$\int \frac{\cosh^2(x)\sinh^3(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx = \frac{a\int \frac{\cosh(x)\sinh^3(x)}{a\cosh(x)+b\sinh(x)}\,dx}{a^2-b^2} - \frac{b\int \frac{\cosh^2(x)\sinh^2(x)}{a\cosh(x)+b\sinh(x)}\,dx}{a^2-b^2} + \frac{(ab)\int \frac{\cosh(x)\sinh^2(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx}{a^2-b^2}$$

$$= \frac{a^2\int \sinh^3(x)\,dx}{\left(a^2-b^2\right)^2} - 2\frac{(ab)\int \cosh(x)\sinh^2(x)\,dx}{\left(a^2-b^2\right)^2} + 2\frac{\left(a^2b\right)\int \frac{\sinh^2(x)}{a\cosh(x)+b\sinh(x)}\,dx}{\left(a^2-b^2\right)^2}$$

$$= -\frac{a^3b^2}{\left(a^2-b^2\right)^3(a\cosh(x)+b\sinh(x))} + 2\left(\frac{a^3b\sinh(x)}{\left(a^2-b^2\right)^3} - \frac{\left(a^4b\right)\int \frac{1}{a\cosh(x)+b\sinh(x)}}{\left(a^2-b^2\right)^3}\right.$$

$$= -\frac{a^2\cosh(x)}{\left(a^2-b^2\right)^2} + \frac{a^2\cosh^3(x)}{3\left(a^2-b^2\right)^2} + \frac{b^2\cosh^3(x)}{3\left(a^2-b^2\right)^2} - \frac{2ab\sinh^3(x)}{3\left(a^2-b^2\right)^2} - \frac{a^3b^2}{\left(a^2-b^2\right)^3(a\cosh(}$$

$$= -\frac{a^2b^3\tan^{-1}\left(\frac{b\cosh(x)+a\sinh(x)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{7/2}} - \frac{a^2\cosh(x)}{\left(a^2-b^2\right)^2} + \frac{a^2\cosh^3(x)}{3\left(a^2-b^2\right)^2} + \frac{b^2\cosh^3(x)}{3\left(a^2-b^2\right)^2} - \frac{2a}{3(}$$

Mathematica [A] time = 3.94, size = 474, normalized size = 1.82

$$\frac{1}{16}\left(\frac{32ab\left(a^2+b^2\right)\sinh(x)}{(a-b)^3(a+b)^3} - \frac{4\left(a^2+b^2\right)\cosh(x)}{(a-b)^2(a+b)^2} + \frac{4\left(a^2+b^2\right)\cosh(3x)}{3(a-b)^2(a+b)^2} - \frac{6b\left(3a^2+b^2\right)\tan^{-1}\left(\frac{a\tanh\left(\frac{x}{2}\right)+b}{\sqrt{a-b}\sqrt{a+b}}\right)}{(a-b)^{5/2}(a+b)^{5/2}} - \frac{}{(a}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cosh[x]^2*Sinh[x]^3)/(a*Cosh[x] + b*Sinh[x])^2,x]
```

```
[Out] ((-6*b*(3*a^2 + b^2)*ArcTan[(b + a*Tanh[x/2])/(Sqrt[a - b]*Sqrt[a + b])])/(
(a - b)^(5/2)*(a + b)^(5/2)) - (10*b*(5*a^4 + 10*a^2*b^2 + b^4)*ArcTan[(b +
 a*Tanh[x/2])/(Sqrt[a - b]*Sqrt[a + b])])/((a - b)^(7/2)*(a + b)^(7/2)) - (
4*(a^2 + b^2)*Cosh[x])/((a - b)^2*(a + b)^2) - (8*(a^4 + 6*a^2*b^2 + b^4)*C
osh[x])/((a - b)^3*(a + b)^3) + (4*(a^2 + b^2)*Cosh[3*x])/(3*(a - b)^2*(a +
 b)^2) + (8*a*b*Sinh[x])/((a - b)^2*(a + b)^2) + (32*a*b*(a^2 + b^2)*Sinh[x
])/((a - b)^3*(a + b)^3) - (a*(a^2 + 3*b^2))/((a - b)^2*(a + b)^2*(a*Cosh[x
] + b*Sinh[x])) - (a*(a^4 + 10*a^2*b^2 + 5*b^4))/((a - b)^3*(a + b)^3*(a*Co
sh[x] + b*Sinh[x])) + (2*(a*Sqrt[a - b]*(a + b) + 2*a*b*Sqrt[a + b]*ArcTan[
(b + a*Tanh[x/2])/(Sqrt[a - b]*Sqrt[a + b])]*Cosh[x] + 2*b^2*Sqrt[a + b]*Ar
cTan[(b + a*Tanh[x/2])/(Sqrt[a - b]*Sqrt[a + b])]*Sinh[x]))/((a - b)^(3/2)*
(a + b)^2*(a*Cosh[x] + b*Sinh[x])) - (8*a*b*Sinh[3*x])/(3*(a - b)^2*(a + b)
^2))/16
```

fricas [B] time = 0.63, size = 5061, normalized size = 19.39

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^2*sinh(x)^3/(a*cosh(x)+b*sinh(x))^2,x, algorithm="fricas"
)
```

```
[Out] [1/24*((a^7 - a^6*b - 3*a^5*b^2 + 3*a^4*b^3 + 3*a^3*b^4 - 3*a^2*b^5 - a*b^6
 + b^7)*cosh(x)^8 + 8*(a^7 - a^6*b - 3*a^5*b^2 + 3*a^4*b^3 + 3*a^3*b^4 - 3*
a^2*b^5 - a*b^6 + b^7)*cosh(x)*sinh(x)^7 + (a^7 - a^6*b - 3*a^5*b^2 + 3*a^4
*b^3 + 3*a^3*b^4 - 3*a^2*b^5 - a*b^6 + b^7)*sinh(x)^8 + a^7 + a^6*b - 3*a^5
*b^2 - 3*a^4*b^3 + 3*a^3*b^4 + 3*a^2*b^5 - a*b^6 - b^7 - 2*(4*a^7 - 9*a^6*b
 - 2*a^5*b^2 + 17*a^4*b^3 - 8*a^3*b^4 - 7*a^2*b^5 + 6*a*b^6 - b^7)*cosh(x)^
6 - 2*(4*a^7 - 9*a^6*b - 2*a^5*b^2 + 17*a^4*b^3 - 8*a^3*b^4 - 7*a^2*b^5 + 6
*a*b^6 - b^7 - 14*(a^7 - a^6*b - 3*a^5*b^2 + 3*a^4*b^3 + 3*a^3*b^4 - 3*a^2*
b^5 - a*b^6 + b^7)*cosh(x)^2)*sinh(x)^6 + 4*(14*(a^7 - a^6*b - 3*a^5*b^2 +
3*a^4*b^3 + 3*a^3*b^4 - 3*a^2*b^5 - a*b^6 + b^7)*cosh(x)^3 - 3*(4*a^7 - 9*a
^6*b - 2*a^5*b^2 + 17*a^4*b^3 - 8*a^3*b^4 - 7*a^2*b^5 + 6*a*b^6 - b^7)*cosh
(x))*sinh(x)^5 - 6*(3*a^7 + 27*a^5*b^2 - 23*a^3*b^4 - 7*a*b^6)*cosh(x)^4 -
2*(9*a^7 + 81*a^5*b^2 - 69*a^3*b^4 - 21*a*b^6 - 35*(a^7 - a^6*b - 3*a^5*b^2
 + 3*a^4*b^3 + 3*a^3*b^4 - 3*a^2*b^5 - a*b^6 + b^7)*cosh(x)^4 + 15*(4*a^7 -
 9*a^6*b - 2*a^5*b^2 + 17*a^4*b^3 - 8*a^3*b^4 - 7*a^2*b^5 + 6*a*b^6 - b^7)*
cosh(x)^2)*sinh(x)^4 + 8*(7*(a^7 - a^6*b - 3*a^5*b^2 + 3*a^4*b^3 + 3*a^3*b^
4 - 3*a^2*b^5 - a*b^6 + b^7)*cosh(x)^5 - 5*(4*a^7 - 9*a^6*b - 2*a^5*b^2 + 1
7*a^4*b^3 - 8*a^3*b^4 - 7*a^2*b^5 + 6*a*b^6 - b^7)*cosh(x)^3 - 3*(3*a^7 + 2
7*a^5*b^2 - 23*a^3*b^4 - 7*a*b^6)*cosh(x))*sinh(x)^3 - 2*(4*a^7 + 9*a^6*b -
 2*a^5*b^2 - 17*a^4*b^3 - 8*a^3*b^4 + 7*a^2*b^5 + 6*a*b^6 + b^7)*cosh(x)^2
- 2*(4*a^7 + 9*a^6*b - 2*a^5*b^2 - 17*a^4*b^3 - 8*a^3*b^4 + 7*a^2*b^5 + 6*a
*b^6 + b^7 - 14*(a^7 - a^6*b - 3*a^5*b^2 + 3*a^4*b^3 + 3*a^3*b^4 - 3*a^2*b^
5 - a*b^6 + b^7)*cosh(x)^6 + 15*(4*a^7 - 9*a^6*b - 2*a^5*b^2 + 17*a^4*b^3 -
 8*a^3*b^4 - 7*a^2*b^5 + 6*a*b^6 - b^7)*cosh(x)^4 + 18*(3*a^7 + 27*a^5*b^2
- 23*a^3*b^4 - 7*a*b^6)*cosh(x)^2)*sinh(x)^2 + 24*((2*a^5*b + 2*a^4*b^2 + 3
*a^3*b^3 + 3*a^2*b^4)*cosh(x)^5 + 5*(2*a^5*b + 2*a^4*b^2 + 3*a^3*b^3 + 3*a^
2*b^4)*cosh(x)*sinh(x)^4 + (2*a^5*b + 2*a^4*b^2 + 3*a^3*b^3 + 3*a^2*b^4)*si
nh(x)^5 + (2*a^5*b - 2*a^4*b^2 + 3*a^3*b^3 - 3*a^2*b^4)*cosh(x)^3 + (2*a^5*
b - 2*a^4*b^2 + 3*a^3*b^3 - 3*a^2*b^4 + 10*(2*a^5*b + 2*a^4*b^2 + 3*a^3*b^3
 + 3*a^2*b^4)*cosh(x)^2)*sinh(x)^3 + (10*(2*a^5*b + 2*a^4*b^2 + 3*a^3*b^3 +
 3*a^2*b^4)*cosh(x)^3 + 3*(2*a^5*b - 2*a^4*b^2 + 3*a^3*b^3 - 3*a^2*b^4)*cos
h(x))*sinh(x)^2 + (5*(2*a^5*b + 2*a^4*b^2 + 3*a^3*b^3 + 3*a^2*b^4)*cosh(x)^
4 + 3*(2*a^5*b - 2*a^4*b^2 + 3*a^3*b^3 - 3*a^2*b^4)*cosh(x)^2)*sinh(x))*sqr
t(-a^2 + b^2)*log(((a + b)*cosh(x)^2 + 2*(a + b)*cosh(x)*sinh(x) + (a + b)*
sinh(x)^2 - 2*sqrt(-a^2 + b^2)*(cosh(x) + sinh(x)) - a + b)/((a + b)*cosh(x
)^2 + 2*(a + b)*cosh(x)*sinh(x) + (a + b)*sinh(x)^2 + a - b)) + 4*(2*(a^7 -
```

a^6*b - 3*a^5*b^2 + 3*a^4*b^3 + 3*a^3*b^4 - 3*a^2*b^5 - a*b^6 + b^7)*cosh(
x)^7 - 3*(4*a^7 - 9*a^6*b - 2*a^5*b^2 + 17*a^4*b^3 - 8*a^3*b^4 - 7*a^2*b^5
+ 6*a*b^6 - b^7)*cosh(x)^5 - 6*(3*a^7 + 27*a^5*b^2 - 23*a^3*b^4 - 7*a*b^6)*
cosh(x)^3 - (4*a^7 + 9*a^6*b - 2*a^5*b^2 - 17*a^4*b^3 - 8*a^3*b^4 + 7*a^2*b
^5 + 6*a*b^6 + b^7)*cosh(x))*sinh(x))/((a^9 + a^8*b - 4*a^7*b^2 - 4*a^6*b^3
+ 6*a^5*b^4 + 6*a^4*b^5 - 4*a^3*b^6 - 4*a^2*b^7 + a*b^8 + b^9)*cosh(x)^5 +
5*(a^9 + a^8*b - 4*a^7*b^2 - 4*a^6*b^3 + 6*a^5*b^4 + 6*a^4*b^5 - 4*a^3*b^6
- 4*a^2*b^7 + a*b^8 + b^9)*cosh(x)*sinh(x)^4 + (a^9 + a^8*b - 4*a^7*b^2 -
4*a^6*b^3 + 6*a^5*b^4 + 6*a^4*b^5 - 4*a^3*b^6 - 4*a^2*b^7 + a*b^8 + b^9)*si
nh(x)^5 + (a^9 - a^8*b - 4*a^7*b^2 + 4*a^6*b^3 + 6*a^5*b^4 - 6*a^4*b^5 - 4*
a^3*b^6 + 4*a^2*b^7 + a*b^8 - b^9)*cosh(x)^3 + (a^9 - a^8*b - 4*a^7*b^2 + 4
*a^6*b^3 + 6*a^5*b^4 - 6*a^4*b^5 - 4*a^3*b^6 + 4*a^2*b^7 + a*b^8 - b^9 + 10
*(a^9 + a^8*b - 4*a^7*b^2 - 4*a^6*b^3 + 6*a^5*b^4 + 6*a^4*b^5 - 4*a^3*b^6 -
4*a^2*b^7 + a*b^8 + b^9)*cosh(x)^2)*sinh(x)^3 + (10*(a^9 + a^8*b - 4*a^7*b
^2 - 4*a^6*b^3 + 6*a^5*b^4 + 6*a^4*b^5 - 4*a^3*b^6 - 4*a^2*b^7 + a*b^8 + b^
9)*cosh(x)^3 + 3*(a^9 - a^8*b - 4*a^7*b^2 + 4*a^6*b^3 + 6*a^5*b^4 - 6*a^4*b
^5 - 4*a^3*b^6 + 4*a^2*b^7 + a*b^8 - b^9)*cosh(x))*sinh(x)^2 + (5*(a^9 + a^
8*b - 4*a^7*b^2 - 4*a^6*b^3 + 6*a^5*b^4 + 6*a^4*b^5 - 4*a^3*b^6 - 4*a^2*b^7
+ a*b^8 + b^9)*cosh(x)^4 + 3*(a^9 - a^8*b - 4*a^7*b^2 + 4*a^6*b^3 + 6*a^5*
b^4 - 6*a^4*b^5 - 4*a^3*b^6 + 4*a^2*b^7 + a*b^8 - b^9)*cosh(x)^2)*sinh(x)),
1/24*((a^7 - a^6*b - 3*a^5*b^2 + 3*a^4*b^3 + 3*a^3*b^4 - 3*a^2*b^5 - a*b^6
+ b^7)*cosh(x)^8 + 8*(a^7 - a^6*b - 3*a^5*b^2 + 3*a^4*b^3 + 3*a^3*b^4 - 3*
a^2*b^5 - a*b^6 + b^7)*cosh(x)*sinh(x)^7 + (a^7 - a^6*b - 3*a^5*b^2 + 3*a^4
*b^3 + 3*a^3*b^4 - 3*a^2*b^5 - a*b^6 + b^7)*sinh(x)^8 + a^7 + a^6*b - 3*a^5
*b^2 - 3*a^4*b^3 + 3*a^3*b^4 + 3*a^2*b^5 - a*b^6 - b^7 - 2*(4*a^7 - 9*a^6*b
- 2*a^5*b^2 + 17*a^4*b^3 - 8*a^3*b^4 - 7*a^2*b^5 + 6*a*b^6 - b^7)*cosh(x)^
6 - 2*(4*a^7 - 9*a^6*b - 2*a^5*b^2 + 17*a^4*b^3 - 8*a^3*b^4 - 7*a^2*b^5 + 6
*a*b^6 - b^7 - 14*(a^7 - a^6*b - 3*a^5*b^2 + 3*a^4*b^3 + 3*a^3*b^4 - 3*a^2*
b^5 - a*b^6 + b^7)*cosh(x)^2)*sinh(x)^6 + 4*(14*(a^7 - a^6*b - 3*a^5*b^2 +
3*a^4*b^3 + 3*a^3*b^4 - 3*a^2*b^5 - a*b^6 + b^7)*cosh(x)^3 - 3*(4*a^7 - 9*a
^6*b - 2*a^5*b^2 + 17*a^4*b^3 - 8*a^3*b^4 - 7*a^2*b^5 + 6*a*b^6 - b^7)*cosh
(x))*sinh(x)^5 - 6*(3*a^7 + 27*a^5*b^2 - 23*a^3*b^4 - 7*a*b^6)*cosh(x)^4 -
2*(9*a^7 + 81*a^5*b^2 - 69*a^3*b^4 - 21*a*b^6 - 35*(a^7 - a^6*b - 3*a^5*b^2
+ 3*a^4*b^3 + 3*a^3*b^4 - 3*a^2*b^5 - a*b^6 + b^7)*cosh(x)^4 + 15*(4*a^7 -
9*a^6*b - 2*a^5*b^2 + 17*a^4*b^3 - 8*a^3*b^4 - 7*a^2*b^5 + 6*a*b^6 - b^7)*
cosh(x)^2)*sinh(x)^4 + 8*(7*(a^7 - a^6*b - 3*a^5*b^2 + 3*a^4*b^3 + 3*a^3*b^
4 - 3*a^2*b^5 - a*b^6 + b^7)*cosh(x)^5 - 5*(4*a^7 - 9*a^6*b - 2*a^5*b^2 + 1
7*a^4*b^3 - 8*a^3*b^4 - 7*a^2*b^5 + 6*a*b^6 - b^7)*cosh(x)^3 - 3*(3*a^7 + 2
7*a^5*b^2 - 23*a^3*b^4 - 7*a*b^6)*cosh(x))*sinh(x)^3 - 2*(4*a^7 + 9*a^6*b -
2*a^5*b^2 - 17*a^4*b^3 - 8*a^3*b^4 + 7*a^2*b^5 + 6*a*b^6 + b^7)*cosh(x)^2
- 2*(4*a^7 + 9*a^6*b - 2*a^5*b^2 - 17*a^4*b^3 - 8*a^3*b^4 + 7*a^2*b^5 + 6*a
b^6 + b^7 - 14(a^7 - a^6*b - 3*a^5*b^2 + 3*a^4*b^3 + 3*a^3*b^4 - 3*a^2*b^
5 - a*b^6 + b^7)*cosh(x)^6 + 15*(4*a^7 - 9*a^6*b - 2*a^5*b^2 + 17*a^4*b^3 -
8*a^3*b^4 - 7*a^2*b^5 + 6*a*b^6 - b^7)*cosh(x)^4 + 18*(3*a^7 + 27*a^5*b^2
- 23*a^3*b^4 - 7*a*b^6)*cosh(x)^2)*sinh(x)^2 + 48*((2*a^5*b + 2*a^4*b^2 + 3

```
*a^3*b^3 + 3*a^2*b^4)*cosh(x)^5 + 5*(2*a^5*b + 2*a^4*b^2 + 3*a^3*b^3 + 3*a^
2*b^4)*cosh(x)*sinh(x)^4 + (2*a^5*b + 2*a^4*b^2 + 3*a^3*b^3 + 3*a^2*b^4)*si
nh(x)^5 + (2*a^5*b - 2*a^4*b^2 + 3*a^3*b^3 - 3*a^2*b^4)*cosh(x)^3 + (2*a^5*
b - 2*a^4*b^2 + 3*a^3*b^3 - 3*a^2*b^4 + 10*(2*a^5*b + 2*a^4*b^2 + 3*a^3*b^3
 + 3*a^2*b^4)*cosh(x)^2)*sinh(x)^3 + (10*(2*a^5*b + 2*a^4*b^2 + 3*a^3*b^3 +
 3*a^2*b^4)*cosh(x)^3 + 3*(2*a^5*b - 2*a^4*b^2 + 3*a^3*b^3 - 3*a^2*b^4)*cos
h(x))*sinh(x)^2 + (5*(2*a^5*b + 2*a^4*b^2 + 3*a^3*b^3 + 3*a^2*b^4)*cosh(x)^
4 + 3*(2*a^5*b - 2*a^4*b^2 + 3*a^3*b^3 - 3*a^2*b^4)*cosh(x)^2)*sinh(x))*sqr
t(a^2 - b^2)*arctan(sqrt(a^2 - b^2)/((a + b)*cosh(x) + (a + b)*sinh(x))) +
4*(2*(a^7 - a^6*b - 3*a^5*b^2 + 3*a^4*b^3 + 3*a^3*b^4 - 3*a^2*b^5 - a*b^6 +
 b^7)*cosh(x)^7 - 3*(4*a^7 - 9*a^6*b - 2*a^5*b^2 + 17*a^4*b^3 - 8*a^3*b^4 -
 7*a^2*b^5 + 6*a*b^6 - b^7)*cosh(x)^5 - 6*(3*a^7 + 27*a^5*b^2 - 23*a^3*b^4
- 7*a*b^6)*cosh(x)^3 - (4*a^7 + 9*a^6*b - 2*a^5*b^2 - 17*a^4*b^3 - 8*a^3*b^
4 + 7*a^2*b^5 + 6*a*b^6 + b^7)*cosh(x))*sinh(x))/((a^9 + a^8*b - 4*a^7*b^2
- 4*a^6*b^3 + 6*a^5*b^4 + 6*a^4*b^5 - 4*a^3*b^6 - 4*a^2*b^7 + a*b^8 + b^9)*
cosh(x)^5 + 5*(a^9 + a^8*b - 4*a^7*b^2 - 4*a^6*b^3 + 6*a^5*b^4 + 6*a^4*b^5
- 4*a^3*b^6 - 4*a^2*b^7 + a*b^8 + b^9)*cosh(x)*sinh(x)^4 + (a^9 + a^8*b - 4
*a^7*b^2 - 4*a^6*b^3 + 6*a^5*b^4 + 6*a^4*b^5 - 4*a^3*b^6 - 4*a^2*b^7 + a*b^
8 + b^9)*sinh(x)^5 + (a^9 - a^8*b - 4*a^7*b^2 + 4*a^6*b^3 + 6*a^5*b^4 - 6*a
^4*b^5 - 4*a^3*b^6 + 4*a^2*b^7 + a*b^8 - b^9)*cosh(x)^3 + (a^9 - a^8*b - 4*
a^7*b^2 + 4*a^6*b^3 + 6*a^5*b^4 - 6*a^4*b^5 - 4*a^3*b^6 + 4*a^2*b^7 + a*b^8
 - b^9 + 10*(a^9 + a^8*b - 4*a^7*b^2 - 4*a^6*b^3 + 6*a^5*b^4 + 6*a^4*b^5 -
4*a^3*b^6 - 4*a^2*b^7 + a*b^8 + b^9)*cosh(x)^2)*sinh(x)^3 + (10*(a^9 + a^8*
b - 4*a^7*b^2 - 4*a^6*b^3 + 6*a^5*b^4 + 6*a^4*b^5 - 4*a^3*b^6 - 4*a^2*b^7 +
 a*b^8 + b^9)*cosh(x)^3 + 3*(a^9 - a^8*b - 4*a^7*b^2 + 4*a^6*b^3 + 6*a^5*b^
4 - 6*a^4*b^5 - 4*a^3*b^6 + 4*a^2*b^7 + a*b^8 - b^9)*cosh(x))*sinh(x)^2 + (
5*(a^9 + a^8*b - 4*a^7*b^2 - 4*a^6*b^3 + 6*a^5*b^4 + 6*a^4*b^5 - 4*a^3*b^6
- 4*a^2*b^7 + a*b^8 + b^9)*cosh(x)^4 + 3*(a^9 - a^8*b - 4*a^7*b^2 + 4*a^6*b
^3 + 6*a^5*b^4 - 6*a^4*b^5 - 4*a^3*b^6 + 4*a^2*b^7 + a*b^8 - b^9)*cosh(x)^2
)*sinh(x))]
```

giac [A] time = 0.15, size = 310, normalized size = 1.19

$$-\frac{2\,a^3b^2e^x}{\left(a^6-3\,a^4b^2+3\,a^2b^4-b^6\right)\left(ae^{(2\,x)}+be^{(2\,x)}+a-b\right)}-\frac{\left(9\,ae^{(2\,x)}+3\,be^{(2\,x)}-a+b\right)e^{(-3\,x)}}{24\left(a^3-3\,a^2b+3\,ab^2-b^3\right)}-\frac{2\left(2\,a^4b+3\,a^2b^3\right)\mathrm{arct}}{\left(a^6-3\,a^4b^2+3\,a^2b^4-\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^2*sinh(x)^3/(a*cosh(x)+b*sinh(x))^2,x, algorithm="giac")
```

```
[Out] -2*a^3*b^2*e^x/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*(a*e^(2*x) + b*e^(2*x)
+ a - b)) - 1/24*(9*a*e^(2*x) + 3*b*e^(2*x) - a + b)*e^(-3*x)/(a^3 - 3*a^2*
b + 3*a*b^2 - b^3) - 2*(2*a^4*b + 3*a^2*b^3)*arctan((a*e^x + b*e^x)/sqrt(a^
2 - b^2))/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*sqrt(a^2 - b^2)) + 1/24*(a^4
```

```
*e^(3*x) + 4*a^3*b*e^(3*x) + 6*a^2*b^2*e^(3*x) + 4*a*b^3*e^(3*x) + b^4*e^(3
*x) - 9*a^4*e^x - 24*a^3*b*e^x - 18*a^2*b^2*e^x + 3*b^4*e^x)/(a^6 + 6*a^5*b
 + 15*a^4*b^2 + 20*a^3*b^3 + 15*a^2*b^4 + 6*a*b^5 + b^6)
```

maple [A] time = 0.27, size = 326, normalized size = 1.25

$$-\frac{1}{3(a+b)^2\left(\tanh\left(\frac{x}{2}\right)-1\right)^3}-\frac{1}{2(a+b)^2\left(\tanh\left(\frac{x}{2}\right)-1\right)^2}+\frac{a}{2(a+b)^3\left(\tanh\left(\frac{x}{2}\right)-1\right)}-\frac{b}{2(a+b)^3\left(\tanh\left(\frac{x}{2}\right)-1\right)}-\frac{}{(a-}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^2*sinh(x)^3/(a*cosh(x)+b*sinh(x))^2,x)
```

```
[Out] -1/3/(a+b)^2/(tanh(1/2*x)-1)^3-1/2/(a+b)^2/(tanh(1/2*x)-1)^2+1/2/(a+b)^3/(t
anh(1/2*x)-1)*a-1/2/(a+b)^3/(tanh(1/2*x)-1)*b-2*a^2/(a-b)^3/(a+b)^3*b^3*tan
h(1/2*x)/(a+2*tanh(1/2*x)*b+a*tanh(1/2*x)^2)-2*a^3*b^2/(a-b)^3/(a+b)^3/(a+2
*tanh(1/2*x)*b+a*tanh(1/2*x)^2)-4*a^4*b/(a-b)^3/(a+b)^3/(a^2-b^2)^(1/2)*arc
tan(1/2*(2*a*tanh(1/2*x)+2*b)/(a^2-b^2)^(1/2))-6*a^2*b^3/(a-b)^3/(a+b)^3/(a
^2-b^2)^(1/2)*arctan(1/2*(2*a*tanh(1/2*x)+2*b)/(a^2-b^2)^(1/2))+1/3/(a-b)^2
/(tanh(1/2*x)+1)^3-1/2/(a-b)^2/(tanh(1/2*x)+1)^2-1/2/(a-b)^3/(tanh(1/2*x)+1
)*a-1/2/(a-b)^3/(tanh(1/2*x)+1)*b
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^2*sinh(x)^3/(a*cosh(x)+b*sinh(x))^2,x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [B] time = 1.95, size = 592, normalized size = 2.27

$$\frac{e^{3x}}{24(a+b)^2}+\frac{e^{-3x}}{24(a-b)^2}-\frac{2\,\mathrm{atan}\left(\frac{e^x\left(2a^4b\sqrt{a^{14}-7a^{12}b^2+21a^{10}b^4-35a^8b^6+35a^6b\right.}{a^7\sqrt{4a^8b^2+12a^6b^4+9a^4b^6}+b^7\sqrt{4a^8b^2+12a^6b^4+9a^4b^6}-3a^2b^5\sqrt{4a^8b^2+12a^6b^4+9a^4b^6}+3a^3b^4\sqrt{}}\right.}{\sqrt{a^{14}-}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cosh(x)^2*sinh(x)^3)/(a*cosh(x) + b*sinh(x))^2,x)
```

```
[Out] exp(3*x)/(24*(a + b)^2) + exp(-3*x)/(24*(a - b)^2) - (2*atan((exp(x)*(2*a^4
*b*(a^14 - b^14 + 7*a^2*b^12 - 21*a^4*b^10 + 35*a^6*b^8 - 35*a^8*b^6 + 21*a
^10*b^4 - 7*a^12*b^2)^(1/2) + 3*a^2*b^3*(a^14 - b^14 + 7*a^2*b^12 - 21*a^4*
b^10 + 35*a^6*b^8 - 35*a^8*b^6 + 21*a^10*b^4 - 7*a^12*b^2)^(1/2)))/(a^7*(9*
a^4*b^6 + 12*a^6*b^4 + 4*a^8*b^2)^(1/2) + b^7*(9*a^4*b^6 + 12*a^6*b^4 + 4*a
^8*b^2)^(1/2) - 3*a^2*b^5*(9*a^4*b^6 + 12*a^6*b^4 + 4*a^8*b^2)^(1/2) + 3*a^
3*b^4*(9*a^4*b^6 + 12*a^6*b^4 + 4*a^8*b^2)^(1/2) + 3*a^4*b^3*(9*a^4*b^6 + 1
2*a^6*b^4 + 4*a^8*b^2)^(1/2) - 3*a^5*b^2*(9*a^4*b^6 + 12*a^6*b^4 + 4*a^8*b^
2)^(1/2) - a*b^6*(9*a^4*b^6 + 12*a^6*b^4 + 4*a^8*b^2)^(1/2) - a^6*b*(9*a^4*
b^6 + 12*a^6*b^4 + 4*a^8*b^2)^(1/2)))*(9*a^4*b^6 + 12*a^6*b^4 + 4*a^8*b^2)^
(1/2))/(a^14 - b^14 + 7*a^2*b^12 - 21*a^4*b^10 + 35*a^6*b^8 - 35*a^8*b^6 +
21*a^10*b^4 - 7*a^12*b^2)^(1/2) - (exp(x)*(3*a - b))/(8*(a + b)^3) - (exp(-
x)*(3*a + b))/(8*(a - b)^3) - (2*a^3*b^2*exp(x))/((a + b)^3*(a - b)^3*(a -
b + exp(2*x)*(a + b)))
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)**2*sinh(x)**3/(a*cosh(x)+b*sinh(x))**2,x)
```

```
[Out] Timed out
```

3.721 $\int \frac{\cosh^3(x)\sinh(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx$

Optimal. Leaf size=215

$$\frac{abx\left(a^2+b^2\right)}{\left(a^2-b^2\right)^3}-\frac{abx}{\left(a^2-b^2\right)^2}+\frac{a^2\sinh^2(x)}{2\left(a^2-b^2\right)^2}+\frac{b^2\cosh^2(x)}{2\left(a^2-b^2\right)^2}+\frac{ab^2}{\left(a^2-b^2\right)^2(a+b\tanh(x))}-\frac{ab\sinh(x)\cosh(x)}{\left(a^2-b^2\right)^2}-\frac{3a^2b^2\log}{}$$

```
[Out] a^3*b*x/(a^2-b^2)^3+a*b^3*x/(a^2-b^2)^3-a*b*x/(a^2-b^2)^2+a*b*(a^2+b^2)*x/(
a^2-b^2)^3+1/2*b^2*cosh(x)^2/(a^2-b^2)^2-3*a^2*b^2*ln(a*cosh(x)+b*sinh(x))/
(a^2-b^2)^3-b^4*ln(a*cosh(x)+b*sinh(x))/(a^2-b^2)^3-a*b*cosh(x)*sinh(x)/(a^
2-b^2)^2+1/2*a^2*sinh(x)^2/(a^2-b^2)^2+a*b^2/(a^2-b^2)^2/(a+b*tanh(x))
```

Rubi [A] time = 0.56, antiderivative size = 215, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 13, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.722, Rules used = {3111, 3100, 2635, 8, 3098, 3133, 3109, 2564, 30, 3086, 3483, 3531, 3530}

$$\frac{ab^3x}{\left(a^2-b^2\right)^3}+\frac{abx\left(a^2+b^2\right)}{\left(a^2-b^2\right)^3}-\frac{abx}{\left(a^2-b^2\right)^2}+\frac{a^3bx}{\left(a^2-b^2\right)^3}+\frac{a^2\sinh^2(x)}{2\left(a^2-b^2\right)^2}+\frac{b^2\cosh^2(x)}{2\left(a^2-b^2\right)^2}+\frac{ab^2}{\left(a^2-b^2\right)^2(a+b\tanh(x))}-\frac{ab\sin}{(a}$$

Antiderivative was successfully verified.

```
[In] Int[(Cosh[x]^3*Sinh[x])/(a*Cosh[x] + b*Sinh[x])^2,x]
```

```
[Out] (a^3*b*x)/(a^2 - b^2)^3 + (a*b^3*x)/(a^2 - b^2)^3 - (a*b*x)/(a^2 - b^2)^2 +
 (a*b*(a^2 + b^2)*x)/(a^2 - b^2)^3 + (b^2*Cosh[x]^2)/(2*(a^2 - b^2)^2) - (3
*a^2*b^2*Log[a*Cosh[x] + b*Sinh[x]])/(a^2 - b^2)^3 - (b^4*Log[a*Cosh[x] + b
*Sinh[x]])/(a^2 - b^2)^3 - (a*b*Cosh[x]*Sinh[x])/(a^2 - b^2)^2 + (a^2*Sinh[
x]^2)/(2*(a^2 - b^2)^2) + (a*b^2)/((a^2 - b^2)^2*(a + b*Tanh[x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
```

```
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 3086

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin
[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[(a + b*Tan[c + d*x])^n, x] /;
 FreeQ[{a, b, c, d}, x] && EqQ[m + n, 0] && IntegerQ[n] && NeQ[a^2 + b^2, 0
]
```

Rule 3098

```
Int[cos[(c_.) + (d_.)*(x_)]/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.
) + (d_.)*(x_)]), x_Symbol] :> Simp[(a*x)/(a^2 + b^2), x] + Dist[b/(a^2 + b
^2), Int[(b*Cos[c + d*x] - a*Sin[c + d*x])/(a*Cos[c + d*x] + b*Sin[c + d*x]
), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3100

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_)/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin
[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[(b*Cos[c + d*x]^(m - 1))/(d*(a^2 +
 b^2)*(m - 1)), x] + (Dist[a/(a^2 + b^2), Int[Cos[c + d*x]^(m - 1), x], x]
+ Dist[b^2/(a^2 + b^2), Int[Cos[c + d*x]^(m - 2)/(a*Cos[c + d*x] + b*Sin[c
+ d*x]), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && GtQ[m, 1
]
```

Rule 3109

```
Int[(cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.))/(cos[(c_.
) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[b
/(a^2 + b^2), Int[Cos[c + d*x]^m*Sin[c + d*x]^(n - 1), x], x] + (Dist[a/(a^
2 + b^2), Int[Cos[c + d*x]^(m - 1)*Sin[c + d*x]^n, x], x] - Dist[(a*b)/(a^2
 + b^2), Int[(Cos[c + d*x]^(m - 1)*Sin[c + d*x]^(n - 1))/(a*Cos[c + d*x] +
b*Sin[c + d*x]), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] &&
IGtQ[m, 0] && IGtQ[n, 0]
```

Rule 3111

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.)*(cos[(c_.)
+ (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol] :> Dis
t[b/(a^2 + b^2), Int[Cos[c + d*x]^m*Sin[c + d*x]^(n - 1)*(a*Cos[c + d*x] +
b*Sin[c + d*x])^(p + 1), x], x] + (Dist[a/(a^2 + b^2), Int[Cos[c + d*x]^(m
- 1)*Sin[c + d*x]^n*(a*Cos[c + d*x] + b*Sin[c + d*x])^(p + 1), x], x] - Dis
t[(a*b)/(a^2 + b^2), Int[Cos[c + d*x]^(m - 1)*Sin[c + d*x]^(n - 1)*(a*Cos[c
 + d*x] + b*Sin[c + d*x])^p, x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 +
 b^2, 0] && IGtQ[m, 0] && IGtQ[n, 0] && ILtQ[p, 0]
```

Rule 3133

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)]), x
_Symbol] :> Simp[((b*B + c*C)*x)/(b^2 + c^2), x] + Simp[((c*B - b*C)*Log[a
+ b*Cos[d + e*x] + c*Sin[d + e*x]])/(e*(b^2 + c^2)), x] /; FreeQ[{a, b, c,
d, e, A, B, C}, x] && NeQ[b^2 + c^2, 0] && EqQ[A*(b^2 + c^2) - a*(b*B + c*C
), 0]
```

Rule 3483

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(a +
b*Tan[c + d*x])^(n + 1))/(d*(n + 1)*(a^2 + b^2)), x] + Dist[1/(a^2 + b^2),
Int[(a - b*Tan[c + d*x])*(a + b*Tan[c + d*x])^(n + 1), x], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 + b^2, 0] && LtQ[n, -1]
```

Rule 3530

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c*Log[RemoveContent[a*Cos[e + f*x] + b*Sin[e + f
*x], x]])/(b*f), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

Rule 3531

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[((a*c + b*d)*x)/(a^2 + b^2), x] + Dist[(b*c - a
*d)/(a^2 + b^2), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && Ne
Q[a*c + b*d, 0]
```

Rubi steps

$$\int \frac{\cosh^3(x)\sinh(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx = \frac{a\int \frac{\cosh^2(x)\sinh(x)}{a\cosh(x)+b\sinh(x)}\,dx}{a^2-b^2} - \frac{b\int \frac{\cosh^3(x)}{a\cosh(x)+b\sinh(x)}\,dx}{a^2-b^2} + \frac{(ab)\int \frac{\cosh^2(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx}{a^2-b^2}$$

$$= \frac{b^2\cosh^2(x)}{2\left(a^2-b^2\right)^2} + \frac{a^2\int \cosh(x)\sinh(x)\,dx}{\left(a^2-b^2\right)^2} - 2\frac{(ab)\int \cosh^2(x)\,dx}{\left(a^2-b^2\right)^2} + \frac{\left(a^2b\right)\int \frac{}{a\cosh}}{\left(a^2-\right.}$$

$$= \frac{a^3bx}{\left(a^2-b^2\right)^3} + \frac{ab^3x}{\left(a^2-b^2\right)^3} + \frac{b^2\cosh^2(x)}{2\left(a^2-b^2\right)^2} + \frac{ab^2}{\left(a^2-b^2\right)^2(a+b\tanh(x))} - \frac{\left(ia^2b^2\right)}{}$$

$$= \frac{a^3bx}{\left(a^2-b^2\right)^3} + \frac{ab^3x}{\left(a^2-b^2\right)^3} + \frac{ab\left(a^2+b^2\right)x}{\left(a^2-b^2\right)^3} + \frac{b^2\cosh^2(x)}{2\left(a^2-b^2\right)^2} - \frac{a^2b^2\log(a\cosh(x)}{\left(a^2-b^2\right)}$$

$$= \frac{a^3bx}{\left(a^2-b^2\right)^3} + \frac{ab^3x}{\left(a^2-b^2\right)^3} + \frac{ab\left(a^2+b^2\right)x}{\left(a^2-b^2\right)^3} + \frac{b^2\cosh^2(x)}{2\left(a^2-b^2\right)^2} - \frac{3a^2b^2\log(a\cosh(x)}{\left(a^2-b^2\right.}$$

Mathematica [A] time = 1.32, size = 183, normalized size = 0.85

$$\frac{a\cosh(x)\left(\left(a^4-b^4\right)\cosh(2x)-4b\left(-ax\left(a^2+3b^2\right)+a\left(a^2-b^2\right)\sinh(x)\cosh(x)+b\left(3a^2+b^2\right)\log(a\cosh(x)+\right.\right.}{4(a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cosh[x]^3*Sinh[x])/(a*Cosh[x] + b*Sinh[x])^2,x]
```

```
[Out] (a*Cosh[x]*((a^4 - b^4)*Cosh[2*x] - 4*b*(-(a*(a^2 + 3*b^2)*x) + b*(3*a^2 +
b^2)*Log[a*Cosh[x] + b*Sinh[x]] + a*(a^2 - b^2)*Cosh[x]*Sinh[x])) + b*Sinh[
x]*((a^4 - b^4)*Cosh[2*x] + 4*b*(-(a^2*b) + b^3 + a^3*x + 3*a*b^2*x - b*(3*
a^2 + b^2)*Log[a*Cosh[x] + b*Sinh[x]]) - 2*a*b*(a^2 - b^2)*Sinh[2*x]))/(4*(
a - b)^3*(a + b)^3*(a*Cosh[x] + b*Sinh[x]))
```

fricas [B] time = 0.52, size = 1661, normalized size = 7.73

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^3*sinh(x)/(a*cosh(x)+b*sinh(x))^2,x, algorithm="fricas")
```

```
[Out] 1/8*((a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^6 + 6*(a^5
 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)*sinh(x)^5 + (a^5 -
```

```
a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*sinh(x)^6 + a^5 + a^4*b - 2*a^
3*b^2 - 2*a^2*b^3 + a*b^4 + b^5 + (a^5 - 3*a^4*b + 2*a^3*b^2 + 2*a^2*b^3 -
3*a*b^4 + b^5 + 8*(a^4*b + 4*a^3*b^2 + 6*a^2*b^3 + 4*a*b^4 + b^5)*x)*cosh(x
)^4 + (a^5 - 3*a^4*b + 2*a^3*b^2 + 2*a^2*b^3 - 3*a*b^4 + b^5 + 15*(a^5 - a^
4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^2 + 8*(a^4*b + 4*a^3*b^2
 + 6*a^2*b^3 + 4*a*b^4 + b^5)*x)*sinh(x)^4 + 4*(5*(a^5 - a^4*b - 2*a^3*b^2
+ 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^3 + (a^5 - 3*a^4*b + 2*a^3*b^2 + 2*a^2*b
^3 - 3*a*b^4 + b^5 + 8*(a^4*b + 4*a^3*b^2 + 6*a^2*b^3 + 4*a*b^4 + b^5)*x)*c
osh(x))*sinh(x)^3 + (a^5 + 3*a^4*b + 2*a^3*b^2 + 14*a^2*b^3 - 19*a*b^4 - b^
5 + 8*(a^4*b + 2*a^3*b^2 - 2*a*b^4 - b^5)*x)*cosh(x)^2 + (a^5 + 3*a^4*b + 2
*a^3*b^2 + 14*a^2*b^3 - 19*a*b^4 - b^5 + 15*(a^5 - a^4*b - 2*a^3*b^2 + 2*a^
2*b^3 + a*b^4 - b^5)*cosh(x)^4 + 6*(a^5 - 3*a^4*b + 2*a^3*b^2 + 2*a^2*b^3 -
 3*a*b^4 + b^5 + 8*(a^4*b + 4*a^3*b^2 + 6*a^2*b^3 + 4*a*b^4 + b^5)*x)*cosh(
x)^2 + 8*(a^4*b + 2*a^3*b^2 - 2*a*b^4 - b^5)*x)*sinh(x)^2 - 8*((3*a^3*b^2 +
 3*a^2*b^3 + a*b^4 + b^5)*cosh(x)^4 + 4*(3*a^3*b^2 + 3*a^2*b^3 + a*b^4 + b^
5)*cosh(x)*sinh(x)^3 + (3*a^3*b^2 + 3*a^2*b^3 + a*b^4 + b^5)*sinh(x)^4 + (3
*a^3*b^2 - 3*a^2*b^3 + a*b^4 - b^5)*cosh(x)^2 + (3*a^3*b^2 - 3*a^2*b^3 + a*
b^4 - b^5 + 6*(3*a^3*b^2 + 3*a^2*b^3 + a*b^4 + b^5)*cosh(x)^2)*sinh(x)^2 +
2*(2*(3*a^3*b^2 + 3*a^2*b^3 + a*b^4 + b^5)*cosh(x)^3 + (3*a^3*b^2 - 3*a^2*b
^3 + a*b^4 - b^5)*cosh(x))*sinh(x))*log(2*(a*cosh(x) + b*sinh(x))/(cosh(x)
- sinh(x))) + 2*(3*(a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh
(x)^5 + 2*(a^5 - 3*a^4*b + 2*a^3*b^2 + 2*a^2*b^3 - 3*a*b^4 + b^5 + 8*(a^4*b
 + 4*a^3*b^2 + 6*a^2*b^3 + 4*a*b^4 + b^5)*x)*cosh(x)^3 + (a^5 + 3*a^4*b + 2
*a^3*b^2 + 14*a^2*b^3 - 19*a*b^4 - b^5 + 8*(a^4*b + 2*a^3*b^2 - 2*a*b^4 - b
^5)*x)*cosh(x))*sinh(x))/((a^7 + a^6*b - 3*a^5*b^2 - 3*a^4*b^3 + 3*a^3*b^4
+ 3*a^2*b^5 - a*b^6 - b^7)*cosh(x)^4 + 4*(a^7 + a^6*b - 3*a^5*b^2 - 3*a^4*b
^3 + 3*a^3*b^4 + 3*a^2*b^5 - a*b^6 - b^7)*cosh(x)*sinh(x)^3 + (a^7 + a^6*b
- 3*a^5*b^2 - 3*a^4*b^3 + 3*a^3*b^4 + 3*a^2*b^5 - a*b^6 - b^7)*sinh(x)^4 +
(a^7 - a^6*b - 3*a^5*b^2 + 3*a^4*b^3 + 3*a^3*b^4 - 3*a^2*b^5 - a*b^6 + b^7)
*cosh(x)^2 + (a^7 - a^6*b - 3*a^5*b^2 + 3*a^4*b^3 + 3*a^3*b^4 - 3*a^2*b^5 -
 a*b^6 + b^7 + 6*(a^7 + a^6*b - 3*a^5*b^2 - 3*a^4*b^3 + 3*a^3*b^4 + 3*a^2*b
^5 - a*b^6 - b^7)*cosh(x)^2)*sinh(x)^2 + 2*(2*(a^7 + a^6*b - 3*a^5*b^2 - 3*
a^4*b^3 + 3*a^3*b^4 + 3*a^2*b^5 - a*b^6 - b^7)*cosh(x)^3 + (a^7 - a^6*b - 3
*a^5*b^2 + 3*a^4*b^3 + 3*a^3*b^4 - 3*a^2*b^5 - a*b^6 + b^7)*cosh(x))*sinh(x
))
```

giac [A] time = 0.13, size = 240, normalized size = 1.12

$$\frac{bx}{a^3-3\,a^2b+3\,ab^2-b^3}-\frac{\left(3\,a^2b^2+b^4\right)\log\left(\left|ae^{(2\,x)}+be^{(2\,x)}+a-b\right|\right)}{a^6-3\,a^4b^2+3\,a^2b^4-b^6}+\frac{e^{(2\,x)}}{8\left(a^2+2\,ab+b^2\right)}-\frac{2\,a^2be^{(4\,x)}-4\,ab^2e^{(4\,x)}+}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^3*sinh(x)/(a*cosh(x)+b*sinh(x))^2,x, algorithm="giac")
```

```
[Out] b*x/(a^3 - 3*a^2*b + 3*a*b^2 - b^3) - (3*a^2*b^2 + b^4)*log(abs(a*e^(2*x) +
 b*e^(2*x) + a - b))/(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6) + 1/8*e^(2*x)/(a^2
 + 2*a*b + b^2) - 1/8*(2*a^2*b*e^(4*x) - 4*a*b^2*e^(4*x) + 2*b^3*e^(4*x) -
a^3*e^(2*x) - a^2*b*e^(2*x) - 11*a*b^2*e^(2*x) - 3*b^3*e^(2*x) - a^3 - a^2*
b + a*b^2 + b^3)/((a^4 - 2*a^2*b^2 + b^4)*(a*e^(4*x) + b*e^(4*x) + a*e^(2*x
) - b*e^(2*x)))
```

maple [A] time = 0.26, size = 253, normalized size = 1.18

$$\frac{1}{2\left(a+b\right)^{2}\left(\tanh\left(\frac{x}{2}\right)-1\right)^{2}}+\frac{1}{2\left(a+b\right)^{2}\left(\tanh\left(\frac{x}{2}\right)-1\right)}-\frac{\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)b}{\left(a+b\right)^{3}}-\frac{2a^{2}b^{3}\tanh\left(\frac{x}{2}\right)}{\left(a-b\right)^{3}\left(a+b\right)^{3}\left(a+2\tanh\left(\frac{x}{2}\right)b+a\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^3*sinh(x)/(a*cosh(x)+b*sinh(x))^2,x)
```

```
[Out] 1/2/(a+b)^2/(tanh(1/2*x)-1)^2+1/2/(a+b)^2/(tanh(1/2*x)-1)-1/(a+b)^3*ln(tanh
(1/2*x)-1)*b-2*a^2/(a-b)^3/(a+b)^3*b^3*tanh(1/2*x)/(a+2*tanh(1/2*x)*b+a*tan
h(1/2*x)^2)+2*b^5/(a-b)^3/(a+b)^3*tanh(1/2*x)/(a+2*tanh(1/2*x)*b+a*tanh(1/2
*x)^2)-3*a^2/(a-b)^3/(a+b)^3*ln(a+2*tanh(1/2*x)*b+a*tanh(1/2*x)^2)*b^2-b^4/
(a-b)^3/(a+b)^3*ln(a+2*tanh(1/2*x)*b+a*tanh(1/2*x)^2)+1/2/(a-b)^2/(tanh(1/2
*x)+1)^2-1/2/(a-b)^2/(tanh(1/2*x)+1)+1/(a-b)^3*ln(tanh(1/2*x)+1)*b
```

maxima [A] time = 0.46, size = 240, normalized size = 1.12

$$\frac{bx}{a^{3}+3\,a^{2}b+3\,ab^{2}+b^{3}}-\frac{\left(3\,a^{2}b^{2}+b^{4}\right)\log\left(-(a-b)e^{(-2\,x)}-a-b\right)}{a^{6}-3\,a^{4}b^{2}+3\,a^{2}b^{4}-b^{6}}+\frac{a^{4}-2\,a^{3}b+2\,ab^{3}-b^{4}+\left(a^{4}-\right.}{8\left(\left(a^{6}-3\,a^{4}b^{2}+3\,a^{2}b^{4}-b^{6}\right)e^{(-2\,x)}+\left(a^{6}-\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^3*sinh(x)/(a*cosh(x)+b*sinh(x))^2,x, algorithm="maxima")
```

```
[Out] b*x/(a^3 + 3*a^2*b + 3*a*b^2 + b^3) - (3*a^2*b^2 + b^4)*log(-(a - b)*e^(-2*
x) - a - b)/(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6) + 1/8*(a^4 - 2*a^3*b + 2*a*
b^3 - b^4 + (a^4 - 4*a^3*b + 6*a^2*b^2 - 20*a*b^3 + b^4)*e^(-2*x))/((a^6 -
3*a^4*b^2 + 3*a^2*b^4 - b^6)*e^(-2*x) + (a^6 - 2*a^5*b - a^4*b^2 + 4*a^3*b^
3 - a^2*b^4 - 2*a*b^5 + b^6)*e^(-4*x)) + 1/8*e^(-2*x)/(a^2 - 2*a*b + b^2)
```

mupad [B] time = 1.83, size = 127, normalized size = 0.59

$$\frac{e^{2\,x}}{8\left(a+b\right)^{2}}+\frac{e^{-2\,x}}{8\left(a-b\right)^{2}}+\frac{b\,x}{\left(a-b\right)^{3}}-\frac{\ln\left(a-b+a\,e^{2\,x}+b\,e^{2\,x}\right)\left(3\,a^{2}\,b^{2}+b^{4}\right)}{a^{6}-3\,a^{4}\,b^{2}+3\,a^{2}\,b^{4}-b^{6}}+\frac{2\,a\,b^{3}}{\left(a+b\right)^{3}\left(a-b\right)^{2}\left(a-b+e^{2\,x}\,(a+b)\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cosh(x)^3*sinh(x))/(a*cosh(x) + b*sinh(x))^2,x)
```

```
[Out] exp(2*x)/(8*(a + b)^2) + exp(-2*x)/(8*(a - b)^2) + (b*x)/(a - b)^3 - (log(a
 - b + a*exp(2*x) + b*exp(2*x))*(b^4 + 3*a^2*b^2))/(a^6 - b^6 + 3*a^2*b^4 -
 3*a^4*b^2) + (2*a*b^3)/((a + b)^3*(a - b)^2*(a - b + exp(2*x)*(a + b)))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)**3*sinh(x)/(a*cosh(x)+b*sinh(x))**2,x)
```

```
[Out] Timed out
```

3.722 $\int \frac{\cosh^3(x)\sinh^2(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx$

Optimal. Leaf size=259

$$\frac{b^2\sinh^3(x)}{3\left(a^2-b^2\right)^2}+\frac{a^2\sinh^3(x)}{3\left(a^2-b^2\right)^2}+\frac{b^2\sinh(x)}{\left(a^2-b^2\right)^2}-\frac{4a^2b^2\sinh(x)}{\left(a^2-b^2\right)^3}-\frac{2ab\cosh^3(x)}{3\left(a^2-b^2\right)^2}+\frac{2ab^4\tan^{-1}\left(\frac{a\sinh(x)+b\cosh(x)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{7/2}}+\frac{2ab^3\cosh}{\left(a^2-b^2\right.}$$

```
[Out] 3*a^3*b^2*arctan((b*cosh(x)+a*sinh(x))/(a^2-b^2)^(1/2))/(a^2-b^2)^(7/2)+2*a
*b^4*arctan((b*cosh(x)+a*sinh(x))/(a^2-b^2)^(1/2))/(a^2-b^2)^(7/2)+2*a^3*b*
cosh(x)/(a^2-b^2)^3+2*a*b^3*cosh(x)/(a^2-b^2)^3-2/3*a*b*cosh(x)^3/(a^2-b^2)
^2-4*a^2*b^2*sinh(x)/(a^2-b^2)^3+b^2*sinh(x)/(a^2-b^2)^2+1/3*a^2*sinh(x)^3/
(a^2-b^2)^2+1/3*b^2*sinh(x)^3/(a^2-b^2)^2+a^2*b^3/(a^2-b^2)^3/(a*cosh(x)+b*
sinh(x))
```

Rubi [A] time = 0.91, antiderivative size = 259, normalized size of antiderivative = 1.00, number of steps used = 33, number of rules used = 12, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {3111, 3109, 2633, 2565, 30, 3100, 2637, 3074, 206, 2564, 2638, 3155}

$$\frac{b^2\sinh^3(x)}{3\left(a^2-b^2\right)^2}+\frac{a^2\sinh^3(x)}{3\left(a^2-b^2\right)^2}+\frac{b^2\sinh(x)}{\left(a^2-b^2\right)^2}-\frac{4a^2b^2\sinh(x)}{\left(a^2-b^2\right)^3}-\frac{2ab\cosh^3(x)}{3\left(a^2-b^2\right)^2}+\frac{2ab^3\cosh(x)}{\left(a^2-b^2\right)^3}+\frac{2a^3b\cosh(x)}{\left(a^2-b^2\right)^3}+\frac{}{\left(a^2-b^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(Cosh[x]^3*Sinh[x]^2)/(a*Cosh[x] + b*Sinh[x])^2,x]
```

```
[Out] (3*a^3*b^2*ArcTan[(b*Cosh[x] + a*Sinh[x])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(7/
2) + (2*a*b^4*ArcTan[(b*Cosh[x] + a*Sinh[x])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^
(7/2) + (2*a^3*b*Cosh[x])/(a^2 - b^2)^3 + (2*a*b^3*Cosh[x])/(a^2 - b^2)^3 -
 (2*a*b*Cosh[x]^3)/(3*(a^2 - b^2)^2) - (4*a^2*b^2*Sinh[x])/(a^2 - b^2)^3 +
(b^2*Sinh[x])/(a^2 - b^2)^2 + (a^2*Sinh[x]^3)/(3*(a^2 - b^2)^2) + (b^2*Sinh
[x]^3)/(3*(a^2 - b^2)^2) + (a^2*b^3)/((a^2 - b^2)^3*(a*Cosh[x] + b*Sinh[x])
)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3074

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> -Dist[d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d
*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3100

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_)/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin
[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[(b*Cos[c + d*x]^(m - 1))/(d*(a^2 +
 b^2)*(m - 1)), x] + (Dist[a/(a^2 + b^2), Int[Cos[c + d*x]^(m - 1), x], x]
+ Dist[b^2/(a^2 + b^2), Int[Cos[c + d*x]^(m - 2)/(a*Cos[c + d*x] + b*Sin[c
+ d*x]), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && GtQ[m, 1
```

]

Rule 3109

```
Int[(cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.))/(cos[(c_.
) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[b
/(a^2 + b^2), Int[Cos[c + d*x]^m*Sin[c + d*x]^(n - 1), x], x] + (Dist[a/(a^
2 + b^2), Int[Cos[c + d*x]^(m - 1)*Sin[c + d*x]^n, x], x] - Dist[(a*b)/(a^2
 + b^2), Int[(Cos[c + d*x]^(m - 1)*Sin[c + d*x]^(n - 1))/(a*Cos[c + d*x] +
b*Sin[c + d*x]), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] &&
IGtQ[m, 0] && IGtQ[n, 0]
```

Rule 3111

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.)*(cos[(c_.)
+ (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol] :> Dis
t[b/(a^2 + b^2), Int[Cos[c + d*x]^m*Sin[c + d*x]^(n - 1)*(a*Cos[c + d*x] +
b*Sin[c + d*x])^(p + 1), x], x] + (Dist[a/(a^2 + b^2), Int[Cos[c + d*x]^(m
- 1)*Sin[c + d*x]^n*(a*Cos[c + d*x] + b*Sin[c + d*x])^(p + 1), x], x] - Dis
t[(a*b)/(a^2 + b^2), Int[Cos[c + d*x]^(m - 1)*Sin[c + d*x]^(n - 1)*(a*Cos[c
 + d*x] + b*Sin[c + d*x])^p, x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 +
 b^2, 0] && IGtQ[m, 0] && IGtQ[n, 0] && ILtQ[p, 0]
```

Rule 3155

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.))/((a_.) + cos[(d_.) + (e_.)*(x_)
]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])^2, x_Symbol] :> Simp[(c*B + c*A*Co
s[d + e*x] + (a*B - b*A)*Sin[d + e*x])/(e*(a^2 - b^2 - c^2)*(a + b*Cos[d +
e*x] + c*Sin[d + e*x])), x] + Dist[(a*A - b*B)/(a^2 - b^2 - c^2), Int[1/(a
+ b*Cos[d + e*x] + c*Sin[d + e*x]), x], x] /; FreeQ[{a, b, c, d, e, A, B},
x] && NeQ[a^2 - b^2 - c^2, 0] && NeQ[a*A - b*B, 0]
```

Rubi steps

$$\int \frac{\cosh^3(x)\sinh^2(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx = \frac{a\int \frac{\cosh^2(x)\sinh^2(x)}{a\cosh(x)+b\sinh(x)}\,dx}{a^2-b^2} - \frac{b\int \frac{\cosh^3(x)\sinh(x)}{a\cosh(x)+b\sinh(x)}\,dx}{a^2-b^2} + \frac{(ab)\int \frac{\cosh^2(x)\sinh(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx}{a^2-b^2}$$

$$= \frac{a^2\int \cosh(x)\sinh^2(x)\,dx}{\left(a^2-b^2\right)^2} - 2\frac{(ab)\int \cosh^2(x)\sinh(x)\,dx}{\left(a^2-b^2\right)^2} + 2\frac{\left(a^2b\right)\int \frac{\cosh(x)\sinh}{a\cosh(x)+b\sin}}{\left(a^2-b^2\right)^2}$$

$$= \frac{a^2b^3}{\left(a^2-b^2\right)^3(a\cosh(x)+b\sinh(x))} + \frac{\left(a^3b^2\right)\int \frac{1}{a\cosh(x)+b\sinh(x)}\,dx}{\left(a^2-b^2\right)^3} + 2\left(\frac{\left(a^3b\right)\int \mathrm{si}}{\left(a^2-\right.}\right.$$

$$= -\frac{2ab\cosh^3(x)}{3\left(a^2-b^2\right)^2} + \frac{b^2\sinh(x)}{\left(a^2-b^2\right)^2} + \frac{a^2\sinh^3(x)}{3\left(a^2-b^2\right)^2} + \frac{b^2\sinh^3(x)}{3\left(a^2-b^2\right)^2} + \frac{a^2b}{\left(a^2-b^2\right)^3(a\cosh}$$

$$= \frac{a^3b^2\tan^{-1}\left(\frac{b\cosh(x)+a\sinh(x)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{7/2}} - \frac{2ab\cosh^3(x)}{3\left(a^2-b^2\right)^2} + \frac{b^2\sinh(x)}{\left(a^2-b^2\right)^2} + \frac{a^2\sinh^3(x)}{3\left(a^2-b^2\right)^2} + \frac{b^2}{3(}$$

Mathematica [**A**] time = 2.17, size = 481, normalized size = 1.86

$$\frac{1}{16}\left(\frac{4\left(a^2+b^2\right)\sinh(x)}{(a-b)^2(a+b)^2} - \frac{6a\left(a^2+3b^2\right)\tan^{-1}\left(\frac{a\tanh\left(\frac{x}{2}\right)+b}{\sqrt{a-b}\sqrt{a+b}}\right)}{(a-b)^{5/2}(a+b)^{5/2}} - \frac{b\left(3a^2+b^2\right)}{(a-b)^2(a+b)^2(a\cosh(x)+b\sinh(x))} - \frac{8ab\cosh}{(a-b)^2(a}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cosh[x]^3*Sinh[x]^2)/(a*Cosh[x] + b*Sinh[x])^2,x]
```

```
[Out] -1/8*(Sqrt[a - b]*b*(a + b) + 2*a^2*Sqrt[a + b]*ArcTan[(b + a*Tanh[x/2])/(S
qrt[a - b]*Sqrt[a + b])]*Cosh[x] + 2*a*b*Sqrt[a + b]*ArcTan[(b + a*Tanh[x/2
])/(Sqrt[a - b]*Sqrt[a + b])]*Sinh[x])/((a - b)^(3/2)*(a + b)^2*(a*Cosh[x]
+ b*Sinh[x])) + ((-6*a*(a^2 + 3*b^2)*ArcTan[(b + a*Tanh[x/2])/(Sqrt[a - b]*
Sqrt[a + b])])/((a - b)^(5/2)*(a + b)^(5/2)) - (8*a*b*Cosh[x])/((a - b)^2*(
a + b)^2) + (4*(a^2 + b^2)*Sinh[x])/((a - b)^2*(a + b)^2) - (b*(3*a^2 + b^2
))/((a - b)^2*(a + b)^2*(a*Cosh[x] + b*Sinh[x])))/16 + ((10*a*(a^4 + 10*a^2
*b^2 + 5*b^4)*ArcTan[(b + a*Tanh[x/2])/(Sqrt[a - b]*Sqrt[a + b])])/((a - b)
^(7/2)*(a + b)^(7/2)) + (32*a*b*(a^2 + b^2)*Cosh[x])/((a - b)^3*(a + b)^3)
- (8*a*b*Cosh[3*x])/(3*(a - b)^2*(a + b)^2) - (8*(a^4 + 6*a^2*b^2 + b^4)*Si
nh[x])/((a - b)^3*(a + b)^3) + (b*(5*a^4 + 10*a^2*b^2 + b^4))/((a - b)^3*(a
 + b)^3*(a*Cosh[x] + b*Sinh[x])) + (4*(a^2 + b^2)*Sinh[3*x])/(3*(a - b)^2*(
a + b)^2))/16
```

fricas [B] time = 0.69, size = 5031, normalized size = 19.42

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^3*sinh(x)^2/(a*cosh(x)+b*sinh(x))^2,x, algorithm="fricas"
)
```

```
[Out] [1/24*((a^7 - a^6*b - 3*a^5*b^2 + 3*a^4*b^3 + 3*a^3*b^4 - 3*a^2*b^5 - a*b^6
 + b^7)*cosh(x)^8 + 8*(a^7 - a^6*b - 3*a^5*b^2 + 3*a^4*b^3 + 3*a^3*b^4 - 3*
a^2*b^5 - a*b^6 + b^7)*cosh(x)*sinh(x)^7 + (a^7 - a^6*b - 3*a^5*b^2 + 3*a^4
*b^3 + 3*a^3*b^4 - 3*a^2*b^5 - a*b^6 + b^7)*sinh(x)^8 - a^7 - a^6*b + 3*a^5
*b^2 + 3*a^4*b^3 - 3*a^3*b^4 - 3*a^2*b^5 + a*b^6 + b^7 - 2*(a^7 - 6*a^6*b +
 7*a^5*b^2 + 8*a^4*b^3 - 17*a^3*b^4 + 2*a^2*b^5 + 9*a*b^6 - 4*b^7)*cosh(x)^
6 - 2*(a^7 - 6*a^6*b + 7*a^5*b^2 + 8*a^4*b^3 - 17*a^3*b^4 + 2*a^2*b^5 + 9*a
*b^6 - 4*b^7 - 14*(a^7 - a^6*b - 3*a^5*b^2 + 3*a^4*b^3 + 3*a^3*b^4 - 3*a^2*
b^5 - a*b^6 + b^7)*cosh(x)^2)*sinh(x)^6 + 4*(14*(a^7 - a^6*b - 3*a^5*b^2 +
3*a^4*b^3 + 3*a^3*b^4 - 3*a^2*b^5 - a*b^6 + b^7)*cosh(x)^3 - 3*(a^7 - 6*a^6
*b + 7*a^5*b^2 + 8*a^4*b^3 - 17*a^3*b^4 + 2*a^2*b^5 + 9*a*b^6 - 4*b^7)*cosh
(x))*sinh(x)^5 + 6*(7*a^6*b + 23*a^4*b^3 - 27*a^2*b^5 - 3*b^7)*cosh(x)^4 +
2*(21*a^6*b + 69*a^4*b^3 - 81*a^2*b^5 - 9*b^7 + 35*(a^7 - a^6*b - 3*a^5*b^2
 + 3*a^4*b^3 + 3*a^3*b^4 - 3*a^2*b^5 - a*b^6 + b^7)*cosh(x)^4 - 15*(a^7 - 6
*a^6*b + 7*a^5*b^2 + 8*a^4*b^3 - 17*a^3*b^4 + 2*a^2*b^5 + 9*a*b^6 - 4*b^7)*
cosh(x)^2)*sinh(x)^4 + 8*(7*(a^7 - a^6*b - 3*a^5*b^2 + 3*a^4*b^3 + 3*a^3*b^
4 - 3*a^2*b^5 - a*b^6 + b^7)*cosh(x)^5 - 5*(a^7 - 6*a^6*b + 7*a^5*b^2 + 8*a
^4*b^3 - 17*a^3*b^4 + 2*a^2*b^5 + 9*a*b^6 - 4*b^7)*cosh(x)^3 + 3*(7*a^6*b +
 23*a^4*b^3 - 27*a^2*b^5 - 3*b^7)*cosh(x))*sinh(x)^3 + 2*(a^7 + 6*a^6*b + 7
*a^5*b^2 - 8*a^4*b^3 - 17*a^3*b^4 - 2*a^2*b^5 + 9*a*b^6 + 4*b^7)*cosh(x)^2
+ 2*(a^7 + 6*a^6*b + 7*a^5*b^2 - 8*a^4*b^3 - 17*a^3*b^4 - 2*a^2*b^5 + 9*a*b
^6 + 4*b^7 + 14*(a^7 - a^6*b - 3*a^5*b^2 + 3*a^4*b^3 + 3*a^3*b^4 - 3*a^2*b^
5 - a*b^6 + b^7)*cosh(x)^6 - 15*(a^7 - 6*a^6*b + 7*a^5*b^2 + 8*a^4*b^3 - 17
*a^3*b^4 + 2*a^2*b^5 + 9*a*b^6 - 4*b^7)*cosh(x)^4 + 18*(7*a^6*b + 23*a^4*b^
3 - 27*a^2*b^5 - 3*b^7)*cosh(x)^2)*sinh(x)^2 + 24*((3*a^4*b^2 + 3*a^3*b^3 +
 2*a^2*b^4 + 2*a*b^5)*cosh(x)^5 + 5*(3*a^4*b^2 + 3*a^3*b^3 + 2*a^2*b^4 + 2*
a*b^5)*cosh(x)*sinh(x)^4 + (3*a^4*b^2 + 3*a^3*b^3 + 2*a^2*b^4 + 2*a*b^5)*si
nh(x)^5 + (3*a^4*b^2 - 3*a^3*b^3 + 2*a^2*b^4 - 2*a*b^5)*cosh(x)^3 + (3*a^4*
b^2 - 3*a^3*b^3 + 2*a^2*b^4 - 2*a*b^5 + 10*(3*a^4*b^2 + 3*a^3*b^3 + 2*a^2*b
^4 + 2*a*b^5)*cosh(x)^2)*sinh(x)^3 + (10*(3*a^4*b^2 + 3*a^3*b^3 + 2*a^2*b^4
 + 2*a*b^5)*cosh(x)^3 + 3*(3*a^4*b^2 - 3*a^3*b^3 + 2*a^2*b^4 - 2*a*b^5)*cos
h(x))*sinh(x)^2 + (5*(3*a^4*b^2 + 3*a^3*b^3 + 2*a^2*b^4 + 2*a*b^5)*cosh(x)^
4 + 3*(3*a^4*b^2 - 3*a^3*b^3 + 2*a^2*b^4 - 2*a*b^5)*cosh(x)^2)*sinh(x))*sqr
t(-a^2 + b^2)*log(((a + b)*cosh(x)^2 + 2*(a + b)*cosh(x)*sinh(x) + (a + b)*
sinh(x)^2 + 2*sqrt(-a^2 + b^2)*(cosh(x) + sinh(x)) - a + b)/((a + b)*cosh(x
)^2 + 2*(a + b)*cosh(x)*sinh(x) + (a + b)*sinh(x)^2 + a - b)) + 4*(2*(a^7 -
```

a^6*b - 3*a^5*b^2 + 3*a^4*b^3 + 3*a^3*b^4 - 3*a^2*b^5 - a*b^6 + b^7)*cosh(
x)^7 - 3*(a^7 - 6*a^6*b + 7*a^5*b^2 + 8*a^4*b^3 - 17*a^3*b^4 + 2*a^2*b^5 +
9*a*b^6 - 4*b^7)*cosh(x)^5 + 6*(7*a^6*b + 23*a^4*b^3 - 27*a^2*b^5 - 3*b^7)*
cosh(x)^3 + (a^7 + 6*a^6*b + 7*a^5*b^2 - 8*a^4*b^3 - 17*a^3*b^4 - 2*a^2*b^5
 + 9*a*b^6 + 4*b^7)*cosh(x))*sinh(x))/((a^9 + a^8*b - 4*a^7*b^2 - 4*a^6*b^3
 + 6*a^5*b^4 + 6*a^4*b^5 - 4*a^3*b^6 - 4*a^2*b^7 + a*b^8 + b^9)*cosh(x)^5 +
 5*(a^9 + a^8*b - 4*a^7*b^2 - 4*a^6*b^3 + 6*a^5*b^4 + 6*a^4*b^5 - 4*a^3*b^6
 - 4*a^2*b^7 + a*b^8 + b^9)*cosh(x)*sinh(x)^4 + (a^9 + a^8*b - 4*a^7*b^2 -
4*a^6*b^3 + 6*a^5*b^4 + 6*a^4*b^5 - 4*a^3*b^6 - 4*a^2*b^7 + a*b^8 + b^9)*si
nh(x)^5 + (a^9 - a^8*b - 4*a^7*b^2 + 4*a^6*b^3 + 6*a^5*b^4 - 6*a^4*b^5 - 4*
a^3*b^6 + 4*a^2*b^7 + a*b^8 - b^9)*cosh(x)^3 + (a^9 - a^8*b - 4*a^7*b^2 + 4
*a^6*b^3 + 6*a^5*b^4 - 6*a^4*b^5 - 4*a^3*b^6 + 4*a^2*b^7 + a*b^8 - b^9 + 10
*(a^9 + a^8*b - 4*a^7*b^2 - 4*a^6*b^3 + 6*a^5*b^4 + 6*a^4*b^5 - 4*a^3*b^6 -
 4*a^2*b^7 + a*b^8 + b^9)*cosh(x)^2)*sinh(x)^3 + (10*(a^9 + a^8*b - 4*a^7*b
^2 - 4*a^6*b^3 + 6*a^5*b^4 + 6*a^4*b^5 - 4*a^3*b^6 - 4*a^2*b^7 + a*b^8 + b^
9)*cosh(x)^3 + 3*(a^9 - a^8*b - 4*a^7*b^2 + 4*a^6*b^3 + 6*a^5*b^4 - 6*a^4*b
^5 - 4*a^3*b^6 + 4*a^2*b^7 + a*b^8 - b^9)*cosh(x))*sinh(x)^2 + (5*(a^9 + a^
8*b - 4*a^7*b^2 - 4*a^6*b^3 + 6*a^5*b^4 + 6*a^4*b^5 - 4*a^3*b^6 - 4*a^2*b^7
 + a*b^8 + b^9)*cosh(x)^4 + 3*(a^9 - a^8*b - 4*a^7*b^2 + 4*a^6*b^3 + 6*a^5*
b^4 - 6*a^4*b^5 - 4*a^3*b^6 + 4*a^2*b^7 + a*b^8 - b^9)*cosh(x)^2)*sinh(x)),
 1/24*((a^7 - a^6*b - 3*a^5*b^2 + 3*a^4*b^3 + 3*a^3*b^4 - 3*a^2*b^5 - a*b^6
 + b^7)*cosh(x)^8 + 8*(a^7 - a^6*b - 3*a^5*b^2 + 3*a^4*b^3 + 3*a^3*b^4 - 3*
a^2*b^5 - a*b^6 + b^7)*cosh(x)*sinh(x)^7 + (a^7 - a^6*b - 3*a^5*b^2 + 3*a^4
*b^3 + 3*a^3*b^4 - 3*a^2*b^5 - a*b^6 + b^7)*sinh(x)^8 - a^7 - a^6*b + 3*a^5
*b^2 + 3*a^4*b^3 - 3*a^3*b^4 - 3*a^2*b^5 + a*b^6 + b^7 - 2*(a^7 - 6*a^6*b +
 7*a^5*b^2 + 8*a^4*b^3 - 17*a^3*b^4 + 2*a^2*b^5 + 9*a*b^6 - 4*b^7)*cosh(x)^
6 - 2*(a^7 - 6*a^6*b + 7*a^5*b^2 + 8*a^4*b^3 - 17*a^3*b^4 + 2*a^2*b^5 + 9*a
*b^6 - 4*b^7 - 14*(a^7 - a^6*b - 3*a^5*b^2 + 3*a^4*b^3 + 3*a^3*b^4 - 3*a^2*
b^5 - a*b^6 + b^7)*cosh(x)^2)*sinh(x)^6 + 4*(14*(a^7 - a^6*b - 3*a^5*b^2 +
3*a^4*b^3 + 3*a^3*b^4 - 3*a^2*b^5 - a*b^6 + b^7)*cosh(x)^3 - 3*(a^7 - 6*a^6
*b + 7*a^5*b^2 + 8*a^4*b^3 - 17*a^3*b^4 + 2*a^2*b^5 + 9*a*b^6 - 4*b^7)*cosh
(x))*sinh(x)^5 + 6*(7*a^6*b + 23*a^4*b^3 - 27*a^2*b^5 - 3*b^7)*cosh(x)^4 +
2*(21*a^6*b + 69*a^4*b^3 - 81*a^2*b^5 - 9*b^7 + 35*(a^7 - a^6*b - 3*a^5*b^2
 + 3*a^4*b^3 + 3*a^3*b^4 - 3*a^2*b^5 - a*b^6 + b^7)*cosh(x)^4 - 15*(a^7 - 6
*a^6*b + 7*a^5*b^2 + 8*a^4*b^3 - 17*a^3*b^4 + 2*a^2*b^5 + 9*a*b^6 - 4*b^7)*
cosh(x)^2)*sinh(x)^4 + 8*(7*(a^7 - a^6*b - 3*a^5*b^2 + 3*a^4*b^3 + 3*a^3*b^
4 - 3*a^2*b^5 - a*b^6 + b^7)*cosh(x)^5 - 5*(a^7 - 6*a^6*b + 7*a^5*b^2 + 8*a
^4*b^3 - 17*a^3*b^4 + 2*a^2*b^5 + 9*a*b^6 - 4*b^7)*cosh(x)^3 + 3*(7*a^6*b +
 23*a^4*b^3 - 27*a^2*b^5 - 3*b^7)*cosh(x))*sinh(x)^3 + 2*(a^7 + 6*a^6*b + 7
*a^5*b^2 - 8*a^4*b^3 - 17*a^3*b^4 - 2*a^2*b^5 + 9*a*b^6 + 4*b^7)*cosh(x)^2
+ 2*(a^7 + 6*a^6*b + 7*a^5*b^2 - 8*a^4*b^3 - 17*a^3*b^4 - 2*a^2*b^5 + 9*a*b
^6 + 4*b^7 + 14*(a^7 - a^6*b - 3*a^5*b^2 + 3*a^4*b^3 + 3*a^3*b^4 - 3*a^2*b^
5 - a*b^6 + b^7)*cosh(x)^6 - 15*(a^7 - 6*a^6*b + 7*a^5*b^2 + 8*a^4*b^3 - 17
*a^3*b^4 + 2*a^2*b^5 + 9*a*b^6 - 4*b^7)*cosh(x)^4 + 18*(7*a^6*b + 23*a^4*b^
3 - 27*a^2*b^5 - 3*b^7)*cosh(x)^2)*sinh(x)^2 - 48*((3*a^4*b^2 + 3*a^3*b^3 +

```
 2*a^2*b^4 + 2*a*b^5)*cosh(x)^5 + 5*(3*a^4*b^2 + 3*a^3*b^3 + 2*a^2*b^4 + 2*
a*b^5)*cosh(x)*sinh(x)^4 + (3*a^4*b^2 + 3*a^3*b^3 + 2*a^2*b^4 + 2*a*b^5)*si
nh(x)^5 + (3*a^4*b^2 - 3*a^3*b^3 + 2*a^2*b^4 - 2*a*b^5)*cosh(x)^3 + (3*a^4*
b^2 - 3*a^3*b^3 + 2*a^2*b^4 - 2*a*b^5 + 10*(3*a^4*b^2 + 3*a^3*b^3 + 2*a^2*b
^4 + 2*a*b^5)*cosh(x)^2)*sinh(x)^3 + (10*(3*a^4*b^2 + 3*a^3*b^3 + 2*a^2*b^4
 + 2*a*b^5)*cosh(x)^3 + 3*(3*a^4*b^2 - 3*a^3*b^3 + 2*a^2*b^4 - 2*a*b^5)*cos
h(x))*sinh(x)^2 + (5*(3*a^4*b^2 + 3*a^3*b^3 + 2*a^2*b^4 + 2*a*b^5)*cosh(x)^
4 + 3*(3*a^4*b^2 - 3*a^3*b^3 + 2*a^2*b^4 - 2*a*b^5)*cosh(x)^2)*sinh(x))*sqr
t(a^2 - b^2)*arctan(sqrt(a^2 - b^2)/((a + b)*cosh(x) + (a + b)*sinh(x))) +
4*(2*(a^7 - a^6*b - 3*a^5*b^2 + 3*a^4*b^3 + 3*a^3*b^4 - 3*a^2*b^5 - a*b^6 +
 b^7)*cosh(x)^7 - 3*(a^7 - 6*a^6*b + 7*a^5*b^2 + 8*a^4*b^3 - 17*a^3*b^4 + 2
*a^2*b^5 + 9*a*b^6 - 4*b^7)*cosh(x)^5 + 6*(7*a^6*b + 23*a^4*b^3 - 27*a^2*b^
5 - 3*b^7)*cosh(x)^3 + (a^7 + 6*a^6*b + 7*a^5*b^2 - 8*a^4*b^3 - 17*a^3*b^4
- 2*a^2*b^5 + 9*a*b^6 + 4*b^7)*cosh(x))*sinh(x))/((a^9 + a^8*b - 4*a^7*b^2
- 4*a^6*b^3 + 6*a^5*b^4 + 6*a^4*b^5 - 4*a^3*b^6 - 4*a^2*b^7 + a*b^8 + b^9)*
cosh(x)^5 + 5*(a^9 + a^8*b - 4*a^7*b^2 - 4*a^6*b^3 + 6*a^5*b^4 + 6*a^4*b^5
- 4*a^3*b^6 - 4*a^2*b^7 + a*b^8 + b^9)*cosh(x)*sinh(x)^4 + (a^9 + a^8*b - 4
*a^7*b^2 - 4*a^6*b^3 + 6*a^5*b^4 + 6*a^4*b^5 - 4*a^3*b^6 - 4*a^2*b^7 + a*b^
8 + b^9)*sinh(x)^5 + (a^9 - a^8*b - 4*a^7*b^2 + 4*a^6*b^3 + 6*a^5*b^4 - 6*a
^4*b^5 - 4*a^3*b^6 + 4*a^2*b^7 + a*b^8 - b^9)*cosh(x)^3 + (a^9 - a^8*b - 4*
a^7*b^2 + 4*a^6*b^3 + 6*a^5*b^4 - 6*a^4*b^5 - 4*a^3*b^6 + 4*a^2*b^7 + a*b^8
 - b^9 + 10*(a^9 + a^8*b - 4*a^7*b^2 - 4*a^6*b^3 + 6*a^5*b^4 + 6*a^4*b^5 -
4*a^3*b^6 - 4*a^2*b^7 + a*b^8 + b^9)*cosh(x)^2)*sinh(x)^3 + (10*(a^9 + a^8*
b - 4*a^7*b^2 - 4*a^6*b^3 + 6*a^5*b^4 + 6*a^4*b^5 - 4*a^3*b^6 - 4*a^2*b^7 +
 a*b^8 + b^9)*cosh(x)^3 + 3*(a^9 - a^8*b - 4*a^7*b^2 + 4*a^6*b^3 + 6*a^5*b^
4 - 6*a^4*b^5 - 4*a^3*b^6 + 4*a^2*b^7 + a*b^8 - b^9)*cosh(x))*sinh(x)^2 + (
5*(a^9 + a^8*b - 4*a^7*b^2 - 4*a^6*b^3 + 6*a^5*b^4 + 6*a^4*b^5 - 4*a^3*b^6
- 4*a^2*b^7 + a*b^8 + b^9)*cosh(x)^4 + 3*(a^9 - a^8*b - 4*a^7*b^2 + 4*a^6*b
^3 + 6*a^5*b^4 - 6*a^4*b^5 - 4*a^3*b^6 + 4*a^2*b^7 + a*b^8 - b^9)*cosh(x)^2
)*sinh(x))]
```

giac [A] time = 0.15, size = 310, normalized size = 1.20

$$\frac{2\,a^2b^3e^x}{\left(a^6-3\,a^4b^2+3\,a^2b^4-b^6\right)\left(ae^{(2\,x)}+be^{(2\,x)}+a-b\right)}+\frac{\left(3\,ae^{(2\,x)}+9\,be^{(2\,x)}-a+b\right)e^{(-3\,x)}}{24\left(a^3-3\,a^2b+3\,ab^2-b^3\right)}+\frac{2\left(3\,a^3b^2+2\,ab^4\right)\text{arcta}}{\left(a^6-3\,a^4b^2+3\,a^2b^4-b\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^3*sinh(x)^2/(a*cosh(x)+b*sinh(x))^2,x, algorithm="giac")
```

```
[Out] 2*a^2*b^3*e^x/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*(a*e^(2*x) + b*e^(2*x) +
 a - b)) + 1/24*(3*a*e^(2*x) + 9*b*e^(2*x) - a + b)*e^(-3*x)/(a^3 - 3*a^2*b
 + 3*a*b^2 - b^3) + 2*(3*a^3*b^2 + 2*a*b^4)*arctan((a*e^x + b*e^x)/sqrt(a^2
 - b^2))/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*sqrt(a^2 - b^2)) + 1/24*(a^4*
```

```
e^(3*x) + 4*a^3*b*e^(3*x) + 6*a^2*b^2*e^(3*x) + 4*a*b^3*e^(3*x) + b^4*e^(3*
x) - 3*a^4*e^x + 18*a^2*b^2*e^x + 24*a*b^3*e^x + 9*b^4*e^x)/(a^6 + 6*a^5*b
+ 15*a^4*b^2 + 20*a^3*b^3 + 15*a^2*b^4 + 6*a*b^5 + b^6)
```

maple [A] time = 0.27, size = 289, normalized size = 1.12

$$-\frac{1}{3\left(a+b\right)^{2}\left(\tanh\left(\frac{x}{2}\right)-1\right)^{3}}-\frac{1}{2\left(a+b\right)^{2}\left(\tanh\left(\frac{x}{2}\right)-1\right)^{2}}-\frac{b}{\left(a+b\right)^{3}\left(\tanh\left(\frac{x}{2}\right)-1\right)}+\frac{2a\,b^{4}\tanh\left(\frac{x}{2}\right)}{\left(a-b\right)^{3}\left(a+b\right)^{3}\left(a+2\tanh\left(\frac{x}{2}\right)\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^3*sinh(x)^2/(a*cosh(x)+b*sinh(x))^2,x)
```

```
[Out] -1/3/(a+b)^2/(tanh(1/2*x)-1)^3-1/2/(a+b)^2/(tanh(1/2*x)-1)^2-1/(a+b)^3/(tan
h(1/2*x)-1)*b+2*a*b^4/(a-b)^3/(a+b)^3*tanh(1/2*x)/(a+2*tanh(1/2*x)*b+a*tanh
(1/2*x)^2)+2*a^2*b^3/(a-b)^3/(a+b)^3/(a+2*tanh(1/2*x)*b+a*tanh(1/2*x)^2)+6*
a^3*b^2/(a-b)^3/(a+b)^3/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tanh(1/2*x)+2*b)/(a
^2-b^2)^(1/2))+4*a*b^4/(a-b)^3/(a+b)^3/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tanh
(1/2*x)+2*b)/(a^2-b^2)^(1/2))-1/3/(a-b)^2/(tanh(1/2*x)+1)^3+1/2/(a-b)^2/(ta
nh(1/2*x)+1)^2+1/(a-b)^3/(tanh(1/2*x)+1)*b
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^3*sinh(x)^2/(a*cosh(x)+b*sinh(x))^2,x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [B] time = 1.97, size = 590, normalized size = 2.28

$$\frac{e^{3x}}{24\left(a+b\right)^{2}}-\frac{e^{-3x}}{24\left(a-b\right)^{2}}-\frac{e^{x}\left(a-3b\right)}{8\left(a+b\right)^{3}}+\frac{2\,\mathrm{atan}\left(\frac{e^{x}\left(2\,a\,b^{4}\,\sqrt{a^{14}-7\,a^{12}\,b^{2}+21\,a^{10}\,b\cdots}\right.}{a^{7}\,\sqrt{9\,a^{6}\,b^{4}+12\,a^{4}\,b^{6}+4\,a^{2}\,b^{8}}+b^{7}\,\sqrt{9\,a^{6}\,b^{4}+12\,a^{4}\,b^{6}+4\,a^{2}\,b^{8}}-3\,a^{2}\,b^{5}\,\sqrt{9\,a^{6}\,b^{4}+12\,a^{4}\,b^{6}+\cdots}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cosh(x)^3*sinh(x)^2)/(a*cosh(x) + b*sinh(x))^2,x)
```

```
[Out] exp(3*x)/(24*(a + b)^2) - exp(-3*x)/(24*(a - b)^2) - (exp(x)*(a - 3*b))/(8*
(a + b)^3) + (2*atan((exp(x)*(2*a*b^4*(a^14 - b^14 + 7*a^2*b^12 - 21*a^4*b^
10 + 35*a^6*b^8 - 35*a^8*b^6 + 21*a^10*b^4 - 7*a^12*b^2)^(1/2) + 3*a^3*b^2*
(a^14 - b^14 + 7*a^2*b^12 - 21*a^4*b^10 + 35*a^6*b^8 - 35*a^8*b^6 + 21*a^10
*b^4 - 7*a^12*b^2)^(1/2)))/(a^7*(4*a^2*b^8 + 12*a^4*b^6 + 9*a^6*b^4)^(1/2)
+ b^7*(4*a^2*b^8 + 12*a^4*b^6 + 9*a^6*b^4)^(1/2) - 3*a^2*b^5*(4*a^2*b^8 + 1
2*a^4*b^6 + 9*a^6*b^4)^(1/2) + 3*a^3*b^4*(4*a^2*b^8 + 12*a^4*b^6 + 9*a^6*b^
4)^(1/2) + 3*a^4*b^3*(4*a^2*b^8 + 12*a^4*b^6 + 9*a^6*b^4)^(1/2) - 3*a^5*b^2
*(4*a^2*b^8 + 12*a^4*b^6 + 9*a^6*b^4)^(1/2) - a*b^6*(4*a^2*b^8 + 12*a^4*b^6
 + 9*a^6*b^4)^(1/2) - a^6*b*(4*a^2*b^8 + 12*a^4*b^6 + 9*a^6*b^4)^(1/2)))*(4
*a^2*b^8 + 12*a^4*b^6 + 9*a^6*b^4)^(1/2))/(a^14 - b^14 + 7*a^2*b^12 - 21*a^
4*b^10 + 35*a^6*b^8 - 35*a^8*b^6 + 21*a^10*b^4 - 7*a^12*b^2)^(1/2) + (exp(-
x)*(a + 3*b))/(8*(a - b)^3) + (2*a^2*b^3*exp(x))/((a + b)^3*(a - b)^3*(a -
b + exp(2*x)*(a + b)))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)**3*sinh(x)**2/(a*cosh(x)+b*sinh(x))**2,x)
```

```
[Out] Timed out
```

3.723 $\int \frac{\cosh^3(x)\sinh^3(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx$

Optimal. Leaf size=314

$$\frac{abx}{4\left(a^2-b^2\right)^2}+\frac{a^2\sinh^4(x)}{4\left(a^2-b^2\right)^2}-\frac{2a^2b^2\sinh^2(x)}{\left(a^2-b^2\right)^3}+\frac{b^2\cosh^4(x)}{4\left(a^2-b^2\right)^2}-\frac{ab\sinh(x)\cosh^3(x)}{2\left(a^2-b^2\right)^2}+\frac{ab\sinh(x)\cosh(x)}{4\left(a^2-b^2\right)^2}+\frac{3a^2b^4\log(a}{(}$$

```
[Out] -6*a^3*b^3*x/(a^2-b^2)^4-a^3*b*x/(a^2-b^2)^3+a*b^3*x/(a^2-b^2)^3+1/4*a*b*x/
(a^2-b^2)^2+1/4*b^2*cosh(x)^4/(a^2-b^2)^2+3*a^4*b^2*ln(a*cosh(x)+b*sinh(x))
/(a^2-b^2)^4+3*a^2*b^4*ln(a*cosh(x)+b*sinh(x))/(a^2-b^2)^4+a^3*b*cosh(x)*si
nh(x)/(a^2-b^2)^3+a*b^3*cosh(x)*sinh(x)/(a^2-b^2)^3+1/4*a*b*cosh(x)*sinh(x)
/(a^2-b^2)^2-1/2*a*b*cosh(x)^3*sinh(x)/(a^2-b^2)^2-2*a^2*b^2*sinh(x)^2/(a^2
-b^2)^3+1/4*a^2*sinh(x)^4/(a^2-b^2)^2+a^2*b^3*sinh(x)/(a^2-b^2)^3/(a*cosh(x
)+b*sinh(x))
```

Rubi [A] time = 1.73, antiderivative size = 314, normalized size of antiderivative = 1.00, number of steps used = 48, number of rules used = 12, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {3111, 3109, 2565, 30, 2568, 2635, 8, 2564, 3098, 3133, 3097, 3075}

$$-\frac{a^3bx}{\left(a^2-b^2\right)^3}-\frac{6a^3b^3x}{\left(a^2-b^2\right)^4}+\frac{abx}{4\left(a^2-b^2\right)^2}+\frac{ab^3x}{\left(a^2-b^2\right)^3}+\frac{a^2\sinh^4(x)}{4\left(a^2-b^2\right)^2}-\frac{2a^2b^2\sinh^2(x)}{\left(a^2-b^2\right)^3}+\frac{b^2\cosh^4(x)}{4\left(a^2-b^2\right)^2}-\frac{ab\sinh(x)\cosh^3}{2\left(a^2-b^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Cosh[x]^3*Sinh[x]^3)/(a*Cosh[x] + b*Sinh[x])^2,x]
```

```
[Out] (-6*a^3*b^3*x)/(a^2 - b^2)^4 - (a^3*b*x)/(a^2 - b^2)^3 + (a*b^3*x)/(a^2 - b
^2)^3 + (a*b*x)/(4*(a^2 - b^2)^2) + (b^2*Cosh[x]^4)/(4*(a^2 - b^2)^2) + (3*
a^4*b^2*Log[a*Cosh[x] + b*Sinh[x]])/(a^2 - b^2)^4 + (3*a^2*b^4*Log[a*Cosh[x
] + b*Sinh[x]])/(a^2 - b^2)^4 + (a^3*b*Cosh[x]*Sinh[x])/(a^2 - b^2)^3 + (a*
b^3*Cosh[x]*Sinh[x])/(a^2 - b^2)^3 + (a*b*Cosh[x]*Sinh[x])/(4*(a^2 - b^2)^2
) - (a*b*Cosh[x]^3*Sinh[x])/(2*(a^2 - b^2)^2) - (2*a^2*b^2*Sinh[x]^2)/(a^2
- b^2)^3 + (a^2*Sinh[x]^4)/(4*(a^2 - b^2)^2) + (a^2*b^3*Sinh[x])/((a^2 - b^
2)^3*(a*Cosh[x] + b*Sinh[x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2568

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m - 1)
)/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Cos[e + f*x])^n*(a
*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 3075

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-2), x
_Symbol] :> Simp[Sin[c + d*x]/(a*d*(a*Cos[c + d*x] + b*Sin[c + d*x])), x] /
; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3097

```
Int[sin[(c_.) + (d_.)*(x_)]/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.
) + (d_.)*(x_)]), x_Symbol] :> Simp[(b*x)/(a^2 + b^2), x] - Dist[a/(a^2 + b
^2), Int[(b*Cos[c + d*x] - a*Sin[c + d*x])/(a*Cos[c + d*x] + b*Sin[c + d*x]
), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3098

```
Int[cos[(c_.) + (d_.)*(x_)]/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.
```

```
) + (d_.)*(x_)]), x_Symbol] :> Simp[(a*x)/(a^2 + b^2), x] + Dist[b/(a^2 + b
^2), Int[(b*Cos[c + d*x] - a*Sin[c + d*x])/(a*Cos[c + d*x] + b*Sin[c + d*x]
), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3109

```
Int[(cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.))/(cos[(c_.
) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[b
/(a^2 + b^2), Int[Cos[c + d*x]^m*Sin[c + d*x]^(n - 1), x], x] + (Dist[a/(a^
2 + b^2), Int[Cos[c + d*x]^(m - 1)*Sin[c + d*x]^n, x], x] - Dist[(a*b)/(a^2
 + b^2), Int[(Cos[c + d*x]^(m - 1)*Sin[c + d*x]^(n - 1))/(a*Cos[c + d*x] +
b*Sin[c + d*x]), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] &&
IGtQ[m, 0] && IGtQ[n, 0]
```

Rule 3111

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.)*(cos[(c_.)
+ (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol] :> Dis
t[b/(a^2 + b^2), Int[Cos[c + d*x]^m*Sin[c + d*x]^(n - 1)*(a*Cos[c + d*x] +
b*Sin[c + d*x])^(p + 1), x], x] + (Dist[a/(a^2 + b^2), Int[Cos[c + d*x]^(m
- 1)*Sin[c + d*x]^n*(a*Cos[c + d*x] + b*Sin[c + d*x])^(p + 1), x], x] - Dis
t[(a*b)/(a^2 + b^2), Int[Cos[c + d*x]^(m - 1)*Sin[c + d*x]^(n - 1)*(a*Cos[c
 + d*x] + b*Sin[c + d*x])^p, x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 +
 b^2, 0] && IGtQ[m, 0] && IGtQ[n, 0] && ILtQ[p, 0]
```

Rule 3133

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)]), x
_Symbol] :> Simp[((b*B + c*C)*x)/(b^2 + c^2), x] + Simp[((c*B - b*C)*Log[a
+ b*Cos[d + e*x] + c*Sin[d + e*x]])/(e*(b^2 + c^2)), x] /; FreeQ[{a, b, c,
d, e, A, B, C}, x] && NeQ[b^2 + c^2, 0] && EqQ[A*(b^2 + c^2) - a*(b*B + c*C
), 0]
```

Rubi steps

$$\int \frac{\cosh^3(x)\sinh^3(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx = \frac{a\int \frac{\cosh^2(x)\sinh^3(x)}{a\cosh(x)+b\sinh(x)}\,dx}{a^2-b^2} - \frac{b\int \frac{\cosh^3(x)\sinh^2(x)}{a\cosh(x)+b\sinh(x)}\,dx}{a^2-b^2} + \frac{(ab)\int \frac{\cosh^2(x)\sinh^2(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx}{a^2-b^2}$$

$$= \frac{a^2\int \cosh(x)\sinh^3(x)\,dx}{\left(a^2-b^2\right)^2} - 2\frac{(ab)\int \cosh^2(x)\sinh^2(x)\,dx}{\left(a^2-b^2\right)^2} + 2\frac{\left(a^2b\right)\int \frac{\cosh(x)\,\mathrm{si}}{a\cosh(x)+b}}{\left(a^2-b^2\right)}$$

$$= \frac{\left(a^3b^2\right)\int \frac{\sinh(x)}{a\cosh(x)+b\sinh(x)}\,dx}{\left(a^2-b^2\right)^3} + 2\left(\frac{\left(a^3b\right)\int \sinh^2(x)\,dx}{\left(a^2-b^2\right)^3} - \frac{\left(a^2b^2\right)\int \cosh(x)\sinh}{\left(a^2-b^2\right)^3}\right.$$

$$= -\frac{2a^3b^3x}{\left(a^2-b^2\right)^4} + \frac{b^2\cosh^4(x)}{4\left(a^2-b^2\right)^2} + \frac{a^2\sinh^4(x)}{4\left(a^2-b^2\right)^2} + \frac{a^2b^3\sinh(x)}{\left(a^2-b^2\right)^3(a\cosh(x)+b\sinh(x))}$$

$$= -\frac{2a^3b^3x}{\left(a^2-b^2\right)^4} + \frac{b^2\cosh^4(x)}{4\left(a^2-b^2\right)^2} + \frac{a^4b^2\log(a\cosh(x)+b\sinh(x))}{\left(a^2-b^2\right)^4} + \frac{a^2b^4\log(a\,\mathrm{co}}{\left(a^2\right.}$$

Mathematica [A] time = 1.49, size = 366, normalized size = 1.17

$$\frac{a^7\cosh(5x)+20a^6b\sinh(x)+9a^6b\sinh(3x)-a^6b\sinh(5x)-48a^5b^2x\sinh(x)-3a^5b^2\cosh(5x)+84a^4b^3\sinh}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cosh[x]^3*Sinh[x]^3)/(a*Cosh[x] + b*Sinh[x])^2,x]
```

```
[Out] (-3*a*(a^2 - b^2)^2*(a^2 + 3*b^2)*Cosh[3*x] + a^7*Cosh[5*x] - 3*a^5*b^2*Cos
h[5*x] + 3*a^3*b^4*Cosh[5*x] - a*b^6*Cosh[5*x] - 4*a*Cosh[x]*(a^6 + 9*a^4*b
^2 - 5*a^2*b^4 - 5*b^6 + 12*a^5*b*x + 72*a^3*b^3*x + 12*a*b^5*x - 48*a^2*b^
2*(a^2 + b^2)*Log[a*Cosh[x] + b*Sinh[x]]) + 20*a^6*b*Sinh[x] + 84*a^4*b^3*S
inh[x] - 100*a^2*b^5*Sinh[x] - 4*b^7*Sinh[x] - 48*a^5*b^2*x*Sinh[x] - 288*a
^3*b^4*x*Sinh[x] - 48*a*b^6*x*Sinh[x] + 192*a^4*b^3*Log[a*Cosh[x] + b*Sinh[
x]]*Sinh[x] + 192*a^2*b^5*Log[a*Cosh[x] + b*Sinh[x]]*Sinh[x] + 9*a^6*b*Sinh
[3*x] - 15*a^4*b^3*Sinh[3*x] + 3*a^2*b^5*Sinh[3*x] + 3*b^7*Sinh[3*x] - a^6*
b*Sinh[5*x] + 3*a^4*b^3*Sinh[5*x] - 3*a^2*b^5*Sinh[5*x] + b^7*Sinh[5*x])/(6
4*(a - b)^4*(a + b)^4*(a*Cosh[x] + b*Sinh[x]))
```

fricas [B] time = 0.60, size = 4001, normalized size = 12.74

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^3*sinh(x)^3/(a*cosh(x)+b*sinh(x))^2,x, algorithm="fricas"
)

[Out] 1/64*((a^7 - a^6*b - 3*a^5*b^2 + 3*a^4*b^3 + 3*a^3*b^4 - 3*a^2*b^5 - a*b^6
+ b^7)*cosh(x)^10 + 10*(a^7 - a^6*b - 3*a^5*b^2 + 3*a^4*b^3 + 3*a^3*b^4 - 3
*a^2*b^5 - a*b^6 + b^7)*cosh(x)*sinh(x)^9 + (a^7 - a^6*b - 3*a^5*b^2 + 3*a^
4*b^3 + 3*a^3*b^4 - 3*a^2*b^5 - a*b^6 + b^7)*sinh(x)^10 - 3*(a^7 - 3*a^6*b
+ a^5*b^2 + 5*a^4*b^3 - 5*a^3*b^4 - a^2*b^5 + 3*a*b^6 - b^7)*cosh(x)^8 - 3*
(a^7 - 3*a^6*b + a^5*b^2 + 5*a^4*b^3 - 5*a^3*b^4 - a^2*b^5 + 3*a*b^6 - b^7
- 15*(a^7 - a^6*b - 3*a^5*b^2 + 3*a^4*b^3 + 3*a^3*b^4 - 3*a^2*b^5 - a*b^6 +
 b^7)*cosh(x)^2)*sinh(x)^8 + 24*(5*(a^7 - a^6*b - 3*a^5*b^2 + 3*a^4*b^3 + 3
*a^3*b^4 - 3*a^2*b^5 - a*b^6 + b^7)*cosh(x)^3 - (a^7 - 3*a^6*b + a^5*b^2 +
5*a^4*b^3 - 5*a^3*b^4 - a^2*b^5 + 3*a*b^6 - b^7)*cosh(x))*sinh(x)^7 + a^7 +
 a^6*b - 3*a^5*b^2 - 3*a^4*b^3 + 3*a^3*b^4 + 3*a^2*b^5 - a*b^6 - b^7 - 4*(a
^7 - 5*a^6*b + 9*a^5*b^2 - 5*a^4*b^3 - 5*a^3*b^4 + 9*a^2*b^5 - 5*a*b^6 + b^
7 + 12*(a^6*b + 5*a^5*b^2 + 10*a^4*b^3 + 10*a^3*b^4 + 5*a^2*b^5 + a*b^6)*x)
*cosh(x)^6 - 2*(2*a^7 - 10*a^6*b + 18*a^5*b^2 - 10*a^4*b^3 - 10*a^3*b^4 + 1
8*a^2*b^5 - 10*a*b^6 + 2*b^7 - 105*(a^7 - a^6*b - 3*a^5*b^2 + 3*a^4*b^3 + 3
*a^3*b^4 - 3*a^2*b^5 - a*b^6 + b^7)*cosh(x)^4 + 42*(a^7 - 3*a^6*b + a^5*b^2
 + 5*a^4*b^3 - 5*a^3*b^4 - a^2*b^5 + 3*a*b^6 - b^7)*cosh(x)^2 + 24*(a^6*b +
 5*a^5*b^2 + 10*a^4*b^3 + 10*a^3*b^4 + 5*a^2*b^5 + a*b^6)*x)*sinh(x)^6 + 12
*(21*(a^7 - a^6*b - 3*a^5*b^2 + 3*a^4*b^3 + 3*a^3*b^4 - 3*a^2*b^5 - a*b^6 +
 b^7)*cosh(x)^5 - 14*(a^7 - 3*a^6*b + a^5*b^2 + 5*a^4*b^3 - 5*a^3*b^4 - a^2
*b^5 + 3*a*b^6 - b^7)*cosh(x)^3 - 2*(a^7 - 5*a^6*b + 9*a^5*b^2 - 5*a^4*b^3
- 5*a^3*b^4 + 9*a^2*b^5 - 5*a*b^6 + b^7 + 12*(a^6*b + 5*a^5*b^2 + 10*a^4*b^
3 + 10*a^3*b^4 + 5*a^2*b^5 + a*b^6)*x)*cosh(x))*sinh(x)^5 - 4*(a^7 + 5*a^6*
b + 9*a^5*b^2 + 37*a^4*b^3 - 37*a^3*b^4 - 9*a^2*b^5 - 5*a*b^6 - b^7 + 12*(a
^6*b + 3*a^5*b^2 + 2*a^4*b^3 - 2*a^3*b^4 - 3*a^2*b^5 - a*b^6)*x)*cosh(x)^4
- 2*(2*a^7 + 10*a^6*b + 18*a^5*b^2 + 74*a^4*b^3 - 74*a^3*b^4 - 18*a^2*b^5 -
 10*a*b^6 - 2*b^7 - 105*(a^7 - a^6*b - 3*a^5*b^2 + 3*a^4*b^3 + 3*a^3*b^4 -
3*a^2*b^5 - a*b^6 + b^7)*cosh(x)^6 + 105*(a^7 - 3*a^6*b + a^5*b^2 + 5*a^4*b
^3 - 5*a^3*b^4 - a^2*b^5 + 3*a*b^6 - b^7)*cosh(x)^4 + 30*(a^7 - 5*a^6*b + 9
*a^5*b^2 - 5*a^4*b^3 - 5*a^3*b^4 + 9*a^2*b^5 - 5*a*b^6 + b^7 + 12*(a^6*b +
5*a^5*b^2 + 10*a^4*b^3 + 10*a^3*b^4 + 5*a^2*b^5 + a*b^6)*x)*cosh(x)^2 + 24*
(a^6*b + 3*a^5*b^2 + 2*a^4*b^3 - 2*a^3*b^4 - 3*a^2*b^5 - a*b^6)*x)*sinh(x)^
4 + 8*(15*(a^7 - a^6*b - 3*a^5*b^2 + 3*a^4*b^3 + 3*a^3*b^4 - 3*a^2*b^5 - a*
b^6 + b^7)*cosh(x)^7 - 21*(a^7 - 3*a^6*b + a^5*b^2 + 5*a^4*b^3 - 5*a^3*b^4
- a^2*b^5 + 3*a*b^6 - b^7)*cosh(x)^5 - 10*(a^7 - 5*a^6*b + 9*a^5*b^2 - 5*a^
4*b^3 - 5*a^3*b^4 + 9*a^2*b^5 - 5*a*b^6 + b^7 + 12*(a^6*b + 5*a^5*b^2 + 10*
a^4*b^3 + 10*a^3*b^4 + 5*a^2*b^5 + a*b^6)*x)*cosh(x)^3 - 2*(a^7 + 5*a^6*b +
 9*a^5*b^2 + 37*a^4*b^3 - 37*a^3*b^4 - 9*a^2*b^5 - 5*a*b^6 - b^7 + 12*(a^6*
b + 3*a^5*b^2 + 2*a^4*b^3 - 2*a^3*b^4 - 3*a^2*b^5 - a*b^6)*x)*cosh(x))*sinh
(x)^3 - 3*(a^7 + 3*a^6*b + a^5*b^2 - 5*a^4*b^3 - 5*a^3*b^4 + a^2*b^5 + 3*a*
b^6 + b^7)*cosh(x)^2 + 3*(15*(a^7 - a^6*b - 3*a^5*b^2 + 3*a^4*b^3 + 3*a^3*b
^4 - 3*a^2*b^5 - a*b^6 + b^7)*cosh(x)^8 - a^7 - 3*a^6*b - a^5*b^2 + 5*a^4*b
```

```
^3 + 5*a^3*b^4 - a^2*b^5 - 3*a*b^6 - b^7 - 28*(a^7 - 3*a^6*b + a^5*b^2 + 5*
a^4*b^3 - 5*a^3*b^4 - a^2*b^5 + 3*a*b^6 - b^7)*cosh(x)^6 - 20*(a^7 - 5*a^6*
b + 9*a^5*b^2 - 5*a^4*b^3 - 5*a^3*b^4 + 9*a^2*b^5 - 5*a*b^6 + b^7 + 12*(a^6
*b + 5*a^5*b^2 + 10*a^4*b^3 + 10*a^3*b^4 + 5*a^2*b^5 + a*b^6)*x)*cosh(x)^4
- 8*(a^7 + 5*a^6*b + 9*a^5*b^2 + 37*a^4*b^3 - 37*a^3*b^4 - 9*a^2*b^5 - 5*a*
b^6 - b^7 + 12*(a^6*b + 3*a^5*b^2 + 2*a^4*b^3 - 2*a^3*b^4 - 3*a^2*b^5 - a*b
^6)*x)*cosh(x)^2)*sinh(x)^2 + 192*((a^5*b^2 + a^4*b^3 + a^3*b^4 + a^2*b^5)*
cosh(x)^6 + 6*(a^5*b^2 + a^4*b^3 + a^3*b^4 + a^2*b^5)*cosh(x)*sinh(x)^5 + (
a^5*b^2 + a^4*b^3 + a^3*b^4 + a^2*b^5)*sinh(x)^6 + (a^5*b^2 - a^4*b^3 + a^3
*b^4 - a^2*b^5)*cosh(x)^4 + (a^5*b^2 - a^4*b^3 + a^3*b^4 - a^2*b^5 + 15*(a^
5*b^2 + a^4*b^3 + a^3*b^4 + a^2*b^5)*cosh(x)^2)*sinh(x)^4 + 4*(5*(a^5*b^2 +
 a^4*b^3 + a^3*b^4 + a^2*b^5)*cosh(x)^3 + (a^5*b^2 - a^4*b^3 + a^3*b^4 - a^
2*b^5)*cosh(x))*sinh(x)^3 + 3*(5*(a^5*b^2 + a^4*b^3 + a^3*b^4 + a^2*b^5)*co
sh(x)^4 + 2*(a^5*b^2 - a^4*b^3 + a^3*b^4 - a^2*b^5)*cosh(x)^2)*sinh(x)^2 +
2*(3*(a^5*b^2 + a^4*b^3 + a^3*b^4 + a^2*b^5)*cosh(x)^5 + 2*(a^5*b^2 - a^4*b
^3 + a^3*b^4 - a^2*b^5)*cosh(x)^3)*sinh(x))*log(2*(a*cosh(x) + b*sinh(x))/(
cosh(x) - sinh(x))) + 2*(5*(a^7 - a^6*b - 3*a^5*b^2 + 3*a^4*b^3 + 3*a^3*b^4
 - 3*a^2*b^5 - a*b^6 + b^7)*cosh(x)^9 - 12*(a^7 - 3*a^6*b + a^5*b^2 + 5*a^4
*b^3 - 5*a^3*b^4 - a^2*b^5 + 3*a*b^6 - b^7)*cosh(x)^7 - 12*(a^7 - 5*a^6*b +
 9*a^5*b^2 - 5*a^4*b^3 - 5*a^3*b^4 + 9*a^2*b^5 - 5*a*b^6 + b^7 + 12*(a^6*b
+ 5*a^5*b^2 + 10*a^4*b^3 + 10*a^3*b^4 + 5*a^2*b^5 + a*b^6)*x)*cosh(x)^5 - 8
*(a^7 + 5*a^6*b + 9*a^5*b^2 + 37*a^4*b^3 - 37*a^3*b^4 - 9*a^2*b^5 - 5*a*b^6
 - b^7 + 12*(a^6*b + 3*a^5*b^2 + 2*a^4*b^3 - 2*a^3*b^4 - 3*a^2*b^5 - a*b^6)
*x)*cosh(x)^3 - 3*(a^7 + 3*a^6*b + a^5*b^2 - 5*a^4*b^3 - 5*a^3*b^4 + a^2*b^
5 + 3*a*b^6 + b^7)*cosh(x))*sinh(x))/((a^9 + a^8*b - 4*a^7*b^2 - 4*a^6*b^3
+ 6*a^5*b^4 + 6*a^4*b^5 - 4*a^3*b^6 - 4*a^2*b^7 + a*b^8 + b^9)*cosh(x)^6 +
6*(a^9 + a^8*b - 4*a^7*b^2 - 4*a^6*b^3 + 6*a^5*b^4 + 6*a^4*b^5 - 4*a^3*b^6
- 4*a^2*b^7 + a*b^8 + b^9)*cosh(x)*sinh(x)^5 + (a^9 + a^8*b - 4*a^7*b^2 - 4
*a^6*b^3 + 6*a^5*b^4 + 6*a^4*b^5 - 4*a^3*b^6 - 4*a^2*b^7 + a*b^8 + b^9)*sin
h(x)^6 + (a^9 - a^8*b - 4*a^7*b^2 + 4*a^6*b^3 + 6*a^5*b^4 - 6*a^4*b^5 - 4*a
^3*b^6 + 4*a^2*b^7 + a*b^8 - b^9)*cosh(x)^4 + (a^9 - a^8*b - 4*a^7*b^2 + 4*
a^6*b^3 + 6*a^5*b^4 - 6*a^4*b^5 - 4*a^3*b^6 + 4*a^2*b^7 + a*b^8 - b^9 + 15*
(a^9 + a^8*b - 4*a^7*b^2 - 4*a^6*b^3 + 6*a^5*b^4 + 6*a^4*b^5 - 4*a^3*b^6 -
4*a^2*b^7 + a*b^8 + b^9)*cosh(x)^2)*sinh(x)^4 + 4*(5*(a^9 + a^8*b - 4*a^7*b
^2 - 4*a^6*b^3 + 6*a^5*b^4 + 6*a^4*b^5 - 4*a^3*b^6 - 4*a^2*b^7 + a*b^8 + b^
9)*cosh(x)^3 + (a^9 - a^8*b - 4*a^7*b^2 + 4*a^6*b^3 + 6*a^5*b^4 - 6*a^4*b^5
 - 4*a^3*b^6 + 4*a^2*b^7 + a*b^8 - b^9)*cosh(x))*sinh(x)^3 + 3*(5*(a^9 + a^
8*b - 4*a^7*b^2 - 4*a^6*b^3 + 6*a^5*b^4 + 6*a^4*b^5 - 4*a^3*b^6 - 4*a^2*b^7
 + a*b^8 + b^9)*cosh(x)^4 + 2*(a^9 - a^8*b - 4*a^7*b^2 + 4*a^6*b^3 + 6*a^5*
b^4 - 6*a^4*b^5 - 4*a^3*b^6 + 4*a^2*b^7 + a*b^8 - b^9)*cosh(x)^2)*sinh(x)^2
 + 2*(3*(a^9 + a^8*b - 4*a^7*b^2 - 4*a^6*b^3 + 6*a^5*b^4 + 6*a^4*b^5 - 4*a^
3*b^6 - 4*a^2*b^7 + a*b^8 + b^9)*cosh(x)^5 + 2*(a^9 - a^8*b - 4*a^7*b^2 + 4
*a^6*b^3 + 6*a^5*b^4 - 6*a^4*b^5 - 4*a^3*b^6 + 4*a^2*b^7 + a*b^8 - b^9)*cos
h(x)^3)*sinh(x))
```

giac [A] time = 0.15, size = 384, normalized size = 1.22

$$-\frac{3\,abx}{4\left(a^4-4\,a^3b+6\,a^2b^2-4\,ab^3+b^4\right)}+\frac{\left(36\,abe^{(4\,x)}-4\,a^2e^{(2\,x)}+4\,b^2e^{(2\,x)}+a^2-2\,ab+b^2\right)e^{(-4\,x)}}{64\left(a^4-4\,a^3b+6\,a^2b^2-4\,ab^3+b^4\right)}+\frac{3\left(a^4b^2+a^2b^4\right)\mathrm{l}}{a^8-4\,a^6b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^3*sinh(x)^3/(a*cosh(x)+b*sinh(x))^2,x, algorithm="giac")
```

```
[Out] -3/4*a*b*x/(a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4) + 1/64*(36*a*b*e^(4*
x) - 4*a^2*e^(2*x) + 4*b^2*e^(2*x) + a^2 - 2*a*b + b^2)*e^(-4*x)/(a^4 - 4*a
^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4) + 3*(a^4*b^2 + a^2*b^4)*log(abs(a*e^(2*x)
 + b*e^(2*x) + a - b))/(a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8) + 1/
64*(a^2*e^(4*x) + 2*a*b*e^(4*x) + b^2*e^(4*x) - 4*a^2*e^(2*x) + 4*b^2*e^(2*
x))/(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4) - (3*a^5*b^2*e^(2*x) + 3*a^
4*b^3*e^(2*x) + 3*a^3*b^4*e^(2*x) + 3*a^2*b^5*e^(2*x) + 3*a^5*b^2 - a^4*b^3
 + a^3*b^4 - 3*a^2*b^5)/((a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8)*(a
*e^(2*x) + b*e^(2*x) + a - b))
```

maple [A] time = 0.28, size = 398, normalized size = 1.27

$$\frac{1}{4\left(a+b\right)^2\left(\tanh\left(\frac{x}{2}\right)-1\right)^4}+\frac{1}{2\left(a+b\right)^2\left(\tanh\left(\frac{x}{2}\right)-1\right)^3}+\frac{a}{8\left(a+b\right)^3\left(\tanh\left(\frac{x}{2}\right)-1\right)^2}+\frac{5b}{8\left(a+b\right)^3\left(\tanh\left(\frac{x}{2}\right)-1\right)^2}-\frac{}{8(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^3*sinh(x)^3/(a*cosh(x)+b*sinh(x))^2,x)
```

```
[Out] 1/4/(a+b)^2/(tanh(1/2*x)-1)^4+1/2/(a+b)^2/(tanh(1/2*x)-1)^3+1/8/(a+b)^3/(ta
nh(1/2*x)-1)^2*a+5/8/(a+b)^3/(tanh(1/2*x)-1)^2*b-1/8/(a+b)^3/(tanh(1/2*x)-1
)*a+3/8/(a+b)^3/(tanh(1/2*x)-1)*b+3/4*a*b/(a+b)^4*ln(tanh(1/2*x)-1)+2*a^4*b
^3/(a-b)^4/(a+b)^4*tanh(1/2*x)/(a+2*tanh(1/2*x)*b+a*tanh(1/2*x)^2)-2*a^2*b^
5/(a-b)^4/(a+b)^4*tanh(1/2*x)/(a+2*tanh(1/2*x)*b+a*tanh(1/2*x)^2)+3*a^4*b^2
/(a-b)^4/(a+b)^4*ln(a+2*tanh(1/2*x)*b+a*tanh(1/2*x)^2)+3*a^2*b^4/(a-b)^4/(a
+b)^4*ln(a+2*tanh(1/2*x)*b+a*tanh(1/2*x)^2)+1/4/(a-b)^2/(tanh(1/2*x)+1)^4-1
/2/(a-b)^2/(tanh(1/2*x)+1)^3+1/8/(a-b)^3/(tanh(1/2*x)+1)*a+3/8/(a-b)^3/(tan
h(1/2*x)+1)*b+1/8/(a-b)^3/(tanh(1/2*x)+1)^2*a-5/8/(a-b)^3/(tanh(1/2*x)+1)^2
*b-3/4*a*b/(a-b)^4*ln(tanh(1/2*x)+1)
```

maxima [A] time = 0.52, size = 384, normalized size = 1.22

$$-\frac{3\,abx}{4\left(a^4+4\,a^3b+6\,a^2b^2+4\,ab^3+b^4\right)}+\frac{3\left(a^4b^2+a^2b^4\right)\log\left(-(a-b)e^{(-2\,x)}-a-b\right)}{a^8-4\,a^6b^2+6\,a^4b^4-4\,a^2b^6+b^8}-\frac{4\,(a+b)e^{(-2\,x)}-(a-b)e^{(-4\,x)}}{64\left(a^3-3\,a^2b+3\,ab^2-b^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^3*sinh(x)^3/(a*cosh(x)+b*sinh(x))^2,x, algorithm="maxima"
)
```

```
[Out] -3/4*a*b*x/(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4) + 3*(a^4*b^2 + a^2*b
^4)*log(-(a - b)*e^(-2*x) - a - b)/(a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6
 + b^8) - 1/64*(4*(a + b)*e^(-2*x) - (a - b)*e^(-4*x))/(a^3 - 3*a^2*b + 3*a
*b^2 - b^3) + 1/64*(a^6 - 2*a^5*b - a^4*b^2 + 4*a^3*b^3 - a^2*b^4 - 2*a*b^5
 + b^6 - 3*(a^6 - 4*a^5*b + 5*a^4*b^2 - 5*a^2*b^4 + 4*a*b^5 - b^6)*e^(-2*x)
 - 4*(a^6 - 6*a^5*b + 15*a^4*b^2 - 52*a^3*b^3 + 15*a^2*b^4 - 6*a*b^5 + b^6)
*e^(-4*x))/((a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8)*e^(-4*x) + (a^8
 - 2*a^7*b - 2*a^6*b^2 + 6*a^5*b^3 - 6*a^3*b^5 + 2*a^2*b^6 + 2*a*b^7 - b^8)
*e^(-6*x))
```

mupad [B] time = 1.92, size = 173, normalized size = 0.55

$$\frac{e^{4x}}{64(a+b)^2}+\frac{e^{-4x}}{64(a-b)^2}+\frac{\ln\left(a-b+a\,e^{2x}+b\,e^{2x}\right)\left(3a^4b^2+3a^2b^4\right)}{a^8-4a^6b^2+6a^4b^4-4a^2b^6+b^8}-\frac{e^{-2x}(a+b)}{16(a-b)^3}-\frac{e^{2x}(a-b)}{16(a+b)^3}-\frac{3abx}{4(a-b)^4}-\frac{}{(a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cosh(x)^3*sinh(x)^3)/(a*cosh(x) + b*sinh(x))^2,x)
```

```
[Out] exp(4*x)/(64*(a + b)^2) + exp(-4*x)/(64*(a - b)^2) + (log(a - b + a*exp(2*x
) + b*exp(2*x))*(3*a^2*b^4 + 3*a^4*b^2))/(a^8 + b^8 - 4*a^2*b^6 + 6*a^4*b^4
 - 4*a^6*b^2) - (exp(-2*x)*(a + b))/(16*(a - b)^3) - (exp(2*x)*(a - b))/(16
*(a + b)^3) - (3*a*b*x)/(4*(a - b)^4) - (2*a^3*b^3)/((a + b)^4*(a - b)^3*(a
 - b + exp(2*x)*(a + b)))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)**3*sinh(x)**3/(a*cosh(x)+b*sinh(x))**2,x)
```

```
[Out] Timed out
```

3.724 $\int \frac{A+C\sinh(x)}{b\cosh(x)+c\sinh(x)}\,dx$

Optimal. Leaf size=80

$$\frac{A\tan^{-1}\left(\frac{b\sinh(x)+c\cosh(x)}{\sqrt{b^2-c^2}}\right)}{\sqrt{b^2-c^2}}-\frac{cCx}{b^2-c^2}+\frac{bC\log(b\cosh(x)+c\sinh(x))}{b^2-c^2}$$

```
[Out] -c*C*x/(b^2-c^2)+b*C*ln(b*cosh(x)+c*sinh(x))/(b^2-c^2)+A*arctan((c*cosh(x)+
b*sinh(x))/(b^2-c^2)^(1/2))/(b^2-c^2)^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3137, 3074, 206}

$$\frac{A\tan^{-1}\left(\frac{b\sinh(x)+c\cosh(x)}{\sqrt{b^2-c^2}}\right)}{\sqrt{b^2-c^2}}-\frac{cCx}{b^2-c^2}+\frac{bC\log(b\cosh(x)+c\sinh(x))}{b^2-c^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + C*Sinh[x])/(b*Cosh[x] + c*Sinh[x]),x]
```

```
[Out] -((c*C*x)/(b^2 - c^2)) + (A*ArcTan[(c*Cosh[x] + b*Sinh[x])/Sqrt[b^2 - c^2]]
)/Sqrt[b^2 - c^2] + (b*C*Log[b*Cosh[x] + c*Sinh[x]])/(b^2 - c^2)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3074

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> -Dist[d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d
*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3137

```
Int[((A_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])/((a_.) + cos[(d_.) + (e_.)*(x_)
]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)]), x_Symbol] :> Simp[(c*C*(d + e*x))
/(e*(b^2 + c^2)), x] + (Dist[(A*(b^2 + c^2) - a*c*C)/(b^2 + c^2), Int[1/(a
+ b*Cos[d + e*x] + c*Sin[d + e*x]), x], x] - Simp[(b*C*Log[a + b*Cos[d + e*
x] + c*Sin[d + e*x]])/(e*(b^2 + c^2)), x]) /; FreeQ[{a, b, c, d, e, A, C},
```

```
x] && NeQ[b^2 + c^2, 0] && NeQ[A*(b^2 + c^2) - a*c*C, 0]
```

Rubi steps

$$\int \frac{A+C\sinh(x)}{b\cosh(x)+c\sinh(x)}\,dx = -\frac{cCx}{b^2-c^2}+\frac{bC\log(b\cosh(x)+c\sinh(x))}{b^2-c^2}+A\int\frac{1}{b\cosh(x)+c\sinh(x)}\,dx$$
$$= -\frac{cCx}{b^2-c^2}+\frac{bC\log(b\cosh(x)+c\sinh(x))}{b^2-c^2}+(iA)\,\text{Subst}\left(\int\frac{1}{b^2-c^2-x^2}\,dx,x,-ic$$
$$= -\frac{cCx}{b^2-c^2}+\frac{A\tan^{-1}\left(\frac{c\cosh(x)+b\sinh(x)}{\sqrt{b^2-c^2}}\right)}{\sqrt{b^2-c^2}}+\frac{bC\log(b\cosh(x)+c\sinh(x))}{b^2-c^2}$$

Mathematica [A] time = 0.24, size = 78, normalized size = 0.98

$$\frac{2A\tan^{-1}\left(\frac{b\tanh\left(\frac{x}{2}\right)+c}{\sqrt{b-c}\sqrt{b+c}}\right)}{\sqrt{b-c}\sqrt{b+c}}+\frac{C(b\log(b\cosh(x)+c\sinh(x))-cx)}{b^2-c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + C*Sinh[x])/(b*Cosh[x] + c*Sinh[x]),x]
```

```
[Out] (2*A*ArcTan[(c + b*Tanh[x/2])/(Sqrt[b - c]*Sqrt[b + c])])/(Sqrt[b - c]*Sqrt
[b + c]) + (C*(-(c*x) + b*Log[b*Cosh[x] + c*Sinh[x]]))/(b^2 - c^2)
```

fricas [A] time = 0.53, size = 233, normalized size = 2.91

$$\left[\frac{Cb\log\left(\frac{2(b\cosh(x)+c\sinh(x))}{\cosh(x)-\sinh(x)}\right)-\sqrt{-b^2+c^2}\,A\log\left(\frac{(b+c)\cosh(x)^2+2(b+c)\cosh(x)\sinh(x)+(b+c)\sinh(x)^2-2\sqrt{-b^2+c^2}(\cosh(x)+\sinh(x)}{(b+c)\cosh(x)^2+2(b+c)\cosh(x)\sinh(x)+(b+c)\sinh(x)^2+b-c}\right.}{b^2-c^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sinh(x))/(b*cosh(x)+c*sinh(x)),x, algorithm="fricas")
```

```
[Out] [(C*b*log(2*(b*cosh(x) + c*sinh(x))/(cosh(x) - sinh(x))) - sqrt(-b^2 + c^2)
*A*log(((b + c)*cosh(x)^2 + 2*(b + c)*cosh(x)*sinh(x) + (b + c)*sinh(x)^2 -
 2*sqrt(-b^2 + c^2)*(cosh(x) + sinh(x)) - b + c)/((b + c)*cosh(x)^2 + 2*(b
+ c)*cosh(x)*sinh(x) + (b + c)*sinh(x)^2 + b - c)) - (C*b + C*c)*x)/(b^2 -
c^2), (C*b*log(2*(b*cosh(x) + c*sinh(x))/(cosh(x) - sinh(x))) - 2*sqrt(b^2
```

```
- c^2)*A*arctan(sqrt(b^2 - c^2)/((b + c)*cosh(x) + (b + c)*sinh(x))) - (C*b
 + C*c)*x)/(b^2 - c^2)]
```

giac [A] time = 0.12, size = 80, normalized size = 1.00

$$\frac{Cb\log\left(be^{(2x)}+ce^{(2x)}+b-c\right)}{b^2-c^2}+\frac{2A\arctan\left(\frac{be^x+ce^x}{\sqrt{b^2-c^2}}\right)}{\sqrt{b^2-c^2}}-\frac{Cx}{b-c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sinh(x))/(b*cosh(x)+c*sinh(x)),x, algorithm="giac")
```

```
[Out] C*b*log(b*e^(2*x) + c*e^(2*x) + b - c)/(b^2 - c^2) + 2*A*arctan((b*e^x + c*
e^x)/sqrt(b^2 - c^2))/sqrt(b^2 - c^2) - C*x/(b - c)
```

maple [B] time = 0.21, size = 181, normalized size = 2.26

$$-\frac{2C\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{2b+2c}-\frac{2C\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)}{2b-2c}+\frac{bC\ln\left(\left(\tanh^2\left(\frac{x}{2}\right)\right)b+2c\tanh\left(\frac{x}{2}\right)+b\right)}{(b-c)(b+c)}+\frac{2\arctan\left(\frac{2\tanh\left(\frac{x}{2}\right)b+2c}{2\sqrt{b^2-c^2}}\right)A}{(b-c)(b+c)\sqrt{b^2-c^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*sinh(x))/(b*cosh(x)+c*sinh(x)),x)
```

```
[Out] -2*C/(2*b+2*c)*ln(tanh(1/2*x)-1)-2*C/(2*b-2*c)*ln(tanh(1/2*x)+1)+1/(b-c)/(b
+c)*b*C*ln(tanh(1/2*x)^2*b+2*c*tanh(1/2*x)+b)+2/(b-c)/(b+c)/(b^2-c^2)^(1/2)
*arctan(1/2*(2*tanh(1/2*x)*b+2*c)/(b^2-c^2)^(1/2))*A*b^2-2/(b-c)/(b+c)/(b^2
-c^2)^(1/2)*arctan(1/2*(2*tanh(1/2*x)*b+2*c)/(b^2-c^2)^(1/2))*A*c^2
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sinh(x))/(b*cosh(x)+c*sinh(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*c^2-4*b^2>0)', see `assume?` for
more details)Is 4*c^2-4*b^2 positive or negative?
```

mupad [B] time = 3.78, size = 178, normalized size = 2.22

$$\frac{2\,\mathrm{atan}\left(\frac{A\,e^x\,\sqrt{b^2-c^2}}{b\,\sqrt{A^2}-c\,\sqrt{A^2}}\right)\sqrt{A^2}}{\sqrt{b^2-c^2}}-\frac{C\,x}{b-c}+\frac{C\,b^3\,\ln\left(4\,A^2\,b-4\,A^2\,c+4\,A^2\,b\,e^{2\,x}+4\,A^2\,c\,e^{2\,x}\right)}{b^4-2\,b^2\,c^2+c^4}-\frac{C\,b\,c^2\,\ln\left(4\,A^2\,b-4\,A^2\,c\right.}{b^4-2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + C*sinh(x))/(b*cosh(x) + c*sinh(x)),x)
```

```
[Out] (2*atan((A*exp(x)*(b^2 - c^2)^(1/2))/(b*(A^2)^(1/2) - c*(A^2)^(1/2)))*(A^2)
^(1/2))/(b^2 - c^2)^(1/2) - (C*x)/(b - c) + (C*b^3*log(4*A^2*b - 4*A^2*c +
4*A^2*b*exp(2*x) + 4*A^2*c*exp(2*x)))/(b^4 + c^4 - 2*b^2*c^2) - (C*b*c^2*lo
g(4*A^2*b - 4*A^2*c + 4*A^2*b*exp(2*x) + 4*A^2*c*exp(2*x)))/(b^4 + c^4 - 2*
b^2*c^2)
```

sympy [A] time = 53.14, size = 367, normalized size = 4.59

$$\begin{cases} \tilde{\infty}\left(A\log\left(\tanh\left(\frac{x}{2}\right)\right)+Cx\right) \\ -\frac{2A}{-2c\sinh(x)+2c\cosh(x)}-\frac{Cx\sinh(x)}{-2c\sinh(x)+2c\cosh(x)}+\frac{Cx\cosh(x)}{-2c\sinh(x)+2c\cosh(x)}-\frac{C\cosh(x)}{-2c\sinh(x)+2c\cosh(x)} \\ -\frac{2A}{2c\sinh(x)+2c\cosh(x)}+\frac{Cx\sinh(x)}{2c\sinh(x)+2c\cosh(x)}+\frac{Cx\cosh(x)}{2c\sinh(x)+2c\cosh(x)}+\frac{C\cosh(x)}{2c\sinh(x)+2c\cosh(x)} \\ \frac{A\log\left(\tanh\left(\frac{x}{2}\right)\right)+Cx}{c} \\ -\frac{A\sqrt{-b^2+c^2}\log\left(\tanh\left(\frac{x}{2}\right)+\frac{c}{b}-\frac{\sqrt{-b^2+c^2}}{b}\right)}{b^2-c^2}+\frac{A\sqrt{-b^2+c^2}\log\left(\tanh\left(\frac{x}{2}\right)+\frac{c}{b}+\frac{\sqrt{-b^2+c^2}}{b}\right)}{b^2-c^2}+\frac{Cbx}{b^2-c^2}-\frac{2Cb\log\left(\tanh\left(\frac{x}{2}\right)+1\right)}{b^2-c^2}+\frac{Cb\log\left(\tanh\left(\frac{x}{2}\right)-\cdots\right.}{b^2-c\cdots} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sinh(x))/(b*cosh(x)+c*sinh(x)),x)
```

```
[Out] Piecewise((zoo*(A*log(tanh(x/2)) + C*x), Eq(b, 0) & Eq(c, 0)), (-2*A/(-2*c*
sinh(x) + 2*c*cosh(x)) - C*x*sinh(x)/(-2*c*sinh(x) + 2*c*cosh(x)) + C*x*cos
h(x)/(-2*c*sinh(x) + 2*c*cosh(x)) - C*cosh(x)/(-2*c*sinh(x) + 2*c*cosh(x)),
 Eq(b, -c)), (-2*A/(2*c*sinh(x) + 2*c*cosh(x)) + C*x*sinh(x)/(2*c*sinh(x) +
 2*c*cosh(x)) + C*x*cosh(x)/(2*c*sinh(x) + 2*c*cosh(x)) + C*cosh(x)/(2*c*si
nh(x) + 2*c*cosh(x)), Eq(b, c)), ((A*log(tanh(x/2)) + C*x)/c, Eq(b, 0)), (-
A*sqrt(-b**2 + c**2)*log(tanh(x/2) + c/b - sqrt(-b**2 + c**2)/b)/(b**2 - c*
*2) + A*sqrt(-b**2 + c**2)*log(tanh(x/2) + c/b + sqrt(-b**2 + c**2)/b)/(b**
2 - c**2) + C*b*x/(b**2 - c**2) - 2*C*b*log(tanh(x/2) + 1)/(b**2 - c**2) +
C*b*log(tanh(x/2) + c/b - sqrt(-b**2 + c**2)/b)/(b**2 - c**2) + C*b*log(tan
h(x/2) + c/b + sqrt(-b**2 + c**2)/b)/(b**2 - c**2) - C*c*x/(b**2 - c**2), T
rue))
```

3.725 $\int \frac{A+C \sinh(x)}{(b \cosh(x)+c \sinh(x))^2} \, dx$

Optimal. Leaf size=82

$$-\frac{-Ab\sinh(x)-Ac\cosh(x)+bC}{\left(b^2-c^2\right)(b\cosh(x)+c\sinh(x))}-\frac{cC\tan^{-1}\left(\frac{b\sinh(x)+c\cosh(x)}{\sqrt{b^2-c^2}}\right)}{\left(b^2-c^2\right)^{3/2}}$$

```
[Out] -c*C*arctan((c*cosh(x)+b*sinh(x))/(b^2-c^2)^(1/2))/(b^2-c^2)^(3/2)+(-b*C+A*
c*cosh(x)+A*b*sinh(x))/(b^2-c^2)/(b*cosh(x)+c*sinh(x))
```

Rubi [A] time = 0.08, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3154, 3074, 206}

$$-\frac{-Ab\sinh(x)-Ac\cosh(x)+bC}{\left(b^2-c^2\right)(b\cosh(x)+c\sinh(x))}-\frac{cC\tan^{-1}\left(\frac{b\sinh(x)+c\cosh(x)}{\sqrt{b^2-c^2}}\right)}{\left(b^2-c^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + C*Sinh[x])/(b*Cosh[x] + c*Sinh[x])^2,x]
```

```
[Out] -((c*C*ArcTan[(c*Cosh[x] + b*Sinh[x])/Sqrt[b^2 - c^2]])/(b^2 - c^2)^(3/2))
- (b*C - A*c*Cosh[x] - A*b*Sinh[x])/((b^2 - c^2)*(b*Cosh[x] + c*Sinh[x]))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3074

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> -Dist[d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d
*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3154

```
Int[((A_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])/((a_.) + cos[(d_.) + (e_.)*(x_)
]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])^2, x_Symbol] :> -Simp[(b*C + (a*C
- c*A)*Cos[d + e*x] + b*A*Sin[d + e*x])/(e*(a^2 - b^2 - c^2)*(a + b*Cos[d +
 e*x] + c*Sin[d + e*x])), x] + Dist[(a*A - c*C)/(a^2 - b^2 - c^2), Int[1/(a
 + b*Cos[d + e*x] + c*Sin[d + e*x]), x], x] /; FreeQ[{a, b, c, d, e, A, C},
```

```
x] && NeQ[a^2 - b^2 - c^2, 0] && NeQ[a*A - c*C, 0]
```

Rubi steps

$$\int \frac{A+C\sinh(x)}{(b\cosh(x)+c\sinh(x))^2}\,dx = -\frac{bC-Ac\cosh(x)-Ab\sinh(x)}{\left(b^2-c^2\right)(b\cosh(x)+c\sinh(x))} - \frac{(cC)\int \frac{1}{b\cosh(x)+c\sinh(x)}\,dx}{b^2-c^2}$$

$$= -\frac{bC-Ac\cosh(x)-Ab\sinh(x)}{\left(b^2-c^2\right)(b\cosh(x)+c\sinh(x))} - \frac{(icC)\,\text{Subst}\left(\int \frac{1}{b^2-c^2-x^2}\,dx, x, -ic\cosh(x)$$

$$= -\frac{cC\tan^{-1}\left(\frac{c\cosh(x)+b\sinh(x)}{\sqrt{b^2-c^2}}\right)}{\left(b^2-c^2\right)^{3/2}} - \frac{bC-Ac\cosh(x)-Ab\sinh(x)}{\left(b^2-c^2\right)(b\cosh(x)+c\sinh(x))}$$

Mathematica [A] time = 0.46, size = 155, normalized size = 1.89

$$-\frac{\sinh(x)\left(2bc^2C\sqrt{b+c}\,\tan^{-1}\left(\frac{b\tanh\left(\frac{x}{2}\right)+c}{\sqrt{b-c}\,\sqrt{b+c}}\right)-A(b-c)^{3/2}(b+c)^2\right)+2b^2cC\sqrt{b+c}\,\cosh(x)\tan^{-1}\left(\frac{b\tanh\left(\frac{x}{2}\right)+c}{\sqrt{b-c}\,\sqrt{b+c}}\right)+b}{b(b-c)^{3/2}(b+c)^2(b\cosh(x)+c\sinh(x))}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + C*Sinh[x])/(b*Cosh[x] + c*Sinh[x])^2,x]
```

```
[Out] -((b^2*Sqrt[b - c]*(b + c)*C + 2*b^2*c*Sqrt[b + c]*C*ArcTan[(c + b*Tanh[x/2
])/(Sqrt[b - c]*Sqrt[b + c])]*Cosh[x] + (-(A*(b - c)^(3/2)*(b + c)^2) + 2*b
*c^2*Sqrt[b + c]*C*ArcTan[(c + b*Tanh[x/2])/(Sqrt[b - c]*Sqrt[b + c])])*Sin
h[x])/(b*(b - c)^(3/2)*(b + c)^2*(b*Cosh[x] + c*Sinh[x])))
```

fricas [B] time = 0.50, size = 679, normalized size = 8.28

$$\left[-\frac{2\,Ab^3-2\,Ab^2c-2\,Abc^2+2\,Ac^3-\left(Cbc-Cc^2+\left(Cbc+Cc^2\right)\cosh(x)^2+2\left(Cbc+Cc^2\right)\cosh(x)\sinh(x)+\left(C\right.\right.}{b^5-b^4c-2\,b^3c^2+2\,b^2c^3+bc^4-c^5+\left(b^5+b^4c-2\,b^3c^2-2\,b^2c^3+bc^4\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sinh(x))/(b*cosh(x)+c*sinh(x))^2,x, algorithm="fricas")
```

```
[Out] [-(2*A*b^3 - 2*A*b^2*c - 2*A*b*c^2 + 2*A*c^3 - (C*b*c - C*c^2 + (C*b*c + C*
c^2)*cosh(x)^2 + 2*(C*b*c + C*c^2)*cosh(x)*sinh(x) + (C*b*c + C*c^2)*sinh(x
```

```
)^2)*sqrt(-b^2 + c^2)*log(((b + c)*cosh(x)^2 + 2*(b + c)*cosh(x)*sinh(x) +
(b + c)*sinh(x)^2 - 2*sqrt(-b^2 + c^2)*(cosh(x) + sinh(x)) - b + c)/((b + c
)*cosh(x)^2 + 2*(b + c)*cosh(x)*sinh(x) + (b + c)*sinh(x)^2 + b - c)) + 2*(
C*b^3 - C*b*c^2)*cosh(x) + 2*(C*b^3 - C*b*c^2)*sinh(x))/(b^5 - b^4*c - 2*b^
3*c^2 + 2*b^2*c^3 + b*c^4 - c^5 + (b^5 + b^4*c - 2*b^3*c^2 - 2*b^2*c^3 + b*
c^4 + c^5)*cosh(x)^2 + 2*(b^5 + b^4*c - 2*b^3*c^2 - 2*b^2*c^3 + b*c^4 + c^5
)*cosh(x)*sinh(x) + (b^5 + b^4*c - 2*b^3*c^2 - 2*b^2*c^3 + b*c^4 + c^5)*sin
h(x)^2), -2*(A*b^3 - A*b^2*c - A*b*c^2 + A*c^3 - (C*b*c - C*c^2 + (C*b*c +
C*c^2)*cosh(x)^2 + 2*(C*b*c + C*c^2)*cosh(x)*sinh(x) + (C*b*c + C*c^2)*sinh
(x)^2)*sqrt(b^2 - c^2)*arctan(sqrt(b^2 - c^2)/((b + c)*cosh(x) + (b + c)*si
nh(x))) + (C*b^3 - C*b*c^2)*cosh(x) + (C*b^3 - C*b*c^2)*sinh(x))/(b^5 - b^4
*c - 2*b^3*c^2 + 2*b^2*c^3 + b*c^4 - c^5 + (b^5 + b^4*c - 2*b^3*c^2 - 2*b^2
*c^3 + b*c^4 + c^5)*cosh(x)^2 + 2*(b^5 + b^4*c - 2*b^3*c^2 - 2*b^2*c^3 + b*
c^4 + c^5)*cosh(x)*sinh(x) + (b^5 + b^4*c - 2*b^3*c^2 - 2*b^2*c^3 + b*c^4 +
 c^5)*sinh(x)^2)]
```

giac [A] time = 0.12, size = 83, normalized size = 1.01

$$-\frac{2\,Cc\arctan\left(\frac{be^{x}+ce^{x}}{\sqrt{b^{2}-c^{2}}}\right)}{\left(b^{2}-c^{2}\right)^{\frac{3}{2}}}-\frac{2\,\left(Cbe^{x}+Ab-Ac\right)}{\left(b^{2}-c^{2}\right)\left(be^{(2\,x)}+ce^{(2\,x)}+b-c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sinh(x))/(b*cosh(x)+c*sinh(x))^2,x, algorithm="giac")
```

```
[Out] -2*C*c*arctan((b*e^x + c*e^x)/sqrt(b^2 - c^2))/(b^2 - c^2)^(3/2) - 2*(C*b*e
^x + A*b - A*c)/((b^2 - c^2)*(b*e^(2*x) + c*e^(2*x) + b - c))
```

maple [A] time = 0.26, size = 115, normalized size = 1.40

$$-\frac{2\left(-\frac{\left(A\,b^{2}-A\,c^{2}-Ccb\right)\tanh\left(\frac{x}{2}\right)}{\left(b^{2}-c^{2}\right)b}+\frac{bC}{b^{2}-c^{2}}\right)}{\left(\tanh^{2}\left(\frac{x}{2}\right)\right)b+2c\tanh\left(\frac{x}{2}\right)+b}-\frac{2Cc\arctan\left(\frac{2\tanh\left(\frac{x}{2}\right)b+2c}{2\sqrt{b^{2}-c^{2}}}\right)}{\left(b^{2}-c^{2}\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*sinh(x))/(b*cosh(x)+c*sinh(x))^2,x)
```

```
[Out] -2*(-(A*b^2-A*c^2-C*b*c)/(b^2-c^2)/b*tanh(1/2*x)+b*C/(b^2-c^2))/(tanh(1/2*x
)^2*b+2*c*tanh(1/2*x)+b)-2*C*c/(b^2-c^2)^(3/2)*arctan(1/2*(2*tanh(1/2*x)*b+
2*c)/(b^2-c^2)^(1/2))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sinh(x))/(b*cosh(x)+c*sinh(x))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*c^2-4*b^2>0)', see `assume?` for
more details)Is 4*c^2-4*b^2 positive or negative?
```

mupad [B] time = 1.85, size = 168, normalized size = 2.05

$$\frac{C\,c\,\ln\left(\frac{2\,C\,c}{{(b+c)}^{5/2}\,\sqrt{c-b}}-\frac{2\,C\,c\,\mathrm{e}^x}{-b^3-b^2\,c+b\,c^2+c^3}\right)}{{(b+c)}^{3/2}\,{(c-b)}^{3/2}}-\frac{C\,c\,\ln\left(-\frac{2\,C\,c}{{(b+c)}^{5/2}\,\sqrt{c-b}}-\frac{2\,C\,c\,\mathrm{e}^x}{-b^3-b^2\,c+b\,c^2+c^3}\right)}{{(b+c)}^{3/2}\,{(c-b)}^{3/2}}-\frac{\frac{2\,A}{b+c}+\frac{2\,C\,b\,\mathrm{e}^x}{(b+c)\,(b-c)}}{b-c+\mathrm{e}^{2\,x}\,(b+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + C*sinh(x))/(b*cosh(x) + c*sinh(x))^2,x)
```

```
[Out] (C*c*log((2*C*c)/((b + c)^(5/2)*(c - b)^(1/2)) - (2*C*c*exp(x))/(b*c^2 - b^
2*c - b^3 + c^3)))/((b + c)^(3/2)*(c - b)^(3/2)) - (C*c*log(- (2*C*c)/((b +
 c)^(5/2)*(c - b)^(1/2)) - (2*C*c*exp(x))/(b*c^2 - b^2*c - b^3 + c^3)))/((b
 + c)^(3/2)*(c - b)^(3/2)) - ((2*A)/(b + c) + (2*C*b*exp(x))/((b + c)*(b -
c)))/(b - c + exp(2*x)*(b + c))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sinh(x))/(b*cosh(x)+c*sinh(x))**2,x)
```

```
[Out] Timed out
```

3.726 $\int \frac{A+C\sinh(x)}{(b\cosh(x)+c\sinh(x))^3}\,dx$

Optimal. Leaf size=123

$$-\frac{-Ab\sinh(x)-Ac\cosh(x)+bC}{2\left(b^2-c^2\right)(b\cosh(x)+c\sinh(x))^2}+\frac{A\tan^{-1}\left(\frac{b\sinh(x)+c\cosh(x)}{\sqrt{b^2-c^2}}\right)}{2\left(b^2-c^2\right)^{3/2}}-\frac{bcC\sinh(x)+c^2C\cosh(x)}{\left(b^2-c^2\right)^2(b\cosh(x)+c\sinh(x))}$$

```
[Out] 1/2*A*arctan((c*cosh(x)+b*sinh(x))/(b^2-c^2)^(1/2))/(b^2-c^2)^(3/2)+1/2*(-b
*C+A*c*cosh(x)+A*b*sinh(x))/(b^2-c^2)/(b*cosh(x)+c*sinh(x))^2+(-c^2*C*cosh(
x)-b*c*C*sinh(x))/(b^2-c^2)^2/(b*cosh(x)+c*sinh(x))
```

Rubi [A] time = 0.13, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {3157, 3153, 3074, 206}

$$-\frac{-Ab\sinh(x)-Ac\cosh(x)+bC}{2\left(b^2-c^2\right)(b\cosh(x)+c\sinh(x))^2}+\frac{A\tan^{-1}\left(\frac{b\sinh(x)+c\cosh(x)}{\sqrt{b^2-c^2}}\right)}{2\left(b^2-c^2\right)^{3/2}}-\frac{bcC\sinh(x)+c^2C\cosh(x)}{\left(b^2-c^2\right)^2(b\cosh(x)+c\sinh(x))}$$

Antiderivative was successfully verified.

```
[In] Int[(A + C*Sinh[x])/(b*Cosh[x] + c*Sinh[x])^3,x]
```

```
[Out] (A*ArcTan[(c*Cosh[x] + b*Sinh[x])/Sqrt[b^2 - c^2]])/(2*(b^2 - c^2)^(3/2)) -
 (b*C - A*c*Cosh[x] - A*b*Sinh[x])/(2*(b^2 - c^2)*(b*Cosh[x] + c*Sinh[x])^2
) - (c^2*C*Cosh[x] + b*c*C*Sinh[x])/((b^2 - c^2)^2*(b*Cosh[x] + c*Sinh[x]))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3074

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> -Dist[d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d
*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3153

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])^2,
 x_Symbol] :> Simp[(c*B - b*C - (a*C - c*A)*Cos[d + e*x] + (a*B - b*A)*Sin[
```

```
d + e*x])/(e*(a^2 - b^2 - c^2)*(a + b*Cos[d + e*x] + c*Sin[d + e*x])), x] +
 Dist[(a*A - b*B - c*C)/(a^2 - b^2 - c^2), Int[1/(a + b*Cos[d + e*x] + c*Si
n[d + e*x]), x], x] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && NeQ[a^2 - b^2
- c^2, 0] && NeQ[a*A - b*B - c*C, 0]
```

Rule 3157

```
Int[((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])
^(n_)*((A_.) + (C_.)*sin[(d_.) + (e_.)*(x_)]), x_Symbol] :> Simp[((b*C + (a
*C - c*A)*Cos[d + e*x] + b*A*Sin[d + e*x])*(a + b*Cos[d + e*x] + c*Sin[d +
e*x])^(n + 1))/(e*(n + 1)*(a^2 - b^2 - c^2)), x] + Dist[1/((n + 1)*(a^2 - b
^2 - c^2)), Int[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1)*Simp[(n + 1)*
(a*A - c*C) - (n + 2)*b*A*Cos[d + e*x] + (n + 2)*(a*C - c*A)*Sin[d + e*x],
x], x], x] /; FreeQ[{a, b, c, d, e, A, C}, x] && LtQ[n, -1] && NeQ[a^2 - b^
2 - c^2, 0] && NeQ[n, -2]
```

Rubi steps

$$\int \frac{A+C\sinh(x)}{(b\cosh(x)+c\sinh(x))^3}\,dx = -\frac{bC-Ac\cosh(x)-Ab\sinh(x)}{2\left(b^2-c^2\right)(b\cosh(x)+c\sinh(x))^2}+\frac{\int \frac{-2cC+Ab\cosh(x)+Ac\sinh(x)}{(b\cosh(x)+c\sinh(x))^2}\,dx}{2\left(b^2-c^2\right)}$$

$$= -\frac{bC-Ac\cosh(x)-Ab\sinh(x)}{2\left(b^2-c^2\right)(b\cosh(x)+c\sinh(x))^2}-\frac{c^2C\cosh(x)+bcC\sinh(x)}{\left(b^2-c^2\right)^2(b\cosh(x)+c\sinh(x))}+\frac{A}{\ldots}$$

$$= -\frac{bC-Ac\cosh(x)-Ab\sinh(x)}{2\left(b^2-c^2\right)(b\cosh(x)+c\sinh(x))^2}-\frac{c^2C\cosh(x)+bcC\sinh(x)}{\left(b^2-c^2\right)^2(b\cosh(x)+c\sinh(x))}+\frac{(i\ldots}{\ldots}$$

$$= \frac{A\tan^{-1}\left(\frac{c\cosh(x)+b\sinh(x)}{\sqrt{b^2-c^2}}\right)}{2\left(b^2-c^2\right)^{3/2}}-\frac{bC-Ac\cosh(x)-Ab\sinh(x)}{2\left(b^2-c^2\right)(b\cosh(x)+c\sinh(x))^2}-\frac{c^2C\cos\ldots}{\left(b^2-c^2\right)^2(\ldots}$$

Mathematica [A] time = 1.19, size = 134, normalized size = 1.09

$$\frac{1}{2}\left(\frac{A\left(b^2-c^2\right)\sinh(x)-b^2C}{b(b-c)(b+c)(b\cosh(x)+c\sinh(x))^2}+\frac{c(A-2C\sinh(x))}{b(b-c)(b+c)(b\cosh(x)+c\sinh(x))}+\frac{2A\tan^{-1}\left(\frac{b\tanh\left(\frac{x}{2}\right)+c}{\sqrt{b-c}\sqrt{b+c}}\right)}{(b-c)^{3/2}(b+c)^{3/2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + C*Sinh[x])/(b*Cosh[x] + c*Sinh[x])^3,x]
```

```
[Out] ((2*A*ArcTan[(c + b*Tanh[x/2])/(Sqrt[b - c]*Sqrt[b + c])])/((b - c)^(3/2)*(
b + c)^(3/2)) + (-(b^2*C) + A*(b^2 - c^2)*Sinh[x])/(b*(b - c)*(b + c)*(b*Co
sh[x] + c*Sinh[x])^2) + (c*(A - 2*C*Sinh[x]))/(b*(b - c)*(b + c)*(b*Cosh[x]
 + c*Sinh[x])))/2
```

<u>fricas</u> [B] time = 0.56, size = 1855, normalized size = 15.08

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sinh(x))/(b*cosh(x)+c*sinh(x))^3,x, algorithm="fricas")
```

```
[Out] [1/2*(4*C*b^2*c - 8*C*b*c^2 + 4*C*c^3 + 2*(A*b^3 + A*b^2*c - A*b*c^2 - A*c^
3)*cosh(x)^3 + 2*(A*b^3 + A*b^2*c - A*b*c^2 - A*c^3)*sinh(x)^3 - 4*(C*b^3 -
 C*b^2*c - C*b*c^2 + C*c^3)*cosh(x)^2 - 2*(2*C*b^3 - 2*C*b^2*c - 2*C*b*c^2
+ 2*C*c^3 - 3*(A*b^3 + A*b^2*c - A*b*c^2 - A*c^3)*cosh(x))*sinh(x)^2 + ((A*
b^2 + 2*A*b*c + A*c^2)*cosh(x)^4 + 4*(A*b^2 + 2*A*b*c + A*c^2)*cosh(x)*sinh
(x)^3 + (A*b^2 + 2*A*b*c + A*c^2)*sinh(x)^4 + A*b^2 - 2*A*b*c + A*c^2 + 2*(
A*b^2 - A*c^2)*cosh(x)^2 + 2*(A*b^2 - A*c^2 + 3*(A*b^2 + 2*A*b*c + A*c^2)*c
osh(x)^2)*sinh(x)^2 + 4*((A*b^2 + 2*A*b*c + A*c^2)*cosh(x)^3 + (A*b^2 - A*c
^2)*cosh(x))*sinh(x))*sqrt(-b^2 + c^2)*log(((b + c)*cosh(x)^2 + 2*(b + c)*c
osh(x)*sinh(x) + (b + c)*sinh(x)^2 + 2*sqrt(-b^2 + c^2)*(cosh(x) + sinh(x))
 - b + c)/((b + c)*cosh(x)^2 + 2*(b + c)*cosh(x)*sinh(x) + (b + c)*sinh(x)^
2 + b - c)) - 2*(A*b^3 - A*b^2*c - A*b*c^2 + A*c^3)*cosh(x) - 2*(A*b^3 - A*
b^2*c - A*b*c^2 + A*c^3 - 3*(A*b^3 + A*b^2*c - A*b*c^2 - A*c^3)*cosh(x)^2 +
 4*(C*b^3 - C*b^2*c - C*b*c^2 + C*c^3)*cosh(x))*sinh(x))/(b^6 - 2*b^5*c - b
^4*c^2 + 4*b^3*c^3 - b^2*c^4 - 2*b*c^5 + c^6 + (b^6 + 2*b^5*c - b^4*c^2 - 4
*b^3*c^3 - b^2*c^4 + 2*b*c^5 + c^6)*cosh(x)^4 + 4*(b^6 + 2*b^5*c - b^4*c^2
- 4*b^3*c^3 - b^2*c^4 + 2*b*c^5 + c^6)*cosh(x)*sinh(x)^3 + (b^6 + 2*b^5*c -
 b^4*c^2 - 4*b^3*c^3 - b^2*c^4 + 2*b*c^5 + c^6)*sinh(x)^4 + 2*(b^6 - 3*b^4*
c^2 + 3*b^2*c^4 - c^6)*cosh(x)^2 + 2*(b^6 - 3*b^4*c^2 + 3*b^2*c^4 - c^6 + 3
*(b^6 + 2*b^5*c - b^4*c^2 - 4*b^3*c^3 - b^2*c^4 + 2*b*c^5 + c^6)*cosh(x)^2)
*sinh(x)^2 + 4*((b^6 + 2*b^5*c - b^4*c^2 - 4*b^3*c^3 - b^2*c^4 + 2*b*c^5 +
c^6)*cosh(x)^3 + (b^6 - 3*b^4*c^2 + 3*b^2*c^4 - c^6)*cosh(x))*sinh(x)), (2*
C*b^2*c - 4*C*b*c^2 + 2*C*c^3 + (A*b^3 + A*b^2*c - A*b*c^2 - A*c^3)*cosh(x)
^3 + (A*b^3 + A*b^2*c - A*b*c^2 - A*c^3)*sinh(x)^3 - 2*(C*b^3 - C*b^2*c - C
*b*c^2 + C*c^3)*cosh(x)^2 - (2*C*b^3 - 2*C*b^2*c - 2*C*b*c^2 + 2*C*c^3 - 3*
(A*b^3 + A*b^2*c - A*b*c^2 - A*c^3)*cosh(x))*sinh(x)^2 - ((A*b^2 + 2*A*b*c
+ A*c^2)*cosh(x)^4 + 4*(A*b^2 + 2*A*b*c + A*c^2)*cosh(x)*sinh(x)^3 + (A*b^2
 + 2*A*b*c + A*c^2)*sinh(x)^4 + A*b^2 - 2*A*b*c + A*c^2 + 2*(A*b^2 - A*c^2)
*cosh(x)^2 + 2*(A*b^2 - A*c^2 + 3*(A*b^2 + 2*A*b*c + A*c^2)*cosh(x)^2)*sinh
(x)^2 + 4*((A*b^2 + 2*A*b*c + A*c^2)*cosh(x)^3 + (A*b^2 - A*c^2)*cosh(x))*s
inh(x))*sqrt(b^2 - c^2)*arctan(sqrt(b^2 - c^2)/((b + c)*cosh(x) + (b + c)*s
```

```
inh(x))) - (A*b^3 - A*b^2*c - A*b*c^2 + A*c^3)*cosh(x) - (A*b^3 - A*b^2*c -
 A*b*c^2 + A*c^3 - 3*(A*b^3 + A*b^2*c - A*b*c^2 - A*c^3)*cosh(x)^2 + 4*(C*b
^3 - C*b^2*c - C*b*c^2 + C*c^3)*cosh(x))*sinh(x))/(b^6 - 2*b^5*c - b^4*c^2
+ 4*b^3*c^3 - b^2*c^4 - 2*b*c^5 + c^6 + (b^6 + 2*b^5*c - b^4*c^2 - 4*b^3*c^
3 - b^2*c^4 + 2*b*c^5 + c^6)*cosh(x)^4 + 4*(b^6 + 2*b^5*c - b^4*c^2 - 4*b^3
*c^3 - b^2*c^4 + 2*b*c^5 + c^6)*cosh(x)*sinh(x)^3 + (b^6 + 2*b^5*c - b^4*c^
2 - 4*b^3*c^3 - b^2*c^4 + 2*b*c^5 + c^6)*sinh(x)^4 + 2*(b^6 - 3*b^4*c^2 + 3
*b^2*c^4 - c^6)*cosh(x)^2 + 2*(b^6 - 3*b^4*c^2 + 3*b^2*c^4 - c^6 + 3*(b^6 +
 2*b^5*c - b^4*c^2 - 4*b^3*c^3 - b^2*c^4 + 2*b*c^5 + c^6)*cosh(x)^2)*sinh(x
)^2 + 4*((b^6 + 2*b^5*c - b^4*c^2 - 4*b^3*c^3 - b^2*c^4 + 2*b*c^5 + c^6)*co
sh(x)^3 + (b^6 - 3*b^4*c^2 + 3*b^2*c^4 - c^6)*cosh(x))*sinh(x))]
```

giac [A] time = 0.13, size = 152, normalized size = 1.24

$$\frac{A \arctan\left(\frac{be^x+ce^x}{\sqrt{b^2-c^2}}\right)}{\left(b^2-c^2\right)^{\frac{3}{2}}}+\frac{Ab^2e^{(3\,x)}+2\,Abce^{(3\,x)}+Ac^2e^{(3\,x)}-2\,Cb^2e^{(2\,x)}+2\,Cc^2e^{(2\,x)}-Ab^2e^x+Ac^2e^x+2\,Cbc-2\,Cc^2}{\left(b^3+b^2c-bc^2-c^3\right)\left(be^{(2\,x)}+ce^{(2\,x)}+b-c\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sinh(x))/(b*cosh(x)+c*sinh(x))^3,x, algorithm="giac")
```

```
[Out] A*arctan((b*e^x + c*e^x)/sqrt(b^2 - c^2))/(b^2 - c^2)^(3/2) + (A*b^2*e^(3*x
) + 2*A*b*c*e^(3*x) + A*c^2*e^(3*x) - 2*C*b^2*e^(2*x) + 2*C*c^2*e^(2*x) - A
*b^2*e^x + A*c^2*e^x + 2*C*b*c - 2*C*c^2)/((b^3 + b^2*c - b*c^2 - c^3)*(b*e
^(2*x) + c*e^(2*x) + b - c)^2)
```

maple [A] time = 0.28, size = 187, normalized size = 1.52

$$\frac{-\frac{A\left(b^2-2c^2\right)\left(\tanh^3\left(\frac{x}{2}\right)\right)}{\left(b^2-c^2\right)b}+\frac{\left(A\,b^2c+2A\,c^3+2C\,b^3-2Cb\,c^2\right)\left(\tanh^2\left(\frac{x}{2}\right)\right)}{\left(b^2-c^2\right)b^2}+\frac{A\left(b^2+2c^2\right)\tanh\left(\frac{x}{2}\right)}{\left(b^2-c^2\right)b}+\frac{2Ac}{2b^2-2c^2}}{\left(\left(\tanh^2\left(\frac{x}{2}\right)\right)b+2c\tanh\left(\frac{x}{2}\right)+b\right)^2}+\frac{A\arctan\left(\frac{2\tanh\left(\frac{x}{2}\right)b+2c}{2\sqrt{b^2-c^2}}\right)}{\left(b^2-c^2\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*sinh(x))/(b*cosh(x)+c*sinh(x))^3,x)
```

```
[Out] 2*(-1/2*A*(b^2-2*c^2)/(b^2-c^2)/b*tanh(1/2*x)^3+1/2*(A*b^2*c+2*A*c^3+2*C*b^
3-2*C*b*c^2)/(b^2-c^2)/b^2*tanh(1/2*x)^2+1/2*A*(b^2+2*c^2)/(b^2-c^2)/b*tanh
(1/2*x)+1/2*A*c/(b^2-c^2))/(tanh(1/2*x)^2*b+2*c*tanh(1/2*x)+b)^2+A/(b^2-c^2
)^(3/2)*arctan(1/2*(2*tanh(1/2*x)*b+2*c)/(b^2-c^2)^(1/2))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sinh(x))/(b*cosh(x)+c*sinh(x))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*c^2-4*b^2>0)', see `assume?` for
more details)Is 4*c^2-4*b^2 positive or negative?
```

mupad [B] time = 1.69, size = 217, normalized size = 1.76

$$\frac{\operatorname{atan}\left(\frac{A\,e^x\,\sqrt{b^6-3\,b^4\,c^2+3\,b^2\,c^4-c^6}}{b^3\,\sqrt{A^2}+c^3\,\sqrt{A^2}-b\,c^2\,\sqrt{A^2}-b^2\,c\,\sqrt{A^2}}\right)\,\sqrt{A^2}}{\sqrt{b^6-3\,b^4\,c^2+3\,b^2\,c^4-c^6}}-\frac{\frac{C}{{(b+c)}^2}-\frac{A\,e^x}{(b+c)\,(b-c)}}{b-c+e^{2\,x}\,(b+c)}-\frac{\frac{2\,A\,e^x}{b+c}-\frac{C}{b+c}+\frac{C\,e^{2\,x}}{b+c}}{e^{4\,x}\,{(b+c)}^2+{(b-c)}^2+2\,e^{2\,x}\,(b+c)\,(b-c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + C*sinh(x))/(b*cosh(x) + c*sinh(x))^3,x)
```

```
[Out] (atan((A*exp(x)*(b^6 - c^6 + 3*b^2*c^4 - 3*b^4*c^2)^(1/2))/(b^3*(A^2)^(1/2)
 + c^3*(A^2)^(1/2) - b*c^2*(A^2)^(1/2) - b^2*c*(A^2)^(1/2)))*(A^2)^(1/2))/(
b^6 - c^6 + 3*b^2*c^4 - 3*b^4*c^2)^(1/2) - (C/(b + c)^2 - (A*exp(x))/((b +
c)*(b - c)))/(b - c + exp(2*x)*(b + c)) - ((2*A*exp(x))/(b + c) - C/(b + c)
 + (C*exp(2*x))/(b + c))/(exp(4*x)*(b + c)^2 + (b - c)^2 + 2*exp(2*x)*(b +
c)*(b - c))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sinh(x))/(b*cosh(x)+c*sinh(x))**3,x)
```

```
[Out] Timed out
```

3.727 $\int \frac{A+B\cosh(x)}{b\cosh(x)+c\sinh(x)}\,dx$

Optimal. Leaf size=80

$$\frac{A\tan^{-1}\left(\frac{b\sinh(x)+c\cosh(x)}{\sqrt{b^2-c^2}}\right)}{\sqrt{b^2-c^2}}+\frac{bBx}{b^2-c^2}-\frac{Bc\log(b\cosh(x)+c\sinh(x))}{b^2-c^2}$$

```
[Out] b*B*x/(b^2-c^2)-B*c*ln(b*cosh(x)+c*sinh(x))/(b^2-c^2)+A*arctan((c*cosh(x)+b
*sinh(x))/(b^2-c^2)^(1/2))/(b^2-c^2)^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3138, 3074, 206}

$$\frac{A\tan^{-1}\left(\frac{b\sinh(x)+c\cosh(x)}{\sqrt{b^2-c^2}}\right)}{\sqrt{b^2-c^2}}+\frac{bBx}{b^2-c^2}-\frac{Bc\log(b\cosh(x)+c\sinh(x))}{b^2-c^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cosh[x])/(b*Cosh[x] + c*Sinh[x]),x]
```

```
[Out] (b*B*x)/(b^2 - c^2) + (A*ArcTan[(c*Cosh[x] + b*Sinh[x])/Sqrt[b^2 - c^2]])/S
qrt[b^2 - c^2] - (B*c*Log[b*Cosh[x] + c*Sinh[x]])/(b^2 - c^2)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3074

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> -Dist[d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d
*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3138

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.))/((a_.) + cos[(d_.) + (e_.)*(x_)
]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)]), x_Symbol] :> Simp[(b*B*(d + e*x))
/(e*(b^2 + c^2)), x] + (Dist[(A*(b^2 + c^2) - a*b*B)/(b^2 + c^2), Int[1/(a
+ b*Cos[d + e*x] + c*Sin[d + e*x]), x], x] + Simp[(c*B*Log[a + b*Cos[d + e*
x] + c*Sin[d + e*x]])/(e*(b^2 + c^2)), x]) /; FreeQ[{a, b, c, d, e, A, B},
```

```
x] && NeQ[b^2 + c^2, 0] && NeQ[A*(b^2 + c^2) - a*b*B, 0]
```

Rubi steps

$$\int \frac{A+B\cosh(x)}{b\cosh(x)+c\sinh(x)}\,dx = \frac{bBx}{b^2-c^2}-\frac{Bc\log(b\cosh(x)+c\sinh(x))}{b^2-c^2}+A\int \frac{1}{b\cosh(x)+c\sinh(x)}\,dx$$
$$= \frac{bBx}{b^2-c^2}-\frac{Bc\log(b\cosh(x)+c\sinh(x))}{b^2-c^2}+(iA)\,\text{Subst}\left(\int \frac{1}{b^2-c^2-x^2}\,dx,x,-ic\,\text{co}\right.$$
$$= \frac{bBx}{b^2-c^2}+\frac{A\tan^{-1}\left(\frac{c\cosh(x)+b\sinh(x)}{\sqrt{b^2-c^2}}\right)}{\sqrt{b^2-c^2}}-\frac{Bc\log(b\cosh(x)+c\sinh(x))}{b^2-c^2}$$

Mathematica [A] time = 0.18, size = 78, normalized size = 0.98

$$\frac{2A\sqrt{b-c}\,\sqrt{b+c}\,\tan^{-1}\left(\frac{b\tanh\left(\frac{x}{2}\right)+c}{\sqrt{b-c}\,\sqrt{b+c}}\right)-Bc\log(b\cosh(x)+c\sinh(x))+bBx}{b^2-c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cosh[x])/(b*Cosh[x] + c*Sinh[x]),x]
```

```
[Out] (b*B*x + 2*A*Sqrt[b - c]*Sqrt[b + c]*ArcTan[(c + b*Tanh[x/2])/(Sqrt[b - c]*
Sqrt[b + c])] - B*c*Log[b*Cosh[x] + c*Sinh[x]])/(b^2 - c^2)
```

fricas [A] time = 0.52, size = 234, normalized size = 2.92

$$\left[-\frac{Bc\log\left(\frac{2\,(b\cosh(x)+c\sinh(x))}{\cosh(x)-\sinh(x)}\right)+\sqrt{-b^2+c^2}\,A\log\left(\frac{(b+c)\cosh(x)^2+2\,(b+c)\cosh(x)\sinh(x)+(b+c)\sinh(x)^2-2\,\sqrt{-b^2+c^2}\,(\cosh(x)+\sinh(\right.}{(b+c)\cosh(x)^2+2\,(b+c)\cosh(x)\sinh(x)+(b+c)\sinh(x)^2+b-c}\right.}{b^2-c^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(b*cosh(x)+c*sinh(x)),x, algorithm="fricas")
```

```
[Out] [-(B*c*log(2*(b*cosh(x) + c*sinh(x))/(cosh(x) - sinh(x))) + sqrt(-b^2 + c^2
)*A*log(((b + c)*cosh(x)^2 + 2*(b + c)*cosh(x)*sinh(x) + (b + c)*sinh(x)^2
- 2*sqrt(-b^2 + c^2)*(cosh(x) + sinh(x)) - b + c)/((b + c)*cosh(x)^2 + 2*(b
 + c)*cosh(x)*sinh(x) + (b + c)*sinh(x)^2 + b - c)) - (B*b + B*c)*x)/(b^2 -
 c^2), -(B*c*log(2*(b*cosh(x) + c*sinh(x))/(cosh(x) - sinh(x))) + 2*sqrt(b^
2 - c^2)*A*arctan(sqrt(b^2 - c^2)/((b + c)*cosh(x) + (b + c)*sinh(x))) - (B
*b + B*c)*x)/(b^2 - c^2)]
```

giac [A] time = 0.13, size = 80, normalized size = 1.00

$$-\frac{Bc\log\left(be^{(2x)}+ce^{(2x)}+b-c\right)}{b^2-c^2}+\frac{2A\arctan\left(\frac{be^x+ce^x}{\sqrt{b^2-c^2}}\right)}{\sqrt{b^2-c^2}}+\frac{Bx}{b-c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(b*cosh(x)+c*sinh(x)),x, algorithm="giac")
```

```
[Out] -B*c*log(b*e^(2*x) + c*e^(2*x) + b - c)/(b^2 - c^2) + 2*A*arctan((b*e^x + c
*e^x)/sqrt(b^2 - c^2))/sqrt(b^2 - c^2) + B*x/(b - c)
```

maple [B] time = 0.21, size = 182, normalized size = 2.28

$$-\frac{2B\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{2b+2c}+\frac{2B\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)}{2b-2c}-\frac{Bc\ln\left(\left(\tanh^2\left(\frac{x}{2}\right)\right)b+2c\tanh\left(\frac{x}{2}\right)+b\right)}{(b-c)(b+c)}+\frac{2\arctan\left(\frac{2\tanh\left(\frac{x}{2}\right)b+2c}{2\sqrt{b^2-c^2}}\right)}{(b-c)(b+c)\sqrt{b^2-c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cosh(x))/(b*cosh(x)+c*sinh(x)),x)
```

```
[Out] -2*B/(2*b+2*c)*ln(tanh(1/2*x)-1)+2*B/(2*b-2*c)*ln(tanh(1/2*x)+1)-1/(b-c)/(b
+c)*B*c*ln(tanh(1/2*x)^2*b+2*c*tanh(1/2*x)+b)+2/(b-c)/(b+c)/(b^2-c^2)^(1/2)
*arctan(1/2*(2*tanh(1/2*x)*b+2*c)/(b^2-c^2)^(1/2))*A*b^2-2/(b-c)/(b+c)/(b^2
-c^2)^(1/2)*arctan(1/2*(2*tanh(1/2*x)*b+2*c)/(b^2-c^2)^(1/2))*A*c^2
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(b*cosh(x)+c*sinh(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*c^2-4*b^2>0)', see `assume?` for
more details)Is 4*c^2-4*b^2 positive or negative?
```

mupad [B] time = 3.05, size = 177, normalized size = 2.21

$$\frac{2\,\mathrm{atan}\left(\frac{A\,e^x\,\sqrt{b^2-c^2}}{b\,\sqrt{A^2}-c\,\sqrt{A^2}}\right)\,\sqrt{A^2}}{\sqrt{b^2-c^2}}+\frac{B\,x}{b-c}+\frac{B\,c^3\,\ln\left(4\,A^2\,b-4\,A^2\,c+4\,A^2\,b\,e^{2\,x}+4\,A^2\,c\,e^{2\,x}\right)}{b^4-2\,b^2\,c^2+c^4}-\frac{B\,b^2\,c\,\ln\left(4\,A^2\,b-4\,A^2\right.}{b^4-2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cosh(x))/(b*cosh(x) + c*sinh(x)),x)
```

```
[Out] (2*atan((A*exp(x)*(b^2 - c^2)^(1/2))/(b*(A^2)^(1/2) - c*(A^2)^(1/2)))*(A^2)
^(1/2))/(b^2 - c^2)^(1/2) + (B*x)/(b - c) + (B*c^3*log(4*A^2*b - 4*A^2*c +
4*A^2*b*exp(2*x) + 4*A^2*c*exp(2*x)))/(b^4 + c^4 - 2*b^2*c^2) - (B*b^2*c*lo
g(4*A^2*b - 4*A^2*c + 4*A^2*b*exp(2*x) + 4*A^2*c*exp(2*x)))/(b^4 + c^4 - 2*
b^2*c^2)
```

sympy [A] time = 50.44, size = 697, normalized size = 8.71

$$\begin{cases} \tilde{\infty}\left(A\log\left(\tanh\left(\frac{x}{2}\right)\right) + Bx - 2B\log\left(\tanh\left(\frac{x}{2}\right)+1\right) + B\log\left(\tanh\left(\frac{x}{2}\right)\right)\right) \\ \frac{A\log\left(\tanh\left(\frac{x}{2}\right)\right)+Bx-2B\log\left(\tanh\left(\frac{x}{2}\right)+1\right)+B\log\left(\tanh\left(\frac{x}{2}\right)\right)}{c} \\ -\frac{2A}{-2c\sinh(x)+2c\cosh(x)} + \frac{Bx\sinh(x)}{-2c\sinh(x)+2c\cosh(x)} - \frac{Bx\cosh(x)}{-2c\sinh(x)+2c\cosh(x)} - \frac{B\cosh(x)}{-2c\sinh(x)+2c\cosh(x)} \\ -\frac{2A}{2c\sinh(x)+2c\cosh(x)} + \frac{Bx\sinh(x)}{2c\sinh(x)+2c\cosh(x)} + \frac{Bx\cosh(x)}{2c\sinh(x)+2c\cosh(x)} - \frac{B\cosh(x)}{2c\sinh(x)+2c\cosh(x)} \\ \frac{Ab^2\log\left(\tanh\left(\frac{x}{2}\right)+\frac{c}{b}-\frac{\sqrt{-b^2+c^2}}{b}\right)}{b^2\sqrt{-b^2+c^2}-c^2\sqrt{-b^2+c^2}} - \frac{Ab^2\log\left(\tanh\left(\frac{x}{2}\right)+\frac{c}{b}+\frac{\sqrt{-b^2+c^2}}{b}\right)}{b^2\sqrt{-b^2+c^2}-c^2\sqrt{-b^2+c^2}} - \frac{Ac^2\log\left(\tanh\left(\frac{x}{2}\right)+\frac{c}{b}-\frac{\sqrt{-b^2+c^2}}{b}\right)}{b^2\sqrt{-b^2+c^2}-c^2\sqrt{-b^2+c^2}} + \frac{Ac^2\log\left(\tanh\left(\frac{x}{2}\right)+\frac{c}{b}+\frac{\sqrt{-b^2+c^2}}{b}\right)}{b^2\sqrt{-b^2+c^2}-c^2\sqrt{-b^2+c^2}} + \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(b*cosh(x)+c*sinh(x)),x)
```

```
[Out] Piecewise((zoo*(A*log(tanh(x/2)) + B*x - 2*B*log(tanh(x/2) + 1) + B*log(tan
h(x/2))), Eq(b, 0) & Eq(c, 0)), ((A*log(tanh(x/2)) + B*x - 2*B*log(tanh(x/2
) + 1) + B*log(tanh(x/2)))/c, Eq(b, 0)), (-2*A/(-2*c*sinh(x) + 2*c*cosh(x))
 + B*x*sinh(x)/(-2*c*sinh(x) + 2*c*cosh(x)) - B*x*cosh(x)/(-2*c*sinh(x) + 2
*c*cosh(x)) - B*cosh(x)/(-2*c*sinh(x) + 2*c*cosh(x)), Eq(b, -c)), (-2*A/(2*
c*sinh(x) + 2*c*cosh(x)) + B*x*sinh(x)/(2*c*sinh(x) + 2*c*cosh(x)) + B*x*co
sh(x)/(2*c*sinh(x) + 2*c*cosh(x)) - B*cosh(x)/(2*c*sinh(x) + 2*c*cosh(x)),
Eq(b, c)), (A*b**2*log(tanh(x/2) + c/b - sqrt(-b**2 + c**2)/b)/(b**2*sqrt(-
b**2 + c**2) - c**2*sqrt(-b**2 + c**2)) - A*b**2*log(tanh(x/2) + c/b + sqrt
(-b**2 + c**2)/b)/(b**2*sqrt(-b**2 + c**2) - c**2*sqrt(-b**2 + c**2)) - A*c
**2*log(tanh(x/2) + c/b - sqrt(-b**2 + c**2)/b)/(b**2*sqrt(-b**2 + c**2) -
c**2*sqrt(-b**2 + c**2)) + A*c**2*log(tanh(x/2) + c/b + sqrt(-b**2 + c**2)/
b)/(b**2*sqrt(-b**2 + c**2) - c**2*sqrt(-b**2 + c**2)) + B*b*x*sqrt(-b**2 +
 c**2)/(b**2*sqrt(-b**2 + c**2) - c**2*sqrt(-b**2 + c**2)) - B*c*x*sqrt(-b*
*2 + c**2)/(b**2*sqrt(-b**2 + c**2) - c**2*sqrt(-b**2 + c**2)) + 2*B*c*sqrt
(-b**2 + c**2)*log(tanh(x/2) + 1)/(b**2*sqrt(-b**2 + c**2) - c**2*sqrt(-b**
2 + c**2)) - B*c*sqrt(-b**2 + c**2)*log(tanh(x/2) + c/b - sqrt(-b**2 + c**2
```

```
)/b)/(b**2*sqrt(-b**2 + c**2) - c**2*sqrt(-b**2 + c**2)) - B*c*sqrt(-b**2 +
 c**2)*log(tanh(x/2) + c/b + sqrt(-b**2 + c**2)/b)/(b**2*sqrt(-b**2 + c**2)
 - c**2*sqrt(-b**2 + c**2)), True))
```

3.728 $\int \frac{A+B\cosh(x)}{(b\cosh(x)+c\sinh(x))^2}\,dx$

Optimal. Leaf size=78

$$\frac{Ab\sinh(x)+Ac\cosh(x)+Bc}{(b^2-c^2)(b\cosh(x)+c\sinh(x))}+\frac{bB\tan^{-1}\left(\frac{b\sinh(x)+c\cosh(x)}{\sqrt{b^2-c^2}}\right)}{(b^2-c^2)^{3/2}}$$

```
[Out] b*B*arctan((c*cosh(x)+b*sinh(x))/(b^2-c^2)^(1/2))/(b^2-c^2)^(3/2)+(B*c+A*c*
cosh(x)+A*b*sinh(x))/(b^2-c^2)/(b*cosh(x)+c*sinh(x))
```

Rubi [A] time = 0.06, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3155, 3074, 206}

$$\frac{Ab\sinh(x)+Ac\cosh(x)+Bc}{(b^2-c^2)(b\cosh(x)+c\sinh(x))}+\frac{bB\tan^{-1}\left(\frac{b\sinh(x)+c\cosh(x)}{\sqrt{b^2-c^2}}\right)}{(b^2-c^2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cosh[x])/(b*Cosh[x] + c*Sinh[x])^2,x]
```

```
[Out] (b*B*ArcTan[(c*Cosh[x] + b*Sinh[x])/Sqrt[b^2 - c^2]])/(b^2 - c^2)^(3/2) + (
B*c + A*c*Cosh[x] + A*b*Sinh[x])/((b^2 - c^2)*(b*Cosh[x] + c*Sinh[x]))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3074

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> -Dist[d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d
*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3155

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.))/((a_.) + cos[(d_.) + (e_.)*(x_)
]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])^2, x_Symbol] :> Simp[(c*B + c*A*Co
s[d + e*x] + (a*B - b*A)*Sin[d + e*x])/(e*(a^2 - b^2 - c^2)*(a + b*Cos[d +
e*x] + c*Sin[d + e*x])), x] + Dist[(a*A - b*B)/(a^2 - b^2 - c^2), Int[1/(a
+ b*Cos[d + e*x] + c*Sin[d + e*x]), x], x] /; FreeQ[{a, b, c, d, e, A, B},
```

```
x] && NeQ[a^2 - b^2 - c^2, 0] && NeQ[a*A - b*B, 0]
```

Rubi steps

$$\int \frac{A+B\cosh(x)}{(b\cosh(x)+c\sinh(x))^2}\,dx = \frac{Bc+Ac\cosh(x)+Ab\sinh(x)}{\left(b^2-c^2\right)(b\cosh(x)+c\sinh(x))}+\frac{(bB)\int \frac{1}{b\cosh(x)+c\sinh(x)}\,dx}{b^2-c^2}$$
$$= \frac{Bc+Ac\cosh(x)+Ab\sinh(x)}{\left(b^2-c^2\right)(b\cosh(x)+c\sinh(x))}+\frac{(ibB)\,\text{Subst}\left(\int \frac{1}{b^2-c^2-x^2}\,dx, x, -ic\cosh(x) -\right.}{b^2-c^2}$$
$$= \frac{bB\tan^{-1}\left(\frac{c\cosh(x)+b\sinh(x)}{\sqrt{b^2-c^2}}\right)}{\left(b^2-c^2\right)^{3/2}}+\frac{Bc+Ac\cosh(x)+Ab\sinh(x)}{\left(b^2-c^2\right)(b\cosh(x)+c\sinh(x))}$$

Mathematica [A] time = 0.30, size = 151, normalized size = 1.94

$$\frac{\sinh(x)\left(A(b-c)^{3/2}(b+c)^2+2b^2Bc\sqrt{b+c}\,\tan^{-1}\left(\frac{b\tanh\left(\frac{x}{2}\right)+c}{\sqrt{b-c}\,\sqrt{b+c}}\right)\right)+2b^3B\sqrt{b+c}\,\cosh(x)\tan^{-1}\left(\frac{b\tanh\left(\frac{x}{2}\right)+c}{\sqrt{b-c}\,\sqrt{b+c}}\right)+bBc}{b(b-c)^{3/2}(b+c)^2(b\cosh(x)+c\sinh(x))}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cosh[x])/(b*Cosh[x] + c*Sinh[x])^2,x]
```

```
[Out] (b*B*Sqrt[b - c]*c*(b + c) + 2*b^3*B*Sqrt[b + c]*ArcTan[(c + b*Tanh[x/2])/(
Sqrt[b - c]*Sqrt[b + c])]*Cosh[x] + (A*(b - c)^(3/2)*(b + c)^2 + 2*b^2*B*c*
Sqrt[b + c]*ArcTan[(c + b*Tanh[x/2])/(Sqrt[b - c]*Sqrt[b + c])])*Sinh[x])/(
b*(b - c)^(3/2)*(b + c)^2*(b*Cosh[x] + c*Sinh[x]))
```

fricas [B] time = 0.53, size = 680, normalized size = 8.72

$$\left[-\frac{2\,Ab^3-2\,Ab^2c-2\,Abc^2+2\,Ac^3-\left(Bb^2-Bbc+\left(Bb^2+Bbc\right)\cosh(x)^2+2\left(Bb^2+Bbc\right)\cosh(x)\sinh(x)+\left(B\right.\right.}{b^5-b^4c-2\,b^3c^2+2\,b^2c^3+bc^4-c^5+\left(b^5+b^4c-2\,b^3c^2-2\,b^2c^3+bc^4+\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(b*cosh(x)+c*sinh(x))^2,x, algorithm="fricas")
```

```
[Out] [-(2*A*b^3 - 2*A*b^2*c - 2*A*b*c^2 + 2*A*c^3 - (B*b^2 - B*b*c + (B*b^2 + B*
b*c)*cosh(x)^2 + 2*(B*b^2 + B*b*c)*cosh(x)*sinh(x) + (B*b^2 + B*b*c)*sinh(x
```

```
)^2)*sqrt(-b^2 + c^2)*log(((b + c)*cosh(x)^2 + 2*(b + c)*cosh(x)*sinh(x) +
(b + c)*sinh(x)^2 + 2*sqrt(-b^2 + c^2)*(cosh(x) + sinh(x)) - b + c)/((b + c
)*cosh(x)^2 + 2*(b + c)*cosh(x)*sinh(x) + (b + c)*sinh(x)^2 + b - c)) - 2*(
B*b^2*c - B*c^3)*cosh(x) - 2*(B*b^2*c - B*c^3)*sinh(x))/(b^5 - b^4*c - 2*b^
3*c^2 + 2*b^2*c^3 + b*c^4 - c^5 + (b^5 + b^4*c - 2*b^3*c^2 - 2*b^2*c^3 + b*
c^4 + c^5)*cosh(x)^2 + 2*(b^5 + b^4*c - 2*b^3*c^2 - 2*b^2*c^3 + b*c^4 + c^5
)*cosh(x)*sinh(x) + (b^5 + b^4*c - 2*b^3*c^2 - 2*b^2*c^3 + b*c^4 + c^5)*sin
h(x)^2), -2*(A*b^3 - A*b^2*c - A*b*c^2 + A*c^3 + (B*b^2 - B*b*c + (B*b^2 +
B*b*c)*cosh(x)^2 + 2*(B*b^2 + B*b*c)*cosh(x)*sinh(x) + (B*b^2 + B*b*c)*sinh
(x)^2)*sqrt(b^2 - c^2)*arctan(sqrt(b^2 - c^2)/((b + c)*cosh(x) + (b + c)*si
nh(x))) - (B*b^2*c - B*c^3)*cosh(x) - (B*b^2*c - B*c^3)*sinh(x))/(b^5 - b^4
*c - 2*b^3*c^2 + 2*b^2*c^3 + b*c^4 - c^5 + (b^5 + b^4*c - 2*b^3*c^2 - 2*b^2
*c^3 + b*c^4 + c^5)*cosh(x)^2 + 2*(b^5 + b^4*c - 2*b^3*c^2 - 2*b^2*c^3 + b*
c^4 + c^5)*cosh(x)*sinh(x) + (b^5 + b^4*c - 2*b^3*c^2 - 2*b^2*c^3 + b*c^4 +
 c^5)*sinh(x)^2)]
```

giac [A] time = 0.13, size = 83, normalized size = 1.06

$$\frac{2\,Bb\arctan\left(\frac{be^{x}+ce^{x}}{\sqrt{b^{2}-c^{2}}}\right)}{\left(b^{2}-c^{2}\right)^{\frac{3}{2}}}+\frac{2\,(Bce^{x}-Ab+Ac)}{\left(b^{2}-c^{2}\right)\left(be^{(2\,x)}+ce^{(2\,x)}+b-c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(b*cosh(x)+c*sinh(x))^2,x, algorithm="giac")
```

```
[Out] 2*B*b*arctan((b*e^x + c*e^x)/sqrt(b^2 - c^2))/(b^2 - c^2)^(3/2) + 2*(B*c*e^
x - A*b + A*c)/((b^2 - c^2)*(b*e^(2*x) + c*e^(2*x) + b - c))
```

maple [A] time = 0.28, size = 116, normalized size = 1.49

$$-\frac{2\left(-\frac{\left(A\,b^{2}-A\,c^{2}+B\,c^{2}\right)\tanh\left(\frac{x}{2}\right)}{b\left(b^{2}-c^{2}\right)}-\frac{Bc}{b^{2}-c^{2}}\right)}{\left(\tanh^{2}\left(\frac{x}{2}\right)\right)b+2c\tanh\left(\frac{x}{2}\right)+b}+\frac{2bB\arctan\left(\frac{2\tanh\left(\frac{x}{2}\right)b+2c}{2\sqrt{b^{2}-c^{2}}}\right)}{\left(b^{2}-c^{2}\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cosh(x))/(b*cosh(x)+c*sinh(x))^2,x)
```

```
[Out] -2*(-(A*b^2-A*c^2+B*c^2)/b/(b^2-c^2)*tanh(1/2*x)-B*c/(b^2-c^2))/(tanh(1/2*x
)^2*b+2*c*tanh(1/2*x)+b)+2*b*B/(b^2-c^2)^(3/2)*arctan(1/2*(2*tanh(1/2*x)*b+
2*c)/(b^2-c^2)^(1/2))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(b*cosh(x)+c*sinh(x))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*c^2-4*b^2>0)', see `assume?` for
more details)Is 4*c^2-4*b^2 positive or negative?
```

mupad [B] time = 1.79, size = 168, normalized size = 2.15

$$\frac{B\,b\,\ln\left(\frac{2\,B\,b}{{(b+c)}^{5/2}\,\sqrt{c-b}}+\frac{2\,B\,b\,{\mathrm{e}}^x}{-b^3-b^2\,c+b\,c^2+c^3}\right)}{{(b+c)}^{3/2}\,{(c-b)}^{3/2}}-\frac{B\,b\,\ln\left(\frac{2\,B\,b\,{\mathrm{e}}^x}{-b^3-b^2\,c+b\,c^2+c^3}-\frac{2\,B\,b}{{(b+c)}^{5/2}\,\sqrt{c-b}}\right)}{{(b+c)}^{3/2}\,{(c-b)}^{3/2}}-\frac{\frac{2\,A}{b+c}-\frac{2\,B\,c\,{\mathrm{e}}^x}{(b+c)\,(b-c)}}{b-c+{\mathrm{e}}^{2\,x}\,(b+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cosh(x))/(b*cosh(x) + c*sinh(x))^2,x)
```

```
[Out] (B*b*log((2*B*b)/((b + c)^(5/2)*(c - b)^(1/2)) + (2*B*b*exp(x))/(b*c^2 - b^
2*c - b^3 + c^3)))/((b + c)^(3/2)*(c - b)^(3/2)) - (B*b*log((2*B*b*exp(x))/
(b*c^2 - b^2*c - b^3 + c^3) - (2*B*b)/((b + c)^(5/2)*(c - b)^(1/2))))/((b +
 c)^(3/2)*(c - b)^(3/2)) - ((2*A)/(b + c) - (2*B*c*exp(x))/((b + c)*(b - c)
))/(b - c + exp(2*x)*(b + c))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(b*cosh(x)+c*sinh(x))**2,x)
```

```
[Out] Timed out
```

3.729 $\int \frac{A+B\cosh(x)}{(b\cosh(x)+c\sinh(x))^3}\,dx$

Optimal. Leaf size=120

$$\frac{Ab\sinh(x)+Ac\cosh(x)+Bc}{2\left(b^2-c^2\right)(b\cosh(x)+c\sinh(x))^2}+\frac{A\tan^{-1}\left(\frac{b\sinh(x)+c\cosh(x)}{\sqrt{b^2-c^2}}\right)}{2\left(b^2-c^2\right)^{3/2}}+\frac{b^2B\sinh(x)+bBc\cosh(x)}{\left(b^2-c^2\right)^2(b\cosh(x)+c\sinh(x))}$$

```
[Out] 1/2*A*arctan((c*cosh(x)+b*sinh(x))/(b^2-c^2)^(1/2))/(b^2-c^2)^(3/2)+1/2*(B*
c+A*c*cosh(x)+A*b*sinh(x))/(b^2-c^2)/(b*cosh(x)+c*sinh(x))^2+(b*B*c*cosh(x)
+b^2*B*sinh(x))/(b^2-c^2)^2/(b*cosh(x)+c*sinh(x))
```

Rubi [A] time = 0.12, antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {3158, 3153, 3074, 206}

$$\frac{Ab\sinh(x)+Ac\cosh(x)+Bc}{2\left(b^2-c^2\right)(b\cosh(x)+c\sinh(x))^2}+\frac{A\tan^{-1}\left(\frac{b\sinh(x)+c\cosh(x)}{\sqrt{b^2-c^2}}\right)}{2\left(b^2-c^2\right)^{3/2}}+\frac{b^2B\sinh(x)+bBc\cosh(x)}{\left(b^2-c^2\right)^2(b\cosh(x)+c\sinh(x))}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cosh[x])/(b*Cosh[x] + c*Sinh[x])^3,x]
```

```
[Out] (A*ArcTan[(c*Cosh[x] + b*Sinh[x])/Sqrt[b^2 - c^2]])/(2*(b^2 - c^2)^(3/2)) +
 (B*c + A*c*Cosh[x] + A*b*Sinh[x])/(2*(b^2 - c^2)*(b*Cosh[x] + c*Sinh[x])^2
) + (b*B*c*Cosh[x] + b^2*B*Sinh[x])/((b^2 - c^2)^2*(b*Cosh[x] + c*Sinh[x]))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3074

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> -Dist[d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d
*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3153

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])^2,
 x_Symbol] :> Simp[(c*B - b*C - (a*C - c*A)*Cos[d + e*x] + (a*B - b*A)*Sin[
```

```
d + e*x])/(e*(a^2 - b^2 - c^2)*(a + b*Cos[d + e*x] + c*Sin[d + e*x])), x] +
 Dist[(a*A - b*B - c*C)/(a^2 - b^2 - c^2), Int[1/(a + b*Cos[d + e*x] + c*Si
n[d + e*x]), x], x] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && NeQ[a^2 - b^2
- c^2, 0] && NeQ[a*A - b*B - c*C, 0]
```

Rule 3158

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.))*((a_.) + cos[(d_.) + (e_.)*(x_)
]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])^(n_), x_Symbol] :> -Simp[((c*B + c
*A*Cos[d + e*x] + (a*B - b*A)*Sin[d + e*x])*(a + b*Cos[d + e*x] + c*Sin[d +
 e*x])^(n + 1))/(e*(n + 1)*(a^2 - b^2 - c^2)), x] + Dist[1/((n + 1)*(a^2 -
b^2 - c^2)), Int[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1)*Simp[(n + 1)
*(a*A - b*B) + (n + 2)*(a*B - b*A)*Cos[d + e*x] - (n + 2)*c*A*Sin[d + e*x],
 x], x], x] /; FreeQ[{a, b, c, d, e, A, B}, x] && LtQ[n, -1] && NeQ[a^2 - b
^2 - c^2, 0] && NeQ[n, -2]
```

Rubi steps

$$\int \frac{A+B\cosh(x)}{(b\cosh(x)+c\sinh(x))^3}\,dx = \frac{Bc+Ac\cosh(x)+Ab\sinh(x)}{2\left(b^2-c^2\right)(b\cosh(x)+c\sinh(x))^2}+\frac{\int \frac{2bB+Ab\cosh(x)+Ac\sinh(x)}{(b\cosh(x)+c\sinh(x))^2}\,dx}{2\left(b^2-c^2\right)}$$

$$= \frac{Bc+Ac\cosh(x)+Ab\sinh(x)}{2\left(b^2-c^2\right)(b\cosh(x)+c\sinh(x))^2}+\frac{bBc\cosh(x)+b^2B\sinh(x)}{\left(b^2-c^2\right)^2(b\cosh(x)+c\sinh(x))}+\frac{A\int$$

$$= \frac{Bc+Ac\cosh(x)+Ab\sinh(x)}{2\left(b^2-c^2\right)(b\cosh(x)+c\sinh(x))^2}+\frac{bBc\cosh(x)+b^2B\sinh(x)}{\left(b^2-c^2\right)^2(b\cosh(x)+c\sinh(x))}+\frac{(iA$$

$$= \frac{A\tan^{-1}\left(\frac{c\cosh(x)+b\sinh(x)}{\sqrt{b^2-c^2}}\right)}{2\left(b^2-c^2\right)^{3/2}}+\frac{Bc+Ac\cosh(x)+Ab\sinh(x)}{2\left(b^2-c^2\right)(b\cosh(x)+c\sinh(x))^2}+\frac{bBc\cos}{\left(b^2-c^2\right)^2(}$$

Mathematica [A] time = 1.09, size = 134, normalized size = 1.12

$$\frac{1}{2}\left(\frac{A\left(b^2-c^2\right)\sinh(x)+bBc}{b(b-c)(b+c)(b\cosh(x)+c\sinh(x))^2}+\frac{Ac+2bB\sinh(x)}{b(b-c)(b+c)(b\cosh(x)+c\sinh(x))}+\frac{2A\tan^{-1}\left(\frac{b\tanh\left(\frac{x}{2}\right)+c}{\sqrt{b-c}\sqrt{b+c}}\right)}{(b-c)^{3/2}(b+c)^{3/2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cosh[x])/(b*Cosh[x] + c*Sinh[x])^3,x]
```

```
[Out] ((2*A*ArcTan[(c + b*Tanh[x/2])/(Sqrt[b - c]*Sqrt[b + c])])/((b - c)^(3/2)*(
b + c)^(3/2)) + (A*c + 2*b*B*Sinh[x])/(b*(b - c)*(b + c)*(b*Cosh[x] + c*Sin
h[x])) + (b*B*c + A*(b^2 - c^2)*Sinh[x])/(b*(b - c)*(b + c)*(b*Cosh[x] + c*
Sinh[x])^2))/2
```

fricas [B] time = 0.52, size = 1855, normalized size = 15.46

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(b*cosh(x)+c*sinh(x))^3,x, algorithm="fricas")
```

```
[Out] [-1/2*(4*B*b^3 - 8*B*b^2*c + 4*B*b*c^2 - 2*(A*b^3 + A*b^2*c - A*b*c^2 - A*c
^3)*cosh(x)^3 - 2*(A*b^3 + A*b^2*c - A*b*c^2 - A*c^3)*sinh(x)^3 + 4*(B*b^3
- B*b^2*c - B*b*c^2 + B*c^3)*cosh(x)^2 + 2*(2*B*b^3 - 2*B*b^2*c - 2*B*b*c^2
 + 2*B*c^3 - 3*(A*b^3 + A*b^2*c - A*b*c^2 - A*c^3)*cosh(x))*sinh(x)^2 - ((A
*b^2 + 2*A*b*c + A*c^2)*cosh(x)^4 + 4*(A*b^2 + 2*A*b*c + A*c^2)*cosh(x)*sin
h(x)^3 + (A*b^2 + 2*A*b*c + A*c^2)*sinh(x)^4 + A*b^2 - 2*A*b*c + A*c^2 + 2*
(A*b^2 - A*c^2)*cosh(x)^2 + 2*(A*b^2 - A*c^2 + 3*(A*b^2 + 2*A*b*c + A*c^2)*
cosh(x)^2)*sinh(x)^2 + 4*((A*b^2 + 2*A*b*c + A*c^2)*cosh(x)^3 + (A*b^2 - A*
c^2)*cosh(x))*sinh(x))*sqrt(-b^2 + c^2)*log(((b + c)*cosh(x)^2 + 2*(b + c)*
cosh(x)*sinh(x) + (b + c)*sinh(x)^2 + 2*sqrt(-b^2 + c^2)*(cosh(x) + sinh(x)
) - b + c)/((b + c)*cosh(x)^2 + 2*(b + c)*cosh(x)*sinh(x) + (b + c)*sinh(x)
^2 + b - c)) + 2*(A*b^3 - A*b^2*c - A*b*c^2 + A*c^3)*cosh(x) + 2*(A*b^3 - A
*b^2*c - A*b*c^2 + A*c^3 - 3*(A*b^3 + A*b^2*c - A*b*c^2 - A*c^3)*cosh(x)^2
+ 4*(B*b^3 - B*b^2*c - B*b*c^2 + B*c^3)*cosh(x))*sinh(x))/(b^6 - 2*b^5*c -
b^4*c^2 + 4*b^3*c^3 - b^2*c^4 - 2*b*c^5 + c^6 + (b^6 + 2*b^5*c - b^4*c^2 -
4*b^3*c^3 - b^2*c^4 + 2*b*c^5 + c^6)*cosh(x)^4 + 4*(b^6 + 2*b^5*c - b^4*c^2
 - 4*b^3*c^3 - b^2*c^4 + 2*b*c^5 + c^6)*cosh(x)*sinh(x)^3 + (b^6 + 2*b^5*c
- b^4*c^2 - 4*b^3*c^3 - b^2*c^4 + 2*b*c^5 + c^6)*sinh(x)^4 + 2*(b^6 - 3*b^4
*c^2 + 3*b^2*c^4 - c^6)*cosh(x)^2 + 2*(b^6 - 3*b^4*c^2 + 3*b^2*c^4 - c^6 +
3*(b^6 + 2*b^5*c - b^4*c^2 - 4*b^3*c^3 - b^2*c^4 + 2*b*c^5 + c^6)*cosh(x)^2
)*sinh(x)^2 + 4*((b^6 + 2*b^5*c - b^4*c^2 - 4*b^3*c^3 - b^2*c^4 + 2*b*c^5 +
 c^6)*cosh(x)^3 + (b^6 - 3*b^4*c^2 + 3*b^2*c^4 - c^6)*cosh(x))*sinh(x)), -(
2*B*b^3 - 4*B*b^2*c + 2*B*b*c^2 - (A*b^3 + A*b^2*c - A*b*c^2 - A*c^3)*cosh(
x)^3 - (A*b^3 + A*b^2*c - A*b*c^2 - A*c^3)*sinh(x)^3 + 2*(B*b^3 - B*b^2*c -
 B*b*c^2 + B*c^3)*cosh(x)^2 + (2*B*b^3 - 2*B*b^2*c - 2*B*b*c^2 + 2*B*c^3 -
3*(A*b^3 + A*b^2*c - A*b*c^2 - A*c^3)*cosh(x))*sinh(x)^2 + ((A*b^2 + 2*A*b*
c + A*c^2)*cosh(x)^4 + 4*(A*b^2 + 2*A*b*c + A*c^2)*cosh(x)*sinh(x)^3 + (A*b
^2 + 2*A*b*c + A*c^2)*sinh(x)^4 + A*b^2 - 2*A*b*c + A*c^2 + 2*(A*b^2 - A*c^
2)*cosh(x)^2 + 2*(A*b^2 - A*c^2 + 3*(A*b^2 + 2*A*b*c + A*c^2)*cosh(x)^2)*si
nh(x)^2 + 4*((A*b^2 + 2*A*b*c + A*c^2)*cosh(x)^3 + (A*b^2 - A*c^2)*cosh(x))
*sinh(x))*sqrt(b^2 - c^2)*arctan(sqrt(b^2 - c^2)/((b + c)*cosh(x) + (b + c)
```

```
*sinh(x))) + (A*b^3 - A*b^2*c - A*b*c^2 + A*c^3)*cosh(x) + (A*b^3 - A*b^2*c
 - A*b*c^2 + A*c^3 - 3*(A*b^3 + A*b^2*c - A*b*c^2 - A*c^3)*cosh(x)^2 + 4*(B
*b^3 - B*b^2*c - B*b*c^2 + B*c^3)*cosh(x))*sinh(x))/(b^6 - 2*b^5*c - b^4*c^
2 + 4*b^3*c^3 - b^2*c^4 - 2*b*c^5 + c^6 + (b^6 + 2*b^5*c - b^4*c^2 - 4*b^3*
c^3 - b^2*c^4 + 2*b*c^5 + c^6)*cosh(x)^4 + 4*(b^6 + 2*b^5*c - b^4*c^2 - 4*b
^3*c^3 - b^2*c^4 + 2*b*c^5 + c^6)*cosh(x)*sinh(x)^3 + (b^6 + 2*b^5*c - b^4*
c^2 - 4*b^3*c^3 - b^2*c^4 + 2*b*c^5 + c^6)*sinh(x)^4 + 2*(b^6 - 3*b^4*c^2 +
 3*b^2*c^4 - c^6)*cosh(x)^2 + 2*(b^6 - 3*b^4*c^2 + 3*b^2*c^4 - c^6 + 3*(b^6
 + 2*b^5*c - b^4*c^2 - 4*b^3*c^3 - b^2*c^4 + 2*b*c^5 + c^6)*cosh(x)^2)*sinh
(x)^2 + 4*((b^6 + 2*b^5*c - b^4*c^2 - 4*b^3*c^3 - b^2*c^4 + 2*b*c^5 + c^6)*
cosh(x)^3 + (b^6 - 3*b^4*c^2 + 3*b^2*c^4 - c^6)*cosh(x))*sinh(x))]
```

giac [A] time = 0.13, size = 152, normalized size = 1.27

$$\frac{A\arctan\left(\frac{be^x+ce^x}{\sqrt{b^2-c^2}}\right)}{\left(b^2-c^2\right)^{\frac{3}{2}}}+\frac{Ab^2e^{(3x)}+2\,Abce^{(3x)}+Ac^2e^{(3x)}-2\,Bb^2e^{(2x)}+2\,Bc^2e^{(2x)}-Ab^2e^x+Ac^2e^x-2\,Bb^2+2\,Bbc}{\left(b^3+b^2c-bc^2-c^3\right)\left(be^{(2x)}+ce^{(2x)}+b-c\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(b*cosh(x)+c*sinh(x))^3,x, algorithm="giac")
```

```
[Out] A*arctan((b*e^x + c*e^x)/sqrt(b^2 - c^2))/(b^2 - c^2)^(3/2) + (A*b^2*e^(3*x
) + 2*A*b*c*e^(3*x) + A*c^2*e^(3*x) - 2*B*b^2*e^(2*x) + 2*B*c^2*e^(2*x) - A
*b^2*e^x + A*c^2*e^x - 2*B*b^2 + 2*B*b*c)/((b^3 + b^2*c - b*c^2 - c^3)*(b*e
^(2*x) + c*e^(2*x) + b - c)^2)
```

maple [A] time = 0.27, size = 214, normalized size = 1.78

$$\frac{-\frac{\left(A\,b^2-2A\,c^2-2B\,b^2+2B\,c^2\right)\left(\tanh^3\left(\frac{x}{2}\right)\right)}{\left(b^2-c^2\right)b}+\frac{c\left(A\,b^2+2A\,c^2+2B\,b^2-2B\,c^2\right)\left(\tanh^2\left(\frac{x}{2}\right)\right)}{\left(b^2-c^2\right)b^2}+\frac{\left(A\,b^2+2A\,c^2+2B\,b^2-2B\,c^2\right)\tanh\left(\frac{x}{2}\right)}{\left(b^2-c^2\right)b}+\frac{2Ac}{2b^2-2c^2}}{\left(\left(\tanh^2\left(\frac{x}{2}\right)\right)b+2c\tanh\left(\frac{x}{2}\right)+b\right)^2}+\frac{A\,\mathrm{arc}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cosh(x))/(b*cosh(x)+c*sinh(x))^3,x)
```

```
[Out] 2*(-1/2*(A*b^2-2*A*c^2-2*B*b^2+2*B*c^2)/(b^2-c^2)/b*tanh(1/2*x)^3+1/2*c*(A*
b^2+2*A*c^2+2*B*b^2-2*B*c^2)/(b^2-c^2)/b^2*tanh(1/2*x)^2+1/2*(A*b^2+2*A*c^2
+2*B*b^2-2*B*c^2)/(b^2-c^2)/b*tanh(1/2*x)+1/2*A*c/(b^2-c^2))/(tanh(1/2*x)^2
*b+2*c*tanh(1/2*x)+b)^2+A/(b^2-c^2)^(3/2)*arctan(1/2*(2*tanh(1/2*x)*b+2*c)/
(b^2-c^2)^(1/2))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(b*cosh(x)+c*sinh(x))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*c^2-4*b^2>0)', see `assume?` for
more details)Is 4*c^2-4*b^2 positive or negative?
```

mupad [B] time = 1.67, size = 216, normalized size = 1.80

$$\frac{\operatorname{atan}\left(\frac{A\,e^x\,\sqrt{b^6-3\,b^4\,c^2+3\,b^2\,c^4-c^6}}{b^3\,\sqrt{A^2}+c^3\,\sqrt{A^2}-b\,c^2\,\sqrt{A^2}-b^2\,c\,\sqrt{A^2}}\right)\sqrt{A^2}}{\sqrt{b^6-3\,b^4\,c^2+3\,b^2\,c^4-c^6}}-\frac{\frac{B}{(b+c)^2}-\frac{A\,e^x}{(b+c)\,(b-c)}}{b-c+e^{2\,x}\,(b+c)}-\frac{\frac{B}{b+c}+\frac{2\,A\,e^x}{b+c}+\frac{B\,e^{2\,x}}{b+c}}{e^{4\,x}\,(b+c)^2+(b-c)^2+2\,e^{2\,x}\,(b+c)\,(b-c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cosh(x))/(b*cosh(x) + c*sinh(x))^3,x)
```

```
[Out] (atan((A*exp(x)*(b^6 - c^6 + 3*b^2*c^4 - 3*b^4*c^2)^(1/2))/(b^3*(A^2)^(1/2)
 + c^3*(A^2)^(1/2) - b*c^2*(A^2)^(1/2) - b^2*c*(A^2)^(1/2)))*(A^2)^(1/2))/(
b^6 - c^6 + 3*b^2*c^4 - 3*b^4*c^2)^(1/2) - (B/(b + c)^2 - (A*exp(x))/((b +
c)*(b - c)))/(b - c + exp(2*x)*(b + c)) - (B/(b + c) + (2*A*exp(x))/(b + c)
 + (B*exp(2*x))/(b + c))/(exp(4*x)*(b + c)^2 + (b - c)^2 + 2*exp(2*x)*(b +
c)*(b - c))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(b*cosh(x)+c*sinh(x))**3,x)
```

```
[Out] Timed out
```

3.730 $\int \frac{\cosh(x)+\sinh(x)}{\cosh(x)-\sinh(x)}\,dx$

Optimal. Leaf size=11

$$\frac{1}{2}(\sinh(x) + \cosh(x))^2$$

```
[Out] 1/2*(cosh(x)+sinh(x))^2
```

Rubi [A] time = 0.04, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {4385}

$$\frac{1}{2}(\sinh(x) + \cosh(x))^2$$

Antiderivative was successfully verified.

```
[In] Int[(Cosh[x] + Sinh[x])/(Cosh[x] - Sinh[x]),x]
```

```
[Out] (Cosh[x] + Sinh[x])^2/2
```

Rule 4385

```
Int[(u_)*(y_)^(m_.), x_Symbol] :> With[{q = DerivativeDivides[ActivateTrig[
y], ActivateTrig[u], x]}, Simp[(q*ActivateTrig[y^(m + 1)])/(m + 1), x] /;
!FalseQ[q]] /; FreeQ[m, x] && NeQ[m, -1] &&  !InertTrigFreeQ[u]
```

Rubi steps

$$\int \frac{\cosh(x) + \sinh(x)}{\cosh(x) - \sinh(x)}\,dx = \frac{1}{2}(\cosh(x) + \sinh(x))^2$$

Mathematica [A] time = 0.00, size = 17, normalized size = 1.55

$$\frac{1}{2}\sinh(2x) + \frac{1}{2}\cosh(2x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cosh[x] + Sinh[x])/(Cosh[x] - Sinh[x]),x]
```

```
[Out] Cosh[2*x]/2 + Sinh[2*x]/2
```

fricas [A] time = 0.53, size = 16, normalized size = 1.45

$$\frac{\cosh(x) + \sinh(x)}{2\,(\cosh(x) - \sinh(x))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cosh(x)+sinh(x))/(cosh(x)-sinh(x)),x, algorithm="fricas")
[Out] 1/2*(cosh(x) + sinh(x))/(cosh(x) - sinh(x))
```

giac [A] time = 0.11, size = 6, normalized size = 0.55

$$\frac{1}{2}\,e^{(2\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cosh(x)+sinh(x))/(cosh(x)-sinh(x)),x, algorithm="giac")
[Out] 1/2*e^(2*x)
```

maple [A] time = 0.02, size = 17, normalized size = 1.55

$$\frac{\cosh(x)+\sinh(x)}{2\cosh(x)-2\sinh(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cosh(x)+sinh(x))/(cosh(x)-sinh(x)),x)
[Out] 1/2*(cosh(x)+sinh(x))/(cosh(x)-sinh(x))
```

maxima [A] time = 0.50, size = 6, normalized size = 0.55

$$\frac{1}{2}\,e^{(2\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cosh(x)+sinh(x))/(cosh(x)-sinh(x)),x, algorithm="maxima")
[Out] 1/2*e^(2*x)
```

mupad [B] time = 0.06, size = 6, normalized size = 0.55

$$\frac{\mathrm{e}^{2\,x}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cosh(x) + sinh(x))/(cosh(x) - sinh(x)),x)
[Out] exp(2*x)/2
```

sympy [A] time = 0.35, size = 8, normalized size = 0.73

$$\frac{\cosh(x)}{-\sinh(x) + \cosh(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cosh(x)+sinh(x))/(cosh(x)-sinh(x)),x)
```

```
[Out] cosh(x)/(-sinh(x) + cosh(x))
```

3.731 $\int \frac{\cosh(x)-\sinh(x)}{\cosh(x)+\sinh(x)}\,dx$

Optimal. Leaf size=11

$$-\frac{1}{2(\sinh(x)+\cosh(x))^2}$$

```
[Out] -1/2/(cosh(x)+sinh(x))^2
```

Rubi [A] time = 0.03, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {4385}

$$-\frac{1}{2(\sinh(x)+\cosh(x))^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Cosh[x] - Sinh[x])/(Cosh[x] + Sinh[x]),x]
```

```
[Out] -1/(2*(Cosh[x] + Sinh[x])^2)
```

Rule 4385

```
Int[(u_)*(y_)^(m_.), x_Symbol] :> With[{q = DerivativeDivides[ActivateTrig[
y], ActivateTrig[u], x]}, Simp[(q*ActivateTrig[y^(m + 1)])/(m + 1), x] /;
!FalseQ[q]] /; FreeQ[m, x] && NeQ[m, -1] &&  !InertTrigFreeQ[u]
```

Rubi steps

$$\int \frac{\cosh(x)-\sinh(x)}{\cosh(x)+\sinh(x)}\,dx = -\frac{1}{2(\cosh(x)+\sinh(x))^2}$$

Mathematica [A] time = 0.00, size = 17, normalized size = 1.55

$$\frac{1}{2}\sinh(2x)-\frac{1}{2}\cosh(2x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cosh[x] - Sinh[x])/(Cosh[x] + Sinh[x]),x]
```

```
[Out] -1/2*Cosh[2*x] + Sinh[2*x]/2
```

fricas [B] time = 0.43, size = 19, normalized size = 1.73

$$-\frac{1}{2\left(\cosh(x)^{2}+2\cosh(x)\sinh(x)+\sinh(x)^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cosh(x)-sinh(x))/(cosh(x)+sinh(x)),x, algorithm="fricas")
[Out] -1/2/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2)
```

giac [A] time = 0.11, size = 6, normalized size = 0.55

$$-\frac{1}{2}\,e^{(-2\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cosh(x)-sinh(x))/(cosh(x)+sinh(x)),x, algorithm="giac")
[Out] -1/2*e^(-2*x)
```

maple [A] time = 0.02, size = 17, normalized size = 1.55

$$-\frac{\cosh(x)-\sinh(x)}{2\left(\cosh(x)+\sinh(x)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cosh(x)-sinh(x))/(cosh(x)+sinh(x)),x)
[Out] -1/2*(cosh(x)-sinh(x))/(cosh(x)+sinh(x))
```

maxima [A] time = 0.40, size = 6, normalized size = 0.55

$$-\frac{1}{2}\,e^{(-2\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cosh(x)-sinh(x))/(cosh(x)+sinh(x)),x, algorithm="maxima")
[Out] -1/2*e^(-2*x)
```

mupad [B] time = 1.53, size = 6, normalized size = 0.55

$$-\frac{\mathrm{e}^{-2\,x}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cosh(x) - sinh(x))/(cosh(x) + sinh(x)),x)
```

```
[Out] -exp(-2*x)/2
```

sympy [A] time = 0.31, size = 10, normalized size = 0.91

$$-\frac{\cosh(x)}{\sinh(x)+\cosh(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cosh(x)-sinh(x))/(cosh(x)+sinh(x)),x)
```

```
[Out] -cosh(x)/(sinh(x) + cosh(x))
```

3.732 $\int \frac{\cosh(x)-i\sinh(x)}{\cosh(x)+i\sinh(x)}\,dx$

Optimal. Leaf size=14

$$-i\log(\cosh(x)+i\sinh(x))$$

```
[Out] -I*ln(cosh(x)+I*sinh(x))
```

Rubi [A] time = 0.03, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {3133}

$$-i\log(\cosh(x)+i\sinh(x))$$

Antiderivative was successfully verified.

```
[In] Int[(Cosh[x] - I*Sinh[x])/(Cosh[x] + I*Sinh[x]),x]
```

```
[Out] (-I)*Log[Cosh[x] + I*Sinh[x]]
```

Rule 3133

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)]), x
_Symbol] :> Simp[((b*B + c*C)*x)/(b^2 + c^2), x] + Simp[((c*B - b*C)*Log[a
+ b*Cos[d + e*x] + c*Sin[d + e*x]])/(e*(b^2 + c^2)), x] /; FreeQ[{a, b, c,
d, e, A, B, C}, x] && NeQ[b^2 + c^2, 0] && EqQ[A*(b^2 + c^2) - a*(b*B + c*C
), 0]
```

Rubi steps

$$\int \frac{\cosh(x)-i\sinh(x)}{\cosh(x)+i\sinh(x)}\,dx = -i\log(\cosh(x)+i\sinh(x))$$

Mathematica [A] time = 0.03, size = 15, normalized size = 1.07

$$\tan^{-1}(\tanh(x))-\frac{1}{2}i\log(\cosh(2x))$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cosh[x] - I*Sinh[x])/(Cosh[x] + I*Sinh[x]),x]
```

```
[Out] ArcTan[Tanh[x]] - (I/2)*Log[Cosh[2*x]]
```

fricas [A] time = 0.49, size = 13, normalized size = 0.93

$$i\,x - i\,\log\left(e^{(2\,x)} - i\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cosh(x)-I*sinh(x))/(cosh(x)+I*sinh(x)),x, algorithm="fricas")
[Out] I*x - I*log(e^(2*x) - I)
```

giac [A] time = 0.12, size = 13, normalized size = 0.93

$$i\,x - i\,\log\left(e^{(2\,x)} - i\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cosh(x)-I*sinh(x))/(cosh(x)+I*sinh(x)),x, algorithm="giac")
[Out] I*x - I*log(e^(2*x) - I)
```

maple [A] time = 0.17, size = 13, normalized size = 0.93

$$-i\ln\left(\cosh(x) + i\sinh(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cosh(x)-I*sinh(x))/(cosh(x)+I*sinh(x)),x)
[Out] -I*ln(cosh(x)+I*sinh(x))
```

maxima [A] time = 0.56, size = 10, normalized size = 0.71

$$-i\,\log\left(\cosh(x) + i\,\sinh(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cosh(x)-I*sinh(x))/(cosh(x)+I*sinh(x)),x, algorithm="maxima")
[Out] -I*log(cosh(x) + I*sinh(x))
```

mupad [B] time = 0.08, size = 16, normalized size = 1.14

$$x\,1\mathrm{i} - \ln\left(\mathrm{e}^{2\,x} - \mathrm{i}\right)\,1\mathrm{i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cosh(x) - sinh(x)*1i)/(cosh(x) + sinh(x)*1i),x)
```

```
[Out] x*1i - log(exp(2*x) - 1i)*1i
```

sympy [A] time = 0.12, size = 14, normalized size = 1.00

$$x(-2-i)+\log\left(e^{2x}-i\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cosh(x)-I*sinh(x))/(cosh(x)+I*sinh(x)),x)
[Out] x*(-2 - I) + log(exp(2*x) - I)
```

3.733 $\int \frac{B\cosh(x)+C\sinh(x)}{b\cosh(x)+c\sinh(x)}\,dx$

Optimal. Leaf size=53

$$\frac{x(bB-cC)}{b^2-c^2}-\frac{(Bc-bC)\log(b\cosh(x)+c\sinh(x))}{b^2-c^2}$$

```
[Out] (B*b-C*c)*x/(b^2-c^2)-(B*c-C*b)*ln(b*cosh(x)+c*sinh(x))/(b^2-c^2)
```

Rubi [A] time = 0.05, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {3133}

$$\frac{x(bB-cC)}{b^2-c^2}-\frac{(Bc-bC)\log(b\cosh(x)+c\sinh(x))}{b^2-c^2}$$

Antiderivative was successfully verified.

```
[In] Int[(B*Cosh[x] + C*Sinh[x])/(b*Cosh[x] + c*Sinh[x]),x]
```

```
[Out] ((b*B - c*C)*x)/(b^2 - c^2) - ((B*c - b*C)*Log[b*Cosh[x] + c*Sinh[x]])/(b^2
 - c^2)
```

Rule 3133

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)]), x
_Symbol] :> Simp[((b*B + c*C)*x)/(b^2 + c^2), x] + Simp[((c*B - b*C)*Log[a
+ b*Cos[d + e*x] + c*Sin[d + e*x]])/(e*(b^2 + c^2)), x] /; FreeQ[{a, b, c,
d, e, A, B, C}, x] && NeQ[b^2 + c^2, 0] && EqQ[A*(b^2 + c^2) - a*(b*B + c*C
), 0]
```

Rubi steps

$$\int \frac{B\cosh(x)+C\sinh(x)}{b\cosh(x)+c\sinh(x)}\,dx=\frac{(bB-cC)x}{b^2-c^2}-\frac{(Bc-bC)\log(b\cosh(x)+c\sinh(x))}{b^2-c^2}$$

Mathematica [A] time = 0.12, size = 43, normalized size = 0.81

$$\frac{x(bB-cC)+(bC-Bc)\log(b\cosh(x)+c\sinh(x))}{(b-c)(b+c)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(B*Cosh[x] + C*Sinh[x])/(b*Cosh[x] + c*Sinh[x]),x]
```

```
[Out] ((b*B - c*C)*x + (-(B*c) + b*C)*Log[b*Cosh[x] + c*Sinh[x]])/((b - c)*(b + c
))
```

fricas [A] time = 0.53, size = 60, normalized size = 1.13

$$\frac{((B-C)b+(B-C)c)x+(Cb-Bc)\log\left(\frac{2\,(b\cosh(x)+c\sinh(x))}{\cosh(x)-\sinh(x)}\right)}{b^2-c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cosh(x)+C*sinh(x))/(b*cosh(x)+c*sinh(x)),x, algorithm="fricas"
)
```

```
[Out] (((B - C)*b + (B - C)*c)*x + (C*b - B*c)*log(2*(b*cosh(x) + c*sinh(x))/(cos
h(x) - sinh(x))))/(b^2 - c^2)
```

giac [A] time = 0.11, size = 54, normalized size = 1.02

$$\frac{(B-C)x}{b-c}+\frac{(Cb-Bc)\log\left(\left|be^{(2\,x)}+ce^{(2\,x)}+b-c\right|\right)}{b^2-c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cosh(x)+C*sinh(x))/(b*cosh(x)+c*sinh(x)),x, algorithm="giac")
```

```
[Out] (B - C)*x/(b - c) + (C*b - B*c)*log(abs(b*e^(2*x) + c*e^(2*x) + b - c))/(b^
2 - c^2)
```

maple [B] time = 0.22, size = 145, normalized size = 2.74

$$-\frac{2B\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{2b+2c}-\frac{2C\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{2b+2c}+\frac{2B\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)}{2b-2c}-\frac{2C\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)}{2b-2c}-\frac{Bc\ln\left(\left(\tanh^2\left(\frac{x}{2}\right)\right)b\right.}{(b-c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*cosh(x)+C*sinh(x))/(b*cosh(x)+c*sinh(x)),x)
```

```
[Out] -2*B/(2*b+2*c)*ln(tanh(1/2*x)-1)-2*C/(2*b+2*c)*ln(tanh(1/2*x)-1)+2*B/(2*b-2
*c)*ln(tanh(1/2*x)+1)-2*C/(2*b-2*c)*ln(tanh(1/2*x)+1)-1/(b-c)/(b+c)*B*c*ln(
tanh(1/2*x)^2*b+2*c*tanh(1/2*x)+b)+1/(b-c)/(b+c)*b*C*ln(tanh(1/2*x)^2*b+2*c
*tanh(1/2*x)+b)
```

maxima [A] time = 0.48, size = 87, normalized size = 1.64

$$C\left(\frac{b\log\left(-(b-c)e^{(-2\,x)}-b-c\right)}{b^2-c^2}+\frac{x}{b+c}\right)-B\left(\frac{c\log\left(-(b-c)e^{(-2\,x)}-b-c\right)}{b^2-c^2}-\frac{x}{b+c}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cosh(x)+C*sinh(x))/(b*cosh(x)+c*sinh(x)),x, algorithm="maxima"
)
```

```
[Out] C*(b*log(-(b - c)*e^(-2*x) - b - c)/(b^2 - c^2) + x/(b + c)) - B*(c*log(-(b
 - c)*e^(-2*x) - b - c)/(b^2 - c^2) - x/(b + c))
```

mupad [B] time = 1.56, size = 53, normalized size = 1.00

$$\frac{x\,\left(B\,b-C\,c\right)}{b^2-c^2}-\frac{\ln\left(b\cosh(x)+c\sinh(x)\right)\,\left(B\,c-C\,b\right)}{b^2-c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*cosh(x) + C*sinh(x))/(b*cosh(x) + c*sinh(x)),x)
```

```
[Out] (x*(B*b - C*c))/(b^2 - c^2) - (log(b*cosh(x) + c*sinh(x))*(B*c - C*b))/(b^2
 - c^2)
```

sympy [A] time = 0.71, size = 326, normalized size = 6.15

$$\begin{cases} \tilde{\infty}\left(B\log\left(\sinh\left(x\right)\right)+Cx\right) \\ \frac{Bx\sinh\left(x\right)}{-2c\sinh\left(x\right)+2c\cosh\left(x\right)}-\frac{Bx\cosh\left(x\right)}{-2c\sinh\left(x\right)+2c\cosh\left(x\right)}-\frac{B\cosh\left(x\right)}{-2c\sinh\left(x\right)+2c\cosh\left(x\right)}-\frac{Cx\sinh\left(x\right)}{-2c\sinh\left(x\right)+2c\cosh\left(x\right)}+\frac{Cx\cosh\left(x\right)}{-2c\sinh\left(x\right)+2c\cosh\left(x\right)}-\ldots \\ \frac{Bx\sinh\left(x\right)}{2c\sinh\left(x\right)+2c\cosh\left(x\right)}+\frac{Bx\cosh\left(x\right)}{2c\sinh\left(x\right)+2c\cosh\left(x\right)}-\frac{B\cosh\left(x\right)}{2c\sinh\left(x\right)+2c\cosh\left(x\right)}+\frac{Cx\sinh\left(x\right)}{2c\sinh\left(x\right)+2c\cosh\left(x\right)}+\frac{Cx\cosh\left(x\right)}{2c\sinh\left(x\right)+2c\cosh\left(x\right)}+\ldots \\ \frac{B\log\left(\sinh\left(x\right)\right)+Cx}{c} \\ \frac{Bbx}{b^2-c^2}-\frac{Bc\log\left(\cosh\left(x\right)+\frac{c\sinh\left(x\right)}{b}\right)}{b^2-c^2}+\frac{Cb\log\left(\cosh\left(x\right)+\frac{c\sinh\left(x\right)}{b}\right)}{b^2-c^2}-\frac{Ccx}{b^2-c^2} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cosh(x)+C*sinh(x))/(b*cosh(x)+c*sinh(x)),x)
```

```
[Out] Piecewise((zoo*(B*log(sinh(x)) + C*x), Eq(b, 0) & Eq(c, 0)), (B*x*sinh(x)/(
-2*c*sinh(x) + 2*c*cosh(x)) - B*x*cosh(x)/(-2*c*sinh(x) + 2*c*cosh(x)) - B*
cosh(x)/(-2*c*sinh(x) + 2*c*cosh(x)) - C*x*sinh(x)/(-2*c*sinh(x) + 2*c*cosh
(x)) + C*x*cosh(x)/(-2*c*sinh(x) + 2*c*cosh(x)) - C*cosh(x)/(-2*c*sinh(x) +
 2*c*cosh(x)), Eq(b, -c)), (B*x*sinh(x)/(2*c*sinh(x) + 2*c*cosh(x)) + B*x*c
osh(x)/(2*c*sinh(x) + 2*c*cosh(x)) - B*cosh(x)/(2*c*sinh(x) + 2*c*cosh(x))
+ C*x*sinh(x)/(2*c*sinh(x) + 2*c*cosh(x)) + C*x*cosh(x)/(2*c*sinh(x) + 2*c*
cosh(x)) + C*cosh(x)/(2*c*sinh(x) + 2*c*cosh(x)), Eq(b, c)), ((B*log(sinh(x
)) + C*x)/c, Eq(b, 0)), (B*b*x/(b**2 - c**2) - B*c*log(cosh(x) + c*sinh(x)/
```

```
b)/(b**2 - c**2) + C*b*log(cosh(x) + c*sinh(x)/b)/(b**2 - c**2) - C*c*x/(b*
*2 - c**2), True))
```

3.734 $\int \frac{B\cosh(x)+C\sinh(x)}{(b\cosh(x)+c\sinh(x))^2}\,dx$

Optimal. Leaf size=78

$$\frac{Bc-bC}{(b^2-c^2)(b\cosh(x)+c\sinh(x))}+\frac{(bB-cC)\tan^{-1}\left(\frac{b\sinh(x)+c\cosh(x)}{\sqrt{b^2-c^2}}\right)}{(b^2-c^2)^{3/2}}$$

```
[Out] (B*b-C*c)*arctan((c*cosh(x)+b*sinh(x))/(b^2-c^2)^(1/2))/(b^2-c^2)^(3/2)+(B*
c-C*b)/(b^2-c^2)/(b*cosh(x)+c*sinh(x))
```

Rubi [A] time = 0.07, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3153, 3074, 206}

$$\frac{Bc-bC}{(b^2-c^2)(b\cosh(x)+c\sinh(x))}+\frac{(bB-cC)\tan^{-1}\left(\frac{b\sinh(x)+c\cosh(x)}{\sqrt{b^2-c^2}}\right)}{(b^2-c^2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(B*Cosh[x] + C*Sinh[x])/(b*Cosh[x] + c*Sinh[x])^2,x]
```

```
[Out] ((b*B - c*C)*ArcTan[(c*Cosh[x] + b*Sinh[x])/Sqrt[b^2 - c^2]])/(b^2 - c^2)^(
3/2) + (B*c - b*C)/((b^2 - c^2)*(b*Cosh[x] + c*Sinh[x]))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3074

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> -Dist[d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d
*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3153

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])^2,
 x_Symbol] :> Simp[(c*B - b*C - (a*C - c*A)*Cos[d + e*x] + (a*B - b*A)*Sin[
d + e*x])/(e*(a^2 - b^2 - c^2)*(a + b*Cos[d + e*x] + c*Sin[d + e*x])), x] +
 Dist[(a*A - b*B - c*C)/(a^2 - b^2 - c^2), Int[1/(a + b*Cos[d + e*x] + c*Si
```

```
n[d + e*x]), x], x] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && NeQ[a^2 - b^2
- c^2, 0] && NeQ[a*A - b*B - c*C, 0]
```

Rubi steps

$$\int \frac{B\cosh(x)+C\sinh(x)}{(b\cosh(x)+c\sinh(x))^2}\,dx = \frac{Bc-bC}{\left(b^2-c^2\right)(b\cosh(x)+c\sinh(x))}+\frac{(bB-cC)\int\frac{1}{b\cosh(x)+c\sinh(x)}\,dx}{b^2-c^2}$$
$$= \frac{Bc-bC}{\left(b^2-c^2\right)(b\cosh(x)+c\sinh(x))}+\frac{(i(bB-cC))\,\text{Subst}\left(\int\frac{1}{b^2-c^2-x^2}\,dx,x,-ic\cos$$
$$= \frac{(bB-cC)\tan^{-1}\left(\frac{c\cosh(x)+b\sinh(x)}{\sqrt{b^2-c^2}}\right)}{\left(b^2-c^2\right)^{3/2}}+\frac{Bc-bC}{\left(b^2-c^2\right)(b\cosh(x)+c\sinh(x))}$$

Mathematica [A] time = 0.25, size = 87, normalized size = 1.12

$$\frac{2(bB-cC)\tan^{-1}\left(\frac{b\tanh\left(\frac{x}{2}\right)+c}{\sqrt{b-c}\sqrt{b+c}}\right)}{(b-c)^{3/2}(b+c)^{3/2}}+\frac{Bc-bC}{(b-c)(b+c)(b\cosh(x)+c\sinh(x))}$$

Antiderivative was successfully verified.

```
[In] Integrate[(B*Cosh[x] + C*Sinh[x])/(b*Cosh[x] + c*Sinh[x])^2,x]
```

```
[Out] (2*(b*B - c*C)*ArcTan[(c + b*Tanh[x/2])/(Sqrt[b - c]*Sqrt[b + c])])/((b - c
)^(3/2)*(b + c)^(3/2)) + (B*c - b*C)/((b - c)*(b + c)*(b*Cosh[x] + c*Sinh[x
]))
```

fricas [B] time = 0.54, size = 749, normalized size = 9.60

$$\left[-\frac{\left(Bb^2-(B+C)bc+Cc^2+\left(Bb^2+(B-C)bc-Cc^2\right)\cosh(x)^2+2\left(Bb^2+(B-C)bc-Cc^2\right)\cosh(x)\sinh(x)+(}{b^5-b^4c-2b^3c^2+2b^2c^3+bc^4-c^5+\left(b^5+b^4c-2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cosh(x)+C*sinh(x))/(b*cosh(x)+c*sinh(x))^2,x, algorithm="frica
s")
```

```
[Out] [-((B*b^2 - (B + C)*b*c + C*c^2 + (B*b^2 + (B - C)*b*c - C*c^2)*cosh(x)^2 +
 2*(B*b^2 + (B - C)*b*c - C*c^2)*cosh(x)*sinh(x) + (B*b^2 + (B - C)*b*c - C
*c^2)*sinh(x)^2)*sqrt(-b^2 + c^2)*log(((b + c)*cosh(x)^2 + 2*(b + c)*cosh(x
)*sinh(x) + (b + c)*sinh(x)^2 - 2*sqrt(-b^2 + c^2)*(cosh(x) + sinh(x)) - b
+ c)/((b + c)*cosh(x)^2 + 2*(b + c)*cosh(x)*sinh(x) + (b + c)*sinh(x)^2 + b
 - c)) + 2*(C*b^3 - B*b^2*c - C*b*c^2 + B*c^3)*cosh(x) + 2*(C*b^3 - B*b^2*c
 - C*b*c^2 + B*c^3)*sinh(x))/(b^5 - b^4*c - 2*b^3*c^2 + 2*b^2*c^3 + b*c^4 -
 c^5 + (b^5 + b^4*c - 2*b^3*c^2 - 2*b^2*c^3 + b*c^4 + c^5)*cosh(x)^2 + 2*(b
^5 + b^4*c - 2*b^3*c^2 - 2*b^2*c^3 + b*c^4 + c^5)*cosh(x)*sinh(x) + (b^5 +
b^4*c - 2*b^3*c^2 - 2*b^2*c^3 + b*c^4 + c^5)*sinh(x)^2), -2*((B*b^2 - (B +
C)*b*c + C*c^2 + (B*b^2 + (B - C)*b*c - C*c^2)*cosh(x)^2 + 2*(B*b^2 + (B -
C)*b*c - C*c^2)*cosh(x)*sinh(x) + (B*b^2 + (B - C)*b*c - C*c^2)*sinh(x)^2)*
sqrt(b^2 - c^2)*arctan(sqrt(b^2 - c^2)/((b + c)*cosh(x) + (b + c)*sinh(x)))
 + (C*b^3 - B*b^2*c - C*b*c^2 + B*c^3)*cosh(x) + (C*b^3 - B*b^2*c - C*b*c^2
 + B*c^3)*sinh(x))/(b^5 - b^4*c - 2*b^3*c^2 + 2*b^2*c^3 + b*c^4 - c^5 + (b^
5 + b^4*c - 2*b^3*c^2 - 2*b^2*c^3 + b*c^4 + c^5)*cosh(x)^2 + 2*(b^5 + b^4*c
 - 2*b^3*c^2 - 2*b^2*c^3 + b*c^4 + c^5)*cosh(x)*sinh(x) + (b^5 + b^4*c - 2*
b^3*c^2 - 2*b^2*c^3 + b*c^4 + c^5)*sinh(x)^2)]
```

giac [A] time = 0.12, size = 88, normalized size = 1.13

$$\frac{2\,(Bb-Cc)\arctan\left(\frac{be^{x}+ce^{x}}{\sqrt{b^{2}-c^{2}}}\right)}{\left(b^{2}-c^{2}\right)^{\frac{3}{2}}}-\frac{2\,(Cbe^{x}-Bce^{x})}{\left(b^{2}-c^{2}\right)\left(be^{(2\,x)}+ce^{(2\,x)}+b-c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cosh(x)+C*sinh(x))/(b*cosh(x)+c*sinh(x))^2,x, algorithm="giac"
)
```

```
[Out] 2*(B*b - C*c)*arctan((b*e^x + c*e^x)/sqrt(b^2 - c^2))/(b^2 - c^2)^(3/2) - 2
*(C*b*e^x - B*c*e^x)/((b^2 - c^2)*(b*e^(2*x) + c*e^(2*x) + b - c))
```

maple [B] time = 0.26, size = 152, normalized size = 1.95

$$\frac{\frac{2c(Bc-bC)\tanh\left(\frac{x}{2}\right)}{\left(b^{2}-c^{2}\right)b}+\frac{2(Bc-bC)}{b^{2}-c^{2}}}{\left(\tanh^{2}\left(\frac{x}{2}\right)\right)b+2c\tanh\left(\frac{x}{2}\right)+b}+\frac{2bB\arctan\left(\frac{2\tanh\left(\frac{x}{2}\right)b+2c}{2\sqrt{b^{2}-c^{2}}}\right)}{\left(b^{2}-c^{2}\right)^{\frac{3}{2}}}-\frac{2Cc\arctan\left(\frac{2\tanh\left(\frac{x}{2}\right)b+2c}{2\sqrt{b^{2}-c^{2}}}\right)}{\left(b^{2}-c^{2}\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*cosh(x)+C*sinh(x))/(b*cosh(x)+c*sinh(x))^2,x)
```

```
[Out] 2*(c*(B*c-C*b)/(b^2-c^2)/b*tanh(1/2*x)+(B*c-C*b)/(b^2-c^2))/(tanh(1/2*x)^2*
b+2*c*tanh(1/2*x)+b)+2*b*B/(b^2-c^2)^(3/2)*arctan(1/2*(2*tanh(1/2*x)*b+2*c)
```

```
/(b^2-c^2)^(1/2))-2*C*c/(b^2-c^2)^(3/2)*arctan(1/2*(2*tanh(1/2*x)*b+2*c)/(b
^2-c^2)^(1/2))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cosh(x)+C*sinh(x))/(b*cosh(x)+c*sinh(x))^2,x, algorithm="maxim
a")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*c^2-4*b^2>0)', see `assume?` for
more details)Is 4*c^2-4*b^2 positive or negative?
```

mupad [B] time = 1.92, size = 199, normalized size = 2.55

$$\frac{\ln\left(\frac{2\,(B\,b-C\,c)}{(b+c)^{5/2}\,\sqrt{c-b}}+\frac{2\,e^x\,(B\,b-C\,c)}{-b^3-b^2\,c+b\,c^2+c^3}\right)\,(B\,b-C\,c)}{(b+c)^{3/2}\,(c-b)^{3/2}}-\frac{\ln\left(\frac{2\,e^x\,(B\,b-C\,c)}{-b^3-b^2\,c+b\,c^2+c^3}-\frac{2\,(B\,b-C\,c)}{(b+c)^{5/2}\,\sqrt{c-b}}\right)\,(B\,b-C\,c)}{(b+c)^{3/2}\,(c-b)^{3/2}}+\frac{2\,e^x\,(B\,c\ \ldots}{(b+c)\,(b-c)\,\left(b-\ldots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*cosh(x) + C*sinh(x))/(b*cosh(x) + c*sinh(x))^2,x)
```

```
[Out] (log((2*(B*b - C*c))/((b + c)^(5/2)*(c - b)^(1/2)) + (2*exp(x)*(B*b - C*c))
/(b*c^2 - b^2*c - b^3 + c^3))*(B*b - C*c))/((b + c)^(3/2)*(c - b)^(3/2)) -
(log((2*exp(x)*(B*b - C*c))/(b*c^2 - b^2*c - b^3 + c^3) - (2*(B*b - C*c))/(
(b + c)^(5/2)*(c - b)^(1/2)))*(B*b - C*c))/((b + c)^(3/2)*(c - b)^(3/2)) +
(2*exp(x)*(B*c - C*b))/((b + c)*(b - c)*(b - c + exp(2*x)*(b + c)))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cosh(x)+C*sinh(x))/(b*cosh(x)+c*sinh(x))**2,x)
```

```
[Out] Timed out
```

3.735 $\int \frac{B\cosh(x)+C\sinh(x)}{(b\cosh(x)+c\sinh(x))^3}\,dx$

Optimal. Leaf size=71

$$\frac{Bc-bC}{2\left(b^2-c^2\right)(b\cosh(x)+c\sinh(x))^2}+\frac{\sinh(x)(bB-cC)}{b\left(b^2-c^2\right)(b\cosh(x)+c\sinh(x))}$$

```
[Out] 1/2*(B*c-C*b)/(b^2-c^2)/(b*cosh(x)+c*sinh(x))^2+(B*b-C*c)*sinh(x)/b/(b^2-c^
2)/(b*cosh(x)+c*sinh(x))
```

Rubi [A] time = 0.07, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3156, 12, 3075}

$$\frac{Bc-bC}{2\left(b^2-c^2\right)(b\cosh(x)+c\sinh(x))^2}+\frac{\sinh(x)(bB-cC)}{b\left(b^2-c^2\right)(b\cosh(x)+c\sinh(x))}$$

Antiderivative was successfully verified.

```
[In] Int[(B*Cosh[x] + C*Sinh[x])/(b*Cosh[x] + c*Sinh[x])^3,x]
```

```
[Out] (B*c - b*C)/(2*(b^2 - c^2)*(b*Cosh[x] + c*Sinh[x])^2) + ((b*B - c*C)*Sinh[x
])/(b*(b^2 - c^2)*(b*Cosh[x] + c*Sinh[x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3075

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-2), x
_Symbol] :> Simp[Sin[c + d*x]/(a*d*(a*Cos[c + d*x] + b*Sin[c + d*x])), x] /
; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3156

```
Int[((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])
^(n_)*((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)
]), x_Symbol] :> -Simp[((c*B - b*C - (a*C - c*A)*Cos[d + e*x] + (a*B - b*A)
*Sin[d + e*x])*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1))/(e*(n + 1)*(a
^2 - b^2 - c^2)), x] + Dist[1/((n + 1)*(a^2 - b^2 - c^2)), Int[(a + b*Cos[d
 + e*x] + c*Sin[d + e*x])^(n + 1)*Simp[(n + 1)*(a*A - b*B - c*C) + (n + 2)*
(a*B - b*A)*Cos[d + e*x] + (n + 2)*(a*C - c*A)*Sin[d + e*x], x], x], x] /;
FreeQ[{a, b, c, d, e, A, B, C}, x] && LtQ[n, -1] && NeQ[a^2 - b^2 - c^2, 0]
```

```
&& NeQ[n, -2]
```

Rubi steps

$$\int \frac{B\cosh(x)+C\sinh(x)}{(b\cosh(x)+c\sinh(x))^3}\,dx = \frac{Bc-bC}{2\left(b^2-c^2\right)(b\cosh(x)+c\sinh(x))^2}+\frac{\int \frac{2(bB-cC)}{(b\cosh(x)+c\sinh(x))^2}\,dx}{2\left(b^2-c^2\right)}$$
$$= \frac{Bc-bC}{2\left(b^2-c^2\right)(b\cosh(x)+c\sinh(x))^2}+\frac{(bB-cC)\int \frac{1}{(b\cosh(x)+c\sinh(x))^2}\,dx}{b^2-c^2}$$
$$= \frac{Bc-bC}{2\left(b^2-c^2\right)(b\cosh(x)+c\sinh(x))^2}+\frac{(bB-cC)\sinh(x)}{b\left(b^2-c^2\right)(b\cosh(x)+c\sinh(x))}$$

Mathematica [A] time = 0.17, size = 70, normalized size = 0.99

$$\frac{C\left(c^2-b^2\right)+b\sinh(2x)(bB-cC)+c\cosh(2x)(bB-cC)}{2b(b-c)(b+c)(b\cosh(x)+c\sinh(x))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(B*Cosh[x] + C*Sinh[x])/(b*Cosh[x] + c*Sinh[x])^3,x]
```

```
[Out] ((-b^2 + c^2)*C + c*(b*B - c*C)*Cosh[2*x] + b*(b*B - c*C)*Sinh[2*x])/(2*b*(
b - c)*(b + c)*(b*Cosh[x] + c*Sinh[x])^2)
```

fricas [B] time = 0.51, size = 232, normalized size = 3.27

$$-\frac{}{\left(b^4+4\,b^3c+6\,b^2c^2+4\,bc^3+c^4\right)\cosh(x)^3+3\left(b^4+4\,b^3c+6\,b^2c^2+4\,bc^3+c^4\right)\cosh(x)\sinh(x)^2+\left(b^4+4\,b^3\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cosh(x)+C*sinh(x))/(b*cosh(x)+c*sinh(x))^3,x, algorithm="frica
s")
```

```
[Out] -2*(((2*B + C)*b + B*c)*cosh(x) + (C*b + (B + 2*C)*c)*sinh(x))/((b^4 + 4*b^
3*c + 6*b^2*c^2 + 4*b*c^3 + c^4)*cosh(x)^3 + 3*(b^4 + 4*b^3*c + 6*b^2*c^2 +
 4*b*c^3 + c^4)*cosh(x)*sinh(x)^2 + (b^4 + 4*b^3*c + 6*b^2*c^2 + 4*b*c^3 +
c^4)*sinh(x)^3 + (3*b^4 + 4*b^3*c - 2*b^2*c^2 - 4*b*c^3 - c^4)*cosh(x) + (b
^4 + 4*b^3*c + 2*b^2*c^2 - 4*b*c^3 - 3*c^4 + 3*(b^4 + 4*b^3*c + 6*b^2*c^2 +
 4*b*c^3 + c^4)*cosh(x)^2)*sinh(x))
```

giac [A] time = 0.14, size = 70, normalized size = 0.99

$$-\frac{2\left(Bbe^{(2x)} + Cbe^{(2x)} + Bce^{(2x)} + Cce^{(2x)} + Bb - Cc\right)}{\left(b^2 + 2bc + c^2\right)\left(be^{(2x)} + ce^{(2x)} + b - c\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cosh(x)+C*sinh(x))/(b*cosh(x)+c*sinh(x))^3,x, algorithm="giac"
)
```

```
[Out] -2*(B*b*e^(2*x) + C*b*e^(2*x) + B*c*e^(2*x) + C*c*e^(2*x) + B*b - C*c)/((b^
2 + 2*b*c + c^2)*(b*e^(2*x) + c*e^(2*x) + b - c)^2)
```

maple [A] time = 0.28, size = 63, normalized size = 0.89

$$-\frac{2\left(-\frac{B\left(\tanh^3\left(\frac{x}{2}\right)\right)}{b} - \frac{(Bc+bC)\left(\tanh^2\left(\frac{x}{2}\right)\right)}{b^2} - \frac{B\tanh\left(\frac{x}{2}\right)}{b}\right)}{\left(\left(\tanh^2\left(\frac{x}{2}\right)\right)b + 2c\tanh\left(\frac{x}{2}\right) + b\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*cosh(x)+C*sinh(x))/(b*cosh(x)+c*sinh(x))^3,x)
```

```
[Out] -2*(-B/b*tanh(1/2*x)^3-(B*c+C*b)/b^2*tanh(1/2*x)^2-B/b*tanh(1/2*x))/(tanh(1
/2*x)^2*b+2*c*tanh(1/2*x)+b)^2
```

maxima [B] time = 0.60, size = 337, normalized size = 4.75

$$2B\left(\frac{(b-c)e^{(-2x)}}{b^4 - 2b^2c^2 + c^4 + 2\left(b^4 - 2b^3c + 2bc^3 - c^4\right)e^{(-2x)} + \left(b^4 - 4b^3c + 6b^2c^2 - 4bc^3 + c^4\right)e^{(-4x)}} + \frac{}{b^4 - 2b^2c^2 + c^4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cosh(x)+C*sinh(x))/(b*cosh(x)+c*sinh(x))^3,x, algorithm="maxim
a")
```

```
[Out] 2*B*((b - c)*e^(-2*x)/(b^4 - 2*b^2*c^2 + c^4 + 2*(b^4 - 2*b^3*c + 2*b*c^3 -
 c^4)*e^(-2*x) + (b^4 - 4*b^3*c + 6*b^2*c^2 - 4*b*c^3 + c^4)*e^(-4*x)) + b/
(b^4 - 2*b^2*c^2 + c^4 + 2*(b^4 - 2*b^3*c + 2*b*c^3 - c^4)*e^(-2*x) + (b^4
- 4*b^3*c + 6*b^2*c^2 - 4*b*c^3 + c^4)*e^(-4*x))) - 2*C*((b - c)*e^(-2*x)/(
b^4 - 2*b^2*c^2 + c^4 + 2*(b^4 - 2*b^3*c + 2*b*c^3 - c^4)*e^(-2*x) + (b^4 -
 4*b^3*c + 6*b^2*c^2 - 4*b*c^3 + c^4)*e^(-4*x)) + c/(b^4 - 2*b^2*c^2 + c^4
+ 2*(b^4 - 2*b^3*c + 2*b*c^3 - c^4)*e^(-2*x) + (b^4 - 4*b^3*c + 6*b^2*c^2 -
 4*b*c^3 + c^4)*e^(-4*x)))
```

mupad [B] time = 1.66, size = 67, normalized size = 0.94

$$-\frac{b\,\left(2\,B+2\,B\,{\mathrm{e}}^{2\,x}+2\,C\,{\mathrm{e}}^{2\,x}\right)+c\,\left(2\,B\,{\mathrm{e}}^{2\,x}-2\,C+2\,C\,{\mathrm{e}}^{2\,x}\right)}{{\left(b+c\right)}^2\,{\left(b-c+b\,{\mathrm{e}}^{2\,x}+c\,{\mathrm{e}}^{2\,x}\right)}^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*cosh(x) + C*sinh(x))/(b*cosh(x) + c*sinh(x))^3,x)
```

```
[Out] -(b*(2*B + 2*B*exp(2*x) + 2*C*exp(2*x)) + c*(2*B*exp(2*x) - 2*C + 2*C*exp(2
*x)))/((b + c)^2*(b - c + b*exp(2*x) + c*exp(2*x))^2)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cosh(x)+C*sinh(x))/(b*cosh(x)+c*sinh(x))**3,x)
```

```
[Out] Timed out
```

3.736 $\int \frac{A+B\cosh(x)+C\sinh(x)}{b\cosh(x)+c\sinh(x)}\,dx$

Optimal. Leaf size=92

$$\frac{A\tan^{-1}\left(\frac{b\sinh(x)+c\cosh(x)}{\sqrt{b^2-c^2}}\right)}{\sqrt{b^2-c^2}}+\frac{x(bB-cC)}{b^2-c^2}-\frac{(Bc-bC)\log(b\cosh(x)+c\sinh(x))}{b^2-c^2}$$

```
[Out] (B*b-C*c)*x/(b^2-c^2)-(B*c-C*b)*ln(b*cosh(x)+c*sinh(x))/(b^2-c^2)+A*arctan(
(c*cosh(x)+b*sinh(x))/(b^2-c^2)^(1/2))/(b^2-c^2)^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {3136, 3074, 206}

$$\frac{A\tan^{-1}\left(\frac{b\sinh(x)+c\cosh(x)}{\sqrt{b^2-c^2}}\right)}{\sqrt{b^2-c^2}}+\frac{x(bB-cC)}{b^2-c^2}-\frac{(Bc-bC)\log(b\cosh(x)+c\sinh(x))}{b^2-c^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cosh[x] + C*Sinh[x])/(b*Cosh[x] + c*Sinh[x]),x]
```

```
[Out] ((b*B - c*C)*x)/(b^2 - c^2) + (A*ArcTan[(c*Cosh[x] + b*Sinh[x])/Sqrt[b^2 -
c^2]])/Sqrt[b^2 - c^2] - ((B*c - b*C)*Log[b*Cosh[x] + c*Sinh[x]])/(b^2 - c^
2)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3074

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> -Dist[d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d
*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3136

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)]), x
_Symbol] :> Simp[((b*B + c*C)*x)/(b^2 + c^2), x] + (Dist[(A*(b^2 + c^2) - a
*(b*B + c*C))/(b^2 + c^2), Int[1/(a + b*Cos[d + e*x] + c*Sin[d + e*x]), x],
```

```
 x] + Simp[((c*B - b*C)*Log[a + b*Cos[d + e*x] + c*Sin[d + e*x]])/(e*(b^2 +
 c^2)), x]) /; FreeQ[{a, b, c, d, e, A, B, C}, x] && NeQ[b^2 + c^2, 0] && N
eQ[A*(b^2 + c^2) - a*(b*B + c*C), 0]
```

Rubi steps

$$\int \frac{A+B\cosh(x)+C\sinh(x)}{b\cosh(x)+c\sinh(x)}\,dx = \frac{(bB-cC)x}{b^2-c^2}-\frac{(Bc-bC)\log(b\cosh(x)+c\sinh(x))}{b^2-c^2}+A\int \frac{1}{b\cosh(x)+c\sin$$

$$= \frac{(bB-cC)x}{b^2-c^2}-\frac{(Bc-bC)\log(b\cosh(x)+c\sinh(x))}{b^2-c^2}+(iA)\,\mathrm{Subst}\left(\int \frac{1}{b^2-c$$

$$= \frac{(bB-cC)x}{b^2-c^2}+\frac{A\tan^{-1}\left(\frac{c\cosh(x)+b\sinh(x)}{\sqrt{b^2-c^2}}\right)}{\sqrt{b^2-c^2}}-\frac{(Bc-bC)\log(b\cosh(x)+c\sinh$$

Mathematica [A] time = 0.27, size = 90, normalized size = 0.98

$$\frac{2A\sqrt{b-c}\,\sqrt{b+c}\,\tan^{-1}\left(\frac{b\tanh\left(\frac{x}{2}\right)+c}{\sqrt{b-c}\,\sqrt{b+c}}\right)+x(bB-cC)+(bC-Bc)\log(b\cosh(x)+c\sinh(x))}{(b-c)(b+c)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cosh[x] + C*Sinh[x])/(b*Cosh[x] + c*Sinh[x]),x]
```

```
[Out] ((b*B - c*C)*x + 2*A*Sqrt[b - c]*Sqrt[b + c]*ArcTan[(c + b*Tanh[x/2])/(Sqrt
[b - c]*Sqrt[b + c])] + (-(B*c) + b*C)*Log[b*Cosh[x] + c*Sinh[x]])/((b - c)
*(b + c))
```

fricas [A] time = 0.53, size = 264, normalized size = 2.87

$$\left[-\frac{\sqrt{-b^2+c^2}\,A\log\left(\frac{(b+c)\cosh(x)^2+2\,(b+c)\cosh(x)\sinh(x)+(b+c)\sinh(x)^2-2\,\sqrt{-b^2+c^2}\,(\cosh(x)+\sinh(x))-b+c}{(b+c)\cosh(x)^2+2\,(b+c)\cosh(x)\sinh(x)+(b+c)\sinh(x)^2+b-c}\right)-((B-C)b+(B-C$$
$$b^2-c^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x)+C*sinh(x))/(b*cosh(x)+c*sinh(x)),x, algorithm="frica
s")
```

```
[Out] [-(sqrt(-b^2 + c^2)*A*log(((b + c)*cosh(x)^2 + 2*(b + c)*cosh(x)*sinh(x) +
(b + c)*sinh(x)^2 - 2*sqrt(-b^2 + c^2)*(cosh(x) + sinh(x)) - b + c)/((b + c
```

```
)*cosh(x)^2 + 2*(b + c)*cosh(x)*sinh(x) + (b + c)*sinh(x)^2 + b - c)) - ((B
 - C)*b + (B - C)*c)*x - (C*b - B*c)*log(2*(b*cosh(x) + c*sinh(x))/(cosh(x)
 - sinh(x))))/(b^2 - c^2), -(2*sqrt(b^2 - c^2)*A*arctan(sqrt(b^2 - c^2)/((b
 + c)*cosh(x) + (b + c)*sinh(x))) - ((B - C)*b + (B - C)*c)*x - (C*b - B*c)
*log(2*(b*cosh(x) + c*sinh(x))/(cosh(x) - sinh(x))))/(b^2 - c^2)]
```

giac [A] time = 0.13, size = 89, normalized size = 0.97

$$\frac{2\,A\arctan\left(\frac{be^x+ce^x}{\sqrt{b^2-c^2}}\right)}{\sqrt{b^2-c^2}}+\frac{(B-C)x}{b-c}+\frac{(Cb-Bc)\log\left(be^{(2\,x)}+ce^{(2\,x)}+b-c\right)}{b^2-c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x)+C*sinh(x))/(b*cosh(x)+c*sinh(x)),x, algorithm="giac"
)
```

```
[Out] 2*A*arctan((b*e^x + c*e^x)/sqrt(b^2 - c^2))/sqrt(b^2 - c^2) + (B - C)*x/(b
- c) + (C*b - B*c)*log(b*e^(2*x) + c*e^(2*x) + b - c)/(b^2 - c^2)
```

maple [B] time = 0.21, size = 253, normalized size = 2.75

$$-\frac{2B\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{2b+2c}-\frac{2C\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{2b+2c}+\frac{2B\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)}{2b-2c}-\frac{2C\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)}{2b-2c}-\frac{Bc\ln\left(\left(\tanh^2\left(\frac{x}{2}\right)\right)b\right.}{(b-c)(b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cosh(x)+C*sinh(x))/(b*cosh(x)+c*sinh(x)),x)
```

```
[Out] -2*B/(2*b+2*c)*ln(tanh(1/2*x)-1)-2*C/(2*b+2*c)*ln(tanh(1/2*x)-1)+2*B/(2*b-2
*c)*ln(tanh(1/2*x)+1)-2*C/(2*b-2*c)*ln(tanh(1/2*x)+1)-1/(b-c)/(b+c)*B*c*ln(
tanh(1/2*x)^2*b+2*c*tanh(1/2*x)+b)+1/(b-c)/(b+c)*b*C*ln(tanh(1/2*x)^2*b+2*c
*tanh(1/2*x)+b)+2/(b-c)/(b+c)/(b^2-c^2)^(1/2)*arctan(1/2*(2*tanh(1/2*x)*b+2
*c)/(b^2-c^2)^(1/2))*A*b^2-2/(b-c)/(b+c)/(b^2-c^2)^(1/2)*arctan(1/2*(2*tanh
(1/2*x)*b+2*c)/(b^2-c^2)^(1/2))*A*c^2
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x)+C*sinh(x))/(b*cosh(x)+c*sinh(x)),x, algorithm="maxim
a")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
```

```
elp (example of legal syntax is 'assume(4*c^2-4*b^2>0)', see `assume?` for
more details)Is 4*c^2-4*b^2 positive or negative?
```

mupad [B] time = 3.81, size = 302, normalized size = 3.28

$$\frac{2\,\mathrm{atan}\left(\frac{A\,\mathrm{e}^x\,\sqrt{b^2-c^2}}{b\,\sqrt{A^2}-c\,\sqrt{A^2}}\right)\,\sqrt{A^2}}{\sqrt{b^2-c^2}}+\frac{B\,x}{b-c}-\frac{C\,x}{b-c}+\frac{B\,c^3\,\ln\left(4\,A^2\,b-4\,A^2\,c+4\,A^2\,b\,\mathrm{e}^{2\,x}+4\,A^2\,c\,\mathrm{e}^{2\,x}\right)}{b^4-2\,b^2\,c^2+c^4}+\frac{C\,b^3\,\ln\left(4\,A^2\,b-\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cosh(x) + C*sinh(x))/(b*cosh(x) + c*sinh(x)),x)
```

```
[Out] (2*atan((A*exp(x)*(b^2 - c^2)^(1/2))/(b*(A^2)^(1/2) - c*(A^2)^(1/2)))*(A^2)
^(1/2))/(b^2 - c^2)^(1/2) + (B*x)/(b - c) - (C*x)/(b - c) + (B*c^3*log(4*A^
2*b - 4*A^2*c + 4*A^2*b*exp(2*x) + 4*A^2*c*exp(2*x)))/(b^4 + c^4 - 2*b^2*c^
2) + (C*b^3*log(4*A^2*b - 4*A^2*c + 4*A^2*b*exp(2*x) + 4*A^2*c*exp(2*x)))/(
b^4 + c^4 - 2*b^2*c^2) - (B*b^2*c*log(4*A^2*b - 4*A^2*c + 4*A^2*b*exp(2*x)
+ 4*A^2*c*exp(2*x)))/(b^4 + c^4 - 2*b^2*c^2) - (C*b*c^2*log(4*A^2*b - 4*A^2
*c + 4*A^2*b*exp(2*x) + 4*A^2*c*exp(2*x)))/(b^4 + c^4 - 2*b^2*c^2)
```

sympy [A] time = 49.36, size = 643, normalized size = 6.99

$$\begin{cases} \tilde{\infty}\left(A\log\left(\tanh\left(\frac{x}{2}\right)\right)+Bx-2B\log\left(\tanh\left(\frac{x}{2}\right)+1\right)+B\log\left(\tanh\left(\frac{x}{2}\right)\right)+Cx\right) \\ -\frac{2A}{-2c\sinh(x)+2c\cosh(x)}+\frac{Bx\sinh(x)}{-2c\sinh(x)+2c\cosh(x)}-\frac{Bx\cosh(x)}{-2c\sinh(x)+2c\cosh(x)}-\frac{B\cosh(x)}{-2c\sinh(x)+2c\cosh(x)}-\frac{Cx\sinh(x)}{-2c\sinh(x)+2c\cosh(x)}+ \\ -\frac{2A}{2c\sinh(x)+2c\cosh(x)}+\frac{Bx\sinh(x)}{2c\sinh(x)+2c\cosh(x)}+\frac{Bx\cosh(x)}{2c\sinh(x)+2c\cosh(x)}-\frac{B\cosh(x)}{2c\sinh(x)+2c\cosh(x)}+\frac{Cx\sinh(x)}{2c\sinh(x)+2c\cosh(x)}+ \\ \frac{A\log\left(\tanh\left(\frac{x}{2}\right)\right)+Bx-2B\log\left(\tanh\left(\frac{x}{2}\right)+1\right)+B\log\left(\tanh\left(\frac{x}{2}\right)\right)+Cx}{c} \\ -\frac{A\sqrt{-b^2+c^2}\log\left(\tanh\left(\frac{x}{2}\right)+\frac{c}{b}-\frac{\sqrt{-b^2+c^2}}{b}\right)}{b^2-c^2}+\frac{A\sqrt{-b^2+c^2}\log\left(\tanh\left(\frac{x}{2}\right)+\frac{c}{b}+\frac{\sqrt{-b^2+c^2}}{b}\right)}{b^2-c^2}+\frac{Bbx}{b^2-c^2}-\frac{Bcx}{b^2-c^2}+\frac{2Bc\log\left(\tanh\left(\frac{x}{2}\right)+1\right)}{b^2-c^2}-\frac{Bc\log(}{} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x)+C*sinh(x))/(b*cosh(x)+c*sinh(x)),x)
```

```
[Out] Piecewise((zoo*(A*log(tanh(x/2)) + B*x - 2*B*log(tanh(x/2) + 1) + B*log(tan
h(x/2)) + C*x), Eq(b, 0) & Eq(c, 0)), (-2*A/(-2*c*sinh(x) + 2*c*cosh(x)) +
B*x*sinh(x)/(-2*c*sinh(x) + 2*c*cosh(x)) - B*x*cosh(x)/(-2*c*sinh(x) + 2*c*
cosh(x)) - B*cosh(x)/(-2*c*sinh(x) + 2*c*cosh(x)) - C*x*sinh(x)/(-2*c*sinh(
x) + 2*c*cosh(x)) + C*x*cosh(x)/(-2*c*sinh(x) + 2*c*cosh(x)) - C*cosh(x)/(-
2*c*sinh(x) + 2*c*cosh(x)), Eq(b, -c)), (-2*A/(2*c*sinh(x) + 2*c*cosh(x)) +
```

```
 B*x*sinh(x)/(2*c*sinh(x) + 2*c*cosh(x)) + B*x*cosh(x)/(2*c*sinh(x) + 2*c*c
osh(x)) - B*cosh(x)/(2*c*sinh(x) + 2*c*cosh(x)) + C*x*sinh(x)/(2*c*sinh(x)
+ 2*c*cosh(x)) + C*x*cosh(x)/(2*c*sinh(x) + 2*c*cosh(x)) + C*cosh(x)/(2*c*s
inh(x) + 2*c*cosh(x)), Eq(b, c)), ((A*log(tanh(x/2)) + B*x - 2*B*log(tanh(x
/2) + 1) + B*log(tanh(x/2)) + C*x)/c, Eq(b, 0)), (-A*sqrt(-b**2 + c**2)*log
(tanh(x/2) + c/b - sqrt(-b**2 + c**2)/b)/(b**2 - c**2) + A*sqrt(-b**2 + c**
2)*log(tanh(x/2) + c/b + sqrt(-b**2 + c**2)/b)/(b**2 - c**2) + B*b*x/(b**2
- c**2) - B*c*x/(b**2 - c**2) + 2*B*c*log(tanh(x/2) + 1)/(b**2 - c**2) - B*
c*log(tanh(x/2) + c/b - sqrt(-b**2 + c**2)/b)/(b**2 - c**2) - B*c*log(tanh(
x/2) + c/b + sqrt(-b**2 + c**2)/b)/(b**2 - c**2) + C*b*x/(b**2 - c**2) - 2*
C*b*log(tanh(x/2) + 1)/(b**2 - c**2) + C*b*log(tanh(x/2) + c/b - sqrt(-b**2
 + c**2)/b)/(b**2 - c**2) + C*b*log(tanh(x/2) + c/b + sqrt(-b**2 + c**2)/b)
/(b**2 - c**2) - C*c*x/(b**2 - c**2), True))
```

3.737 $\int \frac{A+B\cosh(x)+C\sinh(x)}{(b\cosh(x)+c\sinh(x))^2}\,dx$

Optimal. Leaf size=88

$$\frac{Ab\sinh(x)+Ac\cosh(x)-bC+Bc}{(b^2-c^2)(b\cosh(x)+c\sinh(x))}+\frac{(bB-cC)\tan^{-1}\left(\frac{b\sinh(x)+c\cosh(x)}{\sqrt{b^2-c^2}}\right)}{(b^2-c^2)^{3/2}}$$

```
[Out] (B*b-C*c)*arctan((c*cosh(x)+b*sinh(x))/(b^2-c^2)^(1/2))/(b^2-c^2)^(3/2)+(B*
c-b*C+A*c*cosh(x)+A*b*sinh(x))/(b^2-c^2)/(b*cosh(x)+c*sinh(x))
```

Rubi [A] time = 0.07, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {3153, 3074, 206}

$$\frac{Ab\sinh(x)+Ac\cosh(x)-bC+Bc}{(b^2-c^2)(b\cosh(x)+c\sinh(x))}+\frac{(bB-cC)\tan^{-1}\left(\frac{b\sinh(x)+c\cosh(x)}{\sqrt{b^2-c^2}}\right)}{(b^2-c^2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cosh[x] + C*Sinh[x])/(b*Cosh[x] + c*Sinh[x])^2,x]
```

```
[Out] ((b*B - c*C)*ArcTan[(c*Cosh[x] + b*Sinh[x])/Sqrt[b^2 - c^2]])/(b^2 - c^2)^(
3/2) + (B*c - b*C + A*c*Cosh[x] + A*b*Sinh[x])/((b^2 - c^2)*(b*Cosh[x] + c*
Sinh[x]))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3074

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> -Dist[d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d
*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3153

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])^2,
 x_Symbol] :> Simp[(c*B - b*C - (a*C - c*A)*Cos[d + e*x] + (a*B - b*A)*Sin[
d + e*x])/(e*(a^2 - b^2 - c^2)*(a + b*Cos[d + e*x] + c*Sin[d + e*x])), x] +
```

```
Dist[(a*A - b*B - c*C)/(a^2 - b^2 - c^2), Int[1/(a + b*Cos[d + e*x] + c*Si
n[d + e*x]), x], x] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && NeQ[a^2 - b^2
- c^2, 0] && NeQ[a*A - b*B - c*C, 0]
```

Rubi steps

$$\int \frac{A+B\cosh(x)+C\sinh(x)}{(b\cosh(x)+c\sinh(x))^2}\,dx = \frac{Bc-bC+Ac\cosh(x)+Ab\sinh(x)}{\left(b^2-c^2\right)(b\cosh(x)+c\sinh(x))}+\frac{(bB-cC)\int\frac{1}{b\cosh(x)+c\sinh(x)}\,dx}{b^2-c^2}$$

$$=\frac{Bc-bC+Ac\cosh(x)+Ab\sinh(x)}{\left(b^2-c^2\right)(b\cosh(x)+c\sinh(x))}+\frac{(i(bB-cC))\,\text{Subst}\left(\int\frac{1}{b^2-c^2-x^2}\,dx,x,-\right.}{b^2-c^2}$$

$$=\frac{(bB-cC)\tan^{-1}\left(\frac{c\cosh(x)+b\sinh(x)}{\sqrt{b^2-c^2}}\right)}{\left(b^2-c^2\right)^{3/2}}+\frac{Bc-bC+Ac\cosh(x)+Ab\sinh(x)}{\left(b^2-c^2\right)(b\cosh(x)+c\sinh(x))}$$

Mathematica [A] time = 0.29, size = 106, normalized size = 1.20

$$\frac{A\left(b^2-c^2\right)\sinh(x)+b(Bc-bC)}{b(b-c)(b+c)(b\cosh(x)+c\sinh(x))}+\frac{2(bB-cC)\tan^{-1}\left(\frac{b\tanh\left(\frac{x}{2}\right)+c}{\sqrt{b-c}\,\sqrt{b+c}}\right)}{(b-c)^{3/2}(b+c)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cosh[x] + C*Sinh[x])/(b*Cosh[x] + c*Sinh[x])^2,x]
```

```
[Out] (2*(b*B - c*C)*ArcTan[(c + b*Tanh[x/2])/(Sqrt[b - c]*Sqrt[b + c])])/((b - c
)^(3/2)*(b + c)^(3/2)) + (b*(B*c - b*C) + A*(b^2 - c^2)*Sinh[x])/(b*(b - c)
*(b + c)*(b*Cosh[x] + c*Sinh[x]))
```

fricas [B] time = 0.53, size = 799, normalized size = 9.08

$$\left[-\frac{2\,Ab^3-2\,Ab^2c-2\,Abc^2+2\,Ac^3+\left(Bb^2-(B+C)bc+Cc^2+\left(Bb^2+(B-C)bc-Cc^2\right)\cosh(x)^2+2\left(Bb^2+(B\right.\right.}{b^5-b^4c-2\,b^3c^2+2\,b^2c^3+bc^4-c^5}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x)+C*sinh(x))/(b*cosh(x)+c*sinh(x))^2,x, algorithm="fri
cas")
```

```
[Out] [-(2*A*b^3 - 2*A*b^2*c - 2*A*b*c^2 + 2*A*c^3 + (B*b^2 - (B + C)*b*c + C*c^2
 + (B*b^2 + (B - C)*b*c - C*c^2)*cosh(x)^2 + 2*(B*b^2 + (B - C)*b*c - C*c^2
)*cosh(x)*sinh(x) + (B*b^2 + (B - C)*b*c - C*c^2)*sinh(x)^2)*sqrt(-b^2 + c^
2)*log(((b + c)*cosh(x)^2 + 2*(b + c)*cosh(x)*sinh(x) + (b + c)*sinh(x)^2 -
 2*sqrt(-b^2 + c^2)*(cosh(x) + sinh(x)) - b + c)/((b + c)*cosh(x)^2 + 2*(b
+ c)*cosh(x)*sinh(x) + (b + c)*sinh(x)^2 + b - c)) + 2*(C*b^3 - B*b^2*c - C
*b*c^2 + B*c^3)*cosh(x) + 2*(C*b^3 - B*b^2*c - C*b*c^2 + B*c^3)*sinh(x))/(b
^5 - b^4*c - 2*b^3*c^2 + 2*b^2*c^3 + b*c^4 - c^5 + (b^5 + b^4*c - 2*b^3*c^2
 - 2*b^2*c^3 + b*c^4 + c^5)*cosh(x)^2 + 2*(b^5 + b^4*c - 2*b^3*c^2 - 2*b^2*
c^3 + b*c^4 + c^5)*cosh(x)*sinh(x) + (b^5 + b^4*c - 2*b^3*c^2 - 2*b^2*c^3 +
 b*c^4 + c^5)*sinh(x)^2), -2*(A*b^3 - A*b^2*c - A*b*c^2 + A*c^3 + (B*b^2 -
(B + C)*b*c + C*c^2 + (B*b^2 + (B - C)*b*c - C*c^2)*cosh(x)^2 + 2*(B*b^2 +
(B - C)*b*c - C*c^2)*cosh(x)*sinh(x) + (B*b^2 + (B - C)*b*c - C*c^2)*sinh(x
)^2)*sqrt(b^2 - c^2)*arctan(sqrt(b^2 - c^2)/((b + c)*cosh(x) + (b + c)*sinh
(x))) + (C*b^3 - B*b^2*c - C*b*c^2 + B*c^3)*cosh(x) + (C*b^3 - B*b^2*c - C*
b*c^2 + B*c^3)*sinh(x))/(b^5 - b^4*c - 2*b^3*c^2 + 2*b^2*c^3 + b*c^4 - c^5
+ (b^5 + b^4*c - 2*b^3*c^2 - 2*b^2*c^3 + b*c^4 + c^5)*cosh(x)^2 + 2*(b^5 +
b^4*c - 2*b^3*c^2 - 2*b^2*c^3 + b*c^4 + c^5)*cosh(x)*sinh(x) + (b^5 + b^4*c
 - 2*b^3*c^2 - 2*b^2*c^3 + b*c^4 + c^5)*sinh(x)^2)]
```

giac [A] time = 0.14, size = 95, normalized size = 1.08

$$\frac{2\,(Bb-Cc)\arctan\left(\frac{be^{x}+ce^{x}}{\sqrt{b^{2}-c^{2}}}\right)}{\left(b^{2}-c^{2}\right)^{\frac{3}{2}}}-\frac{2\,(Cbe^{x}-Bce^{x}+Ab-Ac)}{\left(b^{2}-c^{2}\right)\left(be^{(2\,x)}+ce^{(2\,x)}+b-c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x)+C*sinh(x))/(b*cosh(x)+c*sinh(x))^2,x, algorithm="gia
c")
```

```
[Out] 2*(B*b - C*c)*arctan((b*e^x + c*e^x)/sqrt(b^2 - c^2))/(b^2 - c^2)^(3/2) - 2
*(C*b*e^x - B*c*e^x + A*b - A*c)/((b^2 - c^2)*(b*e^(2*x) + c*e^(2*x) + b -
c))
```

maple [A] time = 0.27, size = 167, normalized size = 1.90

$$-\frac{2\left(-\frac{\left(A\,b^{2}-A\,c^{2}+B\,c^{2}-Ccb\right)\tanh\left(\frac{x}{2}\right)}{b\left(b^{2}-c^{2}\right)}-\frac{Bc-bC}{b^{2}-c^{2}}\right)}{\left(\tanh^{2}\left(\frac{x}{2}\right)\right)b+2c\tanh\left(\frac{x}{2}\right)+b}+\frac{2bB\arctan\left(\frac{2\tanh\left(\frac{x}{2}\right)b+2c}{2\sqrt{b^{2}-c^{2}}}\right)}{\left(b^{2}-c^{2}\right)^{\frac{3}{2}}}-\frac{2Cc\arctan\left(\frac{2\tanh\left(\frac{x}{2}\right)b+2c}{2\sqrt{b^{2}-c^{2}}}\right)}{\left(b^{2}-c^{2}\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cosh(x)+C*sinh(x))/(b*cosh(x)+c*sinh(x))^2,x)
```

```
[Out] -2*(-(A*b^2-A*c^2+B*c^2-C*b*c)/b/(b^2-c^2)*tanh(1/2*x)-(B*c-C*b)/(b^2-c^2))
/(tanh(1/2*x)^2*b+2*c*tanh(1/2*x)+b)+2*b*B/(b^2-c^2)^(3/2)*arctan(1/2*(2*ta
nh(1/2*x)*b+2*c)/(b^2-c^2)^(1/2))-2*C*c/(b^2-c^2)^(3/2)*arctan(1/2*(2*tanh(
1/2*x)*b+2*c)/(b^2-c^2)^(1/2))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x)+C*sinh(x))/(b*cosh(x)+c*sinh(x))^2,x, algorithm="max
ima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*c^2-4*b^2>0)', see `assume?` for
more details)Is 4*c^2-4*b^2 positive or negative?
```

mupad [B] time = 1.75, size = 210, normalized size = 2.39

$$\frac{\ln\left(\frac{2\,(B\,b-C\,c)}{(b+c)^{5/2}\,\sqrt{c-b}}+\frac{2\,e^x\,(B\,b-C\,c)}{-b^3-b^2\,c+b\,c^2+c^3}\right)\,(B\,b-C\,c)}{(b+c)^{3/2}\,(c-b)^{3/2}}-\frac{\ln\left(\frac{2\,e^x\,(B\,b-C\,c)}{-b^3-b^2\,c+b\,c^2+c^3}-\frac{2\,(B\,b-C\,c)}{(b+c)^{5/2}\,\sqrt{c-b}}\right)\,(B\,b-C\,c)}{(b+c)^{3/2}\,(c-b)^{3/2}}-\frac{\frac{2\,A}{b+c}-\frac{2\,e^x\,(B\,c-C\,b)}{(b+c)\,(b-c)}}{b-c+e^{2\,x}\,(b+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cosh(x) + C*sinh(x))/(b*cosh(x) + c*sinh(x))^2,x)
```

```
[Out] (log((2*(B*b - C*c))/((b + c)^(5/2)*(c - b)^(1/2)) + (2*exp(x)*(B*b - C*c))
/(b*c^2 - b^2*c - b^3 + c^3))*(B*b - C*c))/((b + c)^(3/2)*(c - b)^(3/2)) -
(log((2*exp(x)*(B*b - C*c))/(b*c^2 - b^2*c - b^3 + c^3) - (2*(B*b - C*c))/(
(b + c)^(5/2)*(c - b)^(1/2)))*(B*b - C*c))/((b + c)^(3/2)*(c - b)^(3/2)) -
((2*A)/(b + c) - (2*exp(x)*(B*c - C*b))/((b + c)*(b - c)))/(b - c + exp(2*x
)*(b + c))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x)+C*sinh(x))/(b*cosh(x)+c*sinh(x))**2,x)
```

```
[Out] Timed out
```

3.738 $\int \frac{A+B\cosh(x)+C\sinh(x)}{(b\cosh(x)+c\sinh(x))^3}\,dx$

Optimal. Leaf size=135

$$\frac{Ab\sinh(x)+Ac\cosh(x)-bC+Bc}{2\left(b^2-c^2\right)(b\cosh(x)+c\sinh(x))^2}+\frac{A\tan^{-1}\left(\frac{b\sinh(x)+c\cosh(x)}{\sqrt{b^2-c^2}}\right)}{2\left(b^2-c^2\right)^{3/2}}+\frac{b\sinh(x)(bB-cC)+c\cosh(x)(bB-cC)}{\left(b^2-c^2\right)^2(b\cosh(x)+c\sinh(x))}$$

```
[Out] 1/2*A*arctan((c*cosh(x)+b*sinh(x))/(b^2-c^2)^(1/2))/(b^2-c^2)^(3/2)+1/2*(B*
c-b*C+A*c*cosh(x)+A*b*sinh(x))/(b^2-c^2)/(b*cosh(x)+c*sinh(x))^2+(c*(B*b-C*
c)*cosh(x)+b*(B*b-C*c)*sinh(x))/(b^2-c^2)^2/(b*cosh(x)+c*sinh(x))
```

Rubi [A] time = 0.14, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {3156, 3153, 3074, 206}

$$\frac{Ab\sinh(x)+Ac\cosh(x)-bC+Bc}{2\left(b^2-c^2\right)(b\cosh(x)+c\sinh(x))^2}+\frac{A\tan^{-1}\left(\frac{b\sinh(x)+c\cosh(x)}{\sqrt{b^2-c^2}}\right)}{2\left(b^2-c^2\right)^{3/2}}+\frac{b\sinh(x)(bB-cC)+c\cosh(x)(bB-cC)}{\left(b^2-c^2\right)^2(b\cosh(x)+c\sinh(x))}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cosh[x] + C*Sinh[x])/(b*Cosh[x] + c*Sinh[x])^3,x]
```

```
[Out] (A*ArcTan[(c*Cosh[x] + b*Sinh[x])/Sqrt[b^2 - c^2]])/(2*(b^2 - c^2)^(3/2)) +
 (B*c - b*C + A*c*Cosh[x] + A*b*Sinh[x])/(2*(b^2 - c^2)*(b*Cosh[x] + c*Sinh
[x])^2) + (c*(b*B - c*C)*Cosh[x] + b*(b*B - c*C)*Sinh[x])/((b^2 - c^2)^2*(b
*Cosh[x] + c*Sinh[x]))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3074

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> -Dist[d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d
*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3153

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])^2,
```

```
 x_Symbol] :> Simp[(c*B - b*C - (a*C - c*A)*Cos[d + e*x] + (a*B - b*A)*Sin[
d + e*x])/(e*(a^2 - b^2 - c^2)*(a + b*Cos[d + e*x] + c*Sin[d + e*x])), x] +
 Dist[(a*A - b*B - c*C)/(a^2 - b^2 - c^2), Int[1/(a + b*Cos[d + e*x] + c*Si
n[d + e*x]), x], x] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && NeQ[a^2 - b^2
- c^2, 0] && NeQ[a*A - b*B - c*C, 0]
```

Rule 3156

```
Int[((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])
^(n_)*((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)
]), x_Symbol] :> -Simp[((c*B - b*C - (a*C - c*A)*Cos[d + e*x] + (a*B - b*A)
*Sin[d + e*x])*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1))/(e*(n + 1)*(a
^2 - b^2 - c^2)), x] + Dist[1/((n + 1)*(a^2 - b^2 - c^2)), Int[(a + b*Cos[d
 + e*x] + c*Sin[d + e*x])^(n + 1)*Simp[(n + 1)*(a*A - b*B - c*C) + (n + 2)*
(a*B - b*A)*Cos[d + e*x] + (n + 2)*(a*C - c*A)*Sin[d + e*x], x], x], x] /;
FreeQ[{a, b, c, d, e, A, B, C}, x] && LtQ[n, -1] && NeQ[a^2 - b^2 - c^2, 0]
 && NeQ[n, -2]
```

Rubi steps

$$\int \frac{A+B\cosh(x)+C\sinh(x)}{(b\cosh(x)+c\sinh(x))^3}\,dx = \frac{Bc-bC+Ac\cosh(x)+Ab\sinh(x)}{2\left(b^2-c^2\right)(b\cosh(x)+c\sinh(x))^2}+\frac{\int \frac{2(bB-cC)+Ab\cosh(x)+Ac\sinh(x)}{(b\cosh(x)+c\sinh(x))^2}\,dx}{2\left(b^2-c^2\right)}$$

$$= \frac{Bc-bC+Ac\cosh(x)+Ab\sinh(x)}{2\left(b^2-c^2\right)(b\cosh(x)+c\sinh(x))^2}+\frac{c(bB-cC)\cosh(x)+b(bB-cC)\sinh}{\left(b^2-c^2\right)^2(b\cosh(x)+c\sinh(x))}$$

$$= \frac{Bc-bC+Ac\cosh(x)+Ab\sinh(x)}{2\left(b^2-c^2\right)(b\cosh(x)+c\sinh(x))^2}+\frac{c(bB-cC)\cosh(x)+b(bB-cC)\sinh}{\left(b^2-c^2\right)^2(b\cosh(x)+c\sinh(x))}$$

$$= \frac{A\tan^{-1}\left(\frac{c\cosh(x)+b\sinh(x)}{\sqrt{b^2-c^2}}\right)}{2\left(b^2-c^2\right)^{3/2}}+\frac{Bc-bC+Ac\cosh(x)+Ab\sinh(x)}{2\left(b^2-c^2\right)(b\cosh(x)+c\sinh(x))^2}+\frac{c(bB-c}{\left(b^2-\right.}$$

Mathematica [A] time = 0.84, size = 146, normalized size = 1.08

$$\frac{1}{2}\left(\frac{A\left(b^2-c^2\right)\sinh(x)+b(Bc-bC)}{b(b-c)(b+c)(b\cosh(x)+c\sinh(x))^2}+\frac{Ac+2\sinh(x)(bB-cC)}{b(b-c)(b+c)(b\cosh(x)+c\sinh(x))}+\frac{2A\tan^{-1}\left(\frac{b\tanh\left(\frac{x}{2}\right)+c}{\sqrt{b-c}\sqrt{b+c}}\right)}{(b-c)^{3/2}(b+c)^{3/2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cosh[x] + C*Sinh[x])/(b*Cosh[x] + c*Sinh[x])^3,x]
```

```
[Out] ((2*A*ArcTan[(c + b*Tanh[x/2])/(Sqrt[b - c]*Sqrt[b + c])])/((b - c)^(3/2)*(
b + c)^(3/2)) + (b*(B*c - b*C) + A*(b^2 - c^2)*Sinh[x])/(b*(b - c)*(b + c)*
(b*Cosh[x] + c*Sinh[x])^2) + (A*c + 2*(b*B - c*C)*Sinh[x])/(b*(b - c)*(b +
c)*(b*Cosh[x] + c*Sinh[x])))/2
```

fricas [B] time = 0.52, size = 1931, normalized size = 14.30

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x)+C*sinh(x))/(b*cosh(x)+c*sinh(x))^3,x, algorithm="fri
cas")
```

```
[Out] [-1/2*(4*B*b^3 - 4*(2*B + C)*b^2*c + 4*(B + 2*C)*b*c^2 - 4*C*c^3 - 2*(A*b^3
 + A*b^2*c - A*b*c^2 - A*c^3)*cosh(x)^3 - 2*(A*b^3 + A*b^2*c - A*b*c^2 - A*
c^3)*sinh(x)^3 + 4*((B + C)*b^3 - (B + C)*b^2*c - (B + C)*b*c^2 + (B + C)*c
^3)*cosh(x)^2 + 2*(2*(B + C)*b^3 - 2*(B + C)*b^2*c - 2*(B + C)*b*c^2 + 2*(B
 + C)*c^3 - 3*(A*b^3 + A*b^2*c - A*b*c^2 - A*c^3)*cosh(x))*sinh(x)^2 - ((A*
b^2 + 2*A*b*c + A*c^2)*cosh(x)^4 + 4*(A*b^2 + 2*A*b*c + A*c^2)*cosh(x)*sinh
(x)^3 + (A*b^2 + 2*A*b*c + A*c^2)*sinh(x)^4 + A*b^2 - 2*A*b*c + A*c^2 + 2*(
A*b^2 - A*c^2)*cosh(x)^2 + 2*(A*b^2 - A*c^2 + 3*(A*b^2 + 2*A*b*c + A*c^2)*c
osh(x)^2)*sinh(x)^2 + 4*((A*b^2 + 2*A*b*c + A*c^2)*cosh(x)^3 + (A*b^2 - A*c
^2)*cosh(x))*sinh(x))*sqrt(-b^2 + c^2)*log(((b + c)*cosh(x)^2 + 2*(b + c)*c
osh(x)*sinh(x) + (b + c)*sinh(x)^2 + 2*sqrt(-b^2 + c^2)*(cosh(x) + sinh(x))
 - b + c)/((b + c)*cosh(x)^2 + 2*(b + c)*cosh(x)*sinh(x) + (b + c)*sinh(x)^
2 + b - c)) + 2*(A*b^3 - A*b^2*c - A*b*c^2 + A*c^3)*cosh(x) + 2*(A*b^3 - A*
b^2*c - A*b*c^2 + A*c^3 - 3*(A*b^3 + A*b^2*c - A*b*c^2 - A*c^3)*cosh(x)^2 +
 4*((B + C)*b^3 - (B + C)*b^2*c - (B + C)*b*c^2 + (B + C)*c^3)*cosh(x))*sin
h(x))/(b^6 - 2*b^5*c - b^4*c^2 + 4*b^3*c^3 - b^2*c^4 - 2*b*c^5 + c^6 + (b^6
 + 2*b^5*c - b^4*c^2 - 4*b^3*c^3 - b^2*c^4 + 2*b*c^5 + c^6)*cosh(x)^4 + 4*(
b^6 + 2*b^5*c - b^4*c^2 - 4*b^3*c^3 - b^2*c^4 + 2*b*c^5 + c^6)*cosh(x)*sinh
(x)^3 + (b^6 + 2*b^5*c - b^4*c^2 - 4*b^3*c^3 - b^2*c^4 + 2*b*c^5 + c^6)*sin
h(x)^4 + 2*(b^6 - 3*b^4*c^2 + 3*b^2*c^4 - c^6)*cosh(x)^2 + 2*(b^6 - 3*b^4*c
^2 + 3*b^2*c^4 - c^6 + 3*(b^6 + 2*b^5*c - b^4*c^2 - 4*b^3*c^3 - b^2*c^4 + 2
*b*c^5 + c^6)*cosh(x)^2)*sinh(x)^2 + 4*((b^6 + 2*b^5*c - b^4*c^2 - 4*b^3*c^
3 - b^2*c^4 + 2*b*c^5 + c^6)*cosh(x)^3 + (b^6 - 3*b^4*c^2 + 3*b^2*c^4 - c^6
)*cosh(x))*sinh(x)), -(2*B*b^3 - 2*(2*B + C)*b^2*c + 2*(B + 2*C)*b*c^2 - 2*
C*c^3 - (A*b^3 + A*b^2*c - A*b*c^2 - A*c^3)*cosh(x)^3 - (A*b^3 + A*b^2*c -
A*b*c^2 - A*c^3)*sinh(x)^3 + 2*((B + C)*b^3 - (B + C)*b^2*c - (B + C)*b*c^2
 + (B + C)*c^3)*cosh(x)^2 + (2*(B + C)*b^3 - 2*(B + C)*b^2*c - 2*(B + C)*b*
c^2 + 2*(B + C)*c^3 - 3*(A*b^3 + A*b^2*c - A*b*c^2 - A*c^3)*cosh(x))*sinh(x
```

```
)^2 + ((A*b^2 + 2*A*b*c + A*c^2)*cosh(x)^4 + 4*(A*b^2 + 2*A*b*c + A*c^2)*co
sh(x)*sinh(x)^3 + (A*b^2 + 2*A*b*c + A*c^2)*sinh(x)^4 + A*b^2 - 2*A*b*c + A
*c^2 + 2*(A*b^2 - A*c^2)*cosh(x)^2 + 2*(A*b^2 - A*c^2 + 3*(A*b^2 + 2*A*b*c
+ A*c^2)*cosh(x)^2)*sinh(x)^2 + 4*((A*b^2 + 2*A*b*c + A*c^2)*cosh(x)^3 + (A
*b^2 - A*c^2)*cosh(x))*sinh(x))*sqrt(b^2 - c^2)*arctan(sqrt(b^2 - c^2)/((b
+ c)*cosh(x) + (b + c)*sinh(x))) + (A*b^3 - A*b^2*c - A*b*c^2 + A*c^3)*cosh
(x) + (A*b^3 - A*b^2*c - A*b*c^2 + A*c^3 - 3*(A*b^3 + A*b^2*c - A*b*c^2 - A
*c^3)*cosh(x)^2 + 4*((B + C)*b^3 - (B + C)*b^2*c - (B + C)*b*c^2 + (B + C)*
c^3)*cosh(x))*sinh(x))/(b^6 - 2*b^5*c - b^4*c^2 + 4*b^3*c^3 - b^2*c^4 - 2*b
*c^5 + c^6 + (b^6 + 2*b^5*c - b^4*c^2 - 4*b^3*c^3 - b^2*c^4 + 2*b*c^5 + c^6
)*cosh(x)^4 + 4*(b^6 + 2*b^5*c - b^4*c^2 - 4*b^3*c^3 - b^2*c^4 + 2*b*c^5 +
c^6)*cosh(x)*sinh(x)^3 + (b^6 + 2*b^5*c - b^4*c^2 - 4*b^3*c^3 - b^2*c^4 + 2
*b*c^5 + c^6)*sinh(x)^4 + 2*(b^6 - 3*b^4*c^2 + 3*b^2*c^4 - c^6)*cosh(x)^2 +
 2*(b^6 - 3*b^4*c^2 + 3*b^2*c^4 - c^6 + 3*(b^6 + 2*b^5*c - b^4*c^2 - 4*b^3*
c^3 - b^2*c^4 + 2*b*c^5 + c^6)*cosh(x)^2)*sinh(x)^2 + 4*((b^6 + 2*b^5*c - b
^4*c^2 - 4*b^3*c^3 - b^2*c^4 + 2*b*c^5 + c^6)*cosh(x)^3 + (b^6 - 3*b^4*c^2
+ 3*b^2*c^4 - c^6)*cosh(x))*sinh(x))]
```

giac [A] time = 0.14, size = 183, normalized size = 1.36

$$\frac{A \arctan\left(\frac{be^{x}+ce^{x}}{\sqrt{b^2-c^2}}\right)}{\left(b^2-c^2\right)^{\frac{3}{2}}}+\frac{Ab^2e^{(3\,x)}+2\,Abce^{(3\,x)}+Ac^2e^{(3\,x)}-2\,Bb^2e^{(2\,x)}-2\,Cb^2e^{(2\,x)}+2\,Bc^2e^{(2\,x)}+2\,Cc^2e^{(2\,x)}-Ab^2e^{x}}{\left(b^3+b^2c-bc^2-c^3\right)\left(be^{(2\,x)}+ce^{(2\,x)}+b-c\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x)+C*sinh(x))/(b*cosh(x)+c*sinh(x))^3,x, algorithm="gia
c")
```

```
[Out] A*arctan((b*e^x + c*e^x)/sqrt(b^2 - c^2))/(b^2 - c^2)^(3/2) + (A*b^2*e^(3*x
) + 2*A*b*c*e^(3*x) + A*c^2*e^(3*x) - 2*B*b^2*e^(2*x) - 2*C*b^2*e^(2*x) + 2
*B*c^2*e^(2*x) + 2*C*c^2*e^(2*x) - A*b^2*e^x + A*c^2*e^x - 2*B*b^2 + 2*B*b*
c + 2*C*b*c - 2*C*c^2)/((b^3 + b^2*c - b*c^2 - c^3)*(b*e^(2*x) + c*e^(2*x)
+ b - c)^2)
```

maple [A] time = 0.29, size = 228, normalized size = 1.69

$$\frac{-\frac{\left(A\,b^2-2A\,c^2-2B\,b^2+2B\,c^2\right)\left(\tanh^3\left(\frac{x}{2}\right)\right)}{\left(b^2-c^2\right)b}+\frac{\left(A\,b^2c+2A\,c^3+2Bc\,b^2-2B\,c^3+2C\,b^3-2Cb\,c^2\right)\left(\tanh^2\left(\frac{x}{2}\right)\right)}{\left(b^2-c^2\right)b^2}+\frac{\left(A\,b^2+2A\,c^2+2B\,b^2-2B\,c^2\right)\tanh\left(\frac{x}{2}\right)}{\left(b^2-c^2\right)b}+\frac{2A}{2b^2-}}{\left(\left(\tanh^2\left(\frac{x}{2}\right)\right)b+2c\tanh\left(\frac{x}{2}\right)+b\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cosh(x)+C*sinh(x))/(b*cosh(x)+c*sinh(x))^3,x)
```

```
[Out] 2*(-1/2*(A*b^2-2*A*c^2-2*B*b^2+2*B*c^2)/(b^2-c^2)/b*tanh(1/2*x)^3+1/2*(A*b^
2*c+2*A*c^3+2*B*b^2*c-2*B*c^3+2*C*b^3-2*C*b*c^2)/(b^2-c^2)/b^2*tanh(1/2*x)^
2+1/2*(A*b^2+2*A*c^2+2*B*b^2-2*B*c^2)/(b^2-c^2)/b*tanh(1/2*x)+1/2*A*c/(b^2-
c^2))/(tanh(1/2*x)^2*b+2*c*tanh(1/2*x)+b)^2+A/(b^2-c^2)^(3/2)*arctan(1/2*(2
*tanh(1/2*x)*b+2*c)/(b^2-c^2)^(1/2))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x)+C*sinh(x))/(b*cosh(x)+c*sinh(x))^3,x, algorithm="max
ima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*c^2-4*b^2>0)', see `assume?` for
more details)Is 4*c^2-4*b^2 positive or negative?
```

mupad [B] time = 1.69, size = 224, normalized size = 1.66

$$\frac{\mathrm{atan}\left(\frac{A\,\mathrm{e}^x\,\sqrt{b^6-3\,b^4\,c^2+3\,b^2\,c^4-c^6}}{b^3\,\sqrt{A^2}+c^3\,\sqrt{A^2}-b\,c^2\,\sqrt{A^2}-b^2\,c\,\sqrt{A^2}}\right)\,\sqrt{A^2}}{\sqrt{b^6-3\,b^4\,c^2+3\,b^2\,c^4-c^6}}-\frac{\frac{B-C}{b+c}+\frac{2\,A\,\mathrm{e}^x}{b+c}+\frac{\mathrm{e}^{2\,x}\,(B+C)}{b+c}}{\mathrm{e}^{4\,x}\,{(b+c)}^2+{(b-c)}^2+2\,\mathrm{e}^{2\,x}\,(b+c)\,(b-c)}-\frac{\frac{B+C}{{(b+c)}^2}-\frac{A\,\mathrm{e}^x}{(b+c)\,(b-c)}}{b-c+\mathrm{e}^{2\,x}\,(b+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cosh(x) + C*sinh(x))/(b*cosh(x) + c*sinh(x))^3,x)
```

```
[Out] (atan((A*exp(x)*(b^6 - c^6 + 3*b^2*c^4 - 3*b^4*c^2)^(1/2))/(b^3*(A^2)^(1/2)
 + c^3*(A^2)^(1/2) - b*c^2*(A^2)^(1/2) - b^2*c*(A^2)^(1/2)))*(A^2)^(1/2))/(
b^6 - c^6 + 3*b^2*c^4 - 3*b^4*c^2)^(1/2) - ((B - C)/(b + c) + (2*A*exp(x))/
(b + c) + (exp(2*x)*(B + C))/(b + c))/(exp(4*x)*(b + c)^2 + (b - c)^2 + 2*e
xp(2*x)*(b + c)*(b - c)) - ((B + C)/(b + c)^2 - (A*exp(x))/((b + c)*(b - c)
))/(b - c + exp(2*x)*(b + c))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x)+C*sinh(x))/(b*cosh(x)+c*sinh(x))**3,x)
```

```
[Out] Timed out
```

3.739 $\int (a + b\cosh(x) + c\sinh(x))^3\,dx$

Optimal. Leaf size=119

$$\frac{1}{2}ax\left(2a^2+3b^2-3c^2\right)+\frac{1}{6}b\sinh(x)\left(11a^2+4b^2-4c^2\right)+\frac{1}{6}c\cosh(x)\left(11a^2+4b^2-4c^2\right)+\frac{1}{3}(b\sinh(x)+c\cosh(x))(a$$

```
[Out] 1/2*a*(2*a^2+3*b^2-3*c^2)*x+1/6*c*(11*a^2+4*b^2-4*c^2)*cosh(x)+1/6*b*(11*a^
2+4*b^2-4*c^2)*sinh(x)+5/6*(a*c*cosh(x)+a*b*sinh(x))*(a+b*cosh(x)+c*sinh(x)
)+1/3*(c*cosh(x)+b*sinh(x))*(a+b*cosh(x)+c*sinh(x))^2
```

Rubi [A] time = 0.13, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3120, 3146, 2637, 2638}

$$\frac{1}{2}ax\left(2a^2+3b^2-3c^2\right)+\frac{1}{6}b\sinh(x)\left(11a^2+4b^2-4c^2\right)+\frac{1}{6}c\cosh(x)\left(11a^2+4b^2-4c^2\right)+\frac{1}{3}(b\sinh(x)+c\cosh(x))(a$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cosh[x] + c*Sinh[x])^3,x]
```

```
[Out] (a*(2*a^2 + 3*b^2 - 3*c^2)*x)/2 + (c*(11*a^2 + 4*b^2 - 4*c^2)*Cosh[x])/6 +
(b*(11*a^2 + 4*b^2 - 4*c^2)*Sinh[x])/6 + (5*(a*c*Cosh[x] + a*b*Sinh[x])*(a
+ b*Cosh[x] + c*Sinh[x]))/6 + ((c*Cosh[x] + b*Sinh[x])*(a + b*Cosh[x] + c*S
inh[x])^2)/3
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3120

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> -Simp[((c*Cos[d + e*x] - b*Sin[d + e*x])*(a + b*Cos[d +
e*x] + c*Sin[d + e*x])^(n - 1))/(e*n), x] + Dist[1/n, Int[Simp[n*a^2 + (n -
 1)*(b^2 + c^2) + a*b*(2*n - 1)*Cos[d + e*x] + a*c*(2*n - 1)*Sin[d + e*x],
x]*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n - 2), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && GtQ[n, 1]
```

Rule 3146

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_.)*((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)
]), x_Symbol] :> Simp[((B*c - b*C - a*C*Cos[d + e*x] + a*B*Sin[d + e*x])*(a
 + b*Cos[d + e*x] + c*Sin[d + e*x])^n)/(a*e*(n + 1)), x] + Dist[1/(a*(n + 1
)), Int[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n - 1)*Simp[a*(b*B + c*C)*n
+ a^2*A*(n + 1) + (n*(a^2*B - B*c^2 + b*c*C) + a*b*A*(n + 1))*Cos[d + e*x]
+ (n*(b*B*c + a^2*C - b^2*C) + a*c*A*(n + 1))*Sin[d + e*x], x], x], x] /; F
reeQ[{a, b, c, d, e, A, B, C}, x] && GtQ[n, 0] && NeQ[a^2 - b^2 - c^2, 0]
```

Rubi steps

$$\int (a+b\cosh(x)+c\sinh(x))^3\,dx = \frac{1}{3}(c\cosh(x)+b\sinh(x))(a+b\cosh(x)+c\sinh(x))^2+\frac{1}{3}\int (a+b\cosh(x)$$

$$= \frac{5}{6}(ac\cosh(x)+ab\sinh(x))(a+b\cosh(x)+c\sinh(x))+\frac{1}{3}(c\cosh(x)+b\sin$$

$$= \frac{1}{2}a\left(2a^2+3b^2-3c^2\right)x+\frac{5}{6}(ac\cosh(x)+ab\sinh(x))(a+b\cosh(x)+c\sinh$$

$$= \frac{1}{2}a\left(2a^2+3b^2-3c^2\right)x+\frac{1}{6}c\left(11a^2+4b^2-4c^2\right)\cosh(x)+\frac{1}{6}b\left(11a^2+4b^2-\right.$$

Mathematica [A] time = 0.20, size = 116, normalized size = 0.97

$$\frac{1}{12}\left(6ax\left(2a^2+3b^2-3c^2\right)+9b\sinh(x)\left(4a^2+b^2-c^2\right)+9c\cosh(x)\left(4a^2+b^2-c^2\right)+9a\left(b^2+c^2\right)\sinh(2x)+18\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cosh[x] + c*Sinh[x])^3,x]
```

```
[Out] (6*a*(2*a^2 + 3*b^2 - 3*c^2)*x + 9*c*(4*a^2 + b^2 - c^2)*Cosh[x] + 18*a*b*c
*Cosh[2*x] + c*(3*b^2 + c^2)*Cosh[3*x] + 9*b*(4*a^2 + b^2 - c^2)*Sinh[x] +
9*a*(b^2 + c^2)*Sinh[2*x] + b*(b^2 + 3*c^2)*Sinh[3*x])/12
```

fricas [A] time = 0.50, size = 160, normalized size = 1.34

$$\frac{3}{2}abc\cosh(x)^2+\frac{1}{12}\left(3b^2c+c^3\right)\cosh(x)^3+\frac{1}{12}\left(b^3+3bc^2\right)\sinh(x)^3+\frac{1}{4}\left(6abc+\left(3b^2c+c^3\right)\cosh(x)\right)\sinh(x)^2+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cosh(x)+c*sinh(x))^3,x, algorithm="fricas")
```

```
[Out] 3/2*a*b*c*cosh(x)^2 + 1/12*(3*b^2*c + c^3)*cosh(x)^3 + 1/12*(b^3 + 3*b*c^2)
*sinh(x)^3 + 1/4*(6*a*b*c + (3*b^2*c + c^3)*cosh(x))*sinh(x)^2 + 1/2*(2*a^3
 + 3*a*b^2 - 3*a*c^2)*x - 3/4*(c^3 - (4*a^2 + b^2)*c)*cosh(x) + 1/4*(12*a^2
*b + 3*b^3 - 3*b*c^2 + (b^3 + 3*b*c^2)*cosh(x)^2 + 6*(a*b^2 + a*c^2)*cosh(x
))*sinh(x)
```

giac [B] time = 0.14, size = 219, normalized size = 1.84

$$\frac{1}{24}\,b^3e^{(3\,x)}+\frac{1}{8}\,b^2ce^{(3\,x)}+\frac{1}{8}\,bc^2e^{(3\,x)}+\frac{1}{24}\,c^3e^{(3\,x)}+\frac{3}{8}\,ab^2e^{(2\,x)}+\frac{3}{4}\,abce^{(2\,x)}+\frac{3}{8}\,ac^2e^{(2\,x)}+\frac{3}{2}\,a^2be^x+\frac{3}{8}\,b^3e^x+\frac{3}{2}\,a^2ce^x+\frac{3}{8}\,b^2ce$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cosh(x)+c*sinh(x))^3,x, algorithm="giac")
```

```
[Out] 1/24*b^3*e^(3*x) + 1/8*b^2*c*e^(3*x) + 1/8*b*c^2*e^(3*x) + 1/24*c^3*e^(3*x)
 + 3/8*a*b^2*e^(2*x) + 3/4*a*b*c*e^(2*x) + 3/8*a*c^2*e^(2*x) + 3/2*a^2*b*e^
x + 3/8*b^3*e^x + 3/2*a^2*c*e^x + 3/8*b^2*c*e^x - 3/8*b*c^2*e^x - 3/8*c^3*e
^x + 1/2*(2*a^3 + 3*a*b^2 - 3*a*c^2)*x - 1/24*(b^3 - 3*b^2*c + 3*b*c^2 - c^
3 + 9*(4*a^2*b + b^3 - 4*a^2*c - b^2*c - b*c^2 + c^3)*e^(2*x) + 9*(a*b^2 -
2*a*b*c + a*c^2)*e^x)*e^(-3*x)
```

maple [A] time = 0.42, size = 110, normalized size = 0.92

$$a^3x+3a^2b\sinh(x)+3a^2c\cosh(x)+3a\,b^2\left(\frac{\cosh(x)\sinh(x)}{2}+\frac{x}{2}\right)+3abc\left(\cosh^2(x)\right)+3a\,c^2\left(\frac{\cosh(x)\sinh(x)}{2}-\frac{x}{2}\right)+b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cosh(x)+c*sinh(x))^3,x)
```

```
[Out] a^3*x+3*a^2*b*sinh(x)+3*a^2*c*cosh(x)+3*a*b^2*(1/2*cosh(x)*sinh(x)+1/2*x)+3
*a*b*c*cosh(x)^2+3*a*c^2*(1/2*cosh(x)*sinh(x)-1/2*x)+b^3*(2/3+1/3*cosh(x)^2
)*sinh(x)+cosh(x)^3*b^2*c+b*c^2*sinh(x)^3+c^3*(-2/3+1/3*sinh(x)^2)*cosh(x)
```

maxima [A] time = 0.31, size = 137, normalized size = 1.15

$$b^2c\cosh(x)^3+bc^2\sinh(x)^3+a^3x+\frac{1}{24}\,c^3\left(e^{(3\,x)}-9\,e^{(-x)}+e^{(-3\,x)}-9\,e^x\right)+\frac{1}{24}\,b^3\left(e^{(3\,x)}-9\,e^{(-x)}-e^{(-3\,x)}+9\,e^x\right)+3\,(c\,\mathrm{co}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cosh(x)+c*sinh(x))^3,x, algorithm="maxima")
```

```
[Out] b^2*c*cosh(x)^3 + b*c^2*sinh(x)^3 + a^3*x + 1/24*c^3*(e^(3*x) - 9*e^(-x) +
e^(-3*x) - 9*e^x) + 1/24*b^3*(e^(3*x) - 9*e^(-x) - e^(-3*x) + 9*e^x) + 3*(c
*cosh(x) + b*sinh(x))*a^2 + 3/8*(8*b*c*cosh(x)^2 + b^2*(4*x + e^(2*x) - e^(
-2*x)) - c^2*(4*x - e^(2*x) + e^(-2*x)))*a
```

mupad [B] time = 0.15, size = 131, normalized size = 1.10

$$a^3\,x+\cosh(x)^3\,\left(b^2\,c-\frac{2\,c^3}{3}\right)+\sinh(x)^3\,\left(b\,c^2-\frac{2\,b^3}{3}\right)+b^3\,\cosh(x)^2\,\sinh(x)+c^3\,\cosh(x)\,\sinh(x)^2+3\,a^2\,c\,\cosh(x)+3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*cosh(x) + c*sinh(x))^3,x)
```

```
[Out] a^3*x + cosh(x)^3*(b^2*c - (2*c^3)/3) + sinh(x)^3*(b*c^2 - (2*b^3)/3) + b^3
*cosh(x)^2*sinh(x) + c^3*cosh(x)*sinh(x)^2 + 3*a^2*c*cosh(x) + 3*a^2*b*sinh
(x) + (3*a*cosh(x)*sinh(x)*(b^2 + c^2))/2 + (3*a*x*cosh(x)^2*(b^2 - c^2))/2
 + 3*a*b*c*cosh(x)^2 - (3*a*x*sinh(x)^2*(b^2 - c^2))/2
```

sympy [A] time = 0.42, size = 196, normalized size = 1.65

$$a^3x+3a^2b\sinh{(x)}+3a^2c\cosh{(x)}-\frac{3ab^2x\sinh^2{(x)}}{2}+\frac{3ab^2x\cosh^2{(x)}}{2}+\frac{3ab^2\sinh{(x)}\cosh{(x)}}{2}+3abc\cosh^2{(x)}+\frac{3}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cosh(x)+c*sinh(x))**3,x)
```

```
[Out] a**3*x + 3*a**2*b*sinh(x) + 3*a**2*c*cosh(x) - 3*a*b**2*x*sinh(x)**2/2 + 3*
a*b**2*x*cosh(x)**2/2 + 3*a*b**2*sinh(x)*cosh(x)/2 + 3*a*b*c*cosh(x)**2 + 3
*a*c**2*x*sinh(x)**2/2 - 3*a*c**2*x*cosh(x)**2/2 + 3*a*c**2*sinh(x)*cosh(x)
/2 - 2*b**3*sinh(x)**3/3 + b**3*sinh(x)*cosh(x)**2 + b**2*c*cosh(x)**3 + b*
c**2*sinh(x)**3 + c**3*sinh(x)**2*cosh(x) - 2*c**3*cosh(x)**3/3
```

3.740 $\int (a+b\cosh(x)+c\sinh(x))^2\,dx$

Optimal. Leaf size=59

$$\frac{1}{2}x\left(2a^2+b^2-c^2\right)+\frac{1}{2}(b\sinh(x)+c\cosh(x))(a+b\cosh(x)+c\sinh(x))+\frac{3}{2}ab\sinh(x)+\frac{3}{2}ac\cosh(x)$$

```
[Out] 1/2*(2*a^2+b^2-c^2)*x+3/2*a*c*cosh(x)+3/2*a*b*sinh(x)+1/2*(c*cosh(x)+b*sinh
(x))*(a+b*cosh(x)+c*sinh(x))
```

Rubi [A] time = 0.04, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3120, 2637, 2638}

$$\frac{1}{2}x\left(2a^2+b^2-c^2\right)+\frac{1}{2}(b\sinh(x)+c\cosh(x))(a+b\cosh(x)+c\sinh(x))+\frac{3}{2}ab\sinh(x)+\frac{3}{2}ac\cosh(x)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cosh[x] + c*Sinh[x])^2,x]
```

```
[Out] ((2*a^2 + b^2 - c^2)*x)/2 + (3*a*c*Cosh[x])/2 + (3*a*b*Sinh[x])/2 + ((c*Cos
h[x] + b*Sinh[x])*(a + b*Cosh[x] + c*Sinh[x]))/2
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3120

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> -Simp[((c*Cos[d + e*x] - b*Sin[d + e*x])*(a + b*Cos[d +
e*x] + c*Sin[d + e*x])^(n - 1))/(e*n), x] + Dist[1/n, Int[Simp[n*a^2 + (n -
 1)*(b^2 + c^2) + a*b*(2*n - 1)*Cos[d + e*x] + a*c*(2*n - 1)*Sin[d + e*x],
x]*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n - 2), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && GtQ[n, 1]
```

Rubi steps

$$\int (a + b\cosh(x) + c\sinh(x))^2\,dx = \frac{1}{2}(c\cosh(x) + b\sinh(x))(a + b\cosh(x) + c\sinh(x)) + \frac{1}{2}\int \left(2a^2 + b^2 - c^2 +\right.$$
$$= \frac{1}{2}\left(2a^2 + b^2 - c^2\right)x + \frac{1}{2}(c\cosh(x) + b\sinh(x))(a + b\cosh(x) + c\sinh(x)) +$$
$$= \frac{1}{2}\left(2a^2 + b^2 - c^2\right)x + \frac{3}{2}ac\cosh(x) + \frac{3}{2}ab\sinh(x) + \frac{1}{2}(c\cosh(x) + b\sinh(x)$$

Mathematica [A] time = 0.09, size = 54, normalized size = 0.92

$$\frac{1}{4}\left(2x\left(2a^2 + b^2 - c^2\right) + 8ab\sinh(x) + 8ac\cosh(x) + \left(b^2 + c^2\right)\sinh(2x) + 2bc\cosh(2x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cosh[x] + c*Sinh[x])^2,x]
```

```
[Out] (2*(2*a^2 + b^2 - c^2)*x + 8*a*c*Cosh[x] + 2*b*c*Cosh[2*x] + 8*a*b*Sinh[x]
+ (b^2 + c^2)*Sinh[2*x])/4
```

fricas [A] time = 0.49, size = 59, normalized size = 1.00

$$\frac{1}{2}\,bc\cosh(x)^2 + \frac{1}{2}\,bc\sinh(x)^2 + 2\,ac\cosh(x) + \frac{1}{2}\left(2\,a^2 + b^2 - c^2\right)x + \frac{1}{2}\left(4\,ab + \left(b^2 + c^2\right)\cosh(x)\right)\sinh(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cosh(x)+c*sinh(x))^2,x, algorithm="fricas")
```

```
[Out] 1/2*b*c*cosh(x)^2 + 1/2*b*c*sinh(x)^2 + 2*a*c*cosh(x) + 1/2*(2*a^2 + b^2 -
c^2)*x + 1/2*(4*a*b + (b^2 + c^2)*cosh(x))*sinh(x)
```

giac [A] time = 0.11, size = 83, normalized size = 1.41

$$\frac{1}{8}\,b^2e^{(2\,x)} + \frac{1}{4}\,bce^{(2\,x)} + \frac{1}{8}\,c^2e^{(2\,x)} + abe^x + ace^x + \frac{1}{2}\left(2\,a^2 + b^2 - c^2\right)x - \frac{1}{8}\left(b^2 - 2\,bc + c^2 + 8\,(ab - ac)e^x\right)e^{(-2\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cosh(x)+c*sinh(x))^2,x, algorithm="giac")
```

```
[Out] 1/8*b^2*e^(2*x) + 1/4*b*c*e^(2*x) + 1/8*c^2*e^(2*x) + a*b*e^x + a*c*e^x + 1
/2*(2*a^2 + b^2 - c^2)*x - 1/8*(b^2 - 2*b*c + c^2 + 8*(a*b - a*c)*e^x)*e^(-
2*x)
```

maple [A] time = 0.15, size = 54, normalized size = 0.92

$$a^{2}x+2ab\sinh(x)+2ac\cosh(x)+b^{2}\left(\frac{\cosh(x)\sinh(x)}{2}+\frac{x}{2}\right)+cb\left(\cosh^{2}(x)\right)+c^{2}\left(\frac{\cosh(x)\sinh(x)}{2}-\frac{x}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cosh(x)+c*sinh(x))^2,x)
```

```
[Out] a^2*x+2*a*b*sinh(x)+2*a*c*cosh(x)+b^2*(1/2*cosh(x)*sinh(x)+1/2*x)+c*b*cosh(
x)^2+c^2*(1/2*cosh(x)*sinh(x)-1/2*x)
```

maxima [A] time = 0.50, size = 63, normalized size = 1.07

$$bc\cosh(x)^{2}+\frac{1}{8}b^{2}\left(4x+e^{(2x)}-e^{(-2x)}\right)-\frac{1}{8}c^{2}\left(4x-e^{(2x)}+e^{(-2x)}\right)+a^{2}x+2\left(c\cosh(x)+b\sinh(x)\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cosh(x)+c*sinh(x))^2,x, algorithm="maxima")
```

```
[Out] b*c*cosh(x)^2 + 1/8*b^2*(4*x + e^(2*x) - e^(-2*x)) - 1/8*c^2*(4*x - e^(2*x)
 + e^(-2*x)) + a^2*x + 2*(c*cosh(x) + b*sinh(x))*a
```

mupad [B] time = 1.54, size = 55, normalized size = 0.93

$$x\,a^{2}+2\sinh(x)\,a\,b+2\,a\,c\cosh(x)+\frac{\sinh(x)\,b^{2}\cosh(x)}{2}+\frac{x\,b^{2}}{2}+b\,c\cosh(x)^{2}+\frac{\sinh(x)\,c^{2}\cosh(x)}{2}-\frac{x\,c^{2}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*cosh(x) + c*sinh(x))^2,x)
```

```
[Out] a^2*x + (b^2*x)/2 - (c^2*x)/2 + 2*a*b*sinh(x) + b*c*cosh(x)^2 + (b^2*cosh(x
)*sinh(x))/2 + (c^2*cosh(x)*sinh(x))/2 + 2*a*c*cosh(x)
```

sympy [A] time = 0.21, size = 100, normalized size = 1.69

$$a^{2}x+2ab\sinh(x)+2ac\cosh(x)-\frac{b^{2}x\sinh^{2}(x)}{2}+\frac{b^{2}x\cosh^{2}(x)}{2}+\frac{b^{2}\sinh(x)\cosh(x)}{2}+bc\cosh^{2}(x)+\frac{c^{2}x\sinh^{2}(x)}{2}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cosh(x)+c*sinh(x))**2,x)
```

```
[Out] a**2*x + 2*a*b*sinh(x) + 2*a*c*cosh(x) - b**2*x*sinh(x)**2/2 + b**2*x*cosh(
x)**2/2 + b**2*sinh(x)*cosh(x)/2 + b*c*cosh(x)**2 + c**2*x*sinh(x)**2/2 - c
**2*x*cosh(x)**2/2 + c**2*sinh(x)*cosh(x)/2
```

3.741 $\int (a+b\cosh (x)+c\sinh (x)) \, dx$

Optimal. Leaf size=12

$$ax+b\sinh (x)+c\cosh (x)$$

[Out] a*x+c*cosh(x)+b*sinh(x)

Rubi [A] time = 0.01, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2637, 2638}

$$ax+b\sinh (x)+c\cosh (x)$$

Antiderivative was successfully verified.

[In] Int[a + b*Cosh[x] + c*Sinh[x],x]

[Out] a*x + c*Cosh[x] + b*Sinh[x]

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\begin{aligned} \int (a+b\cosh (x)+c\sinh (x)) \, dx &= ax+b\int \cosh (x) \, dx+c\int \sinh (x) \, dx \\ &= ax+c\cosh (x)+b\sinh (x) \end{aligned}$$

Mathematica [A] time = 0.00, size = 12, normalized size = 1.00

$$ax+b\sinh (x)+c\cosh (x)$$

Antiderivative was successfully verified.

[In] Integrate[a + b*Cosh[x] + c*Sinh[x],x]

[Out] a*x + c*Cosh[x] + b*Sinh[x]

fricas [A] time = 0.53, size = 12, normalized size = 1.00

$$ax + c\cosh(x) + b\sinh(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*cosh(x)+c*sinh(x),x, algorithm="fricas")
[Out] a*x + c*cosh(x) + b*sinh(x)
```

giac [B] time = 0.11, size = 26, normalized size = 2.17

$$ax + \frac{1}{2}c\left(e^{(-x)} + e^x\right) - \frac{1}{2}b\left(e^{(-x)} - e^x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*cosh(x)+c*sinh(x),x, algorithm="giac")
[Out] a*x + 1/2*c*(e^(-x) + e^x) - 1/2*b*(e^(-x) - e^x)
```

maple [A] time = 0.02, size = 13, normalized size = 1.08

$$ax + c\cosh(x) + b\sinh(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a+b*cosh(x)+c*sinh(x),x)
[Out] a*x+c*cosh(x)+b*sinh(x)
```

maxima [A] time = 0.30, size = 12, normalized size = 1.00

$$ax + c\cosh(x) + b\sinh(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*cosh(x)+c*sinh(x),x, algorithm="maxima")
[Out] a*x + c*cosh(x) + b*sinh(x)
```

mupad [B] time = 0.05, size = 12, normalized size = 1.00

$$a\,x + c\cosh(x) + b\sinh(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a + b*cosh(x) + c*sinh(x),x)
```

```
[Out] a*x + c*cosh(x) + b*sinh(x)
```

sympy [A] time = 0.11, size = 12, normalized size = 1.00

$$ax + b\sinh(x) + c\cosh(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*cosh(x)+c*sinh(x),x)
[Out] a*x + b*sinh(x) + c*cosh(x)
```

3.742 $\int \frac{1}{a+b\cosh(x)+c\sinh(x)}\,dx$

Optimal. Leaf size=51

$$-\frac{2\tanh^{-1}\left(\frac{c-(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2+c^2}}\right)}{\sqrt{a^2-b^2+c^2}}$$

```
[Out] -2*arctanh((c-(a-b)*tanh(1/2*x))/(a^2-b^2+c^2)^(1/2))/(a^2-b^2+c^2)^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3124, 618, 206}

$$-\frac{2\tanh^{-1}\left(\frac{c-(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2+c^2}}\right)}{\sqrt{a^2-b^2+c^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cosh[x] + c*Sinh[x])^(-1),x]
```

```
[Out] (-2*ArcTanh[(c - (a - b)*Tanh[x/2])/Sqrt[a^2 - b^2 + c^2]])/Sqrt[a^2 - b^2
+ c^2]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 3124

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Module[{f = FreeFactors[Tan[(d + e*x)/2], x]}, Dist[(2*f
)/e, Subst[Int[1/(a + b + 2*c*f*x + (a - b)*f^2*x^2), x], x, Tan[(d + e*x)/
2]/f], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
```

Rubi steps

$$\int \frac{1}{a+b\cosh(x)+c\sinh(x)}\,dx = 2\,\text{Subst}\left(\int \frac{1}{a+b+2cx-(a-b)x^2}\,dx, x, \tanh\left(\frac{x}{2}\right)\right)$$
$$= -\left(4\,\text{Subst}\left(\int \frac{1}{4\left(a^2-b^2+c^2\right)-x^2}\,dx, x, 2c+2(-a+b)\tanh\left(\frac{x}{2}\right)\right)\right)$$
$$= -\frac{2\tanh^{-1}\left(\frac{c-(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2+c^2}}\right)}{\sqrt{a^2-b^2+c^2}}$$

Mathematica [A] time = 0.08, size = 54, normalized size = 1.06

$$\frac{2\tan^{-1}\left(\frac{(b-a)\tanh\left(\frac{x}{2}\right)+c}{\sqrt{-a^2+b^2-c^2}}\right)}{\sqrt{-a^2+b^2-c^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cosh[x] + c*Sinh[x])^(-1),x]
```

```
[Out] (2*ArcTan[(c + (-a + b)*Tanh[x/2])/Sqrt[-a^2 + b^2 - c^2]])/Sqrt[-a^2 + b^2
 - c^2]
```

fricas [A] time = 0.47, size = 248, normalized size = 4.86

$$\left[\frac{\log\left(\frac{\left(b^2+2\,bc+c^2\right)\cosh(x)^2+\left(b^2+2\,bc+c^2\right)\sinh(x)^2+2\,a^2-b^2+c^2+2\,(ab+ac)\cosh(x)+2\,\left(ab+ac+\left(b^2+2\,bc+c^2\right)\cosh(x)\right)\sinh(x)-2\,\sqrt{a^2-b^2+c^2}\,((b+c}{(b+c)\cosh(x)^2+(b+c)\sinh(x)^2+2\,a\cosh(x)+2\,((b+c)\cosh(x)+a)\sinh(x)+b-c}\right.}{\sqrt{a^2-b^2+c^2}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cosh(x)+c*sinh(x)),x, algorithm="fricas")
```

```
[Out] [log(((b^2 + 2*b*c + c^2)*cosh(x)^2 + (b^2 + 2*b*c + c^2)*sinh(x)^2 + 2*a^2
 - b^2 + c^2 + 2*(a*b + a*c)*cosh(x) + 2*(a*b + a*c + (b^2 + 2*b*c + c^2)*c
osh(x))*sinh(x) - 2*sqrt(a^2 - b^2 + c^2)*((b + c)*cosh(x) + (b + c)*sinh(x
) + a))/((b + c)*cosh(x)^2 + (b + c)*sinh(x)^2 + 2*a*cosh(x) + 2*((b + c)*c
osh(x) + a)*sinh(x) + b - c))/sqrt(a^2 - b^2 + c^2), 2*sqrt(-a^2 + b^2 - c^
2)*arctan(sqrt(-a^2 + b^2 - c^2)*((b + c)*cosh(x) + (b + c)*sinh(x) + a)/(a
^2 - b^2 + c^2))/(a^2 - b^2 + c^2)]
```

giac [A] time = 0.13, size = 46, normalized size = 0.90

$$\frac{2\,\arctan\left(\frac{be^{x}+ce^{x}+a}{\sqrt{-a^2+b^2-c^2}}\right)}{\sqrt{-a^2+b^2-c^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cosh(x)+c*sinh(x)),x, algorithm="giac")
[Out] 2*arctan((b*e^x + c*e^x + a)/sqrt(-a^2 + b^2 - c^2))/sqrt(-a^2 + b^2 - c^2)
```

maple [A] time = 0.19, size = 53, normalized size = 1.04

$$-\frac{2\arctan\left(\frac{2(a-b)\tanh\left(\frac{x}{2}\right)-2c}{2\sqrt{-a^2+b^2-c^2}}\right)}{\sqrt{-a^2+b^2-c^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*cosh(x)+c*sinh(x)),x)
[Out] -2/(-a^2+b^2-c^2)^(1/2)*arctan(1/2*(2*(a-b)*tanh(1/2*x)-2*c)/(-a^2+b^2-c^2)
^(1/2))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cosh(x)+c*sinh(x)),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(c^2-b^2+a^2>0)', see `assume?` for
more details)Is c^2-b^2+a^2 positive or negative?
```

mupad [B] time = 0.21, size = 78, normalized size = 1.53

$$\frac{2\,\mathrm{atan}\left(\frac{a}{\sqrt{-a^2+b^2-c^2}}+\frac{b\,\mathrm{e}^x}{\sqrt{-a^2+b^2-c^2}}+\frac{c\,\mathrm{e}^x}{\sqrt{-a^2+b^2-c^2}}\right)}{\sqrt{-a^2+b^2-c^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*cosh(x) + c*sinh(x)),x)
```

```
[Out] (2*atan(a/(b^2 - a^2 - c^2)^(1/2) + (b*exp(x))/(b^2 - a^2 - c^2)^(1/2) + (c
*exp(x))/(b^2 - a^2 - c^2)^(1/2)))/(b^2 - a^2 - c^2)^(1/2)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cosh(x)+c*sinh(x)),x)
```

```
[Out] Timed out
```

3.743 $\int \frac{1}{(a+b\cosh(x)+c\sinh(x))^2}\,dx$

Optimal. Leaf size=90

$$-\frac{2a\tanh^{-1}\left(\frac{c-(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2+c^2}}\right)}{\left(a^2-b^2+c^2\right)^{3/2}}-\frac{b\sinh(x)+c\cosh(x)}{\left(a^2-b^2+c^2\right)(a+b\cosh(x)+c\sinh(x))}$$

```
[Out] -2*a*arctanh((c-(a-b)*tanh(1/2*x))/(a^2-b^2+c^2)^(1/2))/(a^2-b^2+c^2)^(3/2)
+(-c*cosh(x)-b*sinh(x))/(a^2-b^2+c^2)/(a+b*cosh(x)+c*sinh(x))
```

Rubi [A] time = 0.09, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {3129, 12, 3124, 618, 206}

$$-\frac{2a\tanh^{-1}\left(\frac{c-(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2+c^2}}\right)}{\left(a^2-b^2+c^2\right)^{3/2}}-\frac{b\sinh(x)+c\cosh(x)}{\left(a^2-b^2+c^2\right)(a+b\cosh(x)+c\sinh(x))}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cosh[x] + c*Sinh[x])^(-2),x]
```

```
[Out] (-2*a*ArcTanh[(c - (a - b)*Tanh[x/2])/Sqrt[a^2 - b^2 + c^2]])/(a^2 - b^2 +
c^2)^(3/2) - (c*Cosh[x] + b*Sinh[x])/((a^2 - b^2 + c^2)*(a + b*Cosh[x] + c*
Sinh[x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 3124

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Module[{f = FreeFactors[Tan[(d + e*x)/2], x]}, Dist[(2*f
)/e, Subst[Int[1/(a + b + 2*c*f*x + (a - b)*f^2*x^2), x], x, Tan[(d + e*x)/
2]/f], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
```

Rule 3129

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> Simp[((-(c*Cos[d + e*x]) + b*Sin[d + e*x])*(a + b*Cos[d
+ e*x] + c*Sin[d + e*x])^(n + 1))/(e*(n + 1)*(a^2 - b^2 - c^2)), x] + Dist[
1/((n + 1)*(a^2 - b^2 - c^2)), Int[(a*(n + 1) - b*(n + 2)*Cos[d + e*x] - c*
(n + 2)*Sin[d + e*x])*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1), x], x]
 /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && LtQ[n, -1] && N
eQ[n, -3/2]
```

Rubi steps

$$\int \frac{1}{(a+b\cosh(x)+c\sinh(x))^2}\,dx = -\frac{c\cosh(x)+b\sinh(x)}{\left(a^2-b^2+c^2\right)(a+b\cosh(x)+c\sinh(x))}+\frac{\int \frac{a}{a+b\cosh(x)+c\sinh(x)}\,dx}{a^2-b^2+c^2}$$

$$= -\frac{c\cosh(x)+b\sinh(x)}{\left(a^2-b^2+c^2\right)(a+b\cosh(x)+c\sinh(x))}+\frac{a\int \frac{1}{a+b\cosh(x)+c\sinh(x)}\,dx}{a^2-b^2+c^2}$$

$$= -\frac{c\cosh(x)+b\sinh(x)}{\left(a^2-b^2+c^2\right)(a+b\cosh(x)+c\sinh(x))}+\frac{(2a)\,\mathrm{Subst}\left(\int \frac{1}{a+b+2cx-(a-b)x^2}\right.}{a^2-b^2+c^2}$$

$$= -\frac{c\cosh(x)+b\sinh(x)}{\left(a^2-b^2+c^2\right)(a+b\cosh(x)+c\sinh(x))}-\frac{(4a)\,\mathrm{Subst}\left(\int \frac{1}{4\left(a^2-b^2+c^2\right)-x^2}\right.}{a^2-}$$

$$= -\frac{2a\tanh^{-1}\left(\frac{c-(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2+c^2}}\right)}{\left(a^2-b^2+c^2\right)^{3/2}}-\frac{c\cosh(x)+b\sinh(x)}{\left(a^2-b^2+c^2\right)(a+b\cosh(x)+c\sinh(x))}$$

Mathematica [A] time = 0.28, size = 105, normalized size = 1.17

$$\frac{\left(b^2-c^2\right)\sinh(x)-ac}{b\left(-a^2+b^2-c^2\right)(a+b\cosh(x)+c\sinh(x))}-\frac{2a\tan^{-1}\left(\frac{(b-a)\tanh\left(\frac{x}{2}\right)+c}{\sqrt{-a^2+b^2-c^2}}\right)}{\left(-a^2+b^2-c^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cosh[x] + c*Sinh[x])^(-2),x]
```

```
[Out] (-2*a*ArcTan[(c + (-a + b)*Tanh[x/2])/Sqrt[-a^2 + b^2 - c^2]])/(-a^2 + b^2
- c^2)^(3/2) + (-(a*c) + (b^2 - c^2)*Sinh[x])/(b*(-a^2 + b^2 - c^2)*(a + b*
Cosh[x] + c*Sinh[x]))
```

fricas [B] time = 0.47, size = 1268, normalized size = 14.09

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cosh(x)+c*sinh(x))^2,x, algorithm="fricas")
```

```
[Out] [(2*a^2*b - 2*b^3 + 2*b*c^2 - 2*c^3 + (2*a^2*cosh(x) + (a*b + a*c)*cosh(x)^
2 + (a*b + a*c)*sinh(x)^2 + a*b - a*c + 2*(a^2 + (a*b + a*c)*cosh(x))*sinh(
x))*sqrt(a^2 - b^2 + c^2)*log(((b^2 + 2*b*c + c^2)*cosh(x)^2 + (b^2 + 2*b*c
 + c^2)*sinh(x)^2 + 2*a^2 - b^2 + c^2 + 2*(a*b + a*c)*cosh(x) + 2*(a*b + a*
c + (b^2 + 2*b*c + c^2)*cosh(x))*sinh(x) - 2*sqrt(a^2 - b^2 + c^2)*((b + c)
*cosh(x) + (b + c)*sinh(x) + a))/((b + c)*cosh(x)^2 + (b + c)*sinh(x)^2 + 2
*a*cosh(x) + 2*((b + c)*cosh(x) + a)*sinh(x) + b - c)) - 2*(a^2 - b^2)*c +
2*(a^3 - a*b^2 + a*c^2)*cosh(x) + 2*(a^3 - a*b^2 + a*c^2)*sinh(x))/(a^4*b -
 2*a^2*b^3 + b^5 + b*c^4 - c^5 - 2*(a^2 - b^2)*c^3 + 2*(a^2*b - b^3)*c^2 +
(a^4*b - 2*a^2*b^3 + b^5 + b*c^4 + c^5 + 2*(a^2 - b^2)*c^3 + 2*(a^2*b - b^3
)*c^2 + (a^4 - 2*a^2*b^2 + b^4)*c)*cosh(x)^2 + (a^4*b - 2*a^2*b^3 + b^5 + b
*c^4 + c^5 + 2*(a^2 - b^2)*c^3 + 2*(a^2*b - b^3)*c^2 + (a^4 - 2*a^2*b^2 + b
^4)*c)*sinh(x)^2 - (a^4 - 2*a^2*b^2 + b^4)*c + 2*(a^5 - 2*a^3*b^2 + a*b^4 +
 a*c^4 + 2*(a^3 - a*b^2)*c^2)*cosh(x) + 2*(a^5 - 2*a^3*b^2 + a*b^4 + a*c^4
+ 2*(a^3 - a*b^2)*c^2 + (a^4*b - 2*a^2*b^3 + b^5 + b*c^4 + c^5 + 2*(a^2 - b
^2)*c^3 + 2*(a^2*b - b^3)*c^2 + (a^4 - 2*a^2*b^2 + b^4)*c)*cosh(x))*sinh(x)
), 2*(a^2*b - b^3 + b*c^2 - c^3 + (2*a^2*cosh(x) + (a*b + a*c)*cosh(x)^2 +
(a*b + a*c)*sinh(x)^2 + a*b - a*c + 2*(a^2 + (a*b + a*c)*cosh(x))*sinh(x))*
sqrt(-a^2 + b^2 - c^2)*arctan(sqrt(-a^2 + b^2 - c^2)*((b + c)*cosh(x) + (b
+ c)*sinh(x) + a)/(a^2 - b^2 + c^2)) - (a^2 - b^2)*c + (a^3 - a*b^2 + a*c^2
)*cosh(x) + (a^3 - a*b^2 + a*c^2)*sinh(x))/(a^4*b - 2*a^2*b^3 + b^5 + b*c^4
 - c^5 - 2*(a^2 - b^2)*c^3 + 2*(a^2*b - b^3)*c^2 + (a^4*b - 2*a^2*b^3 + b^5
 + b*c^4 + c^5 + 2*(a^2 - b^2)*c^3 + 2*(a^2*b - b^3)*c^2 + (a^4 - 2*a^2*b^2
 + b^4)*c)*cosh(x)^2 + (a^4*b - 2*a^2*b^3 + b^5 + b*c^4 + c^5 + 2*(a^2 - b^
2)*c^3 + 2*(a^2*b - b^3)*c^2 + (a^4 - 2*a^2*b^2 + b^4)*c)*sinh(x)^2 - (a^4
- 2*a^2*b^2 + b^4)*c + 2*(a^5 - 2*a^3*b^2 + a*b^4 + a*c^4 + 2*(a^3 - a*b^2)
*c^2)*cosh(x) + 2*(a^5 - 2*a^3*b^2 + a*b^4 + a*c^4 + 2*(a^3 - a*b^2)*c^2 +
(a^4*b - 2*a^2*b^3 + b^5 + b*c^4 + c^5 + 2*(a^2 - b^2)*c^3 + 2*(a^2*b - b^3
)*c^2 + (a^4 - 2*a^2*b^2 + b^4)*c)*cosh(x))*sinh(x))]
```

giac [A] time = 0.12, size = 111, normalized size = 1.23

$$\frac{2\,a\arctan\left(\frac{be^{x}+ce^{x}+a}{\sqrt{-a^{2}+b^{2}-c^{2}}}\right)}{\left(a^{2}-b^{2}+c^{2}\right)\sqrt{-a^{2}+b^{2}-c^{2}}}+\frac{2\,(ae^{x}+b-c)}{\left(a^{2}-b^{2}+c^{2}\right)\left(be^{(2\,x)}+ce^{(2\,x)}+2\,ae^{x}+b-c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cosh(x)+c*sinh(x))^2,x, algorithm="giac")
```

```
[Out] 2*a*arctan((b*e^x + c*e^x + a)/sqrt(-a^2 + b^2 - c^2))/((a^2 - b^2 + c^2)*s
qrt(-a^2 + b^2 - c^2)) + 2*(a*e^x + b - c)/((a^2 - b^2 + c^2)*(b*e^(2*x) +
c*e^(2*x) + 2*a*e^x + b - c))
```

maple [B] time = 0.26, size = 191, normalized size = 2.12

$$-\frac{2\left(-\frac{\left(ab-b^{2}+c^{2}\right)\tanh\left(\frac{x}{2}\right)}{a^{3}-a^{2}b-a\,b^{2}+a\,c^{2}+b^{3}-b\,c^{2}}-\frac{ac}{a^{3}-a^{2}b-a\,b^{2}+a\,c^{2}+b^{3}-b\,c^{2}}\right)}{a\left(\tanh^{2}\left(\frac{x}{2}\right)\right)-\left(\tanh^{2}\left(\frac{x}{2}\right)\right)b-2c\tanh\left(\frac{x}{2}\right)-a-b}-\frac{2a\arctan\left(\frac{2(a-b)\tanh\left(\frac{x}{2}\right)-2c}{2\sqrt{-a^{2}+b^{2}-c^{2}}}\right)}{\left(a^{2}-b^{2}+c^{2}\right)\sqrt{-a^{2}+b^{2}-c^{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*cosh(x)+c*sinh(x))^2,x)
```

```
[Out] -2*(-(a*b-b^2+c^2)/(a^3-a^2*b-a*b^2+a*c^2+b^3-b*c^2)*tanh(1/2*x)-a*c/(a^3-a
^2*b-a*b^2+a*c^2+b^3-b*c^2))/(a*tanh(1/2*x)^2-tanh(1/2*x)^2*b-2*c*tanh(1/2*
x)-a-b)-2*a/(a^2-b^2+c^2)/(-a^2+b^2-c^2)^(1/2)*arctan(1/2*(2*(a-b)*tanh(1/2
*x)-2*c)/(-a^2+b^2-c^2)^(1/2))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cosh(x)+c*sinh(x))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(c^2-b^2+a^2>0)', see `assume?` for
more details)Is c^2-b^2+a^2 positive or negative?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(a+b\cosh(x)+c\sinh(x)\right)}^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*cosh(x) + c*sinh(x))^2,x)
```

```
[Out] int(1/(a + b*cosh(x) + c*sinh(x))^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cosh(x)+c*sinh(x))**2,x)
```

```
[Out] Timed out
```

3.744 $\int \frac{1}{(a+b\cosh(x)+c\sinh(x))^3}\,dx$

Optimal. Leaf size=146

$$-\frac{\left(2a^2+b^2-c^2\right)\tanh^{-1}\left(\frac{c-(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2+c^2}}\right)}{\left(a^2-b^2+c^2\right)^{5/2}}-\frac{3(ab\sinh(x)+ac\cosh(x))}{2\left(a^2-b^2+c^2\right)^2(a+b\cosh(x)+c\sinh(x))}-\frac{b\sinh(x)+c}{2\left(a^2-b^2+c^2\right)(a+b\,c}$$

```
[Out] -(2*a^2+b^2-c^2)*arctanh((c-(a-b)*tanh(1/2*x))/(a^2-b^2+c^2)^(1/2))/(a^2-b^
2+c^2)^(5/2)+1/2*(-c*cosh(x)-b*sinh(x))/(a^2-b^2+c^2)/(a+b*cosh(x)+c*sinh(x
))^2-3/2*(a*c*cosh(x)+a*b*sinh(x))/(a^2-b^2+c^2)^2/(a+b*cosh(x)+c*sinh(x))
```

Rubi [A] time = 0.17, antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {3129, 3153, 3124, 618, 206}

$$-\frac{\left(2a^2+b^2-c^2\right)\tanh^{-1}\left(\frac{c-(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2+c^2}}\right)}{\left(a^2-b^2+c^2\right)^{5/2}}-\frac{3(ab\sinh(x)+ac\cosh(x))}{2\left(a^2-b^2+c^2\right)^2(a+b\cosh(x)+c\sinh(x))}-\frac{b\sinh(x)+c}{2\left(a^2-b^2+c^2\right)(a+b\,c}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cosh[x] + c*Sinh[x])^(-3),x]
```

```
[Out] -(((2*a^2 + b^2 - c^2)*ArcTanh[(c - (a - b)*Tanh[x/2])/Sqrt[a^2 - b^2 + c^2
]])/(a^2 - b^2 + c^2)^(5/2)) - (c*Cosh[x] + b*Sinh[x])/(2*(a^2 - b^2 + c^2)
*(a + b*Cosh[x] + c*Sinh[x])^2) - (3*(a*c*Cosh[x] + a*b*Sinh[x]))/(2*(a^2 -
 b^2 + c^2)^2*(a + b*Cosh[x] + c*Sinh[x]))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 3124

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Module[{f = FreeFactors[Tan[(d + e*x)/2], x]}, Dist[(2*f
)/e, Subst[Int[1/(a + b + 2*c*f*x + (a - b)*f^2*x^2), x], x, Tan[(d + e*x)/
2]/f], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
```

Rule 3129

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> Simp[((-(c*Cos[d + e*x]) + b*Sin[d + e*x])*(a + b*Cos[d
+ e*x] + c*Sin[d + e*x])^(n + 1))/(e*(n + 1)*(a^2 - b^2 - c^2)), x] + Dist[
1/((n + 1)*(a^2 - b^2 - c^2)), Int[(a*(n + 1) - b*(n + 2)*Cos[d + e*x] - c*
(n + 2)*Sin[d + e*x])*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1), x], x]
 /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && LtQ[n, -1] && N
eQ[n, -3/2]
```

Rule 3153

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])^2,
 x_Symbol] :> Simp[(c*B - b*C - (a*C - c*A)*Cos[d + e*x] + (a*B - b*A)*Sin[
d + e*x])/(e*(a^2 - b^2 - c^2)*(a + b*Cos[d + e*x] + c*Sin[d + e*x])), x] +
 Dist[(a*A - b*B - c*C)/(a^2 - b^2 - c^2), Int[1/(a + b*Cos[d + e*x] + c*Si
n[d + e*x]), x], x] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && NeQ[a^2 - b^2
- c^2, 0] && NeQ[a*A - b*B - c*C, 0]
```

Rubi steps

$$\int \frac{1}{(a+b\cosh(x)+c\sinh(x))^3}\,dx = -\frac{c\cosh(x)+b\sinh(x)}{2\left(a^2-b^2+c^2\right)(a+b\cosh(x)+c\sinh(x))^2} - \frac{\int \frac{-2a+b\cosh(x)+c\sinh(x)}{(a+b\cosh(x)+c\sinh(x))^2}\,dx}{2\left(a^2-b^2+c^2\right)}$$

$$= -\frac{c\cosh(x)+b\sinh(x)}{2\left(a^2-b^2+c^2\right)(a+b\cosh(x)+c\sinh(x))^2} - \frac{3(ac\cosh(x)+a}{2\left(a^2-b^2+c^2\right)^2(a+b\cosh}$$

$$= -\frac{c\cosh(x)+b\sinh(x)}{2\left(a^2-b^2+c^2\right)(a+b\cosh(x)+c\sinh(x))^2} - \frac{3(ac\cosh(x)+a}{2\left(a^2-b^2+c^2\right)^2(a+b\cosh}$$

$$= -\frac{c\cosh(x)+b\sinh(x)}{2\left(a^2-b^2+c^2\right)(a+b\cosh(x)+c\sinh(x))^2} - \frac{3(ac\cosh(x)+a}{2\left(a^2-b^2+c^2\right)^2(a+b\cosh}$$

$$= -\frac{\left(2a^2+b^2-c^2\right)\tanh^{-1}\left(\frac{c-(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2+c^2}}\right)}{\left(a^2-b^2+c^2\right)^{5/2}} - \frac{c\cosh(x)+b\sinh(x}{2\left(a^2-b^2+c^2\right)(a+b\cosh(x)}$$

Mathematica [A] time = 0.53, size = 183, normalized size = 1.25

$$\frac{1}{2}\left(\frac{2\left(2a^2+b^2-c^2\right)\tan^{-1}\left(\frac{(b-a)\tanh\left(\frac{x}{2}\right)+c}{\sqrt{-a^2+b^2-c^2}}\right)}{\left(-a^2+b^2-c^2\right)^{5/2}} + \frac{\left(b^2-c^2\right)\sinh(x)-ac}{b\left(-a^2+b^2-c^2\right)(a+b\cosh(x)+c\sinh(x))^2} + \frac{c\left(2a^2+b^2-c^2\right.}{b\left(a^2-b^2+c^2\right)^2(}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cosh[x] + c*Sinh[x])^(-3),x]
```

```
[Out] ((2*(2*a^2 + b^2 - c^2)*ArcTan[(c + (-a + b)*Tanh[x/2])/Sqrt[-a^2 + b^2 - c
^2]])/(-a^2 + b^2 - c^2)^(5/2) + (-(a*c) + (b^2 - c^2)*Sinh[x])/(b*(-a^2 +
b^2 - c^2)*(a + b*Cosh[x] + c*Sinh[x])^2) + (c*(2*a^2 + b^2 - c^2) - 3*a*(b
^2 - c^2)*Sinh[x])/(b*(a^2 - b^2 + c^2)^2*(a + b*Cosh[x] + c*Sinh[x])))/2
```

fricas [B] time = 0.60, size = 7379, normalized size = 50.54

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cosh(x)+c*sinh(x))^3,x, algorithm="fricas")
```

```
[Out] [1/2*(6*a^3*b^2 - 6*a*b^4 + 6*a^3*c^2 - 12*a*b*c^3 + 6*a*c^4 + 2*(2*a^4*b -
 a^2*b^3 - b^5 - b*c^4 - c^5 + (a^2 + 2*b^2)*c^3 + (a^2*b + 2*b^3)*c^2 + (2
*a^4 - a^2*b^2 - b^4)*c)*cosh(x)^3 + 2*(2*a^4*b - a^2*b^3 - b^5 - b*c^4 - c
^5 + (a^2 + 2*b^2)*c^3 + (a^2*b + 2*b^3)*c^2 + (2*a^4 - a^2*b^2 - b^4)*c)*s
inh(x)^3 + 6*(2*a^5 - a^3*b^2 - a*b^4 - a*c^4 + (a^3 + 2*a*b^2)*c^2)*cosh(x
)^2 + 6*(2*a^5 - a^3*b^2 - a*b^4 - a*c^4 + (a^3 + 2*a*b^2)*c^2 + (2*a^4*b -
 a^2*b^3 - b^5 - b*c^4 - c^5 + (a^2 + 2*b^2)*c^3 + (a^2*b + 2*b^3)*c^2 + (2
*a^4 - a^2*b^2 - b^4)*c)*cosh(x))*sinh(x)^2 - ((2*a^2*b^2 + b^4 + 2*a^2*c^2
 - 2*b*c^3 - c^4 + 2*(2*a^2*b + b^3)*c)*cosh(x)^4 + (2*a^2*b^2 + b^4 + 2*a^
2*c^2 - 2*b*c^3 - c^4 + 2*(2*a^2*b + b^3)*c)*sinh(x)^4 + 2*a^2*b^2 + b^4 +
2*a^2*c^2 + 2*b*c^3 - c^4 + 4*(2*a^3*b + a*b^3 - a*b*c^2 - a*c^3 + (2*a^3 +
 a*b^2)*c)*cosh(x)^3 + 4*(2*a^3*b + a*b^3 - a*b*c^2 - a*c^3 + (2*a^3 + a*b^
2)*c + (2*a^2*b^2 + b^4 + 2*a^2*c^2 - 2*b*c^3 - c^4 + 2*(2*a^2*b + b^3)*c)*
cosh(x))*sinh(x)^3 + 2*(4*a^4 + 4*a^2*b^2 + b^4 + c^4 - 2*(2*a^2 + b^2)*c^2
)*cosh(x)^2 + 2*(4*a^4 + 4*a^2*b^2 + b^4 + c^4 - 2*(2*a^2 + b^2)*c^2 + 3*(2
*a^2*b^2 + b^4 + 2*a^2*c^2 - 2*b*c^3 - c^4 + 2*(2*a^2*b + b^3)*c)*cosh(x)^2
 + 6*(2*a^3*b + a*b^3 - a*b*c^2 - a*c^3 + (2*a^3 + a*b^2)*c)*cosh(x))*sinh(
x)^2 - 2*(2*a^2*b + b^3)*c + 4*(2*a^3*b + a*b^3 - a*b*c^2 + a*c^3 - (2*a^3
+ a*b^2)*c)*cosh(x) + 4*(2*a^3*b + a*b^3 - a*b*c^2 + a*c^3 + (2*a^2*b^2 + b
^4 + 2*a^2*c^2 - 2*b*c^3 - c^4 + 2*(2*a^2*b + b^3)*c)*cosh(x)^3 + 3*(2*a^3*
b + a*b^3 - a*b*c^2 - a*c^3 + (2*a^3 + a*b^2)*c)*cosh(x)^2 - (2*a^3 + a*b^2
)*c + (4*a^4 + 4*a^2*b^2 + b^4 + c^4 - 2*(2*a^2 + b^2)*c^2)*cosh(x))*sinh(x
))*sqrt(a^2 - b^2 + c^2)*log(((b^2 + 2*b*c + c^2)*cosh(x)^2 + (b^2 + 2*b*c
+ c^2)*sinh(x)^2 + 2*a^2 - b^2 + c^2 + 2*(a*b + a*c)*cosh(x) + 2*(a*b + a*c
 + (b^2 + 2*b*c + c^2)*cosh(x))*sinh(x) + 2*sqrt(a^2 - b^2 + c^2)*((b + c)*
cosh(x) + (b + c)*sinh(x) + a))/((b + c)*cosh(x)^2 + (b + c)*sinh(x)^2 + 2*
a*cosh(x) + 2*((b + c)*cosh(x) + a)*sinh(x) + b - c)) - 12*(a^3*b - a*b^3)*
c + 2*(10*a^4*b - 11*a^2*b^3 + b^5 + b*c^4 - c^5 - (11*a^2 - 2*b^2)*c^3 + (
11*a^2*b - 2*b^3)*c^2 - (10*a^4 - 11*a^2*b^2 + b^4)*c)*cosh(x) + 2*(10*a^4*
b - 11*a^2*b^3 + b^5 + b*c^4 - c^5 - (11*a^2 - 2*b^2)*c^3 + (11*a^2*b - 2*b
^3)*c^2 + 3*(2*a^4*b - a^2*b^3 - b^5 - b*c^4 - c^5 + (a^2 + 2*b^2)*c^3 + (a
^2*b + 2*b^3)*c^2 + (2*a^4 - a^2*b^2 - b^4)*c)*cosh(x)^2 - (10*a^4 - 11*a^2
*b^2 + b^4)*c + 6*(2*a^5 - a^3*b^2 - a*b^4 - a*c^4 + (a^3 + 2*a*b^2)*c^2)*c
osh(x))*sinh(x))/(a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^8 - 2*b*c^7 + c^8 + (
3*a^2 - 2*b^2)*c^6 - 6*(a^2*b - b^3)*c^5 + 3*(a^4 - a^2*b^2)*c^4 + (a^6*b^2
 - 3*a^4*b^4 + 3*a^2*b^6 - b^8 + 2*b*c^7 + c^8 + (3*a^2 - 2*b^2)*c^6 + 6*(a
^2*b - b^3)*c^5 + 3*(a^4 - a^2*b^2)*c^4 + 6*(a^4*b - 2*a^2*b^3 + b^5)*c^3 +
 (a^6 - 3*a^2*b^4 + 2*b^6)*c^2 + 2*(a^6*b - 3*a^4*b^3 + 3*a^2*b^5 - b^7)*c)
*cosh(x)^4 + (a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^8 + 2*b*c^7 + c^8 + (3*a^
2 - 2*b^2)*c^6 + 6*(a^2*b - b^3)*c^5 + 3*(a^4 - a^2*b^2)*c^4 + 6*(a^4*b - 2
*a^2*b^3 + b^5)*c^3 + (a^6 - 3*a^2*b^4 + 2*b^6)*c^2 + 2*(a^6*b - 3*a^4*b^3
+ 3*a^2*b^5 - b^7)*c)*sinh(x)^4 - 6*(a^4*b - 2*a^2*b^3 + b^5)*c^3 + 4*(a^7*
b - 3*a^5*b^3 + 3*a^3*b^5 - a*b^7 + a*b*c^6 + a*c^7 + 3*(a^3 - a*b^2)*c^5 +
 3*(a^3*b - a*b^3)*c^4 + 3*(a^5 - 2*a^3*b^2 + a*b^4)*c^3 + 3*(a^5*b - 2*a^3
*b^3 + a*b^5)*c^2 + (a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6)*c)*cosh(x)^3 + 4*
```

```
(a^7*b - 3*a^5*b^3 + 3*a^3*b^5 - a*b^7 + a*b*c^6 + a*c^7 + 3*(a^3 - a*b^2)*
c^5 + 3*(a^3*b - a*b^3)*c^4 + 3*(a^5 - 2*a^3*b^2 + a*b^4)*c^3 + 3*(a^5*b -
2*a^3*b^3 + a*b^5)*c^2 + (a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6)*c + (a^6*b^2
 - 3*a^4*b^4 + 3*a^2*b^6 - b^8 + 2*b*c^7 + c^8 + (3*a^2 - 2*b^2)*c^6 + 6*(a
^2*b - b^3)*c^5 + 3*(a^4 - a^2*b^2)*c^4 + 6*(a^4*b - 2*a^2*b^3 + b^5)*c^3 +
 (a^6 - 3*a^2*b^4 + 2*b^6)*c^2 + 2*(a^6*b - 3*a^4*b^3 + 3*a^2*b^5 - b^7)*c)
*cosh(x))*sinh(x)^3 + (a^6 - 3*a^2*b^4 + 2*b^6)*c^2 + 2*(2*a^8 - 5*a^6*b^2
+ 3*a^4*b^4 + a^2*b^6 - b^8 - c^8 - (a^2 - 4*b^2)*c^6 + 3*(a^4 + a^2*b^2 -
2*b^4)*c^4 + (5*a^6 - 6*a^4*b^2 - 3*a^2*b^4 + 4*b^6)*c^2)*cosh(x)^2 + 2*(2*
a^8 - 5*a^6*b^2 + 3*a^4*b^4 + a^2*b^6 - b^8 - c^8 - (a^2 - 4*b^2)*c^6 + 3*(
a^4 + a^2*b^2 - 2*b^4)*c^4 + (5*a^6 - 6*a^4*b^2 - 3*a^2*b^4 + 4*b^6)*c^2 +
3*(a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^8 + 2*b*c^7 + c^8 + (3*a^2 - 2*b^2)*
c^6 + 6*(a^2*b - b^3)*c^5 + 3*(a^4 - a^2*b^2)*c^4 + 6*(a^4*b - 2*a^2*b^3 +
b^5)*c^3 + (a^6 - 3*a^2*b^4 + 2*b^6)*c^2 + 2*(a^6*b - 3*a^4*b^3 + 3*a^2*b^5
 - b^7)*c)*cosh(x)^2 + 6*(a^7*b - 3*a^5*b^3 + 3*a^3*b^5 - a*b^7 + a*b*c^6 +
 a*c^7 + 3*(a^3 - a*b^2)*c^5 + 3*(a^3*b - a*b^3)*c^4 + 3*(a^5 - 2*a^3*b^2 +
 a*b^4)*c^3 + 3*(a^5*b - 2*a^3*b^3 + a*b^5)*c^2 + (a^7 - 3*a^5*b^2 + 3*a^3*
b^4 - a*b^6)*c)*cosh(x))*sinh(x)^2 - 2*(a^6*b - 3*a^4*b^3 + 3*a^2*b^5 - b^7
)*c + 4*(a^7*b - 3*a^5*b^3 + 3*a^3*b^5 - a*b^7 + a*b*c^6 - a*c^7 - 3*(a^3 -
 a*b^2)*c^5 + 3*(a^3*b - a*b^3)*c^4 - 3*(a^5 - 2*a^3*b^2 + a*b^4)*c^3 + 3*(
a^5*b - 2*a^3*b^3 + a*b^5)*c^2 - (a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6)*c)*c
osh(x) + 4*(a^7*b - 3*a^5*b^3 + 3*a^3*b^5 - a*b^7 + a*b*c^6 - a*c^7 - 3*(a^
3 - a*b^2)*c^5 + 3*(a^3*b - a*b^3)*c^4 - 3*(a^5 - 2*a^3*b^2 + a*b^4)*c^3 +
(a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^8 + 2*b*c^7 + c^8 + (3*a^2 - 2*b^2)*c^
6 + 6*(a^2*b - b^3)*c^5 + 3*(a^4 - a^2*b^2)*c^4 + 6*(a^4*b - 2*a^2*b^3 + b^
5)*c^3 + (a^6 - 3*a^2*b^4 + 2*b^6)*c^2 + 2*(a^6*b - 3*a^4*b^3 + 3*a^2*b^5 -
 b^7)*c)*cosh(x)^3 + 3*(a^5*b - 2*a^3*b^3 + a*b^5)*c^2 + 3*(a^7*b - 3*a^5*b
^3 + 3*a^3*b^5 - a*b^7 + a*b*c^6 + a*c^7 + 3*(a^3 - a*b^2)*c^5 + 3*(a^3*b -
 a*b^3)*c^4 + 3*(a^5 - 2*a^3*b^2 + a*b^4)*c^3 + 3*(a^5*b - 2*a^3*b^3 + a*b^
5)*c^2 + (a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6)*c)*cosh(x)^2 - (a^7 - 3*a^5*
b^2 + 3*a^3*b^4 - a*b^6)*c + (2*a^8 - 5*a^6*b^2 + 3*a^4*b^4 + a^2*b^6 - b^8
 - c^8 - (a^2 - 4*b^2)*c^6 + 3*(a^4 + a^2*b^2 - 2*b^4)*c^4 + (5*a^6 - 6*a^4
*b^2 - 3*a^2*b^4 + 4*b^6)*c^2)*cosh(x))*sinh(x)), (3*a^3*b^2 - 3*a*b^4 + 3*
a^3*c^2 - 6*a*b*c^3 + 3*a*c^4 + (2*a^4*b - a^2*b^3 - b^5 - b*c^4 - c^5 + (a
^2 + 2*b^2)*c^3 + (a^2*b + 2*b^3)*c^2 + (2*a^4 - a^2*b^2 - b^4)*c)*cosh(x)^
3 + (2*a^4*b - a^2*b^3 - b^5 - b*c^4 - c^5 + (a^2 + 2*b^2)*c^3 + (a^2*b + 2
*b^3)*c^2 + (2*a^4 - a^2*b^2 - b^4)*c)*sinh(x)^3 + 3*(2*a^5 - a^3*b^2 - a*b
^4 - a*c^4 + (a^3 + 2*a*b^2)*c^2)*cosh(x)^2 + 3*(2*a^5 - a^3*b^2 - a*b^4 -
a*c^4 + (a^3 + 2*a*b^2)*c^2 + (2*a^4*b - a^2*b^3 - b^5 - b*c^4 - c^5 + (a^2
 + 2*b^2)*c^3 + (a^2*b + 2*b^3)*c^2 + (2*a^4 - a^2*b^2 - b^4)*c)*cosh(x))*s
inh(x)^2 + ((2*a^2*b^2 + b^4 + 2*a^2*c^2 - 2*b*c^3 - c^4 + 2*(2*a^2*b + b^3
)*c)*cosh(x)^4 + (2*a^2*b^2 + b^4 + 2*a^2*c^2 - 2*b*c^3 - c^4 + 2*(2*a^2*b
+ b^3)*c)*sinh(x)^4 + 2*a^2*b^2 + b^4 + 2*a^2*c^2 + 2*b*c^3 - c^4 + 4*(2*a^
3*b + a*b^3 - a*b*c^2 - a*c^3 + (2*a^3 + a*b^2)*c)*cosh(x)^3 + 4*(2*a^3*b +
 a*b^3 - a*b*c^2 - a*c^3 + (2*a^3 + a*b^2)*c + (2*a^2*b^2 + b^4 + 2*a^2*c^2
```

- 2*b*c^3 - c^4 + 2*(2*a^2*b + b^3)*c)*cosh(x))*sinh(x)^3 + 2*(4*a^4 + 4*a^2*b^2 + b^4 + c^4 - 2*(2*a^2 + b^2)*c^2)*cosh(x)^2 + 2*(4*a^4 + 4*a^2*b^2 + b^4 + c^4 - 2*(2*a^2 + b^2)*c^2 + 3*(2*a^2*b^2 + b^4 + 2*a^2*c^2 - 2*b*c^3 - c^4 + 2*(2*a^2*b + b^3)*c)*cosh(x)^2 + 6*(2*a^3*b + a*b^3 - a*b*c^2 - a*c^3 + (2*a^3 + a*b^2)*c)*cosh(x))*sinh(x)^2 - 2*(2*a^2*b + b^3)*c + 4*(2*a^3*b + a*b^3 - a*b*c^2 + a*c^3 - (2*a^3 + a*b^2)*c)*cosh(x) + 4*(2*a^3*b + a*b^3 - a*b*c^2 + a*c^3 + (2*a^2*b^2 + b^4 + 2*a^2*c^2 - 2*b*c^3 - c^4 + 2*(2*a^2*b + b^3)*c)*cosh(x)^3 + 3*(2*a^3*b + a*b^3 - a*b*c^2 - a*c^3 + (2*a^3 + a*b^2)*c)*cosh(x)^2 - (2*a^3 + a*b^2)*c + (4*a^4 + 4*a^2*b^2 + b^4 + c^4 - 2*(2*a^2 + b^2)*c^2)*cosh(x))*sinh(x))*sqrt(-a^2 + b^2 - c^2)*arctan(sqrt(-a^2 + b^2 - c^2)*((b + c)*cosh(x) + (b + c)*sinh(x) + a)/(a^2 - b^2 + c^2)) - 6*(a^3*b - a*b^3)*c + (10*a^4*b - 11*a^2*b^3 + b^5 + b*c^4 - c^5 - (11*a^2 - 2*b^2)*c^3 + (11*a^2*b - 2*b^3)*c^2 - (10*a^4 - 11*a^2*b^2 + b^4)*c)*cosh(x) + (10*a^4*b - 11*a^2*b^3 + b^5 + b*c^4 - c^5 - (11*a^2 - 2*b^2)*c^3 + (11*a^2*b - 2*b^3)*c^2 + 3*(2*a^4*b - a^2*b^3 - b^5 - b*c^4 - c^5 + (a^2 + 2*b^2)*c^3 + (a^2*b + 2*b^3)*c^2 + (2*a^4 - a^2*b^2 - b^4)*c)*cosh(x)^2 - (10*a^4 - 11*a^2*b^2 + b^4)*c + 6*(2*a^5 - a^3*b^2 - a*b^4 - a*c^4 + (a^3 + 2*a*b^2)*c^2)*cosh(x))*sinh(x))/(a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^8 - 2*b*c^7 + c^8 + (3*a^2 - 2*b^2)*c^6 - 6*(a^2*b - b^3)*c^5 + 3*(a^4 - a^2*b^2)*c^4 + (a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^8 + 2*b*c^7 + c^8 + (3*a^2 - 2*b^2)*c^6 + 6*(a^2*b - b^3)*c^5 + 3*(a^4 - a^2*b^2)*c^4 + 6*(a^4*b - 2*a^2*b^3 + b^5)*c^3 + (a^6 - 3*a^2*b^4 + 2*b^6)*c^2 + 2*(a^6*b - 3*a^4*b^3 + 3*a^2*b^5 - b^7)*c)*cosh(x)^4 + (a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^8 + 2*b*c^7 + c^8 + (3*a^2 - 2*b^2)*c^6 + 6*(a^2*b - b^3)*c^5 + 3*(a^4 - a^2*b^2)*c^4 + 6*(a^4*b - 2*a^2*b^3 + b^5)*c^3 + (a^6 - 3*a^2*b^4 + 2*b^6)*c^2 + 2*(a^6*b - 3*a^4*b^3 + 3*a^2*b^5 - b^7)*c)*sinh(x)^4 - 6*(a^4*b - 2*a^2*b^3 + b^5)*c^3 + 4*(a^7*b - 3*a^5*b^3 + 3*a^3*b^5 - a*b^7 + a*b*c^6 + a*c^7 + 3*(a^3 - a*b^2)*c^5 + 3*(a^3*b - a*b^3)*c^4 + 3*(a^5 - 2*a^3*b^2 + a*b^4)*c^3 + 3*(a^5*b - 2*a^3*b^3 + a*b^5)*c^2 + (a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6)*c)*cosh(x)^3 + 4*(a^7*b - 3*a^5*b^3 + 3*a^3*b^5 - a*b^7 + a*b*c^6 + a*c^7 + 3*(a^3 - a*b^2)*c^5 + 3*(a^3*b - a*b^3)*c^4 + 3*(a^5 - 2*a^3*b^2 + a*b^4)*c^3 + 3*(a^5*b - 2*a^3*b^3 + a*b^5)*c^2 + (a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6)*c + (a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^8 + 2*b*c^7 + c^8 + (3*a^2 - 2*b^2)*c^6 + 6*(a^2*b - b^3)*c^5 + 3*(a^4 - a^2*b^2)*c^4 + 6*(a^4*b - 2*a^2*b^3 + b^5)*c^3 + (a^6 - 3*a^2*b^4 + 2*b^6)*c^2 + 2*(a^6*b - 3*a^4*b^3 + 3*a^2*b^5 - b^7)*c)*cosh(x))*sinh(x)^3 + (a^6 - 3*a^2*b^4 + 2*b^6)*c^2 + 2*(2*a^8 - 5*a^6*b^2 + 3*a^4*b^4 + a^2*b^6 - b^8 - c^8 - (a^2 - 4*b^2)*c^6 + 3*(a^4 + a^2*b^2 - 2*b^4)*c^4 + (5*a^6 - 6*a^4*b^2 - 3*a^2*b^4 + 4*b^6)*c^2)*cosh(x)^2 + 2*(2*a^8 - 5*a^6*b^2 + 3*a^4*b^4 + a^2*b^6 - b^8 - c^8 - (a^2 - 4*b^2)*c^6 + 3*(a^4 + a^2*b^2 - 2*b^4)*c^4 + (5*a^6 - 6*a^4*b^2 - 3*a^2*b^4 + 4*b^6)*c^2 + 3*(a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^8 + 2*b*c^7 + c^8 + (3*a^2 - 2*b^2)*c^6 + 6*(a^2*b - b^3)*c^5 + 3*(a^4 - a^2*b^2)*c^4 + 6*(a^4*b - 2*a^2*b^3 + b^5)*c^3 + (a^6 - 3*a^2*b^4 + 2*b^6)*c^2 + 2*(a^6*b - 3*a^4*b^3 + 3*a^2*b^5 - b^7)*c)*cosh(x)^2 + 6*(a^7*b - 3*a^5*b^3 + 3*a^3*b^5 - a*b^7 + a*b*c^6 + a*c^7 + 3*(a^3 - a*b^2)*c^5 + 3*(a^3*b - a*b^3)*c^4 +

```
3*(a^5 - 2*a^3*b^2 + a*b^4)*c^3 + 3*(a^5*b - 2*a^3*b^3 + a*b^5)*c^2 + (a^7
 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6)*c)*cosh(x))*sinh(x)^2 - 2*(a^6*b - 3*a^4*
b^3 + 3*a^2*b^5 - b^7)*c + 4*(a^7*b - 3*a^5*b^3 + 3*a^3*b^5 - a*b^7 + a*b*c
^6 - a*c^7 - 3*(a^3 - a*b^2)*c^5 + 3*(a^3*b - a*b^3)*c^4 - 3*(a^5 - 2*a^3*b
^2 + a*b^4)*c^3 + 3*(a^5*b - 2*a^3*b^3 + a*b^5)*c^2 - (a^7 - 3*a^5*b^2 + 3*
a^3*b^4 - a*b^6)*c)*cosh(x) + 4*(a^7*b - 3*a^5*b^3 + 3*a^3*b^5 - a*b^7 + a*
b*c^6 - a*c^7 - 3*(a^3 - a*b^2)*c^5 + 3*(a^3*b - a*b^3)*c^4 - 3*(a^5 - 2*a^
3*b^2 + a*b^4)*c^3 + (a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^8 + 2*b*c^7 + c^8
 + (3*a^2 - 2*b^2)*c^6 + 6*(a^2*b - b^3)*c^5 + 3*(a^4 - a^2*b^2)*c^4 + 6*(a
^4*b - 2*a^2*b^3 + b^5)*c^3 + (a^6 - 3*a^2*b^4 + 2*b^6)*c^2 + 2*(a^6*b - 3*
a^4*b^3 + 3*a^2*b^5 - b^7)*c)*cosh(x)^3 + 3*(a^5*b - 2*a^3*b^3 + a*b^5)*c^2
 + 3*(a^7*b - 3*a^5*b^3 + 3*a^3*b^5 - a*b^7 + a*b*c^6 + a*c^7 + 3*(a^3 - a*
b^2)*c^5 + 3*(a^3*b - a*b^3)*c^4 + 3*(a^5 - 2*a^3*b^2 + a*b^4)*c^3 + 3*(a^5
*b - 2*a^3*b^3 + a*b^5)*c^2 + (a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6)*c)*cosh
(x)^2 - (a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6)*c + (2*a^8 - 5*a^6*b^2 + 3*a^
4*b^4 + a^2*b^6 - b^8 - c^8 - (a^2 - 4*b^2)*c^6 + 3*(a^4 + a^2*b^2 - 2*b^4)
*c^4 + (5*a^6 - 6*a^4*b^2 - 3*a^2*b^4 + 4*b^6)*c^2)*cosh(x))*sinh(x))]
```

giac [B] time = 0.13, size = 304, normalized size = 2.08

$$\frac{\left(2\,a^2+b^2-c^2\right)\arctan\left(\frac{be^x+ce^x+a}{\sqrt{-a^2+b^2-c^2}}\right)}{\left(a^4-2\,a^2b^2+b^4+2\,a^2c^2-2\,b^2c^2+c^4\right)\sqrt{-a^2+b^2-c^2}}+\frac{2\,a^2be^{(3\,x)}+b^3e^{(3\,x)}+2\,a^2ce^{(3\,x)}+b^2ce^{(3\,x)}-bc^2e^{(3\,x)}-}{\left(a^4-\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cosh(x)+c*sinh(x))^3,x, algorithm="giac")
```

```
[Out] (2*a^2 + b^2 - c^2)*arctan((b*e^x + c*e^x + a)/sqrt(-a^2 + b^2 - c^2))/((a^
4 - 2*a^2*b^2 + b^4 + 2*a^2*c^2 - 2*b^2*c^2 + c^4)*sqrt(-a^2 + b^2 - c^2))
+ (2*a^2*b*e^(3*x) + b^3*e^(3*x) + 2*a^2*c*e^(3*x) + b^2*c*e^(3*x) - b*c^2*
e^(3*x) - c^3*e^(3*x) + 6*a^3*e^(2*x) + 3*a*b^2*e^(2*x) - 3*a*c^2*e^(2*x) +
 10*a^2*b*e^x - b^3*e^x - 10*a^2*c*e^x + b^2*c*e^x + b*c^2*e^x - c^3*e^x +
3*a*b^2 - 6*a*b*c + 3*a*c^2)/((a^4 - 2*a^2*b^2 + b^4 + 2*a^2*c^2 - 2*b^2*c^
2 + c^4)*(b*e^(2*x) + c*e^(2*x) + 2*a*e^x + b - c)^2)
```

maple [B] time = 0.28, size = 747, normalized size = 5.12

$$-\frac{2\left(-\frac{\left(4a^3b-7a^2b^2+5a^2c^2+2a\,b^3-2a\,c^2b+b^4-3c^2b^2+2c^4\right)\left(\tanh^3\left(\frac{x}{2}\right)\right)}{2(a-b)\left(a^4-2a^2b^2+2a^2c^2+b^4-2c^2b^2+c^4\right)}-\frac{c\left(4a^4-12a^3b+13a^2b^2-7a^2c^2-6a\,b^3+6a\,c^2b+b^4+c^2b^2-2c^4\right)\left(\tanh^2\left(\frac{x}{2}\right)\right)}{2\left(a^4-2a^2b^2+2a^2c^2+b^4-2c^2b^2+c^4\right)\left(a^2-2ab+b^2\right)}+\right.}{\left(a\left(\tanh^2\left(\frac{x}{2}\right)\right)-\left(\tanh^2\left(\frac{x}{2}\right)\right)b\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*cosh(x)+c*sinh(x))^3,x)
```

```
[Out] -2*(-1/2*(4*a^3*b-7*a^2*b^2+5*a^2*c^2+2*a*b^3-2*a*b*c^2+b^4-3*b^2*c^2+2*c^4
)/(a-b)/(a^4-2*a^2*b^2+2*a^2*c^2+b^4-2*b^2*c^2+c^4)*tanh(1/2*x)^3-1/2*c*(4*
a^4-12*a^3*b+13*a^2*b^2-7*a^2*c^2-6*a*b^3+6*a*b*c^2+b^4+b^2*c^2-2*c^4)/(a^4
-2*a^2*b^2+2*a^2*c^2+b^4-2*b^2*c^2+c^4)/(a^2-2*a*b+b^2)*tanh(1/2*x)^2+1/2*(
4*a^4*b-5*a^3*b^2+11*a^3*c^2-3*a^2*b^3-3*a^2*b*c^2+5*a*b^4-7*a*b^2*c^2+2*a*
c^4-b^5-b^3*c^2+2*b*c^4)/(a^4-2*a^2*b^2+2*a^2*c^2+b^4-2*b^2*c^2+c^4)/(a^2-2
*a*b+b^2)*tanh(1/2*x)+1/2*c*(4*a^4-3*a^2*b^2+a^2*c^2-b^4+b^2*c^2)/(a^4-2*a^
2*b^2+2*a^2*c^2+b^4-2*b^2*c^2+c^4)/(a^2-2*a*b+b^2))/(a*tanh(1/2*x)^2-tanh(1
/2*x)^2*b-2*c*tanh(1/2*x)-a-b)^2-2/(a^4-2*a^2*b^2+2*a^2*c^2+b^4-2*b^2*c^2+c
^4)/(-a^2+b^2-c^2)^(1/2)*arctan(1/2*(2*(a-b)*tanh(1/2*x)-2*c)/(-a^2+b^2-c^2
)^(1/2))*a^2-1/(a^4-2*a^2*b^2+2*a^2*c^2+b^4-2*b^2*c^2+c^4)/(-a^2+b^2-c^2)^(
1/2)*arctan(1/2*(2*(a-b)*tanh(1/2*x)-2*c)/(-a^2+b^2-c^2)^(1/2))*b^2+1/(a^4-
2*a^2*b^2+2*a^2*c^2+b^4-2*b^2*c^2+c^4)/(-a^2+b^2-c^2)^(1/2)*arctan(1/2*(2*(
a-b)*tanh(1/2*x)-2*c)/(-a^2+b^2-c^2)^(1/2))*c^2
```

maxima [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cosh(x)+c*sinh(x))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(c^2-b^2+a^2>0)', see `assume?` for
more details)Is c^2-b^2+a^2 positive or negative?
```

mupad [**F**] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(a+b\cosh(x)+c\sinh(x))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*cosh(x) + c*sinh(x))^3,x)
```

```
[Out] int(1/(a + b*cosh(x) + c*sinh(x))^3, x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cosh(x)+c*sinh(x))**3,x)
```

```
[Out] Timed out
```

3.745 $\int \frac{1}{(a+b\cosh(x)+c\sinh(x))^4}\,dx$

Optimal. Leaf size=220

$$-\frac{a\left(2a^2+3b^2-3c^2\right)\tanh^{-1}\left(\frac{c-(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2+c^2}}\right)}{\left(a^2-b^2+c^2\right)^{7/2}}-\frac{b\sinh(x)\left(11a^2+4b^2-4c^2\right)+c\cosh(x)\left(11a^2+4b^2-4c^2\right)}{6\left(a^2-b^2+c^2\right)^3(a+b\cosh(x)+c\sinh(x))}-\frac{}{6\left(a^2\right.}$$

```
[Out] -a*(2*a^2+3*b^2-3*c^2)*arctanh((c-(a-b)*tanh(1/2*x))/(a^2-b^2+c^2)^(1/2))/(
a^2-b^2+c^2)^(7/2)+1/3*(-c*cosh(x)-b*sinh(x))/(a^2-b^2+c^2)/(a+b*cosh(x)+c*
sinh(x))^3-5/6*(a*c*cosh(x)+a*b*sinh(x))/(a^2-b^2+c^2)^2/(a+b*cosh(x)+c*sin
h(x))^2+1/6*(-c*(11*a^2+4*b^2-4*c^2)*cosh(x)-b*(11*a^2+4*b^2-4*c^2)*sinh(x)
)/(a^2-b^2+c^2)^3/(a+b*cosh(x)+c*sinh(x))
```

Rubi [A] time = 0.30, antiderivative size = 220, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3129, 3156, 3153, 3124, 618, 206}

$$-\frac{a\left(2a^2+3b^2-3c^2\right)\tanh^{-1}\left(\frac{c-(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2+c^2}}\right)}{\left(a^2-b^2+c^2\right)^{7/2}}-\frac{b\sinh(x)\left(11a^2+4b^2-4c^2\right)+c\cosh(x)\left(11a^2+4b^2-4c^2\right)}{6\left(a^2-b^2+c^2\right)^3(a+b\cosh(x)+c\sinh(x))}-\frac{}{6\left(a^2\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cosh[x] + c*Sinh[x])^(-4),x]
```

```
[Out] -((a*(2*a^2 + 3*b^2 - 3*c^2)*ArcTanh[(c - (a - b)*Tanh[x/2])/Sqrt[a^2 - b^2
 + c^2]])/(a^2 - b^2 + c^2)^(7/2)) - (c*Cosh[x] + b*Sinh[x])/(3*(a^2 - b^2
+ c^2)*(a + b*Cosh[x] + c*Sinh[x])^3) - (5*(a*c*Cosh[x] + a*b*Sinh[x]))/(6*
(a^2 - b^2 + c^2)^2*(a + b*Cosh[x] + c*Sinh[x])^2) - (c*(11*a^2 + 4*b^2 - 4
*c^2)*Cosh[x] + b*(11*a^2 + 4*b^2 - 4*c^2)*Sinh[x])/(6*(a^2 - b^2 + c^2)^3*
(a + b*Cosh[x] + c*Sinh[x]))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 3124

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Module[{f = FreeFactors[Tan[(d + e*x)/2], x]}, Dist[(2*f
)/e, Subst[Int[1/(a + b + 2*c*f*x + (a - b)*f^2*x^2), x], x, Tan[(d + e*x)/
2]/f], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
```

Rule 3129

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> Simp[((-(c*Cos[d + e*x]) + b*Sin[d + e*x])*(a + b*Cos[d
+ e*x] + c*Sin[d + e*x])^(n + 1))/(e*(n + 1)*(a^2 - b^2 - c^2)), x] + Dist[
1/((n + 1)*(a^2 - b^2 - c^2)), Int[(a*(n + 1) - b*(n + 2)*Cos[d + e*x] - c*
(n + 2)*Sin[d + e*x])*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1), x], x]
 /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && LtQ[n, -1] && N
eQ[n, -3/2]
```

Rule 3153

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])^2,
 x_Symbol] :> Simp[(c*B - b*C - (a*C - c*A)*Cos[d + e*x] + (a*B - b*A)*Sin[
d + e*x])/(e*(a^2 - b^2 - c^2)*(a + b*Cos[d + e*x] + c*Sin[d + e*x])), x] +
 Dist[(a*A - b*B - c*C)/(a^2 - b^2 - c^2), Int[1/(a + b*Cos[d + e*x] + c*Si
n[d + e*x]), x], x] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && NeQ[a^2 - b^2
- c^2, 0] && NeQ[a*A - b*B - c*C, 0]
```

Rule 3156

```
Int[((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])
^(n_)*((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)
]), x_Symbol] :> -Simp[((c*B - b*C - (a*C - c*A)*Cos[d + e*x] + (a*B - b*A)
*Sin[d + e*x])*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1))/(e*(n + 1)*(a
^2 - b^2 - c^2)), x] + Dist[1/((n + 1)*(a^2 - b^2 - c^2)), Int[(a + b*Cos[d
 + e*x] + c*Sin[d + e*x])^(n + 1)*Simp[(n + 1)*(a*A - b*B - c*C) + (n + 2)*
(a*B - b*A)*Cos[d + e*x] + (n + 2)*(a*C - c*A)*Sin[d + e*x], x], x], x] /;
FreeQ[{a, b, c, d, e, A, B, C}, x] && LtQ[n, -1] && NeQ[a^2 - b^2 - c^2, 0]
 && NeQ[n, -2]
```

Rubi steps

$$\int \frac{1}{(a+b\cosh(x)+c\sinh(x))^4}\,dx = -\frac{c\cosh(x)+b\sinh(x)}{3\left(a^2-b^2+c^2\right)(a+b\cosh(x)+c\sinh(x))^3} - \frac{\int \frac{-3a+2b\cosh(x)+2c\sinh(x)}{(a+b\cosh(x)+c\sinh(x))^3}\,dx}{3\left(a^2-b^2+c^2\right)}$$

$$= -\frac{c\cosh(x)+b\sinh(x)}{3\left(a^2-b^2+c^2\right)(a+b\cosh(x)+c\sinh(x))^3} - \frac{5(ac\cosh(x)+a}{6\left(a^2-b^2+c^2\right)^2(a+b\cosh}$$

$$= -\frac{c\cosh(x)+b\sinh(x)}{3\left(a^2-b^2+c^2\right)(a+b\cosh(x)+c\sinh(x))^3} - \frac{5(ac\cosh(x)+a}{6\left(a^2-b^2+c^2\right)^2(a+b\cosh}$$

$$= -\frac{c\cosh(x)+b\sinh(x)}{3\left(a^2-b^2+c^2\right)(a+b\cosh(x)+c\sinh(x))^3} - \frac{5(ac\cosh(x)+a}{6\left(a^2-b^2+c^2\right)^2(a+b\cosh}$$

$$= -\frac{c\cosh(x)+b\sinh(x)}{3\left(a^2-b^2+c^2\right)(a+b\cosh(x)+c\sinh(x))^3} - \frac{5(ac\cosh(x)+a}{6\left(a^2-b^2+c^2\right)^2(a+b\cosh}$$

$$= -\frac{a\left(2a^2+3b^2-3c^2\right)\tanh^{-1}\left(\frac{c-(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2+c^2}}\right)}{\left(a^2-b^2+c^2\right)^{7/2}} - \frac{c\cosh(x)+b\sin}{3\left(a^2-b^2+c^2\right)(a+b\cosh}$$

Mathematica [B] time = 0.96, size = 488, normalized size = 2.22

$$-\frac{a\left(2a^2+3b^2-3c^2\right)\tan^{-1}\left(\frac{(b-a)\tanh\left(\frac{x}{2}\right)+c}{\sqrt{-a^2+b^2-c^2}}\right)}{\left(-a^2+b^2-c^2\right)^{7/2}} - \frac{-44a^5c+72a^4b^2\sinh(x)-132a^4c^2\sinh(x)+54a^3b^3\sinh(2x)-82}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cosh[x] + c*Sinh[x])^(-4),x]
```

```
[Out] -((a*(2*a^2 + 3*b^2 - 3*c^2)*ArcTan[(c + (-a + b)*Tanh[x/2])/Sqrt[-a^2 + b^
2 - c^2]])/(-a^2 + b^2 - c^2)^(7/2)) - (-44*a^5*c - 82*a^3*b^2*c - 24*a*b^4
*c + 82*a^3*c^3 + 48*a*b^2*c^3 - 24*a*c^5 - 30*a^2*b*c*(2*a^2 + 3*b^2 - 3*c
^2)*Cosh[x] - 6*a*c*(a^2*(-7*b^2 + 11*c^2) + 2*(b^4 + b^2*c^2 - 2*c^4))*Cos
h[2*x] + 22*a^2*b^3*c*Cosh[3*x] + 8*b^5*c*Cosh[3*x] - 22*a^2*b*c^3*Cosh[3*x
] - 16*b^3*c^3*Cosh[3*x] + 8*b*c^5*Cosh[3*x] + 72*a^4*b^2*Sinh[x] - 9*a^2*b
^4*Sinh[x] + 12*b^6*Sinh[x] - 132*a^4*c^2*Sinh[x] - 72*a^2*b^2*c^2*Sinh[x]
 - 36*b^4*c^2*Sinh[x] + 81*a^2*c^4*Sinh[x] + 36*b^2*c^4*Sinh[x] - 12*c^6*Sin
```

```
h[x] + 54*a^3*b^3*Sinh[2*x] + 6*a*b^5*Sinh[2*x] - 78*a^3*b*c^2*Sinh[2*x] -
48*a*b^3*c^2*Sinh[2*x] + 42*a*b*c^4*Sinh[2*x] + 11*a^2*b^4*Sinh[3*x] + 4*b^
6*Sinh[3*x] - 4*b^4*c^2*Sinh[3*x] - 11*a^2*c^4*Sinh[3*x] - 4*b^2*c^4*Sinh[3
*x] + 4*c^6*Sinh[3*x])/(24*b*(a^2 - b^2 + c^2)^3*(a + b*Cosh[x] + c*Sinh[x]
)^3)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cosh(x)+c*sinh(x))^4,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [B] time = 0.18, size = 717, normalized size = 3.26

$$\frac{\left(2\,a^3+3\,ab^2-3\,ac^2\right)\arctan\left(\frac{be^x+ce^x+a}{\sqrt{-a^2+b^2-c^2}}\right)}{\left(a^6-3\,a^4b^2+3\,a^2b^4-b^6+3\,a^4c^2-6\,a^2b^2c^2+3\,b^4c^2+3\,a^2c^4-3\,b^2c^4+c^6\right)\sqrt{-a^2+b^2-c^2}}+\frac{6\,a^3b^2e^{(5\,x)}+9\,ab^4}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cosh(x)+c*sinh(x))^4,x, algorithm="giac")
```

```
[Out] (2*a^3 + 3*a*b^2 - 3*a*c^2)*arctan((b*e^x + c*e^x + a)/sqrt(-a^2 + b^2 - c^
2))/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6 + 3*a^4*c^2 - 6*a^2*b^2*c^2 + 3*b^4
*c^2 + 3*a^2*c^4 - 3*b^2*c^4 + c^6)*sqrt(-a^2 + b^2 - c^2)) + 1/3*(6*a^3*b^
2*e^(5*x) + 9*a*b^4*e^(5*x) + 12*a^3*b*c*e^(5*x) + 18*a*b^3*c*e^(5*x) + 6*a
^3*c^2*e^(5*x) - 18*a*b*c^3*e^(5*x) - 9*a*c^4*e^(5*x) + 30*a^4*b*e^(4*x) +
45*a^2*b^3*e^(4*x) + 30*a^4*c*e^(4*x) + 45*a^2*b^2*c*e^(4*x) - 45*a^2*b*c^2
*e^(4*x) - 45*a^2*c^3*e^(4*x) + 44*a^5*e^(3*x) + 82*a^3*b^2*e^(3*x) + 24*a*
b^4*e^(3*x) - 82*a^3*c^2*e^(3*x) - 48*a*b^2*c^2*e^(3*x) + 24*a*c^4*e^(3*x)
+ 102*a^4*b*e^(2*x) + 36*a^2*b^3*e^(2*x) + 12*b^5*e^(2*x) - 102*a^4*c*e^(2*
x) - 36*a^2*b^2*c*e^(2*x) - 12*b^4*c*e^(2*x) - 36*a^2*b*c^2*e^(2*x) - 24*b^
3*c^2*e^(2*x) + 36*a^2*c^3*e^(2*x) + 24*b^2*c^3*e^(2*x) + 12*b*c^4*e^(2*x)
- 12*c^5*e^(2*x) + 60*a^3*b^2*e^x + 15*a*b^4*e^x - 120*a^3*b*c*e^x - 30*a*b
^3*c*e^x + 60*a^3*c^2*e^x + 30*a*b*c^3*e^x - 15*a*c^4*e^x + 11*a^2*b^3 + 4*
b^5 - 33*a^2*b^2*c - 12*b^4*c + 33*a^2*b*c^2 + 8*b^3*c^2 - 11*a^2*c^3 + 8*b
^2*c^3 - 12*b*c^4 + 4*c^5)/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6 + 3*a^4*c^2
- 6*a^2*b^2*c^2 + 3*b^4*c^2 + 3*a^2*c^4 - 3*b^2*c^4 + c^6)*(b*e^(2*x) + c*e
^(2*x) + 2*a*e^x + b - c)^3)
```

maple [B] time = 0.36, size = 1842, normalized size = 8.37

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*cosh(x)+c*sinh(x))^4,x)
```

```
[Out] -2*(-1/2*(6*a^5*b-15*a^4*b^2+9*a^4*c^2+11*a^3*b^3-9*a^3*b*c^2-3*a^2*b^4-3*a
^2*b^2*c^2+6*a^2*c^4+3*a*b^5-3*a*b^3*c^2-2*b^6+6*b^4*c^2-6*b^2*c^4+2*c^6)/(
a^6-3*a^4*b^2+3*a^4*c^2+3*a^2*b^4-6*a^2*b^2*c^2+3*a^2*c^4-b^6+3*b^4*c^2-3*b
^2*c^4+c^6)/(a-b)*tanh(1/2*x)^5-1/2*c*(6*a^6-30*a^5*b+57*a^4*b^2-27*a^4*c^2
-55*a^3*b^3+45*a^3*b*c^2+33*a^2*b^4-21*a^2*b^2*c^2-12*a^2*c^4-15*a*b^5+15*a
*b^3*c^2+4*b^6-12*b^4*c^2+12*b^2*c^4-4*c^6)/(a^6-3*a^4*b^2+3*a^4*c^2+3*a^2*
b^4-6*a^2*b^2*c^2+3*a^2*c^4-b^6+3*b^4*c^2-3*b^2*c^4+c^6)/(a^2-2*a*b+b^2)*ta
nh(1/2*x)^4+1/3*(18*a^7*b-54*a^6*b^2+54*a^6*c^2+38*a^5*b^3-120*a^5*b*c^2+30
*a^4*b^4+81*a^4*b^2*c^2-21*a^4*c^4-50*a^3*b^5-61*a^3*b^3*c^2+81*a^3*b*c^4+2
2*a^2*b^6+87*a^2*b^4*c^2-105*a^2*b^2*c^4-4*a^2*c^6-6*a*b^7-39*a*b^5*c^2+51*
a*b^3*c^4-6*a*b*c^6+2*b^8-2*b^6*c^2-6*b^4*c^4+10*b^2*c^6-4*c^8)/(a^6-3*a^4*
b^2+3*a^4*c^2+3*a^2*b^4-6*a^2*b^2*c^2+3*a^2*c^4-b^6+3*b^4*c^2-3*b^2*c^4+c^6
)/(a^3-3*a^2*b+3*a*b^2-b^3)*tanh(1/2*x)^3+c*(6*a^7-18*a^6*b+18*a^5*b^2-20*a
^5*c^2-2*a^4*b^3+22*a^4*b*c^2-14*a^3*b^4+7*a^3*b^2*c^2-3*a^3*c^4+18*a^2*b^5
-6*a^2*b^3*c^2-12*a^2*b*c^4-10*a*b^6+3*a*b^4*c^2+9*a*b^2*c^4-2*a*c^6+2*b^7-
6*b^5*c^2+6*b^3*c^4-2*b*c^6)/(a^6-3*a^4*b^2+3*a^4*c^2+3*a^2*b^4-6*a^2*b^2*c
^2+3*a^2*c^4-b^6+3*b^4*c^2-3*b^2*c^4+c^6)/(a^2-2*a*b+b^2)/(a-b)*tanh(1/2*x)
^2-1/2*(6*a^7*b-9*a^6*b^2+27*a^6*c^2-7*a^5*b^3-9*a^5*b*c^2+16*a^4*b^4-30*a^
4*b^2*c^2+4*a^4*c^4-4*a^3*b^5+14*a^3*b*c^4-5*a^2*b^6-3*a^2*b^4*c^2+6*a^2*b^
2*c^4+2*a^2*c^6+5*a*b^7+9*a*b^5*c^2-18*a*b^3*c^4+4*a*b*c^6-2*b^8+6*b^6*c^2-
6*b^4*c^4+2*b^2*c^6)/(a^6-3*a^4*b^2+3*a^4*c^2+3*a^2*b^4-6*a^2*b^2*c^2+3*a^2
*c^4-b^6+3*b^4*c^2-3*b^2*c^4+c^6)/(a^3-3*a^2*b+3*a*b^2-b^3)*tanh(1/2*x)-1/6
*a*c*(18*a^6-21*a^4*b^2+5*a^4*c^2-12*a^2*b^4+16*a^2*b^2*c^2+2*a^2*c^4+15*b^
6-21*b^4*c^2+6*b^2*c^4)/(a^6-3*a^4*b^2+3*a^4*c^2+3*a^2*b^4-6*a^2*b^2*c^2+3*
a^2*c^4-b^6+3*b^4*c^2-3*b^2*c^4+c^6)/(a^3-3*a^2*b+3*a*b^2-b^3))/(a*tanh(1/2
*x)^2-tanh(1/2*x)^2*b-2*c*tanh(1/2*x)-a-b)^3-2*a^3/(a^6-3*a^4*b^2+3*a^4*c^2
+3*a^2*b^4-6*a^2*b^2*c^2+3*a^2*c^4-b^6+3*b^4*c^2-3*b^2*c^4+c^6)/(-a^2+b^2-c
^2)^(1/2)*arctan(1/2*(2*(a-b)*tanh(1/2*x)-2*c)/(-a^2+b^2-c^2)^(1/2))-3*a/(a
^6-3*a^4*b^2+3*a^4*c^2+3*a^2*b^4-6*a^2*b^2*c^2+3*a^2*c^4-b^6+3*b^4*c^2-3*b^
2*c^4+c^6)/(-a^2+b^2-c^2)^(1/2)*arctan(1/2*(2*(a-b)*tanh(1/2*x)-2*c)/(-a^2+
b^2-c^2)^(1/2))*b^2+3*a/(a^6-3*a^4*b^2+3*a^4*c^2+3*a^2*b^4-6*a^2*b^2*c^2+3*
a^2*c^4-b^6+3*b^4*c^2-3*b^2*c^4+c^6)/(-a^2+b^2-c^2)^(1/2)*arctan(1/2*(2*(a-
b)*tanh(1/2*x)-2*c)/(-a^2+b^2-c^2)^(1/2))*c^2
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cosh(x)+c*sinh(x))^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(c^2-b^2+a^2>0)', see `assume?` for
more details)Is c^2-b^2+a^2 positive or negative?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{(a + b\cosh(x) + c\sinh(x))^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*cosh(x) + c*sinh(x))^4,x)
```

```
[Out] int(1/(a + b*cosh(x) + c*sinh(x))^4, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cosh(x)+c*sinh(x))**4,x)
```

```
[Out] Timed out
```

3.746 $\int (a + a\cosh(x) + c\sinh(x))^3 \, dx$

Optimal. Leaf size=105

$$\frac{1}{2}ax\left(5a^2-3c^2\right)+\frac{1}{6}a\left(15a^2-4c^2\right)\sinh(x)+\frac{1}{6}c\left(15a^2-4c^2\right)\cosh(x)+\frac{5}{6}\left(a^2\sinh(x)+ac\cosh(x)\right)(a\cosh(x)+a+$$

```
[Out] 1/2*a*(5*a^2-3*c^2)*x+1/6*c*(15*a^2-4*c^2)*cosh(x)+1/6*a*(15*a^2-4*c^2)*sin
h(x)+5/6*(a*c*cosh(x)+a^2*sinh(x))*(a+a*cosh(x)+c*sinh(x))+1/3*(c*cosh(x)+a
*sinh(x))*(a+a*cosh(x)+c*sinh(x))^2
```

Rubi [A] time = 0.12, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3120, 3146, 2637, 2638}

$$\frac{1}{2}ax\left(5a^2-3c^2\right)+\frac{1}{6}a\left(15a^2-4c^2\right)\sinh(x)+\frac{1}{6}c\left(15a^2-4c^2\right)\cosh(x)+\frac{5}{6}\left(a^2\sinh(x)+ac\cosh(x)\right)(a\cosh(x)+a+$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cosh[x] + c*Sinh[x])^3,x]
```

```
[Out] (a*(5*a^2 - 3*c^2)*x)/2 + (c*(15*a^2 - 4*c^2)*Cosh[x])/6 + (a*(15*a^2 - 4*c
^2)*Sinh[x])/6 + (5*(a*c*Cosh[x] + a^2*Sinh[x])*(a + a*Cosh[x] + c*Sinh[x])
)/6 + ((c*Cosh[x] + a*Sinh[x])*(a + a*Cosh[x] + c*Sinh[x])^2)/3
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3120

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> -Simp[((c*Cos[d + e*x] - b*Sin[d + e*x])*(a + b*Cos[d +
e*x] + c*Sin[d + e*x])^(n - 1))/(e*n), x] + Dist[1/n, Int[Simp[n*a^2 + (n -
 1)*(b^2 + c^2) + a*b*(2*n - 1)*Cos[d + e*x] + a*c*(2*n - 1)*Sin[d + e*x],
x]*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n - 2), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && GtQ[n, 1]
```

Rule 3146

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_.)*((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)
]), x_Symbol] :> Simp[((B*c - b*C - a*C*Cos[d + e*x] + a*B*Sin[d + e*x])*(a
 + b*Cos[d + e*x] + c*Sin[d + e*x])^n)/(a*e*(n + 1)), x] + Dist[1/(a*(n + 1
)), Int[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n - 1)*Simp[a*(b*B + c*C)*n
+ a^2*A*(n + 1) + (n*(a^2*B - B*c^2 + b*c*C) + a*b*A*(n + 1))*Cos[d + e*x]
+ (n*(b*B*c + a^2*C - b^2*C) + a*c*A*(n + 1))*Sin[d + e*x], x], x], x] /; F
reeQ[{a, b, c, d, e, A, B, C}, x] && GtQ[n, 0] && NeQ[a^2 - b^2 - c^2, 0]
```

Rubi steps

$$\begin{aligned}\int (a+a\cosh(x)+c\sinh(x))^3\,dx &= \frac{1}{3}(c\cosh(x)+a\sinh(x))(a+a\cosh(x)+c\sinh(x))^2+\frac{1}{3}\int (a+a\cosh(x)+\\ &= \frac{5}{6}\left(ac\cosh(x)+a^2\sinh(x)\right)(a+a\cosh(x)+c\sinh(x))+\frac{1}{3}(c\cosh(x)+a\sin\\ &= \frac{1}{2}a\left(5a^2-3c^2\right)x+\frac{5}{6}\left(ac\cosh(x)+a^2\sinh(x)\right)(a+a\cosh(x)+c\sinh(x))+\\ &= \frac{1}{2}a\left(5a^2-3c^2\right)x+\frac{1}{6}c\left(15a^2-4c^2\right)\cosh(x)+\frac{1}{6}a\left(15a^2-4c^2\right)\sinh(x)+\frac{5}{6}\left(a\right.\end{aligned}$$

Mathematica [A] time = 0.17, size = 112, normalized size = 1.07

$$\frac{1}{12}\left(30a^3x+45a^3\sinh(x)+9a^3\sinh(2x)+a^3\sinh(3x)-9c\left(c^2-5a^2\right)\cosh(x)+18a^2c\cosh(2x)+3a^2c\cosh(3x\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cosh[x] + c*Sinh[x])^3,x]
```

```
[Out] (30*a^3*x - 18*a*c^2*x - 9*c*(-5*a^2 + c^2)*Cosh[x] + 18*a^2*c*Cosh[2*x] +
3*a^2*c*Cosh[3*x] + c^3*Cosh[3*x] + 45*a^3*Sinh[x] - 9*a*c^2*Sinh[x] + 9*a^
3*Sinh[2*x] + 9*a*c^2*Sinh[2*x] + a^3*Sinh[3*x] + 3*a*c^2*Sinh[3*x])/12
```

fricas [A] time = 0.44, size = 144, normalized size = 1.37

$$\frac{3}{2}a^2c\cosh(x)^2+\frac{1}{12}\left(3a^2c+c^3\right)\cosh(x)^3+\frac{1}{12}\left(a^3+3ac^2\right)\sinh(x)^3+\frac{1}{4}\left(6a^2c+\left(3a^2c+c^3\right)\cosh(x)\right)\sinh(x)^2+\frac{1}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cosh(x)+c*sinh(x))^3,x, algorithm="fricas")
```

```
[Out] 3/2*a^2*c*cosh(x)^2 + 1/12*(3*a^2*c + c^3)*cosh(x)^3 + 1/12*(a^3 + 3*a*c^2)
*sinh(x)^3 + 1/4*(6*a^2*c + (3*a^2*c + c^3)*cosh(x))*sinh(x)^2 + 1/2*(5*a^3
```

```
 - 3*a*c^2)*x + 3/4*(5*a^2*c - c^3)*cosh(x) + 1/4*(15*a^3 - 3*a*c^2 + (a^3
+ 3*a*c^2)*cosh(x)^2 + 6*(a^3 + a*c^2)*cosh(x))*sinh(x)
```

giac [A] time = 0.12, size = 186, normalized size = 1.77

$$\frac{1}{24}a^3e^{(3x)}+\frac{1}{8}a^2ce^{(3x)}+\frac{1}{8}ac^2e^{(3x)}+\frac{1}{24}c^3e^{(3x)}+\frac{3}{8}a^3e^{(2x)}+\frac{3}{4}a^2ce^{(2x)}+\frac{3}{8}ac^2e^{(2x)}+\frac{15}{8}a^3e^x+\frac{15}{8}a^2ce^x-\frac{3}{8}ac^2e^x-\frac{3}{8}c^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cosh(x)+c*sinh(x))^3,x, algorithm="giac")
```

```
[Out] 1/24*a^3*e^(3*x) + 1/8*a^2*c*e^(3*x) + 1/8*a*c^2*e^(3*x) + 1/24*c^3*e^(3*x)
 + 3/8*a^3*e^(2*x) + 3/4*a^2*c*e^(2*x) + 3/8*a*c^2*e^(2*x) + 15/8*a^3*e^x +
 15/8*a^2*c*e^x - 3/8*a*c^2*e^x - 3/8*c^3*e^x + 1/2*(5*a^3 - 3*a*c^2)*x - 1
/24*(a^3 - 3*a^2*c + 3*a*c^2 - c^3 + 9*(5*a^3 - 5*a^2*c - a*c^2 + c^3)*e^(2
*x) + 9*(a^3 - 2*a^2*c + a*c^2)*e^x)*e^(-3*x)
```

maple [A] time = 0.50, size = 109, normalized size = 1.04

$$a^3x+3a^3\sinh(x)+3a^2c\cosh(x)+3a^3\left(\frac{\cosh(x)\sinh(x)}{2}+\frac{x}{2}\right)+3a^2c\left(\cosh^2(x)\right)+3a\,c^2\left(\frac{\cosh(x)\sinh(x)}{2}-\frac{x}{2}\right)+a^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cosh(x)+c*sinh(x))^3,x)
```

```
[Out] a^3*x+3*a^3*sinh(x)+3*a^2*c*cosh(x)+3*a^3*(1/2*cosh(x)*sinh(x)+1/2*x)+3*a^2
*c*cosh(x)^2+3*a*c^2*(1/2*cosh(x)*sinh(x)-1/2*x)+a^3*(2/3+1/3*cosh(x)^2)*si
nh(x)+a^2*c*cosh(x)^3+a*c^2*sinh(x)^3+c^3*(-2/3+1/3*sinh(x)^2)*cosh(x)
```

maxima [A] time = 0.31, size = 137, normalized size = 1.30

$$a^2c\cosh(x)^3+ac^2\sinh(x)^3+a^3x+\frac{1}{24}c^3\left(e^{(3x)}-9\,e^{(-x)}+e^{(-3x)}-9\,e^x\right)+\frac{1}{24}a^3\left(e^{(3x)}-9\,e^{(-x)}-e^{(-3x)}+9\,e^x\right)+3\,(c\,c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cosh(x)+c*sinh(x))^3,x, algorithm="maxima")
```

```
[Out] a^2*c*cosh(x)^3 + a*c^2*sinh(x)^3 + a^3*x + 1/24*c^3*(e^(3*x) - 9*e^(-x) +
e^(-3*x) - 9*e^x) + 1/24*a^3*(e^(3*x) - 9*e^(-x) - e^(-3*x) + 9*e^x) + 3*(c
*cosh(x) + a*sinh(x))*a^2 + 3/8*(8*a*c*cosh(x)^2 + a^2*(4*x + e^(2*x) - e^(
-2*x)) - c^2*(4*x - e^(2*x) + e^(-2*x)))*a
```

mupad [B] time = 0.15, size = 131, normalized size = 1.25

$$3\,a^3\sinh(x)+a^3\,x+\cosh(x)^3\,\left(a^2\,c-\frac{2\,c^3}{3}\right)+\sinh(x)^3\,\left(a\,c^2-\frac{2\,a^3}{3}\right)+a^3\cosh(x)^2\sinh(x)+c^3\cosh(x)\sinh(x)^2+3\,a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*cosh(x) + c*sinh(x))^3,x)
```

```
[Out] 3*a^3*sinh(x) + a^3*x + cosh(x)^3*(a^2*c - (2*c^3)/3) + sinh(x)^3*(a*c^2 -
(2*a^3)/3) + a^3*cosh(x)^2*sinh(x) + c^3*cosh(x)*sinh(x)^2 + 3*a^2*c*cosh(x
) + 3*a^2*c*cosh(x)^2 + (3*a*cosh(x)*sinh(x)*(a^2 + c^2))/2 + (3*a*x*cosh(x
)^2*(a^2 - c^2))/2 - (3*a*x*sinh(x)^2*(a^2 - c^2))/2
```

sympy [A] time = 0.42, size = 189, normalized size = 1.80

$$-\frac{3a^3x\sinh^2(x)}{2}+\frac{3a^3x\cosh^2(x)}{2}+a^3x-\frac{2a^3\sinh^3(x)}{3}+a^3\sinh(x)\cosh^2(x)+\frac{3a^3\sinh(x)\cosh(x)}{2}+3a^3\sinh(x)+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cosh(x)+c*sinh(x))**3,x)
```

```
[Out] -3*a**3*x*sinh(x)**2/2 + 3*a**3*x*cosh(x)**2/2 + a**3*x - 2*a**3*sinh(x)**3
/3 + a**3*sinh(x)*cosh(x)**2 + 3*a**3*sinh(x)*cosh(x)/2 + 3*a**3*sinh(x) +
a**2*c*cosh(x)**3 + 3*a**2*c*cosh(x)**2 + 3*a**2*c*cosh(x) + 3*a*c**2*x*sin
h(x)**2/2 - 3*a*c**2*x*cosh(x)**2/2 + a*c**2*sinh(x)**3 + 3*a*c**2*sinh(x)*
cosh(x)/2 + c**3*sinh(x)**2*cosh(x) - 2*c**3*cosh(x)**3/3
```

3.747 $\int (a+a\cosh(x)+c\sinh(x))^2\,dx$

Optimal. Leaf size=57

$$\frac{1}{2}x\left(3a^2-c^2\right)+\frac{3}{2}a^2\sinh(x)+\frac{3}{2}ac\cosh(x)+\frac{1}{2}(a\sinh(x)+c\cosh(x))(a\cosh(x)+a+c\sinh(x))$$

```
[Out] 1/2*(3*a^2-c^2)*x+3/2*a*c*cosh(x)+3/2*a^2*sinh(x)+1/2*(c*cosh(x)+a*sinh(x))
*(a+a*cosh(x)+c*sinh(x))
```

Rubi [A] time = 0.03, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3120, 2637, 2638}

$$\frac{1}{2}x\left(3a^2-c^2\right)+\frac{3}{2}a^2\sinh(x)+\frac{3}{2}ac\cosh(x)+\frac{1}{2}(a\sinh(x)+c\cosh(x))(a\cosh(x)+a+c\sinh(x))$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cosh[x] + c*Sinh[x])^2,x]
```

```
[Out] ((3*a^2 - c^2)*x)/2 + (3*a*c*Cosh[x])/2 + (3*a^2*Sinh[x])/2 + ((c*Cosh[x] +
 a*Sinh[x])*(a + a*Cosh[x] + c*Sinh[x]))/2
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3120

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> -Simp[((c*Cos[d + e*x] - b*Sin[d + e*x])*(a + b*Cos[d +
e*x] + c*Sin[d + e*x])^(n - 1))/(e*n), x] + Dist[1/n, Int[Simp[n*a^2 + (n -
 1)*(b^2 + c^2) + a*b*(2*n - 1)*Cos[d + e*x] + a*c*(2*n - 1)*Sin[d + e*x],
x]*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n - 2), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && GtQ[n, 1]
```

Rubi steps

$$\int (a + a\cosh(x) + c\sinh(x))^2\,dx = \frac{1}{2}(c\cosh(x) + a\sinh(x))(a + a\cosh(x) + c\sinh(x)) + \frac{1}{2}\int \left(3a^2 - c^2 + 3a^2 c\right.$$
$$= \frac{1}{2}\left(3a^2 - c^2\right)x + \frac{1}{2}(c\cosh(x) + a\sinh(x))(a + a\cosh(x) + c\sinh(x)) + \frac{1}{2}\left(3a\right.$$
$$= \frac{1}{2}\left(3a^2 - c^2\right)x + \frac{3}{2}ac\cosh(x) + \frac{3}{2}a^2\sinh(x) + \frac{1}{2}(c\cosh(x) + a\sinh(x))(a + a$$

Mathematica [A] time = 0.07, size = 55, normalized size = 0.96

$$\frac{1}{2}x\left(3a^2 - c^2\right) + \frac{1}{4}\left(a^2 + c^2\right)\sinh(2x) + 2a^2\sinh(x) + 2ac\cosh(x) + \frac{1}{2}ac\cosh(2x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cosh[x] + c*Sinh[x])^2,x]
```

```
[Out] ((3*a^2 - c^2)*x)/2 + 2*a*c*Cosh[x] + (a*c*Cosh[2*x])/2 + 2*a^2*Sinh[x] + (
(a^2 + c^2)*Sinh[2*x])/4
```

fricas [A] time = 0.42, size = 57, normalized size = 1.00

$$\frac{1}{2}ac\cosh(x)^2 + \frac{1}{2}ac\sinh(x)^2 + 2ac\cosh(x) + \frac{1}{2}\left(3a^2 - c^2\right)x + \frac{1}{2}\left(4a^2 + \left(a^2 + c^2\right)\cosh(x)\right)\sinh(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cosh(x)+c*sinh(x))^2,x, algorithm="fricas")
```

```
[Out] 1/2*a*c*cosh(x)^2 + 1/2*a*c*sinh(x)^2 + 2*a*c*cosh(x) + 1/2*(3*a^2 - c^2)*x
 + 1/2*(4*a^2 + (a^2 + c^2)*cosh(x))*sinh(x)
```

giac [A] time = 0.13, size = 81, normalized size = 1.42

$$\frac{1}{8}a^2e^{(2x)} + \frac{1}{4}ace^{(2x)} + \frac{1}{8}c^2e^{(2x)} + a^2e^x + ace^x + \frac{1}{2}\left(3a^2 - c^2\right)x - \frac{1}{8}\left(a^2 - 2ac + c^2 + 8\left(a^2 - ac\right)e^x\right)e^{(-2x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cosh(x)+c*sinh(x))^2,x, algorithm="giac")
```

```
[Out] 1/8*a^2*e^(2*x) + 1/4*a*c*e^(2*x) + 1/8*c^2*e^(2*x) + a^2*e^x + a*c*e^x + 1
/2*(3*a^2 - c^2)*x - 1/8*(a^2 - 2*a*c + c^2 + 8*(a^2 - a*c)*e^x)*e^(-2*x)
```

maple [A] time = 0.14, size = 55, normalized size = 0.96

$$a^2x + 2a^2\sinh(x) + 2ac\cosh(x) + a^2\left(\frac{\cosh(x)\sinh(x)}{2} + \frac{x}{2}\right) + ac\left(\cosh^2(x)\right) + c^2\left(\frac{\cosh(x)\sinh(x)}{2} - \frac{x}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cosh(x)+c*sinh(x))^2,x)
```

```
[Out] a^2*x+2*a^2*sinh(x)+2*a*c*cosh(x)+a^2*(1/2*cosh(x)*sinh(x)+1/2*x)+a*c*cosh(
x)^2+c^2*(1/2*cosh(x)*sinh(x)-1/2*x)
```

maxima [A] time = 0.41, size = 63, normalized size = 1.11

$$ac\cosh(x)^2+\frac{1}{8}\,a^2\left(4\,x+e^{(2\,x)}-e^{(-2\,x)}\right)-\frac{1}{8}\,c^2\left(4\,x-e^{(2\,x)}+e^{(-2\,x)}\right)+a^2x+2\,(c\cosh(x)+a\sinh(x))a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cosh(x)+c*sinh(x))^2,x, algorithm="maxima")
```

```
[Out] a*c*cosh(x)^2 + 1/8*a^2*(4*x + e^(2*x) - e^(-2*x)) - 1/8*c^2*(4*x - e^(2*x)
 + e^(-2*x)) + a^2*x + 2*(c*cosh(x) + a*sinh(x))*a
```

mupad [B] time = 1.53, size = 51, normalized size = 0.89

$$2\,a^2\sinh(x)+\frac{3\,a^2\,x}{2}-\frac{c^2\,x}{2}+a\,c\cosh(x)^2+\frac{a^2\cosh(x)\sinh(x)}{2}+\frac{c^2\cosh(x)\sinh(x)}{2}+2\,a\,c\cosh(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*cosh(x) + c*sinh(x))^2,x)
```

```
[Out] 2*a^2*sinh(x) + (3*a^2*x)/2 - (c^2*x)/2 + a*c*cosh(x)^2 + (a^2*cosh(x)*sinh
(x))/2 + (c^2*cosh(x)*sinh(x))/2 + 2*a*c*cosh(x)
```

sympy [A] time = 0.22, size = 100, normalized size = 1.75

$$-\frac{a^2x\sinh^2(x)}{2}+\frac{a^2x\cosh^2(x)}{2}+a^2x+\frac{a^2\sinh(x)\cosh(x)}{2}+2a^2\sinh(x)+ac\cosh^2(x)+2ac\cosh(x)+\frac{c^2x\sinh^2(x}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cosh(x)+c*sinh(x))**2,x)
```

```
[Out] -a**2*x*sinh(x)**2/2 + a**2*x*cosh(x)**2/2 + a**2*x + a**2*sinh(x)*cosh(x)/
2 + 2*a**2*sinh(x) + a*c*cosh(x)**2 + 2*a*c*cosh(x) + c**2*x*sinh(x)**2/2 -
 c**2*x*cosh(x)**2/2 + c**2*sinh(x)*cosh(x)/2
```

3.748 $\int (a + a\cosh(x) + c\sinh(x))\,dx$

Optimal. Leaf size=12

$$ax + a\sinh(x) + c\cosh(x)$$

[Out] a*x+c*cosh(x)+a*sinh(x)

Rubi [A] time = 0.01, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} =$ 0.200, Rules used = {2637, 2638}

$$ax + a\sinh(x) + c\cosh(x)$$

Antiderivative was successfully verified.

[In] Int[a + a*Cosh[x] + c*Sinh[x],x]

[Out] a*x + c*Cosh[x] + a*Sinh[x]

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int (a + a\cosh(x) + c\sinh(x))\,dx &= ax + a\int \cosh(x)\,dx + c\int \sinh(x)\,dx\\ &= ax + c\cosh(x) + a\sinh(x)\end{aligned}$$

Mathematica [A] time = 0.00, size = 12, normalized size = 1.00

$$ax + a\sinh(x) + c\cosh(x)$$

Antiderivative was successfully verified.

[In] Integrate[a + a*Cosh[x] + c*Sinh[x],x]

[Out] a*x + c*Cosh[x] + a*Sinh[x]

fricas [A] time = 0.44, size = 12, normalized size = 1.00

$$ax + c\cosh(x) + a\sinh(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+a*cosh(x)+c*sinh(x),x, algorithm="fricas")
[Out] a*x + c*cosh(x) + a*sinh(x)
```

giac [B] time = 0.12, size = 26, normalized size = 2.17

$$ax + \frac{1}{2}c\left(e^{(-x)} + e^{x}\right) - \frac{1}{2}a\left(e^{(-x)} - e^{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+a*cosh(x)+c*sinh(x),x, algorithm="giac")
[Out] a*x + 1/2*c*(e^(-x) + e^x) - 1/2*a*(e^(-x) - e^x)
```

maple [A] time = 0.02, size = 13, normalized size = 1.08

$$ax + c\cosh(x) + a\sinh(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a+a*cosh(x)+c*sinh(x),x)
[Out] a*x+c*cosh(x)+a*sinh(x)
```

maxima [A] time = 0.31, size = 12, normalized size = 1.00

$$ax + c\cosh(x) + a\sinh(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+a*cosh(x)+c*sinh(x),x, algorithm="maxima")
[Out] a*x + c*cosh(x) + a*sinh(x)
```

mupad [B] time = 1.49, size = 12, normalized size = 1.00

$$a\,x + c\cosh(x) + a\sinh(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a + a*cosh(x) + c*sinh(x),x)
```

```
[Out] a*x + c*cosh(x) + a*sinh(x)
```

sympy [A] time = 0.11, size = 12, normalized size = 1.00

$$ax + a\sinh(x) + c\cosh(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+a*cosh(x)+c*sinh(x),x)
```

```
[Out] a*x + a*sinh(x) + c*cosh(x)
```

3.749 $\int \frac{1}{a+a\cosh(x)+c\sinh(x)}\,dx$

Optimal. Leaf size=15

$$\frac{\log\left(a+c\tanh\left(\frac{x}{2}\right)\right)}{c}$$

```
[Out] ln(a+c*tanh(1/2*x))/c
```

Rubi [A] time = 0.02, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3124, 31}

$$\frac{\log\left(a+c\tanh\left(\frac{x}{2}\right)\right)}{c}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cosh[x] + c*Sinh[x])^(-1),x]
```

```
[Out] Log[a + c*Tanh[x/2]]/c
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 3124

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Module[{f = FreeFactors[Tan[(d + e*x)/2], x]}, Dist[(2*f
)/e, Subst[Int[1/(a + b + 2*c*f*x + (a - b)*f^2*x^2), x], x, Tan[(d + e*x)/
2]/f], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
```

Rubi steps

$$\begin{aligned} \int \frac{1}{a+a\cosh(x)+c\sinh(x)}\,dx &= 2\,\text{Subst}\left(\int \frac{1}{2a+2cx}\,dx, x, \tanh\left(\frac{x}{2}\right)\right)\\ &= \frac{\log\left(a+c\tanh\left(\frac{x}{2}\right)\right)}{c} \end{aligned}$$

Mathematica [B] time = 0.04, size = 35, normalized size = 2.33

$$\frac{\log\left(a\cosh\left(\frac{x}{2}\right)+c\sinh\left(\frac{x}{2}\right)\right)}{c}-\frac{\log\left(\cosh\left(\frac{x}{2}\right)\right)}{c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cosh[x] + c*Sinh[x])^(-1),x]
```

```
[Out] -(Log[Cosh[x/2]]/c) + Log[a*Cosh[x/2] + c*Sinh[x/2]]/c
```

fricas [B] time = 0.40, size = 32, normalized size = 2.13

$$\frac{\log\left((a+c)\cosh(x)+(a+c)\sinh(x)+a-c\right)-\log\left(\cosh(x)+\sinh(x)+1\right)}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cosh(x)+c*sinh(x)),x, algorithm="fricas")
```

```
[Out] (log((a + c)*cosh(x) + (a + c)*sinh(x) + a - c) - log(cosh(x) + sinh(x) + 1
))/c
```

giac [B] time = 0.13, size = 39, normalized size = 2.60

$$\frac{(a+c)\log\left(|ae^{x}+ce^{x}+a-c|\right)}{ac+c^{2}}-\frac{\log\left(e^{x}+1\right)}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cosh(x)+c*sinh(x)),x, algorithm="giac")
```

```
[Out] (a + c)*log(abs(a*e^x + c*e^x + a - c))/(a*c + c^2) - log(e^x + 1)/c
```

maple [A] time = 0.18, size = 14, normalized size = 0.93

$$\frac{\ln\left(a+c\tanh\left(\frac{x}{2}\right)\right)}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*cosh(x)+c*sinh(x)),x)
```

```
[Out] ln(a+c*tanh(1/2*x))/c
```

maxima [B] time = 0.31, size = 36, normalized size = 2.40

$$\frac{\log\left(-(a-c)e^{(-x)}-a-c\right)}{c}-\frac{\log\left(e^{(-x)}+1\right)}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cosh(x)+c*sinh(x)),x, algorithm="maxima")
```

```
[Out] log(-(a - c)*e^(-x) - a - c)/c - log(e^(-x) + 1)/c
```

mupad [B] time = 0.16, size = 46, normalized size = 3.07

$$-\frac{2\,\mathrm{atan}\left(\frac{a\,\sqrt{-c^2}+a\,\mathrm{e}^x\,\sqrt{-c^2}+c\,\mathrm{e}^x\,\sqrt{-c^2}}{c^2}\right)}{\sqrt{-c^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + a*cosh(x) + c*sinh(x)),x)
```

```
[Out] -(2*atan((a*(-c^2)^(1/2) + a*exp(x)*(-c^2)^(1/2) + c*exp(x)*(-c^2)^(1/2))/c
^2))/(-c^2)^(1/2)
```

sympy [A] time = 0.72, size = 17, normalized size = 1.13

$$\begin{cases} \frac{\log\left(\frac{a}{c}+\tanh\left(\frac{x}{2}\right)\right)}{c} & \text{for}\ c \neq 0 \\ \frac{\tanh\left(\frac{x}{2}\right)}{a} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cosh(x)+c*sinh(x)),x)
```

```
[Out] Piecewise((log(a/c + tanh(x/2))/c, Ne(c, 0)), (tanh(x/2)/a, True))
```

3.750 $\int \frac{1}{(a+a\cosh(x)+c\sinh(x))^2}\,dx$

Optimal. Leaf size=43

$$\frac{a\log\left(a+c\tanh\left(\frac{x}{2}\right)\right)}{c^3}-\frac{a\sinh(x)+c\cosh(x)}{c^2(a\cosh(x)+a+c\sinh(x))}$$

```
[Out] a*ln(a+c*tanh(1/2*x))/c^3+(-c*cosh(x)-a*sinh(x))/c^2/(a+a*cosh(x)+c*sinh(x)
)
```

Rubi [A] time = 0.04, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3129, 12, 3124, 31}

$$\frac{a\log\left(a+c\tanh\left(\frac{x}{2}\right)\right)}{c^3}-\frac{a\sinh(x)+c\cosh(x)}{c^2(a\cosh(x)+a+c\sinh(x))}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cosh[x] + c*Sinh[x])^(-2),x]
```

```
[Out] (a*Log[a + c*Tanh[x/2]])/c^3 - (c*Cosh[x] + a*Sinh[x])/(c^2*(a + a*Cosh[x]
+ c*Sinh[x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 3124

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Module[{f = FreeFactors[Tan[(d + e*x)/2], x]}, Dist[(2*f
)/e, Subst[Int[1/(a + b + 2*c*f*x + (a - b)*f^2*x^2), x], x, Tan[(d + e*x)/
2]/f], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
```

Rule 3129

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> Simp[((-(c*Cos[d + e*x]) + b*Sin[d + e*x])*(a + b*Cos[d
```

```
+ e*x] + c*Sin[d + e*x])^(n + 1))/(e*(n + 1)*(a^2 - b^2 - c^2)), x] + Dist[
1/((n + 1)*(a^2 - b^2 - c^2)), Int[(a*(n + 1) - b*(n + 2)*Cos[d + e*x] - c*
(n + 2)*Sin[d + e*x])*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1), x], x]
 /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && LtQ[n, -1] && N
eQ[n, -3/2]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(a+a\cosh(x)+c\sinh(x))^2}\,dx &= -\frac{c\cosh(x)+a\sinh(x)}{c^2(a+a\cosh(x)+c\sinh(x))}+\frac{\int \frac{a}{a+a\cosh(x)+c\sinh(x)}\,dx}{c^2}\\ &= -\frac{c\cosh(x)+a\sinh(x)}{c^2(a+a\cosh(x)+c\sinh(x))}+\frac{a\int \frac{1}{a+a\cosh(x)+c\sinh(x)}\,dx}{c^2}\\ &= -\frac{c\cosh(x)+a\sinh(x)}{c^2(a+a\cosh(x)+c\sinh(x))}+\frac{(2a)\,\mathrm{Subst}\left(\int \frac{1}{2a+2cx}\,dx,x,\tanh\left(\frac{x}{2}\right)\right)}{c^2}\\ &= \frac{a\log\left(a+c\tanh\left(\frac{x}{2}\right)\right)}{c^3}-\frac{c\cosh(x)+a\sinh(x)}{c^2(a+a\cosh(x)+c\sinh(x))}\end{aligned}$$

Mathematica [B] time = 0.32, size = 87, normalized size = 2.02

$$\frac{\frac{c(c^2-a^2)\sinh\left(\frac{x}{2}\right)}{a\left(a\cosh\left(\frac{x}{2}\right)+c\sinh\left(\frac{x}{2}\right)\right)}+2a\left(\log\left(a\cosh\left(\frac{x}{2}\right)+c\sinh\left(\frac{x}{2}\right)\right)-\log\left(\cosh\left(\frac{x}{2}\right)\right)\right)-c\tanh\left(\frac{x}{2}\right)}{2c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cosh[x] + c*Sinh[x])^(-2),x]
```

```
[Out] (2*a*(-Log[Cosh[x/2]] + Log[a*Cosh[x/2] + c*Sinh[x/2]]) + (c*(-a^2 + c^2)*S
inh[x/2])/(a*(a*Cosh[x/2] + c*Sinh[x/2])) - c*Tanh[x/2])/(2*c^3)
```

fricas [B] time = 0.44, size = 236, normalized size = 5.49

$$\frac{2\,ac\cosh(x)+2\,ac\sinh(x)+2\,ac-2\,c^2+\left(2\,a^2\cosh(x)+\left(a^2+ac\right)\cosh(x)^2+\left(a^2+ac\right)\sinh(x)^2+a^2-ac+2\right.}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cosh(x)+c*sinh(x))^2,x, algorithm="fricas")
```

```
[Out] (2*a*c*cosh(x) + 2*a*c*sinh(x) + 2*a*c - 2*c^2 + (2*a^2*cosh(x) + (a^2 + a*
c)*cosh(x)^2 + (a^2 + a*c)*sinh(x)^2 + a^2 - a*c + 2*(a^2 + (a^2 + a*c)*cos
h(x))*sinh(x))*log((a + c)*cosh(x) + (a + c)*sinh(x) + a - c) - (2*a^2*cosh
(x) + (a^2 + a*c)*cosh(x)^2 + (a^2 + a*c)*sinh(x)^2 + a^2 - a*c + 2*(a^2 +
(a^2 + a*c)*cosh(x))*sinh(x))*log(cosh(x) + sinh(x) + 1))/(2*a*c^3*cosh(x)
+ a*c^3 - c^4 + (a*c^3 + c^4)*cosh(x)^2 + (a*c^3 + c^4)*sinh(x)^2 + 2*(a*c^
3 + (a*c^3 + c^4)*cosh(x))*sinh(x))
```

giac [B] time = 0.14, size = 84, normalized size = 1.95

$$\frac{\left(a^2+ac\right)\log\left(|ae^x+ce^x+a-c|\right)}{ac^3+c^4}-\frac{a\log\left(e^x+1\right)}{c^3}+\frac{2\left(ae^x+a-c\right)}{\left(ae^{(2x)}+ce^{(2x)}+2\,ae^x+a-c\right)c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cosh(x)+c*sinh(x))^2,x, algorithm="giac")
```

```
[Out] (a^2 + a*c)*log(abs(a*e^x + c*e^x + a - c))/(a*c^3 + c^4) - a*log(e^x + 1)/
c^3 + 2*(a*e^x + a - c)/((a*e^(2*x) + c*e^(2*x) + 2*a*e^x + a - c)*c^2)
```

maple [A] time = 0.25, size = 58, normalized size = 1.35

$$-\frac{\tanh\left(\frac{x}{2}\right)}{2c^2}+\frac{a^2}{2c^3\left(a+c\tanh\left(\frac{x}{2}\right)\right)}-\frac{1}{2c\left(a+c\tanh\left(\frac{x}{2}\right)\right)}+\frac{a\ln\left(a+c\tanh\left(\frac{x}{2}\right)\right)}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*cosh(x)+c*sinh(x))^2,x)
```

```
[Out] -1/2/c^2*tanh(1/2*x)+1/2/c^3/(a+c*tanh(1/2*x))*a^2-1/2/c/(a+c*tanh(1/2*x))+
a*ln(a+c*tanh(1/2*x))/c^3
```

maxima [B] time = 0.53, size = 86, normalized size = 2.00

$$-\frac{2\left(ae^{(-x)}+a+c\right)}{2\,ac^2e^{(-x)}+ac^2+c^3+\left(ac^2-c^3\right)e^{(-2x)}}+\frac{a\log\left(-(a-c)e^{(-x)}-a-c\right)}{c^3}-\frac{a\log\left(e^{(-x)}+1\right)}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cosh(x)+c*sinh(x))^2,x, algorithm="maxima")
```

```
[Out] -2*(a*e^(-x) + a + c)/(2*a*c^2*e^(-x) + a*c^2 + c^3 + (a*c^2 - c^3)*e^(-2*x
)) + a*log(-(a - c)*e^(-x) - a - c)/c^3 - a*log(e^(-x) + 1)/c^3
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{(a + a\cosh(x) + c\sinh(x))^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + a*cosh(x) + c*sinh(x))^2,x)
[Out] int(1/(a + a*cosh(x) + c*sinh(x))^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cosh(x)+c*sinh(x))**2,x)
[Out] Timed out
```

3.751 $\int \frac{1}{(a+a\cosh(x)+c\sinh(x))^3}\,dx$

Optimal. Leaf size=89

$$-\frac{3\left(a^2\sinh(x)+ac\cosh(x)\right)}{2c^4(a\cosh(x)+a+c\sinh(x))}+\frac{\left(3a^2-c^2\right)\log\left(a+c\tanh\left(\frac{x}{2}\right)\right)}{2c^5}-\frac{a\sinh(x)+c\cosh(x)}{2c^2(a\cosh(x)+a+c\sinh(x))^2}$$

```
[Out] 1/2*(3*a^2-c^2)*ln(a+c*tanh(1/2*x))/c^5+1/2*(-c*cosh(x)-a*sinh(x))/c^2/(a+a
*cosh(x)+c*sinh(x))^2-3/2*(a*c*cosh(x)+a^2*sinh(x))/c^4/(a+a*cosh(x)+c*sinh
(x))
```

Rubi [A] time = 0.09, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3129, 3153, 3124, 31}

$$\frac{\left(3a^2-c^2\right)\log\left(a+c\tanh\left(\frac{x}{2}\right)\right)}{2c^5}-\frac{3\left(a^2\sinh(x)+ac\cosh(x)\right)}{2c^4(a\cosh(x)+a+c\sinh(x))}-\frac{a\sinh(x)+c\cosh(x)}{2c^2(a\cosh(x)+a+c\sinh(x))^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cosh[x] + c*Sinh[x])^(-3),x]
```

```
[Out] ((3*a^2 - c^2)*Log[a + c*Tanh[x/2]])/(2*c^5) - (c*Cosh[x] + a*Sinh[x])/(2*c
^2*(a + a*Cosh[x] + c*Sinh[x])^2) - (3*(a*c*Cosh[x] + a^2*Sinh[x]))/(2*c^4*
(a + a*Cosh[x] + c*Sinh[x]))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 3124

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Module[{f = FreeFactors[Tan[(d + e*x)/2], x]}, Dist[(2*f
)/e, Subst[Int[1/(a + b + 2*c*f*x + (a - b)*f^2*x^2), x], x, Tan[(d + e*x)/
2]/f], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
```

Rule 3129

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> Simp[((-(c*Cos[d + e*x]) + b*Sin[d + e*x])*(a + b*Cos[d
+ e*x] + c*Sin[d + e*x])^(n + 1))/(e*(n + 1)*(a^2 - b^2 - c^2)), x] + Dist[
1/((n + 1)*(a^2 - b^2 - c^2)), Int[(a*(n + 1) - b*(n + 2)*Cos[d + e*x] - c*
(n + 2)*Sin[d + e*x])*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1), x], x]
```

```
/; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && LtQ[n, -1] && N
eQ[n, -3/2]
```

Rule 3153

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])^2,
 x_Symbol] :> Simp[(c*B - b*C - (a*C - c*A)*Cos[d + e*x] + (a*B - b*A)*Sin[
d + e*x])/(e*(a^2 - b^2 - c^2)*(a + b*Cos[d + e*x] + c*Sin[d + e*x])), x] +
 Dist[(a*A - b*B - c*C)/(a^2 - b^2 - c^2), Int[1/(a + b*Cos[d + e*x] + c*Si
n[d + e*x]), x], x] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && NeQ[a^2 - b^2
- c^2, 0] && NeQ[a*A - b*B - c*C, 0]
```

Rubi steps

$$\int \frac{1}{(a+a\cosh(x)+c\sinh(x))^3}\,dx = -\frac{c\cosh(x)+a\sinh(x)}{2c^2(a+a\cosh(x)+c\sinh(x))^2} - \frac{\int \frac{-2a+a\cosh(x)+c\sinh(x)}{(a+a\cosh(x)+c\sinh(x))^2}\,dx}{2c^2}$$

$$= -\frac{c\cosh(x)+a\sinh(x)}{2c^2(a+a\cosh(x)+c\sinh(x))^2} - \frac{3\left(ac\cosh(x)+a^2\sinh(x)\right)}{2c^4(a+a\cosh(x)+c\sinh(x))} + \frac{\left(3a^2\right.}{}$$

$$= -\frac{c\cosh(x)+a\sinh(x)}{2c^2(a+a\cosh(x)+c\sinh(x))^2} - \frac{3\left(ac\cosh(x)+a^2\sinh(x)\right)}{2c^4(a+a\cosh(x)+c\sinh(x))} + \frac{\left(3a^2\right.}{}$$

$$= \frac{\left(3a^2-c^2\right)\log\left(a+c\tanh\left(\frac{x}{2}\right)\right)}{2c^5} - \frac{c\cosh(x)+a\sinh(x)}{2c^2(a+a\cosh(x)+c\sinh(x))^2} - \frac{3\left(ac\right.}{2c^4(a}$$

Mathematica [A] time = 0.55, size = 148, normalized size = 1.66

$$\frac{4\left(c^2-3a^2\right)\log\left(\cosh\left(\frac{x}{2}\right)\right)+\frac{6c\left(c^2-a^2\right)\sinh\left(\frac{x}{2}\right)}{a\cosh\left(\frac{x}{2}\right)+c\sinh\left(\frac{x}{2}\right)}+4\left(3a^2-c^2\right)\log\left(a\cosh\left(\frac{x}{2}\right)+c\sinh\left(\frac{x}{2}\right)\right)+\frac{c^2(a-c)(a+c)}{\left(a\cosh\left(\frac{x}{2}\right)+c\sinh\left(\frac{x}{2}\right)\right)^2}}{8c^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cosh[x] + c*Sinh[x])^(-3),x]
```

```
[Out] (4*(-3*a^2 + c^2)*Log[Cosh[x/2]] + 4*(3*a^2 - c^2)*Log[a*Cosh[x/2] + c*Sinh
[x/2]] - c^2*Sech[x/2]^2 + ((a - c)*c^2*(a + c))/(a*Cosh[x/2] + c*Sinh[x/2]
)^2 + (6*c*(-a^2 + c^2)*Sinh[x/2])/(a*Cosh[x/2] + c*Sinh[x/2]) - 6*a*c*Tanh
[x/2])/(8*c^5)
```

fricas [B] time = 0.45, size = 1504, normalized size = 16.90

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cosh(x)+c*sinh(x))^3,x, algorithm="fricas")
```

```
[Out] 1/2*(6*a^3*c - 12*a^2*c^2 + 6*a*c^3 + 2*(3*a^3*c + 3*a^2*c^2 - a*c^3 - c^4)
*cosh(x)^3 + 2*(3*a^3*c + 3*a^2*c^2 - a*c^3 - c^4)*sinh(x)^3 + 6*(3*a^3*c -
 a*c^3)*cosh(x)^2 + 6*(3*a^3*c - a*c^3 + (3*a^3*c + 3*a^2*c^2 - a*c^3 - c^4
)*cosh(x))*sinh(x)^2 + 2*(9*a^3*c - 9*a^2*c^2 + a*c^3 - c^4)*cosh(x) + ((3*
a^4 + 6*a^3*c + 2*a^2*c^2 - 2*a*c^3 - c^4)*cosh(x)^4 + (3*a^4 + 6*a^3*c + 2
*a^2*c^2 - 2*a*c^3 - c^4)*sinh(x)^4 + 3*a^4 - 6*a^3*c + 2*a^2*c^2 + 2*a*c^3
 - c^4 + 4*(3*a^4 + 3*a^3*c - a^2*c^2 - a*c^3)*cosh(x)^3 + 4*(3*a^4 + 3*a^3
*c - a^2*c^2 - a*c^3 + (3*a^4 + 6*a^3*c + 2*a^2*c^2 - 2*a*c^3 - c^4)*cosh(x
))*sinh(x)^3 + 2*(9*a^4 - 6*a^2*c^2 + c^4)*cosh(x)^2 + 2*(9*a^4 - 6*a^2*c^2
 + c^4 + 3*(3*a^4 + 6*a^3*c + 2*a^2*c^2 - 2*a*c^3 - c^4)*cosh(x)^2 + 6*(3*a
^4 + 3*a^3*c - a^2*c^2 - a*c^3)*cosh(x))*sinh(x)^2 + 4*(3*a^4 - 3*a^3*c - a
^2*c^2 + a*c^3)*cosh(x) + 4*(3*a^4 - 3*a^3*c - a^2*c^2 + a*c^3 + (3*a^4 + 6
*a^3*c + 2*a^2*c^2 - 2*a*c^3 - c^4)*cosh(x)^3 + 3*(3*a^4 + 3*a^3*c - a^2*c^
2 - a*c^3)*cosh(x)^2 + (9*a^4 - 6*a^2*c^2 + c^4)*cosh(x))*sinh(x))*log((a +
 c)*cosh(x) + (a + c)*sinh(x) + a - c) - ((3*a^4 + 6*a^3*c + 2*a^2*c^2 - 2*
a*c^3 - c^4)*cosh(x)^4 + (3*a^4 + 6*a^3*c + 2*a^2*c^2 - 2*a*c^3 - c^4)*sinh
(x)^4 + 3*a^4 - 6*a^3*c + 2*a^2*c^2 + 2*a*c^3 - c^4 + 4*(3*a^4 + 3*a^3*c -
a^2*c^2 - a*c^3)*cosh(x)^3 + 4*(3*a^4 + 3*a^3*c - a^2*c^2 - a*c^3 + (3*a^4
+ 6*a^3*c + 2*a^2*c^2 - 2*a*c^3 - c^4)*cosh(x))*sinh(x)^3 + 2*(9*a^4 - 6*a^
2*c^2 + c^4)*cosh(x)^2 + 2*(9*a^4 - 6*a^2*c^2 + c^4 + 3*(3*a^4 + 6*a^3*c +
2*a^2*c^2 - 2*a*c^3 - c^4)*cosh(x)^2 + 6*(3*a^4 + 3*a^3*c - a^2*c^2 - a*c^3
)*cosh(x))*sinh(x)^2 + 4*(3*a^4 - 3*a^3*c - a^2*c^2 + a*c^3)*cosh(x) + 4*(3
*a^4 - 3*a^3*c - a^2*c^2 + a*c^3 + (3*a^4 + 6*a^3*c + 2*a^2*c^2 - 2*a*c^3 -
 c^4)*cosh(x)^3 + 3*(3*a^4 + 3*a^3*c - a^2*c^2 - a*c^3)*cosh(x)^2 + (9*a^4
- 6*a^2*c^2 + c^4)*cosh(x))*sinh(x))*log(cosh(x) + sinh(x) + 1) + 2*(9*a^3*
c - 9*a^2*c^2 + a*c^3 - c^4 + 3*(3*a^3*c + 3*a^2*c^2 - a*c^3 - c^4)*cosh(x)
^2 + 6*(3*a^3*c - a*c^3)*cosh(x))*sinh(x))/(a^2*c^5 - 2*a*c^6 + c^7 + (a^2*
c^5 + 2*a*c^6 + c^7)*cosh(x)^4 + (a^2*c^5 + 2*a*c^6 + c^7)*sinh(x)^4 + 4*(a
^2*c^5 + a*c^6)*cosh(x)^3 + 4*(a^2*c^5 + a*c^6 + (a^2*c^5 + 2*a*c^6 + c^7)*
cosh(x))*sinh(x)^3 + 2*(3*a^2*c^5 - c^7)*cosh(x)^2 + 2*(3*a^2*c^5 - c^7 + 3
*(a^2*c^5 + 2*a*c^6 + c^7)*cosh(x)^2 + 6*(a^2*c^5 + a*c^6)*cosh(x))*sinh(x)
^2 + 4*(a^2*c^5 - a*c^6)*cosh(x) + 4*(a^2*c^5 - a*c^6 + (a^2*c^5 + 2*a*c^6
+ c^7)*cosh(x)^3 + 3*(a^2*c^5 + a*c^6)*cosh(x)^2 + (3*a^2*c^5 - c^7)*cosh(x
))*sinh(x))
```

giac [B] time = 0.13, size = 205, normalized size = 2.30

$$\frac{\left(3\,a^{3}+3\,a^{2}c-ac^{2}-c^{3}\right)\log\left(|ae^{x}+ce^{x}+a-c|\right)}{2\left(ac^{5}+c^{6}\right)}-\frac{\left(3\,a^{2}-c^{2}\right)\log\left(e^{x}+1\right)}{2\,c^{5}}+\frac{3\,a^{3}e^{(3\,x)}+3\,a^{2}ce^{(3\,x)}-ac^{2}e^{(3\,x)}-c^{3}e^{(3}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cosh(x)+c*sinh(x))^3,x, algorithm="giac")
```

```
[Out] 1/2*(3*a^3 + 3*a^2*c - a*c^2 - c^3)*log(abs(a*e^x + c*e^x + a - c))/(a*c^5
+ c^6) - 1/2*(3*a^2 - c^2)*log(e^x + 1)/c^5 + (3*a^3*e^(3*x) + 3*a^2*c*e^(3
*x) - a*c^2*e^(3*x) - c^3*e^(3*x) + 9*a^3*e^(2*x) - 3*a*c^2*e^(2*x) + 9*a^3
*e^x - 9*a^2*c*e^x + a*c^2*e^x - c^3*e^x + 3*a^3 - 6*a^2*c + 3*a*c^2)/((a*e
^(2*x) + c*e^(2*x) + 2*a*e^x + a - c)^2*c^4)
```

maple [A] time = 0.27, size = 138, normalized size = 1.55

$$\frac{\tanh^{2}\left(\frac{x}{2}\right)}{8c^{3}}-\frac{3a\tanh\left(\frac{x}{2}\right)}{4c^{4}}-\frac{a^{4}}{8c^{5}\left(a+c\tanh\left(\frac{x}{2}\right)\right)^{2}}+\frac{a^{2}}{4c^{3}\left(a+c\tanh\left(\frac{x}{2}\right)\right)^{2}}-\frac{1}{8c\left(a+c\tanh\left(\frac{x}{2}\right)\right)^{2}}+\frac{a^{3}}{c^{5}\left(a+c\tanh\left(\frac{x}{2}\right)\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*cosh(x)+c*sinh(x))^3,x)
```

```
[Out] 1/8/c^3*tanh(1/2*x)^2-3/4/c^4*a*tanh(1/2*x)-1/8/c^5/(a+c*tanh(1/2*x))^2*a^4
+1/4/c^3/(a+c*tanh(1/2*x))^2*a^2-1/8/c/(a+c*tanh(1/2*x))^2+a^3/c^5/(a+c*tan
h(1/2*x))-a/c^3/(a+c*tanh(1/2*x))+3/2/c^5*ln(a+c*tanh(1/2*x))*a^2-1/2/c^3*l
n(a+c*tanh(1/2*x))
```

maxima [B] time = 0.35, size = 248, normalized size = 2.79

$$-\frac{3\,a^{3}+6\,a^{2}c+3\,ac^{2}+\left(9\,a^{3}+9\,a^{2}c+ac^{2}+c^{3}\right)e^{(-x)}+3\left(3\,a^{3}-ac^{2}\right)e^{(-2\,x)}+\left(3\,a^{3}-3\,a^{2}c-ac^{2}+c^{3}\right)e^{(-3\,x)}}{a^{2}c^{4}+2\,ac^{5}+c^{6}+4\left(a^{2}c^{4}+ac^{5}\right)e^{(-x)}+2\left(3\,a^{2}c^{4}-c^{6}\right)e^{(-2\,x)}+4\left(a^{2}c^{4}-ac^{5}\right)e^{(-3\,x)}+\left(a^{2}c^{4}-2\,ac^{5}+c^{6}\right)e^{(-4\,x)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cosh(x)+c*sinh(x))^3,x, algorithm="maxima")
```

```
[Out] -(3*a^3 + 6*a^2*c + 3*a*c^2 + (9*a^3 + 9*a^2*c + a*c^2 + c^3)*e^(-x) + 3*(3
*a^3 - a*c^2)*e^(-2*x) + (3*a^3 - 3*a^2*c - a*c^2 + c^3)*e^(-3*x))/(a^2*c^4
 + 2*a*c^5 + c^6 + 4*(a^2*c^4 + a*c^5)*e^(-x) + 2*(3*a^2*c^4 - c^6)*e^(-2*x
) + 4*(a^2*c^4 - a*c^5)*e^(-3*x) + (a^2*c^4 - 2*a*c^5 + c^6)*e^(-4*x)) + 1/
2*(3*a^2 - c^2)*log(-(a - c)*e^(-x) - a - c)/c^5 - 1/2*(3*a^2 - c^2)*log(e^
(-x) + 1)/c^5
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(a + a\cosh(x) + c\sinh(x)\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + a*cosh(x) + c*sinh(x))^3,x)
```

```
[Out] int(1/(a + a*cosh(x) + c*sinh(x))^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cosh(x)+c*sinh(x))**3,x)
```

```
[Out] Timed out
```

3.752 $\int \frac{1}{(a+a\cosh(x)+c\sinh(x))^4}\,dx$

Optimal. Leaf size=140

$$-\frac{5\left(a^2\sinh(x)+ac\cosh(x)\right)}{6c^4(a\cosh(x)+a+c\sinh(x))^2}+\frac{a\left(5a^2-3c^2\right)\log\left(a+c\tanh\left(\frac{x}{2}\right)\right)}{2c^7}-\frac{a\left(15a^2-4c^2\right)\sinh(x)+c\left(15a^2-4c^2\right)\mathrm{co}}{6c^6(a\cosh(x)+a+c\sinh(x))}$$

```
[Out] 1/2*a*(5*a^2-3*c^2)*ln(a+c*tanh(1/2*x))/c^7+1/3*(-c*cosh(x)-a*sinh(x))/c^2/
(a+a*cosh(x)+c*sinh(x))^3-5/6*(a*c*cosh(x)+a^2*sinh(x))/c^4/(a+a*cosh(x)+c*
sinh(x))^2+1/6*(-c*(15*a^2-4*c^2)*cosh(x)-a*(15*a^2-4*c^2)*sinh(x))/c^6/(a+
a*cosh(x)+c*sinh(x))
```

Rubi [A] time = 0.21, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {3129, 3156, 3153, 3124, 31}

$$\frac{a\left(5a^2-3c^2\right)\log\left(a+c\tanh\left(\frac{x}{2}\right)\right)}{2c^7}-\frac{a\left(15a^2-4c^2\right)\sinh(x)+c\left(15a^2-4c^2\right)\cosh(x)}{6c^6(a\cosh(x)+a+c\sinh(x))}-\frac{5\left(a^2\sinh(x)+ac\cosh(\right.}{6c^4(a\cosh(x)+a+c\,\mathrm{sinh}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cosh[x] + c*Sinh[x])^(-4),x]
```

```
[Out] (a*(5*a^2 - 3*c^2)*Log[a + c*Tanh[x/2]])/(2*c^7) - (c*Cosh[x] + a*Sinh[x])/
(3*c^2*(a + a*Cosh[x] + c*Sinh[x])^3) - (5*(a*c*Cosh[x] + a^2*Sinh[x]))/(6*
c^4*(a + a*Cosh[x] + c*Sinh[x])^2) - (c*(15*a^2 - 4*c^2)*Cosh[x] + a*(15*a^
2 - 4*c^2)*Sinh[x])/(6*c^6*(a + a*Cosh[x] + c*Sinh[x]))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 3124

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Module[{f = FreeFactors[Tan[(d + e*x)/2], x]}, Dist[(2*f
)/e, Subst[Int[1/(a + b + 2*c*f*x + (a - b)*f^2*x^2), x], x, Tan[(d + e*x)/
2]/f], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
```

Rule 3129

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> Simp[((-(c*Cos[d + e*x]) + b*Sin[d + e*x])*(a + b*Cos[d
+ e*x] + c*Sin[d + e*x])^(n + 1))/(e*(n + 1)*(a^2 - b^2 - c^2)), x] + Dist[
```

```
1/((n + 1)*(a^2 - b^2 - c^2)), Int[(a*(n + 1) - b*(n + 2)*Cos[d + e*x] - c*
(n + 2)*Sin[d + e*x])*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1), x], x]
 /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && LtQ[n, -1] && N
eQ[n, -3/2]
```

Rule 3153

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])^2,
 x_Symbol] :> Simp[(c*B - b*C - (a*C - c*A)*Cos[d + e*x] + (a*B - b*A)*Sin[
d + e*x])/(e*(a^2 - b^2 - c^2)*(a + b*Cos[d + e*x] + c*Sin[d + e*x])), x] +
 Dist[(a*A - b*B - c*C)/(a^2 - b^2 - c^2), Int[1/(a + b*Cos[d + e*x] + c*Si
n[d + e*x]), x], x] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && NeQ[a^2 - b^2
- c^2, 0] && NeQ[a*A - b*B - c*C, 0]
```

Rule 3156

```
Int[((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])
^(n_)*((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)
]), x_Symbol] :> -Simp[((c*B - b*C - (a*C - c*A)*Cos[d + e*x] + (a*B - b*A)
*Sin[d + e*x])*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1))/(e*(n + 1)*(a
^2 - b^2 - c^2)), x] + Dist[1/((n + 1)*(a^2 - b^2 - c^2)), Int[(a + b*Cos[d
 + e*x] + c*Sin[d + e*x])^(n + 1)*Simp[(n + 1)*(a*A - b*B - c*C) + (n + 2)*
(a*B - b*A)*Cos[d + e*x] + (n + 2)*(a*C - c*A)*Sin[d + e*x], x], x], x] /;
FreeQ[{a, b, c, d, e, A, B, C}, x] && LtQ[n, -1] && NeQ[a^2 - b^2 - c^2, 0]
 && NeQ[n, -2]
```

Rubi steps

$$\int \frac{1}{(a+a\cosh(x)+c\sinh(x))^4}\,dx = -\frac{c\cosh(x)+a\sinh(x)}{3c^2(a+a\cosh(x)+c\sinh(x))^3} - \frac{\int \frac{-3a+2a\cosh(x)+2c\sinh(x)}{(a+a\cosh(x)+c\sinh(x))^3}\,dx}{3c^2}$$

$$= -\frac{c\cosh(x)+a\sinh(x)}{3c^2(a+a\cosh(x)+c\sinh(x))^3} - \frac{5\left(ac\cosh(x)+a^2\sinh(x)\right)}{6c^4(a+a\cosh(x)+c\sinh(x))^2} + \frac{\int \frac{2}{-}}{}$$

$$= -\frac{c\cosh(x)+a\sinh(x)}{3c^2(a+a\cosh(x)+c\sinh(x))^3} - \frac{5\left(ac\cosh(x)+a^2\sinh(x)\right)}{6c^4(a+a\cosh(x)+c\sinh(x))^2} - \frac{c\left(1\right.}{}$$

$$= -\frac{c\cosh(x)+a\sinh(x)}{3c^2(a+a\cosh(x)+c\sinh(x))^3} - \frac{5\left(ac\cosh(x)+a^2\sinh(x)\right)}{6c^4(a+a\cosh(x)+c\sinh(x))^2} - \frac{c\left(1\right.}{}$$

$$= -\frac{a\left(3-\frac{5a^2}{c^2}\right)\log\left(a+c\tanh\left(\frac{x}{2}\right)\right)}{2c^5} - \frac{c\cosh(x)+a\sinh(x)}{3c^2(a+a\cosh(x)+c\sinh(x))^3} - \frac{5}{6c^4(}$$

Mathematica [B] time = 0.63, size = 300, normalized size = 2.14

$$\frac{192\left(3ac^2-5a^3\right)\log\left(\cosh\left(\frac{x}{2}\right)\right)+192a\left(5a^2-3c^2\right)\log\left(a\cosh\left(\frac{x}{2}\right)+c\sinh\left(\frac{x}{2}\right)\right)-\frac{c\operatorname{sech}^6\left(\frac{x}{2}\right)\left(150a^6\sinh(x)+120a^6\sinh\right.}{}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cosh[x] + c*Sinh[x])^(-4),x]
```

```
[Out] (192*(-5*a^3 + 3*a*c^2)*Log[Cosh[x/2]] + 192*a*(5*a^2 - 3*c^2)*Log[a*Cosh[x
/2] + c*Sinh[x/2]] - (c*Sech[x/2]^6*(-150*a^5*c + 130*a^3*c^3 - 24*a*c^5 +
(-75*a^5*c + 75*a^3*c^3 + 12*a*c^5)*Cosh[x] + 6*a*c*(25*a^4 - 15*a^2*c^2 +
4*c^4)*Cosh[2*x] + 75*a^5*c*Cosh[3*x] - 35*a^3*c^3*Cosh[3*x] + 4*a*c^5*Cosh
[3*x] + 150*a^6*Sinh[x] - 255*a^4*c^2*Sinh[x] + 129*a^2*c^4*Sinh[x] - 12*c^
6*Sinh[x] + 120*a^6*Sinh[2*x] - 72*a^4*c^2*Sinh[2*x] + 36*a^2*c^4*Sinh[2*x]
 + 30*a^6*Sinh[3*x] + 37*a^4*c^2*Sinh[3*x] - 27*a^2*c^4*Sinh[3*x] + 4*c^6*S
inh[3*x]))/(a*(a + c*Tanh[x/2])^3))/(384*c^7)
```

fricas [B] time = 0.52, size = 4015, normalized size = 28.68

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cosh(x)+c*sinh(x))^4,x, algorithm="fricas")
```

```
[Out] 1/6*(30*a^5*c - 90*a^4*c^2 + 82*a^3*c^3 - 6*a^2*c^4 - 24*a*c^5 + 8*c^6 + 6*
(5*a^5*c + 10*a^4*c^2 + 2*a^3*c^3 - 6*a^2*c^4 - 3*a*c^5)*cosh(x)^5 + 6*(5*a
^5*c + 10*a^4*c^2 + 2*a^3*c^3 - 6*a^2*c^4 - 3*a*c^5)*sinh(x)^5 + 30*(5*a^5*
c + 5*a^4*c^2 - 3*a^3*c^3 - 3*a^2*c^4)*cosh(x)^4 + 30*(5*a^5*c + 5*a^4*c^2
- 3*a^3*c^3 - 3*a^2*c^4 + (5*a^5*c + 10*a^4*c^2 + 2*a^3*c^3 - 6*a^2*c^4 - 3
*a*c^5)*cosh(x))*sinh(x)^4 + 4*(75*a^5*c - 65*a^3*c^3 + 12*a*c^5)*cosh(x)^3
 + 4*(75*a^5*c - 65*a^3*c^3 + 12*a*c^5 + 15*(5*a^5*c + 10*a^4*c^2 + 2*a^3*c
^3 - 6*a^2*c^4 - 3*a*c^5)*cosh(x)^2 + 30*(5*a^5*c + 5*a^4*c^2 - 3*a^3*c^3 -
 3*a^2*c^4)*cosh(x))*sinh(x)^3 + 12*(25*a^5*c - 25*a^4*c^2 - 10*a^3*c^3 + 1
0*a^2*c^4 + 2*a*c^5 - 2*c^6)*cosh(x)^2 + 12*(25*a^5*c - 25*a^4*c^2 - 10*a^3
*c^3 + 10*a^2*c^4 + 2*a*c^5 - 2*c^6 + 5*(5*a^5*c + 10*a^4*c^2 + 2*a^3*c^3 -
 6*a^2*c^4 - 3*a*c^5)*cosh(x)^3 + 15*(5*a^5*c + 5*a^4*c^2 - 3*a^3*c^3 - 3*a
^2*c^4)*cosh(x)^2 + (75*a^5*c - 65*a^3*c^3 + 12*a*c^5)*cosh(x))*sinh(x)^2 +
 30*(5*a^5*c - 10*a^4*c^2 + 4*a^3*c^3 + 2*a^2*c^4 - a*c^5)*cosh(x) + 3*((5*
a^6 + 15*a^5*c + 12*a^4*c^2 - 4*a^3*c^3 - 9*a^2*c^4 - 3*a*c^5)*cosh(x)^6 +
(5*a^6 + 15*a^5*c + 12*a^4*c^2 - 4*a^3*c^3 - 9*a^2*c^4 - 3*a*c^5)*sinh(x)^6
 + 5*a^6 - 15*a^5*c + 12*a^4*c^2 + 4*a^3*c^3 - 9*a^2*c^4 + 3*a*c^5 + 6*(5*a
^6 + 10*a^5*c + 2*a^4*c^2 - 6*a^3*c^3 - 3*a^2*c^4)*cosh(x)^5 + 6*(5*a^6 + 1
0*a^5*c + 2*a^4*c^2 - 6*a^3*c^3 - 3*a^2*c^4 + (5*a^6 + 15*a^5*c + 12*a^4*c^
2 - 4*a^3*c^3 - 9*a^2*c^4 - 3*a*c^5)*cosh(x))*sinh(x)^5 + 3*(25*a^6 + 25*a^
5*c - 20*a^4*c^2 - 20*a^3*c^3 + 3*a^2*c^4 + 3*a*c^5)*cosh(x)^4 + 3*(25*a^6
+ 25*a^5*c - 20*a^4*c^2 - 20*a^3*c^3 + 3*a^2*c^4 + 3*a*c^5 + 5*(5*a^6 + 15*
a^5*c + 12*a^4*c^2 - 4*a^3*c^3 - 9*a^2*c^4 - 3*a*c^5)*cosh(x)^2 + 10*(5*a^6
 + 10*a^5*c + 2*a^4*c^2 - 6*a^3*c^3 - 3*a^2*c^4)*cosh(x))*sinh(x)^4 + 4*(25
*a^6 - 30*a^4*c^2 + 9*a^2*c^4)*cosh(x)^3 + 4*(25*a^6 - 30*a^4*c^2 + 9*a^2*c
^4 + 5*(5*a^6 + 15*a^5*c + 12*a^4*c^2 - 4*a^3*c^3 - 9*a^2*c^4 - 3*a*c^5)*co
sh(x)^3 + 15*(5*a^6 + 10*a^5*c + 2*a^4*c^2 - 6*a^3*c^3 - 3*a^2*c^4)*cosh(x)
^2 + 3*(25*a^6 + 25*a^5*c - 20*a^4*c^2 - 20*a^3*c^3 + 3*a^2*c^4 + 3*a*c^5)*
cosh(x))*sinh(x)^3 + 3*(25*a^6 - 25*a^5*c - 20*a^4*c^2 + 20*a^3*c^3 + 3*a^2
*c^4 - 3*a*c^5)*cosh(x)^2 + 3*(25*a^6 - 25*a^5*c - 20*a^4*c^2 + 20*a^3*c^3
+ 3*a^2*c^4 - 3*a*c^5 + 5*(5*a^6 + 15*a^5*c + 12*a^4*c^2 - 4*a^3*c^3 - 9*a^
2*c^4 - 3*a*c^5)*cosh(x)^4 + 20*(5*a^6 + 10*a^5*c + 2*a^4*c^2 - 6*a^3*c^3 -
 3*a^2*c^4)*cosh(x)^3 + 6*(25*a^6 + 25*a^5*c - 20*a^4*c^2 - 20*a^3*c^3 + 3*
a^2*c^4 + 3*a*c^5)*cosh(x)^2 + 4*(25*a^6 - 30*a^4*c^2 + 9*a^2*c^4)*cosh(x))
*sinh(x)^2 + 6*(5*a^6 - 10*a^5*c + 2*a^4*c^2 + 6*a^3*c^3 - 3*a^2*c^4)*cosh(
x) + 6*(5*a^6 - 10*a^5*c + 2*a^4*c^2 + 6*a^3*c^3 - 3*a^2*c^4 + (5*a^6 + 15*
a^5*c + 12*a^4*c^2 - 4*a^3*c^3 - 9*a^2*c^4 - 3*a*c^5)*cosh(x)^5 + 5*(5*a^6
+ 10*a^5*c + 2*a^4*c^2 - 6*a^3*c^3 - 3*a^2*c^4)*cosh(x)^4 + 2*(25*a^6 + 25*
a^5*c - 20*a^4*c^2 - 20*a^3*c^3 + 3*a^2*c^4 + 3*a*c^5)*cosh(x)^3 + 2*(25*a^
6 - 30*a^4*c^2 + 9*a^2*c^4)*cosh(x)^2 + (25*a^6 - 25*a^5*c - 20*a^4*c^2 + 2
0*a^3*c^3 + 3*a^2*c^4 - 3*a*c^5)*cosh(x))*sinh(x))*log((a + c)*cosh(x) + (a
 + c)*sinh(x) + a - c) - 3*((5*a^6 + 15*a^5*c + 12*a^4*c^2 - 4*a^3*c^3 - 9*
a^2*c^4 - 3*a*c^5)*cosh(x)^6 + (5*a^6 + 15*a^5*c + 12*a^4*c^2 - 4*a^3*c^3 -
 9*a^2*c^4 - 3*a*c^5)*sinh(x)^6 + 5*a^6 - 15*a^5*c + 12*a^4*c^2 + 4*a^3*c^3
 - 9*a^2*c^4 + 3*a*c^5 + 6*(5*a^6 + 10*a^5*c + 2*a^4*c^2 - 6*a^3*c^3 - 3*a^
```

2*c^4)*cosh(x)^5 + 6*(5*a^6 + 10*a^5*c + 2*a^4*c^2 - 6*a^3*c^3 - 3*a^2*c^4 + (5*a^6 + 15*a^5*c + 12*a^4*c^2 - 4*a^3*c^3 - 9*a^2*c^4 - 3*a*c^5)*cosh(x))*sinh(x)^5 + 3*(25*a^6 + 25*a^5*c - 20*a^4*c^2 - 20*a^3*c^3 + 3*a^2*c^4 + 3*a*c^5)*cosh(x)^4 + 3*(25*a^6 + 25*a^5*c - 20*a^4*c^2 - 20*a^3*c^3 + 3*a^2*c^4 + 3*a*c^5 + 5*(5*a^6 + 15*a^5*c + 12*a^4*c^2 - 4*a^3*c^3 - 9*a^2*c^4 - 3*a*c^5)*cosh(x)^2 + 10*(5*a^6 + 10*a^5*c + 2*a^4*c^2 - 6*a^3*c^3 - 3*a^2*c^4)*cosh(x))*sinh(x)^4 + 4*(25*a^6 - 30*a^4*c^2 + 9*a^2*c^4)*cosh(x)^3 + 4*(25*a^6 - 30*a^4*c^2 + 9*a^2*c^4 + 5*(5*a^6 + 15*a^5*c + 12*a^4*c^2 - 4*a^3*c^3 - 9*a^2*c^4 - 3*a*c^5)*cosh(x)^3 + 15*(5*a^6 + 10*a^5*c + 2*a^4*c^2 - 6*a^3*c^3 - 3*a^2*c^4)*cosh(x)^2 + 3*(25*a^6 + 25*a^5*c - 20*a^4*c^2 - 20*a^3*c^3 + 3*a^2*c^4 + 3*a*c^5)*cosh(x))*sinh(x)^3 + 3*(25*a^6 - 25*a^5*c - 20*a^4*c^2 + 20*a^3*c^3 + 3*a^2*c^4 - 3*a*c^5)*cosh(x)^2 + 3*(25*a^6 - 25*a^5*c - 20*a^4*c^2 + 20*a^3*c^3 + 3*a^2*c^4 - 3*a*c^5 + 5*(5*a^6 + 15*a^5*c + 12*a^4*c^2 - 4*a^3*c^3 - 9*a^2*c^4 - 3*a*c^5)*cosh(x)^4 + 20*(5*a^6 + 10*a^5*c + 2*a^4*c^2 - 6*a^3*c^3 - 3*a^2*c^4)*cosh(x)^3 + 6*(25*a^6 + 25*a^5*c - 20*a^4*c^2 - 20*a^3*c^3 + 3*a^2*c^4 + 3*a*c^5)*cosh(x)^2 + 4*(25*a^6 - 30*a^4*c^2 + 9*a^2*c^4)*cosh(x))*sinh(x)^2 + 6*(5*a^6 - 10*a^5*c + 2*a^4*c^2 + 6*a^3*c^3 - 3*a^2*c^4)*cosh(x) + 6*(5*a^6 - 10*a^5*c + 2*a^4*c^2 + 6*a^3*c^3 - 3*a^2*c^4 + (5*a^6 + 15*a^5*c + 12*a^4*c^2 - 4*a^3*c^3 - 9*a^2*c^4 - 3*a*c^5)*cosh(x)^5 + 5*(5*a^6 + 10*a^5*c + 2*a^4*c^2 - 6*a^3*c^3 - 3*a^2*c^4)*cosh(x)^4 + 2*(25*a^6 + 25*a^5*c - 20*a^4*c^2 - 20*a^3*c^3 + 3*a^2*c^4 + 3*a*c^5)*cosh(x)^3 + 2*(25*a^6 - 30*a^4*c^2 + 9*a^2*c^4)*cosh(x)^2 + (25*a^6 - 25*a^5*c - 20*a^4*c^2 + 20*a^3*c^3 + 3*a^2*c^4 - 3*a*c^5)*cosh(x))*sinh(x))*log(cosh(x) + sinh(x) + 1) + 6*(25*a^5*c - 50*a^4*c^2 + 20*a^3*c^3 + 10*a^2*c^4 - 5*a*c^5 + 5*(5*a^5*c + 10*a^4*c^2 + 2*a^3*c^3 - 6*a^2*c^4 - 3*a*c^5)*cosh(x)^4 + 20*(5*a^5*c + 5*a^4*c^2 - 3*a^3*c^3 - 3*a^2*c^4)*cosh(x)^3 + 2*(75*a^5*c - 65*a^3*c^3 + 12*a*c^5)*cosh(x)^2 + 4*(25*a^5*c - 25*a^4*c^2 - 10*a^3*c^3 + 10*a^2*c^4 + 2*a*c^5 - 2*c^6)*cosh(x))*sinh(x))/(a^3*c^7 - 3*a^2*c^8 + 3*a*c^9 - c^10 + (a^3*c^7 + 3*a^2*c^8 + 3*a*c^9 + c^10)*cosh(x)^6 + (a^3*c^7 + 3*a^2*c^8 + 3*a*c^9 + c^10)*sinh(x)^6 + 6*(a^3*c^7 + 2*a^2*c^8 + a*c^9)*cosh(x)^5 + 6*(a^3*c^7 + 2*a^2*c^8 + a*c^9 + (a^3*c^7 + 3*a^2*c^8 + 3*a*c^9 + c^10)*cosh(x))*sinh(x)^5 + 3*(5*a^3*c^7 + 5*a^2*c^8 - a*c^9 - c^10)*cosh(x)^4 + 3*(5*a^3*c^7 + 5*a^2*c^8 - a*c^9 - c^10 + 5*(a^3*c^7 + 3*a^2*c^8 + 3*a*c^9 + c^10)*cosh(x)^2 + 10*(a^3*c^7 + 2*a^2*c^8 + a*c^9)*cosh(x))*sinh(x)^4 + 4*(5*a^3*c^7 - 3*a*c^9)*cosh(x)^3 + 4*(5*a^3*c^7 - 3*a*c^9 + 5*(a^3*c^7 + 3*a^2*c^8 + 3*a*c^9 + c^10)*cosh(x)^3 + 15*(a^3*c^7 + 2*a^2*c^8 + a*c^9)*cosh(x)^2 + 3*(5*a^3*c^7 + 5*a^2*c^8 - a*c^9 - c^10)*cosh(x))*sinh(x)^3 + 3*(5*a^3*c^7 - 5*a^2*c^8 - a*c^9 + c^10)*cosh(x)^2 + 3*(5*a^3*c^7 - 5*a^2*c^8 - a*c^9 + c^10 + 5*(a^3*c^7 + 3*a^2*c^8 + 3*a*c^9 + c^10)*cosh(x)^4 + 20*(a^3*c^7 + 2*a^2*c^8 + a*c^9)*cosh(x)^3 + 6*(5*a^3*c^7 + 5*a^2*c^8 - a*c^9 - c^10)*cosh(x)^2 + 4*(5*a^3*c^7 - 3*a*c^9)*cosh(x))*sinh(x)^2 + 6*(a^3*c^7 - 2*a^2*c^8 + a*c^9)*cosh(x) + 6*(a^3*c^7 - 2*a^2*c^8 + a*c^9 + (a^3*c^7 + 3*a^2*c^8 + 3*a*c^9 + c^10)*cosh(x)^5 + 5*(a^3*c^7 + 2*a^2*c^8 + a*c^9)*cosh(x)^4 + 2*(5*a^3*c^7 + 5*a^2*c^8 - a*c^9 - c^10)*cosh(x)^3 + 2*(5*a^3*c^7 - 3*a*c^9)*cosh(x)^2 + (5*a^3*c^7 - 5*a^2*c^8 - a*c^9

```
+ c^10)*cosh(x))*sinh(x))
```

giac [B] time = 0.16, size = 377, normalized size = 2.69

$$\frac{\left(5\,a^4+5\,a^3c-3\,a^2c^2-3\,ac^3\right)\log\left(|ae^x+ce^x+a-c|\right)}{2\left(ac^7+c^8\right)}-\frac{\left(5\,a^3-3\,ac^2\right)\log\left(e^x+1\right)}{2\,c^7}+\frac{15\,a^5e^{(5\,x)}+30\,a^4ce^{(5\,x)}+6\,a^3c}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cosh(x)+c*sinh(x))^4,x, algorithm="giac")
```

```
[Out] 1/2*(5*a^4 + 5*a^3*c - 3*a^2*c^2 - 3*a*c^3)*log(abs(a*e^x + c*e^x + a - c))
/(a*c^7 + c^8) - 1/2*(5*a^3 - 3*a*c^2)*log(e^x + 1)/c^7 + 1/3*(15*a^5*e^(5*
x) + 30*a^4*c*e^(5*x) + 6*a^3*c^2*e^(5*x) - 18*a^2*c^3*e^(5*x) - 9*a*c^4*e^
(5*x) + 75*a^5*e^(4*x) + 75*a^4*c*e^(4*x) - 45*a^3*c^2*e^(4*x) - 45*a^2*c^3
*e^(4*x) + 150*a^5*e^(3*x) - 130*a^3*c^2*e^(3*x) + 24*a*c^4*e^(3*x) + 150*a
^5*e^(2*x) - 150*a^4*c*e^(2*x) - 60*a^3*c^2*e^(2*x) + 60*a^2*c^3*e^(2*x) +
12*a*c^4*e^(2*x) - 12*c^5*e^(2*x) + 75*a^5*e^x - 150*a^4*c*e^x + 60*a^3*c^2
*e^x + 30*a^2*c^3*e^x - 15*a*c^4*e^x + 15*a^5 - 45*a^4*c + 41*a^3*c^2 - 3*a
^2*c^3 - 12*a*c^4 + 4*c^5)/((a*e^(2*x) + c*e^(2*x) + 2*a*e^x + a - c)^3*c^6
)
```

maple [A] time = 0.30, size = 250, normalized size = 1.79

$$-\frac{\tanh^3\left(\frac{x}{2}\right)}{24c^4}+\frac{a\left(\tanh^2\left(\frac{x}{2}\right)\right)}{4c^5}-\frac{5a^2\tanh\left(\frac{x}{2}\right)}{4c^6}+\frac{3\tanh\left(\frac{x}{2}\right)}{8c^4}+\frac{a^6}{24c^7\left(a+c\tanh\left(\frac{x}{2}\right)\right)^3}-\frac{a^4}{8c^5\left(a+c\tanh\left(\frac{x}{2}\right)\right)^3}+\frac{}{8c^3\left(a+c\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*cosh(x)+c*sinh(x))^4,x)
```

```
[Out] -1/24/c^4*tanh(1/2*x)^3+1/4/c^5*a*tanh(1/2*x)^2-5/4/c^6*a^2*tanh(1/2*x)+3/8
/c^4*tanh(1/2*x)+1/24/c^7/(a+c*tanh(1/2*x))^3*a^6-1/8/c^5/(a+c*tanh(1/2*x))
^3*a^4+1/8/c^3/(a+c*tanh(1/2*x))^3*a^2-1/24/c/(a+c*tanh(1/2*x))^3-3/8*a^5/c
^7/(a+c*tanh(1/2*x))^2+3/4*a^3/c^5/(a+c*tanh(1/2*x))^2-3/8*a/c^3/(a+c*tanh(
1/2*x))^2+5/2*a^3/c^7*ln(a+c*tanh(1/2*x))-3/2*a/c^5*ln(a+c*tanh(1/2*x))+15/
8/c^7/(a+c*tanh(1/2*x))*a^4-9/4/c^5/(a+c*tanh(1/2*x))*a^2+3/8/c^3/(a+c*tanh
(1/2*x))
```

maxima [B] time = 0.38, size = 487, normalized size = 3.48

$$-\frac{15\,a^5+45\,a^4c+41\,a^3c^2+3\,a^2c^3-12\,ac^4-4\,c^5+15\left(5\,a^5+10\,a^4c+4\,a^3c^2-2\,a^2c^3-ac^4\right)e^{(-x)}+6\left(25\,a^5+25\right.}{3\left(a^3c^6+3\,a^2c^7+3\,ac^8+c^9+6\left(a^3c^6+2\,a^2c^7+ac^8\right)e^{(-x)}+3\left(5\,a^3c^6+5\,a^2c^7-ac^8\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cosh(x)+c*sinh(x))^4,x, algorithm="maxima")
```

```
[Out] -1/3*(15*a^5 + 45*a^4*c + 41*a^3*c^2 + 3*a^2*c^3 - 12*a*c^4 - 4*c^5 + 15*(5
*a^5 + 10*a^4*c + 4*a^3*c^2 - 2*a^2*c^3 - a*c^4)*e^(-x) + 6*(25*a^5 + 25*a^
4*c - 10*a^3*c^2 - 10*a^2*c^3 + 2*a*c^4 + 2*c^5)*e^(-2*x) + 2*(75*a^5 - 65*
a^3*c^2 + 12*a*c^4)*e^(-3*x) + 15*(5*a^5 - 5*a^4*c - 3*a^3*c^2 + 3*a^2*c^3)
*e^(-4*x) + 3*(5*a^5 - 10*a^4*c + 2*a^3*c^2 + 6*a^2*c^3 - 3*a*c^4)*e^(-5*x)
)/(a^3*c^6 + 3*a^2*c^7 + 3*a*c^8 + c^9 + 6*(a^3*c^6 + 2*a^2*c^7 + a*c^8)*e^
(-x) + 3*(5*a^3*c^6 + 5*a^2*c^7 - a*c^8 - c^9)*e^(-2*x) + 4*(5*a^3*c^6 - 3*
a*c^8)*e^(-3*x) + 3*(5*a^3*c^6 - 5*a^2*c^7 - a*c^8 + c^9)*e^(-4*x) + 6*(a^3
*c^6 - 2*a^2*c^7 + a*c^8)*e^(-5*x) + (a^3*c^6 - 3*a^2*c^7 + 3*a*c^8 - c^9)*
e^(-6*x)) + 1/2*(5*a^3 - 3*a*c^2)*log(-(a - c)*e^(-x) - a - c)/c^7 - 1/2*(5
*a^3 - 3*a*c^2)*log(e^(-x) + 1)/c^7
```

mupad **[F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(a + a\cosh(x) + c\sinh(x))^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + a*cosh(x) + c*sinh(x))^4,x)
```

```
[Out] int(1/(a + a*cosh(x) + c*sinh(x))^4, x)
```

sympy **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cosh(x)+c*sinh(x))**4,x)
```

```
[Out] Timed out
```

3.753 $\int \left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^4\,dx$

Optimal. Leaf size=188

$$\frac{35}{8}x\left(b^2-c^2\right)^2+\frac{35}{8}b\left(b^2-c^2\right)^{3/2}\sinh(x)+\frac{35}{8}c\left(b^2-c^2\right)^{3/2}\cosh(x)+\frac{1}{4}(b\sinh(x)+c\cosh(x))\left(\sqrt{b^2-c^2}+b\cosh(x\right.$$

```
[Out] 35/8*(b^2-c^2)^2*x+35/8*c*(b^2-c^2)^(3/2)*cosh(x)+35/8*b*(b^2-c^2)^(3/2)*si
nh(x)+35/24*(b^2-c^2)*(c*cosh(x)+b*sinh(x))*(b*cosh(x)+c*sinh(x)+(b^2-c^2)^
(1/2))+7/12*(c*cosh(x)+b*sinh(x))*(b^2-c^2)^(1/2)*(b*cosh(x)+c*sinh(x)+(b^2
-c^2)^(1/2))^2+1/4*(c*cosh(x)+b*sinh(x))*(b*cosh(x)+c*sinh(x)+(b^2-c^2)^(1/
2))^3
```

Rubi [A] time = 0.15, antiderivative size = 188, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3113, 2637, 2638}

$$\frac{35}{8}x\left(b^2-c^2\right)^2+\frac{35}{8}b\left(b^2-c^2\right)^{3/2}\sinh(x)+\frac{35}{8}c\left(b^2-c^2\right)^{3/2}\cosh(x)+\frac{1}{4}(b\sinh(x)+c\cosh(x))\left(\sqrt{b^2-c^2}+b\cosh(x\right.$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[b^2 - c^2] + b*Cosh[x] + c*Sinh[x])^4,x]
```

```
[Out] (35*(b^2 - c^2)^2*x)/8 + (35*c*(b^2 - c^2)^(3/2)*Cosh[x])/8 + (35*b*(b^2 -
c^2)^(3/2)*Sinh[x])/8 + (35*(b^2 - c^2)*(c*Cosh[x] + b*Sinh[x])*(Sqrt[b^2 -
 c^2] + b*Cosh[x] + c*Sinh[x]))/24 + (7*Sqrt[b^2 - c^2]*(c*Cosh[x] + b*Sinh
[x])*(Sqrt[b^2 - c^2] + b*Cosh[x] + c*Sinh[x])^2)/12 + ((c*Cosh[x] + b*Sinh
[x])*(Sqrt[b^2 - c^2] + b*Cosh[x] + c*Sinh[x])^3)/4
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3113

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> -Simp[((c*Cos[d + e*x] - b*Sin[d + e*x])*(a + b*Cos[d +
e*x] + c*Sin[d + e*x])^(n - 1))/(e*n), x] + Dist[(a*(2*n - 1))/n, Int[(a +
```

```
b*Cos[d + e*x] + c*Sin[d + e*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e},
x] && EqQ[a^2 - b^2 - c^2, 0] && GtQ[n, 0]
```

Rubi steps

$$\int \left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^4 dx = \frac{1}{4}(c\cosh(x)+b\sinh(x))\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^3+\frac{1}{4}$$
$$= \frac{7}{12}\sqrt{b^2-c^2}(c\cosh(x)+b\sinh(x))\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh\right.$$
$$= \frac{35}{24}\left(b^2-c^2\right)(c\cosh(x)+b\sinh(x))\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sin\right.$$
$$= \frac{35}{8}\left(b^2-c^2\right)^2 x+\frac{35}{24}\left(b^2-c^2\right)(c\cosh(x)+b\sinh(x))\left(\sqrt{b^2-c^2}+\right.$$
$$= \frac{35}{8}\left(b^2-c^2\right)^2 x+\frac{35}{8}c\left(b^2-c^2\right)^{3/2}\cosh(x)+\frac{35}{8}b\left(b^2-c^2\right)^{3/2}\sinh($$

Mathematica [A] time = 0.51, size = 208, normalized size = 1.11

$$\frac{7}{4}\left(b^4-c^4\right)\sinh(2x)+7b(b-c)\sqrt{b^2-c^2}(b+c)\sinh(x)+\frac{1}{3}b\sqrt{b^2-c^2}\left(b^2+3c^2\right)\sinh(3x)+7c(b-c)\sqrt{b^2-c^2}(b+c)\,c$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[b^2 - c^2] + b*Cosh[x] + c*Sinh[x])^4,x]
```

```
[Out] (35*(b - c)^2*(b + c)^2*x)/8 + 7*(b - c)*c*(b + c)*Sqrt[b^2 - c^2]*Cosh[x]
+ (7*b*c*(b^2 - c^2)*Cosh[2*x])/2 + (c*Sqrt[b^2 - c^2]*(3*b^2 + c^2)*Cosh[3
*x])/3 + (b*c*(b^2 + c^2)*Cosh[4*x])/8 + 7*b*(b - c)*(b + c)*Sqrt[b^2 - c^2
]*Sinh[x] + (7*(b^4 - c^4)*Sinh[2*x])/4 + (b*Sqrt[b^2 - c^2]*(b^2 + 3*c^2)*
Sinh[3*x])/3 + ((b^4 + 6*b^2*c^2 + c^4)*Sinh[4*x])/32
```

fricas [B] time = 0.45, size = 1293, normalized size = 6.88

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cosh(x)+c*sinh(x)+(b^2-c^2)^(1/2))^4,x, algorithm="fricas")
```

```
[Out] 1/192*(3*(b^4 + 4*b^3*c + 6*b^2*c^2 + 4*b*c^3 + c^4)*cosh(x)^8 + 24*(b^4 +
4*b^3*c + 6*b^2*c^2 + 4*b*c^3 + c^4)*cosh(x)*sinh(x)^7 + 3*(b^4 + 4*b^3*c +
 6*b^2*c^2 + 4*b*c^3 + c^4)*sinh(x)^8 + 168*(b^4 + 2*b^3*c - 2*b*c^3 - c^4)
*cosh(x)^6 + 84*(2*b^4 + 4*b^3*c - 4*b*c^3 - 2*c^4 + (b^4 + 4*b^3*c + 6*b^2
```

```
*c^2 + 4*b*c^3 + c^4)*cosh(x)^2)*sinh(x)^6 + 840*(b^4 - 2*b^2*c^2 + c^4)*x*
cosh(x)^4 + 168*((b^4 + 4*b^3*c + 6*b^2*c^2 + 4*b*c^3 + c^4)*cosh(x)^3 + 6*
(b^4 + 2*b^3*c - 2*b*c^3 - c^4)*cosh(x))*sinh(x)^5 + 210*((b^4 + 4*b^3*c +
6*b^2*c^2 + 4*b*c^3 + c^4)*cosh(x)^4 + 12*(b^4 + 2*b^3*c - 2*b*c^3 - c^4)*c
osh(x)^2 + 4*(b^4 - 2*b^2*c^2 + c^4)*x)*sinh(x)^4 - 3*b^4 + 12*b^3*c - 18*b
^2*c^2 + 12*b*c^3 - 3*c^4 + 168*((b^4 + 4*b^3*c + 6*b^2*c^2 + 4*b*c^3 + c^4
)*cosh(x)^5 + 20*(b^4 + 2*b^3*c - 2*b*c^3 - c^4)*cosh(x)^3 + 20*(b^4 - 2*b^
2*c^2 + c^4)*x*cosh(x))*sinh(x)^3 - 168*(b^4 - 2*b^3*c + 2*b*c^3 - c^4)*cos
h(x)^2 + 84*((b^4 + 4*b^3*c + 6*b^2*c^2 + 4*b*c^3 + c^4)*cosh(x)^6 + 30*(b^
4 + 2*b^3*c - 2*b*c^3 - c^4)*cosh(x)^4 - 2*b^4 + 4*b^3*c - 4*b*c^3 + 2*c^4
+ 60*(b^4 - 2*b^2*c^2 + c^4)*x*cosh(x)^2)*sinh(x)^2 + 24*((b^4 + 4*b^3*c +
6*b^2*c^2 + 4*b*c^3 + c^4)*cosh(x)^7 + 42*(b^4 + 2*b^3*c - 2*b*c^3 - c^4)*c
osh(x)^5 + 140*(b^4 - 2*b^2*c^2 + c^4)*x*cosh(x)^3 - 14*(b^4 - 2*b^3*c + 2*
b*c^3 - c^4)*cosh(x))*sinh(x) + 32*((b^3 + 3*b^2*c + 3*b*c^2 + c^3)*cosh(x)
^7 + 7*(b^3 + 3*b^2*c + 3*b*c^2 + c^3)*cosh(x)*sinh(x)^6 + (b^3 + 3*b^2*c +
 3*b*c^2 + c^3)*sinh(x)^7 + 21*(b^3 + b^2*c - b*c^2 - c^3)*cosh(x)^5 + 21*(
b^3 + b^2*c - b*c^2 - c^3 + (b^3 + 3*b^2*c + 3*b*c^2 + c^3)*cosh(x)^2)*sinh
(x)^5 + 35*((b^3 + 3*b^2*c + 3*b*c^2 + c^3)*cosh(x)^3 + 3*(b^3 + b^2*c - b*
c^2 - c^3)*cosh(x))*sinh(x)^4 - 21*(b^3 - b^2*c - b*c^2 + c^3)*cosh(x)^3 +
7*(5*(b^3 + 3*b^2*c + 3*b*c^2 + c^3)*cosh(x)^4 - 3*b^3 + 3*b^2*c + 3*b*c^2
- 3*c^3 + 30*(b^3 + b^2*c - b*c^2 - c^3)*cosh(x)^2)*sinh(x)^3 + 21*((b^3 +
3*b^2*c + 3*b*c^2 + c^3)*cosh(x)^5 + 10*(b^3 + b^2*c - b*c^2 - c^3)*cosh(x)
^3 - 3*(b^3 - b^2*c - b*c^2 + c^3)*cosh(x))*sinh(x)^2 - (b^3 - 3*b^2*c + 3*
b*c^2 - c^3)*cosh(x) + (7*(b^3 + 3*b^2*c + 3*b*c^2 + c^3)*cosh(x)^6 + 105*(
b^3 + b^2*c - b*c^2 - c^3)*cosh(x)^4 - b^3 + 3*b^2*c - 3*b*c^2 + c^3 - 63*(
b^3 - b^2*c - b*c^2 + c^3)*cosh(x)^2)*sinh(x))*sqrt(b^2 - c^2))/(cosh(x)^4
+ 4*cosh(x)^3*sinh(x) + 6*cosh(x)^2*sinh(x)^2 + 4*cosh(x)*sinh(x)^3 + sinh(
x)^4)
```

giac [B] time = 0.16, size = 390, normalized size = 2.07

$$\frac{7}{2}\left(b^3+b^2c-bc^2-c^3\right)\sqrt{b^2-c^2}\,e^x+\frac{35}{8}\left(b^4-2\,b^2c^2+c^4\right)x+\frac{1}{64}\left(b^4+4\,b^3c+6\,b^2c^2+4\,bc^3+c^4\right)e^{(4\,x)}+\frac{1}{6}\left(\sqrt{b^2-c^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cosh(x)+c*sinh(x)+(b^2-c^2)^(1/2))^4,x, algorithm="giac")
```

```
[Out] 7/2*(b^3 + b^2*c - b*c^2 - c^3)*sqrt(b^2 - c^2)*e^x + 35/8*(b^4 - 2*b^2*c^2
 + c^4)*x + 1/64*(b^4 + 4*b^3*c + 6*b^2*c^2 + 4*b*c^3 + c^4)*e^(4*x) + 1/6*
(sqrt(b^2 - c^2)*b^3 + 3*sqrt(b^2 - c^2)*b^2*c + 3*sqrt(b^2 - c^2)*b*c^2 +
sqrt(b^2 - c^2)*c^3)*e^(3*x) + 7/8*(b^4 + 2*b^3*c - 2*b*c^3 - c^4)*e^(2*x)
- 1/192*(3*b^4 - 12*b^3*c + 18*b^2*c^2 - 12*b*c^3 + 3*c^4 + 672*(sqrt(b^2 -
 c^2)*b^3 - sqrt(b^2 - c^2)*b^2*c - sqrt(b^2 - c^2)*b*c^2 + sqrt(b^2 - c^2)
*c^3)*e^(3*x) + 168*(b^4 - 2*b^3*c + 2*b*c^3 - c^4)*e^(2*x) + 32*(sqrt(b^2
```

```
- c^2)*b^3 - 3*sqrt(b^2 - c^2)*b^2*c + 3*sqrt(b^2 - c^2)*b*c^2 - sqrt(b^2 -
 c^2)*c^3)*e^x)*e^(-4*x)
```

maple [B] time = 0.67, size = 363, normalized size = 1.93

$$-2c^2b^2x+4\sqrt{b^2-c^2}\,b^3\left(\frac{2}{3}+\frac{\left(\cosh^2(x)\right)}{3}\right)\sinh(x)-6c^2b^2\left(\frac{\cosh(x)\sinh(x)}{2}+\frac{x}{2}\right)+4\sqrt{b^2-c^2}\,b^3\sinh(x)+4\sqrt{b^2-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cosh(x)+c*sinh(x)+(b^2-c^2)^(1/2))^4,x)
```

```
[Out] -2*c^2*b^2*x+4*(b^2-c^2)^(1/2)*b^3*(2/3+1/3*cosh(x)^2)*sinh(x)-6*c^2*b^2*(1
/2*cosh(x)*sinh(x)+1/2*x)+4*(b^2-c^2)^(1/2)*b^3*sinh(x)+4*(b^2-c^2)^(1/2)*c
^3*(-2/3+1/3*sinh(x)^2)*cosh(x)+6*c^2*b^2*(1/2*cosh(x)*sinh(x)-1/2*x)-4*(b^
2-c^2)^(1/2)*c^3*cosh(x)+b^4*x+c^4*x+4*(b^2-c^2)^(1/2)*b^2*c*cosh(x)^3+4*(b
^2-c^2)^(1/2)*b*c^2*sinh(x)^3+b^4*((1/4*cosh(x)^3+3/8*cosh(x))*sinh(x)+3/8*
x)+6*b^4*(1/2*cosh(x)*sinh(x)+1/2*x)+c^4*((1/4*sinh(x)^3-3/8*sinh(x))*cosh(
x)+3/8*x)-6*c^4*(1/2*cosh(x)*sinh(x)-1/2*x)+b^3*cosh(x)^4*c+6*c^2*b^2*(1/4*
sinh(x)*cosh(x)^3-1/8*cosh(x)*sinh(x)-1/8*x)+b*c^3*sinh(x)^4+6*b^3*cosh(x)^
2*c-6*b*cosh(x)^2*c^3-4*(b^2-c^2)^(1/2)*b*c^2*sinh(x)+4*(b^2-c^2)^(1/2)*b^2
*c*cosh(x)
```

maxima [A] time = 0.34, size = 277, normalized size = 1.47

$$b^3c\cosh(x)^4+bc^3\sinh(x)^4+\frac{1}{64}\,b^4\left(24\,x+e^{(4\,x)}+8\,e^{(2\,x)}-8\,e^{(-2\,x)}-e^{(-4\,x)}\right)+\frac{1}{64}\,c^4\left(24\,x+e^{(4\,x)}-8\,e^{(2\,x)}+8\,e^{(-2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cosh(x)+c*sinh(x)+(b^2-c^2)^(1/2))^4,x, algorithm="maxima")
```

```
[Out] b^3*c*cosh(x)^4 + b*c^3*sinh(x)^4 + 1/64*b^4*(24*x + e^(4*x) + 8*e^(2*x) -
8*e^(-2*x) - e^(-4*x)) + 1/64*c^4*(24*x + e^(4*x) - 8*e^(2*x) + 8*e^(-2*x)
- e^(-4*x)) - 3/32*b^2*c^2*(8*x - e^(4*x) + e^(-4*x)) + (b^2 - c^2)^2*x + 4
*(b^2 - c^2)^(3/2)*(c*cosh(x) + b*sinh(x)) + 3/4*(8*b*c*cosh(x)^2 + b^2*(4*
x + e^(2*x) - e^(-2*x)) - c^2*(4*x - e^(2*x) + e^(-2*x)))*(b^2 - c^2) + 1/6
*(24*b^2*c*cosh(x)^3 + 24*b*c^2*sinh(x)^3 + c^3*(e^(3*x) - 9*e^(-x) + e^(-3
*x) - 9*e^x) + b^3*(e^(3*x) - 9*e^(-x) - e^(-3*x) + 9*e^x))*sqrt(b^2 - c^2)
```

mupad [B] time = 0.45, size = 361, normalized size = 1.92

$$x\left(b^2-c^2\right)^2-\cosh(x)^2\,\left(6\,b\,c^3-6\,b^3\,c\right)-\cosh(x)^4\,\left(b\,c^3-b^3\,c\right)+\cosh(x)\sinh(x)^3\,\left(-\frac{3\,b^4}{8}+\frac{3\,b^2\,c^2}{4}+\frac{5\,c^4}{8}\right)+\cosh$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cosh(x) + (b^2 - c^2)^(1/2) + c*sinh(x))^4,x)
```

```
[Out] x*(b^2 - c^2)^2 - cosh(x)^2*(6*b*c^3 - 6*b^3*c) - cosh(x)^4*(b*c^3 - b^3*c)
 + cosh(x)*sinh(x)^3*((5*c^4)/8 - (3*b^4)/8 + (3*b^2*c^2)/4) + cosh(x)^3*si
nh(x)*((5*b^4)/8 - (3*c^4)/8 + (3*b^2*c^2)/4) + 4*c*cosh(x)*(b^2 - c^2)^(3/
2) + 4*b*sinh(x)*(b^2 - c^2)^(3/2) + 3*x*cosh(x)^2*(b^2 - c^2)^2 + (3*x*cos
h(x)^4*(b^2 - c^2)^2)/8 - 3*x*sinh(x)^2*(b^2 - c^2)^2 + (3*x*sinh(x)^4*(b^2
 - c^2)^2)/8 + cosh(x)*sinh(x)*(3*b^4 - 3*c^4) + 2*b*c^3*cosh(x)^2*sinh(x)^
2 + (4*c*cosh(x)^3*(b^2 - c^2)^(1/2)*(3*b^2 - 2*c^2))/3 - (4*b*sinh(x)^3*(b
^2 - c^2)^(1/2)*(2*b^2 - 3*c^2))/3 + 4*b^3*cosh(x)^2*sinh(x)*(b^2 - c^2)^(1
/2) + 4*c^3*cosh(x)*sinh(x)^2*(b^2 - c^2)^(1/2) - (3*x*cosh(x)^2*sinh(x)^2*
(b^2 - c^2)^2)/4
```

sympy [B] time = 1.47, size = 626, normalized size = 3.33

$$\frac{3b^4x\sinh^4(x)}{8}-\frac{3b^4x\sinh^2(x)\cosh^2(x)}{4}-3b^4x\sinh^2(x)+\frac{3b^4x\cosh^4(x)}{8}+3b^4x\cosh^2(x)+b^4x-\frac{3b^4\sinh^3(x)\cosh}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cosh(x)+c*sinh(x)+(b**2-c**2)**(1/2))**4,x)
```

```
[Out] 3*b**4*x*sinh(x)**4/8 - 3*b**4*x*sinh(x)**2*cosh(x)**2/4 - 3*b**4*x*sinh(x)
**2 + 3*b**4*x*cosh(x)**4/8 + 3*b**4*x*cosh(x)**2 + b**4*x - 3*b**4*sinh(x)
**3*cosh(x)/8 + 5*b**4*sinh(x)*cosh(x)**3/8 + 3*b**4*sinh(x)*cosh(x) + b**3
*c*cosh(x)**4 + 6*b**3*c*cosh(x)**2 - 8*b**3*sqrt(b**2 - c**2)*sinh(x)**3/3
 + 4*b**3*sqrt(b**2 - c**2)*sinh(x)*cosh(x)**2 + 4*b**3*sqrt(b**2 - c**2)*s
inh(x) - 3*b**2*c**2*x*sinh(x)**4/4 + 3*b**2*c**2*x*sinh(x)**2*cosh(x)**2/2
 + 6*b**2*c**2*x*sinh(x)**2 - 3*b**2*c**2*x*cosh(x)**4/4 - 6*b**2*c**2*x*co
sh(x)**2 - 2*b**2*c**2*x + 3*b**2*c**2*sinh(x)**3*cosh(x)/4 + 3*b**2*c**2*s
inh(x)*cosh(x)**3/4 + 4*b**2*c*sqrt(b**2 - c**2)*cosh(x)**3 + 4*b**2*c*sqrt
(b**2 - c**2)*cosh(x) + b*c**3*sinh(x)**4 - 6*b*c**3*cosh(x)**2 + 4*b*c**2*
sqrt(b**2 - c**2)*sinh(x)**3 - 4*b*c**2*sqrt(b**2 - c**2)*sinh(x) + 3*c**4*
x*sinh(x)**4/8 - 3*c**4*x*sinh(x)**2*cosh(x)**2/4 - 3*c**4*x*sinh(x)**2 + 3
*c**4*x*cosh(x)**4/8 + 3*c**4*x*cosh(x)**2 + c**4*x + 5*c**4*sinh(x)**3*cos
h(x)/8 - 3*c**4*sinh(x)*cosh(x)**3/8 - 3*c**4*sinh(x)*cosh(x) + 4*c**3*sqrt
(b**2 - c**2)*sinh(x)**2*cosh(x) - 8*c**3*sqrt(b**2 - c**2)*cosh(x)**3/3 -
4*c**3*sqrt(b**2 - c**2)*cosh(x)
```

3.754 $\int \left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^3\,dx$

Optimal. Leaf size=136

$$\frac{5}{2}x\left(b^2-c^2\right)^{3/2}+\frac{5}{2}b\left(b^2-c^2\right)\sinh(x)+\frac{5}{2}c\left(b^2-c^2\right)\cosh(x)+\frac{1}{3}(b\sinh(x)+c\cosh(x))\left(\sqrt{b^2-c^2}+b\cosh(x)+c\,\mathrm{s}\right.$$

```
[Out] 5/2*(b^2-c^2)^(3/2)*x+5/2*c*(b^2-c^2)*cosh(x)+5/2*b*(b^2-c^2)*sinh(x)+5/6*(
c*cosh(x)+b*sinh(x))*(b^2-c^2)^(1/2)*(b*cosh(x)+c*sinh(x)+(b^2-c^2)^(1/2))+
1/3*(c*cosh(x)+b*sinh(x))*(b*cosh(x)+c*sinh(x)+(b^2-c^2)^(1/2))^2
```

Rubi [A] time = 0.09, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3113, 2637, 2638}

$$\frac{5}{2}x\left(b^2-c^2\right)^{3/2}+\frac{5}{2}b\left(b^2-c^2\right)\sinh(x)+\frac{5}{2}c\left(b^2-c^2\right)\cosh(x)+\frac{1}{3}(b\sinh(x)+c\cosh(x))\left(\sqrt{b^2-c^2}+b\cosh(x)+c\,\mathrm{s}\right.$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[b^2 - c^2] + b*Cosh[x] + c*Sinh[x])^3,x]
```

```
[Out] (5*(b^2 - c^2)^(3/2)*x)/2 + (5*c*(b^2 - c^2)*Cosh[x])/2 + (5*b*(b^2 - c^2)*
Sinh[x])/2 + (5*Sqrt[b^2 - c^2]*(c*Cosh[x] + b*Sinh[x])*(Sqrt[b^2 - c^2] +
b*Cosh[x] + c*Sinh[x]))/6 + ((c*Cosh[x] + b*Sinh[x])*(Sqrt[b^2 - c^2] + b*C
osh[x] + c*Sinh[x])^2)/3
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3113

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> -Simp[((c*Cos[d + e*x] - b*Sin[d + e*x])*(a + b*Cos[d +
e*x] + c*Sin[d + e*x])^(n - 1))/(e*n), x] + Dist[(a*(2*n - 1))/n, Int[(a +
b*Cos[d + e*x] + c*Sin[d + e*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e},
x] && EqQ[a^2 - b^2 - c^2, 0] && GtQ[n, 0]
```

Rubi steps

$$\int \left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^3 dx = \frac{1}{3}(c\cosh(x)+b\sinh(x))\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^2+\frac{1}{3}\Big($$
$$=\frac{5}{6}\sqrt{b^2-c^2}\,(c\cosh(x)+b\sinh(x))\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x\right.$$
$$=\frac{5}{2}\left(b^2-c^2\right)^{3/2}x+\frac{5}{6}\sqrt{b^2-c^2}\,(c\cosh(x)+b\sinh(x))\left(\sqrt{b^2-c^2}+b\,c\right.$$
$$=\frac{5}{2}\left(b^2-c^2\right)^{3/2}x+\frac{5}{2}c\left(b^2-c^2\right)\cosh(x)+\frac{5}{2}b\left(b^2-c^2\right)\sinh(x)+\frac{5}{6}\sqrt{}$$

Mathematica [A] time = 0.27, size = 134, normalized size = 0.99

$$\frac{1}{12}\left(30x(b-c)(b+c)\sqrt{b^2-c^2}+45b\left(b^2-c^2\right)\sinh(x)+9\sqrt{b^2-c^2}\left(b^2+c^2\right)\sinh(2x)+b\left(b^2+3c^2\right)\sinh(3x)+4\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[b^2 - c^2] + b*Cosh[x] + c*Sinh[x])^3,x]
```

```
[Out] (30*(b - c)*(b + c)*Sqrt[b^2 - c^2]*x + 45*c*(b^2 - c^2)*Cosh[x] + 18*b*c*S
qrt[b^2 - c^2]*Cosh[2*x] + c*(3*b^2 + c^2)*Cosh[3*x] + 45*b*(b^2 - c^2)*Sin
h[x] + 9*Sqrt[b^2 - c^2]*(b^2 + c^2)*Sinh[2*x] + b*(b^2 + 3*c^2)*Sinh[3*x])
/12
```

fricas [B] time = 0.42, size = 664, normalized size = 4.88

$$\frac{\left(b^3+3\,b^2c+3\,bc^2+c^3\right)\cosh(x)^6+6\left(b^3+3\,b^2c+3\,bc^2+c^3\right)\cosh(x)\sinh(x)^5+\left(b^3+3\,b^2c+3\,bc^2+c^3\right)\sinh(}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cosh(x)+c*sinh(x)+(b^2-c^2)^(1/2))^3,x, algorithm="fricas")
```

```
[Out] 1/24*((b^3 + 3*b^2*c + 3*b*c^2 + c^3)*cosh(x)^6 + 6*(b^3 + 3*b^2*c + 3*b*c^
2 + c^3)*cosh(x)*sinh(x)^5 + (b^3 + 3*b^2*c + 3*b*c^2 + c^3)*sinh(x)^6 + 45
*(b^3 + b^2*c - b*c^2 - c^3)*cosh(x)^4 + 15*(3*b^3 + 3*b^2*c - 3*b*c^2 - 3*
c^3 + (b^3 + 3*b^2*c + 3*b*c^2 + c^3)*cosh(x)^2)*sinh(x)^4 + 20*((b^3 + 3*b
^2*c + 3*b*c^2 + c^3)*cosh(x)^3 + 9*(b^3 + b^2*c - b*c^2 - c^3)*cosh(x))*si
nh(x)^3 - b^3 + 3*b^2*c - 3*b*c^2 + c^3 - 45*(b^3 - b^2*c - b*c^2 + c^3)*co
sh(x)^2 + 15*((b^3 + 3*b^2*c + 3*b*c^2 + c^3)*cosh(x)^4 - 3*b^3 + 3*b^2*c +
 3*b*c^2 - 3*c^3 + 18*(b^3 + b^2*c - b*c^2 - c^3)*cosh(x)^2)*sinh(x)^2 + 6*
((b^3 + 3*b^2*c + 3*b*c^2 + c^3)*cosh(x)^5 + 30*(b^3 + b^2*c - b*c^2 - c^3)
```

```
*cosh(x)^3 - 15*(b^3 - b^2*c - b*c^2 + c^3)*cosh(x))*sinh(x) + 3*(3*(b^2 +
2*b*c + c^2)*cosh(x)^5 + 15*(b^2 + 2*b*c + c^2)*cosh(x)*sinh(x)^4 + 3*(b^2
+ 2*b*c + c^2)*sinh(x)^5 + 20*(b^2 - c^2)*x*cosh(x)^3 + 10*(3*(b^2 + 2*b*c
+ c^2)*cosh(x)^2 + 2*(b^2 - c^2)*x)*sinh(x)^3 + 30*((b^2 + 2*b*c + c^2)*cos
h(x)^3 + 2*(b^2 - c^2)*x*cosh(x))*sinh(x)^2 - 3*(b^2 - 2*b*c + c^2)*cosh(x)
 + 3*(5*(b^2 + 2*b*c + c^2)*cosh(x)^4 + 20*(b^2 - c^2)*x*cosh(x)^2 - b^2 +
2*b*c - c^2)*sinh(x))*sqrt(b^2 - c^2))/(cosh(x)^3 + 3*cosh(x)^2*sinh(x) + 3
*cosh(x)*sinh(x)^2 + sinh(x)^3)
```

giac [A] time = 0.15, size = 194, normalized size = 1.43

$$\frac{5}{2}\left(b^2-c^2\right)^{\frac{3}{2}}x+\frac{3}{8}\left(b^2+2\,bc+c^2\right)\sqrt{b^2-c^2}\,e^{(2\,x)}+\frac{1}{24}\left(b^3+3\,b^2c+3\,bc^2+c^3\right)e^{(3\,x)}-\frac{1}{24}\left(b^3-3\,b^2c+3\,bc^2-c^3+4\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cosh(x)+c*sinh(x)+(b^2-c^2)^(1/2))^3,x, algorithm="giac")
```

```
[Out] 5/2*(b^2 - c^2)^(3/2)*x + 3/8*(b^2 + 2*b*c + c^2)*sqrt(b^2 - c^2)*e^(2*x) +
 1/24*(b^3 + 3*b^2*c + 3*b*c^2 + c^3)*e^(3*x) - 1/24*(b^3 - 3*b^2*c + 3*b*c
^2 - c^3 + 45*(b^3 - b^2*c - b*c^2 + c^3)*e^(2*x) + 9*(sqrt(b^2 - c^2)*b^2
- 2*sqrt(b^2 - c^2)*b*c + sqrt(b^2 - c^2)*c^2)*e^x)*e^(-3*x) + 15/8*(b^3 +
b^2*c - b*c^2 - c^3)*e^x
```

maple [A] time = 0.42, size = 182, normalized size = 1.34

$$b^3\left(\frac{2}{3}+\frac{\left(\cosh^2(x)\right)}{3}\right)\sinh(x)+\left(\cosh^3(x)\right)b^2c+3\sqrt{b^2-c^2}\;b^2\left(\frac{\cosh(x)\sinh(x)}{2}+\frac{x}{2}\right)+b\,c^2\left(\sinh^3(x)\right)+3\sqrt{b^2-c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cosh(x)+c*sinh(x)+(b^2-c^2)^(1/2))^3,x)
```

```
[Out] b^3*(2/3+1/3*cosh(x)^2)*sinh(x)+cosh(x)^3*b^2*c+3*(b^2-c^2)^(1/2)*b^2*(1/2*
cosh(x)*sinh(x)+1/2*x)+b*c^2*sinh(x)^3+3*(b^2-c^2)^(1/2)*b*c*cosh(x)^2+3*b^
3*sinh(x)-3*b*c^2*sinh(x)+c^3*(-2/3+1/3*sinh(x)^2)*cosh(x)+3*(b^2-c^2)^(1/2
)*c^2*(1/2*cosh(x)*sinh(x)-1/2*x)+3*c*b^2*cosh(x)-3*c^3*cosh(x)+(b^2-c^2)^(
1/2)*b^2*x-(b^2-c^2)^(1/2)*c^2*x
```

maxima [A] time = 0.45, size = 161, normalized size = 1.18

$$b^2c\cosh(x)^3+bc^2\sinh(x)^3+\frac{1}{24}\,c^3\left(e^{(3\,x)}-9\,e^{(-x)}+e^{(-3\,x)}-9\,e^x\right)+\frac{1}{24}\,b^3\left(e^{(3\,x)}-9\,e^{(-x)}-e^{(-3\,x)}+9\,e^x\right)+\left(b^2-c^2\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cosh(x)+c*sinh(x)+(b^2-c^2)^(1/2))^3,x, algorithm="maxima")
```

```
[Out] b^2*c*cosh(x)^3 + b*c^2*sinh(x)^3 + 1/24*c^3*(e^(3*x) - 9*e^(-x) + e^(-3*x)
 - 9*e^x) + 1/24*b^3*(e^(3*x) - 9*e^(-x) - e^(-3*x) + 9*e^x) + (b^2 - c^2)^
(3/2)*x + 3*(b^2 - c^2)*(c*cosh(x) + b*sinh(x)) + 3/8*(8*b*c*cosh(x)^2 + b^
2*(4*x + e^(2*x) - e^(-2*x)) - c^2*(4*x - e^(2*x) + e^(-2*x)))*sqrt(b^2 - c
^2)
```

mupad [B] time = 1.66, size = 144, normalized size = 1.06

$$\frac{11\,b^3\sinh(x)}{3}+\frac{c^3\cosh(x)^3}{3}+\frac{5\,x\left(b^2-c^2\right)^{3/2}}{2}-4\,c^3\cosh(x)+\frac{b^3\cosh(x)^2\sinh(x)}{3}+3\,b^2\,c\cosh(x)-4\,b\,c^2\sinh(x)+b^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cosh(x) + (b^2 - c^2)^(1/2) + c*sinh(x))^3,x)
```

```
[Out] (11*b^3*sinh(x))/3 + (c^3*cosh(x)^3)/3 + (5*x*(b^2 - c^2)^(3/2))/2 - 4*c^3*
cosh(x) + (b^3*cosh(x)^2*sinh(x))/3 + 3*b^2*c*cosh(x) - 4*b*c^2*sinh(x) + b
^2*c*cosh(x)^3 + 3*b*c*cosh(x)^2*(b^2 - c^2)^(1/2) + (3*b^2*cosh(x)*sinh(x)
*(b^2 - c^2)^(1/2))/2 + (3*c^2*cosh(x)*sinh(x)*(b^2 - c^2)^(1/2))/2 + b*c^2
*cosh(x)^2*sinh(x)
```

sympy [B] time = 0.69, size = 298, normalized size = 2.19

$$-\frac{2b^3\sinh^3(x)}{3}+b^3\sinh(x)\cosh^2(x)+3b^3\sinh(x)+b^2c\cosh^3(x)+3b^2c\cosh(x)-\frac{3b^2x\sqrt{b^2-c^2}\,\sinh^2(x)}{2}+\frac{3b^2x\sqrt{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cosh(x)+c*sinh(x)+(b**2-c**2)**(1/2))**3,x)
```

```
[Out] -2*b**3*sinh(x)**3/3 + b**3*sinh(x)*cosh(x)**2 + 3*b**3*sinh(x) + b**2*c*co
sh(x)**3 + 3*b**2*c*cosh(x) - 3*b**2*x*sqrt(b**2 - c**2)*sinh(x)**2/2 + 3*b
**2*x*sqrt(b**2 - c**2)*cosh(x)**2/2 + b**2*x*sqrt(b**2 - c**2) + 3*b**2*sq
rt(b**2 - c**2)*sinh(x)*cosh(x)/2 + b*c**2*sinh(x)**3 - 3*b*c**2*sinh(x) +
3*b*c*sqrt(b**2 - c**2)*cosh(x)**2 + c**3*sinh(x)**2*cosh(x) - 2*c**3*cosh(
x)**3/3 - 3*c**3*cosh(x) + 3*c**2*x*sqrt(b**2 - c**2)*sinh(x)**2/2 - 3*c**2
*x*sqrt(b**2 - c**2)*cosh(x)**2/2 - c**2*x*sqrt(b**2 - c**2) + 3*c**2*sqrt(
b**2 - c**2)*sinh(x)*cosh(x)/2
```

3.755 $\int \left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^2\,dx$

Optimal. Leaf size=90

$$\frac{3}{2}x\left(b^2-c^2\right)+\frac{3}{2}b\sqrt{b^2-c^2}\,\sinh(x)+\frac{3}{2}c\sqrt{b^2-c^2}\,\cosh(x)+\frac{1}{2}(b\sinh(x)+c\cosh(x))\left(\sqrt{b^2-c^2}\,+b\cosh(x)+c\sin\right.$$

```
[Out] 3/2*(b^2-c^2)*x+3/2*c*cosh(x)*(b^2-c^2)^(1/2)+3/2*b*sinh(x)*(b^2-c^2)^(1/2)
+1/2*(c*cosh(x)+b*sinh(x))*(b*cosh(x)+c*sinh(x)+(b^2-c^2)^(1/2))
```

Rubi [A] time = 0.05, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3113, 2637, 2638}

$$\frac{3}{2}x\left(b^2-c^2\right)+\frac{3}{2}b\sqrt{b^2-c^2}\,\sinh(x)+\frac{3}{2}c\sqrt{b^2-c^2}\,\cosh(x)+\frac{1}{2}(b\sinh(x)+c\cosh(x))\left(\sqrt{b^2-c^2}\,+b\cosh(x)+c\sin\right.$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[b^2 - c^2] + b*Cosh[x] + c*Sinh[x])^2,x]
```

```
[Out] (3*(b^2 - c^2)*x)/2 + (3*c*Sqrt[b^2 - c^2]*Cosh[x])/2 + (3*b*Sqrt[b^2 - c^2
]*Sinh[x])/2 + ((c*Cosh[x] + b*Sinh[x])*(Sqrt[b^2 - c^2] + b*Cosh[x] + c*Si
nh[x]))/2
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3113

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> -Simp[((c*Cos[d + e*x] - b*Sin[d + e*x])*(a + b*Cos[d +
e*x] + c*Sin[d + e*x])^(n - 1))/(e*n), x] + Dist[(a*(2*n - 1))/n, Int[(a +
b*Cos[d + e*x] + c*Sin[d + e*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e},
x] && EqQ[a^2 - b^2 - c^2, 0] && GtQ[n, 0]
```

Rubi steps

$$\int \left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^2 dx = \frac{1}{2}(c\cosh(x)+b\sinh(x))\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)+\frac{1}{2}\Big(3$$
$$= \frac{3}{2}\left(b^2-c^2\right)x+\frac{1}{2}(c\cosh(x)+b\sinh(x))\left(\sqrt{b^2-c^2}+b\cosh(x)+c$$
$$= \frac{3}{2}\left(b^2-c^2\right)x+\frac{3}{2}c\sqrt{b^2-c^2}\cosh(x)+\frac{3}{2}b\sqrt{b^2-c^2}\sinh(x)+\frac{1}{2}(c\,\mathrm{co}$$

Mathematica [A] time = 0.12, size = 72, normalized size = 0.80

$$\frac{1}{4}\left(8b\sqrt{b^2-c^2}\sinh(x)+\left(b^2+c^2\right)\sinh(2x)+8c\sqrt{b^2-c^2}\cosh(x)+6x(b-c)(b+c)+2bc\cosh(2x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[b^2 - c^2] + b*Cosh[x] + c*Sinh[x])^2,x]
```

```
[Out] (6*(b - c)*(b + c)*x + 8*c*Sqrt[b^2 - c^2]*Cosh[x] + 2*b*c*Cosh[2*x] + 8*b*
Sqrt[b^2 - c^2]*Sinh[x] + (b^2 + c^2)*Sinh[2*x])/4
```

fricas [B] time = 0.41, size = 238, normalized size = 2.64

$$\frac{\left(b^2+2\,bc+c^2\right)\cosh(x)^4+4\left(b^2+2\,bc+c^2\right)\cosh(x)\sinh(x)^3+\left(b^2+2\,bc+c^2\right)\sinh(x)^4+12\left(b^2-c^2\right)x\cosh(x}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cosh(x)+c*sinh(x)+(b^2-c^2)^(1/2))^2,x, algorithm="fricas")
```

```
[Out] 1/8*((b^2 + 2*b*c + c^2)*cosh(x)^4 + 4*(b^2 + 2*b*c + c^2)*cosh(x)*sinh(x)^
3 + (b^2 + 2*b*c + c^2)*sinh(x)^4 + 12*(b^2 - c^2)*x*cosh(x)^2 + 6*((b^2 +
2*b*c + c^2)*cosh(x)^2 + 2*(b^2 - c^2)*x)*sinh(x)^2 - b^2 + 2*b*c - c^2 + 4
*((b^2 + 2*b*c + c^2)*cosh(x)^3 + 6*(b^2 - c^2)*x*cosh(x))*sinh(x) + 8*((b
+ c)*cosh(x)^3 + 3*(b + c)*cosh(x)*sinh(x)^2 + (b + c)*sinh(x)^3 - (b - c)*
cosh(x) + (3*(b + c)*cosh(x)^2 - b + c)*sinh(x))*sqrt(b^2 - c^2))/(cosh(x)^
2 + 2*cosh(x)*sinh(x) + sinh(x)^2)
```

giac [A] time = 0.14, size = 96, normalized size = 1.07

$$\sqrt{b^2-c^2}\,(b+c)e^x+\frac{3}{2}\left(b^2-c^2\right)x+\frac{1}{8}\left(b^2+2\,bc+c^2\right)e^{(2\,x)}-\frac{1}{8}\left(b^2-2\,bc+c^2+8\left(\sqrt{b^2-c^2}\,b-\sqrt{b^2-c^2}\,c\right)e^x\right)e^{(-2\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cosh(x)+c*sinh(x)+(b^2-c^2)^(1/2))^2,x, algorithm="giac")
```

```
[Out] sqrt(b^2 - c^2)*(b + c)*e^x + 3/2*(b^2 - c^2)*x + 1/8*(b^2 + 2*b*c + c^2)*e
^(2*x) - 1/8*(b^2 - 2*b*c + c^2 + 8*(sqrt(b^2 - c^2)*b - sqrt(b^2 - c^2)*c)
*e^x)*e^(-2*x)
```

maple [A] time = 0.16, size = 80, normalized size = 0.89

$$b^2\left(\frac{\cosh(x)\sinh(x)}{2}+\frac{x}{2}\right)+cb\left(\cosh^2(x)\right)+c^2\left(\frac{\cosh(x)\sinh(x)}{2}-\frac{x}{2}\right)+2b\sinh(x)\sqrt{b^2-c^2}+2c\cosh(x)\sqrt{b^2-c^2}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cosh(x)+c*sinh(x)+(b^2-c^2)^(1/2))^2,x)
```

```
[Out] b^2*(1/2*cosh(x)*sinh(x)+1/2*x)+c*b*cosh(x)^2+c^2*(1/2*cosh(x)*sinh(x)-1/2*
x)+2*b*sinh(x)*(b^2-c^2)^(1/2)+2*c*cosh(x)*(b^2-c^2)^(1/2)+b^2*x-c^2*x
```

maxima [A] time = 0.31, size = 79, normalized size = 0.88

$$bc\cosh(x)^2+\frac{1}{8}b^2\left(4x+e^{(2x)}-e^{(-2x)}\right)-\frac{1}{8}c^2\left(4x-e^{(2x)}+e^{(-2x)}\right)+b^2x-c^2x+2\sqrt{b^2-c^2}\left(c\cosh(x)+b\sinh(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cosh(x)+c*sinh(x)+(b^2-c^2)^(1/2))^2,x, algorithm="maxima")
```

```
[Out] b*c*cosh(x)^2 + 1/8*b^2*(4*x + e^(2*x) - e^(-2*x)) - 1/8*c^2*(4*x - e^(2*x)
 + e^(-2*x)) + b^2*x - c^2*x + 2*sqrt(b^2 - c^2)*(c*cosh(x) + b*sinh(x))
```

mupad [B] time = 1.61, size = 70, normalized size = 0.78

$$\frac{3b^2x}{2}-\frac{3c^2x}{2}+2c\cosh(x)\sqrt{b^2-c^2}+2b\sinh(x)\sqrt{b^2-c^2}+bc\cosh(x)^2+\frac{b^2\cosh(x)\sinh(x)}{2}+\frac{c^2\cosh(x)\sinh(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cosh(x) + (b^2 - c^2)^(1/2) + c*sinh(x))^2,x)
```

```
[Out] (3*b^2*x)/2 - (3*c^2*x)/2 + 2*c*cosh(x)*(b^2 - c^2)^(1/2) + 2*b*sinh(x)*(b^
2 - c^2)^(1/2) + b*c*cosh(x)^2 + (b^2*cosh(x)*sinh(x))/2 + (c^2*cosh(x)*sin
h(x))/2
```

sympy [A] time = 0.29, size = 122, normalized size = 1.36

$$-\frac{b^2x\sinh^2(x)}{2}+\frac{b^2x\cosh^2(x)}{2}+b^2x+\frac{b^2\sinh(x)\cosh(x)}{2}+bc\cosh^2(x)+2b\sqrt{b^2-c^2}\sinh(x)+\frac{c^2x\sinh^2(x)}{2}-\frac{c^2x}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cosh(x)+c*sinh(x)+(b**2-c**2)**(1/2))**2,x)
```

```
[Out] -b**2*x*sinh(x)**2/2 + b**2*x*cosh(x)**2/2 + b**2*x + b**2*sinh(x)*cosh(x)/
2 + b*c*cosh(x)**2 + 2*b*sqrt(b**2 - c**2)*sinh(x) + c**2*x*sinh(x)**2/2 -
c**2*x*cosh(x)**2/2 - c**2*x + c**2*sinh(x)*cosh(x)/2 + 2*c*sqrt(b**2 - c**
2)*cosh(x)
```

3.756 $\int \left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)\,dx$

Optimal. Leaf size=24

$$x\sqrt{b^2-c^2}+b\sinh(x)+c\cosh(x)$$

```
[Out] c*cosh(x)+b*sinh(x)+x*(b^2-c^2)^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {2637, 2638}

$$x\sqrt{b^2-c^2}+b\sinh(x)+c\cosh(x)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b^2 - c^2] + b*Cosh[x] + c*Sinh[x],x]
```

```
[Out] Sqrt[b^2 - c^2]*x + c*Cosh[x] + b*Sinh[x]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)\,dx &= \sqrt{b^2-c^2}\,x+b\int \cosh(x)\,dx+c\int \sinh(x)\,dx\\ &= \sqrt{b^2-c^2}\,x+c\cosh(x)+b\sinh(x)\end{aligned}$$

Mathematica [A] time = 0.01, size = 24, normalized size = 1.00

$$x\sqrt{b^2-c^2}+b\sinh(x)+c\cosh(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b^2 - c^2] + b*Cosh[x] + c*Sinh[x],x]
```

```
[Out] Sqrt[b^2 - c^2]*x + c*Cosh[x] + b*Sinh[x]
```

fricas [B] time = 0.40, size = 61, normalized size = 2.54

$$\frac{(b+c)\cosh(x)^2 + 2\,(b+c)\cosh(x)\sinh(x) + (b+c)\sinh(x)^2 + 2\,\sqrt{b^2-c^2}\,(x\cosh(x) + x\sinh(x)) - b + c}{2\,(\cosh(x) + \sinh(x))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(b*cosh(x)+c*sinh(x)+(b^2-c^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*((b + c)*cosh(x)^2 + 2*(b + c)*cosh(x)*sinh(x) + (b + c)*sinh(x)^2 + 2*
sqrt(b^2 - c^2)*(x*cosh(x) + x*sinh(x)) - b + c)/(cosh(x) + sinh(x))
```

giac [A] time = 0.13, size = 36, normalized size = 1.50

$$\frac{1}{2}\,c\left(e^{(-x)} + e^x\right) - \frac{1}{2}\,b\left(e^{(-x)} - e^x\right) + \sqrt{b^2-c^2}\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(b*cosh(x)+c*sinh(x)+(b^2-c^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*c*(e^(-x) + e^x) - 1/2*b*(e^(-x) - e^x) + sqrt(b^2 - c^2)*x
```

maple [A] time = 0.02, size = 23, normalized size = 0.96

$$c\cosh(x) + b\sinh(x) + x\sqrt{b^2-c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(b*cosh(x)+c*sinh(x)+(b^2-c^2)^(1/2),x)
```

```
[Out] c*cosh(x)+b*sinh(x)+x*(b^2-c^2)^(1/2)
```

maxima [A] time = 0.47, size = 22, normalized size = 0.92

$$c\cosh(x) + b\sinh(x) + \sqrt{b^2-c^2}\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(b*cosh(x)+c*sinh(x)+(b^2-c^2)^(1/2),x, algorithm="maxima")
```

```
[Out] c*cosh(x) + b*sinh(x) + sqrt(b^2 - c^2)*x
```

mupad [B] time = 0.06, size = 22, normalized size = 0.92

$$x\,\sqrt{b^2-c^2}\, + c\cosh(x) + b\sinh(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(b*cosh(x) + (b^2 - c^2)^(1/2) + c*sinh(x),x)
```

```
[Out] x*(b^2 - c^2)^(1/2) + c*cosh(x) + b*sinh(x)
```

sympy [**A**] time = 0.12, size = 20, normalized size = 0.83

$$b\sinh(x) + c\cosh(x) + x\sqrt{b^2 - c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(b*cosh(x)+c*sinh(x)+(b**2-c**2)**(1/2),x)
```

```
[Out] b*sinh(x) + c*cosh(x) + x*sqrt(b**2 - c**2)
```

3.757 $\int \frac{1}{\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)}\,dx$

Optimal. Leaf size=34

$$-\frac{\sqrt{b^2-c^2}\,\sinh(x)+c}{c(b\sinh(x)+c\cosh(x))}$$

```
[Out] (-c-sinh(x)*(b^2-c^2)^(1/2))/c/(c*cosh(x)+b*sinh(x))
```

Rubi [A] time = 0.04, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {3114}

$$-\frac{\sqrt{b^2-c^2}\,\sinh(x)+c}{c(b\sinh(x)+c\cosh(x))}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[b^2 - c^2] + b*Cosh[x] + c*Sinh[x])^(-1),x]
```

```
[Out] -((c + Sqrt[b^2 - c^2]*Sinh[x])/(c*(c*Cosh[x] + b*Sinh[x])))
```

Rule 3114

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> -Simp[(c - a*Sin[d + e*x])/(c*e*(c*Cos[d + e*x] - b*Sin[
d + e*x])), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[a^2 - b^2 - c^2, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)}\,dx=-\frac{c+\sqrt{b^2-c^2}\,\sinh(x)}{c(c\cosh(x)+b\sinh(x))}$$

Mathematica [A] time = 0.08, size = 36, normalized size = 1.06

$$\frac{-\sqrt{b^2-c^2}\,\sinh(x)-c}{c(b\sinh(x)+c\cosh(x))}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[b^2 - c^2] + b*Cosh[x] + c*Sinh[x])^(-1),x]
```

```
[Out] (-c - Sqrt[b^2 - c^2]*Sinh[x])/(c*(c*Cosh[x] + b*Sinh[x]))
```

fricas [B] time = 0.40, size = 88, normalized size = 2.59

$$-\frac{2\left((b+c)\cosh(x)+(b+c)\sinh(x)-\sqrt{b^2-c^2}\right)}{\left(b^2+2\,bc+c^2\right)\cosh(x)^2+2\left(b^2+2\,bc+c^2\right)\cosh(x)\sinh(x)+\left(b^2+2\,bc+c^2\right)\sinh(x)^2-b^2+c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*cosh(x)+c*sinh(x)+(b^2-c^2)^(1/2)),x, algorithm="fricas")
```

```
[Out] -2*((b + c)*cosh(x) + (b + c)*sinh(x) - sqrt(b^2 - c^2))/((b^2 + 2*b*c + c^
2)*cosh(x)^2 + 2*(b^2 + 2*b*c + c^2)*cosh(x)*sinh(x) + (b^2 + 2*b*c + c^2)*
sinh(x)^2 - b^2 + c^2)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*cosh(x)+c*sinh(x)+(b^2-c^2)^(1/2)),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to divide, perhaps due to rounding er
ror%%%{%%%{1,[1,0]%%%}+%%%{1,[0,1]%%%},[2]%%%}+%%%{%%{[2,0]:[1,0,%%%{-1,[2,
0]%%%}+%%%{1,[0,2]%%%}]%%},[1]%%%}+%%%{%%%{1,[1,0]%%%}+%%%{-1,[0,1]%%%},[0]
%%%} / %%%{%%%{1,[2,0]%%%}+%%%{2,[1,1]%%%}+%%%{1,[0,2]%%%},[2]%%%}+%%%{%%{[
%%%{2,[1,0]%%%}+%%%{2,[0,1]%%%},0]:[1,0,%%%{-1,[2,0]%%%}+%%%{1,[0,2]%%%}]%%
},[1]%%%}+%%%{%%%{1,[2,0]%%%}+%%%{-1,[0,2]%%%},[0]%%%} Error: Bad Argument
Value
```

maple [C] time = 0.62, size = 596, normalized size = 17.53

$$\left(\_R=\text{RootOf}\left(\left(|(b-c)(b+c)|\text{signum}((b-c)(b+c))^2-2\sqrt{|(b-c)(b+c)|}\,\text{signum}((b-c)(b+c))b+|(b-c)(b+c)|-2\sqrt{|(b-c)(b+c)|}\,b+2b^2\right)\_Z^4+\left(-4\sqrt{|(b-c)(b+c)|}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*cosh(x)+c*sinh(x)+(b^2-c^2)^(1/2)),x)
```

```
[Out] 1/2*sum((2*b*_R^2+4*_R*c+2*b+abs(b^2-c^2)^(1/2)*(1-I+_R^2*(-1+I-I*signum(b^
2-c^2)-signum(b^2-c^2))+I*signum(b^2-c^2)+signum(b^2-c^2)))/(abs((b-c)*(b+c
))*_R^3*signum((b-c)*(b+c))^2-abs((b-c)*(b+c))*_R*signum((b-c)*(b+c))^2+abs
((b-c)*(b+c))*_R^3+2*b^2*_R^3+6*_R^2*b*c-abs((b-c)*(b+c))*_R+2*b^2*_R+4*c^2
*_R+2*c*b+abs((b-c)*(b+c))^(1/2)*(-2*signum((b-c)*(b+c))*b*_R^3-3*signum((b
```

```
-c)*(b+c))*c*_R^2-2*b*_R^3-3*c*_R^2+signum((b-c)*(b+c))*c+c))*ln(tanh(1/2*x
)-_R),_R=RootOf((abs((b-c)*(b+c))*signum((b-c)*(b+c))^2-2*abs((b-c)*(b+c))^
(1/2)*signum((b-c)*(b+c))*b+abs((b-c)*(b+c))-2*abs((b-c)*(b+c))^(1/2)*b+2*b
^2)*_Z^4+(-4*abs((b-c)*(b+c))^(1/2)*signum((b-c)*(b+c))*c-4*abs((b-c)*(b+c)
)^(1/2)*c+8*c*b)*_Z^3+(-2*abs((b-c)*(b+c))*signum((b-c)*(b+c))^2-2*abs((b-c
)*(b+c))+4*b^2+8*c^2)*_Z^2+(4*abs((b-c)*(b+c))^(1/2)*signum((b-c)*(b+c))*c+
4*abs((b-c)*(b+c))^(1/2)*c+8*c*b)*_Z+abs((b-c)*(b+c))*signum((b-c)*(b+c))^2
+2*abs((b-c)*(b+c))^(1/2)*signum((b-c)*(b+c))*b+abs((b-c)*(b+c))+2*abs((b-c
)*(b+c))^(1/2)*b+2*b^2))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*cosh(x)+c*sinh(x)+(b^2-c^2)^(1/2)),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{b\cosh(x)+\sqrt{b^2-c^2}+c\sinh(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*cosh(x) + (b^2 - c^2)^(1/2) + c*sinh(x)),x)
```

```
[Out] int(1/(b*cosh(x) + (b^2 - c^2)^(1/2) + c*sinh(x)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*cosh(x)+c*sinh(x)+(b**2-c**2)**(1/2)),x)
```

```
[Out] Timed out
```

3.758 $\int \frac{1}{\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^2}\,dx$

Optimal. Leaf size=100

$$\frac{b\sinh(x)+c\cosh(x)}{3\sqrt{b^2-c^2}\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^2}-\frac{\sqrt{b^2-c^2}\sinh(x)+c}{3c\sqrt{b^2-c^2}\,(b\sinh(x)+c\cosh(x))}$$

```
[Out] 1/3*(c*cosh(x)+b*sinh(x))/(b^2-c^2)^(1/2)/(b*cosh(x)+c*sinh(x)+(b^2-c^2)^(1
/2))^2+1/3*(-c-sinh(x)*(b^2-c^2)^(1/2))/c/(c*cosh(x)+b*sinh(x))/(b^2-c^2)^(
1/2)
```

Rubi [A] time = 0.08, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {3116, 3114}

$$\frac{b\sinh(x)+c\cosh(x)}{3\sqrt{b^2-c^2}\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^2}-\frac{\sqrt{b^2-c^2}\sinh(x)+c}{3c\sqrt{b^2-c^2}\,(b\sinh(x)+c\cosh(x))}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[b^2 - c^2] + b*Cosh[x] + c*Sinh[x])^(-2),x]
```

```
[Out] (c*Cosh[x] + b*Sinh[x])/(3*Sqrt[b^2 - c^2]*(Sqrt[b^2 - c^2] + b*Cosh[x] + c
*Sinh[x])^2) - (c + Sqrt[b^2 - c^2]*Sinh[x])/(3*c*Sqrt[b^2 - c^2]*(c*Cosh[x
] + b*Sinh[x]))
```

Rule 3114

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> -Simp[(c - a*Sin[d + e*x])/(c*e*(c*Cos[d + e*x] - b*Sin[
d + e*x])), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[a^2 - b^2 - c^2, 0]
```

Rule 3116

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> Simp[((c*Cos[d + e*x] - b*Sin[d + e*x])*(a + b*Cos[d + e
*x] + c*Sin[d + e*x])^n)/(a*e*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n + 1)),
Int[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1), x], x] /; FreeQ[{a, b, c
, d, e}, x] && EqQ[a^2 - b^2 - c^2, 0] && LtQ[n, -1]
```

Rubi steps

$$\int \frac{1}{\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^2}\,dx = \frac{c\cosh(x)+b\sinh(x)}{3\sqrt{b^2-c^2}\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^2} + \frac{\int \frac{1}{\sqrt{b^2-c^2}+b\cosh(x)}}{3\sqrt{b^2-}}$$

$$= \frac{c\cosh(x)+b\sinh(x)}{3\sqrt{b^2-c^2}\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^2} - \frac{c+\sqrt{b^2-}}{3c\sqrt{b^2-c^2}\,(c\cos}$$

Mathematica [A] time = 0.17, size = 68, normalized size = 0.68

$$-\frac{-2c\sqrt{b^2-c^2}+b^2\sinh^3(x)+2bc\cosh^3(x)+2c^2\sinh(x)+c^2\sinh(x)\cosh^2(x)}{3c(b\sinh(x)+c\cosh(x))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[b^2 - c^2] + b*Cosh[x] + c*Sinh[x])^(-2),x]
```

```
[Out] -1/3*(-2*c*Sqrt[b^2 - c^2] + 2*b*c*Cosh[x]^3 + 2*c^2*Sinh[x] + c^2*Cosh[x]^
2*Sinh[x] + b^2*Sinh[x]^3)/(c*(c*Cosh[x] + b*Sinh[x])^3)
```

fricas [B] time = 0.43, size = 660, normalized size = 6.60

$$-\frac{}{3\left(\left(b^4+4b^3c+6b^2c^2+4bc^3+c^4\right)\cosh(x)^6+6\left(b^4+4b^3c+6b^2c^2+4bc^3+c^4\right)\cosh(x)\sinh(x)^5+\left(b^4+4b\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*cosh(x)+c*sinh(x)+(b^2-c^2)^(1/2))^2,x, algorithm="fricas")
```

```
[Out] -2/3*(3*(b^2 + 2*b*c + c^2)*cosh(x)^4 + 12*(b^2 + 2*b*c + c^2)*cosh(x)*sinh
(x)^3 + 3*(b^2 + 2*b*c + c^2)*sinh(x)^4 + 6*(b^2 - c^2)*cosh(x)^2 + 6*(3*(b
^2 + 2*b*c + c^2)*cosh(x)^2 + b^2 - c^2)*sinh(x)^2 - b^2 + 2*b*c - c^2 + 12
*((b^2 + 2*b*c + c^2)*cosh(x)^3 + (b^2 - c^2)*cosh(x))*sinh(x) - 8*((b + c)
*cosh(x)^3 + 3*(b + c)*cosh(x)^2*sinh(x) + 3*(b + c)*cosh(x)*sinh(x)^2 + (b
 + c)*sinh(x)^3)*sqrt(b^2 - c^2))/((b^4 + 4*b^3*c + 6*b^2*c^2 + 4*b*c^3 + c
^4)*cosh(x)^6 + 6*(b^4 + 4*b^3*c + 6*b^2*c^2 + 4*b*c^3 + c^4)*cosh(x)*sinh(
x)^5 + (b^4 + 4*b^3*c + 6*b^2*c^2 + 4*b*c^3 + c^4)*sinh(x)^6 - 3*(b^4 + 2*b
^3*c - 2*b*c^3 - c^4)*cosh(x)^4 - 3*(b^4 + 2*b^3*c - 2*b*c^3 - c^4 - 5*(b^4
 + 4*b^3*c + 6*b^2*c^2 + 4*b*c^3 + c^4)*cosh(x)^2)*sinh(x)^4 - b^4 + 2*b^3*
c - 2*b*c^3 + c^4 + 4*(5*(b^4 + 4*b^3*c + 6*b^2*c^2 + 4*b*c^3 + c^4)*cosh(x
)^3 - 3*(b^4 + 2*b^3*c - 2*b*c^3 - c^4)*cosh(x))*sinh(x)^3 + 3*(b^4 - 2*b^2
```

```
*c^2 + c^4)*cosh(x)^2 + 3*(5*(b^4 + 4*b^3*c + 6*b^2*c^2 + 4*b*c^3 + c^4)*co
sh(x)^4 + b^4 - 2*b^2*c^2 + c^4 - 6*(b^4 + 2*b^3*c - 2*b*c^3 - c^4)*cosh(x)
^2)*sinh(x)^2 + 6*((b^4 + 4*b^3*c + 6*b^2*c^2 + 4*b*c^3 + c^4)*cosh(x)^5 -
2*(b^4 + 2*b^3*c - 2*b*c^3 - c^4)*cosh(x)^3 + (b^4 - 2*b^2*c^2 + c^4)*cosh(
x))*sinh(x))
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*cosh(x)+c*sinh(x)+(b^2-c^2)^(1/2))^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to divide, perhaps due to rounding er
ror%%%{%%%{1,[2,0]%%%}+%%%{2,[1,1]%%%}+%%%{1,[0,2]%%%},[4]%%%}+%%%{%%{[%%%{
4,[1,0]%%%}+%%%{4,[0,1]%%%},0]:[1,0,%%%{-1,[2,0]%%%}+%%%{1,[0,2]%%%}]%%},[3
]%%%}+%%%{%%%{6,[2,0]%%%}+%%%{-6,[0,2]%%%},[2]%%%}+%%%{%%{[%%%{4,[1,0]%%%}+
%%%{-4,[0,1]%%%},0]:[1,0,%%%{-1,[2,0]%%%}+%%%{1,[0,2]%%%}]%%},[1]%%%}+%%%{%
%%{1,[2,0]%%%}+%%%{-2,[1,1]%%%}+%%%{1,[0,2]%%%},[0]%%%} / %%%{%%%{1,[4,0]%%
%}+%%%{4,[3,1]%%%}+%%%{6,[2,2]%%%}+%%%{4,[1,3]%%%}+%%%{1,[0,4]%%%},[4]%%%}+
%%%{%%{[%%%{4,[3,0]%%%}+%%%{12,[2,1]%%%}+%%%{12,[1,2]%%%}+%%%{4,[0,3]%%%},0
]:[1,0,%%%{-1,[2,0]%%%}+%%%{1,[0,2]%%%}]%%},[3]%%%}+%%%{%%%{6,[4,0]%%%}+%%%
{12,[3,1]%%%}+%%%{-12,[1,3]%%%}+%%%{-6,[0,4]%%%},[2]%%%}+%%%{%%{[%%%{4,[3,0
]%%%}+%%%{4,[2,1]%%%}+%%%{-4,[1,2]%%%}+%%%{-4,[0,3]%%%},0]:[1,0,%%%{-1,[2,0
]%%%}+%%%{1,[0,2]%%%}]%%},[1]%%%}+%%%{%%%{1,[4,0]%%%}+%%%{-2,[2,2]%%%}+%%%{
1,[0,4]%%%},[0]%%%} Error: Bad Argument Value
```

maple [B] time = 0.36, size = 217, normalized size = 2.17

$$\frac{2\left(\sqrt{b^2-c^2}+b\right)\left(\frac{\left(\sqrt{b^2-c^2}+b\right)\left(\tanh^2\left(\frac{x}{2}\right)\right)}{c^2}+\frac{\left(2b^2-c^2+2\sqrt{b^2-c^2}\,b\right)\tanh\left(\frac{x}{2}\right)}{c^3}+\frac{\frac{4\sqrt{b^2-c^2}\,b^2}{3}-\frac{2\sqrt{b^2-c^2}\,c^2}{3}+\frac{4b^3}{3}-\frac{4b\,c^2}{3}}{c^4}\right)}{c^2\left(\tanh^2\left(\frac{x}{2}\right)+\frac{2\sqrt{(b-c)(b+c)}\,\tanh\left(\frac{x}{2}\right)}{c}+\frac{2\tanh\left(\frac{x}{2}\right)b}{c}+\frac{2\sqrt{(b-c)(b+c)}\,b}{c^2}+\frac{2b^2}{c^2}-1\right)\left(\tanh\left(\frac{x}{2}\right)+\frac{\sqrt{(b-c)(b+c)}}{c}+\frac{b}{c}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*cosh(x)+c*sinh(x)+(b^2-c^2)^(1/2))^2,x)
```

```
[Out] 2*((b^2-c^2)^(1/2)+b)/c^2*(((b^2-c^2)^(1/2)+b)/c^2*tanh(1/2*x)^2+(2*b^2-c^2
+2*(b^2-c^2)^(1/2)*b)/c^3*tanh(1/2*x)+2/3*(2*(b^2-c^2)^(1/2)*b^2-(b^2-c^2)^
(1/2)*c^2+2*b^3-2*b*c^2)/c^4)/(tanh(1/2*x)^2+2/c*((b-c)*(b+c))^(1/2)*tanh(1
/2*x)+2/c*tanh(1/2*x)*b+2/c^2*((b-c)*(b+c))^(1/2)*b+2/c^2*b^2-1)/(tanh(1/2*
x)+1/c*((b-c)*(b+c))^(1/2)+b/c)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*cosh(x)+c*sinh(x)+(b^2-c^2)^(1/2))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(b\cosh(x)+\sqrt{b^2-c^2}+c\sinh(x)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*cosh(x) + (b^2 - c^2)^(1/2) + c*sinh(x))^2,x)
```

```
[Out] int(1/(b*cosh(x) + (b^2 - c^2)^(1/2) + c*sinh(x))^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*cosh(x)+c*sinh(x)+(b**2-c**2)**(1/2))**2,x)
```

```
[Out] Timed out
```

3.759 $\int \frac{1}{\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^3}\,dx$

Optimal. Leaf size=146

$$\frac{2(b\sinh(x)+c\cosh(x))}{15\left(b^2-c^2\right)\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^2}+\frac{b\sinh(x)+c\cosh(x)}{5\sqrt{b^2-c^2}\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^3}-\frac{2\left(\sqrt{b\cdots}\right.}{15c\left(b^2-c^2\cdots\right.}$$

```
[Out] 1/5*(c*cosh(x)+b*sinh(x))/(b^2-c^2)^(1/2)/(b*cosh(x)+c*sinh(x)+(b^2-c^2)^(1
/2))^3+2/15*(c*cosh(x)+b*sinh(x))/(b^2-c^2)/(b*cosh(x)+c*sinh(x)+(b^2-c^2)^
(1/2))^2-2/15*(c+sinh(x)*(b^2-c^2)^(1/2))/c/(b^2-c^2)/(c*cosh(x)+b*sinh(x))
```

Rubi [A] time = 0.12, antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {3116, 3114}

$$\frac{2(b\sinh(x)+c\cosh(x))}{15\left(b^2-c^2\right)\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^2}+\frac{b\sinh(x)+c\cosh(x)}{5\sqrt{b^2-c^2}\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^3}-\frac{2\left(\sqrt{b\cdots}\right.}{15c\left(b^2-c^2\cdots\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[b^2 - c^2] + b*Cosh[x] + c*Sinh[x])^(-3),x]
```

```
[Out] (c*Cosh[x] + b*Sinh[x])/(5*Sqrt[b^2 - c^2]*(Sqrt[b^2 - c^2] + b*Cosh[x] + c
*Sinh[x])^3) + (2*(c*Cosh[x] + b*Sinh[x]))/(15*(b^2 - c^2)*(Sqrt[b^2 - c^2]
 + b*Cosh[x] + c*Sinh[x])^2) - (2*(c + Sqrt[b^2 - c^2]*Sinh[x]))/(15*c*(b^2
 - c^2)*(c*Cosh[x] + b*Sinh[x]))
```

Rule 3114

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> -Simp[(c - a*Sin[d + e*x])/(c*e*(c*Cos[d + e*x] - b*Sin[
d + e*x])), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[a^2 - b^2 - c^2, 0]
```

Rule 3116

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> Simp[((c*Cos[d + e*x] - b*Sin[d + e*x])*(a + b*Cos[d + e
*x] + c*Sin[d + e*x])^n)/(a*e*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n + 1)),
Int[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1), x], x] /; FreeQ[{a, b, c
, d, e}, x] && EqQ[a^2 - b^2 - c^2, 0] && LtQ[n, -1]
```

Rubi steps

$$\int \frac{1}{\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^3}\,dx = \frac{c\cosh(x)+b\sinh(x)}{5\sqrt{b^2-c^2}\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^3}+\frac{2\int\frac{1}{\left(\sqrt{b^2-c^2}+b\cosh\ldots\right.}}{5\sqrt{b^2\ldots}}$$

$$= \frac{c\cosh(x)+b\sinh(x)}{5\sqrt{b^2-c^2}\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^3}+\frac{2(c\,c\ldots}{15\left(b^2-c^2\right)\left(\sqrt{b^2\ldots}\right.}$$

$$= \frac{c\cosh(x)+b\sinh(x)}{5\sqrt{b^2-c^2}\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^3}+\frac{2(c\,c\ldots}{15\left(b^2-c^2\right)\left(\sqrt{b^2\ldots}\right.}$$

Mathematica [A] time = 0.40, size = 184, normalized size = 1.26

$$\frac{-12\left(b^2-c^2\right)\left(\sqrt{b^2-c^2}\sinh(x)+c\right)-\frac{2\sqrt{b^2-c^2}\sinh(x)(b\sinh(x)+c\cosh(x))^4}{(b-c)(b+c)}-\frac{b\sqrt{b^2-c^2}(b\sinh(x)+c\cosh(x))^3}{(b-c)(b+c)}+\left(\sqrt{b^2-c^2}\sin\ldots\right.}{15c(b\sinh(x)+c\cosh(x))^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[b^2 - c^2] + b*Cosh[x] + c*Sinh[x])^(-3),x]
```

```
[Out] (12*b*Sqrt[b^2 - c^2]*(c*Cosh[x] + b*Sinh[x]) - (b*Sqrt[b^2 - c^2]*(c*Cosh[
x] + b*Sinh[x])^3)/((b - c)*(b + c)) - (2*Sqrt[b^2 - c^2]*Sinh[x]*(c*Cosh[x
] + b*Sinh[x])^4)/((b - c)*(b + c)) + (c*Cosh[x] + b*Sinh[x])^2*(-5*c + Sqr
t[b^2 - c^2]*Sinh[x]) - 12*(b^2 - c^2)*(c + Sqrt[b^2 - c^2]*Sinh[x]))/(15*c
*(c*Cosh[x] + b*Sinh[x])^5)
```

fricas [B] time = 0.50, size = 3035, normalized size = 20.79

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*cosh(x)+c*sinh(x)+(b^2-c^2)^(1/2))^3,x, algorithm="fricas")
```

```
[Out] -4/15*(10*(b^4 + 4*b^3*c + 6*b^2*c^2 + 4*b*c^3 + c^4)*cosh(x)^7 + 70*(b^4 +
 4*b^3*c + 6*b^2*c^2 + 4*b*c^3 + c^4)*cosh(x)*sinh(x)^6 + 10*(b^4 + 4*b^3*c
 + 6*b^2*c^2 + 4*b*c^3 + c^4)*sinh(x)^7 + 76*(b^4 + 2*b^3*c - 2*b*c^3 - c^4
)*cosh(x)^5 + 2*(38*b^4 + 76*b^3*c - 76*b*c^3 - 38*c^4 + 105*(b^4 + 4*b^3*c
```

+ 6*b^2*c^2 + 4*b*c^3 + c^4)*cosh(x)^2)*sinh(x)^5 + 10*(35*(b^4 + 4*b^3*c + 6*b^2*c^2 + 4*b*c^3 + c^4)*cosh(x)^3 + 38*(b^4 + 2*b^3*c - 2*b*c^3 - c^4)*cosh(x))*sinh(x)^4 + 10*(b^4 - 2*b^2*c^2 + c^4)*cosh(x)^3 + 10*(35*(b^4 + 4*b^3*c + 6*b^2*c^2 + 4*b*c^3 + c^4)*cosh(x)^4 + b^4 - 2*b^2*c^2 + c^4 + 76*(b^4 + 2*b^3*c - 2*b*c^3 - c^4)*cosh(x)^2)*sinh(x)^3 + 10*(21*(b^4 + 4*b^3*c + 6*b^2*c^2 + 4*b*c^3 + c^4)*cosh(x)^5 + 76*(b^4 + 2*b^3*c - 2*b*c^3 - c^4)*cosh(x)^3 + 3*(b^4 - 2*b^2*c^2 + c^4)*cosh(x))*sinh(x)^2 + 10*(7*(b^4 + 4*b^3*c + 6*b^2*c^2 + 4*b*c^3 + c^4)*cosh(x)^6 + 38*(b^4 + 2*b^3*c - 2*b*c^3 - c^4)*cosh(x)^4 + 3*(b^4 - 2*b^2*c^2 + c^4)*cosh(x)^2)*sinh(x) - (45*(b^3 + 3*b^2*c + 3*b*c^2 + c^3)*cosh(x)^6 + 270*(b^3 + 3*b^2*c + 3*b*c^2 + c^3)*cosh(x)*sinh(x)^5 + 45*(b^3 + 3*b^2*c + 3*b*c^2 + c^3)*sinh(x)^6 + 55*(b^3 + b^2*c - b*c^2 - c^3)*cosh(x)^4 + 5*(11*b^3 + 11*b^2*c - 11*b*c^2 - 11*c^3 + 135*(b^3 + 3*b^2*c + 3*b*c^2 + c^3)*cosh(x)^2)*sinh(x)^4 + 20*(45*(b^3 + 3*b^2*c + 3*b*c^2 + c^3)*cosh(x)^3 + 11*(b^3 + b^2*c - b*c^2 - c^3)*cosh(x))*sinh(x)^3 + b^3 - 3*b^2*c + 3*b*c^2 - c^3 - 5*(b^3 - b^2*c - b*c^2 + c^3)*cosh(x)^2 + 5*(135*(b^3 + 3*b^2*c + 3*b*c^2 + c^3)*cosh(x)^4 - b^3 + b^2*c + b*c^2 - c^3 + 66*(b^3 + b^2*c - b*c^2 - c^3)*cosh(x)^2)*sinh(x)^2 + 10*(27*(b^3 + 3*b^2*c + 3*b*c^2 + c^3)*cosh(x)^5 + 22*(b^3 + b^2*c - b*c^2 - c^3)*cosh(x)^3 - (b^3 - b^2*c - b*c^2 + c^3)*cosh(x))*sinh(x))*sqrt(b^2 - c^2))/((b^7 + 7*b^6*c + 21*b^5*c^2 + 35*b^4*c^3 + 35*b^3*c^4 + 21*b^2*c^5 + 7*b*c^6 + c^7)*cosh(x)^10 + 10*(b^7 + 7*b^6*c + 21*b^5*c^2 + 35*b^4*c^3 + 35*b^3*c^4 + 21*b^2*c^5 + 7*b*c^6 + c^7)*cosh(x)*sinh(x)^9 + (b^7 + 7*b^6*c + 21*b^5*c^2 + 35*b^4*c^3 + 35*b^3*c^4 + 21*b^2*c^5 + 7*b*c^6 + c^7)*sinh(x)^10 - 5*(b^7 + 5*b^6*c + 9*b^5*c^2 + 5*b^4*c^3 - 5*b^3*c^4 - 9*b^2*c^5 - 5*b*c^6 - c^7)*cosh(x)^8 - 5*(b^7 + 5*b^6*c + 9*b^5*c^2 + 5*b^4*c^3 - 5*b^3*c^4 - 9*b^2*c^5 - 5*b*c^6 - c^7 - 9*(b^7 + 7*b^6*c + 21*b^5*c^2 + 35*b^4*c^3 + 35*b^3*c^4 + 21*b^2*c^5 + 7*b*c^6 + c^7)*cosh(x)^2)*sinh(x)^8 + 40*(3*(b^7 + 7*b^6*c + 21*b^5*c^2 + 35*b^4*c^3 + 35*b^3*c^4 + 21*b^2*c^5 + 7*b*c^6 + c^7)*cosh(x)^3 - (b^7 + 5*b^6*c + 9*b^5*c^2 + 5*b^4*c^3 - 5*b^3*c^4 - 9*b^2*c^5 - 5*b*c^6 - c^7)*cosh(x))*sinh(x)^7 - b^7 + 3*b^6*c - b^5*c^2 - 5*b^4*c^3 + 5*b^3*c^4 + b^2*c^5 - 3*b*c^6 + c^7 + 10*(b^7 + 3*b^6*c + b^5*c^2 - 5*b^4*c^3 - 5*b^3*c^4 + b^2*c^5 + 3*b*c^6 + c^7)*cosh(x)^6 + 10*(b^7 + 3*b^6*c + b^5*c^2 - 5*b^4*c^3 - 5*b^3*c^4 + b^2*c^5 + 3*b*c^6 + c^7 + 21*(b^7 + 7*b^6*c + 21*b^5*c^2 + 35*b^4*c^3 + 35*b^3*c^4 + 21*b^2*c^5 + 7*b*c^6 + c^7)*cosh(x)^4 - 14*(b^7 + 5*b^6*c + 9*b^5*c^2 + 5*b^4*c^3 - 5*b^3*c^4 - 9*b^2*c^5 - 5*b*c^6 - c^7)*cosh(x)^2)*sinh(x)^6 + 4*(63*(b^7 + 7*b^6*c + 21*b^5*c^2 + 35*b^4*c^3 + 35*b^3*c^4 + 21*b^2*c^5 + 7*b*c^6 + c^7)*cosh(x)^5 - 70*(b^7 + 5*b^6*c + 9*b^5*c^2 + 5*b^4*c^3 - 5*b^3*c^4 - 9*b^2*c^5 - 5*b*c^6 - c^7)*cosh(x)^3 + 15*(b^7 + 3*b^6*c + b^5*c^2 - 5*b^4*c^3 - 5*b^3*c^4 + b^2*c^5 + 3*b*c^6 + c^7)*cosh(x))*sinh(x)^5 - 10*(b^7 + b^6*c - 3*b^5*c^2 - 3*b^4*c^3 + 3*b^3*c^4 + 3*b^2*c^5 - b*c^6 - c^7)*cosh(x)^4 - 10*(b^7 + b^6*c - 3*b^5*c^2 - 3*b^4*c^3 + 3*b^3*c^4 + 3*b^2*c^5 - b*c^6 - c^7 - 21*(b^7 + 7*b^6*c + 21*b^5*c^2 + 35*b^4*c^3 + 35*b^3*c^4 + 21*b^2*c^5 + 7*b*c^6 + c^7)*cosh(x)^6 + 35*(b^7 + 5*b^6*c + 9*b^5*c^2 + 5*b^4*c^3 - 5*b^3*c^4 - 9*b^2*c^5 - 5*b*c^6 - c^7)*cosh(x)^4 - 15*(b^7 + 3*b^6*c + b^5*c^2 - 5*b^4*c^

```
3 - 5*b^3*c^4 + b^2*c^5 + 3*b*c^6 + c^7)*cosh(x)^2)*sinh(x)^4 + 40*(3*(b^7
+ 7*b^6*c + 21*b^5*c^2 + 35*b^4*c^3 + 35*b^3*c^4 + 21*b^2*c^5 + 7*b*c^6 + c
^7)*cosh(x)^7 - 7*(b^7 + 5*b^6*c + 9*b^5*c^2 + 5*b^4*c^3 - 5*b^3*c^4 - 9*b^
2*c^5 - 5*b*c^6 - c^7)*cosh(x)^5 + 5*(b^7 + 3*b^6*c + b^5*c^2 - 5*b^4*c^3 -
 5*b^3*c^4 + b^2*c^5 + 3*b*c^6 + c^7)*cosh(x)^3 - (b^7 + b^6*c - 3*b^5*c^2
- 3*b^4*c^3 + 3*b^3*c^4 + 3*b^2*c^5 - b*c^6 - c^7)*cosh(x))*sinh(x)^3 + 5*(
b^7 - b^6*c - 3*b^5*c^2 + 3*b^4*c^3 + 3*b^3*c^4 - 3*b^2*c^5 - b*c^6 + c^7)*
cosh(x)^2 + 5*(9*(b^7 + 7*b^6*c + 21*b^5*c^2 + 35*b^4*c^3 + 35*b^3*c^4 + 21
*b^2*c^5 + 7*b*c^6 + c^7)*cosh(x)^8 + b^7 - b^6*c - 3*b^5*c^2 + 3*b^4*c^3 +
 3*b^3*c^4 - 3*b^2*c^5 - b*c^6 + c^7 - 28*(b^7 + 5*b^6*c + 9*b^5*c^2 + 5*b^
4*c^3 - 5*b^3*c^4 - 9*b^2*c^5 - 5*b*c^6 - c^7)*cosh(x)^6 + 30*(b^7 + 3*b^6*
c + b^5*c^2 - 5*b^4*c^3 - 5*b^3*c^4 + b^2*c^5 + 3*b*c^6 + c^7)*cosh(x)^4 -
12*(b^7 + b^6*c - 3*b^5*c^2 - 3*b^4*c^3 + 3*b^3*c^4 + 3*b^2*c^5 - b*c^6 - c
^7)*cosh(x)^2)*sinh(x)^2 + 10*((b^7 + 7*b^6*c + 21*b^5*c^2 + 35*b^4*c^3 + 3
5*b^3*c^4 + 21*b^2*c^5 + 7*b*c^6 + c^7)*cosh(x)^9 - 4*(b^7 + 5*b^6*c + 9*b^
5*c^2 + 5*b^4*c^3 - 5*b^3*c^4 - 9*b^2*c^5 - 5*b*c^6 - c^7)*cosh(x)^7 + 6*(b
^7 + 3*b^6*c + b^5*c^2 - 5*b^4*c^3 - 5*b^3*c^4 + b^2*c^5 + 3*b*c^6 + c^7)*c
osh(x)^5 - 4*(b^7 + b^6*c - 3*b^5*c^2 - 3*b^4*c^3 + 3*b^3*c^4 + 3*b^2*c^5 -
 b*c^6 - c^7)*cosh(x)^3 + (b^7 - b^6*c - 3*b^5*c^2 + 3*b^4*c^3 + 3*b^3*c^4
- 3*b^2*c^5 - b*c^6 + c^7)*cosh(x))*sinh(x))
```

giac [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*cosh(x)+c*sinh(x)+(b^2-c^2)^(1/2))^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to divide, perhaps due to rounding er
ror%%%{%%%{1,[3,0]%%%}+%%%{3,[2,1]%%%}+%%%{3,[1,2]%%%}+%%%{1,[0,3]%%%},[6]%
%%}+%%%{%%{[%%%{6,[2,0]%%%}+%%%{12,[1,1]%%%}+%%%{6,[0,2]%%%},0]:[1,0,%%%{-1
,[2,0]%%%}+%%%{1,[0,2]%%%}]%%},[5]%%%}+%%%{%%%{15,[3,0]%%%}+%%%{15,[2,1]%%%
}+%%%{-15,[1,2]%%%}+%%%{-15,[0,3]%%%},[4]%%%}+%%%{%%{[%%%{20,[2,0]%%%}+%%%{
-20,[0,2]%%%},0]:[1,0,%%%{-1,[2,0]%%%}+%%%{1,[0,2]%%%}]%%},[3]%%%}+%%%{%%%{
15,[3,0]%%%}+%%%{-15,[2,1]%%%}+%%%{-15,[1,2]%%%}+%%%{15,[0,3]%%%},[2]%%%}+%
%%{%%{[%%%{6,[2,0]%%%}+%%%{-12,[1,1]%%%}+%%%{6,[0,2]%%%},0]:[1,0,%%%{-1,[2,
0]%%%}+%%%{1,[0,2]%%%}]%%},[1]%%%}+%%%{%%%{1,[3,0]%%%}+%%%{-3,[2,1]%%%}+%%%
{3,[1,2]%%%}+%%%{-1,[0,3]%%%},[0]%%%} / %%%{%%%{1,[6,0]%%%}+%%%{6,[5,1]%%%}
+%%%{15,[4,2]%%%}+%%%{20,[3,3]%%%}+%%%{15,[2,4]%%%}+%%%{6,[1,5]%%%}+%%%{1,[
0,6]%%%},[6]%%%}+%%%{%%{[%%%{6,[5,0]%%%}+%%%{30,[4,1]%%%}+%%%{60,[3,2]%%%}+
%%%{60,[2,3]%%%}+%%%{30,[1,4]%%%}+%%%{6,[0,5]%%%},0]:[1,0,%%%{-1,[2,0]%%%}+
%%%{1,[0,2]%%%}]%%},[5]%%%}+%%%{%%%{15,[6,0]%%%}+%%%{60,[5,1]%%%}+%%%{75,[4
,2]%%%}+%%%{-75,[2,4]%%%}+%%%{-60,[1,5]%%%}+%%%{-15,[0,6]%%%},[4]%%%}+%%%{%
%{[%%%{20,[5,0]%%%}+%%%{60,[4,1]%%%}+%%%{40,[3,2]%%%}+%%%{-40,[2,3]%%%}+%%%
```

```
{-60,[1,4]%%%}+%%%{-20,[0,5]%%%},0]:[1,0,%%%{-1,[2,0]%%%}+%%%{1,[0,2]%%%}]%
%},[3]%%%}+%%%{%%%{15,[6,0]%%%}+%%%{30,[5,1]%%%}+%%%{-15,[4,2]%%%}+%%%{-60,
[3,3]%%%}+%%%{-15,[2,4]%%%}+%%%{30,[1,5]%%%}+%%%{15,[0,6]%%%},[2]%%%}+%%%{%
%{[%%%{6,[5,0]%%%}+%%%{6,[4,1]%%%}+%%%{-12,[3,2]%%%}+%%%{-12,[2,3]%%%}+%%%{
6,[1,4]%%%}+%%%{6,[0,5]%%%},0]:[1,0,%%%{-1,[2,0]%%%}+%%%{1,[0,2]%%%}]%%},[1
]%%%}+%%%{%%%{1,[6,0]%%%}+%%%{-3,[4,2]%%%}+%%%{3,[2,4]%%%}+%%%{-1,[0,6]%%%}
,[0]%%%} Error: Bad Argument Value
```

maple [B] time = 0.47, size = 488, normalized size = 3.34

$$\frac{-\frac{2\left(4\sqrt{b^2-c^2}\,b^2-\sqrt{b^2-c^2}\,c^2+4b^3-3bc^2\right)\left(\tanh^4\left(\frac{x}{2}\right)\right)}{c^2}-\frac{4\left(8b^4-8c^2b^2+c^4+8\sqrt{b^2-c^2}\,b^3-4\sqrt{b^2-c^2}\,c^2b\right)\left(\tanh^3\left(\frac{x}{2}\right)\right)}{c^3}-\frac{8\left(24\sqrt{b^2-c^2}\,b^4-20\sqrt{b^2-c^2}\,b^2\right.}{}}{c^4\left(\tanh^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*cosh(x)+c*sinh(x)+(b^2-c^2)^(1/2))^3,x)
```

```
[Out] 2/c^4*(-(4*(b^2-c^2)^(1/2)*b^2-(b^2-c^2)^(1/2)*c^2+4*b^3-3*b*c^2)/c^2*tanh(
1/2*x)^4-2*(8*b^4-8*c^2*b^2+c^4+8*(b^2-c^2)^(1/2)*b^3-4*(b^2-c^2)^(1/2)*c^2
*b)/c^3*tanh(1/2*x)^3-4/3*(24*(b^2-c^2)^(1/2)*b^4-20*(b^2-c^2)^(1/2)*b^2*c^
2+2*(b^2-c^2)^(1/2)*c^4+24*b^5-32*b^3*c^2+9*b*c^4)/c^4*tanh(1/2*x)^2-2/3*(4
8*b^6-76*b^4*c^2+31*b^2*c^4-2*c^6+48*(b^2-c^2)^(1/2)*b^5-52*(b^2-c^2)^(1/2)
*b^3*c^2+11*(b^2-c^2)^(1/2)*b*c^4)/c^5*tanh(1/2*x)-1/15/c^6*(192*(b^2-c^2)^
(1/2)*b^6-256*(b^2-c^2)^(1/2)*b^4*c^2+96*(b^2-c^2)^(1/2)*b^2*c^4-7*(b^2-c^2
)^(1/2)*c^6+192*b^7-352*b^5*c^2+200*b^3*c^4-35*c^6*b))/(tanh(1/2*x)^2+2/c*(
b^2-c^2)^(1/2)*tanh(1/2*x)+2/c*tanh(1/2*x)*b+2/c^2*(b^2-c^2)^(1/2)*b+2/c^2*
b^2-1)^2/(tanh(1/2*x)+1/c*(b^2-c^2)^(1/2)+b/c)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*cosh(x)+c*sinh(x)+(b^2-c^2)^(1/2))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(b\cosh(x)+\sqrt{b^2-c^2}+c\sinh(x)\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*cosh(x) + (b^2 - c^2)^(1/2) + c*sinh(x))^3,x)
```

```
[Out] int(1/(b*cosh(x) + (b^2 - c^2)^(1/2) + c*sinh(x))^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*cosh(x)+c*sinh(x)+(b**2-c**2)**(1/2))**3,x)
```

```
[Out] Timed out
```

3.760 $\int \frac{1}{\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^4}\,dx$

Optimal. Leaf size=198

$$\frac{2(b\sinh(x)+c\cosh(x))}{35\left(b^2-c^2\right)^{3/2}\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^2}+\frac{3(b\sinh(x)+c\cosh(x))}{35\left(b^2-c^2\right)\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^3}+\frac{}{7\sqrt{b^2-}}$$

```
[Out] 1/7*(c*cosh(x)+b*sinh(x))/(b^2-c^2)^(1/2)/(b*cosh(x)+c*sinh(x)+(b^2-c^2)^(1
/2))^4+3/35*(c*cosh(x)+b*sinh(x))/(b^2-c^2)/(b*cosh(x)+c*sinh(x)+(b^2-c^2)^
(1/2))^3+2/35*(c*cosh(x)+b*sinh(x))/(b^2-c^2)^(3/2)/(b*cosh(x)+c*sinh(x)+(b
^2-c^2)^(1/2))^2-2/35*(c+sinh(x)*(b^2-c^2)^(1/2))/c/(b^2-c^2)^(3/2)/(c*cosh
(x)+b*sinh(x))
```

Rubi [A] time = 0.18, antiderivative size = 198, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {3116, 3114}

$$\frac{2(b\sinh(x)+c\cosh(x))}{35\left(b^2-c^2\right)^{3/2}\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^2}+\frac{3(b\sinh(x)+c\cosh(x))}{35\left(b^2-c^2\right)\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^3}+\frac{}{7\sqrt{b^2-}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[b^2 - c^2] + b*Cosh[x] + c*Sinh[x])^(-4),x]
```

```
[Out] (c*Cosh[x] + b*Sinh[x])/(7*Sqrt[b^2 - c^2]*(Sqrt[b^2 - c^2] + b*Cosh[x] + c
*Sinh[x])^4) + (3*(c*Cosh[x] + b*Sinh[x]))/(35*(b^2 - c^2)*(Sqrt[b^2 - c^2]
 + b*Cosh[x] + c*Sinh[x])^3) + (2*(c*Cosh[x] + b*Sinh[x]))/(35*(b^2 - c^2)^
(3/2)*(Sqrt[b^2 - c^2] + b*Cosh[x] + c*Sinh[x])^2) - (2*(c + Sqrt[b^2 - c^2
]*Sinh[x]))/(35*c*(b^2 - c^2)^(3/2)*(c*Cosh[x] + b*Sinh[x]))
```

Rule 3114

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> -Simp[(c - a*Sin[d + e*x])/(c*e*(c*Cos[d + e*x] - b*Sin[
d + e*x])), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[a^2 - b^2 - c^2, 0]
```

Rule 3116

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> Simp[((c*Cos[d + e*x] - b*Sin[d + e*x])*(a + b*Cos[d + e
```

```
*x] + c*Sin[d + e*x])^n)/(a*e*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n + 1)),
Int[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1), x], x] /; FreeQ[{a, b, c
, d, e}, x] && EqQ[a^2 - b^2 - c^2, 0] && LtQ[n, -1]
```

Rubi steps

$$\int \frac{1}{\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^4}\,dx = \frac{c\cosh(x)+b\sinh(x)}{7\sqrt{b^2-c^2}\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^4} + \frac{3\int \frac{1}{\left(\sqrt{b^2-c^2}+b\cosh\right.}}{7\sqrt{b^2}}$$

$$= \frac{c\cosh(x)+b\sinh(x)}{7\sqrt{b^2-c^2}\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^4} + \frac{3(c\,c}{35\left(b^2-c^2\right)\left(\sqrt{b^2}\right.}$$

$$= \frac{c\cosh(x)+b\sinh(x)}{7\sqrt{b^2-c^2}\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^4} + \frac{3(c\,c}{35\left(b^2-c^2\right)\left(\sqrt{b^2}\right.}$$

$$= \frac{c\cosh(x)+b\sinh(x)}{7\sqrt{b^2-c^2}\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^4} + \frac{3(c\,c}{35\left(b^2-c^2\right)\left(\sqrt{b^2}\right.}$$

Mathematica [B] time = 0.83, size = 425, normalized size = 2.15

$$-\frac{-35b^6\sinh(x)+21b^6\sinh(3x)-7b^6\sinh(5x)+b^6\sinh(7x)+112b^5c\cosh(3x)-28b^5c\cosh(5x)+6b^5c\cosh(7}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[b^2 - c^2] + b*Cosh[x] + c*Sinh[x])^(-4),x]
```

```
[Out] -1/1120*(-832*b^4*c*Sqrt[b^2 - c^2] + 1664*b^2*c^3*Sqrt[b^2 - c^2] - 832*c^
5*Sqrt[b^2 - c^2] + 1190*b*c*(b^2 - c^2)^2*Cosh[x] + 448*c*Sqrt[b^2 - c^2]*
(-b^4 + c^4)*Cosh[2*x] + 112*b^5*c*Cosh[3*x] + 56*b^3*c^3*Cosh[3*x] - 168*b
*c^5*Cosh[3*x] - 28*b^5*c*Cosh[5*x] + 28*b*c^5*Cosh[5*x] + 6*b^5*c*Cosh[7*x
] + 20*b^3*c^3*Cosh[7*x] + 6*b*c^5*Cosh[7*x] - 35*b^6*Sinh[x] + 1295*b^4*c^
2*Sinh[x] - 2485*b^2*c^4*Sinh[x] + 1225*c^6*Sinh[x] - 896*b^3*c^2*Sqrt[b^2
- c^2]*Sinh[2*x] + 896*b*c^4*Sqrt[b^2 - c^2]*Sinh[2*x] + 21*b^6*Sinh[3*x] +
 189*b^4*c^2*Sinh[3*x] - 161*b^2*c^4*Sinh[3*x] - 49*c^6*Sinh[3*x] - 7*b^6*S
```

```
inh[5*x] - 35*b^4*c^2*Sinh[5*x] + 35*b^2*c^4*Sinh[5*x] + 7*c^6*Sinh[5*x] +
b^6*Sinh[7*x] + 15*b^4*c^2*Sinh[7*x] + 15*b^2*c^4*Sinh[7*x] + c^6*Sinh[7*x]
)/((b - c)*c*(b + c)*(c*Cosh[x] + b*Sinh[x])^7)
```

fricas [B] time = 0.59, size = 6590, normalized size = 33.28

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*cosh(x)+c*sinh(x)+(b^2-c^2)^(1/2))^4,x, algorithm="fricas")
```

```
[Out] -4/35*(35*(b^5 + 5*b^4*c + 10*b^3*c^2 + 10*b^2*c^3 + 5*b*c^4 + c^5)*cosh(x)
^10 + 350*(b^5 + 5*b^4*c + 10*b^3*c^2 + 10*b^2*c^3 + 5*b*c^4 + c^5)*cosh(x)
*sinh(x)^9 + 35*(b^5 + 5*b^4*c + 10*b^3*c^2 + 10*b^2*c^3 + 5*b*c^4 + c^5)*s
inh(x)^10 + 595*(b^5 + 3*b^4*c + 2*b^3*c^2 - 2*b^2*c^3 - 3*b*c^4 - c^5)*cos
h(x)^8 + 35*(17*b^5 + 51*b^4*c + 34*b^3*c^2 - 34*b^2*c^3 - 51*b*c^4 - 17*c^
5 + 45*(b^5 + 5*b^4*c + 10*b^3*c^2 + 10*b^2*c^3 + 5*b*c^4 + c^5)*cosh(x)^2)
*sinh(x)^8 + 280*(15*(b^5 + 5*b^4*c + 10*b^3*c^2 + 10*b^2*c^3 + 5*b*c^4 + c
^5)*cosh(x)^3 + 17*(b^5 + 3*b^4*c + 2*b^3*c^2 - 2*b^2*c^3 - 3*b*c^4 - c^5)*
cosh(x))*sinh(x)^7 + 630*(b^5 + b^4*c - 2*b^3*c^2 - 2*b^2*c^3 + b*c^4 + c^5
)*cosh(x)^6 + 70*(9*b^5 + 9*b^4*c - 18*b^3*c^2 - 18*b^2*c^3 + 9*b*c^4 + 9*c
^5 + 105*(b^5 + 5*b^4*c + 10*b^3*c^2 + 10*b^2*c^3 + 5*b*c^4 + c^5)*cosh(x)^
4 + 238*(b^5 + 3*b^4*c + 2*b^3*c^2 - 2*b^2*c^3 - 3*b*c^4 - c^5)*cosh(x)^2)*
sinh(x)^6 + 140*(63*(b^5 + 5*b^4*c + 10*b^3*c^2 + 10*b^2*c^3 + 5*b*c^4 + c^
5)*cosh(x)^5 + 238*(b^5 + 3*b^4*c + 2*b^3*c^2 - 2*b^2*c^3 - 3*b*c^4 - c^5)*
cosh(x)^3 + 27*(b^5 + b^4*c - 2*b^3*c^2 - 2*b^2*c^3 + b*c^4 + c^5)*cosh(x))
*sinh(x)^5 - b^5 + 5*b^4*c - 10*b^3*c^2 + 10*b^2*c^3 - 5*b*c^4 + c^5 + 14*(
b^5 - b^4*c - 2*b^3*c^2 + 2*b^2*c^3 + b*c^4 - c^5)*cosh(x)^4 + 14*(525*(b^5
 + 5*b^4*c + 10*b^3*c^2 + 10*b^2*c^3 + 5*b*c^4 + c^5)*cosh(x)^6 + b^5 - b^4
*c - 2*b^3*c^2 + 2*b^2*c^3 + b*c^4 - c^5 + 2975*(b^5 + 3*b^4*c + 2*b^3*c^2
- 2*b^2*c^3 - 3*b*c^4 - c^5)*cosh(x)^4 + 675*(b^5 + b^4*c - 2*b^3*c^2 - 2*b
^2*c^3 + b*c^4 + c^5)*cosh(x)^2)*sinh(x)^4 + 56*(75*(b^5 + 5*b^4*c + 10*b^3
*c^2 + 10*b^2*c^3 + 5*b*c^4 + c^5)*cosh(x)^7 + 595*(b^5 + 3*b^4*c + 2*b^3*c
^2 - 2*b^2*c^3 - 3*b*c^4 - c^5)*cosh(x)^5 + 225*(b^5 + b^4*c - 2*b^3*c^2 -
2*b^2*c^3 + b*c^4 + c^5)*cosh(x)^3 + (b^5 - b^4*c - 2*b^3*c^2 + 2*b^2*c^3 +
 b*c^4 - c^5)*cosh(x))*sinh(x)^3 + 7*(b^5 - 3*b^4*c + 2*b^3*c^2 + 2*b^2*c^3
 - 3*b*c^4 + c^5)*cosh(x)^2 + 7*(225*(b^5 + 5*b^4*c + 10*b^3*c^2 + 10*b^2*c
^3 + 5*b*c^4 + c^5)*cosh(x)^8 + 2380*(b^5 + 3*b^4*c + 2*b^3*c^2 - 2*b^2*c^3
 - 3*b*c^4 - c^5)*cosh(x)^6 + b^5 - 3*b^4*c + 2*b^3*c^2 + 2*b^2*c^3 - 3*b*c
^4 + c^5 + 1350*(b^5 + b^4*c - 2*b^3*c^2 - 2*b^2*c^3 + b*c^4 + c^5)*cosh(x)
^4 + 12*(b^5 - b^4*c - 2*b^3*c^2 + 2*b^2*c^3 + b*c^4 - c^5)*cosh(x)^2)*sinh
(x)^2 + 14*(25*(b^5 + 5*b^4*c + 10*b^3*c^2 + 10*b^2*c^3 + 5*b*c^4 + c^5)*co
sh(x)^9 + 340*(b^5 + 3*b^4*c + 2*b^3*c^2 - 2*b^2*c^3 - 3*b*c^4 - c^5)*cosh(
x)^7 + 270*(b^5 + b^4*c - 2*b^3*c^2 - 2*b^2*c^3 + b*c^4 + c^5)*cosh(x)^5 +
4*(b^5 - b^4*c - 2*b^3*c^2 + 2*b^2*c^3 + b*c^4 - c^5)*cosh(x)^3 + (b^5 - 3*
```

b^4*c + 2*b^3*c^2 + 2*b^2*c^3 - 3*b*c^4 + c^5)*cosh(x))*sinh(x) - 32*(7*(b^4 + 4*b^3*c + 6*b^2*c^2 + 4*b*c^3 + c^4)*cosh(x)^9 + 63*(b^4 + 4*b^3*c + 6*b^2*c^2 + 4*b*c^3 + c^4)*cosh(x)*sinh(x)^8 + 7*(b^4 + 4*b^3*c + 6*b^2*c^2 + 4*b*c^3 + c^4)*sinh(x)^9 + 26*(b^4 + 2*b^3*c - 2*b*c^3 - c^4)*cosh(x)^7 + 2*(13*b^4 + 26*b^3*c - 26*b*c^3 - 13*c^4 + 126*(b^4 + 4*b^3*c + 6*b^2*c^2 + 4*b*c^3 + c^4)*cosh(x)^2)*sinh(x)^7 + 14*(42*(b^4 + 4*b^3*c + 6*b^2*c^2 + 4*b*c^3 + c^4)*cosh(x)^3 + 13*(b^4 + 2*b^3*c - 2*b*c^3 - c^4)*cosh(x))*sinh(x)^6 + 7*(b^4 - 2*b^2*c^2 + c^4)*cosh(x)^5 + 7*(126*(b^4 + 4*b^3*c + 6*b^2*c^2 + 4*b*c^3 + c^4)*cosh(x)^4 + b^4 - 2*b^2*c^2 + c^4 + 78*(b^4 + 2*b^3*c - 2*b*c^3 - c^4)*cosh(x)^2)*sinh(x)^5 + 7*(126*(b^4 + 4*b^3*c + 6*b^2*c^2 + 4*b*c^3 + c^4)*cosh(x)^5 + 130*(b^4 + 2*b^3*c - 2*b*c^3 - c^4)*cosh(x)^3 + 5*(b^4 - 2*b^2*c^2 + c^4)*cosh(x))*sinh(x)^4 + 14*(42*(b^4 + 4*b^3*c + 6*b^2*c^2 + 4*b*c^3 + c^4)*cosh(x)^6 + 65*(b^4 + 2*b^3*c - 2*b*c^3 - c^4)*cosh(x)^4 + 5*(b^4 - 2*b^2*c^2 + c^4)*cosh(x)^2)*sinh(x)^3 + 14*(18*(b^4 + 4*b^3*c + 6*b^2*c^2 + 4*b*c^3 + c^4)*cosh(x)^7 + 39*(b^4 + 2*b^3*c - 2*b*c^3 - c^4)*cosh(x)^5 + 5*(b^4 - 2*b^2*c^2 + c^4)*cosh(x)^3)*sinh(x)^2 + 7*(9*(b^4 + 4*b^3*c + 6*b^2*c^2 + 4*b*c^3 + c^4)*cosh(x)^8 + 26*(b^4 + 2*b^3*c - 2*b*c^3 - c^4)*cosh(x)^6 + 5*(b^4 - 2*b^2*c^2 + c^4)*cosh(x)^4)*sinh(x))*sqrt(b^2 - c^2))/((b^9 + 9*b^8*c + 36*b^7*c^2 + 84*b^6*c^3 + 126*b^5*c^4 + 126*b^4*c^5 + 84*b^3*c^6 + 36*b^2*c^7 + 9*b*c^8 + c^9)*cosh(x)^14 + 14*(b^9 + 9*b^8*c + 36*b^7*c^2 + 84*b^6*c^3 + 126*b^5*c^4 + 126*b^4*c^5 + 84*b^3*c^6 + 36*b^2*c^7 + 9*b*c^8 + c^9)*cosh(x)*sinh(x)^13 + (b^9 + 9*b^8*c + 36*b^7*c^2 + 84*b^6*c^3 + 126*b^5*c^4 + 126*b^4*c^5 + 84*b^3*c^6 + 36*b^2*c^7 + 9*b*c^8 + c^9)*sinh(x)^14 - 7*(b^9 + 7*b^8*c + 20*b^7*c^2 + 28*b^6*c^3 + 14*b^5*c^4 - 14*b^4*c^5 - 28*b^3*c^6 - 20*b^2*c^7 - 7*b*c^8 - c^9)*cosh(x)^12 - 7*(b^9 + 7*b^8*c + 20*b^7*c^2 + 28*b^6*c^3 + 14*b^5*c^4 - 14*b^4*c^5 - 28*b^3*c^6 - 20*b^2*c^7 - 7*b*c^8 - c^9 - 13*(b^9 + 9*b^8*c + 36*b^7*c^2 + 84*b^6*c^3 + 126*b^5*c^4 + 126*b^4*c^5 + 84*b^3*c^6 + 36*b^2*c^7 + 9*b*c^8 + c^9)*cosh(x)^2)*sinh(x)^12 + 28*(13*(b^9 + 9*b^8*c + 36*b^7*c^2 + 84*b^6*c^3 + 126*b^5*c^4 + 126*b^4*c^5 + 84*b^3*c^6 + 36*b^2*c^7 + 9*b*c^8 + c^9)*cosh(x)^3 - 3*(b^9 + 7*b^8*c + 20*b^7*c^2 + 28*b^6*c^3 + 14*b^5*c^4 - 14*b^4*c^5 - 28*b^3*c^6 - 20*b^2*c^7 - 7*b*c^8 - c^9)*cosh(x))*sinh(x)^11 + 21*(b^9 + 5*b^8*c + 8*b^7*c^2 - 14*b^5*c^4 - 14*b^4*c^5 + 8*b^2*c^7 + 5*b*c^8 + c^9)*cosh(x)^10 + 7*(3*b^9 + 15*b^8*c + 24*b^7*c^2 - 42*b^5*c^4 - 42*b^4*c^5 + 24*b^2*c^7 + 15*b*c^8 + 3*c^9 + 143*(b^9 + 9*b^8*c + 36*b^7*c^2 + 84*b^6*c^3 + 126*b^5*c^4 + 126*b^4*c^5 + 84*b^3*c^6 + 36*b^2*c^7 + 9*b*c^8 + c^9)*cosh(x)^4 - 66*(b^9 + 7*b^8*c + 20*b^7*c^2 + 28*b^6*c^3 + 14*b^5*c^4 - 14*b^4*c^5 - 28*b^3*c^6 - 20*b^2*c^7 - 7*b*c^8 - c^9)*cosh(x)^2)*sinh(x)^10 + 14*(143*(b^9 + 9*b^8*c + 36*b^7*c^2 + 84*b^6*c^3 + 126*b^5*c^4 + 126*b^4*c^5 + 84*b^3*c^6 + 36*b^2*c^7 + 9*b*c^8 + c^9)*cosh(x)^5 - 110*(b^9 + 7*b^8*c + 20*b^7*c^2 + 28*b^6*c^3 + 14*b^5*c^4 - 14*b^4*c^5 - 28*b^3*c^6 - 20*b^2*c^7 - 7*b*c^8 - c^9)*cosh(x)^3 + 15*(b^9 + 5*b^8*c + 8*b^7*c^2 - 14*b^5*c^4 - 14*b^4*c^5 + 8*b^2*c^7 + 5*b*c^8 + c^9)*cosh(x))*sinh(x)^9 - b^9 + 5*b^8*c - 8*b^7*c^2 + 14*b^5*c^4 - 14*b^4*c^5 + 8*b^2*c^7 - 5*b*c^8 + c^9 - 35*(b^9 + 3*b^8*c - 8*b^6*c^3 - 6*b^5*c^4 + 6*b^4*c^5 + 8*b^3*c^6 - 3*b*c^8 - c^9

)*cosh(x)^8 - 7*(5*b^9 + 15*b^8*c - 40*b^6*c^3 - 30*b^5*c^4 + 30*b^4*c^5 +
40*b^3*c^6 - 15*b*c^8 - 5*c^9 - 429*(b^9 + 9*b^8*c + 36*b^7*c^2 + 84*b^6*c^
3 + 126*b^5*c^4 + 126*b^4*c^5 + 84*b^3*c^6 + 36*b^2*c^7 + 9*b*c^8 + c^9)*co
sh(x)^6 + 495*(b^9 + 7*b^8*c + 20*b^7*c^2 + 28*b^6*c^3 + 14*b^5*c^4 - 14*b^
4*c^5 - 28*b^3*c^6 - 20*b^2*c^7 - 7*b*c^8 - c^9)*cosh(x)^4 - 135*(b^9 + 5*b
^8*c + 8*b^7*c^2 - 14*b^5*c^4 - 14*b^4*c^5 + 8*b^2*c^7 + 5*b*c^8 + c^9)*cos
h(x)^2)*sinh(x)^8 + 8*(429*(b^9 + 9*b^8*c + 36*b^7*c^2 + 84*b^6*c^3 + 126*b
^5*c^4 + 126*b^4*c^5 + 84*b^3*c^6 + 36*b^2*c^7 + 9*b*c^8 + c^9)*cosh(x)^7 -
693*(b^9 + 7*b^8*c + 20*b^7*c^2 + 28*b^6*c^3 + 14*b^5*c^4 - 14*b^4*c^5 - 2
8*b^3*c^6 - 20*b^2*c^7 - 7*b*c^8 - c^9)*cosh(x)^5 + 315*(b^9 + 5*b^8*c + 8*
b^7*c^2 - 14*b^5*c^4 - 14*b^4*c^5 + 8*b^2*c^7 + 5*b*c^8 + c^9)*cosh(x)^3 -
35*(b^9 + 3*b^8*c - 8*b^6*c^3 - 6*b^5*c^4 + 6*b^4*c^5 + 8*b^3*c^6 - 3*b*c^8
 - c^9)*cosh(x))*sinh(x)^7 + 35*(b^9 + b^8*c - 4*b^7*c^2 - 4*b^6*c^3 + 6*b^
5*c^4 + 6*b^4*c^5 - 4*b^3*c^6 - 4*b^2*c^7 + b*c^8 + c^9)*cosh(x)^6 + 7*(5*b
^9 + 5*b^8*c - 20*b^7*c^2 - 20*b^6*c^3 + 30*b^5*c^4 + 30*b^4*c^5 - 20*b^3*c
^6 - 20*b^2*c^7 + 5*b*c^8 + 5*c^9 + 429*(b^9 + 9*b^8*c + 36*b^7*c^2 + 84*b^
6*c^3 + 126*b^5*c^4 + 126*b^4*c^5 + 84*b^3*c^6 + 36*b^2*c^7 + 9*b*c^8 + c^9
)*cosh(x)^8 - 924*(b^9 + 7*b^8*c + 20*b^7*c^2 + 28*b^6*c^3 + 14*b^5*c^4 - 1
4*b^4*c^5 - 28*b^3*c^6 - 20*b^2*c^7 - 7*b*c^8 - c^9)*cosh(x)^6 + 630*(b^9 +
 5*b^8*c + 8*b^7*c^2 - 14*b^5*c^4 - 14*b^4*c^5 + 8*b^2*c^7 + 5*b*c^8 + c^9)
cosh(x)^4 - 140(b^9 + 3*b^8*c - 8*b^6*c^3 - 6*b^5*c^4 + 6*b^4*c^5 + 8*b^3
*c^6 - 3*b*c^8 - c^9)*cosh(x)^2)*sinh(x)^6 + 14*(143*(b^9 + 9*b^8*c + 36*b^
7*c^2 + 84*b^6*c^3 + 126*b^5*c^4 + 126*b^4*c^5 + 84*b^3*c^6 + 36*b^2*c^7 +
9*b*c^8 + c^9)*cosh(x)^9 - 396*(b^9 + 7*b^8*c + 20*b^7*c^2 + 28*b^6*c^3 + 1
4*b^5*c^4 - 14*b^4*c^5 - 28*b^3*c^6 - 20*b^2*c^7 - 7*b*c^8 - c^9)*cosh(x)^7
 + 378*(b^9 + 5*b^8*c + 8*b^7*c^2 - 14*b^5*c^4 - 14*b^4*c^5 + 8*b^2*c^7 + 5
*b*c^8 + c^9)*cosh(x)^5 - 140*(b^9 + 3*b^8*c - 8*b^6*c^3 - 6*b^5*c^4 + 6*b^
4*c^5 + 8*b^3*c^6 - 3*b*c^8 - c^9)*cosh(x)^3 + 15*(b^9 + b^8*c - 4*b^7*c^2
- 4*b^6*c^3 + 6*b^5*c^4 + 6*b^4*c^5 - 4*b^3*c^6 - 4*b^2*c^7 + b*c^8 + c^9)*
cosh(x))*sinh(x)^5 - 21*(b^9 - b^8*c - 4*b^7*c^2 + 4*b^6*c^3 + 6*b^5*c^4 -
6*b^4*c^5 - 4*b^3*c^6 + 4*b^2*c^7 + b*c^8 - c^9)*cosh(x)^4 + 7*(143*(b^9 +
9*b^8*c + 36*b^7*c^2 + 84*b^6*c^3 + 126*b^5*c^4 + 126*b^4*c^5 + 84*b^3*c^6
+ 36*b^2*c^7 + 9*b*c^8 + c^9)*cosh(x)^10 - 3*b^9 + 3*b^8*c + 12*b^7*c^2 - 1
2*b^6*c^3 - 18*b^5*c^4 + 18*b^4*c^5 + 12*b^3*c^6 - 12*b^2*c^7 - 3*b*c^8 + 3
c^9 - 495(b^9 + 7*b^8*c + 20*b^7*c^2 + 28*b^6*c^3 + 14*b^5*c^4 - 14*b^4*c
^5 - 28*b^3*c^6 - 20*b^2*c^7 - 7*b*c^8 - c^9)*cosh(x)^8 + 630*(b^9 + 5*b^8*
c + 8*b^7*c^2 - 14*b^5*c^4 - 14*b^4*c^5 + 8*b^2*c^7 + 5*b*c^8 + c^9)*cosh(x
)^6 - 350*(b^9 + 3*b^8*c - 8*b^6*c^3 - 6*b^5*c^4 + 6*b^4*c^5 + 8*b^3*c^6 -
3*b*c^8 - c^9)*cosh(x)^4 + 75*(b^9 + b^8*c - 4*b^7*c^2 - 4*b^6*c^3 + 6*b^5*
c^4 + 6*b^4*c^5 - 4*b^3*c^6 - 4*b^2*c^7 + b*c^8 + c^9)*cosh(x)^2)*sinh(x)^4
 + 28*(13*(b^9 + 9*b^8*c + 36*b^7*c^2 + 84*b^6*c^3 + 126*b^5*c^4 + 126*b^4*
c^5 + 84*b^3*c^6 + 36*b^2*c^7 + 9*b*c^8 + c^9)*cosh(x)^11 - 55*(b^9 + 7*b^8
*c + 20*b^7*c^2 + 28*b^6*c^3 + 14*b^5*c^4 - 14*b^4*c^5 - 28*b^3*c^6 - 20*b^
2*c^7 - 7*b*c^8 - c^9)*cosh(x)^9 + 90*(b^9 + 5*b^8*c + 8*b^7*c^2 - 14*b^5*c
^4 - 14*b^4*c^5 + 8*b^2*c^7 + 5*b*c^8 + c^9)*cosh(x)^7 - 70*(b^9 + 3*b^8*c

```
- 8*b^6*c^3 - 6*b^5*c^4 + 6*b^4*c^5 + 8*b^3*c^6 - 3*b*c^8 - c^9)*cosh(x)^5
+ 25*(b^9 + b^8*c - 4*b^7*c^2 - 4*b^6*c^3 + 6*b^5*c^4 + 6*b^4*c^5 - 4*b^3*c
^6 - 4*b^2*c^7 + b*c^8 + c^9)*cosh(x)^3 - 3*(b^9 - b^8*c - 4*b^7*c^2 + 4*b^
6*c^3 + 6*b^5*c^4 - 6*b^4*c^5 - 4*b^3*c^6 + 4*b^2*c^7 + b*c^8 - c^9)*cosh(x
))*sinh(x)^3 + 7*(b^9 - 3*b^8*c + 8*b^6*c^3 - 6*b^5*c^4 - 6*b^4*c^5 + 8*b^3
*c^6 - 3*b*c^8 + c^9)*cosh(x)^2 + 7*(13*(b^9 + 9*b^8*c + 36*b^7*c^2 + 84*b^
6*c^3 + 126*b^5*c^4 + 126*b^4*c^5 + 84*b^3*c^6 + 36*b^2*c^7 + 9*b*c^8 + c^9
)*cosh(x)^12 - 66*(b^9 + 7*b^8*c + 20*b^7*c^2 + 28*b^6*c^3 + 14*b^5*c^4 - 1
4*b^4*c^5 - 28*b^3*c^6 - 20*b^2*c^7 - 7*b*c^8 - c^9)*cosh(x)^10 + b^9 - 3*b
^8*c + 8*b^6*c^3 - 6*b^5*c^4 - 6*b^4*c^5 + 8*b^3*c^6 - 3*b*c^8 + c^9 + 135*
(b^9 + 5*b^8*c + 8*b^7*c^2 - 14*b^5*c^4 - 14*b^4*c^5 + 8*b^2*c^7 + 5*b*c^8
+ c^9)*cosh(x)^8 - 140*(b^9 + 3*b^8*c - 8*b^6*c^3 - 6*b^5*c^4 + 6*b^4*c^5 +
 8*b^3*c^6 - 3*b*c^8 - c^9)*cosh(x)^6 + 75*(b^9 + b^8*c - 4*b^7*c^2 - 4*b^6
*c^3 + 6*b^5*c^4 + 6*b^4*c^5 - 4*b^3*c^6 - 4*b^2*c^7 + b*c^8 + c^9)*cosh(x)
^4 - 18*(b^9 - b^8*c - 4*b^7*c^2 + 4*b^6*c^3 + 6*b^5*c^4 - 6*b^4*c^5 - 4*b^
3*c^6 + 4*b^2*c^7 + b*c^8 - c^9)*cosh(x)^2)*sinh(x)^2 + 14*((b^9 + 9*b^8*c
+ 36*b^7*c^2 + 84*b^6*c^3 + 126*b^5*c^4 + 126*b^4*c^5 + 84*b^3*c^6 + 36*b^2
*c^7 + 9*b*c^8 + c^9)*cosh(x)^13 - 6*(b^9 + 7*b^8*c + 20*b^7*c^2 + 28*b^6*c
^3 + 14*b^5*c^4 - 14*b^4*c^5 - 28*b^3*c^6 - 20*b^2*c^7 - 7*b*c^8 - c^9)*cos
h(x)^11 + 15*(b^9 + 5*b^8*c + 8*b^7*c^2 - 14*b^5*c^4 - 14*b^4*c^5 + 8*b^2*c
^7 + 5*b*c^8 + c^9)*cosh(x)^9 - 20*(b^9 + 3*b^8*c - 8*b^6*c^3 - 6*b^5*c^4 +
 6*b^4*c^5 + 8*b^3*c^6 - 3*b*c^8 - c^9)*cosh(x)^7 + 15*(b^9 + b^8*c - 4*b^7
*c^2 - 4*b^6*c^3 + 6*b^5*c^4 + 6*b^4*c^5 - 4*b^3*c^6 - 4*b^2*c^7 + b*c^8 +
c^9)*cosh(x)^5 - 6*(b^9 - b^8*c - 4*b^7*c^2 + 4*b^6*c^3 + 6*b^5*c^4 - 6*b^4
*c^5 - 4*b^3*c^6 + 4*b^2*c^7 + b*c^8 - c^9)*cosh(x)^3 + (b^9 - 3*b^8*c + 8*
b^6*c^3 - 6*b^5*c^4 - 6*b^4*c^5 + 8*b^3*c^6 - 3*b*c^8 + c^9)*cosh(x))*sinh(
x))
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*cosh(x)+c*sinh(x)+(b^2-c^2)^(1/2))^4,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to divide, perhaps due to rounding er
ror%%%{%%%{1,[4,0]%%%}+%%%{4,[3,1]%%%}+%%%{6,[2,2]%%%}+%%%{4,[1,3]%%%}+%%%{
1,[0,4]%%%},[8]%%%}+%%%{%%{[%%%{8,[3,0]%%%}+%%%{24,[2,1]%%%}+%%%{24,[1,2]%%
%}+%%%{8,[0,3]%%%},0]:[1,0,%%%{-1,[2,0]%%%}+%%%{1,[0,2]%%%}]%%},[7]%%%}+%%%
{%%%{28,[4,0]%%%}+%%%{56,[3,1]%%%}+%%%{-56,[1,3]%%%}+%%%{-28,[0,4]%%%},[6]%
%%}+%%%{%%{[%%%{56,[3,0]%%%}+%%%{56,[2,1]%%%}+%%%{-56,[1,2]%%%}+%%%{-56,[0,
3]%%%},0]:[1,0,%%%{-1,[2,0]%%%}+%%%{1,[0,2]%%%}]%%},[5]%%%}+%%%{%%%{70,[4,0
]%%%}+%%%{-140,[2,2]%%%}+%%%{70,[0,4]%%%},[4]%%%}+%%%{%%{[%%%{56,[3,0]%%%}+
%%%{-56,[2,1]%%%}+%%%{-56,[1,2]%%%}+%%%{56,[0,3]%%%},0]:[1,0,%%%{-1,[2,0]%%
```

```
%}+%%%{1,[0,2]%%%}]%%},[3]%%%}+%%%{%%%{28,[4,0]%%%}+%%%{-56,[3,1]%%%}+%%%{5
6,[1,3]%%%}+%%%{-28,[0,4]%%%},[2]%%%}+%%%{%%{[%%%{8,[3,0]%%%}+%%%{-24,[2,1]
%%%}+%%%{24,[1,2]%%%}+%%%{-8,[0,3]%%%},0]:[1,0,%%%{-1,[2,0]%%%}+%%%{1,[0,2]
%%%}]%%},[1]%%%}+%%%{%%%{1,[4,0]%%%}+%%%{-4,[3,1]%%%}+%%%{6,[2,2]%%%}+%%%{-
4,[1,3]%%%}+%%%{1,[0,4]%%%},[0]%%%} / %%%{%%%{1,[8,0]%%%}+%%%{8,[7,1]%%%}+%
%%{28,[6,2]%%%}+%%%{56,[5,3]%%%}+%%%{70,[4,4]%%%}+%%%{56,[3,5]%%%}+%%%{28,[
2,6]%%%}+%%%{8,[1,7]%%%}+%%%{1,[0,8]%%%},[8]%%%}+%%%{%%{[%%%{8,[7,0]%%%}+%%
%{56,[6,1]%%%}+%%%{168,[5,2]%%%}+%%%{280,[4,3]%%%}+%%%{280,[3,4]%%%}+%%%{16
8,[2,5]%%%}+%%%{56,[1,6]%%%}+%%%{8,[0,7]%%%},0]:[1,0,%%%{-1,[2,0]%%%}+%%%{1
,[0,2]%%%}]%%},[7]%%%}+%%%{%%%{28,[8,0]%%%}+%%%{168,[7,1]%%%}+%%%{392,[6,2]
%%%}+%%%{392,[5,3]%%%}+%%%{-392,[3,5]%%%}+%%%{-392,[2,6]%%%}+%%%{-168,[1,7]
%%%}+%%%{-28,[0,8]%%%},[6]%%%}+%%%{%%{[%%%{56,[7,0]%%%}+%%%{280,[6,1]%%%}+%
%%{504,[5,2]%%%}+%%%{280,[4,3]%%%}+%%%{-280,[3,4]%%%}+%%%{-504,[2,5]%%%}+%%
%{-280,[1,6]%%%}+%%%{-56,[0,7]%%%},0]:[1,0,%%%{-1,[2,0]%%%}+%%%{1,[0,2]%%%}
]%%},[5]%%%}+%%%{%%%{70,[8,0]%%%}+%%%{280,[7,1]%%%}+%%%{280,[6,2]%%%}+%%%{-
280,[5,3]%%%}+%%%{-700,[4,4]%%%}+%%%{-280,[3,5]%%%}+%%%{280,[2,6]%%%}+%%%{2
80,[1,7]%%%}+%%%{70,[0,8]%%%},[4]%%%}+%%%{%%{[%%%{56,[7,0]%%%}+%%%{168,[6,1
]%%%}+%%%{56,[5,2]%%%}+%%%{-280,[4,3]%%%}+%%%{-280,[3,4]%%%}+%%%{56,[2,5]%%
%}+%%%{168,[1,6]%%%}+%%%{56,[0,7]%%%},0]:[1,0,%%%{-1,[2,0]%%%}+%%%{1,[0,2]%
%%}]%%},[3]%%%}+%%%{%%%{28,[8,0]%%%}+%%%{56,[7,1]%%%}+%%%{-56,[6,2]%%%}+%%%
{-168,[5,3]%%%}+%%%{168,[3,5]%%%}+%%%{56,[2,6]%%%}+%%%{-56,[1,7]%%%}+%%%{-2
8,[0,8]%%%},[2]%%%}+%%%{%%{[%%%{8,[7,0]%%%}+%%%{8,[6,1]%%%}+%%%{-24,[5,2]%%
%}+%%%{-24,[4,3]%%%}+%%%{24,[3,4]%%%}+%%%{24,[2,5]%%%}+%%%{-8,[1,6]%%%}+%%%
{-8,[0,7]%%%},0]:[1,0,%%%{-1,[2,0]%%%}+%%%{1,[0,2]%%%}]%%},[1]%%%}+%%%{%%%{
1,[8,0]%%%}+%%%{-4,[6,2]%%%}+%%%{6,[4,4]%%%}+%%%{-4,[2,6]%%%}+%%%{1,[0,8]%%
%},[0]%%%} Error: Bad Argument Value
```

maple [B] time = 0.66, size = 828, normalized size = 4.18

$$\frac{\frac{2\left(8b^4-8c^2b^2+c^4+8\sqrt{b^2-c^2}\,b^3-4\sqrt{b^2-c^2}\,c^2b\right)\left(\tanh^6\left(\frac{x}{2}\right)\right)}{c^2}+\frac{6\left(16\sqrt{b^2-c^2}\,b^4-12\sqrt{b^2-c^2}\,b^2c^2+\sqrt{b^2-c^2}\,c^4+16b^5-20b^3c^2+5bc^4\right)\left(\tanh^5\left(\frac{x}{2}\right)\right)}{c^3}+\frac{4(80\cdots}{\cdots}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*cosh(x)+c*sinh(x)+(b^2-c^2)^(1/2))^4,x)
```

```
[Out] 2/c^6*((8*b^4-8*c^2*b^2+c^4+8*(b^2-c^2)^(1/2)*b^3-4*(b^2-c^2)^(1/2)*c^2*b)/
c^2*tanh(1/2*x)^6+3*(16*(b^2-c^2)^(1/2)*b^4-12*(b^2-c^2)^(1/2)*b^2*c^2+(b^2
-c^2)^(1/2)*c^4+16*b^5-20*b^3*c^2+5*b*c^4)/c^3*tanh(1/2*x)^5+2*(80*(b^2-c^2
)^(1/2)*b^5-84*(b^2-c^2)^(1/2)*b^3*c^2+17*(b^2-c^2)^(1/2)*b*c^4+80*b^6-124*
b^4*c^2+49*b^2*c^4-3*c^6)/c^4*tanh(1/2*x)^4+2*(160*b^7-288*b^5*c^2+150*b^3*
c^4-20*c^6*b+160*(b^2-c^2)^(1/2)*b^6-208*(b^2-c^2)^(1/2)*b^4*c^2+66*(b^2-c^
2)^(1/2)*b^2*c^4-3*(b^2-c^2)^(1/2)*c^6)/c^5*tanh(1/2*x)^3+3/5*(640*b^7*(b^2
```

```
-c^2)^(1/2)-992*(b^2-c^2)^(1/2)*b^5*c^2+440*(b^2-c^2)^(1/2)*b^3*c^4-50*(b^2
-c^2)^(1/2)*b*c^6+640*b^8-1312*b^6*c^2+856*b^4*c^4-186*c^6*b^2+7*c^8)/c^6*t
anh(1/2*x)^2+1/5*(1280*b^9-2944*b^7*c^2+2288*b^5*c^4-676*b^3*c^6+57*b*c^8+1
280*(b^2-c^2)^(1/2)*b^8-2304*(b^2-c^2)^(1/2)*b^6*c^2+1296*(b^2-c^2)^(1/2)*b
^4*c^4-236*(b^2-c^2)^(1/2)*b^2*c^6+7*(b^2-c^2)^(1/2)*c^8)/c^7*tanh(1/2*x)+4
/35*(640*(b^2-c^2)^(1/2)*b^9-1312*(b^2-c^2)^(1/2)*b^7*c^2+896*(b^2-c^2)^(1/
2)*b^5*c^4-238*(b^2-c^2)^(1/2)*b^3*c^6+21*(b^2-c^2)^(1/2)*b*c^8+640*b^10-16
32*b^8*c^2+1472*b^6*c^4-562*b^4*c^6+85*b^2*c^8-3*c^10)/c^8)/(tanh(1/2*x)^2+
2/c*(b^2-c^2)^(1/2)*tanh(1/2*x)+2/c*tanh(1/2*x)*b+2/c^2*(b^2-c^2)^(1/2)*b+2
/c^2*b^2-1)^3/(tanh(1/2*x)+1/c*(b^2-c^2)^(1/2)+b/c)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*cosh(x)+c*sinh(x)+(b^2-c^2)^(1/2))^4,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(b\cosh(x)+\sqrt{b^2-c^2}\,+c\sinh(x)\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*cosh(x) + (b^2 - c^2)^(1/2) + c*sinh(x))^4,x)
```

```
[Out] int(1/(b*cosh(x) + (b^2 - c^2)^(1/2) + c*sinh(x))^4, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*cosh(x)+c*sinh(x)+(b**2-c**2)**(1/2))**4,x)
```

```
[Out] Timed out
```

3.761 $\int (a+b\cosh(x)+c\sinh(x))^{5/2}\,dx$

Optimal. Leaf size=294

$$\frac{16ia\left(a^2-b^2+c^2\right)\sqrt{\frac{a+b\cosh(x)+c\sinh(x)}{a+\sqrt{b^2-c^2}}}F\left(\frac{1}{2}\left(ix-\tan^{-1}(b,-ic)\right)|\frac{2\sqrt{b^2-c^2}}{a+\sqrt{b^2-c^2}}\right)}{15\sqrt{a+b\cosh(x)+c\sinh(x)}}-\frac{2i\left(23a^2+9b^2-9c^2\right)\sqrt{a+b\cosh(x}}{15\sqrt{}}$$

```
[Out] 2/5*(c*cosh(x)+b*sinh(x))*(a+b*cosh(x)+c*sinh(x))^(3/2)+16/15*(a*c*cosh(x)+
a*b*sinh(x))*(a+b*cosh(x)+c*sinh(x))^(1/2)-2/15*I*(23*a^2+9*b^2-9*c^2)*(cos
(1/2*I*x-1/2*arctan(b,-I*c))^2)^(1/2)/cos(1/2*I*x-1/2*arctan(b,-I*c))*Ellip
ticE(sin(1/2*I*x-1/2*arctan(b,-I*c)),2^(1/2)*((b^2-c^2)^(1/2)/(a+(b^2-c^2)^
(1/2)))^(1/2))*(a+b*cosh(x)+c*sinh(x))^(1/2)/((a+b*cosh(x)+c*sinh(x))/(a+(b
^2-c^2)^(1/2)))^(1/2)+16/15*I*a*(a^2-b^2+c^2)*(cos(1/2*I*x-1/2*arctan(b,-I*
c))^2)^(1/2)/cos(1/2*I*x-1/2*arctan(b,-I*c))*EllipticF(sin(1/2*I*x-1/2*arct
an(b,-I*c)),2^(1/2)*((b^2-c^2)^(1/2)/(a+(b^2-c^2)^(1/2)))^(1/2))*((a+b*cosh
(x)+c*sinh(x))/(a+(b^2-c^2)^(1/2)))^(1/2)/(a+b*cosh(x)+c*sinh(x))^(1/2)
```

Rubi [A] time = 0.48, antiderivative size = 294, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3120, 3146, 3149, 3119, 2653, 3127, 2661}

$$\frac{16ia\left(a^2-b^2+c^2\right)\sqrt{\frac{a+b\cosh(x)+c\sinh(x)}{a+\sqrt{b^2-c^2}}}F\left(\frac{1}{2}\left(ix-\tan^{-1}(b,-ic)\right)|\frac{2\sqrt{b^2-c^2}}{a+\sqrt{b^2-c^2}}\right)}{15\sqrt{a+b\cosh(x)+c\sinh(x)}}-\frac{2i\left(23a^2+9b^2-9c^2\right)\sqrt{a+b\cosh(x}}{15\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cosh[x] + c*Sinh[x])^(5/2),x]
```

```
[Out] (16*(a*c*Cosh[x] + a*b*Sinh[x])*Sqrt[a + b*Cosh[x] + c*Sinh[x]])/15 + (2*(c
*Cosh[x] + b*Sinh[x])*(a + b*Cosh[x] + c*Sinh[x])^(3/2))/5 - (((2*I)/15)*(2
3*a^2 + 9*b^2 - 9*c^2)*EllipticE[(I*x - ArcTan[b, (-I)*c])/2, (2*Sqrt[b^2 -
 c^2])/(a + Sqrt[b^2 - c^2])]*Sqrt[a + b*Cosh[x] + c*Sinh[x]])/Sqrt[(a + b*
Cosh[x] + c*Sinh[x])/(a + Sqrt[b^2 - c^2])] + (((16*I)/15)*a*(a^2 - b^2 + c
^2)*EllipticF[(I*x - ArcTan[b, (-I)*c])/2, (2*Sqrt[b^2 - c^2])/(a + Sqrt[b^
2 - c^2])]*Sqrt[(a + b*Cosh[x] + c*Sinh[x])/(a + Sqrt[b^2 - c^2])])/Sqrt[a
+ b*Cosh[x] + c*Sinh[x]]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
```

```
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3119

```
Int[Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_
)]], x_Symbol] :> Dist[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]]/Sqrt[(a +
b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sqrt[b^2 + c^2])], Int[Sqrt[a/(a + Sq
rt[b^2 + c^2]) + (Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b, c]])/(a + Sqrt[b^
2 + c^2])], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
&& NeQ[b^2 + c^2, 0] &&  !GtQ[a + Sqrt[b^2 + c^2], 0]
```

Rule 3120

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> -Simp[((c*Cos[d + e*x] - b*Sin[d + e*x])*(a + b*Cos[d +
e*x] + c*Sin[d + e*x])^(n - 1))/(e*n), x] + Dist[1/n, Int[Simp[n*a^2 + (n -
 1)*(b^2 + c^2) + a*b*(2*n - 1)*Cos[d + e*x] + a*c*(2*n - 1)*Sin[d + e*x],
x]*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n - 2), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && GtQ[n, 1]
```

Rule 3127

```
Int[1/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(
x_)]], x_Symbol] :> Dist[Sqrt[(a + b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sq
rt[b^2 + c^2])]/Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]], Int[1/Sqrt[a/(a
+ Sqrt[b^2 + c^2]) + (Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b, c]])/(a + Sqr
t[b^2 + c^2])], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2,
 0] && NeQ[b^2 + c^2, 0] &&  !GtQ[a + Sqrt[b^2 + c^2], 0]
```

Rule 3146

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_.)*((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)
]), x_Symbol] :> Simp[((B*c - b*C - a*C*Cos[d + e*x] + a*B*Sin[d + e*x])*(a
 + b*Cos[d + e*x] + c*Sin[d + e*x])^n)/(a*e*(n + 1)), x] + Dist[1/(a*(n + 1
)), Int[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n - 1)*Simp[a*(b*B + c*C)*n
+ a^2*A*(n + 1) + (n*(a^2*B - B*c^2 + b*c*C) + a*b*A*(n + 1))*Cos[d + e*x]
+ (n*(b*B*c + a^2*C - b^2*C) + a*c*A*(n + 1))*Sin[d + e*x], x], x], x] /; F
```

```
reeQ[{a, b, c, d, e, A, B, C}, x] && GtQ[n, 0] && NeQ[a^2 - b^2 - c^2, 0]
```

Rule 3149

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)]]
, x_Symbol] :> Dist[B/b, Int[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]], x],
 x] + Dist[(A*b - a*B)/b, Int[1/Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]],
x], x] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && EqQ[B*c - b*C, 0] && NeQ[A*
b - a*B, 0]
```

Rubi steps

$$\int (a+b\cosh(x)+c\sinh(x))^{5/2}\,dx = \frac{2}{5}(c\cosh(x)+b\sinh(x))(a+b\cosh(x)+c\sinh(x))^{3/2}+\frac{2}{5}\int \sqrt{a+b\cosh}$$

$$= \frac{16}{15}(ac\cosh(x)+ab\sinh(x))\sqrt{a+b\cosh(x)+c\sinh(x)} + \frac{2}{5}(c\cosh(x)+b$$

$$= \frac{16}{15}(ac\cosh(x)+ab\sinh(x))\sqrt{a+b\cosh(x)+c\sinh(x)} + \frac{2}{5}(c\cosh(x)+b$$

$$= \frac{16}{15}(ac\cosh(x)+ab\sinh(x))\sqrt{a+b\cosh(x)+c\sinh(x)} + \frac{2}{5}(c\cosh(x)+b$$

$$= \frac{16}{15}(ac\cosh(x)+ab\sinh(x))\sqrt{a+b\cosh(x)+c\sinh(x)} + \frac{2}{5}(c\cosh(x)+b$$

Mathematica [C] time = 6.36, size = 3775, normalized size = 12.84

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Cosh[x] + c*Sinh[x])^(5/2),x]
```

```
[Out] Sqrt[a + b*Cosh[x] + c*Sinh[x]]*((2*b*(23*a^2 + 9*b^2 - 9*c^2))/(15*c) + (2
2*a*c*Cosh[x])/15 + (2*b*c*Cosh[2*x])/5 + (22*a*b*Sinh[x])/15 + ((b^2 + c^2
```

```
)*Sinh[2*x])/5) + (2*a^3*AppellF1[1/2, 1/2, 1/2, 3/2, ((-I)*(a + Sqrt[1 - b
^2/c^2]*c*Sinh[x + ArcTanh[b/c]]))/(Sqrt[1 - b^2/c^2]*(1 - (I*a)/(Sqrt[1 -
b^2/c^2]*c))*c), ((-I)*(a + Sqrt[1 - b^2/c^2]*c*Sinh[x + ArcTanh[b/c]]))/(S
qrt[1 - b^2/c^2]*(-1 - (I*a)/(Sqrt[1 - b^2/c^2]*c))*c)]*Sech[x + ArcTanh[b/
c]]*Sqrt[-1 + I*Sinh[x + ArcTanh[b/c]]]*Sqrt[(c*Sqrt[(-b^2 + c^2)/c^2] - I*
c*Sqrt[(-b^2 + c^2)/c^2]*Sinh[x + ArcTanh[b/c]])/(I*a + c*Sqrt[(-b^2 + c^2)
/c^2])]*Sqrt[(c*Sqrt[(-b^2 + c^2)/c^2] + I*c*Sqrt[(-b^2 + c^2)/c^2]*Sinh[x
+ ArcTanh[b/c]])/((-I)*a + c*Sqrt[(-b^2 + c^2)/c^2])]*Sqrt[a + c*Sqrt[(-b^2
 + c^2)/c^2]*Sinh[x + ArcTanh[b/c]]])/(Sqrt[1 - b^2/c^2]*c*Sqrt[I*(I + Sinh
[x + ArcTanh[b/c]])]) + (34*a*b^2*AppellF1[1/2, 1/2, 1/2, 3/2, ((-I)*(a + S
qrt[1 - b^2/c^2]*c*Sinh[x + ArcTanh[b/c]]))/(Sqrt[1 - b^2/c^2]*(1 - (I*a)/(
Sqrt[1 - b^2/c^2]*c))*c), ((-I)*(a + Sqrt[1 - b^2/c^2]*c*Sinh[x + ArcTanh[b
/c]]))/(Sqrt[1 - b^2/c^2]*(-1 - (I*a)/(Sqrt[1 - b^2/c^2]*c))*c)]*Sech[x + A
rcTanh[b/c]]*Sqrt[-1 + I*Sinh[x + ArcTanh[b/c]]]*Sqrt[(c*Sqrt[(-b^2 + c^2)/
c^2] - I*c*Sqrt[(-b^2 + c^2)/c^2]*Sinh[x + ArcTanh[b/c]])/(I*a + c*Sqrt[(-b
^2 + c^2)/c^2])]*Sqrt[(c*Sqrt[(-b^2 + c^2)/c^2] + I*c*Sqrt[(-b^2 + c^2)/c^2
]*Sinh[x + ArcTanh[b/c]])/((-I)*a + c*Sqrt[(-b^2 + c^2)/c^2])]*Sqrt[a + c*S
qrt[(-b^2 + c^2)/c^2]*Sinh[x + ArcTanh[b/c]]])/(15*Sqrt[1 - b^2/c^2]*c*Sqrt
[I*(I + Sinh[x + ArcTanh[b/c]])]) - (34*a*c*AppellF1[1/2, 1/2, 1/2, 3/2, ((
-I)*(a + Sqrt[1 - b^2/c^2]*c*Sinh[x + ArcTanh[b/c]]))/(Sqrt[1 - b^2/c^2]*(1
 - (I*a)/(Sqrt[1 - b^2/c^2]*c))*c), ((-I)*(a + Sqrt[1 - b^2/c^2]*c*Sinh[x +
 ArcTanh[b/c]]))/(Sqrt[1 - b^2/c^2]*(-1 - (I*a)/(Sqrt[1 - b^2/c^2]*c))*c)]*
Sech[x + ArcTanh[b/c]]*Sqrt[-1 + I*Sinh[x + ArcTanh[b/c]]]*Sqrt[(c*Sqrt[(-b
^2 + c^2)/c^2] - I*c*Sqrt[(-b^2 + c^2)/c^2]*Sinh[x + ArcTanh[b/c]])/(I*a +
c*Sqrt[(-b^2 + c^2)/c^2])]*Sqrt[(c*Sqrt[(-b^2 + c^2)/c^2] + I*c*Sqrt[(-b^2
+ c^2)/c^2]*Sinh[x + ArcTanh[b/c]])/((-I)*a + c*Sqrt[(-b^2 + c^2)/c^2])]*Sq
rt[a + c*Sqrt[(-b^2 + c^2)/c^2]*Sinh[x + ArcTanh[b/c]]])/(15*Sqrt[1 - b^2/c
^2]*Sqrt[I*(I + Sinh[x + ArcTanh[b/c]])]) - (23*a^2*b^2*((c*AppellF1[-1/2,
-1/2, -1/2, 1/2, (a + b*Sqrt[1 - c^2/b^2]*Cosh[x + ArcTanh[c/b]])/(b*Sqrt[1
 - c^2/b^2]*(1 + a/(b*Sqrt[1 - c^2/b^2]))), (a + b*Sqrt[1 - c^2/b^2]*Cosh[x
 + ArcTanh[c/b]])/(b*Sqrt[1 - c^2/b^2]*(-1 + a/(b*Sqrt[1 - c^2/b^2])))]*Sin
h[x + ArcTanh[c/b]])/(b*Sqrt[1 - c^2/b^2]*Sqrt[(b*Sqrt[(b^2 - c^2)/b^2] - b
*Sqrt[(b^2 - c^2)/b^2]*Cosh[x + ArcTanh[c/b]])/(a + b*Sqrt[(b^2 - c^2)/b^2]
)]*Sqrt[a + b*Sqrt[(b^2 - c^2)/b^2]*Cosh[x + ArcTanh[c/b]]]*Sqrt[(b*Sqrt[(b
^2 - c^2)/b^2] + b*Sqrt[(b^2 - c^2)/b^2]*Cosh[x + ArcTanh[c/b]])/(-a + b*Sq
rt[(b^2 - c^2)/b^2])]) - ((-2*b*(a + b*Sqrt[1 - c^2/b^2]*Cosh[x + ArcTanh[c
/b]]))/(b^2 - c^2) + (c*Sinh[x + ArcTanh[c/b]])/(b*Sqrt[1 - c^2/b^2]))/Sqrt
[a + b*Sqrt[1 - c^2/b^2]*Cosh[x + ArcTanh[c/b]]]))/(15*c) - (3*b^4*((c*Appe
llF1[-1/2, -1/2, -1/2, 1/2, (a + b*Sqrt[1 - c^2/b^2]*Cosh[x + ArcTanh[c/b]]
)/(b*Sqrt[1 - c^2/b^2]*(1 + a/(b*Sqrt[1 - c^2/b^2]))), (a + b*Sqrt[1 - c^2/
b^2]*Cosh[x + ArcTanh[c/b]])/(b*Sqrt[1 - c^2/b^2]*(-1 + a/(b*Sqrt[1 - c^2/b
^2])))]*Sinh[x + ArcTanh[c/b]])/(b*Sqrt[1 - c^2/b^2]*Sqrt[(b*Sqrt[(b^2 - c^
2)/b^2] - b*Sqrt[(b^2 - c^2)/b^2]*Cosh[x + ArcTanh[c/b]])/(a + b*Sqrt[(b^2
- c^2)/b^2])]*Sqrt[a + b*Sqrt[(b^2 - c^2)/b^2]*Cosh[x + ArcTanh[c/b]]]*Sqrt
[(b*Sqrt[(b^2 - c^2)/b^2] + b*Sqrt[(b^2 - c^2)/b^2]*Cosh[x + ArcTanh[c/b]])
```

```
/(-a + b*Sqrt[(b^2 - c^2)/b^2])]) - ((-2*b*(a + b*Sqrt[1 - c^2/b^2]*Cosh[x
+ ArcTanh[c/b]]))/(b^2 - c^2) + (c*Sinh[x + ArcTanh[c/b]])/(b*Sqrt[1 - c^2/
b^2]))/Sqrt[a + b*Sqrt[1 - c^2/b^2]*Cosh[x + ArcTanh[c/b]]]))/(5*c) + (23*a
^2*c*((c*AppellF1[-1/2, -1/2, -1/2, 1/2, (a + b*Sqrt[1 - c^2/b^2]*Cosh[x +
ArcTanh[c/b]])/(b*Sqrt[1 - c^2/b^2]*(1 + a/(b*Sqrt[1 - c^2/b^2]))), (a + b*
Sqrt[1 - c^2/b^2]*Cosh[x + ArcTanh[c/b]])/(b*Sqrt[1 - c^2/b^2]*(-1 + a/(b*S
qrt[1 - c^2/b^2])))]*Sinh[x + ArcTanh[c/b]])/(b*Sqrt[1 - c^2/b^2]*Sqrt[(b*S
qrt[(b^2 - c^2)/b^2] - b*Sqrt[(b^2 - c^2)/b^2]*Cosh[x + ArcTanh[c/b]])/(a +
 b*Sqrt[(b^2 - c^2)/b^2])]*Sqrt[a + b*Sqrt[(b^2 - c^2)/b^2]*Cosh[x + ArcTan
h[c/b]]]*Sqrt[(b*Sqrt[(b^2 - c^2)/b^2] + b*Sqrt[(b^2 - c^2)/b^2]*Cosh[x + A
rcTanh[c/b]])/(-a + b*Sqrt[(b^2 - c^2)/b^2])]) - ((-2*b*(a + b*Sqrt[1 - c^2
/b^2]*Cosh[x + ArcTanh[c/b]]))/(b^2 - c^2) + (c*Sinh[x + ArcTanh[c/b]])/(b*
Sqrt[1 - c^2/b^2]))/Sqrt[a + b*Sqrt[1 - c^2/b^2]*Cosh[x + ArcTanh[c/b]]]))/
15 + (6*b^2*c*((c*AppellF1[-1/2, -1/2, -1/2, 1/2, (a + b*Sqrt[1 - c^2/b^2]*
Cosh[x + ArcTanh[c/b]])/(b*Sqrt[1 - c^2/b^2]*(1 + a/(b*Sqrt[1 - c^2/b^2])))
, (a + b*Sqrt[1 - c^2/b^2]*Cosh[x + ArcTanh[c/b]])/(b*Sqrt[1 - c^2/b^2]*(-1
 + a/(b*Sqrt[1 - c^2/b^2])))]*Sinh[x + ArcTanh[c/b]])/(b*Sqrt[1 - c^2/b^2]*
Sqrt[(b*Sqrt[(b^2 - c^2)/b^2] - b*Sqrt[(b^2 - c^2)/b^2]*Cosh[x + ArcTanh[c/
b]])/(a + b*Sqrt[(b^2 - c^2)/b^2])]*Sqrt[a + b*Sqrt[(b^2 - c^2)/b^2]*Cosh[x
 + ArcTanh[c/b]]]*Sqrt[(b*Sqrt[(b^2 - c^2)/b^2] + b*Sqrt[(b^2 - c^2)/b^2]*C
osh[x + ArcTanh[c/b]])/(-a + b*Sqrt[(b^2 - c^2)/b^2])]) - ((-2*b*(a + b*Sqr
t[1 - c^2/b^2]*Cosh[x + ArcTanh[c/b]]))/(b^2 - c^2) + (c*Sinh[x + ArcTanh[c
/b]])/(b*Sqrt[1 - c^2/b^2]))/Sqrt[a + b*Sqrt[1 - c^2/b^2]*Cosh[x + ArcTanh[
c/b]]]))/5 - (3*c^3*((c*AppellF1[-1/2, -1/2, -1/2, 1/2, (a + b*Sqrt[1 - c^2
/b^2]*Cosh[x + ArcTanh[c/b]])/(b*Sqrt[1 - c^2/b^2]*(1 + a/(b*Sqrt[1 - c^2/b
^2]))), (a + b*Sqrt[1 - c^2/b^2]*Cosh[x + ArcTanh[c/b]])/(b*Sqrt[1 - c^2/b^
2]*(-1 + a/(b*Sqrt[1 - c^2/b^2])))]*Sinh[x + ArcTanh[c/b]])/(b*Sqrt[1 - c^2
/b^2]*Sqrt[(b*Sqrt[(b^2 - c^2)/b^2] - b*Sqrt[(b^2 - c^2)/b^2]*Cosh[x + ArcT
anh[c/b]])/(a + b*Sqrt[(b^2 - c^2)/b^2])]*Sqrt[a + b*Sqrt[(b^2 - c^2)/b^2]*
Cosh[x + ArcTanh[c/b]]]*Sqrt[(b*Sqrt[(b^2 - c^2)/b^2] + b*Sqrt[(b^2 - c^2)/
b^2]*Cosh[x + ArcTanh[c/b]])/(-a + b*Sqrt[(b^2 - c^2)/b^2])]) - ((-2*b*(a +
 b*Sqrt[1 - c^2/b^2]*Cosh[x + ArcTanh[c/b]]))/(b^2 - c^2) + (c*Sinh[x + Arc
Tanh[c/b]])/(b*Sqrt[1 - c^2/b^2]))/Sqrt[a + b*Sqrt[1 - c^2/b^2]*Cosh[x + Ar
cTanh[c/b]]]))/5
```

fricas [F] time = 0.44, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b^2\cosh(x)^2 + c^2\sinh(x)^2 + 2\,ab\cosh(x) + a^2 + 2\,(bc\cosh(x) + ac)\sinh(x)\right)\sqrt{b\cosh(x) + c\sinh(x) + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cosh(x)+c*sinh(x))^(5/2),x, algorithm="fricas")
```

```
[Out] integral((b^2*cosh(x)^2 + c^2*sinh(x)^2 + 2*a*b*cosh(x) + a^2 + 2*(b*c*cosh
(x) + a*c)*sinh(x))*sqrt(b*cosh(x) + c*sinh(x) + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\cosh(x) + c\sinh(x) + a)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cosh(x)+c*sinh(x))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*cosh(x) + c*sinh(x) + a)^(5/2), x)
```

maple [B] time = 1.38, size = 1036, normalized size = 3.52

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cosh(x)+c*sinh(x))^(5/2),x)
```

```
[Out] 1/6/(-b^2*sinh(x)^2+c^2*sinh(x)^2+2*sinh(x)*a*(b^2-c^2)^(1/2)-a^2)/sinh(x)*
(-6*ln((cosh(x)*sinh(x)*(-b^2+c^2)+cosh(x)*(b^2-c^2)^(1/2)*a+((-b^2+c^2)/(b
^2-c^2)^(1/2)*sinh(x)^3+a*sinh(x)^2)^(1/2)*(b^2-c^2)^(1/2)*((-b^2+c^2)/(b^2
-c^2)^(1/2)*sinh(x)+a)^(1/2))/(b^2-c^2)^(1/2)/((-b^2+c^2)/(b^2-c^2)^(1/2)*s
inh(x)+a)^(1/2))*((-b^2+c^2)/(b^2-c^2)^(1/2)*sinh(x)^3+a*sinh(x)^2)^(1/2)*a
^4+3*ln((cosh(x)*sinh(x)*(-b^2+c^2)+cosh(x)*(b^2-c^2)^(1/2)*a+((-b^2+c^2)/(
b^2-c^2)^(1/2)*sinh(x)^3+a*sinh(x)^2)^(1/2)*(b^2-c^2)^(1/2)*((-b^2+c^2)/(b^
2-c^2)^(1/2)*sinh(x)+a)^(1/2))/(b^2-c^2)^(1/2)/((-b^2+c^2)/(b^2-c^2)^(1/2)*
sinh(x)+a)^(1/2))*((-b^2+c^2)/(b^2-c^2)^(1/2)*sinh(x)^3+a*sinh(x)^2)^(1/2)*
a^2*b^2-3*ln((cosh(x)*sinh(x)*(-b^2+c^2)+cosh(x)*(b^2-c^2)^(1/2)*a+((-b^2+c
^2)/(b^2-c^2)^(1/2)*sinh(x)^3+a*sinh(x)^2)^(1/2)*(b^2-c^2)^(1/2)*((-b^2+c^2
)/(b^2-c^2)^(1/2)*sinh(x)+a)^(1/2))/(b^2-c^2)^(1/2)/((-b^2+c^2)/(b^2-c^2)^(
1/2)*sinh(x)+a)^(1/2))*((-b^2+c^2)/(b^2-c^2)^(1/2)*sinh(x)^3+a*sinh(x)^2)^(
1/2)*a^2*c^2+5*sinh(x)^3*cosh(x)*(b^2-c^2)^(3/2)*((-b^2+c^2)/(b^2-c^2)^(1/2
)*sinh(x)+a)^(1/2)*a-2*(b^4-2*b^2*c^2+c^4)*cosh(x)*sinh(x)^4*((-b^2+c^2)/(b
^2-c^2)^(1/2)*sinh(x)+a)^(1/2)+3*(2*a^2-b^2+c^2)*sinh(x)*ln((cosh(x)*sinh(x
)*(-b^2+c^2)+cosh(x)*(b^2-c^2)^(1/2)*a+((-b^2+c^2)/(b^2-c^2)^(1/2)*sinh(x)^
3+a*sinh(x)^2)^(1/2)*(b^2-c^2)^(1/2)*((-b^2+c^2)/(b^2-c^2)^(1/2)*sinh(x)+a)
^(1/2))/(b^2-c^2)^(1/2)/((-b^2+c^2)/(b^2-c^2)^(1/2)*sinh(x)+a)^(1/2))*((-b^
2+c^2)/(b^2-c^2)^(1/2)*sinh(x)^3+a*sinh(x)^2)^(1/2)*(b^2-c^2)^(1/2)*a+2*sin
h(x)*cosh(x)*(9*a^2-2*b^2+2*c^2)*(b^2-c^2)^(1/2)*((-b^2+c^2)/(b^2-c^2)^(1/2
)*sinh(x)+a)^(1/2)*a-(21*a^2*b^2-21*a^2*c^2-4*b^4+8*b^2*c^2-4*c^4)*cosh(x)*
sinh(x)^2*((-b^2+c^2)/(b^2-c^2)^(1/2)*sinh(x)+a)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\cosh(x) + c\sinh(x) + a)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cosh(x)+c*sinh(x))^(5/2),x, algorithm="maxima")
[Out] integrate((b*cosh(x) + c*sinh(x) + a)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int (a + b\cosh(x) + c\sinh(x))^{5/2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*cosh(x) + c*sinh(x))^(5/2),x)
[Out] int((a + b*cosh(x) + c*sinh(x))^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cosh(x)+c*sinh(x))**(5/2),x)
[Out] Timed out
```

3.762 $\int (a+b\cosh(x)+c\sinh(x))^{3/2}\,dx$

Optimal. Leaf size=249

$$\frac{2i\left(a^2-b^2+c^2\right)\sqrt{\frac{a+b\cosh(x)+c\sinh(x)}{a+\sqrt{b^2-c^2}}}\,F\left(\frac{1}{2}\left(ix-\tan^{-1}(b,-ic)\right)|\frac{2\sqrt{b^2-c^2}}{a+\sqrt{b^2-c^2}}\right)}{3\sqrt{a+b\cosh(x)+c\sinh(x)}}-\frac{8ia\sqrt{a+b\cosh(x)+c\sinh(x)}\,E\left(\frac{1}{2}\left(ix\right.\right.}{3\sqrt{\frac{a+b\cosh(x)+c\sin}{a+\sqrt{b^2-c^2}}}}$$

```
[Out] 2/3*(c*cosh(x)+b*sinh(x))*(a+b*cosh(x)+c*sinh(x))^(1/2)-8/3*I*a*(cos(1/2*I*
x-1/2*arctan(b,-I*c))^2)^(1/2)/cos(1/2*I*x-1/2*arctan(b,-I*c))*EllipticE(si
n(1/2*I*x-1/2*arctan(b,-I*c)),2^(1/2)*((b^2-c^2)^(1/2)/(a+(b^2-c^2)^(1/2)))
^(1/2))*(a+b*cosh(x)+c*sinh(x))^(1/2)/((a+b*cosh(x)+c*sinh(x))/(a+(b^2-c^2)
^(1/2)))^(1/2)+2/3*I*(a^2-b^2+c^2)*(cos(1/2*I*x-1/2*arctan(b,-I*c))^2)^(1/2
)/cos(1/2*I*x-1/2*arctan(b,-I*c))*EllipticF(sin(1/2*I*x-1/2*arctan(b,-I*c))
,2^(1/2)*((b^2-c^2)^(1/2)/(a+(b^2-c^2)^(1/2)))^(1/2))*((a+b*cosh(x)+c*sinh(
x))/(a+(b^2-c^2)^(1/2)))^(1/2)/(a+b*cosh(x)+c*sinh(x))^(1/2)
```

Rubi [A] time = 0.27, antiderivative size = 249, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {3120, 3149, 3119, 2653, 3127, 2661}

$$\frac{2i\left(a^2-b^2+c^2\right)\sqrt{\frac{a+b\cosh(x)+c\sinh(x)}{a+\sqrt{b^2-c^2}}}\,F\left(\frac{1}{2}\left(ix-\tan^{-1}(b,-ic)\right)|\frac{2\sqrt{b^2-c^2}}{a+\sqrt{b^2-c^2}}\right)}{3\sqrt{a+b\cosh(x)+c\sinh(x)}}-\frac{8ia\sqrt{a+b\cosh(x)+c\sinh(x)}\,E\left(\frac{1}{2}\left(ix\right.\right.}{3\sqrt{\frac{a+b\cosh(x)+c\sin}{a+\sqrt{b^2-c^2}}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cosh[x] + c*Sinh[x])^(3/2),x]
```

```
[Out] (2*(c*Cosh[x] + b*Sinh[x])*Sqrt[a + b*Cosh[x] + c*Sinh[x]])/3 - (((8*I)/3)*
a*EllipticE[(I*x - ArcTan[b, (-I)*c])/2, (2*Sqrt[b^2 - c^2])/(a + Sqrt[b^2
- c^2])]*Sqrt[a + b*Cosh[x] + c*Sinh[x]])/Sqrt[(a + b*Cosh[x] + c*Sinh[x])/
(a + Sqrt[b^2 - c^2])] + (((2*I)/3)*(a^2 - b^2 + c^2)*EllipticF[(I*x - ArcT
an[b, (-I)*c])/2, (2*Sqrt[b^2 - c^2])/(a + Sqrt[b^2 - c^2])]*Sqrt[(a + b*Co
sh[x] + c*Sinh[x])/(a + Sqrt[b^2 - c^2])])/Sqrt[a + b*Cosh[x] + c*Sinh[x]]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3119

```
Int[Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_
)]], x_Symbol] :> Dist[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]]/Sqrt[(a +
b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sqrt[b^2 + c^2])], Int[Sqrt[a/(a + Sq
rt[b^2 + c^2]) + (Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b, c]])/(a + Sqrt[b^
2 + c^2])], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
&& NeQ[b^2 + c^2, 0] &&  !GtQ[a + Sqrt[b^2 + c^2], 0]
```

Rule 3120

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> -Simp[((c*Cos[d + e*x] - b*Sin[d + e*x])*(a + b*Cos[d +
e*x] + c*Sin[d + e*x])^(n - 1))/(e*n), x] + Dist[1/n, Int[Simp[n*a^2 + (n -
 1)*(b^2 + c^2) + a*b*(2*n - 1)*Cos[d + e*x] + a*c*(2*n - 1)*Sin[d + e*x],
x]*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n - 2), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && GtQ[n, 1]
```

Rule 3127

```
Int[1/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(
x_)]], x_Symbol] :> Dist[Sqrt[(a + b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sq
rt[b^2 + c^2])]/Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]], Int[1/Sqrt[a/(a
+ Sqrt[b^2 + c^2]) + (Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b, c]])/(a + Sqr
t[b^2 + c^2])], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2,
 0] && NeQ[b^2 + c^2, 0] &&  !GtQ[a + Sqrt[b^2 + c^2], 0]
```

Rule 3149

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)]]
, x_Symbol] :> Dist[B/b, Int[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]], x],
 x] + Dist[(A*b - a*B)/b, Int[1/Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]],
x], x] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && EqQ[B*c - b*C, 0] && NeQ[A*
b - a*B, 0]
```

Rubi steps

$$\int (a+b\cosh(x)+c\sinh(x))^{3/2}\,dx = \frac{2}{3}(c\cosh(x)+b\sinh(x))\sqrt{a+b\cosh(x)+c\sinh(x)} + \frac{2}{3}\int \frac{\frac{1}{2}\left(3a^2+b^2-c\right.}{\sqrt{a+}}$$

$$= \frac{2}{3}(c\cosh(x)+b\sinh(x))\sqrt{a+b\cosh(x)+c\sinh(x)} + \frac{1}{3}(4a)\int \sqrt{a+b\cos}$$

$$= \frac{2}{3}(c\cosh(x)+b\sinh(x))\sqrt{a+b\cosh(x)+c\sinh(x)} + \frac{\left(4a\sqrt{a+b\cosh(x)}\right.}{}$$

$$= \frac{2}{3}(c\cosh(x)+b\sinh(x))\sqrt{a+b\cosh(x)+c\sinh(x)} - \frac{8iaE\left(\frac{1}{2}\left(ix-\tan^{-1}(\right.\right.}{}$$

Mathematica [C] time = 6.16, size = 2292, normalized size = 9.20

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Cosh[x] + c*Sinh[x])^(3/2),x]
```

```
[Out] ((8*a*b)/(3*c) + (2*c*Cosh[x])/3 + (2*b*Sinh[x])/3)*Sqrt[a + b*Cosh[x] + c*
Sinh[x]] + (2*a^2*AppellF1[1/2, 1/2, 1/2, 3/2, ((-I)*(a + Sqrt[1 - b^2/c^2]
*c*Sinh[x + ArcTanh[b/c]]))/(Sqrt[1 - b^2/c^2]*(1 - (I*a)/(Sqrt[1 - b^2/c^2
]*c))*c), ((-I)*(a + Sqrt[1 - b^2/c^2]*c*Sinh[x + ArcTanh[b/c]]))/(Sqrt[1 -
 b^2/c^2]*(-1 - (I*a)/(Sqrt[1 - b^2/c^2]*c))*c)]*Sech[x + ArcTanh[b/c]]*Sqr
t[-1 + I*Sinh[x + ArcTanh[b/c]]]*Sqrt[(c*Sqrt[(-b^2 + c^2)/c^2] - I*c*Sqrt[
(-b^2 + c^2)/c^2]*Sinh[x + ArcTanh[b/c]])/(I*a + c*Sqrt[(-b^2 + c^2)/c^2])]
*Sqrt[(c*Sqrt[(-b^2 + c^2)/c^2] + I*c*Sqrt[(-b^2 + c^2)/c^2]*Sinh[x + ArcTa
nh[b/c]])/((-I)*a + c*Sqrt[(-b^2 + c^2)/c^2])]*Sqrt[a + c*Sqrt[(-b^2 + c^2)
/c^2]*Sinh[x + ArcTanh[b/c]]])/(Sqrt[1 - b^2/c^2]*c*Sqrt[I*(I + Sinh[x + Ar
cTanh[b/c]])]) + (2*b^2*AppellF1[1/2, 1/2, 1/2, 3/2, ((-I)*(a + Sqrt[1 - b^
2/c^2]*c*Sinh[x + ArcTanh[b/c]]))/(Sqrt[1 - b^2/c^2]*(1 - (I*a)/(Sqrt[1 - b
^2/c^2]*c))*c), ((-I)*(a + Sqrt[1 - b^2/c^2]*c*Sinh[x + ArcTanh[b/c]]))/(Sq
rt[1 - b^2/c^2]*(-1 - (I*a)/(Sqrt[1 - b^2/c^2]*c))*c)]*Sech[x + ArcTanh[b/c
]]*Sqrt[-1 + I*Sinh[x + ArcTanh[b/c]]]*Sqrt[(c*Sqrt[(-b^2 + c^2)/c^2] - I*c
*Sqrt[(-b^2 + c^2)/c^2]*Sinh[x + ArcTanh[b/c]])/(I*a + c*Sqrt[(-b^2 + c^2)/
c^2])]*Sqrt[(c*Sqrt[(-b^2 + c^2)/c^2] + I*c*Sqrt[(-b^2 + c^2)/c^2]*Sinh[x +
 ArcTanh[b/c]])/((-I)*a + c*Sqrt[(-b^2 + c^2)/c^2])]*Sqrt[a + c*Sqrt[(-b^2
```

```
+ c^2)/c^2]*Sinh[x + ArcTanh[b/c]]])/(3*Sqrt[1 - b^2/c^2]*c*Sqrt[I*(I + Sin
h[x + ArcTanh[b/c]])]) - (2*c*AppellF1[1/2, 1/2, 1/2, 3/2, ((-I)*(a + Sqrt[
1 - b^2/c^2]*c*Sinh[x + ArcTanh[b/c]]))/(Sqrt[1 - b^2/c^2]*(1 - (I*a)/(Sqrt
[1 - b^2/c^2]*c))*c), ((-I)*(a + Sqrt[1 - b^2/c^2]*c*Sinh[x + ArcTanh[b/c]]
))/(Sqrt[1 - b^2/c^2]*(-1 - (I*a)/(Sqrt[1 - b^2/c^2]*c))*c)]*Sech[x + ArcTa
nh[b/c]]*Sqrt[-1 + I*Sinh[x + ArcTanh[b/c]]]*Sqrt[(c*Sqrt[(-b^2 + c^2)/c^2]
 - I*c*Sqrt[(-b^2 + c^2)/c^2]*Sinh[x + ArcTanh[b/c]])/(I*a + c*Sqrt[(-b^2 +
 c^2)/c^2])]*Sqrt[(c*Sqrt[(-b^2 + c^2)/c^2] + I*c*Sqrt[(-b^2 + c^2)/c^2]*Si
nh[x + ArcTanh[b/c]])/((-I)*a + c*Sqrt[(-b^2 + c^2)/c^2])]*Sqrt[a + c*Sqrt[
(-b^2 + c^2)/c^2]*Sinh[x + ArcTanh[b/c]]])/(3*Sqrt[1 - b^2/c^2]*Sqrt[I*(I +
 Sinh[x + ArcTanh[b/c]])]) - (4*a*b^2*((c*AppellF1[-1/2, -1/2, -1/2, 1/2, (
a + b*Sqrt[1 - c^2/b^2]*Cosh[x + ArcTanh[c/b]])/(b*Sqrt[1 - c^2/b^2]*(1 + a
/(b*Sqrt[1 - c^2/b^2]))), (a + b*Sqrt[1 - c^2/b^2]*Cosh[x + ArcTanh[c/b]])/
(b*Sqrt[1 - c^2/b^2]*(-1 + a/(b*Sqrt[1 - c^2/b^2])))]*Sinh[x + ArcTanh[c/b]
])/(b*Sqrt[1 - c^2/b^2]*Sqrt[(b*Sqrt[(b^2 - c^2)/b^2] - b*Sqrt[(b^2 - c^2)/
b^2]*Cosh[x + ArcTanh[c/b]])/(a + b*Sqrt[(b^2 - c^2)/b^2])]*Sqrt[a + b*Sqrt
[(b^2 - c^2)/b^2]*Cosh[x + ArcTanh[c/b]]]*Sqrt[(b*Sqrt[(b^2 - c^2)/b^2] + b
*Sqrt[(b^2 - c^2)/b^2]*Cosh[x + ArcTanh[c/b]])/(-a + b*Sqrt[(b^2 - c^2)/b^2
])]) - ((-2*b*(a + b*Sqrt[1 - c^2/b^2]*Cosh[x + ArcTanh[c/b]]))/(b^2 - c^2)
 + (c*Sinh[x + ArcTanh[c/b]])/(b*Sqrt[1 - c^2/b^2]))/Sqrt[a + b*Sqrt[1 - c^
2/b^2]*Cosh[x + ArcTanh[c/b]]]))/(3*c) + (4*a*c*((c*AppellF1[-1/2, -1/2, -1
/2, 1/2, (a + b*Sqrt[1 - c^2/b^2]*Cosh[x + ArcTanh[c/b]])/(b*Sqrt[1 - c^2/b
^2]*(1 + a/(b*Sqrt[1 - c^2/b^2]))), (a + b*Sqrt[1 - c^2/b^2]*Cosh[x + ArcTa
nh[c/b]])/(b*Sqrt[1 - c^2/b^2]*(-1 + a/(b*Sqrt[1 - c^2/b^2])))]*Sinh[x + Ar
cTanh[c/b]])/(b*Sqrt[1 - c^2/b^2]*Sqrt[(b*Sqrt[(b^2 - c^2)/b^2] - b*Sqrt[(b
^2 - c^2)/b^2]*Cosh[x + ArcTanh[c/b]])/(a + b*Sqrt[(b^2 - c^2)/b^2])]*Sqrt[
a + b*Sqrt[(b^2 - c^2)/b^2]*Cosh[x + ArcTanh[c/b]]]*Sqrt[(b*Sqrt[(b^2 - c^2
)/b^2] + b*Sqrt[(b^2 - c^2)/b^2]*Cosh[x + ArcTanh[c/b]])/(-a + b*Sqrt[(b^2
- c^2)/b^2])]) - ((-2*b*(a + b*Sqrt[1 - c^2/b^2]*Cosh[x + ArcTanh[c/b]]))/(
b^2 - c^2) + (c*Sinh[x + ArcTanh[c/b]])/(b*Sqrt[1 - c^2/b^2]))/Sqrt[a + b*S
qrt[1 - c^2/b^2]*Cosh[x + ArcTanh[c/b]]]))/3
```

fricas [F] time = 0.44, size = 0, normalized size = 0.00

$$\text{integral}\left((b\cosh(x)+c\sinh(x)+a)^{\frac{3}{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cosh(x)+c*sinh(x))^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b*cosh(x) + c*sinh(x) + a)^(3/2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\cosh(x)+c\sinh(x)+a)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cosh(x)+c*sinh(x))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*cosh(x) + c*sinh(x) + a)^(3/2), x)
```

maple [A] time = 0.79, size = 318, normalized size = 1.28

$$\frac{2a\left(-b^2+c^2\right)\cosh(x)}{\sqrt{b^2-c^2}\,\sqrt{\frac{-\sinh(x)b^2+\sinh(x)c^2+a\sqrt{b^2-c^2}}{\sqrt{b^2-c^2}}}}+\frac{\sqrt{\frac{\left(-\sinh(x)b^2+\sinh(x)c^2+a\sqrt{b^2-c^2}\right)\left(\sinh^2(x)\right)}{\sqrt{b^2-c^2}}}\;a^2\ln\left(\frac{\cosh(x)\sinh(x)\left(-b^2+c^2\right)+\cosh(x)\sqrt{b\cdots}}{\sqrt{b\cdots}}\right.}{\left(-\sinh(x)b^2+\sinh(x)c^2+\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cosh(x)+c*sinh(x))^(3/2),x)
```

```
[Out] 2*a/(b^2-c^2)^(1/2)*(-b^2+c^2)/((-sinh(x)*b^2+sinh(x)*c^2+a*(b^2-c^2)^(1/2)
)/(b^2-c^2)^(1/2))^(1/2)*cosh(x)+((-sinh(x)*b^2+sinh(x)*c^2+a*(b^2-c^2)^(1/
2))/(b^2-c^2)^(1/2)*sinh(x)^2)^(1/2)*a^2*ln((cosh(x)*sinh(x)*(-b^2+c^2)+cos
h(x)*(b^2-c^2)^(1/2)*a+((-b^2+c^2)/(b^2-c^2)^(1/2)*sinh(x)^3+a*sinh(x)^2)^(
1/2)*(b^2-c^2)^(1/2)*((-b^2+c^2)/(b^2-c^2)^(1/2)*sinh(x)+a)^(1/2))/(b^2-c^2
)^(1/2)/((-sinh(x)*b^2+sinh(x)*c^2+a*(b^2-c^2)^(1/2))/(b^2-c^2)^(1/2))^(1/2
))/(-sinh(x)*b^2+sinh(x)*c^2+a*(b^2-c^2)^(1/2))*(b^2-c^2)^(1/2)/sinh(x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\cosh(x)+c\sinh(x)+a)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cosh(x)+c*sinh(x))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*cosh(x) + c*sinh(x) + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int (a+b\cosh(x)+c\sinh(x))^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*cosh(x) + c*sinh(x))^(3/2),x)
```

```
[Out] int((a + b*cosh(x) + c*sinh(x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b\cosh(x) + c\sinh(x))^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cosh(x)+c*sinh(x))**(3/2),x)
[Out] Integral((a + b*cosh(x) + c*sinh(x))**(3/2), x)
```

3.763 $\int \sqrt{a+b\cosh(x)+c\sinh(x)}\,dx$

Optimal. Leaf size=102

$$-\frac{2i\sqrt{a+b\cosh(x)+c\sinh(x)}\,E\left(\frac{1}{2}\left(ix-\tan^{-1}(b,-ic)\right)|\frac{2\sqrt{b^2-c^2}}{a+\sqrt{b^2-c^2}}\right)}{\sqrt{\frac{a+b\cosh(x)+c\sinh(x)}{a+\sqrt{b^2-c^2}}}}$$

```
[Out] -2*I*(cos(1/2*I*x-1/2*arctan(b,-I*c))^2)^(1/2)/cos(1/2*I*x-1/2*arctan(b,-I*
c))*EllipticE(sin(1/2*I*x-1/2*arctan(b,-I*c)),2^(1/2)*((b^2-c^2)^(1/2)/(a+(
b^2-c^2)^(1/2)))^(1/2))*(a+b*cosh(x)+c*sinh(x))^(1/2)/((a+b*cosh(x)+c*sinh(
x))/(a+(b^2-c^2)^(1/2)))^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3119, 2653}

$$-\frac{2i\sqrt{a+b\cosh(x)+c\sinh(x)}\,E\left(\frac{1}{2}\left(ix-\tan^{-1}(b,-ic)\right)|\frac{2\sqrt{b^2-c^2}}{a+\sqrt{b^2-c^2}}\right)}{\sqrt{\frac{a+b\cosh(x)+c\sinh(x)}{a+\sqrt{b^2-c^2}}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Cosh[x] + c*Sinh[x]],x]
```

```
[Out] ((-2*I)*EllipticE[(I*x - ArcTan[b, (-I)*c])/2, (2*Sqrt[b^2 - c^2])/(a + Sqr
t[b^2 - c^2])]*Sqrt[a + b*Cosh[x] + c*Sinh[x]])/Sqrt[(a + b*Cosh[x] + c*Sin
h[x])/(a + Sqrt[b^2 - c^2])]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3119

```
Int[Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_
)]], x_Symbol] :> Dist[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]]/Sqrt[(a +
b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sqrt[b^2 + c^2])], Int[Sqrt[a/(a + Sq
rt[b^2 + c^2]) + (Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b, c]])/(a + Sqrt[b^
2 + c^2])], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
```

```
&& NeQ[b^2 + c^2, 0] &&  !GtQ[a + Sqrt[b^2 + c^2], 0]
```

Rubi steps

$$\int \sqrt{a+b\cosh(x)+c\sinh(x)}\,dx = \frac{\sqrt{a+b\cosh(x)+c\sinh(x)}\int\sqrt{\frac{a}{a+\sqrt{b^2-c^2}}+\frac{\sqrt{b^2-c^2}\cosh\left(x+i\tan^{-1}(b,-ic)\right)}{a+\sqrt{b^2-c^2}}}\,dx}{\sqrt{\frac{a+b\cosh(x)+c\sinh(x)}{a+\sqrt{b^2-c^2}}}}$$

$$= -\frac{2iE\left(\frac{1}{2}\left(ix-\tan^{-1}(b,-ic)\right)|\frac{2\sqrt{b^2-c^2}}{a+\sqrt{b^2-c^2}}\right)\sqrt{a+b\cosh(x)+c\sinh(x)}}{\sqrt{\frac{a+b\cosh(x)+c\sinh(x)}{a+\sqrt{b^2-c^2}}}}$$

Mathematica [C] time = 6.11, size = 1401, normalized size = 13.74

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[a + b*Cosh[x] + c*Sinh[x]],x]
```

```
[Out] (2*b*Sqrt[a + b*Cosh[x] + c*Sinh[x]])/c + (2*a*AppellF1[1/2, 1/2, 1/2, 3/2,
 ((-I)*(a + Sqrt[1 - b^2/c^2]*c*Sinh[x + ArcTanh[b/c]]))/(Sqrt[1 - b^2/c^2]
*(1 - (I*a)/(Sqrt[1 - b^2/c^2]*c))*c), ((-I)*(a + Sqrt[1 - b^2/c^2]*c*Sinh[
x + ArcTanh[b/c]]))/(Sqrt[1 - b^2/c^2]*(-1 - (I*a)/(Sqrt[1 - b^2/c^2]*c))*c
)]*Sech[x + ArcTanh[b/c]]*Sqrt[-1 + I*Sinh[x + ArcTanh[b/c]]]*Sqrt[(c*Sqrt[
(-b^2 + c^2)/c^2] - I*c*Sqrt[(-b^2 + c^2)/c^2]*Sinh[x + ArcTanh[b/c]])/(I*a
 + c*Sqrt[(-b^2 + c^2)/c^2])]*Sqrt[(c*Sqrt[(-b^2 + c^2)/c^2] + I*c*Sqrt[(-b
^2 + c^2)/c^2]*Sinh[x + ArcTanh[b/c]])/((-I)*a + c*Sqrt[(-b^2 + c^2)/c^2])]
*Sqrt[a + c*Sqrt[(-b^2 + c^2)/c^2]*Sinh[x + ArcTanh[b/c]]])/(Sqrt[1 - b^2/c
^2]*c*Sqrt[I*(I + Sinh[x + ArcTanh[b/c]])]) - (b^2*((c*AppellF1[-1/2, -1/2,
 -1/2, 1/2, (a + b*Sqrt[1 - c^2/b^2]*Cosh[x + ArcTanh[c/b]])/(b*Sqrt[1 - c^
2/b^2]*(1 + a/(b*Sqrt[1 - c^2/b^2]))), (a + b*Sqrt[1 - c^2/b^2]*Cosh[x + Ar
cTanh[c/b]])/(b*Sqrt[1 - c^2/b^2]*(-1 + a/(b*Sqrt[1 - c^2/b^2])))]*Sinh[x +
 ArcTanh[c/b]])/(b*Sqrt[1 - c^2/b^2]*Sqrt[(b*Sqrt[(b^2 - c^2)/b^2] - b*Sqrt
[(b^2 - c^2)/b^2]*Cosh[x + ArcTanh[c/b]])/(a + b*Sqrt[(b^2 - c^2)/b^2])]*Sq
rt[a + b*Sqrt[(b^2 - c^2)/b^2]*Cosh[x + ArcTanh[c/b]]]*Sqrt[(b*Sqrt[(b^2 -
c^2)/b^2] + b*Sqrt[(b^2 - c^2)/b^2]*Cosh[x + ArcTanh[c/b]])/(-a + b*Sqrt[(b
^2 - c^2)/b^2])]) - ((-2*b*(a + b*Sqrt[1 - c^2/b^2]*Cosh[x + ArcTanh[c/b]])
)/(b^2 - c^2) + (c*Sinh[x + ArcTanh[c/b]])/(b*Sqrt[1 - c^2/b^2]))/Sqrt[a +
b*Sqrt[1 - c^2/b^2]*Cosh[x + ArcTanh[c/b]]]))/c + c*((c*AppellF1[-1/2, -1/2
```

```
, -1/2, 1/2, (a + b*Sqrt[1 - c^2/b^2]*Cosh[x + ArcTanh[c/b]])/(b*Sqrt[1 - c
^2/b^2]*(1 + a/(b*Sqrt[1 - c^2/b^2]))), (a + b*Sqrt[1 - c^2/b^2]*Cosh[x + A
rcTanh[c/b]])/(b*Sqrt[1 - c^2/b^2]*(-1 + a/(b*Sqrt[1 - c^2/b^2])))]*Sinh[x
+ ArcTanh[c/b]])/(b*Sqrt[1 - c^2/b^2]*Sqrt[(b*Sqrt[(b^2 - c^2)/b^2] - b*Sqr
t[(b^2 - c^2)/b^2]*Cosh[x + ArcTanh[c/b]])/(a + b*Sqrt[(b^2 - c^2)/b^2])]*S
qrt[a + b*Sqrt[(b^2 - c^2)/b^2]*Cosh[x + ArcTanh[c/b]]]*Sqrt[(b*Sqrt[(b^2 -
 c^2)/b^2] + b*Sqrt[(b^2 - c^2)/b^2]*Cosh[x + ArcTanh[c/b]])/(-a + b*Sqrt[(
b^2 - c^2)/b^2])]) - ((-2*b*(a + b*Sqrt[1 - c^2/b^2]*Cosh[x + ArcTanh[c/b]]
))/(b^2 - c^2) + (c*Sinh[x + ArcTanh[c/b]])/(b*Sqrt[1 - c^2/b^2]))/Sqrt[a +
 b*Sqrt[1 - c^2/b^2]*Cosh[x + ArcTanh[c/b]]])
```

fricas [F] time = 0.42, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{b\cosh(x) + c\sinh(x) + a}\,, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cosh(x)+c*sinh(x))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*cosh(x) + c*sinh(x) + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b\cosh(x) + c\sinh(x) + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cosh(x)+c*sinh(x))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*cosh(x) + c*sinh(x) + a), x)
```

maple [B] time = 0.80, size = 314, normalized size = 3.08

$$\frac{\left(-b^2+c^2\right)\cosh(x)}{\sqrt{b^2-c^2}\,\sqrt{\frac{-\sinh(x)b^2+\sinh(x)c^2+a\sqrt{b^2-c^2}}{\sqrt{b^2-c^2}}}}+\frac{\sqrt{\frac{\left(-\sinh(x)b^2+\sinh(x)c^2+a\sqrt{b^2-c^2}\right)\left(\sinh^2(x)\right)}{\sqrt{b^2-c^2}}}\;a\ln\left(\frac{\cosh(x)\sinh(x)\left(-b^2+c^2\right)+\cosh(x)\sqrt{b^2\cdots}}{\sqrt{b^2\cdots}}\cdots\right.}{\left(-\sinh(x)b^2+\sinh(x)c^2+\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cosh(x)+c*sinh(x))^(1/2),x)
```

```
[Out] (-b^2+c^2)/(b^2-c^2)^(1/2)/((-sinh(x)*b^2+sinh(x)*c^2+a*(b^2-c^2)^(1/2))/(b
^2-c^2)^(1/2))^(1/2)*cosh(x)+((-sinh(x)*b^2+sinh(x)*c^2+a*(b^2-c^2)^(1/2))/
```

```
(b^2-c^2)^(1/2)*sinh(x)^2)^(1/2)*a*ln((cosh(x)*sinh(x)*(-b^2+c^2)+cosh(x)*(
b^2-c^2)^(1/2)*a+((-b^2+c^2)/(b^2-c^2)^(1/2)*sinh(x)^3+a*sinh(x)^2)^(1/2)*(
b^2-c^2)^(1/2)*((-b^2+c^2)/(b^2-c^2)^(1/2)*sinh(x)+a)^(1/2))/(b^2-c^2)^(1/2
)/((-sinh(x)*b^2+sinh(x)*c^2+a*(b^2-c^2)^(1/2))/(b^2-c^2)^(1/2))^(1/2))/(-s
inh(x)*b^2+sinh(x)*c^2+a*(b^2-c^2)^(1/2))*(b^2-c^2)^(1/2)/sinh(x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b\cosh(x) + c\sinh(x) + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cosh(x)+c*sinh(x))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(b*cosh(x) + c*sinh(x) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{a + b\cosh(x) + c\sinh(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*cosh(x) + c*sinh(x))^(1/2),x)
[Out] int((a + b*cosh(x) + c*sinh(x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + b\cosh{\left(x \right)} + c\sinh{\left(x \right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cosh(x)+c*sinh(x))**(1/2),x)
[Out] Integral(sqrt(a + b*cosh(x) + c*sinh(x)), x)
```

3.764 $\int \frac{1}{\sqrt{a+b\cosh(x)+c\sinh(x)}}\,dx$

Optimal. Leaf size=102

$$-\frac{2i\sqrt{\frac{a+b\cosh(x)+c\sinh(x)}{a+\sqrt{b^2-c^2}}}\,F\left(\frac{1}{2}\left(ix-\tan^{-1}(b,-ic)\right)|\frac{2\sqrt{b^2-c^2}}{a+\sqrt{b^2-c^2}}\right)}{\sqrt{a+b\cosh(x)+c\sinh(x)}}$$

```
[Out] -2*I*(cos(1/2*I*x-1/2*arctan(b,-I*c))^2)^(1/2)/cos(1/2*I*x-1/2*arctan(b,-I*
c))*EllipticF(sin(1/2*I*x-1/2*arctan(b,-I*c)),2^(1/2)*((b^2-c^2)^(1/2)/(a+(
b^2-c^2)^(1/2)))^(1/2))*((a+b*cosh(x)+c*sinh(x))/(a+(b^2-c^2)^(1/2)))^(1/2)
/(a+b*cosh(x)+c*sinh(x))^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3127, 2661}

$$-\frac{2i\sqrt{\frac{a+b\cosh(x)+c\sinh(x)}{a+\sqrt{b^2-c^2}}}\,F\left(\frac{1}{2}\left(ix-\tan^{-1}(b,-ic)\right)|\frac{2\sqrt{b^2-c^2}}{a+\sqrt{b^2-c^2}}\right)}{\sqrt{a+b\cosh(x)+c\sinh(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a + b*Cosh[x] + c*Sinh[x]],x]
```

```
[Out] ((-2*I)*EllipticF[(I*x - ArcTan[b, (-I)*c])/2, (2*Sqrt[b^2 - c^2])/(a + Sqr
t[b^2 - c^2])]*Sqrt[(a + b*Cosh[x] + c*Sinh[x])/(a + Sqrt[b^2 - c^2])])/Sqr
t[a + b*Cosh[x] + c*Sinh[x]]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3127

```
Int[1/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(
x_)]], x_Symbol] :> Dist[Sqrt[(a + b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sq
rt[b^2 + c^2])]/Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]], Int[1/Sqrt[a/(a
+ Sqrt[b^2 + c^2]) + (Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b, c]])/(a + Sqr
t[b^2 + c^2])], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2,
 0] && NeQ[b^2 + c^2, 0] &&  !GtQ[a + Sqrt[b^2 + c^2], 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+b\cosh(x)+c\sinh(x)}}\,dx = \frac{\sqrt{\frac{a+b\cosh(x)+c\sinh(x)}{a+\sqrt{b^2-c^2}}}\int \frac{1}{\sqrt{\frac{a}{a+\sqrt{b^2-c^2}}+\frac{\sqrt{b^2-c^2}\cosh\left(x+i\tan^{-1}(b,-ic)\right)}{a+\sqrt{b^2-c^2}}}}\,dx}{\sqrt{a+b\cosh(x)+c\sinh(x)}}$$
$$= -\frac{2iF\left(\frac{1}{2}\left(ix-\tan^{-1}(b,-ic)\right)|\frac{2\sqrt{b^2-c^2}}{a+\sqrt{b^2-c^2}}\right)\sqrt{\frac{a+b\cosh(x)+c\sinh(x)}{a+\sqrt{b^2-c^2}}}}{\sqrt{a+b\cosh(x)+c\sinh(x)}}$$

Mathematica [C] time = 0.52, size = 237, normalized size = 2.32

$$\frac{2\operatorname{sech}\left(\tanh^{-1}\left(\frac{b}{c}\right)+x\right)\sqrt{a+b\cosh(x)+c\sinh(x)}\sqrt{-\frac{-ic\sqrt{1-\frac{b^2}{c^2}}+b\cosh(x)+c\sinh(x)}{a+ic\sqrt{1-\frac{b^2}{c^2}}}}\sqrt{-\frac{ic\sqrt{1-\frac{b^2}{c^2}}+b\cosh(x)+c\sinh(x)}{a-ic\sqrt{1-\frac{b^2}{c^2}}}}F_1}{c\sqrt{1-\frac{b^2}{c^2}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/Sqrt[a + b*Cosh[x] + c*Sinh[x]],x]
```

```
[Out] (2*AppellF1[1/2, 1/2, 1/2, 3/2, (a + b*Cosh[x] + c*Sinh[x])/(a + I*Sqrt[1 -
 b^2/c^2]*c), (a + b*Cosh[x] + c*Sinh[x])/(a - I*Sqrt[1 - b^2/c^2]*c)]*Sech
[x + ArcTanh[b/c]]*Sqrt[a + b*Cosh[x] + c*Sinh[x]]*Sqrt[-(((-I)*Sqrt[1 - b^
2/c^2]*c + b*Cosh[x] + c*Sinh[x])/(a + I*Sqrt[1 - b^2/c^2]*c))]*Sqrt[-((I*S
qrt[1 - b^2/c^2]*c + b*Cosh[x] + c*Sinh[x])/(a - I*Sqrt[1 - b^2/c^2]*c))])/
(Sqrt[1 - b^2/c^2]*c)
```

fricas [F] time = 0.42, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{1}{\sqrt{b\cosh(x)+c\sinh(x)+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cosh(x)+c*sinh(x))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(1/sqrt(b*cosh(x) + c*sinh(x) + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{b\cosh(x)+c\sinh(x)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cosh(x)+c*sinh(x))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(b*cosh(x) + c*sinh(x) + a), x)
```

maple [A] time = 0.64, size = 248, normalized size = 2.43

$$\frac{\sqrt{\frac{\left(-\sinh(x)b^2+\sinh(x)c^2+a\sqrt{b^2-c^2}\right)\left(\sinh^2(x)\right)}{\sqrt{b^2-c^2}}}\ \ln\left(\frac{\cosh(x)\sinh(x)\left(-b^2+c^2\right)+\cosh(x)\sqrt{b^2-c^2}\ a+\sqrt{\frac{\left(-b^2+c^2\right)\left(\sinh^3(x)\right)}{\sqrt{b^2-c^2}}+a\left(\sinh^2(x)\right)}\ \sqrt{b^2-c^2}\ \sqrt{\left(-\cdots\right.}}{\sqrt{b^2-c^2}\ \sqrt{\frac{-\sinh(x)b^2+\sinh(x)c^2+a\sqrt{b^2-c^2}}{\sqrt{b^2-c^2}}}}\right.}{\left(-\sinh(x)b^2+\sinh(x)c^2+a\sqrt{b^2-c^2}\right)\sinh(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*cosh(x)+c*sinh(x))^(1/2),x)
```

```
[Out] ((-sinh(x)*b^2+sinh(x)*c^2+a*(b^2-c^2)^(1/2))/(b^2-c^2)^(1/2)*sinh(x)^2)^(1
/2)*ln((cosh(x)*sinh(x)*(-b^2+c^2)+cosh(x)*(b^2-c^2)^(1/2)*a+((-b^2+c^2)/(b
^2-c^2)^(1/2)*sinh(x)^3+a*sinh(x)^2)^(1/2)*(b^2-c^2)^(1/2)*((-b^2+c^2)/(b^2
-c^2)^(1/2)*sinh(x)+a)^(1/2))/(b^2-c^2)^(1/2)/((-sinh(x)*b^2+sinh(x)*c^2+a*
(b^2-c^2)^(1/2))/(b^2-c^2)^(1/2))^(1/2))/(-sinh(x)*b^2+sinh(x)*c^2+a*(b^2-c
^2)^(1/2))*(b^2-c^2)^(1/2)/sinh(x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{b\cosh(x) + c\sinh(x) + a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cosh(x)+c*sinh(x))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(b*cosh(x) + c*sinh(x) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{a + b\cosh(x) + c\sinh(x)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*cosh(x) + c*sinh(x))^(1/2),x)
```

```
[Out] int(1/(a + b*cosh(x) + c*sinh(x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a + b\cosh(x) + c\sinh(x)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cosh(x)+c*sinh(x))**(1/2),x)
```

```
[Out] Integral(1/sqrt(a + b*cosh(x) + c*sinh(x)), x)
```

3.765 $\int \frac{1}{(a+b\cosh(x)+c\sinh(x))^{3/2}}\,dx$

Optimal. Leaf size=156

$$-\frac{2(b\sinh(x)+c\cosh(x))}{\left(a^2-b^2+c^2\right)\sqrt{a+b\cosh(x)+c\sinh(x)}}-\frac{2i\sqrt{a+b\cosh(x)+c\sinh(x)}\,E\left(\frac{1}{2}\left(ix-\tan^{-1}(b,-ic)\right)|\frac{2\sqrt{b^2-c^2}}{a+\sqrt{b^2-c^2}}\right)}{\left(a^2-b^2+c^2\right)\sqrt{\frac{a+b\cosh(x)+c\sinh(x)}{a+\sqrt{b^2-c^2}}}}$$

```
[Out] -2*(c*cosh(x)+b*sinh(x))/(a^2-b^2+c^2)/(a+b*cosh(x)+c*sinh(x))^(1/2)-2*I*(c
os(1/2*I*x-1/2*arctan(b,-I*c))^2)^(1/2)/cos(1/2*I*x-1/2*arctan(b,-I*c))*Ell
ipticE(sin(1/2*I*x-1/2*arctan(b,-I*c)),2^(1/2)*((b^2-c^2)^(1/2)/(a+(b^2-c^2
)^(1/2)))^(1/2))*(a+b*cosh(x)+c*sinh(x))^(1/2)/(a^2-b^2+c^2)/((a+b*cosh(x)+
c*sinh(x))/(a+(b^2-c^2)^(1/2)))^(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3128, 3119, 2653}

$$-\frac{2(b\sinh(x)+c\cosh(x))}{\left(a^2-b^2+c^2\right)\sqrt{a+b\cosh(x)+c\sinh(x)}}-\frac{2i\sqrt{a+b\cosh(x)+c\sinh(x)}\,E\left(\frac{1}{2}\left(ix-\tan^{-1}(b,-ic)\right)|\frac{2\sqrt{b^2-c^2}}{a+\sqrt{b^2-c^2}}\right)}{\left(a^2-b^2+c^2\right)\sqrt{\frac{a+b\cosh(x)+c\sinh(x)}{a+\sqrt{b^2-c^2}}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cosh[x] + c*Sinh[x])^(-3/2),x]
```

```
[Out] (-2*(c*Cosh[x] + b*Sinh[x]))/((a^2 - b^2 + c^2)*Sqrt[a + b*Cosh[x] + c*Sinh
[x]]) - ((2*I)*EllipticE[(I*x - ArcTan[b, (-I)*c])/2, (2*Sqrt[b^2 - c^2])/(
a + Sqrt[b^2 - c^2])]*Sqrt[a + b*Cosh[x] + c*Sinh[x]])/((a^2 - b^2 + c^2)*S
qrt[(a + b*Cosh[x] + c*Sinh[x])/(a + Sqrt[b^2 - c^2])])
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3119

```
Int[Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_
)]], x_Symbol] :> Dist[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]]/Sqrt[(a +
b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sqrt[b^2 + c^2])], Int[Sqrt[a/(a + Sq
```

```
rt[b^2 + c^2]) + (Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b, c]])/(a + Sqrt[b^
2 + c^2])], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
&& NeQ[b^2 + c^2, 0] &&  !GtQ[a + Sqrt[b^2 + c^2], 0]
```

Rule 3128

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-3/2), x_Symbol] :> Simp[(2*(c*Cos[d + e*x] - b*Sin[d + e*x]))/(e*(a^2 - b
^2 - c^2)*Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]]), x] + Dist[1/(a^2 - b^
2 - c^2), Int[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]], x], x] /; FreeQ[{a
, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
```

Rubi steps

$$\int \frac{1}{(a+b\cosh(x)+c\sinh(x))^{3/2}}\,dx = -\frac{2(c\cosh(x)+b\sinh(x))}{\left(a^2-b^2+c^2\right)\sqrt{a+b\cosh(x)+c\sinh(x)}}+\frac{\int\sqrt{a+b\cosh(x)+c\sinh}}{a^2-b^2+c^2}$$

$$= -\frac{2(c\cosh(x)+b\sinh(x))}{\left(a^2-b^2+c^2\right)\sqrt{a+b\cosh(x)+c\sinh(x)}}+\frac{\sqrt{a+b\cosh(x)+c\sinh(x}}{\left(a^2-b\right.}$$

$$= -\frac{2(c\cosh(x)+b\sinh(x))}{\left(a^2-b^2+c^2\right)\sqrt{a+b\cosh(x)+c\sinh(x)}}-\frac{2iE\left(\frac{1}{2}\left(ix-\tan^{-1}(b,-ic)\right)\right.}{\left(a^2-b^2+\right.}$$

Mathematica [C] time = 6.21, size = 1522, normalized size = 9.76

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Cosh[x] + c*Sinh[x])^(-3/2),x]
```

```
[Out] Sqrt[a + b*Cosh[x] + c*Sinh[x]]*((-2*(b^2 - c^2))/(b*c*(-a^2 + b^2 - c^2))
- (2*(a*c - b^2*Sinh[x] + c^2*Sinh[x]))/(b*(-a^2 + b^2 - c^2)*(a + b*Cosh[x
] + c*Sinh[x]))) + (2*a*AppellF1[1/2, 1/2, 1/2, 3/2, ((-I)*(a + Sqrt[1 - b^
2/c^2]*c*Sinh[x + ArcTanh[b/c]]))/(Sqrt[1 - b^2/c^2]*(1 - (I*a)/(Sqrt[1 - b
^2/c^2]*c))*c), ((-I)*(a + Sqrt[1 - b^2/c^2]*c*Sinh[x + ArcTanh[b/c]]))/(Sq
rt[1 - b^2/c^2]*(-1 - (I*a)/(Sqrt[1 - b^2/c^2]*c))*c)]*Sech[x + ArcTanh[b/c
]]*Sqrt[-1 + I*Sinh[x + ArcTanh[b/c]]]*Sqrt[(c*Sqrt[(-b^2 + c^2)/c^2] - I*c
```

```
*Sqrt[(-b^2 + c^2)/c^2]*Sinh[x + ArcTanh[b/c]])/(I*a + c*Sqrt[(-b^2 + c^2)/
c^2])]*Sqrt[(c*Sqrt[(-b^2 + c^2)/c^2] + I*c*Sqrt[(-b^2 + c^2)/c^2]*Sinh[x +
 ArcTanh[b/c]])/((-I)*a + c*Sqrt[(-b^2 + c^2)/c^2])]*Sqrt[a + c*Sqrt[(-b^2
+ c^2)/c^2]*Sinh[x + ArcTanh[b/c]]])/(Sqrt[1 - b^2/c^2]*c*(a^2 - b^2 + c^2)
*Sqrt[I*(I + Sinh[x + ArcTanh[b/c]])]) - (b^2*((c*AppellF1[-1/2, -1/2, -1/2
, 1/2, (a + b*Sqrt[1 - c^2/b^2]*Cosh[x + ArcTanh[c/b]])/(b*Sqrt[1 - c^2/b^2
]*(1 + a/(b*Sqrt[1 - c^2/b^2]))), (a + b*Sqrt[1 - c^2/b^2]*Cosh[x + ArcTanh
[c/b]])/(b*Sqrt[1 - c^2/b^2]*(-1 + a/(b*Sqrt[1 - c^2/b^2])))]*Sinh[x + ArcT
anh[c/b]])/(b*Sqrt[1 - c^2/b^2]*Sqrt[(b*Sqrt[(b^2 - c^2)/b^2] - b*Sqrt[(b^2
 - c^2)/b^2]*Cosh[x + ArcTanh[c/b]])/(a + b*Sqrt[(b^2 - c^2)/b^2])]*Sqrt[a
+ b*Sqrt[(b^2 - c^2)/b^2]*Cosh[x + ArcTanh[c/b]]]*Sqrt[(b*Sqrt[(b^2 - c^2)/
b^2] + b*Sqrt[(b^2 - c^2)/b^2]*Cosh[x + ArcTanh[c/b]])/(-a + b*Sqrt[(b^2 -
c^2)/b^2])]) - ((-2*b*(a + b*Sqrt[1 - c^2/b^2]*Cosh[x + ArcTanh[c/b]]))/(b^
2 - c^2) + (c*Sinh[x + ArcTanh[c/b]])/(b*Sqrt[1 - c^2/b^2]))/Sqrt[a + b*Sqr
t[1 - c^2/b^2]*Cosh[x + ArcTanh[c/b]]]))/(c*(a^2 - b^2 + c^2)) + (c*((c*App
ellF1[-1/2, -1/2, -1/2, 1/2, (a + b*Sqrt[1 - c^2/b^2]*Cosh[x + ArcTanh[c/b]
])/(b*Sqrt[1 - c^2/b^2]*(1 + a/(b*Sqrt[1 - c^2/b^2]))), (a + b*Sqrt[1 - c^2
/b^2]*Cosh[x + ArcTanh[c/b]])/(b*Sqrt[1 - c^2/b^2]*(-1 + a/(b*Sqrt[1 - c^2/
b^2])))]*Sinh[x + ArcTanh[c/b]])/(b*Sqrt[1 - c^2/b^2]*Sqrt[(b*Sqrt[(b^2 - c
^2)/b^2] - b*Sqrt[(b^2 - c^2)/b^2]*Cosh[x + ArcTanh[c/b]])/(a + b*Sqrt[(b^2
 - c^2)/b^2])]*Sqrt[a + b*Sqrt[(b^2 - c^2)/b^2]*Cosh[x + ArcTanh[c/b]]]*Sqr
t[(b*Sqrt[(b^2 - c^2)/b^2] + b*Sqrt[(b^2 - c^2)/b^2]*Cosh[x + ArcTanh[c/b]]
)/(-a + b*Sqrt[(b^2 - c^2)/b^2])]) - ((-2*b*(a + b*Sqrt[1 - c^2/b^2]*Cosh[x
 + ArcTanh[c/b]]))/(b^2 - c^2) + (c*Sinh[x + ArcTanh[c/b]])/(b*Sqrt[1 - c^2
/b^2]))/Sqrt[a + b*Sqrt[1 - c^2/b^2]*Cosh[x + ArcTanh[c/b]]]))/(a^2 - b^2 +
 c^2)
```

fricas [F] time = 0.46, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{b\cosh(x)+c\sinh(x)+a}}{b^2\cosh(x)^2+c^2\sinh(x)^2+2\,ab\cosh(x)+a^2+2\,(bc\cosh(x)+ac)\sinh(x)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cosh(x)+c*sinh(x))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*cosh(x) + c*sinh(x) + a)/(b^2*cosh(x)^2 + c^2*sinh(x)^2 + 2
*a*b*cosh(x) + a^2 + 2*(b*c*cosh(x) + a*c)*sinh(x)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{(b\cosh(x)+c\sinh(x)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cosh(x)+c*sinh(x))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*cosh(x) + c*sinh(x) + a)^(-3/2), x)
```

maple [B] time = 1.74, size = 1430, normalized size = 9.17

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*cosh(x)+c*sinh(x))^(3/2),x)
```

```
[Out] 1/2*(2*(b^2-c^2)^(1/2)*arctanh((b^2-c^2)*cosh(x)/((a^2+b^2-c^2)*(b^2-c^2))^
(1/2))*(-a^2/(b^2-c^2)^(1/2)*sinh(x)+a^3/(b^2-c^2))^(1/2)*((a^2+b^2-c^2)*(b
-c)*(b+c))^(1/2)*sinh(x)+a*((-b^2+c^2)/(b^2-c^2)^(1/2)*sinh(x)^3+a*sinh(x)^
2)^(1/2)*((a^2+b^2-c^2)*(b^2-c^2))^(1/2)*ln((cosh(x)*sinh(x)*(2*((a^2+b^2-c
^2)*(b-c)*(b+c))^(1/2)*b^4-4*((a^2+b^2-c^2)*(b-c)*(b+c))^(1/2)*b^2*c^2+2*((
a^2+b^2-c^2)*(b-c)*(b+c))^(1/2)*c^4)+cosh(x)*(-2*(b^2-c^2)^(1/2)*((a^2+b^2-
c^2)*(b-c)*(b+c))^(1/2)*a*b^2+2*(b^2-c^2)^(1/2)*((a^2+b^2-c^2)*(b-c)*(b+c))
^(1/2)*a*c^2)+sinh(x)*(-2*b^6+6*b^4*c^2-6*b^2*c^4+2*c^6)-2*(b^2-c^2)^(3/2)*
(-a^2/(b^2-c^2)^(1/2)*sinh(x)+a^3/(b^2-c^2))^(1/2)*((-b^2+c^2)/(b^2-c^2)^(1
/2)*sinh(x)^3+a*sinh(x)^2)^(1/2)*b^2+2*(b^2-c^2)^(3/2)*(-a^2/(b^2-c^2)^(1/2
)*sinh(x)+a^3/(b^2-c^2))^(1/2)*((-b^2+c^2)/(b^2-c^2)^(1/2)*sinh(x)^3+a*sinh
(x)^2)^(1/2)*c^2-2*(b^2-c^2)^(3/2)*a^3+2*a^3*b^2*(b^2-c^2)^(1/2)-2*a^3*c^2*
(b^2-c^2)^(1/2)+2*(b^2-c^2)^(1/2)*a*b^4-4*(b^2-c^2)^(1/2)*a*b^2*c^2+2*(b^2-
c^2)^(1/2)*a*c^4)/(-b^2*cosh(x)+c^2*cosh(x)+((a^2+b^2-c^2)*(b-c)*(b+c))^(1/
2))/(b^2-c^2)^(3/2))-a*((-b^2+c^2)/(b^2-c^2)^(1/2)*sinh(x)^3+a*sinh(x)^2)^(
1/2)*((a^2+b^2-c^2)*(b^2-c^2))^(1/2)*ln((cosh(x)*sinh(x)*(2*((a^2+b^2-c^2)*
(b-c)*(b+c))^(1/2)*b^4-4*((a^2+b^2-c^2)*(b-c)*(b+c))^(1/2)*b^2*c^2+2*((a^2+
b^2-c^2)*(b-c)*(b+c))^(1/2)*c^4)+cosh(x)*(-2*(b^2-c^2)^(1/2)*((a^2+b^2-c^2)
*(b-c)*(b+c))^(1/2)*a*b^2+2*(b^2-c^2)^(1/2)*((a^2+b^2-c^2)*(b-c)*(b+c))^(1/
2)*a*c^2)+sinh(x)*(2*b^6-6*b^4*c^2+6*b^2*c^4-2*c^6)+2*(b^2-c^2)^(3/2)*(-a^2
/(b^2-c^2)^(1/2)*sinh(x)+a^3/(b^2-c^2))^(1/2)*((-b^2+c^2)/(b^2-c^2)^(1/2)*s
inh(x)^3+a*sinh(x)^2)^(1/2)*b^2-2*(b^2-c^2)^(3/2)*(-a^2/(b^2-c^2)^(1/2)*sin
h(x)+a^3/(b^2-c^2))^(1/2)*((-b^2+c^2)/(b^2-c^2)^(1/2)*sinh(x)^3+a*sinh(x)^2
)^(1/2)*c^2+2*(b^2-c^2)^(3/2)*a^3-2*a^3*b^2*(b^2-c^2)^(1/2)+2*a^3*c^2*(b^2-
c^2)^(1/2)-2*(b^2-c^2)^(1/2)*a*b^4+4*(b^2-c^2)^(1/2)*a*b^2*c^2-2*(b^2-c^2)^
(1/2)*a*c^4)/(b^2*cosh(x)-c^2*cosh(x)+((a^2+b^2-c^2)*(b-c)*(b+c))^(1/2))/(b
^2-c^2)^(3/2)))/((-b^2+c^2)/(b^2-c^2)^(1/2)*sinh(x)+a)^(1/2)/((a^2+b^2-c^2)
*(b^2-c^2))^(1/2)/(-a^2/(b^2-c^2)^(1/2)*sinh(x)+a^3/(b^2-c^2))^(1/2)/((a^2+
b^2-c^2)*(b-c)*(b+c))^(1/2)/sinh(x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\cosh\left(x\right) + c\sinh\left(x\right) + a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cosh(x)+c*sinh(x))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*cosh(x) + c*sinh(x) + a)^(-3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(a + b \cosh(x) + c \sinh(x))^{3/2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*cosh(x) + c*sinh(x))^(3/2),x)
```

```
[Out] int(1/(a + b*cosh(x) + c*sinh(x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a + b \cosh (x) + c \sinh (x))^{\frac{3}{2}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cosh(x)+c*sinh(x))**(3/2),x)
```

```
[Out] Integral((a + b*cosh(x) + c*sinh(x))**(-3/2), x)
```

3.766 $\int \frac{1}{(a+b\cosh(x)+c\sinh(x))^{5/2}}\,dx$

Optimal. Leaf size=322

$$\frac{2i\sqrt{\frac{a+b\cosh(x)+c\sinh(x)}{a+\sqrt{b^2-c^2}}}\,F\left(\frac{1}{2}\left(ix-\tan^{-1}(b,-ic)\right)|\frac{2\sqrt{b^2-c^2}}{a+\sqrt{b^2-c^2}}\right)}{3\left(a^2-b^2+c^2\right)\sqrt{a+b\cosh(x)+c\sinh(x)}}-\frac{8ia\sqrt{a+b\cosh(x)+c\sinh(x)}\,E\left(\frac{1}{2}\left(ix-\tan^{-1}(b,-ic\right.\right.}{3\left(a^2-b^2+c^2\right)^2\sqrt{\frac{a+b\cosh(x)+c\sinh(x)}{a+\sqrt{b^2-c^2}}}}$$

```
[Out] -2/3*(c*cosh(x)+b*sinh(x))/(a^2-b^2+c^2)/(a+b*cosh(x)+c*sinh(x))^(3/2)-8/3*
(a*c*cosh(x)+a*b*sinh(x))/(a^2-b^2+c^2)^2/(a+b*cosh(x)+c*sinh(x))^(1/2)-8/3
*I*a*(cos(1/2*I*x-1/2*arctan(b,-I*c))^2)^(1/2)/cos(1/2*I*x-1/2*arctan(b,-I*
c))*EllipticE(sin(1/2*I*x-1/2*arctan(b,-I*c)),2^(1/2)*((b^2-c^2)^(1/2)/(a+(
b^2-c^2)^(1/2)))^(1/2))*(a+b*cosh(x)+c*sinh(x))^(1/2)/(a^2-b^2+c^2)^2/((a+b
*cosh(x)+c*sinh(x))/(a+(b^2-c^2)^(1/2)))^(1/2)+2/3*I*(cos(1/2*I*x-1/2*arcta
n(b,-I*c))^2)^(1/2)/cos(1/2*I*x-1/2*arctan(b,-I*c))*EllipticF(sin(1/2*I*x-1
/2*arctan(b,-I*c)),2^(1/2)*((b^2-c^2)^(1/2)/(a+(b^2-c^2)^(1/2)))^(1/2))*((a
+b*cosh(x)+c*sinh(x))/(a+(b^2-c^2)^(1/2)))^(1/2)/(a^2-b^2+c^2)/(a+b*cosh(x)
+c*sinh(x))^(1/2)
```

Rubi [A] time = 0.34, antiderivative size = 322, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3129, 3156, 3149, 3119, 2653, 3127, 2661}

$$\frac{2i\sqrt{\frac{a+b\cosh(x)+c\sinh(x)}{a+\sqrt{b^2-c^2}}}\,F\left(\frac{1}{2}\left(ix-\tan^{-1}(b,-ic)\right)|\frac{2\sqrt{b^2-c^2}}{a+\sqrt{b^2-c^2}}\right)}{3\left(a^2-b^2+c^2\right)\sqrt{a+b\cosh(x)+c\sinh(x)}}-\frac{8ia\sqrt{a+b\cosh(x)+c\sinh(x)}\,E\left(\frac{1}{2}\left(ix-\tan^{-1}(b,-ic\right.\right.}{3\left(a^2-b^2+c^2\right)^2\sqrt{\frac{a+b\cosh(x)+c\sinh(x)}{a+\sqrt{b^2-c^2}}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cosh[x] + c*Sinh[x])^(-5/2),x]
```

```
[Out] (-2*(c*Cosh[x] + b*Sinh[x]))/(3*(a^2 - b^2 + c^2)*(a + b*Cosh[x] + c*Sinh[x
])^(3/2)) - (8*(a*c*Cosh[x] + a*b*Sinh[x]))/(3*(a^2 - b^2 + c^2)^2*Sqrt[a +
 b*Cosh[x] + c*Sinh[x]]) - (((8*I)/3)*a*EllipticE[(I*x - ArcTan[b, (-I)*c])
/2, (2*Sqrt[b^2 - c^2])/(a + Sqrt[b^2 - c^2])]*Sqrt[a + b*Cosh[x] + c*Sinh[
x]])/((a^2 - b^2 + c^2)^2*Sqrt[(a + b*Cosh[x] + c*Sinh[x])/(a + Sqrt[b^2 -
c^2])]) + (((2*I)/3)*EllipticF[(I*x - ArcTan[b, (-I)*c])/2, (2*Sqrt[b^2 - c
^2])/(a + Sqrt[b^2 - c^2])]*Sqrt[(a + b*Cosh[x] + c*Sinh[x])/(a + Sqrt[b^2
- c^2])])/((a^2 - b^2 + c^2)*Sqrt[a + b*Cosh[x] + c*Sinh[x]])
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3119

```
Int[Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_
)]], x_Symbol] :> Dist[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]]/Sqrt[(a +
b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sqrt[b^2 + c^2])], Int[Sqrt[a/(a + Sq
rt[b^2 + c^2]) + (Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b, c]])/(a + Sqrt[b^
2 + c^2])], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
&& NeQ[b^2 + c^2, 0] &&  !GtQ[a + Sqrt[b^2 + c^2], 0]
```

Rule 3127

```
Int[1/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(
x_)]], x_Symbol] :> Dist[Sqrt[(a + b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sq
rt[b^2 + c^2])]/Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]], Int[1/Sqrt[a/(a
+ Sqrt[b^2 + c^2]) + (Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b, c]])/(a + Sqr
t[b^2 + c^2])], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2,
 0] && NeQ[b^2 + c^2, 0] &&  !GtQ[a + Sqrt[b^2 + c^2], 0]
```

Rule 3129

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> Simp[((-(c*Cos[d + e*x]) + b*Sin[d + e*x])*(a + b*Cos[d
+ e*x] + c*Sin[d + e*x])^(n + 1))/(e*(n + 1)*(a^2 - b^2 - c^2)), x] + Dist[
1/((n + 1)*(a^2 - b^2 - c^2)), Int[(a*(n + 1) - b*(n + 2)*Cos[d + e*x] - c*
(n + 2)*Sin[d + e*x])*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1), x], x]
 /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && LtQ[n, -1] && N
eQ[n, -3/2]
```

Rule 3149

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)]]
, x_Symbol] :> Dist[B/b, Int[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]], x],
 x] + Dist[(A*b - a*B)/b, Int[1/Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]],
x], x] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && EqQ[B*c - b*C, 0] && NeQ[A*
```

```
b - a*B, 0]
```

Rule 3156

```
Int[((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])
^(n_)*((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)
]), x_Symbol] :> -Simp[((c*B - b*C - (a*C - c*A)*Cos[d + e*x] + (a*B - b*A)
*Sin[d + e*x])*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1))/(e*(n + 1)*(a
^2 - b^2 - c^2)), x] + Dist[1/((n + 1)*(a^2 - b^2 - c^2)), Int[(a + b*Cos[d
 + e*x] + c*Sin[d + e*x])^(n + 1)*Simp[(n + 1)*(a*A - b*B - c*C) + (n + 2)*
(a*B - b*A)*Cos[d + e*x] + (n + 2)*(a*C - c*A)*Sin[d + e*x], x], x], x] /;
FreeQ[{a, b, c, d, e, A, B, C}, x] && LtQ[n, -1] && NeQ[a^2 - b^2 - c^2, 0]
 && NeQ[n, -2]
```

Rubi steps

$$\int \frac{1}{(a+b\cosh(x)+c\sinh(x))^{5/2}}\,dx = -\frac{2(c\cosh(x)+b\sinh(x))}{3\left(a^2-b^2+c^2\right)(a+b\cosh(x)+c\sinh(x))^{3/2}} - \frac{2\int \frac{-\frac{3a}{2}+\frac{1}{2}b\cosh(x)+\frac{1}{2}c\sin}{(a+b\cosh(x)+c\sinh(x))}}{3\left(a^2-b^2+c^2\right)}$$

$$= -\frac{2(c\cosh(x)+b\sinh(x))}{3\left(a^2-b^2+c^2\right)(a+b\cosh(x)+c\sinh(x))^{3/2}} - \frac{8(ac\cosh(x)}{3\left(a^2-b^2+c^2\right)^2\sqrt{a+}}$$

$$= -\frac{2(c\cosh(x)+b\sinh(x))}{3\left(a^2-b^2+c^2\right)(a+b\cosh(x)+c\sinh(x))^{3/2}} - \frac{8(ac\cosh(x)}{3\left(a^2-b^2+c^2\right)^2\sqrt{a+}}$$

$$= -\frac{2(c\cosh(x)+b\sinh(x))}{3\left(a^2-b^2+c^2\right)(a+b\cosh(x)+c\sinh(x))^{3/2}} - \frac{8(ac\cosh(x)}{3\left(a^2-b^2+c^2\right)^2\sqrt{a+}}$$

$$= -\frac{2(c\cosh(x)+b\sinh(x))}{3\left(a^2-b^2+c^2\right)(a+b\cosh(x)+c\sinh(x))^{3/2}} - \frac{8(ac\cosh(x)}{3\left(a^2-b^2+c^2\right)^2\sqrt{a+}}$$

Mathematica [C] time = 6.26, size = 2492, normalized size = 7.74

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Cosh[x] + c*Sinh[x])^(-5/2),x]
```

```
[Out] Sqrt[a + b*Cosh[x] + c*Sinh[x]]*((8*a*(b^2 - c^2))/(3*b*c*(a^2 - b^2 + c^2)
^2) - (2*(a*c - b^2*Sinh[x] + c^2*Sinh[x]))/(3*b*(-a^2 + b^2 - c^2)*(a + b*
Cosh[x] + c*Sinh[x])^2) - (2*(-3*a^2*c - b^2*c + c^3 + 4*a*b^2*Sinh[x] - 4*
a*c^2*Sinh[x]))/(3*b*(-a^2 + b^2 - c^2)^2*(a + b*Cosh[x] + c*Sinh[x]))) + (
2*a^2*AppellF1[1/2, 1/2, 1/2, 3/2, ((-I)*(a + Sqrt[1 - b^2/c^2]*c*Sinh[x +
ArcTanh[b/c]]))/(Sqrt[1 - b^2/c^2]*(1 - (I*a)/(Sqrt[1 - b^2/c^2]*c))*c), ((
-I)*(a + Sqrt[1 - b^2/c^2]*c*Sinh[x + ArcTanh[b/c]]))/(Sqrt[1 - b^2/c^2]*(-
1 - (I*a)/(Sqrt[1 - b^2/c^2]*c))*c)]*Sech[x + ArcTanh[b/c]]*Sqrt[-1 + I*Sin
h[x + ArcTanh[b/c]]]*Sqrt[(c*Sqrt[(-b^2 + c^2)/c^2] - I*c*Sqrt[(-b^2 + c^2)
/c^2]*Sinh[x + ArcTanh[b/c]])/(I*a + c*Sqrt[(-b^2 + c^2)/c^2])]*Sqrt[(c*Sqr
t[(-b^2 + c^2)/c^2] + I*c*Sqrt[(-b^2 + c^2)/c^2]*Sinh[x + ArcTanh[b/c]])/((
-I)*a + c*Sqrt[(-b^2 + c^2)/c^2])]*Sqrt[a + c*Sqrt[(-b^2 + c^2)/c^2]*Sinh[x
 + ArcTanh[b/c]]])/(Sqrt[1 - b^2/c^2]*c*(a^2 - b^2 + c^2)^2*Sqrt[I*(I + Sin
h[x + ArcTanh[b/c]])]) + (2*b^2*AppellF1[1/2, 1/2, 1/2, 3/2, ((-I)*(a + Sqr
t[1 - b^2/c^2]*c*Sinh[x + ArcTanh[b/c]]))/(Sqrt[1 - b^2/c^2]*(1 - (I*a)/(Sq
rt[1 - b^2/c^2]*c))*c), ((-I)*(a + Sqrt[1 - b^2/c^2]*c*Sinh[x + ArcTanh[b/c
]]))/(Sqrt[1 - b^2/c^2]*(-1 - (I*a)/(Sqrt[1 - b^2/c^2]*c))*c)]*Sech[x + Arc
Tanh[b/c]]*Sqrt[-1 + I*Sinh[x + ArcTanh[b/c]]]*Sqrt[(c*Sqrt[(-b^2 + c^2)/c^
2] - I*c*Sqrt[(-b^2 + c^2)/c^2]*Sinh[x + ArcTanh[b/c]])/(I*a + c*Sqrt[(-b^2
 + c^2)/c^2])]*Sqrt[(c*Sqrt[(-b^2 + c^2)/c^2] + I*c*Sqrt[(-b^2 + c^2)/c^2]*
Sinh[x + ArcTanh[b/c]])/((-I)*a + c*Sqrt[(-b^2 + c^2)/c^2])]*Sqrt[a + c*Sqr
t[(-b^2 + c^2)/c^2]*Sinh[x + ArcTanh[b/c]]])/(3*Sqrt[1 - b^2/c^2]*c*(a^2 -
b^2 + c^2)^2*Sqrt[I*(I + Sinh[x + ArcTanh[b/c]])]) - (2*c*AppellF1[1/2, 1/2
, 1/2, 3/2, ((-I)*(a + Sqrt[1 - b^2/c^2]*c*Sinh[x + ArcTanh[b/c]]))/(Sqrt[1
 - b^2/c^2]*(1 - (I*a)/(Sqrt[1 - b^2/c^2]*c))*c), ((-I)*(a + Sqrt[1 - b^2/c
^2]*c*Sinh[x + ArcTanh[b/c]]))/(Sqrt[1 - b^2/c^2]*(-1 - (I*a)/(Sqrt[1 - b^2
/c^2]*c))*c)]*Sech[x + ArcTanh[b/c]]*Sqrt[-1 + I*Sinh[x + ArcTanh[b/c]]]*Sq
rt[(c*Sqrt[(-b^2 + c^2)/c^2] - I*c*Sqrt[(-b^2 + c^2)/c^2]*Sinh[x + ArcTanh[
b/c]])/(I*a + c*Sqrt[(-b^2 + c^2)/c^2])]*Sqrt[(c*Sqrt[(-b^2 + c^2)/c^2] + I
*c*Sqrt[(-b^2 + c^2)/c^2]*Sinh[x + ArcTanh[b/c]])/((-I)*a + c*Sqrt[(-b^2 +
c^2)/c^2])]*Sqrt[a + c*Sqrt[(-b^2 + c^2)/c^2]*Sinh[x + ArcTanh[b/c]]])/(3*S
qrt[1 - b^2/c^2]*(a^2 - b^2 + c^2)^2*Sqrt[I*(I + Sinh[x + ArcTanh[b/c]])])
- (4*a*b^2*((c*AppellF1[-1/2, -1/2, -1/2, 1/2, (a + b*Sqrt[1 - c^2/b^2]*Cos
h[x + ArcTanh[c/b]])/(b*Sqrt[1 - c^2/b^2]*(1 + a/(b*Sqrt[1 - c^2/b^2]))), (
a + b*Sqrt[1 - c^2/b^2]*Cosh[x + ArcTanh[c/b]])/(b*Sqrt[1 - c^2/b^2]*(-1 +
a/(b*Sqrt[1 - c^2/b^2])))]*Sinh[x + ArcTanh[c/b]])/(b*Sqrt[1 - c^2/b^2]*Sqr
t[(b*Sqrt[(b^2 - c^2)/b^2] - b*Sqrt[(b^2 - c^2)/b^2]*Cosh[x + ArcTanh[c/b]]
)/(a + b*Sqrt[(b^2 - c^2)/b^2])]*Sqrt[a + b*Sqrt[(b^2 - c^2)/b^2]*Cosh[x +
ArcTanh[c/b]]]*Sqrt[(b*Sqrt[(b^2 - c^2)/b^2] + b*Sqrt[(b^2 - c^2)/b^2]*Cosh
[x + ArcTanh[c/b]])/(-a + b*Sqrt[(b^2 - c^2)/b^2])]) - ((-2*b*(a + b*Sqrt[1
 - c^2/b^2]*Cosh[x + ArcTanh[c/b]]))/(b^2 - c^2) + (c*Sinh[x + ArcTanh[c/b]
```

```
])/(b*Sqrt[1 - c^2/b^2]))/Sqrt[a + b*Sqrt[1 - c^2/b^2]*Cosh[x + ArcTanh[c/b
]]]))/(3*c*(a^2 - b^2 + c^2)^2) + (4*a*c*((c*AppellF1[-1/2, -1/2, -1/2, 1/2
, (a + b*Sqrt[1 - c^2/b^2]*Cosh[x + ArcTanh[c/b]])/(b*Sqrt[1 - c^2/b^2]*(1
+ a/(b*Sqrt[1 - c^2/b^2]))), (a + b*Sqrt[1 - c^2/b^2]*Cosh[x + ArcTanh[c/b]
])/(b*Sqrt[1 - c^2/b^2]*(-1 + a/(b*Sqrt[1 - c^2/b^2])))]*Sinh[x + ArcTanh[c
/b]])/(b*Sqrt[1 - c^2/b^2]*Sqrt[(b*Sqrt[(b^2 - c^2)/b^2] - b*Sqrt[(b^2 - c^
2)/b^2]*Cosh[x + ArcTanh[c/b]])/(a + b*Sqrt[(b^2 - c^2)/b^2])]*Sqrt[a + b*S
qrt[(b^2 - c^2)/b^2]*Cosh[x + ArcTanh[c/b]]]*Sqrt[(b*Sqrt[(b^2 - c^2)/b^2]
+ b*Sqrt[(b^2 - c^2)/b^2]*Cosh[x + ArcTanh[c/b]])/(-a + b*Sqrt[(b^2 - c^2)/
b^2])]) - ((-2*b*(a + b*Sqrt[1 - c^2/b^2]*Cosh[x + ArcTanh[c/b]]))/(b^2 - c
^2) + (c*Sinh[x + ArcTanh[c/b]])/(b*Sqrt[1 - c^2/b^2]))/Sqrt[a + b*Sqrt[1 -
 c^2/b^2]*Cosh[x + ArcTanh[c/b]]]))/(3*(a^2 - b^2 + c^2)^2)
```

fricas [F] time = 0.46, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{b\cosh(x)+c\sinh(x)+a}}{b^3\cosh(x)^3+c^3\sinh(x)^3+3\,ab^2\cosh(x)^2+3\,a^2b\cosh(x)+a^3+3\left(bc^2\cosh(x)+ac^2\right)\sinh(x)^2+3\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cosh(x)+c*sinh(x))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*cosh(x) + c*sinh(x) + a)/(b^3*cosh(x)^3 + c^3*sinh(x)^3 + 3
*a*b^2*cosh(x)^2 + 3*a^2*b*cosh(x) + a^3 + 3*(b*c^2*cosh(x) + a*c^2)*sinh(x
)^2 + 3*(b^2*c*cosh(x)^2 + 2*a*b*c*cosh(x) + a^2*c)*sinh(x)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b\cosh(x)+c\sinh(x)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cosh(x)+c*sinh(x))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*cosh(x) + c*sinh(x) + a)^(-5/2), x)
```

maple [B] time = 2.96, size = 6075, normalized size = 18.87

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*cosh(x)+c*sinh(x))^(5/2),x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b\cosh(x) + c\sinh(x) + a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cosh(x)+c*sinh(x))^(5/2),x, algorithm="maxima")
[Out] integrate((b*cosh(x) + c*sinh(x) + a)^(-5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{(a + b\cosh(x) + c\sinh(x))^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*cosh(x) + c*sinh(x))^(5/2),x)
[Out] int(1/(a + b*cosh(x) + c*sinh(x))^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cosh(x)+c*sinh(x))**(5/2),x)
[Out] Timed out
```

3.767 $\int \frac{1}{(a+b\cosh(x)+c\sinh(x))^{7/2}}\,dx$

Optimal. Leaf size=411

$$\frac{16ia\sqrt{\frac{a+b\cosh(x)+c\sinh(x)}{a+\sqrt{b^2-c^2}}}\,F\left(\frac{1}{2}\left(ix-\tan^{-1}(b,-ic)\right)|\frac{2\sqrt{b^2-c^2}}{a+\sqrt{b^2-c^2}}\right)}{15\left(a^2-b^2+c^2\right)^2\sqrt{a+b\cosh(x)+c\sinh(x)}}-\frac{2i\left(23a^2+9b^2-9c^2\right)\sqrt{a+b\cosh(x)+c\sinh(x)}\,E}{15\left(a^2-b^2+c^2\right)^3\sqrt{\frac{a+b\cosh}{a+}}}$$

```
[Out] -2/5*(c*cosh(x)+b*sinh(x))/(a^2-b^2+c^2)/(a+b*cosh(x)+c*sinh(x))^(5/2)-16/1
5*(a*c*cosh(x)+a*b*sinh(x))/(a^2-b^2+c^2)^2/(a+b*cosh(x)+c*sinh(x))^(3/2)-2
/15*(c*(23*a^2+9*b^2-9*c^2)*cosh(x)+b*(23*a^2+9*b^2-9*c^2)*sinh(x))/(a^2-b^
2+c^2)^3/(a+b*cosh(x)+c*sinh(x))^(1/2)-2/15*I*(23*a^2+9*b^2-9*c^2)*(cos(1/2
*I*x-1/2*arctan(b,-I*c))^2)^(1/2)/cos(1/2*I*x-1/2*arctan(b,-I*c))*EllipticE
(sin(1/2*I*x-1/2*arctan(b,-I*c)),2^(1/2)*((b^2-c^2)^(1/2)/(a+(b^2-c^2)^(1/2
)))^(1/2))*(a+b*cosh(x)+c*sinh(x))^(1/2)/(a^2-b^2+c^2)^3/((a+b*cosh(x)+c*si
nh(x))/(a+(b^2-c^2)^(1/2)))^(1/2)+16/15*I*a*(cos(1/2*I*x-1/2*arctan(b,-I*c)
)^2)^(1/2)/cos(1/2*I*x-1/2*arctan(b,-I*c))*EllipticF(sin(1/2*I*x-1/2*arctan
(b,-I*c)),2^(1/2)*((b^2-c^2)^(1/2)/(a+(b^2-c^2)^(1/2)))^(1/2))*((a+b*cosh(x
)+c*sinh(x))/(a+(b^2-c^2)^(1/2)))^(1/2)/(a^2-b^2+c^2)^2/(a+b*cosh(x)+c*sinh
(x))^(1/2)
```

Rubi [A] time = 0.54, antiderivative size = 411, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3129, 3156, 3149, 3119, 2653, 3127, 2661}

$$\frac{16ia\sqrt{\frac{a+b\cosh(x)+c\sinh(x)}{a+\sqrt{b^2-c^2}}}\,F\left(\frac{1}{2}\left(ix-\tan^{-1}(b,-ic)\right)|\frac{2\sqrt{b^2-c^2}}{a+\sqrt{b^2-c^2}}\right)}{15\left(a^2-b^2+c^2\right)^2\sqrt{a+b\cosh(x)+c\sinh(x)}}-\frac{2i\left(23a^2+9b^2-9c^2\right)\sqrt{a+b\cosh(x)+c\sinh(x)}\,E}{15\left(a^2-b^2+c^2\right)^3\sqrt{\frac{a+b\cosh}{a+}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cosh[x] + c*Sinh[x])^(-7/2),x]
```

```
[Out] (-2*(c*Cosh[x] + b*Sinh[x]))/(5*(a^2 - b^2 + c^2)*(a + b*Cosh[x] + c*Sinh[x
])^(5/2)) - (16*(a*c*Cosh[x] + a*b*Sinh[x]))/(15*(a^2 - b^2 + c^2)^2*(a + b
*Cosh[x] + c*Sinh[x])^(3/2)) - (((2*I)/15)*(23*a^2 + 9*b^2 - 9*c^2)*Ellipti
cE[(I*x - ArcTan[b, (-I)*c])/2, (2*Sqrt[b^2 - c^2])/(a + Sqrt[b^2 - c^2])]*
Sqrt[a + b*Cosh[x] + c*Sinh[x]])/((a^2 - b^2 + c^2)^3*Sqrt[(a + b*Cosh[x] +
 c*Sinh[x])/(a + Sqrt[b^2 - c^2])]) + (((16*I)/15)*a*EllipticF[(I*x - ArcTa
n[b, (-I)*c])/2, (2*Sqrt[b^2 - c^2])/(a + Sqrt[b^2 - c^2])]*Sqrt[(a + b*Cos
h[x] + c*Sinh[x])/(a + Sqrt[b^2 - c^2])])/((a^2 - b^2 + c^2)^2*Sqrt[a + b*C
```

```
osh[x] + c*Sinh[x]]) - (2*(c*(23*a^2 + 9*b^2 - 9*c^2)*Cosh[x] + b*(23*a^2 +
 9*b^2 - 9*c^2)*Sinh[x]))/(15*(a^2 - b^2 + c^2)^3*Sqrt[a + b*Cosh[x] + c*Si
nh[x]])
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3119

```
Int[Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_
)]], x_Symbol] :> Dist[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]]/Sqrt[(a +
b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sqrt[b^2 + c^2])], Int[Sqrt[a/(a + Sq
rt[b^2 + c^2]) + (Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b, c]])/(a + Sqrt[b^
2 + c^2])], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
&& NeQ[b^2 + c^2, 0] &&  !GtQ[a + Sqrt[b^2 + c^2], 0]
```

Rule 3127

```
Int[1/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(
x_)]], x_Symbol] :> Dist[Sqrt[(a + b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sq
rt[b^2 + c^2])]/Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]], Int[1/Sqrt[a/(a
+ Sqrt[b^2 + c^2]) + (Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b, c]])/(a + Sqr
t[b^2 + c^2])], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2,
 0] && NeQ[b^2 + c^2, 0] &&  !GtQ[a + Sqrt[b^2 + c^2], 0]
```

Rule 3129

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> Simp[((-(c*Cos[d + e*x]) + b*Sin[d + e*x])*(a + b*Cos[d
+ e*x] + c*Sin[d + e*x])^(n + 1))/(e*(n + 1)*(a^2 - b^2 - c^2)), x] + Dist[
1/((n + 1)*(a^2 - b^2 - c^2)), Int[(a*(n + 1) - b*(n + 2)*Cos[d + e*x] - c*
(n + 2)*Sin[d + e*x])*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1), x], x]
 /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && LtQ[n, -1] && N
eQ[n, -3/2]
```

Rule 3149

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)]]
, x_Symbol] :> Dist[B/b, Int[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]], x],
 x] + Dist[(A*b - a*B)/b, Int[1/Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]],
x], x] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && EqQ[B*c - b*C, 0] && NeQ[A*
b - a*B, 0]
```

Rule 3156

```
Int[((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])
^(n_)*((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)
]), x_Symbol] :> -Simp[((c*B - b*C - (a*C - c*A)*Cos[d + e*x] + (a*B - b*A)
*Sin[d + e*x])*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1))/(e*(n + 1)*(a
^2 - b^2 - c^2)), x] + Dist[1/((n + 1)*(a^2 - b^2 - c^2)), Int[(a + b*Cos[d
 + e*x] + c*Sin[d + e*x])^(n + 1)*Simp[(n + 1)*(a*A - b*B - c*C) + (n + 2)*
(a*B - b*A)*Cos[d + e*x] + (n + 2)*(a*C - c*A)*Sin[d + e*x], x], x], x] /;
FreeQ[{a, b, c, d, e, A, B, C}, x] && LtQ[n, -1] && NeQ[a^2 - b^2 - c^2, 0]
 && NeQ[n, -2]
```

Rubi steps

$$\int \frac{1}{(a+b\cosh(x)+c\sinh(x))^{7/2}}\,dx = -\frac{2(c\cosh(x)+b\sinh(x))}{5\left(a^2-b^2+c^2\right)(a+b\cosh(x)+c\sinh(x))^{5/2}} - \frac{2\int \frac{-\frac{5a}{2}+\frac{3}{2}b\cosh(x)+\frac{3}{2}c\sinh(}{(a+b\cosh(x)+c\sinh(x))^{5/}}}{5\left(a^2-b^2+c^2\right)}$$

$$= -\frac{2(c\cosh(x)+b\sinh(x))}{5\left(a^2-b^2+c^2\right)(a+b\cosh(x)+c\sinh(x))^{5/2}} - \frac{16(ac\cosh(x)}{15\left(a^2-b^2+c^2\right)^2(a+b}$$

$$= -\frac{2(c\cosh(x)+b\sinh(x))}{5\left(a^2-b^2+c^2\right)(a+b\cosh(x)+c\sinh(x))^{5/2}} - \frac{16(ac\cosh(x)}{15\left(a^2-b^2+c^2\right)^2(a+b}$$

$$= -\frac{2(c\cosh(x)+b\sinh(x))}{5\left(a^2-b^2+c^2\right)(a+b\cosh(x)+c\sinh(x))^{5/2}} - \frac{16(ac\cosh(x)}{15\left(a^2-b^2+c^2\right)^2(a+b}$$

$$= -\frac{2(c\cosh(x)+b\sinh(x))}{5\left(a^2-b^2+c^2\right)(a+b\cosh(x)+c\sinh(x))^{5/2}} - \frac{16(ac\cosh(x)}{15\left(a^2-b^2+c^2\right)^2(a+b}$$

$$= -\frac{2(c\cosh(x)+b\sinh(x))}{5\left(a^2-b^2+c^2\right)(a+b\cosh(x)+c\sinh(x))^{5/2}} - \frac{16(ac\cosh(x)}{15\left(a^2-b^2+c^2\right)^2(a+b}$$

Mathematica [C] time = 6.57, size = 4093, normalized size = 9.96

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Cosh[x] + c*Sinh[x])^(-7/2),x]
```

```
[Out] Sqrt[a + b*Cosh[x] + c*Sinh[x]]*((-2*(23*a^2 + 9*b^2 - 9*c^2)*(b^2 - c^2))/
(15*b*c*(-a^2 + b^2 - c^2)^3) - (2*(a*c - b^2*Sinh[x] + c^2*Sinh[x]))/(5*b*
(-a^2 + b^2 - c^2)*(a + b*Cosh[x] + c*Sinh[x])^3) - (2*(-5*a^2*c - 3*b^2*c
+ 3*c^3 + 8*a*b^2*Sinh[x] - 8*a*c^2*Sinh[x]))/(15*b*(-a^2 + b^2 - c^2)^2*(a
 + b*Cosh[x] + c*Sinh[x])^2) + (2*(-15*a^3*c - 17*a*b^2*c + 17*a*c^3 + 23*a
^2*b^2*Sinh[x] + 9*b^4*Sinh[x] - 23*a^2*c^2*Sinh[x] - 18*b^2*c^2*Sinh[x] +
9*c^4*Sinh[x]))/(15*b*(-a^2 + b^2 - c^2)^3*(a + b*Cosh[x] + c*Sinh[x]))) +
(2*a^3*AppellF1[1/2, 1/2, 1/2, 3/2, ((-I)*(a + Sqrt[1 - b^2/c^2]*c*Sinh[x +
```

ArcTanh[b/c]]))/(Sqrt[1 - b^2/c^2]*(1 - (I*a)/(Sqrt[1 - b^2/c^2]*c))*c), ((-I)*(a + Sqrt[1 - b^2/c^2]*c*Sinh[x + ArcTanh[b/c]]))/(Sqrt[1 - b^2/c^2]*(-1 - (I*a)/(Sqrt[1 - b^2/c^2]*c))*c)]*Sech[x + ArcTanh[b/c]]*Sqrt[-1 + I*Sinh[x + ArcTanh[b/c]]]*Sqrt[(c*Sqrt[(-b^2 + c^2)/c^2] - I*c*Sqrt[(-b^2 + c^2)/c^2]*Sinh[x + ArcTanh[b/c]])/(I*a + c*Sqrt[(-b^2 + c^2)/c^2])]*Sqrt[(c*Sqrt[(-b^2 + c^2)/c^2] + I*c*Sqrt[(-b^2 + c^2)/c^2]*Sinh[x + ArcTanh[b/c]])/((-I)*a + c*Sqrt[(-b^2 + c^2)/c^2])]*Sqrt[a + c*Sqrt[(-b^2 + c^2)/c^2]*Sinh[x + ArcTanh[b/c]]])/(Sqrt[1 - b^2/c^2]*c*(a^2 - b^2 + c^2)^3*Sqrt[I*(I + Sinh[x + ArcTanh[b/c]])]) + (34*a*b^2*AppellF1[1/2, 1/2, 1/2, 3/2, ((-I)*(a + Sqrt[1 - b^2/c^2]*c*Sinh[x + ArcTanh[b/c]]))/(Sqrt[1 - b^2/c^2]*(1 - (I*a)/(Sqrt[1 - b^2/c^2]*c))*c), ((-I)*(a + Sqrt[1 - b^2/c^2]*c*Sinh[x + ArcTanh[b/c]]))/(Sqrt[1 - b^2/c^2]*(-1 - (I*a)/(Sqrt[1 - b^2/c^2]*c))*c)]*Sech[x + ArcTanh[b/c]]*Sqrt[-1 + I*Sinh[x + ArcTanh[b/c]]]*Sqrt[(c*Sqrt[(-b^2 + c^2)/c^2] - I*c*Sqrt[(-b^2 + c^2)/c^2]*Sinh[x + ArcTanh[b/c]])/(I*a + c*Sqrt[(-b^2 + c^2)/c^2])]*Sqrt[(c*Sqrt[(-b^2 + c^2)/c^2] + I*c*Sqrt[(-b^2 + c^2)/c^2]*Sinh[x + ArcTanh[b/c]])/((-I)*a + c*Sqrt[(-b^2 + c^2)/c^2])]*Sqrt[a + c*Sqrt[(-b^2 + c^2)/c^2]*Sinh[x + ArcTanh[b/c]]])/(15*Sqrt[1 - b^2/c^2]*c*(a^2 - b^2 + c^2)^3*Sqrt[I*(I + Sinh[x + ArcTanh[b/c]])]) - (34*a*c*AppellF1[1/2, 1/2, 1/2, 3/2, ((-I)*(a + Sqrt[1 - b^2/c^2]*c*Sinh[x + ArcTanh[b/c]]))/(Sqrt[1 - b^2/c^2]*(1 - (I*a)/(Sqrt[1 - b^2/c^2]*c))*c), ((-I)*(a + Sqrt[1 - b^2/c^2]*c*Sinh[x + ArcTanh[b/c]]))/(Sqrt[1 - b^2/c^2]*(-1 - (I*a)/(Sqrt[1 - b^2/c^2]*c))*c)]*Sech[x + ArcTanh[b/c]]*Sqrt[-1 + I*Sinh[x + ArcTanh[b/c]]]*Sqrt[(c*Sqrt[(-b^2 + c^2)/c^2] - I*c*Sqrt[(-b^2 + c^2)/c^2]*Sinh[x + ArcTanh[b/c]])/(I*a + c*Sqrt[(-b^2 + c^2)/c^2])]*Sqrt[(c*Sqrt[(-b^2 + c^2)/c^2] + I*c*Sqrt[(-b^2 + c^2)/c^2]*Sinh[x + ArcTanh[b/c]])/((-I)*a + c*Sqrt[(-b^2 + c^2)/c^2])]*Sqrt[a + c*Sqrt[(-b^2 + c^2)/c^2]*Sinh[x + ArcTanh[b/c]]])/(15*Sqrt[1 - b^2/c^2]*(a^2 - b^2 + c^2)^3*Sqrt[I*(I + Sinh[x + ArcTanh[b/c]])]) - (23*a^2*b^2*((c*AppellF1[-1/2, -1/2, -1/2, 1/2, (a + b*Sqrt[1 - c^2/b^2]*Cosh[x + ArcTanh[c/b]])/(b*Sqrt[1 - c^2/b^2]*(1 + a/(b*Sqrt[1 - c^2/b^2]))), (a + b*Sqrt[1 - c^2/b^2]*Cosh[x + ArcTanh[c/b]])/(b*Sqrt[1 - c^2/b^2]*(-1 + a/(b*Sqrt[1 - c^2/b^2])))]*Sinh[x + ArcTanh[c/b]])/(b*Sqrt[1 - c^2/b^2]*Sqrt[(b*Sqrt[(b^2 - c^2)/b^2] - b*Sqrt[(b^2 - c^2)/b^2]*Cosh[x + ArcTanh[c/b]])/(a + b*Sqrt[(b^2 - c^2)/b^2])]*Sqrt[a + b*Sqrt[(b^2 - c^2)/b^2]*Cosh[x + ArcTanh[c/b]]]*Sqrt[(b*Sqrt[(b^2 - c^2)/b^2] + b*Sqrt[(b^2 - c^2)/b^2]*Cosh[x + ArcTanh[c/b]])/(-a + b*Sqrt[(b^2 - c^2)/b^2])]) - ((-2*b*(a + b*Sqrt[1 - c^2/b^2]*Cosh[x + ArcTanh[c/b]]))/(b^2 - c^2) + (c*Sinh[x + ArcTanh[c/b]])/(b*Sqrt[1 - c^2/b^2]))/Sqrt[a + b*Sqrt[1 - c^2/b^2]*Cosh[x + ArcTanh[c/b]]]))/(15*c*(a^2 - b^2 + c^2)^3) - (3*b^4*((c*AppellF1[-1/2, -1/2, -1/2, 1/2, (a + b*Sqrt[1 - c^2/b^2]*Cosh[x + ArcTanh[c/b]])/(b*Sqrt[1 - c^2/b^2]*(1 + a/(b*Sqrt[1 - c^2/b^2]))), (a + b*Sqrt[1 - c^2/b^2]*Cosh[x + ArcTanh[c/b]])/(b*Sqrt[1 - c^2/b^2]*(-1 + a/(b*Sqrt[1 - c^2/b^2])))]*Sinh[x + ArcTanh[c/b]])/(b*Sqrt[1 - c^2/b^2]*Sqrt[(b*Sqrt[(b^2 - c^2)/b^2] - b*Sqrt[(b^2 - c^2)/b^2]*Cosh[x + ArcTanh[c/b]])/(a + b*Sqrt[(b^2 - c^2)/b^2])]*Sqrt[a + b*Sqrt[(b^2 - c^2)/b^2]*Cosh[x + ArcTanh[c/b]]]*Sqrt[(b*Sqrt[(b^2 - c^2)/b^2] + b*Sqrt[(b^2 - c^2)/b^2]*Cosh[x + ArcTanh[c/b]])/(-a + b*Sqrt

```
[(b^2 - c^2)/b^2])]) - ((-2*b*(a + b*Sqrt[1 - c^2/b^2]*Cosh[x + ArcTanh[c/b
]]))/(b^2 - c^2) + (c*Sinh[x + ArcTanh[c/b]])/(b*Sqrt[1 - c^2/b^2]))/Sqrt[a
 + b*Sqrt[1 - c^2/b^2]*Cosh[x + ArcTanh[c/b]]]))/(5*c*(a^2 - b^2 + c^2)^3)
+ (23*a^2*c*((c*AppellF1[-1/2, -1/2, -1/2, 1/2, (a + b*Sqrt[1 - c^2/b^2]*Co
sh[x + ArcTanh[c/b]])/(b*Sqrt[1 - c^2/b^2]*(1 + a/(b*Sqrt[1 - c^2/b^2]))),
(a + b*Sqrt[1 - c^2/b^2]*Cosh[x + ArcTanh[c/b]])/(b*Sqrt[1 - c^2/b^2]*(-1 +
 a/(b*Sqrt[1 - c^2/b^2])))]*Sinh[x + ArcTanh[c/b]])/(b*Sqrt[1 - c^2/b^2]*Sq
rt[(b*Sqrt[(b^2 - c^2)/b^2] - b*Sqrt[(b^2 - c^2)/b^2]*Cosh[x + ArcTanh[c/b]
])/(a + b*Sqrt[(b^2 - c^2)/b^2])]*Sqrt[a + b*Sqrt[(b^2 - c^2)/b^2]*Cosh[x +
 ArcTanh[c/b]]]*Sqrt[(b*Sqrt[(b^2 - c^2)/b^2] + b*Sqrt[(b^2 - c^2)/b^2]*Cos
h[x + ArcTanh[c/b]])/(-a + b*Sqrt[(b^2 - c^2)/b^2])]) - ((-2*b*(a + b*Sqrt[
1 - c^2/b^2]*Cosh[x + ArcTanh[c/b]]))/(b^2 - c^2) + (c*Sinh[x + ArcTanh[c/b
]])/(b*Sqrt[1 - c^2/b^2]))/Sqrt[a + b*Sqrt[1 - c^2/b^2]*Cosh[x + ArcTanh[c/
b]]]))/(15*(a^2 - b^2 + c^2)^3) + (6*b^2*c*((c*AppellF1[-1/2, -1/2, -1/2, 1
/2, (a + b*Sqrt[1 - c^2/b^2]*Cosh[x + ArcTanh[c/b]])/(b*Sqrt[1 - c^2/b^2]*(
1 + a/(b*Sqrt[1 - c^2/b^2]))), (a + b*Sqrt[1 - c^2/b^2]*Cosh[x + ArcTanh[c/
b]])/(b*Sqrt[1 - c^2/b^2]*(-1 + a/(b*Sqrt[1 - c^2/b^2])))]*Sinh[x + ArcTanh
[c/b]])/(b*Sqrt[1 - c^2/b^2]*Sqrt[(b*Sqrt[(b^2 - c^2)/b^2] - b*Sqrt[(b^2 -
c^2)/b^2]*Cosh[x + ArcTanh[c/b]])/(a + b*Sqrt[(b^2 - c^2)/b^2])]*Sqrt[a + b
*Sqrt[(b^2 - c^2)/b^2]*Cosh[x + ArcTanh[c/b]]]*Sqrt[(b*Sqrt[(b^2 - c^2)/b^2
] + b*Sqrt[(b^2 - c^2)/b^2]*Cosh[x + ArcTanh[c/b]])/(-a + b*Sqrt[(b^2 - c^2
)/b^2])]) - ((-2*b*(a + b*Sqrt[1 - c^2/b^2]*Cosh[x + ArcTanh[c/b]]))/(b^2 -
 c^2) + (c*Sinh[x + ArcTanh[c/b]])/(b*Sqrt[1 - c^2/b^2]))/Sqrt[a + b*Sqrt[1
 - c^2/b^2]*Cosh[x + ArcTanh[c/b]]]))/(5*(a^2 - b^2 + c^2)^3) - (3*c^3*((c*
AppellF1[-1/2, -1/2, -1/2, 1/2, (a + b*Sqrt[1 - c^2/b^2]*Cosh[x + ArcTanh[c
/b]])/(b*Sqrt[1 - c^2/b^2]*(1 + a/(b*Sqrt[1 - c^2/b^2]))), (a + b*Sqrt[1 -
c^2/b^2]*Cosh[x + ArcTanh[c/b]])/(b*Sqrt[1 - c^2/b^2]*(-1 + a/(b*Sqrt[1 - c
^2/b^2])))]*Sinh[x + ArcTanh[c/b]])/(b*Sqrt[1 - c^2/b^2]*Sqrt[(b*Sqrt[(b^2
- c^2)/b^2] - b*Sqrt[(b^2 - c^2)/b^2]*Cosh[x + ArcTanh[c/b]])/(a + b*Sqrt[(
b^2 - c^2)/b^2])]*Sqrt[a + b*Sqrt[(b^2 - c^2)/b^2]*Cosh[x + ArcTanh[c/b]]]*
Sqrt[(b*Sqrt[(b^2 - c^2)/b^2] + b*Sqrt[(b^2 - c^2)/b^2]*Cosh[x + ArcTanh[c/
b]])/(-a + b*Sqrt[(b^2 - c^2)/b^2])]) - ((-2*b*(a + b*Sqrt[1 - c^2/b^2]*Cos
h[x + ArcTanh[c/b]]))/(b^2 - c^2) + (c*Sinh[x + ArcTanh[c/b]])/(b*Sqrt[1 -
c^2/b^2]))/Sqrt[a + b*Sqrt[1 - c^2/b^2]*Cosh[x + ArcTanh[c/b]]]))/(5*(a^2 -
 b^2 + c^2)^3)
```

fricas [F] time = 0.47, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{}{b^4\cosh(x)^4 + c^4\sinh(x)^4 + 4\,ab^3\cosh(x)^3 + 6\,a^2b^2\cosh(x)^2 + 4\,a^3b\cosh(x) + a^4 + 4\left(bc^3\cosh(x) + a\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cosh(x)+c*sinh(x))^(7/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*cosh(x) + c*sinh(x) + a)/(b^4*cosh(x)^4 + c^4*sinh(x)^4 + 4
*a*b^3*cosh(x)^3 + 6*a^2*b^2*cosh(x)^2 + 4*a^3*b*cosh(x) + a^4 + 4*(b*c^3*c
osh(x) + a*c^3)*sinh(x)^3 + 6*(b^2*c^2*cosh(x)^2 + 2*a*b*c^2*cosh(x) + a^2*
c^2)*sinh(x)^2 + 4*(b^3*c*cosh(x)^3 + 3*a*b^2*c*cosh(x)^2 + 3*a^2*b*c*cosh(
x) + a^3*c)*sinh(x)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b\cosh(x) + c\sinh(x) + a)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cosh(x)+c*sinh(x))^(7/2),x, algorithm="giac")
```

```
[Out] integrate((b*cosh(x) + c*sinh(x) + a)^(-7/2), x)
```

maple [B] time = 12.72, size = 57909, normalized size = 140.90

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*cosh(x)+c*sinh(x))^(7/2),x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b\cosh(x) + c\sinh(x) + a)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cosh(x)+c*sinh(x))^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((b*cosh(x) + c*sinh(x) + a)^(-7/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{(a + b\cosh(x) + c\sinh(x))^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*cosh(x) + c*sinh(x))^(7/2),x)
```

```
[Out] int(1/(a + b*cosh(x) + c*sinh(x))^(7/2), x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cosh(x)+c*sinh(x))**(7/2),x)
```

```
[Out] Timed out
```

3.768 $\int \left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{5/2}\,dx$

Optimal. Leaf size=140

$$\frac{2}{5}(b\sinh(x)+c\cosh(x))\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{3/2}+\frac{16}{15}\sqrt{b^2-c^2}\,(b\sinh(x)+c\cosh(x))\sqrt{\sqrt{b^2-c^2}+$$

```
[Out] 2/5*(c*cosh(x)+b*sinh(x))*(b*cosh(x)+c*sinh(x)+(b^2-c^2)^(1/2))^(3/2)+64/15
*(b^2-c^2)*(c*cosh(x)+b*sinh(x))/(b*cosh(x)+c*sinh(x)+(b^2-c^2)^(1/2))^(1/2
)+16/15*(c*cosh(x)+b*sinh(x))*(b^2-c^2)^(1/2)*(b*cosh(x)+c*sinh(x)+(b^2-c^2
)^(1/2))^(1/2)
```

Rubi [A] time = 0.12, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {3113, 3112}

$$\frac{2}{5}(b\sinh(x)+c\cosh(x))\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{3/2}+\frac{16}{15}\sqrt{b^2-c^2}\,(b\sinh(x)+c\cosh(x))\sqrt{\sqrt{b^2-c^2}+$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[b^2 - c^2] + b*Cosh[x] + c*Sinh[x])^(5/2),x]
```

```
[Out] (64*(b^2 - c^2)*(c*Cosh[x] + b*Sinh[x]))/(15*Sqrt[Sqrt[b^2 - c^2] + b*Cosh[
x] + c*Sinh[x]]) + (16*Sqrt[b^2 - c^2]*(c*Cosh[x] + b*Sinh[x])*Sqrt[Sqrt[b^
2 - c^2] + b*Cosh[x] + c*Sinh[x]])/15 + (2*(c*Cosh[x] + b*Sinh[x])*(Sqrt[b^
2 - c^2] + b*Cosh[x] + c*Sinh[x])^(3/2))/5
```

Rule 3112

```
Int[Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_
)]], x_Symbol] :> Simp[(-2*(c*Cos[d + e*x] - b*Sin[d + e*x]))/(e*Sqrt[a + b
*Cos[d + e*x] + c*Sin[d + e*x]]), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[a^
2 - b^2 - c^2, 0]
```

Rule 3113

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> -Simp[((c*Cos[d + e*x] - b*Sin[d + e*x])*(a + b*Cos[d +
e*x] + c*Sin[d + e*x])^(n - 1))/(e*n), x] + Dist[(a*(2*n - 1))/n, Int[(a +
b*Cos[d + e*x] + c*Sin[d + e*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e},
x] && EqQ[a^2 - b^2 - c^2, 0] && GtQ[n, 0]
```

Rubi steps

$$\int \left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{5/2}dx=\frac{2}{5}(c\cosh(x)+b\sinh(x))\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{3/2}+$$
$$=\frac{16}{15}\sqrt{b^2-c^2}\,(c\cosh(x)+b\sinh(x))\sqrt{\sqrt{b^2-c^2}+b\cosh(x)+c\sin}$$
$$=\frac{64\left(b^2-c^2\right)(c\cosh(x)+b\sinh(x))}{15\sqrt{\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)}}+\frac{16}{15}\sqrt{b^2-c^2}\,(c\cosh(x)+$$

Mathematica [C] time = 75.52, size = 10223, normalized size = 73.02

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[b^2 - c^2] + b*Cosh[x] + c*Sinh[x])^(5/2),x]
```

```
[Out] Result too large to show
```

fricas [B] time = 0.44, size = 784, normalized size = 5.60

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cosh(x)+c*sinh(x)+(b^2-c^2)^(1/2))^(5/2),x, algorithm="fricas"
)
```

```
[Out] 1/30*sqrt(1/2)*(3*(b^3 + 3*b^2*c + 3*b*c^2 + c^3)*cosh(x)^6 + 18*(b^3 + 3*b
^2*c + 3*b*c^2 + c^3)*cosh(x)*sinh(x)^5 + 3*(b^3 + 3*b^2*c + 3*b*c^2 + c^3)
*sinh(x)^6 + 125*(b^3 + b^2*c - b*c^2 - c^3)*cosh(x)^4 + 5*(25*b^3 + 25*b^2
*c - 25*b*c^2 - 25*c^3 + 9*(b^3 + 3*b^2*c + 3*b*c^2 + c^3)*cosh(x)^2)*sinh(
x)^4 + 20*(3*(b^3 + 3*b^2*c + 3*b*c^2 + c^3)*cosh(x)^3 + 25*(b^3 + b^2*c -
b*c^2 - c^3)*cosh(x))*sinh(x)^3 + 3*b^3 - 9*b^2*c + 9*b*c^2 - 3*c^3 + 125*(
b^3 - b^2*c - b*c^2 + c^3)*cosh(x)^2 + 5*(9*(b^3 + 3*b^2*c + 3*b*c^2 + c^3)
*cosh(x)^4 + 25*b^3 - 25*b^2*c - 25*b*c^2 + 25*c^3 + 150*(b^3 + b^2*c - b*c
^2 - c^3)*cosh(x)^2)*sinh(x)^2 + 2*(9*(b^3 + 3*b^2*c + 3*b*c^2 + c^3)*cosh(
x)^5 + 250*(b^3 + b^2*c - b*c^2 - c^3)*cosh(x)^3 + 125*(b^3 - b^2*c - b*c^2
 + c^3)*cosh(x))*sinh(x) + 2*(11*(b^2 + 2*b*c + c^2)*cosh(x)^5 + 55*(b^2 +
2*b*c + c^2)*cosh(x)*sinh(x)^4 + 11*(b^2 + 2*b*c + c^2)*sinh(x)^5 - 150*(b^
2 - c^2)*cosh(x)^3 + 10*(11*(b^2 + 2*b*c + c^2)*cosh(x)^2 - 15*b^2 + 15*c^2
)*sinh(x)^3 + 10*(11*(b^2 + 2*b*c + c^2)*cosh(x)^3 - 45*(b^2 - c^2)*cosh(x)
```

```
)*sinh(x)^2 + 11*(b^2 - 2*b*c + c^2)*cosh(x) + (55*(b^2 + 2*b*c + c^2)*cosh
(x)^4 - 450*(b^2 - c^2)*cosh(x)^2 + 11*b^2 - 22*b*c + 11*c^2)*sinh(x))*sqrt
(b^2 - c^2))*sqrt(((b + c)*cosh(x)^2 + 2*(b + c)*cosh(x)*sinh(x) + (b + c)*
sinh(x)^2 + 2*sqrt(b^2 - c^2)*(cosh(x) + sinh(x)) + b - c)/(cosh(x) + sinh(
x)))/((b + c)*cosh(x)^4 + 4*(b + c)*cosh(x)*sinh(x)^3 + (b + c)*sinh(x)^4 -
 (b - c)*cosh(x)^2 + (6*(b + c)*cosh(x)^2 - b + c)*sinh(x)^2 + 2*(2*(b + c)
*cosh(x)^3 - (b - c)*cosh(x))*sinh(x))
```

giac [B] time = 0.33, size = 657, normalized size = 4.69

$$-\frac{\sqrt{2}\left(3\left(\sqrt{b^2-c^2}\,b^2\mathrm{sgn}\left(-\sqrt{b^2-c^2}\,e^x-b+c\right)+2\,\sqrt{b^2-c^2}\,bc\mathrm{sgn}\left(-\sqrt{b^2-c^2}\,e^x-b+c\right)+\sqrt{b^2-c^2}\,c^2\mathrm{sgn}\left(-\sqrt{\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cosh(x)+c*sinh(x)+(b^2-c^2)^(1/2))^(5/2),x, algorithm="giac")
```

```
[Out] -1/60*sqrt(2)*(3*(sqrt(b^2 - c^2)*b^2*sgn(-sqrt(b^2 - c^2)*e^x - b + c) + 2
*sqrt(b^2 - c^2)*b*c*sgn(-sqrt(b^2 - c^2)*e^x - b + c) + sqrt(b^2 - c^2)*c^
2*sgn(-sqrt(b^2 - c^2)*e^x - b + c))*e^(5/2*x) + 25*(b^3*sgn(-sqrt(b^2 - c^
2)*e^x - b + c) + b^2*c*sgn(-sqrt(b^2 - c^2)*e^x - b + c) - b*c^2*sgn(-sqrt
(b^2 - c^2)*e^x - b + c) - c^3*sgn(-sqrt(b^2 - c^2)*e^x - b + c))*e^(3/2*x)
 + 150*(sqrt(b^2 - c^2)*b^2*sgn(-sqrt(b^2 - c^2)*e^x - b + c) - sqrt(b^2 -
c^2)*c^2*sgn(-sqrt(b^2 - c^2)*e^x - b + c))*e^(1/2*x) - 150*(b^3*sgn(-sqrt(
b^2 - c^2)*e^x - b + c) - b^2*c*sgn(-sqrt(b^2 - c^2)*e^x - b + c) - b*c^2*s
gn(-sqrt(b^2 - c^2)*e^x - b + c) + c^3*sgn(-sqrt(b^2 - c^2)*e^x - b + c))*e
^(-1/2*x) - 25*(sqrt(b^2 - c^2)*b^2*sgn(-sqrt(b^2 - c^2)*e^x - b + c) - 2*s
qrt(b^2 - c^2)*b*c*sgn(-sqrt(b^2 - c^2)*e^x - b + c) + sqrt(b^2 - c^2)*c^2*
sgn(-sqrt(b^2 - c^2)*e^x - b + c))*e^(-3/2*x) - 3*(b^3*sgn(-sqrt(b^2 - c^2)
*e^x - b + c) - 3*b^2*c*sgn(-sqrt(b^2 - c^2)*e^x - b + c) + 3*b*c^2*sgn(-sq
rt(b^2 - c^2)*e^x - b + c) - c^3*sgn(-sqrt(b^2 - c^2)*e^x - b + c))*e^(-5/2
*x))/sqrt(b - c)
```

maple [B] time = 1.13, size = 518, normalized size = 3.70

$$\frac{-\frac{\left(b^2-c^2\right)^{\frac{3}{2}}\left(\cosh^3(x)\right)}{3}-\frac{\left(-2b^2+2c^2\right)\left(-b^2+c^2\right)\cosh(x)}{\sqrt{b^2-c^2}}}{\sqrt{-\frac{\sinh(x)b^2-\sinh(x)c^2-b^2+c^2}{\sqrt{b^2-c^2}}}}-\frac{\left(\cosh(x)\sqrt{\sinh(x)\sqrt{b^2-c^2}-\sqrt{b^2-c^2}}\,\sqrt{-\sqrt{b^2-c^2}}\,\left(\sinh^3(x)\right)+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cosh(x)+c*sinh(x)+(b^2-c^2)^(1/2))^(5/2),x)
```

```
[Out] 1/(-(sinh(x)*b^2-sinh(x)*c^2-b^2+c^2)/(b^2-c^2)^(1/2))^(1/2)*(-1/3*(b^2-c^2
)^(3/2)*cosh(x)^3-(-2*b^2+2*c^2)*(-b^2+c^2)/(b^2-c^2)^(1/2)*cosh(x))-1/2*(c
osh(x)*(sinh(x)*(b^2-c^2)^(1/2)-(b^2-c^2)^(1/2))^(1/2)*(-(b^2-c^2)^(1/2)*si
nh(x)^3+(b^2-c^2)^(1/2)*sinh(x)^2)^(1/2)*(b^2-c^2)-sinh(x)*(b^2-c^2)^(3/2)*
arctan((sinh(x)*(b^2-c^2)^(1/2)-(b^2-c^2)^(1/2))^(1/2)*cosh(x)/(-(b^2-c^2)^
(1/2)*sinh(x)^3+(b^2-c^2)^(1/2)*sinh(x)^2)^(1/2))+(b^2-c^2)^(1/2)*arctan((s
inh(x)*(b^2-c^2)^(1/2)-(b^2-c^2)^(1/2))^(1/2)*cosh(x)/(-(b^2-c^2)^(1/2)*sin
h(x)^3+(b^2-c^2)^(1/2)*sinh(x)^2)^(1/2))*b^2-(b^2-c^2)^(1/2)*arctan((sinh(x
)*(b^2-c^2)^(1/2)-(b^2-c^2)^(1/2))^(1/2)*cosh(x)/(-(b^2-c^2)^(1/2)*sinh(x)^
3+(b^2-c^2)^(1/2)*sinh(x)^2)^(1/2))*c^2)*(-(b^2-c^2)^(1/2)*(sinh(x)-1)*sinh
(x)^2)^(1/2)/((b^2-c^2)^(1/2)*(sinh(x)-1))^(1/2)/(sinh(x)-1)/sinh(x)/(-(sin
h(x)*b^2-sinh(x)*c^2-b^2+c^2)/(b^2-c^2)^(1/2))^(1/2)
```

maxima [B] time = 4.42, size = 1783, normalized size = 12.74

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cosh(x)+c*sinh(x)+(b^2-c^2)^(1/2))^(5/2),x, algorithm="maxima"
)
```

```
[Out] 1/20*sqrt(2)*(sqrt(b + c)*sqrt(b - c)*b^2 + 2*sqrt(b + c)*sqrt(b - c)*b*c +
 sqrt(b + c)*sqrt(b - c)*c^2)*(2*sqrt(b + c)*sqrt(b - c)*e^(-x) + (b - c)*e
^(-2*x) + b + c)^(5/2)*e^(5/2*x)/(sqrt(b + c)*sqrt(b - c)*b^2 + 2*sqrt(b +
c)*sqrt(b - c)*b*c + sqrt(b + c)*sqrt(b - c)*c^2 + 5*(b^3 + b^2*c - b*c^2 -
 c^3)*e^(-x) + 10*(sqrt(b + c)*sqrt(b - c)*b^2 - sqrt(b + c)*sqrt(b - c)*c^
2)*e^(-2*x) + 10*(b^3 - b^2*c - b*c^2 + c^3)*e^(-3*x) + 5*(sqrt(b + c)*sqrt
(b - c)*b^2 - 2*sqrt(b + c)*sqrt(b - c)*b*c + sqrt(b + c)*sqrt(b - c)*c^2)*
e^(-4*x) + (b^3 - 3*b^2*c + 3*b*c^2 - c^3)*e^(-5*x)) + 5/12*sqrt(2)*(b^3 +
b^2*c - b*c^2 - c^3)*(2*sqrt(b + c)*sqrt(b - c)*e^(-x) + (b - c)*e^(-2*x) +
 b + c)^(5/2)*e^(3/2*x)/(sqrt(b + c)*sqrt(b - c)*b^2 + 2*sqrt(b + c)*sqrt(b
 - c)*b*c + sqrt(b + c)*sqrt(b - c)*c^2 + 5*(b^3 + b^2*c - b*c^2 - c^3)*e^(
-x) + 10*(sqrt(b + c)*sqrt(b - c)*b^2 - sqrt(b + c)*sqrt(b - c)*c^2)*e^(-2*
x) + 10*(b^3 - b^2*c - b*c^2 + c^3)*e^(-3*x) + 5*(sqrt(b + c)*sqrt(b - c)*b
^2 - 2*sqrt(b + c)*sqrt(b - c)*b*c + sqrt(b + c)*sqrt(b - c)*c^2)*e^(-4*x)
+ (b^3 - 3*b^2*c + 3*b*c^2 - c^3)*e^(-5*x)) + 5/2*sqrt(2)*(sqrt(b + c)*sqrt
(b - c)*b^2 - sqrt(b + c)*sqrt(b - c)*c^2)*(2*sqrt(b + c)*sqrt(b - c)*e^(-x
) + (b - c)*e^(-2*x) + b + c)^(5/2)*e^(1/2*x)/(sqrt(b + c)*sqrt(b - c)*b^2
+ 2*sqrt(b + c)*sqrt(b - c)*b*c + sqrt(b + c)*sqrt(b - c)*c^2 + 5*(b^3 + b^
2*c - b*c^2 - c^3)*e^(-x) + 10*(sqrt(b + c)*sqrt(b - c)*b^2 - sqrt(b + c)*s
qrt(b - c)*c^2)*e^(-2*x) + 10*(b^3 - b^2*c - b*c^2 + c^3)*e^(-3*x) + 5*(sqr
t(b + c)*sqrt(b - c)*b^2 - 2*sqrt(b + c)*sqrt(b - c)*b*c + sqrt(b + c)*sqrt
(b - c)*c^2)*e^(-4*x) + (b^3 - 3*b^2*c + 3*b*c^2 - c^3)*e^(-5*x)) - 5/2*sqr
t(2)*(b^3 - b^2*c - b*c^2 + c^3)*(2*sqrt(b + c)*sqrt(b - c)*e^(-x) + (b - c
)*e^(-2*x) + b + c)^(5/2)*e^(-1/2*x)/(sqrt(b + c)*sqrt(b - c)*b^2 + 2*sqrt(
```

```
b + c)*sqrt(b - c)*b*c + sqrt(b + c)*sqrt(b - c)*c^2 + 5*(b^3 + b^2*c - b*c
^2 - c^3)*e^(-x) + 10*(sqrt(b + c)*sqrt(b - c)*b^2 - sqrt(b + c)*sqrt(b - c
)*c^2)*e^(-2*x) + 10*(b^3 - b^2*c - b*c^2 + c^3)*e^(-3*x) + 5*(sqrt(b + c)*
sqrt(b - c)*b^2 - 2*sqrt(b + c)*sqrt(b - c)*b*c + sqrt(b + c)*sqrt(b - c)*c
^2)*e^(-4*x) + (b^3 - 3*b^2*c + 3*b*c^2 - c^3)*e^(-5*x)) - 5/12*sqrt(2)*(sq
rt(b + c)*sqrt(b - c)*b^2 - 2*sqrt(b + c)*sqrt(b - c)*b*c + sqrt(b + c)*sqr
t(b - c)*c^2)*(2*sqrt(b + c)*sqrt(b - c)*e^(-x) + (b - c)*e^(-2*x) + b + c)
^(5/2)*e^(-3/2*x)/(sqrt(b + c)*sqrt(b - c)*b^2 + 2*sqrt(b + c)*sqrt(b - c)*
b*c + sqrt(b + c)*sqrt(b - c)*c^2 + 5*(b^3 + b^2*c - b*c^2 - c^3)*e^(-x) +
10*(sqrt(b + c)*sqrt(b - c)*b^2 - sqrt(b + c)*sqrt(b - c)*c^2)*e^(-2*x) + 1
0*(b^3 - b^2*c - b*c^2 + c^3)*e^(-3*x) + 5*(sqrt(b + c)*sqrt(b - c)*b^2 - 2
*sqrt(b + c)*sqrt(b - c)*b*c + sqrt(b + c)*sqrt(b - c)*c^2)*e^(-4*x) + (b^3
 - 3*b^2*c + 3*b*c^2 - c^3)*e^(-5*x)) - 1/20*sqrt(2)*(b^3 - 3*b^2*c + 3*b*c
^2 - c^3)*(2*sqrt(b + c)*sqrt(b - c)*e^(-x) + (b - c)*e^(-2*x) + b + c)^(5/
2)*e^(-5/2*x)/(sqrt(b + c)*sqrt(b - c)*b^2 + 2*sqrt(b + c)*sqrt(b - c)*b*c
+ sqrt(b + c)*sqrt(b - c)*c^2 + 5*(b^3 + b^2*c - b*c^2 - c^3)*e^(-x) + 10*(
sqrt(b + c)*sqrt(b - c)*b^2 - sqrt(b + c)*sqrt(b - c)*c^2)*e^(-2*x) + 10*(b
^3 - b^2*c - b*c^2 + c^3)*e^(-3*x) + 5*(sqrt(b + c)*sqrt(b - c)*b^2 - 2*sqr
t(b + c)*sqrt(b - c)*b*c + sqrt(b + c)*sqrt(b - c)*c^2)*e^(-4*x) + (b^3 - 3
*b^2*c + 3*b*c^2 - c^3)*e^(-5*x))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(b\cosh(x) + \sqrt{b^2 - c^2} + c\sinh(x)\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cosh(x) + (b^2 - c^2)^(1/2) + c*sinh(x))^(5/2),x)
[Out] int((b*cosh(x) + (b^2 - c^2)^(1/2) + c*sinh(x))^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cosh(x)+c*sinh(x)+(b**2-c**2)**(1/2))**(5/2),x)
[Out] Timed out
```

3.769 $\int \left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{3/2}\,dx$

Optimal. Leaf size=92

$$\frac{2}{3}\sqrt{\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)}\,(b\sinh(x)+c\cosh(x))+\frac{8\sqrt{b^2-c^2}\,(b\sinh(x)+c\cosh(x))}{3\sqrt{\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)}}$$

```
[Out] 8/3*(c*cosh(x)+b*sinh(x))*(b^2-c^2)^(1/2)/(b*cosh(x)+c*sinh(x)+(b^2-c^2)^(1
/2))^(1/2)+2/3*(c*cosh(x)+b*sinh(x))*(b*cosh(x)+c*sinh(x)+(b^2-c^2)^(1/2))^
(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {3113, 3112}

$$\frac{2}{3}\sqrt{\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)}\,(b\sinh(x)+c\cosh(x))+\frac{8\sqrt{b^2-c^2}\,(b\sinh(x)+c\cosh(x))}{3\sqrt{\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[b^2 - c^2] + b*Cosh[x] + c*Sinh[x])^(3/2),x]
```

```
[Out] (8*Sqrt[b^2 - c^2]*(c*Cosh[x] + b*Sinh[x]))/(3*Sqrt[Sqrt[b^2 - c^2] + b*Cos
h[x] + c*Sinh[x]]) + (2*(c*Cosh[x] + b*Sinh[x])*Sqrt[Sqrt[b^2 - c^2] + b*Co
sh[x] + c*Sinh[x]])/3
```

Rule 3112

```
Int[Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_
)]], x_Symbol] :> Simp[(-2*(c*Cos[d + e*x] - b*Sin[d + e*x]))/(e*Sqrt[a + b
*Cos[d + e*x] + c*Sin[d + e*x]]), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[a^
2 - b^2 - c^2, 0]
```

Rule 3113

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> -Simp[((c*Cos[d + e*x] - b*Sin[d + e*x])*(a + b*Cos[d +
e*x] + c*Sin[d + e*x])^(n - 1))/(e*n), x] + Dist[(a*(2*n - 1))/n, Int[(a +
b*Cos[d + e*x] + c*Sin[d + e*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e},
x] && EqQ[a^2 - b^2 - c^2, 0] && GtQ[n, 0]
```

Rubi steps

$$\int \left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{3/2}\,dx = \frac{2}{3}(c\cosh(x)+b\sinh(x))\sqrt{\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)}+\frac{1}{3}$$

$$= \frac{8\sqrt{b^2-c^2}\,(c\cosh(x)+b\sinh(x))}{3\sqrt{\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)}}+\frac{2}{3}(c\cosh(x)+b\sinh(x)$$

Mathematica [C] time = 73.41, size = 10141, normalized size = 110.23

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[b^2 - c^2] + b*Cosh[x] + c*Sinh[x])^(3/2),x]
```

```
[Out] Result too large to show
```

fricas [B] time = 0.44, size = 329, normalized size = 3.58

$$\frac{\sqrt{\frac{1}{2}}\left(\left(b^2+2\,bc+c^2\right)\cosh(x)^4+4\left(b^2+2\,bc+c^2\right)\cosh(x)\sinh(x)^3+\left(b^2+2\,bc+c^2\right)\sinh(x)^4-18\left(b^2-c^2\right)\mathrm{co}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cosh(x)+c*sinh(x)+(b^2-c^2)^(1/2))^(3/2),x, algorithm="fricas"
)
```

```
[Out] 1/3*sqrt(1/2)*((b^2 + 2*b*c + c^2)*cosh(x)^4 + 4*(b^2 + 2*b*c + c^2)*cosh(x
)*sinh(x)^3 + (b^2 + 2*b*c + c^2)*sinh(x)^4 - 18*(b^2 - c^2)*cosh(x)^2 + 6*
((b^2 + 2*b*c + c^2)*cosh(x)^2 - 3*b^2 + 3*c^2)*sinh(x)^2 + b^2 - 2*b*c + c
^2 + 4*((b^2 + 2*b*c + c^2)*cosh(x)^3 - 9*(b^2 - c^2)*cosh(x))*sinh(x) + 8*
((b + c)*cosh(x)^3 + 3*(b + c)*cosh(x)*sinh(x)^2 + (b + c)*sinh(x)^3 + (b -
 c)*cosh(x) + (3*(b + c)*cosh(x)^2 + b - c)*sinh(x))*sqrt(b^2 - c^2))*sqrt(
((b + c)*cosh(x)^2 + 2*(b + c)*cosh(x)*sinh(x) + (b + c)*sinh(x)^2 + 2*sqrt
(b^2 - c^2)*(cosh(x) + sinh(x)) + b - c)/(cosh(x) + sinh(x)))/((b + c)*cosh
(x)^3 + 3*(b + c)*cosh(x)*sinh(x)^2 + (b + c)*sinh(x)^3 - (b - c)*cosh(x) +
 (3*(b + c)*cosh(x)^2 - b + c)*sinh(x))
```

giac [B] time = 0.25, size = 303, normalized size = 3.29

$$-\frac{\sqrt{2}\left(\left(\sqrt{b^2-c^2}\,b\,\mathrm{sgn}\left(-\sqrt{b^2-c^2}\,e^x-b+c\right)+\sqrt{b^2-c^2}\,c\,\mathrm{sgn}\left(-\sqrt{b^2-c^2}\,e^x-b+c\right)\right)e^{\left(\frac{3}{2}\,x\right)}+9\left(b^2\mathrm{sgn}\left(-\sqrt{b^2-\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cosh(x)+c*sinh(x)+(b^2-c^2)^(1/2))^(3/2),x, algorithm="giac")
```

```
[Out] -1/6*sqrt(2)*((sqrt(b^2 - c^2)*b*sgn(-sqrt(b^2 - c^2)*e^x - b + c) + sqrt(b
^2 - c^2)*c*sgn(-sqrt(b^2 - c^2)*e^x - b + c))*e^(3/2*x) + 9*(b^2*sgn(-sqrt
(b^2 - c^2)*e^x - b + c) - c^2*sgn(-sqrt(b^2 - c^2)*e^x - b + c))*e^(1/2*x)
 - 9*(sqrt(b^2 - c^2)*b*sgn(-sqrt(b^2 - c^2)*e^x - b + c) - sqrt(b^2 - c^2)
*c*sgn(-sqrt(b^2 - c^2)*e^x - b + c))*e^(-1/2*x) - (b^2*sgn(-sqrt(b^2 - c^2
)*e^x - b + c) - 2*b*c*sgn(-sqrt(b^2 - c^2)*e^x - b + c) + c^2*sgn(-sqrt(b^
2 - c^2)*e^x - b + c))*e^(-3/2*x))/sqrt(b - c)
```

maple [B] time = 0.81, size = 190, normalized size = 2.07

$$\frac{\left(-2b^{2}+2c^{2}\right)\cosh(x)}{\sqrt{-\frac{\sinh(x)b^{2}-\sinh(x)c^{2}-b^{2}+c^{2}}{\sqrt{b^{2}-c^{2}}}}}+\frac{\sqrt{-\sqrt{b^{2}-c^{2}}\ (\sinh(x)-1)\left(\sinh^{2}(x)\right)}\ \arctan\left(\frac{\sqrt{\sqrt{b^{2}-c^{2}}\ (\sinh(x)-1)}\ \cosh(x)}{\sqrt{-\sqrt{b^{2}-c^{2}}\ (\sinh(x)-1)\left(\sinh^{2}(x)\right)}}\right)\left(b^{2}-c^{2}\right)}{\sqrt{\sqrt{b^{2}-c^{2}}\ (\sinh(x)-1)}\ \sinh(x)\sqrt{-\frac{\sinh(x)b^{2}-\sinh(x)c^{2}-b^{2}+c^{2}}{\sqrt{b^{2}-c^{2}}}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cosh(x)+c*sinh(x)+(b^2-c^2)^(1/2))^(3/2),x)
```

```
[Out] (-2*b^2+2*c^2)/(-(sinh(x)*b^2-sinh(x)*c^2-b^2+c^2)/(b^2-c^2)^(1/2))^(1/2)*c
osh(x)+(-(b^2-c^2)^(1/2)*(sinh(x)-1)*sinh(x)^2)^(1/2)*arctan(((b^2-c^2)^(1/
2)*(sinh(x)-1))^(1/2)*cosh(x)/(-(b^2-c^2)^(1/2)*(sinh(x)-1)*sinh(x)^2)^(1/2
))*(b^2-c^2)/((b^2-c^2)^(1/2)*(sinh(x)-1))^(1/2)/sinh(x)/(-(sinh(x)*b^2-sin
h(x)*c^2-b^2+c^2)/(b^2-c^2)^(1/2))^(1/2)
```

maxima [B] time = 0.94, size = 640, normalized size = 6.96

$$\frac{\sqrt{2}\left(\sqrt{b+c}\sqrt{b-c}\,b+\sqrt{b+c}\sqrt{b-c}\,c\right)\left(2\sqrt{b+c}\sqrt{b-c}\,e^{(-x)}+(b-c)e^{(-2\,x)}+b+c\right)^{\frac{3}{2}}e^{\left(\frac{3}{2}\,x\right)}}{6\left(\sqrt{b+c}\sqrt{b-c}\,b+\sqrt{b+c}\sqrt{b-c}\,c+3\left(b^{2}-c^{2}\right)e^{(-x)}+3\left(\sqrt{b+c}\sqrt{b-c}\,b-\sqrt{b+c}\sqrt{b-c}\,c\right)e^{(-2\,x)}+\left(b^{2}-2\,bc\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cosh(x)+c*sinh(x)+(b^2-c^2)^(1/2))^(3/2),x, algorithm="maxima"
)
```

```
[Out] 1/6*sqrt(2)*(sqrt(b + c)*sqrt(b - c)*b + sqrt(b + c)*sqrt(b - c)*c)*(2*sqrt
(b + c)*sqrt(b - c)*e^(-x) + (b - c)*e^(-2*x) + b + c)^(3/2)*e^(3/2*x)/(sqr
t(b + c)*sqrt(b - c)*b + sqrt(b + c)*sqrt(b - c)*c + 3*(b^2 - c^2)*e^(-x) +
 3*(sqrt(b + c)*sqrt(b - c)*b - sqrt(b + c)*sqrt(b - c)*c)*e^(-2*x) + (b^2
- 2*b*c + c^2)*e^(-3*x)) + 3/2*sqrt(2)*(b^2 - c^2)*(2*sqrt(b + c)*sqrt(b -
c)*e^(-x) + (b - c)*e^(-2*x) + b + c)^(3/2)*e^(1/2*x)/(sqrt(b + c)*sqrt(b -
```

```
 c)*b + sqrt(b + c)*sqrt(b - c)*c + 3*(b^2 - c^2)*e^(-x) + 3*(sqrt(b + c)*s
qrt(b - c)*b - sqrt(b + c)*sqrt(b - c)*c)*e^(-2*x) + (b^2 - 2*b*c + c^2)*e^
(-3*x)) - 3/2*sqrt(2)*(sqrt(b + c)*sqrt(b - c)*b - sqrt(b + c)*sqrt(b - c)*
c)*(2*sqrt(b + c)*sqrt(b - c)*e^(-x) + (b - c)*e^(-2*x) + b + c)^(3/2)*e^(-
1/2*x)/(sqrt(b + c)*sqrt(b - c)*b + sqrt(b + c)*sqrt(b - c)*c + 3*(b^2 - c^
2)*e^(-x) + 3*(sqrt(b + c)*sqrt(b - c)*b - sqrt(b + c)*sqrt(b - c)*c)*e^(-2
*x) + (b^2 - 2*b*c + c^2)*e^(-3*x)) - 1/6*sqrt(2)*(b^2 - 2*b*c + c^2)*(2*sq
rt(b + c)*sqrt(b - c)*e^(-x) + (b - c)*e^(-2*x) + b + c)^(3/2)*e^(-3/2*x)/(
sqrt(b + c)*sqrt(b - c)*b + sqrt(b + c)*sqrt(b - c)*c + 3*(b^2 - c^2)*e^(-x
) + 3*(sqrt(b + c)*sqrt(b - c)*b - sqrt(b + c)*sqrt(b - c)*c)*e^(-2*x) + (b
^2 - 2*b*c + c^2)*e^(-3*x))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(b\cosh(x)+\sqrt{b^2-c^2}+c\sinh(x)\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cosh(x) + (b^2 - c^2)^(1/2) + c*sinh(x))^(3/2),x)
[Out] int((b*cosh(x) + (b^2 - c^2)^(1/2) + c*sinh(x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\cosh\left(x\right)+c\sinh\left(x\right)+\sqrt{b^2-c^2}\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cosh(x)+c*sinh(x)+(b**2-c**2)**(1/2))**(3/2),x)
[Out] Integral((b*cosh(x) + c*sinh(x) + sqrt(b**2 - c**2))**(3/2), x)
```

3.770 $\int \sqrt{\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)}\,dx$

Optimal. Leaf size=37

$$\frac{2(b\sinh(x)+c\cosh(x))}{\sqrt{\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)}}$$

```
[Out] 2*(c*cosh(x)+b*sinh(x))/(b*cosh(x)+c*sinh(x)+(b^2-c^2)^(1/2))^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.038, Rules used = {3112}

$$\frac{2(b\sinh(x)+c\cosh(x))}{\sqrt{\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Sqrt[b^2 - c^2] + b*Cosh[x] + c*Sinh[x]],x]
```

```
[Out] (2*(c*Cosh[x] + b*Sinh[x]))/Sqrt[Sqrt[b^2 - c^2] + b*Cosh[x] + c*Sinh[x]]
```

Rule 3112

```
Int[Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_
)]], x_Symbol] :> Simp[(-2*(c*Cos[d + e*x] - b*Sin[d + e*x]))/(e*Sqrt[a + b
*Cos[d + e*x] + c*Sin[d + e*x]]), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[a^
2 - b^2 - c^2, 0]
```

Rubi steps

$$\int \sqrt{\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)}\,dx=\frac{2(c\cosh(x)+b\sinh(x))}{\sqrt{\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)}}$$

Mathematica [C] time = 74.46, size = 10054, normalized size = 271.73

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[Sqrt[b^2 - c^2] + b*Cosh[x] + c*Sinh[x]],x]
```

[Out] Result too large to show

fricas [B] time = 0.45, size = 143, normalized size = 3.86

$$\frac{2\sqrt{\frac{1}{2}}\left((b+c)\cosh(x)^2+2\,(b+c)\cosh(x)\sinh(x)+(b+c)\sinh(x)^2-2\,\sqrt{b^2-c^2}\,(\cosh(x)+\sinh(x))+b-c\right)\cdots}{(b+c)\cosh(x)^2+2\,(b+c)\cosh(x)\sinh(x)+(b+c)\sin\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cosh(x)+c*sinh(x)+(b^2-c^2)^(1/2))^(1/2),x, algorithm="fricas"
)
```

```
[Out] 2*sqrt(1/2)*((b + c)*cosh(x)^2 + 2*(b + c)*cosh(x)*sinh(x) + (b + c)*sinh(x
)^2 - 2*sqrt(b^2 - c^2)*(cosh(x) + sinh(x)) + b - c)*sqrt(((b + c)*cosh(x)^
2 + 2*(b + c)*cosh(x)*sinh(x) + (b + c)*sinh(x)^2 + 2*sqrt(b^2 - c^2)*(cosh
(x) + sinh(x)) + b - c)/(cosh(x) + sinh(x)))/((b + c)*cosh(x)^2 + 2*(b + c)
*cosh(x)*sinh(x) + (b + c)*sinh(x)^2 - b + c)
```

giac [B] time = 0.14, size = 104, normalized size = 2.81

$$-\frac{\sqrt{2}\left(\sqrt{b^2-c^2}\,e^{\left(\frac{1}{2}\,x\right)}\mathrm{sgn}\left(-\sqrt{b^2-c^2}\,e^x-b+c\right)-\left(b\mathrm{sgn}\left(-\sqrt{b^2-c^2}\,e^x-b+c\right)-c\mathrm{sgn}\left(-\sqrt{b^2-c^2}\,e^x-b+c\right)\right)e^{\cdots}\right.}{\sqrt{b-c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cosh(x)+c*sinh(x)+(b^2-c^2)^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] -sqrt(2)*(sqrt(b^2 - c^2)*e^(1/2*x)*sgn(-sqrt(b^2 - c^2)*e^x - b + c) - (b*
sgn(-sqrt(b^2 - c^2)*e^x - b + c) - c*sgn(-sqrt(b^2 - c^2)*e^x - b + c))*e^
(-1/2*x))/sqrt(b - c)
```

maple [B] time = 0.81, size = 201, normalized size = 5.43

$$\frac{\left(-b^2+c^2\right)\cosh(x)}{\sqrt{b^2-c^2}\,\sqrt{-\frac{\sinh(x)b^2-\sinh(x)c^2-b^2+c^2}{\sqrt{b^2-c^2}}}}+\frac{\sqrt{-\sqrt{b^2-c^2}\,\left(\sinh(x)-1\right)\left(\sinh^2(x)\right)}\,\sqrt{b^2-c^2}\,\arctan\left(\frac{\sqrt{\sqrt{b^2-c^2}\,(\sinh(x)-\cdots}}{\sqrt{-\sqrt{b^2-c^2}\,(\sinh(x)-\cdots}}\right.}{\sqrt{\sqrt{b^2-c^2}\,\left(\sinh(x)-1\right)}\,\sinh(x)\sqrt{-\frac{\sinh(x)b^2-\sinh(x)c^2-b^2+c^2}{\sqrt{b^2-c^2}}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cosh(x)+c*sinh(x)+(b^2-c^2)^(1/2))^(1/2),x)
```

```
[Out] (-b^2+c^2)/(b^2-c^2)^(1/2)/(-(sinh(x)*b^2-sinh(x)*c^2-b^2+c^2)/(b^2-c^2)^(1
/2))^(1/2)*cosh(x)+(-(b^2-c^2)^(1/2)*(sinh(x)-1)*sinh(x)^2)^(1/2)*(b^2-c^2)
```

```
^(1/2)/((b^2-c^2)^(1/2)*(sinh(x)-1))^(1/2)*arctan(((b^2-c^2)^(1/2)*(sinh(x)
-1))^(1/2)*cosh(x)/(-(b^2-c^2)^(1/2)*(sinh(x)-1)*sinh(x)^2)^(1/2))/sinh(x)/
(-(sinh(x)*b^2-sinh(x)*c^2-b^2+c^2)/(b^2-c^2)^(1/2))^(1/2)
```

maxima [B] time = 0.83, size = 153, normalized size = 4.14

$$\frac{\sqrt{2}\sqrt{2\sqrt{b+c}\sqrt{b-c}\,e^{(-x)}+(b-c)e^{(-2\,x)}+b+c}\,\sqrt{b+c}\,\sqrt{b-c}\,e^{\left(\frac{1}{2}\,x\right)}}{(b-c)e^{(-x)}+\sqrt{b+c}\,\sqrt{b-c}}-\frac{\sqrt{2}\sqrt{2\sqrt{b+c}\sqrt{b-c}\,e^{(-x)}+(b-c)e^{(-2\,x)}+}{(b-c)e^{(-x)}+\sqrt{b+c}\,\sqrt{b-}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cosh(x)+c*sinh(x)+(b^2-c^2)^(1/2))^(1/2),x, algorithm="maxima"
)
```

```
[Out] sqrt(2)*sqrt(2*sqrt(b + c)*sqrt(b - c)*e^(-x) + (b - c)*e^(-2*x) + b + c)*s
qrt(b + c)*sqrt(b - c)*e^(1/2*x)/((b - c)*e^(-x) + sqrt(b + c)*sqrt(b - c))
 - sqrt(2)*sqrt(2*sqrt(b + c)*sqrt(b - c)*e^(-x) + (b - c)*e^(-2*x) + b + c
)*(b - c)*e^(-1/2*x)/((b - c)*e^(-x) + sqrt(b + c)*sqrt(b - c))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \sqrt{b\cosh(x)+\sqrt{b^2-c^2}+c\sinh(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cosh(x) + (b^2 - c^2)^(1/2) + c*sinh(x))^(1/2),x)
```

```
[Out] int((b*cosh(x) + (b^2 - c^2)^(1/2) + c*sinh(x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b\cosh{(x)}+c\sinh{(x)}+\sqrt{b^2-c^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cosh(x)+c*sinh(x)+(b**2-c**2)**(1/2))**(1/2),x)
```

```
[Out] Integral(sqrt(b*cosh(x) + c*sinh(x) + sqrt(b**2 - c**2)), x)
```

3.771 $\int \frac{1}{\sqrt{\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)}}\,dx$

Optimal. Leaf size=99

$$\frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt[4]{b^2-c^2}\sinh\left(x+i\tan^{-1}(b,-ic)\right)}{\sqrt{2}\sqrt{\sqrt{b^2-c^2}+\sqrt{b^2-c^2}\cosh\left(x+i\tan^{-1}(b,-ic)\right)}}\right)}{\sqrt[4]{b^2-c^2}}$$

```
[Out] arctan(1/2*(b^2-c^2)^(1/4)*sinh(x+I*arctan(b,-I*c))*2^(1/2)/((b^2-c^2)^(1/2
)+cosh(x+I*arctan(b,-I*c))*(b^2-c^2)^(1/2))^(1/2))*2^(1/2)/(b^2-c^2)^(1/4)
```

Rubi [A] time = 0.11, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {3115, 2649, 206}

$$\frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt[4]{b^2-c^2}\sinh\left(x+i\tan^{-1}(b,-ic)\right)}{\sqrt{2}\sqrt{\sqrt{b^2-c^2}+\sqrt{b^2-c^2}\cosh\left(x+i\tan^{-1}(b,-ic)\right)}}\right)}{\sqrt[4]{b^2-c^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[Sqrt[b^2 - c^2] + b*Cosh[x] + c*Sinh[x]],x]
```

```
[Out] (Sqrt[2]*ArcTan[((b^2 - c^2)^(1/4)*Sinh[x + I*ArcTan[b, (-I)*c]])/(Sqrt[2]*
Sqrt[Sqrt[b^2 - c^2] + Sqrt[b^2 - c^2]*Cosh[x + I*ArcTan[b, (-I)*c]]])])/(b
^2 - c^2)^(1/4)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3115

```
Int[1/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(
x_)]], x_Symbol] :> Int[1/Sqrt[a + Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b,
```

```
c]]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[a^2 - b^2 - c^2, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)}}\,dx = \int \frac{1}{\sqrt{\sqrt{b^2-c^2}+\sqrt{b^2-c^2}\cosh\left(x+i\tan^{-1}(b,-ic)\right)}}\,dx$$

$$= 2i\,\mathrm{Subst}\left(\int \frac{1}{2\sqrt{b^2-c^2}-x^2}\,dx, x, -\frac{i\sqrt{b^2-c^2}\sinh\left(x+i\tan\right.}{\sqrt{\sqrt{b^2-c^2}+\sqrt{b^2-c^2}\cosh\left(x\right.}}\right.$$

$$= \frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt[4]{b^2-c^2}\sinh\left(x+i\tan^{-1}(b,-ic)\right)}{\sqrt{2}\sqrt{\sqrt{b^2-c^2}+\sqrt{b^2-c^2}\cosh\left(x+i\tan^{-1}(b,-ic)\right)}}\right)}{\sqrt[4]{b^2-c^2}}$$

Mathematica [C] time = 32.69, size = 211, normalized size = 2.13

$$-\frac{\sqrt{2}\left(c\sqrt{b^2-c^2}\sinh(x)+b\sqrt{b^2-c^2}\cosh(x)+b^2-c^2\right)\sqrt{-\frac{c\sqrt{b^2-c^2}\sinh(x)+b\sqrt{b^2-c^2}\cosh(x)-b^2+c^2}{b^2-c^2}}\,F\left(\sin^{-1}\left(\frac{\sqrt{\frac{-b\cosh(x}{}}}{}\right.\right.}{\sqrt{b^2-c^2}\,(b\sinh(x)+c\cosh(x))\sqrt{\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[Sqrt[b^2 - c^2] + b*Cosh[x] + c*Sinh[x]],x]
```

```
[Out] -((Sqrt[2]*EllipticF[ArcSin[Sqrt[(Sqrt[b^2 - c^2] - b*Cosh[x] - c*Sinh[x])/
Sqrt[b^2 - c^2]]/Sqrt[2]], 1]*(b^2 - c^2 + b*Sqrt[b^2 - c^2]*Cosh[x] + c*Sq
rt[b^2 - c^2]*Sinh[x])*Sqrt[-((-b^2 + c^2 + b*Sqrt[b^2 - c^2]*Cosh[x] + c*S
qrt[b^2 - c^2]*Sinh[x])/(b^2 - c^2))])/(Sqrt[b^2 - c^2]*(c*Cosh[x] + b*Sinh
[x])*Sqrt[Sqrt[b^2 - c^2] + b*Cosh[x] + c*Sinh[x]]))
```

fricas [B] time = 0.50, size = 681, normalized size = 6.88

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*cosh(x)+c*sinh(x)+(b^2-c^2)^(1/2))^(1/2),x, algorithm="frica
s")
```

```
[Out] [sqrt(2)*sqrt(-1/sqrt(b^2 - c^2))*log(-((b^2 + 2*b*c + c^2)*cosh(x)^4 + 4*(
b^2 + 2*b*c + c^2)*cosh(x)^3*sinh(x) + 6*(b^2 + 2*b*c + c^2)*cosh(x)^2*sinh
(x)^2 + 4*(b^2 + 2*b*c + c^2)*cosh(x)*sinh(x)^3 + (b^2 + 2*b*c + c^2)*sinh(
x)^4 - 2*sqrt(2)*sqrt(1/2)*(2*(b^2 - c^2)*cosh(x)^2 + 4*(b^2 - c^2)*cosh(x)
*sinh(x) + 2*(b^2 - c^2)*sinh(x)^2 - ((b + c)*cosh(x)^3 + 3*(b + c)*cosh(x)
*sinh(x)^2 + (b + c)*sinh(x)^3 + (b - c)*cosh(x) + (3*(b + c)*cosh(x)^2 + b
 - c)*sinh(x))*sqrt(b^2 - c^2))*sqrt(((b + c)*cosh(x)^2 + 2*(b + c)*cosh(x)
*sinh(x) + (b + c)*sinh(x)^2 + 2*sqrt(b^2 - c^2)*(cosh(x) + sinh(x)) + b -
c)/(cosh(x) + sinh(x)))*sqrt(-1/sqrt(b^2 - c^2)) - b^2 + 2*b*c - c^2 - 2*((
b + c)*cosh(x)^3 + 3*(b + c)*cosh(x)*sinh(x)^2 + (b + c)*sinh(x)^3 - (b - c
)*cosh(x) + (3*(b + c)*cosh(x)^2 - b + c)*sinh(x))*sqrt(b^2 - c^2))/((b^2 +
 2*b*c + c^2)*cosh(x)^4 + 4*(b^2 + 2*b*c + c^2)*cosh(x)*sinh(x)^3 + (b^2 +
2*b*c + c^2)*sinh(x)^4 - 2*(b^2 - c^2)*cosh(x)^2 + 2*(3*(b^2 + 2*b*c + c^2)
*cosh(x)^2 - b^2 + c^2)*sinh(x)^2 + b^2 - 2*b*c + c^2 + 4*((b^2 + 2*b*c + c
^2)*cosh(x)^3 - (b^2 - c^2)*cosh(x))*sinh(x))), -2*sqrt(2)*arctan(sqrt(2)*s
qrt(1/2)*(sqrt(b^2 - c^2)*(cosh(x) + sinh(x)) - b + c)*sqrt(((b + c)*cosh(x
)^2 + 2*(b + c)*cosh(x)*sinh(x) + (b + c)*sinh(x)^2 + 2*sqrt(b^2 - c^2)*(co
sh(x) + sinh(x)) + b - c)/(cosh(x) + sinh(x)))/(((b + c)*cosh(x)^2 + 2*(b +
 c)*cosh(x)*sinh(x) + (b + c)*sinh(x)^2 - b + c)*(b^2 - c^2)^(1/4)))/(b^2 -
 c^2)^(1/4)]
```

giac [B] time = 1.48, size = 297, normalized size = 3.00

$$\frac{2\sqrt{2}\left(b^2-c^2-b+c\right)\sqrt{b-c}\,\arctan\left(\frac{b^3e^{\left(-\frac{1}{2}x\right)}-b^2ce^{\left(-\frac{1}{2}x\right)}-bc^2e^{\left(-\frac{1}{2}x\right)}+c^3e^{\left(-\frac{1}{2}x\right)}-b^2e^{\left(-\frac{1}{2}x\right)}+2\,bce^{\left(-\frac{1}{2}x\right)}-c^2e^{\left(-\frac{1}{2}x\right)}}{\sqrt{\left(b^5-b^4c-2\,b^3c^2+2\,b^2c^3+bc^4-c^5-2\,b^4+4\,b^3c-4\,bc^3+2\,c^4+b^3-3\,b^2c+3\,bc^2-c^3\right)\sqrt{b^2}}}\right.}{\sqrt{\left(b^5-b^4c-2\,b^3c^2+2\,b^2c^3+bc^4-c^5-2\,b^4+4\,b^3c-4\,bc^3+2\,c^4+b^3-3\,b^2c+3\,bc^2-c^3\right)\sqrt{b^2-c^2}}\,\mathrm{sgn}\left(-\sqrt{\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*cosh(x)+c*sinh(x)+(b^2-c^2)^(1/2))^(1/2),x, algorithm="giac"
)
```

```
[Out] 2*sqrt(2)*(b^2 - c^2 - b + c)*sqrt(b - c)*arctan((b^3*e^(-1/2*x) - b^2*c*e^
(-1/2*x) - b*c^2*e^(-1/2*x) + c^3*e^(-1/2*x) - b^2*e^(-1/2*x) + 2*b*c*e^(-1
/2*x) - c^2*e^(-1/2*x))/sqrt((b^5 - b^4*c - 2*b^3*c^2 + 2*b^2*c^3 + b*c^4 -
 c^5 - 2*b^4 + 4*b^3*c - 4*b*c^3 + 2*c^4 + b^3 - 3*b^2*c + 3*b*c^2 - c^3)*s
qrt(b^2 - c^2)))/(sqrt((b^5 - b^4*c - 2*b^3*c^2 + 2*b^2*c^3 + b*c^4 - c^5 -
 2*b^4 + 4*b^3*c - 4*b*c^3 + 2*c^4 + b^3 - 3*b^2*c + 3*b*c^2 - c^3)*sqrt(b^
2 - c^2))*sgn(-sqrt(b^2 - c^2)*e^x - b + c))
```

maple [A] time = 0.63, size = 129, normalized size = 1.30

$$\frac{\sqrt{-\sqrt{b^2-c^2}\,\left(\sinh(x)-1\right)\left(\sinh^2(x)\right)}\,\arctan\left(\frac{\sqrt{\sqrt{b^2-c^2}\,\left(\sinh(x)-1\right)}\,\cosh(x)}{\sqrt{-\sqrt{b^2-c^2}\,\left(\sinh(x)-1\right)\left(\sinh^2(x)\right)}}\right)}{\sqrt{\sqrt{b^2-c^2}\,\left(\sinh(x)-1\right)}\,\sinh(x)\sqrt{-\frac{\sinh(x)b^2-\sinh(x)c^2-b^2+c^2}{\sqrt{b^2-c^2}}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*cosh(x)+c*sinh(x)+(b^2-c^2)^(1/2))^(1/2),x)
```

```
[Out] (-(b^2-c^2)^(1/2)*(sinh(x)-1)*sinh(x)^2)^(1/2)/((b^2-c^2)^(1/2)*(sinh(x)-1)
)^(1/2)*arctan(((b^2-c^2)^(1/2)*(sinh(x)-1))^(1/2)*cosh(x)/(-(b^2-c^2)^(1/2
)*(sinh(x)-1)*sinh(x)^2)^(1/2))/sinh(x)/(-(sinh(x)*b^2-sinh(x)*c^2-b^2+c^2)
/(b^2-c^2)^(1/2))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{b\cosh(x)+c\sinh(x)+\sqrt{b^2-c^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*cosh(x)+c*sinh(x)+(b^2-c^2)^(1/2))^(1/2),x, algorithm="maxim
a")
```

```
[Out] integrate(1/sqrt(b*cosh(x) + c*sinh(x) + sqrt(b^2 - c^2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{b\cosh(x)+\sqrt{b^2-c^2}+c\sinh(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*cosh(x) + (b^2 - c^2)^(1/2) + c*sinh(x))^(1/2),x)
```

```
[Out] int(1/(b*cosh(x) + (b^2 - c^2)^(1/2) + c*sinh(x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{b\cosh{(x)}+c\sinh{(x)}+\sqrt{b^2-c^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*cosh(x)+c*sinh(x)+(b**2-c**2)**(1/2))**(1/2),x)
```

```
[Out] Integral(1/sqrt(b*cosh(x) + c*sinh(x) + sqrt(b**2 - c**2)), x)
```

3.772 $\int \frac{1}{\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{3/2}}\,dx$

Optimal. Leaf size=155

$$\frac{b\sinh(x)+c\cosh(x)}{2\sqrt{b^2-c^2}\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{3/2}}+\frac{\tan^{-1}\left(\frac{\sqrt[4]{b^2-c^2}\sinh\left(x+i\tan^{-1}(b,-ic)\right)}{\sqrt{2}\sqrt{\sqrt{b^2-c^2}+\sqrt{b^2-c^2}\cosh\left(x+i\tan^{-1}(b,-ic)\right)}}\right)}{2\sqrt{2}\left(b^2-c^2\right)^{3/4}}$$

```
[Out] 1/4*arctan(1/2*(b^2-c^2)^(1/4)*sinh(x+I*arctan(b,-I*c))*2^(1/2)/((b^2-c^2)^
(1/2)+cosh(x+I*arctan(b,-I*c))*(b^2-c^2)^(1/2))^(1/2))/(b^2-c^2)^(3/4)*2^(1
/2)+1/2*(c*cosh(x)+b*sinh(x))/(b^2-c^2)^(1/2)/(b*cosh(x)+c*sinh(x)+(b^2-c^2
)^(1/2))^(3/2)
```

Rubi [A] time = 0.13, antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3116, 3115, 2649, 206}

$$\frac{b\sinh(x)+c\cosh(x)}{2\sqrt{b^2-c^2}\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{3/2}}+\frac{\tan^{-1}\left(\frac{\sqrt[4]{b^2-c^2}\sinh\left(x+i\tan^{-1}(b,-ic)\right)}{\sqrt{2}\sqrt{\sqrt{b^2-c^2}+\sqrt{b^2-c^2}\cosh\left(x+i\tan^{-1}(b,-ic)\right)}}\right)}{2\sqrt{2}\left(b^2-c^2\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[b^2 - c^2] + b*Cosh[x] + c*Sinh[x])^(-3/2),x]
```

```
[Out] ArcTan[((b^2 - c^2)^(1/4)*Sinh[x + I*ArcTan[b, (-I)*c]])/(Sqrt[2]*Sqrt[Sqrt
[b^2 - c^2] + Sqrt[b^2 - c^2]*Cosh[x + I*ArcTan[b, (-I)*c]]])]/(2*Sqrt[2]*(
b^2 - c^2)^(3/4)) + (c*Cosh[x] + b*Sinh[x])/(2*Sqrt[b^2 - c^2]*(Sqrt[b^2 -
c^2] + b*Cosh[x] + c*Sinh[x])^(3/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
```

```
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3115

```
Int[1/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(
x_)]], x_Symbol] :> Int[1/Sqrt[a + Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b,
c]]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[a^2 - b^2 - c^2, 0]
```

Rule 3116

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> Simp[((c*Cos[d + e*x] - b*Sin[d + e*x])*(a + b*Cos[d + e
*x] + c*Sin[d + e*x])^n)/(a*e*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n + 1)),
Int[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1), x], x] /; FreeQ[{a, b, c
, d, e}, x] && EqQ[a^2 - b^2 - c^2, 0] && LtQ[n, -1]
```

Rubi steps

$$\int \frac{1}{\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{3/2}}\,dx = \frac{c\cosh(x)+b\sinh(x)}{2\sqrt{b^2-c^2}\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{3/2}}+\frac{\int \frac{}{\sqrt{\sqrt{b^2-c^2}+b}}}{4\sqrt{}}$$

$$=\frac{c\cosh(x)+b\sinh(x)}{2\sqrt{b^2-c^2}\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{3/2}}+\frac{\int \frac{}{\sqrt{\sqrt{b^2-c^2}+}}}{}$$

$$=\frac{c\cosh(x)+b\sinh(x)}{2\sqrt{b^2-c^2}\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{3/2}}+\frac{i\,\text{Subst}\left(\int \frac{}{2}\right.}{}$$

$$=\frac{\tan^{-1}\left(\frac{\sqrt[4]{b^2-c^2}\,\sinh\left(x+i\tan^{-1}(b,-ic)\right)}{\sqrt{2}\sqrt{\sqrt{b^2-c^2}+\sqrt{b^2-c^2}\,\cosh\left(x+i\tan^{-1}(b,-ic)\right)}}\right)}{2\sqrt{2}\left(b^2-c^2\right)^{3/4}}+\frac{c\,c}{2\sqrt{b^2-c^2}\left(\sqrt{b^2\right.}}$$

Mathematica [F] time = 180.00, size = 0, normalized size = 0.00

$Aborted

Verification is Not applicable to the result.

```
[In] Integrate[(Sqrt[b^2 - c^2] + b*Cosh[x] + c*Sinh[x])^(-3/2),x]
```

```
[Out] $Aborted
```

fricas [B] time = 0.51, size = 1801, normalized size = 11.62

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*cosh(x)+c*sinh(x)+(b^2-c^2)^(1/2))^(3/2),x, algorithm="frica
s")
```

```
[Out] 1/2*((sqrt(2)*(b^3 + 3*b^2*c + 3*b*c^2 + c^3)*cosh(x)^6 + 6*sqrt(2)*(b^3 +
3*b^2*c + 3*b*c^2 + c^3)*cosh(x)*sinh(x)^5 + sqrt(2)*(b^3 + 3*b^2*c + 3*b*c
^2 + c^3)*sinh(x)^6 - 3*sqrt(2)*(b^3 + b^2*c - b*c^2 - c^3)*cosh(x)^4 + 3*(
5*sqrt(2)*(b^3 + 3*b^2*c + 3*b*c^2 + c^3)*cosh(x)^2 - sqrt(2)*(b^3 + b^2*c
- b*c^2 - c^3))*sinh(x)^4 + 4*(5*sqrt(2)*(b^3 + 3*b^2*c + 3*b*c^2 + c^3)*co
sh(x)^3 - 3*sqrt(2)*(b^3 + b^2*c - b*c^2 - c^3)*cosh(x))*sinh(x)^3 + 3*sqrt
(2)*(b^3 - b^2*c - b*c^2 + c^3)*cosh(x)^2 + 3*(5*sqrt(2)*(b^3 + 3*b^2*c + 3
*b*c^2 + c^3)*cosh(x)^4 - 6*sqrt(2)*(b^3 + b^2*c - b*c^2 - c^3)*cosh(x)^2 +
 sqrt(2)*(b^3 - b^2*c - b*c^2 + c^3))*sinh(x)^2 + 6*(sqrt(2)*(b^3 + 3*b^2*c
 + 3*b*c^2 + c^3)*cosh(x)^5 - 2*sqrt(2)*(b^3 + b^2*c - b*c^2 - c^3)*cosh(x)
^3 + sqrt(2)*(b^3 - b^2*c - b*c^2 + c^3)*cosh(x))*sinh(x) - sqrt(2)*(b^3 -
3*b^2*c + 3*b*c^2 - c^3))*(b^2 - c^2)^(1/4)*arctan(-sqrt(1/2)*(sqrt(2)*(b +
 c)*cosh(x) + sqrt(2)*(b + c)*sinh(x) - sqrt(2)*sqrt(b^2 - c^2))*(b^2 - c^2
)^(1/4)*sqrt(((b + c)*cosh(x)^2 + 2*(b + c)*cosh(x)*sinh(x) + (b + c)*sinh(
x)^2 + 2*sqrt(b^2 - c^2)*(cosh(x) + sinh(x)) + b - c)/(cosh(x) + sinh(x)))/
((b^2 + 2*b*c + c^2)*cosh(x)^2 + 2*(b^2 + 2*b*c + c^2)*cosh(x)*sinh(x) + (b
^2 + 2*b*c + c^2)*sinh(x)^2 - b^2 + c^2)) - 2*sqrt(1/2)*(4*(b^3 + b^2*c - b
*c^2 - c^3)*cosh(x)^4 + 16*(b^3 + b^2*c - b*c^2 - c^3)*cosh(x)*sinh(x)^3 +
4*(b^3 + b^2*c - b*c^2 - c^3)*sinh(x)^4 + 4*(b^3 - b^2*c - b*c^2 + c^3)*cos
h(x)^2 + 4*(b^3 - b^2*c - b*c^2 + c^3 + 6*(b^3 + b^2*c - b*c^2 - c^3)*cosh(
x)^2)*sinh(x)^2 + 8*(2*(b^3 + b^2*c - b*c^2 - c^3)*cosh(x)^3 + (b^3 - b^2*c
 - b*c^2 + c^3)*cosh(x))*sinh(x) - ((b^2 + 2*b*c + c^2)*cosh(x)^5 + 5*(b^2
+ 2*b*c + c^2)*cosh(x)*sinh(x)^4 + (b^2 + 2*b*c + c^2)*sinh(x)^5 + 6*(b^2 -
 c^2)*cosh(x)^3 + 2*(5*(b^2 + 2*b*c + c^2)*cosh(x)^2 + 3*b^2 - 3*c^2)*sinh(
x)^3 + 2*(5*(b^2 + 2*b*c + c^2)*cosh(x)^3 + 9*(b^2 - c^2)*cosh(x))*sinh(x)^
2 + (b^2 - 2*b*c + c^2)*cosh(x) + (5*(b^2 + 2*b*c + c^2)*cosh(x)^4 + 18*(b^
2 - c^2)*cosh(x)^2 + b^2 - 2*b*c + c^2)*sinh(x))*sqrt(b^2 - c^2))*sqrt(((b
+ c)*cosh(x)^2 + 2*(b + c)*cosh(x)*sinh(x) + (b + c)*sinh(x)^2 + 2*sqrt(b^2
 - c^2)*(cosh(x) + sinh(x)) + b - c)/(cosh(x) + sinh(x))))/((b^5 + 3*b^4*c
+ 2*b^3*c^2 - 2*b^2*c^3 - 3*b*c^4 - c^5)*cosh(x)^6 + 6*(b^5 + 3*b^4*c + 2*b
^3*c^2 - 2*b^2*c^3 - 3*b*c^4 - c^5)*cosh(x)*sinh(x)^5 + (b^5 + 3*b^4*c + 2*
b^3*c^2 - 2*b^2*c^3 - 3*b*c^4 - c^5)*sinh(x)^6 - b^5 + 3*b^4*c - 2*b^3*c^2
- 2*b^2*c^3 + 3*b*c^4 - c^5 - 3*(b^5 + b^4*c - 2*b^3*c^2 - 2*b^2*c^3 + b*c^
```

```
4 + c^5)*cosh(x)^4 - 3*(b^5 + b^4*c - 2*b^3*c^2 - 2*b^2*c^3 + b*c^4 + c^5 -
 5*(b^5 + 3*b^4*c + 2*b^3*c^2 - 2*b^2*c^3 - 3*b*c^4 - c^5)*cosh(x)^2)*sinh(
x)^4 + 4*(5*(b^5 + 3*b^4*c + 2*b^3*c^2 - 2*b^2*c^3 - 3*b*c^4 - c^5)*cosh(x)
^3 - 3*(b^5 + b^4*c - 2*b^3*c^2 - 2*b^2*c^3 + b*c^4 + c^5)*cosh(x))*sinh(x)
^3 + 3*(b^5 - b^4*c - 2*b^3*c^2 + 2*b^2*c^3 + b*c^4 - c^5)*cosh(x)^2 + 3*(b
^5 - b^4*c - 2*b^3*c^2 + 2*b^2*c^3 + b*c^4 - c^5 + 5*(b^5 + 3*b^4*c + 2*b^3
*c^2 - 2*b^2*c^3 - 3*b*c^4 - c^5)*cosh(x)^4 - 6*(b^5 + b^4*c - 2*b^3*c^2 -
2*b^2*c^3 + b*c^4 + c^5)*cosh(x)^2)*sinh(x)^2 + 6*((b^5 + 3*b^4*c + 2*b^3*c
^2 - 2*b^2*c^3 - 3*b*c^4 - c^5)*cosh(x)^5 - 2*(b^5 + b^4*c - 2*b^3*c^2 - 2*
b^2*c^3 + b*c^4 + c^5)*cosh(x)^3 + (b^5 - b^4*c - 2*b^3*c^2 + 2*b^2*c^3 + b
*c^4 - c^5)*cosh(x))*sinh(x))
```

giac [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*cosh(x)+c*sinh(x)+(b^2-c^2)^(1/2))^(3/2),x, algorithm="giac"
)
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(t_n
ostep),abs((-sqrt(b^2-c^2))*t_nostep-b+c)]Evaluation time: 0.58Unable to di
vide, perhaps due to rounding error%%%{%%%{8,[4,0]%%%}+%%%{16,[3,1]%%%}+%%%
{-8,[3,0]%%%}+%%%{-8,[2,1]%%%}+%%%{-16,[1,3]%%%}+%%%{8,[1,2]%%%}+%%%{-8,[0,
4]%%%}+%%%{8,[0,3]%%%},[6,1]%%%}+%%%{%%{[%%%{-12,[1,0]%%%}+%%%{-12,[0,1]%%%
},0,%%%{36,[3,0]%%%}+%%%{36,[2,1]%%%}+%%%{12,[2,0]%%%}+%%%{-36,[1,2]%%%}+%%
%{-36,[0,3]%%%}+%%%{-12,[0,2]%%%},0]:[1,0,%%%{-2,[2,0]%%%}+%%%{-2,[1,0]%%%}
+%%%{2,[0,2]%%%}+%%%{2,[0,1]%%%},0,%%%{1,[4,0]%%%}+%%%{-2,[3,0]%%%}+%%%{-2,
[2,2]%%%}+%%%{2,[2,1]%%%}+%%%{1,[2,0]%%%}+%%%{2,[1,2]%%%}+%%%{-2,[1,1]%%%}+
%%%{1,[0,4]%%%}+%%%{-2,[0,3]%%%}+%%%{1,[0,2]%%%}]%%},[4,1]%%%}+%%%{%%%{24,[
4,0]%%%}+%%%{-24,[3,0]%%%}+%%%{-48,[2,2]%%%}+%%%{24,[2,1]%%%}+%%%{24,[1,2]%
%%}+%%%{24,[0,4]%%%}+%%%{-24,[0,3]%%%},[2,1]%%%}+%%%{%%{[%%%{-4,[1,0]%%%}+%
%%{4,[0,1]%%%},0,%%%{12,[3,0]%%%}+%%%{-12,[2,1]%%%}+%%%{4,[2,0]%%%}+%%%{-12
,[1,2]%%%}+%%%{-8,[1,1]%%%}+%%%{12,[0,3]%%%}+%%%{4,[0,2]%%%},0]:[1,0,%%%{-2
,[2,0]%%%}+%%%{-2,[1,0]%%%}+%%%{2,[0,2]%%%}+%%%{2,[0,1]%%%},0,%%%{1,[4,0]%%
%}+%%%{-2,[3,0]%%%}+%%%{-2,[2,2]%%%}+%%%{2,[2,1]%%%}+%%%{1,[2,0]%%%}+%%%{2,
[1,2]%%%}+%%%{-2,[1,1]%%%}+%%%{1,[0,4]%%%}+%%%{-2,[0,3]%%%}+%%%{1,[0,2]%%%}
]%%},[0,1]%%%} / %%%{%%%{8,[9,0]%%%}+%%%{24,[8,1]%%%}+%%%{-24,[8,0]%%%}+%%%
{-48,[7,1]%%%}+%%%{24,[7,0]%%%}+%%%{-64,[6,3]%%%}+%%%{48,[6,2]%%%}+%%%{24,[
6,1]%%%}+%%%{-8,[6,0]%%%}+%%%{-48,[5,4]%%%}+%%%{144,[5,3]%%%}+%%%{-72,[5,2]
%%%}+%%%{48,[4,5]%%%}+%%%{-72,[4,3]%%%}+%%%{24,[4,2]%%%}+%%%{64,[3,6]%%%}+%
%%{-144,[3,5]%%%}+%%%{72,[3,4]%%%}+%%%{-48,[2,6]%%%}+%%%{72,[2,5]%%%}+%%%{-
24,[2,4]%%%}+%%%{-24,[1,8]%%%}+%%%{48,[1,7]%%%}+%%%{-24,[1,6]%%%}+%%%{-8,[0
```

```
,9]%%%}+%%%{24,[0,8]%%%}+%%%{-24,[0,7]%%%}+%%%{8,[0,6]%%%},[6,0]%%%}+%%%{%%
{[%%%{-12,[6,0]%%%}+%%%{-24,[5,1]%%%}+%%%{24,[5,0]%%%}+%%%{12,[4,2]%%%}+%%%
{24,[4,1]%%%}+%%%{-12,[4,0]%%%}+%%%{48,[3,3]%%%}+%%%{-48,[3,2]%%%}+%%%{12,[
2,4]%%%}+%%%{-48,[2,3]%%%}+%%%{24,[2,2]%%%}+%%%{-24,[1,5]%%%}+%%%{24,[1,4]%
%%}+%%%{-12,[0,6]%%%}+%%%{24,[0,5]%%%}+%%%{-12,[0,4]%%%},0,%%%{36,[8,0]%%%}
+%%%{72,[7,1]%%%}+%%%{-60,[7,0]%%%}+%%%{-72,[6,2]%%%}+%%%{-60,[6,1]%%%}+%%%
{12,[6,0]%%%}+%%%{-216,[5,3]%%%}+%%%{180,[5,2]%%%}+%%%{12,[5,0]%%%}+%%%{180
,[4,3]%%%}+%%%{-36,[4,2]%%%}+%%%{-12,[4,1]%%%}+%%%{216,[3,5]%%%}+%%%{-180,[
3,4]%%%}+%%%{-24,[3,2]%%%}+%%%{72,[2,6]%%%}+%%%{-180,[2,5]%%%}+%%%{36,[2,4]
%%%}+%%%{24,[2,3]%%%}+%%%{-72,[1,7]%%%}+%%%{60,[1,6]%%%}+%%%{12,[1,4]%%%}+%
%%{-36,[0,8]%%%}+%%%{60,[0,7]%%%}+%%%{-12,[0,6]%%%}+%%%{-12,[0,5]%%%},0]:[1
,0,%%%{-2,[2,0]%%%}+%%%{-2,[1,0]%%%}+%%%{2,[0,2]%%%}+%%%{2,[0,1]%%%},0,%%%{
1,[4,0]%%%}+%%%{-2,[3,0]%%%}+%%%{-2,[2,2]%%%}+%%%{2,[2,1]%%%}+%%%{1,[2,0]%%
%}+%%%{2,[1,2]%%%}+%%%{-2,[1,1]%%%}+%%%{1,[0,4]%%%}+%%%{-2,[0,3]%%%}+%%%{1,
[0,2]%%%}]%%},[4,0]%%%}+%%%{%%%{24,[9,0]%%%}+%%%{24,[8,1]%%%}+%%%{-72,[8,0]
%%%}+%%%{-96,[7,2]%%%}+%%%{72,[7,0]%%%}+%%%{-96,[6,3]%%%}+%%%{288,[6,2]%%%}
+%%%{-72,[6,1]%%%}+%%%{-24,[6,0]%%%}+%%%{144,[5,4]%%%}+%%%{-216,[5,2]%%%}+%
%%{48,[5,1]%%%}+%%%{144,[4,5]%%%}+%%%{-432,[4,4]%%%}+%%%{216,[4,3]%%%}+%%%{
24,[4,2]%%%}+%%%{-96,[3,6]%%%}+%%%{216,[3,4]%%%}+%%%{-96,[3,3]%%%}+%%%{-96,
[2,7]%%%}+%%%{288,[2,6]%%%}+%%%{-216,[2,5]%%%}+%%%{24,[2,4]%%%}+%%%{24,[1,8
]%%%}+%%%{-72,[1,6]%%%}+%%%{48,[1,5]%%%}+%%%{24,[0,9]%%%}+%%%{-72,[0,8]%%%}
+%%%{72,[0,7]%%%}+%%%{-24,[0,6]%%%},[2,0]%%%}+%%%{%%{[%%%{-4,[6,0]%%%}+%%%{
8,[5,0]%%%}+%%%{12,[4,2]%%%}+%%%{-8,[4,1]%%%}+%%%{-4,[4,0]%%%}+%%%{-16,[3,2
]%%%}+%%%{8,[3,1]%%%}+%%%{-12,[2,4]%%%}+%%%{16,[2,3]%%%}+%%%{8,[1,4]%%%}+%%
%{-8,[1,3]%%%}+%%%{4,[0,6]%%%}+%%%{-8,[0,5]%%%}+%%%{4,[0,4]%%%},0,%%%{12,[8
,0]%%%}+%%%{-20,[7,0]%%%}+%%%{-48,[6,2]%%%}+%%%{20,[6,1]%%%}+%%%{4,[6,0]%%%
}+%%%{60,[5,2]%%%}+%%%{-8,[5,1]%%%}+%%%{4,[5,0]%%%}+%%%{72,[4,4]%%%}+%%%{-6
0,[4,3]%%%}+%%%{-4,[4,2]%%%}+%%%{-12,[4,1]%%%}+%%%{-60,[3,4]%%%}+%%%{16,[3,
3]%%%}+%%%{8,[3,2]%%%}+%%%{-48,[2,6]%%%}+%%%{60,[2,5]%%%}+%%%{-4,[2,4]%%%}+
%%%{8,[2,3]%%%}+%%%{20,[1,6]%%%}+%%%{-8,[1,5]%%%}+%%%{-12,[1,4]%%%}+%%%{12,
[0,8]%%%}+%%%{-20,[0,7]%%%}+%%%{4,[0,6]%%%}+%%%{4,[0,5]%%%},0]:[1,0,%%%{-2,
[2,0]%%%}+%%%{-2,[1,0]%%%}+%%%{2,[0,2]%%%}+%%%{2,[0,1]%%%},0,%%%{1,[4,0]%%%
}+%%%{-2,[3,0]%%%}+%%%{-2,[2,2]%%%}+%%%{2,[2,1]%%%}+%%%{1,[2,0]%%%}+%%%{2,[
1,2]%%%}+%%%{-2,[1,1]%%%}+%%%{1,[0,4]%%%}+%%%{-2,[0,3]%%%}+%%%{1,[0,2]%%%}]
%%},[0,0]%%%} Error: Bad Argument Value
```

maple [B] time = 1.58, size = 417, normalized size = 2.69

$$\frac{\sqrt{2}\,\operatorname{arctanh}\left(\frac{\cosh(x)\sqrt{2}}{2}\right)}{2\sqrt{b^2-c^2}\,\sqrt{-\frac{\sinh(x)b^2-\sinh(x)c^2-b^2+c^2}{\sqrt{b^2-c^2}}}}+\frac{\sqrt{-\sqrt{b^2-c^2}\,\left(\sinh(x)-1\right)\left(\sinh^2(x)\right)}\,\sqrt{2}\,\sqrt{b^2-c^2}\,\left(\ln\left(-\frac{2\left(\cosh(x)\sqrt{b^2-c^2}\ \sqrt{\cdots}\right.}{\cdots}\right.\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*cosh(x)+c*sinh(x)+(b^2-c^2)^(1/2))^(3/2),x)
```

```
[Out] 1/2/(b^2-c^2)^(1/2)/(-(sinh(x)*b^2-sinh(x)*c^2-b^2+c^2)/(b^2-c^2)^(1/2))^(1
/2)*2^(1/2)*arctanh(1/2*cosh(x)*2^(1/2))+1/4*(-(b^2-c^2)^(1/2)*(sinh(x)-1)*
sinh(x)^2)^(1/2)*2^(1/2)*(b^2-c^2)^(1/2)*(ln(-2*(cosh(x)*(b^2-c^2)^(1/2)*2^
(1/2)*sinh(x)-sinh(x)*(b^2-c^2)^(1/2)-cosh(x)*(b^2-c^2)^(1/2)*2^(1/2)+(b^2-
c^2)^(1/2)-(-(b^2-c^2)^(1/2)*(sinh(x)-1))^(1/2)*(-(b^2-c^2)^(1/2)*(sinh(x)-
1)*sinh(x)^2)^(1/2))/(cosh(x)-2^(1/2)))-ln(2*(cosh(x)*(b^2-c^2)^(1/2)*2^(1/
2)*sinh(x)+sinh(x)*(b^2-c^2)^(1/2)-cosh(x)*(b^2-c^2)^(1/2)*2^(1/2)-(b^2-c^2
)^(1/2)+(-(b^2-c^2)^(1/2)*(sinh(x)-1))^(1/2)*(-(b^2-c^2)^(1/2)*(sinh(x)-1)*
sinh(x)^2)^(1/2))/(cosh(x)+2^(1/2))))/(b-c)/(b+c)/(-(b^2-c^2)^(1/2)*(sinh(x
)-1))^(1/2)/sinh(x)/(-(sinh(x)*b^2-sinh(x)*c^2-b^2+c^2)/(b^2-c^2)^(1/2))^(1
/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\cosh(x)+c\sinh(x)+\sqrt{b^2-c^2}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*cosh(x)+c*sinh(x)+(b^2-c^2)^(1/2))^(3/2),x, algorithm="maxim
a")
```

```
[Out] integrate((b*cosh(x) + c*sinh(x) + sqrt(b^2 - c^2))^(-3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(b\cosh(x)+\sqrt{b^2-c^2}+c\sinh(x)\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*cosh(x) + (b^2 - c^2)^(1/2) + c*sinh(x))^(3/2),x)
```

```
[Out] int(1/(b*cosh(x) + (b^2 - c^2)^(1/2) + c*sinh(x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\cosh(x)+c\sinh(x)+\sqrt{b^2-c^2}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*cosh(x)+c*sinh(x)+(b**2-c**2)**(1/2))**(3/2),x)
[Out] Integral((b*cosh(x) + c*sinh(x) + sqrt(b**2 - c**2))**(-3/2), x)
```

3.773 $\int \frac{1}{\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{5/2}}\,dx$

Optimal. Leaf size=205

$$\frac{3\tan^{-1}\left(\frac{\sqrt[4]{b^2-c^2}\,\sinh\left(x+i\tan^{-1}(b,-ic)\right)}{\sqrt{2}\sqrt{\sqrt{b^2-c^2}+\sqrt{b^2-c^2}\,\cosh\left(x+i\tan^{-1}(b,-ic)\right)}}\right)}{16\sqrt{2}\left(b^2-c^2\right)^{5/4}}+\frac{3(b\sinh(x)+c\cosh(x))}{16\left(b^2-c^2\right)\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{3/2}}+\frac{}{4\sqrt{b^2-c^2}}$$

```
[Out] 3/32*arctan(1/2*(b^2-c^2)^(1/4)*sinh(x+I*arctan(b,-I*c))*2^(1/2)/((b^2-c^2)
^(1/2)+cosh(x+I*arctan(b,-I*c))*(b^2-c^2)^(1/2))^(1/2))/(b^2-c^2)^(5/4)*2^(
1/2)+1/4*(c*cosh(x)+b*sinh(x))/(b^2-c^2)^(1/2)/(b*cosh(x)+c*sinh(x)+(b^2-c^
2)^(1/2))^(5/2)+3/16*(c*cosh(x)+b*sinh(x))/(b^2-c^2)/(b*cosh(x)+c*sinh(x)+(
b^2-c^2)^(1/2))^(3/2)
```

Rubi [A] time = 0.18, antiderivative size = 205, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3116, 3115, 2649, 206}

$$\frac{3\tan^{-1}\left(\frac{\sqrt[4]{b^2-c^2}\,\sinh\left(x+i\tan^{-1}(b,-ic)\right)}{\sqrt{2}\sqrt{\sqrt{b^2-c^2}+\sqrt{b^2-c^2}\,\cosh\left(x+i\tan^{-1}(b,-ic)\right)}}\right)}{16\sqrt{2}\left(b^2-c^2\right)^{5/4}}+\frac{3(b\sinh(x)+c\cosh(x))}{16\left(b^2-c^2\right)\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{3/2}}+\frac{}{4\sqrt{b^2-c^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[b^2 - c^2] + b*Cosh[x] + c*Sinh[x])^(-5/2),x]
```

```
[Out] (3*ArcTan[((b^2 - c^2)^(1/4)*Sinh[x + I*ArcTan[b, (-I)*c]])/(Sqrt[2]*Sqrt[S
qrt[b^2 - c^2] + Sqrt[b^2 - c^2]*Cosh[x + I*ArcTan[b, (-I)*c]]])])/(16*Sqrt
[2]*(b^2 - c^2)^(5/4)) + (c*Cosh[x] + b*Sinh[x])/(4*Sqrt[b^2 - c^2]*(Sqrt[b
^2 - c^2] + b*Cosh[x] + c*Sinh[x])^(5/2)) + (3*(c*Cosh[x] + b*Sinh[x]))/(16
*(b^2 - c^2)*(Sqrt[b^2 - c^2] + b*Cosh[x] + c*Sinh[x])^(3/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3115

```
Int[1/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(
x_)]], x_Symbol] :> Int[1/Sqrt[a + Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b,
c]]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[a^2 - b^2 - c^2, 0]
```

Rule 3116

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> Simp[((c*Cos[d + e*x] - b*Sin[d + e*x])*(a + b*Cos[d + e
*x] + c*Sin[d + e*x])^n)/(a*e*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n + 1)),
Int[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1), x], x] /; FreeQ[{a, b, c
, d, e}, x] && EqQ[a^2 - b^2 - c^2, 0] && LtQ[n, -1]
```

Rubi steps

$$\int \frac{1}{\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{5/2}}\,dx = \frac{c\cosh(x)+b\sinh(x)}{4\sqrt{b^2-c^2}\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{5/2}} + \frac{3\int \frac{}{\left(\sqrt{b^2-c^2}+\right.}}{8}$$

$$= \frac{c\cosh(x)+b\sinh(x)}{4\sqrt{b^2-c^2}\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{5/2}} + \frac{}{16\left(b^2-c^2\right)}$$

$$= \frac{c\cosh(x)+b\sinh(x)}{4\sqrt{b^2-c^2}\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{5/2}} + \frac{}{16\left(b^2-c^2\right)}$$

$$= \frac{c\cosh(x)+b\sinh(x)}{4\sqrt{b^2-c^2}\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{5/2}} + \frac{}{16\left(b^2-c^2\right)}$$

$$= \frac{3\tan^{-1}\left(\frac{\sqrt[4]{b^2-c^2}\,\sinh\left(x+i\tan^{-1}(b,-ic)\right)}{\sqrt{2}\sqrt{\sqrt{b^2-c^2}+\sqrt{b^2-c^2}\cosh\left(x+i\tan^{-1}(b,-ic)\right)}}\right)}{16\sqrt{2}\left(b^2-c^2\right)^{5/4}} + \frac{c}{4\sqrt{b^2-c^2}\left(\sqrt{b\right.}}$$

Mathematica [F] time = 180.01, size = 0, normalized size = 0.00

$Aborted

Verification is Not applicable to the result.

```
[In] Integrate[(Sqrt[b^2 - c^2] + b*Cosh[x] + c*Sinh[x])^(-5/2),x]
[Out] $Aborted
```

fricas [B] time = 0.69, size = 5297, normalized size = 25.84

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*cosh(x)+c*sinh(x)+(b^2-c^2)^(1/2))^(5/2),x, algorithm="frica
s")
```

```
[Out] -1/8*(3*sqrt(1/2)*((b^5 + 5*b^4*c + 10*b^3*c^2 + 10*b^2*c^3 + 5*b*c^4 + c^5
)*cosh(x)^10 + 10*(b^5 + 5*b^4*c + 10*b^3*c^2 + 10*b^2*c^3 + 5*b*c^4 + c^5)
*cosh(x)*sinh(x)^9 + (b^5 + 5*b^4*c + 10*b^3*c^2 + 10*b^2*c^3 + 5*b*c^4 + c
^5)*sinh(x)^10 - 5*(b^5 + 3*b^4*c + 2*b^3*c^2 - 2*b^2*c^3 - 3*b*c^4 - c^5)*
cosh(x)^8 - 5*(b^5 + 3*b^4*c + 2*b^3*c^2 - 2*b^2*c^3 - 3*b*c^4 - c^5 - 9*(b
^5 + 5*b^4*c + 10*b^3*c^2 + 10*b^2*c^3 + 5*b*c^4 + c^5)*cosh(x)^2)*sinh(x)^
8 + 40*(3*(b^5 + 5*b^4*c + 10*b^3*c^2 + 10*b^2*c^3 + 5*b*c^4 + c^5)*cosh(x)
^3 - (b^5 + 3*b^4*c + 2*b^3*c^2 - 2*b^2*c^3 - 3*b*c^4 - c^5)*cosh(x))*sinh(
x)^7 + 10*(b^5 + b^4*c - 2*b^3*c^2 - 2*b^2*c^3 + b*c^4 + c^5)*cosh(x)^6 + 1
0*(b^5 + b^4*c - 2*b^3*c^2 - 2*b^2*c^3 + b*c^4 + c^5 + 21*(b^5 + 5*b^4*c +
10*b^3*c^2 + 10*b^2*c^3 + 5*b*c^4 + c^5)*cosh(x)^4 - 14*(b^5 + 3*b^4*c + 2*
b^3*c^2 - 2*b^2*c^3 - 3*b*c^4 - c^5)*cosh(x)^2)*sinh(x)^6 + 4*(63*(b^5 + 5*
b^4*c + 10*b^3*c^2 + 10*b^2*c^3 + 5*b*c^4 + c^5)*cosh(x)^5 - 70*(b^5 + 3*b^
4*c + 2*b^3*c^2 - 2*b^2*c^3 - 3*b*c^4 - c^5)*cosh(x)^3 + 15*(b^5 + b^4*c -
2*b^3*c^2 - 2*b^2*c^3 + b*c^4 + c^5)*cosh(x))*sinh(x)^5 - b^5 + 5*b^4*c - 1
0*b^3*c^2 + 10*b^2*c^3 - 5*b*c^4 + c^5 - 10*(b^5 - b^4*c - 2*b^3*c^2 + 2*b^
2*c^3 + b*c^4 - c^5)*cosh(x)^4 + 10*(21*(b^5 + 5*b^4*c + 10*b^3*c^2 + 10*b^
2*c^3 + 5*b*c^4 + c^5)*cosh(x)^6 - b^5 + b^4*c + 2*b^3*c^2 - 2*b^2*c^3 - b*
c^4 + c^5 - 35*(b^5 + 3*b^4*c + 2*b^3*c^2 - 2*b^2*c^3 - 3*b*c^4 - c^5)*cosh
(x)^4 + 15*(b^5 + b^4*c - 2*b^3*c^2 - 2*b^2*c^3 + b*c^4 + c^5)*cosh(x)^2)*s
inh(x)^4 + 40*(3*(b^5 + 5*b^4*c + 10*b^3*c^2 + 10*b^2*c^3 + 5*b*c^4 + c^5)*
cosh(x)^7 - 7*(b^5 + 3*b^4*c + 2*b^3*c^2 - 2*b^2*c^3 - 3*b*c^4 - c^5)*cosh(
x)^5 + 5*(b^5 + b^4*c - 2*b^3*c^2 - 2*b^2*c^3 + b*c^4 + c^5)*cosh(x)^3 - (b
^5 - b^4*c - 2*b^3*c^2 + 2*b^2*c^3 + b*c^4 - c^5)*cosh(x))*sinh(x)^3 + 5*(b
^5 - 3*b^4*c + 2*b^3*c^2 + 2*b^2*c^3 - 3*b*c^4 + c^5)*cosh(x)^2 + 5*(9*(b^5
 + 5*b^4*c + 10*b^3*c^2 + 10*b^2*c^3 + 5*b*c^4 + c^5)*cosh(x)^8 - 28*(b^5 +
 3*b^4*c + 2*b^3*c^2 - 2*b^2*c^3 - 3*b*c^4 - c^5)*cosh(x)^6 + b^5 - 3*b^4*c
 + 2*b^3*c^2 + 2*b^2*c^3 - 3*b*c^4 + c^5 + 30*(b^5 + b^4*c - 2*b^3*c^2 - 2*
b^2*c^3 + b*c^4 + c^5)*cosh(x)^4 - 12*(b^5 - b^4*c - 2*b^3*c^2 + 2*b^2*c^3
+ b*c^4 - c^5)*cosh(x)^2)*sinh(x)^2 + 10*((b^5 + 5*b^4*c + 10*b^3*c^2 + 10*
b^2*c^3 + 5*b*c^4 + c^5)*cosh(x)^9 - 4*(b^5 + 3*b^4*c + 2*b^3*c^2 - 2*b^2*c
^3 - 3*b*c^4 - c^5)*cosh(x)^7 + 6*(b^5 + b^4*c - 2*b^3*c^2 - 2*b^2*c^3 + b*
c^4 + c^5)*cosh(x)^5 - 4*(b^5 - b^4*c - 2*b^3*c^2 + 2*b^2*c^3 + b*c^4 - c^5
)*cosh(x)^3 + (b^5 - 3*b^4*c + 2*b^3*c^2 + 2*b^2*c^3 - 3*b*c^4 + c^5)*cosh(
x))*sinh(x))*arctan((sqrt(b^2 - c^2)*(cosh(x) + sinh(x)) - b + c)*sqrt(((b
+ c)*cosh(x)^2 + 2*(b + c)*cosh(x)*sinh(x) + (b + c)*sinh(x)^2 + 2*sqrt(b^2
 - c^2)*(cosh(x) + sinh(x)) + b - c)/(cosh(x) + sinh(x)))/(((b + c)*cosh(x)
^2 + 2*(b + c)*cosh(x)*sinh(x) + (b + c)*sinh(x)^2 - b + c)*(b^2 - c^2)^(1/
4)))/(b^2 - c^2)^(1/4) - sqrt(1/2)*(3*(b^4 + 4*b^3*c + 6*b^2*c^2 + 4*b*c^3
+ c^4)*cosh(x)^9 + 27*(b^4 + 4*b^3*c + 6*b^2*c^2 + 4*b*c^3 + c^4)*cosh(x)*s
inh(x)^8 + 3*(b^4 + 4*b^3*c + 6*b^2*c^2 + 4*b*c^3 + c^4)*sinh(x)^9 - 36*(b^
4 + 2*b^3*c - 2*b*c^3 - c^4)*cosh(x)^7 - 36*(b^4 + 2*b^3*c - 2*b*c^3 - c^4
- 3*(b^4 + 4*b^3*c + 6*b^2*c^2 + 4*b*c^3 + c^4)*cosh(x)^2)*sinh(x)^7 + 252*
((b^4 + 4*b^3*c + 6*b^2*c^2 + 4*b*c^3 + c^4)*cosh(x)^3 - (b^4 + 2*b^3*c - 2
*b*c^3 - c^4)*cosh(x))*sinh(x)^6 - 190*(b^4 - 2*b^2*c^2 + c^4)*cosh(x)^5 +
```

2*(189*(b^4 + 4*b^3*c + 6*b^2*c^2 + 4*b*c^3 + c^4)*cosh(x)^4 - 95*b^4 + 190
*b^2*c^2 - 95*c^4 - 378*(b^4 + 2*b^3*c - 2*b*c^3 - c^4)*cosh(x)^2)*sinh(x)^
5 + 2*(189*(b^4 + 4*b^3*c + 6*b^2*c^2 + 4*b*c^3 + c^4)*cosh(x)^5 - 630*(b^4
 + 2*b^3*c - 2*b*c^3 - c^4)*cosh(x)^3 - 475*(b^4 - 2*b^2*c^2 + c^4)*cosh(x)
)*sinh(x)^4 - 36*(b^4 - 2*b^3*c + 2*b*c^3 - c^4)*cosh(x)^3 + 4*(63*(b^4 + 4
*b^3*c + 6*b^2*c^2 + 4*b*c^3 + c^4)*cosh(x)^6 - 315*(b^4 + 2*b^3*c - 2*b*c^
3 - c^4)*cosh(x)^4 - 9*b^4 + 18*b^3*c - 18*b*c^3 + 9*c^4 - 475*(b^4 - 2*b^2
*c^2 + c^4)*cosh(x)^2)*sinh(x)^3 + 4*(27*(b^4 + 4*b^3*c + 6*b^2*c^2 + 4*b*c
^3 + c^4)*cosh(x)^7 - 189*(b^4 + 2*b^3*c - 2*b*c^3 - c^4)*cosh(x)^5 - 475*(
b^4 - 2*b^2*c^2 + c^4)*cosh(x)^3 - 27*(b^4 - 2*b^3*c + 2*b*c^3 - c^4)*cosh(
x))*sinh(x)^2 + 3*(b^4 - 4*b^3*c + 6*b^2*c^2 - 4*b*c^3 + c^4)*cosh(x) + (27
*(b^4 + 4*b^3*c + 6*b^2*c^2 + 4*b*c^3 + c^4)*cosh(x)^8 - 252*(b^4 + 2*b^3*c
 - 2*b*c^3 - c^4)*cosh(x)^6 - 950*(b^4 - 2*b^2*c^2 + c^4)*cosh(x)^4 + 3*b^4
 - 12*b^3*c + 18*b^2*c^2 - 12*b*c^3 + 3*c^4 - 108*(b^4 - 2*b^3*c + 2*b*c^3
- c^4)*cosh(x)^2)*sinh(x) - 4*((b^3 + 3*b^2*c + 3*b*c^2 + c^3)*cosh(x)^8 +
8*(b^3 + 3*b^2*c + 3*b*c^2 + c^3)*cosh(x)*sinh(x)^7 + (b^3 + 3*b^2*c + 3*b*
c^2 + c^3)*sinh(x)^8 - 33*(b^3 + b^2*c - b*c^2 - c^3)*cosh(x)^6 - (33*b^3 +
 33*b^2*c - 33*b*c^2 - 33*c^3 - 28*(b^3 + 3*b^2*c + 3*b*c^2 + c^3)*cosh(x)^
2)*sinh(x)^6 + 2*(28*(b^3 + 3*b^2*c + 3*b*c^2 + c^3)*cosh(x)^3 - 99*(b^3 +
b^2*c - b*c^2 - c^3)*cosh(x))*sinh(x)^5 - 33*(b^3 - b^2*c - b*c^2 + c^3)*co
sh(x)^4 + (70*(b^3 + 3*b^2*c + 3*b*c^2 + c^3)*cosh(x)^4 - 33*b^3 + 33*b^2*c
 + 33*b*c^2 - 33*c^3 - 495*(b^3 + b^2*c - b*c^2 - c^3)*cosh(x)^2)*sinh(x)^4
 + 4*(14*(b^3 + 3*b^2*c + 3*b*c^2 + c^3)*cosh(x)^5 - 165*(b^3 + b^2*c - b*c
^2 - c^3)*cosh(x)^3 - 33*(b^3 - b^2*c - b*c^2 + c^3)*cosh(x))*sinh(x)^3 + (
b^3 - 3*b^2*c + 3*b*c^2 - c^3)*cosh(x)^2 + (28*(b^3 + 3*b^2*c + 3*b*c^2 + c
^3)*cosh(x)^6 - 495*(b^3 + b^2*c - b*c^2 - c^3)*cosh(x)^4 + b^3 - 3*b^2*c +
 3*b*c^2 - c^3 - 198*(b^3 - b^2*c - b*c^2 + c^3)*cosh(x)^2)*sinh(x)^2 + 2*(
4*(b^3 + 3*b^2*c + 3*b*c^2 + c^3)*cosh(x)^7 - 99*(b^3 + b^2*c - b*c^2 - c^3
)*cosh(x)^5 - 66*(b^3 - b^2*c - b*c^2 + c^3)*cosh(x)^3 + (b^3 - 3*b^2*c + 3
*b*c^2 - c^3)*cosh(x))*sinh(x))*sqrt(b^2 - c^2))*sqrt(((b + c)*cosh(x)^2 +
2*(b + c)*cosh(x)*sinh(x) + (b + c)*sinh(x)^2 + 2*sqrt(b^2 - c^2)*(cosh(x)
+ sinh(x)) + b - c)/(cosh(x) + sinh(x))))/((b^7 + 5*b^6*c + 9*b^5*c^2 + 5*b
^4*c^3 - 5*b^3*c^4 - 9*b^2*c^5 - 5*b*c^6 - c^7)*cosh(x)^10 + 10*(b^7 + 5*b^
6*c + 9*b^5*c^2 + 5*b^4*c^3 - 5*b^3*c^4 - 9*b^2*c^5 - 5*b*c^6 - c^7)*cosh(x
)*sinh(x)^9 + (b^7 + 5*b^6*c + 9*b^5*c^2 + 5*b^4*c^3 - 5*b^3*c^4 - 9*b^2*c^
5 - 5*b*c^6 - c^7)*sinh(x)^10 - 5*(b^7 + 3*b^6*c + b^5*c^2 - 5*b^4*c^3 - 5*
b^3*c^4 + b^2*c^5 + 3*b*c^6 + c^7)*cosh(x)^8 - 5*(b^7 + 3*b^6*c + b^5*c^2 -
 5*b^4*c^3 - 5*b^3*c^4 + b^2*c^5 + 3*b*c^6 + c^7 - 9*(b^7 + 5*b^6*c + 9*b^5
*c^2 + 5*b^4*c^3 - 5*b^3*c^4 - 9*b^2*c^5 - 5*b*c^6 - c^7)*cosh(x)^2)*sinh(x
)^8 + 40*(3*(b^7 + 5*b^6*c + 9*b^5*c^2 + 5*b^4*c^3 - 5*b^3*c^4 - 9*b^2*c^5
- 5*b*c^6 - c^7)*cosh(x)^3 - (b^7 + 3*b^6*c + b^5*c^2 - 5*b^4*c^3 - 5*b^3*c
^4 + b^2*c^5 + 3*b*c^6 + c^7)*cosh(x))*sinh(x)^7 - b^7 + 5*b^6*c - 9*b^5*c^
2 + 5*b^4*c^3 + 5*b^3*c^4 - 9*b^2*c^5 + 5*b*c^6 - c^7 + 10*(b^7 + b^6*c - 3
*b^5*c^2 - 3*b^4*c^3 + 3*b^3*c^4 + 3*b^2*c^5 - b*c^6 - c^7)*cosh(x)^6 + 10*
(b^7 + b^6*c - 3*b^5*c^2 - 3*b^4*c^3 + 3*b^3*c^4 + 3*b^2*c^5 - b*c^6 - c^7

```
+ 21*(b^7 + 5*b^6*c + 9*b^5*c^2 + 5*b^4*c^3 - 5*b^3*c^4 - 9*b^2*c^5 - 5*b*c
^6 - c^7)*cosh(x)^4 - 14*(b^7 + 3*b^6*c + b^5*c^2 - 5*b^4*c^3 - 5*b^3*c^4 +
 b^2*c^5 + 3*b*c^6 + c^7)*cosh(x)^2)*sinh(x)^6 + 4*(63*(b^7 + 5*b^6*c + 9*b
^5*c^2 + 5*b^4*c^3 - 5*b^3*c^4 - 9*b^2*c^5 - 5*b*c^6 - c^7)*cosh(x)^5 - 70*
(b^7 + 3*b^6*c + b^5*c^2 - 5*b^4*c^3 - 5*b^3*c^4 + b^2*c^5 + 3*b*c^6 + c^7)
*cosh(x)^3 + 15*(b^7 + b^6*c - 3*b^5*c^2 - 3*b^4*c^3 + 3*b^3*c^4 + 3*b^2*c^
5 - b*c^6 - c^7)*cosh(x))*sinh(x)^5 - 10*(b^7 - b^6*c - 3*b^5*c^2 + 3*b^4*c
^3 + 3*b^3*c^4 - 3*b^2*c^5 - b*c^6 + c^7)*cosh(x)^4 - 10*(b^7 - b^6*c - 3*b
^5*c^2 + 3*b^4*c^3 + 3*b^3*c^4 - 3*b^2*c^5 - b*c^6 + c^7 - 21*(b^7 + 5*b^6*
c + 9*b^5*c^2 + 5*b^4*c^3 - 5*b^3*c^4 - 9*b^2*c^5 - 5*b*c^6 - c^7)*cosh(x)^
6 + 35*(b^7 + 3*b^6*c + b^5*c^2 - 5*b^4*c^3 - 5*b^3*c^4 + b^2*c^5 + 3*b*c^6
 + c^7)*cosh(x)^4 - 15*(b^7 + b^6*c - 3*b^5*c^2 - 3*b^4*c^3 + 3*b^3*c^4 + 3
*b^2*c^5 - b*c^6 - c^7)*cosh(x)^2)*sinh(x)^4 + 40*(3*(b^7 + 5*b^6*c + 9*b^5
*c^2 + 5*b^4*c^3 - 5*b^3*c^4 - 9*b^2*c^5 - 5*b*c^6 - c^7)*cosh(x)^7 - 7*(b^
7 + 3*b^6*c + b^5*c^2 - 5*b^4*c^3 - 5*b^3*c^4 + b^2*c^5 + 3*b*c^6 + c^7)*co
sh(x)^5 + 5*(b^7 + b^6*c - 3*b^5*c^2 - 3*b^4*c^3 + 3*b^3*c^4 + 3*b^2*c^5 -
b*c^6 - c^7)*cosh(x)^3 - (b^7 - b^6*c - 3*b^5*c^2 + 3*b^4*c^3 + 3*b^3*c^4 -
 3*b^2*c^5 - b*c^6 + c^7)*cosh(x))*sinh(x)^3 + 5*(b^7 - 3*b^6*c + b^5*c^2 +
 5*b^4*c^3 - 5*b^3*c^4 - b^2*c^5 + 3*b*c^6 - c^7)*cosh(x)^2 + 5*(9*(b^7 + 5
*b^6*c + 9*b^5*c^2 + 5*b^4*c^3 - 5*b^3*c^4 - 9*b^2*c^5 - 5*b*c^6 - c^7)*cos
h(x)^8 + b^7 - 3*b^6*c + b^5*c^2 + 5*b^4*c^3 - 5*b^3*c^4 - b^2*c^5 + 3*b*c^
6 - c^7 - 28*(b^7 + 3*b^6*c + b^5*c^2 - 5*b^4*c^3 - 5*b^3*c^4 + b^2*c^5 + 3
*b*c^6 + c^7)*cosh(x)^6 + 30*(b^7 + b^6*c - 3*b^5*c^2 - 3*b^4*c^3 + 3*b^3*c
^4 + 3*b^2*c^5 - b*c^6 - c^7)*cosh(x)^4 - 12*(b^7 - b^6*c - 3*b^5*c^2 + 3*b
^4*c^3 + 3*b^3*c^4 - 3*b^2*c^5 - b*c^6 + c^7)*cosh(x)^2)*sinh(x)^2 + 10*((b
^7 + 5*b^6*c + 9*b^5*c^2 + 5*b^4*c^3 - 5*b^3*c^4 - 9*b^2*c^5 - 5*b*c^6 - c^
7)*cosh(x)^9 - 4*(b^7 + 3*b^6*c + b^5*c^2 - 5*b^4*c^3 - 5*b^3*c^4 + b^2*c^5
 + 3*b*c^6 + c^7)*cosh(x)^7 + 6*(b^7 + b^6*c - 3*b^5*c^2 - 3*b^4*c^3 + 3*b^
3*c^4 + 3*b^2*c^5 - b*c^6 - c^7)*cosh(x)^5 - 4*(b^7 - b^6*c - 3*b^5*c^2 + 3
*b^4*c^3 + 3*b^3*c^4 - 3*b^2*c^5 - b*c^6 + c^7)*cosh(x)^3 + (b^7 - 3*b^6*c
+ b^5*c^2 + 5*b^4*c^3 - 5*b^3*c^4 - b^2*c^5 + 3*b*c^6 - c^7)*cosh(x))*sinh(
x))
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*cosh(x)+c*sinh(x)+(b^2-c^2)^(1/2))^(5/2),x, algorithm="giac"
)
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(t_n
ostep),abs((-sqrt(b^2-c^2))*t_nostep-b+c)]Evaluation time: 1.36Unable to di
```

vide, perhaps due to rounding error%%%{%%%{32,[5,0]%%%}+%%%{96,[4,1]%%%}+%%
%{-32,[4,0]%%%}+%%%{64,[3,2]%%%}+%%%{-64,[3,1]%%%}+%%%{-64,[2,3]%%%}+%%%{-9
6,[1,4]%%%}+%%%{64,[1,3]%%%}+%%%{-32,[0,5]%%%}+%%%{32,[0,4]%%%},[10,1]%%%}+
%%%{%%{[%%%{-80,[2,0]%%%}+%%%{-160,[1,1]%%%}+%%%{-80,[0,2]%%%},0,%%%{240,[4
,0]%%%}+%%%{480,[3,1]%%%}+%%%{80,[3,0]%%%}+%%%{80,[2,1]%%%}+%%%{-480,[1,3]%
%%}+%%%{-80,[1,2]%%%}+%%%{-240,[0,4]%%%}+%%%{-80,[0,3]%%%},0]:[1,0,%%%{-2,[
2,0]%%%}+%%%{-2,[1,0]%%%}+%%%{2,[0,2]%%%}+%%%{2,[0,1]%%%},0,%%%{1,[4,0]%%%}
+%%%{-2,[3,0]%%%}+%%%{-2,[2,2]%%%}+%%%{2,[2,1]%%%}+%%%{1,[2,0]%%%}+%%%{2,[1
,2]%%%}+%%%{-2,[1,1]%%%}+%%%{1,[0,4]%%%}+%%%{-2,[0,3]%%%}+%%%{1,[0,2]%%%}]%
%},[8,1]%%%}+%%%{%%%{320,[5,0]%%%}+%%%{320,[4,1]%%%}+%%%{-320,[4,0]%%%}+%%%
{-640,[3,2]%%%}+%%%{-640,[2,3]%%%}+%%%{640,[2,2]%%%}+%%%{320,[1,4]%%%}+%%%{
320,[0,5]%%%}+%%%{-320,[0,4]%%%},[6,1]%%%}+%%%{%%{[%%%{-160,[2,0]%%%}+%%%{1
60,[0,2]%%%},0,%%%{480,[4,0]%%%}+%%%{160,[3,0]%%%}+%%%{-960,[2,2]%%%}+%%%{-
160,[2,1]%%%}+%%%{-160,[1,2]%%%}+%%%{480,[0,4]%%%}+%%%{160,[0,3]%%%},0]:[1,
0,%%%{-2,[2,0]%%%}+%%%{-2,[1,0]%%%}+%%%{2,[0,2]%%%}+%%%{2,[0,1]%%%},0,%%%{1
,[4,0]%%%}+%%%{-2,[3,0]%%%}+%%%{-2,[2,2]%%%}+%%%{2,[2,1]%%%}+%%%{1,[2,0]%%%
}+%%%{2,[1,2]%%%}+%%%{-2,[1,1]%%%}+%%%{1,[0,4]%%%}+%%%{-2,[0,3]%%%}+%%%{1,[
0,2]%%%}]%%},[4,1]%%%}+%%%{%%%{160,[5,0]%%%}+%%%{-160,[4,1]%%%}+%%%{-160,[4
,0]%%%}+%%%{-320,[3,2]%%%}+%%%{320,[3,1]%%%}+%%%{320,[2,3]%%%}+%%%{160,[1,4
]%%%}+%%%{-320,[1,3]%%%}+%%%{-160,[0,5]%%%}+%%%{160,[0,4]%%%},[2,1]%%%}+%%%
{%%{[%%%{-16,[2,0]%%%}+%%%{32,[1,1]%%%}+%%%{-16,[0,2]%%%},0,%%%{48,[4,0]%%%
}+%%%{-96,[3,1]%%%}+%%%{16,[3,0]%%%}+%%%{-48,[2,1]%%%}+%%%{96,[1,3]%%%}+%%%
{48,[1,2]%%%}+%%%{-48,[0,4]%%%}+%%%{-16,[0,3]%%%},0]:[1,0,%%%{-2,[2,0]%%%}+
%%%{-2,[1,0]%%%}+%%%{2,[0,2]%%%}+%%%{2,[0,1]%%%},0,%%%{1,[4,0]%%%}+%%%{-2,[
3,0]%%%}+%%%{-2,[2,2]%%%}+%%%{2,[2,1]%%%}+%%%{1,[2,0]%%%}+%%%{2,[1,2]%%%}+%
%%{-2,[1,1]%%%}+%%%{1,[0,4]%%%}+%%%{-2,[0,3]%%%}+%%%{1,[0,2]%%%}]%%},[0,1]%
%%} / %%%{%%%{32,[15,0]%%%}+%%%{160,[14,1]%%%}+%%%{-160,[14,0]%%%}+%%%{160,
[13,2]%%%}+%%%{-640,[13,1]%%%}+%%%{320,[13,0]%%%}+%%%{-480,[12,3]%%%}+%%%{-
160,[12,2]%%%}+%%%{960,[12,1]%%%}+%%%{-320,[12,0]%%%}+%%%{-1120,[11,4]%%%}+
%%%{2560,[11,3]%%%}+%%%{-640,[11,2]%%%}+%%%{-640,[11,1]%%%}+%%%{160,[11,0]%
%%}+%%%{32,[10,5]%%%}+%%%{3040,[10,4]%%%}+%%%{-4480,[10,3]%%%}+%%%{1280,[10
,2]%%%}+%%%{160,[10,1]%%%}+%%%{-32,[10,0]%%%}+%%%{2080,[9,6]%%%}+%%%{-3200,
[9,5]%%%}+%%%{-1600,[9,4]%%%}+%%%{3200,[9,3]%%%}+%%%{-800,[9,2]%%%}+%%%{144
0,[8,7]%%%}+%%%{-7200,[8,6]%%%}+%%%{8000,[8,5]%%%}+%%%{-1600,[8,4]%%%}+%%%{
-800,[8,3]%%%}+%%%{160,[8,2]%%%}+%%%{-1440,[7,8]%%%}+%%%{6400,[7,6]%%%}+%%%
{-6400,[7,5]%%%}+%%%{1600,[7,4]%%%}+%%%{-2080,[6,9]%%%}+%%%{7200,[6,8]%%%}+
%%%{-6400,[6,7]%%%}+%%%{1600,[6,5]%%%}+%%%{-320,[6,4]%%%}+%%%{-32,[5,10]%%%
}+%%%{3200,[5,9]%%%}+%%%{-8000,[5,8]%%%}+%%%{6400,[5,7]%%%}+%%%{-1600,[5,6]
%%%}+%%%{1120,[4,11]%%%}+%%%{-3040,[4,10]%%%}+%%%{1600,[4,9]%%%}+%%%{1600,[
4,8]%%%}+%%%{-1600,[4,7]%%%}+%%%{320,[4,6]%%%}+%%%{480,[3,12]%%%}+%%%{-2560
,[3,11]%%%}+%%%{4480,[3,10]%%%}+%%%{-3200,[3,9]%%%}+%%%{800,[3,8]%%%}+%%%{-
160,[2,13]%%%}+%%%{160,[2,12]%%%}+%%%{640,[2,11]%%%}+%%%{-1280,[2,10]%%%}+%
%%{800,[2,9]%%%}+%%%{-160,[2,8]%%%}+%%%{-160,[1,14]%%%}+%%%{640,[1,13]%%%}+
%%%{-960,[1,12]%%%}+%%%{640,[1,11]%%%}+%%%{-160,[1,10]%%%}+%%%{-32,[0,15]%%
%}+%%%{160,[0,14]%%%}+%%%{-320,[0,13]%%%}+%%%{320,[0,12]%%%}+%%%{-160,[0,11

]%%%}+%%%{32,[0,10]%%%},[10,0]%%%}+%%%{%%{[%%%{-80,[12,0]%%%}+%%%{-320,[11,
1]%%%}+%%%{320,[11,0]%%%}+%%%{-160,[10,2]%%%}+%%%{960,[10,1]%%%}+%%%{-480,[
10,0]%%%}+%%%{960,[9,3]%%%}+%%%{-320,[9,2]%%%}+%%%{-960,[9,1]%%%}+%%%{320,[
9,0]%%%}+%%%{1360,[8,4]%%%}+%%%{-3520,[8,3]%%%}+%%%{1440,[8,2]%%%}+%%%{320,
[8,1]%%%}+%%%{-80,[8,0]%%%}+%%%{-640,[7,5]%%%}+%%%{-1920,[7,4]%%%}+%%%{3840
,[7,3]%%%}+%%%{-1280,[7,2]%%%}+%%%{-2240,[6,6]%%%}+%%%{4480,[6,5]%%%}+%%%{-
960,[6,4]%%%}+%%%{-1280,[6,3]%%%}+%%%{320,[6,2]%%%}+%%%{-640,[5,7]%%%}+%%%{
4480,[5,6]%%%}+%%%{-5760,[5,5]%%%}+%%%{1920,[5,4]%%%}+%%%{1360,[4,8]%%%}+%%
%{-1920,[4,7]%%%}+%%%{-960,[4,6]%%%}+%%%{1920,[4,5]%%%}+%%%{-480,[4,4]%%%}+
%%%{960,[3,9]%%%}+%%%{-3520,[3,8]%%%}+%%%{3840,[3,7]%%%}+%%%{-1280,[3,6]%%%
}+%%%{-160,[2,10]%%%}+%%%{-320,[2,9]%%%}+%%%{1440,[2,8]%%%}+%%%{-1280,[2,7]
%%%}+%%%{320,[2,6]%%%}+%%%{-320,[1,11]%%%}+%%%{960,[1,10]%%%}+%%%{-960,[1,9
]%%%}+%%%{320,[1,8]%%%}+%%%{-80,[0,12]%%%}+%%%{320,[0,11]%%%}+%%%{-480,[0,1
0]%%%}+%%%{320,[0,9]%%%}+%%%{-80,[0,8]%%%},0,%%%{240,[14,0]%%%}+%%%{960,[13
,1]%%%}+%%%{-880,[13,0]%%%}+%%%{240,[12,2]%%%}+%%%{-2640,[12,1]%%%}+%%%{112
0,[12,0]%%%}+%%%{-3840,[11,3]%%%}+%%%{1760,[11,2]%%%}+%%%{2240,[11,1]%%%}+%
%%{-480,[11,0]%%%}+%%%{-4560,[10,4]%%%}+%%%{12320,[10,3]%%%}+%%%{-4480,[10,
2]%%%}+%%%{-480,[10,1]%%%}+%%%{-80,[10,0]%%%}+%%%{4800,[9,5]%%%}+%%%{4400,[
9,4]%%%}+%%%{-11200,[9,3]%%%}+%%%{2400,[9,2]%%%}+%%%{80,[9,0]%%%}+%%%{10800
,[8,6]%%%}+%%%{-22000,[8,5]%%%}+%%%{5600,[8,4]%%%}+%%%{2400,[8,3]%%%}+%%%{4
00,[8,2]%%%}+%%%{-80,[8,1]%%%}+%%%{-17600,[7,6]%%%}+%%%{22400,[7,5]%%%}+%%%
{-4800,[7,4]%%%}+%%%{-320,[7,2]%%%}+%%%{-10800,[6,8]%%%}+%%%{17600,[6,7]%%%
}+%%%{-4800,[6,5]%%%}+%%%{-800,[6,4]%%%}+%%%{320,[6,3]%%%}+%%%{-4800,[5,9]%
%%}+%%%{22000,[5,8]%%%}+%%%{-22400,[5,7]%%%}+%%%{4800,[5,6]%%%}+%%%{480,[5,
4]%%%}+%%%{4560,[4,10]%%%}+%%%{-4400,[4,9]%%%}+%%%{-5600,[4,8]%%%}+%%%{4800
,[4,7]%%%}+%%%{800,[4,6]%%%}+%%%{-480,[4,5]%%%}+%%%{3840,[3,11]%%%}+%%%{-12
320,[3,10]%%%}+%%%{11200,[3,9]%%%}+%%%{-2400,[3,8]%%%}+%%%{-320,[3,6]%%%}+%
%%{-240,[2,12]%%%}+%%%{-1760,[2,11]%%%}+%%%{4480,[2,10]%%%}+%%%{-2400,[2,9]
%%%}+%%%{-400,[2,8]%%%}+%%%{320,[2,7]%%%}+%%%{-960,[1,13]%%%}+%%%{2640,[1,1
2]%%%}+%%%{-2240,[1,11]%%%}+%%%{480,[1,10]%%%}+%%%{80,[1,8]%%%}+%%%{-240,[0
,14]%%%}+%%%{880,[0,13]%%%}+%%%{-1120,[0,12]%%%}+%%%{480,[0,11]%%%}+%%%{80,
[0,10]%%%}+%%%{-80,[0,9]%%%},0]:[1,0,%%%{-2,[2,0]%%%}+%%%{-2,[1,0]%%%}+%%%{
2,[0,2]%%%}+%%%{2,[0,1]%%%},0,%%%{1,[4,0]%%%}+%%%{-2,[3,0]%%%}+%%%{-2,[2,2]
%%%}+%%%{2,[2,1]%%%}+%%%{1,[2,0]%%%}+%%%{2,[1,2]%%%}+%%%{-2,[1,1]%%%}+%%%{1
,[0,4]%%%}+%%%{-2,[0,3]%%%}+%%%{1,[0,2]%%%}]%%},[8,0]%%%}+%%%{%%%{320,[15,0
]%%%}+%%%{960,[14,1]%%%}+%%%{-1600,[14,0]%%%}+%%%{-960,[13,2]%%%}+%%%{-3200
,[13,1]%%%}+%%%{3200,[13,0]%%%}+%%%{-5440,[12,3]%%%}+%%%{8000,[12,2]%%%}+%%
%{3200,[12,1]%%%}+%%%{-3200,[12,0]%%%}+%%%{-960,[11,4]%%%}+%%%{19200,[11,3]
%%%}+%%%{-19200,[11,2]%%%}+%%%{1600,[11,0]%%%}+%%%{12480,[10,5]%%%}+%%%{-14
400,[10,4]%%%}+%%%{-19200,[10,3]%%%}+%%%{19200,[10,2]%%%}+%%%{-1600,[10,1]%
%%}+%%%{-320,[10,0]%%%}+%%%{8000,[9,6]%%%}+%%%{-48000,[9,5]%%%}+%%%{48000,[
9,4]%%%}+%%%{-8000,[9,2]%%%}+%%%{640,[9,1]%%%}+%%%{-14400,[8,7]%%%}+%%%{800
0,[8,6]%%%}+%%%{48000,[8,5]%%%}+%%%{-48000,[8,4]%%%}+%%%{8000,[8,3]%%%}+%%%
{960,[8,2]%%%}+%%%{-14400,[7,8]%%%}+%%%{64000,[7,7]%%%}+%%%{-64000,[7,6]%%%
}+%%%{16000,[7,4]%%%}+%%%{-2560,[7,3]%%%}+%%%{8000,[6,9]%%%}+%%%{8000,[6,8]

%%%}+%%%{-64000,[6,7]%%%}+%%%{64000,[6,6]%%%}+%%%{-16000,[6,5]%%%}+%%%{-640
,[6,4]%%%}+%%%{12480,[5,10]%%%}+%%%{-48000,[5,9]%%%}+%%%{48000,[5,8]%%%}+%%
%{-16000,[5,6]%%%}+%%%{3840,[5,5]%%%}+%%%{-960,[4,11]%%%}+%%%{-14400,[4,10]
%%%}+%%%{48000,[4,9]%%%}+%%%{-48000,[4,8]%%%}+%%%{16000,[4,7]%%%}+%%%{-640,
[4,6]%%%}+%%%{-5440,[3,12]%%%}+%%%{19200,[3,11]%%%}+%%%{-19200,[3,10]%%%}+%
%%{8000,[3,8]%%%}+%%%{-2560,[3,7]%%%}+%%%{-960,[2,13]%%%}+%%%{8000,[2,12]%%
%}+%%%{-19200,[2,11]%%%}+%%%{19200,[2,10]%%%}+%%%{-8000,[2,9]%%%}+%%%{960,[
2,8]%%%}+%%%{960,[1,14]%%%}+%%%{-3200,[1,13]%%%}+%%%{3200,[1,12]%%%}+%%%{-1
600,[1,10]%%%}+%%%{640,[1,9]%%%}+%%%{320,[0,15]%%%}+%%%{-1600,[0,14]%%%}+%%
%{3200,[0,13]%%%}+%%%{-3200,[0,12]%%%}+%%%{1600,[0,11]%%%}+%%%{-320,[0,10]%
%%},[6,0]%%%}+%%%{%%{[%%%{-160,[12,0]%%%}+%%%{-320,[11,1]%%%}+%%%{640,[11,0
]%%%}+%%%{640,[10,2]%%%}+%%%{640,[10,1]%%%}+%%%{-960,[10,0]%%%}+%%%{1600,[9
,3]%%%}+%%%{-3200,[9,2]%%%}+%%%{640,[9,0]%%%}+%%%{-800,[8,4]%%%}+%%%{-3200,
[8,3]%%%}+%%%{4800,[8,2]%%%}+%%%{-640,[8,1]%%%}+%%%{-160,[8,0]%%%}+%%%{-320
0,[7,5]%%%}+%%%{6400,[7,4]%%%}+%%%{-2560,[7,2]%%%}+%%%{320,[7,1]%%%}+%%%{64
00,[6,5]%%%}+%%%{-9600,[6,4]%%%}+%%%{2560,[6,3]%%%}+%%%{320,[6,2]%%%}+%%%{3
200,[5,7]%%%}+%%%{-6400,[5,6]%%%}+%%%{3840,[5,4]%%%}+%%%{-960,[5,3]%%%}+%%%
{800,[4,8]%%%}+%%%{-6400,[4,7]%%%}+%%%{9600,[4,6]%%%}+%%%{-3840,[4,5]%%%}+%
%%{-1600,[3,9]%%%}+%%%{3200,[3,8]%%%}+%%%{-2560,[3,6]%%%}+%%%{960,[3,5]%%%}
+%%%{-640,[2,10]%%%}+%%%{3200,[2,9]%%%}+%%%{-4800,[2,8]%%%}+%%%{2560,[2,7]%
%%}+%%%{-320,[2,6]%%%}+%%%{320,[1,11]%%%}+%%%{-640,[1,10]%%%}+%%%{640,[1,8]
%%%}+%%%{-320,[1,7]%%%}+%%%{160,[0,12]%%%}+%%%{-640,[0,11]%%%}+%%%{960,[0,1
0]%%%}+%%%{-640,[0,9]%%%}+%%%{160,[0,8]%%%},0,%%%{480,[14,0]%%%}+%%%{960,[1
3,1]%%%}+%%%{-1760,[13,0]%%%}+%%%{-2400,[12,2]%%%}+%%%{-1760,[12,1]%%%}+%%%
{2240,[12,0]%%%}+%%%{-5760,[11,3]%%%}+%%%{10560,[11,2]%%%}+%%%{-960,[11,0]%
%%}+%%%{4320,[10,4]%%%}+%%%{10560,[10,3]%%%}+%%%{-13440,[10,2]%%%}+%%%{960,
[10,1]%%%}+%%%{-160,[10,0]%%%}+%%%{14400,[9,5]%%%}+%%%{-26400,[9,4]%%%}+%%%
{4800,[9,2]%%%}+%%%{320,[9,1]%%%}+%%%{160,[9,0]%%%}+%%%{-2400,[8,6]%%%}+%%%
{-26400,[8,5]%%%}+%%%{33600,[8,4]%%%}+%%%{-4800,[8,3]%%%}+%%%{480,[8,2]%%%}
+%%%{-480,[8,1]%%%}+%%%{-19200,[7,7]%%%}+%%%{35200,[7,6]%%%}+%%%{-9600,[7,4
]%%%}+%%%{-1280,[7,3]%%%}+%%%{-2400,[6,8]%%%}+%%%{35200,[6,7]%%%}+%%%{-4480
0,[6,6]%%%}+%%%{9600,[6,5]%%%}+%%%{-320,[6,4]%%%}+%%%{1280,[6,3]%%%}+%%%{14
400,[5,9]%%%}+%%%{-26400,[5,8]%%%}+%%%{9600,[5,6]%%%}+%%%{1920,[5,5]%%%}+%%
%{-960,[5,4]%%%}+%%%{4320,[4,10]%%%}+%%%{-26400,[4,9]%%%}+%%%{33600,[4,8]%%
%}+%%%{-9600,[4,7]%%%}+%%%{-320,[4,6]%%%}+%%%{-960,[4,5]%%%}+%%%{-5760,[3,1
1]%%%}+%%%{10560,[3,10]%%%}+%%%{-4800,[3,8]%%%}+%%%{-1280,[3,7]%%%}+%%%{128
0,[3,6]%%%}+%%%{-2400,[2,12]%%%}+%%%{10560,[2,11]%%%}+%%%{-13440,[2,10]%%%}
+%%%{4800,[2,9]%%%}+%%%{480,[2,8]%%%}+%%%{960,[1,13]%%%}+%%%{-1760,[1,12]%%
%}+%%%{960,[1,10]%%%}+%%%{320,[1,9]%%%}+%%%{-480,[1,8]%%%}+%%%{480,[0,14]%%
%}+%%%{-1760,[0,13]%%%}+%%%{2240,[0,12]%%%}+%%%{-960,[0,11]%%%}+%%%{-160,[0
,10]%%%}+%%%{160,[0,9]%%%},0]:[1,0,%%%{-2,[2,0]%%%}+%%%{-2,[1,0]%%%}+%%%{2,
[0,2]%%%}+%%%{2,[0,1]%%%},0,%%%{1,[4,0]%%%}+%%%{-2,[3,0]%%%}+%%%{-2,[2,2]%%
%}+%%%{2,[2,1]%%%}+%%%{1,[2,0]%%%}+%%%{2,[1,2]%%%}+%%%{-2,[1,1]%%%}+%%%{1,[
0,4]%%%}+%%%{-2,[0,3]%%%}+%%%{1,[0,2]%%%}]%%},[4,0]%%%}+%%%{%%%{160,[15,0]%
%%}+%%%{160,[14,1]%%%}+%%%{-800,[14,0]%%%}+%%%{-1120,[13,2]%%%}+%%%{1600,[1

3,0]%%%}+%%%{-1120,[12,3]%%%}+%%%{5600,[12,2]%%%}+%%%{-1600,[12,1]%%%}+%%%{
-1600,[12,0]%%%}+%%%{3360,[11,4]%%%}+%%%{-9600,[11,2]%%%}+%%%{3200,[11,1]%%
%}+%%%{800,[11,0]%%%}+%%%{3360,[10,5]%%%}+%%%{-16800,[10,4]%%%}+%%%{9600,[1
0,3]%%%}+%%%{6400,[10,2]%%%}+%%%{-2400,[10,1]%%%}+%%%{-160,[10,0]%%%}+%%%{-
5600,[9,6]%%%}+%%%{24000,[9,4]%%%}+%%%{-16000,[9,3]%%%}+%%%{-800,[9,2]%%%}+
%%%{640,[9,1]%%%}+%%%{-5600,[8,7]%%%}+%%%{28000,[8,6]%%%}+%%%{-24000,[8,5]%
%%}+%%%{-8000,[8,4]%%%}+%%%{8800,[8,3]%%%}+%%%{-480,[8,2]%%%}+%%%{5600,[7,8
]%%%}+%%%{-32000,[7,6]%%%}+%%%{32000,[7,5]%%%}+%%%{-4800,[7,4]%%%}+%%%{-128
0,[7,3]%%%}+%%%{5600,[6,9]%%%}+%%%{-28000,[6,8]%%%}+%%%{32000,[6,7]%%%}+%%%
{-11200,[6,5]%%%}+%%%{2240,[6,4]%%%}+%%%{-3360,[5,10]%%%}+%%%{24000,[5,8]%%
%}+%%%{-32000,[5,7]%%%}+%%%{11200,[5,6]%%%}+%%%{-3360,[4,11]%%%}+%%%{16800,
[4,10]%%%}+%%%{-24000,[4,9]%%%}+%%%{8000,[4,8]%%%}+%%%{4800,[4,7]%%%}+%%%{-
2240,[4,6]%%%}+%%%{1120,[3,12]%%%}+%%%{-9600,[3,10]%%%}+%%%{16000,[3,9]%%%}
+%%%{-8800,[3,8]%%%}+%%%{1280,[3,7]%%%}+%%%{1120,[2,13]%%%}+%%%{-5600,[2,12
]%%%}+%%%{9600,[2,11]%%%}+%%%{-6400,[2,10]%%%}+%%%{800,[2,9]%%%}+%%%{480,[2
,8]%%%}+%%%{-160,[1,14]%%%}+%%%{1600,[1,12]%%%}+%%%{-3200,[1,11]%%%}+%%%{24
00,[1,10]%%%}+%%%{-640,[1,9]%%%}+%%%{-160,[0,15]%%%}+%%%{800,[0,14]%%%}+%%%
{-1600,[0,13]%%%}+%%%{1600,[0,12]%%%}+%%%{-800,[0,11]%%%}+%%%{160,[0,10]%%%
},[2,0]%%%}+%%%{%%{[%%%{-16,[12,0]%%%}+%%%{64,[11,0]%%%}+%%%{96,[10,2]%%%}+
%%%{-64,[10,1]%%%}+%%%{-96,[10,0]%%%}+%%%{-320,[9,2]%%%}+%%%{192,[9,1]%%%}+
%%%{64,[9,0]%%%}+%%%{-240,[8,4]%%%}+%%%{320,[8,3]%%%}+%%%{288,[8,2]%%%}+%%%
{-192,[8,1]%%%}+%%%{-16,[8,0]%%%}+%%%{640,[7,4]%%%}+%%%{-768,[7,3]%%%}+%%%{
64,[7,1]%%%}+%%%{320,[6,6]%%%}+%%%{-640,[6,5]%%%}+%%%{-192,[6,4]%%%}+%%%{51
2,[6,3]%%%}+%%%{-64,[6,2]%%%}+%%%{-640,[5,6]%%%}+%%%{1152,[5,5]%%%}+%%%{-38
4,[5,4]%%%}+%%%{-64,[5,3]%%%}+%%%{-240,[4,8]%%%}+%%%{640,[4,7]%%%}+%%%{-192
,[4,6]%%%}+%%%{-384,[4,5]%%%}+%%%{160,[4,4]%%%}+%%%{320,[3,8]%%%}+%%%{-768,
[3,7]%%%}+%%%{512,[3,6]%%%}+%%%{-64,[3,5]%%%}+%%%{96,[2,10]%%%}+%%%{-320,[2
,9]%%%}+%%%{288,[2,8]%%%}+%%%{-64,[2,6]%%%}+%%%{-64,[1,10]%%%}+%%%{192,[1,9
]%%%}+%%%{-192,[1,8]%%%}+%%%{64,[1,7]%%%}+%%%{-16,[0,12]%%%}+%%%{64,[0,11]%
%%}+%%%{-96,[0,10]%%%}+%%%{64,[0,9]%%%}+%%%{-16,[0,8]%%%},0,%%%{48,[14,0]%%
%}+%%%{-176,[13,0]%%%}+%%%{-336,[12,2]%%%}+%%%{176,[12,1]%%%}+%%%{224,[12,0
]%%%}+%%%{1056,[11,2]%%%}+%%%{-448,[11,1]%%%}+%%%{-96,[11,0]%%%}+%%%{1008,[
10,4]%%%}+%%%{-1056,[10,3]%%%}+%%%{-896,[10,2]%%%}+%%%{288,[10,1]%%%}+%%%{-
16,[10,0]%%%}+%%%{-2640,[9,4]%%%}+%%%{2240,[9,3]%%%}+%%%{96,[9,2]%%%}+%%%{6
4,[9,1]%%%}+%%%{16,[9,0]%%%}+%%%{-1680,[8,6]%%%}+%%%{2640,[8,5]%%%}+%%%{112
0,[8,4]%%%}+%%%{-1056,[8,3]%%%}+%%%{-48,[8,2]%%%}+%%%{-80,[8,1]%%%}+%%%{352
0,[7,6]%%%}+%%%{-4480,[7,5]%%%}+%%%{576,[7,4]%%%}+%%%{-128,[7,3]%%%}+%%%{12
8,[7,2]%%%}+%%%{1680,[6,8]%%%}+%%%{-3520,[6,7]%%%}+%%%{1344,[6,5]%%%}+%%%{2
24,[6,4]%%%}+%%%{-2640,[5,8]%%%}+%%%{4480,[5,7]%%%}+%%%{-1344,[5,6]%%%}+%%%
{-224,[5,4]%%%}+%%%{-1008,[4,10]%%%}+%%%{2640,[4,9]%%%}+%%%{-1120,[4,8]%%%}
+%%%{-576,[4,7]%%%}+%%%{-224,[4,6]%%%}+%%%{224,[4,5]%%%}+%%%{1056,[3,10]%%%
}+%%%{-2240,[3,9]%%%}+%%%{1056,[3,8]%%%}+%%%{128,[3,7]%%%}+%%%{336,[2,12]%%
%}+%%%{-1056,[2,11]%%%}+%%%{896,[2,10]%%%}+%%%{-96,[2,9]%%%}+%%%{48,[2,8]%%
%}+%%%{-128,[2,7]%%%}+%%%{-176,[1,12]%%%}+%%%{448,[1,11]%%%}+%%%{-288,[1,10
]%%%}+%%%{-64,[1,9]%%%}+%%%{80,[1,8]%%%}+%%%{-48,[0,14]%%%}+%%%{176,[0,13]%

```
%%}+%%%{-224,[0,12]%%%}+%%%{96,[0,11]%%%}+%%%{16,[0,10]%%%}+%%%{-16,[0,9]%%
%},0]:[1,0,%%%{-2,[2,0]%%%}+%%%{-2,[1,0]%%%}+%%%{2,[0,2]%%%}+%%%{2,[0,1]%%%
},0,%%%{1,[4,0]%%%}+%%%{-2,[3,0]%%%}+%%%{-2,[2,2]%%%}+%%%{2,[2,1]%%%}+%%%{1
,[2,0]%%%}+%%%{2,[1,2]%%%}+%%%{-2,[1,1]%%%}+%%%{1,[0,4]%%%}+%%%{-2,[0,3]%%%
}+%%%{1,[0,2]%%%}]%%},[0,0]%%%} Error: Bad Argument Value
```

maple [B] time = 1.90, size = 954, normalized size = 4.65

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*cosh(x)+c*sinh(x)+(b^2-c^2)^(1/2))^(5/2),x)
```

```
[Out] 1/8/(sinh(x)-1)/sinh(x)/((-b^2+c^2)/(b^2-c^2)^(1/2)*sinh(x)+(b^2-c^2)^(1/2)
)^(1/2)/(-sinh(x)*(b^2-c^2)^(1/2)+(b^2-c^2)^(1/2))^(1/2)/(b^2-c^2)*(2*2^(1/
2)*arctanh(1/2*cosh(x)*2^(1/2))*(-sinh(x)*(b^2-c^2)^(1/2)+(b^2-c^2)^(1/2))^
(1/2)*sinh(x)^2+2^(1/2)*ln(2/(-cosh(x)+2^(1/2))*(cosh(x)*(b^2-c^2)^(1/2)*2^
(1/2)*sinh(x)-cosh(x)*(b^2-c^2)^(1/2)*2^(1/2)-sinh(x)*(b^2-c^2)^(1/2)-(-sin
h(x)*(b^2-c^2)^(1/2)+(b^2-c^2)^(1/2))^(1/2)*(-(b^2-c^2)^(1/2)*sinh(x)^3+(b^
2-c^2)^(1/2)*sinh(x)^2)^(1/2)+(b^2-c^2)^(1/2)))*(-(b^2-c^2)^(1/2)*sinh(x)^3
+(b^2-c^2)^(1/2)*sinh(x)^2)^(1/2)*sinh(x)-2^(1/2)*ln(2/(cosh(x)+2^(1/2))*(c
osh(x)*(b^2-c^2)^(1/2)*2^(1/2)*sinh(x)-cosh(x)*(b^2-c^2)^(1/2)*2^(1/2)+sinh
(x)*(b^2-c^2)^(1/2)+(-sinh(x)*(b^2-c^2)^(1/2)+(b^2-c^2)^(1/2))^(1/2)*(-(b^2
-c^2)^(1/2)*sinh(x)^3+(b^2-c^2)^(1/2)*sinh(x)^2)^(1/2)-(b^2-c^2)^(1/2)))*(-
(b^2-c^2)^(1/2)*sinh(x)^3+(b^2-c^2)^(1/2)*sinh(x)^2)^(1/2)*sinh(x)-2*2^(1/2
)*arctanh(1/2*cosh(x)*2^(1/2))*(-sinh(x)*(b^2-c^2)^(1/2)+(b^2-c^2)^(1/2))^(
1/2)*sinh(x)-2^(1/2)*ln(2/(-cosh(x)+2^(1/2))*(cosh(x)*(b^2-c^2)^(1/2)*2^(1/
2)*sinh(x)-cosh(x)*(b^2-c^2)^(1/2)*2^(1/2)-sinh(x)*(b^2-c^2)^(1/2)-(-sinh(x
)*(b^2-c^2)^(1/2)+(b^2-c^2)^(1/2))^(1/2)*(-(b^2-c^2)^(1/2)*sinh(x)^3+(b^2-c
^2)^(1/2)*sinh(x)^2)^(1/2)+(b^2-c^2)^(1/2)))*(-(b^2-c^2)^(1/2)*sinh(x)^3+(b
^2-c^2)^(1/2)*sinh(x)^2)^(1/2)+2^(1/2)*ln(2/(cosh(x)+2^(1/2))*(cosh(x)*(b^2
-c^2)^(1/2)*2^(1/2)*sinh(x)-cosh(x)*(b^2-c^2)^(1/2)*2^(1/2)+sinh(x)*(b^2-c^
2)^(1/2)+(-sinh(x)*(b^2-c^2)^(1/2)+(b^2-c^2)^(1/2))^(1/2)*(-(b^2-c^2)^(1/2)
*sinh(x)^3+(b^2-c^2)^(1/2)*sinh(x)^2)^(1/2)-(b^2-c^2)^(1/2)))*(-(b^2-c^2)^(
1/2)*sinh(x)^3+(b^2-c^2)^(1/2)*sinh(x)^2)^(1/2)-4*(-sinh(x)*(b^2-c^2)^(1/2)
+(b^2-c^2)^(1/2))^(1/2)*cosh(x)*sinh(x))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\cosh(x) + c\sinh(x) + \sqrt{b^2 - c^2}\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*cosh(x)+c*sinh(x)+(b^2-c^2)^(1/2))^(5/2),x, algorithm="maxim
a")
```

```
[Out] integrate((b*cosh(x) + c*sinh(x) + sqrt(b^2 - c^2))^(-5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(b\cosh(x)+\sqrt{b^2-c^2}+c\sinh(x)\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*cosh(x) + (b^2 - c^2)^(1/2) + c*sinh(x))^(5/2),x)
```

```
[Out] int(1/(b*cosh(x) + (b^2 - c^2)^(1/2) + c*sinh(x))^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*cosh(x)+c*sinh(x)+(b**2-c**2)**(1/2))**(5/2),x)
```

```
[Out] Timed out
```

3.774 $\int \left(-\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{5/2}\,dx$

Optimal. Leaf size=146

$$\frac{2}{5}(b\sinh(x)+c\cosh(x))\left(-\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{3/2}-\frac{16}{15}\sqrt{b^2-c^2}\,(b\sinh(x)+c\cosh(x))\sqrt{-\sqrt{b^2-c^2}$$

```
[Out] 2/5*(c*cosh(x)+b*sinh(x))*(b*cosh(x)+c*sinh(x)-(b^2-c^2)^(1/2))^(3/2)+64/15
*(b^2-c^2)*(c*cosh(x)+b*sinh(x))/(b*cosh(x)+c*sinh(x)-(b^2-c^2)^(1/2))^(1/2
)-16/15*(c*cosh(x)+b*sinh(x))*(b^2-c^2)^(1/2)*(b*cosh(x)+c*sinh(x)-(b^2-c^2
)^(1/2))^(1/2)
```

Rubi [A] time = 0.12, antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {3113, 3112}

$$\frac{2}{5}(b\sinh(x)+c\cosh(x))\left(-\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{3/2}-\frac{16}{15}\sqrt{b^2-c^2}\,(b\sinh(x)+c\cosh(x))\sqrt{-\sqrt{b^2-c^2}$$

Antiderivative was successfully verified.

```
[In] Int[(-Sqrt[b^2 - c^2] + b*Cosh[x] + c*Sinh[x])^(5/2),x]
```

```
[Out] (64*(b^2 - c^2)*(c*Cosh[x] + b*Sinh[x]))/(15*Sqrt[-Sqrt[b^2 - c^2] + b*Cosh
[x] + c*Sinh[x]]) - (16*Sqrt[b^2 - c^2]*(c*Cosh[x] + b*Sinh[x])*Sqrt[-Sqrt[
b^2 - c^2] + b*Cosh[x] + c*Sinh[x]])/15 + (2*(c*Cosh[x] + b*Sinh[x])*(-Sqrt
[b^2 - c^2] + b*Cosh[x] + c*Sinh[x])^(3/2))/5
```

Rule 3112

```
Int[Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_
)]], x_Symbol] :> Simp[(-2*(c*Cos[d + e*x] - b*Sin[d + e*x]))/(e*Sqrt[a + b
*Cos[d + e*x] + c*Sin[d + e*x]]), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[a^
2 - b^2 - c^2, 0]
```

Rule 3113

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> -Simp[((c*Cos[d + e*x] - b*Sin[d + e*x])*(a + b*Cos[d +
e*x] + c*Sin[d + e*x])^(n - 1))/(e*n), x] + Dist[(a*(2*n - 1))/n, Int[(a +
b*Cos[d + e*x] + c*Sin[d + e*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e},
x] && EqQ[a^2 - b^2 - c^2, 0] && GtQ[n, 0]
```

Rubi steps

$$\int \left(-\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{5/2}dx = \frac{2}{5}(c\cosh(x)+b\sinh(x))\left(-\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{3/2}$$
$$= -\frac{16}{15}\sqrt{b^2-c^2}(c\cosh(x)+b\sinh(x))\sqrt{-\sqrt{b^2-c^2}+b\cosh(x)+$$
$$= \frac{64(b^2-c^2)(c\cosh(x)+b\sinh(x))}{15\sqrt{-\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)}}-\frac{16}{15}\sqrt{b^2-c^2}(c\cosh(x$$

Mathematica [C] time = 76.23, size = 9943, normalized size = 68.10

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(-Sqrt[b^2 - c^2] + b*Cosh[x] + c*Sinh[x])^(5/2),x]
```

```
[Out] Result too large to show
```

fricas [B] time = 0.50, size = 784, normalized size = 5.37

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cosh(x)+c*sinh(x)-(b^2-c^2)^(1/2))^(5/2),x, algorithm="fricas"
)
```

```
[Out] 1/30*sqrt(1/2)*(3*(b^3 + 3*b^2*c + 3*b*c^2 + c^3)*cosh(x)^6 + 18*(b^3 + 3*b
^2*c + 3*b*c^2 + c^3)*cosh(x)*sinh(x)^5 + 3*(b^3 + 3*b^2*c + 3*b*c^2 + c^3)
*sinh(x)^6 + 125*(b^3 + b^2*c - b*c^2 - c^3)*cosh(x)^4 + 5*(25*b^3 + 25*b^2
*c - 25*b*c^2 - 25*c^3 + 9*(b^3 + 3*b^2*c + 3*b*c^2 + c^3)*cosh(x)^2)*sinh(
x)^4 + 20*(3*(b^3 + 3*b^2*c + 3*b*c^2 + c^3)*cosh(x)^3 + 25*(b^3 + b^2*c -
b*c^2 - c^3)*cosh(x))*sinh(x)^3 + 3*b^3 - 9*b^2*c + 9*b*c^2 - 3*c^3 + 125*(
b^3 - b^2*c - b*c^2 + c^3)*cosh(x)^2 + 5*(9*(b^3 + 3*b^2*c + 3*b*c^2 + c^3)
*cosh(x)^4 + 25*b^3 - 25*b^2*c - 25*b*c^2 + 25*c^3 + 150*(b^3 + b^2*c - b*c
^2 - c^3)*cosh(x)^2)*sinh(x)^2 + 2*(9*(b^3 + 3*b^2*c + 3*b*c^2 + c^3)*cosh(
x)^5 + 250*(b^3 + b^2*c - b*c^2 - c^3)*cosh(x)^3 + 125*(b^3 - b^2*c - b*c^2
 + c^3)*cosh(x))*sinh(x) - 2*(11*(b^2 + 2*b*c + c^2)*cosh(x)^5 + 55*(b^2 +
2*b*c + c^2)*cosh(x)*sinh(x)^4 + 11*(b^2 + 2*b*c + c^2)*sinh(x)^5 - 150*(b^
2 - c^2)*cosh(x)^3 + 10*(11*(b^2 + 2*b*c + c^2)*cosh(x)^2 - 15*b^2 + 15*c^2
)*sinh(x)^3 + 10*(11*(b^2 + 2*b*c + c^2)*cosh(x)^3 - 45*(b^2 - c^2)*cosh(x)
```

```
)*sinh(x)^2 + 11*(b^2 - 2*b*c + c^2)*cosh(x) + (55*(b^2 + 2*b*c + c^2)*cosh
(x)^4 - 450*(b^2 - c^2)*cosh(x)^2 + 11*b^2 - 22*b*c + 11*c^2)*sinh(x))*sqrt
(b^2 - c^2))*sqrt(((b + c)*cosh(x)^2 + 2*(b + c)*cosh(x)*sinh(x) + (b + c)*
sinh(x)^2 - 2*sqrt(b^2 - c^2)*(cosh(x) + sinh(x)) + b - c)/(cosh(x) + sinh(
x)))/((b + c)*cosh(x)^4 + 4*(b + c)*cosh(x)*sinh(x)^3 + (b + c)*sinh(x)^4 -
 (b - c)*cosh(x)^2 + (6*(b + c)*cosh(x)^2 - b + c)*sinh(x)^2 + 2*(2*(b + c)
*cosh(x)^3 - (b - c)*cosh(x))*sinh(x))
```

giac [B] time = 0.31, size = 657, normalized size = 4.50

$$-\frac{\sqrt{2}\left(3\left(\sqrt{b^2-c^2}\,b^2\mathrm{sgn}\left(-\sqrt{b^2-c^2}\,e^x+b-c\right)+2\sqrt{b^2-c^2}\,bc\mathrm{sgn}\left(-\sqrt{b^2-c^2}\,e^x+b-c\right)+\sqrt{b^2-c^2}\,c^2\mathrm{sgn}\left(-\cdots\right.\right.\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cosh(x)+c*sinh(x)-(b^2-c^2)^(1/2))^(5/2),x, algorithm="giac")
```

```
[Out] -1/60*sqrt(2)*(3*(sqrt(b^2 - c^2)*b^2*sgn(-sqrt(b^2 - c^2)*e^x + b - c) + 2
*sqrt(b^2 - c^2)*b*c*sgn(-sqrt(b^2 - c^2)*e^x + b - c) + sqrt(b^2 - c^2)*c^
2*sgn(-sqrt(b^2 - c^2)*e^x + b - c))*e^(5/2*x) - 25*(b^3*sgn(-sqrt(b^2 - c^
2)*e^x + b - c) + b^2*c*sgn(-sqrt(b^2 - c^2)*e^x + b - c) - b*c^2*sgn(-sqrt
(b^2 - c^2)*e^x + b - c) - c^3*sgn(-sqrt(b^2 - c^2)*e^x + b - c))*e^(3/2*x)
 + 150*(sqrt(b^2 - c^2)*b^2*sgn(-sqrt(b^2 - c^2)*e^x + b - c) - sqrt(b^2 -
c^2)*c^2*sgn(-sqrt(b^2 - c^2)*e^x + b - c))*e^(1/2*x) + 150*(b^3*sgn(-sqrt(
b^2 - c^2)*e^x + b - c) - b^2*c*sgn(-sqrt(b^2 - c^2)*e^x + b - c) - b*c^2*s
gn(-sqrt(b^2 - c^2)*e^x + b - c) + c^3*sgn(-sqrt(b^2 - c^2)*e^x + b - c))*e
^(-1/2*x) - 25*(sqrt(b^2 - c^2)*b^2*sgn(-sqrt(b^2 - c^2)*e^x + b - c) - 2*s
qrt(b^2 - c^2)*b*c*sgn(-sqrt(b^2 - c^2)*e^x + b - c) + sqrt(b^2 - c^2)*c^2*
sgn(-sqrt(b^2 - c^2)*e^x + b - c))*e^(-3/2*x) + 3*(b^3*sgn(-sqrt(b^2 - c^2)
*e^x + b - c) - 3*b^2*c*sgn(-sqrt(b^2 - c^2)*e^x + b - c) + 3*b*c^2*sgn(-sq
rt(b^2 - c^2)*e^x + b - c) - c^3*sgn(-sqrt(b^2 - c^2)*e^x + b - c))*e^(-5/2
*x))/sqrt(b - c)
```

maple [B] time = 1.09, size = 288, normalized size = 1.97

$$\frac{-\frac{\left(b^2-c^2\right)^{\frac{3}{2}}\left(\cosh^3(x)\right)}{3}-\left(2b^2-2c^2\right)\sqrt{b^2-c^2}\;\cosh(x)}{\sqrt{-\frac{\sinh(x)b^2-\sinh(x)c^2+b^2-c^2}{\sqrt{b^2-c^2}}}}+\frac{\sqrt{-\sqrt{b^2-c^2}\;(\sinh(x)+1)\left(\sinh^2(x)\right)}\left(\frac{\left(b^2-c^2\right)^2\cosh(x)\sqrt{-\sqrt{b^2-\cdots}}}{2\sinh(x)b^2-2\sinh\cdots}\cdots\right.}{\sinh(x)\sqrt{-\frac{\sinh\cdots}{\cdots}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cosh(x)+c*sinh(x)-(b^2-c^2)^(1/2))^(5/2),x)
```

```
[Out] 1/(-(sinh(x)*b^2-sinh(x)*c^2+b^2-c^2)/(b^2-c^2)^(1/2))^(1/2)*(-1/3*(b^2-c^2
)^(3/2)*cosh(x)^3-(2*b^2-2*c^2)*(b^2-c^2)^(1/2)*cosh(x))+(-(b^2-c^2)^(1/2)*
(sinh(x)+1)*sinh(x)^2)^(1/2)*(1/2*(b^2-c^2)^2*cosh(x)/(sinh(x)*b^2-sinh(x)*
c^2+b^2-c^2)*(-(b^2-c^2)^(1/2)*(sinh(x)+1)*sinh(x)^2)^(1/2)-1/2*(b^2-c^2)^(
3/2)/((b^2-c^2)^(1/2)*(sinh(x)+1))^(1/2)*arctan(((b^2-c^2)^(1/2)*(sinh(x)+1
))^(1/2)*cosh(x)/(-(b^2-c^2)^(1/2)*(sinh(x)+1)*sinh(x)^2)^(1/2)))/sinh(x)/(
-(sinh(x)*b^2-sinh(x)*c^2+b^2-c^2)/(b^2-c^2)^(1/2))^(1/2)
```

maxima [B] time = 4.38, size = 1789, normalized size = 12.25

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cosh(x)+c*sinh(x)-(b^2-c^2)^(1/2))^(5/2),x, algorithm="maxima"
)
```

```
[Out] 1/20*sqrt(2)*(sqrt(b + c)*sqrt(b - c)*b^2 + 2*sqrt(b + c)*sqrt(b - c)*b*c +
 sqrt(b + c)*sqrt(b - c)*c^2)*(-2*sqrt(b + c)*sqrt(b - c)*e^(-x) + (b - c)*
e^(-2*x) + b + c)^(5/2)*e^(5/2*x)/(sqrt(b + c)*sqrt(b - c)*b^2 + 2*sqrt(b +
 c)*sqrt(b - c)*b*c + sqrt(b + c)*sqrt(b - c)*c^2 - 5*(b^3 + b^2*c - b*c^2
- c^3)*e^(-x) + 10*(sqrt(b + c)*sqrt(b - c)*b^2 - sqrt(b + c)*sqrt(b - c)*c
^2)*e^(-2*x) - 10*(b^3 - b^2*c - b*c^2 + c^3)*e^(-3*x) + 5*(sqrt(b + c)*sqr
t(b - c)*b^2 - 2*sqrt(b + c)*sqrt(b - c)*b*c + sqrt(b + c)*sqrt(b - c)*c^2)
*e^(-4*x) - (b^3 - 3*b^2*c + 3*b*c^2 - c^3)*e^(-5*x)) - 5/12*sqrt(2)*(b^3 +
 b^2*c - b*c^2 - c^3)*(-2*sqrt(b + c)*sqrt(b - c)*e^(-x) + (b - c)*e^(-2*x)
 + b + c)^(5/2)*e^(3/2*x)/(sqrt(b + c)*sqrt(b - c)*b^2 + 2*sqrt(b + c)*sqrt
(b - c)*b*c + sqrt(b + c)*sqrt(b - c)*c^2 - 5*(b^3 + b^2*c - b*c^2 - c^3)*e
^(-x) + 10*(sqrt(b + c)*sqrt(b - c)*b^2 - sqrt(b + c)*sqrt(b - c)*c^2)*e^(-
2*x) - 10*(b^3 - b^2*c - b*c^2 + c^3)*e^(-3*x) + 5*(sqrt(b + c)*sqrt(b - c)
*b^2 - 2*sqrt(b + c)*sqrt(b - c)*b*c + sqrt(b + c)*sqrt(b - c)*c^2)*e^(-4*x
) - (b^3 - 3*b^2*c + 3*b*c^2 - c^3)*e^(-5*x)) + 5/2*sqrt(2)*(sqrt(b + c)*sq
rt(b - c)*b^2 - sqrt(b + c)*sqrt(b - c)*c^2)*(-2*sqrt(b + c)*sqrt(b - c)*e^
(-x) + (b - c)*e^(-2*x) + b + c)^(5/2)*e^(1/2*x)/(sqrt(b + c)*sqrt(b - c)*b
^2 + 2*sqrt(b + c)*sqrt(b - c)*b*c + sqrt(b + c)*sqrt(b - c)*c^2 - 5*(b^3 +
 b^2*c - b*c^2 - c^3)*e^(-x) + 10*(sqrt(b + c)*sqrt(b - c)*b^2 - sqrt(b + c
)*sqrt(b - c)*c^2)*e^(-2*x) - 10*(b^3 - b^2*c - b*c^2 + c^3)*e^(-3*x) + 5*(
sqrt(b + c)*sqrt(b - c)*b^2 - 2*sqrt(b + c)*sqrt(b - c)*b*c + sqrt(b + c)*s
qrt(b - c)*c^2)*e^(-4*x) - (b^3 - 3*b^2*c + 3*b*c^2 - c^3)*e^(-5*x)) + 5/2*
sqrt(2)*(b^3 - b^2*c - b*c^2 + c^3)*(-2*sqrt(b + c)*sqrt(b - c)*e^(-x) + (b
 - c)*e^(-2*x) + b + c)^(5/2)*e^(-1/2*x)/(sqrt(b + c)*sqrt(b - c)*b^2 + 2*s
qrt(b + c)*sqrt(b - c)*b*c + sqrt(b + c)*sqrt(b - c)*c^2 - 5*(b^3 + b^2*c -
 b*c^2 - c^3)*e^(-x) + 10*(sqrt(b + c)*sqrt(b - c)*b^2 - sqrt(b + c)*sqrt(b
 - c)*c^2)*e^(-2*x) - 10*(b^3 - b^2*c - b*c^2 + c^3)*e^(-3*x) + 5*(sqrt(b +
```

```
c)*sqrt(b - c)*b^2 - 2*sqrt(b + c)*sqrt(b - c)*b*c + sqrt(b + c)*sqrt(b -
c)*c^2)*e^(-4*x) - (b^3 - 3*b^2*c + 3*b*c^2 - c^3)*e^(-5*x)) - 5/12*sqrt(2)
*(sqrt(b + c)*sqrt(b - c)*b^2 - 2*sqrt(b + c)*sqrt(b - c)*b*c + sqrt(b + c)
*sqrt(b - c)*c^2)*(-2*sqrt(b + c)*sqrt(b - c)*e^(-x) + (b - c)*e^(-2*x) + b
 + c)^(5/2)*e^(-3/2*x)/(sqrt(b + c)*sqrt(b - c)*b^2 + 2*sqrt(b + c)*sqrt(b
- c)*b*c + sqrt(b + c)*sqrt(b - c)*c^2 - 5*(b^3 + b^2*c - b*c^2 - c^3)*e^(-
x) + 10*(sqrt(b + c)*sqrt(b - c)*b^2 - sqrt(b + c)*sqrt(b - c)*c^2)*e^(-2*x
) - 10*(b^3 - b^2*c - b*c^2 + c^3)*e^(-3*x) + 5*(sqrt(b + c)*sqrt(b - c)*b^
2 - 2*sqrt(b + c)*sqrt(b - c)*b*c + sqrt(b + c)*sqrt(b - c)*c^2)*e^(-4*x) -
 (b^3 - 3*b^2*c + 3*b*c^2 - c^3)*e^(-5*x)) + 1/20*sqrt(2)*(b^3 - 3*b^2*c +
3*b*c^2 - c^3)*(-2*sqrt(b + c)*sqrt(b - c)*e^(-x) + (b - c)*e^(-2*x) + b +
c)^(5/2)*e^(-5/2*x)/(sqrt(b + c)*sqrt(b - c)*b^2 + 2*sqrt(b + c)*sqrt(b - c
)*b*c + sqrt(b + c)*sqrt(b - c)*c^2 - 5*(b^3 + b^2*c - b*c^2 - c^3)*e^(-x)
+ 10*(sqrt(b + c)*sqrt(b - c)*b^2 - sqrt(b + c)*sqrt(b - c)*c^2)*e^(-2*x) -
 10*(b^3 - b^2*c - b*c^2 + c^3)*e^(-3*x) + 5*(sqrt(b + c)*sqrt(b - c)*b^2 -
 2*sqrt(b + c)*sqrt(b - c)*b*c + sqrt(b + c)*sqrt(b - c)*c^2)*e^(-4*x) - (b
^3 - 3*b^2*c + 3*b*c^2 - c^3)*e^(-5*x))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(b\cosh(x)-\sqrt{b^2-c^2}+c\sinh(x)\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cosh(x) - (b^2 - c^2)^(1/2) + c*sinh(x))^(5/2),x)
[Out] int((b*cosh(x) - (b^2 - c^2)^(1/2) + c*sinh(x))^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cosh(x)+c*sinh(x)-(b**2-c**2)**(1/2))**(5/2),x)
[Out] Timed out
```

3.775 $\int \left(-\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{3/2}\,dx$

Optimal. Leaf size=96

$$\frac{2}{3}(b\sinh(x)+c\cosh(x))\sqrt{-\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)}-\frac{8\sqrt{b^2-c^2}\,(b\sinh(x)+c\cosh(x))}{3\sqrt{-\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)}}$$

```
[Out] -8/3*(c*cosh(x)+b*sinh(x))*(b^2-c^2)^(1/2)/(b*cosh(x)+c*sinh(x)-(b^2-c^2)^(
1/2))^(1/2)+2/3*(c*cosh(x)+b*sinh(x))*(b*cosh(x)+c*sinh(x)-(b^2-c^2)^(1/2))
^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {3113, 3112}

$$\frac{2}{3}(b\sinh(x)+c\cosh(x))\sqrt{-\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)}-\frac{8\sqrt{b^2-c^2}\,(b\sinh(x)+c\cosh(x))}{3\sqrt{-\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[(-Sqrt[b^2 - c^2] + b*Cosh[x] + c*Sinh[x])^(3/2),x]
```

```
[Out] (-8*Sqrt[b^2 - c^2]*(c*Cosh[x] + b*Sinh[x]))/(3*Sqrt[-Sqrt[b^2 - c^2] + b*C
osh[x] + c*Sinh[x]]) + (2*(c*Cosh[x] + b*Sinh[x])*Sqrt[-Sqrt[b^2 - c^2] + b
*Cosh[x] + c*Sinh[x]])/3
```

Rule 3112

```
Int[Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_
)]], x_Symbol] :> Simp[(-2*(c*Cos[d + e*x] - b*Sin[d + e*x]))/(e*Sqrt[a + b
*Cos[d + e*x] + c*Sin[d + e*x]]), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[a^
2 - b^2 - c^2, 0]
```

Rule 3113

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> -Simp[((c*Cos[d + e*x] - b*Sin[d + e*x])*(a + b*Cos[d +
e*x] + c*Sin[d + e*x])^(n - 1))/(e*n), x] + Dist[(a*(2*n - 1))/n, Int[(a +
b*Cos[d + e*x] + c*Sin[d + e*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e},
x] && EqQ[a^2 - b^2 - c^2, 0] && GtQ[n, 0]
```

Rubi steps

$$\int \left(-\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{3/2}\,dx=\frac{2}{3}(c\cosh(x)+b\sinh(x))\sqrt{-\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)}$$

$$=-\frac{8\sqrt{b^2-c^2}\,(c\cosh(x)+b\sinh(x))}{3\sqrt{-\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)}}+\frac{2}{3}(c\cosh(x)+b\sin$$

Mathematica [C] time = 73.74, size = 9861, normalized size = 102.72

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(-Sqrt[b^2 - c^2] + b*Cosh[x] + c*Sinh[x])^(3/2),x]
```

```
[Out] Result too large to show
```

fricas [B] time = 0.44, size = 329, normalized size = 3.43

$$\frac{\sqrt{\frac{1}{2}}\left(\left(b^2+2\,bc+c^2\right)\cosh(x)^4+4\left(b^2+2\,bc+c^2\right)\cosh(x)\sinh(x)^3+\left(b^2+2\,bc+c^2\right)\sinh(x)^4-18\left(b^2-c^2\right)\mathrm{co}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cosh(x)+c*sinh(x)-(b^2-c^2)^(1/2))^(3/2),x, algorithm="fricas"
)
```

```
[Out] 1/3*sqrt(1/2)*((b^2 + 2*b*c + c^2)*cosh(x)^4 + 4*(b^2 + 2*b*c + c^2)*cosh(x
)*sinh(x)^3 + (b^2 + 2*b*c + c^2)*sinh(x)^4 - 18*(b^2 - c^2)*cosh(x)^2 + 6*
((b^2 + 2*b*c + c^2)*cosh(x)^2 - 3*b^2 + 3*c^2)*sinh(x)^2 + b^2 - 2*b*c + c
^2 + 4*((b^2 + 2*b*c + c^2)*cosh(x)^3 - 9*(b^2 - c^2)*cosh(x))*sinh(x) - 8*
((b + c)*cosh(x)^3 + 3*(b + c)*cosh(x)*sinh(x)^2 + (b + c)*sinh(x)^3 + (b -
 c)*cosh(x) + (3*(b + c)*cosh(x)^2 + b - c)*sinh(x))*sqrt(b^2 - c^2))*sqrt(
((b + c)*cosh(x)^2 + 2*(b + c)*cosh(x)*sinh(x) + (b + c)*sinh(x)^2 - 2*sqrt
(b^2 - c^2)*(cosh(x) + sinh(x)) + b - c)/(cosh(x) + sinh(x)))/((b + c)*cosh
(x)^3 + 3*(b + c)*cosh(x)*sinh(x)^2 + (b + c)*sinh(x)^3 - (b - c)*cosh(x) +
 (3*(b + c)*cosh(x)^2 - b + c)*sinh(x))
```

giac [B] time = 0.20, size = 302, normalized size = 3.15

$$-\frac{\sqrt{2}\left(\left(\sqrt{b^2-c^2}\,b\,\mathrm{sgn}\left(-\sqrt{b^2-c^2}\,e^x+b-c\right)+\sqrt{b^2-c^2}\,c\,\mathrm{sgn}\left(-\sqrt{b^2-c^2}\,e^x+b-c\right)\right)e^{\left(\frac{3}{2}\,x\right)}-9\left(b^2\mathrm{sgn}\left(-\sqrt{b^2-\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cosh(x)+c*sinh(x)-(b^2-c^2)^(1/2))^(3/2),x, algorithm="giac")
```

```
[Out] -1/6*sqrt(2)*((sqrt(b^2 - c^2)*b*sgn(-sqrt(b^2 - c^2)*e^x + b - c) + sqrt(b
^2 - c^2)*c*sgn(-sqrt(b^2 - c^2)*e^x + b - c))*e^(3/2*x) - 9*(b^2*sgn(-sqrt
(b^2 - c^2)*e^x + b - c) - c^2*sgn(-sqrt(b^2 - c^2)*e^x + b - c))*e^(1/2*x)
 - 9*(sqrt(b^2 - c^2)*b*sgn(-sqrt(b^2 - c^2)*e^x + b - c) - sqrt(b^2 - c^2)
*c*sgn(-sqrt(b^2 - c^2)*e^x + b - c))*e^(-1/2*x) + (b^2*sgn(-sqrt(b^2 - c^2
)*e^x + b - c) - 2*b*c*sgn(-sqrt(b^2 - c^2)*e^x + b - c) + c^2*sgn(-sqrt(b^
2 - c^2)*e^x + b - c))*e^(-3/2*x))/sqrt(b - c)
```

maple [B] time = 0.81, size = 190, normalized size = 1.98

$$\frac{\left(2b^2-2c^2\right)\cosh(x)}{\sqrt{-\frac{\sinh(x)b^2-\sinh(x)c^2+b^2-c^2}{\sqrt{b^2-c^2}}}}+\frac{\sqrt{-\sqrt{b^2-c^2}\,(\sinh(x)+1)\left(\sinh^2(x)\right)}\ \arctan\left(\frac{\sqrt{\sqrt{b^2-c^2}\,(\sinh(x)+1)}\ \cosh(x)}{\sqrt{-\sqrt{b^2-c^2}\,(\sinh(x)+1)\left(\sinh^2(x)\right)}}\right)\left(b^2-c^2\right)}{\sqrt{\sqrt{b^2-c^2}\,(\sinh(x)+1)}\ \sinh(x)\sqrt{-\frac{\sinh(x)b^2-\sinh(x)c^2+b^2-c^2}{\sqrt{b^2-c^2}}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cosh(x)+c*sinh(x)-(b^2-c^2)^(1/2))^(3/2),x)
```

```
[Out] (2*b^2-2*c^2)/(-(sinh(x)*b^2-sinh(x)*c^2+b^2-c^2)/(b^2-c^2)^(1/2))^(1/2)*co
sh(x)+(-(b^2-c^2)^(1/2)*(sinh(x)+1)*sinh(x)^2)^(1/2)*arctan(((b^2-c^2)^(1/2
)*(sinh(x)+1))^(1/2)*cosh(x)/(-(b^2-c^2)^(1/2)*(sinh(x)+1)*sinh(x)^2)^(1/2)
)*(b^2-c^2)/((b^2-c^2)^(1/2)*(sinh(x)+1))^(1/2)/sinh(x)/(-(sinh(x)*b^2-sinh
(x)*c^2+b^2-c^2)/(b^2-c^2)^(1/2))^(1/2)
```

maxima [B] time = 0.98, size = 644, normalized size = 6.71

$$\frac{\sqrt{2}\left(\sqrt{b+c}\,\sqrt{b-c}\,b+\sqrt{b+c}\,\sqrt{b-c}\,c\right)\left(-2\,\sqrt{b+c}\,\sqrt{b-c}\,e^{(-x)}+(b-c)e^{(-2\,x)}+b+c\right)^{\frac{3}{2}}e^{\left(\frac{3}{2}\,x\right)}}{6\left(\sqrt{b+c}\,\sqrt{b-c}\,b+\sqrt{b+c}\,\sqrt{b-c}\,c-3\left(b^2-c^2\right)e^{(-x)}+3\left(\sqrt{b+c}\,\sqrt{b-c}\,b-\sqrt{b+c}\,\sqrt{b-c}\,c\right)e^{(-2\,x)}-\left(b^2-2\,bc\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cosh(x)+c*sinh(x)-(b^2-c^2)^(1/2))^(3/2),x, algorithm="maxima"
)
```

```
[Out] 1/6*sqrt(2)*(sqrt(b + c)*sqrt(b - c)*b + sqrt(b + c)*sqrt(b - c)*c)*(-2*sqr
t(b + c)*sqrt(b - c)*e^(-x) + (b - c)*e^(-2*x) + b + c)^(3/2)*e^(3/2*x)/(sq
rt(b + c)*sqrt(b - c)*b + sqrt(b + c)*sqrt(b - c)*c - 3*(b^2 - c^2)*e^(-x)
+ 3*(sqrt(b + c)*sqrt(b - c)*b - sqrt(b + c)*sqrt(b - c)*c)*e^(-2*x) - (b^2
 - 2*b*c + c^2)*e^(-3*x)) - 3/2*sqrt(2)*(b^2 - c^2)*(-2*sqrt(b + c)*sqrt(b
- c)*e^(-x) + (b - c)*e^(-2*x) + b + c)^(3/2)*e^(1/2*x)/(sqrt(b + c)*sqrt(b
```

```
 - c)*b + sqrt(b + c)*sqrt(b - c)*c - 3*(b^2 - c^2)*e^(-x) + 3*(sqrt(b + c)
*sqrt(b - c)*b - sqrt(b + c)*sqrt(b - c)*c)*e^(-2*x) - (b^2 - 2*b*c + c^2)*
e^(-3*x)) - 3/2*sqrt(2)*(sqrt(b + c)*sqrt(b - c)*b - sqrt(b + c)*sqrt(b - c
)*c)*(-2*sqrt(b + c)*sqrt(b - c)*e^(-x) + (b - c)*e^(-2*x) + b + c)^(3/2)*e
^(-1/2*x)/(sqrt(b + c)*sqrt(b - c)*b + sqrt(b + c)*sqrt(b - c)*c - 3*(b^2 -
 c^2)*e^(-x) + 3*(sqrt(b + c)*sqrt(b - c)*b - sqrt(b + c)*sqrt(b - c)*c)*e^
(-2*x) - (b^2 - 2*b*c + c^2)*e^(-3*x)) + 1/6*sqrt(2)*(b^2 - 2*b*c + c^2)*(-
2*sqrt(b + c)*sqrt(b - c)*e^(-x) + (b - c)*e^(-2*x) + b + c)^(3/2)*e^(-3/2*
x)/(sqrt(b + c)*sqrt(b - c)*b + sqrt(b + c)*sqrt(b - c)*c - 3*(b^2 - c^2)*e
^(-x) + 3*(sqrt(b + c)*sqrt(b - c)*b - sqrt(b + c)*sqrt(b - c)*c)*e^(-2*x)
- (b^2 - 2*b*c + c^2)*e^(-3*x))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(b\cosh(x) - \sqrt{b^2 - c^2} + c\sinh(x)\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cosh(x) - (b^2 - c^2)^(1/2) + c*sinh(x))^(3/2),x)
```

```
[Out] int((b*cosh(x) - (b^2 - c^2)^(1/2) + c*sinh(x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\cosh(x) + c\sinh(x) - \sqrt{b^2 - c^2}\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cosh(x)+c*sinh(x)-(b**2-c**2)**(1/2))**(3/2),x)
```

```
[Out] Integral((b*cosh(x) + c*sinh(x) - sqrt(b**2 - c**2))**(3/2), x)
```

3.776 $\int \sqrt{-\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)}\,dx$

Optimal. Leaf size=39

$$\frac{2(b\sinh(x)+c\cosh(x))}{\sqrt{-\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)}}$$

```
[Out] 2*(c*cosh(x)+b*sinh(x))/(b*cosh(x)+c*sinh(x)-(b^2-c^2)^(1/2))^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.036, Rules used = {3112}

$$\frac{2(b\sinh(x)+c\cosh(x))}{\sqrt{-\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-Sqrt[b^2 - c^2] + b*Cosh[x] + c*Sinh[x]],x]
```

```
[Out] (2*(c*Cosh[x] + b*Sinh[x]))/Sqrt[-Sqrt[b^2 - c^2] + b*Cosh[x] + c*Sinh[x]]
```

Rule 3112

```
Int[Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_
)]], x_Symbol] :> Simp[(-2*(c*Cos[d + e*x] - b*Sin[d + e*x]))/(e*Sqrt[a + b
*Cos[d + e*x] + c*Sin[d + e*x]]), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[a^
2 - b^2 - c^2, 0]
```

Rubi steps

$$\int \sqrt{-\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)}\,dx=\frac{2(c\cosh(x)+b\sinh(x))}{\sqrt{-\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)}}$$

Mathematica [C] time = 74.76, size = 9771, normalized size = 250.54

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[-Sqrt[b^2 - c^2] + b*Cosh[x] + c*Sinh[x]],x]
```

```
[Out] Result too large to show
```

fricas [B] time = 0.45, size = 143, normalized size = 3.67

$$\frac{2\sqrt{\frac{1}{2}}\left((b+c)\cosh(x)^2+2(b+c)\cosh(x)\sinh(x)+(b+c)\sinh(x)^2+2\sqrt{b^2-c^2}(\cosh(x)+\sinh(x))+b-c\right)\cdots}{(b+c)\cosh(x)^2+2(b+c)\cosh(x)\sinh(x)+(b+c)\sin\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cosh(x)+c*sinh(x)-(b^2-c^2)^(1/2))^(1/2),x, algorithm="fricas"
)
```

```
[Out] 2*sqrt(1/2)*((b + c)*cosh(x)^2 + 2*(b + c)*cosh(x)*sinh(x) + (b + c)*sinh(x
)^2 + 2*sqrt(b^2 - c^2)*(cosh(x) + sinh(x)) + b - c)*sqrt(((b + c)*cosh(x)^
2 + 2*(b + c)*cosh(x)*sinh(x) + (b + c)*sinh(x)^2 - 2*sqrt(b^2 - c^2)*(cosh
(x) + sinh(x)) + b - c)/(cosh(x) + sinh(x)))/((b + c)*cosh(x)^2 + 2*(b + c)
*cosh(x)*sinh(x) + (b + c)*sinh(x)^2 - b + c)
```

giac [B] time = 0.15, size = 103, normalized size = 2.64

$$-\frac{\sqrt{2}\left(\sqrt{b^2-c^2}\,e^{\left(\frac{1}{2}x\right)}\operatorname{sgn}\left(-\sqrt{b^2-c^2}\,e^x+b-c\right)+\left(b\operatorname{sgn}\left(-\sqrt{b^2-c^2}\,e^x+b-c\right)-c\operatorname{sgn}\left(-\sqrt{b^2-c^2}\,e^x+b-c\right)\right)e\cdots}{\sqrt{b-c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cosh(x)+c*sinh(x)-(b^2-c^2)^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] -sqrt(2)*(sqrt(b^2 - c^2)*e^(1/2*x)*sgn(-sqrt(b^2 - c^2)*e^x + b - c) + (b*
sgn(-sqrt(b^2 - c^2)*e^x + b - c) - c*sgn(-sqrt(b^2 - c^2)*e^x + b - c))*e^
(-1/2*x))/sqrt(b - c)
```

maple [B] time = 0.89, size = 202, normalized size = 5.18

$$\frac{\left(-b^2+c^2\right)\cosh(x)}{\sqrt{b^2-c^2}\,\sqrt{-\frac{\sinh(x)b^2-\sinh(x)c^2+b^2-c^2}{\sqrt{b^2-c^2}}}}-\frac{\sqrt{-\sqrt{b^2-c^2}\,\left(\sinh(x)+1\right)\left(\sinh^2(x)\right)}\,\arctan\left(\frac{\sqrt{\sqrt{b^2-c^2}\,(\sinh(x)+1)}\,\cosh(x)}{\sqrt{-\sqrt{b^2-c^2}\,(\sinh(x)+1)\left(\sinh^2(x)\right)}}\cdots\right.}{\sqrt{\sqrt{b^2-c^2}\,\left(\sinh(x)+1\right)}\,\sinh(x)\sqrt{-\frac{\sinh(x)b^2-\sinh(x)c^2+b^2-c^2}{\sqrt{b^2-c^2}}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cosh(x)+c*sinh(x)-(b^2-c^2)^(1/2))^(1/2),x)
```

```
[Out] (-b^2+c^2)/(b^2-c^2)^(1/2)/(-(sinh(x)*b^2-sinh(x)*c^2+b^2-c^2)/(b^2-c^2)^(1
/2))^(1/2)*cosh(x)-(-(b^2-c^2)^(1/2)*(sinh(x)+1)*sinh(x)^2)^(1/2)*arctan(((
b^2-c^2)^(1/2)*(sinh(x)+1))^(1/2)*cosh(x)/(-(b^2-c^2)^(1/2)*(sinh(x)+1)*sin
h(x)^2)^(1/2))*(b^2-c^2)^(1/2)/((b^2-c^2)^(1/2)*(sinh(x)+1))^(1/2)/sinh(x)/
(-(sinh(x)*b^2-sinh(x)*c^2+b^2-c^2)/(b^2-c^2)^(1/2))^(1/2)
```

maxima [B] time = 0.83, size = 156, normalized size = 4.00

$$-\frac{\sqrt{2}\sqrt{-2\sqrt{b+c}\sqrt{b-c}e^{(-x)}+(b-c)e^{(-2x)}+b+c}\sqrt{b+c}\sqrt{b-c}e^{\left(\frac{1}{2}x\right)}}{(b-c)e^{(-x)}-\sqrt{b+c}\sqrt{b-c}}-\frac{\sqrt{2}\sqrt{-2\sqrt{b+c}\sqrt{b-c}e^{(-x)}+(b-c)e^{(-}}{(b-c)e^{(-x)}-\sqrt{b+c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cosh(x)+c*sinh(x)-(b^2-c^2)^(1/2))^(1/2),x, algorithm="maxima"
)
```

```
[Out] -sqrt(2)*sqrt(-2*sqrt(b + c)*sqrt(b - c)*e^(-x) + (b - c)*e^(-2*x) + b + c)
*sqrt(b + c)*sqrt(b - c)*e^(1/2*x)/((b - c)*e^(-x) - sqrt(b + c)*sqrt(b - c
)) - sqrt(2)*sqrt(-2*sqrt(b + c)*sqrt(b - c)*e^(-x) + (b - c)*e^(-2*x) + b
+ c)*(b - c)*e^(-1/2*x)/((b - c)*e^(-x) - sqrt(b + c)*sqrt(b - c))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \sqrt{b\cosh(x)-\sqrt{b^2-c^2}+c\sinh(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cosh(x) - (b^2 - c^2)^(1/2) + c*sinh(x))^(1/2),x)
```

```
[Out] int((b*cosh(x) - (b^2 - c^2)^(1/2) + c*sinh(x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b\cosh{(x)}+c\sinh{(x)}-\sqrt{b^2-c^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cosh(x)+c*sinh(x)-(b**2-c**2)**(1/2))**(1/2),x)
```

```
[Out] Integral(sqrt(b*cosh(x) + c*sinh(x) - sqrt(b**2 - c**2)), x)
```

3.777 $\int \frac{1}{\sqrt{-\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)}}\,dx$

Optimal. Leaf size=102

$$-\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt[4]{b^2-c^2}\sinh\left(x+i\tan^{-1}(b,-ic)\right)}{\sqrt{2}\sqrt{-\sqrt{b^2-c^2}+\sqrt{b^2-c^2}\cosh\left(x+i\tan^{-1}(b,-ic)\right)}}\right)}{\sqrt[4]{b^2-c^2}}$$

```
[Out] -arctanh(1/2*(b^2-c^2)^(1/4)*sinh(x+I*arctan(b,-I*c))*2^(1/2)/(-(b^2-c^2)^(
1/2)+cosh(x+I*arctan(b,-I*c))*(b^2-c^2)^(1/2))^(1/2))*2^(1/2)/(b^2-c^2)^(1/
4)
```

Rubi [A] time = 0.09, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {3115, 2649, 204}

$$-\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt[4]{b^2-c^2}\sinh\left(x+i\tan^{-1}(b,-ic)\right)}{\sqrt{2}\sqrt{-\sqrt{b^2-c^2}+\sqrt{b^2-c^2}\cosh\left(x+i\tan^{-1}(b,-ic)\right)}}\right)}{\sqrt[4]{b^2-c^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[-Sqrt[b^2 - c^2] + b*Cosh[x] + c*Sinh[x]],x]
```

```
[Out] -((Sqrt[2]*ArcTanh[((b^2 - c^2)^(1/4)*Sinh[x + I*ArcTan[b, (-I)*c]])/(Sqrt[
2]*Sqrt[-Sqrt[b^2 - c^2] + Sqrt[b^2 - c^2]*Cosh[x + I*ArcTan[b, (-I)*c]]])]
)/(b^2 - c^2)^(1/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3115

```
Int[1/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(
x_)]], x_Symbol] :> Int[1/Sqrt[a + Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b,
```

```
c]]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[a^2 - b^2 - c^2, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{-\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)}}\,dx = \int \frac{1}{\sqrt{-\sqrt{b^2-c^2}+\sqrt{b^2-c^2}\cosh\left(x+i\tan^{-1}(b,-ic)\right)}}\,dx$$

$$= 2i\,\text{Subst}\left(\int \frac{1}{-2\sqrt{b^2-c^2}-x^2}\,dx, x, -\frac{i\sqrt{b^2-c^2}\sinh\left(x+\right.}{\sqrt{-\sqrt{b^2-c^2}+\sqrt{b^2-c^2}\cos}}\right.$$

$$= -\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt[4]{b^2-c^2}\sinh\left(x+i\tan^{-1}(b,-ic)\right)}{\sqrt{2}\sqrt{-\sqrt{b^2-c^2}+\sqrt{b^2-c^2}\cosh\left(x+i\tan^{-1}(b,-ic)\right)}}\right)}{\sqrt[4]{b^2-c^2}}$$

Mathematica [C] time = 31.29, size = 52609, normalized size = 515.77

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/Sqrt[-Sqrt[b^2 - c^2] + b*Cosh[x] + c*Sinh[x]],x]
```

```
[Out] Result too large to show
```

fricas [B] time = 0.53, size = 680, normalized size = 6.67

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*cosh(x)+c*sinh(x)-(b^2-c^2)^(1/2))^(1/2),x, algorithm="frica
s")
```

```
[Out] [sqrt(2)*log(-((b^2 + 2*b*c + c^2)*cosh(x)^4 + 4*(b^2 + 2*b*c + c^2)*cosh(x
)^3*sinh(x) + 6*(b^2 + 2*b*c + c^2)*cosh(x)^2*sinh(x)^2 + 4*(b^2 + 2*b*c +
c^2)*cosh(x)*sinh(x)^3 + (b^2 + 2*b*c + c^2)*sinh(x)^4 - 2*sqrt(2)*sqrt(1/2
)*(2*(b^2 - c^2)*cosh(x)^2 + 4*(b^2 - c^2)*cosh(x)*sinh(x) + 2*(b^2 - c^2)*
sinh(x)^2 + ((b + c)*cosh(x)^3 + 3*(b + c)*cosh(x)*sinh(x)^2 + (b + c)*sinh
(x)^3 + (b - c)*cosh(x) + (3*(b + c)*cosh(x)^2 + b - c)*sinh(x))*sqrt(b^2 -
 c^2))*sqrt(((b + c)*cosh(x)^2 + 2*(b + c)*cosh(x)*sinh(x) + (b + c)*sinh(x
)^2 - 2*sqrt(b^2 - c^2)*(cosh(x) + sinh(x)) + b - c)/(cosh(x) + sinh(x)))/(
b^2 - c^2)^(1/4) - b^2 + 2*b*c - c^2 + 2*((b + c)*cosh(x)^3 + 3*(b + c)*cos
```

```
h(x)*sinh(x)^2 + (b + c)*sinh(x)^3 - (b - c)*cosh(x) + (3*(b + c)*cosh(x)^2
 - b + c)*sinh(x))*sqrt(b^2 - c^2))/((b^2 + 2*b*c + c^2)*cosh(x)^4 + 4*(b^2
 + 2*b*c + c^2)*cosh(x)*sinh(x)^3 + (b^2 + 2*b*c + c^2)*sinh(x)^4 - 2*(b^2
- c^2)*cosh(x)^2 + 2*(3*(b^2 + 2*b*c + c^2)*cosh(x)^2 - b^2 + c^2)*sinh(x)^
2 + b^2 - 2*b*c + c^2 + 4*((b^2 + 2*b*c + c^2)*cosh(x)^3 - (b^2 - c^2)*cosh
(x))*sinh(x)))/(b^2 - c^2)^(1/4), 2*sqrt(2)*sqrt(-1/sqrt(b^2 - c^2))*arctan
(sqrt(2)*sqrt(1/2)*(sqrt(b^2 - c^2)*(cosh(x) + sinh(x)) + b - c)*sqrt(((b +
 c)*cosh(x)^2 + 2*(b + c)*cosh(x)*sinh(x) + (b + c)*sinh(x)^2 - 2*sqrt(b^2
- c^2)*(cosh(x) + sinh(x)) + b - c)/(cosh(x) + sinh(x)))*sqrt(-1/sqrt(b^2 -
 c^2))/((b + c)*cosh(x)^2 + 2*(b + c)*cosh(x)*sinh(x) + (b + c)*sinh(x)^2 -
 b + c))]
```

giac [B] time = 1.43, size = 299, normalized size = 2.93

$$-\frac{2\sqrt{2}\left(b^2-c^2-b+c\right)\sqrt{b-c}\,\arctan\left(\frac{b^3e^{\left(-\frac{1}{2}x\right)}-b^2ce^{\left(-\frac{1}{2}x\right)}-bc^2e^{\left(-\frac{1}{2}x\right)}+c^3e^{\left(-\frac{1}{2}x\right)}-b^2e^{\left(-\frac{1}{2}x\right)}+2\,bce^{\left(-\frac{1}{2}x\right)}-c^2e^{\left(-\frac{1}{2}x\right)}}{\sqrt{-\left(b^5-b^4c-2\,b^3c^2+2\,b^2c^3+bc^4-c^5-2\,b^4+4\,b^3c-4\,bc^3+2\,c^4+b^3-3\,b^2c+3\,bc^2-c^3\right)\cdots}}\right)}{\sqrt{-\left(b^5-b^4c-2\,b^3c^2+2\,b^2c^3+bc^4-c^5-2\,b^4+4\,b^3c-4\,bc^3+2\,c^4+b^3-3\,b^2c+3\,bc^2-c^3\right)\sqrt{b^2-c^2}}\,\mathrm{sgn}\left(\cdots\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*cosh(x)+c*sinh(x)-(b^2-c^2)^(1/2))^(1/2),x, algorithm="giac"
)
```

```
[Out] -2*sqrt(2)*(b^2 - c^2 - b + c)*sqrt(b - c)*arctan((b^3*e^(-1/2*x) - b^2*c*e
^(-1/2*x) - b*c^2*e^(-1/2*x) + c^3*e^(-1/2*x) - b^2*e^(-1/2*x) + 2*b*c*e^(-
1/2*x) - c^2*e^(-1/2*x))/sqrt(-(b^5 - b^4*c - 2*b^3*c^2 + 2*b^2*c^3 + b*c^4
 - c^5 - 2*b^4 + 4*b^3*c - 4*b*c^3 + 2*c^4 + b^3 - 3*b^2*c + 3*b*c^2 - c^3)
*sqrt(b^2 - c^2)))/(sqrt(-(b^5 - b^4*c - 2*b^3*c^2 + 2*b^2*c^3 + b*c^4 - c^
5 - 2*b^4 + 4*b^3*c - 4*b*c^3 + 2*c^4 + b^3 - 3*b^2*c + 3*b*c^2 - c^3)*sqrt
(b^2 - c^2))*sgn(-sqrt(b^2 - c^2)*e^x + b - c))
```

maple [A] time = 0.58, size = 129, normalized size = 1.26

$$\frac{\sqrt{-\sqrt{b^2-c^2}\,\left(\sinh(x)+1\right)\left(\sinh^2(x)\right)}\,\arctan\left(\frac{\sqrt{\sqrt{b^2-c^2}\,\left(\sinh(x)+1\right)}\,\cosh(x)}{\sqrt{-\sqrt{b^2-c^2}\,\left(\sinh(x)+1\right)\left(\sinh^2(x)\right)}}\right)}{\sqrt{\sqrt{b^2-c^2}\,\left(\sinh(x)+1\right)}\,\sinh(x)\sqrt{-\frac{\sinh(x)b^2-\sinh(x)c^2+b^2-c^2}{\sqrt{b^2-c^2}}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*cosh(x)+c*sinh(x)-(b^2-c^2)^(1/2))^(1/2),x)
```

```
[Out] (-(b^2-c^2)^(1/2)*(sinh(x)+1)*sinh(x)^2)^(1/2)/((b^2-c^2)^(1/2)*(sinh(x)+1)
)^(1/2)*arctan(((b^2-c^2)^(1/2)*(sinh(x)+1))^(1/2)*cosh(x)/(-(b^2-c^2)^(1/2
```

```
)*(sinh(x)+1)*sinh(x)^2)^(1/2))/sinh(x)/(-(sinh(x)*b^2-sinh(x)*c^2+b^2-c^2)
/(b^2-c^2)^(1/2))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{b\cosh(x) + c\sinh(x) - \sqrt{b^2 - c^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*cosh(x)+c*sinh(x)-(b^2-c^2)^(1/2))^(1/2),x, algorithm="maxim
a")
```

```
[Out] integrate(1/sqrt(b*cosh(x) + c*sinh(x) - sqrt(b^2 - c^2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{b\cosh(x) - \sqrt{b^2 - c^2} + c\sinh(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*cosh(x) - (b^2 - c^2)^(1/2) + c*sinh(x))^(1/2),x)
```

```
[Out] int(1/(b*cosh(x) - (b^2 - c^2)^(1/2) + c*sinh(x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{b\cosh(x) + c\sinh(x) - \sqrt{b^2 - c^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*cosh(x)+c*sinh(x)-(b**2-c**2)**(1/2))**(1/2),x)
```

```
[Out] Integral(1/sqrt(b*cosh(x) + c*sinh(x) - sqrt(b**2 - c**2)), x)
```

3.778 $\int \frac{1}{\left(-\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{3/2}}\,dx$

Optimal. Leaf size=159

$$-\frac{b\sinh(x)+c\cosh(x)}{2\sqrt{b^2-c^2}\left(-\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{3/2}}+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{b^2-c^2}\sinh\left(x+i\tan^{-1}(b,-ic)\right)}{\sqrt{2}\sqrt{-\sqrt{b^2-c^2}+\sqrt{b^2-c^2}\cosh\left(x+i\tan^{-1}(b,-ic)\right)}}\right)}{2\sqrt{2}\left(b^2-c^2\right)^{3/4}}$$

```
[Out] 1/4*arctanh(1/2*(b^2-c^2)^(1/4)*sinh(x+I*arctan(b,-I*c))*2^(1/2)/(-(b^2-c^2
)^(1/2)+cosh(x+I*arctan(b,-I*c))*(b^2-c^2)^(1/2))^(1/2))/(b^2-c^2)^(3/4)*2^
(1/2)+1/2*(-c*cosh(x)-b*sinh(x))/(b*cosh(x)+c*sinh(x)-(b^2-c^2)^(1/2))^(3/2
)/(b^2-c^2)^(1/2)
```

Rubi [A] time = 0.12, antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3116, 3115, 2649, 204}

$$-\frac{b\sinh(x)+c\cosh(x)}{2\sqrt{b^2-c^2}\left(-\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{3/2}}+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{b^2-c^2}\sinh\left(x+i\tan^{-1}(b,-ic)\right)}{\sqrt{2}\sqrt{-\sqrt{b^2-c^2}+\sqrt{b^2-c^2}\cosh\left(x+i\tan^{-1}(b,-ic)\right)}}\right)}{2\sqrt{2}\left(b^2-c^2\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(-Sqrt[b^2 - c^2] + b*Cosh[x] + c*Sinh[x])^(-3/2),x]
```

```
[Out] ArcTanh[((b^2 - c^2)^(1/4)*Sinh[x + I*ArcTan[b, (-I)*c]])/(Sqrt[2]*Sqrt[-Sq
rt[b^2 - c^2] + Sqrt[b^2 - c^2]*Cosh[x + I*ArcTan[b, (-I)*c]]])]/(2*Sqrt[2]
*(b^2 - c^2)^(3/4)) - (c*Cosh[x] + b*Sinh[x])/(2*Sqrt[b^2 - c^2]*(-Sqrt[b^2
 - c^2] + b*Cosh[x] + c*Sinh[x])^(3/2))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
```

```
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3115

```
Int[1/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(
x_)]], x_Symbol] :> Int[1/Sqrt[a + Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b,
c]]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[a^2 - b^2 - c^2, 0]
```

Rule 3116

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> Simp[((c*Cos[d + e*x] - b*Sin[d + e*x])*(a + b*Cos[d + e
*x] + c*Sin[d + e*x])^n)/(a*e*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n + 1)),
Int[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1), x], x] /; FreeQ[{a, b, c
, d, e}, x] && EqQ[a^2 - b^2 - c^2, 0] && LtQ[n, -1]
```

Rubi steps

$$\int \frac{1}{\left(-\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{3/2}}\,dx = -\frac{c\cosh(x)+b\sinh(x)}{2\sqrt{b^2-c^2}\left(-\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{3/2}} - \frac{\int \frac{}{\sqrt{-\sqrt{b^2-}}}}{}$$

$$= -\frac{c\cosh(x)+b\sinh(x)}{2\sqrt{b^2-c^2}\left(-\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{3/2}} - \frac{\int \frac{}{\sqrt{-\sqrt{b^2-}}}}{}$$

$$= -\frac{c\cosh(x)+b\sinh(x)}{2\sqrt{b^2-c^2}\left(-\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{3/2}} - \frac{i\,\mathrm{Subst}\Big(}{}$$

$$= \frac{\tanh^{-1}\left(\frac{\sqrt[4]{b^2-c^2}\,\sinh\left(x+i\tan^{-1}(b,-ic)\right)}{\sqrt{2}\sqrt{-\sqrt{b^2-c^2}+\sqrt{b^2-c^2}\cosh\left(x+i\tan^{-1}(b,-ic)\right)}}\right)}{2\sqrt{2}\left(b^2-c^2\right)^{3/4}} - \frac{}{2\sqrt{b^2-c^2}\,\Big(-}$$

Mathematica [F] time = 180.00, size = 0, normalized size = 0.00

$Aborted

Verification is Not applicable to the result.

```
[In] Integrate[(-Sqrt[b^2 - c^2] + b*Cosh[x] + c*Sinh[x])^(-3/2),x]
```

```
[Out] $Aborted
```

fricas [B] time = 0.59, size = 2137, normalized size = 13.44

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*cosh(x)+c*sinh(x)-(b^2-c^2)^(1/2))^(3/2),x, algorithm="frica
s")
```

```
[Out] -1/4*((sqrt(2)*(b^3 + 3*b^2*c + 3*b*c^2 + c^3)*cosh(x)^6 + 6*sqrt(2)*(b^3 +
 3*b^2*c + 3*b*c^2 + c^3)*cosh(x)*sinh(x)^5 + sqrt(2)*(b^3 + 3*b^2*c + 3*b*
c^2 + c^3)*sinh(x)^6 - 3*sqrt(2)*(b^3 + b^2*c - b*c^2 - c^3)*cosh(x)^4 + 3*
(5*sqrt(2)*(b^3 + 3*b^2*c + 3*b*c^2 + c^3)*cosh(x)^2 - sqrt(2)*(b^3 + b^2*c
 - b*c^2 - c^3))*sinh(x)^4 + 4*(5*sqrt(2)*(b^3 + 3*b^2*c + 3*b*c^2 + c^3)*c
osh(x)^3 - 3*sqrt(2)*(b^3 + b^2*c - b*c^2 - c^3)*cosh(x))*sinh(x)^3 + 3*sqr
t(2)*(b^3 - b^2*c - b*c^2 + c^3)*cosh(x)^2 + 3*(5*sqrt(2)*(b^3 + 3*b^2*c +
3*b*c^2 + c^3)*cosh(x)^4 - 6*sqrt(2)*(b^3 + b^2*c - b*c^2 - c^3)*cosh(x)^2
+ sqrt(2)*(b^3 - b^2*c - b*c^2 + c^3))*sinh(x)^2 + 6*(sqrt(2)*(b^3 + 3*b^2*
c + 3*b*c^2 + c^3)*cosh(x)^5 - 2*sqrt(2)*(b^3 + b^2*c - b*c^2 - c^3)*cosh(x
)^3 + sqrt(2)*(b^3 - b^2*c - b*c^2 + c^3)*cosh(x))*sinh(x) - sqrt(2)*(b^3 -
 3*b^2*c + 3*b*c^2 - c^3))*(b^2 - c^2)^(1/4)*log(-((b^2 + 2*b*c + c^2)*cosh
(x)^4 + 4*(b^2 + 2*b*c + c^2)*cosh(x)^3*sinh(x) + 6*(b^2 + 2*b*c + c^2)*cos
h(x)^2*sinh(x)^2 + 4*(b^2 + 2*b*c + c^2)*cosh(x)*sinh(x)^3 + (b^2 + 2*b*c +
 c^2)*sinh(x)^4 - 2*sqrt(1/2)*(sqrt(2)*(b + c)*cosh(x)^3 + 3*sqrt(2)*(b + c
)*cosh(x)*sinh(x)^2 + sqrt(2)*(b + c)*sinh(x)^3 + sqrt(2)*(b - c)*cosh(x) +
 (3*sqrt(2)*(b + c)*cosh(x)^2 + sqrt(2)*(b - c))*sinh(x) + 2*(sqrt(2)*cosh(
x)^2 + 2*sqrt(2)*cosh(x)*sinh(x) + sqrt(2)*sinh(x)^2)*sqrt(b^2 - c^2))*(b^2
 - c^2)^(1/4)*sqrt(((b + c)*cosh(x)^2 + 2*(b + c)*cosh(x)*sinh(x) + (b + c)
*sinh(x)^2 - 2*sqrt(b^2 - c^2)*(cosh(x) + sinh(x)) + b - c)/(cosh(x) + sinh
(x))) - b^2 + 2*b*c - c^2 + 2*((b + c)*cosh(x)^3 + 3*(b + c)*cosh(x)*sinh(x
)^2 + (b + c)*sinh(x)^3 - (b - c)*cosh(x) + (3*(b + c)*cosh(x)^2 - b + c)*s
inh(x))*sqrt(b^2 - c^2))/((b^2 + 2*b*c + c^2)*cosh(x)^4 + 4*(b^2 + 2*b*c +
c^2)*cosh(x)*sinh(x)^3 + (b^2 + 2*b*c + c^2)*sinh(x)^4 - 2*(b^2 - c^2)*cosh
(x)^2 + 2*(3*(b^2 + 2*b*c + c^2)*cosh(x)^2 - b^2 + c^2)*sinh(x)^2 + b^2 - 2
*b*c + c^2 + 4*((b^2 + 2*b*c + c^2)*cosh(x)^3 - (b^2 - c^2)*cosh(x))*sinh(x
))) + 4*sqrt(1/2)*(4*(b^3 + b^2*c - b*c^2 - c^3)*cosh(x)^4 + 16*(b^3 + b^2*
c - b*c^2 - c^3)*cosh(x)*sinh(x)^3 + 4*(b^3 + b^2*c - b*c^2 - c^3)*sinh(x)^
4 + 4*(b^3 - b^2*c - b*c^2 + c^3)*cosh(x)^2 + 4*(b^3 - b^2*c - b*c^2 + c^3
+ 6*(b^3 + b^2*c - b*c^2 - c^3)*cosh(x)^2)*sinh(x)^2 + 8*(2*(b^3 + b^2*c -
b*c^2 - c^3)*cosh(x)^3 + (b^3 - b^2*c - b*c^2 + c^3)*cosh(x))*sinh(x) + ((b
^2 + 2*b*c + c^2)*cosh(x)^5 + 5*(b^2 + 2*b*c + c^2)*cosh(x)*sinh(x)^4 + (b^
2 + 2*b*c + c^2)*sinh(x)^5 + 6*(b^2 - c^2)*cosh(x)^3 + 2*(5*(b^2 + 2*b*c +
```

```
c^2)*cosh(x)^2 + 3*b^2 - 3*c^2)*sinh(x)^3 + 2*(5*(b^2 + 2*b*c + c^2)*cosh(x
)^3 + 9*(b^2 - c^2)*cosh(x))*sinh(x)^2 + (b^2 - 2*b*c + c^2)*cosh(x) + (5*(
b^2 + 2*b*c + c^2)*cosh(x)^4 + 18*(b^2 - c^2)*cosh(x)^2 + b^2 - 2*b*c + c^2
)*sinh(x))*sqrt(b^2 - c^2))*sqrt(((b + c)*cosh(x)^2 + 2*(b + c)*cosh(x)*sin
h(x) + (b + c)*sinh(x)^2 - 2*sqrt(b^2 - c^2)*(cosh(x) + sinh(x)) + b - c)/(
cosh(x) + sinh(x))))/((b^5 + 3*b^4*c + 2*b^3*c^2 - 2*b^2*c^3 - 3*b*c^4 - c^
5)*cosh(x)^6 + 6*(b^5 + 3*b^4*c + 2*b^3*c^2 - 2*b^2*c^3 - 3*b*c^4 - c^5)*co
sh(x)*sinh(x)^5 + (b^5 + 3*b^4*c + 2*b^3*c^2 - 2*b^2*c^3 - 3*b*c^4 - c^5)*s
inh(x)^6 - b^5 + 3*b^4*c - 2*b^3*c^2 - 2*b^2*c^3 + 3*b*c^4 - c^5 - 3*(b^5 +
 b^4*c - 2*b^3*c^2 - 2*b^2*c^3 + b*c^4 + c^5)*cosh(x)^4 - 3*(b^5 + b^4*c -
2*b^3*c^2 - 2*b^2*c^3 + b*c^4 + c^5 - 5*(b^5 + 3*b^4*c + 2*b^3*c^2 - 2*b^2*
c^3 - 3*b*c^4 - c^5)*cosh(x)^2)*sinh(x)^4 + 4*(5*(b^5 + 3*b^4*c + 2*b^3*c^2
 - 2*b^2*c^3 - 3*b*c^4 - c^5)*cosh(x)^3 - 3*(b^5 + b^4*c - 2*b^3*c^2 - 2*b^
2*c^3 + b*c^4 + c^5)*cosh(x))*sinh(x)^3 + 3*(b^5 - b^4*c - 2*b^3*c^2 + 2*b^
2*c^3 + b*c^4 - c^5)*cosh(x)^2 + 3*(b^5 - b^4*c - 2*b^3*c^2 + 2*b^2*c^3 + b
*c^4 - c^5 + 5*(b^5 + 3*b^4*c + 2*b^3*c^2 - 2*b^2*c^3 - 3*b*c^4 - c^5)*cosh
(x)^4 - 6*(b^5 + b^4*c - 2*b^3*c^2 - 2*b^2*c^3 + b*c^4 + c^5)*cosh(x)^2)*si
nh(x)^2 + 6*((b^5 + 3*b^4*c + 2*b^3*c^2 - 2*b^2*c^3 - 3*b*c^4 - c^5)*cosh(x
)^5 - 2*(b^5 + b^4*c - 2*b^3*c^2 - 2*b^2*c^3 + b*c^4 + c^5)*cosh(x)^3 + (b^
5 - b^4*c - 2*b^3*c^2 + 2*b^2*c^3 + b*c^4 - c^5)*cosh(x))*sinh(x))
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*cosh(x)+c*sinh(x)-(b^2-c^2)^(1/2))^(3/2),x, algorithm="giac"
)
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(t_n
ostep),abs((-sqrt(b^2-c^2))*t_nostep+b-c)]Evaluation time: 0.6Unable to div
ide, perhaps due to rounding error%%%{%%%{8,[4,0]%%%}+%%%{16,[3,1]%%%}+%%%{
-8,[3,0]%%%}+%%%{-8,[2,1]%%%}+%%%{-16,[1,3]%%%}+%%%{8,[1,2]%%%}+%%%{-8,[0,4
]%%%}+%%%{8,[0,3]%%%},[6,1]%%%}+%%%{%%{[%%%{12,[1,0]%%%}+%%%{12,[0,1]%%%},0
,%%%{-36,[3,0]%%%}+%%%{-36,[2,1]%%%}+%%%{-12,[2,0]%%%}+%%%{36,[1,2]%%%}+%%%
{36,[0,3]%%%}+%%%{12,[0,2]%%%},0]:[1,0,%%%{-2,[2,0]%%%}+%%%{-2,[1,0]%%%}+%%
%{2,[0,2]%%%}+%%%{2,[0,1]%%%},0,%%%{1,[4,0]%%%}+%%%{-2,[3,0]%%%}+%%%{-2,[2,
2]%%%}+%%%{2,[2,1]%%%}+%%%{1,[2,0]%%%}+%%%{2,[1,2]%%%}+%%%{-2,[1,1]%%%}+%%%
{1,[0,4]%%%}+%%%{-2,[0,3]%%%}+%%%{1,[0,2]%%%}]%%},[4,1]%%%}+%%%{%%%{24,[4,0
]%%%}+%%%{-24,[3,0]%%%}+%%%{-48,[2,2]%%%}+%%%{24,[2,1]%%%}+%%%{24,[1,2]%%%}
+%%%{24,[0,4]%%%}+%%%{-24,[0,3]%%%},[2,1]%%%}+%%%{%%{[%%%{4,[1,0]%%%}+%%%{-
4,[0,1]%%%},0,%%%{-12,[3,0]%%%}+%%%{12,[2,1]%%%}+%%%{-4,[2,0]%%%}+%%%{12,[1
,2]%%%}+%%%{8,[1,1]%%%}+%%%{-12,[0,3]%%%}+%%%{-4,[0,2]%%%},0]:[1,0,%%%{-2,[
```

```
2,0]%%%}+%%%{-2,[1,0]%%%}+%%%{2,[0,2]%%%}+%%%{2,[0,1]%%%},0,%%%{1,[4,0]%%%}
+%%%{-2,[3,0]%%%}+%%%{-2,[2,2]%%%}+%%%{2,[2,1]%%%}+%%%{1,[2,0]%%%}+%%%{2,[1
,2]%%%}+%%%{-2,[1,1]%%%}+%%%{1,[0,4]%%%}+%%%{-2,[0,3]%%%}+%%%{1,[0,2]%%%}]%
%},[0,1]%%%} / %%%{%%%{8,[9,0]%%%}+%%%{24,[8,1]%%%}+%%%{-24,[8,0]%%%}+%%%{-
48,[7,1]%%%}+%%%{24,[7,0]%%%}+%%%{-64,[6,3]%%%}+%%%{48,[6,2]%%%}+%%%{24,[6,
1]%%%}+%%%{-8,[6,0]%%%}+%%%{-48,[5,4]%%%}+%%%{144,[5,3]%%%}+%%%{-72,[5,2]%%
%}+%%%{48,[4,5]%%%}+%%%{-72,[4,3]%%%}+%%%{24,[4,2]%%%}+%%%{64,[3,6]%%%}+%%%
{-144,[3,5]%%%}+%%%{72,[3,4]%%%}+%%%{-48,[2,6]%%%}+%%%{72,[2,5]%%%}+%%%{-24
,[2,4]%%%}+%%%{-24,[1,8]%%%}+%%%{48,[1,7]%%%}+%%%{-24,[1,6]%%%}+%%%{-8,[0,9
]%%%}+%%%{24,[0,8]%%%}+%%%{-24,[0,7]%%%}+%%%{8,[0,6]%%%},[6,0]%%%}+%%%{%%{[
%%%{12,[6,0]%%%}+%%%{24,[5,1]%%%}+%%%{-24,[5,0]%%%}+%%%{-12,[4,2]%%%}+%%%{-
24,[4,1]%%%}+%%%{12,[4,0]%%%}+%%%{-48,[3,3]%%%}+%%%{48,[3,2]%%%}+%%%{-12,[2
,4]%%%}+%%%{48,[2,3]%%%}+%%%{-24,[2,2]%%%}+%%%{24,[1,5]%%%}+%%%{-24,[1,4]%%
%}+%%%{12,[0,6]%%%}+%%%{-24,[0,5]%%%}+%%%{12,[0,4]%%%},0,%%%{-36,[8,0]%%%}+
%%%{-72,[7,1]%%%}+%%%{60,[7,0]%%%}+%%%{72,[6,2]%%%}+%%%{60,[6,1]%%%}+%%%{-1
2,[6,0]%%%}+%%%{216,[5,3]%%%}+%%%{-180,[5,2]%%%}+%%%{-12,[5,0]%%%}+%%%{-180
,[4,3]%%%}+%%%{36,[4,2]%%%}+%%%{12,[4,1]%%%}+%%%{-216,[3,5]%%%}+%%%{180,[3,
4]%%%}+%%%{24,[3,2]%%%}+%%%{-72,[2,6]%%%}+%%%{180,[2,5]%%%}+%%%{-36,[2,4]%%
%}+%%%{-24,[2,3]%%%}+%%%{72,[1,7]%%%}+%%%{-60,[1,6]%%%}+%%%{-12,[1,4]%%%}+%
%%{36,[0,8]%%%}+%%%{-60,[0,7]%%%}+%%%{12,[0,6]%%%}+%%%{12,[0,5]%%%},0]:[1,0
,%%%{-2,[2,0]%%%}+%%%{-2,[1,0]%%%}+%%%{2,[0,2]%%%}+%%%{2,[0,1]%%%},0,%%%{1,
[4,0]%%%}+%%%{-2,[3,0]%%%}+%%%{-2,[2,2]%%%}+%%%{2,[2,1]%%%}+%%%{1,[2,0]%%%}
+%%%{2,[1,2]%%%}+%%%{-2,[1,1]%%%}+%%%{1,[0,4]%%%}+%%%{-2,[0,3]%%%}+%%%{1,[0
,2]%%%}]%%},[4,0]%%%}+%%%{%%%{24,[9,0]%%%}+%%%{24,[8,1]%%%}+%%%{-72,[8,0]%%
%}+%%%{-96,[7,2]%%%}+%%%{72,[7,0]%%%}+%%%{-96,[6,3]%%%}+%%%{288,[6,2]%%%}+%
%%{-72,[6,1]%%%}+%%%{-24,[6,0]%%%}+%%%{144,[5,4]%%%}+%%%{-216,[5,2]%%%}+%%%
{48,[5,1]%%%}+%%%{144,[4,5]%%%}+%%%{-432,[4,4]%%%}+%%%{216,[4,3]%%%}+%%%{24
,[4,2]%%%}+%%%{-96,[3,6]%%%}+%%%{216,[3,4]%%%}+%%%{-96,[3,3]%%%}+%%%{-96,[2
,7]%%%}+%%%{288,[2,6]%%%}+%%%{-216,[2,5]%%%}+%%%{24,[2,4]%%%}+%%%{24,[1,8]%
%%}+%%%{-72,[1,6]%%%}+%%%{48,[1,5]%%%}+%%%{24,[0,9]%%%}+%%%{-72,[0,8]%%%}+%
%%{72,[0,7]%%%}+%%%{-24,[0,6]%%%},[2,0]%%%}+%%%{%%{[%%%{4,[6,0]%%%}+%%%{-8,
[5,0]%%%}+%%%{-12,[4,2]%%%}+%%%{8,[4,1]%%%}+%%%{4,[4,0]%%%}+%%%{16,[3,2]%%%
}+%%%{-8,[3,1]%%%}+%%%{12,[2,4]%%%}+%%%{-16,[2,3]%%%}+%%%{-8,[1,4]%%%}+%%%{
8,[1,3]%%%}+%%%{-4,[0,6]%%%}+%%%{8,[0,5]%%%}+%%%{-4,[0,4]%%%},0,%%%{-12,[8,
0]%%%}+%%%{20,[7,0]%%%}+%%%{48,[6,2]%%%}+%%%{-20,[6,1]%%%}+%%%{-4,[6,0]%%%}
+%%%{-60,[5,2]%%%}+%%%{8,[5,1]%%%}+%%%{-4,[5,0]%%%}+%%%{-72,[4,4]%%%}+%%%{6
0,[4,3]%%%}+%%%{4,[4,2]%%%}+%%%{12,[4,1]%%%}+%%%{60,[3,4]%%%}+%%%{-16,[3,3]
%%%}+%%%{-8,[3,2]%%%}+%%%{48,[2,6]%%%}+%%%{-60,[2,5]%%%}+%%%{4,[2,4]%%%}+%%
%{-8,[2,3]%%%}+%%%{-20,[1,6]%%%}+%%%{8,[1,5]%%%}+%%%{12,[1,4]%%%}+%%%{-12,[
0,8]%%%}+%%%{20,[0,7]%%%}+%%%{-4,[0,6]%%%}+%%%{-4,[0,5]%%%},0]:[1,0,%%%{-2,
[2,0]%%%}+%%%{-2,[1,0]%%%}+%%%{2,[0,2]%%%}+%%%{2,[0,1]%%%},0,%%%{1,[4,0]%%%
}+%%%{-2,[3,0]%%%}+%%%{-2,[2,2]%%%}+%%%{2,[2,1]%%%}+%%%{1,[2,0]%%%}+%%%{2,[
1,2]%%%}+%%%{-2,[1,1]%%%}+%%%{1,[0,4]%%%}+%%%{-2,[0,3]%%%}+%%%{1,[0,2]%%%}]
%%},[0,0]%%%} Error: Bad Argument Value
```

maple [B] time = 1.48, size = 415, normalized size = 2.61

$$\frac{\sqrt{2}\ \operatorname{arctanh}\left(\frac{\cosh(x)\sqrt{2}}{2}\right)}{2\sqrt{b^2-c^2}\ \sqrt{-\frac{\sinh(x)b^2-\sinh(x)c^2+b^2-c^2}{\sqrt{b^2-c^2}}}}-\frac{\sqrt{-\sqrt{b^2-c^2}\ (\sinh(x)+1)\left(\sinh^2(x)\right)}\ \sqrt{2}\ \sqrt{b^2-c^2}\ \left(\ln\left(-\frac{2\left(\cosh(x)\sqrt{b^2-c^2}\ \cdots\right.}{\cdots}\right.\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*cosh(x)+c*sinh(x)-(b^2-c^2)^(1/2))^(3/2),x)
```

```
[Out] 1/2/(b^2-c^2)^(1/2)/(-(sinh(x)*b^2-sinh(x)*c^2+b^2-c^2)/(b^2-c^2)^(1/2))^(1
/2)*2^(1/2)*arctanh(1/2*cosh(x)*2^(1/2))-1/4*(-(b^2-c^2)^(1/2)*(sinh(x)+1)*
sinh(x)^2)^(1/2)*2^(1/2)*(b^2-c^2)^(1/2)*(ln(-2*(cosh(x)*(b^2-c^2)^(1/2)*2^
(1/2)*sinh(x)-sinh(x)*(b^2-c^2)^(1/2)+cosh(x)*(b^2-c^2)^(1/2)*2^(1/2)-(b^2-
c^2)^(1/2)-(-(b^2-c^2)^(1/2)*(sinh(x)+1))^(1/2)*(-(b^2-c^2)^(1/2)*(sinh(x)+
1)*sinh(x)^2)^(1/2))/(cosh(x)-2^(1/2)))-ln(2*(cosh(x)*(b^2-c^2)^(1/2)*2^(1/
2)*sinh(x)+sinh(x)*(b^2-c^2)^(1/2)+cosh(x)*(b^2-c^2)^(1/2)*2^(1/2)+(b^2-c^2
)^(1/2)+(-(b^2-c^2)^(1/2)*(sinh(x)+1))^(1/2)*(-(b^2-c^2)^(1/2)*(sinh(x)+1)*
sinh(x)^2)^(1/2))/(cosh(x)+2^(1/2))))/(b-c)/(b+c)/(-(b^2-c^2)^(1/2)*(sinh(x
)+1))^(1/2)/sinh(x)/(-(sinh(x)*b^2-sinh(x)*c^2+b^2-c^2)/(b^2-c^2)^(1/2))^(1
/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\cosh(x)+c\sinh(x)-\sqrt{b^2-c^2}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*cosh(x)+c*sinh(x)-(b^2-c^2)^(1/2))^(3/2),x, algorithm="maxim
a")
```

```
[Out] integrate((b*cosh(x) + c*sinh(x) - sqrt(b^2 - c^2))^(-3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(b\cosh(x)-\sqrt{b^2-c^2}+c\sinh(x)\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*cosh(x) - (b^2 - c^2)^(1/2) + c*sinh(x))^(3/2),x)
```

[Out] int(1/(b*cosh(x) - (b^2 - c^2)^(1/2) + c*sinh(x))^(3/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\cosh\left(x\right) + c\sinh\left(x\right) - \sqrt{b^2 - c^2}\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*cosh(x)+c*sinh(x)-(b**2-c**2)**(1/2))**(3/2),x)

[Out] Integral((b*cosh(x) + c*sinh(x) - sqrt(b**2 - c**2))**(-3/2), x)

3.779 $\int \frac{1}{\left(-\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{5/2}}\,dx$

Optimal. Leaf size=211

$$-\frac{3\tanh^{-1}\left(\frac{\sqrt[4]{b^2-c^2}\,\sinh\left(x+i\tan^{-1}(b,-ic)\right)}{\sqrt{2}\sqrt{-\sqrt{b^2-c^2}+\sqrt{b^2-c^2}\,\cosh\left(x+i\tan^{-1}(b,-ic)\right)}}\right)}{16\sqrt{2}\left(b^2-c^2\right)^{5/4}}+\frac{3(b\sinh(x)+c\cosh(x))}{16\left(b^2-c^2\right)\left(-\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{3/2}}-\frac{}{4\sqrt{b^2-}}$$

```
[Out] -3/32*arctanh(1/2*(b^2-c^2)^(1/4)*sinh(x+I*arctan(b,-I*c))*2^(1/2)/(-(b^2-c
^2)^(1/2)+cosh(x+I*arctan(b,-I*c))*(b^2-c^2)^(1/2))^(1/2))/(b^2-c^2)^(5/4)*
2^(1/2)+3/16*(c*cosh(x)+b*sinh(x))/(b^2-c^2)/(b*cosh(x)+c*sinh(x)-(b^2-c^2)
^(1/2))^(3/2)+1/4*(-c*cosh(x)-b*sinh(x))/(b*cosh(x)+c*sinh(x)-(b^2-c^2)^(1/
2))^(5/2)/(b^2-c^2)^(1/2)
```

Rubi [A] time = 0.17, antiderivative size = 211, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3116, 3115, 2649, 204}

$$-\frac{3\tanh^{-1}\left(\frac{\sqrt[4]{b^2-c^2}\,\sinh\left(x+i\tan^{-1}(b,-ic)\right)}{\sqrt{2}\sqrt{-\sqrt{b^2-c^2}+\sqrt{b^2-c^2}\,\cosh\left(x+i\tan^{-1}(b,-ic)\right)}}\right)}{16\sqrt{2}\left(b^2-c^2\right)^{5/4}}+\frac{3(b\sinh(x)+c\cosh(x))}{16\left(b^2-c^2\right)\left(-\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{3/2}}-\frac{}{4\sqrt{b^2-}}$$

Antiderivative was successfully verified.

```
[In] Int[(-Sqrt[b^2 - c^2] + b*Cosh[x] + c*Sinh[x])^(-5/2),x]
```

```
[Out] (-3*ArcTanh[((b^2 - c^2)^(1/4)*Sinh[x + I*ArcTan[b, (-I)*c]])/(Sqrt[2]*Sqrt
[-Sqrt[b^2 - c^2] + Sqrt[b^2 - c^2]*Cosh[x + I*ArcTan[b, (-I)*c]]])])/(16*S
qrt[2]*(b^2 - c^2)^(5/4)) - (c*Cosh[x] + b*Sinh[x])/(4*Sqrt[b^2 - c^2]*(-Sq
rt[b^2 - c^2] + b*Cosh[x] + c*Sinh[x])^(5/2)) + (3*(c*Cosh[x] + b*Sinh[x]))
/(16*(b^2 - c^2)*(-Sqrt[b^2 - c^2] + b*Cosh[x] + c*Sinh[x])^(3/2))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3115

```
Int[1/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(
x_)]], x_Symbol] :> Int[1/Sqrt[a + Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b,
c]]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[a^2 - b^2 - c^2, 0]
```

Rule 3116

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> Simp[((c*Cos[d + e*x] - b*Sin[d + e*x])*(a + b*Cos[d + e
*x] + c*Sin[d + e*x])^n)/(a*e*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n + 1)),
Int[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1), x], x] /; FreeQ[{a, b, c
, d, e}, x] && EqQ[a^2 - b^2 - c^2, 0] && LtQ[n, -1]
```

Rubi steps

$$\int \frac{1}{\left(-\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{5/2}}\,dx = -\frac{c\cosh(x)+b\sinh(x)}{4\sqrt{b^2-c^2}\left(-\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{5/2}} - \frac{3\int \frac{}{\left(-\sqrt{b^2}\right.}}{}$$

$$= -\frac{c\cosh(x)+b\sinh(x)}{4\sqrt{b^2-c^2}\left(-\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{5/2}} + \frac{}{16\left(b^2-c\right.}$$

$$= -\frac{c\cosh(x)+b\sinh(x)}{4\sqrt{b^2-c^2}\left(-\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{5/2}} + \frac{}{16\left(b^2-c\right.}$$

$$= -\frac{c\cosh(x)+b\sinh(x)}{4\sqrt{b^2-c^2}\left(-\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{5/2}} + \frac{}{16\left(b^2-c\right.}$$

$$= -\frac{3\tanh^{-1}\left(\frac{\sqrt[4]{b^2-c^2}\sinh\left(x+i\tan^{-1}(b,-ic)\right)}{\sqrt{2}\sqrt{-\sqrt{b^2-c^2}+\sqrt{b^2-c^2}\cosh\left(x+i\tan^{-1}(b,-ic)\right)}}\right)}{16\sqrt{2}\left(b^2-c^2\right)^{5/4}} - \frac{}{4\sqrt{b^2-c^2}}$$

Mathematica [F] time = 180.00, size = 0, normalized size = 0.00

$Aborted

Verification is Not applicable to the result.

```
[In] Integrate[(-Sqrt[b^2 - c^2] + b*Cosh[x] + c*Sinh[x])^(-5/2),x]
[Out] $Aborted
```

fricas [B] time = 0.69, size = 5675, normalized size = 26.90

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*cosh(x)+c*sinh(x)-(b^2-c^2)^(1/2))^(5/2),x, algorithm="frica
s")
```

```
[Out] -1/16*(3*sqrt(1/2)*((b^5 + 5*b^4*c + 10*b^3*c^2 + 10*b^2*c^3 + 5*b*c^4 + c^
5)*cosh(x)^10 + 10*(b^5 + 5*b^4*c + 10*b^3*c^2 + 10*b^2*c^3 + 5*b*c^4 + c^5
)*cosh(x)*sinh(x)^9 + (b^5 + 5*b^4*c + 10*b^3*c^2 + 10*b^2*c^3 + 5*b*c^4 +
c^5)*sinh(x)^10 - 5*(b^5 + 3*b^4*c + 2*b^3*c^2 - 2*b^2*c^3 - 3*b*c^4 - c^5)
*cosh(x)^8 - 5*(b^5 + 3*b^4*c + 2*b^3*c^2 - 2*b^2*c^3 - 3*b*c^4 - c^5 - 9*(
b^5 + 5*b^4*c + 10*b^3*c^2 + 10*b^2*c^3 + 5*b*c^4 + c^5)*cosh(x)^2)*sinh(x)
^8 + 40*(3*(b^5 + 5*b^4*c + 10*b^3*c^2 + 10*b^2*c^3 + 5*b*c^4 + c^5)*cosh(x
)^3 - (b^5 + 3*b^4*c + 2*b^3*c^2 - 2*b^2*c^3 - 3*b*c^4 - c^5)*cosh(x))*sinh
(x)^7 + 10*(b^5 + b^4*c - 2*b^3*c^2 - 2*b^2*c^3 + b*c^4 + c^5)*cosh(x)^6 +
10*(b^5 + b^4*c - 2*b^3*c^2 - 2*b^2*c^3 + b*c^4 + c^5 + 21*(b^5 + 5*b^4*c +
 10*b^3*c^2 + 10*b^2*c^3 + 5*b*c^4 + c^5)*cosh(x)^4 - 14*(b^5 + 3*b^4*c + 2
*b^3*c^2 - 2*b^2*c^3 - 3*b*c^4 - c^5)*cosh(x)^2)*sinh(x)^6 + 4*(63*(b^5 + 5
*b^4*c + 10*b^3*c^2 + 10*b^2*c^3 + 5*b*c^4 + c^5)*cosh(x)^5 - 70*(b^5 + 3*b
^4*c + 2*b^3*c^2 - 2*b^2*c^3 - 3*b*c^4 - c^5)*cosh(x)^3 + 15*(b^5 + b^4*c -
 2*b^3*c^2 - 2*b^2*c^3 + b*c^4 + c^5)*cosh(x))*sinh(x)^5 - b^5 + 5*b^4*c -
10*b^3*c^2 + 10*b^2*c^3 - 5*b*c^4 + c^5 - 10*(b^5 - b^4*c - 2*b^3*c^2 + 2*b
^2*c^3 + b*c^4 - c^5)*cosh(x)^4 + 10*(21*(b^5 + 5*b^4*c + 10*b^3*c^2 + 10*b
^2*c^3 + 5*b*c^4 + c^5)*cosh(x)^6 - b^5 + b^4*c + 2*b^3*c^2 - 2*b^2*c^3 - b
*c^4 + c^5 - 35*(b^5 + 3*b^4*c + 2*b^3*c^2 - 2*b^2*c^3 - 3*b*c^4 - c^5)*cos
h(x)^4 + 15*(b^5 + b^4*c - 2*b^3*c^2 - 2*b^2*c^3 + b*c^4 + c^5)*cosh(x)^2)*
sinh(x)^4 + 40*(3*(b^5 + 5*b^4*c + 10*b^3*c^2 + 10*b^2*c^3 + 5*b*c^4 + c^5)
*cosh(x)^7 - 7*(b^5 + 3*b^4*c + 2*b^3*c^2 - 2*b^2*c^3 - 3*b*c^4 - c^5)*cosh
(x)^5 + 5*(b^5 + b^4*c - 2*b^3*c^2 - 2*b^2*c^3 + b*c^4 + c^5)*cosh(x)^3 - (
b^5 - b^4*c - 2*b^3*c^2 + 2*b^2*c^3 + b*c^4 - c^5)*cosh(x))*sinh(x)^3 + 5*(
b^5 - 3*b^4*c + 2*b^3*c^2 + 2*b^2*c^3 - 3*b*c^4 + c^5)*cosh(x)^2 + 5*(9*(b^
5 + 5*b^4*c + 10*b^3*c^2 + 10*b^2*c^3 + 5*b*c^4 + c^5)*cosh(x)^8 - 28*(b^5
+ 3*b^4*c + 2*b^3*c^2 - 2*b^2*c^3 - 3*b*c^4 - c^5)*cosh(x)^6 + b^5 - 3*b^4*
c + 2*b^3*c^2 + 2*b^2*c^3 - 3*b*c^4 + c^5 + 30*(b^5 + b^4*c - 2*b^3*c^2 - 2
*b^2*c^3 + b*c^4 + c^5)*cosh(x)^4 - 12*(b^5 - b^4*c - 2*b^3*c^2 + 2*b^2*c^3
 + b*c^4 - c^5)*cosh(x)^2)*sinh(x)^2 + 10*((b^5 + 5*b^4*c + 10*b^3*c^2 + 10
*b^2*c^3 + 5*b*c^4 + c^5)*cosh(x)^9 - 4*(b^5 + 3*b^4*c + 2*b^3*c^2 - 2*b^2*
c^3 - 3*b*c^4 - c^5)*cosh(x)^7 + 6*(b^5 + b^4*c - 2*b^3*c^2 - 2*b^2*c^3 + b
*c^4 + c^5)*cosh(x)^5 - 4*(b^5 - b^4*c - 2*b^3*c^2 + 2*b^2*c^3 + b*c^4 - c^
5)*cosh(x)^3 + (b^5 - 3*b^4*c + 2*b^3*c^2 + 2*b^2*c^3 - 3*b*c^4 + c^5)*cosh
(x))*sinh(x))*log(-((b^2 + 2*b*c + c^2)*cosh(x)^4 + 4*(b^2 + 2*b*c + c^2)*c
osh(x)^3*sinh(x) + 6*(b^2 + 2*b*c + c^2)*cosh(x)^2*sinh(x)^2 + 4*(b^2 + 2*b
*c + c^2)*cosh(x)*sinh(x)^3 + (b^2 + 2*b*c + c^2)*sinh(x)^4 - b^2 + 2*b*c -
 c^2 + 2*((b + c)*cosh(x)^3 + 3*(b + c)*cosh(x)*sinh(x)^2 + (b + c)*sinh(x)
^3 - (b - c)*cosh(x) + (3*(b + c)*cosh(x)^2 - b + c)*sinh(x))*sqrt(b^2 - c^
2) + 2*(2*(b^2 - c^2)*cosh(x)^2 + 4*(b^2 - c^2)*cosh(x)*sinh(x) + 2*(b^2 -
c^2)*sinh(x)^2 + ((b + c)*cosh(x)^3 + 3*(b + c)*cosh(x)*sinh(x)^2 + (b + c)
*sinh(x)^3 + (b - c)*cosh(x) + (3*(b + c)*cosh(x)^2 + b - c)*sinh(x))*sqrt(
b^2 - c^2))*sqrt(((b + c)*cosh(x)^2 + 2*(b + c)*cosh(x)*sinh(x) + (b + c)*s
inh(x)^2 - 2*sqrt(b^2 - c^2)*(cosh(x) + sinh(x)) + b - c)/(cosh(x) + sinh(x
)))/(b^2 - c^2)^(1/4))/((b^2 + 2*b*c + c^2)*cosh(x)^4 + 4*(b^2 + 2*b*c + c^
```

2)*cosh(x)*sinh(x)^3 + (b^2 + 2*b*c + c^2)*sinh(x)^4 - 2*(b^2 - c^2)*cosh(x)^2 + 2*(3*(b^2 + 2*b*c + c^2)*cosh(x)^2 - b^2 + c^2)*sinh(x)^2 + b^2 - 2*b*c + c^2 + 4*((b^2 + 2*b*c + c^2)*cosh(x)^3 - (b^2 - c^2)*cosh(x))*sinh(x)))/(b^2 - c^2)^(1/4) - 2*sqrt(1/2)*(3*(b^4 + 4*b^3*c + 6*b^2*c^2 + 4*b*c^3 + c^4)*cosh(x)^9 + 27*(b^4 + 4*b^3*c + 6*b^2*c^2 + 4*b*c^3 + c^4)*cosh(x)*sinh(x)^8 + 3*(b^4 + 4*b^3*c + 6*b^2*c^2 + 4*b*c^3 + c^4)*sinh(x)^9 - 36*(b^4 + 2*b^3*c - 2*b*c^3 - c^4)*cosh(x)^7 - 36*(b^4 + 2*b^3*c - 2*b*c^3 - c^4 - 3*(b^4 + 4*b^3*c + 6*b^2*c^2 + 4*b*c^3 + c^4)*cosh(x)^2)*sinh(x)^7 + 252*((b^4 + 4*b^3*c + 6*b^2*c^2 + 4*b*c^3 + c^4)*cosh(x)^3 - (b^4 + 2*b^3*c - 2*b*c^3 - c^4)*cosh(x))*sinh(x)^6 - 190*(b^4 - 2*b^2*c^2 + c^4)*cosh(x)^5 + 2*(189*(b^4 + 4*b^3*c + 6*b^2*c^2 + 4*b*c^3 + c^4)*cosh(x)^4 - 95*b^4 + 190*b^2*c^2 - 95*c^4 - 378*(b^4 + 2*b^3*c - 2*b*c^3 - c^4)*cosh(x)^2)*sinh(x)^5 + 2*(189*(b^4 + 4*b^3*c + 6*b^2*c^2 + 4*b*c^3 + c^4)*cosh(x)^5 - 630*(b^4 + 2*b^3*c - 2*b*c^3 - c^4)*cosh(x)^3 - 475*(b^4 - 2*b^2*c^2 + c^4)*cosh(x))*sinh(x)^4 - 36*(b^4 - 2*b^3*c + 2*b*c^3 - c^4)*cosh(x)^3 + 4*(63*(b^4 + 4*b^3*c + 6*b^2*c^2 + 4*b*c^3 + c^4)*cosh(x)^6 - 315*(b^4 + 2*b^3*c - 2*b*c^3 - c^4)*cosh(x)^4 - 9*b^4 + 18*b^3*c - 18*b*c^3 + 9*c^4 - 475*(b^4 - 2*b^2*c^2 + c^4)*cosh(x)^2)*sinh(x)^3 + 4*(27*(b^4 + 4*b^3*c + 6*b^2*c^2 + 4*b*c^3 + c^4)*cosh(x)^7 - 189*(b^4 + 2*b^3*c - 2*b*c^3 - c^4)*cosh(x)^5 - 475*(b^4 - 2*b^2*c^2 + c^4)*cosh(x)^3 - 27*(b^4 - 2*b^3*c + 2*b*c^3 - c^4)*cosh(x))*sinh(x)^2 + 3*(b^4 - 4*b^3*c + 6*b^2*c^2 - 4*b*c^3 + c^4)*cosh(x) + (27*(b^4 + 4*b^3*c + 6*b^2*c^2 + 4*b*c^3 + c^4)*cosh(x)^8 - 252*(b^4 + 2*b^3*c - 2*b*c^3 - c^4)*cosh(x)^6 - 950*(b^4 - 2*b^2*c^2 + c^4)*cosh(x)^4 + 3*b^4 - 12*b^3*c + 18*b^2*c^2 - 12*b*c^3 + 3*c^4 - 108*(b^4 - 2*b^3*c + 2*b*c^3 - c^4)*cosh(x)^2)*sinh(x) + 4*((b^3 + 3*b^2*c + 3*b*c^2 + c^3)*cosh(x)^8 + 8*(b^3 + 3*b^2*c + 3*b*c^2 + c^3)*cosh(x)*sinh(x)^7 + (b^3 + 3*b^2*c + 3*b*c^2 + c^3)*sinh(x)^8 - 33*(b^3 + b^2*c - b*c^2 - c^3)*cosh(x)^6 - (33*b^3 + 33*b^2*c - 33*b*c^2 - 33*c^3 - 28*(b^3 + 3*b^2*c + 3*b*c^2 + c^3)*cosh(x)^2)*sinh(x)^6 + 2*(28*(b^3 + 3*b^2*c + 3*b*c^2 + c^3)*cosh(x)^3 - 99*(b^3 + b^2*c - b*c^2 - c^3)*cosh(x))*sinh(x)^5 - 33*(b^3 - b^2*c - b*c^2 + c^3)*cosh(x)^4 + (70*(b^3 + 3*b^2*c + 3*b*c^2 + c^3)*cosh(x)^4 - 33*b^3 + 33*b^2*c + 33*b*c^2 - 33*c^3 - 495*(b^3 + b^2*c - b*c^2 - c^3)*cosh(x)^2)*sinh(x)^4 + 4*(14*(b^3 + 3*b^2*c + 3*b*c^2 + c^3)*cosh(x)^5 - 165*(b^3 + b^2*c - b*c^2 - c^3)*cosh(x)^3 - 33*(b^3 - b^2*c - b*c^2 + c^3)*cosh(x))*sinh(x)^3 + (b^3 - 3*b^2*c + 3*b*c^2 - c^3)*cosh(x)^2 + (28*(b^3 + 3*b^2*c + 3*b*c^2 + c^3)*cosh(x)^6 - 495*(b^3 + b^2*c - b*c^2 - c^3)*cosh(x)^4 + b^3 - 3*b^2*c + 3*b*c^2 - c^3 - 198*(b^3 - b^2*c - b*c^2 + c^3)*cosh(x)^2)*sinh(x)^2 + 2*(4*(b^3 + 3*b^2*c + 3*b*c^2 + c^3)*cosh(x)^7 - 99*(b^3 + b^2*c - b*c^2 - c^3)*cosh(x)^5 - 66*(b^3 - b^2*c - b*c^2 + c^3)*cosh(x)^3 + (b^3 - 3*b^2*c + 3*b*c^2 - c^3)*cosh(x))*sinh(x))*sqrt(b^2 - c^2))*sqrt(((b + c)*cosh(x)^2 + 2*(b + c)*cosh(x)*sinh(x) + (b + c)*sinh(x)^2 - 2*sqrt(b^2 - c^2)*(cosh(x) + sinh(x)) + b - c)/(cosh(x) + sinh(x))))/((b^7 + 5*b^6*c + 9*b^5*c^2 + 5*b^4*c^3 - 5*b^3*c^4 - 9*b^2*c^5 - 5*b*c^6 - c^7)*cosh(x)^10 + 10*(b^7 + 5*b^6*c + 9*b^5*c^2 + 5*b^4*c^3 - 5*b^3*c^4 - 9*b^2*c^5 - 5*b*c^6 - c^7)*cosh(x)*sinh(x)^9 + (b^7 + 5*b^6*c + 9*b^5*c^2 + 5*b^4*c^3 - 5*b^3*c^4 - 9*b^2*c^5

```
 - 5*b*c^6 - c^7)*sinh(x)^10 - 5*(b^7 + 3*b^6*c + b^5*c^2 - 5*b^4*c^3 - 5*b
^3*c^4 + b^2*c^5 + 3*b*c^6 + c^7)*cosh(x)^8 - 5*(b^7 + 3*b^6*c + b^5*c^2 -
5*b^4*c^3 - 5*b^3*c^4 + b^2*c^5 + 3*b*c^6 + c^7 - 9*(b^7 + 5*b^6*c + 9*b^5*
c^2 + 5*b^4*c^3 - 5*b^3*c^4 - 9*b^2*c^5 - 5*b*c^6 - c^7)*cosh(x)^2)*sinh(x)
^8 + 40*(3*(b^7 + 5*b^6*c + 9*b^5*c^2 + 5*b^4*c^3 - 5*b^3*c^4 - 9*b^2*c^5 -
 5*b*c^6 - c^7)*cosh(x)^3 - (b^7 + 3*b^6*c + b^5*c^2 - 5*b^4*c^3 - 5*b^3*c^
4 + b^2*c^5 + 3*b*c^6 + c^7)*cosh(x))*sinh(x)^7 - b^7 + 5*b^6*c - 9*b^5*c^2
 + 5*b^4*c^3 + 5*b^3*c^4 - 9*b^2*c^5 + 5*b*c^6 - c^7 + 10*(b^7 + b^6*c - 3*
b^5*c^2 - 3*b^4*c^3 + 3*b^3*c^4 + 3*b^2*c^5 - b*c^6 - c^7)*cosh(x)^6 + 10*(
b^7 + b^6*c - 3*b^5*c^2 - 3*b^4*c^3 + 3*b^3*c^4 + 3*b^2*c^5 - b*c^6 - c^7 +
 21*(b^7 + 5*b^6*c + 9*b^5*c^2 + 5*b^4*c^3 - 5*b^3*c^4 - 9*b^2*c^5 - 5*b*c^
6 - c^7)*cosh(x)^4 - 14*(b^7 + 3*b^6*c + b^5*c^2 - 5*b^4*c^3 - 5*b^3*c^4 +
b^2*c^5 + 3*b*c^6 + c^7)*cosh(x)^2)*sinh(x)^6 + 4*(63*(b^7 + 5*b^6*c + 9*b^
5*c^2 + 5*b^4*c^3 - 5*b^3*c^4 - 9*b^2*c^5 - 5*b*c^6 - c^7)*cosh(x)^5 - 70*(
b^7 + 3*b^6*c + b^5*c^2 - 5*b^4*c^3 - 5*b^3*c^4 + b^2*c^5 + 3*b*c^6 + c^7)*
cosh(x)^3 + 15*(b^7 + b^6*c - 3*b^5*c^2 - 3*b^4*c^3 + 3*b^3*c^4 + 3*b^2*c^5
 - b*c^6 - c^7)*cosh(x))*sinh(x)^5 - 10*(b^7 - b^6*c - 3*b^5*c^2 + 3*b^4*c^
3 + 3*b^3*c^4 - 3*b^2*c^5 - b*c^6 + c^7)*cosh(x)^4 - 10*(b^7 - b^6*c - 3*b^
5*c^2 + 3*b^4*c^3 + 3*b^3*c^4 - 3*b^2*c^5 - b*c^6 + c^7 - 21*(b^7 + 5*b^6*c
 + 9*b^5*c^2 + 5*b^4*c^3 - 5*b^3*c^4 - 9*b^2*c^5 - 5*b*c^6 - c^7)*cosh(x)^6
 + 35*(b^7 + 3*b^6*c + b^5*c^2 - 5*b^4*c^3 - 5*b^3*c^4 + b^2*c^5 + 3*b*c^6
+ c^7)*cosh(x)^4 - 15*(b^7 + b^6*c - 3*b^5*c^2 - 3*b^4*c^3 + 3*b^3*c^4 + 3*
b^2*c^5 - b*c^6 - c^7)*cosh(x)^2)*sinh(x)^4 + 40*(3*(b^7 + 5*b^6*c + 9*b^5*
c^2 + 5*b^4*c^3 - 5*b^3*c^4 - 9*b^2*c^5 - 5*b*c^6 - c^7)*cosh(x)^7 - 7*(b^7
 + 3*b^6*c + b^5*c^2 - 5*b^4*c^3 - 5*b^3*c^4 + b^2*c^5 + 3*b*c^6 + c^7)*cos
h(x)^5 + 5*(b^7 + b^6*c - 3*b^5*c^2 - 3*b^4*c^3 + 3*b^3*c^4 + 3*b^2*c^5 - b
*c^6 - c^7)*cosh(x)^3 - (b^7 - b^6*c - 3*b^5*c^2 + 3*b^4*c^3 + 3*b^3*c^4 -
3*b^2*c^5 - b*c^6 + c^7)*cosh(x))*sinh(x)^3 + 5*(b^7 - 3*b^6*c + b^5*c^2 +
5*b^4*c^3 - 5*b^3*c^4 - b^2*c^5 + 3*b*c^6 - c^7)*cosh(x)^2 + 5*(9*(b^7 + 5*
b^6*c + 9*b^5*c^2 + 5*b^4*c^3 - 5*b^3*c^4 - 9*b^2*c^5 - 5*b*c^6 - c^7)*cosh
(x)^8 + b^7 - 3*b^6*c + b^5*c^2 + 5*b^4*c^3 - 5*b^3*c^4 - b^2*c^5 + 3*b*c^6
 - c^7 - 28*(b^7 + 3*b^6*c + b^5*c^2 - 5*b^4*c^3 - 5*b^3*c^4 + b^2*c^5 + 3*
b*c^6 + c^7)*cosh(x)^6 + 30*(b^7 + b^6*c - 3*b^5*c^2 - 3*b^4*c^3 + 3*b^3*c^
4 + 3*b^2*c^5 - b*c^6 - c^7)*cosh(x)^4 - 12*(b^7 - b^6*c - 3*b^5*c^2 + 3*b^
4*c^3 + 3*b^3*c^4 - 3*b^2*c^5 - b*c^6 + c^7)*cosh(x)^2)*sinh(x)^2 + 10*((b^
7 + 5*b^6*c + 9*b^5*c^2 + 5*b^4*c^3 - 5*b^3*c^4 - 9*b^2*c^5 - 5*b*c^6 - c^7
)*cosh(x)^9 - 4*(b^7 + 3*b^6*c + b^5*c^2 - 5*b^4*c^3 - 5*b^3*c^4 + b^2*c^5
+ 3*b*c^6 + c^7)*cosh(x)^7 + 6*(b^7 + b^6*c - 3*b^5*c^2 - 3*b^4*c^3 + 3*b^3
*c^4 + 3*b^2*c^5 - b*c^6 - c^7)*cosh(x)^5 - 4*(b^7 - b^6*c - 3*b^5*c^2 + 3*
b^4*c^3 + 3*b^3*c^4 - 3*b^2*c^5 - b*c^6 + c^7)*cosh(x)^3 + (b^7 - 3*b^6*c +
 b^5*c^2 + 5*b^4*c^3 - 5*b^3*c^4 - b^2*c^5 + 3*b*c^6 - c^7)*cosh(x))*sinh(x
))
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*cosh(x)+c*sinh(x)-(b^2-c^2)^(1/2))^(5/2),x, algorithm="giac"
)
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(t_n
ostep),abs((-sqrt(b^2-c^2))*t_nostep+b-c)]Evaluation time: 1.45Unable to di
vide, perhaps due to rounding error%%%{%%%{32,[5,0]%%%}+%%%{96,[4,1]%%%}+%%
%{-32,[4,0]%%%}+%%%{64,[3,2]%%%}+%%%{-64,[3,1]%%%}+%%%{-64,[2,3]%%%}+%%%{-9
6,[1,4]%%%}+%%%{64,[1,3]%%%}+%%%{-32,[0,5]%%%}+%%%{32,[0,4]%%%},[10,1]%%%}+
%%%{%%{[%%%{80,[2,0]%%%}+%%%{160,[1,1]%%%}+%%%{80,[0,2]%%%},0,%%%{-240,[4,0
]%%%}+%%%{-480,[3,1]%%%}+%%%{-80,[3,0]%%%}+%%%{-80,[2,1]%%%}+%%%{480,[1,3]%
%%}+%%%{80,[1,2]%%%}+%%%{240,[0,4]%%%}+%%%{80,[0,3]%%%},0]:[1,0,%%%{-2,[2,0
]%%%}+%%%{-2,[1,0]%%%}+%%%{2,[0,2]%%%}+%%%{2,[0,1]%%%},0,%%%{1,[4,0]%%%}+%%
%{-2,[3,0]%%%}+%%%{-2,[2,2]%%%}+%%%{2,[2,1]%%%}+%%%{1,[2,0]%%%}+%%%{2,[1,2]
%%%}+%%%{-2,[1,1]%%%}+%%%{1,[0,4]%%%}+%%%{-2,[0,3]%%%}+%%%{1,[0,2]%%%}]%%},
[8,1]%%%}+%%%{%%%{320,[5,0]%%%}+%%%{320,[4,1]%%%}+%%%{-320,[4,0]%%%}+%%%{-6
40,[3,2]%%%}+%%%{-640,[2,3]%%%}+%%%{640,[2,2]%%%}+%%%{320,[1,4]%%%}+%%%{320
,[0,5]%%%}+%%%{-320,[0,4]%%%},[6,1]%%%}+%%%{%%{[%%%{160,[2,0]%%%}+%%%{-160,
[0,2]%%%},0,%%%{-480,[4,0]%%%}+%%%{-160,[3,0]%%%}+%%%{960,[2,2]%%%}+%%%{160
,[2,1]%%%}+%%%{160,[1,2]%%%}+%%%{-480,[0,4]%%%}+%%%{-160,[0,3]%%%},0]:[1,0,
%%%{-2,[2,0]%%%}+%%%{-2,[1,0]%%%}+%%%{2,[0,2]%%%}+%%%{2,[0,1]%%%},0,%%%{1,[
4,0]%%%}+%%%{-2,[3,0]%%%}+%%%{-2,[2,2]%%%}+%%%{2,[2,1]%%%}+%%%{1,[2,0]%%%}+
%%%{2,[1,2]%%%}+%%%{-2,[1,1]%%%}+%%%{1,[0,4]%%%}+%%%{-2,[0,3]%%%}+%%%{1,[0,
2]%%%}]%%},[4,1]%%%}+%%%{%%%{160,[5,0]%%%}+%%%{-160,[4,1]%%%}+%%%{-160,[4,0
]%%%}+%%%{-320,[3,2]%%%}+%%%{320,[3,1]%%%}+%%%{320,[2,3]%%%}+%%%{160,[1,4]%
%%}+%%%{-320,[1,3]%%%}+%%%{-160,[0,5]%%%}+%%%{160,[0,4]%%%},[2,1]%%%}+%%%{%
%{[%%%{16,[2,0]%%%}+%%%{-32,[1,1]%%%}+%%%{16,[0,2]%%%},0,%%%{-48,[4,0]%%%}+
%%%{96,[3,1]%%%}+%%%{-16,[3,0]%%%}+%%%{48,[2,1]%%%}+%%%{-96,[1,3]%%%}+%%%{-
48,[1,2]%%%}+%%%{48,[0,4]%%%}+%%%{16,[0,3]%%%},0]:[1,0,%%%{-2,[2,0]%%%}+%%%
{-2,[1,0]%%%}+%%%{2,[0,2]%%%}+%%%{2,[0,1]%%%},0,%%%{1,[4,0]%%%}+%%%{-2,[3,0
]%%%}+%%%{-2,[2,2]%%%}+%%%{2,[2,1]%%%}+%%%{1,[2,0]%%%}+%%%{2,[1,2]%%%}+%%%{
-2,[1,1]%%%}+%%%{1,[0,4]%%%}+%%%{-2,[0,3]%%%}+%%%{1,[0,2]%%%}]%%},[0,1]%%%}
 / %%%{%%%{32,[15,0]%%%}+%%%{160,[14,1]%%%}+%%%{-160,[14,0]%%%}+%%%{160,[13
,2]%%%}+%%%{-640,[13,1]%%%}+%%%{320,[13,0]%%%}+%%%{-480,[12,3]%%%}+%%%{-160
,[12,2]%%%}+%%%{960,[12,1]%%%}+%%%{-320,[12,0]%%%}+%%%{-1120,[11,4]%%%}+%%%
{2560,[11,3]%%%}+%%%{-640,[11,2]%%%}+%%%{-640,[11,1]%%%}+%%%{160,[11,0]%%%}
+%%%{32,[10,5]%%%}+%%%{3040,[10,4]%%%}+%%%{-4480,[10,3]%%%}+%%%{1280,[10,2]
%%%}+%%%{160,[10,1]%%%}+%%%{-32,[10,0]%%%}+%%%{2080,[9,6]%%%}+%%%{-3200,[9,
5]%%%}+%%%{-1600,[9,4]%%%}+%%%{3200,[9,3]%%%}+%%%{-800,[9,2]%%%}+%%%{1440,[
8,7]%%%}+%%%{-7200,[8,6]%%%}+%%%{8000,[8,5]%%%}+%%%{-1600,[8,4]%%%}+%%%{-80
0,[8,3]%%%}+%%%{160,[8,2]%%%}+%%%{-1440,[7,8]%%%}+%%%{6400,[7,6]%%%}+%%%{-6
400,[7,5]%%%}+%%%{1600,[7,4]%%%}+%%%{-2080,[6,9]%%%}+%%%{7200,[6,8]%%%}+%%%
```

{-6400,[6,7]%%%}+%%%{1600,[6,5]%%%}+%%%{-320,[6,4]%%%}+%%%{-32,[5,10]%%%}+%
%%{3200,[5,9]%%%}+%%%{-8000,[5,8]%%%}+%%%{6400,[5,7]%%%}+%%%{-1600,[5,6]%%%
}+%%%{1120,[4,11]%%%}+%%%{-3040,[4,10]%%%}+%%%{1600,[4,9]%%%}+%%%{1600,[4,8
]%%%}+%%%{-1600,[4,7]%%%}+%%%{320,[4,6]%%%}+%%%{480,[3,12]%%%}+%%%{-2560,[3
,11]%%%}+%%%{4480,[3,10]%%%}+%%%{-3200,[3,9]%%%}+%%%{800,[3,8]%%%}+%%%{-160
,[2,13]%%%}+%%%{160,[2,12]%%%}+%%%{640,[2,11]%%%}+%%%{-1280,[2,10]%%%}+%%%{
800,[2,9]%%%}+%%%{-160,[2,8]%%%}+%%%{-160,[1,14]%%%}+%%%{640,[1,13]%%%}+%%%
{-960,[1,12]%%%}+%%%{640,[1,11]%%%}+%%%{-160,[1,10]%%%}+%%%{-32,[0,15]%%%}+
%%%{160,[0,14]%%%}+%%%{-320,[0,13]%%%}+%%%{320,[0,12]%%%}+%%%{-160,[0,11]%%
%}+%%%{32,[0,10]%%%},[10,0]%%%}+%%%{%%{[%%%{80,[12,0]%%%}+%%%{320,[11,1]%%%
}+%%%{-320,[11,0]%%%}+%%%{160,[10,2]%%%}+%%%{-960,[10,1]%%%}+%%%{480,[10,0]
%%%}+%%%{-960,[9,3]%%%}+%%%{320,[9,2]%%%}+%%%{960,[9,1]%%%}+%%%{-320,[9,0]%
%%}+%%%{-1360,[8,4]%%%}+%%%{3520,[8,3]%%%}+%%%{-1440,[8,2]%%%}+%%%{-320,[8,
1]%%%}+%%%{80,[8,0]%%%}+%%%{640,[7,5]%%%}+%%%{1920,[7,4]%%%}+%%%{-3840,[7,3
]%%%}+%%%{1280,[7,2]%%%}+%%%{2240,[6,6]%%%}+%%%{-4480,[6,5]%%%}+%%%{960,[6,
4]%%%}+%%%{1280,[6,3]%%%}+%%%{-320,[6,2]%%%}+%%%{640,[5,7]%%%}+%%%{-4480,[5
,6]%%%}+%%%{5760,[5,5]%%%}+%%%{-1920,[5,4]%%%}+%%%{-1360,[4,8]%%%}+%%%{1920
,[4,7]%%%}+%%%{960,[4,6]%%%}+%%%{-1920,[4,5]%%%}+%%%{480,[4,4]%%%}+%%%{-960
,[3,9]%%%}+%%%{3520,[3,8]%%%}+%%%{-3840,[3,7]%%%}+%%%{1280,[3,6]%%%}+%%%{16
0,[2,10]%%%}+%%%{320,[2,9]%%%}+%%%{-1440,[2,8]%%%}+%%%{1280,[2,7]%%%}+%%%{-
320,[2,6]%%%}+%%%{320,[1,11]%%%}+%%%{-960,[1,10]%%%}+%%%{960,[1,9]%%%}+%%%{
-320,[1,8]%%%}+%%%{80,[0,12]%%%}+%%%{-320,[0,11]%%%}+%%%{480,[0,10]%%%}+%%%
{-320,[0,9]%%%}+%%%{80,[0,8]%%%},0,%%%{-240,[14,0]%%%}+%%%{-960,[13,1]%%%}+
%%%{880,[13,0]%%%}+%%%{-240,[12,2]%%%}+%%%{2640,[12,1]%%%}+%%%{-1120,[12,0]
%%%}+%%%{3840,[11,3]%%%}+%%%{-1760,[11,2]%%%}+%%%{-2240,[11,1]%%%}+%%%{480,
[11,0]%%%}+%%%{4560,[10,4]%%%}+%%%{-12320,[10,3]%%%}+%%%{4480,[10,2]%%%}+%%
%{480,[10,1]%%%}+%%%{80,[10,0]%%%}+%%%{-4800,[9,5]%%%}+%%%{-4400,[9,4]%%%}+
%%%{11200,[9,3]%%%}+%%%{-2400,[9,2]%%%}+%%%{-80,[9,0]%%%}+%%%{-10800,[8,6]%
%%}+%%%{22000,[8,5]%%%}+%%%{-5600,[8,4]%%%}+%%%{-2400,[8,3]%%%}+%%%{-400,[8
,2]%%%}+%%%{80,[8,1]%%%}+%%%{17600,[7,6]%%%}+%%%{-22400,[7,5]%%%}+%%%{4800,
[7,4]%%%}+%%%{320,[7,2]%%%}+%%%{10800,[6,8]%%%}+%%%{-17600,[6,7]%%%}+%%%{48
00,[6,5]%%%}+%%%{800,[6,4]%%%}+%%%{-320,[6,3]%%%}+%%%{4800,[5,9]%%%}+%%%{-2
2000,[5,8]%%%}+%%%{22400,[5,7]%%%}+%%%{-4800,[5,6]%%%}+%%%{-480,[5,4]%%%}+%
%%{-4560,[4,10]%%%}+%%%{4400,[4,9]%%%}+%%%{5600,[4,8]%%%}+%%%{-4800,[4,7]%%
%}+%%%{-800,[4,6]%%%}+%%%{480,[4,5]%%%}+%%%{-3840,[3,11]%%%}+%%%{12320,[3,1
0]%%%}+%%%{-11200,[3,9]%%%}+%%%{2400,[3,8]%%%}+%%%{320,[3,6]%%%}+%%%{240,[2
,12]%%%}+%%%{1760,[2,11]%%%}+%%%{-4480,[2,10]%%%}+%%%{2400,[2,9]%%%}+%%%{40
0,[2,8]%%%}+%%%{-320,[2,7]%%%}+%%%{960,[1,13]%%%}+%%%{-2640,[1,12]%%%}+%%%{
2240,[1,11]%%%}+%%%{-480,[1,10]%%%}+%%%{-80,[1,8]%%%}+%%%{240,[0,14]%%%}+%%
%{-880,[0,13]%%%}+%%%{1120,[0,12]%%%}+%%%{-480,[0,11]%%%}+%%%{-80,[0,10]%%%
}+%%%{80,[0,9]%%%},0]:[1,0,%%%{-2,[2,0]%%%}+%%%{-2,[1,0]%%%}+%%%{2,[0,2]%%%
}+%%%{2,[0,1]%%%},0,%%%{1,[4,0]%%%}+%%%{-2,[3,0]%%%}+%%%{-2,[2,2]%%%}+%%%{2
,[2,1]%%%}+%%%{1,[2,0]%%%}+%%%{2,[1,2]%%%}+%%%{-2,[1,1]%%%}+%%%{1,[0,4]%%%}
+%%%{-2,[0,3]%%%}+%%%{1,[0,2]%%%}]%%},[8,0]%%%}+%%%{%%%{320,[15,0]%%%}+%%%{
960,[14,1]%%%}+%%%{-1600,[14,0]%%%}+%%%{-960,[13,2]%%%}+%%%{-3200,[13,1]%%%

}+%%%{3200,[13,0]%%%}+%%%{-5440,[12,3]%%%}+%%%{8000,[12,2]%%%}+%%%{3200,[12
,1]%%%}+%%%{-3200,[12,0]%%%}+%%%{-960,[11,4]%%%}+%%%{19200,[11,3]%%%}+%%%{-
19200,[11,2]%%%}+%%%{1600,[11,0]%%%}+%%%{12480,[10,5]%%%}+%%%{-14400,[10,4]
%%%}+%%%{-19200,[10,3]%%%}+%%%{19200,[10,2]%%%}+%%%{-1600,[10,1]%%%}+%%%{-3
20,[10,0]%%%}+%%%{8000,[9,6]%%%}+%%%{-48000,[9,5]%%%}+%%%{48000,[9,4]%%%}+%
%%{-8000,[9,2]%%%}+%%%{640,[9,1]%%%}+%%%{-14400,[8,7]%%%}+%%%{8000,[8,6]%%%
}+%%%{48000,[8,5]%%%}+%%%{-48000,[8,4]%%%}+%%%{8000,[8,3]%%%}+%%%{960,[8,2]
%%%}+%%%{-14400,[7,8]%%%}+%%%{64000,[7,7]%%%}+%%%{-64000,[7,6]%%%}+%%%{1600
0,[7,4]%%%}+%%%{-2560,[7,3]%%%}+%%%{8000,[6,9]%%%}+%%%{8000,[6,8]%%%}+%%%{-
64000,[6,7]%%%}+%%%{64000,[6,6]%%%}+%%%{-16000,[6,5]%%%}+%%%{-640,[6,4]%%%}
+%%%{12480,[5,10]%%%}+%%%{-48000,[5,9]%%%}+%%%{48000,[5,8]%%%}+%%%{-16000,[
5,6]%%%}+%%%{3840,[5,5]%%%}+%%%{-960,[4,11]%%%}+%%%{-14400,[4,10]%%%}+%%%{4
8000,[4,9]%%%}+%%%{-48000,[4,8]%%%}+%%%{16000,[4,7]%%%}+%%%{-640,[4,6]%%%}+
%%%{-5440,[3,12]%%%}+%%%{19200,[3,11]%%%}+%%%{-19200,[3,10]%%%}+%%%{8000,[3
,8]%%%}+%%%{-2560,[3,7]%%%}+%%%{-960,[2,13]%%%}+%%%{8000,[2,12]%%%}+%%%{-19
200,[2,11]%%%}+%%%{19200,[2,10]%%%}+%%%{-8000,[2,9]%%%}+%%%{960,[2,8]%%%}+%
%%{960,[1,14]%%%}+%%%{-3200,[1,13]%%%}+%%%{3200,[1,12]%%%}+%%%{-1600,[1,10]
%%%}+%%%{640,[1,9]%%%}+%%%{320,[0,15]%%%}+%%%{-1600,[0,14]%%%}+%%%{3200,[0,
13]%%%}+%%%{-3200,[0,12]%%%}+%%%{1600,[0,11]%%%}+%%%{-320,[0,10]%%%},[6,0]%
%%}+%%%{%%{[%%%{160,[12,0]%%%}+%%%{320,[11,1]%%%}+%%%{-640,[11,0]%%%}+%%%{-
640,[10,2]%%%}+%%%{-640,[10,1]%%%}+%%%{960,[10,0]%%%}+%%%{-1600,[9,3]%%%}+%
%%{3200,[9,2]%%%}+%%%{-640,[9,0]%%%}+%%%{800,[8,4]%%%}+%%%{3200,[8,3]%%%}+%
%%{-4800,[8,2]%%%}+%%%{640,[8,1]%%%}+%%%{160,[8,0]%%%}+%%%{3200,[7,5]%%%}+%
%%{-6400,[7,4]%%%}+%%%{2560,[7,2]%%%}+%%%{-320,[7,1]%%%}+%%%{-6400,[6,5]%%%
}+%%%{9600,[6,4]%%%}+%%%{-2560,[6,3]%%%}+%%%{-320,[6,2]%%%}+%%%{-3200,[5,7]
%%%}+%%%{6400,[5,6]%%%}+%%%{-3840,[5,4]%%%}+%%%{960,[5,3]%%%}+%%%{-800,[4,8
]%%%}+%%%{6400,[4,7]%%%}+%%%{-9600,[4,6]%%%}+%%%{3840,[4,5]%%%}+%%%{1600,[3
,9]%%%}+%%%{-3200,[3,8]%%%}+%%%{2560,[3,6]%%%}+%%%{-960,[3,5]%%%}+%%%{640,[
2,10]%%%}+%%%{-3200,[2,9]%%%}+%%%{4800,[2,8]%%%}+%%%{-2560,[2,7]%%%}+%%%{32
0,[2,6]%%%}+%%%{-320,[1,11]%%%}+%%%{640,[1,10]%%%}+%%%{-640,[1,8]%%%}+%%%{3
20,[1,7]%%%}+%%%{-160,[0,12]%%%}+%%%{640,[0,11]%%%}+%%%{-960,[0,10]%%%}+%%%
{640,[0,9]%%%}+%%%{-160,[0,8]%%%},0,%%%{-480,[14,0]%%%}+%%%{-960,[13,1]%%%}
+%%%{1760,[13,0]%%%}+%%%{2400,[12,2]%%%}+%%%{1760,[12,1]%%%}+%%%{-2240,[12,
0]%%%}+%%%{5760,[11,3]%%%}+%%%{-10560,[11,2]%%%}+%%%{960,[11,0]%%%}+%%%{-43
20,[10,4]%%%}+%%%{-10560,[10,3]%%%}+%%%{13440,[10,2]%%%}+%%%{-960,[10,1]%%%
}+%%%{160,[10,0]%%%}+%%%{-14400,[9,5]%%%}+%%%{26400,[9,4]%%%}+%%%{-4800,[9,
2]%%%}+%%%{-320,[9,1]%%%}+%%%{-160,[9,0]%%%}+%%%{2400,[8,6]%%%}+%%%{26400,[
8,5]%%%}+%%%{-33600,[8,4]%%%}+%%%{4800,[8,3]%%%}+%%%{-480,[8,2]%%%}+%%%{480
,[8,1]%%%}+%%%{19200,[7,7]%%%}+%%%{-35200,[7,6]%%%}+%%%{9600,[7,4]%%%}+%%%{
1280,[7,3]%%%}+%%%{2400,[6,8]%%%}+%%%{-35200,[6,7]%%%}+%%%{44800,[6,6]%%%}+
%%%{-9600,[6,5]%%%}+%%%{320,[6,4]%%%}+%%%{-1280,[6,3]%%%}+%%%{-14400,[5,9]%
%%}+%%%{26400,[5,8]%%%}+%%%{-9600,[5,6]%%%}+%%%{-1920,[5,5]%%%}+%%%{960,[5,
4]%%%}+%%%{-4320,[4,10]%%%}+%%%{26400,[4,9]%%%}+%%%{-33600,[4,8]%%%}+%%%{96
00,[4,7]%%%}+%%%{320,[4,6]%%%}+%%%{960,[4,5]%%%}+%%%{5760,[3,11]%%%}+%%%{-1
0560,[3,10]%%%}+%%%{4800,[3,8]%%%}+%%%{1280,[3,7]%%%}+%%%{-1280,[3,6]%%%}+%

%%{2400,[2,12]%%%}+%%%{-10560,[2,11]%%%}+%%%{13440,[2,10]%%%}+%%%{-4800,[2,
9]%%%}+%%%{-480,[2,8]%%%}+%%%{-960,[1,13]%%%}+%%%{1760,[1,12]%%%}+%%%{-960,
[1,10]%%%}+%%%{-320,[1,9]%%%}+%%%{480,[1,8]%%%}+%%%{-480,[0,14]%%%}+%%%{176
0,[0,13]%%%}+%%%{-2240,[0,12]%%%}+%%%{960,[0,11]%%%}+%%%{160,[0,10]%%%}+%%%
{-160,[0,9]%%%},0]:[1,0,%%%{-2,[2,0]%%%}+%%%{-2,[1,0]%%%}+%%%{2,[0,2]%%%}+%
%%{2,[0,1]%%%},0,%%%{1,[4,0]%%%}+%%%{-2,[3,0]%%%}+%%%{-2,[2,2]%%%}+%%%{2,[2
,1]%%%}+%%%{1,[2,0]%%%}+%%%{2,[1,2]%%%}+%%%{-2,[1,1]%%%}+%%%{1,[0,4]%%%}+%%
%{-2,[0,3]%%%}+%%%{1,[0,2]%%%}]%%},[4,0]%%%}+%%%{%%%{160,[15,0]%%%}+%%%{160
,[14,1]%%%}+%%%{-800,[14,0]%%%}+%%%{-1120,[13,2]%%%}+%%%{1600,[13,0]%%%}+%%
%{-1120,[12,3]%%%}+%%%{5600,[12,2]%%%}+%%%{-1600,[12,1]%%%}+%%%{-1600,[12,0
]%%%}+%%%{3360,[11,4]%%%}+%%%{-9600,[11,2]%%%}+%%%{3200,[11,1]%%%}+%%%{800,
[11,0]%%%}+%%%{3360,[10,5]%%%}+%%%{-16800,[10,4]%%%}+%%%{9600,[10,3]%%%}+%%
%{6400,[10,2]%%%}+%%%{-2400,[10,1]%%%}+%%%{-160,[10,0]%%%}+%%%{-5600,[9,6]%
%%}+%%%{24000,[9,4]%%%}+%%%{-16000,[9,3]%%%}+%%%{-800,[9,2]%%%}+%%%{640,[9,
1]%%%}+%%%{-5600,[8,7]%%%}+%%%{28000,[8,6]%%%}+%%%{-24000,[8,5]%%%}+%%%{-80
00,[8,4]%%%}+%%%{8800,[8,3]%%%}+%%%{-480,[8,2]%%%}+%%%{5600,[7,8]%%%}+%%%{-
32000,[7,6]%%%}+%%%{32000,[7,5]%%%}+%%%{-4800,[7,4]%%%}+%%%{-1280,[7,3]%%%}
+%%%{5600,[6,9]%%%}+%%%{-28000,[6,8]%%%}+%%%{32000,[6,7]%%%}+%%%{-11200,[6,
5]%%%}+%%%{2240,[6,4]%%%}+%%%{-3360,[5,10]%%%}+%%%{24000,[5,8]%%%}+%%%{-320
00,[5,7]%%%}+%%%{11200,[5,6]%%%}+%%%{-3360,[4,11]%%%}+%%%{16800,[4,10]%%%}+
%%%{-24000,[4,9]%%%}+%%%{8000,[4,8]%%%}+%%%{4800,[4,7]%%%}+%%%{-2240,[4,6]%
%%}+%%%{1120,[3,12]%%%}+%%%{-9600,[3,10]%%%}+%%%{16000,[3,9]%%%}+%%%{-8800,
[3,8]%%%}+%%%{1280,[3,7]%%%}+%%%{1120,[2,13]%%%}+%%%{-5600,[2,12]%%%}+%%%{9
600,[2,11]%%%}+%%%{-6400,[2,10]%%%}+%%%{800,[2,9]%%%}+%%%{480,[2,8]%%%}+%%%
{-160,[1,14]%%%}+%%%{1600,[1,12]%%%}+%%%{-3200,[1,11]%%%}+%%%{2400,[1,10]%%
%}+%%%{-640,[1,9]%%%}+%%%{-160,[0,15]%%%}+%%%{800,[0,14]%%%}+%%%{-1600,[0,1
3]%%%}+%%%{1600,[0,12]%%%}+%%%{-800,[0,11]%%%}+%%%{160,[0,10]%%%},[2,0]%%%}
+%%%{%%{[%%%{16,[12,0]%%%}+%%%{-64,[11,0]%%%}+%%%{-96,[10,2]%%%}+%%%{64,[10
,1]%%%}+%%%{96,[10,0]%%%}+%%%{320,[9,2]%%%}+%%%{-192,[9,1]%%%}+%%%{-64,[9,0
]%%%}+%%%{240,[8,4]%%%}+%%%{-320,[8,3]%%%}+%%%{-288,[8,2]%%%}+%%%{192,[8,1]
%%%}+%%%{16,[8,0]%%%}+%%%{-640,[7,4]%%%}+%%%{768,[7,3]%%%}+%%%{-64,[7,1]%%%
}+%%%{-320,[6,6]%%%}+%%%{640,[6,5]%%%}+%%%{192,[6,4]%%%}+%%%{-512,[6,3]%%%}
+%%%{64,[6,2]%%%}+%%%{640,[5,6]%%%}+%%%{-1152,[5,5]%%%}+%%%{384,[5,4]%%%}+%
%%{64,[5,3]%%%}+%%%{240,[4,8]%%%}+%%%{-640,[4,7]%%%}+%%%{192,[4,6]%%%}+%%%{
384,[4,5]%%%}+%%%{-160,[4,4]%%%}+%%%{-320,[3,8]%%%}+%%%{768,[3,7]%%%}+%%%{-
512,[3,6]%%%}+%%%{64,[3,5]%%%}+%%%{-96,[2,10]%%%}+%%%{320,[2,9]%%%}+%%%{-28
8,[2,8]%%%}+%%%{64,[2,6]%%%}+%%%{64,[1,10]%%%}+%%%{-192,[1,9]%%%}+%%%{192,[
1,8]%%%}+%%%{-64,[1,7]%%%}+%%%{16,[0,12]%%%}+%%%{-64,[0,11]%%%}+%%%{96,[0,1
0]%%%}+%%%{-64,[0,9]%%%}+%%%{16,[0,8]%%%},0,%%%{-48,[14,0]%%%}+%%%{176,[13,
0]%%%}+%%%{336,[12,2]%%%}+%%%{-176,[12,1]%%%}+%%%{-224,[12,0]%%%}+%%%{-1056
,[11,2]%%%}+%%%{448,[11,1]%%%}+%%%{96,[11,0]%%%}+%%%{-1008,[10,4]%%%}+%%%{1
056,[10,3]%%%}+%%%{896,[10,2]%%%}+%%%{-288,[10,1]%%%}+%%%{16,[10,0]%%%}+%%%
{2640,[9,4]%%%}+%%%{-2240,[9,3]%%%}+%%%{-96,[9,2]%%%}+%%%{-64,[9,1]%%%}+%%%
{-16,[9,0]%%%}+%%%{1680,[8,6]%%%}+%%%{-2640,[8,5]%%%}+%%%{-1120,[8,4]%%%}+%
%%{1056,[8,3]%%%}+%%%{48,[8,2]%%%}+%%%{80,[8,1]%%%}+%%%{-3520,[7,6]%%%}+%%%

```
{4480,[7,5]%%%}+%%%{-576,[7,4]%%%}+%%%{128,[7,3]%%%}+%%%{-128,[7,2]%%%}+%%%
{-1680,[6,8]%%%}+%%%{3520,[6,7]%%%}+%%%{-1344,[6,5]%%%}+%%%{-224,[6,4]%%%}+
%%%{2640,[5,8]%%%}+%%%{-4480,[5,7]%%%}+%%%{1344,[5,6]%%%}+%%%{224,[5,4]%%%}
+%%%{1008,[4,10]%%%}+%%%{-2640,[4,9]%%%}+%%%{1120,[4,8]%%%}+%%%{576,[4,7]%%
%}+%%%{224,[4,6]%%%}+%%%{-224,[4,5]%%%}+%%%{-1056,[3,10]%%%}+%%%{2240,[3,9]
%%%}+%%%{-1056,[3,8]%%%}+%%%{-128,[3,7]%%%}+%%%{-336,[2,12]%%%}+%%%{1056,[2
,11]%%%}+%%%{-896,[2,10]%%%}+%%%{96,[2,9]%%%}+%%%{-48,[2,8]%%%}+%%%{128,[2,
7]%%%}+%%%{176,[1,12]%%%}+%%%{-448,[1,11]%%%}+%%%{288,[1,10]%%%}+%%%{64,[1,
9]%%%}+%%%{-80,[1,8]%%%}+%%%{48,[0,14]%%%}+%%%{-176,[0,13]%%%}+%%%{224,[0,1
2]%%%}+%%%{-96,[0,11]%%%}+%%%{-16,[0,10]%%%}+%%%{16,[0,9]%%%},0]:[1,0,%%%{-
2,[2,0]%%%}+%%%{-2,[1,0]%%%}+%%%{2,[0,2]%%%}+%%%{2,[0,1]%%%},0,%%%{1,[4,0]%
%%}+%%%{-2,[3,0]%%%}+%%%{-2,[2,2]%%%}+%%%{2,[2,1]%%%}+%%%{1,[2,0]%%%}+%%%{2
,[1,2]%%%}+%%%{-2,[1,1]%%%}+%%%{1,[0,4]%%%}+%%%{-2,[0,3]%%%}+%%%{1,[0,2]%%%
}]%%},[0,0]%%%} Error: Bad Argument Value
```

maple [B] time = 1.92, size = 984, normalized size = 4.66

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*cosh(x)+c*sinh(x)-(b^2-c^2)^(1/2))^(5/2),x)
```

```
[Out] -1/8/(sinh(x)+1)/sinh(x)/((-b^2+c^2)/(b^2-c^2)^(1/2)*sinh(x)+(-b^2+c^2)/(b^
2-c^2)^(1/2))^(1/2)/(-sinh(x)*(b^2-c^2)^(1/2)-(b^2-c^2)^(1/2))^(1/2)/(b^2-c
^2)*(2*2^(1/2)*arctanh(1/2*cosh(x)*2^(1/2))*(-sinh(x)*(b^2-c^2)^(1/2)-(b^2-
c^2)^(1/2))^(1/2)*sinh(x)^2+2^(1/2)*(-(b^2-c^2)^(1/2)*sinh(x)^3-(b^2-c^2)^(
1/2)*sinh(x)^2)^(1/2)*ln(2/(cosh(x)+2^(1/2))*(cosh(x)*(b^2-c^2)^(1/2)*2^(1/
2)*sinh(x)+cosh(x)*(b^2-c^2)^(1/2)*2^(1/2)+sinh(x)*(b^2-c^2)^(1/2)+(-sinh(x
)*(b^2-c^2)^(1/2)-(b^2-c^2)^(1/2))^(1/2)*(-(b^2-c^2)^(1/2)*sinh(x)^3-(b^2-c
^2)^(1/2)*sinh(x)^2)^(1/2)+(b^2-c^2)^(1/2)))*sinh(x)-2^(1/2)*(-(b^2-c^2)^(1
/2)*sinh(x)^3-(b^2-c^2)^(1/2)*sinh(x)^2)^(1/2)*ln(2/(-cosh(x)+2^(1/2))*(cos
h(x)*(b^2-c^2)^(1/2)*2^(1/2)*sinh(x)+cosh(x)*(b^2-c^2)^(1/2)*2^(1/2)-sinh(x
)*(b^2-c^2)^(1/2)-(-sinh(x)*(b^2-c^2)^(1/2)-(b^2-c^2)^(1/2))^(1/2)*(-(b^2-c
^2)^(1/2)*sinh(x)^3-(b^2-c^2)^(1/2)*sinh(x)^2)^(1/2)-(b^2-c^2)^(1/2)))*sinh
(x)+2*2^(1/2)*arctanh(1/2*cosh(x)*2^(1/2))*(-sinh(x)*(b^2-c^2)^(1/2)-(b^2-c
^2)^(1/2))^(1/2)*sinh(x)+2^(1/2)*(-(b^2-c^2)^(1/2)*sinh(x)^3-(b^2-c^2)^(1/2
)*sinh(x)^2)^(1/2)*ln(2/(cosh(x)+2^(1/2))*(cosh(x)*(b^2-c^2)^(1/2)*2^(1/2)*
sinh(x)+cosh(x)*(b^2-c^2)^(1/2)*2^(1/2)+sinh(x)*(b^2-c^2)^(1/2)+(-sinh(x)*(
b^2-c^2)^(1/2)-(b^2-c^2)^(1/2))^(1/2)*(-(b^2-c^2)^(1/2)*sinh(x)^3-(b^2-c^2)
^(1/2)*sinh(x)^2)^(1/2)+(b^2-c^2)^(1/2)))-2^(1/2)*(-(b^2-c^2)^(1/2)*sinh(x)
^3-(b^2-c^2)^(1/2)*sinh(x)^2)^(1/2)*ln(2/(-cosh(x)+2^(1/2))*(cosh(x)*(b^2-c
^2)^(1/2)*2^(1/2)*sinh(x)+cosh(x)*(b^2-c^2)^(1/2)*2^(1/2)-sinh(x)*(b^2-c^2)
^(1/2)-(-sinh(x)*(b^2-c^2)^(1/2)-(b^2-c^2)^(1/2))^(1/2)*(-(b^2-c^2)^(1/2)*s
inh(x)^3-(b^2-c^2)^(1/2)*sinh(x)^2)^(1/2)-(b^2-c^2)^(1/2)))+4*(-sinh(x)*(b^
2-c^2)^(1/2)-(b^2-c^2)^(1/2))^(1/2)*cosh(x)*sinh(x))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\cosh(x)+c\sinh(x)-\sqrt{b^2-c^2}\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*cosh(x)+c*sinh(x)-(b^2-c^2)^(1/2))^(5/2),x, algorithm="maxim
a")
[Out] integrate((b*cosh(x) + c*sinh(x) - sqrt(b^2 - c^2))^(-5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(b\cosh(x)-\sqrt{b^2-c^2}+c\sinh(x)\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*cosh(x) - (b^2 - c^2)^(1/2) + c*sinh(x))^(5/2),x)
[Out] int(1/(b*cosh(x) - (b^2 - c^2)^(1/2) + c*sinh(x))^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\cosh(x)+c\sinh(x)-\sqrt{b^2-c^2}\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*cosh(x)+c*sinh(x)-(b**2-c**2)**(1/2))**(5/2),x)
[Out] Integral((b*cosh(x) + c*sinh(x) - sqrt(b**2 - c**2))**(-5/2), x)
```

3.780 $\int \frac{1}{a+c\,\mathrm{sech}(x)+b\tanh(x)}\,dx$

Optimal. Leaf size=107

$$-\frac{2ac\tan^{-1}\left(\frac{(a-c)\tanh\left(\frac{x}{2}\right)+b}{\sqrt{a^2-b^2-c^2}}\right)}{\left(a^2-b^2\right)\sqrt{a^2-b^2-c^2}}-\frac{b\log(a\cosh(x)+b\sinh(x)+c)}{a^2-b^2}+\frac{ax}{a^2-b^2}$$

```
[Out] a*x/(a^2-b^2)-b*ln(c+a*cosh(x)+b*sinh(x))/(a^2-b^2)-2*a*c*arctan((b+(a-c)*t
anh(1/2*x))/(a^2-b^2-c^2)^(1/2))/(a^2-b^2)/(a^2-b^2-c^2)^(1/2)
```

Rubi [A] time = 0.14, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {3159, 3138, 3124, 618, 204}

$$-\frac{2ac\tan^{-1}\left(\frac{(a-c)\tanh\left(\frac{x}{2}\right)+b}{\sqrt{a^2-b^2-c^2}}\right)}{\left(a^2-b^2\right)\sqrt{a^2-b^2-c^2}}-\frac{b\log(a\cosh(x)+b\sinh(x)+c)}{a^2-b^2}+\frac{ax}{a^2-b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*Sech[x] + b*Tanh[x])^(-1),x]
```

```
[Out] (a*x)/(a^2 - b^2) - (2*a*c*ArcTan[(b + (a - c)*Tanh[x/2])/Sqrt[a^2 - b^2 -
c^2]])/((a^2 - b^2)*Sqrt[a^2 - b^2 - c^2]) - (b*Log[c + a*Cosh[x] + b*Sinh[
x]])/(a^2 - b^2)
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 3124

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Module[{f = FreeFactors[Tan[(d + e*x)/2], x]}, Dist[(2*f
)/e, Subst[Int[1/(a + b + 2*c*f*x + (a - b)*f^2*x^2), x], x, Tan[(d + e*x)/
```

```
2]/f], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
```

Rule 3138

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.))/((a_.) + cos[(d_.) + (e_.)*(x_)
]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)]), x_Symbol] :> Simp[(b*B*(d + e*x))
/(e*(b^2 + c^2)), x] + (Dist[(A*(b^2 + c^2) - a*b*B)/(b^2 + c^2), Int[1/(a
+ b*Cos[d + e*x] + c*Sin[d + e*x]), x], x] + Simp[(c*B*Log[a + b*Cos[d + e*
x] + c*Sin[d + e*x]])/(e*(b^2 + c^2)), x]) /; FreeQ[{a, b, c, d, e, A, B},
x] && NeQ[b^2 + c^2, 0] && NeQ[A*(b^2 + c^2) - a*b*B, 0]
```

Rule 3159

```
Int[((a_.) + (b_.)*sec[(d_.) + (e_.)*(x_)] + (c_.)*tan[(d_.) + (e_.)*(x_)])
^(-1), x_Symbol] :> Int[Cos[d + e*x]/(b + a*Cos[d + e*x] + c*Sin[d + e*x]),
 x] /; FreeQ[{a, b, c, d, e}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{a+c\operatorname{sech}(x)+b\tanh(x)}\,dx &= \int \frac{\cosh(x)}{c+a\cosh(x)+b\sinh(x)}\,dx \\
&= \frac{ax}{a^2-b^2}-\frac{b\log(c+a\cosh(x)+b\sinh(x))}{a^2-b^2}-\frac{(ac)\int \frac{1}{c+a\cosh(x)+b\sinh(x)}\,dx}{a^2-b^2} \\
&= \frac{ax}{a^2-b^2}-\frac{b\log(c+a\cosh(x)+b\sinh(x))}{a^2-b^2}-\frac{(2ac)\operatorname{Subst}\left(\int \frac{1}{a+c+2bx-(-a+c)x^2}\,d\right.}{a^2-b^2} \\
&= \frac{ax}{a^2-b^2}-\frac{b\log(c+a\cosh(x)+b\sinh(x))}{a^2-b^2}+\frac{(4ac)\operatorname{Subst}\left(\int \frac{1}{-4\left(a^2-b^2-c^2\right)-x^2}\,dx\right.}{a^2-b} \\
&= \frac{ax}{a^2-b^2}-\frac{2ac\tan^{-1}\left(\frac{b+(a-c)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2-c^2}}\right)}{\left(a^2-b^2\right)\sqrt{a^2-b^2-c^2}}-\frac{b\log(c+a\cosh(x)+b\sinh(x))}{a^2-b^2}
\end{aligned}
$$

Mathematica [A] time = 0.20, size = 86, normalized size = 0.80

$$
\frac{-\frac{2ac\tan^{-1}\left(\frac{(a-c)\tanh\left(\frac{x}{2}\right)+b}{\sqrt{a^2-b^2-c^2}}\right)}{\sqrt{a^2-b^2-c^2}}-b\log(a\cosh(x)+b\sinh(x)+c)+ax}{a^2-b^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + c*Sech[x] + b*Tanh[x])^(-1),x]
```

```
[Out] (a*x - (2*a*c*ArcTan[(b + (a - c)*Tanh[x/2])/Sqrt[a^2 - b^2 - c^2]])/Sqrt[a
^2 - b^2 - c^2] - b*Log[c + a*Cosh[x] + b*Sinh[x]])/(a^2 - b^2)
```

fricas [A] time = 0.46, size = 429, normalized size = 4.01

$$\left[\frac{\sqrt{-a^2+b^2+c^2}\,ac\log\left(\frac{2\,(a+b)c\cosh(x)+\left(a^2+2\,ab+b^2\right)\cosh(x)^2+\left(a^2+2\,ab+b^2\right)\sinh(x)^2-a^2+b^2+2\,c^2+2\,\left((a+b)c+\left(a^2+2\,ab+b^2\right)\cosh(x)\right)\sinh}{(a+b)\cosh(x)^2+(a+b)\sinh(x)^2+2\,c\cosh(x)+2\,((a+b)\cosh(x)+c)\sinh(x}\right.}{a^4-2\,a^2b}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+c*sech(x)+b*tanh(x)),x, algorithm="fricas")
```

```
[Out] [(sqrt(-a^2 + b^2 + c^2)*a*c*log((2*(a + b)*c*cosh(x) + (a^2 + 2*a*b + b^2)
*cosh(x)^2 + (a^2 + 2*a*b + b^2)*sinh(x)^2 - a^2 + b^2 + 2*c^2 + 2*((a + b)
*c + (a^2 + 2*a*b + b^2)*cosh(x))*sinh(x) - 2*sqrt(-a^2 + b^2 + c^2)*((a +
b)*cosh(x) + (a + b)*sinh(x) + c))/((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 +
 2*c*cosh(x) + 2*((a + b)*cosh(x) + c)*sinh(x) + a - b)) + (a^3 + a^2*b - a
*b^2 - b^3 - (a + b)*c^2)*x - (a^2*b - b^3 - b*c^2)*log(2*(a*cosh(x) + b*si
nh(x) + c)/(cosh(x) - sinh(x))))/(a^4 - 2*a^2*b^2 + b^4 - (a^2 - b^2)*c^2),
 (2*sqrt(a^2 - b^2 - c^2)*a*c*arctan(-((a + b)*cosh(x) + (a + b)*sinh(x) +
c)/sqrt(a^2 - b^2 - c^2)) + (a^3 + a^2*b - a*b^2 - b^3 - (a + b)*c^2)*x - (
a^2*b - b^3 - b*c^2)*log(2*(a*cosh(x) + b*sinh(x) + c)/(cosh(x) - sinh(x)))
)/(a^4 - 2*a^2*b^2 + b^4 - (a^2 - b^2)*c^2)]
```

giac [A] time = 0.13, size = 106, normalized size = 0.99

$$-\frac{2\,ac\arctan\left(\frac{ae^x+be^x+c}{\sqrt{a^2-b^2-c^2}}\right)}{\sqrt{a^2-b^2-c^2}\left(a^2-b^2\right)}-\frac{b\log\left(ae^{(2\,x)}+be^{(2\,x)}+2\,ce^x+a-b\right)}{a^2-b^2}+\frac{x}{a-b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+c*sech(x)+b*tanh(x)),x, algorithm="giac")
```

```
[Out] -2*a*c*arctan((a*e^x + b*e^x + c)/sqrt(a^2 - b^2 - c^2))/(sqrt(a^2 - b^2 -
c^2)*(a^2 - b^2)) - b*log(a*e^(2*x) + b*e^(2*x) + 2*c*e^x + a - b)/(a^2 - b
^2) + x/(a - b)
```

maple [B] time = 0.24, size = 422, normalized size = 3.94

$$-\frac{2\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{2b+2a}+\frac{2\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)}{2a-2b}-\frac{\ln\left(a\left(\tanh^2\left(\frac{x}{2}\right)\right)-\left(\tanh^2\left(\frac{x}{2}\right)\right)c+2\tanh\left(\frac{x}{2}\right)b+a+c\right)ab}{(a+b)\,(a-b)\,(a-c)}+\frac{\ln\left(a\left(\text{ta}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+c*sech(x)+b*tanh(x)),x)
```

```
[Out] -2/(2*b+2*a)*ln(tanh(1/2*x)-1)+2/(2*a-2*b)*ln(tanh(1/2*x)+1)-1/(a+b)/(a-b)/
(a-c)*ln(a*tanh(1/2*x)^2-tanh(1/2*x)^2*c+2*tanh(1/2*x)*b+a+c)*a*b+1/(a+b)/(
a-b)/(a-c)*ln(a*tanh(1/2*x)^2-tanh(1/2*x)^2*c+2*tanh(1/2*x)*b+a+c)*c*b-2/(a
+b)/(a-b)/(a^2-b^2-c^2)^(1/2)*arctan(1/2*(2*(a-c)*tanh(1/2*x)+2*b)/(a^2-b^2
-c^2)^(1/2))*a*c-2/(a+b)/(a-b)/(a^2-b^2-c^2)^(1/2)*arctan(1/2*(2*(a-c)*tanh
(1/2*x)+2*b)/(a^2-b^2-c^2)^(1/2))*b^2+2/(a+b)/(a-b)/(a^2-b^2-c^2)^(1/2)*arc
tan(1/2*(2*(a-c)*tanh(1/2*x)+2*b)/(a^2-b^2-c^2)^(1/2))*b^2/(a-c)*a-2/(a+b)/
(a-b)/(a^2-b^2-c^2)^(1/2)*arctan(1/2*(2*(a-c)*tanh(1/2*x)+2*b)/(a^2-b^2-c^2
)^(1/2))*b^2/(a-c)*c
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+c*sech(x)+b*tanh(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(c^2+b^2-a^2>0)', see `assume?` for
more details)Is c^2+b^2-a^2 positive or negative?
```

mupad [B] time = 6.28, size = 472, normalized size = 4.41

$$\frac{x}{a-b}+\frac{\ln\left(a-b+2\,c\,\mathrm{e}^x+a\,\mathrm{e}^{2\,x}+b\,\mathrm{e}^{2\,x}\right)\,\left(-2\,a^2\,b+2\,b^3+2\,b\,c^2\right)}{2\,\left(a^4-2\,a^2\,b^2-a^2\,c^2+b^4+b^2\,c^2\right)}-\frac{2\,\mathrm{atan}\left(\left(\mathrm{e}^x\,\left(\frac{2\,a\,c}{{\left(a+b\right)}^2\,\left(a^2-b^2\right)\,{\left(a-b\right)}^2\,\sqrt{a^2\,c^2}}-\frac{2\,(}{a\,{(a+b)}^2}\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*tanh(x) + c/cosh(x)),x)
```

```
[Out] x/(a - b) + (log(a - b + 2*c*exp(x) + a*exp(2*x) + b*exp(2*x))*(2*b*c^2 - 2
*a^2*b + 2*b^3))/(2*(a^4 + b^4 - 2*a^2*b^2 - a^2*c^2 + b^2*c^2)) - (2*atan(
(exp(x)*((2*a*c)/((a + b)^2*(a^2 - b^2)*(a - b)^2*(a^2*c^2)^(1/2)) - (2*(a^
2*c*(a^2*c^2)^(1/2) - b^2*c*(a^2*c^2)^(1/2)))/(a*(a + b)^2*(a^2 - b^2)^2*(a
 - b)^2*(b^2 - a^2 + c^2))) - (2*(a^3*(a^2*c^2)^(1/2) + b^3*(a^2*c^2)^(1/2)
 - a*b^2*(a^2*c^2)^(1/2) - a^2*b*(a^2*c^2)^(1/2)))/(a*(a + b)^2*(a^2 - b^2)
^2*(a - b)^2*(b^2 - a^2 + c^2)))*((a^3*(-(a^2 - b^2)^2*(b^2 - a^2 + c^2))^(
1/2))/2 - (b^3*(-(a^2 - b^2)^2*(b^2 - a^2 + c^2))^(1/2))/2 - (a*b^2*(-(a^2
```

```
- b^2)^2*(b^2 - a^2 + c^2))^(1/2))/2 + (a^2*b*(-(a^2 - b^2)^2*(b^2 - a^2 +
c^2))^(1/2))/2))*(a^2*c^2)^(1/2))/(-(a^2 - b^2)^2*(b^2 - a^2 + c^2))^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{a + b \tanh(x) + c \operatorname{sech}(x)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+c*sech(x)+b*tanh(x)),x)
```

```
[Out] Integral(1/(a + b*tanh(x) + c*sech(x)), x)
```

3.781 $\int \frac{1}{a+b\coth(x)+c\,\mathbf{csch}(x)}\,dx$

Optimal. Leaf size=113

$$\frac{2ac\tanh^{-1}\left(\frac{a+(b-c)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2+c^2}}\right)}{\left(a^2-b^2\right)\sqrt{a^2-b^2+c^2}}-\frac{b\log(ia\sinh(x)+ib\cosh(x)+ic)}{a^2-b^2}+\frac{ax}{a^2-b^2}$$

```
[Out] a*x/(a^2-b^2)-b*ln(I*c+I*b*cosh(x)+I*a*sinh(x))/(a^2-b^2)+2*a*c*arctanh((a+
(b-c)*tanh(1/2*x))/(a^2-b^2+c^2)^(1/2))/(a^2-b^2)/(a^2-b^2+c^2)^(1/2)
```

Rubi [A] time = 0.16, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {3160, 3137, 3124, 618, 204}

$$\frac{2ac\tanh^{-1}\left(\frac{a+(b-c)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2+c^2}}\right)}{\left(a^2-b^2\right)\sqrt{a^2-b^2+c^2}}-\frac{b\log(ia\sinh(x)+ib\cosh(x)+ic)}{a^2-b^2}+\frac{ax}{a^2-b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Coth[x] + c*Csch[x])^(-1),x]
```

```
[Out] (a*x)/(a^2 - b^2) + (2*a*c*ArcTanh[(a + (b - c)*Tanh[x/2])/Sqrt[a^2 - b^2 +
 c^2]])/((a^2 - b^2)*Sqrt[a^2 - b^2 + c^2]) - (b*Log[I*c + I*b*Cosh[x] + I*
a*Sinh[x]])/(a^2 - b^2)
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 3124

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Module[{f = FreeFactors[Tan[(d + e*x)/2], x]}, Dist[(2*f
)/e, Subst[Int[1/(a + b + 2*c*f*x + (a - b)*f^2*x^2), x], x, Tan[(d + e*x)/
```

```
2]/f], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
```

Rule 3137

```
Int[((A_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])/((a_.) + cos[(d_.) + (e_.)*(x_)
]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)]), x_Symbol] :> Simp[(c*C*(d + e*x))
/(e*(b^2 + c^2)), x] + (Dist[(A*(b^2 + c^2) - a*c*C)/(b^2 + c^2), Int[1/(a
+ b*Cos[d + e*x] + c*Sin[d + e*x]), x], x] - Simp[(b*C*Log[a + b*Cos[d + e*
x] + c*Sin[d + e*x]])/(e*(b^2 + c^2)), x]) /; FreeQ[{a, b, c, d, e, A, C},
x] && NeQ[b^2 + c^2, 0] && NeQ[A*(b^2 + c^2) - a*c*C, 0]
```

Rule 3160

```
Int[((a_.) + csc[(d_.) + (e_.)*(x_)]*(b_.) + cot[(d_.) + (e_.)*(x_)]*(c_.))
^(-1), x_Symbol] :> Int[Sin[d + e*x]/(b + a*Sin[d + e*x] + c*Cos[d + e*x]),
 x] /; FreeQ[{a, b, c, d, e}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{a+b\coth(x)+c\,\mathrm{csch}(x)}\,dx &= i\int \frac{\sinh(x)}{ic+ib\cosh(x)+ia\sinh(x)}\,dx \\
&= \frac{ax}{a^2-b^2}-\frac{b\log(ic+ib\cosh(x)+ia\sinh(x))}{a^2-b^2}-\frac{(iac)\int \frac{1}{ic+ib\cosh(x)+ia\sinh(x)}\,dx}{a^2-b^2} \\
&= \frac{ax}{a^2-b^2}-\frac{b\log(ic+ib\cosh(x)+ia\sinh(x))}{a^2-b^2}-\frac{(2iac)\,\mathrm{Subst}\left(\int \frac{1}{ib+ic+2iax-(-ib+ic)}\right.}{a^2-b^2} \\
&= \frac{ax}{a^2-b^2}-\frac{b\log(ic+ib\cosh(x)+ia\sinh(x))}{a^2-b^2}+\frac{(4iac)\,\mathrm{Subst}\left(\int \frac{1}{-4\left(a^2-b^2+c^2\right)-x^2}\right.}{a^2-} \\
&= \frac{ax}{a^2-b^2}+\frac{2ac\tanh^{-1}\left(\frac{a+(b-c)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2+c^2}}\right)}{\left(a^2-b^2\right)\sqrt{a^2-b^2+c^2}}-\frac{b\log(ic+ib\cosh(x)+ia\sinh(x))}{a^2-b^2}
\end{aligned}
$$

Mathematica [A] time = 0.22, size = 86, normalized size = 0.76

$$
\frac{-\frac{2ac\tan^{-1}\left(\frac{a+(b-c)\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2+b^2-c^2}}\right)}{\sqrt{-a^2+b^2-c^2}}-b\log(a\sinh(x)+b\cosh(x)+c)+ax}{a^2-b^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Coth[x] + c*Csch[x])^(-1),x]
```

```
[Out] (a*x - (2*a*c*ArcTan[(a + (b - c)*Tanh[x/2])/Sqrt[-a^2 + b^2 - c^2]])/Sqrt[
-a^2 + b^2 - c^2] - b*Log[c + b*Cosh[x] + a*Sinh[x]])/(a^2 - b^2)
```

fricas [A] time = 0.46, size = 438, normalized size = 3.88

$$\left[-\frac{\sqrt{a^2-b^2+c^2}\,ac\log\left(\frac{2\,(a+b)c\cosh(x)+\left(a^2+2\,ab+b^2\right)\cosh(x)^2+\left(a^2+2\,ab+b^2\right)\sinh(x)^2+a^2-b^2+2\,c^2+2\,\left((a+b)c+\left(a^2+2\,ab+b^2\right)\cosh(x)\right)\sin}{(a+b)\cosh(x)^2+(a+b)\sinh(x)^2+2\,c\cosh(x)+2\,((a+b)\cosh(x)+c)\sinh(}\right.}{a^4-2\,a}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*coth(x)+c*csch(x)),x, algorithm="fricas")
```

```
[Out] [-(sqrt(a^2 - b^2 + c^2)*a*c*log((2*(a + b)*c*cosh(x) + (a^2 + 2*a*b + b^2)
*cosh(x)^2 + (a^2 + 2*a*b + b^2)*sinh(x)^2 + a^2 - b^2 + 2*c^2 + 2*((a + b)
*c + (a^2 + 2*a*b + b^2)*cosh(x))*sinh(x) - 2*sqrt(a^2 - b^2 + c^2)*((a + b
)*cosh(x) + (a + b)*sinh(x) + c))/((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 +
2*c*cosh(x) + 2*((a + b)*cosh(x) + c)*sinh(x) - a + b)) - (a^3 + a^2*b - a*
b^2 - b^3 + (a + b)*c^2)*x + (a^2*b - b^3 + b*c^2)*log(2*(b*cosh(x) + a*sin
h(x) + c)/(cosh(x) - sinh(x))))/(a^4 - 2*a^2*b^2 + b^4 + (a^2 - b^2)*c^2),
-(2*sqrt(-a^2 + b^2 - c^2)*a*c*arctan(sqrt(-a^2 + b^2 - c^2)*((a + b)*cosh(
x) + (a + b)*sinh(x) + c)/(a^2 - b^2 + c^2)) - (a^3 + a^2*b - a*b^2 - b^3 +
 (a + b)*c^2)*x + (a^2*b - b^3 + b*c^2)*log(2*(b*cosh(x) + a*sinh(x) + c)/(
cosh(x) - sinh(x))))/(a^4 - 2*a^2*b^2 + b^4 + (a^2 - b^2)*c^2)]
```

giac [A] time = 0.14, size = 106, normalized size = 0.94

$$-\frac{2\,ac\arctan\left(\frac{ae^x+be^x+c}{\sqrt{-a^2+b^2-c^2}}\right)}{\left(a^2-b^2\right)\sqrt{-a^2+b^2-c^2}}-\frac{b\log\left(ae^{(2\,x)}+be^{(2\,x)}+2\,ce^x-a+b\right)}{a^2-b^2}+\frac{x}{a-b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*coth(x)+c*csch(x)),x, algorithm="giac")
```

```
[Out] -2*a*c*arctan((a*e^x + b*e^x + c)/sqrt(-a^2 + b^2 - c^2))/((a^2 - b^2)*sqrt
(-a^2 + b^2 - c^2)) - b*log(a*e^(2*x) + b*e^(2*x) + 2*c*e^x - a + b)/(a^2 -
 b^2) + x/(a - b)
```

maple [B] time = 0.22, size = 421, normalized size = 3.73

$$-\frac{4\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{4a+4b}+\frac{4\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)}{4a-4b}-\frac{\ln\left(\left(\tanh^2\left(\frac{x}{2}\right)\right)b-\left(\tanh^2\left(\frac{x}{2}\right)\right)c+2a\tanh\left(\frac{x}{2}\right)+b+c\right)b^2}{(a+b)(a-b)(b-c)}+\frac{\ln\left(\left(\tan\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*coth(x)+c*csch(x)),x)
```

```
[Out] -4/(4*a+4*b)*ln(tanh(1/2*x)-1)+4/(4*a-4*b)*ln(tanh(1/2*x)+1)-1/(a+b)/(a-b)/
(b-c)*ln(tanh(1/2*x)^2*b-tanh(1/2*x)^2*c+2*a*tanh(1/2*x)+b+c)*b^2+1/(a+b)/(
a-b)/(b-c)*ln(tanh(1/2*x)^2*b-tanh(1/2*x)^2*c+2*a*tanh(1/2*x)+b+c)*c*b-2/(a
+b)/(a-b)/(-a^2+b^2-c^2)^(1/2)*arctan(1/2*(2*(b-c)*tanh(1/2*x)+2*a)/(-a^2+b
^2-c^2)^(1/2))*a*b-2/(a+b)/(a-b)/(-a^2+b^2-c^2)^(1/2)*arctan(1/2*(2*(b-c)*t
anh(1/2*x)+2*a)/(-a^2+b^2-c^2)^(1/2))*a*c+2/(a+b)/(a-b)/(-a^2+b^2-c^2)^(1/2
)*arctan(1/2*(2*(b-c)*tanh(1/2*x)+2*a)/(-a^2+b^2-c^2)^(1/2))*a/(b-c)*b^2-2/
(a+b)/(a-b)/(-a^2+b^2-c^2)^(1/2)*arctan(1/2*(2*(b-c)*tanh(1/2*x)+2*a)/(-a^2
+b^2-c^2)^(1/2))*a/(b-c)*c*b
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*coth(x)+c*csch(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(c^2-b^2+a^2>0)', see `assume?` for
more details)Is c^2-b^2+a^2 positive or negative?
```

mupad [B] time = 0.70, size = 324, normalized size = 2.87

$$\frac{x}{a-b}-\frac{\ln\left(\frac{2\,(b+c\,\mathrm{e}^x)}{(a+b)^2}+\frac{2\,(b-a+c\,\mathrm{e}^x)\left(a^2\,b+b\,c^2-b^3+a\,c\,\sqrt{a^2-b^2+c^2}\right)}{(a+b)\left(a^2-b^2\right)\left(a^2-b^2+c^2\right)}\right)\left(a^2\,b+b\,c^2-b^3+a\,c\,\sqrt{a^2-b^2+c^2}\right)}{a^4-2\,a^2\,b^2+a^2\,c^2+b^4-b^2\,c^2}-\frac{\ln\left(\frac{2\,(b+c\,\mathrm{e}^x)}{(a+b)^2}+\frac{2\,(b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + c/sinh(x) + b*coth(x)),x)
```

```
[Out] x/(a - b) - (log((2*(b + c*exp(x)))/(a + b)^2 + (2*(b - a + c*exp(x))*(a^2*
b + b*c^2 - b^3 + a*c*(a^2 - b^2 + c^2)^(1/2)))/((a + b)*(a^2 - b^2)*(a^2 -
 b^2 + c^2)))*(a^2*b + b*c^2 - b^3 + a*c*(a^2 - b^2 + c^2)^(1/2)))/(a^4 + b
^4 - 2*a^2*b^2 + a^2*c^2 - b^2*c^2) - (log((2*(b + c*exp(x)))/(a + b)^2 + (
2*(b - a + c*exp(x))*(a^2*b + b*c^2 - b^3 - a*c*(a^2 - b^2 + c^2)^(1/2)))/(
(a + b)*(a^2 - b^2)*(a^2 - b^2 + c^2)))*(a^2*b + b*c^2 - b^3 - a*c*(a^2 - b
^2 + c^2)^(1/2)))/(a^4 + b^4 - 2*a^2*b^2 + a^2*c^2 - b^2*c^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{a+b\coth(x)+c\operatorname{csch}(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*coth(x)+c*csch(x)),x)
```

```
[Out] Integral(1/(a + b*coth(x) + c*csch(x)), x)
```

3.782 $\int \frac{\sinh(x)}{a+b\cosh(x)+c\sinh(x)}\,dx$

Optimal. Leaf size=104

$$-\frac{2ac\tanh^{-1}\left(\frac{c-(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2+c^2}}\right)}{\left(b^2-c^2\right)\sqrt{a^2-b^2+c^2}}+\frac{b\log(a+b\cosh(x)+c\sinh(x))}{b^2-c^2}-\frac{cx}{b^2-c^2}$$

```
[Out] -c*x/(b^2-c^2)+b*ln(a+b*cosh(x)+c*sinh(x))/(b^2-c^2)-2*a*c*arctanh((c-(a-b)
*tanh(1/2*x))/(a^2-b^2+c^2)^(1/2))/(b^2-c^2)/(a^2-b^2+c^2)^(1/2)
```

Rubi [A] time = 0.11, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {3137, 3124, 618, 206}

$$-\frac{2ac\tanh^{-1}\left(\frac{c-(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2+c^2}}\right)}{\left(b^2-c^2\right)\sqrt{a^2-b^2+c^2}}+\frac{b\log(a+b\cosh(x)+c\sinh(x))}{b^2-c^2}-\frac{cx}{b^2-c^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[x]/(a + b*Cosh[x] + c*Sinh[x]),x]
```

```
[Out] -((c*x)/(b^2 - c^2)) - (2*a*c*ArcTanh[(c - (a - b)*Tanh[x/2])/Sqrt[a^2 - b^
2 + c^2]])/((b^2 - c^2)*Sqrt[a^2 - b^2 + c^2]) + (b*Log[a + b*Cosh[x] + c*S
inh[x]])/(b^2 - c^2)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 3124

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Module[{f = FreeFactors[Tan[(d + e*x)/2], x]}, Dist[(2*f
)/e, Subst[Int[1/(a + b + 2*c*f*x + (a - b)*f^2*x^2), x], x, Tan[(d + e*x)/
```

```
2]/f], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
```

Rule 3137

```
Int[((A_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])/((a_.) + cos[(d_.) + (e_.)*(x_)
]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)]), x_Symbol] :> Simp[(c*C*(d + e*x))
/(e*(b^2 + c^2)), x] + (Dist[(A*(b^2 + c^2) - a*c*C)/(b^2 + c^2), Int[1/(a
+ b*Cos[d + e*x] + c*Sin[d + e*x]), x], x] - Simp[(b*C*Log[a + b*Cos[d + e*
x] + c*Sin[d + e*x]])/(e*(b^2 + c^2)), x]) /; FreeQ[{a, b, c, d, e, A, C},
x] && NeQ[b^2 + c^2, 0] && NeQ[A*(b^2 + c^2) - a*c*C, 0]
```

Rubi steps

$$\int \frac{\sinh(x)}{a+b\cosh(x)+c\sinh(x)}\,dx = -\frac{cx}{b^2-c^2}+\frac{b\log(a+b\cosh(x)+c\sinh(x))}{b^2-c^2}+\frac{(ac)\int \frac{1}{a+b\cosh(x)+c\sinh(x)}\,dx}{b^2-c^2}$$

$$= -\frac{cx}{b^2-c^2}+\frac{b\log(a+b\cosh(x)+c\sinh(x))}{b^2-c^2}+\frac{(2ac)\,\mathrm{Subst}\left(\int \frac{1}{a+b+2cx-(a-b)x^2}\right.}{b^2-c^2}$$

$$= -\frac{cx}{b^2-c^2}+\frac{b\log(a+b\cosh(x)+c\sinh(x))}{b^2-c^2}-\frac{(4ac)\,\mathrm{Subst}\left(\int \frac{1}{4\left(a^2-b^2+c^2\right)-x^2}\,d\right.}{b^2-}$$

$$= -\frac{cx}{b^2-c^2}-\frac{2ac\tanh^{-1}\left(\frac{c-(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2+c^2}}\right)}{\left(b^2-c^2\right)\sqrt{a^2-b^2+c^2}}+\frac{b\log(a+b\cosh(x)+c\sinh(x))}{b^2-c^2}$$

Mathematica [A] time = 0.21, size = 86, normalized size = 0.83

$$\frac{\frac{2ac\tan^{-1}\left(\frac{(b-a)\tanh\left(\frac{x}{2}\right)+c}{\sqrt{-a^2+b^2-c^2}}\right)}{\sqrt{-a^2+b^2-c^2}}+b\log(a+b\cosh(x)+c\sinh(x))-cx}{b^2-c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[x]/(a + b*Cosh[x] + c*Sinh[x]),x]
```

```
[Out] (-(c*x) + (2*a*c*ArcTan[(c + (-a + b)*Tanh[x/2])/Sqrt[-a^2 + b^2 - c^2]])/S
qrt[-a^2 + b^2 - c^2] + b*Log[a + b*Cosh[x] + c*Sinh[x]])/(b^2 - c^2)
```

fricas [A] time = 0.46, size = 455, normalized size = 4.38

$$\left[-\frac{\sqrt{a^2-b^2+c^2}\,ac\log\left(\frac{(b^2+2\,bc+c^2)\cosh(x)^2+(b^2+2\,bc+c^2)\sinh(x)^2+2\,a^2-b^2+c^2+2\,(ab+ac)\cosh(x)+2\,(ab+ac+(b^2+2\,bc+c^2)\cosh(x))\sinh(}{(b+c)\cosh(x)^2+(b+c)\sinh(x)^2+2\,a\cosh(x)+2\,((b+c)\cosh(x)+a)\sinh(x)}\right.}{a^2b^2-b^4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)/(a+b*cosh(x)+c*sinh(x)),x, algorithm="fricas")
```

```
[Out] [-(sqrt(a^2 - b^2 + c^2)*a*c*log(((b^2 + 2*b*c + c^2)*cosh(x)^2 + (b^2 + 2*
b*c + c^2)*sinh(x)^2 + 2*a^2 - b^2 + c^2 + 2*(a*b + a*c)*cosh(x) + 2*(a*b +
 a*c + (b^2 + 2*b*c + c^2)*cosh(x))*sinh(x) + 2*sqrt(a^2 - b^2 + c^2)*((b +
 c)*cosh(x) + (b + c)*sinh(x) + a))/((b + c)*cosh(x)^2 + (b + c)*sinh(x)^2
+ 2*a*cosh(x) + 2*((b + c)*cosh(x) + a)*sinh(x) + b - c)) + (a^2*b - b^3 +
b*c^2 + c^3 + (a^2 - b^2)*c)*x - (a^2*b - b^3 + b*c^2)*log(2*(b*cosh(x) + c
*sinh(x) + a)/(cosh(x) - sinh(x))))/(a^2*b^2 - b^4 - c^4 - (a^2 - 2*b^2)*c^
2), (2*sqrt(-a^2 + b^2 - c^2)*a*c*arctan(sqrt(-a^2 + b^2 - c^2)*((b + c)*co
sh(x) + (b + c)*sinh(x) + a)/(a^2 - b^2 + c^2)) - (a^2*b - b^3 + b*c^2 + c^
3 + (a^2 - b^2)*c)*x + (a^2*b - b^3 + b*c^2)*log(2*(b*cosh(x) + c*sinh(x) +
 a)/(cosh(x) - sinh(x))))/(a^2*b^2 - b^4 - c^4 - (a^2 - 2*b^2)*c^2)]
```

giac [A] time = 0.12, size = 106, normalized size = 1.02

$$\frac{2\,ac\arctan\left(\frac{be^x+ce^x+a}{\sqrt{-a^2+b^2-c^2}}\right)}{\sqrt{-a^2+b^2-c^2}\,(b^2-c^2)}+\frac{b\log\left(be^{(2\,x)}+ce^{(2\,x)}+2\,ae^x+b-c\right)}{b^2-c^2}-\frac{x}{b-c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)/(a+b*cosh(x)+c*sinh(x)),x, algorithm="giac")
```

```
[Out] 2*a*c*arctan((b*e^x + c*e^x + a)/sqrt(-a^2 + b^2 - c^2))/(sqrt(-a^2 + b^2 -
 c^2)*(b^2 - c^2)) + b*log(b*e^(2*x) + c*e^(2*x) + 2*a*e^x + b - c)/(b^2 -
c^2) - x/(b - c)
```

maple [B] time = 0.20, size = 429, normalized size = 4.12

$$-\frac{4\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{4b+4c}+\frac{\ln\left(a\left(\tanh^2\left(\frac{x}{2}\right)\right)-\left(\tanh^2\left(\frac{x}{2}\right)\right)b-2c\tanh\left(\frac{x}{2}\right)-a-b\right)ab}{(b-c)(b+c)(a-b)}-\frac{\ln\left(a\left(\tanh^2\left(\frac{x}{2}\right)\right)-\left(\tanh^2\left(\frac{x}{2}\right.\right.\right.}{(b-c)(b+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)/(a+b*cosh(x)+c*sinh(x)),x)
```

```
[Out] -4/(4*b+4*c)*ln(tanh(1/2*x)-1)+1/(b-c)/(b+c)/(a-b)*ln(a*tanh(1/2*x)^2-tanh(
1/2*x)^2*b-2*c*tanh(1/2*x)-a-b)*a*b-1/(b-c)/(b+c)/(a-b)*ln(a*tanh(1/2*x)^2-
tanh(1/2*x)^2*b-2*c*tanh(1/2*x)-a-b)*b^2-2/(b-c)/(b+c)/(-a^2+b^2-c^2)^(1/2)
*arctan(1/2*(2*(a-b)*tanh(1/2*x)-2*c)/(-a^2+b^2-c^2)^(1/2))*a*c-2/(b-c)/(b+
c)/(-a^2+b^2-c^2)^(1/2)*arctan(1/2*(2*(a-b)*tanh(1/2*x)-2*c)/(-a^2+b^2-c^2)
^(1/2))*c*b+2/(b-c)/(b+c)/(-a^2+b^2-c^2)^(1/2)*arctan(1/2*(2*(a-b)*tanh(1/2
*x)-2*c)/(-a^2+b^2-c^2)^(1/2))*c/(a-b)*a*b-2/(b-c)/(b+c)/(-a^2+b^2-c^2)^(1/
2)*arctan(1/2*(2*(a-b)*tanh(1/2*x)-2*c)/(-a^2+b^2-c^2)^(1/2))*c/(a-b)*b^2-4
/(4*b-4*c)*ln(tanh(1/2*x)+1)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)/(a+b*cosh(x)+c*sinh(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(c^2-b^2+a^2>0)', see `assume?` for
more details)Is c^2-b^2+a^2 positive or negative?
```

mupad [B] time = 2.16, size = 325, normalized size = 3.12

$$-\frac{x}{b-c}-\frac{\ln\left(\frac{2\,(b+a\,\mathrm{e}^x)}{(b+c)^2}-\frac{2\,(b-c+a\,\mathrm{e}^x)\left(a^2\,b+b\,c^2-b^3+a\,c\,\sqrt{a^2-b^2+c^2}\right)}{(b+c)\left(b^2-c^2\right)\left(a^2-b^2+c^2\right)}\right)\left(a^2\,b+b\,c^2-b^3+a\,c\,\sqrt{a^2-b^2+c^2}\right)}{-a^2\,b^2+a^2\,c^2+b^4-2\,b^2\,c^2+c^4}-\frac{\ln\left(\frac{2\,(b+a\,\mathrm{e}^x)}{(b+c)^2}-\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)/(a + b*cosh(x) + c*sinh(x)),x)
```

```
[Out] - x/(b - c) - (log((2*(b + a*exp(x)))/(b + c)^2 - (2*(b - c + a*exp(x))*(a^
2*b + b*c^2 - b^3 + a*c*(a^2 - b^2 + c^2)^(1/2)))/((b + c)*(b^2 - c^2)*(a^2
 - b^2 + c^2)))*(a^2*b + b*c^2 - b^3 + a*c*(a^2 - b^2 + c^2)^(1/2)))/(b^4 +
 c^4 - a^2*b^2 + a^2*c^2 - 2*b^2*c^2) - (log((2*(b + a*exp(x)))/(b + c)^2 -
 (2*(b - c + a*exp(x))*(a^2*b + b*c^2 - b^3 - a*c*(a^2 - b^2 + c^2)^(1/2)))
/((b + c)*(b^2 - c^2)*(a^2 - b^2 + c^2)))*(a^2*b + b*c^2 - b^3 - a*c*(a^2 -
 b^2 + c^2)^(1/2)))/(b^4 + c^4 - a^2*b^2 + a^2*c^2 - 2*b^2*c^2)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)/(a+b*cosh(x)+c*sinh(x)),x)
[Out] Timed out
```

3.783 $\int \frac{\sinh(x)}{1+\cosh(x)+\sinh(x)} \, dx$

Optimal. Leaf size=18

$$\frac{x}{2} - \frac{\sinh(x)}{2} + \frac{\cosh(x)}{2}$$

```
[Out] 1/2*x+1/2*cosh(x)-1/2*sinh(x)
```

Rubi [A] time = 0.03, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {3131}

$$\frac{x}{2} - \frac{\sinh(x)}{2} + \frac{\cosh(x)}{2}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[x]/(1 + Cosh[x] + Sinh[x]),x]
```

```
[Out] x/2 + Cosh[x]/2 - Sinh[x]/2
```

Rule 3131

```
Int[((A_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])/(cos[(d_.) + (e_.)*(x_)]*(b_.)
+ (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)]), x_Symbol] :> Simp[((2*a*A - c*C)*x
)/(2*a^2), x] + (-Simp[(C*Cos[d + e*x])/(2*a*e), x] + Simp[(c*C*Sin[d + e*x
])/(2*a*b*e), x] + Simp[((-(a^2*C) + 2*a*c*A + b^2*C)*Log[RemoveContent[a +
 b*Cos[d + e*x] + c*Sin[d + e*x], x]])/(2*a^2*b*e), x]) /; FreeQ[{a, b, c,
d, e, A, C}, x] && EqQ[b^2 + c^2, 0]
```

Rubi steps

$$\int \frac{\sinh(x)}{1 + \cosh(x) + \sinh(x)} \, dx = \frac{x}{2} + \frac{\cosh(x)}{2} - \frac{\sinh(x)}{2}$$

Mathematica [A] time = 0.05, size = 18, normalized size = 1.00

$$\frac{x}{2} - \frac{\sinh(x)}{2} + \frac{\cosh(x)}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[x]/(1 + Cosh[x] + Sinh[x]),x]
```

```
[Out] x/2 + Cosh[x]/2 - Sinh[x]/2
```

fricas [A] time = 0.41, size = 19, normalized size = 1.06

$$\frac{x\cosh(x) + x\sinh(x) + 1}{2\left(\cosh(x) + \sinh(x)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)/(1+cosh(x)+sinh(x)),x, algorithm="fricas")
[Out] 1/2*(x*cosh(x) + x*sinh(x) + 1)/(cosh(x) + sinh(x))
```

giac [A] time = 0.14, size = 10, normalized size = 0.56

$$\frac{1}{2}\,x + \frac{1}{2}\,e^{(-x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)/(1+cosh(x)+sinh(x)),x, algorithm="giac")
[Out] 1/2*x + 1/2*e^(-x)
```

maple [B] time = 0.17, size = 28, normalized size = 1.56

$$-\frac{\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{2} + \frac{1}{\tanh\left(\frac{x}{2}\right)+1} + \frac{\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)/(1+cosh(x)+sinh(x)),x)
[Out] -1/2*ln(tanh(1/2*x)-1)+1/(tanh(1/2*x)+1)+1/2*ln(tanh(1/2*x)+1)
```

maxima [A] time = 0.74, size = 10, normalized size = 0.56

$$\frac{1}{2}\,x + \frac{1}{2}\,e^{(-x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)/(1+cosh(x)+sinh(x)),x, algorithm="maxima")
[Out] 1/2*x + 1/2*e^(-x)
```

mupad [B] time = 0.05, size = 10, normalized size = 0.56

$$\frac{x}{2} + \frac{\mathrm{e}^{-x}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)/(cosh(x) + sinh(x) + 1),x)
```

```
[Out] x/2 + exp(-x)/2
```

sympy [B] time = 0.56, size = 34, normalized size = 1.89

$$\frac{x\tanh\left(\frac{x}{2}\right)}{2\tanh\left(\frac{x}{2}\right)+2}+\frac{x}{2\tanh\left(\frac{x}{2}\right)+2}+\frac{2}{2\tanh\left(\frac{x}{2}\right)+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)/(1+cosh(x)+sinh(x)),x)
```

```
[Out] x*tanh(x/2)/(2*tanh(x/2) + 2) + x/(2*tanh(x/2) + 2) + 2/(2*tanh(x/2) + 2)
```

3.784 $\int \frac{\operatorname{sech}(x)}{a+c\operatorname{sech}(x)+b\tanh(x)}\,dx$

Optimal. Leaf size=54

$$\frac{2\tan^{-1}\left(\frac{(a-c)\tanh\left(\frac{x}{2}\right)+b}{\sqrt{a^2-b^2-c^2}}\right)}{\sqrt{a^2-b^2-c^2}}$$

```
[Out] 2*arctan((b+(a-c)*tanh(1/2*x))/(a^2-b^2-c^2)^(1/2))/(a^2-b^2-c^2)^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {3165, 3124, 618, 204}

$$\frac{2\tan^{-1}\left(\frac{(a-c)\tanh\left(\frac{x}{2}\right)+b}{\sqrt{a^2-b^2-c^2}}\right)}{\sqrt{a^2-b^2-c^2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[x]/(a + c*Sech[x] + b*Tanh[x]),x]
```

```
[Out] (2*ArcTan[(b + (a - c)*Tanh[x/2])/Sqrt[a^2 - b^2 - c^2]])/Sqrt[a^2 - b^2 -
c^2]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 3124

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Module[{f = FreeFactors[Tan[(d + e*x)/2], x]}, Dist[(2*f
)/e, Subst[Int[1/(a + b + 2*c*f*x + (a - b)*f^2*x^2), x], x, Tan[(d + e*x)/
2]/f], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
```

Rule 3165

```
Int[sec[(d_.) + (e_.)*(x_)]^(n_.)*((a_.) + (b_.)*sec[(d_.) + (e_.)*(x_)] +
(c_.)*tan[(d_.) + (e_.)*(x_)])^(m_), x_Symbol] :> Int[1/(b + a*Cos[d + e*x]
 + c*Sin[d + e*x])^n, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[m + n, 0] && I
ntegerQ[n]
```

Rubi steps

$$\begin{aligned}\int \frac{\operatorname{sech}(x)}{a+c\operatorname{sech}(x)+b\tanh(x)}\,dx &= \int \frac{1}{c+a\cosh(x)+b\sinh(x)}\,dx\\ &= 2\,\text{Subst}\left(\int \frac{1}{a+c+2bx-(-a+c)x^2}\,dx, x, \tanh\left(\frac{x}{2}\right)\right)\\ &= -\left(4\,\text{Subst}\left(\int \frac{1}{-4\left(a^2-b^2-c^2\right)-x^2}\,dx, x, 2b+2(a-c)\tanh\left(\frac{x}{2}\right)\right)\right)\\ &= \frac{2\tan^{-1}\left(\frac{b+(a-c)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2-c^2}}\right)}{\sqrt{a^2-b^2-c^2}}\end{aligned}$$

Mathematica [A] time = 0.06, size = 54, normalized size = 1.00

$$\frac{2\tan^{-1}\left(\frac{(a-c)\tanh\left(\frac{x}{2}\right)+b}{\sqrt{a^2-b^2-c^2}}\right)}{\sqrt{a^2-b^2-c^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[x]/(a + c*Sech[x] + b*Tanh[x]),x]
```

```
[Out] (2*ArcTan[(b + (a - c)*Tanh[x/2])/Sqrt[a^2 - b^2 - c^2]])/Sqrt[a^2 - b^2 -
c^2]
```

fricas [A] time = 0.44, size = 234, normalized size = 4.33

$$\left[-\frac{\sqrt{-a^2+b^2+c^2}\,\log\left(\frac{2\,(a+b)c\cosh(x)+\left(a^2+2\,ab+b^2\right)\cosh(x)^2+\left(a^2+2\,ab+b^2\right)\sinh(x)^2-a^2+b^2+2\,c^2+2\,\left((a+b)c+\left(a^2+2\,ab+b^2\right)\cosh(x)\right)\sin}{(a+b)\cosh(x)^2+(a+b)\sinh(x)^2+2\,c\cosh(x)+2\,((a+b)\cosh(x)+c)\sinh(}\right.}{a^2-b^2-c^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)/(a+c*sech(x)+b*tanh(x)),x, algorithm="fricas")
```

```
[Out] [-sqrt(-a^2 + b^2 + c^2)*log((2*(a + b)*c*cosh(x) + (a^2 + 2*a*b + b^2)*cos
h(x)^2 + (a^2 + 2*a*b + b^2)*sinh(x)^2 - a^2 + b^2 + 2*c^2 + 2*((a + b)*c +
 (a^2 + 2*a*b + b^2)*cosh(x))*sinh(x) - 2*sqrt(-a^2 + b^2 + c^2)*((a + b)*c
osh(x) + (a + b)*sinh(x) + c))/((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + 2*c
*cosh(x) + 2*((a + b)*cosh(x) + c)*sinh(x) + a - b))/(a^2 - b^2 - c^2), -2*
arctan(-((a + b)*cosh(x) + (a + b)*sinh(x) + c)/sqrt(a^2 - b^2 - c^2))/sqrt
(a^2 - b^2 - c^2)]
```

giac [A] time = 0.11, size = 46, normalized size = 0.85

$$\frac{2\arctan\left(\frac{ae^{x}+be^{x}+c}{\sqrt{a^{2}-b^{2}-c^{2}}}\right)}{\sqrt{a^{2}-b^{2}-c^{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)/(a+c*sech(x)+b*tanh(x)),x, algorithm="giac")
```

```
[Out] 2*arctan((a*e^x + b*e^x + c)/sqrt(a^2 - b^2 - c^2))/sqrt(a^2 - b^2 - c^2)
```

maple [A] time = 0.24, size = 53, normalized size = 0.98

$$\frac{2\arctan\left(\frac{2(a-c)\tanh\left(\frac{x}{2}\right)+2b}{2\sqrt{a^{2}-b^{2}-c^{2}}}\right)}{\sqrt{a^{2}-b^{2}-c^{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(x)/(a+c*sech(x)+b*tanh(x)),x)
```

```
[Out] 2/(a^2-b^2-c^2)^(1/2)*arctan(1/2*(2*(a-c)*tanh(1/2*x)+2*b)/(a^2-b^2-c^2)^(1
/2))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)/(a+c*sech(x)+b*tanh(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(c^2+b^2-a^2>0)', see `assume?` for
more details)Is c^2+b^2-a^2 positive or negative?
```

mupad [B] time = 1.82, size = 78, normalized size = 1.44

$$\frac{2\,\mathrm{atan}\left(\frac{c}{\sqrt{a^2-b^2-c^2}}+\frac{a\,\mathrm{e}^x}{\sqrt{a^2-b^2-c^2}}+\frac{b\,\mathrm{e}^x}{\sqrt{a^2-b^2-c^2}}\right)}{\sqrt{a^2-b^2-c^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(x)*(a + b*tanh(x) + c/cosh(x))),x)
```

```
[Out] (2*atan(c/(a^2 - b^2 - c^2)^(1/2) + (a*exp(x))/(a^2 - b^2 - c^2)^(1/2) + (b
*exp(x))/(a^2 - b^2 - c^2)^(1/2)))/(a^2 - b^2 - c^2)^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}(x)}{a+b\tanh(x)+c\operatorname{sech}(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)/(a+c*sech(x)+b*tanh(x)),x)
```

```
[Out] Integral(sech(x)/(a + b*tanh(x) + c*sech(x)), x)
```

3.785 $\int \frac{\mathbf{sech}^2(x)}{a+c\mathbf{sech}(x)+b\tanh(x)}\,dx$

Optimal. Leaf size=146

$$-\frac{2ac\tan^{-1}\left(\frac{(a-c)\tanh\left(\frac{x}{2}\right)+b}{\sqrt{a^2-b^2-c^2}}\right)}{\left(b^2+c^2\right)\sqrt{a^2-b^2-c^2}}+\frac{b\log\left((a-c)\tanh^2\left(\frac{x}{2}\right)+a+2b\tanh\left(\frac{x}{2}\right)+c\right)}{b^2+c^2}-\frac{b\log\left(\tanh^2\left(\frac{x}{2}\right)+1\right)}{b^2+c^2}+\frac{2c\tan^{-1}\left(\tan\ldots\right.}{b^2+c\ldots}$$

[Out] 2*c*arctan(tanh(1/2*x))/(b^2+c^2)-b*ln(1+tanh(1/2*x)^2)/(b^2+c^2)+b*ln(a+c+2*b*tanh(1/2*x)+(a-c)*tanh(1/2*x)^2)/(b^2+c^2)-2*a*c*arctan((b+(a-c)*tanh(1/2*x))/(a^2-b^2-c^2)^(1/2))/(b^2+c^2)/(a^2-b^2-c^2)^(1/2)

Rubi [A] time = 0.48, antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.529, Rules used = {4397, 1075, 634, 618, 204, 628, 635, 203, 260}

$$-\frac{2ac\tan^{-1}\left(\frac{(a-c)\tanh\left(\frac{x}{2}\right)+b}{\sqrt{a^2-b^2-c^2}}\right)}{\left(b^2+c^2\right)\sqrt{a^2-b^2-c^2}}+\frac{b\log\left((a-c)\tanh^2\left(\frac{x}{2}\right)+a+2b\tanh\left(\frac{x}{2}\right)+c\right)}{b^2+c^2}-\frac{b\log\left(\tanh^2\left(\frac{x}{2}\right)+1\right)}{b^2+c^2}+\frac{2c\tan^{-1}\left(\tan\ldots\right.}{b^2+c\ldots}$$

Antiderivative was successfully verified.

[In] Int[Sech[x]^2/(a + c*Sech[x] + b*Tanh[x]),x]

[Out] (2*c*ArcTan[Tanh[x/2]])/(b^2 + c^2) - (2*a*c*ArcTan[(b + (a - c)*Tanh[x/2])/Sqrt[a^2 - b^2 - c^2]])/(Sqrt[a^2 - b^2 - c^2]*(b^2 + c^2)) - (b*Log[1 + Tanh[x/2]^2])/(b^2 + c^2) + (b*Log[a + c + 2*b*Tanh[x/2] + (a - c)*Tanh[x/2]^2])/(b^2 + c^2)

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 1075

```
Int[((A_.) + (C_.)*(x_)^2)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*((d_) + (f_.
)*(x_)^2)), x_Symbol] :> With[{q = c^2*d^2 + b^2*d*f - 2*a*c*d*f + a^2*f^2}
, Dist[1/q, Int[(A*c^2*d - a*c*C*d + A*b^2*f - a*A*c*f + a^2*C*f + c*(-(b*C
*d) + A*b*f)*x)/(a + b*x + c*x^2), x], x] + Dist[1/q, Int[(c*C*d^2 - A*c*d*
f - a*C*d*f + a*A*f^2 - f*(-(b*C*d) + A*b*f)*x)/(d + f*x^2), x], x] /; NeQ[
q, 0]] /; FreeQ[{a, b, c, d, f, A, C}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 4397

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rubi steps

$$\int \frac{\operatorname{sech}^2(x)}{a+c\operatorname{sech}(x)+b\tanh(x)}\,dx = \int \frac{\operatorname{sech}(x)}{c+a\cosh(x)+b\sinh(x)}\,dx$$

$$= 2\,\text{Subst}\left(\int \frac{1-x^2}{(1+x^2)\left(a+c+2bx+(a-c)x^2\right)}\,dx, x, \tanh\left(\frac{x}{2}\right)\right)$$

$$= \frac{\text{Subst}\left(\int \frac{4c-4bx}{1+x^2}\,dx, x, \tanh\left(\frac{x}{2}\right)\right)}{2\left(b^2+c^2\right)} + \frac{\text{Subst}\left(\int \frac{4b^2+(a-c)^2-(a+c)^2+4b(a-c)x}{a+c+2bx+(a-c)x^2}\,dx, x, \tanh\right.}{2\left(b^2+c^2\right)}$$

$$= \frac{b\,\text{Subst}\left(\int \frac{2b+2(a-c)x}{a+c+2bx+(a-c)x^2}\,dx, x, \tanh\left(\frac{x}{2}\right)\right)}{b^2+c^2} - \frac{(2b)\,\text{Subst}\left(\int \frac{x}{1+x^2}\,dx, x, \tanh\left(\frac{x}{2}\right)\right.}{b^2+c^2}$$

$$= \frac{2c\tan^{-1}\left(\tanh\left(\frac{x}{2}\right)\right)}{b^2+c^2} - \frac{b\log\left(1+\tanh^2\left(\frac{x}{2}\right)\right)}{b^2+c^2} + \frac{b\log\left(a+c+2b\tanh\left(\frac{x}{2}\right)+(a\right.}{b^2+c^2}$$

$$= \frac{2c\tan^{-1}\left(\tanh\left(\frac{x}{2}\right)\right)}{b^2+c^2} - \frac{2ac\tan^{-1}\left(\frac{b+(a-c)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2-c^2}}\right)}{\sqrt{a^2-b^2-c^2}\left(b^2+c^2\right)} - \frac{b\log\left(1+\tanh^2\left(\frac{x}{2}\right)\right)}{b^2+c^2} + \frac{b\log}{}$$

Mathematica [A] time = 0.27, size = 96, normalized size = 0.66

$$\frac{-\frac{2ac\tan^{-1}\left(\frac{(a-c)\tanh\left(\frac{x}{2}\right)+b}{\sqrt{a^2-b^2-c^2}}\right)}{\sqrt{a^2-b^2-c^2}} + b(\log(a\cosh(x)+b\sinh(x)+c)-\log(\cosh(x))) + 2c\tan^{-1}\left(\tanh\left(\frac{x}{2}\right)\right)}{b^2+c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[x]^2/(a + c*Sech[x] + b*Tanh[x]),x]
```

```
[Out] (2*c*ArcTan[Tanh[x/2]] - (2*a*c*ArcTan[(b + (a - c)*Tanh[x/2])/Sqrt[a^2 - b
^2 - c^2]])/Sqrt[a^2 - b^2 - c^2] + b*(-Log[Cosh[x]] + Log[c + a*Cosh[x] +
b*Sinh[x]]))/(b^2 + c^2)
```

fricas [A] time = 1.54, size = 486, normalized size = 3.33

$$\left[-\frac{\sqrt{-a^2+b^2+c^2}\,ac\log\left(\frac{2\,(a+b)c\cosh(x)+\left(a^2+2\,ab+b^2\right)\cosh(x)^2+\left(a^2+2\,ab+b^2\right)\sinh(x)^2-a^2+b^2+2\,c^2+2\,\left((a+b)c+\left(a^2+2\,ab+b^2\right)\cosh(x)\right)\sin}{(a+b)\cosh(x)^2+(a+b)\sinh(x)^2+2\,c\cosh(x)+2\,((a+b)\cosh(x)+c)\sinh}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^2/(a+c*sech(x)+b*tanh(x)),x, algorithm="fricas")
```

```
[Out] [-(sqrt(-a^2 + b^2 + c^2)*a*c*log((2*(a + b)*c*cosh(x) + (a^2 + 2*a*b + b^2
)*cosh(x)^2 + (a^2 + 2*a*b + b^2)*sinh(x)^2 - a^2 + b^2 + 2*c^2 + 2*((a + b
)*c + (a^2 + 2*a*b + b^2)*cosh(x))*sinh(x) + 2*sqrt(-a^2 + b^2 + c^2)*((a +
 b)*cosh(x) + (a + b)*sinh(x) + c))/((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2
+ 2*c*cosh(x) + 2*((a + b)*cosh(x) + c)*sinh(x) + a - b)) + 2*(c^3 - (a^2 -
 b^2)*c)*arctan(cosh(x) + sinh(x)) - (a^2*b - b^3 - b*c^2)*log(2*(a*cosh(x)
 + b*sinh(x) + c)/(cosh(x) - sinh(x))) + (a^2*b - b^3 - b*c^2)*log(2*cosh(x
)/(cosh(x) - sinh(x))))/(a^2*b^2 - b^4 - c^4 + (a^2 - 2*b^2)*c^2), (2*sqrt(
a^2 - b^2 - c^2)*a*c*arctan(-((a + b)*cosh(x) + (a + b)*sinh(x) + c)/sqrt(a
^2 - b^2 - c^2)) - 2*(c^3 - (a^2 - b^2)*c)*arctan(cosh(x) + sinh(x)) + (a^2
*b - b^3 - b*c^2)*log(2*(a*cosh(x) + b*sinh(x) + c)/(cosh(x) - sinh(x))) -
(a^2*b - b^3 - b*c^2)*log(2*cosh(x)/(cosh(x) - sinh(x))))/(a^2*b^2 - b^4 -
c^4 + (a^2 - 2*b^2)*c^2)]
```

giac [A] time = 0.14, size = 126, normalized size = 0.86

$$-\frac{2\,ac\arctan\left(\frac{ae^{x}+be^{x}+c}{\sqrt{a^{2}-b^{2}-c^{2}}}\right)}{\sqrt{a^{2}-b^{2}-c^{2}}\left(b^{2}+c^{2}\right)}+\frac{2\,c\arctan\left(e^{x}\right)}{b^{2}+c^{2}}+\frac{b\log\left(ae^{(2\,x)}+be^{(2\,x)}+2\,ce^{x}+a-b\right)}{b^{2}+c^{2}}-\frac{b\log\left(e^{(2\,x)}+1\right)}{b^{2}+c^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^2/(a+c*sech(x)+b*tanh(x)),x, algorithm="giac")
```

```
[Out] -2*a*c*arctan((a*e^x + b*e^x + c)/sqrt(a^2 - b^2 - c^2))/(sqrt(a^2 - b^2 -
c^2)*(b^2 + c^2)) + 2*c*arctan(e^x)/(b^2 + c^2) + b*log(a*e^(2*x) + b*e^(2*
x) + 2*c*e^x + a - b)/(b^2 + c^2) - b*log(e^(2*x) + 1)/(b^2 + c^2)
```

maple [B] time = 0.22, size = 406, normalized size = 2.78

$$\frac{\ln\left(a\left(\tanh^{2}\left(\frac{x}{2}\right)\right)-\left(\tanh^{2}\left(\frac{x}{2}\right)\right)c+2\tanh\left(\frac{x}{2}\right)b+a+c\right)ab}{\left(b^{2}+c^{2}\right)\left(a-c\right)}-\frac{\ln\left(a\left(\tanh^{2}\left(\frac{x}{2}\right)\right)-\left(\tanh^{2}\left(\frac{x}{2}\right)\right)c+2\tanh\left(\frac{x}{2}\right)b+\right.}{\left(b^{2}+c^{2}\right)\left(a-c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(x)^2/(a+c*sech(x)+b*tanh(x)),x)
```

```
[Out] 1/(b^2+c^2)/(a-c)*ln(a*tanh(1/2*x)^2-tanh(1/2*x)^2*c+2*tanh(1/2*x)*b+a+c)*a
*b-1/(b^2+c^2)/(a-c)*ln(a*tanh(1/2*x)^2-tanh(1/2*x)^2*c+2*tanh(1/2*x)*b+a+c
)*c*b-2/(b^2+c^2)/(a^2-b^2-c^2)^(1/2)*arctan(1/2*(2*(a-c)*tanh(1/2*x)+2*b)/
(a^2-b^2-c^2)^(1/2))*a*c+2/(b^2+c^2)/(a^2-b^2-c^2)^(1/2)*arctan(1/2*(2*(a-c
)*tanh(1/2*x)+2*b)/(a^2-b^2-c^2)^(1/2))*b^2-2/(b^2+c^2)/(a^2-b^2-c^2)^(1/2)
*arctan(1/2*(2*(a-c)*tanh(1/2*x)+2*b)/(a^2-b^2-c^2)^(1/2))*b^2/(a-c)*a+2/(b
```

```
^2+c^2)/(a^2-b^2-c^2)^(1/2)*arctan(1/2*(2*(a-c)*tanh(1/2*x)+2*b)/(a^2-b^2-c
^2)^(1/2))*b^2/(a-c)*c-b*ln(tanh(1/2*x)^2+1)/(b^2+c^2)+2*c*arctan(tanh(1/2*
x))/(b^2+c^2)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^2/(a+c*sech(x)+b*tanh(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(c^2+b^2-a^2>0)', see `assume?` for
more details)Is c^2+b^2-a^2 positive or negative?
```

mupad [B] time = 28.99, size = 1069, normalized size = 7.32

$$\ln\left(\frac{64\,(a-b+2\,c\,e^x)}{(a+b)^4}+\frac{\left(\frac{32\left(2\,a^3+3\,e^x\,a^2\,c-2\,a\,b^2+6\,e^x\,a\,b\,c-2\,a\,c^2+3\,e^x\,b^2\,c+2\,b\,c^2-4\,e^x\,c^3\right)}{(a+b)^5}+\frac{\left(\frac{32\,(a-b)\left(-2\,b^3+6\,e^x\,b^2\,c-2\,a\,b^2+b\,c^2+6\,a\,e^x\,b\,c+3\,e^x\,c^3+2\,a\,c^2\right)}{(a+b)^5}-\frac{32\left(b\,c^2-a^2\right.}{}\right.}{}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(x)^2*(a + b*tanh(x) + c/cosh(x))),x)
```

```
[Out] (log((64*(a - b + 2*c*exp(x)))/(a + b)^4 + (((32*(2*b*c^2 - 2*a*c^2 - 2*a*b
^2 + 2*a^3 - 4*c^3*exp(x) + 3*a^2*c*exp(x) + 3*b^2*c*exp(x) + 6*a*b*c*exp(x
)))/(a + b)^5 + (((32*(a - b)*(2*a*c^2 - 2*a*b^2 + b*c^2 - 2*b^3 + 3*c^3*ex
p(x) + 6*b^2*c*exp(x) + 6*a*b*c*exp(x)))/(a + b)^5 - (32*(b*c^2 - a^2*b + b
^3 + a*c*(b^2 - a^2 + c^2)^(1/2))*(3*a*c^4 - 2*a*b^4 - 3*b*c^4 + 2*a^3*b^2
- 2*a^3*c^2 - 3*b^3*c^2 + 4*c^5*exp(x) + a*b^2*c^2 + 4*a^2*b*c^2 + b^4*c*ex
p(x) - 3*a^2*c^3*exp(x) + 5*b^2*c^3*exp(x) + a^2*b^2*c*exp(x) + 6*a*b*c^3*e
xp(x) + 6*a*b^3*c*exp(x) - 4*a^3*b*c*exp(x)))/((a + b)^5*(b^2 + c^2)*(b^2 -
 a^2 + c^2)))*(b*c^2 - a^2*b + b^3 + a*c*(b^2 - a^2 + c^2)^(1/2)))/((b^2 +
c^2)*(b^2 - a^2 + c^2)))*(b*c^2 - a^2*b + b^3 + a*c*(b^2 - a^2 + c^2)^(1/2)
))/((b^2 + c^2)*(b^2 - a^2 + c^2)))*(b*c^2 - a^2*b + b^3 + a*c*(b^2 - a^2 +
 c^2)^(1/2)))/(b^4 + c^4 - a^2*b^2 - a^2*c^2 + 2*b^2*c^2) - (log(exp(x) + 1
i)*1i)/(b*1i - c) - (log((64*(a - b + 2*c*exp(x)))/(a + b)^4 - (((32*(2*b*c
```

```
^2 - 2*a*c^2 - 2*a*b^2 + 2*a^3 - 4*c^3*exp(x) + 3*a^2*c*exp(x) + 3*b^2*c*ex
p(x) + 6*a*b*c*exp(x)))/(a + b)^5 - (((32*(a - b)*(2*a*c^2 - 2*a*b^2 + b*c^
2 - 2*b^3 + 3*c^3*exp(x) + 6*b^2*c*exp(x) + 6*a*b*c*exp(x)))/(a + b)^5 + (3
2*(a^2*b - b*c^2 - b^3 + a*c*(b^2 - a^2 + c^2)^(1/2))*(3*a*c^4 - 2*a*b^4 -
3*b*c^4 + 2*a^3*b^2 - 2*a^3*c^2 - 3*b^3*c^2 + 4*c^5*exp(x) + a*b^2*c^2 + 4*
a^2*b*c^2 + b^4*c*exp(x) - 3*a^2*c^3*exp(x) + 5*b^2*c^3*exp(x) + a^2*b^2*c*
exp(x) + 6*a*b*c^3*exp(x) + 6*a*b^3*c*exp(x) - 4*a^3*b*c*exp(x)))/((a + b)^
5*(b^2 + c^2)*(b^2 - a^2 + c^2)))*(a^2*b - b*c^2 - b^3 + a*c*(b^2 - a^2 + c
^2)^(1/2)))/((b^2 + c^2)*(b^2 - a^2 + c^2)))*(a^2*b - b*c^2 - b^3 + a*c*(b^
2 - a^2 + c^2)^(1/2)))/((b^2 + c^2)*(b^2 - a^2 + c^2)))*(a^2*b - b*c^2 - b^
3 + a*c*(b^2 - a^2 + c^2)^(1/2)))/(b^4 + c^4 - a^2*b^2 - a^2*c^2 + 2*b^2*c^
2) - log(exp(x)*1i + 1)/(b - c*1i)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}^2(x)}{a + b\tanh(x) + c\operatorname{sech}(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)**2/(a+c*sech(x)+b*tanh(x)),x)
```

```
[Out] Integral(sech(x)**2/(a + b*tanh(x) + c*sech(x)), x)
```

3.786 $\int \frac{\operatorname{csch}(x)}{2+2\coth(x)+3\operatorname{csch}(x)}\,dx$

Optimal. Leaf size=19

$$-\frac{2}{3}\tanh^{-1}\left(\frac{1}{3}\left(2-\tanh\left(\frac{x}{2}\right)\right)\right)$$

```
[Out] 2/3*arctanh(-2/3+1/3*tanh(1/2*x))
```

Rubi [A] time = 0.05, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {3166, 3124, 618, 204}

$$-\frac{2}{3}\tanh^{-1}\left(\frac{1}{3}\left(2-\tanh\left(\frac{x}{2}\right)\right)\right)$$

Antiderivative was successfully verified.

```
[In] Int[Csch[x]/(2 + 2*Coth[x] + 3*Csch[x]),x]
```

```
[Out] (-2*ArcTanh[(2 - Tanh[x/2])/3])/3
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 3124

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Module[{f = FreeFactors[Tan[(d + e*x)/2], x]}, Dist[(2*f
)/e, Subst[Int[1/(a + b + 2*c*f*x + (a - b)*f^2*x^2), x], x, Tan[(d + e*x)/
2]/f], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
```

Rule 3166

```
Int[csc[(d_.) + (e_.)*(x_)]^(n_.)*((a_.) + csc[(d_.) + (e_.)*(x_)]*(b_.) +
cot[(d_.) + (e_.)*(x_)]*(c_.))^(m_), x_Symbol] :> Int[1/(b + a*Sin[d + e*x]
 + c*Cos[d + e*x])^n, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[m + n, 0] && I
```

```
ntegerQ[n]
```

Rubi steps

$$\begin{aligned}\int \frac{\operatorname{csch}(x)}{2+2\coth(x)+3\operatorname{csch}(x)}\,dx &= i\int \frac{1}{3i+2i\cosh(x)+2i\sinh(x)}\,dx\\ &= 2i\,\text{Subst}\left(\int \frac{1}{5i+4ix-ix^2}\,dx, x, \tanh\left(\frac{x}{2}\right)\right)\\ &= -\left(4i\,\text{Subst}\left(\int \frac{1}{-36-x^2}\,dx, x, 4i-2i\tanh\left(\frac{x}{2}\right)\right)\right)\\ &= -\frac{2}{3}\tanh^{-1}\left(\frac{1}{3}\left(2-\tanh\left(\frac{x}{2}\right)\right)\right)\end{aligned}$$

Mathematica [A] time = 0.07, size = 28, normalized size = 1.47

$$\frac{x}{6}-\frac{1}{3}\log\left(5\cosh\left(\frac{x}{2}\right)-\sinh\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[x]/(2 + 2*Coth[x] + 3*Csch[x]),x]
[Out] x/6 - Log[5*Cosh[x/2] - Sinh[x/2]]/3
```

fricas [A] time = 0.41, size = 17, normalized size = 0.89

$$\frac{1}{3}\,x-\frac{1}{3}\,\log\left(2\,\cosh(x)+2\,\sinh(x)+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)/(2+2*coth(x)+3*csch(x)),x, algorithm="fricas")
[Out] 1/3*x - 1/3*log(2*cosh(x) + 2*sinh(x) + 3)
```

giac [A] time = 0.11, size = 13, normalized size = 0.68

$$\frac{1}{3}\,x-\frac{1}{3}\,\log\left(2\,e^{x}+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)/(2+2*coth(x)+3*csch(x)),x, algorithm="giac")
```

```
[Out] 1/3*x - 1/3*log(2*e^x + 3)
```

maple [A] time = 0.20, size = 20, normalized size = 1.05

$$\frac{\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)}{3}-\frac{\ln\left(\tanh\left(\frac{x}{2}\right)-5\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(x)/(2+2*coth(x)+3*csch(x)),x)
[Out] 1/3*ln(tanh(1/2*x)+1)-1/3*ln(tanh(1/2*x)-5)
```

maxima [A] time = 0.83, size = 11, normalized size = 0.58

$$-\frac{1}{3}\log\left(3e^{(-x)}+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)/(2+2*coth(x)+3*csch(x)),x, algorithm="maxima")
[Out] -1/3*log(3*e^(-x) + 2)
```

mupad [B] time = 0.06, size = 11, normalized size = 0.58

$$\frac{x}{3}-\frac{\ln\left(e^x+\frac{3}{2}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sinh(x)*(2*coth(x) + 3/sinh(x) + 2)),x)
[Out] x/3 - log(exp(x) + 3/2)/3
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\operatorname{csch}(x)}{2\coth(x)+3\operatorname{csch}(x)+2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)/(2+2*coth(x)+3*csch(x)),x)
[Out] Integral(csch(x)/(2*coth(x) + 3*csch(x) + 2), x)
```

3.787 $\int \frac{\mathbf{csch}(x)}{a+b\coth(x)+c\mathbf{csch}(x)}\,dx$

Optimal. Leaf size=50

$$-\frac{2\tanh^{-1}\left(\frac{a+(b-c)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2+c^2}}\right)}{\sqrt{a^2-b^2+c^2}}$$

```
[Out] -2*arctanh((a+(b-c)*tanh(1/2*x))/(a^2-b^2+c^2)^(1/2))/(a^2-b^2+c^2)^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {3166, 3124, 618, 204}

$$-\frac{2\tanh^{-1}\left(\frac{a+(b-c)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2+c^2}}\right)}{\sqrt{a^2-b^2+c^2}}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[x]/(a + b*Coth[x] + c*Csch[x]),x]
```

```
[Out] (-2*ArcTanh[(a + (b - c)*Tanh[x/2])/Sqrt[a^2 - b^2 + c^2]])/Sqrt[a^2 - b^2
+ c^2]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 3124

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Module[{f = FreeFactors[Tan[(d + e*x)/2], x]}, Dist[(2*f
)/e, Subst[Int[1/(a + b + 2*c*f*x + (a - b)*f^2*x^2), x], x, Tan[(d + e*x)/
2]/f], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
```

Rule 3166

```
Int[csc[(d_.) + (e_.)*(x_)]^(n_.)*((a_.) + csc[(d_.) + (e_.)*(x_)]*(b_.) +
cot[(d_.) + (e_.)*(x_)]*(c_.))^(m_), x_Symbol] :> Int[1/(b + a*Sin[d + e*x]
 + c*Cos[d + e*x])^n, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[m + n, 0] && I
ntegerQ[n]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\operatorname{csch}(x)}{a+b\coth(x)+c\operatorname{csch}(x)}\,dx &= i\int \frac{1}{ic+ib\cosh(x)+ia\sinh(x)}\,dx \\
&= 2i\,\mathrm{Subst}\left(\int \frac{1}{ib+ic+2iax-(-ib+ic)x^2}\,dx, x, \tanh\left(\frac{x}{2}\right)\right) \\
&= -\left(4i\,\mathrm{Subst}\left(\int \frac{1}{-4\left(a^2-b^2+c^2\right)-x^2}\,dx, x, 2ia+2(ib-ic)\tanh\left(\frac{x}{2}\right)\right)\right) \\
&= -\frac{2\tanh^{-1}\left(\frac{a+(b-c)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2+c^2}}\right)}{\sqrt{a^2-b^2+c^2}}
\end{aligned}
$$

Mathematica [A] time = 0.05, size = 54, normalized size = 1.08

$$
\frac{2\tan^{-1}\left(\frac{a+(b-c)\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2+b^2-c^2}}\right)}{\sqrt{-a^2+b^2-c^2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[x]/(a + b*Coth[x] + c*Csch[x]),x]
```

```
[Out] (2*ArcTan[(a + (b - c)*Tanh[x/2])/Sqrt[-a^2 + b^2 - c^2]])/Sqrt[-a^2 + b^2
- c^2]
```

fricas [A] time = 0.45, size = 244, normalized size = 4.88

$$
\left[\frac{\log\left(\frac{2\,(a+b)c\cosh(x)+\left(a^2+2\,ab+b^2\right)\cosh(x)^2+\left(a^2+2\,ab+b^2\right)\sinh(x)^2+a^2-b^2+2\,c^2+2\,\left((a+b)c+\left(a^2+2\,ab+b^2\right)\cosh(x)\right)\sinh(x)-2\,\sqrt{a^2-b^2+c^2}\,((a+b)}{(a+b)\cosh(x)^2+(a+b)\sinh(x)^2+2\,c\cosh(x)+2\,((a+b)\cosh(x)+c)\sinh(x)-a+b}\right.}{\sqrt{a^2-b^2+c^2}}\right.
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)/(a+b*coth(x)+c*csch(x)),x, algorithm="fricas")
```

```
[Out] [log((2*(a + b)*c*cosh(x) + (a^2 + 2*a*b + b^2)*cosh(x)^2 + (a^2 + 2*a*b +
b^2)*sinh(x)^2 + a^2 - b^2 + 2*c^2 + 2*((a + b)*c + (a^2 + 2*a*b + b^2)*cos
h(x))*sinh(x) - 2*sqrt(a^2 - b^2 + c^2)*((a + b)*cosh(x) + (a + b)*sinh(x)
+ c))/((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + 2*c*cosh(x) + 2*((a + b)*cos
h(x) + c)*sinh(x) - a + b))/sqrt(a^2 - b^2 + c^2), 2*sqrt(-a^2 + b^2 - c^2)
*arctan(sqrt(-a^2 + b^2 - c^2)*((a + b)*cosh(x) + (a + b)*sinh(x) + c)/(a^2
 - b^2 + c^2))/(a^2 - b^2 + c^2)]
```

giac [A] time = 0.13, size = 46, normalized size = 0.92

$$\frac{2 \arctan\left(\frac{ae^x+be^x+c}{\sqrt{-a^2+b^2-c^2}}\right)}{\sqrt{-a^2+b^2-c^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)/(a+b*coth(x)+c*csch(x)),x, algorithm="giac")
```

```
[Out] 2*arctan((a*e^x + b*e^x + c)/sqrt(-a^2 + b^2 - c^2))/sqrt(-a^2 + b^2 - c^2)
```

maple [A] time = 0.20, size = 53, normalized size = 1.06

$$\frac{2 \arctan\left(\frac{2(b-c)\tanh\left(\frac{x}{2}\right)+2a}{2\sqrt{-a^2+b^2-c^2}}\right)}{\sqrt{-a^2+b^2-c^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(x)/(a+b*coth(x)+c*csch(x)),x)
```

```
[Out] 2/(-a^2+b^2-c^2)^(1/2)*arctan(1/2*(2*(b-c)*tanh(1/2*x)+2*a)/(-a^2+b^2-c^2)^
(1/2))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)/(a+b*coth(x)+c*csch(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(c^2-b^2+a^2>0)', see `assume?` for
more details)Is c^2-b^2+a^2 positive or negative?
```

mupad [B] time = 0.21, size = 78, normalized size = 1.56

$$\frac{2\,\mathrm{atan}\left(\frac{c}{\sqrt{-a^2+b^2-c^2}}+\frac{a\,\mathrm{e}^x}{\sqrt{-a^2+b^2-c^2}}+\frac{b\,\mathrm{e}^x}{\sqrt{-a^2+b^2-c^2}}\right)}{\sqrt{-a^2+b^2-c^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sinh(x)*(a + c/sinh(x) + b*coth(x))),x)
```

```
[Out] (2*atan(c/(b^2 - a^2 - c^2)^(1/2) + (a*exp(x))/(b^2 - a^2 - c^2)^(1/2) + (b
*exp(x))/(b^2 - a^2 - c^2)^(1/2)))/(b^2 - a^2 - c^2)^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}(x)}{a+b\coth(x)+c\operatorname{csch}(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)/(a+b*coth(x)+c*csch(x)),x)
```

```
[Out] Integral(csch(x)/(a + b*coth(x) + c*csch(x)), x)
```

3.788 $\int \frac{\operatorname{csch}^2(x)}{a+b\coth(x)+c\operatorname{csch}(x)}\,dx$

Optimal. Leaf size=118

$$-\frac{2ac\tanh^{-1}\left(\frac{a+(b-c)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2+c^2}}\right)}{\left(b^2-c^2\right)\sqrt{a^2-b^2+c^2}}-\frac{b\log\left(2a\tanh\left(\frac{x}{2}\right)+(b-c)\tanh^2\left(\frac{x}{2}\right)+b+c\right)}{b^2-c^2}+\frac{\log\left(\tanh\left(\frac{x}{2}\right)\right)}{b+c}$$

```
[Out] ln(tanh(1/2*x))/(b+c)-b*ln(b+c+2*a*tanh(1/2*x)+(b-c)*tanh(1/2*x)^2)/(b^2-c^
2)-2*a*c*arctanh((a+(b-c)*tanh(1/2*x))/(a^2-b^2+c^2)^(1/2))/(b^2-c^2)/(a^2-
b^2+c^2)^(1/2)
```

Rubi [A] time = 0.58, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.412, Rules used = {4397, 12, 1628, 634, 618, 206, 628}

$$-\frac{2ac\tanh^{-1}\left(\frac{a+(b-c)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2+c^2}}\right)}{\left(b^2-c^2\right)\sqrt{a^2-b^2+c^2}}-\frac{b\log\left(2a\tanh\left(\frac{x}{2}\right)+(b-c)\tanh^2\left(\frac{x}{2}\right)+b+c\right)}{b^2-c^2}+\frac{\log\left(\tanh\left(\frac{x}{2}\right)\right)}{b+c}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[x]^2/(a + b*Coth[x] + c*Csch[x]),x]
```

```
[Out] (-2*a*c*ArcTanh[(a + (b - c)*Tanh[x/2])/Sqrt[a^2 - b^2 + c^2]])/((b^2 - c^2
)*Sqrt[a^2 - b^2 + c^2]) + Log[Tanh[x/2]]/(b + c) - (b*Log[b + c + 2*a*Tanh
[x/2] + (b - c)*Tanh[x/2]^2])/(b^2 - c^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 4397

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\operatorname{csch}^2(x)}{a+b\coth(x)+c\operatorname{csch}(x)}\,dx &= i\int \frac{\operatorname{csch}(x)}{ic+ib\cosh(x)+ia\sinh(x)}\,dx \\
&= -\left(2\,\text{Subst}\left(\int \frac{-1+x^2}{2x\left(b+c+2ax+(b-c)x^2\right)}\,dx,x,\tanh\left(\frac{x}{2}\right)\right)\right) \\
&= -\text{Subst}\left(\int \frac{-1+x^2}{x\left(b+c+2ax+(b-c)x^2\right)}\,dx,x,\tanh\left(\frac{x}{2}\right)\right) \\
&= -\text{Subst}\left(\int \left(-\frac{1}{(b+c)x}+\frac{2(a+bx)}{(b+c)\left(b+c+2ax+(b-c)x^2\right)}\right)dx,x,\tanh\left(\frac{x}{2}\right)\right) \\
&= \frac{\log\left(\tanh\left(\frac{x}{2}\right)\right)}{b+c}-\frac{2\,\text{Subst}\left(\int \frac{a+bx}{b+c+2ax+(b-c)x^2}\,dx,x,\tanh\left(\frac{x}{2}\right)\right)}{b+c} \\
&= \frac{\log\left(\tanh\left(\frac{x}{2}\right)\right)}{b+c}-\frac{b\,\text{Subst}\left(\int \frac{2a+2(b-c)x}{b+c+2ax+(b-c)x^2}\,dx,x,\tanh\left(\frac{x}{2}\right)\right)}{b^2-c^2}+\frac{(2ac)\,\text{Subst}\left(\int\right.}{} \\
&= \frac{\log\left(\tanh\left(\frac{x}{2}\right)\right)}{b+c}-\frac{b\log\left(b+c+2a\tanh\left(\frac{x}{2}\right)+(b-c)\tanh^2\left(\frac{x}{2}\right)\right)}{b^2-c^2}-\frac{(4ac)\,\text{Sub}}{} \\
&= -\frac{2ac\tanh^{-1}\left(\frac{a+(b-c)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2+c^2}}\right)}{\left(b^2-c^2\right)\sqrt{a^2-b^2+c^2}}+\frac{\log\left(\tanh\left(\frac{x}{2}\right)\right)}{b+c}-\frac{b\log\left(b+c+2a\tanh\left(\frac{x}{2}\right)+\right.}{b^2-c^2}
\end{aligned}
$$

Mathematica [A] time = 0.24, size = 97, normalized size = 0.82

$$
\frac{-\frac{2ac\tan^{-1}\left(\frac{a+(b-c)\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2+b^2-c^2}}\right)}{\sqrt{-a^2+b^2-c^2}}+b\log(a\sinh(x)+b\cosh(x)+c)-b\log(\sinh(x))+c\log\left(\tanh\left(\frac{x}{2}\right)\right)}{c^2-b^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[x]^2/(a + b*Coth[x] + c*Csch[x]),x]
```

```
[Out] ((-2*a*c*ArcTan[(a + (b - c)*Tanh[x/2])/Sqrt[-a^2 + b^2 - c^2]])/Sqrt[-a^2
+ b^2 - c^2] - b*Log[Sinh[x]] + b*Log[c + b*Cosh[x] + a*Sinh[x]] + c*Log[Ta
nh[x/2]])/(-b^2 + c^2)
```

fricas [B] time = 1.55, size = 546, normalized size = 4.63

$$\left[-\frac{\sqrt{a^2-b^2+c^2}\,ac\log\left(\frac{2\,(a+b)c\cosh(x)+\left(a^2+2\,ab+b^2\right)\cosh(x)^2+\left(a^2+2\,ab+b^2\right)\sinh(x)^2+a^2-b^2+2\,c^2+2\,\left((a+b)c+\left(a^2+2\,ab+b^2\right)\cosh(x)\right)\sinh}{(a+b)\cosh(x)^2+(a+b)\sinh(x)^2+2\,c\cosh(x)+2\,((a+b)\cosh(x)+c)\sinh(x)}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^2/(a+b*coth(x)+c*csch(x)),x, algorithm="fricas")
```

```
[Out] [-(sqrt(a^2 - b^2 + c^2)*a*c*log((2*(a + b)*c*cosh(x) + (a^2 + 2*a*b + b^2)
*cosh(x)^2 + (a^2 + 2*a*b + b^2)*sinh(x)^2 + a^2 - b^2 + 2*c^2 + 2*((a + b)
*c + (a^2 + 2*a*b + b^2)*cosh(x))*sinh(x) + 2*sqrt(a^2 - b^2 + c^2)*((a + b
)*cosh(x) + (a + b)*sinh(x) + c))/((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 +
2*c*cosh(x) + 2*((a + b)*cosh(x) + c)*sinh(x) - a + b)) + (a^2*b - b^3 + b*
c^2)*log(2*(b*cosh(x) + a*sinh(x) + c)/(cosh(x) - sinh(x))) - (a^2*b - b^3
+ b*c^2 + c^3 + (a^2 - b^2)*c)*log(cosh(x) + sinh(x) + 1) - (a^2*b - b^3 +
b*c^2 - c^3 - (a^2 - b^2)*c)*log(cosh(x) + sinh(x) - 1))/(a^2*b^2 - b^4 - c
^4 - (a^2 - 2*b^2)*c^2), (2*sqrt(-a^2 + b^2 - c^2)*a*c*arctan(sqrt(-a^2 + b
^2 - c^2)*((a + b)*cosh(x) + (a + b)*sinh(x) + c)/(a^2 - b^2 + c^2)) - (a^2
*b - b^3 + b*c^2)*log(2*(b*cosh(x) + a*sinh(x) + c)/(cosh(x) - sinh(x))) +
(a^2*b - b^3 + b*c^2 + c^3 + (a^2 - b^2)*c)*log(cosh(x) + sinh(x) + 1) + (a
^2*b - b^3 + b*c^2 - c^3 - (a^2 - b^2)*c)*log(cosh(x) + sinh(x) - 1))/(a^2*
b^2 - b^4 - c^4 - (a^2 - 2*b^2)*c^2)]
```

giac [A] time = 0.12, size = 122, normalized size = 1.03

$$\frac{2\,ac\arctan\left(\frac{ae^x+be^x+c}{\sqrt{-a^2+b^2-c^2}}\right)}{\sqrt{-a^2+b^2-c^2}\left(b^2-c^2\right)}-\frac{b\log\left(ae^{(2\,x)}+be^{(2\,x)}+2\,ce^x-a+b\right)}{b^2-c^2}+\frac{\log\left(e^x+1\right)}{b-c}+\frac{\log\left(|e^x-1|\right)}{b+c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^2/(a+b*coth(x)+c*csch(x)),x, algorithm="giac")
```

```
[Out] 2*a*c*arctan((a*e^x + b*e^x + c)/sqrt(-a^2 + b^2 - c^2))/(sqrt(-a^2 + b^2 -
 c^2)*(b^2 - c^2)) - b*log(a*e^(2*x) + b*e^(2*x) + 2*c*e^x - a + b)/(b^2 -
c^2) + log(e^x + 1)/(b - c) + log(abs(e^x - 1))/(b + c)
```

maple [A] time = 0.20, size = 180, normalized size = 1.53

$$-\frac{b\ln\left(\left(\tanh^2\left(\frac{x}{2}\right)\right)b-\left(\tanh^2\left(\frac{x}{2}\right)\right)c+2a\tanh\left(\frac{x}{2}\right)+b+c\right)}{(b+c)\,(b-c)}-\frac{2\arctan\left(\frac{2(b-c)\tanh\left(\frac{x}{2}\right)+2a}{2\sqrt{-a^2+b^2-c^2}}\right)a}{(b+c)\sqrt{-a^2+b^2-c^2}}+\frac{2\arctan\left(\frac{2(b-c)\tanh\left(\frac{x}{2}\right.}{2\sqrt{-a^2+b^2-}}\right.}{(b+c)\sqrt{-a^2+b^2-c^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(x)^2/(a+b*coth(x)+c*csch(x)),x)
```

```
[Out] -1/(b+c)*b/(b-c)*ln(tanh(1/2*x)^2*b-tanh(1/2*x)^2*c+2*a*tanh(1/2*x)+b+c)-2/
(b+c)/(-a^2+b^2-c^2)^(1/2)*arctan(1/2*(2*(b-c)*tanh(1/2*x)+2*a)/(-a^2+b^2-c
^2)^(1/2))*a+2/(b+c)/(-a^2+b^2-c^2)^(1/2)*arctan(1/2*(2*(b-c)*tanh(1/2*x)+2
*a)/(-a^2+b^2-c^2)^(1/2))*b*a/(b-c)+ln(tanh(1/2*x))/(b+c)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^2/(a+b*coth(x)+c*csch(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(c^2-b^2+a^2>0)', see `assume?` for
more details)Is c^2-b^2+a^2 positive or negative?
```

mupad [B] time = 7.18, size = 1069, normalized size = 9.06

$$\frac{\ln\left(e^x-1\right)}{b+c}+\frac{\ln\left(e^x+1\right)}{b-c}+\frac{\ln\left(-\frac{64\,\left(b-a+2\,c\,e^x\right)}{{\left(a+b\right)}^4}-\frac{\left(\frac{32\,\left(-2\,a^3+3\,e^x\,a^2\,c+2\,a\,b^2+6\,e^x\,a\,b\,c-2\,a\,c^2+3\,e^x\,b^2\,c+2\,b\,c^2+4\,e^x\,c^3\right)}{{\left(a+b\right)}^5}+\left(\frac{32\,\left(a-b\right)\,\left(2\,b^3+6\,e^x\,b^2\,c+2\,a\,b^2 \cdots\right.}{\left(a+ \cdots\right.} \cdots\right.\right.}{\cdots}\right)}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sinh(x)^2*(a + c/sinh(x) + b*coth(x))),x)
```

```
[Out] log(exp(x) - 1)/(b + c) + log(exp(x) + 1)/(b - c) + (log(- (64*(b - a + 2*c
*exp(x)))/(a + b)^4 - (((32*(2*a*b^2 - 2*a*c^2 + 2*b*c^2 - 2*a^3 + 4*c^3*ex
p(x) + 3*a^2*c*exp(x) + 3*b^2*c*exp(x) + 6*a*b*c*exp(x)))/(a + b)^5 + (((32
*(a - b)*(2*a*b^2 + 2*a*c^2 + b*c^2 + 2*b^3 - 3*c^3*exp(x) + 6*b^2*c*exp(x)
 + 6*a*b*c*exp(x)))/(a + b)^5 - (32*(a^2*b + b*c^2 - b^3 + a*c*(a^2 - b^2 +
 c^2)^(1/2))*(2*a*b^4 - 3*a*c^4 + 3*b*c^4 - 2*a^3*b^2 - 2*a^3*c^2 - 3*b^3*c
^2 + 4*c^5*exp(x) + a*b^2*c^2 + 4*a^2*b*c^2 + b^4*c*exp(x) + 3*a^2*c^3*exp(
x) - 5*b^2*c^3*exp(x) + a^2*b^2*c*exp(x) - 6*a*b*c^3*exp(x) + 6*a*b^3*c*exp
```

```
(x) - 4*a^3*b*c*exp(x)))/((a + b)^5*(b^2 - c^2)*(a^2 - b^2 + c^2)))*(a^2*b
+ b*c^2 - b^3 + a*c*(a^2 - b^2 + c^2)^(1/2)))/((b^2 - c^2)*(a^2 - b^2 + c^2
)))*(a^2*b + b*c^2 - b^3 + a*c*(a^2 - b^2 + c^2)^(1/2)))/((b^2 - c^2)*(a^2
- b^2 + c^2)))*(a^2*b + b*c^2 - b^3 + a*c*(a^2 - b^2 + c^2)^(1/2)))/(b^4 +
c^4 - a^2*b^2 + a^2*c^2 - 2*b^2*c^2) + (log(- (64*(b - a + 2*c*exp(x)))/(a
+ b)^4 - (((32*(2*a*b^2 - 2*a*c^2 + 2*b*c^2 - 2*a^3 + 4*c^3*exp(x) + 3*a^2*
c*exp(x) + 3*b^2*c*exp(x) + 6*a*b*c*exp(x)))/(a + b)^5 + (((32*(a - b)*(2*a
*b^2 + 2*a*c^2 + b*c^2 + 2*b^3 - 3*c^3*exp(x) + 6*b^2*c*exp(x) + 6*a*b*c*ex
p(x)))/(a + b)^5 - (32*(a^2*b + b*c^2 - b^3 - a*c*(a^2 - b^2 + c^2)^(1/2))*
(2*a*b^4 - 3*a*c^4 + 3*b*c^4 - 2*a^3*b^2 - 2*a^3*c^2 - 3*b^3*c^2 + 4*c^5*ex
p(x) + a*b^2*c^2 + 4*a^2*b*c^2 + b^4*c*exp(x) + 3*a^2*c^3*exp(x) - 5*b^2*c^
3*exp(x) + a^2*b^2*c*exp(x) - 6*a*b*c^3*exp(x) + 6*a*b^3*c*exp(x) - 4*a^3*b
*c*exp(x)))/((a + b)^5*(b^2 - c^2)*(a^2 - b^2 + c^2)))*(a^2*b + b*c^2 - b^3
 - a*c*(a^2 - b^2 + c^2)^(1/2)))/((b^2 - c^2)*(a^2 - b^2 + c^2)))*(a^2*b +
b*c^2 - b^3 - a*c*(a^2 - b^2 + c^2)^(1/2)))/((b^2 - c^2)*(a^2 - b^2 + c^2))
)*(a^2*b + b*c^2 - b^3 - a*c*(a^2 - b^2 + c^2)^(1/2)))/(b^4 + c^4 - a^2*b^2
 + a^2*c^2 - 2*b^2*c^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}^2(x)}{a + b\coth(x) + c\operatorname{csch}(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)**2/(a+b*coth(x)+c*csch(x)),x)
```

```
[Out] Integral(csch(x)**2/(a + b*coth(x) + c*csch(x)), x)
```

3.789 $\int \frac{A+C\sinh(x)}{a+b\cosh(x)+c\sinh(x)}\,dx$

Optimal. Leaf size=120

$$-\frac{2\left(acC+A\left(b^2-c^2\right)\right)\tanh^{-1}\left(\frac{c-(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2+c^2}}\right)}{\left(b^2-c^2\right)\sqrt{a^2-b^2+c^2}}+\frac{bC\log(a+b\cosh(x)+c\sinh(x))}{b^2-c^2}-\frac{cCx}{b^2-c^2}$$

```
[Out] -c*C*x/(b^2-c^2)+b*C*ln(a+b*cosh(x)+c*sinh(x))/(b^2-c^2)-2*(A*(b^2-c^2)+a*c
*C)*arctanh((c-(a-b)*tanh(1/2*x))/(a^2-b^2+c^2)^(1/2))/(b^2-c^2)/(a^2-b^2+c
^2)^(1/2)
```

Rubi [A] time = 0.12, antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {3137, 3124, 618, 206}

$$-\frac{2\left(acC+A\left(b^2-c^2\right)\right)\tanh^{-1}\left(\frac{c-(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2+c^2}}\right)}{\left(b^2-c^2\right)\sqrt{a^2-b^2+c^2}}+\frac{bC\log(a+b\cosh(x)+c\sinh(x))}{b^2-c^2}-\frac{cCx}{b^2-c^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + C*Sinh[x])/(a + b*Cosh[x] + c*Sinh[x]),x]
```

```
[Out] -((c*C*x)/(b^2 - c^2)) - (2*(A*(b^2 - c^2) + a*c*C)*ArcTanh[(c - (a - b)*Ta
nh[x/2])/Sqrt[a^2 - b^2 + c^2]])/((b^2 - c^2)*Sqrt[a^2 - b^2 + c^2]) + (b*C
*Log[a + b*Cosh[x] + c*Sinh[x]])/(b^2 - c^2)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 3124

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Module[{f = FreeFactors[Tan[(d + e*x)/2], x]}, Dist[(2*f
```

```
)/e, Subst[Int[1/(a + b + 2*c*f*x + (a - b)*f^2*x^2), x], x, Tan[(d + e*x)/
2]/f], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
```

Rule 3137

```
Int[((A_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])/((a_.) + cos[(d_.) + (e_.)*(x_)
]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)]), x_Symbol] :> Simp[(c*C*(d + e*x))
/(e*(b^2 + c^2)), x] + (Dist[(A*(b^2 + c^2) - a*c*C)/(b^2 + c^2), Int[1/(a
+ b*Cos[d + e*x] + c*Sin[d + e*x]), x], x] - Simp[(b*C*Log[a + b*Cos[d + e*
x] + c*Sin[d + e*x]])/(e*(b^2 + c^2)), x]) /; FreeQ[{a, b, c, d, e, A, C},
x] && NeQ[b^2 + c^2, 0] && NeQ[A*(b^2 + c^2) - a*c*C, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{A+C\sinh(x)}{a+b\cosh(x)+c\sinh(x)}\,dx &= -\frac{cCx}{b^2-c^2}+\frac{bC\log(a+b\cosh(x)+c\sinh(x))}{b^2-c^2}+\left(A+\frac{acC}{b^2-c^2}\right)\int \frac{}{a+b\cosh(x}\\ &= -\frac{cCx}{b^2-c^2}+\frac{bC\log(a+b\cosh(x)+c\sinh(x))}{b^2-c^2}+\left(2\left(A+\frac{acC}{b^2-c^2}\right)\right)\mathrm{Subst}\left(\int\right.\\ &= -\frac{cCx}{b^2-c^2}+\frac{bC\log(a+b\cosh(x)+c\sinh(x))}{b^2-c^2}-\left(4\left(A+\frac{acC}{b^2-c^2}\right)\right)\mathrm{Subst}\left(\int\right.\\ &= -\frac{cCx}{b^2-c^2}-\frac{2\left(A+\frac{acC}{b^2-c^2}\right)\tanh^{-1}\left(\frac{c-(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2+c^2}}\right)}{\sqrt{a^2-b^2+c^2}}+\frac{bC\log(a+b\cosh(x)+c\,\mathrm{si}}{b^2-c^2}\end{aligned}$$

Mathematica [A] time = 0.26, size = 104, normalized size = 0.87

$$\frac{\frac{2\left(acC+A\left(b^2-c^2\right)\right)\tan^{-1}\left(\frac{(b-a)\tanh\left(\frac{x}{2}\right)+c}{\sqrt{-a^2+b^2-c^2}}\right)}{\sqrt{-a^2+b^2-c^2}}+C(b\log(a+b\cosh(x)+c\sinh(x))-cx)}{(b-c)(b+c)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + C*Sinh[x])/(a + b*Cosh[x] + c*Sinh[x]),x]
```

```
[Out] ((2*(A*(b^2 - c^2) + a*c*C)*ArcTan[(c + (-a + b)*Tanh[x/2])/Sqrt[-a^2 + b^2
 - c^2]])/Sqrt[-a^2 + b^2 - c^2] + C*(-(c*x) + b*Log[a + b*Cosh[x] + c*Sinh
[x]]))/((b - c)*(b + c))
```

fricas [A] time = 0.48, size = 505, normalized size = 4.21

$$\left[-\frac{\left(Ab^{2}+Cac-Ac^{2}\right)\sqrt{a^{2}-b^{2}+c^{2}}\,\log\left(\frac{\left(b^{2}+2\,bc+c^{2}\right)\cosh(x)^{2}+\left(b^{2}+2\,bc+c^{2}\right)\sinh(x)^{2}+2\,a^{2}-b^{2}+c^{2}+2\,(ab+ac)\cosh(x)+2\left(ab+ac+\left(b^{2}\cdots\right.\right.}{(b+c)\cosh(x)^{2}+(b+c)\sinh(x)^{2}+2\,a\cosh(x)+2\,(\cdots}\right.}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sinh(x))/(a+b*cosh(x)+c*sinh(x)),x, algorithm="fricas")
```

```
[Out] [-((A*b^2 + C*a*c - A*c^2)*sqrt(a^2 - b^2 + c^2)*log(((b^2 + 2*b*c + c^2)*c
osh(x)^2 + (b^2 + 2*b*c + c^2)*sinh(x)^2 + 2*a^2 - b^2 + c^2 + 2*(a*b + a*c
)*cosh(x) + 2*(a*b + a*c + (b^2 + 2*b*c + c^2)*cosh(x))*sinh(x) + 2*sqrt(a^
2 - b^2 + c^2)*((b + c)*cosh(x) + (b + c)*sinh(x) + a))/((b + c)*cosh(x)^2
+ (b + c)*sinh(x)^2 + 2*a*cosh(x) + 2*((b + c)*cosh(x) + a)*sinh(x) + b - c
)) + (C*a^2*b - C*b^3 + C*b*c^2 + C*c^3 + (C*a^2 - C*b^2)*c)*x - (C*a^2*b -
 C*b^3 + C*b*c^2)*log(2*(b*cosh(x) + c*sinh(x) + a)/(cosh(x) - sinh(x))))/(
a^2*b^2 - b^4 - c^4 - (a^2 - 2*b^2)*c^2), (2*(A*b^2 + C*a*c - A*c^2)*sqrt(-
a^2 + b^2 - c^2)*arctan(sqrt(-a^2 + b^2 - c^2)*((b + c)*cosh(x) + (b + c)*s
inh(x) + a)/(a^2 - b^2 + c^2)) - (C*a^2*b - C*b^3 + C*b*c^2 + C*c^3 + (C*a^
2 - C*b^2)*c)*x + (C*a^2*b - C*b^3 + C*b*c^2)*log(2*(b*cosh(x) + c*sinh(x)
+ a)/(cosh(x) - sinh(x))))/(a^2*b^2 - b^4 - c^4 - (a^2 - 2*b^2)*c^2)]
```

giac [A] time = 0.12, size = 122, normalized size = 1.02

$$\frac{Cb\log\left(be^{(2\,x)}+ce^{(2\,x)}+2\,ae^{x}+b-c\right)}{b^{2}-c^{2}}-\frac{Cx}{b-c}+\frac{2\left(Ab^{2}+Cac-Ac^{2}\right)\arctan\left(\frac{be^{x}+ce^{x}+a}{\sqrt{-a^{2}+b^{2}-c^{2}}}\right)}{\sqrt{-a^{2}+b^{2}-c^{2}}\left(b^{2}-c^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sinh(x))/(a+b*cosh(x)+c*sinh(x)),x, algorithm="giac")
```

```
[Out] C*b*log(b*e^(2*x) + c*e^(2*x) + 2*a*e^x + b - c)/(b^2 - c^2) - C*x/(b - c)
+ 2*(A*b^2 + C*a*c - A*c^2)*arctan((b*e^x + c*e^x + a)/sqrt(-a^2 + b^2 - c^
2))/(sqrt(-a^2 + b^2 - c^2)*(b^2 - c^2))
```

maple [B] time = 0.20, size = 573, normalized size = 4.78

$$-\frac{2C\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{2b+2c}+\frac{\ln\left(a\left(\tanh^{2}\left(\frac{x}{2}\right)\right)-\left(\tanh^{2}\left(\frac{x}{2}\right)\right)b-2c\tanh\left(\frac{x}{2}\right)-a-b\right)abC}{(b-c)(b+c)(a-b)}-\frac{\ln\left(a\left(\tanh^{2}\left(\frac{x}{2}\right)\right)-\left(\tanh\cdots\right.\right.}{(b-\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*sinh(x))/(a+b*cosh(x)+c*sinh(x)),x)
```

```
[Out] -2*C/(2*b+2*c)*ln(tanh(1/2*x)-1)+1/(b-c)/(b+c)/(a-b)*ln(a*tanh(1/2*x)^2-tan
h(1/2*x)^2*b-2*c*tanh(1/2*x)-a-b)*a*b*C-1/(b-c)/(b+c)/(a-b)*ln(a*tanh(1/2*x
)^2-tanh(1/2*x)^2*b-2*c*tanh(1/2*x)-a-b)*C*b^2-2/(b-c)/(b+c)/(-a^2+b^2-c^2)
^(1/2)*arctan(1/2*(2*(a-b)*tanh(1/2*x)-2*c)/(-a^2+b^2-c^2)^(1/2))*A*b^2+2/(
b-c)/(b+c)/(-a^2+b^2-c^2)^(1/2)*arctan(1/2*(2*(a-b)*tanh(1/2*x)-2*c)/(-a^2+
b^2-c^2)^(1/2))*A*c^2-2/(b-c)/(b+c)/(-a^2+b^2-c^2)^(1/2)*arctan(1/2*(2*(a-b
)*tanh(1/2*x)-2*c)/(-a^2+b^2-c^2)^(1/2))*a*c*C-2/(b-c)/(b+c)/(-a^2+b^2-c^2)
^(1/2)*arctan(1/2*(2*(a-b)*tanh(1/2*x)-2*c)/(-a^2+b^2-c^2)^(1/2))*C*c*b+2/(
b-c)/(b+c)/(-a^2+b^2-c^2)^(1/2)*arctan(1/2*(2*(a-b)*tanh(1/2*x)-2*c)/(-a^2+
b^2-c^2)^(1/2))*c/(a-b)*a*b*C-2/(b-c)/(b+c)/(-a^2+b^2-c^2)^(1/2)*arctan(1/2
*(2*(a-b)*tanh(1/2*x)-2*c)/(-a^2+b^2-c^2)^(1/2))*c/(a-b)*C*b^2-2*C/(2*b-2*c
)*ln(tanh(1/2*x)+1)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sinh(x))/(a+b*cosh(x)+c*sinh(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(c^2-b^2+a^2>0)', see `assume?` for
more details)Is c^2-b^2+a^2 positive or negative?
```

mupad [B] time = 0.75, size = 377, normalized size = 3.14

$$\frac{\ln\left(b\,\sqrt{a^2-b^2+c^2}-c\,\sqrt{a^2-b^2+c^2}-a^2\,{\mathrm{e}}^x+b^2\,{\mathrm{e}}^x-c^2\,{\mathrm{e}}^x+a\,{\mathrm{e}}^x\,\sqrt{a^2-b^2+c^2}\right)\,\left(C\,b^3+A\,b^2\,\sqrt{a^2-b^2+c^2}-\right.}{-a^2\,b^2+a^2\,c^2+b^4-2\,b^2\,c^2+c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + C*sinh(x))/(a + b*cosh(x) + c*sinh(x)),x)
```

```
[Out] (log(b*(a^2 - b^2 + c^2)^(1/2) - c*(a^2 - b^2 + c^2)^(1/2) - a^2*exp(x) + b
^2*exp(x) - c^2*exp(x) + a*exp(x)*(a^2 - b^2 + c^2)^(1/2))*(C*b^3 + A*b^2*(
a^2 - b^2 + c^2)^(1/2) - C*a^2*b - A*c^2*(a^2 - b^2 + c^2)^(1/2) - C*b*c^2
+ C*a*c*(a^2 - b^2 + c^2)^(1/2)))/(b^4 + c^4 - a^2*b^2 + a^2*c^2 - 2*b^2*c^
2) - (log(b*(a^2 - b^2 + c^2)^(1/2) - c*(a^2 - b^2 + c^2)^(1/2) + a^2*exp(x
) - b^2*exp(x) + c^2*exp(x) + a*exp(x)*(a^2 - b^2 + c^2)^(1/2))*(A*b^2*(a^2
 - b^2 + c^2)^(1/2) - C*b^3 + C*a^2*b - A*c^2*(a^2 - b^2 + c^2)^(1/2) + C*b
*c^2 + C*a*c*(a^2 - b^2 + c^2)^(1/2)))/(b^4 + c^4 - a^2*b^2 + a^2*c^2 - 2*b
^2*c^2) - (C*x)/(b - c)
```

sympy **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sinh(x))/(a+b*cosh(x)+c*sinh(x)),x)
[Out] Timed out
```

3.790 $\int \frac{A+C\sinh(x)}{(a+b\cosh(x)+c\sinh(x))^2}\,dx$

Optimal. Leaf size=108

$$\frac{-\cosh(x)(Ac-aC)-Ab\sinh(x)+bC}{\left(a^2-b^2+c^2\right)(a+b\cosh(x)+c\sinh(x))}-\frac{2(aA+cC)\tanh^{-1}\left(\frac{c-(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2+c^2}}\right)}{\left(a^2-b^2+c^2\right)^{3/2}}$$

```
[Out] -2*(A*a+C*c)*arctanh((c-(a-b)*tanh(1/2*x))/(a^2-b^2+c^2)^(1/2))/(a^2-b^2+c^
2)^(3/2)+(b*C-(A*c-C*a)*cosh(x)-A*b*sinh(x))/(a^2-b^2+c^2)/(a+b*cosh(x)+c*s
inh(x))
```

Rubi [A] time = 0.12, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {3154, 3124, 618, 206}

$$\frac{-\cosh(x)(Ac-aC)-Ab\sinh(x)+bC}{\left(a^2-b^2+c^2\right)(a+b\cosh(x)+c\sinh(x))}-\frac{2(aA+cC)\tanh^{-1}\left(\frac{c-(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2+c^2}}\right)}{\left(a^2-b^2+c^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + C*Sinh[x])/(a + b*Cosh[x] + c*Sinh[x])^2,x]
```

```
[Out] (-2*(a*A + c*C)*ArcTanh[(c - (a - b)*Tanh[x/2])/Sqrt[a^2 - b^2 + c^2]])/(a^
2 - b^2 + c^2)^(3/2) + (b*C - (A*c - a*C)*Cosh[x] - A*b*Sinh[x])/((a^2 - b^
2 + c^2)*(a + b*Cosh[x] + c*Sinh[x]))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 3124

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Module[{f = FreeFactors[Tan[(d + e*x)/2], x]}, Dist[(2*f
```

```
)/e, Subst[Int[1/(a + b + 2*c*f*x + (a - b)*f^2*x^2), x], x, Tan[(d + e*x)/
2]/f], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
```

Rule 3154

```
Int[((A_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])/((a_.) + cos[(d_.) + (e_.)*(x_)
]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])^2, x_Symbol] :> -Simp[(b*C + (a*C
- c*A)*Cos[d + e*x] + b*A*Sin[d + e*x])/(e*(a^2 - b^2 - c^2)*(a + b*Cos[d +
 e*x] + c*Sin[d + e*x])), x] + Dist[(a*A - c*C)/(a^2 - b^2 - c^2), Int[1/(a
 + b*Cos[d + e*x] + c*Sin[d + e*x]), x], x] /; FreeQ[{a, b, c, d, e, A, C},
 x] && NeQ[a^2 - b^2 - c^2, 0] && NeQ[a*A - c*C, 0]
```

Rubi steps

$$\int \frac{A+C\sinh(x)}{(a+b\cosh(x)+c\sinh(x))^2}\,dx = \frac{bC-(Ac-aC)\cosh(x)-Ab\sinh(x)}{\left(a^2-b^2+c^2\right)(a+b\cosh(x)+c\sinh(x))} + \frac{(aA+cC)\int \frac{1}{a+b\cosh(x)+c\sinh(x)}}{a^2-b^2+c^2}$$

$$= \frac{bC-(Ac-aC)\cosh(x)-Ab\sinh(x)}{\left(a^2-b^2+c^2\right)(a+b\cosh(x)+c\sinh(x))} + \frac{(2(aA+cC))\,\mathrm{Subst}\left(\int \frac{}{a+b+2c}\right.}{a^2-b^2+}$$

$$= \frac{bC-(Ac-aC)\cosh(x)-Ab\sinh(x)}{\left(a^2-b^2+c^2\right)(a+b\cosh(x)+c\sinh(x))} - \frac{(4(aA+cC))\,\mathrm{Subst}\left(\int \frac{}{4\left(a^2-b^2\right.}\right.}{a^2}$$

$$= -\frac{2(aA+cC)\tanh^{-1}\left(\frac{c-(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2+c^2}}\right)}{\left(a^2-b^2+c^2\right)^{3/2}} + \frac{bC-(Ac-aC)\cosh(x)-Ab\sin}{\left(a^2-b^2+c^2\right)(a+b\cosh(x)+c\,\mathrm{si}}$$

Mathematica [A] time = 0.33, size = 130, normalized size = 1.20

$$\frac{a^2C+\sinh(x)\left(acC+A\left(b^2-c^2\right)\right)-aAc-b^2C}{b\left(-a^2+b^2-c^2\right)(a+b\cosh(x)+c\sinh(x))} - \frac{2(aA+cC)\tan^{-1}\left(\frac{(b-a)\tanh\left(\frac{x}{2}\right)+c}{\sqrt{-a^2+b^2-c^2}}\right)}{\left(-a^2+b^2-c^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + C*Sinh[x])/(a + b*Cosh[x] + c*Sinh[x])^2,x]
```

```
[Out] (-2*(a*A + c*C)*ArcTan[(c + (-a + b)*Tanh[x/2])/Sqrt[-a^2 + b^2 - c^2]])/(-
a^2 + b^2 - c^2)^(3/2) + (-(a*A*c) + a^2*C - b^2*C + (A*(b^2 - c^2) + a*c*C
)*Sinh[x])/(b*(-a^2 + b^2 - c^2)*(a + b*Cosh[x] + c*Sinh[x]))
```

fricas [B] time = 0.51, size = 2211, normalized size = 20.47

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sinh(x))/(a+b*cosh(x)+c*sinh(x))^2,x, algorithm="fricas")
```

```
[Out] [(2*A*a^2*b^2 - 2*A*b^4 + 2*C*a*c^3 - 2*A*c^4 - 2*(A*a^2 - 2*A*b^2)*c^2 + (
A*a*b^2 + C*b^2*c - A*a*c^2 - C*c^3 + (A*a*b^2 + C*c^3 + (A*a + 2*C*b)*c^2
+ (2*A*a*b + C*b^2)*c)*cosh(x)^2 + (A*a*b^2 + C*c^3 + (A*a + 2*C*b)*c^2 + (
2*A*a*b + C*b^2)*c)*sinh(x)^2 + 2*(A*a^2*b + C*a*c^2 + (A*a^2 + C*a*b)*c)*c
osh(x) + 2*(A*a^2*b + C*a*c^2 + (A*a^2 + C*a*b)*c + (A*a*b^2 + C*c^3 + (A*a
 + 2*C*b)*c^2 + (2*A*a*b + C*b^2)*c)*cosh(x))*sinh(x))*sqrt(a^2 - b^2 + c^2
)*log(((b^2 + 2*b*c + c^2)*cosh(x)^2 + (b^2 + 2*b*c + c^2)*sinh(x)^2 + 2*a^
2 - b^2 + c^2 + 2*(a*b + a*c)*cosh(x) + 2*(a*b + a*c + (b^2 + 2*b*c + c^2)*
cosh(x))*sinh(x) - 2*sqrt(a^2 - b^2 + c^2)*((b + c)*cosh(x) + (b + c)*sinh(
x) + a))/((b + c)*cosh(x)^2 + (b + c)*sinh(x)^2 + 2*a*cosh(x) + 2*((b + c)*
cosh(x) + a)*sinh(x) + b - c)) + 2*(C*a^3 - C*a*b^2)*c - 2*(C*a^4 - A*a^3*b
 - 2*C*a^2*b^2 + A*a*b^3 + C*b^4 - (A*a + C*b)*c^3 + (C*a^2 - A*a*b - C*b^2
)*c^2 - (A*a^3 + C*a^2*b - A*a*b^2 - C*b^3)*c)*cosh(x) - 2*(C*a^4 - A*a^3*b
 - 2*C*a^2*b^2 + A*a*b^3 + C*b^4 - (A*a + C*b)*c^3 + (C*a^2 - A*a*b - C*b^2
)*c^2 - (A*a^3 + C*a^2*b - A*a*b^2 - C*b^3)*c)*sinh(x))/(a^4*b^2 - 2*a^2*b^
4 + b^6 - c^6 - (2*a^2 - 3*b^2)*c^4 - (a^4 - 4*a^2*b^2 + 3*b^4)*c^2 + (a^4*
b^2 - 2*a^2*b^4 + b^6 + 2*b*c^5 + c^6 + (2*a^2 - b^2)*c^4 + 4*(a^2*b - b^3)
*c^3 + (a^4 - b^4)*c^2 + 2*(a^4*b - 2*a^2*b^3 + b^5)*c)*cosh(x)^2 + (a^4*b^
2 - 2*a^2*b^4 + b^6 + 2*b*c^5 + c^6 + (2*a^2 - b^2)*c^4 + 4*(a^2*b - b^3)*c
^3 + (a^4 - b^4)*c^2 + 2*(a^4*b - 2*a^2*b^3 + b^5)*c)*sinh(x)^2 + 2*(a^5*b
- 2*a^3*b^3 + a*b^5 + a*b*c^4 + a*c^5 + 2*(a^3 - a*b^2)*c^3 + 2*(a^3*b - a*
b^3)*c^2 + (a^5 - 2*a^3*b^2 + a*b^4)*c)*cosh(x) + 2*(a^5*b - 2*a^3*b^3 + a*
b^5 + a*b*c^4 + a*c^5 + 2*(a^3 - a*b^2)*c^3 + 2*(a^3*b - a*b^3)*c^2 + (a^5
- 2*a^3*b^2 + a*b^4)*c + (a^4*b^2 - 2*a^2*b^4 + b^6 + 2*b*c^5 + c^6 + (2*a^
2 - b^2)*c^4 + 4*(a^2*b - b^3)*c^3 + (a^4 - b^4)*c^2 + 2*(a^4*b - 2*a^2*b^3
 + b^5)*c)*cosh(x))*sinh(x)), 2*(A*a^2*b^2 - A*b^4 + C*a*c^3 - A*c^4 - (A*a
^2 - 2*A*b^2)*c^2 + (A*a*b^2 + C*b^2*c - A*a*c^2 - C*c^3 + (A*a*b^2 + C*c^3
 + (A*a + 2*C*b)*c^2 + (2*A*a*b + C*b^2)*c)*cosh(x)^2 + (A*a*b^2 + C*c^3 +
(A*a + 2*C*b)*c^2 + (2*A*a*b + C*b^2)*c)*sinh(x)^2 + 2*(A*a^2*b + C*a*c^2 +
 (A*a^2 + C*a*b)*c)*cosh(x) + 2*(A*a^2*b + C*a*c^2 + (A*a^2 + C*a*b)*c + (A
*a*b^2 + C*c^3 + (A*a + 2*C*b)*c^2 + (2*A*a*b + C*b^2)*c)*cosh(x))*sinh(x))
*sqrt(-a^2 + b^2 - c^2)*arctan(sqrt(-a^2 + b^2 - c^2)*((b + c)*cosh(x) + (b
 + c)*sinh(x) + a)/(a^2 - b^2 + c^2)) + (C*a^3 - C*a*b^2)*c - (C*a^4 - A*a^
3*b - 2*C*a^2*b^2 + A*a*b^3 + C*b^4 - (A*a + C*b)*c^3 + (C*a^2 - A*a*b - C*
b^2)*c^2 - (A*a^3 + C*a^2*b - A*a*b^2 - C*b^3)*c)*cosh(x) - (C*a^4 - A*a^3*
b - 2*C*a^2*b^2 + A*a*b^3 + C*b^4 - (A*a + C*b)*c^3 + (C*a^2 - A*a*b - C*b^
2)*c^2 - (A*a^3 + C*a^2*b - A*a*b^2 - C*b^3)*c)*sinh(x))/(a^4*b^2 - 2*a^2*b
```

```
^4 + b^6 - c^6 - (2*a^2 - 3*b^2)*c^4 - (a^4 - 4*a^2*b^2 + 3*b^4)*c^2 + (a^4
*b^2 - 2*a^2*b^4 + b^6 + 2*b*c^5 + c^6 + (2*a^2 - b^2)*c^4 + 4*(a^2*b - b^3
)*c^3 + (a^4 - b^4)*c^2 + 2*(a^4*b - 2*a^2*b^3 + b^5)*c)*cosh(x)^2 + (a^4*b
^2 - 2*a^2*b^4 + b^6 + 2*b*c^5 + c^6 + (2*a^2 - b^2)*c^4 + 4*(a^2*b - b^3)*
c^3 + (a^4 - b^4)*c^2 + 2*(a^4*b - 2*a^2*b^3 + b^5)*c)*sinh(x)^2 + 2*(a^5*b
 - 2*a^3*b^3 + a*b^5 + a*b*c^4 + a*c^5 + 2*(a^3 - a*b^2)*c^3 + 2*(a^3*b - a
*b^3)*c^2 + (a^5 - 2*a^3*b^2 + a*b^4)*c)*cosh(x) + 2*(a^5*b - 2*a^3*b^3 + a
*b^5 + a*b*c^4 + a*c^5 + 2*(a^3 - a*b^2)*c^3 + 2*(a^3*b - a*b^3)*c^2 + (a^5
 - 2*a^3*b^2 + a*b^4)*c + (a^4*b^2 - 2*a^2*b^4 + b^6 + 2*b*c^5 + c^6 + (2*a
^2 - b^2)*c^4 + 4*(a^2*b - b^3)*c^3 + (a^4 - b^4)*c^2 + 2*(a^4*b - 2*a^2*b^
3 + b^5)*c)*cosh(x))*sinh(x))]
```

giac [A] time = 0.12, size = 179, normalized size = 1.66

$$\frac{2\,(Aa+Cc)\arctan\left(\frac{be^{x}+ce^{x}+a}{\sqrt{-a^{2}+b^{2}-c^{2}}}\right)}{\left(a^{2}-b^{2}+c^{2}\right)\sqrt{-a^{2}+b^{2}-c^{2}}}-\frac{2\left(Ca^{2}e^{x}-Aabe^{x}-Cb^{2}e^{x}-Aace^{x}-Cbce^{x}-Ab^{2}-Cac+Ac^{2}\right)}{\left(a^{2}b-b^{3}+a^{2}c-b^{2}c+bc^{2}+c^{3}\right)\left(be^{(2\,x)}+ce^{(2\,x)}+2\,ae^{x}+b-c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sinh(x))/(a+b*cosh(x)+c*sinh(x))^2,x, algorithm="giac")
```

```
[Out] 2*(A*a + C*c)*arctan((b*e^x + c*e^x + a)/sqrt(-a^2 + b^2 - c^2))/((a^2 - b^
2 + c^2)*sqrt(-a^2 + b^2 - c^2)) - 2*(C*a^2*e^x - A*a*b*e^x - C*b^2*e^x - A
*a*c*e^x - C*b*c*e^x - A*b^2 - C*a*c + A*c^2)/((a^2*b - b^3 + a^2*c - b^2*c
 + b*c^2 + c^3)*(b*e^(2*x) + c*e^(2*x) + 2*a*e^x + b - c))
```

maple [B] time = 0.26, size = 287, normalized size = 2.66

$$-\frac{2\left(-\frac{\left(Aab-A\,b^{2}+A\,c^{2}-acC+Ccb\right)\tanh\left(\frac{x}{2}\right)}{a^{3}-a^{2}b-a\,b^{2}+a\,c^{2}+b^{3}-b\,c^{2}}-\frac{aAc-a^{2}C+C\,b^{2}}{a^{3}-a^{2}b-a\,b^{2}+a\,c^{2}+b^{3}-b\,c^{2}}\right)}{a\left(\tanh^{2}\left(\frac{x}{2}\right)\right)-\left(\tanh^{2}\left(\frac{x}{2}\right)\right)b-2c\tanh\left(\frac{x}{2}\right)-a-b}-\frac{2\arctan\left(\frac{2(a-b)\tanh\left(\frac{x}{2}\right)-2c}{2\sqrt{-a^{2}+b^{2}-c^{2}}}\right)Aa}{\left(a^{2}-b^{2}+c^{2}\right)\sqrt{-a^{2}+b^{2}-c^{2}}}-\frac{2\arctan\left(\frac{2(a-b)\tanh\left(\frac{x}{2}\right)-2c}{2\sqrt{-a^{2}+}\cdots}\right.}{\left(a^{2}-b^{2}+c^{2}\right)\sqrt{-a}\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*sinh(x))/(a+b*cosh(x)+c*sinh(x))^2,x)
```

```
[Out] -2*(-(A*a*b-A*b^2+A*c^2-C*a*c+C*b*c)/(a^3-a^2*b-a*b^2+a*c^2+b^3-b*c^2)*tanh
(1/2*x)-(A*a*c-C*a^2+C*b^2)/(a^3-a^2*b-a*b^2+a*c^2+b^3-b*c^2))/(a*tanh(1/2*
x)^2-tanh(1/2*x)^2*b-2*c*tanh(1/2*x)-a-b)-2/(a^2-b^2+c^2)/(-a^2+b^2-c^2)^(1
/2)*arctan(1/2*(2*(a-b)*tanh(1/2*x)-2*c)/(-a^2+b^2-c^2)^(1/2))*A*a-2/(a^2-b
^2+c^2)/(-a^2+b^2-c^2)^(1/2)*arctan(1/2*(2*(a-b)*tanh(1/2*x)-2*c)/(-a^2+b^2
-c^2)^(1/2))*C*c
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sinh(x))/(a+b*cosh(x)+c*sinh(x))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(c^2-b^2+a^2>0)', see `assume?` for
more details)Is c^2-b^2+a^2 positive or negative?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A + C \sinh(x)}{(a + b \cosh(x) + c \sinh(x))^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + C*sinh(x))/(a + b*cosh(x) + c*sinh(x))^2,x)
```

```
[Out] int((A + C*sinh(x))/(a + b*cosh(x) + c*sinh(x))^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sinh(x))/(a+b*cosh(x)+c*sinh(x))**2,x)
```

```
[Out] Timed out
```

3.791 $\int \frac{A+C \sinh(x)}{(a+b \cosh(x)+c \sinh(x))^3} \, dx$

Optimal. Leaf size=198

$$-\frac{\left(2a^2A+3acC+A\left(b^2-c^2\right)\right)\tanh^{-1}\left(\frac{c-(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2+c^2}}\right)}{\left(a^2-b^2+c^2\right)^{5/2}}+\frac{-\cosh(x)\left(a^2(-C)+3aAc+2c^2C\right)-b\sinh(x)(3aA+ \cdots}{2\left(a^2-b^2+c^2\right)^2(a+b\cosh(x)+c\sinh(x))}$$

```
[Out] -(2*a^2*A+A*(b^2-c^2)+3*a*c*C)*arctanh((c-(a-b)*tanh(1/2*x))/(a^2-b^2+c^2)^
(1/2))/(a^2-b^2+c^2)^(5/2)+1/2*(b*C-(A*c-C*a)*cosh(x)-A*b*sinh(x))/(a^2-b^2
+c^2)/(a+b*cosh(x)+c*sinh(x))^2+1/2*(a*b*C-(3*A*a*c-C*a^2+2*C*c^2)*cosh(x)-
b*(3*A*a+2*C*c)*sinh(x))/(a^2-b^2+c^2)^2/(a+b*cosh(x)+c*sinh(x))
```

Rubi [A] time = 0.29, antiderivative size = 198, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {3157, 3153, 3124, 618, 206}

$$-\frac{\left(2a^2A+3acC+A\left(b^2-c^2\right)\right)\tanh^{-1}\left(\frac{c-(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2+c^2}}\right)}{\left(a^2-b^2+c^2\right)^{5/2}}+\frac{-\cosh(x)\left(a^2(-C)+3aAc+2c^2C\right)-b\sinh(x)(3aA+ \cdots}{2\left(a^2-b^2+c^2\right)^2(a+b\cosh(x)+c\sinh(x))}$$

Antiderivative was successfully verified.

```
[In] Int[(A + C*Sinh[x])/(a + b*Cosh[x] + c*Sinh[x])^3,x]
```

```
[Out] -(((2*a^2*A + A*(b^2 - c^2) + 3*a*c*C)*ArcTanh[(c - (a - b)*Tanh[x/2])/Sqrt
[a^2 - b^2 + c^2]])/(a^2 - b^2 + c^2)^(5/2)) + (b*C - (A*c - a*C)*Cosh[x] -
 A*b*Sinh[x])/(2*(a^2 - b^2 + c^2)*(a + b*Cosh[x] + c*Sinh[x])^2) + (a*b*C
- (3*a*A*c - a^2*C + 2*c^2*C)*Cosh[x] - b*(3*a*A + 2*c*C)*Sinh[x])/(2*(a^2
- b^2 + c^2)^2*(a + b*Cosh[x] + c*Sinh[x]))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 3124

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Module[{f = FreeFactors[Tan[(d + e*x)/2], x]}, Dist[(2*f
)/e, Subst[Int[1/(a + b + 2*c*f*x + (a - b)*f^2*x^2), x], x, Tan[(d + e*x)/
2]/f], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
```

Rule 3153

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])^2,
 x_Symbol] :> Simp[(c*B - b*C - (a*C - c*A)*Cos[d + e*x] + (a*B - b*A)*Sin[
d + e*x])/(e*(a^2 - b^2 - c^2)*(a + b*Cos[d + e*x] + c*Sin[d + e*x])), x] +
 Dist[(a*A - b*B - c*C)/(a^2 - b^2 - c^2), Int[1/(a + b*Cos[d + e*x] + c*Si
n[d + e*x]), x], x] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && NeQ[a^2 - b^2
- c^2, 0] && NeQ[a*A - b*B - c*C, 0]
```

Rule 3157

```
Int[((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])
^(n_)*((A_.) + (C_.)*sin[(d_.) + (e_.)*(x_)]), x_Symbol] :> Simp[((b*C + (a
*C - c*A)*Cos[d + e*x] + b*A*Sin[d + e*x])*(a + b*Cos[d + e*x] + c*Sin[d +
e*x])^(n + 1))/(e*(n + 1)*(a^2 - b^2 - c^2)), x] + Dist[1/((n + 1)*(a^2 - b
^2 - c^2)), Int[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1)*Simp[(n + 1)*
(a*A - c*C) - (n + 2)*b*A*Cos[d + e*x] + (n + 2)*(a*C - c*A)*Sin[d + e*x],
x], x], x] /; FreeQ[{a, b, c, d, e, A, C}, x] && LtQ[n, -1] && NeQ[a^2 - b^
2 - c^2, 0] && NeQ[n, -2]
```

Rubi steps

$$\int \frac{A+C\sinh(x)}{(a+b\cosh(x)+c\sinh(x))^3}\,dx = \frac{bC-(Ac-aC)\cosh(x)-Ab\sinh(x)}{2\left(a^2-b^2+c^2\right)(a+b\cosh(x)+c\sinh(x))^2} - \frac{\int \frac{-2(aA+cC)+Ab\cosh(x)+(Ac-a\text{C}}{(a+b\cosh(x)+c\sinh(x))}}{2\left(a^2-b^2+c^2\right)}$$

$$= \frac{bC-(Ac-aC)\cosh(x)-Ab\sinh(x)}{2\left(a^2-b^2+c^2\right)(a+b\cosh(x)+c\sinh(x))^2} + \frac{abC-\left(3aAc-a^2C+2c^2C\right.}{2\left(a^2-b^2+c^2\right)^2(a}$$

$$= \frac{bC-(Ac-aC)\cosh(x)-Ab\sinh(x)}{2\left(a^2-b^2+c^2\right)(a+b\cosh(x)+c\sinh(x))^2} + \frac{abC-\left(3aAc-a^2C+2c^2C\right.}{2\left(a^2-b^2+c^2\right)^2(a}$$

$$= \frac{bC-(Ac-aC)\cosh(x)-Ab\sinh(x)}{2\left(a^2-b^2+c^2\right)(a+b\cosh(x)+c\sinh(x))^2} + \frac{abC-\left(3aAc-a^2C+2c^2C\right.}{2\left(a^2-b^2+c^2\right)^2(a}$$

$$= -\frac{\left(2a^2A+A\left(b^2-c^2\right)+3acC\right)\tanh^{-1}\left(\frac{c-(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2+c^2}}\right)}{\left(a^2-b^2+c^2\right)^{5/2}} + \frac{bC-(Ac-a\text{C}}{2\left(a^2-b^2+c^2\right)(}$$

Mathematica [A] time = 0.88, size = 373, normalized size = 1.88

$$\frac{\left(2a^2A+3acC+A\left(b^2-c^2\right)\right)\tan^{-1}\left(\frac{(b-a)\tanh\left(\frac{x}{2}\right)+c}{\sqrt{-a^2+b^2-c^2}}\right)}{\left(-a^2+b^2-c^2\right)^{5/2}} + \frac{-2a^4C+6a^3Ac-4a^3cC\sinh(x)+2bc\cosh(x)\left(2a^2A+3\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + C*Sinh[x])/(a + b*Cosh[x] + c*Sinh[x])^3,x]
```

```
[Out] ((2*a^2*A + A*(b^2 - c^2) + 3*a*c*C)*ArcTan[(c + (-a + b)*Tanh[x/2])/Sqrt[-
a^2 + b^2 - c^2]])/(-a^2 + b^2 - c^2)^(5/2) + (6*a^3*A*c + 3*a*A*b^2*c - 3*
a*A*c^3 - 2*a^4*C + 4*a^2*b^2*C - 2*b^4*C + 5*a^2*c^2*C + 4*b^2*c^2*C - 2*c
^4*C + 2*b*c*(2*a^2*A + A*(b^2 - c^2) + 3*a*c*C)*Cosh[x] + c*(3*a*A*(-b^2 +
 c^2) - a^2*c*C + 2*c*(-b^2 + c^2)*C)*Cosh[2*x] - 8*a^2*A*b^2*Sinh[x] + 2*A
*b^4*Sinh[x] + 12*a^2*A*c^2*Sinh[x] - 2*A*b^2*c^2*Sinh[x] - 4*a^3*c*C*Sinh[
x] - 2*a*b^2*c*C*Sinh[x] + 8*a*c^3*C*Sinh[x] - 3*a*A*b^3*Sinh[2*x] + 3*a*A*
b*c^2*Sinh[2*x] - a^2*b*c*C*Sinh[2*x] - 2*b^3*c*C*Sinh[2*x] + 2*b*c^3*C*Sin
h[2*x])/(4*b*(a^2 - b^2 + c^2)^2*(a + b*Cosh[x] + c*Sinh[x])^2)
```

fricas [B] time = 0.68, size = 12285, normalized size = 62.05

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+C*sinh(x))/(a+b*cosh(x)+c*sinh(x))^3,x, algorithm="fricas")

[Out] [1/2*(6*A*a^3*b^3 - 6*A*a*b^5 + 4*C*c^6 + 2*(3*A*a - 2*C*b)*c^5 + 2*(C*a^2 - 3*A*a*b - 4*C*b^2)*c^4 + 2*(3*A*a^3 - C*a^2*b - 6*A*a*b^2 + 4*C*b^3)*c^3 + 2*(2*A*a^4*b^2 - A*a^2*b^4 - A*b^6 - A*c^6 + (3*C*a - 2*A*b)*c^5 + (A*a^2 + 6*C*a*b + A*b^2)*c^4 + (3*C*a^3 + 2*A*a^2*b + 4*A*b^3)*c^3 + (2*A*a^4 + 6*C*a^3*b - 6*C*a*b^3 + A*b^4)*c^2 + (4*A*a^4*b + 3*C*a^3*b^2 - 2*A*a^2*b^3 - 3*C*a*b^4 - 2*A*b^5)*c)*cosh(x)^3 + 2*(2*A*a^4*b^2 - A*a^2*b^4 - A*b^6 - A*c^6 + (3*C*a - 2*A*b)*c^5 + (A*a^2 + 6*C*a*b + A*b^2)*c^4 + (3*C*a^3 + 2*A*a^2*b + 4*A*b^3)*c^3 + (2*A*a^4 + 6*C*a^3*b - 6*C*a*b^3 + A*b^4)*c^2 + (4*A*a^4*b + 3*C*a^3*b^2 - 2*A*a^2*b^3 - 3*C*a*b^4 - 2*A*b^5)*c)*sinh(x)^3 - 2*(C*a^4 + 3*A*a^3*b + C*a^2*b^2 - 6*A*a*b^3 - 2*C*b^4)*c^2 - 2*(2*C*a^6 - 6*A*a^5*b - 6*C*a^4*b^2 + 3*A*a^3*b^3 + 6*C*a^2*b^4 + 3*A*a*b^5 - 2*C*b^6 + 3*A*a*c^5 + 2*C*c^6 - 3*(C*a^2 - A*a*b + 2*C*b^2)*c^4 - 3*(A*a^3 + 3*C*a^2*b + 2*A*a*b^2)*c^3 - 3*(C*a^4 + A*a^3*b + C*a^2*b^2 + 2*A*a*b^3 - 2*C*b^4)*c^2 - 3*(2*A*a^5 + 3*C*a^4*b - A*a^3*b^2 - 3*C*a^2*b^3 - A*a*b^4)*c)*cosh(x)^2 - 2*(2*C*a^6 - 6*A*a^5*b - 6*C*a^4*b^2 + 3*A*a^3*b^3 + 6*C*a^2*b^4 + 3*A*a*b^5 - 2*C*b^6 + 3*A*a*c^5 + 2*C*c^6 - 3*(C*a^2 - A*a*b + 2*C*b^2)*c^4 - 3*(A*a^3 + 3*C*a^2*b + 2*A*a*b^2)*c^3 - 3*(C*a^4 + A*a^3*b + C*a^2*b^2 + 2*A*a*b^3 - 2*C*b^4)*c^2 - 3*(2*A*a^5 + 3*C*a^4*b - A*a^3*b^2 - 3*C*a^2*b^3 - A*a*b^4)*c - 3*(2*A*a^4*b^2 - A*a^2*b^4 - A*b^6 - A*c^6 + (3*C*a - 2*A*b)*c^5 + (A*a^2 + 6*C*a*b + A*b^2)*c^4 + (3*C*a^3 + 2*A*a^2*b + 4*A*b^3)*c^3 + (2*A*a^4 + 6*C*a^3*b - 6*C*a*b^3 + A*b^4)*c^2 + (4*A*a^4*b + 3*C*a^3*b^2 - 2*A*a^2*b^3 - 3*C*a*b^4 - 2*A*b^5)*c)*cosh(x))*sinh(x)^2 - (2*A*a^2*b^3 + A*b^5 - A*c^5 + (3*C*a + A*b)*c^4 + (2*A*a^2*b^3 + A*b^5 - A*c^5 + 3*(C*a - A*b)*c^4 + (2*A*a^2 + 9*C*a*b - 2*A*b^2)*c^3 + (6*A*a^2*b + 9*C*a*b^2 + 2*A*b^3)*c^2 + 3*(2*A*a^2*b^2 + C*a*b^3 + A*b^4)*c)*cosh(x)^4 + (2*A*a^2*b^3 + A*b^5 - A*c^5 + 3*(C*a - A*b)*c^4 + (2*A*a^2 + 9*C*a*b - 2*A*b^2)*c^3 + (6*A*a^2*b + 9*C*a*b^2 + 2*A*b^3)*c^2 + 3*(2*A*a^2*b^2 + C*a*b^3 + A*b^4)*c)*sinh(x)^4 + (2*A*a^2 - 3*C*a*b + 2*A*b^2)*c^3 + 4*(2*A*a^3*b^2 + A*a*b^4 - A*a*c^4 + (3*C*a^2 - 2*A*a*b)*c^3 + 2*(A*a^3 + 3*C*a^2*b)*c^2 + (4*A*a^3*b + 3*C*a^2*b^2 + 2*A*a*b^3)*c)*cosh(x)^3 + 4*(2*A*a^3*b^2 + A*a*b^4 - A*a*c^4 + (3*C*a^2 - 2*A*a*b)*c^3 + 2*(A*a^3 + 3*C*a^2*b)*c^2 + (4*A*a^3*b + 3*C*a^2*b^2 + 2*A*a*b^3)*c + (2*A*a^2*b^3 + A*b^5 - A*c^5 + 3*(C*a - A*b)*c^4 + (2*A*a^2 + 9*C*a*b - 2*A*b^2)*c^3 + (6*A*a^2*b + 9*C*a*b^2 + 2*A*b^3)*c^2 + 3*(2*A*a^2*b^2 + C*a*b^3 + A*b^4)*c)*cosh(x))*sinh(x)^3 - (2*A*a^2*b + 3*C*a*b^2 + 2*A*b^3)*c^2 + 2*(4*A*a^4*b + 4*A*a^2*b^3 + A*b^5 + A*c^5 - (3*C*a - A*b)*c^4 - (4*A*a^2 + 3*C*a*b + 2*A*b^2)*c^3 + (6*C*a^3 - 4*A*a^2*b + 3*C*a*b^2 - 2*A*b^3)*c^2 + (4*A*a^4 + 6*C*a^3*b + 4*A*a^2*b^2 + 3*C*a*b^3 + A*b^4)*c)*cosh(x)^2 + 2*(4*A*a^4*b + 4*A*a^2*b^3 + A*b^5 + A*c^5 - (3*C*a - A*b)*c^4 - (4*A*a^2 + 3*C*a*b + 2*A*b^2)*c^3 + (6*C*a^3 - 4*A*a^2*b + 3*C*a*b^2 - 2*A*b^3)*c^2 + 3*(2*A*a^2*b^3 + A*b^5 - A*c^5 + 3*(C*a - A*b)*c^4 + (2*A*a^2 + 9*C*a*b - 2*A*b^2)*c^3 + (6*A*a^2*b + 9*C*a*b^2 + 2*A*b^3)*

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c^2 + 3*(2*A*a^2*b^2 + C*a*b^3 + A*b^4)*c)*cosh(x)^2 + (4*A*a^4 + 6*C*a^3*b
 + 4*A*a^2*b^2 + 3*C*a*b^3 + A*b^4)*c + 6*(2*A*a^3*b^2 + A*a*b^4 - A*a*c^4
+ (3*C*a^2 - 2*A*a*b)*c^3 + 2*(A*a^3 + 3*C*a^2*b)*c^2 + (4*A*a^3*b + 3*C*a^
2*b^2 + 2*A*a*b^3)*c)*cosh(x))*sinh(x)^2 - (2*A*a^2*b^2 - 3*C*a*b^3 + A*b^4
)*c + 4*(2*A*a^3*b^2 + A*a*b^4 + 3*C*a^2*b^2*c - 3*C*a^2*c^3 + A*a*c^4 - 2*
(A*a^3 + A*a*b^2)*c^2)*cosh(x) + 4*(2*A*a^3*b^2 + A*a*b^4 + 3*C*a^2*b^2*c -
 3*C*a^2*c^3 + A*a*c^4 + (2*A*a^2*b^3 + A*b^5 - A*c^5 + 3*(C*a - A*b)*c^4 +
 (2*A*a^2 + 9*C*a*b - 2*A*b^2)*c^3 + (6*A*a^2*b + 9*C*a*b^2 + 2*A*b^3)*c^2
+ 3*(2*A*a^2*b^2 + C*a*b^3 + A*b^4)*c)*cosh(x)^3 - 2*(A*a^3 + A*a*b^2)*c^2
+ 3*(2*A*a^3*b^2 + A*a*b^4 - A*a*c^4 + (3*C*a^2 - 2*A*a*b)*c^3 + 2*(A*a^3 +
 3*C*a^2*b)*c^2 + (4*A*a^3*b + 3*C*a^2*b^2 + 2*A*a*b^3)*c)*cosh(x)^2 + (4*A
*a^4*b + 4*A*a^2*b^3 + A*b^5 + A*c^5 - (3*C*a - A*b)*c^4 - (4*A*a^2 + 3*C*a
*b + 2*A*b^2)*c^3 + (6*C*a^3 - 4*A*a^2*b + 3*C*a*b^2 - 2*A*b^3)*c^2 + (4*A*
a^4 + 6*C*a^3*b + 4*A*a^2*b^2 + 3*C*a*b^3 + A*b^4)*c)*cosh(x))*sinh(x))*sqr
t(a^2 - b^2 + c^2)*log(((b^2 + 2*b*c + c^2)*cosh(x)^2 + (b^2 + 2*b*c + c^2)
*sinh(x)^2 + 2*a^2 - b^2 + c^2 + 2*(a*b + a*c)*cosh(x) + 2*(a*b + a*c + (b^
2 + 2*b*c + c^2)*cosh(x))*sinh(x) + 2*sqrt(a^2 - b^2 + c^2)*((b + c)*cosh(x
) + (b + c)*sinh(x) + a))/((b + c)*cosh(x)^2 + (b + c)*sinh(x)^2 + 2*a*cosh
(x) + 2*((b + c)*cosh(x) + a)*sinh(x) + b - c)) + 2*(C*a^4*b - 3*A*a^3*b^2
+ C*a^2*b^3 + 3*A*a*b^4 - 2*C*b^5)*c + 2*(10*A*a^4*b^2 - 11*A*a^2*b^4 + A*b
^6 - 5*C*a*c^5 - A*c^6 - (11*A*a^2 - 3*A*b^2)*c^4 - (C*a^3 - 10*C*a*b^2)*c^
3 - (10*A*a^4 - 22*A*a^2*b^2 + 3*A*b^4)*c^2 + (4*C*a^5 + C*a^3*b^2 - 5*C*a*
b^4)*c)*cosh(x) + 2*(10*A*a^4*b^2 - 11*A*a^2*b^4 + A*b^6 - 5*C*a*c^5 - A*c^
6 - (11*A*a^2 - 3*A*b^2)*c^4 - (C*a^3 - 10*C*a*b^2)*c^3 - (10*A*a^4 - 22*A*
a^2*b^2 + 3*A*b^4)*c^2 + 3*(2*A*a^4*b^2 - A*a^2*b^4 - A*b^6 - A*c^6 + (3*C*
a - 2*A*b)*c^5 + (A*a^2 + 6*C*a*b + A*b^2)*c^4 + (3*C*a^3 + 2*A*a^2*b + 4*A
*b^3)*c^3 + (2*A*a^4 + 6*C*a^3*b - 6*C*a*b^3 + A*b^4)*c^2 + (4*A*a^4*b + 3*
C*a^3*b^2 - 2*A*a^2*b^3 - 3*C*a*b^4 - 2*A*b^5)*c)*cosh(x)^2 + (4*C*a^5 + C*
a^3*b^2 - 5*C*a*b^4)*c - 2*(2*C*a^6 - 6*A*a^5*b - 6*C*a^4*b^2 + 3*A*a^3*b^3
 + 6*C*a^2*b^4 + 3*A*a*b^5 - 2*C*b^6 + 3*A*a*c^5 + 2*C*c^6 - 3*(C*a^2 - A*a
*b + 2*C*b^2)*c^4 - 3*(A*a^3 + 3*C*a^2*b + 2*A*a*b^2)*c^3 - 3*(C*a^4 + A*a^
3*b + C*a^2*b^2 + 2*A*a*b^3 - 2*C*b^4)*c^2 - 3*(2*A*a^5 + 3*C*a^4*b - A*a^3
*b^2 - 3*C*a^2*b^3 - A*a*b^4)*c)*cosh(x))*sinh(x))/(a^6*b^3 - 3*a^4*b^5 + 3
*a^2*b^7 - b^9 - b*c^8 + c^9 + (3*a^2 - 4*b^2)*c^7 - (3*a^2*b - 4*b^3)*c^6
+ 3*(a^4 - 3*a^2*b^2 + 2*b^4)*c^5 - 3*(a^4*b - 3*a^2*b^3 + 2*b^5)*c^4 + (a^
6*b^3 - 3*a^4*b^5 + 3*a^2*b^7 - b^9 + 3*a^2*c^7 + 3*b*c^8 + c^9 + (9*a^2*b
- 8*b^3)*c^6 + 3*(a^4 + a^2*b^2 - 2*b^4)*c^5 + 3*(3*a^4*b - 5*a^2*b^3 + 2*b
^5)*c^4 + (a^6 + 6*a^4*b^2 - 15*a^2*b^4 + 8*b^6)*c^3 + 3*(a^6*b - 2*a^4*b^3
 + a^2*b^5)*c^2 + 3*(a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^8)*c)*cosh(x)^4 +
(a^6*b^3 - 3*a^4*b^5 + 3*a^2*b^7 - b^9 + 3*a^2*c^7 + 3*b*c^8 + c^9 + (9*a^2
*b - 8*b^3)*c^6 + 3*(a^4 + a^2*b^2 - 2*b^4)*c^5 + 3*(3*a^4*b - 5*a^2*b^3 +
2*b^5)*c^4 + (a^6 + 6*a^4*b^2 - 15*a^2*b^4 + 8*b^6)*c^3 + 3*(a^6*b - 2*a^4*
b^3 + a^2*b^5)*c^2 + 3*(a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^8)*c)*sinh(x)^4
 + (a^6 - 6*a^4*b^2 + 9*a^2*b^4 - 4*b^6)*c^3 + 4*(a^7*b^2 - 3*a^5*b^4 + 3*a
^3*b^6 - a*b^8 + 2*a*b*c^7 + a*c^8 + (3*a^3 - 2*a*b^2)*c^6 + 6*(a^3*b - a*b
```

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^3)*c^5 + 3*(a^5 - a^3*b^2)*c^4 + 6*(a^5*b - 2*a^3*b^3 + a*b^5)*c^3 + (a^7
- 3*a^3*b^4 + 2*a*b^6)*c^2 + 2*(a^7*b - 3*a^5*b^3 + 3*a^3*b^5 - a*b^7)*c)*c
osh(x)^3 + 4*(a^7*b^2 - 3*a^5*b^4 + 3*a^3*b^6 - a*b^8 + 2*a*b*c^7 + a*c^8 +
 (3*a^3 - 2*a*b^2)*c^6 + 6*(a^3*b - a*b^3)*c^5 + 3*(a^5 - a^3*b^2)*c^4 + 6*
(a^5*b - 2*a^3*b^3 + a*b^5)*c^3 + (a^7 - 3*a^3*b^4 + 2*a*b^6)*c^2 + 2*(a^7*
b - 3*a^5*b^3 + 3*a^3*b^5 - a*b^7)*c + (a^6*b^3 - 3*a^4*b^5 + 3*a^2*b^7 - b
^9 + 3*a^2*c^7 + 3*b*c^8 + c^9 + (9*a^2*b - 8*b^3)*c^6 + 3*(a^4 + a^2*b^2 -
 2*b^4)*c^5 + 3*(3*a^4*b - 5*a^2*b^3 + 2*b^5)*c^4 + (a^6 + 6*a^4*b^2 - 15*a
^2*b^4 + 8*b^6)*c^3 + 3*(a^6*b - 2*a^4*b^3 + a^2*b^5)*c^2 + 3*(a^6*b^2 - 3*
a^4*b^4 + 3*a^2*b^6 - b^8)*c)*cosh(x))*sinh(x)^3 - (a^6*b - 6*a^4*b^3 + 9*a
^2*b^5 - 4*b^7)*c^2 + 2*(2*a^8*b - 5*a^6*b^3 + 3*a^4*b^5 + a^2*b^7 - b^9 -
b*c^8 - c^9 - (a^2 - 4*b^2)*c^7 - (a^2*b - 4*b^3)*c^6 + 3*(a^4 + a^2*b^2 -
2*b^4)*c^5 + 3*(a^4*b + a^2*b^3 - 2*b^5)*c^4 + (5*a^6 - 6*a^4*b^2 - 3*a^2*b
^4 + 4*b^6)*c^3 + (5*a^6*b - 6*a^4*b^3 - 3*a^2*b^5 + 4*b^7)*c^2 + (2*a^8 -
5*a^6*b^2 + 3*a^4*b^4 + a^2*b^6 - b^8)*c)*cosh(x)^2 + 2*(2*a^8*b - 5*a^6*b^
3 + 3*a^4*b^5 + a^2*b^7 - b^9 - b*c^8 - c^9 - (a^2 - 4*b^2)*c^7 - (a^2*b -
4*b^3)*c^6 + 3*(a^4 + a^2*b^2 - 2*b^4)*c^5 + 3*(a^4*b + a^2*b^3 - 2*b^5)*c^
4 + (5*a^6 - 6*a^4*b^2 - 3*a^2*b^4 + 4*b^6)*c^3 + (5*a^6*b - 6*a^4*b^3 - 3*
a^2*b^5 + 4*b^7)*c^2 + 3*(a^6*b^3 - 3*a^4*b^5 + 3*a^2*b^7 - b^9 + 3*a^2*c^7
 + 3*b*c^8 + c^9 + (9*a^2*b - 8*b^3)*c^6 + 3*(a^4 + a^2*b^2 - 2*b^4)*c^5 +
3*(3*a^4*b - 5*a^2*b^3 + 2*b^5)*c^4 + (a^6 + 6*a^4*b^2 - 15*a^2*b^4 + 8*b^6
)*c^3 + 3*(a^6*b - 2*a^4*b^3 + a^2*b^5)*c^2 + 3*(a^6*b^2 - 3*a^4*b^4 + 3*a^
2*b^6 - b^8)*c)*cosh(x)^2 + (2*a^8 - 5*a^6*b^2 + 3*a^4*b^4 + a^2*b^6 - b^8)
*c + 6*(a^7*b^2 - 3*a^5*b^4 + 3*a^3*b^6 - a*b^8 + 2*a*b*c^7 + a*c^8 + (3*a^
3 - 2*a*b^2)*c^6 + 6*(a^3*b - a*b^3)*c^5 + 3*(a^5 - a^3*b^2)*c^4 + 6*(a^5*b
 - 2*a^3*b^3 + a*b^5)*c^3 + (a^7 - 3*a^3*b^4 + 2*a*b^6)*c^2 + 2*(a^7*b - 3*
a^5*b^3 + 3*a^3*b^5 - a*b^7)*c)*cosh(x))*sinh(x)^2 - (a^6*b^2 - 3*a^4*b^4 +
 3*a^2*b^6 - b^8)*c + 4*(a^7*b^2 - 3*a^5*b^4 + 3*a^3*b^6 - a*b^8 - a*c^8 -
(3*a^3 - 4*a*b^2)*c^6 - 3*(a^5 - 3*a^3*b^2 + 2*a*b^4)*c^4 - (a^7 - 6*a^5*b^
2 + 9*a^3*b^4 - 4*a*b^6)*c^2)*cosh(x) + 4*(a^7*b^2 - 3*a^5*b^4 + 3*a^3*b^6
- a*b^8 - a*c^8 - (3*a^3 - 4*a*b^2)*c^6 - 3*(a^5 - 3*a^3*b^2 + 2*a*b^4)*c^4
 + (a^6*b^3 - 3*a^4*b^5 + 3*a^2*b^7 - b^9 + 3*a^2*c^7 + 3*b*c^8 + c^9 + (9*
a^2*b - 8*b^3)*c^6 + 3*(a^4 + a^2*b^2 - 2*b^4)*c^5 + 3*(3*a^4*b - 5*a^2*b^3
 + 2*b^5)*c^4 + (a^6 + 6*a^4*b^2 - 15*a^2*b^4 + 8*b^6)*c^3 + 3*(a^6*b - 2*a
^4*b^3 + a^2*b^5)*c^2 + 3*(a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^8)*c)*cosh(x
)^3 - (a^7 - 6*a^5*b^2 + 9*a^3*b^4 - 4*a*b^6)*c^2 + 3*(a^7*b^2 - 3*a^5*b^4
+ 3*a^3*b^6 - a*b^8 + 2*a*b*c^7 + a*c^8 + (3*a^3 - 2*a*b^2)*c^6 + 6*(a^3*b
- a*b^3)*c^5 + 3*(a^5 - a^3*b^2)*c^4 + 6*(a^5*b - 2*a^3*b^3 + a*b^5)*c^3 +
(a^7 - 3*a^3*b^4 + 2*a*b^6)*c^2 + 2*(a^7*b - 3*a^5*b^3 + 3*a^3*b^5 - a*b^7)
*c)*cosh(x)^2 + (2*a^8*b - 5*a^6*b^3 + 3*a^4*b^5 + a^2*b^7 - b^9 - b*c^8 -
c^9 - (a^2 - 4*b^2)*c^7 - (a^2*b - 4*b^3)*c^6 + 3*(a^4 + a^2*b^2 - 2*b^4)*c
^5 + 3*(a^4*b + a^2*b^3 - 2*b^5)*c^4 + (5*a^6 - 6*a^4*b^2 - 3*a^2*b^4 + 4*b
^6)*c^3 + (5*a^6*b - 6*a^4*b^3 - 3*a^2*b^5 + 4*b^7)*c^2 + (2*a^8 - 5*a^6*b^
2 + 3*a^4*b^4 + a^2*b^6 - b^8)*c)*cosh(x))*sinh(x)), (3*A*a^3*b^3 - 3*A*a*b
^5 + 2*C*c^6 + (3*A*a - 2*C*b)*c^5 + (C*a^2 - 3*A*a*b - 4*C*b^2)*c^4 + (3*A
```

*a^3 - C*a^2*b - 6*A*a*b^2 + 4*C*b^3)*c^3 + (2*A*a^4*b^2 - A*a^2*b^4 - A*b^
6 - A*c^6 + (3*C*a - 2*A*b)*c^5 + (A*a^2 + 6*C*a*b + A*b^2)*c^4 + (3*C*a^3
+ 2*A*a^2*b + 4*A*b^3)*c^3 + (2*A*a^4 + 6*C*a^3*b - 6*C*a*b^3 + A*b^4)*c^2
+ (4*A*a^4*b + 3*C*a^3*b^2 - 2*A*a^2*b^3 - 3*C*a*b^4 - 2*A*b^5)*c)*cosh(x)^
3 + (2*A*a^4*b^2 - A*a^2*b^4 - A*b^6 - A*c^6 + (3*C*a - 2*A*b)*c^5 + (A*a^2
+ 6*C*a*b + A*b^2)*c^4 + (3*C*a^3 + 2*A*a^2*b + 4*A*b^3)*c^3 + (2*A*a^4 +
6*C*a^3*b - 6*C*a*b^3 + A*b^4)*c^2 + (4*A*a^4*b + 3*C*a^3*b^2 - 2*A*a^2*b^3
- 3*C*a*b^4 - 2*A*b^5)*c)*sinh(x)^3 - (C*a^4 + 3*A*a^3*b + C*a^2*b^2 - 6*A
*a*b^3 - 2*C*b^4)*c^2 - (2*C*a^6 - 6*A*a^5*b - 6*C*a^4*b^2 + 3*A*a^3*b^3 +
6*C*a^2*b^4 + 3*A*a*b^5 - 2*C*b^6 + 3*A*a*c^5 + 2*C*c^6 - 3*(C*a^2 - A*a*b
+ 2*C*b^2)*c^4 - 3*(A*a^3 + 3*C*a^2*b + 2*A*a*b^2)*c^3 - 3*(C*a^4 + A*a^3*b
+ C*a^2*b^2 + 2*A*a*b^3 - 2*C*b^4)*c^2 - 3*(2*A*a^5 + 3*C*a^4*b - A*a^3*b^
2 - 3*C*a^2*b^3 - A*a*b^4)*c)*cosh(x)^2 - (2*C*a^6 - 6*A*a^5*b - 6*C*a^4*b^
2 + 3*A*a^3*b^3 + 6*C*a^2*b^4 + 3*A*a*b^5 - 2*C*b^6 + 3*A*a*c^5 + 2*C*c^6 -
3*(C*a^2 - A*a*b + 2*C*b^2)*c^4 - 3*(A*a^3 + 3*C*a^2*b + 2*A*a*b^2)*c^3 -
3*(C*a^4 + A*a^3*b + C*a^2*b^2 + 2*A*a*b^3 - 2*C*b^4)*c^2 - 3*(2*A*a^5 + 3*
C*a^4*b - A*a^3*b^2 - 3*C*a^2*b^3 - A*a*b^4)*c - 3*(2*A*a^4*b^2 - A*a^2*b^4
- A*b^6 - A*c^6 + (3*C*a - 2*A*b)*c^5 + (A*a^2 + 6*C*a*b + A*b^2)*c^4 + (3
*C*a^3 + 2*A*a^2*b + 4*A*b^3)*c^3 + (2*A*a^4 + 6*C*a^3*b - 6*C*a*b^3 + A*b^
4)*c^2 + (4*A*a^4*b + 3*C*a^3*b^2 - 2*A*a^2*b^3 - 3*C*a*b^4 - 2*A*b^5)*c)*c
osh(x))*sinh(x)^2 + (2*A*a^2*b^3 + A*b^5 - A*c^5 + (3*C*a + A*b)*c^4 + (2*A
*a^2*b^3 + A*b^5 - A*c^5 + 3*(C*a - A*b)*c^4 + (2*A*a^2 + 9*C*a*b - 2*A*b^2
)*c^3 + (6*A*a^2*b + 9*C*a*b^2 + 2*A*b^3)*c^2 + 3*(2*A*a^2*b^2 + C*a*b^3 +
A*b^4)*c)*cosh(x)^4 + (2*A*a^2*b^3 + A*b^5 - A*c^5 + 3*(C*a - A*b)*c^4 + (2
*A*a^2 + 9*C*a*b - 2*A*b^2)*c^3 + (6*A*a^2*b + 9*C*a*b^2 + 2*A*b^3)*c^2 + 3
*(2*A*a^2*b^2 + C*a*b^3 + A*b^4)*c)*sinh(x)^4 + (2*A*a^2 - 3*C*a*b + 2*A*b^
2)*c^3 + 4*(2*A*a^3*b^2 + A*a*b^4 - A*a*c^4 + (3*C*a^2 - 2*A*a*b)*c^3 + 2*(
A*a^3 + 3*C*a^2*b)*c^2 + (4*A*a^3*b + 3*C*a^2*b^2 + 2*A*a*b^3)*c)*cosh(x)^3
+ 4*(2*A*a^3*b^2 + A*a*b^4 - A*a*c^4 + (3*C*a^2 - 2*A*a*b)*c^3 + 2*(A*a^3
+ 3*C*a^2*b)*c^2 + (4*A*a^3*b + 3*C*a^2*b^2 + 2*A*a*b^3)*c + (2*A*a^2*b^3 +
A*b^5 - A*c^5 + 3*(C*a - A*b)*c^4 + (2*A*a^2 + 9*C*a*b - 2*A*b^2)*c^3 + (6
*A*a^2*b + 9*C*a*b^2 + 2*A*b^3)*c^2 + 3*(2*A*a^2*b^2 + C*a*b^3 + A*b^4)*c)*
cosh(x))*sinh(x)^3 - (2*A*a^2*b + 3*C*a*b^2 + 2*A*b^3)*c^2 + 2*(4*A*a^4*b +
4*A*a^2*b^3 + A*b^5 + A*c^5 - (3*C*a - A*b)*c^4 - (4*A*a^2 + 3*C*a*b + 2*A
*b^2)*c^3 + (6*C*a^3 - 4*A*a^2*b + 3*C*a*b^2 - 2*A*b^3)*c^2 + (4*A*a^4 + 6*
C*a^3*b + 4*A*a^2*b^2 + 3*C*a*b^3 + A*b^4)*c)*cosh(x)^2 + 2*(4*A*a^4*b + 4*
A*a^2*b^3 + A*b^5 + A*c^5 - (3*C*a - A*b)*c^4 - (4*A*a^2 + 3*C*a*b + 2*A*b^
2)*c^3 + (6*C*a^3 - 4*A*a^2*b + 3*C*a*b^2 - 2*A*b^3)*c^2 + 3*(2*A*a^2*b^3 +
A*b^5 - A*c^5 + 3*(C*a - A*b)*c^4 + (2*A*a^2 + 9*C*a*b - 2*A*b^2)*c^3 + (6
*A*a^2*b + 9*C*a*b^2 + 2*A*b^3)*c^2 + 3*(2*A*a^2*b^2 + C*a*b^3 + A*b^4)*c)*
cosh(x)^2 + (4*A*a^4 + 6*C*a^3*b + 4*A*a^2*b^2 + 3*C*a*b^3 + A*b^4)*c + 6*(
2*A*a^3*b^2 + A*a*b^4 - A*a*c^4 + (3*C*a^2 - 2*A*a*b)*c^3 + 2*(A*a^3 + 3*C*
a^2*b)*c^2 + (4*A*a^3*b + 3*C*a^2*b^2 + 2*A*a*b^3)*c)*cosh(x))*sinh(x)^2 -
(2*A*a^2*b^2 - 3*C*a*b^3 + A*b^4)*c + 4*(2*A*a^3*b^2 + A*a*b^4 + 3*C*a^2*b^
2*c - 3*C*a^2*c^3 + A*a*c^4 - 2*(A*a^3 + A*a*b^2)*c^2)*cosh(x) + 4*(2*A*a^3

```
*b^2 + A*a*b^4 + 3*C*a^2*b^2*c - 3*C*a^2*c^3 + A*a*c^4 + (2*A*a^2*b^3 + A*b
^5 - A*c^5 + 3*(C*a - A*b)*c^4 + (2*A*a^2 + 9*C*a*b - 2*A*b^2)*c^3 + (6*A*a
^2*b + 9*C*a*b^2 + 2*A*b^3)*c^2 + 3*(2*A*a^2*b^2 + C*a*b^3 + A*b^4)*c)*cosh
(x)^3 - 2*(A*a^3 + A*a*b^2)*c^2 + 3*(2*A*a^3*b^2 + A*a*b^4 - A*a*c^4 + (3*C
*a^2 - 2*A*a*b)*c^3 + 2*(A*a^3 + 3*C*a^2*b)*c^2 + (4*A*a^3*b + 3*C*a^2*b^2
+ 2*A*a*b^3)*c)*cosh(x)^2 + (4*A*a^4*b + 4*A*a^2*b^3 + A*b^5 + A*c^5 - (3*C
*a - A*b)*c^4 - (4*A*a^2 + 3*C*a*b + 2*A*b^2)*c^3 + (6*C*a^3 - 4*A*a^2*b +
3*C*a*b^2 - 2*A*b^3)*c^2 + (4*A*a^4 + 6*C*a^3*b + 4*A*a^2*b^2 + 3*C*a*b^3 +
 A*b^4)*c)*cosh(x))*sinh(x))*sqrt(-a^2 + b^2 - c^2)*arctan(sqrt(-a^2 + b^2
- c^2)*((b + c)*cosh(x) + (b + c)*sinh(x) + a)/(a^2 - b^2 + c^2)) + (C*a^4*
b - 3*A*a^3*b^2 + C*a^2*b^3 + 3*A*a*b^4 - 2*C*b^5)*c + (10*A*a^4*b^2 - 11*A
*a^2*b^4 + A*b^6 - 5*C*a*c^5 - A*c^6 - (11*A*a^2 - 3*A*b^2)*c^4 - (C*a^3 -
10*C*a*b^2)*c^3 - (10*A*a^4 - 22*A*a^2*b^2 + 3*A*b^4)*c^2 + (4*C*a^5 + C*a^
3*b^2 - 5*C*a*b^4)*c)*cosh(x) + (10*A*a^4*b^2 - 11*A*a^2*b^4 + A*b^6 - 5*C*
a*c^5 - A*c^6 - (11*A*a^2 - 3*A*b^2)*c^4 - (C*a^3 - 10*C*a*b^2)*c^3 - (10*A
*a^4 - 22*A*a^2*b^2 + 3*A*b^4)*c^2 + 3*(2*A*a^4*b^2 - A*a^2*b^4 - A*b^6 - A
*c^6 + (3*C*a - 2*A*b)*c^5 + (A*a^2 + 6*C*a*b + A*b^2)*c^4 + (3*C*a^3 + 2*A
*a^2*b + 4*A*b^3)*c^3 + (2*A*a^4 + 6*C*a^3*b - 6*C*a*b^3 + A*b^4)*c^2 + (4*
A*a^4*b + 3*C*a^3*b^2 - 2*A*a^2*b^3 - 3*C*a*b^4 - 2*A*b^5)*c)*cosh(x)^2 + (
4*C*a^5 + C*a^3*b^2 - 5*C*a*b^4)*c - 2*(2*C*a^6 - 6*A*a^5*b - 6*C*a^4*b^2 +
 3*A*a^3*b^3 + 6*C*a^2*b^4 + 3*A*a*b^5 - 2*C*b^6 + 3*A*a*c^5 + 2*C*c^6 - 3*
(C*a^2 - A*a*b + 2*C*b^2)*c^4 - 3*(A*a^3 + 3*C*a^2*b + 2*A*a*b^2)*c^3 - 3*(
C*a^4 + A*a^3*b + C*a^2*b^2 + 2*A*a*b^3 - 2*C*b^4)*c^2 - 3*(2*A*a^5 + 3*C*a
^4*b - A*a^3*b^2 - 3*C*a^2*b^3 - A*a*b^4)*c)*cosh(x))*sinh(x))/(a^6*b^3 - 3
*a^4*b^5 + 3*a^2*b^7 - b^9 - b*c^8 + c^9 + (3*a^2 - 4*b^2)*c^7 - (3*a^2*b -
 4*b^3)*c^6 + 3*(a^4 - 3*a^2*b^2 + 2*b^4)*c^5 - 3*(a^4*b - 3*a^2*b^3 + 2*b^
5)*c^4 + (a^6*b^3 - 3*a^4*b^5 + 3*a^2*b^7 - b^9 + 3*a^2*c^7 + 3*b*c^8 + c^9
 + (9*a^2*b - 8*b^3)*c^6 + 3*(a^4 + a^2*b^2 - 2*b^4)*c^5 + 3*(3*a^4*b - 5*a
^2*b^3 + 2*b^5)*c^4 + (a^6 + 6*a^4*b^2 - 15*a^2*b^4 + 8*b^6)*c^3 + 3*(a^6*b
 - 2*a^4*b^3 + a^2*b^5)*c^2 + 3*(a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^8)*c)*
cosh(x)^4 + (a^6*b^3 - 3*a^4*b^5 + 3*a^2*b^7 - b^9 + 3*a^2*c^7 + 3*b*c^8 +
c^9 + (9*a^2*b - 8*b^3)*c^6 + 3*(a^4 + a^2*b^2 - 2*b^4)*c^5 + 3*(3*a^4*b -
5*a^2*b^3 + 2*b^5)*c^4 + (a^6 + 6*a^4*b^2 - 15*a^2*b^4 + 8*b^6)*c^3 + 3*(a^
6*b - 2*a^4*b^3 + a^2*b^5)*c^2 + 3*(a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^8)*
c)*sinh(x)^4 + (a^6 - 6*a^4*b^2 + 9*a^2*b^4 - 4*b^6)*c^3 + 4*(a^7*b^2 - 3*a
^5*b^4 + 3*a^3*b^6 - a*b^8 + 2*a*b*c^7 + a*c^8 + (3*a^3 - 2*a*b^2)*c^6 + 6*
(a^3*b - a*b^3)*c^5 + 3*(a^5 - a^3*b^2)*c^4 + 6*(a^5*b - 2*a^3*b^3 + a*b^5)
*c^3 + (a^7 - 3*a^3*b^4 + 2*a*b^6)*c^2 + 2*(a^7*b - 3*a^5*b^3 + 3*a^3*b^5 -
 a*b^7)*c)*cosh(x)^3 + 4*(a^7*b^2 - 3*a^5*b^4 + 3*a^3*b^6 - a*b^8 + 2*a*b*c
^7 + a*c^8 + (3*a^3 - 2*a*b^2)*c^6 + 6*(a^3*b - a*b^3)*c^5 + 3*(a^5 - a^3*b
^2)*c^4 + 6*(a^5*b - 2*a^3*b^3 + a*b^5)*c^3 + (a^7 - 3*a^3*b^4 + 2*a*b^6)*c
^2 + 2*(a^7*b - 3*a^5*b^3 + 3*a^3*b^5 - a*b^7)*c + (a^6*b^3 - 3*a^4*b^5 + 3
*a^2*b^7 - b^9 + 3*a^2*c^7 + 3*b*c^8 + c^9 + (9*a^2*b - 8*b^3)*c^6 + 3*(a^4
 + a^2*b^2 - 2*b^4)*c^5 + 3*(3*a^4*b - 5*a^2*b^3 + 2*b^5)*c^4 + (a^6 + 6*a^
4*b^2 - 15*a^2*b^4 + 8*b^6)*c^3 + 3*(a^6*b - 2*a^4*b^3 + a^2*b^5)*c^2 + 3*(
```

```
a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^8)*c)*cosh(x))*sinh(x)^3 - (a^6*b - 6*a
^4*b^3 + 9*a^2*b^5 - 4*b^7)*c^2 + 2*(2*a^8*b - 5*a^6*b^3 + 3*a^4*b^5 + a^2*
b^7 - b^9 - b*c^8 - c^9 - (a^2 - 4*b^2)*c^7 - (a^2*b - 4*b^3)*c^6 + 3*(a^4
+ a^2*b^2 - 2*b^4)*c^5 + 3*(a^4*b + a^2*b^3 - 2*b^5)*c^4 + (5*a^6 - 6*a^4*b
^2 - 3*a^2*b^4 + 4*b^6)*c^3 + (5*a^6*b - 6*a^4*b^3 - 3*a^2*b^5 + 4*b^7)*c^2
 + (2*a^8 - 5*a^6*b^2 + 3*a^4*b^4 + a^2*b^6 - b^8)*c)*cosh(x)^2 + 2*(2*a^8*
b - 5*a^6*b^3 + 3*a^4*b^5 + a^2*b^7 - b^9 - b*c^8 - c^9 - (a^2 - 4*b^2)*c^7
 - (a^2*b - 4*b^3)*c^6 + 3*(a^4 + a^2*b^2 - 2*b^4)*c^5 + 3*(a^4*b + a^2*b^3
 - 2*b^5)*c^4 + (5*a^6 - 6*a^4*b^2 - 3*a^2*b^4 + 4*b^6)*c^3 + (5*a^6*b - 6*
a^4*b^3 - 3*a^2*b^5 + 4*b^7)*c^2 + 3*(a^6*b^3 - 3*a^4*b^5 + 3*a^2*b^7 - b^9
 + 3*a^2*c^7 + 3*b*c^8 + c^9 + (9*a^2*b - 8*b^3)*c^6 + 3*(a^4 + a^2*b^2 - 2
*b^4)*c^5 + 3*(3*a^4*b - 5*a^2*b^3 + 2*b^5)*c^4 + (a^6 + 6*a^4*b^2 - 15*a^2
*b^4 + 8*b^6)*c^3 + 3*(a^6*b - 2*a^4*b^3 + a^2*b^5)*c^2 + 3*(a^6*b^2 - 3*a^
4*b^4 + 3*a^2*b^6 - b^8)*c)*cosh(x)^2 + (2*a^8 - 5*a^6*b^2 + 3*a^4*b^4 + a^
2*b^6 - b^8)*c + 6*(a^7*b^2 - 3*a^5*b^4 + 3*a^3*b^6 - a*b^8 + 2*a*b*c^7 + a
*c^8 + (3*a^3 - 2*a*b^2)*c^6 + 6*(a^3*b - a*b^3)*c^5 + 3*(a^5 - a^3*b^2)*c^
4 + 6*(a^5*b - 2*a^3*b^3 + a*b^5)*c^3 + (a^7 - 3*a^3*b^4 + 2*a*b^6)*c^2 + 2
*(a^7*b - 3*a^5*b^3 + 3*a^3*b^5 - a*b^7)*c)*cosh(x))*sinh(x)^2 - (a^6*b^2 -
 3*a^4*b^4 + 3*a^2*b^6 - b^8)*c + 4*(a^7*b^2 - 3*a^5*b^4 + 3*a^3*b^6 - a*b^
8 - a*c^8 - (3*a^3 - 4*a*b^2)*c^6 - 3*(a^5 - 3*a^3*b^2 + 2*a*b^4)*c^4 - (a^
7 - 6*a^5*b^2 + 9*a^3*b^4 - 4*a*b^6)*c^2)*cosh(x) + 4*(a^7*b^2 - 3*a^5*b^4
+ 3*a^3*b^6 - a*b^8 - a*c^8 - (3*a^3 - 4*a*b^2)*c^6 - 3*(a^5 - 3*a^3*b^2 +
2*a*b^4)*c^4 + (a^6*b^3 - 3*a^4*b^5 + 3*a^2*b^7 - b^9 + 3*a^2*c^7 + 3*b*c^8
 + c^9 + (9*a^2*b - 8*b^3)*c^6 + 3*(a^4 + a^2*b^2 - 2*b^4)*c^5 + 3*(3*a^4*b
 - 5*a^2*b^3 + 2*b^5)*c^4 + (a^6 + 6*a^4*b^2 - 15*a^2*b^4 + 8*b^6)*c^3 + 3*
(a^6*b - 2*a^4*b^3 + a^2*b^5)*c^2 + 3*(a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^
8)*c)*cosh(x)^3 - (a^7 - 6*a^5*b^2 + 9*a^3*b^4 - 4*a*b^6)*c^2 + 3*(a^7*b^2
- 3*a^5*b^4 + 3*a^3*b^6 - a*b^8 + 2*a*b*c^7 + a*c^8 + (3*a^3 - 2*a*b^2)*c^6
 + 6*(a^3*b - a*b^3)*c^5 + 3*(a^5 - a^3*b^2)*c^4 + 6*(a^5*b - 2*a^3*b^3 + a
*b^5)*c^3 + (a^7 - 3*a^3*b^4 + 2*a*b^6)*c^2 + 2*(a^7*b - 3*a^5*b^3 + 3*a^3*
b^5 - a*b^7)*c)*cosh(x)^2 + (2*a^8*b - 5*a^6*b^3 + 3*a^4*b^5 + a^2*b^7 - b^
9 - b*c^8 - c^9 - (a^2 - 4*b^2)*c^7 - (a^2*b - 4*b^3)*c^6 + 3*(a^4 + a^2*b^
2 - 2*b^4)*c^5 + 3*(a^4*b + a^2*b^3 - 2*b^5)*c^4 + (5*a^6 - 6*a^4*b^2 - 3*a
^2*b^4 + 4*b^6)*c^3 + (5*a^6*b - 6*a^4*b^3 - 3*a^2*b^5 + 4*b^7)*c^2 + (2*a^
8 - 5*a^6*b^2 + 3*a^4*b^4 + a^2*b^6 - b^8)*c)*cosh(x))*sinh(x))]
```

giac [B] time = 0.19, size = 625, normalized size = 3.16

$$\frac{\left(2\,Aa^2+Ab^2+3\,Cac-Ac^2\right)\arctan\left(\frac{be^x+ce^x+a}{\sqrt{-a^2+b^2-c^2}}\right)}{\left(a^4-2\,a^2b^2+b^4+2\,a^2c^2-2\,b^2c^2+c^4\right)\sqrt{-a^2+b^2-c^2}}+\frac{2\,Aa^2b^2e^{(3\,x)}+Ab^4e^{(3\,x)}+4\,Aa^2bce^{(3\,x)}+3\,Cab^2ce^{(3\,x)}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sinh(x))/(a+b*cosh(x)+c*sinh(x))^3,x, algorithm="giac")
```

```
[Out] (2*A*a^2 + A*b^2 + 3*C*a*c - A*c^2)*arctan((b*e^x + c*e^x + a)/sqrt(-a^2 +
b^2 - c^2))/((a^4 - 2*a^2*b^2 + b^4 + 2*a^2*c^2 - 2*b^2*c^2 + c^4)*sqrt(-a^
2 + b^2 - c^2)) + (2*A*a^2*b^2*e^(3*x) + A*b^4*e^(3*x) + 4*A*a^2*b*c*e^(3*x
) + 3*C*a*b^2*c*e^(3*x) + 2*A*b^3*c*e^(3*x) + 2*A*a^2*c^2*e^(3*x) + 6*C*a*b
*c^2*e^(3*x) + 3*C*a*c^3*e^(3*x) - 2*A*b*c^3*e^(3*x) - A*c^4*e^(3*x) - 2*C*
a^4*e^(2*x) + 6*A*a^3*b*e^(2*x) + 4*C*a^2*b^2*e^(2*x) + 3*A*a*b^3*e^(2*x) -
 2*C*b^4*e^(2*x) + 6*A*a^3*c*e^(2*x) + 9*C*a^2*b*c*e^(2*x) + 3*A*a*b^2*c*e^
(2*x) + 5*C*a^2*c^2*e^(2*x) - 3*A*a*b*c^2*e^(2*x) + 4*C*b^2*c^2*e^(2*x) - 3
*A*a*c^3*e^(2*x) - 2*C*c^4*e^(2*x) + 10*A*a^2*b^2*e^x - A*b^4*e^x + 4*C*a^3
*c*e^x + 5*C*a*b^2*c*e^x - 10*A*a^2*c^2*e^x + 2*A*b^2*c^2*e^x - 5*C*a*c^3*e
^x - A*c^4*e^x + 3*A*a*b^3 + C*a^2*b*c - 3*A*a*b^2*c + 2*C*b^3*c - C*a^2*c^
2 - 3*A*a*b*c^2 - 2*C*b^2*c^2 + 3*A*a*c^3 - 2*C*b*c^3 + 2*C*c^4)/((a^4*b -
2*a^2*b^3 + b^5 + a^4*c - 2*a^2*b^2*c + b^4*c + 2*a^2*b*c^2 - 2*b^3*c^2 + 2
*a^2*c^3 - 2*b^2*c^3 + b*c^4 + c^5)*(b*e^(2*x) + c*e^(2*x) + 2*a*e^x + b -
c)^2)
```

maple [B] time = 0.30, size = 1091, normalized size = 5.51

$$-\frac{2\left(-\frac{\left(4A\,a^3b-7A\,a^2b^2+5A\,a^2c^2+2Aa\,b^3-2Aab\,c^2+A\,b^4-3A\,b^2c^2+2A\,c^4-3C\,a^3c+6C\,a^2bc-3Ca\,b^2c\right)\left(\tanh^3\left(\frac{x}{2}\right)\right)}{2\left(a^4-2a^2b^2+2a^2c^2+b^4-2c^2b^2+c^4\right)(a-b)}-\frac{\left(4A\,a^4c-12A\,a^3bc+13A\,a^2b^2c-7\cdots\right.}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*sinh(x))/(a+b*cosh(x)+c*sinh(x))^3,x)
```

```
[Out] -2*(-1/2*(4*A*a^3*b-7*A*a^2*b^2+5*A*a^2*c^2+2*A*a*b^3-2*A*a*b*c^2+A*b^4-3*A
*b^2*c^2+2*A*c^4-3*C*a^3*c+6*C*a^2*b*c-3*C*a*b^2*c)/(a^4-2*a^2*b^2+2*a^2*c^
2+b^4-2*b^2*c^2+c^4)/(a-b)*tanh(1/2*x)^3-1/2*(4*A*a^4*c-12*A*a^3*b*c+13*A*a
^2*b^2*c-7*A*a^2*c^3-6*A*a*b^3*c+6*A*a*b*c^3+A*b^4*c+A*b^2*c^3-2*A*c^5-2*C*
a^5+2*C*a^4*b+4*C*a^3*b^2+5*C*a^3*c^2-4*C*a^2*b^3-14*C*a^2*b*c^2-2*C*a*b^4+
13*C*a*b^2*c^2-2*C*a*c^4+2*C*b^5-4*C*b^3*c^2+2*C*b*c^4)/(a^4-2*a^2*b^2+2*a^
2*c^2+b^4-2*b^2*c^2+c^4)/(a^2-2*a*b+b^2)*tanh(1/2*x)^2+1/2*(4*A*a^4*b-5*A*a
^3*b^2+11*A*a^3*c^2-3*A*a^2*b^3-3*A*a^2*b*c^2+5*A*a*b^4-7*A*a*b^2*c^2+2*A*a
*c^4-A*b^5-A*b^3*c^2+2*A*b*c^4-5*C*a^4*c+5*C*a^3*b*c+5*C*a^2*b^2*c+4*C*a^2*
c^3-5*C*a*b^3*c-4*C*a*b*c^3)/(a^4-2*a^2*b^2+2*a^2*c^2+b^4-2*b^2*c^2+c^4)/(a
^2-2*a*b+b^2)*tanh(1/2*x)+1/2*(4*A*a^4*c-3*A*a^2*b^2*c+A*a^2*c^3-A*b^4*c+A*
b^2*c^3-2*C*a^5+4*C*a^3*b^2+C*a^3*c^2-2*C*a*b^4-C*a*b^2*c^2)/(a^4-2*a^2*b^2
+2*a^2*c^2+b^4-2*b^2*c^2+c^4)/(a^2-2*a*b+b^2))/(a*tanh(1/2*x)^2-tanh(1/2*x)
^2*b-2*c*tanh(1/2*x)-a-b)^2-2/(a^4-2*a^2*b^2+2*a^2*c^2+b^4-2*b^2*c^2+c^4)/(
-a^2+b^2-c^2)^(1/2)*arctan(1/2*(2*(a-b)*tanh(1/2*x)-2*c)/(-a^2+b^2-c^2)^(1/
2))*a^2*A-1/(a^4-2*a^2*b^2+2*a^2*c^2+b^4-2*b^2*c^2+c^4)/(-a^2+b^2-c^2)^(1/2
)*arctan(1/2*(2*(a-b)*tanh(1/2*x)-2*c)/(-a^2+b^2-c^2)^(1/2))*A*b^2+1/(a^4-2
*a^2*b^2+2*a^2*c^2+b^4-2*b^2*c^2+c^4)/(-a^2+b^2-c^2)^(1/2)*arctan(1/2*(2*(a
-b)*tanh(1/2*x)-2*c)/(-a^2+b^2-c^2)^(1/2))*A*c^2-3/(a^4-2*a^2*b^2+2*a^2*c^2
```

```
+b^4-2*b^2*c^2+c^4)/(-a^2+b^2-c^2)^(1/2)*arctan(1/2*(2*(a-b)*tanh(1/2*x)-2*
c)/(-a^2+b^2-c^2)^(1/2))*a*c*C
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sinh(x))/(a+b*cosh(x)+c*sinh(x))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(c^2-b^2+a^2>0)', see `assume?` for
more details)Is c^2-b^2+a^2 positive or negative?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A + C\sinh(x)}{(a + b\cosh(x) + c\sinh(x))^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + C*sinh(x))/(a + b*cosh(x) + c*sinh(x))^3,x)
```

```
[Out] int((A + C*sinh(x))/(a + b*cosh(x) + c*sinh(x))^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sinh(x))/(a+b*cosh(x)+c*sinh(x))**3,x)
```

```
[Out] Timed out
```

3.792 $\int \frac{A+B\cosh(x)}{a+b\cosh(x)+c\sinh(x)}\,dx$

Optimal. Leaf size=121

$$\frac{2\left(abB-A\left(b^2-c^2\right)\right)\tanh^{-1}\left(\frac{c-(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2+c^2}}\right)}{\left(b^2-c^2\right)\sqrt{a^2-b^2+c^2}}-\frac{Bc\log(a+b\cosh(x)+c\sinh(x))}{b^2-c^2}+\frac{bBx}{b^2-c^2}$$

```
[Out] b*B*x/(b^2-c^2)-B*c*ln(a+b*cosh(x)+c*sinh(x))/(b^2-c^2)+2*(a*b*B-A*(b^2-c^2
))*arctanh((c-(a-b)*tanh(1/2*x))/(a^2-b^2+c^2)^(1/2))/(b^2-c^2)/(a^2-b^2+c^
2)^(1/2)
```

Rubi [A] time = 0.12, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {3138, 3124, 618, 206}

$$\frac{2\left(abB-A\left(b^2-c^2\right)\right)\tanh^{-1}\left(\frac{c-(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2+c^2}}\right)}{\left(b^2-c^2\right)\sqrt{a^2-b^2+c^2}}-\frac{Bc\log(a+b\cosh(x)+c\sinh(x))}{b^2-c^2}+\frac{bBx}{b^2-c^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cosh[x])/(a + b*Cosh[x] + c*Sinh[x]),x]
```

```
[Out] (b*B*x)/(b^2 - c^2) + (2*(a*b*B - A*(b^2 - c^2))*ArcTanh[(c - (a - b)*Tanh[
x/2])/Sqrt[a^2 - b^2 + c^2]])/((b^2 - c^2)*Sqrt[a^2 - b^2 + c^2]) - (B*c*Lo
g[a + b*Cosh[x] + c*Sinh[x]])/(b^2 - c^2)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 3124

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Module[{f = FreeFactors[Tan[(d + e*x)/2], x]}, Dist[(2*f
```

```
)/e, Subst[Int[1/(a + b + 2*c*f*x + (a - b)*f^2*x^2), x], x, Tan[(d + e*x)/
2]/f], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
```

Rule 3138

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.))/((a_.) + cos[(d_.) + (e_.)*(x_)
]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)]), x_Symbol] :> Simp[(b*B*(d + e*x))
/(e*(b^2 + c^2)), x] + (Dist[(A*(b^2 + c^2) - a*b*B)/(b^2 + c^2), Int[1/(a
+ b*Cos[d + e*x] + c*Sin[d + e*x]), x], x] + Simp[(c*B*Log[a + b*Cos[d + e*
x] + c*Sin[d + e*x]])/(e*(b^2 + c^2)), x]) /; FreeQ[{a, b, c, d, e, A, B},
x] && NeQ[b^2 + c^2, 0] && NeQ[A*(b^2 + c^2) - a*b*B, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{A+B\cosh(x)}{a+b\cosh(x)+c\sinh(x)}\,dx &= \frac{bBx}{b^2-c^2}-\frac{Bc\log(a+b\cosh(x)+c\sinh(x))}{b^2-c^2}+\left(A-\frac{abB}{b^2-c^2}\right)\int \frac{1}{a+b\cosh(x \\ &= \frac{bBx}{b^2-c^2}-\frac{Bc\log(a+b\cosh(x)+c\sinh(x))}{b^2-c^2}+\left(2\left(A-\frac{abB}{b^2-c^2}\right)\right)\mathrm{Subst}\left(\int \frac{}{a} \\ &= \frac{bBx}{b^2-c^2}-\frac{Bc\log(a+b\cosh(x)+c\sinh(x))}{b^2-c^2}-\left(4\left(A-\frac{abB}{b^2-c^2}\right)\right)\mathrm{Subst}\left(\int \frac{}{4} \\ &= \frac{bBx}{b^2-c^2}-\frac{2\left(A-\frac{abB}{b^2-c^2}\right)\tanh^{-1}\left(\frac{c-(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2+c^2}}\right)}{\sqrt{a^2-b^2+c^2}}-\frac{Bc\log(a+b\cosh(x)+c\sin}{b^2-c^2}\end{aligned}$$

Mathematica [A] time = 0.22, size = 104, normalized size = 0.86

$$\frac{B(bx-c\log(a+b\cosh(x)+c\sinh(x)))-\frac{2\left(abB+A\left(c^2-b^2\right)\right)\tan^{-1}\left(\frac{(b-a)\tanh\left(\frac{x}{2}\right)+c}{\sqrt{-a^2+b^2-c^2}}\right)}{\sqrt{-a^2+b^2-c^2}}}{(b-c)(b+c)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cosh[x])/(a + b*Cosh[x] + c*Sinh[x]),x]
```

```
[Out] ((-2*(a*b*B + A*(-b^2 + c^2))*ArcTan[(c + (-a + b)*Tanh[x/2])/Sqrt[-a^2 + b
^2 - c^2]])/Sqrt[-a^2 + b^2 - c^2] + B*(b*x - c*Log[a + b*Cosh[x] + c*Sinh[
x]]))/((b - c)*(b + c))
```

fricas [A] time = 0.46, size = 508, normalized size = 4.20

$$\left[-\frac{\left(Bab - Ab^2 + Ac^2\right)\sqrt{a^2 - b^2 + c^2}\,\log\left(\frac{\left(b^2+2\,bc+c^2\right)\cosh(x)^2+\left(b^2+2\,bc+c^2\right)\sinh(x)^2+2\,a^2-b^2+c^2+2\,(ab+ac)\cosh(x)+2\,\left(ab+ac+\left(b^2+\right.\right.}{(b+c)\cosh(x)^2+(b+c)\sinh(x)^2+2\,a\cosh(x)+2\,((b\right.}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(a+b*cosh(x)+c*sinh(x)),x, algorithm="fricas")
```

```
[Out] [-((B*a*b - A*b^2 + A*c^2)*sqrt(a^2 - b^2 + c^2)*log(((b^2 + 2*b*c + c^2)*c
osh(x)^2 + (b^2 + 2*b*c + c^2)*sinh(x)^2 + 2*a^2 - b^2 + c^2 + 2*(a*b + a*c
)*cosh(x) + 2*(a*b + a*c + (b^2 + 2*b*c + c^2)*cosh(x))*sinh(x) - 2*sqrt(a^
2 - b^2 + c^2)*((b + c)*cosh(x) + (b + c)*sinh(x) + a))/((b + c)*cosh(x)^2
+ (b + c)*sinh(x)^2 + 2*a*cosh(x) + 2*((b + c)*cosh(x) + a)*sinh(x) + b - c
)) - (B*a^2*b - B*b^3 + B*b*c^2 + B*c^3 + (B*a^2 - B*b^2)*c)*x + (B*c^3 + (
B*a^2 - B*b^2)*c)*log(2*(b*cosh(x) + c*sinh(x) + a)/(cosh(x) - sinh(x))))/(
a^2*b^2 - b^4 - c^4 - (a^2 - 2*b^2)*c^2), -(2*(B*a*b - A*b^2 + A*c^2)*sqrt(
-a^2 + b^2 - c^2)*arctan(sqrt(-a^2 + b^2 - c^2)*((b + c)*cosh(x) + (b + c)*
sinh(x) + a)/(a^2 - b^2 + c^2)) - (B*a^2*b - B*b^3 + B*b*c^2 + B*c^3 + (B*a
^2 - B*b^2)*c)*x + (B*c^3 + (B*a^2 - B*b^2)*c)*log(2*(b*cosh(x) + c*sinh(x)
 + a)/(cosh(x) - sinh(x))))/(a^2*b^2 - b^4 - c^4 - (a^2 - 2*b^2)*c^2)]
```

giac [A] time = 0.14, size = 122, normalized size = 1.01

$$-\frac{Bc\log\left(be^{(2\,x)} + ce^{(2\,x)} + 2\,ae^{x} + b - c\right)}{b^2 - c^2} + \frac{Bx}{b - c} - \frac{2\left(Bab - Ab^2 + Ac^2\right)\arctan\left(\frac{be^{x}+ce^{x}+a}{\sqrt{-a^2+b^2-c^2}}\right)}{\sqrt{-a^2 + b^2 - c^2}\left(b^2 - c^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(a+b*cosh(x)+c*sinh(x)),x, algorithm="giac")
```

```
[Out] -B*c*log(b*e^(2*x) + c*e^(2*x) + 2*a*e^x + b - c)/(b^2 - c^2) + B*x/(b - c)
 - 2*(B*a*b - A*b^2 + A*c^2)*arctan((b*e^x + c*e^x + a)/sqrt(-a^2 + b^2 - c
^2))/(sqrt(-a^2 + b^2 - c^2)*(b^2 - c^2))
```

maple [B] time = 0.20, size = 574, normalized size = 4.74

$$-\frac{2B\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{2b+2c}-\frac{\ln\left(a\left(\tanh^2\left(\frac{x}{2}\right)\right)-\left(\tanh^2\left(\frac{x}{2}\right)\right)b-2c\tanh\left(\frac{x}{2}\right)-a-b\right)aBc}{(b-c)\,(b+c)\,(a-b)}+\frac{\ln\left(a\left(\tanh^2\left(\frac{x}{2}\right)\right)-\left(\tanh^2\right.\right.}{(b-c)\,(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cosh(x))/(a+b*cosh(x)+c*sinh(x)),x)
```

```
[Out] -2*B/(2*b+2*c)*ln(tanh(1/2*x)-1)-1/(b-c)/(b+c)/(a-b)*ln(a*tanh(1/2*x)^2-tan
h(1/2*x)^2*b-2*c*tanh(1/2*x)-a-b)*a*B*c+1/(b-c)/(b+c)/(a-b)*ln(a*tanh(1/2*x
)^2-tanh(1/2*x)^2*b-2*c*tanh(1/2*x)-a-b)*b*B*c-2/(b-c)/(b+c)/(-a^2+b^2-c^2)
^(1/2)*arctan(1/2*(2*(a-b)*tanh(1/2*x)-2*c)/(-a^2+b^2-c^2)^(1/2))*A*b^2+2/(
b-c)/(b+c)/(-a^2+b^2-c^2)^(1/2)*arctan(1/2*(2*(a-b)*tanh(1/2*x)-2*c)/(-a^2+
b^2-c^2)^(1/2))*A*c^2+2/(b-c)/(b+c)/(-a^2+b^2-c^2)^(1/2)*arctan(1/2*(2*(a-b
)*tanh(1/2*x)-2*c)/(-a^2+b^2-c^2)^(1/2))*b*B*a+2/(b-c)/(b+c)/(-a^2+b^2-c^2)
^(1/2)*arctan(1/2*(2*(a-b)*tanh(1/2*x)-2*c)/(-a^2+b^2-c^2)^(1/2))*B*c^2-2/(
b-c)/(b+c)/(-a^2+b^2-c^2)^(1/2)*arctan(1/2*(2*(a-b)*tanh(1/2*x)-2*c)/(-a^2+
b^2-c^2)^(1/2))*c^2/(a-b)*a*B+2/(b-c)/(b+c)/(-a^2+b^2-c^2)^(1/2)*arctan(1/2
*(2*(a-b)*tanh(1/2*x)-2*c)/(-a^2+b^2-c^2)^(1/2))*c^2/(a-b)*b*B+2*B/(2*b-2*c
)*ln(tanh(1/2*x)+1)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(a+b*cosh(x)+c*sinh(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(c^2-b^2+a^2>0)', see `assume?` for
more details)Is c^2-b^2+a^2 positive or negative?
```

mupad [B] time = 0.67, size = 375, normalized size = 3.10

$$\frac{\ln\left(b\,\sqrt{a^2-b^2+c^2}\,-c\,\sqrt{a^2-b^2+c^2}\,+a^2\,e^x-b^2\,e^x+c^2\,e^x+a\,e^x\,\sqrt{a^2-b^2+c^2}\right)\,\left(B\,c^3-A\,b^2\,\sqrt{a^2-b^2+c^2}\,+\right.}{-a^2\,b^2+a^2\,c^2+b^4-2\,b^2\,c^2+c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cosh(x))/(a + b*cosh(x) + c*sinh(x)),x)
```

```
[Out] (log(b*(a^2 - b^2 + c^2)^(1/2) - c*(a^2 - b^2 + c^2)^(1/2) + a^2*exp(x) - b
^2*exp(x) + c^2*exp(x) + a*exp(x)*(a^2 - b^2 + c^2)^(1/2))*(B*c^3 - A*b^2*(
a^2 - b^2 + c^2)^(1/2) + B*a^2*c + A*c^2*(a^2 - b^2 + c^2)^(1/2) - B*b^2*c
+ B*a*b*(a^2 - b^2 + c^2)^(1/2)))/(b^4 + c^4 - a^2*b^2 + a^2*c^2 - 2*b^2*c^
2) + (log(b*(a^2 - b^2 + c^2)^(1/2) - c*(a^2 - b^2 + c^2)^(1/2) - a^2*exp(x
) + b^2*exp(x) - c^2*exp(x) + a*exp(x)*(a^2 - b^2 + c^2)^(1/2))*(B*c^3 + A*
b^2*(a^2 - b^2 + c^2)^(1/2) + B*a^2*c - A*c^2*(a^2 - b^2 + c^2)^(1/2) - B*b
^2*c - B*a*b*(a^2 - b^2 + c^2)^(1/2)))/(b^4 + c^4 - a^2*b^2 + a^2*c^2 - 2*b
^2*c^2) + (B*x)/(b - c)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(a+b*cosh(x)+c*sinh(x)),x)
```

```
[Out] Timed out
```

3.793 $\int \frac{A+B\cosh(x)}{(a+b\cosh(x)+c\sinh(x))^2}\,dx$

Optimal. Leaf size=108

$$-\frac{2(aA-bB)\tanh^{-1}\left(\frac{c-(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2+c^2}}\right)}{\left(a^2-b^2+c^2\right)^{3/2}}-\frac{\sinh(x)(Ab-aB)+Ac\cosh(x)+Bc}{\left(a^2-b^2+c^2\right)(a+b\cosh(x)+c\sinh(x))}$$

```
[Out] -2*(A*a-B*b)*arctanh((c-(a-b)*tanh(1/2*x))/(a^2-b^2+c^2)^(1/2))/(a^2-b^2+c^
2)^(3/2)+(-B*c-A*c*cosh(x)-(A*b-B*a)*sinh(x))/(a^2-b^2+c^2)/(a+b*cosh(x)+c*
sinh(x))
```

Rubi [A] time = 0.13, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {3155, 3124, 618, 206}

$$-\frac{2(aA-bB)\tanh^{-1}\left(\frac{c-(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2+c^2}}\right)}{\left(a^2-b^2+c^2\right)^{3/2}}-\frac{\sinh(x)(Ab-aB)+Ac\cosh(x)+Bc}{\left(a^2-b^2+c^2\right)(a+b\cosh(x)+c\sinh(x))}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cosh[x])/(a + b*Cosh[x] + c*Sinh[x])^2,x]
```

```
[Out] (-2*(a*A - b*B)*ArcTanh[(c - (a - b)*Tanh[x/2])/Sqrt[a^2 - b^2 + c^2]])/(a^
2 - b^2 + c^2)^(3/2) - (B*c + A*c*Cosh[x] + (A*b - a*B)*Sinh[x])/((a^2 - b^
2 + c^2)*(a + b*Cosh[x] + c*Sinh[x]))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 3124

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Module[{f = FreeFactors[Tan[(d + e*x)/2], x]}, Dist[(2*f
```

```
)/e, Subst[Int[1/(a + b + 2*c*f*x + (a - b)*f^2*x^2), x], x, Tan[(d + e*x)/
2]/f], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
```

Rule 3155

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.))/((a_.) + cos[(d_.) + (e_.)*(x_)
]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])^2, x_Symbol] :> Simp[(c*B + c*A*Co
s[d + e*x] + (a*B - b*A)*Sin[d + e*x])/(e*(a^2 - b^2 - c^2)*(a + b*Cos[d +
e*x] + c*Sin[d + e*x])), x] + Dist[(a*A - b*B)/(a^2 - b^2 - c^2), Int[1/(a
+ b*Cos[d + e*x] + c*Sin[d + e*x]), x], x] /; FreeQ[{a, b, c, d, e, A, B},
x] && NeQ[a^2 - b^2 - c^2, 0] && NeQ[a*A - b*B, 0]
```

Rubi steps

$$\int \frac{A+B\cosh(x)}{(a+b\cosh(x)+c\sinh(x))^2}\,dx = -\frac{Bc+Ac\cosh(x)+(Ab-aB)\sinh(x)}{\left(a^2-b^2+c^2\right)(a+b\cosh(x)+c\sinh(x))} + \frac{(aA-bB)\int \frac{1}{a+b\cosh(x)+c\sinh(x)}$$

$$= -\frac{Bc+Ac\cosh(x)+(Ab-aB)\sinh(x)}{\left(a^2-b^2+c^2\right)(a+b\cosh(x)+c\sinh(x))} + \frac{(2(aA-bB))\,\mathrm{Subst}\left(\int \frac{}{a+b+2cx}\right.}{a^2-b^2+}$$

$$= -\frac{Bc+Ac\cosh(x)+(Ab-aB)\sinh(x)}{\left(a^2-b^2+c^2\right)(a+b\cosh(x)+c\sinh(x))} - \frac{(4(aA-bB))\,\mathrm{Subst}\left(\int \frac{1}{4\left(a^2-b^2\right.}\right.}{a^2}$$

$$= -\frac{2(aA-bB)\tanh^{-1}\left(\frac{c-(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2+c^2}}\right)}{\left(a^2-b^2+c^2\right)^{3/2}} - \frac{Bc+Ac\cosh(x)+(Ab-aB)\sinh}{\left(a^2-b^2+c^2\right)(a+b\cosh(x)+c\sinh}$$

Mathematica [A] time = 0.28, size = 125, normalized size = 1.16

$$\frac{\sinh(x)\left(A\left(b^2-c^2\right)-abB\right)-aAc+bBc}{b\left(-a^2+b^2-c^2\right)(a+b\cosh(x)+c\sinh(x))} - \frac{2(aA-bB)\tan^{-1}\left(\frac{(b-a)\tanh\left(\frac{x}{2}\right)+c}{\sqrt{-a^2+b^2-c^2}}\right)}{\left(-a^2+b^2-c^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cosh[x])/(a + b*Cosh[x] + c*Sinh[x])^2,x]
```

```
[Out] (-2*(a*A - b*B)*ArcTan[(c + (-a + b)*Tanh[x/2])/Sqrt[-a^2 + b^2 - c^2]])/(-
a^2 + b^2 - c^2)^(3/2) + (-(a*A*c) + b*B*c + (-(a*b*B) + A*(b^2 - c^2))*Sin
h[x])/(b*(-a^2 + b^2 - c^2)*(a + b*Cosh[x] + c*Sinh[x]))
```

fricas [B] time = 0.48, size = 2228, normalized size = 20.63

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(a+b*cosh(x)+c*sinh(x))^2,x, algorithm="fricas")
```

```
[Out] [-(2*B*a^3*b - 2*A*a^2*b^2 - 2*B*a*b^3 + 2*A*b^4 + 2*A*c^4 + 2*(A*a^2 + B*a
*b - 2*A*b^2)*c^2 + (A*a*b^2 - B*b^3 - (A*a - B*b)*c^2 + (A*a*b^2 - B*b^3 +
 (A*a - B*b)*c^2 + 2*(A*a*b - B*b^2)*c)*cosh(x)^2 + (A*a*b^2 - B*b^3 + (A*a
 - B*b)*c^2 + 2*(A*a*b - B*b^2)*c)*sinh(x)^2 + 2*(A*a^2*b - B*a*b^2 + (A*a^
2 - B*a*b)*c)*cosh(x) + 2*(A*a^2*b - B*a*b^2 + (A*a^2 - B*a*b)*c + (A*a*b^2
 - B*b^3 + (A*a - B*b)*c^2 + 2*(A*a*b - B*b^2)*c)*cosh(x))*sinh(x))*sqrt(a^
2 - b^2 + c^2)*log(((b^2 + 2*b*c + c^2)*cosh(x)^2 + (b^2 + 2*b*c + c^2)*sin
h(x)^2 + 2*a^2 - b^2 + c^2 + 2*(a*b + a*c)*cosh(x) + 2*(a*b + a*c + (b^2 +
2*b*c + c^2)*cosh(x))*sinh(x) + 2*sqrt(a^2 - b^2 + c^2)*((b + c)*cosh(x) +
(b + c)*sinh(x) + a))/((b + c)*cosh(x)^2 + (b + c)*sinh(x)^2 + 2*a*cosh(x)
+ 2*((b + c)*cosh(x) + a)*sinh(x) + b - c)) + 2*(B*a^4 - A*a^3*b - B*a^2*b^
2 + A*a*b^3 + B*c^4 - (A*a - B*b)*c^3 + (2*B*a^2 - A*a*b - B*b^2)*c^2 - (A*
a^3 - B*a^2*b - A*a*b^2 + B*b^3)*c)*cosh(x) + 2*(B*a^4 - A*a^3*b - B*a^2*b^
2 + A*a*b^3 + B*c^4 - (A*a - B*b)*c^3 + (2*B*a^2 - A*a*b - B*b^2)*c^2 - (A*
a^3 - B*a^2*b - A*a*b^2 + B*b^3)*c)*sinh(x))/(a^4*b^2 - 2*a^2*b^4 + b^6 - c
^6 - (2*a^2 - 3*b^2)*c^4 - (a^4 - 4*a^2*b^2 + 3*b^4)*c^2 + (a^4*b^2 - 2*a^2
*b^4 + b^6 + 2*b*c^5 + c^6 + (2*a^2 - b^2)*c^4 + 4*(a^2*b - b^3)*c^3 + (a^4
 - b^4)*c^2 + 2*(a^4*b - 2*a^2*b^3 + b^5)*c)*cosh(x)^2 + (a^4*b^2 - 2*a^2*b
^4 + b^6 + 2*b*c^5 + c^6 + (2*a^2 - b^2)*c^4 + 4*(a^2*b - b^3)*c^3 + (a^4 -
 b^4)*c^2 + 2*(a^4*b - 2*a^2*b^3 + b^5)*c)*sinh(x)^2 + 2*(a^5*b - 2*a^3*b^3
 + a*b^5 + a*b*c^4 + a*c^5 + 2*(a^3 - a*b^2)*c^3 + 2*(a^3*b - a*b^3)*c^2 +
(a^5 - 2*a^3*b^2 + a*b^4)*c)*cosh(x) + 2*(a^5*b - 2*a^3*b^3 + a*b^5 + a*b*c
^4 + a*c^5 + 2*(a^3 - a*b^2)*c^3 + 2*(a^3*b - a*b^3)*c^2 + (a^5 - 2*a^3*b^2
 + a*b^4)*c + (a^4*b^2 - 2*a^2*b^4 + b^6 + 2*b*c^5 + c^6 + (2*a^2 - b^2)*c^
4 + 4*(a^2*b - b^3)*c^3 + (a^4 - b^4)*c^2 + 2*(a^4*b - 2*a^2*b^3 + b^5)*c)*
cosh(x))*sinh(x)), -2*(B*a^3*b - A*a^2*b^2 - B*a*b^3 + A*b^4 + A*c^4 + (A*a
^2 + B*a*b - 2*A*b^2)*c^2 - (A*a*b^2 - B*b^3 - (A*a - B*b)*c^2 + (A*a*b^2 -
 B*b^3 + (A*a - B*b)*c^2 + 2*(A*a*b - B*b^2)*c)*cosh(x)^2 + (A*a*b^2 - B*b^
3 + (A*a - B*b)*c^2 + 2*(A*a*b - B*b^2)*c)*sinh(x)^2 + 2*(A*a^2*b - B*a*b^2
 + (A*a^2 - B*a*b)*c)*cosh(x) + 2*(A*a^2*b - B*a*b^2 + (A*a^2 - B*a*b)*c +
(A*a*b^2 - B*b^3 + (A*a - B*b)*c^2 + 2*(A*a*b - B*b^2)*c)*cosh(x))*sinh(x))
*sqrt(-a^2 + b^2 - c^2)*arctan(sqrt(-a^2 + b^2 - c^2)*((b + c)*cosh(x) + (b
 + c)*sinh(x) + a)/(a^2 - b^2 + c^2)) + (B*a^4 - A*a^3*b - B*a^2*b^2 + A*a*
b^3 + B*c^4 - (A*a - B*b)*c^3 + (2*B*a^2 - A*a*b - B*b^2)*c^2 - (A*a^3 - B*
a^2*b - A*a*b^2 + B*b^3)*c)*cosh(x) + (B*a^4 - A*a^3*b - B*a^2*b^2 + A*a*b^
3 + B*c^4 - (A*a - B*b)*c^3 + (2*B*a^2 - A*a*b - B*b^2)*c^2 - (A*a^3 - B*a^
2*b - A*a*b^2 + B*b^3)*c)*sinh(x))/(a^4*b^2 - 2*a^2*b^4 + b^6 - c^6 - (2*a^
```

```
2 - 3*b^2)*c^4 - (a^4 - 4*a^2*b^2 + 3*b^4)*c^2 + (a^4*b^2 - 2*a^2*b^4 + b^6
 + 2*b*c^5 + c^6 + (2*a^2 - b^2)*c^4 + 4*(a^2*b - b^3)*c^3 + (a^4 - b^4)*c^
2 + 2*(a^4*b - 2*a^2*b^3 + b^5)*c)*cosh(x)^2 + (a^4*b^2 - 2*a^2*b^4 + b^6 +
 2*b*c^5 + c^6 + (2*a^2 - b^2)*c^4 + 4*(a^2*b - b^3)*c^3 + (a^4 - b^4)*c^2
+ 2*(a^4*b - 2*a^2*b^3 + b^5)*c)*sinh(x)^2 + 2*(a^5*b - 2*a^3*b^3 + a*b^5 +
 a*b*c^4 + a*c^5 + 2*(a^3 - a*b^2)*c^3 + 2*(a^3*b - a*b^3)*c^2 + (a^5 - 2*a
^3*b^2 + a*b^4)*c)*cosh(x) + 2*(a^5*b - 2*a^3*b^3 + a*b^5 + a*b*c^4 + a*c^5
 + 2*(a^3 - a*b^2)*c^3 + 2*(a^3*b - a*b^3)*c^2 + (a^5 - 2*a^3*b^2 + a*b^4)*
c + (a^4*b^2 - 2*a^2*b^4 + b^6 + 2*b*c^5 + c^6 + (2*a^2 - b^2)*c^4 + 4*(a^2
*b - b^3)*c^3 + (a^4 - b^4)*c^2 + 2*(a^4*b - 2*a^2*b^3 + b^5)*c)*cosh(x))*s
inh(x))]
```

giac [A] time = 0.12, size = 177, normalized size = 1.64

$$\frac{2\,(Aa-Bb)\arctan\left(\frac{be^{x}+ce^{x}+a}{\sqrt{-a^{2}+b^{2}-c^{2}}}\right)}{\left(a^{2}-b^{2}+c^{2}\right)\sqrt{-a^{2}+b^{2}-c^{2}}}-\frac{2\left(Ba^{2}e^{x}-Aabe^{x}-Aace^{x}+Bbce^{x}+Bc^{2}e^{x}+Bab-Ab^{2}+Ac^{2}\right)}{\left(a^{2}b-b^{3}+a^{2}c-b^{2}c+bc^{2}+c^{3}\right)\left(be^{(2\,x)}+ce^{(2\,x)}+2\,ae^{x}+b-c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(a+b*cosh(x)+c*sinh(x))^2,x, algorithm="giac")
```

```
[Out] 2*(A*a - B*b)*arctan((b*e^x + c*e^x + a)/sqrt(-a^2 + b^2 - c^2))/((a^2 - b^
2 + c^2)*sqrt(-a^2 + b^2 - c^2)) - 2*(B*a^2*e^x - A*a*b*e^x - A*a*c*e^x + B
*b*c*e^x + B*c^2*e^x + B*a*b - A*b^2 + A*c^2)/((a^2*b - b^3 + a^2*c - b^2*c
 + b*c^2 + c^3)*(b*e^(2*x) + c*e^(2*x) + 2*a*e^x + b - c))
```

maple [B] time = 0.26, size = 287, normalized size = 2.66

$$-\frac{2\left(-\frac{\left(Aab-A\,b^{2}+A\,c^{2}-B\,a^{2}+bBa-B\,c^{2}\right)\tanh\left(\frac{x}{2}\right)}{a^{3}-a^{2}b-a\,b^{2}+a\,c^{2}+b^{3}-b\,c^{2}}-\frac{(Aa-Bb)c}{a^{3}-a^{2}b-a\,b^{2}+a\,c^{2}+b^{3}-b\,c^{2}}\right)}{a\left(\tanh^{2}\left(\frac{x}{2}\right)\right)-\left(\tanh^{2}\left(\frac{x}{2}\right)\right)b-2c\tanh\left(\frac{x}{2}\right)-a-b}-\frac{2\arctan\left(\frac{2(a-b)\tanh\left(\frac{x}{2}\right)-2c}{2\sqrt{-a^{2}+b^{2}-c^{2}}}\right)Aa}{\left(a^{2}-b^{2}+c^{2}\right)\sqrt{-a^{2}+b^{2}-c^{2}}}+\frac{2\arctan\left(\frac{2(a-b)}{2\sqrt{-}}\right.}{\left(a^{2}-b^{2}+c^{2}\right)\sqrt{}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cosh(x))/(a+b*cosh(x)+c*sinh(x))^2,x)
```

```
[Out] -2*(-(A*a*b-A*b^2+A*c^2-B*a^2+B*a*b-B*c^2)/(a^3-a^2*b-a*b^2+a*c^2+b^3-b*c^2
)*tanh(1/2*x)-(A*a-B*b)*c/(a^3-a^2*b-a*b^2+a*c^2+b^3-b*c^2))/(a*tanh(1/2*x)
^2-tanh(1/2*x)^2*b-2*c*tanh(1/2*x)-a-b)-2/(a^2-b^2+c^2)/(-a^2+b^2-c^2)^(1/2
)*arctan(1/2*(2*(a-b)*tanh(1/2*x)-2*c)/(-a^2+b^2-c^2)^(1/2))*A*a+2/(a^2-b^2
+c^2)/(-a^2+b^2-c^2)^(1/2)*arctan(1/2*(2*(a-b)*tanh(1/2*x)-2*c)/(-a^2+b^2-c
^2)^(1/2))*B*b
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(a+b*cosh(x)+c*sinh(x))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(c^2-b^2+a^2>0)', see `assume?` for
more details)Is c^2-b^2+a^2 positive or negative?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A+B\cosh(x)}{(a+b\cosh(x)+c\sinh(x))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cosh(x))/(a + b*cosh(x) + c*sinh(x))^2,x)
```

```
[Out] int((A + B*cosh(x))/(a + b*cosh(x) + c*sinh(x))^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(a+b*cosh(x)+c*sinh(x))**2,x)
```

```
[Out] Timed out
```

3.794 $\int \frac{A+B\cosh(x)}{(a+b\cosh(x)+c\sinh(x))^3}\,dx$

Optimal. Leaf size=194

$$-\frac{\left(2a^2A-3abB+A\left(b^2-c^2\right)\right)\tanh^{-1}\left(\frac{c-(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2+c^2}}\right)}{\left(a^2-b^2+c^2\right)^{5/2}}-\frac{\sinh(x)\left(a^2(-B)+3aAb-2b^2B\right)+c\cosh(x)(3aA-2bB)}{2\left(a^2-b^2+c^2\right)^2(a+b\cosh(x)+c\sinh(x))}$$

```
[Out] -(2*a^2*A-3*a*b*B+A*(b^2-c^2))*arctanh((c-(a-b)*tanh(1/2*x))/(a^2-b^2+c^2)^
(1/2))/(a^2-b^2+c^2)^(5/2)+1/2*(-B*c-A*c*cosh(x)-(A*b-B*a)*sinh(x))/(a^2-b^
2+c^2)/(a+b*cosh(x)+c*sinh(x))^2+1/2*(-a*B*c-(3*A*a-2*B*b)*c*cosh(x)-(3*A*a
*b-B*a^2-2*B*b^2)*sinh(x))/(a^2-b^2+c^2)^2/(a+b*cosh(x)+c*sinh(x))
```

Rubi [A] time = 0.28, antiderivative size = 194, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {3158, 3153, 3124, 618, 206}

$$-\frac{\left(2a^2A-3abB+A\left(b^2-c^2\right)\right)\tanh^{-1}\left(\frac{c-(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2+c^2}}\right)}{\left(a^2-b^2+c^2\right)^{5/2}}-\frac{\sinh(x)\left(a^2(-B)+3aAb-2b^2B\right)+c\cosh(x)(3aA-2bB)}{2\left(a^2-b^2+c^2\right)^2(a+b\cosh(x)+c\sinh(x))}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cosh[x])/(a + b*Cosh[x] + c*Sinh[x])^3,x]
```

```
[Out] -(((2*a^2*A - 3*a*b*B + A*(b^2 - c^2))*ArcTanh[(c - (a - b)*Tanh[x/2])/Sqrt
[a^2 - b^2 + c^2]])/(a^2 - b^2 + c^2)^(5/2)) - (B*c + A*c*Cosh[x] + (A*b -
a*B)*Sinh[x])/(2*(a^2 - b^2 + c^2)*(a + b*Cosh[x] + c*Sinh[x])^2) - (a*B*c
+ (3*a*A - 2*b*B)*c*Cosh[x] + (3*a*A*b - a^2*B - 2*b^2*B)*Sinh[x])/(2*(a^2
- b^2 + c^2)^2*(a + b*Cosh[x] + c*Sinh[x]))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 3124

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Module[{f = FreeFactors[Tan[(d + e*x)/2], x]}, Dist[(2*f
)/e, Subst[Int[1/(a + b + 2*c*f*x + (a - b)*f^2*x^2), x], x, Tan[(d + e*x)/
2]/f], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
```

Rule 3153

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])^2,
 x_Symbol] :> Simp[(c*B - b*C - (a*C - c*A)*Cos[d + e*x] + (a*B - b*A)*Sin[
d + e*x])/(e*(a^2 - b^2 - c^2)*(a + b*Cos[d + e*x] + c*Sin[d + e*x])), x] +
 Dist[(a*A - b*B - c*C)/(a^2 - b^2 - c^2), Int[1/(a + b*Cos[d + e*x] + c*Si
n[d + e*x]), x], x] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && NeQ[a^2 - b^2
- c^2, 0] && NeQ[a*A - b*B - c*C, 0]
```

Rule 3158

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.))*((a_.) + cos[(d_.) + (e_.)*(x_)
]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])^(n_), x_Symbol] :> -Simp[((c*B + c
*A*Cos[d + e*x] + (a*B - b*A)*Sin[d + e*x])*(a + b*Cos[d + e*x] + c*Sin[d +
 e*x])^(n + 1))/(e*(n + 1)*(a^2 - b^2 - c^2)), x] + Dist[1/((n + 1)*(a^2 -
b^2 - c^2)), Int[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1)*Simp[(n + 1)
*(a*A - b*B) + (n + 2)*(a*B - b*A)*Cos[d + e*x] - (n + 2)*c*A*Sin[d + e*x],
 x], x], x] /; FreeQ[{a, b, c, d, e, A, B}, x] && LtQ[n, -1] && NeQ[a^2 - b
^2 - c^2, 0] && NeQ[n, -2]
```

Rubi steps

$$\int \frac{A+B\cosh(x)}{(a+b\cosh(x)+c\sinh(x))^3}\,dx = -\frac{Bc+Ac\cosh(x)+(Ab-aB)\sinh(x)}{2\left(a^2-b^2+c^2\right)(a+b\cosh(x)+c\sinh(x))^2} - \frac{\int \frac{-2(aA-bB)+(Ab-aB)\cosh(x)+A}{(a+b\cosh(x)+c\sinh(x))}}{2\left(a^2-b^2+c^2\right)}$$

$$= -\frac{Bc+Ac\cosh(x)+(Ab-aB)\sinh(x)}{2\left(a^2-b^2+c^2\right)(a+b\cosh(x)+c\sinh(x))^2} - \frac{aBc+(3aA-2bB)c\cosh(x}{2\left(a^2-b^2+c^2\right)^2(a}$$

$$= -\frac{Bc+Ac\cosh(x)+(Ab-aB)\sinh(x)}{2\left(a^2-b^2+c^2\right)(a+b\cosh(x)+c\sinh(x))^2} - \frac{aBc+(3aA-2bB)c\cosh(x}{2\left(a^2-b^2+c^2\right)^2(a}$$

$$= -\frac{Bc+Ac\cosh(x)+(Ab-aB)\sinh(x)}{2\left(a^2-b^2+c^2\right)(a+b\cosh(x)+c\sinh(x))^2} - \frac{aBc+(3aA-2bB)c\cosh(x}{2\left(a^2-b^2+c^2\right)^2(a}$$

$$= -\frac{\left(2a^2A-3abB+A\left(b^2-c^2\right)\right)\tanh^{-1}\left(\frac{c-(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2+c^2}}\right)}{\left(a^2-b^2+c^2\right)^{5/2}} - \frac{Bc+Ac\cosh(x}{2\left(a^2-b^2+c^2\right)(a}$$

Mathematica [A] time = 0.73, size = 336, normalized size = 1.73

$$\frac{\left(2a^2A-3abB+A\left(b^2-c^2\right)\right)\tan^{-1}\left(\frac{(b-a)\tanh\left(\frac{x}{2}\right)+c}{\sqrt{-a^2+b^2-c^2}}\right)}{\left(-a^2+b^2-c^2\right)^{5/2}} + \frac{6a^3Ac+4a^3bB\sinh(x)+2bc\cosh(x)\left(2a^2A-3abB+A\left(b^2\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cosh[x])/(a + b*Cosh[x] + c*Sinh[x])^3,x]
```

```
[Out] ((2*a^2*A - 3*a*b*B + A*(b^2 - c^2))*ArcTan[(c + (-a + b)*Tanh[x/2])/Sqrt[-
a^2 + b^2 - c^2]])/(-a^2 + b^2 - c^2)^(5/2) + (6*a^3*A*c + 3*a*A*b^2*c - 9*
a^2*b*B*c - 3*a*A*c^3 + 2*b*c*(2*a^2*A - 3*a*b*B + A*(b^2 - c^2))*Cosh[x] +
 c*(a^2*b*B + 2*b*B*(b^2 - c^2) + 3*a*A*(-b^2 + c^2))*Cosh[2*x] - 8*a^2*A*b
^2*Sinh[x] + 2*A*b^4*Sinh[x] + 4*a^3*b*B*Sinh[x] + 2*a*b^3*B*Sinh[x] + 12*a
^2*A*c^2*Sinh[x] - 2*A*b^2*c^2*Sinh[x] - 8*a*b*B*c^2*Sinh[x] - 3*a*A*b^3*Si
nh[2*x] + a^2*b^2*B*Sinh[2*x] + 2*b^4*B*Sinh[2*x] + 3*a*A*b*c^2*Sinh[2*x] -
 2*b^2*B*c^2*Sinh[2*x])/(4*b*(a^2 - b^2 + c^2)^2*(a + b*Cosh[x] + c*Sinh[x]
)^2)
```

fricas [B] time = 0.70, size = 12366, normalized size = 63.74

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(a+b*cosh(x)+c*sinh(x))^3,x, algorithm="fricas")
```

```
[Out] [-1/2*(2*B*a^4*b^2 - 6*A*a^3*b^3 + 2*B*a^2*b^4 + 6*A*a*b^5 - 4*B*b^6 - 2*(3
*A*a - 2*B*b)*c^5 + 2*(3*A*a*b - 2*B*b^2)*c^4 - 2*(3*A*a^3 - B*a^2*b - 6*A*
a*b^2 + 4*B*b^3)*c^3 - 2*(2*A*a^4*b^2 - 3*B*a^3*b^3 - A*a^2*b^4 + 3*B*a*b^5
 - A*b^6 - 2*A*b*c^5 - A*c^6 + (A*a^2 - 3*B*a*b + A*b^2)*c^4 + 2*(A*a^2*b -
 3*B*a*b^2 + 2*A*b^3)*c^3 + (2*A*a^4 - 3*B*a^3*b + A*b^4)*c^2 + 2*(2*A*a^4*
b - 3*B*a^3*b^2 - A*a^2*b^3 + 3*B*a*b^4 - A*b^5)*c)*cosh(x)^3 - 2*(2*A*a^4*
b^2 - 3*B*a^3*b^3 - A*a^2*b^4 + 3*B*a*b^5 - A*b^6 - 2*A*b*c^5 - A*c^6 + (A*
a^2 - 3*B*a*b + A*b^2)*c^4 + 2*(A*a^2*b - 3*B*a*b^2 + 2*A*b^3)*c^3 + (2*A*a
^4 - 3*B*a^3*b + A*b^4)*c^2 + 2*(2*A*a^4*b - 3*B*a^3*b^2 - A*a^2*b^3 + 3*B*
a*b^4 - A*b^5)*c)*sinh(x)^3 + 2*(3*A*a^3*b - B*a^2*b^2 - 6*A*a*b^3 + 4*B*b^
4)*c^2 + 2*(2*B*a^6 - 6*A*a^5*b + 3*B*a^4*b^2 + 3*A*a^3*b^3 - 3*B*a^2*b^4 +
 3*A*a*b^5 - 2*B*b^6 + 3*A*a*c^5 + 2*B*c^6 + 3*(2*B*a^2 + A*a*b - 2*B*b^2)*
c^4 - 3*(A*a^3 - 3*B*a^2*b + 2*A*a*b^2)*c^3 + 3*(2*B*a^4 - A*a^3*b - B*a^2*
b^2 - 2*A*a*b^3 + 2*B*b^4)*c^2 - 3*(2*A*a^5 - 3*B*a^4*b - A*a^3*b^2 + 3*B*a
^2*b^3 - A*a*b^4)*c)*cosh(x)^2 + 2*(2*B*a^6 - 6*A*a^5*b + 3*B*a^4*b^2 + 3*A
*a^3*b^3 - 3*B*a^2*b^4 + 3*A*a*b^5 - 2*B*b^6 + 3*A*a*c^5 + 2*B*c^6 + 3*(2*B
*a^2 + A*a*b - 2*B*b^2)*c^4 - 3*(A*a^3 - 3*B*a^2*b + 2*A*a*b^2)*c^3 + 3*(2*
B*a^4 - A*a^3*b - B*a^2*b^2 - 2*A*a*b^3 + 2*B*b^4)*c^2 - 3*(2*A*a^5 - 3*B*a
^4*b - A*a^3*b^2 + 3*B*a^2*b^3 - A*a*b^4)*c - 3*(2*A*a^4*b^2 - 3*B*a^3*b^3
- A*a^2*b^4 + 3*B*a*b^5 - A*b^6 - 2*A*b*c^5 - A*c^6 + (A*a^2 - 3*B*a*b + A*
b^2)*c^4 + 2*(A*a^2*b - 3*B*a*b^2 + 2*A*b^3)*c^3 + (2*A*a^4 - 3*B*a^3*b + A
*b^4)*c^2 + 2*(2*A*a^4*b - 3*B*a^3*b^2 - A*a^2*b^3 + 3*B*a*b^4 - A*b^5)*c)*
cosh(x))*sinh(x)^2 + (2*A*a^2*b^3 - 3*B*a*b^4 + A*b^5 + A*b*c^4 - A*c^5 + (
2*A*a^2*b^3 - 3*B*a*b^4 + A*b^5 - 3*A*b*c^4 - A*c^5 + (2*A*a^2 - 3*B*a*b -
2*A*b^2)*c^3 + (6*A*a^2*b - 9*B*a*b^2 + 2*A*b^3)*c^2 + 3*(2*A*a^2*b^2 - 3*B
*a*b^3 + A*b^4)*c)*cosh(x)^4 + (2*A*a^2*b^3 - 3*B*a*b^4 + A*b^5 - 3*A*b*c^4
 - A*c^5 + (2*A*a^2 - 3*B*a*b - 2*A*b^2)*c^3 + (6*A*a^2*b - 9*B*a*b^2 + 2*A
*b^3)*c^2 + 3*(2*A*a^2*b^2 - 3*B*a*b^3 + A*b^4)*c)*sinh(x)^4 + (2*A*a^2 - 3
*B*a*b + 2*A*b^2)*c^3 + 4*(2*A*a^3*b^2 - 3*B*a^2*b^3 + A*a*b^4 - 2*A*a*b*c^
3 - A*a*c^4 + (2*A*a^3 - 3*B*a^2*b)*c^2 + 2*(2*A*a^3*b - 3*B*a^2*b^2 + A*a*
b^3)*c)*cosh(x)^3 + 4*(2*A*a^3*b^2 - 3*B*a^2*b^3 + A*a*b^4 - 2*A*a*b*c^3 -
A*a*c^4 + (2*A*a^3 - 3*B*a^2*b)*c^2 + 2*(2*A*a^3*b - 3*B*a^2*b^2 + A*a*b^3)
*c + (2*A*a^2*b^3 - 3*B*a*b^4 + A*b^5 - 3*A*b*c^4 - A*c^5 + (2*A*a^2 - 3*B*
a*b - 2*A*b^2)*c^3 + (6*A*a^2*b - 9*B*a*b^2 + 2*A*b^3)*c^2 + 3*(2*A*a^2*b^2
 - 3*B*a*b^3 + A*b^4)*c)*cosh(x))*sinh(x)^3 - (2*A*a^2*b - 3*B*a*b^2 + 2*A*
b^3)*c^2 + 2*(4*A*a^4*b - 6*B*a^3*b^2 + 4*A*a^2*b^3 - 3*B*a*b^4 + A*b^5 + A
*b*c^4 + A*c^5 - (4*A*a^2 - 3*B*a*b + 2*A*b^2)*c^3 - (4*A*a^2*b - 3*B*a*b^2
 + 2*A*b^3)*c^2 + (4*A*a^4 - 6*B*a^3*b + 4*A*a^2*b^2 - 3*B*a*b^3 + A*b^4)*c
)*cosh(x)^2 + 2*(4*A*a^4*b - 6*B*a^3*b^2 + 4*A*a^2*b^3 - 3*B*a*b^4 + A*b^5
+ A*b*c^4 + A*c^5 - (4*A*a^2 - 3*B*a*b + 2*A*b^2)*c^3 - (4*A*a^2*b - 3*B*a*
b^2 + 2*A*b^3)*c^2 + 3*(2*A*a^2*b^3 - 3*B*a*b^4 + A*b^5 - 3*A*b*c^4 - A*c^5
```

```
 + (2*A*a^2 - 3*B*a*b - 2*A*b^2)*c^3 + (6*A*a^2*b - 9*B*a*b^2 + 2*A*b^3)*c^
2 + 3*(2*A*a^2*b^2 - 3*B*a*b^3 + A*b^4)*c)*cosh(x)^2 + (4*A*a^4 - 6*B*a^3*b
 + 4*A*a^2*b^2 - 3*B*a*b^3 + A*b^4)*c + 6*(2*A*a^3*b^2 - 3*B*a^2*b^3 + A*a*
b^4 - 2*A*a*b*c^3 - A*a*c^4 + (2*A*a^3 - 3*B*a^2*b)*c^2 + 2*(2*A*a^3*b - 3*
B*a^2*b^2 + A*a*b^3)*c)*cosh(x))*sinh(x)^2 - (2*A*a^2*b^2 - 3*B*a*b^3 + A*b
^4)*c + 4*(2*A*a^3*b^2 - 3*B*a^2*b^3 + A*a*b^4 + A*a*c^4 - (2*A*a^3 - 3*B*a
^2*b + 2*A*a*b^2)*c^2)*cosh(x) + 4*(2*A*a^3*b^2 - 3*B*a^2*b^3 + A*a*b^4 + A
*a*c^4 + (2*A*a^2*b^3 - 3*B*a*b^4 + A*b^5 - 3*A*b*c^4 - A*c^5 + (2*A*a^2 -
3*B*a*b - 2*A*b^2)*c^3 + (6*A*a^2*b - 9*B*a*b^2 + 2*A*b^3)*c^2 + 3*(2*A*a^2
*b^2 - 3*B*a*b^3 + A*b^4)*c)*cosh(x)^3 - (2*A*a^3 - 3*B*a^2*b + 2*A*a*b^2)*
c^2 + 3*(2*A*a^3*b^2 - 3*B*a^2*b^3 + A*a*b^4 - 2*A*a*b*c^3 - A*a*c^4 + (2*A
*a^3 - 3*B*a^2*b)*c^2 + 2*(2*A*a^3*b - 3*B*a^2*b^2 + A*a*b^3)*c)*cosh(x)^2
+ (4*A*a^4*b - 6*B*a^3*b^2 + 4*A*a^2*b^3 - 3*B*a*b^4 + A*b^5 + A*b*c^4 + A*
c^5 - (4*A*a^2 - 3*B*a*b + 2*A*b^2)*c^3 - (4*A*a^2*b - 3*B*a*b^2 + 2*A*b^3)
*c^2 + (4*A*a^4 - 6*B*a^3*b + 4*A*a^2*b^2 - 3*B*a*b^3 + A*b^4)*c)*cosh(x))*
sinh(x))*sqrt(a^2 - b^2 + c^2)*log(((b^2 + 2*b*c + c^2)*cosh(x)^2 + (b^2 +
2*b*c + c^2)*sinh(x)^2 + 2*a^2 - b^2 + c^2 + 2*(a*b + a*c)*cosh(x) + 2*(a*b
 + a*c + (b^2 + 2*b*c + c^2)*cosh(x))*sinh(x) + 2*sqrt(a^2 - b^2 + c^2)*((b
 + c)*cosh(x) + (b + c)*sinh(x) + a))/((b + c)*cosh(x)^2 + (b + c)*sinh(x)^
2 + 2*a*cosh(x) + 2*((b + c)*cosh(x) + a)*sinh(x) + b - c)) - 2*(B*a^4*b -
3*A*a^3*b^2 + B*a^2*b^3 + 3*A*a*b^4 - 2*B*b^5)*c + 2*(4*B*a^5*b - 10*A*a^4*
b^2 + B*a^3*b^3 + 11*A*a^2*b^4 - 5*B*a*b^5 - A*b^6 + A*c^6 + (11*A*a^2 - 5*
B*a*b - 3*A*b^2)*c^4 + (10*A*a^4 - B*a^3*b - 22*A*a^2*b^2 + 10*B*a*b^3 + 3*
A*b^4)*c^2)*cosh(x) + 2*(4*B*a^5*b - 10*A*a^4*b^2 + B*a^3*b^3 + 11*A*a^2*b^
4 - 5*B*a*b^5 - A*b^6 + A*c^6 + (11*A*a^2 - 5*B*a*b - 3*A*b^2)*c^4 + (10*A*
a^4 - B*a^3*b - 22*A*a^2*b^2 + 10*B*a*b^3 + 3*A*b^4)*c^2 - 3*(2*A*a^4*b^2 -
 3*B*a^3*b^3 - A*a^2*b^4 + 3*B*a*b^5 - A*b^6 - 2*A*b*c^5 - A*c^6 + (A*a^2 -
 3*B*a*b + A*b^2)*c^4 + 2*(A*a^2*b - 3*B*a*b^2 + 2*A*b^3)*c^3 + (2*A*a^4 -
3*B*a^3*b + A*b^4)*c^2 + 2*(2*A*a^4*b - 3*B*a^3*b^2 - A*a^2*b^3 + 3*B*a*b^4
 - A*b^5)*c)*cosh(x)^2 + 2*(2*B*a^6 - 6*A*a^5*b + 3*B*a^4*b^2 + 3*A*a^3*b^3
 - 3*B*a^2*b^4 + 3*A*a*b^5 - 2*B*b^6 + 3*A*a*c^5 + 2*B*c^6 + 3*(2*B*a^2 + A
*a*b - 2*B*b^2)*c^4 - 3*(A*a^3 - 3*B*a^2*b + 2*A*a*b^2)*c^3 + 3*(2*B*a^4 -
A*a^3*b - B*a^2*b^2 - 2*A*a*b^3 + 2*B*b^4)*c^2 - 3*(2*A*a^5 - 3*B*a^4*b - A
*a^3*b^2 + 3*B*a^2*b^3 - A*a*b^4)*c)*cosh(x))*sinh(x))/(a^6*b^3 - 3*a^4*b^5
 + 3*a^2*b^7 - b^9 - b*c^8 + c^9 + (3*a^2 - 4*b^2)*c^7 - (3*a^2*b - 4*b^3)*
c^6 + 3*(a^4 - 3*a^2*b^2 + 2*b^4)*c^5 - 3*(a^4*b - 3*a^2*b^3 + 2*b^5)*c^4 +
 (a^6*b^3 - 3*a^4*b^5 + 3*a^2*b^7 - b^9 + 3*a^2*c^7 + 3*b*c^8 + c^9 + (9*a^
2*b - 8*b^3)*c^6 + 3*(a^4 + a^2*b^2 - 2*b^4)*c^5 + 3*(3*a^4*b - 5*a^2*b^3 +
 2*b^5)*c^4 + (a^6 + 6*a^4*b^2 - 15*a^2*b^4 + 8*b^6)*c^3 + 3*(a^6*b - 2*a^4
*b^3 + a^2*b^5)*c^2 + 3*(a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^8)*c)*cosh(x)^
4 + (a^6*b^3 - 3*a^4*b^5 + 3*a^2*b^7 - b^9 + 3*a^2*c^7 + 3*b*c^8 + c^9 + (9
*a^2*b - 8*b^3)*c^6 + 3*(a^4 + a^2*b^2 - 2*b^4)*c^5 + 3*(3*a^4*b - 5*a^2*b^
3 + 2*b^5)*c^4 + (a^6 + 6*a^4*b^2 - 15*a^2*b^4 + 8*b^6)*c^3 + 3*(a^6*b - 2*
a^4*b^3 + a^2*b^5)*c^2 + 3*(a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^8)*c)*sinh(
x)^4 + (a^6 - 6*a^4*b^2 + 9*a^2*b^4 - 4*b^6)*c^3 + 4*(a^7*b^2 - 3*a^5*b^4 +
```

```
3*a^3*b^6 - a*b^8 + 2*a*b*c^7 + a*c^8 + (3*a^3 - 2*a*b^2)*c^6 + 6*(a^3*b -
 a*b^3)*c^5 + 3*(a^5 - a^3*b^2)*c^4 + 6*(a^5*b - 2*a^3*b^3 + a*b^5)*c^3 + (
a^7 - 3*a^3*b^4 + 2*a*b^6)*c^2 + 2*(a^7*b - 3*a^5*b^3 + 3*a^3*b^5 - a*b^7)*
c)*cosh(x)^3 + 4*(a^7*b^2 - 3*a^5*b^4 + 3*a^3*b^6 - a*b^8 + 2*a*b*c^7 + a*c
^8 + (3*a^3 - 2*a*b^2)*c^6 + 6*(a^3*b - a*b^3)*c^5 + 3*(a^5 - a^3*b^2)*c^4
+ 6*(a^5*b - 2*a^3*b^3 + a*b^5)*c^3 + (a^7 - 3*a^3*b^4 + 2*a*b^6)*c^2 + 2*(
a^7*b - 3*a^5*b^3 + 3*a^3*b^5 - a*b^7)*c + (a^6*b^3 - 3*a^4*b^5 + 3*a^2*b^7
 - b^9 + 3*a^2*c^7 + 3*b*c^8 + c^9 + (9*a^2*b - 8*b^3)*c^6 + 3*(a^4 + a^2*b
^2 - 2*b^4)*c^5 + 3*(3*a^4*b - 5*a^2*b^3 + 2*b^5)*c^4 + (a^6 + 6*a^4*b^2 -
15*a^2*b^4 + 8*b^6)*c^3 + 3*(a^6*b - 2*a^4*b^3 + a^2*b^5)*c^2 + 3*(a^6*b^2
- 3*a^4*b^4 + 3*a^2*b^6 - b^8)*c)*cosh(x))*sinh(x)^3 - (a^6*b - 6*a^4*b^3 +
 9*a^2*b^5 - 4*b^7)*c^2 + 2*(2*a^8*b - 5*a^6*b^3 + 3*a^4*b^5 + a^2*b^7 - b^
9 - b*c^8 - c^9 - (a^2 - 4*b^2)*c^7 - (a^2*b - 4*b^3)*c^6 + 3*(a^4 + a^2*b^
2 - 2*b^4)*c^5 + 3*(a^4*b + a^2*b^3 - 2*b^5)*c^4 + (5*a^6 - 6*a^4*b^2 - 3*a
^2*b^4 + 4*b^6)*c^3 + (5*a^6*b - 6*a^4*b^3 - 3*a^2*b^5 + 4*b^7)*c^2 + (2*a^
8 - 5*a^6*b^2 + 3*a^4*b^4 + a^2*b^6 - b^8)*c)*cosh(x)^2 + 2*(2*a^8*b - 5*a^
6*b^3 + 3*a^4*b^5 + a^2*b^7 - b^9 - b*c^8 - c^9 - (a^2 - 4*b^2)*c^7 - (a^2*
b - 4*b^3)*c^6 + 3*(a^4 + a^2*b^2 - 2*b^4)*c^5 + 3*(a^4*b + a^2*b^3 - 2*b^5
)*c^4 + (5*a^6 - 6*a^4*b^2 - 3*a^2*b^4 + 4*b^6)*c^3 + (5*a^6*b - 6*a^4*b^3
- 3*a^2*b^5 + 4*b^7)*c^2 + 3*(a^6*b^3 - 3*a^4*b^5 + 3*a^2*b^7 - b^9 + 3*a^2
*c^7 + 3*b*c^8 + c^9 + (9*a^2*b - 8*b^3)*c^6 + 3*(a^4 + a^2*b^2 - 2*b^4)*c^
5 + 3*(3*a^4*b - 5*a^2*b^3 + 2*b^5)*c^4 + (a^6 + 6*a^4*b^2 - 15*a^2*b^4 + 8
*b^6)*c^3 + 3*(a^6*b - 2*a^4*b^3 + a^2*b^5)*c^2 + 3*(a^6*b^2 - 3*a^4*b^4 +
3*a^2*b^6 - b^8)*c)*cosh(x)^2 + (2*a^8 - 5*a^6*b^2 + 3*a^4*b^4 + a^2*b^6 -
b^8)*c + 6*(a^7*b^2 - 3*a^5*b^4 + 3*a^3*b^6 - a*b^8 + 2*a*b*c^7 + a*c^8 + (
3*a^3 - 2*a*b^2)*c^6 + 6*(a^3*b - a*b^3)*c^5 + 3*(a^5 - a^3*b^2)*c^4 + 6*(a
^5*b - 2*a^3*b^3 + a*b^5)*c^3 + (a^7 - 3*a^3*b^4 + 2*a*b^6)*c^2 + 2*(a^7*b
- 3*a^5*b^3 + 3*a^3*b^5 - a*b^7)*c)*cosh(x))*sinh(x)^2 - (a^6*b^2 - 3*a^4*b
^4 + 3*a^2*b^6 - b^8)*c + 4*(a^7*b^2 - 3*a^5*b^4 + 3*a^3*b^6 - a*b^8 - a*c^
8 - (3*a^3 - 4*a*b^2)*c^6 - 3*(a^5 - 3*a^3*b^2 + 2*a*b^4)*c^4 - (a^7 - 6*a^
5*b^2 + 9*a^3*b^4 - 4*a*b^6)*c^2)*cosh(x) + 4*(a^7*b^2 - 3*a^5*b^4 + 3*a^3*
b^6 - a*b^8 - a*c^8 - (3*a^3 - 4*a*b^2)*c^6 - 3*(a^5 - 3*a^3*b^2 + 2*a*b^4)
*c^4 + (a^6*b^3 - 3*a^4*b^5 + 3*a^2*b^7 - b^9 + 3*a^2*c^7 + 3*b*c^8 + c^9 +
 (9*a^2*b - 8*b^3)*c^6 + 3*(a^4 + a^2*b^2 - 2*b^4)*c^5 + 3*(3*a^4*b - 5*a^2
*b^3 + 2*b^5)*c^4 + (a^6 + 6*a^4*b^2 - 15*a^2*b^4 + 8*b^6)*c^3 + 3*(a^6*b -
 2*a^4*b^3 + a^2*b^5)*c^2 + 3*(a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^8)*c)*co
sh(x)^3 - (a^7 - 6*a^5*b^2 + 9*a^3*b^4 - 4*a*b^6)*c^2 + 3*(a^7*b^2 - 3*a^5*
b^4 + 3*a^3*b^6 - a*b^8 + 2*a*b*c^7 + a*c^8 + (3*a^3 - 2*a*b^2)*c^6 + 6*(a^
3*b - a*b^3)*c^5 + 3*(a^5 - a^3*b^2)*c^4 + 6*(a^5*b - 2*a^3*b^3 + a*b^5)*c^
3 + (a^7 - 3*a^3*b^4 + 2*a*b^6)*c^2 + 2*(a^7*b - 3*a^5*b^3 + 3*a^3*b^5 - a*
b^7)*c)*cosh(x)^2 + (2*a^8*b - 5*a^6*b^3 + 3*a^4*b^5 + a^2*b^7 - b^9 - b*c^
8 - c^9 - (a^2 - 4*b^2)*c^7 - (a^2*b - 4*b^3)*c^6 + 3*(a^4 + a^2*b^2 - 2*b^
4)*c^5 + 3*(a^4*b + a^2*b^3 - 2*b^5)*c^4 + (5*a^6 - 6*a^4*b^2 - 3*a^2*b^4 +
 4*b^6)*c^3 + (5*a^6*b - 6*a^4*b^3 - 3*a^2*b^5 + 4*b^7)*c^2 + (2*a^8 - 5*a^
6*b^2 + 3*a^4*b^4 + a^2*b^6 - b^8)*c)*cosh(x))*sinh(x)), -(B*a^4*b^2 - 3*A*
```

a^3*b^3 + B*a^2*b^4 + 3*A*a*b^5 - 2*B*b^6 - (3*A*a - 2*B*b)*c^5 + (3*A*a*b - 2*B*b^2)*c^4 - (3*A*a^3 - B*a^2*b - 6*A*a*b^2 + 4*B*b^3)*c^3 - (2*A*a^4*b ^2 - 3*B*a^3*b^3 - A*a^2*b^4 + 3*B*a*b^5 - A*b^6 - 2*A*b*c^5 - A*c^6 + (A*a ^2 - 3*B*a*b + A*b^2)*c^4 + 2*(A*a^2*b - 3*B*a*b^2 + 2*A*b^3)*c^3 + (2*A*a^ 4 - 3*B*a^3*b + A*b^4)*c^2 + 2*(2*A*a^4*b - 3*B*a^3*b^2 - A*a^2*b^3 + 3*B*a *b^4 - A*b^5)*c)*cosh(x)^3 - (2*A*a^4*b^2 - 3*B*a^3*b^3 - A*a^2*b^4 + 3*B*a *b^5 - A*b^6 - 2*A*b*c^5 - A*c^6 + (A*a^2 - 3*B*a*b + A*b^2)*c^4 + 2*(A*a^2 *b - 3*B*a*b^2 + 2*A*b^3)*c^3 + (2*A*a^4 - 3*B*a^3*b + A*b^4)*c^2 + 2*(2*A* a^4*b - 3*B*a^3*b^2 - A*a^2*b^3 + 3*B*a*b^4 - A*b^5)*c)*sinh(x)^3 + (3*A*a^ 3*b - B*a^2*b^2 - 6*A*a*b^3 + 4*B*b^4)*c^2 + (2*B*a^6 - 6*A*a^5*b + 3*B*a^4 *b^2 + 3*A*a^3*b^3 - 3*B*a^2*b^4 + 3*A*a*b^5 - 2*B*b^6 + 3*A*a*c^5 + 2*B*c^ 6 + 3*(2*B*a^2 + A*a*b - 2*B*b^2)*c^4 - 3*(A*a^3 - 3*B*a^2*b + 2*A*a*b^2)*c ^3 + 3*(2*B*a^4 - A*a^3*b - B*a^2*b^2 - 2*A*a*b^3 + 2*B*b^4)*c^2 - 3*(2*A*a ^5 - 3*B*a^4*b - A*a^3*b^2 + 3*B*a^2*b^3 - A*a*b^4)*c)*cosh(x)^2 + (2*B*a^6 - 6*A*a^5*b + 3*B*a^4*b^2 + 3*A*a^3*b^3 - 3*B*a^2*b^4 + 3*A*a*b^5 - 2*B*b^ 6 + 3*A*a*c^5 + 2*B*c^6 + 3*(2*B*a^2 + A*a*b - 2*B*b^2)*c^4 - 3*(A*a^3 - 3* B*a^2*b + 2*A*a*b^2)*c^3 + 3*(2*B*a^4 - A*a^3*b - B*a^2*b^2 - 2*A*a*b^3 + 2 *B*b^4)*c^2 - 3*(2*A*a^5 - 3*B*a^4*b - A*a^3*b^2 + 3*B*a^2*b^3 - A*a*b^4)*c - 3*(2*A*a^4*b^2 - 3*B*a^3*b^3 - A*a^2*b^4 + 3*B*a*b^5 - A*b^6 - 2*A*b*c^5 - A*c^6 + (A*a^2 - 3*B*a*b + A*b^2)*c^4 + 2*(A*a^2*b - 3*B*a*b^2 + 2*A*b^3)*c^3 + (2*A*a^4 - 3*B*a^3*b + A*b^4)*c^2 + 2*(2*A*a^4*b - 3*B*a^3*b^2 - A* a^2*b^3 + 3*B*a*b^4 - A*b^5)*c)*cosh(x))*sinh(x)^2 - (2*A*a^2*b^3 - 3*B*a*b ^4 + A*b^5 + A*b*c^4 - A*c^5 + (2*A*a^2*b^3 - 3*B*a*b^4 + A*b^5 - 3*A*b*c^4 - A*c^5 + (2*A*a^2 - 3*B*a*b - 2*A*b^2)*c^3 + (6*A*a^2*b - 9*B*a*b^2 + 2*A *b^3)*c^2 + 3*(2*A*a^2*b^2 - 3*B*a*b^3 + A*b^4)*c)*cosh(x)^4 + (2*A*a^2*b^3 - 3*B*a*b^4 + A*b^5 - 3*A*b*c^4 - A*c^5 + (2*A*a^2 - 3*B*a*b - 2*A*b^2)*c^ 3 + (6*A*a^2*b - 9*B*a*b^2 + 2*A*b^3)*c^2 + 3*(2*A*a^2*b^2 - 3*B*a*b^3 + A* b^4)*c)*sinh(x)^4 + (2*A*a^2 - 3*B*a*b + 2*A*b^2)*c^3 + 4*(2*A*a^3*b^2 - 3* B*a^2*b^3 + A*a*b^4 - 2*A*a*b*c^3 - A*a*c^4 + (2*A*a^3 - 3*B*a^2*b)*c^2 + 2 *(2*A*a^3*b - 3*B*a^2*b^2 + A*a*b^3)*c)*cosh(x)^3 + 4*(2*A*a^3*b^2 - 3*B*a^ 2*b^3 + A*a*b^4 - 2*A*a*b*c^3 - A*a*c^4 + (2*A*a^3 - 3*B*a^2*b)*c^2 + 2*(2* A*a^3*b - 3*B*a^2*b^2 + A*a*b^3)*c + (2*A*a^2*b^3 - 3*B*a*b^4 + A*b^5 - 3*A *b*c^4 - A*c^5 + (2*A*a^2 - 3*B*a*b - 2*A*b^2)*c^3 + (6*A*a^2*b - 9*B*a*b^2 + 2*A*b^3)*c^2 + 3*(2*A*a^2*b^2 - 3*B*a*b^3 + A*b^4)*c)*cosh(x))*sinh(x)^3 - (2*A*a^2*b - 3*B*a*b^2 + 2*A*b^3)*c^2 + 2*(4*A*a^4*b - 6*B*a^3*b^2 + 4*A *a^2*b^3 - 3*B*a*b^4 + A*b^5 + A*b*c^4 + A*c^5 - (4*A*a^2 - 3*B*a*b + 2*A*b ^2)*c^3 - (4*A*a^2*b - 3*B*a*b^2 + 2*A*b^3)*c^2 + (4*A*a^4 - 6*B*a^3*b + 4* A*a^2*b^2 - 3*B*a*b^3 + A*b^4)*c)*cosh(x)^2 + 2*(4*A*a^4*b - 6*B*a^3*b^2 + 4*A*a^2*b^3 - 3*B*a*b^4 + A*b^5 + A*b*c^4 + A*c^5 - (4*A*a^2 - 3*B*a*b + 2* A*b^2)*c^3 - (4*A*a^2*b - 3*B*a*b^2 + 2*A*b^3)*c^2 + 3*(2*A*a^2*b^3 - 3*B*a *b^4 + A*b^5 - 3*A*b*c^4 - A*c^5 + (2*A*a^2 - 3*B*a*b - 2*A*b^2)*c^3 + (6*A *a^2*b - 9*B*a*b^2 + 2*A*b^3)*c^2 + 3*(2*A*a^2*b^2 - 3*B*a*b^3 + A*b^4)*c)* cosh(x)^2 + (4*A*a^4 - 6*B*a^3*b + 4*A*a^2*b^2 - 3*B*a*b^3 + A*b^4)*c + 6*(2*A*a^3*b^2 - 3*B*a^2*b^3 + A*a*b^4 - 2*A*a*b*c^3 - A*a*c^4 + (2*A*a^3 - 3* B*a^2*b)*c^2 + 2*(2*A*a^3*b - 3*B*a^2*b^2 + A*a*b^3)*c)*cosh(x))*sinh(x)^2

- (2*A*a^2*b^2 - 3*B*a*b^3 + A*b^4)*c + 4*(2*A*a^3*b^2 - 3*B*a^2*b^3 + A*a*b^4 + A*a*c^4 - (2*A*a^3 - 3*B*a^2*b + 2*A*a*b^2)*c^2)*cosh(x) + 4*(2*A*a^3*b^2 - 3*B*a^2*b^3 + A*a*b^4 + A*a*c^4 + (2*A*a^2*b^3 - 3*B*a*b^4 + A*b^5 - 3*A*b*c^4 - A*c^5 + (2*A*a^2 - 3*B*a*b - 2*A*b^2)*c^3 + (6*A*a^2*b - 9*B*a*b^2 + 2*A*b^3)*c^2 + 3*(2*A*a^2*b^2 - 3*B*a*b^3 + A*b^4)*c)*cosh(x)^3 - (2*A*a^3 - 3*B*a^2*b + 2*A*a*b^2)*c^2 + 3*(2*A*a^3*b^2 - 3*B*a^2*b^3 + A*a*b^4 - 2*A*a*b*c^3 - A*a*c^4 + (2*A*a^3 - 3*B*a^2*b)*c^2 + 2*(2*A*a^3*b - 3*B*a^2*b^2 + A*a*b^3)*c)*cosh(x)^2 + (4*A*a^4*b - 6*B*a^3*b^2 + 4*A*a^2*b^3 - 3*B*a*b^4 + A*b^5 + A*b*c^4 + A*c^5 - (4*A*a^2 - 3*B*a*b + 2*A*b^2)*c^3 - (4*A*a^2*b - 3*B*a*b^2 + 2*A*b^3)*c^2 + (4*A*a^4 - 6*B*a^3*b + 4*A*a^2*b^2 - 3*B*a*b^3 + A*b^4)*c)*cosh(x))*sinh(x))*sqrt(-a^2 + b^2 - c^2)*arctan(sqrt(-a^2 + b^2 - c^2)*((b + c)*cosh(x) + (b + c)*sinh(x) + a)/(a^2 - b^2 + c^2)) - (B*a^4*b - 3*A*a^3*b^2 + B*a^2*b^3 + 3*A*a*b^4 - 2*B*b^5)*c + (4*B*a^5*b - 10*A*a^4*b^2 + B*a^3*b^3 + 11*A*a^2*b^4 - 5*B*a*b^5 - A*b^6 + A*c^6 + (11*A*a^2 - 5*B*a*b - 3*A*b^2)*c^4 + (10*A*a^4 - B*a^3*b - 22*A*a^2*b^2 + 10*B*a*b^3 + 3*A*b^4)*c^2)*cosh(x) + (4*B*a^5*b - 10*A*a^4*b^2 + B*a^3*b^3 + 11*A*a^2*b^4 - 5*B*a*b^5 - A*b^6 + A*c^6 + (11*A*a^2 - 5*B*a*b - 3*A*b^2)*c^4 + (10*A*a^4 - B*a^3*b - 22*A*a^2*b^2 + 10*B*a*b^3 + 3*A*b^4)*c^2 - 3*(2*A*a^4*b^2 - 3*B*a^3*b^3 - A*a^2*b^4 + 3*B*a*b^5 - A*b^6 - 2*A*b*c^5 - A*c^6 + (A*a^2 - 3*B*a*b + A*b^2)*c^4 + 2*(A*a^2*b - 3*B*a*b^2 + 2*A*b^3)*c^3 + (2*A*a^4 - 3*B*a^3*b + A*b^4)*c^2 + 2*(2*A*a^4*b - 3*B*a^3*b^2 - A*a^2*b^3 + 3*B*a*b^4 - A*b^5)*c)*cosh(x)^2 + 2*(2*B*a^6 - 6*A*a^5*b + 3*B*a^4*b^2 + 3*A*a^3*b^3 - 3*B*a^2*b^4 + 3*A*a*b^5 - 2*B*b^6 + 3*A*a*c^5 + 2*B*c^6 + 3*(2*B*a^2 + A*a*b - 2*B*b^2)*c^4 - 3*(A*a^3 - 3*B*a^2*b + 2*A*a*b^2)*c^3 + 3*(2*B*a^4 - A*a^3*b - B*a^2*b^2 - 2*A*a*b^3 + 2*B*b^4)*c^2 - 3*(2*A*a^5 - 3*B*a^4*b - A*a^3*b^2 + 3*B*a^2*b^3 - A*a*b^4)*c)*cosh(x))*sinh(x))/(a^6*b^3 - 3*a^4*b^5 + 3*a^2*b^7 - b^9 - b*c^8 + c^9 + (3*a^2 - 4*b^2)*c^7 - (3*a^2*b - 4*b^3)*c^6 + 3*(a^4 - 3*a^2*b^2 + 2*b^4)*c^5 - 3*(a^4*b - 3*a^2*b^3 + 2*b^5)*c^4 + (a^6*b^3 - 3*a^4*b^5 + 3*a^2*b^7 - b^9 + 3*a^2*c^7 + 3*b*c^8 + c^9 + (9*a^2*b - 8*b^3)*c^6 + 3*(a^4 + a^2*b^2 - 2*b^4)*c^5 + 3*(3*a^4*b - 5*a^2*b^3 + 2*b^5)*c^4 + (a^6 + 6*a^4*b^2 - 15*a^2*b^4 + 8*b^6)*c^3 + 3*(a^6*b - 2*a^4*b^3 + a^2*b^5)*c^2 + 3*(a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^8)*c)*cosh(x)^4 + (a^6*b^3 - 3*a^4*b^5 + 3*a^2*b^7 - b^9 + 3*a^2*c^7 + 3*b*c^8 + c^9 + (9*a^2*b - 8*b^3)*c^6 + 3*(a^4 + a^2*b^2 - 2*b^4)*c^5 + 3*(3*a^4*b - 5*a^2*b^3 + 2*b^5)*c^4 + (a^6 + 6*a^4*b^2 - 15*a^2*b^4 + 8*b^6)*c^3 + 3*(a^6*b - 2*a^4*b^3 + a^2*b^5)*c^2 + 3*(a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^8)*c)*sinh(x)^4 + (a^6 - 6*a^4*b^2 + 9*a^2*b^4 - 4*b^6)*c^3 + 4*(a^7*b^2 - 3*a^5*b^4 + 3*a^3*b^6 - a*b^8 + 2*a*b*c^7 + a*c^8 + (3*a^3 - 2*a*b^2)*c^6 + 6*(a^3*b - a*b^3)*c^5 + 3*(a^5 - a^3*b^2)*c^4 + 6*(a^5*b - 2*a^3*b^3 + a*b^5)*c^3 + (a^7 - 3*a^3*b^4 + 2*a*b^6)*c^2 + 2*(a^7*b - 3*a^5*b^3 + 3*a^3*b^5 - a*b^7)*c)*cosh(x)^3 + 4*(a^7*b^2 - 3*a^5*b^4 + 3*a^3*b^6 - a*b^8 + 2*a*b*c^7 + a*c^8 + (3*a^3 - 2*a*b^2)*c^6 + 6*(a^3*b - a*b^3)*c^5 + 3*(a^5 - a^3*b^2)*c^4 + 6*(a^5*b - 2*a^3*b^3 + a*b^5)*c^3 + (a^7 - 3*a^3*b^4 + 2*a*b^6)*c^2 + 2*(a^7*b - 3*a^5*b^3 + 3*a^3*b^5 - a*b^7)*c + (a^6*b^3 - 3*a^4*b^5 + 3*a^2*b^7 - b^9 + 3*a^2*c^7 + 3*b*c^8 + c^9 + (9*a^2*b - 8*b^3)*c^6 + 3*

```
(a^4 + a^2*b^2 - 2*b^4)*c^5 + 3*(3*a^4*b - 5*a^2*b^3 + 2*b^5)*c^4 + (a^6 +
6*a^4*b^2 - 15*a^2*b^4 + 8*b^6)*c^3 + 3*(a^6*b - 2*a^4*b^3 + a^2*b^5)*c^2 +
 3*(a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^8)*c)*cosh(x))*sinh(x)^3 - (a^6*b -
 6*a^4*b^3 + 9*a^2*b^5 - 4*b^7)*c^2 + 2*(2*a^8*b - 5*a^6*b^3 + 3*a^4*b^5 +
a^2*b^7 - b^9 - b*c^8 - c^9 - (a^2 - 4*b^2)*c^7 - (a^2*b - 4*b^3)*c^6 + 3*(
a^4 + a^2*b^2 - 2*b^4)*c^5 + 3*(a^4*b + a^2*b^3 - 2*b^5)*c^4 + (5*a^6 - 6*a
^4*b^2 - 3*a^2*b^4 + 4*b^6)*c^3 + (5*a^6*b - 6*a^4*b^3 - 3*a^2*b^5 + 4*b^7)
*c^2 + (2*a^8 - 5*a^6*b^2 + 3*a^4*b^4 + a^2*b^6 - b^8)*c)*cosh(x)^2 + 2*(2*
a^8*b - 5*a^6*b^3 + 3*a^4*b^5 + a^2*b^7 - b^9 - b*c^8 - c^9 - (a^2 - 4*b^2)
*c^7 - (a^2*b - 4*b^3)*c^6 + 3*(a^4 + a^2*b^2 - 2*b^4)*c^5 + 3*(a^4*b + a^2
*b^3 - 2*b^5)*c^4 + (5*a^6 - 6*a^4*b^2 - 3*a^2*b^4 + 4*b^6)*c^3 + (5*a^6*b
- 6*a^4*b^3 - 3*a^2*b^5 + 4*b^7)*c^2 + 3*(a^6*b^3 - 3*a^4*b^5 + 3*a^2*b^7 -
 b^9 + 3*a^2*c^7 + 3*b*c^8 + c^9 + (9*a^2*b - 8*b^3)*c^6 + 3*(a^4 + a^2*b^2
 - 2*b^4)*c^5 + 3*(3*a^4*b - 5*a^2*b^3 + 2*b^5)*c^4 + (a^6 + 6*a^4*b^2 - 15
*a^2*b^4 + 8*b^6)*c^3 + 3*(a^6*b - 2*a^4*b^3 + a^2*b^5)*c^2 + 3*(a^6*b^2 -
3*a^4*b^4 + 3*a^2*b^6 - b^8)*c)*cosh(x)^2 + (2*a^8 - 5*a^6*b^2 + 3*a^4*b^4
+ a^2*b^6 - b^8)*c + 6*(a^7*b^2 - 3*a^5*b^4 + 3*a^3*b^6 - a*b^8 + 2*a*b*c^7
 + a*c^8 + (3*a^3 - 2*a*b^2)*c^6 + 6*(a^3*b - a*b^3)*c^5 + 3*(a^5 - a^3*b^2
)*c^4 + 6*(a^5*b - 2*a^3*b^3 + a*b^5)*c^3 + (a^7 - 3*a^3*b^4 + 2*a*b^6)*c^2
 + 2*(a^7*b - 3*a^5*b^3 + 3*a^3*b^5 - a*b^7)*c)*cosh(x))*sinh(x)^2 - (a^6*b
^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^8)*c + 4*(a^7*b^2 - 3*a^5*b^4 + 3*a^3*b^6 -
a*b^8 - a*c^8 - (3*a^3 - 4*a*b^2)*c^6 - 3*(a^5 - 3*a^3*b^2 + 2*a*b^4)*c^4 -
 (a^7 - 6*a^5*b^2 + 9*a^3*b^4 - 4*a*b^6)*c^2)*cosh(x) + 4*(a^7*b^2 - 3*a^5*
b^4 + 3*a^3*b^6 - a*b^8 - a*c^8 - (3*a^3 - 4*a*b^2)*c^6 - 3*(a^5 - 3*a^3*b^
2 + 2*a*b^4)*c^4 + (a^6*b^3 - 3*a^4*b^5 + 3*a^2*b^7 - b^9 + 3*a^2*c^7 + 3*b
*c^8 + c^9 + (9*a^2*b - 8*b^3)*c^6 + 3*(a^4 + a^2*b^2 - 2*b^4)*c^5 + 3*(3*a
^4*b - 5*a^2*b^3 + 2*b^5)*c^4 + (a^6 + 6*a^4*b^2 - 15*a^2*b^4 + 8*b^6)*c^3
+ 3*(a^6*b - 2*a^4*b^3 + a^2*b^5)*c^2 + 3*(a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^6
- b^8)*c)*cosh(x)^3 - (a^7 - 6*a^5*b^2 + 9*a^3*b^4 - 4*a*b^6)*c^2 + 3*(a^7*
b^2 - 3*a^5*b^4 + 3*a^3*b^6 - a*b^8 + 2*a*b*c^7 + a*c^8 + (3*a^3 - 2*a*b^2)
*c^6 + 6*(a^3*b - a*b^3)*c^5 + 3*(a^5 - a^3*b^2)*c^4 + 6*(a^5*b - 2*a^3*b^3
 + a*b^5)*c^3 + (a^7 - 3*a^3*b^4 + 2*a*b^6)*c^2 + 2*(a^7*b - 3*a^5*b^3 + 3*
a^3*b^5 - a*b^7)*c)*cosh(x)^2 + (2*a^8*b - 5*a^6*b^3 + 3*a^4*b^5 + a^2*b^7
- b^9 - b*c^8 - c^9 - (a^2 - 4*b^2)*c^7 - (a^2*b - 4*b^3)*c^6 + 3*(a^4 + a^
2*b^2 - 2*b^4)*c^5 + 3*(a^4*b + a^2*b^3 - 2*b^5)*c^4 + (5*a^6 - 6*a^4*b^2 -
 3*a^2*b^4 + 4*b^6)*c^3 + (5*a^6*b - 6*a^4*b^3 - 3*a^2*b^5 + 4*b^7)*c^2 + (
2*a^8 - 5*a^6*b^2 + 3*a^4*b^4 + a^2*b^6 - b^8)*c)*cosh(x))*sinh(x))]
```

giac [B] time = 0.17, size = 625, normalized size = 3.22

$$\frac{\left(2\,Aa^2 - 3\,Bab + Ab^2 - Ac^2\right)\arctan\left(\frac{be^x+ce^x+a}{\sqrt{-a^2+b^2-c^2}}\right)}{\left(a^4 - 2\,a^2b^2 + b^4 + 2\,a^2c^2 - 2\,b^2c^2 + c^4\right)\sqrt{-a^2+b^2-c^2}} + \frac{2\,Aa^2b^2e^{(3\,x)} - 3\,Bab^3e^{(3\,x)} + Ab^4e^{(3\,x)} + 4\,Aa^2bce^{(3\,x)} - \cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(a+b*cosh(x)+c*sinh(x))^3,x, algorithm="giac")
```

```
[Out] (2*A*a^2 - 3*B*a*b + A*b^2 - A*c^2)*arctan((b*e^x + c*e^x + a)/sqrt(-a^2 +
b^2 - c^2))/((a^4 - 2*a^2*b^2 + b^4 + 2*a^2*c^2 - 2*b^2*c^2 + c^4)*sqrt(-a^
2 + b^2 - c^2)) + (2*A*a^2*b^2*e^(3*x) - 3*B*a*b^3*e^(3*x) + A*b^4*e^(3*x)
+ 4*A*a^2*b*c*e^(3*x) - 6*B*a*b^2*c*e^(3*x) + 2*A*b^3*c*e^(3*x) + 2*A*a^2*c
^2*e^(3*x) - 3*B*a*b*c^2*e^(3*x) - 2*A*b*c^3*e^(3*x) - A*c^4*e^(3*x) - 2*B*
a^4*e^(2*x) + 6*A*a^3*b*e^(2*x) - 5*B*a^2*b^2*e^(2*x) + 3*A*a*b^3*e^(2*x) -
 2*B*b^4*e^(2*x) + 6*A*a^3*c*e^(2*x) - 9*B*a^2*b*c*e^(2*x) + 3*A*a*b^2*c*e^
(2*x) - 4*B*a^2*c^2*e^(2*x) - 3*A*a*b*c^2*e^(2*x) + 4*B*b^2*c^2*e^(2*x) - 3
*A*a*c^3*e^(2*x) - 2*B*c^4*e^(2*x) - 4*B*a^3*b*e^x + 10*A*a^2*b^2*e^x - 5*B
*a*b^3*e^x - A*b^4*e^x - 10*A*a^2*c^2*e^x + 5*B*a*b*c^2*e^x + 2*A*b^2*c^2*e
^x - A*c^4*e^x - B*a^2*b^2 + 3*A*a*b^3 - 2*B*b^4 + B*a^2*b*c - 3*A*a*b^2*c
+ 2*B*b^3*c - 3*A*a*b*c^2 + 2*B*b^2*c^2 + 3*A*a*c^3 - 2*B*b*c^3)/((a^4*b -
2*a^2*b^3 + b^5 + a^4*c - 2*a^2*b^2*c + b^4*c + 2*a^2*b*c^2 - 2*b^3*c^2 + 2
*a^2*c^3 - 2*b^2*c^3 + b*c^4 + c^5)*(b*e^(2*x) + c*e^(2*x) + 2*a*e^x + b -
c)^2)
```

maple [B] time = 0.30, size = 1112, normalized size = 5.73

$$-\frac{2\left(-\frac{\left(4A\,a^{3}b-7A\,a^{2}b^{2}+5A\,a^{2}c^{2}+2Aa\,b^{3}-2Aab\,c^{2}+A\,b^{4}-3A\,b^{2}c^{2}+2A\,c^{4}-2B\,a^{4}+3B\,a^{3}b-2B\,a^{2}b^{2}-4B\,a^{2}c^{2}+3Ba\,b^{3}-2B\,b^{4}+4B\,b^{2}c^{2}-2B\,c^{4}\right)\left(\tanh^{3}\left(\frac{x}{2}\right.\right.}{2(a-b)\left(a^{4}-2a^{2}b^{2}+2a^{2}c^{2}+b^{4}-2c^{2}b^{2}+c^{4}\right)}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cosh(x))/(a+b*cosh(x)+c*sinh(x))^3,x)
```

```
[Out] -2*(-1/2*(4*A*a^3*b-7*A*a^2*b^2+5*A*a^2*c^2+2*A*a*b^3-2*A*a*b*c^2+A*b^4-3*A
*b^2*c^2+2*A*c^4-2*B*a^4+3*B*a^3*b-2*B*a^2*b^2-4*B*a^2*c^2+3*B*a*b^3-2*B*b^
4+4*B*b^2*c^2-2*B*c^4)/(a-b)/(a^4-2*a^2*b^2+2*a^2*c^2+b^4-2*b^2*c^2+c^4)*ta
nh(1/2*x)^3-1/2*c*(4*A*a^4-12*A*a^3*b+13*A*a^2*b^2-7*A*a^2*c^2-6*A*a*b^3+6*
A*a*b*c^2+A*b^4+A*b^2*c^2-2*A*c^4+2*B*a^4-9*B*a^3*b+14*B*a^2*b^2+4*B*a^2*c^
2-9*B*a*b^3+2*B*b^4-4*B*b^2*c^2+2*B*c^4)/(a^4-2*a^2*b^2+2*a^2*c^2+b^4-2*b^2
*c^2+c^4)/(a^2-2*a*b+b^2)*tanh(1/2*x)^2+1/2*(4*A*a^4*b-5*A*a^3*b^2+11*A*a^3
*c^2-3*A*a^2*b^3-3*A*a^2*b*c^2+5*A*a*b^4-7*A*a*b^2*c^2+2*A*a*c^4-A*b^5-A*b^
3*c^2+2*A*b*c^4-2*B*a^5+3*B*a^4*b-B*a^3*b^2-4*B*a^3*c^2-B*a^2*b^3-8*B*a^2*b
*c^2+3*B*a*b^4+8*B*a*b^2*c^2-2*B*a*c^4-2*B*b^5+4*B*b^3*c^2-2*B*b*c^4)/(a^4-
2*a^2*b^2+2*a^2*c^2+b^4-2*b^2*c^2+c^4)/(a^2-2*a*b+b^2)*tanh(1/2*x)+1/2*c*(4
*A*a^4-3*A*a^2*b^2+A*a^2*c^2-A*b^4+A*b^2*c^2-5*B*a^3*b+5*B*a*b^3-2*B*a*b*c^
2)/(a^4-2*a^2*b^2+2*a^2*c^2+b^4-2*b^2*c^2+c^4)/(a^2-2*a*b+b^2))/(a*tanh(1/2
*x)^2-tanh(1/2*x)^2*b-2*c*tanh(1/2*x)-a-b)^2-2/(a^4-2*a^2*b^2+2*a^2*c^2+b^4
-2*b^2*c^2+c^4)/(-a^2+b^2-c^2)^(1/2)*arctan(1/2*(2*(a-b)*tanh(1/2*x)-2*c)/(
-a^2+b^2-c^2)^(1/2))*a^2*A-1/(a^4-2*a^2*b^2+2*a^2*c^2+b^4-2*b^2*c^2+c^4)/(-
a^2+b^2-c^2)^(1/2)*arctan(1/2*(2*(a-b)*tanh(1/2*x)-2*c)/(-a^2+b^2-c^2)^(1/2
```

```
))*A*b^2+1/(a^4-2*a^2*b^2+2*a^2*c^2+b^4-2*b^2*c^2+c^4)/(-a^2+b^2-c^2)^(1/2)
*arctan(1/2*(2*(a-b)*tanh(1/2*x)-2*c)/(-a^2+b^2-c^2)^(1/2))*A*c^2+3/(a^4-2*
a^2*b^2+2*a^2*c^2+b^4-2*b^2*c^2+c^4)/(-a^2+b^2-c^2)^(1/2)*arctan(1/2*(2*(a-
b)*tanh(1/2*x)-2*c)/(-a^2+b^2-c^2)^(1/2))*b*B*a
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(a+b*cosh(x)+c*sinh(x))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(c^2-b^2+a^2>0)', see `assume?` for
more details)Is c^2-b^2+a^2 positive or negative?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A + B\cosh(x)}{(a + b\cosh(x) + c\sinh(x))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cosh(x))/(a + b*cosh(x) + c*sinh(x))^3,x)
```

```
[Out] int((A + B*cosh(x))/(a + b*cosh(x) + c*sinh(x))^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(a+b*cosh(x)+c*sinh(x))**3,x)
```

```
[Out] Timed out
```

3.795 $\int \frac{B\cosh(x)+C\sinh(x)}{a+b\cosh(x)+c\sinh(x)}\,dx$

Optimal. Leaf size=125

$$\frac{2a(bB-cC)\tanh^{-1}\left(\frac{c-(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2+c^2}}\right)}{\left(b^2-c^2\right)\sqrt{a^2-b^2+c^2}}-\frac{(Bc-bC)\log(a+b\cosh(x)+c\sinh(x))}{b^2-c^2}+\frac{x(bB-cC)}{b^2-c^2}$$

```
[Out] (B*b-C*c)*x/(b^2-c^2)-(B*c-C*b)*ln(a+b*cosh(x)+c*sinh(x))/(b^2-c^2)+2*a*(B*
b-C*c)*arctanh((c-(a-b)*tanh(1/2*x))/(a^2-b^2+c^2)^(1/2))/(b^2-c^2)/(a^2-b^
2+c^2)^(1/2)
```

Rubi [A] time = 0.14, antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {3136, 3124, 618, 206}

$$\frac{2a(bB-cC)\tanh^{-1}\left(\frac{c-(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2+c^2}}\right)}{\left(b^2-c^2\right)\sqrt{a^2-b^2+c^2}}-\frac{(Bc-bC)\log(a+b\cosh(x)+c\sinh(x))}{b^2-c^2}+\frac{x(bB-cC)}{b^2-c^2}$$

Antiderivative was successfully verified.

```
[In] Int[(B*Cosh[x] + C*Sinh[x])/(a + b*Cosh[x] + c*Sinh[x]),x]
```

```
[Out] ((b*B - c*C)*x)/(b^2 - c^2) + (2*a*(b*B - c*C)*ArcTanh[(c - (a - b)*Tanh[x/
2])/Sqrt[a^2 - b^2 + c^2]])/((b^2 - c^2)*Sqrt[a^2 - b^2 + c^2]) - ((B*c - b
*C)*Log[a + b*Cosh[x] + c*Sinh[x]])/(b^2 - c^2)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 3124

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Module[{f = FreeFactors[Tan[(d + e*x)/2], x]}, Dist[(2*f
```

```
)/e, Subst[Int[1/(a + b + 2*c*f*x + (a - b)*f^2*x^2), x], x, Tan[(d + e*x)/
2]/f], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
```

Rule 3136

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)]), x
_Symbol] :> Simp[((b*B + c*C)*x)/(b^2 + c^2), x] + (Dist[(A*(b^2 + c^2) - a
*(b*B + c*C))/(b^2 + c^2), Int[1/(a + b*Cos[d + e*x] + c*Sin[d + e*x]), x],
 x] + Simp[((c*B - b*C)*Log[a + b*Cos[d + e*x] + c*Sin[d + e*x]])/(e*(b^2 +
 c^2)), x]) /; FreeQ[{a, b, c, d, e, A, B, C}, x] && NeQ[b^2 + c^2, 0] && N
eQ[A*(b^2 + c^2) - a*(b*B + c*C), 0]
```

Rubi steps

$$\int \frac{B\cosh(x)+C\sinh(x)}{a+b\cosh(x)+c\sinh(x)}\,dx = \frac{(bB-cC)x}{b^2-c^2} - \frac{(Bc-bC)\log(a+b\cosh(x)+c\sinh(x))}{b^2-c^2} - \frac{(a(bB-cC))\int \frac{}{a+b\,\mathrm{co}}}{b^2-c^2}$$

$$= \frac{(bB-cC)x}{b^2-c^2} - \frac{(Bc-bC)\log(a+b\cosh(x)+c\sinh(x))}{b^2-c^2} - \frac{(2a(bB-cC))\,\mathrm{Subst}}{}$$

$$= \frac{(bB-cC)x}{b^2-c^2} - \frac{(Bc-bC)\log(a+b\cosh(x)+c\sinh(x))}{b^2-c^2} + \frac{(4a(bB-cC))\,\mathrm{Subst}}{}$$

$$= \frac{(bB-cC)x}{b^2-c^2} + \frac{2a(bB-cC)\tanh^{-1}\left(\frac{c-(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2+c^2}}\right)}{\left(b^2-c^2\right)\sqrt{a^2-b^2+c^2}} - \frac{(Bc-bC)\log(a+b\cosh(}{b^2-c^2}$$

Mathematica [A] time = 0.35, size = 107, normalized size = 0.86

$$\frac{-\frac{2a(bB-cC)\tan^{-1}\left(\frac{(b-a)\tanh\left(\frac{x}{2}\right)+c}{\sqrt{-a^2+b^2-c^2}}\right)}{\sqrt{-a^2+b^2-c^2}} + (bC-Bc)\log(a+b\cosh(x)+c\sinh(x)) + x(bB-cC)}{(b-c)(b+c)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(B*Cosh[x] + C*Sinh[x])/(a + b*Cosh[x] + c*Sinh[x]),x]
```

```
[Out] ((b*B - c*C)*x - (2*a*(b*B - c*C)*ArcTan[(c + (-a + b)*Tanh[x/2])/Sqrt[-a^2
 + b^2 - c^2]])/Sqrt[-a^2 + b^2 - c^2] + (-(B*c) + b*C)*Log[a + b*Cosh[x] +
 c*Sinh[x]])/((b - c)*(b + c))
```

fricas [A] time = 0.48, size = 583, normalized size = 4.66

$$\left[\frac{(Bab-Cac)\sqrt{a^2-b^2+c^2}\,\log\left(\frac{\left(b^2+2\,bc+c^2\right)\cosh(x)^2+\left(b^2+2\,bc+c^2\right)\sinh(x)^2+2\,a^2-b^2+c^2+2\,(ab+ac)\cosh(x)+2\left(ab+ac+\left(b^2+2\,bc+c^2\right)\cos\right.}{(b+c)\cosh(x)^2+(b+c)\sinh(x)^2+2\,a\cosh(x)+2\,((b+c)\cosh(\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cosh(x)+C*sinh(x))/(a+b*cosh(x)+c*sinh(x)),x, algorithm="frica
s")
```

```
[Out] [((B*a*b - C*a*c)*sqrt(a^2 - b^2 + c^2)*log(((b^2 + 2*b*c + c^2)*cosh(x)^2
+ (b^2 + 2*b*c + c^2)*sinh(x)^2 + 2*a^2 - b^2 + c^2 + 2*(a*b + a*c)*cosh(x)
 + 2*(a*b + a*c + (b^2 + 2*b*c + c^2)*cosh(x))*sinh(x) + 2*sqrt(a^2 - b^2 +
 c^2)*((b + c)*cosh(x) + (b + c)*sinh(x) + a))/((b + c)*cosh(x)^2 + (b + c)
*sinh(x)^2 + 2*a*cosh(x) + 2*((b + c)*cosh(x) + a)*sinh(x) + b - c)) + ((B
- C)*a^2*b - (B - C)*b^3 + (B - C)*b*c^2 + (B - C)*c^3 + ((B - C)*a^2 - (B
- C)*b^2)*c)*x + (C*a^2*b - C*b^3 + C*b*c^2 - B*c^3 - (B*a^2 - B*b^2)*c)*lo
g(2*(b*cosh(x) + c*sinh(x) + a)/(cosh(x) - sinh(x))))/(a^2*b^2 - b^4 - c^4
- (a^2 - 2*b^2)*c^2), -(2*(B*a*b - C*a*c)*sqrt(-a^2 + b^2 - c^2)*arctan(sqr
t(-a^2 + b^2 - c^2)*((b + c)*cosh(x) + (b + c)*sinh(x) + a)/(a^2 - b^2 + c^
2)) - ((B - C)*a^2*b - (B - C)*b^3 + (B - C)*b*c^2 + (B - C)*c^3 + ((B - C)
*a^2 - (B - C)*b^2)*c)*x - (C*a^2*b - C*b^3 + C*b*c^2 - B*c^3 - (B*a^2 - B*
b^2)*c)*log(2*(b*cosh(x) + c*sinh(x) + a)/(cosh(x) - sinh(x))))/(a^2*b^2 -
b^4 - c^4 - (a^2 - 2*b^2)*c^2)]
```

giac [A] time = 0.14, size = 125, normalized size = 1.00

$$\frac{(B-C)x}{b-c}+\frac{(Cb-Bc)\log\left(be^{(2\,x)}+ce^{(2\,x)}+2\,ae^x+b-c\right)}{b^2-c^2}-\frac{2\,(Bab-Cac)\arctan\left(\frac{be^x+ce^x+a}{\sqrt{-a^2+b^2-c^2}}\right)}{\sqrt{-a^2+b^2-c^2}\left(b^2-c^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cosh(x)+C*sinh(x))/(a+b*cosh(x)+c*sinh(x)),x, algorithm="giac"
)
```

```
[Out] (B - C)*x/(b - c) + (C*b - B*c)*log(b*e^(2*x) + c*e^(2*x) + 2*a*e^x + b - c
)/(b^2 - c^2) - 2*(B*a*b - C*a*c)*arctan((b*e^x + c*e^x + a)/sqrt(-a^2 + b^
2 - c^2))/(sqrt(-a^2 + b^2 - c^2)*(b^2 - c^2))
```

maple [B] time = 0.20, size = 873, normalized size = 6.98

$$-\frac{2B\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{2b+2c}-\frac{2C\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{2b+2c}-\frac{\ln\left(a\left(\tanh^2\left(\frac{x}{2}\right)\right)-\left(\tanh^2\left(\frac{x}{2}\right)\right)b-2c\tanh\left(\frac{x}{2}\right)-a-b\right)aBc}{(b-c)\,(b+c)\,(a-b)}+\frac{\ln}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*cosh(x)+C*sinh(x))/(a+b*cosh(x)+c*sinh(x)),x)
```

```
[Out] -2*B/(2*b+2*c)*ln(tanh(1/2*x)-1)-2*C/(2*b+2*c)*ln(tanh(1/2*x)-1)-1/(b-c)/(b
+c)/(a-b)*ln(a*tanh(1/2*x)^2-tanh(1/2*x)^2*b-2*c*tanh(1/2*x)-a-b)*a*B*c+1/(
b-c)/(b+c)/(a-b)*ln(a*tanh(1/2*x)^2-tanh(1/2*x)^2*b-2*c*tanh(1/2*x)-a-b)*b*
B*c+1/(b-c)/(b+c)/(a-b)*ln(a*tanh(1/2*x)^2-tanh(1/2*x)^2*b-2*c*tanh(1/2*x)-
a-b)*a*b*C-1/(b-c)/(b+c)/(a-b)*ln(a*tanh(1/2*x)^2-tanh(1/2*x)^2*b-2*c*tanh(
1/2*x)-a-b)*C*b^2+2/(b-c)/(b+c)/(-a^2+b^2-c^2)^(1/2)*arctan(1/2*(2*(a-b)*ta
nh(1/2*x)-2*c)/(-a^2+b^2-c^2)^(1/2))*b*B*a+2/(b-c)/(b+c)/(-a^2+b^2-c^2)^(1/
2)*arctan(1/2*(2*(a-b)*tanh(1/2*x)-2*c)/(-a^2+b^2-c^2)^(1/2))*B*c^2-2/(b-c)
/(b+c)/(-a^2+b^2-c^2)^(1/2)*arctan(1/2*(2*(a-b)*tanh(1/2*x)-2*c)/(-a^2+b^2-
c^2)^(1/2))*a*c*C-2/(b-c)/(b+c)/(-a^2+b^2-c^2)^(1/2)*arctan(1/2*(2*(a-b)*ta
nh(1/2*x)-2*c)/(-a^2+b^2-c^2)^(1/2))*C*c*b-2/(b-c)/(b+c)/(-a^2+b^2-c^2)^(1/
2)*arctan(1/2*(2*(a-b)*tanh(1/2*x)-2*c)/(-a^2+b^2-c^2)^(1/2))*c^2/(a-b)*a*B
+2/(b-c)/(b+c)/(-a^2+b^2-c^2)^(1/2)*arctan(1/2*(2*(a-b)*tanh(1/2*x)-2*c)/(-
a^2+b^2-c^2)^(1/2))*c^2/(a-b)*b*B+2/(b-c)/(b+c)/(-a^2+b^2-c^2)^(1/2)*arctan
(1/2*(2*(a-b)*tanh(1/2*x)-2*c)/(-a^2+b^2-c^2)^(1/2))*c/(a-b)*a*b*C-2/(b-c)/
(b+c)/(-a^2+b^2-c^2)^(1/2)*arctan(1/2*(2*(a-b)*tanh(1/2*x)-2*c)/(-a^2+b^2-c
^2)^(1/2))*c/(a-b)*C*b^2+2*B/(2*b-2*c)*ln(tanh(1/2*x)+1)-2*C/(2*b-2*c)*ln(t
anh(1/2*x)+1)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cosh(x)+C*sinh(x))/(a+b*cosh(x)+c*sinh(x)),x, algorithm="maxim
a")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(c^2-b^2+a^2>0)', see `assume?` for
more details)Is c^2-b^2+a^2 positive or negative?
```

mupad [B] time = 0.70, size = 376, normalized size = 3.01

$$\frac{\ln\Big(b\,\sqrt{a^2-b^2+c^2}\;-c\,\sqrt{a^2-b^2+c^2}\;+a^2\,\mathrm{e}^x-b^2\,\mathrm{e}^x+c^2\,\mathrm{e}^x+a\,\mathrm{e}^x\,\sqrt{a^2-b^2+c^2}\Big)\,\Big(B\,c^3+C\,b^3+B\,a^2\,c-C\,a^2\,b}{-a^2\,b^2+a^2\,c^2+b^4-2\,b^2\,c^2+c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*cosh(x) + C*sinh(x))/(a + b*cosh(x) + c*sinh(x)),x)
```

```
[Out] (log(b*(a^2 - b^2 + c^2)^(1/2) - c*(a^2 - b^2 + c^2)^(1/2) + a^2*exp(x) - b
^2*exp(x) + c^2*exp(x) + a*exp(x)*(a^2 - b^2 + c^2)^(1/2))*(B*c^3 + C*b^3 +
 B*a^2*c - C*a^2*b - B*b^2*c - C*b*c^2 + B*a*b*(a^2 - b^2 + c^2)^(1/2) - C*
a*c*(a^2 - b^2 + c^2)^(1/2)))/(b^4 + c^4 - a^2*b^2 + a^2*c^2 - 2*b^2*c^2) +
 (log(b*(a^2 - b^2 + c^2)^(1/2) - c*(a^2 - b^2 + c^2)^(1/2) - a^2*exp(x) +
b^2*exp(x) - c^2*exp(x) + a*exp(x)*(a^2 - b^2 + c^2)^(1/2))*(B*c^3 + C*b^3
+ B*a^2*c - C*a^2*b - B*b^2*c - C*b*c^2 - B*a*b*(a^2 - b^2 + c^2)^(1/2) + C
*a*c*(a^2 - b^2 + c^2)^(1/2)))/(b^4 + c^4 - a^2*b^2 + a^2*c^2 - 2*b^2*c^2)
+ (x*(B - C))/(b - c)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cosh(x)+C*sinh(x))/(a+b*cosh(x)+c*sinh(x)),x)
```

```
[Out] Timed out
```

3.796 $\int \frac{B\cosh(x)+C\sinh(x)}{(a+b\cosh(x)+c\sinh(x))^2}\,dx$

Optimal. Leaf size=108

$$\frac{2(bB-cC)\tanh^{-1}\left(\frac{c-(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2+c^2}}\right)}{\left(a^2-b^2+c^2\right)^{3/2}}-\frac{-aB\sinh(x)-aC\cosh(x)-bC+Bc}{\left(a^2-b^2+c^2\right)(a+b\cosh(x)+c\sinh(x))}$$

```
[Out] 2*(B*b-C*c)*arctanh((c-(a-b)*tanh(1/2*x))/(a^2-b^2+c^2)^(1/2))/(a^2-b^2+c^2
)^(3/2)+(-B*c+b*C+a*C*cosh(x)+a*B*sinh(x))/(a^2-b^2+c^2)/(a+b*cosh(x)+c*sin
h(x))
```

Rubi [A] time = 0.13, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {3153, 3124, 618, 206}

$$\frac{2(bB-cC)\tanh^{-1}\left(\frac{c-(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2+c^2}}\right)}{\left(a^2-b^2+c^2\right)^{3/2}}-\frac{-aB\sinh(x)-aC\cosh(x)-bC+Bc}{\left(a^2-b^2+c^2\right)(a+b\cosh(x)+c\sinh(x))}$$

Antiderivative was successfully verified.

```
[In] Int[(B*Cosh[x] + C*Sinh[x])/(a + b*Cosh[x] + c*Sinh[x])^2,x]
```

```
[Out] (2*(b*B - c*C)*ArcTanh[(c - (a - b)*Tanh[x/2])/Sqrt[a^2 - b^2 + c^2]])/(a^2
 - b^2 + c^2)^(3/2) - (B*c - b*C - a*C*Cosh[x] - a*B*Sinh[x])/((a^2 - b^2 +
 c^2)*(a + b*Cosh[x] + c*Sinh[x]))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 3124

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Module[{f = FreeFactors[Tan[(d + e*x)/2], x]}, Dist[(2*f
```

```
)/e, Subst[Int[1/(a + b + 2*c*f*x + (a - b)*f^2*x^2), x], x, Tan[(d + e*x)/
2]/f], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
```

Rule 3153

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])^2,
 x_Symbol] :> Simp[(c*B - b*C - (a*C - c*A)*Cos[d + e*x] + (a*B - b*A)*Sin[
d + e*x])/(e*(a^2 - b^2 - c^2)*(a + b*Cos[d + e*x] + c*Sin[d + e*x])), x] +
 Dist[(a*A - b*B - c*C)/(a^2 - b^2 - c^2), Int[1/(a + b*Cos[d + e*x] + c*Si
n[d + e*x]), x], x] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && NeQ[a^2 - b^2
- c^2, 0] && NeQ[a*A - b*B - c*C, 0]
```

Rubi steps

$$\int \frac{B\cosh(x)+C\sinh(x)}{(a+b\cosh(x)+c\sinh(x))^2}\,dx = -\frac{Bc-bC-aC\cosh(x)-aB\sinh(x)}{\left(a^2-b^2+c^2\right)(a+b\cosh(x)+c\sinh(x))} - \frac{(bB-cC)\int \frac{1}{a+b\cosh(x)+c\sinh(x}}{a^2-b^2+c^2}$$

$$= -\frac{Bc-bC-aC\cosh(x)-aB\sinh(x)}{\left(a^2-b^2+c^2\right)(a+b\cosh(x)+c\sinh(x))} - \frac{(2(bB-cC))\,\mathrm{Subst}\left(\int \frac{}{a+b+2}\right.}{a^2-b^2}$$

$$= -\frac{Bc-bC-aC\cosh(x)-aB\sinh(x)}{\left(a^2-b^2+c^2\right)(a+b\cosh(x)+c\sinh(x))} + \frac{(4(bB-cC))\,\mathrm{Subst}\left(\int \frac{}{4\left(a^2-b\right.}\right.}{a}$$

$$= \frac{2(bB-cC)\tanh^{-1}\left(\frac{c-(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2+c^2}}\right)}{\left(a^2-b^2+c^2\right)^{3/2}} - \frac{Bc-bC-aC\cosh(x)-aB\sinh(x}{\left(a^2-b^2+c^2\right)(a+b\cosh(x)+c\sinh}$$

Mathematica [A] time = 0.34, size = 123, normalized size = 1.14

$$\frac{2(bB-cC)\tan^{-1}\left(\frac{(b-a)\tanh\left(\frac{x}{2}\right)+c}{\sqrt{-a^2+b^2-c^2}}\right)}{\left(-a^2+b^2-c^2\right)^{3/2}} + \frac{a^2C+a\sinh(x)(cC-bB)-b^2C+bBc}{b\left(-a^2+b^2-c^2\right)(a+b\cosh(x)+c\sinh(x))}$$

Antiderivative was successfully verified.

```
[In] Integrate[(B*Cosh[x] + C*Sinh[x])/(a + b*Cosh[x] + c*Sinh[x])^2,x]
```

```
[Out] (2*(b*B - c*C)*ArcTan[(c + (-a + b)*Tanh[x/2])/Sqrt[-a^2 + b^2 - c^2]])/(-a
^2 + b^2 - c^2)^(3/2) + (b*B*c + a^2*C - b^2*C + a*(-(b*B) + c*C)*Sinh[x])/
(b*(-a^2 + b^2 - c^2)*(a + b*Cosh[x] + c*Sinh[x]))
```

fricas [B] time = 0.49, size = 2119, normalized size = 19.62

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cosh(x)+C*sinh(x))/(a+b*cosh(x)+c*sinh(x))^2,x, algorithm="fri
cas")
```

```
[Out] [-(2*B*a^3*b - 2*B*a*b^3 + 2*B*a*b*c^2 - 2*C*a*c^3 + (B*b^3 - C*b^2*c - B*b
*c^2 + C*c^3 + (B*b^3 + (2*B - C)*b^2*c + (B - 2*C)*b*c^2 - C*c^3)*cosh(x)^
2 + (B*b^3 + (2*B - C)*b^2*c + (B - 2*C)*b*c^2 - C*c^3)*sinh(x)^2 + 2*(B*a*
b^2 + (B - C)*a*b*c - C*a*c^2)*cosh(x) + 2*(B*a*b^2 + (B - C)*a*b*c - C*a*c
^2 + (B*b^3 + (2*B - C)*b^2*c + (B - 2*C)*b*c^2 - C*c^3)*cosh(x))*sinh(x))*
sqrt(a^2 - b^2 + c^2)*log(((b^2 + 2*b*c + c^2)*cosh(x)^2 + (b^2 + 2*b*c + c
^2)*sinh(x)^2 + 2*a^2 - b^2 + c^2 + 2*(a*b + a*c)*cosh(x) + 2*(a*b + a*c +
(b^2 + 2*b*c + c^2)*cosh(x))*sinh(x) - 2*sqrt(a^2 - b^2 + c^2)*((b + c)*cos
h(x) + (b + c)*sinh(x) + a))/((b + c)*cosh(x)^2 + (b + c)*sinh(x)^2 + 2*a*c
osh(x) + 2*((b + c)*cosh(x) + a)*sinh(x) + b - c)) - 2*(C*a^3 - C*a*b^2)*c
+ 2*((B + C)*a^4 - (B + 2*C)*a^2*b^2 + C*b^4 + (B - C)*b*c^3 + B*c^4 + ((2*
B + C)*a^2 - (B + C)*b^2)*c^2 + ((B - C)*a^2*b - (B - C)*b^3)*c)*cosh(x) +
2*((B + C)*a^4 - (B + 2*C)*a^2*b^2 + C*b^4 + (B - C)*b*c^3 + B*c^4 + ((2*B
+ C)*a^2 - (B + C)*b^2)*c^2 + ((B - C)*a^2*b - (B - C)*b^3)*c)*sinh(x))/(a^
4*b^2 - 2*a^2*b^4 + b^6 - c^6 - (2*a^2 - 3*b^2)*c^4 - (a^4 - 4*a^2*b^2 + 3*
b^4)*c^2 + (a^4*b^2 - 2*a^2*b^4 + b^6 + 2*b*c^5 + c^6 + (2*a^2 - b^2)*c^4 +
 4*(a^2*b - b^3)*c^3 + (a^4 - b^4)*c^2 + 2*(a^4*b - 2*a^2*b^3 + b^5)*c)*cos
h(x)^2 + (a^4*b^2 - 2*a^2*b^4 + b^6 + 2*b*c^5 + c^6 + (2*a^2 - b^2)*c^4 + 4
*(a^2*b - b^3)*c^3 + (a^4 - b^4)*c^2 + 2*(a^4*b - 2*a^2*b^3 + b^5)*c)*sinh(
x)^2 + 2*(a^5*b - 2*a^3*b^3 + a*b^5 + a*b*c^4 + a*c^5 + 2*(a^3 - a*b^2)*c^3
 + 2*(a^3*b - a*b^3)*c^2 + (a^5 - 2*a^3*b^2 + a*b^4)*c)*cosh(x) + 2*(a^5*b
- 2*a^3*b^3 + a*b^5 + a*b*c^4 + a*c^5 + 2*(a^3 - a*b^2)*c^3 + 2*(a^3*b - a*
b^3)*c^2 + (a^5 - 2*a^3*b^2 + a*b^4)*c + (a^4*b^2 - 2*a^2*b^4 + b^6 + 2*b*c
^5 + c^6 + (2*a^2 - b^2)*c^4 + 4*(a^2*b - b^3)*c^3 + (a^4 - b^4)*c^2 + 2*(a
^4*b - 2*a^2*b^3 + b^5)*c)*cosh(x))*sinh(x)), -2*(B*a^3*b - B*a*b^3 + B*a*b
*c^2 - C*a*c^3 + (B*b^3 - C*b^2*c - B*b*c^2 + C*c^3 + (B*b^3 + (2*B - C)*b^
2*c + (B - 2*C)*b*c^2 - C*c^3)*cosh(x)^2 + (B*b^3 + (2*B - C)*b^2*c + (B -
2*C)*b*c^2 - C*c^3)*sinh(x)^2 + 2*(B*a*b^2 + (B - C)*a*b*c - C*a*c^2)*cosh(
x) + 2*(B*a*b^2 + (B - C)*a*b*c - C*a*c^2 + (B*b^3 + (2*B - C)*b^2*c + (B -
 2*C)*b*c^2 - C*c^3)*cosh(x))*sinh(x))*sqrt(-a^2 + b^2 - c^2)*arctan(sqrt(-
a^2 + b^2 - c^2)*((b + c)*cosh(x) + (b + c)*sinh(x) + a)/(a^2 - b^2 + c^2))
 - (C*a^3 - C*a*b^2)*c + ((B + C)*a^4 - (B + 2*C)*a^2*b^2 + C*b^4 + (B - C)
*b*c^3 + B*c^4 + ((2*B + C)*a^2 - (B + C)*b^2)*c^2 + ((B - C)*a^2*b - (B -
```

```
C)*b^3)*c)*cosh(x) + ((B + C)*a^4 - (B + 2*C)*a^2*b^2 + C*b^4 + (B - C)*b*c
^3 + B*c^4 + ((2*B + C)*a^2 - (B + C)*b^2)*c^2 + ((B - C)*a^2*b - (B - C)*b
^3)*c)*sinh(x))/(a^4*b^2 - 2*a^2*b^4 + b^6 - c^6 - (2*a^2 - 3*b^2)*c^4 - (a
^4 - 4*a^2*b^2 + 3*b^4)*c^2 + (a^4*b^2 - 2*a^2*b^4 + b^6 + 2*b*c^5 + c^6 +
(2*a^2 - b^2)*c^4 + 4*(a^2*b - b^3)*c^3 + (a^4 - b^4)*c^2 + 2*(a^4*b - 2*a^
2*b^3 + b^5)*c)*cosh(x)^2 + (a^4*b^2 - 2*a^2*b^4 + b^6 + 2*b*c^5 + c^6 + (2
*a^2 - b^2)*c^4 + 4*(a^2*b - b^3)*c^3 + (a^4 - b^4)*c^2 + 2*(a^4*b - 2*a^2*
b^3 + b^5)*c)*sinh(x)^2 + 2*(a^5*b - 2*a^3*b^3 + a*b^5 + a*b*c^4 + a*c^5 +
2*(a^3 - a*b^2)*c^3 + 2*(a^3*b - a*b^3)*c^2 + (a^5 - 2*a^3*b^2 + a*b^4)*c)*
cosh(x) + 2*(a^5*b - 2*a^3*b^3 + a*b^5 + a*b*c^4 + a*c^5 + 2*(a^3 - a*b^2)*
c^3 + 2*(a^3*b - a*b^3)*c^2 + (a^5 - 2*a^3*b^2 + a*b^4)*c + (a^4*b^2 - 2*a^
2*b^4 + b^6 + 2*b*c^5 + c^6 + (2*a^2 - b^2)*c^4 + 4*(a^2*b - b^3)*c^3 + (a^
4 - b^4)*c^2 + 2*(a^4*b - 2*a^2*b^3 + b^5)*c)*cosh(x))*sinh(x))]
```

giac [A] time = 0.12, size = 179, normalized size = 1.66

$$-\frac{2\,(Bb-Cc)\arctan\left(\frac{be^{x}+ce^{x}+a}{\sqrt{-a^{2}+b^{2}-c^{2}}}\right)}{\left(a^{2}-b^{2}+c^{2}\right)\sqrt{-a^{2}+b^{2}-c^{2}}}-\frac{2\left(Ba^{2}e^{x}+Ca^{2}e^{x}-Cb^{2}e^{x}+Bbce^{x}-Cbce^{x}+Bc^{2}e^{x}+Bab-Cac\right)}{\left(a^{2}b-b^{3}+a^{2}c-b^{2}c+bc^{2}+c^{3}\right)\left(be^{(2\,x)}+ce^{(2\,x)}+2\,ae^{x}+b-c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cosh(x)+C*sinh(x))/(a+b*cosh(x)+c*sinh(x))^2,x, algorithm="gia
c")
```

```
[Out] -2*(B*b - C*c)*arctan((b*e^x + c*e^x + a)/sqrt(-a^2 + b^2 - c^2))/((a^2 - b
^2 + c^2)*sqrt(-a^2 + b^2 - c^2)) - 2*(B*a^2*e^x + C*a^2*e^x - C*b^2*e^x +
B*b*c*e^x - C*b*c*e^x + B*c^2*e^x + B*a*b - C*a*c)/((a^2*b - b^3 + a^2*c -
b^2*c + b*c^2 + c^3)*(b*e^(2*x) + c*e^(2*x) + 2*a*e^x + b - c))
```

maple [B] time = 0.27, size = 287, normalized size = 2.66

$$\frac{-\frac{2\left(B\,a^{2}-bBa+B\,c^{2}+acC-Ccb\right)\tanh\left(\frac{x}{2}\right)}{a^{3}-a^{2}b-a\,b^{2}+a\,c^{2}+b^{3}-b\,c^{2}}-\frac{2\left(bBc+a^{2}C-C\,b^{2}\right)}{a^{3}-a^{2}b-a\,b^{2}+a\,c^{2}+b^{3}-b\,c^{2}}}{a\left(\tanh^{2}\left(\frac{x}{2}\right)\right)-\left(\tanh^{2}\left(\frac{x}{2}\right)\right)b-2c\tanh\left(\frac{x}{2}\right)-a-b}+\frac{2\arctan\left(\frac{2(a-b)\tanh\left(\frac{x}{2}\right)-2c}{2\sqrt{-a^{2}+b^{2}-c^{2}}}\right)Bb}{\left(a^{2}-b^{2}+c^{2}\right)\sqrt{-a^{2}+b^{2}-c^{2}}}-\frac{2\arctan\left(\frac{2(a-b)\tanh\left(\frac{x}{2}\right)-2c}{2\sqrt{-a^{2}+b^{2}-c^{2}}}\right)}{\left(a^{2}-b^{2}+c^{2}\right)\sqrt{-a^{2}+b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*cosh(x)+C*sinh(x))/(a+b*cosh(x)+c*sinh(x))^2,x)
```

```
[Out] 2*(-(B*a^2-B*a*b+B*c^2+C*a*c-C*b*c)/(a^3-a^2*b-a*b^2+a*c^2+b^3-b*c^2)*tanh(
1/2*x)-(B*b*c+C*a^2-C*b^2)/(a^3-a^2*b-a*b^2+a*c^2+b^3-b*c^2))/(a*tanh(1/2*x
)^2-tanh(1/2*x)^2*b-2*c*tanh(1/2*x)-a-b)+2/(a^2-b^2+c^2)/(-a^2+b^2-c^2)^(1/
2)*arctan(1/2*(2*(a-b)*tanh(1/2*x)-2*c)/(-a^2+b^2-c^2)^(1/2))*B*b-2/(a^2-b^
2+c^2)/(-a^2+b^2-c^2)^(1/2)*arctan(1/2*(2*(a-b)*tanh(1/2*x)-2*c)/(-a^2+b^2-
c^2)^(1/2))*C*c
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cosh(x)+C*sinh(x))/(a+b*cosh(x)+c*sinh(x))^2,x, algorithm="max
ima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(c^2-b^2+a^2>0)', see `assume?` for
more details)Is c^2-b^2+a^2 positive or negative?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{B\cosh(x) + C\sinh(x)}{(a + b\cosh(x) + c\sinh(x))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*cosh(x) + C*sinh(x))/(a + b*cosh(x) + c*sinh(x))^2,x)
```

```
[Out] int((B*cosh(x) + C*sinh(x))/(a + b*cosh(x) + c*sinh(x))^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cosh(x)+C*sinh(x))/(a+b*cosh(x)+c*sinh(x))**2,x)
```

```
[Out] Timed out
```

3.797 $\int \frac{B\cosh(x)+C\sinh(x)}{(a+b\cosh(x)+c\sinh(x))^3}\,dx$

Optimal. Leaf size=194

$$\frac{3a(bB-cC)\tanh^{-1}\left(\frac{c-(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2+c^2}}\right)}{\left(a^2-b^2+c^2\right)^{5/2}}-\frac{-\cosh(x)\left(C\left(a^2-2c^2\right)+2bBc\right)-\sinh(x)\left(a^2B+2b(bB-cC)\right)+a(Bc-b}{2\left(a^2-b^2+c^2\right)^2(a+b\cosh(x)+c\sinh(x))}$$

```
[Out] 3*a*(B*b-C*c)*arctanh((c-(a-b)*tanh(1/2*x))/(a^2-b^2+c^2)^(1/2))/(a^2-b^2+c
^2)^(5/2)+1/2*(-B*c+b*C+a*C*cosh(x)+a*B*sinh(x))/(a^2-b^2+c^2)/(a+b*cosh(x)
+c*sinh(x))^2+1/2*(-a*(B*c-C*b)+(2*b*B*c+(a^2-2*c^2)*C)*cosh(x)+(a^2*B+2*b*
(B*b-C*c))*sinh(x))/(a^2-b^2+c^2)^2/(a+b*cosh(x)+c*sinh(x))
```

Rubi [A] time = 0.26, antiderivative size = 194, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {3156, 3153, 3124, 618, 206}

$$\frac{3a(bB-cC)\tanh^{-1}\left(\frac{c-(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2+c^2}}\right)}{\left(a^2-b^2+c^2\right)^{5/2}}-\frac{-\cosh(x)\left(C\left(a^2-2c^2\right)+2bBc\right)-\sinh(x)\left(a^2B+2b(bB-cC)\right)+a(Bc-b}{2\left(a^2-b^2+c^2\right)^2(a+b\cosh(x)+c\sinh(x))}$$

Antiderivative was successfully verified.

```
[In] Int[(B*Cosh[x] + C*Sinh[x])/(a + b*Cosh[x] + c*Sinh[x])^3,x]
```

```
[Out] (3*a*(b*B - c*C)*ArcTanh[(c - (a - b)*Tanh[x/2])/Sqrt[a^2 - b^2 + c^2]])/(a
^2 - b^2 + c^2)^(5/2) - (B*c - b*C - a*C*Cosh[x] - a*B*Sinh[x])/(2*(a^2 - b
^2 + c^2)*(a + b*Cosh[x] + c*Sinh[x])^2) - (a*(B*c - b*C) - (2*b*B*c + (a^2
 - 2*c^2)*C)*Cosh[x] - (a^2*B + 2*b*(b*B - c*C))*Sinh[x])/(2*(a^2 - b^2 + c
^2)^2*(a + b*Cosh[x] + c*Sinh[x]))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 3124

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Module[{f = FreeFactors[Tan[(d + e*x)/2], x]}, Dist[(2*f
)/e, Subst[Int[1/(a + b + 2*c*f*x + (a - b)*f^2*x^2), x], x, Tan[(d + e*x)/
2]/f], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
```

Rule 3153

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])^2,
 x_Symbol] :> Simp[(c*B - b*C - (a*C - c*A)*Cos[d + e*x] + (a*B - b*A)*Sin[
d + e*x])/(e*(a^2 - b^2 - c^2)*(a + b*Cos[d + e*x] + c*Sin[d + e*x])), x] +
 Dist[(a*A - b*B - c*C)/(a^2 - b^2 - c^2), Int[1/(a + b*Cos[d + e*x] + c*Si
n[d + e*x]), x], x] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && NeQ[a^2 - b^2
- c^2, 0] && NeQ[a*A - b*B - c*C, 0]
```

Rule 3156

```
Int[((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])
^(n_)*((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)
]), x_Symbol] :> -Simp[((c*B - b*C - (a*C - c*A)*Cos[d + e*x] + (a*B - b*A)
*Sin[d + e*x])*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1))/(e*(n + 1)*(a
^2 - b^2 - c^2)), x] + Dist[1/((n + 1)*(a^2 - b^2 - c^2)), Int[(a + b*Cos[d
 + e*x] + c*Sin[d + e*x])^(n + 1)*Simp[(n + 1)*(a*A - b*B - c*C) + (n + 2)*
(a*B - b*A)*Cos[d + e*x] + (n + 2)*(a*C - c*A)*Sin[d + e*x], x], x], x] /;
FreeQ[{a, b, c, d, e, A, B, C}, x] && LtQ[n, -1] && NeQ[a^2 - b^2 - c^2, 0]
 && NeQ[n, -2]
```

Rubi steps

$$\int \frac{B\cosh(x)+C\sinh(x)}{(a+b\cosh(x)+c\sinh(x))^3}\,dx = -\frac{Bc-bC-aC\cosh(x)-aB\sinh(x)}{2\left(a^2-b^2+c^2\right)(a+b\cosh(x)+c\sinh(x))^2} - \frac{\int \frac{2(bB-cC)-aB\cosh(x)-aC\sinh}{(a+b\cosh(x)+c\sinh(x))^2}}{2\left(a^2-b^2+c^2\right)}$$

$$= -\frac{Bc-bC-aC\cosh(x)-aB\sinh(x)}{2\left(a^2-b^2+c^2\right)(a+b\cosh(x)+c\sinh(x))^2} - \frac{a(Bc-bC)-\left(2bBc+\left(a^2\right.\right.}{2\left(a^2-b^2\right.}$$

$$= -\frac{Bc-bC-aC\cosh(x)-aB\sinh(x)}{2\left(a^2-b^2+c^2\right)(a+b\cosh(x)+c\sinh(x))^2} - \frac{a(Bc-bC)-\left(2bBc+\left(a^2\right.\right.}{2\left(a^2-b^2\right.}$$

$$= -\frac{Bc-bC-aC\cosh(x)-aB\sinh(x)}{2\left(a^2-b^2+c^2\right)(a+b\cosh(x)+c\sinh(x))^2} - \frac{a(Bc-bC)-\left(2bBc+\left(a^2\right.\right.}{2\left(a^2-b^2\right.}$$

$$= \frac{3a(bB-cC)\tanh^{-1}\left(\frac{c-(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2+c^2}}\right)}{\left(a^2-b^2+c^2\right)^{5/2}} - \frac{Bc-bC-aC\cosh(x)-aB\sin}{2\left(a^2-b^2+c^2\right)(a+b\cosh(x)+c\,\mathrm{s}}$$

Mathematica [A] time = 0.75, size = 319, normalized size = 1.64

$$\frac{-2a^4C+4a^3bB\sinh(x)-4a^3cC\sinh(x)+c\cosh(2x)\left(a^2+2b^2-2c^2\right)(bB-cC)+a^2b^2B\sinh(2x)+4a^2b^2C-}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(B*Cosh[x] + C*Sinh[x])/(a + b*Cosh[x] + c*Sinh[x])^3,x]
```

```
[Out] (-3*a*(b*B - c*C)*ArcTan[(c + (-a + b)*Tanh[x/2])/Sqrt[-a^2 + b^2 - c^2]])/
(-a^2 + b^2 - c^2)^(5/2) + (-9*a^2*b*B*c - 2*a^4*C + 4*a^2*b^2*C - 2*b^4*C
+ 5*a^2*c^2*C + 4*b^2*c^2*C - 2*c^4*C - 6*a*b*c*(b*B - c*C)*Cosh[x] + c*(a^
2 + 2*b^2 - 2*c^2)*(b*B - c*C)*Cosh[2*x] + 4*a^3*b*B*Sinh[x] + 2*a*b^3*B*Si
nh[x] - 8*a*b*B*c^2*Sinh[x] - 4*a^3*c*C*Sinh[x] - 2*a*b^2*c*C*Sinh[x] + 8*a
*c^3*C*Sinh[x] + a^2*b^2*B*Sinh[2*x] + 2*b^4*B*Sinh[2*x] - 2*b^2*B*c^2*Sinh
[2*x] - a^2*b*c*C*Sinh[2*x] - 2*b^3*c*C*Sinh[2*x] + 2*b*c^3*C*Sinh[2*x])/(4
*b*(a^2 - b^2 + c^2)^2*(a + b*Cosh[x] + c*Sinh[x])^2)
```

fricas [B] time = 0.65, size = 10107, normalized size = 52.10

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cosh(x)+C*sinh(x))/(a+b*cosh(x)+c*sinh(x))^3,x, algorithm="fri
cas")
```

```
[Out] [-1/2*(2*B*a^4*b^2 + 2*B*a^2*b^4 - 4*B*b^6 + 4*(B + C)*b*c^5 - 4*C*c^6 - 2*
(C*a^2 + 2*(B - 2*C)*b^2)*c^4 + 2*((B + C)*a^2*b - 4*(B + C)*b^3)*c^3 + 6*(
B*a^3*b^3 - B*a*b^5 + (B - 2*C)*a*b*c^4 - C*a*c^5 - (C*a^3 - 2*B*a*b^2)*c^3
 + ((B - 2*C)*a^3*b + 2*C*a*b^3)*c^2 + ((2*B - C)*a^3*b^2 - (2*B - C)*a*b^4
)*c)*cosh(x)^3 + 6*(B*a^3*b^3 - B*a*b^5 + (B - 2*C)*a*b*c^4 - C*a*c^5 - (C*
a^3 - 2*B*a*b^2)*c^3 + ((B - 2*C)*a^3*b + 2*C*a*b^3)*c^2 + ((2*B - C)*a^3*b
^2 - (2*B - C)*a*b^4)*c)*sinh(x)^3 + 2*(C*a^4 - (B - C)*a^2*b^2 + 2*(2*B -
C)*b^4)*c^2 + 2*(2*(B + C)*a^6 + 3*(B - 2*C)*a^4*b^2 - 3*(B - 2*C)*a^2*b^4
- 2*(B + C)*b^6 + 9*(B - C)*a^2*b*c^3 + 2*(B + C)*c^6 + 3*((2*B - C)*a^2 -
2*(B + C)*b^2)*c^4 + 3*((2*B - C)*a^4 - (B + C)*a^2*b^2 + 2*(B + C)*b^4)*c^
2 + 9*((B - C)*a^4*b - (B - C)*a^2*b^3)*c)*cosh(x)^2 + 2*(2*(B + C)*a^6 + 3
*(B - 2*C)*a^4*b^2 - 3*(B - 2*C)*a^2*b^4 - 2*(B + C)*b^6 + 9*(B - C)*a^2*b*
c^3 + 2*(B + C)*c^6 + 3*((2*B - C)*a^2 - 2*(B + C)*b^2)*c^4 + 3*((2*B - C)*
a^4 - (B + C)*a^2*b^2 + 2*(B + C)*b^4)*c^2 + 9*((B - C)*a^4*b - (B - C)*a^2
*b^3)*c + 9*(B*a^3*b^3 - B*a*b^5 + (B - 2*C)*a*b*c^4 - C*a*c^5 - (C*a^3 - 2
*B*a*b^2)*c^3 + ((B - 2*C)*a^3*b + 2*C*a*b^3)*c^2 + ((2*B - C)*a^3*b^2 - (2
*B - C)*a*b^4)*c)*cosh(x))*sinh(x)^2 + 3*(B*a*b^4 - (B + C)*a*b^3*c - (B -
C)*a*b^2*c^2 + (B + C)*a*b*c^3 - C*a*c^4 + (B*a*b^4 + (3*B - C)*a*b^3*c + 3
*(B - C)*a*b^2*c^2 + (B - 3*C)*a*b*c^3 - C*a*c^4)*cosh(x)^4 + (B*a*b^4 + (3
*B - C)*a*b^3*c + 3*(B - C)*a*b^2*c^2 + (B - 3*C)*a*b*c^3 - C*a*c^4)*sinh(x
)^4 + 4*(B*a^2*b^3 + (2*B - C)*a^2*b^2*c + (B - 2*C)*a^2*b*c^2 - C*a^2*c^3)
*cosh(x)^3 + 4*(B*a^2*b^3 + (2*B - C)*a^2*b^2*c + (B - 2*C)*a^2*b*c^2 - C*a
^2*c^3 + (B*a*b^4 + (3*B - C)*a*b^3*c + 3*(B - C)*a*b^2*c^2 + (B - 3*C)*a*b
*c^3 - C*a*c^4)*cosh(x))*sinh(x)^3 + 2*(2*B*a^3*b^2 + B*a*b^4 - (B - C)*a*b
*c^3 + C*a*c^4 - (2*C*a^3 + (B + C)*a*b^2)*c^2 + (2*(B - C)*a^3*b + (B - C)
*a*b^3)*c)*cosh(x)^2 + 2*(2*B*a^3*b^2 + B*a*b^4 - (B - C)*a*b*c^3 + C*a*c^4
 - (2*C*a^3 + (B + C)*a*b^2)*c^2 + 3*(B*a*b^4 + (3*B - C)*a*b^3*c + 3*(B -
C)*a*b^2*c^2 + (B - 3*C)*a*b*c^3 - C*a*c^4)*cosh(x)^2 + (2*(B - C)*a^3*b +
(B - C)*a*b^3)*c + 6*(B*a^2*b^3 + (2*B - C)*a^2*b^2*c + (B - 2*C)*a^2*b*c^2
 - C*a^2*c^3)*cosh(x))*sinh(x)^2 + 4*(B*a^2*b^3 - C*a^2*b^2*c - B*a^2*b*c^2
 + C*a^2*c^3)*cosh(x) + 4*(B*a^2*b^3 - C*a^2*b^2*c - B*a^2*b*c^2 + C*a^2*c^
3 + (B*a*b^4 + (3*B - C)*a*b^3*c + 3*(B - C)*a*b^2*c^2 + (B - 3*C)*a*b*c^3
- C*a*c^4)*cosh(x)^3 + 3*(B*a^2*b^3 + (2*B - C)*a^2*b^2*c + (B - 2*C)*a^2*b
*c^2 - C*a^2*c^3)*cosh(x)^2 + (2*B*a^3*b^2 + B*a*b^4 - (B - C)*a*b*c^3 + C*
a*c^4 - (2*C*a^3 + (B + C)*a*b^2)*c^2 + (2*(B - C)*a^3*b + (B - C)*a*b^3)*c
)*cosh(x))*sinh(x))*sqrt(a^2 - b^2 + c^2)*log(((b^2 + 2*b*c + c^2)*cosh(x)^
2 + (b^2 + 2*b*c + c^2)*sinh(x)^2 + 2*a^2 - b^2 + c^2 + 2*(a*b + a*c)*cosh(
x) + 2*(a*b + a*c + (b^2 + 2*b*c + c^2)*cosh(x))*sinh(x) - 2*sqrt(a^2 - b^2
 + c^2)*((b + c)*cosh(x) + (b + c)*sinh(x) + a))/((b + c)*cosh(x)^2 + (b +
c)*sinh(x)^2 + 2*a*cosh(x) + 2*((b + c)*cosh(x) + a)*sinh(x) + b - c)) - 2*
```

```
((B + C)*a^4*b + (B + C)*a^2*b^3 - 2*(B + C)*b^5)*c + 2*(4*B*a^5*b + B*a^3*
b^3 - 5*B*a*b^5 - 5*B*a*b*c^4 + 5*C*a*c^5 + (C*a^3 - 10*C*a*b^2)*c^3 - (B*a
^3*b - 10*B*a*b^3)*c^2 - (4*C*a^5 + C*a^3*b^2 - 5*C*a*b^4)*c)*cosh(x) + 2*(
4*B*a^5*b + B*a^3*b^3 - 5*B*a*b^5 - 5*B*a*b*c^4 + 5*C*a*c^5 + (C*a^3 - 10*C
*a*b^2)*c^3 - (B*a^3*b - 10*B*a*b^3)*c^2 + 9*(B*a^3*b^3 - B*a*b^5 + (B - 2*
C)*a*b*c^4 - C*a*c^5 - (C*a^3 - 2*B*a*b^2)*c^3 + ((B - 2*C)*a^3*b + 2*C*a*b
^3)*c^2 + ((2*B - C)*a^3*b^2 - (2*B - C)*a*b^4)*c)*cosh(x)^2 - (4*C*a^5 + C
*a^3*b^2 - 5*C*a*b^4)*c + 2*(2*(B + C)*a^6 + 3*(B - 2*C)*a^4*b^2 - 3*(B - 2
*C)*a^2*b^4 - 2*(B + C)*b^6 + 9*(B - C)*a^2*b*c^3 + 2*(B + C)*c^6 + 3*((2*B
 - C)*a^2 - 2*(B + C)*b^2)*c^4 + 3*((2*B - C)*a^4 - (B + C)*a^2*b^2 + 2*(B
+ C)*b^4)*c^2 + 9*((B - C)*a^4*b - (B - C)*a^2*b^3)*c)*cosh(x))*sinh(x))/(a
^6*b^3 - 3*a^4*b^5 + 3*a^2*b^7 - b^9 - b*c^8 + c^9 + (3*a^2 - 4*b^2)*c^7 -
(3*a^2*b - 4*b^3)*c^6 + 3*(a^4 - 3*a^2*b^2 + 2*b^4)*c^5 - 3*(a^4*b - 3*a^2*
b^3 + 2*b^5)*c^4 + (a^6*b^3 - 3*a^4*b^5 + 3*a^2*b^7 - b^9 + 3*a^2*c^7 + 3*b
*c^8 + c^9 + (9*a^2*b - 8*b^3)*c^6 + 3*(a^4 + a^2*b^2 - 2*b^4)*c^5 + 3*(3*a
^4*b - 5*a^2*b^3 + 2*b^5)*c^4 + (a^6 + 6*a^4*b^2 - 15*a^2*b^4 + 8*b^6)*c^3
+ 3*(a^6*b - 2*a^4*b^3 + a^2*b^5)*c^2 + 3*(a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^6
- b^8)*c)*cosh(x)^4 + (a^6*b^3 - 3*a^4*b^5 + 3*a^2*b^7 - b^9 + 3*a^2*c^7 +
3*b*c^8 + c^9 + (9*a^2*b - 8*b^3)*c^6 + 3*(a^4 + a^2*b^2 - 2*b^4)*c^5 + 3*(
3*a^4*b - 5*a^2*b^3 + 2*b^5)*c^4 + (a^6 + 6*a^4*b^2 - 15*a^2*b^4 + 8*b^6)*c
^3 + 3*(a^6*b - 2*a^4*b^3 + a^2*b^5)*c^2 + 3*(a^6*b^2 - 3*a^4*b^4 + 3*a^2*b
^6 - b^8)*c)*sinh(x)^4 + (a^6 - 6*a^4*b^2 + 9*a^2*b^4 - 4*b^6)*c^3 + 4*(a^7
*b^2 - 3*a^5*b^4 + 3*a^3*b^6 - a*b^8 + 2*a*b*c^7 + a*c^8 + (3*a^3 - 2*a*b^2
)*c^6 + 6*(a^3*b - a*b^3)*c^5 + 3*(a^5 - a^3*b^2)*c^4 + 6*(a^5*b - 2*a^3*b^
3 + a*b^5)*c^3 + (a^7 - 3*a^3*b^4 + 2*a*b^6)*c^2 + 2*(a^7*b - 3*a^5*b^3 + 3
*a^3*b^5 - a*b^7)*c)*cosh(x)^3 + 4*(a^7*b^2 - 3*a^5*b^4 + 3*a^3*b^6 - a*b^8
 + 2*a*b*c^7 + a*c^8 + (3*a^3 - 2*a*b^2)*c^6 + 6*(a^3*b - a*b^3)*c^5 + 3*(a
^5 - a^3*b^2)*c^4 + 6*(a^5*b - 2*a^3*b^3 + a*b^5)*c^3 + (a^7 - 3*a^3*b^4 +
2*a*b^6)*c^2 + 2*(a^7*b - 3*a^5*b^3 + 3*a^3*b^5 - a*b^7)*c + (a^6*b^3 - 3*a
^4*b^5 + 3*a^2*b^7 - b^9 + 3*a^2*c^7 + 3*b*c^8 + c^9 + (9*a^2*b - 8*b^3)*c^
6 + 3*(a^4 + a^2*b^2 - 2*b^4)*c^5 + 3*(3*a^4*b - 5*a^2*b^3 + 2*b^5)*c^4 + (
a^6 + 6*a^4*b^2 - 15*a^2*b^4 + 8*b^6)*c^3 + 3*(a^6*b - 2*a^4*b^3 + a^2*b^5)
*c^2 + 3*(a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^8)*c)*cosh(x))*sinh(x)^3 - (a
^6*b - 6*a^4*b^3 + 9*a^2*b^5 - 4*b^7)*c^2 + 2*(2*a^8*b - 5*a^6*b^3 + 3*a^4*
b^5 + a^2*b^7 - b^9 - b*c^8 - c^9 - (a^2 - 4*b^2)*c^7 - (a^2*b - 4*b^3)*c^6
 + 3*(a^4 + a^2*b^2 - 2*b^4)*c^5 + 3*(a^4*b + a^2*b^3 - 2*b^5)*c^4 + (5*a^6
 - 6*a^4*b^2 - 3*a^2*b^4 + 4*b^6)*c^3 + (5*a^6*b - 6*a^4*b^3 - 3*a^2*b^5 +
4*b^7)*c^2 + (2*a^8 - 5*a^6*b^2 + 3*a^4*b^4 + a^2*b^6 - b^8)*c)*cosh(x)^2 +
 2*(2*a^8*b - 5*a^6*b^3 + 3*a^4*b^5 + a^2*b^7 - b^9 - b*c^8 - c^9 - (a^2 -
4*b^2)*c^7 - (a^2*b - 4*b^3)*c^6 + 3*(a^4 + a^2*b^2 - 2*b^4)*c^5 + 3*(a^4*b
 + a^2*b^3 - 2*b^5)*c^4 + (5*a^6 - 6*a^4*b^2 - 3*a^2*b^4 + 4*b^6)*c^3 + (5*
a^6*b - 6*a^4*b^3 - 3*a^2*b^5 + 4*b^7)*c^2 + 3*(a^6*b^3 - 3*a^4*b^5 + 3*a^2
*b^7 - b^9 + 3*a^2*c^7 + 3*b*c^8 + c^9 + (9*a^2*b - 8*b^3)*c^6 + 3*(a^4 + a
^2*b^2 - 2*b^4)*c^5 + 3*(3*a^4*b - 5*a^2*b^3 + 2*b^5)*c^4 + (a^6 + 6*a^4*b^
2 - 15*a^2*b^4 + 8*b^6)*c^3 + 3*(a^6*b - 2*a^4*b^3 + a^2*b^5)*c^2 + 3*(a^6*
```

b^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^8)*c)*cosh(x)^2 + (2*a^8 - 5*a^6*b^2 + 3*a^
4*b^4 + a^2*b^6 - b^8)*c + 6*(a^7*b^2 - 3*a^5*b^4 + 3*a^3*b^6 - a*b^8 + 2*a
*b*c^7 + a*c^8 + (3*a^3 - 2*a*b^2)*c^6 + 6*(a^3*b - a*b^3)*c^5 + 3*(a^5 - a
^3*b^2)*c^4 + 6*(a^5*b - 2*a^3*b^3 + a*b^5)*c^3 + (a^7 - 3*a^3*b^4 + 2*a*b^
6)*c^2 + 2*(a^7*b - 3*a^5*b^3 + 3*a^3*b^5 - a*b^7)*c)*cosh(x))*sinh(x)^2 -
(a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^8)*c + 4*(a^7*b^2 - 3*a^5*b^4 + 3*a^3*
b^6 - a*b^8 - a*c^8 - (3*a^3 - 4*a*b^2)*c^6 - 3*(a^5 - 3*a^3*b^2 + 2*a*b^4)
*c^4 - (a^7 - 6*a^5*b^2 + 9*a^3*b^4 - 4*a*b^6)*c^2)*cosh(x) + 4*(a^7*b^2 -
3*a^5*b^4 + 3*a^3*b^6 - a*b^8 - a*c^8 - (3*a^3 - 4*a*b^2)*c^6 - 3*(a^5 - 3*
a^3*b^2 + 2*a*b^4)*c^4 + (a^6*b^3 - 3*a^4*b^5 + 3*a^2*b^7 - b^9 + 3*a^2*c^7
 + 3*b*c^8 + c^9 + (9*a^2*b - 8*b^3)*c^6 + 3*(a^4 + a^2*b^2 - 2*b^4)*c^5 +
3*(3*a^4*b - 5*a^2*b^3 + 2*b^5)*c^4 + (a^6 + 6*a^4*b^2 - 15*a^2*b^4 + 8*b^6
)*c^3 + 3*(a^6*b - 2*a^4*b^3 + a^2*b^5)*c^2 + 3*(a^6*b^2 - 3*a^4*b^4 + 3*a^
2*b^6 - b^8)*c)*cosh(x)^3 - (a^7 - 6*a^5*b^2 + 9*a^3*b^4 - 4*a*b^6)*c^2 + 3
*(a^7*b^2 - 3*a^5*b^4 + 3*a^3*b^6 - a*b^8 + 2*a*b*c^7 + a*c^8 + (3*a^3 - 2*
a*b^2)*c^6 + 6*(a^3*b - a*b^3)*c^5 + 3*(a^5 - a^3*b^2)*c^4 + 6*(a^5*b - 2*a
^3*b^3 + a*b^5)*c^3 + (a^7 - 3*a^3*b^4 + 2*a*b^6)*c^2 + 2*(a^7*b - 3*a^5*b^
3 + 3*a^3*b^5 - a*b^7)*c)*cosh(x)^2 + (2*a^8*b - 5*a^6*b^3 + 3*a^4*b^5 + a^
2*b^7 - b^9 - b*c^8 - c^9 - (a^2 - 4*b^2)*c^7 - (a^2*b - 4*b^3)*c^6 + 3*(a^
4 + a^2*b^2 - 2*b^4)*c^5 + 3*(a^4*b + a^2*b^3 - 2*b^5)*c^4 + (5*a^6 - 6*a^4
*b^2 - 3*a^2*b^4 + 4*b^6)*c^3 + (5*a^6*b - 6*a^4*b^3 - 3*a^2*b^5 + 4*b^7)*c
^2 + (2*a^8 - 5*a^6*b^2 + 3*a^4*b^4 + a^2*b^6 - b^8)*c)*cosh(x))*sinh(x)),
-(B*a^4*b^2 + B*a^2*b^4 - 2*B*b^6 + 2*(B + C)*b*c^5 - 2*C*c^6 - (C*a^2 + 2*
(B - 2*C)*b^2)*c^4 + ((B + C)*a^2*b - 4*(B + C)*b^3)*c^3 + 3*(B*a^3*b^3 - B
*a*b^5 + (B - 2*C)*a*b*c^4 - C*a*c^5 - (C*a^3 - 2*B*a*b^2)*c^3 + ((B - 2*C)
*a^3*b + 2*C*a*b^3)*c^2 + ((2*B - C)*a^3*b^2 - (2*B - C)*a*b^4)*c)*cosh(x)^
3 + 3*(B*a^3*b^3 - B*a*b^5 + (B - 2*C)*a*b*c^4 - C*a*c^5 - (C*a^3 - 2*B*a*b
^2)*c^3 + ((B - 2*C)*a^3*b + 2*C*a*b^3)*c^2 + ((2*B - C)*a^3*b^2 - (2*B - C
)*a*b^4)*c)*sinh(x)^3 + (C*a^4 - (B - C)*a^2*b^2 + 2*(2*B - C)*b^4)*c^2 + (
2*(B + C)*a^6 + 3*(B - 2*C)*a^4*b^2 - 3*(B - 2*C)*a^2*b^4 - 2*(B + C)*b^6 +
 9*(B - C)*a^2*b*c^3 + 2*(B + C)*c^6 + 3*((2*B - C)*a^2 - 2*(B + C)*b^2)*c^
4 + 3*((2*B - C)*a^4 - (B + C)*a^2*b^2 + 2*(B + C)*b^4)*c^2 + 9*((B - C)*a^
4*b - (B - C)*a^2*b^3)*c)*cosh(x)^2 + (2*(B + C)*a^6 + 3*(B - 2*C)*a^4*b^2
- 3*(B - 2*C)*a^2*b^4 - 2*(B + C)*b^6 + 9*(B - C)*a^2*b*c^3 + 2*(B + C)*c^6
 + 3*((2*B - C)*a^2 - 2*(B + C)*b^2)*c^4 + 3*((2*B - C)*a^4 - (B + C)*a^2*b
^2 + 2*(B + C)*b^4)*c^2 + 9*((B - C)*a^4*b - (B - C)*a^2*b^3)*c + 9*(B*a^3*
b^3 - B*a*b^5 + (B - 2*C)*a*b*c^4 - C*a*c^5 - (C*a^3 - 2*B*a*b^2)*c^3 + ((B
 - 2*C)*a^3*b + 2*C*a*b^3)*c^2 + ((2*B - C)*a^3*b^2 - (2*B - C)*a*b^4)*c)*c
osh(x))*sinh(x)^2 + 3*(B*a*b^4 - (B + C)*a*b^3*c - (B - C)*a*b^2*c^2 + (B +
 C)*a*b*c^3 - C*a*c^4 + (B*a*b^4 + (3*B - C)*a*b^3*c + 3*(B - C)*a*b^2*c^2
+ (B - 3*C)*a*b*c^3 - C*a*c^4)*cosh(x)^4 + (B*a*b^4 + (3*B - C)*a*b^3*c + 3
*(B - C)*a*b^2*c^2 + (B - 3*C)*a*b*c^3 - C*a*c^4)*sinh(x)^4 + 4*(B*a^2*b^3
+ (2*B - C)*a^2*b^2*c + (B - 2*C)*a^2*b*c^2 - C*a^2*c^3)*cosh(x)^3 + 4*(B*a
^2*b^3 + (2*B - C)*a^2*b^2*c + (B - 2*C)*a^2*b*c^2 - C*a^2*c^3 + (B*a*b^4 +
 (3*B - C)*a*b^3*c + 3*(B - C)*a*b^2*c^2 + (B - 3*C)*a*b*c^3 - C*a*c^4)*cos

h(x))*sinh(x)^3 + 2*(2*B*a^3*b^2 + B*a*b^4 - (B - C)*a*b*c^3 + C*a*c^4 - (2
*C*a^3 + (B + C)*a*b^2)*c^2 + (2*(B - C)*a^3*b + (B - C)*a*b^3)*c)*cosh(x)^
2 + 2*(2*B*a^3*b^2 + B*a*b^4 - (B - C)*a*b*c^3 + C*a*c^4 - (2*C*a^3 + (B +
C)*a*b^2)*c^2 + 3*(B*a*b^4 + (3*B - C)*a*b^3*c + 3*(B - C)*a*b^2*c^2 + (B -
3*C)*a*b*c^3 - C*a*c^4)*cosh(x)^2 + (2*(B - C)*a^3*b + (B - C)*a*b^3)*c +
6*(B*a^2*b^3 + (2*B - C)*a^2*b^2*c + (B - 2*C)*a^2*b*c^2 - C*a^2*c^3)*cosh(
x))*sinh(x)^2 + 4*(B*a^2*b^3 - C*a^2*b^2*c - B*a^2*b*c^2 + C*a^2*c^3)*cosh(
x) + 4*(B*a^2*b^3 - C*a^2*b^2*c - B*a^2*b*c^2 + C*a^2*c^3 + (B*a*b^4 + (3*B
- C)*a*b^3*c + 3*(B - C)*a*b^2*c^2 + (B - 3*C)*a*b*c^3 - C*a*c^4)*cosh(x)^
3 + 3*(B*a^2*b^3 + (2*B - C)*a^2*b^2*c + (B - 2*C)*a^2*b*c^2 - C*a^2*c^3)*c
osh(x)^2 + (2*B*a^3*b^2 + B*a*b^4 - (B - C)*a*b*c^3 + C*a*c^4 - (2*C*a^3 +
(B + C)*a*b^2)*c^2 + (2*(B - C)*a^3*b + (B - C)*a*b^3)*c)*cosh(x))*sinh(x))
*sqrt(-a^2 + b^2 - c^2)*arctan(sqrt(-a^2 + b^2 - c^2)*((b + c)*cosh(x) + (b
+ c)*sinh(x) + a)/(a^2 - b^2 + c^2)) - ((B + C)*a^4*b + (B + C)*a^2*b^3 -
2*(B + C)*b^5)*c + (4*B*a^5*b + B*a^3*b^3 - 5*B*a*b^5 - 5*B*a*b*c^4 + 5*C*a
*c^5 + (C*a^3 - 10*C*a*b^2)*c^3 - (B*a^3*b - 10*B*a*b^3)*c^2 - (4*C*a^5 + C
*a^3*b^2 - 5*C*a*b^4)*c)*cosh(x) + (4*B*a^5*b + B*a^3*b^3 - 5*B*a*b^5 - 5*B
*a*b*c^4 + 5*C*a*c^5 + (C*a^3 - 10*C*a*b^2)*c^3 - (B*a^3*b - 10*B*a*b^3)*c^
2 + 9*(B*a^3*b^3 - B*a*b^5 + (B - 2*C)*a*b*c^4 - C*a*c^5 - (C*a^3 - 2*B*a*b
^2)*c^3 + ((B - 2*C)*a^3*b + 2*C*a*b^3)*c^2 + ((2*B - C)*a^3*b^2 - (2*B - C
)*a*b^4)*c)*cosh(x)^2 - (4*C*a^5 + C*a^3*b^2 - 5*C*a*b^4)*c + 2*(2*(B + C)*
a^6 + 3*(B - 2*C)*a^4*b^2 - 3*(B - 2*C)*a^2*b^4 - 2*(B + C)*b^6 + 9*(B - C)
*a^2*b*c^3 + 2*(B + C)*c^6 + 3*((2*B - C)*a^2 - 2*(B + C)*b^2)*c^4 + 3*((2*
B - C)*a^4 - (B + C)*a^2*b^2 + 2*(B + C)*b^4)*c^2 + 9*((B - C)*a^4*b - (B -
C)*a^2*b^3)*c)*cosh(x))*sinh(x))/(a^6*b^3 - 3*a^4*b^5 + 3*a^2*b^7 - b^9 -
b*c^8 + c^9 + (3*a^2 - 4*b^2)*c^7 - (3*a^2*b - 4*b^3)*c^6 + 3*(a^4 - 3*a^2*
b^2 + 2*b^4)*c^5 - 3*(a^4*b - 3*a^2*b^3 + 2*b^5)*c^4 + (a^6*b^3 - 3*a^4*b^5
+ 3*a^2*b^7 - b^9 + 3*a^2*c^7 + 3*b*c^8 + c^9 + (9*a^2*b - 8*b^3)*c^6 + 3*
(a^4 + a^2*b^2 - 2*b^4)*c^5 + 3*(3*a^4*b - 5*a^2*b^3 + 2*b^5)*c^4 + (a^6 +
6*a^4*b^2 - 15*a^2*b^4 + 8*b^6)*c^3 + 3*(a^6*b - 2*a^4*b^3 + a^2*b^5)*c^2 +
3*(a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^8)*c)*cosh(x)^4 + (a^6*b^3 - 3*a^4*
b^5 + 3*a^2*b^7 - b^9 + 3*a^2*c^7 + 3*b*c^8 + c^9 + (9*a^2*b - 8*b^3)*c^6 +
3*(a^4 + a^2*b^2 - 2*b^4)*c^5 + 3*(3*a^4*b - 5*a^2*b^3 + 2*b^5)*c^4 + (a^6
+ 6*a^4*b^2 - 15*a^2*b^4 + 8*b^6)*c^3 + 3*(a^6*b - 2*a^4*b^3 + a^2*b^5)*c^
2 + 3*(a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^8)*c)*sinh(x)^4 + (a^6 - 6*a^4*b
^2 + 9*a^2*b^4 - 4*b^6)*c^3 + 4*(a^7*b^2 - 3*a^5*b^4 + 3*a^3*b^6 - a*b^8 +
2*a*b*c^7 + a*c^8 + (3*a^3 - 2*a*b^2)*c^6 + 6*(a^3*b - a*b^3)*c^5 + 3*(a^5
- a^3*b^2)*c^4 + 6*(a^5*b - 2*a^3*b^3 + a*b^5)*c^3 + (a^7 - 3*a^3*b^4 + 2*a
*b^6)*c^2 + 2*(a^7*b - 3*a^5*b^3 + 3*a^3*b^5 - a*b^7)*c)*cosh(x)^3 + 4*(a^7
*b^2 - 3*a^5*b^4 + 3*a^3*b^6 - a*b^8 + 2*a*b*c^7 + a*c^8 + (3*a^3 - 2*a*b^2
)*c^6 + 6*(a^3*b - a*b^3)*c^5 + 3*(a^5 - a^3*b^2)*c^4 + 6*(a^5*b - 2*a^3*b^
3 + a*b^5)*c^3 + (a^7 - 3*a^3*b^4 + 2*a*b^6)*c^2 + 2*(a^7*b - 3*a^5*b^3 + 3
*a^3*b^5 - a*b^7)*c + (a^6*b^3 - 3*a^4*b^5 + 3*a^2*b^7 - b^9 + 3*a^2*c^7 +
3*b*c^8 + c^9 + (9*a^2*b - 8*b^3)*c^6 + 3*(a^4 + a^2*b^2 - 2*b^4)*c^5 + 3*(
3*a^4*b - 5*a^2*b^3 + 2*b^5)*c^4 + (a^6 + 6*a^4*b^2 - 15*a^2*b^4 + 8*b^6)*c

```
^3 + 3*(a^6*b - 2*a^4*b^3 + a^2*b^5)*c^2 + 3*(a^6*b^2 - 3*a^4*b^4 + 3*a^2*b
^6 - b^8)*c)*cosh(x))*sinh(x)^3 - (a^6*b - 6*a^4*b^3 + 9*a^2*b^5 - 4*b^7)*c
^2 + 2*(2*a^8*b - 5*a^6*b^3 + 3*a^4*b^5 + a^2*b^7 - b^9 - b*c^8 - c^9 - (a^
2 - 4*b^2)*c^7 - (a^2*b - 4*b^3)*c^6 + 3*(a^4 + a^2*b^2 - 2*b^4)*c^5 + 3*(a
^4*b + a^2*b^3 - 2*b^5)*c^4 + (5*a^6 - 6*a^4*b^2 - 3*a^2*b^4 + 4*b^6)*c^3 +
 (5*a^6*b - 6*a^4*b^3 - 3*a^2*b^5 + 4*b^7)*c^2 + (2*a^8 - 5*a^6*b^2 + 3*a^4
*b^4 + a^2*b^6 - b^8)*c)*cosh(x)^2 + 2*(2*a^8*b - 5*a^6*b^3 + 3*a^4*b^5 + a
^2*b^7 - b^9 - b*c^8 - c^9 - (a^2 - 4*b^2)*c^7 - (a^2*b - 4*b^3)*c^6 + 3*(a
^4 + a^2*b^2 - 2*b^4)*c^5 + 3*(a^4*b + a^2*b^3 - 2*b^5)*c^4 + (5*a^6 - 6*a^
4*b^2 - 3*a^2*b^4 + 4*b^6)*c^3 + (5*a^6*b - 6*a^4*b^3 - 3*a^2*b^5 + 4*b^7)*
c^2 + 3*(a^6*b^3 - 3*a^4*b^5 + 3*a^2*b^7 - b^9 + 3*a^2*c^7 + 3*b*c^8 + c^9
+ (9*a^2*b - 8*b^3)*c^6 + 3*(a^4 + a^2*b^2 - 2*b^4)*c^5 + 3*(3*a^4*b - 5*a^
2*b^3 + 2*b^5)*c^4 + (a^6 + 6*a^4*b^2 - 15*a^2*b^4 + 8*b^6)*c^3 + 3*(a^6*b
- 2*a^4*b^3 + a^2*b^5)*c^2 + 3*(a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^8)*c)*c
osh(x)^2 + (2*a^8 - 5*a^6*b^2 + 3*a^4*b^4 + a^2*b^6 - b^8)*c + 6*(a^7*b^2 -
 3*a^5*b^4 + 3*a^3*b^6 - a*b^8 + 2*a*b*c^7 + a*c^8 + (3*a^3 - 2*a*b^2)*c^6
+ 6*(a^3*b - a*b^3)*c^5 + 3*(a^5 - a^3*b^2)*c^4 + 6*(a^5*b - 2*a^3*b^3 + a*
b^5)*c^3 + (a^7 - 3*a^3*b^4 + 2*a*b^6)*c^2 + 2*(a^7*b - 3*a^5*b^3 + 3*a^3*b
^5 - a*b^7)*c)*cosh(x))*sinh(x)^2 - (a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^8)
*c + 4*(a^7*b^2 - 3*a^5*b^4 + 3*a^3*b^6 - a*b^8 - a*c^8 - (3*a^3 - 4*a*b^2)
*c^6 - 3*(a^5 - 3*a^3*b^2 + 2*a*b^4)*c^4 - (a^7 - 6*a^5*b^2 + 9*a^3*b^4 - 4
*a*b^6)*c^2)*cosh(x) + 4*(a^7*b^2 - 3*a^5*b^4 + 3*a^3*b^6 - a*b^8 - a*c^8 -
 (3*a^3 - 4*a*b^2)*c^6 - 3*(a^5 - 3*a^3*b^2 + 2*a*b^4)*c^4 + (a^6*b^3 - 3*a
^4*b^5 + 3*a^2*b^7 - b^9 + 3*a^2*c^7 + 3*b*c^8 + c^9 + (9*a^2*b - 8*b^3)*c^
6 + 3*(a^4 + a^2*b^2 - 2*b^4)*c^5 + 3*(3*a^4*b - 5*a^2*b^3 + 2*b^5)*c^4 + (
a^6 + 6*a^4*b^2 - 15*a^2*b^4 + 8*b^6)*c^3 + 3*(a^6*b - 2*a^4*b^3 + a^2*b^5)
*c^2 + 3*(a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^8)*c)*cosh(x)^3 - (a^7 - 6*a^
5*b^2 + 9*a^3*b^4 - 4*a*b^6)*c^2 + 3*(a^7*b^2 - 3*a^5*b^4 + 3*a^3*b^6 - a*b
^8 + 2*a*b*c^7 + a*c^8 + (3*a^3 - 2*a*b^2)*c^6 + 6*(a^3*b - a*b^3)*c^5 + 3*
(a^5 - a^3*b^2)*c^4 + 6*(a^5*b - 2*a^3*b^3 + a*b^5)*c^3 + (a^7 - 3*a^3*b^4
+ 2*a*b^6)*c^2 + 2*(a^7*b - 3*a^5*b^3 + 3*a^3*b^5 - a*b^7)*c)*cosh(x)^2 + (
2*a^8*b - 5*a^6*b^3 + 3*a^4*b^5 + a^2*b^7 - b^9 - b*c^8 - c^9 - (a^2 - 4*b^
2)*c^7 - (a^2*b - 4*b^3)*c^6 + 3*(a^4 + a^2*b^2 - 2*b^4)*c^5 + 3*(a^4*b + a
^2*b^3 - 2*b^5)*c^4 + (5*a^6 - 6*a^4*b^2 - 3*a^2*b^4 + 4*b^6)*c^3 + (5*a^6*
b - 6*a^4*b^3 - 3*a^2*b^5 + 4*b^7)*c^2 + (2*a^8 - 5*a^6*b^2 + 3*a^4*b^4 + a
^2*b^6 - b^8)*c)*cosh(x))*sinh(x))]
```

giac [B] time = 0.18, size = 577, normalized size = 2.97

$$-\frac{3\,(Bab-Cac)\arctan\left(\frac{be^{x}+ce^{x}+a}{\sqrt{-a^{2}+b^{2}-c^{2}}}\right)}{\left(a^{4}-2\,a^{2}b^{2}+b^{4}+2\,a^{2}c^{2}-2\,b^{2}c^{2}+c^{4}\right)\sqrt{-a^{2}+b^{2}-c^{2}}}-\frac{3\,Bab^{3}e^{(3\,x)}+6\,Bab^{2}ce^{(3\,x)}-3\,Cab^{2}ce^{(3\,x)}+3\,Babc^{2}e^{(3}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cosh(x)+C*sinh(x))/(a+b*cosh(x)+c*sinh(x))^3,x, algorithm="gia
c")
```

```
[Out] -3*(B*a*b - C*a*c)*arctan((b*e^x + c*e^x + a)/sqrt(-a^2 + b^2 - c^2))/((a^4
 - 2*a^2*b^2 + b^4 + 2*a^2*c^2 - 2*b^2*c^2 + c^4)*sqrt(-a^2 + b^2 - c^2)) -
 (3*B*a*b^3*e^(3*x) + 6*B*a*b^2*c*e^(3*x) - 3*C*a*b^2*c*e^(3*x) + 3*B*a*b*c
^2*e^(3*x) - 6*C*a*b*c^2*e^(3*x) - 3*C*a*c^3*e^(3*x) + 2*B*a^4*e^(2*x) + 2*
C*a^4*e^(2*x) + 5*B*a^2*b^2*e^(2*x) - 4*C*a^2*b^2*e^(2*x) + 2*B*b^4*e^(2*x)
 + 2*C*b^4*e^(2*x) + 9*B*a^2*b*c*e^(2*x) - 9*C*a^2*b*c*e^(2*x) + 4*B*a^2*c^
2*e^(2*x) - 5*C*a^2*c^2*e^(2*x) - 4*B*b^2*c^2*e^(2*x) - 4*C*b^2*c^2*e^(2*x)
 + 2*B*c^4*e^(2*x) + 2*C*c^4*e^(2*x) + 4*B*a^3*b*e^x + 5*B*a*b^3*e^x - 4*C*
a^3*c*e^x - 5*C*a*b^2*c*e^x - 5*B*a*b*c^2*e^x + 5*C*a*c^3*e^x + B*a^2*b^2 +
 2*B*b^4 - B*a^2*b*c - C*a^2*b*c - 2*B*b^3*c - 2*C*b^3*c + C*a^2*c^2 - 2*B*
b^2*c^2 + 2*C*b^2*c^2 + 2*B*b*c^3 + 2*C*b*c^3 - 2*C*c^4)/((a^4*b - 2*a^2*b^
3 + b^5 + a^4*c - 2*a^2*b^2*c + b^4*c + 2*a^2*b*c^2 - 2*b^3*c^2 + 2*a^2*c^3
 - 2*b^2*c^3 + b*c^4 + c^5)*(b*e^(2*x) + c*e^(2*x) + 2*a*e^x + b - c)^2)
```

maple [B] time = 0.31, size = 885, normalized size = 4.56

$$-\frac{\left(2B a^4-3B a^3b+2B a^2b^2+4B a^2c^2-3Ba b^3+2B b^4-4B b^2c^2+2B c^4+3C a^3c-6C a^2bc+3Ca b^2c\right)\left(\tanh^3\left(\frac{x}{2}\right)\right)}{\left(a^4-2a^2b^2+2a^2c^2+b^4-2c^2b^2+c^4\right)(a-b)} + \frac{\left(2B a^4c-9B a^3bc+14B a^2b^2c+4B a^2c^3-9\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*cosh(x)+C*sinh(x))/(a+b*cosh(x)+c*sinh(x))^3,x)
```

```
[Out] 2*(-1/2*(2*B*a^4-3*B*a^3*b+2*B*a^2*b^2+4*B*a^2*c^2-3*B*a*b^3+2*B*b^4-4*B*b^
2*c^2+2*B*c^4+3*C*a^3*c-6*C*a^2*b*c+3*C*a*b^2*c)/(a^4-2*a^2*b^2+2*a^2*c^2+b
^4-2*b^2*c^2+c^4)/(a-b)*tanh(1/2*x)^3+1/2*(2*B*a^4*c-9*B*a^3*b*c+14*B*a^2*b
^2*c+4*B*a^2*c^3-9*B*a*b^3*c+2*B*b^4*c-4*B*b^2*c^3+2*B*c^5-2*C*a^5+2*C*a^4*
b+4*C*a^3*b^2+5*C*a^3*c^2-4*C*a^2*b^3-14*C*a^2*b*c^2-2*C*a*b^4+13*C*a*b^2*c
^2-2*C*a*c^4+2*C*b^5-4*C*b^3*c^2+2*C*b*c^4)/(a^4-2*a^2*b^2+2*a^2*c^2+b^4-2*
b^2*c^2+c^4)/(a^2-2*a*b+b^2)*tanh(1/2*x)^2+1/2*(2*B*a^5-3*B*a^4*b+B*a^3*b^2
+4*B*a^3*c^2+B*a^2*b^3+8*B*a^2*b*c^2-3*B*a*b^4-8*B*a*b^2*c^2+2*B*a*c^4+2*B*
b^5-4*B*b^3*c^2+2*B*b*c^4+5*C*a^4*c-5*C*a^3*b*c-5*C*a^2*b^2*c-4*C*a^2*c^3+5
*C*a*b^3*c+4*C*a*b*c^3)/(a^4-2*a^2*b^2+2*a^2*c^2+b^4-2*b^2*c^2+c^4)/(a^2-2*
a*b+b^2)*tanh(1/2*x)+1/2*a*(5*B*a^2*b*c-5*B*b^3*c+2*B*b*c^3+2*C*a^4-4*C*a^2
*b^2-C*a^2*c^2+2*C*b^4+C*b^2*c^2)/(a^4-2*a^2*b^2+2*a^2*c^2+b^4-2*b^2*c^2+c^
4)/(a^2-2*a*b+b^2))/(a*tanh(1/2*x)^2-tanh(1/2*x)^2*b-2*c*tanh(1/2*x)-a-b)^2
+3/(a^4-2*a^2*b^2+2*a^2*c^2+b^4-2*b^2*c^2+c^4)/(-a^2+b^2-c^2)^(1/2)*arctan(
1/2*(2*(a-b)*tanh(1/2*x)-2*c)/(-a^2+b^2-c^2)^(1/2))*b*B*a-3/(a^4-2*a^2*b^2+
2*a^2*c^2+b^4-2*b^2*c^2+c^4)/(-a^2+b^2-c^2)^(1/2)*arctan(1/2*(2*(a-b)*tanh(
1/2*x)-2*c)/(-a^2+b^2-c^2)^(1/2))*a*c*C
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cosh(x)+C*sinh(x))/(a+b*cosh(x)+c*sinh(x))^3,x, algorithm="max
ima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(c^2-b^2+a^2>0)', see `assume?` for
more details)Is c^2-b^2+a^2 positive or negative?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{B\cosh(x) + C\sinh(x)}{(a + b\cosh(x) + c\sinh(x))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*cosh(x) + C*sinh(x))/(a + b*cosh(x) + c*sinh(x))^3,x)
```

```
[Out] int((B*cosh(x) + C*sinh(x))/(a + b*cosh(x) + c*sinh(x))^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cosh(x)+C*sinh(x))/(a+b*cosh(x)+c*sinh(x))**3,x)
```

```
[Out] Timed out
```

3.798 $\int \frac{A+B\cosh(x)+C\sinh(x)}{a+b\cosh(x)+c\sinh(x)}\,dx$

Optimal. Leaf size=137

$$-\frac{2\tanh^{-1}\left(\frac{c-(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2+c^2}}\right)\left(-abB+acC+Ab^2-Ac^2\right)}{\left(b^2-c^2\right)\sqrt{a^2-b^2+c^2}}-\frac{(Bc-bC)\log(a+b\cosh(x)+c\sinh(x))}{b^2-c^2}+\frac{x(bB-cC)}{b^2-c^2}$$

```
[Out] (B*b-C*c)*x/(b^2-c^2)-(B*c-C*b)*ln(a+b*cosh(x)+c*sinh(x))/(b^2-c^2)-2*(A*b^
2-A*c^2-B*a*b+C*a*c)*arctanh((c-(a-b)*tanh(1/2*x))/(a^2-b^2+c^2)^(1/2))/(b^
2-c^2)/(a^2-b^2+c^2)^(1/2)
```

Rubi [A] time = 0.22, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3136, 3124, 618, 206}

$$-\frac{2\tanh^{-1}\left(\frac{c-(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2+c^2}}\right)\left(-abB+acC+Ab^2-Ac^2\right)}{\left(b^2-c^2\right)\sqrt{a^2-b^2+c^2}}-\frac{(Bc-bC)\log(a+b\cosh(x)+c\sinh(x))}{b^2-c^2}+\frac{x(bB-cC)}{b^2-c^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cosh[x] + C*Sinh[x])/(a + b*Cosh[x] + c*Sinh[x]),x]
```

```
[Out] ((b*B - c*C)*x)/(b^2 - c^2) - (2*(A*b^2 - a*b*B - A*c^2 + a*c*C)*ArcTanh[(c
 - (a - b)*Tanh[x/2])/Sqrt[a^2 - b^2 + c^2]])/((b^2 - c^2)*Sqrt[a^2 - b^2 +
 c^2]) - ((B*c - b*C)*Log[a + b*Cosh[x] + c*Sinh[x]])/(b^2 - c^2)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 3124

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Module[{f = FreeFactors[Tan[(d + e*x)/2], x]}, Dist[(2*f
```

```
)/e, Subst[Int[1/(a + b + 2*c*f*x + (a - b)*f^2*x^2), x], x, Tan[(d + e*x)/
2]/f], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
```

Rule 3136

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)]), x
_Symbol] :> Simp[((b*B + c*C)*x)/(b^2 + c^2), x] + (Dist[(A*(b^2 + c^2) - a
*(b*B + c*C))/(b^2 + c^2), Int[1/(a + b*Cos[d + e*x] + c*Sin[d + e*x]), x],
 x] + Simp[((c*B - b*C)*Log[a + b*Cos[d + e*x] + c*Sin[d + e*x]])/(e*(b^2 +
 c^2)), x]) /; FreeQ[{a, b, c, d, e, A, B, C}, x] && NeQ[b^2 + c^2, 0] && N
eQ[A*(b^2 + c^2) - a*(b*B + c*C), 0]
```

Rubi steps

$$\int \frac{A+B\cosh(x)+C\sinh(x)}{a+b\cosh(x)+c\sinh(x)}\,dx = \frac{(bB-cC)x}{b^2-c^2} - \frac{(Bc-bC)\log(a+b\cosh(x)+c\sinh(x))}{b^2-c^2} + \frac{\left(Ab^2-abB-Ac^2\right.}{}$$

$$= \frac{(bB-cC)x}{b^2-c^2} - \frac{(Bc-bC)\log(a+b\cosh(x)+c\sinh(x))}{b^2-c^2} + \frac{\left(2\left(Ab^2-abB-A\right.\right.}{}$$

$$= \frac{(bB-cC)x}{b^2-c^2} - \frac{(Bc-bC)\log(a+b\cosh(x)+c\sinh(x))}{b^2-c^2} - \frac{\left(4\left(Ab^2-abB-A\right.\right.}{}$$

$$= \frac{(bB-cC)x}{b^2-c^2} - \frac{2\left(Ab^2-abB-Ac^2+acC\right)\tanh^{-1}\left(\frac{c-(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2+c^2}}\right)}{\left(b^2-c^2\right)\sqrt{a^2-b^2+c^2}} - \frac{(Bc-bC}{}$$

Mathematica [A] time = 0.34, size = 119, normalized size = 0.87

$$\frac{\frac{2\left(-abB+acC+Ab^2-Ac^2\right)\tan^{-1}\left(\frac{(b-a)\tanh\left(\frac{x}{2}\right)+c}{\sqrt{-a^2+b^2-c^2}}\right)}{\sqrt{-a^2+b^2-c^2}}+(bC-Bc)\log(a+b\cosh(x)+c\sinh(x))+x(bB-cC)}{(b-c)(b+c)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cosh[x] + C*Sinh[x])/(a + b*Cosh[x] + c*Sinh[x]),x]
```

```
[Out] ((b*B - c*C)*x + (2*(A*b^2 - a*b*B - A*c^2 + a*c*C)*ArcTan[(c + (-a + b)*Ta
nh[x/2])/Sqrt[-a^2 + b^2 - c^2]])/Sqrt[-a^2 + b^2 - c^2] + (-(B*c) + b*C)*L
og[a + b*Cosh[x] + c*Sinh[x]])/((b - c)*(b + c))
```

fricas [A] time = 0.51, size = 605, normalized size = 4.42

$$\left[\frac{\left(Bab - Ab^2 - Cac + Ac^2\right)\sqrt{a^2 - b^2 + c^2}\,\log\left(\frac{\left(b^2+2\,bc+c^2\right)\cosh(x)^2+\left(b^2+2\,bc+c^2\right)\sinh(x)^2+2\,a^2-b^2+c^2+2\,(ab+ac)\cosh(x)+2\,\left(ab+\right.}{(b+c)\cosh(x)^2+(b+c)\sinh(x)^2+2\,a\cosh(}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x)+C*sinh(x))/(a+b*cosh(x)+c*sinh(x)),x, algorithm="fri
cas")
```

```
[Out] [((B*a*b - A*b^2 - C*a*c + A*c^2)*sqrt(a^2 - b^2 + c^2)*log(((b^2 + 2*b*c +
 c^2)*cosh(x)^2 + (b^2 + 2*b*c + c^2)*sinh(x)^2 + 2*a^2 - b^2 + c^2 + 2*(a*
b + a*c)*cosh(x) + 2*(a*b + a*c + (b^2 + 2*b*c + c^2)*cosh(x))*sinh(x) + 2*
sqrt(a^2 - b^2 + c^2)*((b + c)*cosh(x) + (b + c)*sinh(x) + a))/((b + c)*cos
h(x)^2 + (b + c)*sinh(x)^2 + 2*a*cosh(x) + 2*((b + c)*cosh(x) + a)*sinh(x)
+ b - c)) + ((B - C)*a^2*b - (B - C)*b^3 + (B - C)*b*c^2 + (B - C)*c^3 + ((
B - C)*a^2 - (B - C)*b^2)*c)*x + (C*a^2*b - C*b^3 + C*b*c^2 - B*c^3 - (B*a^
2 - B*b^2)*c)*log(2*(b*cosh(x) + c*sinh(x) + a)/(cosh(x) - sinh(x))))/(a^2*
b^2 - b^4 - c^4 - (a^2 - 2*b^2)*c^2), -(2*(B*a*b - A*b^2 - C*a*c + A*c^2)*s
qrt(-a^2 + b^2 - c^2)*arctan(sqrt(-a^2 + b^2 - c^2)*((b + c)*cosh(x) + (b +
 c)*sinh(x) + a)/(a^2 - b^2 + c^2)) - ((B - C)*a^2*b - (B - C)*b^3 + (B - C
)*b*c^2 + (B - C)*c^3 + ((B - C)*a^2 - (B - C)*b^2)*c)*x - (C*a^2*b - C*b^3
 + C*b*c^2 - B*c^3 - (B*a^2 - B*b^2)*c)*log(2*(b*cosh(x) + c*sinh(x) + a)/(
cosh(x) - sinh(x))))/(a^2*b^2 - b^4 - c^4 - (a^2 - 2*b^2)*c^2)]
```

giac [A] time = 0.14, size = 136, normalized size = 0.99

$$\frac{(B-C)x}{b-c}+\frac{(Cb-Bc)\log\left(be^{(2\,x)}+ce^{(2\,x)}+2\,ae^{x}+b-c\right)}{b^2-c^2}-\frac{2\left(Bab-Ab^2-Cac+Ac^2\right)\arctan\left(\frac{be^{x}+ce^{x}+a}{\sqrt{-a^2+b^2-c^2}}\right)}{\sqrt{-a^2+b^2-c^2}\left(b^2-c^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x)+C*sinh(x))/(a+b*cosh(x)+c*sinh(x)),x, algorithm="gia
c")
```

```
[Out] (B - C)*x/(b - c) + (C*b - B*c)*log(b*e^(2*x) + c*e^(2*x) + 2*a*e^x + b - c
)/(b^2 - c^2) - 2*(B*a*b - A*b^2 - C*a*c + A*c^2)*arctan((b*e^x + c*e^x + a
)/sqrt(-a^2 + b^2 - c^2))/(sqrt(-a^2 + b^2 - c^2)*(b^2 - c^2))
```

maple [B] time = 0.20, size = 1009, normalized size = 7.36

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cosh(x)+C*sinh(x))/(a+b*cosh(x)+c*sinh(x)),x)
```

```
[Out] -2*B/(2*b+2*c)*ln(tanh(1/2*x)-1)-2*C/(2*b+2*c)*ln(tanh(1/2*x)-1)-1/(b-c)/(b
+c)/(a-b)*ln(a*tanh(1/2*x)^2-tanh(1/2*x)^2*b-2*c*tanh(1/2*x)-a-b)*a*B*c+1/(
b-c)/(b+c)/(a-b)*ln(a*tanh(1/2*x)^2-tanh(1/2*x)^2*b-2*c*tanh(1/2*x)-a-b)*b*
B*c+1/(b-c)/(b+c)/(a-b)*ln(a*tanh(1/2*x)^2-tanh(1/2*x)^2*b-2*c*tanh(1/2*x)-
a-b)*a*b*C-1/(b-c)/(b+c)/(a-b)*ln(a*tanh(1/2*x)^2-tanh(1/2*x)^2*b-2*c*tanh(
1/2*x)-a-b)*C*b^2-2/(b-c)/(b+c)/(-a^2+b^2-c^2)^(1/2)*arctan(1/2*(2*(a-b)*ta
nh(1/2*x)-2*c)/(-a^2+b^2-c^2)^(1/2))*A*b^2+2/(b-c)/(b+c)/(-a^2+b^2-c^2)^(1/
2)*arctan(1/2*(2*(a-b)*tanh(1/2*x)-2*c)/(-a^2+b^2-c^2)^(1/2))*A*c^2+2/(b-c)
/(b+c)/(-a^2+b^2-c^2)^(1/2)*arctan(1/2*(2*(a-b)*tanh(1/2*x)-2*c)/(-a^2+b^2-
c^2)^(1/2))*b*B*a+2/(b-c)/(b+c)/(-a^2+b^2-c^2)^(1/2)*arctan(1/2*(2*(a-b)*ta
nh(1/2*x)-2*c)/(-a^2+b^2-c^2)^(1/2))*B*c^2-2/(b-c)/(b+c)/(-a^2+b^2-c^2)^(1/
2)*arctan(1/2*(2*(a-b)*tanh(1/2*x)-2*c)/(-a^2+b^2-c^2)^(1/2))*a*c*C-2/(b-c)
/(b+c)/(-a^2+b^2-c^2)^(1/2)*arctan(1/2*(2*(a-b)*tanh(1/2*x)-2*c)/(-a^2+b^2-
c^2)^(1/2))*C*c*b-2/(b-c)/(b+c)/(-a^2+b^2-c^2)^(1/2)*arctan(1/2*(2*(a-b)*ta
nh(1/2*x)-2*c)/(-a^2+b^2-c^2)^(1/2))*c^2/(a-b)*a*B+2/(b-c)/(b+c)/(-a^2+b^2-
c^2)^(1/2)*arctan(1/2*(2*(a-b)*tanh(1/2*x)-2*c)/(-a^2+b^2-c^2)^(1/2))*c^2/(
a-b)*b*B+2/(b-c)/(b+c)/(-a^2+b^2-c^2)^(1/2)*arctan(1/2*(2*(a-b)*tanh(1/2*x)
-2*c)/(-a^2+b^2-c^2)^(1/2))*c/(a-b)*a*b*C-2/(b-c)/(b+c)/(-a^2+b^2-c^2)^(1/2
)*arctan(1/2*(2*(a-b)*tanh(1/2*x)-2*c)/(-a^2+b^2-c^2)^(1/2))*c/(a-b)*C*b^2+
2*B/(2*b-2*c)*ln(tanh(1/2*x)+1)-2*C/(2*b-2*c)*ln(tanh(1/2*x)+1)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x)+C*sinh(x))/(a+b*cosh(x)+c*sinh(x)),x, algorithm="max
ima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(c^2-b^2+a^2>0)', see `assume?` for
more details)Is c^2-b^2+a^2 positive or negative?
```

mupad [B] time = 2.45, size = 454, normalized size = 3.31

$$\frac{\ln\left(b\sqrt{a^2-b^2+c^2}-c\sqrt{a^2-b^2+c^2}+a^2\,\mathrm{e}^x-b^2\,\mathrm{e}^x+c^2\,\mathrm{e}^x+a\,\mathrm{e}^x\sqrt{a^2-b^2+c^2}\right)\left(B\,c^3+C\,b^3-A\,b^2\sqrt{a^2-b^2-}\right.}{-a^2\,b^2+a^2\,c^2+b^4-}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cosh(x) + C*sinh(x))/(a + b*cosh(x) + c*sinh(x)),x)
```

```
[Out] (log(b*(a^2 - b^2 + c^2)^(1/2) - c*(a^2 - b^2 + c^2)^(1/2) + a^2*exp(x) - b
^2*exp(x) + c^2*exp(x) + a*exp(x)*(a^2 - b^2 + c^2)^(1/2))*(B*c^3 + C*b^3 -
 A*b^2*(a^2 - b^2 + c^2)^(1/2) + B*a^2*c - C*a^2*b + A*c^2*(a^2 - b^2 + c^2
)^(1/2) - B*b^2*c - C*b*c^2 + B*a*b*(a^2 - b^2 + c^2)^(1/2) - C*a*c*(a^2 -
b^2 + c^2)^(1/2)))/(b^4 + c^4 - a^2*b^2 + a^2*c^2 - 2*b^2*c^2) + (log(b*(a^
2 - b^2 + c^2)^(1/2) - c*(a^2 - b^2 + c^2)^(1/2) - a^2*exp(x) + b^2*exp(x)
- c^2*exp(x) + a*exp(x)*(a^2 - b^2 + c^2)^(1/2))*(B*c^3 + C*b^3 + A*b^2*(a^
2 - b^2 + c^2)^(1/2) + B*a^2*c - C*a^2*b - A*c^2*(a^2 - b^2 + c^2)^(1/2) -
B*b^2*c - C*b*c^2 - B*a*b*(a^2 - b^2 + c^2)^(1/2) + C*a*c*(a^2 - b^2 + c^2)
^(1/2)))/(b^4 + c^4 - a^2*b^2 + a^2*c^2 - 2*b^2*c^2) + (x*(B - C))/(b - c)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x)+C*sinh(x))/(a+b*cosh(x)+c*sinh(x)),x)
```

```
[Out] Timed out
```

3.799 $\int \frac{A+B\cosh(x)+C\sinh(x)}{(a+b\cosh(x)+c\sinh(x))^2}\,dx$

Optimal. Leaf size=121

$$-\frac{2\tanh^{-1}\left(\frac{c-(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2+c^2}}\right)(aA-bB+cC)}{\left(a^2-b^2+c^2\right)^{3/2}}-\frac{\sinh(x)(Ab-aB)+\cosh(x)(Ac-aC)-bC+Bc}{\left(a^2-b^2+c^2\right)(a+b\cosh(x)+c\sinh(x))}$$

```
[Out] -2*(A*a-B*b+C*c)*arctanh((c-(a-b)*tanh(1/2*x))/(a^2-b^2+c^2)^(1/2))/(a^2-b^
2+c^2)^(3/2)+(-B*c+b*C-(A*c-C*a)*cosh(x)-(A*b-B*a)*sinh(x))/(a^2-b^2+c^2)/(
a+b*cosh(x)+c*sinh(x))
```

Rubi [A] time = 0.15, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3153, 3124, 618, 206}

$$-\frac{2\tanh^{-1}\left(\frac{c-(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2+c^2}}\right)(aA-bB+cC)}{\left(a^2-b^2+c^2\right)^{3/2}}-\frac{\sinh(x)(Ab-aB)+\cosh(x)(Ac-aC)-bC+Bc}{\left(a^2-b^2+c^2\right)(a+b\cosh(x)+c\sinh(x))}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cosh[x] + C*Sinh[x])/(a + b*Cosh[x] + c*Sinh[x])^2,x]
```

```
[Out] (-2*(a*A - b*B + c*C)*ArcTanh[(c - (a - b)*Tanh[x/2])/Sqrt[a^2 - b^2 + c^2]
])/(a^2 - b^2 + c^2)^(3/2) - (B*c - b*C + (A*c - a*C)*Cosh[x] + (A*b - a*B)
*Sinh[x])/((a^2 - b^2 + c^2)*(a + b*Cosh[x] + c*Sinh[x]))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 3124

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Module[{f = FreeFactors[Tan[(d + e*x)/2], x]}, Dist[(2*f
```

```
)/e, Subst[Int[1/(a + b + 2*c*f*x + (a - b)*f^2*x^2), x], x, Tan[(d + e*x)/
2]/f], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
```

Rule 3153

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])^2,
 x_Symbol] :> Simp[(c*B - b*C - (a*C - c*A)*Cos[d + e*x] + (a*B - b*A)*Sin[
d + e*x])/(e*(a^2 - b^2 - c^2)*(a + b*Cos[d + e*x] + c*Sin[d + e*x])), x] +
 Dist[(a*A - b*B - c*C)/(a^2 - b^2 - c^2), Int[1/(a + b*Cos[d + e*x] + c*Si
n[d + e*x]), x], x] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && NeQ[a^2 - b^2
- c^2, 0] && NeQ[a*A - b*B - c*C, 0]
```

Rubi steps

$$\int \frac{A+B\cosh(x)+C\sinh(x)}{(a+b\cosh(x)+c\sinh(x))^2}\,dx = -\frac{Bc-bC+(Ac-aC)\cosh(x)+(Ab-aB)\sinh(x)}{\left(a^2-b^2+c^2\right)(a+b\cosh(x)+c\sinh(x))}+\frac{(aA-bB+cC)\int \frac{}{a}}{a^2-b}$$

$$= -\frac{Bc-bC+(Ac-aC)\cosh(x)+(Ab-aB)\sinh(x)}{\left(a^2-b^2+c^2\right)(a+b\cosh(x)+c\sinh(x))}+\frac{(2(aA-bB+cC))}{}$$

$$= -\frac{Bc-bC+(Ac-aC)\cosh(x)+(Ab-aB)\sinh(x)}{\left(a^2-b^2+c^2\right)(a+b\cosh(x)+c\sinh(x))}-\frac{(4(aA-bB+cC))}{}$$

$$= -\frac{2(aA-bB+cC)\tanh^{-1}\left(\frac{c-(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2+c^2}}\right)}{\left(a^2-b^2+c^2\right)^{3/2}}-\frac{Bc-bC+(Ac-aC)\cosh(x)}{\left(a^2-b^2+c^2\right)(a+b\cosh}$$

Mathematica [A] time = 0.42, size = 143, normalized size = 1.18

$$\frac{a^2C+\sinh(x)\left(-abB+acC+A\left(b^2-c^2\right)\right)-aAc+b(Bc-bC)}{b\left(-a^2+b^2-c^2\right)(a+b\cosh(x)+c\sinh(x))}-\frac{2(aA-bB+cC)\tan^{-1}\left(\frac{(b-a)\tanh\left(\frac{x}{2}\right)+c}{\sqrt{-a^2+b^2-c^2}}\right)}{\left(-a^2+b^2-c^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cosh[x] + C*Sinh[x])/(a + b*Cosh[x] + c*Sinh[x])^2,x]
```

```
[Out] (-2*(a*A - b*B + c*C)*ArcTan[(c + (-a + b)*Tanh[x/2])/Sqrt[-a^2 + b^2 - c^2
]])/(-a^2 + b^2 - c^2)^(3/2) + (-(a*A*c) + a^2*C + b*(B*c - b*C) + (-(a*b*B
```

```
) + A*(b^2 - c^2) + a*c*C)*Sinh[x])/(b*(-a^2 + b^2 - c^2)*(a + b*Cosh[x] +
c*Sinh[x]))
```

fricas [B] time = 0.50, size = 2541, normalized size = 21.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x)+C*sinh(x))/(a+b*cosh(x)+c*sinh(x))^2,x, algorithm="f
ricas")
```

```
[Out] [-(2*B*a^3*b - 2*A*a^2*b^2 - 2*B*a*b^3 + 2*A*b^4 - 2*C*a*c^3 + 2*A*c^4 + 2*
(A*a^2 + B*a*b - 2*A*b^2)*c^2 - (A*a*b^2 - B*b^3 + C*b^2*c - C*c^3 - (A*a -
 B*b)*c^2 + (A*a*b^2 - B*b^3 + C*c^3 + (A*a - (B - 2*C)*b)*c^2 + (2*A*a*b -
 (2*B - C)*b^2)*c)*cosh(x)^2 + (A*a*b^2 - B*b^3 + C*c^3 + (A*a - (B - 2*C)*
b)*c^2 + (2*A*a*b - (2*B - C)*b^2)*c)*sinh(x)^2 + 2*(A*a^2*b - B*a*b^2 + C*
a*c^2 + (A*a^2 - (B - C)*a*b)*c)*cosh(x) + 2*(A*a^2*b - B*a*b^2 + C*a*c^2 +
 (A*a^2 - (B - C)*a*b)*c + (A*a*b^2 - B*b^3 + C*c^3 + (A*a - (B - 2*C)*b)*c
^2 + (2*A*a*b - (2*B - C)*b^2)*c)*cosh(x))*sinh(x))*sqrt(a^2 - b^2 + c^2)*l
og(((b^2 + 2*b*c + c^2)*cosh(x)^2 + (b^2 + 2*b*c + c^2)*sinh(x)^2 + 2*a^2 -
 b^2 + c^2 + 2*(a*b + a*c)*cosh(x) + 2*(a*b + a*c + (b^2 + 2*b*c + c^2)*cos
h(x))*sinh(x) - 2*sqrt(a^2 - b^2 + c^2)*((b + c)*cosh(x) + (b + c)*sinh(x)
+ a))/((b + c)*cosh(x)^2 + (b + c)*sinh(x)^2 + 2*a*cosh(x) + 2*((b + c)*cos
h(x) + a)*sinh(x) + b - c)) - 2*(C*a^3 - C*a*b^2)*c + 2*((B + C)*a^4 - A*a^
3*b - (B + 2*C)*a^2*b^2 + A*a*b^3 + C*b^4 + B*c^4 - (A*a - (B - C)*b)*c^3 +
 ((2*B + C)*a^2 - A*a*b - (B + C)*b^2)*c^2 - (A*a^3 - (B - C)*a^2*b - A*a*b
^2 + (B - C)*b^3)*c)*cosh(x) + 2*((B + C)*a^4 - A*a^3*b - (B + 2*C)*a^2*b^2
 + A*a*b^3 + C*b^4 + B*c^4 - (A*a - (B - C)*b)*c^3 + ((2*B + C)*a^2 - A*a*b
 - (B + C)*b^2)*c^2 - (A*a^3 - (B - C)*a^2*b - A*a*b^2 + (B - C)*b^3)*c)*si
nh(x))/(a^4*b^2 - 2*a^2*b^4 + b^6 - c^6 - (2*a^2 - 3*b^2)*c^4 - (a^4 - 4*a^
2*b^2 + 3*b^4)*c^2 + (a^4*b^2 - 2*a^2*b^4 + b^6 + 2*b*c^5 + c^6 + (2*a^2 -
b^2)*c^4 + 4*(a^2*b - b^3)*c^3 + (a^4 - b^4)*c^2 + 2*(a^4*b - 2*a^2*b^3 + b
^5)*c)*cosh(x)^2 + (a^4*b^2 - 2*a^2*b^4 + b^6 + 2*b*c^5 + c^6 + (2*a^2 - b^
2)*c^4 + 4*(a^2*b - b^3)*c^3 + (a^4 - b^4)*c^2 + 2*(a^4*b - 2*a^2*b^3 + b^5
)*c)*sinh(x)^2 + 2*(a^5*b - 2*a^3*b^3 + a*b^5 + a*b*c^4 + a*c^5 + 2*(a^3 -
a*b^2)*c^3 + 2*(a^3*b - a*b^3)*c^2 + (a^5 - 2*a^3*b^2 + a*b^4)*c)*cosh(x) +
 2*(a^5*b - 2*a^3*b^3 + a*b^5 + a*b*c^4 + a*c^5 + 2*(a^3 - a*b^2)*c^3 + 2*(
a^3*b - a*b^3)*c^2 + (a^5 - 2*a^3*b^2 + a*b^4)*c + (a^4*b^2 - 2*a^2*b^4 + b
^6 + 2*b*c^5 + c^6 + (2*a^2 - b^2)*c^4 + 4*(a^2*b - b^3)*c^3 + (a^4 - b^4)*
c^2 + 2*(a^4*b - 2*a^2*b^3 + b^5)*c)*cosh(x))*sinh(x)), -2*(B*a^3*b - A*a^2
*b^2 - B*a*b^3 + A*b^4 - C*a*c^3 + A*c^4 + (A*a^2 + B*a*b - 2*A*b^2)*c^2 -
(A*a*b^2 - B*b^3 + C*b^2*c - C*c^3 - (A*a - B*b)*c^2 + (A*a*b^2 - B*b^3 + C
*c^3 + (A*a - (B - 2*C)*b)*c^2 + (2*A*a*b - (2*B - C)*b^2)*c)*cosh(x)^2 + (
A*a*b^2 - B*b^3 + C*c^3 + (A*a - (B - 2*C)*b)*c^2 + (2*A*a*b - (2*B - C)*b^
2)*c)*sinh(x)^2 + 2*(A*a^2*b - B*a*b^2 + C*a*c^2 + (A*a^2 - (B - C)*a*b)*c)
```

```
*cosh(x) + 2*(A*a^2*b - B*a*b^2 + C*a*c^2 + (A*a^2 - (B - C)*a*b)*c + (A*a*
b^2 - B*b^3 + C*c^3 + (A*a - (B - 2*C)*b)*c^2 + (2*A*a*b - (2*B - C)*b^2)*c
)*cosh(x))*sinh(x))*sqrt(-a^2 + b^2 - c^2)*arctan(sqrt(-a^2 + b^2 - c^2)*((
b + c)*cosh(x) + (b + c)*sinh(x) + a)/(a^2 - b^2 + c^2)) - (C*a^3 - C*a*b^2
)*c + ((B + C)*a^4 - A*a^3*b - (B + 2*C)*a^2*b^2 + A*a*b^3 + C*b^4 + B*c^4
- (A*a - (B - C)*b)*c^3 + ((2*B + C)*a^2 - A*a*b - (B + C)*b^2)*c^2 - (A*a^
3 - (B - C)*a^2*b - A*a*b^2 + (B - C)*b^3)*c)*cosh(x) + ((B + C)*a^4 - A*a^
3*b - (B + 2*C)*a^2*b^2 + A*a*b^3 + C*b^4 + B*c^4 - (A*a - (B - C)*b)*c^3 +
 ((2*B + C)*a^2 - A*a*b - (B + C)*b^2)*c^2 - (A*a^3 - (B - C)*a^2*b - A*a*b
^2 + (B - C)*b^3)*c)*sinh(x))/(a^4*b^2 - 2*a^2*b^4 + b^6 - c^6 - (2*a^2 - 3
*b^2)*c^4 - (a^4 - 4*a^2*b^2 + 3*b^4)*c^2 + (a^4*b^2 - 2*a^2*b^4 + b^6 + 2*
b*c^5 + c^6 + (2*a^2 - b^2)*c^4 + 4*(a^2*b - b^3)*c^3 + (a^4 - b^4)*c^2 + 2
*(a^4*b - 2*a^2*b^3 + b^5)*c)*cosh(x)^2 + (a^4*b^2 - 2*a^2*b^4 + b^6 + 2*b*
c^5 + c^6 + (2*a^2 - b^2)*c^4 + 4*(a^2*b - b^3)*c^3 + (a^4 - b^4)*c^2 + 2*(
a^4*b - 2*a^2*b^3 + b^5)*c)*sinh(x)^2 + 2*(a^5*b - 2*a^3*b^3 + a*b^5 + a*b*
c^4 + a*c^5 + 2*(a^3 - a*b^2)*c^3 + 2*(a^3*b - a*b^3)*c^2 + (a^5 - 2*a^3*b^
2 + a*b^4)*c)*cosh(x) + 2*(a^5*b - 2*a^3*b^3 + a*b^5 + a*b*c^4 + a*c^5 + 2*
(a^3 - a*b^2)*c^3 + 2*(a^3*b - a*b^3)*c^2 + (a^5 - 2*a^3*b^2 + a*b^4)*c + (
a^4*b^2 - 2*a^2*b^4 + b^6 + 2*b*c^5 + c^6 + (2*a^2 - b^2)*c^4 + 4*(a^2*b -
b^3)*c^3 + (a^4 - b^4)*c^2 + 2*(a^4*b - 2*a^2*b^3 + b^5)*c)*cosh(x))*sinh(x
))]
```

giac [A] time = 0.13, size = 207, normalized size = 1.71

$$\frac{2\,(Aa - Bb + Cc)\arctan\left(\frac{be^x+ce^x+a}{\sqrt{-a^2+b^2-c^2}}\right)}{\left(a^2-b^2+c^2\right)\sqrt{-a^2+b^2-c^2}} - \frac{2\left(Ba^2e^x + Ca^2e^x - Aabe^x - Cb^2e^x - Aace^x + Bbce^x - Cbce^x + Bc^2e^x + \right.}{\left(a^2b - b^3 + a^2c - b^2c + bc^2 + c^3\right)\left(be^{(2\,x)} + ce^{(2\,x)} + 2\,ae^x\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x)+C*sinh(x))/(a+b*cosh(x)+c*sinh(x))^2,x, algorithm="g
iac")
```

```
[Out] 2*(A*a - B*b + C*c)*arctan((b*e^x + c*e^x + a)/sqrt(-a^2 + b^2 - c^2))/((a^
2 - b^2 + c^2)*sqrt(-a^2 + b^2 - c^2)) - 2*(B*a^2*e^x + C*a^2*e^x - A*a*b*e
^x - C*b^2*e^x - A*a*c*e^x + B*b*c*e^x - C*b*c*e^x + B*c^2*e^x + B*a*b - A*
b^2 - C*a*c + A*c^2)/((a^2*b - b^3 + a^2*c - b^2*c + b*c^2 + c^3)*(b*e^(2*x
) + c*e^(2*x) + 2*a*e^x + b - c))
```

maple [B] time = 0.27, size = 376, normalized size = 3.11

$$-\frac{2\left(-\frac{\left(Aab-A\,b^2+A\,c^2-B\,a^2+bBa-B\,c^2-acC+Ccb\right)\tanh\left(\frac{x}{2}\right)}{a^3-a^2b-a\,b^2+a\,c^2+b^3-b\,c^2}-\frac{aAc-bBc-a^2C+C\,b^2}{a^3-a^2b-a\,b^2+a\,c^2+b^3-b\,c^2}\right)}{a\left(\tanh^2\left(\frac{x}{2}\right)\right)-\left(\tanh^2\left(\frac{x}{2}\right)\right)b-2c\tanh\left(\frac{x}{2}\right)-a-b}-\frac{2\arctan\left(\frac{2(a-b)\tanh\left(\frac{x}{2}\right)-2c}{2\sqrt{-a^2+b^2-c^2}}\right)Aa}{\left(a^2-b^2+c^2\right)\sqrt{-a^2+b^2-c^2}}+\frac{2\,\mathrm{arct}}{\left(a^2-\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cosh(x)+C*sinh(x))/(a+b*cosh(x)+c*sinh(x))^2,x)
```

```
[Out] -2*(-(A*a*b-A*b^2+A*c^2-B*a^2+B*a*b-B*c^2-C*a*c+C*b*c)/(a^3-a^2*b-a*b^2+a*c
^2+b^3-b*c^2)*tanh(1/2*x)-(A*a*c-B*b*c-C*a^2+C*b^2)/(a^3-a^2*b-a*b^2+a*c^2+
b^3-b*c^2))/(a*tanh(1/2*x)^2-tanh(1/2*x)^2*b-2*c*tanh(1/2*x)-a-b)-2/(a^2-b^
2+c^2)/(-a^2+b^2-c^2)^(1/2)*arctan(1/2*(2*(a-b)*tanh(1/2*x)-2*c)/(-a^2+b^2-
c^2)^(1/2))*A*a+2/(a^2-b^2+c^2)/(-a^2+b^2-c^2)^(1/2)*arctan(1/2*(2*(a-b)*ta
nh(1/2*x)-2*c)/(-a^2+b^2-c^2)^(1/2))*B*b-2/(a^2-b^2+c^2)/(-a^2+b^2-c^2)^(1/
2)*arctan(1/2*(2*(a-b)*tanh(1/2*x)-2*c)/(-a^2+b^2-c^2)^(1/2))*C*c
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x)+C*sinh(x))/(a+b*cosh(x)+c*sinh(x))^2,x, algorithm="m
axima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(c^2-b^2+a^2>0)', see `assume?` for
more details)Is c^2-b^2+a^2 positive or negative?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A + B\cosh(x) + C\sinh(x)}{(a + b\cosh(x) + c\sinh(x))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cosh(x) + C*sinh(x))/(a + b*cosh(x) + c*sinh(x))^2,x)
```

```
[Out] int((A + B*cosh(x) + C*sinh(x))/(a + b*cosh(x) + c*sinh(x))^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x)+C*sinh(x))/(a+b*cosh(x)+c*sinh(x))**2,x)
```

```
[Out] Timed out
```

3.800 $\int \frac{A+B\cosh(x)+C\sinh(x)}{(a+b\cosh(x)+c\sinh(x))^3}\,dx$

Optimal. Leaf size=233

$$-\frac{\tanh^{-1}\left(\frac{c-(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2+c^2}}\right)\left(2a^2A-3abB+3acC+Ab^2-Ac^2\right)}{\left(a^2-b^2+c^2\right)^{5/2}}-\frac{\sinh(x)\left(a^2(-B)+3aAb-2b(bB-cC)\right)+\cosh(}{2\left(a^2-b^2+c^2\right)^2(a+}$$

```
[Out] -(2*A*a^2+A*b^2-A*c^2-3*B*a*b+3*C*a*c)*arctanh((c-(a-b)*tanh(1/2*x))/(a^2-b
^2+c^2)^(1/2))/(a^2-b^2+c^2)^(5/2)+1/2*(-B*c+b*C-(A*c-C*a)*cosh(x)-(A*b-B*a
)*sinh(x))/(a^2-b^2+c^2)/(a+b*cosh(x)+c*sinh(x))^2+1/2*(-a*(B*c-C*b)-(3*a*A
*c-a^2*C-2*c*(B*b-C*c))*cosh(x)-(3*a*A*b-a^2*B-2*b*(B*b-C*c))*sinh(x))/(a^2
-b^2+c^2)^2/(a+b*cosh(x)+c*sinh(x))
```

Rubi [A] time = 0.51, antiderivative size = 233, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3156, 3153, 3124, 618, 206}

$$-\frac{\tanh^{-1}\left(\frac{c-(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2+c^2}}\right)\left(2a^2A-3abB+3acC+Ab^2-Ac^2\right)}{\left(a^2-b^2+c^2\right)^{5/2}}-\frac{\sinh(x)\left(a^2(-B)+3aAb-2b(bB-cC)\right)+\cosh(}{2\left(a^2-b^2+c^2\right)^2(a+}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cosh[x] + C*Sinh[x])/(a + b*Cosh[x] + c*Sinh[x])^3,x]
```

```
[Out] -(((2*a^2*A + A*b^2 - 3*a*b*B - A*c^2 + 3*a*c*C)*ArcTanh[(c - (a - b)*Tanh[
x/2])/Sqrt[a^2 - b^2 + c^2]])/(a^2 - b^2 + c^2)^(5/2)) - (B*c - b*C + (A*c
- a*C)*Cosh[x] + (A*b - a*B)*Sinh[x])/(2*(a^2 - b^2 + c^2)*(a + b*Cosh[x] +
 c*Sinh[x])^2) - (a*(B*c - b*C) + (3*a*A*c - a^2*C - 2*c*(b*B - c*C))*Cosh[
x] + (3*a*A*b - a^2*B - 2*b*(b*B - c*C))*Sinh[x])/(2*(a^2 - b^2 + c^2)^2*(a
 + b*Cosh[x] + c*Sinh[x]))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 3124

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Module[{f = FreeFactors[Tan[(d + e*x)/2], x]}, Dist[(2*f
)/e, Subst[Int[1/(a + b + 2*c*f*x + (a - b)*f^2*x^2), x], x, Tan[(d + e*x)/
2]/f], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
```

Rule 3153

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])^2,
 x_Symbol] :> Simp[(c*B - b*C - (a*C - c*A)*Cos[d + e*x] + (a*B - b*A)*Sin[
d + e*x])/(e*(a^2 - b^2 - c^2)*(a + b*Cos[d + e*x] + c*Sin[d + e*x])), x] +
 Dist[(a*A - b*B - c*C)/(a^2 - b^2 - c^2), Int[1/(a + b*Cos[d + e*x] + c*Si
n[d + e*x]), x], x] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && NeQ[a^2 - b^2
- c^2, 0] && NeQ[a*A - b*B - c*C, 0]
```

Rule 3156

```
Int[((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])
^(n_)*((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)
]), x_Symbol] :> -Simp[((c*B - b*C - (a*C - c*A)*Cos[d + e*x] + (a*B - b*A)
*Sin[d + e*x])*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1))/(e*(n + 1)*(a
^2 - b^2 - c^2)), x] + Dist[1/((n + 1)*(a^2 - b^2 - c^2)), Int[(a + b*Cos[d
 + e*x] + c*Sin[d + e*x])^(n + 1)*Simp[(n + 1)*(a*A - b*B - c*C) + (n + 2)*
(a*B - b*A)*Cos[d + e*x] + (n + 2)*(a*C - c*A)*Sin[d + e*x], x], x], x] /;
FreeQ[{a, b, c, d, e, A, B, C}, x] && LtQ[n, -1] && NeQ[a^2 - b^2 - c^2, 0]
 && NeQ[n, -2]
```

Rubi steps

$$\int \frac{A+B\cosh(x)+C\sinh(x)}{(a+b\cosh(x)+c\sinh(x))^3}\,dx = -\frac{Bc-bC+(Ac-aC)\cosh(x)+(Ab-aB)\sinh(x)}{2\left(a^2-b^2+c^2\right)(a+b\cosh(x)+c\sinh(x))^2} - \frac{\int \frac{-2(aA-bB+cC)+(Ab-\dots}{(a+b\cosh\dots}}{2\left(a^2\dots\right.}$$

$$= -\frac{Bc-bC+(Ac-aC)\cosh(x)+(Ab-aB)\sinh(x)}{2\left(a^2-b^2+c^2\right)(a+b\cosh(x)+c\sinh(x))^2} - \frac{a(Bc-bC)+\left(3aA\dots\right.}{\dots}$$

$$= -\frac{Bc-bC+(Ac-aC)\cosh(x)+(Ab-aB)\sinh(x)}{2\left(a^2-b^2+c^2\right)(a+b\cosh(x)+c\sinh(x))^2} - \frac{a(Bc-bC)+\left(3aA\dots\right.}{\dots}$$

$$= -\frac{Bc-bC+(Ac-aC)\cosh(x)+(Ab-aB)\sinh(x)}{2\left(a^2-b^2+c^2\right)(a+b\cosh(x)+c\sinh(x))^2} - \frac{a(Bc-bC)+\left(3aA\dots\right.}{\dots}$$

$$= -\frac{\left(2a^2A+Ab^2-3abB-Ac^2+3acC\right)\tanh^{-1}\left(\frac{c-(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2+c^2}}\right)}{\left(a^2-b^2+c^2\right)^{5/2}} - \frac{Bc-bC+\dots}{2\left(a^2-\dots\right.}$$

Mathematica [A] time = 1.10, size = 465, normalized size = 2.00

$$\frac{\left(2a^2A-3abB+3acC+Ab^2-Ac^2\right)\tan^{-1}\left(\frac{(b-a)\tanh\left(\frac{x}{2}\right)+c}{\sqrt{-a^2+b^2-c^2}}\right)}{\left(-a^2+b^2-c^2\right)^{5/2}} + \frac{-2a^4C+6a^3Ac+4a^3bB\sinh(x)-4a^3cC\sinh(x)-\dots}{\dots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cosh[x] + C*Sinh[x])/(a + b*Cosh[x] + c*Sinh[x])^3,x]
```

```
[Out] ((2*a^2*A + A*b^2 - 3*a*b*B - A*c^2 + 3*a*c*C)*ArcTan[(c + (-a + b)*Tanh[x/
2])/Sqrt[-a^2 + b^2 - c^2]])/(-a^2 + b^2 - c^2)^(5/2) + (6*a^3*A*c + 3*a*A*
b^2*c - 9*a^2*b*B*c - 3*a*A*c^3 - 2*a^4*C + 4*a^2*b^2*C - 2*b^4*C + 5*a^2*c
^2*C + 4*b^2*c^2*C - 2*c^4*C + 2*b*c*(2*a^2*A + A*b^2 - 3*a*b*B - A*c^2 + 3
*a*c*C)*Cosh[x] + c*(3*a*A*(-b^2 + c^2) + a^2*(b*B - c*C) + 2*(b^2 - c^2)*(
b*B - c*C))*Cosh[2*x] - 8*a^2*A*b^2*Sinh[x] + 2*A*b^4*Sinh[x] + 4*a^3*b*B*S
inh[x] + 2*a*b^3*B*Sinh[x] + 12*a^2*A*c^2*Sinh[x] - 2*A*b^2*c^2*Sinh[x] - 8
*a*b*B*c^2*Sinh[x] - 4*a^3*c*C*Sinh[x] - 2*a*b^2*c*C*Sinh[x] + 8*a*c^3*C*Si
nh[x] - 3*a*A*b^3*Sinh[2*x] + a^2*b^2*B*Sinh[2*x] + 2*b^4*B*Sinh[2*x] + 3*a
*A*b*c^2*Sinh[2*x] - 2*b^2*B*c^2*Sinh[2*x] - a^2*b*c*C*Sinh[2*x] - 2*b^3*c*
C*Sinh[2*x] + 2*b*c^3*C*Sinh[2*x])/(4*b*(a^2 - b^2 + c^2)^2*(a + b*Cosh[x]
+ c*Sinh[x])^2)
```

fricas [B] time = 0.70, size = 13813, normalized size = 59.28

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x)+C*sinh(x))/(a+b*cosh(x)+c*sinh(x))^3,x, algorithm="f
ricas")
```

```
[Out] [-1/2*(2*B*a^4*b^2 - 6*A*a^3*b^3 + 2*B*a^2*b^4 + 6*A*a*b^5 - 4*B*b^6 - 4*C*
c^6 - 2*(3*A*a - 2*(B + C)*b)*c^5 - 2*(C*a^2 - 3*A*a*b + 2*(B - 2*C)*b^2)*c
^4 - 2*(3*A*a^3 - (B + C)*a^2*b - 6*A*a*b^2 + 4*(B + C)*b^3)*c^3 - 2*(2*A*a
^4*b^2 - 3*B*a^3*b^3 - A*a^2*b^4 + 3*B*a*b^5 - A*b^6 - A*c^6 + (3*C*a - 2*A
*b)*c^5 + (A*a^2 - 3*(B - 2*C)*a*b + A*b^2)*c^4 + (3*C*a^3 + 2*A*a^2*b - 6*
B*a*b^2 + 4*A*b^3)*c^3 + (2*A*a^4 - 3*(B - 2*C)*a^3*b - 6*C*a*b^3 + A*b^4)*
c^2 + (4*A*a^4*b - 3*(2*B - C)*a^3*b^2 - 2*A*a^2*b^3 + 3*(2*B - C)*a*b^4 -
2*A*b^5)*c)*cosh(x)^3 - 2*(2*A*a^4*b^2 - 3*B*a^3*b^3 - A*a^2*b^4 + 3*B*a*b^
5 - A*b^6 - A*c^6 + (3*C*a - 2*A*b)*c^5 + (A*a^2 - 3*(B - 2*C)*a*b + A*b^2)
*c^4 + (3*C*a^3 + 2*A*a^2*b - 6*B*a*b^2 + 4*A*b^3)*c^3 + (2*A*a^4 - 3*(B -
2*C)*a^3*b - 6*C*a*b^3 + A*b^4)*c^2 + (4*A*a^4*b - 3*(2*B - C)*a^3*b^2 - 2*
A*a^2*b^3 + 3*(2*B - C)*a*b^4 - 2*A*b^5)*c)*sinh(x)^3 + 2*(C*a^4 + 3*A*a^3*
b - (B - C)*a^2*b^2 - 6*A*a*b^3 + 2*(2*B - C)*b^4)*c^2 + 2*(2*(B + C)*a^6 -
 6*A*a^5*b + 3*(B - 2*C)*a^4*b^2 + 3*A*a^3*b^3 - 3*(B - 2*C)*a^2*b^4 + 3*A*
a*b^5 - 2*(B + C)*b^6 + 3*A*a*c^5 + 2*(B + C)*c^6 + 3*((2*B - C)*a^2 + A*a*
b - 2*(B + C)*b^2)*c^4 - 3*(A*a^3 - 3*(B - C)*a^2*b + 2*A*a*b^2)*c^3 + 3*((
2*B - C)*a^4 - A*a^3*b - (B + C)*a^2*b^2 - 2*A*a*b^3 + 2*(B + C)*b^4)*c^2 -
 3*(2*A*a^5 - 3*(B - C)*a^4*b - A*a^3*b^2 + 3*(B - C)*a^2*b^3 - A*a*b^4)*c)
*cosh(x)^2 + 2*(2*(B + C)*a^6 - 6*A*a^5*b + 3*(B - 2*C)*a^4*b^2 + 3*A*a^3*b
^3 - 3*(B - 2*C)*a^2*b^4 + 3*A*a*b^5 - 2*(B + C)*b^6 + 3*A*a*c^5 + 2*(B + C
)*c^6 + 3*((2*B - C)*a^2 + A*a*b - 2*(B + C)*b^2)*c^4 - 3*(A*a^3 - 3*(B - C
)*a^2*b + 2*A*a*b^2)*c^3 + 3*((2*B - C)*a^4 - A*a^3*b - (B + C)*a^2*b^2 - 2
*A*a*b^3 + 2*(B + C)*b^4)*c^2 - 3*(2*A*a^5 - 3*(B - C)*a^4*b - A*a^3*b^2 +
3*(B - C)*a^2*b^3 - A*a*b^4)*c - 3*(2*A*a^4*b^2 - 3*B*a^3*b^3 - A*a^2*b^4 +
 3*B*a*b^5 - A*b^6 - A*c^6 + (3*C*a - 2*A*b)*c^5 + (A*a^2 - 3*(B - 2*C)*a*b
 + A*b^2)*c^4 + (3*C*a^3 + 2*A*a^2*b - 6*B*a*b^2 + 4*A*b^3)*c^3 + (2*A*a^4
- 3*(B - 2*C)*a^3*b - 6*C*a*b^3 + A*b^4)*c^2 + (4*A*a^4*b - 3*(2*B - C)*a^3
*b^2 - 2*A*a^2*b^3 + 3*(2*B - C)*a*b^4 - 2*A*b^5)*c)*cosh(x))*sinh(x)^2 + (
2*A*a^2*b^3 - 3*B*a*b^4 + A*b^5 - A*c^5 + (3*C*a + A*b)*c^4 + (2*A*a^2*b^3
- 3*B*a*b^4 + A*b^5 - A*c^5 + 3*(C*a - A*b)*c^4 + (2*A*a^2 - 3*(B - 3*C)*a*
b - 2*A*b^2)*c^3 + (6*A*a^2*b - 9*(B - C)*a*b^2 + 2*A*b^3)*c^2 + 3*(2*A*a^2
*b^2 - (3*B - C)*a*b^3 + A*b^4)*c)*cosh(x)^4 + (2*A*a^2*b^3 - 3*B*a*b^4 + A
*b^5 - A*c^5 + 3*(C*a - A*b)*c^4 + (2*A*a^2 - 3*(B - 3*C)*a*b - 2*A*b^2)*c^
3 + (6*A*a^2*b - 9*(B - C)*a*b^2 + 2*A*b^3)*c^2 + 3*(2*A*a^2*b^2 - (3*B - C
)*a*b^3 + A*b^4)*c)*sinh(x)^4 + (2*A*a^2 - 3*(B + C)*a*b + 2*A*b^2)*c^3 + 4
*(2*A*a^3*b^2 - 3*B*a^2*b^3 + A*a*b^4 - A*a*c^4 + (3*C*a^2 - 2*A*a*b)*c^3 +
```

```
(2*A*a^3 - 3*(B - 2*C)*a^2*b)*c^2 + (4*A*a^3*b - 3*(2*B - C)*a^2*b^2 + 2*A
*a*b^3)*c)*cosh(x)^3 + 4*(2*A*a^3*b^2 - 3*B*a^2*b^3 + A*a*b^4 - A*a*c^4 + (
3*C*a^2 - 2*A*a*b)*c^3 + (2*A*a^3 - 3*(B - 2*C)*a^2*b)*c^2 + (4*A*a^3*b - 3
*(2*B - C)*a^2*b^2 + 2*A*a*b^3)*c + (2*A*a^2*b^3 - 3*B*a*b^4 + A*b^5 - A*c^
5 + 3*(C*a - A*b)*c^4 + (2*A*a^2 - 3*(B - 3*C)*a*b - 2*A*b^2)*c^3 + (6*A*a^
2*b - 9*(B - C)*a*b^2 + 2*A*b^3)*c^2 + 3*(2*A*a^2*b^2 - (3*B - C)*a*b^3 + A
*b^4)*c)*cosh(x))*sinh(x)^3 - (2*A*a^2*b - 3*(B - C)*a*b^2 + 2*A*b^3)*c^2 +
 2*(4*A*a^4*b - 6*B*a^3*b^2 + 4*A*a^2*b^3 - 3*B*a*b^4 + A*b^5 + A*c^5 - (3*
C*a - A*b)*c^4 - (4*A*a^2 - 3*(B - C)*a*b + 2*A*b^2)*c^3 + (6*C*a^3 - 4*A*a
^2*b + 3*(B + C)*a*b^2 - 2*A*b^3)*c^2 + (4*A*a^4 - 6*(B - C)*a^3*b + 4*A*a^
2*b^2 - 3*(B - C)*a*b^3 + A*b^4)*c)*cosh(x)^2 + 2*(4*A*a^4*b - 6*B*a^3*b^2
+ 4*A*a^2*b^3 - 3*B*a*b^4 + A*b^5 + A*c^5 - (3*C*a - A*b)*c^4 - (4*A*a^2 -
3*(B - C)*a*b + 2*A*b^2)*c^3 + (6*C*a^3 - 4*A*a^2*b + 3*(B + C)*a*b^2 - 2*A
*b^3)*c^2 + 3*(2*A*a^2*b^3 - 3*B*a*b^4 + A*b^5 - A*c^5 + 3*(C*a - A*b)*c^4
+ (2*A*a^2 - 3*(B - 3*C)*a*b - 2*A*b^2)*c^3 + (6*A*a^2*b - 9*(B - C)*a*b^2
+ 2*A*b^3)*c^2 + 3*(2*A*a^2*b^2 - (3*B - C)*a*b^3 + A*b^4)*c)*cosh(x)^2 + (
4*A*a^4 - 6*(B - C)*a^3*b + 4*A*a^2*b^2 - 3*(B - C)*a*b^3 + A*b^4)*c + 6*(2
*A*a^3*b^2 - 3*B*a^2*b^3 + A*a*b^4 - A*a*c^4 + (3*C*a^2 - 2*A*a*b)*c^3 + (2
*A*a^3 - 3*(B - 2*C)*a^2*b)*c^2 + (4*A*a^3*b - 3*(2*B - C)*a^2*b^2 + 2*A*a*
b^3)*c)*cosh(x))*sinh(x)^2 - (2*A*a^2*b^2 - 3*(B + C)*a*b^3 + A*b^4)*c + 4*
(2*A*a^3*b^2 - 3*B*a^2*b^3 + A*a*b^4 + 3*C*a^2*b^2*c - 3*C*a^2*c^3 + A*a*c^
4 - (2*A*a^3 - 3*B*a^2*b + 2*A*a*b^2)*c^2)*cosh(x) + 4*(2*A*a^3*b^2 - 3*B*a
^2*b^3 + A*a*b^4 + 3*C*a^2*b^2*c - 3*C*a^2*c^3 + A*a*c^4 + (2*A*a^2*b^3 - 3
*B*a*b^4 + A*b^5 - A*c^5 + 3*(C*a - A*b)*c^4 + (2*A*a^2 - 3*(B - 3*C)*a*b -
 2*A*b^2)*c^3 + (6*A*a^2*b - 9*(B - C)*a*b^2 + 2*A*b^3)*c^2 + 3*(2*A*a^2*b^
2 - (3*B - C)*a*b^3 + A*b^4)*c)*cosh(x)^3 - (2*A*a^3 - 3*B*a^2*b + 2*A*a*b^
2)*c^2 + 3*(2*A*a^3*b^2 - 3*B*a^2*b^3 + A*a*b^4 - A*a*c^4 + (3*C*a^2 - 2*A*
a*b)*c^3 + (2*A*a^3 - 3*(B - 2*C)*a^2*b)*c^2 + (4*A*a^3*b - 3*(2*B - C)*a^2
*b^2 + 2*A*a*b^3)*c)*cosh(x)^2 + (4*A*a^4*b - 6*B*a^3*b^2 + 4*A*a^2*b^3 - 3
*B*a*b^4 + A*b^5 + A*c^5 - (3*C*a - A*b)*c^4 - (4*A*a^2 - 3*(B - C)*a*b + 2
*A*b^2)*c^3 + (6*C*a^3 - 4*A*a^2*b + 3*(B + C)*a*b^2 - 2*A*b^3)*c^2 + (4*A*
a^4 - 6*(B - C)*a^3*b + 4*A*a^2*b^2 - 3*(B - C)*a*b^3 + A*b^4)*c)*cosh(x))*
sinh(x))*sqrt(a^2 - b^2 + c^2)*log(((b^2 + 2*b*c + c^2)*cosh(x)^2 + (b^2 +
2*b*c + c^2)*sinh(x)^2 + 2*a^2 - b^2 + c^2 + 2*(a*b + a*c)*cosh(x) + 2*(a*b
 + a*c + (b^2 + 2*b*c + c^2)*cosh(x))*sinh(x) + 2*sqrt(a^2 - b^2 + c^2)*((b
 + c)*cosh(x) + (b + c)*sinh(x) + a))/((b + c)*cosh(x)^2 + (b + c)*sinh(x)^
2 + 2*a*cosh(x) + 2*((b + c)*cosh(x) + a)*sinh(x) + b - c)) - 2*((B + C)*a^
4*b - 3*A*a^3*b^2 + (B + C)*a^2*b^3 + 3*A*a*b^4 - 2*(B + C)*b^5)*c + 2*(4*B
*a^5*b - 10*A*a^4*b^2 + B*a^3*b^3 + 11*A*a^2*b^4 - 5*B*a*b^5 - A*b^6 + 5*C*
a*c^5 + A*c^6 + (11*A*a^2 - 5*B*a*b - 3*A*b^2)*c^4 + (C*a^3 - 10*C*a*b^2)*c
^3 + (10*A*a^4 - B*a^3*b - 22*A*a^2*b^2 + 10*B*a*b^3 + 3*A*b^4)*c^2 - (4*C*
a^5 + C*a^3*b^2 - 5*C*a*b^4)*c)*cosh(x) + 2*(4*B*a^5*b - 10*A*a^4*b^2 + B*a
^3*b^3 + 11*A*a^2*b^4 - 5*B*a*b^5 - A*b^6 + 5*C*a*c^5 + A*c^6 + (11*A*a^2 -
 5*B*a*b - 3*A*b^2)*c^4 + (C*a^3 - 10*C*a*b^2)*c^3 + (10*A*a^4 - B*a^3*b -
22*A*a^2*b^2 + 10*B*a*b^3 + 3*A*b^4)*c^2 - 3*(2*A*a^4*b^2 - 3*B*a^3*b^3 - A
```

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*a^2*b^4 + 3*B*a*b^5 - A*b^6 - A*c^6 + (3*C*a - 2*A*b)*c^5 + (A*a^2 - 3*(B
- 2*C)*a*b + A*b^2)*c^4 + (3*C*a^3 + 2*A*a^2*b - 6*B*a*b^2 + 4*A*b^3)*c^3 +
 (2*A*a^4 - 3*(B - 2*C)*a^3*b - 6*C*a*b^3 + A*b^4)*c^2 + (4*A*a^4*b - 3*(2*
B - C)*a^3*b^2 - 2*A*a^2*b^3 + 3*(2*B - C)*a*b^4 - 2*A*b^5)*c)*cosh(x)^2 -
(4*C*a^5 + C*a^3*b^2 - 5*C*a*b^4)*c + 2*(2*(B + C)*a^6 - 6*A*a^5*b + 3*(B -
 2*C)*a^4*b^2 + 3*A*a^3*b^3 - 3*(B - 2*C)*a^2*b^4 + 3*A*a*b^5 - 2*(B + C)*b
^6 + 3*A*a*c^5 + 2*(B + C)*c^6 + 3*((2*B - C)*a^2 + A*a*b - 2*(B + C)*b^2)*
c^4 - 3*(A*a^3 - 3*(B - C)*a^2*b + 2*A*a*b^2)*c^3 + 3*((2*B - C)*a^4 - A*a^
3*b - (B + C)*a^2*b^2 - 2*A*a*b^3 + 2*(B + C)*b^4)*c^2 - 3*(2*A*a^5 - 3*(B
- C)*a^4*b - A*a^3*b^2 + 3*(B - C)*a^2*b^3 - A*a*b^4)*c)*cosh(x))*sinh(x))/
(a^6*b^3 - 3*a^4*b^5 + 3*a^2*b^7 - b^9 - b*c^8 + c^9 + (3*a^2 - 4*b^2)*c^7
- (3*a^2*b - 4*b^3)*c^6 + 3*(a^4 - 3*a^2*b^2 + 2*b^4)*c^5 - 3*(a^4*b - 3*a^
2*b^3 + 2*b^5)*c^4 + (a^6*b^3 - 3*a^4*b^5 + 3*a^2*b^7 - b^9 + 3*a^2*c^7 + 3
*b*c^8 + c^9 + (9*a^2*b - 8*b^3)*c^6 + 3*(a^4 + a^2*b^2 - 2*b^4)*c^5 + 3*(3
*a^4*b - 5*a^2*b^3 + 2*b^5)*c^4 + (a^6 + 6*a^4*b^2 - 15*a^2*b^4 + 8*b^6)*c^
3 + 3*(a^6*b - 2*a^4*b^3 + a^2*b^5)*c^2 + 3*(a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^
6 - b^8)*c)*cosh(x)^4 + (a^6*b^3 - 3*a^4*b^5 + 3*a^2*b^7 - b^9 + 3*a^2*c^7
+ 3*b*c^8 + c^9 + (9*a^2*b - 8*b^3)*c^6 + 3*(a^4 + a^2*b^2 - 2*b^4)*c^5 + 3
*(3*a^4*b - 5*a^2*b^3 + 2*b^5)*c^4 + (a^6 + 6*a^4*b^2 - 15*a^2*b^4 + 8*b^6)
*c^3 + 3*(a^6*b - 2*a^4*b^3 + a^2*b^5)*c^2 + 3*(a^6*b^2 - 3*a^4*b^4 + 3*a^2
*b^6 - b^8)*c)*sinh(x)^4 + (a^6 - 6*a^4*b^2 + 9*a^2*b^4 - 4*b^6)*c^3 + 4*(a
^7*b^2 - 3*a^5*b^4 + 3*a^3*b^6 - a*b^8 + 2*a*b*c^7 + a*c^8 + (3*a^3 - 2*a*b
^2)*c^6 + 6*(a^3*b - a*b^3)*c^5 + 3*(a^5 - a^3*b^2)*c^4 + 6*(a^5*b - 2*a^3*
b^3 + a*b^5)*c^3 + (a^7 - 3*a^3*b^4 + 2*a*b^6)*c^2 + 2*(a^7*b - 3*a^5*b^3 +
 3*a^3*b^5 - a*b^7)*c)*cosh(x)^3 + 4*(a^7*b^2 - 3*a^5*b^4 + 3*a^3*b^6 - a*b
^8 + 2*a*b*c^7 + a*c^8 + (3*a^3 - 2*a*b^2)*c^6 + 6*(a^3*b - a*b^3)*c^5 + 3*
(a^5 - a^3*b^2)*c^4 + 6*(a^5*b - 2*a^3*b^3 + a*b^5)*c^3 + (a^7 - 3*a^3*b^4
+ 2*a*b^6)*c^2 + 2*(a^7*b - 3*a^5*b^3 + 3*a^3*b^5 - a*b^7)*c + (a^6*b^3 - 3
*a^4*b^5 + 3*a^2*b^7 - b^9 + 3*a^2*c^7 + 3*b*c^8 + c^9 + (9*a^2*b - 8*b^3)*
c^6 + 3*(a^4 + a^2*b^2 - 2*b^4)*c^5 + 3*(3*a^4*b - 5*a^2*b^3 + 2*b^5)*c^4 +
 (a^6 + 6*a^4*b^2 - 15*a^2*b^4 + 8*b^6)*c^3 + 3*(a^6*b - 2*a^4*b^3 + a^2*b^
5)*c^2 + 3*(a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^8)*c)*cosh(x))*sinh(x)^3 -
(a^6*b - 6*a^4*b^3 + 9*a^2*b^5 - 4*b^7)*c^2 + 2*(2*a^8*b - 5*a^6*b^3 + 3*a^
4*b^5 + a^2*b^7 - b^9 - b*c^8 - c^9 - (a^2 - 4*b^2)*c^7 - (a^2*b - 4*b^3)*c
^6 + 3*(a^4 + a^2*b^2 - 2*b^4)*c^5 + 3*(a^4*b + a^2*b^3 - 2*b^5)*c^4 + (5*a
^6 - 6*a^4*b^2 - 3*a^2*b^4 + 4*b^6)*c^3 + (5*a^6*b - 6*a^4*b^3 - 3*a^2*b^5
+ 4*b^7)*c^2 + (2*a^8 - 5*a^6*b^2 + 3*a^4*b^4 + a^2*b^6 - b^8)*c)*cosh(x)^2
 + 2*(2*a^8*b - 5*a^6*b^3 + 3*a^4*b^5 + a^2*b^7 - b^9 - b*c^8 - c^9 - (a^2
- 4*b^2)*c^7 - (a^2*b - 4*b^3)*c^6 + 3*(a^4 + a^2*b^2 - 2*b^4)*c^5 + 3*(a^4
*b + a^2*b^3 - 2*b^5)*c^4 + (5*a^6 - 6*a^4*b^2 - 3*a^2*b^4 + 4*b^6)*c^3 + (
5*a^6*b - 6*a^4*b^3 - 3*a^2*b^5 + 4*b^7)*c^2 + 3*(a^6*b^3 - 3*a^4*b^5 + 3*a
^2*b^7 - b^9 + 3*a^2*c^7 + 3*b*c^8 + c^9 + (9*a^2*b - 8*b^3)*c^6 + 3*(a^4 +
 a^2*b^2 - 2*b^4)*c^5 + 3*(3*a^4*b - 5*a^2*b^3 + 2*b^5)*c^4 + (a^6 + 6*a^4*
b^2 - 15*a^2*b^4 + 8*b^6)*c^3 + 3*(a^6*b - 2*a^4*b^3 + a^2*b^5)*c^2 + 3*(a^
6*b^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^8)*c)*cosh(x)^2 + (2*a^8 - 5*a^6*b^2 + 3*
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a^4*b^4 + a^2*b^6 - b^8)*c + 6*(a^7*b^2 - 3*a^5*b^4 + 3*a^3*b^6 - a*b^8 + 2
*a*b*c^7 + a*c^8 + (3*a^3 - 2*a*b^2)*c^6 + 6*(a^3*b - a*b^3)*c^5 + 3*(a^5 -
 a^3*b^2)*c^4 + 6*(a^5*b - 2*a^3*b^3 + a*b^5)*c^3 + (a^7 - 3*a^3*b^4 + 2*a*
b^6)*c^2 + 2*(a^7*b - 3*a^5*b^3 + 3*a^3*b^5 - a*b^7)*c)*cosh(x))*sinh(x)^2
- (a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^8)*c + 4*(a^7*b^2 - 3*a^5*b^4 + 3*a^
3*b^6 - a*b^8 - a*c^8 - (3*a^3 - 4*a*b^2)*c^6 - 3*(a^5 - 3*a^3*b^2 + 2*a*b^
4)*c^4 - (a^7 - 6*a^5*b^2 + 9*a^3*b^4 - 4*a*b^6)*c^2)*cosh(x) + 4*(a^7*b^2
- 3*a^5*b^4 + 3*a^3*b^6 - a*b^8 - a*c^8 - (3*a^3 - 4*a*b^2)*c^6 - 3*(a^5 -
3*a^3*b^2 + 2*a*b^4)*c^4 + (a^6*b^3 - 3*a^4*b^5 + 3*a^2*b^7 - b^9 + 3*a^2*c
^7 + 3*b*c^8 + c^9 + (9*a^2*b - 8*b^3)*c^6 + 3*(a^4 + a^2*b^2 - 2*b^4)*c^5
+ 3*(3*a^4*b - 5*a^2*b^3 + 2*b^5)*c^4 + (a^6 + 6*a^4*b^2 - 15*a^2*b^4 + 8*b
^6)*c^3 + 3*(a^6*b - 2*a^4*b^3 + a^2*b^5)*c^2 + 3*(a^6*b^2 - 3*a^4*b^4 + 3*
a^2*b^6 - b^8)*c)*cosh(x)^3 - (a^7 - 6*a^5*b^2 + 9*a^3*b^4 - 4*a*b^6)*c^2 +
 3*(a^7*b^2 - 3*a^5*b^4 + 3*a^3*b^6 - a*b^8 + 2*a*b*c^7 + a*c^8 + (3*a^3 -
2*a*b^2)*c^6 + 6*(a^3*b - a*b^3)*c^5 + 3*(a^5 - a^3*b^2)*c^4 + 6*(a^5*b - 2
*a^3*b^3 + a*b^5)*c^3 + (a^7 - 3*a^3*b^4 + 2*a*b^6)*c^2 + 2*(a^7*b - 3*a^5*
b^3 + 3*a^3*b^5 - a*b^7)*c)*cosh(x)^2 + (2*a^8*b - 5*a^6*b^3 + 3*a^4*b^5 +
a^2*b^7 - b^9 - b*c^8 - c^9 - (a^2 - 4*b^2)*c^7 - (a^2*b - 4*b^3)*c^6 + 3*(
a^4 + a^2*b^2 - 2*b^4)*c^5 + 3*(a^4*b + a^2*b^3 - 2*b^5)*c^4 + (5*a^6 - 6*a
^4*b^2 - 3*a^2*b^4 + 4*b^6)*c^3 + (5*a^6*b - 6*a^4*b^3 - 3*a^2*b^5 + 4*b^7)
*c^2 + (2*a^8 - 5*a^6*b^2 + 3*a^4*b^4 + a^2*b^6 - b^8)*c)*cosh(x))*sinh(x))
, -(B*a^4*b^2 - 3*A*a^3*b^3 + B*a^2*b^4 + 3*A*a*b^5 - 2*B*b^6 - 2*C*c^6 - (
3*A*a - 2*(B + C)*b)*c^5 - (C*a^2 - 3*A*a*b + 2*(B - 2*C)*b^2)*c^4 - (3*A*a
^3 - (B + C)*a^2*b - 6*A*a*b^2 + 4*(B + C)*b^3)*c^3 - (2*A*a^4*b^2 - 3*B*a^
3*b^3 - A*a^2*b^4 + 3*B*a*b^5 - A*b^6 - A*c^6 + (3*C*a - 2*A*b)*c^5 + (A*a^
2 - 3*(B - 2*C)*a*b + A*b^2)*c^4 + (3*C*a^3 + 2*A*a^2*b - 6*B*a*b^2 + 4*A*b
^3)*c^3 + (2*A*a^4 - 3*(B - 2*C)*a^3*b - 6*C*a*b^3 + A*b^4)*c^2 + (4*A*a^4*
b - 3*(2*B - C)*a^3*b^2 - 2*A*a^2*b^3 + 3*(2*B - C)*a*b^4 - 2*A*b^5)*c)*cos
h(x)^3 - (2*A*a^4*b^2 - 3*B*a^3*b^3 - A*a^2*b^4 + 3*B*a*b^5 - A*b^6 - A*c^6
 + (3*C*a - 2*A*b)*c^5 + (A*a^2 - 3*(B - 2*C)*a*b + A*b^2)*c^4 + (3*C*a^3 +
 2*A*a^2*b - 6*B*a*b^2 + 4*A*b^3)*c^3 + (2*A*a^4 - 3*(B - 2*C)*a^3*b - 6*C*
a*b^3 + A*b^4)*c^2 + (4*A*a^4*b - 3*(2*B - C)*a^3*b^2 - 2*A*a^2*b^3 + 3*(2*
B - C)*a*b^4 - 2*A*b^5)*c)*sinh(x)^3 + (C*a^4 + 3*A*a^3*b - (B - C)*a^2*b^2
 - 6*A*a*b^3 + 2*(2*B - C)*b^4)*c^2 + (2*(B + C)*a^6 - 6*A*a^5*b + 3*(B - 2
*C)*a^4*b^2 + 3*A*a^3*b^3 - 3*(B - 2*C)*a^2*b^4 + 3*A*a*b^5 - 2*(B + C)*b^6
 + 3*A*a*c^5 + 2*(B + C)*c^6 + 3*((2*B - C)*a^2 + A*a*b - 2*(B + C)*b^2)*c^
4 - 3*(A*a^3 - 3*(B - C)*a^2*b + 2*A*a*b^2)*c^3 + 3*((2*B - C)*a^4 - A*a^3*
b - (B + C)*a^2*b^2 - 2*A*a*b^3 + 2*(B + C)*b^4)*c^2 - 3*(2*A*a^5 - 3*(B -
C)*a^4*b - A*a^3*b^2 + 3*(B - C)*a^2*b^3 - A*a*b^4)*c)*cosh(x)^2 + (2*(B +
C)*a^6 - 6*A*a^5*b + 3*(B - 2*C)*a^4*b^2 + 3*A*a^3*b^3 - 3*(B - 2*C)*a^2*b^
4 + 3*A*a*b^5 - 2*(B + C)*b^6 + 3*A*a*c^5 + 2*(B + C)*c^6 + 3*((2*B - C)*a^
2 + A*a*b - 2*(B + C)*b^2)*c^4 - 3*(A*a^3 - 3*(B - C)*a^2*b + 2*A*a*b^2)*c^
3 + 3*((2*B - C)*a^4 - A*a^3*b - (B + C)*a^2*b^2 - 2*A*a*b^3 + 2*(B + C)*b^
4)*c^2 - 3*(2*A*a^5 - 3*(B - C)*a^4*b - A*a^3*b^2 + 3*(B - C)*a^2*b^3 - A*a
*b^4)*c - 3*(2*A*a^4*b^2 - 3*B*a^3*b^3 - A*a^2*b^4 + 3*B*a*b^5 - A*b^6 - A*
```

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c^6 + (3*C*a - 2*A*b)*c^5 + (A*a^2 - 3*(B - 2*C)*a*b + A*b^2)*c^4 + (3*C*a^
3 + 2*A*a^2*b - 6*B*a*b^2 + 4*A*b^3)*c^3 + (2*A*a^4 - 3*(B - 2*C)*a^3*b - 6
*C*a*b^3 + A*b^4)*c^2 + (4*A*a^4*b - 3*(2*B - C)*a^3*b^2 - 2*A*a^2*b^3 + 3*
(2*B - C)*a*b^4 - 2*A*b^5)*c)*cosh(x))*sinh(x)^2 - (2*A*a^2*b^3 - 3*B*a*b^4
 + A*b^5 - A*c^5 + (3*C*a + A*b)*c^4 + (2*A*a^2*b^3 - 3*B*a*b^4 + A*b^5 - A
*c^5 + 3*(C*a - A*b)*c^4 + (2*A*a^2 - 3*(B - 3*C)*a*b - 2*A*b^2)*c^3 + (6*A
*a^2*b - 9*(B - C)*a*b^2 + 2*A*b^3)*c^2 + 3*(2*A*a^2*b^2 - (3*B - C)*a*b^3
+ A*b^4)*c)*cosh(x)^4 + (2*A*a^2*b^3 - 3*B*a*b^4 + A*b^5 - A*c^5 + 3*(C*a -
 A*b)*c^4 + (2*A*a^2 - 3*(B - 3*C)*a*b - 2*A*b^2)*c^3 + (6*A*a^2*b - 9*(B -
 C)*a*b^2 + 2*A*b^3)*c^2 + 3*(2*A*a^2*b^2 - (3*B - C)*a*b^3 + A*b^4)*c)*sin
h(x)^4 + (2*A*a^2 - 3*(B + C)*a*b + 2*A*b^2)*c^3 + 4*(2*A*a^3*b^2 - 3*B*a^2
*b^3 + A*a*b^4 - A*a*c^4 + (3*C*a^2 - 2*A*a*b)*c^3 + (2*A*a^3 - 3*(B - 2*C)
*a^2*b)*c^2 + (4*A*a^3*b - 3*(2*B - C)*a^2*b^2 + 2*A*a*b^3)*c)*cosh(x)^3 +
4*(2*A*a^3*b^2 - 3*B*a^2*b^3 + A*a*b^4 - A*a*c^4 + (3*C*a^2 - 2*A*a*b)*c^3
+ (2*A*a^3 - 3*(B - 2*C)*a^2*b)*c^2 + (4*A*a^3*b - 3*(2*B - C)*a^2*b^2 + 2*
A*a*b^3)*c + (2*A*a^2*b^3 - 3*B*a*b^4 + A*b^5 - A*c^5 + 3*(C*a - A*b)*c^4 +
 (2*A*a^2 - 3*(B - 3*C)*a*b - 2*A*b^2)*c^3 + (6*A*a^2*b - 9*(B - C)*a*b^2 +
 2*A*b^3)*c^2 + 3*(2*A*a^2*b^2 - (3*B - C)*a*b^3 + A*b^4)*c)*cosh(x))*sinh(
x)^3 - (2*A*a^2*b - 3*(B - C)*a*b^2 + 2*A*b^3)*c^2 + 2*(4*A*a^4*b - 6*B*a^3
*b^2 + 4*A*a^2*b^3 - 3*B*a*b^4 + A*b^5 + A*c^5 - (3*C*a - A*b)*c^4 - (4*A*a
^2 - 3*(B - C)*a*b + 2*A*b^2)*c^3 + (6*C*a^3 - 4*A*a^2*b + 3*(B + C)*a*b^2
- 2*A*b^3)*c^2 + (4*A*a^4 - 6*(B - C)*a^3*b + 4*A*a^2*b^2 - 3*(B - C)*a*b^3
 + A*b^4)*c)*cosh(x)^2 + 2*(4*A*a^4*b - 6*B*a^3*b^2 + 4*A*a^2*b^3 - 3*B*a*b
^4 + A*b^5 + A*c^5 - (3*C*a - A*b)*c^4 - (4*A*a^2 - 3*(B - C)*a*b + 2*A*b^2
)*c^3 + (6*C*a^3 - 4*A*a^2*b + 3*(B + C)*a*b^2 - 2*A*b^3)*c^2 + 3*(2*A*a^2*
b^3 - 3*B*a*b^4 + A*b^5 - A*c^5 + 3*(C*a - A*b)*c^4 + (2*A*a^2 - 3*(B - 3*C
)*a*b - 2*A*b^2)*c^3 + (6*A*a^2*b - 9*(B - C)*a*b^2 + 2*A*b^3)*c^2 + 3*(2*A
*a^2*b^2 - (3*B - C)*a*b^3 + A*b^4)*c)*cosh(x)^2 + (4*A*a^4 - 6*(B - C)*a^3
*b + 4*A*a^2*b^2 - 3*(B - C)*a*b^3 + A*b^4)*c + 6*(2*A*a^3*b^2 - 3*B*a^2*b^
3 + A*a*b^4 - A*a*c^4 + (3*C*a^2 - 2*A*a*b)*c^3 + (2*A*a^3 - 3*(B - 2*C)*a^
2*b)*c^2 + (4*A*a^3*b - 3*(2*B - C)*a^2*b^2 + 2*A*a*b^3)*c)*cosh(x))*sinh(x
)^2 - (2*A*a^2*b^2 - 3*(B + C)*a*b^3 + A*b^4)*c + 4*(2*A*a^3*b^2 - 3*B*a^2*
b^3 + A*a*b^4 + 3*C*a^2*b^2*c - 3*C*a^2*c^3 + A*a*c^4 - (2*A*a^3 - 3*B*a^2*
b + 2*A*a*b^2)*c^2)*cosh(x) + 4*(2*A*a^3*b^2 - 3*B*a^2*b^3 + A*a*b^4 + 3*C*
a^2*b^2*c - 3*C*a^2*c^3 + A*a*c^4 + (2*A*a^2*b^3 - 3*B*a*b^4 + A*b^5 - A*c^
5 + 3*(C*a - A*b)*c^4 + (2*A*a^2 - 3*(B - 3*C)*a*b - 2*A*b^2)*c^3 + (6*A*a^
2*b - 9*(B - C)*a*b^2 + 2*A*b^3)*c^2 + 3*(2*A*a^2*b^2 - (3*B - C)*a*b^3 + A
*b^4)*c)*cosh(x)^3 - (2*A*a^3 - 3*B*a^2*b + 2*A*a*b^2)*c^2 + 3*(2*A*a^3*b^2
 - 3*B*a^2*b^3 + A*a*b^4 - A*a*c^4 + (3*C*a^2 - 2*A*a*b)*c^3 + (2*A*a^3 - 3
*(B - 2*C)*a^2*b)*c^2 + (4*A*a^3*b - 3*(2*B - C)*a^2*b^2 + 2*A*a*b^3)*c)*co
sh(x)^2 + (4*A*a^4*b - 6*B*a^3*b^2 + 4*A*a^2*b^3 - 3*B*a*b^4 + A*b^5 + A*c^
5 - (3*C*a - A*b)*c^4 - (4*A*a^2 - 3*(B - C)*a*b + 2*A*b^2)*c^3 + (6*C*a^3
- 4*A*a^2*b + 3*(B + C)*a*b^2 - 2*A*b^3)*c^2 + (4*A*a^4 - 6*(B - C)*a^3*b +
 4*A*a^2*b^2 - 3*(B - C)*a*b^3 + A*b^4)*c)*cosh(x))*sinh(x))*sqrt(-a^2 + b^
2 - c^2)*arctan(sqrt(-a^2 + b^2 - c^2)*((b + c)*cosh(x) + (b + c)*sinh(x) +
```

```
a)/(a^2 - b^2 + c^2)) - ((B + C)*a^4*b - 3*A*a^3*b^2 + (B + C)*a^2*b^3 + 3
*A*a*b^4 - 2*(B + C)*b^5)*c + (4*B*a^5*b - 10*A*a^4*b^2 + B*a^3*b^3 + 11*A*
a^2*b^4 - 5*B*a*b^5 - A*b^6 + 5*C*a*c^5 + A*c^6 + (11*A*a^2 - 5*B*a*b - 3*A
*b^2)*c^4 + (C*a^3 - 10*C*a*b^2)*c^3 + (10*A*a^4 - B*a^3*b - 22*A*a^2*b^2 +
 10*B*a*b^3 + 3*A*b^4)*c^2 - (4*C*a^5 + C*a^3*b^2 - 5*C*a*b^4)*c)*cosh(x) +
 (4*B*a^5*b - 10*A*a^4*b^2 + B*a^3*b^3 + 11*A*a^2*b^4 - 5*B*a*b^5 - A*b^6 +
 5*C*a*c^5 + A*c^6 + (11*A*a^2 - 5*B*a*b - 3*A*b^2)*c^4 + (C*a^3 - 10*C*a*b
^2)*c^3 + (10*A*a^4 - B*a^3*b - 22*A*a^2*b^2 + 10*B*a*b^3 + 3*A*b^4)*c^2 -
3*(2*A*a^4*b^2 - 3*B*a^3*b^3 - A*a^2*b^4 + 3*B*a*b^5 - A*b^6 - A*c^6 + (3*C
*a - 2*A*b)*c^5 + (A*a^2 - 3*(B - 2*C)*a*b + A*b^2)*c^4 + (3*C*a^3 + 2*A*a^
2*b - 6*B*a*b^2 + 4*A*b^3)*c^3 + (2*A*a^4 - 3*(B - 2*C)*a^3*b - 6*C*a*b^3 +
 A*b^4)*c^2 + (4*A*a^4*b - 3*(2*B - C)*a^3*b^2 - 2*A*a^2*b^3 + 3*(2*B - C)*
a*b^4 - 2*A*b^5)*c)*cosh(x)^2 - (4*C*a^5 + C*a^3*b^2 - 5*C*a*b^4)*c + 2*(2*
(B + C)*a^6 - 6*A*a^5*b + 3*(B - 2*C)*a^4*b^2 + 3*A*a^3*b^3 - 3*(B - 2*C)*a
^2*b^4 + 3*A*a*b^5 - 2*(B + C)*b^6 + 3*A*a*c^5 + 2*(B + C)*c^6 + 3*((2*B -
C)*a^2 + A*a*b - 2*(B + C)*b^2)*c^4 - 3*(A*a^3 - 3*(B - C)*a^2*b + 2*A*a*b^
2)*c^3 + 3*((2*B - C)*a^4 - A*a^3*b - (B + C)*a^2*b^2 - 2*A*a*b^3 + 2*(B +
C)*b^4)*c^2 - 3*(2*A*a^5 - 3*(B - C)*a^4*b - A*a^3*b^2 + 3*(B - C)*a^2*b^3
- A*a*b^4)*c)*cosh(x))*sinh(x))/(a^6*b^3 - 3*a^4*b^5 + 3*a^2*b^7 - b^9 - b*
c^8 + c^9 + (3*a^2 - 4*b^2)*c^7 - (3*a^2*b - 4*b^3)*c^6 + 3*(a^4 - 3*a^2*b^
2 + 2*b^4)*c^5 - 3*(a^4*b - 3*a^2*b^3 + 2*b^5)*c^4 + (a^6*b^3 - 3*a^4*b^5 +
 3*a^2*b^7 - b^9 + 3*a^2*c^7 + 3*b*c^8 + c^9 + (9*a^2*b - 8*b^3)*c^6 + 3*(a
^4 + a^2*b^2 - 2*b^4)*c^5 + 3*(3*a^4*b - 5*a^2*b^3 + 2*b^5)*c^4 + (a^6 + 6*
a^4*b^2 - 15*a^2*b^4 + 8*b^6)*c^3 + 3*(a^6*b - 2*a^4*b^3 + a^2*b^5)*c^2 + 3
*(a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^8)*c)*cosh(x)^4 + (a^6*b^3 - 3*a^4*b^
5 + 3*a^2*b^7 - b^9 + 3*a^2*c^7 + 3*b*c^8 + c^9 + (9*a^2*b - 8*b^3)*c^6 + 3
*(a^4 + a^2*b^2 - 2*b^4)*c^5 + 3*(3*a^4*b - 5*a^2*b^3 + 2*b^5)*c^4 + (a^6 +
 6*a^4*b^2 - 15*a^2*b^4 + 8*b^6)*c^3 + 3*(a^6*b - 2*a^4*b^3 + a^2*b^5)*c^2
+ 3*(a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^8)*c)*sinh(x)^4 + (a^6 - 6*a^4*b^2
 + 9*a^2*b^4 - 4*b^6)*c^3 + 4*(a^7*b^2 - 3*a^5*b^4 + 3*a^3*b^6 - a*b^8 + 2*
a*b*c^7 + a*c^8 + (3*a^3 - 2*a*b^2)*c^6 + 6*(a^3*b - a*b^3)*c^5 + 3*(a^5 -
a^3*b^2)*c^4 + 6*(a^5*b - 2*a^3*b^3 + a*b^5)*c^3 + (a^7 - 3*a^3*b^4 + 2*a*b
^6)*c^2 + 2*(a^7*b - 3*a^5*b^3 + 3*a^3*b^5 - a*b^7)*c)*cosh(x)^3 + 4*(a^7*b
^2 - 3*a^5*b^4 + 3*a^3*b^6 - a*b^8 + 2*a*b*c^7 + a*c^8 + (3*a^3 - 2*a*b^2)*
c^6 + 6*(a^3*b - a*b^3)*c^5 + 3*(a^5 - a^3*b^2)*c^4 + 6*(a^5*b - 2*a^3*b^3
+ a*b^5)*c^3 + (a^7 - 3*a^3*b^4 + 2*a*b^6)*c^2 + 2*(a^7*b - 3*a^5*b^3 + 3*a
^3*b^5 - a*b^7)*c + (a^6*b^3 - 3*a^4*b^5 + 3*a^2*b^7 - b^9 + 3*a^2*c^7 + 3*
b*c^8 + c^9 + (9*a^2*b - 8*b^3)*c^6 + 3*(a^4 + a^2*b^2 - 2*b^4)*c^5 + 3*(3*
a^4*b - 5*a^2*b^3 + 2*b^5)*c^4 + (a^6 + 6*a^4*b^2 - 15*a^2*b^4 + 8*b^6)*c^3
 + 3*(a^6*b - 2*a^4*b^3 + a^2*b^5)*c^2 + 3*(a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^6
 - b^8)*c)*cosh(x))*sinh(x)^3 - (a^6*b - 6*a^4*b^3 + 9*a^2*b^5 - 4*b^7)*c^2
 + 2*(2*a^8*b - 5*a^6*b^3 + 3*a^4*b^5 + a^2*b^7 - b^9 - b*c^8 - c^9 - (a^2
- 4*b^2)*c^7 - (a^2*b - 4*b^3)*c^6 + 3*(a^4 + a^2*b^2 - 2*b^4)*c^5 + 3*(a^4
*b + a^2*b^3 - 2*b^5)*c^4 + (5*a^6 - 6*a^4*b^2 - 3*a^2*b^4 + 4*b^6)*c^3 + (
5*a^6*b - 6*a^4*b^3 - 3*a^2*b^5 + 4*b^7)*c^2 + (2*a^8 - 5*a^6*b^2 + 3*a^4*b
```

```
^4 + a^2*b^6 - b^8)*c)*cosh(x)^2 + 2*(2*a^8*b - 5*a^6*b^3 + 3*a^4*b^5 + a^2
*b^7 - b^9 - b*c^8 - c^9 - (a^2 - 4*b^2)*c^7 - (a^2*b - 4*b^3)*c^6 + 3*(a^4
 + a^2*b^2 - 2*b^4)*c^5 + 3*(a^4*b + a^2*b^3 - 2*b^5)*c^4 + (5*a^6 - 6*a^4*
b^2 - 3*a^2*b^4 + 4*b^6)*c^3 + (5*a^6*b - 6*a^4*b^3 - 3*a^2*b^5 + 4*b^7)*c^
2 + 3*(a^6*b^3 - 3*a^4*b^5 + 3*a^2*b^7 - b^9 + 3*a^2*c^7 + 3*b*c^8 + c^9 +
(9*a^2*b - 8*b^3)*c^6 + 3*(a^4 + a^2*b^2 - 2*b^4)*c^5 + 3*(3*a^4*b - 5*a^2*
b^3 + 2*b^5)*c^4 + (a^6 + 6*a^4*b^2 - 15*a^2*b^4 + 8*b^6)*c^3 + 3*(a^6*b -
2*a^4*b^3 + a^2*b^5)*c^2 + 3*(a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^8)*c)*cos
h(x)^2 + (2*a^8 - 5*a^6*b^2 + 3*a^4*b^4 + a^2*b^6 - b^8)*c + 6*(a^7*b^2 - 3
*a^5*b^4 + 3*a^3*b^6 - a*b^8 + 2*a*b*c^7 + a*c^8 + (3*a^3 - 2*a*b^2)*c^6 +
6*(a^3*b - a*b^3)*c^5 + 3*(a^5 - a^3*b^2)*c^4 + 6*(a^5*b - 2*a^3*b^3 + a*b^
5)*c^3 + (a^7 - 3*a^3*b^4 + 2*a*b^6)*c^2 + 2*(a^7*b - 3*a^5*b^3 + 3*a^3*b^5
 - a*b^7)*c)*cosh(x))*sinh(x)^2 - (a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^8)*c
 + 4*(a^7*b^2 - 3*a^5*b^4 + 3*a^3*b^6 - a*b^8 - a*c^8 - (3*a^3 - 4*a*b^2)*c
^6 - 3*(a^5 - 3*a^3*b^2 + 2*a*b^4)*c^4 - (a^7 - 6*a^5*b^2 + 9*a^3*b^4 - 4*a
*b^6)*c^2)*cosh(x) + 4*(a^7*b^2 - 3*a^5*b^4 + 3*a^3*b^6 - a*b^8 - a*c^8 - (
3*a^3 - 4*a*b^2)*c^6 - 3*(a^5 - 3*a^3*b^2 + 2*a*b^4)*c^4 + (a^6*b^3 - 3*a^4
*b^5 + 3*a^2*b^7 - b^9 + 3*a^2*c^7 + 3*b*c^8 + c^9 + (9*a^2*b - 8*b^3)*c^6
+ 3*(a^4 + a^2*b^2 - 2*b^4)*c^5 + 3*(3*a^4*b - 5*a^2*b^3 + 2*b^5)*c^4 + (a^
6 + 6*a^4*b^2 - 15*a^2*b^4 + 8*b^6)*c^3 + 3*(a^6*b - 2*a^4*b^3 + a^2*b^5)*c
^2 + 3*(a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^8)*c)*cosh(x)^3 - (a^7 - 6*a^5*
b^2 + 9*a^3*b^4 - 4*a*b^6)*c^2 + 3*(a^7*b^2 - 3*a^5*b^4 + 3*a^3*b^6 - a*b^8
 + 2*a*b*c^7 + a*c^8 + (3*a^3 - 2*a*b^2)*c^6 + 6*(a^3*b - a*b^3)*c^5 + 3*(a
^5 - a^3*b^2)*c^4 + 6*(a^5*b - 2*a^3*b^3 + a*b^5)*c^3 + (a^7 - 3*a^3*b^4 +
2*a*b^6)*c^2 + 2*(a^7*b - 3*a^5*b^3 + 3*a^3*b^5 - a*b^7)*c)*cosh(x)^2 + (2*
a^8*b - 5*a^6*b^3 + 3*a^4*b^5 + a^2*b^7 - b^9 - b*c^8 - c^9 - (a^2 - 4*b^2)
*c^7 - (a^2*b - 4*b^3)*c^6 + 3*(a^4 + a^2*b^2 - 2*b^4)*c^5 + 3*(a^4*b + a^2
*b^3 - 2*b^5)*c^4 + (5*a^6 - 6*a^4*b^2 - 3*a^2*b^4 + 4*b^6)*c^3 + (5*a^6*b
- 6*a^4*b^3 - 3*a^2*b^5 + 4*b^7)*c^2 + (2*a^8 - 5*a^6*b^2 + 3*a^4*b^4 + a^2
*b^6 - b^8)*c)*cosh(x))*sinh(x))]
```

giac [B] time = 0.19, size = 819, normalized size = 3.52

$$\frac{\left(2\,Aa^2-3\,Bab+Ab^2+3\,Cac-Ac^2\right)\arctan\left(\frac{be^x+ce^x+a}{\sqrt{-a^2+b^2-c^2}}\right)}{\left(a^4-2\,a^2b^2+b^4+2\,a^2c^2-2\,b^2c^2+c^4\right)\sqrt{-a^2+b^2-c^2}}+\frac{2\,Aa^2b^2e^{(3\,x)}-3\,Bab^3e^{(3\,x)}+Ab^4e^{(3\,x)}+4\,Aa^2bce^{(3\,x)}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x)+C*sinh(x))/(a+b*cosh(x)+c*sinh(x))^3,x, algorithm="g
iac")
```

```
[Out] (2*A*a^2 - 3*B*a*b + A*b^2 + 3*C*a*c - A*c^2)*arctan((b*e^x + c*e^x + a)/sq
rt(-a^2 + b^2 - c^2))/((a^4 - 2*a^2*b^2 + b^4 + 2*a^2*c^2 - 2*b^2*c^2 + c^4
)*sqrt(-a^2 + b^2 - c^2)) + (2*A*a^2*b^2*e^(3*x) - 3*B*a*b^3*e^(3*x) + A*b^
```

```
4*e^(3*x) + 4*A*a^2*b*c*e^(3*x) - 6*B*a*b^2*c*e^(3*x) + 3*C*a*b^2*c*e^(3*x)
 + 2*A*b^3*c*e^(3*x) + 2*A*a^2*c^2*e^(3*x) - 3*B*a*b*c^2*e^(3*x) + 6*C*a*b*
c^2*e^(3*x) + 3*C*a*c^3*e^(3*x) - 2*A*b*c^3*e^(3*x) - A*c^4*e^(3*x) - 2*B*a
^4*e^(2*x) - 2*C*a^4*e^(2*x) + 6*A*a^3*b*e^(2*x) - 5*B*a^2*b^2*e^(2*x) + 4*
C*a^2*b^2*e^(2*x) + 3*A*a*b^3*e^(2*x) - 2*B*b^4*e^(2*x) - 2*C*b^4*e^(2*x) +
 6*A*a^3*c*e^(2*x) - 9*B*a^2*b*c*e^(2*x) + 9*C*a^2*b*c*e^(2*x) + 3*A*a*b^2*
c*e^(2*x) - 4*B*a^2*c^2*e^(2*x) + 5*C*a^2*c^2*e^(2*x) - 3*A*a*b*c^2*e^(2*x)
 + 4*B*b^2*c^2*e^(2*x) + 4*C*b^2*c^2*e^(2*x) - 3*A*a*c^3*e^(2*x) - 2*B*c^4*
e^(2*x) - 2*C*c^4*e^(2*x) - 4*B*a^3*b*e^x + 10*A*a^2*b^2*e^x - 5*B*a*b^3*e^
x - A*b^4*e^x + 4*C*a^3*c*e^x + 5*C*a*b^2*c*e^x - 10*A*a^2*c^2*e^x + 5*B*a*
b*c^2*e^x + 2*A*b^2*c^2*e^x - 5*C*a*c^3*e^x - A*c^4*e^x - B*a^2*b^2 + 3*A*a
*b^3 - 2*B*b^4 + B*a^2*b*c + C*a^2*b*c - 3*A*a*b^2*c + 2*B*b^3*c + 2*C*b^3*
c - C*a^2*c^2 - 3*A*a*b*c^2 + 2*B*b^2*c^2 - 2*C*b^2*c^2 + 3*A*a*c^3 - 2*B*b
*c^3 - 2*C*b*c^3 + 2*C*c^4)/((a^4*b - 2*a^2*b^3 + b^5 + a^4*c - 2*a^2*b^2*c
 + b^4*c + 2*a^2*b*c^2 - 2*b^3*c^2 + 2*a^2*c^3 - 2*b^2*c^3 + b*c^4 + c^5)*(
b*e^(2*x) + c*e^(2*x) + 2*a*e^x + b - c)^2)
```

maple [B] time = 0.36, size = 1425, normalized size = 6.12

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cosh(x)+C*sinh(x))/(a+b*cosh(x)+c*sinh(x))^3,x)
```

```
[Out] -2*(-1/2*(4*A*a^3*b-7*A*a^2*b^2+5*A*a^2*c^2+2*A*a*b^3-2*A*a*b*c^2+A*b^4-3*A
*b^2*c^2+2*A*c^4-2*B*a^4+3*B*a^3*b-2*B*a^2*b^2-4*B*a^2*c^2+3*B*a*b^3-2*B*b^
4+4*B*b^2*c^2-2*B*c^4-3*C*a^3*c+6*C*a^2*b*c-3*C*a*b^2*c)/(a-b)/(a^4-2*a^2*b
^2+2*a^2*c^2+b^4-2*b^2*c^2+c^4)*tanh(1/2*x)^3-1/2*(4*A*a^4*c-12*A*a^3*b*c+1
3*A*a^2*b^2*c-7*A*a^2*c^3-6*A*a*b^3*c+6*A*a*b*c^3+A*b^4*c+A*b^2*c^3-2*A*c^5
+2*B*a^4*c-9*B*a^3*b*c+14*B*a^2*b^2*c+4*B*a^2*c^3-9*B*a*b^3*c+2*B*b^4*c-4*B
*b^2*c^3+2*B*c^5-2*C*a^5+2*C*a^4*b+4*C*a^3*b^2+5*C*a^3*c^2-4*C*a^2*b^3-14*C
*a^2*b*c^2-2*C*a*b^4+13*C*a*b^2*c^2-2*C*a*c^4+2*C*b^5-4*C*b^3*c^2+2*C*b*c^4
)/(a^4-2*a^2*b^2+2*a^2*c^2+b^4-2*b^2*c^2+c^4)/(a^2-2*a*b+b^2)*tanh(1/2*x)^2
+1/2*(4*A*a^4*b-5*A*a^3*b^2+11*A*a^3*c^2-3*A*a^2*b^3-3*A*a^2*b*c^2+5*A*a*b^
4-7*A*a*b^2*c^2+2*A*a*c^4-A*b^5-A*b^3*c^2+2*A*b*c^4-2*B*a^5+3*B*a^4*b-B*a^3
*b^2-4*B*a^3*c^2-B*a^2*b^3-8*B*a^2*b*c^2+3*B*a*b^4+8*B*a*b^2*c^2-2*B*a*c^4-
2*B*b^5+4*B*b^3*c^2-2*B*b*c^4-5*C*a^4*c+5*C*a^3*b*c+5*C*a^2*b^2*c+4*C*a^2*c
^3-5*C*a*b^3*c-4*C*a*b*c^3)/(a^4-2*a^2*b^2+2*a^2*c^2+b^4-2*b^2*c^2+c^4)/(a^
2-2*a*b+b^2)*tanh(1/2*x)+1/2*(4*A*a^4*c-3*A*a^2*b^2*c+A*a^2*c^3-A*b^4*c+A*b
^2*c^3-5*B*a^3*b*c+5*B*a*b^3*c-2*B*a*b*c^3-2*C*a^5+4*C*a^3*b^2+C*a^3*c^2-2*
C*a*b^4-C*a*b^2*c^2)/(a^4-2*a^2*b^2+2*a^2*c^2+b^4-2*b^2*c^2+c^4)/(a^2-2*a*b
+b^2))/(a*tanh(1/2*x)^2-tanh(1/2*x)^2*b-2*c*tanh(1/2*x)-a-b)^2-2/(a^4-2*a^2
*b^2+2*a^2*c^2+b^4-2*b^2*c^2+c^4)/(-a^2+b^2-c^2)^(1/2)*arctan(1/2*(2*(a-b)*
tanh(1/2*x)-2*c)/(-a^2+b^2-c^2)^(1/2))*a^2*A-1/(a^4-2*a^2*b^2+2*a^2*c^2+b^4
-2*b^2*c^2+c^4)/(-a^2+b^2-c^2)^(1/2)*arctan(1/2*(2*(a-b)*tanh(1/2*x)-2*c)/(
```

```
-a^2+b^2-c^2)^(1/2))*A*b^2+1/(a^4-2*a^2*b^2+2*a^2*c^2+b^4-2*b^2*c^2+c^4)/(-
a^2+b^2-c^2)^(1/2)*arctan(1/2*(2*(a-b)*tanh(1/2*x)-2*c)/(-a^2+b^2-c^2)^(1/2
))*A*c^2+3/(a^4-2*a^2*b^2+2*a^2*c^2+b^4-2*b^2*c^2+c^4)/(-a^2+b^2-c^2)^(1/2)
*arctan(1/2*(2*(a-b)*tanh(1/2*x)-2*c)/(-a^2+b^2-c^2)^(1/2))*b*B*a-3/(a^4-2*
a^2*b^2+2*a^2*c^2+b^4-2*b^2*c^2+c^4)/(-a^2+b^2-c^2)^(1/2)*arctan(1/2*(2*(a-
b)*tanh(1/2*x)-2*c)/(-a^2+b^2-c^2)^(1/2))*a*c*C
```

maxima [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x)+C*sinh(x))/(a+b*cosh(x)+c*sinh(x))^3,x, algorithm="m
axima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(c^2-b^2+a^2>0)', see `assume?` for
more details)Is c^2-b^2+a^2 positive or negative?
```

mupad [**F**] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A + B\cosh(x) + C\sinh(x)}{(a + b\cosh(x) + c\sinh(x))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cosh(x) + C*sinh(x))/(a + b*cosh(x) + c*sinh(x))^3,x)
```

```
[Out] int((A + B*cosh(x) + C*sinh(x))/(a + b*cosh(x) + c*sinh(x))^3, x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x)+C*sinh(x))/(a+b*cosh(x)+c*sinh(x))**3,x)
```

```
[Out] Timed out
```

3.801 $\int \frac{b^2-c^2+ab\cosh(x)+ac\sinh(x)}{(a+b\cosh(x)+c\sinh(x))^2}\,dx$

Optimal. Leaf size=22

$$\frac{b\sinh(x)+c\cosh(x)}{a+b\cosh(x)+c\sinh(x)}$$

```
[Out] (c*cosh(x)+b*sinh(x))/(a+b*cosh(x)+c*sinh(x))
```

Rubi [A] time = 0.08, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.031, Rules used = {3150}

$$\frac{b\sinh(x)+c\cosh(x)}{a+b\cosh(x)+c\sinh(x)}$$

Antiderivative was successfully verified.

```
[In] Int[(b^2 - c^2 + a*b*Cosh[x] + a*c*Sinh[x])/(a + b*Cosh[x] + c*Sinh[x])^2,x
]
```

```
[Out] (c*Cosh[x] + b*Sinh[x])/(a + b*Cosh[x] + c*Sinh[x])
```

Rule 3150

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])^2,
 x_Symbol] :> Simp[(c*B - b*C - (a*C - c*A)*Cos[d + e*x] + (a*B - b*A)*Sin[
d + e*x])/(e*(a^2 - b^2 - c^2)*(a + b*Cos[d + e*x] + c*Sin[d + e*x])), x] /
; FreeQ[{a, b, c, d, e, A, B, C}, x] && NeQ[a^2 - b^2 - c^2, 0] && EqQ[a*A
- b*B - c*C, 0]
```

Rubi steps

$$\int \frac{b^2-c^2+ab\cosh(x)+ac\sinh(x)}{(a+b\cosh(x)+c\sinh(x))^2}\,dx=\frac{c\cosh(x)+b\sinh(x)}{a+b\cosh(x)+c\sinh(x)}$$

Mathematica [A] time = 0.11, size = 34, normalized size = 1.55

$$\frac{-ac+b^2\sinh(x)-c^2\sinh(x)}{b(a+b\cosh(x)+c\sinh(x))}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b^2 - c^2 + a*b*Cosh[x] + a*c*Sinh[x])/(a + b*Cosh[x] + c*Sinh[x
])^2,x]
```

```
[Out] (-(a*c) + b^2*Sinh[x] - c^2*Sinh[x])/(b*(a + b*Cosh[x] + c*Sinh[x]))
```

fricas [B] time = 0.45, size = 55, normalized size = 2.50

$$-\frac{2\,(a\cosh(x) + a\sinh(x) + b - c)}{(b+c)\cosh(x)^2 + (b+c)\sinh(x)^2 + 2\,a\cosh(x) + 2\,((b+c)\cosh(x) + a)\sinh(x) + b - c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2-c^2+a*b*cosh(x)+a*c*sinh(x))/(a+b*cosh(x)+c*sinh(x))^2,x, al
gorithm="fricas")
```

```
[Out] -2*(a*cosh(x) + a*sinh(x) + b - c)/((b + c)*cosh(x)^2 + (b + c)*sinh(x)^2 +
 2*a*cosh(x) + 2*((b + c)*cosh(x) + a)*sinh(x) + b - c)
```

giac [A] time = 0.14, size = 35, normalized size = 1.59

$$-\frac{2\,(ae^x + b - c)}{be^{(2\,x)} + ce^{(2\,x)} + 2\,ae^x + b - c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2-c^2+a*b*cosh(x)+a*c*sinh(x))/(a+b*cosh(x)+c*sinh(x))^2,x, al
gorithm="giac")
```

```
[Out] -2*(a*e^x + b - c)/(b*e^(2*x) + c*e^(2*x) + 2*a*e^x + b - c)
```

maple [B] time = 0.27, size = 73, normalized size = 3.32

$$\frac{-\frac{2\left(ab-b^2+c^2\right)\tanh\left(\frac{x}{2}\right)}{a-b} - \frac{2ac}{a-b}}{a\left(\tanh^2\left(\frac{x}{2}\right)\right) - \left(\tanh^2\left(\frac{x}{2}\right)\right)b - 2c\tanh\left(\frac{x}{2}\right) - a - b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2-c^2+a*b*cosh(x)+a*c*sinh(x))/(a+b*cosh(x)+c*sinh(x))^2,x)
```

```
[Out] 2*(-(a*b-b^2+c^2)/(a-b)*tanh(1/2*x)-a*c/(a-b))/(a*tanh(1/2*x)^2-tanh(1/2*x)
^2*b-2*c*tanh(1/2*x)-a-b)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2-c^2+a*b*cosh(x)+a*c*sinh(x))/(a+b*cosh(x)+c*sinh(x))^2,x, al
gorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(c^2-b^2+a^2>0)', see `assume?` for
more details)Is c^2-b^2+a^2 positive or negative?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{b^2 + a\cosh(x)\,b - c^2 + a\sinh(x)\,c}{(a + b\cosh(x) + c\sinh(x))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2 - c^2 + a*c*sinh(x) + a*b*cosh(x))/(a + b*cosh(x) + c*sinh(x))^2,x
)
```

```
[Out] int((b^2 - c^2 + a*c*sinh(x) + a*b*cosh(x))/(a + b*cosh(x) + c*sinh(x))^2,
x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2-c**2+a*b*cosh(x)+a*c*sinh(x))/(a+b*cosh(x)+c*sinh(x))**2,x)
```

```
[Out] Timed out
```

3.802 $\int \frac{A+C\sinh(x)}{a+b\cosh(x)+b\sinh(x)}\,dx$

Optimal. Leaf size=71

$$\frac{x(2aA+bC)}{2a^2}-\frac{1}{2}\left(\frac{bC}{a^2}+\frac{2A}{a}-\frac{C}{b}\right)\log(a+b\sinh(x)+b\cosh(x))-\frac{C\sinh(x)}{2a}+\frac{C\cosh(x)}{2a}$$

```
[Out] 1/2*(2*A*a+C*b)*x/a^2+1/2*C*cosh(x)/a-1/2*(2*A*a*b-C*a^2+C*b^2)*ln(a+b*cosh
(x)+b*sinh(x))/a^2/b-1/2*C*sinh(x)/a
```

Rubi [A] time = 0.06, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {3131}

$$\frac{x(2aA+bC)}{2a^2}-\frac{1}{2}\left(\frac{bC}{a^2}+\frac{2A}{a}-\frac{C}{b}\right)\log(a+b\sinh(x)+b\cosh(x))-\frac{C\sinh(x)}{2a}+\frac{C\cosh(x)}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[(A + C*Sinh[x])/(a + b*Cosh[x] + b*Sinh[x]),x]
```

```
[Out] ((2*a*A + b*C)*x)/(2*a^2) + (C*Cosh[x])/(2*a) - (((2*A)/a - C/b + (b*C)/a^2
)*Log[a + b*Cosh[x] + b*Sinh[x]])/2 - (C*Sinh[x])/(2*a)
```

Rule 3131

```
Int[((A_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])/(cos[(d_.) + (e_.)*(x_)]*(b_.)
+ (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)]), x_Symbol] :> Simp[((2*a*A - c*C)*x
)/(2*a^2), x] + (-Simp[(C*Cos[d + e*x])/(2*a*e), x] + Simp[(c*C*Sin[d + e*x
])/(2*a*b*e), x] + Simp[((-(a^2*C) + 2*a*c*A + b^2*C)*Log[RemoveContent[a +
 b*Cos[d + e*x] + c*Sin[d + e*x], x]])/(2*a^2*b*e), x]) /; FreeQ[{a, b, c,
d, e, A, C}, x] && EqQ[b^2 + c^2, 0]
```

Rubi steps

$$\int \frac{A+C\sinh(x)}{a+b\cosh(x)+b\sinh(x)}\,dx=\frac{(2aA+bC)x}{2a^2}+\frac{C\cosh(x)}{2a}-\frac{1}{2}\left(\frac{2A}{a}-\frac{C}{b}+\frac{bC}{a^2}\right)\log(a+b\cosh(x)+b\sinh(x)$$

Mathematica [A] time = 0.24, size = 86, normalized size = 1.21

$$\frac{x\left(a^2C+2aAb+b^2C\right)+2\left(a^2C-2aAb-b^2C\right)\log\left((b-a)\sinh\left(\frac{x}{2}\right)+(a+b)\cosh\left(\frac{x}{2}\right)\right)-2abC\sinh(x)+2abC}{4a^2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + C*Sinh[x])/(a + b*Cosh[x] + b*Sinh[x]),x]
```

```
[Out] ((2*a*A*b + a^2*C + b^2*C)*x + 2*a*b*C*Cosh[x] + 2*(-2*a*A*b + a^2*C - b^2*
C)*Log[(a + b)*Cosh[x/2] + (-a + b)*Sinh[x/2]] - 2*a*b*C*Sinh[x])/(4*a^2*b)
```

fricas [A] time = 0.45, size = 107, normalized size = 1.51

$$\frac{Cab + \left(2\,Aab + Cb^{2}\right)x\cosh(x) + \left(2\,Aab + Cb^{2}\right)x\sinh(x) + \left(\left(Ca^{2} - 2\,Aab - Cb^{2}\right)\cosh(x) + \left(Ca^{2} - 2\,Aab - C\right.\right.}{2\left(a^{2}b\cosh(x) + a^{2}b\sinh(x)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sinh(x))/(a+b*cosh(x)+b*sinh(x)),x, algorithm="fricas")
```

```
[Out] 1/2*(C*a*b + (2*A*a*b + C*b^2)*x*cosh(x) + (2*A*a*b + C*b^2)*x*sinh(x) + ((
C*a^2 - 2*A*a*b - C*b^2)*cosh(x) + (C*a^2 - 2*A*a*b - C*b^2)*sinh(x))*log(b
*cosh(x) + b*sinh(x) + a))/(a^2*b*cosh(x) + a^2*b*sinh(x))
```

giac [A] time = 0.14, size = 58, normalized size = 0.82

$$\frac{Ce^{(-x)}}{2\,a} + \frac{(2\,Aa + Cb)x}{2\,a^{2}} + \frac{\left(Ca^{2} - 2\,Aab - Cb^{2}\right)\log\left(|be^{x} + a|\right)}{2\,a^{2}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sinh(x))/(a+b*cosh(x)+b*sinh(x)),x, algorithm="giac")
```

```
[Out] 1/2*C*e^(-x)/a + 1/2*(2*A*a + C*b)*x/a^2 + 1/2*(C*a^2 - 2*A*a*b - C*b^2)*lo
g(abs(b*e^x + a))/(a^2*b)
```

maple [B] time = 0.19, size = 136, normalized size = 1.92

$$-\frac{C\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{2b}-\frac{\ln\left(a\tanh\left(\frac{x}{2}\right)-\tanh\left(\frac{x}{2}\right)b-a-b\right)A}{a}+\frac{\ln\left(a\tanh\left(\frac{x}{2}\right)-\tanh\left(\frac{x}{2}\right)b-a-b\right)C}{2b}-\frac{b\ln\left(a\,\mathrm{ta}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*sinh(x))/(a+b*cosh(x)+b*sinh(x)),x)
```

```
[Out] -1/2*C/b*ln(tanh(1/2*x)-1)-1/a*ln(a*tanh(1/2*x)-tanh(1/2*x)*b-a-b)*A+1/2/b*
ln(a*tanh(1/2*x)-tanh(1/2*x)*b-a-b)*C-1/2/a^2*b*ln(a*tanh(1/2*x)-tanh(1/2*x
)*b-a-b)*C+C/a/(tanh(1/2*x)+1)+1/a*ln(tanh(1/2*x)+1)*A+1/2/a^2*ln(tanh(1/2*
x)+1)*b*C
```

maxima [A] time = 0.32, size = 58, normalized size = 0.82

$$\frac{1}{2}\,C\left(\frac{x}{b} + \frac{e^{(-x)}}{a} + \frac{\left(a^2 - b^2\right)\log\left(ae^{(-x)} + b\right)}{a^2 b}\right) - \frac{A\log\left(ae^{(-x)} + b\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sinh(x))/(a+b*cosh(x)+b*sinh(x)),x, algorithm="maxima")
```

```
[Out] 1/2*C*(x/b + e^(-x)/a + (a^2 - b^2)*log(a*e^(-x) + b)/(a^2*b)) - A*log(a*e^
(-x) + b)/a
```

mupad [B] time = 1.66, size = 57, normalized size = 0.80

$$\frac{C\,e^{-x}}{2\,a} + \frac{x\,\left(2\,A\,a + C\,b\right)}{2\,a^2} - \frac{\ln\left(a + b\,e^{x}\right)\,\left(-C\,a^2 + 2\,A\,a\,b + C\,b^2\right)}{2\,a^2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + C*sinh(x))/(a + b*cosh(x) + b*sinh(x)),x)
```

```
[Out] (C*exp(-x))/(2*a) + (x*(2*A*a + C*b))/(2*a^2) - (log(a + b*exp(x))*(C*b^2 -
 C*a^2 + 2*A*a*b))/(2*a^2*b)
```

sympy [A] time = 5.11, size = 753, normalized size = 10.61

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sinh(x))/(a+b*cosh(x)+b*sinh(x)),x)
```

```
[Out] Piecewise((zoo*(A*x + C*cosh(x)), Eq(a, 0) & Eq(b, 0)), (-2*A/(2*b*sinh(x)
+ 2*b*cosh(x)) + C*x*sinh(x)/(2*b*sinh(x) + 2*b*cosh(x)) + C*x*cosh(x)/(2*b
*sinh(x) + 2*b*cosh(x)) + C*cosh(x)/(2*b*sinh(x) + 2*b*cosh(x)), Eq(a, 0)),
 ((A*x + C*cosh(x))/a, Eq(b, 0)), (2*A*log(tanh(x/2) + 1)*tanh(x/2)/(2*b*ta
nh(x/2) + 2*b) + 2*A*log(tanh(x/2) + 1)/(2*b*tanh(x/2) + 2*b) + C*x*tanh(x/
2)/(2*b*tanh(x/2) + 2*b) + C*x/(2*b*tanh(x/2) + 2*b) + 2*C/(2*b*tanh(x/2) +
 2*b), Eq(a, b)), (2*A*a*b*log(tanh(x/2) + 1)*tanh(x/2)/(2*a**2*b*tanh(x/2)
 + 2*a**2*b) + 2*A*a*b*log(tanh(x/2) + 1)/(2*a**2*b*tanh(x/2) + 2*a**2*b) -
 2*A*a*b*log(-a/(a - b) - b/(a - b) + tanh(x/2))*tanh(x/2)/(2*a**2*b*tanh(x
/2) + 2*a**2*b) - 2*A*a*b*log(-a/(a - b) - b/(a - b) + tanh(x/2))/(2*a**2*b
*tanh(x/2) + 2*a**2*b) + C*a**2*x*tanh(x/2)/(2*a**2*b*tanh(x/2) + 2*a**2*b)
 + C*a**2*x/(2*a**2*b*tanh(x/2) + 2*a**2*b) - C*a**2*log(tanh(x/2) + 1)*tan
h(x/2)/(2*a**2*b*tanh(x/2) + 2*a**2*b) - C*a**2*log(tanh(x/2) + 1)/(2*a**2*
b*tanh(x/2) + 2*a**2*b) + C*a**2*log(-a/(a - b) - b/(a - b) + tanh(x/2))*ta
```

```
nh(x/2)/(2*a**2*b*tanh(x/2) + 2*a**2*b) + C*a**2*log(-a/(a - b) - b/(a - b)
 + tanh(x/2))/(2*a**2*b*tanh(x/2) + 2*a**2*b) + 2*C*a*b/(2*a**2*b*tanh(x/2)
 + 2*a**2*b) + C*b**2*log(tanh(x/2) + 1)*tanh(x/2)/(2*a**2*b*tanh(x/2) + 2*
a**2*b) + C*b**2*log(tanh(x/2) + 1)/(2*a**2*b*tanh(x/2) + 2*a**2*b) - C*b**
2*log(-a/(a - b) - b/(a - b) + tanh(x/2))*tanh(x/2)/(2*a**2*b*tanh(x/2) + 2
*a**2*b) - C*b**2*log(-a/(a - b) - b/(a - b) + tanh(x/2))/(2*a**2*b*tanh(x/
2) + 2*a**2*b), True))
```

3.803 $\int \frac{A+B\cosh(x)}{a+b\cosh(x)+b\sinh(x)}\,dx$

Optimal. Leaf size=77

$$-\frac{\left(a^2(-B)+2aAb-b^2B\right)\log(a+b\sinh(x)+b\cosh(x))}{2a^2b}+\frac{x(2aA-bB)}{2a^2}+\frac{B\sinh(x)}{2a}-\frac{B\cosh(x)}{2a}$$

```
[Out] 1/2*(2*A*a-B*b)*x/a^2-1/2*B*cosh(x)/a-1/2*(2*A*a*b-B*a^2-B*b^2)*ln(a+b*cosh
(x)+b*sinh(x))/a^2/b+1/2*B*sinh(x)/a
```

Rubi [A] time = 0.05, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {3132}

$$-\frac{\left(a^2(-B)+2aAb-b^2B\right)\log(a+b\sinh(x)+b\cosh(x))}{2a^2b}+\frac{x(2aA-bB)}{2a^2}+\frac{B\sinh(x)}{2a}-\frac{B\cosh(x)}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cosh[x])/(a + b*Cosh[x] + b*Sinh[x]),x]
```

```
[Out] ((2*a*A - b*B)*x)/(2*a^2) - (B*Cosh[x])/(2*a) - ((2*a*A*b - a^2*B - b^2*B)*
Log[a + b*Cosh[x] + b*Sinh[x]])/(2*a^2*b) + (B*Sinh[x])/(2*a)
```

Rule 3132

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.))/(cos[(d_.) + (e_.)*(x_)]*(b_.)
+ (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)]), x_Symbol] :> Simp[((2*a*A - b*B)*x
)/(2*a^2), x] + (Simp[(B*Sin[d + e*x])/(2*a*e), x] - Simp[(b*B*Cos[d + e*x]
)/(2*a*c*e), x] + Simp[((a^2*B - 2*a*b*A + b^2*B)*Log[RemoveContent[a + b*C
os[d + e*x] + c*Sin[d + e*x], x]])/(2*a^2*c*e), x]) /; FreeQ[{a, b, c, d, e
, A, B}, x] && EqQ[b^2 + c^2, 0]
```

Rubi steps

$$\int \frac{A+B\cosh(x)}{a+b\cosh(x)+b\sinh(x)}\,dx=\frac{(2aA-bB)x}{2a^2}-\frac{B\cosh(x)}{2a}-\frac{\left(2aAb-a^2B-b^2B\right)\log(a+b\cosh(x)+b\sinh(x$$

Mathematica [A] time = 0.19, size = 84, normalized size = 1.09

$$\frac{\frac{2\left(a^2B-2aAb+b^2B\right)\log\left((b-a)\sinh\left(\frac{x}{2}\right)+(a+b)\cosh\left(\frac{x}{2}\right)\right)}{b}+x\left(\frac{a^2B}{b}+2aA-bB\right)+2aB\sinh(x)-2aB\cosh(x)}{4a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cosh[x])/(a + b*Cosh[x] + b*Sinh[x]),x]
```

```
[Out] ((2*a*A + (a^2*B)/b - b*B)*x - 2*a*B*Cosh[x] + (2*(-2*a*A*b + a^2*B + b^2*B
)*Log[(a + b)*Cosh[x/2] + (-a + b)*Sinh[x/2]])/b + 2*a*B*Sinh[x])/(4*a^2)
```

fricas [A] time = 0.44, size = 110, normalized size = 1.43

$$-\frac{Bab - \left(2\,Aab - Bb^2\right)x\cosh(x) - \left(2\,Aab - Bb^2\right)x\sinh(x) - \left(\left(Ba^2 - 2\,Aab + Bb^2\right)\cosh(x) + \left(Ba^2 - 2\,Aab + B\right.\right.}{2\left(a^2b\cosh(x) + a^2b\sinh(x)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(a+b*cosh(x)+b*sinh(x)),x, algorithm="fricas")
```

```
[Out] -1/2*(B*a*b - (2*A*a*b - B*b^2)*x*cosh(x) - (2*A*a*b - B*b^2)*x*sinh(x) - (
(B*a^2 - 2*A*a*b + B*b^2)*cosh(x) + (B*a^2 - 2*A*a*b + B*b^2)*sinh(x))*log(
b*cosh(x) + b*sinh(x) + a))/(a^2*b*cosh(x) + a^2*b*sinh(x))
```

giac [A] time = 0.13, size = 58, normalized size = 0.75

$$-\frac{Be^{(-x)}}{2\,a} + \frac{(2\,Aa - Bb)x}{2\,a^2} + \frac{\left(Ba^2 - 2\,Aab + Bb^2\right)\log\left(|be^x + a|\right)}{2\,a^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(a+b*cosh(x)+b*sinh(x)),x, algorithm="giac")
```

```
[Out] -1/2*B*e^(-x)/a + 1/2*(2*A*a - B*b)*x/a^2 + 1/2*(B*a^2 - 2*A*a*b + B*b^2)*l
og(abs(b*e^x + a))/(a^2*b)
```

maple [A] time = 0.20, size = 137, normalized size = 1.78

$$-\frac{B\ln\left(\tanh\left(\frac{x}{2}\right) - 1\right)}{2b} - \frac{\ln\left(a\tanh\left(\frac{x}{2}\right) - \tanh\left(\frac{x}{2}\right)b - a - b\right)A}{a} + \frac{\ln\left(a\tanh\left(\frac{x}{2}\right) - \tanh\left(\frac{x}{2}\right)b - a - b\right)B}{2b} + \frac{b\ln\left(a\,\mathrm{ta}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cosh(x))/(a+b*cosh(x)+b*sinh(x)),x)
```

```
[Out] -1/2*B/b*ln(tanh(1/2*x)-1)-1/a*ln(a*tanh(1/2*x)-tanh(1/2*x)*b-a-b)*A+1/2/b*
ln(a*tanh(1/2*x)-tanh(1/2*x)*b-a-b)*B+1/2/a^2*b*ln(a*tanh(1/2*x)-tanh(1/2*x
)*b-a-b)*B-B/a/(tanh(1/2*x)+1)+1/a*ln(tanh(1/2*x)+1)*A-1/2/a^2*ln(tanh(1/2*
x)+1)*B*b
```

maxima [A] time = 0.44, size = 57, normalized size = 0.74

$$\frac{1}{2}B\left(\frac{x}{b}-\frac{e^{(-x)}}{a}+\frac{\left(a^2+b^2\right)\log\left(ae^{(-x)}+b\right)}{a^2b}\right)-\frac{A\log\left(ae^{(-x)}+b\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(a+b*cosh(x)+b*sinh(x)),x, algorithm="maxima")
```

```
[Out] 1/2*B*(x/b - e^(-x)/a + (a^2 + b^2)*log(a*e^(-x) + b)/(a^2*b)) - A*log(a*e^
(-x) + b)/a
```

mupad [B] time = 1.69, size = 57, normalized size = 0.74

$$\frac{x\,\left(2\,A\,a-B\,b\right)}{2\,a^2}-\frac{B\,\mathrm{e}^{-x}}{2\,a}+\frac{\ln\left(a+b\,\mathrm{e}^x\right)\,\left(B\,a^2-2\,A\,a\,b+B\,b^2\right)}{2\,a^2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cosh(x))/(a + b*cosh(x) + b*sinh(x)),x)
```

```
[Out] (x*(2*A*a - B*b))/(2*a^2) - (B*exp(-x))/(2*a) + (log(a + b*exp(x))*(B*a^2 +
 B*b^2 - 2*A*a*b))/(2*a^2*b)
```

sympy [A] time = 5.15, size = 806, normalized size = 10.47

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(a+b*cosh(x)+b*sinh(x)),x)
```

```
[Out] Piecewise((zoo*(A*x + B*sinh(x)), Eq(a, 0) & Eq(b, 0)), (-2*A/(2*b*sinh(x)
+ 2*b*cosh(x)) + B*x*sinh(x)/(2*b*sinh(x) + 2*b*cosh(x)) + B*x*cosh(x)/(2*b
*sinh(x) + 2*b*cosh(x)) - B*cosh(x)/(2*b*sinh(x) + 2*b*cosh(x)), Eq(a, 0)),
 ((A*x + B*sinh(x))/a, Eq(b, 0)), (2*A*log(tanh(x/2) + 1)*tanh(x/2)/(2*b*ta
nh(x/2) + 2*b) + 2*A*log(tanh(x/2) + 1)/(2*b*tanh(x/2) + 2*b) + B*x*tanh(x/
2)/(2*b*tanh(x/2) + 2*b) + B*x/(2*b*tanh(x/2) + 2*b) - 2*B*log(tanh(x/2) +
1)*tanh(x/2)/(2*b*tanh(x/2) + 2*b) - 2*B*log(tanh(x/2) + 1)/(2*b*tanh(x/2)
+ 2*b) - 2*B/(2*b*tanh(x/2) + 2*b), Eq(a, b)), (2*A*a*b*log(tanh(x/2) + 1)*
tanh(x/2)/(2*a**2*b*tanh(x/2) + 2*a**2*b) + 2*A*a*b*log(tanh(x/2) + 1)/(2*a
**2*b*tanh(x/2) + 2*a**2*b) - 2*A*a*b*log(-a/(a - b) - b/(a - b) + tanh(x/2
))*tanh(x/2)/(2*a**2*b*tanh(x/2) + 2*a**2*b) - 2*A*a*b*log(-a/(a - b) - b/(
a - b) + tanh(x/2))/(2*a**2*b*tanh(x/2) + 2*a**2*b) + B*a**2*x*tanh(x/2)/(2
*a**2*b*tanh(x/2) + 2*a**2*b) + B*a**2*x/(2*a**2*b*tanh(x/2) + 2*a**2*b) -
B*a**2*log(tanh(x/2) + 1)*tanh(x/2)/(2*a**2*b*tanh(x/2) + 2*a**2*b) - B*a**
```

```
2*log(tanh(x/2) + 1)/(2*a**2*b*tanh(x/2) + 2*a**2*b) + B*a**2*log(-a/(a - b
) - b/(a - b) + tanh(x/2))*tanh(x/2)/(2*a**2*b*tanh(x/2) + 2*a**2*b) + B*a*
*2*log(-a/(a - b) - b/(a - b) + tanh(x/2))/(2*a**2*b*tanh(x/2) + 2*a**2*b)
- 2*B*a*b/(2*a**2*b*tanh(x/2) + 2*a**2*b) - B*b**2*log(tanh(x/2) + 1)*tanh(
x/2)/(2*a**2*b*tanh(x/2) + 2*a**2*b) - B*b**2*log(tanh(x/2) + 1)/(2*a**2*b*
tanh(x/2) + 2*a**2*b) + B*b**2*log(-a/(a - b) - b/(a - b) + tanh(x/2))*tanh
(x/2)/(2*a**2*b*tanh(x/2) + 2*a**2*b) + B*b**2*log(-a/(a - b) - b/(a - b) +
 tanh(x/2))/(2*a**2*b*tanh(x/2) + 2*a**2*b), True))
```

3.804 $\int \frac{A+B\cosh(x)+C\sinh(x)}{a+b\cosh(x)+b\sinh(x)}\,dx$

Optimal. Leaf size=86

$$-\frac{\left(-\left(a^2(B+C)\right)+2aAb-b^2(B-C)\right)\log(a+b\sinh(x)+b\cosh(x))}{2a^2b}+\frac{x(2aA-b(B-C))}{2a^2}-\frac{(B-C)(\cosh(x)-\sin}{2a}$$

```
[Out] 1/2*(2*a*A-b*(B-C))*x/a^2-1/2*(2*a*A*b-b^2*(B-C)-a^2*(B+C))*ln(a+b*cosh(x)+
b*sinh(x))/a^2/b-1/2*(B-C)*(cosh(x)-sinh(x))/a
```

Rubi [A] time = 0.08, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {3130}

$$-\frac{\left(a^2(-(B+C))+2aAb-b^2(B-C)\right)\log(a+b\sinh(x)+b\cosh(x))}{2a^2b}+\frac{x(2aA-b(B-C))}{2a^2}-\frac{(B-C)(\cosh(x)-\sin}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cosh[x] + C*Sinh[x])/(a + b*Cosh[x] + b*Sinh[x]),x]
```

```
[Out] ((2*a*A - b*(B - C))*x)/(2*a^2) - ((2*a*A*b - b^2*(B - C) - a^2*(B + C))*Lo
g[a + b*Cosh[x] + b*Sinh[x]])/(2*a^2*b) - ((B - C)*(Cosh[x] - Sinh[x]))/(2*
a)
```

Rule 3130

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)]), x_
Symbol] :> Simp[((2*a*A - b*B - c*C)*x)/(2*a^2), x] + (-Simp[((b*B + c*C)*(
b*Cos[d + e*x] - c*Sin[d + e*x]))/(2*a*b*c*e), x] + Simp[((a^2*(b*B - c*C)
- 2*a*A*b^2 + b^2*(b*B + c*C))*Log[RemoveContent[a + b*Cos[d + e*x] + c*Sin
[d + e*x], x]])/(2*a^2*b*c*e), x]) /; FreeQ[{a, b, c, d, e, A, B, C}, x] &&
 EqQ[b^2 + c^2, 0]
```

Rubi steps

$$\int \frac{A+B\cosh(x)+C\sinh(x)}{a+b\cosh(x)+b\sinh(x)}\,dx=\frac{(2aA-b(B-C))x}{2a^2}-\frac{\left(2aAb-b^2(B-C)-a^2(B+C)\right)\log(a+b\cosh(x)+b}{2a^2b}$$

Mathematica [A] time = 0.31, size = 103, normalized size = 1.20

$$\frac{\frac{2\left(a^2(B+C)-2aAb+b^2(B-C)\right)\log\left((b-a)\sinh\left(\frac{x}{2}\right)+(a+b)\cosh\left(\frac{x}{2}\right)\right)}{b}+x\left(\frac{a^2(B+C)}{b}+2aA+b(C-B)\right)+2a(B-C)\sinh(x)-2a(B-C}{4a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cosh[x] + C*Sinh[x])/(a + b*Cosh[x] + b*Sinh[x]),x]
```

```
[Out] ((2*a*A + b*(-B + C) + (a^2*(B + C))/b)*x - 2*a*(B - C)*Cosh[x] + (2*(-2*a*
A*b + b^2*(B - C) + a^2*(B + C))*Log[(a + b)*Cosh[x/2] + (-a + b)*Sinh[x/2]
])/b + 2*a*(B - C)*Sinh[x])/(4*a^2)
```

fricas [A] time = 0.43, size = 134, normalized size = 1.56

$$-\frac{(B-C)ab-\left(2\,Aab-(B-C)b^2\right)x\cosh(x)-\left(2\,Aab-(B-C)b^2\right)x\sinh(x)-\left(\left((B+C)a^2-2\,Aab+(B-C)b^2\right.\right.}{2\left(a^2b\cosh(x)+a^2b\sinh(\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x)+C*sinh(x))/(a+b*cosh(x)+b*sinh(x)),x, algorithm="fri
cas")
```

```
[Out] -1/2*((B - C)*a*b - (2*A*a*b - (B - C)*b^2)*x*cosh(x) - (2*A*a*b - (B - C)*
b^2)*x*sinh(x) - (((B + C)*a^2 - 2*A*a*b + (B - C)*b^2)*cosh(x) + ((B + C)*
a^2 - 2*A*a*b + (B - C)*b^2)*sinh(x))*log(b*cosh(x) + b*sinh(x) + a))/(a^2*
b*cosh(x) + a^2*b*sinh(x))
```

giac [A] time = 0.12, size = 79, normalized size = 0.92

$$\frac{(2\,Aa-Bb+Cb)x}{2\,a^2}-\frac{(Ba-Ca)e^{(-x)}}{2\,a^2}+\frac{\left(Ba^2+Ca^2-2\,Aab+Bb^2-Cb^2\right)\log\left(|be^x+a|\right)}{2\,a^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x)+C*sinh(x))/(a+b*cosh(x)+b*sinh(x)),x, algorithm="gia
c")
```

```
[Out] 1/2*(2*A*a - B*b + C*b)*x/a^2 - 1/2*(B*a - C*a)*e^(-x)/a^2 + 1/2*(B*a^2 + C
*a^2 - 2*A*a*b + B*b^2 - C*b^2)*log(abs(b*e^x + a))/(a^2*b)
```

maple [B] time = 0.19, size = 232, normalized size = 2.70

$$-\frac{B\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{2b}-\frac{C\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{2b}-\frac{\ln\left(a\tanh\left(\frac{x}{2}\right)-\tanh\left(\frac{x}{2}\right)b-a-b\right)A}{a}+\frac{\ln\left(a\tanh\left(\frac{x}{2}\right)-\tanh\left(\frac{x}{2}\right)b\right.}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cosh(x)+C*sinh(x))/(a+b*cosh(x)+b*sinh(x)),x)
```

```
[Out] -1/2*B/b*ln(tanh(1/2*x)-1)-1/2*C/b*ln(tanh(1/2*x)-1)-1/a*ln(a*tanh(1/2*x)-t
anh(1/2*x)*b-a-b)*A+1/2/b*ln(a*tanh(1/2*x)-tanh(1/2*x)*b-a-b)*B+1/2/a^2*b*l
```

```
n(a*tanh(1/2*x)-tanh(1/2*x)*b-a-b)*B+1/2/b*ln(a*tanh(1/2*x)-tanh(1/2*x)*b-a
-b)*C-1/2/a^2*b*ln(a*tanh(1/2*x)-tanh(1/2*x)*b-a-b)*C-B/a/(tanh(1/2*x)+1)+C
/a/(tanh(1/2*x)+1)+1/a*ln(tanh(1/2*x)+1)*A-1/2/a^2*ln(tanh(1/2*x)+1)*B*b+1/
2/a^2*ln(tanh(1/2*x)+1)*b*C
```

maxima [A] time = 0.32, size = 99, normalized size = 1.15

$$\frac{1}{2}\,C\left(\frac{x}{b}+\frac{e^{(-x)}}{a}+\frac{\left(a^2-b^2\right)\log\left(ae^{(-x)}+b\right)}{a^2b}\right)+\frac{1}{2}\,B\left(\frac{x}{b}-\frac{e^{(-x)}}{a}+\frac{\left(a^2+b^2\right)\log\left(ae^{(-x)}+b\right)}{a^2b}\right)-\frac{A\log\left(ae^{(-x)}+b\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x)+C*sinh(x))/(a+b*cosh(x)+b*sinh(x)),x, algorithm="max
ima")
```

```
[Out] 1/2*C*(x/b + e^(-x)/a + (a^2 - b^2)*log(a*e^(-x) + b)/(a^2*b)) + 1/2*B*(x/b
 - e^(-x)/a + (a^2 + b^2)*log(a*e^(-x) + b)/(a^2*b)) - A*log(a*e^(-x) + b)/
a
```

mupad [B] time = 1.72, size = 75, normalized size = 0.87

$$\frac{x\,\left(2\,A\,a-B\,b+C\,b\right)}{2\,a^2}-\frac{{\mathrm{e}}^{-x}\,\left(B-C\right)}{2\,a}+\frac{\ln\left(a+b\,{\mathrm{e}}^x\right)\,\left(B\,a^2+B\,b^2+C\,a^2-C\,b^2-2\,A\,a\,b\right)}{2\,a^2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cosh(x) + C*sinh(x))/(a + b*cosh(x) + b*sinh(x)),x)
```

```
[Out] (x*(2*A*a - B*b + C*b))/(2*a^2) - (exp(-x)*(B - C))/(2*a) + (log(a + b*exp(
x))*(B*a^2 + B*b^2 + C*a^2 - C*b^2 - 2*A*a*b))/(2*a^2*b)
```

sympy [A] time = 5.94, size = 1321, normalized size = 15.36

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x)+C*sinh(x))/(a+b*cosh(x)+b*sinh(x)),x)
```

```
[Out] Piecewise((zoo*(A*x + B*sinh(x) + C*cosh(x)), Eq(a, 0) & Eq(b, 0)), (-2*A/(
2*b*sinh(x) + 2*b*cosh(x)) + B*x*sinh(x)/(2*b*sinh(x) + 2*b*cosh(x)) + B*x*
cosh(x)/(2*b*sinh(x) + 2*b*cosh(x)) - B*cosh(x)/(2*b*sinh(x) + 2*b*cosh(x))
 + C*x*sinh(x)/(2*b*sinh(x) + 2*b*cosh(x)) + C*x*cosh(x)/(2*b*sinh(x) + 2*b
*cosh(x)) + C*cosh(x)/(2*b*sinh(x) + 2*b*cosh(x)), Eq(a, 0)), ((A*x + B*sin
h(x) + C*cosh(x))/a, Eq(b, 0)), (2*A*log(tanh(x/2) + 1)*tanh(x/2)/(2*b*tanh
(x/2) + 2*b) + 2*A*log(tanh(x/2) + 1)/(2*b*tanh(x/2) + 2*b) + B*x*tanh(x/2)
```

```
/(2*b*tanh(x/2) + 2*b) + B*x/(2*b*tanh(x/2) + 2*b) - 2*B*log(tanh(x/2) + 1)
*tanh(x/2)/(2*b*tanh(x/2) + 2*b) - 2*B*log(tanh(x/2) + 1)/(2*b*tanh(x/2) +
2*b) - 2*B/(2*b*tanh(x/2) + 2*b) + C*x*tanh(x/2)/(2*b*tanh(x/2) + 2*b) + C*
x/(2*b*tanh(x/2) + 2*b) + 2*C/(2*b*tanh(x/2) + 2*b), Eq(a, b)), (2*A*a*b*lo
g(tanh(x/2) + 1)*tanh(x/2)/(2*a**2*b*tanh(x/2) + 2*a**2*b) + 2*A*a*b*log(ta
nh(x/2) + 1)/(2*a**2*b*tanh(x/2) + 2*a**2*b) - 2*A*a*b*log(-a/(a - b) - b/(
a - b) + tanh(x/2))*tanh(x/2)/(2*a**2*b*tanh(x/2) + 2*a**2*b) - 2*A*a*b*log
(-a/(a - b) - b/(a - b) + tanh(x/2))/(2*a**2*b*tanh(x/2) + 2*a**2*b) + B*a*
*2*x*tanh(x/2)/(2*a**2*b*tanh(x/2) + 2*a**2*b) + B*a**2*x/(2*a**2*b*tanh(x/
2) + 2*a**2*b) - B*a**2*log(tanh(x/2) + 1)*tanh(x/2)/(2*a**2*b*tanh(x/2) +
2*a**2*b) - B*a**2*log(tanh(x/2) + 1)/(2*a**2*b*tanh(x/2) + 2*a**2*b) + B*a
**2*log(-a/(a - b) - b/(a - b) + tanh(x/2))*tanh(x/2)/(2*a**2*b*tanh(x/2) +
 2*a**2*b) + B*a**2*log(-a/(a - b) - b/(a - b) + tanh(x/2))/(2*a**2*b*tanh(
x/2) + 2*a**2*b) - 2*B*a*b/(2*a**2*b*tanh(x/2) + 2*a**2*b) - B*b**2*log(tan
h(x/2) + 1)*tanh(x/2)/(2*a**2*b*tanh(x/2) + 2*a**2*b) - B*b**2*log(tanh(x/2
) + 1)/(2*a**2*b*tanh(x/2) + 2*a**2*b) + B*b**2*log(-a/(a - b) - b/(a - b)
+ tanh(x/2))*tanh(x/2)/(2*a**2*b*tanh(x/2) + 2*a**2*b) + B*b**2*log(-a/(a -
 b) - b/(a - b) + tanh(x/2))/(2*a**2*b*tanh(x/2) + 2*a**2*b) + C*a**2*x*tan
h(x/2)/(2*a**2*b*tanh(x/2) + 2*a**2*b) + C*a**2*x/(2*a**2*b*tanh(x/2) + 2*a
**2*b) - C*a**2*log(tanh(x/2) + 1)*tanh(x/2)/(2*a**2*b*tanh(x/2) + 2*a**2*b
) - C*a**2*log(tanh(x/2) + 1)/(2*a**2*b*tanh(x/2) + 2*a**2*b) + C*a**2*log(
-a/(a - b) - b/(a - b) + tanh(x/2))*tanh(x/2)/(2*a**2*b*tanh(x/2) + 2*a**2*
b) + C*a**2*log(-a/(a - b) - b/(a - b) + tanh(x/2))/(2*a**2*b*tanh(x/2) + 2
*a**2*b) + 2*C*a*b/(2*a**2*b*tanh(x/2) + 2*a**2*b) + C*b**2*log(tanh(x/2) +
 1)*tanh(x/2)/(2*a**2*b*tanh(x/2) + 2*a**2*b) + C*b**2*log(tanh(x/2) + 1)/(
2*a**2*b*tanh(x/2) + 2*a**2*b) - C*b**2*log(-a/(a - b) - b/(a - b) + tanh(x
/2))*tanh(x/2)/(2*a**2*b*tanh(x/2) + 2*a**2*b) - C*b**2*log(-a/(a - b) - b/
(a - b) + tanh(x/2))/(2*a**2*b*tanh(x/2) + 2*a**2*b), True))
```

3.805 $\int \frac{A+C\sinh(x)}{a+b\cosh(x)-b\sinh(x)}\,dx$

Optimal. Leaf size=77

$$\frac{\left(a^2C+2aAb-b^2C\right)\log(a-b\sinh(x)+b\cosh(x))}{2a^2b}+\frac{x(2aA-bC)}{2a^2}+\frac{C\sinh(x)}{2a}+\frac{C\cosh(x)}{2a}$$

```
[Out] 1/2*(2*A*a-C*b)*x/a^2+1/2*C*cosh(x)/a+1/2*(2*A*a*b+C*a^2-C*b^2)*ln(a+b*cosh
(x)-b*sinh(x))/a^2/b+1/2*C*sinh(x)/a
```

Rubi [A] time = 0.05, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {3131}

$$\frac{\left(a^2C+2aAb-b^2C\right)\log(a-b\sinh(x)+b\cosh(x))}{2a^2b}+\frac{x(2aA-bC)}{2a^2}+\frac{C\sinh(x)}{2a}+\frac{C\cosh(x)}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[(A + C*Sinh[x])/(a + b*Cosh[x] - b*Sinh[x]),x]
```

```
[Out] ((2*a*A - b*C)*x)/(2*a^2) + (C*Cosh[x])/(2*a) + ((2*a*A*b + a^2*C - b^2*C)*
Log[a + b*Cosh[x] - b*Sinh[x]])/(2*a^2*b) + (C*Sinh[x])/(2*a)
```

Rule 3131

```
Int[((A_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])/(cos[(d_.) + (e_.)*(x_)]*(b_.)
+ (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)]), x_Symbol] :> Simp[((2*a*A - c*C)*x
)/(2*a^2), x] + (-Simp[(C*Cos[d + e*x])/(2*a*e), x] + Simp[(c*C*Sin[d + e*x
])/(2*a*b*e), x] + Simp[((-(a^2*C) + 2*a*c*A + b^2*C)*Log[RemoveContent[a +
 b*Cos[d + e*x] + c*Sin[d + e*x], x]])/(2*a^2*b*e), x]) /; FreeQ[{a, b, c,
d, e, A, C}, x] && EqQ[b^2 + c^2, 0]
```

Rubi steps

$$\int \frac{A+C\sinh(x)}{a+b\cosh(x)-b\sinh(x)}\,dx=\frac{(2aA-bC)x}{2a^2}+\frac{C\cosh(x)}{2a}+\frac{\left(2aAb+a^2C-b^2C\right)\log(a+b\cosh(x)-b\sinh($$

Mathematica [A] time = 0.25, size = 86, normalized size = 1.12

$$\frac{\frac{2\left(a^2C+2aAb-b^2C\right)\log\left((a-b)\sinh\left(\frac{x}{2}\right)+(a+b)\cosh\left(\frac{x}{2}\right)\right)}{b}+x\left(-\frac{a^2C}{b}+2aA-bC\right)+2aC\sinh(x)+2aC\cosh(x)}{4a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + C*Sinh[x])/(a + b*Cosh[x] - b*Sinh[x]),x]
```

```
[Out] ((2*a*A - (a^2*C)/b - b*C)*x + 2*a*C*Cosh[x] + (2*(2*a*A*b + a^2*C - b^2*C)
*Log[(a + b)*Cosh[x/2] + (a - b)*Sinh[x/2]])/b + 2*a*C*Sinh[x])/(4*a^2)
```

fricas [A] time = 0.45, size = 59, normalized size = 0.77

$$-\frac{Ca^2x - Cab\cosh(x) - Cab\sinh(x) - \left(Ca^2 + 2\,Aab - Cb^2\right)\log\left(a\cosh(x) + a\sinh(x) + b\right)}{2\,a^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sinh(x))/(a+b*cosh(x)-b*sinh(x)),x, algorithm="fricas")
```

```
[Out] -1/2*(C*a^2*x - C*a*b*cosh(x) - C*a*b*sinh(x) - (C*a^2 + 2*A*a*b - C*b^2)*l
og(a*cosh(x) + a*sinh(x) + b))/(a^2*b)
```

giac [A] time = 0.13, size = 49, normalized size = 0.64

$$-\frac{Cx}{2\,b} + \frac{Ce^x}{2\,a} + \frac{\left(Ca^2 + 2\,Aab - Cb^2\right)\log\left(|ae^x + b|\right)}{2\,a^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sinh(x))/(a+b*cosh(x)-b*sinh(x)),x, algorithm="giac")
```

```
[Out] -1/2*C*x/b + 1/2*C*e^x/a + 1/2*(C*a^2 + 2*A*a*b - C*b^2)*log(abs(a*e^x + b)
)/(a^2*b)
```

maple [A] time = 0.19, size = 125, normalized size = 1.62

$$-\frac{C}{a\left(\tanh\left(\frac{x}{2}\right)-1\right)} - \frac{\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)A}{a} + \frac{\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)bC}{2a^2} - \frac{C\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)}{2b} + \frac{\ln\left(a\tanh\left(\frac{x}{2}\right)-\tanh\left(\frac{x}{2}\right)\right.}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*sinh(x))/(a+b*cosh(x)-b*sinh(x)),x)
```

```
[Out] -C/a/(tanh(1/2*x)-1)-1/a*ln(tanh(1/2*x)-1)*A+1/2/a^2*ln(tanh(1/2*x)-1)*b*C-
1/2*C/b*ln(tanh(1/2*x)+1)+1/a*ln(a*tanh(1/2*x)-tanh(1/2*x)*b+a+b)*A+1/2/b*l
n(a*tanh(1/2*x)-tanh(1/2*x)*b+a+b)*C-1/2/a^2*b*ln(a*tanh(1/2*x)-tanh(1/2*x)
*b+a+b)*C
```

maxima [A] time = 0.33, size = 65, normalized size = 0.84

$$A\left(\frac{x}{a} + \frac{\log\left(be^{(-x)} + a\right)}{a}\right) - \frac{1}{2}\,C\left(\frac{bx}{a^2} - \frac{e^x}{a} - \frac{\left(a^2 - b^2\right)\log\left(be^{(-x)} + a\right)}{a^2b}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sinh(x))/(a+b*cosh(x)-b*sinh(x)),x, algorithm="maxima")
```

```
[Out] A*(x/a + log(b*e^(-x) + a)/a) - 1/2*C*(b*x/a^2 - e^x/a - (a^2 - b^2)*log(b*
e^(-x) + a)/(a^2*b))
```

mupad [B] time = 0.12, size = 48, normalized size = 0.62

$$\frac{C\,e^x}{2\,a}-\frac{C\,x}{2\,b}+\frac{\ln\left(b+a\,e^x\right)\,\left(C\,a^2+2\,A\,a\,b-C\,b^2\right)}{2\,a^2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + C*sinh(x))/(a + b*cosh(x) - b*sinh(x)),x)
```

```
[Out] (C*exp(x))/(2*a) - (C*x)/(2*b) + (log(b + a*exp(x))*(C*a^2 - C*b^2 + 2*A*a*
b))/(2*a^2*b)
```

sympy [A] time = 5.26, size = 852, normalized size = 11.06

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sinh(x))/(a+b*cosh(x)-b*sinh(x)),x)
```

```
[Out] Piecewise((zoo*(A*x + C*cosh(x)), Eq(a, 0) & Eq(b, 0)), (2*A/(-2*b*sinh(x)
+ 2*b*cosh(x)) + C*x*sinh(x)/(-2*b*sinh(x) + 2*b*cosh(x)) - C*x*cosh(x)/(-2
*b*sinh(x) + 2*b*cosh(x)) + C*cosh(x)/(-2*b*sinh(x) + 2*b*cosh(x)), Eq(a, 0
)), ((A*x + C*cosh(x))/a, Eq(b, 0)), (2*A*x*tanh(x/2)/(2*b*tanh(x/2) - 2*b)
 - 2*A*x/(2*b*tanh(x/2) - 2*b) - 2*A*log(tanh(x/2) + 1)*tanh(x/2)/(2*b*tanh
(x/2) - 2*b) + 2*A*log(tanh(x/2) + 1)/(2*b*tanh(x/2) - 2*b) - C*x*tanh(x/2)
/(2*b*tanh(x/2) - 2*b) + C*x/(2*b*tanh(x/2) - 2*b) - 2*C/(2*b*tanh(x/2) - 2
*b), Eq(a, b)), (2*A*a*b*x*tanh(x/2)/(2*a**2*b*tanh(x/2) - 2*a**2*b) - 2*A*
a*b*x/(2*a**2*b*tanh(x/2) - 2*a**2*b) - 2*A*a*b*log(tanh(x/2) + 1)*tanh(x/2
)/(2*a**2*b*tanh(x/2) - 2*a**2*b) + 2*A*a*b*log(tanh(x/2) + 1)/(2*a**2*b*ta
nh(x/2) - 2*a**2*b) + 2*A*a*b*log(a/(a - b) + b/(a - b) + tanh(x/2))*tanh(x
/2)/(2*a**2*b*tanh(x/2) - 2*a**2*b) - 2*A*a*b*log(a/(a - b) + b/(a - b) + t
anh(x/2))/(2*a**2*b*tanh(x/2) - 2*a**2*b) - C*a**2*log(tanh(x/2) + 1)*tanh(
x/2)/(2*a**2*b*tanh(x/2) - 2*a**2*b) + C*a**2*log(tanh(x/2) + 1)/(2*a**2*b*
tanh(x/2) - 2*a**2*b) + C*a**2*log(a/(a - b) + b/(a - b) + tanh(x/2))*tanh(
x/2)/(2*a**2*b*tanh(x/2) - 2*a**2*b) - C*a**2*log(a/(a - b) + b/(a - b) + t
anh(x/2))/(2*a**2*b*tanh(x/2) - 2*a**2*b) - 2*C*a*b/(2*a**2*b*tanh(x/2) - 2
*a**2*b) - C*b**2*x*tanh(x/2)/(2*a**2*b*tanh(x/2) - 2*a**2*b) + C*b**2*x/(2
*a**2*b*tanh(x/2) - 2*a**2*b) + C*b**2*log(tanh(x/2) + 1)*tanh(x/2)/(2*a**2
*b*tanh(x/2) - 2*a**2*b) - C*b**2*log(tanh(x/2) + 1)/(2*a**2*b*tanh(x/2) -
```

```
2*a**2*b) - C*b**2*log(a/(a - b) + b/(a - b) + tanh(x/2))*tanh(x/2)/(2*a**2
*b*tanh(x/2) - 2*a**2*b) + C*b**2*log(a/(a - b) + b/(a - b) + tanh(x/2))/(2
*a**2*b*tanh(x/2) - 2*a**2*b), True))
```

3.806 $\int \frac{A+B\cosh(x)}{a+b\cosh(x)-b\sinh(x)}\,dx$

Optimal. Leaf size=78

$$\frac{\left(a^2(-B)+2aAb-b^2B\right)\log(a-b\sinh(x)+b\cosh(x))}{2a^2b}+\frac{x(2aA-bB)}{2a^2}+\frac{B\sinh(x)}{2a}+\frac{B\cosh(x)}{2a}$$

```
[Out] 1/2*(2*A*a-B*b)*x/a^2+1/2*B*cosh(x)/a+1/2*(2*A*a*b-B*a^2-B*b^2)*ln(a+b*cosh
(x)-b*sinh(x))/a^2/b+1/2*B*sinh(x)/a
```

Rubi [A] time = 0.05, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {3132}

$$\frac{\left(a^2(-B)+2aAb-b^2B\right)\log(a-b\sinh(x)+b\cosh(x))}{2a^2b}+\frac{x(2aA-bB)}{2a^2}+\frac{B\sinh(x)}{2a}+\frac{B\cosh(x)}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cosh[x])/(a + b*Cosh[x] - b*Sinh[x]),x]
```

```
[Out] ((2*a*A - b*B)*x)/(2*a^2) + (B*Cosh[x])/(2*a) + ((2*a*A*b - a^2*B - b^2*B)*
Log[a + b*Cosh[x] - b*Sinh[x]])/(2*a^2*b) + (B*Sinh[x])/(2*a)
```

Rule 3132

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.))/(cos[(d_.) + (e_.)*(x_)]*(b_.)
+ (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)]), x_Symbol] :> Simp[((2*a*A - b*B)*x
)/(2*a^2), x] + (Simp[(B*Sin[d + e*x])/(2*a*e), x] - Simp[(b*B*Cos[d + e*x]
)/(2*a*c*e), x] + Simp[((a^2*B - 2*a*b*A + b^2*B)*Log[RemoveContent[a + b*C
os[d + e*x] + c*Sin[d + e*x], x]])/(2*a^2*c*e), x]) /; FreeQ[{a, b, c, d, e
, A, B}, x] && EqQ[b^2 + c^2, 0]
```

Rubi steps

$$\int \frac{A+B\cosh(x)}{a+b\cosh(x)-b\sinh(x)}\,dx=\frac{(2aA-bB)x}{2a^2}+\frac{B\cosh(x)}{2a}+\frac{\left(2aAb-a^2B-b^2B\right)\log(a+b\cosh(x)-b\sinh(x$$

Mathematica [A] time = 0.16, size = 86, normalized size = 1.10

$$\frac{x\left(a^2B+2aAb-b^2B\right)-2\left(a^2B-2aAb+b^2B\right)\log\left((a-b)\sinh\left(\frac{x}{2}\right)+(a+b)\cosh\left(\frac{x}{2}\right)\right)+2abB\sinh(x)+2abB\,\mathrm{c}}{4a^2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cosh[x])/(a + b*Cosh[x] - b*Sinh[x]),x]
```

```
[Out] ((2*a*A*b + a^2*B - b^2*B)*x + 2*a*b*B*Cosh[x] - 2*(-2*a*A*b + a^2*B + b^2*
B)*Log[(a + b)*Cosh[x/2] + (a - b)*Sinh[x/2]] + 2*a*b*B*Sinh[x])/(4*a^2*b)
```

fricas [A] time = 0.44, size = 56, normalized size = 0.72

$$\frac{Ba^2x + Bab\cosh(x) + Bab\sinh(x) - \left(Ba^2 - 2\,Aab + Bb^2\right)\log\left(a\cosh(x) + a\sinh(x) + b\right)}{2\,a^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(a+b*cosh(x)-b*sinh(x)),x, algorithm="fricas")
```

```
[Out] 1/2*(B*a^2*x + B*a*b*cosh(x) + B*a*b*sinh(x) - (B*a^2 - 2*A*a*b + B*b^2)*lo
g(a*cosh(x) + a*sinh(x) + b))/(a^2*b)
```

giac [A] time = 0.13, size = 48, normalized size = 0.62

$$\frac{Bx}{2\,b} + \frac{Be^x}{2\,a} - \frac{\left(Ba^2 - 2\,Aab + Bb^2\right)\log\left(|ae^x + b|\right)}{2\,a^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(a+b*cosh(x)-b*sinh(x)),x, algorithm="giac")
```

```
[Out] 1/2*B*x/b + 1/2*B*e^x/a - 1/2*(B*a^2 - 2*A*a*b + B*b^2)*log(abs(a*e^x + b))
/(a^2*b)
```

maple [A] time = 0.19, size = 125, normalized size = 1.60

$$-\frac{B}{a\left(\tanh\left(\frac{x}{2}\right)-1\right)} - \frac{\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)A}{a} + \frac{\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)Bb}{2a^2} + \frac{B\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)}{2b} + \frac{\ln\left(a\tanh\left(\frac{x}{2}\right) - \tanh\left(\frac{x}{2}\right)\right.}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cosh(x))/(a+b*cosh(x)-b*sinh(x)),x)
```

```
[Out] -B/a/(tanh(1/2*x)-1)-1/a*ln(tanh(1/2*x)-1)*A+1/2/a^2*ln(tanh(1/2*x)-1)*B*b+
1/2*B/b*ln(tanh(1/2*x)+1)+1/a*ln(a*tanh(1/2*x)-tanh(1/2*x)*b+a+b)*A-1/2/b*l
n(a*tanh(1/2*x)-tanh(1/2*x)*b+a+b)*B-1/2/a^2*b*ln(a*tanh(1/2*x)-tanh(1/2*x)
*b+a+b)*B
```

maxima [A] time = 0.32, size = 62, normalized size = 0.79

$$A\left(\frac{x}{a} + \frac{\log\left(be^{(-x)} + a\right)}{a}\right) - \frac{1}{2}\,B\left(\frac{bx}{a^2} - \frac{e^x}{a} + \frac{\left(a^2 + b^2\right)\log\left(be^{(-x)} + a\right)}{a^2b}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(a+b*cosh(x)-b*sinh(x)),x, algorithm="maxima")
```

```
[Out] A*(x/a + log(b*e^(-x) + a)/a) - 1/2*B*(b*x/a^2 - e^x/a + (a^2 + b^2)*log(b*
e^(-x) + a)/(a^2*b))
```

mupad [B] time = 1.59, size = 47, normalized size = 0.60

$$\frac{B\,\mathrm{e}^x}{2\,a}+\frac{B\,x}{2\,b}-\frac{\ln\left(b+a\,\mathrm{e}^x\right)\,\left(B\,a^2-2\,A\,a\,b+B\,b^2\right)}{2\,a^2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cosh(x))/(a + b*cosh(x) - b*sinh(x)),x)
```

```
[Out] (B*exp(x))/(2*a) + (B*x)/(2*b) - (log(b + a*exp(x))*(B*a^2 + B*b^2 - 2*A*a*
b))/(2*a^2*b)
```

sympy [A] time = 5.26, size = 904, normalized size = 11.59

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(a+b*cosh(x)-b*sinh(x)),x)
```

```
[Out] Piecewise((zoo*(A*x + B*sinh(x)), Eq(a, 0) & Eq(b, 0)), (2*A/(-2*b*sinh(x)
+ 2*b*cosh(x)) - B*x*sinh(x)/(-2*b*sinh(x) + 2*b*cosh(x)) + B*x*cosh(x)/(-2
*b*sinh(x) + 2*b*cosh(x)) + B*cosh(x)/(-2*b*sinh(x) + 2*b*cosh(x)), Eq(a, 0
)), ((A*x + B*sinh(x))/a, Eq(b, 0)), (2*A*x*tanh(x/2)/(2*b*tanh(x/2) - 2*b)
 - 2*A*x/(2*b*tanh(x/2) - 2*b) - 2*A*log(tanh(x/2) + 1)*tanh(x/2)/(2*b*tanh
(x/2) - 2*b) + 2*A*log(tanh(x/2) + 1)/(2*b*tanh(x/2) - 2*b) - B*x*tanh(x/2)
/(2*b*tanh(x/2) - 2*b) + B*x/(2*b*tanh(x/2) - 2*b) + 2*B*log(tanh(x/2) + 1)
*tanh(x/2)/(2*b*tanh(x/2) - 2*b) - 2*B*log(tanh(x/2) + 1)/(2*b*tanh(x/2) -
2*b) - 2*B/(2*b*tanh(x/2) - 2*b), Eq(a, b)), (2*A*a*b*x*tanh(x/2)/(2*a**2*b
*tanh(x/2) - 2*a**2*b) - 2*A*a*b*x/(2*a**2*b*tanh(x/2) - 2*a**2*b) - 2*A*a*
b*log(tanh(x/2) + 1)*tanh(x/2)/(2*a**2*b*tanh(x/2) - 2*a**2*b) + 2*A*a*b*lo
g(tanh(x/2) + 1)/(2*a**2*b*tanh(x/2) - 2*a**2*b) + 2*A*a*b*log(a/(a - b) +
b/(a - b) + tanh(x/2))*tanh(x/2)/(2*a**2*b*tanh(x/2) - 2*a**2*b) - 2*A*a*b*
log(a/(a - b) + b/(a - b) + tanh(x/2))/(2*a**2*b*tanh(x/2) - 2*a**2*b) + B*
a**2*log(tanh(x/2) + 1)*tanh(x/2)/(2*a**2*b*tanh(x/2) - 2*a**2*b) - B*a**2*
log(tanh(x/2) + 1)/(2*a**2*b*tanh(x/2) - 2*a**2*b) - B*a**2*log(a/(a - b) +
 b/(a - b) + tanh(x/2))*tanh(x/2)/(2*a**2*b*tanh(x/2) - 2*a**2*b) + B*a**2*
log(a/(a - b) + b/(a - b) + tanh(x/2))/(2*a**2*b*tanh(x/2) - 2*a**2*b) - 2*
B*a*b/(2*a**2*b*tanh(x/2) - 2*a**2*b) - B*b**2*x*tanh(x/2)/(2*a**2*b*tanh(x
/2) - 2*a**2*b) + B*b**2*x/(2*a**2*b*tanh(x/2) - 2*a**2*b) + B*b**2*log(tan
```

```
h(x/2) + 1)*tanh(x/2)/(2*a**2*b*tanh(x/2) - 2*a**2*b) - B*b**2*log(tanh(x/2
) + 1)/(2*a**2*b*tanh(x/2) - 2*a**2*b) - B*b**2*log(a/(a - b) + b/(a - b) +
 tanh(x/2))*tanh(x/2)/(2*a**2*b*tanh(x/2) - 2*a**2*b) + B*b**2*log(a/(a - b
) + b/(a - b) + tanh(x/2))/(2*a**2*b*tanh(x/2) - 2*a**2*b), True))
```

3.807 $\int \frac{A+B\cosh(x)+C\sinh(x)}{a+b\cosh(x)-b\sinh(x)}\,dx$

Optimal. Leaf size=81

$$\frac{\left(-\left(a^2(B-C)\right)+2aAb-b^2(B+C)\right)\log(a-b\sinh(x)+b\cosh(x))}{2a^2b}+\frac{x(2aA-b(B+C))}{2a^2}+\frac{(B+C)(\sinh(x)+\cos$$

```
[Out] 1/2*(2*a*A-b*(B+C))*x/a^2+1/2*(2*a*A*b-a^2*(B-C)-b^2*(B+C))*ln(a+b*cosh(x)-
b*sinh(x))/a^2/b+1/2*(B+C)*(cosh(x)+sinh(x))/a
```

Rubi [A] time = 0.08, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {3130}

$$\frac{\left(a^2(-(B-C))+2aAb-b^2(B+C)\right)\log(a-b\sinh(x)+b\cosh(x))}{2a^2b}+\frac{x(2aA-b(B+C))}{2a^2}+\frac{(B+C)(\sinh(x)+\cosh$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cosh[x] + C*Sinh[x])/(a + b*Cosh[x] - b*Sinh[x]),x]
```

```
[Out] ((2*a*A - b*(B + C))*x)/(2*a^2) + ((2*a*A*b - a^2*(B - C) - b^2*(B + C))*Lo
g[a + b*Cosh[x] - b*Sinh[x]])/(2*a^2*b) + ((B + C)*(Cosh[x] + Sinh[x]))/(2*
a)
```

Rule 3130

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)]), x_
Symbol] :> Simp[((2*a*A - b*B - c*C)*x)/(2*a^2), x] + (-Simp[((b*B + c*C)*(
b*Cos[d + e*x] - c*Sin[d + e*x]))/(2*a*b*c*e), x] + Simp[((a^2*(b*B - c*C)
- 2*a*A*b^2 + b^2*(b*B + c*C))*Log[RemoveContent[a + b*Cos[d + e*x] + c*Sin
[d + e*x], x]])/(2*a^2*b*c*e), x]) /; FreeQ[{a, b, c, d, e, A, B, C}, x] &&
 EqQ[b^2 + c^2, 0]
```

Rubi steps

$$\int \frac{A+B\cosh(x)+C\sinh(x)}{a+b\cosh(x)-b\sinh(x)}\,dx=\frac{(2aA-b(B+C))x}{2a^2}+\frac{\left(2aAb-a^2(B-C)-b^2(B+C)\right)\log(a+b\cosh(x)-b$$

Mathematica [A] time = 0.30, size = 102, normalized size = 1.26

$$\frac{x\left(a^2(B-C)+2aAb-b^2(B+C)\right)-2\left(a^2(B-C)-2aAb+b^2(B+C)\right)\log\left((a-b)\sinh\left(\frac{x}{2}\right)+(a+b)\cosh\left(\frac{x}{2}\right)\right)-}{4a^2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cosh[x] + C*Sinh[x])/(a + b*Cosh[x] - b*Sinh[x]),x]
```

```
[Out] ((2*a*A*b + a^2*(B - C) - b^2*(B + C))*x + 2*a*b*(B + C)*Cosh[x] - 2*(-2*a*
A*b + a^2*(B - C) + b^2*(B + C))*Log[(a + b)*Cosh[x/2] + (a - b)*Sinh[x/2]]
 + 2*a*b*(B + C)*Sinh[x])/(4*a^2*b)
```

fricas [A] time = 0.46, size = 70, normalized size = 0.86

$$\frac{(B-C)a^2x+(B+C)ab\cosh(x)+(B+C)ab\sinh(x)-\left((B-C)a^2-2\,Aab+(B+C)b^2\right)\log\left(a\cosh(x)+a\,\mathrm{sinl}\right.}{2\,a^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x)+C*sinh(x))/(a+b*cosh(x)-b*sinh(x)),x, algorithm="fri
cas")
```

```
[Out] 1/2*((B - C)*a^2*x + (B + C)*a*b*cosh(x) + (B + C)*a*b*sinh(x) - ((B - C)*a
^2 - 2*A*a*b + (B + C)*b^2)*log(a*cosh(x) + a*sinh(x) + b))/(a^2*b)
```

giac [A] time = 0.13, size = 69, normalized size = 0.85

$$\frac{(B-C)x}{2\,b}+\frac{Be^x+Ce^x}{2\,a}-\frac{\left(Ba^2-Ca^2-2\,Aab+Bb^2+Cb^2\right)\log\left(|ae^x+b|\right)}{2\,a^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x)+C*sinh(x))/(a+b*cosh(x)-b*sinh(x)),x, algorithm="gia
c")
```

```
[Out] 1/2*(B - C)*x/b + 1/2*(B*e^x + C*e^x)/a - 1/2*(B*a^2 - C*a^2 - 2*A*a*b + B*
b^2 + C*b^2)*log(abs(a*e^x + b))/(a^2*b)
```

maple [B] time = 0.21, size = 213, normalized size = 2.63

$$-\frac{B}{a\left(\tanh\left(\frac{x}{2}\right)-1\right)}-\frac{C}{a\left(\tanh\left(\frac{x}{2}\right)-1\right)}-\frac{\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)A}{a}+\frac{\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)Bb}{2a^2}+\frac{\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)bC}{2a^2}+\frac{B\ln\left(\mathrm{ta}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cosh(x)+C*sinh(x))/(a+b*cosh(x)-b*sinh(x)),x)
```

```
[Out] -B/a/(tanh(1/2*x)-1)-C/a/(tanh(1/2*x)-1)-1/a*ln(tanh(1/2*x)-1)*A+1/2/a^2*ln
(tanh(1/2*x)-1)*B*b+1/2/a^2*ln(tanh(1/2*x)-1)*b*C+1/2*B/b*ln(tanh(1/2*x)+1)
-1/2*C/b*ln(tanh(1/2*x)+1)+1/a*ln(a*tanh(1/2*x)-tanh(1/2*x)*b+a+b)*A-1/2/b*
ln(a*tanh(1/2*x)-tanh(1/2*x)*b+a+b)*B-1/2/a^2*b*ln(a*tanh(1/2*x)-tanh(1/2*x
```

```
)*b+a+b)*B+1/2/b*ln(a*tanh(1/2*x)-tanh(1/2*x)*b+a+b)*C-1/2/a^2*b*ln(a*tanh(
1/2*x)-tanh(1/2*x)*b+a+b)*C
```

maxima [A] time = 0.33, size = 105, normalized size = 1.30

$$A\left(\frac{x}{a}+\frac{\log\left(be^{(-x)}+a\right)}{a}\right)-\frac{1}{2}B\left(\frac{bx}{a^2}-\frac{e^x}{a}+\frac{\left(a^2+b^2\right)\log\left(be^{(-x)}+a\right)}{a^2b}\right)-\frac{1}{2}C\left(\frac{bx}{a^2}-\frac{e^x}{a}-\frac{\left(a^2-b^2\right)\log\left(be^{(-x)}+a\right)}{a^2b}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x)+C*sinh(x))/(a+b*cosh(x)-b*sinh(x)),x, algorithm="max
ima")
```

```
[Out] A*(x/a + log(b*e^(-x) + a)/a) - 1/2*B*(b*x/a^2 - e^x/a + (a^2 + b^2)*log(b*
e^(-x) + a)/(a^2*b)) - 1/2*C*(b*x/a^2 - e^x/a - (a^2 - b^2)*log(b*e^(-x) +
a)/(a^2*b))
```

mupad [B] time = 1.63, size = 64, normalized size = 0.79

$$\frac{x\,(B-C)}{2\,b}+\frac{e^x\,(B+C)}{2\,a}-\frac{\ln\left(b+a\,e^x\right)\,\left(B\,a^2+B\,b^2-C\,a^2+C\,b^2-2\,A\,a\,b\right)}{2\,a^2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cosh(x) + C*sinh(x))/(a + b*cosh(x) - b*sinh(x)),x)
```

```
[Out] (x*(B - C))/(2*b) + (exp(x)*(B + C))/(2*a) - (log(b + a*exp(x))*(B*a^2 + B*
b^2 - C*a^2 + C*b^2 - 2*A*a*b))/(2*a^2*b)
```

sympy [A] time = 5.97, size = 1420, normalized size = 17.53

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x)+C*sinh(x))/(a+b*cosh(x)-b*sinh(x)),x)
```

```
[Out] Piecewise((zoo*(A*x + B*sinh(x) + C*cosh(x)), Eq(a, 0) & Eq(b, 0)), (2*A/(-
2*b*sinh(x) + 2*b*cosh(x)) - B*x*sinh(x)/(-2*b*sinh(x) + 2*b*cosh(x)) + B*x
*cosh(x)/(-2*b*sinh(x) + 2*b*cosh(x)) + B*cosh(x)/(-2*b*sinh(x) + 2*b*cosh(
x)) + C*x*sinh(x)/(-2*b*sinh(x) + 2*b*cosh(x)) - C*x*cosh(x)/(-2*b*sinh(x)
+ 2*b*cosh(x)) + C*cosh(x)/(-2*b*sinh(x) + 2*b*cosh(x)), Eq(a, 0)), ((A*x +
 B*sinh(x) + C*cosh(x))/a, Eq(b, 0)), (2*A*x*tanh(x/2)/(2*b*tanh(x/2) - 2*b
) - 2*A*x/(2*b*tanh(x/2) - 2*b) - 2*A*log(tanh(x/2) + 1)*tanh(x/2)/(2*b*tan
h(x/2) - 2*b) + 2*A*log(tanh(x/2) + 1)/(2*b*tanh(x/2) - 2*b) - B*x*tanh(x/2
)/(2*b*tanh(x/2) - 2*b) + B*x/(2*b*tanh(x/2) - 2*b) + 2*B*log(tanh(x/2) + 1
```

```
)*tanh(x/2)/(2*b*tanh(x/2) - 2*b) - 2*B*log(tanh(x/2) + 1)/(2*b*tanh(x/2) -
 2*b) - 2*B/(2*b*tanh(x/2) - 2*b) - C*x*tanh(x/2)/(2*b*tanh(x/2) - 2*b) + C
*x/(2*b*tanh(x/2) - 2*b) - 2*C/(2*b*tanh(x/2) - 2*b), Eq(a, b)), (2*A*a*b*x
*tanh(x/2)/(2*a**2*b*tanh(x/2) - 2*a**2*b) - 2*A*a*b*x/(2*a**2*b*tanh(x/2)
- 2*a**2*b) - 2*A*a*b*log(tanh(x/2) + 1)*tanh(x/2)/(2*a**2*b*tanh(x/2) - 2*
a**2*b) + 2*A*a*b*log(tanh(x/2) + 1)/(2*a**2*b*tanh(x/2) - 2*a**2*b) + 2*A*
a*b*log(a/(a - b) + b/(a - b) + tanh(x/2))*tanh(x/2)/(2*a**2*b*tanh(x/2) -
2*a**2*b) - 2*A*a*b*log(a/(a - b) + b/(a - b) + tanh(x/2))/(2*a**2*b*tanh(x
/2) - 2*a**2*b) + B*a**2*log(tanh(x/2) + 1)*tanh(x/2)/(2*a**2*b*tanh(x/2) -
 2*a**2*b) - B*a**2*log(tanh(x/2) + 1)/(2*a**2*b*tanh(x/2) - 2*a**2*b) - B*
a**2*log(a/(a - b) + b/(a - b) + tanh(x/2))*tanh(x/2)/(2*a**2*b*tanh(x/2) -
 2*a**2*b) + B*a**2*log(a/(a - b) + b/(a - b) + tanh(x/2))/(2*a**2*b*tanh(x
/2) - 2*a**2*b) - 2*B*a*b/(2*a**2*b*tanh(x/2) - 2*a**2*b) - B*b**2*x*tanh(x
/2)/(2*a**2*b*tanh(x/2) - 2*a**2*b) + B*b**2*x/(2*a**2*b*tanh(x/2) - 2*a**2
*b) + B*b**2*log(tanh(x/2) + 1)*tanh(x/2)/(2*a**2*b*tanh(x/2) - 2*a**2*b) -
 B*b**2*log(tanh(x/2) + 1)/(2*a**2*b*tanh(x/2) - 2*a**2*b) - B*b**2*log(a/(
a - b) + b/(a - b) + tanh(x/2))*tanh(x/2)/(2*a**2*b*tanh(x/2) - 2*a**2*b) +
 B*b**2*log(a/(a - b) + b/(a - b) + tanh(x/2))/(2*a**2*b*tanh(x/2) - 2*a**2
*b) - C*a**2*log(tanh(x/2) + 1)*tanh(x/2)/(2*a**2*b*tanh(x/2) - 2*a**2*b) +
 C*a**2*log(tanh(x/2) + 1)/(2*a**2*b*tanh(x/2) - 2*a**2*b) + C*a**2*log(a/(
a - b) + b/(a - b) + tanh(x/2))*tanh(x/2)/(2*a**2*b*tanh(x/2) - 2*a**2*b) -
 C*a**2*log(a/(a - b) + b/(a - b) + tanh(x/2))/(2*a**2*b*tanh(x/2) - 2*a**2
*b) - 2*C*a*b/(2*a**2*b*tanh(x/2) - 2*a**2*b) - C*b**2*x*tanh(x/2)/(2*a**2*
b*tanh(x/2) - 2*a**2*b) + C*b**2*x/(2*a**2*b*tanh(x/2) - 2*a**2*b) + C*b**2
*log(tanh(x/2) + 1)*tanh(x/2)/(2*a**2*b*tanh(x/2) - 2*a**2*b) - C*b**2*log(
tanh(x/2) + 1)/(2*a**2*b*tanh(x/2) - 2*a**2*b) - C*b**2*log(a/(a - b) + b/(
a - b) + tanh(x/2))*tanh(x/2)/(2*a**2*b*tanh(x/2) - 2*a**2*b) + C*b**2*log(
a/(a - b) + b/(a - b) + tanh(x/2))/(2*a**2*b*tanh(x/2) - 2*a**2*b), True))
```

3.808 $\int \frac{1}{\cosh^2(x)+\sinh^2(x)}\,dx$

Optimal. Leaf size=3

$$\tan^{-1}(\tanh(x))$$

```
[Out] arctan(tanh(x))
```

Rubi [A] time = 0.02, antiderivative size = 3, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {203}

$$\tan^{-1}(\tanh(x))$$

Antiderivative was successfully verified.

```
[In] Int[(Cosh[x]^2 + Sinh[x]^2)^(-1),x]
```

```
[Out] ArcTan[Tanh[x]]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\cosh^2(x)+\sinh^2(x)}\,dx &= \text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \tanh(x)\right)\\ &= \tan^{-1}(\tanh(x))\end{aligned}$$

Mathematica [A] time = 0.00, size = 3, normalized size = 1.00

$$\tan^{-1}(\tanh(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cosh[x]^2 + Sinh[x]^2)^(-1),x]
```

```
[Out] ArcTan[Tanh[x]]
```

fricas [B] time = 0.43, size = 19, normalized size = 6.33

$$-\arctan\left(-\frac{\cosh(x)+\sinh(x)}{\cosh(x)-\sinh(x)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cosh(x)^2+sinh(x)^2),x, algorithm="fricas")
```

```
[Out] -arctan(-(cosh(x) + sinh(x))/(cosh(x) - sinh(x)))
```

giac [**A**] time = 0.12, size = 5, normalized size = 1.67

$$\arctan\left(e^{(2x)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cosh(x)^2+sinh(x)^2),x, algorithm="giac")
```

```
[Out] arctan(e^(2*x))
```

maple [**B**] time = 0.23, size = 116, normalized size = 38.67

$$\frac{2\sqrt{2}\ \arctan\left(\frac{2\tanh\left(\frac{x}{2}\right)}{-2+2\sqrt{2}}\right)}{-2+2\sqrt{2}}-\frac{2\arctan\left(\frac{2\tanh\left(\frac{x}{2}\right)}{-2+2\sqrt{2}}\right)}{-2+2\sqrt{2}}-\frac{2\sqrt{2}\ \arctan\left(\frac{2\tanh\left(\frac{x}{2}\right)}{2+2\sqrt{2}}\right)}{2+2\sqrt{2}}-\frac{2\arctan\left(\frac{2\tanh\left(\frac{x}{2}\right)}{2+2\sqrt{2}}\right)}{2+2\sqrt{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(x)^2+sinh(x)^2),x)
```

```
[Out] 2*2^(1/2)/(-2+2*2^(1/2))*arctan(2*tanh(1/2*x)/(-2+2*2^(1/2)))-2/(-2+2*2^(1/
2))*arctan(2*tanh(1/2*x)/(-2+2*2^(1/2)))-2*2^(1/2)/(2+2*2^(1/2))*arctan(2*t
anh(1/2*x)/(2+2*2^(1/2)))-2/(2+2*2^(1/2))*arctan(2*tanh(1/2*x)/(2+2*2^(1/2)
))
```

maxima [**B**] time = 0.41, size = 35, normalized size = 11.67

$$\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2e^{(-x)}\right)\right)-\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2e^{(-x)}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cosh(x)^2+sinh(x)^2),x, algorithm="maxima")
```

```
[Out] arctan(1/2*sqrt(2)*(sqrt(2) + 2*e^(-x))) - arctan(-1/2*sqrt(2)*(sqrt(2) - 2
*e^(-x)))
```

mupad [**B**] time = 0.04, size = 5, normalized size = 1.67

$$\mathrm{atan}\left(\mathrm{e}^{2x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(x)^2 + sinh(x)^2),x)
```

```
[Out] atan(exp(2*x))
```

sympy [B] time = 7.66, size = 172, normalized size = 57.33

$$\frac{47321\sqrt{3-2\sqrt{2}}\ \operatorname{atan}\left(\frac{\tanh\left(\frac{x}{2}\right)}{\sqrt{3-2\sqrt{2}}}\right)}{13860\sqrt{2}+19601}+\frac{33461\sqrt{2}\sqrt{3-2\sqrt{2}}\ \operatorname{atan}\left(\frac{\tanh\left(\frac{x}{2}\right)}{\sqrt{3-2\sqrt{2}}}\right)}{13860\sqrt{2}+19601}-\frac{5741\sqrt{2}\sqrt{2\sqrt{2}+3}\ \operatorname{atan}\left(\frac{\tanh\left(\frac{x}{2}\right)}{\sqrt{2\sqrt{2}+3}}\right)}{13860\sqrt{2}+19601}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cosh(x)**2+sinh(x)**2),x)
```

```
[Out] 47321*sqrt(3 - 2*sqrt(2))*atan(tanh(x/2)/sqrt(3 - 2*sqrt(2)))/(13860*sqrt(2
) + 19601) + 33461*sqrt(2)*sqrt(3 - 2*sqrt(2))*atan(tanh(x/2)/sqrt(3 - 2*sq
rt(2)))/(13860*sqrt(2) + 19601) - 5741*sqrt(2)*sqrt(2*sqrt(2) + 3)*atan(tan
h(x/2)/sqrt(2*sqrt(2) + 3))/(13860*sqrt(2) + 19601) - 8119*sqrt(2*sqrt(2) +
 3)*atan(tanh(x/2)/sqrt(2*sqrt(2) + 3))/(13860*sqrt(2) + 19601)
```

3.809 $\int \frac{1}{\left(\cosh^2(x)+\sinh^2(x)\right)^2} \, dx$

Optimal. Leaf size=11

$$\frac{\tanh(x)}{\tanh^2(x)+1}$$

```
[Out] tanh(x)/(1+tanh(x)^2)
```

Rubi [A] time = 0.02, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {383}

$$\frac{\tanh(x)}{\tanh^2(x)+1}$$

Antiderivative was successfully verified.

```
[In] Int[(Cosh[x]^2 + Sinh[x]^2)^(-2),x]
```

```
[Out] Tanh[x]/(1 + Tanh[x]^2)
```

Rule 383

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> S
imp[(c*x*(a + b*x^n)^(p + 1))/a, x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[
b*c - a*d, 0] && EqQ[a*d - b*c*(n*(p + 1) + 1), 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\left(\cosh^2(x)+\sinh^2(x)\right)^2} \, dx &= \text{Subst}\left(\int \frac{1-x^2}{\left(1+x^2\right)^2} \, dx, x, \tanh(x)\right) \\
&= \frac{\tanh(x)}{1+\tanh^2(x)}
\end{aligned}$$

Mathematica [A] time = 0.00, size = 8, normalized size = 0.73

$$\frac{1}{2} \tanh(2x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cosh[x]^2 + Sinh[x]^2)^(-2),x]
```

[Out] Tanh[2*x]/2

fricas [B] time = 0.41, size = 40, normalized size = 3.64

$$-\frac{1}{\cosh(x)^4+4\,\cosh(x)^3\sinh(x)+6\,\cosh(x)^2\sinh(x)^2+4\,\cosh(x)\sinh(x)^3+\sinh(x)^4+1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(cosh(x)^2+sinh(x)^2)^2,x, algorithm="fricas")

[Out] -1/(cosh(x)^4 + 4*cosh(x)^3*sinh(x) + 6*cosh(x)^2*sinh(x)^2 + 4*cosh(x)*sin h(x)^3 + sinh(x)^4 + 1)

giac [A] time = 0.13, size = 10, normalized size = 0.91

$$-\frac{1}{e^{(4\,x)}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(cosh(x)^2+sinh(x)^2)^2,x, algorithm="giac")

[Out] -1/(e^(4*x) + 1)

maple [B] time = 0.19, size = 36, normalized size = 3.27

$$-\frac{2\left(-\left(\tanh^{3}\left(\frac{x}{2}\right)\right)-\tanh\left(\frac{x}{2}\right)\right)}{\tanh^{4}\left(\frac{x}{2}\right)+6\left(\tanh^{2}\left(\frac{x}{2}\right)\right)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(cosh(x)^2+sinh(x)^2)^2,x)

[Out] -2*(-tanh(1/2*x)^3-tanh(1/2*x))/(tanh(1/2*x)^4+6*tanh(1/2*x)^2+1)

maxima [A] time = 0.31, size = 8, normalized size = 0.73

$$\frac{1}{e^{(-4\,x)}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(cosh(x)^2+sinh(x)^2)^2,x, algorithm="maxima")

[Out] 1/(e^(-4*x) + 1)

mupad [B] time = 1.55, size = 10, normalized size = 0.91

$$-\frac{1}{e^{4x}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(x)^2 + sinh(x)^2)^2,x)
```

```
[Out] -1/(exp(4*x) + 1)
```

sympy [B] time = 1.37, size = 48, normalized size = 4.36

$$\frac{2\tanh^3\left(\frac{x}{2}\right)}{\tanh^4\left(\frac{x}{2}\right)+6\tanh^2\left(\frac{x}{2}\right)+1}+\frac{2\tanh\left(\frac{x}{2}\right)}{\tanh^4\left(\frac{x}{2}\right)+6\tanh^2\left(\frac{x}{2}\right)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cosh(x)**2+sinh(x)**2)**2,x)
```

```
[Out] 2*tanh(x/2)**3/(tanh(x/2)**4 + 6*tanh(x/2)**2 + 1) + 2*tanh(x/2)/(tanh(x/2)
**4 + 6*tanh(x/2)**2 + 1)
```

3.810 $\int \frac{1}{\left(\cosh^2(x)+\sinh^2(x)\right)^3}\,dx$

Optimal. Leaf size=26

$$\frac{1}{2}\tan^{-1}(\tanh(x))+\frac{\tanh(x)\operatorname{sech}^2(x)}{2\left(\tanh^2(x)+1\right)^2}$$

```
[Out] 1/2*arctan(tanh(x))+1/2*sech(x)^2*tanh(x)/(1+tanh(x)^2)^2
```

Rubi [A] time = 0.03, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {413, 21, 203}

$$\frac{1}{2}\tan^{-1}(\tanh(x))+\frac{\tanh(x)\operatorname{sech}^2(x)}{2\left(\tanh^2(x)+1\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Cosh[x]^2 + Sinh[x]^2)^(-3),x]
```

```
[Out] ArcTan[Tanh[x]]/2 + (Sech[x]^2*Tanh[x])/(2*(1 + Tanh[x]^2)^2)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 413

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[((a*d - c*b)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(a*b*n*(p +
 1)), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
```

```
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{\left(\cosh^2(x)+\sinh^2(x)\right)^3}\,dx &= \text{Subst}\left(\int \frac{\left(1-x^2\right)^2}{\left(1+x^2\right)^3}\,dx, x, \tanh(x)\right)\\
&= \frac{\operatorname{sech}^2(x)\tanh(x)}{2\left(1+\tanh^2(x)\right)^2}+\frac{1}{4}\text{Subst}\left(\int \frac{2+2x^2}{\left(1+x^2\right)^2}\,dx, x, \tanh(x)\right)\\
&= \frac{\operatorname{sech}^2(x)\tanh(x)}{2\left(1+\tanh^2(x)\right)^2}+\frac{1}{2}\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \tanh(x)\right)\\
&= \frac{1}{2}\tan^{-1}(\tanh(x))+\frac{\operatorname{sech}^2(x)\tanh(x)}{2\left(1+\tanh^2(x)\right)^2}
\end{aligned}
$$

Mathematica [A] time = 0.01, size = 22, normalized size = 0.85

$$\frac{1}{4}\tan^{-1}(\sinh(2x))+\frac{1}{4}\tanh(2x)\operatorname{sech}(2x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cosh[x]^2 + Sinh[x]^2)^(-3),x]
[Out] ArcTan[Sinh[2*x]]/4 + (Sech[2*x]*Tanh[2*x])/4
```

fricas [B] time = 0.43, size = 304, normalized size = 11.69

$$\frac{\cosh(x)^6+20\cosh(x)^3\sinh(x)^3+15\cosh(x)^2\sinh(x)^4+6\cosh(x)\sinh(x)^5+\sinh(x)^6+\left(15\cosh(x)^4-1\right)}{2\left(\cosh(x)^8+5\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cosh(x)^2+sinh(x)^2)^3,x, algorithm="fricas")
[Out] 1/2*(cosh(x)^6 + 20*cosh(x)^3*sinh(x)^3 + 15*cosh(x)^2*sinh(x)^4 + 6*cosh(x
)*sinh(x)^5 + sinh(x)^6 + (15*cosh(x)^4 - 1)*sinh(x)^2 - (cosh(x)^8 + 56*co
sh(x)^3*sinh(x)^5 + 28*cosh(x)^2*sinh(x)^6 + 8*cosh(x)*sinh(x)^7 + sinh(x)^
8 + 2*(35*cosh(x)^4 + 1)*sinh(x)^4 + 2*cosh(x)^4 + 8*(7*cosh(x)^5 + cosh(x)
)*sinh(x)^3 + 4*(7*cosh(x)^6 + 3*cosh(x)^2)*sinh(x)^2 + 8*(cosh(x)^7 + cosh
```

```
(x)^3)*sinh(x) + 1)*arctan(-(cosh(x) + sinh(x))/(cosh(x) - sinh(x))) - cosh
(x)^2 + 2*(3*cosh(x)^5 - cosh(x))*sinh(x))/(cosh(x)^8 + 56*cosh(x)^3*sinh(x
)^5 + 28*cosh(x)^2*sinh(x)^6 + 8*cosh(x)*sinh(x)^7 + sinh(x)^8 + 2*(35*cosh
(x)^4 + 1)*sinh(x)^4 + 2*cosh(x)^4 + 8*(7*cosh(x)^5 + cosh(x))*sinh(x)^3 +
4*(7*cosh(x)^6 + 3*cosh(x)^2)*sinh(x)^2 + 8*(cosh(x)^7 + cosh(x)^3)*sinh(x)
 + 1)
```

giac [B] time = 0.13, size = 46, normalized size = 1.77

$$\frac{e^{(2x)} - e^{(-2x)}}{2\left(\left(e^{(2x)} - e^{(-2x)}\right)^2 + 4\right)} + \frac{1}{4}\arctan\left(\frac{1}{2}\left(e^{(4x)} - 1\right)e^{(-2x)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cosh(x)^2+sinh(x)^2)^3,x, algorithm="giac")
```

```
[Out] 1/2*(e^(2*x) - e^(-2*x))/((e^(2*x) - e^(-2*x))^2 + 4) + 1/4*arctan(1/2*(e^(
4*x) - 1)*e^(-2*x))
```

maple [B] time = 0.23, size = 166, normalized size = 6.38

$$-\frac{2\left(-\frac{\left(\tanh^7\left(\frac{x}{2}\right)\right)}{2} + \frac{\left(\tanh^5\left(\frac{x}{2}\right)\right)}{2} + \frac{\left(\tanh^3\left(\frac{x}{2}\right)\right)}{2} - \frac{\tanh\left(\frac{x}{2}\right)}{2}\right)}{\left(\tanh^4\left(\frac{x}{2}\right) + 6\left(\tanh^2\left(\frac{x}{2}\right)\right) + 1\right)^2} + \frac{\sqrt{2}\,\arctan\left(\frac{2\tanh\left(\frac{x}{2}\right)}{-2+2\sqrt{2}}\right)}{-2+2\sqrt{2}} - \frac{\arctan\left(\frac{2\tanh\left(\frac{x}{2}\right)}{-2+2\sqrt{2}}\right)}{-2+2\sqrt{2}} - \frac{\sqrt{2}\,\arctan\left(\frac{2\tanh}{2+2}\right.}{2+2\sqrt{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(x)^2+sinh(x)^2)^3,x)
```

```
[Out] -2*(-1/2*tanh(1/2*x)^7+1/2*tanh(1/2*x)^5+1/2*tanh(1/2*x)^3-1/2*tanh(1/2*x))
/(tanh(1/2*x)^4+6*tanh(1/2*x)^2+1)^2+2^(1/2)/(-2+2*2^(1/2))*arctan(2*tanh(1
/2*x)/(-2+2*2^(1/2)))-1/(-2+2*2^(1/2))*arctan(2*tanh(1/2*x)/(-2+2*2^(1/2)))
-2^(1/2)/(2+2*2^(1/2))*arctan(2*tanh(1/2*x)/(2+2*2^(1/2)))-1/(2+2*2^(1/2))*
arctan(2*tanh(1/2*x)/(2+2*2^(1/2)))
```

maxima [B] time = 0.41, size = 64, normalized size = 2.46

$$\frac{e^{(-2x)} - e^{(-6x)}}{2\left(2e^{(-4x)} + e^{(-8x)} + 1\right)} + \frac{1}{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2} + 2e^{(-x)}\right)\right) - \frac{1}{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2} - 2e^{(-x)}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cosh(x)^2+sinh(x)^2)^3,x, algorithm="maxima")
```

```
[Out] 1/2*(e^(-2*x) - e^(-6*x))/(2*e^(-4*x) + e^(-8*x) + 1) + 1/2*arctan(1/2*sqrt
(2)*(sqrt(2) + 2*e^(-x))) - 1/2*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*e^(-x)))
```

mupad [B] time = 1.56, size = 28, normalized size = 1.08

$$\frac{\operatorname{atan}\left(e^{2x}\right)}{2}-\frac{e^{-2x}}{4\cosh\left(2x\right)^2}+\frac{1}{4\cosh\left(2x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(x)^2 + sinh(x)^2)^3,x)
[Out] atan(exp(2*x))/2 - exp(-2*x)/(4*cosh(2*x)^2) + 1/(4*cosh(2*x))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cosh(x)**2+sinh(x)**2)**3,x)
[Out] Timed out
```

3.811 $\int \frac{1}{\cosh^2(x)-\sinh^2(x)}\,dx$

Optimal. Leaf size=1

$$x$$

```
[Out] x
```

Rubi [A] time = 0.01, antiderivative size = 1, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {4380, 8}

$$x$$

Antiderivative was successfully verified.

```
[In] Int[(Cosh[x]^2 - Sinh[x]^2)^(-1),x]
[Out] x
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 4380

```
Int[(u_.)*((a_.) + cos[(d_.) + (e_.)*(x_)]^2*(b_.) + (c_.)*sin[(d_.) + (e_.
)*(x_)]^2)^(p_.), x_Symbol] :> Dist[(a + c)^p, Int[ActivateTrig[u], x], x]
/; FreeQ[{a, b, c, d, e, p}, x] && EqQ[b - c, 0]
```

Rubi steps

$$\begin{aligned} \int \frac{1}{\cosh^2(x)-\sinh^2(x)}\,dx &= \int 1\,dx \\ &= x \end{aligned}$$

Mathematica [A] time = 0.00, size = 1, normalized size = 1.00

$$x$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cosh[x]^2 - Sinh[x]^2)^(-1),x]
[Out] x
```

fricas [A] time = 0.40, size = 1, normalized size = 1.00

$$x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cosh(x)^2-sinh(x)^2),x, algorithm="fricas")
[Out] x
```

giac [A] time = 0.11, size = 1, normalized size = 1.00

$$x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cosh(x)^2-sinh(x)^2),x, algorithm="giac")
[Out] x
```

maple [C] time = 0.10, size = 8, normalized size = 8.00

$$2 \operatorname{arctanh}\left(\tanh\left(\frac{x}{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(x)^2-sinh(x)^2),x)
[Out] 2*arctanh(tanh(1/2*x))
```

maxima [A] time = 0.31, size = 1, normalized size = 1.00

$$x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cosh(x)^2-sinh(x)^2),x, algorithm="maxima")
[Out] x
```

mupad [B] time = 1.52, size = 1, normalized size = 1.00

$$x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(x)^2 - sinh(x)^2),x)
```

```
[Out] x
```

sympy [B] time = 0.39, size = 10, normalized size = 10.00

$$\frac{x}{-\sinh^2(x) + \cosh^2(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cosh(x)**2-sinh(x)**2),x)
[Out] x/(-sinh(x)**2 + cosh(x)**2)
```

3.812 $\int \frac{1}{\left(\cosh^2(x)-\sinh^2(x)\right)^2} \, dx$

Optimal. Leaf size=1

$$x$$

```
[Out] x
```

Rubi [A] time = 0.01, antiderivative size = 1, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {4380, 8}

$$x$$

Antiderivative was successfully verified.

```
[In] Int[(Cosh[x]^2 - Sinh[x]^2)^(-2),x]
```

```
[Out] x
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 4380

```
Int[(u_.)*((a_.) + cos[(d_.) + (e_.)*(x_)]^2*(b_.) + (c_.)*sin[(d_.) + (e_.
)*(x_)]^2)^(p_.), x_Symbol] :> Dist[(a + c)^p, Int[ActivateTrig[u], x], x]
/; FreeQ[{a, b, c, d, e, p}, x] && EqQ[b - c, 0]
```

Rubi steps

$$\begin{aligned} \int \frac{1}{\left(\cosh^2(x)-\sinh^2(x)\right)^2} \, dx &= \int 1 \, dx \\ &= x \end{aligned}$$

Mathematica [A] time = 0.00, size = 1, normalized size = 1.00

$$x$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cosh[x]^2 - Sinh[x]^2)^(-2),x]
```

[Out] x

fricas [A] time = 0.40, size = 1, normalized size = 1.00

$$x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cosh(x)^2-sinh(x)^2)^2,x, algorithm="fricas")
[Out] x
```

giac [A] time = 0.11, size = 1, normalized size = 1.00

$$x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cosh(x)^2-sinh(x)^2)^2,x, algorithm="giac")
[Out] x
```

maple [C] time = 0.10, size = 8, normalized size = 8.00

$$2 \operatorname{arctanh}\left(\tanh\left(\frac{x}{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(x)^2-sinh(x)^2)^2,x)
[Out] 2*arctanh(tanh(1/2*x))
```

maxima [A] time = 0.31, size = 1, normalized size = 1.00

$$x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cosh(x)^2-sinh(x)^2)^2,x, algorithm="maxima")
[Out] x
```

mupad [B] time = 0.04, size = 1, normalized size = 1.00

$$x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(x)^2 - sinh(x)^2)^2,x)
```

```
[Out] x
```

sympy [B] time = 1.11, size = 22, normalized size = 22.00

$$\frac{x}{\sinh^4(x) - 2\sinh^2(x)\cosh^2(x) + \cosh^4(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cosh(x)**2-sinh(x)**2)**2,x)
```

```
[Out] x/(sinh(x)**4 - 2*sinh(x)**2*cosh(x)**2 + cosh(x)**4)
```

3.813 $\int \frac{1}{\left(\cosh^2(x)-\sinh^2(x)\right)^3} \, dx$

Optimal. Leaf size=1

$$x$$

[Out] x

Rubi [A] time = 0.01, antiderivative size = 1, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {4380, 8}

$$x$$

Antiderivative was successfully verified.

[In] Int[(Cosh[x]^2 - Sinh[x]^2)^(-3),x]

[Out] x

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 4380

```
Int[(u_.)*((a_.) + cos[(d_.) + (e_.)*(x_)]^2*(b_.) + (c_.)*sin[(d_.) + (e_.
)*(x_)]^2)^(p_.), x_Symbol] :> Dist[(a + c)^p, Int[ActivateTrig[u], x], x]
/; FreeQ[{a, b, c, d, e, p}, x] && EqQ[b - c, 0]
```

Rubi steps

$$\begin{aligned} \int \frac{1}{\left(\cosh^2(x)-\sinh^2(x)\right)^3} \, dx &= \int 1 \, dx \\ &= x \end{aligned}$$

Mathematica [A] time = 0.00, size = 1, normalized size = 1.00

$$x$$

Antiderivative was successfully verified.

[In] Integrate[(Cosh[x]^2 - Sinh[x]^2)^(-3),x]

[Out] x

fricas [A] time = 0.39, size = 1, normalized size = 1.00

$$x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cosh(x)^2-sinh(x)^2)^3,x, algorithm="fricas")
[Out] x
```

giac [A] time = 0.11, size = 1, normalized size = 1.00

$$x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cosh(x)^2-sinh(x)^2)^3,x, algorithm="giac")
[Out] x
```

maple [C] time = 0.10, size = 8, normalized size = 8.00

$$2 \operatorname{arctanh}\left(\tanh\left(\frac{x}{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(x)^2-sinh(x)^2)^3,x)
[Out] 2*arctanh(tanh(1/2*x))
```

maxima [A] time = 0.32, size = 1, normalized size = 1.00

$$x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cosh(x)^2-sinh(x)^2)^3,x, algorithm="maxima")
[Out] x
```

mupad [B] time = 0.02, size = 1, normalized size = 1.00

$$x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(x)^2 - sinh(x)^2)^3,x)
```

```
[Out] x
```

sympy [B] time = 2.31, size = 34, normalized size = 34.00

$$\frac{x}{-\sinh^{6}(x)+3\sinh^{4}(x)\cosh^{2}(x)-3\sinh^{2}(x)\cosh^{4}(x)+\cosh^{6}(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cosh(x)**2-sinh(x)**2)**3,x)
```

```
[Out] x/(-sinh(x)**6 + 3*sinh(x)**4*cosh(x)**2 - 3*sinh(x)**2*cosh(x)**4 + cosh(x
)**6)
```

3.814 $\int \frac{1}{\operatorname{sech}^2(x)+\tanh^2(x)}\,dx$

Optimal. Leaf size=1

$$x$$

```
[Out] x
```

Rubi [A] time = 0.01, antiderivative size = 1, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {4381, 8}

$$x$$

Antiderivative was successfully verified.

```
[In] Int[(Sech[x]^2 + Tanh[x]^2)^(-1),x]
```

```
[Out] x
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 4381

```
Int[(u_.)*((a_.) + (c_.)*sec[(d_.) + (e_.)*(x_)]^2 + (b_.)*tan[(d_.) + (e_.
)*(x_)]^2)^(p_.), x_Symbol] :> Dist[(a + c)^p, Int[ActivateTrig[u], x], x]
/; FreeQ[{a, b, c, d, e, p}, x] && EqQ[b + c, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\operatorname{sech}^2(x)+\tanh^2(x)}\,dx &= \int 1\,dx\\ &= x\end{aligned}$$

Mathematica [A] time = 0.00, size = 1, normalized size = 1.00

$$x$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sech[x]^2 + Tanh[x]^2)^(-1),x]
```

```
[Out] x
```

fricas [A] time = 0.39, size = 1, normalized size = 1.00

$$x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sech(x)^2+tanh(x)^2),x, algorithm="fricas")
[Out] x
```

giac [A] time = 0.11, size = 1, normalized size = 1.00

$$x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sech(x)^2+tanh(x)^2),x, algorithm="giac")
[Out] x
```

maple [C] time = 0.11, size = 8, normalized size = 8.00

$$2\,\mathrm{arctanh}\left(\tanh\left(\frac{x}{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sech(x)^2+tanh(x)^2),x)
[Out] 2*arctanh(tanh(1/2*x))
```

maxima [A] time = 0.31, size = 1, normalized size = 1.00

$$x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sech(x)^2+tanh(x)^2),x, algorithm="maxima")
[Out] x
```

mupad [B] time = 1.57, size = 1, normalized size = 1.00

$$x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1/cosh(x)^2 + tanh(x)^2),x)
```

```
[Out] x
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\tanh^2(x) + \operatorname{sech}^2(x)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sech(x)**2+tanh(x)**2),x)
[Out] Integral(1/(tanh(x)**2 + sech(x)**2), x)
```

3.815 $\int \frac{1}{\left(\mathbf{sech}^2(x)+\tanh^2(x)\right)^2}\,dx$

Optimal. Leaf size=1

$$x$$

[Out] x

Rubi [A] time = 0.01, antiderivative size = 1, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {4381, 8}

$$x$$

Antiderivative was successfully verified.

[In] Int[(Sech[x]^2 + Tanh[x]^2)^(-2),x]

[Out] x

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 4381

```
Int[(u_.)*((a_.) + (c_.)*sec[(d_.) + (e_.)*(x_)]^2 + (b_.)*tan[(d_.) + (e_.
)*(x_)]^2)^(p_.), x_Symbol] :> Dist[(a + c)^p, Int[ActivateTrig[u], x], x]
/; FreeQ[{a, b, c, d, e, p}, x] && EqQ[b + c, 0]
```

Rubi steps

$$\begin{aligned} \int \frac{1}{\left(\operatorname{sech}^2(x)+\tanh^2(x)\right)^2}\,dx &= \int 1\,dx\\ &= x \end{aligned}$$

Mathematica [A] time = 0.00, size = 1, normalized size = 1.00

$$x$$

Antiderivative was successfully verified.

[In] Integrate[(Sech[x]^2 + Tanh[x]^2)^(-2),x]

```
[Out] x
```

fricas [A] time = 0.38, size = 1, normalized size = 1.00

$$x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sech(x)^2+tanh(x)^2)^2,x, algorithm="fricas")
[Out] x
```

giac [A] time = 0.11, size = 1, normalized size = 1.00

$$x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sech(x)^2+tanh(x)^2)^2,x, algorithm="giac")
[Out] x
```

maple [C] time = 0.13, size = 8, normalized size = 8.00

$$2 \operatorname{arctanh}\left(\tanh\left(\frac{x}{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sech(x)^2+tanh(x)^2)^2,x)
[Out] 2*arctanh(tanh(1/2*x))
```

maxima [A] time = 0.31, size = 1, normalized size = 1.00

$$x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sech(x)^2+tanh(x)^2)^2,x, algorithm="maxima")
[Out] x
```

mupad [B] time = 1.57, size = 1, normalized size = 1.00

$$x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1/cosh(x)^2 + tanh(x)^2)^2,x)
```

```
[Out] x
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(\tanh^2\left(x\right) + \operatorname{sech}^2\left(x\right)\right)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sech(x)**2+tanh(x)**2)**2,x)
[Out] Integral((tanh(x)**2 + sech(x)**2)**(-2), x)
```

3.816 $\int \frac{1}{\left(\operatorname{sech}^2(x)+\tanh^2(x)\right)^3} \, dx$

Optimal. Leaf size=1

$$x$$

```
[Out] x
```

Rubi [A] time = 0.01, antiderivative size = 1, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {4381, 8}

$$x$$

Antiderivative was successfully verified.

```
[In] Int[(Sech[x]^2 + Tanh[x]^2)^(-3),x]
```

```
[Out] x
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 4381

```
Int[(u_.)*((a_.) + (c_.)*sec[(d_.) + (e_.)*(x_)]^2 + (b_.)*tan[(d_.) + (e_.
)*(x_)]^2)^(p_.), x_Symbol] :> Dist[(a + c)^p, Int[ActivateTrig[u], x], x]
/; FreeQ[{a, b, c, d, e, p}, x] && EqQ[b + c, 0]
```

Rubi steps

$$\begin{aligned} \int \frac{1}{\left(\operatorname{sech}^2(x)+\tanh^2(x)\right)^3} \, dx &= \int 1 \, dx \\ &= x \end{aligned}$$

Mathematica [A] time = 0.00, size = 1, normalized size = 1.00

$$x$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sech[x]^2 + Tanh[x]^2)^(-3),x]
```

```
[Out] x
```

fricas [A] time = 0.40, size = 1, normalized size = 1.00

$$x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sech(x)^2+tanh(x)^2)^3,x, algorithm="fricas")
[Out] x
```

giac [A] time = 0.11, size = 1, normalized size = 1.00

$$x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sech(x)^2+tanh(x)^2)^3,x, algorithm="giac")
[Out] x
```

maple [C] time = 0.12, size = 8, normalized size = 8.00

$$2\,\mathrm{arctanh}\left(\tanh\left(\frac{x}{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sech(x)^2+tanh(x)^2)^3,x)
[Out] 2*arctanh(tanh(1/2*x))
```

maxima [A] time = 0.46, size = 1, normalized size = 1.00

$$x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sech(x)^2+tanh(x)^2)^3,x, algorithm="maxima")
[Out] x
```

mupad [B] time = 1.51, size = 1, normalized size = 1.00

$$x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1/cosh(x)^2 + tanh(x)^2)^3,x)
```

```
[Out] x
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(\tanh^2(x) + \mathrm{sech}^2(x)\right)^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sech(x)**2+tanh(x)**2)**3,x)
```

```
[Out] Integral((tanh(x)**2 + sech(x)**2)**(-3), x)
```

3.817 $\int \frac{1}{\mathrm{sech}^2(x)-\tanh^2(x)} \, dx$

Optimal. Leaf size=19

$$\sqrt{2} \tanh^{-1}\left(\sqrt{2} \tanh(x)\right) - x$$

```
[Out] -x+arctanh(2^(1/2)*tanh(x))*2^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1093, 207}

$$\sqrt{2} \tanh^{-1}\left(\sqrt{2} \tanh(x)\right) - x$$

Antiderivative was successfully verified.

```
[In] Int[(Sech[x]^2 - Tanh[x]^2)^(-1),x]
```

```
[Out] -x + Sqrt[2]*ArcTanh[Sqrt[2]*Tanh[x]]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 1093

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^2), x], x] - Dist[c/q, Int
[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
0] && PosQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\mathrm{sech}^2(x) - \tanh^2(x)} \, dx &= \mathrm{Subst}\left(\int \frac{1}{1-3x^2+2x^4} \, dx, x, \tanh(x)\right)\\
&= 2\,\mathrm{Subst}\left(\int \frac{1}{-2+2x^2} \, dx, x, \tanh(x)\right) - 2\,\mathrm{Subst}\left(\int \frac{1}{-1+2x^2} \, dx, x, \tanh(x)\right)\\
&= -x + \sqrt{2} \tanh^{-1}\left(\sqrt{2} \tanh(x)\right)
\end{aligned}$$

Mathematica [A] time = 0.04, size = 19, normalized size = 1.00

$$\sqrt{2}\tanh^{-1}\left(\sqrt{2}\tanh(x)\right)-x$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sech[x]^2 - Tanh[x]^2)^(-1),x]
```

```
[Out] -x + Sqrt[2]*ArcTanh[Sqrt[2]*Tanh[x]]
```

fricas [B] time = 0.44, size = 70, normalized size = 3.68

$$\frac{1}{2}\sqrt{2}\log\left(-\frac{3\left(2\sqrt{2}-3\right)\cosh(x)^2-4\left(3\sqrt{2}-4\right)\cosh(x)\sinh(x)+3\left(2\sqrt{2}-3\right)\sinh(x)^2-2\sqrt{2}+3}{\cosh(x)^2+\sinh(x)^2-3}\right)-x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sech(x)^2-tanh(x)^2),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(2)*log(-(3*(2*sqrt(2) - 3)*cosh(x)^2 - 4*(3*sqrt(2) - 4)*cosh(x)*s
inh(x) + 3*(2*sqrt(2) - 3)*sinh(x)^2 - 2*sqrt(2) + 3)/(cosh(x)^2 + sinh(x)^
2 - 3)) - x
```

giac [B] time = 0.14, size = 41, normalized size = 2.16

$$-\frac{1}{2}\sqrt{2}\log\left(\frac{\left|-4\sqrt{2}+2e^{(2x)}-6\right|}{\left|4\sqrt{2}+2e^{(2x)}-6\right|}\right)-x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sech(x)^2-tanh(x)^2),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(2)*log(abs(-4*sqrt(2) + 2*e^(2*x) - 6)/abs(4*sqrt(2) + 2*e^(2*x)
- 6)) - x
```

maple [B] time = 0.22, size = 54, normalized size = 2.84

$$\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)-\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)+\sqrt{2}\ \operatorname{arctanh}\left(\frac{\left(2\tanh\left(\frac{x}{2}\right)-2\right)\sqrt{2}}{4}\right)+\sqrt{2}\ \operatorname{arctanh}\left(\frac{\left(2\tanh\left(\frac{x}{2}\right)+2\right)\sqrt{2}}{4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sech(x)^2-tanh(x)^2),x)
```

```
[Out] ln(tanh(1/2*x)-1)-ln(tanh(1/2*x)+1)+2^(1/2)*arctanh(1/4*(2*tanh(1/2*x)-2)*2
^(1/2))+2^(1/2)*arctanh(1/4*(2*tanh(1/2*x)+2)*2^(1/2))
```

maxima [B] time = 0.43, size = 64, normalized size = 3.37

$$\frac{1}{2}\sqrt{2}\log\left(-\frac{\sqrt{2}-e^{(-x)}+1}{\sqrt{2}+e^{(-x)}-1}\right)-\frac{1}{2}\sqrt{2}\log\left(-\frac{\sqrt{2}-e^{(-x)}-1}{\sqrt{2}+e^{(-x)}+1}\right)-x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sech(x)^2-tanh(x)^2),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(2)*log(-(sqrt(2) - e^(-x) + 1)/(sqrt(2) + e^(-x) - 1)) - 1/2*sqrt(
2)*log(-(sqrt(2) - e^(-x) - 1)/(sqrt(2) + e^(-x) + 1)) - x
```

mupad [B] time = 0.17, size = 56, normalized size = 2.95

$$\frac{\sqrt{2}\ \ln\left(8\,e^{2\,x}+\frac{\sqrt{2}\,\left(12\,e^{2\,x}-4\right)}{2}\right)}{2}-\frac{\sqrt{2}\ \ln\left(8\,e^{2\,x}-\frac{\sqrt{2}\,\left(12\,e^{2\,x}-4\right)}{2}\right)}{2}-x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1/cosh(x)^2 - tanh(x)^2),x)
```

```
[Out] (2^(1/2)*log(8*exp(2*x) + (2^(1/2)*(12*exp(2*x) - 4))/2))/2 - (2^(1/2)*log(
8*exp(2*x) - (2^(1/2)*(12*exp(2*x) - 4))/2))/2 - x
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\left(-\tanh\left(x\right)+\operatorname{sech}\left(x\right)\right)\left(\tanh\left(x\right)+\operatorname{sech}\left(x\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sech(x)**2-tanh(x)**2),x)
```

```
[Out] Integral(1/((-tanh(x) + sech(x))*(tanh(x) + sech(x))), x)
```

3.818 $\int \frac{1}{\left(\operatorname{sech}^2(x)-\tanh^2(x)\right)^2} \, dx$

Optimal. Leaf size=31

$$x-\frac{\tanh^{-1}\left(\sqrt{2}\tanh(x)\right)}{\sqrt{2}}+\frac{\tanh(x)}{1-2\tanh^2(x)}$$

```
[Out] x-1/2*arctanh(2^(1/2)*tanh(x))*2^(1/2)+tanh(x)/(1-2*tanh(x)^2)
```

Rubi [A] time = 0.05, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {414, 12, 481, 206}

$$x-\frac{\tanh^{-1}\left(\sqrt{2}\tanh(x)\right)}{\sqrt{2}}+\frac{\tanh(x)}{1-2\tanh^2(x)}$$

Antiderivative was successfully verified.

```
[In] Int[(Sech[x]^2 - Tanh[x]^2)^(-2),x]
```

```
[Out] x - ArcTanh[Sqrt[2]*Tanh[x]]/Sqrt[2] + Tanh[x]/(1 - 2*Tanh[x]^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 481

```
Int[((e_.)*(x_))^(m_.)/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))),
 x_Symbol] :> -Dist[(a*e^n)/(b*c - a*d), Int[(e*x)^(m - n)/(a + b*x^n), x],
 x] + Dist[(c*e^n)/(b*c - a*d), Int[(e*x)^(m - n)/(c + d*x^n), x], x] /; Fr
eeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LeQ[n, m,
2*n - 1]
```

Rubi steps

$$\int \frac{1}{\left(\operatorname{sech}^2(x)-\tanh^2(x)\right)^2}\,dx = \text{Subst}\left(\int \frac{1}{\left(1-2x^2\right)^2\left(1-x^2\right)}\,dx, x, \tanh(x)\right)$$
$$= \frac{\tanh(x)}{1-2\tanh^2(x)} + \frac{1}{2}\text{Subst}\left(\int -\frac{2x^2}{\left(1-2x^2\right)\left(1-x^2\right)}\,dx, x, \tanh(x)\right)$$
$$= \frac{\tanh(x)}{1-2\tanh^2(x)} - \text{Subst}\left(\int \frac{x^2}{\left(1-2x^2\right)\left(1-x^2\right)}\,dx, x, \tanh(x)\right)$$
$$= \frac{\tanh(x)}{1-2\tanh^2(x)} - \text{Subst}\left(\int \frac{1}{1-2x^2}\,dx, x, \tanh(x)\right) + \text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \tan\right.$$
$$= x - \frac{\tanh^{-1}\left(\sqrt{2}\tanh(x)\right)}{\sqrt{2}} + \frac{\tanh(x)}{1-2\tanh^2(x)}$$

Mathematica [A] time = 0.12, size = 42, normalized size = 1.35

$$\frac{-3x-\sinh(2x)+x\cosh(2x)}{\cosh(2x)-3} - \frac{\tanh^{-1}\left(\sqrt{2}\tanh(x)\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sech[x]^2 - Tanh[x]^2)^(-2),x]
```

```
[Out] -(ArcTanh[Sqrt[2]*Tanh[x]]/Sqrt[2]) + (-3*x + x*Cosh[2*x] - Sinh[2*x])/(-3
+ Cosh[2*x])
```

fricas [B] time = 0.44, size = 266, normalized size = 8.58

$$4\,x\cosh(x)^4 + 16\,x\cosh(x)\sinh(x)^3 + 4\,x\sinh(x)^4 - 24\,(x+1)\cosh(x)^2 + 24\left(x\cosh(x)^2 - x - 1\right)\sinh(x)^2 + \left(\sqrt{\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sech(x)^2-tanh(x)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/4*(4*x*cosh(x)^4 + 16*x*cosh(x)*sinh(x)^3 + 4*x*sinh(x)^4 - 24*(x + 1)*co
sh(x)^2 + 24*(x*cosh(x)^2 - x - 1)*sinh(x)^2 + (sqrt(2)*cosh(x)^4 + 4*sqrt(
2)*cosh(x)*sinh(x)^3 + sqrt(2)*sinh(x)^4 + 6*(sqrt(2)*cosh(x)^2 - sqrt(2))*
sinh(x)^2 - 6*sqrt(2)*cosh(x)^2 + 4*(sqrt(2)*cosh(x)^3 - 3*sqrt(2)*cosh(x))
*sinh(x) + sqrt(2))*log((3*(2*sqrt(2) + 3)*cosh(x)^2 - 4*(3*sqrt(2) + 4)*co
sh(x)*sinh(x) + 3*(2*sqrt(2) + 3)*sinh(x)^2 - 2*sqrt(2) - 3)/(cosh(x)^2 + s
inh(x)^2 - 3)) + 16*(x*cosh(x)^3 - 3*(x + 1)*cosh(x))*sinh(x) + 4*x + 8)/(c
osh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + 6*(cosh(x)^2 - 1)*sinh(x)^2 -
6*cosh(x)^2 + 4*(cosh(x)^3 - 3*cosh(x))*sinh(x) + 1)
```

giac [B] time = 0.14, size = 63, normalized size = 2.03

$$\frac{1}{4}\sqrt{2}\log\left(\frac{\left|-4\sqrt{2}+2e^{(2x)}-6\right|}{\left|4\sqrt{2}+2e^{(2x)}-6\right|}\right)+x-\frac{2\left(3e^{(2x)}-1\right)}{e^{(4x)}-6e^{(2x)}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sech(x)^2-tanh(x)^2)^2,x, algorithm="giac")
```

```
[Out] 1/4*sqrt(2)*log(abs(-4*sqrt(2) + 2*e^(2*x) - 6)/abs(4*sqrt(2) + 2*e^(2*x) -
 6)) + x - 2*(3*e^(2*x) - 1)/(e^(4*x) - 6*e^(2*x) + 1)
```

maple [B] time = 0.24, size = 108, normalized size = 3.48

$$\frac{2\tanh\left(\frac{x}{2}\right)+2}{2\left(\tanh^{2}\left(\frac{x}{2}\right)\right)-4\tanh\left(\frac{x}{2}\right)-2}-\frac{\sqrt{2}\,\operatorname{arctanh}\left(\frac{\left(2\tanh\left(\frac{x}{2}\right)-2\right)\sqrt{2}}{4}\right)}{2}-\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)+\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)-\frac{2}{2\left(\tanh^{2}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sech(x)^2-tanh(x)^2)^2,x)
```

```
[Out] 1/2*(2*tanh(1/2*x)+2)/(tanh(1/2*x)^2-2*tanh(1/2*x)-1)-1/2*2^(1/2)*arctanh(1
/4*(2*tanh(1/2*x)-2)*2^(1/2))-ln(tanh(1/2*x)-1)+ln(tanh(1/2*x)+1)-1/2*(2-2*
tanh(1/2*x))/(tanh(1/2*x)^2+2*tanh(1/2*x)-1)-1/2*2^(1/2)*arctanh(1/4*(2*tan
h(1/2*x)+2)*2^(1/2))
```

maxima [B] time = 0.42, size = 88, normalized size = 2.84

$$-\frac{1}{4}\sqrt{2}\log\left(-\frac{\sqrt{2}-e^{(-x)}+1}{\sqrt{2}+e^{(-x)}-1}\right)+\frac{1}{4}\sqrt{2}\log\left(-\frac{\sqrt{2}-e^{(-x)}-1}{\sqrt{2}+e^{(-x)}+1}\right)+x-\frac{2\left(3e^{(-2x)}-1\right)}{6e^{(-2x)}-e^{(-4x)}-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sech(x)^2-tanh(x)^2)^2,x, algorithm="maxima")
```

```
[Out] -1/4*sqrt(2)*log(-(sqrt(2) - e^(-x) + 1)/(sqrt(2) + e^(-x) - 1)) + 1/4*sqrt
(2)*log(-(sqrt(2) - e^(-x) - 1)/(sqrt(2) + e^(-x) + 1)) + x - 2*(3*e^(-2*x)
 - 1)/(6*e^(-2*x) - e^(-4*x) - 1)
```

mupad [B] time = 1.61, size = 78, normalized size = 2.52

$$x-\frac{\sqrt{2}\,\ln\left(-4\,{\mathrm{e}}^{2\,x}-\frac{\sqrt{2}\,\left(12\,{\mathrm{e}}^{2\,x}-4\right)}{4}\right)}{4}+\frac{\sqrt{2}\,\ln\left(\frac{\sqrt{2}\,\left(12\,{\mathrm{e}}^{2\,x}-4\right)}{4}-4\,{\mathrm{e}}^{2\,x}\right)}{4}-\frac{6\,{\mathrm{e}}^{2\,x}-2}{{\mathrm{e}}^{4\,x}-6\,{\mathrm{e}}^{2\,x}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1/cosh(x)^2 - tanh(x)^2)^2,x)
```

```
[Out] x - (2^(1/2)*log(- 4*exp(2*x) - (2^(1/2)*(12*exp(2*x) - 4))/4))/4 + (2^(1/2
)*log((2^(1/2)*(12*exp(2*x) - 4))/4 - 4*exp(2*x)))/4 - (6*exp(2*x) - 2)/(ex
p(4*x) - 6*exp(2*x) + 1)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(- \tanh{\left(x \right)} + \operatorname{sech}{\left(x \right)}\right)^{2} \left(\tanh{\left(x \right)} + \operatorname{sech}{\left(x \right)}\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sech(x)**2-tanh(x)**2)**2,x)
```

```
[Out] Integral(1/((-tanh(x) + sech(x))**2*(tanh(x) + sech(x))**2), x)
```

3.819 $\int \frac{1}{\left(\operatorname{sech}^2(x)-\tanh^2(x)\right)^3}\,dx$

Optimal. Leaf size=54

$$-x+\frac{7\tanh^{-1}\left(\sqrt{2}\tanh(x)\right)}{4\sqrt{2}}-\frac{\tanh(x)}{4\left(1-2\tanh^2(x)\right)}+\frac{\tanh(x)}{2\left(1-2\tanh^2(x)\right)^2}$$

```
[Out] -x+7/8*arctanh(2^(1/2)*tanh(x))*2^(1/2)+1/2*tanh(x)/(1-2*tanh(x)^2)^2-1/4*t
anh(x)/(1-2*tanh(x)^2)
```

Rubi [A] time = 0.06, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {414, 527, 522, 206}

$$-x+\frac{7\tanh^{-1}\left(\sqrt{2}\tanh(x)\right)}{4\sqrt{2}}-\frac{\tanh(x)}{4\left(1-2\tanh^2(x)\right)}+\frac{\tanh(x)}{2\left(1-2\tanh^2(x)\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Sech[x]^2 - Tanh[x]^2)^(-3),x]
```

```
[Out] -x + (7*ArcTanh[Sqrt[2]*Tanh[x]])/(4*Sqrt[2]) + Tanh[x]/(2*(1 - 2*Tanh[x]^2
)^2) - Tanh[x]/(4*(1 - 2*Tanh[x]^2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{\left(\operatorname{sech}^2(x)-\tanh^2(x)\right)^3}\,dx &= \text{Subst}\left(\int \frac{1}{\left(1-2x^2\right)^3\left(1-x^2\right)}\,dx, x, \tanh(x)\right)\\
&= \frac{\tanh(x)}{2\left(1-2\tanh^2(x)\right)^2}+\frac{1}{4}\text{Subst}\left(\int \frac{2-6x^2}{\left(1-2x^2\right)^2\left(1-x^2\right)}\,dx, x, \tanh(x)\right)\\
&= \frac{\tanh(x)}{2\left(1-2\tanh^2(x)\right)^2}-\frac{\tanh(x)}{4\left(1-2\tanh^2(x)\right)}+\frac{1}{8}\text{Subst}\left(\int \frac{6+2x^2}{\left(1-2x^2\right)\left(1-x^2\right)}\,dx, x,\right.\\
&= \frac{\tanh(x)}{2\left(1-2\tanh^2(x)\right)^2}-\frac{\tanh(x)}{4\left(1-2\tanh^2(x)\right)}+\frac{7}{4}\text{Subst}\left(\int \frac{1}{1-2x^2}\,dx, x, \tanh(x)\right)-\\
&= -x+\frac{7\tanh^{-1}\left(\sqrt{2}\,\tanh(x)\right)}{4\sqrt{2}}+\frac{\tanh(x)}{2\left(1-2\tanh^2(x)\right)^2}-\frac{\tanh(x)}{4\left(1-2\tanh^2(x)\right)}
\end{aligned}
$$

Mathematica [A] time = 0.21, size = 66, normalized size = 1.22

$$
\frac{-76x-2\sinh(2x)+3\sinh(4x)+48x\cosh(2x)-4x\cosh(4x)+7\sqrt{2}(\cosh(2x)-3)^2\tanh^{-1}\left(\sqrt{2}\,\tanh(x)\right)}{8(\cosh(2x)-3)^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sech[x]^2 - Tanh[x]^2)^(-3),x]
```

```
[Out] (-76*x + 7*Sqrt[2]*ArcTanh[Sqrt[2]*Tanh[x]]*(-3 + Cosh[2*x])^2 + 48*x*Cosh[
2*x] - 4*x*Cosh[4*x] - 2*Sinh[2*x] + 3*Sinh[4*x])/(8*(-3 + Cosh[2*x])^2)
```

fricas [B] time = 0.45, size = 717, normalized size = 13.28

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sech(x)^2-tanh(x)^2)^3,x, algorithm="fricas")
```

```
[Out] -1/16*(16*x*cosh(x)^8 + 128*x*cosh(x)*sinh(x)^7 + 16*x*sinh(x)^8 - 8*(24*x
+ 17)*cosh(x)^6 + 8*(56*x*cosh(x)^2 - 24*x - 17)*sinh(x)^6 + 16*(56*x*cosh(
x)^3 - 3*(24*x + 17)*cosh(x))*sinh(x)^5 + 152*(4*x + 3)*cosh(x)^4 + 8*(140*
x*cosh(x)^4 - 15*(24*x + 17)*cosh(x)^2 + 76*x + 57)*sinh(x)^4 + 32*(28*x*co
sh(x)^5 - 5*(24*x + 17)*cosh(x)^3 + 19*(4*x + 3)*cosh(x))*sinh(x)^3 - 8*(24
*x + 19)*cosh(x)^2 + 8*(56*x*cosh(x)^6 - 15*(24*x + 17)*cosh(x)^4 + 114*(4*
x + 3)*cosh(x)^2 - 24*x - 19)*sinh(x)^2 - 7*(sqrt(2)*cosh(x)^8 + 8*sqrt(2)*
cosh(x)*sinh(x)^7 + sqrt(2)*sinh(x)^8 + 4*(7*sqrt(2)*cosh(x)^2 - 3*sqrt(2))
*sinh(x)^6 - 12*sqrt(2)*cosh(x)^6 + 8*(7*sqrt(2)*cosh(x)^3 - 9*sqrt(2)*cosh
(x))*sinh(x)^5 + 2*(35*sqrt(2)*cosh(x)^4 - 90*sqrt(2)*cosh(x)^2 + 19*sqrt(2
))*sinh(x)^4 + 38*sqrt(2)*cosh(x)^4 + 8*(7*sqrt(2)*cosh(x)^5 - 30*sqrt(2)*c
osh(x)^3 + 19*sqrt(2)*cosh(x))*sinh(x)^3 + 4*(7*sqrt(2)*cosh(x)^6 - 45*sqrt
(2)*cosh(x)^4 + 57*sqrt(2)*cosh(x)^2 - 3*sqrt(2))*sinh(x)^2 - 12*sqrt(2)*co
sh(x)^2 + 8*(sqrt(2)*cosh(x)^7 - 9*sqrt(2)*cosh(x)^5 + 19*sqrt(2)*cosh(x)^3
 - 3*sqrt(2)*cosh(x))*sinh(x) + sqrt(2))*log(-(3*(2*sqrt(2) - 3)*cosh(x)^2
- 4*(3*sqrt(2) - 4)*cosh(x)*sinh(x) + 3*(2*sqrt(2) - 3)*sinh(x)^2 - 2*sqrt(
2) + 3)/(cosh(x)^2 + sinh(x)^2 - 3)) + 16*(8*x*cosh(x)^7 - 3*(24*x + 17)*co
sh(x)^5 + 38*(4*x + 3)*cosh(x)^3 - (24*x + 19)*cosh(x))*sinh(x) + 16*x + 24
)/(cosh(x)^8 + 8*cosh(x)*sinh(x)^7 + sinh(x)^8 + 4*(7*cosh(x)^2 - 3)*sinh(x
)^6 - 12*cosh(x)^6 + 8*(7*cosh(x)^3 - 9*cosh(x))*sinh(x)^5 + 2*(35*cosh(x)^
4 - 90*cosh(x)^2 + 19)*sinh(x)^4 + 38*cosh(x)^4 + 8*(7*cosh(x)^5 - 30*cosh(
x)^3 + 19*cosh(x))*sinh(x)^3 + 4*(7*cosh(x)^6 - 45*cosh(x)^4 + 57*cosh(x)^2
 - 3)*sinh(x)^2 - 12*cosh(x)^2 + 8*(cosh(x)^7 - 9*cosh(x)^5 + 19*cosh(x)^3
- 3*cosh(x))*sinh(x) + 1)
```

giac [A] time = 0.12, size = 77, normalized size = 1.43

$$-\frac{7}{16}\sqrt{2}\log\left(\frac{\left|-4\sqrt{2}+2e^{(2x)}-6\right|}{\left|4\sqrt{2}+2e^{(2x)}-6\right|}\right)-x+\frac{17e^{(6x)}-57e^{(4x)}+19e^{(2x)}-3}{2\left(e^{(4x)}-6e^{(2x)}+1\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sech(x)^2-tanh(x)^2)^3,x, algorithm="giac")
```

```
[Out] -7/16*sqrt(2)*log(abs(-4*sqrt(2) + 2*e^(2*x) - 6)/abs(4*sqrt(2) + 2*e^(2*x)
 - 6)) - x + 1/2*(17*e^(6*x) - 57*e^(4*x) + 19*e^(2*x) - 3)/(e^(4*x) - 6*e^
(2*x) + 1)^2
```

maple [B] time = 0.25, size = 140, normalized size = 2.59

$$\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)-\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)-\frac{2\left(-\frac{\left(\tanh^{3}\left(\frac{x}{2}\right)\right)}{8}-\frac{\left(\tanh^{2}\left(\frac{x}{2}\right)\right)}{8}-\frac{5\tanh\left(\frac{x}{2}\right)}{8}-\frac{1}{8}\right)}{\left(\tanh^{2}\left(\frac{x}{2}\right)-2\tanh\left(\frac{x}{2}\right)-1\right)^{2}}+\frac{7\sqrt{2}\ \operatorname{arctanh}\left(\frac{\left(2\tanh\left(\frac{x}{2}\right)-2\right)\sqrt{2}}{4}\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sech(x)^2-tanh(x)^2)^3,x)
```

```
[Out] ln(tanh(1/2*x)-1)-ln(tanh(1/2*x)+1)-2*(-1/8*tanh(1/2*x)^3-1/8*tanh(1/2*x)^2
-5/8*tanh(1/2*x)-1/8)/(tanh(1/2*x)^2-2*tanh(1/2*x)-1)^2+7/8*2^(1/2)*arctanh
(1/4*(2*tanh(1/2*x)-2)*2^(1/2))-2*(-1/8*tanh(1/2*x)^3+1/8*tanh(1/2*x)^2-5/8
*tanh(1/2*x)+1/8)/(tanh(1/2*x)^2+2*tanh(1/2*x)-1)^2+7/8*2^(1/2)*arctanh(1/4
*(2*tanh(1/2*x)+2)*2^(1/2))
```

maxima [B] time = 0.44, size = 114, normalized size = 2.11

$$\frac{7}{16}\sqrt{2}\log\left(-\frac{\sqrt{2}-e^{(-x)}+1}{\sqrt{2}+e^{(-x)}-1}\right)-\frac{7}{16}\sqrt{2}\log\left(-\frac{\sqrt{2}-e^{(-x)}-1}{\sqrt{2}+e^{(-x)}+1}\right)-x+\frac{19\,e^{(-2\,x)}-57\,e^{(-4\,x)}+17\,e^{(-6\,x)}-3}{2\left(12\,e^{(-2\,x)}-38\,e^{(-4\,x)}+12\,e^{(-6\,x)}-e^{(-8\,x)}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sech(x)^2-tanh(x)^2)^3,x, algorithm="maxima")
```

```
[Out] 7/16*sqrt(2)*log(-(sqrt(2) - e^(-x) + 1)/(sqrt(2) + e^(-x) - 1)) - 7/16*sqr
t(2)*log(-(sqrt(2) - e^(-x) - 1)/(sqrt(2) + e^(-x) + 1)) - x + 1/2*(19*e^(-
2*x) - 57*e^(-4*x) + 17*e^(-6*x) - 3)/(12*e^(-2*x) - 38*e^(-4*x) + 12*e^(-6
*x) - e^(-8*x) - 1)
```

mupad [B] time = 0.07, size = 114, normalized size = 2.11

$$\frac{136\,e^{2\,x}-24}{38\,e^{4\,x}-12\,e^{2\,x}-12\,e^{6\,x}+e^{8\,x}+1}-x-\frac{7\sqrt{2}\ \ln\left(7\,e^{2\,x}-\frac{7\sqrt{2}\left(12\,e^{2\,x}-4\right)}{16}\right)}{16}+\frac{7\sqrt{2}\ \ln\left(7\,e^{2\,x}+\frac{7\sqrt{2}\left(12\,e^{2\,x}-4\right)}{16}\right)}{16}+\frac{\frac{17\,e^{2}}{2}}{e^{4\,x}-}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1/cosh(x)^2 - tanh(x)^2)^3,x)
```

```
[Out] (136*exp(2*x) - 24)/(38*exp(4*x) - 12*exp(2*x) - 12*exp(6*x) + exp(8*x) + 1
) - x - (7*2^(1/2)*log(7*exp(2*x) - (7*2^(1/2)*(12*exp(2*x) - 4))/16))/16 +
```

```
 (7*2^(1/2)*log(7*exp(2*x) + (7*2^(1/2)*(12*exp(2*x) - 4))/16))/16 + ((17*e
xp(2*x))/2 + 45/2)/(exp(4*x) - 6*exp(2*x) + 1)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(-\tanh (x) + \operatorname{sech}(x))^3 (\tanh (x) + \operatorname{sech}(x))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sech(x)**2-tanh(x)**2)**3,x)
[Out] Integral(1/((-tanh(x) + sech(x))**3*(tanh(x) + sech(x))**3), x)
```

3.820 $\int \frac{1}{\coth^2(x)+\mathbf{csch}^2(x)}\,dx$

Optimal. Leaf size=18

$$x-\sqrt{2}\tanh^{-1}\left(\frac{\tanh(x)}{\sqrt{2}}\right)$$

```
[Out] x-arctanh(1/2*2^(1/2)*tanh(x))*2^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {1130, 207}

$$x-\sqrt{2}\tanh^{-1}\left(\frac{\tanh(x)}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(Coth[x]^2 + Csch[x]^2)^(-1),x]
```

```
[Out] x - Sqrt[2]*ArcTanh[Tanh[x]/Sqrt[2]]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 1130

```
Int[((d_.)*(x_))^(m_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Wi
th[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(d^2*(b/q + 1))/2, Int[(d*x)^(m - 2)/(b/2
 + q/2 + c*x^2), x], x] - Dist[(d^2*(b/q - 1))/2, Int[(d*x)^(m - 2)/(b/2 -
q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && G
eQ[m, 2]
```

Rubi steps

$$\int \frac{1}{\coth^2(x)+\operatorname{csch}^2(x)}\,dx = \text{Subst}\left(\int \frac{x^2}{2-3x^2+x^4}\,dx, x, \tanh(x)\right)$$
$$= 2\,\text{Subst}\left(\int \frac{1}{-2+x^2}\,dx, x, \tanh(x)\right) - \text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \tanh(x)\right)$$
$$= x - \sqrt{2}\tanh^{-1}\left(\frac{\tanh(x)}{\sqrt{2}}\right)$$

Mathematica [A] time = 0.04, size = 18, normalized size = 1.00

$$x - \sqrt{2}\tanh^{-1}\left(\frac{\tanh(x)}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Coth[x]^2 + Csch[x]^2)^(-1),x]
[Out] x - Sqrt[2]*ArcTanh[Tanh[x]/Sqrt[2]]
```

fricas [B] time = 0.45, size = 67, normalized size = 3.72

$$\frac{1}{2}\sqrt{2}\log\left(\frac{3\left(2\sqrt{2}+3\right)\cosh(x)^2 - 4\left(3\sqrt{2}+4\right)\cosh(x)\sinh(x) + 3\left(2\sqrt{2}+3\right)\sinh(x)^2 + 2\sqrt{2}+3}{\cosh(x)^2+\sinh(x)^2+3}\right)+x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(coth(x)^2+csch(x)^2),x, algorithm="fricas")
[Out] 1/2*sqrt(2)*log((3*(2*sqrt(2) + 3)*cosh(x)^2 - 4*(3*sqrt(2) + 4)*cosh(x)*si
nh(x) + 3*(2*sqrt(2) + 3)*sinh(x)^2 + 2*sqrt(2) + 3)/(cosh(x)^2 + sinh(x)^2
 + 3)) + x
```

giac [B] time = 0.15, size = 36, normalized size = 2.00

$$-\frac{1}{2}\sqrt{2}\log\left(-\frac{2\sqrt{2}-e^{(2x)}-3}{2\sqrt{2}+e^{(2x)}+3}\right)+x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(coth(x)^2+csch(x)^2),x, algorithm="giac")
[Out] -1/2*sqrt(2)*log(-(2*sqrt(2) - e^(2*x) - 3)/(2*sqrt(2) + e^(2*x) + 3)) + x
```

maple [B] time = 0.21, size = 102, normalized size = 5.67

$$-\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)-\frac{\sqrt{2}\,\ln\left(\frac{\tanh^{2}\left(\frac{x}{2}\right)+\sqrt{2}\,\tanh\left(\frac{x}{2}\right)+1}{\tanh^{2}\left(\frac{x}{2}\right)-\sqrt{2}\,\tanh\left(\frac{x}{2}\right)+1}\right)}{4}+\frac{\sqrt{2}\,\ln\left(\frac{\tanh^{2}\left(\frac{x}{2}\right)-\sqrt{2}\,\tanh\left(\frac{x}{2}\right)+1}{\tanh^{2}\left(\frac{x}{2}\right)+\sqrt{2}\,\tanh\left(\frac{x}{2}\right)+1}\right)}{4}+\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(coth(x)^2+csch(x)^2),x)
```

```
[Out] -ln(tanh(1/2*x)-1)-1/4*2^(1/2)*ln((tanh(1/2*x)^2+2^(1/2)*tanh(1/2*x)+1)/(ta
nh(1/2*x)^2-2^(1/2)*tanh(1/2*x)+1))+1/4*2^(1/2)*ln((tanh(1/2*x)^2-2^(1/2)*t
anh(1/2*x)+1)/(tanh(1/2*x)^2+2^(1/2)*tanh(1/2*x)+1))+ln(tanh(1/2*x)+1)
```

maxima [B] time = 0.41, size = 36, normalized size = 2.00

$$\frac{1}{2}\sqrt{2}\log\left(-\frac{2\sqrt{2}-e^{(-2x)}-3}{2\sqrt{2}+e^{(-2x)}+3}\right)+x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(coth(x)^2+csch(x)^2),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(2)*log(-(2*sqrt(2) - e^(-2*x) - 3)/(2*sqrt(2) + e^(-2*x) + 3)) + x
```

mupad [B] time = 0.15, size = 54, normalized size = 3.00

$$x+\frac{\sqrt{2}\,\ln\left(8\,\mathrm{e}^{2x}-\frac{\sqrt{2}\,\left(12\,\mathrm{e}^{2x}+4\right)}{2}\right)}{2}-\frac{\sqrt{2}\,\ln\left(8\,\mathrm{e}^{2x}+\frac{\sqrt{2}\,\left(12\,\mathrm{e}^{2x}+4\right)}{2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(coth(x)^2 + 1/sinh(x)^2),x)
```

```
[Out] x + (2^(1/2)*log(8*exp(2*x) - (2^(1/2)*(12*exp(2*x) + 4))/2))/2 - (2^(1/2)*
log(8*exp(2*x) + (2^(1/2)*(12*exp(2*x) + 4))/2))/2
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\coth^{2}(x)+\operatorname{csch}^{2}(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(coth(x)**2+csch(x)**2),x)
```

```
[Out] Integral(1/(coth(x)**2 + csch(x)**2), x)
```

3.821 $\int \frac{1}{\left(\coth^2(x)+\mathbf{csch}^2(x)\right)^2} dx$

Optimal. Leaf size=32

$$x-\frac{\tanh^{-1}\left(\frac{\tanh(x)}{\sqrt{2}}\right)}{\sqrt{2}}-\frac{\tanh(x)}{2-\tanh^2(x)}$$

[Out] x-1/2*arctanh(1/2*2^(1/2)*tanh(x))*2^(1/2)-tanh(x)/(2-tanh(x)^2)

Rubi [A] time = 0.05, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {470, 12, 391, 206}

$$x-\frac{\tanh^{-1}\left(\frac{\tanh(x)}{\sqrt{2}}\right)}{\sqrt{2}}-\frac{\tanh(x)}{2-\tanh^2(x)}$$

Antiderivative was successfully verified.

[In] Int[(Coth[x]^2 + Csch[x]^2)^(-2),x]

[Out] x - ArcTanh[Tanh[x]/Sqrt[2]]/Sqrt[2] - Tanh[x]/(2 - Tanh[x]^2)

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 391

```
Int[1/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))), x_Symbol] :> Dis
t[b/(b*c - a*d), Int[1/(a + b*x^n), x], x] - Dist[d/(b*c - a*d), Int[1/(c +
 d*x^n), x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^
(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(
```

```
b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^
n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n,
p, q, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\left(\coth^2(x)+\operatorname{csch}^2(x)\right)^2}\,dx &= \operatorname{Subst}\left(\int \frac{x^4}{\left(1-x^2\right)\left(2-x^2\right)^2}\,dx, x, \tanh(x)\right)\\
&= -\frac{\tanh(x)}{2-\tanh^2(x)}+\frac{1}{2}\operatorname{Subst}\left(\int \frac{2}{\left(1-x^2\right)\left(2-x^2\right)}\,dx, x, \tanh(x)\right)\\
&= -\frac{\tanh(x)}{2-\tanh^2(x)}+\operatorname{Subst}\left(\int \frac{1}{\left(1-x^2\right)\left(2-x^2\right)}\,dx, x, \tanh(x)\right)\\
&= -\frac{\tanh(x)}{2-\tanh^2(x)}+\operatorname{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \tanh(x)\right)-\operatorname{Subst}\left(\int \frac{1}{2-x^2}\,dx, x, \tanh\right.\\
&= x-\frac{\tanh^{-1}\left(\frac{\tanh(x)}{\sqrt{2}}\right)}{\sqrt{2}}-\frac{\tanh(x)}{2-\tanh^2(x)}
\end{aligned}$$

Mathematica [A] time = 0.10, size = 64, normalized size = 2.00

$$\frac{(\cosh(2x)+3)\operatorname{csch}^4(x)\left(6x-2\sinh(2x)+2x\cosh(2x)-\sqrt{2}(\cosh(2x)+3)\tanh^{-1}\left(\frac{\tanh(x)}{\sqrt{2}}\right)\right)}{8\left(\coth^2(x)+\operatorname{csch}^2(x)\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Coth[x]^2 + Csch[x]^2)^(-2),x]
```

```
[Out] ((3 + Cosh[2*x])*Csch[x]^4*(6*x + 2*x*Cosh[2*x] - Sqrt[2]*ArcTanh[Tanh[x]/S
qrt[2]]*(3 + Cosh[2*x]) - 2*Sinh[2*x]))/(8*(Coth[x]^2 + Csch[x]^2)^2)
```

fricas [B] time = 0.43, size = 262, normalized size = 8.19

$$\frac{4x\cosh(x)^4+16x\cosh(x)\sinh(x)^3+4x\sinh(x)^4+24(x+1)\cosh(x)^2+24\left(x\cosh(x)^2+x+1\right)\sinh(x)^2+\left(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(coth(x)^2+csch(x)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/4*(4*x*cosh(x)^4 + 16*x*cosh(x)*sinh(x)^3 + 4*x*sinh(x)^4 + 24*(x + 1)*co
sh(x)^2 + 24*(x*cosh(x)^2 + x + 1)*sinh(x)^2 + (sqrt(2)*cosh(x)^4 + 4*sqrt(
2)*cosh(x)*sinh(x)^3 + sqrt(2)*sinh(x)^4 + 6*(sqrt(2)*cosh(x)^2 + sqrt(2))*
sinh(x)^2 + 6*sqrt(2)*cosh(x)^2 + 4*(sqrt(2)*cosh(x)^3 + 3*sqrt(2)*cosh(x))
*sinh(x) + sqrt(2))*log((3*(2*sqrt(2) + 3)*cosh(x)^2 - 4*(3*sqrt(2) + 4)*co
sh(x)*sinh(x) + 3*(2*sqrt(2) + 3)*sinh(x)^2 + 2*sqrt(2) + 3)/(cosh(x)^2 + s
inh(x)^2 + 3)) + 16*(x*cosh(x)^3 + 3*(x + 1)*cosh(x))*sinh(x) + 4*x + 8)/(c
osh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + 6*(cosh(x)^2 + 1)*sinh(x)^2 +
6*cosh(x)^2 + 4*(cosh(x)^3 + 3*cosh(x))*sinh(x) + 1)
```

giac [B] time = 0.12, size = 60, normalized size = 1.88

$$-\frac{1}{4}\sqrt{2}\log\left(-\frac{2\sqrt{2}-e^{(2x)}-3}{2\sqrt{2}+e^{(2x)}+3}\right)+x+\frac{2\left(3e^{(2x)}+1\right)}{e^{(4x)}+6e^{(2x)}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(coth(x)^2+csch(x)^2)^2,x, algorithm="giac")
```

```
[Out] -1/4*sqrt(2)*log(-(2*sqrt(2) - e^(2*x) - 3)/(2*sqrt(2) + e^(2*x) + 3)) + x
+ 2*(3*e^(2*x) + 1)/(e^(4*x) + 6*e^(2*x) + 1)
```

maple [B] time = 0.23, size = 129, normalized size = 4.03

$$-\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)+\frac{-\left(\tanh^{3}\left(\frac{x}{2}\right)\right)-\tanh\left(\frac{x}{2}\right)}{\tanh^{4}\left(\frac{x}{2}\right)+1}-\frac{\sqrt{2}\ \ln\left(\frac{\tanh^{2}\left(\frac{x}{2}\right)+\sqrt{2}\ \tanh\left(\frac{x}{2}\right)+1}{\tanh^{2}\left(\frac{x}{2}\right)-\sqrt{2}\ \tanh\left(\frac{x}{2}\right)+1}\right)}{8}+\frac{\sqrt{2}\ \ln\left(\frac{\tanh^{2}\left(\frac{x}{2}\right)-\sqrt{2}\ \tanh\left(\frac{x}{2}\right)+1}{\tanh^{2}\left(\frac{x}{2}\right)+\sqrt{2}\ \tanh\left(\frac{x}{2}\right)+1}\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(coth(x)^2+csch(x)^2)^2,x)
```

```
[Out] -ln(tanh(1/2*x)-1)+2*(-1/2*tanh(1/2*x)^3-1/2*tanh(1/2*x))/(tanh(1/2*x)^4+1)
-1/8*2^(1/2)*ln((tanh(1/2*x)^2+2^(1/2)*tanh(1/2*x)+1)/(tanh(1/2*x)^2-2^(1/2
)*tanh(1/2*x)+1))+1/8*2^(1/2)*ln((tanh(1/2*x)^2-2^(1/2)*tanh(1/2*x)+1)/(tan
h(1/2*x)^2+2^(1/2)*tanh(1/2*x)+1))+ln(tanh(1/2*x)+1)
```

maxima [B] time = 0.43, size = 60, normalized size = 1.88

$$\frac{1}{4}\sqrt{2}\log\left(-\frac{2\sqrt{2}-e^{(-2x)}-3}{2\sqrt{2}+e^{(-2x)}+3}\right)+x-\frac{2\left(3e^{(-2x)}+1\right)}{6e^{(-2x)}+e^{(-4x)}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(coth(x)^2+csch(x)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/4*sqrt(2)*log(-(2*sqrt(2) - e^(-2*x) - 3)/(2*sqrt(2) + e^(-2*x) + 3)) + x
 - 2*(3*e^(-2*x) + 1)/(6*e^(-2*x) + e^(-4*x) + 1)
```

mupad [B] time = 0.06, size = 77, normalized size = 2.41

$$x+\frac{\sqrt{2}\ \ln\left(4\,e^{2\,x}-\frac{\sqrt{2}\,\left(12\,e^{2\,x}+4\right)}{4}\right)}{4}-\frac{\sqrt{2}\ \ln\left(4\,e^{2\,x}+\frac{\sqrt{2}\,\left(12\,e^{2\,x}+4\right)}{4}\right)}{4}+\frac{6\,e^{2\,x}+2}{6\,e^{2\,x}+e^{4\,x}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(coth(x)^2 + 1/sinh(x)^2)^2,x)
```

```
[Out] x + (2^(1/2)*log(4*exp(2*x) - (2^(1/2)*(12*exp(2*x) + 4))/4))/4 - (2^(1/2)*
log(4*exp(2*x) + (2^(1/2)*(12*exp(2*x) + 4))/4))/4 + (6*exp(2*x) + 2)/(6*ex
p(2*x) + exp(4*x) + 1)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\left(\coth^{2}(x)+\operatorname{csch}^{2}(x)\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(coth(x)**2+csch(x)**2)**2,x)
```

```
[Out] Integral((coth(x)**2 + csch(x)**2)**(-2), x)
```

3.822 $\int \frac{1}{\left(\coth^2(x)+\mathbf{csch}^2(x)\right)^3} \, dx$

Optimal. Leaf size=54

$$x-\frac{7 \tanh^{-1}\left(\frac{\tanh(x)}{\sqrt{2}}\right)}{4\sqrt{2}}-\frac{\tanh(x)}{4\left(2-\tanh^2(x)\right)}-\frac{\tanh^3(x)}{2\left(2-\tanh^2(x)\right)^2}$$

```
[Out] x-7/8*arctanh(1/2*2^(1/2)*tanh(x))*2^(1/2)-1/2*tanh(x)^3/(2-tanh(x)^2)^2-1/
4*tanh(x)/(2-tanh(x)^2)
```

Rubi [A] time = 0.09, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {470, 578, 522, 206}

$$x-\frac{\tanh^3(x)}{2\left(2-\tanh^2(x)\right)^2}-\frac{\tanh(x)}{4\left(2-\tanh^2(x)\right)}-\frac{7 \tanh^{-1}\left(\frac{\tanh(x)}{\sqrt{2}}\right)}{4\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Coth[x]^2 + Csch[x]^2)^(-3),x]
```

```
[Out] x - (7*ArcTanh[Tanh[x]/Sqrt[2]])/(4*Sqrt[2]) - Tanh[x]^3/(2*(2 - Tanh[x]^2)
^2) - Tanh[x]/(4*(2 - Tanh[x]^2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^
(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(
b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^
n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n,
p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 578

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(g^(n - 1)*(b*e - a*f)*
(g*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)
*(p + 1)), x] - Dist[g^n/(b*n*(b*c - a*d)*(p + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f)*(m - n + 1) + (d*(b*e - a*f
)*(m + n*q + 1) - b*n*(c*f - d*e)*(p + 1))*x^n, x], x], x] /; FreeQ[{a, b,
c, d, e, f, g, q}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m - n + 1, 0]
```

Rubi steps

$$\int \frac{1}{\left(\coth^2(x)+\operatorname{csch}^2(x)\right)^3}\,dx = \text{Subst}\left(\int \frac{x^6}{\left(1-x^2\right)\left(2-x^2\right)^3}\,dx,x,\tanh(x)\right)$$

$$= -\frac{\tanh^3(x)}{2\left(2-\tanh^2(x)\right)^2}+\frac{1}{4}\text{Subst}\left(\int \frac{x^2\left(6-2x^2\right)}{\left(1-x^2\right)\left(2-x^2\right)^2}\,dx,x,\tanh(x)\right)$$

$$= -\frac{\tanh^3(x)}{2\left(2-\tanh^2(x)\right)^2}-\frac{\tanh(x)}{4\left(2-\tanh^2(x)\right)}-\frac{1}{8}\text{Subst}\left(\int \frac{-2-6x^2}{\left(1-x^2\right)\left(2-x^2\right)}\,dx,x,\tan$$

$$= -\frac{\tanh^3(x)}{2\left(2-\tanh^2(x)\right)^2}-\frac{\tanh(x)}{4\left(2-\tanh^2(x)\right)}-\frac{7}{4}\text{Subst}\left(\int \frac{1}{2-x^2}\,dx,x,\tanh(x)\right)+\text{S}$$

$$= x-\frac{7\tanh^{-1}\left(\frac{\tanh(x)}{\sqrt{2}}\right)}{4\sqrt{2}}-\frac{\tanh^3(x)}{2\left(2-\tanh^2(x)\right)^2}-\frac{\tanh(x)}{4\left(2-\tanh^2(x)\right)}$$

Mathematica [A] time = 0.21, size = 66, normalized size = 1.22

$$\frac{76x-2\sinh(2x)-3\sinh(4x)+48x\cosh(2x)+4x\cosh(4x)-7\sqrt{2}(\cosh(2x)+3)^2\tanh^{-1}\left(\frac{\tanh(x)}{\sqrt{2}}\right)}{8(\cosh(2x)+3)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Coth[x]^2 + Csch[x]^2)^(-3),x]
```

```
[Out] (76*x + 48*x*Cosh[2*x] - 7*Sqrt[2]*ArcTanh[Tanh[x]/Sqrt[2]]*(3 + Cosh[2*x])
^2 + 4*x*Cosh[4*x] - 2*Sinh[2*x] - 3*Sinh[4*x])/(8*(3 + Cosh[2*x])^2)
```

fricas [B] time = 0.43, size = 715, normalized size = 13.24

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(coth(x)^2+csch(x)^2)^3,x, algorithm="fricas")
```

```
[Out] 1/16*(16*x*cosh(x)^8 + 128*x*cosh(x)*sinh(x)^7 + 16*x*sinh(x)^8 + 8*(24*x +
 17)*cosh(x)^6 + 8*(56*x*cosh(x)^2 + 24*x + 17)*sinh(x)^6 + 16*(56*x*cosh(x
)^3 + 3*(24*x + 17)*cosh(x))*sinh(x)^5 + 152*(4*x + 3)*cosh(x)^4 + 8*(140*x
*cosh(x)^4 + 15*(24*x + 17)*cosh(x)^2 + 76*x + 57)*sinh(x)^4 + 32*(28*x*cos
h(x)^5 + 5*(24*x + 17)*cosh(x)^3 + 19*(4*x + 3)*cosh(x))*sinh(x)^3 + 8*(24*
x + 19)*cosh(x)^2 + 8*(56*x*cosh(x)^6 + 15*(24*x + 17)*cosh(x)^4 + 114*(4*x
 + 3)*cosh(x)^2 + 24*x + 19)*sinh(x)^2 + 7*(sqrt(2)*cosh(x)^8 + 8*sqrt(2)*c
osh(x)*sinh(x)^7 + sqrt(2)*sinh(x)^8 + 4*(7*sqrt(2)*cosh(x)^2 + 3*sqrt(2))*
sinh(x)^6 + 12*sqrt(2)*cosh(x)^6 + 8*(7*sqrt(2)*cosh(x)^3 + 9*sqrt(2)*cosh(
x))*sinh(x)^5 + 2*(35*sqrt(2)*cosh(x)^4 + 90*sqrt(2)*cosh(x)^2 + 19*sqrt(2)
)*sinh(x)^4 + 38*sqrt(2)*cosh(x)^4 + 8*(7*sqrt(2)*cosh(x)^5 + 30*sqrt(2)*co
sh(x)^3 + 19*sqrt(2)*cosh(x))*sinh(x)^3 + 4*(7*sqrt(2)*cosh(x)^6 + 45*sqrt(
2)*cosh(x)^4 + 57*sqrt(2)*cosh(x)^2 + 3*sqrt(2))*sinh(x)^2 + 12*sqrt(2)*cos
h(x)^2 + 8*(sqrt(2)*cosh(x)^7 + 9*sqrt(2)*cosh(x)^5 + 19*sqrt(2)*cosh(x)^3
+ 3*sqrt(2)*cosh(x))*sinh(x) + sqrt(2))*log((3*(2*sqrt(2) + 3)*cosh(x)^2 -
4*(3*sqrt(2) + 4)*cosh(x)*sinh(x) + 3*(2*sqrt(2) + 3)*sinh(x)^2 + 2*sqrt(2)
 + 3)/(cosh(x)^2 + sinh(x)^2 + 3)) + 16*(8*x*cosh(x)^7 + 3*(24*x + 17)*cosh
(x)^5 + 38*(4*x + 3)*cosh(x)^3 + (24*x + 19)*cosh(x))*sinh(x) + 16*x + 24)/
(cosh(x)^8 + 8*cosh(x)*sinh(x)^7 + sinh(x)^8 + 4*(7*cosh(x)^2 + 3)*sinh(x)^
6 + 12*cosh(x)^6 + 8*(7*cosh(x)^3 + 9*cosh(x))*sinh(x)^5 + 2*(35*cosh(x)^4
+ 90*cosh(x)^2 + 19)*sinh(x)^4 + 38*cosh(x)^4 + 8*(7*cosh(x)^5 + 30*cosh(x)
^3 + 19*cosh(x))*sinh(x)^3 + 4*(7*cosh(x)^6 + 45*cosh(x)^4 + 57*cosh(x)^2 +
 3)*sinh(x)^2 + 12*cosh(x)^2 + 8*(cosh(x)^7 + 9*cosh(x)^5 + 19*cosh(x)^3 +
3*cosh(x))*sinh(x) + 1)
```

giac [A] time = 0.13, size = 72, normalized size = 1.33

$$-\frac{7}{16}\,\sqrt{2}\,\log\left(-\frac{2\,\sqrt{2}-e^{(2\,x)}-3}{2\,\sqrt{2}+e^{(2\,x)}+3}\right)+x+\frac{17\,e^{(6\,x)}+57\,e^{(4\,x)}+19\,e^{(2\,x)}+3}{2\,\left(e^{(4\,x)}+6\,e^{(2\,x)}+1\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(coth(x)^2+csch(x)^2)^3,x, algorithm="giac")
```

```
[Out] -7/16*sqrt(2)*log(-(2*sqrt(2) - e^(2*x) - 3)/(2*sqrt(2) + e^(2*x) + 3)) + x
 + 1/2*(17*e^(6*x) + 57*e^(4*x) + 19*e^(2*x) + 3)/(e^(4*x) + 6*e^(2*x) + 1)
^2
```

maple [B] time = 0.23, size = 145, normalized size = 2.69

$$-\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)+\frac{-\frac{\left(\tanh^7\left(\frac{x}{2}\right)\right)}{4}-\frac{5\left(\tanh^5\left(\frac{x}{2}\right)\right)}{4}-\frac{5\left(\tanh^3\left(\frac{x}{2}\right)\right)}{4}-\frac{\tanh\left(\frac{x}{2}\right)}{4}}{\left(\tanh^4\left(\frac{x}{2}\right)+1\right)^2}-\frac{7\sqrt{2}\ \ln\left(\frac{\tanh^2\left(\frac{x}{2}\right)+\sqrt{2}\ \tanh\left(\frac{x}{2}\right)+1}{\tanh^2\left(\frac{x}{2}\right)-\sqrt{2}\ \tanh\left(\frac{x}{2}\right)+1}\right)}{32}+\frac{7\sqrt{2}\ \ln\left(\frac{\mathrm{ta}}{\mathrm{ta}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(coth(x)^2+csch(x)^2)^3,x)
```

```
[Out] -ln(tanh(1/2*x)-1)+2*(-1/8*tanh(1/2*x)^7-5/8*tanh(1/2*x)^5-5/8*tanh(1/2*x)^
3-1/8*tanh(1/2*x))/(tanh(1/2*x)^4+1)^2-7/32*2^(1/2)*ln((tanh(1/2*x)^2+2^(1/
2)*tanh(1/2*x)+1)/(tanh(1/2*x)^2-2^(1/2)*tanh(1/2*x)+1))+7/32*2^(1/2)*ln((t
anh(1/2*x)^2-2^(1/2)*tanh(1/2*x)+1)/(tanh(1/2*x)^2+2^(1/2)*tanh(1/2*x)+1))+
ln(tanh(1/2*x)+1)
```

maxima [B] time = 0.46, size = 84, normalized size = 1.56

$$\frac{7}{16}\sqrt{2}\log\left(-\frac{2\sqrt{2}-e^{(-2x)}-3}{2\sqrt{2}+e^{(-2x)}+3}\right)+x-\frac{19\,e^{(-2x)}+57\,e^{(-4x)}+17\,e^{(-6x)}+3}{2\left(12\,e^{(-2x)}+38\,e^{(-4x)}+12\,e^{(-6x)}+e^{(-8x)}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(coth(x)^2+csch(x)^2)^3,x, algorithm="maxima")
```

```
[Out] 7/16*sqrt(2)*log(-(2*sqrt(2) - e^(-2*x) - 3)/(2*sqrt(2) + e^(-2*x) + 3)) +
x - 1/2*(19*e^(-2*x) + 57*e^(-4*x) + 17*e^(-6*x) + 3)/(12*e^(-2*x) + 38*e^(
-4*x) + 12*e^(-6*x) + e^(-8*x) + 1)
```

mupad [B] time = 1.67, size = 112, normalized size = 2.07

$$x+\frac{136\,e^{2x}+24}{12\,e^{2x}+38\,e^{4x}+12\,e^{6x}+e^{8x}+1}+\frac{7\sqrt{2}\ \ln\left(7\,e^{2x}-\frac{7\sqrt{2}\left(12\,e^{2x}+4\right)}{16}\right)}{16}-\frac{7\sqrt{2}\ \ln\left(7\,e^{2x}+\frac{7\sqrt{2}\left(12\,e^{2x}+4\right)}{16}\right)}{16}+\frac{\frac{17\,e}{2}}{6\,e^{2x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(coth(x)^2 + 1/sinh(x)^2)^3,x)
```

```
[Out] x + (136*exp(2*x) + 24)/(12*exp(2*x) + 38*exp(4*x) + 12*exp(6*x) + exp(8*x)
 + 1) + (7*2^(1/2)*log(7*exp(2*x) - (7*2^(1/2)*(12*exp(2*x) + 4))/16))/16 -
```

```
 (7*2^(1/2)*log(7*exp(2*x) + (7*2^(1/2)*(12*exp(2*x) + 4))/16))/16 + ((17*e
xp(2*x))/2 - 45/2)/(6*exp(2*x) + exp(4*x) + 1)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(\coth^2(x)+\operatorname{csch}^2(x)\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(coth(x)**2+csch(x)**2)**3,x)
```

```
[Out] Integral((coth(x)**2 + csch(x)**2)**(-3), x)
```

3.823 $\int \frac{1}{\coth^2(x)-\mathbf{csch}^2(x)}\,dx$

Optimal. Leaf size=1

$$x$$

```
[Out] x
```

Rubi [A] time = 0.02, antiderivative size = 1, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {4382, 8}

$$x$$

Antiderivative was successfully verified.

```
[In] Int[(Coth[x]^2 - Csch[x]^2)^(-1),x]
```

```
[Out] x
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 4382

```
Int[((a_.) + cot[(d_.) + (e_.)*(x_)]^2*(b_.) + csc[(d_.) + (e_.)*(x_)]^2*(c
_.))^(p_.)*(u_.), x_Symbol] :> Dist[(a + c)^p, Int[ActivateTrig[u], x], x]
/; FreeQ[{a, b, c, d, e, p}, x] && EqQ[b + c, 0]
```

Rubi steps

$$\begin{aligned} \int \frac{1}{\coth^2(x)-\operatorname{csch}^2(x)}\,dx &= \int 1\,dx\\ &= x \end{aligned}$$

Mathematica [A] time = 0.00, size = 1, normalized size = 1.00

$$x$$

Antiderivative was successfully verified.

```
[In] Integrate[(Coth[x]^2 - Csch[x]^2)^(-1),x]
```

```
[Out] x
```

fricas [A] time = 0.39, size = 1, normalized size = 1.00

$$x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(coth(x)^2-csch(x)^2),x, algorithm="fricas")
[Out] x
```

giac [A] time = 0.11, size = 1, normalized size = 1.00

$$x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(coth(x)^2-csch(x)^2),x, algorithm="giac")
[Out] x
```

maple [C] time = 0.11, size = 8, normalized size = 8.00

$$2 \operatorname{arctanh}\left(\tanh\left(\frac{x}{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(coth(x)^2-csch(x)^2),x)
[Out] 2*arctanh(tanh(1/2*x))
```

maxima [A] time = 0.30, size = 1, normalized size = 1.00

$$x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(coth(x)^2-csch(x)^2),x, algorithm="maxima")
[Out] x
```

mupad [B] time = 0.07, size = 1, normalized size = 1.00

$$x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(coth(x)^2 - 1/sinh(x)^2),x)
```

```
[Out] x
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(\coth (x) - \operatorname{csch}(x)) (\coth (x) + \operatorname{csch}(x))}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(coth(x)**2-csch(x)**2),x)
```

```
[Out] Integral(1/((coth(x) - csch(x))*(coth(x) + csch(x))), x)
```

3.824 $\int \frac{1}{\left(\coth^2(x)-\mathbf{csch}^2(x)\right)^2}\,dx$

Optimal. Leaf size=1

$$x$$

```
[Out] x
```

Rubi [A] time = 0.02, antiderivative size = 1, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {4382, 8}

$$x$$

Antiderivative was successfully verified.

```
[In] Int[(Coth[x]^2 - Csch[x]^2)^(-2),x]
```

```
[Out] x
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 4382

```
Int[((a_.) + cot[(d_.) + (e_.)*(x_)]^2*(b_.) + csc[(d_.) + (e_.)*(x_)]^2*(c
_.))^(p_.)*(u_.), x_Symbol] :> Dist[(a + c)^p, Int[ActivateTrig[u], x], x]
/; FreeQ[{a, b, c, d, e, p}, x] && EqQ[b + c, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(\coth^2(x)-\operatorname{csch}^2(x)\right)^2}\,dx &= \int 1\,dx\\ &= x\end{aligned}$$

Mathematica [A] time = 0.00, size = 1, normalized size = 1.00

$$x$$

Antiderivative was successfully verified.

```
[In] Integrate[(Coth[x]^2 - Csch[x]^2)^(-2),x]
```

```
[Out] x
```

fricas [A] time = 0.38, size = 1, normalized size = 1.00

$$x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(coth(x)^2-csch(x)^2)^2,x, algorithm="fricas")
[Out] x
```

giac [A] time = 0.11, size = 1, normalized size = 1.00

$$x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(coth(x)^2-csch(x)^2)^2,x, algorithm="giac")
[Out] x
```

maple [C] time = 0.13, size = 8, normalized size = 8.00

$$2 \operatorname{arctanh}\left(\tanh\left(\frac{x}{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(coth(x)^2-csch(x)^2)^2,x)
[Out] 2*arctanh(tanh(1/2*x))
```

maxima [A] time = 0.32, size = 1, normalized size = 1.00

$$x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(coth(x)^2-csch(x)^2)^2,x, algorithm="maxima")
[Out] x
```

mupad [B] time = 0.02, size = 1, normalized size = 1.00

$$x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(coth(x)^2 - 1/sinh(x)^2)^2,x)
```

[Out] x

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(\coth\left(x\right)-\operatorname{csch}\left(x\right)\right)^{2}\left(\coth\left(x\right)+\operatorname{csch}\left(x\right)\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(coth(x)**2-csch(x)**2)**2,x)
```

```
[Out] Integral(1/((coth(x) - csch(x))**2*(coth(x) + csch(x))**2), x)
```

3.825 $\int \frac{1}{\left(\coth^2(x)-\mathbf{csch}^2(x)\right)^3}\,dx$

Optimal. Leaf size=1

$$x$$

[Out] x

Rubi [A] time = 0.02, antiderivative size = 1, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {4382, 8}

$$x$$

Antiderivative was successfully verified.

```
[In] Int[(Coth[x]^2 - Csch[x]^2)^(-3),x]
```

```
[Out] x
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 4382

```
Int[((a_.) + cot[(d_.) + (e_.)*(x_)]^2*(b_.) + csc[(d_.) + (e_.)*(x_)]^2*(c
_.))^(p_.)*(u_.), x_Symbol] :> Dist[(a + c)^p, Int[ActivateTrig[u], x], x]
/; FreeQ[{a, b, c, d, e, p}, x] && EqQ[b + c, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(\coth^2(x)-\operatorname{csch}^2(x)\right)^3}\,dx &= \int 1\,dx\\ &= x\end{aligned}$$

Mathematica [A] time = 0.00, size = 1, normalized size = 1.00

$$x$$

Antiderivative was successfully verified.

```
[In] Integrate[(Coth[x]^2 - Csch[x]^2)^(-3),x]
```

[Out] x

fricas [A] time = 0.38, size = 1, normalized size = 1.00

$$x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(coth(x)^2-csch(x)^2)^3,x, algorithm="fricas")
[Out] x
```

giac [A] time = 0.11, size = 1, normalized size = 1.00

$$x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(coth(x)^2-csch(x)^2)^3,x, algorithm="giac")
[Out] x
```

maple [C] time = 0.12, size = 8, normalized size = 8.00

$$2\,\mathrm{arctanh}\left(\tanh\left(\frac{x}{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(coth(x)^2-csch(x)^2)^3,x)
[Out] 2*arctanh(tanh(1/2*x))
```

maxima [A] time = 0.32, size = 1, normalized size = 1.00

$$x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(coth(x)^2-csch(x)^2)^3,x, algorithm="maxima")
[Out] x
```

mupad [B] time = 1.54, size = 1, normalized size = 1.00

$$x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(coth(x)^2 - 1/sinh(x)^2)^3,x)
```

```
[Out] x
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(\coth\left(x\right) - \operatorname{csch}\left(x\right)\right)^{3}\left(\coth\left(x\right) + \operatorname{csch}\left(x\right)\right)^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(coth(x)**2-csch(x)**2)**3,x)
[Out] Integral(1/((coth(x) - csch(x))**3*(coth(x) + csch(x))**3), x)
```

3.826 $\int \frac{1}{a+b\sinh(x)+c\sinh^2(x)}\,dx$

Optimal. Leaf size=271

$$\frac{2\sqrt{2}\,c\tan^{-1}\left(\frac{2ic-\tanh\left(\frac{x}{2}\right)\left(\sqrt{4ac-b^2}+ib\right)}{\sqrt{2}\sqrt{-ib\sqrt{4ac-b^2}-2c(a-c)+b^2}}\right)}{\sqrt{4ac-b^2}\sqrt{-ib\sqrt{4ac-b^2}-2c(a-c)+b^2}}-\frac{2\sqrt{2}\,c\tan^{-1}\left(\frac{\tanh\left(\frac{x}{2}\right)\sqrt{4ac-b^2}-ib\tanh\left(\frac{x}{2}\right)+2ic}{\sqrt{2}\sqrt{ib\sqrt{4ac-b^2}-2c(a-c)+b^2}}\right)}{\sqrt{4ac-b^2}\sqrt{ib\sqrt{4ac-b^2}-2c(a-c)+b^2}}$$

```
[Out] 2*c*arctan(1/2*(2*I*c-(I*b+(4*a*c-b^2)^(1/2))*tanh(1/2*x))*2^(1/2)/(b^2-2*(
a-c)*c-I*b*(4*a*c-b^2)^(1/2))^(1/2))*2^(1/2)/(4*a*c-b^2)^(1/2)/(b^2-2*(a-c)
*c-I*b*(4*a*c-b^2)^(1/2))^(1/2)-2*c*arctan(1/2*(2*I*c-I*b*tanh(1/2*x)+(4*a*
c-b^2)^(1/2)*tanh(1/2*x))*2^(1/2)/(b^2-2*(a-c)*c+I*b*(4*a*c-b^2)^(1/2))^(1/
2))*2^(1/2)/(4*a*c-b^2)^(1/2)/(b^2-2*(a-c)*c+I*b*(4*a*c-b^2)^(1/2))^(1/2)
```

Rubi [A] time = 0.92, antiderivative size = 271, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3248, 2660, 618, 204}

$$\frac{2\sqrt{2}\,c\tan^{-1}\left(\frac{2ic-\tanh\left(\frac{x}{2}\right)\left(\sqrt{4ac-b^2}+ib\right)}{\sqrt{2}\sqrt{-ib\sqrt{4ac-b^2}-2c(a-c)+b^2}}\right)}{\sqrt{4ac-b^2}\sqrt{-ib\sqrt{4ac-b^2}-2c(a-c)+b^2}}-\frac{2\sqrt{2}\,c\tan^{-1}\left(\frac{\tanh\left(\frac{x}{2}\right)\sqrt{4ac-b^2}-ib\tanh\left(\frac{x}{2}\right)+2ic}{\sqrt{2}\sqrt{ib\sqrt{4ac-b^2}-2c(a-c)+b^2}}\right)}{\sqrt{4ac-b^2}\sqrt{ib\sqrt{4ac-b^2}-2c(a-c)+b^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sinh[x] + c*Sinh[x]^2)^(-1),x]
```

```
[Out] (-2*Sqrt[2]*c*ArcTan[((2*I)*c - I*b*Tanh[x/2] + Sqrt[-b^2 + 4*a*c]*Tanh[x/2
])/(Sqrt[2]*Sqrt[b^2 - 2*(a - c)*c + I*b*Sqrt[-b^2 + 4*a*c]])])/(Sqrt[-b^2
+ 4*a*c]*Sqrt[b^2 - 2*(a - c)*c + I*b*Sqrt[-b^2 + 4*a*c]]) + (2*Sqrt[2]*c*A
rcTan[((2*I)*c - (I*b + Sqrt[-b^2 + 4*a*c])*Tanh[x/2])/(Sqrt[2]*Sqrt[b^2 -
2*(a - c)*c - I*b*Sqrt[-b^2 + 4*a*c]])])/(Sqrt[-b^2 + 4*a*c]*Sqrt[b^2 - 2*(
a - c)*c - I*b*Sqrt[-b^2 + 4*a*c]])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 3248

```
Int[((a_.) + (b_.)*sin[(d_.) + (e_.)*(x_)]^(n_.) + (c_.)*sin[(d_.) + (e_.)*
(x_)]^(n2_.))^(-1), x_Symbol] :> Module[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c
)/q, Int[1/(b - q + 2*c*Sin[d + e*x]^n), x], x] - Dist[(2*c)/q, Int[1/(b +
q + 2*c*Sin[d + e*x]^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2,
 2*n] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{1}{a+b\sinh(x)+c\sinh^2(x)}\,dx = -\frac{(2c)\int \frac{1}{-ib-\sqrt{-b^2+4ac}-2ic\sinh(x)}\,dx}{\sqrt{-b^2+4ac}} + \frac{(2c)\int \frac{1}{-ib+\sqrt{-b^2+4ac}-2ic\sinh(x)}\,dx}{\sqrt{-b^2+4ac}}$$

$$= -\frac{(4c)\,\mathrm{Subst}\left(\int \frac{1}{-ib-\sqrt{-b^2+4ac}-4icx-\left(-ib-\sqrt{-b^2+4ac}\right)x^2}\,dx, x, \tanh\left(\frac{x}{2}\right)\right)}{\sqrt{-b^2+4ac}} + \frac{(4c)\,\mathrm{Subs}}{}$$

$$= \frac{(8c)\,\mathrm{Subst}\left(\int \frac{1}{-8\left(b^2-2(a-c)c-ib\sqrt{-b^2+4ac}\right)-x^2}\,dx, x, -4ic+2\left(ib+\sqrt{-b^2+4ac}\right)\tan}{\sqrt{-b^2+4ac}}$$

$$= -\frac{2\sqrt{2}\,c\tan^{-1}\left(\frac{2ic-\left(ib-\sqrt{-b^2+4ac}\right)\tanh\left(\frac{x}{2}\right)}{\sqrt{2}\sqrt{b^2-2(a-c)c+ib\sqrt{-b^2+4ac}}}\right)}{\sqrt{-b^2+4ac}\sqrt{b^2-2(a-c)c+ib\sqrt{-b^2+4ac}}} + \frac{2\sqrt{2}\,c\tan^{-1}\left(\frac{2ic-\left(ib+\sqrt{-b}\right.}{\sqrt{2}\sqrt{b^2-2(a-}}\right.}{\sqrt{-b^2+4ac}\sqrt{b^2-2(a-c)c}}$$

Mathematica [A] time = 0.73, size = 217, normalized size = 0.80

$$\frac{2\sqrt{2}\,c\left(\frac{\tan^{-1}\left(\frac{\tanh\left(\frac{x}{2}\right)\left(\sqrt{b^2-4ac}-b\right)+2c}{\sqrt{2b\sqrt{b^2-4ac}+4c(a-c)-2b^2}}\right)}{\sqrt{b\sqrt{b^2-4ac}+2c(a-c)-b^2}}-\frac{\tan^{-1}\left(\frac{2c-\tanh\left(\frac{x}{2}\right)\left(\sqrt{b^2-4ac}+b\right)}{\sqrt{2}\sqrt{-b\sqrt{b^2-4ac}+2c(a-c)-b^2}}\right)}{\sqrt{-b\sqrt{b^2-4ac}+2c(a-c)-b^2}}\right)}{\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sinh[x] + c*Sinh[x]^2)^(-1),x]
```

```
[Out] (2*Sqrt[2]*c*(ArcTan[(2*c + (-b + Sqrt[b^2 - 4*a*c])*Tanh[x/2])/Sqrt[-2*b^2
 + 4*(a - c)*c + 2*b*Sqrt[b^2 - 4*a*c]]]/Sqrt[-b^2 + 2*(a - c)*c + b*Sqrt[b
^2 - 4*a*c]] - ArcTan[(2*c - (b + Sqrt[b^2 - 4*a*c])*Tanh[x/2])/(Sqrt[2]*Sq
rt[-b^2 + 2*(a - c)*c - b*Sqrt[b^2 - 4*a*c]])]/Sqrt[-b^2 + 2*(a - c)*c - b*
Sqrt[b^2 - 4*a*c]]))/Sqrt[b^2 - 4*a*c]
```

fricas [B] time = 0.59, size = 3313, normalized size = 12.23

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sinh(x)+c*sinh(x)^2),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(2)*sqrt((b^2 - 2*a*c + 2*c^2 + (a^2*b^2 + b^4 - 4*a*c^3 + (8*a^2 +
 b^2)*c^2 - 2*(2*a^3 + 3*a*b^2)*c)*sqrt(b^2/(a^4*b^2 + 2*a^2*b^4 + b^6 - 4*
a*c^5 + (16*a^2 + b^2)*c^4 - 12*(2*a^3 + a*b^2)*c^3 + 2*(8*a^4 + 11*a^2*b^2
 + b^4)*c^2 - 4*(a^5 + 3*a^3*b^2 + 2*a*b^4)*c)))/(a^2*b^2 + b^4 - 4*a*c^3 +
 (8*a^2 + b^2)*c^2 - 2*(2*a^3 + 3*a*b^2)*c))*log(4*b*c^2*cosh(x) + 4*b*c^2*
sinh(x) + 2*b^2*c + sqrt(2)*(b^4 - 4*a*b^2*c - (a^2*b^4 + b^6 - 8*a*c^5 + 2
*(12*a^2 + b^2)*c^4 - 6*(4*a^3 + 3*a*b^2)*c^3 + (8*a^4 + 22*a^2*b^2 + 3*b^4
)*c^2 - 2*(3*a^3*b^2 + 4*a*b^4)*c)*sqrt(b^2/(a^4*b^2 + 2*a^2*b^4 + b^6 - 4*
a*c^5 + (16*a^2 + b^2)*c^4 - 12*(2*a^3 + a*b^2)*c^3 + 2*(8*a^4 + 11*a^2*b^2
 + b^4)*c^2 - 4*(a^5 + 3*a^3*b^2 + 2*a*b^4)*c)))*sqrt((b^2 - 2*a*c + 2*c^2
+ (a^2*b^2 + b^4 - 4*a*c^3 + (8*a^2 + b^2)*c^2 - 2*(2*a^3 + 3*a*b^2)*c)*sqr
t(b^2/(a^4*b^2 + 2*a^2*b^4 + b^6 - 4*a*c^5 + (16*a^2 + b^2)*c^4 - 12*(2*a^3
 + a*b^2)*c^3 + 2*(8*a^4 + 11*a^2*b^2 + b^4)*c^2 - 4*(a^5 + 3*a^3*b^2 + 2*a
*b^4)*c)))/(a^2*b^2 + b^4 - 4*a*c^3 + (8*a^2 + b^2)*c^2 - 2*(2*a^3 + 3*a*b^
2)*c)) + 2*(4*a*c^4 - (8*a^2 + b^2)*c^3 + 2*(2*a^3 + 3*a*b^2)*c^2 - (a^2*b^
2 + b^4)*c)*sqrt(b^2/(a^4*b^2 + 2*a^2*b^4 + b^6 - 4*a*c^5 + (16*a^2 + b^2)*
c^4 - 12*(2*a^3 + a*b^2)*c^3 + 2*(8*a^4 + 11*a^2*b^2 + b^4)*c^2 - 4*(a^5 +
3*a^3*b^2 + 2*a*b^4)*c))) - 1/2*sqrt(2)*sqrt((b^2 - 2*a*c + 2*c^2 + (a^2*b^
2 + b^4 - 4*a*c^3 + (8*a^2 + b^2)*c^2 - 2*(2*a^3 + 3*a*b^2)*c)*sqrt(b^2/(a^
4*b^2 + 2*a^2*b^4 + b^6 - 4*a*c^5 + (16*a^2 + b^2)*c^4 - 12*(2*a^3 + a*b^2)
```

c^3 + 2(8*a^4 + 11*a^2*b^2 + b^4)*c^2 - 4*(a^5 + 3*a^3*b^2 + 2*a*b^4)*c))
)/(a^2*b^2 + b^4 - 4*a*c^3 + (8*a^2 + b^2)*c^2 - 2*(2*a^3 + 3*a*b^2)*c))*lo
g(4*b*c^2*cosh(x) + 4*b*c^2*sinh(x) + 2*b^2*c - sqrt(2)*(b^4 - 4*a*b^2*c -
(a^2*b^4 + b^6 - 8*a*c^5 + 2*(12*a^2 + b^2)*c^4 - 6*(4*a^3 + 3*a*b^2)*c^3 +
(8*a^4 + 22*a^2*b^2 + 3*b^4)*c^2 - 2*(3*a^3*b^2 + 4*a*b^4)*c)*sqrt(b^2/(a^
4*b^2 + 2*a^2*b^4 + b^6 - 4*a*c^5 + (16*a^2 + b^2)*c^4 - 12*(2*a^3 + a*b^2)
c^3 + 2(8*a^4 + 11*a^2*b^2 + b^4)*c^2 - 4*(a^5 + 3*a^3*b^2 + 2*a*b^4)*c))
)*sqrt((b^2 - 2*a*c + 2*c^2 + (a^2*b^2 + b^4 - 4*a*c^3 + (8*a^2 + b^2)*c^2
- 2*(2*a^3 + 3*a*b^2)*c)*sqrt(b^2/(a^4*b^2 + 2*a^2*b^4 + b^6 - 4*a*c^5 + (1
6*a^2 + b^2)*c^4 - 12*(2*a^3 + a*b^2)*c^3 + 2*(8*a^4 + 11*a^2*b^2 + b^4)*c^
2 - 4*(a^5 + 3*a^3*b^2 + 2*a*b^4)*c)))/(a^2*b^2 + b^4 - 4*a*c^3 + (8*a^2 +
b^2)*c^2 - 2*(2*a^3 + 3*a*b^2)*c)) + 2*(4*a*c^4 - (8*a^2 + b^2)*c^3 + 2*(2*
a^3 + 3*a*b^2)*c^2 - (a^2*b^2 + b^4)*c)*sqrt(b^2/(a^4*b^2 + 2*a^2*b^4 + b^6
- 4*a*c^5 + (16*a^2 + b^2)*c^4 - 12*(2*a^3 + a*b^2)*c^3 + 2*(8*a^4 + 11*a^
2*b^2 + b^4)*c^2 - 4*(a^5 + 3*a^3*b^2 + 2*a*b^4)*c))) + 1/2*sqrt(2)*sqrt((b
^2 - 2*a*c + 2*c^2 - (a^2*b^2 + b^4 - 4*a*c^3 + (8*a^2 + b^2)*c^2 - 2*(2*a^
3 + 3*a*b^2)*c)*sqrt(b^2/(a^4*b^2 + 2*a^2*b^4 + b^6 - 4*a*c^5 + (16*a^2 + b
^2)*c^4 - 12*(2*a^3 + a*b^2)*c^3 + 2*(8*a^4 + 11*a^2*b^2 + b^4)*c^2 - 4*(a^
5 + 3*a^3*b^2 + 2*a*b^4)*c)))/(a^2*b^2 + b^4 - 4*a*c^3 + (8*a^2 + b^2)*c^2
- 2*(2*a^3 + 3*a*b^2)*c))*log(4*b*c^2*cosh(x) + 4*b*c^2*sinh(x) + 2*b^2*c +
sqrt(2)*(b^4 - 4*a*b^2*c + (a^2*b^4 + b^6 - 8*a*c^5 + 2*(12*a^2 + b^2)*c^4
- 6*(4*a^3 + 3*a*b^2)*c^3 + (8*a^4 + 22*a^2*b^2 + 3*b^4)*c^2 - 2*(3*a^3*b^
2 + 4*a*b^4)*c)*sqrt(b^2/(a^4*b^2 + 2*a^2*b^4 + b^6 - 4*a*c^5 + (16*a^2 + b
^2)*c^4 - 12*(2*a^3 + a*b^2)*c^3 + 2*(8*a^4 + 11*a^2*b^2 + b^4)*c^2 - 4*(a^
5 + 3*a^3*b^2 + 2*a*b^4)*c)))*sqrt((b^2 - 2*a*c + 2*c^2 - (a^2*b^2 + b^4 -
4*a*c^3 + (8*a^2 + b^2)*c^2 - 2*(2*a^3 + 3*a*b^2)*c)*sqrt(b^2/(a^4*b^2 + 2*
a^2*b^4 + b^6 - 4*a*c^5 + (16*a^2 + b^2)*c^4 - 12*(2*a^3 + a*b^2)*c^3 + 2*(
8*a^4 + 11*a^2*b^2 + b^4)*c^2 - 4*(a^5 + 3*a^3*b^2 + 2*a*b^4)*c)))/(a^2*b^2
+ b^4 - 4*a*c^3 + (8*a^2 + b^2)*c^2 - 2*(2*a^3 + 3*a*b^2)*c)) - 2*(4*a*c^4
- (8*a^2 + b^2)*c^3 + 2*(2*a^3 + 3*a*b^2)*c^2 - (a^2*b^2 + b^4)*c)*sqrt(b^
2/(a^4*b^2 + 2*a^2*b^4 + b^6 - 4*a*c^5 + (16*a^2 + b^2)*c^4 - 12*(2*a^3 + a
*b^2)*c^3 + 2*(8*a^4 + 11*a^2*b^2 + b^4)*c^2 - 4*(a^5 + 3*a^3*b^2 + 2*a*b^4
)*c))) - 1/2*sqrt(2)*sqrt((b^2 - 2*a*c + 2*c^2 - (a^2*b^2 + b^4 - 4*a*c^3 +
(8*a^2 + b^2)*c^2 - 2*(2*a^3 + 3*a*b^2)*c)*sqrt(b^2/(a^4*b^2 + 2*a^2*b^4 +
b^6 - 4*a*c^5 + (16*a^2 + b^2)*c^4 - 12*(2*a^3 + a*b^2)*c^3 + 2*(8*a^4 + 1
1*a^2*b^2 + b^4)*c^2 - 4*(a^5 + 3*a^3*b^2 + 2*a*b^4)*c)))/(a^2*b^2 + b^4 -
4*a*c^3 + (8*a^2 + b^2)*c^2 - 2*(2*a^3 + 3*a*b^2)*c))*log(4*b*c^2*cosh(x) +
4*b*c^2*sinh(x) + 2*b^2*c - sqrt(2)*(b^4 - 4*a*b^2*c + (a^2*b^4 + b^6 - 8*
a*c^5 + 2*(12*a^2 + b^2)*c^4 - 6*(4*a^3 + 3*a*b^2)*c^3 + (8*a^4 + 22*a^2*b^
2 + 3*b^4)*c^2 - 2*(3*a^3*b^2 + 4*a*b^4)*c)*sqrt(b^2/(a^4*b^2 + 2*a^2*b^4 +
b^6 - 4*a*c^5 + (16*a^2 + b^2)*c^4 - 12*(2*a^3 + a*b^2)*c^3 + 2*(8*a^4 + 1
1*a^2*b^2 + b^4)*c^2 - 4*(a^5 + 3*a^3*b^2 + 2*a*b^4)*c)))*sqrt((b^2 - 2*a*c
+ 2*c^2 - (a^2*b^2 + b^4 - 4*a*c^3 + (8*a^2 + b^2)*c^2 - 2*(2*a^3 + 3*a*b^
2)*c)*sqrt(b^2/(a^4*b^2 + 2*a^2*b^4 + b^6 - 4*a*c^5 + (16*a^2 + b^2)*c^4 -
12*(2*a^3 + a*b^2)*c^3 + 2*(8*a^4 + 11*a^2*b^2 + b^4)*c^2 - 4*(a^5 + 3*a^3*

```
b^2 + 2*a*b^4)*c)))/(a^2*b^2 + b^4 - 4*a*c^3 + (8*a^2 + b^2)*c^2 - 2*(2*a^3
 + 3*a*b^2)*c)) - 2*(4*a*c^4 - (8*a^2 + b^2)*c^3 + 2*(2*a^3 + 3*a*b^2)*c^2
- (a^2*b^2 + b^4)*c)*sqrt(b^2/(a^4*b^2 + 2*a^2*b^4 + b^6 - 4*a*c^5 + (16*a^
2 + b^2)*c^4 - 12*(2*a^3 + a*b^2)*c^3 + 2*(8*a^4 + 11*a^2*b^2 + b^4)*c^2 -
4*(a^5 + 3*a^3*b^2 + 2*a*b^4)*c)))
```

giac [A] time = 94.70, size = 1, normalized size = 0.00

$$0$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sinh(x)+c*sinh(x)^2),x, algorithm="giac")
[Out] 0
```

maple [C] time = 0.24, size = 74, normalized size = 0.27

$$\sum_{\_R=\mathrm{RootOf}\left(a\_Z^4-2b\_Z^3+(-2a+4c)\_Z^2+2b\_Z+a\right)} \frac{\left(-\_R^2+1\right)\ln\left(\tanh\left(\frac{x}{2}\right)-\_R\right)}{2\_R^3a-3b\_R^2-2\_Ra+4\_Rc+b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sinh(x)+c*sinh(x)^2),x)
[Out] sum((-_R^2+1)/(2*_R^3*a-3*_R^2*b-2*_R*a+4*_R*c+b)*ln(tanh(1/2*x)-_R),_R=Roo
tOf(a*_Z^4-2*b*_Z^3+(-2*a+4*c)*_Z^2+2*b*_Z+a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{c\sinh(x)^2 + b\sinh(x) + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sinh(x)+c*sinh(x)^2),x, algorithm="maxima")
[Out] integrate(1/(c*sinh(x)^2 + b*sinh(x) + a), x)
```

mupad [F(-1)] time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + c*sinh(x)^2 + b*sinh(x)),x)
```

```
[Out] \text{Hanged}
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sinh(x)+c*sinh(x)**2),x)
```

```
[Out] Timed out
```

3.827 $\int \frac{\sinh(x)}{a+b\sinh(x)+c\sinh^2(x)}\,dx$

Optimal. Leaf size=280

$$\frac{\sqrt{2}\left(\frac{b}{\sqrt{4ac-b^2}}+i\right)\tan^{-1}\left(\frac{\tanh\left(\frac{x}{2}\right)\sqrt{4ac-b^2}-ib\tanh\left(\frac{x}{2}\right)+2ic}{\sqrt{2}\sqrt{ib\sqrt{4ac-b^2}-2c(a-c)+b^2}}\right)}{\sqrt{ib\sqrt{4ac-b^2}-2c(a-c)+b^2}}+\frac{\sqrt{2}\left(-\frac{b}{\sqrt{4ac-b^2}}+i\right)\tan^{-1}\left(\frac{2ic-\tanh\left(\frac{x}{2}\right)\left(\sqrt{4ac-b^2}+ib\right)}{\sqrt{2}\sqrt{-ib\sqrt{4ac-b^2}-2c(a-c)+b^2}}\right)}{\sqrt{-ib\sqrt{4ac-b^2}-2c(a-c)+b^2}}$$

```
[Out] arctan(1/2*(2*I*c-(I*b+(4*a*c-b^2)^(1/2))*tanh(1/2*x))*2^(1/2)/(b^2-2*(a-c)
*c-I*b*(4*a*c-b^2)^(1/2))^(1/2))*2^(1/2)*(I-b/(4*a*c-b^2)^(1/2))/(b^2-2*(a-
c)*c-I*b*(4*a*c-b^2)^(1/2))^(1/2)+arctan(1/2*(2*I*c-I*b*tanh(1/2*x)+(4*a*c-
b^2)^(1/2)*tanh(1/2*x))*2^(1/2)/(b^2-2*(a-c)*c+I*b*(4*a*c-b^2)^(1/2))^(1/2)
)*2^(1/2)*(I+b/(4*a*c-b^2)^(1/2))/(b^2-2*(a-c)*c+I*b*(4*a*c-b^2)^(1/2))^(1/
2)
```

Rubi [A] time = 0.72, antiderivative size = 280, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {3256, 2660, 618, 204}

$$\frac{\sqrt{2}\left(\frac{b}{\sqrt{4ac-b^2}}+i\right)\tan^{-1}\left(\frac{\tanh\left(\frac{x}{2}\right)\sqrt{4ac-b^2}-ib\tanh\left(\frac{x}{2}\right)+2ic}{\sqrt{2}\sqrt{ib\sqrt{4ac-b^2}-2c(a-c)+b^2}}\right)}{\sqrt{ib\sqrt{4ac-b^2}-2c(a-c)+b^2}}+\frac{\sqrt{2}\left(-\frac{b}{\sqrt{4ac-b^2}}+i\right)\tan^{-1}\left(\frac{2ic-\tanh\left(\frac{x}{2}\right)\left(\sqrt{4ac-b^2}+ib\right)}{\sqrt{2}\sqrt{-ib\sqrt{4ac-b^2}-2c(a-c)+b^2}}\right)}{\sqrt{-ib\sqrt{4ac-b^2}-2c(a-c)+b^2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[x]/(a + b*Sinh[x] + c*Sinh[x]^2),x]
```

```
[Out] (Sqrt[2]*(I + b/Sqrt[-b^2 + 4*a*c])*ArcTan[((2*I)*c - I*b*Tanh[x/2] + Sqrt[
-b^2 + 4*a*c]*Tanh[x/2])/(Sqrt[2]*Sqrt[b^2 - 2*(a - c)*c + I*b*Sqrt[-b^2 +
4*a*c]])])/Sqrt[b^2 - 2*(a - c)*c + I*b*Sqrt[-b^2 + 4*a*c]] + (Sqrt[2]*(I -
 b/Sqrt[-b^2 + 4*a*c])*ArcTan[((2*I)*c - (I*b + Sqrt[-b^2 + 4*a*c])*Tanh[x/
2])/(Sqrt[2]*Sqrt[b^2 - 2*(a - c)*c - I*b*Sqrt[-b^2 + 4*a*c]])])/Sqrt[b^2 -
 2*(a - c)*c - I*b*Sqrt[-b^2 + 4*a*c]]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 3256

```
Int[sin[(d_.) + (e_.)*(x_)]^(m_.)*((a_.) + (b_.)*sin[(d_.) + (e_.)*(x_)]^(n
_.) + (c_.)*sin[(d_.) + (e_.)*(x_)]^(n2_.))^(p_), x_Symbol] :> Int[ExpandTr
ig[sin[d + e*x]^m*(a + b*sin[d + e*x]^n + c*sin[d + e*x]^(2*n))^p, x], x] /
; FreeQ[{a, b, c, d, e}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && Integ
ersQ[m, n, p]
```

Rubi steps

$$\int \frac{\sinh(x)}{a+b\sinh(x)+c\sinh^2(x)}\,dx = -\left(i\int\left(\frac{1+\frac{ib}{\sqrt{-b^2+4ac}}}{-ib-\sqrt{-b^2+4ac}-2ic\sinh(x)}+\frac{1-\frac{ib}{\sqrt{-b^2+4ac}}}{-ib+\sqrt{-b^2+4ac}-2ic\sinh(x)}\right)\right.$$

$$= -\left(\left(i-\frac{b}{\sqrt{-b^2+4ac}}\right)\int\frac{1}{-ib-\sqrt{-b^2+4ac}-2ic\sinh(x)}\,dx\right)-\left(i+\frac{b}{\sqrt{-b^2+4}}\right.$$

$$= -\left(\left(2\left(i-\frac{b}{\sqrt{-b^2+4ac}}\right)\right)\text{Subst}\left(\int\frac{1}{-ib-\sqrt{-b^2+4ac}-4icx-\left(-ib-\sqrt{-b^2}\right.}\right.\right.$$

$$= \left(4\left(i-\frac{b}{\sqrt{-b^2+4ac}}\right)\right)\text{Subst}\left(\int\frac{1}{-8\left(b^2-2(a-c)c-ib\sqrt{-b^2+4ac}\right)-x^2}\,dx,\right.$$

$$= \frac{\sqrt{2}\left(i+\frac{b}{\sqrt{-b^2+4ac}}\right)\tan^{-1}\left(\frac{2ic-\left(ib-\sqrt{-b^2+4ac}\right)\tanh\left(\frac{x}{2}\right)}{\sqrt{2}\sqrt{b^2-2(a-c)c+ib\sqrt{-b^2+4ac}}}\right)}{\sqrt{b^2-2(a-c)c+ib\sqrt{-b^2+4ac}}}+\frac{\sqrt{2}\left(i-\frac{b}{\sqrt{-b^2+4ac}}\right)\tan^{-1}}{\sqrt{b^2-2(a-c}}$$

Mathematica [A] time = 0.59, size = 244, normalized size = 0.87

$$\frac{\sqrt{2}\left(\frac{\left(\sqrt{b^2-4ac}-b\right)\tan^{-1}\left(\frac{\tanh\left(\frac{x}{2}\right)\left(\sqrt{b^2-4ac}-b\right)+2c}{\sqrt{2b\sqrt{b^2-4ac}+4c(a-c)-2b^2}}\right)}{\sqrt{b\sqrt{b^2-4ac}+2c(a-c)-b^2}}+\frac{\left(\sqrt{b^2-4ac}+b\right)\tan^{-1}\left(\frac{2c-\tanh\left(\frac{x}{2}\right)\left(\sqrt{b^2-4ac}+b\right)}{\sqrt{2}\sqrt{-b\sqrt{b^2-4ac}+2c(a-c)-b^2}}\right)}{\sqrt{-b\sqrt{b^2-4ac}+2c(a-c)-b^2}}\right)}{\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[x]/(a + b*Sinh[x] + c*Sinh[x]^2),x]
```

```
[Out] (Sqrt[2]*(((-b + Sqrt[b^2 - 4*a*c])*ArcTan[(2*c + (-b + Sqrt[b^2 - 4*a*c])*
Tanh[x/2])/Sqrt[-2*b^2 + 4*(a - c)*c + 2*b*Sqrt[b^2 - 4*a*c]]])/Sqrt[-b^2 +
 2*(a - c)*c + b*Sqrt[b^2 - 4*a*c]] + ((b + Sqrt[b^2 - 4*a*c])*ArcTan[(2*c
- (b + Sqrt[b^2 - 4*a*c])*Tanh[x/2])/(Sqrt[2]*Sqrt[-b^2 + 2*(a - c)*c - b*S
qrt[b^2 - 4*a*c]])])/Sqrt[-b^2 + 2*(a - c)*c - b*Sqrt[b^2 - 4*a*c]]))/Sqrt[
b^2 - 4*a*c]
```

fricas [B] time = 0.59, size = 3309, normalized size = 11.82

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)/(a+b*sinh(x)+c*sinh(x)^2),x, algorithm="fricas")
```

```
[Out] -1/2*sqrt(2)*sqrt((2*a^2 + b^2 - 2*a*c + (a^2*b^2 + b^4 - 4*a*c^3 + (8*a^2
+ b^2)*c^2 - 2*(2*a^3 + 3*a*b^2)*c)*sqrt(b^2/(a^4*b^2 + 2*a^2*b^4 + b^6 - 4
*a*c^5 + (16*a^2 + b^2)*c^4 - 12*(2*a^3 + a*b^2)*c^3 + 2*(8*a^4 + 11*a^2*b^
2 + b^4)*c^2 - 4*(a^5 + 3*a^3*b^2 + 2*a*b^4)*c)))/(a^2*b^2 + b^4 - 4*a*c^3
+ (8*a^2 + b^2)*c^2 - 2*(2*a^3 + 3*a*b^2)*c))*log(4*a*b*c*cosh(x) + 4*a*b*c
*sinh(x) + 2*a*b^2 + sqrt(2)*(a*b^3 + 4*a*b*c^2 - (4*a^2*b + b^3)*c + (a^3*
b^3 + a*b^5 - 4*a*b*c^4 + (4*a^2*b + b^3)*c^3 + (4*a^3*b - 5*a*b^3)*c^2 - (
4*a^4*b + 5*a^2*b^3 - b^5)*c)*sqrt(b^2/(a^4*b^2 + 2*a^2*b^4 + b^6 - 4*a*c^5
 + (16*a^2 + b^2)*c^4 - 12*(2*a^3 + a*b^2)*c^3 + 2*(8*a^4 + 11*a^2*b^2 + b^
4)*c^2 - 4*(a^5 + 3*a^3*b^2 + 2*a*b^4)*c)))*sqrt((2*a^2 + b^2 - 2*a*c + (a^
2*b^2 + b^4 - 4*a*c^3 + (8*a^2 + b^2)*c^2 - 2*(2*a^3 + 3*a*b^2)*c)*sqrt(b^2
/(a^4*b^2 + 2*a^2*b^4 + b^6 - 4*a*c^5 + (16*a^2 + b^2)*c^4 - 12*(2*a^3 + a*
b^2)*c^3 + 2*(8*a^4 + 11*a^2*b^2 + b^4)*c^2 - 4*(a^5 + 3*a^3*b^2 + 2*a*b^4)
*c)))/(a^2*b^2 + b^4 - 4*a*c^3 + (8*a^2 + b^2)*c^2 - 2*(2*a^3 + 3*a*b^2)*c)
) + 2*(a^3*b^2 + a*b^4 - 4*a^2*c^3 + (8*a^3 + a*b^2)*c^2 - 2*(2*a^4 + 3*a^2
*b^2)*c)*sqrt(b^2/(a^4*b^2 + 2*a^2*b^4 + b^6 - 4*a*c^5 + (16*a^2 + b^2)*c^4
 - 12*(2*a^3 + a*b^2)*c^3 + 2*(8*a^4 + 11*a^2*b^2 + b^4)*c^2 - 4*(a^5 + 3*a
^3*b^2 + 2*a*b^4)*c))) + 1/2*sqrt(2)*sqrt((2*a^2 + b^2 - 2*a*c + (a^2*b^2 +
 b^4 - 4*a*c^3 + (8*a^2 + b^2)*c^2 - 2*(2*a^3 + 3*a*b^2)*c)*sqrt(b^2/(a^4*b
```

```
^2 + 2*a^2*b^4 + b^6 - 4*a*c^5 + (16*a^2 + b^2)*c^4 - 12*(2*a^3 + a*b^2)*c^
3 + 2*(8*a^4 + 11*a^2*b^2 + b^4)*c^2 - 4*(a^5 + 3*a^3*b^2 + 2*a*b^4)*c)))/(
a^2*b^2 + b^4 - 4*a*c^3 + (8*a^2 + b^2)*c^2 - 2*(2*a^3 + 3*a*b^2)*c))*log(4
*a*b*c*cosh(x) + 4*a*b*c*sinh(x) + 2*a*b^2 - sqrt(2)*(a*b^3 + 4*a*b*c^2 - (
4*a^2*b + b^3)*c + (a^3*b^3 + a*b^5 - 4*a*b*c^4 + (4*a^2*b + b^3)*c^3 + (4*
a^3*b - 5*a*b^3)*c^2 - (4*a^4*b + 5*a^2*b^3 - b^5)*c)*sqrt(b^2/(a^4*b^2 + 2
*a^2*b^4 + b^6 - 4*a*c^5 + (16*a^2 + b^2)*c^4 - 12*(2*a^3 + a*b^2)*c^3 + 2*
(8*a^4 + 11*a^2*b^2 + b^4)*c^2 - 4*(a^5 + 3*a^3*b^2 + 2*a*b^4)*c)))*sqrt((2
*a^2 + b^2 - 2*a*c + (a^2*b^2 + b^4 - 4*a*c^3 + (8*a^2 + b^2)*c^2 - 2*(2*a^
3 + 3*a*b^2)*c)*sqrt(b^2/(a^4*b^2 + 2*a^2*b^4 + b^6 - 4*a*c^5 + (16*a^2 + b
^2)*c^4 - 12*(2*a^3 + a*b^2)*c^3 + 2*(8*a^4 + 11*a^2*b^2 + b^4)*c^2 - 4*(a^
5 + 3*a^3*b^2 + 2*a*b^4)*c)))/(a^2*b^2 + b^4 - 4*a*c^3 + (8*a^2 + b^2)*c^2
- 2*(2*a^3 + 3*a*b^2)*c)) + 2*(a^3*b^2 + a*b^4 - 4*a^2*c^3 + (8*a^3 + a*b^2
)*c^2 - 2*(2*a^4 + 3*a^2*b^2)*c)*sqrt(b^2/(a^4*b^2 + 2*a^2*b^4 + b^6 - 4*a*
c^5 + (16*a^2 + b^2)*c^4 - 12*(2*a^3 + a*b^2)*c^3 + 2*(8*a^4 + 11*a^2*b^2 +
 b^4)*c^2 - 4*(a^5 + 3*a^3*b^2 + 2*a*b^4)*c))) - 1/2*sqrt(2)*sqrt((2*a^2 +
b^2 - 2*a*c - (a^2*b^2 + b^4 - 4*a*c^3 + (8*a^2 + b^2)*c^2 - 2*(2*a^3 + 3*a
*b^2)*c)*sqrt(b^2/(a^4*b^2 + 2*a^2*b^4 + b^6 - 4*a*c^5 + (16*a^2 + b^2)*c^4
 - 12*(2*a^3 + a*b^2)*c^3 + 2*(8*a^4 + 11*a^2*b^2 + b^4)*c^2 - 4*(a^5 + 3*a
^3*b^2 + 2*a*b^4)*c)))/(a^2*b^2 + b^4 - 4*a*c^3 + (8*a^2 + b^2)*c^2 - 2*(2*
a^3 + 3*a*b^2)*c))*log(4*a*b*c*cosh(x) + 4*a*b*c*sinh(x) + 2*a*b^2 + sqrt(2
)*(a*b^3 + 4*a*b*c^2 - (4*a^2*b + b^3)*c - (a^3*b^3 + a*b^5 - 4*a*b*c^4 + (
4*a^2*b + b^3)*c^3 + (4*a^3*b - 5*a*b^3)*c^2 - (4*a^4*b + 5*a^2*b^3 - b^5)*
c)*sqrt(b^2/(a^4*b^2 + 2*a^2*b^4 + b^6 - 4*a*c^5 + (16*a^2 + b^2)*c^4 - 12*
(2*a^3 + a*b^2)*c^3 + 2*(8*a^4 + 11*a^2*b^2 + b^4)*c^2 - 4*(a^5 + 3*a^3*b^2
 + 2*a*b^4)*c)))*sqrt((2*a^2 + b^2 - 2*a*c - (a^2*b^2 + b^4 - 4*a*c^3 + (8*
a^2 + b^2)*c^2 - 2*(2*a^3 + 3*a*b^2)*c)*sqrt(b^2/(a^4*b^2 + 2*a^2*b^4 + b^6
 - 4*a*c^5 + (16*a^2 + b^2)*c^4 - 12*(2*a^3 + a*b^2)*c^3 + 2*(8*a^4 + 11*a^
2*b^2 + b^4)*c^2 - 4*(a^5 + 3*a^3*b^2 + 2*a*b^4)*c)))/(a^2*b^2 + b^4 - 4*a*
c^3 + (8*a^2 + b^2)*c^2 - 2*(2*a^3 + 3*a*b^2)*c)) - 2*(a^3*b^2 + a*b^4 - 4*
a^2*c^3 + (8*a^3 + a*b^2)*c^2 - 2*(2*a^4 + 3*a^2*b^2)*c)*sqrt(b^2/(a^4*b^2
+ 2*a^2*b^4 + b^6 - 4*a*c^5 + (16*a^2 + b^2)*c^4 - 12*(2*a^3 + a*b^2)*c^3 +
 2*(8*a^4 + 11*a^2*b^2 + b^4)*c^2 - 4*(a^5 + 3*a^3*b^2 + 2*a*b^4)*c))) + 1/
2*sqrt(2)*sqrt((2*a^2 + b^2 - 2*a*c - (a^2*b^2 + b^4 - 4*a*c^3 + (8*a^2 + b
^2)*c^2 - 2*(2*a^3 + 3*a*b^2)*c)*sqrt(b^2/(a^4*b^2 + 2*a^2*b^4 + b^6 - 4*a*
c^5 + (16*a^2 + b^2)*c^4 - 12*(2*a^3 + a*b^2)*c^3 + 2*(8*a^4 + 11*a^2*b^2 +
 b^4)*c^2 - 4*(a^5 + 3*a^3*b^2 + 2*a*b^4)*c)))/(a^2*b^2 + b^4 - 4*a*c^3 + (
8*a^2 + b^2)*c^2 - 2*(2*a^3 + 3*a*b^2)*c))*log(4*a*b*c*cosh(x) + 4*a*b*c*si
nh(x) + 2*a*b^2 - sqrt(2)*(a*b^3 + 4*a*b*c^2 - (4*a^2*b + b^3)*c - (a^3*b^3
 + a*b^5 - 4*a*b*c^4 + (4*a^2*b + b^3)*c^3 + (4*a^3*b - 5*a*b^3)*c^2 - (4*a
^4*b + 5*a^2*b^3 - b^5)*c)*sqrt(b^2/(a^4*b^2 + 2*a^2*b^4 + b^6 - 4*a*c^5 +
(16*a^2 + b^2)*c^4 - 12*(2*a^3 + a*b^2)*c^3 + 2*(8*a^4 + 11*a^2*b^2 + b^4)*
c^2 - 4*(a^5 + 3*a^3*b^2 + 2*a*b^4)*c)))*sqrt((2*a^2 + b^2 - 2*a*c - (a^2*b
^2 + b^4 - 4*a*c^3 + (8*a^2 + b^2)*c^2 - 2*(2*a^3 + 3*a*b^2)*c)*sqrt(b^2/(a
^4*b^2 + 2*a^2*b^4 + b^6 - 4*a*c^5 + (16*a^2 + b^2)*c^4 - 12*(2*a^3 + a*b^2
```

```
)*c^3 + 2*(8*a^4 + 11*a^2*b^2 + b^4)*c^2 - 4*(a^5 + 3*a^3*b^2 + 2*a*b^4)*c)
))/(a^2*b^2 + b^4 - 4*a*c^3 + (8*a^2 + b^2)*c^2 - 2*(2*a^3 + 3*a*b^2)*c)) -
 2*(a^3*b^2 + a*b^4 - 4*a^2*c^3 + (8*a^3 + a*b^2)*c^2 - 2*(2*a^4 + 3*a^2*b^
2)*c)*sqrt(b^2/(a^4*b^2 + 2*a^2*b^4 + b^6 - 4*a*c^5 + (16*a^2 + b^2)*c^4 -
12*(2*a^3 + a*b^2)*c^3 + 2*(8*a^4 + 11*a^2*b^2 + b^4)*c^2 - 4*(a^5 + 3*a^3*
b^2 + 2*a*b^4)*c)))
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)/(a+b*sinh(x)+c*sinh(x)^2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [C] time = 0.24, size = 70, normalized size = 0.25

$$2\left(\sum_{\_R=\mathrm{RootOf}\left(a\_Z^4-2b\_Z^3+(-2a+4c)\_Z^2+2b\_Z+a\right)}\frac{\_R\ln\left(\tanh\left(\frac{x}{2}\right)-\_R\right)}{2\_R^3a-3b\_R^2-2\_Ra+4\_Rc+b}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)/(a+b*sinh(x)+c*sinh(x)^2),x)
```

```
[Out] 2*sum(_R/(2*_R^3*a-3*_R^2*b-2*_R*a+4*_R*c+b)*ln(tanh(1/2*x)-_R),_R=RootOf(a
*_Z^4-2*b*_Z^3+(-2*a+4*c)*_Z^2+2*b*_Z+a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh(x)}{c\sinh(x)^2+b\sinh(x)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)/(a+b*sinh(x)+c*sinh(x)^2),x, algorithm="maxima")
```

```
[Out] integrate(sinh(x)/(c*sinh(x)^2 + b*sinh(x) + a), x)
```

mupad [F(-1)] time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)/(a + c*sinh(x)^2 + b*sinh(x)),x)
```

```
[Out] \text{Hanged}
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)/(a+b*sinh(x)+c*sinh(x)**2),x)
```

```
[Out] Timed out
```

3.828 $\int \frac{\sinh^2(x)}{a+b\sinh(x)+c\sinh^2(x)}\,dx$

Optimal. Leaf size=309

$$-\frac{\sqrt{2}\left(\frac{b^2-2ac}{\sqrt{4ac-b^2}}+ib\right)\tan^{-1}\left(\frac{2ic-\tanh\left(\frac{x}{2}\right)\left(-\sqrt{4ac-b^2}+ib\right)}{\sqrt{2}\sqrt{ib\sqrt{4ac-b^2}-2c(a-c)+b^2}}\right)}{c\sqrt{ib\sqrt{4ac-b^2}-2c(a-c)+b^2}}-\frac{\sqrt{2}\left(-\frac{b^2-2ac}{\sqrt{4ac-b^2}}+ib\right)\tan^{-1}\left(\frac{2ic-\tanh\left(\frac{x}{2}\right)\left(\sqrt{4ac-b^2}+ib\right)}{\sqrt{2}\sqrt{-ib\sqrt{4ac-b^2}-2c(a-c)+b^2}}\right)}{c\sqrt{-ib\sqrt{4ac-b^2}-2c(a-c)+b^2}}+\frac{x}{c}$$

```
[Out] x/c-arctan(1/2*(2*I*c-(I*b+(4*a*c-b^2)^(1/2))*tanh(1/2*x))*2^(1/2)/(b^2-2*(
a-c)*c-I*b*(4*a*c-b^2)^(1/2))^(1/2))*2^(1/2)*(I*b+(2*a*c-b^2)/(4*a*c-b^2)^(
1/2))/c/(b^2-2*(a-c)*c-I*b*(4*a*c-b^2)^(1/2))^(1/2)-arctan(1/2*(2*I*c-(I*b-
(4*a*c-b^2)^(1/2))*tanh(1/2*x))*2^(1/2)/(b^2-2*(a-c)*c+I*b*(4*a*c-b^2)^(1/2
))^(1/2))*2^(1/2)*(I*b+(-2*a*c+b^2)/(4*a*c-b^2)^(1/2))/c/(b^2-2*(a-c)*c+I*b
*(4*a*c-b^2)^(1/2))^(1/2)
```

Rubi [A] time = 1.08, antiderivative size = 309, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {3256, 3292, 2660, 618, 204}

$$-\frac{\sqrt{2}\left(\frac{b^2-2ac}{\sqrt{4ac-b^2}}+ib\right)\tan^{-1}\left(\frac{2ic-\tanh\left(\frac{x}{2}\right)\left(-\sqrt{4ac-b^2}+ib\right)}{\sqrt{2}\sqrt{ib\sqrt{4ac-b^2}-2c(a-c)+b^2}}\right)}{c\sqrt{ib\sqrt{4ac-b^2}-2c(a-c)+b^2}}-\frac{\sqrt{2}\left(-\frac{b^2-2ac}{\sqrt{4ac-b^2}}+ib\right)\tan^{-1}\left(\frac{2ic-\tanh\left(\frac{x}{2}\right)\left(\sqrt{4ac-b^2}+ib\right)}{\sqrt{2}\sqrt{-ib\sqrt{4ac-b^2}-2c(a-c)+b^2}}\right)}{c\sqrt{-ib\sqrt{4ac-b^2}-2c(a-c)+b^2}}+\frac{x}{c}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[x]^2/(a + b*Sinh[x] + c*Sinh[x]^2),x]
```

```
[Out] x/c - (Sqrt[2]*(I*b + (b^2 - 2*a*c)/Sqrt[-b^2 + 4*a*c])*ArcTan[((2*I)*c - (
I*b - Sqrt[-b^2 + 4*a*c])*Tanh[x/2])/(Sqrt[2]*Sqrt[b^2 - 2*(a - c)*c + I*b*
Sqrt[-b^2 + 4*a*c]])])/(c*Sqrt[b^2 - 2*(a - c)*c + I*b*Sqrt[-b^2 + 4*a*c]])
 - (Sqrt[2]*(I*b - (b^2 - 2*a*c)/Sqrt[-b^2 + 4*a*c])*ArcTan[((2*I)*c - (I*b
 + Sqrt[-b^2 + 4*a*c])*Tanh[x/2])/(Sqrt[2]*Sqrt[b^2 - 2*(a - c)*c - I*b*Sqr
t[-b^2 + 4*a*c]])])/(c*Sqrt[b^2 - 2*(a - c)*c - I*b*Sqrt[-b^2 + 4*a*c]])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 3256

```
Int[sin[(d_.) + (e_.)*(x_)]^(m_.)*((a_.) + (b_.)*sin[(d_.) + (e_.)*(x_)]^(n
_.) + (c_.)*sin[(d_.) + (e_.)*(x_)]^(n2_.))^(p_), x_Symbol] :> Int[ExpandTr
ig[sin[d + e*x]^m*(a + b*sin[d + e*x]^n + c*sin[d + e*x]^(2*n))^p, x], x] /
; FreeQ[{a, b, c, d, e}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && Integ
ersQ[m, n, p]
```

Rule 3292

```
Int[((A_) + (B_.)*sin[(d_.) + (e_.)*(x_)])/((a_.) + (b_.)*sin[(d_.) + (e_.)
*(x_)] + (c_.)*sin[(d_.) + (e_.)*(x_)]^2), x_Symbol] :> Module[{q = Rt[b^2
- 4*a*c, 2]}, Dist[B + (b*B - 2*A*c)/q, Int[1/(b + q + 2*c*Sin[d + e*x]), x
], x] + Dist[B - (b*B - 2*A*c)/q, Int[1/(b - q + 2*c*Sin[d + e*x]), x], x]]
 /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{\sinh^2(x)}{a+b\sinh(x)+c\sinh^2(x)}\,dx = -\int \left(-\frac{1}{c}+\frac{-a-b\sinh(x)}{c\left(-a-b\sinh(x)-c\sinh^2(x)\right)}\right)dx$$

$$= \frac{x}{c} - \frac{\int \frac{-a-b\sinh(x)}{-a-b\sinh(x)-c\sinh^2(x)}\,dx}{c}$$

$$= \frac{x}{c} - \frac{\left(ib-\frac{b^2-2ac}{\sqrt{-b^2+4ac}}\right)\int \frac{1}{ib+\sqrt{-b^2+4ac}+2ic\sinh(x)}\,dx}{c} - \frac{\left(ib+\frac{b^2-2ac}{\sqrt{-b^2+4ac}}\right)\int \frac{}{ib-\sqrt{-b^2+4}}}{c}$$

$$= \frac{x}{c} - \frac{\left(2\left(ib-\frac{b^2-2ac}{\sqrt{-b^2+4ac}}\right)\right)\text{Subst}\left(\int \frac{1}{ib+\sqrt{-b^2+4ac}+4icx-\left(ib+\sqrt{-b^2+4ac}\right)x^2}\,dx, x, \tanh\right.}{c}$$

$$= \frac{x}{c} + \frac{\left(4\left(ib-\frac{b^2-2ac}{\sqrt{-b^2+4ac}}\right)\right)\text{Subst}\left(\int \frac{1}{-8\left(b^2-2(a-c)c-ib\sqrt{-b^2+4ac}\right)-x^2}\,dx, x, 4ic+2\left(-\right.\right.}{c}$$

$$= \frac{x}{c} - \frac{\sqrt{2}\left(ib+\frac{b^2-2ac}{\sqrt{-b^2+4ac}}\right)\tan^{-1}\left(\frac{2ic-\left(ib-\sqrt{-b^2+4ac}\right)\tanh\left(\frac{x}{2}\right)}{\sqrt{2}\sqrt{b^2-2(a-c)c+ib\sqrt{-b^2+4ac}}}\right)}{c\sqrt{b^2-2(a-c)c+ib\sqrt{-b^2+4ac}}} - \frac{\sqrt{2}\left(ib-\frac{b^2-2a}{\sqrt{-b^2+4}}\right.}{c\sqrt{b^2-}}$$

Mathematica [A] time = 0.61, size = 283, normalized size = 0.92

$$\frac{-\frac{\sqrt{2}\left(b\sqrt{b^2-4ac}+2ac-b^2\right)\tan^{-1}\left(\frac{\tanh\left(\frac{x}{2}\right)\left(\sqrt{b^2-4ac}-b\right)+2c}{\sqrt{2b\sqrt{b^2-4ac}+4c(a-c)-2b^2}}\right)}{\sqrt{b^2-4ac}\sqrt{b\sqrt{b^2-4ac}+2c(a-c)-b^2}} - \frac{\sqrt{2}\left(b\sqrt{b^2-4ac}-2ac+b^2\right)\tan^{-1}\left(\frac{2c-\tanh\left(\frac{x}{2}\right)\left(\sqrt{b^2-4ac}+b\right)}{\sqrt{2}\sqrt{-b\sqrt{b^2-4ac}+2c(a-c)-b^2}}\right)}{\sqrt{b^2-4ac}\sqrt{-b\sqrt{b^2-4ac}+2c(a-c)-b^2}}+x}{c}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[x]^2/(a + b*Sinh[x] + c*Sinh[x]^2),x]
```

```
[Out] (x - (Sqrt[2]*(-b^2 + 2*a*c + b*Sqrt[b^2 - 4*a*c])*ArcTan[(2*c + (-b + Sqrt
[b^2 - 4*a*c])*Tanh[x/2])/Sqrt[-2*b^2 + 4*(a - c)*c + 2*b*Sqrt[b^2 - 4*a*c]
]])/(Sqrt[b^2 - 4*a*c]*Sqrt[-b^2 + 2*(a - c)*c + b*Sqrt[b^2 - 4*a*c]]) - (S
qrt[2]*(b^2 - 2*a*c + b*Sqrt[b^2 - 4*a*c])*ArcTan[(2*c - (b + Sqrt[b^2 - 4*
a*c])*Tanh[x/2])/(Sqrt[2]*Sqrt[-b^2 + 2*(a - c)*c - b*Sqrt[b^2 - 4*a*c]])])
/(Sqrt[b^2 - 4*a*c]*Sqrt[-b^2 + 2*(a - c)*c - b*Sqrt[b^2 - 4*a*c]]))/c
```

fricas [B] time = 0.84, size = 4943, normalized size = 16.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^2/(a+b*sinh(x)+c*sinh(x)^2),x, algorithm="fricas")
```

```
[Out] 1/2*(sqrt(2)*c*sqrt(-(a^2*b^2 + b^4 + 2*a^2*c^2 - 2*(a^3 + 2*a*b^2)*c + (4*
a*c^5 - (8*a^2 + b^2)*c^4 + 2*(2*a^3 + 3*a*b^2)*c^3 - (a^2*b^2 + b^4)*c^2)*
sqrt(-(a^4*b^2 + 2*a^2*b^4 + b^6 + 4*a^2*b^2*c^2 - 4*(a^3*b^2 + a*b^4)*c)/(
4*a*c^9 - (16*a^2 + b^2)*c^8 + 12*(2*a^3 + a*b^2)*c^7 - 2*(8*a^4 + 11*a^2*b
^2 + b^4)*c^6 + 4*(a^5 + 3*a^3*b^2 + 2*a*b^4)*c^5 - (a^4*b^2 + 2*a^2*b^4 +
b^6)*c^4)))/(4*a*c^5 - (8*a^2 + b^2)*c^4 + 2*(2*a^3 + 3*a*b^2)*c^3 - (a^2*b
^2 + b^4)*c^2))*log(-2*a^4*b^2 - 2*a^2*b^4 + 4*a^3*b^2*c + sqrt(2)*(8*a^2*b
^2*c^3 - 2*(2*a^3*b^2 + 3*a*b^4)*c^2 + (a^2*b^4 + b^6)*c + (8*a^2*c^7 - 6*(
4*a^3 + a*b^2)*c^6 + (24*a^4 + 22*a^2*b^2 + b^4)*c^5 - 2*(4*a^5 + 9*a^3*b^2
 + 4*a*b^4)*c^4 + (2*a^4*b^2 + 3*a^2*b^4 + b^6)*c^3)*sqrt(-(a^4*b^2 + 2*a^2
*b^4 + b^6 + 4*a^2*b^2*c^2 - 4*(a^3*b^2 + a*b^4)*c)/(4*a*c^9 - (16*a^2 + b^
2)*c^8 + 12*(2*a^3 + a*b^2)*c^7 - 2*(8*a^4 + 11*a^2*b^2 + b^4)*c^6 + 4*(a^5
 + 3*a^3*b^2 + 2*a*b^4)*c^5 - (a^4*b^2 + 2*a^2*b^4 + b^6)*c^4)))*sqrt(-(a^2
*b^2 + b^4 + 2*a^2*c^2 - 2*(a^3 + 2*a*b^2)*c + (4*a*c^5 - (8*a^2 + b^2)*c^4
 + 2*(2*a^3 + 3*a*b^2)*c^3 - (a^2*b^2 + b^4)*c^2)*sqrt(-(a^4*b^2 + 2*a^2*b^
4 + b^6 + 4*a^2*b^2*c^2 - 4*(a^3*b^2 + a*b^4)*c)/(4*a*c^9 - (16*a^2 + b^2)*
c^8 + 12*(2*a^3 + a*b^2)*c^7 - 2*(8*a^4 + 11*a^2*b^2 + b^4)*c^6 + 4*(a^5 +
3*a^3*b^2 + 2*a*b^4)*c^5 - (a^4*b^2 + 2*a^2*b^4 + b^6)*c^4)))/(4*a*c^5 - (8
*a^2 + b^2)*c^4 + 2*(2*a^3 + 3*a*b^2)*c^3 - (a^2*b^2 + b^4)*c^2)) + 4*(2*a^
3*b*c^2 - (a^4*b + a^2*b^3)*c)*cosh(x) + 4*(2*a^3*b*c^2 - (a^4*b + a^2*b^3)
*c)*sinh(x) - 2*(4*a^3*c^5 - (8*a^4 + a^2*b^2)*c^4 + 2*(2*a^5 + 3*a^3*b^2)*
c^3 - (a^4*b^2 + a^2*b^4)*c^2)*sqrt(-(a^4*b^2 + 2*a^2*b^4 + b^6 + 4*a^2*b^2
*c^2 - 4*(a^3*b^2 + a*b^4)*c)/(4*a*c^9 - (16*a^2 + b^2)*c^8 + 12*(2*a^3 + a
*b^2)*c^7 - 2*(8*a^4 + 11*a^2*b^2 + b^4)*c^6 + 4*(a^5 + 3*a^3*b^2 + 2*a*b^4
)*c^5 - (a^4*b^2 + 2*a^2*b^4 + b^6)*c^4))) - sqrt(2)*c*sqrt(-(a^2*b^2 + b^4
 + 2*a^2*c^2 - 2*(a^3 + 2*a*b^2)*c + (4*a*c^5 - (8*a^2 + b^2)*c^4 + 2*(2*a^
3 + 3*a*b^2)*c^3 - (a^2*b^2 + b^4)*c^2)*sqrt(-(a^4*b^2 + 2*a^2*b^4 + b^6 +
4*a^2*b^2*c^2 - 4*(a^3*b^2 + a*b^4)*c)/(4*a*c^9 - (16*a^2 + b^2)*c^8 + 12*(
2*a^3 + a*b^2)*c^7 - 2*(8*a^4 + 11*a^2*b^2 + b^4)*c^6 + 4*(a^5 + 3*a^3*b^2
+ 2*a*b^4)*c^5 - (a^4*b^2 + 2*a^2*b^4 + b^6)*c^4)))/(4*a*c^5 - (8*a^2 + b^2
)*c^4 + 2*(2*a^3 + 3*a*b^2)*c^3 - (a^2*b^2 + b^4)*c^2))*log(-2*a^4*b^2 - 2*
a^2*b^4 + 4*a^3*b^2*c - sqrt(2)*(8*a^2*b^2*c^3 - 2*(2*a^3*b^2 + 3*a*b^4)*c^
2 + (a^2*b^4 + b^6)*c + (8*a^2*c^7 - 6*(4*a^3 + a*b^2)*c^6 + (24*a^4 + 22*a
^2*b^2 + b^4)*c^5 - 2*(4*a^5 + 9*a^3*b^2 + 4*a*b^4)*c^4 + (2*a^4*b^2 + 3*a^
2*b^4 + b^6)*c^3)*sqrt(-(a^4*b^2 + 2*a^2*b^4 + b^6 + 4*a^2*b^2*c^2 - 4*(a^3
*b^2 + a*b^4)*c)/(4*a*c^9 - (16*a^2 + b^2)*c^8 + 12*(2*a^3 + a*b^2)*c^7 - 2
*(8*a^4 + 11*a^2*b^2 + b^4)*c^6 + 4*(a^5 + 3*a^3*b^2 + 2*a*b^4)*c^5 - (a^4*
b^2 + 2*a^2*b^4 + b^6)*c^4)))*sqrt(-(a^2*b^2 + b^4 + 2*a^2*c^2 - 2*(a^3 + 2
*a*b^2)*c + (4*a*c^5 - (8*a^2 + b^2)*c^4 + 2*(2*a^3 + 3*a*b^2)*c^3 - (a^2*b
^2 + b^4)*c^2)*sqrt(-(a^4*b^2 + 2*a^2*b^4 + b^6 + 4*a^2*b^2*c^2 - 4*(a^3*b^
2 + a*b^4)*c)/(4*a*c^9 - (16*a^2 + b^2)*c^8 + 12*(2*a^3 + a*b^2)*c^7 - 2*(8
```

*a^4 + 11*a^2*b^2 + b^4)*c^6 + 4*(a^5 + 3*a^3*b^2 + 2*a*b^4)*c^5 - (a^4*b^2 + 2*a^2*b^4 + b^6)*c^4)))/(4*a*c^5 - (8*a^2 + b^2)*c^4 + 2*(2*a^3 + 3*a*b^2)*c^3 - (a^2*b^2 + b^4)*c^2)) + 4*(2*a^3*b*c^2 - (a^4*b + a^2*b^3)*c)*cosh(x) + 4*(2*a^3*b*c^2 - (a^4*b + a^2*b^3)*c)*sinh(x) - 2*(4*a^3*c^5 - (8*a^4 + a^2*b^2)*c^4 + 2*(2*a^5 + 3*a^3*b^2)*c^3 - (a^4*b^2 + a^2*b^4)*c^2)*sqrt(-(a^4*b^2 + 2*a^2*b^4 + b^6 + 4*a^2*b^2*c^2 - 4*(a^3*b^2 + a*b^4)*c)/(4*a*c^9 - (16*a^2 + b^2)*c^8 + 12*(2*a^3 + a*b^2)*c^7 - 2*(8*a^4 + 11*a^2*b^2 + b^4)*c^6 + 4*(a^5 + 3*a^3*b^2 + 2*a*b^4)*c^5 - (a^4*b^2 + 2*a^2*b^4 + b^6)*c^4))) + sqrt(2)*c*sqrt(-(a^2*b^2 + b^4 + 2*a^2*c^2 - 2*(a^3 + 2*a*b^2)*c - (4*a*c^5 - (8*a^2 + b^2)*c^4 + 2*(2*a^3 + 3*a*b^2)*c^3 - (a^2*b^2 + b^4)*c^2)*sqrt(-(a^4*b^2 + 2*a^2*b^4 + b^6 + 4*a^2*b^2*c^2 - 4*(a^3*b^2 + a*b^4)*c)/(4*a*c^9 - (16*a^2 + b^2)*c^8 + 12*(2*a^3 + a*b^2)*c^7 - 2*(8*a^4 + 11*a^2*b^2 + b^4)*c^6 + 4*(a^5 + 3*a^3*b^2 + 2*a*b^4)*c^5 - (a^4*b^2 + 2*a^2*b^4 + b^6)*c^4)))/(4*a*c^5 - (8*a^2 + b^2)*c^4 + 2*(2*a^3 + 3*a*b^2)*c^3 - (a^2*b^2 + b^4)*c^2))*log(-2*a^4*b^2 - 2*a^2*b^4 + 4*a^3*b^2*c + sqrt(2)*(8*a^2*b^2*c^3 - 2*(2*a^3*b^2 + 3*a*b^4)*c^2 + (a^2*b^4 + b^6)*c - (8*a^2*c^7 - 6*(4*a^3 + a*b^2)*c^6 + (24*a^4 + 22*a^2*b^2 + b^4)*c^5 - 2*(4*a^5 + 9*a^3*b^2 + 4*a*b^4)*c^4 + (2*a^4*b^2 + 3*a^2*b^4 + b^6)*c^3)*sqrt(-(a^4*b^2 + 2*a^2*b^4 + b^6 + 4*a^2*b^2*c^2 - 4*(a^3*b^2 + a*b^4)*c)/(4*a*c^9 - (16*a^2 + b^2)*c^8 + 12*(2*a^3 + a*b^2)*c^7 - 2*(8*a^4 + 11*a^2*b^2 + b^4)*c^6 + 4*(a^5 + 3*a^3*b^2 + 2*a*b^4)*c^5 - (a^4*b^2 + 2*a^2*b^4 + b^6)*c^4)))*sqrt(-(a^2*b^2 + b^4 + 2*a^2*c^2 - 2*(a^3 + 2*a*b^2)*c - (4*a*c^5 - (8*a^2 + b^2)*c^4 + 2*(2*a^3 + 3*a*b^2)*c^3 - (a^2*b^2 + b^4)*c^2)*sqrt(-(a^4*b^2 + 2*a^2*b^4 + b^6 + 4*a^2*b^2*c^2 - 4*(a^3*b^2 + a*b^4)*c)/(4*a*c^9 - (16*a^2 + b^2)*c^8 + 12*(2*a^3 + a*b^2)*c^7 - 2*(8*a^4 + 11*a^2*b^2 + b^4)*c^6 + 4*(a^5 + 3*a^3*b^2 + 2*a*b^4)*c^5 - (a^4*b^2 + 2*a^2*b^4 + b^6)*c^4)))/(4*a*c^5 - (8*a^2 + b^2)*c^4 + 2*(2*a^3 + 3*a*b^2)*c^3 - (a^2*b^2 + b^4)*c^2)) + 4*(2*a^3*b*c^2 - (a^4*b + a^2*b^3)*c)*cosh(x) + 4*(2*a^3*b*c^2 - (a^4*b + a^2*b^3)*c)*sinh(x) + 2*(4*a^3*c^5 - (8*a^4 + a^2*b^2)*c^4 + 2*(2*a^5 + 3*a^3*b^2)*c^3 - (a^4*b^2 + a^2*b^4)*c^2)*sqrt(-(a^4*b^2 + 2*a^2*b^4 + b^6 + 4*a^2*b^2*c^2 - 4*(a^3*b^2 + a*b^4)*c)/(4*a*c^9 - (16*a^2 + b^2)*c^8 + 12*(2*a^3 + a*b^2)*c^7 - 2*(8*a^4 + 11*a^2*b^2 + b^4)*c^6 + 4*(a^5 + 3*a^3*b^2 + 2*a*b^4)*c^5 - (a^4*b^2 + 2*a^2*b^4 + b^6)*c^4))) - sqrt(2)*c*sqrt(-(a^2*b^2 + b^4 + 2*a^2*c^2 - 2*(a^3 + 2*a*b^2)*c - (4*a*c^5 - (8*a^2 + b^2)*c^4 + 2*(2*a^3 + 3*a*b^2)*c^3 - (a^2*b^2 + b^4)*c^2)*sqrt(-(a^4*b^2 + 2*a^2*b^4 + b^6 + 4*a^2*b^2*c^2 - 4*(a^3*b^2 + a*b^4)*c)/(4*a*c^9 - (16*a^2 + b^2)*c^8 + 12*(2*a^3 + a*b^2)*c^7 - 2*(8*a^4 + 11*a^2*b^2 + b^4)*c^6 + 4*(a^5 + 3*a^3*b^2 + 2*a*b^4)*c^5 - (a^4*b^2 + 2*a^2*b^4 + b^6)*c^4)))/(4*a*c^5 - (8*a^2 + b^2)*c^4 + 2*(2*a^3 + 3*a*b^2)*c^3 - (a^2*b^2 + b^4)*c^2))*log(-2*a^4*b^2 - 2*a^2*b^4 + 4*a^3*b^2*c - sqrt(2)*(8*a^2*b^2*c^3 - 2*(2*a^3*b^2 + 3*a*b^4)*c^2 + (a^2*b^4 + b^6)*c - (8*a^2*c^7 - 6*(4*a^3 + a*b^2)*c^6 + (24*a^4 + 22*a^2*b^2 + b^4)*c^5 - 2*(4*a^5 + 9*a^3*b^2 + 4*a*b^4)*c^4 + (2*a^4*b^2 + 3*a^2*b^4 + b^6)*c^3)*sqrt(-(a^4*b^2 + 2*a^2*b^4 + b^6 + 4*a^2*b^2*c^2 - 4*(a^3*b^2 + a*b^4)*c)/(4*a*c^9 - (16*a^2 + b^2)*c^8 + 12*(2*a^3 + a*b^2)*c^7 - 2*(8*a^4 + 11*a^2*b^2 + b^4)*c^6 + 4*(a^5 + 3*a^3*b^2 + 2*a*b^4)*c^5 -

```
(a^4*b^2 + 2*a^2*b^4 + b^6)*c^4)))*sqrt(-(a^2*b^2 + b^4 + 2*a^2*c^2 - 2*(a^
3 + 2*a*b^2)*c - (4*a*c^5 - (8*a^2 + b^2)*c^4 + 2*(2*a^3 + 3*a*b^2)*c^3 - (
a^2*b^2 + b^4)*c^2)*sqrt(-(a^4*b^2 + 2*a^2*b^4 + b^6 + 4*a^2*b^2*c^2 - 4*(a
^3*b^2 + a*b^4)*c)/(4*a*c^9 - (16*a^2 + b^2)*c^8 + 12*(2*a^3 + a*b^2)*c^7 -
 2*(8*a^4 + 11*a^2*b^2 + b^4)*c^6 + 4*(a^5 + 3*a^3*b^2 + 2*a*b^4)*c^5 - (a^
4*b^2 + 2*a^2*b^4 + b^6)*c^4)))/(4*a*c^5 - (8*a^2 + b^2)*c^4 + 2*(2*a^3 + 3
*a*b^2)*c^3 - (a^2*b^2 + b^4)*c^2)) + 4*(2*a^3*b*c^2 - (a^4*b + a^2*b^3)*c)
*cosh(x) + 4*(2*a^3*b*c^2 - (a^4*b + a^2*b^3)*c)*sinh(x) + 2*(4*a^3*c^5 - (
8*a^4 + a^2*b^2)*c^4 + 2*(2*a^5 + 3*a^3*b^2)*c^3 - (a^4*b^2 + a^2*b^4)*c^2)
*sqrt(-(a^4*b^2 + 2*a^2*b^4 + b^6 + 4*a^2*b^2*c^2 - 4*(a^3*b^2 + a*b^4)*c)/
(4*a*c^9 - (16*a^2 + b^2)*c^8 + 12*(2*a^3 + a*b^2)*c^7 - 2*(8*a^4 + 11*a^2*
b^2 + b^4)*c^6 + 4*(a^5 + 3*a^3*b^2 + 2*a*b^4)*c^5 - (a^4*b^2 + 2*a^2*b^4 +
 b^6)*c^4))) + 2*x)/c
```

giac [A] time = 2.11, size = 5, normalized size = 0.02

$$\frac{x}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^2/(a+b*sinh(x)+c*sinh(x)^2),x, algorithm="giac")
```

```
[Out] x/c
```

maple [C] time = 0.25, size = 108, normalized size = 0.35

$$-\frac{\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{c}+\frac{\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)}{c}+\frac{\sum_{\_R=\mathrm{RootOf}\left(a\_Z^4-2b\_Z^3+(-2a+4c)\_Z^2+2b\_Z+a\right)}\frac{\left(\_R^2a-2\_Rb-a\right)\ln\left(\tanh\left(\frac{x}{2}\right)-\_R\right)}{2\_R^3a-3b\_R^2-2\_Ra+4\_Rc+b}}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)^2/(a+b*sinh(x)+c*sinh(x)^2),x)
```

```
[Out] -1/c*ln(tanh(1/2*x)-1)+1/c*ln(tanh(1/2*x)+1)+1/c*sum((_R^2*a-2*_R*b-a)/(2*_
R^3*a-3*_R^2*b-2*_R*a+4*_R*c+b)*ln(tanh(1/2*x)-_R),_R=RootOf(a*_Z^4-2*b*_Z^
3+(-2*a+4*c)*_Z^2+2*b*_Z+a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{x}{c}-\frac{1}{4}\int\frac{8\left(be^{(3x)}+2\,ae^{(2x)}-be^{x}\right)}{c^2e^{(4x)}+2\,bce^{(3x)}-2\,bce^{x}+c^2+2\left(2\,ac-c^2\right)e^{(2x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^2/(a+b*sinh(x)+c*sinh(x)^2),x, algorithm="maxima")
```

```
[Out] x/c - 1/4*integrate(8*(b*e^(3*x) + 2*a*e^(2*x) - b*e^x)/(c^2*e^(4*x) + 2*b*
c*e^(3*x) - 2*b*c*e^x + c^2 + 2*(2*a*c - c^2)*e^(2*x)), x)
```

mupad [**F(-1)**] time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)^2/(a + c*sinh(x)^2 + b*sinh(x)),x)
```

```
[Out] \text{Hanged}
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)**2/(a+b*sinh(x)+c*sinh(x)**2),x)
```

```
[Out] Timed out
```

3.829 $\int \frac{\sinh^3(x)}{a+b\sinh(x)+c\sinh^2(x)}\,dx$

Optimal. Leaf size=363

$$\frac{\sqrt{2}\left(\frac{b^3}{\sqrt{4ac-b^2}}+i\left(\frac{3iabc}{\sqrt{4ac-b^2}}-ac+b^2\right)\right)\tan^{-1}\left(\frac{\tanh\left(\frac{x}{2}\right)\sqrt{4ac-b^2}-ib\tanh\left(\frac{x}{2}\right)+2ic}{\sqrt{2}\sqrt{ib\sqrt{4ac-b^2}-2c(a-c)+b^2}}\right)}{c^2\sqrt{ib\sqrt{4ac-b^2}-2c(a-c)+b^2}}-\frac{\sqrt{2}\left(\frac{b^3}{\sqrt{4ac-b^2}}-i\left(-\frac{3iabc}{\sqrt{4ac-b^2}}-ac+b^2\right)\right)}{c^2\sqrt{-ib\sqrt{4ac-b^2}-}}$$

```
[Out] -b*x/c^2+cosh(x)/c-arctan(1/2*(2*I*c-(I*b+(4*a*c-b^2)^(1/2))*tanh(1/2*x))*2
^(1/2)/(b^2-2*(a-c)*c-I*b*(4*a*c-b^2)^(1/2))^(1/2))*2^(1/2)*(-I*(b^2-a*c-3*
I*a*b*c/(4*a*c-b^2)^(1/2))+b^3/(4*a*c-b^2)^(1/2))/c^2/(b^2-2*(a-c)*c-I*b*(4
*a*c-b^2)^(1/2))^(1/2)+arctan(1/2*(2*I*c-I*b*tanh(1/2*x)+(4*a*c-b^2)^(1/2)*
tanh(1/2*x))*2^(1/2)/(b^2-2*(a-c)*c+I*b*(4*a*c-b^2)^(1/2))^(1/2))*2^(1/2)*(
I*(b^2-a*c+3*I*a*b*c/(4*a*c-b^2)^(1/2))+b^3/(4*a*c-b^2)^(1/2))/c^2/(b^2-2*(
a-c)*c+I*b*(4*a*c-b^2)^(1/2))^(1/2)
```

Rubi [A] time = 4.68, antiderivative size = 363, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {3256, 2638, 3292, 2660, 618, 204}

$$\frac{\sqrt{2}\left(\frac{b^3}{\sqrt{4ac-b^2}}+i\left(\frac{3iabc}{\sqrt{4ac-b^2}}-ac+b^2\right)\right)\tan^{-1}\left(\frac{\tanh\left(\frac{x}{2}\right)\sqrt{4ac-b^2}-ib\tanh\left(\frac{x}{2}\right)+2ic}{\sqrt{2}\sqrt{ib\sqrt{4ac-b^2}-2c(a-c)+b^2}}\right)}{c^2\sqrt{ib\sqrt{4ac-b^2}-2c(a-c)+b^2}}-\frac{\sqrt{2}\left(\frac{b^3}{\sqrt{4ac-b^2}}-i\left(-\frac{3iabc}{\sqrt{4ac-b^2}}-ac+b^2\right)\right)}{c^2\sqrt{-ib\sqrt{4ac-b^2}-}}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[x]^3/(a + b*Sinh[x] + c*Sinh[x]^2),x]
```

```
[Out] -((b*x)/c^2) + (Sqrt[2]*(b^3/Sqrt[-b^2 + 4*a*c] + I*(b^2 - a*c + ((3*I)*a*b
*c)/Sqrt[-b^2 + 4*a*c]))*ArcTan[((2*I)*c - I*b*Tanh[x/2] + Sqrt[-b^2 + 4*a*
c]*Tanh[x/2])/(Sqrt[2]*Sqrt[b^2 - 2*(a - c)*c + I*b*Sqrt[-b^2 + 4*a*c]])])/
(c^2*Sqrt[b^2 - 2*(a - c)*c + I*b*Sqrt[-b^2 + 4*a*c]]) - (Sqrt[2]*(b^3/Sqrt
[-b^2 + 4*a*c] - I*(b^2 - a*c - ((3*I)*a*b*c)/Sqrt[-b^2 + 4*a*c]))*ArcTan[(
(2*I)*c - (I*b + Sqrt[-b^2 + 4*a*c])*Tanh[x/2])/(Sqrt[2]*Sqrt[b^2 - 2*(a -
c)*c - I*b*Sqrt[-b^2 + 4*a*c]])])/(c^2*Sqrt[b^2 - 2*(a - c)*c - I*b*Sqrt[-b
^2 + 4*a*c]]) + Cosh[x]/c
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
```

```
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 3256

```
Int[sin[(d_.) + (e_.)*(x_)]^(m_.)*((a_.) + (b_.)*sin[(d_.) + (e_.)*(x_)]^(n
_.) + (c_.)*sin[(d_.) + (e_.)*(x_)]^(n2_.))^(p_), x_Symbol] :> Int[ExpandTr
ig[sin[d + e*x]^m*(a + b*sin[d + e*x]^n + c*sin[d + e*x]^(2*n))^p, x], x] /
; FreeQ[{a, b, c, d, e}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && Integ
ersQ[m, n, p]
```

Rule 3292

```
Int[((A_) + (B_.)*sin[(d_.) + (e_.)*(x_)])/((a_.) + (b_.)*sin[(d_.) + (e_.)
*(x_)] + (c_.)*sin[(d_.) + (e_.)*(x_)]^2), x_Symbol] :> Module[{q = Rt[b^2
- 4*a*c, 2]}, Dist[B + (b*B - 2*A*c)/q, Int[1/(b + q + 2*c*Sin[d + e*x]), x
], x] + Dist[B - (b*B - 2*A*c)/q, Int[1/(b - q + 2*c*Sin[d + e*x]), x], x]]
 /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{\sinh^3(x)}{a+b\sinh(x)+c\sinh^2(x)}\,dx = i\int\left(\frac{ib}{c^2}-\frac{i\sinh(x)}{c}+\frac{-iab-ib^2\left(1-\frac{ac}{b^2}\right)\sinh(x)}{c^2\left(a+b\sinh(x)+c\sinh^2(x)\right)}\right)dx$$

$$= -\frac{bx}{c^2}+\frac{i\int\frac{-iab-ib^2\left(1-\frac{ac}{b^2}\right)\sinh(x)}{a+b\sinh(x)+c\sinh^2(x)}dx}{c^2}+\frac{\int\sinh(x)\,dx}{c}$$

$$= -\frac{bx}{c^2}+\frac{\cosh(x)}{c}-\frac{\left(i\left(b^2-ac+\frac{ib^3}{\sqrt{-b^2+4ac}}-\frac{3iabc}{\sqrt{-b^2+4ac}}\right)\right)\int\frac{1}{-ib-\sqrt{-b^2+4ac}-2ic\sinh(x)}}{c^2}$$

$$= -\frac{bx}{c^2}+\frac{\cosh(x)}{c}-\frac{\left(2i\left(b^2-ac+\frac{ib^3}{\sqrt{-b^2+4ac}}-\frac{3iabc}{\sqrt{-b^2+4ac}}\right)\right)\text{Subst}\left(\int\frac{}{-ib-\sqrt{-b^2+4ac}}\right.}{c^2}$$

$$= -\frac{bx}{c^2}+\frac{\cosh(x)}{c}+\frac{\left(4i\left(b^2-ac+\frac{ib^3}{\sqrt{-b^2+4ac}}-\frac{3iabc}{\sqrt{-b^2+4ac}}\right)\right)\text{Subst}\left(\int\frac{}{-8\left(b^2-2(a-c)c-\right.}\right.}{c^2}$$

$$= -\frac{bx}{c^2}+\frac{\sqrt{2}\left(\frac{b^3}{\sqrt{-b^2+4ac}}+i\left(b^2-ac+\frac{3iabc}{\sqrt{-b^2+4ac}}\right)\right)\tan^{-1}\left(\frac{2ic-\left(ib-\sqrt{-b^2+4ac}\right)\tanh\left(\frac{x}{2}\right)}{\sqrt{2}\sqrt{b^2-2(a-c)c+ib\sqrt{-b^2+4ac}}}\right)}{c^2\sqrt{b^2-2(a-c)c+ib\sqrt{-b^2+4ac}}}$$

Mathematica [A] time = 0.94, size = 326, normalized size = 0.90

$$\frac{\frac{\sqrt{2}\left(b^2\sqrt{b^2-4ac}-ac\sqrt{b^2-4ac}+3abc-b^3\right)\tan^{-1}\left(\frac{\tanh\left(\frac{x}{2}\right)\left(\sqrt{b^2-4ac}-b\right)+2c}{\sqrt{2b\sqrt{b^2-4ac}+4c(a-c)-2b^2}}\right)}{\sqrt{b^2-4ac}\sqrt{b\sqrt{b^2-4ac}+2c(a-c)-b^2}}+\frac{\sqrt{2}\left(b^2\sqrt{b^2-4ac}-ac\sqrt{b^2-4ac}-3abc+b^3\right)\tan^{-1}\left(\frac{2c-\tanh\left(\frac{x}{2}\right)\left(\sqrt{b^2-4ac}+b\right)}{\sqrt{2}\sqrt{-b\sqrt{b^2-4ac}+2c(a-c)-b^2}}\right)}{\sqrt{b^2-4ac}\sqrt{-b\sqrt{b^2-4ac}+2c(a-c)-b^2}}}{c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[x]^3/(a + b*Sinh[x] + c*Sinh[x]^2),x]
```

```
[Out] (-(b*x) + (Sqrt[2]*(-b^3 + 3*a*b*c + b^2*Sqrt[b^2 - 4*a*c] - a*c*Sqrt[b^2 -
 4*a*c])*ArcTan[(2*c + (-b + Sqrt[b^2 - 4*a*c])*Tanh[x/2])/Sqrt[-2*b^2 + 4*
(a - c)*c + 2*b*Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[-b^2 + 2*(a -
c)*c + b*Sqrt[b^2 - 4*a*c]]) + (Sqrt[2]*(b^3 - 3*a*b*c + b^2*Sqrt[b^2 - 4*a
*c] - a*c*Sqrt[b^2 - 4*a*c])*ArcTan[(2*c - (b + Sqrt[b^2 - 4*a*c])*Tanh[x/2
])/(Sqrt[2]*Sqrt[-b^2 + 2*(a - c)*c - b*Sqrt[b^2 - 4*a*c]])])/(Sqrt[b^2 - 4
*a*c]*Sqrt[-b^2 + 2*(a - c)*c - b*Sqrt[b^2 - 4*a*c]]) + c*Cosh[x])/c^2
```

fricas [B] time = 1.39, size = 6680, normalized size = 18.40

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^3/(a+b*sinh(x)+c*sinh(x)^2),x, algorithm="fricas")
```

```
[Out] -1/2*(2*b*x*cosh(x) - c*cosh(x)^2 - sqrt(2)*(c^2*cosh(x) + c^2*sinh(x))*sqr
t(-(a^2*b^4 + b^6 - 2*a^3*c^3 + (2*a^4 + 9*a^2*b^2)*c^2 - 2*(2*a^3*b^2 + 3*
a*b^4)*c + (4*a*c^7 - (8*a^2 + b^2)*c^6 + 2*(2*a^3 + 3*a*b^2)*c^5 - (a^2*b^
2 + b^4)*c^4)*sqrt(-(a^4*b^6 + 2*a^2*b^8 + b^10 + 9*a^4*b^2*c^4 - 12*(a^5*b
^2 + 2*a^3*b^4)*c^3 + 2*(2*a^6*b^2 + 11*a^4*b^4 + 11*a^2*b^6)*c^2 - 4*(a^5*
b^4 + 3*a^3*b^6 + 2*a*b^8)*c)/(4*a*c^13 - (16*a^2 + b^2)*c^12 + 12*(2*a^3 +
 a*b^2)*c^11 - 2*(8*a^4 + 11*a^2*b^2 + b^4)*c^10 + 4*(a^5 + 3*a^3*b^2 + 2*a
*b^4)*c^9 - (a^4*b^2 + 2*a^2*b^4 + b^6)*c^8)))/(4*a*c^7 - (8*a^2 + b^2)*c^6
 + 2*(2*a^3 + 3*a*b^2)*c^5 - (a^2*b^2 + b^4)*c^4))*log(2*a^5*b^4 + 2*a^3*b^
6 + 6*a^5*b^2*c^2 - 4*(a^6*b^2 + 2*a^4*b^4)*c + sqrt(2)*(12*a^4*b*c^5 - (20
*a^5*b + 31*a^3*b^3)*c^4 + (8*a^6*b + 33*a^4*b^3 + 27*a^2*b^5)*c^3 - 3*(2*a
^5*b^3 + 5*a^3*b^5 + 3*a*b^7)*c^2 + (a^4*b^5 + 2*a^2*b^7 + b^9)*c + (12*a^2
*b*c^9 - 7*(4*a^3*b + a*b^3)*c^8 + (20*a^4*b + 27*a^2*b^3 + b^5)*c^7 - (4*a
^5*b + 13*a^3*b^3 + 9*a*b^5)*c^6 + (a^4*b^3 + 2*a^2*b^5 + b^7)*c^5)*sqrt(-(
a^4*b^6 + 2*a^2*b^8 + b^10 + 9*a^4*b^2*c^4 - 12*(a^5*b^2 + 2*a^3*b^4)*c^3 +
 2*(2*a^6*b^2 + 11*a^4*b^4 + 11*a^2*b^6)*c^2 - 4*(a^5*b^4 + 3*a^3*b^6 + 2*a
*b^8)*c)/(4*a*c^13 - (16*a^2 + b^2)*c^12 + 12*(2*a^3 + a*b^2)*c^11 - 2*(8*a
^4 + 11*a^2*b^2 + b^4)*c^10 + 4*(a^5 + 3*a^3*b^2 + 2*a*b^4)*c^9 - (a^4*b^2
+ 2*a^2*b^4 + b^6)*c^8)))*sqrt(-(a^2*b^4 + b^6 - 2*a^3*c^3 + (2*a^4 + 9*a^2
*b^2)*c^2 - 2*(2*a^3*b^2 + 3*a*b^4)*c + (4*a*c^7 - (8*a^2 + b^2)*c^6 + 2*(2
*a^3 + 3*a*b^2)*c^5 - (a^2*b^2 + b^4)*c^4)*sqrt(-(a^4*b^6 + 2*a^2*b^8 + b^1
0 + 9*a^4*b^2*c^4 - 12*(a^5*b^2 + 2*a^3*b^4)*c^3 + 2*(2*a^6*b^2 + 11*a^4*b^
4 + 11*a^2*b^6)*c^2 - 4*(a^5*b^4 + 3*a^3*b^6 + 2*a*b^8)*c)/(4*a*c^13 - (16*
a^2 + b^2)*c^12 + 12*(2*a^3 + a*b^2)*c^11 - 2*(8*a^4 + 11*a^2*b^2 + b^4)*c^
10 + 4*(a^5 + 3*a^3*b^2 + 2*a*b^4)*c^9 - (a^4*b^2 + 2*a^2*b^4 + b^6)*c^8)))
/(4*a*c^7 - (8*a^2 + b^2)*c^6 + 2*(2*a^3 + 3*a*b^2)*c^5 - (a^2*b^2 + b^4)*c
^4)) + 4*(3*a^5*b*c^3 - 2*(a^6*b + 2*a^4*b^3)*c^2 + (a^5*b^3 + a^3*b^5)*c)*
cosh(x) + 4*(3*a^5*b*c^3 - 2*(a^6*b + 2*a^4*b^3)*c^2 + (a^5*b^3 + a^3*b^5)*
c)*sinh(x) + 2*(4*a^4*c^7 - (8*a^5 + a^3*b^2)*c^6 + 2*(2*a^6 + 3*a^4*b^2)*c
^5 - (a^5*b^2 + a^3*b^4)*c^4)*sqrt(-(a^4*b^6 + 2*a^2*b^8 + b^10 + 9*a^4*b^2
*c^4 - 12*(a^5*b^2 + 2*a^3*b^4)*c^3 + 2*(2*a^6*b^2 + 11*a^4*b^4 + 11*a^2*b^
6)*c^2 - 4*(a^5*b^4 + 3*a^3*b^6 + 2*a*b^8)*c)/(4*a*c^13 - (16*a^2 + b^2)*c^
12 + 12*(2*a^3 + a*b^2)*c^11 - 2*(8*a^4 + 11*a^2*b^2 + b^4)*c^10 + 4*(a^5 +
 3*a^3*b^2 + 2*a*b^4)*c^9 - (a^4*b^2 + 2*a^2*b^4 + b^6)*c^8))) + sqrt(2)*(c
^2*cosh(x) + c^2*sinh(x))*sqrt(-(a^2*b^4 + b^6 - 2*a^3*c^3 + (2*a^4 + 9*a^2
*b^2)*c^2 - 2*(2*a^3*b^2 + 3*a*b^4)*c + (4*a*c^7 - (8*a^2 + b^2)*c^6 + 2*(2
*a^3 + 3*a*b^2)*c^5 - (a^2*b^2 + b^4)*c^4)*sqrt(-(a^4*b^6 + 2*a^2*b^8 + b^1
```

0 + 9*a^4*b^2*c^4 - 12*(a^5*b^2 + 2*a^3*b^4)*c^3 + 2*(2*a^6*b^2 + 11*a^4*b^
4 + 11*a^2*b^6)*c^2 - 4*(a^5*b^4 + 3*a^3*b^6 + 2*a*b^8)*c)/(4*a*c^13 - (16*
a^2 + b^2)*c^12 + 12*(2*a^3 + a*b^2)*c^11 - 2*(8*a^4 + 11*a^2*b^2 + b^4)*c^
10 + 4*(a^5 + 3*a^3*b^2 + 2*a*b^4)*c^9 - (a^4*b^2 + 2*a^2*b^4 + b^6)*c^8)))
/(4*a*c^7 - (8*a^2 + b^2)*c^6 + 2*(2*a^3 + 3*a*b^2)*c^5 - (a^2*b^2 + b^4)*c
^4))*log(2*a^5*b^4 + 2*a^3*b^6 + 6*a^5*b^2*c^2 - 4*(a^6*b^2 + 2*a^4*b^4)*c
- sqrt(2)*(12*a^4*b*c^5 - (20*a^5*b + 31*a^3*b^3)*c^4 + (8*a^6*b + 33*a^4*b
^3 + 27*a^2*b^5)*c^3 - 3*(2*a^5*b^3 + 5*a^3*b^5 + 3*a*b^7)*c^2 + (a^4*b^5 +
2*a^2*b^7 + b^9)*c + (12*a^2*b*c^9 - 7*(4*a^3*b + a*b^3)*c^8 + (20*a^4*b +
27*a^2*b^3 + b^5)*c^7 - (4*a^5*b + 13*a^3*b^3 + 9*a*b^5)*c^6 + (a^4*b^3 +
2*a^2*b^5 + b^7)*c^5)*sqrt(-(a^4*b^6 + 2*a^2*b^8 + b^10 + 9*a^4*b^2*c^4 - 1
2*(a^5*b^2 + 2*a^3*b^4)*c^3 + 2*(2*a^6*b^2 + 11*a^4*b^4 + 11*a^2*b^6)*c^2 -
4*(a^5*b^4 + 3*a^3*b^6 + 2*a*b^8)*c)/(4*a*c^13 - (16*a^2 + b^2)*c^12 + 12*
(2*a^3 + a*b^2)*c^11 - 2*(8*a^4 + 11*a^2*b^2 + b^4)*c^10 + 4*(a^5 + 3*a^3*b
^2 + 2*a*b^4)*c^9 - (a^4*b^2 + 2*a^2*b^4 + b^6)*c^8)))*sqrt(-(a^2*b^4 + b^6
- 2*a^3*c^3 + (2*a^4 + 9*a^2*b^2)*c^2 - 2*(2*a^3*b^2 + 3*a*b^4)*c + (4*a*c
^7 - (8*a^2 + b^2)*c^6 + 2*(2*a^3 + 3*a*b^2)*c^5 - (a^2*b^2 + b^4)*c^4)*sqr
t(-(a^4*b^6 + 2*a^2*b^8 + b^10 + 9*a^4*b^2*c^4 - 12*(a^5*b^2 + 2*a^3*b^4)*c
^3 + 2*(2*a^6*b^2 + 11*a^4*b^4 + 11*a^2*b^6)*c^2 - 4*(a^5*b^4 + 3*a^3*b^6 +
2*a*b^8)*c)/(4*a*c^13 - (16*a^2 + b^2)*c^12 + 12*(2*a^3 + a*b^2)*c^11 - 2*
(8*a^4 + 11*a^2*b^2 + b^4)*c^10 + 4*(a^5 + 3*a^3*b^2 + 2*a*b^4)*c^9 - (a^4*
b^2 + 2*a^2*b^4 + b^6)*c^8)))/(4*a*c^7 - (8*a^2 + b^2)*c^6 + 2*(2*a^3 + 3*a
*b^2)*c^5 - (a^2*b^2 + b^4)*c^4)) + 4*(3*a^5*b*c^3 - 2*(a^6*b + 2*a^4*b^3)*
c^2 + (a^5*b^3 + a^3*b^5)*c)*cosh(x) + 4*(3*a^5*b*c^3 - 2*(a^6*b + 2*a^4*b^
3)*c^2 + (a^5*b^3 + a^3*b^5)*c)*sinh(x) + 2*(4*a^4*c^7 - (8*a^5 + a^3*b^2)*
c^6 + 2*(2*a^6 + 3*a^4*b^2)*c^5 - (a^5*b^2 + a^3*b^4)*c^4)*sqrt(-(a^4*b^6 +
2*a^2*b^8 + b^10 + 9*a^4*b^2*c^4 - 12*(a^5*b^2 + 2*a^3*b^4)*c^3 + 2*(2*a^6
*b^2 + 11*a^4*b^4 + 11*a^2*b^6)*c^2 - 4*(a^5*b^4 + 3*a^3*b^6 + 2*a*b^8)*c)/
(4*a*c^13 - (16*a^2 + b^2)*c^12 + 12*(2*a^3 + a*b^2)*c^11 - 2*(8*a^4 + 11*a
^2*b^2 + b^4)*c^10 + 4*(a^5 + 3*a^3*b^2 + 2*a*b^4)*c^9 - (a^4*b^2 + 2*a^2*b
^4 + b^6)*c^8))) - sqrt(2)*(c^2*cosh(x) + c^2*sinh(x))*sqrt(-(a^2*b^4 + b^6
- 2*a^3*c^3 + (2*a^4 + 9*a^2*b^2)*c^2 - 2*(2*a^3*b^2 + 3*a*b^4)*c - (4*a*c
^7 - (8*a^2 + b^2)*c^6 + 2*(2*a^3 + 3*a*b^2)*c^5 - (a^2*b^2 + b^4)*c^4)*sqr
t(-(a^4*b^6 + 2*a^2*b^8 + b^10 + 9*a^4*b^2*c^4 - 12*(a^5*b^2 + 2*a^3*b^4)*c
^3 + 2*(2*a^6*b^2 + 11*a^4*b^4 + 11*a^2*b^6)*c^2 - 4*(a^5*b^4 + 3*a^3*b^6 +
2*a*b^8)*c)/(4*a*c^13 - (16*a^2 + b^2)*c^12 + 12*(2*a^3 + a*b^2)*c^11 - 2*
(8*a^4 + 11*a^2*b^2 + b^4)*c^10 + 4*(a^5 + 3*a^3*b^2 + 2*a*b^4)*c^9 - (a^4*
b^2 + 2*a^2*b^4 + b^6)*c^8)))/(4*a*c^7 - (8*a^2 + b^2)*c^6 + 2*(2*a^3 + 3*a
*b^2)*c^5 - (a^2*b^2 + b^4)*c^4))*log(2*a^5*b^4 + 2*a^3*b^6 + 6*a^5*b^2*c^2
- 4*(a^6*b^2 + 2*a^4*b^4)*c + sqrt(2)*(12*a^4*b*c^5 - (20*a^5*b + 31*a^3*b
^3)*c^4 + (8*a^6*b + 33*a^4*b^3 + 27*a^2*b^5)*c^3 - 3*(2*a^5*b^3 + 5*a^3*b^
5 + 3*a*b^7)*c^2 + (a^4*b^5 + 2*a^2*b^7 + b^9)*c - (12*a^2*b*c^9 - 7*(4*a^3
*b + a*b^3)*c^8 + (20*a^4*b + 27*a^2*b^3 + b^5)*c^7 - (4*a^5*b + 13*a^3*b^3
+ 9*a*b^5)*c^6 + (a^4*b^3 + 2*a^2*b^5 + b^7)*c^5)*sqrt(-(a^4*b^6 + 2*a^2*b
^8 + b^10 + 9*a^4*b^2*c^4 - 12*(a^5*b^2 + 2*a^3*b^4)*c^3 + 2*(2*a^6*b^2 + 1

1*a^4*b^4 + 11*a^2*b^6)*c^2 - 4*(a^5*b^4 + 3*a^3*b^6 + 2*a*b^8)*c)/(4*a*c^1
3 - (16*a^2 + b^2)*c^12 + 12*(2*a^3 + a*b^2)*c^11 - 2*(8*a^4 + 11*a^2*b^2 +
b^4)*c^10 + 4*(a^5 + 3*a^3*b^2 + 2*a*b^4)*c^9 - (a^4*b^2 + 2*a^2*b^4 + b^6
)*c^8)))*sqrt(-(a^2*b^4 + b^6 - 2*a^3*c^3 + (2*a^4 + 9*a^2*b^2)*c^2 - 2*(2*
a^3*b^2 + 3*a*b^4)*c - (4*a*c^7 - (8*a^2 + b^2)*c^6 + 2*(2*a^3 + 3*a*b^2)*c
^5 - (a^2*b^2 + b^4)*c^4)*sqrt(-(a^4*b^6 + 2*a^2*b^8 + b^10 + 9*a^4*b^2*c^4
- 12*(a^5*b^2 + 2*a^3*b^4)*c^3 + 2*(2*a^6*b^2 + 11*a^4*b^4 + 11*a^2*b^6)*c
^2 - 4*(a^5*b^4 + 3*a^3*b^6 + 2*a*b^8)*c)/(4*a*c^13 - (16*a^2 + b^2)*c^12 +
12*(2*a^3 + a*b^2)*c^11 - 2*(8*a^4 + 11*a^2*b^2 + b^4)*c^10 + 4*(a^5 + 3*a
^3*b^2 + 2*a*b^4)*c^9 - (a^4*b^2 + 2*a^2*b^4 + b^6)*c^8)))/(4*a*c^7 - (8*a^
2 + b^2)*c^6 + 2*(2*a^3 + 3*a*b^2)*c^5 - (a^2*b^2 + b^4)*c^4)) + 4*(3*a^5*b
c^3 - 2(a^6*b + 2*a^4*b^3)*c^2 + (a^5*b^3 + a^3*b^5)*c)*cosh(x) + 4*(3*a^
5*b*c^3 - 2*(a^6*b + 2*a^4*b^3)*c^2 + (a^5*b^3 + a^3*b^5)*c)*sinh(x) - 2*(4
*a^4*c^7 - (8*a^5 + a^3*b^2)*c^6 + 2*(2*a^6 + 3*a^4*b^2)*c^5 - (a^5*b^2 + a
^3*b^4)*c^4)*sqrt(-(a^4*b^6 + 2*a^2*b^8 + b^10 + 9*a^4*b^2*c^4 - 12*(a^5*b^
2 + 2*a^3*b^4)*c^3 + 2*(2*a^6*b^2 + 11*a^4*b^4 + 11*a^2*b^6)*c^2 - 4*(a^5*b
^4 + 3*a^3*b^6 + 2*a*b^8)*c)/(4*a*c^13 - (16*a^2 + b^2)*c^12 + 12*(2*a^3 +
a*b^2)*c^11 - 2*(8*a^4 + 11*a^2*b^2 + b^4)*c^10 + 4*(a^5 + 3*a^3*b^2 + 2*a*
b^4)*c^9 - (a^4*b^2 + 2*a^2*b^4 + b^6)*c^8))) + sqrt(2)*(c^2*cosh(x) + c^2*
sinh(x))*sqrt(-(a^2*b^4 + b^6 - 2*a^3*c^3 + (2*a^4 + 9*a^2*b^2)*c^2 - 2*(2*
a^3*b^2 + 3*a*b^4)*c - (4*a*c^7 - (8*a^2 + b^2)*c^6 + 2*(2*a^3 + 3*a*b^2)*c
^5 - (a^2*b^2 + b^4)*c^4)*sqrt(-(a^4*b^6 + 2*a^2*b^8 + b^10 + 9*a^4*b^2*c^4
- 12*(a^5*b^2 + 2*a^3*b^4)*c^3 + 2*(2*a^6*b^2 + 11*a^4*b^4 + 11*a^2*b^6)*c
^2 - 4*(a^5*b^4 + 3*a^3*b^6 + 2*a*b^8)*c)/(4*a*c^13 - (16*a^2 + b^2)*c^12 +
12*(2*a^3 + a*b^2)*c^11 - 2*(8*a^4 + 11*a^2*b^2 + b^4)*c^10 + 4*(a^5 + 3*a
^3*b^2 + 2*a*b^4)*c^9 - (a^4*b^2 + 2*a^2*b^4 + b^6)*c^8)))/(4*a*c^7 - (8*a^
2 + b^2)*c^6 + 2*(2*a^3 + 3*a*b^2)*c^5 - (a^2*b^2 + b^4)*c^4))*log(2*a^5*b^
4 + 2*a^3*b^6 + 6*a^5*b^2*c^2 - 4*(a^6*b^2 + 2*a^4*b^4)*c - sqrt(2)*(12*a^4
*b*c^5 - (20*a^5*b + 31*a^3*b^3)*c^4 + (8*a^6*b + 33*a^4*b^3 + 27*a^2*b^5)*
c^3 - 3*(2*a^5*b^3 + 5*a^3*b^5 + 3*a*b^7)*c^2 + (a^4*b^5 + 2*a^2*b^7 + b^9)
*c - (12*a^2*b*c^9 - 7*(4*a^3*b + a*b^3)*c^8 + (20*a^4*b + 27*a^2*b^3 + b^5
)*c^7 - (4*a^5*b + 13*a^3*b^3 + 9*a*b^5)*c^6 + (a^4*b^3 + 2*a^2*b^5 + b^7)*
c^5)*sqrt(-(a^4*b^6 + 2*a^2*b^8 + b^10 + 9*a^4*b^2*c^4 - 12*(a^5*b^2 + 2*a^
3*b^4)*c^3 + 2*(2*a^6*b^2 + 11*a^4*b^4 + 11*a^2*b^6)*c^2 - 4*(a^5*b^4 + 3*a
^3*b^6 + 2*a*b^8)*c)/(4*a*c^13 - (16*a^2 + b^2)*c^12 + 12*(2*a^3 + a*b^2)*c
^11 - 2*(8*a^4 + 11*a^2*b^2 + b^4)*c^10 + 4*(a^5 + 3*a^3*b^2 + 2*a*b^4)*c^9
- (a^4*b^2 + 2*a^2*b^4 + b^6)*c^8)))*sqrt(-(a^2*b^4 + b^6 - 2*a^3*c^3 + (2
*a^4 + 9*a^2*b^2)*c^2 - 2*(2*a^3*b^2 + 3*a*b^4)*c - (4*a*c^7 - (8*a^2 + b^2
)*c^6 + 2*(2*a^3 + 3*a*b^2)*c^5 - (a^2*b^2 + b^4)*c^4)*sqrt(-(a^4*b^6 + 2*a
^2*b^8 + b^10 + 9*a^4*b^2*c^4 - 12*(a^5*b^2 + 2*a^3*b^4)*c^3 + 2*(2*a^6*b^2
+ 11*a^4*b^4 + 11*a^2*b^6)*c^2 - 4*(a^5*b^4 + 3*a^3*b^6 + 2*a*b^8)*c)/(4*a
*c^13 - (16*a^2 + b^2)*c^12 + 12*(2*a^3 + a*b^2)*c^11 - 2*(8*a^4 + 11*a^2*b
^2 + b^4)*c^10 + 4*(a^5 + 3*a^3*b^2 + 2*a*b^4)*c^9 - (a^4*b^2 + 2*a^2*b^4 +
b^6)*c^8)))/(4*a*c^7 - (8*a^2 + b^2)*c^6 + 2*(2*a^3 + 3*a*b^2)*c^5 - (a^2*
b^2 + b^4)*c^4)) + 4*(3*a^5*b*c^3 - 2*(a^6*b + 2*a^4*b^3)*c^2 + (a^5*b^3 +

```
a^3*b^5)*c)*cosh(x) + 4*(3*a^5*b*c^3 - 2*(a^6*b + 2*a^4*b^3)*c^2 + (a^5*b^3
 + a^3*b^5)*c)*sinh(x) - 2*(4*a^4*c^7 - (8*a^5 + a^3*b^2)*c^6 + 2*(2*a^6 +
3*a^4*b^2)*c^5 - (a^5*b^2 + a^3*b^4)*c^4)*sqrt(-(a^4*b^6 + 2*a^2*b^8 + b^10
 + 9*a^4*b^2*c^4 - 12*(a^5*b^2 + 2*a^3*b^4)*c^3 + 2*(2*a^6*b^2 + 11*a^4*b^4
 + 11*a^2*b^6)*c^2 - 4*(a^5*b^4 + 3*a^3*b^6 + 2*a*b^8)*c)/(4*a*c^13 - (16*a
^2 + b^2)*c^12 + 12*(2*a^3 + a*b^2)*c^11 - 2*(8*a^4 + 11*a^2*b^2 + b^4)*c^1
0 + 4*(a^5 + 3*a^3*b^2 + 2*a*b^4)*c^9 - (a^4*b^2 + 2*a^2*b^4 + b^6)*c^8)))
- c*sinh(x)^2 + 2*(b*x - c*cosh(x))*sinh(x) - c)/(c^2*cosh(x) + c^2*sinh(x)
)
```

giac [A] time = 6.46, size = 24, normalized size = 0.07

$$-\frac{bx}{c^2} + \frac{e^{(-x)}}{2\,c} + \frac{e^x}{2\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^3/(a+b*sinh(x)+c*sinh(x)^2),x, algorithm="giac")
```

```
[Out] -b*x/c^2 + 1/2*e^(-x)/c + 1/2*e^x/c
```

maple [C] time = 0.23, size = 144, normalized size = 0.40

$$-\frac{1}{c\left(\tanh\left(\frac{x}{2}\right)-1\right)}+\frac{b\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{c^2}+\frac{1}{c\left(\tanh\left(\frac{x}{2}\right)+1\right)}-\frac{b\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)}{c^2}+\frac{\sum_{\_R=\mathrm{RootOf}\left(a\_Z^4-2b\_Z^3+(-2a+4c)\_Z^2+2\right.}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)^3/(a+b*sinh(x)+c*sinh(x)^2),x)
```

```
[Out] -1/c/(tanh(1/2*x)-1)+b/c^2*ln(tanh(1/2*x)-1)+1/c/(tanh(1/2*x)+1)-b/c^2*ln(t
anh(1/2*x)+1)+1/c^2*sum((-_R^2*a*b+2*(-a*c+b^2)*_R+a*b)/(2*_R^3*a-3*_R^2*b-
2*_R*a+4*_R*c+b)*ln(tanh(1/2*x)-_R),_R=RootOf(a*_Z^4-2*b*_Z^3+(-2*a+4*c)*_Z
^2+2*b*_Z+a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\left(2\,bxe^x - ce^{(2\,x)} - c\right)e^{(-x)}}{2\,c^2} - \frac{1}{8}\int -\frac{16\left(2\,abe^{(2\,x)} + \left(b^2 - ac\right)e^{(3\,x)} - \left(b^2 - ac\right)e^x\right)}{c^3e^{(4\,x)} + 2\,bc^2e^{(3\,x)} - 2\,bc^2e^x + c^3 + 2\left(2\,ac^2 - c^3\right)e^{(2\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^3/(a+b*sinh(x)+c*sinh(x)^2),x, algorithm="maxima")
```

```
[Out] -1/2*(2*b*x*e^x - c*e^(2*x) - c)*e^(-x)/c^2 - 1/8*integrate(-16*(2*a*b*e^(2
*x) + (b^2 - a*c)*e^(3*x) - (b^2 - a*c)*e^x)/(c^3*e^(4*x) + 2*b*c^2*e^(3*x)
 - 2*b*c^2*e^x + c^3 + 2*(2*a*c^2 - c^3)*e^(2*x)), x)
```

mupad [F(-1)] time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)^3/(a + c*sinh(x)^2 + b*sinh(x)),x)
```

```
[Out] \text{Hanged}
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)**3/(a+b*sinh(x)+c*sinh(x)**2),x)
```

```
[Out] Timed out
```

3.830 $\int \frac{a+b\sinh(x)}{b^2-2ab\sinh(x)+a^2\sinh^2(x)}\,dx$

Optimal. Leaf size=12

$$\frac{\cosh(x)}{b-a\sinh(x)}$$

[Out] cosh(x)/(b-a*sinh(x))

Rubi [A] time = 0.09, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {3288, 2754, 8}

$$\frac{\cosh(x)}{b-a\sinh(x)}$$

Antiderivative was successfully verified.

[In] Int[(a + b*Sinh[x])/(b^2 - 2*a*b*Sinh[x] + a^2*Sinh[x]^2),x]

[Out] Cosh[x]/(b - a*Sinh[x])

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2754

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 3288

```
Int[((A_) + (B_.)*sin[(d_.) + (e_.)*(x_)])*((a_) + (b_.)*sin[(d_.) + (e_.)*
(x_)] + (c_.)*sin[(d_.) + (e_.)*(x_)]^2)^(n_), x_Symbol] :> Dist[1/(4^n*c^n
), Int[(A + B*Sin[d + e*x])*(b + 2*c*Sin[d + e*x])^(2*n), x], x] /; FreeQ[{
a, b, c, d, e, A, B}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[n]
```

Rubi steps

$$\int \frac{a+b\sinh(x)}{b^2-2ab\sinh(x)+a^2\sinh^2(x)}\,dx = -\left((4a^2)\int \frac{a+b\sinh(x)}{\left(2iab-2ia^2\sinh(x)\right)^2}\,dx\right)$$
$$= \frac{\cosh(x)}{b-a\sinh(x)} - \frac{\int 0\,dx}{a^2+b^2}$$
$$= \frac{\cosh(x)}{b-a\sinh(x)}$$

Mathematica [A] time = 0.04, size = 14, normalized size = 1.17

$$-\frac{\cosh(x)}{a\sinh(x)-b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sinh[x])/(b^2 - 2*a*b*Sinh[x] + a^2*Sinh[x]^2),x]
```

```
[Out] -(Cosh[x]/(-b + a*Sinh[x]))
```

fricas [B] time = 0.43, size = 57, normalized size = 4.75

$$-\frac{2\,(b\cosh(x)+b\sinh(x)+a)}{a^2\cosh(x)^2+a^2\sinh(x)^2-2\,ab\cosh(x)-a^2+2\left(a^2\cosh(x)-ab\right)\sinh(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(x))/(b^2-2*a*b*sinh(x)+a^2*sinh(x)^2),x, algorithm="fri
cas")
```

```
[Out] -2*(b*cosh(x) + b*sinh(x) + a)/(a^2*cosh(x)^2 + a^2*sinh(x)^2 - 2*a*b*cosh(
x) - a^2 + 2*(a^2*cosh(x) - a*b)*sinh(x))
```

giac [A] time = 0.13, size = 28, normalized size = 2.33

$$-\frac{2\,(be^x+a)}{\left(ae^{(2\,x)}-2\,be^x-a\right)a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(x))/(b^2-2*a*b*sinh(x)+a^2*sinh(x)^2),x, algorithm="gia
c")
```

```
[Out] -2*(b*e^x + a)/((a*e^(2*x) - 2*b*e^x - a)*a)
```

maple [B] time = 0.17, size = 36, normalized size = 3.00

$$-\frac{2\left(-\frac{a\tanh\left(\frac{x}{2}\right)}{b}+1\right)}{\left(\tanh^{2}\left(\frac{x}{2}\right)\right)b+2a\tanh\left(\frac{x}{2}\right)-b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sinh(x))/(b^2-2*a*b*sinh(x)+a^2*sinh(x)^2),x)
```

```
[Out] -2*(-a/b*tanh(1/2*x)+1)/(tanh(1/2*x)^2*b+2*a*tanh(1/2*x)-b)
```

maxima [B] time = 0.42, size = 225, normalized size = 18.75

$$b\left(\frac{a\log\left(\frac{ae^{(-x)}+b-\sqrt{a^2+b^2}}{ae^{(-x)}+b+\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{\frac{3}{2}}}+\frac{2\left(b^2e^{(-x)}-ab\right)}{a^4+a^2b^2-2\left(a^3b+ab^3\right)e^{(-x)}-\left(a^4+a^2b^2\right)e^{(-2x)}}\right)-a\left(\frac{b\log\left(\frac{ae^{(-x)}+b-\sqrt{a^2+b^2}}{ae^{(-x)}+b+\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{\frac{3}{2}}}-\frac{}{a^3+a}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(x))/(b^2-2*a*b*sinh(x)+a^2*sinh(x)^2),x, algorithm="max
ima")
```

```
[Out] b*(a*log((a*e^(-x) + b - sqrt(a^2 + b^2))/(a*e^(-x) + b + sqrt(a^2 + b^2)))
/(a^2 + b^2)^(3/2) + 2*(b^2*e^(-x) - a*b)/(a^4 + a^2*b^2 - 2*(a^3*b + a*b^3
)*e^(-x) - (a^4 + a^2*b^2)*e^(-2*x))) - a*(b*log((a*e^(-x) + b - sqrt(a^2 +
 b^2))/(a*e^(-x) + b + sqrt(a^2 + b^2)))/(a^2 + b^2)^(3/2) - 2*(b*e^(-x) -
a)/(a^3 + a*b^2 - 2*(a^2*b + b^3)*e^(-x) - (a^3 + a*b^2)*e^(-2*x)))
```

mupad [B] time = 1.91, size = 48, normalized size = 4.00

$$\frac{\frac{2\,\mathrm{e}^{x}\,\left(a^3\,b+a\,b^3\right)}{a\,\left(a^3+a\,b^2\right)}+2}{a+2\,b\,\mathrm{e}^{x}-a\,\mathrm{e}^{2\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sinh(x))/(a^2*sinh(x)^2 + b^2 - 2*a*b*sinh(x)),x)
```

```
[Out] ((2*exp(x)*(a*b^3 + a^3*b))/(a*(a*b^2 + a^3)) + 2)/(a + 2*b*exp(x) - a*exp(
2*x))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(x))/(b**2-2*a*b*sinh(x)+a**2*sinh(x)**2),x)
```

```
[Out] Timed out
```

3.831 $\int \frac{d+e\sinh(x)}{a+b\sinh(x)+c\sinh^2(x)}\,dx$

Optimal. Leaf size=300

$$\frac{\sqrt{2}\left(-\frac{2cd-be}{\sqrt{4ac-b^2}}+ie\right)\tan^{-1}\left(\frac{\tanh\left(\frac{x}{2}\right)\sqrt{4ac-b^2}-ib\tanh\left(\frac{x}{2}\right)+2ic}{\sqrt{2}\sqrt{ib\sqrt{4ac-b^2}-2c(a-c)+b^2}}\right)}{\sqrt{ib\sqrt{4ac-b^2}-2c(a-c)+b^2}}+\frac{\sqrt{2}\left(\frac{2cd-be}{\sqrt{4ac-b^2}}+ie\right)\tan^{-1}\left(\frac{2ic-\tanh\left(\frac{x}{2}\right)\left(\sqrt{4ac-b^2}+ib\right)}{\sqrt{2}\sqrt{-ib\sqrt{4ac-b^2}-2c(a-c)+b^2}}\right)}{\sqrt{-ib\sqrt{4ac-b^2}-2c(a-c)+b^2}}$$

```
[Out] arctan(1/2*(2*I*c-(I*b+(4*a*c-b^2)^(1/2))*tanh(1/2*x))*2^(1/2)/(b^2-2*(a-c)
*c-I*b*(4*a*c-b^2)^(1/2))^(1/2))*2^(1/2)*(I*e+(-b*e+2*c*d)/(4*a*c-b^2)^(1/2
))/(b^2-2*(a-c)*c-I*b*(4*a*c-b^2)^(1/2))^(1/2)+arctan(1/2*(2*I*c-I*b*tanh(1
/2*x)+(4*a*c-b^2)^(1/2)*tanh(1/2*x))*2^(1/2)/(b^2-2*(a-c)*c+I*b*(4*a*c-b^2)
^(1/2))^(1/2))*2^(1/2)*(I*e+(b*e-2*c*d)/(4*a*c-b^2)^(1/2))/(b^2-2*(a-c)*c+I
*b*(4*a*c-b^2)^(1/2))^(1/2)
```

Rubi [A] time = 0.76, antiderivative size = 300, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3292, 2660, 618, 204}

$$\frac{\sqrt{2}\left(-\frac{2cd-be}{\sqrt{4ac-b^2}}+ie\right)\tan^{-1}\left(\frac{\tanh\left(\frac{x}{2}\right)\sqrt{4ac-b^2}-ib\tanh\left(\frac{x}{2}\right)+2ic}{\sqrt{2}\sqrt{ib\sqrt{4ac-b^2}-2c(a-c)+b^2}}\right)}{\sqrt{ib\sqrt{4ac-b^2}-2c(a-c)+b^2}}+\frac{\sqrt{2}\left(\frac{2cd-be}{\sqrt{4ac-b^2}}+ie\right)\tan^{-1}\left(\frac{2ic-\tanh\left(\frac{x}{2}\right)\left(\sqrt{4ac-b^2}+ib\right)}{\sqrt{2}\sqrt{-ib\sqrt{4ac-b^2}-2c(a-c)+b^2}}\right)}{\sqrt{-ib\sqrt{4ac-b^2}-2c(a-c)+b^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*Sinh[x])/(a + b*Sinh[x] + c*Sinh[x]^2),x]
```

```
[Out] (Sqrt[2]*(I*e - (2*c*d - b*e)/Sqrt[-b^2 + 4*a*c])*ArcTan[((2*I)*c - I*b*Tan
h[x/2] + Sqrt[-b^2 + 4*a*c]*Tanh[x/2])/(Sqrt[2]*Sqrt[b^2 - 2*(a - c)*c + I*
b*Sqrt[-b^2 + 4*a*c]])])/Sqrt[b^2 - 2*(a - c)*c + I*b*Sqrt[-b^2 + 4*a*c]] +
 (Sqrt[2]*(I*e + (2*c*d - b*e)/Sqrt[-b^2 + 4*a*c])*ArcTan[((2*I)*c - (I*b +
 Sqrt[-b^2 + 4*a*c])*Tanh[x/2])/(Sqrt[2]*Sqrt[b^2 - 2*(a - c)*c - I*b*Sqrt[
-b^2 + 4*a*c]])])/Sqrt[b^2 - 2*(a - c)*c - I*b*Sqrt[-b^2 + 4*a*c]]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 3292

```
Int[((A_) + (B_.)*sin[(d_.) + (e_.)*(x_)])/((a_.) + (b_.)*sin[(d_.) + (e_.)
*(x_)] + (c_.)*sin[(d_.) + (e_.)*(x_)]^2), x_Symbol] :> Module[{q = Rt[b^2
- 4*a*c, 2]}, Dist[B + (b*B - 2*A*c)/q, Int[1/(b + q + 2*c*Sin[d + e*x]), x
], x] + Dist[B - (b*B - 2*A*c)/q, Int[1/(b - q + 2*c*Sin[d + e*x]), x], x]]
 /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{d+e\sinh(x)}{a+b\sinh(x)+c\sinh^2(x)}\,dx = \left(-ie-\frac{2cd-be}{\sqrt{-b^2+4ac}}\right)\int \frac{1}{-ib-\sqrt{-b^2+4ac}-2ic\sinh(x)}\,dx + \left(-ie+\frac{2cd-}{\sqrt{-b^2}}\right.$$

$$= -\left(\left(2\left(ie-\frac{2cd-be}{\sqrt{-b^2+4ac}}\right)\right)\text{Subst}\left(\int \frac{1}{-ib+\sqrt{-b^2+4ac}-4icx-\left(-ib+\sqrt{-b}\right.}\right.\right.$$

$$= \left(4\left(ie-\frac{2cd-be}{\sqrt{-b^2+4ac}}\right)\right)\text{Subst}\left(\int \frac{1}{-8\left(b^2-2(a-c)c+ib\sqrt{-b^2+4ac}\right)-x^2}\right.$$

$$= \frac{\sqrt{2}\left(ie-\frac{2cd-be}{\sqrt{-b^2+4ac}}\right)\tan^{-1}\left(\frac{2ic-\left(ib-\sqrt{-b^2+4ac}\right)\tanh\left(\frac{x}{2}\right)}{\sqrt{2}\sqrt{b^2-2(a-c)c+ib\sqrt{-b^2+4ac}}}\right)}{\sqrt{b^2-2(a-c)c+ib\sqrt{-b^2+4ac}}} + \frac{\sqrt{2}\left(ie+\frac{2cd-be}{\sqrt{-b^2+4ac}}\right)\text{t}}{\sqrt{b^2-2(a}}$$

Mathematica [A] time = 0.59, size = 258, normalized size = 0.86

$$\frac{\sqrt{2}\left(\frac{\left(e\left(\sqrt{b^2-4ac}-b\right)+2cd\right)\tan^{-1}\left(\frac{\tanh\left(\frac{x}{2}\right)\left(\sqrt{b^2-4ac}-b\right)+2c}{\sqrt{2b\sqrt{b^2-4ac}+4c(a-c)-2b^2}}\right)}{\sqrt{b\sqrt{b^2-4ac}+2c(a-c)-b^2}}+\frac{\left(e\left(\sqrt{b^2-4ac}+b\right)-2cd\right)\tan^{-1}\left(\frac{2c-\tanh\left(\frac{x}{2}\right)\left(\sqrt{b^2-4ac}+b\right)}{\sqrt{2}\sqrt{-b\sqrt{b^2-4ac}+2c(a-c)-b^2}}\right)}{\sqrt{-b\sqrt{b^2-4ac}+2c(a-c)-b^2}}\right)}{\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*Sinh[x])/(a + b*Sinh[x] + c*Sinh[x]^2),x]
```

```
[Out] (Sqrt[2]*(((2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e)*ArcTan[(2*c + (-b + Sqrt[b^
2 - 4*a*c])*Tanh[x/2])/Sqrt[-2*b^2 + 4*(a - c)*c + 2*b*Sqrt[b^2 - 4*a*c]]])
/Sqrt[-b^2 + 2*(a - c)*c + b*Sqrt[b^2 - 4*a*c]] + ((-2*c*d + (b + Sqrt[b^2
- 4*a*c])*e)*ArcTan[(2*c - (b + Sqrt[b^2 - 4*a*c])*Tanh[x/2])/(Sqrt[2]*Sqrt
[-b^2 + 2*(a - c)*c - b*Sqrt[b^2 - 4*a*c]])])/Sqrt[-b^2 + 2*(a - c)*c - b*S
qrt[b^2 - 4*a*c]]))/Sqrt[b^2 - 4*a*c]
```

fricas [B] time = 4.50, size = 6841, normalized size = 22.80

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*sinh(x))/(a+b*sinh(x)+c*sinh(x)^2),x, algorithm="fricas")
```

```
[Out] -1/2*sqrt(2)*sqrt(((b^2 - 2*a*c + 2*c^2)*d^2 - 2*(a*b + b*c)*d*e + (2*a^2 +
 b^2 - 2*a*c)*e^2 + (a^2*b^2 + b^4 - 4*a*c^3 + (8*a^2 + b^2)*c^2 - 2*(2*a^3
 + 3*a*b^2)*c)*sqrt((b^2*d^4 + b^2*e^4 - 4*(a*b - b*c)*d^3*e + 2*(2*a^2 - b
^2 - 4*a*c + 2*c^2)*d^2*e^2 + 4*(a*b - b*c)*d*e^3)/(a^4*b^2 + 2*a^2*b^4 + b
^6 - 4*a*c^5 + (16*a^2 + b^2)*c^4 - 12*(2*a^3 + a*b^2)*c^3 + 2*(8*a^4 + 11*
a^2*b^2 + b^4)*c^2 - 4*(a^5 + 3*a^3*b^2 + 2*a*b^4)*c)))/(a^2*b^2 + b^4 - 4*
a*c^3 + (8*a^2 + b^2)*c^2 - 2*(2*a^3 + 3*a*b^2)*c))*log(-2*b^2*c*d^4 + 2*a*
b^2*e^4 + 2*(b^3 + 2*a*b*c - 2*b*c^2)*d^3*e - 6*(a*b^2 - b^2*c)*d^2*e^2 + 2
*(2*a^2*b - b^3 - 2*a*b*c)*d*e^3 + sqrt(2)*((b^4 - 4*a*b^2*c)*d^3 - 3*(a*b^
3 + 4*a*b*c^2 - (4*a^2*b + b^3)*c)*d^2*e + (2*a^2*b^2 - b^4 - 8*a^3*c - 8*a
*c^3 + 2*(8*a^2 + b^2)*c^2)*d*e^2 + (a*b^3 + 4*a*b*c^2 - (4*a^2*b + b^3)*c)
*e^3 - ((a^2*b^4 + b^6 - 8*a*c^5 + 2*(12*a^2 + b^2)*c^4 - 6*(4*a^3 + 3*a*b^
2)*c^3 + (8*a^4 + 22*a^2*b^2 + 3*b^4)*c^2 - 2*(3*a^3*b^2 + 4*a*b^4)*c)*d -
(a^3*b^3 + a*b^5 - 4*a*b*c^4 + (4*a^2*b + b^3)*c^3 + (4*a^3*b - 5*a*b^3)*c^
2 - (4*a^4*b + 5*a^2*b^3 - b^5)*c)*e)*sqrt((b^2*d^4 + b^2*e^4 - 4*(a*b - b*
c)*d^3*e + 2*(2*a^2 - b^2 - 4*a*c + 2*c^2)*d^2*e^2 + 4*(a*b - b*c)*d*e^3)/(
a^4*b^2 + 2*a^2*b^4 + b^6 - 4*a*c^5 + (16*a^2 + b^2)*c^4 - 12*(2*a^3 + a*b^
2)*c^3 + 2*(8*a^4 + 11*a^2*b^2 + b^4)*c^2 - 4*(a^5 + 3*a^3*b^2 + 2*a*b^4)*c
)))*sqrt(((b^2 - 2*a*c + 2*c^2)*d^2 - 2*(a*b + b*c)*d*e + (2*a^2 + b^2 - 2*
```

```
a*c)*e^2 + (a^2*b^2 + b^4 - 4*a*c^3 + (8*a^2 + b^2)*c^2 - 2*(2*a^3 + 3*a*b^
2)*c)*sqrt((b^2*d^4 + b^2*e^4 - 4*(a*b - b*c)*d^3*e + 2*(2*a^2 - b^2 - 4*a*
c + 2*c^2)*d^2*e^2 + 4*(a*b - b*c)*d*e^3)/(a^4*b^2 + 2*a^2*b^4 + b^6 - 4*a*
c^5 + (16*a^2 + b^2)*c^4 - 12*(2*a^3 + a*b^2)*c^3 + 2*(8*a^4 + 11*a^2*b^2 +
 b^4)*c^2 - 4*(a^5 + 3*a^3*b^2 + 2*a*b^4)*c)))/(a^2*b^2 + b^4 - 4*a*c^3 + (
8*a^2 + b^2)*c^2 - 2*(2*a^3 + 3*a*b^2)*c)) - 4*(b*c^2*d^4 - a*b*c*e^4 - (b^
2*c + 2*a*c^2 - 2*c^3)*d^3*e + 3*(a*b*c - b*c^2)*d^2*e^2 + (2*a*c^2 - (2*a^
2 - b^2)*c)*d*e^3)*cosh(x) - 4*(b*c^2*d^4 - a*b*c*e^4 - (b^2*c + 2*a*c^2 -
2*c^3)*d^3*e + 3*(a*b*c - b*c^2)*d^2*e^2 + (2*a*c^2 - (2*a^2 - b^2)*c)*d*e^
3)*sinh(x) - 2*((4*a*c^4 - (8*a^2 + b^2)*c^3 + 2*(2*a^3 + 3*a*b^2)*c^2 - (a
^2*b^2 + b^4)*c)*d^2 + (a^2*b^3 + b^5 - 4*a*b*c^3 + (8*a^2*b + b^3)*c^2 - 2
*(2*a^3*b + 3*a*b^3)*c)*d*e - (a^3*b^2 + a*b^4 - 4*a^2*c^3 + (8*a^3 + a*b^2
)*c^2 - 2*(2*a^4 + 3*a^2*b^2)*c)*e^2)*sqrt((b^2*d^4 + b^2*e^4 - 4*(a*b - b*
c)*d^3*e + 2*(2*a^2 - b^2 - 4*a*c + 2*c^2)*d^2*e^2 + 4*(a*b - b*c)*d*e^3)/(
a^4*b^2 + 2*a^2*b^4 + b^6 - 4*a*c^5 + (16*a^2 + b^2)*c^4 - 12*(2*a^3 + a*b^
2)*c^3 + 2*(8*a^4 + 11*a^2*b^2 + b^4)*c^2 - 4*(a^5 + 3*a^3*b^2 + 2*a*b^4)*c
))) + 1/2*sqrt(2)*sqrt(((b^2 - 2*a*c + 2*c^2)*d^2 - 2*(a*b + b*c)*d*e + (2*
a^2 + b^2 - 2*a*c)*e^2 + (a^2*b^2 + b^4 - 4*a*c^3 + (8*a^2 + b^2)*c^2 - 2*(
2*a^3 + 3*a*b^2)*c)*sqrt((b^2*d^4 + b^2*e^4 - 4*(a*b - b*c)*d^3*e + 2*(2*a^
2 - b^2 - 4*a*c + 2*c^2)*d^2*e^2 + 4*(a*b - b*c)*d*e^3)/(a^4*b^2 + 2*a^2*b^
4 + b^6 - 4*a*c^5 + (16*a^2 + b^2)*c^4 - 12*(2*a^3 + a*b^2)*c^3 + 2*(8*a^4
+ 11*a^2*b^2 + b^4)*c^2 - 4*(a^5 + 3*a^3*b^2 + 2*a*b^4)*c)))/(a^2*b^2 + b^4
 - 4*a*c^3 + (8*a^2 + b^2)*c^2 - 2*(2*a^3 + 3*a*b^2)*c))*log(-2*b^2*c*d^4 +
 2*a*b^2*e^4 + 2*(b^3 + 2*a*b*c - 2*b*c^2)*d^3*e - 6*(a*b^2 - b^2*c)*d^2*e^
2 + 2*(2*a^2*b - b^3 - 2*a*b*c)*d*e^3 - sqrt(2)*((b^4 - 4*a*b^2*c)*d^3 - 3*
(a*b^3 + 4*a*b*c^2 - (4*a^2*b + b^3)*c)*d^2*e + (2*a^2*b^2 - b^4 - 8*a^3*c
- 8*a*c^3 + 2*(8*a^2 + b^2)*c^2)*d*e^2 + (a*b^3 + 4*a*b*c^2 - (4*a^2*b + b^
3)*c)*e^3 - ((a^2*b^4 + b^6 - 8*a*c^5 + 2*(12*a^2 + b^2)*c^4 - 6*(4*a^3 + 3
*a*b^2)*c^3 + (8*a^4 + 22*a^2*b^2 + 3*b^4)*c^2 - 2*(3*a^3*b^2 + 4*a*b^4)*c)
*d - (a^3*b^3 + a*b^5 - 4*a*b*c^4 + (4*a^2*b + b^3)*c^3 + (4*a^3*b - 5*a*b^
3)*c^2 - (4*a^4*b + 5*a^2*b^3 - b^5)*c)*e)*sqrt((b^2*d^4 + b^2*e^4 - 4*(a*b
 - b*c)*d^3*e + 2*(2*a^2 - b^2 - 4*a*c + 2*c^2)*d^2*e^2 + 4*(a*b - b*c)*d*e
^3)/(a^4*b^2 + 2*a^2*b^4 + b^6 - 4*a*c^5 + (16*a^2 + b^2)*c^4 - 12*(2*a^3 +
 a*b^2)*c^3 + 2*(8*a^4 + 11*a^2*b^2 + b^4)*c^2 - 4*(a^5 + 3*a^3*b^2 + 2*a*b
^4)*c)))*sqrt(((b^2 - 2*a*c + 2*c^2)*d^2 - 2*(a*b + b*c)*d*e + (2*a^2 + b^2
 - 2*a*c)*e^2 + (a^2*b^2 + b^4 - 4*a*c^3 + (8*a^2 + b^2)*c^2 - 2*(2*a^3 + 3
*a*b^2)*c)*sqrt((b^2*d^4 + b^2*e^4 - 4*(a*b - b*c)*d^3*e + 2*(2*a^2 - b^2 -
 4*a*c + 2*c^2)*d^2*e^2 + 4*(a*b - b*c)*d*e^3)/(a^4*b^2 + 2*a^2*b^4 + b^6 -
 4*a*c^5 + (16*a^2 + b^2)*c^4 - 12*(2*a^3 + a*b^2)*c^3 + 2*(8*a^4 + 11*a^2*
b^2 + b^4)*c^2 - 4*(a^5 + 3*a^3*b^2 + 2*a*b^4)*c)))/(a^2*b^2 + b^4 - 4*a*c^
3 + (8*a^2 + b^2)*c^2 - 2*(2*a^3 + 3*a*b^2)*c)) - 4*(b*c^2*d^4 - a*b*c*e^4
- (b^2*c + 2*a*c^2 - 2*c^3)*d^3*e + 3*(a*b*c - b*c^2)*d^2*e^2 + (2*a*c^2 -
(2*a^2 - b^2)*c)*d*e^3)*cosh(x) - 4*(b*c^2*d^4 - a*b*c*e^4 - (b^2*c + 2*a*c
^2 - 2*c^3)*d^3*e + 3*(a*b*c - b*c^2)*d^2*e^2 + (2*a*c^2 - (2*a^2 - b^2)*c)
*d*e^3)*sinh(x) - 2*((4*a*c^4 - (8*a^2 + b^2)*c^3 + 2*(2*a^3 + 3*a*b^2)*c^2
```

- (a^2*b^2 + b^4)*c)*d^2 + (a^2*b^3 + b^5 - 4*a*b*c^3 + (8*a^2*b + b^3)*c^
2 - 2*(2*a^3*b + 3*a*b^3)*c)*d*e - (a^3*b^2 + a*b^4 - 4*a^2*c^3 + (8*a^3 +
a*b^2)*c^2 - 2*(2*a^4 + 3*a^2*b^2)*c)*e^2)*sqrt((b^2*d^4 + b^2*e^4 - 4*(a*b
 - b*c)*d^3*e + 2*(2*a^2 - b^2 - 4*a*c + 2*c^2)*d^2*e^2 + 4*(a*b - b*c)*d*e
^3)/(a^4*b^2 + 2*a^2*b^4 + b^6 - 4*a*c^5 + (16*a^2 + b^2)*c^4 - 12*(2*a^3 +
 a*b^2)*c^3 + 2*(8*a^4 + 11*a^2*b^2 + b^4)*c^2 - 4*(a^5 + 3*a^3*b^2 + 2*a*b
^4)*c))) - 1/2*sqrt(2)*sqrt(((b^2 - 2*a*c + 2*c^2)*d^2 - 2*(a*b + b*c)*d*e
+ (2*a^2 + b^2 - 2*a*c)*e^2 - (a^2*b^2 + b^4 - 4*a*c^3 + (8*a^2 + b^2)*c^2
- 2*(2*a^3 + 3*a*b^2)*c)*sqrt((b^2*d^4 + b^2*e^4 - 4*(a*b - b*c)*d^3*e + 2*
(2*a^2 - b^2 - 4*a*c + 2*c^2)*d^2*e^2 + 4*(a*b - b*c)*d*e^3)/(a^4*b^2 + 2*a
^2*b^4 + b^6 - 4*a*c^5 + (16*a^2 + b^2)*c^4 - 12*(2*a^3 + a*b^2)*c^3 + 2*(8
*a^4 + 11*a^2*b^2 + b^4)*c^2 - 4*(a^5 + 3*a^3*b^2 + 2*a*b^4)*c)))/(a^2*b^2
+ b^4 - 4*a*c^3 + (8*a^2 + b^2)*c^2 - 2*(2*a^3 + 3*a*b^2)*c))*log(-2*b^2*c*
d^4 + 2*a*b^2*e^4 + 2*(b^3 + 2*a*b*c - 2*b*c^2)*d^3*e - 6*(a*b^2 - b^2*c)*d
^2*e^2 + 2*(2*a^2*b - b^3 - 2*a*b*c)*d*e^3 + sqrt(2)*((b^4 - 4*a*b^2*c)*d^3
 - 3*(a*b^3 + 4*a*b*c^2 - (4*a^2*b + b^3)*c)*d^2*e + (2*a^2*b^2 - b^4 - 8*a
^3*c - 8*a*c^3 + 2*(8*a^2 + b^2)*c^2)*d*e^2 + (a*b^3 + 4*a*b*c^2 - (4*a^2*b
 + b^3)*c)*e^3 + ((a^2*b^4 + b^6 - 8*a*c^5 + 2*(12*a^2 + b^2)*c^4 - 6*(4*a^
3 + 3*a*b^2)*c^3 + (8*a^4 + 22*a^2*b^2 + 3*b^4)*c^2 - 2*(3*a^3*b^2 + 4*a*b^
4)*c)*d - (a^3*b^3 + a*b^5 - 4*a*b*c^4 + (4*a^2*b + b^3)*c^3 + (4*a^3*b - 5
*a*b^3)*c^2 - (4*a^4*b + 5*a^2*b^3 - b^5)*c)*e)*sqrt((b^2*d^4 + b^2*e^4 - 4
*(a*b - b*c)*d^3*e + 2*(2*a^2 - b^2 - 4*a*c + 2*c^2)*d^2*e^2 + 4*(a*b - b*c
)*d*e^3)/(a^4*b^2 + 2*a^2*b^4 + b^6 - 4*a*c^5 + (16*a^2 + b^2)*c^4 - 12*(2*
a^3 + a*b^2)*c^3 + 2*(8*a^4 + 11*a^2*b^2 + b^4)*c^2 - 4*(a^5 + 3*a^3*b^2 +
2*a*b^4)*c)))*sqrt(((b^2 - 2*a*c + 2*c^2)*d^2 - 2*(a*b + b*c)*d*e + (2*a^2
+ b^2 - 2*a*c)*e^2 - (a^2*b^2 + b^4 - 4*a*c^3 + (8*a^2 + b^2)*c^2 - 2*(2*a^
3 + 3*a*b^2)*c)*sqrt((b^2*d^4 + b^2*e^4 - 4*(a*b - b*c)*d^3*e + 2*(2*a^2 -
b^2 - 4*a*c + 2*c^2)*d^2*e^2 + 4*(a*b - b*c)*d*e^3)/(a^4*b^2 + 2*a^2*b^4 +
b^6 - 4*a*c^5 + (16*a^2 + b^2)*c^4 - 12*(2*a^3 + a*b^2)*c^3 + 2*(8*a^4 + 11
*a^2*b^2 + b^4)*c^2 - 4*(a^5 + 3*a^3*b^2 + 2*a*b^4)*c)))/(a^2*b^2 + b^4 - 4
*a*c^3 + (8*a^2 + b^2)*c^2 - 2*(2*a^3 + 3*a*b^2)*c)) - 4*(b*c^2*d^4 - a*b*c
*e^4 - (b^2*c + 2*a*c^2 - 2*c^3)*d^3*e + 3*(a*b*c - b*c^2)*d^2*e^2 + (2*a*c
^2 - (2*a^2 - b^2)*c)*d*e^3)*cosh(x) - 4*(b*c^2*d^4 - a*b*c*e^4 - (b^2*c +
2*a*c^2 - 2*c^3)*d^3*e + 3*(a*b*c - b*c^2)*d^2*e^2 + (2*a*c^2 - (2*a^2 - b^
2)*c)*d*e^3)*sinh(x) + 2*((4*a*c^4 - (8*a^2 + b^2)*c^3 + 2*(2*a^3 + 3*a*b^2
)*c^2 - (a^2*b^2 + b^4)*c)*d^2 + (a^2*b^3 + b^5 - 4*a*b*c^3 + (8*a^2*b + b^
3)*c^2 - 2*(2*a^3*b + 3*a*b^3)*c)*d*e - (a^3*b^2 + a*b^4 - 4*a^2*c^3 + (8*a
^3 + a*b^2)*c^2 - 2*(2*a^4 + 3*a^2*b^2)*c)*e^2)*sqrt((b^2*d^4 + b^2*e^4 - 4
*(a*b - b*c)*d^3*e + 2*(2*a^2 - b^2 - 4*a*c + 2*c^2)*d^2*e^2 + 4*(a*b - b*c
)*d*e^3)/(a^4*b^2 + 2*a^2*b^4 + b^6 - 4*a*c^5 + (16*a^2 + b^2)*c^4 - 12*(2*
a^3 + a*b^2)*c^3 + 2*(8*a^4 + 11*a^2*b^2 + b^4)*c^2 - 4*(a^5 + 3*a^3*b^2 +
2*a*b^4)*c))) + 1/2*sqrt(2)*sqrt(((b^2 - 2*a*c + 2*c^2)*d^2 - 2*(a*b + b*c)
*d*e + (2*a^2 + b^2 - 2*a*c)*e^2 - (a^2*b^2 + b^4 - 4*a*c^3 + (8*a^2 + b^2)
c^2 - 2(2*a^3 + 3*a*b^2)*c)*sqrt((b^2*d^4 + b^2*e^4 - 4*(a*b - b*c)*d^3*e
 + 2*(2*a^2 - b^2 - 4*a*c + 2*c^2)*d^2*e^2 + 4*(a*b - b*c)*d*e^3)/(a^4*b^2

```
+ 2*a^2*b^4 + b^6 - 4*a*c^5 + (16*a^2 + b^2)*c^4 - 12*(2*a^3 + a*b^2)*c^3 +
 2*(8*a^4 + 11*a^2*b^2 + b^4)*c^2 - 4*(a^5 + 3*a^3*b^2 + 2*a*b^4)*c)))/(a^2
*b^2 + b^4 - 4*a*c^3 + (8*a^2 + b^2)*c^2 - 2*(2*a^3 + 3*a*b^2)*c))*log(-2*b
^2*c*d^4 + 2*a*b^2*e^4 + 2*(b^3 + 2*a*b*c - 2*b*c^2)*d^3*e - 6*(a*b^2 - b^2
*c)*d^2*e^2 + 2*(2*a^2*b - b^3 - 2*a*b*c)*d*e^3 - sqrt(2)*((b^4 - 4*a*b^2*c
)*d^3 - 3*(a*b^3 + 4*a*b*c^2 - (4*a^2*b + b^3)*c)*d^2*e + (2*a^2*b^2 - b^4
- 8*a^3*c - 8*a*c^3 + 2*(8*a^2 + b^2)*c^2)*d*e^2 + (a*b^3 + 4*a*b*c^2 - (4*
a^2*b + b^3)*c)*e^3 + ((a^2*b^4 + b^6 - 8*a*c^5 + 2*(12*a^2 + b^2)*c^4 - 6*
(4*a^3 + 3*a*b^2)*c^3 + (8*a^4 + 22*a^2*b^2 + 3*b^4)*c^2 - 2*(3*a^3*b^2 + 4
*a*b^4)*c)*d - (a^3*b^3 + a*b^5 - 4*a*b*c^4 + (4*a^2*b + b^3)*c^3 + (4*a^3*
b - 5*a*b^3)*c^2 - (4*a^4*b + 5*a^2*b^3 - b^5)*c)*e)*sqrt((b^2*d^4 + b^2*e^
4 - 4*(a*b - b*c)*d^3*e + 2*(2*a^2 - b^2 - 4*a*c + 2*c^2)*d^2*e^2 + 4*(a*b
- b*c)*d*e^3)/(a^4*b^2 + 2*a^2*b^4 + b^6 - 4*a*c^5 + (16*a^2 + b^2)*c^4 - 1
2*(2*a^3 + a*b^2)*c^3 + 2*(8*a^4 + 11*a^2*b^2 + b^4)*c^2 - 4*(a^5 + 3*a^3*b
^2 + 2*a*b^4)*c)))*sqrt(((b^2 - 2*a*c + 2*c^2)*d^2 - 2*(a*b + b*c)*d*e + (2
*a^2 + b^2 - 2*a*c)*e^2 - (a^2*b^2 + b^4 - 4*a*c^3 + (8*a^2 + b^2)*c^2 - 2*
(2*a^3 + 3*a*b^2)*c)*sqrt((b^2*d^4 + b^2*e^4 - 4*(a*b - b*c)*d^3*e + 2*(2*a
^2 - b^2 - 4*a*c + 2*c^2)*d^2*e^2 + 4*(a*b - b*c)*d*e^3)/(a^4*b^2 + 2*a^2*b
^4 + b^6 - 4*a*c^5 + (16*a^2 + b^2)*c^4 - 12*(2*a^3 + a*b^2)*c^3 + 2*(8*a^4
 + 11*a^2*b^2 + b^4)*c^2 - 4*(a^5 + 3*a^3*b^2 + 2*a*b^4)*c)))/(a^2*b^2 + b^
4 - 4*a*c^3 + (8*a^2 + b^2)*c^2 - 2*(2*a^3 + 3*a*b^2)*c)) - 4*(b*c^2*d^4 -
a*b*c*e^4 - (b^2*c + 2*a*c^2 - 2*c^3)*d^3*e + 3*(a*b*c - b*c^2)*d^2*e^2 + (
2*a*c^2 - (2*a^2 - b^2)*c)*d*e^3)*cosh(x) - 4*(b*c^2*d^4 - a*b*c*e^4 - (b^2
*c + 2*a*c^2 - 2*c^3)*d^3*e + 3*(a*b*c - b*c^2)*d^2*e^2 + (2*a*c^2 - (2*a^2
 - b^2)*c)*d*e^3)*sinh(x) + 2*((4*a*c^4 - (8*a^2 + b^2)*c^3 + 2*(2*a^3 + 3*
a*b^2)*c^2 - (a^2*b^2 + b^4)*c)*d^2 + (a^2*b^3 + b^5 - 4*a*b*c^3 + (8*a^2*b
 + b^3)*c^2 - 2*(2*a^3*b + 3*a*b^3)*c)*d*e - (a^3*b^2 + a*b^4 - 4*a^2*c^3 +
 (8*a^3 + a*b^2)*c^2 - 2*(2*a^4 + 3*a^2*b^2)*c)*e^2)*sqrt((b^2*d^4 + b^2*e^
4 - 4*(a*b - b*c)*d^3*e + 2*(2*a^2 - b^2 - 4*a*c + 2*c^2)*d^2*e^2 + 4*(a*b
- b*c)*d*e^3)/(a^4*b^2 + 2*a^2*b^4 + b^6 - 4*a*c^5 + (16*a^2 + b^2)*c^4 - 1
2*(2*a^3 + a*b^2)*c^3 + 2*(8*a^4 + 11*a^2*b^2 + b^4)*c^2 - 4*(a^5 + 3*a^3*b
^2 + 2*a*b^4)*c)))
```

giac [A] time = 4.28, size = 1, normalized size = 0.00

$$0$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*sinh(x))/(a+b*sinh(x)+c*sinh(x)^2),x, algorithm="giac")
[Out] 0
```

maple [C] time = 0.26, size = 79, normalized size = 0.26

$$\sum_{\_R=\mathrm{RootOf}\left(a\_Z^4-2b\_Z^3+(-2a+4c)\_Z^2+2b\_Z+a\right)}\frac{\left(-\_R^2d+2\_Re+d\right)\ln\left(\tanh\left(\frac{x}{2}\right)-\_R\right)}{2\_R^3a-3b\_R^2-2\_Ra+4\_Rc+b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d+e*sinh(x))/(a+b*sinh(x)+c*sinh(x)^2),x)
```

```
[Out] sum((-_R^2*d+2*_R*e+d)/(2*_R^3*a-3*_R^2*b-2*_R*a+4*_R*c+b)*ln(tanh(1/2*x)-_
R),_R=RootOf(a*_Z^4-2*b*_Z^3+(-2*a+4*c)*_Z^2+2*b*_Z+a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{e\sinh(x)+d}{c\sinh(x)^2+b\sinh(x)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*sinh(x))/(a+b*sinh(x)+c*sinh(x)^2),x, algorithm="maxima")
```

```
[Out] integrate((e*sinh(x) + d)/(c*sinh(x)^2 + b*sinh(x) + a), x)
```

mupad [F(-1)] time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*sinh(x))/(a + c*sinh(x)^2 + b*sinh(x)),x)
```

```
[Out] \text{Hanged}
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*sinh(x))/(a+b*sinh(x)+c*sinh(x)**2),x)
```

```
[Out] Timed out
```

3.832 $\int \frac{1}{a+b\cosh(x)+c\cosh^2(x)}\,dx$

Optimal. Leaf size=223

$$\frac{4c\tanh^{-1}\left(\frac{\tanh\left(\frac{x}{2}\right)\sqrt{-\sqrt{b^2-4ac}+b-2c}}{\sqrt{-\sqrt{b^2-4ac}+b+2c}}\right)}{\sqrt{b^2-4ac}\sqrt{-\sqrt{b^2-4ac}+b-2c}\sqrt{-\sqrt{b^2-4ac}+b+2c}}-\frac{4c\tanh^{-1}\left(\frac{\tanh\left(\frac{x}{2}\right)\sqrt{\sqrt{b^2-4ac}+b-2c}}{\sqrt{\sqrt{b^2-4ac}+b+2c}}\right)}{\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}+b-2c}\sqrt{\sqrt{b^2-4ac}+b+\cdots}}$$

```
[Out] 4*c*arctanh((b-2*c-(-4*a*c+b^2)^(1/2))^(1/2)*tanh(1/2*x)/(b+2*c-(-4*a*c+b^2
)^(1/2))^(1/2))/(-4*a*c+b^2)^(1/2)/(b-2*c-(-4*a*c+b^2)^(1/2))^(1/2)/(b+2*c-
(-4*a*c+b^2)^(1/2))^(1/2)-4*c*arctanh((b-2*c+(-4*a*c+b^2)^(1/2))^(1/2)*tanh
(1/2*x)/(b+2*c+(-4*a*c+b^2)^(1/2))^(1/2))/(-4*a*c+b^2)^(1/2)/(b-2*c+(-4*a*c
+b^2)^(1/2))^(1/2)/(b+2*c+(-4*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A] time = 0.63, antiderivative size = 223, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3249, 2659, 208}

$$\frac{4c\tanh^{-1}\left(\frac{\tanh\left(\frac{x}{2}\right)\sqrt{-\sqrt{b^2-4ac}+b-2c}}{\sqrt{-\sqrt{b^2-4ac}+b+2c}}\right)}{\sqrt{b^2-4ac}\sqrt{-\sqrt{b^2-4ac}+b-2c}\sqrt{-\sqrt{b^2-4ac}+b+2c}}-\frac{4c\tanh^{-1}\left(\frac{\tanh\left(\frac{x}{2}\right)\sqrt{\sqrt{b^2-4ac}+b-2c}}{\sqrt{\sqrt{b^2-4ac}+b+2c}}\right)}{\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}+b-2c}\sqrt{\sqrt{b^2-4ac}+b+\cdots}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cosh[x] + c*Cosh[x]^2)^(-1),x]
```

```
[Out] (4*c*ArcTanh[(Sqrt[b - 2*c - Sqrt[b^2 - 4*a*c]]*Tanh[x/2])/Sqrt[b + 2*c - S
qrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[b - 2*c - Sqrt[b^2 - 4*a*c]]*Sq
rt[b + 2*c - Sqrt[b^2 - 4*a*c]]) - (4*c*ArcTanh[(Sqrt[b - 2*c + Sqrt[b^2 -
4*a*c]]*Tanh[x/2])/Sqrt[b + 2*c + Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*S
qrt[b - 2*c + Sqrt[b^2 - 4*a*c]]*Sqrt[b + 2*c + Sqrt[b^2 - 4*a*c]])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
```

```
&& NeQ[a^2 - b^2, 0]
```

Rule 3249

```
Int[((a_.) + cos[(d_.) + (e_.)*(x_)]^(n_.)*(b_.) + cos[(d_.) + (e_.)*(x_)]^
(n2_.)*(c_.))^(-1), x_Symbol] :> Module[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c
)/q, Int[1/(b - q + 2*c*Cos[d + e*x]^n), x], x] - Dist[(2*c)/q, Int[1/(b +
q + 2*c*Cos[d + e*x]^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2,
 2*n] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{1}{a+b\cosh(x)+c\cosh^2(x)}\,dx = \frac{(2c)\int \frac{1}{b-\sqrt{b^2-4ac}+2c\cosh(x)}\,dx}{\sqrt{b^2-4ac}} - \frac{(2c)\int \frac{1}{b+\sqrt{b^2-4ac}+2c\cosh(x)}\,dx}{\sqrt{b^2-4ac}}$$

$$= \frac{(4c)\,\mathrm{Subst}\left(\int \frac{1}{b+2c-\sqrt{b^2-4ac}-\left(b-2c-\sqrt{b^2-4ac}\right)x^2}\,dx, x, \tanh\left(\frac{x}{2}\right)\right)}{\sqrt{b^2-4ac}} - \frac{(4c)\,\mathrm{Subst}\left(\int \frac{}{b}\right.}{}$$

$$= \frac{4c\tanh^{-1}\left(\frac{\sqrt{b-2c-\sqrt{b^2-4ac}}\,\tanh\left(\frac{x}{2}\right)}{\sqrt{b+2c-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\,\sqrt{b-2c-\sqrt{b^2-4ac}}\,\sqrt{b+2c-\sqrt{b^2-4ac}}} - \frac{4c\tanh^{-}}{\sqrt{b^2-4ac}\,\sqrt{b-2c}}$$

Mathematica [A] time = 0.61, size = 198, normalized size = 0.89

$$\frac{2\sqrt{2}\,c\left(\frac{\tan^{-1}\left(\frac{\tanh\left(\frac{x}{2}\right)\left(\sqrt{b^2-4ac}+b-2c\right)}{\sqrt{-2b\sqrt{b^2-4ac}+4c(a+c)-2b^2}}\right)}{\sqrt{-b\sqrt{b^2-4ac}+2c(a+c)-b^2}}+\frac{\tan^{-1}\left(\frac{\tanh\left(\frac{x}{2}\right)\left(\sqrt{b^2-4ac}-b+2c\right)}{\sqrt{2b\sqrt{b^2-4ac}+4c(a+c)-2b^2}}\right)}{\sqrt{b\sqrt{b^2-4ac}+2c(a+c)-b^2}}\right)}{\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cosh[x] + c*Cosh[x]^2)^(-1),x]
```

```
[Out] (2*Sqrt[2]*c*(ArcTan[((b - 2*c + Sqrt[b^2 - 4*a*c])*Tanh[x/2])/Sqrt[-2*b^2
+ 4*c*(a + c) - 2*b*Sqrt[b^2 - 4*a*c]]]/Sqrt[-b^2 + 2*c*(a + c) - b*Sqrt[b^
2 - 4*a*c]] + ArcTan[((-b + 2*c + Sqrt[b^2 - 4*a*c])*Tanh[x/2])/Sqrt[-2*b^2
 + 4*c*(a + c) + 2*b*Sqrt[b^2 - 4*a*c]]]/Sqrt[-b^2 + 2*c*(a + c) + b*Sqrt[b
^2 - 4*a*c]]))/Sqrt[b^2 - 4*a*c]
```

fricas [B] time = 0.60, size = 3485, normalized size = 15.63

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cosh(x)+c*cosh(x)^2),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(2)*sqrt((b^2 - 2*a*c - 2*c^2 + (a^2*b^2 - b^4 - 4*a*c^3 - (8*a^2 -
 b^2)*c^2 - 2*(2*a^3 - 3*a*b^2)*c)*sqrt(b^2/(a^4*b^2 - 2*a^2*b^4 + b^6 - 4*
a*c^5 - (16*a^2 - b^2)*c^4 - 12*(2*a^3 - a*b^2)*c^3 - 2*(8*a^4 - 11*a^2*b^2
 + b^4)*c^2 - 4*(a^5 - 3*a^3*b^2 + 2*a*b^4)*c)))/(a^2*b^2 - b^4 - 4*a*c^3 -
 (8*a^2 - b^2)*c^2 - 2*(2*a^3 - 3*a*b^2)*c))*log(4*b*c^2*cosh(x) + 4*b*c^2*
sinh(x) + 2*b^2*c + sqrt(2)*(b^4 - 4*a*b^2*c - (a^2*b^4 - b^6 + 8*a*c^5 + 2
*(12*a^2 - b^2)*c^4 + 6*(4*a^3 - 3*a*b^2)*c^3 + (8*a^4 - 22*a^2*b^2 + 3*b^4
)*c^2 - 2*(3*a^3*b^2 - 4*a*b^4)*c)*sqrt(b^2/(a^4*b^2 - 2*a^2*b^4 + b^6 - 4*
a*c^5 - (16*a^2 - b^2)*c^4 - 12*(2*a^3 - a*b^2)*c^3 - 2*(8*a^4 - 11*a^2*b^2
 + b^4)*c^2 - 4*(a^5 - 3*a^3*b^2 + 2*a*b^4)*c)))*sqrt((b^2 - 2*a*c - 2*c^2
+ (a^2*b^2 - b^4 - 4*a*c^3 - (8*a^2 - b^2)*c^2 - 2*(2*a^3 - 3*a*b^2)*c)*sqr
t(b^2/(a^4*b^2 - 2*a^2*b^4 + b^6 - 4*a*c^5 - (16*a^2 - b^2)*c^4 - 12*(2*a^3
 - a*b^2)*c^3 - 2*(8*a^4 - 11*a^2*b^2 + b^4)*c^2 - 4*(a^5 - 3*a^3*b^2 + 2*a
*b^4)*c)))/(a^2*b^2 - b^4 - 4*a*c^3 - (8*a^2 - b^2)*c^2 - 2*(2*a^3 - 3*a*b^
2)*c)) + 2*(4*a*c^4 + (8*a^2 - b^2)*c^3 + 2*(2*a^3 - 3*a*b^2)*c^2 - (a^2*b^
2 - b^4)*c)*sqrt(b^2/(a^4*b^2 - 2*a^2*b^4 + b^6 - 4*a*c^5 - (16*a^2 - b^2)*
c^4 - 12*(2*a^3 - a*b^2)*c^3 - 2*(8*a^4 - 11*a^2*b^2 + b^4)*c^2 - 4*(a^5 -
3*a^3*b^2 + 2*a*b^4)*c))) - 1/2*sqrt(2)*sqrt((b^2 - 2*a*c - 2*c^2 + (a^2*b^
2 - b^4 - 4*a*c^3 - (8*a^2 - b^2)*c^2 - 2*(2*a^3 - 3*a*b^2)*c)*sqrt(b^2/(a^
4*b^2 - 2*a^2*b^4 + b^6 - 4*a*c^5 - (16*a^2 - b^2)*c^4 - 12*(2*a^3 - a*b^2)
*c^3 - 2*(8*a^4 - 11*a^2*b^2 + b^4)*c^2 - 4*(a^5 - 3*a^3*b^2 + 2*a*b^4)*c))
)/(a^2*b^2 - b^4 - 4*a*c^3 - (8*a^2 - b^2)*c^2 - 2*(2*a^3 - 3*a*b^2)*c))*lo
g(4*b*c^2*cosh(x) + 4*b*c^2*sinh(x) + 2*b^2*c - sqrt(2)*(b^4 - 4*a*b^2*c -
(a^2*b^4 - b^6 + 8*a*c^5 + 2*(12*a^2 - b^2)*c^4 + 6*(4*a^3 - 3*a*b^2)*c^3 +
 (8*a^4 - 22*a^2*b^2 + 3*b^4)*c^2 - 2*(3*a^3*b^2 - 4*a*b^4)*c)*sqrt(b^2/(a^
4*b^2 - 2*a^2*b^4 + b^6 - 4*a*c^5 - (16*a^2 - b^2)*c^4 - 12*(2*a^3 - a*b^2)
*c^3 - 2*(8*a^4 - 11*a^2*b^2 + b^4)*c^2 - 4*(a^5 - 3*a^3*b^2 + 2*a*b^4)*c))
)*sqrt((b^2 - 2*a*c - 2*c^2 + (a^2*b^2 - b^4 - 4*a*c^3 - (8*a^2 - b^2)*c^2
- 2*(2*a^3 - 3*a*b^2)*c)*sqrt(b^2/(a^4*b^2 - 2*a^2*b^4 + b^6 - 4*a*c^5 - (1
6*a^2 - b^2)*c^4 - 12*(2*a^3 - a*b^2)*c^3 - 2*(8*a^4 - 11*a^2*b^2 + b^4)*c^
2 - 4*(a^5 - 3*a^3*b^2 + 2*a*b^4)*c)))/(a^2*b^2 - b^4 - 4*a*c^3 - (8*a^2 -
b^2)*c^2 - 2*(2*a^3 - 3*a*b^2)*c)) + 2*(4*a*c^4 + (8*a^2 - b^2)*c^3 + 2*(2*
a^3 - 3*a*b^2)*c^2 - (a^2*b^2 - b^4)*c)*sqrt(b^2/(a^4*b^2 - 2*a^2*b^4 + b^6
 - 4*a*c^5 - (16*a^2 - b^2)*c^4 - 12*(2*a^3 - a*b^2)*c^3 - 2*(8*a^4 - 11*a^
2*b^2 + b^4)*c^2 - 4*(a^5 - 3*a^3*b^2 + 2*a*b^4)*c))) + 1/2*sqrt(2)*sqrt((b
^2 - 2*a*c - 2*c^2 - (a^2*b^2 - b^4 - 4*a*c^3 - (8*a^2 - b^2)*c^2 - 2*(2*a^
3 - 3*a*b^2)*c)*sqrt(b^2/(a^4*b^2 - 2*a^2*b^4 + b^6 - 4*a*c^5 - (16*a^2 - b
```

```
^2)*c^4 - 12*(2*a^3 - a*b^2)*c^3 - 2*(8*a^4 - 11*a^2*b^2 + b^4)*c^2 - 4*(a^
5 - 3*a^3*b^2 + 2*a*b^4)*c)))/(a^2*b^2 - b^4 - 4*a*c^3 - (8*a^2 - b^2)*c^2
- 2*(2*a^3 - 3*a*b^2)*c))*log(4*b*c^2*cosh(x) + 4*b*c^2*sinh(x) + 2*b^2*c +
 sqrt(2)*(b^4 - 4*a*b^2*c + (a^2*b^4 - b^6 + 8*a*c^5 + 2*(12*a^2 - b^2)*c^4
 + 6*(4*a^3 - 3*a*b^2)*c^3 + (8*a^4 - 22*a^2*b^2 + 3*b^4)*c^2 - 2*(3*a^3*b^
2 - 4*a*b^4)*c)*sqrt(b^2/(a^4*b^2 - 2*a^2*b^4 + b^6 - 4*a*c^5 - (16*a^2 - b
^2)*c^4 - 12*(2*a^3 - a*b^2)*c^3 - 2*(8*a^4 - 11*a^2*b^2 + b^4)*c^2 - 4*(a^
5 - 3*a^3*b^2 + 2*a*b^4)*c)))*sqrt((b^2 - 2*a*c - 2*c^2 - (a^2*b^2 - b^4 -
4*a*c^3 - (8*a^2 - b^2)*c^2 - 2*(2*a^3 - 3*a*b^2)*c)*sqrt(b^2/(a^4*b^2 - 2*
a^2*b^4 + b^6 - 4*a*c^5 - (16*a^2 - b^2)*c^4 - 12*(2*a^3 - a*b^2)*c^3 - 2*(
8*a^4 - 11*a^2*b^2 + b^4)*c^2 - 4*(a^5 - 3*a^3*b^2 + 2*a*b^4)*c)))/(a^2*b^2
 - b^4 - 4*a*c^3 - (8*a^2 - b^2)*c^2 - 2*(2*a^3 - 3*a*b^2)*c)) - 2*(4*a*c^4
 + (8*a^2 - b^2)*c^3 + 2*(2*a^3 - 3*a*b^2)*c^2 - (a^2*b^2 - b^4)*c)*sqrt(b^
2/(a^4*b^2 - 2*a^2*b^4 + b^6 - 4*a*c^5 - (16*a^2 - b^2)*c^4 - 12*(2*a^3 - a
*b^2)*c^3 - 2*(8*a^4 - 11*a^2*b^2 + b^4)*c^2 - 4*(a^5 - 3*a^3*b^2 + 2*a*b^4
)*c))) - 1/2*sqrt(2)*sqrt((b^2 - 2*a*c - 2*c^2 - (a^2*b^2 - b^4 - 4*a*c^3 -
 (8*a^2 - b^2)*c^2 - 2*(2*a^3 - 3*a*b^2)*c)*sqrt(b^2/(a^4*b^2 - 2*a^2*b^4 +
 b^6 - 4*a*c^5 - (16*a^2 - b^2)*c^4 - 12*(2*a^3 - a*b^2)*c^3 - 2*(8*a^4 - 1
1*a^2*b^2 + b^4)*c^2 - 4*(a^5 - 3*a^3*b^2 + 2*a*b^4)*c)))/(a^2*b^2 - b^4 -
4*a*c^3 - (8*a^2 - b^2)*c^2 - 2*(2*a^3 - 3*a*b^2)*c))*log(4*b*c^2*cosh(x) +
 4*b*c^2*sinh(x) + 2*b^2*c - sqrt(2)*(b^4 - 4*a*b^2*c + (a^2*b^4 - b^6 + 8*
a*c^5 + 2*(12*a^2 - b^2)*c^4 + 6*(4*a^3 - 3*a*b^2)*c^3 + (8*a^4 - 22*a^2*b^
2 + 3*b^4)*c^2 - 2*(3*a^3*b^2 - 4*a*b^4)*c)*sqrt(b^2/(a^4*b^2 - 2*a^2*b^4 +
 b^6 - 4*a*c^5 - (16*a^2 - b^2)*c^4 - 12*(2*a^3 - a*b^2)*c^3 - 2*(8*a^4 - 1
1*a^2*b^2 + b^4)*c^2 - 4*(a^5 - 3*a^3*b^2 + 2*a*b^4)*c)))*sqrt((b^2 - 2*a*c
 - 2*c^2 - (a^2*b^2 - b^4 - 4*a*c^3 - (8*a^2 - b^2)*c^2 - 2*(2*a^3 - 3*a*b^
2)*c)*sqrt(b^2/(a^4*b^2 - 2*a^2*b^4 + b^6 - 4*a*c^5 - (16*a^2 - b^2)*c^4 -
12*(2*a^3 - a*b^2)*c^3 - 2*(8*a^4 - 11*a^2*b^2 + b^4)*c^2 - 4*(a^5 - 3*a^3*
b^2 + 2*a*b^4)*c)))/(a^2*b^2 - b^4 - 4*a*c^3 - (8*a^2 - b^2)*c^2 - 2*(2*a^3
 - 3*a*b^2)*c)) - 2*(4*a*c^4 + (8*a^2 - b^2)*c^3 + 2*(2*a^3 - 3*a*b^2)*c^2
- (a^2*b^2 - b^4)*c)*sqrt(b^2/(a^4*b^2 - 2*a^2*b^4 + b^6 - 4*a*c^5 - (16*a^
2 - b^2)*c^4 - 12*(2*a^3 - a*b^2)*c^3 - 2*(8*a^4 - 11*a^2*b^2 + b^4)*c^2 -
4*(a^5 - 3*a^3*b^2 + 2*a*b^4)*c)))
```

giac [A] time = 93.58, size = 1, normalized size = 0.00

$$0$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cosh(x)+c*cosh(x)^2),x, algorithm="giac")
```

```
[Out] 0
```

maple [B] time = 0.19, size = 1264, normalized size = 5.67

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*cosh(x)+c*cosh(x)^2),x)
```

```
[Out] -a/(-4*a*c+b^2)^(1/2)/(a-b+c)/(((-4*a*c+b^2)^(1/2)+a-c)*(a-b+c))^(1/2)*arct
anh((-a+b-c)*tanh(1/2*x)/(((-4*a*c+b^2)^(1/2)+a-c)*(a-b+c))^(1/2))*b+2*a/(-
4*a*c+b^2)^(1/2)/(a-b+c)/(((-4*a*c+b^2)^(1/2)+a-c)*(a-b+c))^(1/2)*arctanh((
-a+b-c)*tanh(1/2*x)/(((-4*a*c+b^2)^(1/2)+a-c)*(a-b+c))^(1/2))*c-a/(a-b+c)/(
((-4*a*c+b^2)^(1/2)+a-c)*(a-b+c))^(1/2)*arctanh((-a+b-c)*tanh(1/2*x)/(((-4*
a*c+b^2)^(1/2)+a-c)*(a-b+c))^(1/2))+a/(-4*a*c+b^2)^(1/2)/(a-b+c)/(((-4*a*c+
b^2)^(1/2)-a+c)*(a-b+c))^(1/2)*arctan((a-b+c)*tanh(1/2*x)/(((-4*a*c+b^2)^(1
/2)-a+c)*(a-b+c))^(1/2))*b-2*a/(-4*a*c+b^2)^(1/2)/(a-b+c)/(((-4*a*c+b^2)^(1
/2)-a+c)*(a-b+c))^(1/2)*arctan((a-b+c)*tanh(1/2*x)/(((-4*a*c+b^2)^(1/2)-a+c
)*(a-b+c))^(1/2))*c-a/(a-b+c)/(((-4*a*c+b^2)^(1/2)-a+c)*(a-b+c))^(1/2)*arct
an((a-b+c)*tanh(1/2*x)/(((-4*a*c+b^2)^(1/2)-a+c)*(a-b+c))^(1/2))+1/(-4*a*c+
b^2)^(1/2)/(a-b+c)/(((-4*a*c+b^2)^(1/2)+a-c)*(a-b+c))^(1/2)*arctanh((-a+b-c
)*tanh(1/2*x)/(((-4*a*c+b^2)^(1/2)+a-c)*(a-b+c))^(1/2))*b^2-3*b/(-4*a*c+b^2
)^(1/2)/(a-b+c)/(((-4*a*c+b^2)^(1/2)+a-c)*(a-b+c))^(1/2)*arctanh((-a+b-c)*t
anh(1/2*x)/(((-4*a*c+b^2)^(1/2)+a-c)*(a-b+c))^(1/2))*c+b/(a-b+c)/(((-4*a*c+
b^2)^(1/2)+a-c)*(a-b+c))^(1/2)*arctanh((-a+b-c)*tanh(1/2*x)/(((-4*a*c+b^2)^
(1/2)+a-c)*(a-b+c))^(1/2))-1/(-4*a*c+b^2)^(1/2)/(a-b+c)/(((-4*a*c+b^2)^(1/2
)-a+c)*(a-b+c))^(1/2)*arctan((a-b+c)*tanh(1/2*x)/(((-4*a*c+b^2)^(1/2)-a+c)*
(a-b+c))^(1/2))*b^2+3*b/(-4*a*c+b^2)^(1/2)/(a-b+c)/(((-4*a*c+b^2)^(1/2)-a+c
)*(a-b+c))^(1/2)*arctan((a-b+c)*tanh(1/2*x)/(((-4*a*c+b^2)^(1/2)-a+c)*(a-b+
c))^(1/2))*c+b/(a-b+c)/(((-4*a*c+b^2)^(1/2)-a+c)*(a-b+c))^(1/2)*arctan((a-b
+c)*tanh(1/2*x)/(((-4*a*c+b^2)^(1/2)-a+c)*(a-b+c))^(1/2))+2/(-4*a*c+b^2)^(1
/2)/(a-b+c)/(((-4*a*c+b^2)^(1/2)+a-c)*(a-b+c))^(1/2)*arctanh((-a+b-c)*tanh(
1/2*x)/(((-4*a*c+b^2)^(1/2)+a-c)*(a-b+c))^(1/2))*c^2-c/(a-b+c)/(((-4*a*c+b^
2)^(1/2)+a-c)*(a-b+c))^(1/2)*arctanh((-a+b-c)*tanh(1/2*x)/(((-4*a*c+b^2)^(1
/2)+a-c)*(a-b+c))^(1/2))-2/(-4*a*c+b^2)^(1/2)/(a-b+c)/(((-4*a*c+b^2)^(1/2)-
a+c)*(a-b+c))^(1/2)*arctan((a-b+c)*tanh(1/2*x)/(((-4*a*c+b^2)^(1/2)-a+c)*(a
-b+c))^(1/2))*c^2-c/(a-b+c)/(((-4*a*c+b^2)^(1/2)-a+c)*(a-b+c))^(1/2)*arctan
((a-b+c)*tanh(1/2*x)/(((-4*a*c+b^2)^(1/2)-a+c)*(a-b+c))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{c\cosh(x)^2 + b\cosh(x) + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cosh(x)+c*cosh(x)^2),x, algorithm="maxima")
```

```
[Out] integrate(1/(c*cosh(x)^2 + b*cosh(x) + a), x)
```

mupad [F(-1)] time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*cosh(x) + c*cosh(x)^2),x)
```

```
[Out] \text{Hanged}
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cosh(x)+c*cosh(x)**2),x)
```

```
[Out] Timed out
```

3.833 $\int \frac{\cosh(x)}{a+b\cosh(x)+c\cosh^2(x)}\,dx$

Optimal. Leaf size=230

$$\frac{2\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\tanh^{-1}\left(\frac{\tanh\left(\frac{x}{2}\right)\sqrt{-\sqrt{b^2-4ac}+b-2c}}{\sqrt{-\sqrt{b^2-4ac}+b+2c}}\right)}{\sqrt{-\sqrt{b^2-4ac}+b-2c}\sqrt{-\sqrt{b^2-4ac}+b+2c}}+\frac{2\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\tanh^{-1}\left(\frac{\tanh\left(\frac{x}{2}\right)\sqrt{\sqrt{b^2-4ac}+b-2c}}{\sqrt{\sqrt{b^2-4ac}+b+2c}}\right)}{\sqrt{\sqrt{b^2-4ac}+b-2c}\sqrt{\sqrt{b^2-4ac}+b+2c}}$$

```
[Out] 2*arctanh((b-2*c-(-4*a*c+b^2)^(1/2))^(1/2)*tanh(1/2*x)/(b+2*c-(-4*a*c+b^2)^
(1/2))^(1/2))*(1-b/(-4*a*c+b^2)^(1/2))/(b-2*c-(-4*a*c+b^2)^(1/2))^(1/2)/(b+
2*c-(-4*a*c+b^2)^(1/2))^(1/2)+2*arctanh((b-2*c+(-4*a*c+b^2)^(1/2))^(1/2)*ta
nh(1/2*x)/(b+2*c+(-4*a*c+b^2)^(1/2))^(1/2))*(1+b/(-4*a*c+b^2)^(1/2))/(b-2*c
+(-4*a*c+b^2)^(1/2))^(1/2)/(b+2*c+(-4*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A] time = 0.58, antiderivative size = 230, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {3257, 2659, 208}

$$\frac{2\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\tanh^{-1}\left(\frac{\tanh\left(\frac{x}{2}\right)\sqrt{-\sqrt{b^2-4ac}+b-2c}}{\sqrt{-\sqrt{b^2-4ac}+b+2c}}\right)}{\sqrt{-\sqrt{b^2-4ac}+b-2c}\sqrt{-\sqrt{b^2-4ac}+b+2c}}+\frac{2\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\tanh^{-1}\left(\frac{\tanh\left(\frac{x}{2}\right)\sqrt{\sqrt{b^2-4ac}+b-2c}}{\sqrt{\sqrt{b^2-4ac}+b+2c}}\right)}{\sqrt{\sqrt{b^2-4ac}+b-2c}\sqrt{\sqrt{b^2-4ac}+b+2c}}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]/(a + b*Cosh[x] + c*Cosh[x]^2),x]
```

```
[Out] (2*(1 - b/Sqrt[b^2 - 4*a*c])*ArcTanh[(Sqrt[b - 2*c - Sqrt[b^2 - 4*a*c]]*Tan
h[x/2])/Sqrt[b + 2*c - Sqrt[b^2 - 4*a*c]]])/(Sqrt[b - 2*c - Sqrt[b^2 - 4*a*
c]]*Sqrt[b + 2*c - Sqrt[b^2 - 4*a*c]]) + (2*(1 + b/Sqrt[b^2 - 4*a*c])*ArcTa
nh[(Sqrt[b - 2*c + Sqrt[b^2 - 4*a*c]]*Tanh[x/2])/Sqrt[b + 2*c + Sqrt[b^2 -
4*a*c]]])/(Sqrt[b - 2*c + Sqrt[b^2 - 4*a*c]]*Sqrt[b + 2*c + Sqrt[b^2 - 4*a*
c]])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
```

```
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 3257

```
Int[cos[(d_.) + (e_.)*(x_)]^(m_.)*((a_.) + cos[(d_.) + (e_.)*(x_)]^(n_.)*(b
_.) + cos[(d_.) + (e_.)*(x_)]^(n2_.)*(c_.))^(p_), x_Symbol] :> Int[ExpandTr
ig[cos[d + e*x]^m*(a + b*cos[d + e*x]^n + c*cos[d + e*x]^(2*n))^p, x], x] /
; FreeQ[{a, b, c, d, e}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && Integ
ersQ[m, n, p]
```

Rubi steps

$$\int \frac{\cosh(x)}{a+b\cosh(x)+c\cosh^2(x)}\,dx = \int\left(\frac{1-\frac{b}{\sqrt{b^2-4ac}}}{b-\sqrt{b^2-4ac}+2c\cosh(x)}+\frac{1+\frac{b}{\sqrt{b^2-4ac}}}{b+\sqrt{b^2-4ac}+2c\cosh(x)}\right)dx$$

$$=\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\int\frac{1}{b-\sqrt{b^2-4ac}+2c\cosh(x)}\,dx+\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\int\frac{}{b}$$

$$=\left(2\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\right)\text{Subst}\left(\int\frac{1}{b+2c-\sqrt{b^2-4ac}-\left(b-2c-\sqrt{b^2-4ac}\right)x}\right.$$

$$=\frac{2\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\tanh^{-1}\left(\frac{\sqrt{b-2c-\sqrt{b^2-4ac}}\tanh\left(\frac{x}{2}\right)}{\sqrt{b+2c-\sqrt{b^2-4ac}}}\right)}{\sqrt{b-2c-\sqrt{b^2-4ac}}\sqrt{b+2c-\sqrt{b^2-4ac}}}+\frac{2\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\tanh^{-1}\left(\frac{\sqrt{b}}{}\right.}{\sqrt{b-2c+\sqrt{b^2-4ac}}\sqrt{b}}$$

Mathematica [A] time = 0.55, size = 227, normalized size = 0.99

$$\frac{\sqrt{2}\left(\frac{\left(\sqrt{b^2-4ac}-b\right)\tan^{-1}\left(\frac{\tanh\left(\frac{x}{2}\right)\left(\sqrt{b^2-4ac}-b+2c\right)}{\sqrt{2b\sqrt{b^2-4ac}+4c(a+c)-2b^2}}\right)}{\sqrt{b\sqrt{b^2-4ac}+2c(a+c)-b^2}}-\frac{\left(\sqrt{b^2-4ac}+b\right)\tan^{-1}\left(\frac{\tanh\left(\frac{x}{2}\right)\left(\sqrt{b^2-4ac}+b-2c\right)}{\sqrt{-2b\sqrt{b^2-4ac}+4c(a+c)-2b^2}}\right)}{\sqrt{-b\sqrt{b^2-4ac}+2c(a+c)-b^2}}\right)}{\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]/(a + b*Cosh[x] + c*Cosh[x]^2),x]
```

```
[Out] (Sqrt[2]*(-(((b + Sqrt[b^2 - 4*a*c])*ArcTan[((b - 2*c + Sqrt[b^2 - 4*a*c])*
Tanh[x/2])/Sqrt[-2*b^2 + 4*c*(a + c) - 2*b*Sqrt[b^2 - 4*a*c]]])/Sqrt[-b^2 +
```

```
 2*c*(a + c) - b*Sqrt[b^2 - 4*a*c]]) + ((-b + Sqrt[b^2 - 4*a*c])*ArcTan[((-
b + 2*c + Sqrt[b^2 - 4*a*c])*Tanh[x/2])/Sqrt[-2*b^2 + 4*c*(a + c) + 2*b*Sqr
t[b^2 - 4*a*c]]])/Sqrt[-b^2 + 2*c*(a + c) + b*Sqrt[b^2 - 4*a*c]]))/Sqrt[b^2
 - 4*a*c]
```

fricas [B] time = 0.61, size = 3505, normalized size = 15.24

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)/(a+b*cosh(x)+c*cosh(x)^2),x, algorithm="fricas")
```

```
[Out] -1/2*sqrt(2)*sqrt((2*a^2 - b^2 + 2*a*c + (a^2*b^2 - b^4 - 4*a*c^3 - (8*a^2
- b^2)*c^2 - 2*(2*a^3 - 3*a*b^2)*c)*sqrt(b^2/(a^4*b^2 - 2*a^2*b^4 + b^6 - 4
*a*c^5 - (16*a^2 - b^2)*c^4 - 12*(2*a^3 - a*b^2)*c^3 - 2*(8*a^4 - 11*a^2*b^
2 + b^4)*c^2 - 4*(a^5 - 3*a^3*b^2 + 2*a*b^4)*c)))/(a^2*b^2 - b^4 - 4*a*c^3
- (8*a^2 - b^2)*c^2 - 2*(2*a^3 - 3*a*b^2)*c))*log(4*a*b*c*cosh(x) + 4*a*b*c
*sinh(x) + 2*a*b^2 + sqrt(2)*(a*b^3 - 4*a*b*c^2 - (4*a^2*b - b^3)*c - (a^3*
b^3 - a*b^5 + 4*a*b*c^4 + (4*a^2*b - b^3)*c^3 - (4*a^3*b + 5*a*b^3)*c^2 - (
4*a^4*b - 5*a^2*b^3 - b^5)*c)*sqrt(b^2/(a^4*b^2 - 2*a^2*b^4 + b^6 - 4*a*c^5
 - (16*a^2 - b^2)*c^4 - 12*(2*a^3 - a*b^2)*c^3 - 2*(8*a^4 - 11*a^2*b^2 + b^
4)*c^2 - 4*(a^5 - 3*a^3*b^2 + 2*a*b^4)*c)))*sqrt((2*a^2 - b^2 + 2*a*c + (a^
2*b^2 - b^4 - 4*a*c^3 - (8*a^2 - b^2)*c^2 - 2*(2*a^3 - 3*a*b^2)*c)*sqrt(b^2
/(a^4*b^2 - 2*a^2*b^4 + b^6 - 4*a*c^5 - (16*a^2 - b^2)*c^4 - 12*(2*a^3 - a*
b^2)*c^3 - 2*(8*a^4 - 11*a^2*b^2 + b^4)*c^2 - 4*(a^5 - 3*a^3*b^2 + 2*a*b^4)
*c)))/(a^2*b^2 - b^4 - 4*a*c^3 - (8*a^2 - b^2)*c^2 - 2*(2*a^3 - 3*a*b^2)*c)
) - 2*(a^3*b^2 - a*b^4 - 4*a^2*c^3 - (8*a^3 - a*b^2)*c^2 - 2*(2*a^4 - 3*a^2
*b^2)*c)*sqrt(b^2/(a^4*b^2 - 2*a^2*b^4 + b^6 - 4*a*c^5 - (16*a^2 - b^2)*c^4
 - 12*(2*a^3 - a*b^2)*c^3 - 2*(8*a^4 - 11*a^2*b^2 + b^4)*c^2 - 4*(a^5 - 3*a
^3*b^2 + 2*a*b^4)*c))) + 1/2*sqrt(2)*sqrt((2*a^2 - b^2 + 2*a*c + (a^2*b^2 -
 b^4 - 4*a*c^3 - (8*a^2 - b^2)*c^2 - 2*(2*a^3 - 3*a*b^2)*c)*sqrt(b^2/(a^4*b
^2 - 2*a^2*b^4 + b^6 - 4*a*c^5 - (16*a^2 - b^2)*c^4 - 12*(2*a^3 - a*b^2)*c^
3 - 2*(8*a^4 - 11*a^2*b^2 + b^4)*c^2 - 4*(a^5 - 3*a^3*b^2 + 2*a*b^4)*c)))/(
a^2*b^2 - b^4 - 4*a*c^3 - (8*a^2 - b^2)*c^2 - 2*(2*a^3 - 3*a*b^2)*c))*log(4
*a*b*c*cosh(x) + 4*a*b*c*sinh(x) + 2*a*b^2 - sqrt(2)*(a*b^3 - 4*a*b*c^2 - (
4*a^2*b - b^3)*c - (a^3*b^3 - a*b^5 + 4*a*b*c^4 + (4*a^2*b - b^3)*c^3 - (4*
a^3*b + 5*a*b^3)*c^2 - (4*a^4*b - 5*a^2*b^3 - b^5)*c)*sqrt(b^2/(a^4*b^2 - 2
*a^2*b^4 + b^6 - 4*a*c^5 - (16*a^2 - b^2)*c^4 - 12*(2*a^3 - a*b^2)*c^3 - 2*
(8*a^4 - 11*a^2*b^2 + b^4)*c^2 - 4*(a^5 - 3*a^3*b^2 + 2*a*b^4)*c)))*sqrt((2
*a^2 - b^2 + 2*a*c + (a^2*b^2 - b^4 - 4*a*c^3 - (8*a^2 - b^2)*c^2 - 2*(2*a^
3 - 3*a*b^2)*c)*sqrt(b^2/(a^4*b^2 - 2*a^2*b^4 + b^6 - 4*a*c^5 - (16*a^2 - b
^2)*c^4 - 12*(2*a^3 - a*b^2)*c^3 - 2*(8*a^4 - 11*a^2*b^2 + b^4)*c^2 - 4*(a^
5 - 3*a^3*b^2 + 2*a*b^4)*c)))/(a^2*b^2 - b^4 - 4*a*c^3 - (8*a^2 - b^2)*c^2
- 2*(2*a^3 - 3*a*b^2)*c)) - 2*(a^3*b^2 - a*b^4 - 4*a^2*c^3 - (8*a^3 - a*b^2
)*c^2 - 2*(2*a^4 - 3*a^2*b^2)*c)*sqrt(b^2/(a^4*b^2 - 2*a^2*b^4 + b^6 - 4*a*
```

```
c^5 - (16*a^2 - b^2)*c^4 - 12*(2*a^3 - a*b^2)*c^3 - 2*(8*a^4 - 11*a^2*b^2 +
 b^4)*c^2 - 4*(a^5 - 3*a^3*b^2 + 2*a*b^4)*c))) - 1/2*sqrt(2)*sqrt((2*a^2 -
b^2 + 2*a*c - (a^2*b^2 - b^4 - 4*a*c^3 - (8*a^2 - b^2)*c^2 - 2*(2*a^3 - 3*a
*b^2)*c)*sqrt(b^2/(a^4*b^2 - 2*a^2*b^4 + b^6 - 4*a*c^5 - (16*a^2 - b^2)*c^4
 - 12*(2*a^3 - a*b^2)*c^3 - 2*(8*a^4 - 11*a^2*b^2 + b^4)*c^2 - 4*(a^5 - 3*a
^3*b^2 + 2*a*b^4)*c)))/(a^2*b^2 - b^4 - 4*a*c^3 - (8*a^2 - b^2)*c^2 - 2*(2*
a^3 - 3*a*b^2)*c))*log(4*a*b*c*cosh(x) + 4*a*b*c*sinh(x) + 2*a*b^2 + sqrt(2
)*(a*b^3 - 4*a*b*c^2 - (4*a^2*b - b^3)*c + (a^3*b^3 - a*b^5 + 4*a*b*c^4 + (
4*a^2*b - b^3)*c^3 - (4*a^3*b + 5*a*b^3)*c^2 - (4*a^4*b - 5*a^2*b^3 - b^5)*
c)*sqrt(b^2/(a^4*b^2 - 2*a^2*b^4 + b^6 - 4*a*c^5 - (16*a^2 - b^2)*c^4 - 12*
(2*a^3 - a*b^2)*c^3 - 2*(8*a^4 - 11*a^2*b^2 + b^4)*c^2 - 4*(a^5 - 3*a^3*b^2
 + 2*a*b^4)*c)))*sqrt((2*a^2 - b^2 + 2*a*c - (a^2*b^2 - b^4 - 4*a*c^3 - (8*
a^2 - b^2)*c^2 - 2*(2*a^3 - 3*a*b^2)*c)*sqrt(b^2/(a^4*b^2 - 2*a^2*b^4 + b^6
 - 4*a*c^5 - (16*a^2 - b^2)*c^4 - 12*(2*a^3 - a*b^2)*c^3 - 2*(8*a^4 - 11*a^
2*b^2 + b^4)*c^2 - 4*(a^5 - 3*a^3*b^2 + 2*a*b^4)*c)))/(a^2*b^2 - b^4 - 4*a*
c^3 - (8*a^2 - b^2)*c^2 - 2*(2*a^3 - 3*a*b^2)*c)) + 2*(a^3*b^2 - a*b^4 - 4*
a^2*c^3 - (8*a^3 - a*b^2)*c^2 - 2*(2*a^4 - 3*a^2*b^2)*c)*sqrt(b^2/(a^4*b^2
- 2*a^2*b^4 + b^6 - 4*a*c^5 - (16*a^2 - b^2)*c^4 - 12*(2*a^3 - a*b^2)*c^3 -
 2*(8*a^4 - 11*a^2*b^2 + b^4)*c^2 - 4*(a^5 - 3*a^3*b^2 + 2*a*b^4)*c))) + 1/
2*sqrt(2)*sqrt((2*a^2 - b^2 + 2*a*c - (a^2*b^2 - b^4 - 4*a*c^3 - (8*a^2 - b
^2)*c^2 - 2*(2*a^3 - 3*a*b^2)*c)*sqrt(b^2/(a^4*b^2 - 2*a^2*b^4 + b^6 - 4*a*
c^5 - (16*a^2 - b^2)*c^4 - 12*(2*a^3 - a*b^2)*c^3 - 2*(8*a^4 - 11*a^2*b^2 +
 b^4)*c^2 - 4*(a^5 - 3*a^3*b^2 + 2*a*b^4)*c)))/(a^2*b^2 - b^4 - 4*a*c^3 - (
8*a^2 - b^2)*c^2 - 2*(2*a^3 - 3*a*b^2)*c))*log(4*a*b*c*cosh(x) + 4*a*b*c*si
nh(x) + 2*a*b^2 - sqrt(2)*(a*b^3 - 4*a*b*c^2 - (4*a^2*b - b^3)*c + (a^3*b^3
 - a*b^5 + 4*a*b*c^4 + (4*a^2*b - b^3)*c^3 - (4*a^3*b + 5*a*b^3)*c^2 - (4*a
^4*b - 5*a^2*b^3 - b^5)*c)*sqrt(b^2/(a^4*b^2 - 2*a^2*b^4 + b^6 - 4*a*c^5 -
(16*a^2 - b^2)*c^4 - 12*(2*a^3 - a*b^2)*c^3 - 2*(8*a^4 - 11*a^2*b^2 + b^4)*
c^2 - 4*(a^5 - 3*a^3*b^2 + 2*a*b^4)*c)))*sqrt((2*a^2 - b^2 + 2*a*c - (a^2*b
^2 - b^4 - 4*a*c^3 - (8*a^2 - b^2)*c^2 - 2*(2*a^3 - 3*a*b^2)*c)*sqrt(b^2/(a
^4*b^2 - 2*a^2*b^4 + b^6 - 4*a*c^5 - (16*a^2 - b^2)*c^4 - 12*(2*a^3 - a*b^2
)*c^3 - 2*(8*a^4 - 11*a^2*b^2 + b^4)*c^2 - 4*(a^5 - 3*a^3*b^2 + 2*a*b^4)*c)
))/(a^2*b^2 - b^4 - 4*a*c^3 - (8*a^2 - b^2)*c^2 - 2*(2*a^3 - 3*a*b^2)*c)) +
 2*(a^3*b^2 - a*b^4 - 4*a^2*c^3 - (8*a^3 - a*b^2)*c^2 - 2*(2*a^4 - 3*a^2*b^
2)*c)*sqrt(b^2/(a^4*b^2 - 2*a^2*b^4 + b^6 - 4*a*c^5 - (16*a^2 - b^2)*c^4 -
12*(2*a^3 - a*b^2)*c^3 - 2*(8*a^4 - 11*a^2*b^2 + b^4)*c^2 - 4*(a^5 - 3*a^3*
b^2 + 2*a*b^4)*c)))
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)/(a+b*cosh(x)+c*cosh(x)^2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.14, size = 1262, normalized size = 5.49

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)/(a+b*cosh(x)+c*cosh(x)^2),x)
```

```
[Out] a/(a-b+c)/(((-4*a*c+b^2)^(1/2)+a-c)*(a-b+c))^(1/2)*arctanh((-a+b-c)*tanh(1/
2*x)/(((-4*a*c+b^2)^(1/2)+a-c)*(a-b+c))^(1/2))+2/(-4*a*c+b^2)^(1/2)/(a-b+c)
/(((-4*a*c+b^2)^(1/2)+a-c)*(a-b+c))^(1/2)*arctanh((-a+b-c)*tanh(1/2*x)/(((-
4*a*c+b^2)^(1/2)+a-c)*(a-b+c))^(1/2))*a^2-3*a/(-4*a*c+b^2)^(1/2)/(a-b+c)/((
(-4*a*c+b^2)^(1/2)+a-c)*(a-b+c))^(1/2)*arctanh((-a+b-c)*tanh(1/2*x)/(((-4*a
*c+b^2)^(1/2)+a-c)*(a-b+c))^(1/2))*b-2/(-4*a*c+b^2)^(1/2)/(a-b+c)/(((-4*a*c
+b^2)^(1/2)-a+c)*(a-b+c))^(1/2)*arctan((a-b+c)*tanh(1/2*x)/(((-4*a*c+b^2)^(
1/2)-a+c)*(a-b+c))^(1/2))*a^2+3*a/(-4*a*c+b^2)^(1/2)/(a-b+c)/(((-4*a*c+b^2)
^(1/2)-a+c)*(a-b+c))^(1/2)*arctan((a-b+c)*tanh(1/2*x)/(((-4*a*c+b^2)^(1/2)-
a+c)*(a-b+c))^(1/2))*b+a/(a-b+c)/(((-4*a*c+b^2)^(1/2)-a+c)*(a-b+c))^(1/2)*a
rctan((a-b+c)*tanh(1/2*x)/(((-4*a*c+b^2)^(1/2)-a+c)*(a-b+c))^(1/2))-b/(a-b+
c)/(((-4*a*c+b^2)^(1/2)+a-c)*(a-b+c))^(1/2)*arctanh((-a+b-c)*tanh(1/2*x)/((
(-4*a*c+b^2)^(1/2)+a-c)*(a-b+c))^(1/2))+1/(-4*a*c+b^2)^(1/2)/(a-b+c)/(((-4*
a*c+b^2)^(1/2)+a-c)*(a-b+c))^(1/2)*arctanh((-a+b-c)*tanh(1/2*x)/(((-4*a*c+b
^2)^(1/2)+a-c)*(a-b+c))^(1/2))*b^2-1/(-4*a*c+b^2)^(1/2)/(a-b+c)/(((-4*a*c+b
^2)^(1/2)-a+c)*(a-b+c))^(1/2)*arctan((a-b+c)*tanh(1/2*x)/(((-4*a*c+b^2)^(1/
2)-a+c)*(a-b+c))^(1/2))*b^2-b/(a-b+c)/(((-4*a*c+b^2)^(1/2)-a+c)*(a-b+c))^(1
/2)*arctan((a-b+c)*tanh(1/2*x)/(((-4*a*c+b^2)^(1/2)-a+c)*(a-b+c))^(1/2))+c/
(a-b+c)/(((-4*a*c+b^2)^(1/2)+a-c)*(a-b+c))^(1/2)*arctanh((-a+b-c)*tanh(1/2*
x)/(((-4*a*c+b^2)^(1/2)+a-c)*(a-b+c))^(1/2))+2*a/(-4*a*c+b^2)^(1/2)/(a-b+c)
/(((-4*a*c+b^2)^(1/2)+a-c)*(a-b+c))^(1/2)*arctanh((-a+b-c)*tanh(1/2*x)/(((-
4*a*c+b^2)^(1/2)+a-c)*(a-b+c))^(1/2))*c-b/(-4*a*c+b^2)^(1/2)/(a-b+c)/(((-4*
a*c+b^2)^(1/2)+a-c)*(a-b+c))^(1/2)*arctanh((-a+b-c)*tanh(1/2*x)/(((-4*a*c+b
^2)^(1/2)+a-c)*(a-b+c))^(1/2))*c-2*a/(-4*a*c+b^2)^(1/2)/(a-b+c)/(((-4*a*c+b
^2)^(1/2)-a+c)*(a-b+c))^(1/2)*arctan((a-b+c)*tanh(1/2*x)/(((-4*a*c+b^2)^(1/
2)-a+c)*(a-b+c))^(1/2))*c+b/(-4*a*c+b^2)^(1/2)/(a-b+c)/(((-4*a*c+b^2)^(1/2)
-a+c)*(a-b+c))^(1/2)*arctan((a-b+c)*tanh(1/2*x)/(((-4*a*c+b^2)^(1/2)-a+c)*(
a-b+c))^(1/2))*c+c/(a-b+c)/(((-4*a*c+b^2)^(1/2)-a+c)*(a-b+c))^(1/2)*arctan(
(a-b+c)*tanh(1/2*x)/(((-4*a*c+b^2)^(1/2)-a+c)*(a-b+c))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh(x)}{c\cosh(x)^2 + b\cosh(x) + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)/(a+b*cosh(x)+c*cosh(x)^2),x, algorithm="maxima")
```

```
[Out] integrate(cosh(x)/(c*cosh(x)^2 + b*cosh(x) + a), x)
```

mupad [F(-1)] time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)/(a + b*cosh(x) + c*cosh(x)^2),x)
```

```
[Out] \text{Hanged}
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)/(a+b*cosh(x)+c*cosh(x)**2),x)
```

```
[Out] Timed out
```

3.834 $\int \frac{\cosh^2(x)}{a+b\cosh(x)+c\cosh^2(x)}\,dx$

Optimal. Leaf size=255

$$-\frac{2\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tanh^{-1}\left(\frac{\tanh\left(\frac{x}{2}\right)\sqrt{-\sqrt{b^2-4ac}+b-2c}}{\sqrt{-\sqrt{b^2-4ac}+b+2c}}\right)}{c\sqrt{-\sqrt{b^2-4ac}+b-2c}\sqrt{-\sqrt{b^2-4ac}+b+2c}}-\frac{2\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\tanh^{-1}\left(\frac{\tanh\left(\frac{x}{2}\right)\sqrt{\sqrt{b^2-4ac}+b-2c}}{\sqrt{\sqrt{b^2-4ac}+b+2c}}\right)}{c\sqrt{\sqrt{b^2-4ac}+b-2c}\sqrt{\sqrt{b^2-4ac}+b+2c}}+\frac{x}{c}$$

```
[Out] x/c-2*arctanh((b-2*c-(-4*a*c+b^2)^(1/2))^(1/2)*tanh(1/2*x)/(b+2*c-(-4*a*c+b
^2)^(1/2))^(1/2))*(b+(2*a*c-b^2)/(-4*a*c+b^2)^(1/2))/c/(b-2*c-(-4*a*c+b^2)^
(1/2))^(1/2)/(b+2*c-(-4*a*c+b^2)^(1/2))^(1/2)-2*arctanh((b-2*c+(-4*a*c+b^2)
^(1/2))^(1/2)*tanh(1/2*x)/(b+2*c+(-4*a*c+b^2)^(1/2))^(1/2))*(b+(-2*a*c+b^2)
/(-4*a*c+b^2)^(1/2))/c/(b-2*c+(-4*a*c+b^2)^(1/2))^(1/2)/(b+2*c+(-4*a*c+b^2)
^(1/2))^(1/2)
```

Rubi [A] time = 1.29, antiderivative size = 255, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {3257, 3293, 2659, 208}

$$-\frac{2\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tanh^{-1}\left(\frac{\tanh\left(\frac{x}{2}\right)\sqrt{-\sqrt{b^2-4ac}+b-2c}}{\sqrt{-\sqrt{b^2-4ac}+b+2c}}\right)}{c\sqrt{-\sqrt{b^2-4ac}+b-2c}\sqrt{-\sqrt{b^2-4ac}+b+2c}}-\frac{2\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\tanh^{-1}\left(\frac{\tanh\left(\frac{x}{2}\right)\sqrt{\sqrt{b^2-4ac}+b-2c}}{\sqrt{\sqrt{b^2-4ac}+b+2c}}\right)}{c\sqrt{\sqrt{b^2-4ac}+b-2c}\sqrt{\sqrt{b^2-4ac}+b+2c}}+\frac{x}{c}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]^2/(a + b*Cosh[x] + c*Cosh[x]^2),x]
```

```
[Out] x/c - (2*(b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTanh[(Sqrt[b - 2*c - Sqrt
[b^2 - 4*a*c]]*Tanh[x/2])/Sqrt[b + 2*c - Sqrt[b^2 - 4*a*c]]])/(c*Sqrt[b - 2
*c - Sqrt[b^2 - 4*a*c]]*Sqrt[b + 2*c - Sqrt[b^2 - 4*a*c]]) - (2*(b + (b^2 -
 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTanh[(Sqrt[b - 2*c + Sqrt[b^2 - 4*a*c]]*Tanh[
x/2])/Sqrt[b + 2*c + Sqrt[b^2 - 4*a*c]]])/(c*Sqrt[b - 2*c + Sqrt[b^2 - 4*a*
c]]*Sqrt[b + 2*c + Sqrt[b^2 - 4*a*c]])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 3257

```
Int[cos[(d_.) + (e_.)*(x_)]^(m_.)*((a_.) + cos[(d_.) + (e_.)*(x_)]^(n_.)*(b
_.) + cos[(d_.) + (e_.)*(x_)]^(n2_.)*(c_.))^(p_), x_Symbol] :> Int[ExpandTr
ig[cos[d + e*x]^m*(a + b*cos[d + e*x]^n + c*cos[d + e*x]^(2*n))^p, x], x] /
; FreeQ[{a, b, c, d, e}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && Integ
ersQ[m, n, p]
```

Rule 3293

```
Int[(cos[(d_.) + (e_.)*(x_)]*(B_.) + (A_))/((a_.) + cos[(d_.) + (e_.)*(x_)]
*(b_.) + cos[(d_.) + (e_.)*(x_)]^2*(c_.)), x_Symbol] :> Module[{q = Rt[b^2
- 4*a*c, 2]}, Dist[B + (b*B - 2*A*c)/q, Int[1/(b + q + 2*c*Cos[d + e*x]), x
], x] + Dist[B - (b*B - 2*A*c)/q, Int[1/(b - q + 2*c*Cos[d + e*x]), x], x]]
 /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cosh^2(x)}{a+b\cosh(x)+c\cosh^2(x)}\,dx &= \int \left(\frac{1}{c}+\frac{-a-b\cosh(x)}{c\left(a+b\cosh(x)+c\cosh^2(x)\right)}\right)dx\\
&= \frac{x}{c}+\frac{\int \frac{-a-b\cosh(x)}{a+b\cosh(x)+c\cosh^2(x)}\,dx}{c}\\
&= \frac{x}{c}-\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\int \frac{1}{b-\sqrt{b^2-4ac}+2c\cosh(x)}\,dx}{c}-\frac{\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\int \frac{1}{b+\sqrt{b^2-4ac}+2c\cosh(x)}
\\
&= \frac{x}{c}-\frac{\left(2\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\right)\mathrm{Subst}\left(\int \frac{1}{b+2c-\sqrt{b^2-4ac}-\left(b-2c-\sqrt{b^2-4ac}\right)x^2}\,dx,x,\tanh\left(\frac{x}{2}\right)\right)}{c}\\
&= \frac{x}{c}-\frac{2\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tanh^{-1}\left(\frac{\sqrt{b-2c-\sqrt{b^2-4ac}}\,\tanh\left(\frac{x}{2}\right)}{\sqrt{b+2c-\sqrt{b^2-4ac}}}\right)}{c\sqrt{b-2c-\sqrt{b^2-4ac}}\,\sqrt{b+2c-\sqrt{b^2-4ac}}}-\frac{2\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tanh^{-1}}{c\sqrt{b-2c+\sqrt{b^2-4ac}}}
\end{aligned}$$

Mathematica [A] time = 0.59, size = 264, normalized size = 1.04

$$\frac{\frac{\sqrt{2}\left(b\sqrt{b^2-4ac}-2ac+b^2\right)\tan^{-1}\left(\frac{\tanh\left(\frac{x}{2}\right)\left(\sqrt{b^2-4ac}+b-2c\right)}{\sqrt{-2b\sqrt{b^2-4ac}+4c(a+c)-2b^2}}\right)}{\sqrt{b^2-4ac}\sqrt{-b\sqrt{b^2-4ac}+2c(a+c)-b^2}}-\frac{\sqrt{2}\left(b\sqrt{b^2-4ac}+2ac-b^2\right)\tan^{-1}\left(\frac{\tanh\left(\frac{x}{2}\right)\left(\sqrt{b^2-4ac}-b+2c\right)}{\sqrt{2b\sqrt{b^2-4ac}+4c(a+c)-2b^2}}\right)}{\sqrt{b^2-4ac}\sqrt{b\sqrt{b^2-4ac}+2c(a+c)-b^2}}+x}{c}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]^2/(a + b*Cosh[x] + c*Cosh[x]^2),x]
```

```
[Out] (x + (Sqrt[2]*(b^2 - 2*a*c + b*Sqrt[b^2 - 4*a*c])*ArcTan[((b - 2*c + Sqrt[b
^2 - 4*a*c])*Tanh[x/2])/Sqrt[-2*b^2 + 4*c*(a + c) - 2*b*Sqrt[b^2 - 4*a*c]]]
)/(Sqrt[b^2 - 4*a*c]*Sqrt[-b^2 + 2*c*(a + c) - b*Sqrt[b^2 - 4*a*c]]) - (Sqr
t[2]*(-b^2 + 2*a*c + b*Sqrt[b^2 - 4*a*c])*ArcTan[((-b + 2*c + Sqrt[b^2 - 4*
a*c])*Tanh[x/2])/Sqrt[-2*b^2 + 4*c*(a + c) + 2*b*Sqrt[b^2 - 4*a*c]]])/(Sqrt
[b^2 - 4*a*c]*Sqrt[-b^2 + 2*c*(a + c) + b*Sqrt[b^2 - 4*a*c]]))/c
```

fricas [B] time = 0.86, size = 5079, normalized size = 19.92

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^2/(a+b*cosh(x)+c*cosh(x)^2),x, algorithm="fricas")
```

```
[Out] -1/2*(sqrt(2)*c*sqrt(-(a^2*b^2 - b^4 - 2*a^2*c^2 - 2*(a^3 - 2*a*b^2)*c + (4
*a*c^5 + (8*a^2 - b^2)*c^4 + 2*(2*a^3 - 3*a*b^2)*c^3 - (a^2*b^2 - b^4)*c^2)
*sqrt(-(a^4*b^2 - 2*a^2*b^4 + b^6 + 4*a^2*b^2*c^2 + 4*(a^3*b^2 - a*b^4)*c)/
(4*a*c^9 + (16*a^2 - b^2)*c^8 + 12*(2*a^3 - a*b^2)*c^7 + 2*(8*a^4 - 11*a^2*
b^2 + b^4)*c^6 + 4*(a^5 - 3*a^3*b^2 + 2*a*b^4)*c^5 - (a^4*b^2 - 2*a^2*b^4 +
 b^6)*c^4)))/(4*a*c^5 + (8*a^2 - b^2)*c^4 + 2*(2*a^3 - 3*a*b^2)*c^3 - (a^2*
b^2 - b^4)*c^2))*log(2*a^4*b^2 - 2*a^2*b^4 + 4*a^3*b^2*c + sqrt(2)*(8*a^2*b
^2*c^3 + 2*(2*a^3*b^2 - 3*a*b^4)*c^2 - (a^2*b^4 - b^6)*c - (8*a^2*c^7 + 6*(
4*a^3 - a*b^2)*c^6 + (24*a^4 - 22*a^2*b^2 + b^4)*c^5 + 2*(4*a^5 - 9*a^3*b^2
 + 4*a*b^4)*c^4 - (2*a^4*b^2 - 3*a^2*b^4 + b^6)*c^3)*sqrt(-(a^4*b^2 - 2*a^2
*b^4 + b^6 + 4*a^2*b^2*c^2 + 4*(a^3*b^2 - a*b^4)*c)/(4*a*c^9 + (16*a^2 - b^
2)*c^8 + 12*(2*a^3 - a*b^2)*c^7 + 2*(8*a^4 - 11*a^2*b^2 + b^4)*c^6 + 4*(a^5
 - 3*a^3*b^2 + 2*a*b^4)*c^5 - (a^4*b^2 - 2*a^2*b^4 + b^6)*c^4)))*sqrt(-(a^2
*b^2 - b^4 - 2*a^2*c^2 - 2*(a^3 - 2*a*b^2)*c + (4*a*c^5 + (8*a^2 - b^2)*c^4
 + 2*(2*a^3 - 3*a*b^2)*c^3 - (a^2*b^2 - b^4)*c^2)*sqrt(-(a^4*b^2 - 2*a^2*b^
4 + b^6 + 4*a^2*b^2*c^2 + 4*(a^3*b^2 - a*b^4)*c)/(4*a*c^9 + (16*a^2 - b^2)*
c^8 + 12*(2*a^3 - a*b^2)*c^7 + 2*(8*a^4 - 11*a^2*b^2 + b^4)*c^6 + 4*(a^5 -
3*a^3*b^2 + 2*a*b^4)*c^5 - (a^4*b^2 - 2*a^2*b^4 + b^6)*c^4)))/(4*a*c^5 + (8
*a^2 - b^2)*c^4 + 2*(2*a^3 - 3*a*b^2)*c^3 - (a^2*b^2 - b^4)*c^2)) + 4*(2*a^
3*b*c^2 + (a^4*b - a^2*b^3)*c)*cosh(x) + 4*(2*a^3*b*c^2 + (a^4*b - a^2*b^3)
```

*c)*sinh(x) + 2*(4*a^3*c^5 + (8*a^4 - a^2*b^2)*c^4 + 2*(2*a^5 - 3*a^3*b^2)*
c^3 - (a^4*b^2 - a^2*b^4)*c^2)*sqrt(-(a^4*b^2 - 2*a^2*b^4 + b^6 + 4*a^2*b^2
c^2 + 4(a^3*b^2 - a*b^4)*c)/(4*a*c^9 + (16*a^2 - b^2)*c^8 + 12*(2*a^3 - a
*b^2)*c^7 + 2*(8*a^4 - 11*a^2*b^2 + b^4)*c^6 + 4*(a^5 - 3*a^3*b^2 + 2*a*b^4
)*c^5 - (a^4*b^2 - 2*a^2*b^4 + b^6)*c^4))) - sqrt(2)*c*sqrt(-(a^2*b^2 - b^4
 - 2*a^2*c^2 - 2*(a^3 - 2*a*b^2)*c + (4*a*c^5 + (8*a^2 - b^2)*c^4 + 2*(2*a^
3 - 3*a*b^2)*c^3 - (a^2*b^2 - b^4)*c^2)*sqrt(-(a^4*b^2 - 2*a^2*b^4 + b^6 +
4*a^2*b^2*c^2 + 4*(a^3*b^2 - a*b^4)*c)/(4*a*c^9 + (16*a^2 - b^2)*c^8 + 12*(
2*a^3 - a*b^2)*c^7 + 2*(8*a^4 - 11*a^2*b^2 + b^4)*c^6 + 4*(a^5 - 3*a^3*b^2
+ 2*a*b^4)*c^5 - (a^4*b^2 - 2*a^2*b^4 + b^6)*c^4)))/(4*a*c^5 + (8*a^2 - b^2
)*c^4 + 2*(2*a^3 - 3*a*b^2)*c^3 - (a^2*b^2 - b^4)*c^2))*log(2*a^4*b^2 - 2*a
^2*b^4 + 4*a^3*b^2*c - sqrt(2)*(8*a^2*b^2*c^3 + 2*(2*a^3*b^2 - 3*a*b^4)*c^2
 - (a^2*b^4 - b^6)*c - (8*a^2*c^7 + 6*(4*a^3 - a*b^2)*c^6 + (24*a^4 - 22*a^
2*b^2 + b^4)*c^5 + 2*(4*a^5 - 9*a^3*b^2 + 4*a*b^4)*c^4 - (2*a^4*b^2 - 3*a^2
*b^4 + b^6)*c^3)*sqrt(-(a^4*b^2 - 2*a^2*b^4 + b^6 + 4*a^2*b^2*c^2 + 4*(a^3*
b^2 - a*b^4)*c)/(4*a*c^9 + (16*a^2 - b^2)*c^8 + 12*(2*a^3 - a*b^2)*c^7 + 2*
(8*a^4 - 11*a^2*b^2 + b^4)*c^6 + 4*(a^5 - 3*a^3*b^2 + 2*a*b^4)*c^5 - (a^4*b
^2 - 2*a^2*b^4 + b^6)*c^4)))*sqrt(-(a^2*b^2 - b^4 - 2*a^2*c^2 - 2*(a^3 - 2*
a*b^2)*c + (4*a*c^5 + (8*a^2 - b^2)*c^4 + 2*(2*a^3 - 3*a*b^2)*c^3 - (a^2*b^
2 - b^4)*c^2)*sqrt(-(a^4*b^2 - 2*a^2*b^4 + b^6 + 4*a^2*b^2*c^2 + 4*(a^3*b^2
 - a*b^4)*c)/(4*a*c^9 + (16*a^2 - b^2)*c^8 + 12*(2*a^3 - a*b^2)*c^7 + 2*(8*
a^4 - 11*a^2*b^2 + b^4)*c^6 + 4*(a^5 - 3*a^3*b^2 + 2*a*b^4)*c^5 - (a^4*b^2
- 2*a^2*b^4 + b^6)*c^4)))/(4*a*c^5 + (8*a^2 - b^2)*c^4 + 2*(2*a^3 - 3*a*b^2
)*c^3 - (a^2*b^2 - b^4)*c^2)) + 4*(2*a^3*b*c^2 + (a^4*b - a^2*b^3)*c)*cosh(
x) + 4*(2*a^3*b*c^2 + (a^4*b - a^2*b^3)*c)*sinh(x) + 2*(4*a^3*c^5 + (8*a^4
- a^2*b^2)*c^4 + 2*(2*a^5 - 3*a^3*b^2)*c^3 - (a^4*b^2 - a^2*b^4)*c^2)*sqrt(
-(a^4*b^2 - 2*a^2*b^4 + b^6 + 4*a^2*b^2*c^2 + 4*(a^3*b^2 - a*b^4)*c)/(4*a*c
^9 + (16*a^2 - b^2)*c^8 + 12*(2*a^3 - a*b^2)*c^7 + 2*(8*a^4 - 11*a^2*b^2 +
b^4)*c^6 + 4*(a^5 - 3*a^3*b^2 + 2*a*b^4)*c^5 - (a^4*b^2 - 2*a^2*b^4 + b^6)*
c^4))) + sqrt(2)*c*sqrt(-(a^2*b^2 - b^4 - 2*a^2*c^2 - 2*(a^3 - 2*a*b^2)*c -
 (4*a*c^5 + (8*a^2 - b^2)*c^4 + 2*(2*a^3 - 3*a*b^2)*c^3 - (a^2*b^2 - b^4)*c
^2)*sqrt(-(a^4*b^2 - 2*a^2*b^4 + b^6 + 4*a^2*b^2*c^2 + 4*(a^3*b^2 - a*b^4)*
c)/(4*a*c^9 + (16*a^2 - b^2)*c^8 + 12*(2*a^3 - a*b^2)*c^7 + 2*(8*a^4 - 11*a
^2*b^2 + b^4)*c^6 + 4*(a^5 - 3*a^3*b^2 + 2*a*b^4)*c^5 - (a^4*b^2 - 2*a^2*b^
4 + b^6)*c^4)))/(4*a*c^5 + (8*a^2 - b^2)*c^4 + 2*(2*a^3 - 3*a*b^2)*c^3 - (a
^2*b^2 - b^4)*c^2))*log(2*a^4*b^2 - 2*a^2*b^4 + 4*a^3*b^2*c + sqrt(2)*(8*a^
2*b^2*c^3 + 2*(2*a^3*b^2 - 3*a*b^4)*c^2 - (a^2*b^4 - b^6)*c + (8*a^2*c^7 +
6*(4*a^3 - a*b^2)*c^6 + (24*a^4 - 22*a^2*b^2 + b^4)*c^5 + 2*(4*a^5 - 9*a^3*
b^2 + 4*a*b^4)*c^4 - (2*a^4*b^2 - 3*a^2*b^4 + b^6)*c^3)*sqrt(-(a^4*b^2 - 2*
a^2*b^4 + b^6 + 4*a^2*b^2*c^2 + 4*(a^3*b^2 - a*b^4)*c)/(4*a*c^9 + (16*a^2 -
 b^2)*c^8 + 12*(2*a^3 - a*b^2)*c^7 + 2*(8*a^4 - 11*a^2*b^2 + b^4)*c^6 + 4*(
a^5 - 3*a^3*b^2 + 2*a*b^4)*c^5 - (a^4*b^2 - 2*a^2*b^4 + b^6)*c^4)))*sqrt(-(
a^2*b^2 - b^4 - 2*a^2*c^2 - 2*(a^3 - 2*a*b^2)*c - (4*a*c^5 + (8*a^2 - b^2)*
c^4 + 2*(2*a^3 - 3*a*b^2)*c^3 - (a^2*b^2 - b^4)*c^2)*sqrt(-(a^4*b^2 - 2*a^2
*b^4 + b^6 + 4*a^2*b^2*c^2 + 4*(a^3*b^2 - a*b^4)*c)/(4*a*c^9 + (16*a^2 - b^

```
2)*c^8 + 12*(2*a^3 - a*b^2)*c^7 + 2*(8*a^4 - 11*a^2*b^2 + b^4)*c^6 + 4*(a^5
 - 3*a^3*b^2 + 2*a*b^4)*c^5 - (a^4*b^2 - 2*a^2*b^4 + b^6)*c^4)))/(4*a*c^5 +
 (8*a^2 - b^2)*c^4 + 2*(2*a^3 - 3*a*b^2)*c^3 - (a^2*b^2 - b^4)*c^2)) + 4*(2
*a^3*b*c^2 + (a^4*b - a^2*b^3)*c)*cosh(x) + 4*(2*a^3*b*c^2 + (a^4*b - a^2*b
^3)*c)*sinh(x) - 2*(4*a^3*c^5 + (8*a^4 - a^2*b^2)*c^4 + 2*(2*a^5 - 3*a^3*b^
2)*c^3 - (a^4*b^2 - a^2*b^4)*c^2)*sqrt(-(a^4*b^2 - 2*a^2*b^4 + b^6 + 4*a^2*
b^2*c^2 + 4*(a^3*b^2 - a*b^4)*c)/(4*a*c^9 + (16*a^2 - b^2)*c^8 + 12*(2*a^3
- a*b^2)*c^7 + 2*(8*a^4 - 11*a^2*b^2 + b^4)*c^6 + 4*(a^5 - 3*a^3*b^2 + 2*a*
b^4)*c^5 - (a^4*b^2 - 2*a^2*b^4 + b^6)*c^4))) - sqrt(2)*c*sqrt(-(a^2*b^2 -
b^4 - 2*a^2*c^2 - 2*(a^3 - 2*a*b^2)*c - (4*a*c^5 + (8*a^2 - b^2)*c^4 + 2*(2
*a^3 - 3*a*b^2)*c^3 - (a^2*b^2 - b^4)*c^2)*sqrt(-(a^4*b^2 - 2*a^2*b^4 + b^6
 + 4*a^2*b^2*c^2 + 4*(a^3*b^2 - a*b^4)*c)/(4*a*c^9 + (16*a^2 - b^2)*c^8 + 1
2*(2*a^3 - a*b^2)*c^7 + 2*(8*a^4 - 11*a^2*b^2 + b^4)*c^6 + 4*(a^5 - 3*a^3*b
^2 + 2*a*b^4)*c^5 - (a^4*b^2 - 2*a^2*b^4 + b^6)*c^4)))/(4*a*c^5 + (8*a^2 -
b^2)*c^4 + 2*(2*a^3 - 3*a*b^2)*c^3 - (a^2*b^2 - b^4)*c^2))*log(2*a^4*b^2 -
2*a^2*b^4 + 4*a^3*b^2*c - sqrt(2)*(8*a^2*b^2*c^3 + 2*(2*a^3*b^2 - 3*a*b^4)*
c^2 - (a^2*b^4 - b^6)*c + (8*a^2*c^7 + 6*(4*a^3 - a*b^2)*c^6 + (24*a^4 - 22
*a^2*b^2 + b^4)*c^5 + 2*(4*a^5 - 9*a^3*b^2 + 4*a*b^4)*c^4 - (2*a^4*b^2 - 3*
a^2*b^4 + b^6)*c^3)*sqrt(-(a^4*b^2 - 2*a^2*b^4 + b^6 + 4*a^2*b^2*c^2 + 4*(a
^3*b^2 - a*b^4)*c)/(4*a*c^9 + (16*a^2 - b^2)*c^8 + 12*(2*a^3 - a*b^2)*c^7 +
 2*(8*a^4 - 11*a^2*b^2 + b^4)*c^6 + 4*(a^5 - 3*a^3*b^2 + 2*a*b^4)*c^5 - (a^
4*b^2 - 2*a^2*b^4 + b^6)*c^4)))*sqrt(-(a^2*b^2 - b^4 - 2*a^2*c^2 - 2*(a^3 -
 2*a*b^2)*c - (4*a*c^5 + (8*a^2 - b^2)*c^4 + 2*(2*a^3 - 3*a*b^2)*c^3 - (a^2
*b^2 - b^4)*c^2)*sqrt(-(a^4*b^2 - 2*a^2*b^4 + b^6 + 4*a^2*b^2*c^2 + 4*(a^3*
b^2 - a*b^4)*c)/(4*a*c^9 + (16*a^2 - b^2)*c^8 + 12*(2*a^3 - a*b^2)*c^7 + 2*
(8*a^4 - 11*a^2*b^2 + b^4)*c^6 + 4*(a^5 - 3*a^3*b^2 + 2*a*b^4)*c^5 - (a^4*b
^2 - 2*a^2*b^4 + b^6)*c^4)))/(4*a*c^5 + (8*a^2 - b^2)*c^4 + 2*(2*a^3 - 3*a*
b^2)*c^3 - (a^2*b^2 - b^4)*c^2)) + 4*(2*a^3*b*c^2 + (a^4*b - a^2*b^3)*c)*co
sh(x) + 4*(2*a^3*b*c^2 + (a^4*b - a^2*b^3)*c)*sinh(x) - 2*(4*a^3*c^5 + (8*a
^4 - a^2*b^2)*c^4 + 2*(2*a^5 - 3*a^3*b^2)*c^3 - (a^4*b^2 - a^2*b^4)*c^2)*sq
rt(-(a^4*b^2 - 2*a^2*b^4 + b^6 + 4*a^2*b^2*c^2 + 4*(a^3*b^2 - a*b^4)*c)/(4*
a*c^9 + (16*a^2 - b^2)*c^8 + 12*(2*a^3 - a*b^2)*c^7 + 2*(8*a^4 - 11*a^2*b^2
 + b^4)*c^6 + 4*(a^5 - 3*a^3*b^2 + 2*a*b^4)*c^5 - (a^4*b^2 - 2*a^2*b^4 + b^
6)*c^4))) - 2*x)/c
```

giac [A] time = 1.18, size = 5, normalized size = 0.02

$$\frac{x}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^2/(a+b*cosh(x)+c*cosh(x)^2),x, algorithm="giac")
```

```
[Out] x/c
```

maple [B] time = 0.16, size = 1957, normalized size = 7.67

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^2/(a+b*cosh(x)+c*cosh(x)^2),x)
```

```
[Out] 1/c/(-4*a*c+b^2)^(1/2)/(a-b+c)/(((-4*a*c+b^2)^(1/2)-a+c)*(a-b+c))^(1/2)*arc
tan((a-b+c)*tanh(1/2*x)/(((-4*a*c+b^2)^(1/2)-a+c)*(a-b+c))^(1/2))*a^2*b-2/c
*a/(-4*a*c+b^2)^(1/2)/(a-b+c)/(((-4*a*c+b^2)^(1/2)-a+c)*(a-b+c))^(1/2)*arct
an((a-b+c)*tanh(1/2*x)/(((-4*a*c+b^2)^(1/2)-a+c)*(a-b+c))^(1/2))*b^2-1/c/(-
4*a*c+b^2)^(1/2)/(a-b+c)/(((-4*a*c+b^2)^(1/2)+a-c)*(a-b+c))^(1/2)*arctanh((
-a+b-c)*tanh(1/2*x)/(((-4*a*c+b^2)^(1/2)+a-c)*(a-b+c))^(1/2))*a^2*b+2/c*a/(
-4*a*c+b^2)^(1/2)/(a-b+c)/(((-4*a*c+b^2)^(1/2)+a-c)*(a-b+c))^(1/2)*arctanh(
(-a+b-c)*tanh(1/2*x)/(((-4*a*c+b^2)^(1/2)+a-c)*(a-b+c))^(1/2))*b^2-2/(-4*a*
c+b^2)^(1/2)/(a-b+c)/(((-4*a*c+b^2)^(1/2)+a-c)*(a-b+c))^(1/2)*arctanh((-a+b
-c)*tanh(1/2*x)/(((-4*a*c+b^2)^(1/2)+a-c)*(a-b+c))^(1/2))*a^2+a/(a-b+c)/(((
-4*a*c+b^2)^(1/2)-a+c)*(a-b+c))^(1/2)*arctan((a-b+c)*tanh(1/2*x)/(((-4*a*c+
b^2)^(1/2)-a+c)*(a-b+c))^(1/2))-b/(a-b+c)/(((-4*a*c+b^2)^(1/2)+a-c)*(a-b+c)
)^(1/2)*arctanh((-a+b-c)*tanh(1/2*x)/(((-4*a*c+b^2)^(1/2)+a-c)*(a-b+c))^(1/
2))-b/(a-b+c)/(((-4*a*c+b^2)^(1/2)-a+c)*(a-b+c))^(1/2)*arctan((a-b+c)*tanh(
1/2*x)/(((-4*a*c+b^2)^(1/2)-a+c)*(a-b+c))^(1/2))+a/(a-b+c)/(((-4*a*c+b^2)^(
1/2)+a-c)*(a-b+c))^(1/2)*arctanh((-a+b-c)*tanh(1/2*x)/(((-4*a*c+b^2)^(1/2)+
a-c)*(a-b+c))^(1/2))-a/(-4*a*c+b^2)^(1/2)/(a-b+c)/(((-4*a*c+b^2)^(1/2)-a+c)
*(a-b+c))^(1/2)*arctan((a-b+c)*tanh(1/2*x)/(((-4*a*c+b^2)^(1/2)-a+c)*(a-b+c
))^(1/2))*b+2*a/(-4*a*c+b^2)^(1/2)/(a-b+c)/(((-4*a*c+b^2)^(1/2)-a+c)*(a-b+c
))^(1/2)*arctan((a-b+c)*tanh(1/2*x)/(((-4*a*c+b^2)^(1/2)-a+c)*(a-b+c))^(1/2
))*c+1/c/(a-b+c)/(((-4*a*c+b^2)^(1/2)+a-c)*(a-b+c))^(1/2)*arctanh((-a+b-c)*
tanh(1/2*x)/(((-4*a*c+b^2)^(1/2)+a-c)*(a-b+c))^(1/2))*a^2+1/c/(a-b+c)/(((-4
*a*c+b^2)^(1/2)-a+c)*(a-b+c))^(1/2)*arctan((a-b+c)*tanh(1/2*x)/(((-4*a*c+b^
2)^(1/2)-a+c)*(a-b+c))^(1/2))*a^2+1/c/(a-b+c)/(((-4*a*c+b^2)^(1/2)+a-c)*(a-
b+c))^(1/2)*arctanh((-a+b-c)*tanh(1/2*x)/(((-4*a*c+b^2)^(1/2)+a-c)*(a-b+c))
^(1/2))*b^2+1/c/(a-b+c)/(((-4*a*c+b^2)^(1/2)-a+c)*(a-b+c))^(1/2)*arctan((a-
b+c)*tanh(1/2*x)/(((-4*a*c+b^2)^(1/2)-a+c)*(a-b+c))^(1/2))*b^2+2/(-4*a*c+b^
2)^(1/2)/(a-b+c)/(((-4*a*c+b^2)^(1/2)-a+c)*(a-b+c))^(1/2)*arctan((a-b+c)*ta
nh(1/2*x)/(((-4*a*c+b^2)^(1/2)-a+c)*(a-b+c))^(1/2))*a^2-1/c*ln(tanh(1/2*x)-
1)+1/c*ln(tanh(1/2*x)+1)-2/c*a/(a-b+c)/(((-4*a*c+b^2)^(1/2)+a-c)*(a-b+c))^(
1/2)*arctanh((-a+b-c)*tanh(1/2*x)/(((-4*a*c+b^2)^(1/2)+a-c)*(a-b+c))^(1/2))
*b-2/c*a/(a-b+c)/(((-4*a*c+b^2)^(1/2)-a+c)*(a-b+c))^(1/2)*arctan((a-b+c)*ta
nh(1/2*x)/(((-4*a*c+b^2)^(1/2)-a+c)*(a-b+c))^(1/2))*b-1/c/(-4*a*c+b^2)^(1/2
)/(a-b+c)/(((-4*a*c+b^2)^(1/2)+a-c)*(a-b+c))^(1/2)*arctanh((-a+b-c)*tanh(1/
2*x)/(((-4*a*c+b^2)^(1/2)+a-c)*(a-b+c))^(1/2))*b^3+1/c/(-4*a*c+b^2)^(1/2)/(
a-b+c)/(((-4*a*c+b^2)^(1/2)-a+c)*(a-b+c))^(1/2)*arctan((a-b+c)*tanh(1/2*x)/
(((-4*a*c+b^2)^(1/2)-a+c)*(a-b+c))^(1/2))*b^3+1/(-4*a*c+b^2)^(1/2)/(a-b+c)/
```

```
(((-4*a*c+b^2)^(1/2)+a-c)*(a-b+c))^(1/2)*arctanh((-a+b-c)*tanh(1/2*x)/(((-4
*a*c+b^2)^(1/2)+a-c)*(a-b+c))^(1/2))*b^2-1/(-4*a*c+b^2)^(1/2)/(a-b+c)/(((-4
*a*c+b^2)^(1/2)-a+c)*(a-b+c))^(1/2)*arctan((a-b+c)*tanh(1/2*x)/(((-4*a*c+b^
2)^(1/2)-a+c)*(a-b+c))^(1/2))*b^2+a/(-4*a*c+b^2)^(1/2)/(a-b+c)/(((-4*a*c+b^
2)^(1/2)+a-c)*(a-b+c))^(1/2)*arctanh((-a+b-c)*tanh(1/2*x)/(((-4*a*c+b^2)^(1
/2)+a-c)*(a-b+c))^(1/2))*b-2*a/(-4*a*c+b^2)^(1/2)/(a-b+c)/(((-4*a*c+b^2)^(1
/2)+a-c)*(a-b+c))^(1/2)*arctanh((-a+b-c)*tanh(1/2*x)/(((-4*a*c+b^2)^(1/2)+a
-c)*(a-b+c))^(1/2))*c
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{x}{c} - \frac{1}{4} \int \frac{8 \left(be^{(3x)} + 2\,ae^{(2x)} + be^{x}\right)}{c^2e^{(4x)} + 2\,bce^{(3x)} + 2\,bce^{x} + c^2 + 2\left(2\,ac + c^2\right)e^{(2x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^2/(a+b*cosh(x)+c*cosh(x)^2),x, algorithm="maxima")
```

```
[Out] x/c - 1/4*integrate(8*(b*e^(3*x) + 2*a*e^(2*x) + b*e^x)/(c^2*e^(4*x) + 2*b*
c*e^(3*x) + 2*b*c*e^x + c^2 + 2*(2*a*c + c^2)*e^(2*x)), x)
```

mupad [F(-1)] time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^2/(a + b*cosh(x) + c*cosh(x)^2),x)
```

```
[Out] \text{Hanged}
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)**2/(a+b*cosh(x)+c*cosh(x)**2),x)
```

```
[Out] Timed out
```

3.835 $\int \frac{\cosh^3(x)}{a+b\cosh(x)+c\cosh^2(x)}\,dx$

Optimal. Leaf size=299

$$\frac{2\left(\frac{3abc}{\sqrt{b^2-4ac}}-\frac{b^3}{\sqrt{b^2-4ac}}-ac+b^2\right)\tanh^{-1}\left(\frac{\tanh\left(\frac{x}{2}\right)\sqrt{-\sqrt{b^2-4ac}+b-2c}}{\sqrt{-\sqrt{b^2-4ac}+b+2c}}\right)}{c^2\sqrt{-\sqrt{b^2-4ac}+b-2c}\,\sqrt{-\sqrt{b^2-4ac}+b+2c}}+\frac{2\left(-\frac{3abc}{\sqrt{b^2-4ac}}+\frac{b^3}{\sqrt{b^2-4ac}}-ac+b^2\right)\tanh^{-1}\left(\frac{\tanh\left(\frac{x}{2}\right)\cdots}{\sqrt{\cdots}}\right)}{c^2\sqrt{\sqrt{b^2-4ac}+b-2c}\,\sqrt{\sqrt{b^2-4ac}+\cdots}}$$

```
[Out] -b*x/c^2+sinh(x)/c+2*arctanh((b-2*c-(-4*a*c+b^2)^(1/2))^(1/2)*tanh(1/2*x)/(
b+2*c-(-4*a*c+b^2)^(1/2))^(1/2))*(b^2-a*c-b^3/(-4*a*c+b^2)^(1/2)+3*a*b*c/(-
4*a*c+b^2)^(1/2))/c^2/(b-2*c-(-4*a*c+b^2)^(1/2))^(1/2)/(b+2*c-(-4*a*c+b^2)^
(1/2))^(1/2)+2*arctanh((b-2*c+(-4*a*c+b^2)^(1/2))^(1/2)*tanh(1/2*x)/(b+2*c+
(-4*a*c+b^2)^(1/2))^(1/2))*(b^2-a*c+b^3/(-4*a*c+b^2)^(1/2)-3*a*b*c/(-4*a*c+
b^2)^(1/2))/c^2/(b-2*c+(-4*a*c+b^2)^(1/2))^(1/2)/(b+2*c+(-4*a*c+b^2)^(1/2))
^(1/2)
```

Rubi [A] time = 6.39, antiderivative size = 299, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {3257, 2637, 3293, 2659, 208}

$$\frac{2\left(-\frac{b^3}{\sqrt{b^2-4ac}}+\frac{3abc}{\sqrt{b^2-4ac}}-ac+b^2\right)\tanh^{-1}\left(\frac{\tanh\left(\frac{x}{2}\right)\sqrt{-\sqrt{b^2-4ac}+b-2c}}{\sqrt{-\sqrt{b^2-4ac}+b+2c}}\right)}{c^2\sqrt{-\sqrt{b^2-4ac}+b-2c}\,\sqrt{-\sqrt{b^2-4ac}+b+2c}}+\frac{2\left(\frac{b^3}{\sqrt{b^2-4ac}}-\frac{3abc}{\sqrt{b^2-4ac}}-ac+b^2\right)\tanh^{-1}\left(\frac{\tanh\left(\frac{x}{2}\right)\cdots}{\sqrt{\cdots}}\right)}{c^2\sqrt{\sqrt{b^2-4ac}+b-2c}\,\sqrt{\sqrt{b^2-4ac}+\cdots}}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]^3/(a + b*Cosh[x] + c*Cosh[x]^2),x]
```

```
[Out] -((b*x)/c^2) + (2*(b^2 - a*c - b^3/Sqrt[b^2 - 4*a*c] + (3*a*b*c)/Sqrt[b^2 -
 4*a*c])*ArcTanh[(Sqrt[b - 2*c - Sqrt[b^2 - 4*a*c]]*Tanh[x/2])/Sqrt[b + 2*c
 - Sqrt[b^2 - 4*a*c]]])/(c^2*Sqrt[b - 2*c - Sqrt[b^2 - 4*a*c]]*Sqrt[b + 2*c
 - Sqrt[b^2 - 4*a*c]]) + (2*(b^2 - a*c + b^3/Sqrt[b^2 - 4*a*c] - (3*a*b*c)/
Sqrt[b^2 - 4*a*c])*ArcTanh[(Sqrt[b - 2*c + Sqrt[b^2 - 4*a*c]]*Tanh[x/2])/Sq
rt[b + 2*c + Sqrt[b^2 - 4*a*c]]])/(c^2*Sqrt[b - 2*c + Sqrt[b^2 - 4*a*c]]*Sq
rt[b + 2*c + Sqrt[b^2 - 4*a*c]]) + Sinh[x]/c
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 3257

```
Int[cos[(d_.) + (e_.)*(x_)]^(m_.)*((a_.) + cos[(d_.) + (e_.)*(x_)]^(n_.)*(b
_.) + cos[(d_.) + (e_.)*(x_)]^(n2_.)*(c_.))^(p_), x_Symbol] :> Int[ExpandTr
ig[cos[d + e*x]^m*(a + b*cos[d + e*x]^n + c*cos[d + e*x]^(2*n))^p, x], x] /
; FreeQ[{a, b, c, d, e}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && Integ
ersQ[m, n, p]
```

Rule 3293

```
Int[(cos[(d_.) + (e_.)*(x_)]*(B_.) + (A_))/((a_.) + cos[(d_.) + (e_.)*(x_)]
*(b_.) + cos[(d_.) + (e_.)*(x_)]^2*(c_.)), x_Symbol] :> Module[{q = Rt[b^2
- 4*a*c, 2]}, Dist[B + (b*B - 2*A*c)/q, Int[1/(b + q + 2*c*Cos[d + e*x]), x
], x] + Dist[B - (b*B - 2*A*c)/q, Int[1/(b - q + 2*c*Cos[d + e*x]), x], x]]
 /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{\cosh^3(x)}{a+b\cosh(x)+c\cosh^2(x)}\,dx = \int \left(-\frac{b}{c^2}+\frac{\cosh(x)}{c}+\frac{ab+b^2\left(1-\frac{ac}{b^2}\right)\cosh(x)}{c^2\left(a+b\cosh(x)+c\cosh^2(x)\right)}\right)dx$$

$$= -\frac{bx}{c^2}+\frac{\int \frac{ab+b^2\left(1-\frac{ac}{b^2}\right)\cosh(x)}{a+b\cosh(x)+c\cosh^2(x)}\,dx}{c^2}+\frac{\int \cosh(x)\,dx}{c}$$

$$= -\frac{bx}{c^2}+\frac{\sinh(x)}{c}+\frac{\left(b^2-ac+\frac{b^3}{\sqrt{b^2-4ac}}-\frac{3abc}{\sqrt{b^2-4ac}}\right)\int \frac{1}{b+\sqrt{b^2-4ac}+2c\cosh(x)}\,dx}{c^2}+ \frac{(b}{}$$

$$= -\frac{bx}{c^2}+\frac{\sinh(x)}{c}+\frac{\left(2\left(b^2-ac+\frac{b^3}{\sqrt{b^2-4ac}}-\frac{3abc}{\sqrt{b^2-4ac}}\right)\right)\text{Subst}\left(\int \frac{1}{b+2c+\sqrt{b^2-4ac}-(b}\right.}{c^2}$$

$$= -\frac{bx}{c^2}+\frac{2\left(b^2-ac-\frac{b^3}{\sqrt{b^2-4ac}}+\frac{3abc}{\sqrt{b^2-4ac}}\right)\tanh^{-1}\left(\frac{\sqrt{b-2c-\sqrt{b^2-4ac}}\,\tanh\left(\frac{x}{2}\right)}{\sqrt{b+2c-\sqrt{b^2-4ac}}}\right)}{c^2\sqrt{b-2c-\sqrt{b^2-4ac}}\,\sqrt{b+2c-\sqrt{b^2-4ac}}}+\frac{2(b}{}$$

Mathematica [A] time = 0.80, size = 309, normalized size = 1.03

$$\frac{-\frac{\sqrt{2}\left(b^2\sqrt{b^2-4ac}-ac\sqrt{b^2-4ac}-3abc+b^3\right)\tan^{-1}\left(\frac{\tanh\left(\frac{x}{2}\right)\left(\sqrt{b^2-4ac}+b-2c\right)}{\sqrt{-2b\sqrt{b^2-4ac}+4c(a+c)-2b^2}}\right)}{\sqrt{b^2-4ac}\sqrt{-b\sqrt{b^2-4ac}+2c(a+c)-b^2}}+\frac{\sqrt{2}\left(b^2\sqrt{b^2-4ac}-ac\sqrt{b^2-4ac}+3abc-b^3\right)\tan^{-1}\left(\frac{\tanh\left(\frac{x}{2}\right)\left(\sqrt{b^2-4ac}-b+2\right.}{\sqrt{2b\sqrt{b^2-4ac}+4c(a+c)-2}}\right.}{\sqrt{b^2-4ac}\sqrt{b\sqrt{b^2-4ac}+2c(a+c)-b^2}}}{c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]^3/(a + b*Cosh[x] + c*Cosh[x]^2),x]
```

```
[Out] (-(b*x) - (Sqrt[2]*(b^3 - 3*a*b*c + b^2*Sqrt[b^2 - 4*a*c] - a*c*Sqrt[b^2 -
4*a*c])*ArcTan[((b - 2*c + Sqrt[b^2 - 4*a*c])*Tanh[x/2])/Sqrt[-2*b^2 + 4*c*
(a + c) - 2*b*Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[-b^2 + 2*c*(a +
c) - b*Sqrt[b^2 - 4*a*c]]) + (Sqrt[2]*(-b^3 + 3*a*b*c + b^2*Sqrt[b^2 - 4*a*
c] - a*c*Sqrt[b^2 - 4*a*c])*ArcTan[((-b + 2*c + Sqrt[b^2 - 4*a*c])*Tanh[x/2
])/Sqrt[-2*b^2 + 4*c*(a + c) + 2*b*Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*
Sqrt[-b^2 + 2*c*(a + c) + b*Sqrt[b^2 - 4*a*c]]) + c*Sinh[x])/c^2
```

fricas [B] time = 1.40, size = 6794, normalized size = 22.72

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^3/(a+b*cosh(x)+c*cosh(x)^2),x, algorithm="fricas")

[Out] -1/2*(2*b*x*cosh(x) - c*cosh(x)^2 + sqrt(2)*(c^2*cosh(x) + c^2*sinh(x))*sqr
t(-(a^2*b^4 - b^6 + 2*a^3*c^3 + (2*a^4 - 9*a^2*b^2)*c^2 - 2*(2*a^3*b^2 - 3*
a*b^4)*c + (4*a*c^7 + (8*a^2 - b^2)*c^6 + 2*(2*a^3 - 3*a*b^2)*c^5 - (a^2*b^
2 - b^4)*c^4)*sqrt(-(a^4*b^6 - 2*a^2*b^8 + b^10 + 9*a^4*b^2*c^4 + 12*(a^5*b
^2 - 2*a^3*b^4)*c^3 + 2*(2*a^6*b^2 - 11*a^4*b^4 + 11*a^2*b^6)*c^2 - 4*(a^5*
b^4 - 3*a^3*b^6 + 2*a*b^8)*c)/(4*a*c^13 + (16*a^2 - b^2)*c^12 + 12*(2*a^3 -
 a*b^2)*c^11 + 2*(8*a^4 - 11*a^2*b^2 + b^4)*c^10 + 4*(a^5 - 3*a^3*b^2 + 2*a
*b^4)*c^9 - (a^4*b^2 - 2*a^2*b^4 + b^6)*c^8)))/(4*a*c^7 + (8*a^2 - b^2)*c^6
 + 2*(2*a^3 - 3*a*b^2)*c^5 - (a^2*b^2 - b^4)*c^4))*log(-2*a^5*b^4 + 2*a^3*b
^6 + 6*a^5*b^2*c^2 + 4*(a^6*b^2 - 2*a^4*b^4)*c + sqrt(2)*(12*a^4*b*c^5 + (2
0*a^5*b - 31*a^3*b^3)*c^4 + (8*a^6*b - 33*a^4*b^3 + 27*a^2*b^5)*c^3 - 3*(2*
a^5*b^3 - 5*a^3*b^5 + 3*a*b^7)*c^2 + (a^4*b^5 - 2*a^2*b^7 + b^9)*c - (12*a^
2*b*c^9 + 7*(4*a^3*b - a*b^3)*c^8 + (20*a^4*b - 27*a^2*b^3 + b^5)*c^7 + (4*
a^5*b - 13*a^3*b^3 + 9*a*b^5)*c^6 - (a^4*b^3 - 2*a^2*b^5 + b^7)*c^5)*sqrt(-
(a^4*b^6 - 2*a^2*b^8 + b^10 + 9*a^4*b^2*c^4 + 12*(a^5*b^2 - 2*a^3*b^4)*c^3
+ 2*(2*a^6*b^2 - 11*a^4*b^4 + 11*a^2*b^6)*c^2 - 4*(a^5*b^4 - 3*a^3*b^6 + 2*
a*b^8)*c)/(4*a*c^13 + (16*a^2 - b^2)*c^12 + 12*(2*a^3 - a*b^2)*c^11 + 2*(8*
a^4 - 11*a^2*b^2 + b^4)*c^10 + 4*(a^5 - 3*a^3*b^2 + 2*a*b^4)*c^9 - (a^4*b^2
 - 2*a^2*b^4 + b^6)*c^8)))*sqrt(-(a^2*b^4 - b^6 + 2*a^3*c^3 + (2*a^4 - 9*a^
2*b^2)*c^2 - 2*(2*a^3*b^2 - 3*a*b^4)*c + (4*a*c^7 + (8*a^2 - b^2)*c^6 + 2*(
2*a^3 - 3*a*b^2)*c^5 - (a^2*b^2 - b^4)*c^4)*sqrt(-(a^4*b^6 - 2*a^2*b^8 + b^
10 + 9*a^4*b^2*c^4 + 12*(a^5*b^2 - 2*a^3*b^4)*c^3 + 2*(2*a^6*b^2 - 11*a^4*b
^4 + 11*a^2*b^6)*c^2 - 4*(a^5*b^4 - 3*a^3*b^6 + 2*a*b^8)*c)/(4*a*c^13 + (16
*a^2 - b^2)*c^12 + 12*(2*a^3 - a*b^2)*c^11 + 2*(8*a^4 - 11*a^2*b^2 + b^4)*c
^10 + 4*(a^5 - 3*a^3*b^2 + 2*a*b^4)*c^9 - (a^4*b^2 - 2*a^2*b^4 + b^6)*c^8))
)/(4*a*c^7 + (8*a^2 - b^2)*c^6 + 2*(2*a^3 - 3*a*b^2)*c^5 - (a^2*b^2 - b^4)*
c^4)) + 4*(3*a^5*b*c^3 + 2*(a^6*b - 2*a^4*b^3)*c^2 - (a^5*b^3 - a^3*b^5)*c)
*cosh(x) + 4*(3*a^5*b*c^3 + 2*(a^6*b - 2*a^4*b^3)*c^2 - (a^5*b^3 - a^3*b^5)
*c)*sinh(x) - 2*(4*a^4*c^7 + (8*a^5 - a^3*b^2)*c^6 + 2*(2*a^6 - 3*a^4*b^2)*
c^5 - (a^5*b^2 - a^3*b^4)*c^4)*sqrt(-(a^4*b^6 - 2*a^2*b^8 + b^10 + 9*a^4*b^
2*c^4 + 12*(a^5*b^2 - 2*a^3*b^4)*c^3 + 2*(2*a^6*b^2 - 11*a^4*b^4 + 11*a^2*b
^6)*c^2 - 4*(a^5*b^4 - 3*a^3*b^6 + 2*a*b^8)*c)/(4*a*c^13 + (16*a^2 - b^2)*c
^12 + 12*(2*a^3 - a*b^2)*c^11 + 2*(8*a^4 - 11*a^2*b^2 + b^4)*c^10 + 4*(a^5
- 3*a^3*b^2 + 2*a*b^4)*c^9 - (a^4*b^2 - 2*a^2*b^4 + b^6)*c^8))) - sqrt(2)*(
c^2*cosh(x) + c^2*sinh(x))*sqrt(-(a^2*b^4 - b^6 + 2*a^3*c^3 + (2*a^4 - 9*a^
2*b^2)*c^2 - 2*(2*a^3*b^2 - 3*a*b^4)*c + (4*a*c^7 + (8*a^2 - b^2)*c^6 + 2*(
2*a^3 - 3*a*b^2)*c^5 - (a^2*b^2 - b^4)*c^4)*sqrt(-(a^4*b^6 - 2*a^2*b^8 + b^
10 + 9*a^4*b^2*c^4 + 12*(a^5*b^2 - 2*a^3*b^4)*c^3 + 2*(2*a^6*b^2 - 11*a^4*b
^4 + 11*a^2*b^6)*c^2 - 4*(a^5*b^4 - 3*a^3*b^6 + 2*a*b^8)*c)/(4*a*c^13 + (16
*a^2 - b^2)*c^12 + 12*(2*a^3 - a*b^2)*c^11 + 2*(8*a^4 - 11*a^2*b^2 + b^4)*c
^10 + 4*(a^5 - 3*a^3*b^2 + 2*a*b^4)*c^9 - (a^4*b^2 - 2*a^2*b^4 + b^6)*c^8))
)/(4*a*c^7 + (8*a^2 - b^2)*c^6 + 2*(2*a^3 - 3*a*b^2)*c^5 - (a^2*b^2 - b^4)*
c^4))*log(-2*a^5*b^4 + 2*a^3*b^6 + 6*a^5*b^2*c^2 + 4*(a^6*b^2 - 2*a^4*b^4)*
```

c - sqrt(2)*(12*a^4*b*c^5 + (20*a^5*b - 31*a^3*b^3)*c^4 + (8*a^6*b - 33*a^4*b^3 + 27*a^2*b^5)*c^3 - 3*(2*a^5*b^3 - 5*a^3*b^5 + 3*a*b^7)*c^2 + (a^4*b^5 - 2*a^2*b^7 + b^9)*c - (12*a^2*b*c^9 + 7*(4*a^3*b - a*b^3)*c^8 + (20*a^4*b - 27*a^2*b^3 + b^5)*c^7 + (4*a^5*b - 13*a^3*b^3 + 9*a*b^5)*c^6 - (a^4*b^3 - 2*a^2*b^5 + b^7)*c^5)*sqrt(-(a^4*b^6 - 2*a^2*b^8 + b^10 + 9*a^4*b^2*c^4 + 12*(a^5*b^2 - 2*a^3*b^4)*c^3 + 2*(2*a^6*b^2 - 11*a^4*b^4 + 11*a^2*b^6)*c^2 - 4*(a^5*b^4 - 3*a^3*b^6 + 2*a*b^8)*c)/(4*a*c^13 + (16*a^2 - b^2)*c^12 + 12*(2*a^3 - a*b^2)*c^11 + 2*(8*a^4 - 11*a^2*b^2 + b^4)*c^10 + 4*(a^5 - 3*a^3*b^2 + 2*a*b^4)*c^9 - (a^4*b^2 - 2*a^2*b^4 + b^6)*c^8)))*sqrt(-(a^2*b^4 - b^6 + 2*a^3*c^3 + (2*a^4 - 9*a^2*b^2)*c^2 - 2*(2*a^3*b^2 - 3*a*b^4)*c + (4*a*c^7 + (8*a^2 - b^2)*c^6 + 2*(2*a^3 - 3*a*b^2)*c^5 - (a^2*b^2 - b^4)*c^4)*sqrt(-(a^4*b^6 - 2*a^2*b^8 + b^10 + 9*a^4*b^2*c^4 + 12*(a^5*b^2 - 2*a^3*b^4)*c^3 + 2*(2*a^6*b^2 - 11*a^4*b^4 + 11*a^2*b^6)*c^2 - 4*(a^5*b^4 - 3*a^3*b^6 + 2*a*b^8)*c)/(4*a*c^13 + (16*a^2 - b^2)*c^12 + 12*(2*a^3 - a*b^2)*c^11 + 2*(8*a^4 - 11*a^2*b^2 + b^4)*c^10 + 4*(a^5 - 3*a^3*b^2 + 2*a*b^4)*c^9 - (a^4*b^2 - 2*a^2*b^4 + b^6)*c^8)))/(4*a*c^7 + (8*a^2 - b^2)*c^6 + 2*(2*a^3 - 3*a*b^2)*c^5 - (a^2*b^2 - b^4)*c^4)) + 4*(3*a^5*b*c^3 + 2*(a^6*b - 2*a^4*b^3)*c^2 - (a^5*b^3 - a^3*b^5)*c)*cosh(x) + 4*(3*a^5*b*c^3 + 2*(a^6*b - 2*a^4*b^3)*c^2 - (a^5*b^3 - a^3*b^5)*c)*sinh(x) - 2*(4*a^4*c^7 + (8*a^5 - a^3*b^2)*c^6 + 2*(2*a^6 - 3*a^4*b^2)*c^5 - (a^5*b^2 - a^3*b^4)*c^4)*sqrt(-(a^4*b^6 - 2*a^2*b^8 + b^10 + 9*a^4*b^2*c^4 + 12*(a^5*b^2 - 2*a^3*b^4)*c^3 + 2*(2*a^6*b^2 - 11*a^4*b^4 + 11*a^2*b^6)*c^2 - 4*(a^5*b^4 - 3*a^3*b^6 + 2*a*b^8)*c)/(4*a*c^13 + (16*a^2 - b^2)*c^12 + 12*(2*a^3 - a*b^2)*c^11 + 2*(8*a^4 - 11*a^2*b^2 + b^4)*c^10 + 4*(a^5 - 3*a^3*b^2 + 2*a*b^4)*c^9 - (a^4*b^2 - 2*a^2*b^4 + b^6)*c^8))) + sqrt(2)*(c^2*cosh(x) + c^2*sinh(x))*sqrt(-(a^2*b^4 - b^6 + 2*a^3*c^3 + (2*a^4 - 9*a^2*b^2)*c^2 - 2*(2*a^3*b^2 - 3*a*b^4)*c - (4*a*c^7 + (8*a^2 - b^2)*c^6 + 2*(2*a^3 - 3*a*b^2)*c^5 - (a^2*b^2 - b^4)*c^4)*sqrt(-(a^4*b^6 - 2*a^2*b^8 + b^10 + 9*a^4*b^2*c^4 + 12*(a^5*b^2 - 2*a^3*b^4)*c^3 + 2*(2*a^6*b^2 - 11*a^4*b^4 + 11*a^2*b^6)*c^2 - 4*(a^5*b^4 - 3*a^3*b^6 + 2*a*b^8)*c)/(4*a*c^13 + (16*a^2 - b^2)*c^12 + 12*(2*a^3 - a*b^2)*c^11 + 2*(8*a^4 - 11*a^2*b^2 + b^4)*c^10 + 4*(a^5 - 3*a^3*b^2 + 2*a*b^4)*c^9 - (a^4*b^2 - 2*a^2*b^4 + b^6)*c^8)))/(4*a*c^7 + (8*a^2 - b^2)*c^6 + 2*(2*a^3 - 3*a*b^2)*c^5 - (a^2*b^2 - b^4)*c^4))*log(-2*a^5*b^4 + 2*a^3*b^6 + 6*a^5*b^2*c^2 + 4*(a^6*b^2 - 2*a^4*b^4)*c + sqrt(2)*(12*a^4*b*c^5 + (20*a^5*b - 31*a^3*b^3)*c^4 + (8*a^6*b - 33*a^4*b^3 + 27*a^2*b^5)*c^3 - 3*(2*a^5*b^3 - 5*a^3*b^5 + 3*a*b^7)*c^2 + (a^4*b^5 - 2*a^2*b^7 + b^9)*c + (12*a^2*b*c^9 + 7*(4*a^3*b - a*b^3)*c^8 + (20*a^4*b - 27*a^2*b^3 + b^5)*c^7 + (4*a^5*b - 13*a^3*b^3 + 9*a*b^5)*c^6 - (a^4*b^3 - 2*a^2*b^5 + b^7)*c^5)*sqrt(-(a^4*b^6 - 2*a^2*b^8 + b^10 + 9*a^4*b^2*c^4 + 12*(a^5*b^2 - 2*a^3*b^4)*c^3 + 2*(2*a^6*b^2 - 11*a^4*b^4 + 11*a^2*b^6)*c^2 - 4*(a^5*b^4 - 3*a^3*b^6 + 2*a*b^8)*c)/(4*a*c^13 + (16*a^2 - b^2)*c^12 + 12*(2*a^3 - a*b^2)*c^11 + 2*(8*a^4 - 11*a^2*b^2 + b^4)*c^10 + 4*(a^5 - 3*a^3*b^2 + 2*a*b^4)*c^9 - (a^4*b^2 - 2*a^2*b^4 + b^6)*c^8)))*sqrt(-(a^2*b^4 - b^6 + 2*a^3*c^3 + (2*a^4 - 9*a^2*b^2)*c^2 - 2*(2*a^3*b^2 - 3*a*b^4)*c - (4*a*c^7 + (8*a^2 - b^2)*c^6 + 2*(2*a^3 - 3*a*b^2)*c^5 - (a^2*b^2 - b^4)*c^4)*sqrt(-(a^4*b^6 - 2*a^2*b^8 + b^10 + 9*a^4*b^2*

c^4 + 12*(a^5*b^2 - 2*a^3*b^4)*c^3 + 2*(2*a^6*b^2 - 11*a^4*b^4 + 11*a^2*b^6
)*c^2 - 4*(a^5*b^4 - 3*a^3*b^6 + 2*a*b^8)*c)/(4*a*c^13 + (16*a^2 - b^2)*c^1
2 + 12*(2*a^3 - a*b^2)*c^11 + 2*(8*a^4 - 11*a^2*b^2 + b^4)*c^10 + 4*(a^5 -
3*a^3*b^2 + 2*a*b^4)*c^9 - (a^4*b^2 - 2*a^2*b^4 + b^6)*c^8)))/(4*a*c^7 + (8
*a^2 - b^2)*c^6 + 2*(2*a^3 - 3*a*b^2)*c^5 - (a^2*b^2 - b^4)*c^4)) + 4*(3*a^
5*b*c^3 + 2*(a^6*b - 2*a^4*b^3)*c^2 - (a^5*b^3 - a^3*b^5)*c)*cosh(x) + 4*(3
*a^5*b*c^3 + 2*(a^6*b - 2*a^4*b^3)*c^2 - (a^5*b^3 - a^3*b^5)*c)*sinh(x) + 2
*(4*a^4*c^7 + (8*a^5 - a^3*b^2)*c^6 + 2*(2*a^6 - 3*a^4*b^2)*c^5 - (a^5*b^2
- a^3*b^4)*c^4)*sqrt(-(a^4*b^6 - 2*a^2*b^8 + b^10 + 9*a^4*b^2*c^4 + 12*(a^5
*b^2 - 2*a^3*b^4)*c^3 + 2*(2*a^6*b^2 - 11*a^4*b^4 + 11*a^2*b^6)*c^2 - 4*(a^
5*b^4 - 3*a^3*b^6 + 2*a*b^8)*c)/(4*a*c^13 + (16*a^2 - b^2)*c^12 + 12*(2*a^3
- a*b^2)*c^11 + 2*(8*a^4 - 11*a^2*b^2 + b^4)*c^10 + 4*(a^5 - 3*a^3*b^2 + 2
*a*b^4)*c^9 - (a^4*b^2 - 2*a^2*b^4 + b^6)*c^8))) - sqrt(2)*(c^2*cosh(x) + c
^2*sinh(x))*sqrt(-(a^2*b^4 - b^6 + 2*a^3*c^3 + (2*a^4 - 9*a^2*b^2)*c^2 - 2*
(2*a^3*b^2 - 3*a*b^4)*c - (4*a*c^7 + (8*a^2 - b^2)*c^6 + 2*(2*a^3 - 3*a*b^2
)*c^5 - (a^2*b^2 - b^4)*c^4)*sqrt(-(a^4*b^6 - 2*a^2*b^8 + b^10 + 9*a^4*b^2*
c^4 + 12*(a^5*b^2 - 2*a^3*b^4)*c^3 + 2*(2*a^6*b^2 - 11*a^4*b^4 + 11*a^2*b^6
)*c^2 - 4*(a^5*b^4 - 3*a^3*b^6 + 2*a*b^8)*c)/(4*a*c^13 + (16*a^2 - b^2)*c^1
2 + 12*(2*a^3 - a*b^2)*c^11 + 2*(8*a^4 - 11*a^2*b^2 + b^4)*c^10 + 4*(a^5 -
3*a^3*b^2 + 2*a*b^4)*c^9 - (a^4*b^2 - 2*a^2*b^4 + b^6)*c^8)))/(4*a*c^7 + (8
*a^2 - b^2)*c^6 + 2*(2*a^3 - 3*a*b^2)*c^5 - (a^2*b^2 - b^4)*c^4))*log(-2*a^
5*b^4 + 2*a^3*b^6 + 6*a^5*b^2*c^2 + 4*(a^6*b^2 - 2*a^4*b^4)*c - sqrt(2)*(12
*a^4*b*c^5 + (20*a^5*b - 31*a^3*b^3)*c^4 + (8*a^6*b - 33*a^4*b^3 + 27*a^2*b
^5)*c^3 - 3*(2*a^5*b^3 - 5*a^3*b^5 + 3*a*b^7)*c^2 + (a^4*b^5 - 2*a^2*b^7 +
b^9)*c + (12*a^2*b*c^9 + 7*(4*a^3*b - a*b^3)*c^8 + (20*a^4*b - 27*a^2*b^3 +
b^5)*c^7 + (4*a^5*b - 13*a^3*b^3 + 9*a*b^5)*c^6 - (a^4*b^3 - 2*a^2*b^5 + b
^7)*c^5)*sqrt(-(a^4*b^6 - 2*a^2*b^8 + b^10 + 9*a^4*b^2*c^4 + 12*(a^5*b^2 -
2*a^3*b^4)*c^3 + 2*(2*a^6*b^2 - 11*a^4*b^4 + 11*a^2*b^6)*c^2 - 4*(a^5*b^4 -
3*a^3*b^6 + 2*a*b^8)*c)/(4*a*c^13 + (16*a^2 - b^2)*c^12 + 12*(2*a^3 - a*b^
2)*c^11 + 2*(8*a^4 - 11*a^2*b^2 + b^4)*c^10 + 4*(a^5 - 3*a^3*b^2 + 2*a*b^4)
*c^9 - (a^4*b^2 - 2*a^2*b^4 + b^6)*c^8)))*sqrt(-(a^2*b^4 - b^6 + 2*a^3*c^3
+ (2*a^4 - 9*a^2*b^2)*c^2 - 2*(2*a^3*b^2 - 3*a*b^4)*c - (4*a*c^7 + (8*a^2 -
b^2)*c^6 + 2*(2*a^3 - 3*a*b^2)*c^5 - (a^2*b^2 - b^4)*c^4)*sqrt(-(a^4*b^6 -
2*a^2*b^8 + b^10 + 9*a^4*b^2*c^4 + 12*(a^5*b^2 - 2*a^3*b^4)*c^3 + 2*(2*a^6
*b^2 - 11*a^4*b^4 + 11*a^2*b^6)*c^2 - 4*(a^5*b^4 - 3*a^3*b^6 + 2*a*b^8)*c)/
(4*a*c^13 + (16*a^2 - b^2)*c^12 + 12*(2*a^3 - a*b^2)*c^11 + 2*(8*a^4 - 11*a
^2*b^2 + b^4)*c^10 + 4*(a^5 - 3*a^3*b^2 + 2*a*b^4)*c^9 - (a^4*b^2 - 2*a^2*b
^4 + b^6)*c^8)))/(4*a*c^7 + (8*a^2 - b^2)*c^6 + 2*(2*a^3 - 3*a*b^2)*c^5 - (
a^2*b^2 - b^4)*c^4)) + 4*(3*a^5*b*c^3 + 2*(a^6*b - 2*a^4*b^3)*c^2 - (a^5*b^
3 - a^3*b^5)*c)*cosh(x) + 4*(3*a^5*b*c^3 + 2*(a^6*b - 2*a^4*b^3)*c^2 - (a^5
*b^3 - a^3*b^5)*c)*sinh(x) + 2*(4*a^4*c^7 + (8*a^5 - a^3*b^2)*c^6 + 2*(2*a^
6 - 3*a^4*b^2)*c^5 - (a^5*b^2 - a^3*b^4)*c^4)*sqrt(-(a^4*b^6 - 2*a^2*b^8 +
b^10 + 9*a^4*b^2*c^4 + 12*(a^5*b^2 - 2*a^3*b^4)*c^3 + 2*(2*a^6*b^2 - 11*a^4
*b^4 + 11*a^2*b^6)*c^2 - 4*(a^5*b^4 - 3*a^3*b^6 + 2*a*b^8)*c)/(4*a*c^13 + (
16*a^2 - b^2)*c^12 + 12*(2*a^3 - a*b^2)*c^11 + 2*(8*a^4 - 11*a^2*b^2 + b^4)

```
*c^10 + 4*(a^5 - 3*a^3*b^2 + 2*a*b^4)*c^9 - (a^4*b^2 - 2*a^2*b^4 + b^6)*c^8
))) - c*sinh(x)^2 + 2*(b*x - c*cosh(x))*sinh(x) + c)/(c^2*cosh(x) + c^2*sin
h(x))
```

giac [A] time = 5.38, size = 24, normalized size = 0.08

$$-\frac{bx}{c^2} - \frac{e^{(-x)}}{2c} + \frac{e^x}{2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^3/(a+b*cosh(x)+c*cosh(x)^2),x, algorithm="giac")
```

```
[Out] -b*x/c^2 - 1/2*e^(-x)/c + 1/2*e^x/c
```

maple [B] time = 0.18, size = 2530, normalized size = 8.46

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^3/(a+b*cosh(x)+c*cosh(x)^2),x)
```

```
[Out] -5/c/(-4*a*c+b^2)^(1/2)/(a-b+c)/(((-4*a*c+b^2)^(1/2)-a+c)*(a-b+c))^(1/2)*ar
ctan((a-b+c)*tanh(1/2*x)/(((-4*a*c+b^2)^(1/2)-a+c)*(a-b+c))^(1/2))*a^2*b+2/
c*a/(-4*a*c+b^2)^(1/2)/(a-b+c)/(((-4*a*c+b^2)^(1/2)-a+c)*(a-b+c))^(1/2)*arc
tan((a-b+c)*tanh(1/2*x)/(((-4*a*c+b^2)^(1/2)-a+c)*(a-b+c))^(1/2))*b^2+5/c/(
-4*a*c+b^2)^(1/2)/(a-b+c)/(((-4*a*c+b^2)^(1/2)+a-c)*(a-b+c))^(1/2)*arctanh(
(-a+b-c)*tanh(1/2*x)/(((-4*a*c+b^2)^(1/2)+a-c)*(a-b+c))^(1/2))*a^2*b-2/c*a/
(-4*a*c+b^2)^(1/2)/(a-b+c)/(((-4*a*c+b^2)^(1/2)+a-c)*(a-b+c))^(1/2)*arctanh
((-a+b-c)*tanh(1/2*x)/(((-4*a*c+b^2)^(1/2)+a-c)*(a-b+c))^(1/2))*b^2-2/(-4*a
*c+b^2)^(1/2)/(a-b+c)/(((-4*a*c+b^2)^(1/2)+a-c)*(a-b+c))^(1/2)*arctanh((-a+
b-c)*tanh(1/2*x)/(((-4*a*c+b^2)^(1/2)+a-c)*(a-b+c))^(1/2))*a^2-a/(a-b+c)/((
(-4*a*c+b^2)^(1/2)-a+c)*(a-b+c))^(1/2)*arctan((a-b+c)*tanh(1/2*x)/(((-4*a*c
+b^2)^(1/2)-a+c)*(a-b+c))^(1/2))-a/(a-b+c)/(((-4*a*c+b^2)^(1/2)+a-c)*(a-b+c
))^(1/2)*arctanh((-a+b-c)*tanh(1/2*x)/(((-4*a*c+b^2)^(1/2)+a-c)*(a-b+c))^(1
/2))-3*a/(-4*a*c+b^2)^(1/2)/(a-b+c)/(((-4*a*c+b^2)^(1/2)-a+c)*(a-b+c))^(1/2
)*arctan((a-b+c)*tanh(1/2*x)/(((-4*a*c+b^2)^(1/2)-a+c)*(a-b+c))^(1/2))*b-2/
c^2*a/(-4*a*c+b^2)^(1/2)/(a-b+c)/(((-4*a*c+b^2)^(1/2)+a-c)*(a-b+c))^(1/2)*a
rctanh((-a+b-c)*tanh(1/2*x)/(((-4*a*c+b^2)^(1/2)+a-c)*(a-b+c))^(1/2))*b^3+1
/c^2/(-4*a*c+b^2)^(1/2)/(a-b+c)/(((-4*a*c+b^2)^(1/2)+a-c)*(a-b+c))^(1/2)*ar
ctanh((-a+b-c)*tanh(1/2*x)/(((-4*a*c+b^2)^(1/2)+a-c)*(a-b+c))^(1/2))*a^2*b^
2-1/c^2/(-4*a*c+b^2)^(1/2)/(a-b+c)/(((-4*a*c+b^2)^(1/2)-a+c)*(a-b+c))^(1/2)
*arctan((a-b+c)*tanh(1/2*x)/(((-4*a*c+b^2)^(1/2)-a+c)*(a-b+c))^(1/2))*a^2*b
^2+2/c^2*a/(-4*a*c+b^2)^(1/2)/(a-b+c)/(((-4*a*c+b^2)^(1/2)-a+c)*(a-b+c))^(1
/2)*arctan((a-b+c)*tanh(1/2*x)/(((-4*a*c+b^2)^(1/2)-a+c)*(a-b+c))^(1/2))*b^
```

```
3-1/c^2/(a-b+c)/(((-4*a*c+b^2)^(1/2)+a-c)*(a-b+c))^(1/2)*arctanh((-a+b-c)*t
anh(1/2*x)/(((-4*a*c+b^2)^(1/2)+a-c)*(a-b+c))^(1/2))*b^3+b/c^2*ln(tanh(1/2*
x)-1)-b/c^2*ln(tanh(1/2*x)+1)-1/c/(a-b+c)/(((-4*a*c+b^2)^(1/2)+a-c)*(a-b+c)
)^(1/2)*arctanh((-a+b-c)*tanh(1/2*x)/(((-4*a*c+b^2)^(1/2)+a-c)*(a-b+c))^(1/
2))*a^2-1/c/(a-b+c)/(((-4*a*c+b^2)^(1/2)-a+c)*(a-b+c))^(1/2)*arctan((a-b+c)
*tanh(1/2*x)/(((-4*a*c+b^2)^(1/2)-a+c)*(a-b+c))^(1/2))*a^2+1/c/(a-b+c)/(((-
4*a*c+b^2)^(1/2)+a-c)*(a-b+c))^(1/2)*arctanh((-a+b-c)*tanh(1/2*x)/(((-4*a*c
+b^2)^(1/2)+a-c)*(a-b+c))^(1/2))*b^2+1/c/(a-b+c)/(((-4*a*c+b^2)^(1/2)-a+c)*
(a-b+c))^(1/2)*arctan((a-b+c)*tanh(1/2*x)/(((-4*a*c+b^2)^(1/2)-a+c)*(a-b+c)
)^(1/2))*b^2+2/(-4*a*c+b^2)^(1/2)/(a-b+c)/(((-4*a*c+b^2)^(1/2)-a+c)*(a-b+c)
)^(1/2)*arctan((a-b+c)*tanh(1/2*x)/(((-4*a*c+b^2)^(1/2)-a+c)*(a-b+c))^(1/2)
)*a^2-1/c^2/(a-b+c)/(((-4*a*c+b^2)^(1/2)-a+c)*(a-b+c))^(1/2)*arctan((a-b+c)
*tanh(1/2*x)/(((-4*a*c+b^2)^(1/2)-a+c)*(a-b+c))^(1/2))*b^3-1/c/(-4*a*c+b^2)
^(1/2)/(a-b+c)/(((-4*a*c+b^2)^(1/2)+a-c)*(a-b+c))^(1/2)*arctanh((-a+b-c)*ta
nh(1/2*x)/(((-4*a*c+b^2)^(1/2)+a-c)*(a-b+c))^(1/2))*b^3+1/c/(-4*a*c+b^2)^(1
/2)/(a-b+c)/(((-4*a*c+b^2)^(1/2)-a+c)*(a-b+c))^(1/2)*arctan((a-b+c)*tanh(1/
2*x)/(((-4*a*c+b^2)^(1/2)-a+c)*(a-b+c))^(1/2))*b^3+3*a/(-4*a*c+b^2)^(1/2)/(
a-b+c)/(((-4*a*c+b^2)^(1/2)+a-c)*(a-b+c))^(1/2)*arctanh((-a+b-c)*tanh(1/2*x
)/(((-4*a*c+b^2)^(1/2)+a-c)*(a-b+c))^(1/2))*b-1/c/(tanh(1/2*x)-1)-1/c/(tanh
(1/2*x)+1)-1/c^2/(a-b+c)/(((-4*a*c+b^2)^(1/2)+a-c)*(a-b+c))^(1/2)*arctanh((
-a+b-c)*tanh(1/2*x)/(((-4*a*c+b^2)^(1/2)+a-c)*(a-b+c))^(1/2))*a^2*b+2/c^2*a
/(a-b+c)/(((-4*a*c+b^2)^(1/2)+a-c)*(a-b+c))^(1/2)*arctanh((-a+b-c)*tanh(1/2
*x)/(((-4*a*c+b^2)^(1/2)+a-c)*(a-b+c))^(1/2))*b^2-1/c^2/(a-b+c)/(((-4*a*c+b
^2)^(1/2)-a+c)*(a-b+c))^(1/2)*arctan((a-b+c)*tanh(1/2*x)/(((-4*a*c+b^2)^(1/
2)-a+c)*(a-b+c))^(1/2))*a^2*b+2/c/(-4*a*c+b^2)^(1/2)/(a-b+c)/(((-4*a*c+b^2)
^(1/2)-a+c)*(a-b+c))^(1/2)*arctan((a-b+c)*tanh(1/2*x)/(((-4*a*c+b^2)^(1/2)-
a+c)*(a-b+c))^(1/2))*a^3+2/c^2*a/(a-b+c)/(((-4*a*c+b^2)^(1/2)-a+c)*(a-b+c))
^(1/2)*arctan((a-b+c)*tanh(1/2*x)/(((-4*a*c+b^2)^(1/2)-a+c)*(a-b+c))^(1/2))
*b^2+1/c^2/(-4*a*c+b^2)^(1/2)/(a-b+c)/(((-4*a*c+b^2)^(1/2)+a-c)*(a-b+c))^(1
/2)*arctanh((-a+b-c)*tanh(1/2*x)/(((-4*a*c+b^2)^(1/2)+a-c)*(a-b+c))^(1/2))*
b^4-1/c^2/(-4*a*c+b^2)^(1/2)/(a-b+c)/(((-4*a*c+b^2)^(1/2)-a+c)*(a-b+c))^(1/
2)*arctan((a-b+c)*tanh(1/2*x)/(((-4*a*c+b^2)^(1/2)-a+c)*(a-b+c))^(1/2))*b^4
-2/c/(-4*a*c+b^2)^(1/2)/(a-b+c)/(((-4*a*c+b^2)^(1/2)+a-c)*(a-b+c))^(1/2)*ar
ctanh((-a+b-c)*tanh(1/2*x)/(((-4*a*c+b^2)^(1/2)+a-c)*(a-b+c))^(1/2))*a^3
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\left(2\,bxe^{x}-ce^{(2\,x)}+c\right)e^{(-x)}}{2\,c^{2}}-\frac{1}{8}\int-\frac{16\left(2\,abe^{(2\,x)}+\left(b^{2}-ac\right)e^{(3\,x)}+\left(b^{2}-ac\right)e^{x}\right)}{c^{3}e^{(4\,x)}+2\,bc^{2}e^{(3\,x)}+2\,bc^{2}e^{x}+c^{3}+2\left(2\,ac^{2}+c^{3}\right)e^{(2\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^3/(a+b*cosh(x)+c*cosh(x)^2),x, algorithm="maxima")
```

```
[Out] -1/2*(2*b*x*e^x - c*e^(2*x) + c)*e^(-x)/c^2 - 1/8*integrate(-16*(2*a*b*e^(2
```

```
*x) + (b^2 - a*c)*e^(3*x) + (b^2 - a*c)*e^x)/(c^3*e^(4*x) + 2*b*c^2*e^(3*x)
 + 2*b*c^2*e^x + c^3 + 2*(2*a*c^2 + c^3)*e^(2*x)), x)
```

mupad [F(-1)] time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^3/(a + b*cosh(x) + c*cosh(x)^2),x)
[Out] \text{Hanged}
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)**3/(a+b*cosh(x)+c*cosh(x)**2),x)
[Out] Timed out
```

3.836 $\int \frac{a+b\cosh(x)}{b^2+2ab\cosh(x)+a^2\cosh^2(x)}\,dx$

Optimal. Leaf size=11

$$\frac{\sinh(x)}{a\cosh(x)+b}$$

```
[Out] sinh(x)/(b+a*cosh(x))
```

Rubi [A] time = 0.09, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {3289, 2754, 8}

$$\frac{\sinh(x)}{a\cosh(x)+b}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cosh[x])/(b^2 + 2*a*b*Cosh[x] + a^2*Cosh[x]^2),x]
```

```
[Out] Sinh[x]/(b + a*Cosh[x])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2754

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 3289

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + cos[(d_.) + (e_.)*(x_)]^2*(c_.) + (a_)
)^(n_)*(cos[(d_.) + (e_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Dist[1/(4^n*c^n
), Int[(A + B*Cos[d + e*x])*(b + 2*c*Cos[d + e*x])^(2*n), x], x] /; FreeQ[{
a, b, c, d, e, A, B}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[n]
```

Rubi steps

$$\int \frac{a+b\cosh(x)}{b^2+2ab\cosh(x)+a^2\cosh^2(x)}\,dx = \left(4a^2\right)\int \frac{a+b\cosh(x)}{\left(2ab+2a^2\cosh(x)\right)^2}\,dx$$
$$= \frac{\sinh(x)}{b+a\cosh(x)} + \frac{\int 0\,dx}{a^2-b^2}$$
$$= \frac{\sinh(x)}{b+a\cosh(x)}$$

Mathematica [A] time = 0.06, size = 11, normalized size = 1.00

$$\frac{\sinh(x)}{a\cosh(x)+b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cosh[x])/(b^2 + 2*a*b*Cosh[x] + a^2*Cosh[x]^2),x]
```

```
[Out] Sinh[x]/(b + a*Cosh[x])
```

fricas [B] time = 0.42, size = 54, normalized size = 4.91

$$-\frac{2\,(b\cosh(x)+b\sinh(x)+a)}{a^2\cosh(x)^2+a^2\sinh(x)^2+2\,ab\cosh(x)+a^2+2\left(a^2\cosh(x)+ab\right)\sinh(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cosh(x))/(b^2+2*a*b*cosh(x)+a^2*cosh(x)^2),x, algorithm="fri
cas")
```

```
[Out] -2*(b*cosh(x) + b*sinh(x) + a)/(a^2*cosh(x)^2 + a^2*sinh(x)^2 + 2*a*b*cosh(
x) + a^2 + 2*(a^2*cosh(x) + a*b)*sinh(x))
```

giac [B] time = 0.14, size = 26, normalized size = 2.36

$$-\frac{2\,(be^x+a)}{\left(ae^{(2\,x)}+2\,be^x+a\right)a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cosh(x))/(b^2+2*a*b*cosh(x)+a^2*cosh(x)^2),x, algorithm="gia
c")
```

```
[Out] -2*(b*e^x + a)/((a*e^(2*x) + 2*b*e^x + a)*a)
```

maple [B] time = 0.16, size = 29, normalized size = 2.64

$$\frac{2\tanh\left(\frac{x}{2}\right)}{a\left(\tanh^2\left(\frac{x}{2}\right)\right)-\left(\tanh^2\left(\frac{x}{2}\right)\right)b+a+b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cosh(x))/(b^2+2*a*b*cosh(x)+a^2*cosh(x)^2),x)
```

```
[Out] 2*tanh(1/2*x)/(a*tanh(1/2*x)^2-tanh(1/2*x)^2*b+a+b)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cosh(x))/(b^2+2*a*b*cosh(x)+a^2*cosh(x)^2),x, algorithm="max
ima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [B] time = 1.93, size = 51, normalized size = 4.64

$$-\frac{\frac{2\,e^x\,\left(a\,b^3-a^3\,b\right)}{a\,\left(a\,b^2-a^3\right)}+2}{a+2\,b\,e^x+a\,e^{2\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*cosh(x))/(a^2*cosh(x)^2 + b^2 + 2*a*b*cosh(x)),x)
```

```
[Out] -((2*exp(x)*(a*b^3 - a^3*b))/(a*(a*b^2 - a^3)) + 2)/(a + 2*b*exp(x) + a*exp
(2*x))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cosh(x))/(b**2+2*a*b*cosh(x)+a**2*cosh(x)**2),x)
```

```
[Out] Timed out
```

3.837 $\int \frac{d+e\cosh(x)}{a+b\cosh(x)+c\cosh^2(x)}\,dx$

Optimal. Leaf size=246

$$\frac{2\left(\frac{2cd-be}{\sqrt{b^2-4ac}}+e\right)\tanh^{-1}\left(\frac{\tanh\left(\frac{x}{2}\right)\sqrt{-\sqrt{b^2-4ac}+b-2c}}{\sqrt{-\sqrt{b^2-4ac}+b+2c}}\right)}{\sqrt{-\sqrt{b^2-4ac}+b-2c}\,\sqrt{-\sqrt{b^2-4ac}+b+2c}}+\frac{2\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\tanh^{-1}\left(\frac{\tanh\left(\frac{x}{2}\right)\sqrt{\sqrt{b^2-4ac}+b-2c}}{\sqrt{\sqrt{b^2-4ac}+b+2c}}\right)}{\sqrt{\sqrt{b^2-4ac}+b-2c}\,\sqrt{\sqrt{b^2-4ac}+b+2c}}$$

```
[Out] 2*arctanh((b-2*c-(-4*a*c+b^2)^(1/2))^(1/2)*tanh(1/2*x)/(b+2*c-(-4*a*c+b^2)^
(1/2))^(1/2))*(e+(-b*e+2*c*d)/(-4*a*c+b^2)^(1/2))/(b-2*c-(-4*a*c+b^2)^(1/2)
)^(1/2)/(b+2*c-(-4*a*c+b^2)^(1/2))^(1/2)+2*arctanh((b-2*c+(-4*a*c+b^2)^(1/2
))^(1/2)*tanh(1/2*x)/(b+2*c+(-4*a*c+b^2)^(1/2))^(1/2))*(e+(b*e-2*c*d)/(-4*a
*c+b^2)^(1/2))/(b-2*c+(-4*a*c+b^2)^(1/2))^(1/2)/(b+2*c+(-4*a*c+b^2)^(1/2))^
(1/2)
```

Rubi [A] time = 0.68, antiderivative size = 246, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3293, 2659, 208}

$$\frac{2\left(\frac{2cd-be}{\sqrt{b^2-4ac}}+e\right)\tanh^{-1}\left(\frac{\tanh\left(\frac{x}{2}\right)\sqrt{-\sqrt{b^2-4ac}+b-2c}}{\sqrt{-\sqrt{b^2-4ac}+b+2c}}\right)}{\sqrt{-\sqrt{b^2-4ac}+b-2c}\,\sqrt{-\sqrt{b^2-4ac}+b+2c}}+\frac{2\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\tanh^{-1}\left(\frac{\tanh\left(\frac{x}{2}\right)\sqrt{\sqrt{b^2-4ac}+b-2c}}{\sqrt{\sqrt{b^2-4ac}+b+2c}}\right)}{\sqrt{\sqrt{b^2-4ac}+b-2c}\,\sqrt{\sqrt{b^2-4ac}+b+2c}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*Cosh[x])/(a + b*Cosh[x] + c*Cosh[x]^2),x]
```

```
[Out] (2*(e + (2*c*d - b*e)/Sqrt[b^2 - 4*a*c])*ArcTanh[(Sqrt[b - 2*c - Sqrt[b^2 -
 4*a*c]]*Tanh[x/2])/Sqrt[b + 2*c - Sqrt[b^2 - 4*a*c]]])/(Sqrt[b - 2*c - Sqr
t[b^2 - 4*a*c]]*Sqrt[b + 2*c - Sqrt[b^2 - 4*a*c]]) + (2*(e - (2*c*d - b*e)/
Sqrt[b^2 - 4*a*c])*ArcTanh[(Sqrt[b - 2*c + Sqrt[b^2 - 4*a*c]]*Tanh[x/2])/Sq
rt[b + 2*c + Sqrt[b^2 - 4*a*c]]])/(Sqrt[b - 2*c + Sqrt[b^2 - 4*a*c]]*Sqrt[b
 + 2*c + Sqrt[b^2 - 4*a*c]])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
```

```
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 3293

```
Int[(cos[(d_.) + (e_.)*(x_)]*(B_.) + (A_))/((a_.) + cos[(d_.) + (e_.)*(x_)]
*(b_.) + cos[(d_.) + (e_.)*(x_)]^2*(c_.)), x_Symbol] :> Module[{q = Rt[b^2
- 4*a*c, 2]}, Dist[B + (b*B - 2*A*c)/q, Int[1/(b + q + 2*c*Cos[d + e*x]), x
], x] + Dist[B - (b*B - 2*A*c)/q, Int[1/(b - q + 2*c*Cos[d + e*x]), x], x]]
 /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{d+e\cosh(x)}{a+b\cosh(x)+c\cosh^2(x)}\,dx = \left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\int \frac{1}{b+\sqrt{b^2-4ac}+2c\cosh(x)}\,dx + \left(e+\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\int \frac{}{b}$$

$$= \left(2\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\right)\text{Subst}\left(\int \frac{1}{b+2c+\sqrt{b^2-4ac}-\left(b-2c+\sqrt{b^2-4ac}\right)}\right.$$

$$= \frac{2\left(e+\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\tanh^{-1}\left(\frac{\sqrt{b-2c-\sqrt{b^2-4ac}}\,\tanh\left(\frac{x}{2}\right)}{\sqrt{b+2c-\sqrt{b^2-4ac}}}\right)}{\sqrt{b-2c-\sqrt{b^2-4ac}}\,\sqrt{b+2c-\sqrt{b^2-4ac}}} + \frac{2\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\tanh^{-1}\left(\frac{\sqrt{}}{}\right.}{\sqrt{b-2c+\sqrt{b^2-4ac}}\,\sqrt{b}}$$

Mathematica [A] time = 0.48, size = 241, normalized size = 0.98

$$\frac{\sqrt{2}\left(\frac{\left(e\left(\sqrt{b^2-4ac}-b\right)+2cd\right)\tan^{-1}\left(\frac{\tanh\left(\frac{x}{2}\right)\left(\sqrt{b^2-4ac}-b+2c\right)}{\sqrt{2b\sqrt{b^2-4ac}+4c(a+c)-2b^2}}\right)}{\sqrt{b\sqrt{b^2-4ac}+2c(a+c)-b^2}}-\frac{\left(e\left(\sqrt{b^2-4ac}+b\right)-2cd\right)\tan^{-1}\left(\frac{\tanh\left(\frac{x}{2}\right)\left(\sqrt{b^2-4ac}+b-2c\right)}{\sqrt{-2b\sqrt{b^2-4ac}+4c(a+c)-2b^2}}\right)}{\sqrt{-b\sqrt{b^2-4ac}+2c(a+c)-b^2}}\right)}{\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*Cosh[x])/(a + b*Cosh[x] + c*Cosh[x]^2),x]
```

```
[Out] (Sqrt[2]*(-(((-2*c*d + (b + Sqrt[b^2 - 4*a*c])*e)*ArcTan[((b - 2*c + Sqrt[b
^2 - 4*a*c])*Tanh[x/2])/Sqrt[-2*b^2 + 4*c*(a + c) - 2*b*Sqrt[b^2 - 4*a*c]]]
)/Sqrt[-b^2 + 2*c*(a + c) - b*Sqrt[b^2 - 4*a*c]]) + ((2*c*d + (-b + Sqrt[b^
2 - 4*a*c])*e)*ArcTan[((-b + 2*c + Sqrt[b^2 - 4*a*c])*Tanh[x/2])/Sqrt[-2*b^
```

```
2 + 4*c*(a + c) + 2*b*Sqrt[b^2 - 4*a*c]]])/Sqrt[-b^2 + 2*c*(a + c) + b*Sqrt
[b^2 - 4*a*c]]))/Sqrt[b^2 - 4*a*c]
```

<u>fricas</u> [B] time = 4.59, size = 6997, normalized size = 28.44

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*cosh(x))/(a+b*cosh(x)+c*cosh(x)^2),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(2)*sqrt(((b^2 - 2*a*c - 2*c^2)*d^2 - 2*(a*b - b*c)*d*e + (2*a^2 -
b^2 + 2*a*c)*e^2 + (a^2*b^2 - b^4 - 4*a*c^3 - (8*a^2 - b^2)*c^2 - 2*(2*a^3
- 3*a*b^2)*c)*sqrt((b^2*d^4 + b^2*e^4 - 4*(a*b + b*c)*d^3*e + 2*(2*a^2 + b^
2 + 4*a*c + 2*c^2)*d^2*e^2 - 4*(a*b + b*c)*d*e^3)/(a^4*b^2 - 2*a^2*b^4 + b^
6 - 4*a*c^5 - (16*a^2 - b^2)*c^4 - 12*(2*a^3 - a*b^2)*c^3 - 2*(8*a^4 - 11*a
^2*b^2 + b^4)*c^2 - 4*(a^5 - 3*a^3*b^2 + 2*a*b^4)*c)))/(a^2*b^2 - b^4 - 4*a
*c^3 - (8*a^2 - b^2)*c^2 - 2*(2*a^3 - 3*a*b^2)*c))*log(2*b^2*c*d^4 + 2*a*b^
2*e^4 - 2*(b^3 + 2*a*b*c + 2*b*c^2)*d^3*e + 6*(a*b^2 + b^2*c)*d^2*e^2 - 2*(
2*a^2*b + b^3 + 2*a*b*c)*d*e^3 + sqrt(2)*((b^4 - 4*a*b^2*c)*d^3 - 3*(a*b^3
- 4*a*b*c^2 - (4*a^2*b - b^3)*c)*d^2*e + (2*a^2*b^2 + b^4 - 8*a^3*c - 8*a*c
^3 - 2*(8*a^2 - b^2)*c^2)*d*e^2 - (a*b^3 - 4*a*b*c^2 - (4*a^2*b - b^3)*c)*e
^3 - ((a^2*b^4 - b^6 + 8*a*c^5 + 2*(12*a^2 - b^2)*c^4 + 6*(4*a^3 - 3*a*b^2)
*c^3 + (8*a^4 - 22*a^2*b^2 + 3*b^4)*c^2 - 2*(3*a^3*b^2 - 4*a*b^4)*c)*d - (a
^3*b^3 - a*b^5 + 4*a*b*c^4 + (4*a^2*b - b^3)*c^3 - (4*a^3*b + 5*a*b^3)*c^2
- (4*a^4*b - 5*a^2*b^3 - b^5)*c)*e)*sqrt((b^2*d^4 + b^2*e^4 - 4*(a*b + b*c)
*d^3*e + 2*(2*a^2 + b^2 + 4*a*c + 2*c^2)*d^2*e^2 - 4*(a*b + b*c)*d*e^3)/(a^
4*b^2 - 2*a^2*b^4 + b^6 - 4*a*c^5 - (16*a^2 - b^2)*c^4 - 12*(2*a^3 - a*b^2)
*c^3 - 2*(8*a^4 - 11*a^2*b^2 + b^4)*c^2 - 4*(a^5 - 3*a^3*b^2 + 2*a*b^4)*c))
)*sqrt(((b^2 - 2*a*c - 2*c^2)*d^2 - 2*(a*b - b*c)*d*e + (2*a^2 - b^2 + 2*a*
c)*e^2 + (a^2*b^2 - b^4 - 4*a*c^3 - (8*a^2 - b^2)*c^2 - 2*(2*a^3 - 3*a*b^2)
*c)*sqrt((b^2*d^4 + b^2*e^4 - 4*(a*b + b*c)*d^3*e + 2*(2*a^2 + b^2 + 4*a*c
+ 2*c^2)*d^2*e^2 - 4*(a*b + b*c)*d*e^3)/(a^4*b^2 - 2*a^2*b^4 + b^6 - 4*a*c^
5 - (16*a^2 - b^2)*c^4 - 12*(2*a^3 - a*b^2)*c^3 - 2*(8*a^4 - 11*a^2*b^2 + b
^4)*c^2 - 4*(a^5 - 3*a^3*b^2 + 2*a*b^4)*c)))/(a^2*b^2 - b^4 - 4*a*c^3 - (8*
a^2 - b^2)*c^2 - 2*(2*a^3 - 3*a*b^2)*c)) + 4*(b*c^2*d^4 + a*b*c*e^4 - (b^2*
c + 2*a*c^2 + 2*c^3)*d^3*e + 3*(a*b*c + b*c^2)*d^2*e^2 - (2*a*c^2 + (2*a^2
+ b^2)*c)*d*e^3)*cosh(x) + 4*(b*c^2*d^4 + a*b*c*e^4 - (b^2*c + 2*a*c^2 + 2*
c^3)*d^3*e + 3*(a*b*c + b*c^2)*d^2*e^2 - (2*a*c^2 + (2*a^2 + b^2)*c)*d*e^3)
*sinh(x) + 2*((4*a*c^4 + (8*a^2 - b^2)*c^3 + 2*(2*a^3 - 3*a*b^2)*c^2 - (a^2
*b^2 - b^4)*c)*d^2 + (a^2*b^3 - b^5 - 4*a*b*c^3 - (8*a^2*b - b^3)*c^2 - 2*(
2*a^3*b - 3*a*b^3)*c)*d*e - (a^3*b^2 - a*b^4 - 4*a^2*c^3 - (8*a^3 - a*b^2)*
c^2 - 2*(2*a^4 - 3*a^2*b^2)*c)*e^2)*sqrt((b^2*d^4 + b^2*e^4 - 4*(a*b + b*c)
*d^3*e + 2*(2*a^2 + b^2 + 4*a*c + 2*c^2)*d^2*e^2 - 4*(a*b + b*c)*d*e^3)/(a^
4*b^2 - 2*a^2*b^4 + b^6 - 4*a*c^5 - (16*a^2 - b^2)*c^4 - 12*(2*a^3 - a*b^2)
*c^3 - 2*(8*a^4 - 11*a^2*b^2 + b^4)*c^2 - 4*(a^5 - 3*a^3*b^2 + 2*a*b^4)*c))
```

```
) - 1/2*sqrt(2)*sqrt(((b^2 - 2*a*c - 2*c^2)*d^2 - 2*(a*b - b*c)*d*e + (2*a^
2 - b^2 + 2*a*c)*e^2 + (a^2*b^2 - b^4 - 4*a*c^3 - (8*a^2 - b^2)*c^2 - 2*(2*
a^3 - 3*a*b^2)*c)*sqrt((b^2*d^4 + b^2*e^4 - 4*(a*b + b*c)*d^3*e + 2*(2*a^2
+ b^2 + 4*a*c + 2*c^2)*d^2*e^2 - 4*(a*b + b*c)*d*e^3)/(a^4*b^2 - 2*a^2*b^4
+ b^6 - 4*a*c^5 - (16*a^2 - b^2)*c^4 - 12*(2*a^3 - a*b^2)*c^3 - 2*(8*a^4 -
11*a^2*b^2 + b^4)*c^2 - 4*(a^5 - 3*a^3*b^2 + 2*a*b^4)*c)))/(a^2*b^2 - b^4 -
 4*a*c^3 - (8*a^2 - b^2)*c^2 - 2*(2*a^3 - 3*a*b^2)*c))*log(2*b^2*c*d^4 + 2*
a*b^2*e^4 - 2*(b^3 + 2*a*b*c + 2*b*c^2)*d^3*e + 6*(a*b^2 + b^2*c)*d^2*e^2 -
 2*(2*a^2*b + b^3 + 2*a*b*c)*d*e^3 - sqrt(2)*((b^4 - 4*a*b^2*c)*d^3 - 3*(a*
b^3 - 4*a*b*c^2 - (4*a^2*b - b^3)*c)*d^2*e + (2*a^2*b^2 + b^4 - 8*a^3*c - 8
*a*c^3 - 2*(8*a^2 - b^2)*c^2)*d*e^2 - (a*b^3 - 4*a*b*c^2 - (4*a^2*b - b^3)*
c)*e^3 - ((a^2*b^4 - b^6 + 8*a*c^5 + 2*(12*a^2 - b^2)*c^4 + 6*(4*a^3 - 3*a*
b^2)*c^3 + (8*a^4 - 22*a^2*b^2 + 3*b^4)*c^2 - 2*(3*a^3*b^2 - 4*a*b^4)*c)*d
- (a^3*b^3 - a*b^5 + 4*a*b*c^4 + (4*a^2*b - b^3)*c^3 - (4*a^3*b + 5*a*b^3)*
c^2 - (4*a^4*b - 5*a^2*b^3 - b^5)*c)*e)*sqrt((b^2*d^4 + b^2*e^4 - 4*(a*b +
b*c)*d^3*e + 2*(2*a^2 + b^2 + 4*a*c + 2*c^2)*d^2*e^2 - 4*(a*b + b*c)*d*e^3)
/(a^4*b^2 - 2*a^2*b^4 + b^6 - 4*a*c^5 - (16*a^2 - b^2)*c^4 - 12*(2*a^3 - a*
b^2)*c^3 - 2*(8*a^4 - 11*a^2*b^2 + b^4)*c^2 - 4*(a^5 - 3*a^3*b^2 + 2*a*b^4)
*c)))*sqrt(((b^2 - 2*a*c - 2*c^2)*d^2 - 2*(a*b - b*c)*d*e + (2*a^2 - b^2 +
2*a*c)*e^2 + (a^2*b^2 - b^4 - 4*a*c^3 - (8*a^2 - b^2)*c^2 - 2*(2*a^3 - 3*a*
b^2)*c)*sqrt((b^2*d^4 + b^2*e^4 - 4*(a*b + b*c)*d^3*e + 2*(2*a^2 + b^2 + 4*
a*c + 2*c^2)*d^2*e^2 - 4*(a*b + b*c)*d*e^3)/(a^4*b^2 - 2*a^2*b^4 + b^6 - 4*
a*c^5 - (16*a^2 - b^2)*c^4 - 12*(2*a^3 - a*b^2)*c^3 - 2*(8*a^4 - 11*a^2*b^2
 + b^4)*c^2 - 4*(a^5 - 3*a^3*b^2 + 2*a*b^4)*c)))/(a^2*b^2 - b^4 - 4*a*c^3 -
 (8*a^2 - b^2)*c^2 - 2*(2*a^3 - 3*a*b^2)*c)) + 4*(b*c^2*d^4 + a*b*c*e^4 - (
b^2*c + 2*a*c^2 + 2*c^3)*d^3*e + 3*(a*b*c + b*c^2)*d^2*e^2 - (2*a*c^2 + (2*
a^2 + b^2)*c)*d*e^3)*cosh(x) + 4*(b*c^2*d^4 + a*b*c*e^4 - (b^2*c + 2*a*c^2
+ 2*c^3)*d^3*e + 3*(a*b*c + b*c^2)*d^2*e^2 - (2*a*c^2 + (2*a^2 + b^2)*c)*d*
e^3)*sinh(x) + 2*((4*a*c^4 + (8*a^2 - b^2)*c^3 + 2*(2*a^3 - 3*a*b^2)*c^2 -
(a^2*b^2 - b^4)*c)*d^2 + (a^2*b^3 - b^5 - 4*a*b*c^3 - (8*a^2*b - b^3)*c^2 -
 2*(2*a^3*b - 3*a*b^3)*c)*d*e - (a^3*b^2 - a*b^4 - 4*a^2*c^3 - (8*a^3 - a*b
^2)*c^2 - 2*(2*a^4 - 3*a^2*b^2)*c)*e^2)*sqrt((b^2*d^4 + b^2*e^4 - 4*(a*b +
b*c)*d^3*e + 2*(2*a^2 + b^2 + 4*a*c + 2*c^2)*d^2*e^2 - 4*(a*b + b*c)*d*e^3)
/(a^4*b^2 - 2*a^2*b^4 + b^6 - 4*a*c^5 - (16*a^2 - b^2)*c^4 - 12*(2*a^3 - a*
b^2)*c^3 - 2*(8*a^4 - 11*a^2*b^2 + b^4)*c^2 - 4*(a^5 - 3*a^3*b^2 + 2*a*b^4)
*c))) + 1/2*sqrt(2)*sqrt(((b^2 - 2*a*c - 2*c^2)*d^2 - 2*(a*b - b*c)*d*e + (
2*a^2 - b^2 + 2*a*c)*e^2 - (a^2*b^2 - b^4 - 4*a*c^3 - (8*a^2 - b^2)*c^2 - 2
*(2*a^3 - 3*a*b^2)*c)*sqrt((b^2*d^4 + b^2*e^4 - 4*(a*b + b*c)*d^3*e + 2*(2*
a^2 + b^2 + 4*a*c + 2*c^2)*d^2*e^2 - 4*(a*b + b*c)*d*e^3)/(a^4*b^2 - 2*a^2*
b^4 + b^6 - 4*a*c^5 - (16*a^2 - b^2)*c^4 - 12*(2*a^3 - a*b^2)*c^3 - 2*(8*a^
4 - 11*a^2*b^2 + b^4)*c^2 - 4*(a^5 - 3*a^3*b^2 + 2*a*b^4)*c)))/(a^2*b^2 - b
^4 - 4*a*c^3 - (8*a^2 - b^2)*c^2 - 2*(2*a^3 - 3*a*b^2)*c))*log(2*b^2*c*d^4
+ 2*a*b^2*e^4 - 2*(b^3 + 2*a*b*c + 2*b*c^2)*d^3*e + 6*(a*b^2 + b^2*c)*d^2*e
^2 - 2*(2*a^2*b + b^3 + 2*a*b*c)*d*e^3 + sqrt(2)*((b^4 - 4*a*b^2*c)*d^3 - 3
*(a*b^3 - 4*a*b*c^2 - (4*a^2*b - b^3)*c)*d^2*e + (2*a^2*b^2 + b^4 - 8*a^3*c
```

```
- 8*a*c^3 - 2*(8*a^2 - b^2)*c^2)*d*e^2 - (a*b^3 - 4*a*b*c^2 - (4*a^2*b - b
^3)*c)*e^3 + ((a^2*b^4 - b^6 + 8*a*c^5 + 2*(12*a^2 - b^2)*c^4 + 6*(4*a^3 -
3*a*b^2)*c^3 + (8*a^4 - 22*a^2*b^2 + 3*b^4)*c^2 - 2*(3*a^3*b^2 - 4*a*b^4)*c
)*d - (a^3*b^3 - a*b^5 + 4*a*b*c^4 + (4*a^2*b - b^3)*c^3 - (4*a^3*b + 5*a*b
^3)*c^2 - (4*a^4*b - 5*a^2*b^3 - b^5)*c)*e)*sqrt((b^2*d^4 + b^2*e^4 - 4*(a*
b + b*c)*d^3*e + 2*(2*a^2 + b^2 + 4*a*c + 2*c^2)*d^2*e^2 - 4*(a*b + b*c)*d*
e^3)/(a^4*b^2 - 2*a^2*b^4 + b^6 - 4*a*c^5 - (16*a^2 - b^2)*c^4 - 12*(2*a^3
- a*b^2)*c^3 - 2*(8*a^4 - 11*a^2*b^2 + b^4)*c^2 - 4*(a^5 - 3*a^3*b^2 + 2*a*
b^4)*c)))*sqrt(((b^2 - 2*a*c - 2*c^2)*d^2 - 2*(a*b - b*c)*d*e + (2*a^2 - b^
2 + 2*a*c)*e^2 - (a^2*b^2 - b^4 - 4*a*c^3 - (8*a^2 - b^2)*c^2 - 2*(2*a^3 -
3*a*b^2)*c)*sqrt((b^2*d^4 + b^2*e^4 - 4*(a*b + b*c)*d^3*e + 2*(2*a^2 + b^2
+ 4*a*c + 2*c^2)*d^2*e^2 - 4*(a*b + b*c)*d*e^3)/(a^4*b^2 - 2*a^2*b^4 + b^6
- 4*a*c^5 - (16*a^2 - b^2)*c^4 - 12*(2*a^3 - a*b^2)*c^3 - 2*(8*a^4 - 11*a^2
*b^2 + b^4)*c^2 - 4*(a^5 - 3*a^3*b^2 + 2*a*b^4)*c)))/(a^2*b^2 - b^4 - 4*a*c
^3 - (8*a^2 - b^2)*c^2 - 2*(2*a^3 - 3*a*b^2)*c)) + 4*(b*c^2*d^4 + a*b*c*e^4
 - (b^2*c + 2*a*c^2 + 2*c^3)*d^3*e + 3*(a*b*c + b*c^2)*d^2*e^2 - (2*a*c^2 +
 (2*a^2 + b^2)*c)*d*e^3)*cosh(x) + 4*(b*c^2*d^4 + a*b*c*e^4 - (b^2*c + 2*a*
c^2 + 2*c^3)*d^3*e + 3*(a*b*c + b*c^2)*d^2*e^2 - (2*a*c^2 + (2*a^2 + b^2)*c
)*d*e^3)*sinh(x) - 2*((4*a*c^4 + (8*a^2 - b^2)*c^3 + 2*(2*a^3 - 3*a*b^2)*c^
2 - (a^2*b^2 - b^4)*c)*d^2 + (a^2*b^3 - b^5 - 4*a*b*c^3 - (8*a^2*b - b^3)*c
^2 - 2*(2*a^3*b - 3*a*b^3)*c)*d*e - (a^3*b^2 - a*b^4 - 4*a^2*c^3 - (8*a^3 -
 a*b^2)*c^2 - 2*(2*a^4 - 3*a^2*b^2)*c)*e^2)*sqrt((b^2*d^4 + b^2*e^4 - 4*(a*
b + b*c)*d^3*e + 2*(2*a^2 + b^2 + 4*a*c + 2*c^2)*d^2*e^2 - 4*(a*b + b*c)*d*
e^3)/(a^4*b^2 - 2*a^2*b^4 + b^6 - 4*a*c^5 - (16*a^2 - b^2)*c^4 - 12*(2*a^3
- a*b^2)*c^3 - 2*(8*a^4 - 11*a^2*b^2 + b^4)*c^2 - 4*(a^5 - 3*a^3*b^2 + 2*a*
b^4)*c))) - 1/2*sqrt(2)*sqrt(((b^2 - 2*a*c - 2*c^2)*d^2 - 2*(a*b - b*c)*d*e
 + (2*a^2 - b^2 + 2*a*c)*e^2 - (a^2*b^2 - b^4 - 4*a*c^3 - (8*a^2 - b^2)*c^2
 - 2*(2*a^3 - 3*a*b^2)*c)*sqrt((b^2*d^4 + b^2*e^4 - 4*(a*b + b*c)*d^3*e + 2
*(2*a^2 + b^2 + 4*a*c + 2*c^2)*d^2*e^2 - 4*(a*b + b*c)*d*e^3)/(a^4*b^2 - 2*
a^2*b^4 + b^6 - 4*a*c^5 - (16*a^2 - b^2)*c^4 - 12*(2*a^3 - a*b^2)*c^3 - 2*(
8*a^4 - 11*a^2*b^2 + b^4)*c^2 - 4*(a^5 - 3*a^3*b^2 + 2*a*b^4)*c)))/(a^2*b^2
 - b^4 - 4*a*c^3 - (8*a^2 - b^2)*c^2 - 2*(2*a^3 - 3*a*b^2)*c))*log(2*b^2*c*
d^4 + 2*a*b^2*e^4 - 2*(b^3 + 2*a*b*c + 2*b*c^2)*d^3*e + 6*(a*b^2 + b^2*c)*d
^2*e^2 - 2*(2*a^2*b + b^3 + 2*a*b*c)*d*e^3 - sqrt(2)*((b^4 - 4*a*b^2*c)*d^3
 - 3*(a*b^3 - 4*a*b*c^2 - (4*a^2*b - b^3)*c)*d^2*e + (2*a^2*b^2 + b^4 - 8*a
^3*c - 8*a*c^3 - 2*(8*a^2 - b^2)*c^2)*d*e^2 - (a*b^3 - 4*a*b*c^2 - (4*a^2*b
 - b^3)*c)*e^3 + ((a^2*b^4 - b^6 + 8*a*c^5 + 2*(12*a^2 - b^2)*c^4 + 6*(4*a^
3 - 3*a*b^2)*c^3 + (8*a^4 - 22*a^2*b^2 + 3*b^4)*c^2 - 2*(3*a^3*b^2 - 4*a*b^
4)*c)*d - (a^3*b^3 - a*b^5 + 4*a*b*c^4 + (4*a^2*b - b^3)*c^3 - (4*a^3*b + 5
*a*b^3)*c^2 - (4*a^4*b - 5*a^2*b^3 - b^5)*c)*e)*sqrt((b^2*d^4 + b^2*e^4 - 4
*(a*b + b*c)*d^3*e + 2*(2*a^2 + b^2 + 4*a*c + 2*c^2)*d^2*e^2 - 4*(a*b + b*c
)*d*e^3)/(a^4*b^2 - 2*a^2*b^4 + b^6 - 4*a*c^5 - (16*a^2 - b^2)*c^4 - 12*(2*
a^3 - a*b^2)*c^3 - 2*(8*a^4 - 11*a^2*b^2 + b^4)*c^2 - 4*(a^5 - 3*a^3*b^2 +
2*a*b^4)*c)))*sqrt(((b^2 - 2*a*c - 2*c^2)*d^2 - 2*(a*b - b*c)*d*e + (2*a^2
- b^2 + 2*a*c)*e^2 - (a^2*b^2 - b^4 - 4*a*c^3 - (8*a^2 - b^2)*c^2 - 2*(2*a^
```

```
3 - 3*a*b^2)*c)*sqrt((b^2*d^4 + b^2*e^4 - 4*(a*b + b*c)*d^3*e + 2*(2*a^2 +
b^2 + 4*a*c + 2*c^2)*d^2*e^2 - 4*(a*b + b*c)*d*e^3)/(a^4*b^2 - 2*a^2*b^4 +
b^6 - 4*a*c^5 - (16*a^2 - b^2)*c^4 - 12*(2*a^3 - a*b^2)*c^3 - 2*(8*a^4 - 11
*a^2*b^2 + b^4)*c^2 - 4*(a^5 - 3*a^3*b^2 + 2*a*b^4)*c)))/(a^2*b^2 - b^4 - 4
*a*c^3 - (8*a^2 - b^2)*c^2 - 2*(2*a^3 - 3*a*b^2)*c)) + 4*(b*c^2*d^4 + a*b*c
*e^4 - (b^2*c + 2*a*c^2 + 2*c^3)*d^3*e + 3*(a*b*c + b*c^2)*d^2*e^2 - (2*a*c
^2 + (2*a^2 + b^2)*c)*d*e^3)*cosh(x) + 4*(b*c^2*d^4 + a*b*c*e^4 - (b^2*c +
2*a*c^2 + 2*c^3)*d^3*e + 3*(a*b*c + b*c^2)*d^2*e^2 - (2*a*c^2 + (2*a^2 + b^
2)*c)*d*e^3)*sinh(x) - 2*((4*a*c^4 + (8*a^2 - b^2)*c^3 + 2*(2*a^3 - 3*a*b^2
)*c^2 - (a^2*b^2 - b^4)*c)*d^2 + (a^2*b^3 - b^5 - 4*a*b*c^3 - (8*a^2*b - b^
3)*c^2 - 2*(2*a^3*b - 3*a*b^3)*c)*d*e - (a^3*b^2 - a*b^4 - 4*a^2*c^3 - (8*a
^3 - a*b^2)*c^2 - 2*(2*a^4 - 3*a^2*b^2)*c)*e^2)*sqrt((b^2*d^4 + b^2*e^4 - 4
*(a*b + b*c)*d^3*e + 2*(2*a^2 + b^2 + 4*a*c + 2*c^2)*d^2*e^2 - 4*(a*b + b*c
)*d*e^3)/(a^4*b^2 - 2*a^2*b^4 + b^6 - 4*a*c^5 - (16*a^2 - b^2)*c^4 - 12*(2*
a^3 - a*b^2)*c^3 - 2*(8*a^4 - 11*a^2*b^2 + b^4)*c^2 - 4*(a^5 - 3*a^3*b^2 +
2*a*b^4)*c)))
```

giac [A] time = 4.30, size = 1, normalized size = 0.00

$$0$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*cosh(x))/(a+b*cosh(x)+c*cosh(x)^2),x, algorithm="giac")
```

```
[Out] 0
```

maple [B] time = 0.16, size = 2556, normalized size = 10.39

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d+e*cosh(x))/(a+b*cosh(x)+c*cosh(x)^2),x)
```

```
[Out] -a/(-4*a*c+b^2)^(1/2)/(a-b+c)/(((-4*a*c+b^2)^(1/2)+a-c)*(a-b+c))^(1/2)*arct
anh((-a+b-c)*tanh(1/2*x)/(((-4*a*c+b^2)^(1/2)+a-c)*(a-b+c))^(1/2))*b*d-3*a/
(-4*a*c+b^2)^(1/2)/(a-b+c)/(((-4*a*c+b^2)^(1/2)+a-c)*(a-b+c))^(1/2)*arctanh
((-a+b-c)*tanh(1/2*x)/(((-4*a*c+b^2)^(1/2)+a-c)*(a-b+c))^(1/2))*b*e+2*a/(-4
*a*c+b^2)^(1/2)/(a-b+c)/(((-4*a*c+b^2)^(1/2)+a-c)*(a-b+c))^(1/2)*arctanh((-
a+b-c)*tanh(1/2*x)/(((-4*a*c+b^2)^(1/2)+a-c)*(a-b+c))^(1/2))*c*d+a/(-4*a*c+
b^2)^(1/2)/(a-b+c)/(((-4*a*c+b^2)^(1/2)-a+c)*(a-b+c))^(1/2)*arctan((a-b+c)*
tanh(1/2*x)/(((-4*a*c+b^2)^(1/2)-a+c)*(a-b+c))^(1/2))*b*d+3*a/(-4*a*c+b^2)^
(1/2)/(a-b+c)/(((-4*a*c+b^2)^(1/2)-a+c)*(a-b+c))^(1/2)*arctan((a-b+c)*tanh(
1/2*x)/(((-4*a*c+b^2)^(1/2)-a+c)*(a-b+c))^(1/2))*b*e-2*a/(-4*a*c+b^2)^(1/2)
/(a-b+c)/(((-4*a*c+b^2)^(1/2)-a+c)*(a-b+c))^(1/2)*arctan((a-b+c)*tanh(1/2*x
```

)/(((-4*a*c+b^2)^(1/2)-a+c)*(a-b+c))^(1/2))*c*d-2*c/(-4*a*c+b^2)^(1/2)/(a-b+c)/(((-4*a*c+b^2)^(1/2)-a+c)*(a-b+c))^(1/2)*arctan((a-b+c)*tanh(1/2*x)/(((-4*a*c+b^2)^(1/2)-a+c)*(a-b+c))^(1/2))*e*a+c/(-4*a*c+b^2)^(1/2)/(a-b+c)/(((-4*a*c+b^2)^(1/2)-a+c)*(a-b+c))^(1/2)*arctan((a-b+c)*tanh(1/2*x)/(((-4*a*c+b^2)^(1/2)-a+c)*(a-b+c))^(1/2))*b*e+2*c/(-4*a*c+b^2)^(1/2)/(a-b+c)/(((-4*a*c+b^2)^(1/2)+a-c)*(a-b+c))^(1/2)*arctanh((-a+b-c)*tanh(1/2*x)/(((-4*a*c+b^2)^(1/2)+a-c)*(a-b+c))^(1/2))*e*a-c/(-4*a*c+b^2)^(1/2)/(a-b+c)/(((-4*a*c+b^2)^(1/2)+a-c)*(a-b+c))^(1/2)*arctanh((-a+b-c)*tanh(1/2*x)/(((-4*a*c+b^2)^(1/2)+a-c)*(a-b+c))^(1/2))*b*e-3*b/(-4*a*c+b^2)^(1/2)/(a-b+c)/(((-4*a*c+b^2)^(1/2)+a-c)*(a-b+c))^(1/2)*arctanh((-a+b-c)*tanh(1/2*x)/(((-4*a*c+b^2)^(1/2)+a-c)*(a-b+c))^(1/2))*c*d+3*b/(-4*a*c+b^2)^(1/2)/(a-b+c)/(((-4*a*c+b^2)^(1/2)-a+c)*(a-b+c))^(1/2)*arctan((a-b+c)*tanh(1/2*x)/(((-4*a*c+b^2)^(1/2)-a+c)*(a-b+c))^(1/2))*c*d+a/(a-b+c)/(((-4*a*c+b^2)^(1/2)+a-c)*(a-b+c))^(1/2)*arctanh((-a+b-c)*tanh(1/2*x)/(((-4*a*c+b^2)^(1/2)+a-c)*(a-b+c))^(1/2))*e-c/(a-b+c)/(((-4*a*c+b^2)^(1/2)-a+c)*(a-b+c))^(1/2)*arctan((a-b+c)*tanh(1/2*x)/(((-4*a*c+b^2)^(1/2)-a+c)*(a-b+c))^(1/2))*d+c/(a-b+c)/(((-4*a*c+b^2)^(1/2)-a+c)*(a-b+c))^(1/2)*arctan((a-b+c)*tanh(1/2*x)/(((-4*a*c+b^2)^(1/2)-a+c)*(a-b+c))^(1/2))*e+1/(-4*a*c+b^2)^(1/2)/(a-b+c)/(((-4*a*c+b^2)^(1/2)+a-c)*(a-b+c))^(1/2)*arctanh((-a+b-c)*tanh(1/2*x)/(((-4*a*c+b^2)^(1/2)+a-c)*(a-b+c))^(1/2))*b^2*e-1/(-4*a*c+b^2)^(1/2)/(a-b+c)/(((-4*a*c+b^2)^(1/2)-a+c)*(a-b+c))^(1/2)*arctan((a-b+c)*tanh(1/2*x)/(((-4*a*c+b^2)^(1/2)-a+c)*(a-b+c))^(1/2))*b^2*d-1/(-4*a*c+b^2)^(1/2)/(a-b+c)/(((-4*a*c+b^2)^(1/2)-a+c)*(a-b+c))^(1/2)*arctan((a-b+c)*tanh(1/2*x)/(((-4*a*c+b^2)^(1/2)-a+c)*(a-b+c))^(1/2))*b^2*e+2/(-4*a*c+b^2)^(1/2)/(a-b+c)/(((-4*a*c+b^2)^(1/2)+a-c)*(a-b+c))^(1/2)*arctanh((-a+b-c)*tanh(1/2*x)/(((-4*a*c+b^2)^(1/2)+a-c)*(a-b+c))^(1/2))*c^2*d-a/(a-b+c)/(((-4*a*c+b^2)^(1/2)-a+c)*(a-b+c))^(1/2)*arctan((a-b+c)*tanh(1/2*x)/(((-4*a*c+b^2)^(1/2)-a+c)*(a-b+c))^(1/2))*d+a/(a-b+c)/(((-4*a*c+b^2)^(1/2)-a+c)*(a-b+c))^(1/2)*arctan((a-b+c)*tanh(1/2*x)/(((-4*a*c+b^2)^(1/2)-a+c)*(a-b+c))^(1/2))*e+b/(a-b+c)/(((-4*a*c+b^2)^(1/2)+a-c)*(a-b+c))^(1/2)*arctanh((-a+b-c)*tanh(1/2*x)/(((-4*a*c+b^2)^(1/2)+a-c)*(a-b+c))^(1/2))*d-b/(a-b+c)/(((-4*a*c+b^2)^(1/2)+a-c)*(a-b+c))^(1/2)*arctanh((-a+b-c)*tanh(1/2*x)/(((-4*a*c+b^2)^(1/2)+a-c)*(a-b+c))^(1/2))*e+b/(a-b+c)/(((-4*a*c+b^2)^(1/2)-a+c)*(a-b+c))^(1/2)*arctan((a-b+c)*tanh(1/2*x)/(((-4*a*c+b^2)^(1/2)-a+c)*(a-b+c))^(1/2))*d-b/(a-b+c)/(((-4*a*c+b^2)^(1/2)-a+c)*(a-b+c))^(1/2)*arctan((a-b+c)*tanh(1/2*x)/(((-4*a*c+b^2)^(1/2)-a+c)*(a-b+c))^(1/2))*e-c/(a-b+c)/(((-4*a*c+b^2)^(1/2)+a-c)*(a-b+c))^(1/2)*arctanh((-a+b-c)*tanh(1/2*x)/(((-4*a*c+b^2)^(1/2)+a-c)*(a-b+c))^(1/2))*d+c/(a-b+c)/(((-4*a*c+b^2)^(1/2)+a-c)*(a-b+c))^(1/2)*arctanh((-a+b-c)*tanh(1/2*x)/(((-4*a*c+b^2)^(1/2)+a-c)*(a-b+c))^(1/2))*e-a/(a-b+c)/(((-4*a*c+b^2)^(1/2)+a-c)*(a-b+c))^(1/2)*arctanh((-a+b-c)*tanh(1/2*x)/(((-4*a*c+b^2)^(1/2)+a-c)*(a-b+c))^(1/2))*d-2/(-4*a*c+b^2)^(1/2)/(a-b+c)/(((-4*a*c+b^2)^(1/2)-a+c)*(a-b+c))^(1/2)*arctan((a-b+c)*tanh(1/2*x)/(((-4*a*c+b^2)^(1/2)-a+c)*(a-b+c))^(1/2))*c^2*d+2/(-4*a*c+b^2)^(1/2)/(a-b+c)/(((-4*a*c+b^2)^(1/2)+a-c)*(a-b+c))^(1/2)*arctanh((-a+b-c)*tanh(1/2*x)/(((-4*a*c+b^2)^(1/2)+a-c)*(a-b+c))^(1/2))*e*a^2-2/(-4*a*c+b^2)^(1/2)/(a-b+c)/(((-4*a*c+b^2)^(1/2)-a+c)*(a-b+c))^(1/2)*arctan((a-b+c)*tanh(1/2*x)/(((-4*

```
a*c+b^2)^(1/2)-a+c)*(a-b+c))^(1/2))*e*a^2+1/(-4*a*c+b^2)^(1/2)/(a-b+c)/(((-
4*a*c+b^2)^(1/2)+a-c)*(a-b+c))^(1/2)*arctanh((-a+b-c)*tanh(1/2*x)/(((-4*a*c
+b^2)^(1/2)+a-c)*(a-b+c))^(1/2))*b^2*d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{e\cosh(x) + d}{c\cosh(x)^2 + b\cosh(x) + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*cosh(x))/(a+b*cosh(x)+c*cosh(x)^2),x, algorithm="maxima")
[Out] integrate((e*cosh(x) + d)/(c*cosh(x)^2 + b*cosh(x) + a), x)
```

mupad [F(-1)] time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*cosh(x))/(a + b*cosh(x) + c*cosh(x)^2),x)
[Out] \text{Hanged}
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*cosh(x))/(a+b*cosh(x)+c*cosh(x)**2),x)
[Out] Timed out
```

3.838 $\int \frac{\sinh^2(x)}{a\cosh^2(x)+b\sinh^2(x)}\,dx$

Optimal. Leaf size=39

$$\frac{x}{a+b}-\frac{\sqrt{a}\,\tan^{-1}\left(\frac{\sqrt{b}\,\tanh(x)}{\sqrt{a}}\right)}{\sqrt{b}\,(a+b)}$$

```
[Out] x/(a+b)-arctan(b^(1/2)*tanh(x)/a^(1/2))*a^(1/2)/(a+b)/b^(1/2)
```

Rubi [A] time = 0.15, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {481, 207, 205}

$$\frac{x}{a+b}-\frac{\sqrt{a}\,\tan^{-1}\left(\frac{\sqrt{b}\,\tanh(x)}{\sqrt{a}}\right)}{\sqrt{b}\,(a+b)}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[x]^2/(a*Cosh[x]^2 + b*Sinh[x]^2),x]
```

```
[Out] x/(a + b) - (Sqrt[a]*ArcTan[(Sqrt[b]*Tanh[x])/Sqrt[a]])/(Sqrt[b]*(a + b))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 481

```
Int[((e_.)*(x_))^(m_.)/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))),
 x_Symbol] :> -Dist[(a*e^n)/(b*c - a*d), Int[(e*x)^(m - n)/(a + b*x^n), x],
 x] + Dist[(c*e^n)/(b*c - a*d), Int[(e*x)^(m - n)/(c + d*x^n), x], x] /; Fr
eeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LeQ[n, m,
2*n - 1]
```

Rubi steps

$$\int \frac{\sinh^2(x)}{a\cosh^2(x)+b\sinh^2(x)}\,dx = -\text{Subst}\left(\int \frac{x^2}{(-1+x^2)\,(a+bx^2)}\,dx, x, \tanh(x)\right)$$
$$= -\frac{\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \tanh(x)\right)}{a+b} - \frac{a\,\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, \tanh(x)\right)}{a+b}$$
$$= \frac{x}{a+b} - \frac{\sqrt{a}\,\tan^{-1}\left(\frac{\sqrt{b}\tanh(x)}{\sqrt{a}}\right)}{\sqrt{b}\,(a+b)}$$

Mathematica [A] time = 0.11, size = 34, normalized size = 0.87

$$\frac{x - \frac{\sqrt{a}\tan^{-1}\left(\frac{\sqrt{b}\tanh(x)}{\sqrt{a}}\right)}{\sqrt{b}}}{a+b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[x]^2/(a*Cosh[x]^2 + b*Sinh[x]^2),x]
```

```
[Out] (x - (Sqrt[a]*ArcTan[(Sqrt[b]*Tanh[x])/Sqrt[a]])/Sqrt[b])/(a + b)
```

fricas [A] time = 0.47, size = 367, normalized size = 9.41

$$\left[\frac{\sqrt{-\frac{a}{b}}\log\left(\frac{(a^2+2ab+b^2)\cosh(x)^4+4(a^2+2ab+b^2)\cosh(x)\sinh(x)^3+(a^2+2ab+b^2)\sinh(x)^4+2(a^2-b^2)\cosh(x)^2+2(3(a^2+2ab+b^2)\cosh(x)^2+a}{(a+b)\cosh(x)^4+4(a+b)\cosh(x)\sinh(x)^3+(a+b)\sinh(x)^4+2(a-b}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^2/(a*cosh(x)^2+b*sinh(x)^2),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(-a/b)*log(((a^2 + 2*a*b + b^2)*cosh(x)^4 + 4*(a^2 + 2*a*b + b^2)
*cosh(x)*sinh(x)^3 + (a^2 + 2*a*b + b^2)*sinh(x)^4 + 2*(a^2 - b^2)*cosh(x)^
2 + 2*(3*(a^2 + 2*a*b + b^2)*cosh(x)^2 + a^2 - b^2)*sinh(x)^2 + a^2 - 6*a*b
 + b^2 + 4*((a^2 + 2*a*b + b^2)*cosh(x)^3 + (a^2 - b^2)*cosh(x))*sinh(x) -
4*((a*b + b^2)*cosh(x)^2 + 2*(a*b + b^2)*cosh(x)*sinh(x) + (a*b + b^2)*sinh
(x)^2 + a*b - b^2)*sqrt(-a/b))/((a + b)*cosh(x)^4 + 4*(a + b)*cosh(x)*sinh(
x)^3 + (a + b)*sinh(x)^4 + 2*(a - b)*cosh(x)^2 + 2*(3*(a + b)*cosh(x)^2 + a
 - b)*sinh(x)^2 + 4*((a + b)*cosh(x)^3 + (a - b)*cosh(x))*sinh(x) + a + b))
```

```
 + 2*x)/(a + b), -(sqrt(a/b)*arctan(1/2*((a + b)*cosh(x)^2 + 2*(a + b)*cosh
(x)*sinh(x) + (a + b)*sinh(x)^2 + a - b)*sqrt(a/b)/a) - x)/(a + b)]
```

giac [A] time = 0.12, size = 46, normalized size = 1.18

$$-\frac{a \arctan\left(\frac{ae^{(2x)}+be^{(2x)}+a-b}{2\sqrt{ab}}\right)}{\sqrt{ab}\,(a+b)}+\frac{x}{a+b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^2/(a*cosh(x)^2+b*sinh(x)^2),x, algorithm="giac")
```

```
[Out] -a*arctan(1/2*(a*e^(2*x) + b*e^(2*x) + a - b)/sqrt(a*b))/(sqrt(a*b)*(a + b)
) + x/(a + b)
```

maple [B] time = 0.20, size = 414, normalized size = 10.62

$$-\frac{8\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{8a+8b}+\frac{8\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)}{8a+8b}+\frac{4a^2\,\mathrm{arctanh}\left(\frac{a\tanh\left(\frac{x}{2}\right)}{\sqrt{\left(2\sqrt{b(a+b)}-a-2b\right)a}}\right)}{(4a+4b)\sqrt{b\,(a+b)}\,\sqrt{\left(2\sqrt{b\,(a+b)}-a-2b\right)a}}-\frac{4a\,\mathrm{arctanh}\left(\frac{}{\sqrt{(2}}\right)}{(4a+4b)\sqrt{\left(2\sqrt{b\,(a}\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)^2/(a*cosh(x)^2+b*sinh(x)^2),x)
```

```
[Out] -8/(8*a+8*b)*ln(tanh(1/2*x)-1)+8/(8*a+8*b)*ln(tanh(1/2*x)+1)+4*a^2/(4*a+4*b
)/(b*(a+b))^(1/2)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*x)
/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2))-4*a/(4*a+4*b)/((2*(b*(a+b))^(1/2)-a-2
*b)*a)^(1/2)*arctanh(a*tanh(1/2*x)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2))+4*a
/(4*a+4*b)/(b*(a+b))^(1/2)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2)*arctanh(a*ta
nh(1/2*x)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2))*b+4*a^2/(4*a+4*b)/(b*(a+b))^
(1/2)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*x)/((2*(b*(a+b)
)^(1/2)+a+2*b)*a)^(1/2))+4*a/(4*a+4*b)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2)*
arctan(a*tanh(1/2*x)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2))+4*a/(4*a+4*b)/(b*
(a+b))^(1/2)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*x)/((2*(
b*(a+b))^(1/2)+a+2*b)*a)^(1/2))*b
```

maxima [B] time = 0.43, size = 74, normalized size = 1.90

$$-\frac{(a-b)\arctan\left(\frac{(a+b)e^{(2x)}+a-b}{2\sqrt{ab}}\right)}{2\sqrt{ab}\,(a+b)}+\frac{\arctan\left(\frac{(a+b)e^{(-2x)}+a-b}{2\sqrt{ab}}\right)}{2\sqrt{ab}}+\frac{x}{a+b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^2/(a*cosh(x)^2+b*sinh(x)^2),x, algorithm="maxima")
```

```
[Out] -1/2*(a - b)*arctan(1/2*((a + b)*e^(2*x) + a - b)/sqrt(a*b))/(sqrt(a*b)*(a
+ b)) + 1/2*arctan(1/2*((a + b)*e^(-2*x) + a - b)/sqrt(a*b))/sqrt(a*b) + x/
(a + b)
```

mupad [B] time = 2.29, size = 209, normalized size = 5.36

$$\frac{x}{a+b}-\frac{\sqrt{a}\,\mathrm{atan}\left(\frac{\left(e^{2x}\left(\frac{4a}{(a+b)^4}+\frac{\left(a^2-b^2\right)(a-b)}{(a+b)^3\sqrt{b\,(a+b)^2}\,\sqrt{a^2\,b+2\,a\,b^2+b^3}}\right)+\frac{(a-b)\left(a^2+2\,a\,b+b^2\right)}{(a+b)^3\sqrt{b\,(a+b)^2}\,\sqrt{a^2\,b+2\,a\,b^2+b^3}}\right)\left(a^2\sqrt{a^2\,b+2\,a\,b^2+b^3}+b^2\sqrt{a^2\,b+2\,a\,b^2+b^3}+2\right.}{2\sqrt{a}}\right)}{\sqrt{a^2\,b+2\,a\,b^2+b^3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)^2/(b*sinh(x)^2 + a*cosh(x)^2),x)
```

```
[Out] x/(a + b) - (a^(1/2)*atan(((exp(2*x)*((4*a)/(a + b)^4 + ((a^2 - b^2)*(a - b
))/((a + b)^3*(b*(a + b)^2)^(1/2)*(2*a*b^2 + a^2*b + b^3)^(1/2))) + ((a - b
)*(2*a*b + a^2 + b^2))/((a + b)^3*(b*(a + b)^2)^(1/2)*(2*a*b^2 + a^2*b + b^
3)^(1/2)))*(a^2*(2*a*b^2 + a^2*b + b^3)^(1/2) + b^2*(2*a*b^2 + a^2*b + b^3)
^(1/2) + 2*a*b*(2*a*b^2 + a^2*b + b^3)^(1/2)))/(2*a^(1/2))))/(2*a*b^2 + a^2
*b + b^3)^(1/2)
```

sympy [A] time = 1.54, size = 243, normalized size = 6.23

$$\begin{cases} \tilde{\infty}x & \text{for } a=0 \wedge b=0 \\ \frac{x}{b} & \text{for } a=0 \\ -\frac{x\sinh^2(x)}{-2b\sinh^2(x)+2b\cosh^2(x)}+\frac{x\cosh^2(x)}{-2b\sinh^2(x)+2b\cosh^2(x)}-\frac{\sinh(x)\cosh(x)}{-2b\sinh^2(x)+2b\cosh^2(x)} & \text{for } a=-b \\ \frac{x-\frac{\sinh(x)}{\cosh(x)}}{a} & \text{for } b=0 \\ \frac{2i\sqrt{b}\,x\sqrt{\frac{1}{a}}}{2ia\sqrt{b}\sqrt{\frac{1}{a}}+2ib^{\frac{3}{2}}\sqrt{\frac{1}{a}}}+\frac{\log\left(-i\sqrt{b}\sqrt{\frac{1}{a}}\sinh(x)+\cosh(x)\right)}{2ia\sqrt{b}\sqrt{\frac{1}{a}}+2ib^{\frac{3}{2}}\sqrt{\frac{1}{a}}}-\frac{\log\left(i\sqrt{b}\sqrt{\frac{1}{a}}\sinh(x)+\cosh(x)\right)}{2ia\sqrt{b}\sqrt{\frac{1}{a}}+2ib^{\frac{3}{2}}\sqrt{\frac{1}{a}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)**2/(a*cosh(x)**2+b*sinh(x)**2),x)
```

```
[Out] Piecewise((zoo*x, Eq(a, 0) & Eq(b, 0)), (x/b, Eq(a, 0)), (-x*sinh(x)**2/(-2
*b*sinh(x)**2 + 2*b*cosh(x)**2) + x*cosh(x)**2/(-2*b*sinh(x)**2 + 2*b*cosh(
```

```
x)**2) - sinh(x)*cosh(x)/(-2*b*sinh(x)**2 + 2*b*cosh(x)**2), Eq(a, -b)), ((
x - sinh(x)/cosh(x))/a, Eq(b, 0)), (2*I*sqrt(b)*x*sqrt(1/a)/(2*I*a*sqrt(b)*
sqrt(1/a) + 2*I*b**(3/2)*sqrt(1/a)) + log(-I*sqrt(b)*sqrt(1/a)*sinh(x) + co
sh(x))/(2*I*a*sqrt(b)*sqrt(1/a) + 2*I*b**(3/2)*sqrt(1/a)) - log(I*sqrt(b)*s
qrt(1/a)*sinh(x) + cosh(x))/(2*I*a*sqrt(b)*sqrt(1/a) + 2*I*b**(3/2)*sqrt(1/
a)), True))
```

3.839 $\int \frac{\cosh^2(x)}{a\cosh^2(x)+b\sinh^2(x)}\,dx$

Optimal. Leaf size=38

$$\frac{x}{a+b}+\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\tanh(x)}{\sqrt{a}}\right)}{\sqrt{a}\,(a+b)}$$

```
[Out] x/(a+b)+arctan(b^(1/2)*tanh(x)/a^(1/2))*b^(1/2)/(a+b)/a^(1/2)
```

Rubi [A] time = 0.11, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {391, 206, 205}

$$\frac{x}{a+b}+\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\tanh(x)}{\sqrt{a}}\right)}{\sqrt{a}\,(a+b)}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]^2/(a*Cosh[x]^2 + b*Sinh[x]^2),x]
```

```
[Out] x/(a + b) + (Sqrt[b]*ArcTan[(Sqrt[b]*Tanh[x])/Sqrt[a]])/(Sqrt[a]*(a + b))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 391

```
Int[1/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))), x_Symbol] :> Dis
t[b/(b*c - a*d), Int[1/(a + b*x^n), x], x] - Dist[d/(b*c - a*d), Int[1/(c +
 d*x^n), x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\int \frac{\cosh^2(x)}{a\cosh^2(x)+b\sinh^2(x)}\,dx = \text{Subst}\left(\int \frac{1}{(1-x^2)(a+bx^2)}\,dx, x, \tanh(x)\right)$$
$$= \frac{\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \tanh(x)\right)}{a+b} + \frac{b\,\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, \tanh(x)\right)}{a+b}$$
$$= \frac{x}{a+b} + \frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\tanh(x)}{\sqrt{a}}\right)}{\sqrt{a}\,(a+b)}$$

Mathematica [A] time = 0.06, size = 33, normalized size = 0.87

$$\frac{\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\tanh(x)}{\sqrt{a}}\right)}{\sqrt{a}}+x}{a+b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]^2/(a*Cosh[x]^2 + b*Sinh[x]^2),x]
[Out] (x + (Sqrt[b]*ArcTan[(Sqrt[b]*Tanh[x])/Sqrt[a]])/Sqrt[a])/(a + b)
```

fricas [A] time = 0.49, size = 363, normalized size = 9.55

$$\left[\frac{\sqrt{-\frac{b}{a}}\log\left(\frac{(a^2+2ab+b^2)\cosh(x)^4+4(a^2+2ab+b^2)\cosh(x)\sinh(x)^3+(a^2+2ab+b^2)\sinh(x)^4+2(a^2-b^2)\cosh(x)^2+2\left(3(a^2+2ab+b^2)\cosh(x)^2+a^2-\right.}{(a+b)\cosh(x)^4+4(a+b)\cosh(x)\sinh(x)^3+(a+b)\sinh(x)^4+2(a-b)}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^2/(a*cosh(x)^2+b*sinh(x)^2),x, algorithm="fricas")
[Out] [1/2*(sqrt(-b/a)*log(((a^2 + 2*a*b + b^2)*cosh(x)^4 + 4*(a^2 + 2*a*b + b^2)
*cosh(x)*sinh(x)^3 + (a^2 + 2*a*b + b^2)*sinh(x)^4 + 2*(a^2 - b^2)*cosh(x)^
2 + 2*(3*(a^2 + 2*a*b + b^2)*cosh(x)^2 + a^2 - b^2)*sinh(x)^2 + a^2 - 6*a*b
 + b^2 + 4*((a^2 + 2*a*b + b^2)*cosh(x)^3 + (a^2 - b^2)*cosh(x))*sinh(x) +
4*((a^2 + a*b)*cosh(x)^2 + 2*(a^2 + a*b)*cosh(x)*sinh(x) + (a^2 + a*b)*sinh
(x)^2 + a^2 - a*b)*sqrt(-b/a))/((a + b)*cosh(x)^4 + 4*(a + b)*cosh(x)*sinh(
x)^3 + (a + b)*sinh(x)^4 + 2*(a - b)*cosh(x)^2 + 2*(3*(a + b)*cosh(x)^2 + a
 - b)*sinh(x)^2 + 4*((a + b)*cosh(x)^3 + (a - b)*cosh(x))*sinh(x) + a + b))
```

```
 + 2*x)/(a + b), (sqrt(b/a)*arctan(1/2*((a + b)*cosh(x)^2 + 2*(a + b)*cosh(
x)*sinh(x) + (a + b)*sinh(x)^2 + a - b)*sqrt(b/a)/b) + x)/(a + b)]
```

giac [A] time = 0.14, size = 45, normalized size = 1.18

$$\frac{b \arctan\left(\frac{ae^{(2\,x)}+be^{(2\,x)}+a-b}{2\,\sqrt{ab}}\right)}{\sqrt{ab}\,(a+b)}+\frac{x}{a+b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^2/(a*cosh(x)^2+b*sinh(x)^2),x, algorithm="giac")
```

```
[Out] b*arctan(1/2*(a*e^(2*x) + b*e^(2*x) + a - b)/sqrt(a*b))/(sqrt(a*b)*(a + b))
 + x/(a + b)
```

maple [B] time = 0.21, size = 389, normalized size = 10.24

$$-\frac{2\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{2b+2a}+\frac{2\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)}{2b+2a}-\frac{ba\,\mathrm{arctanh}\left(\frac{a\tanh\left(\frac{x}{2}\right)}{\sqrt{\left(2\sqrt{b(a+b)}-a-2b\right)a}}\right)}{(a+b)\sqrt{b\,(a+b)}\,\sqrt{\left(2\sqrt{b\,(a+b)}-a-2b\right)a}}+\frac{b\,\mathrm{arctanh}\left(\frac{a\,\mathrm{t}}{\sqrt{(2\sqrt{b(}}}\right)}{(a+b)\sqrt{\left(2\sqrt{b\,(a+}\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^2/(a*cosh(x)^2+b*sinh(x)^2),x)
```

```
[Out] -2/(2*b+2*a)*ln(tanh(1/2*x)-1)+2/(2*b+2*a)*ln(tanh(1/2*x)+1)-b/(a+b)*a/(b*(
a+b))^(1/2)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*x)/((2*(
b*(a+b))^(1/2)-a-2*b)*a)^(1/2))+b/(a+b)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2)
*arctanh(a*tanh(1/2*x)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2))-b^2/(a+b)/(b*(a
+b))^(1/2)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*x)/((2*(b
*(a+b))^(1/2)-a-2*b)*a)^(1/2))-b/(a+b)*a/(b*(a+b))^(1/2)/((2*(b*(a+b))^(1/2
)+a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*x)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2))
-b/(a+b)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*x)/((2*(b*(a
+b))^(1/2)+a+2*b)*a)^(1/2))-b^2/(a+b)/(b*(a+b))^(1/2)/((2*(b*(a+b))^(1/2)+a
+2*b)*a)^(1/2)*arctan(a*tanh(1/2*x)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2))
```

maxima [B] time = 0.44, size = 74, normalized size = 1.95

$$-\frac{(a-b)\arctan\left(\frac{(a+b)e^{(2\,x)}+a-b}{2\,\sqrt{ab}}\right)}{2\,\sqrt{ab}\,(a+b)}-\frac{\arctan\left(\frac{(a+b)e^{(-2\,x)}+a-b}{2\,\sqrt{ab}}\right)}{2\,\sqrt{ab}}+\frac{x}{a+b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^2/(a*cosh(x)^2+b*sinh(x)^2),x, algorithm="maxima")
```

[Out] -1/2*(a - b)*arctan(1/2*((a + b)*e^(2*x) + a - b)/sqrt(a*b))/(sqrt(a*b)*(a + b)) - 1/2*arctan(1/2*((a + b)*e^(-2*x) + a - b)/sqrt(a*b))/sqrt(a*b) + x/(a + b)

mupad [B] time = 1.95, size = 208, normalized size = 5.47

$$\frac{x}{a+b}+\frac{\sqrt{b}\,\mathrm{atan}\left(\frac{\left(e^{2x}\left(\frac{4b}{(a+b)^4}+\frac{(a^2-b^2)(a-b)}{(a+b)^3\sqrt{a(a+b)^2}\sqrt{a^3+2a^2b+ab^2}}\right)+\frac{(a-b)(a^2+2ab+b^2)}{(a+b)^3\sqrt{a(a+b)^2}\sqrt{a^3+2a^2b+ab^2}}\right)\left(a^2\sqrt{a^3+2a^2b+ab^2}+b^2\sqrt{a^3+2a^2b+ab^2}+2a\right.}{2\sqrt{b}}\right.}{\sqrt{a^3+2a^2b+ab^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cosh(x)^2/(b*sinh(x)^2 + a*cosh(x)^2),x)

[Out] x/(a + b) + (b^(1/2)*atan(((exp(2*x)*((4*b)/(a + b)^4 + ((a^2 - b^2)*(a - b))/((a + b)^3*(a*(a + b)^2)^(1/2)*(a*b^2 + 2*a^2*b + a^3)^(1/2))) + ((a - b)*(2*a*b + a^2 + b^2))/((a + b)^3*(a*(a + b)^2)^(1/2)*(a*b^2 + 2*a^2*b + a^3)^(1/2)))*(a^2*(a*b^2 + 2*a^2*b + a^3)^(1/2) + b^2*(a*b^2 + 2*a^2*b + a^3)^(1/2) + 2*a*b*(a*b^2 + 2*a^2*b + a^3)^(1/2)))/(2*b^(1/2))))/(a*b^2 + 2*a^2*b + a^3)^(1/2)

sympy [A] time = 1.69, size = 265, normalized size = 6.97

$$\begin{cases} \tilde{\infty}\left(x-\frac{\cosh(x)}{\sinh(x)}\right) & \text{for } a=0\wedge b=0 \\ \frac{x-\frac{\cosh(x)}{\sinh(x)}}{b} & \text{for } a=0 \\ \frac{x\sinh^2(x)}{-2b\sinh^2(x)+2b\cosh^2(x)}-\frac{x\cosh^2(x)}{-2b\sinh^2(x)+2b\cosh^2(x)}-\frac{\sinh(x)\cosh(x)}{-2b\sinh^2(x)+2b\cosh^2(x)} & \text{for } a=-b \\ \frac{x}{a} & \text{for } b=0 \\ \frac{2ia\sqrt{b}\,x\sqrt{\frac{1}{a}}}{2ia^2\sqrt{b}\sqrt{\frac{1}{a}}+2iab^{\frac{3}{2}}\sqrt{\frac{1}{a}}}-\frac{b\log\left(-i\sqrt{b}\sqrt{\frac{1}{a}}\sinh(x)+\cosh(x)\right)}{2ia^2\sqrt{b}\sqrt{\frac{1}{a}}+2iab^{\frac{3}{2}}\sqrt{\frac{1}{a}}}+\frac{b\log\left(i\sqrt{b}\sqrt{\frac{1}{a}}\sinh(x)+\cosh(x)\right)}{2ia^2\sqrt{b}\sqrt{\frac{1}{a}}+2iab^{\frac{3}{2}}\sqrt{\frac{1}{a}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(x)**2/(a*cosh(x)**2+b*sinh(x)**2),x)

[Out] Piecewise((zoo*(x - cosh(x)/sinh(x)), Eq(a, 0) & Eq(b, 0)), ((x - cosh(x)/sinh(x))/b, Eq(a, 0)), (x*sinh(x)**2/(-2*b*sinh(x)**2 + 2*b*cosh(x)**2) - x*

```
cosh(x)**2/(-2*b*sinh(x)**2 + 2*b*cosh(x)**2) - sinh(x)*cosh(x)/(-2*b*sinh(
x)**2 + 2*b*cosh(x)**2), Eq(a, -b)), (x/a, Eq(b, 0)), (2*I*a*sqrt(b)*x*sqrt
(1/a)/(2*I*a**2*sqrt(b)*sqrt(1/a) + 2*I*a*b**(3/2)*sqrt(1/a)) - b*log(-I*sq
rt(b)*sqrt(1/a)*sinh(x) + cosh(x))/(2*I*a**2*sqrt(b)*sqrt(1/a) + 2*I*a*b**(
3/2)*sqrt(1/a)) + b*log(I*sqrt(b)*sqrt(1/a)*sinh(x) + cosh(x))/(2*I*a**2*sq
rt(b)*sqrt(1/a) + 2*I*a*b**(3/2)*sqrt(1/a)), True))
```

3.840 $\int \frac{\sinh^3(x)}{\cosh^3(x)+\sinh^3(x)}\,dx$

Optimal. Leaf size=38

$$\frac{x}{2}+\frac{1}{6(\tanh(x)+1)}+\frac{2\tan^{-1}\left(\frac{1-2\tanh(x)}{\sqrt{3}}\right)}{3\sqrt{3}}$$

```
[Out] 1/2*x+2/9*arctan(1/3*(1-2*tanh(x))*3^(1/2))*3^(1/2)+1/6/(1+tanh(x))
```

Rubi [A] time = 0.15, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2074, 207, 618, 204}

$$\frac{x}{2}+\frac{1}{6(\tanh(x)+1)}+\frac{2\tan^{-1}\left(\frac{1-2\tanh(x)}{\sqrt{3}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[x]^3/(Cosh[x]^3 + Sinh[x]^3),x]
```

```
[Out] x/2 + (2*ArcTan[(1 - 2*Tanh[x])/Sqrt[3]])/(3*Sqrt[3]) + 1/(6*(1 + Tanh[x]))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2074

```
Int[(P_)^(p_)*(Q_)^(q_.), x_Symbol] :> With[{PP = Factor[P]}, Int[ExpandInt
egrand[PP^p*Q^q, x], x] /;  !SumQ[NonfreeFactors[PP, x]]] /; FreeQ[q, x] &&
```

```
PolyQ[P, x] && PolyQ[Q, x] && IntegerQ[p] && NeQ[P, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sinh^3(x)}{\cosh^3(x)+\sinh^3(x)}\,dx &= -\operatorname{Subst}\left(\int \frac{x^3}{-1+x^2-x^3+x^5}\,dx, x, \tanh(x)\right)\\
&= -\operatorname{Subst}\left(\int \left(\frac{1}{6(1+x)^2}+\frac{1}{2\left(-1+x^2\right)}+\frac{1}{3\left(1-x+x^2\right)}\right)dx, x, \tanh(x)\right)\\
&= \frac{1}{6(1+\tanh(x))}-\frac{1}{3}\operatorname{Subst}\left(\int \frac{1}{1-x+x^2}\,dx, x, \tanh(x)\right)-\frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{-1+x^2}\,dx\right.\\
&= \frac{x}{2}+\frac{1}{6(1+\tanh(x))}+\frac{2}{3}\operatorname{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, -1+2\tanh(x)\right)\\
&= \frac{x}{2}+\frac{2\tan^{-1}\left(\frac{1-2\tanh(x)}{\sqrt{3}}\right)}{3\sqrt{3}}+\frac{1}{6(1+\tanh(x))}
\end{aligned}$$

Mathematica [A] time = 0.14, size = 40, normalized size = 1.05

$$\frac{1}{36}\left(18x-3\sinh(2x)+3\cosh(2x)-8\sqrt{3}\,\tan^{-1}\left(\frac{2\tanh(x)-1}{\sqrt{3}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[x]^3/(Cosh[x]^3 + Sinh[x]^3),x]
```

```
[Out] (18*x - 8*Sqrt[3]*ArcTan[(-1 + 2*Tanh[x])/Sqrt[3]] + 3*Cosh[2*x] - 3*Sinh[2
*x])/36
```

fricas [B] time = 0.43, size = 95, normalized size = 2.50

$$\frac{18\,x\cosh(x)^2+36\,x\cosh(x)\sinh(x)+18\,x\sinh(x)^2+8\left(\sqrt{3}\,\cosh(x)^2+2\,\sqrt{3}\,\cosh(x)\sinh(x)+\sqrt{3}\,\sinh(x)^2\right)}{36\left(\cosh(x)^2+2\,\cosh(x)\sinh(x)+\sinh(x)^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^3/(cosh(x)^3+sinh(x)^3),x, algorithm="fricas")
```

```
[Out] 1/36*(18*x*cosh(x)^2 + 36*x*cosh(x)*sinh(x) + 18*x*sinh(x)^2 + 8*(sqrt(3)*c
osh(x)^2 + 2*sqrt(3)*cosh(x)*sinh(x) + sqrt(3)*sinh(x)^2)*arctan(-1/3*(sqrt
```

```
(3)*cosh(x) + sqrt(3)*sinh(x))/(cosh(x) - sinh(x))) + 3)/(cosh(x)^2 + 2*cos
h(x)*sinh(x) + sinh(x)^2)
```

giac [A] time = 0.13, size = 33, normalized size = 0.87

$$-\frac{1}{12}\left(3\,e^{(2\,x)}-1\right)e^{(-2\,x)}-\frac{2}{9}\,\sqrt{3}\,\arctan\left(\frac{1}{3}\,\sqrt{3}\,e^{(2\,x)}\right)+\frac{1}{2}\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^3/(cosh(x)^3+sinh(x)^3),x, algorithm="giac")
```

```
[Out] -1/12*(3*e^(2*x) - 1)*e^(-2*x) - 2/9*sqrt(3)*arctan(1/3*sqrt(3)*e^(2*x)) +
1/2*x
```

maple [C] time = 0.24, size = 96, normalized size = 2.53

$$-\frac{\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{2}+\frac{i\sqrt{3}\,\ln\left(\tanh^{2}\left(\frac{x}{2}\right)+\left(i\sqrt{3}\,-1\right)\tanh\left(\frac{x}{2}\right)+1\right)}{9}-\frac{i\sqrt{3}\,\ln\left(\tanh^{2}\left(\frac{x}{2}\right)+\left(-i\sqrt{3}\,-1\right)\tanh\left(\frac{x}{2}\right)+\right.}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)^3/(cosh(x)^3+sinh(x)^3),x)
```

```
[Out] -1/2*ln(tanh(1/2*x)-1)+1/9*I*3^(1/2)*ln(tanh(1/2*x)^2+(I*3^(1/2)-1)*tanh(1/
2*x)+1)-1/9*I*3^(1/2)*ln(tanh(1/2*x)^2+(-I*3^(1/2)-1)*tanh(1/2*x)+1)+1/3/(t
anh(1/2*x)+1)^2-1/3/(tanh(1/2*x)+1)+1/2*ln(tanh(1/2*x)+1)
```

maxima [B] time = 0.46, size = 73, normalized size = 1.92

$$-\frac{2}{9}\,\sqrt{3}\,\arctan\left(\frac{1}{6}\cdot 3^{\frac{3}{4}}\sqrt{2}\left(2\,\sqrt{3}\,e^{(-x)}+3^{\frac{1}{4}}\sqrt{2}\right)\right)+\frac{2}{9}\,\sqrt{3}\,\arctan\left(\frac{1}{6}\cdot 3^{\frac{3}{4}}\sqrt{2}\left(2\,\sqrt{3}\,e^{(-x)}-3^{\frac{1}{4}}\sqrt{2}\right)\right)+\frac{1}{2}\,x+\frac{1}{12}\,e^{(-2\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^3/(cosh(x)^3+sinh(x)^3),x, algorithm="maxima")
```

```
[Out] -2/9*sqrt(3)*arctan(1/6*3^(3/4)*sqrt(2)*(2*sqrt(3)*e^(-x) + 3^(1/4)*sqrt(2)
)) + 2/9*sqrt(3)*arctan(1/6*3^(3/4)*sqrt(2)*(2*sqrt(3)*e^(-x) - 3^(1/4)*sqr
t(2))) + 1/2*x + 1/12*e^(-2*x)
```

mupad [B] time = 1.74, size = 25, normalized size = 0.66

$$\frac{x}{2}+\frac{\mathrm{e}^{-2\,x}}{12}-\frac{2\,\sqrt{3}\,\mathrm{atan}\left(\frac{\sqrt{3}\,\mathrm{e}^{2\,x}}{3}\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)^3/(cosh(x)^3 + sinh(x)^3),x)
```

```
[Out] x/2 + exp(-2*x)/12 - (2*3^(1/2)*atan((3^(1/2)*exp(2*x))/3))/9
```

sympy [B] time = 1.55, size = 136, normalized size = 3.58

$$\frac{9x\sinh(x)}{18\sinh(x)+18\cosh(x)}+\frac{9x\cosh(x)}{18\sinh(x)+18\cosh(x)}-\frac{4\sqrt{3}\,\sinh(x)\,\mathrm{atan}\left(\frac{\sqrt{3}}{3}-\frac{2\sqrt{3}\,\cosh(x)}{3\sinh(x)}\right)}{18\sinh(x)+18\cosh(x)}-\frac{4\sqrt{3}\,\cosh(x)\,\mathrm{atan}}{18\sinh(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)**3/(cosh(x)**3+sinh(x)**3),x)
```

```
[Out] 9*x*sinh(x)/(18*sinh(x) + 18*cosh(x)) + 9*x*cosh(x)/(18*sinh(x) + 18*cosh(x
)) - 4*sqrt(3)*sinh(x)*atan(sqrt(3)/3 - 2*sqrt(3)*cosh(x)/(3*sinh(x)))/(18*
sinh(x) + 18*cosh(x)) - 4*sqrt(3)*cosh(x)*atan(sqrt(3)/3 - 2*sqrt(3)*cosh(x
)/(3*sinh(x)))/(18*sinh(x) + 18*cosh(x)) + 3*cosh(x)/(18*sinh(x) + 18*cosh(
x))
```

3.841 $\int \frac{\cosh^3(x)}{\cosh^3(x)+\sinh^3(x)}\,dx$

Optimal. Leaf size=38

$$\frac{x}{2}-\frac{1}{6(\tanh(x)+1)}-\frac{2\tan^{-1}\left(\frac{1-2\tanh(x)}{\sqrt{3}}\right)}{3\sqrt{3}}$$

```
[Out] 1/2*x-2/9*arctan(1/3*(1-2*tanh(x))*3^(1/2))*3^(1/2)-1/6/(1+tanh(x))
```

Rubi [A] time = 0.09, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2058, 207, 618, 204}

$$\frac{x}{2}-\frac{1}{6(\tanh(x)+1)}-\frac{2\tan^{-1}\left(\frac{1-2\tanh(x)}{\sqrt{3}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]^3/(Cosh[x]^3 + Sinh[x]^3),x]
```

```
[Out] x/2 - (2*ArcTan[(1 - 2*Tanh[x])/Sqrt[3]])/(3*Sqrt[3]) - 1/(6*(1 + Tanh[x]))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2058

```
Int[(P_)^(p_), x_Symbol] :> With[{u = Factor[P]}, Int[ExpandIntegrand[u^p,
x], x] /;  !SumQ[NonfreeFactors[u, x]]] /; PolyQ[P, x] && ILtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cosh^3(x)}{\cosh^3(x)+\sinh^3(x)}\,dx &= \text{Subst}\left(\int \frac{1}{1-x^2+x^3-x^5}\,dx, x, \tanh(x)\right)\\
&= \text{Subst}\left(\int \left(\frac{1}{6(1+x)^2}-\frac{1}{2\left(-1+x^2\right)}+\frac{1}{3\left(1-x+x^2\right)}\right)dx, x, \tanh(x)\right)\\
&= -\frac{1}{6(1+\tanh(x))}+\frac{1}{3}\text{Subst}\left(\int \frac{1}{1-x+x^2}\,dx, x, \tanh(x)\right)-\frac{1}{2}\text{Subst}\left(\int \frac{1}{-1+x^2}\right.\\
&= \frac{x}{2}-\frac{1}{6(1+\tanh(x))}-\frac{2}{3}\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, -1+2\tanh(x)\right)\\
&= \frac{x}{2}-\frac{2\tan^{-1}\left(\frac{1-2\tanh(x)}{\sqrt{3}}\right)}{3\sqrt{3}}-\frac{1}{6(1+\tanh(x))}
\end{aligned}$$

Mathematica [A] time = 0.10, size = 40, normalized size = 1.05

$$\frac{1}{36}\left(18x+3\sinh(2x)-3\cosh(2x)+8\sqrt{3}\,\tan^{-1}\left(\frac{2\tanh(x)-1}{\sqrt{3}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]^3/(Cosh[x]^3 + Sinh[x]^3),x]
```

```
[Out] (18*x + 8*Sqrt[3]*ArcTan[(-1 + 2*Tanh[x])/Sqrt[3]] - 3*Cosh[2*x] + 3*Sinh[2
*x])/36
```

fricas [B] time = 0.42, size = 95, normalized size = 2.50

$$\frac{18\,x\cosh(x)^2+36\,x\cosh(x)\sinh(x)+18\,x\sinh(x)^2-8\left(\sqrt{3}\,\cosh(x)^2+2\,\sqrt{3}\,\cosh(x)\sinh(x)+\sqrt{3}\,\sinh(x)^2\right)}{36\left(\cosh(x)^2+2\,\cosh(x)\sinh(x)+\sinh(x)^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^3/(cosh(x)^3+sinh(x)^3),x, algorithm="fricas")
```

```
[Out] 1/36*(18*x*cosh(x)^2 + 36*x*cosh(x)*sinh(x) + 18*x*sinh(x)^2 - 8*(sqrt(3)*c
osh(x)^2 + 2*sqrt(3)*cosh(x)*sinh(x) + sqrt(3)*sinh(x)^2)*arctan(-1/3*(sqrt
```

```
(3)*cosh(x) + sqrt(3)*sinh(x))/(cosh(x) - sinh(x))) - 3)/(cosh(x)^2 + 2*cos
h(x)*sinh(x) + sinh(x)^2)
```

giac [A] time = 0.11, size = 33, normalized size = 0.87

$$-\frac{1}{12}\left(3e^{(2x)}+1\right)e^{(-2x)}+\frac{2}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}e^{(2x)}\right)+\frac{1}{2}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^3/(cosh(x)^3+sinh(x)^3),x, algorithm="giac")
```

```
[Out] -1/12*(3*e^(2*x) + 1)*e^(-2*x) + 2/9*sqrt(3)*arctan(1/3*sqrt(3)*e^(2*x)) +
1/2*x
```

maple [C] time = 0.24, size = 96, normalized size = 2.53

$$-\frac{\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{2}+\frac{i\sqrt{3}\,\ln\left(\tanh^2\left(\frac{x}{2}\right)+\left(-i\sqrt{3}-1\right)\tanh\left(\frac{x}{2}\right)+1\right)}{9}-\frac{i\sqrt{3}\,\ln\left(\tanh^2\left(\frac{x}{2}\right)+\left(i\sqrt{3}-1\right)\tanh\left(\frac{x}{2}\right)+\right.}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^3/(cosh(x)^3+sinh(x)^3),x)
```

```
[Out] -1/2*ln(tanh(1/2*x)-1)+1/9*I*3^(1/2)*ln(tanh(1/2*x)^2+(-I*3^(1/2)-1)*tanh(1
/2*x)+1)-1/9*I*3^(1/2)*ln(tanh(1/2*x)^2+(I*3^(1/2)-1)*tanh(1/2*x)+1)-1/3/(t
anh(1/2*x)+1)^2+1/3/(tanh(1/2*x)+1)+1/2*ln(tanh(1/2*x)+1)
```

maxima [B] time = 0.42, size = 73, normalized size = 1.92

$$\frac{2}{9}\sqrt{3}\arctan\left(\frac{1}{6}\cdot 3^{\frac{3}{4}}\sqrt{2}\left(2\sqrt{3}e^{(-x)}+3^{\frac{1}{4}}\sqrt{2}\right)\right)-\frac{2}{9}\sqrt{3}\arctan\left(\frac{1}{6}\cdot 3^{\frac{3}{4}}\sqrt{2}\left(2\sqrt{3}e^{(-x)}-3^{\frac{1}{4}}\sqrt{2}\right)\right)+\frac{1}{2}x-\frac{1}{12}e^{(-2x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^3/(cosh(x)^3+sinh(x)^3),x, algorithm="maxima")
```

```
[Out] 2/9*sqrt(3)*arctan(1/6*3^(3/4)*sqrt(2)*(2*sqrt(3)*e^(-x) + 3^(1/4)*sqrt(2))
) - 2/9*sqrt(3)*arctan(1/6*3^(3/4)*sqrt(2)*(2*sqrt(3)*e^(-x) - 3^(1/4)*sqrt
(2))) + 1/2*x - 1/12*e^(-2*x)
```

mupad [B] time = 1.69, size = 25, normalized size = 0.66

$$\frac{x}{2}-\frac{\mathrm{e}^{-2x}}{12}+\frac{2\sqrt{3}\,\mathrm{atan}\left(\frac{\sqrt{3}\,\mathrm{e}^{2x}}{3}\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^3/(cosh(x)^3 + sinh(x)^3),x)
```

```
[Out] x/2 - exp(-2*x)/12 + (2*3^(1/2)*atan((3^(1/2)*exp(2*x))/3))/9
```

sympy [B] time = 1.53, size = 136, normalized size = 3.58

$$\frac{9x\sinh(x)}{18\sinh(x)+18\cosh(x)}+\frac{9x\cosh(x)}{18\sinh(x)+18\cosh(x)}+\frac{4\sqrt{3}\,\sinh(x)\operatorname{atan}\left(\frac{\sqrt{3}}{3}-\frac{2\sqrt{3}\,\cosh(x)}{3\sinh(x)}\right)}{18\sinh(x)+18\cosh(x)}+\frac{4\sqrt{3}\,\cosh(x)\operatorname{atan}}{18\sinh(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)**3/(cosh(x)**3+sinh(x)**3),x)
```

```
[Out] 9*x*sinh(x)/(18*sinh(x) + 18*cosh(x)) + 9*x*cosh(x)/(18*sinh(x) + 18*cosh(x
)) + 4*sqrt(3)*sinh(x)*atan(sqrt(3)/3 - 2*sqrt(3)*cosh(x)/(3*sinh(x)))/(18*
sinh(x) + 18*cosh(x)) + 4*sqrt(3)*cosh(x)*atan(sqrt(3)/3 - 2*sqrt(3)*cosh(x
)/(3*sinh(x)))/(18*sinh(x) + 18*cosh(x)) - 3*cosh(x)/(18*sinh(x) + 18*cosh(
x))
```

3.842 $\int \frac{x\mathbf{csch}(x)\mathbf{sech}(x)}{\sqrt{a\mathbf{sech}^2(x)}}\,dx$

Optimal. Leaf size=59

$$-\frac{\text{Li}_2\left(-e^x\right)\text{sech}(x)}{\sqrt{a\text{sech}^2(x)}}+\frac{\text{Li}_2\left(e^x\right)\text{sech}(x)}{\sqrt{a\text{sech}^2(x)}}-\frac{2x\tanh^{-1}\left(e^x\right)\text{sech}(x)}{\sqrt{a\text{sech}^2(x)}}$$

```
[Out] -2*x*arctanh(exp(x))*sech(x)/(a*sech(x)^2)^(1/2)-polylog(2,-exp(x))*sech(x)
/(a*sech(x)^2)^(1/2)+polylog(2,exp(x))*sech(x)/(a*sech(x)^2)^(1/2)
```

Rubi [A] time = 0.70, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6720, 4182, 2279, 2391}

$$-\frac{\text{sech}(x)\text{PolyLog}\left(2,-e^x\right)}{\sqrt{a\text{sech}^2(x)}}+\frac{\text{sech}(x)\text{PolyLog}\left(2,e^x\right)}{\sqrt{a\text{sech}^2(x)}}-\frac{2x\tanh^{-1}\left(e^x\right)\text{sech}(x)}{\sqrt{a\text{sech}^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Csch[x]*Sech[x])/Sqrt[a*Sech[x]^2],x]
```

```
[Out] (-2*x*ArcTanh[E^x]*Sech[x])/Sqrt[a*Sech[x]^2] - (PolyLog[2, -E^x]*Sech[x])/
Sqrt[a*Sech[x]^2] + (PolyLog[2, E^x]*Sech[x])/Sqrt[a*Sech[x]^2]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 6720

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a*v^m)^
FracPart[p])/v^(m*FracPart[p]), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rubi steps

$$\int \frac{x\text{csch}(x)\text{sech}(x)}{\sqrt{a\text{sech}^2(x)}}\,dx = \frac{\text{sech}(x)\int x\text{csch}(x)\,dx}{\sqrt{a\text{sech}^2(x)}}$$

$$= -\frac{2x\tanh^{-1}(e^x)\,\text{sech}(x)}{\sqrt{a\text{sech}^2(x)}} - \frac{\text{sech}(x)\int \log(1-e^x)\,dx}{\sqrt{a\text{sech}^2(x)}} + \frac{\text{sech}(x)\int \log(1+e^x)\,dx}{\sqrt{a\text{sech}^2(x)}}$$

$$= -\frac{2x\tanh^{-1}(e^x)\,\text{sech}(x)}{\sqrt{a\text{sech}^2(x)}} - \frac{\text{sech}(x)\,\text{Subst}\left(\int \frac{\log(1-x)}{x}\,dx, x, e^x\right)}{\sqrt{a\text{sech}^2(x)}} + \frac{\text{sech}(x)\,\text{Subst}\left(\int \frac{\log}{}\right.}{\sqrt{a\text{sech}^2(}}$$

$$= -\frac{2x\tanh^{-1}(e^x)\,\text{sech}(x)}{\sqrt{a\text{sech}^2(x)}} - \frac{\text{Li}_2(-e^x)\,\text{sech}(x)}{\sqrt{a\text{sech}^2(x)}} + \frac{\text{Li}_2(e^x)\,\text{sech}(x)}{\sqrt{a\text{sech}^2(x)}}$$

Mathematica [A] time = 0.06, size = 55, normalized size = 0.93

$$\frac{\text{sech}(x)\left(\text{Li}_2(-e^{-x}) - \text{Li}_2(e^{-x}) + x\left(\log(1-e^{-x}) - \log(e^{-x}+1)\right)\right)}{\sqrt{a\text{sech}^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Csch[x]*Sech[x])/Sqrt[a*Sech[x]^2],x]
```

```
[Out] ((x*(Log[1 - E^(-x)] - Log[1 + E^(-x)]) + PolyLog[2, -E^(-x)] - PolyLog[2,
E^(-x)])*Sech[x])/Sqrt[a*Sech[x]^2]
```

fricas [A] time = 0.42, size = 91, normalized size = 1.54

$$\frac{\left(\left(e^{(2x)}+1\right)\text{Li}_2(\cosh(x)+\sinh(x)) - \left(e^{(2x)}+1\right)\text{Li}_2(-\cosh(x)-\sinh(x)) - \left(xe^{(2x)}+x\right)\log(\cosh(x)+\sinh(x)\right.}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*csch(x)*sech(x)/(a*sech(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] ((e^(2*x) + 1)*dilog(cosh(x) + sinh(x)) - (e^(2*x) + 1)*dilog(-cosh(x) - si
nh(x)) - (x*e^(2*x) + x)*log(cosh(x) + sinh(x) + 1) + (x*e^(2*x) + x)*log(-
cosh(x) - sinh(x) + 1))*sqrt(a/(e^(4*x) + 2*e^(2*x) + 1))/a
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\,\mathrm{csch}(x)\,\mathrm{sech}(x)}{\sqrt{a\,\mathrm{sech}(x)^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*csch(x)*sech(x)/(a*sech(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x*csch(x)*sech(x)/sqrt(a*sech(x)^2), x)
```

maple [B] time = 0.34, size = 136, normalized size = 2.31

$$-\frac{e^{x} x \ln\left(e^{x}+1\right)}{\sqrt{\frac{a\,e^{2x}}{\left(1+e^{2x}\right)^{2}}}\,\left(1+e^{2x}\right)}-\frac{e^{x}\,\mathrm{polylog}\left(2,-e^{x}\right)}{\sqrt{\frac{a\,e^{2x}}{\left(1+e^{2x}\right)^{2}}}\,\left(1+e^{2x}\right)}+\frac{e^{x} x \ln\left(1-e^{x}\right)}{\sqrt{\frac{a\,e^{2x}}{\left(1+e^{2x}\right)^{2}}}\,\left(1+e^{2x}\right)}+\frac{e^{x}\,\mathrm{polylog}\left(2,e^{x}\right)}{\sqrt{\frac{a\,e^{2x}}{\left(1+e^{2x}\right)^{2}}}\,\left(1+e^{2x}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*csch(x)*sech(x)/(a*sech(x)^2)^(1/2),x)
```

```
[Out] -1/(a*exp(2*x)/(1+exp(2*x))^2)^(1/2)/(1+exp(2*x))*exp(x)*x*ln(exp(x)+1)-1/(
a*exp(2*x)/(1+exp(2*x))^2)^(1/2)/(1+exp(2*x))*exp(x)*polylog(2,-exp(x))+1/(
a*exp(2*x)/(1+exp(2*x))^2)^(1/2)/(1+exp(2*x))*exp(x)*x*ln(1-exp(x))+1/(a*ex
p(2*x)/(1+exp(2*x))^2)^(1/2)/(1+exp(2*x))*exp(x)*polylog(2,exp(x))
```

maxima [A] time = 0.44, size = 36, normalized size = 0.61

$$-\frac{x \log\left(e^{x}+1\right)+\mathrm{Li}_2\left(-e^{x}\right)}{\sqrt{a}}+\frac{x \log\left(-e^{x}+1\right)+\mathrm{Li}_2\left(e^{x}\right)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*csch(x)*sech(x)/(a*sech(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -(x*log(e^x + 1) + dilog(-e^x))/sqrt(a) + (x*log(-e^x + 1) + dilog(e^x))/sq
rt(a)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x}{\cosh(x)\,\sinh(x)\,\sqrt{\frac{a}{\cosh(x)^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(cosh(x)*sinh(x)*(a/cosh(x)^2)^(1/2)),x)
```

```
[Out] int(x/(cosh(x)*sinh(x)*(a/cosh(x)^2)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x \operatorname{csch}(x) \operatorname{sech}(x)}{\sqrt{a \operatorname{sech}^2(x)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*csch(x)*sech(x)/(a*sech(x)**2)**(1/2),x)
```

```
[Out] Integral(x*csch(x)*sech(x)/sqrt(a*sech(x)**2), x)
```

3.843 $\int \frac{x^2\mathbf{csch}(x)\mathbf{sech}(x)}{\sqrt{a\mathbf{sech}^2(x)}}\,dx$

Optimal. Leaf size=104

$$-\frac{2x\text{Li}_2(-e^x)\operatorname{sech}(x)}{\sqrt{a\operatorname{sech}^2(x)}}+\frac{2x\text{Li}_2(e^x)\operatorname{sech}(x)}{\sqrt{a\operatorname{sech}^2(x)}}+\frac{2\text{Li}_3(-e^x)\operatorname{sech}(x)}{\sqrt{a\operatorname{sech}^2(x)}}-\frac{2\text{Li}_3(e^x)\operatorname{sech}(x)}{\sqrt{a\operatorname{sech}^2(x)}}-\frac{2x^2\tanh^{-1}(e^x)\operatorname{sech}(x)}{\sqrt{a\operatorname{sech}^2(x)}}$$

```
[Out] -2*x^2*arctanh(exp(x))*sech(x)/(a*sech(x)^2)^(1/2)-2*x*polylog(2,-exp(x))*s
ech(x)/(a*sech(x)^2)^(1/2)+2*x*polylog(2,exp(x))*sech(x)/(a*sech(x)^2)^(1/2
)+2*polylog(3,-exp(x))*sech(x)/(a*sech(x)^2)^(1/2)-2*polylog(3,exp(x))*sech
(x)/(a*sech(x)^2)^(1/2)
```

Rubi [A] time = 0.81, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {6720, 4182, 2531, 2282, 6589}

$$-\frac{2x\operatorname{sech}(x)\text{PolyLog}(2,-e^x)}{\sqrt{a\operatorname{sech}^2(x)}}+\frac{2x\operatorname{sech}(x)\text{PolyLog}(2,e^x)}{\sqrt{a\operatorname{sech}^2(x)}}+\frac{2\operatorname{sech}(x)\text{PolyLog}(3,-e^x)}{\sqrt{a\operatorname{sech}^2(x)}}-\frac{2\operatorname{sech}(x)\text{PolyLog}(3,e^x)}{\sqrt{a\operatorname{sech}^2(x)}}-$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*Csch[x]*Sech[x])/Sqrt[a*Sech[x]^2],x]
```

```
[Out] (-2*x^2*ArcTanh[E^x]*Sech[x])/Sqrt[a*Sech[x]^2] - (2*x*PolyLog[2, -E^x]*Sec
h[x])/Sqrt[a*Sech[x]^2] + (2*x*PolyLog[2, E^x]*Sech[x])/Sqrt[a*Sech[x]^2] +
 (2*PolyLog[3, -E^x]*Sech[x])/Sqrt[a*Sech[x]^2] - (2*PolyLog[3, E^x]*Sech[x
])/Sqrt[a*Sech[x]^2]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
```

```
, g, n}, x] && GtQ[m, 0]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6720

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a*v^m)^
FracPart[p])/v^(m*FracPart[p]), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] &&  !IntegerQ[p] &&  !FreeQ[v, x] &&  !(EqQ[a, 1] && EqQ[m, 1]) &&  !(EqQ
[v, x] && EqQ[m, 1])
```

Rubi steps

$$\int \frac{x^2\text{csch}(x)\text{sech}(x)}{\sqrt{a\text{sech}^2(x)}}\,dx = \frac{\text{sech}(x)\int x^2\text{csch}(x)\,dx}{\sqrt{a\text{sech}^2(x)}}$$

$$= -\frac{2x^2\tanh^{-1}(e^x)\,\text{sech}(x)}{\sqrt{a\text{sech}^2(x)}} - \frac{(2\text{sech}(x))\int x\log(1-e^x)\,dx}{\sqrt{a\text{sech}^2(x)}} + \frac{(2\text{sech}(x))\int x\log(1+$$

$$= -\frac{2x^2\tanh^{-1}(e^x)\,\text{sech}(x)}{\sqrt{a\text{sech}^2(x)}} - \frac{2x\text{Li}_2(-e^x)\,\text{sech}(x)}{\sqrt{a\text{sech}^2(x)}} + \frac{2x\text{Li}_2(e^x)\,\text{sech}(x)}{\sqrt{a\text{sech}^2(x)}} + \frac{(2\text{sech}(x))\int}{\sqrt{ase}}$$

$$= -\frac{2x^2\tanh^{-1}(e^x)\,\text{sech}(x)}{\sqrt{a\text{sech}^2(x)}} - \frac{2x\text{Li}_2(-e^x)\,\text{sech}(x)}{\sqrt{a\text{sech}^2(x)}} + \frac{2x\text{Li}_2(e^x)\,\text{sech}(x)}{\sqrt{a\text{sech}^2(x)}} + \frac{(2\text{sech}(x))\,\text{S}}{}$$

$$= -\frac{2x^2\tanh^{-1}(e^x)\,\text{sech}(x)}{\sqrt{a\text{sech}^2(x)}} - \frac{2x\text{Li}_2(-e^x)\,\text{sech}(x)}{\sqrt{a\text{sech}^2(x)}} + \frac{2x\text{Li}_2(e^x)\,\text{sech}(x)}{\sqrt{a\text{sech}^2(x)}} + \frac{2\text{Li}_3(-e^x)\,\text{se}}{\sqrt{a\text{sech}^2(}}$$

Mathematica [A] time = 0.07, size = 83, normalized size = 0.80

$$\frac{\mathrm{sech}(x)\left(2x\mathrm{Li}_2\left(-e^{-x}\right)-2x\mathrm{Li}_2\left(e^{-x}\right)+2\mathrm{Li}_3\left(-e^{-x}\right)-2\mathrm{Li}_3\left(e^{-x}\right)+x^2\log\left(1-e^{-x}\right)-x^2\log\left(e^{-x}+1\right)\right)}{\sqrt{a\mathrm{sech}^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*Csch[x]*Sech[x])/Sqrt[a*Sech[x]^2],x]
```

```
[Out] ((x^2*Log[1 - E^(-x)] - x^2*Log[1 + E^(-x)] + 2*x*PolyLog[2, -E^(-x)] - 2*x
*PolyLog[2, E^(-x)] + 2*PolyLog[3, -E^(-x)] - 2*PolyLog[3, E^(-x)])*Sech[x]
)/Sqrt[a*Sech[x]^2]
```

fricas [C] time = 0.43, size = 188, normalized size = 1.81

$$-\frac{\left(2\sqrt{\frac{a}{e^{(4x)}+2e^{(2x)}+1}}\left(e^{(2x)}+1\right)e^x\mathrm{polylog}\left(3,\cosh(x)+\sinh(x)\right)-2\sqrt{\frac{a}{e^{(4x)}+2e^{(2x)}+1}}\left(e^{(2x)}+1\right)e^x\mathrm{polylog}\left(3,-\cosh(\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*csch(x)*sech(x)/(a*sech(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -(2*sqrt(a/(e^(4*x) + 2*e^(2*x) + 1))*(e^(2*x) + 1)*e^x*polylog(3, cosh(x)
+ sinh(x)) - 2*sqrt(a/(e^(4*x) + 2*e^(2*x) + 1))*(e^(2*x) + 1)*e^x*polylog(
3, -cosh(x) - sinh(x)) - (2*(x*e^(2*x) + x)*dilog(cosh(x) + sinh(x)) - 2*(x
*e^(2*x) + x)*dilog(-cosh(x) - sinh(x)) - (x^2*e^(2*x) + x^2)*log(cosh(x) +
 sinh(x) + 1) + (x^2*e^(2*x) + x^2)*log(-cosh(x) - sinh(x) + 1))*sqrt(a/(e^
(4*x) + 2*e^(2*x) + 1))*e^x)*e^(-x)/a
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^2\,\mathrm{csch}(x)\,\mathrm{sech}(x)}{\sqrt{a\,\mathrm{sech}(x)^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*csch(x)*sech(x)/(a*sech(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^2*csch(x)*sech(x)/sqrt(a*sech(x)^2), x)
```

maple [B] time = 0.35, size = 209, normalized size = 2.01

$$-\frac{e^x x^2\ln\left(e^x+1\right)}{\sqrt{\frac{a\,e^{2x}}{\left(1+e^{2x}\right)^2}}\left(1+e^{2x}\right)}-\frac{2\,e^x x\,\mathrm{polylog}\left(2,-e^x\right)}{\sqrt{\frac{a\,e^{2x}}{\left(1+e^{2x}\right)^2}}\left(1+e^{2x}\right)}+\frac{2\,e^x\,\mathrm{polylog}\left(3,-e^x\right)}{\sqrt{\frac{a\,e^{2x}}{\left(1+e^{2x}\right)^2}}\left(1+e^{2x}\right)}+\frac{e^x x^2\ln\left(1-e^x\right)}{\sqrt{\frac{a\,e^{2x}}{\left(1+e^{2x}\right)^2}}\left(1+e^{2x}\right)}+\frac{2\,e^x x\,\mathrm{polylog}\left(2,e^{x}\right)}{\sqrt{\frac{a\,e^{2x}}{\left(1+e^{2x}\right)^2}}\left(1+e^{2x}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*csch(x)*sech(x)/(a*sech(x)^2)^(1/2),x)
```

```
[Out] -1/(a*exp(2*x)/(1+exp(2*x))^2)^(1/2)/(1+exp(2*x))*exp(x)*x^2*ln(exp(x)+1)-2
/(a*exp(2*x)/(1+exp(2*x))^2)^(1/2)/(1+exp(2*x))*exp(x)*x*polylog(2,-exp(x))
+2/(a*exp(2*x)/(1+exp(2*x))^2)^(1/2)/(1+exp(2*x))*exp(x)*polylog(3,-exp(x))
+1/(a*exp(2*x)/(1+exp(2*x))^2)^(1/2)/(1+exp(2*x))*exp(x)*x^2*ln(1-exp(x))+2
/(a*exp(2*x)/(1+exp(2*x))^2)^(1/2)/(1+exp(2*x))*exp(x)*x*polylog(2,exp(x))-
2/(a*exp(2*x)/(1+exp(2*x))^2)^(1/2)/(1+exp(2*x))*exp(x)*polylog(3,exp(x))
```

maxima [A] time = 0.44, size = 60, normalized size = 0.58

$$-\frac{x^2 \log\left(e^x+1\right)+2\,x\mathrm{Li}_2\left(-e^x\right)-2\,\mathrm{Li}_3(-e^x)}{\sqrt{a}}+\frac{x^2 \log\left(-e^x+1\right)+2\,x\mathrm{Li}_2\left(e^x\right)-2\,\mathrm{Li}_3(e^x)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*csch(x)*sech(x)/(a*sech(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -(x^2*log(e^x + 1) + 2*x*dilog(-e^x) - 2*polylog(3, -e^x))/sqrt(a) + (x^2*l
og(-e^x + 1) + 2*x*dilog(e^x) - 2*polylog(3, e^x))/sqrt(a)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2}{\cosh(x)\sinh(x)\sqrt{\frac{a}{\cosh(x)^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(cosh(x)*sinh(x)*(a/cosh(x)^2)^(1/2)),x)
```

```
[Out] int(x^2/(cosh(x)*sinh(x)*(a/cosh(x)^2)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2\operatorname{csch}(x)\operatorname{sech}(x)}{\sqrt{a\operatorname{sech}^2(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*csch(x)*sech(x)/(a*sech(x)**2)**(1/2),x)
```

```
[Out] Integral(x**2*csch(x)*sech(x)/sqrt(a*sech(x)**2), x)
```

3.844 $\int \frac{x^3\operatorname{csch}(x)\operatorname{sech}(x)}{\sqrt{a\operatorname{sech}^2(x)}}\,dx$

Optimal. Leaf size=150

$$-\frac{3x^2\text{Li}_2\left(-e^x\right)\text{sech}(x)}{\sqrt{a\text{sech}^2(x)}}+\frac{3x^2\text{Li}_2\left(e^x\right)\text{sech}(x)}{\sqrt{a\text{sech}^2(x)}}+\frac{6x\text{Li}_3\left(-e^x\right)\text{sech}(x)}{\sqrt{a\text{sech}^2(x)}}-\frac{6x\text{Li}_3\left(e^x\right)\text{sech}(x)}{\sqrt{a\text{sech}^2(x)}}-\frac{6\text{Li}_4\left(-e^x\right)\text{sech}(x)}{\sqrt{a\text{sech}^2(x)}}+\frac{6\text{Li}_4\left(\right.}{\sqrt{a}}$$

```
[Out] -2*x^3*arctanh(exp(x))*sech(x)/(a*sech(x)^2)^(1/2)-3*x^2*polylog(2,-exp(x))
*sech(x)/(a*sech(x)^2)^(1/2)+3*x^2*polylog(2,exp(x))*sech(x)/(a*sech(x)^2)^
(1/2)+6*x*polylog(3,-exp(x))*sech(x)/(a*sech(x)^2)^(1/2)-6*x*polylog(3,exp(
x))*sech(x)/(a*sech(x)^2)^(1/2)-6*polylog(4,-exp(x))*sech(x)/(a*sech(x)^2)^
(1/2)+6*polylog(4,exp(x))*sech(x)/(a*sech(x)^2)^(1/2)
```

Rubi [A] time = 0.83, antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6720, 4182, 2531, 6609, 2282, 6589}

$$-\frac{3x^2\text{sech}(x)\text{PolyLog}\left(2,-e^x\right)}{\sqrt{a\text{sech}^2(x)}}+\frac{3x^2\text{sech}(x)\text{PolyLog}\left(2,e^x\right)}{\sqrt{a\text{sech}^2(x)}}+\frac{6x\text{sech}(x)\text{PolyLog}\left(3,-e^x\right)}{\sqrt{a\text{sech}^2(x)}}-\frac{6x\text{sech}(x)\text{PolyLog}(3,}{\sqrt{a\text{sech}^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*Csch[x]*Sech[x])/Sqrt[a*Sech[x]^2],x]
```

```
[Out] (-2*x^3*ArcTanh[E^x]*Sech[x])/Sqrt[a*Sech[x]^2] - (3*x^2*PolyLog[2, -E^x]*S
ech[x])/Sqrt[a*Sech[x]^2] + (3*x^2*PolyLog[2, E^x]*Sech[x])/Sqrt[a*Sech[x]^
2] + (6*x*PolyLog[3, -E^x]*Sech[x])/Sqrt[a*Sech[x]^2] - (6*x*PolyLog[3, E^x
]*Sech[x])/Sqrt[a*Sech[x]^2] - (6*PolyLog[4, -E^x]*Sech[x])/Sqrt[a*Sech[x]^
2] + (6*PolyLog[4, E^x]*Sech[x])/Sqrt[a*Sech[x]^2]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
```

```
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 6720

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a*v^m)^
FracPart[p])/v^(m*FracPart[p]), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rubi steps

$$\int \frac{x^3\text{csch}(x)\text{sech}(x)}{\sqrt{a\text{sech}^2(x)}}\,dx = \frac{\text{sech}(x)\int x^3\text{csch}(x)\,dx}{\sqrt{a\text{sech}^2(x)}}$$
$$= -\frac{2x^3\tanh^{-1}(e^x)\,\text{sech}(x)}{\sqrt{a\text{sech}^2(x)}} - \frac{(3\text{sech}(x))\int x^2\log(1-e^x)\,dx}{\sqrt{a\text{sech}^2(x)}} + \frac{(3\text{sech}(x))\int x^2\log(1+}{\sqrt{a\text{sech}^2(x)}}$$
$$= -\frac{2x^3\tanh^{-1}(e^x)\,\text{sech}(x)}{\sqrt{a\text{sech}^2(x)}} - \frac{3x^2\text{Li}_2(-e^x)\,\text{sech}(x)}{\sqrt{a\text{sech}^2(x)}} + \frac{3x^2\text{Li}_2(e^x)\,\text{sech}(x)}{\sqrt{a\text{sech}^2(x)}} + \frac{(6\text{sech}(x))}{\sqrt{as}}$$
$$= -\frac{2x^3\tanh^{-1}(e^x)\,\text{sech}(x)}{\sqrt{a\text{sech}^2(x)}} - \frac{3x^2\text{Li}_2(-e^x)\,\text{sech}(x)}{\sqrt{a\text{sech}^2(x)}} + \frac{3x^2\text{Li}_2(e^x)\,\text{sech}(x)}{\sqrt{a\text{sech}^2(x)}} + \frac{6x\text{Li}_3(-e^x)}{\sqrt{a\text{sech}}}$$
$$= -\frac{2x^3\tanh^{-1}(e^x)\,\text{sech}(x)}{\sqrt{a\text{sech}^2(x)}} - \frac{3x^2\text{Li}_2(-e^x)\,\text{sech}(x)}{\sqrt{a\text{sech}^2(x)}} + \frac{3x^2\text{Li}_2(e^x)\,\text{sech}(x)}{\sqrt{a\text{sech}^2(x)}} + \frac{6x\text{Li}_3(-e^x)}{\sqrt{a\text{sech}}}$$
$$= -\frac{2x^3\tanh^{-1}(e^x)\,\text{sech}(x)}{\sqrt{a\text{sech}^2(x)}} - \frac{3x^2\text{Li}_2(-e^x)\,\text{sech}(x)}{\sqrt{a\text{sech}^2(x)}} + \frac{3x^2\text{Li}_2(e^x)\,\text{sech}(x)}{\sqrt{a\text{sech}^2(x)}} + \frac{6x\text{Li}_3(-e^x)}{\sqrt{a\text{sech}}}$$

Mathematica [A] time = 0.09, size = 113, normalized size = 0.75

$$\frac{\text{sech}(x)\left(24x^2\text{Li}_2(-e^{-x}) + 24x^2\text{Li}_2(e^x) + 48x\text{Li}_3(-e^{-x}) - 48x\text{Li}_3(e^x) + 48\text{Li}_4(-e^{-x}) + 48\text{Li}_4(e^x) - 2x^4 - 8x^3\log\right.}{8\sqrt{a\text{sech}^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*Csch[x]*Sech[x])/Sqrt[a*Sech[x]^2],x]
```

```
[Out] ((Pi^4 - 2*x^4 - 8*x^3*Log[1 + E^(-x)] + 8*x^3*Log[1 - E^x] + 24*x^2*PolyLo
g[2, -E^(-x)] + 24*x^2*PolyLog[2, E^x] + 48*x*PolyLog[3, -E^(-x)] - 48*x*Po
lyLog[3, E^x] + 48*PolyLog[4, -E^(-x)] + 48*PolyLog[4, E^x])*Sech[x])/(8*Sq
rt[a*Sech[x]^2])
```

fricas [C] time = 0.43, size = 272, normalized size = 1.81

$$\frac{\left(6\sqrt{\frac{a}{e^{(4x)}+2e^{(2x)}+1}}\left(e^{(2x)}+1\right)e^x\text{polylog}(4, \cosh(x)+\sinh(x)) - 6\sqrt{\frac{a}{e^{(4x)}+2e^{(2x)}+1}}\left(e^{(2x)}+1\right)e^x\text{polylog}(4, -\cosh(x\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*csch(x)*sech(x)/(a*sech(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] (6*sqrt(a/(e^(4*x) + 2*e^(2*x) + 1))*(e^(2*x) + 1)*e^x*polylog(4, cosh(x) +
 sinh(x)) - 6*sqrt(a/(e^(4*x) + 2*e^(2*x) + 1))*(e^(2*x) + 1)*e^x*polylog(4
, -cosh(x) - sinh(x)) - 6*(x*e^(2*x) + x)*sqrt(a/(e^(4*x) + 2*e^(2*x) + 1))
*e^x*polylog(3, cosh(x) + sinh(x)) + 6*(x*e^(2*x) + x)*sqrt(a/(e^(4*x) + 2*
e^(2*x) + 1))*e^x*polylog(3, -cosh(x) - sinh(x)) + (3*(x^2*e^(2*x) + x^2)*d
ilog(cosh(x) + sinh(x)) - 3*(x^2*e^(2*x) + x^2)*dilog(-cosh(x) - sinh(x)) -
 (x^3*e^(2*x) + x^3)*log(cosh(x) + sinh(x) + 1) + (x^3*e^(2*x) + x^3)*log(-
cosh(x) - sinh(x) + 1))*sqrt(a/(e^(4*x) + 2*e^(2*x) + 1))*e^x)*e^(-x)/a
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3 \operatorname{csch}(x) \operatorname{sech}(x)}{\sqrt{a \operatorname{sech}(x)^2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*csch(x)*sech(x)/(a*sech(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^3*csch(x)*sech(x)/sqrt(a*sech(x)^2), x)
```

maple [B] time = 0.35, size = 281, normalized size = 1.87

$$-\frac{e^{x} x^{3} \ln \left(e^{x}+1\right)}{\sqrt{\frac{a e^{2 x}}{\left(1+e^{2 x}\right)^{2}}}\, \left(1+e^{2 x}\right)}-\frac{3 e^{x} x^{2} \operatorname{polylog}\left(2,-e^{x}\right)}{\sqrt{\frac{a e^{2 x}}{\left(1+e^{2 x}\right)^{2}}}\, \left(1+e^{2 x}\right)}+\frac{6 e^{x} x \operatorname{polylog}\left(3,-e^{x}\right)}{\sqrt{\frac{a e^{2 x}}{\left(1+e^{2 x}\right)^{2}}}\, \left(1+e^{2 x}\right)}-\frac{6 e^{x} \operatorname{polylog}\left(4,-e^{x}\right)}{\sqrt{\frac{a e^{2 x}}{\left(1+e^{2 x}\right)^{2}}}\, \left(1+e^{2 x}\right)}+\frac{e^{x} x^{3} \ln \left(1-e\right.}{\sqrt{\frac{a e^{2 x}}{\left(1+e^{2 x}\right)^{2}}}\, \left(1+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*csch(x)*sech(x)/(a*sech(x)^2)^(1/2),x)
```

```
[Out] -1/(a*exp(2*x)/(1+exp(2*x))^2)^(1/2)/(1+exp(2*x))*exp(x)*x^3*ln(exp(x)+1)-3
/(a*exp(2*x)/(1+exp(2*x))^2)^(1/2)/(1+exp(2*x))*exp(x)*x^2*polylog(2,-exp(x
))+6/(a*exp(2*x)/(1+exp(2*x))^2)^(1/2)/(1+exp(2*x))*exp(x)*x*polylog(3,-exp
(x))-6/(a*exp(2*x)/(1+exp(2*x))^2)^(1/2)/(1+exp(2*x))*exp(x)*polylog(4,-exp
(x))+1/(a*exp(2*x)/(1+exp(2*x))^2)^(1/2)/(1+exp(2*x))*exp(x)*x^3*ln(1-exp(x
))+3/(a*exp(2*x)/(1+exp(2*x))^2)^(1/2)/(1+exp(2*x))*exp(x)*x^2*polylog(2,ex
p(x))-6/(a*exp(2*x)/(1+exp(2*x))^2)^(1/2)/(1+exp(2*x))*exp(x)*x*polylog(3,e
xp(x))+6/(a*exp(2*x)/(1+exp(2*x))^2)^(1/2)/(1+exp(2*x))*exp(x)*polylog(4,ex
p(x))
```

maxima [A] time = 0.46, size = 80, normalized size = 0.53

$$-\frac{x^{3} \log \left(e^{x}+1\right)+3 x^{2} \mathrm{Li}_{2}\left(-e^{x}\right)-6 x \mathrm{Li}_{3}(-e^{x})+6 \mathrm{Li}_{4}(-e^{x})}{\sqrt{a}}+\frac{x^{3} \log \left(-e^{x}+1\right)+3 x^{2} \mathrm{Li}_{2}\left(e^{x}\right)-6 x \mathrm{Li}_{3}(e^{x})+6 \mathrm{Li}_{4}(e}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*csch(x)*sech(x)/(a*sech(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -(x^3*log(e^x + 1) + 3*x^2*dilog(-e^x) - 6*x*polylog(3, -e^x) + 6*polylog(4
, -e^x))/sqrt(a) + (x^3*log(-e^x + 1) + 3*x^2*dilog(e^x) - 6*x*polylog(3, e
^x) + 6*polylog(4, e^x))/sqrt(a)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3}{\cosh(x)\sinh(x)\sqrt{\frac{a}{\cosh(x)^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(cosh(x)*sinh(x)*(a/cosh(x)^2)^(1/2)),x)
```

```
[Out] int(x^3/(cosh(x)*sinh(x)*(a/cosh(x)^2)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3 \operatorname{csch}(x) \operatorname{sech}(x)}{\sqrt{a \operatorname{sech}^2(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*csch(x)*sech(x)/(a*sech(x)**2)**(1/2),x)
```

```
[Out] Integral(x**3*csch(x)*sech(x)/sqrt(a*sech(x)**2), x)
```

3.845 $\int \frac{x\text{csch}(x)\text{sech}(x)}{\sqrt{a\text{sech}^4(x)}}\,dx$

Optimal. Leaf size=73

$$\frac{\text{Li}_2\left(e^{2x}\right)\text{sech}^2(x)}{2\sqrt{a\text{sech}^4(x)}}-\frac{x^2\text{sech}^2(x)}{2\sqrt{a\text{sech}^4(x)}}+\frac{x\log\left(1-e^{2x}\right)\text{sech}^2(x)}{\sqrt{a\text{sech}^4(x)}}$$

```
[Out] -1/2*x^2*sech(x)^2/(a*sech(x)^4)^(1/2)+x*ln(1-exp(2*x))*sech(x)^2/(a*sech(x
)^4)^(1/2)+1/2*polylog(2,exp(2*x))*sech(x)^2/(a*sech(x)^4)^(1/2)
```

Rubi [A] time = 0.56, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {6720, 3716, 2190, 2279, 2391}

$$\frac{\text{sech}^2(x)\text{PolyLog}\left(2,e^{2x}\right)}{2\sqrt{a\text{sech}^4(x)}}-\frac{x^2\text{sech}^2(x)}{2\sqrt{a\text{sech}^4(x)}}+\frac{x\log\left(1-e^{2x}\right)\text{sech}^2(x)}{\sqrt{a\text{sech}^4(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Csch[x]*Sech[x])/Sqrt[a*Sech[x]^4],x]
```

```
[Out] -(x^2*Sech[x]^2)/(2*Sqrt[a*Sech[x]^4]) + (x*Log[1 - E^(2*x)]*Sech[x]^2)/Sqr
t[a*Sech[x]^4] + (PolyLog[2, E^(2*x)]*Sech[x]^2)/(2*Sqrt[a*Sech[x]^4])
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3716

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2
*I, Int[((c + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(E^(2*I*k*Pi)*(1 + E^(2*(-(I*
e) + f*fz*x))/E^(2*I*k*Pi))), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Integ
erQ[4*k] && IGtQ[m, 0]
```

Rule 6720

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a*v^m)^
FracPart[p])/v^(m*FracPart[p]), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rubi steps

$$\begin{aligned}
\int \frac{x\text{csch}(x)\text{sech}(x)}{\sqrt{a\text{sech}^4(x)}}\,dx &= \frac{\text{sech}^2(x)\int x\coth(x)\,dx}{\sqrt{a\text{sech}^4(x)}}\\
&= -\frac{x^2\text{sech}^2(x)}{2\sqrt{a\text{sech}^4(x)}}-\frac{\left(2\text{sech}^2(x)\right)\int \frac{e^{2x}x}{1-e^{2x}}\,dx}{\sqrt{a\text{sech}^4(x)}}\\
&= -\frac{x^2\text{sech}^2(x)}{2\sqrt{a\text{sech}^4(x)}}+\frac{x\log\left(1-e^{2x}\right)\text{sech}^2(x)}{\sqrt{a\text{sech}^4(x)}}-\frac{\text{sech}^2(x)\int \log\left(1-e^{2x}\right)\,dx}{\sqrt{a\text{sech}^4(x)}}\\
&= -\frac{x^2\text{sech}^2(x)}{2\sqrt{a\text{sech}^4(x)}}+\frac{x\log\left(1-e^{2x}\right)\text{sech}^2(x)}{\sqrt{a\text{sech}^4(x)}}-\frac{\text{sech}^2(x)\,\text{Subst}\left(\int \frac{\log(1-x)}{x}\,dx,x,e^{2x}\right)}{2\sqrt{a\text{sech}^4(x)}}\\
&= -\frac{x^2\text{sech}^2(x)}{2\sqrt{a\text{sech}^4(x)}}+\frac{x\log\left(1-e^{2x}\right)\text{sech}^2(x)}{\sqrt{a\text{sech}^4(x)}}+\frac{\text{Li}_2\left(e^{2x}\right)\text{sech}^2(x)}{2\sqrt{a\text{sech}^4(x)}}
\end{aligned}$$

Mathematica [A] time = 0.05, size = 44, normalized size = 0.60

$$\frac{\text{sech}^2(x)\left(x\left(x+2\log\left(1-e^{-2x}\right)\right)-\text{Li}_2\left(e^{-2x}\right)\right)}{2\sqrt{a\text{sech}^4(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Csch[x]*Sech[x])/Sqrt[a*Sech[x]^4],x]
```

```
[Out] ((x*(x + 2*Log[1 - E^(-2*x)]) - PolyLog[2, E^(-2*x)])*Sech[x]^2)/(2*Sqrt[a*
Sech[x]^4])
```

fricas [B] time = 0.45, size = 152, normalized size = 2.08

$$-\frac{\left(x^{2} e^{(4\,x)} + 2\,x^{2} e^{(2\,x)} + x^{2} - 2\left(e^{(4\,x)} + 2\,e^{(2\,x)} + 1\right){\rm Li}_2\left(\cosh(x) + \sinh(x)\right) - 2\left(e^{(4\,x)} + 2\,e^{(2\,x)} + 1\right){\rm Li}_2\left(-\cosh(x)\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*csch(x)*sech(x)/(a*sech(x)^4)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/2*(x^2*e^(4*x) + 2*x^2*e^(2*x) + x^2 - 2*(e^(4*x) + 2*e^(2*x) + 1)*dilog
(cosh(x) + sinh(x)) - 2*(e^(4*x) + 2*e^(2*x) + 1)*dilog(-cosh(x) - sinh(x))
 - 2*(x*e^(4*x) + 2*x*e^(2*x) + x)*log(cosh(x) + sinh(x) + 1) - 2*(x*e^(4*x
) + 2*x*e^(2*x) + x)*log(-cosh(x) - sinh(x) + 1))*sqrt(a/(e^(8*x) + 4*e^(6*
x) + 6*e^(4*x) + 4*e^(2*x) + 1))/a
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\operatorname{csch}(x)\operatorname{sech}(x)}{\sqrt{a\operatorname{sech}(x)^{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*csch(x)*sech(x)/(a*sech(x)^4)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x*csch(x)*sech(x)/sqrt(a*sech(x)^4), x)
```

maple [B] time = 0.37, size = 175, normalized size = 2.40

$$-\frac{{\rm e}^{2x}x^{2}}{2\sqrt{\frac{a\,{\rm e}^{4x}}{\left(1+{\rm e}^{2x}\right)^{4}}}\,\left(1+{\rm e}^{2x}\right)^{2}}+\frac{{\rm e}^{2x}x\ln\left({\rm e}^{x}+1\right)}{\sqrt{\frac{a\,{\rm e}^{4x}}{\left(1+{\rm e}^{2x}\right)^{4}}}\,\left(1+{\rm e}^{2x}\right)^{2}}+\frac{{\rm e}^{2x}\operatorname{polylog}\left(2,-{\rm e}^{x}\right)}{\sqrt{\frac{a\,{\rm e}^{4x}}{\left(1+{\rm e}^{2x}\right)^{4}}}\,\left(1+{\rm e}^{2x}\right)^{2}}+\frac{{\rm e}^{2x}x\ln\left(1-{\rm e}^{x}\right)}{\sqrt{\frac{a\,{\rm e}^{4x}}{\left(1+{\rm e}^{2x}\right)^{4}}}\,\left(1+{\rm e}^{2x}\right)^{2}}+\frac{{\rm e}^{2x}\operatorname{polylog}(2}{\sqrt{\frac{a\,{\rm e}^{4x}}{\left(1+{\rm e}^{2x}\right)^{4}}}\,\left(1+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*csch(x)*sech(x)/(a*sech(x)^4)^(1/2),x)
```

```
[Out] -1/2/(a*exp(4*x)/(1+exp(2*x))^4)^(1/2)/(1+exp(2*x))^2*exp(2*x)*x^2+1/(a*exp
(4*x)/(1+exp(2*x))^4)^(1/2)/(1+exp(2*x))^2*exp(2*x)*x*ln(exp(x)+1)+1/(a*exp
(4*x)/(1+exp(2*x))^4)^(1/2)/(1+exp(2*x))^2*exp(2*x)*polylog(2,-exp(x))+1/(a
*exp(4*x)/(1+exp(2*x))^4)^(1/2)/(1+exp(2*x))^2*exp(2*x)*x*ln(1-exp(x))+1/(a
*exp(4*x)/(1+exp(2*x))^4)^(1/2)/(1+exp(2*x))^2*exp(2*x)*polylog(2,exp(x))
```

maxima [A] time = 0.46, size = 43, normalized size = 0.59

$$-\frac{x^2}{2\sqrt{a}} + \frac{x\log\left(e^x+1\right) + \mathrm{Li}_2\left(-e^x\right)}{\sqrt{a}} + \frac{x\log\left(-e^x+1\right) + \mathrm{Li}_2\left(e^x\right)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*csch(x)*sech(x)/(a*sech(x)^4)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/2*x^2/sqrt(a) + (x*log(e^x + 1) + dilog(-e^x))/sqrt(a) + (x*log(-e^x + 1
) + dilog(e^x))/sqrt(a)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x}{\cosh(x)\sinh(x)\sqrt{\frac{a}{\cosh(x)^4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(cosh(x)*sinh(x)*(a/cosh(x)^4)^(1/2)),x)
```

```
[Out] int(x/(cosh(x)*sinh(x)*(a/cosh(x)^4)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\,\mathrm{csch}\,(x)\,\mathrm{sech}\,(x)}{\sqrt{a\,\mathrm{sech}^4\,(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*csch(x)*sech(x)/(a*sech(x)**4)**(1/2),x)
```

```
[Out] Integral(x*csch(x)*sech(x)/sqrt(a*sech(x)**4), x)
```

3.846 $\int \frac{x^2\mathbf{csch}(x)\mathbf{sech}(x)}{\sqrt{a\mathbf{sech}^4(x)}}\,dx$

Optimal. Leaf size=98

$$\frac{x\text{Li}_2\left(e^{2x}\right)\text{sech}^2(x)}{\sqrt{a\text{sech}^4(x)}}-\frac{\text{Li}_3\left(e^{2x}\right)\text{sech}^2(x)}{2\sqrt{a\text{sech}^4(x)}}-\frac{x^3\text{sech}^2(x)}{3\sqrt{a\text{sech}^4(x)}}+\frac{x^2\log\left(1-e^{2x}\right)\text{sech}^2(x)}{\sqrt{a\text{sech}^4(x)}}$$

```
[Out] -1/3*x^3*sech(x)^2/(a*sech(x)^4)^(1/2)+x^2*ln(1-exp(2*x))*sech(x)^2/(a*sech
(x)^4)^(1/2)+x*polylog(2,exp(2*x))*sech(x)^2/(a*sech(x)^4)^(1/2)-1/2*polylo
g(3,exp(2*x))*sech(x)^2/(a*sech(x)^4)^(1/2)
```

Rubi [A] time = 0.61, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6720, 3716, 2190, 2531, 2282, 6589}

$$\frac{x\text{sech}^2(x)\text{PolyLog}\left(2,e^{2x}\right)}{\sqrt{a\text{sech}^4(x)}}-\frac{\text{sech}^2(x)\text{PolyLog}\left(3,e^{2x}\right)}{2\sqrt{a\text{sech}^4(x)}}-\frac{x^3\text{sech}^2(x)}{3\sqrt{a\text{sech}^4(x)}}+\frac{x^2\log\left(1-e^{2x}\right)\text{sech}^2(x)}{\sqrt{a\text{sech}^4(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*Csch[x]*Sech[x])/Sqrt[a*Sech[x]^4],x]
```

```
[Out] -(x^3*Sech[x]^2)/(3*Sqrt[a*Sech[x]^4]) + (x^2*Log[1 - E^(2*x)]*Sech[x]^2)/S
qrt[a*Sech[x]^4] + (x*PolyLog[2, E^(2*x)]*Sech[x]^2)/Sqrt[a*Sech[x]^4] - (P
olyLog[3, E^(2*x)]*Sech[x]^2)/(2*Sqrt[a*Sech[x]^4])
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 3716

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2
*I, Int[((c + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(E^(2*I*k*Pi)*(1 + E^(2*(-(I*
e) + f*fz*x))/E^(2*I*k*Pi))), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Integ
erQ[4*k] && IGtQ[m, 0]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6720

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a*v^m)^
FracPart[p])/v^(m*FracPart[p]), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rubi steps

$$\int \frac{x^2\text{csch}(x)\text{sech}(x)}{\sqrt{a\text{sech}^4(x)}}\,dx = \frac{\text{sech}^2(x)\int x^2\coth(x)\,dx}{\sqrt{a\text{sech}^4(x)}}$$
$$= -\frac{x^3\text{sech}^2(x)}{3\sqrt{a\text{sech}^4(x)}} - \frac{\left(2\text{sech}^2(x)\right)\int \frac{e^{2x}x^2}{1-e^{2x}}\,dx}{\sqrt{a\text{sech}^4(x)}}$$
$$= -\frac{x^3\text{sech}^2(x)}{3\sqrt{a\text{sech}^4(x)}} + \frac{x^2\log\left(1-e^{2x}\right)\text{sech}^2(x)}{\sqrt{a\text{sech}^4(x)}} - \frac{\left(2\text{sech}^2(x)\right)\int x\log\left(1-e^{2x}\right)\,dx}{\sqrt{a\text{sech}^4(x)}}$$
$$= -\frac{x^3\text{sech}^2(x)}{3\sqrt{a\text{sech}^4(x)}} + \frac{x^2\log\left(1-e^{2x}\right)\text{sech}^2(x)}{\sqrt{a\text{sech}^4(x)}} + \frac{x\text{Li}_2\left(e^{2x}\right)\text{sech}^2(x)}{\sqrt{a\text{sech}^4(x)}} - \frac{\text{sech}^2(x)\int \text{Li}_2\left(\right.}{\sqrt{a\text{sech}^4(x}}$$
$$= -\frac{x^3\text{sech}^2(x)}{3\sqrt{a\text{sech}^4(x)}} + \frac{x^2\log\left(1-e^{2x}\right)\text{sech}^2(x)}{\sqrt{a\text{sech}^4(x)}} + \frac{x\text{Li}_2\left(e^{2x}\right)\text{sech}^2(x)}{\sqrt{a\text{sech}^4(x)}} - \frac{\text{sech}^2(x)\,\text{Subst}\,(}{2\sqrt{as}}$$
$$= -\frac{x^3\text{sech}^2(x)}{3\sqrt{a\text{sech}^4(x)}} + \frac{x^2\log\left(1-e^{2x}\right)\text{sech}^2(x)}{\sqrt{a\text{sech}^4(x)}} + \frac{x\text{Li}_2\left(e^{2x}\right)\text{sech}^2(x)}{\sqrt{a\text{sech}^4(x)}} - \frac{\text{Li}_3\left(e^{2x}\right)\text{sech}^2(}{2\sqrt{a\text{sech}^4(x)}}$$

Mathematica [A] time = 0.05, size = 57, normalized size = 0.58

$$\frac{\text{sech}^2(x)\left(6x\text{Li}_2\left(e^{2x}\right) - 3\text{Li}_3\left(e^{2x}\right) - 2x^2\left(x - 3\log\left(1-e^{2x}\right)\right)\right)}{6\sqrt{a\text{sech}^4(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*Csch[x]*Sech[x])/Sqrt[a*Sech[x]^4],x]
```

```
[Out] ((-2*x^2*(x - 3*Log[1 - E^(2*x)]) + 6*x*PolyLog[2, E^(2*x)] - 3*PolyLog[3,
E^(2*x)])*Sech[x]^2)/(6*Sqrt[a*Sech[x]^4])
```

fricas [C] time = 0.47, size = 294, normalized size = 3.00

$$-\frac{\left(6\sqrt{\frac{a}{e^{(8x)}+4e^{(6x)}+6e^{(4x)}+4e^{(2x)}+1}}\left(e^{(4x)}+2e^{(2x)}+1\right)e^{(2x)}\text{polylog}\left(3,\cosh(x)+\sinh(x)\right)+6\sqrt{\frac{a}{e^{(8x)}+4e^{(6x)}+6e^{(4x)}+4e^{(}}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*csch(x)*sech(x)/(a*sech(x)^4)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/3*(6*sqrt(a/(e^(8*x) + 4*e^(6*x) + 6*e^(4*x) + 4*e^(2*x) + 1))*(e^(4*x)
+ 2*e^(2*x) + 1)*e^(2*x)*polylog(3, cosh(x) + sinh(x)) + 6*sqrt(a/(e^(8*x)
+ 4*e^(6*x) + 6*e^(4*x) + 4*e^(2*x) + 1))*(e^(4*x) + 2*e^(2*x) + 1)*e^(2*x)
*polylog(3, -cosh(x) - sinh(x)) + (x^3*e^(4*x) + 2*x^3*e^(2*x) + x^3 - 6*(x
*e^(4*x) + 2*x*e^(2*x) + x)*dilog(cosh(x) + sinh(x)) - 6*(x*e^(4*x) + 2*x*e
^(2*x) + x)*dilog(-cosh(x) - sinh(x)) - 3*(x^2*e^(4*x) + 2*x^2*e^(2*x) + x^
2)*log(cosh(x) + sinh(x) + 1) - 3*(x^2*e^(4*x) + 2*x^2*e^(2*x) + x^2)*log(-
cosh(x) - sinh(x) + 1))*sqrt(a/(e^(8*x) + 4*e^(6*x) + 6*e^(4*x) + 4*e^(2*x)
 + 1))*e^(2*x))*e^(-2*x)/a
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 \operatorname{csch}(x) \operatorname{sech}(x)}{\sqrt{a \operatorname{sech}(x)^4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*csch(x)*sech(x)/(a*sech(x)^4)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^2*csch(x)*sech(x)/sqrt(a*sech(x)^4), x)
```

maple [B] time = 0.37, size = 253, normalized size = 2.58

$$-\frac{e^{2x} x^3}{3\sqrt{\frac{a\,e^{4x}}{(1+e^{2x})^4}}\,\left(1+e^{2x}\right)^2}+\frac{e^{2x} x^2 \ln\left(e^x+1\right)}{\sqrt{\frac{a\,e^{4x}}{(1+e^{2x})^4}}\,\left(1+e^{2x}\right)^2}+\frac{2\,e^{2x} x \operatorname{polylog}\left(2,-e^x\right)}{\sqrt{\frac{a\,e^{4x}}{(1+e^{2x})^4}}\,\left(1+e^{2x}\right)^2}-\frac{2\,e^{2x} \operatorname{polylog}\left(3,-e^x\right)}{\sqrt{\frac{a\,e^{4x}}{(1+e^{2x})^4}}\,\left(1+e^{2x}\right)^2}+\frac{e^{2x} x^2 \ln\left(1-\right.}{\sqrt{\frac{a\,e^{4x}}{(1+e^{2x})^4}}\,\left(1+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*csch(x)*sech(x)/(a*sech(x)^4)^(1/2),x)
```

```
[Out] -1/3/(a*exp(4*x)/(1+exp(2*x))^4)^(1/2)/(1+exp(2*x))^2*exp(2*x)*x^3+1/(a*exp
(4*x)/(1+exp(2*x))^4)^(1/2)/(1+exp(2*x))^2*exp(2*x)*x^2*ln(exp(x)+1)+2/(a*e
xp(4*x)/(1+exp(2*x))^4)^(1/2)/(1+exp(2*x))^2*exp(2*x)*x*polylog(2,-exp(x))-
2/(a*exp(4*x)/(1+exp(2*x))^4)^(1/2)/(1+exp(2*x))^2*exp(2*x)*polylog(3,-exp(
x))+1/(a*exp(4*x)/(1+exp(2*x))^4)^(1/2)/(1+exp(2*x))^2*exp(2*x)*x^2*ln(1-ex
p(x))+2/(a*exp(4*x)/(1+exp(2*x))^4)^(1/2)/(1+exp(2*x))^2*exp(2*x)*x*polylog
(2,exp(x))-2/(a*exp(4*x)/(1+exp(2*x))^4)^(1/2)/(1+exp(2*x))^2*exp(2*x)*poly
log(3,exp(x))
```

maxima [A] time = 1.07, size = 67, normalized size = 0.68

$$-\frac{x^3}{3\sqrt{a}}+\frac{x^2 \log\left(e^x+1\right)+2\,x \operatorname{Li}_2\left(-e^x\right)-2\operatorname{Li}_3(-e^x)}{\sqrt{a}}+\frac{x^2 \log\left(-e^x+1\right)+2\,x \operatorname{Li}_2\left(e^x\right)-2\operatorname{Li}_3(e^x)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*csch(x)*sech(x)/(a*sech(x)^4)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/3*x^3/sqrt(a) + (x^2*log(e^x + 1) + 2*x*dilog(-e^x) - 2*polylog(3, -e^x)
)/sqrt(a) + (x^2*log(-e^x + 1) + 2*x*dilog(e^x) - 2*polylog(3, e^x))/sqrt(a
)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2}{\cosh(x)\sinh(x)\sqrt{\frac{a}{\cosh(x)^4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(cosh(x)*sinh(x)*(a/cosh(x)^4)^(1/2)),x)
```

```
[Out] int(x^2/(cosh(x)*sinh(x)*(a/cosh(x)^4)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 \operatorname{csch}(x)\operatorname{sech}(x)}{\sqrt{a\operatorname{sech}^4(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*csch(x)*sech(x)/(a*sech(x)**4)**(1/2),x)
```

```
[Out] Integral(x**2*csch(x)*sech(x)/sqrt(a*sech(x)**4), x)
```

3.847 $\int \frac{x^3\text{csch}(x)\text{sech}(x)}{\sqrt{a\text{sech}^4(x)}}\,dx$

Optimal. Leaf size=129

$$\frac{3x^2\text{Li}_2\left(e^{2x}\right)\text{sech}^2(x)}{2\sqrt{a\text{sech}^4(x)}}-\frac{3x\text{Li}_3\left(e^{2x}\right)\text{sech}^2(x)}{2\sqrt{a\text{sech}^4(x)}}+\frac{3\text{Li}_4\left(e^{2x}\right)\text{sech}^2(x)}{4\sqrt{a\text{sech}^4(x)}}-\frac{x^4\text{sech}^2(x)}{4\sqrt{a\text{sech}^4(x)}}+\frac{x^3\log\left(1-e^{2x}\right)\text{sech}^2(x)}{\sqrt{a\text{sech}^4(x)}}$$

```
[Out] -1/4*x^4*sech(x)^2/(a*sech(x)^4)^(1/2)+x^3*ln(1-exp(2*x))*sech(x)^2/(a*sech
(x)^4)^(1/2)+3/2*x^2*polylog(2,exp(2*x))*sech(x)^2/(a*sech(x)^4)^(1/2)-3/2*
x*polylog(3,exp(2*x))*sech(x)^2/(a*sech(x)^4)^(1/2)+3/4*polylog(4,exp(2*x))
*sech(x)^2/(a*sech(x)^4)^(1/2)
```

Rubi [A] time = 0.55, antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {6720, 3716, 2190, 2531, 6609, 2282, 6589}

$$\frac{3x^2\text{sech}^2(x)\text{PolyLog}\left(2,e^{2x}\right)}{2\sqrt{a\text{sech}^4(x)}}-\frac{3x\text{sech}^2(x)\text{PolyLog}\left(3,e^{2x}\right)}{2\sqrt{a\text{sech}^4(x)}}+\frac{3\text{sech}^2(x)\text{PolyLog}\left(4,e^{2x}\right)}{4\sqrt{a\text{sech}^4(x)}}-\frac{x^4\text{sech}^2(x)}{4\sqrt{a\text{sech}^4(x)}}+\frac{x^3\log(}{\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*Csch[x]*Sech[x])/Sqrt[a*Sech[x]^4],x]
```

```
[Out] -(x^4*Sech[x]^2)/(4*Sqrt[a*Sech[x]^4]) + (x^3*Log[1 - E^(2*x)]*Sech[x]^2)/S
qrt[a*Sech[x]^4] + (3*x^2*PolyLog[2, E^(2*x)]*Sech[x]^2)/(2*Sqrt[a*Sech[x]^
4]) - (3*x*PolyLog[3, E^(2*x)]*Sech[x]^2)/(2*Sqrt[a*Sech[x]^4]) + (3*PolyLo
g[4, E^(2*x)]*Sech[x]^2)/(4*Sqrt[a*Sech[x]^4])
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
```

```
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 3716

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2
*I, Int[((c + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(E^(2*I*k*Pi)*(1 + E^(2*(-(I*
e) + f*fz*x))/E^(2*I*k*Pi))), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Integ
erQ[4*k] && IGtQ[m, 0]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 6720

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a*v^m)^
FracPart[p])/v^(m*FracPart[p]), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rubi steps

$$\int \frac{x^3\text{csch}(x)\text{sech}(x)}{\sqrt{a\text{sech}^4(x)}}\,dx = \frac{\text{sech}^2(x)\int x^3\coth(x)\,dx}{\sqrt{a\text{sech}^4(x)}}$$
$$= -\frac{x^4\text{sech}^2(x)}{4\sqrt{a\text{sech}^4(x)}} - \frac{\left(2\text{sech}^2(x)\right)\int \frac{e^{2x}x^3}{1-e^{2x}}\,dx}{\sqrt{a\text{sech}^4(x)}}$$
$$= -\frac{x^4\text{sech}^2(x)}{4\sqrt{a\text{sech}^4(x)}} + \frac{x^3\log\left(1-e^{2x}\right)\text{sech}^2(x)}{\sqrt{a\text{sech}^4(x)}} - \frac{\left(3\text{sech}^2(x)\right)\int x^2\log\left(1-e^{2x}\right)\,dx}{\sqrt{a\text{sech}^4(x)}}$$
$$= -\frac{x^4\text{sech}^2(x)}{4\sqrt{a\text{sech}^4(x)}} + \frac{x^3\log\left(1-e^{2x}\right)\text{sech}^2(x)}{\sqrt{a\text{sech}^4(x)}} + \frac{3x^2\text{Li}_2\left(e^{2x}\right)\text{sech}^2(x)}{2\sqrt{a\text{sech}^4(x)}} - \frac{\left(3\text{sech}^2(x)\right)\int x}{\sqrt{a\text{sech}}}$$
$$= -\frac{x^4\text{sech}^2(x)}{4\sqrt{a\text{sech}^4(x)}} + \frac{x^3\log\left(1-e^{2x}\right)\text{sech}^2(x)}{\sqrt{a\text{sech}^4(x)}} + \frac{3x^2\text{Li}_2\left(e^{2x}\right)\text{sech}^2(x)}{2\sqrt{a\text{sech}^4(x)}} - \frac{3x\text{Li}_3\left(e^{2x}\right)\text{sec}}{2\sqrt{a\text{sech}^4(x}}$$
$$= -\frac{x^4\text{sech}^2(x)}{4\sqrt{a\text{sech}^4(x)}} + \frac{x^3\log\left(1-e^{2x}\right)\text{sech}^2(x)}{\sqrt{a\text{sech}^4(x)}} + \frac{3x^2\text{Li}_2\left(e^{2x}\right)\text{sech}^2(x)}{2\sqrt{a\text{sech}^4(x)}} - \frac{3x\text{Li}_3\left(e^{2x}\right)\text{sec}}{2\sqrt{a\text{sech}^4(x}}$$
$$= -\frac{x^4\text{sech}^2(x)}{4\sqrt{a\text{sech}^4(x)}} + \frac{x^3\log\left(1-e^{2x}\right)\text{sech}^2(x)}{\sqrt{a\text{sech}^4(x)}} + \frac{3x^2\text{Li}_2\left(e^{2x}\right)\text{sech}^2(x)}{2\sqrt{a\text{sech}^4(x)}} - \frac{3x\text{Li}_3\left(e^{2x}\right)\text{sec}}{2\sqrt{a\text{sech}^4(x}}$$

Mathematica [A] time = 0.06, size = 68, normalized size = 0.53

$$-\frac{\text{sech}^2(x)\left(-6x^2\text{Li}_2\left(e^{2x}\right)+6x\text{Li}_3\left(e^{2x}\right)-3\text{Li}_4\left(e^{2x}\right)+x^4-4x^3\log\left(1-e^{2x}\right)\right)}{4\sqrt{a\text{sech}^4(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*Csch[x]*Sech[x])/Sqrt[a*Sech[x]^4],x]
```

```
[Out] -1/4*((x^4 - 4*x^3*Log[1 - E^(2*x)] - 6*x^2*PolyLog[2, E^(2*x)] + 6*x*PolyL
og[3, E^(2*x)] - 3*PolyLog[4, E^(2*x)])*Sech[x]^2)/Sqrt[a*Sech[x]^4]
```

fricas [C] time = 0.48, size = 427, normalized size = 3.31

$$\left(24\sqrt{\frac{a}{e^{(8x)}+4e^{(6x)}+6e^{(4x)}+4e^{(2x)}+1}}\left(e^{(4x)}+2e^{(2x)}+1\right)e^{(2x)}\text{polylog}\left(4,\cosh(x)+\sinh(x)\right)+24\sqrt{\frac{a}{e^{(8x)}+4e^{(6x)}+6e^{(4x)}+4e^{(2}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*csch(x)*sech(x)/(a*sech(x)^4)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/4*(24*sqrt(a/(e^(8*x) + 4*e^(6*x) + 6*e^(4*x) + 4*e^(2*x) + 1))*(e^(4*x)
+ 2*e^(2*x) + 1)*e^(2*x)*polylog(4, cosh(x) + sinh(x)) + 24*sqrt(a/(e^(8*x)
 + 4*e^(6*x) + 6*e^(4*x) + 4*e^(2*x) + 1))*(e^(4*x) + 2*e^(2*x) + 1)*e^(2*x
)*polylog(4, -cosh(x) - sinh(x)) - 24*(x*e^(4*x) + 2*x*e^(2*x) + x)*sqrt(a/
(e^(8*x) + 4*e^(6*x) + 6*e^(4*x) + 4*e^(2*x) + 1))*e^(2*x)*polylog(3, cosh(
x) + sinh(x)) - 24*(x*e^(4*x) + 2*x*e^(2*x) + x)*sqrt(a/(e^(8*x) + 4*e^(6*x
) + 6*e^(4*x) + 4*e^(2*x) + 1))*e^(2*x)*polylog(3, -cosh(x) - sinh(x)) - (x
^4*e^(4*x) + 2*x^4*e^(2*x) + x^4 - 12*(x^2*e^(4*x) + 2*x^2*e^(2*x) + x^2)*d
ilog(cosh(x) + sinh(x)) - 12*(x^2*e^(4*x) + 2*x^2*e^(2*x) + x^2)*dilog(-cos
h(x) - sinh(x)) - 4*(x^3*e^(4*x) + 2*x^3*e^(2*x) + x^3)*log(cosh(x) + sinh(
x) + 1) - 4*(x^3*e^(4*x) + 2*x^3*e^(2*x) + x^3)*log(-cosh(x) - sinh(x) + 1)
)*sqrt(a/(e^(8*x) + 4*e^(6*x) + 6*e^(4*x) + 4*e^(2*x) + 1))*e^(2*x))*e^(-2*
x)/a
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3 \operatorname{csch}(x) \operatorname{sech}(x)}{\sqrt{a \operatorname{sech}(x)^4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*csch(x)*sech(x)/(a*sech(x)^4)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^3*csch(x)*sech(x)/sqrt(a*sech(x)^4), x)
```

maple [B] time = 0.37, size = 329, normalized size = 2.55

$$-\frac{e^{2x} x^4}{4\sqrt{\frac{a\,e^{4x}}{\left(1+e^{2x}\right)^4}}\,\left(1+e^{2x}\right)^2}+\frac{e^{2x} x^3 \ln\left(e^x+1\right)}{\sqrt{\frac{a\,e^{4x}}{\left(1+e^{2x}\right)^4}}\,\left(1+e^{2x}\right)^2}+\frac{3\,e^{2x} x^2 \operatorname{polylog}\left(2,-e^x\right)}{\sqrt{\frac{a\,e^{4x}}{\left(1+e^{2x}\right)^4}}\,\left(1+e^{2x}\right)^2}-\frac{6\,e^{2x} x \operatorname{polylog}\left(3,-e^x\right)}{\sqrt{\frac{a\,e^{4x}}{\left(1+e^{2x}\right)^4}}\,\left(1+e^{2x}\right)^2}+\frac{6\,e^{2x} \operatorname{polyl}}{\sqrt{\frac{a\,e^{4x}}{\left(1+e^{2x}\right)^4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*csch(x)*sech(x)/(a*sech(x)^4)^(1/2),x)
```

```
[Out] -1/4/(a*exp(4*x)/(1+exp(2*x))^4)^(1/2)/(1+exp(2*x))^2*exp(2*x)*x^4+1/(a*exp
(4*x)/(1+exp(2*x))^4)^(1/2)/(1+exp(2*x))^2*exp(2*x)*x^3*ln(exp(x)+1)+3/(a*e
xp(4*x)/(1+exp(2*x))^4)^(1/2)/(1+exp(2*x))^2*exp(2*x)*x^2*polylog(2,-exp(x)
)-6/(a*exp(4*x)/(1+exp(2*x))^4)^(1/2)/(1+exp(2*x))^2*exp(2*x)*x*polylog(3,-
exp(x))+6/(a*exp(4*x)/(1+exp(2*x))^4)^(1/2)/(1+exp(2*x))^2*exp(2*x)*polylog
(4,-exp(x))+1/(a*exp(4*x)/(1+exp(2*x))^4)^(1/2)/(1+exp(2*x))^2*exp(2*x)*x^3
*ln(1-exp(x))+3/(a*exp(4*x)/(1+exp(2*x))^4)^(1/2)/(1+exp(2*x))^2*exp(2*x)*x
```

```
^2*polylog(2,exp(x))-6/(a*exp(4*x)/(1+exp(2*x))^4)^(1/2)/(1+exp(2*x))^2*exp
(2*x)*x*polylog(3,exp(x))+6/(a*exp(4*x)/(1+exp(2*x))^4)^(1/2)/(1+exp(2*x))^
2*exp(2*x)*polylog(4,exp(x))
```

maxima [A] time = 0.46, size = 87, normalized size = 0.67

$$-\frac{x^4}{4\sqrt{a}}+\frac{x^3\log(e^x+1)+3x^2\text{Li}_2(-e^x)-6x\text{Li}_3(-e^x)+6\,\text{Li}_4(-e^x)}{\sqrt{a}}+\frac{x^3\log(-e^x+1)+3x^2\text{Li}_2(e^x)-6x\text{Li}_3(e^x)+6}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*csch(x)*sech(x)/(a*sech(x)^4)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/4*x^4/sqrt(a) + (x^3*log(e^x + 1) + 3*x^2*dilog(-e^x) - 6*x*polylog(3, -
e^x) + 6*polylog(4, -e^x))/sqrt(a) + (x^3*log(-e^x + 1) + 3*x^2*dilog(e^x)
- 6*x*polylog(3, e^x) + 6*polylog(4, e^x))/sqrt(a)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3}{\cosh(x)\sinh(x)\sqrt{\frac{a}{\cosh(x)^4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(cosh(x)*sinh(x)*(a/cosh(x)^4)^(1/2)),x)
```

```
[Out] int(x^3/(cosh(x)*sinh(x)*(a/cosh(x)^4)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3\operatorname{csch}(x)\operatorname{sech}(x)}{\sqrt{a\operatorname{sech}^4(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*csch(x)*sech(x)/(a*sech(x)**4)**(1/2),x)
```

```
[Out] Integral(x**3*csch(x)*sech(x)/sqrt(a*sech(x)**4), x)
```

3.848 $\int x\operatorname{csch}(x)\operatorname{sech}(x)\sqrt{a\operatorname{sech}^2(x)}\,dx$

Optimal. Leaf size=88

$$-\operatorname{Li}_2(-e^x)\cosh(x)\sqrt{a\operatorname{sech}^2(x)}+\operatorname{Li}_2(e^x)\cosh(x)\sqrt{a\operatorname{sech}^2(x)}+x\sqrt{a\operatorname{sech}^2(x)}-2x\cosh(x)\tanh^{-1}(e^x)\sqrt{a\operatorname{sech}^2(x)}$$

```
[Out] x*(a*sech(x)^2)^(1/2)-arctan(sinh(x))*cosh(x)*(a*sech(x)^2)^(1/2)-2*x*arcta
nh(exp(x))*cosh(x)*(a*sech(x)^2)^(1/2)-cosh(x)*polylog(2,-exp(x))*(a*sech(x
)^2)^(1/2)+cosh(x)*polylog(2,exp(x))*(a*sech(x)^2)^(1/2)
```

Rubi [A] time = 0.36, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {6720, 2622, 321, 207, 5462, 6271, 4182, 2279, 2391, 3770}

$$-\cosh(x)\text{PolyLog}(2,-e^x)\sqrt{a\operatorname{sech}^2(x)}+\cosh(x)\text{PolyLog}(2,e^x)\sqrt{a\operatorname{sech}^2(x)}+x\sqrt{a\operatorname{sech}^2(x)}-2x\cosh(x)\tanh^{-1}$$

Antiderivative was successfully verified.

```
[In] Int[x*Csch[x]*Sech[x]*Sqrt[a*Sech[x]^2],x]
```

```
[Out] x*Sqrt[a*Sech[x]^2] - ArcTan[Sinh[x]]*Cosh[x]*Sqrt[a*Sech[x]^2] - 2*x*ArcTa
nh[E^x]*Cosh[x]*Sqrt[a*Sech[x]^2] - Cosh[x]*PolyLog[2, -E^x]*Sqrt[a*Sech[x]
^2] + Cosh[x]*PolyLog[2, E^x]*Sqrt[a*Sech[x]^2]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5462

```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> With[{u = IntHide[Csch[a + b*x]^n*Sech[a +
b*x]^p, x]}, Dist[(c + d*x)^m, u, x] - Dist[d*m, Int[(c + d*x)^(m - 1)*u, x
], x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n,
p]
```

Rule 6271

```
Int[ArcTanh[u_], x_Symbol] :> Simp[x*ArcTanh[u], x] - Int[SimplifyIntegrand
[(x*D[u, x])/(1 - u^2), x], x] /; InverseFunctionFreeQ[u, x]
```

Rule 6720

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a*v^m)^
FracPart[p])/v^(m*FracPart[p]), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] &&  !IntegerQ[p] &&  !FreeQ[v, x] &&  !(EqQ[a, 1] && EqQ[m, 1]) &&  !(EqQ
[v, x] && EqQ[m, 1])
```

Rubi steps

$$\int x\text{csch}(x)\text{sech}(x)\sqrt{a\text{sech}^2(x)}\,dx = \left(\cosh(x)\sqrt{a\text{sech}^2(x)}\right)\int x\text{csch}(x)\text{sech}^2(x)\,dx$$
$$= x\sqrt{a\text{sech}^2(x)} - x\tanh^{-1}(\cosh(x))\cosh(x)\sqrt{a\text{sech}^2(x)} - \left(\cosh(x)\sqrt{a\text{sec}\right.$$
$$= x\sqrt{a\text{sech}^2(x)} - x\tanh^{-1}(\cosh(x))\cosh(x)\sqrt{a\text{sech}^2(x)} + \left(\cosh(x)\sqrt{a\text{sec}\right.$$
$$= x\sqrt{a\text{sech}^2(x)} - \tan^{-1}(\sinh(x))\cosh(x)\sqrt{a\text{sech}^2(x)} + \left(\cosh(x)\sqrt{a\text{sech}^2(}\right.$$
$$= x\sqrt{a\text{sech}^2(x)} - \tan^{-1}(\sinh(x))\cosh(x)\sqrt{a\text{sech}^2(x)} - 2x\tanh^{-1}(e^x)\cosh$$
$$= x\sqrt{a\text{sech}^2(x)} - \tan^{-1}(\sinh(x))\cosh(x)\sqrt{a\text{sech}^2(x)} - 2x\tanh^{-1}(e^x)\cosh$$
$$= x\sqrt{a\text{sech}^2(x)} - \tan^{-1}(\sinh(x))\cosh(x)\sqrt{a\text{sech}^2(x)} - 2x\tanh^{-1}(e^x)\cosh$$

Mathematica [A] time = 0.06, size = 74, normalized size = 0.84

$$\sqrt{a\text{sech}^2(x)}\left(\text{Li}_2(-e^{-x})\cosh(x) - \text{Li}_2(e^{-x})\cosh(x) + x + x\log(1-e^{-x})\cosh(x) - x\log(e^{-x}+1)\cosh(x) - 2\,\text{c}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Csch[x]*Sech[x]*Sqrt[a*Sech[x]^2],x]
```

```
[Out] (x - 2*ArcTan[Tanh[x/2]]*Cosh[x] + x*Cosh[x]*Log[1 - E^(-x)] - x*Cosh[x]*Lo
g[1 + E^(-x)] + Cosh[x]*PolyLog[2, -E^(-x)] - Cosh[x]*PolyLog[2, E^(-x)])*S
qrt[a*Sech[x]^2]
```

fricas [B] time = 0.49, size = 351, normalized size = 3.99

$$\frac{\left(2\,x\cosh(x)e^{(2\,x)} - 2\left(\left(e^{(2\,x)}+1\right)\sinh(x)^2 + \cosh(x)^2 + \left(\cosh(x)^2+1\right)e^{(2\,x)} + 2\left(\cosh(x)e^{(2\,x)} + \cosh(x)\right)\sinh(x\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*csch(x)*sech(x)*(a*sech(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] (2*x*cosh(x)*e^(2*x) - 2*((e^(2*x) + 1)*sinh(x)^2 + cosh(x)^2 + (cosh(x)^2
+ 1)*e^(2*x) + 2*(cosh(x)*e^(2*x) + cosh(x))*sinh(x) + 1)*arctan(cosh(x) +
```

```
sinh(x)) + 2*x*cosh(x) + ((e^(2*x) + 1)*sinh(x)^2 + cosh(x)^2 + (cosh(x)^2
+ 1)*e^(2*x) + 2*(cosh(x)*e^(2*x) + cosh(x))*sinh(x) + 1)*dilog(cosh(x) + s
inh(x)) - ((e^(2*x) + 1)*sinh(x)^2 + cosh(x)^2 + (cosh(x)^2 + 1)*e^(2*x) +
2*(cosh(x)*e^(2*x) + cosh(x))*sinh(x) + 1)*dilog(-cosh(x) - sinh(x)) - (x*c
osh(x)^2 + (x*e^(2*x) + x)*sinh(x)^2 + (x*cosh(x)^2 + x)*e^(2*x) + 2*(x*cos
h(x)*e^(2*x) + x*cosh(x))*sinh(x) + x)*log(cosh(x) + sinh(x) + 1) + (x*cosh
(x)^2 + (x*e^(2*x) + x)*sinh(x)^2 + (x*cosh(x)^2 + x)*e^(2*x) + 2*(x*cosh(x
)*e^(2*x) + x*cosh(x))*sinh(x) + x)*log(-cosh(x) - sinh(x) + 1) + 2*(x*e^(2
*x) + x)*sinh(x))*sqrt(a/(e^(4*x) + 2*e^(2*x) + 1))*e^x/(2*cosh(x)*e^x*sinh
(x) + e^x*sinh(x)^2 + (cosh(x)^2 + 1)*e^x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a\,\mathrm{sech}(x)^2}\,x\,\mathrm{csch}(x)\,\mathrm{sech}(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*csch(x)*sech(x)*(a*sech(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a*sech(x)^2)*x*csch(x)*sech(x), x)
```

maple [A] time = 0.38, size = 150, normalized size = 1.70

$$2\sqrt{\frac{a\,\mathrm{e}^{2x}}{\left(1+\mathrm{e}^{2x}\right)^2}}\,x-2\sqrt{\frac{a\,\mathrm{e}^{2x}}{\left(1+\mathrm{e}^{2x}\right)^2}}\,\mathrm{e}^{-x}\left(1+\mathrm{e}^{2x}\right)\arctan\left(\mathrm{e}^{x}\right)-\sqrt{\frac{a\,\mathrm{e}^{2x}}{\left(1+\mathrm{e}^{2x}\right)^2}}\,\mathrm{e}^{-x}\left(1+\mathrm{e}^{2x}\right)\mathrm{dilog}\left(\mathrm{e}^{x}\right)-\sqrt{\frac{a\,\mathrm{e}^{2x}}{\left(1+\mathrm{e}^{2x}\right)^2}}\,\mathrm{e}^{-}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*csch(x)*sech(x)*(a*sech(x)^2)^(1/2),x)
```

```
[Out] 2*(a*exp(2*x)/(1+exp(2*x))^2)^(1/2)*x-2*(a*exp(2*x)/(1+exp(2*x))^2)^(1/2)*e
xp(-x)*(1+exp(2*x))*arctan(exp(x))-(a*exp(2*x)/(1+exp(2*x))^2)^(1/2)*exp(-x
)*(1+exp(2*x))*dilog(exp(x))-(a*exp(2*x)/(1+exp(2*x))^2)^(1/2)*exp(-x)*(1+e
xp(2*x))*dilog(exp(x)+1)-(a*exp(2*x)/(1+exp(2*x))^2)^(1/2)*exp(-x)*(1+exp(2
*x))*x*ln(exp(x)+1)
```

maxima [A] time = 0.44, size = 60, normalized size = 0.68

$$-\left(x\log\left(e^{x}+1\right)+\mathrm{Li}_2\left(-e^{x}\right)\right)\sqrt{a}+\left(x\log\left(-e^{x}+1\right)+\mathrm{Li}_2\left(e^{x}\right)\right)\sqrt{a}-2\sqrt{a}\,\arctan\left(e^{x}\right)+\frac{2\sqrt{a}\,xe^{x}}{e^{(2x)}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*csch(x)*sech(x)*(a*sech(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -(x*log(e^x + 1) + dilog(-e^x))*sqrt(a) + (x*log(-e^x + 1) + dilog(e^x))*sq
rt(a) - 2*sqrt(a)*arctan(e^x) + 2*sqrt(a)*x*e^x/(e^(2*x) + 1)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x\sqrt{\frac{a}{\cosh(x)^2}}}{\cosh(x)\sinh(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a/cosh(x)^2)^(1/2))/(cosh(x)*sinh(x)),x)
[Out] int((x*(a/cosh(x)^2)^(1/2))/(cosh(x)*sinh(x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\sqrt{a\,\mathrm{sech}^2(x)}\,\mathrm{csch}(x)\,\mathrm{sech}(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*csch(x)*sech(x)*(a*sech(x)**2)**(1/2),x)
[Out] Integral(x*sqrt(a*sech(x)**2)*csch(x)*sech(x), x)
```

3.849 $\int x^2 \operatorname{csch}(x)\operatorname{sech}(x)\sqrt{a\operatorname{sech}^2(x)}\,dx$

Optimal. Leaf size=187

$$-2x\text{Li}_2(-e^x)\cosh(x)\sqrt{a\text{sech}^2(x)}+2x\text{Li}_2(e^x)\cosh(x)\sqrt{a\text{sech}^2(x)}+2i\text{Li}_2(-ie^x)\cosh(x)\sqrt{a\text{sech}^2(x)}-2i\text{Li}_2(ie^x)\text{ c}$$

```
[Out] x^2*(a*sech(x)^2)^(1/2)-4*x*arctan(exp(x))*cosh(x)*(a*sech(x)^2)^(1/2)-2*x^
2*arctanh(exp(x))*cosh(x)*(a*sech(x)^2)^(1/2)-2*x*cosh(x)*polylog(2,-exp(x)
)*(a*sech(x)^2)^(1/2)+2*I*cosh(x)*polylog(2,-I*exp(x))*(a*sech(x)^2)^(1/2)-
2*I*cosh(x)*polylog(2,I*exp(x))*(a*sech(x)^2)^(1/2)+2*x*cosh(x)*polylog(2,e
xp(x))*(a*sech(x)^2)^(1/2)+2*cosh(x)*polylog(3,-exp(x))*(a*sech(x)^2)^(1/2)
-2*cosh(x)*polylog(3,exp(x))*(a*sech(x)^2)^(1/2)
```

Rubi [A] time = 0.51, antiderivative size = 187, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 14, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.778, Rules used = {6720, 2622, 321, 207, 5462, 14, 6273, 4182, 2531, 2282, 6589, 4180, 2279, 2391}

$$-2x\cosh(x)\text{PolyLog}(2,-e^x)\sqrt{a\text{sech}^2(x)}+2x\cosh(x)\text{PolyLog}(2,e^x)\sqrt{a\text{sech}^2(x)}+2i\cosh(x)\text{PolyLog}(2,-ie^x)$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Csch[x]*Sech[x]*Sqrt[a*Sech[x]^2],x]
```

```
[Out] x^2*Sqrt[a*Sech[x]^2] - 4*x*ArcTan[E^x]*Cosh[x]*Sqrt[a*Sech[x]^2] - 2*x^2*A
rcTanh[E^x]*Cosh[x]*Sqrt[a*Sech[x]^2] - 2*x*Cosh[x]*PolyLog[2, -E^x]*Sqrt[a
*Sech[x]^2] + (2*I)*Cosh[x]*PolyLog[2, (-I)*E^x]*Sqrt[a*Sech[x]^2] - (2*I)*
Cosh[x]*PolyLog[2, I*E^x]*Sqrt[a*Sech[x]^2] + 2*x*Cosh[x]*PolyLog[2, E^x]*S
qrt[a*Sech[x]^2] + 2*Cosh[x]*PolyLog[3, -E^x]*Sqrt[a*Sech[x]^2] - 2*Cosh[x]
*PolyLog[3, E^x]*Sqrt[a*Sech[x]^2]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
```

```
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5462

```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> With[{u = IntHide[Csch[a + b*x]^n*Sech[a +
b*x]^p, x]}, Dist[(c + d*x)^m, u, x] - Dist[d*m, Int[(c + d*x)^(m - 1)*u, x
], x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n,
p]
```

Rule 6273

```
Int[((a_.) + ArcTanh[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[((c + d*x)^(m + 1)*(a + b*ArcTanh[u]))/(d*(m + 1)), x] - Dist[b/(d*(m +
1)), Int[SimplifyIntegrand[((c + d*x)^(m + 1)*D[u, x])/(1 - u^2), x], x], x
] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && InverseFunctionFreeQ[u, x]
&&  !FunctionOfQ[(c + d*x)^(m + 1), u, x] && FalseQ[PowerVariableExpn[u, m
+ 1, x]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6720

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a*v^m)^
FracPart[p])/v^(m*FracPart[p]), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] &&  !IntegerQ[p] &&  !FreeQ[v, x] &&  !(EqQ[a, 1] && EqQ[m, 1]) &&  !(EqQ
[v, x] && EqQ[m, 1])
```

Rubi steps

$$\int x^2\text{csch}(x)\text{sech}(x)\sqrt{a\text{sech}^2(x)}\,dx = \left(\cosh(x)\sqrt{a\text{sech}^2(x)}\right)\int x^2\text{csch}(x)\text{sech}^2(x)\,dx$$

$$= x^2\sqrt{a\text{sech}^2(x)} - x^2\tanh^{-1}(\cosh(x))\cosh(x)\sqrt{a\text{sech}^2(x)} - \Big(2\cosh(x)\sqrt{}$$

$$= x^2\sqrt{a\text{sech}^2(x)} - x^2\tanh^{-1}(\cosh(x))\cosh(x)\sqrt{a\text{sech}^2(x)} - \Big(2\cosh(x)\sqrt{}$$

$$= x^2\sqrt{a\text{sech}^2(x)} - x^2\tanh^{-1}(\cosh(x))\cosh(x)\sqrt{a\text{sech}^2(x)} + \Big(2\cosh(x)\sqrt{}$$

$$= x^2\sqrt{a\text{sech}^2(x)} - 4x\tan^{-1}(e^x)\cosh(x)\sqrt{a\text{sech}^2(x)} + \Big(2i\cosh(x)\sqrt{a\text{sech}}$$

$$= x^2\sqrt{a\text{sech}^2(x)} - 4x\tan^{-1}(e^x)\cosh(x)\sqrt{a\text{sech}^2(x)} - 2x^2\tanh^{-1}(e^x)\cos$$

$$= x^2\sqrt{a\text{sech}^2(x)} - 4x\tan^{-1}(e^x)\cosh(x)\sqrt{a\text{sech}^2(x)} - 2x^2\tanh^{-1}(e^x)\cos$$

$$= x^2\sqrt{a\text{sech}^2(x)} - 4x\tan^{-1}(e^x)\cosh(x)\sqrt{a\text{sech}^2(x)} - 2x^2\tanh^{-1}(e^x)\cos$$

$$= x^2\sqrt{a\text{sech}^2(x)} - 4x\tan^{-1}(e^x)\cosh(x)\sqrt{a\text{sech}^2(x)} - 2x^2\tanh^{-1}(e^x)\cos$$

Mathematica [A] time = 0.20, size = 154, normalized size = 0.82

$$\sqrt{a\text{sech}^2(x)}\,\Big(2x\left(\text{Li}_2\left(-e^{-x}\right) - \text{Li}_2\left(e^{-x}\right)\right)\cosh(x) + 2i\left(\text{Li}_2\left(-ie^{-x}\right) - \text{Li}_2\left(ie^{-x}\right)\right)\cosh(x) + 2\left(\text{Li}_3\left(-e^{-x}\right) - \text{Li}_3\left(e^{-x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Csch[x]*Sech[x]*Sqrt[a*Sech[x]^2],x]
```

```
[Out] (x^2 + (2*I)*x*Cosh[x]*(Log[1 - I/E^x] - Log[1 + I/E^x]) + x^2*Cosh[x]*(Log
[1 - E^(-x)] - Log[1 + E^(-x)]) + (2*I)*Cosh[x]*(PolyLog[2, (-I)/E^x] - Pol
yLog[2, I/E^x]) + 2*x*Cosh[x]*(PolyLog[2, -E^(-x)] - PolyLog[2, E^(-x)]) +
2*Cosh[x]*(PolyLog[3, -E^(-x)] - PolyLog[3, E^(-x)]))*Sqrt[a*Sech[x]^2]
```

fricas [C] time = 0.49, size = 778, normalized size = 4.16

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*csch(x)*sech(x)*(a*sech(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -(2*((e^(2*x) + 1)*sinh(x)^2 + cosh(x)^2 + (cosh(x)^2 + 1)*e^(2*x) + 2*(cos
h(x)*e^(2*x) + cosh(x))*sinh(x) + 1)*sqrt(a/(e^(4*x) + 2*e^(2*x) + 1))*e^x*
polylog(3, cosh(x) + sinh(x)) - 2*((e^(2*x) + 1)*sinh(x)^2 + cosh(x)^2 + (c
osh(x)^2 + 1)*e^(2*x) + 2*(cosh(x)*e^(2*x) + cosh(x))*sinh(x) + 1)*sqrt(a/(
e^(4*x) + 2*e^(2*x) + 1))*e^x*polylog(3, -cosh(x) - sinh(x)) - (2*x^2*cosh(
x)*e^(2*x) + 2*x^2*cosh(x) + 2*(x*cosh(x)^2 + (x*e^(2*x) + x)*sinh(x)^2 + (
x*cosh(x)^2 + x)*e^(2*x) + 2*(x*cosh(x)*e^(2*x) + x*cosh(x))*sinh(x) + x)*d
ilog(cosh(x) + sinh(x)) + ((-2*I*e^(2*x) - 2*I)*sinh(x)^2 - 2*I*cosh(x)^2 +
 (-2*I*cosh(x)^2 - 2*I)*e^(2*x) + (-4*I*cosh(x)*e^(2*x) - 4*I*cosh(x))*sinh
(x) - 2*I)*dilog(I*cosh(x) + I*sinh(x)) + ((2*I*e^(2*x) + 2*I)*sinh(x)^2 +
2*I*cosh(x)^2 + (2*I*cosh(x)^2 + 2*I)*e^(2*x) + (4*I*cosh(x)*e^(2*x) + 4*I*
cosh(x))*sinh(x) + 2*I)*dilog(-I*cosh(x) - I*sinh(x)) - 2*(x*cosh(x)^2 + (x
*e^(2*x) + x)*sinh(x)^2 + (x*cosh(x)^2 + x)*e^(2*x) + 2*(x*cosh(x)*e^(2*x)
+ x*cosh(x))*sinh(x) + x)*dilog(-cosh(x) - sinh(x)) - (x^2*cosh(x)^2 + (x^2
*e^(2*x) + x^2)*sinh(x)^2 + x^2 + (x^2*cosh(x)^2 + x^2)*e^(2*x) + 2*(x^2*co
sh(x)*e^(2*x) + x^2*cosh(x))*sinh(x))*log(cosh(x) + sinh(x) + 1) + (2*I*x*c
osh(x)^2 + (2*I*x*e^(2*x) + 2*I*x)*sinh(x)^2 + (2*I*x*cosh(x)^2 + 2*I*x)*e^
(2*x) + (4*I*x*cosh(x)*e^(2*x) + 4*I*x*cosh(x))*sinh(x) + 2*I*x)*log(I*cosh
(x) + I*sinh(x) + 1) + (-2*I*x*cosh(x)^2 + (-2*I*x*e^(2*x) - 2*I*x)*sinh(x)
^2 + (-2*I*x*cosh(x)^2 - 2*I*x)*e^(2*x) + (-4*I*x*cosh(x)*e^(2*x) - 4*I*x*c
osh(x))*sinh(x) - 2*I*x)*log(-I*cosh(x) - I*sinh(x) + 1) + (x^2*cosh(x)^2 +
 (x^2*e^(2*x) + x^2)*sinh(x)^2 + x^2 + (x^2*cosh(x)^2 + x^2)*e^(2*x) + 2*(x
^2*cosh(x)*e^(2*x) + x^2*cosh(x))*sinh(x))*log(-cosh(x) - sinh(x) + 1) + 2*
(x^2*e^(2*x) + x^2)*sinh(x))*sqrt(a/(e^(4*x) + 2*e^(2*x) + 1))*e^x)/(2*cosh
(x)*e^x*sinh(x) + e^x*sinh(x)^2 + (cosh(x)^2 + 1)*e^x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a \operatorname{sech}(x)^2}\, x^2 \operatorname{csch}(x) \operatorname{sech}(x)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*csch(x)*sech(x)*(a*sech(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a*sech(x)^2)*x^2*csch(x)*sech(x), x)
```

maple [F] time = 0.54, size = 0, normalized size = 0.00

$$\int x^{2} \operatorname{csch}(x) \operatorname{sech}(x) \sqrt{a \operatorname{sech}(x)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*csch(x)*sech(x)*(a*sech(x)^2)^(1/2),x)
```

```
[Out] int(x^2*csch(x)*sech(x)*(a*sech(x)^2)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{2\sqrt{a}\,x^2e^x}{e^{(2x)}+1}-\left(x^2\log\left(e^x+1\right)+2\,x\mathrm{Li}_2\left(-e^x\right)-2\,\mathrm{Li}_3(-e^x)\right)\sqrt{a}+\left(x^2\log\left(-e^x+1\right)+2\,x\mathrm{Li}_2\left(e^x\right)-2\,\mathrm{Li}_3(e^x)\right)\sqrt{a}-4\,\sqrt{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*csch(x)*sech(x)*(a*sech(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 2*sqrt(a)*x^2*e^x/(e^(2*x) + 1) - (x^2*log(e^x + 1) + 2*x*dilog(-e^x) - 2*p
olylog(3, -e^x))*sqrt(a) + (x^2*log(-e^x + 1) + 2*x*dilog(e^x) - 2*polylog(
3, e^x))*sqrt(a) - 4*sqrt(a)*integrate(x*e^x/(e^(2*x) + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2\,\sqrt{\frac{a}{{\mathrm{cosh}\left(x\right)}^2}}}{\mathrm{cosh}\left(x\right)\,\mathrm{sinh}\left(x\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(a/cosh(x)^2)^(1/2))/(cosh(x)*sinh(x)),x)
```

```
[Out] int((x^2*(a/cosh(x)^2)^(1/2))/(cosh(x)*sinh(x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^{2} \sqrt{a \operatorname{sech}^{2}{\left (x \right)}} \operatorname{csch}{\left (x \right)} \operatorname{sech}{\left (x \right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*csch(x)*sech(x)*(a*sech(x)**2)**(1/2),x)
```

```
[Out] Integral(x**2*sqrt(a*sech(x)**2)*csch(x)*sech(x), x)
```

3.850 $\int x^3 \operatorname{csch}(x)\operatorname{sech}(x)\sqrt{a\operatorname{sech}^2(x)}\,dx$

Optimal. Leaf size=287

$$-3x^2\text{Li}_2\left(-e^x\right)\cosh(x)\sqrt{a\operatorname{sech}^2(x)}+3x^2\text{Li}_2\left(e^x\right)\cosh(x)\sqrt{a\operatorname{sech}^2(x)}+6ix\text{Li}_2\left(-ie^x\right)\cosh(x)\sqrt{a\operatorname{sech}^2(x)}-6ix\text{Li}_2\left(i\right.$$

```
[Out] x^3*(a*sech(x)^2)^(1/2)-6*x^2*arctan(exp(x))*cosh(x)*(a*sech(x)^2)^(1/2)-2*
x^3*arctanh(exp(x))*cosh(x)*(a*sech(x)^2)^(1/2)-3*x^2*cosh(x)*polylog(2,-ex
p(x))*(a*sech(x)^2)^(1/2)+6*I*x*cosh(x)*polylog(2,-I*exp(x))*(a*sech(x)^2)^
(1/2)-6*I*x*cosh(x)*polylog(2,I*exp(x))*(a*sech(x)^2)^(1/2)+3*x^2*cosh(x)*p
olylog(2,exp(x))*(a*sech(x)^2)^(1/2)+6*x*cosh(x)*polylog(3,-exp(x))*(a*sech
(x)^2)^(1/2)-6*I*cosh(x)*polylog(3,-I*exp(x))*(a*sech(x)^2)^(1/2)+6*I*cosh(
x)*polylog(3,I*exp(x))*(a*sech(x)^2)^(1/2)-6*x*cosh(x)*polylog(3,exp(x))*(a
*sech(x)^2)^(1/2)-6*cosh(x)*polylog(4,-exp(x))*(a*sech(x)^2)^(1/2)+6*cosh(x
)*polylog(4,exp(x))*(a*sech(x)^2)^(1/2)
```

Rubi [A] time = 0.67, antiderivative size = 287, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 13, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.722, Rules used = {6720, 2622, 321, 207, 5462, 14, 6273, 4182, 2531, 6609, 2282, 6589, 4180}

$$-3x^2\cosh(x)\text{PolyLog}\left(2,-e^x\right)\sqrt{a\operatorname{sech}^2(x)}+3x^2\cosh(x)\text{PolyLog}\left(2,e^x\right)\sqrt{a\operatorname{sech}^2(x)}+6ix\cosh(x)\text{PolyLog}\left(2,-i\right.$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Csch[x]*Sech[x]*Sqrt[a*Sech[x]^2],x]
```

```
[Out] x^3*Sqrt[a*Sech[x]^2] - 6*x^2*ArcTan[E^x]*Cosh[x]*Sqrt[a*Sech[x]^2] - 2*x^3
*ArcTanh[E^x]*Cosh[x]*Sqrt[a*Sech[x]^2] - 3*x^2*Cosh[x]*PolyLog[2, -E^x]*Sq
rt[a*Sech[x]^2] + (6*I)*x*Cosh[x]*PolyLog[2, (-I)*E^x]*Sqrt[a*Sech[x]^2] -
(6*I)*x*Cosh[x]*PolyLog[2, I*E^x]*Sqrt[a*Sech[x]^2] + 3*x^2*Cosh[x]*PolyLog
[2, E^x]*Sqrt[a*Sech[x]^2] + 6*x*Cosh[x]*PolyLog[3, -E^x]*Sqrt[a*Sech[x]^2]
 - (6*I)*Cosh[x]*PolyLog[3, (-I)*E^x]*Sqrt[a*Sech[x]^2] + (6*I)*Cosh[x]*Pol
yLog[3, I*E^x]*Sqrt[a*Sech[x]^2] - 6*x*Cosh[x]*PolyLog[3, E^x]*Sqrt[a*Sech[
x]^2] - 6*Cosh[x]*PolyLog[4, -E^x]*Sqrt[a*Sech[x]^2] + 6*Cosh[x]*PolyLog[4,
 E^x]*Sqrt[a*Sech[x]^2]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5462

```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> With[{u = IntHide[Csch[a + b*x]^n*Sech[a +
b*x]^p, x]}, Dist[(c + d*x)^m, u, x] - Dist[d*m, Int[(c + d*x)^(m - 1)*u, x
], x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n,
p]
```

Rule 6273

```
Int[((a_.) + ArcTanh[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[((c + d*x)^(m + 1)*(a + b*ArcTanh[u]))/(d*(m + 1)), x] - Dist[b/(d*(m +
1)), Int[SimplifyIntegrand[((c + d*x)^(m + 1)*D[u, x])/(1 - u^2), x], x], x
] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && InverseFunctionFreeQ[u, x]
&&  !FunctionOfQ[(c + d*x)^(m + 1), u, x] && FalseQ[PowerVariableExpn[u, m
+ 1, x]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 6720

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a*v^m)^
FracPart[p])/v^(m*FracPart[p]), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] &&  !IntegerQ[p] &&  !FreeQ[v, x] &&  !(EqQ[a, 1] && EqQ[m, 1]) &&  !(EqQ
```

[v, x] && EqQ[m, 1])

Rubi steps

$$\int x^3\text{csch}(x)\text{sech}(x)\sqrt{a\text{sech}^2(x)}\,dx = \left(\cosh(x)\sqrt{a\text{sech}^2(x)}\right)\int x^3\text{csch}(x)\text{sech}^2(x)\,dx$$
$$= x^3\sqrt{a\text{sech}^2(x)} - x^3\tanh^{-1}(\cosh(x))\cosh(x)\sqrt{a\text{sech}^2(x)} - \Big(3\cosh(x)\sqrt{}$$
$$= x^3\sqrt{a\text{sech}^2(x)} - x^3\tanh^{-1}(\cosh(x))\cosh(x)\sqrt{a\text{sech}^2(x)} - \Big(3\cosh(x)\sqrt{}$$
$$= x^3\sqrt{a\text{sech}^2(x)} - x^3\tanh^{-1}(\cosh(x))\cosh(x)\sqrt{a\text{sech}^2(x)} + \Big(3\cosh(x)\sqrt{}$$
$$= x^3\sqrt{a\text{sech}^2(x)} - 6x^2\tan^{-1}(e^x)\cosh(x)\sqrt{a\text{sech}^2(x)} + \Big(6i\cosh(x)\sqrt{a\text{sec}}$$
$$= x^3\sqrt{a\text{sech}^2(x)} - 6x^2\tan^{-1}(e^x)\cosh(x)\sqrt{a\text{sech}^2(x)} - 2x^3\tanh^{-1}(e^x)\,\text{co}$$
$$= x^3\sqrt{a\text{sech}^2(x)} - 6x^2\tan^{-1}(e^x)\cosh(x)\sqrt{a\text{sech}^2(x)} - 2x^3\tanh^{-1}(e^x)\,\text{co}$$
$$= x^3\sqrt{a\text{sech}^2(x)} - 6x^2\tan^{-1}(e^x)\cosh(x)\sqrt{a\text{sech}^2(x)} - 2x^3\tanh^{-1}(e^x)\,\text{co}$$
$$= x^3\sqrt{a\text{sech}^2(x)} - 6x^2\tan^{-1}(e^x)\cosh(x)\sqrt{a\text{sech}^2(x)} - 2x^3\tanh^{-1}(e^x)\,\text{co}$$
$$= x^3\sqrt{a\text{sech}^2(x)} - 6x^2\tan^{-1}(e^x)\cosh(x)\sqrt{a\text{sech}^2(x)} - 2x^3\tanh^{-1}(e^x)\,\text{co}$$

Mathematica [A] time = 0.66, size = 249, normalized size = 0.87

$$\frac{1}{8}\sqrt{a\text{sech}^2(x)}\,\Big(24x^2\text{Li}_2\left(-e^{-x}\right)\cosh(x) + 24x^2\text{Li}_2\left(e^x\right)\cosh(x) + 48ix\text{Li}_2\left(-ie^{-x}\right)\cosh(x) - 48ix\text{Li}_2\left(ie^{-x}\right)\cosh(x$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Csch[x]*Sech[x]*Sqrt[a*Sech[x]^2],x]
```

```
[Out] ((8*x^3 + Pi^4*Cosh[x] - 2*x^4*Cosh[x] + (24*I)*x^2*Cosh[x]*Log[1 - I/E^x]
- (24*I)*x^2*Cosh[x]*Log[1 + I/E^x] - 8*x^3*Cosh[x]*Log[1 + E^(-x)] + 8*x^3
*Cosh[x]*Log[1 - E^x] + 24*x^2*Cosh[x]*PolyLog[2, -E^(-x)] + (48*I)*x*Cosh[
x]*PolyLog[2, (-I)/E^x] - (48*I)*x*Cosh[x]*PolyLog[2, I/E^x] + 24*x^2*Cosh[
x]*PolyLog[2, E^x] + 48*x*Cosh[x]*PolyLog[3, -E^(-x)] + (48*I)*Cosh[x]*Poly
```

```
Log[3, (-I)/E^x] - (48*I)*Cosh[x]*PolyLog[3, I/E^x] - 48*x*Cosh[x]*PolyLog[
3, E^x] + 48*Cosh[x]*PolyLog[4, -E^(-x)] + 48*Cosh[x]*PolyLog[4, E^x])*Sqrt
[a*Sech[x]^2])/8
```

fricas [C] time = 0.54, size = 1190, normalized size = 4.15

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*csch(x)*sech(x)*(a*sech(x)^2)^(1/2),x, algorithm="fricas")

[Out] (6*((e^(2*x) + 1)*sinh(x)^2 + cosh(x)^2 + (cosh(x)^2 + 1)*e^(2*x) + 2*(cosh
(x)*e^(2*x) + cosh(x))*sinh(x) + 1)*sqrt(a/(e^(4*x) + 2*e^(2*x) + 1))*e^x*p
olylog(4, cosh(x) + sinh(x)) - 6*((e^(2*x) + 1)*sinh(x)^2 + cosh(x)^2 + (co
sh(x)^2 + 1)*e^(2*x) + 2*(cosh(x)*e^(2*x) + cosh(x))*sinh(x) + 1)*sqrt(a/(e
^(4*x) + 2*e^(2*x) + 1))*e^x*polylog(4, -cosh(x) - sinh(x)) - 6*(x*cosh(x)^
2 + (x*e^(2*x) + x)*sinh(x)^2 + (x*cosh(x)^2 + x)*e^(2*x) + 2*(x*cosh(x)*e^
(2*x) + x*cosh(x))*sinh(x) + x)*sqrt(a/(e^(4*x) + 2*e^(2*x) + 1))*e^x*polyl
og(3, cosh(x) + sinh(x)) + ((6*I*e^(2*x) + 6*I)*sinh(x)^2 + 6*I*cosh(x)^2 +
 (6*I*cosh(x)^2 + 6*I)*e^(2*x) + (12*I*cosh(x)*e^(2*x) + 12*I*cosh(x))*sinh
(x) + 6*I)*sqrt(a/(e^(4*x) + 2*e^(2*x) + 1))*e^x*polylog(3, I*cosh(x) + I*s
inh(x)) + ((-6*I*e^(2*x) - 6*I)*sinh(x)^2 - 6*I*cosh(x)^2 + (-6*I*cosh(x)^2
 - 6*I)*e^(2*x) + (-12*I*cosh(x)*e^(2*x) - 12*I*cosh(x))*sinh(x) - 6*I)*sqr
t(a/(e^(4*x) + 2*e^(2*x) + 1))*e^x*polylog(3, -I*cosh(x) - I*sinh(x)) + 6*(
x*cosh(x)^2 + (x*e^(2*x) + x)*sinh(x)^2 + (x*cosh(x)^2 + x)*e^(2*x) + 2*(x*
cosh(x)*e^(2*x) + x*cosh(x))*sinh(x) + x)*sqrt(a/(e^(4*x) + 2*e^(2*x) + 1))
*e^x*polylog(3, -cosh(x) - sinh(x)) + (2*x^3*cosh(x)*e^(2*x) + 2*x^3*cosh(x
) + 3*(x^2*cosh(x)^2 + (x^2*e^(2*x) + x^2)*sinh(x)^2 + x^2 + (x^2*cosh(x)^2
 + x^2)*e^(2*x) + 2*(x^2*cosh(x)*e^(2*x) + x^2*cosh(x))*sinh(x))*dilog(cosh
(x) + sinh(x)) + (-6*I*x*cosh(x)^2 + (-6*I*x*e^(2*x) - 6*I*x)*sinh(x)^2 + (
-6*I*x*cosh(x)^2 - 6*I*x)*e^(2*x) + (-12*I*x*cosh(x)*e^(2*x) - 12*I*x*cosh(
x))*sinh(x) - 6*I*x)*dilog(I*cosh(x) + I*sinh(x)) + (6*I*x*cosh(x)^2 + (6*I
*x*e^(2*x) + 6*I*x)*sinh(x)^2 + (6*I*x*cosh(x)^2 + 6*I*x)*e^(2*x) + (12*I*x
*cosh(x)*e^(2*x) + 12*I*x*cosh(x))*sinh(x) + 6*I*x)*dilog(-I*cosh(x) - I*si
nh(x)) - 3*(x^2*cosh(x)^2 + (x^2*e^(2*x) + x^2)*sinh(x)^2 + x^2 + (x^2*cosh
(x)^2 + x^2)*e^(2*x) + 2*(x^2*cosh(x)*e^(2*x) + x^2*cosh(x))*sinh(x))*dilog
(-cosh(x) - sinh(x)) - (x^3*cosh(x)^2 + x^3 + (x^3*e^(2*x) + x^3)*sinh(x)^2
 + (x^3*cosh(x)^2 + x^3)*e^(2*x) + 2*(x^3*cosh(x)*e^(2*x) + x^3*cosh(x))*si
nh(x))*log(cosh(x) + sinh(x) + 1) + (3*I*x^2*cosh(x)^2 + (3*I*x^2*e^(2*x) +
 3*I*x^2)*sinh(x)^2 + 3*I*x^2 + (3*I*x^2*cosh(x)^2 + 3*I*x^2)*e^(2*x) + (6*
I*x^2*cosh(x)*e^(2*x) + 6*I*x^2*cosh(x))*sinh(x))*log(I*cosh(x) + I*sinh(x)
 + 1) + (-3*I*x^2*cosh(x)^2 + (-3*I*x^2*e^(2*x) - 3*I*x^2)*sinh(x)^2 - 3*I*
x^2 + (-3*I*x^2*cosh(x)^2 - 3*I*x^2)*e^(2*x) + (-6*I*x^2*cosh(x)*e^(2*x) -
6*I*x^2*cosh(x))*sinh(x))*log(-I*cosh(x) - I*sinh(x) + 1) + (x^3*cosh(x)^2
+ x^3 + (x^3*e^(2*x) + x^3)*sinh(x)^2 + (x^3*cosh(x)^2 + x^3)*e^(2*x) + 2*(
```

```
x^3*cosh(x)*e^(2*x) + x^3*cosh(x))*sinh(x))*log(-cosh(x) - sinh(x) + 1) + 2
*(x^3*e^(2*x) + x^3)*sinh(x))*sqrt(a/(e^(4*x) + 2*e^(2*x) + 1))*e^x)/(2*cos
h(x)*e^x*sinh(x) + e^x*sinh(x)^2 + (cosh(x)^2 + 1)*e^x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a\,\mathrm{sech}(x)^2}\,x^3\,\mathrm{csch}(x)\,\mathrm{sech}(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*csch(x)*sech(x)*(a*sech(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a*sech(x)^2)*x^3*csch(x)*sech(x), x)
```

maple [F] time = 0.53, size = 0, normalized size = 0.00

$$\int x^3\mathrm{csch}(x)\mathrm{sech}(x)\sqrt{a\mathrm{sech}(x)^2}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*csch(x)*sech(x)*(a*sech(x)^2)^(1/2),x)
```

```
[Out] int(x^3*csch(x)*sech(x)*(a*sech(x)^2)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{2\,\sqrt{a}\,x^3e^x}{e^{(2\,x)}+1}-\left(x^3\log\left(e^x+1\right)+3\,x^2\mathrm{Li}_2\left(-e^x\right)-6\,x\mathrm{Li}_3(-e^x)+6\,\mathrm{Li}_4(-e^x)\right)\sqrt{a}+\left(x^3\log\left(-e^x+1\right)+3\,x^2\mathrm{Li}_2\left(e^x\right)-6\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*csch(x)*sech(x)*(a*sech(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 2*sqrt(a)*x^3*e^x/(e^(2*x) + 1) - (x^3*log(e^x + 1) + 3*x^2*dilog(-e^x) - 6
*x*polylog(3, -e^x) + 6*polylog(4, -e^x))*sqrt(a) + (x^3*log(-e^x + 1) + 3*
x^2*dilog(e^x) - 6*x*polylog(3, e^x) + 6*polylog(4, e^x))*sqrt(a) - 12*sqrt
(a)*integrate(1/2*x^2*e^x/(e^(2*x) + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3\,\sqrt{\frac{a}{{\mathrm{cosh}\left(x\right)}^2}}}{\mathrm{cosh}\left(x\right)\,\mathrm{sinh}\left(x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(a/cosh(x)^2)^(1/2))/(cosh(x)*sinh(x)),x)
[Out] int((x^3*(a/cosh(x)^2)^(1/2))/(cosh(x)*sinh(x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3\sqrt{a\,\mathrm{sech}^2(x)}\,\mathrm{csch}(x)\,\mathrm{sech}(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*csch(x)*sech(x)*(a*sech(x)**2)**(1/2),x)
[Out] Integral(x**3*sqrt(a*sech(x)**2)*csch(x)*sech(x), x)
```

3.851 $\int x\operatorname{csch}(x)\operatorname{sech}(x)\sqrt{a\operatorname{sech}^4(x)}\,dx$

Optimal. Leaf size=132

$$-\frac{1}{2}\text{Li}_2\left(-e^{2x}\right)\cosh^2(x)\sqrt{a\operatorname{sech}^4(x)}+\frac{1}{2}\text{Li}_2\left(e^{2x}\right)\cosh^2(x)\sqrt{a\operatorname{sech}^4(x)}+\frac{1}{2}x\cosh^2(x)\sqrt{a\operatorname{sech}^4(x)}-\frac{1}{2}x\sinh^2(x)\sqrt{}$$

```
[Out] 1/2*x*cosh(x)^2*(a*sech(x)^4)^(1/2)-2*x*arctanh(exp(2*x))*cosh(x)^2*(a*sech
(x)^4)^(1/2)-1/2*cosh(x)^2*polylog(2,-exp(2*x))*(a*sech(x)^4)^(1/2)+1/2*cos
h(x)^2*polylog(2,exp(2*x))*(a*sech(x)^4)^(1/2)-1/2*cosh(x)*sinh(x)*(a*sech(
x)^4)^(1/2)-1/2*x*sinh(x)^2*(a*sech(x)^4)^(1/2)
```

Rubi [A] time = 0.41, antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 11, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.688, Rules used = {6720, 2620, 14, 5462, 2548, 5461, 4182, 2279, 2391, 3473, 8}

$$-\frac{1}{2}\cosh^2(x)\text{PolyLog}\left(2,-e^{2x}\right)\sqrt{a\operatorname{sech}^4(x)}+\frac{1}{2}\cosh^2(x)\text{PolyLog}\left(2,e^{2x}\right)\sqrt{a\operatorname{sech}^4(x)}+\frac{1}{2}x\cosh^2(x)\sqrt{a\operatorname{sech}^4(x}$$

Antiderivative was successfully verified.

```
[In] Int[x*Csch[x]*Sech[x]*Sqrt[a*Sech[x]^4],x]
```

```
[Out] (x*Cosh[x]^2*Sqrt[a*Sech[x]^4])/2 - 2*x*ArcTanh[E^(2*x)]*Cosh[x]^2*Sqrt[a*S
ech[x]^4] - (Cosh[x]^2*PolyLog[2, -E^(2*x)]*Sqrt[a*Sech[x]^4])/2 + (Cosh[x]
^2*PolyLog[2, E^(2*x)]*Sqrt[a*Sech[x]^4])/2 - (Cosh[x]*Sqrt[a*Sech[x]^4]*Si
nh[x])/2 - (x*Sqrt[a*Sech[x]^4]*Sinh[x]^2)/2
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2548

```
Int[Log[u_], x_Symbol] :> Simp[x*Log[u], x] - Int[SimplifyIntegrand[(x*D[u,
 x])/u, x], x] /; InverseFunctionFreeQ[u, x]
```

Rule 2620

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5461

```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[2^n, Int[(c + d*x)^m*Csch[2*a + 2*b*x]
^n, x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[m] && IntegerQ[n]
```

Rule 5462

```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> With[{u = IntHide[Csch[a + b*x]^n*Sech[a +
b*x]^p, x]}, Dist[(c + d*x)^m, u, x] - Dist[d*m, Int[(c + d*x)^(m - 1)*u, x
], x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n,
p]
```

Rule 6720

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a*v^m)^
FracPart[p])/v^(m*FracPart[p]), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rubi steps

$$\int x\text{csch}(x)\text{sech}(x)\sqrt{a\text{sech}^4(x)}\,dx = \left(\cosh^2(x)\sqrt{a\text{sech}^4(x)}\right)\int x\text{csch}(x)\text{sech}^3(x)\,dx$$

$$= x\cosh^2(x)\log(\tanh(x))\sqrt{a\text{sech}^4(x)} - \frac{1}{2}x\sqrt{a\text{sech}^4(x)}\,\sinh^2(x) - \Big(\cosh^2$$

$$= x\cosh^2(x)\log(\tanh(x))\sqrt{a\text{sech}^4(x)} - \frac{1}{2}x\sqrt{a\text{sech}^4(x)}\,\sinh^2(x) + \frac{1}{2}\Big(\cos$$

$$= -\frac{1}{2}\cosh(x)\sqrt{a\text{sech}^4(x)}\,\sinh(x) - \frac{1}{2}x\sqrt{a\text{sech}^4(x)}\,\sinh^2(x) + \frac{1}{2}\Big(\cosh^2(x$$

$$= \frac{1}{2}x\cosh^2(x)\sqrt{a\text{sech}^4(x)} - \frac{1}{2}\cosh(x)\sqrt{a\text{sech}^4(x)}\,\sinh(x) - \frac{1}{2}x\sqrt{a\text{sech}^4(}$$

$$= \frac{1}{2}x\cosh^2(x)\sqrt{a\text{sech}^4(x)} - 2x\tanh^{-1}\left(e^{2x}\right)\cosh^2(x)\sqrt{a\text{sech}^4(x)} - \frac{1}{2}\text{cosl}$$

$$= \frac{1}{2}x\cosh^2(x)\sqrt{a\text{sech}^4(x)} - 2x\tanh^{-1}\left(e^{2x}\right)\cosh^2(x)\sqrt{a\text{sech}^4(x)} - \frac{1}{2}\text{cosl}$$

$$= \frac{1}{2}x\cosh^2(x)\sqrt{a\text{sech}^4(x)} - 2x\tanh^{-1}\left(e^{2x}\right)\cosh^2(x)\sqrt{a\text{sech}^4(x)} - \frac{1}{2}\text{cosl}$$

Mathematica [A] time = 0.23, size = 71, normalized size = 0.54

$$\frac{1}{2}\cosh^2(x)\sqrt{a\text{sech}^4(x)}\left(\text{Li}_2\left(-e^{-2x}\right) - \text{Li}_2\left(e^{-2x}\right) + 2x\log\left(1-e^{-2x}\right) - 2x\log\left(e^{-2x}+1\right) - \tanh(x) + x\text{sech}^2(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Csch[x]*Sech[x]*Sqrt[a*Sech[x]^4],x]
```

```
[Out] (Cosh[x]^2*Sqrt[a*Sech[x]^4]*(2*x*Log[1 - E^(-2*x)] - 2*x*Log[1 + E^(-2*x)]
 + PolyLog[2, -E^(-2*x)] - PolyLog[2, E^(-2*x)] + x*Sech[x]^2 - Tanh[x]))/2
```

fricas [C] time = 0.53, size = 1757, normalized size = 13.31

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*csch(x)*sech(x)*(a*sech(x)^4)^(1/2),x, algorithm="fricas")

[Out] ((2*x + 1)*cosh(x)^2 + ((2*x + 1)*e^(4*x) + 2*(2*x + 1)*e^(2*x) + 2*x + 1)*
sinh(x)^2 + ((e^(4*x) + 2*e^(2*x) + 1)*sinh(x)^4 + cosh(x)^4 + 4*(cosh(x)*e
^(4*x) + 2*cosh(x)*e^(2*x) + cosh(x))*sinh(x)^3 + 2*(3*cosh(x)^2 + (3*cosh(
x)^2 + 1)*e^(4*x) + 2*(3*cosh(x)^2 + 1)*e^(2*x) + 1)*sinh(x)^2 + 2*cosh(x)^
2 + (cosh(x)^4 + 2*cosh(x)^2 + 1)*e^(4*x) + 2*(cosh(x)^4 + 2*cosh(x)^2 + 1)
*e^(2*x) + 4*(cosh(x)^3 + (cosh(x)^3 + cosh(x))*e^(4*x) + 2*(cosh(x)^3 + co
sh(x))*e^(2*x) + cosh(x))*sinh(x) + 1)*dilog(cosh(x) + sinh(x)) - ((e^(4*x)
 + 2*e^(2*x) + 1)*sinh(x)^4 + cosh(x)^4 + 4*(cosh(x)*e^(4*x) + 2*cosh(x)*e^
(2*x) + cosh(x))*sinh(x)^3 + 2*(3*cosh(x)^2 + (3*cosh(x)^2 + 1)*e^(4*x) + 2
*(3*cosh(x)^2 + 1)*e^(2*x) + 1)*sinh(x)^2 + 2*cosh(x)^2 + (cosh(x)^4 + 2*co
sh(x)^2 + 1)*e^(4*x) + 2*(cosh(x)^4 + 2*cosh(x)^2 + 1)*e^(2*x) + 4*(cosh(x)
^3 + (cosh(x)^3 + cosh(x))*e^(4*x) + 2*(cosh(x)^3 + cosh(x))*e^(2*x) + cosh
(x))*sinh(x) + 1)*dilog(I*cosh(x) + I*sinh(x)) - ((e^(4*x) + 2*e^(2*x) + 1)
*sinh(x)^4 + cosh(x)^4 + 4*(cosh(x)*e^(4*x) + 2*cosh(x)*e^(2*x) + cosh(x))*
sinh(x)^3 + 2*(3*cosh(x)^2 + (3*cosh(x)^2 + 1)*e^(4*x) + 2*(3*cosh(x)^2 + 1
)*e^(2*x) + 1)*sinh(x)^2 + 2*cosh(x)^2 + (cosh(x)^4 + 2*cosh(x)^2 + 1)*e^(4
*x) + 2*(cosh(x)^4 + 2*cosh(x)^2 + 1)*e^(2*x) + 4*(cosh(x)^3 + (cosh(x)^3 +
 cosh(x))*e^(4*x) + 2*(cosh(x)^3 + cosh(x))*e^(2*x) + cosh(x))*sinh(x) + 1)
*dilog(-I*cosh(x) - I*sinh(x)) + ((e^(4*x) + 2*e^(2*x) + 1)*sinh(x)^4 + cos
h(x)^4 + 4*(cosh(x)*e^(4*x) + 2*cosh(x)*e^(2*x) + cosh(x))*sinh(x)^3 + 2*(3
*cosh(x)^2 + (3*cosh(x)^2 + 1)*e^(4*x) + 2*(3*cosh(x)^2 + 1)*e^(2*x) + 1)*s
inh(x)^2 + 2*cosh(x)^2 + (cosh(x)^4 + 2*cosh(x)^2 + 1)*e^(4*x) + 2*(cosh(x)
^4 + 2*cosh(x)^2 + 1)*e^(2*x) + 4*(cosh(x)^3 + (cosh(x)^3 + cosh(x))*e^(4*x
) + 2*(cosh(x)^3 + cosh(x))*e^(2*x) + cosh(x))*sinh(x) + 1)*dilog(-cosh(x)
- sinh(x)) + ((2*x + 1)*cosh(x)^2 + 1)*e^(4*x) + 2*((2*x + 1)*cosh(x)^2 + 1
)*e^(2*x) + (x*cosh(x)^4 + (x*e^(4*x) + 2*x*e^(2*x) + x)*sinh(x)^4 + 4*(x*c
osh(x)*e^(4*x) + 2*x*cosh(x)*e^(2*x) + x*cosh(x))*sinh(x)^3 + 2*x*cosh(x)^2
 + 2*(3*x*cosh(x)^2 + (3*x*cosh(x)^2 + x)*e^(4*x) + 2*(3*x*cosh(x)^2 + x)*e
^(2*x) + x)*sinh(x)^2 + (x*cosh(x)^4 + 2*x*cosh(x)^2 + x)*e^(4*x) + 2*(x*co
sh(x)^4 + 2*x*cosh(x)^2 + x)*e^(2*x) + 4*(x*cosh(x)^3 + x*cosh(x) + (x*cosh
(x)^3 + x*cosh(x))*e^(4*x) + 2*(x*cosh(x)^3 + x*cosh(x))*e^(2*x))*sinh(x) +
 x)*log(cosh(x) + sinh(x) + 1) - (x*cosh(x)^4 + (x*e^(4*x) + 2*x*e^(2*x) +
x)*sinh(x)^4 + 4*(x*cosh(x)*e^(4*x) + 2*x*cosh(x)*e^(2*x) + x*cosh(x))*sinh
(x)^3 + 2*x*cosh(x)^2 + 2*(3*x*cosh(x)^2 + (3*x*cosh(x)^2 + x)*e^(4*x) + 2*
(3*x*cosh(x)^2 + x)*e^(2*x) + x)*sinh(x)^2 + (x*cosh(x)^4 + 2*x*cosh(x)^2 +
 x)*e^(4*x) + 2*(x*cosh(x)^4 + 2*x*cosh(x)^2 + x)*e^(2*x) + 4*(x*cosh(x)^3
+ x*cosh(x) + (x*cosh(x)^3 + x*cosh(x))*e^(4*x) + 2*(x*cosh(x)^3 + x*cosh(x
))*e^(2*x))*sinh(x) + x)*log(I*cosh(x) + I*sinh(x) + 1) - (x*cosh(x)^4 + (x
*e^(4*x) + 2*x*e^(2*x) + x)*sinh(x)^4 + 4*(x*cosh(x)*e^(4*x) + 2*x*cosh(x)*
e^(2*x) + x*cosh(x))*sinh(x)^3 + 2*x*cosh(x)^2 + 2*(3*x*cosh(x)^2 + (3*x*co
sh(x)^2 + x)*e^(4*x) + 2*(3*x*cosh(x)^2 + x)*e^(2*x) + x)*sinh(x)^2 + (x*co
sh(x)^4 + 2*x*cosh(x)^2 + x)*e^(4*x) + 2*(x*cosh(x)^4 + 2*x*cosh(x)^2 + x)*
e^(2*x) + 4*(x*cosh(x)^3 + x*cosh(x) + (x*cosh(x)^3 + x*cosh(x))*e^(4*x) +
```

```
2*(x*cosh(x)^3 + x*cosh(x))*e^(2*x))*sinh(x) + x)*log(-I*cosh(x) - I*sinh(x
) + 1) + (x*cosh(x)^4 + (x*e^(4*x) + 2*x*e^(2*x) + x)*sinh(x)^4 + 4*(x*cosh
(x)*e^(4*x) + 2*x*cosh(x)*e^(2*x) + x*cosh(x))*sinh(x)^3 + 2*x*cosh(x)^2 +
2*(3*x*cosh(x)^2 + (3*x*cosh(x)^2 + x)*e^(4*x) + 2*(3*x*cosh(x)^2 + x)*e^(2
*x) + x)*sinh(x)^2 + (x*cosh(x)^4 + 2*x*cosh(x)^2 + x)*e^(4*x) + 2*(x*cosh(
x)^4 + 2*x*cosh(x)^2 + x)*e^(2*x) + 4*(x*cosh(x)^3 + x*cosh(x) + (x*cosh(x)
^3 + x*cosh(x))*e^(4*x) + 2*(x*cosh(x)^3 + x*cosh(x))*e^(2*x))*sinh(x) + x)
*log(-cosh(x) - sinh(x) + 1) + 2*((2*x + 1)*cosh(x)*e^(4*x) + 2*(2*x + 1)*c
osh(x)*e^(2*x) + (2*x + 1)*cosh(x))*sinh(x) + 1)*sqrt(a/(e^(8*x) + 4*e^(6*x
) + 6*e^(4*x) + 4*e^(2*x) + 1))*e^(2*x)/(4*cosh(x)*e^(2*x)*sinh(x)^3 + e^(2
*x)*sinh(x)^4 + 2*(3*cosh(x)^2 + 1)*e^(2*x)*sinh(x)^2 + 4*(cosh(x)^3 + cosh
(x))*e^(2*x)*sinh(x) + (cosh(x)^4 + 2*cosh(x)^2 + 1)*e^(2*x))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a\,\mathrm{sech}(x)^4}\,x\,\mathrm{csch}(x)\,\mathrm{sech}(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*csch(x)*sech(x)*(a*sech(x)^4)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a*sech(x)^4)*x*csch(x)*sech(x), x)
```

maple [B] time = 0.34, size = 252, normalized size = 1.91

$$\sqrt{\frac{a\,\mathrm{e}^{4x}}{\left(1+\mathrm{e}^{2x}\right)^4}}\;\mathrm{e}^{-2x}\left(2x\,\mathrm{e}^{2x}+\mathrm{e}^{2x}+1\right)+\sqrt{\frac{a\,\mathrm{e}^{4x}}{\left(1+\mathrm{e}^{2x}\right)^4}}\;\mathrm{e}^{-2x}\left(1+\mathrm{e}^{2x}\right)^2 x\ln\left(\mathrm{e}^{x}+1\right)+\sqrt{\frac{a\,\mathrm{e}^{4x}}{\left(1+\mathrm{e}^{2x}\right)^4}}\;\mathrm{e}^{-2x}\left(1+\mathrm{e}^{2x}\right)^2 \mathrm{po}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*csch(x)*sech(x)*(a*sech(x)^4)^(1/2),x)
```

```
[Out] (a*exp(4*x)/(1+exp(2*x))^4)^(1/2)*exp(-2*x)*(2*x*exp(2*x)+exp(2*x)+1)+(a*ex
p(4*x)/(1+exp(2*x))^4)^(1/2)*exp(-2*x)*(1+exp(2*x))^2*x*ln(exp(x)+1)+(a*exp
(4*x)/(1+exp(2*x))^4)^(1/2)*exp(-2*x)*(1+exp(2*x))^2*polylog(2,-exp(x))-(a*
exp(4*x)/(1+exp(2*x))^4)^(1/2)*exp(-2*x)*(1+exp(2*x))^2*x*ln(1+exp(2*x))-1/
2*(a*exp(4*x)/(1+exp(2*x))^4)^(1/2)*exp(-2*x)*(1+exp(2*x))^2*polylog(2,-exp
(2*x))+(a*exp(4*x)/(1+exp(2*x))^4)^(1/2)*exp(-2*x)*(1+exp(2*x))^2*x*ln(1-ex
p(x))+(a*exp(4*x)/(1+exp(2*x))^4)^(1/2)*exp(-2*x)*(1+exp(2*x))^2*polylog(2,
exp(x))
```

maxima [A] time = 0.46, size = 92, normalized size = 0.70

$$-\frac{1}{2}\left(2\,x\log\left(e^{(2\,x)}+1\right)+\mathrm{Li}_2\left(-e^{(2\,x)}\right)\right)\sqrt{a}+\left(x\log\left(e^{x}+1\right)+\mathrm{Li}_2\left(-e^{x}\right)\right)\sqrt{a}+\left(x\log\left(-e^{x}+1\right)+\mathrm{Li}_2\left(e^{x}\right)\right)\sqrt{a}+\frac{(2\,\sqrt{}}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*csch(x)*sech(x)*(a*sech(x)^4)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/2*(2*x*log(e^(2*x) + 1) + dilog(-e^(2*x)))*sqrt(a) + (x*log(e^x + 1) + d
ilog(-e^x))*sqrt(a) + (x*log(-e^x + 1) + dilog(e^x))*sqrt(a) + ((2*sqrt(a)*
x + sqrt(a))*e^(2*x) + sqrt(a))/(e^(4*x) + 2*e^(2*x) + 1)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x\sqrt{\frac{a}{\cosh(x)^4}}}{\cosh(x)\sinh(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a/cosh(x)^4)^(1/2))/(cosh(x)*sinh(x)),x)
```

```
[Out] int((x*(a/cosh(x)^4)^(1/2))/(cosh(x)*sinh(x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\sqrt{a\,\mathrm{sech}^4\,(x)}\,\,\mathrm{csch}\,(x)\,\mathrm{sech}\,(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*csch(x)*sech(x)*(a*sech(x)**4)**(1/2),x)
```

```
[Out] Integral(x*sqrt(a*sech(x)**4)*csch(x)*sech(x), x)
```

3.852 $\int x^2 \operatorname{csch}(x)\operatorname{sech}(x)\sqrt{a\operatorname{sech}^4(x)}\,dx$

Optimal. Leaf size=204

$$-x\text{Li}_2\left(-e^{2x}\right)\cosh^2(x)\sqrt{a\text{sech}^4(x)}+x\text{Li}_2\left(e^{2x}\right)\cosh^2(x)\sqrt{a\text{sech}^4(x)}+\frac{1}{2}\text{Li}_3\left(-e^{2x}\right)\cosh^2(x)\sqrt{a\text{sech}^4(x)}-\frac{1}{2}\text{Li}_3\left(\right.$$

```
[Out] 1/2*x^2*cosh(x)^2*(a*sech(x)^4)^(1/2)-2*x^2*arctanh(exp(2*x))*cosh(x)^2*(a*
sech(x)^4)^(1/2)+cosh(x)^2*ln(cosh(x))*(a*sech(x)^4)^(1/2)-x*cosh(x)^2*poly
log(2,-exp(2*x))*(a*sech(x)^4)^(1/2)+x*cosh(x)^2*polylog(2,exp(2*x))*(a*sec
h(x)^4)^(1/2)+1/2*cosh(x)^2*polylog(3,-exp(2*x))*(a*sech(x)^4)^(1/2)-1/2*co
sh(x)^2*polylog(3,exp(2*x))*(a*sech(x)^4)^(1/2)-x*cosh(x)*sinh(x)*(a*sech(x
)^4)^(1/2)-1/2*x^2*sinh(x)^2*(a*sech(x)^4)^(1/2)
```

Rubi [A] time = 0.64, antiderivative size = 204, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 13, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.722, Rules used = {6720, 2620, 14, 5462, 2551, 5461, 4182, 2531, 2282, 6589, 3720, 3475, 30}

$$-x\cosh^2(x)\text{PolyLog}\left(2,-e^{2x}\right)\sqrt{a\text{sech}^4(x)}+x\cosh^2(x)\text{PolyLog}\left(2,e^{2x}\right)\sqrt{a\text{sech}^4(x)}+\frac{1}{2}\cosh^2(x)\text{PolyLog}\left(3,-\right.$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Csch[x]*Sech[x]*Sqrt[a*Sech[x]^4],x]
```

```
[Out] (x^2*Cosh[x]^2*Sqrt[a*Sech[x]^4])/2 - 2*x^2*ArcTanh[E^(2*x)]*Cosh[x]^2*Sqrt
[a*Sech[x]^4] + Cosh[x]^2*Log[Cosh[x]]*Sqrt[a*Sech[x]^4] - x*Cosh[x]^2*Poly
Log[2, -E^(2*x)]*Sqrt[a*Sech[x]^4] + x*Cosh[x]^2*PolyLog[2, E^(2*x)]*Sqrt[a
*Sech[x]^4] + (Cosh[x]^2*PolyLog[3, -E^(2*x)]*Sqrt[a*Sech[x]^4])/2 - (Cosh[
x]^2*PolyLog[3, E^(2*x)]*Sqrt[a*Sech[x]^4])/2 - x*Cosh[x]*Sqrt[a*Sech[x]^4]
*Sinh[x] - (x^2*Sqrt[a*Sech[x]^4]*Sinh[x]^2)/2
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2551

```
Int[Log[u_]*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[((a + b*x)^(m + 1
)*Log[u])/(b*(m + 1)), x] - Dist[1/(b*(m + 1)), Int[SimplifyIntegrand[((a +
 b*x)^(m + 1)*D[u, x])/u, x], x], x] /; FreeQ[{a, b, m}, x] && InverseFunct
ionFreeQ[u, x] && NeQ[m, -1]
```

Rule 2620

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3720

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[(b*(c + d*x)^m*(b*Tan[e + f*x])^(n - 1))/(f*(n - 1)), x] + (-Di
st[(b*d*m)/(f*(n - 1)), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x],
 x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[
{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
```

```
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5461

```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[2^n, Int[(c + d*x)^m*Csch[2*a + 2*b*x]
^n, x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[m] && IntegerQ[n]
```

Rule 5462

```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> With[{u = IntHide[Csch[a + b*x]^n*Sech[a +
b*x]^p, x]}, Dist[(c + d*x)^m, u, x] - Dist[d*m, Int[(c + d*x)^(m - 1)*u, x
], x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n,
p]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6720

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a*v^m)^
FracPart[p])/v^(m*FracPart[p]), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rubi steps

$$\int x^2\text{csch}(x)\text{sech}(x)\sqrt{a\text{sech}^4(x)}\,dx = \left(\cosh^2(x)\sqrt{a\text{sech}^4(x)}\right)\int x^2\text{csch}(x)\text{sech}^3(x)\,dx$$
$$= x^2\cosh^2(x)\log(\tanh(x))\sqrt{a\text{sech}^4(x)} - \frac{1}{2}x^2\sqrt{a\text{sech}^4(x)}\sinh^2(x) - \Big(2\cos$$
$$= x^2\cosh^2(x)\log(\tanh(x))\sqrt{a\text{sech}^4(x)} - \frac{1}{2}x^2\sqrt{a\text{sech}^4(x)}\sinh^2(x) - \Big(2\cos$$
$$= x^2\cosh^2(x)\log(\tanh(x))\sqrt{a\text{sech}^4(x)} - \frac{1}{2}x^2\sqrt{a\text{sech}^4(x)}\sinh^2(x) + \Big(\cosh$$
$$= -x\cosh(x)\sqrt{a\text{sech}^4(x)}\sinh(x) - \frac{1}{2}x^2\sqrt{a\text{sech}^4(x)}\sinh^2(x) + \Big(\cosh^2(x)$$
$$= \frac{1}{2}x^2\cosh^2(x)\sqrt{a\text{sech}^4(x)} + \cosh^2(x)\log(\cosh(x))\sqrt{a\text{sech}^4(x)} - x\cosh($$
$$= \frac{1}{2}x^2\cosh^2(x)\sqrt{a\text{sech}^4(x)} - 2x^2\tanh^{-1}\left(e^{2x}\right)\cosh^2(x)\sqrt{a\text{sech}^4(x)} + \cosh$$
$$= \frac{1}{2}x^2\cosh^2(x)\sqrt{a\text{sech}^4(x)} - 2x^2\tanh^{-1}\left(e^{2x}\right)\cosh^2(x)\sqrt{a\text{sech}^4(x)} + \cosh$$
$$= \frac{1}{2}x^2\cosh^2(x)\sqrt{a\text{sech}^4(x)} - 2x^2\tanh^{-1}\left(e^{2x}\right)\cosh^2(x)\sqrt{a\text{sech}^4(x)} + \cosh$$
$$= \frac{1}{2}x^2\cosh^2(x)\sqrt{a\text{sech}^4(x)} - 2x^2\tanh^{-1}\left(e^{2x}\right)\cosh^2(x)\sqrt{a\text{sech}^4(x)} + \cosh$$

Mathematica [C] time = 0.65, size = 120, normalized size = 0.59

$$\frac{1}{24}\cosh^2(x)\sqrt{a\text{sech}^4(x)}\left(24x\text{Li}_2\left(-e^{-2x}\right) + 24x\text{Li}_2\left(e^{2x}\right) + 12\text{Li}_3\left(-e^{-2x}\right) - 12\text{Li}_3\left(e^{2x}\right) - 16x^3 - 24x^2\log\left(e^{-2x} + \right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Csch[x]*Sech[x]*Sqrt[a*Sech[x]^4],x]
```

```
[Out] (Cosh[x]^2*Sqrt[a*Sech[x]^4]*(I*Pi^3 - 16*x^3 - 24*x^2*Log[1 + E^(-2*x)] +
24*x^2*Log[1 - E^(2*x)] + 24*Log[Cosh[x]] + 24*x*PolyLog[2, -E^(-2*x)] + 24
*x*PolyLog[2, E^(2*x)] + 12*PolyLog[3, -E^(-2*x)] - 12*PolyLog[3, E^(2*x)]
+ 12*x^2*Sech[x]^2 - 24*x*Tanh[x]))/24
```

fricas [C] time = 0.84, size = 3431, normalized size = 16.82

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*csch(x)*sech(x)*(a*sech(x)^4)^(1/2),x, algorithm="fricas")

[Out] -(2*((e^(4*x) + 2*e^(2*x) + 1)*sinh(x)^4 + cosh(x)^4 + 4*(cosh(x)*e^(4*x) +
 2*cosh(x)*e^(2*x) + cosh(x))*sinh(x)^3 + 2*(3*cosh(x)^2 + (3*cosh(x)^2 + 1
)*e^(4*x) + 2*(3*cosh(x)^2 + 1)*e^(2*x) + 1)*sinh(x)^2 + 2*cosh(x)^2 + (cos
h(x)^4 + 2*cosh(x)^2 + 1)*e^(4*x) + 2*(cosh(x)^4 + 2*cosh(x)^2 + 1)*e^(2*x)
 + 4*(cosh(x)^3 + (cosh(x)^3 + cosh(x))*e^(4*x) + 2*(cosh(x)^3 + cosh(x))*e
^(2*x) + cosh(x))*sinh(x) + 1)*sqrt(a/(e^(8*x) + 4*e^(6*x) + 6*e^(4*x) + 4*
e^(2*x) + 1))*e^(2*x)*polylog(3, cosh(x) + sinh(x)) - 2*((e^(4*x) + 2*e^(2*
x) + 1)*sinh(x)^4 + cosh(x)^4 + 4*(cosh(x)*e^(4*x) + 2*cosh(x)*e^(2*x) + co
sh(x))*sinh(x)^3 + 2*(3*cosh(x)^2 + (3*cosh(x)^2 + 1)*e^(4*x) + 2*(3*cosh(x
)^2 + 1)*e^(2*x) + 1)*sinh(x)^2 + 2*cosh(x)^2 + (cosh(x)^4 + 2*cosh(x)^2 +
1)*e^(4*x) + 2*(cosh(x)^4 + 2*cosh(x)^2 + 1)*e^(2*x) + 4*(cosh(x)^3 + (cosh
(x)^3 + cosh(x))*e^(4*x) + 2*(cosh(x)^3 + cosh(x))*e^(2*x) + cosh(x))*sinh(
x) + 1)*sqrt(a/(e^(8*x) + 4*e^(6*x) + 6*e^(4*x) + 4*e^(2*x) + 1))*e^(2*x)*p
olylog(3, I*cosh(x) + I*sinh(x)) - 2*((e^(4*x) + 2*e^(2*x) + 1)*sinh(x)^4 +
 cosh(x)^4 + 4*(cosh(x)*e^(4*x) + 2*cosh(x)*e^(2*x) + cosh(x))*sinh(x)^3 +
2*(3*cosh(x)^2 + (3*cosh(x)^2 + 1)*e^(4*x) + 2*(3*cosh(x)^2 + 1)*e^(2*x) +
1)*sinh(x)^2 + 2*cosh(x)^2 + (cosh(x)^4 + 2*cosh(x)^2 + 1)*e^(4*x) + 2*(cos
h(x)^4 + 2*cosh(x)^2 + 1)*e^(2*x) + 4*(cosh(x)^3 + (cosh(x)^3 + cosh(x))*e^
(4*x) + 2*(cosh(x)^3 + cosh(x))*e^(2*x) + cosh(x))*sinh(x) + 1)*sqrt(a/(e^(
8*x) + 4*e^(6*x) + 6*e^(4*x) + 4*e^(2*x) + 1))*e^(2*x)*polylog(3, -I*cosh(x
) - I*sinh(x)) + 2*((e^(4*x) + 2*e^(2*x) + 1)*sinh(x)^4 + cosh(x)^4 + 4*(co
sh(x)*e^(4*x) + 2*cosh(x)*e^(2*x) + cosh(x))*sinh(x)^3 + 2*(3*cosh(x)^2 + (
3*cosh(x)^2 + 1)*e^(4*x) + 2*(3*cosh(x)^2 + 1)*e^(2*x) + 1)*sinh(x)^2 + 2*c
osh(x)^2 + (cosh(x)^4 + 2*cosh(x)^2 + 1)*e^(4*x) + 2*(cosh(x)^4 + 2*cosh(x)
^2 + 1)*e^(2*x) + 4*(cosh(x)^3 + (cosh(x)^3 + cosh(x))*e^(4*x) + 2*(cosh(x)
^3 + cosh(x))*e^(2*x) + cosh(x))*sinh(x) + 1)*sqrt(a/(e^(8*x) + 4*e^(6*x) +
 6*e^(4*x) + 4*e^(2*x) + 1))*e^(2*x)*polylog(3, -cosh(x) - sinh(x)) + (2*x*
cosh(x)^4 + 2*(x*e^(4*x) + 2*x*e^(2*x) + x)*sinh(x)^4 + 8*(x*cosh(x)*e^(4*x
) + 2*x*cosh(x)*e^(2*x) + x*cosh(x))*sinh(x)^3 - 2*(x^2 - x)*cosh(x)^2 + 2*
(6*x*cosh(x)^2 - x^2 + (6*x*cosh(x)^2 - x^2 + x)*e^(4*x) + 2*(6*x*cosh(x)^2
 - x^2 + x)*e^(2*x) + x)*sinh(x)^2 - 2*(x*cosh(x)^4 + (x*e^(4*x) + 2*x*e^(2
*x) + x)*sinh(x)^4 + 4*(x*cosh(x)*e^(4*x) + 2*x*cosh(x)*e^(2*x) + x*cosh(x)
)*sinh(x)^3 + 2*x*cosh(x)^2 + 2*(3*x*cosh(x)^2 + (3*x*cosh(x)^2 + x)*e^(4*x
) + 2*(3*x*cosh(x)^2 + x)*e^(2*x) + x)*sinh(x)^2 + (x*cosh(x)^4 + 2*x*cosh(
x)^2 + x)*e^(4*x) + 2*(x*cosh(x)^4 + 2*x*cosh(x)^2 + x)*e^(2*x) + 4*(x*cosh
(x)^3 + x*cosh(x) + (x*cosh(x)^3 + x*cosh(x))*e^(4*x) + 2*(x*cosh(x)^3 + x*
cosh(x))*e^(2*x))*sinh(x) + x)*dilog(cosh(x) + sinh(x)) + 2*(x*cosh(x)^4 +
(x*e^(4*x) + 2*x*e^(2*x) + x)*sinh(x)^4 + 4*(x*cosh(x)*e^(4*x) + 2*x*cosh(x
)*e^(2*x) + x*cosh(x))*sinh(x)^3 + 2*x*cosh(x)^2 + 2*(3*x*cosh(x)^2 + (3*x*
cosh(x)^2 + x)*e^(4*x) + 2*(3*x*cosh(x)^2 + x)*e^(2*x) + x)*sinh(x)^2 + (x*
cosh(x)^4 + 2*x*cosh(x)^2 + x)*e^(4*x) + 2*(x*cosh(x)^4 + 2*x*cosh(x)^2 + x
)*e^(2*x) + 4*(x*cosh(x)^3 + x*cosh(x) + (x*cosh(x)^3 + x*cosh(x))*e^(4*x)
+ 2*(x*cosh(x)^3 + x*cosh(x))*e^(2*x))*sinh(x) + x)*dilog(I*cosh(x) + I*sin
```

```
h(x)) + 2*(x*cosh(x)^4 + (x*e^(4*x) + 2*x*e^(2*x) + x)*sinh(x)^4 + 4*(x*cos
h(x)*e^(4*x) + 2*x*cosh(x)*e^(2*x) + x*cosh(x))*sinh(x)^3 + 2*x*cosh(x)^2 +
 2*(3*x*cosh(x)^2 + (3*x*cosh(x)^2 + x)*e^(4*x) + 2*(3*x*cosh(x)^2 + x)*e^(
2*x) + x)*sinh(x)^2 + (x*cosh(x)^4 + 2*x*cosh(x)^2 + x)*e^(4*x) + 2*(x*cosh
(x)^4 + 2*x*cosh(x)^2 + x)*e^(2*x) + 4*(x*cosh(x)^3 + x*cosh(x) + (x*cosh(x
)^3 + x*cosh(x))*e^(4*x) + 2*(x*cosh(x)^3 + x*cosh(x))*e^(2*x))*sinh(x) + x
)*dilog(-I*cosh(x) - I*sinh(x)) - 2*(x*cosh(x)^4 + (x*e^(4*x) + 2*x*e^(2*x)
 + x)*sinh(x)^4 + 4*(x*cosh(x)*e^(4*x) + 2*x*cosh(x)*e^(2*x) + x*cosh(x))*s
inh(x)^3 + 2*x*cosh(x)^2 + 2*(3*x*cosh(x)^2 + (3*x*cosh(x)^2 + x)*e^(4*x) +
 2*(3*x*cosh(x)^2 + x)*e^(2*x) + x)*sinh(x)^2 + (x*cosh(x)^4 + 2*x*cosh(x)^
2 + x)*e^(4*x) + 2*(x*cosh(x)^4 + 2*x*cosh(x)^2 + x)*e^(2*x) + 4*(x*cosh(x)
^3 + x*cosh(x) + (x*cosh(x)^3 + x*cosh(x))*e^(4*x) + 2*(x*cosh(x)^3 + x*cos
h(x))*e^(2*x))*sinh(x) + x)*dilog(-cosh(x) - sinh(x)) + 2*(x*cosh(x)^4 - (x
^2 - x)*cosh(x)^2)*e^(4*x) + 4*(x*cosh(x)^4 - (x^2 - x)*cosh(x)^2)*e^(2*x)
 - (x^2*cosh(x)^4 + (x^2*e^(4*x) + 2*x^2*e^(2*x) + x^2)*sinh(x)^4 + 2*x^2*co
sh(x)^2 + 4*(x^2*cosh(x)*e^(4*x) + 2*x^2*cosh(x)*e^(2*x) + x^2*cosh(x))*sin
h(x)^3 + 2*(3*x^2*cosh(x)^2 + x^2 + (3*x^2*cosh(x)^2 + x^2)*e^(4*x) + 2*(3*
x^2*cosh(x)^2 + x^2)*e^(2*x))*sinh(x)^2 + x^2 + (x^2*cosh(x)^4 + 2*x^2*cosh
(x)^2 + x^2)*e^(4*x) + 2*(x^2*cosh(x)^4 + 2*x^2*cosh(x)^2 + x^2)*e^(2*x) +
4*(x^2*cosh(x)^3 + x^2*cosh(x) + (x^2*cosh(x)^3 + x^2*cosh(x))*e^(4*x) + 2*
(x^2*cosh(x)^3 + x^2*cosh(x))*e^(2*x))*sinh(x))*log(cosh(x) + sinh(x) + 1)
 - ((e^(4*x) + 2*e^(2*x) + 1)*sinh(x)^4 + cosh(x)^4 + 4*(cosh(x)*e^(4*x) + 2
*cosh(x)*e^(2*x) + cosh(x))*sinh(x)^3 + 2*(3*cosh(x)^2 + (3*cosh(x)^2 + 1)*
e^(4*x) + 2*(3*cosh(x)^2 + 1)*e^(2*x) + 1)*sinh(x)^2 + 2*cosh(x)^2 + (cosh(
x)^4 + 2*cosh(x)^2 + 1)*e^(4*x) + 2*(cosh(x)^4 + 2*cosh(x)^2 + 1)*e^(2*x) +
 4*(cosh(x)^3 + (cosh(x)^3 + cosh(x))*e^(4*x) + 2*(cosh(x)^3 + cosh(x))*e^(
2*x) + cosh(x))*sinh(x) + 1)*log(cosh(x) + sinh(x) + I) - ((e^(4*x) + 2*e^(
2*x) + 1)*sinh(x)^4 + cosh(x)^4 + 4*(cosh(x)*e^(4*x) + 2*cosh(x)*e^(2*x) +
cosh(x))*sinh(x)^3 + 2*(3*cosh(x)^2 + (3*cosh(x)^2 + 1)*e^(4*x) + 2*(3*cosh
(x)^2 + 1)*e^(2*x) + 1)*sinh(x)^2 + 2*cosh(x)^2 + (cosh(x)^4 + 2*cosh(x)^2
+ 1)*e^(4*x) + 2*(cosh(x)^4 + 2*cosh(x)^2 + 1)*e^(2*x) + 4*(cosh(x)^3 + (co
sh(x)^3 + cosh(x))*e^(4*x) + 2*(cosh(x)^3 + cosh(x))*e^(2*x) + cosh(x))*sin
h(x) + 1)*log(cosh(x) + sinh(x) - I) + (x^2*cosh(x)^4 + (x^2*e^(4*x) + 2*x^
2*e^(2*x) + x^2)*sinh(x)^4 + 2*x^2*cosh(x)^2 + 4*(x^2*cosh(x)*e^(4*x) + 2*x
^2*cosh(x)*e^(2*x) + x^2*cosh(x))*sinh(x)^3 + 2*(3*x^2*cosh(x)^2 + x^2 + (3
*x^2*cosh(x)^2 + x^2)*e^(4*x) + 2*(3*x^2*cosh(x)^2 + x^2)*e^(2*x))*sinh(x)^
2 + x^2 + (x^2*cosh(x)^4 + 2*x^2*cosh(x)^2 + x^2)*e^(4*x) + 2*(x^2*cosh(x)^
4 + 2*x^2*cosh(x)^2 + x^2)*e^(2*x) + 4*(x^2*cosh(x)^3 + x^2*cosh(x) + (x^2*
cosh(x)^3 + x^2*cosh(x))*e^(4*x) + 2*(x^2*cosh(x)^3 + x^2*cosh(x))*e^(2*x))
*sinh(x))*log(I*cosh(x) + I*sinh(x) + 1) + (x^2*cosh(x)^4 + (x^2*e^(4*x) +
2*x^2*e^(2*x) + x^2)*sinh(x)^4 + 2*x^2*cosh(x)^2 + 4*(x^2*cosh(x)*e^(4*x) +
 2*x^2*cosh(x)*e^(2*x) + x^2*cosh(x))*sinh(x)^3 + 2*(3*x^2*cosh(x)^2 + x^2
+ (3*x^2*cosh(x)^2 + x^2)*e^(4*x) + 2*(3*x^2*cosh(x)^2 + x^2)*e^(2*x))*sinh
(x)^2 + x^2 + (x^2*cosh(x)^4 + 2*x^2*cosh(x)^2 + x^2)*e^(4*x) + 2*(x^2*cosh
(x)^4 + 2*x^2*cosh(x)^2 + x^2)*e^(2*x) + 4*(x^2*cosh(x)^3 + x^2*cosh(x) + (
```

```
x^2*cosh(x)^3 + x^2*cosh(x))*e^(4*x) + 2*(x^2*cosh(x)^3 + x^2*cosh(x))*e^(2
*x))*sinh(x))*log(-I*cosh(x) - I*sinh(x) + 1) - (x^2*cosh(x)^4 + (x^2*e^(4*
x) + 2*x^2*e^(2*x) + x^2)*sinh(x)^4 + 2*x^2*cosh(x)^2 + 4*(x^2*cosh(x)*e^(4
*x) + 2*x^2*cosh(x)*e^(2*x) + x^2*cosh(x))*sinh(x)^3 + 2*(3*x^2*cosh(x)^2 +
 x^2 + (3*x^2*cosh(x)^2 + x^2)*e^(4*x) + 2*(3*x^2*cosh(x)^2 + x^2)*e^(2*x))
*sinh(x)^2 + x^2 + (x^2*cosh(x)^4 + 2*x^2*cosh(x)^2 + x^2)*e^(4*x) + 2*(x^2
*cosh(x)^4 + 2*x^2*cosh(x)^2 + x^2)*e^(2*x) + 4*(x^2*cosh(x)^3 + x^2*cosh(x
) + (x^2*cosh(x)^3 + x^2*cosh(x))*e^(4*x) + 2*(x^2*cosh(x)^3 + x^2*cosh(x))
*e^(2*x))*sinh(x))*log(-cosh(x) - sinh(x) + 1) + 4*(2*x*cosh(x)^3 - (x^2 -
x)*cosh(x) + (2*x*cosh(x)^3 - (x^2 - x)*cosh(x))*e^(4*x) + 2*(2*x*cosh(x)^3
 - (x^2 - x)*cosh(x))*e^(2*x))*sinh(x))*sqrt(a/(e^(8*x) + 4*e^(6*x) + 6*e^(
4*x) + 4*e^(2*x) + 1))*e^(2*x))/(4*cosh(x)*e^(2*x)*sinh(x)^3 + e^(2*x)*sinh
(x)^4 + 2*(3*cosh(x)^2 + 1)*e^(2*x)*sinh(x)^2 + 4*(cosh(x)^3 + cosh(x))*e^(
2*x)*sinh(x) + (cosh(x)^4 + 2*cosh(x)^2 + 1)*e^(2*x))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a\,\mathrm{sech}(x)^4}\,x^2\,\mathrm{csch}(x)\,\mathrm{sech}(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*csch(x)*sech(x)*(a*sech(x)^4)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a*sech(x)^4)*x^2*csch(x)*sech(x), x)
```

maple [B] time = 0.36, size = 441, normalized size = 2.16

$$2\sqrt{\frac{a\,e^{4x}}{\left(1+e^{2x}\right)^4}}\;e^{-2x}x\left(x\,e^{2x}+e^{2x}+1\right)-2\sqrt{\frac{a\,e^{4x}}{\left(1+e^{2x}\right)^4}}\;e^{-2x}\left(1+e^{2x}\right)^2\ln\left(e^{x}\right)+\sqrt{\frac{a\,e^{4x}}{\left(1+e^{2x}\right)^4}}\;e^{-2x}\left(1+e^{2x}\right)^2\ln\left(1\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*csch(x)*sech(x)*(a*sech(x)^4)^(1/2),x)
```

```
[Out] 2*(a*exp(4*x)/(1+exp(2*x))^4)^(1/2)*exp(-2*x)*x*(x*exp(2*x)+exp(2*x)+1)-2*(
a*exp(4*x)/(1+exp(2*x))^4)^(1/2)*exp(-2*x)*(1+exp(2*x))^2*ln(exp(x))+(a*exp
(4*x)/(1+exp(2*x))^4)^(1/2)*exp(-2*x)*(1+exp(2*x))^2*ln(1+exp(2*x))+(a*exp(
4*x)/(1+exp(2*x))^4)^(1/2)*exp(-2*x)*(1+exp(2*x))^2*x^2*ln(exp(x)+1)+2*(a*e
xp(4*x)/(1+exp(2*x))^4)^(1/2)*exp(-2*x)*(1+exp(2*x))^2*x*polylog(2,-exp(x))
-2*(a*exp(4*x)/(1+exp(2*x))^4)^(1/2)*exp(-2*x)*(1+exp(2*x))^2*polylog(3,-ex
p(x))-(a*exp(4*x)/(1+exp(2*x))^4)^(1/2)*exp(-2*x)*(1+exp(2*x))^2*x^2*ln(1+e
xp(2*x))-(a*exp(4*x)/(1+exp(2*x))^4)^(1/2)*exp(-2*x)*(1+exp(2*x))^2*x*polyl
og(2,-exp(2*x))+1/2*(a*exp(4*x)/(1+exp(2*x))^4)^(1/2)*exp(-2*x)*(1+exp(2*x)
```

```
)^2*polylog(3,-exp(2*x))+(a*exp(4*x)/(1+exp(2*x))^4)^(1/2)*exp(-2*x)*(1+exp
(2*x))^2*x^2*ln(1-exp(x))+2*(a*exp(4*x)/(1+exp(2*x))^4)^(1/2)*exp(-2*x)*(1+
exp(2*x))^2*x*polylog(2,exp(x))-2*(a*exp(4*x)/(1+exp(2*x))^4)^(1/2)*exp(-2*
x)*(1+exp(2*x))^2*polylog(3,exp(x))
```

maxima [A] time = 0.45, size = 154, normalized size = 0.75

$$-\frac{1}{2}\left(2\,x^2\log\left(e^{(2\,x)}+1\right)+2\,x\mathrm{Li}_2\left(-e^{(2\,x)}\right)-\mathrm{Li}_3(-e^{(2\,x)})\right)\sqrt{a}+\left(x^2\log\left(e^x+1\right)+2\,x\mathrm{Li}_2\left(-e^x\right)-2\,\mathrm{Li}_3(-e^x)\right)\sqrt{a}+\left(x^2\,\mathrm{l}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*csch(x)*sech(x)*(a*sech(x)^4)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/2*(2*x^2*log(e^(2*x) + 1) + 2*x*dilog(-e^(2*x)) - polylog(3, -e^(2*x)))*
sqrt(a) + (x^2*log(e^x + 1) + 2*x*dilog(-e^x) - 2*polylog(3, -e^x))*sqrt(a)
 + (x^2*log(-e^x + 1) + 2*x*dilog(e^x) - 2*polylog(3, e^x))*sqrt(a) - 2*sqr
t(a)*x + sqrt(a)*log(e^(2*x) + 1) + 2*((sqrt(a)*x^2 + sqrt(a)*x)*e^(2*x) +
sqrt(a)*x)/(e^(4*x) + 2*e^(2*x) + 1)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2\,\sqrt{\frac{a}{{\cosh(x)}^4}}}{\cosh(x)\,\sinh(x)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(a/cosh(x)^4)^(1/2))/(cosh(x)*sinh(x)),x)
```

```
[Out] int((x^2*(a/cosh(x)^4)^(1/2))/(cosh(x)*sinh(x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^{2} \sqrt{a \operatorname{sech}^{4}{\left (x \right)}} \operatorname{csch}{\left (x \right)} \operatorname{sech}{\left (x \right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*csch(x)*sech(x)*(a*sech(x)**4)**(1/2),x)
```

```
[Out] Integral(x**2*sqrt(a*sech(x)**4)*csch(x)*sech(x), x)
```

3.853 $\int x^3\operatorname{csch}(x)\operatorname{sech}(x)\sqrt{a\operatorname{sech}^4(x)}\,dx$

Optimal. Leaf size=326

$$-\frac{3}{2}x^2\text{Li}_2\left(-e^{2x}\right)\cosh^2(x)\sqrt{a\text{sech}^4(x)}+\frac{3}{2}x^2\text{Li}_2\left(e^{2x}\right)\cosh^2(x)\sqrt{a\text{sech}^4(x)}+\frac{3}{2}x\text{Li}_3\left(-e^{2x}\right)\cosh^2(x)\sqrt{a\text{sech}^4(x)}-$$

```
[Out] -3/2*x^2*cosh(x)^2*(a*sech(x)^4)^(1/2)+1/2*x^3*cosh(x)^2*(a*sech(x)^4)^(1/2
)-2*x^3*arctanh(exp(2*x))*cosh(x)^2*(a*sech(x)^4)^(1/2)+3*x*cosh(x)^2*ln(ex
p(2*x)+1)*(a*sech(x)^4)^(1/2)+3/2*cosh(x)^2*polylog(2,-exp(2*x))*(a*sech(x)
^4)^(1/2)-3/2*x^2*cosh(x)^2*polylog(2,-exp(2*x))*(a*sech(x)^4)^(1/2)+3/2*x^
2*cosh(x)^2*polylog(2,exp(2*x))*(a*sech(x)^4)^(1/2)+3/2*x*cosh(x)^2*polylog
(3,-exp(2*x))*(a*sech(x)^4)^(1/2)-3/2*x*cosh(x)^2*polylog(3,exp(2*x))*(a*se
ch(x)^4)^(1/2)-3/4*cosh(x)^2*polylog(4,-exp(2*x))*(a*sech(x)^4)^(1/2)+3/4*c
osh(x)^2*polylog(4,exp(2*x))*(a*sech(x)^4)^(1/2)-3/2*x^2*cosh(x)*sinh(x)*(a
*sech(x)^4)^(1/2)-1/2*x^3*sinh(x)^2*(a*sech(x)^4)^(1/2)
```

Rubi [A] time = 0.68, antiderivative size = 326, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 17, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.944, Rules used = {6720, 2620, 14, 5462, 2551, 5461, 4182, 2531, 6609, 2282, 6589, 3720, 3718, 2190, 2279, 2391, 30}

$$-\frac{3}{2}x^2\cosh^2(x)\text{PolyLog}\left(2,-e^{2x}\right)\sqrt{a\text{sech}^4(x)}+\frac{3}{2}x^2\cosh^2(x)\text{PolyLog}\left(2,e^{2x}\right)\sqrt{a\text{sech}^4(x)}+\frac{3}{2}x\cosh^2(x)\text{PolyLo}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Csch[x]*Sech[x]*Sqrt[a*Sech[x]^4],x]
```

```
[Out] (-3*x^2*Cosh[x]^2*Sqrt[a*Sech[x]^4])/2 + (x^3*Cosh[x]^2*Sqrt[a*Sech[x]^4])/
2 - 2*x^3*ArcTanh[E^(2*x)]*Cosh[x]^2*Sqrt[a*Sech[x]^4] + 3*x*Cosh[x]^2*Log[
1 + E^(2*x)]*Sqrt[a*Sech[x]^4] + (3*Cosh[x]^2*PolyLog[2, -E^(2*x)]*Sqrt[a*S
ech[x]^4])/2 - (3*x^2*Cosh[x]^2*PolyLog[2, -E^(2*x)]*Sqrt[a*Sech[x]^4])/2 +
 (3*x^2*Cosh[x]^2*PolyLog[2, E^(2*x)]*Sqrt[a*Sech[x]^4])/2 + (3*x*Cosh[x]^2
*PolyLog[3, -E^(2*x)]*Sqrt[a*Sech[x]^4])/2 - (3*x*Cosh[x]^2*PolyLog[3, E^(2
*x)]*Sqrt[a*Sech[x]^4])/2 - (3*Cosh[x]^2*PolyLog[4, -E^(2*x)]*Sqrt[a*Sech[x
]^4])/4 + (3*Cosh[x]^2*PolyLog[4, E^(2*x)]*Sqrt[a*Sech[x]^4])/4 - (3*x^2*Co
sh[x]*Sqrt[a*Sech[x]^4]*Sinh[x])/2 - (x^3*Sqrt[a*Sech[x]^4]*Sinh[x]^2)/2
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2551

```
Int[Log[u_]*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[((a + b*x)^(m + 1
)*Log[u])/(b*(m + 1)), x] - Dist[1/(b*(m + 1)), Int[SimplifyIntegrand[((a +
 b*x)^(m + 1)*D[u, x])/u, x], x], x] /; FreeQ[{a, b, m}, x] && InverseFunct
```

```
ionFreeQ[u, x] && NeQ[m, -1]
```

Rule 2620

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 3718

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c
 + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /;
 FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 3720

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[(b*(c + d*x)^m*(b*Tan[e + f*x])^(n - 1))/(f*(n - 1)), x] + (-Di
st[(b*d*m)/(f*(n - 1)), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x],
 x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[
{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5461

```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[2^n, Int[(c + d*x)^m*Csch[2*a + 2*b*x]
^n, x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[m] && IntegerQ[n]
```

Rule 5462

```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> With[{u = IntHide[Csch[a + b*x]^n*Sech[a +
b*x]^p, x]}, Dist[(c + d*x)^m, u, x] - Dist[d*m, Int[(c + d*x)^(m - 1)*u, x
], x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n,
p]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 6720

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a*v^m)^
FracPart[p])/v^(m*FracPart[p]), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rubi steps

$$\int x^3\text{csch}(x)\text{sech}(x)\sqrt{a\text{sech}^4(x)}\,dx = \left(\cosh^2(x)\sqrt{a\text{sech}^4(x)}\right)\int x^3\text{csch}(x)\text{sech}^3(x)\,dx$$
$$= x^3\cosh^2(x)\log(\tanh(x))\sqrt{a\text{sech}^4(x)} - \frac{1}{2}x^3\sqrt{a\text{sech}^4(x)}\sinh^2(x) - \Big(3\,\text{c}$$
$$= x^3\cosh^2(x)\log(\tanh(x))\sqrt{a\text{sech}^4(x)} - \frac{1}{2}x^3\sqrt{a\text{sech}^4(x)}\sinh^2(x) - \Big(3\,\text{c}$$
$$= x^3\cosh^2(x)\log(\tanh(x))\sqrt{a\text{sech}^4(x)} - \frac{1}{2}x^3\sqrt{a\text{sech}^4(x)}\sinh^2(x) + \frac{1}{2}\Big($$
$$= -\frac{3}{2}x^2\cosh(x)\sqrt{a\text{sech}^4(x)}\sinh(x) - \frac{1}{2}x^3\sqrt{a\text{sech}^4(x)}\sinh^2(x) + \Big(\cosh^2$$
$$= -\frac{3}{2}x^2\cosh^2(x)\sqrt{a\text{sech}^4(x)} + \frac{1}{2}x^3\cosh^2(x)\sqrt{a\text{sech}^4(x)} - \frac{3}{2}x^2\cosh(x)$$
$$= -\frac{3}{2}x^2\cosh^2(x)\sqrt{a\text{sech}^4(x)} + \frac{1}{2}x^3\cosh^2(x)\sqrt{a\text{sech}^4(x)} - 2x^3\tanh^{-1}\big(e$$
$$= -\frac{3}{2}x^2\cosh^2(x)\sqrt{a\text{sech}^4(x)} + \frac{1}{2}x^3\cosh^2(x)\sqrt{a\text{sech}^4(x)} - 2x^3\tanh^{-1}\big(e$$
$$= -\frac{3}{2}x^2\cosh^2(x)\sqrt{a\text{sech}^4(x)} + \frac{1}{2}x^3\cosh^2(x)\sqrt{a\text{sech}^4(x)} - 2x^3\tanh^{-1}\big(e$$
$$= -\frac{3}{2}x^2\cosh^2(x)\sqrt{a\text{sech}^4(x)} + \frac{1}{2}x^3\cosh^2(x)\sqrt{a\text{sech}^4(x)} - 2x^3\tanh^{-1}\big(e$$
$$= -\frac{3}{2}x^2\cosh^2(x)\sqrt{a\text{sech}^4(x)} + \frac{1}{2}x^3\cosh^2(x)\sqrt{a\text{sech}^4(x)} - 2x^3\tanh^{-1}\big(e$$

Mathematica [A] time = 1.04, size = 157, normalized size = 0.48

$$\frac{1}{64}\cosh^2(x)\sqrt{a\text{sech}^4(x)}\left(96x^2\text{Li}_2\left(e^{2x}\right) + 96\left(x^2-1\right)\text{Li}_2\left(-e^{-2x}\right) + 96x\text{Li}_3\left(-e^{-2x}\right) - 96x\text{Li}_3\left(e^{2x}\right) + 48\text{Li}_4\left(-e^{-2}\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Csch[x]*Sech[x]*Sqrt[a*Sech[x]^4],x]
```

```
[Out] (Cosh[x]^2*Sqrt[a*Sech[x]^4]*(Pi^4 + 96*x^2 - 32*x^4 + 192*x*Log[1 + E^(-2*
x)] - 64*x^3*Log[1 + E^(-2*x)] + 64*x^3*Log[1 - E^(2*x)] + 96*(-1 + x^2)*Po
lyLog[2, -E^(-2*x)] + 96*x^2*PolyLog[2, E^(2*x)] + 96*x*PolyLog[3, -E^(-2*x
)] - 96*x*PolyLog[3, E^(2*x)] + 48*PolyLog[4, -E^(-2*x)] + 48*PolyLog[4, E^
(2*x)] + 32*x^3*Sech[x]^2 - 96*x^2*Tanh[x]))/64
```

fricas [C] time = 0.75, size = 4629, normalized size = 14.20

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*csch(x)*sech(x)*(a*sech(x)^4)^(1/2),x, algorithm="fricas")

[Out] (6*((e^(4*x) + 2*e^(2*x) + 1)*sinh(x)^4 + cosh(x)^4 + 4*(cosh(x)*e^(4*x) + 2*cosh(x)*e^(2*x) + cosh(x))*sinh(x)^3 + 2*(3*cosh(x)^2 + (3*cosh(x)^2 + 1)*e^(4*x) + 2*(3*cosh(x)^2 + 1)*e^(2*x) + 1)*sinh(x)^2 + 2*cosh(x)^2 + (cosh(x)^4 + 2*cosh(x)^2 + 1)*e^(4*x) + 2*(cosh(x)^4 + 2*cosh(x)^2 + 1)*e^(2*x) + 4*(cosh(x)^3 + (cosh(x)^3 + cosh(x))*e^(4*x) + 2*(cosh(x)^3 + cosh(x))*e^(2*x) + cosh(x))*sinh(x) + 1)*sqrt(a/(e^(8*x) + 4*e^(6*x) + 6*e^(4*x) + 4*e^(2*x) + 1))*e^(2*x)*polylog(4, cosh(x) + sinh(x)) - 6*((e^(4*x) + 2*e^(2*x) + 1)*sinh(x)^4 + cosh(x)^4 + 4*(cosh(x)*e^(4*x) + 2*cosh(x)*e^(2*x) + cosh(x))*sinh(x)^3 + 2*(3*cosh(x)^2 + (3*cosh(x)^2 + 1)*e^(4*x) + 2*(3*cosh(x)^2 + 1)*e^(2*x) + 1)*sinh(x)^2 + 2*cosh(x)^2 + (cosh(x)^4 + 2*cosh(x)^2 + 1)*e^(4*x) + 2*(cosh(x)^4 + 2*cosh(x)^2 + 1)*e^(2*x) + 4*(cosh(x)^3 + (cosh(x)^3 + cosh(x))*e^(4*x) + 2*(cosh(x)^3 + cosh(x))*e^(2*x) + cosh(x))*sinh(x) + 1)*sqrt(a/(e^(8*x) + 4*e^(6*x) + 6*e^(4*x) + 4*e^(2*x) + 1))*e^(2*x)*polylog(4, I*cosh(x) + I*sinh(x)) - 6*((e^(4*x) + 2*e^(2*x) + 1)*sinh(x)^4 + cosh(x)^4 + 4*(cosh(x)*e^(4*x) + 2*cosh(x)*e^(2*x) + cosh(x))*sinh(x)^3 + 2*(3*cosh(x)^2 + (3*cosh(x)^2 + 1)*e^(4*x) + 2*(3*cosh(x)^2 + 1)*e^(2*x) + 1)*sinh(x)^2 + 2*cosh(x)^2 + (cosh(x)^4 + 2*cosh(x)^2 + 1)*e^(4*x) + 2*(cosh(x)^4 + 2*cosh(x)^2 + 1)*e^(2*x) + 4*(cosh(x)^3 + (cosh(x)^3 + cosh(x))*e^(4*x) + 2*(cosh(x)^3 + cosh(x))*e^(2*x) + cosh(x))*sinh(x) + 1)*sqrt(a/(e^(8*x) + 4*e^(6*x) + 6*e^(4*x) + 4*e^(2*x) + 1))*e^(2*x)*polylog(4, -I*cosh(x) - I*sinh(x)) + 6*((e^(4*x) + 2*e^(2*x) + 1)*sinh(x)^4 + cosh(x)^4 + 4*(cosh(x)*e^(4*x) + 2*cosh(x)*e^(2*x) + cosh(x))*sinh(x)^3 + 2*(3*cosh(x)^2 + (3*cosh(x)^2 + 1)*e^(4*x) + 2*(3*cosh(x)^2 + 1)*e^(2*x) + 1)*sinh(x)^2 + 2*cosh(x)^2 + (cosh(x)^4 + 2*cosh(x)^2 + 1)*e^(4*x) + 2*(cosh(x)^4 + 2*cosh(x)^2 + 1)*e^(2*x) + 4*(cosh(x)^3 + (cosh(x)^3 + cosh(x))*e^(4*x) + 2*(cosh(x)^3 + cosh(x))*e^(2*x) + cosh(x))*sinh(x) + 1)*sqrt(a/(e^(8*x) + 4*e^(6*x) + 6*e^(4*x) + 4*e^(2*x) + 1))*e^(2*x)*polylog(4, -cosh(x) - sinh(x)) - 6*(x*cosh(x)^4 + (x*e^(4*x) + 2*x*e^(2*x) + x)*sinh(x)^4 + 4*(x*cosh(x)*e^(4*x) + 2*x*cosh(x)*e^(2*x) + x*cosh(x))*sinh(x)^3 + 2*x*cosh(x)^2 + 2*(3*x*cosh(x)^2 + (3*x*cosh(x)^2 + x)*e^(4*x) + 2*(3*x*cosh(x)^2 + x)*e^(2*x) + x)*sinh(x)^2 + (x*cosh(x)^4 + 2*x*cosh(x)^2 + x)*e^(4*x) + 2*(x*cosh(x)^4 + 2*x*cosh(x)^2 + x)*e^(2*x) + 4*(x*cosh(x)^3 + x*cosh(x) + (x*cosh(x)^3 + x*cosh(x))*e^(4*x) + 2*(x*cosh(x)^3 + x*cosh(x))*e^(2*x))*sinh(x) + x)*sqrt(a/(e^(8*x) + 4*e^(6*x) + 6*e^(4*x) + 4*e^(2*x) + 1))*e^(2*x)*polylog(3, cosh(x) + sinh(x)) + 6*(x*cosh(x)^4 + (x*e^(4*x) + 2*x*e^(2*x) + x)*sinh(x)^4 + 4*(x*cosh(x)*e^(4*x) + 2*x*cosh(x)*e^(2*x) + x*cosh(x))*sinh(x)^3 + 2*x*cosh(x)^2 + 2*(3*x*cosh(x)^2 + (3*x*cosh(x)^2 + x)*e^(4*x) + 2*(3*x*cosh(x)^2 + x)*e^(2*x) + x)*sinh(x)^2 + (x*cosh(x)^4 + 2*x*cosh(x)^2 + x)*e^(4*x) + 2*(x*cosh(x)^4 + 2*x*cosh(x)^2 + x)*e^(2*x) + 4*(x*cosh(x)^3 + x*cosh(x) + (x*cosh(x)^3 + x*cosh(x))*e^(4*x) + 2*(x*cosh(x)^3 + x*cosh(x))*e^(2*x))*sinh(x) + x)*sqrt(a/(e^(8*x) + 4*e^(6*x) + 6*e^(4*x) + 4*e^(2*x) + 1))*e^(2*x)*poly

```
log(3, I*cosh(x) + I*sinh(x)) + 6*(x*cosh(x)^4 + (x*e^(4*x) + 2*x*e^(2*x) +
 x)*sinh(x)^4 + 4*(x*cosh(x)*e^(4*x) + 2*x*cosh(x)*e^(2*x) + x*cosh(x))*sin
h(x)^3 + 2*x*cosh(x)^2 + 2*(3*x*cosh(x)^2 + (3*x*cosh(x)^2 + x)*e^(4*x) + 2
*(3*x*cosh(x)^2 + x)*e^(2*x) + x)*sinh(x)^2 + (x*cosh(x)^4 + 2*x*cosh(x)^2
+ x)*e^(4*x) + 2*(x*cosh(x)^4 + 2*x*cosh(x)^2 + x)*e^(2*x) + 4*(x*cosh(x)^3
 + x*cosh(x) + (x*cosh(x)^3 + x*cosh(x))*e^(4*x) + 2*(x*cosh(x)^3 + x*cosh(
x))*e^(2*x))*sinh(x) + x)*sqrt(a/(e^(8*x) + 4*e^(6*x) + 6*e^(4*x) + 4*e^(2*
x) + 1))*e^(2*x)*polylog(3, -I*cosh(x) - I*sinh(x)) - 6*(x*cosh(x)^4 + (x*e
^(4*x) + 2*x*e^(2*x) + x)*sinh(x)^4 + 4*(x*cosh(x)*e^(4*x) + 2*x*cosh(x)*e^
(2*x) + x*cosh(x))*sinh(x)^3 + 2*x*cosh(x)^2 + 2*(3*x*cosh(x)^2 + (3*x*cosh
(x)^2 + x)*e^(4*x) + 2*(3*x*cosh(x)^2 + x)*e^(2*x) + x)*sinh(x)^2 + (x*cosh
(x)^4 + 2*x*cosh(x)^2 + x)*e^(4*x) + 2*(x*cosh(x)^4 + 2*x*cosh(x)^2 + x)*e^
(2*x) + 4*(x*cosh(x)^3 + x*cosh(x) + (x*cosh(x)^3 + x*cosh(x))*e^(4*x) + 2*
(x*cosh(x)^3 + x*cosh(x))*e^(2*x))*sinh(x) + x)*sqrt(a/(e^(8*x) + 4*e^(6*x)
 + 6*e^(4*x) + 4*e^(2*x) + 1))*e^(2*x)*polylog(3, -cosh(x) - sinh(x)) - (3*
x^2*cosh(x)^4 + 3*(x^2*e^(4*x) + 2*x^2*e^(2*x) + x^2)*sinh(x)^4 + 12*(x^2*c
osh(x)*e^(4*x) + 2*x^2*cosh(x)*e^(2*x) + x^2*cosh(x))*sinh(x)^3 - (2*x^3 -
3*x^2)*cosh(x)^2 + (18*x^2*cosh(x)^2 - 2*x^3 + 3*x^2 + (18*x^2*cosh(x)^2 -
2*x^3 + 3*x^2)*e^(4*x) + 2*(18*x^2*cosh(x)^2 - 2*x^3 + 3*x^2)*e^(2*x))*sinh
(x)^2 - 3*(x^2*cosh(x)^4 + (x^2*e^(4*x) + 2*x^2*e^(2*x) + x^2)*sinh(x)^4 +
2*x^2*cosh(x)^2 + 4*(x^2*cosh(x)*e^(4*x) + 2*x^2*cosh(x)*e^(2*x) + x^2*cosh
(x))*sinh(x)^3 + 2*(3*x^2*cosh(x)^2 + x^2 + (3*x^2*cosh(x)^2 + x^2)*e^(4*x)
 + 2*(3*x^2*cosh(x)^2 + x^2)*e^(2*x))*sinh(x)^2 + x^2 + (x^2*cosh(x)^4 + 2*
x^2*cosh(x)^2 + x^2)*e^(4*x) + 2*(x^2*cosh(x)^4 + 2*x^2*cosh(x)^2 + x^2)*e^
(2*x) + 4*(x^2*cosh(x)^3 + x^2*cosh(x) + (x^2*cosh(x)^3 + x^2*cosh(x))*e^(4
*x) + 2*(x^2*cosh(x)^3 + x^2*cosh(x))*e^(2*x))*sinh(x))*dilog(cosh(x) + sin
h(x)) + 3*((x^2 - 1)*cosh(x)^4 + (x^2 + (x^2 - 1)*e^(4*x) + 2*(x^2 - 1)*e^(
2*x) - 1)*sinh(x)^4 + 4*((x^2 - 1)*cosh(x)*e^(4*x) + 2*(x^2 - 1)*cosh(x)*e^
(2*x) + (x^2 - 1)*cosh(x))*sinh(x)^3 + 2*(x^2 - 1)*cosh(x)^2 + 2*(3*(x^2 -
1)*cosh(x)^2 + x^2 + (3*(x^2 - 1)*cosh(x)^2 + x^2 - 1)*e^(4*x) + 2*(3*(x^2
- 1)*cosh(x)^2 + x^2 - 1)*e^(2*x) - 1)*sinh(x)^2 + x^2 + ((x^2 - 1)*cosh(x)
^4 + 2*(x^2 - 1)*cosh(x)^2 + x^2 - 1)*e^(4*x) + 2*((x^2 - 1)*cosh(x)^4 + 2*
(x^2 - 1)*cosh(x)^2 + x^2 - 1)*e^(2*x) + 4*((x^2 - 1)*cosh(x)^3 + (x^2 - 1)
*cosh(x) + ((x^2 - 1)*cosh(x)^3 + (x^2 - 1)*cosh(x))*e^(4*x) + 2*((x^2 - 1)
*cosh(x)^3 + (x^2 - 1)*cosh(x))*e^(2*x))*sinh(x) - 1)*dilog(I*cosh(x) + I*s
inh(x)) + 3*((x^2 - 1)*cosh(x)^4 + (x^2 + (x^2 - 1)*e^(4*x) + 2*(x^2 - 1)*e
^(2*x) - 1)*sinh(x)^4 + 4*((x^2 - 1)*cosh(x)*e^(4*x) + 2*(x^2 - 1)*cosh(x)*
e^(2*x) + (x^2 - 1)*cosh(x))*sinh(x)^3 + 2*(x^2 - 1)*cosh(x)^2 + 2*(3*(x^2
- 1)*cosh(x)^2 + x^2 + (3*(x^2 - 1)*cosh(x)^2 + x^2 - 1)*e^(4*x) + 2*(3*(x^
2 - 1)*cosh(x)^2 + x^2 - 1)*e^(2*x) - 1)*sinh(x)^2 + x^2 + ((x^2 - 1)*cosh(
x)^4 + 2*(x^2 - 1)*cosh(x)^2 + x^2 - 1)*e^(4*x) + 2*((x^2 - 1)*cosh(x)^4 +
2*(x^2 - 1)*cosh(x)^2 + x^2 - 1)*e^(2*x) + 4*((x^2 - 1)*cosh(x)^3 + (x^2 -
1)*cosh(x) + ((x^2 - 1)*cosh(x)^3 + (x^2 - 1)*cosh(x))*e^(4*x) + 2*((x^2 -
1)*cosh(x)^3 + (x^2 - 1)*cosh(x))*e^(2*x))*sinh(x) - 1)*dilog(-I*cosh(x) -
I*sinh(x)) - 3*(x^2*cosh(x)^4 + (x^2*e^(4*x) + 2*x^2*e^(2*x) + x^2)*sinh(x)
```

^4 + 2*x^2*cosh(x)^2 + 4*(x^2*cosh(x)*e^(4*x) + 2*x^2*cosh(x)*e^(2*x) + x^2
*cosh(x))*sinh(x)^3 + 2*(3*x^2*cosh(x)^2 + x^2 + (3*x^2*cosh(x)^2 + x^2)*e^
(4*x) + 2*(3*x^2*cosh(x)^2 + x^2)*e^(2*x))*sinh(x)^2 + x^2 + (x^2*cosh(x)^4
 + 2*x^2*cosh(x)^2 + x^2)*e^(4*x) + 2*(x^2*cosh(x)^4 + 2*x^2*cosh(x)^2 + x^
2)*e^(2*x) + 4*(x^2*cosh(x)^3 + x^2*cosh(x) + (x^2*cosh(x)^3 + x^2*cosh(x))
*e^(4*x) + 2*(x^2*cosh(x)^3 + x^2*cosh(x))*e^(2*x))*sinh(x))*dilog(-cosh(x)
 - sinh(x)) + (3*x^2*cosh(x)^4 - (2*x^3 - 3*x^2)*cosh(x)^2)*e^(4*x) + 2*(3*
x^2*cosh(x)^4 - (2*x^3 - 3*x^2)*cosh(x)^2)*e^(2*x) - (x^3*cosh(x)^4 + 2*x^3
*cosh(x)^2 + (x^3*e^(4*x) + 2*x^3*e^(2*x) + x^3)*sinh(x)^4 + 4*(x^3*cosh(x)
*e^(4*x) + 2*x^3*cosh(x)*e^(2*x) + x^3*cosh(x))*sinh(x)^3 + x^3 + 2*(3*x^3*
cosh(x)^2 + x^3 + (3*x^3*cosh(x)^2 + x^3)*e^(4*x) + 2*(3*x^3*cosh(x)^2 + x^
3)*e^(2*x))*sinh(x)^2 + (x^3*cosh(x)^4 + 2*x^3*cosh(x)^2 + x^3)*e^(4*x) + 2
*(x^3*cosh(x)^4 + 2*x^3*cosh(x)^2 + x^3)*e^(2*x) + 4*(x^3*cosh(x)^3 + x^3*c
osh(x) + (x^3*cosh(x)^3 + x^3*cosh(x))*e^(4*x) + 2*(x^3*cosh(x)^3 + x^3*cos
h(x))*e^(2*x))*sinh(x))*log(cosh(x) + sinh(x) + 1) + ((x^3 - 3*x)*cosh(x)^4
 + (x^3 + (x^3 - 3*x)*e^(4*x) + 2*(x^3 - 3*x)*e^(2*x) - 3*x)*sinh(x)^4 + 4*
((x^3 - 3*x)*cosh(x)*e^(4*x) + 2*(x^3 - 3*x)*cosh(x)*e^(2*x) + (x^3 - 3*x)*
cosh(x))*sinh(x)^3 + x^3 + 2*(x^3 - 3*x)*cosh(x)^2 + 2*(x^3 + 3*(x^3 - 3*x)
cosh(x)^2 + (x^3 + 3(x^3 - 3*x)*cosh(x)^2 - 3*x)*e^(4*x) + 2*(x^3 + 3*(x^
3 - 3*x)*cosh(x)^2 - 3*x)*e^(2*x) - 3*x)*sinh(x)^2 + ((x^3 - 3*x)*cosh(x)^4
 + x^3 + 2*(x^3 - 3*x)*cosh(x)^2 - 3*x)*e^(4*x) + 2*((x^3 - 3*x)*cosh(x)^4
+ x^3 + 2*(x^3 - 3*x)*cosh(x)^2 - 3*x)*e^(2*x) + 4*((x^3 - 3*x)*cosh(x)^3 +
 (x^3 - 3*x)*cosh(x) + ((x^3 - 3*x)*cosh(x)^3 + (x^3 - 3*x)*cosh(x))*e^(4*x
) + 2*((x^3 - 3*x)*cosh(x)^3 + (x^3 - 3*x)*cosh(x))*e^(2*x))*sinh(x) - 3*x)
*log(I*cosh(x) + I*sinh(x) + 1) + ((x^3 - 3*x)*cosh(x)^4 + (x^3 + (x^3 - 3*
x)*e^(4*x) + 2*(x^3 - 3*x)*e^(2*x) - 3*x)*sinh(x)^4 + 4*((x^3 - 3*x)*cosh(x
)*e^(4*x) + 2*(x^3 - 3*x)*cosh(x)*e^(2*x) + (x^3 - 3*x)*cosh(x))*sinh(x)^3
+ x^3 + 2*(x^3 - 3*x)*cosh(x)^2 + 2*(x^3 + 3*(x^3 - 3*x)*cosh(x)^2 + (x^3 +
 3*(x^3 - 3*x)*cosh(x)^2 - 3*x)*e^(4*x) + 2*(x^3 + 3*(x^3 - 3*x)*cosh(x)^2
- 3*x)*e^(2*x) - 3*x)*sinh(x)^2 + ((x^3 - 3*x)*cosh(x)^4 + x^3 + 2*(x^3 - 3
*x)*cosh(x)^2 - 3*x)*e^(4*x) + 2*((x^3 - 3*x)*cosh(x)^4 + x^3 + 2*(x^3 - 3*
x)*cosh(x)^2 - 3*x)*e^(2*x) + 4*((x^3 - 3*x)*cosh(x)^3 + (x^3 - 3*x)*cosh(x
) + ((x^3 - 3*x)*cosh(x)^3 + (x^3 - 3*x)*cosh(x))*e^(4*x) + 2*((x^3 - 3*x)*
cosh(x)^3 + (x^3 - 3*x)*cosh(x))*e^(2*x))*sinh(x) - 3*x)*log(-I*cosh(x) - I
*sinh(x) + 1) - (x^3*cosh(x)^4 + 2*x^3*cosh(x)^2 + (x^3*e^(4*x) + 2*x^3*e^(
2*x) + x^3)*sinh(x)^4 + 4*(x^3*cosh(x)*e^(4*x) + 2*x^3*cosh(x)*e^(2*x) + x^
3*cosh(x))*sinh(x)^3 + x^3 + 2*(3*x^3*cosh(x)^2 + x^3 + (3*x^3*cosh(x)^2 +
x^3)*e^(4*x) + 2*(3*x^3*cosh(x)^2 + x^3)*e^(2*x))*sinh(x)^2 + (x^3*cosh(x)^
4 + 2*x^3*cosh(x)^2 + x^3)*e^(4*x) + 2*(x^3*cosh(x)^4 + 2*x^3*cosh(x)^2 + x
^3)*e^(2*x) + 4*(x^3*cosh(x)^3 + x^3*cosh(x) + (x^3*cosh(x)^3 + x^3*cosh(x)
)*e^(4*x) + 2*(x^3*cosh(x)^3 + x^3*cosh(x))*e^(2*x))*sinh(x))*log(-cosh(x)
- sinh(x) + 1) + 2*(6*x^2*cosh(x)^3 - (2*x^3 - 3*x^2)*cosh(x) + (6*x^2*cosh
(x)^3 - (2*x^3 - 3*x^2)*cosh(x))*e^(4*x) + 2*(6*x^2*cosh(x)^3 - (2*x^3 - 3*
x^2)*cosh(x))*e^(2*x))*sinh(x))*sqrt(a/(e^(8*x) + 4*e^(6*x) + 6*e^(4*x) + 4
*e^(2*x) + 1))*e^(2*x))/(4*cosh(x)*e^(2*x)*sinh(x)^3 + e^(2*x)*sinh(x)^4 +

```
2*(3*cosh(x)^2 + 1)*e^(2*x)*sinh(x)^2 + 4*(cosh(x)^3 + cosh(x))*e^(2*x)*sin
h(x) + (cosh(x)^4 + 2*cosh(x)^2 + 1)*e^(2*x))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a\,\mathrm{sech}(x)^4}\,x^3\,\mathrm{csch}(x)\,\mathrm{sech}(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*csch(x)*sech(x)*(a*sech(x)^4)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a*sech(x)^4)*x^3*csch(x)*sech(x), x)
```

maple [B] time = 0.37, size = 602, normalized size = 1.85

$$\sqrt{\frac{a\,\mathrm{e}^{4x}}{\left(1+\mathrm{e}^{2x}\right)^4}}\;\mathrm{e}^{-2x}x^2\left(2x\,\mathrm{e}^{2x}+3\,\mathrm{e}^{2x}+3\right)-3\sqrt{\frac{a\,\mathrm{e}^{4x}}{\left(1+\mathrm{e}^{2x}\right)^4}}\;\mathrm{e}^{-2x}\left(1+\mathrm{e}^{2x}\right)^2x^2+3\sqrt{\frac{a\,\mathrm{e}^{4x}}{\left(1+\mathrm{e}^{2x}\right)^4}}\;\mathrm{e}^{-2x}\left(1+\mathrm{e}^{2x}\right)^2x\ln\left(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*csch(x)*sech(x)*(a*sech(x)^4)^(1/2),x)
```

```
[Out] (a*exp(4*x)/(1+exp(2*x))^4)^(1/2)*exp(-2*x)*x^2*(2*x*exp(2*x)+3*exp(2*x)+3)
-3*(a*exp(4*x)/(1+exp(2*x))^4)^(1/2)*exp(-2*x)*(1+exp(2*x))^2*x^2+3*(a*exp(
4*x)/(1+exp(2*x))^4)^(1/2)*exp(-2*x)*(1+exp(2*x))^2*x*ln(1+exp(2*x))+3/2*(a
*exp(4*x)/(1+exp(2*x))^4)^(1/2)*exp(-2*x)*(1+exp(2*x))^2*polylog(2,-exp(2*x
))+(a*exp(4*x)/(1+exp(2*x))^4)^(1/2)*exp(-2*x)*(1+exp(2*x))^2*x^3*ln(exp(x)
+1)+3*(a*exp(4*x)/(1+exp(2*x))^4)^(1/2)*exp(-2*x)*(1+exp(2*x))^2*x^2*polylo
g(2,-exp(x))-6*(a*exp(4*x)/(1+exp(2*x))^4)^(1/2)*exp(-2*x)*(1+exp(2*x))^2*x
*polylog(3,-exp(x))+6*(a*exp(4*x)/(1+exp(2*x))^4)^(1/2)*exp(-2*x)*(1+exp(2*
x))^2*polylog(4,-exp(x))-(a*exp(4*x)/(1+exp(2*x))^4)^(1/2)*exp(-2*x)*(1+exp
(2*x))^2*x^3*ln(1+exp(2*x))-3/2*(a*exp(4*x)/(1+exp(2*x))^4)^(1/2)*exp(-2*x)
*(1+exp(2*x))^2*x^2*polylog(2,-exp(2*x))+3/2*(a*exp(4*x)/(1+exp(2*x))^4)^(1
/2)*exp(-2*x)*(1+exp(2*x))^2*x*polylog(3,-exp(2*x))-3/4*(a*exp(4*x)/(1+exp(
2*x))^4)^(1/2)*exp(-2*x)*(1+exp(2*x))^2*polylog(4,-exp(2*x))+(a*exp(4*x)/(1
+exp(2*x))^4)^(1/2)*exp(-2*x)*(1+exp(2*x))^2*x^3*ln(1-exp(x))+3*(a*exp(4*x)
/(1+exp(2*x))^4)^(1/2)*exp(-2*x)*(1+exp(2*x))^2*x^2*polylog(2,exp(x))-6*(a*
exp(4*x)/(1+exp(2*x))^4)^(1/2)*exp(-2*x)*(1+exp(2*x))^2*x*polylog(3,exp(x))
+6*(a*exp(4*x)/(1+exp(2*x))^4)^(1/2)*exp(-2*x)*(1+exp(2*x))^2*polylog(4,exp
(x))
```

maxima [A] time = 0.44, size = 207, normalized size = 0.63

$$-3\sqrt{a}\,x^2-\frac{1}{3}\left(4\,x^3\log\left(e^{(2\,x)}+1\right)+6\,x^2\mathrm{Li}_2\left(-e^{(2\,x)}\right)-6\,x\mathrm{Li}_3(-e^{(2\,x)})+3\,\mathrm{Li}_4(-e^{(2\,x)})\right)\sqrt{a}+\left(x^3\log\left(e^x+1\right)+3\,x^2\mathrm{L}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*csch(x)*sech(x)*(a*sech(x)^4)^(1/2),x, algorithm="maxima")
```

```
[Out] -3*sqrt(a)*x^2 - 1/3*(4*x^3*log(e^(2*x) + 1) + 6*x^2*dilog(-e^(2*x)) - 6*x*
polylog(3, -e^(2*x)) + 3*polylog(4, -e^(2*x)))*sqrt(a) + (x^3*log(e^x + 1)
+ 3*x^2*dilog(-e^x) - 6*x*polylog(3, -e^x) + 6*polylog(4, -e^x))*sqrt(a) +
(x^3*log(-e^x + 1) + 3*x^2*dilog(e^x) - 6*x*polylog(3, e^x) + 6*polylog(4,
e^x))*sqrt(a) + 3/2*(2*x*log(e^(2*x) + 1) + dilog(-e^(2*x)))*sqrt(a) + (3*s
qrt(a)*x^2 + (2*sqrt(a)*x^3 + 3*sqrt(a)*x^2)*e^(2*x))/(e^(4*x) + 2*e^(2*x)
+ 1)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3 \sqrt{\frac{a}{\cosh(x)^4}}}{\cosh(x)\sinh(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(a/cosh(x)^4)^(1/2))/(cosh(x)*sinh(x)),x)
```

```
[Out] int((x^3*(a/cosh(x)^4)^(1/2))/(cosh(x)*sinh(x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3 \sqrt{a \operatorname{sech}^4(x)}\, \operatorname{csch}(x) \operatorname{sech}(x)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*csch(x)*sech(x)*(a*sech(x)**4)**(1/2),x)
```

```
[Out] Integral(x**3*sqrt(a*sech(x)**4)*csch(x)*sech(x), x)
```

3.854 $\int (a+b\cosh(c+dx)\sinh(c+dx))^m\,dx$

Optimal. Leaf size=147

$$\frac{i\cosh(2c+2dx)\left(a+\frac{1}{2}b\sinh(2c+2dx)\right)^m\left(\frac{2a+b\sinh(2c+2dx)}{2a-ib}\right)^{-m}F_1\left(\frac{1}{2};\frac{1}{2},-m;\frac{3}{2};\frac{1}{2}(1-i\sinh(2c+2dx)),\frac{b(1-i\sinh(2c+2dx))}{2ia+b}\right)}{\sqrt{2}\,d\sqrt{1+i\sinh(2c+2dx)}}$$

```
[Out] 1/2*I*AppellF1(1/2,-m,1/2,3/2,b*(1-I*sinh(2*d*x+2*c))/(2*I*a+b),1/2-1/2*I*s
inh(2*d*x+2*c))*cosh(2*d*x+2*c)*(a+1/2*b*sinh(2*d*x+2*c))^m/d/(((2*a+b*sinh
(2*d*x+2*c))/(2*a-I*b))^m)*2^(1/2)/(1+I*sinh(2*d*x+2*c))^(1/2)
```

Rubi [A] time = 0.13, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2666, 2665, 139, 138}

$$\frac{i\cosh(2c+2dx)\left(a+\frac{1}{2}b\sinh(2c+2dx)\right)^m\left(\frac{2a+b\sinh(2c+2dx)}{2a-ib}\right)^{-m}F_1\left(\frac{1}{2};\frac{1}{2},-m;\frac{3}{2};\frac{1}{2}(1-i\sinh(2c+2dx)),\frac{b(1-i\sinh(2c+2dx))}{2ia+b}\right)}{\sqrt{2}\,d\sqrt{1+i\sinh(2c+2dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cosh[c + d*x]*Sinh[c + d*x])^m,x]
```

```
[Out] (I*AppellF1[1/2, 1/2, -m, 3/2, (1 - I*Sinh[2*c + 2*d*x])/2, (b*(1 - I*Sinh[
2*c + 2*d*x]))/((2*I)*a + b)]*Cosh[2*c + 2*d*x]*(a + (b*Sinh[2*c + 2*d*x])/
2)^m)/(Sqrt[2]*d*Sqrt[1 + I*Sinh[2*c + 2*d*x]]*((2*a + b*Sinh[2*c + 2*d*x])
/(2*a - I*b))^m)
```

Rule 138

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2,
-((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(b*(m + 1)*(b/
(b*c - a*d))^n*(b/(b*e - a*f))^p), x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] && !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) && !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 139

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
((b*(e + f*x))/(b*e - a*f))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*((b*e
)/(b*e - a*f) + (b*f*x)/(b*e - a*f))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
```

```
m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] &&  !GtQ[b/(b*e - a*f), 0]
```

Rule 2665

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[Cos[c +
d*x]/(d*Sqrt[1 + Sin[c + d*x]]*Sqrt[1 - Sin[c + d*x]]), Subst[Int[(a + b*x)
^n/(Sqrt[1 + x]*Sqrt[1 - x]), x], x, Sin[c + d*x]], x] /; FreeQ[{a, b, c, d
, n}, x] && NeQ[a^2 - b^2, 0] &&  !IntegerQ[2*n]
```

Rule 2666

```
Int[((a_) + cos[(c_.) + (d_.)*(x_)]*(b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_
Symbol] :> Int[(a + (b*Sin[2*c + 2*d*x])/2)^n, x] /; FreeQ[{a, b, c, d, n},
 x]
```

Rubi steps

$$\int (a+b\cosh(c+dx)\sinh(c+dx))^m\,dx = \int \left(a+\frac{1}{2}b\sinh(2c+2dx)\right)^m dx$$

$$= -\frac{(i\cosh(2c+2dx))\,\text{Subst}\left(\int \frac{\left(a-\frac{ibx}{2}\right)^m}{\sqrt{1-x}\,\sqrt{1+x}}\,dx, x, i\sinh(2c+2dx)\right)}{2d\sqrt{1-i\sinh(2c+2dx)}\,\sqrt{1+i\sinh(2c+2dx)}}$$

$$= -\frac{\left(i\cosh(2c+2dx)\left(a+\frac{1}{2}b\sinh(2c+2dx)\right)^m\left(-\frac{a+\frac{1}{2}b\sinh(2c+2dx)}{-a+\frac{ib}{2}}\right)^{-m}\right)\text{S}}{2d\sqrt{1-i\sinh(2c+2dx)}\,\sqrt{1+i\,\text{s}}}$$

$$= \frac{iF_1\left(\frac{1}{2};\frac{1}{2},-m;\frac{3}{2};\frac{1}{2}(1-i\sinh(2c+2dx)),\frac{b(1-i\sinh(2c+2dx))}{2ia+b}\right)\cosh(2c+2}{\sqrt{2}\,d\sqrt{1+i\sinh(2c+2}}$$

Mathematica [A] time = 0.69, size = 162, normalized size = 1.10

$$\frac{\text{sech}(2(c+dx))\sqrt{\frac{b(1-i\sinh(2(c+dx)))}{b+2ia}}\sqrt{\frac{b(1+i\sinh(2(c+dx)))}{b-2ia}}\left(a+\frac{1}{2}b\sinh(2(c+dx))\right)^{m+1}F_1\left(m+1;\frac{1}{2},\frac{1}{2};m+2;\frac{2a+b\sinh(}{2a+}\right.}{bd(m+1)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Cosh[c + d*x]*Sinh[c + d*x])^m,x]
```

```
[Out] (AppellF1[1 + m, 1/2, 1/2, 2 + m, (2*a + b*Sinh[2*(c + d*x)])/(2*a + I*b),
(2*a + b*Sinh[2*(c + d*x)])/(2*a - I*b)]*Sech[2*(c + d*x)]*Sqrt[(b*(1 - I*S
inh[2*(c + d*x)]))/((2*I)*a + b)]*Sqrt[(b*(1 + I*Sinh[2*(c + d*x)]))/((-2*I
)*a + b)]*(a + (b*Sinh[2*(c + d*x)])/2)^(1 + m))/(b*d*(1 + m))
```

fricas [F] time = 0.55, size = 0, normalized size = 0.00

$$\text{integral}\left((b\cosh(dx+c)\sinh(dx+c)+a)^m,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cosh(d*x+c)*sinh(d*x+c))^m,x, algorithm="fricas")
```

```
[Out] integral((b*cosh(d*x + c)*sinh(d*x + c) + a)^m, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\cosh(dx+c)\sinh(dx+c)+a)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cosh(d*x+c)*sinh(d*x+c))^m,x, algorithm="giac")
```

```
[Out] integrate((b*cosh(d*x + c)*sinh(d*x + c) + a)^m, x)
```

maple [F] time = 1.02, size = 0, normalized size = 0.00

$$\int (a+b\cosh(dx+c)\sinh(dx+c))^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cosh(d*x+c)*sinh(d*x+c))^m,x)
```

```
[Out] int((a+b*cosh(d*x+c)*sinh(d*x+c))^m,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\cosh(dx+c)\sinh(dx+c)+a)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cosh(d*x+c)*sinh(d*x+c))^m,x, algorithm="maxima")
```

```
[Out] integrate((b*cosh(d*x + c)*sinh(d*x + c) + a)^m, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (a + b \cosh (c + d x) \sinh (c + d x))^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*cosh(c + d*x)*sinh(c + d*x))^m,x)
[Out] int((a + b*cosh(c + d*x)*sinh(c + d*x))^m, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cosh(d*x+c)*sinh(d*x+c))**m,x)
[Out] Timed out
```

3.855 $\int (a+b\cosh(c+dx)\sinh(c+dx))^3\,dx$

Optimal. Leaf size=109

$$\frac{b(16a^2-b^2)\cosh(2c+2dx)}{24d}+\frac{1}{8}ax(8a^2-3b^2)+\frac{5ab^2\sinh(2c+2dx)\cosh(2c+2dx)}{48d}+\frac{b\cosh(2c+2dx)(2a+b}{48d}$$

```
[Out] 1/8*a*(8*a^2-3*b^2)*x+1/24*b*(16*a^2-b^2)*cosh(2*d*x+2*c)/d+5/48*a*b^2*cosh
(2*d*x+2*c)*sinh(2*d*x+2*c)/d+1/48*b*cosh(2*d*x+2*c)*(2*a+b*sinh(2*d*x+2*c)
)^2/d
```

Rubi [A] time = 0.10, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2666, 2656, 2734}

$$\frac{b(16a^2-b^2)\cosh(2c+2dx)}{24d}+\frac{1}{8}ax(8a^2-3b^2)+\frac{5ab^2\sinh(2c+2dx)\cosh(2c+2dx)}{48d}+\frac{b\cosh(2c+2dx)(2a+b}{48d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cosh[c + d*x]*Sinh[c + d*x])^3,x]
```

```
[Out] (a*(8*a^2 - 3*b^2)*x)/8 + (b*(16*a^2 - b^2)*Cosh[2*c + 2*d*x])/(24*d) + (5*
a*b^2*Cosh[2*c + 2*d*x]*Sinh[2*c + 2*d*x])/(48*d) + (b*Cosh[2*c + 2*d*x]*(2
*a + b*Sinh[2*c + 2*d*x])^2)/(48*d)
```

Rule 2656

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[1/n, Int[(a + b*Sin
[c + d*x])^(n - 2)*Simp[a^2*n + b^2*(n - 1) + a*b*(2*n - 1)*Sin[c + d*x], x
], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[n, 1] && In
tegerQ[2*n]
```

Rule 2666

```
Int[((a_) + cos[(c_.) + (d_.)*(x_)]*(b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_
Symbol] :> Int[(a + (b*Sin[2*c + 2*d*x])/2)^n, x] /; FreeQ[{a, b, c, d, n},
 x]
```

Rule 2734

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[((2*a*c + b*d)*x)/2, x] + (-Simp[((b*c + a*d)*Co
s[e + f*x])/f, x] - Simp[(b*d*Cos[e + f*x]*Sin[e + f*x])/(2*f), x]) /; Free
```

```
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\int (a+b\cosh(c+dx)\sinh(c+dx))^3\,dx = \int \left(a+\frac{1}{2}b\sinh(2c+2dx)\right)^3\,dx$$
$$= \frac{b\cosh(2c+2dx)(2a+b\sinh(2c+2dx))^2}{48d} + \frac{1}{3}\int \left(a+\frac{1}{2}b\sinh(2c+2\right.$$
$$= \frac{1}{8}a\left(8a^2-3b^2\right)x + \frac{b\left(16a^2-b^2\right)\cosh(2c+2dx)}{24d} + \frac{5ab^2\cosh(2c+2d}{48}$$

Mathematica [A] time = 0.27, size = 77, normalized size = 0.71

$$\frac{9\left(16a^2b-b^3\right)\cosh(2(c+dx)) + 6a\left(4\left(8a^2-3b^2\right)(c+dx)+3b^2\sinh(4(c+dx))\right) + b^3\cosh(6(c+dx))}{192d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cosh[c + d*x]*Sinh[c + d*x])^3,x]
```

```
[Out] (9*(16*a^2*b - b^3)*Cosh[2*(c + d*x)] + b^3*Cosh[6*(c + d*x)] + 6*a*(4*(8*a
^2 - 3*b^2)*(c + d*x) + 3*b^2*Sinh[4*(c + d*x)]))/(192*d)
```

fricas [A] time = 0.68, size = 164, normalized size = 1.50

$$\frac{b^3\cosh(dx+c)^6 + 15\,b^3\cosh(dx+c)^2\sinh(dx+c)^4 + b^3\sinh(dx+c)^6 + 72\,ab^2\cosh(dx+c)^3\sinh(dx+c) +}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cosh(d*x+c)*sinh(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/192*(b^3*cosh(d*x + c)^6 + 15*b^3*cosh(d*x + c)^2*sinh(d*x + c)^4 + b^3*s
inh(d*x + c)^6 + 72*a*b^2*cosh(d*x + c)^3*sinh(d*x + c) + 72*a*b^2*cosh(d*x
 + c)*sinh(d*x + c)^3 + 24*(8*a^3 - 3*a*b^2)*d*x + 9*(16*a^2*b - b^3)*cosh(
d*x + c)^2 + 3*(5*b^3*cosh(d*x + c)^4 + 48*a^2*b - 3*b^3)*sinh(d*x + c)^2)/
d
```

giac [A] time = 0.12, size = 138, normalized size = 1.27

$$\frac{b^3e^{(6\,dx+6\,c)}}{384\,d} + \frac{3\,ab^2e^{(4\,dx+4\,c)}}{64\,d} - \frac{3\,ab^2e^{(-4\,dx-4\,c)}}{64\,d} + \frac{b^3e^{(-6\,dx-6\,c)}}{384\,d} + \frac{1}{8}\left(8\,a^3-3\,ab^2\right)x + \frac{3\left(16\,a^2b-b^3\right)e^{(2\,dx+2\,c)}}{128\,d} + \frac{3\left(16\,a^2b-\right.}{1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cosh(d*x+c)*sinh(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/384*b^3*e^(6*d*x + 6*c)/d + 3/64*a*b^2*e^(4*d*x + 4*c)/d - 3/64*a*b^2*e^(
-4*d*x - 4*c)/d + 1/384*b^3*e^(-6*d*x - 6*c)/d + 1/8*(8*a^3 - 3*a*b^2)*x +
3/128*(16*a^2*b - b^3)*e^(2*d*x + 2*c)/d + 3/128*(16*a^2*b - b^3)*e^(-2*d*x
 - 2*c)/d
```

maple [A] time = 0.44, size = 106, normalized size = 0.97

$$\frac{b^3\left(\frac{\left(\sinh^2(dx+c)\right)\left(\cosh^4(dx+c)\right)}{6}-\frac{\left(\cosh^4(dx+c)\right)}{12}\right)+3a\,b^2\left(\frac{\sinh(dx+c)\left(\cosh^3(dx+c)\right)}{4}-\frac{\cosh(dx+c)\sinh(dx+c)}{8}-\frac{dx}{8}-\frac{c}{8}\right)+\frac{3a^2b\left(\cosh\right.}{2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cosh(d*x+c)*sinh(d*x+c))^3,x)
```

```
[Out] 1/d*(b^3*(1/6*sinh(d*x+c)^2*cosh(d*x+c)^4-1/12*cosh(d*x+c)^4)+3*a*b^2*(1/4*
sinh(d*x+c)*cosh(d*x+c)^3-1/8*cosh(d*x+c)*sinh(d*x+c)-1/8*d*x-1/8*c)+3/2*a^
2*b*cosh(d*x+c)^2+a^3*(d*x+c))
```

maxima [A] time = 0.33, size = 126, normalized size = 1.16

$$a^3x-\frac{1}{384}\,b^3\left(\frac{\left(9\,e^{(-4\,dx-4\,c)}-1\right)e^{(6\,dx+6\,c)}}{d}+\frac{9\,e^{(-2\,dx-2\,c)}-e^{(-6\,dx-6\,c)}}{d}\right)-\frac{3}{64}\,ab^2\left(\frac{8\,(dx+c)}{d}-\frac{e^{(4\,dx+4\,c)}}{d}+\frac{e^{(-4\,dx-4\,c)}}{d}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cosh(d*x+c)*sinh(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] a^3*x - 1/384*b^3*((9*e^(-4*d*x - 4*c) - 1)*e^(6*d*x + 6*c)/d + (9*e^(-2*d*
x - 2*c) - e^(-6*d*x - 6*c))/d) - 3/64*a*b^2*(8*(d*x + c)/d - e^(4*d*x + 4*
c)/d + e^(-4*d*x - 4*c)/d) + 3/2*a^2*b*cosh(d*x + c)^2/d
```

mupad [B] time = 1.87, size = 79, normalized size = 0.72

$$\frac{\frac{b^3\cosh(6\,c+6\,d\,x)}{8}-\frac{9\,b^3\cosh(2\,c+2\,d\,x)}{8}+18\,a^2\,b\cosh\left(2\,c+2\,d\,x\right)+\frac{9\,a\,b^2\sinh(4\,c+4\,d\,x)}{4}+24\,a^3\,d\,x-9\,a\,b^2\,d\,x}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*cosh(c + d*x)*sinh(c + d*x))^3,x)
```

```
[Out] ((b^3*cosh(6*c + 6*d*x))/8 - (9*b^3*cosh(2*c + 2*d*x))/8 + 18*a^2*b*cosh(2*
c + 2*d*x) + (9*a*b^2*sinh(4*c + 4*d*x))/4 + 24*a^3*d*x - 9*a*b^2*d*x)/(24*
d)
```

sympy [A] time = 3.25, size = 190, normalized size = 1.74

$$\begin{cases} a^3x + \frac{3a^2b\sinh^2(c+dx)}{2d} - \frac{3ab^2x\sinh^4(c+dx)}{8} + \frac{3ab^2x\sinh^2(c+dx)\cosh^2(c+dx)}{4} - \frac{3ab^2x\cosh^4(c+dx)}{8} + \frac{3ab^2\sinh^3(c+dx)\cosh(c+dx)}{8d} + \\ x(a+b\sinh(c)\cosh(c))^3 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cosh(d*x+c)*sinh(d*x+c))**3,x)
```

```
[Out] Piecewise((a**3*x + 3*a**2*b*sinh(c + d*x)**2/(2*d) - 3*a*b**2*x*sinh(c + d
*x)**4/8 + 3*a*b**2*x*sinh(c + d*x)**2*cosh(c + d*x)**2/4 - 3*a*b**2*x*cosh
(c + d*x)**4/8 + 3*a*b**2*sinh(c + d*x)**3*cosh(c + d*x)/(8*d) + 3*a*b**2*s
inh(c + d*x)*cosh(c + d*x)**3/(8*d) + b**3*sinh(c + d*x)**2*cosh(c + d*x)**
4/(4*d) - b**3*cosh(c + d*x)**6/(12*d), Ne(d, 0)), (x*(a + b*sinh(c)*cosh(c
))**3, True))
```

3.856 $\int (a+b\cosh (c+dx)\sinh (c+dx))^2\,dx$

Optimal. Leaf size=63

$$\frac{1}{8}x\left(8a^2-b^2\right)+\frac{ab\cosh (2c+2dx)}{2d}+\frac{b^2\sinh (2c+2dx)\cosh (2c+2dx)}{16d}$$

```
[Out] 1/8*(8*a^2-b^2)*x+1/2*a*b*cosh(2*d*x+2*c)/d+1/16*b^2*cosh(2*d*x+2*c)*sinh(2
*d*x+2*c)/d
```

Rubi [A] time = 0.04, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2666, 2644}

$$\frac{1}{8}x\left(8a^2-b^2\right)+\frac{ab\cosh (2c+2dx)}{2d}+\frac{b^2\sinh (2c+2dx)\cosh (2c+2dx)}{16d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cosh[c + d*x]*Sinh[c + d*x])^2,x]
```

```
[Out] ((8*a^2 - b^2)*x)/8 + (a*b*Cosh[2*c + 2*d*x])/(2*d) + (b^2*Cosh[2*c + 2*d*x
]*Sinh[2*c + 2*d*x])/(16*d)
```

Rule 2644

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^2, x_Symbol] :> Simp[((2*a^2 + b
^2)*x)/2, x] + (-Simp[(2*a*b*Cos[c + d*x])/d, x] - Simp[(b^2*Cos[c + d*x]*S
in[c + d*x])/(2*d), x]) /; FreeQ[{a, b, c, d}, x]
```

Rule 2666

```
Int[((a_) + cos[(c_.) + (d_.)*(x_)]*(b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_
Symbol] :> Int[(a + (b*Sin[2*c + 2*d*x])/2)^n, x] /; FreeQ[{a, b, c, d, n},
 x]
```

Rubi steps

$$\begin{aligned}\int (a+b\cosh (c+dx)\sinh (c+dx))^2\,dx &= \int \left(a+\frac{1}{2}b\sinh (2c+2dx)\right)^2\,dx\\ &= \frac{1}{8}\left(8a^2-b^2\right)x+\frac{ab\cosh (2c+2dx)}{2d}+\frac{b^2\cosh (2c+2dx)\sinh (2c+2}{16d}\end{aligned}$$

Mathematica [A] time = 0.12, size = 50, normalized size = 0.79

$$\frac{4\left(8a^2-b^2\right)(c+dx)+16ab\cosh(2(c+dx))+b^2\sinh(4(c+dx))}{32d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cosh[c + d*x]*Sinh[c + d*x])^2,x]
```

```
[Out] (4*(8*a^2 - b^2)*(c + d*x) + 16*a*b*Cosh[2*(c + d*x)] + b^2*Sinh[4*(c + d*x
)])/(32*d)
```

fricas [A] time = 0.49, size = 80, normalized size = 1.27

$$\frac{b^2\cosh\left(dx+c\right)^3\sinh\left(dx+c\right)+b^2\cosh\left(dx+c\right)\sinh\left(dx+c\right)^3+4\,ab\cosh\left(dx+c\right)^2+4\,ab\sinh\left(dx+c\right)^2+\left(8\,a\right.}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cosh(d*x+c)*sinh(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/8*(b^2*cosh(d*x + c)^3*sinh(d*x + c) + b^2*cosh(d*x + c)*sinh(d*x + c)^3
+ 4*a*b*cosh(d*x + c)^2 + 4*a*b*sinh(d*x + c)^2 + (8*a^2 - b^2)*d*x)/d
```

giac [A] time = 0.12, size = 81, normalized size = 1.29

$$\frac{1}{8}\left(8\,a^2-b^2\right)x+\frac{b^2e^{(4\,dx+4\,c)}}{64\,d}+\frac{abe^{(2\,dx+2\,c)}}{4\,d}+\frac{abe^{(-2\,dx-2\,c)}}{4\,d}-\frac{b^2e^{(-4\,dx-4\,c)}}{64\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cosh(d*x+c)*sinh(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/8*(8*a^2 - b^2)*x + 1/64*b^2*e^(4*d*x + 4*c)/d + 1/4*a*b*e^(2*d*x + 2*c)/
d + 1/4*a*b*e^(-2*d*x - 2*c)/d - 1/64*b^2*e^(-4*d*x - 4*c)/d
```

maple [A] time = 0.43, size = 68, normalized size = 1.08

$$\frac{b^2\left(\frac{\sinh(dx+c)\left(\cosh^3(dx+c)\right)}{4}-\frac{\cosh(dx+c)\sinh(dx+c)}{8}-\frac{dx}{8}-\frac{c}{8}\right)+a\left(\cosh^2\left(dx+c\right)\right)b+a^2\left(dx+c\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cosh(d*x+c)*sinh(d*x+c))^2,x)
```

```
[Out] 1/d*(b^2*(1/4*sinh(d*x+c)*cosh(d*x+c)^3-1/8*cosh(d*x+c)*sinh(d*x+c)-1/8*d*x
-1/8*c)+a*cosh(d*x+c)^2*b+a^2*(d*x+c))
```

maxima [A] time = 0.39, size = 63, normalized size = 1.00

$$a^2x - \frac{1}{64}b^2\left(\frac{8(dx+c)}{d} - \frac{e^{(4dx+4c)}}{d} + \frac{e^{(-4dx-4c)}}{d}\right) + \frac{ab\cosh(dx+c)^2}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cosh(d*x+c)*sinh(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] a^2*x - 1/64*b^2*(8*(d*x + c)/d - e^(4*d*x + 4*c)/d + e^(-4*d*x - 4*c)/d) +
 a*b*cosh(d*x + c)^2/d
```

mupad [B] time = 0.14, size = 44, normalized size = 0.70

$$\frac{\frac{\sinh(4c+4dx)b^2}{32} + \frac{a\cosh(2c+2dx)b}{2}}{d} + a^2x - \frac{b^2x}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*cosh(c + d*x)*sinh(c + d*x))^2,x)
```

```
[Out] ((b^2*sinh(4*c + 4*d*x))/32 + (a*b*cosh(2*c + 2*d*x))/2)/d + a^2*x - (b^2*x
)/8
```

sympy [A] time = 0.93, size = 129, normalized size = 2.05

$$\begin{cases} a^2x + \frac{ab\sinh^2(c+dx)}{d} - \frac{b^2x\sinh^4(c+dx)}{8} + \frac{b^2x\sinh^2(c+dx)\cosh^2(c+dx)}{4} - \frac{b^2x\cosh^4(c+dx)}{8} + \frac{b^2\sinh^3(c+dx)\cosh(c+dx)}{8d} + \frac{b^2\sinh(c- \\ x(a+b\sinh(c)\cosh(c))^2 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cosh(d*x+c)*sinh(d*x+c))**2,x)
```

```
[Out] Piecewise((a**2*x + a*b*sinh(c + d*x)**2/d - b**2*x*sinh(c + d*x)**4/8 + b*
*2*x*sinh(c + d*x)**2*cosh(c + d*x)**2/4 - b**2*x*cosh(c + d*x)**4/8 + b**2
*sinh(c + d*x)**3*cosh(c + d*x)/(8*d) + b**2*sinh(c + d*x)*cosh(c + d*x)**3
/(8*d), Ne(d, 0)), (x*(a + b*sinh(c)*cosh(c))**2, True))
```

3.857 $\int (a + b\cosh(c + dx)\sinh(c + dx))\,dx$

Optimal. Leaf size=20

$$ax + \frac{b\sinh^2(c + dx)}{2d}$$

[Out] a*x+1/2*b*sinh(d*x+c)^2/d

Rubi [A] time = 0.02, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2564, 30}

$$ax + \frac{b\sinh^2(c + dx)}{2d}$$

Antiderivative was successfully verified.

[In] Int[a + b*Cosh[c + d*x]*Sinh[c + d*x],x]

[Out] a*x + (b*Sinh[c + d*x]^2)/(2*d)

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rubi steps

$$\begin{aligned}\int (a + b\cosh(c + dx)\sinh(c + dx))\,dx &= ax + b\int \cosh(c + dx)\sinh(c + dx)\,dx\\ &= ax - \frac{b\,\text{Subst}\left(\int x\,dx, x, i\sinh(c + dx)\right)}{d}\\ &= ax + \frac{b\sinh^2(c + dx)}{2d}\end{aligned}$$

Mathematica [A] time = 0.01, size = 38, normalized size = 1.90

$$ax+\frac{b\sinh(2c)\sinh(2dx)}{4d}+\frac{b\cosh(2c)\cosh(2dx)}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[a + b*Cosh[c + d*x]*Sinh[c + d*x],x]
[Out] a*x + (b*Cosh[2*c]*Cosh[2*d*x])/(4*d) + (b*Sinh[2*c]*Sinh[2*d*x])/(4*d)
```

fricas [A] time = 0.56, size = 31, normalized size = 1.55

$$\frac{4\,adx+b\cosh\left(dx+c\right)^2+b\sinh\left(dx+c\right)^2}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*cosh(d*x+c)*sinh(d*x+c),x, algorithm="fricas")
[Out] 1/4*(4*a*d*x + b*cosh(d*x + c)^2 + b*sinh(d*x + c)^2)/d
```

giac [A] time = 0.13, size = 34, normalized size = 1.70

$$ax+\frac{1}{8}\,b\left(\frac{e^{(2\,dx+2\,c)}}{d}+\frac{e^{(-2\,dx-2\,c)}}{d}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*cosh(d*x+c)*sinh(d*x+c),x, algorithm="giac")
[Out] a*x + 1/8*b*(e^(2*d*x + 2*c)/d + e^(-2*d*x - 2*c)/d)
```

maple [A] time = 0.02, size = 19, normalized size = 0.95

$$ax+\frac{b\left(\cosh^2\left(dx+c\right)\right)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a+b*cosh(d*x+c)*sinh(d*x+c),x)
[Out] a*x+1/2*b*cosh(d*x+c)^2/d
```

maxima [A] time = 0.30, size = 18, normalized size = 0.90

$$ax+\frac{b\cosh\left(dx+c\right)^2}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*cosh(d*x+c)*sinh(d*x+c),x, algorithm="maxima")
```

```
[Out] a*x + 1/2*b*cosh(d*x + c)^2/d
```

mupad [B] time = 1.64, size = 18, normalized size = 0.90

$$a\,x+\frac{b\,{\mathrm{cosh}\left(c+d\,x\right)}^2}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a + b*cosh(c + d*x)*sinh(c + d*x),x)
```

```
[Out] a*x + (b*cosh(c + d*x)^2)/(2*d)
```

sympy [A] time = 0.19, size = 24, normalized size = 1.20

$$ax + b\left(\begin{cases} \frac{\sinh^{2}{\left(c + dx \right)}}{2d} & \text{for}\: d \neq 0 \\ x \sinh{\left(c \right)} \cosh{\left(c \right)} & \text{otherwise} \end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*cosh(d*x+c)*sinh(d*x+c),x)
```

```
[Out] a*x + b*Piecewise((sinh(c + d*x)**2/(2*d), Ne(d, 0)), (x*sinh(c)*cosh(c), T
rue))
```

3.858 $\int \frac{1}{a+b\cosh(c+dx)\sinh(c+dx)}\,dx$

Optimal. Leaf size=44

$$-\frac{2\tanh^{-1}\left(\frac{b-2a\tanh(c+dx)}{\sqrt{4a^2+b^2}}\right)}{d\sqrt{4a^2+b^2}}$$

```
[Out] -2*arctanh((b-2*a*tanh(d*x+c))/(4*a^2+b^2)^(1/2))/d/(4*a^2+b^2)^(1/2)
```

Rubi [A] time = 0.12, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2666, 2660, 618, 204}

$$-\frac{2\tanh^{-1}\left(\frac{b-2a\tanh(c+dx)}{\sqrt{4a^2+b^2}}\right)}{d\sqrt{4a^2+b^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cosh[c + d*x]*Sinh[c + d*x])^(-1),x]
```

```
[Out] (-2*ArcTanh[(b - 2*a*Tanh[c + d*x])/Sqrt[4*a^2 + b^2]])/(Sqrt[4*a^2 + b^2]*
d)
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2666

```
Int[((a_) + cos[(c_.) + (d_.)*(x_)]*(b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_
Symbol] :> Int[(a + (b*Sin[2*c + 2*d*x])/2)^n, x] /; FreeQ[{a, b, c, d, n},
 x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{a+b\cosh(c+dx)\sinh(c+dx)}\,dx &= \int \frac{1}{a+\frac{1}{2}b\sinh(2c+2dx)}\,dx\\ &= -\frac{i\,\text{Subst}\left(\int \frac{1}{a-ibx+ax^2}\,dx, x, \tan\left(\frac{1}{2}(2ic+2idx)\right)\right)}{d}\\ &= \frac{(2i)\,\text{Subst}\left(\int \frac{1}{-4a^2-b^2-x^2}\,dx, x, -ib+2a\tan\left(\frac{1}{2}(2ic+2idx)\right)\right)}{d}\\ &= -\frac{2\tanh^{-1}\left(\frac{b-2a\tanh(c+dx)}{\sqrt{4a^2+b^2}}\right)}{\sqrt{4a^2+b^2}\,d}\end{aligned}$$

Mathematica [A] time = 0.08, size = 48, normalized size = 1.09

$$\frac{2\tan^{-1}\left(\frac{b-2a\tanh(c+dx)}{\sqrt{-4a^2-b^2}}\right)}{d\sqrt{-4a^2-b^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cosh[c + d*x]*Sinh[c + d*x])^(-1),x]
```

```
[Out] (2*ArcTan[(b - 2*a*Tanh[c + d*x])/Sqrt[-4*a^2 - b^2]])/(Sqrt[-4*a^2 - b^2]*
d)
```

fricas [B] time = 0.76, size = 299, normalized size = 6.80

$$\frac{\log\left(\frac{b^2\cosh(dx+c)^4+4\,b^2\cosh(dx+c)\sinh(dx+c)^3+b^2\sinh(dx+c)^4+4\,ab\cosh(dx+c)^2+2\left(3\,b^2\cosh(dx+c)^2+2\,ab\right)\sinh(dx+c)^2+8\,a^2+b^2+4\left(b^2\cosh(\cdots\right.}{b\cosh(dx+c)^4+4\,b\cosh(dx+c)\sinh(dx+c)^3+b\sinh(dx+c)^4+4\,a\cosh(dx+c)^2+2\left(3\,b\cosh(dx+c)^2+\cdots\right.}\cdots}{\sqrt{4\,a^2+b^2}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cosh(d*x+c)*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] log((b^2*cosh(d*x + c)^4 + 4*b^2*cosh(d*x + c)*sinh(d*x + c)^3 + b^2*sinh(d
*x + c)^4 + 4*a*b*cosh(d*x + c)^2 + 2*(3*b^2*cosh(d*x + c)^2 + 2*a*b)*sinh(
```

```
d*x + c)^2 + 8*a^2 + b^2 + 4*(b^2*cosh(d*x + c)^3 + 2*a*b*cosh(d*x + c))*si
nh(d*x + c) - 2*(b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*si
nh(d*x + c)^2 + 2*a)*sqrt(4*a^2 + b^2))/(b*cosh(d*x + c)^4 + 4*b*cosh(d*x +
 c)*sinh(d*x + c)^3 + b*sinh(d*x + c)^4 + 4*a*cosh(d*x + c)^2 + 2*(3*b*cosh
(d*x + c)^2 + 2*a)*sinh(d*x + c)^2 + 4*(b*cosh(d*x + c)^3 + 2*a*cosh(d*x +
c))*sinh(d*x + c) - b))/(sqrt(4*a^2 + b^2)*d)
```

giac [A] time = 0.41, size = 79, normalized size = 1.80

$$\frac{\log\left(\frac{\left|2\,be^{(2\,dx+2\,c)}+4\,a-2\,\sqrt{4\,a^2+b^2}\right|}{\left|2\,be^{(2\,dx+2\,c)}+4\,a+2\,\sqrt{4\,a^2+b^2}\right|}\right)}{\sqrt{4\,a^2+b^2}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cosh(d*x+c)*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] log(abs(2*b*e^(2*d*x + 2*c) + 4*a - 2*sqrt(4*a^2 + b^2))/abs(2*b*e^(2*d*x +
 2*c) + 4*a + 2*sqrt(4*a^2 + b^2)))/(sqrt(4*a^2 + b^2)*d)
```

maple [B] time = 0.66, size = 207, normalized size = 4.70

$$-\frac{4a^2\ln\left(-\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a+\sqrt{4a^2+b^2}\,\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)b-a\right)}{d\left(4a^2+b^2\right)^{\frac{3}{2}}}-\frac{\ln\left(-\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a+\cdots\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*cosh(d*x+c)*sinh(d*x+c)),x)
```

```
[Out] -4/d*a^2/(4*a^2+b^2)^(3/2)*ln(-tanh(1/2*d*x+1/2*c)^2*a+(4*a^2+b^2)^(1/2)*ta
nh(1/2*d*x+1/2*c)-tanh(1/2*d*x+1/2*c)*b-a)-1/d/(4*a^2+b^2)^(3/2)*ln(-tanh(1
/2*d*x+1/2*c)^2*a+(4*a^2+b^2)^(1/2)*tanh(1/2*d*x+1/2*c)-tanh(1/2*d*x+1/2*c)
*b-a)*b^2+1/d/(4*a^2+b^2)^(1/2)*ln(tanh(1/2*d*x+1/2*c)^2*a+(4*a^2+b^2)^(1/2
)*tanh(1/2*d*x+1/2*c)+tanh(1/2*d*x+1/2*c)*b+a)
```

maxima [A] time = 0.42, size = 73, normalized size = 1.66

$$\frac{\log\left(\frac{be^{(-2\,dx-2\,c)}-2\,a-\sqrt{4\,a^2+b^2}}{be^{(-2\,dx-2\,c)}-2\,a+\sqrt{4\,a^2+b^2}}\right)}{\sqrt{4\,a^2+b^2}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cosh(d*x+c)*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] log((b*e^(-2*d*x - 2*c) - 2*a - sqrt(4*a^2 + b^2))/(b*e^(-2*d*x - 2*c) - 2*
a + sqrt(4*a^2 + b^2)))/(sqrt(4*a^2 + b^2)*d)
```

mupad [B] time = 2.30, size = 343, normalized size = 7.80

$$\frac{2\,\mathrm{atan}\left(\left(\frac{b^4\,\sqrt{-4\,a^2\,d^2-b^2\,d^2}}{16}+\frac{a^2\,b^2\,\sqrt{-4\,a^2\,d^2-b^2\,d^2}}{4}\right)\left(\frac{32\,a\,\left(8\,a^2+b^2\right)}{b^4\,d\,\left(4\,a^2+b^2\right)^2}-\mathrm{e}^{2\,c}\,\mathrm{e}^{2\,d\,x}\,\left(\frac{64\,a\,\left(16\,d\,a^3+4\,d\,a\,b^2\right)}{b^5\,\sqrt{-4\,a^2\,d^2-b^2\,d^2}\,\left(4\,a^2+b^2\right)\,\sqrt{-d^2\,\left(4\,a^2+b^2\right)}}+\frac{16\left(\cdots\right.}{\cdots}\right.\right.\right.}{\sqrt{-4\,a^2\,d^2-b^2\,d^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*cosh(c + d*x)*sinh(c + d*x)),x)
```

```
[Out] (2*atan(((b^4*(- 4*a^2*d^2 - b^2*d^2)^(1/2))/16 + (a^2*b^2*(- 4*a^2*d^2 - b
^2*d^2)^(1/2))/4)*((32*a*(8*a^2 + b^2))/(b^4*d*(4*a^2 + b^2)^2) - exp(2*c)*
exp(2*d*x)*((64*a*(16*a^3*d + 4*a*b^2*d))/(b^5*(- 4*a^2*d^2 - b^2*d^2)^(1/2
)*(4*a^2 + b^2)*(-d^2*(4*a^2 + b^2))^(1/2)) + (16*(8*a^2 + b^2)*(8*a^2*(- 4
*a^2*d^2 - b^2*d^2)^(1/2) + b^2*(- 4*a^2*d^2 - b^2*d^2)^(1/2)))/(b^5*d*(- 4
*a^2*d^2 - b^2*d^2)^(1/2)*(4*a^2 + b^2)^2)) + (64*a*(b^3*d + 4*a^2*b*d))/(b
^5*(- 4*a^2*d^2 - b^2*d^2)^(1/2)*(4*a^2 + b^2)*(-d^2*(4*a^2 + b^2))^(1/2)))
))/(- 4*a^2*d^2 - b^2*d^2)^(1/2)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cosh(d*x+c)*sinh(d*x+c)),x)
```

```
[Out] Timed out
```

3.859 $\int \frac{1}{(a+b\cosh(c+dx)\sinh(c+dx))^2}\,dx$

Optimal. Leaf size=89

$$-\frac{8a\tanh^{-1}\left(\frac{b-2a\tanh(c+dx)}{\sqrt{4a^2+b^2}}\right)}{d\left(4a^2+b^2\right)^{3/2}}-\frac{2b\cosh(2c+2dx)}{d\left(4a^2+b^2\right)(2a+b\sinh(2c+2dx))}$$

```
[Out] -8*a*arctanh((b-2*a*tanh(d*x+c))/(4*a^2+b^2)^(1/2))/(4*a^2+b^2)^(3/2)/d-2*b
*cosh(2*d*x+2*c)/(4*a^2+b^2)/d/(2*a+b*sinh(2*d*x+2*c))
```

Rubi [A] time = 0.10, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2666, 2664, 12, 2660, 618, 204}

$$-\frac{8a\tanh^{-1}\left(\frac{b-2a\tanh(c+dx)}{\sqrt{4a^2+b^2}}\right)}{d\left(4a^2+b^2\right)^{3/2}}-\frac{2b\cosh(2c+2dx)}{d\left(4a^2+b^2\right)(2a+b\sinh(2c+2dx))}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cosh[c + d*x]*Sinh[c + d*x])^(-2),x]
```

```
[Out] (-8*a*ArcTanh[(b - 2*a*Tanh[c + d*x])/Sqrt[4*a^2 + b^2]])/((4*a^2 + b^2)^(3
/2)*d) - (2*b*Cosh[2*c + 2*d*x])/((4*a^2 + b^2)*d*(2*a + b*Sinh[2*c + 2*d*x
]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2664

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n + 1))/(d*(n + 1)*(a^2 - b^2)), x] + Dist[1
/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) - b
*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^
2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2666

```
Int[((a_) + cos[(c_.) + (d_.)*(x_)]*(b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_
Symbol] :> Int[(a + (b*Sin[2*c + 2*d*x])/2)^n, x] /; FreeQ[{a, b, c, d, n},
 x]
```

Rubi steps

$$\int \frac{1}{(a+b\cosh(c+dx)\sinh(c+dx))^2}\,dx = \int \frac{1}{\left(a+\frac{1}{2}b\sinh(2c+2dx)\right)^2}\,dx$$

$$= -\frac{2b\cosh(2c+2dx)}{\left(4a^2+b^2\right)d(2a+b\sinh(2c+2dx))} + \frac{4\int \frac{a}{a+\frac{1}{2}b\sinh(2c+2dx)}\,dx}{4a^2+b^2}$$

$$= -\frac{2b\cosh(2c+2dx)}{\left(4a^2+b^2\right)d(2a+b\sinh(2c+2dx))} + \frac{(4a)\int \frac{1}{a+\frac{1}{2}b\sinh(2c+2dx)}\,dx}{4a^2+b^2}$$

$$= -\frac{2b\cosh(2c+2dx)}{\left(4a^2+b^2\right)d(2a+b\sinh(2c+2dx))} - \frac{(4ia)\,\text{Subst}\left(\int \frac{1}{a-ibx+ax^2}\,dx,\right.}{\left(4a^2+b\right.}$$

$$= -\frac{2b\cosh(2c+2dx)}{\left(4a^2+b^2\right)d(2a+b\sinh(2c+2dx))} + \frac{(8ia)\,\text{Subst}\left(\int \frac{1}{-4a^2-b^2-x^2}\,dx\right.}{\left(4a\right.}$$

$$= -\frac{8a\tanh^{-1}\left(\frac{b-2a\tanh(c+dx)}{\sqrt{4a^2+b^2}}\right)}{\left(4a^2+b^2\right)^{3/2}d} - \frac{2b\cosh(2c+2dx)}{\left(4a^2+b^2\right)d(2a+b\sinh(2c+2dx))}$$

Mathematica [A] time = 0.35, size = 90, normalized size = 1.01

$$\frac{2\left(-\frac{4a\tan^{-1}\left(\frac{b-2a\tanh(c+dx)}{\sqrt{-4a^2-b^2}}\right)}{\left(-4a^2-b^2\right)^{3/2}} - \frac{b\cosh(2(c+dx))}{\left(4a^2+b^2\right)(2a+b\sinh(2(c+dx)))}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cosh[c + d*x]*Sinh[c + d*x])^(-2),x]
```

```
[Out] (2*((-4*a*ArcTan[(b - 2*a*Tanh[c + d*x])/Sqrt[-4*a^2 - b^2]])/(-4*a^2 - b^2
)^(3/2) - (b*Cosh[2*(c + d*x)])/((4*a^2 + b^2)*(2*a + b*Sinh[2*(c + d*x)]))
))/d
```

fricas [B] time = 0.55, size = 765, normalized size = 8.60

$$-\frac{4\left(4a^2b+b^3-2\left(4a^3+ab^2\right)\cosh(dx+c)^2-4\left(4a^3+ab^2\right)\cosh(dx+c)\sinh(dx+c)-2\left(4a^3+ab^2\right)\sinh\right.}{\left(16a^4b+8a^2b^3+b^5\right)d\cosh(dx+c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cosh(d*x+c)*sinh(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -4*(4*a^2*b + b^3 - 2*(4*a^3 + a*b^2)*cosh(d*x + c)^2 - 4*(4*a^3 + a*b^2)*c
osh(d*x + c)*sinh(d*x + c) - 2*(4*a^3 + a*b^2)*sinh(d*x + c)^2 - (a*b*cosh(
d*x + c)^4 + 4*a*b*cosh(d*x + c)*sinh(d*x + c)^3 + a*b*sinh(d*x + c)^4 + 4*
a^2*cosh(d*x + c)^2 + 2*(3*a*b*cosh(d*x + c)^2 + 2*a^2)*sinh(d*x + c)^2 - a
*b + 4*(a*b*cosh(d*x + c)^3 + 2*a^2*cosh(d*x + c))*sinh(d*x + c))*sqrt(4*a^
2 + b^2)*log((b^2*cosh(d*x + c)^4 + 4*b^2*cosh(d*x + c)*sinh(d*x + c)^3 + b
^2*sinh(d*x + c)^4 + 4*a*b*cosh(d*x + c)^2 + 2*(3*b^2*cosh(d*x + c)^2 + 2*a
*b)*sinh(d*x + c)^2 + 8*a^2 + b^2 + 4*(b^2*cosh(d*x + c)^3 + 2*a*b*cosh(d*x
 + c))*sinh(d*x + c) - 2*(b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x +
c) + b*sinh(d*x + c)^2 + 2*a)*sqrt(4*a^2 + b^2))/(b*cosh(d*x + c)^4 + 4*b*c
osh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x + c)^4 + 4*a*cosh(d*x + c)^2 + 2*
(3*b*cosh(d*x + c)^2 + 2*a)*sinh(d*x + c)^2 + 4*(b*cosh(d*x + c)^3 + 2*a*co
sh(d*x + c))*sinh(d*x + c) - b)))/((16*a^4*b + 8*a^2*b^3 + b^5)*d*cosh(d*x
+ c)^4 + 4*(16*a^4*b + 8*a^2*b^3 + b^5)*d*cosh(d*x + c)*sinh(d*x + c)^3 + (
16*a^4*b + 8*a^2*b^3 + b^5)*d*sinh(d*x + c)^4 + 4*(16*a^5 + 8*a^3*b^2 + a*b
^4)*d*cosh(d*x + c)^2 + 2*(3*(16*a^4*b + 8*a^2*b^3 + b^5)*d*cosh(d*x + c)^2
 + 2*(16*a^5 + 8*a^3*b^2 + a*b^4)*d)*sinh(d*x + c)^2 - (16*a^4*b + 8*a^2*b^
3 + b^5)*d + 4*((16*a^4*b + 8*a^2*b^3 + b^5)*d*cosh(d*x + c)^3 + 2*(16*a^5
+ 8*a^3*b^2 + a*b^4)*d*cosh(d*x + c))*sinh(d*x + c))
```

giac [A] time = 0.51, size = 140, normalized size = 1.57

$$\frac{4\left(\frac{a\log\left(\frac{\left|2\,be^{(2\,dx+2\,c)}+4\,a-2\,\sqrt{4\,a^2+b^2}\right|}{\left|2\,be^{(2\,dx+2\,c)}+4\,a+2\,\sqrt{4\,a^2+b^2}\right|}\right)}{\left(4\,a^2+b^2\right)^{\frac{3}{2}}}+\frac{2\,ae^{(2\,dx+2\,c)}-b}{\left(4\,a^2+b^2\right)\left(be^{(4\,dx+4\,c)}+4\,ae^{(2\,dx+2\,c)}-b\right)}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cosh(d*x+c)*sinh(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 4*(a*log(abs(2*b*e^(2*d*x + 2*c) + 4*a - 2*sqrt(4*a^2 + b^2))/abs(2*b*e^(2*
d*x + 2*c) + 4*a + 2*sqrt(4*a^2 + b^2)))/(4*a^2 + b^2)^(3/2) + (2*a*e^(2*d*
x + 2*c) - b)/((4*a^2 + b^2)*(b*e^(4*d*x + 4*c) + 4*a*e^(2*d*x + 2*c) - b))
)/d
```

maple [B] time = 0.80, size = 469, normalized size = 5.27

$$\frac{2b^2\left(\tanh^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d\left(\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a+2\left(\tanh^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b-2\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)b+a\right)a\left(4a^2+b^2\right)}-\frac{}{d\left(\left(\tanh\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*cosh(d*x+c)*sinh(d*x+c))^2,x)
```

```
[Out] 2/d/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^3*b-2*tanh(1/2*d*x+1/2*c
)^2*a+2*tanh(1/2*d*x+1/2*c)*b+a)*b^2/a/(4*a^2+b^2)*tanh(1/2*d*x+1/2*c)^3-8/
d/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^3*b-2*tanh(1/2*d*x+1/2*c)^
2*a+2*tanh(1/2*d*x+1/2*c)*b+a)*b/(4*a^2+b^2)*tanh(1/2*d*x+1/2*c)^2+2/d/(tan
h(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^3*b-2*tanh(1/2*d*x+1/2*c)^2*a+2*
tanh(1/2*d*x+1/2*c)*b+a)*b^2/a/(4*a^2+b^2)*tanh(1/2*d*x+1/2*c)-4/d*a/(4*a^2
+b^2)^(3/2)*ln(-tanh(1/2*d*x+1/2*c)^2*a+(4*a^2+b^2)^(1/2)*tanh(1/2*d*x+1/2*
c)-tanh(1/2*d*x+1/2*c)*b-a)+16/d*a^3/(4*a^2+b^2)^(5/2)*ln(tanh(1/2*d*x+1/2*
c)^2*a+(4*a^2+b^2)^(1/2)*tanh(1/2*d*x+1/2*c)+tanh(1/2*d*x+1/2*c)*b+a)+4/d*a
/(4*a^2+b^2)^(5/2)*ln(tanh(1/2*d*x+1/2*c)^2*a+(4*a^2+b^2)^(1/2)*tanh(1/2*d*
x+1/2*c)+tanh(1/2*d*x+1/2*c)*b+a)*b^2
```

maxima [A] time = 0.43, size = 150, normalized size = 1.69

$$\frac{4\,a\log\left(\frac{be^{(-2\,dx-2\,c)}-2\,a-\sqrt{4\,a^2+b^2}}{be^{(-2\,dx-2\,c)}-2\,a+\sqrt{4\,a^2+b^2}}\right)}{\left(4\,a^2+b^2\right)^{\frac{3}{2}}d}-\frac{4\left(2\,ae^{(-2\,dx-2\,c)}+b\right)}{\left(4\,a^2b+b^3+4\left(4\,a^3+ab^2\right)e^{(-2\,dx-2\,c)}-\left(4\,a^2b+b^3\right)e^{(-4\,dx-4\,c)}\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cosh(d*x+c)*sinh(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 4*a*log((b*e^(-2*d*x - 2*c) - 2*a - sqrt(4*a^2 + b^2))/(b*e^(-2*d*x - 2*c)
- 2*a + sqrt(4*a^2 + b^2)))/((4*a^2 + b^2)^(3/2)*d) - 4*(2*a*e^(-2*d*x - 2*
c) + b)/((4*a^2*b + b^3 + 4*(4*a^3 + a*b^2)*e^(-2*d*x - 2*c) - (4*a^2*b + b
^3)*e^(-4*d*x - 4*c))*d)
```

mupad [B] time = 2.11, size = 229, normalized size = 2.57

$$\frac{4\,a\,\ln\left(\frac{16\,a\left(b-2\,a\,e^{2\,c+2\,d\,x}\right)}{b\left(4\,a^2+b^2\right)^{3/2}}-\frac{16\,a\,e^{2\,c+2\,d\,x}}{4\,a^2\,b+b^3}\right)}{d\left(4\,a^2+b^2\right)^{3/2}}-\frac{4\,a\,\ln\left(-\frac{16\,a\,e^{2\,c+2\,d\,x}}{4\,a^2\,b+b^3}-\frac{16\,a\left(b-2\,a\,e^{2\,c+2\,d\,x}\right)}{b\left(4\,a^2+b^2\right)^{3/2}}\right)}{d\left(4\,a^2+b^2\right)^{3/2}}-\frac{\frac{4\,b^2}{d\left(4\,a^2\,b+b^3\right)}-\frac{8\,a\,b\,e^{2\,c+2\,d\,x}}{d\left(4\,a^2\,b+b^3\right)}}{4\,a\,e^{2\,c+2\,d\,x}-b+b\,e^{4\,c+4\,d\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*cosh(c + d*x)*sinh(c + d*x))^2,x)
```

```
[Out] (4*a*log((16*a*(b - 2*a*exp(2*c + 2*d*x)))/(b*(4*a^2 + b^2)^(3/2)) - (16*a*
exp(2*c + 2*d*x))/(4*a^2*b + b^3)))/(d*(4*a^2 + b^2)^(3/2)) - (4*a*log(- (1
6*a*exp(2*c + 2*d*x))/(4*a^2*b + b^3) - (16*a*(b - 2*a*exp(2*c + 2*d*x)))/(
b*(4*a^2 + b^2)^(3/2))))/(d*(4*a^2 + b^2)^(3/2)) - ((4*b^2)/(d*(4*a^2*b + b
^3)) - (8*a*b*exp(2*c + 2*d*x))/(d*(4*a^2*b + b^3)))/(4*a*exp(2*c + 2*d*x)
- b + b*exp(4*c + 4*d*x))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cosh(d*x+c)*sinh(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.860 $\int \frac{1}{(a+b\cosh(c+dx)\sinh(c+dx))^3}\,dx$

Optimal. Leaf size=143

$$-\frac{4\left(8a^2-b^2\right)\tanh^{-1}\left(\frac{b-2a\tanh(c+dx)}{\sqrt{4a^2+b^2}}\right)}{d\left(4a^2+b^2\right)^{5/2}}-\frac{12ab\cosh(2c+2dx)}{d\left(4a^2+b^2\right)^2(2a+b\sinh(2c+2dx))}-\frac{2b\cosh(2c+2dx)}{d\left(4a^2+b^2\right)(2a+b\sinh(2c+2dx))^2}$$

```
[Out] -4*(8*a^2-b^2)*arctanh((b-2*a*tanh(d*x+c))/(4*a^2+b^2)^(1/2))/(4*a^2+b^2)^(
5/2)/d-2*b*cosh(2*d*x+2*c)/(4*a^2+b^2)/d/(2*a+b*sinh(2*d*x+2*c))^2-12*a*b*c
osh(2*d*x+2*c)/(4*a^2+b^2)^2/d/(2*a+b*sinh(2*d*x+2*c))
```

Rubi [A] time = 0.17, antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {2666, 2664, 2754, 12, 2660, 618, 204}

$$-\frac{4\left(8a^2-b^2\right)\tanh^{-1}\left(\frac{b-2a\tanh(c+dx)}{\sqrt{4a^2+b^2}}\right)}{d\left(4a^2+b^2\right)^{5/2}}-\frac{12ab\cosh(2c+2dx)}{d\left(4a^2+b^2\right)^2(2a+b\sinh(2c+2dx))}-\frac{2b\cosh(2c+2dx)}{d\left(4a^2+b^2\right)(2a+b\sinh(2c+2dx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cosh[c + d*x]*Sinh[c + d*x])^(-3),x]
```

```
[Out] (-4*(8*a^2 - b^2)*ArcTanh[(b - 2*a*Tanh[c + d*x])/Sqrt[4*a^2 + b^2]])/((4*a
^2 + b^2)^(5/2)*d) - (2*b*Cosh[2*c + 2*d*x])/((4*a^2 + b^2)*d*(2*a + b*Sinh
[2*c + 2*d*x])^2) - (12*a*b*Cosh[2*c + 2*d*x])/((4*a^2 + b^2)^2*d*(2*a + b*
Sinh[2*c + 2*d*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2664

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n + 1))/(d*(n + 1)*(a^2 - b^2)), x] + Dist[1
/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) - b
*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^
2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2666

```
Int[((a_) + cos[(c_.) + (d_.)*(x_)]*(b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_
Symbol] :> Int[(a + (b*Sin[2*c + 2*d*x])/2)^n, x] /; FreeQ[{a, b, c, d, n},
 x]
```

Rule 2754

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rubi steps

$$\int \frac{1}{(a+b\cosh(c+dx)\sinh(c+dx))^3}\,dx = \int \frac{1}{\left(a+\frac{1}{2}b\sinh(2c+2dx)\right)^3}\,dx$$

$$= -\frac{2b\cosh(2c+2dx)}{\left(4a^2+b^2\right)d(2a+b\sinh(2c+2dx))^2} - \frac{2\int \frac{-2a+\frac{1}{2}b\sinh(2c+2dx)}{\left(a+\frac{1}{2}b\sinh(2c+2dx)\right)^2}\,dx}{4a^2+b^2}$$

$$= -\frac{2b\cosh(2c+2dx)}{\left(4a^2+b^2\right)d(2a+b\sinh(2c+2dx))^2} - \frac{12ab\cosh(2c+2d}{\left(4a^2+b^2\right)^2 d(2a+b\sinh(}$$

$$= -\frac{2b\cosh(2c+2dx)}{\left(4a^2+b^2\right)d(2a+b\sinh(2c+2dx))^2} - \frac{12ab\cosh(2c+2d}{\left(4a^2+b^2\right)^2 d(2a+b\sinh(}$$

$$= -\frac{2b\cosh(2c+2dx)}{\left(4a^2+b^2\right)d(2a+b\sinh(2c+2dx))^2} - \frac{12ab\cosh(2c+2d}{\left(4a^2+b^2\right)^2 d(2a+b\sinh(}$$

$$= -\frac{2b\cosh(2c+2dx)}{\left(4a^2+b^2\right)d(2a+b\sinh(2c+2dx))^2} - \frac{12ab\cosh(2c+2d}{\left(4a^2+b^2\right)^2 d(2a+b\sinh(}$$

$$= -\frac{4\left(8a^2-b^2\right)\tanh^{-1}\left(\frac{b-2a\tanh(c+dx)}{\sqrt{4a^2+b^2}}\right)}{\left(4a^2+b^2\right)^{5/2} d} - \frac{2b\cosh(2c+2dx)}{\left(4a^2+b^2\right)d(2a+b\sinh(2c}$$

Mathematica [A] time = 0.66, size = 121, normalized size = 0.85

$$\frac{2\left(\frac{2\left(8a^2-b^2\right)\tan^{-1}\left(\frac{b-2a\tanh(c+dx)}{\sqrt{-4a^2-b^2}}\right)}{\sqrt{-4a^2-b^2}} - \frac{b\cosh(2(c+dx))\left(16a^2+6ab\sinh(2(c+dx))+b^2\right)}{(2a+b\sinh(2(c+dx)))^2}\right)}{d\left(4a^2+b^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cosh[c + d*x]*Sinh[c + d*x])^(-3),x]
```

```
[Out] (2*((2*(8*a^2 - b^2)*ArcTan[(b - 2*a*Tanh[c + d*x])/Sqrt[-4*a^2 - b^2]])/Sq
rt[-4*a^2 - b^2] - (b*Cosh[2*(c + d*x)]*(16*a^2 + b^2 + 6*a*b*Sinh[2*(c + d
*x)]))/(2*a + b*Sinh[2*(c + d*x)])^2))/((4*a^2 + b^2)^2*d)
```

fricas [B] time = 0.48, size = 2439, normalized size = 17.06

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cosh(d*x+c)*sinh(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 2*(2*(32*a^4*b + 4*a^2*b^3 - b^5)*cosh(d*x + c)^6 + 12*(32*a^4*b + 4*a^2*b^
3 - b^5)*cosh(d*x + c)*sinh(d*x + c)^5 + 2*(32*a^4*b + 4*a^2*b^3 - b^5)*sin
h(d*x + c)^6 + 48*a^3*b^2 + 12*a*b^4 + 12*(32*a^5 + 4*a^3*b^2 - a*b^4)*cosh
(d*x + c)^4 + 6*(64*a^5 + 8*a^3*b^2 - 2*a*b^4 + 5*(32*a^4*b + 4*a^2*b^3 - b
^5)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 8*(5*(32*a^4*b + 4*a^2*b^3 - b^5)*co
sh(d*x + c)^3 + 6*(32*a^5 + 4*a^3*b^2 - a*b^4)*cosh(d*x + c))*sinh(d*x + c)
^3 - 2*(160*a^4*b + 44*a^2*b^3 + b^5)*cosh(d*x + c)^2 - 2*(160*a^4*b + 44*a
^2*b^3 + b^5 - 15*(32*a^4*b + 4*a^2*b^3 - b^5)*cosh(d*x + c)^4 - 36*(32*a^5
 + 4*a^3*b^2 - a*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^2 - ((8*a^2*b^2 - b^4)
*cosh(d*x + c)^8 + 8*(8*a^2*b^2 - b^4)*cosh(d*x + c)*sinh(d*x + c)^7 + (8*a
^2*b^2 - b^4)*sinh(d*x + c)^8 + 8*(8*a^3*b - a*b^3)*cosh(d*x + c)^6 + 4*(16
*a^3*b - 2*a*b^3 + 7*(8*a^2*b^2 - b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 8
*(7*(8*a^2*b^2 - b^4)*cosh(d*x + c)^3 + 6*(8*a^3*b - a*b^3)*cosh(d*x + c))*
sinh(d*x + c)^5 + 2*(64*a^4 - 16*a^2*b^2 + b^4)*cosh(d*x + c)^4 + 2*(35*(8*
a^2*b^2 - b^4)*cosh(d*x + c)^4 + 64*a^4 - 16*a^2*b^2 + b^4 + 60*(8*a^3*b -
a*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 8*a^2*b^2 - b^4 + 8*(7*(8*a^2*b^2
 - b^4)*cosh(d*x + c)^5 + 20*(8*a^3*b - a*b^3)*cosh(d*x + c)^3 + (64*a^4 -
16*a^2*b^2 + b^4)*cosh(d*x + c))*sinh(d*x + c)^3 - 8*(8*a^3*b - a*b^3)*cosh
(d*x + c)^2 + 4*(7*(8*a^2*b^2 - b^4)*cosh(d*x + c)^6 + 30*(8*a^3*b - a*b^3)
*cosh(d*x + c)^4 - 16*a^3*b + 2*a*b^3 + 3*(64*a^4 - 16*a^2*b^2 + b^4)*cosh(
d*x + c)^2)*sinh(d*x + c)^2 + 8*((8*a^2*b^2 - b^4)*cosh(d*x + c)^7 + 6*(8*a
^3*b - a*b^3)*cosh(d*x + c)^5 + (64*a^4 - 16*a^2*b^2 + b^4)*cosh(d*x + c)^3
 - 2*(8*a^3*b - a*b^3)*cosh(d*x + c))*sinh(d*x + c))*sqrt(4*a^2 + b^2)*log(
(b^2*cosh(d*x + c)^4 + 4*b^2*cosh(d*x + c)*sinh(d*x + c)^3 + b^2*sinh(d*x +
 c)^4 + 4*a*b*cosh(d*x + c)^2 + 2*(3*b^2*cosh(d*x + c)^2 + 2*a*b)*sinh(d*x
+ c)^2 + 8*a^2 + b^2 + 4*(b^2*cosh(d*x + c)^3 + 2*a*b*cosh(d*x + c))*sinh(d
*x + c) + 2*(b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d
*x + c)^2 + 2*a)*sqrt(4*a^2 + b^2))/(b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)*
sinh(d*x + c)^3 + b*sinh(d*x + c)^4 + 4*a*cosh(d*x + c)^2 + 2*(3*b*cosh(d*x
 + c)^2 + 2*a)*sinh(d*x + c)^2 + 4*(b*cosh(d*x + c)^3 + 2*a*cosh(d*x + c))*
sinh(d*x + c) - b)) + 4*(3*(32*a^4*b + 4*a^2*b^3 - b^5)*cosh(d*x + c)^5 + 1
2*(32*a^5 + 4*a^3*b^2 - a*b^4)*cosh(d*x + c)^3 - (160*a^4*b + 44*a^2*b^3 +
b^5)*cosh(d*x + c))*sinh(d*x + c))/((64*a^6*b^2 + 48*a^4*b^4 + 12*a^2*b^6 +
 b^8)*d*cosh(d*x + c)^8 + 8*(64*a^6*b^2 + 48*a^4*b^4 + 12*a^2*b^6 + b^8)*d*
cosh(d*x + c)*sinh(d*x + c)^7 + (64*a^6*b^2 + 48*a^4*b^4 + 12*a^2*b^6 + b^8
)*d*sinh(d*x + c)^8 + 8*(64*a^7*b + 48*a^5*b^3 + 12*a^3*b^5 + a*b^7)*d*cosh
(d*x + c)^6 + 4*(7*(64*a^6*b^2 + 48*a^4*b^4 + 12*a^2*b^6 + b^8)*d*cosh(d*x
```

```
+ c)^2 + 2*(64*a^7*b + 48*a^5*b^3 + 12*a^3*b^5 + a*b^7)*d)*sinh(d*x + c)^6
+ 2*(512*a^8 + 320*a^6*b^2 + 48*a^4*b^4 - 4*a^2*b^6 - b^8)*d*cosh(d*x + c)^
4 + 8*(7*(64*a^6*b^2 + 48*a^4*b^4 + 12*a^2*b^6 + b^8)*d*cosh(d*x + c)^3 + 6
*(64*a^7*b + 48*a^5*b^3 + 12*a^3*b^5 + a*b^7)*d*cosh(d*x + c))*sinh(d*x + c
)^5 + 2*(35*(64*a^6*b^2 + 48*a^4*b^4 + 12*a^2*b^6 + b^8)*d*cosh(d*x + c)^4
+ 60*(64*a^7*b + 48*a^5*b^3 + 12*a^3*b^5 + a*b^7)*d*cosh(d*x + c)^2 + (512*
a^8 + 320*a^6*b^2 + 48*a^4*b^4 - 4*a^2*b^6 - b^8)*d)*sinh(d*x + c)^4 - 8*(6
4*a^7*b + 48*a^5*b^3 + 12*a^3*b^5 + a*b^7)*d*cosh(d*x + c)^2 + 8*(7*(64*a^6
*b^2 + 48*a^4*b^4 + 12*a^2*b^6 + b^8)*d*cosh(d*x + c)^5 + 20*(64*a^7*b + 48
*a^5*b^3 + 12*a^3*b^5 + a*b^7)*d*cosh(d*x + c)^3 + (512*a^8 + 320*a^6*b^2 +
 48*a^4*b^4 - 4*a^2*b^6 - b^8)*d*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(7*(64*
a^6*b^2 + 48*a^4*b^4 + 12*a^2*b^6 + b^8)*d*cosh(d*x + c)^6 + 30*(64*a^7*b +
 48*a^5*b^3 + 12*a^3*b^5 + a*b^7)*d*cosh(d*x + c)^4 + 3*(512*a^8 + 320*a^6*
b^2 + 48*a^4*b^4 - 4*a^2*b^6 - b^8)*d*cosh(d*x + c)^2 - 2*(64*a^7*b + 48*a^
5*b^3 + 12*a^3*b^5 + a*b^7)*d)*sinh(d*x + c)^2 + (64*a^6*b^2 + 48*a^4*b^4 +
 12*a^2*b^6 + b^8)*d + 8*((64*a^6*b^2 + 48*a^4*b^4 + 12*a^2*b^6 + b^8)*d*co
sh(d*x + c)^7 + 6*(64*a^7*b + 48*a^5*b^3 + 12*a^3*b^5 + a*b^7)*d*cosh(d*x +
 c)^5 + (512*a^8 + 320*a^6*b^2 + 48*a^4*b^4 - 4*a^2*b^6 - b^8)*d*cosh(d*x +
 c)^3 - 2*(64*a^7*b + 48*a^5*b^3 + 12*a^3*b^5 + a*b^7)*d*cosh(d*x + c))*sin
h(d*x + c))
```

giac [A] time = 0.90, size = 256, normalized size = 1.79

$$\frac{2\left(\frac{(8a^2-b^2)\log\left(\frac{\left|2be^{(2dx+2c)}+4a-2\sqrt{4a^2+b^2}\right|}{\left|2be^{(2dx+2c)}+4a+2\sqrt{4a^2+b^2}\right|}\right)}{(16a^4+8a^2b^2+b^4)\sqrt{4a^2+b^2}}+\frac{2\left(8a^2be^{(6dx+6c)}-b^3e^{(6dx+6c)}+48a^3e^{(4dx+4c)}-6ab^2e^{(4dx+4c)}-40a^2be^{(2dx+2c)}-b^3e^{(2dx+2c)}+6ab\right.}{(16a^4+8a^2b^2+b^4)\left(be^{(4dx+4c)}+4ae^{(2dx+2c)}-b\right)^2}\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cosh(d*x+c)*sinh(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 2*((8*a^2 - b^2)*log(abs(2*b*e^(2*d*x + 2*c) + 4*a - 2*sqrt(4*a^2 + b^2))/a
bs(2*b*e^(2*d*x + 2*c) + 4*a + 2*sqrt(4*a^2 + b^2)))/((16*a^4 + 8*a^2*b^2 +
 b^4)*sqrt(4*a^2 + b^2)) + 2*(8*a^2*b*e^(6*d*x + 6*c) - b^3*e^(6*d*x + 6*c)
 + 48*a^3*e^(4*d*x + 4*c) - 6*a*b^2*e^(4*d*x + 4*c) - 40*a^2*b*e^(2*d*x + 2
*c) - b^3*e^(2*d*x + 2*c) + 6*a*b^2)/((16*a^4 + 8*a^2*b^2 + b^4)*(b*e^(4*d*
x + 4*c) + 4*a*e^(2*d*x + 2*c) - b)^2))/d
```

maple [B] time = 0.86, size = 2082, normalized size = 14.56

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*cosh(d*x+c)*sinh(d*x+c))^3,x)
```

```
[Out] 20/d/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^3*b-2*tanh(1/2*d*x+1/2*
c)^2*a+2*tanh(1/2*d*x+1/2*c)*b+a)^2*a*b^2/(16*a^4+8*a^2*b^2+b^4)*tanh(1/2*d
*x+1/2*c)^7+2/d/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^3*b-2*tanh(1
/2*d*x+1/2*c)^2*a+2*tanh(1/2*d*x+1/2*c)*b+a)^2/a*b^4/(16*a^4+8*a^2*b^2+b^4)
*tanh(1/2*d*x+1/2*c)^7-64/d/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^
3*b-2*tanh(1/2*d*x+1/2*c)^2*a+2*tanh(1/2*d*x+1/2*c)*b+a)^2*b*a^2/(16*a^4+8*
a^2*b^2+b^4)*tanh(1/2*d*x+1/2*c)^6+28/d/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2
*d*x+1/2*c)^3*b-2*tanh(1/2*d*x+1/2*c)^2*a+2*tanh(1/2*d*x+1/2*c)*b+a)^2*b^3/
(16*a^4+8*a^2*b^2+b^4)*tanh(1/2*d*x+1/2*c)^6+2/d/(tanh(1/2*d*x+1/2*c)^4*a+2
*tanh(1/2*d*x+1/2*c)^3*b-2*tanh(1/2*d*x+1/2*c)^2*a+2*tanh(1/2*d*x+1/2*c)*b+
a)^2*b^5/a^2/(16*a^4+8*a^2*b^2+b^4)*tanh(1/2*d*x+1/2*c)^6-116/d/(tanh(1/2*d
*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^3*b-2*tanh(1/2*d*x+1/2*c)^2*a+2*tanh(1/
2*d*x+1/2*c)*b+a)^2*a*b^2/(16*a^4+8*a^2*b^2+b^4)*tanh(1/2*d*x+1/2*c)^5-2/d/
(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^3*b-2*tanh(1/2*d*x+1/2*c)^2*
a+2*tanh(1/2*d*x+1/2*c)*b+a)^2/a*b^4/(16*a^4+8*a^2*b^2+b^4)*tanh(1/2*d*x+1/
2*c)^5+128/d/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^3*b-2*tanh(1/2*
d*x+1/2*c)^2*a+2*tanh(1/2*d*x+1/2*c)*b+a)^2*a^2*b/(16*a^4+8*a^2*b^2+b^4)*ta
nh(1/2*d*x+1/2*c)^4+72/d/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^3*b
-2*tanh(1/2*d*x+1/2*c)^2*a+2*tanh(1/2*d*x+1/2*c)*b+a)^2*b^3/(16*a^4+8*a^2*b
^2+b^4)*tanh(1/2*d*x+1/2*c)^4+4/d/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1
/2*c)^3*b-2*tanh(1/2*d*x+1/2*c)^2*a+2*tanh(1/2*d*x+1/2*c)*b+a)^2/a^2*b^5/(1
6*a^4+8*a^2*b^2+b^4)*tanh(1/2*d*x+1/2*c)^4-116/d/(tanh(1/2*d*x+1/2*c)^4*a+2
*tanh(1/2*d*x+1/2*c)^3*b-2*tanh(1/2*d*x+1/2*c)^2*a+2*tanh(1/2*d*x+1/2*c)*b+
a)^2*a*b^2/(16*a^4+8*a^2*b^2+b^4)*tanh(1/2*d*x+1/2*c)^3-2/d/(tanh(1/2*d*x+1
/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^3*b-2*tanh(1/2*d*x+1/2*c)^2*a+2*tanh(1/2*d*
x+1/2*c)*b+a)^2/a*b^4/(16*a^4+8*a^2*b^2+b^4)*tanh(1/2*d*x+1/2*c)^3-64/d/(ta
nh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^3*b-2*tanh(1/2*d*x+1/2*c)^2*a+2
*tanh(1/2*d*x+1/2*c)*b+a)^2*b*a^2/(16*a^4+8*a^2*b^2+b^4)*tanh(1/2*d*x+1/2*c
)^2+28/d/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^3*b-2*tanh(1/2*d*x+
1/2*c)^2*a+2*tanh(1/2*d*x+1/2*c)*b+a)^2*b^3/(16*a^4+8*a^2*b^2+b^4)*tanh(1/2
*d*x+1/2*c)^2+2/d/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^3*b-2*tanh
(1/2*d*x+1/2*c)^2*a+2*tanh(1/2*d*x+1/2*c)*b+a)^2*b^5/a^2/(16*a^4+8*a^2*b^2+
b^4)*tanh(1/2*d*x+1/2*c)^2+20/d/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2
*c)^3*b-2*tanh(1/2*d*x+1/2*c)^2*a+2*tanh(1/2*d*x+1/2*c)*b+a)^2*a*b^2/(16*a^
4+8*a^2*b^2+b^4)*tanh(1/2*d*x+1/2*c)+2/d/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/
2*d*x+1/2*c)^3*b-2*tanh(1/2*d*x+1/2*c)^2*a+2*tanh(1/2*d*x+1/2*c)*b+a)^2/a*b
^4/(16*a^4+8*a^2*b^2+b^4)*tanh(1/2*d*x+1/2*c)-16/d/(16*a^4+8*a^2*b^2+b^4)*a
^2/(4*a^2+b^2)^(1/2)*ln(-tanh(1/2*d*x+1/2*c)^2*a+(4*a^2+b^2)^(1/2)*tanh(1/2
*d*x+1/2*c)-tanh(1/2*d*x+1/2*c)*b-a)+2/d/(16*a^4+8*a^2*b^2+b^4)/(4*a^2+b^2)
^(1/2)*ln(-tanh(1/2*d*x+1/2*c)^2*a+(4*a^2+b^2)^(1/2)*tanh(1/2*d*x+1/2*c)-ta
nh(1/2*d*x+1/2*c)*b-a)*b^2+64/d/(16*a^4+8*a^2*b^2+b^4)*a^4/(4*a^2+b^2)^(3/2
)*ln(tanh(1/2*d*x+1/2*c)^2*a+(4*a^2+b^2)^(1/2)*tanh(1/2*d*x+1/2*c)+tanh(1/2
*d*x+1/2*c)*b+a)+8/d/(16*a^4+8*a^2*b^2+b^4)*a^2/(4*a^2+b^2)^(3/2)*ln(tanh(1
```

```
/2*d*x+1/2*c)^2*a+(4*a^2+b^2)^(1/2)*tanh(1/2*d*x+1/2*c)+tanh(1/2*d*x+1/2*c)
*b+a)*b^2-2/d/(16*a^4+8*a^2*b^2+b^4)/(4*a^2+b^2)^(3/2)*ln(tanh(1/2*d*x+1/2*
c)^2*a+(4*a^2+b^2)^(1/2)*tanh(1/2*d*x+1/2*c)+tanh(1/2*d*x+1/2*c)*b+a)*b^4
```

maxima [B] time = 0.49, size = 327, normalized size = 2.29

$$\frac{2\left(8\,a^{2}-b^{2}\right)\log\left(\frac{be^{(-2\,dx-2\,c)}-2\,a-\sqrt{4\,a^{2}+b^{2}}}{be^{(-2\,dx-2\,c)}-2\,a+\sqrt{4\,a^{2}+b^{2}}}\right)}{\left(16\,a^{4}+8\,a^{2}b^{2}+b^{4}\right)\sqrt{4\,a^{2}+b^{2}}\,d}-\frac{4\left(6\,ab^{2}+\left(40\,a^{2}b+b^{3}\right)e^{(-}}{\left(16\,a^{4}b^{2}+8\,a^{2}b^{4}+b^{6}+8\left(16\,a^{5}b+8\,a^{3}b^{3}+ab^{5}\right)e^{(-2\,dx-2\,c)}+2\left(128\,a\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cosh(d*x+c)*sinh(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 2*(8*a^2 - b^2)*log((b*e^(-2*d*x - 2*c) - 2*a - sqrt(4*a^2 + b^2))/(b*e^(-2
*d*x - 2*c) - 2*a + sqrt(4*a^2 + b^2)))/((16*a^4 + 8*a^2*b^2 + b^4)*sqrt(4*
a^2 + b^2)*d) - 4*(6*a*b^2 + (40*a^2*b + b^3)*e^(-2*d*x - 2*c) + 6*(8*a^3 -
 a*b^2)*e^(-4*d*x - 4*c) - (8*a^2*b - b^3)*e^(-6*d*x - 6*c))/((16*a^4*b^2 +
 8*a^2*b^4 + b^6 + 8*(16*a^5*b + 8*a^3*b^3 + a*b^5)*e^(-2*d*x - 2*c) + 2*(1
28*a^6 + 48*a^4*b^2 - b^6)*e^(-4*d*x - 4*c) - 8*(16*a^5*b + 8*a^3*b^3 + a*b
^5)*e^(-6*d*x - 6*c) + (16*a^4*b^2 + 8*a^2*b^4 + b^6)*e^(-8*d*x - 8*c))*d)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{1}{\left(a+b\cosh\left(c+d\,x\right)\,\sinh\left(c+d\,x\right)\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*cosh(c + d*x)*sinh(c + d*x))^3,x)
```

```
[Out] int(1/(a + b*cosh(c + d*x)*sinh(c + d*x))^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cosh(d*x+c)*sinh(d*x+c))**3,x)
```

```
[Out] Timed out
```

3.861 $\int (a + b\cosh(c + dx)\sinh(c + dx))^{5/2}\,dx$

Optimal. Leaf size=301

$$\frac{2i\sqrt{2}\,a\left(4a^2+b^2\right)\sqrt{\frac{2a+b\sinh(2c+2dx)}{2a-ib}}\,F\left(\frac{1}{2}\left(2ic+2idx-\frac{\pi}{2}\right)|\frac{2b}{2ia+b}\right)}{15d\sqrt{2a+b\sinh(2c+2dx)}}-\frac{i\left(92a^2-9b^2\right)\sqrt{2a+b\sinh(2c+2dx)}\,E\left(\frac{1}{2}\left(2ic\right.\right.}{60\sqrt{2}\,d\sqrt{\frac{2a+b\sinh(2c+2dx)}{2a-ib}}}$$

```
[Out] 1/40*b*cosh(2*d*x+2*c)*(2*a+b*sinh(2*d*x+2*c))^(3/2)/d*2^(1/2)+2/15*a*b*cos
h(2*d*x+2*c)*2^(1/2)*(2*a+b*sinh(2*d*x+2*c))^(1/2)/d+1/120*I*(92*a^2-9*b^2)
*(sin(I*c+1/4*Pi+I*d*x)^2)^(1/2)/sin(I*c+1/4*Pi+I*d*x)*EllipticE(cos(I*c+1/
4*Pi+I*d*x),2^(1/2)*(b/(2*I*a+b))^(1/2))*(2*a+b*sinh(2*d*x+2*c))^(1/2)/d*2^
(1/2)/((2*a+b*sinh(2*d*x+2*c))/(2*a-I*b))^(1/2)-2/15*I*a*(4*a^2+b^2)*(sin(I
*c+1/4*Pi+I*d*x)^2)^(1/2)/sin(I*c+1/4*Pi+I*d*x)*EllipticF(cos(I*c+1/4*Pi+I*
d*x),2^(1/2)*(b/(2*I*a+b))^(1/2))*2^(1/2)*((2*a+b*sinh(2*d*x+2*c))/(2*a-I*b
))^(1/2)/d/(2*a+b*sinh(2*d*x+2*c))^(1/2)
```

Rubi [A] time = 0.39, antiderivative size = 301, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {2666, 2656, 2753, 2752, 2663, 2661, 2655, 2653}

$$\frac{2i\sqrt{2}\,a\left(4a^2+b^2\right)\sqrt{\frac{2a+b\sinh(2c+2dx)}{2a-ib}}\,F\left(\frac{1}{2}\left(2ic+2idx-\frac{\pi}{2}\right)|\frac{2b}{2ia+b}\right)}{15d\sqrt{2a+b\sinh(2c+2dx)}}-\frac{i\left(92a^2-9b^2\right)\sqrt{2a+b\sinh(2c+2dx)}\,E\left(\frac{1}{2}\left(2ic\right.\right.}{60\sqrt{2}\,d\sqrt{\frac{2a+b\sinh(2c+2dx)}{2a-ib}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cosh[c + d*x]*Sinh[c + d*x])^(5/2),x]
```

```
[Out] (2*Sqrt[2]*a*b*Cosh[2*c + 2*d*x]*Sqrt[2*a + b*Sinh[2*c + 2*d*x]])/(15*d) +
(b*Cosh[2*c + 2*d*x]*(2*a + b*Sinh[2*c + 2*d*x])^(3/2))/(20*Sqrt[2]*d) - ((
I/60)*(92*a^2 - 9*b^2)*EllipticE[((2*I)*c - Pi/2 + (2*I)*d*x)/2, (2*b)/((2*
I)*a + b)]*Sqrt[2*a + b*Sinh[2*c + 2*d*x]])/(Sqrt[2]*d*Sqrt[(2*a + b*Sinh[2
*c + 2*d*x])/(2*a - I*b)]) + (((2*I)/15)*Sqrt[2]*a*(4*a^2 + b^2)*EllipticF[
((2*I)*c - Pi/2 + (2*I)*d*x)/2, (2*b)/((2*I)*a + b)]*Sqrt[(2*a + b*Sinh[2*c
 + 2*d*x])/(2*a - I*b)])/(d*Sqrt[2*a + b*Sinh[2*c + 2*d*x]])
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2656

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[1/n, Int[(a + b*Sin
[c + d*x])^(n - 2)*Simp[a^2*n + b^2*(n - 1) + a*b*(2*n - 1)*Sin[c + d*x], x
], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[n, 1] && In
tegerQ[2*n]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2666

```
Int[((a_) + cos[(c_.) + (d_.)*(x_)]*(b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_
Symbol] :> Int[(a + (b*Sin[2*c + 2*d*x])/2)^n, x] /; FreeQ[{a, b, c, d, n},
 x]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2753

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d*m
 + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a
```

```
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
 && IntegerQ[2*m]
```

Rubi steps

$$\int (a+b\cosh(c+dx)\sinh(c+dx))^{5/2}\,dx=\int \left(a+\frac{1}{2}b\sinh(2c+2dx)\right)^{5/2}\,dx$$

$$=\frac{b\cosh(2c+2dx)(2a+b\sinh(2c+2dx))^{3/2}}{20\sqrt{2}\,d}+\frac{2}{5}\int \sqrt{a+\frac{1}{2}b\sinh(2c$$

$$=\frac{2\sqrt{2}\,ab\cosh(2c+2dx)\sqrt{2a+b\sinh(2c+2dx)}}{15d}+\frac{b\cosh(2c+2dx)(}{}$$

$$=\frac{2\sqrt{2}\,ab\cosh(2c+2dx)\sqrt{2a+b\sinh(2c+2dx)}}{15d}+\frac{b\cosh(2c+2dx)(}{}$$

$$=\frac{2\sqrt{2}\,ab\cosh(2c+2dx)\sqrt{2a+b\sinh(2c+2dx)}}{15d}+\frac{b\cosh(2c+2dx)(}{}$$

$$=\frac{2\sqrt{2}\,ab\cosh(2c+2dx)\sqrt{2a+b\sinh(2c+2dx)}}{15d}+\frac{b\cosh(2c+2dx)(}{}$$

Mathematica [A] time = 1.41, size = 239, normalized size = 0.79

$$\frac{-32ia\left(4a^2+b^2\right)\sqrt{\frac{2a+b\sinh(2(c+dx))}{2a-ib}}\,F\left(\frac{1}{4}(-4ic-4idx+\pi)|-\frac{2ib}{2a-ib}\right)+b\left(88a^2\cosh(2(c+dx))+b\sinh(4(c+dx))(\right.}{120d\sqrt{4a+2b\,s}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cosh[c + d*x]*Sinh[c + d*x])^(5/2),x]
```

```
[Out] (2*((184*I)*a^3 + 92*a^2*b - (18*I)*a*b^2 - 9*b^3)*EllipticE[((-4*I)*c + Pi
 - (4*I)*d*x)/4, ((-2*I)*b)/(2*a - I*b)]*Sqrt[(2*a + b*Sinh[2*(c + d*x)])/(
2*a - I*b)] - (32*I)*a*(4*a^2 + b^2)*EllipticF[((-4*I)*c + Pi - (4*I)*d*x)/
4, ((-2*I)*b)/(2*a - I*b)]*Sqrt[(2*a + b*Sinh[2*(c + d*x)])/(2*a - I*b)] +
```

```
b*(88*a^2*Cosh[2*(c + d*x)] + b*(28*a + 3*b*Sinh[2*(c + d*x)])*Sinh[4*(c +
d*x)]))/(120*d*Sqrt[4*a + 2*b*Sinh[2*(c + d*x)]])
```

fricas [F] time = 0.48, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b^2\cosh\left(dx+c\right)^2\sinh\left(dx+c\right)^2+2\,ab\cosh\left(dx+c\right)\sinh\left(dx+c\right)+a^2\right)\sqrt{b\cosh\left(dx+c\right)\sinh\left(dx+c\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cosh(d*x+c)*sinh(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] integral((b^2*cosh(d*x + c)^2*sinh(d*x + c)^2 + 2*a*b*cosh(d*x + c)*sinh(d*
x + c) + a^2)*sqrt(b*cosh(d*x + c)*sinh(d*x + c) + a), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cosh(d*x+c)*sinh(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:index.cc index_m i_lex_is_greater Error: Bad
 Argument Value
```

maple [B] time = 1.11, size = 1260, normalized size = 4.19

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cosh(d*x+c)*sinh(d*x+c))^(5/2),x)
```

```
[Out] 1/60*(64*I*(-(2*a+b*sinh(2*d*x+2*c))/(I*b-2*a))^(1/2)*((-sinh(2*d*x+2*c)+I)
*b/(I*b+2*a))^(1/2)*((sinh(2*d*x+2*c)+I)*b/(I*b-2*a))^(1/2)*EllipticF((-(2*
a+b*sinh(2*d*x+2*c))/(I*b-2*a))^(1/2),(-(I*b-2*a)/(I*b+2*a))^(1/2))*a^3*b+1
6*I*(-(2*a+b*sinh(2*d*x+2*c))/(I*b-2*a))^(1/2)*((-sinh(2*d*x+2*c)+I)*b/(I*b
+2*a))^(1/2)*((sinh(2*d*x+2*c)+I)*b/(I*b-2*a))^(1/2)*EllipticF((-(2*a+b*sin
h(2*d*x+2*c))/(I*b-2*a))^(1/2),(-(I*b-2*a)/(I*b+2*a))^(1/2))*a*b^3+240*(-(2
*a+b*sinh(2*d*x+2*c))/(I*b-2*a))^(1/2)*((-sinh(2*d*x+2*c)+I)*b/(I*b+2*a))^(
1/2)*((sinh(2*d*x+2*c)+I)*b/(I*b-2*a))^(1/2)*EllipticF((-(2*a+b*sinh(2*d*x+
2*c))/(I*b-2*a))^(1/2),(-(I*b-2*a)/(I*b+2*a))^(1/2))*a^4+24*(-(2*a+b*sinh(2
*d*x+2*c))/(I*b-2*a))^(1/2)*((-sinh(2*d*x+2*c)+I)*b/(I*b+2*a))^(1/2)*((sinh
(2*d*x+2*c)+I)*b/(I*b-2*a))^(1/2)*EllipticF((-(2*a+b*sinh(2*d*x+2*c))/(I*b-
2*a))^(1/2),(-(I*b-2*a)/(I*b+2*a))^(1/2))*a^2*b^2-9*(-(2*a+b*sinh(2*d*x+2*c
))/(I*b-2*a))^(1/2)*((-sinh(2*d*x+2*c)+I)*b/(I*b+2*a))^(1/2)*((sinh(2*d*x+2
```

```
*c)+I)*b/(I*b-2*a))^(1/2)*EllipticF((-(2*a+b*sinh(2*d*x+2*c))/(I*b-2*a))^(1
/2),(-(I*b-2*a)/(I*b+2*a))^(1/2))*b^4-368*(-(2*a+b*sinh(2*d*x+2*c))/(I*b-2*
a))^(1/2)*((-sinh(2*d*x+2*c)+I)*b/(I*b+2*a))^(1/2)*((sinh(2*d*x+2*c)+I)*b/(
I*b-2*a))^(1/2)*EllipticE((-(2*a+b*sinh(2*d*x+2*c))/(I*b-2*a))^(1/2),(-(I*b
-2*a)/(I*b+2*a))^(1/2))*a^4-56*(-(2*a+b*sinh(2*d*x+2*c))/(I*b-2*a))^(1/2)*(
(-sinh(2*d*x+2*c)+I)*b/(I*b+2*a))^(1/2)*((sinh(2*d*x+2*c)+I)*b/(I*b-2*a))^(
1/2)*EllipticE((-(2*a+b*sinh(2*d*x+2*c))/(I*b-2*a))^(1/2),(-(I*b-2*a)/(I*b+
2*a))^(1/2))*a^2*b^2+9*(-(2*a+b*sinh(2*d*x+2*c))/(I*b-2*a))^(1/2)*((-sinh(2
*d*x+2*c)+I)*b/(I*b+2*a))^(1/2)*((sinh(2*d*x+2*c)+I)*b/(I*b-2*a))^(1/2)*Ell
ipticE((-(2*a+b*sinh(2*d*x+2*c))/(I*b-2*a))^(1/2),(-(I*b-2*a)/(I*b+2*a))^(1
/2))*b^4+3*b^4*sinh(2*d*x+2*c)^4+28*a*b^3*sinh(2*d*x+2*c)^3+44*a^2*b^2*sinh
(2*d*x+2*c)^2+3*b^4*sinh(2*d*x+2*c)^2+28*a*b^3*sinh(2*d*x+2*c)+44*a^2*b^2)/
b/cosh(2*d*x+2*c)/(4*a+2*b*sinh(2*d*x+2*c))^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\cosh (dx+c)\sinh (dx+c)+a)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cosh(d*x+c)*sinh(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*cosh(d*x + c)*sinh(d*x + c) + a)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int {\left(a+b\,\mathrm{cosh}\left(c+d\,x\right)\,\mathrm{sinh}\left(c+d\,x\right)\right)}^{5/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*cosh(c + d*x)*sinh(c + d*x))^(5/2),x)
```

```
[Out] int((a + b*cosh(c + d*x)*sinh(c + d*x))^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cosh(d*x+c)*sinh(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

3.862 $\int (a+b\cosh(c+dx)\sinh(c+dx))^{3/2}\,dx$

Optimal. Leaf size=248

$$\frac{i\left(4a^2+b^2\right)\sqrt{\frac{2a+b\sinh(2c+2dx)}{2a-ib}}\,F\left(\frac{1}{2}\left(2ic+2idx-\frac{\pi}{2}\right)|\frac{2b}{2ia+b}\right)}{6\sqrt{2}\,d\sqrt{2a+b\sinh(2c+2dx)}}+\frac{b\cosh(2c+2dx)\sqrt{2a+b\sinh(2c+2dx)}}{6\sqrt{2}\,d}-\frac{2i\sqrt{2}\,a\sqrt{}}{}$$

```
[Out] 1/12*b*cosh(2*d*x+2*c)*(2*a+b*sinh(2*d*x+2*c))^(1/2)/d*2^(1/2)+2/3*I*a*(sin
(I*c+1/4*Pi+I*d*x)^2)^(1/2)/sin(I*c+1/4*Pi+I*d*x)*EllipticE(cos(I*c+1/4*Pi+
I*d*x),2^(1/2)*(b/(2*I*a+b))^(1/2))*2^(1/2)*(2*a+b*sinh(2*d*x+2*c))^(1/2)/d
/((2*a+b*sinh(2*d*x+2*c))/(2*a-I*b))^(1/2)-1/12*I*(4*a^2+b^2)*(sin(I*c+1/4*
Pi+I*d*x)^2)^(1/2)/sin(I*c+1/4*Pi+I*d*x)*EllipticF(cos(I*c+1/4*Pi+I*d*x),2^
(1/2)*(b/(2*I*a+b))^(1/2))*((2*a+b*sinh(2*d*x+2*c))/(2*a-I*b))^(1/2)/d*2^(1
/2)/(2*a+b*sinh(2*d*x+2*c))^(1/2)
```

Rubi [A] time = 0.25, antiderivative size = 248, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {2666, 2656, 2752, 2663, 2661, 2655, 2653}

$$\frac{i\left(4a^2+b^2\right)\sqrt{\frac{2a+b\sinh(2c+2dx)}{2a-ib}}\,F\left(\frac{1}{2}\left(2ic+2idx-\frac{\pi}{2}\right)|\frac{2b}{2ia+b}\right)}{6\sqrt{2}\,d\sqrt{2a+b\sinh(2c+2dx)}}+\frac{b\cosh(2c+2dx)\sqrt{2a+b\sinh(2c+2dx)}}{6\sqrt{2}\,d}-\frac{2i\sqrt{2}\,a\sqrt{}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cosh[c + d*x]*Sinh[c + d*x])^(3/2),x]
```

```
[Out] (b*Cosh[2*c + 2*d*x]*Sqrt[2*a + b*Sinh[2*c + 2*d*x]])/(6*Sqrt[2]*d) - (((2*
I)/3)*Sqrt[2]*a*EllipticE[((2*I)*c - Pi/2 + (2*I)*d*x)/2, (2*b)/((2*I)*a +
b)]*Sqrt[2*a + b*Sinh[2*c + 2*d*x]])/(d*Sqrt[(2*a + b*Sinh[2*c + 2*d*x])/(2
*a - I*b)]) + ((I/6)*(4*a^2 + b^2)*EllipticF[((2*I)*c - Pi/2 + (2*I)*d*x)/2
, (2*b)/((2*I)*a + b)]*Sqrt[(2*a + b*Sinh[2*c + 2*d*x])/(2*a - I*b)])/(Sqrt
[2]*d*Sqrt[2*a + b*Sinh[2*c + 2*d*x]])
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
```

```
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2656

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[1/n, Int[(a + b*Sin
[c + d*x])^(n - 2)*Simp[a^2*n + b^2*(n - 1) + a*b*(2*n - 1)*Sin[c + d*x], x
], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[n, 1] && In
tegerQ[2*n]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2666

```
Int[((a_) + cos[(c_.) + (d_.)*(x_)]*(b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_
Symbol] :> Int[(a + (b*Sin[2*c + 2*d*x])/2)^n, x] /; FreeQ[{a, b, c, d, n},
 x]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int (a+b\cosh(c+dx)\sinh(c+dx))^{3/2}\,dx = \int \left(a+\frac{1}{2}b\sinh(2c+2dx)\right)^{3/2} dx$$

$$= \frac{b\cosh(2c+2dx)\sqrt{2a+b\sinh(2c+2dx)}}{6\sqrt{2}\,d} + \frac{2}{3}\int \frac{\frac{1}{8}\left(12a^2-b^2\right)+a}{\sqrt{a+\frac{1}{2}b\sinh}}$$

$$= \frac{b\cosh(2c+2dx)\sqrt{2a+b\sinh(2c+2dx)}}{6\sqrt{2}\,d} + \frac{1}{3}(4a)\int \sqrt{a+\frac{1}{2}b\sinh}$$

$$= \frac{b\cosh(2c+2dx)\sqrt{2a+b\sinh(2c+2dx)}}{6\sqrt{2}\,d} + \frac{\left(4a\sqrt{a+\frac{1}{2}b\sinh(2c+\right.}{3\sqrt{}}$$

$$= \frac{b\cosh(2c+2dx)\sqrt{2a+b\sinh(2c+2dx)}}{6\sqrt{2}\,d} - \frac{2i\sqrt{2}\,aE\left(\frac{1}{2}\left(2ic-\frac{\pi}{2}+\right.\right.}{3d}$$

Mathematica [A] time = 0.85, size = 202, normalized size = 0.81

$$\frac{-2i\left(4a^2+b^2\right)\sqrt{\frac{2a+b\sinh(2(c+dx))}{2a-ib}}\,F\left(\frac{1}{4}(-4ic-4idx+\pi)|-\frac{2ib}{2a-ib}\right)+b(4a\cosh(2(c+dx))+b\sinh(4(c+dx)))+1}{12d\sqrt{4a+2b\sinh(2(c+dx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cosh[c + d*x]*Sinh[c + d*x])^(3/2),x]
```

```
[Out] (16*a*((2*I)*a + b)*EllipticE[((-4*I)*c + Pi - (4*I)*d*x)/4, ((-2*I)*b)/(2*
a - I*b)]*Sqrt[(2*a + b*Sinh[2*(c + d*x)])/(2*a - I*b)] - (2*I)*(4*a^2 + b^
2)*EllipticF[((-4*I)*c + Pi - (4*I)*d*x)/4, ((-2*I)*b)/(2*a - I*b)]*Sqrt[(2
*a + b*Sinh[2*(c + d*x)])/(2*a - I*b)] + b*(4*a*Cosh[2*(c + d*x)] + b*Sinh[
4*(c + d*x)]))/(12*d*Sqrt[4*a + 2*b*Sinh[2*(c + d*x)]])
```

fricas [F] time = 0.47, size = 0, normalized size = 0.00

$$\text{integral}\left((b\cosh(dx+c)\sinh(dx+c)+a)^{\frac{3}{2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cosh(d*x+c)*sinh(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b*cosh(d*x + c)*sinh(d*x + c) + a)^(3/2), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cosh(d*x+c)*sinh(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:index.cc index_m i_lex_is_greater Error: Bad
 Argument Value
```

maple [B] time = 1.08, size = 935, normalized size = 3.77

$$\frac{4i\sqrt{-\frac{2a+b\sinh(2dx+2c)}{ib-2a}}\sqrt{\frac{(-\sinh(2dx+2c)+i)b}{ib+2a}}\sqrt{\frac{(\sinh(2dx+2c)+i)b}{ib-2a}}\ \mathrm{EllipticF}\left(\sqrt{-\frac{2a+b\sinh(2dx+2c)}{ib-2a}},\sqrt{-\frac{ib-2a}{ib+2a}}\right)a^2b+i\sqrt{-\frac{2a+}{}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cosh(d*x+c)*sinh(d*x+c))^(3/2),x)
```

```
[Out] 1/6*(4*I*(-(2*a+b*sinh(2*d*x+2*c))/(I*b-2*a))^(1/2)*((-sinh(2*d*x+2*c)+I)*b
/(I*b+2*a))^(1/2)*((sinh(2*d*x+2*c)+I)*b/(I*b-2*a))^(1/2)*EllipticF((-(2*a+
b*sinh(2*d*x+2*c))/(I*b-2*a))^(1/2),(-(I*b-2*a)/(I*b+2*a))^(1/2))*a^2*b+I*(
-(2*a+b*sinh(2*d*x+2*c))/(I*b-2*a))^(1/2)*((-sinh(2*d*x+2*c)+I)*b/(I*b+2*a)
)^(1/2)*((sinh(2*d*x+2*c)+I)*b/(I*b-2*a))^(1/2)*EllipticF((-(2*a+b*sinh(2*d
*x+2*c))/(I*b-2*a))^(1/2),(-(I*b-2*a)/(I*b+2*a))^(1/2))*b^3+24*(-(2*a+b*sin
h(2*d*x+2*c))/(I*b-2*a))^(1/2)*((-sinh(2*d*x+2*c)+I)*b/(I*b+2*a))^(1/2)*((s
inh(2*d*x+2*c)+I)*b/(I*b-2*a))^(1/2)*EllipticF((-(2*a+b*sinh(2*d*x+2*c))/(I
*b-2*a))^(1/2),(-(I*b-2*a)/(I*b+2*a))^(1/2))*a^3+6*(-(2*a+b*sinh(2*d*x+2*c)
)/(I*b-2*a))^(1/2)*((-sinh(2*d*x+2*c)+I)*b/(I*b+2*a))^(1/2)*((sinh(2*d*x+2*
c)+I)*b/(I*b-2*a))^(1/2)*EllipticF((-(2*a+b*sinh(2*d*x+2*c))/(I*b-2*a))^(1/
2),(-(I*b-2*a)/(I*b+2*a))^(1/2))*a*b^2-32*(-(2*a+b*sinh(2*d*x+2*c))/(I*b-2*
a))^(1/2)*((-sinh(2*d*x+2*c)+I)*b/(I*b+2*a))^(1/2)*((sinh(2*d*x+2*c)+I)*b/(
I*b-2*a))^(1/2)*EllipticE((-(2*a+b*sinh(2*d*x+2*c))/(I*b-2*a))^(1/2),(-(I*b
-2*a)/(I*b+2*a))^(1/2))*a^3-8*(-(2*a+b*sinh(2*d*x+2*c))/(I*b-2*a))^(1/2)*((
-sinh(2*d*x+2*c)+I)*b/(I*b+2*a))^(1/2)*((sinh(2*d*x+2*c)+I)*b/(I*b-2*a))^(1
/2)*EllipticE((-(2*a+b*sinh(2*d*x+2*c))/(I*b-2*a))^(1/2),(-(I*b-2*a)/(I*b+2
*a))^(1/2))*a*b^2+b^3*sinh(2*d*x+2*c)^3+2*a*b^2*sinh(2*d*x+2*c)^2+b^3*sinh(
2*d*x+2*c)+2*a*b^2)/b/cosh(2*d*x+2*c)/(4*a+2*b*sinh(2*d*x+2*c))^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\cosh(dx+c)\sinh(dx+c)+a)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cosh(d*x+c)*sinh(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*cosh(d*x + c)*sinh(d*x + c) + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int (a + b\cosh(c + d\,x)\,\sinh(c + d\,x))^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*cosh(c + d*x)*sinh(c + d*x))^(3/2),x)
```

```
[Out] int((a + b*cosh(c + d*x)*sinh(c + d*x))^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cosh(d*x+c)*sinh(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

3.863 $\int \sqrt{a+b\cosh(c+dx)\sinh(c+dx)}\,dx$

Optimal. Leaf size=96

$$-\frac{i\sqrt{2a+b\sinh(2c+2dx)}\,E\left(\frac{1}{2}\left(2ic+2idx-\frac{\pi}{2}\right)|\frac{2b}{2ia+b}\right)}{\sqrt{2}\,d\sqrt{\frac{2a+b\sinh(2c+2dx)}{2a-ib}}}$$

```
[Out] 1/2*I*(sin(I*c+1/4*Pi+I*d*x)^2)^(1/2)/sin(I*c+1/4*Pi+I*d*x)*EllipticE(cos(I
*c+1/4*Pi+I*d*x),2^(1/2)*(b/(2*I*a+b))^(1/2))*(2*a+b*sinh(2*d*x+2*c))^(1/2)
/d*2^(1/2)/((2*a+b*sinh(2*d*x+2*c))/(2*a-I*b))^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {2666, 2655, 2653}

$$-\frac{i\sqrt{2a+b\sinh(2c+2dx)}\,E\left(\frac{1}{2}\left(2ic+2idx-\frac{\pi}{2}\right)|\frac{2b}{2ia+b}\right)}{\sqrt{2}\,d\sqrt{\frac{2a+b\sinh(2c+2dx)}{2a-ib}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Cosh[c + d*x]*Sinh[c + d*x]],x]
```

```
[Out] ((-I)*EllipticE[((2*I)*c - Pi/2 + (2*I)*d*x)/2, (2*b)/((2*I)*a + b)]*Sqrt[2
*a + b*Sinh[2*c + 2*d*x]])/(Sqrt[2]*d*Sqrt[(2*a + b*Sinh[2*c + 2*d*x])/(2*a
 - I*b)])
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2666

```
Int[((a_) + cos[(c_.) + (d_.)*(x_)]*(b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_
Symbol] :> Int[(a + (b*Sin[2*c + 2*d*x])/2)^n, x] /; FreeQ[{a, b, c, d, n},
```

```
x]
```

Rubi steps

$$\int \sqrt{a+b\cosh(c+dx)\sinh(c+dx)}\,dx = \int \sqrt{a+\frac{1}{2}b\sinh(2c+2dx)}\,dx$$
$$= \frac{\sqrt{a+\frac{1}{2}b\sinh(2c+2dx)}\int \sqrt{\frac{a}{a-\frac{ib}{2}}+\frac{b\sinh(2c+2dx)}{2\left(a-\frac{ib}{2}\right)}}\,dx}{\sqrt{\frac{a+\frac{1}{2}b\sinh(2c+2dx)}{a-\frac{ib}{2}}}}$$
$$= -\frac{iE\left(\frac{1}{2}\left(2ic-\frac{\pi}{2}+2idx\right)|\frac{2b}{2ia+b}\right)\sqrt{2a+b\sinh(2c+2dx)}}{\sqrt{2}\,d\sqrt{\frac{2a+b\sinh(2c+2dx)}{2a-ib}}}$$

Mathematica [A] time = 0.12, size = 94, normalized size = 0.98

$$\frac{(b+2ia)\sqrt{\frac{2a+b\sinh(2(c+dx))}{2a-ib}}\,E\left(\frac{1}{4}(-4ic-4idx+\pi)|-\frac{2ib}{2a-ib}\right)}{d\sqrt{4a+2b\sinh(2(c+dx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Cosh[c + d*x]*Sinh[c + d*x]],x]
```

```
[Out] (((2*I)*a + b)*EllipticE[((-4*I)*c + Pi - (4*I)*d*x)/4, ((-2*I)*b)/(2*a - I
*b)]*Sqrt[(2*a + b*Sinh[2*(c + d*x)])/(2*a - I*b)])/(d*Sqrt[4*a + 2*b*Sinh[
2*(c + d*x)]])
```

fricas [F] time = 0.46, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{b\cosh\left(dx+c\right)\sinh\left(dx+c\right)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cosh(d*x+c)*sinh(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*cosh(d*x + c)*sinh(d*x + c) + a), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cosh(d*x+c)*sinh(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:index.cc index_m i_lex_is_greater Error: Bad
 Argument Value
```

maple [B] time = 1.17, size = 351, normalized size = 3.66

$$\frac{(ib-2a)\sqrt{-\frac{2a+b\sinh(2dx+2c)}{ib-2a}}\sqrt{\frac{(-\sinh(2dx+2c)+i)b}{ib+2a}}\sqrt{\frac{(\sinh(2dx+2c)+i)b}{ib-2a}}\left(i\,\mathrm{EllipticE}\left(\sqrt{-\frac{2a+b\sinh(2dx+2c)}{ib-2a}},\sqrt{-\frac{ib-2a}{ib+2a}}\right)b-i}{b\cosh(2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cosh(d*x+c)*sinh(d*x+c))^(1/2),x)
```

```
[Out] 1/b*(I*b-2*a)*(-(2*a+b*sinh(2*d*x+2*c))/(I*b-2*a))^(1/2)*((-sinh(2*d*x+2*c)
+I)*b/(I*b+2*a))^(1/2)*((sinh(2*d*x+2*c)+I)*b/(I*b-2*a))^(1/2)*(I*EllipticE
((-(2*a+b*sinh(2*d*x+2*c))/(I*b-2*a))^(1/2),(-(I*b-2*a)/(I*b+2*a))^(1/2))*b
-I*EllipticF((-(2*a+b*sinh(2*d*x+2*c))/(I*b-2*a))^(1/2),(-(I*b-2*a)/(I*b+2*
a))^(1/2))*b+2*EllipticE((-(2*a+b*sinh(2*d*x+2*c))/(I*b-2*a))^(1/2),(-(I*b-
2*a)/(I*b+2*a))^(1/2))*a-2*a*EllipticF((-(2*a+b*sinh(2*d*x+2*c))/(I*b-2*a))
^(1/2),(-(I*b-2*a)/(I*b+2*a))^(1/2)))/cosh(2*d*x+2*c)/(4*a+2*b*sinh(2*d*x+2
*c))^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b\cosh(dx+c)\sinh(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cosh(d*x+c)*sinh(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*cosh(d*x + c)*sinh(d*x + c) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{a+b\cosh(c+d\,x)\,\sinh(c+d\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*cosh(c + d*x)*sinh(c + d*x))^(1/2),x)
```

```
[Out] int((a + b*cosh(c + d*x)*sinh(c + d*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + b\sinh(c + dx)\cosh(c + dx)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cosh(d*x+c)*sinh(d*x+c))**(1/2),x)
[Out] Integral(sqrt(a + b*sinh(c + d*x)*cosh(c + d*x)), x)
```

3.864 $\int \frac{1}{\sqrt{a+b\cosh(c+dx)\sinh(c+dx)}}\,dx$

Optimal. Leaf size=96

$$-\frac{i\sqrt{2}\sqrt{\frac{2a+b\sinh(2c+2dx)}{2a-ib}}\,F\left(\frac{1}{2}\left(2ic+2idx-\frac{\pi}{2}\right)|\frac{2b}{2ia+b}\right)}{d\sqrt{2a+b\sinh(2c+2dx)}}$$

```
[Out] I*(sin(I*c+1/4*Pi+I*d*x)^2)^(1/2)/sin(I*c+1/4*Pi+I*d*x)*EllipticF(cos(I*c+1
/4*Pi+I*d*x),2^(1/2)*(b/(2*I*a+b))^(1/2))*2^(1/2)*((2*a+b*sinh(2*d*x+2*c))/
(2*a-I*b))^(1/2)/d/(2*a+b*sinh(2*d*x+2*c))^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {2666, 2663, 2661}

$$-\frac{i\sqrt{2}\sqrt{\frac{2a+b\sinh(2c+2dx)}{2a-ib}}\,F\left(\frac{1}{2}\left(2ic+2idx-\frac{\pi}{2}\right)|\frac{2b}{2ia+b}\right)}{d\sqrt{2a+b\sinh(2c+2dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a + b*Cosh[c + d*x]*Sinh[c + d*x]],x]
```

```
[Out] ((-I)*Sqrt[2]*EllipticF[((2*I)*c - Pi/2 + (2*I)*d*x)/2, (2*b)/((2*I)*a + b)
]*Sqrt[(2*a + b*Sinh[2*c + 2*d*x])/(2*a - I*b)])/(d*Sqrt[2*a + b*Sinh[2*c +
 2*d*x]])
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2666

```
Int[((a_) + cos[(c_.) + (d_.)*(x_)]*(b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_
Symbol] :> Int[(a + (b*Sin[2*c + 2*d*x])/2)^n, x] /; FreeQ[{a, b, c, d, n},
```

```
x]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+b\cosh(c+dx)\sinh(c+dx)}}\,dx = \int \frac{1}{\sqrt{a+\frac{1}{2}b\sinh(2c+2dx)}}\,dx$$
$$= \frac{\sqrt{\frac{a+\frac{1}{2}b\sinh(2c+2dx)}{a-\frac{ib}{2}}}\int \frac{1}{\sqrt{\frac{a}{a-\frac{ib}{2}}+\frac{b\sinh(2c+2dx)}{2\left(a-\frac{ib}{2}\right)}}}\,dx}{\sqrt{a+\frac{1}{2}b\sinh(2c+2dx)}}$$
$$= -\frac{i\sqrt{2}\,F\left(\frac{1}{2}\left(2ic-\frac{\pi}{2}+2idx\right)|\frac{2b}{2ia+b}\right)\sqrt{\frac{2a+b\sinh(2c+2dx)}{2a-ib}}}{d\sqrt{2a+b\sinh(2c+2dx)}}$$

Mathematica [A] time = 0.14, size = 90, normalized size = 0.94

$$\frac{i\sqrt{\frac{2a+b\sinh(2(c+dx))}{2a-ib}}\,F\left(\frac{1}{4}(-4ic-4idx+\pi)|-\frac{2ib}{2a-ib}\right)}{d\sqrt{a+\frac{1}{2}b\sinh(2(c+dx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a + b*Cosh[c + d*x]*Sinh[c + d*x]],x]
```

```
[Out] (I*EllipticF[((-4*I)*c + Pi - (4*I)*d*x)/4, ((-2*I)*b)/(2*a - I*b)]*Sqrt[(2
*a + b*Sinh[2*(c + d*x)])/(2*a - I*b)])/(d*Sqrt[a + (b*Sinh[2*(c + d*x)])/2
])
```

fricas [F] time = 0.43, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{\sqrt{b\cosh(dx+c)\sinh(dx+c)+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cosh(d*x+c)*sinh(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(1/sqrt(b*cosh(d*x + c)*sinh(d*x + c) + a), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cosh(d*x+c)*sinh(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:index.cc index_m i_lex_is_greater Error: Bad
 Argument Value
```

maple [A] time = 0.90, size = 181, normalized size = 1.89

$$-\frac{2\left(ib-2a\right)\sqrt{-\frac{2a+b\sinh(2dx+2c)}{ib-2a}}\,\sqrt{\frac{(-\sinh(2dx+2c)+i)b}{ib+2a}}\,\sqrt{\frac{(\sinh(2dx+2c)+i)b}{ib-2a}}\,\mathrm{EllipticF}\left(\sqrt{-\frac{2a+b\sinh(2dx+2c)}{ib-2a}},\sqrt{-\frac{ib-2a}{ib+2a}}\right)}{b\cosh\left(2dx+2c\right)\sqrt{4a+2b\sinh\left(2dx+2c\right)}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*cosh(d*x+c)*sinh(d*x+c))^(1/2),x)
```

```
[Out] -2*(I*b-2*a)*(-(2*a+b*sinh(2*d*x+2*c))/(I*b-2*a))^(1/2)*((-sinh(2*d*x+2*c)+
I)*b/(I*b+2*a))^(1/2)*((sinh(2*d*x+2*c)+I)*b/(I*b-2*a))^(1/2)*EllipticF((-(
2*a+b*sinh(2*d*x+2*c))/(I*b-2*a))^(1/2),(-(I*b-2*a)/(I*b+2*a))^(1/2))/b/cos
h(2*d*x+2*c)/(4*a+2*b*sinh(2*d*x+2*c))^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{b\cosh\left(dx+c\right)\sinh\left(dx+c\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cosh(d*x+c)*sinh(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(b*cosh(d*x + c)*sinh(d*x + c) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{a+b\cosh\left(c+d\,x\right)\,\sinh\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*cosh(c + d*x)*sinh(c + d*x))^(1/2),x)
```

```
[Out] int(1/(a + b*cosh(c + d*x)*sinh(c + d*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a+b\sinh\left(c+dx\right)\cosh\left(c+dx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cosh(d*x+c)*sinh(d*x+c))**(1/2),x)
```

```
[Out] Integral(1/sqrt(a + b*sinh(c + d*x)*cosh(c + d*x)), x)
```

3.865 $\int \frac{1}{(a+b\cosh(c+dx)\sinh(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=158

$$-\frac{2\sqrt{2}\,b\cosh(2c+2dx)}{d\left(4a^2+b^2\right)\sqrt{2a+b\sinh(2c+2dx)}}-\frac{2i\sqrt{2}\sqrt{2a+b\sinh(2c+2dx)}\,E\left(\frac{1}{2}\left(2ic+2idx-\frac{\pi}{2}\right)|\frac{2b}{2ia+b}\right)}{d\left(4a^2+b^2\right)\sqrt{\frac{2a+b\sinh(2c+2dx)}{2a-ib}}}$$

```
[Out] -2*b*cosh(2*d*x+2*c)*2^(1/2)/(4*a^2+b^2)/d/(2*a+b*sinh(2*d*x+2*c))^(1/2)+2*
I*(sin(I*c+1/4*Pi+I*d*x)^2)^(1/2)/sin(I*c+1/4*Pi+I*d*x)*EllipticE(cos(I*c+1
/4*Pi+I*d*x),2^(1/2)*(b/(2*I*a+b))^(1/2))*2^(1/2)*(2*a+b*sinh(2*d*x+2*c))^(
1/2)/(4*a^2+b^2)/d/((2*a+b*sinh(2*d*x+2*c))/(2*a-I*b))^(1/2)
```

Rubi [A] time = 0.11, antiderivative size = 158, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2666, 2664, 21, 2655, 2653}

$$-\frac{2\sqrt{2}\,b\cosh(2c+2dx)}{d\left(4a^2+b^2\right)\sqrt{2a+b\sinh(2c+2dx)}}-\frac{2i\sqrt{2}\sqrt{2a+b\sinh(2c+2dx)}\,E\left(\frac{1}{2}\left(2ic+2idx-\frac{\pi}{2}\right)|\frac{2b}{2ia+b}\right)}{d\left(4a^2+b^2\right)\sqrt{\frac{2a+b\sinh(2c+2dx)}{2a-ib}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cosh[c + d*x]*Sinh[c + d*x])^(-3/2),x]
```

```
[Out] (-2*Sqrt[2]*b*Cosh[2*c + 2*d*x])/((4*a^2 + b^2)*d*Sqrt[2*a + b*Sinh[2*c + 2
*d*x]]) - ((2*I)*Sqrt[2]*EllipticE[((2*I)*c - Pi/2 + (2*I)*d*x)/2, (2*b)/((
2*I)*a + b)]*Sqrt[2*a + b*Sinh[2*c + 2*d*x]])/((4*a^2 + b^2)*d*Sqrt[(2*a +
b*Sinh[2*c + 2*d*x])/(2*a - I*b)])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2664

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n + 1))/(d*(n + 1)*(a^2 - b^2)), x] + Dist[1
/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) - b
*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^
2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2666

```
Int[((a_) + cos[(c_.) + (d_.)*(x_)]*(b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_
Symbol] :> Int[(a + (b*Sin[2*c + 2*d*x])/2)^n, x] /; FreeQ[{a, b, c, d, n},
 x]
```

Rubi steps

$$\int \frac{1}{(a+b\cosh(c+dx)\sinh(c+dx))^{3/2}}\,dx = \int \frac{1}{\left(a+\frac{1}{2}b\sinh(2c+2dx)\right)^{3/2}}\,dx$$

$$= -\frac{2\sqrt{2}\,b\cosh(2c+2dx)}{\left(4a^2+b^2\right)d\sqrt{2a+b\sinh(2c+2dx)}} - \frac{8\int \frac{-\frac{a}{2}-\frac{1}{4}b\sinh(2c+2dx)}{\sqrt{a+\frac{1}{2}b\sinh(2c+2dx)}}\,dx}{4a^2+b^2}$$

$$= -\frac{2\sqrt{2}\,b\cosh(2c+2dx)}{\left(4a^2+b^2\right)d\sqrt{2a+b\sinh(2c+2dx)}} + \frac{4\int \sqrt{a+\frac{1}{2}b\sinh(2c+2}}{4a^2+b^2}$$

$$= -\frac{2\sqrt{2}\,b\cosh(2c+2dx)}{\left(4a^2+b^2\right)d\sqrt{2a+b\sinh(2c+2dx)}} + \frac{\left(4\sqrt{a+\frac{1}{2}b\sinh(2c+2dx}\right.}{\left(4a^2+b^2\right)\sqrt{}}$$

$$= -\frac{2\sqrt{2}\,b\cosh(2c+2dx)}{\left(4a^2+b^2\right)d\sqrt{2a+b\sinh(2c+2dx)}} - \frac{2i\sqrt{2}\,E\left(\frac{1}{2}\left(2ic-\frac{\pi}{2}+2idx\right.\right.}{\left(4a^2+b^2\right)}$$

Mathematica [A] time = 0.44, size = 119, normalized size = 0.75

$$\frac{2\left(-b\cosh(2(c+dx))+(b+2ia)\sqrt{\frac{2a+b\sinh(2(c+dx))}{2a-ib}}E\left(\frac{1}{4}(-4ic-4idx+\pi)|-\frac{2ib}{2a-ib}\right)\right)}{d\left(4a^2+b^2\right)\sqrt{a+\frac{1}{2}b\sinh(2(c+dx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cosh[c + d*x]*Sinh[c + d*x])^(-3/2),x]
```

```
[Out] (2*(-(b*Cosh[2*(c + d*x)]) + ((2*I)*a + b)*EllipticE[((-4*I)*c + Pi - (4*I)
*d*x)/4, ((-2*I)*b)/(2*a - I*b)]*Sqrt[(2*a + b*Sinh[2*(c + d*x)])/(2*a - I*
b)]))/((4*a^2 + b^2)*d*Sqrt[a + (b*Sinh[2*(c + d*x)])/2])
```

fricas [F] time = 0.47, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{b\cosh(dx+c)\sinh(dx+c)+a}}{b^2\cosh(dx+c)^2\sinh(dx+c)^2+2\,ab\cosh(dx+c)\sinh(dx+c)+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cosh(d*x+c)*sinh(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*cosh(d*x + c)*sinh(d*x + c) + a)/(b^2*cosh(d*x + c)^2*sinh(
d*x + c)^2 + 2*a*b*cosh(d*x + c)*sinh(d*x + c) + a^2), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cosh(d*x+c)*sinh(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:index.cc index_m i_lex_is_greater Error: Bad
 Argument Value
```

maple [B] time = 1.09, size = 630, normalized size = 3.99

$$16\sqrt{-\frac{2a+b\sinh(2dx+2c)}{ib-2a}}\sqrt{\frac{(-\sinh(2dx+2c)+i)b}{ib+2a}}\sqrt{\frac{(\sinh(2dx+2c)+i)b}{ib-2a}}\ \text{EllipticF}\left(\sqrt{-\frac{2a+b\sinh(2dx+2c)}{ib-2a}},\sqrt{-\frac{ib-2a}{ib+2a}}\right)a^2+4\sqrt{-\frac{2a+}{}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a+b*cosh(d*x+c)*sinh(d*x+c))^(3/2),x)

[Out] 4*(4*(-(2*a+b*sinh(2*d*x+2*c))/(I*b-2*a))^(1/2)*((-sinh(2*d*x+2*c)+I)*b/(I*b+2*a))^(1/2)*((sinh(2*d*x+2*c)+I)*b/(I*b-2*a))^(1/2)*EllipticF((-(2*a+b*sinh(2*d*x+2*c))/(I*b-2*a))^(1/2),(-(I*b-2*a)/(I*b+2*a))^(1/2))*a^2+(-(2*a+b*sinh(2*d*x+2*c))/(I*b-2*a))^(1/2)*((-sinh(2*d*x+2*c)+I)*b/(I*b+2*a))^(1/2)*((sinh(2*d*x+2*c)+I)*b/(I*b-2*a))^(1/2)*EllipticF((-(2*a+b*sinh(2*d*x+2*c))/(I*b-2*a))^(1/2),(-(I*b-2*a)/(I*b+2*a))^(1/2))*b^2-4*(-(2*a+b*sinh(2*d*x+2*c))/(I*b-2*a))^(1/2)*((-sinh(2*d*x+2*c)+I)*b/(I*b+2*a))^(1/2)*((sinh(2*d*x+2*c)+I)*b/(I*b-2*a))^(1/2)*EllipticE((-(2*a+b*sinh(2*d*x+2*c))/(I*b-2*a))^(1/2),(-(I*b-2*a)/(I*b+2*a))^(1/2))*a^2-(-(2*a+b*sinh(2*d*x+2*c))/(I*b-2*a))^(1/2)*((-sinh(2*d*x+2*c)+I)*b/(I*b+2*a))^(1/2)*((sinh(2*d*x+2*c)+I)*b/(I*b-2*a))^(1/2)*EllipticE((-(2*a+b*sinh(2*d*x+2*c))/(I*b-2*a))^(1/2),(-(I*b-2*a)/(I*b+2*a))^(1/2))*b^2-b^2*sinh(2*d*x+2*c)^2-b^2)/(4*a^2+b^2)/b/cosh(2*d*x+2*c)/(4*a+2*b*sinh(2*d*x+2*c))^(1/2)/d

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b\cosh(dx+c)\sinh(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b*cosh(d*x+c)*sinh(d*x+c))^(3/2),x, algorithm="maxima")

[Out] integrate((b*cosh(d*x + c)*sinh(d*x + c) + a)^(-3/2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(a+b\cosh(c+d\,x)\,\sinh(c+d\,x))^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a + b*cosh(c + d*x)*sinh(c + d*x))^(3/2),x)

[Out] int(1/(a + b*cosh(c + d*x)*sinh(c + d*x))^(3/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+b\sinh(c+dx)\cosh(c+dx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b*cosh(d*x+c)*sinh(d*x+c))**(3/2),x)

[Out] Integral((a + b*sinh(c + d*x)*cosh(c + d*x))**(-3/2), x)

3.866 $\int \frac{1}{(a+b\cosh(c+dx)\sinh(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=325

$$-\frac{32\sqrt{2}\,ab\cosh(2c+2dx)}{3d\left(4a^2+b^2\right)^2\sqrt{2a+b\sinh(2c+2dx)}}-\frac{4\sqrt{2}\,b\cosh(2c+2dx)}{3d\left(4a^2+b^2\right)(2a+b\sinh(2c+2dx))^{3/2}}+\frac{4i\sqrt{2}\sqrt{\frac{2a+b\sinh(2c+2dx)}{2a-ib}}\,F\left(\frac{1}{2}\left(\right.\right.}{3d\left(4a^2+b^2\right)\sqrt{2a+b}}$$

```
[Out] -4/3*b*cosh(2*d*x+2*c)*2^(1/2)/(4*a^2+b^2)/d/(2*a+b*sinh(2*d*x+2*c))^(3/2)-
32/3*a*b*cosh(2*d*x+2*c)*2^(1/2)/(4*a^2+b^2)^2/d/(2*a+b*sinh(2*d*x+2*c))^(1
/2)+32/3*I*a*(sin(I*c+1/4*Pi+I*d*x)^2)^(1/2)/sin(I*c+1/4*Pi+I*d*x)*Elliptic
E(cos(I*c+1/4*Pi+I*d*x),2^(1/2)*(b/(2*I*a+b))^(1/2))*2^(1/2)*(2*a+b*sinh(2*
d*x+2*c))^(1/2)/(4*a^2+b^2)^2/d/((2*a+b*sinh(2*d*x+2*c))/(2*a-I*b))^(1/2)-4
/3*I*(sin(I*c+1/4*Pi+I*d*x)^2)^(1/2)/sin(I*c+1/4*Pi+I*d*x)*EllipticF(cos(I*
c+1/4*Pi+I*d*x),2^(1/2)*(b/(2*I*a+b))^(1/2))*2^(1/2)*((2*a+b*sinh(2*d*x+2*c
))/(2*a-I*b))^(1/2)/(4*a^2+b^2)/d/(2*a+b*sinh(2*d*x+2*c))^(1/2)
```

Rubi [A] time = 0.38, antiderivative size = 325, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {2666, 2664, 2754, 2752, 2663, 2661, 2655, 2653}

$$-\frac{32\sqrt{2}\,ab\cosh(2c+2dx)}{3d\left(4a^2+b^2\right)^2\sqrt{2a+b\sinh(2c+2dx)}}-\frac{4\sqrt{2}\,b\cosh(2c+2dx)}{3d\left(4a^2+b^2\right)(2a+b\sinh(2c+2dx))^{3/2}}+\frac{4i\sqrt{2}\sqrt{\frac{2a+b\sinh(2c+2dx)}{2a-ib}}\,F\left(\frac{1}{2}\left(\right.\right.}{3d\left(4a^2+b^2\right)\sqrt{2a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cosh[c + d*x]*Sinh[c + d*x])^(-5/2),x]
```

```
[Out] (-4*Sqrt[2]*b*Cosh[2*c + 2*d*x])/(3*(4*a^2 + b^2)*d*(2*a + b*Sinh[2*c + 2*d
*x])^(3/2)) - (32*Sqrt[2]*a*b*Cosh[2*c + 2*d*x])/(3*(4*a^2 + b^2)^2*d*Sqrt[
2*a + b*Sinh[2*c + 2*d*x]]) - (((32*I)/3)*Sqrt[2]*a*EllipticE[((2*I)*c - Pi
/2 + (2*I)*d*x)/2, (2*b)/((2*I)*a + b)]*Sqrt[2*a + b*Sinh[2*c + 2*d*x]])/((
4*a^2 + b^2)^2*d*Sqrt[(2*a + b*Sinh[2*c + 2*d*x])/(2*a - I*b)]) + (((4*I)/3
)*Sqrt[2]*EllipticF[((2*I)*c - Pi/2 + (2*I)*d*x)/2, (2*b)/((2*I)*a + b)]*Sq
rt[(2*a + b*Sinh[2*c + 2*d*x])/(2*a - I*b)])/((4*a^2 + b^2)*d*Sqrt[2*a + b*
Sinh[2*c + 2*d*x]])
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2664

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n + 1))/(d*(n + 1)*(a^2 - b^2)), x] + Dist[1
/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) - b
*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^
2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2666

```
Int[((a_) + cos[(c_.) + (d_.)*(x_)]*(b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_
Symbol] :> Int[(a + (b*Sin[2*c + 2*d*x])/2)^n, x] /; FreeQ[{a, b, c, d, n},
 x]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2754

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
```

```
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rubi steps

$$\int \frac{1}{(a+b\cosh(c+dx)\sinh(c+dx))^{5/2}}\,dx = \int \frac{1}{\left(a+\frac{1}{2}b\sinh(2c+2dx)\right)^{5/2}}\,dx$$

$$= -\frac{4\sqrt{2}\,b\cosh(2c+2dx)}{3\left(4a^2+b^2\right)d(2a+b\sinh(2c+2dx))^{3/2}} - \frac{8\int \frac{-\frac{3a}{2}+\frac{1}{4}b\sinh(2c+2dx)}{\left(a+\frac{1}{2}b\sinh(2c+2dx)\right)^{3/2}}}{3\left(4a^2+b^2\right)}$$

$$= -\frac{4\sqrt{2}\,b\cosh(2c+2dx)}{3\left(4a^2+b^2\right)d(2a+b\sinh(2c+2dx))^{3/2}} - \frac{32\sqrt{2}\,ab\cosh(2}{3\left(4a^2+b^2\right)^2 d\sqrt{2a+b}}$$

$$= -\frac{4\sqrt{2}\,b\cosh(2c+2dx)}{3\left(4a^2+b^2\right)d(2a+b\sinh(2c+2dx))^{3/2}} - \frac{32\sqrt{2}\,ab\cosh(2}{3\left(4a^2+b^2\right)^2 d\sqrt{2a+b}}$$

$$= -\frac{4\sqrt{2}\,b\cosh(2c+2dx)}{3\left(4a^2+b^2\right)d(2a+b\sinh(2c+2dx))^{3/2}} - \frac{32\sqrt{2}\,ab\cosh(2}{3\left(4a^2+b^2\right)^2 d\sqrt{2a+b}}$$

$$= -\frac{4\sqrt{2}\,b\cosh(2c+2dx)}{3\left(4a^2+b^2\right)d(2a+b\sinh(2c+2dx))^{3/2}} - \frac{32\sqrt{2}\,ab\cosh(2}{3\left(4a^2+b^2\right)^2 d\sqrt{2a+b}}$$

Mathematica [A] time = 1.65, size = 237, normalized size = 0.73

$$\frac{4\sqrt{2}\left(-b\cosh(2(c+dx))\left(20a^2+8ab\sinh(2(c+dx))+b^2\right)+(b-2ia)(2a-ib)^2\left(\frac{2a+b\sinh(2(c+dx))}{2a-ib}\right)^{3/2}F\left(\frac{1}{4}(-4ic-\right.\right.}{3d\left(4a^2+b^2\right)^2(2a+b\sinh(2(c+}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cosh[c + d*x]*Sinh[c + d*x])^(-5/2),x]
```

```
[Out] (4*Sqrt[2]*((8*I)*a*(2*a - I*b)^2*EllipticE[((-4*I)*c + Pi - (4*I)*d*x)/4,
((-2*I)*b)/(2*a - I*b)]*((2*a + b*Sinh[2*(c + d*x)])/(2*a - I*b))^(3/2) + (
2*a - I*b)^2*((-2*I)*a + b)*EllipticF[((-4*I)*c + Pi - (4*I)*d*x)/4, ((-2*I
)*b)/(2*a - I*b)]*((2*a + b*Sinh[2*(c + d*x)])/(2*a - I*b))^(3/2) - b*Cosh[
2*(c + d*x)]*(20*a^2 + b^2 + 8*a*b*Sinh[2*(c + d*x)])))/(3*(4*a^2 + b^2)^2*
d*(2*a + b*Sinh[2*(c + d*x)])^(3/2))
```

fricas [F] time = 0.48, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{b\cosh\left(dx+c\right)\sinh\left(dx+c\right)+a}}{b^{3}\cosh\left(dx+c\right)^{3}\sinh\left(dx+c\right)^{3}+3\,ab^{2}\cosh\left(dx+c\right)^{2}\sinh\left(dx+c\right)^{2}+3\,a^{2}b\cosh\left(dx+c\right)\sinh\left(dx+\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cosh(d*x+c)*sinh(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*cosh(d*x + c)*sinh(d*x + c) + a)/(b^3*cosh(d*x + c)^3*sinh(
d*x + c)^3 + 3*a*b^2*cosh(d*x + c)^2*sinh(d*x + c)^2 + 3*a^2*b*cosh(d*x + c
)*sinh(d*x + c) + a^3), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cosh(d*x+c)*sinh(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:index.cc index_m i_lex_is_greater Error: Bad
 Argument Value
```

maple [A] time = 1.86, size = 641, normalized size = 1.97

$$\frac{4\sqrt{\left(2a+b\sinh\left(2dx+2c\right)\right)\left(\cosh^{2}\left(2dx+2c\right)\right)}\left(-\frac{2\sqrt{\left(2a+b\sinh(2dx+2c)\right)\left(\cosh^{2}(2dx+2c)\right)}}{3b\left(4a^{2}+b^{2}\right)\left(\sinh(2dx+2c)+\frac{2a}{b}\right)^{2}}-\frac{16b\left(\cosh^{2}(2dx+2c\right.}{3\left(4a^{2}+b^{2}\right)^{2}\sqrt{\left(2a+b\sinh(2dx+2c)\right)}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*cosh(d*x+c)*sinh(d*x+c))^(5/2),x)
```

```
[Out] 4*((2*a+b*sinh(2*d*x+2*c))*cosh(2*d*x+2*c)^2)^(1/2)*(-2/3/b/(4*a^2+b^2)*((2
*a+b*sinh(2*d*x+2*c))*cosh(2*d*x+2*c)^2)^(1/2)/(sinh(2*d*x+2*c)+2*a/b)^2-16
```

```
/3*b*cosh(2*d*x+2*c)^2/(4*a^2+b^2)^2*a/((2*a+b*sinh(2*d*x+2*c))*cosh(2*d*x+
2*c)^2)^(1/2)+2*(12*a^2-b^2)/(48*a^4+24*a^2*b^2+3*b^4)*(2*a/b-I)*((-b*sinh(
2*d*x+2*c)-2*a)/(I*b-2*a))^(1/2)*((-sinh(2*d*x+2*c)+I)*b/(I*b+2*a))^(1/2)*(
(sinh(2*d*x+2*c)+I)*b/(I*b-2*a))^(1/2)/((2*a+b*sinh(2*d*x+2*c))*cosh(2*d*x+
2*c)^2)^(1/2)*EllipticF(((-b*sinh(2*d*x+2*c)-2*a)/(I*b-2*a))^(1/2),((2*a-I*
b)/(I*b+2*a))^(1/2))+16/3*a*b/(4*a^2+b^2)^2*(2*a/b-I)*((-b*sinh(2*d*x+2*c)-
2*a)/(I*b-2*a))^(1/2)*((-sinh(2*d*x+2*c)+I)*b/(I*b+2*a))^(1/2)*((sinh(2*d*x
+2*c)+I)*b/(I*b-2*a))^(1/2)/((2*a+b*sinh(2*d*x+2*c))*cosh(2*d*x+2*c)^2)^(1/
2)*((-2*a/b-I)*EllipticE(((-b*sinh(2*d*x+2*c)-2*a)/(I*b-2*a))^(1/2),((2*a-I
*b)/(I*b+2*a))^(1/2))+I*EllipticF(((-b*sinh(2*d*x+2*c)-2*a)/(I*b-2*a))^(1/2
),((2*a-I*b)/(I*b+2*a))^(1/2))))/cosh(2*d*x+2*c)/(4*a+2*b*sinh(2*d*x+2*c))^
(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\cosh\left(dx+c\right)\sinh\left(dx+c\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cosh(d*x+c)*sinh(d*x+c))^(5/2),x, algorithm="maxima")
[Out] integrate((b*cosh(d*x + c)*sinh(d*x + c) + a)^(-5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\left(a+b\,\mathrm{cosh}\left(c+d\,x\right)\,\mathrm{sinh}\left(c+d\,x\right)\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*cosh(c + d*x)*sinh(c + d*x))^(5/2),x)
[Out] int(1/(a + b*cosh(c + d*x)*sinh(c + d*x))^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cosh(d*x+c)*sinh(d*x+c))**(5/2),x)
[Out] Timed out
```

3.867 $\int \frac{x^3}{a+b\cosh(x)\sinh(x)}\,dx$

Optimal. Leaf size=386

$$\frac{3x^2\text{Li}_2\left(-\frac{be^{2x}}{2a-\sqrt{4a^2+b^2}}\right)}{2\sqrt{4a^2+b^2}}-\frac{3x^2\text{Li}_2\left(-\frac{be^{2x}}{2a+\sqrt{4a^2+b^2}}\right)}{2\sqrt{4a^2+b^2}}-\frac{3x\text{Li}_3\left(-\frac{be^{2x}}{2a-\sqrt{4a^2+b^2}}\right)}{2\sqrt{4a^2+b^2}}+\frac{3x\text{Li}_3\left(-\frac{be^{2x}}{2a+\sqrt{4a^2+b^2}}\right)}{2\sqrt{4a^2+b^2}}+\frac{3\text{Li}_4\left(-\frac{be^{2x}}{2a-\sqrt{4a^2+b^2}}\right)}{4\sqrt{4a^2+b^2}}$$

```
[Out] x^3*ln(1+b*exp(2*x)/(2*a-(4*a^2+b^2)^(1/2)))/(4*a^2+b^2)^(1/2)-x^3*ln(1+b*e
xp(2*x)/(2*a+(4*a^2+b^2)^(1/2)))/(4*a^2+b^2)^(1/2)+3/2*x^2*polylog(2,-b*exp
(2*x)/(2*a-(4*a^2+b^2)^(1/2)))/(4*a^2+b^2)^(1/2)-3/2*x^2*polylog(2,-b*exp(2
*x)/(2*a+(4*a^2+b^2)^(1/2)))/(4*a^2+b^2)^(1/2)-3/2*x*polylog(3,-b*exp(2*x)/
(2*a-(4*a^2+b^2)^(1/2)))/(4*a^2+b^2)^(1/2)+3/2*x*polylog(3,-b*exp(2*x)/(2*a
+(4*a^2+b^2)^(1/2)))/(4*a^2+b^2)^(1/2)+3/4*polylog(4,-b*exp(2*x)/(2*a-(4*a^
2+b^2)^(1/2)))/(4*a^2+b^2)^(1/2)-3/4*polylog(4,-b*exp(2*x)/(2*a+(4*a^2+b^2)
^(1/2)))/(4*a^2+b^2)^(1/2)
```

Rubi [A] time = 0.60, antiderivative size = 386, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 8, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {5628, 3322, 2264, 2190, 2531, 6609, 2282, 6589}

$$\frac{3x^2\text{PolyLog}\left(2,-\frac{be^{2x}}{2a-\sqrt{4a^2+b^2}}\right)}{2\sqrt{4a^2+b^2}}-\frac{3x^2\text{PolyLog}\left(2,-\frac{be^{2x}}{\sqrt{4a^2+b^2}+2a}\right)}{2\sqrt{4a^2+b^2}}-\frac{3x\text{PolyLog}\left(3,-\frac{be^{2x}}{2a-\sqrt{4a^2+b^2}}\right)}{2\sqrt{4a^2+b^2}}+\frac{3x\text{PolyLog}\left(3,-\right.}{2\sqrt{4a^2+}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(a + b*Cosh[x]*Sinh[x]),x]
```

```
[Out] (x^3*Log[1 + (b*E^(2*x))/(2*a - Sqrt[4*a^2 + b^2])])/Sqrt[4*a^2 + b^2] - (x
^3*Log[1 + (b*E^(2*x))/(2*a + Sqrt[4*a^2 + b^2])])/Sqrt[4*a^2 + b^2] + (3*x
^2*PolyLog[2, -((b*E^(2*x))/(2*a - Sqrt[4*a^2 + b^2]))])/(2*Sqrt[4*a^2 + b^
2]) - (3*x^2*PolyLog[2, -((b*E^(2*x))/(2*a + Sqrt[4*a^2 + b^2]))])/(2*Sqrt[
4*a^2 + b^2]) - (3*x*PolyLog[3, -((b*E^(2*x))/(2*a - Sqrt[4*a^2 + b^2]))])/
(2*Sqrt[4*a^2 + b^2]) + (3*x*PolyLog[3, -((b*E^(2*x))/(2*a + Sqrt[4*a^2 + b
^2]))])/(2*Sqrt[4*a^2 + b^2]) + (3*PolyLog[4, -((b*E^(2*x))/(2*a - Sqrt[4*a
^2 + b^2]))])/(4*Sqrt[4*a^2 + b^2]) - (3*PolyLog[4, -((b*E^(2*x))/(2*a + Sq
rt[4*a^2 + b^2]))])/(4*Sqrt[4*a^2 + b^2])
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
```

```
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2264

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[
((f + g*x)^m*F^u)/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[((f + g*x)^
m*F^u)/(b + q + 2*c*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 3322

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[((c + d*x)^m*E^(-(I*e) + f*fz*x))/(-
(I*b) + 2*a*E^(-(I*e) + f*fz*x) + I*b*E^(2*(-(I*e) + f*fz*x))), x], x] /; F
reeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 5628

```
Int[((e_.) + (f_.)*(x_))^(m_.)*((a_) + Cosh[(c_.) + (d_.)*(x_)]*(b_.)*Sinh[
(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[(e + f*x)^m*(a + (b*Sinh[2*c +
 2*d*x])/2)^n, x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int \frac{x^3}{a+b\cosh(x)\sinh(x)}\,dx = \int \frac{x^3}{a+\frac{1}{2}b\sinh(2x)}\,dx$$

$$= 2\int \frac{e^{2x}x^3}{-\frac{b}{2}+2ae^{2x}+\frac{1}{2}be^{4x}}\,dx$$

$$= \frac{(2b)\int \frac{e^{2x}x^3}{2a-\sqrt{4a^2+b^2}+be^{2x}}\,dx}{\sqrt{4a^2+b^2}} - \frac{(2b)\int \frac{e^{2x}x^3}{2a+\sqrt{4a^2+b^2}+be^{2x}}\,dx}{\sqrt{4a^2+b^2}}$$

$$= \frac{x^3\log\left(1+\frac{be^{2x}}{2a-\sqrt{4a^2+b^2}}\right)}{\sqrt{4a^2+b^2}} - \frac{x^3\log\left(1+\frac{be^{2x}}{2a+\sqrt{4a^2+b^2}}\right)}{\sqrt{4a^2+b^2}} - \frac{3\int x^2\log\left(1+\frac{be^{2x}}{2a-\sqrt{4a^2+b^2}}\right)}{\sqrt{4a^2+b^2}}$$

$$= \frac{x^3\log\left(1+\frac{be^{2x}}{2a-\sqrt{4a^2+b^2}}\right)}{\sqrt{4a^2+b^2}} - \frac{x^3\log\left(1+\frac{be^{2x}}{2a+\sqrt{4a^2+b^2}}\right)}{\sqrt{4a^2+b^2}} + \frac{3x^2\mathrm{Li}_2\left(-\frac{be^{2x}}{2a-\sqrt{4a^2+b^2}}\right)}{2\sqrt{4a^2+b^2}} - \frac{3x}{}$$

$$= \frac{x^3\log\left(1+\frac{be^{2x}}{2a-\sqrt{4a^2+b^2}}\right)}{\sqrt{4a^2+b^2}} - \frac{x^3\log\left(1+\frac{be^{2x}}{2a+\sqrt{4a^2+b^2}}\right)}{\sqrt{4a^2+b^2}} + \frac{3x^2\mathrm{Li}_2\left(-\frac{be^{2x}}{2a-\sqrt{4a^2+b^2}}\right)}{2\sqrt{4a^2+b^2}} - \frac{3x}{}$$

$$= \frac{x^3\log\left(1+\frac{be^{2x}}{2a-\sqrt{4a^2+b^2}}\right)}{\sqrt{4a^2+b^2}} - \frac{x^3\log\left(1+\frac{be^{2x}}{2a+\sqrt{4a^2+b^2}}\right)}{\sqrt{4a^2+b^2}} + \frac{3x^2\mathrm{Li}_2\left(-\frac{be^{2x}}{2a-\sqrt{4a^2+b^2}}\right)}{2\sqrt{4a^2+b^2}} - \frac{3x}{}$$

$$= \frac{x^3\log\left(1+\frac{be^{2x}}{2a-\sqrt{4a^2+b^2}}\right)}{\sqrt{4a^2+b^2}} - \frac{x^3\log\left(1+\frac{be^{2x}}{2a+\sqrt{4a^2+b^2}}\right)}{\sqrt{4a^2+b^2}} + \frac{3x^2\mathrm{Li}_2\left(-\frac{be^{2x}}{2a-\sqrt{4a^2+b^2}}\right)}{2\sqrt{4a^2+b^2}} - \frac{3x}{}$$

Mathematica [A] time = 0.41, size = 279, normalized size = 0.72

$$\frac{6x^2\mathrm{Li}_2\left(\frac{be^{2x}}{\sqrt{4a^2+b^2}-2a}\right) - 6x^2\mathrm{Li}_2\left(-\frac{be^{2x}}{2a+\sqrt{4a^2+b^2}}\right) - 6x\mathrm{Li}_3\left(\frac{be^{2x}}{\sqrt{4a^2+b^2}-2a}\right) + 6x\mathrm{Li}_3\left(-\frac{be^{2x}}{2a+\sqrt{4a^2+b^2}}\right) + 3\mathrm{Li}_4\left(\frac{be^{2x}}{\sqrt{4a^2+b^2}-2a}\right) -}{4\sqrt{4a^2+b^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(a + b*Cosh[x]*Sinh[x]),x]
```

```
[Out] (4*x^3*Log[1 + (b*E^(2*x))/(2*a - Sqrt[4*a^2 + b^2])] - 4*x^3*Log[1 + (b*E^
(2*x))/(2*a + Sqrt[4*a^2 + b^2])] + 6*x^2*PolyLog[2, (b*E^(2*x))/(-2*a + Sq
rt[4*a^2 + b^2])] - 6*x^2*PolyLog[2, -((b*E^(2*x))/(2*a + Sqrt[4*a^2 + b^2]
))] - 6*x*PolyLog[3, (b*E^(2*x))/(-2*a + Sqrt[4*a^2 + b^2])] + 6*x*PolyLog[
3, -((b*E^(2*x))/(2*a + Sqrt[4*a^2 + b^2]))] + 3*PolyLog[4, (b*E^(2*x))/(-2
*a + Sqrt[4*a^2 + b^2])] - 3*PolyLog[4, -((b*E^(2*x))/(2*a + Sqrt[4*a^2 + b
^2]))])/(4*Sqrt[4*a^2 + b^2])
```

fricas [C] time = 0.54, size = 1488, normalized size = 3.85

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+b*cosh(x)*sinh(x)),x, algorithm="fricas")
```

```
[Out] -(b*x^3*sqrt((4*a^2 + b^2)/b^2)*log(((2*a*cosh(x) + 2*a*sinh(x) - (b*cosh(x
) + b*sinh(x))*sqrt((4*a^2 + b^2)/b^2))*sqrt(-(b*sqrt((4*a^2 + b^2)/b^2) +
2*a)/b) + b)/b) + b*x^3*sqrt((4*a^2 + b^2)/b^2)*log(-((2*a*cosh(x) + 2*a*si
nh(x) - (b*cosh(x) + b*sinh(x))*sqrt((4*a^2 + b^2)/b^2))*sqrt(-(b*sqrt((4*a
^2 + b^2)/b^2) + 2*a)/b) - b)/b) - b*x^3*sqrt((4*a^2 + b^2)/b^2)*log(((2*a*
cosh(x) + 2*a*sinh(x) + (b*cosh(x) + b*sinh(x))*sqrt((4*a^2 + b^2)/b^2))*sq
rt((b*sqrt((4*a^2 + b^2)/b^2) - 2*a)/b) + b)/b) - b*x^3*sqrt((4*a^2 + b^2)/
b^2)*log(-((2*a*cosh(x) + 2*a*sinh(x) + (b*cosh(x) + b*sinh(x))*sqrt((4*a^2
 + b^2)/b^2))*sqrt((b*sqrt((4*a^2 + b^2)/b^2) - 2*a)/b) - b)/b) + 3*b*x^2*s
qrt((4*a^2 + b^2)/b^2)*dilog(-((2*a*cosh(x) + 2*a*sinh(x) - (b*cosh(x) + b*
sinh(x))*sqrt((4*a^2 + b^2)/b^2))*sqrt(-(b*sqrt((4*a^2 + b^2)/b^2) + 2*a)/b
) + b)/b + 1) + 3*b*x^2*sqrt((4*a^2 + b^2)/b^2)*dilog(((2*a*cosh(x) + 2*a*s
inh(x) - (b*cosh(x) + b*sinh(x))*sqrt((4*a^2 + b^2)/b^2))*sqrt(-(b*sqrt((4*
a^2 + b^2)/b^2) + 2*a)/b) - b)/b + 1) - 3*b*x^2*sqrt((4*a^2 + b^2)/b^2)*dil
og(-((2*a*cosh(x) + 2*a*sinh(x) + (b*cosh(x) + b*sinh(x))*sqrt((4*a^2 + b^2
)/b^2))*sqrt((b*sqrt((4*a^2 + b^2)/b^2) - 2*a)/b) + b)/b + 1) - 3*b*x^2*sqr
t((4*a^2 + b^2)/b^2)*dilog(((2*a*cosh(x) + 2*a*sinh(x) + (b*cosh(x) + b*sin
h(x))*sqrt((4*a^2 + b^2)/b^2))*sqrt((b*sqrt((4*a^2 + b^2)/b^2) - 2*a)/b) -
b)/b + 1) - 6*b*x*sqrt((4*a^2 + b^2)/b^2)*polylog(3, (2*a*cosh(x) + 2*a*sin
h(x) - (b*cosh(x) + b*sinh(x))*sqrt((4*a^2 + b^2)/b^2))*sqrt(-(b*sqrt((4*a^
2 + b^2)/b^2) + 2*a)/b)/b) - 6*b*x*sqrt((4*a^2 + b^2)/b^2)*polylog(3, -(2*a
*cosh(x) + 2*a*sinh(x) - (b*cosh(x) + b*sinh(x))*sqrt((4*a^2 + b^2)/b^2))*s
qrt(-(b*sqrt((4*a^2 + b^2)/b^2) + 2*a)/b)/b) + 6*b*x*sqrt((4*a^2 + b^2)/b^2
)*polylog(3, (2*a*cosh(x) + 2*a*sinh(x) + (b*cosh(x) + b*sinh(x))*sqrt((4*a
^2 + b^2)/b^2))*sqrt((b*sqrt((4*a^2 + b^2)/b^2) - 2*a)/b)/b) + 6*b*x*sqrt((
4*a^2 + b^2)/b^2)*polylog(3, -(2*a*cosh(x) + 2*a*sinh(x) + (b*cosh(x) + b*s
```

```
inh(x))*sqrt((4*a^2 + b^2)/b^2))*sqrt((b*sqrt((4*a^2 + b^2)/b^2) - 2*a)/b)/
b) + 6*b*sqrt((4*a^2 + b^2)/b^2)*polylog(4, (2*a*cosh(x) + 2*a*sinh(x) - (b
*cosh(x) + b*sinh(x))*sqrt((4*a^2 + b^2)/b^2))*sqrt(-(b*sqrt((4*a^2 + b^2)/
b^2) + 2*a)/b)/b) + 6*b*sqrt((4*a^2 + b^2)/b^2)*polylog(4, -(2*a*cosh(x) +
2*a*sinh(x) - (b*cosh(x) + b*sinh(x))*sqrt((4*a^2 + b^2)/b^2))*sqrt(-(b*sqr
t((4*a^2 + b^2)/b^2) + 2*a)/b)/b) - 6*b*sqrt((4*a^2 + b^2)/b^2)*polylog(4,
(2*a*cosh(x) + 2*a*sinh(x) + (b*cosh(x) + b*sinh(x))*sqrt((4*a^2 + b^2)/b^2
))*sqrt((b*sqrt((4*a^2 + b^2)/b^2) - 2*a)/b)/b) - 6*b*sqrt((4*a^2 + b^2)/b^
2)*polylog(4, -(2*a*cosh(x) + 2*a*sinh(x) + (b*cosh(x) + b*sinh(x))*sqrt((4
*a^2 + b^2)/b^2))*sqrt((b*sqrt((4*a^2 + b^2)/b^2) - 2*a)/b)/b))/(4*a^2 + b^
2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{b\cosh(x)\sinh(x) + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+b*cosh(x)*sinh(x)),x, algorithm="giac")
```

```
[Out] integrate(x^3/(b*cosh(x)*sinh(x) + a), x)
```

maple [B] time = 0.32, size = 687, normalized size = 1.78

$$\frac{\ln\left(1-\frac{b\,e^{2x}}{-2a-\sqrt{4a^2+b^2}}\right)x^3}{-2a-\sqrt{4a^2+b^2}} - \frac{x^4}{2\left(-2a-\sqrt{4a^2+b^2}\right)} + \frac{2\ln\left(1-\frac{b\,e^{2x}}{-2a-\sqrt{4a^2+b^2}}\right)a\,x^3}{\sqrt{4a^2+b^2}\left(-2a-\sqrt{4a^2+b^2}\right)} - \frac{a\,x^4}{\sqrt{4a^2+b^2}\left(-2a-\sqrt{4a^2+b^2\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a+b*cosh(x)*sinh(x)),x)
```

```
[Out] 1/(-2*a-(4*a^2+b^2)^(1/2))*ln(1-b*exp(2*x)/(-2*a-(4*a^2+b^2)^(1/2)))*x^3-1/
2/(-2*a-(4*a^2+b^2)^(1/2))*x^4+2/(4*a^2+b^2)^(1/2)/(-2*a-(4*a^2+b^2)^(1/2))
*ln(1-b*exp(2*x)/(-2*a-(4*a^2+b^2)^(1/2)))*a*x^3-1/(4*a^2+b^2)^(1/2)/(-2*a-
(4*a^2+b^2)^(1/2))*a*x^4+3/2/(-2*a-(4*a^2+b^2)^(1/2))*polylog(2,b*exp(2*x)/
(-2*a-(4*a^2+b^2)^(1/2)))*x^2+3/(4*a^2+b^2)^(1/2)/(-2*a-(4*a^2+b^2)^(1/2))*
polylog(2,b*exp(2*x)/(-2*a-(4*a^2+b^2)^(1/2)))*a*x^2-3/2/(-2*a-(4*a^2+b^2)^
(1/2))*polylog(3,b*exp(2*x)/(-2*a-(4*a^2+b^2)^(1/2)))*x-3/(4*a^2+b^2)^(1/2)
/(-2*a-(4*a^2+b^2)^(1/2))*polylog(3,b*exp(2*x)/(-2*a-(4*a^2+b^2)^(1/2)))*a*
x+3/4/(-2*a-(4*a^2+b^2)^(1/2))*polylog(4,b*exp(2*x)/(-2*a-(4*a^2+b^2)^(1/2)
))+3/2/(4*a^2+b^2)^(1/2)/(-2*a-(4*a^2+b^2)^(1/2))*polylog(4,b*exp(2*x)/(-2*
a-(4*a^2+b^2)^(1/2)))*a+1/(4*a^2+b^2)^(1/2)*x^3*ln(1-b*exp(2*x)/((4*a^2+b^2
)^(1/2)-2*a))-1/2/(4*a^2+b^2)^(1/2)*x^4+3/2/(4*a^2+b^2)^(1/2)*x^2*polylog(2
,b*exp(2*x)/((4*a^2+b^2)^(1/2)-2*a))-3/2/(4*a^2+b^2)^(1/2)*x*polylog(3,b*ex
```

```
p(2*x)/((4*a^2+b^2)^(1/2)-2*a))+3/4/(4*a^2+b^2)^(1/2)*polylog(4,b*exp(2*x)/
((4*a^2+b^2)^(1/2)-2*a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{b\cosh(x)\sinh(x) + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+b*cosh(x)*sinh(x)),x, algorithm="maxima")
[Out] integrate(x^3/(b*cosh(x)*sinh(x) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3}{a + b\cosh(x)\sinh(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a + b*cosh(x)*sinh(x)),x)
[Out] int(x^3/(a + b*cosh(x)*sinh(x)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(a+b*cosh(x)*sinh(x)),x)
[Out] Timed out
```

3.868 $\int \frac{x^2}{a+b\cosh(x)\sinh(x)}\,dx$

Optimal. Leaf size=281

$$\frac{x\text{Li}_2\left(-\frac{be^{2x}}{2a-\sqrt{4a^2+b^2}}\right)}{\sqrt{4a^2+b^2}}-\frac{x\text{Li}_2\left(-\frac{be^{2x}}{2a+\sqrt{4a^2+b^2}}\right)}{\sqrt{4a^2+b^2}}-\frac{\text{Li}_3\left(-\frac{be^{2x}}{2a-\sqrt{4a^2+b^2}}\right)}{2\sqrt{4a^2+b^2}}+\frac{\text{Li}_3\left(-\frac{be^{2x}}{2a+\sqrt{4a^2+b^2}}\right)}{2\sqrt{4a^2+b^2}}+\frac{x^2\log\left(\frac{be^{2x}}{2a-\sqrt{4a^2+b^2}}+1\right)}{\sqrt{4a^2+b^2}}-\frac{x^2}{}$$

```
[Out] x^2*ln(1+b*exp(2*x)/(2*a-(4*a^2+b^2)^(1/2)))/(4*a^2+b^2)^(1/2)-x^2*ln(1+b*e
xp(2*x)/(2*a+(4*a^2+b^2)^(1/2)))/(4*a^2+b^2)^(1/2)+x*polylog(2,-b*exp(2*x)/
(2*a-(4*a^2+b^2)^(1/2)))/(4*a^2+b^2)^(1/2)-x*polylog(2,-b*exp(2*x)/(2*a+(4*
a^2+b^2)^(1/2)))/(4*a^2+b^2)^(1/2)-1/2*polylog(3,-b*exp(2*x)/(2*a-(4*a^2+b^
2)^(1/2)))/(4*a^2+b^2)^(1/2)+1/2*polylog(3,-b*exp(2*x)/(2*a+(4*a^2+b^2)^(1/
2)))/(4*a^2+b^2)^(1/2)
```

Rubi [A] time = 0.52, antiderivative size = 281, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5628, 3322, 2264, 2190, 2531, 2282, 6589}

$$\frac{x\text{PolyLog}\left(2,-\frac{be^{2x}}{2a-\sqrt{4a^2+b^2}}\right)}{\sqrt{4a^2+b^2}}-\frac{x\text{PolyLog}\left(2,-\frac{be^{2x}}{\sqrt{4a^2+b^2}+2a}\right)}{\sqrt{4a^2+b^2}}-\frac{\text{PolyLog}\left(3,-\frac{be^{2x}}{2a-\sqrt{4a^2+b^2}}\right)}{2\sqrt{4a^2+b^2}}+\frac{\text{PolyLog}\left(3,-\frac{be^{2x}}{\sqrt{4a^2+b^2}+2}\right)}{2\sqrt{4a^2+b^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a + b*Cosh[x]*Sinh[x]),x]
```

```
[Out] (x^2*Log[1 + (b*E^(2*x))/(2*a - Sqrt[4*a^2 + b^2])])/Sqrt[4*a^2 + b^2] - (x
^2*Log[1 + (b*E^(2*x))/(2*a + Sqrt[4*a^2 + b^2])])/Sqrt[4*a^2 + b^2] + (x*P
olyLog[2, -((b*E^(2*x))/(2*a - Sqrt[4*a^2 + b^2]))])/Sqrt[4*a^2 + b^2] - (x
*PolyLog[2, -((b*E^(2*x))/(2*a + Sqrt[4*a^2 + b^2]))])/Sqrt[4*a^2 + b^2] -
PolyLog[3, -((b*E^(2*x))/(2*a - Sqrt[4*a^2 + b^2]))]/(2*Sqrt[4*a^2 + b^2])
+ PolyLog[3, -((b*E^(2*x))/(2*a + Sqrt[4*a^2 + b^2]))]/(2*Sqrt[4*a^2 + b^2]
)
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2264

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[
((f + g*x)^m*F^u)/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[((f + g*x)^
m*F^u)/(b + q + 2*c*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 3322

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[((c + d*x)^m*E^(-(I*e) + f*fz*x))/(-
(I*b) + 2*a*E^(-(I*e) + f*fz*x) + I*b*E^(2*(-(I*e) + f*fz*x))), x], x] /; F
reeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 5628

```
Int[((e_.) + (f_.)*(x_))^(m_.)*((a_) + Cosh[(c_.) + (d_.)*(x_)]*(b_.)*Sinh[
(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[(e + f*x)^m*(a + (b*Sinh[2*c +
 2*d*x])/2)^n, x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{x^2}{a+b\cosh(x)\sinh(x)}\,dx = \int \frac{x^2}{a+\frac{1}{2}b\sinh(2x)}\,dx$$
$$= 2\int \frac{e^{2x}x^2}{-\frac{b}{2}+2ae^{2x}+\frac{1}{2}be^{4x}}\,dx$$
$$= \frac{(2b)\int \frac{e^{2x}x^2}{2a-\sqrt{4a^2+b^2}+be^{2x}}\,dx}{\sqrt{4a^2+b^2}} - \frac{(2b)\int \frac{e^{2x}x^2}{2a+\sqrt{4a^2+b^2}+be^{2x}}\,dx}{\sqrt{4a^2+b^2}}$$
$$= \frac{x^2\log\left(1+\frac{be^{2x}}{2a-\sqrt{4a^2+b^2}}\right)}{\sqrt{4a^2+b^2}} - \frac{x^2\log\left(1+\frac{be^{2x}}{2a+\sqrt{4a^2+b^2}}\right)}{\sqrt{4a^2+b^2}} - \frac{2\int x\log\left(1+\frac{be^{2x}}{2a-\sqrt{4a^2+b^2}}\right)\,d}{\sqrt{4a^2+b^2}}$$
$$= \frac{x^2\log\left(1+\frac{be^{2x}}{2a-\sqrt{4a^2+b^2}}\right)}{\sqrt{4a^2+b^2}} - \frac{x^2\log\left(1+\frac{be^{2x}}{2a+\sqrt{4a^2+b^2}}\right)}{\sqrt{4a^2+b^2}} + \frac{x\text{Li}_2\left(-\frac{be^{2x}}{2a-\sqrt{4a^2+b^2}}\right)}{\sqrt{4a^2+b^2}} - \frac{x\text{Li}_2}{}$$
$$= \frac{x^2\log\left(1+\frac{be^{2x}}{2a-\sqrt{4a^2+b^2}}\right)}{\sqrt{4a^2+b^2}} - \frac{x^2\log\left(1+\frac{be^{2x}}{2a+\sqrt{4a^2+b^2}}\right)}{\sqrt{4a^2+b^2}} + \frac{x\text{Li}_2\left(-\frac{be^{2x}}{2a-\sqrt{4a^2+b^2}}\right)}{\sqrt{4a^2+b^2}} - \frac{x\text{Li}_2}{}$$
$$= \frac{x^2\log\left(1+\frac{be^{2x}}{2a-\sqrt{4a^2+b^2}}\right)}{\sqrt{4a^2+b^2}} - \frac{x^2\log\left(1+\frac{be^{2x}}{2a+\sqrt{4a^2+b^2}}\right)}{\sqrt{4a^2+b^2}} + \frac{x\text{Li}_2\left(-\frac{be^{2x}}{2a-\sqrt{4a^2+b^2}}\right)}{\sqrt{4a^2+b^2}} - \frac{x\text{Li}_2}{}$$

Mathematica [A] time = 0.28, size = 210, normalized size = 0.75

$$\frac{2x\text{Li}_2\left(\frac{be^{2x}}{\sqrt{4a^2+b^2}-2a}\right)-2x\text{Li}_2\left(-\frac{be^{2x}}{2a+\sqrt{4a^2+b^2}}\right)-\text{Li}_3\left(\frac{be^{2x}}{\sqrt{4a^2+b^2}-2a}\right)+\text{Li}_3\left(-\frac{be^{2x}}{2a+\sqrt{4a^2+b^2}}\right)+2x^2\log\left(\frac{be^{2x}}{2a-\sqrt{4a^2+b^2}}+1\right)-}{2\sqrt{4a^2+b^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a + b*Cosh[x]*Sinh[x]),x]
```

```
[Out] (2*x^2*Log[1 + (b*E^(2*x))/(2*a - Sqrt[4*a^2 + b^2])] - 2*x^2*Log[1 + (b*E^
(2*x))/(2*a + Sqrt[4*a^2 + b^2])] + 2*x*PolyLog[2, (b*E^(2*x))/(-2*a + Sqrt
[4*a^2 + b^2])] - 2*x*PolyLog[2, -((b*E^(2*x))/(2*a + Sqrt[4*a^2 + b^2]))]
- PolyLog[3, (b*E^(2*x))/(-2*a + Sqrt[4*a^2 + b^2])] + PolyLog[3, -((b*E^(2
*x))/(2*a + Sqrt[4*a^2 + b^2]))])/(2*Sqrt[4*a^2 + b^2])
```

fricas [C] time = 0.54, size = 1122, normalized size = 3.99

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*cosh(x)*sinh(x)),x, algorithm="fricas")
```

```
[Out] -(b*x^2*sqrt((4*a^2 + b^2)/b^2)*log(((2*a*cosh(x) + 2*a*sinh(x) - (b*cosh(x
) + b*sinh(x))*sqrt((4*a^2 + b^2)/b^2))*sqrt(-(b*sqrt((4*a^2 + b^2)/b^2) +
2*a)/b) + b)/b) + b*x^2*sqrt((4*a^2 + b^2)/b^2)*log(-((2*a*cosh(x) + 2*a*si
nh(x) - (b*cosh(x) + b*sinh(x))*sqrt((4*a^2 + b^2)/b^2))*sqrt(-(b*sqrt((4*a
^2 + b^2)/b^2) + 2*a)/b) - b)/b) - b*x^2*sqrt((4*a^2 + b^2)/b^2)*log(((2*a*
cosh(x) + 2*a*sinh(x) + (b*cosh(x) + b*sinh(x))*sqrt((4*a^2 + b^2)/b^2))*sq
rt((b*sqrt((4*a^2 + b^2)/b^2) - 2*a)/b) + b)/b) - b*x^2*sqrt((4*a^2 + b^2)/
b^2)*log(-((2*a*cosh(x) + 2*a*sinh(x) + (b*cosh(x) + b*sinh(x))*sqrt((4*a^2
 + b^2)/b^2))*sqrt((b*sqrt((4*a^2 + b^2)/b^2) - 2*a)/b) - b)/b) + 2*b*x*sqr
t((4*a^2 + b^2)/b^2)*dilog(-((2*a*cosh(x) + 2*a*sinh(x) - (b*cosh(x) + b*si
nh(x))*sqrt((4*a^2 + b^2)/b^2))*sqrt(-(b*sqrt((4*a^2 + b^2)/b^2) + 2*a)/b)
+ b)/b + 1) + 2*b*x*sqrt((4*a^2 + b^2)/b^2)*dilog(((2*a*cosh(x) + 2*a*sinh(
x) - (b*cosh(x) + b*sinh(x))*sqrt((4*a^2 + b^2)/b^2))*sqrt(-(b*sqrt((4*a^2
+ b^2)/b^2) + 2*a)/b) - b)/b + 1) - 2*b*x*sqrt((4*a^2 + b^2)/b^2)*dilog(-((
2*a*cosh(x) + 2*a*sinh(x) + (b*cosh(x) + b*sinh(x))*sqrt((4*a^2 + b^2)/b^2)
)*sqrt((b*sqrt((4*a^2 + b^2)/b^2) - 2*a)/b) + b)/b + 1) - 2*b*x*sqrt((4*a^2
 + b^2)/b^2)*dilog(((2*a*cosh(x) + 2*a*sinh(x) + (b*cosh(x) + b*sinh(x))*sq
rt((4*a^2 + b^2)/b^2))*sqrt((b*sqrt((4*a^2 + b^2)/b^2) - 2*a)/b) - b)/b + 1
) - 2*b*sqrt((4*a^2 + b^2)/b^2)*polylog(3, (2*a*cosh(x) + 2*a*sinh(x) - (b*
cosh(x) + b*sinh(x))*sqrt((4*a^2 + b^2)/b^2))*sqrt(-(b*sqrt((4*a^2 + b^2)/b
^2) + 2*a)/b)/b) - 2*b*sqrt((4*a^2 + b^2)/b^2)*polylog(3, -(2*a*cosh(x) + 2
*a*sinh(x) - (b*cosh(x) + b*sinh(x))*sqrt((4*a^2 + b^2)/b^2))*sqrt(-(b*sqrt
((4*a^2 + b^2)/b^2) + 2*a)/b)/b) + 2*b*sqrt((4*a^2 + b^2)/b^2)*polylog(3, (
2*a*cosh(x) + 2*a*sinh(x) + (b*cosh(x) + b*sinh(x))*sqrt((4*a^2 + b^2)/b^2)
)*sqrt((b*sqrt((4*a^2 + b^2)/b^2) - 2*a)/b)/b) + 2*b*sqrt((4*a^2 + b^2)/b^2
)*polylog(3, -(2*a*cosh(x) + 2*a*sinh(x) + (b*cosh(x) + b*sinh(x))*sqrt((4*
a^2 + b^2)/b^2))*sqrt((b*sqrt((4*a^2 + b^2)/b^2) - 2*a)/b)/b))/(4*a^2 + b^2
)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{b\cosh(x)\sinh(x) + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*cosh(x)*sinh(x)),x, algorithm="giac")
```

```
[Out] integrate(x^2/(b*cosh(x)*sinh(x) + a), x)
```

maple [B] time = 0.31, size = 530, normalized size = 1.89

$$-\frac{2x^3}{3\left(-2a-\sqrt{4a^2+b^2}\right)}+\frac{x^2\ln\left(1-\frac{b\,e^{2x}}{-2a-\sqrt{4a^2+b^2}}\right)}{-2a-\sqrt{4a^2+b^2}}+\frac{x\,\mathrm{polylog}\left(2,\frac{b\,e^{2x}}{-2a-\sqrt{4a^2+b^2}}\right)}{-2a-\sqrt{4a^2+b^2}}-\frac{\mathrm{polylog}\left(3,\frac{b\,e^{2x}}{-2a-\sqrt{4a^2+b^2}}\right)}{2\left(-2a-\sqrt{4a^2+b^2}\right)}-\frac{}{3\sqrt{4a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a+b*cosh(x)*sinh(x)),x)
```

```
[Out] -2/3/(-2*a-(4*a^2+b^2)^(1/2))*x^3+1/(-2*a-(4*a^2+b^2)^(1/2))*x^2*ln(1-b*exp
(2*x)/(-2*a-(4*a^2+b^2)^(1/2)))+1/(-2*a-(4*a^2+b^2)^(1/2))*x*polylog(2,b*ex
p(2*x)/(-2*a-(4*a^2+b^2)^(1/2)))-1/2/(-2*a-(4*a^2+b^2)^(1/2))*polylog(3,b*e
xp(2*x)/(-2*a-(4*a^2+b^2)^(1/2)))-4/3/(4*a^2+b^2)^(1/2)/(-2*a-(4*a^2+b^2)^(
1/2))*a*x^3+2/(4*a^2+b^2)^(1/2)/(-2*a-(4*a^2+b^2)^(1/2))*a*x^2*ln(1-b*exp(2
*x)/(-2*a-(4*a^2+b^2)^(1/2)))+2/(4*a^2+b^2)^(1/2)/(-2*a-(4*a^2+b^2)^(1/2))*
a*x*polylog(2,b*exp(2*x)/(-2*a-(4*a^2+b^2)^(1/2)))-1/(4*a^2+b^2)^(1/2)/(-2*
a-(4*a^2+b^2)^(1/2))*a*polylog(3,b*exp(2*x)/(-2*a-(4*a^2+b^2)^(1/2)))-2/3/(
4*a^2+b^2)^(1/2)*x^3+1/(4*a^2+b^2)^(1/2)*x^2*ln(1-b*exp(2*x)/((4*a^2+b^2)^(
1/2)-2*a))+1/(4*a^2+b^2)^(1/2)*x*polylog(2,b*exp(2*x)/((4*a^2+b^2)^(1/2)-2*
a))-1/2/(4*a^2+b^2)^(1/2)*polylog(3,b*exp(2*x)/((4*a^2+b^2)^(1/2)-2*a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{b\cosh(x)\sinh(x)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*cosh(x)*sinh(x)),x, algorithm="maxima")
```

```
[Out] integrate(x^2/(b*cosh(x)*sinh(x) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2}{a+b\cosh(x)\sinh(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a + b*cosh(x)*sinh(x)),x)
```

```
[Out] int(x^2/(a + b*cosh(x)*sinh(x)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(a+b*cosh(x)*sinh(x)),x)
```

```
[Out] Timed out
```

3.869 $\int \frac{x}{a+b\cosh(x)\sinh(x)}\,dx$

Optimal. Leaf size=186

$$\frac{\text{Li}_2\left(-\frac{be^{2x}}{2a-\sqrt{4a^2+b^2}}\right)}{2\sqrt{4a^2+b^2}}-\frac{\text{Li}_2\left(-\frac{be^{2x}}{2a+\sqrt{4a^2+b^2}}\right)}{2\sqrt{4a^2+b^2}}+\frac{x\log\left(\frac{be^{2x}}{2a-\sqrt{4a^2+b^2}}+1\right)}{\sqrt{4a^2+b^2}}-\frac{x\log\left(\frac{be^{2x}}{\sqrt{4a^2+b^2}+2a}+1\right)}{\sqrt{4a^2+b^2}}$$

```
[Out] x*ln(1+b*exp(2*x)/(2*a-(4*a^2+b^2)^(1/2)))/(4*a^2+b^2)^(1/2)-x*ln(1+b*exp(2
*x)/(2*a+(4*a^2+b^2)^(1/2)))/(4*a^2+b^2)^(1/2)+1/2*polylog(2,-b*exp(2*x)/(2
*a-(4*a^2+b^2)^(1/2)))/(4*a^2+b^2)^(1/2)-1/2*polylog(2,-b*exp(2*x)/(2*a+(4*
a^2+b^2)^(1/2)))/(4*a^2+b^2)^(1/2)
```

Rubi [A] time = 0.30, antiderivative size = 186, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5628, 3322, 2264, 2190, 2279, 2391}

$$\frac{\text{PolyLog}\left(2,-\frac{be^{2x}}{2a-\sqrt{4a^2+b^2}}\right)}{2\sqrt{4a^2+b^2}}-\frac{\text{PolyLog}\left(2,-\frac{be^{2x}}{\sqrt{4a^2+b^2}+2a}\right)}{2\sqrt{4a^2+b^2}}+\frac{x\log\left(\frac{be^{2x}}{2a-\sqrt{4a^2+b^2}}+1\right)}{\sqrt{4a^2+b^2}}-\frac{x\log\left(\frac{be^{2x}}{\sqrt{4a^2+b^2}+2a}+1\right)}{\sqrt{4a^2+b^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a + b*Cosh[x]*Sinh[x]),x]
```

```
[Out] (x*Log[1 + (b*E^(2*x))/(2*a - Sqrt[4*a^2 + b^2])])/Sqrt[4*a^2 + b^2] - (x*L
og[1 + (b*E^(2*x))/(2*a + Sqrt[4*a^2 + b^2])])/Sqrt[4*a^2 + b^2] + PolyLog[
2, -((b*E^(2*x))/(2*a - Sqrt[4*a^2 + b^2]))]/(2*Sqrt[4*a^2 + b^2]) - PolyLo
g[2, -((b*E^(2*x))/(2*a + Sqrt[4*a^2 + b^2]))]/(2*Sqrt[4*a^2 + b^2])
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2264

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[
((f + g*x)^m*F^u)/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[((f + g*x)^
m*F^u)/(b + q + 2*c*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3322

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[((c + d*x)^m*E^(-(I*e) + f*fz*x))/(-
(I*b) + 2*a*E^(-(I*e) + f*fz*x) + I*b*E^(2*(-(I*e) + f*fz*x))), x], x] /; F
reeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 5628

```
Int[((e_.) + (f_.)*(x_))^(m_.)*((a_) + Cosh[(c_.) + (d_.)*(x_)]*(b_.)*Sinh[
(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[(e + f*x)^m*(a + (b*Sinh[2*c +
 2*d*x])/2)^n, x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{x}{a+b\cosh(x)\sinh(x)}\,dx = \int \frac{x}{a+\frac{1}{2}b\sinh(2x)}\,dx$$
$$= 2\int \frac{e^{2x}x}{-\frac{b}{2}+2ae^{2x}+\frac{1}{2}be^{4x}}\,dx$$
$$= \frac{(2b)\int \frac{e^{2x}x}{2a-\sqrt{4a^2+b^2}+be^{2x}}\,dx}{\sqrt{4a^2+b^2}} - \frac{(2b)\int \frac{e^{2x}x}{2a+\sqrt{4a^2+b^2}+be^{2x}}\,dx}{\sqrt{4a^2+b^2}}$$
$$= \frac{x\log\left(1+\frac{be^{2x}}{2a-\sqrt{4a^2+b^2}}\right)}{\sqrt{4a^2+b^2}} - \frac{x\log\left(1+\frac{be^{2x}}{2a+\sqrt{4a^2+b^2}}\right)}{\sqrt{4a^2+b^2}} - \frac{\int\log\left(1+\frac{be^{2x}}{2a-\sqrt{4a^2+b^2}}\right)dx}{\sqrt{4a^2+b^2}} +$$
$$= \frac{x\log\left(1+\frac{be^{2x}}{2a-\sqrt{4a^2+b^2}}\right)}{\sqrt{4a^2+b^2}} - \frac{x\log\left(1+\frac{be^{2x}}{2a+\sqrt{4a^2+b^2}}\right)}{\sqrt{4a^2+b^2}} - \frac{\text{Subst}\left(\int\frac{\log\left(1+\frac{bx}{2a-\sqrt{4a^2+b^2}}\right)}{x}dx,\right.}{2\sqrt{4a^2+b^2}}$$
$$= \frac{x\log\left(1+\frac{be^{2x}}{2a-\sqrt{4a^2+b^2}}\right)}{\sqrt{4a^2+b^2}} - \frac{x\log\left(1+\frac{be^{2x}}{2a+\sqrt{4a^2+b^2}}\right)}{\sqrt{4a^2+b^2}} + \frac{\text{Li}_2\left(-\frac{be^{2x}}{2a-\sqrt{4a^2+b^2}}\right)}{2\sqrt{4a^2+b^2}} - \frac{\text{Li}_2\left(-\right.}{2\sqrt{4a}}$$

Mathematica [C] time = 1.87, size = 956, normalized size = 5.14

$$\frac{1}{2}\left(-\frac{i\pi\tanh^{-1}\left(\frac{2a\tanh(x)-b}{\sqrt{4a^2+b^2}}\right)}{\sqrt{4a^2+b^2}} - \frac{2\cos^{-1}\left(-\frac{2ia}{b}\right)\tanh^{-1}\left(\frac{(2a+ib)\cot\left(\frac{1}{4}(4ix+\pi)\right)}{\sqrt{-4a^2-b^2}}\right)+(\pi-4ix)\tanh^{-1}\left(\frac{(2a-ib)\tan\left(\frac{1}{4}(4ix+\pi)\right)}{\sqrt{-4a^2-b^2}}\right)}{}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a + b*Cosh[x]*Sinh[x]),x]
```

```
[Out] (((-I)*Pi*ArcTanh[(-b + 2*a*Tanh[x])/Sqrt[4*a^2 + b^2]])/Sqrt[4*a^2 + b^2]
- (2*ArcCos[((-2*I)*a)/b]*ArcTanh[((2*a + I*b)*Cot[(Pi + (4*I)*x)/4])/Sqrt[
-4*a^2 - b^2]] + (Pi - (4*I)*x)*ArcTanh[((2*a - I*b)*Tan[(Pi + (4*I)*x)/4])
/Sqrt[-4*a^2 - b^2]] - (ArcCos[((-2*I)*a)/b] + (2*I)*ArcTanh[((2*a + I*b)*C
ot[(Pi + (4*I)*x)/4])/Sqrt[-4*a^2 - b^2]])*Log[(((2*I)*a + b)*((-2*I)*a + b
 + Sqrt[-4*a^2 - b^2])*(1 + I*Cot[(Pi + (4*I)*x)/4]))/(b*((2*I)*a + b + I*S
qrt[-4*a^2 - b^2]*Cot[(Pi + (4*I)*x)/4]))] - (ArcCos[((-2*I)*a)/b] - (2*I)*
ArcTanh[((2*a + I*b)*Cot[(Pi + (4*I)*x)/4])/Sqrt[-4*a^2 - b^2]])*Log[(((2*I
)*a + b)*((2*I)*a - b + Sqrt[-4*a^2 - b^2])*(I + Cot[(Pi + (4*I)*x)/4]))/(b
*(2*a - I*b + Sqrt[-4*a^2 - b^2]*Cot[(Pi + (4*I)*x)/4]))] + (ArcCos[((-2*I)
```

```
*a)/b] - (2*I)*ArcTanh[((2*a + I*b)*Cot[(Pi + (4*I)*x)/4])/Sqrt[-4*a^2 - b^
2]] - (2*I)*ArcTanh[((2*a - I*b)*Tan[(Pi + (4*I)*x)/4])/Sqrt[-4*a^2 - b^2]]
)*Log[-1/2*((-1)^(3/4)*Sqrt[-4*a^2 - b^2])/(Sqrt[(-I)*b]*E^x*Sqrt[a + b*Cos
h[x]*Sinh[x]])] + (ArcCos[((-2*I)*a)/b] + (2*I)*(ArcTanh[((2*a + I*b)*Cot[(
Pi + (4*I)*x)/4])/Sqrt[-4*a^2 - b^2]] + ArcTanh[((2*a - I*b)*Tan[(Pi + (4*I
)*x)/4])/Sqrt[-4*a^2 - b^2]]))*Log[((-1)^(1/4)*Sqrt[-4*a^2 - b^2]*E^x)/(2*S
qrt[(-I)*b]*Sqrt[a + b*Cosh[x]*Sinh[x]])] + I*(PolyLog[2, (((2*I)*a + Sqrt[
-4*a^2 - b^2])*((2*I)*a + b - I*Sqrt[-4*a^2 - b^2]*Cot[(Pi + (4*I)*x)/4]))/
(b*((2*I)*a + b + I*Sqrt[-4*a^2 - b^2]*Cot[(Pi + (4*I)*x)/4]))] - PolyLog[2
, ((2*a + I*Sqrt[-4*a^2 - b^2])*(-2*a + I*b + Sqrt[-4*a^2 - b^2]*Cot[(Pi +
(4*I)*x)/4]))/(b*((2*I)*a + b + I*Sqrt[-4*a^2 - b^2]*Cot[(Pi + (4*I)*x)/4])
)]))/Sqrt[-4*a^2 - b^2])/2
```

fricas [B] time = 0.48, size = 754, normalized size = 4.05

$$-\frac{bx\sqrt{\frac{4a^2+b^2}{b^2}}\,\log\left(\frac{\left(2a\cosh(x)+2a\sinh(x)-(b\cosh(x)+b\sinh(x))\sqrt{\frac{4a^2+b^2}{b^2}}\right)\sqrt{-\frac{b\sqrt{\frac{4a^2+b^2}{b^2}}+2a}{b}}+b}{b}\right)+bx\sqrt{\frac{4a^2+b^2}{b^2}}\,\log\left(-\frac{\left(2a\cosh(x)+2\right.}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*cosh(x)*sinh(x)),x, algorithm="fricas")
```

```
[Out] -(b*x*sqrt((4*a^2 + b^2)/b^2)*log(((2*a*cosh(x) + 2*a*sinh(x) - (b*cosh(x)
+ b*sinh(x))*sqrt((4*a^2 + b^2)/b^2))*sqrt(-(b*sqrt((4*a^2 + b^2)/b^2) + 2*
a)/b) + b)/b) + b*x*sqrt((4*a^2 + b^2)/b^2)*log(-((2*a*cosh(x) + 2*a*sinh(x
) - (b*cosh(x) + b*sinh(x))*sqrt((4*a^2 + b^2)/b^2))*sqrt(-(b*sqrt((4*a^2 +
 b^2)/b^2) + 2*a)/b) - b)/b) - b*x*sqrt((4*a^2 + b^2)/b^2)*log(((2*a*cosh(x
) + 2*a*sinh(x) + (b*cosh(x) + b*sinh(x))*sqrt((4*a^2 + b^2)/b^2))*sqrt((b*
sqrt((4*a^2 + b^2)/b^2) - 2*a)/b) + b)/b) - b*x*sqrt((4*a^2 + b^2)/b^2)*log
(-((2*a*cosh(x) + 2*a*sinh(x) + (b*cosh(x) + b*sinh(x))*sqrt((4*a^2 + b^2)/
b^2))*sqrt((b*sqrt((4*a^2 + b^2)/b^2) - 2*a)/b) - b)/b) + b*sqrt((4*a^2 + b
^2)/b^2)*dilog(-((2*a*cosh(x) + 2*a*sinh(x) - (b*cosh(x) + b*sinh(x))*sqrt(
(4*a^2 + b^2)/b^2))*sqrt(-(b*sqrt((4*a^2 + b^2)/b^2) + 2*a)/b) + b)/b + 1)
+ b*sqrt((4*a^2 + b^2)/b^2)*dilog(((2*a*cosh(x) + 2*a*sinh(x) - (b*cosh(x)
+ b*sinh(x))*sqrt((4*a^2 + b^2)/b^2))*sqrt(-(b*sqrt((4*a^2 + b^2)/b^2) + 2*
a)/b) - b)/b + 1) - b*sqrt((4*a^2 + b^2)/b^2)*dilog(-((2*a*cosh(x) + 2*a*si
nh(x) + (b*cosh(x) + b*sinh(x))*sqrt((4*a^2 + b^2)/b^2))*sqrt((b*sqrt((4*a^
2 + b^2)/b^2) - 2*a)/b) + b)/b + 1) - b*sqrt((4*a^2 + b^2)/b^2)*dilog(((2*a
*cosh(x) + 2*a*sinh(x) + (b*cosh(x) + b*sinh(x))*sqrt((4*a^2 + b^2)/b^2))*s
qrt((b*sqrt((4*a^2 + b^2)/b^2) - 2*a)/b) - b)/b + 1))/(4*a^2 + b^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{b\cosh(x)\sinh(x)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*cosh(x)*sinh(x)),x, algorithm="giac")
```

```
[Out] integrate(x/(b*cosh(x)*sinh(x) + a), x)
```

maple [B] time = 0.30, size = 376, normalized size = 2.02

$$\frac{\ln\left(1-\frac{b\,e^{2x}}{-2a-\sqrt{4a^2+b^2}}\right)x}{-2a-\sqrt{4a^2+b^2}}+\frac{2\ln\left(1-\frac{b\,e^{2x}}{-2a-\sqrt{4a^2+b^2}}\right)ax}{\sqrt{4a^2+b^2}\left(-2a-\sqrt{4a^2+b^2}\right)}-\frac{x^2}{-2a-\sqrt{4a^2+b^2}}-\frac{2a\,x^2}{\sqrt{4a^2+b^2}\left(-2a-\sqrt{4a^2+b^2}\right)}+\frac{\mathrm{p}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a+b*cosh(x)*sinh(x)),x)
```

```
[Out] 1/(-2*a-(4*a^2+b^2)^(1/2))*ln(1-b*exp(2*x)/(-2*a-(4*a^2+b^2)^(1/2)))*x+2/(4
*a^2+b^2)^(1/2)/(-2*a-(4*a^2+b^2)^(1/2))*ln(1-b*exp(2*x)/(-2*a-(4*a^2+b^2)^
(1/2)))*a*x-1/(-2*a-(4*a^2+b^2)^(1/2))*x^2-2/(4*a^2+b^2)^(1/2)/(-2*a-(4*a^2
+b^2)^(1/2))*a*x^2+1/2/(-2*a-(4*a^2+b^2)^(1/2))*polylog(2,b*exp(2*x)/(-2*a-
(4*a^2+b^2)^(1/2)))+1/(4*a^2+b^2)^(1/2)/(-2*a-(4*a^2+b^2)^(1/2))*polylog(2,
b*exp(2*x)/(-2*a-(4*a^2+b^2)^(1/2)))*a+1/(4*a^2+b^2)^(1/2)*x*ln(1-b*exp(2*x
)/((4*a^2+b^2)^(1/2)-2*a))-1/(4*a^2+b^2)^(1/2)*x^2+1/2/(4*a^2+b^2)^(1/2)*po
lylog(2,b*exp(2*x)/((4*a^2+b^2)^(1/2)-2*a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{b\cosh(x)\sinh(x)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*cosh(x)*sinh(x)),x, algorithm="maxima")
```

```
[Out] integrate(x/(b*cosh(x)*sinh(x) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x}{a+b\cosh(x)\sinh(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a + b*cosh(x)*sinh(x)),x)
```

```
[Out] int(x/(a + b*cosh(x)*sinh(x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{a + b \sinh(x) \cosh(x)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*cosh(x)*sinh(x)),x)
```

```
[Out] Integral(x/(a + b*sinh(x)*cosh(x)), x)
```

3.870 $\int \frac{1}{x(a+b\cosh(x)\sinh(x))}\,dx$

Optimal. Leaf size=20

$$\text{Int}\left(\frac{1}{x\left(a+\frac{1}{2}b\sinh(2x)\right)},x\right)$$

```
[Out] Unintegrable(1/x/(a+1/2*b*sinh(2*x)),x)
```

Rubi [A] time = 0.09, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{x(a+b\cosh(x)\sinh(x))}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x*(a + b*Cosh[x]*Sinh[x])),x]
[Out] Defer[Int][1/(x*(a + (b*Sinh[2*x])/2)), x]
```

Rubi steps

$$\int \frac{1}{x(a+b\cosh(x)\sinh(x))}\,dx=\int \frac{1}{x\left(a+\frac{1}{2}b\sinh(2x)\right)}\,dx$$

Mathematica [A] time = 1.08, size = 0, normalized size = 0.00

$$\int \frac{1}{x(a+b\cosh(x)\sinh(x))}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(x*(a + b*Cosh[x]*Sinh[x])),x]
[Out] Integrate[1/(x*(a + b*Cosh[x]*Sinh[x])), x]
```

fricas [A] time = 0.54, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{bx\cosh(x)\sinh(x)+ax},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*cosh(x)*sinh(x)),x, algorithm="fricas")
[Out] integral(1/(b*x*cosh(x)*sinh(x) + a*x), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b\cosh(x)\sinh(x)+a)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*cosh(x)*sinh(x)),x, algorithm="giac")
[Out] integrate(1/((b*cosh(x)*sinh(x) + a)*x), x)
```

maple [A] time = 0.34, size = 0, normalized size = 0.00

$$\int \frac{1}{x\,(a+b\cosh(x)\sinh(x))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(a+b*cosh(x)*sinh(x)),x)
[Out] int(1/x/(a+b*cosh(x)*sinh(x)),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b\cosh(x)\sinh(x)+a)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*cosh(x)*sinh(x)),x, algorithm="maxima")
[Out] integrate(1/((b*cosh(x)*sinh(x) + a)*x), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{1}{x\,\left(a+b\cosh(x)\sinh(x)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*cosh(x)*sinh(x))),x)
[Out] int(1/(x*(a + b*cosh(x)*sinh(x))), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(a+b\sinh\left(x\right)\cosh\left(x\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*cosh(x)*sinh(x)),x)
```

```
[Out] Integral(1/(x*(a + b*sinh(x)*cosh(x))), x)
```

3.871 $\int F^{c(a+bx)} \sinh^n(d+ex)\,dx$

Optimal. Leaf size=95

$$-\frac{\left(1-e^{2(d+ex)}\right)^{-n} F^{c(a+bx)} \sinh^n(d+ex)\,{}_2F_1\left(-n,-\frac{en-bc\log(F)}{2e};\frac{1}{2}\left(-n+\frac{bc\log(F)}{e}+2\right);e^{2(d+ex)}\right)}{en-bc\log(F)}$$

```
[Out] -F^(c*(b*x+a))*hypergeom([-n, 1/2*(-e*n+b*c*ln(F))/e],[1-1/2*n+1/2*b*c*ln(F
)/e],exp(2*e*x+2*d))*sinh(e*x+d)^n/((1-exp(2*e*x+2*d))^n)/(e*n-b*c*ln(F))
```

Rubi [A] time = 0.15, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {5482, 2259}

$$-\frac{\left(1-e^{2(d+ex)}\right)^{-n} F^{c(a+bx)} \sinh^n(d+ex)\,{}_2F_1\left(-n,-\frac{en-bc\log(F)}{2e};\frac{1}{2}\left(-n+\frac{bc\log(F)}{e}+2\right);e^{2(d+ex)}\right)}{en-bc\log(F)}$$

Antiderivative was successfully verified.

```
[In] Int[F^(c*(a + b*x))*Sinh[d + e*x]^n,x]
```

```
[Out] -((F^(c*(a + b*x))*Hypergeometric2F1[-n, -(e*n - b*c*Log[F])/(2*e), (2 - n
+ (b*c*Log[F])/e)/2, E^(2*(d + e*x))]*Sinh[d + e*x]^n)/((1 - E^(2*(d + e*x)
))^n*(e*n - b*c*Log[F])))
```

Rule 2259

```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_)*(G_)^((h_.)*((f_.
) + (g_.)*(x_)))*(H_)^((t_.)*((r_.) + (s_.)*(x_))), x_Symbol] :> Simp[(G^(h
*(f + g*x))*H^(t*(r + s*x))*(a + b*F^(e*(c + d*x)))^p*Hypergeometric2F1[-p,
 (g*h*Log[G] + s*t*Log[H])/(d*e*Log[F]), (g*h*Log[G] + s*t*Log[H])/(d*e*Log
[F]) + 1, Simplify[-((b*F^(e*(c + d*x)))/a)]])/((g*h*Log[G] + s*t*Log[H])*(
(a + b*F^(e*(c + d*x)))/a)^p), x] /; FreeQ[{F, G, H, a, b, c, d, e, f, g, h
, r, s, t, p}, x] &&  !IntegerQ[p]
```

Rule 5482

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sinh[(d_.) + (e_.)*(x_)]^(n_), x_Symb
ol] :> Dist[(E^(n*(d + e*x))*Sinh[d + e*x]^n)/(-1 + E^(2*(d + e*x)))^n, Int
[(F^(c*(a + b*x))*(-1 + E^(2*(d + e*x)))^n)/E^(n*(d + e*x)), x], x] /; Free
Q[{F, a, b, c, d, e, n}, x] &&  !IntegerQ[n]
```

Rubi steps

$$\int F^{c(a+bx)}\sinh^n(d+ex)\,dx=\left(e^{n(d+ex)}\left(-1+e^{2(d+ex)}\right)^{-n}\sinh^n(d+ex)\right)\int e^{-n(d+ex)}\left(-1+e^{2(d+ex)}\right)^n F^{c(a+bx)}\,dx$$
$$=-\frac{\left(1-e^{2(d+ex)}\right)^{-n}F^{c(a+bx)}\,{}_2F_1\left(-n,-\frac{en-bc\log(F)}{2e};\frac{1}{2}\left(2-n+\frac{bc\log(F)}{e}\right);e^{2(d+ex)}\right)\sinh^n}{en-bc\log(F)}$$

Mathematica [A] time = 0.07, size = 96, normalized size = 1.01

$$\frac{\left(1-e^{2(d+ex)}\right)^{-n}F^{c(a+bx)}\sinh^n(d+ex)\,{}_2F_1\left(-n,\frac{bc\log(F)-en}{2e};\frac{bc\log(F)-en}{2e}+1;e^{2(d+ex)}\right)}{bc\log(F)-en}$$

Antiderivative was successfully verified.

```
[In] Integrate[F^(c*(a + b*x))*Sinh[d + e*x]^n,x]
```

```
[Out] (F^(c*(a + b*x))*Hypergeometric2F1[-n, (-(e*n) + b*c*Log[F])/(2*e), 1 + (-(
e*n) + b*c*Log[F])/(2*e), E^(2*(d + e*x))]*Sinh[d + e*x]^n)/((1 - E^(2*(d +
 e*x)))^n*(-(e*n) + b*c*Log[F]))
```

fricas [F] time = 0.49, size = 0, normalized size = 0.00

$$\text{integral}\left(F^{bcx+ac}\sinh\left(ex+d\right)^n,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*sinh(e*x+d)^n,x, algorithm="fricas")
```

```
[Out] integral(F^(b*c*x + a*c)*sinh(e*x + d)^n, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int F^{(bx+a)c}\sinh\left(ex+d\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*sinh(e*x+d)^n,x, algorithm="giac")
```

```
[Out] integrate(F^((b*x + a)*c)*sinh(e*x + d)^n, x)
```

maple [F] time = 0.11, size = 0, normalized size = 0.00

$$\int F^{c(bx+a)}\left(\sinh^n\left(ex+d\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(F^(c*(b*x+a))*sinh(e*x+d)^n,x)
```

```
[Out] int(F^(c*(b*x+a))*sinh(e*x+d)^n,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int F^{(bx+a)c} \sinh\left(ex+d\right)^{n}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*sinh(e*x+d)^n,x, algorithm="maxima")
```

```
[Out] integrate(F^((b*x + a)*c)*sinh(e*x + d)^n, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int F^{c\,(a+b\,x)}\,\sinh\left(d+e\,x\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(F^(c*(a + b*x))*sinh(d + e*x)^n,x)
```

```
[Out] int(F^(c*(a + b*x))*sinh(d + e*x)^n, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int F^{c(a+bx)} \sinh^{n}\left(d + ex\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F**(c*(b*x+a))*sinh(e*x+d)**n,x)
```

```
[Out] Integral(F**(c*(a + b*x))*sinh(d + e*x)**n, x)
```

3.872 $\int e^{2(a+bx)} \sinh^3(a+bx)\,dx$

Optimal. Leaf size=66

$$\frac{e^{-a-bx}}{8b}+\frac{3e^{a+bx}}{8b}-\frac{e^{3a+3bx}}{8b}+\frac{e^{5a+5bx}}{40b}$$

```
[Out] 1/8*exp(-b*x-a)/b+3/8*exp(b*x+a)/b-1/8*exp(3*b*x+3*a)/b+1/40*exp(5*b*x+5*a)
/b
```

Rubi [A] time = 0.04, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2282, 12, 270}

$$\frac{e^{-a-bx}}{8b}+\frac{3e^{a+bx}}{8b}-\frac{e^{3a+3bx}}{8b}+\frac{e^{5a+5bx}}{40b}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*(a + b*x))*Sinh[a + b*x]^3,x]
```

```
[Out] E^(-a - b*x)/(8*b) + (3*E^(a + b*x))/(8*b) - E^(3*a + 3*b*x)/(8*b) + E^(5*a
 + 5*b*x)/(40*b)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$\int e^{2(a+bx)}\sinh^3(a+bx)\,dx = \frac{\text{Subst}\left(\int \frac{(-1+x^2)^3}{8x^2}\,dx, x, e^{a+bx}\right)}{b}$$
$$= \frac{\text{Subst}\left(\int \frac{(-1+x^2)^3}{x^2}\,dx, x, e^{a+bx}\right)}{8b}$$
$$= \frac{\text{Subst}\left(\int \left(3-\frac{1}{x^2}-3x^2+x^4\right)dx, x, e^{a+bx}\right)}{8b}$$
$$= \frac{e^{-a-bx}}{8b}+\frac{3e^{a+bx}}{8b}-\frac{e^{3a+3bx}}{8b}+\frac{e^{5a+5bx}}{40b}$$

Mathematica [A] time = 0.03, size = 50, normalized size = 0.76

$$\frac{e^{-a-bx}\left(15e^{2(a+bx)}-5e^{4(a+bx)}+e^{6(a+bx)}+5\right)}{40b}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*(a + b*x))*Sinh[a + b*x]^3,x]
```

```
[Out] (E^(-a - b*x)*(5 + 15*E^(2*(a + b*x)) - 5*E^(4*(a + b*x)) + E^(6*(a + b*x))
))/(40*b)
```

fricas [A] time = 0.49, size = 105, normalized size = 1.59

$$\frac{3\cosh(bx+a)^3+9\cosh(bx+a)\sinh(bx+a)^2-2\sinh(bx+a)^3-2\left(3\cosh(bx+a)^2+5\right)\sinh(bx+a)+5}{20\left(b\cosh(bx+a)^2-2\,b\cosh(bx+a)\sinh(bx+a)+b\sinh(bx+a)^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*b*x+2*a)*sinh(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/20*(3*cosh(b*x + a)^3 + 9*cosh(b*x + a)*sinh(b*x + a)^2 - 2*sinh(b*x + a)
^3 - 2*(3*cosh(b*x + a)^2 + 5)*sinh(b*x + a) + 5*cosh(b*x + a))/(b*cosh(b*x
 + a)^2 - 2*b*cosh(b*x + a)*sinh(b*x + a) + b*sinh(b*x + a)^2)
```

giac [A] time = 0.14, size = 53, normalized size = 0.80

$$\frac{\left(e^{(5\,bx+10\,a)}-5\,e^{(3\,bx+8\,a)}+15\,e^{(bx+6\,a)}\right)e^{(-5\,a)}+5\,e^{(-bx-a)}}{40\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*b*x+2*a)*sinh(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/40*((e^(5*b*x + 10*a) - 5*e^(3*b*x + 8*a) + 15*e^(b*x + 6*a))*e^(-5*a) +
5*e^(-b*x - a))/b
```

maple [A] time = 0.28, size = 80, normalized size = 1.21

$$\frac{\sinh(bx+a)}{4b}-\frac{\sinh(3bx+3a)}{8b}+\frac{\sinh(5bx+5a)}{40b}+\frac{\cosh(bx+a)}{2b}-\frac{\cosh(3bx+3a)}{8b}+\frac{\cosh(5bx+5a)}{40b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(2*b*x+2*a)*sinh(b*x+a)^3,x)
```

```
[Out] 1/4*sinh(b*x+a)/b-1/8/b*sinh(3*b*x+3*a)+1/40/b*sinh(5*b*x+5*a)+1/2*cosh(b*x
+a)/b-1/8*cosh(3*b*x+3*a)/b+1/40*cosh(5*b*x+5*a)/b
```

maxima [A] time = 0.33, size = 53, normalized size = 0.80

$$-\frac{\left(5\,e^{(-2\,bx-2\,a)}-15\,e^{(-4\,bx-4\,a)}-1\right)e^{(5\,bx+5\,a)}}{40\,b}+\frac{e^{(-bx-a)}}{8\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*b*x+2*a)*sinh(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -1/40*(5*e^(-2*b*x - 2*a) - 15*e^(-4*b*x - 4*a) - 1)*e^(5*b*x + 5*a)/b + 1/
8*e^(-b*x - a)/b
```

mupad [B] time = 0.28, size = 45, normalized size = 0.68

$$\frac{15\,\mathrm{e}^{a+b\,x}+5\,\mathrm{e}^{-a-b\,x}-5\,\mathrm{e}^{3\,a+3\,b\,x}+\mathrm{e}^{5\,a+5\,b\,x}}{40\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(2*a + 2*b*x)*sinh(a + b*x)^3,x)
```

```
[Out] (15*exp(a + b*x) + 5*exp(- a - b*x) - 5*exp(3*a + 3*b*x) + exp(5*a + 5*b*x)
)/(40*b)
```

sympy [A] time = 16.58, size = 124, normalized size = 1.88

$$\begin{cases} \frac{2e^{2a}e^{2bx}\sinh^3(a+bx)}{5b}+\frac{e^{2a}e^{2bx}\sinh^2(a+bx)\cosh(a+bx)}{5b}-\frac{4e^{2a}e^{2bx}\sinh(a+bx)\cosh^2(a+bx)}{5b}+\frac{2e^{2a}e^{2bx}\cosh^3(a+bx)}{5b} & \text{for } b\neq 0 \\ xe^{2a}\sinh^3(a) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*b*x+2*a)*sinh(b*x+a)**3,x)
```

```
[Out] Piecewise((2*exp(2*a)*exp(2*b*x)*sinh(a + b*x)**3/(5*b) + exp(2*a)*exp(2*b*
x)*sinh(a + b*x)**2*cosh(a + b*x)/(5*b) - 4*exp(2*a)*exp(2*b*x)*sinh(a + b*
x)*cosh(a + b*x)**2/(5*b) + 2*exp(2*a)*exp(2*b*x)*cosh(a + b*x)**3/(5*b), N
e(b, 0)), (x*exp(2*a)*sinh(a)**3, True))
```

3.873 $\int e^{2(a+bx)} \sinh^2(a+bx)\,dx$

Optimal. Leaf size=40

$$-\frac{e^{2a+2bx}}{4b}+\frac{e^{4a+4bx}}{16b}+\frac{x}{4}$$

```
[Out] -1/4*exp(2*b*x+2*a)/b+1/16*exp(4*b*x+4*a)/b+1/4*x
```

Rubi [A] time = 0.04, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2282, 12, 266, 43}

$$-\frac{e^{2a+2bx}}{4b}+\frac{e^{4a+4bx}}{16b}+\frac{x}{4}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*(a + b*x))*Sinh[a + b*x]^2,x]
```

```
[Out] -E^(2*a + 2*b*x)/(4*b) + E^(4*a + 4*b*x)/(16*b) + x/4
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
```

```
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$\begin{aligned}\int e^{2(a+bx)}\sinh^2(a+bx)\,dx &= \frac{\text{Subst}\left(\int \frac{\left(1-x^2\right)^2}{4x}\,dx, x, e^{a+bx}\right)}{b}\\ &= \frac{\text{Subst}\left(\int \frac{\left(1-x^2\right)^2}{x}\,dx, x, e^{a+bx}\right)}{4b}\\ &= \frac{\text{Subst}\left(\int \frac{(1-x)^2}{x}\,dx, x, e^{2a+2bx}\right)}{8b}\\ &= \frac{\text{Subst}\left(\int \left(-2+\frac{1}{x}+x\right)dx, x, e^{2a+2bx}\right)}{8b}\\ &= -\frac{e^{2a+2bx}}{4b}+\frac{e^{4a+4bx}}{16b}+\frac{x}{4}\end{aligned}$$

Mathematica [A] time = 0.02, size = 32, normalized size = 0.80

$$\frac{-4e^{2(a+bx)}+e^{4(a+bx)}+4bx}{16b}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*(a + b*x))*Sinh[a + b*x]^2,x]
```

```
[Out] (-4*E^(2*(a + b*x)) + E^(4*(a + b*x)) + 4*b*x)/(16*b)
```

fricas [B] time = 0.45, size = 92, normalized size = 2.30

$$\frac{(4\,bx+1)\cosh(bx+a)^2-2\,(4\,bx-1)\cosh(bx+a)\sinh(bx+a)+(4\,bx+1)\sinh(bx+a)^2-4}{16\left(b\cosh(bx+a)^2-2\,b\cosh(bx+a)\sinh(bx+a)+b\sinh(bx+a)^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*b*x+2*a)*sinh(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/16*((4*b*x + 1)*cosh(b*x + a)^2 - 2*(4*b*x - 1)*cosh(b*x + a)*sinh(b*x +
a) + (4*b*x + 1)*sinh(b*x + a)^2 - 4)/(b*cosh(b*x + a)^2 - 2*b*cosh(b*x + a
)*sinh(b*x + a) + b*sinh(b*x + a)^2)
```

giac [A] time = 0.13, size = 30, normalized size = 0.75

$$\frac{4\,bx + e^{(4\,bx+4\,a)} - 4\,e^{(2\,bx+2\,a)}}{16\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*b*x+2*a)*sinh(b*x+a)^2,x, algorithm="giac")
[Out] 1/16*(4*b*x + e^(4*b*x + 4*a) - 4*e^(2*b*x + 2*a))/b
```

maple [A] time = 0.20, size = 61, normalized size = 1.52

$$\frac{x}{4} - \frac{\sinh(2bx + 2a)}{4b} + \frac{\sinh(4bx + 4a)}{16b} - \frac{\cosh(2bx + 2a)}{4b} + \frac{\cosh(4bx + 4a)}{16b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(2*b*x+2*a)*sinh(b*x+a)^2,x)
[Out] 1/4*x-1/4*sinh(2*b*x+2*a)/b+1/16/b*sinh(4*b*x+4*a)-1/4*cosh(2*b*x+2*a)/b+1/
16*cosh(4*b*x+4*a)/b
```

maxima [A] time = 0.31, size = 37, normalized size = 0.92

$$\frac{1}{4}\,x - \frac{\left(4\,e^{(-2\,bx-2\,a)} - 1\right)e^{(4\,bx+4\,a)}}{16\,b} + \frac{a}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*b*x+2*a)*sinh(b*x+a)^2,x, algorithm="maxima")
[Out] 1/4*x - 1/16*(4*e^(-2*b*x - 2*a) - 1)*e^(4*b*x + 4*a)/b + 1/4*a/b
```

mupad [B] time = 1.86, size = 32, normalized size = 0.80

$$\frac{x}{4} - \frac{\frac{e^{2\,a+2\,b\,x}}{4} - \frac{e^{4\,a+4\,b\,x}}{16}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(2*a + 2*b*x)*sinh(a + b*x)^2,x)
[Out] x/4 - (exp(2*a + 2*b*x)/4 - exp(4*a + 4*b*x)/16)/b
```

sympy [A] time = 4.58, size = 139, normalized size = 3.48

$$\begin{cases} \frac{xe^{2a}e^{2bx}\sinh^2(a+bx)}{4} - \frac{xe^{2a}e^{2bx}\sinh(a+bx)\cosh(a+bx)}{2} + \frac{xe^{2a}e^{2bx}\cosh^2(a+bx)}{4} + \frac{e^{2a}e^{2bx}\sinh^2(a+bx)}{2b} - \frac{e^{2a}e^{2bx}\sinh(a+bx)\cosh(a+bx)}{4b} \\ xe^{2a}\sinh^2(a) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*b*x+2*a)*sinh(b*x+a)**2,x)
```

```
[Out] Piecewise((x*exp(2*a)*exp(2*b*x)*sinh(a + b*x)**2/4 - x*exp(2*a)*exp(2*b*x)
*sinh(a + b*x)*cosh(a + b*x)/2 + x*exp(2*a)*exp(2*b*x)*cosh(a + b*x)**2/4 +
 exp(2*a)*exp(2*b*x)*sinh(a + b*x)**2/(2*b) - exp(2*a)*exp(2*b*x)*sinh(a +
b*x)*cosh(a + b*x)/(4*b), Ne(b, 0)), (x*exp(2*a)*sinh(a)**2, True))
```

3.874 $\int e^{2(a+bx)} \sinh(a+bx)\,dx$

Optimal. Leaf size=32

$$\frac{e^{3a+3bx}}{6b}-\frac{e^{a+bx}}{2b}$$

```
[Out] -1/2*exp(b*x+a)/b+1/6*exp(3*b*x+3*a)/b
```

Rubi [A] time = 0.02, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2282, 12}

$$\frac{e^{3a+3bx}}{6b}-\frac{e^{a+bx}}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*(a + b*x))*Sinh[a + b*x],x]
```

```
[Out] -E^(a + b*x)/(2*b) + E^(3*a + 3*b*x)/(6*b)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$\begin{aligned}
\int e^{2(a+bx)} \sinh(a+bx)\,dx &= \frac{\operatorname{Subst}\left(\int \frac{1}{2}\left(-1+x^2\right)\,dx, x, e^{a+bx}\right)}{b}\\
&= \frac{\operatorname{Subst}\left(\int \left(-1+x^2\right)\,dx, x, e^{a+bx}\right)}{2b}\\
&= -\frac{e^{a+bx}}{2b}+\frac{e^{3a+3bx}}{6b}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 25, normalized size = 0.78

$$\frac{e^{a+bx}\left(e^{2(a+bx)}-3\right)}{6b}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*(a + b*x))*Sinh[a + b*x],x]
```

```
[Out] (E^(a + b*x)*(-3 + E^(2*(a + b*x))))/(6*b)
```

fricas [B] time = 0.51, size = 54, normalized size = 1.69

$$\frac{\cosh(bx+a)^2+2\cosh(bx+a)\sinh(bx+a)+\sinh(bx+a)^2-3}{6(b\cosh(bx+a)-b\sinh(bx+a))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*b*x+2*a)*sinh(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/6*(cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 - 3)
/(b*cosh(b*x + a) - b*sinh(b*x + a))
```

giac [A] time = 0.12, size = 23, normalized size = 0.72

$$\frac{e^{(3bx+3a)}-3e^{(bx+a)}}{6b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*b*x+2*a)*sinh(b*x+a),x, algorithm="giac")
```

```
[Out] 1/6*(e^(3*b*x + 3*a) - 3*e^(b*x + a))/b
```

maple [A] time = 0.15, size = 52, normalized size = 1.62

$$-\frac{\sinh(bx+a)}{2b}+\frac{\sinh(3bx+3a)}{6b}-\frac{\cosh(bx+a)}{2b}+\frac{\cosh(3bx+3a)}{6b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(2*b*x+2*a)*sinh(b*x+a),x)
```

```
[Out] -1/2*sinh(b*x+a)/b+1/6/b*sinh(3*b*x+3*a)-1/2*cosh(b*x+a)/b+1/6*cosh(3*b*x+3
*a)/b
```

maxima [A] time = 0.32, size = 26, normalized size = 0.81

$$\frac{e^{(3bx+3a)}}{6b}-\frac{e^{(bx+a)}}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*b*x+2*a)*sinh(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/6*e^(3*b*x + 3*a)/b - 1/2*e^(b*x + a)/b
```

mupad [**B**] time = 0.07, size = 25, normalized size = 0.78

$$-\frac{3\,\mathrm{e}^{a+b\,x}-\mathrm{e}^{3\,a+3\,b\,x}}{6\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(2*a + 2*b*x)*sinh(a + b*x),x)
```

```
[Out] -(3*exp(a + b*x) - exp(3*a + 3*b*x))/(6*b)
```

sympy [**A**] time = 1.08, size = 54, normalized size = 1.69

$$\begin{cases} \frac{2e^{2a}e^{2bx}\sinh\left(a+bx\right)}{3b}-\frac{e^{2a}e^{2bx}\cosh\left(a+bx\right)}{3b} & \text{for}\ b\neq 0 \\ xe^{2a}\sinh\left(a\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*b*x+2*a)*sinh(b*x+a),x)
```

```
[Out] Piecewise((2*exp(2*a)*exp(2*b*x)*sinh(a + b*x)/(3*b) - exp(2*a)*exp(2*b*x)*
cosh(a + b*x)/(3*b), Ne(b, 0)), (x*exp(2*a)*sinh(a), True))
```

3.875 $\int e^{2(a+bx)}\operatorname{csch}(a+bx)\,dx$

Optimal. Leaf size=26

$$\frac{2e^{a+bx}}{b}-\frac{2\tanh^{-1}\left(e^{a+bx}\right)}{b}$$

```
[Out] 2*exp(b*x+a)/b-2*arctanh(exp(b*x+a))/b
```

Rubi [A] time = 0.02, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2282, 12, 321, 207}

$$\frac{2e^{a+bx}}{b}-\frac{2\tanh^{-1}\left(e^{a+bx}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*(a + b*x))*Csch[a + b*x],x]
```

```
[Out] (2*E^(a + b*x))/b - (2*ArcTanh[E^(a + b*x)])/b
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
```

```
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$\begin{aligned}\int e^{2(a+bx)}\text{csch}(a+bx)\,dx &= \frac{\text{Subst}\left(\int \frac{2x^2}{-1+x^2}\,dx, x, e^{a+bx}\right)}{b}\\ &= \frac{2\,\text{Subst}\left(\int \frac{x^2}{-1+x^2}\,dx, x, e^{a+bx}\right)}{b}\\ &= \frac{2e^{a+bx}}{b} + \frac{2\,\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, e^{a+bx}\right)}{b}\\ &= \frac{2e^{a+bx}}{b} - \frac{2\tanh^{-1}\left(e^{a+bx}\right)}{b}\end{aligned}$$

Mathematica [A] time = 0.02, size = 23, normalized size = 0.88

$$\frac{2\left(e^{a+bx} - \tanh^{-1}\left(e^{a+bx}\right)\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*(a + b*x))*Csch[a + b*x],x]
```

```
[Out] (2*(E^(a + b*x) - ArcTanh[E^(a + b*x)]))/b
```

fricas [B] time = 0.51, size = 53, normalized size = 2.04

$$\frac{2\cosh(bx+a) - \log(\cosh(bx+a) + \sinh(bx+a) + 1) + \log(\cosh(bx+a) + \sinh(bx+a) - 1) + 2\sinh(bx}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*b*x+2*a)*csch(b*x+a),x, algorithm="fricas")
```

```
[Out] (2*cosh(b*x + a) - log(cosh(b*x + a) + sinh(b*x + a) + 1) + log(cosh(b*x +
a) + sinh(b*x + a) - 1) + 2*sinh(b*x + a))/b
```

giac [B] time = 0.12, size = 50, normalized size = 1.92

$$-\frac{\left(e^{(-2a)}\log\left(e^{(bx+a)}+1\right) - e^{(-2a)}\log\left(\left|e^{(bx+a)}-1\right|\right) - 2e^{(bx-a)}\right)e^{(2a)}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*b*x+2*a)*csch(b*x+a),x, algorithm="giac")
```

```
[Out] -(e^(-2*a)*log(e^(b*x + a) + 1) - e^(-2*a)*log(abs(e^(b*x + a) - 1)) - 2*e^
(b*x - a))*e^(2*a)/b
```

maple [A] time = 0.30, size = 40, normalized size = 1.54

$$\frac{2\,\mathrm{e}^{bx+a}}{b}+\frac{\ln\left(\mathrm{e}^{bx+a}-1\right)}{b}-\frac{\ln\left(1+\mathrm{e}^{bx+a}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(2*b*x+2*a)*csch(b*x+a),x)
```

```
[Out] 2*exp(b*x+a)/b+1/b*ln(exp(b*x+a)-1)-1/b*ln(1+exp(b*x+a))
```

maxima [A] time = 0.31, size = 45, normalized size = 1.73

$$\frac{2\,e^{(bx+a)}}{b}-\frac{\log\left(e^{(-bx-a)}+1\right)}{b}+\frac{\log\left(e^{(-bx-a)}-1\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*b*x+2*a)*csch(b*x+a),x, algorithm="maxima")
```

```
[Out] 2*e^(b*x + a)/b - log(e^(-b*x - a) + 1)/b + log(e^(-b*x - a) - 1)/b
```

mupad [B] time = 0.08, size = 39, normalized size = 1.50

$$\frac{2\,\mathrm{e}^{a+b\,x}}{b}-\frac{2\,\mathrm{atan}\left(\frac{\mathrm{e}^{b\,x}\,\mathrm{e}^{a}\,\sqrt{-b^2}}{b}\right)}{\sqrt{-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(2*a + 2*b*x)/sinh(a + b*x),x)
```

```
[Out] (2*exp(a + b*x))/b - (2*atan((exp(b*x)*exp(a)*(-b^2)^(1/2))/b))/(-b^2)^(1/2
)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$e^{2a}\int e^{2bx}\operatorname{csch}\left(a+bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*b*x+2*a)*csch(b*x+a),x)
```

```
[Out] exp(2*a)*Integral(exp(2*b*x)*csch(a + b*x), x)
```

3.876 $\int e^{2(a+bx)}\mathrm{csch}^2(a+bx)\,dx$

Optimal. Leaf size=42

$$\frac{2}{b\left(1-e^{2a+2bx}\right)}+\frac{2\log\left(1-e^{2a+2bx}\right)}{b}$$

```
[Out] 2/b/(1-exp(2*b*x+2*a))+2*ln(1-exp(2*b*x+2*a))/b
```

Rubi [A] time = 0.04, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2282, 12, 266, 43}

$$\frac{2}{b\left(1-e^{2a+2bx}\right)}+\frac{2\log\left(1-e^{2a+2bx}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*(a + b*x))*Csch[a + b*x]^2,x]
```

```
[Out] 2/(b*(1 - E^(2*a + 2*b*x))) + (2*Log[1 - E^(2*a + 2*b*x)])/b
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
```

```
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$\begin{aligned}
\int e^{2(a+bx)}\operatorname{csch}^2(a+bx)\,dx &= \frac{\operatorname{Subst}\left(\int \frac{4x^3}{\left(1-x^2\right)^2}\,dx, x, e^{a+bx}\right)}{b}\\
&= \frac{4\operatorname{Subst}\left(\int \frac{x^3}{\left(1-x^2\right)^2}\,dx, x, e^{a+bx}\right)}{b}\\
&= \frac{2\operatorname{Subst}\left(\int \frac{x}{(1-x)^2}\,dx, x, e^{2a+2bx}\right)}{b}\\
&= \frac{2\operatorname{Subst}\left(\int \left(\frac{1}{(-1+x)^2}+\frac{1}{-1+x}\right)dx, x, e^{2a+2bx}\right)}{b}\\
&= \frac{2}{b\left(1-e^{2a+2bx}\right)}+\frac{2\log\left(1-e^{2a+2bx}\right)}{b}
\end{aligned}$$

Mathematica [A] time = 0.05, size = 37, normalized size = 0.88

$$\frac{2\left(\frac{1}{1-e^{2a+2bx}}+\log\left(1-e^{2a+2bx}\right)\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*(a + b*x))*Csch[a + b*x]^2,x]
[Out] (2*((1 - E^(2*a + 2*b*x))^(-1) + Log[1 - E^(2*a + 2*b*x)]))/b
```

fricas [B] time = 0.48, size = 104, normalized size = 2.48

$$\frac{2\left(\left(\cosh(bx+a)^2+2\cosh(bx+a)\sinh(bx+a)+\sinh(bx+a)^2-1\right)\log\left(\frac{2\sinh(bx+a)}{\cosh(bx+a)-\sinh(bx+a)}\right)-1\right)}{b\cosh(bx+a)^2+2\,b\cosh(bx+a)\sinh(bx+a)+b\sinh(bx+a)^2-b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*b*x+2*a)*csch(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 2*((cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 - 1)*
log(2*sinh(b*x + a)/(cosh(b*x + a) - sinh(b*x + a))) - 1)/(b*cosh(b*x + a)^
2 + 2*b*cosh(b*x + a)*sinh(b*x + a) + b*sinh(b*x + a)^2 - b)
```

giac [A] time = 0.15, size = 48, normalized size = 1.14

$$\frac{2\left(e^{(-2a)}\log\left(\left|e^{(2bx+2a)}-1\right|\right)-\frac{e^{(2bx)}}{e^{(2bx+2a)}-1}\right)e^{(2a)}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*b*x+2*a)*csch(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 2*(e^(-2*a)*log(abs(e^(2*b*x + 2*a) - 1)) - e^(2*b*x)/(e^(2*b*x + 2*a) - 1)
)*e^(2*a)/b
```

maple [A] time = 0.32, size = 43, normalized size = 1.02

$$-\frac{4a}{b}-\frac{2}{b\left(e^{2bx+2a}-1\right)}+\frac{2\ln\left(e^{2bx+2a}-1\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(2*b*x+2*a)*csch(b*x+a)^2,x)
```

```
[Out] -4*a/b-2/b/(exp(2*b*x+2*a)-1)+2/b*ln(exp(2*b*x+2*a)-1)
```

maxima [A] time = 0.32, size = 62, normalized size = 1.48

$$4x+\frac{4a}{b}+\frac{2\log\left(e^{(-bx-a)}+1\right)}{b}+\frac{2\log\left(e^{(-bx-a)}-1\right)}{b}+\frac{2}{b\left(e^{(-2bx-2a)}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*b*x+2*a)*csch(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 4*x + 4*a/b + 2*log(e^(-b*x - a) + 1)/b + 2*log(e^(-b*x - a) - 1)/b + 2/(b*
(e^(-2*b*x - 2*a) - 1))
```

mupad [B] time = 1.79, size = 37, normalized size = 0.88

$$\frac{2\ln\left(e^{2a}e^{2bx}-1\right)}{b}-\frac{2}{b\left(e^{2a+2bx}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(2*a + 2*b*x)/sinh(a + b*x)^2,x)
```

```
[Out] (2*log(exp(2*a)*exp(2*b*x) - 1))/b - 2/(b*(exp(2*a + 2*b*x) - 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$e^{2a} \int e^{2bx} \operatorname{csch}^2(a + bx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*b*x+2*a)*csch(b*x+a)**2,x)
```

```
[Out] exp(2*a)*Integral(exp(2*b*x)*csch(a + b*x)**2, x)
```

3.877 $\int e^{2(a+bx)}\operatorname{csch}^3(a+bx)\,dx$

Optimal. Leaf size=73

$$\frac{3e^{a+bx}}{b\left(1-e^{2a+2bx}\right)}-\frac{2e^{3a+3bx}}{b\left(1-e^{2a+2bx}\right)^2}-\frac{3\tanh^{-1}\left(e^{a+bx}\right)}{b}$$

```
[Out] -2*exp(3*b*x+3*a)/b/(1-exp(2*b*x+2*a))^2+3*exp(b*x+a)/b/(1-exp(2*b*x+2*a))-
3*arctanh(exp(b*x+a))/b
```

Rubi [A] time = 0.04, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2282, 12, 288, 207}

$$\frac{3e^{a+bx}}{b\left(1-e^{2a+2bx}\right)}-\frac{2e^{3a+3bx}}{b\left(1-e^{2a+2bx}\right)^2}-\frac{3\tanh^{-1}\left(e^{a+bx}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*(a + b*x))*Csch[a + b*x]^3,x]
```

```
[Out] (-2*E^(3*a + 3*b*x))/(b*(1 - E^(2*a + 2*b*x))^2) + (3*E^(a + b*x))/(b*(1 -
E^(2*a + 2*b*x))) - (3*ArcTanh[E^(a + b*x)])/b
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$
\begin{aligned}
\int e^{2(a+bx)}\operatorname{csch}^3(a+bx)\,dx &= \frac{\operatorname{Subst}\left(\int \frac{8x^4}{\left(-1+x^2\right)^3}\,dx, x, e^{a+bx}\right)}{b}\\
&= \frac{8\operatorname{Subst}\left(\int \frac{x^4}{\left(-1+x^2\right)^3}\,dx, x, e^{a+bx}\right)}{b}\\
&= -\frac{2e^{3a+3bx}}{b\left(1-e^{2a+2bx}\right)^2}+\frac{6\operatorname{Subst}\left(\int \frac{x^2}{\left(-1+x^2\right)^2}\,dx, x, e^{a+bx}\right)}{b}\\
&= -\frac{2e^{3a+3bx}}{b\left(1-e^{2a+2bx}\right)^2}+\frac{3e^{a+bx}}{b\left(1-e^{2a+2bx}\right)}+\frac{3\operatorname{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, e^{a+bx}\right)}{b}\\
&= -\frac{2e^{3a+3bx}}{b\left(1-e^{2a+2bx}\right)^2}+\frac{3e^{a+bx}}{b\left(1-e^{2a+2bx}\right)}-\frac{3\tanh^{-1}\left(e^{a+bx}\right)}{b}
\end{aligned}
$$

Mathematica [A] time = 0.06, size = 61, normalized size = 0.84

$$
\frac{3e^{a+bx}-5e^{3(a+bx)}-3\left(e^{2(a+bx)}-1\right)^2\tanh^{-1}\left(e^{a+bx}\right)}{b\left(e^{2(a+bx)}-1\right)^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*(a + b*x))*Csch[a + b*x]^3,x]
```

```
[Out] (3*E^(a + b*x) - 5*E^(3*(a + b*x)) - 3*(-1 + E^(2*(a + b*x)))^2*ArcTanh[E^(
a + b*x)])/(b*(-1 + E^(2*(a + b*x)))^2)
```

fricas [B] time = 0.45, size = 388, normalized size = 5.32

$$
-\frac{10\cosh(bx+a)^3+30\cosh(bx+a)\sinh(bx+a)^2+10\sinh(bx+a)^3+3\left(\cosh(bx+a)^4+4\cosh(bx+a\right.}{}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*b*x+2*a)*csch(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -1/2*(10*cosh(b*x + a)^3 + 30*cosh(b*x + a)*sinh(b*x + a)^2 + 10*sinh(b*x +
 a)^3 + 3*(cosh(b*x + a)^4 + 4*cosh(b*x + a)*sinh(b*x + a)^3 + sinh(b*x + a
)^4 + 2*(3*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^2 - 2*cosh(b*x + a)^2 + 4*(co
sh(b*x + a)^3 - cosh(b*x + a))*sinh(b*x + a) + 1)*log(cosh(b*x + a) + sinh(
b*x + a) + 1) - 3*(cosh(b*x + a)^4 + 4*cosh(b*x + a)*sinh(b*x + a)^3 + sinh
(b*x + a)^4 + 2*(3*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^2 - 2*cosh(b*x + a)^2
 + 4*(cosh(b*x + a)^3 - cosh(b*x + a))*sinh(b*x + a) + 1)*log(cosh(b*x + a)
 + sinh(b*x + a) - 1) + 6*(5*cosh(b*x + a)^2 - 1)*sinh(b*x + a) - 6*cosh(b*
x + a))/(b*cosh(b*x + a)^4 + 4*b*cosh(b*x + a)*sinh(b*x + a)^3 + b*sinh(b*x
 + a)^4 - 2*b*cosh(b*x + a)^2 + 2*(3*b*cosh(b*x + a)^2 - b)*sinh(b*x + a)^2
 + 4*(b*cosh(b*x + a)^3 - b*cosh(b*x + a))*sinh(b*x + a) + b)
```

giac [A] time = 0.13, size = 78, normalized size = 1.07

$$-\frac{\left(3\,e^{(-2\,a)}\log\left(e^{(bx+a)}+1\right)-3\,e^{(-2\,a)}\log\left(\left|e^{(bx+a)}-1\right|\right)+\frac{2\left(5\,e^{(3\,bx+2\,a)}-3\,e^{(bx)}\right)e^{(-a)}}{\left(e^{(2\,bx+2\,a)}-1\right)^{2}}\right)e^{(2\,a)}}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*b*x+2*a)*csch(b*x+a)^3,x, algorithm="giac")
```

```
[Out] -1/2*(3*e^(-2*a)*log(e^(b*x + a) + 1) - 3*e^(-2*a)*log(abs(e^(b*x + a) - 1)
) + 2*(5*e^(3*b*x + 2*a) - 3*e^(b*x))*e^(-a)/(e^(2*b*x + 2*a) - 1)^2)*e^(2*
a)/b
```

maple [A] time = 0.34, size = 67, normalized size = 0.92

$$-\frac{\mathrm{e}^{bx+a}\left(5\,\mathrm{e}^{2bx+2a}-3\right)}{b\left(\mathrm{e}^{2bx+2a}-1\right)^{2}}-\frac{3\ln\left(1+\mathrm{e}^{bx+a}\right)}{2b}+\frac{3\ln\left(\mathrm{e}^{bx+a}-1\right)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(2*b*x+2*a)*csch(b*x+a)^3,x)
```

```
[Out] -exp(b*x+a)*(5*exp(2*b*x+2*a)-3)/b/(exp(2*b*x+2*a)-1)^2-3/2/b*ln(1+exp(b*x+
a))+3/2/b*ln(exp(b*x+a)-1)
```

maxima [A] time = 0.31, size = 88, normalized size = 1.21

$$-\frac{3\,\log\left(e^{(-bx-a)}+1\right)}{2\,b}+\frac{3\,\log\left(e^{(-bx-a)}-1\right)}{2\,b}+\frac{5\,e^{(-bx-a)}-3\,e^{(-3\,bx-3\,a)}}{b\left(2\,e^{(-2\,bx-2\,a)}-e^{(-4\,bx-4\,a)}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*b*x+2*a)*csch(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -3/2*log(e^(-b*x - a) + 1)/b + 3/2*log(e^(-b*x - a) - 1)/b + (5*e^(-b*x - a
) - 3*e^(-3*b*x - 3*a))/(b*(2*e^(-2*b*x - 2*a) - e^(-4*b*x - 4*a) - 1))
```

mupad [B] time = 1.81, size = 90, normalized size = 1.23

$$-\frac{3\,\mathrm{atan}\left(\frac{\mathrm{e}^{b\,x}\,\mathrm{e}^{a}\,\sqrt{-b^2}}{b}\right)}{\sqrt{-b^2}}-\frac{2\,\mathrm{e}^{3\,a+3\,b\,x}}{b\,\left(\mathrm{e}^{4\,a+4\,b\,x}-2\,\mathrm{e}^{2\,a+2\,b\,x}+1\right)}-\frac{3\,\mathrm{e}^{a+b\,x}}{b\,\left(\mathrm{e}^{2\,a+2\,b\,x}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(2*a + 2*b*x)/sinh(a + b*x)^3,x)
```

```
[Out] - (3*atan((exp(b*x)*exp(a)*(-b^2)^(1/2))/b))/(-b^2)^(1/2) - (2*exp(3*a + 3*
b*x))/(b*(exp(4*a + 4*b*x) - 2*exp(2*a + 2*b*x) + 1)) - (3*exp(a + b*x))/(b
*(exp(2*a + 2*b*x) - 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$e^{2a}\int e^{2bx}\operatorname{csch}^{3}(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*b*x+2*a)*csch(b*x+a)**3,x)
```

```
[Out] exp(2*a)*Integral(exp(2*b*x)*csch(a + b*x)**3, x)
```

3.878 $\int e^{a+bx} \sinh^3(c + dx)\, dx$

Optimal. Leaf size=139

$$\frac{be^{a+bx}\sinh^3(c+dx)}{b^2-9d^2}-\frac{3de^{a+bx}\sinh^2(c+dx)\cosh(c+dx)}{b^2-9d^2}+\frac{6bd^2e^{a+bx}\sinh(c+dx)}{b^4-10b^2d^2+9d^4}-\frac{6d^3e^{a+bx}\cosh(c+dx)}{b^4-10b^2d^2+9d^4}$$

```
[Out] -6*d^3*exp(b*x+a)*cosh(d*x+c)/(b^4-10*b^2*d^2+9*d^4)+6*b*d^2*exp(b*x+a)*sin
h(d*x+c)/(b^4-10*b^2*d^2+9*d^4)-3*d*exp(b*x+a)*cosh(d*x+c)*sinh(d*x+c)^2/(b
^2-9*d^2)+b*exp(b*x+a)*sinh(d*x+c)^3/(b^2-9*d^2)
```

Rubi [A] time = 0.06, antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {5476, 5474}

$$\frac{be^{a+bx}\sinh^3(c+dx)}{b^2-9d^2}+\frac{6bd^2e^{a+bx}\sinh(c+dx)}{-10b^2d^2+b^4+9d^4}-\frac{6d^3e^{a+bx}\cosh(c+dx)}{-10b^2d^2+b^4+9d^4}-\frac{3de^{a+bx}\sinh^2(c+dx)\cosh(c+dx)}{b^2-9d^2}$$

Antiderivative was successfully verified.

```
[In] Int[E^(a + b*x)*Sinh[c + d*x]^3,x]
```

```
[Out] (-6*d^3*E^(a + b*x)*Cosh[c + d*x])/(b^4 - 10*b^2*d^2 + 9*d^4) + (6*b*d^2*E^
(a + b*x)*Sinh[c + d*x])/(b^4 - 10*b^2*d^2 + 9*d^4) - (3*d*E^(a + b*x)*Cosh
[c + d*x]*Sinh[c + d*x]^2)/(b^2 - 9*d^2) + (b*E^(a + b*x)*Sinh[c + d*x]^3)/
(b^2 - 9*d^2)
```

Rule 5474

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sinh[(d_.) + (e_.)*(x_)], x_Symbol] :
> -Simp[(b*c*Log[F]*F^(c*(a + b*x))*Sinh[d + e*x])/(e^2 - b^2*c^2*Log[F]^2)
, x] + Simp[(e*F^(c*(a + b*x))*Cosh[d + e*x])/(e^2 - b^2*c^2*Log[F]^2), x]
/; FreeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 - b^2*c^2*Log[F]^2, 0]
```

Rule 5476

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sinh[(d_.) + (e_.)*(x_)]^(n_), x_Symb
ol] :> -Simp[(b*c*Log[F]*F^(c*(a + b*x))*Sinh[d + e*x]^n)/(e^2*n^2 - b^2*c^
2*Log[F]^2), x] + (-Dist[(n*(n - 1)*e^2)/(e^2*n^2 - b^2*c^2*Log[F]^2), Int[
F^(c*(a + b*x))*Sinh[d + e*x]^(n - 2), x], x] + Simp[(e*n*F^(c*(a + b*x))*C
osh[d + e*x]*Sinh[d + e*x]^(n - 1))/(e^2*n^2 - b^2*c^2*Log[F]^2), x]) /; Fr
eeQ[{F, a, b, c, d, e}, x] && NeQ[e^2*n^2 - b^2*c^2*Log[F]^2, 0] && GtQ[n,
1]
```

Rubi steps

$$\int e^{a+bx}\sinh^3(c+dx)\,dx = -\frac{3de^{a+bx}\cosh(c+dx)\sinh^2(c+dx)}{b^2-9d^2} + \frac{be^{a+bx}\sinh^3(c+dx)}{b^2-9d^2} + \frac{\left(6d^2\right)\int e^{a+bx}\sinh(c}{b^2-9d^2}$$
$$= -\frac{6d^3e^{a+bx}\cosh(c+dx)}{b^4-10b^2d^2+9d^4} + \frac{6bd^2e^{a+bx}\sinh(c+dx)}{b^4-10b^2d^2+9d^4} - \frac{3de^{a+bx}\cosh(c+dx)\sinh^2(c+}{b^2-9d^2}$$

Mathematica [A] time = 0.50, size = 108, normalized size = 0.78

$$\frac{e^{a+bx}\left(\left(3d^3-3b^2d\right)\cosh(3(c+dx))+3d\left(b^2-9d^2\right)\cosh(c+dx)+2b\sinh(c+dx)\left(\left(b^2-d^2\right)\cosh(2(c+dx))-\right.\right.}{4\left(b^4-10b^2d^2+9d^4\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(a + b*x)*Sinh[c + d*x]^3,x]
```

```
[Out] (E^(a + b*x)*(3*d*(b^2 - 9*d^2)*Cosh[c + d*x] + (-3*b^2*d + 3*d^3)*Cosh[3*(
c + d*x)] + 2*b*(-b^2 + 13*d^2 + (b^2 - d^2)*Cosh[2*(c + d*x)])*Sinh[c + d*
x]))/(4*(b^4 - 10*b^2*d^2 + 9*d^4))
```

fricas [B] time = 0.45, size = 316, normalized size = 2.27

$$-\frac{3\left(b^2d-d^3\right)\cosh(bx+a)\cosh(dx+c)^3-\left(\left(b^3-bd^2\right)\cosh(bx+a)+\left(b^3-bd^2\right)\sinh(bx+a)\right)\sinh(dx+c)^3}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*sinh(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] -1/4*(3*(b^2*d - d^3)*cosh(b*x + a)*cosh(d*x + c)^3 - ((b^3 - b*d^2)*cosh(b
*x + a) + (b^3 - b*d^2)*sinh(b*x + a))*sinh(d*x + c)^3 - 3*(b^2*d - 9*d^3)*
cosh(b*x + a)*cosh(d*x + c) + 9*((b^2*d - d^3)*cosh(b*x + a)*cosh(d*x + c)
+ (b^2*d - d^3)*cosh(d*x + c)*sinh(b*x + a))*sinh(d*x + c)^2 + 3*((b^2*d -
d^3)*cosh(d*x + c)^3 - (b^2*d - 9*d^3)*cosh(d*x + c))*sinh(b*x + a) - 3*((b
^3 - b*d^2)*cosh(b*x + a)*cosh(d*x + c)^2 - (b^3 - 9*b*d^2)*cosh(b*x + a) -
 (b^3 - 9*b*d^2 - (b^3 - b*d^2)*cosh(d*x + c)^2)*sinh(b*x + a))*sinh(d*x +
c))/(b^4 - 10*b^2*d^2 + 9*d^4)
```

giac [A] time = 0.12, size = 84, normalized size = 0.60

$$\frac{e^{(bx+3\,dx+a+3\,c)}}{8\,(b+3\,d)} - \frac{3\,e^{(bx+dx+a+c)}}{8\,(b+d)} + \frac{3\,e^{(bx-dx+a-c)}}{8\,(b-d)} - \frac{e^{(bx-3\,dx+a-3\,c)}}{8\,(b-3\,d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*sinh(d*x+c)^3,x, algorithm="giac")
```

```
[Out] 1/8*e^(b*x + 3*d*x + a + 3*c)/(b + 3*d) - 3/8*e^(b*x + d*x + a + c)/(b + d)
 + 3/8*e^(b*x - d*x + a - c)/(b - d) - 1/8*e^(b*x - 3*d*x + a - 3*c)/(b - 3
*d)
```

maple [A] time = 0.33, size = 166, normalized size = 1.19

$$-\frac{\sinh(a-3c+(b-3d)x)}{8(b-3d)}+\frac{3\sinh(a-c+(b-d)x)}{8(b-d)}-\frac{3\sinh(a+c+(b+d)x)}{8(b+d)}+\frac{\sinh(a+3c+(b+3d)x)}{8b+24d}-\cos$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(b*x+a)*sinh(d*x+c)^3,x)
```

```
[Out] -1/8*sinh(a-3*c+(b-3*d)*x)/(b-3*d)+3/8*sinh(a-c+(b-d)*x)/(b-d)-3/8*sinh(a+c
+(b+d)*x)/(b+d)+1/8*sinh(a+3*c+(b+3*d)*x)/(b+3*d)-1/8*cosh(a-3*c+(b-3*d)*x)
/(b-3*d)+3/8*cosh(a-c+(b-d)*x)/(b-d)-3/8*cosh(a+c+(b+d)*x)/(b+d)+1/8*cosh(a
+3*c+(b+3*d)*x)/(b+3*d)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*sinh(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(-(3*d)/b>0)', see `assume?` for mor
e details)Is -(3*d)/b equal to -1?
```

mupad [B] time = 2.32, size = 127, normalized size = 0.91

$$-\frac{e^{a+b\,x}\,\left(-b^3\,\sinh\left(c+d\,x\right)^3+3\,b^2\,d\,\cosh\left(c+d\,x\right)\,\sinh\left(c+d\,x\right)^2-6\,b\,d^2\,\cosh\left(c+d\,x\right)^2\,\sinh\left(c+d\,x\right)+7\,b\,d^2\,\mathrm{si}}{b^4-10\,b^2\,d^2+9\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(a + b*x)*sinh(c + d*x)^3,x)
```

```
[Out] -(exp(a + b*x)*(6*d^3*cosh(c + d*x)^3 - b^3*sinh(c + d*x)^3 - 9*d^3*cosh(c
+ d*x)*sinh(c + d*x)^2 + 7*b*d^2*sinh(c + d*x)^3 - 6*b*d^2*cosh(c + d*x)^2*
sinh(c + d*x) + 3*b^2*d*cosh(c + d*x)*sinh(c + d*x)^2))/(b^4 + 9*d^4 - 10*b
^2*d^2)
```

sympy [A] time = 44.61, size = 976, normalized size = 7.02

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*sinh(d*x+c)**3,x)
```

```
[Out] Piecewise((x*exp(a)*sinh(c)**3, Eq(b, 0) & Eq(d, 0)), (x*exp(a)*exp(-3*d*x)
*sinh(c + d*x)**3/8 + 3*x*exp(a)*exp(-3*d*x)*sinh(c + d*x)**2*cosh(c + d*x)
/8 + 3*x*exp(a)*exp(-3*d*x)*sinh(c + d*x)*cosh(c + d*x)**2/8 + x*exp(a)*exp
(-3*d*x)*cosh(c + d*x)**3/8 - 3*exp(a)*exp(-3*d*x)*sinh(c + d*x)**3/(8*d) -
 exp(a)*exp(-3*d*x)*sinh(c + d*x)**2*cosh(c + d*x)/(4*d) + exp(a)*exp(-3*d*
x)*cosh(c + d*x)**3/(24*d), Eq(b, -3*d)), (3*x*exp(a)*exp(-d*x)*sinh(c + d*
x)**3/8 + 3*x*exp(a)*exp(-d*x)*sinh(c + d*x)**2*cosh(c + d*x)/8 - 3*x*exp(a
)*exp(-d*x)*sinh(c + d*x)*cosh(c + d*x)**2/8 - 3*x*exp(a)*exp(-d*x)*cosh(c
+ d*x)**3/8 + exp(a)*exp(-d*x)*sinh(c + d*x)**3/(8*d) + 3*exp(a)*exp(-d*x)*
sinh(c + d*x)**2*cosh(c + d*x)/(4*d) - 3*exp(a)*exp(-d*x)*cosh(c + d*x)**3/
(8*d), Eq(b, -d)), (3*x*exp(a)*exp(d*x)*sinh(c + d*x)**3/8 - 3*x*exp(a)*exp
(d*x)*sinh(c + d*x)**2*cosh(c + d*x)/8 - 3*x*exp(a)*exp(d*x)*sinh(c + d*x)*
cosh(c + d*x)**2/8 + 3*x*exp(a)*exp(d*x)*cosh(c + d*x)**3/8 - exp(a)*exp(d*
x)*sinh(c + d*x)**3/(8*d) + 3*exp(a)*exp(d*x)*sinh(c + d*x)**2*cosh(c + d*x
)/(4*d) - 3*exp(a)*exp(d*x)*cosh(c + d*x)**3/(8*d), Eq(b, d)), (x*exp(a)*ex
p(3*d*x)*sinh(c + d*x)**3/8 - 3*x*exp(a)*exp(3*d*x)*sinh(c + d*x)**2*cosh(c
 + d*x)/8 + 3*x*exp(a)*exp(3*d*x)*sinh(c + d*x)*cosh(c + d*x)**2/8 - x*exp(
a)*exp(3*d*x)*cosh(c + d*x)**3/8 + 3*exp(a)*exp(3*d*x)*sinh(c + d*x)**3/(8*
d) - exp(a)*exp(3*d*x)*sinh(c + d*x)**2*cosh(c + d*x)/(4*d) + exp(a)*exp(3*
d*x)*cosh(c + d*x)**3/(24*d), Eq(b, 3*d)), (b**3*exp(a)*exp(b*x)*sinh(c + d
*x)**3/(b**4 - 10*b**2*d**2 + 9*d**4) - 3*b**2*d*exp(a)*exp(b*x)*sinh(c + d
*x)**2*cosh(c + d*x)/(b**4 - 10*b**2*d**2 + 9*d**4) - 7*b*d**2*exp(a)*exp(b
*x)*sinh(c + d*x)**3/(b**4 - 10*b**2*d**2 + 9*d**4) + 6*b*d**2*exp(a)*exp(b
*x)*sinh(c + d*x)*cosh(c + d*x)**2/(b**4 - 10*b**2*d**2 + 9*d**4) + 9*d**3*
exp(a)*exp(b*x)*sinh(c + d*x)**2*cosh(c + d*x)/(b**4 - 10*b**2*d**2 + 9*d**
4) - 6*d**3*exp(a)*exp(b*x)*cosh(c + d*x)**3/(b**4 - 10*b**2*d**2 + 9*d**4)
, True))
```

3.879 $\int e^{a+bx} \sinh^2(c+dx)\,dx$

Optimal. Leaf size=88

$$\frac{be^{a+bx}\sinh^2(c+dx)}{b^2-4d^2}-\frac{2de^{a+bx}\sinh(c+dx)\cosh(c+dx)}{b^2-4d^2}+\frac{2d^2e^{a+bx}}{b\left(b^2-4d^2\right)}$$

```
[Out] 2*d^2*exp(b*x+a)/b/(b^2-4*d^2)-2*d*exp(b*x+a)*cosh(d*x+c)*sinh(d*x+c)/(b^2-
4*d^2)+b*exp(b*x+a)*sinh(d*x+c)^2/(b^2-4*d^2)
```

Rubi [A] time = 0.04, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {5476, 2194}

$$\frac{be^{a+bx}\sinh^2(c+dx)}{b^2-4d^2}-\frac{2de^{a+bx}\sinh(c+dx)\cosh(c+dx)}{b^2-4d^2}+\frac{2d^2e^{a+bx}}{b\left(b^2-4d^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[E^(a + b*x)*Sinh[c + d*x]^2,x]
```

```
[Out] (2*d^2*E^(a + b*x))/(b*(b^2 - 4*d^2)) - (2*d*E^(a + b*x)*Cosh[c + d*x]*Sinh
[c + d*x])/(b^2 - 4*d^2) + (b*E^(a + b*x)*Sinh[c + d*x]^2)/(b^2 - 4*d^2)
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 5476

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sinh[(d_.) + (e_.)*(x_)]^(n_), x_Symb
ol] :> -Simp[(b*c*Log[F]*F^(c*(a + b*x))*Sinh[d + e*x]^n)/(e^2*n^2 - b^2*c^
2*Log[F]^2), x] + (-Dist[(n*(n - 1)*e^2)/(e^2*n^2 - b^2*c^2*Log[F]^2), Int[
F^(c*(a + b*x))*Sinh[d + e*x]^(n - 2), x], x] + Simp[(e*n*F^(c*(a + b*x))*C
osh[d + e*x]*Sinh[d + e*x]^(n - 1))/(e^2*n^2 - b^2*c^2*Log[F]^2), x]) /; Fr
eeQ[{F, a, b, c, d, e}, x] && NeQ[e^2*n^2 - b^2*c^2*Log[F]^2, 0] && GtQ[n,
1]
```

Rubi steps

$$\int e^{a+bx}\sinh^2(c+dx)\,dx = -\frac{2de^{a+bx}\cosh(c+dx)\sinh(c+dx)}{b^2-4d^2} + \frac{be^{a+bx}\sinh^2(c+dx)}{b^2-4d^2} + \frac{\left(2d^2\right)\int e^{a+bx}\,dx}{b^2-4d^2}$$
$$= \frac{2d^2e^{a+bx}}{b\left(b^2-4d^2\right)} - \frac{2de^{a+bx}\cosh(c+dx)\sinh(c+dx)}{b^2-4d^2} + \frac{be^{a+bx}\sinh^2(c+dx)}{b^2-4d^2}$$

Mathematica [A] time = 0.16, size = 58, normalized size = 0.66

$$\frac{e^{a+bx}\left(b^2\cosh(2(c+dx)) - b^2 - 2bd\sinh(2(c+dx)) + 4d^2\right)}{2\left(b^3-4bd^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(a + b*x)*Sinh[c + d*x]^2,x]
```

```
[Out] (E^(a + b*x)*(-b^2 + 4*d^2 + b^2*Cosh[2*(c + d*x)] - 2*b*d*Sinh[2*(c + d*x)
]))/(2*(b^3 - 4*b*d^2))
```

fricas [A] time = 0.45, size = 149, normalized size = 1.69

$$\frac{b^2\cosh(bx+a)\cosh(dx+c)^2 + \left(b^2\cosh(bx+a) + b^2\sinh(bx+a)\right)\sinh(dx+c)^2 - \left(b^2-4\,d^2\right)\cosh(bx+a}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*sinh(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(b^2*cosh(b*x + a)*cosh(d*x + c)^2 + (b^2*cosh(b*x + a) + b^2*sinh(b*x
+ a))*sinh(d*x + c)^2 - (b^2 - 4*d^2)*cosh(b*x + a) + (b^2*cosh(d*x + c)^2
- b^2 + 4*d^2)*sinh(b*x + a) - 4*(b*d*cosh(b*x + a)*cosh(d*x + c) + b*d*cos
h(d*x + c)*sinh(b*x + a))*sinh(d*x + c))/(b^3 - 4*b*d^2)
```

giac [A] time = 0.12, size = 56, normalized size = 0.64

$$\frac{e^{(bx+2\,dx+a+2\,c)}}{4\,(b+2\,d)} + \frac{e^{(bx-2\,dx+a-2\,c)}}{4\,(b-2\,d)} - \frac{e^{(bx+a)}}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*sinh(d*x+c)^2,x, algorithm="giac")
```

```
[Out] 1/4*e^(b*x + 2*d*x + a + 2*c)/(b + 2*d) + 1/4*e^(b*x - 2*d*x + a - 2*c)/(b
- 2*d) - 1/2*e^(b*x + a)/b
```

maple [A] time = 0.23, size = 112, normalized size = 1.27

$$-\frac{\sinh\left(bx+a\right)}{2b}+\frac{\sinh\left(a-2c+\left(b-2d\right)x\right)}{4b-8d}+\frac{\sinh\left(a+2c+\left(b+2d\right)x\right)}{4b+8d}-\frac{\cosh\left(bx+a\right)}{2b}+\frac{\cosh\left(a-2c+\left(b-2d\right)x\right)}{4b-8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(b*x+a)*sinh(d*x+c)^2,x)
```

```
[Out] -1/2*sinh(b*x+a)/b+1/4*sinh(a-2*c+(b-2*d)*x)/(b-2*d)+1/4*sinh(a+2*c+(b+2*d)
*x)/(b+2*d)-1/2*cosh(b*x+a)/b+1/4*cosh(a-2*c+(b-2*d)*x)/(b-2*d)+1/4*cosh(a+
2*c+(b+2*d)*x)/(b+2*d)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*sinh(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(-(2*d)/b>0)', see `assume?` for mor
e details)Is -(2*d)/b equal to -1?
```

mupad [B] time = 0.24, size = 105, normalized size = 1.19

$$-\frac{2\,d^2\,\mathrm{e}^{a+b\,x}-b^2\,\left(\frac{\mathrm{e}^{a+b\,x}}{2}-\mathrm{e}^{a+b\,x}\,\left(\frac{\mathrm{e}^{-2\,c-2\,d\,x}}{4}+\frac{\mathrm{e}^{2\,c+2\,d\,x}}{4}\right)\right)+b\,d\,\mathrm{e}^{a+b\,x}\,\left(\frac{\mathrm{e}^{-2\,c-2\,d\,x}}{2}-\frac{\mathrm{e}^{2\,c+2\,d\,x}}{2}\right)}{4\,b\,d^2-b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(a + b*x)*sinh(c + d*x)^2,x)
```

```
[Out] -(2*d^2*exp(a + b*x) - b^2*(exp(a + b*x)/2 - exp(a + b*x)*(exp(- 2*c - 2*d*
x)/4 + exp(2*c + 2*d*x)/4)) + b*d*exp(a + b*x)*(exp(- 2*c - 2*d*x)/2 - exp(
2*c + 2*d*x)/2))/(4*b*d^2 - b^3)
```

sympy [A] time = 9.12, size = 428, normalized size = 4.86

$$\begin{cases} xe^a\sinh^2(c) \\ \left(\frac{x\sinh^2(c+dx)}{2}-\frac{x\cosh^2(c+dx)}{2}+\frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right)e^a \\ \frac{xe^ae^{-2dx}\sinh^2(c+dx)}{4}+\frac{xe^ae^{-2dx}\sinh(c+dx)\cosh(c+dx)}{2}+\frac{xe^ae^{-2dx}\cosh^2(c+dx)}{4}-\frac{e^ae^{-2dx}\sinh^2(c+dx)}{2d}-\frac{e^ae^{-2dx}\sinh(c+dx)\cosh(c+dx)}{4d} \\ \frac{xe^ae^{2dx}\sinh^2(c+dx)}{4}-\frac{xe^ae^{2dx}\sinh(c+dx)\cosh(c+dx)}{2}+\frac{xe^ae^{2dx}\cosh^2(c+dx)}{4}+\frac{e^ae^{2dx}\sinh^2(c+dx)}{2d}-\frac{e^ae^{2dx}\sinh(c+dx)\cosh(c+dx)}{4d} \\ \frac{b^2e^ae^{bx}\sinh^2(c+dx)}{b^3-4bd^2}-\frac{2bde^ae^{bx}\sinh(c+dx)\cosh(c+dx)}{b^3-4bd^2}-\frac{2d^2e^ae^{bx}\sinh^2(c+dx)}{b^3-4bd^2}+\frac{2d^2e^ae^{bx}\cosh^2(c+dx)}{b^3-4bd^2} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*sinh(d*x+c)**2,x)
```

```
[Out] Piecewise((x*exp(a)*sinh(c)**2, Eq(b, 0) & Eq(d, 0)), ((x*sinh(c + d*x)**2/
2 - x*cosh(c + d*x)**2/2 + sinh(c + d*x)*cosh(c + d*x)/(2*d))*exp(a), Eq(b,
 0)), (x*exp(a)*exp(-2*d*x)*sinh(c + d*x)**2/4 + x*exp(a)*exp(-2*d*x)*sinh(
c + d*x)*cosh(c + d*x)/2 + x*exp(a)*exp(-2*d*x)*cosh(c + d*x)**2/4 - exp(a)
*exp(-2*d*x)*sinh(c + d*x)**2/(2*d) - exp(a)*exp(-2*d*x)*sinh(c + d*x)*cosh
(c + d*x)/(4*d), Eq(b, -2*d)), (x*exp(a)*exp(2*d*x)*sinh(c + d*x)**2/4 - x*
exp(a)*exp(2*d*x)*sinh(c + d*x)*cosh(c + d*x)/2 + x*exp(a)*exp(2*d*x)*cosh(
c + d*x)**2/4 + exp(a)*exp(2*d*x)*sinh(c + d*x)**2/(2*d) - exp(a)*exp(2*d*x
)*sinh(c + d*x)*cosh(c + d*x)/(4*d), Eq(b, 2*d)), (b**2*exp(a)*exp(b*x)*sin
h(c + d*x)**2/(b**3 - 4*b*d**2) - 2*b*d*exp(a)*exp(b*x)*sinh(c + d*x)*cosh(
c + d*x)/(b**3 - 4*b*d**2) - 2*d**2*exp(a)*exp(b*x)*sinh(c + d*x)**2/(b**3
- 4*b*d**2) + 2*d**2*exp(a)*exp(b*x)*cosh(c + d*x)**2/(b**3 - 4*b*d**2), Tr
ue))
```

3.880 $\int e^{a+bx}\sinh(c+dx)\,dx$

Optimal. Leaf size=54

$$\frac{be^{a+bx}\sinh(c+dx)}{b^2-d^2}-\frac{de^{a+bx}\cosh(c+dx)}{b^2-d^2}$$

```
[Out] -d*exp(b*x+a)*cosh(d*x+c)/(b^2-d^2)+b*exp(b*x+a)*sinh(d*x+c)/(b^2-d^2)
```

Rubi [A] time = 0.02, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {5474}

$$\frac{be^{a+bx}\sinh(c+dx)}{b^2-d^2}-\frac{de^{a+bx}\cosh(c+dx)}{b^2-d^2}$$

Antiderivative was successfully verified.

```
[In] Int[E^(a + b*x)*Sinh[c + d*x],x]
```

```
[Out] -((d*E^(a + b*x)*Cosh[c + d*x])/(b^2 - d^2)) + (b*E^(a + b*x)*Sinh[c + d*x]
)/(b^2 - d^2)
```

Rule 5474

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sinh[(d_.) + (e_.)*(x_)], x_Symbol] :
> -Simp[(b*c*Log[F]*F^(c*(a + b*x))*Sinh[d + e*x])/(e^2 - b^2*c^2*Log[F]^2)
, x] + Simp[(e*F^(c*(a + b*x))*Cosh[d + e*x])/(e^2 - b^2*c^2*Log[F]^2), x]
/; FreeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 - b^2*c^2*Log[F]^2, 0]
```

Rubi steps

$$\int e^{a+bx}\sinh(c+dx)\,dx=-\frac{de^{a+bx}\cosh(c+dx)}{b^2-d^2}+\frac{be^{a+bx}\sinh(c+dx)}{b^2-d^2}$$

Mathematica [A] time = 0.08, size = 38, normalized size = 0.70

$$\frac{e^{a+bx}(b\sinh(c+dx)-d\cosh(c+dx))}{(b-d)(b+d)}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(a + b*x)*Sinh[c + d*x],x]
```

```
[Out] (E^(a + b*x)*(-(d*Cosh[c + d*x]) + b*Sinh[c + d*x]))/((b - d)*(b + d))
```

fricas [A] time = 0.49, size = 67, normalized size = 1.24

$$-\frac{d\cosh(bx+a)\cosh(dx+c)+d\cosh(dx+c)\sinh(bx+a)-(b\cosh(bx+a)+b\sinh(bx+a))\sinh(dx+c)}{b^2-d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*sinh(d*x+c),x, algorithm="fricas")
```

```
[Out] -(d*cosh(b*x + a)*cosh(d*x + c) + d*cosh(d*x + c)*sinh(b*x + a) - (b*cosh(b
*x + a) + b*sinh(b*x + a))*sinh(d*x + c))/(b^2 - d^2)
```

giac [A] time = 0.14, size = 40, normalized size = 0.74

$$\frac{e^{(bx+dx+a+c)}}{2(b+d)}-\frac{e^{(bx-dx+a-c)}}{2(b-d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*sinh(d*x+c),x, algorithm="giac")
```

```
[Out] 1/2*e^(b*x + d*x + a + c)/(b + d) - 1/2*e^(b*x - d*x + a - c)/(b - d)
```

maple [A] time = 0.17, size = 78, normalized size = 1.44

$$-\frac{\sinh(a-c+(b-d)x)}{2(b-d)}+\frac{\sinh(a+c+(b+d)x)}{2b+2d}-\frac{\cosh(a-c+(b-d)x)}{2(b-d)}+\frac{\cosh(a+c+(b+d)x)}{2b+2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(b*x+a)*sinh(d*x+c),x)
```

```
[Out] -1/2*sinh(a-c+(b-d)*x)/(b-d)+1/2*sinh(a+c+(b+d)*x)/(b+d)-1/2*cosh(a-c+(b-d)
*x)/(b-d)+1/2*cosh(a+c+(b+d)*x)/(b+d)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*sinh(d*x+c),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(-d/b>0)', see `assume?` for more de
tails)Is -d/b equal to -1?
```

mupad [B] time = 0.11, size = 54, normalized size = 1.00

$$-\frac{e^{a-c+b\,x-d\,x}\,\left(b+d-b\,e^{2\,c+2\,d\,x}+d\,e^{2\,c+2\,d\,x}\right)}{2\,\left(b^2-d^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(a + b*x)*sinh(c + d*x),x)
```

```
[Out] -(exp(a - c + b*x - d*x)*(b + d - b*exp(2*c + 2*d*x) + d*exp(2*c + 2*d*x)))
/(2*(b^2 - d^2))
```

sympy [A] time = 2.31, size = 201, normalized size = 3.72

$$\begin{cases} xe^a\sinh(c) & \text{for } b=0\wedge d=0 \\ \frac{xe^ae^{-dx}\sinh(c+dx)}{2}+\frac{xe^ae^{-dx}\cosh(c+dx)}{2}+\frac{e^ae^{-dx}\sinh(c+dx)}{2d}+\frac{e^ae^{-dx}\cosh(c+dx)}{d} & \text{for } b=-d \\ \frac{xe^ae^{dx}\sinh(c+dx)}{2}-\frac{xe^ae^{dx}\cosh(c+dx)}{2}-\frac{e^ae^{dx}\sinh(c+dx)}{2d}+\frac{e^ae^{dx}\cosh(c+dx)}{d} & \text{for } b=d \\ \frac{be^ae^{bx}\sinh(c+dx)}{b^2-d^2}-\frac{de^ae^{bx}\cosh(c+dx)}{b^2-d^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*sinh(d*x+c),x)
```

```
[Out] Piecewise((x*exp(a)*sinh(c), Eq(b, 0) & Eq(d, 0)), (x*exp(a)*exp(-d*x)*sinh
(c + d*x)/2 + x*exp(a)*exp(-d*x)*cosh(c + d*x)/2 + exp(a)*exp(-d*x)*sinh(c
+ d*x)/(2*d) + exp(a)*exp(-d*x)*cosh(c + d*x)/d, Eq(b, -d)), (x*exp(a)*exp(
d*x)*sinh(c + d*x)/2 - x*exp(a)*exp(d*x)*cosh(c + d*x)/2 - exp(a)*exp(d*x)*
sinh(c + d*x)/(2*d) + exp(a)*exp(d*x)*cosh(c + d*x)/d, Eq(b, d)), (b*exp(a)
*exp(b*x)*sinh(c + d*x)/(b**2 - d**2) - d*exp(a)*exp(b*x)*cosh(c + d*x)/(b*
*2 - d**2), True))
```

3.881 $\int e^{a+bx}\mathrm{csch}(c+dx)\,dx$

Optimal. Leaf size=50

$$-\frac{2e^{a+bx+c+dx}\,{}_2F_1\left(1,\frac{b+d}{2d};\frac{1}{2}\left(\frac{b}{d}+3\right);e^{2(c+dx)}\right)}{b+d}$$

```
[Out] -2*exp(b*x+d*x+a+c)*hypergeom([1, 1/2*(b+d)/d],[3/2+1/2*b/d],exp(2*d*x+2*c)
)/(b+d)
```

Rubi [A] time = 0.02, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {5493}

$$-\frac{2e^{a+bx+c+dx}\,{}_2F_1\left(1,\frac{b+d}{2d};\frac{1}{2}\left(\frac{b}{d}+3\right);e^{2(c+dx)}\right)}{b+d}$$

Antiderivative was successfully verified.

```
[In] Int[E^(a + b*x)*Csch[c + d*x],x]
```

```
[Out] (-2*E^(a + c + b*x + d*x)*Hypergeometric2F1[1, (b + d)/(2*d), (3 + b/d)/2,
E^(2*(c + d*x))])/(b + d)
```

Rule 5493

```
Int[Csch[(d_.) + (e_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Sym
bol] :> Simp[((-2)^n*E^(n*(d + e*x))*F^(c*(a + b*x))*Hypergeometric2F1[n, n
/2 + (b*c*Log[F])/(2*e), 1 + n/2 + (b*c*Log[F])/(2*e), E^(2*(d + e*x))])/(e
*n + b*c*Log[F]), x] /; FreeQ[{F, a, b, c, d, e}, x] && IntegerQ[n]
```

Rubi steps

$$\int e^{a+bx}\mathrm{csch}(c+dx)\,dx=-\frac{2e^{a+c+bx+dx}\,{}_2F_1\left(1,\frac{b+d}{2d};\frac{1}{2}\left(3+\frac{b}{d}\right);e^{2(c+dx)}\right)}{b+d}$$

Mathematica [A] time = 0.13, size = 59, normalized size = 1.18

$$-\frac{2(\sinh(c)+\cosh(c))e^{a+x(b+d)}\,{}_2F_1\left(1,\frac{b+d}{2d};\frac{b+3d}{2d};e^{2dx}(\cosh(c)+\sinh(c))^2\right)}{b+d}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(a + b*x)*Csch[c + d*x],x]
```

```
[Out] (-2*E^(a + (b + d)*x)*Hypergeometric2F1[1, (b + d)/(2*d), (b + 3*d)/(2*d),
E^(2*d*x)*(Cosh[c] + Sinh[c])^2]*(Cosh[c] + Sinh[c]))/(b + d)
```

fricas [F] time = 0.47, size = 0, normalized size = 0.00

$$\text{integral}\left(\text{csch}\,(dx+c)\,e^{(bx+a)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*csch(d*x+c),x, algorithm="fricas")
```

```
[Out] integral(csch(d*x + c)*e^(b*x + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \text{csch}\,(dx+c)\,e^{(bx+a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*csch(d*x+c),x, algorithm="giac")
```

```
[Out] integrate(csch(d*x + c)*e^(b*x + a), x)
```

maple [F] time = 0.27, size = 0, normalized size = 0.00

$$\int \mathrm{e}^{bx+a}\text{csch}\,(dx+c)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(b*x+a)*csch(d*x+c),x)
```

```
[Out] int(exp(b*x+a)*csch(d*x+c),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \text{csch}\,(dx+c)\,e^{(bx+a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*csch(d*x+c),x, algorithm="maxima")
```

```
[Out] integrate(csch(d*x + c)*e^(b*x + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{e^{a+b\,x}}{\sinh\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(a + b*x)/sinh(c + d*x),x)
[Out] int(exp(a + b*x)/sinh(c + d*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$e^{a}\int e^{bx}\operatorname{csch}\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*csch(d*x+c),x)
[Out] exp(a)*Integral(exp(b*x)*csch(c + d*x), x)
```

3.882 $\int e^{c+dx}\operatorname{csch}^2(a+bx)\,dx$

Optimal. Leaf size=54

$$\frac{4e^{2(a+bx)+c+dx}\,{}_2F_1\left(2,\frac{d}{2b}+1;\frac{d}{2b}+2;e^{2(a+bx)}\right)}{2b+d}$$

```
[Out] 4*exp(2*b*x+d*x+2*a+c)*hypergeom([2, 1+1/2*d/b],[2+1/2*d/b],exp(2*b*x+2*a))
/(2*b+d)
```

Rubi [A] time = 0.03, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {5493}

$$\frac{4e^{2(a+bx)+c+dx}\,{}_2F_1\left(2,\frac{d}{2b}+1;\frac{d}{2b}+2;e^{2(a+bx)}\right)}{2b+d}$$

Antiderivative was successfully verified.

```
[In] Int[E^(c + d*x)*Csch[a + b*x]^2,x]
```

```
[Out] (4*E^(c + d*x + 2*(a + b*x))*Hypergeometric2F1[2, 1 + d/(2*b), 2 + d/(2*b),
 E^(2*(a + b*x))])/(2*b + d)
```

Rule 5493

```
Int[Csch[(d_.) + (e_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Sym
bol] :> Simp[((-2)^n*E^(n*(d + e*x))*F^(c*(a + b*x))*Hypergeometric2F1[n, n
/2 + (b*c*Log[F])/(2*e), 1 + n/2 + (b*c*Log[F])/(2*e), E^(2*(d + e*x))])/(e
*n + b*c*Log[F]), x] /; FreeQ[{F, a, b, c, d, e}, x] && IntegerQ[n]
```

Rubi steps

$$\int e^{c+dx}\operatorname{csch}^2(a+bx)\,dx=\frac{4e^{c+dx+2(a+bx)}\,{}_2F_1\left(2,1+\frac{d}{2b};2+\frac{d}{2b};e^{2(a+bx)}\right)}{2b+d}$$

Mathematica [B] time = 3.37, size = 131, normalized size = 2.43

$$\frac{e^c\left(\operatorname{csch}(a)e^{dx}\sinh(bx)\operatorname{csch}(a+bx)-\frac{2e^{2a}d\left(\frac{e^{dx}\,{}_2F_1\left(1,\frac{d}{2b};\frac{d}{2b}+1;e^{2(a+bx)}\right)}{d}-\frac{e^{x(2b+d)}\,{}_2F_1\left(1,\frac{d}{2b}+1;\frac{d}{2b}+2;e^{2(a+bx)}\right)}{2b+d}\right)}{e^{2a}-1}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(c + d*x)*Csch[a + b*x]^2,x]
```

```
[Out] (E^c*((-2*d*E^(2*a)*((E^(d*x)*Hypergeometric2F1[1, d/(2*b), 1 + d/(2*b), E^
(2*(a + b*x))])/d - (E^((2*b + d)*x)*Hypergeometric2F1[1, 1 + d/(2*b), 2 +
d/(2*b), E^(2*(a + b*x))])/(2*b + d)))/(-1 + E^(2*a)) + E^(d*x)*Csch[a]*Csc
h[a + b*x]*Sinh[b*x]))/b
```

fricas [F] time = 0.50, size = 0, normalized size = 0.00

$$\text{integral}\left(\operatorname{csch}\left(bx+a\right)^{2} e^{\left(dx+c\right)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x+c)*csch(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] integral(csch(b*x + a)^2*e^(d*x + c), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{csch}\left(bx+a\right)^{2} e^{\left(dx+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x+c)*csch(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate(csch(b*x + a)^2*e^(d*x + c), x)
```

maple [F] time = 0.34, size = 0, normalized size = 0.00

$$\int \mathrm{e}^{dx+c}\operatorname{csch}\left(bx+a\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(d*x+c)*csch(b*x+a)^2,x)
```

```
[Out] int(exp(d*x+c)*csch(b*x+a)^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$16\,bd\int -\frac{e^{\left(dx+c\right)}}{8\,b^{2}-6\,bd+d^{2}-\left(8\,b^{2}-6\,bd+d^{2}\right)e^{\left(6\,bx+6\,a\right)}+3\left(8\,b^{2}-6\,bd+d^{2}\right)e^{\left(4\,bx+4\,a\right)}-3\left(8\,b^{2}-6\,bd+d^{2}\right)e^{\left(2\,bx\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x+c)*csch(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 16*b*d*integrate(-e^(d*x + c)/(8*b^2 - 6*b*d + d^2 - (8*b^2 - 6*b*d + d^2)*
e^(6*b*x + 6*a) + 3*(8*b^2 - 6*b*d + d^2)*e^(4*b*x + 4*a) - 3*(8*b^2 - 6*b*
d + d^2)*e^(2*b*x + 2*a)), x) - 4*((4*b*e^c - d*e^c)*e^(2*b*x + 2*a) - 4*b*
e^c)*e^(d*x)/(8*b^2 - 6*b*d + d^2 + (8*b^2 - 6*b*d + d^2)*e^(4*b*x + 4*a) -
 2*(8*b^2 - 6*b*d + d^2)*e^(2*b*x + 2*a))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{e^{c+d\,x}}{\sinh\left(a+b\,x\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(c + d*x)/sinh(a + b*x)^2,x)
```

```
[Out] int(exp(c + d*x)/sinh(a + b*x)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$e^{c}\int e^{dx}\operatorname{csch}^{2}\left(a+bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x+c)*csch(b*x+a)**2,x)
```

```
[Out] exp(c)*Integral(exp(d*x)*csch(a + b*x)**2, x)
```

3.883 $\int e^{c+dx}\mathbf{csch}^3(a+bx)\,dx$

Optimal. Leaf size=100

$$\frac{(b-d)e^{a+bx+c+dx}\,{}_2F_1\left(1,\frac{b+d}{2b};\frac{1}{2}\left(\frac{d}{b}+3\right);e^{2(a+bx)}\right)}{b^2}-\frac{de^{c+dx}\text{csch}(a+bx)}{2b^2}-\frac{e^{c+dx}\coth(a+bx)\text{csch}(a+bx)}{2b}$$

```
[Out] -1/2*d*exp(d*x+c)*csch(b*x+a)/b^2-1/2*exp(d*x+c)*coth(b*x+a)*csch(b*x+a)/b+
(b-d)*exp(b*x+d*x+a+c)*hypergeom([1, 1/2*(b+d)/b],[3/2+1/2*d/b],exp(2*b*x+2
*a))/b^2
```

Rubi [A] time = 0.05, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {5491, 5493}

$$\frac{(b-d)e^{a+bx+c+dx}\,{}_2F_1\left(1,\frac{b+d}{2b};\frac{1}{2}\left(\frac{d}{b}+3\right);e^{2(a+bx)}\right)}{b^2}-\frac{de^{c+dx}\text{csch}(a+bx)}{2b^2}-\frac{e^{c+dx}\coth(a+bx)\text{csch}(a+bx)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[E^(c + d*x)*Csch[a + b*x]^3,x]
```

```
[Out] -(d*E^(c + d*x)*Csch[a + b*x])/(2*b^2) - (E^(c + d*x)*Coth[a + b*x]*Csch[a
+ b*x])/(2*b) + ((b - d)*E^(a + c + b*x + d*x)*Hypergeometric2F1[1, (b + d)
/(2*b), (3 + d/b)/2, E^(2*(a + b*x))])/b^2
```

Rule 5491

```
Int[Csch[(d_.) + (e_.)*(x_)]^(n_)*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symb
ol] :> -Simp[(b*c*Log[F]*F^(c*(a + b*x))*Csch[d + e*x]^(n - 2))/(e^2*(n - 1
)*(n - 2)), x] + (-Dist[(e^2*(n - 2)^2 - b^2*c^2*Log[F]^2)/(e^2*(n - 1)*(n
- 2)), Int[F^(c*(a + b*x))*Csch[d + e*x]^(n - 2), x], x] - Simp[(F^(c*(a +
b*x))*Csch[d + e*x]^(n - 1)*Cosh[d + e*x])/(e*(n - 1)), x]) /; FreeQ[{F, a,
 b, c, d, e}, x] && NeQ[e^2*(n - 2)^2 - b^2*c^2*Log[F]^2, 0] && GtQ[n, 1] &
& NeQ[n, 2]
```

Rule 5493

```
Int[Csch[(d_.) + (e_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Sym
bol] :> Simp[((-2)^n*E^(n*(d + e*x))*F^(c*(a + b*x))*Hypergeometric2F1[n, n
/2 + (b*c*Log[F])/(2*e), 1 + n/2 + (b*c*Log[F])/(2*e), E^(2*(d + e*x))])/(e
*n + b*c*Log[F]), x] /; FreeQ[{F, a, b, c, d, e}, x] && IntegerQ[n]
```

Rubi steps

$$\int e^{c+dx}\text{csch}^3(a+bx)\,dx = -\frac{de^{c+dx}\text{csch}(a+bx)}{2b^2} - \frac{e^{c+dx}\coth(a+bx)\text{csch}(a+bx)}{2b} - \frac{1}{2}\left(1-\frac{d^2}{b^2}\right)\int e^{c+dx}\text{csch}(a+$$

$$= -\frac{de^{c+dx}\text{csch}(a+bx)}{2b^2} - \frac{e^{c+dx}\coth(a+bx)\text{csch}(a+bx)}{2b} + \frac{(b-d)e^{a+c+bx+dx}\,{}_2F_1\left(1,\frac{b+d}{2b}\right.}{b^2}$$

Mathematica [A] time = 2.49, size = 94, normalized size = 0.94

$$\frac{e^c\left(\frac{2\text{csch}(a)(b-d)e^{x(b+d)}\,{}_2F_1\left(1,\frac{b+d}{2b};\frac{3b+d}{2b};e^{2bx}(\cosh(a)+\sinh(a))^2\right)}{\coth(a)-1} - e^{dx}\text{csch}(a+bx)(b\coth(a+bx)+d)\right)}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(c + d*x)*Csch[a + b*x]^3,x]
```

```
[Out] (E^c*(-(E^(d*x)*(d + b*Coth[a + b*x])*Csch[a + b*x]) + (2*(b - d)*E^((b + d
)*x)*Csch[a]*Hypergeometric2F1[1, (b + d)/(2*b), (3*b + d)/(2*b), E^(2*b*x)
*(Cosh[a] + Sinh[a])^2])/(-1 + Coth[a])))/(2*b^2)
```

fricas [F] time = 0.55, size = 0, normalized size = 0.00

$$\text{integral}\left(\text{csch}\left(bx+a\right)^3 e^{(dx+c)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x+c)*csch(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] integral(csch(b*x + a)^3*e^(d*x + c), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \text{csch}\left(bx+a\right)^3 e^{(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x+c)*csch(b*x+a)^3,x, algorithm="giac")
```

```
[Out] integrate(csch(b*x + a)^3*e^(d*x + c), x)
```

maple [F] time = 0.35, size = 0, normalized size = 0.00

$$\int e^{dx+c}\text{csch}\left(bx+a\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(d*x+c)*csch(b*x+a)^3,x)
```

```
[Out] int(exp(d*x+c)*csch(b*x+a)^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$48\left(b^{2}e^{c}+bde^{c}\right)\int\frac{e^{(bx+dx+a)}}{15\,b^{2}-8\,bd+d^{2}+\left(15\,b^{2}-8\,bd+d^{2}\right)e^{(8\,bx+8\,a)}-4\left(15\,b^{2}-8\,bd+d^{2}\right)e^{(6\,bx+6\,a)}+6\left(15\,b^{2}-8\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x+c)*csch(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 48*(b^2*e^c + b*d*e^c)*integrate(e^(b*x + d*x + a)/(15*b^2 - 8*b*d + d^2 +
(15*b^2 - 8*b*d + d^2)*e^(8*b*x + 8*a) - 4*(15*b^2 - 8*b*d + d^2)*e^(6*b*x
+ 6*a) + 6*(15*b^2 - 8*b*d + d^2)*e^(4*b*x + 4*a) - 4*(15*b^2 - 8*b*d + d^2
)*e^(2*b*x + 2*a)), x) + 8*((5*b*e^c - d*e^c)*e^(3*b*x + 3*a) - 6*b*e^(b*x
+ a + c))*e^(d*x)/(15*b^2 - 8*b*d + d^2 - (15*b^2 - 8*b*d + d^2)*e^(6*b*x +
 6*a) + 3*(15*b^2 - 8*b*d + d^2)*e^(4*b*x + 4*a) - 3*(15*b^2 - 8*b*d + d^2)
*e^(2*b*x + 2*a))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{\mathrm{e}^{c+d\,x}}{{\mathrm{sinh}\left(a+b\,x\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(c + d*x)/sinh(a + b*x)^3,x)
```

```
[Out] int(exp(c + d*x)/sinh(a + b*x)^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$e^{c}\int e^{dx}\operatorname{csch}^{3}\left(a+bx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x+c)*csch(b*x+a)**3,x)
```

```
[Out] exp(c)*Integral(exp(d*x)*csch(a + b*x)**3, x)
```

3.884 $\int F^{c(a+bx)} \cosh^n(d+ex)\,dx$

Optimal. Leaf size=95

$$-\frac{\left(e^{2(d+ex)}+1\right)^{-n} F^{c(a+bx)} \cosh^n(d+ex)\,{}_2F_1\left(-n,-\frac{en-bc\log(F)}{2e};\frac{1}{2}\left(-n+\frac{bc\log(F)}{e}+2\right);-e^{2(d+ex)}\right)}{en-bc\log(F)}$$

```
[Out] -F^(c*(b*x+a))*cosh(e*x+d)^n*hypergeom([-n, 1/2*(-e*n+b*c*ln(F))/e],[1-1/2*
n+1/2*b*c*ln(F)/e],-exp(2*e*x+2*d))/((1+exp(2*e*x+2*d))^n)/(e*n-b*c*ln(F))
```

Rubi [A] time = 0.12, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {5483, 2259}

$$-\frac{\left(e^{2(d+ex)}+1\right)^{-n} F^{c(a+bx)} \cosh^n(d+ex)\,{}_2F_1\left(-n,-\frac{en-bc\log(F)}{2e};\frac{1}{2}\left(-n+\frac{bc\log(F)}{e}+2\right);-e^{2(d+ex)}\right)}{en-bc\log(F)}$$

Antiderivative was successfully verified.

```
[In] Int[F^(c*(a + b*x))*Cosh[d + e*x]^n,x]
```

```
[Out] -((F^(c*(a + b*x))*Cosh[d + e*x]^n*Hypergeometric2F1[-n, -(e*n - b*c*Log[F]
)/(2*e), (2 - n + (b*c*Log[F])/e)/2, -E^(2*(d + e*x))])/((1 + E^(2*(d + e*x
)))^n*(e*n - b*c*Log[F])))
```

Rule 2259

```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_)*(G_)^((h_.)*((f_.
) + (g_.)*(x_)))*(H_)^((t_.)*((r_.) + (s_.)*(x_))), x_Symbol] :> Simp[(G^(h
*(f + g*x))*H^(t*(r + s*x))*(a + b*F^(e*(c + d*x)))^p*Hypergeometric2F1[-p,
 (g*h*Log[G] + s*t*Log[H])/(d*e*Log[F]), (g*h*Log[G] + s*t*Log[H])/(d*e*Log
[F]) + 1, Simplify[-((b*F^(e*(c + d*x)))/a)]])/((g*h*Log[G] + s*t*Log[H])*(
(a + b*F^(e*(c + d*x)))/a)^p), x] /; FreeQ[{F, G, H, a, b, c, d, e, f, g, h
, r, s, t, p}, x] &&  !IntegerQ[p]
```

Rule 5483

```
Int[Cosh[(d_.) + (e_.)*(x_)]^(n_)*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symb
ol] :> Dist[(E^(n*(d + e*x))*Cosh[d + e*x]^n)/(1 + E^(2*(d + e*x)))^n, Int[
(F^(c*(a + b*x))*(1 + E^(2*(d + e*x)))^n)/E^(n*(d + e*x)), x], x] /; FreeQ[
{F, a, b, c, d, e, n}, x] &&  !IntegerQ[n]
```

Rubi steps

$$\int F^{c(a+bx)}\cosh^n(d+ex)\,dx = \left(e^{n(d+ex)}\left(1+e^{2(d+ex)}\right)^{-n}\cosh^n(d+ex)\right)\int e^{-n(d+ex)}\left(1+e^{2(d+ex)}\right)^n F^{c(a+bx)}\,dx$$
$$= -\frac{\left(1+e^{2(d+ex)}\right)^{-n}F^{c(a+bx)}\cosh^n(d+ex)\,{}_2F_1\left(-n,-\frac{en-bc\log(F)}{2e};\frac{1}{2}\left(2-n+\frac{bc\log(F)}{e}\right);\right.}{en-bc\log(F)}$$

Mathematica [A] time = 0.07, size = 96, normalized size = 1.01

$$\frac{\left(e^{2(d+ex)}+1\right)^{-n}F^{c(a+bx)}\cosh^n(d+ex)\,{}_2F_1\left(-n,\frac{bc\log(F)-en}{2e};\frac{bc\log(F)-en}{2e}+1;-e^{2(d+ex)}\right)}{bc\log(F)-en}$$

Antiderivative was successfully verified.

```
[In] Integrate[F^(c*(a + b*x))*Cosh[d + e*x]^n,x]
```

```
[Out] (F^(c*(a + b*x))*Cosh[d + e*x]^n*Hypergeometric2F1[-n, (-(e*n) + b*c*Log[F]
)/(2*e), 1 + (-(e*n) + b*c*Log[F])/(2*e), -E^(2*(d + e*x))])/((1 + E^(2*(d
+ e*x)))^n*(-(e*n) + b*c*Log[F]))
```

fricas [F] time = 0.54, size = 0, normalized size = 0.00

$$\text{integral}\left(F^{bcx+ac}\cosh\left(ex+d\right)^n, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*cosh(e*x+d)^n,x, algorithm="fricas")
```

```
[Out] integral(F^(b*c*x + a*c)*cosh(e*x + d)^n, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int F^{(bx+a)c}\cosh\left(ex+d\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*cosh(e*x+d)^n,x, algorithm="giac")
```

```
[Out] integrate(F^((b*x + a)*c)*cosh(e*x + d)^n, x)
```

maple [F] time = 0.19, size = 0, normalized size = 0.00

$$\int F^{c(bx+a)}\left(\cosh^n\left(ex+d\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(F^(c*(b*x+a))*cosh(e*x+d)^n,x)
```

```
[Out] int(F^(c*(b*x+a))*cosh(e*x+d)^n,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int F^{(bx+a)c}\cosh\left(ex+d\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*cosh(e*x+d)^n,x, algorithm="maxima")
```

```
[Out] integrate(F^((b*x + a)*c)*cosh(e*x + d)^n, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int F^{c\,(a+b\,x)}\,\cosh\left(d+e\,x\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(F^(c*(a + b*x))*cosh(d + e*x)^n,x)
```

```
[Out] int(F^(c*(a + b*x))*cosh(d + e*x)^n, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int F^{c(a+bx)}\cosh^{n}\left(d+ex\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F**(c*(b*x+a))*cosh(e*x+d)**n,x)
```

```
[Out] Integral(F**(c*(a + b*x))*cosh(d + e*x)**n, x)
```

3.885 $\int e^{a+bx}\cosh^3(c+dx)\,dx$

Optimal. Leaf size=139

$$\frac{be^{a+bx}\cosh^3(c+dx)}{b^2-9d^2}-\frac{3de^{a+bx}\sinh(c+dx)\cosh^2(c+dx)}{b^2-9d^2}-\frac{6bd^2e^{a+bx}\cosh(c+dx)}{b^4-10b^2d^2+9d^4}+\frac{6d^3e^{a+bx}\sinh(c+dx)}{b^4-10b^2d^2+9d^4}$$

```
[Out] -6*b*d^2*exp(b*x+a)*cosh(d*x+c)/(b^4-10*b^2*d^2+9*d^4)+b*exp(b*x+a)*cosh(d*
x+c)^3/(b^2-9*d^2)+6*d^3*exp(b*x+a)*sinh(d*x+c)/(b^4-10*b^2*d^2+9*d^4)-3*d*
exp(b*x+a)*cosh(d*x+c)^2*sinh(d*x+c)/(b^2-9*d^2)
```

Rubi [A] time = 0.05, antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {5477, 5475}

$$\frac{6d^3e^{a+bx}\sinh(c+dx)}{-10b^2d^2+b^4+9d^4}+\frac{be^{a+bx}\cosh^3(c+dx)}{b^2-9d^2}-\frac{6bd^2e^{a+bx}\cosh(c+dx)}{-10b^2d^2+b^4+9d^4}-\frac{3de^{a+bx}\sinh(c+dx)\cosh^2(c+dx)}{b^2-9d^2}$$

Antiderivative was successfully verified.

```
[In] Int[E^(a + b*x)*Cosh[c + d*x]^3,x]
```

```
[Out] (-6*b*d^2*E^(a + b*x)*Cosh[c + d*x])/(b^4 - 10*b^2*d^2 + 9*d^4) + (b*E^(a +
 b*x)*Cosh[c + d*x]^3)/(b^2 - 9*d^2) + (6*d^3*E^(a + b*x)*Sinh[c + d*x])/(b
^4 - 10*b^2*d^2 + 9*d^4) - (3*d*E^(a + b*x)*Cosh[c + d*x]^2*Sinh[c + d*x])/
(b^2 - 9*d^2)
```

Rule 5475

```
Int[Cosh[(d_.) + (e_.)*(x_)]*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symbol] :
> -Simp[(b*c*Log[F]*F^(c*(a + b*x))*Cosh[d + e*x])/(e^2 - b^2*c^2*Log[F]^2)
, x] + Simp[(e*F^(c*(a + b*x))*Sinh[d + e*x])/(e^2 - b^2*c^2*Log[F]^2), x]
/; FreeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 - b^2*c^2*Log[F]^2, 0]
```

Rule 5477

```
Int[Cosh[(d_.) + (e_.)*(x_)]^(n_)*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symb
ol] :> -Simp[(b*c*Log[F]*F^(c*(a + b*x))*Cosh[d + e*x]^n)/(e^2*n^2 - b^2*c^
2*Log[F]^2), x] + (Dist[(n*(n - 1)*e^2)/(e^2*n^2 - b^2*c^2*Log[F]^2), Int[F
^(c*(a + b*x))*Cosh[d + e*x]^(n - 2), x], x] + Simp[(e*n*F^(c*(a + b*x))*Si
nh[d + e*x]*Cosh[d + e*x]^(n - 1))/(e^2*n^2 - b^2*c^2*Log[F]^2), x]) /; Fre
eQ[{F, a, b, c, d, e}, x] && NeQ[e^2*n^2 - b^2*c^2*Log[F]^2, 0] && GtQ[n, 1
]
```

Rubi steps

$$\int e^{a+bx}\cosh^3(c+dx)\,dx = \frac{be^{a+bx}\cosh^3(c+dx)}{b^2-9d^2} - \frac{3de^{a+bx}\cosh^2(c+dx)\sinh(c+dx)}{b^2-9d^2} - \frac{(6d^2)\int e^{a+bx}\cosh(c+}{b^2-9d^2}$$
$$= -\frac{6bd^2e^{a+bx}\cosh(c+dx)}{b^4-10b^2d^2+9d^4} + \frac{be^{a+bx}\cosh^3(c+dx)}{b^2-9d^2} + \frac{6d^3e^{a+bx}\sinh(c+dx)}{b^4-10b^2d^2+9d^4} - \frac{3de^{a+bx}\,c}{}$$

Mathematica [A] time = 0.51, size = 106, normalized size = 0.76

$$\frac{e^{a+bx}\left(\left(b^3-bd^2\right)\cosh(3(c+dx)) + 3b\left(b^2-9d^2\right)\cosh(c+dx) + 6d\sinh(c+dx)\left(\left(d^2-b^2\right)\cosh(2(c+dx)) - b^2\right.\right.}{4\left(b^4-10b^2d^2+9d^4\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(a + b*x)*Cosh[c + d*x]^3,x]
```

```
[Out] (E^(a + b*x)*(3*b*(b^2 - 9*d^2)*Cosh[c + d*x] + (b^3 - b*d^2)*Cosh[3*(c + d
*x)] + 6*d*(-b^2 + 5*d^2 + (-b^2 + d^2)*Cosh[2*(c + d*x)])*Sinh[c + d*x]))/
(4*(b^4 - 10*b^2*d^2 + 9*d^4))
```

fricas [B] time = 0.56, size = 313, normalized size = 2.25

$$\frac{\left(b^3-bd^2\right)\cosh(bx+a)\cosh(dx+c)^3 - 3\left(\left(b^2d-d^3\right)\cosh(bx+a) + \left(b^2d-d^3\right)\sinh(bx+a)\right)\sinh(dx+c)^3 +}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*cosh(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] 1/4*((b^3 - b*d^2)*cosh(b*x + a)*cosh(d*x + c)^3 - 3*((b^2*d - d^3)*cosh(b*
x + a) + (b^2*d - d^3)*sinh(b*x + a))*sinh(d*x + c)^3 + 3*(b^3 - 9*b*d^2)*c
osh(b*x + a)*cosh(d*x + c) + 3*((b^3 - b*d^2)*cosh(b*x + a)*cosh(d*x + c) +
 (b^3 - b*d^2)*cosh(d*x + c)*sinh(b*x + a))*sinh(d*x + c)^2 + ((b^3 - b*d^2
)*cosh(d*x + c)^3 + 3*(b^3 - 9*b*d^2)*cosh(d*x + c))*sinh(b*x + a) - 3*(3*(
b^2*d - d^3)*cosh(b*x + a)*cosh(d*x + c)^2 + (b^2*d - 9*d^3)*cosh(b*x + a)
+ (b^2*d - 9*d^3 + 3*(b^2*d - d^3)*cosh(d*x + c)^2)*sinh(b*x + a))*sinh(d*x
 + c))/(b^4 - 10*b^2*d^2 + 9*d^4)
```

giac [A] time = 0.12, size = 84, normalized size = 0.60

$$\frac{e^{(bx+3\,dx+a+3\,c)}}{8\,(b+3\,d)} + \frac{3\,e^{(bx+dx+a+c)}}{8\,(b+d)} + \frac{3\,e^{(bx-dx+a-c)}}{8\,(b-d)} + \frac{e^{(bx-3\,dx+a-3\,c)}}{8\,(b-3\,d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*cosh(d*x+c)^3,x, algorithm="giac")
```

```
[Out] 1/8*e^(b*x + 3*d*x + a + 3*c)/(b + 3*d) + 3/8*e^(b*x + d*x + a + c)/(b + d)
 + 3/8*e^(b*x - d*x + a - c)/(b - d) + 1/8*e^(b*x - 3*d*x + a - 3*c)/(b - 3
*d)
```

maple [A] time = 0.36, size = 166, normalized size = 1.19

$$\frac{\sinh(a-3c+(b-3d)x)}{8b-24d}+\frac{3\sinh(a-c+(b-d)x)}{8(b-d)}+\frac{3\sinh(a+c+(b+d)x)}{8(b+d)}+\frac{\sinh(a+3c+(b+3d)x)}{8b+24d}+\cos$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(b*x+a)*cosh(d*x+c)^3,x)
```

```
[Out] 1/8*sinh(a-3*c+(b-3*d)*x)/(b-3*d)+3/8*sinh(a-c+(b-d)*x)/(b-d)+3/8*sinh(a+c+
(b+d)*x)/(b+d)+1/8*sinh(a+3*c+(b+3*d)*x)/(b+3*d)+1/8*cosh(a-3*c+(b-3*d)*x)/
(b-3*d)+3/8*cosh(a-c+(b-d)*x)/(b-d)+3/8*cosh(a+c+(b+d)*x)/(b+d)+1/8*cosh(a+
3*c+(b+3*d)*x)/(b+3*d)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*cosh(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(-(3*d)/b>0)', see `assume?` for mor
e details)Is -(3*d)/b equal to -1?
```

mupad [B] time = 2.21, size = 125, normalized size = 0.90

$$\frac{e^{a+b\,x}\,\left(b^3\cosh\left(c+d\,x\right)^3-3\,b^2\,d\cosh\left(c+d\,x\right)^2\sinh\left(c+d\,x\right)-7\,b\,d^2\cosh\left(c+d\,x\right)^3+6\,b\,d^2\cosh\left(c+d\,x\right)\sin\right.}{b^4-10\,b^2\,d^2+9\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(c + d*x)^3*exp(a + b*x),x)
```

```
[Out] (exp(a + b*x)*(b^3*cosh(c + d*x)^3 - 6*d^3*sinh(c + d*x)^3 - 7*b*d^2*cosh(c
 + d*x)^3 + 9*d^3*cosh(c + d*x)^2*sinh(c + d*x) + 6*b*d^2*cosh(c + d*x)*sin
h(c + d*x)^2 - 3*b^2*d*cosh(c + d*x)^2*sinh(c + d*x)))/(b^4 + 9*d^4 - 10*b^
2*d^2)
```

sympy [A] time = 45.78, size = 1085, normalized size = 7.81

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*cosh(d*x+c)**3,x)
```

```
[Out] Piecewise((x*exp(a)*cosh(c)**3, Eq(b, 0) & Eq(d, 0)), (x*exp(a)*exp(-3*d*x)
*sinh(c + d*x)**3/8 + 3*x*exp(a)*exp(-3*d*x)*sinh(c + d*x)**2*cosh(c + d*x)
/8 + 3*x*exp(a)*exp(-3*d*x)*sinh(c + d*x)*cosh(c + d*x)**2/8 + x*exp(a)*exp
(-3*d*x)*cosh(c + d*x)**3/8 + 11*exp(a)*exp(-3*d*x)*sinh(c + d*x)**3/(24*d)
 + 5*exp(a)*exp(-3*d*x)*sinh(c + d*x)**2*cosh(c + d*x)/(4*d) + exp(a)*exp(-
3*d*x)*sinh(c + d*x)*cosh(c + d*x)**2/d + exp(a)*exp(-3*d*x)*cosh(c + d*x)*
*3/(24*d), Eq(b, -3*d)), (-3*x*exp(a)*exp(-d*x)*sinh(c + d*x)**3/8 - 3*x*ex
p(a)*exp(-d*x)*sinh(c + d*x)**2*cosh(c + d*x)/8 + 3*x*exp(a)*exp(-d*x)*sinh
(c + d*x)*cosh(c + d*x)**2/8 + 3*x*exp(a)*exp(-d*x)*cosh(c + d*x)**3/8 - 5*
exp(a)*exp(-d*x)*sinh(c + d*x)**3/(8*d) - exp(a)*exp(-d*x)*sinh(c + d*x)**2
*cosh(c + d*x)/(4*d) + exp(a)*exp(-d*x)*sinh(c + d*x)*cosh(c + d*x)**2/d +
3*exp(a)*exp(-d*x)*cosh(c + d*x)**3/(8*d), Eq(b, -d)), (3*x*exp(a)*exp(d*x)
*sinh(c + d*x)**3/8 - 3*x*exp(a)*exp(d*x)*sinh(c + d*x)**2*cosh(c + d*x)/8
- 3*x*exp(a)*exp(d*x)*sinh(c + d*x)*cosh(c + d*x)**2/8 + 3*x*exp(a)*exp(d*x
)*cosh(c + d*x)**3/8 - 5*exp(a)*exp(d*x)*sinh(c + d*x)**3/(8*d) + exp(a)*ex
p(d*x)*sinh(c + d*x)**2*cosh(c + d*x)/(4*d) + exp(a)*exp(d*x)*sinh(c + d*x)
*cosh(c + d*x)**2/d - 3*exp(a)*exp(d*x)*cosh(c + d*x)**3/(8*d), Eq(b, d)),
(-x*exp(a)*exp(3*d*x)*sinh(c + d*x)**3/8 + 3*x*exp(a)*exp(3*d*x)*sinh(c + d
*x)**2*cosh(c + d*x)/8 - 3*x*exp(a)*exp(3*d*x)*sinh(c + d*x)*cosh(c + d*x)*
*2/8 + x*exp(a)*exp(3*d*x)*cosh(c + d*x)**3/8 + 11*exp(a)*exp(3*d*x)*sinh(c
 + d*x)**3/(24*d) - 5*exp(a)*exp(3*d*x)*sinh(c + d*x)**2*cosh(c + d*x)/(4*d
) + exp(a)*exp(3*d*x)*sinh(c + d*x)*cosh(c + d*x)**2/d - exp(a)*exp(3*d*x)*
cosh(c + d*x)**3/(24*d), Eq(b, 3*d)), (b**3*exp(a)*exp(b*x)*cosh(c + d*x)**
3/(b**4 - 10*b**2*d**2 + 9*d**4) - 3*b**2*d*exp(a)*exp(b*x)*sinh(c + d*x)*c
osh(c + d*x)**2/(b**4 - 10*b**2*d**2 + 9*d**4) + 6*b*d**2*exp(a)*exp(b*x)*s
inh(c + d*x)**2*cosh(c + d*x)/(b**4 - 10*b**2*d**2 + 9*d**4) - 7*b*d**2*exp
(a)*exp(b*x)*cosh(c + d*x)**3/(b**4 - 10*b**2*d**2 + 9*d**4) - 6*d**3*exp(a
)*exp(b*x)*sinh(c + d*x)**3/(b**4 - 10*b**2*d**2 + 9*d**4) + 9*d**3*exp(a)*
exp(b*x)*sinh(c + d*x)*cosh(c + d*x)**2/(b**4 - 10*b**2*d**2 + 9*d**4), Tru
e))
```

3.886 $\int e^{a+bx}\cosh^2(c+dx)\,dx$

Optimal. Leaf size=88

$$\frac{be^{a+bx}\cosh^2(c+dx)}{b^2-4d^2}-\frac{2de^{a+bx}\sinh(c+dx)\cosh(c+dx)}{b^2-4d^2}-\frac{2d^2e^{a+bx}}{b\left(b^2-4d^2\right)}$$

```
[Out] -2*d^2*exp(b*x+a)/b/(b^2-4*d^2)+b*exp(b*x+a)*cosh(d*x+c)^2/(b^2-4*d^2)-2*d*
exp(b*x+a)*cosh(d*x+c)*sinh(d*x+c)/(b^2-4*d^2)
```

Rubi [A] time = 0.03, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {5477, 2194}

$$\frac{be^{a+bx}\cosh^2(c+dx)}{b^2-4d^2}-\frac{2de^{a+bx}\sinh(c+dx)\cosh(c+dx)}{b^2-4d^2}-\frac{2d^2e^{a+bx}}{b\left(b^2-4d^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[E^(a + b*x)*Cosh[c + d*x]^2,x]
```

```
[Out] (-2*d^2*E^(a + b*x))/(b*(b^2 - 4*d^2)) + (b*E^(a + b*x)*Cosh[c + d*x]^2)/(b
^2 - 4*d^2) - (2*d*E^(a + b*x)*Cosh[c + d*x]*Sinh[c + d*x])/(b^2 - 4*d^2)
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 5477

```
Int[Cosh[(d_.) + (e_.)*(x_)]^(n_)*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symb
ol] :> -Simp[(b*c*Log[F]*F^(c*(a + b*x))*Cosh[d + e*x]^n)/(e^2*n^2 - b^2*c^
2*Log[F]^2), x] + (Dist[(n*(n - 1)*e^2)/(e^2*n^2 - b^2*c^2*Log[F]^2), Int[F
^(c*(a + b*x))*Cosh[d + e*x]^(n - 2), x], x] + Simp[(e*n*F^(c*(a + b*x))*Si
nh[d + e*x]*Cosh[d + e*x]^(n - 1))/(e^2*n^2 - b^2*c^2*Log[F]^2), x]) /; Fre
eQ[{F, a, b, c, d, e}, x] && NeQ[e^2*n^2 - b^2*c^2*Log[F]^2, 0] && GtQ[n, 1
]
```

Rubi steps

$$\int e^{a+bx}\cosh^2(c+dx)\,dx = \frac{be^{a+bx}\cosh^2(c+dx)}{b^2-4d^2} - \frac{2de^{a+bx}\cosh(c+dx)\sinh(c+dx)}{b^2-4d^2} - \frac{\left(2d^2\right)\int e^{a+bx}\,dx}{b^2-4d^2}$$
$$= -\frac{2d^2e^{a+bx}}{b\left(b^2-4d^2\right)} + \frac{be^{a+bx}\cosh^2(c+dx)}{b^2-4d^2} - \frac{2de^{a+bx}\cosh(c+dx)\sinh(c+dx)}{b^2-4d^2}$$

Mathematica [A] time = 0.16, size = 56, normalized size = 0.64

$$\frac{e^{a+bx}\left(b^2\cosh(2(c+dx))+b^2-2bd\sinh(2(c+dx))-4d^2\right)}{2\left(b^3-4bd^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(a + b*x)*Cosh[c + d*x]^2,x]
```

```
[Out] (E^(a + b*x)*(b^2 - 4*d^2 + b^2*Cosh[2*(c + d*x)] - 2*b*d*Sinh[2*(c + d*x)]
))/(2*(b^3 - 4*b*d^2))
```

fricas [A] time = 0.45, size = 146, normalized size = 1.66

$$\frac{b^2\cosh(bx+a)\cosh(dx+c)^2 + \left(b^2\cosh(bx+a)+b^2\sinh(bx+a)\right)\sinh(dx+c)^2 + \left(b^2-4\,d^2\right)\cosh(bx+a)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*cosh(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(b^2*cosh(b*x + a)*cosh(d*x + c)^2 + (b^2*cosh(b*x + a) + b^2*sinh(b*x
+ a))*sinh(d*x + c)^2 + (b^2 - 4*d^2)*cosh(b*x + a) + (b^2*cosh(d*x + c)^2
+ b^2 - 4*d^2)*sinh(b*x + a) - 4*(b*d*cosh(b*x + a)*cosh(d*x + c) + b*d*cos
h(d*x + c)*sinh(b*x + a))*sinh(d*x + c))/(b^3 - 4*b*d^2)
```

giac [A] time = 0.12, size = 56, normalized size = 0.64

$$\frac{e^{(bx+2\,dx+a+2\,c)}}{4\,(b+2\,d)} + \frac{e^{(bx-2\,dx+a-2\,c)}}{4\,(b-2\,d)} + \frac{e^{(bx+a)}}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*cosh(d*x+c)^2,x, algorithm="giac")
```

```
[Out] 1/4*e^(b*x + 2*d*x + a + 2*c)/(b + 2*d) + 1/4*e^(b*x - 2*d*x + a - 2*c)/(b
- 2*d) + 1/2*e^(b*x + a)/b
```

maple [A] time = 0.29, size = 112, normalized size = 1.27

$$\frac{\sinh(bx+a)}{2b}+\frac{\sinh(a-2c+(b-2d)x)}{4b-8d}+\frac{\sinh(a+2c+(b+2d)x)}{4b+8d}+\frac{\cosh(bx+a)}{2b}+\frac{\cosh(a-2c+(b-2d)x}{4b-8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(b*x+a)*cosh(d*x+c)^2,x)
```

```
[Out] 1/2*sinh(b*x+a)/b+1/4*sinh(a-2*c+(b-2*d)*x)/(b-2*d)+1/4*sinh(a+2*c+(b+2*d)*
x)/(b+2*d)+1/2*cosh(b*x+a)/b+1/4*cosh(a-2*c+(b-2*d)*x)/(b-2*d)+1/4*cosh(a+2
*c+(b+2*d)*x)/(b+2*d)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*cosh(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(-(2*d)/b>0)', see `assume?` for mor
e details)Is -(2*d)/b equal to -1?
```

mupad [B] time = 0.23, size = 68, normalized size = 0.77

$$\frac{2\,d^2\,e^{a+b\,x}-b^2\cosh(c+d\,x)^2\,e^{a+b\,x}+2\,b\,d\cosh(c+d\,x)\,e^{a+b\,x}\sinh(c+d\,x)}{4\,b\,d^2-b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(c + d*x)^2*exp(a + b*x),x)
```

```
[Out] (2*d^2*exp(a + b*x) - b^2*cosh(c + d*x)^2*exp(a + b*x) + 2*b*d*cosh(c + d*x
)*exp(a + b*x)*sinh(c + d*x))/(4*b*d^2 - b^3)
```

sympy [A] time = 9.02, size = 432, normalized size = 4.91

$$\begin{cases} xe^{a}\cosh^{2}(c) \\ \left(-\frac{x\sinh^{2}(c+dx)}{2}+\frac{x\cosh^{2}(c+dx)}{2}+\frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right)e^{a} \\ \frac{xe^{a}e^{-2dx}\sinh^{2}(c+dx)}{4}+\frac{xe^{a}e^{-2dx}\sinh(c+dx)\cosh(c+dx)}{2}+\frac{xe^{a}e^{-2dx}\cosh^{2}(c+dx)}{4}+\frac{e^{a}e^{-2dx}\sinh^{2}(c+dx)}{2d}+\frac{3e^{a}e^{-2dx}\sinh(c+dx)\cosh(c+dx)}{4d} \\ \frac{xe^{a}e^{2dx}\sinh^{2}(c+dx)}{4}-\frac{xe^{a}e^{2dx}\sinh(c+dx)\cosh(c+dx)}{2}+\frac{xe^{a}e^{2dx}\cosh^{2}(c+dx)}{4}-\frac{e^{a}e^{2dx}\sinh^{2}(c+dx)}{2d}+\frac{3e^{a}e^{2dx}\sinh(c+dx)\cosh(c+dx)}{4d} \\ \frac{b^{2}e^{a}e^{bx}\cosh^{2}(c+dx)}{b^{3}-4bd^{2}}-\frac{2bde^{a}e^{bx}\sinh(c+dx)\cosh(c+dx)}{b^{3}-4bd^{2}}+\frac{2d^{2}e^{a}e^{bx}\sinh^{2}(c+dx)}{b^{3}-4bd^{2}}-\frac{2d^{2}e^{a}e^{bx}\cosh^{2}(c+dx)}{b^{3}-4bd^{2}} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*cosh(d*x+c)**2,x)
```

```
[Out] Piecewise((x*exp(a)*cosh(c)**2, Eq(b, 0) & Eq(d, 0)), ((-x*sinh(c + d*x)**2
/2 + x*cosh(c + d*x)**2/2 + sinh(c + d*x)*cosh(c + d*x)/(2*d))*exp(a), Eq(b
, 0)), (x*exp(a)*exp(-2*d*x)*sinh(c + d*x)**2/4 + x*exp(a)*exp(-2*d*x)*sinh
(c + d*x)*cosh(c + d*x)/2 + x*exp(a)*exp(-2*d*x)*cosh(c + d*x)**2/4 + exp(a
)*exp(-2*d*x)*sinh(c + d*x)**2/(2*d) + 3*exp(a)*exp(-2*d*x)*sinh(c + d*x)*c
osh(c + d*x)/(4*d), Eq(b, -2*d)), (x*exp(a)*exp(2*d*x)*sinh(c + d*x)**2/4 -
 x*exp(a)*exp(2*d*x)*sinh(c + d*x)*cosh(c + d*x)/2 + x*exp(a)*exp(2*d*x)*co
sh(c + d*x)**2/4 - exp(a)*exp(2*d*x)*sinh(c + d*x)**2/(2*d) + 3*exp(a)*exp(
2*d*x)*sinh(c + d*x)*cosh(c + d*x)/(4*d), Eq(b, 2*d)), (b**2*exp(a)*exp(b*x
)*cosh(c + d*x)**2/(b**3 - 4*b*d**2) - 2*b*d*exp(a)*exp(b*x)*sinh(c + d*x)*
cosh(c + d*x)/(b**3 - 4*b*d**2) + 2*d**2*exp(a)*exp(b*x)*sinh(c + d*x)**2/(
b**3 - 4*b*d**2) - 2*d**2*exp(a)*exp(b*x)*cosh(c + d*x)**2/(b**3 - 4*b*d**2
), True))
```

3.887 $\int e^{a+bx}\cosh(c+dx)\,dx$

Optimal. Leaf size=54

$$\frac{be^{a+bx}\cosh(c+dx)}{b^2-d^2}-\frac{de^{a+bx}\sinh(c+dx)}{b^2-d^2}$$

```
[Out] b*exp(b*x+a)*cosh(d*x+c)/(b^2-d^2)-d*exp(b*x+a)*sinh(d*x+c)/(b^2-d^2)
```

Rubi [A] time = 0.02, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {5475}

$$\frac{be^{a+bx}\cosh(c+dx)}{b^2-d^2}-\frac{de^{a+bx}\sinh(c+dx)}{b^2-d^2}$$

Antiderivative was successfully verified.

```
[In] Int[E^(a + b*x)*Cosh[c + d*x],x]
```

```
[Out] (b*E^(a + b*x)*Cosh[c + d*x])/(b^2 - d^2) - (d*E^(a + b*x)*Sinh[c + d*x])/(
b^2 - d^2)
```

Rule 5475

```
Int[Cosh[(d_.) + (e_.)*(x_)]*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symbol] :
> -Simp[(b*c*Log[F]*F^(c*(a + b*x))*Cosh[d + e*x])/(e^2 - b^2*c^2*Log[F]^2)
, x] + Simp[(e*F^(c*(a + b*x))*Sinh[d + e*x])/(e^2 - b^2*c^2*Log[F]^2), x]
/; FreeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 - b^2*c^2*Log[F]^2, 0]
```

Rubi steps

$$\int e^{a+bx}\cosh(c+dx)\,dx=\frac{be^{a+bx}\cosh(c+dx)}{b^2-d^2}-\frac{de^{a+bx}\sinh(c+dx)}{b^2-d^2}$$

Mathematica [A] time = 0.08, size = 38, normalized size = 0.70

$$\frac{e^{a+bx}(b\cosh(c+dx)-d\sinh(c+dx))}{(b-d)(b+d)}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(a + b*x)*Cosh[c + d*x],x]
```

```
[Out] (E^(a + b*x)*(b*Cosh[c + d*x] - d*Sinh[c + d*x]))/((b - d)*(b + d))
```

fricas [A] time = 0.53, size = 66, normalized size = 1.22

$$\frac{b\cosh(bx+a)\cosh(dx+c)+b\cosh(dx+c)\sinh(bx+a)-(d\cosh(bx+a)+d\sinh(bx+a))\sinh(dx+c)}{b^2-d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*cosh(d*x+c),x, algorithm="fricas")
```

```
[Out] (b*cosh(b*x + a)*cosh(d*x + c) + b*cosh(d*x + c)*sinh(b*x + a) - (d*cosh(b*
x + a) + d*sinh(b*x + a))*sinh(d*x + c))/(b^2 - d^2)
```

giac [A] time = 0.11, size = 40, normalized size = 0.74

$$\frac{e^{(bx+dx+a+c)}}{2(b+d)}+\frac{e^{(bx-dx+a-c)}}{2(b-d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*cosh(d*x+c),x, algorithm="giac")
```

```
[Out] 1/2*e^(b*x + d*x + a + c)/(b + d) + 1/2*e^(b*x - d*x + a - c)/(b - d)
```

maple [A] time = 0.19, size = 78, normalized size = 1.44

$$\frac{\sinh(a-c+(b-d)x)}{2b-2d}+\frac{\sinh(a+c+(b+d)x)}{2b+2d}+\frac{\cosh(a-c+(b-d)x)}{2b-2d}+\frac{\cosh(a+c+(b+d)x)}{2b+2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(b*x+a)*cosh(d*x+c),x)
```

```
[Out] 1/2*sinh(a-c+(b-d)*x)/(b-d)+1/2*sinh(a+c+(b+d)*x)/(b+d)+1/2*cosh(a-c+(b-d)*
x)/(b-d)+1/2*cosh(a+c+(b+d)*x)/(b+d)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*cosh(d*x+c),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(-d/b>0)', see `assume?` for more de
tails)Is -d/b equal to -1?
```

mupad [B] time = 0.08, size = 53, normalized size = 0.98

$$\frac{\mathrm{e}^{a-c+b\,x-d\,x}\,\left(b+d+b\,\mathrm{e}^{2\,c+2\,d\,x}-d\,\mathrm{e}^{2\,c+2\,d\,x}\right)}{2\,\left(b^2-d^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(c + d*x)*exp(a + b*x),x)
```

```
[Out] (exp(a - c + b*x - d*x)*(b + d + b*exp(2*c + 2*d*x) - d*exp(2*c + 2*d*x)))/
(2*(b^2 - d^2))
```

sympy [A] time = 2.10, size = 167, normalized size = 3.09

$$\begin{cases} xe^{a}\cosh{(c)} & \text{for } b=0 \wedge d=0 \\ \frac{xe^{a}e^{-dx}\sinh{(c+dx)}}{2}+\frac{xe^{a}e^{-dx}\cosh{(c+dx)}}{2}+\frac{e^{a}e^{-dx}\sinh{(c+dx)}}{2d} & \text{for } b=-d \\ -\frac{xe^{a}e^{dx}\sinh{(c+dx)}}{2}+\frac{xe^{a}e^{dx}\cosh{(c+dx)}}{2}+\frac{e^{a}e^{dx}\sinh{(c+dx)}}{2d} & \text{for } b=d \\ \frac{be^{a}e^{bx}\cosh{(c+dx)}}{b^2-d^2}-\frac{de^{a}e^{bx}\sinh{(c+dx)}}{b^2-d^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*cosh(d*x+c),x)
```

```
[Out] Piecewise((x*exp(a)*cosh(c), Eq(b, 0) & Eq(d, 0)), (x*exp(a)*exp(-d*x)*sinh
(c + d*x)/2 + x*exp(a)*exp(-d*x)*cosh(c + d*x)/2 + exp(a)*exp(-d*x)*sinh(c
+ d*x)/(2*d), Eq(b, -d)), (-x*exp(a)*exp(d*x)*sinh(c + d*x)/2 + x*exp(a)*ex
p(d*x)*cosh(c + d*x)/2 + exp(a)*exp(d*x)*sinh(c + d*x)/(2*d), Eq(b, d)), (b
*exp(a)*exp(b*x)*cosh(c + d*x)/(b**2 - d**2) - d*exp(a)*exp(b*x)*sinh(c + d
*x)/(b**2 - d**2), True))
```

3.888 $\int e^{a+bx}\text{sech}(c+dx)\,dx$

Optimal. Leaf size=52

$$\frac{2e^{a+bx+c+dx}\,{}_2F_1\left(1,\frac{b+d}{2d};\frac{1}{2}\left(\frac{b}{d}+3\right);-e^{2(c+dx)}\right)}{b+d}$$

```
[Out] 2*exp(b*x+d*x+a+c)*hypergeom([1, 1/2*(b+d)/d],[3/2+1/2*b/d],-exp(2*d*x+2*c)
)/(b+d)
```

Rubi [A] time = 0.02, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {5492}

$$\frac{2e^{a+bx+c+dx}\,{}_2F_1\left(1,\frac{b+d}{2d};\frac{1}{2}\left(\frac{b}{d}+3\right);-e^{2(c+dx)}\right)}{b+d}$$

Antiderivative was successfully verified.

```
[In] Int[E^(a + b*x)*Sech[c + d*x],x]
```

```
[Out] (2*E^(a + c + b*x + d*x)*Hypergeometric2F1[1, (b + d)/(2*d), (3 + b/d)/2, -
E^(2*(c + d*x))])/(b + d)
```

Rule 5492

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sech[(d_.) + (e_.)*(x_)]^(n_.), x_Sym
bol] :> Simp[(2^n*E^(n*(d + e*x))*F^(c*(a + b*x))*Hypergeometric2F1[n, n/2
+ (b*c*Log[F])/(2*e), 1 + n/2 + (b*c*Log[F])/(2*e), -E^(2*(d + e*x))])/(e*n
 + b*c*Log[F]), x] /; FreeQ[{F, a, b, c, d, e}, x] && IntegerQ[n]
```

Rubi steps

$$\int e^{a+bx}\text{sech}(c+dx)\,dx=\frac{2e^{a+c+bx+dx}\,{}_2F_1\left(1,\frac{b+d}{2d};\frac{1}{2}\left(3+\frac{b}{d}\right);-e^{2(c+dx)}\right)}{b+d}$$

Mathematica [A] time = 0.02, size = 51, normalized size = 0.98

$$\frac{2e^{a+x(b+d)+c}\,{}_2F_1\left(1,\frac{b+d}{2d};\frac{1}{2}\left(\frac{b}{d}+3\right);-e^{2(c+dx)}\right)}{b+d}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(a + b*x)*Sech[c + d*x],x]
```

```
[Out] (2*E^(a + c + (b + d)*x)*Hypergeometric2F1[1, (b + d)/(2*d), (3 + b/d)/2, -
E^(2*(c + d*x))])/(b + d)
```

fricas [F] time = 0.53, size = 0, normalized size = 0.00

$$\text{integral}\left(e^{(bx+a)}\operatorname{sech}\left(dx+c\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*sech(d*x+c),x, algorithm="fricas")
```

```
[Out] integral(e^(b*x + a)*sech(d*x + c), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int e^{(bx+a)}\operatorname{sech}\left(dx+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*sech(d*x+c),x, algorithm="giac")
```

```
[Out] integrate(e^(b*x + a)*sech(d*x + c), x)
```

maple [F] time = 0.26, size = 0, normalized size = 0.00

$$\int \mathrm{e}^{bx+a}\operatorname{sech}\left(dx+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(b*x+a)*sech(d*x+c),x)
```

```
[Out] int(exp(b*x+a)*sech(d*x+c),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int e^{(bx+a)}\operatorname{sech}\left(dx+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*sech(d*x+c),x, algorithm="maxima")
```

```
[Out] integrate(e^(b*x + a)*sech(d*x + c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\mathrm{e}^{a+b\,x}}{\cosh\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(a + b*x)/cosh(c + d*x),x)
[Out] int(exp(a + b*x)/cosh(c + d*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$e^{a}\int e^{bx}\operatorname{sech}\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*sech(d*x+c),x)
[Out] exp(a)*Integral(exp(b*x)*sech(c + d*x), x)
```

3.889 $\int e^{a+bx}\operatorname{sech}^2(c+dx)\,dx$

Optimal. Leaf size=56

$$\frac{4e^{a+bx+2(c+dx)}\,{}_2F_1\left(2,\frac{b}{2d}+1;\frac{b}{2d}+2;-e^{2(c+dx)}\right)}{b+2d}$$

```
[Out] 4*exp(b*x+2*d*x+a+2*c)*hypergeom([2, 1+1/2*b/d],[2+1/2*b/d],-exp(2*d*x+2*c)
)/(b+2*d)
```

Rubi [A] time = 0.03, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {5492}

$$\frac{4e^{a+bx+2(c+dx)}\,{}_2F_1\left(2,\frac{b}{2d}+1;\frac{b}{2d}+2;-e^{2(c+dx)}\right)}{b+2d}$$

Antiderivative was successfully verified.

```
[In] Int[E^(a + b*x)*Sech[c + d*x]^2,x]
```

```
[Out] (4*E^(a + b*x + 2*(c + d*x))*Hypergeometric2F1[2, 1 + b/(2*d), 2 + b/(2*d),
 -E^(2*(c + d*x))])/(b + 2*d)
```

Rule 5492

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sech[(d_.) + (e_.)*(x_)]^(n_.), x_Sym
bol] :> Simp[(2^n*E^(n*(d + e*x))*F^(c*(a + b*x))*Hypergeometric2F1[n, n/2
+ (b*c*Log[F])/(2*e), 1 + n/2 + (b*c*Log[F])/(2*e), -E^(2*(d + e*x))])/(e*n
 + b*c*Log[F]), x] /; FreeQ[{F, a, b, c, d, e}, x] && IntegerQ[n]
```

Rubi steps

$$\int e^{a+bx}\operatorname{sech}^2(c+dx)\,dx=\frac{4e^{a+bx+2(c+dx)}\,{}_2F_1\left(2,1+\frac{b}{2d};2+\frac{b}{2d};-e^{2(c+dx)}\right)}{b+2d}$$

Mathematica [A] time = 0.02, size = 56, normalized size = 1.00

$$\frac{4e^{a+bx+2(c+dx)}\,{}_2F_1\left(2,\frac{b}{2d}+1;\frac{b}{2d}+2;-e^{2(c+dx)}\right)}{b+2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(a + b*x)*Sech[c + d*x]^2,x]
```

```
[Out] (4*E^(a + b*x + 2*(c + d*x))*Hypergeometric2F1[2, 1 + b/(2*d), 2 + b/(2*d),
 -E^(2*(c + d*x))])/(b + 2*d)
```

fricas [F] time = 0.41, size = 0, normalized size = 0.00

$$\text{integral}\left(e^{(bx+a)}\,\text{sech}\,(dx+c)^2\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*sech(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] integral(e^(b*x + a)*sech(d*x + c)^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int e^{(bx+a)}\,\text{sech}\,(dx+c)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*sech(d*x+c)^2,x, algorithm="giac")
```

```
[Out] integrate(e^(b*x + a)*sech(d*x + c)^2, x)
```

maple [F] time = 0.34, size = 0, normalized size = 0.00

$$\int \mathrm{e}^{bx+a}\text{sech}\,(dx+c)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(b*x+a)*sech(d*x+c)^2,x)
```

```
[Out] int(exp(b*x+a)*sech(d*x+c)^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$4\,b\int \frac{e^{(bx+a)}}{2\left(de^{(2\,dx+2\,c)}+d\right)}\,dx - \frac{2\,e^{(bx+a)}}{de^{(2\,dx+2\,c)}+d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*sech(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] 4*b*integrate(1/2*e^(b*x + a)/(d*e^(2*d*x + 2*c) + d), x) - 2*e^(b*x + a)/(
d*e^(2*d*x + 2*c) + d)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\mathrm{e}^{a+b\,x}}{{\mathrm{cosh}\left(c+d\,x\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(a + b*x)/cosh(c + d*x)^2,x)
```

```
[Out] int(exp(a + b*x)/cosh(c + d*x)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$e^{a} \int e^{bx} \operatorname{sech}^{2}{\left(c + dx \right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*sech(d*x+c)**2,x)
```

```
[Out] exp(a)*Integral(exp(b*x)*sech(c + d*x)**2, x)
```

3.890 $\int e^{a+bx}\mathbf{sech}^3(c+dx)\,dx$

Optimal. Leaf size=103

$$-\frac{(b-d)e^{a+bx+c+dx}\,{}_2F_1\left(1,\frac{b+d}{2d};\frac{1}{2}\left(\frac{b}{d}+3\right);-e^{2(c+dx)}\right)}{d^2}+\frac{be^{a+bx}\text{sech}(c+dx)}{2d^2}+\frac{e^{a+bx}\tanh(c+dx)\text{sech}(c+dx)}{2d}$$

```
[Out] -(b-d)*exp(b*x+d*x+a+c)*hypergeom([1, 1/2*(b+d)/d],[3/2+1/2*b/d],-exp(2*d*x
+2*c))/d^2+1/2*b*exp(b*x+a)*sech(d*x+c)/d^2+1/2*exp(b*x+a)*sech(d*x+c)*tanh
(d*x+c)/d
```

Rubi [A] time = 0.05, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {5490, 5492}

$$-\frac{(b-d)e^{a+bx+c+dx}\,{}_2F_1\left(1,\frac{b+d}{2d};\frac{1}{2}\left(\frac{b}{d}+3\right);-e^{2(c+dx)}\right)}{d^2}+\frac{be^{a+bx}\text{sech}(c+dx)}{2d^2}+\frac{e^{a+bx}\tanh(c+dx)\text{sech}(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[E^(a + b*x)*Sech[c + d*x]^3,x]
```

```
[Out] -(((b - d)*E^(a + c + b*x + d*x)*Hypergeometric2F1[1, (b + d)/(2*d), (3 + b
/d)/2, -E^(2*(c + d*x))])/d^2) + (b*E^(a + b*x)*Sech[c + d*x])/(2*d^2) + (E
^(a + b*x)*Sech[c + d*x]*Tanh[c + d*x])/(2*d)
```

Rule 5490

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sech[(d_.) + (e_.)*(x_)]^(n_), x_Symb
ol] :> Simp[(b*c*Log[F]*F^(c*(a + b*x))*Sech[d + e*x]^(n - 2))/(e^2*(n - 1)
*(n - 2)), x] + (Dist[(e^2*(n - 2)^2 - b^2*c^2*Log[F]^2)/(e^2*(n - 1)*(n -
2)), Int[F^(c*(a + b*x))*Sech[d + e*x]^(n - 2), x], x] + Simp[(F^(c*(a + b*
x))*Sech[d + e*x]^(n - 1)*Sinh[d + e*x])/(e*(n - 1)), x]) /; FreeQ[{F, a, b
, c, d, e}, x] && NeQ[e^2*(n - 2)^2 - b^2*c^2*Log[F]^2, 0] && GtQ[n, 1] &&
NeQ[n, 2]
```

Rule 5492

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sech[(d_.) + (e_.)*(x_)]^(n_.), x_Sym
bol] :> Simp[(2^n*E^(n*(d + e*x))*F^(c*(a + b*x))*Hypergeometric2F1[n, n/2
+ (b*c*Log[F])/(2*e), 1 + n/2 + (b*c*Log[F])/(2*e), -E^(2*(d + e*x))])/(e*n
 + b*c*Log[F]), x] /; FreeQ[{F, a, b, c, d, e}, x] && IntegerQ[n]
```

Rubi steps

$$\int e^{a+bx}\text{sech}^3(c+dx)\,dx = \frac{be^{a+bx}\text{sech}(c+dx)}{2d^2} + \frac{e^{a+bx}\text{sech}(c+dx)\tanh(c+dx)}{2d} + \frac{1}{2}\left(1-\frac{b^2}{d^2}\right)\int e^{a+bx}\text{sech}(c+$$

$$= -\frac{(b-d)e^{a+c+bx+dx}\,{}_2F_1\left(1,\frac{b+d}{2d};\frac{1}{2}\left(3+\frac{b}{d}\right);-e^{2(c+dx)}\right)}{d^2} + \frac{be^{a+bx}\text{sech}(c+dx)}{2d^2} + \frac{e^{a+bx}\text{sec}}{}$$

Mathematica [A] time = 0.15, size = 80, normalized size = 0.78

$$\frac{e^{a+bx}\left(\text{sech}(c+dx)(b+d\tanh(c+dx))-2(b-d)e^{c+dx}\,{}_2F_1\left(1,\frac{b+d}{2d};\frac{1}{2}\left(\frac{b}{d}+3\right);-e^{2(c+dx)}\right)\right)}{2d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(a + b*x)*Sech[c + d*x]^3,x]
[Out] (E^(a + b*x)*(-2*(b - d)*E^(c + d*x)*Hypergeometric2F1[1, (b + d)/(2*d), (3
 + b/d)/2, -E^(2*(c + d*x))] + Sech[c + d*x]*(b + d*Tanh[c + d*x])))/(2*d^2
)
```

fricas [F] time = 0.47, size = 0, normalized size = 0.00

$$\text{integral}\left(e^{(bx+a)}\,\text{sech}\,(dx+c)^3\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*sech(d*x+c)^3,x, algorithm="fricas")
[Out] integral(e^(b*x + a)*sech(d*x + c)^3, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int e^{(bx+a)}\,\text{sech}\,(dx+c)^3\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*sech(d*x+c)^3,x, algorithm="giac")
[Out] integrate(e^(b*x + a)*sech(d*x + c)^3, x)
```

maple [F] time = 0.27, size = 0, normalized size = 0.00

$$\int e^{bx+a}\text{sech}\,(dx+c)^3\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(b*x+a)*sech(d*x+c)^3,x)
```

```
[Out] int(exp(b*x+a)*sech(d*x+c)^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-8\left(b^2e^c-d^2e^c\right)\int\frac{e^{(bx+dx+a)}}{8\left(d^2e^{(2\,dx+2\,c)}+d^2\right)}\,dx+\frac{\left(be^{(3\,c)}+de^{(3\,c)}\right)e^{(bx+3\,dx+a)}+\left(be^c-de^c\right)e^{(bx+dx+a)}}{d^2e^{(4\,dx+4\,c)}+2\,d^2e^{(2\,dx+2\,c)}+d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*sech(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] -8*(b^2*e^c - d^2*e^c)*integrate(1/8*e^(b*x + d*x + a)/(d^2*e^(2*d*x + 2*c)
 + d^2), x) + ((b*e^(3*c) + d*e^(3*c))*e^(b*x + 3*d*x + a) + (b*e^c - d*e^c
)*e^(b*x + d*x + a))/(d^2*e^(4*d*x + 4*c) + 2*d^2*e^(2*d*x + 2*c) + d^2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{\mathrm{e}^{a+b\,x}}{\cosh\left(c+d\,x\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(a + b*x)/cosh(c + d*x)^3,x)
```

```
[Out] int(exp(a + b*x)/cosh(c + d*x)^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$e^a\int e^{bx}\operatorname{sech}^3\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*sech(d*x+c)**3,x)
```

```
[Out] exp(a)*Integral(exp(b*x)*sech(c + d*x)**3, x)
```

3.891 $\int F^{c(a+bx)}\text{sech}^n(d+ex)\,dx$

Optimal. Leaf size=90

$$\frac{\left(e^{2(d+ex)}+1\right)^n F^{ac+bcx}\text{sech}^n(d+ex)\,{}_2F_1\left(n,\frac{en+bc\log(F)}{2e};\frac{en+bc\log(F)}{2e}+1;-e^{2(d+ex)}\right)}{bc\log(F)+en}$$

```
[Out] (1+exp(2*e*x+2*d))^n*F^(b*c*x+a*c)*hypergeom([n, 1/2*(e*n+b*c*ln(F))/e],[1+
1/2*(e*n+b*c*ln(F))/e],-exp(2*e*x+2*d))*sech(e*x+d)^n/(e*n+b*c*ln(F))
```

Rubi [A] time = 0.14, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {5494, 2259}

$$\frac{\left(e^{2(d+ex)}+1\right)^n F^{ac+bcx}\text{sech}^n(d+ex)\,{}_2F_1\left(n,\frac{en+bc\log(F)}{2e};\frac{en+bc\log(F)}{2e}+1;-e^{2(d+ex)}\right)}{bc\log(F)+en}$$

Antiderivative was successfully verified.

```
[In] Int[F^(c*(a + b*x))*Sech[d + e*x]^n,x]
```

```
[Out] ((1 + E^(2*(d + e*x)))^n*F^(a*c + b*c*x)*Hypergeometric2F1[n, (e*n + b*c*Lo
g[F])/(2*e), 1 + (e*n + b*c*Log[F])/(2*e), -E^(2*(d + e*x))]*Sech[d + e*x]^
n)/(e*n + b*c*Log[F])
```

Rule 2259

```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_)*(G_)^((h_.)*((f_.
) + (g_.)*(x_)))*(H_)^((t_.)*((r_.) + (s_.)*(x_))), x_Symbol] :> Simp[(G^(h
*(f + g*x))*H^(t*(r + s*x))*(a + b*F^(e*(c + d*x)))^p*Hypergeometric2F1[-p,
 (g*h*Log[G] + s*t*Log[H])/(d*e*Log[F]), (g*h*Log[G] + s*t*Log[H])/(d*e*Log
[F]) + 1, Simplify[-((b*F^(e*(c + d*x)))/a)]])/((g*h*Log[G] + s*t*Log[H])*(
(a + b*F^(e*(c + d*x)))/a)^p), x] /; FreeQ[{F, G, H, a, b, c, d, e, f, g, h
, r, s, t, p}, x] &&  !IntegerQ[p]
```

Rule 5494

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sech[(d_.) + (e_.)*(x_)]^(n_.), x_Sym
bol] :> Dist[((1 + E^(2*(d + e*x)))^n*Sech[d + e*x]^n)/E^(n*(d + e*x)), Int
[SimplifyIntegrand[(F^(c*(a + b*x))*E^(n*(d + e*x)))/(1 + E^(2*(d + e*x)))^
n, x], x], x] /; FreeQ[{F, a, b, c, d, e}, x] &&  !IntegerQ[n]
```

Rubi steps

$$\int F^{c(a+bx)}\text{sech}^n(d+ex)\,dx = \left(e^{-n(d+ex)}\left(1+e^{2(d+ex)}\right)^n\text{sech}^n(d+ex)\right)\int e^{dn+enx}\left(1+e^{2(d+ex)}\right)^{-n}F^{ac+bcx}\,dx$$
$$= \frac{\left(1+e^{2(d+ex)}\right)^n F^{ac+bcx}\,{}_2F_1\left(n,\frac{en+bc\log(F)}{2e};1+\frac{en+bc\log(F)}{2e};-e^{2(d+ex)}\right)\text{sech}^n(d+ex)}{en+bc\log(F)}$$

Mathematica [A] time = 0.08, size = 89, normalized size = 0.99

$$\frac{\left(e^{2(d+ex)}+1\right)^n F^{c(a+bx)}\text{sech}^n(d+ex)\,{}_2F_1\left(n,\frac{en+bc\log(F)}{2e};\frac{en+bc\log(F)}{2e}+1;-e^{2(d+ex)}\right)}{bc\log(F)+en}$$

Antiderivative was successfully verified.

```
[In] Integrate[F^(c*(a + b*x))*Sech[d + e*x]^n,x]
```

```
[Out] ((1 + E^(2*(d + e*x)))^n*F^(c*(a + b*x))*Hypergeometric2F1[n, (e*n + b*c*Lo
g[F])/(2*e), 1 + (e*n + b*c*Log[F])/(2*e), -E^(2*(d + e*x))]*Sech[d + e*x]^
n)/(e*n + b*c*Log[F])
```

fricas [F] time = 0.55, size = 0, normalized size = 0.00

$$\text{integral}\left(F^{bcx+ac}\,\text{sech}\,(ex+d)^n\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*sech(e*x+d)^n,x, algorithm="fricas")
```

```
[Out] integral(F^(b*c*x + a*c)*sech(e*x + d)^n, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int F^{(bx+a)c}\,\text{sech}\,(ex+d)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*sech(e*x+d)^n,x, algorithm="giac")
```

```
[Out] integrate(F^((b*x + a)*c)*sech(e*x + d)^n, x)
```

maple [F] time = 0.33, size = 0, normalized size = 0.00

$$\int F^{c(bx+a)}\text{sech}\,(ex+d)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(F^(c*(b*x+a))*sech(e*x+d)^n,x)
```

```
[Out] int(F^(c*(b*x+a))*sech(e*x+d)^n,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int F^{(bx+a)c}\operatorname{sech}(ex+d)^n\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*sech(e*x+d)^n,x, algorithm="maxima")
```

```
[Out] integrate(F^((b*x + a)*c)*sech(e*x + d)^n, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int F^{c\,(a+b\,x)}\,{\left(\frac{1}{\cosh\left(d+e\,x\right)}\right)}^n \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(F^(c*(a + b*x))*(1/cosh(d + e*x))^n,x)
```

```
[Out] int(F^(c*(a + b*x))*(1/cosh(d + e*x))^n, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int F^{c(a+bx)}\operatorname{sech}^{n}(d+ex)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F**(c*(b*x+a))*sech(e*x+d)**n,x)
```

```
[Out] Integral(F**(c*(a + b*x))*sech(d + e*x)**n, x)
```

3.892 $\int F^{c(a+bx)}\operatorname{csch}^n(d+ex)\,dx$

Optimal. Leaf size=91

$$-\frac{\left(1-e^{-2(d+ex)}\right)^n F^{ac+bcx}\operatorname{csch}^n(d+ex)\,{}_2F_1\left(n,\frac{en-bc\log(F)}{2e};\frac{1}{2}\left(n-\frac{bc\log(F)}{e}+2\right);e^{-2(d+ex)}\right)}{en-bc\log(F)}$$

```
[Out] -(1-1/exp(2*e*x+2*d))^n*F^(b*c*x+a*c)*csch(e*x+d)^n*hypergeom([n, 1/2*(e*n-
b*c*ln(F))/e],[1+1/2*n-1/2*b*c*ln(F)/e],exp(-2*e*x-2*d))/(e*n-b*c*ln(F))
```

Rubi [A] time = 0.17, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {5495, 2259}

$$-\frac{\left(1-e^{-2(d+ex)}\right)^n F^{ac+bcx}\operatorname{csch}^n(d+ex)\,{}_2F_1\left(n,\frac{en-bc\log(F)}{2e};\frac{1}{2}\left(n-\frac{bc\log(F)}{e}+2\right);e^{-2(d+ex)}\right)}{en-bc\log(F)}$$

Antiderivative was successfully verified.

```
[In] Int[F^(c*(a + b*x))*Csch[d + e*x]^n,x]
```

```
[Out] -(((1 - E^(-2*(d + e*x)))^n*F^(a*c + b*c*x)*Csch[d + e*x]^n*Hypergeometric2
F1[n, (e*n - b*c*Log[F])/(2*e), (2 + n - (b*c*Log[F])/e)/2, E^(-2*(d + e*x)
)])/(e*n - b*c*Log[F]))
```

Rule 2259

```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_)*(G_)^((h_.)*((f_.
) + (g_.)*(x_)))*(H_)^((t_.)*((r_.) + (s_.)*(x_))), x_Symbol] :> Simp[(G^(h
*(f + g*x))*H^(t*(r + s*x))*(a + b*F^(e*(c + d*x)))^p*Hypergeometric2F1[-p,
 (g*h*Log[G] + s*t*Log[H])/(d*e*Log[F]), (g*h*Log[G] + s*t*Log[H])/(d*e*Log
[F]) + 1, Simplify[-((b*F^(e*(c + d*x)))/a)]])/((g*h*Log[G] + s*t*Log[H])*(
(a + b*F^(e*(c + d*x)))/a)^p), x] /; FreeQ[{F, G, H, a, b, c, d, e, f, g, h
, r, s, t, p}, x] &&  !IntegerQ[p]
```

Rule 5495

```
Int[Csch[(d_.) + (e_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Sym
bol] :> Dist[((1 - E^(-2*(d + e*x)))^n*Csch[d + e*x]^n)/E^(-(n*(d + e*x))),
 Int[SimplifyIntegrand[F^(c*(a + b*x))/(E^(n*(d + e*x))*(1 - E^(-2*(d + e*x
)))^n), x], x], x] /; FreeQ[{F, a, b, c, d, e}, x] &&  !IntegerQ[n]
```

Rubi steps

$$\int F^{c(a+bx)}\text{csch}^n(d+ex)\,dx = \left(e^{n(d+ex)}\left(1-e^{-2(d+ex)}\right)^n \text{csch}^n(d+ex)\right)\int e^{-dn-enx}\left(1-e^{-2(d+ex)}\right)^{-n} F^{ac+bcx}\,dx$$
$$= -\frac{\left(1-e^{-2(d+ex)}\right)^n F^{ac+bcx}\text{csch}^n(d+ex)\,{}_2F_1\left(n,\frac{en-bc\log(F)}{2e};\frac{1}{2}\left(2+n-\frac{bc\log(F)}{e}\right);e^{-2(d}\right.}{en-bc\log(F)}$$

Mathematica [A] time = 0.10, size = 90, normalized size = 0.99

$$-\frac{\left(1-e^{-2(d+ex)}\right)^n F^{c(a+bx)}\text{csch}^n(d+ex)\,{}_2F_1\left(n,\frac{en-bc\log(F)}{2e};\frac{1}{2}\left(n-\frac{bc\log(F)}{e}+2\right);e^{-2(d+ex)}\right)}{en-bc\log(F)}$$

Antiderivative was successfully verified.

```
[In] Integrate[F^(c*(a + b*x))*Csch[d + e*x]^n,x]
```

```
[Out] -((((1 - E^(-2*(d + e*x)))^n*F^(c*(a + b*x))*Csch[d + e*x]^n*Hypergeometric2
F1[n, (e*n - b*c*Log[F])/(2*e), (2 + n - (b*c*Log[F])/e)/2, E^(-2*(d + e*x)
)])/(e*n - b*c*Log[F]))
```

fricas [F] time = 0.53, size = 0, normalized size = 0.00

$$\text{integral}\left(F^{bcx+ac}\,\text{csch}\,(ex+d)^n\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*csch(e*x+d)^n,x, algorithm="fricas")
```

```
[Out] integral(F^(b*c*x + a*c)*csch(e*x + d)^n, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int F^{(bx+a)c}\,\text{csch}\,(ex+d)^n\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*csch(e*x+d)^n,x, algorithm="giac")
```

```
[Out] integrate(F^((b*x + a)*c)*csch(e*x + d)^n, x)
```

maple [F] time = 0.24, size = 0, normalized size = 0.00

$$\int F^{c(bx+a)}\text{csch}\,(ex+d)^n\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(F^(c*(b*x+a))*csch(e*x+d)^n,x)
```

```
[Out] int(F^(c*(b*x+a))*csch(e*x+d)^n,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int F^{(bx+a)c}\operatorname{csch}(ex+d)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*csch(e*x+d)^n,x, algorithm="maxima")
```

```
[Out] integrate(F^((b*x + a)*c)*csch(e*x + d)^n, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int F^{c\,(a+b\,x)}\left(\frac{1}{\sinh\left(d+e\,x\right)}\right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(F^(c*(a + b*x))*(1/sinh(d + e*x))^n,x)
```

```
[Out] int(F^(c*(a + b*x))*(1/sinh(d + e*x))^n, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int F^{c(a+bx)}\operatorname{csch}^n(d+ex)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F**(c*(b*x+a))*csch(e*x+d)**n,x)
```

```
[Out] Integral(F**(c*(a + b*x))*csch(d + e*x)**n, x)
```

3.893 $\int F^{c(a+bx)}(f+if\sinh(d+ex))^2\,dx$

Optimal. Leaf size=254

$$\frac{bcf^2\log(F)\sinh^2(d+ex)F^{ac+bcx}}{4e^2-b^2c^2\log^2(F)}-\frac{2ibcf^2\log(F)\sinh(d+ex)F^{ac+bcx}}{e^2-b^2c^2\log^2(F)}+\frac{2ief^2\cosh(d+ex)F^{ac+bcx}}{e^2-b^2c^2\log^2(F)}-\frac{2ef^2\sinh(d+}{4e^2}$$

```
[Out] f^2*F^(b*c*x+a*c)/b/c/ln(F)+2*I*e*f^2*F^(b*c*x+a*c)*cosh(e*x+d)/(e^2-b^2*c^
2*ln(F)^2)+2*e^2*f^2*F^(b*c*x+a*c)/b/c/ln(F)/(4*e^2-b^2*c^2*ln(F)^2)-2*I*b*
c*f^2*F^(b*c*x+a*c)*ln(F)*sinh(e*x+d)/(e^2-b^2*c^2*ln(F)^2)-2*e*f^2*F^(b*c*
x+a*c)*cosh(e*x+d)*sinh(e*x+d)/(4*e^2-b^2*c^2*ln(F)^2)+b*c*f^2*F^(b*c*x+a*c
)*ln(F)*sinh(e*x+d)^2/(4*e^2-b^2*c^2*ln(F)^2)
```

Rubi [A] time = 0.41, antiderivative size = 254, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {6741, 12, 6742, 2194, 5474, 5476}

$$\frac{bcf^2\log(F)\sinh^2(d+ex)F^{ac+bcx}}{4e^2-b^2c^2\log^2(F)}-\frac{2ibcf^2\log(F)\sinh(d+ex)F^{ac+bcx}}{e^2-b^2c^2\log^2(F)}+\frac{2ief^2\cosh(d+ex)F^{ac+bcx}}{e^2-b^2c^2\log^2(F)}-\frac{2ef^2\sinh(d+}{4e^2}$$

Antiderivative was successfully verified.

```
[In] Int[F^(c*(a + b*x))*(f + I*f*Sinh[d + e*x])^2,x]
```

```
[Out] (f^2*F^(a*c + b*c*x))/(b*c*Log[F]) + ((2*I)*e*f^2*F^(a*c + b*c*x)*Cosh[d +
e*x])/(e^2 - b^2*c^2*Log[F]^2) + (2*e^2*f^2*F^(a*c + b*c*x))/(b*c*Log[F]*(4
*e^2 - b^2*c^2*Log[F]^2)) - ((2*I)*b*c*f^2*F^(a*c + b*c*x)*Log[F]*Sinh[d +
e*x])/(e^2 - b^2*c^2*Log[F]^2) - (2*e*f^2*F^(a*c + b*c*x)*Cosh[d + e*x]*Sin
h[d + e*x])/(4*e^2 - b^2*c^2*Log[F]^2) + (b*c*f^2*F^(a*c + b*c*x)*Log[F]*Si
nh[d + e*x]^2)/(4*e^2 - b^2*c^2*Log[F]^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 5474

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sinh[(d_.) + (e_.)*(x_)], x_Symbol] :
> -Simp[(b*c*Log[F]*F^(c*(a + b*x))*Sinh[d + e*x])/(e^2 - b^2*c^2*Log[F]^2)
```

```
, x] + Simp[(e*F^(c*(a + b*x))*Cosh[d + e*x])/(e^2 - b^2*c^2*Log[F]^2), x]
/; FreeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 - b^2*c^2*Log[F]^2, 0]
```

Rule 5476

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sinh[(d_.) + (e_.)*(x_)]^(n_), x_Symb
ol] :> -Simp[(b*c*Log[F]*F^(c*(a + b*x))*Sinh[d + e*x]^n)/(e^2*n^2 - b^2*c^
2*Log[F]^2), x] + (-Dist[(n*(n - 1)*e^2)/(e^2*n^2 - b^2*c^2*Log[F]^2), Int[
F^(c*(a + b*x))*Sinh[d + e*x]^(n - 2), x], x] + Simp[(e*n*F^(c*(a + b*x))*C
osh[d + e*x]*Sinh[d + e*x]^(n - 1))/(e^2*n^2 - b^2*c^2*Log[F]^2), x]) /; Fr
eeQ[{F, a, b, c, d, e}, x] && NeQ[e^2*n^2 - b^2*c^2*Log[F]^2, 0] && GtQ[n,
1]
```

Rule 6741

```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}
\int F^{c(a+bx)}(f+if\sinh(d+ex))^2\,dx &= \int f^2F^{ac+bcx}(1+i\sinh(d+ex))^2\,dx \\
&= f^2\int F^{ac+bcx}(1+i\sinh(d+ex))^2\,dx \\
&= f^2\int \left(F^{ac+bcx}+2iF^{ac+bcx}\sinh(d+ex)-F^{ac+bcx}\sinh^2(d+ex)\right)\,dx \\
&= \left(2if^2\right)\int F^{ac+bcx}\sinh(d+ex)\,dx+f^2\int F^{ac+bcx}\,dx-f^2\int F^{ac+bcx}\sinh^2(d \\
&= \frac{f^2F^{ac+bcx}}{bc\log(F)}+\frac{2ief^2F^{ac+bcx}\cosh(d+ex)}{e^2-b^2c^2\log^2(F)}-\frac{2ibcf^2F^{ac+bcx}\log(F)\sinh(d+ex)}{e^2-b^2c^2\log^2(F)} \\
&= \frac{f^2F^{ac+bcx}}{bc\log(F)}+\frac{2ief^2F^{ac+bcx}\cosh(d+ex)}{e^2-b^2c^2\log^2(F)}+\frac{2e^2f^2F^{ac+bcx}}{bc\log(F)\left(4e^2-b^2c^2\log^2(F)\right)}-
\end{aligned}$$

Mathematica [A] time = 1.92, size = 196, normalized size = 0.77

$$\frac{F^{c(a+bx)}(f+if\sinh(d+ex))^2\left(-\frac{2e\sinh(2(d+ex))}{4e^2-b^2c^2\log^2(F)}-\frac{bc\log(F)\cosh(2(d+ex))}{b^2c^2\log^2(F)-4e^2}+\frac{4ibc\log(F)\sinh(d+ex)}{(bc\log(F)-e)(bc\log(F)+e)}+\frac{4ie\cosh(d+ex)}{(e-bc\log(F))(bc\log(F)+e)}\right.}{2\left(\cosh\left(\frac{1}{2}(d+ex)\right)+i\sinh\left(\frac{1}{2}(d+ex)\right)\right)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[F^(c*(a + b*x))*(f + I*f*Sinh[d + e*x])^2,x]
```

```
[Out] (F^(c*(a + b*x))*(f + I*f*Sinh[d + e*x])^2*(3/(b*c*Log[F]) + ((4*I)*e*Cosh[
d + e*x])/((e - b*c*Log[F])*(e + b*c*Log[F])) - (b*c*Cosh[2*(d + e*x)]*Log[
F])/(-4*e^2 + b^2*c^2*Log[F]^2) + ((4*I)*b*c*Log[F]*Sinh[d + e*x])/((-e + b
*c*Log[F])*(e + b*c*Log[F])) - (2*e*Sinh[2*(d + e*x)])/(4*e^2 - b^2*c^2*Log
[F]^2)))/(2*(Cosh[(d + e*x)/2] + I*Sinh[(d + e*x)/2])^4)
```

fricas [A] time = 0.54, size = 442, normalized size = 1.74

$$\frac{\left(24\,e^4f^2e^{(2\,ex+2\,d)}-\left(b^4c^4f^2e^{(4\,ex+4\,d)}-4i\,b^4c^4f^2e^{(3\,ex+3\,d)}-6\,b^4c^4f^2e^{(2\,ex+2\,d)}+4i\,b^4c^4f^2e^{(ex+d)}+b^4c^4f^2\right)\log(F)^4}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*(f+I*f*sinh(e*x+d))^2,x, algorithm="fricas")
```

```
[Out] 1/4*(24*e^4*f^2*e^(2*e*x + 2*d) - (b^4*c^4*f^2*e^(4*e*x + 4*d) - 4*I*b^4*c^
4*f^2*e^(3*e*x + 3*d) - 6*b^4*c^4*f^2*e^(2*e*x + 2*d) + 4*I*b^4*c^4*f^2*e^(
e*x + d) + b^4*c^4*f^2)*log(F)^4 + (2*b^3*c^3*e*f^2*e^(4*e*x + 4*d) - 4*I*b
^3*c^3*e*f^2*e^(3*e*x + 3*d) - 4*I*b^3*c^3*e*f^2*e^(e*x + d) - 2*b^3*c^3*e*
f^2)*log(F)^3 + (b^2*c^2*e^2*f^2*e^(4*e*x + 4*d) - 16*I*b^2*c^2*e^2*f^2*e^(
3*e*x + 3*d) - 30*b^2*c^2*e^2*f^2*e^(2*e*x + 2*d) + 16*I*b^2*c^2*e^2*f^2*e^
(e*x + d) + b^2*c^2*e^2*f^2)*log(F)^2 - (2*b*c*e^3*f^2*e^(4*e*x + 4*d) - 16
*I*b*c*e^3*f^2*e^(3*e*x + 3*d) - 16*I*b*c*e^3*f^2*e^(e*x + d) - 2*b*c*e^3*f
^2)*log(F))*F^(b*c*x + a*c)/(b^5*c^5*e^(2*e*x + 2*d)*log(F)^5 - 5*b^3*c^3*e
^2*e^(2*e*x + 2*d)*log(F)^3 + 4*b*c*e^4*e^(2*e*x + 2*d)*log(F))
```

giac [B] time = 0.30, size = 1574, normalized size = 6.20

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*(f+I*f*sinh(e*x+d))^2,x, algorithm="giac")
```

```
[Out] 3*(2*b*c*f^2*cos(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*b*c*x - 1/2*pi*a*c*sgn(F) +
1/2*pi*a*c)*log(abs(F))/(4*b^2*c^2*log(abs(F))^2 + (pi*b*c*sgn(F) - pi*b*c)
```

```
^2) - (pi*b*c*sgn(F) - pi*b*c)*f^2*sin(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*b*c*x
- 1/2*pi*a*c*sgn(F) + 1/2*pi*a*c)/(4*b^2*c^2*log(abs(F))^2 + (pi*b*c*sgn(F)
 - pi*b*c)^2))*e^(b*c*x*log(abs(F)) + a*c*log(abs(F))) - 1/2*I*(-6*I*f^2*e^
(1/2*I*pi*b*c*x*sgn(F) - 1/2*I*pi*b*c*x + 1/2*I*pi*a*c*sgn(F) - 1/2*I*pi*a*
c)/(2*I*pi*b*c*sgn(F) - 2*I*pi*b*c + 4*b*c*log(abs(F))) + 6*I*f^2*e^(-1/2*I
*pi*b*c*x*sgn(F) + 1/2*I*pi*b*c*x - 1/2*I*pi*a*c*sgn(F) + 1/2*I*pi*a*c)/(-2
*I*pi*b*c*sgn(F) + 2*I*pi*b*c + 4*b*c*log(abs(F))))*e^(b*c*x*log(abs(F)) +
a*c*log(abs(F))) - 1/2*(2*(b*c*log(abs(F)) + 2*e)*f^2*cos(-1/2*pi*b*c*x*sgn
(F) + 1/2*pi*b*c*x - 1/2*pi*a*c*sgn(F) + 1/2*pi*a*c)/((pi*b*c*sgn(F) - pi*b
*c)^2 + 4*(b*c*log(abs(F)) + 2*e)^2) - (pi*b*c*sgn(F) - pi*b*c)*f^2*sin(-1/
2*pi*b*c*x*sgn(F) + 1/2*pi*b*c*x - 1/2*pi*a*c*sgn(F) + 1/2*pi*a*c)/((pi*b*c
*sgn(F) - pi*b*c)^2 + 4*(b*c*log(abs(F)) + 2*e)^2))*e^(a*c*log(abs(F)) + (b
*c*log(abs(F)) + 2*e)*x + 2*d) - 1/2*I*(2*I*f^2*e^(1/2*I*pi*b*c*x*sgn(F) -
1/2*I*pi*b*c*x + 1/2*I*pi*a*c*sgn(F) - 1/2*I*pi*a*c)/(4*I*pi*b*c*sgn(F) - 4
*I*pi*b*c + 8*b*c*log(abs(F)) + 16*e) - 2*I*f^2*e^(-1/2*I*pi*b*c*x*sgn(F) +
 1/2*I*pi*b*c*x - 1/2*I*pi*a*c*sgn(F) + 1/2*I*pi*a*c)/(-4*I*pi*b*c*sgn(F) +
 4*I*pi*b*c + 8*b*c*log(abs(F)) + 16*e))*e^(a*c*log(abs(F)) + (b*c*log(abs(
F)) + 2*e)*x + 2*d) - 2*((pi*b*c*sgn(F) - pi*b*c)*f^2*cos(-1/2*pi*b*c*x*sgn
(F) + 1/2*pi*b*c*x - 1/2*pi*a*c*sgn(F) + 1/2*pi*a*c)/((pi*b*c*sgn(F) - pi*b
*c)^2 + 4*(b*c*log(abs(F)) + e)^2) + 2*(b*c*log(abs(F)) + e)*f^2*sin(-1/2*p
i*b*c*x*sgn(F) + 1/2*pi*b*c*x - 1/2*pi*a*c*sgn(F) + 1/2*pi*a*c)/((pi*b*c*sg
n(F) - pi*b*c)^2 + 4*(b*c*log(abs(F)) + e)^2))*e^(a*c*log(abs(F)) + (b*c*lo
g(abs(F)) + e)*x + d) + 1/2*(2*I*f^2*e^(1/2*I*pi*b*c*x*sgn(F) - 1/2*I*pi*b*
c*x + 1/2*I*pi*a*c*sgn(F) - 1/2*I*pi*a*c)/(I*pi*b*c*sgn(F) - I*pi*b*c + 2*b
*c*log(abs(F)) + 2*e) + 2*I*f^2*e^(-1/2*I*pi*b*c*x*sgn(F) + 1/2*I*pi*b*c*x
- 1/2*I*pi*a*c*sgn(F) + 1/2*I*pi*a*c)/(-I*pi*b*c*sgn(F) + I*pi*b*c + 2*b*c*
log(abs(F)) + 2*e))*e^(a*c*log(abs(F)) + (b*c*log(abs(F)) + e)*x + d) + 2*(
(pi*b*c*sgn(F) - pi*b*c)*f^2*cos(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*b*c*x - 1/2*
pi*a*c*sgn(F) + 1/2*pi*a*c)/((pi*b*c*sgn(F) - pi*b*c)^2 + 4*(b*c*log(abs(F)
) - e)^2) + 2*(b*c*log(abs(F)) - e)*f^2*sin(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*b
*c*x - 1/2*pi*a*c*sgn(F) + 1/2*pi*a*c)/((pi*b*c*sgn(F) - pi*b*c)^2 + 4*(b*c
*log(abs(F)) - e)^2))*e^(a*c*log(abs(F)) + (b*c*log(abs(F)) - e)*x - d) + 1
/2*(-2*I*f^2*e^(1/2*I*pi*b*c*x*sgn(F) - 1/2*I*pi*b*c*x + 1/2*I*pi*a*c*sgn(F
) - 1/2*I*pi*a*c)/(I*pi*b*c*sgn(F) - I*pi*b*c + 2*b*c*log(abs(F)) - 2*e) -
2*I*f^2*e^(-1/2*I*pi*b*c*x*sgn(F) + 1/2*I*pi*b*c*x - 1/2*I*pi*a*c*sgn(F) +
1/2*I*pi*a*c)/(-I*pi*b*c*sgn(F) + I*pi*b*c + 2*b*c*log(abs(F)) - 2*e))*e^(a
*c*log(abs(F)) + (b*c*log(abs(F)) - e)*x - d) - 1/2*(2*(b*c*log(abs(F)) - 2
*e)*f^2*cos(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*b*c*x - 1/2*pi*a*c*sgn(F) + 1/2*p
i*a*c)/((pi*b*c*sgn(F) - pi*b*c)^2 + 4*(b*c*log(abs(F)) - 2*e)^2) - (pi*b*c
*sgn(F) - pi*b*c)*f^2*sin(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*b*c*x - 1/2*pi*a*c*
sgn(F) + 1/2*pi*a*c)/((pi*b*c*sgn(F) - pi*b*c)^2 + 4*(b*c*log(abs(F)) - 2*e
)^2))*e^(a*c*log(abs(F)) + (b*c*log(abs(F)) - 2*e)*x - 2*d) - 1/2*I*(2*I*f^
2*e^(1/2*I*pi*b*c*x*sgn(F) - 1/2*I*pi*b*c*x + 1/2*I*pi*a*c*sgn(F) - 1/2*I*p
i*a*c)/(4*I*pi*b*c*sgn(F) - 4*I*pi*b*c + 8*b*c*log(abs(F)) - 16*e) - 2*I*f^
2*e^(-1/2*I*pi*b*c*x*sgn(F) + 1/2*I*pi*b*c*x - 1/2*I*pi*a*c*sgn(F) + 1/2*I*
```

```
pi*a*c)/(-4*I*pi*b*c*sgn(F) + 4*I*pi*b*c + 8*b*c*log(abs(F)) - 16*e))*e^(a*
c*log(abs(F)) + (b*c*log(abs(F)) - 2*e)*x - 2*d)
```

maple [A] time = 0.65, size = 434, normalized size = 1.71

$$\frac{f^2\left(-4i\ln(F)^3b^3c^3e\,\mathrm{e}^{3ex+3d}-\ln(F)^4b^4c^4\mathrm{e}^{4ex+4d}+16i\ln(F)bc\,e^3\mathrm{e}^{3ex+3d}+6\ln(F)^4b^4c^4\mathrm{e}^{2ex+2d}+16i\ln(F)bc\,e^3\mathrm{e}^{ex+}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(F^(c*(b*x+a))*(f+I*f*sinh(e*x+d))^2,x)
```

```
[Out] 1/4*f^2*(-4*I*ln(F)^3*b^3*c^3*e*exp(3*e*x+3*d)-ln(F)^4*b^4*c^4*exp(4*e*x+4*
d)+16*I*ln(F)*b*c*e^3*exp(3*e*x+3*d)+6*ln(F)^4*b^4*c^4*exp(2*e*x+2*d)+16*I*
ln(F)*b*c*e^3*exp(e*x+d)+2*ln(F)^3*b^3*c^3*e*exp(4*e*x+4*d)-b^4*c^4*ln(F)^4
+16*I*ln(F)^2*b^2*c^2*e^2*exp(e*x+d)-4*I*ln(F)^3*b^3*c^3*e*exp(e*x+d)+ln(F)
^2*b^2*c^2*e^2*exp(4*e*x+4*d)-2*ln(F)^3*b^3*c^3*e+4*I*ln(F)^4*b^4*c^4*exp(3
*e*x+3*d)-30*ln(F)^2*b^2*c^2*e^2*exp(2*e*x+2*d)-16*I*ln(F)^2*b^2*c^2*e^2*ex
p(3*e*x+3*d)-2*ln(F)*b*c*e^3*exp(4*e*x+4*d)+b^2*c^2*e^2*ln(F)^2-4*I*ln(F)^4
*b^4*c^4*exp(e*x+d)+2*ln(F)*b*c*e^3+24*e^4*exp(2*e*x+2*d))/b/c/ln(F)/(e-b*c
*ln(F))*exp(-2*e*x-2*d)/(2*e-b*c*ln(F))/(e+b*c*ln(F))/(b*c*ln(F)+2*e)*F^(c*
(b*x+a))
```

maxima [A] time = 0.35, size = 189, normalized size = 0.74

$$-\frac{1}{4}f^2\left(\frac{F^{ac}e^{\left(bcx\log(F)+2\,ex+2\,d\right)}}{bc\log(F)+2\,e}+\frac{F^{ac}e^{\left(bcx\log(F)-2\,ex\right)}}{bce^{(2\,d)}\log(F)-2\,ee^{(2\,d)}}-\frac{2\,F^{bcx+ac}}{bc\log(F)}\right)+i\,f^2\left(\frac{F^{ac}e^{\left(bcx\log(F)+ex+d\right)}}{bc\log(F)+e}-\frac{F^{ac}e^{\left(bcx\log(F)-ex\right.}}{bce^{d}\log(F)-ee}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*(f+I*f*sinh(e*x+d))^2,x, algorithm="maxima")
```

```
[Out] -1/4*f^2*(F^(a*c)*e^(b*c*x*log(F) + 2*e*x + 2*d)/(b*c*log(F) + 2*e) + F^(a*
c)*e^(b*c*x*log(F) - 2*e*x)/(b*c*e^(2*d)*log(F) - 2*e*e^(2*d)) - 2*F^(b*c*x
 + a*c)/(b*c*log(F))) + I*f^2*(F^(a*c)*e^(b*c*x*log(F) + e*x + d)/(b*c*log(
F) + e) - F^(a*c)*e^(b*c*x*log(F) - e*x)/(b*c*e^d*log(F) - e*e^d)) + F^(b*c
*x + a*c)*f^2/(b*c*log(F))
```

mupad [B] time = 2.78, size = 252, normalized size = 0.99

$$\frac{F^{c\,(a+b\,x)}\,f^2\,\left(12\,e^4+3\,b^4\,c^4\,\ln(F)^4+b^4\,c^4\,\mathrm{sinh}\,(d+e\,x)\,\ln(F)^4\,4\mathrm{i}-b^4\,c^4\,\ln(F)^4\,\mathrm{cosh}\,(2\,d+2\,e\,x)-15\,b^2\,c^2\,e^2\,\ln(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(F^(c*(a + b*x))*(f + f*sinh(d + e*x)*1i)^2,x)
```

```
[Out] (F^(c*(a + b*x))*f^2*(12*e^4 + 3*b^4*c^4*log(F)^4 + b^4*c^4*sinh(d + e*x)*l
og(F)^4*4i - b^4*c^4*log(F)^4*cosh(2*d + 2*e*x) - 15*b^2*c^2*e^2*log(F)^2 +
 2*b^3*c^3*e*log(F)^3*sinh(2*d + 2*e*x) - b^2*c^2*e^2*sinh(d + e*x)*log(F)^
2*16i - 2*b*c*e^3*log(F)*sinh(2*d + 2*e*x) + b^2*c^2*e^2*log(F)^2*cosh(2*d
+ 2*e*x) - b^3*c^3*e*cosh(d + e*x)*log(F)^3*4i + b*c*e^3*cosh(d + e*x)*log(
F)*16i))/(2*b*c*log(F)*(4*e^4 + b^4*c^4*log(F)^4 - 5*b^2*c^2*e^2*log(F)^2))
```

sympy [A] time = 119.10, size = 1731, normalized size = 6.81

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F**(c*(b*x+a))*(f+I*f*sinh(e*x+d))**2,x)
```

```
[Out] Piecewise((-f**2*x*sinh(d + e*x)**2/2 + f**2*x*cosh(d + e*x)**2/2 + f**2*x
- f**2*sinh(d + e*x)*cosh(d + e*x)/(2*e) + 2*I*f**2*cosh(d + e*x)/e, Eq(F,
1)), (zoo*e**4*f**2*exp(-2*e/(b*c))**(a*c)*exp(-2*e/(b*c))**(b*c*x)*sinh(d
+ e*x)**2 + zoo*e**4*f**2*exp(-2*e/(b*c))**(a*c)*exp(-2*e/(b*c))**(b*c*x)*s
inh(d + e*x)*cosh(d + e*x) + zoo*e**4*f**2*exp(-2*e/(b*c))**(a*c)*exp(-2*e/
(b*c))**(b*c*x)*cosh(d + e*x)**2, Eq(F, exp(-2*e/(b*c)))), (zoo*e**4*f**2*e
xp(-e/(b*c))**(a*c)*exp(-e/(b*c))**(b*c*x)*sinh(d + e*x) + zoo*e**4*f**2*ex
p(-e/(b*c))**(a*c)*exp(-e/(b*c))**(b*c*x)*cosh(d + e*x), Eq(F, exp(-e/(b*c)
))), (zoo*e**4*f**2*exp(e/(b*c))**(a*c)*exp(e/(b*c))**(b*c*x)*sinh(d + e*x)
 + zoo*e**4*f**2*exp(e/(b*c))**(a*c)*exp(e/(b*c))**(b*c*x)*cosh(d + e*x), E
q(F, exp(e/(b*c)))), (zoo*e**4*f**2*exp(2*e/(b*c))**(a*c)*exp(2*e/(b*c))**(
b*c*x)*sinh(d + e*x)**2 + zoo*e**4*f**2*exp(2*e/(b*c))**(a*c)*exp(2*e/(b*c)
)**(b*c*x)*sinh(d + e*x)*cosh(d + e*x) + zoo*e**4*f**2*exp(2*e/(b*c))**(a*c
)*exp(2*e/(b*c))**(b*c*x)*cosh(d + e*x)**2, Eq(F, exp(2*e/(b*c)))), (F**(a*
c)*(-f**2*x*sinh(d + e*x)**2/2 + f**2*x*cosh(d + e*x)**2/2 + f**2*x - f**2*
sinh(d + e*x)*cosh(d + e*x)/(2*e) + 2*I*f**2*cosh(d + e*x)/e), Eq(b, 0)), (
-f**2*x*sinh(d + e*x)**2/2 + f**2*x*cosh(d + e*x)**2/2 + f**2*x - f**2*sinh
(d + e*x)*cosh(d + e*x)/(2*e) + 2*I*f**2*cosh(d + e*x)/e, Eq(c, 0)), (-F**(
a*c)*F**(b*c*x)*b**4*c**4*f**2*log(F)**4*sinh(d + e*x)**2/(b**5*c**5*log(F)
**5 - 5*b**3*c**3*e**2*log(F)**3 + 4*b*c*e**4*log(F)) + 2*I*F**(a*c)*F**(b*
c*x)*b**4*c**4*f**2*log(F)**4*sinh(d + e*x)/(b**5*c**5*log(F)**5 - 5*b**3*c
**3*e**2*log(F)**3 + 4*b*c*e**4*log(F)) + F**(a*c)*F**(b*c*x)*b**4*c**4*f**
2*log(F)**4/(b**5*c**5*log(F)**5 - 5*b**3*c**3*e**2*log(F)**3 + 4*b*c*e**4*
log(F)) + 2*F**(a*c)*F**(b*c*x)*b**3*c**3*e*f**2*log(F)**3*sinh(d + e*x)*co
sh(d + e*x)/(b**5*c**5*log(F)**5 - 5*b**3*c**3*e**2*log(F)**3 + 4*b*c*e**4*
log(F)) - 2*I*F**(a*c)*F**(b*c*x)*b**3*c**3*e*f**2*log(F)**3*cosh(d + e*x)/
(b**5*c**5*log(F)**5 - 5*b**3*c**3*e**2*log(F)**3 + 4*b*c*e**4*log(F)) + 3*
F**(a*c)*F**(b*c*x)*b**2*c**2*e**2*f**2*log(F)**2*sinh(d + e*x)**2/(b**5*c*
*5*log(F)**5 - 5*b**3*c**3*e**2*log(F)**3 + 4*b*c*e**4*log(F)) - 8*I*F**(a*
```

```
c)*F**(b*c*x)*b**2*c**2*e**2*f**2*log(F)**2*sinh(d + e*x)/(b**5*c**5*log(F)
**5 - 5*b**3*c**3*e**2*log(F)**3 + 4*b*c*e**4*log(F)) - 2*F**(a*c)*F**(b*c*
x)*b**2*c**2*e**2*f**2*log(F)**2*cosh(d + e*x)**2/(b**5*c**5*log(F)**5 - 5*
b**3*c**3*e**2*log(F)**3 + 4*b*c*e**4*log(F)) - 5*F**(a*c)*F**(b*c*x)*b**2*
c**2*e**2*f**2*log(F)**2/(b**5*c**5*log(F)**5 - 5*b**3*c**3*e**2*log(F)**3
+ 4*b*c*e**4*log(F)) - 2*F**(a*c)*F**(b*c*x)*b*c*e**3*f**2*log(F)*sinh(d +
e*x)*cosh(d + e*x)/(b**5*c**5*log(F)**5 - 5*b**3*c**3*e**2*log(F)**3 + 4*b*
c*e**4*log(F)) + 8*I*F**(a*c)*F**(b*c*x)*b*c*e**3*f**2*log(F)*cosh(d + e*x)
/(b**5*c**5*log(F)**5 - 5*b**3*c**3*e**2*log(F)**3 + 4*b*c*e**4*log(F)) - 2
*F**(a*c)*F**(b*c*x)*e**4*f**2*sinh(d + e*x)**2/(b**5*c**5*log(F)**5 - 5*b*
*3*c**3*e**2*log(F)**3 + 4*b*c*e**4*log(F)) + 2*F**(a*c)*F**(b*c*x)*e**4*f*
*2*cosh(d + e*x)**2/(b**5*c**5*log(F)**5 - 5*b**3*c**3*e**2*log(F)**3 + 4*b
*c*e**4*log(F)) + 4*F**(a*c)*F**(b*c*x)*e**4*f**2/(b**5*c**5*log(F)**5 - 5*
b**3*c**3*e**2*log(F)**3 + 4*b*c*e**4*log(F)), True))
```

3.894 $\int F^{c(a+bx)}(f+if\sinh(d+ex))\,dx$

Optimal. Leaf size=106

$$-\frac{ibcf\log(F)\sinh(d+ex)F^{ac+bcx}}{e^2-b^2c^2\log^2(F)}+\frac{ief\cosh(d+ex)F^{ac+bcx}}{e^2-b^2c^2\log^2(F)}+\frac{fF^{ac+bcx}}{bc\log(F)}$$

```
[Out] f*F^(b*c*x+a*c)/b/c/ln(F)+I*e*f*F^(b*c*x+a*c)*cosh(e*x+d)/(e^2-b^2*c^2*ln(F
)^2)-I*b*c*f*F^(b*c*x+a*c)*ln(F)*sinh(e*x+d)/(e^2-b^2*c^2*ln(F)^2)
```

Rubi [A] time = 0.18, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {6741, 12, 6742, 2194, 5474}

$$-\frac{ibcf\log(F)\sinh(d+ex)F^{ac+bcx}}{e^2-b^2c^2\log^2(F)}+\frac{ief\cosh(d+ex)F^{ac+bcx}}{e^2-b^2c^2\log^2(F)}+\frac{fF^{ac+bcx}}{bc\log(F)}$$

Antiderivative was successfully verified.

```
[In] Int[F^(c*(a + b*x))*(f + I*f*Sinh[d + e*x]),x]
```

```
[Out] (f*F^(a*c + b*c*x))/(b*c*Log[F]) + (I*e*f*F^(a*c + b*c*x)*Cosh[d + e*x])/(e
^2 - b^2*c^2*Log[F]^2) - (I*b*c*f*F^(a*c + b*c*x)*Log[F]*Sinh[d + e*x])/(e^
2 - b^2*c^2*Log[F]^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 5474

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sinh[(d_.) + (e_.)*(x_)], x_Symbol] :
> -Simp[(b*c*Log[F]*F^(c*(a + b*x))*Sinh[d + e*x])/(e^2 - b^2*c^2*Log[F]^2)
, x] + Simp[(e*F^(c*(a + b*x))*Cosh[d + e*x])/(e^2 - b^2*c^2*Log[F]^2), x]
/; FreeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 - b^2*c^2*Log[F]^2, 0]
```

Rule 6741

```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}\int F^{c(a+bx)}(f+if\sinh(d+ex))\,dx &= \int fF^{ac+bcx}(1+i\sinh(d+ex))\,dx\\ &= f\int F^{ac+bcx}(1+i\sinh(d+ex))\,dx\\ &= f\int \left(F^{ac+bcx}+iF^{ac+bcx}\sinh(d+ex)\right)\,dx\\ &= (if)\int F^{ac+bcx}\sinh(d+ex)\,dx+f\int F^{ac+bcx}\,dx\\ &= \frac{fF^{ac+bcx}}{bc\log(F)}+\frac{iefF^{ac+bcx}\cosh(d+ex)}{e^2-b^2c^2\log^2(F)}-\frac{ibcfF^{ac+bcx}\log(F)\sinh(d+ex)}{e^2-b^2c^2\log^2(F)}\end{aligned}$$

Mathematica [A] time = 0.66, size = 93, normalized size = 0.88

$$\frac{fF^{c(a+bx)}\left(ib^2c^2\log^2(F)\sinh(d+ex)+b^2c^2\log^2(F)-ibce\log(F)\cosh(d+ex)-e^2\right)}{bc\log(F)(bc\log(F)-e)(bc\log(F)+e)}$$

Antiderivative was successfully verified.

```
[In] Integrate[F^(c*(a + b*x))*(f + I*f*Sinh[d + e*x]),x]
```

```
[Out] (f*F^(c*(a + b*x))*(-e^2 - I*b*c*e*Cosh[d + e*x]*Log[F] + b^2*c^2*Log[F]^2
+ I*b^2*c^2*Log[F]^2*Sinh[d + e*x]))/(b*c*Log[F]*(-e + b*c*Log[F])*(e + b*c
*Log[F]))
```

fricas [A] time = 0.49, size = 135, normalized size = 1.27

$$-\frac{\left(2e^2fe^{(ex+d)}-\left(ib^2c^2fe^{(2ex+2d)}+2b^2c^2fe^{(ex+d)}-ib^2c^2f\right)\log(F)^2-\left(-ibcefe^{(2ex+2d)}-ibcef\right)\log(F)\right)F^{bcx+ac}}{2\left(b^3c^3e^{(ex+d)}\log(F)^3-bce^2e^{(ex+d)}\log(F)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*(f+I*f*sinh(e*x+d)),x, algorithm="fricas")
```

```
[Out] -1/2*(2*e^2*f*e^(e*x + d) - (I*b^2*c^2*f*e^(2*e*x + 2*d) + 2*b^2*c^2*f*e^(e
*x + d) - I*b^2*c^2*f)*log(F)^2 - (-I*b*c*e*f*e^(2*e*x + 2*d) - I*b*c*e*f)*
log(F))*F^(b*c*x + a*c)/(b^3*c^3*e^(e*x + d)*log(F)^3 - b*c*e^2*e^(e*x + d)
*log(F))
```

giac [B] time = 0.21, size = 899, normalized size = 8.48

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*(f+I*f*sinh(e*x+d)),x, algorithm="giac")
```

```
[Out] 2*(2*b*c*f*cos(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*b*c*x - 1/2*pi*a*c*sgn(F) + 1/
2*pi*a*c)*log(abs(F))/(4*b^2*c^2*log(abs(F))^2 + (pi*b*c*sgn(F) - pi*b*c)^2
) - (pi*b*c*sgn(F) - pi*b*c)*f*sin(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*b*c*x - 1/
2*pi*a*c*sgn(F) + 1/2*pi*a*c)/(4*b^2*c^2*log(abs(F))^2 + (pi*b*c*sgn(F) - p
i*b*c)^2))*e^(b*c*x*log(abs(F)) + a*c*log(abs(F))) - 1/2*I*(-2*I*f*e^(1/2*I
*pi*b*c*x*sgn(F) - 1/2*I*pi*b*c*x + 1/2*I*pi*a*c*sgn(F) - 1/2*I*pi*a*c)/(I*
pi*b*c*sgn(F) - I*pi*b*c + 2*b*c*log(abs(F))) + 2*I*f*e^(-1/2*I*pi*b*c*x*sg
n(F) + 1/2*I*pi*b*c*x - 1/2*I*pi*a*c*sgn(F) + 1/2*I*pi*a*c)/(-I*pi*b*c*sgn(
F) + I*pi*b*c + 2*b*c*log(abs(F))))*e^(b*c*x*log(abs(F)) + a*c*log(abs(F)))
 - ((pi*b*c*sgn(F) - pi*b*c)*f*cos(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*b*c*x - 1/
2*pi*a*c*sgn(F) + 1/2*pi*a*c)/((pi*b*c*sgn(F) - pi*b*c)^2 + 4*(b*c*log(abs(
F)) + e)^2) + 2*(b*c*log(abs(F)) + e)*f*sin(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*b
*c*x - 1/2*pi*a*c*sgn(F) + 1/2*pi*a*c)/((pi*b*c*sgn(F) - pi*b*c)^2 + 4*(b*c
*log(abs(F)) + e)^2))*e^(a*c*log(abs(F)) + (b*c*log(abs(F)) + e)*x + d) + 1
/2*(2*I*f*e^(1/2*I*pi*b*c*x*sgn(F) - 1/2*I*pi*b*c*x + 1/2*I*pi*a*c*sgn(F) -
 1/2*I*pi*a*c)/(2*I*pi*b*c*sgn(F) - 2*I*pi*b*c + 4*b*c*log(abs(F)) + 4*e) +
 2*I*f*e^(-1/2*I*pi*b*c*x*sgn(F) + 1/2*I*pi*b*c*x - 1/2*I*pi*a*c*sgn(F) + 1
/2*I*pi*a*c)/(-2*I*pi*b*c*sgn(F) + 2*I*pi*b*c + 4*b*c*log(abs(F)) + 4*e))*e
^(a*c*log(abs(F)) + (b*c*log(abs(F)) + e)*x + d) + ((pi*b*c*sgn(F) - pi*b*c
)*f*cos(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*b*c*x - 1/2*pi*a*c*sgn(F) + 1/2*pi*a*
c)/((pi*b*c*sgn(F) - pi*b*c)^2 + 4*(b*c*log(abs(F)) - e)^2) + 2*(b*c*log(ab
s(F)) - e)*f*sin(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*b*c*x - 1/2*pi*a*c*sgn(F) +
1/2*pi*a*c)/((pi*b*c*sgn(F) - pi*b*c)^2 + 4*(b*c*log(abs(F)) - e)^2))*e^(a*
c*log(abs(F)) + (b*c*log(abs(F)) - e)*x - d) + 1/2*(-2*I*f*e^(1/2*I*pi*b*c*
x*sgn(F) - 1/2*I*pi*b*c*x + 1/2*I*pi*a*c*sgn(F) - 1/2*I*pi*a*c)/(2*I*pi*b*c
*sgn(F) - 2*I*pi*b*c + 4*b*c*log(abs(F)) - 4*e) - 2*I*f*e^(-1/2*I*pi*b*c*x*
sgn(F) + 1/2*I*pi*b*c*x - 1/2*I*pi*a*c*sgn(F) + 1/2*I*pi*a*c)/(-2*I*pi*b*c*
sgn(F) + 2*I*pi*b*c + 4*b*c*log(abs(F)) - 4*e))*e^(a*c*log(abs(F)) + (b*c*l
og(abs(F)) - e)*x - d)
```

maple [A] time = 0.18, size = 141, normalized size = 1.33

$$\frac{f\left(-i\ln(F)^2b^2c^2\mathrm{e}^{2ex+2d}+i\ln(F)^2b^2c^2-2\ln(F)^2b^2c^2\mathrm{e}^{ex+d}+i\ln(F)bce\,\mathrm{e}^{2ex+2d}+i\ln(F)bce+2e^2\mathrm{e}^{ex+d}\right)\mathrm{e}^{-ex-d}F^{c(bx+}}{2bc\ln(F)\left(e-bc\ln(F)\right)\left(e+bc\ln(F)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(F^(c*(b*x+a))*(f+I*f*sinh(e*x+d)),x)
```

```
[Out] 1/2*f*(-I*ln(F)^2*b^2*c^2*exp(2*e*x+2*d)+I*ln(F)^2*b^2*c^2-2*ln(F)^2*b^2*c^
2*exp(e*x+d)+I*ln(F)*b*c*e*exp(2*e*x+2*d)+I*ln(F)*b*c*e+2*e^2*exp(e*x+d))/b
/c/ln(F)/(e-b*c*ln(F))*exp(-e*x-d)/(e+b*c*ln(F))*F^(c*(b*x+a))
```

maxima [A] time = 0.34, size = 88, normalized size = 0.83

$$\frac{1}{2}if\left(\frac{F^{ac}e^{\left(bcx\log(F)+ex+d\right)}}{bc\log(F)+e}-\frac{F^{ac}e^{\left(bcx\log(F)-ex\right)}}{bce^{d}\log(F)-ee^{d}}\right)+\frac{F^{bcx+ac}f}{bc\log(F)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*(f+I*f*sinh(e*x+d)),x, algorithm="maxima")
```

```
[Out] 1/2*I*f*(F^(a*c)*e^(b*c*x*log(F) + e*x + d)/(b*c*log(F) + e) - F^(a*c)*e^(b
*c*x*log(F) - e*x)/(b*c*e^d*log(F) - e*e^d)) + F^(b*c*x + a*c)*f/(b*c*log(F
))
```

mupad [B] time = 1.98, size = 88, normalized size = 0.83

$$\frac{F^{c\,(a+b\,x)}\,f\,\left(e^2-b^2\,c^2\,\ln(F)^2-b^2\,c^2\,\sinh\left(d+e\,x\right)\,\ln(F)^2\,1\mathrm{i}+b\,c\,e\cosh\left(d+e\,x\right)\,\ln(F)\,1\mathrm{i}\right)}{b\,c\,\ln(F)\,\left(e^2-b^2\,c^2\ln(F)^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(F^(c*(a + b*x))*(f + f*sinh(d + e*x)*1i),x)
```

```
[Out] (F^(c*(a + b*x))*f*(e^2 - b^2*c^2*log(F)^2 - b^2*c^2*sinh(d + e*x)*log(F)^2
*1i + b*c*e*cosh(d + e*x)*log(F)*1i))/(b*c*log(F)*(e^2 - b^2*c^2*log(F)^2))
```

sympy [A] time = 9.90, size = 400, normalized size = 3.77

$$\begin{cases} fx + \frac{if\cosh(d+ex)}{e} & \text{for } F = \\ \tilde{\infty}e^2 f\left(e^{-\frac{e}{bc}}\right)^{ac}\left(e^{-\frac{e}{bc}}\right)^{bcx}\sinh(d+ex) + \tilde{\infty}e^2 f\left(e^{-\frac{e}{bc}}\right)^{ac}\left(e^{-\frac{e}{bc}}\right)^{bcx}\cosh(d+ex) & \text{for } F = \\ \tilde{\infty}e^2 f\left(e^{\frac{e}{bc}}\right)^{ac}\left(e^{\frac{e}{bc}}\right)^{bcx}\sinh(d+ex) + \tilde{\infty}e^2 f\left(e^{\frac{e}{bc}}\right)^{ac}\left(e^{\frac{e}{bc}}\right)^{bcx}\cosh(d+ex) & \text{for } F = \\ F^{ac}\left(fx + \frac{if\cosh(d+ex)}{e}\right) & \text{for } b = \\ fx + \frac{if\cosh(d+ex)}{e} & \text{for } c = \\ -\frac{iF^{ac}F^{bcx}b^2c^2f\log(F)^2\sinh(d+ex)}{-b^3c^3\log(F)^3+bce^2\log(F)} - \frac{F^{ac}F^{bcx}b^2c^2f\log(F)^2}{-b^3c^3\log(F)^3+bce^2\log(F)} + \frac{iF^{ac}F^{bcx}bcef\log(F)\cosh(d+ex)}{-b^3c^3\log(F)^3+bce^2\log(F)} + \frac{F^{ac}F^{bcx}e^2f}{-b^3c^3\log(F)^3+bce^2\log(F)} & \text{otherw} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F**(c*(b*x+a))*(f+I*f*sinh(e*x+d)),x)
```

```
[Out] Piecewise((f*x + I*f*cosh(d + e*x)/e, Eq(F, 1)), (zoo*e**2*f*exp(-e/(b*c))*
*(a*c)*exp(-e/(b*c))**(b*c*x)*sinh(d + e*x) + zoo*e**2*f*exp(-e/(b*c))**(a*
c)*exp(-e/(b*c))**(b*c*x)*cosh(d + e*x), Eq(F, exp(-e/(b*c)))), (zoo*e**2*f
*exp(e/(b*c))**(a*c)*exp(e/(b*c))**(b*c*x)*sinh(d + e*x) + zoo*e**2*f*exp(e
/(b*c))**(a*c)*exp(e/(b*c))**(b*c*x)*cosh(d + e*x), Eq(F, exp(e/(b*c)))), (
F**(a*c)*(f*x + I*f*cosh(d + e*x)/e), Eq(b, 0)), (f*x + I*f*cosh(d + e*x)/e
, Eq(c, 0)), (-I*F**(a*c)*F**(b*c*x)*b**2*c**2*f*log(F)**2*sinh(d + e*x)/(-
b**3*c**3*log(F)**3 + b*c*e**2*log(F)) - F**(a*c)*F**(b*c*x)*b**2*c**2*f*lo
g(F)**2/(-b**3*c**3*log(F)**3 + b*c*e**2*log(F)) + I*F**(a*c)*F**(b*c*x)*b*
c*e*f*log(F)*cosh(d + e*x)/(-b**3*c**3*log(F)**3 + b*c*e**2*log(F)) + F**(a
*c)*F**(b*c*x)*e**2*f/(-b**3*c**3*log(F)**3 + b*c*e**2*log(F)), True))
```

3.895 $\int \frac{F^{c(a+bx)}}{f+if\sinh(d+ex)}\,dx$

Optimal. Leaf size=85

$$\frac{2e^{\frac{1}{2}(2d+2ex+i\pi)}F^{c(a+bx)}\,{}_2F_1\left(2,\frac{bc\log(F)}{e}+1;\frac{bc\log(F)}{e}+2;-e^{\frac{1}{2}(2d+2ex+i\pi)}\right)}{f(bc\log(F)+e)}$$

```
[Out] 2*exp(d+1/2*I*Pi+e*x)*F^(c*(b*x+a))*hypergeom([2, 1+b*c*ln(F)/e],[2+b*c*ln(
F)/e],-exp(d+1/2*I*Pi+e*x))/f/(e+b*c*ln(F))
```

Rubi [A] time = 0.08, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {5496, 5492}

$$\frac{2e^{\frac{1}{2}(2d+2ex+i\pi)}F^{c(a+bx)}\,{}_2F_1\left(2,\frac{bc\log(F)}{e}+1;\frac{bc\log(F)}{e}+2;-e^{\frac{1}{2}(2d+2ex+i\pi)}\right)}{f(bc\log(F)+e)}$$

Antiderivative was successfully verified.

```
[In] Int[F^(c*(a + b*x))/(f + I*f*Sinh[d + e*x]),x]
```

```
[Out] (2*E^((2*d + I*Pi + 2*e*x)/2)*F^(c*(a + b*x))*Hypergeometric2F1[2, 1 + (b*c
*Log[F])/e, 2 + (b*c*Log[F])/e, -E^((2*d + I*Pi + 2*e*x)/2)])/(f*(e + b*c*L
og[F]))
```

Rule 5492

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sech[(d_.) + (e_.)*(x_)]^(n_.), x_Sym
bol] :> Simp[(2^n*E^(n*(d + e*x))*F^(c*(a + b*x))*Hypergeometric2F1[n, n/2
+ (b*c*Log[F])/(2*e), 1 + n/2 + (b*c*Log[F])/(2*e), -E^(2*(d + e*x))])/(e*n
 + b*c*Log[F]), x] /; FreeQ[{F, a, b, c, d, e}, x] && IntegerQ[n]
```

Rule 5496

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*((f_) + (g_.)*Sinh[(d_.) + (e_.)*(x_)
])^(n_.), x_Symbol] :> Dist[2^n*f^n, Int[F^(c*(a + b*x))*Cosh[d/2 + (e*x)/2
 - (f*Pi)/(4*g)]^(2*n), x], x] /; FreeQ[{F, a, b, c, d, e, f, g}, x] && EqQ
[f^2 + g^2, 0] && ILtQ[n, 0]
```

Rubi steps

$$\int \frac{F^{c(a+bx)}}{f+if\sinh(d+ex)}\,dx = \frac{\int F^{c(a+bx)}\operatorname{sech}^2\left(\frac{d}{2}+\frac{i\pi}{4}+\frac{ex}{2}\right)dx}{2f}$$
$$= \frac{2e^{\frac{1}{2}(2d+i\pi+2ex)}F^{c(a+bx)}\,{}_2F_1\left(2,1+\frac{bc\log(F)}{e};2+\frac{bc\log(F)}{e};-e^{\frac{1}{2}(2d+i\pi+2ex)}\right)}{f(e+bc\log(F))}$$

Mathematica [A] time = 3.74, size = 104, normalized size = 1.22

$$\frac{2F^{c(a+bx)}\left({}_2F_1\left(1,\frac{bc\log(F)}{e};\frac{bc\log(F)}{e}+1;-ie^{d+ex}\right)+\frac{\cosh\left(\frac{ex}{2}\right)-\sinh\left(\frac{ex}{2}\right)}{\left(1-ie^d\right)\sinh\left(\frac{ex}{2}\right)+\left(-1-ie^d\right)\cosh\left(\frac{ex}{2}\right)}\right)}{ef}$$

Antiderivative was successfully verified.

```
[In] Integrate[F^(c*(a + b*x))/(f + I*f*Sinh[d + e*x]),x]
```

```
[Out] (2*F^(c*(a + b*x))*(Hypergeometric2F1[1, (b*c*Log[F])/e, 1 + (b*c*Log[F])/e
, (-I)*E^(d + e*x)] + (Cosh[(e*x)/2] - Sinh[(e*x)/2])/((-1 - I*E^d)*Cosh[(e
*x)/2] + (1 - I*E^d)*Sinh[(e*x)/2])))/(e*f)
```

fricas [F] time = 0.48, size = 0, normalized size = 0.00

$$\frac{\left(efe^{(ex+d)}-ief\right)\mathrm{integral}\left(-\frac{2iF^{bcx+ac}bc\log(F)}{efe^{(ex+d)}-ief},x\right)+2iF^{bcx+ac}}{efe^{(ex+d)}-ief}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))/(f+I*f*sinh(e*x+d)),x, algorithm="fricas")
```

```
[Out] ((e*f*e^(e*x + d) - I*e*f)*integral(-2*I*F^(b*c*x + a*c)*b*c*log(F)/(e*f*e^
(e*x + d) - I*e*f), x) + 2*I*F^(b*c*x + a*c))/(e*f*e^(e*x + d) - I*e*f)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{F^{(bx+a)c}}{if\sinh(ex+d)+f}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))/(f+I*f*sinh(e*x+d)),x, algorithm="giac")
```

```
[Out] integrate(F^((b*x + a)*c)/(I*f*sinh(e*x + d) + f), x)
```

maple [F] time = 0.16, size = 0, normalized size = 0.00

$$\int \frac{F^{c(bx+a)}}{f + if \sinh(ex+d)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(F^(c*(b*x+a))/(f+I*f*sinh(e*x+d)),x)
```

```
[Out] int(F^(c*(b*x+a))/(f+I*f*sinh(e*x+d)),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-4\,F^{ac}bce \int \frac{}{i\,b^2c^2f\log(F)^2 - 3i\,bcef\log(F) + 2i\,e^2f + \left(b^2c^2fe^{(3\,d)}\log(F)^2 - 3\,bcefe^{(3\,d)}\log(F) + 2\,e^2fe^{(3\,d)}\right)e^{(3\,e}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))/(f+I*f*sinh(e*x+d)),x, algorithm="maxima")
```

```
[Out] -4*F^(a*c)*b*c*e*integrate(F^(b*c*x)/(I*b^2*c^2*f*log(F)^2 - 3*I*b*c*e*f*lo
g(F) + 2*I*e^2*f + (b^2*c^2*f*e^(3*d)*log(F)^2 - 3*b*c*e*f*e^(3*d)*log(F) +
 2*e^2*f*e^(3*d))*e^(3*e*x) + (-3*I*b^2*c^2*f*e^(2*d)*log(F)^2 + 9*I*b*c*e*
f*e^(2*d)*log(F) - 6*I*e^2*f*e^(2*d))*e^(2*e*x) - 3*(b^2*c^2*f*e^d*log(F)^2
 - 3*b*c*e*f*e^d*log(F) + 2*e^2*f*e^d)*e^(e*x)), x)*log(F) - 2*(-2*I*F^(a*c
)*e - (F^(a*c)*b*c*e^d*log(F) - 2*F^(a*c)*e*e^d)*e^(e*x))*F^(b*c*x)/(-I*b^2
*c^2*f*log(F)^2 + 3*I*b*c*e*f*log(F) - 2*I*e^2*f + (I*b^2*c^2*f*e^(2*d)*log
(F)^2 - 3*I*b*c*e*f*e^(2*d)*log(F) + 2*I*e^2*f*e^(2*d))*e^(2*e*x) + 2*(b^2*
c^2*f*e^d*log(F)^2 - 3*b*c*e*f*e^d*log(F) + 2*e^2*f*e^d)*e^(e*x))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{F^{c\,(a+b\,x)}}{f + f\sinh(d+e\,x)\,1\mathrm{i}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(F^(c*(a + b*x))/(f + f*sinh(d + e*x)*1i),x)
```

```
[Out] int(F^(c*(a + b*x))/(f + f*sinh(d + e*x)*1i), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{i \int \frac{F^{ac}F^{bcx}}{\sinh(d+ex)-i}\, dx}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F**(c*(b*x+a))/(f+I*f*sinh(e*x+d)),x)
```

```
[Out] -I*Integral(F**(a*c)*F**(b*c*x)/(sinh(d + e*x) - I), x)/f
```

3.896 $\int \frac{F^{c(a+bx)}}{(f+if\sinh(d+ex))^2}\,dx$

Optimal. Leaf size=196

$$\frac{2e^{\frac{1}{2}(2d+2ex+i\pi)}F^{c(a+bx)}(e-bc\log(F))\,{}_2F_1\left(2,\frac{bc\log(F)}{e}+1;\frac{bc\log(F)}{e}+2;-e^{\frac{1}{2}(2d+2ex+i\pi)}\right)}{3e^2f^2}+\frac{bc\log(F)\operatorname{sech}^2\left(\frac{d}{2}+\frac{ex}{2}+\frac{i\pi}{4}\right.}{6e^2f^2}$$

```
[Out] 2/3*exp(d+1/2*I*Pi+e*x)*F^(c*(b*x+a))*hypergeom([2, 1+b*c*ln(F)/e],[2+b*c*l
n(F)/e],-exp(d+1/2*I*Pi+e*x))*(e-b*c*ln(F))/e^2/f^2+1/6*b*c*F^(c*(b*x+a))*l
n(F)*sech(1/2*d+1/4*I*Pi+1/2*e*x)^2/e^2/f^2+1/6*F^(c*(b*x+a))*sech(1/2*d+1/
4*I*Pi+1/2*e*x)^2*tanh(1/2*d+1/4*I*Pi+1/2*e*x)/e/f^2
```

Rubi [A] time = 0.11, antiderivative size = 196, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {5496, 5490, 5492}

$$\frac{2e^{\frac{1}{2}(2d+2ex+i\pi)}F^{c(a+bx)}(e-bc\log(F))\,{}_2F_1\left(2,\frac{bc\log(F)}{e}+1;\frac{bc\log(F)}{e}+2;-e^{\frac{1}{2}(2d+2ex+i\pi)}\right)}{3e^2f^2}+\frac{bc\log(F)\operatorname{sech}^2\left(\frac{d}{2}+\frac{ex}{2}+\frac{i\pi}{4}\right.}{6e^2f^2}$$

Antiderivative was successfully verified.

```
[In] Int[F^(c*(a + b*x))/(f + I*f*Sinh[d + e*x])^2,x]
```

```
[Out] (2*E^((2*d + I*Pi + 2*e*x)/2)*F^(c*(a + b*x))*Hypergeometric2F1[2, 1 + (b*c
*Log[F])/e, 2 + (b*c*Log[F])/e, -E^((2*d + I*Pi + 2*e*x)/2)]*(e - b*c*Log[F
]))/(3*e^2*f^2) + (b*c*F^(c*(a + b*x))*Log[F]*Sech[d/2 + (I/4)*Pi + (e*x)/2
]^2)/(6*e^2*f^2) + (F^(c*(a + b*x))*Sech[d/2 + (I/4)*Pi + (e*x)/2]^2*Tanh[d
/2 + (I/4)*Pi + (e*x)/2])/(6*e*f^2)
```

Rule 5490

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sech[(d_.) + (e_.)*(x_)]^(n_), x_Symb
ol] :> Simp[(b*c*Log[F]*F^(c*(a + b*x))*Sech[d + e*x]^(n - 2))/(e^2*(n - 1)
*(n - 2)), x] + (Dist[(e^2*(n - 2)^2 - b^2*c^2*Log[F]^2)/(e^2*(n - 1)*(n -
2)), Int[F^(c*(a + b*x))*Sech[d + e*x]^(n - 2), x], x] + Simp[(F^(c*(a + b*
x))*Sech[d + e*x]^(n - 1)*Sinh[d + e*x])/(e*(n - 1)), x]) /; FreeQ[{F, a, b
, c, d, e}, x] && NeQ[e^2*(n - 2)^2 - b^2*c^2*Log[F]^2, 0] && GtQ[n, 1] &&
NeQ[n, 2]
```

Rule 5492

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sech[(d_.) + (e_.)*(x_)]^(n_.), x_Sym
bol] :> Simp[(2^n*E^(n*(d + e*x))*F^(c*(a + b*x))*Hypergeometric2F1[n, n/2
```

```
+ (b*c*Log[F])/(2*e), 1 + n/2 + (b*c*Log[F])/(2*e), -E^(2*(d + e*x))])/(e*n
 + b*c*Log[F]), x] /; FreeQ[{F, a, b, c, d, e}, x] && IntegerQ[n]
```

Rule 5496

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*((f_) + (g_.)*Sinh[(d_.) + (e_.)*(x_)
])^(n_.), x_Symbol] :> Dist[2^n*f^n, Int[F^(c*(a + b*x))*Cosh[d/2 + (e*x)/2
 - (f*Pi)/(4*g)]^(2*n), x], x] /; FreeQ[{F, a, b, c, d, e, f, g}, x] && EqQ
[f^2 + g^2, 0] && ILtQ[n, 0]
```

Rubi steps

$$\int \frac{F^{c(a+bx)}}{(f+if\sinh(d+ex))^2}\,dx = \frac{\int F^{c(a+bx)}\operatorname{sech}^4\left(\frac{d}{2}+\frac{i\pi}{4}+\frac{ex}{2}\right)dx}{4f^2}$$

$$= \frac{bcF^{c(a+bx)}\log(F)\operatorname{sech}^2\left(\frac{d}{2}+\frac{i\pi}{4}+\frac{ex}{2}\right)}{6e^2f^2} + \frac{F^{c(a+bx)}\operatorname{sech}^2\left(\frac{d}{2}+\frac{i\pi}{4}+\frac{ex}{2}\right)\tanh\left(\frac{d}{2}+\frac{i\pi}{4}+\right.}{6ef^2}$$

$$= \frac{2e^{\frac{1}{2}(2d+i\pi+2ex)}F^{c(a+bx)}\,{}_2F_1\left(2,1+\frac{bc\log(F)}{e};2+\frac{bc\log(F)}{e};-e^{\frac{1}{2}(2d+i\pi+2ex)}\right)(e-bc\log(F)}{3e^2f^2}$$

Mathematica [A] time = 3.34, size = 255, normalized size = 1.30

$$\frac{F^{c(a+bx)}\left(\cosh\left(\frac{1}{2}(d+ex)\right)+i\sinh\left(\frac{1}{2}(d+ex)\right)\right)\left((1-i)\left(e^2-b^2c^2\log^2(F)\right)\left(\cosh\left(\frac{1}{2}(d+ex)\right)+i\sinh\left(\frac{1}{2}(d+ex)\right)\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[F^(c*(a + b*x))/(f + I*f*Sinh[d + e*x])^2,x]
```

```
[Out] (F^(c*(a + b*x))*(Cosh[(d + e*x)/2] + I*Sinh[(d + e*x)/2])*(e*(I*e + b*c*Lo
g[F])*(Cosh[(d + e*x)/2] + I*Sinh[(d + e*x)/2]) + (1 - I)*(-1 + (1 + I)*Hyp
ergeometric2F1[1, (b*c*Log[F])/e, 1 + (b*c*Log[F])/e, (-I)*(Cosh[d + e*x] +
 Sinh[d + e*x])])*(e^2 - b^2*c^2*Log[F]^2)*(Cosh[(d + e*x)/2] + I*Sinh[(d +
 e*x)/2])^3 + 2*e^2*Sinh[(d + e*x)/2] + 2*(e^2 - b^2*c^2*Log[F]^2)*(Cosh[(d
 + e*x)/2] + I*Sinh[(d + e*x)/2])^2*Sinh[(d + e*x)/2]))/(3*e^3*(f + I*f*Sin
h[d + e*x])^2)
```

fricas [F] time = 0.49, size = 0, normalized size = 0.00

$$\frac{\left(6\,e^{2}e^{(ex+d)}+\left(-2i\,b^{2}c^{2}e^{(2\,ex+2\,d)}-4\,b^{2}c^{2}e^{(ex+d)}+2i\,b^{2}c^{2}\right)\log(F)^{2}-2i\,e^{2}-2\left(i\,bcee^{(2\,ex+2\,d)}+bcee^{(ex+d)}\right)\log(F)\right)F}{3\,e^{3}f^{2}e^{(3\,ex+3\,d)}-9i\,e^{3}f^{2}e^{(2\,ex+}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))/(f+I*f*sinh(e*x+d))^2,x, algorithm="fricas")
```

```
[Out] ((6*e^2*e^(e*x + d) + (-2*I*b^2*c^2*e^(2*e*x + 2*d) - 4*b^2*c^2*e^(e*x + d)
 + 2*I*b^2*c^2)*log(F)^2 - 2*I*e^2 - 2*(I*b*c*e*e^(2*e*x + 2*d) + b*c*e*e^(
e*x + d))*log(F))*F^(b*c*x + a*c) + (3*e^3*f^2*e^(3*e*x + 3*d) - 9*I*e^3*f^
2*e^(2*e*x + 2*d) - 9*e^3*f^2*e^(e*x + d) + 3*I*e^3*f^2)*integral((2*I*b^3*
c^3*log(F)^3 - 2*I*b*c*e^2*log(F))*F^(b*c*x + a*c)/(3*e^3*f^2*e^(e*x + d) -
 3*I*e^3*f^2), x))/(3*e^3*f^2*e^(3*e*x + 3*d) - 9*I*e^3*f^2*e^(2*e*x + 2*d)
 - 9*e^3*f^2*e^(e*x + d) + 3*I*e^3*f^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{F^{(bx+a)c}}{\left(i\,f\sinh\left(ex+d\right)+f\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))/(f+I*f*sinh(e*x+d))^2,x, algorithm="giac")
```

```
[Out] integrate(F^((b*x + a)*c)/(I*f*sinh(e*x + d) + f)^2, x)
```

maple [F] time = 0.59, size = 0, normalized size = 0.00

$$\int \frac{F^{c(bx+a)}}{\left(f+if\sinh\left(ex+d\right)\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(F^(c*(b*x+a))/(f+I*f*sinh(e*x+d))^2,x)
```

```
[Out] int(F^(c*(b*x+a))/(f+I*f*sinh(e*x+d))^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))/(f+I*f*sinh(e*x+d))^2,x, algorithm="maxima")
```

```
[Out] (16*I*F^(a*c)*b^2*c^2*e*log(F)^2 + 16*I*F^(a*c)*b*c*e^2*log(F))*integrate(F
^(b*c*x)/(-I*b^3*c^3*f^2*log(F)^3 + 9*I*b^2*c^2*e*f^2*log(F)^2 - 26*I*b*c*e
^2*f^2*log(F) + 24*I*e^3*f^2 + (b^3*c^3*f^2*e^(5*d)*log(F)^3 - 9*b^2*c^2*e*
f^2*e^(5*d)*log(F)^2 + 26*b*c*e^2*f^2*e^(5*d)*log(F) - 24*e^3*f^2*e^(5*d))*
e^(5*e*x) + (-5*I*b^3*c^3*f^2*e^(4*d)*log(F)^3 + 45*I*b^2*c^2*e*f^2*e^(4*d)
*log(F)^2 - 130*I*b*c*e^2*f^2*e^(4*d)*log(F) + 120*I*e^3*f^2*e^(4*d))*e^(4*
e*x) - 10*(b^3*c^3*f^2*e^(3*d)*log(F)^3 - 9*b^2*c^2*e*f^2*e^(3*d)*log(F)^2
+ 26*b*c*e^2*f^2*e^(3*d)*log(F) - 24*e^3*f^2*e^(3*d))*e^(3*e*x) + (10*I*b^3
*c^3*f^2*e^(2*d)*log(F)^3 - 90*I*b^2*c^2*e*f^2*e^(2*d)*log(F)^2 + 260*I*b*c
*e^2*f^2*e^(2*d)*log(F) - 240*I*e^3*f^2*e^(2*d))*e^(2*e*x) + 5*(b^3*c^3*f^2
*e^d*log(F)^3 - 9*b^2*c^2*e*f^2*e^d*log(F)^2 + 26*b*c*e^2*f^2*e^d*log(F) -
24*e^3*f^2*e^d)*e^(e*x)), x) + (16*F^(a*c)*b*c*e*log(F) + 16*F^(a*c)*e^2 -
4*(F^(a*c)*b^2*c^2*e^(2*d)*log(F)^2 - 7*F^(a*c)*b*c*e*e^(2*d)*log(F) + 12*F
^(a*c)*e^2*e^(2*d))*e^(2*e*x) - (16*I*F^(a*c)*b*c*e*e^d*log(F) - 64*I*F^(a*
c)*e^2*e^d)*e^(e*x))*F^(b*c*x)/(b^3*c^3*f^2*log(F)^3 - 9*b^2*c^2*e*f^2*log(
F)^2 + 26*b*c*e^2*f^2*log(F) - 24*e^3*f^2 + (b^3*c^3*f^2*e^(4*d)*log(F)^3 -
 9*b^2*c^2*e*f^2*e^(4*d)*log(F)^2 + 26*b*c*e^2*f^2*e^(4*d)*log(F) - 24*e^3*
f^2*e^(4*d))*e^(4*e*x) + (-4*I*b^3*c^3*f^2*e^(3*d)*log(F)^3 + 36*I*b^2*c^2*
e*f^2*e^(3*d)*log(F)^2 - 104*I*b*c*e^2*f^2*e^(3*d)*log(F) + 96*I*e^3*f^2*e^
(3*d))*e^(3*e*x) - 6*(b^3*c^3*f^2*e^(2*d)*log(F)^3 - 9*b^2*c^2*e*f^2*e^(2*d
)*log(F)^2 + 26*b*c*e^2*f^2*e^(2*d)*log(F) - 24*e^3*f^2*e^(2*d))*e^(2*e*x)
+ (4*I*b^3*c^3*f^2*e^d*log(F)^3 - 36*I*b^2*c^2*e*f^2*e^d*log(F)^2 + 104*I*b
*c*e^2*f^2*e^d*log(F) - 96*I*e^3*f^2*e^d)*e^(e*x))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{F^{c\,(a+b\,x)}}{\left(f + f\sinh\left(d + e\,x\right)\,1\mathrm{i}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(F^(c*(a + b*x))/(f + f*sinh(d + e*x)*1i)^2,x)
```

```
[Out] int(F^(c*(a + b*x))/(f + f*sinh(d + e*x)*1i)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{F^{ac}F^{bcx}}{\sinh^2(d+ex)-2i\sinh(d+ex)-1}\,dx}{f^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F**(c*(b*x+a))/(f+I*f*sinh(e*x+d))**2,x)
```

```
[Out] -Integral(F**(a*c)*F**(b*c*x)/(sinh(d + e*x)**2 - 2*I*sinh(d + e*x) - 1), x
)/f**2
```

3.897 $\int F^{c(a+bx)}(f+f\cosh(d+ex))^2\,dx$

Optimal. Leaf size=251

$$\frac{2ef^2\sinh(d+ex)F^{ac+bcx}}{e^2-b^2c^2\log^2(F)}-\frac{bcf^2\log(F)\cosh^2(d+ex)F^{ac+bcx}}{4e^2-b^2c^2\log^2(F)}-\frac{2bcf^2\log(F)\cosh(d+ex)F^{ac+bcx}}{e^2-b^2c^2\log^2(F)}+\frac{2ef^2\sinh(d+ex)\ldots}{4e^2-\ldots}$$

```
[Out] f^2*F^(b*c*x+a*c)/b/c/ln(F)-2*b*c*f^2*F^(b*c*x+a*c)*cosh(e*x+d)*ln(F)/(e^2-
b^2*c^2*ln(F)^2)+2*e^2*f^2*F^(b*c*x+a*c)/b/c/ln(F)/(4*e^2-b^2*c^2*ln(F)^2)-
b*c*f^2*F^(b*c*x+a*c)*cosh(e*x+d)^2*ln(F)/(4*e^2-b^2*c^2*ln(F)^2)+2*e*f^2*F
^(b*c*x+a*c)*sinh(e*x+d)/(e^2-b^2*c^2*ln(F)^2)+2*e*f^2*F^(b*c*x+a*c)*cosh(e
*x+d)*sinh(e*x+d)/(4*e^2-b^2*c^2*ln(F)^2)
```

Rubi [A] time = 0.31, antiderivative size = 251, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {6741, 12, 6742, 2194, 5475, 5477}

$$\frac{2ef^2\sinh(d+ex)F^{ac+bcx}}{e^2-b^2c^2\log^2(F)}-\frac{bcf^2\log(F)\cosh^2(d+ex)F^{ac+bcx}}{4e^2-b^2c^2\log^2(F)}-\frac{2bcf^2\log(F)\cosh(d+ex)F^{ac+bcx}}{e^2-b^2c^2\log^2(F)}+\frac{2ef^2\sinh(d+ex)\ldots}{4e^2-\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[F^(c*(a + b*x))*(f + f*Cosh[d + e*x])^2,x]
```

```
[Out] (f^2*F^(a*c + b*c*x))/(b*c*Log[F]) - (2*b*c*f^2*F^(a*c + b*c*x)*Cosh[d + e*
x]*Log[F])/(e^2 - b^2*c^2*Log[F]^2) + (2*e^2*f^2*F^(a*c + b*c*x))/(b*c*Log[
F]*(4*e^2 - b^2*c^2*Log[F]^2)) - (b*c*f^2*F^(a*c + b*c*x)*Cosh[d + e*x]^2*L
og[F])/(4*e^2 - b^2*c^2*Log[F]^2) + (2*e*f^2*F^(a*c + b*c*x)*Sinh[d + e*x])
/(e^2 - b^2*c^2*Log[F]^2) + (2*e*f^2*F^(a*c + b*c*x)*Cosh[d + e*x]*Sinh[d +
 e*x])/(4*e^2 - b^2*c^2*Log[F]^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 5475

```
Int[Cosh[(d_.) + (e_.)*(x_)]*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symbol] :
> -Simp[(b*c*Log[F]*F^(c*(a + b*x))*Cosh[d + e*x])/(e^2 - b^2*c^2*Log[F]^2)
```

```
, x] + Simp[(e*F^(c*(a + b*x))*Sinh[d + e*x])/(e^2 - b^2*c^2*Log[F]^2), x]
/; FreeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 - b^2*c^2*Log[F]^2, 0]
```

Rule 5477

```
Int[Cosh[(d_.) + (e_.)*(x_)]^(n_)*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symb
ol] :> -Simp[(b*c*Log[F]*F^(c*(a + b*x))*Cosh[d + e*x]^n)/(e^2*n^2 - b^2*c^
2*Log[F]^2), x] + (Dist[(n*(n - 1)*e^2)/(e^2*n^2 - b^2*c^2*Log[F]^2), Int[F
^(c*(a + b*x))*Cosh[d + e*x]^(n - 2), x], x] + Simp[(e*n*F^(c*(a + b*x))*Si
nh[d + e*x]*Cosh[d + e*x]^(n - 1))/(e^2*n^2 - b^2*c^2*Log[F]^2), x]) /; Fre
eQ[{F, a, b, c, d, e}, x] && NeQ[e^2*n^2 - b^2*c^2*Log[F]^2, 0] && GtQ[n, 1
]
```

Rule 6741

```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}
\int F^{c(a+bx)}(f+f\cosh(d+ex))^2\,dx &= \int f^2F^{ac+bcx}(1+\cosh(d+ex))^2\,dx \\
&= f^2\int F^{ac+bcx}(1+\cosh(d+ex))^2\,dx \\
&= f^2\int \left(F^{ac+bcx}+2F^{ac+bcx}\cosh(d+ex)+F^{ac+bcx}\cosh^2(d+ex)\right)dx \\
&= f^2\int F^{ac+bcx}\,dx+f^2\int F^{ac+bcx}\cosh^2(d+ex)\,dx+\left(2f^2\right)\int F^{ac+bcx}\cosh(d \\
&= \frac{f^2F^{ac+bcx}}{bc\log(F)}-\frac{2bcf^2F^{ac+bcx}\cosh(d+ex)\log(F)}{e^2-b^2c^2\log^2(F)}-\frac{bcf^2F^{ac+bcx}\cosh^2(d+ex}{4e^2-b^2c^2\log^2(F} \\
&= \frac{f^2F^{ac+bcx}}{bc\log(F)}-\frac{2bcf^2F^{ac+bcx}\cosh(d+ex)\log(F)}{e^2-b^2c^2\log^2(F)}+\frac{2e^2f^2F^{ac+bcx}}{bc\log(F)\left(4e^2-b^2c^2\log\right.}
\end{aligned}$$

Mathematica [A] time = 0.60, size = 230, normalized size = 0.92

$$\frac{f^2F^{c(a+bx)}\left(3b^4c^4\log^4(F)-4b^3c^3e\log^3(F)\sinh(d+ex)-2b^3c^3e\log^3(F)\sinh(2(d+ex))-15b^2c^2e^2\log^2(F)+4\,\mathrm{co}\ldots\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[F^(c*(a + b*x))*(f + f*Cosh[d + e*x])^2,x]
```

```
[Out] (f^2*F^(c*(a + b*x))*(12*e^4 - 15*b^2*c^2*e^2*Log[F]^2 + 3*b^4*c^4*Log[F]^4
 + 4*Cosh[d + e*x]*(-4*b^2*c^2*e^2*Log[F]^2 + b^4*c^4*Log[F]^4) + Cosh[2*(d
 + e*x)]*(-(b^2*c^2*e^2*Log[F]^2) + b^4*c^4*Log[F]^4) + 16*b*c*e^3*Log[F]*S
inh[d + e*x] - 4*b^3*c^3*e*Log[F]^3*Sinh[d + e*x] + 2*b*c*e^3*Log[F]*Sinh[2
*(d + e*x)] - 2*b^3*c^3*e*Log[F]^3*Sinh[2*(d + e*x)]))/(2*(4*b*c*e^4*Log[F]
 - 5*b^3*c^3*e^2*Log[F]^3 + b^5*c^5*Log[F]^5))
```

fricas [B] time = 0.49, size = 2340, normalized size = 9.32

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*(f+f*cosh(e*x+d))^2,x, algorithm="fricas")
```

```
[Out] 1/4*((24*e^4*f^2*cosh(e*x + d)^2 + (b^4*c^4*f^2*cosh(e*x + d)^4 + 4*b^4*c^4
*f^2*cosh(e*x + d)^3 + 6*b^4*c^4*f^2*cosh(e*x + d)^2 + 4*b^4*c^4*f^2*cosh(e
*x + d) + b^4*c^4*f^2)*log(F)^4 + (b^4*c^4*f^2*log(F)^4 - 2*b^3*c^3*e*f^2*l
og(F)^3 - b^2*c^2*e^2*f^2*log(F)^2 + 2*b*c*e^3*f^2*log(F))*sinh(e*x + d)^4
- 2*(b^3*c^3*e*f^2*cosh(e*x + d)^4 + 2*b^3*c^3*e*f^2*cosh(e*x + d)^3 - 2*b^
3*c^3*e*f^2*cosh(e*x + d) - b^3*c^3*e*f^2)*log(F)^3 + 4*((b^4*c^4*f^2*cosh(
e*x + d) + b^4*c^4*f^2)*log(F)^4 - (2*b^3*c^3*e*f^2*cosh(e*x + d) + b^3*c^3
*e*f^2)*log(F)^3 - (b^2*c^2*e^2*f^2*cosh(e*x + d) + 4*b^2*c^2*e^2*f^2)*log(
F)^2 + 2*(b*c*e^3*f^2*cosh(e*x + d) + 2*b*c*e^3*f^2)*log(F))*sinh(e*x + d)^
3 - (b^2*c^2*e^2*f^2*cosh(e*x + d)^4 + 16*b^2*c^2*e^2*f^2*cosh(e*x + d)^3 +
 30*b^2*c^2*e^2*f^2*cosh(e*x + d)^2 + 16*b^2*c^2*e^2*f^2*cosh(e*x + d) + b^
2*c^2*e^2*f^2)*log(F)^2 + 6*(4*e^4*f^2 + (b^4*c^4*f^2*cosh(e*x + d)^2 + 2*b
^4*c^4*f^2*cosh(e*x + d) + b^4*c^4*f^2)*log(F)^4 - 2*(b^3*c^3*e*f^2*cosh(e*
x + d)^2 + b^3*c^3*e*f^2*cosh(e*x + d))*log(F)^3 - (b^2*c^2*e^2*f^2*cosh(e*
x + d)^2 + 8*b^2*c^2*e^2*f^2*cosh(e*x + d) + 5*b^2*c^2*e^2*f^2)*log(F)^2 +
2*(b*c*e^3*f^2*cosh(e*x + d)^2 + 4*b*c*e^3*f^2*cosh(e*x + d))*log(F))*sinh(
e*x + d)^2 + 2*(b*c*e^3*f^2*cosh(e*x + d)^4 + 8*b*c*e^3*f^2*cosh(e*x + d)^3
 - 8*b*c*e^3*f^2*cosh(e*x + d) - b*c*e^3*f^2)*log(F) + 4*(12*e^4*f^2*cosh(e
*x + d) + (b^4*c^4*f^2*cosh(e*x + d)^3 + 3*b^4*c^4*f^2*cosh(e*x + d)^2 + 3*
b^4*c^4*f^2*cosh(e*x + d) + b^4*c^4*f^2)*log(F)^4 - (2*b^3*c^3*e*f^2*cosh(e
*x + d)^3 + 3*b^3*c^3*e*f^2*cosh(e*x + d)^2 - b^3*c^3*e*f^2)*log(F)^3 - (b^
2*c^2*e^2*f^2*cosh(e*x + d)^3 + 12*b^2*c^2*e^2*f^2*cosh(e*x + d)^2 + 15*b^2
```

```
*c^2*e^2*f^2*cosh(e*x + d) + 4*b^2*c^2*e^2*f^2)*log(F)^2 + 2*(b*c*e^3*f^2*c
osh(e*x + d)^3 + 6*b*c*e^3*f^2*cosh(e*x + d)^2 - 2*b*c*e^3*f^2)*log(F))*sin
h(e*x + d))*cosh((b*c*x + a*c)*log(F)) + (24*e^4*f^2*cosh(e*x + d)^2 + (b^4
*c^4*f^2*cosh(e*x + d)^4 + 4*b^4*c^4*f^2*cosh(e*x + d)^3 + 6*b^4*c^4*f^2*co
sh(e*x + d)^2 + 4*b^4*c^4*f^2*cosh(e*x + d) + b^4*c^4*f^2)*log(F)^4 + (b^4*
c^4*f^2*log(F)^4 - 2*b^3*c^3*e*f^2*log(F)^3 - b^2*c^2*e^2*f^2*log(F)^2 + 2*
b*c*e^3*f^2*log(F))*sinh(e*x + d)^4 - 2*(b^3*c^3*e*f^2*cosh(e*x + d)^4 + 2*
b^3*c^3*e*f^2*cosh(e*x + d)^3 - 2*b^3*c^3*e*f^2*cosh(e*x + d) - b^3*c^3*e*f
^2)*log(F)^3 + 4*((b^4*c^4*f^2*cosh(e*x + d) + b^4*c^4*f^2)*log(F)^4 - (2*b
^3*c^3*e*f^2*cosh(e*x + d) + b^3*c^3*e*f^2)*log(F)^3 - (b^2*c^2*e^2*f^2*cos
h(e*x + d) + 4*b^2*c^2*e^2*f^2)*log(F)^2 + 2*(b*c*e^3*f^2*cosh(e*x + d) + 2
*b*c*e^3*f^2)*log(F))*sinh(e*x + d)^3 - (b^2*c^2*e^2*f^2*cosh(e*x + d)^4 +
16*b^2*c^2*e^2*f^2*cosh(e*x + d)^3 + 30*b^2*c^2*e^2*f^2*cosh(e*x + d)^2 + 1
6*b^2*c^2*e^2*f^2*cosh(e*x + d) + b^2*c^2*e^2*f^2)*log(F)^2 + 6*(4*e^4*f^2
+ (b^4*c^4*f^2*cosh(e*x + d)^2 + 2*b^4*c^4*f^2*cosh(e*x + d) + b^4*c^4*f^2)
*log(F)^4 - 2*(b^3*c^3*e*f^2*cosh(e*x + d)^2 + b^3*c^3*e*f^2*cosh(e*x + d))
*log(F)^3 - (b^2*c^2*e^2*f^2*cosh(e*x + d)^2 + 8*b^2*c^2*e^2*f^2*cosh(e*x +
 d) + 5*b^2*c^2*e^2*f^2)*log(F)^2 + 2*(b*c*e^3*f^2*cosh(e*x + d)^2 + 4*b*c*
e^3*f^2*cosh(e*x + d))*log(F))*sinh(e*x + d)^2 + 2*(b*c*e^3*f^2*cosh(e*x +
d)^4 + 8*b*c*e^3*f^2*cosh(e*x + d)^3 - 8*b*c*e^3*f^2*cosh(e*x + d) - b*c*e^
3*f^2)*log(F) + 4*(12*e^4*f^2*cosh(e*x + d) + (b^4*c^4*f^2*cosh(e*x + d)^3
+ 3*b^4*c^4*f^2*cosh(e*x + d)^2 + 3*b^4*c^4*f^2*cosh(e*x + d) + b^4*c^4*f^2
)*log(F)^4 - (2*b^3*c^3*e*f^2*cosh(e*x + d)^3 + 3*b^3*c^3*e*f^2*cosh(e*x +
d)^2 - b^3*c^3*e*f^2)*log(F)^3 - (b^2*c^2*e^2*f^2*cosh(e*x + d)^3 + 12*b^2*
c^2*e^2*f^2*cosh(e*x + d)^2 + 15*b^2*c^2*e^2*f^2*cosh(e*x + d) + 4*b^2*c^2*
e^2*f^2)*log(F)^2 + 2*(b*c*e^3*f^2*cosh(e*x + d)^3 + 6*b*c*e^3*f^2*cosh(e*x
 + d)^2 - 2*b*c*e^3*f^2)*log(F))*sinh(e*x + d))*sinh((b*c*x + a*c)*log(F)))
/(b^5*c^5*cosh(e*x + d)^2*log(F)^5 - 5*b^3*c^3*e^2*cosh(e*x + d)^2*log(F)^3
 + 4*b*c*e^4*cosh(e*x + d)^2*log(F) + (b^5*c^5*log(F)^5 - 5*b^3*c^3*e^2*log
(F)^3 + 4*b*c*e^4*log(F))*sinh(e*x + d)^2 + 2*(b^5*c^5*cosh(e*x + d)*log(F)
^5 - 5*b^3*c^3*e^2*cosh(e*x + d)*log(F)^3 + 4*b*c*e^4*cosh(e*x + d)*log(F))
*sinh(e*x + d))
```

giac [C] time = 0.29, size = 1576, normalized size = 6.28

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*(f+f*cosh(e*x+d))^2,x, algorithm="giac")
```

```
[Out] 3*(2*b*c*f^2*cos(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*b*c*x - 1/2*pi*a*c*sgn(F) +
1/2*pi*a*c)*log(abs(F))/(4*b^2*c^2*log(abs(F))^2 + (pi*b*c*sgn(F) - pi*b*c)
^2) - (pi*b*c*sgn(F) - pi*b*c)*f^2*sin(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*b*c*x
- 1/2*pi*a*c*sgn(F) + 1/2*pi*a*c)/(4*b^2*c^2*log(abs(F))^2 + (pi*b*c*sgn(F)
 - pi*b*c)^2))*e^(b*c*x*log(abs(F)) + a*c*log(abs(F))) - 1/2*I*(-6*I*f^2*e^
```

```
(1/2*I*pi*b*c*x*sgn(F) - 1/2*I*pi*b*c*x + 1/2*I*pi*a*c*sgn(F) - 1/2*I*pi*a*
c)/(2*I*pi*b*c*sgn(F) - 2*I*pi*b*c + 4*b*c*log(abs(F))) + 6*I*f^2*e^(-1/2*I
*pi*b*c*x*sgn(F) + 1/2*I*pi*b*c*x - 1/2*I*pi*a*c*sgn(F) + 1/2*I*pi*a*c)/(-2
*I*pi*b*c*sgn(F) + 2*I*pi*b*c + 4*b*c*log(abs(F))))*e^(b*c*x*log(abs(F)) +
a*c*log(abs(F))) + 1/2*(2*(b*c*log(abs(F)) + 2*e)*f^2*cos(-1/2*pi*b*c*x*sgn
(F) + 1/2*pi*b*c*x - 1/2*pi*a*c*sgn(F) + 1/2*pi*a*c)/((pi*b*c*sgn(F) - pi*b
*c)^2 + 4*(b*c*log(abs(F)) + 2*e)^2) - (pi*b*c*sgn(F) - pi*b*c)*f^2*sin(-1/
2*pi*b*c*x*sgn(F) + 1/2*pi*b*c*x - 1/2*pi*a*c*sgn(F) + 1/2*pi*a*c)/((pi*b*c
*sgn(F) - pi*b*c)^2 + 4*(b*c*log(abs(F)) + 2*e)^2))*e^(a*c*log(abs(F)) + (b
*c*log(abs(F)) + 2*e)*x + 2*d) - 1/2*I*(-2*I*f^2*e^(1/2*I*pi*b*c*x*sgn(F) -
 1/2*I*pi*b*c*x + 1/2*I*pi*a*c*sgn(F) - 1/2*I*pi*a*c)/(4*I*pi*b*c*sgn(F) -
4*I*pi*b*c + 8*b*c*log(abs(F)) + 16*e) + 2*I*f^2*e^(-1/2*I*pi*b*c*x*sgn(F)
+ 1/2*I*pi*b*c*x - 1/2*I*pi*a*c*sgn(F) + 1/2*I*pi*a*c)/(-4*I*pi*b*c*sgn(F)
+ 4*I*pi*b*c + 8*b*c*log(abs(F)) + 16*e))*e^(a*c*log(abs(F)) + (b*c*log(abs
(F)) + 2*e)*x + 2*d) + 2*(2*(b*c*log(abs(F)) + e)*f^2*cos(-1/2*pi*b*c*x*sgn
(F) + 1/2*pi*b*c*x - 1/2*pi*a*c*sgn(F) + 1/2*pi*a*c)/((pi*b*c*sgn(F) - pi*b
*c)^2 + 4*(b*c*log(abs(F)) + e)^2) - (pi*b*c*sgn(F) - pi*b*c)*f^2*sin(-1/2*
pi*b*c*x*sgn(F) + 1/2*pi*b*c*x - 1/2*pi*a*c*sgn(F) + 1/2*pi*a*c)/((pi*b*c*s
gn(F) - pi*b*c)^2 + 4*(b*c*log(abs(F)) + e)^2))*e^(a*c*log(abs(F)) + (b*c*l
og(abs(F)) + e)*x + d) - 1/2*I*(-2*I*f^2*e^(1/2*I*pi*b*c*x*sgn(F) - 1/2*I*p
i*b*c*x + 1/2*I*pi*a*c*sgn(F) - 1/2*I*pi*a*c)/(I*pi*b*c*sgn(F) - I*pi*b*c +
 2*b*c*log(abs(F)) + 2*e) + 2*I*f^2*e^(-1/2*I*pi*b*c*x*sgn(F) + 1/2*I*pi*b*
c*x - 1/2*I*pi*a*c*sgn(F) + 1/2*I*pi*a*c)/(-I*pi*b*c*sgn(F) + I*pi*b*c + 2*
b*c*log(abs(F)) + 2*e))*e^(a*c*log(abs(F)) + (b*c*log(abs(F)) + e)*x + d) +
 2*(2*(b*c*log(abs(F)) - e)*f^2*cos(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*b*c*x - 1
/2*pi*a*c*sgn(F) + 1/2*pi*a*c)/((pi*b*c*sgn(F) - pi*b*c)^2 + 4*(b*c*log(abs
(F)) - e)^2) - (pi*b*c*sgn(F) - pi*b*c)*f^2*sin(-1/2*pi*b*c*x*sgn(F) + 1/2*
pi*b*c*x - 1/2*pi*a*c*sgn(F) + 1/2*pi*a*c)/((pi*b*c*sgn(F) - pi*b*c)^2 + 4*
(b*c*log(abs(F)) - e)^2))*e^(a*c*log(abs(F)) + (b*c*log(abs(F)) - e)*x - d)
 - 1/2*I*(-2*I*f^2*e^(1/2*I*pi*b*c*x*sgn(F) - 1/2*I*pi*b*c*x + 1/2*I*pi*a*c
*sgn(F) - 1/2*I*pi*a*c)/(I*pi*b*c*sgn(F) - I*pi*b*c + 2*b*c*log(abs(F)) - 2
*e) + 2*I*f^2*e^(-1/2*I*pi*b*c*x*sgn(F) + 1/2*I*pi*b*c*x - 1/2*I*pi*a*c*sgn
(F) + 1/2*I*pi*a*c)/(-I*pi*b*c*sgn(F) + I*pi*b*c + 2*b*c*log(abs(F)) - 2*e)
)*e^(a*c*log(abs(F)) + (b*c*log(abs(F)) - e)*x - d) + 1/2*(2*(b*c*log(abs(F
)) - 2*e)*f^2*cos(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*b*c*x - 1/2*pi*a*c*sgn(F) +
 1/2*pi*a*c)/((pi*b*c*sgn(F) - pi*b*c)^2 + 4*(b*c*log(abs(F)) - 2*e)^2) - (
pi*b*c*sgn(F) - pi*b*c)*f^2*sin(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*b*c*x - 1/2*p
i*a*c*sgn(F) + 1/2*pi*a*c)/((pi*b*c*sgn(F) - pi*b*c)^2 + 4*(b*c*log(abs(F))
 - 2*e)^2))*e^(a*c*log(abs(F)) + (b*c*log(abs(F)) - 2*e)*x - 2*d) - 1/2*I*(
-2*I*f^2*e^(1/2*I*pi*b*c*x*sgn(F) - 1/2*I*pi*b*c*x + 1/2*I*pi*a*c*sgn(F) -
1/2*I*pi*a*c)/(4*I*pi*b*c*sgn(F) - 4*I*pi*b*c + 8*b*c*log(abs(F)) - 16*e) +
 2*I*f^2*e^(-1/2*I*pi*b*c*x*sgn(F) + 1/2*I*pi*b*c*x - 1/2*I*pi*a*c*sgn(F) +
 1/2*I*pi*a*c)/(-4*I*pi*b*c*sgn(F) + 4*I*pi*b*c + 8*b*c*log(abs(F)) - 16*e)
)*e^(a*c*log(abs(F)) + (b*c*log(abs(F)) - 2*e)*x - 2*d)
```

maple [A] time = 0.60, size = 426, normalized size = 1.70

$$\frac{f^2\left(\ln(F)^4b^4c^4\mathrm{e}^{4ex+4d}+4\ln(F)^4b^4c^4\mathrm{e}^{3ex+3d}+6\ln(F)^4b^4c^4\mathrm{e}^{2ex+2d}-2\ln(F)^3b^3c^3e\,\mathrm{e}^{4ex+4d}+4\ln(F)^4b^4c^4\mathrm{e}^{ex+d}-4\cdots\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(F^(c*(b*x+a))*(f+f*cosh(e*x+d))^2,x)
```

```
[Out] 1/4*f^2*(ln(F)^4*b^4*c^4*exp(4*e*x+4*d)+4*ln(F)^4*b^4*c^4*exp(3*e*x+3*d)+6*
ln(F)^4*b^4*c^4*exp(2*e*x+2*d)-2*ln(F)^3*b^3*c^3*e*exp(4*e*x+4*d)+4*ln(F)^4
*b^4*c^4*exp(e*x+d)-4*ln(F)^3*b^3*c^3*e*exp(3*e*x+3*d)+b^4*c^4*ln(F)^4-ln(F
)^2*b^2*c^2*e^2*exp(4*e*x+4*d)+4*ln(F)^3*b^3*c^3*e*exp(e*x+d)-16*ln(F)^2*b^
2*c^2*e^2*exp(3*e*x+3*d)+2*ln(F)^3*b^3*c^3*e-30*ln(F)^2*b^2*c^2*e^2*exp(2*e
*x+2*d)+2*ln(F)*b*c*e^3*exp(4*e*x+4*d)-16*ln(F)^2*b^2*c^2*e^2*exp(e*x+d)+16
*ln(F)*b*c*e^3*exp(3*e*x+3*d)-b^2*c^2*e^2*ln(F)^2-16*ln(F)*b*c*e^3*exp(e*x+
d)-2*ln(F)*b*c*e^3+24*e^4*exp(2*e*x+2*d))/b/c/ln(F)/(b*c*ln(F)-e)*exp(-2*e*
x-2*d)/(b*c*ln(F)-2*e)/(e+b*c*ln(F))/(b*c*ln(F)+2*e)*F^(c*(b*x+a))
```

maxima [A] time = 0.37, size = 187, normalized size = 0.75

$$\frac{1}{4}f^2\left(\frac{F^{ac}e^{\left(bcx\log(F)+2ex+2d\right)}}{bc\log(F)+2e}+\frac{F^{ac}e^{\left(bcx\log(F)-2ex\right)}}{bce^{(2d)}\log(F)-2ee^{(2d)}}+\frac{2F^{bcx+ac}}{bc\log(F)}\right)+f^2\left(\frac{F^{ac}e^{\left(bcx\log(F)+ex+d\right)}}{bc\log(F)+e}+\frac{F^{ac}e^{\left(bcx\log(F)-ex\right)}}{bce^{d}\log(F)-ee^{d}}\cdots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*(f+f*cosh(e*x+d))^2,x, algorithm="maxima")
```

```
[Out] 1/4*f^2*(F^(a*c)*e^(b*c*x*log(F) + 2*e*x + 2*d)/(b*c*log(F) + 2*e) + F^(a*c
)*e^(b*c*x*log(F) - 2*e*x)/(b*c*e^(2*d)*log(F) - 2*e*e^(2*d)) + 2*F^(b*c*x
+ a*c)/(b*c*log(F))) + f^2*(F^(a*c)*e^(b*c*x*log(F) + e*x + d)/(b*c*log(F)
+ e) + F^(a*c)*e^(b*c*x*log(F) - e*x)/(b*c*e^d*log(F) - e*e^d)) + F^(b*c*x
+ a*c)*f^2/(b*c*log(F))
```

mupad [B] time = 1.99, size = 288, normalized size = 1.15

$$\frac{2F^{bcx}F^{ac}ef^2\sinh(d+ex)}{e^2-b^2c^2\ln(F)^2}+\frac{F^{bcx}F^{ac}f^2}{bc\ln(F)}+\frac{2F^{bcx}F^{ac}ef^2\cosh(d+ex)\sinh(d+ex)}{4e^2-b^2c^2\ln(F)^2}-\frac{2F^{bcx}F^{ac}bcf^2\cosh(\cdots}{e^2-b^2c^2\ln(\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(F^(c*(a + b*x))*(f + f*cosh(d + e*x))^2,x)
```

```
[Out] (2*F^(b*c*x)*F^(a*c)*e*f^2*sinh(d + e*x))/(e^2 - b^2*c^2*log(F)^2) + (F^(b*
c*x)*F^(a*c)*f^2)/(b*c*log(F)) + (2*F^(b*c*x)*F^(a*c)*e*f^2*cosh(d + e*x)*s
```

```
inh(d + e*x))/(4*e^2 - b^2*c^2*log(F)^2) - (2*F^(b*c*x)*F^(a*c)*b*c*f^2*cos
h(d + e*x)*log(F))/(e^2 - b^2*c^2*log(F)^2) - (2*F^(b*c*x)*F^(a*c)*e^2*f^2*
sinh(d + e*x)^2)/(b*c*log(F)*(4*e^2 - b^2*c^2*log(F)^2)) + (F^(b*c*x)*F^(a*
c)*f^2*cosh(d + e*x)^2*(2*e^2 - b^2*c^2*log(F)^2))/(b*c*log(F)*(4*e^2 - b^2
*c^2*log(F)^2))
```

sympy [A] time = 61.57, size = 1719, normalized size = 6.85

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F**(c*(b*x+a))*(f+f*cosh(e*x+d))**2,x)
```

```
[Out] Piecewise((-f**2*x*sinh(d + e*x)**2/2 + f**2*x*cosh(d + e*x)**2/2 + f**2*x
+ f**2*sinh(d + e*x)*cosh(d + e*x)/(2*e) + 2*f**2*sinh(d + e*x)/e, Eq(F, 1)
), (zoo*e**4*f**2*exp(-2*e/(b*c))**(a*c)*exp(-2*e/(b*c))**(b*c*x)*sinh(d +
e*x)**2 + zoo*e**4*f**2*exp(-2*e/(b*c))**(a*c)*exp(-2*e/(b*c))**(b*c*x)*sin
h(d + e*x)*cosh(d + e*x) + zoo*e**4*f**2*exp(-2*e/(b*c))**(a*c)*exp(-2*e/(b
*c))**(b*c*x)*cosh(d + e*x)**2, Eq(F, exp(-2*e/(b*c)))), (zoo*e**4*f**2*exp
(-e/(b*c))**(a*c)*exp(-e/(b*c))**(b*c*x)*sinh(d + e*x) + zoo*e**4*f**2*exp(
-e/(b*c))**(a*c)*exp(-e/(b*c))**(b*c*x)*cosh(d + e*x), Eq(F, exp(-e/(b*c)))
), (zoo*e**4*f**2*exp(e/(b*c))**(a*c)*exp(e/(b*c))**(b*c*x)*sinh(d + e*x) +
 zoo*e**4*f**2*exp(e/(b*c))**(a*c)*exp(e/(b*c))**(b*c*x)*cosh(d + e*x), Eq(
F, exp(e/(b*c)))), (zoo*e**4*f**2*exp(2*e/(b*c))**(a*c)*exp(2*e/(b*c))**(b*
c*x)*sinh(d + e*x)**2 + zoo*e**4*f**2*exp(2*e/(b*c))**(a*c)*exp(2*e/(b*c))*
*(b*c*x)*sinh(d + e*x)*cosh(d + e*x) + zoo*e**4*f**2*exp(2*e/(b*c))**(a*c)*
exp(2*e/(b*c))**(b*c*x)*cosh(d + e*x)**2, Eq(F, exp(2*e/(b*c)))), (F**(a*c)
*(-f**2*x*sinh(d + e*x)**2/2 + f**2*x*cosh(d + e*x)**2/2 + f**2*x + f**2*si
nh(d + e*x)*cosh(d + e*x)/(2*e) + 2*f**2*sinh(d + e*x)/e), Eq(b, 0)), (-f**
2*x*sinh(d + e*x)**2/2 + f**2*x*cosh(d + e*x)**2/2 + f**2*x + f**2*sinh(d +
 e*x)*cosh(d + e*x)/(2*e) + 2*f**2*sinh(d + e*x)/e, Eq(c, 0)), (F**(a*c)*F*
*(b*c*x)*b**4*c**4*f**2*log(F)**4*cosh(d + e*x)**2/(b**5*c**5*log(F)**5 - 5
*b**3*c**3*e**2*log(F)**3 + 4*b*c*e**4*log(F)) + 2*F**(a*c)*F**(b*c*x)*b**4
*c**4*f**2*log(F)**4*cosh(d + e*x)/(b**5*c**5*log(F)**5 - 5*b**3*c**3*e**2*
log(F)**3 + 4*b*c*e**4*log(F)) + F**(a*c)*F**(b*c*x)*b**4*c**4*f**2*log(F)*
*4/(b**5*c**5*log(F)**5 - 5*b**3*c**3*e**2*log(F)**3 + 4*b*c*e**4*log(F)) -
 2*F**(a*c)*F**(b*c*x)*b**3*c**3*e*f**2*log(F)**3*sinh(d + e*x)*cosh(d + e*
x)/(b**5*c**5*log(F)**5 - 5*b**3*c**3*e**2*log(F)**3 + 4*b*c*e**4*log(F)) -
 2*F**(a*c)*F**(b*c*x)*b**3*c**3*e*f**2*log(F)**3*sinh(d + e*x)/(b**5*c**5*
log(F)**5 - 5*b**3*c**3*e**2*log(F)**3 + 4*b*c*e**4*log(F)) + 2*F**(a*c)*F*
*(b*c*x)*b**2*c**2*e**2*f**2*log(F)**2*sinh(d + e*x)**2/(b**5*c**5*log(F)**
5 - 5*b**3*c**3*e**2*log(F)**3 + 4*b*c*e**4*log(F)) - 3*F**(a*c)*F**(b*c*x)
*b**2*c**2*e**2*f**2*log(F)**2*cosh(d + e*x)**2/(b**5*c**5*log(F)**5 - 5*b*
*3*c**3*e**2*log(F)**3 + 4*b*c*e**4*log(F)) - 8*F**(a*c)*F**(b*c*x)*b**2*c*
*2*e**2*f**2*log(F)**2*cosh(d + e*x)/(b**5*c**5*log(F)**5 - 5*b**3*c**3*e**
```

```
2*log(F)**3 + 4*b*c*e**4*log(F)) - 5*F**(a*c)*F**(b*c*x)*b**2*c**2*e**2*f**
2*log(F)**2/(b**5*c**5*log(F)**5 - 5*b**3*c**3*e**2*log(F)**3 + 4*b*c*e**4*
log(F)) + 2*F**(a*c)*F**(b*c*x)*b*c*e**3*f**2*log(F)*sinh(d + e*x)*cosh(d +
 e*x)/(b**5*c**5*log(F)**5 - 5*b**3*c**3*e**2*log(F)**3 + 4*b*c*e**4*log(F)
) + 8*F**(a*c)*F**(b*c*x)*b*c*e**3*f**2*log(F)*sinh(d + e*x)/(b**5*c**5*log
(F)**5 - 5*b**3*c**3*e**2*log(F)**3 + 4*b*c*e**4*log(F)) - 2*F**(a*c)*F**(b
*c*x)*e**4*f**2*sinh(d + e*x)**2/(b**5*c**5*log(F)**5 - 5*b**3*c**3*e**2*lo
g(F)**3 + 4*b*c*e**4*log(F)) + 2*F**(a*c)*F**(b*c*x)*e**4*f**2*cosh(d + e*x
)**2/(b**5*c**5*log(F)**5 - 5*b**3*c**3*e**2*log(F)**3 + 4*b*c*e**4*log(F))
 + 4*F**(a*c)*F**(b*c*x)*e**4*f**2/(b**5*c**5*log(F)**5 - 5*b**3*c**3*e**2*
log(F)**3 + 4*b*c*e**4*log(F)), True))
```

3.898 $\int F^{c(a+bx)}(f+f\cosh(d+ex))\,dx$

Optimal. Leaf size=101

$$\frac{ef\sinh(d+ex)F^{ac+bcx}}{e^2-b^2c^2\log^2(F)}-\frac{bcf\log(F)\cosh(d+ex)F^{ac+bcx}}{e^2-b^2c^2\log^2(F)}+\frac{fF^{ac+bcx}}{bc\log(F)}$$

```
[Out] f*F^(b*c*x+a*c)/b/c/ln(F)-b*c*f*F^(b*c*x+a*c)*cosh(e*x+d)*ln(F)/(e^2-b^2*c^
2*ln(F)^2)+e*f*F^(b*c*x+a*c)*sinh(e*x+d)/(e^2-b^2*c^2*ln(F)^2)
```

Rubi [A] time = 0.15, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6741, 12, 6742, 2194, 5475}

$$\frac{ef\sinh(d+ex)F^{ac+bcx}}{e^2-b^2c^2\log^2(F)}-\frac{bcf\log(F)\cosh(d+ex)F^{ac+bcx}}{e^2-b^2c^2\log^2(F)}+\frac{fF^{ac+bcx}}{bc\log(F)}$$

Antiderivative was successfully verified.

```
[In] Int[F^(c*(a + b*x))*(f + f*Cosh[d + e*x]),x]
```

```
[Out] (f*F^(a*c + b*c*x))/(b*c*Log[F]) - (b*c*f*F^(a*c + b*c*x)*Cosh[d + e*x]*Log
[F])/(e^2 - b^2*c^2*Log[F]^2) + (e*f*F^(a*c + b*c*x)*Sinh[d + e*x])/(e^2 -
b^2*c^2*Log[F]^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 5475

```
Int[Cosh[(d_.) + (e_.)*(x_)]*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symbol] :
> -Simp[(b*c*Log[F]*F^(c*(a + b*x))*Cosh[d + e*x])/(e^2 - b^2*c^2*Log[F]^2)
, x] + Simp[(e*F^(c*(a + b*x))*Sinh[d + e*x])/(e^2 - b^2*c^2*Log[F]^2), x]
/; FreeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 - b^2*c^2*Log[F]^2, 0]
```

Rule 6741

```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}\int F^{c(a+bx)}(f+f\cosh(d+ex))\,dx &= \int fF^{ac+bcx}(1+\cosh(d+ex))\,dx\\ &= f\int F^{ac+bcx}(1+\cosh(d+ex))\,dx\\ &= f\int \left(F^{ac+bcx}+F^{ac+bcx}\cosh(d+ex)\right)\,dx\\ &= f\int F^{ac+bcx}\,dx+f\int F^{ac+bcx}\cosh(d+ex)\,dx\\ &= \frac{fF^{ac+bcx}}{bc\log(F)}-\frac{bcfF^{ac+bcx}\cosh(d+ex)\log(F)}{e^2-b^2c^2\log^2(F)}+\frac{efF^{ac+bcx}\sinh(d+ex)}{e^2-b^2c^2\log^2(F)}\end{aligned}$$

Mathematica [A] time = 0.24, size = 88, normalized size = 0.87

$$\frac{fF^{c(a+bx)}\left(b^2c^2\log^2(F)\cosh(d+ex)+b^2c^2\log^2(F)-bce\log(F)\sinh(d+ex)-e^2\right)}{bc\log(F)(bc\log(F)-e)(bc\log(F)+e)}$$

Antiderivative was successfully verified.

```
[In] Integrate[F^(c*(a + b*x))*(f + f*Cosh[d + e*x]),x]
```

```
[Out] (f*F^(c*(a + b*x))*(-e^2 + b^2*c^2*Log[F]^2 + b^2*c^2*Cosh[d + e*x]*Log[F]^
2 - b*c*e*Log[F]*Sinh[d + e*x]))/(b*c*Log[F]*(-e + b*c*Log[F])*(e + b*c*Log
[F]))
```

fricas [B] time = 0.42, size = 430, normalized size = 4.26

$$-\frac{\left(2\,e^2f\cosh\left(ex+d\right)-\left(b^2c^2f\cosh\left(ex+d\right)^2+2\,b^2c^2f\cosh\left(ex+d\right)+b^2c^2f\right)\log(F)^2-\left(b^2c^2f\log(F)^2-bcef\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*(f+f*cosh(e*x+d)),x, algorithm="fricas")
```

```
[Out] -1/2*((2*e^2*f*cosh(e*x + d) - (b^2*c^2*f*cosh(e*x + d)^2 + 2*b^2*c^2*f*cos
h(e*x + d) + b^2*c^2*f)*log(F)^2 - (b^2*c^2*f*log(F)^2 - b*c*e*f*log(F))*si
nh(e*x + d)^2 + (b*c*e*f*cosh(e*x + d)^2 - b*c*e*f)*log(F) + 2*(b*c*e*f*cos
h(e*x + d)*log(F) + e^2*f - (b^2*c^2*f*cosh(e*x + d) + b^2*c^2*f)*log(F)^2)
*sinh(e*x + d))*cosh((b*c*x + a*c)*log(F)) + (2*e^2*f*cosh(e*x + d) - (b^2*
c^2*f*cosh(e*x + d)^2 + 2*b^2*c^2*f*cosh(e*x + d) + b^2*c^2*f)*log(F)^2 - (
b^2*c^2*f*log(F)^2 - b*c*e*f*log(F))*sinh(e*x + d)^2 + (b*c*e*f*cosh(e*x +
d)^2 - b*c*e*f)*log(F) + 2*(b*c*e*f*cosh(e*x + d)*log(F) + e^2*f - (b^2*c^2
*f*cosh(e*x + d) + b^2*c^2*f)*log(F)^2)*sinh(e*x + d))*sinh((b*c*x + a*c)*l
og(F)))/(b^3*c^3*cosh(e*x + d)*log(F)^3 - b*c*e^2*cosh(e*x + d)*log(F) + (b
^3*c^3*log(F)^3 - b*c*e^2*log(F))*sinh(e*x + d))
```

giac [C] time = 0.23, size = 900, normalized size = 8.91

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*(f+f*cosh(e*x+d)),x, algorithm="giac")
```

```
[Out] 2*(2*b*c*f*cos(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*b*c*x - 1/2*pi*a*c*sgn(F) + 1/
2*pi*a*c)*log(abs(F))/(4*b^2*c^2*log(abs(F))^2 + (pi*b*c*sgn(F) - pi*b*c)^2
) - (pi*b*c*sgn(F) - pi*b*c)*f*sin(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*b*c*x - 1/
2*pi*a*c*sgn(F) + 1/2*pi*a*c)/(4*b^2*c^2*log(abs(F))^2 + (pi*b*c*sgn(F) - p
i*b*c)^2))*e^(b*c*x*log(abs(F)) + a*c*log(abs(F))) - 1/2*I*(-2*I*f*e^(1/2*I
*pi*b*c*x*sgn(F) - 1/2*I*pi*b*c*x + 1/2*I*pi*a*c*sgn(F) - 1/2*I*pi*a*c)/(I*
pi*b*c*sgn(F) - I*pi*b*c + 2*b*c*log(abs(F))) + 2*I*f*e^(-1/2*I*pi*b*c*x*sg
n(F) + 1/2*I*pi*b*c*x - 1/2*I*pi*a*c*sgn(F) + 1/2*I*pi*a*c)/(-I*pi*b*c*sgn(
F) + I*pi*b*c + 2*b*c*log(abs(F))))*e^(b*c*x*log(abs(F)) + a*c*log(abs(F)))
 + (2*(b*c*log(abs(F)) + e)*f*cos(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*b*c*x - 1/2
*pi*a*c*sgn(F) + 1/2*pi*a*c)/((pi*b*c*sgn(F) - pi*b*c)^2 + 4*(b*c*log(abs(F
)) + e)^2) - (pi*b*c*sgn(F) - pi*b*c)*f*sin(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*b
*c*x - 1/2*pi*a*c*sgn(F) + 1/2*pi*a*c)/((pi*b*c*sgn(F) - pi*b*c)^2 + 4*(b*c
*log(abs(F)) + e)^2))*e^(a*c*log(abs(F)) + (b*c*log(abs(F)) + e)*x + d) - 1
/2*I*(-2*I*f*e^(1/2*I*pi*b*c*x*sgn(F) - 1/2*I*pi*b*c*x + 1/2*I*pi*a*c*sgn(F
) - 1/2*I*pi*a*c)/(2*I*pi*b*c*sgn(F) - 2*I*pi*b*c + 4*b*c*log(abs(F)) + 4*e
) + 2*I*f*e^(-1/2*I*pi*b*c*x*sgn(F) + 1/2*I*pi*b*c*x - 1/2*I*pi*a*c*sgn(F)
+ 1/2*I*pi*a*c)/(-2*I*pi*b*c*sgn(F) + 2*I*pi*b*c + 4*b*c*log(abs(F)) + 4*e)
)*e^(a*c*log(abs(F)) + (b*c*log(abs(F)) + e)*x + d) + (2*(b*c*log(abs(F)) -
 e)*f*cos(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*b*c*x - 1/2*pi*a*c*sgn(F) + 1/2*pi*
a*c)/((pi*b*c*sgn(F) - pi*b*c)^2 + 4*(b*c*log(abs(F)) - e)^2) - (pi*b*c*sgn
(F) - pi*b*c)*f*sin(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*b*c*x - 1/2*pi*a*c*sgn(F)
 + 1/2*pi*a*c)/((pi*b*c*sgn(F) - pi*b*c)^2 + 4*(b*c*log(abs(F)) - e)^2))*e^
(a*c*log(abs(F)) + (b*c*log(abs(F)) - e)*x - d) - 1/2*I*(-2*I*f*e^(1/2*I*pi
*b*c*x*sgn(F) - 1/2*I*pi*b*c*x + 1/2*I*pi*a*c*sgn(F) - 1/2*I*pi*a*c)/(2*I*p
i*b*c*sgn(F) - 2*I*pi*b*c + 4*b*c*log(abs(F)) - 4*e) + 2*I*f*e^(-1/2*I*pi*b
```

```
*c*x*sgn(F) + 1/2*I*pi*b*c*x - 1/2*I*pi*a*c*sgn(F) + 1/2*I*pi*a*c)/(-2*I*pi
*b*c*sgn(F) + 2*I*pi*b*c + 4*b*c*log(abs(F)) - 4*e))*e^(a*c*log(abs(F)) + (
b*c*log(abs(F)) - e)*x - d)
```

maple [A] time = 0.22, size = 135, normalized size = 1.34

$$\frac{f\left(\ln(F)^2b^2c^2\mathrm{e}^{2ex+2d}+2\ln(F)^2b^2c^2\mathrm{e}^{ex+d}+b^2c^2\ln(F)^2-\ln(F)bce\,\mathrm{e}^{2ex+2d}+\ln(F)bce-2e^2\mathrm{e}^{ex+d}\right)\mathrm{e}^{-ex-d}F^{c(bx+a)}}{2bc\ln(F)\left(bc\ln(F)-e\right)\left(e+bc\ln(F)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(F^(c*(b*x+a))*(f+f*cosh(e*x+d)),x)
```

```
[Out] 1/2*f*(ln(F)^2*b^2*c^2*exp(2*e*x+2*d)+2*ln(F)^2*b^2*c^2*exp(e*x+d)+b^2*c^2*
ln(F)^2-ln(F)*b*c*e*exp(2*e*x+2*d)+ln(F)*b*c*e-2*e^2*exp(e*x+d))/b/c/ln(F)/
(b*c*ln(F)-e)*exp(-e*x-d)/(e+b*c*ln(F))*F^(c*(b*x+a))
```

maxima [A] time = 0.33, size = 87, normalized size = 0.86

$$\frac{1}{2}f\left(\frac{F^{ac}e^{\left(bcx\log(F)+ex+d\right)}}{bc\log(F)+e}+\frac{F^{ac}e^{\left(bcx\log(F)-ex\right)}}{bce^d\log(F)-ee^d}\right)+\frac{F^{bcx+ac}f}{bc\log(F)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*(f+f*cosh(e*x+d)),x, algorithm="maxima")
```

```
[Out] 1/2*f*(F^(a*c)*e^(b*c*x*log(F) + e*x + d)/(b*c*log(F) + e) + F^(a*c)*e^(b*c
*x*log(F) - e*x)/(b*c*e^d*log(F) - e*e^d)) + F^(b*c*x + a*c)*f/(b*c*log(F))
```

mupad [B] time = 1.74, size = 134, normalized size = 1.33

$$-\frac{F^{b\,c\,x}\,F^{a\,c}\,f\,\mathrm{e}^{-d-e\,x}\,\left(b^2\,c^2\,\ln(F)^2-2\,e^2\,\mathrm{e}^{d+e\,x}+b\,c\,e\,\ln(F)+2\,b^2\,c^2\,\mathrm{e}^{d+e\,x}\ln(F)^2+b^2\,c^2\,\mathrm{e}^{2\,d+2\,e\,x}\ln(F)^2-b\,c\,e\,\mathrm{e}^{2\,d-}\right.}{2\,b\,c\,\ln(F)\,\left(e^2-b^2\,c^2\ln(F)^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(F^(c*(a + b*x))*(f + f*cosh(d + e*x)),x)
```

```
[Out] -(F^(b*c*x)*F^(a*c)*f*exp(- d - e*x)*(b^2*c^2*log(F)^2 - 2*e^2*exp(d + e*x)
 + b*c*e*log(F) + 2*b^2*c^2*exp(d + e*x)*log(F)^2 + b^2*c^2*exp(2*d + 2*e*x
)*log(F)^2 - b*c*e*exp(2*d + 2*e*x)*log(F)))/(2*b*c*log(F)*(e^2 - b^2*c^2*l
og(F)^2))
```

sympy [A] time = 8.46, size = 391, normalized size = 3.87

$$\begin{cases} fx + \frac{f\sinh(d+ex)}{e} & \text{for } F = 1 \\ \tilde{\infty} e^2 f \left(e^{-\frac{e}{bc}}\right)^{ac} \left(e^{-\frac{e}{bc}}\right)^{bcx} \sinh(d+ex) + \tilde{\infty} e^2 f \left(e^{-\frac{e}{bc}}\right)^{ac} \left(e^{-\frac{e}{bc}}\right)^{bcx} \cosh(d+ex) & \text{for } F = e^{-\frac{e}{bc}} \\ \tilde{\infty} e^2 f \left(e^{\frac{e}{bc}}\right)^{ac} \left(e^{\frac{e}{bc}}\right)^{bcx} \sinh(d+ex) + \tilde{\infty} e^2 f \left(e^{\frac{e}{bc}}\right)^{ac} \left(e^{\frac{e}{bc}}\right)^{bcx} \cosh(d+ex) & \text{for } F = e^{\frac{e}{bc}} \\ F^{ac}\left(fx + \frac{f\sinh(d+ex)}{e}\right) & \text{for } b = 0 \\ fx + \frac{f\sinh(d+ex)}{e} & \text{for } c = 0 \\ \frac{F^{ac}F^{bcx}b^2c^2f\log(F)^2\cosh(d+ex)}{b^3c^3\log(F)^3 - bce^2\log(F)} + \frac{F^{ac}F^{bcx}b^2c^2f\log(F)^2}{b^3c^3\log(F)^3 - bce^2\log(F)} - \frac{F^{ac}F^{bcx}bcef\log(F)\sinh(d+ex)}{b^3c^3\log(F)^3 - bce^2\log(F)} - \frac{F^{ac}F^{bcx}e^2f}{b^3c^3\log(F)^3 - bce^2\log(F)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F**(c*(b*x+a))*(f+f*cosh(e*x+d)),x)
```

```
[Out] Piecewise((f*x + f*sinh(d + e*x)/e, Eq(F, 1)), (zoo*e**2*f*exp(-e/(b*c))**(
a*c)*exp(-e/(b*c))**(b*c*x)*sinh(d + e*x) + zoo*e**2*f*exp(-e/(b*c))**(a*c)
*exp(-e/(b*c))**(b*c*x)*cosh(d + e*x), Eq(F, exp(-e/(b*c)))), (zoo*e**2*f*e
xp(e/(b*c))**(a*c)*exp(e/(b*c))**(b*c*x)*sinh(d + e*x) + zoo*e**2*f*exp(e/(
b*c))**(a*c)*exp(e/(b*c))**(b*c*x)*cosh(d + e*x), Eq(F, exp(e/(b*c)))), (F*
*(a*c)*(f*x + f*sinh(d + e*x)/e), Eq(b, 0)), (f*x + f*sinh(d + e*x)/e, Eq(c
, 0)), (F**(a*c)*F**(b*c*x)*b**2*c**2*f*log(F)**2*cosh(d + e*x)/(b**3*c**3*
log(F)**3 - b*c*e**2*log(F)) + F**(a*c)*F**(b*c*x)*b**2*c**2*f*log(F)**2/(b
**3*c**3*log(F)**3 - b*c*e**2*log(F)) - F**(a*c)*F**(b*c*x)*b*c*e*f*log(F)*
sinh(d + e*x)/(b**3*c**3*log(F)**3 - b*c*e**2*log(F)) - F**(a*c)*F**(b*c*x)
*e**2*f/(b**3*c**3*log(F)**3 - b*c*e**2*log(F)), True))
```

3.899 $\int \frac{F^{c(a+bx)}}{f+f\cosh(d+ex)}\,dx$

Optimal. Leaf size=61

$$\frac{2e^{d+ex}F^{c(a+bx)}\,{}_2F_1\left(2,\frac{bc\log(F)}{e}+1;\frac{bc\log(F)}{e}+2;-e^{d+ex}\right)}{f(bc\log(F)+e)}$$

```
[Out] 2*exp(e*x+d)*F^(c*(b*x+a))*hypergeom([2, 1+b*c*ln(F)/e],[2+b*c*ln(F)/e],-ex
p(e*x+d))/f/(e+b*c*ln(F))
```

Rubi [A] time = 0.06, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {5497, 5492}

$$\frac{2e^{d+ex}F^{c(a+bx)}\,{}_2F_1\left(2,\frac{bc\log(F)}{e}+1;\frac{bc\log(F)}{e}+2;-e^{d+ex}\right)}{f(bc\log(F)+e)}$$

Antiderivative was successfully verified.

```
[In] Int[F^(c*(a + b*x))/(f + f*Cosh[d + e*x]),x]
```

```
[Out] (2*E^(d + e*x)*F^(c*(a + b*x))*Hypergeometric2F1[2, 1 + (b*c*Log[F])/e, 2 +
 (b*c*Log[F])/e, -E^(d + e*x)])/(f*(e + b*c*Log[F]))
```

Rule 5492

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sech[(d_.) + (e_.)*(x_)]^(n_.), x_Sym
bol] :> Simp[(2^n*E^(n*(d + e*x))*F^(c*(a + b*x))*Hypergeometric2F1[n, n/2
+ (b*c*Log[F])/(2*e), 1 + n/2 + (b*c*Log[F])/(2*e), -E^(2*(d + e*x))])/(e*n
 + b*c*Log[F]), x] /; FreeQ[{F, a, b, c, d, e}, x] && IntegerQ[n]
```

Rule 5497

```
Int[(Cosh[(d_.) + (e_.)*(x_)]*(g_.) + (f_))^(n_.)*(F_)^((c_.)*((a_.) + (b_.
)*(x_))), x_Symbol] :> Dist[2^n*g^n, Int[F^(c*(a + b*x))*Cosh[d/2 + (e*x)/2
]^(2*n), x], x] /; FreeQ[{F, a, b, c, d, e, f, g}, x] && EqQ[f - g, 0] && I
LtQ[n, 0]
```

Rubi steps

$$\int \frac{F^{c(a+bx)}}{f+f\cosh(d+ex)}\,dx = \frac{\int F^{c(a+bx)}\operatorname{sech}^2\left(\frac{d}{2}+\frac{ex}{2}\right)dx}{2f}$$
$$= \frac{2e^{d+ex}F^{c(a+bx)}\,{}_2F_1\left(2,1+\frac{bc\log(F)}{e};2+\frac{bc\log(F)}{e};-e^{d+ex}\right)}{f(e+bc\log(F))}$$

Mathematica [A] time = 0.05, size = 61, normalized size = 1.00

$$\frac{2e^{d+ex}F^{c(a+bx)}\,{}_2F_1\left(2,\frac{bc\log(F)}{e}+1;\frac{bc\log(F)}{e}+2;-e^{d+ex}\right)}{bcf\log(F)+ef}$$

Antiderivative was successfully verified.

```
[In] Integrate[F^(c*(a + b*x))/(f + f*Cosh[d + e*x]),x]
```

```
[Out] (2*E^(d + e*x)*F^(c*(a + b*x))*Hypergeometric2F1[2, 1 + (b*c*Log[F])/e, 2 +
 (b*c*Log[F])/e, -E^(d + e*x)])/(e*f + b*c*f*Log[F])
```

fricas [F] time = 0.51, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{F^{bcx+ac}}{f\cosh(ex+d)+f},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))/(f+f*cosh(e*x+d)),x, algorithm="fricas")
```

```
[Out] integral(F^(b*c*x + a*c)/(f*cosh(e*x + d) + f), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{F^{(bx+a)c}}{f\cosh(ex+d)+f}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))/(f+f*cosh(e*x+d)),x, algorithm="giac")
```

```
[Out] integrate(F^((b*x + a)*c)/(f*cosh(e*x + d) + f), x)
```

maple [F] time = 0.19, size = 0, normalized size = 0.00

$$\int \frac{F^{c(bx+a)}}{f+f\cosh(ex+d)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(F^(c*(b*x+a))/(f+f*cosh(e*x+d)),x)
```

```
[Out] int(F^(c*(b*x+a))/(f+f*cosh(e*x+d)),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$4 F^{ac}bce \int \frac{}{b^2c^2 f \log(F)^2 - 3 bcef \log(F) + 2 e^2 f + \left(b^2c^2 f e^{(3d)} \log(F)^2 - 3 bcef e^{(3d)} \log(F) + 2 e^2 f e^{(3d)}\right) e^{(3ex)} +}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))/(f+f*cosh(e*x+d)),x, algorithm="maxima")
```

```
[Out] 4*F^(a*c)*b*c*e*integrate(F^(b*c*x)/(b^2*c^2*f*log(F)^2 - 3*b*c*e*f*log(F)
+ 2*e^2*f + (b^2*c^2*f*e^(3*d)*log(F)^2 - 3*b*c*e*f*e^(3*d)*log(F) + 2*e^2*
f*e^(3*d))*e^(3*e*x) + 3*(b^2*c^2*f*e^(2*d)*log(F)^2 - 3*b*c*e*f*e^(2*d)*lo
g(F) + 2*e^2*f*e^(2*d))*e^(2*e*x) + 3*(b^2*c^2*f*e^d*log(F)^2 - 3*b*c*e*f*e
^d*log(F) + 2*e^2*f*e^d)*e^(e*x)), x)*log(F) - 2*(2*F^(a*c)*e - (F^(a*c)*b*
c*e^d*log(F) - 2*F^(a*c)*e*e^d)*e^(e*x))*F^(b*c*x)/(b^2*c^2*f*log(F)^2 - 3*
b*c*e*f*log(F) + 2*e^2*f + (b^2*c^2*f*e^(2*d)*log(F)^2 - 3*b*c*e*f*e^(2*d)*
log(F) + 2*e^2*f*e^(2*d))*e^(2*e*x) + 2*(b^2*c^2*f*e^d*log(F)^2 - 3*b*c*e*f
*e^d*log(F) + 2*e^2*f*e^d)*e^(e*x))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{F^{c\,(a+b\,x)}}{f+f\cosh\left(d+e\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(F^(c*(a + b*x))/(f + f*cosh(d + e*x)),x)
```

```
[Out] int(F^(c*(a + b*x))/(f + f*cosh(d + e*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{F^{ac}F^{bcx}}{\cosh\left(d+ex\right)+1}\,dx}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F**(c*(b*x+a))/(f+f*cosh(e*x+d)),x)
```

```
[Out] Integral(F**(a*c)*F**(b*c*x)/(cosh(d + e*x) + 1), x)/f
```

3.900 $\int \frac{F^{c(a+bx)}}{(f+f\cosh(d+ex))^2}\,dx$

Optimal. Leaf size=151

$$\frac{2e^{d+ex}F^{c(a+bx)}(e-bc\log(F))\,{}_2F_1\left(2,\frac{bc\log(F)}{e}+1;\frac{bc\log(F)}{e}+2;-e^{d+ex}\right)}{3e^2f^2}+\frac{bc\log(F)\mathrm{sech}^2\left(\frac{d}{2}+\frac{ex}{2}\right)F^{c(a+bx)}}{6e^2f^2}+\frac{\tanh\left(\frac{d}{2}+\right.}{}$$

```
[Out] 2/3*exp(e*x+d)*F^(c*(b*x+a))*hypergeom([2, 1+b*c*ln(F)/e],[2+b*c*ln(F)/e],-
exp(e*x+d))*(e-b*c*ln(F))/e^2/f^2+1/6*b*c*F^(c*(b*x+a))*ln(F)*sech(1/2*e*x+
1/2*d)^2/e^2/f^2+1/6*F^(c*(b*x+a))*sech(1/2*e*x+1/2*d)^2*tanh(1/2*e*x+1/2*d
)/e/f^2
```

Rubi [A] time = 0.10, antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {5497, 5490, 5492}

$$\frac{2e^{d+ex}F^{c(a+bx)}(e-bc\log(F))\,{}_2F_1\left(2,\frac{bc\log(F)}{e}+1;\frac{bc\log(F)}{e}+2;-e^{d+ex}\right)}{3e^2f^2}+\frac{bc\log(F)\mathrm{sech}^2\left(\frac{d}{2}+\frac{ex}{2}\right)F^{c(a+bx)}}{6e^2f^2}+\frac{\tanh\left(\frac{d}{2}+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[F^(c*(a + b*x))/(f + f*Cosh[d + e*x])^2,x]
```

```
[Out] (2*E^(d + e*x)*F^(c*(a + b*x))*Hypergeometric2F1[2, 1 + (b*c*Log[F])/e, 2 +
 (b*c*Log[F])/e, -E^(d + e*x)]*(e - b*c*Log[F]))/(3*e^2*f^2) + (b*c*F^(c*(a
 + b*x))*Log[F]*Sech[d/2 + (e*x)/2]^2)/(6*e^2*f^2) + (F^(c*(a + b*x))*Sech[
d/2 + (e*x)/2]^2*Tanh[d/2 + (e*x)/2])/(6*e*f^2)
```

Rule 5490

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sech[(d_.) + (e_.)*(x_)]^(n_), x_Symb
ol] :> Simp[(b*c*Log[F]*F^(c*(a + b*x))*Sech[d + e*x]^(n - 2))/(e^2*(n - 1)
*(n - 2)), x] + (Dist[(e^2*(n - 2)^2 - b^2*c^2*Log[F]^2)/(e^2*(n - 1)*(n -
2)), Int[F^(c*(a + b*x))*Sech[d + e*x]^(n - 2), x], x] + Simp[(F^(c*(a + b*
x))*Sech[d + e*x]^(n - 1)*Sinh[d + e*x])/(e*(n - 1)), x]) /; FreeQ[{F, a, b
, c, d, e}, x] && NeQ[e^2*(n - 2)^2 - b^2*c^2*Log[F]^2, 0] && GtQ[n, 1] &&
NeQ[n, 2]
```

Rule 5492

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sech[(d_.) + (e_.)*(x_)]^(n_.), x_Sym
bol] :> Simp[(2^n*E^(n*(d + e*x))*F^(c*(a + b*x))*Hypergeometric2F1[n, n/2
+ (b*c*Log[F])/(2*e), 1 + n/2 + (b*c*Log[F])/(2*e), -E^(2*(d + e*x))])/(e*n
```

```
+ b*c*Log[F]), x] /; FreeQ[{F, a, b, c, d, e}, x] && IntegerQ[n]
```

Rule 5497

```
Int[(Cosh[(d_.) + (e_.)*(x_)]*(g_.) + (f_))^(n_.)*(F_)^((c_.)*((a_.) + (b_.
)*(x_))), x_Symbol] :> Dist[2^n*g^n, Int[F^(c*(a + b*x))*Cosh[d/2 + (e*x)/2
]^(2*n), x], x] /; FreeQ[{F, a, b, c, d, e, f, g}, x] && EqQ[f - g, 0] && I
LtQ[n, 0]
```

Rubi steps

$$\int \frac{F^{c(a+bx)}}{(f+f\cosh(d+ex))^2}\,dx = \frac{\int F^{c(a+bx)}\mathrm{sech}^4\left(\frac{d}{2}+\frac{ex}{2}\right)dx}{4f^2}$$

$$= \frac{bcF^{c(a+bx)}\log(F)\mathrm{sech}^2\left(\frac{d}{2}+\frac{ex}{2}\right)}{6e^2f^2} + \frac{F^{c(a+bx)}\mathrm{sech}^2\left(\frac{d}{2}+\frac{ex}{2}\right)\tanh\left(\frac{d}{2}+\frac{ex}{2}\right)}{6ef^2} + \frac{\left(1-\frac{b^2}{\cdots}\right.}{\cdots}$$

$$= \frac{2e^{d+ex}F^{c(a+bx)}\,{}_2F_1\left(2,1+\frac{bc\log(F)}{e};2+\frac{bc\log(F)}{e};-e^{d+ex}\right)(e-bc\log(F))}{3e^2f^2} + \frac{bcF^{c(a+bx)}}{\cdots}$$

Mathematica [A] time = 0.35, size = 127, normalized size = 0.84

$$\frac{2\cosh\left(\frac{1}{2}(d+ex)\right)F^{c(a+bx)}\left(4e^{d+ex}\cosh^3\left(\frac{1}{2}(d+ex)\right)(e-bc\log(F))\,{}_2F_1\left(2,\frac{bc\log(F)}{e}+1;\frac{bc\log(F)}{e}+2;-e^{d+ex}\right)+bc\cdots\right.}{3e^2f^2(\cosh(d+ex)+1)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[F^(c*(a + b*x))/(f + f*Cosh[d + e*x])^2,x]
```

```
[Out] (2*F^(c*(a + b*x))*Cosh[(d + e*x)/2]*(b*c*Cosh[(d + e*x)/2]*Log[F] + 4*E^(d
 + e*x)*Cosh[(d + e*x)/2]^3*Hypergeometric2F1[2, 1 + (b*c*Log[F])/e, 2 + (b
*c*Log[F])/e, -E^(d + e*x)]*(e - b*c*Log[F]) + e*Sinh[(d + e*x)/2]))/(3*e^2
*f^2*(1 + Cosh[d + e*x])^2)
```

fricas [F] time = 0.49, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{F^{bcx+ac}}{f^2\cosh(ex+d)^2+2f^2\cosh(ex+d)+f^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))/(f+f*cosh(e*x+d))^2,x, algorithm="fricas")
```

```
[Out] integral(F^(b*c*x + a*c)/(f^2*cosh(e*x + d)^2 + 2*f^2*cosh(e*x + d) + f^2),
 x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{F^{(bx+a)c}}{\left(f\cosh\left(ex+d\right)+f\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))/(f+f*cosh(e*x+d))^2,x, algorithm="giac")
```

```
[Out] integrate(F^((b*x + a)*c)/(f*cosh(e*x + d) + f)^2, x)
```

maple [F] time = 0.39, size = 0, normalized size = 0.00

$$\int \frac{F^{c(bx+a)}}{\left(f+f\cosh\left(ex+d\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(F^(c*(b*x+a))/(f+f*cosh(e*x+d))^2,x)
```

```
[Out] int(F^(c*(b*x+a))/(f+f*cosh(e*x+d))^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))/(f+f*cosh(e*x+d))^2,x, algorithm="maxima")
```

```
[Out] -16*(F^(a*c)*b^2*c^2*e*log(F)^2 + F^(a*c)*b*c*e^2*log(F))*integrate(F^(b*c*
x)/(b^3*c^3*f^2*log(F)^3 - 9*b^2*c^2*e*f^2*log(F)^2 + 26*b*c*e^2*f^2*log(F)
 - 24*e^3*f^2 + (b^3*c^3*f^2*e^(5*d)*log(F)^3 - 9*b^2*c^2*e*f^2*e^(5*d)*log
(F)^2 + 26*b*c*e^2*f^2*e^(5*d)*log(F) - 24*e^3*f^2*e^(5*d))*e^(5*e*x) + 5*(
b^3*c^3*f^2*e^(4*d)*log(F)^3 - 9*b^2*c^2*e*f^2*e^(4*d)*log(F)^2 + 26*b*c*e^
2*f^2*e^(4*d)*log(F) - 24*e^3*f^2*e^(4*d))*e^(4*e*x) + 10*(b^3*c^3*f^2*e^(3
*d)*log(F)^3 - 9*b^2*c^2*e*f^2*e^(3*d)*log(F)^2 + 26*b*c*e^2*f^2*e^(3*d)*lo
g(F) - 24*e^3*f^2*e^(3*d))*e^(3*e*x) + 10*(b^3*c^3*f^2*e^(2*d)*log(F)^3 - 9
*b^2*c^2*e*f^2*e^(2*d)*log(F)^2 + 26*b*c*e^2*f^2*e^(2*d)*log(F) - 24*e^3*f^
2*e^(2*d))*e^(2*e*x) + 5*(b^3*c^3*f^2*e^d*log(F)^3 - 9*b^2*c^2*e*f^2*e^d*lo
g(F)^2 + 26*b*c*e^2*f^2*e^d*log(F) - 24*e^3*f^2*e^d)*e^(e*x)), x) + 4*(4*F^
```

```
(a*c)*b*c*e*log(F) + 4*F^(a*c)*e^2 + (F^(a*c)*b^2*c^2*e^(2*d)*log(F)^2 - 7*
F^(a*c)*b*c*e*e^(2*d)*log(F) + 12*F^(a*c)*e^2*e^(2*d))*e^(2*e*x) - 4*(F^(a*
c)*b*c*e*e^d*log(F) - 4*F^(a*c)*e^2*e^d)*e^(e*x))*F^(b*c*x)/(b^3*c^3*f^2*lo
g(F)^3 - 9*b^2*c^2*e*f^2*log(F)^2 + 26*b*c*e^2*f^2*log(F) - 24*e^3*f^2 + (b
^3*c^3*f^2*e^(4*d)*log(F)^3 - 9*b^2*c^2*e*f^2*e^(4*d)*log(F)^2 + 26*b*c*e^2
*f^2*e^(4*d)*log(F) - 24*e^3*f^2*e^(4*d))*e^(4*e*x) + 4*(b^3*c^3*f^2*e^(3*d
)*log(F)^3 - 9*b^2*c^2*e*f^2*e^(3*d)*log(F)^2 + 26*b*c*e^2*f^2*e^(3*d)*log(
F) - 24*e^3*f^2*e^(3*d))*e^(3*e*x) + 6*(b^3*c^3*f^2*e^(2*d)*log(F)^3 - 9*b^
2*c^2*e*f^2*e^(2*d)*log(F)^2 + 26*b*c*e^2*f^2*e^(2*d)*log(F) - 24*e^3*f^2*e
^(2*d))*e^(2*e*x) + 4*(b^3*c^3*f^2*e^d*log(F)^3 - 9*b^2*c^2*e*f^2*e^d*log(F
)^2 + 26*b*c*e^2*f^2*e^d*log(F) - 24*e^3*f^2*e^d)*e^(e*x))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{F^{c\,(a+b\,x)}}{\left(f+f\cosh\left(d+e\,x\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(F^(c*(a + b*x))/(f + f*cosh(d + e*x))^2,x)
[Out] int(F^(c*(a + b*x))/(f + f*cosh(d + e*x))^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{F^{ac}F^{bcx}}{\cosh^2(d+ex)+2\cosh(d+ex)+1}\,dx}{f^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F**(c*(b*x+a))/(f+f*cosh(e*x+d))**2,x)
[Out] Integral(F**(a*c)*F**(b*c*x)/(cosh(d + e*x)**2 + 2*cosh(d + e*x) + 1), x)/f
**2
```

3.901 $\int e^{a+bx}\cosh(a+bx)\sinh^3(a+bx)\,dx$

Optimal. Leaf size=69

$$\frac{e^{-3a-3bx}}{48b}-\frac{e^{-a-bx}}{8b}-\frac{e^{3a+3bx}}{24b}+\frac{e^{5a+5bx}}{80b}$$

```
[Out] 1/48*exp(-3*b*x-3*a)/b-1/8*exp(-b*x-a)/b-1/24*exp(3*b*x+3*a)/b+1/80*exp(5*b
*x+5*a)/b
```

Rubi [A] time = 0.05, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {2282, 12, 448}

$$\frac{e^{-3a-3bx}}{48b}-\frac{e^{-a-bx}}{8b}-\frac{e^{3a+3bx}}{24b}+\frac{e^{5a+5bx}}{80b}$$

Antiderivative was successfully verified.

```
[In] Int[E^(a + b*x)*Cosh[a + b*x]*Sinh[a + b*x]^3,x]
```

```
[Out] E^(-3*a - 3*b*x)/(48*b) - E^(-a - b*x)/(8*b) - E^(3*a + 3*b*x)/(24*b) + E^(
5*a + 5*b*x)/(80*b)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$\int e^{a+bx}\cosh(a+bx)\sinh^3(a+bx)\,dx = \frac{\text{Subst}\left(\int \frac{(-1-x^2)(1-x^2)^3}{16x^4}\,dx, x, e^{a+bx}\right)}{b}$$
$$= \frac{\text{Subst}\left(\int \frac{(-1-x^2)(1-x^2)^3}{x^4}\,dx, x, e^{a+bx}\right)}{16b}$$
$$= \frac{\text{Subst}\left(\int \left(-\frac{1}{x^4}+\frac{2}{x^2}-2x^2+x^4\right)dx, x, e^{a+bx}\right)}{16b}$$
$$= \frac{e^{-3a-3bx}}{48b}-\frac{e^{-a-bx}}{8b}-\frac{e^{3a+3bx}}{24b}+\frac{e^{5a+5bx}}{80b}$$

Mathematica [A] time = 0.07, size = 51, normalized size = 0.74

$$\frac{e^{-3(a+bx)}\left(-30e^{2(a+bx)}-10e^{6(a+bx)}+3e^{8(a+bx)}+5\right)}{240b}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(a + b*x)*Cosh[a + b*x]*Sinh[a + b*x]^3,x]
```

```
[Out] (5 - 30*E^(2*(a + b*x)) - 10*E^(6*(a + b*x)) + 3*E^(8*(a + b*x)))/(240*b*E^
(3*(a + b*x)))
```

fricas [A] time = 0.48, size = 111, normalized size = 1.61

$$\frac{\cosh(bx+a)^4-\cosh(bx+a)\sinh(bx+a)^3+\sinh(bx+a)^4+\left(6\cosh(bx+a)^2-5\right)\sinh(bx+a)^2-5\cosh}{30\,(b\cosh(bx+a)-b\sinh(bx+a))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*cosh(b*x+a)*sinh(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/30*(cosh(b*x + a)^4 - cosh(b*x + a)*sinh(b*x + a)^3 + sinh(b*x + a)^4 + (
6*cosh(b*x + a)^2 - 5)*sinh(b*x + a)^2 - 5*cosh(b*x + a)^2 - (cosh(b*x + a)
^3 - 5*cosh(b*x + a))*sinh(b*x + a))/(b*cosh(b*x + a) - b*sinh(b*x + a))
```

giac [A] time = 0.12, size = 52, normalized size = 0.75

$$-\frac{5\left(6\,e^{(2\,bx+2\,a)}-1\right)e^{(-3\,bx-3\,a)}-3\,e^{(5\,bx+5\,a)}+10\,e^{(3\,bx+3\,a)}}{240\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*cosh(b*x+a)*sinh(b*x+a)^3,x, algorithm="giac")
```

```
[Out] -1/240*(5*(6*e^(2*b*x + 2*a) - 1)*e^(-3*b*x - 3*a) - 3*e^(5*b*x + 5*a) + 10
*e^(3*b*x + 3*a))/b
```

maple [A] time = 0.16, size = 44, normalized size = 0.64

$$\frac{\frac{\left(\sinh^5(bx+a)\right)}{5}+\frac{\left(\cosh^3(bx+a)\right)\left(\sinh^2(bx+a)\right)}{5}-\frac{2\left(\cosh^3(bx+a)\right)}{15}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(b*x+a)*cosh(b*x+a)*sinh(b*x+a)^3,x)
```

```
[Out] 1/b*(1/5*sinh(b*x+a)^5+1/5*cosh(b*x+a)^3*sinh(b*x+a)^2-2/15*cosh(b*x+a)^3)
```

maxima [A] time = 0.32, size = 56, normalized size = 0.81

$$-\frac{\left(6\,e^{(2\,bx+2\,a)}-1\right)e^{(-3\,bx-3\,a)}}{48\,b}+\frac{3\,e^{(5\,bx+5\,a)}-10\,e^{(3\,bx+3\,a)}}{240\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*cosh(b*x+a)*sinh(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -1/48*(6*e^(2*b*x + 2*a) - 1)*e^(-3*b*x - 3*a)/b + 1/240*(3*e^(5*b*x + 5*a)
 - 10*e^(3*b*x + 3*a))/b
```

mupad [B] time = 0.55, size = 50, normalized size = 0.72

$$-\frac{30\,\mathrm{e}^{-a-b\,x}-5\,\mathrm{e}^{-3\,a-3\,b\,x}+10\,\mathrm{e}^{3\,a+3\,b\,x}-3\,\mathrm{e}^{5\,a+5\,b\,x}}{240\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a + b*x)*exp(a + b*x)*sinh(a + b*x)^3,x)
```

```
[Out] -(30*exp(- a - b*x) - 5*exp(- 3*a - 3*b*x) + 10*exp(3*a + 3*b*x) - 3*exp(5*
a + 5*b*x))/(240*b)
```

sympy [A] time = 60.11, size = 139, normalized size = 2.01

$$\begin{cases} \frac{e^a e^{bx}\sinh^4(a+bx)}{5b}-\frac{e^a e^{bx}\sinh^3(a+bx)\cosh(a+bx)}{5b}+\frac{e^a e^{bx}\sinh^2(a+bx)\cosh^2(a+bx)}{5b}+\frac{2e^a e^{bx}\sinh(a+bx)\cosh^3(a+bx)}{15b}-\frac{2e^a e^{bx}\cosh^4(a+bx}{15b} \\ xe^a\sinh^3(a)\cosh(a) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*cosh(b*x+a)*sinh(b*x+a)**3,x)
```

```
[Out] Piecewise((exp(a)*exp(b*x)*sinh(a + b*x)**4/(5*b) - exp(a)*exp(b*x)*sinh(a
+ b*x)**3*cosh(a + b*x)/(5*b) + exp(a)*exp(b*x)*sinh(a + b*x)**2*cosh(a + b
*x)**2/(5*b) + 2*exp(a)*exp(b*x)*sinh(a + b*x)*cosh(a + b*x)**3/(15*b) - 2*
exp(a)*exp(b*x)*cosh(a + b*x)**4/(15*b), Ne(b, 0)), (x*exp(a)*sinh(a)**3*co
sh(a), True))
```

3.902 $\int e^{a+bx}\cosh(a+bx)\sinh^2(a+bx)\,dx$

Optimal. Leaf size=57

$$-\frac{e^{-2a-2bx}}{16b}-\frac{e^{2a+2bx}}{16b}+\frac{e^{4a+4bx}}{32b}-\frac{x}{8}$$

```
[Out] -1/16*exp(-2*b*x-2*a)/b-1/16*exp(2*b*x+2*a)/b+1/32*exp(4*b*x+4*a)/b-1/8*x
```

Rubi [A] time = 0.05, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2282, 12, 446, 75}

$$-\frac{e^{-2a-2bx}}{16b}-\frac{e^{2a+2bx}}{16b}+\frac{e^{4a+4bx}}{32b}-\frac{x}{8}$$

Antiderivative was successfully verified.

```
[In] Int[E^(a + b*x)*Cosh[a + b*x]*Sinh[a + b*x]^2,x]
```

```
[Out] -E^(-2*a - 2*b*x)/(16*b) - E^(2*a + 2*b*x)/(16*b) + E^(4*a + 4*b*x)/(32*b)
- x/8
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 75

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && EqQ[b*e + a*f, 0] &&  !(ILtQ[n + p
+ 2, 0] && GtQ[n + 2*p, 0])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
```

```
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$
\begin{aligned}
\int e^{a+bx}\cosh(a+bx)\sinh^2(a+bx)\,dx &= \frac{\operatorname{Subst}\left(\int \frac{(1-x^2)^2(1+x^2)}{8x^3}\,dx, x, e^{a+bx}\right)}{b}\\
&= \frac{\operatorname{Subst}\left(\int \frac{(1-x^2)^2(1+x^2)}{x^3}\,dx, x, e^{a+bx}\right)}{8b}\\
&= \frac{\operatorname{Subst}\left(\int \frac{(1-x)^2(1+x)}{x^2}\,dx, x, e^{2a+2bx}\right)}{16b}\\
&= \frac{\operatorname{Subst}\left(\int \left(-1+\frac{1}{x^2}-\frac{1}{x}+x\right)dx, x, e^{2a+2bx}\right)}{16b}\\
&= -\frac{e^{-2a-2bx}}{16b}-\frac{e^{2a+2bx}}{16b}+\frac{e^{4a+4bx}}{32b}-\frac{x}{8}
\end{aligned}
$$

Mathematica [A] time = 0.06, size = 45, normalized size = 0.79

$$
-\frac{2e^{-2(a+bx)}+2e^{2(a+bx)}-e^{4(a+bx)}+4bx}{32b}
$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(a + b*x)*Cosh[a + b*x]*Sinh[a + b*x]^2,x]
```

```
[Out] -1/32*(2/E^(2*(a + b*x)) + 2*E^(2*(a + b*x)) - E^(4*(a + b*x)) + 4*b*x)/b
```

fricas [B] time = 0.43, size = 95, normalized size = 1.67

$$
-\frac{\cosh(bx+a)^3+3\cosh(bx+a)\sinh(bx+a)^2-3\sinh(bx+a)^3+2(2bx+1)\cosh(bx+a)-\left(4bx+9\cos\right.}{32(b\cosh(bx+a)-b\sinh(bx+a))}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*cosh(b*x+a)*sinh(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/32*(cosh(b*x + a)^3 + 3*cosh(b*x + a)*sinh(b*x + a)^2 - 3*sinh(b*x + a)^
3 + 2*(2*b*x + 1)*cosh(b*x + a) - (4*b*x + 9*cosh(b*x + a)^2 - 2)*sinh(b*x
+ a))/(b*cosh(b*x + a) - b*sinh(b*x + a))
```

giac [A] time = 0.12, size = 57, normalized size = 1.00

$$-\frac{4\,bx - 2\left(e^{(2\,bx+2\,a)} - 1\right)e^{(-2\,bx-2\,a)} + 4\,a - e^{(4\,bx+4\,a)} + 2\,e^{(2\,bx+2\,a)}}{32\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*cosh(b*x+a)*sinh(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -1/32*(4*b*x - 2*(e^(2*b*x + 2*a) - 1)*e^(-2*b*x - 2*a) + 4*a - e^(4*b*x +
4*a) + 2*e^(2*b*x + 2*a))/b
```

maple [A] time = 0.13, size = 53, normalized size = 0.93

$$\frac{\frac{\left(\sinh^4(bx+a)\right)}{4} + \frac{\left(\cosh^3(bx+a)\right)\sinh(bx+a)}{4} - \frac{\cosh(bx+a)\sinh(bx+a)}{8} - \frac{bx}{8} - \frac{a}{8}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(b*x+a)*cosh(b*x+a)*sinh(b*x+a)^2,x)
```

```
[Out] 1/b*(1/4*sinh(b*x+a)^4+1/4*cosh(b*x+a)^3*sinh(b*x+a)-1/8*cosh(b*x+a)*sinh(b
*x+a)-1/8*b*x-1/8*a)
```

maxima [A] time = 0.34, size = 50, normalized size = 0.88

$$-\frac{1}{8}\,x - \frac{a}{8\,b} + \frac{e^{(4\,bx+4\,a)} - 2\,e^{(2\,bx+2\,a)}}{32\,b} - \frac{e^{(-2\,bx-2\,a)}}{16\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*cosh(b*x+a)*sinh(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -1/8*x - 1/8*a/b + 1/32*(e^(4*b*x + 4*a) - 2*e^(2*b*x + 2*a))/b - 1/16*e^(-
2*b*x - 2*a)/b
```

mupad [B] time = 0.28, size = 43, normalized size = 0.75

$$-\frac{x}{8} - \frac{\frac{e^{-2\,a-2\,b\,x}}{16} + \frac{e^{2\,a+2\,b\,x}}{16} - \frac{e^{4\,a+4\,b\,x}}{32}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a + b*x)*exp(a + b*x)*sinh(a + b*x)^2,x)
```

```
[Out] - x/8 - (exp(- 2*a - 2*b*x)/16 + exp(2*a + 2*b*x)/16 - exp(4*a + 4*b*x)/32)
/b
```

sympy [A] time = 18.39, size = 177, normalized size = 3.11

$$\begin{cases} -\frac{xe^{a}e^{bx}\sinh^{3}(a+bx)}{8} + \frac{xe^{a}e^{bx}\sinh^{2}(a+bx)\cosh(a+bx)}{8} + \frac{xe^{a}e^{bx}\sinh(a+bx)\cosh^{2}(a+bx)}{8} - \frac{xe^{a}e^{bx}\cosh^{3}(a+bx)}{8} + \frac{3e^{a}e^{bx}\sinh^{3}(a+bx)}{8b} - \frac{e \\ xe^{a}\sinh^{2}(a)\cosh(a) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*cosh(b*x+a)*sinh(b*x+a)**2,x)
```

```
[Out] Piecewise((-x*exp(a)*exp(b*x)*sinh(a + b*x)**3/8 + x*exp(a)*exp(b*x)*sinh(a
 + b*x)**2*cosh(a + b*x)/8 + x*exp(a)*exp(b*x)*sinh(a + b*x)*cosh(a + b*x)*
*2/8 - x*exp(a)*exp(b*x)*cosh(a + b*x)**3/8 + 3*exp(a)*exp(b*x)*sinh(a + b*
x)**3/(8*b) - exp(a)*exp(b*x)*sinh(a + b*x)**2*cosh(a + b*x)/(4*b) + exp(a)
*exp(b*x)*cosh(a + b*x)**3/(8*b), Ne(b, 0)), (x*exp(a)*sinh(a)**2*cosh(a),
True))
```

3.903 $\int e^{a+bx} \cosh(a+bx) \sinh(a+bx) \, dx$

Optimal. Leaf size=35

$$\frac{e^{-a-bx}}{4b} + \frac{e^{3a+3bx}}{12b}$$

```
[Out] 1/4*exp(-b*x-a)/b+1/12*exp(3*b*x+3*a)/b
```

Rubi [A] time = 0.03, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {2282, 12, 14}

$$\frac{e^{-a-bx}}{4b} + \frac{e^{3a+3bx}}{12b}$$

Antiderivative was successfully verified.

```
[In] Int[E^(a + b*x)*Cosh[a + b*x]*Sinh[a + b*x],x]
```

```
[Out] E^(-a - b*x)/(4*b) + E^(3*a + 3*b*x)/(12*b)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$\int e^{a+bx}\cosh(a+bx)\sinh(a+bx)\,dx = \frac{\text{Subst}\left(\int \frac{-1+x^4}{4x^2}\,dx, x, e^{a+bx}\right)}{b}$$
$$= \frac{\text{Subst}\left(\int \frac{-1+x^4}{x^2}\,dx, x, e^{a+bx}\right)}{4b}$$
$$= \frac{\text{Subst}\left(\int \left(-\frac{1}{x^2}+x^2\right)\,dx, x, e^{a+bx}\right)}{4b}$$
$$= \frac{e^{-a-bx}}{4b} + \frac{e^{3a+3bx}}{12b}$$

Mathematica [A] time = 0.01, size = 28, normalized size = 0.80

$$\frac{e^{-a-bx}\left(e^{4(a+bx)}+3\right)}{12b}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(a + b*x)*Cosh[a + b*x]*Sinh[a + b*x],x]
```

```
[Out] (E^(-a - b*x)*(3 + E^(4*(a + b*x))))/(12*b)
```

fricas [A] time = 0.45, size = 53, normalized size = 1.51

$$\frac{\cosh(bx+a)^2 - \cosh(bx+a)\sinh(bx+a) + \sinh(bx+a)^2}{3\,(b\cosh(bx+a) - b\sinh(bx+a))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*cosh(b*x+a)*sinh(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/3*(cosh(b*x + a)^2 - cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2)/(b*co
sh(b*x + a) - b*sinh(b*x + a))
```

giac [A] time = 0.11, size = 26, normalized size = 0.74

$$\frac{e^{(3\,bx+3\,a)} + 3\,e^{(-bx-a)}}{12\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*cosh(b*x+a)*sinh(b*x+a),x, algorithm="giac")
```

```
[Out] 1/12*(e^(3*b*x + 3*a) + 3*e^(-b*x - a))/b
```

maple [A] time = 0.03, size = 26, normalized size = 0.74

$$\frac{\frac{\left(\sinh^{3}(bx+a)\right)}{3}+\frac{\left(\cosh^{3}(bx+a)\right)}{3}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(b*x+a)*cosh(b*x+a)*sinh(b*x+a),x)
```

```
[Out] 1/b*(1/3*sinh(b*x+a)^3+1/3*cosh(b*x+a)^3)
```

maxima [A] time = 0.34, size = 29, normalized size = 0.83

$$\frac{e^{(3\,bx+3\,a)}}{12\,b}+\frac{e^{(-bx-a)}}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*cosh(b*x+a)*sinh(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/12*e^(3*b*x + 3*a)/b + 1/4*e^(-b*x - a)/b
```

mupad [B] time = 1.75, size = 26, normalized size = 0.74

$$\frac{3\,\mathrm{e}^{-a-b\,x}+\mathrm{e}^{3\,a+3\,b\,x}}{12\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a + b*x)*exp(a + b*x)*sinh(a + b*x),x)
```

```
[Out] (3*exp(- a - b*x) + exp(3*a + 3*b*x))/(12*b)
```

sympy [A] time = 4.99, size = 76, normalized size = 2.17

$$\begin{cases} \frac{e^{a}e^{bx}\sinh^{2}(a+bx)}{3b}-\frac{e^{a}e^{bx}\sinh(a+bx)\cosh(a+bx)}{3b}+\frac{e^{a}e^{bx}\cosh^{2}(a+bx)}{3b} & \text{for } b\neq 0 \\ xe^{a}\sinh(a)\cosh(a) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*cosh(b*x+a)*sinh(b*x+a),x)
```

```
[Out] Piecewise((exp(a)*exp(b*x)*sinh(a + b*x)**2/(3*b) - exp(a)*exp(b*x)*sinh(a
+ b*x)*cosh(a + b*x)/(3*b) + exp(a)*exp(b*x)*cosh(a + b*x)**2/(3*b), Ne(b,
0)), (x*exp(a)*sinh(a)*cosh(a), True))
```

3.904 $\int e^{a+bx}\coth(a+bx)\,dx$

Optimal. Leaf size=25

$$\frac{e^{a+bx}}{b}-\frac{2\tanh^{-1}\left(e^{a+bx}\right)}{b}$$

```
[Out] exp(b*x+a)/b-2*arctanh(exp(b*x+a))/b
```

Rubi [A] time = 0.02, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {2282, 388, 206}

$$\frac{e^{a+bx}}{b}-\frac{2\tanh^{-1}\left(e^{a+bx}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[E^(a + b*x)*Coth[a + b*x],x]
```

```
[Out] E^(a + b*x)/b - (2*ArcTanh[E^(a + b*x)])/b
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$\int e^{a+bx}\coth(a+bx)\,dx = \frac{\text{Subst}\left(\int \frac{-1-x^2}{1-x^2}\,dx, x, e^{a+bx}\right)}{b}$$
$$= \frac{e^{a+bx}}{b} - \frac{2\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, e^{a+bx}\right)}{b}$$
$$= \frac{e^{a+bx}}{b} - \frac{2\tanh^{-1}\left(e^{a+bx}\right)}{b}$$

Mathematica [A] time = 0.02, size = 22, normalized size = 0.88

$$\frac{e^{a+bx} - 2\tanh^{-1}\left(e^{a+bx}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(a + b*x)*Coth[a + b*x],x]
[Out] (E^(a + b*x) - 2*ArcTanh[E^(a + b*x)])/b
```

fricas [B] time = 0.43, size = 49, normalized size = 1.96

$$\frac{\cosh(bx+a) - \log(\cosh(bx+a) + \sinh(bx+a) + 1) + \log(\cosh(bx+a) + \sinh(bx+a) - 1) + \sinh(bx+a)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*cosh(b*x+a)*csch(b*x+a),x, algorithm="fricas")
[Out] (cosh(b*x + a) - log(cosh(b*x + a) + sinh(b*x + a) + 1) + log(cosh(b*x + a)
 + sinh(b*x + a) - 1) + sinh(b*x + a))/b
```

giac [A] time = 0.13, size = 32, normalized size = 1.28

$$\frac{e^{(bx+a)} - \log\left(e^{(bx+a)} + 1\right) + \log\left(\left|e^{(bx+a)} - 1\right|\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*cosh(b*x+a)*csch(b*x+a),x, algorithm="giac")
[Out] (e^(b*x + a) - log(e^(b*x + a) + 1) + log(abs(e^(b*x + a) - 1)))/b
```

maple [A] time = 0.16, size = 27, normalized size = 1.08

$$\frac{\sinh(bx+a)+\cosh(bx+a)-2\operatorname{arctanh}\left(e^{bx+a}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(b*x+a)*cosh(b*x+a)*csch(b*x+a),x)
[Out] 1/b*(sinh(b*x+a)+cosh(b*x+a)-2*arctanh(exp(b*x+a)))
```

maxima [A] time = 0.32, size = 38, normalized size = 1.52

$$\frac{e^{(bx+a)}}{b}-\frac{\log\left(e^{(bx+a)}+1\right)}{b}+\frac{\log\left(e^{(bx+a)}-1\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*cosh(b*x+a)*csch(b*x+a),x, algorithm="maxima")
[Out] e^(b*x + a)/b - log(e^(b*x + a) + 1)/b + log(e^(b*x + a) - 1)/b
```

mupad [B] time = 0.06, size = 38, normalized size = 1.52

$$\frac{e^{a+b\,x}}{b}-\frac{2\,\mathrm{atan}\left(\frac{e^{b\,x}\,e^{a}\,\sqrt{-b^2}}{b}\right)}{\sqrt{-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cosh(a + b*x)*exp(a + b*x))/sinh(a + b*x),x)
[Out] exp(a + b*x)/b - (2*atan((exp(b*x)*exp(a)*(-b^2)^(1/2))/b))/(-b^2)^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$e^{a}\int e^{bx}\cosh(a+bx)\operatorname{csch}(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*cosh(b*x+a)*csch(b*x+a),x)
[Out] exp(a)*Integral(exp(b*x)*cosh(a + b*x)*csch(a + b*x), x)
```

3.905 $\int e^{a+bx} \coth(a+bx)\mathbf{csch}(a+bx)\,dx$

Optimal. Leaf size=41

$$\frac{2}{b\left(1-e^{2a+2bx}\right)}+\frac{\log\left(1-e^{2a+2bx}\right)}{b}$$

```
[Out] 2/b/(1-exp(2*b*x+2*a))+ln(1-exp(2*b*x+2*a))/b
```

Rubi [A] time = 0.04, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2282, 12, 444, 43}

$$\frac{2}{b\left(1-e^{2a+2bx}\right)}+\frac{\log\left(1-e^{2a+2bx}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[E^(a + b*x)*Coth[a + b*x]*Csch[a + b*x],x]
```

```
[Out] 2/(b*(1 - E^(2*a + 2*b*x))) + Log[1 - E^(2*a + 2*b*x)]/b
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
```

```
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$
\begin{aligned}
\int e^{a+bx}\coth(a+bx)\operatorname{csch}(a+bx)\,dx &= \frac{\operatorname{Subst}\left(\int \frac{2x(1+x^2)}{(1-x^2)^2}\,dx, x, e^{a+bx}\right)}{b}\\
&= \frac{2\operatorname{Subst}\left(\int \frac{x(1+x^2)}{(1-x^2)^2}\,dx, x, e^{a+bx}\right)}{b}\\
&= \frac{\operatorname{Subst}\left(\int \frac{1+x}{(1-x)^2}\,dx, x, e^{2a+2bx}\right)}{b}\\
&= \frac{\operatorname{Subst}\left(\int \left(\frac{2}{(-1+x)^2}+\frac{1}{-1+x}\right)dx, x, e^{2a+2bx}\right)}{b}\\
&= \frac{2}{b\left(1-e^{2a+2bx}\right)}+\frac{\log\left(1-e^{2a+2bx}\right)}{b}
\end{aligned}
$$

Mathematica [A] time = 0.05, size = 34, normalized size = 0.83

$$
\frac{\log\left(1-e^{2(a+bx)}\right)-\frac{2}{e^{2(a+bx)}-1}}{b}
$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(a + b*x)*Coth[a + b*x]*Csch[a + b*x],x]
[Out] (-2/(-1 + E^(2*(a + b*x))) + Log[1 - E^(2*(a + b*x))])/b
```

fricas [B] time = 0.43, size = 103, normalized size = 2.51

$$
\frac{\left(\cosh(bx+a)^2+2\cosh(bx+a)\sinh(bx+a)+\sinh(bx+a)^2-1\right)\log\left(\frac{2\sinh(bx+a)}{\cosh(bx+a)-\sinh(bx+a)}\right)-2}{b\cosh(bx+a)^2+2b\cosh(bx+a)\sinh(bx+a)+b\sinh(bx+a)^2-b}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*cosh(b*x+a)*csch(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] ((cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 - 1)*lo
g(2*sinh(b*x + a)/(cosh(b*x + a) - sinh(b*x + a))) - 2)/(b*cosh(b*x + a)^2
+ 2*b*cosh(b*x + a)*sinh(b*x + a) + b*sinh(b*x + a)^2 - b)
```

giac [A] time = 0.14, size = 46, normalized size = 1.12

$$-\frac{\frac{e^{(2bx+2a)}+1}{e^{(2bx+2a)}-1} - \log\left(\left|e^{(2bx+2a)} - 1\right|\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*cosh(b*x+a)*csch(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -((e^(2*b*x + 2*a) + 1)/(e^(2*b*x + 2*a) - 1) - log(abs(e^(2*b*x + 2*a) - 1
)))/b
```

maple [A] time = 0.16, size = 30, normalized size = 0.73

$$x - \frac{\coth(bx+a)}{b} + \frac{\ln(\sinh(bx+a))}{b} + \frac{a}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(b*x+a)*cosh(b*x+a)*csch(b*x+a)^2,x)
```

```
[Out] x-coth(b*x+a)/b+ln(sinh(b*x+a))/b+a/b
```

maxima [A] time = 0.32, size = 45, normalized size = 1.10

$$\frac{\log\left(e^{(bx+a)} + 1\right)}{b} + \frac{\log\left(e^{(bx+a)} - 1\right)}{b} - \frac{2}{b\left(e^{(2bx+2a)} - 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*cosh(b*x+a)*csch(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] log(e^(b*x + a) + 1)/b + log(e^(b*x + a) - 1)/b - 2/(b*(e^(2*b*x + 2*a) - 1
))
```

mupad [B] time = 1.77, size = 36, normalized size = 0.88

$$\frac{\ln\left(e^{2a}\,e^{2bx} - 1\right)}{b} - \frac{2}{b\,\left(e^{2a+2bx} - 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cosh(a + b*x)*exp(a + b*x))/sinh(a + b*x)^2,x)
```

```
[Out] log(exp(2*a)*exp(2*b*x) - 1)/b - 2/(b*(exp(2*a + 2*b*x) - 1))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*cosh(b*x+a)*csch(b*x+a)**2,x)
```

```
[Out] Timed out
```

3.906 $\int e^{a+bx} \coth(a+bx)\mathbf{csch}^2(a+bx)\,dx$

Optimal. Leaf size=70

$$\frac{3e^{a+bx}}{b\left(1-e^{2a+2bx}\right)}-\frac{2e^{a+bx}}{b\left(1-e^{2a+2bx}\right)^2}-\frac{\tanh^{-1}\left(e^{a+bx}\right)}{b}$$

```
[Out] -2*exp(b*x+a)/b/(1-exp(2*b*x+2*a))^2+3*exp(b*x+a)/b/(1-exp(2*b*x+2*a))-arct
anh(exp(b*x+a))/b
```

Rubi [A] time = 0.06, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {2282, 12, 455, 385, 206}

$$\frac{3e^{a+bx}}{b\left(1-e^{2a+2bx}\right)}-\frac{2e^{a+bx}}{b\left(1-e^{2a+2bx}\right)^2}-\frac{\tanh^{-1}\left(e^{a+bx}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[E^(a + b*x)*Coth[a + b*x]*Csch[a + b*x]^2,x]
```

```
[Out] (-2*E^(a + b*x))/(b*(1 - E^(2*a + 2*b*x))^2) + (3*E^(a + b*x))/(b*(1 - E^(2
*a + 2*b*x))) - ArcTanh[E^(a + b*x)]/b
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 455

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[((-a)^(m/2 - 1)*(b*c - a*d)*x*(a + b*x^2)^(p + 1))/(2*b^(m/2 + 1)*(p
 + 1)), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[(a + b*x^2)^(p + 1)*Expand
ToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2 -
 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /; F
reeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &&
 (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$
\begin{aligned}
\int e^{a+bx}\coth(a+bx)\operatorname{csch}^2(a+bx)\,dx &= \frac{\text{Subst}\left(\int \frac{4x^2(-1-x^2)}{(1-x^2)^3}\,dx, x, e^{a+bx}\right)}{b} \\
&= \frac{4\,\text{Subst}\left(\int \frac{x^2(-1-x^2)}{(1-x^2)^3}\,dx, x, e^{a+bx}\right)}{b} \\
&= -\frac{2e^{a+bx}}{b\left(1-e^{2a+2bx}\right)^2} - \frac{\text{Subst}\left(\int \frac{-2-4x^2}{(1-x^2)^2}\,dx, x, e^{a+bx}\right)}{b} \\
&= -\frac{2e^{a+bx}}{b\left(1-e^{2a+2bx}\right)^2} + \frac{3e^{a+bx}}{b\left(1-e^{2a+2bx}\right)} - \frac{\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, e^{a+bx}\right)}{b} \\
&= -\frac{2e^{a+bx}}{b\left(1-e^{2a+2bx}\right)^2} + \frac{3e^{a+bx}}{b\left(1-e^{2a+2bx}\right)} - \frac{\tanh^{-1}\left(e^{a+bx}\right)}{b}
\end{aligned}
$$

Mathematica [A] time = 0.08, size = 59, normalized size = 0.84

$$
\frac{e^{a+bx} - 3e^{3(a+bx)} + \left(e^{2(a+bx)}-1\right)^2\left(-\tanh^{-1}\left(e^{a+bx}\right)\right)}{b\left(e^{2(a+bx)}-1\right)^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(a + b*x)*Coth[a + b*x]*Csch[a + b*x]^2,x]
```

```
[Out] (E^(a + b*x) - 3*E^(3*(a + b*x)) - (-1 + E^(2*(a + b*x)))^2*ArcTanh[E^(a +
b*x)])/(b*(-1 + E^(2*(a + b*x)))^2)
```

fricas [B] time = 0.45, size = 387, normalized size = 5.53

$$-\frac{6\cosh(bx+a)^3+18\cosh(bx+a)\sinh(bx+a)^2+6\sinh(bx+a)^3+\left(\cosh(bx+a)^4+4\cosh(bx+a)\sinh\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*cosh(b*x+a)*csch(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -1/2*(6*cosh(b*x + a)^3 + 18*cosh(b*x + a)*sinh(b*x + a)^2 + 6*sinh(b*x + a
)^3 + (cosh(b*x + a)^4 + 4*cosh(b*x + a)*sinh(b*x + a)^3 + sinh(b*x + a)^4
+ 2*(3*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^2 - 2*cosh(b*x + a)^2 + 4*(cosh(b
*x + a)^3 - cosh(b*x + a))*sinh(b*x + a) + 1)*log(cosh(b*x + a) + sinh(b*x
+ a) + 1) - (cosh(b*x + a)^4 + 4*cosh(b*x + a)*sinh(b*x + a)^3 + sinh(b*x +
 a)^4 + 2*(3*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^2 - 2*cosh(b*x + a)^2 + 4*(
cosh(b*x + a)^3 - cosh(b*x + a))*sinh(b*x + a) + 1)*log(cosh(b*x + a) + sin
h(b*x + a) - 1) + 2*(9*cosh(b*x + a)^2 - 1)*sinh(b*x + a) - 2*cosh(b*x + a)
)/(b*cosh(b*x + a)^4 + 4*b*cosh(b*x + a)*sinh(b*x + a)^3 + b*sinh(b*x + a)^
4 - 2*b*cosh(b*x + a)^2 + 2*(3*b*cosh(b*x + a)^2 - b)*sinh(b*x + a)^2 + 4*(
b*cosh(b*x + a)^3 - b*cosh(b*x + a))*sinh(b*x + a) + b)
```

giac [A] time = 0.12, size = 62, normalized size = 0.89

$$-\frac{\frac{2\left(3\,e^{(3\,bx+3\,a)}-e^{(bx+a)}\right)}{\left(e^{(2\,bx+2\,a)}-1\right)^2}+\log\left(e^{(bx+a)}+1\right)-\log\left(\left|e^{(bx+a)}-1\right|\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*cosh(b*x+a)*csch(b*x+a)^3,x, algorithm="giac")
```

```
[Out] -1/2*(2*(3*e^(3*b*x + 3*a) - e^(b*x + a))/(e^(2*b*x + 2*a) - 1)^2 + log(e^(
b*x + a) + 1) - log(abs(e^(b*x + a) - 1)))/b
```

maple [A] time = 0.42, size = 55, normalized size = 0.79

$$\frac{-\frac{1}{\sinh(bx+a)}-\frac{\cosh(bx+a)}{\sinh(bx+a)^2}+\frac{\operatorname{csch}(bx+a)\coth(bx+a)}{2}-\operatorname{arctanh}\left(e^{bx+a}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(b*x+a)*cosh(b*x+a)*csch(b*x+a)^3,x)
```

```
[Out] 1/b*(-1/sinh(b*x+a)-cosh(b*x+a)/sinh(b*x+a)^2+1/2*csch(b*x+a)*coth(b*x+a)-a
rctanh(exp(b*x+a)))
```

maxima [A] time = 0.32, size = 78, normalized size = 1.11

$$-\frac{\log\left(e^{(bx+a)}+1\right)}{2\,b}+\frac{\log\left(e^{(bx+a)}-1\right)}{2\,b}-\frac{3\,e^{(3\,bx+3\,a)}-e^{(bx+a)}}{b\left(e^{(4\,bx+4\,a)}-2\,e^{(2\,bx+2\,a)}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*cosh(b*x+a)*csch(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -1/2*log(e^(b*x + a) + 1)/b + 1/2*log(e^(b*x + a) - 1)/b - (3*e^(3*b*x + 3*
a) - e^(b*x + a))/(b*(e^(4*b*x + 4*a) - 2*e^(2*b*x + 2*a) + 1))
```

mupad [B] time = 1.76, size = 102, normalized size = 1.46

$$-\frac{\operatorname{atan}\left(\frac{e^{b\,x}\,e^{a}\,\sqrt{-b^2}}{b}\right)}{\sqrt{-b^2}}-\frac{\frac{e^{a+b\,x}}{b}+\frac{e^{3\,a+3\,b\,x}}{b}}{e^{4\,a+4\,b\,x}-2\,e^{2\,a+2\,b\,x}+1}-\frac{2\,e^{a+b\,x}}{b\,\left(e^{2\,a+2\,b\,x}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cosh(a + b*x)*exp(a + b*x))/sinh(a + b*x)^3,x)
```

```
[Out] - atan((exp(b*x)*exp(a)*(-b^2)^(1/2))/b)/(-b^2)^(1/2) - (exp(a + b*x)/b + e
xp(3*a + 3*b*x)/b)/(exp(4*a + 4*b*x) - 2*exp(2*a + 2*b*x) + 1) - (2*exp(a +
 b*x))/(b*(exp(2*a + 2*b*x) - 1))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*cosh(b*x+a)*csch(b*x+a)**3,x)
```

```
[Out] Timed out
```

3.907 $\int e^{a+bx} \cosh^2(a+bx) \sinh^3(a+bx)\,dx$

Optimal. Leaf size=91

$$\frac{e^{-4a-4bx}}{128b} - \frac{e^{-2a-2bx}}{64b} - \frac{e^{2a+2bx}}{32b} - \frac{e^{4a+4bx}}{128b} + \frac{e^{6a+6bx}}{192b} + \frac{x}{16}$$

```
[Out] 1/128*exp(-4*b*x-4*a)/b-1/64*exp(-2*b*x-2*a)/b-1/32*exp(2*b*x+2*a)/b-1/128*
exp(4*b*x+4*a)/b+1/192*exp(6*b*x+6*a)/b+1/16*x
```

Rubi [A] time = 0.07, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2282, 12, 446, 88}

$$\frac{e^{-4a-4bx}}{128b} - \frac{e^{-2a-2bx}}{64b} - \frac{e^{2a+2bx}}{32b} - \frac{e^{4a+4bx}}{128b} + \frac{e^{6a+6bx}}{192b} + \frac{x}{16}$$

Antiderivative was successfully verified.

```
[In] Int[E^(a + b*x)*Cosh[a + b*x]^2*Sinh[a + b*x]^3,x]
```

```
[Out] E^(-4*a - 4*b*x)/(128*b) - E^(-2*a - 2*b*x)/(64*b) - E^(2*a + 2*b*x)/(32*b)
 - E^(4*a + 4*b*x)/(128*b) + E^(6*a + 6*b*x)/(192*b) + x/16
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
```

```
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$\begin{aligned}
\int e^{a+bx}\cosh^2(a+bx)\sinh^3(a+bx)\,dx &= \frac{\text{Subst}\left(\int \frac{\left(-1+x^2\right)^3\left(1+x^2\right)^2}{32x^5}\,dx, x, e^{a+bx}\right)}{b}\\
&= \frac{\text{Subst}\left(\int \frac{\left(-1+x^2\right)^3\left(1+x^2\right)^2}{x^5}\,dx, x, e^{a+bx}\right)}{32b}\\
&= \frac{\text{Subst}\left(\int \frac{(-1+x)^3(1+x)^2}{x^3}\,dx, x, e^{2a+2bx}\right)}{64b}\\
&= \frac{\text{Subst}\left(\int \left(-2-\frac{1}{x^3}+\frac{1}{x^2}+\frac{2}{x}-x+x^2\right)dx, x, e^{2a+2bx}\right)}{64b}\\
&= \frac{e^{-4a-4bx}}{128b}-\frac{e^{-2a-2bx}}{64b}-\frac{e^{2a+2bx}}{32b}-\frac{e^{4a+4bx}}{128b}+\frac{e^{6a+6bx}}{192b}+\frac{x}{16}
\end{aligned}$$

Mathematica [A] time = 0.10, size = 67, normalized size = 0.74

$$\frac{3e^{-4(a+bx)}-6e^{-2(a+bx)}-12e^{2(a+bx)}-3e^{4(a+bx)}+2e^{6(a+bx)}+24bx}{384b}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(a + b*x)*Cosh[a + b*x]^2*Sinh[a + b*x]^3,x]
```

```
[Out] (3/E^(4*(a + b*x)) - 6/E^(2*(a + b*x)) - 12*E^(2*(a + b*x)) - 3*E^(4*(a + b
*x)) + 2*E^(6*(a + b*x)) + 24*b*x)/(384*b)
```

fricas [B] time = 0.47, size = 167, normalized size = 1.84

$$\frac{5\cosh(bx+a)^5+25\cosh(bx+a)\sinh(bx+a)^4-\sinh(bx+a)^5-\left(10\cosh(bx+a)^2-3\right)\sinh(bx+a)^3-9}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*cosh(b*x+a)^2*sinh(b*x+a)^3,x, algorithm="fricas")
```

[Out] 1/384*(5*cosh(b*x + a)^5 + 25*cosh(b*x + a)*sinh(b*x + a)^4 - sinh(b*x + a)
^5 - (10*cosh(b*x + a)^2 - 3)*sinh(b*x + a)^3 - 9*cosh(b*x + a)^3 + (50*cos
h(b*x + a)^3 - 27*cosh(b*x + a))*sinh(b*x + a)^2 + 12*(2*b*x - 1)*cosh(b*x
+ a) - (5*cosh(b*x + a)^4 + 24*b*x - 9*cosh(b*x + a)^2 + 12)*sinh(b*x + a))
/(b*cosh(b*x + a) - b*sinh(b*x + a))

giac [A] time = 0.12, size = 81, normalized size = 0.89

$$\frac{24\,bx - 3\left(6\,e^{(4\,bx+4\,a)} + 2\,e^{(2\,bx+2\,a)} - 1\right)e^{(-4\,bx-4\,a)} + 24\,a + 2\,e^{(6\,bx+6\,a)} - 3\,e^{(4\,bx+4\,a)} - 12\,e^{(2\,bx+2\,a)}}{384\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(b*x+a)*cosh(b*x+a)^2*sinh(b*x+a)^3,x, algorithm="giac")

[Out] 1/384*(24*b*x - 3*(6*e^(4*b*x + 4*a) + 2*e^(2*b*x + 2*a) - 1)*e^(-4*b*x - 4
*a) + 24*a + 2*e^(6*b*x + 6*a) - 3*e^(4*b*x + 4*a) - 12*e^(2*b*x + 2*a))/b

maple [A] time = 0.18, size = 89, normalized size = 0.98

$$\frac{\frac{\left(\cosh^{3}(bx+a)\right)\left(\sinh^{3}(bx+a)\right)}{6} - \frac{\left(\cosh^{3}(bx+a)\right)\sinh(bx+a)}{8} + \frac{\cosh(bx+a)\sinh(bx+a)}{16} + \frac{bx}{16} + \frac{a}{16} + \frac{\left(\cosh^{4}(bx+a)\right)\left(\sinh^{2}(bx+a)\right)}{6} - \frac{\left(\cosh^{4}(bx+\right.}{12}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(exp(b*x+a)*cosh(b*x+a)^2*sinh(b*x+a)^3,x)

[Out] 1/b*(1/6*cosh(b*x+a)^3*sinh(b*x+a)^3-1/8*cosh(b*x+a)^3*sinh(b*x+a)+1/16*cos
h(b*x+a)*sinh(b*x+a)+1/16*b*x+1/16*a+1/6*cosh(b*x+a)^4*sinh(b*x+a)^2-1/12*c
osh(b*x+a)^4)

maxima [A] time = 0.32, size = 77, normalized size = 0.85

$$-\frac{\left(2\,e^{(2\,bx+2\,a)} - 1\right)e^{(-4\,bx-4\,a)}}{128\,b} + \frac{bx + a}{16\,b} + \frac{2\,e^{(6\,bx+6\,a)} - 3\,e^{(4\,bx+4\,a)} - 12\,e^{(2\,bx+2\,a)}}{384\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(b*x+a)*cosh(b*x+a)^2*sinh(b*x+a)^3,x, algorithm="maxima")

[Out] -1/128*(2*e^(2*b*x + 2*a) - 1)*e^(-4*b*x - 4*a)/b + 1/16*(b*x + a)/b + 1/38
4*(2*e^(6*b*x + 6*a) - 3*e^(4*b*x + 4*a) - 12*e^(2*b*x + 2*a))/b

mupad [B] time = 0.63, size = 65, normalized size = 0.71

$$-\frac{6\,\mathrm{e}^{-2\,a-2\,b\,x} + 12\,\mathrm{e}^{2\,a+2\,b\,x} - 3\,\mathrm{e}^{-4\,a-4\,b\,x} + 3\,\mathrm{e}^{4\,a+4\,b\,x} - 2\,\mathrm{e}^{6\,a+6\,b\,x} - 24\,b\,x}{384\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a + b*x)^2*exp(a + b*x)*sinh(a + b*x)^3,x)
```

```
[Out] -(6*exp(- 2*a - 2*b*x) + 12*exp(2*a + 2*b*x) - 3*exp(- 4*a - 4*b*x) + 3*exp
(4*a + 4*b*x) - 2*exp(6*a + 6*b*x) - 24*b*x)/(384*b)
```

sympy [A] time = 176.09, size = 294, normalized size = 3.23

$$\begin{cases} -\frac{xe^{a}e^{bx}\sinh^{5}(a+bx)}{16} + \frac{xe^{a}e^{bx}\sinh^{4}(a+bx)\cosh(a+bx)}{16} + \frac{xe^{a}e^{bx}\sinh^{3}(a+bx)\cosh^{2}(a+bx)}{8} - \frac{xe^{a}e^{bx}\sinh^{2}(a+bx)\cosh^{3}(a+bx)}{8} - \frac{xe^{a}e^{bx}\sinh \cdots}{} \\ xe^{a}\sinh^{3}(a)\cosh^{2}(a) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*cosh(b*x+a)**2*sinh(b*x+a)**3,x)
```

```
[Out] Piecewise((-x*exp(a)*exp(b*x)*sinh(a + b*x)**5/16 + x*exp(a)*exp(b*x)*sinh(
a + b*x)**4*cosh(a + b*x)/16 + x*exp(a)*exp(b*x)*sinh(a + b*x)**3*cosh(a +
b*x)**2/8 - x*exp(a)*exp(b*x)*sinh(a + b*x)**2*cosh(a + b*x)**3/8 - x*exp(a
)*exp(b*x)*sinh(a + b*x)*cosh(a + b*x)**4/16 + x*exp(a)*exp(b*x)*cosh(a + b
*x)**5/16 - exp(a)*exp(b*x)*sinh(a + b*x)**5/(32*b) + 3*exp(a)*exp(b*x)*sin
h(a + b*x)**4*cosh(a + b*x)/(32*b) + exp(a)*exp(b*x)*sinh(a + b*x)**2*cosh(
a + b*x)**3/(6*b) - exp(a)*exp(b*x)*sinh(a + b*x)*cosh(a + b*x)**4/(96*b) -
 5*exp(a)*exp(b*x)*cosh(a + b*x)**5/(96*b), Ne(b, 0)), (x*exp(a)*sinh(a)**3
*cosh(a)**2, True))
```

3.908 $\int e^{a+bx}\cosh^2(a+bx)\sinh^2(a+bx)\,dx$

Optimal. Leaf size=49

$$-\frac{e^{-3a-3bx}}{48b}-\frac{e^{a+bx}}{8b}+\frac{e^{5a+5bx}}{80b}$$

```
[Out] -1/48*exp(-3*b*x-3*a)/b-1/8*exp(b*x+a)/b+1/80*exp(5*b*x+5*a)/b
```

Rubi [A] time = 0.05, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2282, 12, 270}

$$-\frac{e^{-3a-3bx}}{48b}-\frac{e^{a+bx}}{8b}+\frac{e^{5a+5bx}}{80b}$$

Antiderivative was successfully verified.

```
[In] Int[E^(a + b*x)*Cosh[a + b*x]^2*Sinh[a + b*x]^2,x]
```

```
[Out] -E^(-3*a - 3*b*x)/(48*b) - E^(a + b*x)/(8*b) + E^(5*a + 5*b*x)/(80*b)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$\int e^{a+bx}\cosh^2(a+bx)\sinh^2(a+bx)\,dx = \frac{\text{Subst}\left(\int \frac{(1-x^4)^2}{16x^4}\,dx, x, e^{a+bx}\right)}{b}$$
$$= \frac{\text{Subst}\left(\int \frac{(1-x^4)^2}{x^4}\,dx, x, e^{a+bx}\right)}{16b}$$
$$= \frac{\text{Subst}\left(\int \left(-2+\frac{1}{x^4}+x^4\right)dx, x, e^{a+bx}\right)}{16b}$$
$$= -\frac{e^{-3a-3bx}}{48b} - \frac{e^{a+bx}}{8b} + \frac{e^{5a+5bx}}{80b}$$

Mathematica [A] time = 0.03, size = 40, normalized size = 0.82

$$\frac{e^{-3(a+bx)}\left(-30e^{4(a+bx)}+3e^{8(a+bx)}-5\right)}{240b}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(a + b*x)*Cosh[a + b*x]^2*Sinh[a + b*x]^2,x]
[Out] (-5 - 30*E^(4*(a + b*x)) + 3*E^(8*(a + b*x)))/(240*b*E^(3*(a + b*x)))
```

fricas [B] time = 0.44, size = 90, normalized size = 1.84

$$-\frac{\cosh(bx+a)^4 - 16\cosh(bx+a)^3\sinh(bx+a) + 6\cosh(bx+a)^2\sinh(bx+a)^2 - 16\cosh(bx+a)\sinh(bx}{120(b\cosh(bx+a) - b\sinh(bx+a))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*cosh(b*x+a)^2*sinh(b*x+a)^2,x, algorithm="fricas")
[Out] -1/120*(cosh(b*x + a)^4 - 16*cosh(b*x + a)^3*sinh(b*x + a) + 6*cosh(b*x + a
)^2*sinh(b*x + a)^2 - 16*cosh(b*x + a)*sinh(b*x + a)^3 + sinh(b*x + a)^4 +
15)/(b*cosh(b*x + a) - b*sinh(b*x + a))
```

giac [A] time = 0.13, size = 36, normalized size = 0.73

$$\frac{3\,e^{(5\,bx+5\,a)} - 30\,e^{(bx+a)} - 5\,e^{(-3\,bx-3\,a)}}{240\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*cosh(b*x+a)^2*sinh(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/240*(3*e^(5*b*x + 5*a) - 30*e^(b*x + a) - 5*e^(-3*b*x - 3*a))/b
```

maple [A] time = 0.36, size = 70, normalized size = 1.43

$$\frac{\frac{\left(\cosh^{3}(bx+a)\right)\left(\sinh^{2}(bx+a)\right)}{5}-\frac{2\left(\cosh^{3}(bx+a)\right)}{15}+\frac{\sinh(bx+a)\left(\cosh^{4}(bx+a)\right)}{5}-\frac{\left(\frac{2}{3}+\frac{\left(\cosh^{2}(bx+a)\right)}{3}\right)\sinh(bx+a)}{5}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(b*x+a)*cosh(b*x+a)^2*sinh(b*x+a)^2,x)
```

```
[Out] 1/b*(1/5*cosh(b*x+a)^3*sinh(b*x+a)^2-2/15*cosh(b*x+a)^3+1/5*sinh(b*x+a)*cos
h(b*x+a)^4-1/5*(2/3+1/3*cosh(b*x+a)^2)*sinh(b*x+a))
```

maxima [A] time = 0.34, size = 38, normalized size = 0.78

$$\frac{e^{(5\,bx+5\,a)}-10\,e^{(bx+a)}}{80\,b}-\frac{e^{(-3\,bx-3\,a)}}{48\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*cosh(b*x+a)^2*sinh(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/80*(e^(5*b*x + 5*a) - 10*e^(b*x + a))/b - 1/48*e^(-3*b*x - 3*a)/b
```

mupad [B] time = 0.52, size = 36, normalized size = 0.73

$$-\frac{30\,\mathrm{e}^{a+b\,x}+5\,\mathrm{e}^{-3\,a-3\,b\,x}-3\,\mathrm{e}^{5\,a+5\,b\,x}}{240\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a + b*x)^2*exp(a + b*x)*sinh(a + b*x)^2,x)
```

```
[Out] -(30*exp(a + b*x) + 5*exp(- 3*a - 3*b*x) - 3*exp(5*a + 5*b*x))/(240*b)
```

sympy [A] time = 62.53, size = 144, normalized size = 2.94

$$\begin{cases} -\frac{2e^{a}e^{bx}\sinh^{4}(a+bx)}{15b}+\frac{2e^{a}e^{bx}\sinh^{3}(a+bx)\cosh(a+bx)}{15b}+\frac{e^{a}e^{bx}\sinh^{2}(a+bx)\cosh^{2}(a+bx)}{5b}+\frac{2e^{a}e^{bx}\sinh(a+bx)\cosh^{3}(a+bx)}{15b}-\frac{2e^{a}e^{bx}\cosh^{4}(a}{15b} \\ xe^{a}\sinh^{2}(a)\cosh^{2}(a) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*cosh(b*x+a)**2*sinh(b*x+a)**2,x)
```

```
[Out] Piecewise((-2*exp(a)*exp(b*x)*sinh(a + b*x)**4/(15*b) + 2*exp(a)*exp(b*x)*s
inh(a + b*x)**3*cosh(a + b*x)/(15*b) + exp(a)*exp(b*x)*sinh(a + b*x)**2*cos
h(a + b*x)**2/(5*b) + 2*exp(a)*exp(b*x)*sinh(a + b*x)*cosh(a + b*x)**3/(15*
b) - 2*exp(a)*exp(b*x)*cosh(a + b*x)**4/(15*b), Ne(b, 0)), (x*exp(a)*sinh(a
)**2*cosh(a)**2, True))
```

3.909 $\int e^{a+bx} \cosh^2(a+bx) \sinh(a+bx)\, dx$

Optimal. Leaf size=57

$$\frac{e^{-2a-2bx}}{16b}+\frac{e^{2a+2bx}}{16b}+\frac{e^{4a+4bx}}{32b}-\frac{x}{8}$$

```
[Out] 1/16*exp(-2*b*x-2*a)/b+1/16*exp(2*b*x+2*a)/b+1/32*exp(4*b*x+4*a)/b-1/8*x
```

Rubi [A] time = 0.05, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2282, 12, 446, 75}

$$\frac{e^{-2a-2bx}}{16b}+\frac{e^{2a+2bx}}{16b}+\frac{e^{4a+4bx}}{32b}-\frac{x}{8}$$

Antiderivative was successfully verified.

```
[In] Int[E^(a + b*x)*Cosh[a + b*x]^2*Sinh[a + b*x],x]
```

```
[Out] E^(-2*a - 2*b*x)/(16*b) + E^(2*a + 2*b*x)/(16*b) + E^(4*a + 4*b*x)/(32*b) -
 x/8
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 75

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && EqQ[b*e + a*f, 0] &&  !(ILtQ[n + p
+ 2, 0] && GtQ[n + 2*p, 0])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
```

```
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$\int e^{a+bx}\cosh^2(a+bx)\sinh(a+bx)\,dx = \frac{\text{Subst}\left(\int \frac{(-1+x^2)(1+x^2)^2}{8x^3}\,dx, x, e^{a+bx}\right)}{b}$$
$$= \frac{\text{Subst}\left(\int \frac{(-1+x^2)(1+x^2)^2}{x^3}\,dx, x, e^{a+bx}\right)}{8b}$$
$$= \frac{\text{Subst}\left(\int \frac{(-1+x)(1+x)^2}{x^2}\,dx, x, e^{2a+2bx}\right)}{16b}$$
$$= \frac{\text{Subst}\left(\int \left(1-\frac{1}{x^2}-\frac{1}{x}+x\right)dx, x, e^{2a+2bx}\right)}{16b}$$
$$= \frac{e^{-2a-2bx}}{16b}+\frac{e^{2a+2bx}}{16b}+\frac{e^{4a+4bx}}{32b}-\frac{x}{8}$$

Mathematica [A] time = 0.05, size = 43, normalized size = 0.75

$$\frac{2e^{-2(a+bx)}+2e^{2(a+bx)}+e^{4(a+bx)}-4bx}{32b}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(a + b*x)*Cosh[a + b*x]^2*Sinh[a + b*x],x]
[Out] (2/E^(2*(a + b*x)) + 2*E^(2*(a + b*x)) + E^(4*(a + b*x)) - 4*b*x)/(32*b)
```

fricas [B] time = 0.53, size = 96, normalized size = 1.68

$$\frac{3\cosh(bx+a)^3+9\cosh(bx+a)\sinh(bx+a)^2-\sinh(bx+a)^3-2(2bx-1)\cosh(bx+a)+\left(4bx-3\cosh\right.}{32(b\cosh(bx+a)-b\sinh(bx+a))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*cosh(b*x+a)^2*sinh(b*x+a),x, algorithm="fricas")
[Out] 1/32*(3*cosh(b*x + a)^3 + 9*cosh(b*x + a)*sinh(b*x + a)^2 - sinh(b*x + a)^3
 - 2*(2*b*x - 1)*cosh(b*x + a) + (4*b*x - 3*cosh(b*x + a)^2 + 2)*sinh(b*x +
 a))/(b*cosh(b*x + a) - b*sinh(b*x + a))
```

giac [A] time = 0.12, size = 57, normalized size = 1.00

$$-\frac{4\,bx - 2\,\left(e^{(2\,bx+2\,a)} + 1\right)e^{(-2\,bx-2\,a)} + 4\,a - e^{(4\,bx+4\,a)} - 2\,e^{(2\,bx+2\,a)}}{32\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*cosh(b*x+a)^2*sinh(b*x+a),x, algorithm="giac")
```

```
[Out] -1/32*(4*b*x - 2*(e^(2*b*x + 2*a) + 1)*e^(-2*b*x - 2*a) + 4*a - e^(4*b*x +
4*a) - 2*e^(2*b*x + 2*a))/b
```

maple [A] time = 0.10, size = 53, normalized size = 0.93

$$\frac{\frac{\left(\cosh^3(bx+a)\right)\sinh(bx+a)}{4} - \frac{\cosh(bx+a)\sinh(bx+a)}{8} - \frac{bx}{8} - \frac{a}{8} + \frac{\left(\cosh^4(bx+a)\right)}{4}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(b*x+a)*cosh(b*x+a)^2*sinh(b*x+a),x)
```

```
[Out] 1/b*(1/4*cosh(b*x+a)^3*sinh(b*x+a)-1/8*cosh(b*x+a)*sinh(b*x+a)-1/8*b*x-1/8*
a+1/4*cosh(b*x+a)^4)
```

maxima [A] time = 0.32, size = 50, normalized size = 0.88

$$-\frac{1}{8}\,x - \frac{a}{8\,b} + \frac{e^{(4\,bx+4\,a)} + 2\,e^{(2\,bx+2\,a)}}{32\,b} + \frac{e^{(-2\,bx-2\,a)}}{16\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*cosh(b*x+a)^2*sinh(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/8*x - 1/8*a/b + 1/32*(e^(4*b*x + 4*a) + 2*e^(2*b*x + 2*a))/b + 1/16*e^(-
2*b*x - 2*a)/b
```

mupad [B] time = 0.27, size = 42, normalized size = 0.74

$$\frac{\frac{e^{-2\,a-2\,b\,x}}{16} + \frac{e^{2\,a+2\,b\,x}}{16} + \frac{e^{4\,a+4\,b\,x}}{32}}{b} - \frac{x}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a + b*x)^2*exp(a + b*x)*sinh(a + b*x),x)
```

```
[Out] (exp(- 2*a - 2*b*x)/16 + exp(2*a + 2*b*x)/16 + exp(4*a + 4*b*x)/32)/b - x/8
```

sympy [A] time = 18.66, size = 175, normalized size = 3.07

$$\begin{cases} -\frac{xe^{a}e^{bx}\sinh^{3}(a+bx)}{8} + \frac{xe^{a}e^{bx}\sinh^{2}(a+bx)\cosh(a+bx)}{8} + \frac{xe^{a}e^{bx}\sinh(a+bx)\cosh^{2}(a+bx)}{8} - \frac{xe^{a}e^{bx}\cosh^{3}(a+bx)}{8} - \frac{e^{a}e^{bx}\sinh^{3}(a+bx)}{8b} + \frac{e^{a} \\ xe^{a}\sinh(a)\cosh^{2}(a) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*cosh(b*x+a)**2*sinh(b*x+a),x)
```

```
[Out] Piecewise((-x*exp(a)*exp(b*x)*sinh(a + b*x)**3/8 + x*exp(a)*exp(b*x)*sinh(a
 + b*x)**2*cosh(a + b*x)/8 + x*exp(a)*exp(b*x)*sinh(a + b*x)*cosh(a + b*x)*
*2/8 - x*exp(a)*exp(b*x)*cosh(a + b*x)**3/8 - exp(a)*exp(b*x)*sinh(a + b*x)
**3/(8*b) + exp(a)*exp(b*x)*sinh(a + b*x)**2*cosh(a + b*x)/(4*b) + exp(a)*e
xp(b*x)*cosh(a + b*x)**3/(8*b), Ne(b, 0)), (x*exp(a)*sinh(a)*cosh(a)**2, Tr
ue))
```

3.910 $\int e^{a+bx}\cosh(a+bx)\coth(a+bx)\,dx$

Optimal. Leaf size=42

$$\frac{e^{2a+2bx}}{4b}+\frac{\log\left(1-e^{2a+2bx}\right)}{b}-\frac{x}{2}$$

[Out] 1/4*exp(2*b*x+2*a)/b-1/2*x+ln(1-exp(2*b*x+2*a))/b

Rubi [A] time = 0.04, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2282, 12, 446, 72}

$$\frac{e^{2a+2bx}}{4b}+\frac{\log\left(1-e^{2a+2bx}\right)}{b}-\frac{x}{2}$$

Antiderivative was successfully verified.

[In] Int[E^(a + b*x)*Cosh[a + b*x]*Coth[a + b*x],x]

[Out] E^(2*a + 2*b*x)/(4*b) - x/2 + Log[1 - E^(2*a + 2*b*x)]/b

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
```

```
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$\int e^{a+bx}\cosh(a+bx)\coth(a+bx)\,dx = \frac{\text{Subst}\left(\int \frac{\left(1+x^2\right)^2}{2x\left(-1+x^2\right)}\,dx, x, e^{a+bx}\right)}{b}$$
$$= \frac{\text{Subst}\left(\int \frac{\left(1+x^2\right)^2}{x\left(-1+x^2\right)}\,dx, x, e^{a+bx}\right)}{2b}$$
$$= \frac{\text{Subst}\left(\int \frac{(1+x)^2}{(-1+x)x}\,dx, x, e^{2a+2bx}\right)}{4b}$$
$$= \frac{\text{Subst}\left(\int \left(1+\frac{4}{-1+x}-\frac{1}{x}\right)dx, x, e^{2a+2bx}\right)}{4b}$$
$$= \frac{e^{2a+2bx}}{4b}-\frac{x}{2}+\frac{\log\left(1-e^{2a+2bx}\right)}{b}$$

Mathematica [A] time = 0.03, size = 39, normalized size = 0.93

$$\frac{e^{2a+2bx}+4\log\left(1-e^{2a+2bx}\right)-2bx}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(a + b*x)*Cosh[a + b*x]*Coth[a + b*x],x]
[Out] (E^(2*a + 2*b*x) - 2*b*x + 4*Log[1 - E^(2*a + 2*b*x)])/(4*b)
```

fricas [A] time = 0.48, size = 72, normalized size = 1.71

$$-\frac{2\,bx-\cosh\left(bx+a\right)^2-2\,\cosh\left(bx+a\right)\sinh\left(bx+a\right)-\sinh\left(bx+a\right)^2-4\,\log\left(\frac{2\,\sinh(bx+a)}{\cosh(bx+a)-\sinh(bx+a)}\right)}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*cosh(b*x+a)^2*csch(b*x+a),x, algorithm="fricas")
[Out] -1/4*(2*b*x - cosh(b*x + a)^2 - 2*cosh(b*x + a)*sinh(b*x + a) - sinh(b*x +
a)^2 - 4*log(2*sinh(b*x + a)/(cosh(b*x + a) - sinh(b*x + a))))/b
```

giac [A] time = 0.14, size = 39, normalized size = 0.93

$$-\frac{2\,bx + 2\,a - e^{(2\,bx+2\,a)} - 4\,\log\left(\left|e^{(2\,bx+2\,a)} - 1\right|\right)}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*cosh(b*x+a)^2*csch(b*x+a),x, algorithm="giac")
```

```
[Out] -1/4*(2*b*x + 2*a - e^(2*b*x + 2*a) - 4*log(abs(e^(2*b*x + 2*a) - 1)))/b
```

maple [A] time = 0.20, size = 52, normalized size = 1.24

$$\frac{\cosh(bx+a)\sinh(bx+a)}{2b} + \frac{x}{2} + \frac{a}{2b} + \frac{\cosh^2(bx+a)}{2b} + \frac{\ln(\sinh(bx+a))}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(b*x+a)*cosh(b*x+a)^2*csch(b*x+a),x)
```

```
[Out] 1/2*cosh(b*x+a)*sinh(b*x+a)/b+1/2*x+1/2*a/b+1/2*cosh(b*x+a)^2/b+ln(sinh(b*x
+a))/b
```

maxima [A] time = 0.33, size = 50, normalized size = 1.19

$$-\frac{1}{2}\,x - \frac{a}{2\,b} + \frac{e^{(2\,bx+2\,a)}}{4\,b} + \frac{\log\left(e^{(bx+a)} + 1\right)}{b} + \frac{\log\left(e^{(bx+a)} - 1\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*cosh(b*x+a)^2*csch(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/2*x - 1/2*a/b + 1/4*e^(2*b*x + 2*a)/b + log(e^(b*x + a) + 1)/b + log(e^(
b*x + a) - 1)/b
```

mupad [B] time = 1.71, size = 35, normalized size = 0.83

$$\frac{\ln\left(\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x} - 1\right)}{b} - \frac{x}{2} + \frac{\mathrm{e}^{2\,a+2\,b\,x}}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cosh(a + b*x)^2*exp(a + b*x))/sinh(a + b*x),x)
```

```
[Out] log(exp(2*a)*exp(2*b*x) - 1)/b - x/2 + exp(2*a + 2*b*x)/(4*b)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*cosh(b*x+a)**2*csch(b*x+a),x)
```

```
[Out] Timed out
```

3.911 $\int e^{a+bx}\coth^2(a+bx)\,dx$

Optimal. Leaf size=53

$$\frac{e^{a+bx}}{b}+\frac{2e^{a+bx}}{b\left(1-e^{2a+2bx}\right)}-\frac{2\tanh^{-1}\left(e^{a+bx}\right)}{b}$$

```
[Out] exp(b*x+a)/b+2*exp(b*x+a)/b/(1-exp(2*b*x+2*a))-2*arctanh(exp(b*x+a))/b
```

Rubi [A] time = 0.04, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2282, 390, 288, 206}

$$\frac{e^{a+bx}}{b}+\frac{2e^{a+bx}}{b\left(1-e^{2a+2bx}\right)}-\frac{2\tanh^{-1}\left(e^{a+bx}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[E^(a + b*x)*Coth[a + b*x]^2,x]
```

```
[Out] E^(a + b*x)/b + (2*E^(a + b*x))/(b*(1 - E^(2*a + 2*b*x))) - (2*ArcTanh[E^(a
 + b*x)])/b
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$\int e^{a+bx}\coth^2(a+bx)\,dx=\frac{\text{Subst}\left(\int\frac{(1+x^2)^2}{(1-x^2)^2}\,dx,x,e^{a+bx}\right)}{b}$$
$$=\frac{\text{Subst}\left(\int\left(1+\frac{4x^2}{(1-x^2)^2}\right)dx,x,e^{a+bx}\right)}{b}$$
$$=\frac{e^{a+bx}}{b}+\frac{4\,\text{Subst}\left(\int\frac{x^2}{(1-x^2)^2}\,dx,x,e^{a+bx}\right)}{b}$$
$$=\frac{e^{a+bx}}{b}+\frac{2e^{a+bx}}{b\left(1-e^{2a+2bx}\right)}-\frac{2\,\text{Subst}\left(\int\frac{1}{1-x^2}\,dx,x,e^{a+bx}\right)}{b}$$
$$=\frac{e^{a+bx}}{b}+\frac{2e^{a+bx}}{b\left(1-e^{2a+2bx}\right)}-\frac{2\tanh^{-1}\left(e^{a+bx}\right)}{b}$$

Mathematica [C] time = 1.76, size = 179, normalized size = 3.38

$$\frac{e^{a+bx}\left(\frac{4}{105}\left(e^{a+bx}+e^{3(a+bx)}\right)^2\,{}_4F_3\left(\frac{3}{2},2,2,2;1,1,\frac{9}{2};e^{2(a+bx)}\right)+\frac{1}{48}e^{-4(a+bx)}\left(-713e^{2(a+bx)}-181e^{4(a+bx)}+61e^{6(a+bx)}+\right.\right.}{b}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(a + b*x)*Coth[a + b*x]^2,x]
```

```
[Out] (E^(a + b*x)*((-375 - 713*E^(2*(a + b*x)) - 181*E^(4*(a + b*x)) + 61*E^(6*(
a + b*x)) + (3*(125 + 196*E^(2*(a + b*x)) - 14*E^(4*(a + b*x)) - 52*E^(6*(a
 + b*x)) + E^(8*(a + b*x)))*ArcTanh[Sqrt[E^(2*(a + b*x))]])/Sqrt[E^(2*(a +
b*x))])/(48*E^(4*(a + b*x))) + (4*(E^(a + b*x) + E^(3*(a + b*x)))^2*Hyperge
ometricPFQ[{3/2, 2, 2, 2}, {1, 1, 9/2}, E^(2*(a + b*x))])/105))/b
```

fricas [B] time = 0.51, size = 198, normalized size = 3.74

$$\frac{\cosh(bx+a)^3 + 3\cosh(bx+a)\sinh(bx+a)^2 + \sinh(bx+a)^3 - \left(\cosh(bx+a)^2 + 2\cosh(bx+a)\sinh(bx+a\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*cosh(b*x+a)^2*csch(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] (cosh(b*x + a)^3 + 3*cosh(b*x + a)*sinh(b*x + a)^2 + sinh(b*x + a)^3 - (cos
h(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 - 1)*log(cos
h(b*x + a) + sinh(b*x + a) + 1) + (cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b
*x + a) + sinh(b*x + a)^2 - 1)*log(cosh(b*x + a) + sinh(b*x + a) - 1) + 3*(
cosh(b*x + a)^2 - 1)*sinh(b*x + a) - 3*cosh(b*x + a))/(b*cosh(b*x + a)^2 +
2*b*cosh(b*x + a)*sinh(b*x + a) + b*sinh(b*x + a)^2 - b)
```

giac [A] time = 0.12, size = 56, normalized size = 1.06

$$-\frac{\frac{2e^{(bx+a)}}{e^{(2bx+2a)}-1} - e^{(bx+a)} + \log\left(e^{(bx+a)} + 1\right) - \log\left(\left|e^{(bx+a)} - 1\right|\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*cosh(b*x+a)^2*csch(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -(2*e^(b*x + a)/(e^(2*b*x + 2*a) - 1) - e^(b*x + a) + log(e^(b*x + a) + 1)
- log(abs(e^(b*x + a) - 1)))/b
```

maple [A] time = 0.14, size = 48, normalized size = 0.91

$$\frac{\cosh(bx+a) - 2\operatorname{arctanh}\left(e^{bx+a}\right) + \frac{\cosh^2(bx+a)}{\sinh(bx+a)} - \frac{2}{\sinh(bx+a)}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(b*x+a)*cosh(b*x+a)^2*csch(b*x+a)^2,x)
```

```
[Out] 1/b*(cosh(b*x+a)-2*arctanh(exp(b*x+a))+cosh(b*x+a)^2/sinh(b*x+a)-2/sinh(b*x
+a))
```

maxima [A] time = 0.32, size = 62, normalized size = 1.17

$$\frac{e^{(bx+a)}}{b} - \frac{\log\left(e^{(bx+a)} + 1\right)}{b} + \frac{\log\left(e^{(bx+a)} - 1\right)}{b} - \frac{2e^{(bx+a)}}{b\left(e^{(2bx+2a)} - 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(b*x+a)*cosh(b*x+a)^2*csch(b*x+a)^2,x, algorithm="maxima")

[Out] e^(b*x + a)/b - log(e^(b*x + a) + 1)/b + log(e^(b*x + a) - 1)/b - 2*e^(b*x + a)/(b*(e^(2*b*x + 2*a) - 1))

mupad [B] time = 1.84, size = 62, normalized size = 1.17

$$\frac{e^{a+b\,x}}{b}-\frac{2\,\mathrm{atan}\left(\frac{e^{b\,x}\,e^{a}\,\sqrt{-b^2}}{b}\right)}{\sqrt{-b^2}}-\frac{2\,e^{a+b\,x}}{b\,\left(e^{2\,a+2\,b\,x}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((cosh(a + b*x)^2*exp(a + b*x))/sinh(a + b*x)^2,x)

[Out] exp(a + b*x)/b - (2*atan((exp(b*x)*exp(a)*(-b^2)^(1/2))/b))/(-b^2)^(1/2) - (2*exp(a + b*x))/(b*(exp(2*a + 2*b*x) - 1))

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(b*x+a)*cosh(b*x+a)**2*csch(b*x+a)**2,x)

[Out] Timed out

3.912 $\int e^{a+bx}\coth^2(a+bx)\mathbf{csch}(a+bx)\,dx$

Optimal. Leaf size=62

$$\frac{4}{b\left(1-e^{2a+2bx}\right)}-\frac{2}{b\left(1-e^{2a+2bx}\right)^2}+\frac{\log\left(1-e^{2a+2bx}\right)}{b}$$

```
[Out] -2/b/(1-exp(2*b*x+2*a))^2+4/b/(1-exp(2*b*x+2*a))+ln(1-exp(2*b*x+2*a))/b
```

Rubi [A] time = 0.07, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2282, 12, 444, 43}

$$\frac{4}{b\left(1-e^{2a+2bx}\right)}-\frac{2}{b\left(1-e^{2a+2bx}\right)^2}+\frac{\log\left(1-e^{2a+2bx}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[E^(a + b*x)*Coth[a + b*x]^2*Csch[a + b*x],x]
```

```
[Out] -2/(b*(1 - E^(2*a + 2*b*x))^2) + 4/(b*(1 - E^(2*a + 2*b*x))) + Log[1 - E^(2
*a + 2*b*x)]/b
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$\begin{aligned}\int e^{a+bx}\coth^2(a+bx)\operatorname{csch}(a+bx)\,dx &= \frac{\operatorname{Subst}\left(\int \frac{2x\left(1+x^2\right)^2}{\left(-1+x^2\right)^3}\,dx,x,e^{a+bx}\right)}{b}\\ &= \frac{2\operatorname{Subst}\left(\int \frac{x\left(1+x^2\right)^2}{\left(-1+x^2\right)^3}\,dx,x,e^{a+bx}\right)}{b}\\ &= \frac{\operatorname{Subst}\left(\int \frac{(1+x)^2}{(-1+x)^3}\,dx,x,e^{2a+2bx}\right)}{b}\\ &= \frac{\operatorname{Subst}\left(\int \left(\frac{4}{(-1+x)^3}+\frac{4}{(-1+x)^2}+\frac{1}{-1+x}\right)dx,x,e^{2a+2bx}\right)}{b}\\ &= -\frac{2}{b\left(1-e^{2a+2bx}\right)^2}+\frac{4}{b\left(1-e^{2a+2bx}\right)}+\frac{\log\left(1-e^{2a+2bx}\right)}{b}\end{aligned}$$

Mathematica [A] time = 0.09, size = 46, normalized size = 0.74

$$\frac{\frac{2-4e^{2(a+bx)}}{\left(e^{2(a+bx)}-1\right)^2}+\log\left(1-e^{2(a+bx)}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(a + b*x)*Coth[a + b*x]^2*Csch[a + b*x],x]
```

```
[Out] ((2 - 4*E^(2*(a + b*x)))/(-1 + E^(2*(a + b*x)))^2 + Log[1 - E^(2*(a + b*x))
])/b
```

fricas [B] time = 0.47, size = 262, normalized size = 4.23

$$-\frac{4\cosh(bx+a)^2-\left(\cosh(bx+a)^4+4\cosh(bx+a)\sinh(bx+a)^3+\sinh(bx+a)^4+2\left(3\cosh(bx+a)^2-1\right.}{b\cosh(bx+a)^4+4b\cosh(bx+a)\sinh(bx+a)^3+b\sinh(bx-}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*cosh(b*x+a)^2*csch(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -(4*cosh(b*x + a)^2 - (cosh(b*x + a)^4 + 4*cosh(b*x + a)*sinh(b*x + a)^3 +
sinh(b*x + a)^4 + 2*(3*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^2 - 2*cosh(b*x +
a)^2 + 4*(cosh(b*x + a)^3 - cosh(b*x + a))*sinh(b*x + a) + 1)*log(2*sinh(b*
x + a)/(cosh(b*x + a) - sinh(b*x + a))) + 8*cosh(b*x + a)*sinh(b*x + a) + 4
*sinh(b*x + a)^2 - 2)/(b*cosh(b*x + a)^4 + 4*b*cosh(b*x + a)*sinh(b*x + a)^
3 + b*sinh(b*x + a)^4 - 2*b*cosh(b*x + a)^2 + 2*(3*b*cosh(b*x + a)^2 - b)*s
inh(b*x + a)^2 + 4*(b*cosh(b*x + a)^3 - b*cosh(b*x + a))*sinh(b*x + a) + b)
```

giac [A] time = 0.13, size = 59, normalized size = 0.95

$$-\frac{\frac{3\,e^{(4\,bx+4\,a)}+2\,e^{(2\,bx+2\,a)}-1}{\left(e^{(2\,bx+2\,a)}-1\right)^{2}}-2\,\log\left(\left|e^{(2\,bx+2\,a)}-1\right|\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*cosh(b*x+a)^2*csch(b*x+a)^3,x, algorithm="giac")
```

```
[Out] -1/2*((3*e^(4*b*x + 4*a) + 2*e^(2*b*x + 2*a) - 1)/(e^(2*b*x + 2*a) - 1)^2 -
 2*log(abs(e^(2*b*x + 2*a) - 1)))/b
```

maple [A] time = 0.17, size = 43, normalized size = 0.69

$$x+\frac{a}{b}-\frac{\coth\left(bx+a\right)}{b}+\frac{\ln\left(\sinh\left(bx+a\right)\right)}{b}-\frac{\coth^{2}\left(bx+a\right)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(b*x+a)*cosh(b*x+a)^2*csch(b*x+a)^3,x)
```

```
[Out] x+a/b-coth(b*x+a)/b+ln(sinh(b*x+a))/b-1/2*coth(b*x+a)^2/b
```

maxima [A] time = 0.34, size = 69, normalized size = 1.11

$$\frac{\log\left(e^{(bx+a)}+1\right)}{b}+\frac{\log\left(e^{(bx+a)}-1\right)}{b}-\frac{2\left(2\,e^{(2\,bx+2\,a)}-1\right)}{b\left(e^{(4\,bx+4\,a)}-2\,e^{(2\,bx+2\,a)}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*cosh(b*x+a)^2*csch(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] log(e^(b*x + a) + 1)/b + log(e^(b*x + a) - 1)/b - 2*(2*e^(2*b*x + 2*a) - 1)
/(b*(e^(4*b*x + 4*a) - 2*e^(2*b*x + 2*a) + 1))
```

mupad [B] time = 1.84, size = 65, normalized size = 1.05

$$\frac{\ln \left(e^{2a}\,e^{2bx}-1\right)}{b}-\frac{4}{b\,\left(e^{2a+2bx}-1\right)}-\frac{2}{b\,\left(e^{4a+4bx}-2\,e^{2a+2bx}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cosh(a + b*x)^2*exp(a + b*x))/sinh(a + b*x)^3,x)
```

```
[Out] log(exp(2*a)*exp(2*b*x) - 1)/b - 4/(b*(exp(2*a + 2*b*x) - 1)) - 2/(b*(exp(4
*a + 4*b*x) - 2*exp(2*a + 2*b*x) + 1))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*cosh(b*x+a)**2*csch(b*x+a)**3,x)
```

```
[Out] Timed out
```

3.913 $\int e^{a+bx}\cosh^3(a+bx)\sinh^3(a+bx)\,dx$

Optimal. Leaf size=69

$$\frac{e^{-5a-5bx}}{320b}-\frac{3e^{-a-bx}}{64b}-\frac{e^{3a+3bx}}{64b}+\frac{e^{7a+7bx}}{448b}$$

```
[Out] 1/320*exp(-5*b*x-5*a)/b-3/64*exp(-b*x-a)/b-1/64*exp(3*b*x+3*a)/b+1/448*exp(
7*b*x+7*a)/b
```

Rubi [A] time = 0.06, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2282, 12, 270}

$$\frac{e^{-5a-5bx}}{320b}-\frac{3e^{-a-bx}}{64b}-\frac{e^{3a+3bx}}{64b}+\frac{e^{7a+7bx}}{448b}$$

Antiderivative was successfully verified.

```
[In] Int[E^(a + b*x)*Cosh[a + b*x]^3*Sinh[a + b*x]^3,x]
```

```
[Out] E^(-5*a - 5*b*x)/(320*b) - (3*E^(-a - b*x))/(64*b) - E^(3*a + 3*b*x)/(64*b)
 + E^(7*a + 7*b*x)/(448*b)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$\int e^{a+bx}\cosh^3(a+bx)\sinh^3(a+bx)\,dx = \frac{\text{Subst}\left(\int \frac{(-1+x^4)^3}{64x^6}\,dx, x, e^{a+bx}\right)}{b}$$
$$= \frac{\text{Subst}\left(\int \frac{(-1+x^4)^3}{x^6}\,dx, x, e^{a+bx}\right)}{64b}$$
$$= \frac{\text{Subst}\left(\int \left(-\frac{1}{x^6}+\frac{3}{x^2}-3x^2+x^6\right)dx, x, e^{a+bx}\right)}{64b}$$
$$= \frac{e^{-5a-5bx}}{320b} - \frac{3e^{-a-bx}}{64b} - \frac{e^{3a+3bx}}{64b} + \frac{e^{7a+7bx}}{448b}$$

Mathematica [A] time = 0.04, size = 51, normalized size = 0.74

$$\frac{e^{-5(a+bx)}\left(-105e^{4(a+bx)}-35e^{8(a+bx)}+5e^{12(a+bx)}+7\right)}{2240b}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(a + b*x)*Cosh[a + b*x]^3*Sinh[a + b*x]^3,x]
```

```
[Out] (7 - 105*E^(4*(a + b*x)) - 35*E^(8*(a + b*x)) + 5*E^(12*(a + b*x)))/(2240*b
*E^(5*(a + b*x)))
```

fricas [B] time = 0.45, size = 154, normalized size = 2.23

$$\frac{3\cosh(bx+a)^6 - 10\cosh(bx+a)^3\sinh(bx+a)^3 + 45\cosh(bx+a)^2\sinh(bx+a)^4 - 3\cosh(bx+a)\sinh(}{560}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*cosh(b*x+a)^3*sinh(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/560*(3*cosh(b*x + a)^6 - 10*cosh(b*x + a)^3*sinh(b*x + a)^3 + 45*cosh(b*x
 + a)^2*sinh(b*x + a)^4 - 3*cosh(b*x + a)*sinh(b*x + a)^5 + 3*sinh(b*x + a)
^6 + 5*(9*cosh(b*x + a)^4 - 7)*sinh(b*x + a)^2 - 35*cosh(b*x + a)^2 - (3*co
sh(b*x + a)^5 - 35*cosh(b*x + a))*sinh(b*x + a))/(b*cosh(b*x + a) - b*sinh(
b*x + a))
```

giac [A] time = 0.13, size = 52, normalized size = 0.75

$$-\frac{7\left(15\,e^{(4\,bx+4\,a)}-1\right)e^{(-5\,bx-5\,a)}-5\,e^{(7\,bx+7\,a)}+35\,e^{(3\,bx+3\,a)}}{2240\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*cosh(b*x+a)^3*sinh(b*x+a)^3,x, algorithm="giac")
```

```
[Out] -1/2240*(7*(15*e^(4*b*x + 4*a) - 1)*e^(-5*b*x - 5*a) - 5*e^(7*b*x + 7*a) +
35*e^(3*b*x + 3*a))/b
```

maple [A] time = 0.37, size = 88, normalized size = 1.28

$$\frac{\frac{\left(\cosh^{4}(bx+a)\right)\left(\sinh^{3}(bx+a)\right)}{7}-\frac{3\sinh(bx+a)\left(\cosh^{4}(bx+a)\right)}{35}+\frac{3\left(\frac{2}{3}+\frac{\left(\cosh^{2}(bx+a)\right)}{3}\right)\sinh(bx+a)}{35}+\frac{\left(\sinh^{2}(bx+a)\right)\left(\cosh^{5}(bx+a)\right)}{7}-\frac{2\left(\cosh^{5}(bx+a)\right)}{35}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(b*x+a)*cosh(b*x+a)^3*sinh(b*x+a)^3,x)
```

```
[Out] 1/b*(1/7*cosh(b*x+a)^4*sinh(b*x+a)^3-3/35*sinh(b*x+a)*cosh(b*x+a)^4+3/35*(2
/3+1/3*cosh(b*x+a)^2)*sinh(b*x+a)+1/7*sinh(b*x+a)^2*cosh(b*x+a)^5-2/35*cosh
(b*x+a)^5)
```

maxima [A] time = 0.37, size = 54, normalized size = 0.78

$$-\frac{\left(15\,e^{(4\,bx+4\,a)}-1\right)e^{(-5\,bx-5\,a)}}{320\,b}+\frac{e^{(7\,bx+7\,a)}-7\,e^{(3\,bx+3\,a)}}{448\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*cosh(b*x+a)^3*sinh(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -1/320*(15*e^(4*b*x + 4*a) - 1)*e^(-5*b*x - 5*a)/b + 1/448*(e^(7*b*x + 7*a)
 - 7*e^(3*b*x + 3*a))/b
```

mupad [B] time = 2.10, size = 50, normalized size = 0.72

$$-\frac{105\,e^{-a-b\,x}+35\,e^{3\,a+3\,b\,x}-7\,e^{-5\,a-5\,b\,x}-5\,e^{7\,a+7\,b\,x}}{2240\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a + b*x)^3*exp(a + b*x)*sinh(a + b*x)^3,x)
```

```
[Out] -(105*exp(- a - b*x) + 35*exp(3*a + 3*b*x) - 7*exp(- 5*a - 5*b*x) - 5*exp(7
*a + 7*b*x))/(2240*b)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*cosh(b*x+a)**3*sinh(b*x+a)**3,x)
```

```
[Out] Timed out
```

3.914 $\int e^{a+bx}\cosh^3(a+bx)\sinh^2(a+bx)\,dx$

Optimal. Leaf size=91

$$-\frac{e^{-4a-4bx}}{128b}-\frac{e^{-2a-2bx}}{64b}-\frac{e^{2a+2bx}}{32b}+\frac{e^{4a+4bx}}{128b}+\frac{e^{6a+6bx}}{192b}-\frac{x}{16}$$

```
[Out] -1/128*exp(-4*b*x-4*a)/b-1/64*exp(-2*b*x-2*a)/b-1/32*exp(2*b*x+2*a)/b+1/128
*exp(4*b*x+4*a)/b+1/192*exp(6*b*x+6*a)/b-1/16*x
```

Rubi [A] time = 0.08, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2282, 12, 446, 88}

$$-\frac{e^{-4a-4bx}}{128b}-\frac{e^{-2a-2bx}}{64b}-\frac{e^{2a+2bx}}{32b}+\frac{e^{4a+4bx}}{128b}+\frac{e^{6a+6bx}}{192b}-\frac{x}{16}$$

Antiderivative was successfully verified.

```
[In] Int[E^(a + b*x)*Cosh[a + b*x]^3*Sinh[a + b*x]^2,x]
```

```
[Out] -E^(-4*a - 4*b*x)/(128*b) - E^(-2*a - 2*b*x)/(64*b) - E^(2*a + 2*b*x)/(32*b
) + E^(4*a + 4*b*x)/(128*b) + E^(6*a + 6*b*x)/(192*b) - x/16
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
```

```
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$
\begin{aligned}
\int e^{a+bx}\cosh^3(a+bx)\sinh^2(a+bx)\,dx &= \frac{\text{Subst}\left(\int \frac{\left(1-x^2\right)^2\left(1+x^2\right)^3}{32x^5}\,dx, x, e^{a+bx}\right)}{b}\\
&= \frac{\text{Subst}\left(\int \frac{\left(1-x^2\right)^2\left(1+x^2\right)^3}{x^5}\,dx, x, e^{a+bx}\right)}{32b}\\
&= \frac{\text{Subst}\left(\int \frac{(1-x)^2(1+x)^3}{x^3}\,dx, x, e^{2a+2bx}\right)}{64b}\\
&= \frac{\text{Subst}\left(\int \left(-2+\frac{1}{x^3}+\frac{1}{x^2}-\frac{2}{x}+x+x^2\right)dx, x, e^{2a+2bx}\right)}{64b}\\
&= -\frac{e^{-4a-4bx}}{128b}-\frac{e^{-2a-2bx}}{64b}-\frac{e^{2a+2bx}}{32b}+\frac{e^{4a+4bx}}{128b}+\frac{e^{6a+6bx}}{192b}-\frac{x}{16}
\end{aligned}
$$

Mathematica [A] time = 0.11, size = 67, normalized size = 0.74

$$-\frac{3e^{-4(a+bx)}+6e^{-2(a+bx)}+12e^{2(a+bx)}-3e^{4(a+bx)}-2e^{6(a+bx)}+24bx}{384b}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(a + b*x)*Cosh[a + b*x]^3*Sinh[a + b*x]^2,x]
```

```
[Out] -1/384*(3/E^(4*(a + b*x)) + 6/E^(2*(a + b*x)) + 12*E^(2*(a + b*x)) - 3*E^(4
*(a + b*x)) - 2*E^(6*(a + b*x)) + 24*b*x)/b
```

fricas [B] time = 0.47, size = 165, normalized size = 1.81

$$-\frac{\cosh(bx+a)^5+5\cosh(bx+a)\sinh(bx+a)^4-5\sinh(bx+a)^5-\left(50\cosh(bx+a)^2+9\right)\sinh(bx+a)^3+}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*cosh(b*x+a)^3*sinh(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/384*(cosh(b*x + a)^5 + 5*cosh(b*x + a)*sinh(b*x + a)^4 - 5*sinh(b*x + a)
^5 - (50*cosh(b*x + a)^2 + 9)*sinh(b*x + a)^3 + 3*cosh(b*x + a)^3 + (10*cos
h(b*x + a)^3 + 9*cosh(b*x + a))*sinh(b*x + a)^2 + 12*(2*b*x + 1)*cosh(b*x +
 a) - (25*cosh(b*x + a)^4 + 24*b*x + 27*cosh(b*x + a)^2 - 12)*sinh(b*x + a)
)/(b*cosh(b*x + a) - b*sinh(b*x + a))
```

giac [A] time = 0.14, size = 81, normalized size = 0.89

$$-\frac{24\,bx-3\,\left(6\,e^{(4\,bx+4\,a)}-2\,e^{(2\,bx+2\,a)}-1\right)e^{(-4\,bx-4\,a)}+24\,a-2\,e^{(6\,bx+6\,a)}-3\,e^{(4\,bx+4\,a)}+12\,e^{(2\,bx+2\,a)}}{384\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*cosh(b*x+a)^3*sinh(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -1/384*(24*b*x - 3*(6*e^(4*b*x + 4*a) - 2*e^(2*b*x + 2*a) - 1)*e^(-4*b*x -
4*a) + 24*a - 2*e^(6*b*x + 6*a) - 3*e^(4*b*x + 4*a) + 12*e^(2*b*x + 2*a))/b
```

maple [A] time = 0.38, size = 84, normalized size = 0.92

$$\frac{\frac{\left(\cosh^4(bx+a)\right)\left(\sinh^2(bx+a)\right)}{6}-\frac{\left(\cosh^4(bx+a)\right)}{12}+\frac{\sinh(bx+a)\left(\cosh^5(bx+a)\right)}{6}-\frac{\left(\frac{\left(\cosh^3(bx+a)\right)}{4}+\frac{3\cosh(bx+a)}{8}\right)\sinh(bx+a)}{6}-\frac{bx}{16}-\frac{a}{16}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(b*x+a)*cosh(b*x+a)^3*sinh(b*x+a)^2,x)
```

```
[Out] 1/b*(1/6*cosh(b*x+a)^4*sinh(b*x+a)^2-1/12*cosh(b*x+a)^4+1/6*sinh(b*x+a)*cos
h(b*x+a)^5-1/6*(1/4*cosh(b*x+a)^3+3/8*cosh(b*x+a))*sinh(b*x+a)-1/16*b*x-1/1
6*a)
```

maxima [A] time = 0.33, size = 77, normalized size = 0.85

$$-\frac{\left(2\,e^{(2\,bx+2\,a)}+1\right)e^{(-4\,bx-4\,a)}}{128\,b}-\frac{bx+a}{16\,b}+\frac{2\,e^{(6\,bx+6\,a)}+3\,e^{(4\,bx+4\,a)}-12\,e^{(2\,bx+2\,a)}}{384\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*cosh(b*x+a)^3*sinh(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -1/128*(2*e^(2*b*x + 2*a) + 1)*e^(-4*b*x - 4*a)/b - 1/16*(b*x + a)/b + 1/38
4*(2*e^(6*b*x + 6*a) + 3*e^(4*b*x + 4*a) - 12*e^(2*b*x + 2*a))/b
```

mupad [B] time = 0.60, size = 65, normalized size = 0.71

$$-\frac{6\,\mathrm{e}^{-2\,a-2\,b\,x}+12\,\mathrm{e}^{2\,a+2\,b\,x}+3\,\mathrm{e}^{-4\,a-4\,b\,x}-3\,\mathrm{e}^{4\,a+4\,b\,x}-2\,\mathrm{e}^{6\,a+6\,b\,x}+24\,b\,x}{384\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a + b*x)^3*exp(a + b*x)*sinh(a + b*x)^2,x)
```

```
[Out] -(6*exp(- 2*a - 2*b*x) + 12*exp(2*a + 2*b*x) + 3*exp(- 4*a - 4*b*x) - 3*exp
(4*a + 4*b*x) - 2*exp(6*a + 6*b*x) + 24*b*x)/(384*b)
```

sympy [A] time = 170.93, size = 325, normalized size = 3.57

$$\begin{cases} \frac{xe^{a}e^{bx}\sinh^{5}(a+bx)}{16} - \frac{xe^{a}e^{bx}\sinh^{4}(a+bx)\cosh(a+bx)}{16} - \frac{xe^{a}e^{bx}\sinh^{3}(a+bx)\cosh^{2}(a+bx)}{8} + \frac{xe^{a}e^{bx}\sinh^{2}(a+bx)\cosh^{3}(a+bx)}{8} + \frac{xe^{a}e^{bx}\sinh(\ldots}{\ldots} \\ xe^{a}\sinh^{2}(a)\cosh^{3}(a) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*cosh(b*x+a)**3*sinh(b*x+a)**2,x)
```

```
[Out] Piecewise((x*exp(a)*exp(b*x)*sinh(a + b*x)**5/16 - x*exp(a)*exp(b*x)*sinh(a
 + b*x)**4*cosh(a + b*x)/16 - x*exp(a)*exp(b*x)*sinh(a + b*x)**3*cosh(a + b
*x)**2/8 + x*exp(a)*exp(b*x)*sinh(a + b*x)**2*cosh(a + b*x)**3/8 + x*exp(a)
*exp(b*x)*sinh(a + b*x)*cosh(a + b*x)**4/16 - x*exp(a)*exp(b*x)*cosh(a + b*
x)**5/16 - 13*exp(a)*exp(b*x)*sinh(a + b*x)**5/(96*b) + 7*exp(a)*exp(b*x)*s
inh(a + b*x)**4*cosh(a + b*x)/(96*b) + exp(a)*exp(b*x)*sinh(a + b*x)**3*cos
h(a + b*x)**2/(3*b) - exp(a)*exp(b*x)*sinh(a + b*x)**2*cosh(a + b*x)**3/(6*
b) + exp(a)*exp(b*x)*sinh(a + b*x)*cosh(a + b*x)**4/(96*b) + 5*exp(a)*exp(b
*x)*cosh(a + b*x)**5/(96*b), Ne(b, 0)), (x*exp(a)*sinh(a)**2*cosh(a)**3, Tr
ue))
```

3.915 $\int e^{a+bx} \cosh^3(a+bx) \sinh(a+bx) \, dx$

Optimal. Leaf size=69

$$\frac{e^{-3a-3bx}}{48b} + \frac{e^{-a-bx}}{8b} + \frac{e^{3a+3bx}}{24b} + \frac{e^{5a+5bx}}{80b}$$

```
[Out] 1/48*exp(-3*b*x-3*a)/b+1/8*exp(-b*x-a)/b+1/24*exp(3*b*x+3*a)/b+1/80*exp(5*b
*x+5*a)/b
```

Rubi [A] time = 0.05, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {2282, 12, 448}

$$\frac{e^{-3a-3bx}}{48b} + \frac{e^{-a-bx}}{8b} + \frac{e^{3a+3bx}}{24b} + \frac{e^{5a+5bx}}{80b}$$

Antiderivative was successfully verified.

```
[In] Int[E^(a + b*x)*Cosh[a + b*x]^3*Sinh[a + b*x],x]
```

```
[Out] E^(-3*a - 3*b*x)/(48*b) + E^(-a - b*x)/(8*b) + E^(3*a + 3*b*x)/(24*b) + E^(
5*a + 5*b*x)/(80*b)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$\int e^{a+bx}\cosh^3(a+bx)\sinh(a+bx)\,dx = \frac{\text{Subst}\left(\int \frac{(-1+x^2)(1+x^2)^3}{16x^4}\,dx, x, e^{a+bx}\right)}{b}$$
$$= \frac{\text{Subst}\left(\int \frac{(-1+x^2)(1+x^2)^3}{x^4}\,dx, x, e^{a+bx}\right)}{16b}$$
$$= \frac{\text{Subst}\left(\int \left(-\frac{1}{x^4}-\frac{2}{x^2}+2x^2+x^4\right)dx, x, e^{a+bx}\right)}{16b}$$
$$= \frac{e^{-3a-3bx}}{48b}+\frac{e^{-a-bx}}{8b}+\frac{e^{3a+3bx}}{24b}+\frac{e^{5a+5bx}}{80b}$$

Mathematica [A] time = 0.06, size = 51, normalized size = 0.74

$$\frac{e^{-3(a+bx)}\left(30e^{2(a+bx)}+10e^{6(a+bx)}+3e^{8(a+bx)}+5\right)}{240b}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(a + b*x)*Cosh[a + b*x]^3*Sinh[a + b*x],x]
```

```
[Out] (5 + 30*E^(2*(a + b*x)) + 10*E^(6*(a + b*x)) + 3*E^(8*(a + b*x)))/(240*b*E^
(3*(a + b*x)))
```

fricas [A] time = 0.44, size = 111, normalized size = 1.61

$$\frac{\cosh(bx+a)^4-\cosh(bx+a)\sinh(bx+a)^3+\sinh(bx+a)^4+\left(6\cosh(bx+a)^2+5\right)\sinh(bx+a)^2+5\cosh}{30\,(b\cosh(bx+a)-b\sinh(bx+a))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*cosh(b*x+a)^3*sinh(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/30*(cosh(b*x + a)^4 - cosh(b*x + a)*sinh(b*x + a)^3 + sinh(b*x + a)^4 + (
6*cosh(b*x + a)^2 + 5)*sinh(b*x + a)^2 + 5*cosh(b*x + a)^2 - (cosh(b*x + a)
^3 + 5*cosh(b*x + a))*sinh(b*x + a))/(b*cosh(b*x + a) - b*sinh(b*x + a))
```

giac [A] time = 0.14, size = 52, normalized size = 0.75

$$\frac{5\left(6\,e^{(2\,bx+2\,a)}+1\right)e^{(-3\,bx-3\,a)}+3\,e^{(5\,bx+5\,a)}+10\,e^{(3\,bx+3\,a)}}{240\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*cosh(b*x+a)^3*sinh(b*x+a),x, algorithm="giac")
```

```
[Out] 1/240*(5*(6*e^(2*b*x + 2*a) + 1)*e^(-3*b*x - 3*a) + 3*e^(5*b*x + 5*a) + 10*
e^(3*b*x + 3*a))/b
```

maple [A] time = 0.37, size = 52, normalized size = 0.75

$$\frac{\frac{\sinh(bx+a)\left(\cosh^4(bx+a)\right)}{5}-\frac{\left(\frac{2}{3}+\frac{\left(\cosh^2(bx+a)\right)}{3}\right)\sinh(bx+a)}{5}+\frac{\left(\cosh^5(bx+a)\right)}{5}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(b*x+a)*cosh(b*x+a)^3*sinh(b*x+a),x)
```

```
[Out] 1/b*(1/5*sinh(b*x+a)*cosh(b*x+a)^4-1/5*(2/3+1/3*cosh(b*x+a)^2)*sinh(b*x+a)+
1/5*cosh(b*x+a)^5)
```

maxima [A] time = 0.33, size = 56, normalized size = 0.81

$$\frac{\left(6\,e^{(2\,bx+2\,a)}+1\right)e^{(-3\,bx-3\,a)}}{48\,b}+\frac{3\,e^{(5\,bx+5\,a)}+10\,e^{(3\,bx+3\,a)}}{240\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*cosh(b*x+a)^3*sinh(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/48*(6*e^(2*b*x + 2*a) + 1)*e^(-3*b*x - 3*a)/b + 1/240*(3*e^(5*b*x + 5*a)
+ 10*e^(3*b*x + 3*a))/b
```

mupad [B] time = 0.52, size = 50, normalized size = 0.72

$$\frac{30\,\mathrm{e}^{-a-b\,x}+5\,\mathrm{e}^{-3\,a-3\,b\,x}+10\,\mathrm{e}^{3\,a+3\,b\,x}+3\,\mathrm{e}^{5\,a+5\,b\,x}}{240\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a + b*x)^3*exp(a + b*x)*sinh(a + b*x),x)
```

```
[Out] (30*exp(- a - b*x) + 5*exp(- 3*a - 3*b*x) + 10*exp(3*a + 3*b*x) + 3*exp(5*a
 + 5*b*x))/(240*b)
```

sympy [A] time = 59.48, size = 139, normalized size = 2.01

$$\begin{cases} -\frac{2e^{a}e^{bx}\sinh^4(a+bx)}{15b}+\frac{2e^{a}e^{bx}\sinh^3(a+bx)\cosh(a+bx)}{15b}+\frac{e^{a}e^{bx}\sinh^2(a+bx)\cosh^2(a+bx)}{5b}-\frac{e^{a}e^{bx}\sinh(a+bx)\cosh^3(a+bx)}{5b}+\frac{e^{a}e^{bx}\cosh^4(a+\ldots}{5b} \\ xe^{a}\sinh(a)\cosh^3(a) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*cosh(b*x+a)**3*sinh(b*x+a),x)
```

```
[Out] Piecewise((-2*exp(a)*exp(b*x)*sinh(a + b*x)**4/(15*b) + 2*exp(a)*exp(b*x)*s
inh(a + b*x)**3*cosh(a + b*x)/(15*b) + exp(a)*exp(b*x)*sinh(a + b*x)**2*cos
h(a + b*x)**2/(5*b) - exp(a)*exp(b*x)*sinh(a + b*x)*cosh(a + b*x)**3/(5*b)
+ exp(a)*exp(b*x)*cosh(a + b*x)**4/(5*b), Ne(b, 0)), (x*exp(a)*sinh(a)*cosh
(a)**3, True))
```

3.916 $\int e^{a+bx}\cosh^2(a+bx)\coth(a+bx)\,dx$

Optimal. Leaf size=59

$$\frac{e^{-a-bx}}{4b}+\frac{e^{a+bx}}{b}+\frac{e^{3a+3bx}}{12b}-\frac{2\tanh^{-1}\left(e^{a+bx}\right)}{b}$$

```
[Out] 1/4*exp(-b*x-a)/b+exp(b*x+a)/b+1/12*exp(3*b*x+3*a)/b-2*arctanh(exp(b*x+a))/
b
```

Rubi [A] time = 0.05, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2282, 12, 461, 207}

$$\frac{e^{-a-bx}}{4b}+\frac{e^{a+bx}}{b}+\frac{e^{3a+3bx}}{12b}-\frac{2\tanh^{-1}\left(e^{a+bx}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[E^(a + b*x)*Cosh[a + b*x]^2*Coth[a + b*x],x]
```

```
[Out] E^(-a - b*x)/(4*b) + E^(a + b*x)/b + E^(3*a + 3*b*x)/(12*b) - (2*ArcTanh[E^
(a + b*x)])/b
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 461

```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(
n_)), x_Symbol] :> Int[ExpandIntegrand[((e*x)^m*(a + b*x^n)^p)/(c + d*x^n),
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
&& IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
```

```
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$\begin{aligned}
\int e^{a+bx}\cosh^2(a+bx)\coth(a+bx)\,dx &= \frac{\text{Subst}\left(\int \frac{\left(1+x^2\right)^3}{4x^2\left(-1+x^2\right)}\,dx, x, e^{a+bx}\right)}{b}\\
&= \frac{\text{Subst}\left(\int \frac{\left(1+x^2\right)^3}{x^2\left(-1+x^2\right)}\,dx, x, e^{a+bx}\right)}{4b}\\
&= \frac{\text{Subst}\left(\int \left(4-\frac{1}{x^2}+x^2+\frac{8}{-1+x^2}\right)dx, x, e^{a+bx}\right)}{4b}\\
&= \frac{e^{-a-bx}}{4b}+\frac{e^{a+bx}}{b}+\frac{e^{3a+3bx}}{12b}+\frac{2\,\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, e^{a+bx}\right)}{b}\\
&= \frac{e^{-a-bx}}{4b}+\frac{e^{a+bx}}{b}+\frac{e^{3a+3bx}}{12b}-\frac{2\tanh^{-1}\left(e^{a+bx}\right)}{b}
\end{aligned}$$

Mathematica [A] time = 0.08, size = 68, normalized size = 1.15

$$\frac{e^{-a-bx}\left(12e^{2(a+bx)}+e^{4(a+bx)}-24\sqrt{e^{2(a+bx)}}\tanh^{-1}\left(\sqrt{e^{2(a+bx)}}\right)+3\right)}{12b}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(a + b*x)*Cosh[a + b*x]^2*Coth[a + b*x],x]
```

```
[Out] (E^(-a - b*x)*(3 + 12*E^(2*(a + b*x)) + E^(4*(a + b*x)) - 24*Sqrt[E^(2*(a +
 b*x))]*ArcTanh[Sqrt[E^(2*(a + b*x))]]))/(12*b)
```

fricas [B] time = 0.54, size = 170, normalized size = 2.88

$$\cosh(bx+a)^4+4\cosh(bx+a)\sinh(bx+a)^3+\sinh(bx+a)^4+6\left(\cosh(bx+a)^2+2\right)\sinh(bx+a)^2+12\,\text{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*cosh(b*x+a)^3*csch(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/12*(cosh(b*x + a)^4 + 4*cosh(b*x + a)*sinh(b*x + a)^3 + sinh(b*x + a)^4 +
 6*(cosh(b*x + a)^2 + 2)*sinh(b*x + a)^2 + 12*cosh(b*x + a)^2 - 12*(cosh(b*
x + a) + sinh(b*x + a))*log(cosh(b*x + a) + sinh(b*x + a) + 1) + 12*(cosh(b
*x + a) + sinh(b*x + a))*log(cosh(b*x + a) + sinh(b*x + a) - 1) + 4*(cosh(b
*x + a)^3 + 6*cosh(b*x + a))*sinh(b*x + a) + 3)/(b*cosh(b*x + a) + b*sinh(b
*x + a))
```

giac [A] time = 0.12, size = 57, normalized size = 0.97

$$\frac{e^{(3\,bx+3\,a)}+12\,e^{(bx+a)}+3\,e^{(-bx-a)}-12\,\log\left(e^{(bx+a)}+1\right)+12\,\log\left(\left|e^{(bx+a)}-1\right|\right)}{12\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*cosh(b*x+a)^3*csch(b*x+a),x, algorithm="giac")
```

```
[Out] 1/12*(e^(3*b*x + 3*a) + 12*e^(b*x + a) + 3*e^(-b*x - a) - 12*log(e^(b*x + a
) + 1) + 12*log(abs(e^(b*x + a) - 1)))/b
```

maple [A] time = 0.37, size = 50, normalized size = 0.85

$$\frac{\left(\frac{2}{3}+\frac{\left(\cosh^{2}(bx+a)\right)}{3}\right)\sinh\left(bx+a\right)+\frac{\left(\cosh^{3}(bx+a)\right)}{3}+\cosh\left(bx+a\right)-2\,\mathrm{arctanh}\left(\mathrm{e}^{bx+a}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(b*x+a)*cosh(b*x+a)^3*csch(b*x+a),x)
```

```
[Out] 1/b*((2/3+1/3*cosh(b*x+a)^2)*sinh(b*x+a)+1/3*cosh(b*x+a)^3+cosh(b*x+a)-2*ar
ctanh(exp(b*x+a)))
```

maxima [A] time = 0.33, size = 65, normalized size = 1.10

$$\frac{e^{(3\,bx+3\,a)}+12\,e^{(bx+a)}}{12\,b}+\frac{e^{(-bx-a)}}{4\,b}-\frac{\log\left(e^{(bx+a)}+1\right)}{b}+\frac{\log\left(e^{(bx+a)}-1\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*cosh(b*x+a)^3*csch(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/12*(e^(3*b*x + 3*a) + 12*e^(b*x + a))/b + 1/4*e^(-b*x - a)/b - log(e^(b*x
 + a) + 1)/b + log(e^(b*x + a) - 1)/b
```

mupad [B] time = 0.11, size = 66, normalized size = 1.12

$$\frac{\mathrm{e}^{a+b\,x}}{b}-\frac{2\,\mathrm{atan}\left(\frac{\mathrm{e}^{b\,x}\,\mathrm{e}^{a}\,\sqrt{-b^2}}{b}\right)}{\sqrt{-b^2}}+\frac{\mathrm{e}^{-a-b\,x}}{4\,b}+\frac{\mathrm{e}^{3\,a+3\,b\,x}}{12\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cosh(a + b*x)^3*exp(a + b*x))/sinh(a + b*x),x)
```

```
[Out] exp(a + b*x)/b - (2*atan((exp(b*x)*exp(a)*(-b^2)^(1/2))/b))/(-b^2)^(1/2) +
exp(- a - b*x)/(4*b) + exp(3*a + 3*b*x)/(12*b)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*cosh(b*x+a)**3*csch(b*x+a),x)
```

```
[Out] Timed out
```

3.917 $\int e^{a+bx} \cosh(a+bx) \coth^2(a+bx)\, dx$

Optimal. Leaf size=63

$$\frac{e^{2a+2bx}}{4b}+\frac{2}{b\left(1-e^{2a+2bx}\right)}+\frac{\log\left(1-e^{2a+2bx}\right)}{b}+\frac{x}{2}$$

```
[Out] 1/4*exp(2*b*x+2*a)/b+2/b/(1-exp(2*b*x+2*a))+1/2*x+ln(1-exp(2*b*x+2*a))/b
```

Rubi [A] time = 0.06, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2282, 12, 446, 88}

$$\frac{e^{2a+2bx}}{4b}+\frac{2}{b\left(1-e^{2a+2bx}\right)}+\frac{\log\left(1-e^{2a+2bx}\right)}{b}+\frac{x}{2}$$

Antiderivative was successfully verified.

```
[In] Int[E^(a + b*x)*Cosh[a + b*x]*Coth[a + b*x]^2,x]
```

```
[Out] E^(2*a + 2*b*x)/(4*b) + 2/(b*(1 - E^(2*a + 2*b*x))) + x/2 + Log[1 - E^(2*a
+ 2*b*x)]/b
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$\begin{aligned}\int e^{a+bx}\cosh(a+bx)\coth^2(a+bx)\,dx &= \frac{\text{Subst}\left(\int \frac{\left(1+x^2\right)^3}{2x\left(1-x^2\right)^2}\,dx,x,e^{a+bx}\right)}{b}\\ &= \frac{\text{Subst}\left(\int \frac{\left(1+x^2\right)^3}{x\left(1-x^2\right)^2}\,dx,x,e^{a+bx}\right)}{2b}\\ &= \frac{\text{Subst}\left(\int \frac{(1+x)^3}{(1-x)^2x}\,dx,x,e^{2a+2bx}\right)}{4b}\\ &= \frac{\text{Subst}\left(\int \left(1+\frac{8}{(-1+x)^2}+\frac{4}{-1+x}+\frac{1}{x}\right)dx,x,e^{2a+2bx}\right)}{4b}\\ &= \frac{e^{2a+2bx}}{4b}+\frac{2}{b\left(1-e^{2a+2bx}\right)}+\frac{x}{2}+\frac{\log\left(1-e^{2a+2bx}\right)}{b}\end{aligned}$$

Mathematica [A] time = 0.08, size = 52, normalized size = 0.83

$$\frac{e^{2(a+bx)}-\frac{8}{e^{2(a+bx)}-1}+4\log\left(1-e^{2(a+bx)}\right)+2bx}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(a + b*x)*Cosh[a + b*x]*Coth[a + b*x]^2,x]
```

```
[Out] (E^(2*(a + b*x)) - 8/(-1 + E^(2*(a + b*x))) + 2*b*x + 4*Log[1 - E^(2*(a + b
*x))])/(4*b)
```

fricas [B] time = 0.46, size = 213, normalized size = 3.38

$$\cosh(bx+a)^4+4\cosh(bx+a)\sinh(bx+a)^3+\sinh(bx+a)^4+(2bx-1)\cosh(bx+a)^2+\left(2bx+6\cosh(b\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*cosh(b*x+a)^3*csch(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/4*(cosh(b*x + a)^4 + 4*cosh(b*x + a)*sinh(b*x + a)^3 + sinh(b*x + a)^4 +
(2*b*x - 1)*cosh(b*x + a)^2 + (2*b*x + 6*cosh(b*x + a)^2 - 1)*sinh(b*x + a)
^2 - 2*b*x + 4*(cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x
+ a)^2 - 1)*log(2*sinh(b*x + a)/(cosh(b*x + a) - sinh(b*x + a))) + 2*(2*cos
h(b*x + a)^3 + (2*b*x - 1)*cosh(b*x + a))*sinh(b*x + a) - 8)/(b*cosh(b*x +
a)^2 + 2*b*cosh(b*x + a)*sinh(b*x + a) + b*sinh(b*x + a)^2 - b)
```

giac [A] time = 0.15, size = 63, normalized size = 1.00

$$\frac{2\,bx + 2\,a - \frac{4\left(e^{(2\,bx+2\,a)}+1\right)}{e^{(2\,bx+2\,a)}-1} + e^{(2\,bx+2\,a)} + 4\,\log\left(\left|e^{(2\,bx+2\,a)} - 1\right|\right)}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*cosh(b*x+a)^3*csch(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/4*(2*b*x + 2*a - 4*(e^(2*b*x + 2*a) + 1)/(e^(2*b*x + 2*a) - 1) + e^(2*b*x
 + 2*a) + 4*log(abs(e^(2*b*x + 2*a) - 1)))/b
```

maple [A] time = 0.19, size = 67, normalized size = 1.06

$$\frac{\cosh^2(bx+a)}{2b} + \frac{\ln(\sinh(bx+a))}{b} + \frac{\cosh^3(bx+a)}{2b\sinh(bx+a)} + \frac{3x}{2} + \frac{3a}{2b} - \frac{3\coth(bx+a)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(b*x+a)*cosh(b*x+a)^3*csch(b*x+a)^2,x)
```

```
[Out] 1/2*cosh(b*x+a)^2/b+ln(sinh(b*x+a))/b+1/2/b*cosh(b*x+a)^3/sinh(b*x+a)+3/2*x
+3/2*a/b-3/2*coth(b*x+a)/b
```

maxima [A] time = 0.34, size = 68, normalized size = 1.08

$$\frac{1}{2}\,x + \frac{a}{2\,b} + \frac{e^{(2\,bx+2\,a)}}{4\,b} + \frac{\log\left(e^{(bx+a)} + 1\right)}{b} + \frac{\log\left(e^{(bx+a)} - 1\right)}{b} - \frac{2}{b\left(e^{(2\,bx+2\,a)} - 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*cosh(b*x+a)^3*csch(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*x + 1/2*a/b + 1/4*e^(2*b*x + 2*a)/b + log(e^(b*x + a) + 1)/b + log(e^(b
*x + a) - 1)/b - 2/(b*(e^(2*b*x + 2*a) - 1))
```

mupad [B] time = 1.82, size = 53, normalized size = 0.84

$$\frac{x}{2}+\frac{\ln\left(\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}-1\right)}{b}-\frac{2}{b\,\left(\mathrm{e}^{2\,a+2\,b\,x}-1\right)}+\frac{\mathrm{e}^{2\,a+2\,b\,x}}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cosh(a + b*x)^3*exp(a + b*x))/sinh(a + b*x)^2,x)
```

```
[Out] x/2 + log(exp(2*a)*exp(2*b*x) - 1)/b - 2/(b*(exp(2*a + 2*b*x) - 1)) + exp(2
*a + 2*b*x)/(4*b)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*cosh(b*x+a)**3*csch(b*x+a)**2,x)
```

```
[Out] Timed out
```

3.918 $\int e^{a+bx}\coth^3(a+bx)\,dx$

Optimal. Leaf size=81

$$\frac{e^{a+bx}}{b}+\frac{3e^{a+bx}}{b\left(1-e^{2a+2bx}\right)}-\frac{2e^{a+bx}}{b\left(1-e^{2a+2bx}\right)^2}-\frac{3\tanh^{-1}\left(e^{a+bx}\right)}{b}$$

```
[Out] exp(b*x+a)/b-2*exp(b*x+a)/b/(1-exp(2*b*x+2*a))^2+3*exp(b*x+a)/b/(1-exp(2*b*
x+2*a))-3*arctanh(exp(b*x+a))/b
```

Rubi [A] time = 0.05, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {2282, 390, 1158, 12, 288, 207}

$$\frac{e^{a+bx}}{b}+\frac{3e^{a+bx}}{b\left(1-e^{2a+2bx}\right)}-\frac{2e^{a+bx}}{b\left(1-e^{2a+2bx}\right)^2}-\frac{3\tanh^{-1}\left(e^{a+bx}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[E^(a + b*x)*Coth[a + b*x]^3,x]
```

```
[Out] E^(a + b*x)/b - (2*E^(a + b*x))/(b*(1 - E^(2*a + 2*b*x))^2) + (3*E^(a + b*x
))/(b*(1 - E^(2*a + 2*b*x))) - (3*ArcTanh[E^(a + b*x)])/b
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 1158

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Wi
th[{Qx = PolynomialQuotient[(a + c*x^4)^p, d + e*x^2, x], R = Coeff[Polynom
ialRemainder[(a + c*x^4)^p, d + e*x^2, x], x, 0]}, -Simp[(R*x*(d + e*x^2)^(
q + 1))/(2*d*(q + 1)), x] + Dist[1/(2*d*(q + 1)), Int[(d + e*x^2)^(q + 1)*E
xpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x], x], x]] /; FreeQ[{a, c, d, e},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$\int e^{a+bx}\coth^3(a+bx)\,dx = \frac{\text{Subst}\left(\int \frac{(1+x^2)^3}{(-1+x^2)^3}\,dx, x, e^{a+bx}\right)}{b}$$
$$= \frac{\text{Subst}\left(\int \left(1+\frac{2(1+3x^4)}{(-1+x^2)^3}\right)dx, x, e^{a+bx}\right)}{b}$$
$$= \frac{e^{a+bx}}{b} + \frac{2\,\text{Subst}\left(\int \frac{1+3x^4}{(-1+x^2)^3}\,dx, x, e^{a+bx}\right)}{b}$$
$$= \frac{e^{a+bx}}{b} - \frac{2e^{a+bx}}{b\left(1-e^{2a+2bx}\right)^2} + \frac{\text{Subst}\left(\int \frac{12x^2}{(-1+x^2)^2}\,dx, x, e^{a+bx}\right)}{2b}$$
$$= \frac{e^{a+bx}}{b} - \frac{2e^{a+bx}}{b\left(1-e^{2a+2bx}\right)^2} + \frac{6\,\text{Subst}\left(\int \frac{x^2}{(-1+x^2)^2}\,dx, x, e^{a+bx}\right)}{b}$$
$$= \frac{e^{a+bx}}{b} - \frac{2e^{a+bx}}{b\left(1-e^{2a+2bx}\right)^2} + \frac{3e^{a+bx}}{b\left(1-e^{2a+2bx}\right)} + \frac{3\,\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, e^{a+bx}\right)}{b}$$
$$= \frac{e^{a+bx}}{b} - \frac{2e^{a+bx}}{b\left(1-e^{2a+2bx}\right)^2} + \frac{3e^{a+bx}}{b\left(1-e^{2a+2bx}\right)} - \frac{3\tanh^{-1}\left(e^{a+bx}\right)}{b}$$

Mathematica [C] time = 2.54, size = 286, normalized size = 3.53

$$-\frac{e^{-5(a+bx)}\left(256e^{8(a+bx)}\left(e^{2(a+bx)}+1\right)^3 {}_6F_5\left(\frac{3}{2},2,2,2,2,2;1,1,1,1,\frac{11}{2};e^{2(a+bx)}\right)+384e^{8(a+bx)}\left(5e^{2(a+bx)}+7\right)\left(e^{2(a+bx)}\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(a + b*x)*Coth[a + b*x]^3,x]
```

```
[Out] -1/60480*(-21*(252105 + 507305*E^(2*(a + b*x)) + 173916*E^(4*(a + b*x)) - 1
54296*E^(6*(a + b*x)) - 73885*E^(8*(a + b*x)) + 4887*E^(10*(a + b*x))) - (3
15*(-16807 - 28218*E^(2*(a + b*x)) + 1173*E^(4*(a + b*x)) + 17748*E^(6*(a +
 b*x)) + 4299*E^(8*(a + b*x)) - 1434*E^(10*(a + b*x)) + 7*E^(12*(a + b*x)))
*ArcTanh[Sqrt[E^(2*(a + b*x))]])/Sqrt[E^(2*(a + b*x))] + 384*E^(8*(a + b*x)
```

```
)*(1 + E^(2*(a + b*x)))^2*(7 + 5*E^(2*(a + b*x)))*HypergeometricPFQ[{3/2, 2
, 2, 2, 2}, {1, 1, 1, 11/2}, E^(2*(a + b*x))] + 256*E^(8*(a + b*x))*(1 + E^
(2*(a + b*x)))^3*HypergeometricPFQ[{3/2, 2, 2, 2, 2, 2}, {1, 1, 1, 1, 11/2}
, E^(2*(a + b*x))])/(b*E^(5*(a + b*x)))
```

fricas [B] time = 0.49, size = 459, normalized size = 5.67

$$\frac{2\,\cosh\left(bx+a\right)^{5}+10\,\cosh\left(bx+a\right)\sinh\left(bx+a\right)^{4}+2\,\sinh\left(bx+a\right)^{5}+10\left(2\,\cosh\left(bx+a\right)^{2}-1\right)\sinh\left(bx+a\right)^{3}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*cosh(b*x+a)^3*csch(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/2*(2*cosh(b*x + a)^5 + 10*cosh(b*x + a)*sinh(b*x + a)^4 + 2*sinh(b*x + a)
^5 + 10*(2*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^3 - 10*cosh(b*x + a)^3 + 10*(
2*cosh(b*x + a)^3 - 3*cosh(b*x + a))*sinh(b*x + a)^2 - 3*(cosh(b*x + a)^4 +
 4*cosh(b*x + a)*sinh(b*x + a)^3 + sinh(b*x + a)^4 + 2*(3*cosh(b*x + a)^2 -
 1)*sinh(b*x + a)^2 - 2*cosh(b*x + a)^2 + 4*(cosh(b*x + a)^3 - cosh(b*x + a
))*sinh(b*x + a) + 1)*log(cosh(b*x + a) + sinh(b*x + a) + 1) + 3*(cosh(b*x
+ a)^4 + 4*cosh(b*x + a)*sinh(b*x + a)^3 + sinh(b*x + a)^4 + 2*(3*cosh(b*x
+ a)^2 - 1)*sinh(b*x + a)^2 - 2*cosh(b*x + a)^2 + 4*(cosh(b*x + a)^3 - cosh
(b*x + a))*sinh(b*x + a) + 1)*log(cosh(b*x + a) + sinh(b*x + a) - 1) + 2*(5
*cosh(b*x + a)^4 - 15*cosh(b*x + a)^2 + 2)*sinh(b*x + a) + 4*cosh(b*x + a))
/(b*cosh(b*x + a)^4 + 4*b*cosh(b*x + a)*sinh(b*x + a)^3 + b*sinh(b*x + a)^4
 - 2*b*cosh(b*x + a)^2 + 2*(3*b*cosh(b*x + a)^2 - b)*sinh(b*x + a)^2 + 4*(b
*cosh(b*x + a)^3 - b*cosh(b*x + a))*sinh(b*x + a) + b)
```

giac [A] time = 0.15, size = 72, normalized size = 0.89

$$-\frac{\frac{2\left(3\,e^{(3\,bx+3\,a)}-e^{(bx+a)}\right)}{\left(e^{(2\,bx+2\,a)}-1\right)^{2}}-2\,e^{(bx+a)}+3\,\log\left(e^{(bx+a)}+1\right)-3\,\log\left(\left|e^{(bx+a)}-1\right|\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*cosh(b*x+a)^3*csch(b*x+a)^3,x, algorithm="giac")
```

```
[Out] -1/2*(2*(3*e^(3*b*x + 3*a) - e^(b*x + a))/(e^(2*b*x + 2*a) - 1)^2 - 2*e^(b*
x + a) + 3*log(e^(b*x + a) + 1) - 3*log(abs(e^(b*x + a) - 1)))/b
```

maple [A] time = 0.38, size = 89, normalized size = 1.10

$$\frac{\frac{\cosh^{2}(bx+a)}{\sinh(bx+a)}-\frac{2}{\sinh(bx+a)}+\frac{\cosh^{3}(bx+a)}{\sinh(bx+a)^{2}}-\frac{3\cosh(bx+a)}{\sinh(bx+a)^{2}}+\frac{3\,\mathrm{csch}(bx+a)\coth(bx+a)}{2}-3\,\mathrm{arctanh}\left(e^{bx+a}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(b*x+a)*cosh(b*x+a)^3*csch(b*x+a)^3,x)
```

```
[Out] 1/b*(cosh(b*x+a)^2/sinh(b*x+a)-2/sinh(b*x+a)+cosh(b*x+a)^3/sinh(b*x+a)^2-3*
cosh(b*x+a)/sinh(b*x+a)^2+3/2*csch(b*x+a)*coth(b*x+a)-3*arctanh(exp(b*x+a))
)
```

maxima [A] time = 0.35, size = 88, normalized size = 1.09

$$\frac{e^{(bx+a)}}{b} - \frac{3\,\log\left(e^{(bx+a)}+1\right)}{2\,b} + \frac{3\,\log\left(e^{(bx+a)}-1\right)}{2\,b} - \frac{3\,e^{(3\,bx+3\,a)} - e^{(bx+a)}}{b\left(e^{(4\,bx+4\,a)} - 2\,e^{(2\,bx+2\,a)} + 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*cosh(b*x+a)^3*csch(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] e^(b*x + a)/b - 3/2*log(e^(b*x + a) + 1)/b + 3/2*log(e^(b*x + a) - 1)/b - (
3*e^(3*b*x + 3*a) - e^(b*x + a))/(b*(e^(4*b*x + 4*a) - 2*e^(2*b*x + 2*a) +
1))
```

mupad [B] time = 1.79, size = 97, normalized size = 1.20

$$\frac{e^{a+b\,x}}{b} - \frac{3\,\mathrm{atan}\left(\frac{e^{b\,x}\,e^{a}\,\sqrt{-b^2}}{b}\right)}{\sqrt{-b^2}} - \frac{2\,e^{a+b\,x}}{b\,\left(e^{4\,a+4\,b\,x} - 2\,e^{2\,a+2\,b\,x} + 1\right)} - \frac{3\,e^{a+b\,x}}{b\,\left(e^{2\,a+2\,b\,x} - 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cosh(a + b*x)^3*exp(a + b*x))/sinh(a + b*x)^3,x)
```

```
[Out] exp(a + b*x)/b - (3*atan((exp(b*x)*exp(a)*(-b^2)^(1/2))/b))/(-b^2)^(1/2) -
(2*exp(a + b*x))/(b*(exp(4*a + 4*b*x) - 2*exp(2*a + 2*b*x) + 1)) - (3*exp(a
 + b*x))/(b*(exp(2*a + 2*b*x) - 1))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*cosh(b*x+a)**3*csch(b*x+a)**3,x)
```

```
[Out] Timed out
```

3.919 $\int e^{2(a+bx)}\cosh(a+bx)\sinh^3(a+bx)\,dx$

Optimal. Leaf size=57

$$\frac{e^{-2a-2bx}}{32b}-\frac{e^{4a+4bx}}{32b}+\frac{e^{6a+6bx}}{96b}+\frac{x}{8}$$

```
[Out] 1/32*exp(-2*b*x-2*a)/b-1/32*exp(4*b*x+4*a)/b+1/96*exp(6*b*x+6*a)/b+1/8*x
```

Rubi [A] time = 0.05, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2282, 12, 446, 75}

$$\frac{e^{-2a-2bx}}{32b}-\frac{e^{4a+4bx}}{32b}+\frac{e^{6a+6bx}}{96b}+\frac{x}{8}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*(a + b*x))*Cosh[a + b*x]*Sinh[a + b*x]^3,x]
```

```
[Out] E^(-2*a - 2*b*x)/(32*b) - E^(4*a + 4*b*x)/(32*b) + E^(6*a + 6*b*x)/(96*b) +
 x/8
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 75

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && EqQ[b*e + a*f, 0] &&  !(ILtQ[n + p
+ 2, 0] && GtQ[n + 2*p, 0])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
```

```
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$\begin{aligned}\int e^{2(a+bx)}\cosh(a+bx)\sinh^3(a+bx)\,dx &= \frac{\text{Subst}\left(\int \frac{\left(-1-x^2\right)\left(1-x^2\right)^3}{16x^3}\,dx, x, e^{a+bx}\right)}{b}\\ &= \frac{\text{Subst}\left(\int \frac{\left(-1-x^2\right)\left(1-x^2\right)^3}{x^3}\,dx, x, e^{a+bx}\right)}{16b}\\ &= \frac{\text{Subst}\left(\int \frac{(-1-x)(1-x)^3}{x^2}\,dx, x, e^{2a+2bx}\right)}{32b}\\ &= \frac{\text{Subst}\left(\int \left(-\frac{1}{x^2}+\frac{2}{x}-2x+x^2\right)dx, x, e^{2a+2bx}\right)}{32b}\\ &= \frac{e^{-2a-2bx}}{32b}-\frac{e^{4a+4bx}}{32b}+\frac{e^{6a+6bx}}{96b}+\frac{x}{8}\end{aligned}$$

Mathematica [A] time = 0.08, size = 43, normalized size = 0.75

$$\frac{3e^{-2(a+bx)}-3e^{4(a+bx)}+e^{6(a+bx)}+12bx}{96b}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*(a + b*x))*Cosh[a + b*x]*Sinh[a + b*x]^3,x]
[Out] (3/E^(2*(a + b*x)) - 3*E^(4*(a + b*x)) + E^(6*(a + b*x)) + 12*b*x)/(96*b)
```

fricas [B] time = 0.45, size = 152, normalized size = 2.67

$$\frac{4\cosh(bx+a)^4-8\cosh(bx+a)\sinh(bx+a)^3+4\sinh(bx+a)^4+3(4bx-1)\cosh(bx+a)^2+3\left(4bx+8\cosh\right.}{96\left(b\cosh(bx+a)^2-2b\cosh(bx+a)\sinh\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*b*x+2*a)*cosh(b*x+a)*sinh(b*x+a)^3,x, algorithm="fricas")
[Out] 1/96*(4*cosh(b*x + a)^4 - 8*cosh(b*x + a)*sinh(b*x + a)^3 + 4*sinh(b*x + a)
^4 + 3*(4*b*x - 1)*cosh(b*x + a)^2 + 3*(4*b*x + 8*cosh(b*x + a)^2 - 1)*sinh
```

```
(b*x + a)^2 - 2*(4*cosh(b*x + a)^3 + 3*(4*b*x + 1)*cosh(b*x + a))*sinh(b*x
+ a))/(b*cosh(b*x + a)^2 - 2*b*cosh(b*x + a)*sinh(b*x + a) + b*sinh(b*x + a
)^2)
```

giac [A] time = 0.12, size = 60, normalized size = 1.05

$$\frac{12\,bx - 3\left(2\,e^{(2\,bx+2\,a)} - 1\right)e^{(-2\,bx-2\,a)} + \left(e^{(6\,bx+12\,a)} - 3\,e^{(4\,bx+10\,a)}\right)e^{(-6\,a)}}{96\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*b*x+2*a)*cosh(b*x+a)*sinh(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/96*(12*b*x - 3*(2*e^(2*b*x + 2*a) - 1)*e^(-2*b*x - 2*a) + (e^(6*b*x + 12*
a) - 3*e^(4*b*x + 10*a))*e^(-6*a))/b
```

maple [A] time = 0.33, size = 89, normalized size = 1.56

$$\frac{x}{8} - \frac{\sinh(2bx+2a)}{32b} - \frac{\sinh(4bx+4a)}{32b} + \frac{\sinh(6bx+6a)}{96b} + \frac{\cosh(2bx+2a)}{32b} - \frac{\cosh(4bx+4a)}{32b} + \frac{\cosh(6bx+6a)}{96b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(2*b*x+2*a)*cosh(b*x+a)*sinh(b*x+a)^3,x)
```

```
[Out] 1/8*x-1/32*sinh(2*b*x+2*a)/b-1/32/b*sinh(4*b*x+4*a)+1/96/b*sinh(6*b*x+6*a)+
1/32*cosh(2*b*x+2*a)/b-1/32*cosh(4*b*x+4*a)/b+1/96*cosh(6*b*x+6*a)/b
```

maxima [A] time = 0.34, size = 52, normalized size = 0.91

$$-\frac{\left(3\,e^{(-2\,bx-2\,a)} - 1\right)e^{(6\,bx+6\,a)}}{96\,b} + \frac{bx+a}{8\,b} + \frac{e^{(-2\,bx-2\,a)}}{32\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*b*x+2*a)*cosh(b*x+a)*sinh(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -1/96*(3*e^(-2*b*x - 2*a) - 1)*e^(6*b*x + 6*a)/b + 1/8*(b*x + a)/b + 1/32*e
^(-2*b*x - 2*a)/b
```

mupad [B] time = 0.56, size = 42, normalized size = 0.74

$$\frac{x}{8} + \frac{\frac{e^{-2\,a-2\,b\,x}}{32} - \frac{e^{4\,a+4\,b\,x}}{32} + \frac{e^{6\,a+6\,b\,x}}{96}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a + b*x)*exp(2*a + 2*b*x)*sinh(a + b*x)^3,x)
```

```
[Out] x/8 + (exp(- 2*a - 2*b*x)/32 - exp(4*a + 4*b*x)/32 + exp(6*a + 6*b*x)/96)/b
```

sympy [A] time = 58.18, size = 235, normalized size = 4.12

$$\begin{cases} -\frac{xe^{2a}e^{2bx}\sinh^4(a+bx)}{8} + \frac{xe^{2a}e^{2bx}\sinh^3(a+bx)\cosh(a+bx)}{4} - \frac{xe^{2a}e^{2bx}\sinh(a+bx)\cosh^3(a+bx)}{4} + \frac{xe^{2a}e^{2bx}\cosh^4(a+bx)}{8} + \frac{7e^{2a}e^{2bx}\sinh^4(a+ \\ xe^{2a}\sinh^3(a)\cosh(a) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*b*x+2*a)*cosh(b*x+a)*sinh(b*x+a)**3,x)
```

```
[Out] Piecewise((-x*exp(2*a)*exp(2*b*x)*sinh(a + b*x)**4/8 + x*exp(2*a)*exp(2*b*x
)*sinh(a + b*x)**3*cosh(a + b*x)/4 - x*exp(2*a)*exp(2*b*x)*sinh(a + b*x)*co
sh(a + b*x)**3/4 + x*exp(2*a)*exp(2*b*x)*cosh(a + b*x)**4/8 + 7*exp(2*a)*ex
p(2*b*x)*sinh(a + b*x)**4/(48*b) - exp(2*a)*exp(2*b*x)*sinh(a + b*x)**3*cos
h(a + b*x)/(6*b) + exp(2*a)*exp(2*b*x)*sinh(a + b*x)**2*cosh(a + b*x)**2/(4
*b) - exp(2*a)*exp(2*b*x)*cosh(a + b*x)**4/(16*b), Ne(b, 0)), (x*exp(2*a)*s
inh(a)**3*cosh(a), True))
```

3.920 $\int e^{2(a+bx)} \cosh(a+bx) \sinh^2(a+bx) \, dx$

Optimal. Leaf size=66

$$-\frac{e^{-a-bx}}{8b}-\frac{e^{a+bx}}{8b}-\frac{e^{3a+3bx}}{24b}+\frac{e^{5a+5bx}}{40b}$$

```
[Out] -1/8*exp(-b*x-a)/b-1/8*exp(b*x+a)/b-1/24*exp(3*b*x+3*a)/b+1/40*exp(5*b*x+5*
a)/b
```

Rubi [A] time = 0.05, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2282, 12, 448}

$$-\frac{e^{-a-bx}}{8b}-\frac{e^{a+bx}}{8b}-\frac{e^{3a+3bx}}{24b}+\frac{e^{5a+5bx}}{40b}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*(a + b*x))*Cosh[a + b*x]*Sinh[a + b*x]^2,x]
```

```
[Out] -E^(-a - b*x)/(8*b) - E^(a + b*x)/(8*b) - E^(3*a + 3*b*x)/(24*b) + E^(5*a +
 5*b*x)/(40*b)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$\int e^{2(a+bx)}\cosh(a+bx)\sinh^2(a+bx)\,dx = \frac{\text{Subst}\left(\int \frac{\left(1-x^2\right)^2\left(1+x^2\right)}{8x^2}\,dx, x, e^{a+bx}\right)}{b}$$
$$= \frac{\text{Subst}\left(\int \frac{\left(1-x^2\right)^2\left(1+x^2\right)}{x^2}\,dx, x, e^{a+bx}\right)}{8b}$$
$$= \frac{\text{Subst}\left(\int \left(-1+\frac{1}{x^2}-x^2+x^4\right)dx, x, e^{a+bx}\right)}{8b}$$
$$= -\frac{e^{-a-bx}}{8b}-\frac{e^{a+bx}}{8b}-\frac{e^{3a+3bx}}{24b}+\frac{e^{5a+5bx}}{40b}$$

Mathematica [**A**] time = 0.08, size = 51, normalized size = 0.77

$$\frac{3e^{-a-bx}\left(e^{6(a+bx)}-5\right)-5e^{a+bx}\left(e^{2(a+bx)}+3\right)}{120b}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*(a + b*x))*Cosh[a + b*x]*Sinh[a + b*x]^2,x]
```

```
[Out] (-5*E^(a + b*x)*(3 + E^(2*(a + b*x))) + 3*E^(-a - b*x)*(-5 + E^(6*(a + b*x)
)))/(120*b)
```

fricas [**A**] time = 0.43, size = 105, normalized size = 1.59

$$-\frac{6\cosh(bx+a)^3+18\cosh(bx+a)\sinh(bx+a)^2-9\sinh(bx+a)^3-\left(27\cosh(bx+a)^2+5\right)\sinh(bx+a)+}{60\left(b\cosh(bx+a)^2-2b\cosh(bx+a)\sinh(bx+a)+b\sinh(bx+a)^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*b*x+2*a)*cosh(b*x+a)*sinh(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/60*(6*cosh(b*x + a)^3 + 18*cosh(b*x + a)*sinh(b*x + a)^2 - 9*sinh(b*x +
a)^3 - (27*cosh(b*x + a)^2 + 5)*sinh(b*x + a) + 10*cosh(b*x + a))/(b*cosh(b
*x + a)^2 - 2*b*cosh(b*x + a)*sinh(b*x + a) + b*sinh(b*x + a)^2)
```

giac [**A**] time = 0.12, size = 55, normalized size = 0.83

$$\frac{\left(3\,e^{(5\,bx+10\,a)}-5\,e^{(3\,bx+8\,a)}-15\,e^{(bx+6\,a)}\right)e^{(-5\,a)}-15\,e^{(-bx-a)}}{120\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*b*x+2*a)*cosh(b*x+a)*sinh(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/120*((3*e^(5*b*x + 10*a) - 5*e^(3*b*x + 8*a) - 15*e^(b*x + 6*a))*e^(-5*a)
 - 15*e^(-b*x - a))/b
```

maple [A] time = 0.22, size = 69, normalized size = 1.05

$$-\frac{\sinh(3bx+3a)}{24b}+\frac{\sinh(5bx+5a)}{40b}-\frac{\cosh(bx+a)}{4b}-\frac{\cosh(3bx+3a)}{24b}+\frac{\cosh(5bx+5a)}{40b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(2*b*x+2*a)*cosh(b*x+a)*sinh(b*x+a)^2,x)
```

```
[Out] -1/24/b*sinh(3*b*x+3*a)+1/40/b*sinh(5*b*x+5*a)-1/4*cosh(b*x+a)/b-1/24*cosh(
3*b*x+3*a)/b+1/40*cosh(5*b*x+5*a)/b
```

maxima [A] time = 0.33, size = 53, normalized size = 0.80

$$-\frac{\left(5\,e^{(-2\,bx-2\,a)}+15\,e^{(-4\,bx-4\,a)}-3\right)e^{(5\,bx+5\,a)}}{120\,b}-\frac{e^{(-bx-a)}}{8\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*b*x+2*a)*cosh(b*x+a)*sinh(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -1/120*(5*e^(-2*b*x - 2*a) + 15*e^(-4*b*x - 4*a) - 3)*e^(5*b*x + 5*a)/b - 1
/8*e^(-b*x - a)/b
```

mupad [B] time = 0.25, size = 47, normalized size = 0.71

$$-\frac{15\,e^{a+b\,x}+15\,e^{-a-b\,x}+5\,e^{3\,a+3\,b\,x}-3\,e^{5\,a+5\,b\,x}}{120\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a + b*x)*exp(2*a + 2*b*x)*sinh(a + b*x)^2,x)
```

```
[Out] -(15*exp(a + b*x) + 15*exp(- a - b*x) + 5*exp(3*a + 3*b*x) - 3*exp(5*a + 5*
b*x))/(120*b)
```

sympy [A] time = 16.53, size = 128, normalized size = 1.94

$$\begin{cases} \frac{e^{2a}e^{2bx}\sinh^3(a+bx)}{15b}-\frac{2e^{2a}e^{2bx}\sinh^2(a+bx)\cosh(a+bx)}{15b}+\frac{8e^{2a}e^{2bx}\sinh(a+bx)\cosh^2(a+bx)}{15b}-\frac{4e^{2a}e^{2bx}\cosh^3(a+bx)}{15b} & \text{for } b\neq 0 \\ xe^{2a}\sinh^2(a)\cosh(a) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*b*x+2*a)*cosh(b*x+a)*sinh(b*x+a)**2,x)
```

```
[Out] Piecewise((exp(2*a)*exp(2*b*x)*sinh(a + b*x)**3/(15*b) - 2*exp(2*a)*exp(2*b
*x)*sinh(a + b*x)**2*cosh(a + b*x)/(15*b) + 8*exp(2*a)*exp(2*b*x)*sinh(a +
b*x)*cosh(a + b*x)**2/(15*b) - 4*exp(2*a)*exp(2*b*x)*cosh(a + b*x)**3/(15*b
), Ne(b, 0)), (x*exp(2*a)*sinh(a)**2*cosh(a), True))
```

3.921 $\int e^{2(a+bx)} \cosh(a+bx) \sinh(a+bx) \, dx$

Optimal. Leaf size=23

$$\frac{e^{4a+4bx}}{16b} - \frac{x}{4}$$

```
[Out] 1/16*exp(4*b*x+4*a)/b-1/4*x
```

Rubi [A] time = 0.02, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {2282, 12, 14}

$$\frac{e^{4a+4bx}}{16b} - \frac{x}{4}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*(a + b*x))*Cosh[a + b*x]*Sinh[a + b*x],x]
```

```
[Out] E^(4*a + 4*b*x)/(16*b) - x/4
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$\int e^{2(a+bx)}\cosh(a+bx)\sinh(a+bx)\,dx = \frac{\text{Subst}\left(\int \frac{-1+x^4}{4x}\,dx, x, e^{a+bx}\right)}{b}$$
$$= \frac{\text{Subst}\left(\int \frac{-1+x^4}{x}\,dx, x, e^{a+bx}\right)}{4b}$$
$$= \frac{\text{Subst}\left(\int \left(-\frac{1}{x}+x^3\right)\,dx, x, e^{a+bx}\right)}{4b}$$
$$= \frac{e^{4a+4bx}}{16b} - \frac{x}{4}$$

Mathematica [**A**] time = 0.01, size = 25, normalized size = 1.09

$$\frac{1}{4}\left(\frac{e^{4a+4bx}}{4b} - x\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*(a + b*x))*Cosh[a + b*x]*Sinh[a + b*x],x]
```

```
[Out] (E^(4*a + 4*b*x)/(4*b) - x)/4
```

fricas [**B**] time = 0.47, size = 91, normalized size = 3.96

$$-\frac{(4\,bx-1)\cosh\left(bx+a\right)^2 - 2\,(4\,bx+1)\cosh\left(bx+a\right)\sinh\left(bx+a\right) + (4\,bx-1)\sinh\left(bx+a\right)^2}{16\left(b\cosh\left(bx+a\right)^2 - 2\,b\cosh\left(bx+a\right)\sinh\left(bx+a\right) + b\sinh\left(bx+a\right)^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*b*x+2*a)*cosh(b*x+a)*sinh(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/16*((4*b*x - 1)*cosh(b*x + a)^2 - 2*(4*b*x + 1)*cosh(b*x + a)*sinh(b*x +
 a) + (4*b*x - 1)*sinh(b*x + a)^2)/(b*cosh(b*x + a)^2 - 2*b*cosh(b*x + a)*s
inh(b*x + a) + b*sinh(b*x + a)^2)
```

giac [**A**] time = 0.12, size = 18, normalized size = 0.78

$$-\frac{1}{4}\,x + \frac{e^{(4\,bx+4\,a)}}{16\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*b*x+2*a)*cosh(b*x+a)*sinh(b*x+a),x, algorithm="giac")
```

```
[Out] -1/4*x + 1/16*e^(4*b*x + 4*a)/b
```

maple [A] time = 0.10, size = 33, normalized size = 1.43

$$-\frac{x}{4}+\frac{\sinh\left(4bx+4a\right)}{16b}+\frac{\cosh\left(4bx+4a\right)}{16b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(2*b*x+2*a)*cosh(b*x+a)*sinh(b*x+a),x)
```

```
[Out] -1/4*x+1/16/b*sinh(4*b*x+4*a)+1/16*cosh(4*b*x+4*a)/b
```

maxima [A] time = 0.32, size = 24, normalized size = 1.04

$$-\frac{1}{4}\,x-\frac{a}{4\,b}+\frac{e^{\left(4\,bx+4\,a\right)}}{16\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*b*x+2*a)*cosh(b*x+a)*sinh(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/4*x - 1/4*a/b + 1/16*e^(4*b*x + 4*a)/b
```

mupad [B] time = 1.81, size = 18, normalized size = 0.78

$$\frac{\mathrm{e}^{4\,a+4\,b\,x}}{16\,b}-\frac{x}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a + b*x)*exp(2*a + 2*b*x)*sinh(a + b*x),x)
```

```
[Out] exp(4*a + 4*b*x)/(16*b) - x/4
```

sympy [A] time = 4.56, size = 117, normalized size = 5.09

$$\begin{cases} -\frac{xe^{2a}e^{2bx}\sinh^{2}\left(a+bx\right)}{4}+\frac{xe^{2a}e^{2bx}\sinh\left(a+bx\right)\cosh\left(a+bx\right)}{2}-\frac{xe^{2a}e^{2bx}\cosh^{2}\left(a+bx\right)}{4}+\frac{e^{2a}e^{2bx}\sinh\left(a+bx\right)\cosh\left(a+bx\right)}{4b} & \text{for}\ b\neq 0 \\ xe^{2a}\sinh\left(a\right)\cosh\left(a\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*b*x+2*a)*cosh(b*x+a)*sinh(b*x+a),x)
```

```
[Out] Piecewise((-x*exp(2*a)*exp(2*b*x)*sinh(a + b*x)**2/4 + x*exp(2*a)*exp(2*b*x
)*sinh(a + b*x)*cosh(a + b*x)/2 - x*exp(2*a)*exp(2*b*x)*cosh(a + b*x)**2/4
+ exp(2*a)*exp(2*b*x)*sinh(a + b*x)*cosh(a + b*x)/(4*b), Ne(b, 0)), (x*exp(
2*a)*sinh(a)*cosh(a), True))
```

3.922 $\int e^{2(a+bx)} \coth(a+bx)\,dx$

Optimal. Leaf size=37

$$\frac{e^{2a+2bx}}{2b}+\frac{\log\left(1-e^{2a+2bx}\right)}{b}$$

```
[Out] 1/2*exp(2*b*x+2*a)/b+ln(1-exp(2*b*x+2*a))/b
```

Rubi [A] time = 0.03, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {2282, 444, 43}

$$\frac{e^{2a+2bx}}{2b}+\frac{\log\left(1-e^{2a+2bx}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*(a + b*x))*Coth[a + b*x],x]
```

```
[Out] E^(2*a + 2*b*x)/(2*b) + Log[1 - E^(2*a + 2*b*x)]/b
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$\int e^{2(a+bx)}\coth(a+bx)\,dx = \frac{\text{Subst}\left(\int \frac{x(-1-x^2)}{1-x^2}\,dx, x, e^{a+bx}\right)}{b}$$
$$= \frac{\text{Subst}\left(\int \frac{-1-x}{1-x}\,dx, x, e^{2a+2bx}\right)}{2b}$$
$$= \frac{\text{Subst}\left(\int \left(1+\frac{2}{-1+x}\right)dx, x, e^{2a+2bx}\right)}{2b}$$
$$= \frac{e^{2a+2bx}}{2b} + \frac{\log\left(1-e^{2a+2bx}\right)}{b}$$

Mathematica [**A**] time = 0.03, size = 35, normalized size = 0.95

$$\frac{e^{2a+2bx}+2\log\left(1-e^{2a+2bx}\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*(a + b*x))*Coth[a + b*x],x]
```

```
[Out] (E^(2*a + 2*b*x) + 2*Log[1 - E^(2*a + 2*b*x)])/(2*b)
```

fricas [**A**] time = 0.53, size = 64, normalized size = 1.73

$$\frac{\cosh(bx+a)^2 + 2\cosh(bx+a)\sinh(bx+a) + \sinh(bx+a)^2 + 2\log\left(\frac{2\sinh(bx+a)}{\cosh(bx+a)-\sinh(bx+a)}\right)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*b*x+2*a)*cosh(b*x+a)*csch(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/2*(cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 + 2*
log(2*sinh(b*x + a)/(cosh(b*x + a) - sinh(b*x + a))))/b
```

giac [**A**] time = 0.11, size = 30, normalized size = 0.81

$$\frac{e^{(2bx+2a)} + 2\log\left(\left|e^{(2bx+2a)}-1\right|\right)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*b*x+2*a)*cosh(b*x+a)*csch(b*x+a),x, algorithm="giac")
[Out] 1/2*(e^(2*b*x + 2*a) + 2*log(abs(e^(2*b*x + 2*a) - 1)))/b
```

maple [A] time = 0.26, size = 38, normalized size = 1.03

$$\frac{e^{2bx+2a}}{2b} - \frac{2a}{b} + \frac{\ln\left(e^{2bx+2a} - 1\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(2*b*x+2*a)*cosh(b*x+a)*csch(b*x+a),x)
[Out] 1/2*exp(2*b*x+2*a)/b-2*a/b+1/b*ln(exp(2*b*x+2*a)-1)
```

maxima [A] time = 0.32, size = 57, normalized size = 1.54

$$\frac{2(bx+a)}{b} + \frac{e^{(2bx+2a)}}{2b} + \frac{\log\left(e^{(-bx-a)}+1\right)}{b} + \frac{\log\left(e^{(-bx-a)}-1\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*b*x+2*a)*cosh(b*x+a)*csch(b*x+a),x, algorithm="maxima")
[Out] 2*(b*x + a)/b + 1/2*e^(2*b*x + 2*a)/b + log(e^(-b*x - a) + 1)/b + log(e^(-b
*x - a) - 1)/b
```

mupad [B] time = 0.06, size = 30, normalized size = 0.81

$$\frac{e^{2a+2bx} + 2\ln\left(e^{2a}e^{2bx} - 1\right)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cosh(a + b*x)*exp(2*a + 2*b*x))/sinh(a + b*x),x)
[Out] (exp(2*a + 2*b*x) + 2*log(exp(2*a)*exp(2*b*x) - 1))/(2*b)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*b*x+2*a)*cosh(b*x+a)*csch(b*x+a),x)
[Out] Timed out
```

3.923 $\int e^{2(a+bx)} \coth(a+bx)\mathbf{csch}(a+bx)\,dx$

Optimal. Leaf size=54

$$\frac{2e^{a+bx}}{b}+\frac{2e^{a+bx}}{b\left(1-e^{2a+2bx}\right)}-\frac{4\tanh^{-1}\left(e^{a+bx}\right)}{b}$$

```
[Out] 2*exp(b*x+a)/b+2*exp(b*x+a)/b/(1-exp(2*b*x+2*a))-4*arctanh(exp(b*x+a))/b
```

Rubi [A] time = 0.04, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {2282, 12, 455, 388, 206}

$$\frac{2e^{a+bx}}{b}+\frac{2e^{a+bx}}{b\left(1-e^{2a+2bx}\right)}-\frac{4\tanh^{-1}\left(e^{a+bx}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*(a + b*x))*Coth[a + b*x]*Csch[a + b*x],x]
```

```
[Out] (2*E^(a + b*x))/b + (2*E^(a + b*x))/(b*(1 - E^(2*a + 2*b*x))) - (4*ArcTanh[
E^(a + b*x)])/b
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 455

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[((-a)^(m/2 - 1)*(b*c - a*d)*x*(a + b*x^2)^(p + 1))/(2*b^(m/2 + 1)*(p
```

```
 + 1)), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[(a + b*x^2)^(p + 1)*Expand
ToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2 -
 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /; F
reeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &&
 (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$\begin{aligned}\int e^{2(a+bx)}\coth(a+bx)\operatorname{csch}(a+bx)\,dx &= \frac{\operatorname{Subst}\left(\int \frac{2x^2(1+x^2)}{(1-x^2)^2}\,dx,x,e^{a+bx}\right)}{b}\\ &= \frac{2\operatorname{Subst}\left(\int \frac{x^2(1+x^2)}{(1-x^2)^2}\,dx,x,e^{a+bx}\right)}{b}\\ &= \frac{2e^{a+bx}}{b\left(1-e^{2a+2bx}\right)}-\frac{\operatorname{Subst}\left(\int \frac{2+2x^2}{1-x^2}\,dx,x,e^{a+bx}\right)}{b}\\ &= \frac{2e^{a+bx}}{b}+\frac{2e^{a+bx}}{b\left(1-e^{2a+2bx}\right)}-\frac{4\operatorname{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,e^{a+bx}\right)}{b}\\ &= \frac{2e^{a+bx}}{b}+\frac{2e^{a+bx}}{b\left(1-e^{2a+2bx}\right)}-\frac{4\tanh^{-1}\left(e^{a+bx}\right)}{b}\end{aligned}$$

Mathematica [A] time = 0.12, size = 62, normalized size = 1.15

$$\frac{2\left(\frac{e^{a+bx}\left(e^{2(a+bx)}-2\right)}{e^{2(a+bx)}-1}+\log\left(1-e^{a+bx}\right)-\log\left(e^{a+bx}+1\right)\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*(a + b*x))*Coth[a + b*x]*Csch[a + b*x],x]
```

[Out] (2*((E^(a + b*x)*(-2 + E^(2*(a + b*x))))/(-1 + E^(2*(a + b*x))) + Log[1 - E
^(a + b*x)] - Log[1 + E^(a + b*x)]))/b

fricas [B] time = 0.48, size = 200, normalized size = 3.70

$$\frac{2\left(\cosh\left(bx+a\right)^{3}+3\cosh\left(bx+a\right)\sinh\left(bx+a\right)^{2}+\sinh\left(bx+a\right)^{3}-\left(\cosh\left(bx+a\right)^{2}+2\cosh\left(bx+a\right)\sinh\left(bx+a\right)\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(2*b*x+2*a)*cosh(b*x+a)*csch(b*x+a)^2,x, algorithm="fricas")

[Out] 2*(cosh(b*x + a)^3 + 3*cosh(b*x + a)*sinh(b*x + a)^2 + sinh(b*x + a)^3 - (c
osh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 - 1)*log(c
osh(b*x + a) + sinh(b*x + a) + 1) + (cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh
(b*x + a) + sinh(b*x + a)^2 - 1)*log(cosh(b*x + a) + sinh(b*x + a) - 1) + (
3*cosh(b*x + a)^2 - 2)*sinh(b*x + a) - 2*cosh(b*x + a))/(b*cosh(b*x + a)^2
+ 2*b*cosh(b*x + a)*sinh(b*x + a) + b*sinh(b*x + a)^2 - b)

giac [A] time = 0.15, size = 55, normalized size = 1.02

$$-\frac{2\left(\frac{e^{(bx+a)}}{e^{(2bx+2a)}-1}-e^{(bx+a)}+\log\left(e^{(bx+a)}+1\right)-\log\left(\left|e^{(bx+a)}-1\right|\right)\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(2*b*x+2*a)*cosh(b*x+a)*csch(b*x+a)^2,x, algorithm="giac")

[Out] -2*(e^(b*x + a)/(e^(2*b*x + 2*a) - 1) - e^(b*x + a) + log(e^(b*x + a) + 1)
- log(abs(e^(b*x + a) - 1)))/b

maple [A] time = 0.22, size = 65, normalized size = 1.20

$$\frac{2\,\mathrm{e}^{bx+a}}{b}-\frac{2\,\mathrm{e}^{bx+a}}{b\left(\mathrm{e}^{2bx+2a}-1\right)}+\frac{2\ln\left(\mathrm{e}^{bx+a}-1\right)}{b}-\frac{2\ln\left(1+\mathrm{e}^{bx+a}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(exp(2*b*x+2*a)*cosh(b*x+a)*csch(b*x+a)^2,x)

[Out] 2*exp(b*x+a)/b-2/b*exp(b*x+a)/(exp(2*b*x+2*a)-1)+2/b*ln(exp(b*x+a)-1)-2/b*l
n(1+exp(b*x+a))

maxima [A] time = 0.32, size = 76, normalized size = 1.41

$$-\frac{2\log\left(e^{(-bx-a)}+1\right)}{b}+\frac{2\log\left(e^{(-bx-a)}-1\right)}{b}-\frac{2\left(2\,e^{(-2bx-2a)}-1\right)}{b\left(e^{(-bx-a)}-e^{(-3bx-3a)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*b*x+2*a)*cosh(b*x+a)*csch(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -2*log(e^(-b*x - a) + 1)/b + 2*log(e^(-b*x - a) - 1)/b - 2*(2*e^(-2*b*x - 2
*a) - 1)/(b*(e^(-b*x - a) - e^(-3*b*x - 3*a)))
```

mupad [B] time = 1.76, size = 63, normalized size = 1.17

$$\frac{2\,{\mathrm{e}}^{a+b\,x}}{b}-\frac{4\,\mathrm{atan}\left(\frac{{\mathrm{e}}^{b\,x}\,{\mathrm{e}}^a\,\sqrt{-b^2}}{b}\right)}{\sqrt{-b^2}}-\frac{2\,{\mathrm{e}}^{a+b\,x}}{b\,\left({\mathrm{e}}^{2\,a+2\,b\,x}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cosh(a + b*x)*exp(2*a + 2*b*x))/sinh(a + b*x)^2,x)
```

```
[Out] (2*exp(a + b*x))/b - (4*atan((exp(b*x)*exp(a)*(-b^2)^(1/2))/b))/(-b^2)^(1/2
) - (2*exp(a + b*x))/(b*(exp(2*a + 2*b*x) - 1))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*b*x+2*a)*cosh(b*x+a)*csch(b*x+a)**2,x)
```

```
[Out] Timed out
```

3.924 $\int e^{2(a+bx)} \coth(a+bx)\mathbf{csch}^2(a+bx)\,dx$

Optimal. Leaf size=63

$$\frac{6}{b\left(1-e^{2a+2bx}\right)}-\frac{2}{b\left(1-e^{2a+2bx}\right)^2}+\frac{2\log\left(1-e^{2a+2bx}\right)}{b}$$

[Out] -2/b/(1-exp(2*b*x+2*a))^2+6/b/(1-exp(2*b*x+2*a))+2*ln(1-exp(2*b*x+2*a))/b

Rubi [A] time = 0.07, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2282, 12, 446, 77}

$$\frac{6}{b\left(1-e^{2a+2bx}\right)}-\frac{2}{b\left(1-e^{2a+2bx}\right)^2}+\frac{2\log\left(1-e^{2a+2bx}\right)}{b}$$

Antiderivative was successfully verified.

[In] Int[E^(2*(a + b*x))*Coth[a + b*x]*Csch[a + b*x]^2,x]

[Out] -2/(b*(1 - E^(2*a + 2*b*x))^2) + 6/(b*(1 - E^(2*a + 2*b*x))) + (2*Log[1 - E^(2*a + 2*b*x)])/b

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$
\begin{aligned}
\int e^{2(a+bx)}\coth(a+bx)\operatorname{csch}^2(a+bx)\,dx &= \frac{\operatorname{Subst}\left(\int \frac{4x^3(-1-x^2)}{(1-x^2)^3}\,dx, x, e^{a+bx}\right)}{b}\\
&= \frac{4\operatorname{Subst}\left(\int \frac{x^3(-1-x^2)}{(1-x^2)^3}\,dx, x, e^{a+bx}\right)}{b}\\
&= \frac{2\operatorname{Subst}\left(\int \frac{(-1-x)x}{(1-x)^3}\,dx, x, e^{2a+2bx}\right)}{b}\\
&= \frac{2\operatorname{Subst}\left(\int \left(\frac{2}{(-1+x)^3}+\frac{3}{(-1+x)^2}+\frac{1}{-1+x}\right)dx, x, e^{2a+2bx}\right)}{b}\\
&= -\frac{2}{b\left(1-e^{2a+2bx}\right)^2}+\frac{6}{b\left(1-e^{2a+2bx}\right)}+\frac{2\log\left(1-e^{2a+2bx}\right)}{b}
\end{aligned}
$$

Mathematica [A] time = 0.08, size = 47, normalized size = 0.75

$$
\frac{2\left(\frac{2-3e^{2(a+bx)}}{\left(e^{2(a+bx)}-1\right)^2}+\log\left(1-e^{2(a+bx)}\right)\right)}{b}
$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*(a + b*x))*Coth[a + b*x]*Csch[a + b*x]^2,x]
```

```
[Out] (2*((2 - 3*E^(2*(a + b*x)))/(-1 + E^(2*(a + b*x)))^2 + Log[1 - E^(2*(a + b*
x))]))/b
```

fricas [B] time = 0.52, size = 262, normalized size = 4.16

$$
-\frac{2\left(3\cosh(bx+a)^2-\left(\cosh(bx+a)^4+4\cosh(bx+a)\sinh(bx+a)^3+\sinh(bx+a)^4+2\left(3\cosh(bx+a)^2-\right.\right.\right.}{b\cosh(bx+a)^4+4b\cosh(bx+a)\sinh(bx+a)^3+b\sinh(bx\cdot}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*b*x+2*a)*cosh(b*x+a)*csch(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -2*(3*cosh(b*x + a)^2 - (cosh(b*x + a)^4 + 4*cosh(b*x + a)*sinh(b*x + a)^3
+ sinh(b*x + a)^4 + 2*(3*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^2 - 2*cosh(b*x
+ a)^2 + 4*(cosh(b*x + a)^3 - cosh(b*x + a))*sinh(b*x + a) + 1)*log(2*sinh(
b*x + a)/(cosh(b*x + a) - sinh(b*x + a))) + 6*cosh(b*x + a)*sinh(b*x + a) +
 3*sinh(b*x + a)^2 - 2)/(b*cosh(b*x + a)^4 + 4*b*cosh(b*x + a)*sinh(b*x + a
)^3 + b*sinh(b*x + a)^4 - 2*b*cosh(b*x + a)^2 + 2*(3*b*cosh(b*x + a)^2 - b)
*sinh(b*x + a)^2 + 4*(b*cosh(b*x + a)^3 - b*cosh(b*x + a))*sinh(b*x + a) +
b)
```

giac [A] time = 0.15, size = 48, normalized size = 0.76

$$-\frac{\frac{3\,e^{(4\,bx+4\,a)}-1}{\left(e^{(2\,bx+2\,a)}-1\right)^{2}}-2\,\log\left(\left|e^{(2\,bx+2\,a)}-1\right|\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*b*x+2*a)*cosh(b*x+a)*csch(b*x+a)^3,x, algorithm="giac")
```

```
[Out] -((3*e^(4*b*x + 4*a) - 1)/(e^(2*b*x + 2*a) - 1)^2 - 2*log(abs(e^(2*b*x + 2*
a) - 1)))/b
```

maple [A] time = 0.22, size = 56, normalized size = 0.89

$$-\frac{4a}{b}-\frac{2\left(3\,\mathrm{e}^{2bx+2a}-2\right)}{b\left(\mathrm{e}^{2bx+2a}-1\right)^{2}}+\frac{2\ln\left(\mathrm{e}^{2bx+2a}-1\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(2*b*x+2*a)*cosh(b*x+a)*csch(b*x+a)^3,x)
```

```
[Out] -4*a/b-2*(3*exp(2*b*x+2*a)-2)/b/(exp(2*b*x+2*a)-1)^2+2/b*ln(exp(2*b*x+2*a)-
1)
```

maxima [A] time = 0.32, size = 86, normalized size = 1.37

$$4\,x+\frac{4\,a}{b}+\frac{2\,\log\left(e^{(-bx-a)}+1\right)}{b}+\frac{2\,\log\left(e^{(-bx-a)}-1\right)}{b}-\frac{2\left(e^{(-2\,bx-2\,a)}-2\right)}{b\left(2\,e^{(-2\,bx-2\,a)}-e^{(-4\,bx-4\,a)}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*b*x+2*a)*cosh(b*x+a)*csch(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 4*x + 4*a/b + 2*log(e^(-b*x - a) + 1)/b + 2*log(e^(-b*x - a) - 1)/b - 2*(e^
(-2*b*x - 2*a) - 2)/(b*(2*e^(-2*b*x - 2*a) - e^(-4*b*x - 4*a) - 1))
```

mupad [B] time = 1.86, size = 66, normalized size = 1.05

$$\frac{2\ln\left(e^{2a}e^{2bx}-1\right)}{b}-\frac{6}{b\left(e^{2a+2bx}-1\right)}-\frac{2}{b\left(e^{4a+4bx}-2e^{2a+2bx}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cosh(a + b*x)*exp(2*a + 2*b*x))/sinh(a + b*x)^3,x)
```

```
[Out] (2*log(exp(2*a)*exp(2*b*x) - 1))/b - 6/(b*(exp(2*a + 2*b*x) - 1)) - 2/(b*(e
xp(4*a + 4*b*x) - 2*exp(2*a + 2*b*x) + 1))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*b*x+2*a)*cosh(b*x+a)*csch(b*x+a)**3,x)
```

```
[Out] Timed out
```

3.925 $\int e^{2(a+bx)} \cosh^2(a+bx) \sinh^3(a+bx) \, dx$

Optimal. Leaf size=100

$$\frac{e^{-3a-3bx}}{96b} - \frac{e^{-a-bx}}{32b} + \frac{e^{a+bx}}{16b} - \frac{e^{3a+3bx}}{48b} - \frac{e^{5a+5bx}}{160b} + \frac{e^{7a+7bx}}{224b}$$

```
[Out] 1/96*exp(-3*b*x-3*a)/b-1/32*exp(-b*x-a)/b+1/16*exp(b*x+a)/b-1/48*exp(3*b*x+
3*a)/b-1/160*exp(5*b*x+5*a)/b+1/224*exp(7*b*x+7*a)/b
```

Rubi [A] time = 0.08, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {2282, 12, 448}

$$\frac{e^{-3a-3bx}}{96b} - \frac{e^{-a-bx}}{32b} + \frac{e^{a+bx}}{16b} - \frac{e^{3a+3bx}}{48b} - \frac{e^{5a+5bx}}{160b} + \frac{e^{7a+7bx}}{224b}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*(a + b*x))*Cosh[a + b*x]^2*Sinh[a + b*x]^3,x]
```

```
[Out] E^(-3*a - 3*b*x)/(96*b) - E^(-a - b*x)/(32*b) + E^(a + b*x)/(16*b) - E^(3*a
 + 3*b*x)/(48*b) - E^(5*a + 5*b*x)/(160*b) + E^(7*a + 7*b*x)/(224*b)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$\int e^{2(a+bx)}\cosh^2(a+bx)\sinh^3(a+bx)\,dx = \frac{\text{Subst}\left(\int \frac{\left(-1+x^2\right)^3\left(1+x^2\right)^2}{32x^4}\,dx, x, e^{a+bx}\right)}{b}$$
$$= \frac{\text{Subst}\left(\int \frac{\left(-1+x^2\right)^3\left(1+x^2\right)^2}{x^4}\,dx, x, e^{a+bx}\right)}{32b}$$
$$= \frac{\text{Subst}\left(\int \left(2-\frac{1}{x^4}+\frac{1}{x^2}-2x^2-x^4+x^6\right)dx, x, e^{a+bx}\right)}{32b}$$
$$= \frac{e^{-3a-3bx}}{96b}-\frac{e^{-a-bx}}{32b}+\frac{e^{a+bx}}{16b}-\frac{e^{3a+3bx}}{48b}-\frac{e^{5a+5bx}}{160b}+\frac{e^{7a+7bx}}{224b}$$

Mathematica [A] time = 0.11, size = 73, normalized size = 0.73

$$\frac{e^{-3(a+bx)}\left(-105e^{2(a+bx)}+210e^{4(a+bx)}-70e^{6(a+bx)}-21e^{8(a+bx)}+15e^{10(a+bx)}+35\right)}{3360b}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*(a + b*x))*Cosh[a + b*x]^2*Sinh[a + b*x]^3,x]
```

```
[Out] (35 - 105*E^(2*(a + b*x)) + 210*E^(4*(a + b*x)) - 70*E^(6*(a + b*x)) - 21*E
^(8*(a + b*x)) + 15*E^(10*(a + b*x)))/(3360*b*E^(3*(a + b*x)))
```

fricas [B] time = 0.45, size = 175, normalized size = 1.75

$$\frac{25\cosh(bx+a)^5+125\cosh(bx+a)\sinh(bx+a)^4-10\sinh(bx+a)^5-2\left(50\cosh(bx+a)^2-21\right)\sinh(bx+}{1680\left(b\cos\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*b*x+2*a)*cosh(b*x+a)^2*sinh(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/1680*(25*cosh(b*x + a)^5 + 125*cosh(b*x + a)*sinh(b*x + a)^4 - 10*sinh(b*
x + a)^5 - 2*(50*cosh(b*x + a)^2 - 21)*sinh(b*x + a)^3 - 63*cosh(b*x + a)^3
 + (250*cosh(b*x + a)^3 - 189*cosh(b*x + a))*sinh(b*x + a)^2 - 2*(25*cosh(b
*x + a)^4 - 63*cosh(b*x + a)^2 + 70)*sinh(b*x + a) + 70*cosh(b*x + a))/(b*c
osh(b*x + a)^2 - 2*b*cosh(b*x + a)*sinh(b*x + a) + b*sinh(b*x + a)^2)
```

giac [A] time = 0.15, size = 80, normalized size = 0.80

$$-\frac{35\left(3e^{(2bx+2a)}-1\right)e^{(-3bx-3a)}-\left(15e^{(7bx+28a)}-21e^{(5bx+26a)}-70e^{(3bx+24a)}+210e^{(bx+22a)}\right)e^{(-21a)}}{3360b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*b*x+2*a)*cosh(b*x+a)^2*sinh(b*x+a)^3,x, algorithm="giac")
```

```
[Out] -1/3360*(35*(3*e^(2*b*x + 2*a) - 1)*e^(-3*b*x - 3*a) - (15*e^(7*b*x + 28*a)
 - 21*e^(5*b*x + 26*a) - 70*e^(3*b*x + 24*a) + 210*e^(b*x + 22*a))*e^(-21*a
))/b
```

maple [A] time = 0.45, size = 108, normalized size = 1.08

$$\frac{3\sinh(bx+a)}{32b}-\frac{\sinh(3bx+3a)}{32b}-\frac{\sinh(5bx+5a)}{160b}+\frac{\sinh(7bx+7a)}{224b}+\frac{\cosh(bx+a)}{32b}-\frac{\cosh(3bx+3a)}{96b}-\frac{\cosh(5\ldots}{16\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(2*b*x+2*a)*cosh(b*x+a)^2*sinh(b*x+a)^3,x)
```

```
[Out] 3/32*sinh(b*x+a)/b-1/32/b*sinh(3*b*x+3*a)-1/160/b*sinh(5*b*x+5*a)+1/224/b*s
inh(7*b*x+7*a)+1/32*cosh(b*x+a)/b-1/96*cosh(3*b*x+3*a)/b-1/160*cosh(5*b*x+5
*a)/b+1/224*cosh(7*b*x+7*a)/b
```

maxima [A] time = 0.32, size = 78, normalized size = 0.78

$$-\frac{\left(21\,e^{(-2\,bx-2\,a)}+70\,e^{(-4\,bx-4\,a)}-210\,e^{(-6\,bx-6\,a)}-15\right)e^{(7\,bx+7\,a)}}{3360\,b}-\frac{3\,e^{(-bx-a)}-e^{(-3\,bx-3\,a)}}{96\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*b*x+2*a)*cosh(b*x+a)^2*sinh(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -1/3360*(21*e^(-2*b*x - 2*a) + 70*e^(-4*b*x - 4*a) - 210*e^(-6*b*x - 6*a) -
 15)*e^(7*b*x + 7*a)/b - 1/96*(3*e^(-b*x - a) - e^(-3*b*x - 3*a))/b
```

mupad [B] time = 0.59, size = 69, normalized size = 0.69

$$\frac{210\,\mathrm{e}^{a+b\,x}-105\,\mathrm{e}^{-a-b\,x}+35\,\mathrm{e}^{-3\,a-3\,b\,x}-70\,\mathrm{e}^{3\,a+3\,b\,x}-21\,\mathrm{e}^{5\,a+5\,b\,x}+15\,\mathrm{e}^{7\,a+7\,b\,x}}{3360\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a + b*x)^2*exp(2*a + 2*b*x)*sinh(a + b*x)^3,x)
```

```
[Out] (210*exp(a + b*x) - 105*exp(- a - b*x) + 35*exp(- 3*a - 3*b*x) - 70*exp(3*a
 + 3*b*x) - 21*exp(5*a + 5*b*x) + 15*exp(7*a + 7*b*x))/(3360*b)
```

sympy [A] time = 163.25, size = 197, normalized size = 1.97

$$\begin{cases}-\frac{4e^{2a}e^{2bx}\sinh^{5}(a+bx)}{35b}+\frac{8e^{2a}e^{2bx}\sinh^{4}(a+bx)\cosh(a+bx)}{35b}+\frac{2e^{2a}e^{2bx}\sinh^{3}(a+bx)\cosh^{2}(a+bx)}{35b}-\frac{e^{2a}e^{2bx}\sinh^{2}(a+bx)\cosh^{3}(a+bx)}{105b}-\frac{4e^{2a}\ldots}{\ldots}\\ xe^{2a}\sinh^{3}(a)\cosh^{2}(a)\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*b*x+2*a)*cosh(b*x+a)**2*sinh(b*x+a)**3,x)
```

```
[Out] Piecewise((-4*exp(2*a)*exp(2*b*x)*sinh(a + b*x)**5/(35*b) + 8*exp(2*a)*exp(
2*b*x)*sinh(a + b*x)**4*cosh(a + b*x)/(35*b) + 2*exp(2*a)*exp(2*b*x)*sinh(a
 + b*x)**3*cosh(a + b*x)**2/(35*b) - exp(2*a)*exp(2*b*x)*sinh(a + b*x)**2*c
osh(a + b*x)**3/(105*b) - 4*exp(2*a)*exp(2*b*x)*sinh(a + b*x)*cosh(a + b*x)
**4/(105*b) + 2*exp(2*a)*exp(2*b*x)*cosh(a + b*x)**5/(105*b), Ne(b, 0)), (x
*exp(2*a)*sinh(a)**3*cosh(a)**2, True))
```

3.926 $\int e^{2(a+bx)} \cosh^2(a+bx) \sinh^2(a+bx)\,dx$

Optimal. Leaf size=52

$$-\frac{e^{-2a-2bx}}{32b}-\frac{e^{2a+2bx}}{16b}+\frac{e^{6a+6bx}}{96b}$$

```
[Out] -1/32*exp(-2*b*x-2*a)/b-1/16*exp(2*b*x+2*a)/b+1/96*exp(6*b*x+6*a)/b
```

Rubi [A] time = 0.06, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {2282, 12, 270}

$$-\frac{e^{-2a-2bx}}{32b}-\frac{e^{2a+2bx}}{16b}+\frac{e^{6a+6bx}}{96b}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*(a + b*x))*Cosh[a + b*x]^2*Sinh[a + b*x]^2,x]
```

```
[Out] -E^(-2*a - 2*b*x)/(32*b) - E^(2*a + 2*b*x)/(16*b) + E^(6*a + 6*b*x)/(96*b)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$\int e^{2(a+bx)}\cosh^2(a+bx)\sinh^2(a+bx)\,dx=\frac{\text{Subst}\left(\int\frac{(1-x^4)^2}{16x^3}\,dx,x,e^{a+bx}\right)}{b}$$
$$=\frac{\text{Subst}\left(\int\frac{(1-x^4)^2}{x^3}\,dx,x,e^{a+bx}\right)}{16b}$$
$$=\frac{\text{Subst}\left(\int\left(\frac{1}{x^3}-2x+x^5\right)dx,x,e^{a+bx}\right)}{16b}$$
$$=-\frac{e^{-2a-2bx}}{32b}-\frac{e^{2a+2bx}}{16b}+\frac{e^{6a+6bx}}{96b}$$

Mathematica [**A**] time = 0.03, size = 38, normalized size = 0.73

$$\frac{e^{-2(a+bx)}\left(-6e^{4(a+bx)}+e^{8(a+bx)}-3\right)}{96b}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*(a + b*x))*Cosh[a + b*x]^2*Sinh[a + b*x]^2,x]
[Out] (-3 - 6*E^(4*(a + b*x)) + E^(8*(a + b*x)))/(96*b*E^(2*(a + b*x)))
```

fricas [**B**] time = 0.47, size = 108, normalized size = 2.08

$$-\frac{\cosh(bx+a)^4-8\cosh(bx+a)^3\sinh(bx+a)+6\cosh(bx+a)^2\sinh(bx+a)^2-8\cosh(bx+a)\sinh(bx+}{48\left(b\cosh(bx+a)^2-2b\cosh(bx+a)\sinh(bx+a)+b\sinh(bx+a)^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*b*x+2*a)*cosh(b*x+a)^2*sinh(b*x+a)^2,x, algorithm="fricas")
[Out] -1/48*(cosh(b*x + a)^4 - 8*cosh(b*x + a)^3*sinh(b*x + a) + 6*cosh(b*x + a)^
2*sinh(b*x + a)^2 - 8*cosh(b*x + a)*sinh(b*x + a)^3 + sinh(b*x + a)^4 + 3)/
(b*cosh(b*x + a)^2 - 2*b*cosh(b*x + a)*sinh(b*x + a) + b*sinh(b*x + a)^2)
```

giac [**A**] time = 0.14, size = 43, normalized size = 0.83

$$\frac{\left(e^{(6bx+12a)}-6e^{(2bx+8a)}\right)e^{(-6a)}-3e^{(-2bx-2a)}}{96b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*b*x+2*a)*cosh(b*x+a)^2*sinh(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/96*((e^(6*b*x + 12*a) - 6*e^(2*b*x + 8*a))*e^(-6*a) - 3*e^(-2*b*x - 2*a))
/b
```

maple [A] time = 0.26, size = 58, normalized size = 1.12

$$-\frac{\sinh\left(2bx+2a\right)}{32b}+\frac{\sinh\left(6bx+6a\right)}{96b}-\frac{3\cosh\left(2bx+2a\right)}{32b}+\frac{\cosh\left(6bx+6a\right)}{96b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(2*b*x+2*a)*cosh(b*x+a)^2*sinh(b*x+a)^2,x)
```

```
[Out] -1/32*sinh(2*b*x+2*a)/b+1/96/b*sinh(6*b*x+6*a)-3/32*cosh(2*b*x+2*a)/b+1/96*
cosh(6*b*x+6*a)/b
```

maxima [A] time = 0.32, size = 42, normalized size = 0.81

$$-\frac{\left(6\,e^{(-4\,bx-4\,a)}-1\right)e^{(6\,bx+6\,a)}}{96\,b}-\frac{e^{(-2\,bx-2\,a)}}{32\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*b*x+2*a)*cosh(b*x+a)^2*sinh(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -1/96*(6*e^(-4*b*x - 4*a) - 1)*e^(6*b*x + 6*a)/b - 1/32*e^(-2*b*x - 2*a)/b
```

mupad [B] time = 0.53, size = 39, normalized size = 0.75

$$-\frac{3\,\mathrm{e}^{-2\,a-2\,b\,x}+6\,\mathrm{e}^{2\,a+2\,b\,x}-\mathrm{e}^{6\,a+6\,b\,x}}{96\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a + b*x)^2*exp(2*a + 2*b*x)*sinh(a + b*x)^2,x)
```

```
[Out] -(3*exp(- 2*a - 2*b*x) + 6*exp(2*a + 2*b*x) - exp(6*a + 6*b*x))/(96*b)
```

sympy [A] time = 57.10, size = 128, normalized size = 2.46

$$\begin{cases} -\frac{5e^{2a}e^{2bx}\sinh^{4}\left(a+bx\right)}{48b}+\frac{5e^{2a}e^{2bx}\sinh^{3}\left(a+bx\right)\cosh\left(a+bx\right)}{24b}+\frac{e^{2a}e^{2bx}\sinh\left(a+bx\right)\cosh^{3}\left(a+bx\right)}{8b}-\frac{e^{2a}e^{2bx}\cosh^{4}\left(a+bx\right)}{16b} & \text{for}\ b\neq 0\\ xe^{2a}\sinh^{2}\left(a\right)\cosh^{2}\left(a\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*b*x+2*a)*cosh(b*x+a)**2*sinh(b*x+a)**2,x)
```

```
[Out] Piecewise((-5*exp(2*a)*exp(2*b*x)*sinh(a + b*x)**4/(48*b) + 5*exp(2*a)*exp(
2*b*x)*sinh(a + b*x)**3*cosh(a + b*x)/(24*b) + exp(2*a)*exp(2*b*x)*sinh(a +
 b*x)*cosh(a + b*x)**3/(8*b) - exp(2*a)*exp(2*b*x)*cosh(a + b*x)**4/(16*b),
 Ne(b, 0)), (x*exp(2*a)*sinh(a)**2*cosh(a)**2, True))
```

3.927 $\int e^{2(a+bx)} \cosh^2(a+bx) \sinh(a+bx)\, dx$

Optimal. Leaf size=66

$$\frac{e^{-a-bx}}{8b} - \frac{e^{a+bx}}{8b} + \frac{e^{3a+3bx}}{24b} + \frac{e^{5a+5bx}}{40b}$$

```
[Out] 1/8*exp(-b*x-a)/b-1/8*exp(b*x+a)/b+1/24*exp(3*b*x+3*a)/b+1/40*exp(5*b*x+5*a
)/b
```

Rubi [A] time = 0.05, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2282, 12, 448}

$$\frac{e^{-a-bx}}{8b} - \frac{e^{a+bx}}{8b} + \frac{e^{3a+3bx}}{24b} + \frac{e^{5a+5bx}}{40b}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*(a + b*x))*Cosh[a + b*x]^2*Sinh[a + b*x],x]
```

```
[Out] E^(-a - b*x)/(8*b) - E^(a + b*x)/(8*b) + E^(3*a + 3*b*x)/(24*b) + E^(5*a +
5*b*x)/(40*b)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$\int e^{2(a+bx)}\cosh^2(a+bx)\sinh(a+bx)\,dx = \frac{\text{Subst}\left(\int \frac{(-1+x^2)(1+x^2)^2}{8x^2}\,dx, x, e^{a+bx}\right)}{b}$$
$$= \frac{\text{Subst}\left(\int \frac{(-1+x^2)(1+x^2)^2}{x^2}\,dx, x, e^{a+bx}\right)}{8b}$$
$$= \frac{\text{Subst}\left(\int \left(-1-\frac{1}{x^2}+x^2+x^4\right)dx, x, e^{a+bx}\right)}{8b}$$
$$= \frac{e^{-a-bx}}{8b} - \frac{e^{a+bx}}{8b} + \frac{e^{3a+3bx}}{24b} + \frac{e^{5a+5bx}}{40b}$$

Mathematica [A] time = 0.05, size = 54, normalized size = 0.82

$$\frac{e^{a+bx}\left(e^{2(a+bx)}-3\right)}{24b} + \frac{e^{-a-bx}\left(e^{6(a+bx)}+5\right)}{40b}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*(a + b*x))*Cosh[a + b*x]^2*Sinh[a + b*x],x]
```

```
[Out] (E^(a + b*x)*(-3 + E^(2*(a + b*x))))/(24*b) + (E^(-a - b*x)*(5 + E^(6*(a +
b*x))))/(40*b)
```

fricas [A] time = 0.54, size = 105, normalized size = 1.59

$$\frac{9\cosh(bx+a)^3 + 27\cosh(bx+a)\sinh(bx+a)^2 - 6\sinh(bx+a)^3 - 2\left(9\cosh(bx+a)^2-5\right)\sinh(bx+a) - 5}{60\left(b\cosh(bx+a)^2 - 2b\cosh(bx+a)\sinh(bx+a) + b\sinh(bx+a)^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*b*x+2*a)*cosh(b*x+a)^2*sinh(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/60*(9*cosh(b*x + a)^3 + 27*cosh(b*x + a)*sinh(b*x + a)^2 - 6*sinh(b*x + a
)^3 - 2*(9*cosh(b*x + a)^2 - 5)*sinh(b*x + a) - 5*cosh(b*x + a))/(b*cosh(b*
x + a)^2 - 2*b*cosh(b*x + a)*sinh(b*x + a) + b*sinh(b*x + a)^2)
```

giac [A] time = 0.12, size = 55, normalized size = 0.83

$$\frac{\left(3\,e^{(5\,bx+10\,a)} + 5\,e^{(3\,bx+8\,a)} - 15\,e^{(bx+6\,a)}\right)e^{(-5\,a)} + 15\,e^{(-bx-a)}}{120\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*b*x+2*a)*cosh(b*x+a)^2*sinh(b*x+a),x, algorithm="giac")
```

```
[Out] 1/120*((3*e^(5*b*x + 10*a) + 5*e^(3*b*x + 8*a) - 15*e^(b*x + 6*a))*e^(-5*a)
 + 15*e^(-b*x - a))/b
```

maple [A] time = 0.16, size = 69, normalized size = 1.05

$$-\frac{\sinh(bx+a)}{4b}+\frac{\sinh(3bx+3a)}{24b}+\frac{\sinh(5bx+5a)}{40b}+\frac{\cosh(3bx+3a)}{24b}+\frac{\cosh(5bx+5a)}{40b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(2*b*x+2*a)*cosh(b*x+a)^2*sinh(b*x+a),x)
```

```
[Out] -1/4*sinh(b*x+a)/b+1/24/b*sinh(3*b*x+3*a)+1/40/b*sinh(5*b*x+5*a)+1/24*cosh(
3*b*x+3*a)/b+1/40*cosh(5*b*x+5*a)/b
```

maxima [A] time = 0.32, size = 53, normalized size = 0.80

$$\frac{\left(5\,e^{(-2\,bx-2\,a)}-15\,e^{(-4\,bx-4\,a)}+3\right)e^{(5\,bx+5\,a)}}{120\,b}+\frac{e^{(-bx-a)}}{8\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*b*x+2*a)*cosh(b*x+a)^2*sinh(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/120*(5*e^(-2*b*x - 2*a) - 15*e^(-4*b*x - 4*a) + 3)*e^(5*b*x + 5*a)/b + 1/
8*e^(-b*x - a)/b
```

mupad [B] time = 0.24, size = 47, normalized size = 0.71

$$\frac{15\,\mathrm{e}^{-a-b\,x}-15\,\mathrm{e}^{a+b\,x}+5\,\mathrm{e}^{3\,a+3\,b\,x}+3\,\mathrm{e}^{5\,a+5\,b\,x}}{120\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a + b*x)^2*exp(2*a + 2*b*x)*sinh(a + b*x),x)
```

```
[Out] (15*exp(- a - b*x) - 15*exp(a + b*x) + 5*exp(3*a + 3*b*x) + 3*exp(5*a + 5*b
*x))/(120*b)
```

sympy [A] time = 16.23, size = 128, normalized size = 1.94

$$\begin{cases} -\frac{4e^{2a}e^{2bx}\sinh^3(a+bx)}{15b}+\frac{8e^{2a}e^{2bx}\sinh^2(a+bx)\cosh(a+bx)}{15b}-\frac{2e^{2a}e^{2bx}\sinh(a+bx)\cosh^2(a+bx)}{15b}+\frac{e^{2a}e^{2bx}\cosh^3(a+bx)}{15b} & \text{for } b\neq 0 \\ xe^{2a}\sinh(a)\cosh^2(a) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*b*x+2*a)*cosh(b*x+a)**2*sinh(b*x+a),x)
```

```
[Out] Piecewise((-4*exp(2*a)*exp(2*b*x)*sinh(a + b*x)**3/(15*b) + 8*exp(2*a)*exp(
2*b*x)*sinh(a + b*x)**2*cosh(a + b*x)/(15*b) - 2*exp(2*a)*exp(2*b*x)*sinh(a
 + b*x)*cosh(a + b*x)**2/(15*b) + exp(2*a)*exp(2*b*x)*cosh(a + b*x)**3/(15*
b), Ne(b, 0)), (x*exp(2*a)*sinh(a)*cosh(a)**2, True))
```

3.928 $\int e^{2(a+bx)} \cosh(a+bx) \coth(a+bx) \, dx$

Optimal. Leaf size=45

$$\frac{3e^{a+bx}}{2b} + \frac{e^{3a+3bx}}{6b} - \frac{2\tanh^{-1}\left(e^{a+bx}\right)}{b}$$

```
[Out] 3/2*exp(b*x+a)/b+1/6*exp(3*b*x+3*a)/b-2*arctanh(exp(b*x+a))/b
```

Rubi [A] time = 0.03, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2282, 12, 390, 207}

$$\frac{3e^{a+bx}}{2b} + \frac{e^{3a+3bx}}{6b} - \frac{2\tanh^{-1}\left(e^{a+bx}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*(a + b*x))*Cosh[a + b*x]*Coth[a + b*x],x]
```

```
[Out] (3*E^(a + b*x))/(2*b) + E^(3*a + 3*b*x)/(6*b) - (2*ArcTanh[E^(a + b*x)])/b
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
```

```
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$\int e^{2(a+bx)}\cosh(a+bx)\coth(a+bx)\,dx = \frac{\text{Subst}\left(\int \frac{(1+x^2)^2}{2(-1+x^2)}\,dx,x,e^{a+bx}\right)}{b}$$
$$= \frac{\text{Subst}\left(\int \frac{(1+x^2)^2}{-1+x^2}\,dx,x,e^{a+bx}\right)}{2b}$$
$$= \frac{\text{Subst}\left(\int \left(3+x^2+\frac{4}{-1+x^2}\right)dx,x,e^{a+bx}\right)}{2b}$$
$$= \frac{3e^{a+bx}}{2b}+\frac{e^{3a+3bx}}{6b}+\frac{2\,\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx,x,e^{a+bx}\right)}{b}$$
$$= \frac{3e^{a+bx}}{2b}+\frac{e^{3a+3bx}}{6b}-\frac{2\tanh^{-1}\left(e^{a+bx}\right)}{b}$$

Mathematica [A] time = 0.13, size = 58, normalized size = 1.29

$$-\frac{e^{a+bx}\left(-\frac{1}{3}e^{2(a+bx)}+\frac{4\tanh^{-1}\left(\sqrt{e^{2(a+bx)}}\right)}{\sqrt{e^{2(a+bx)}}}-3\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*(a + b*x))*Cosh[a + b*x]*Coth[a + b*x],x]
[Out] -1/2*(E^(a + b*x)*(-3 - E^(2*(a + b*x))/3 + (4*ArcTanh[Sqrt[E^(2*(a + b*x))
]])/Sqrt[E^(2*(a + b*x))]))/b
```

fricas [B] time = 0.50, size = 98, normalized size = 2.18

$$\frac{\cosh(bx+a)^3+3\cosh(bx+a)\sinh(bx+a)^2+\sinh(bx+a)^3+3\left(\cosh(bx+a)^2+3\right)\sinh(bx+a)+9\cosh}{6b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*b*x+2*a)*cosh(b*x+a)^2*csch(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/6*(cosh(b*x + a)^3 + 3*cosh(b*x + a)*sinh(b*x + a)^2 + sinh(b*x + a)^3 +
3*(cosh(b*x + a)^2 + 3)*sinh(b*x + a) + 9*cosh(b*x + a) - 6*log(cosh(b*x +
a) + sinh(b*x + a) + 1) + 6*log(cosh(b*x + a) + sinh(b*x + a) - 1))/b
```

giac [A] time = 0.12, size = 54, normalized size = 1.20

$$\frac{\left(e^{(3\,bx+9\,a)}+9\,e^{(bx+7\,a)}\right)e^{(-6\,a)}-6\,\log\left(e^{(bx+a)}+1\right)+6\,\log\left(\left|e^{(bx+a)}-1\right|\right)}{6\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*b*x+2*a)*cosh(b*x+a)^2*csch(b*x+a),x, algorithm="giac")
```

```
[Out] 1/6*((e^(3*b*x + 9*a) + 9*e^(b*x + 7*a))*e^(-6*a) - 6*log(e^(b*x + a) + 1)
+ 6*log(abs(e^(b*x + a) - 1)))/b
```

maple [A] time = 0.66, size = 54, normalized size = 1.20

$$\frac{e^{3bx+3a}}{6b}+\frac{3\,e^{bx+a}}{2b}+\frac{\ln\left(e^{bx+a}-1\right)}{b}-\frac{\ln\left(1+e^{bx+a}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(2*b*x+2*a)*cosh(b*x+a)^2*csch(b*x+a),x)
```

```
[Out] 1/6*exp(3*b*x+3*a)/b+3/2*exp(b*x+a)/b+1/b*ln(exp(b*x+a)-1)-1/b*ln(1+exp(b*x
+a))
```

maxima [A] time = 0.32, size = 61, normalized size = 1.36

$$\frac{\left(9\,e^{(-2\,bx-2\,a)}+1\right)e^{(3\,bx+3\,a)}}{6\,b}-\frac{\log\left(e^{(-bx-a)}+1\right)}{b}+\frac{\log\left(e^{(-bx-a)}-1\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*b*x+2*a)*cosh(b*x+a)^2*csch(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/6*(9*e^(-2*b*x - 2*a) + 1)*e^(3*b*x + 3*a)/b - log(e^(-b*x - a) + 1)/b +
log(e^(-b*x - a) - 1)/b
```

mupad [B] time = 0.09, size = 53, normalized size = 1.18

$$\frac{3\,e^{a+b\,x}}{2\,b}-\frac{2\,\mathrm{atan}\left(\frac{e^{b\,x}\,e^{a}\,\sqrt{-b^2}}{b}\right)}{\sqrt{-b^2}}+\frac{e^{3\,a+3\,b\,x}}{6\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cosh(a + b*x)^2*exp(2*a + 2*b*x))/sinh(a + b*x),x)
```

```
[Out] (3*exp(a + b*x))/(2*b) - (2*atan((exp(b*x)*exp(a)*(-b^2)^(1/2))/b))/(-b^2)^
(1/2) + exp(3*a + 3*b*x)/(6*b)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*b*x+2*a)*cosh(b*x+a)**2*csch(b*x+a),x)
```

```
[Out] Timed out
```

3.929 $\int e^{2(a+bx)} \coth^2(a+bx)\,dx$

Optimal. Leaf size=59

$$\frac{e^{2a+2bx}}{2b}+\frac{2}{b\left(1-e^{2a+2bx}\right)}+\frac{2\log\left(1-e^{2a+2bx}\right)}{b}$$

```
[Out] 1/2*exp(2*b*x+2*a)/b+2/b/(1-exp(2*b*x+2*a))+2*ln(1-exp(2*b*x+2*a))/b
```

Rubi [A] time = 0.05, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2282, 444, 43}

$$\frac{e^{2a+2bx}}{2b}+\frac{2}{b\left(1-e^{2a+2bx}\right)}+\frac{2\log\left(1-e^{2a+2bx}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*(a + b*x))*Coth[a + b*x]^2,x]
```

```
[Out] E^(2*a + 2*b*x)/(2*b) + 2/(b*(1 - E^(2*a + 2*b*x))) + (2*Log[1 - E^(2*a + 2
*b*x)])/b
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$\int e^{2(a+bx)}\coth^2(a+bx)\,dx = \frac{\text{Subst}\left(\int \frac{x(1+x^2)^2}{(1-x^2)^2}\,dx, x, e^{a+bx}\right)}{b}$$
$$= \frac{\text{Subst}\left(\int \frac{(1+x)^2}{(1-x)^2}\,dx, x, e^{2a+2bx}\right)}{2b}$$
$$= \frac{\text{Subst}\left(\int \left(1+\frac{4}{(-1+x)^2}+\frac{4}{-1+x}\right)dx, x, e^{2a+2bx}\right)}{2b}$$
$$= \frac{e^{2a+2bx}}{2b}+\frac{2}{b\left(1-e^{2a+2bx}\right)}+\frac{2\log\left(1-e^{2a+2bx}\right)}{b}$$

Mathematica [A] time = 0.07, size = 48, normalized size = 0.81

$$\frac{e^{2(a+bx)}-\frac{4}{e^{2(a+bx)}-1}+4\log\left(1-e^{2(a+bx)}\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*(a + b*x))*Coth[a + b*x]^2,x]
```

```
[Out] (E^(2*(a + b*x)) - 4/(-1 + E^(2*(a + b*x))) + 4*Log[1 - E^(2*(a + b*x))])/(
2*b)
```

fricas [B] time = 0.48, size = 195, normalized size = 3.31

$$\frac{\cosh(bx+a)^4+4\cosh(bx+a)\sinh(bx+a)^3+\sinh(bx+a)^4+\left(6\cosh(bx+a)^2-1\right)\sinh(bx+a)^2-\cosh(}{2\left(b\cosh(bx\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*b*x+2*a)*cosh(b*x+a)^2*csch(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(cosh(b*x + a)^4 + 4*cosh(b*x + a)*sinh(b*x + a)^3 + sinh(b*x + a)^4 +
(6*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^2 - cosh(b*x + a)^2 + 4*(cosh(b*x + a
)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 - 1)*log(2*sinh(b*x +
 a)/(cosh(b*x + a) - sinh(b*x + a))) + 2*(2*cosh(b*x + a)^3 - cosh(b*x + a)
)*sinh(b*x + a) - 4)/(b*cosh(b*x + a)^2 + 2*b*cosh(b*x + a)*sinh(b*x + a) +
 b*sinh(b*x + a)^2 - b)
```

giac [A] time = 0.13, size = 56, normalized size = 0.95

$$-\frac{\frac{4\,e^{(2\,bx+2\,a)}}{e^{(2\,bx+2\,a)}-1}-e^{(2\,bx+2\,a)}-4\,\log\left(\left|e^{(2\,bx+2\,a)}-1\right|\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*b*x+2*a)*cosh(b*x+a)^2*csch(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -1/2*(4*e^(2*b*x + 2*a)/(e^(2*b*x + 2*a) - 1) - e^(2*b*x + 2*a) - 4*log(abs
(e^(2*b*x + 2*a) - 1)))/b
```

maple [A] time = 0.60, size = 57, normalized size = 0.97

$$\frac{\mathrm{e}^{2bx+2a}}{2b}-\frac{4a}{b}-\frac{2}{b\left(\mathrm{e}^{2bx+2a}-1\right)}+\frac{2\ln\left(\mathrm{e}^{2bx+2a}-1\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(2*b*x+2*a)*cosh(b*x+a)^2*csch(b*x+a)^2,x)
```

```
[Out] 1/2*exp(2*b*x+2*a)/b-4*a/b-2/b/(exp(2*b*x+2*a)-1)+2/b*ln(exp(2*b*x+2*a)-1)
```

maxima [A] time = 0.32, size = 86, normalized size = 1.46

$$\frac{4\,(bx+a)}{b}+\frac{2\,\log\left(e^{(-bx-a)}+1\right)}{b}+\frac{2\,\log\left(e^{(-bx-a)}-1\right)}{b}-\frac{5\,e^{(-2\,bx-2\,a)}-1}{2\,b\left(e^{(-2\,bx-2\,a)}-e^{(-4\,bx-4\,a)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*b*x+2*a)*cosh(b*x+a)^2*csch(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 4*(b*x + a)/b + 2*log(e^(-b*x - a) + 1)/b + 2*log(e^(-b*x - a) - 1)/b - 1/2
*(5*e^(-2*b*x - 2*a) - 1)/(b*(e^(-2*b*x - 2*a) - e^(-4*b*x - 4*a)))
```

mupad [B] time = 1.85, size = 51, normalized size = 0.86

$$\frac{2\,\ln\left(\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}-1\right)}{b}-\frac{2}{b\,\left(\mathrm{e}^{2\,a+2\,b\,x}-1\right)}+\frac{\mathrm{e}^{2\,a+2\,b\,x}}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cosh(a + b*x)^2*exp(2*a + 2*b*x))/sinh(a + b*x)^2,x)
```

```
[Out] (2*log(exp(2*a)*exp(2*b*x) - 1))/b - 2/(b*(exp(2*a + 2*b*x) - 1)) + exp(2*a
 + 2*b*x)/(2*b)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*b*x+2*a)*cosh(b*x+a)**2*csch(b*x+a)**2,x)
[Out] Timed out
```

3.930 $\int e^{2(a+bx)} \coth^2(a+bx)\mathbf{csch}(a+bx)\,dx$

Optimal. Leaf size=85

$$\frac{2e^{a+bx}}{b}+\frac{3e^{a+bx}}{b\left(1-e^{2a+2bx}\right)}-\frac{2e^{3a+3bx}}{b\left(1-e^{2a+2bx}\right)^2}-\frac{5\tanh^{-1}\left(e^{a+bx}\right)}{b}$$

```
[Out] 2*exp(b*x+a)/b-2*exp(3*b*x+3*a)/b/(1-exp(2*b*x+2*a))^2+3*exp(b*x+a)/b/(1-ex
p(2*b*x+2*a))-5*arctanh(exp(b*x+a))/b
```

Rubi [A] time = 0.07, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2282, 12, 463, 455, 388, 207}

$$\frac{2e^{a+bx}}{b}+\frac{3e^{a+bx}}{b\left(1-e^{2a+2bx}\right)}-\frac{2e^{3a+3bx}}{b\left(1-e^{2a+2bx}\right)^2}-\frac{5\tanh^{-1}\left(e^{a+bx}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*(a + b*x))*Coth[a + b*x]^2*Csch[a + b*x],x]
```

```
[Out] (2*E^(a + b*x))/b - (2*E^(3*a + 3*b*x))/(b*(1 - E^(2*a + 2*b*x))^2) + (3*E^
(a + b*x))/(b*(1 - E^(2*a + 2*b*x))) - (5*ArcTanh[E^(a + b*x)])/b
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 455

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[((-a)^(m/2 - 1)*(b*c - a*d)*x*(a + b*x^2)^(p + 1))/(2*b^(m/2 + 1)*(p
 + 1)), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[(a + b*x^2)^(p + 1)*Expand
ToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2 -
 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /; F
reeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &&
 (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 463

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^2, x_Symbol] :> -Simp[((b*c - a*d)^2*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/
(a*b^2*e*n*(p + 1)), x] + Dist[1/(a*b^2*n*(p + 1)), Int[(e*x)^m*(a + b*x^n)
^(p + 1)*Simp[(b*c - a*d)^2*(m + 1) + b^2*c^2*n*(p + 1) + a*b*d^2*n*(p + 1)
*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] &&
 IGtQ[n, 0] && LtQ[p, -1]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$\int e^{2(a+bx)}\coth^2(a+bx)\text{csch}(a+bx)\,dx=\frac{\text{Subst}\left(\int\frac{2x^2(1+x^2)^2}{(-1+x^2)^3}\,dx,x,e^{a+bx}\right)}{b}$$

$$=\frac{2\,\text{Subst}\left(\int\frac{x^2(1+x^2)^2}{(-1+x^2)^3}\,dx,x,e^{a+bx}\right)}{b}$$

$$=-\frac{2e^{3a+3bx}}{b\left(1-e^{2a+2bx}\right)^2}+\frac{\text{Subst}\left(\int\frac{x^2(8+4x^2)}{(-1+x^2)^2}\,dx,x,e^{a+bx}\right)}{2b}$$

$$=-\frac{2e^{3a+3bx}}{b\left(1-e^{2a+2bx}\right)^2}+\frac{3e^{a+bx}}{b\left(1-e^{2a+2bx}\right)}-\frac{\text{Subst}\left(\int\frac{-12-8x^2}{-1+x^2}\,dx,x,e^{a+bx}\right)}{4b}$$

$$=\frac{2e^{a+bx}}{b}-\frac{2e^{3a+3bx}}{b\left(1-e^{2a+2bx}\right)^2}+\frac{3e^{a+bx}}{b\left(1-e^{2a+2bx}\right)}+\frac{5\,\text{Subst}\left(\int\frac{1}{-1+x^2}\,dx,x,e^{a+bx}\right)}{b}$$

$$=\frac{2e^{a+bx}}{b}-\frac{2e^{3a+3bx}}{b\left(1-e^{2a+2bx}\right)^2}+\frac{3e^{a+bx}}{b\left(1-e^{2a+2bx}\right)}-\frac{5\tanh^{-1}\left(e^{a+bx}\right)}{b}$$

Mathematica [C] time = 4.51, size = 247, normalized size = 2.91

$$-\frac{e^{-3(a+bx)}\left(128e^{8(a+bx)}\left(e^{2(a+bx)}+1\right)^2\,{}_5F_4\left(2,2,2,2,\frac{5}{2};1,1,1,\frac{11}{2};e^{2(a+bx)}\right)+128e^{8(a+bx)}\left(16e^{2(a+bx)}+7e^{4(a+bx)}+9\right)\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(2*(a + b*x))*Coth[a + b*x]^2*Csch[a + b*x],x]
```

```
[Out] -1/10080*(-21*(56595 + 62725*E^(2*(a + b*x)) - 12071*E^(4*(a + b*x)) - 1935
3*E^(6*(a + b*x)) + 768*E^(8*(a + b*x))) + (315*(3773 + 2924*E^(2*(a + b*x)
) - 2534*E^(4*(a + b*x)) - 1548*E^(6*(a + b*x)) + 297*E^(8*(a + b*x)))*ArcT
anh[Sqrt[E^(2*(a + b*x))]])/Sqrt[E^(2*(a + b*x))] + 128*E^(8*(a + b*x))*(9
+ 16*E^(2*(a + b*x)) + 7*E^(4*(a + b*x)))*HypergeometricPFQ[{2, 2, 2, 5/2},
 {1, 1, 11/2}, E^(2*(a + b*x))] + 128*E^(8*(a + b*x))*(1 + E^(2*(a + b*x)))
^2*HypergeometricPFQ[{2, 2, 2, 2, 5/2}, {1, 1, 1, 11/2}, E^(2*(a + b*x))])/
(b*E^(3*(a + b*x)))
```

fricas [B] time = 0.51, size = 459, normalized size = 5.40

$$\frac{4\,\cosh\left(bx+a\right)^{5}+20\,\cosh\left(bx+a\right)\sinh\left(bx+a\right)^{4}+4\,\sinh\left(bx+a\right)^{5}+2\left(20\,\cosh\left(bx+a\right)^{2}-9\right)\sinh\left(bx+a\right)^{3}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*b*x+2*a)*cosh(b*x+a)^2*csch(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/2*(4*cosh(b*x + a)^5 + 20*cosh(b*x + a)*sinh(b*x + a)^4 + 4*sinh(b*x + a)
^5 + 2*(20*cosh(b*x + a)^2 - 9)*sinh(b*x + a)^3 - 18*cosh(b*x + a)^3 + 2*(2
0*cosh(b*x + a)^3 - 27*cosh(b*x + a))*sinh(b*x + a)^2 - 5*(cosh(b*x + a)^4
+ 4*cosh(b*x + a)*sinh(b*x + a)^3 + sinh(b*x + a)^4 + 2*(3*cosh(b*x + a)^2
- 1)*sinh(b*x + a)^2 - 2*cosh(b*x + a)^2 + 4*(cosh(b*x + a)^3 - cosh(b*x +
a))*sinh(b*x + a) + 1)*log(cosh(b*x + a) + sinh(b*x + a) + 1) + 5*(cosh(b*x
 + a)^4 + 4*cosh(b*x + a)*sinh(b*x + a)^3 + sinh(b*x + a)^4 + 2*(3*cosh(b*x
 + a)^2 - 1)*sinh(b*x + a)^2 - 2*cosh(b*x + a)^2 + 4*(cosh(b*x + a)^3 - cos
h(b*x + a))*sinh(b*x + a) + 1)*log(cosh(b*x + a) + sinh(b*x + a) - 1) + 2*(
10*cosh(b*x + a)^4 - 27*cosh(b*x + a)^2 + 5)*sinh(b*x + a) + 10*cosh(b*x +
a))/(b*cosh(b*x + a)^4 + 4*b*cosh(b*x + a)*sinh(b*x + a)^3 + b*sinh(b*x + a
)^4 - 2*b*cosh(b*x + a)^2 + 2*(3*b*cosh(b*x + a)^2 - b)*sinh(b*x + a)^2 + 4
*(b*cosh(b*x + a)^3 - b*cosh(b*x + a))*sinh(b*x + a) + b)
```

giac [A] time = 0.13, size = 72, normalized size = 0.85

$$-\frac{\frac{2\left(5\,e^{(3\,bx+3\,a)}-3\,e^{(bx+a)}\right)}{\left(e^{(2\,bx+2\,a)}-1\right)^{2}}-4\,e^{(bx+a)}+5\,\log\left(e^{(bx+a)}+1\right)-5\,\log\left(\left|e^{(bx+a)}-1\right|\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*b*x+2*a)*cosh(b*x+a)^2*csch(b*x+a)^3,x, algorithm="giac")
```

```
[Out] -1/2*(2*(5*e^(3*b*x + 3*a) - 3*e^(b*x + a))/(e^(2*b*x + 2*a) - 1)^2 - 4*e^(
b*x + a) + 5*log(e^(b*x + a) + 1) - 5*log(abs(e^(b*x + a) - 1)))/b
```

maple [A] time = 0.64, size = 78, normalized size = 0.92

$$\frac{2\,{\rm e}^{bx+a}}{b}-\frac{{\rm e}^{bx+a}\left(5\,{\rm e}^{2bx+2a}-3\right)}{b\left({\rm e}^{2bx+2a}-1\right)^{2}}-\frac{5\ln\left(1+{\rm e}^{bx+a}\right)}{2b}+\frac{5\ln\left({\rm e}^{bx+a}-1\right)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(2*b*x+2*a)*cosh(b*x+a)^2*csch(b*x+a)^3,x)
```

```
[Out] 2*exp(b*x+a)/b-exp(b*x+a)*(5*exp(2*b*x+2*a)-3)/b/(exp(2*b*x+2*a)-1)^2-5/2/b
*ln(1+exp(b*x+a))+5/2/b*ln(exp(b*x+a)-1)
```

maxima [**A**] time = 0.33, size = 96, normalized size = 1.13

$$-\frac{5\,\log\left(e^{(-bx-a)}+1\right)}{2\,b}+\frac{5\,\log\left(e^{(-bx-a)}-1\right)}{2\,b}-\frac{9\,e^{(-2\,bx-2\,a)}-5\,e^{(-4\,bx-4\,a)}-2}{b\left(e^{(-bx-a)}-2\,e^{(-3\,bx-3\,a)}+e^{(-5\,bx-5\,a)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*b*x+2*a)*cosh(b*x+a)^2*csch(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -5/2*log(e^(-b*x - a) + 1)/b + 5/2*log(e^(-b*x - a) - 1)/b - (9*e^(-2*b*x -
 2*a) - 5*e^(-4*b*x - 4*a) - 2)/(b*(e^(-b*x - a) - 2*e^(-3*b*x - 3*a) + e^(
-5*b*x - 5*a)))
```

mupad [**B**] time = 1.90, size = 98, normalized size = 1.15

$$\frac{2\,e^{a+b\,x}}{b}-\frac{5\,\mathrm{atan}\left(\frac{e^{b\,x}\,e^{a}\,\sqrt{-b^2}}{b}\right)}{\sqrt{-b^2}}-\frac{2\,e^{a+b\,x}}{b\,\left(e^{4\,a+4\,b\,x}-2\,e^{2\,a+2\,b\,x}+1\right)}-\frac{5\,e^{a+b\,x}}{b\,\left(e^{2\,a+2\,b\,x}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cosh(a + b*x)^2*exp(2*a + 2*b*x))/sinh(a + b*x)^3,x)
```

```
[Out] (2*exp(a + b*x))/b - (5*atan((exp(b*x)*exp(a)*(-b^2)^(1/2))/b))/(-b^2)^(1/2
) - (2*exp(a + b*x))/(b*(exp(4*a + 4*b*x) - 2*exp(2*a + 2*b*x) + 1)) - (5*e
xp(a + b*x))/(b*(exp(2*a + 2*b*x) - 1))
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*b*x+2*a)*cosh(b*x+a)**2*csch(b*x+a)**3,x)
```

```
[Out] Timed out
```

3.931 $\int e^{2(a+bx)}\cosh^3(a+bx)\sinh^3(a+bx)\,dx$

Optimal. Leaf size=57

$$\frac{e^{-4a-4bx}}{256b}-\frac{3e^{4a+4bx}}{256b}+\frac{e^{8a+8bx}}{512b}+\frac{3x}{64}$$

```
[Out] 1/256*exp(-4*b*x-4*a)/b-3/256*exp(4*b*x+4*a)/b+1/512*exp(8*b*x+8*a)/b+3/64*
x
```

Rubi [A] time = 0.06, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2282, 12, 266, 43}

$$\frac{e^{-4a-4bx}}{256b}-\frac{3e^{4a+4bx}}{256b}+\frac{e^{8a+8bx}}{512b}+\frac{3x}{64}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*(a + b*x))*Cosh[a + b*x]^3*Sinh[a + b*x]^3,x]
```

```
[Out] E^(-4*a - 4*b*x)/(256*b) - (3*E^(4*a + 4*b*x))/(256*b) + E^(8*a + 8*b*x)/(5
12*b) + (3*x)/64
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
```

```
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$\int e^{2(a+bx)}\cosh^3(a+bx)\sinh^3(a+bx)\,dx = \frac{\text{Subst}\left(\int \frac{(-1+x^4)^3}{64x^5}\,dx, x, e^{a+bx}\right)}{b}$$
$$= \frac{\text{Subst}\left(\int \frac{(-1+x^4)^3}{x^5}\,dx, x, e^{a+bx}\right)}{64b}$$
$$= \frac{\text{Subst}\left(\int \frac{(-1+x)^3}{x^2}\,dx, x, e^{4a+4bx}\right)}{256b}$$
$$= \frac{\text{Subst}\left(\int \left(-3-\frac{1}{x^2}+\frac{3}{x}+x\right)dx, x, e^{4a+4bx}\right)}{256b}$$
$$= \frac{e^{-4a-4bx}}{256b} - \frac{3e^{4a+4bx}}{256b} + \frac{e^{8a+8bx}}{512b} + \frac{3x}{64}$$

Mathematica [A] time = 0.05, size = 45, normalized size = 0.79

$$\frac{e^{-4(a+bx)} - 3e^{4(a+bx)} + \frac{1}{2}e^{8(a+bx)} + 12bx}{256b}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*(a + b*x))*Cosh[a + b*x]^3*Sinh[a + b*x]^3,x]
```

```
[Out] (E^(-4*(a + b*x)) - 3*E^(4*(a + b*x)) + E^(8*(a + b*x))/2 + 12*b*x)/(256*b)
```

fricas [B] time = 0.46, size = 186, normalized size = 3.26

$$\frac{3\cosh(bx+a)^6 - 20\cosh(bx+a)^3\sinh(bx+a)^3 + 45\cosh(bx+a)^2\sinh(bx+a)^4 - 6\cosh(bx+a)\sinh(}{512\left(b\,\mathrm{co}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*b*x+2*a)*cosh(b*x+a)^3*sinh(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/512*(3*cosh(b*x + a)^6 - 20*cosh(b*x + a)^3*sinh(b*x + a)^3 + 45*cosh(b*x
 + a)^2*sinh(b*x + a)^4 - 6*cosh(b*x + a)*sinh(b*x + a)^5 + 3*sinh(b*x + a)
```

```
^6 + 6*(4*b*x - 1)*cosh(b*x + a)^2 + 3*(15*cosh(b*x + a)^4 + 8*b*x - 2)*sin
h(b*x + a)^2 - 6*(cosh(b*x + a)^5 + 2*(4*b*x + 1)*cosh(b*x + a))*sinh(b*x +
 a))/(b*cosh(b*x + a)^2 - 2*b*cosh(b*x + a)*sinh(b*x + a) + b*sinh(b*x + a)
^2)
```

giac [A] time = 0.15, size = 60, normalized size = 1.05

$$\frac{24\,bx - 2\left(3\,e^{(4\,bx+4\,a)} - 1\right)e^{(-4\,bx-4\,a)} + \left(e^{(8\,bx+16\,a)} - 6\,e^{(4\,bx+12\,a)}\right)e^{(-8\,a)}}{512\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*b*x+2*a)*cosh(b*x+a)^3*sinh(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/512*(24*b*x - 2*(3*e^(4*b*x + 4*a) - 1)*e^(-4*b*x - 4*a) + (e^(8*b*x + 16
*a) - 6*e^(4*b*x + 12*a))*e^(-8*a))/b
```

maple [A] time = 0.20, size = 61, normalized size = 1.07

$$\frac{3x}{64} - \frac{\sinh(4bx + 4a)}{64b} + \frac{\sinh(8bx + 8a)}{512b} - \frac{\cosh(4bx + 4a)}{128b} + \frac{\cosh(8bx + 8a)}{512b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(2*b*x+2*a)*cosh(b*x+a)^3*sinh(b*x+a)^3,x)
```

```
[Out] 3/64*x-1/64/b*sinh(4*b*x+4*a)+1/512/b*sinh(8*b*x+8*a)-1/128*cosh(4*b*x+4*a)
/b+1/512*cosh(8*b*x+8*a)/b
```

maxima [A] time = 0.34, size = 52, normalized size = 0.91

$$-\frac{\left(6\,e^{(-4\,bx-4\,a)} - 1\right)e^{(8\,bx+8\,a)}}{512\,b} + \frac{3\,(bx + a)}{64\,b} + \frac{e^{(-4\,bx-4\,a)}}{256\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*b*x+2*a)*cosh(b*x+a)^3*sinh(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -1/512*(6*e^(-4*b*x - 4*a) - 1)*e^(8*b*x + 8*a)/b + 3/64*(b*x + a)/b + 1/25
6*e^(-4*b*x - 4*a)/b
```

mupad [B] time = 1.93, size = 46, normalized size = 0.81

$$\frac{3\,x}{64} + \frac{\mathrm{e}^{-4\,a-4\,b\,x}}{256\,b} - \frac{3\,\mathrm{e}^{4\,a+4\,b\,x}}{256\,b} + \frac{\mathrm{e}^{8\,a+8\,b\,x}}{512\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a + b*x)^3*exp(2*a + 2*b*x)*sinh(a + b*x)^3,x)
```

```
[Out] (3*x)/64 + exp(- 4*a - 4*b*x)/(256*b) - (3*exp(4*a + 4*b*x))/(256*b) + exp(
8*a + 8*b*x)/(512*b)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*b*x+2*a)*cosh(b*x+a)**3*sinh(b*x+a)**3,x)
```

```
[Out] Timed out
```

3.932 $\int e^{2(a+bx)} \cosh^3(a+bx) \sinh^2(a+bx)\, dx$

Optimal. Leaf size=100

$$-\frac{e^{-3a-3bx}}{96b}-\frac{e^{-a-bx}}{32b}-\frac{e^{a+bx}}{16b}-\frac{e^{3a+3bx}}{48b}+\frac{e^{5a+5bx}}{160b}+\frac{e^{7a+7bx}}{224b}$$

```
[Out] -1/96*exp(-3*b*x-3*a)/b-1/32*exp(-b*x-a)/b-1/16*exp(b*x+a)/b-1/48*exp(3*b*x
+3*a)/b+1/160*exp(5*b*x+5*a)/b+1/224*exp(7*b*x+7*a)/b
```

Rubi [A] time = 0.07, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {2282, 12, 448}

$$-\frac{e^{-3a-3bx}}{96b}-\frac{e^{-a-bx}}{32b}-\frac{e^{a+bx}}{16b}-\frac{e^{3a+3bx}}{48b}+\frac{e^{5a+5bx}}{160b}+\frac{e^{7a+7bx}}{224b}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*(a + b*x))*Cosh[a + b*x]^3*Sinh[a + b*x]^2,x]
```

```
[Out] -E^(-3*a - 3*b*x)/(96*b) - E^(-a - b*x)/(32*b) - E^(a + b*x)/(16*b) - E^(3*
a + 3*b*x)/(48*b) + E^(5*a + 5*b*x)/(160*b) + E^(7*a + 7*b*x)/(224*b)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$\int e^{2(a+bx)}\cosh^3(a+bx)\sinh^2(a+bx)\,dx = \frac{\text{Subst}\left(\int \frac{(1-x^2)^2(1+x^2)^3}{32x^4}\,dx, x, e^{a+bx}\right)}{b}$$
$$= \frac{\text{Subst}\left(\int \frac{(1-x^2)^2(1+x^2)^3}{x^4}\,dx, x, e^{a+bx}\right)}{32b}$$
$$= \frac{\text{Subst}\left(\int \left(-2+\frac{1}{x^4}+\frac{1}{x^2}-2x^2+x^4+x^6\right)dx, x, e^{a+bx}\right)}{32b}$$
$$= -\frac{e^{-3a-3bx}}{96b}-\frac{e^{-a-bx}}{32b}-\frac{e^{a+bx}}{16b}-\frac{e^{3a+3bx}}{48b}+\frac{e^{5a+5bx}}{160b}+\frac{e^{7a+7bx}}{224b}$$

Mathematica [A] time = 0.11, size = 73, normalized size = 0.73

$$\frac{e^{-3(a+bx)}\left(-105e^{2(a+bx)}-210e^{4(a+bx)}-70e^{6(a+bx)}+21e^{8(a+bx)}+15e^{10(a+bx)}-35\right)}{3360b}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*(a + b*x))*Cosh[a + b*x]^3*Sinh[a + b*x]^2,x]
```

```
[Out] (-35 - 105*E^(2*(a + b*x)) - 210*E^(4*(a + b*x)) - 70*E^(6*(a + b*x)) + 21*
E^(8*(a + b*x)) + 15*E^(10*(a + b*x)))/(3360*b*E^(3*(a + b*x)))
```

fricas [B] time = 0.46, size = 176, normalized size = 1.76

$$-\frac{10\cosh(bx+a)^5+50\cosh(bx+a)\sinh(bx+a)^4-25\sinh(bx+a)^5-\left(250\cosh(bx+a)^2+63\right)\sinh(bx}{1680\,(b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*b*x+2*a)*cosh(b*x+a)^3*sinh(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/1680*(10*cosh(b*x + a)^5 + 50*cosh(b*x + a)*sinh(b*x + a)^4 - 25*sinh(b*
x + a)^5 - (250*cosh(b*x + a)^2 + 63)*sinh(b*x + a)^3 + 42*cosh(b*x + a)^3
+ 2*(50*cosh(b*x + a)^3 + 63*cosh(b*x + a))*sinh(b*x + a)^2 - (125*cosh(b*x
 + a)^4 + 189*cosh(b*x + a)^2 + 70)*sinh(b*x + a) + 140*cosh(b*x + a))/(b*c
osh(b*x + a)^2 - 2*b*cosh(b*x + a)*sinh(b*x + a) + b*sinh(b*x + a)^2)
```

giac [A] time = 0.14, size = 80, normalized size = 0.80

$$-\frac{35\left(3e^{(2bx+2a)}+1\right)e^{(-3bx-3a)}-\left(15e^{(7bx+28a)}+21e^{(5bx+26a)}-70e^{(3bx+24a)}-210e^{(bx+22a)}\right)e^{(-21a)}}{3360\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*b*x+2*a)*cosh(b*x+a)^3*sinh(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -1/3360*(35*(3*e^(2*b*x + 2*a) + 1)*e^(-3*b*x - 3*a) - (15*e^(7*b*x + 28*a)
 + 21*e^(5*b*x + 26*a) - 70*e^(3*b*x + 24*a) - 210*e^(b*x + 22*a))*e^(-21*a
))/b
```

maple [A] time = 0.23, size = 108, normalized size = 1.08

$$-\frac{\sinh(bx+a)}{32b}-\frac{\sinh(3bx+3a)}{96b}+\frac{\sinh(5bx+5a)}{160b}+\frac{\sinh(7bx+7a)}{224b}-\frac{3\cosh(bx+a)}{32b}-\frac{\cosh(3bx+3a)}{32b}+\frac{\cosh(5}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(2*b*x+2*a)*cosh(b*x+a)^3*sinh(b*x+a)^2,x)
```

```
[Out] -1/32*sinh(b*x+a)/b-1/96/b*sinh(3*b*x+3*a)+1/160/b*sinh(5*b*x+5*a)+1/224/b*
sinh(7*b*x+7*a)-3/32*cosh(b*x+a)/b-1/32*cosh(3*b*x+3*a)/b+1/160*cosh(5*b*x+
5*a)/b+1/224*cosh(7*b*x+7*a)/b
```

maxima [A] time = 0.32, size = 76, normalized size = 0.76

$$\frac{\left(21\,e^{(-2\,bx-2\,a)}-70\,e^{(-4\,bx-4\,a)}-210\,e^{(-6\,bx-6\,a)}+15\right)e^{(7\,bx+7\,a)}}{3360\,b}-\frac{3\,e^{(-bx-a)}+e^{(-3\,bx-3\,a)}}{96\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*b*x+2*a)*cosh(b*x+a)^3*sinh(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/3360*(21*e^(-2*b*x - 2*a) - 70*e^(-4*b*x - 4*a) - 210*e^(-6*b*x - 6*a) +
15)*e^(7*b*x + 7*a)/b - 1/96*(3*e^(-b*x - a) + e^(-3*b*x - 3*a))/b
```

mupad [B] time = 0.60, size = 69, normalized size = 0.69

$$-\frac{210\,\mathrm{e}^{a+b\,x}+105\,\mathrm{e}^{-a-b\,x}+35\,\mathrm{e}^{-3\,a-3\,b\,x}+70\,\mathrm{e}^{3\,a+3\,b\,x}-21\,\mathrm{e}^{5\,a+5\,b\,x}-15\,\mathrm{e}^{7\,a+7\,b\,x}}{3360\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a + b*x)^3*exp(2*a + 2*b*x)*sinh(a + b*x)^2,x)
```

```
[Out] -(210*exp(a + b*x) + 105*exp(- a - b*x) + 35*exp(- 3*a - 3*b*x) + 70*exp(3*
a + 3*b*x) - 21*exp(5*a + 5*b*x) - 15*exp(7*a + 7*b*x))/(3360*b)
```

sympy [A] time = 159.71, size = 197, normalized size = 1.97

$$\begin{cases} \frac{2e^{2a}e^{2bx}\sinh^5(a+bx)}{105b}-\frac{4e^{2a}e^{2bx}\sinh^4(a+bx)\cosh(a+bx)}{105b}-\frac{e^{2a}e^{2bx}\sinh^3(a+bx)\cosh^2(a+bx)}{105b}+\frac{2e^{2a}e^{2bx}\sinh^2(a+bx)\cosh^3(a+bx)}{35b}+\frac{8e^{2a}e^{2b}}{} \\ xe^{2a}\sinh^2(a)\cosh^3(a) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*b*x+2*a)*cosh(b*x+a)**3*sinh(b*x+a)**2,x)
```

```
[Out] Piecewise((2*exp(2*a)*exp(2*b*x)*sinh(a + b*x)**5/(105*b) - 4*exp(2*a)*exp(
2*b*x)*sinh(a + b*x)**4*cosh(a + b*x)/(105*b) - exp(2*a)*exp(2*b*x)*sinh(a
+ b*x)**3*cosh(a + b*x)**2/(105*b) + 2*exp(2*a)*exp(2*b*x)*sinh(a + b*x)**2
*cosh(a + b*x)**3/(35*b) + 8*exp(2*a)*exp(2*b*x)*sinh(a + b*x)*cosh(a + b*x
)**4/(35*b) - 4*exp(2*a)*exp(2*b*x)*cosh(a + b*x)**5/(35*b), Ne(b, 0)), (x*
exp(2*a)*sinh(a)**2*cosh(a)**3, True))
```

3.933 $\int e^{2(a+bx)} \cosh^3(a+bx)\sinh(a+bx)\,dx$

Optimal. Leaf size=57

$$\frac{e^{-2a-2bx}}{32b}+\frac{e^{4a+4bx}}{32b}+\frac{e^{6a+6bx}}{96b}-\frac{x}{8}$$

```
[Out] 1/32*exp(-2*b*x-2*a)/b+1/32*exp(4*b*x+4*a)/b+1/96*exp(6*b*x+6*a)/b-1/8*x
```

Rubi [A] time = 0.05, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2282, 12, 446, 75}

$$\frac{e^{-2a-2bx}}{32b}+\frac{e^{4a+4bx}}{32b}+\frac{e^{6a+6bx}}{96b}-\frac{x}{8}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*(a + b*x))*Cosh[a + b*x]^3*Sinh[a + b*x],x]
```

```
[Out] E^(-2*a - 2*b*x)/(32*b) + E^(4*a + 4*b*x)/(32*b) + E^(6*a + 6*b*x)/(96*b) -
 x/8
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 75

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && EqQ[b*e + a*f, 0] &&  !(ILtQ[n + p
+ 2, 0] && GtQ[n + 2*p, 0])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
```

```
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$\begin{aligned}\int e^{2(a+bx)}\cosh^3(a+bx)\sinh(a+bx)\,dx &= \frac{\text{Subst}\left(\int \frac{(-1+x^2)(1+x^2)^3}{16x^3}\,dx, x, e^{a+bx}\right)}{b}\\ &= \frac{\text{Subst}\left(\int \frac{(-1+x^2)(1+x^2)^3}{x^3}\,dx, x, e^{a+bx}\right)}{16b}\\ &= \frac{\text{Subst}\left(\int \frac{(-1+x)(1+x)^3}{x^2}\,dx, x, e^{2a+2bx}\right)}{32b}\\ &= \frac{\text{Subst}\left(\int \left(-\frac{1}{x^2}-\frac{2}{x}+2x+x^2\right)\,dx, x, e^{2a+2bx}\right)}{32b}\\ &= \frac{e^{-2a-2bx}}{32b}+\frac{e^{4a+4bx}}{32b}+\frac{e^{6a+6bx}}{96b}-\frac{x}{8}\end{aligned}$$

Mathematica [A] time = 0.07, size = 43, normalized size = 0.75

$$\frac{3e^{-2(a+bx)}+3e^{4(a+bx)}+e^{6(a+bx)}-12bx}{96b}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*(a + b*x))*Cosh[a + b*x]^3*Sinh[a + b*x],x]
```

```
[Out] (3/E^(2*(a + b*x)) + 3*E^(4*(a + b*x)) + E^(6*(a + b*x)) - 12*b*x)/(96*b)
```

fricas [B] time = 0.58, size = 152, normalized size = 2.67

$$\frac{4\cosh(bx+a)^4-8\cosh(bx+a)\sinh(bx+a)^3+4\sinh(bx+a)^4-3(4bx-1)\cosh(bx+a)^2-3\left(4bx-8\right.}{96\left(b\cosh(bx+a)^2-2b\cosh(bx+a)\sin\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*b*x+2*a)*cosh(b*x+a)^3*sinh(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/96*(4*cosh(b*x + a)^4 - 8*cosh(b*x + a)*sinh(b*x + a)^3 + 4*sinh(b*x + a)
^4 - 3*(4*b*x - 1)*cosh(b*x + a)^2 - 3*(4*b*x - 8*cosh(b*x + a)^2 - 1)*sinh
```

```
(b*x + a)^2 - 2*(4*cosh(b*x + a)^3 - 3*(4*b*x + 1)*cosh(b*x + a))*sinh(b*x
+ a))/(b*cosh(b*x + a)^2 - 2*b*cosh(b*x + a)*sinh(b*x + a) + b*sinh(b*x + a
)^2)
```

giac [A] time = 0.12, size = 61, normalized size = 1.07

$$-\frac{12\,bx - 3\left(2\,e^{(2\,bx+2\,a)} + 1\right)e^{(-2\,bx-2\,a)} - \left(e^{(6\,bx+12\,a)} + 3\,e^{(4\,bx+10\,a)}\right)e^{(-6\,a)}}{96\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*b*x+2*a)*cosh(b*x+a)^3*sinh(b*x+a),x, algorithm="giac")
```

```
[Out] -1/96*(12*b*x - 3*(2*e^(2*b*x + 2*a) + 1)*e^(-2*b*x - 2*a) - (e^(6*b*x + 12
*a) + 3*e^(4*b*x + 10*a))*e^(-6*a))/b
```

maple [A] time = 0.26, size = 89, normalized size = 1.56

$$-\frac{x}{8}-\frac{\sinh\left(2bx+2a\right)}{32b}+\frac{\sinh\left(4bx+4a\right)}{32b}+\frac{\sinh\left(6bx+6a\right)}{96b}+\frac{\cosh\left(2bx+2a\right)}{32b}+\frac{\cosh\left(4bx+4a\right)}{32b}+\frac{\cosh\left(6bx+6a\right)}{96b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(2*b*x+2*a)*cosh(b*x+a)^3*sinh(b*x+a),x)
```

```
[Out] -1/8*x-1/32*sinh(2*b*x+2*a)/b+1/32/b*sinh(4*b*x+4*a)+1/96/b*sinh(6*b*x+6*a)
+1/32*cosh(2*b*x+2*a)/b+1/32*cosh(4*b*x+4*a)/b+1/96*cosh(6*b*x+6*a)/b
```

maxima [A] time = 0.33, size = 52, normalized size = 0.91

$$\frac{\left(3\,e^{(-2\,bx-2\,a)} + 1\right)e^{(6\,bx+6\,a)}}{96\,b} - \frac{bx + a}{8\,b} + \frac{e^{(-2\,bx-2\,a)}}{32\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*b*x+2*a)*cosh(b*x+a)^3*sinh(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/96*(3*e^(-2*b*x - 2*a) + 1)*e^(6*b*x + 6*a)/b - 1/8*(b*x + a)/b + 1/32*e^
(-2*b*x - 2*a)/b
```

mupad [B] time = 0.56, size = 42, normalized size = 0.74

$$\frac{\frac{\mathrm{e}^{-2\,a-2\,b\,x}}{32}+\frac{\mathrm{e}^{4\,a+4\,b\,x}}{32}+\frac{\mathrm{e}^{6\,a+6\,b\,x}}{96}}{b}-\frac{x}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a + b*x)^3*exp(2*a + 2*b*x)*sinh(a + b*x),x)
```

```
[Out] (exp(- 2*a - 2*b*x)/32 + exp(4*a + 4*b*x)/32 + exp(6*a + 6*b*x)/96)/b - x/8
```

sympy [A] time = 52.94, size = 233, normalized size = 4.09

$$\begin{cases} \frac{xe^{2a}e^{2bx}\sinh^4(a+bx)}{8} - \frac{xe^{2a}e^{2bx}\sinh^3(a+bx)\cosh(a+bx)}{4} + \frac{xe^{2a}e^{2bx}\sinh(a+bx)\cosh^3(a+bx)}{4} - \frac{xe^{2a}e^{2bx}\cosh^4(a+bx)}{8} + \frac{e^{2a}e^{2bx}\sinh^4(a+b}{48b} \\ xe^{2a}\sinh(a)\cosh^3(a) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*b*x+2*a)*cosh(b*x+a)**3*sinh(b*x+a),x)
```

```
[Out] Piecewise((x*exp(2*a)*exp(2*b*x)*sinh(a + b*x)**4/8 - x*exp(2*a)*exp(2*b*x)
*sinh(a + b*x)**3*cosh(a + b*x)/4 + x*exp(2*a)*exp(2*b*x)*sinh(a + b*x)*cos
h(a + b*x)**3/4 - x*exp(2*a)*exp(2*b*x)*cosh(a + b*x)**4/8 + exp(2*a)*exp(2
*b*x)*sinh(a + b*x)**4/(48*b) - exp(2*a)*exp(2*b*x)*sinh(a + b*x)**3*cosh(a
 + b*x)/(6*b) + exp(2*a)*exp(2*b*x)*sinh(a + b*x)**2*cosh(a + b*x)**2/(4*b)
 + exp(2*a)*exp(2*b*x)*cosh(a + b*x)**4/(16*b), Ne(b, 0)), (x*exp(2*a)*sinh
(a)*cosh(a)**3, True))
```

3.934 $\int e^{2(a+bx)} \cosh^2(a+bx) \coth(a+bx) \, dx$

Optimal. Leaf size=59

$$\frac{e^{2a+2bx}}{2b} + \frac{e^{4a+4bx}}{16b} + \frac{\log\left(1-e^{2a+2bx}\right)}{b} - \frac{x}{4}$$

```
[Out] 1/2*exp(2*b*x+2*a)/b+1/16*exp(4*b*x+4*a)/b-1/4*x+ln(1-exp(2*b*x+2*a))/b
```

Rubi [A] time = 0.06, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2282, 12, 446, 72}

$$\frac{e^{2a+2bx}}{2b} + \frac{e^{4a+4bx}}{16b} + \frac{\log\left(1-e^{2a+2bx}\right)}{b} - \frac{x}{4}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*(a + b*x))*Cosh[a + b*x]^2*Coth[a + b*x],x]
```

```
[Out] E^(2*a + 2*b*x)/(2*b) + E^(4*a + 4*b*x)/(16*b) - x/4 + Log[1 - E^(2*a + 2*b
*x)]/b
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
```

```
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$\begin{aligned}
\int e^{2(a+bx)}\cosh^2(a+bx)\coth(a+bx)\,dx &= \frac{\text{Subst}\left(\int \frac{\left(1+x^2\right)^3}{4x\left(-1+x^2\right)}\,dx, x, e^{a+bx}\right)}{b}\\
&= \frac{\text{Subst}\left(\int \frac{\left(1+x^2\right)^3}{x\left(-1+x^2\right)}\,dx, x, e^{a+bx}\right)}{4b}\\
&= \frac{\text{Subst}\left(\int \frac{(1+x)^3}{(-1+x)x}\,dx, x, e^{2a+2bx}\right)}{8b}\\
&= \frac{\text{Subst}\left(\int \left(4+\frac{8}{-1+x}-\frac{1}{x}+x\right)dx, x, e^{2a+2bx}\right)}{8b}\\
&= \frac{e^{2a+2bx}}{2b}+\frac{e^{4a+4bx}}{16b}-\frac{x}{4}+\frac{\log\left(1-e^{2a+2bx}\right)}{b}
\end{aligned}$$

Mathematica [A] time = 0.05, size = 48, normalized size = 0.81

$$\frac{8e^{2(a+bx)}+e^{4(a+bx)}+16\log\left(1-e^{2(a+bx)}\right)-4bx}{16b}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*(a + b*x))*Cosh[a + b*x]^2*Coth[a + b*x],x]
```

```
[Out] (8*E^(2*(a + b*x)) + E^(4*(a + b*x)) - 4*b*x + 16*Log[1 - E^(2*(a + b*x))])
/(16*b)
```

fricas [B] time = 0.45, size = 127, normalized size = 2.15

$$\frac{\cosh(bx+a)^4+4\cosh(bx+a)\sinh(bx+a)^3+\sinh(bx+a)^4+2\left(3\cosh(bx+a)^2+4\right)\sinh(bx+a)^2-4b}{16b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*b*x+2*a)*cosh(b*x+a)^3*csch(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/16*(cosh(b*x + a)^4 + 4*cosh(b*x + a)*sinh(b*x + a)^3 + sinh(b*x + a)^4 +
 2*(3*cosh(b*x + a)^2 + 4)*sinh(b*x + a)^2 - 4*b*x + 8*cosh(b*x + a)^2 + 4*
(cosh(b*x + a)^3 + 4*cosh(b*x + a))*sinh(b*x + a) + 16*log(2*sinh(b*x + a)/
(cosh(b*x + a) - sinh(b*x + a))))/b
```

giac [A] time = 0.14, size = 52, normalized size = 0.88

$$-\frac{4\,bx - \left(e^{(4\,bx+8\,a)} + 8\,e^{(2\,bx+6\,a)}\right)e^{(-4\,a)} - 16\,\log\left(\left|e^{(2\,bx+2\,a)} - 1\right|\right)}{16\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*b*x+2*a)*cosh(b*x+a)^3*csch(b*x+a),x, algorithm="giac")
```

```
[Out] -1/16*(4*b*x - (e^(4*b*x + 8*a) + 8*e^(2*b*x + 6*a))*e^(-4*a) - 16*log(abs(
e^(2*b*x + 2*a) - 1)))/b
```

maple [A] time = 0.66, size = 55, normalized size = 0.93

$$-\frac{x}{4} + \frac{e^{4bx+4a}}{16b} + \frac{e^{2bx+2a}}{2b} - \frac{2a}{b} + \frac{\ln\left(e^{2bx+2a} - 1\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(2*b*x+2*a)*cosh(b*x+a)^3*csch(b*x+a),x)
```

```
[Out] -1/4*x+1/16*exp(4*b*x+4*a)/b+1/2*exp(2*b*x+2*a)/b-2*a/b+1/b*ln(exp(2*b*x+2*
a)-1)
```

maxima [A] time = 0.33, size = 70, normalized size = 1.19

$$\frac{\left(8\,e^{(-2\,bx-2\,a)} + 1\right)e^{(4\,bx+4\,a)}}{16\,b} + \frac{7\,(bx + a)}{4\,b} + \frac{\log\left(e^{(-bx-a)} + 1\right)}{b} + \frac{\log\left(e^{(-bx-a)} - 1\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*b*x+2*a)*cosh(b*x+a)^3*csch(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/16*(8*e^(-2*b*x - 2*a) + 1)*e^(4*b*x + 4*a)/b + 7/4*(b*x + a)/b + log(e^(
-b*x - a) + 1)/b + log(e^(-b*x - a) - 1)/b
```

mupad [B] time = 0.10, size = 49, normalized size = 0.83

$$\frac{\ln\left(e^{2\,a}\,e^{2\,b\,x} - 1\right)}{b} - \frac{x}{4} + \frac{e^{2\,a+2\,b\,x}}{2\,b} + \frac{e^{4\,a+4\,b\,x}}{16\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cosh(a + b*x)^3*exp(2*a + 2*b*x))/sinh(a + b*x),x)
```

```
[Out] log(exp(2*a)*exp(2*b*x) - 1)/b - x/4 + exp(2*a + 2*b*x)/(2*b) + exp(4*a + 4
*b*x)/(16*b)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*b*x+2*a)*cosh(b*x+a)**3*csch(b*x+a),x)
```

```
[Out] Timed out
```

3.935 $\int e^{2(a+bx)} \cosh(a+bx) \coth^2(a+bx)\, dx$

Optimal. Leaf size=73

$$\frac{5e^{a+bx}}{2b}+\frac{e^{3a+3bx}}{6b}+\frac{2e^{a+bx}}{b\left(1-e^{2a+2bx}\right)}-\frac{4\tanh^{-1}\left(e^{a+bx}\right)}{b}$$

```
[Out] 5/2*exp(b*x+a)/b+1/6*exp(3*b*x+3*a)/b+2*exp(b*x+a)/b/(1-exp(2*b*x+2*a))-4*a
rctanh(exp(b*x+a))/b
```

Rubi [A] time = 0.05, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {2282, 12, 390, 385, 206}

$$\frac{5e^{a+bx}}{2b}+\frac{e^{3a+3bx}}{6b}+\frac{2e^{a+bx}}{b\left(1-e^{2a+2bx}\right)}-\frac{4\tanh^{-1}\left(e^{a+bx}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*(a + b*x))*Cosh[a + b*x]*Coth[a + b*x]^2,x]
```

```
[Out] (5*E^(a + b*x))/(2*b) + E^(3*a + 3*b*x)/(6*b) + (2*E^(a + b*x))/(b*(1 - E^(
2*a + 2*b*x))) - (4*ArcTanh[E^(a + b*x)])/b
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$\begin{aligned}
\int e^{2(a+bx)}\cosh(a+bx)\coth^2(a+bx)\,dx &= \frac{\text{Subst}\left(\int \frac{\left(1+x^2\right)^3}{2\left(1-x^2\right)^2}\,dx, x, e^{a+bx}\right)}{b}\\
&= \frac{\text{Subst}\left(\int \frac{\left(1+x^2\right)^3}{\left(1-x^2\right)^2}\,dx, x, e^{a+bx}\right)}{2b}\\
&= \frac{\text{Subst}\left(\int \left(5+x^2-\frac{4\left(1-3x^2\right)}{\left(1-x^2\right)^2}\right)dx, x, e^{a+bx}\right)}{2b}\\
&= \frac{5e^{a+bx}}{2b}+\frac{e^{3a+3bx}}{6b}-\frac{2\,\text{Subst}\left(\int \frac{1-3x^2}{\left(1-x^2\right)^2}\,dx, x, e^{a+bx}\right)}{b}\\
&= \frac{5e^{a+bx}}{2b}+\frac{e^{3a+3bx}}{6b}+\frac{2e^{a+bx}}{b\left(1-e^{2a+2bx}\right)}-\frac{4\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, e^{a+bx}\right)}{b}\\
&= \frac{5e^{a+bx}}{2b}+\frac{e^{3a+3bx}}{6b}+\frac{2e^{a+bx}}{b\left(1-e^{2a+2bx}\right)}-\frac{4\tanh^{-1}\left(e^{a+bx}\right)}{b}
\end{aligned}$$

Mathematica [C] time = 0.96, size = 220, normalized size = 3.01

$$e^{-5(a+bx)}\left(256e^{8(a+bx)}\left(e^{2(a+bx)}+1\right)^3 {}_5F_4\left(\frac{3}{2},2,2,2,2;1,1,1,\frac{11}{2};e^{2(a+bx)}\right)-21\left(91925e^{2(a+bx)}+61158e^{4(a+bx)}-201\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(2*(a + b*x))*Cosh[a + b*x]*Coth[a + b*x]^2,x]
```

```
[Out] (-21*(36015 + 91925*E^(2*(a + b*x)) + 61158*E^(4*(a + b*x)) - 20166*E^(6*(a
 + b*x)) - 15061*E^(8*(a + b*x)) + 753*E^(10*(a + b*x))) - (315*(-2401 - 53
28*E^(2*(a + b*x)) - 1821*E^(4*(a + b*x)) + 3264*E^(6*(a + b*x)) + 1149*E^(
8*(a + b*x)) - 240*E^(10*(a + b*x)) + E^(12*(a + b*x)))*ArcTanh[Sqrt[E^(2*(
a + b*x))]])/Sqrt[E^(2*(a + b*x))] + 256*E^(8*(a + b*x))*(1 + E^(2*(a + b*x
)))^3*HypergeometricPFQ[{3/2, 2, 2, 2, 2}, {1, 1, 1, 11/2}, E^(2*(a + b*x))
])/(60480*b*E^(5*(a + b*x)))
```

fricas [B] time = 0.52, size = 272, normalized size = 3.73

$$\frac{\cosh(bx+a)^5 + 5\cosh(bx+a)\sinh(bx+a)^4 + \sinh(bx+a)^5 + 2\left(5\cosh(bx+a)^2 + 7\right)\sinh(bx+a)^3 + 14\,\mathrm{c}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*b*x+2*a)*cosh(b*x+a)^3*csch(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/6*(cosh(b*x + a)^5 + 5*cosh(b*x + a)*sinh(b*x + a)^4 + sinh(b*x + a)^5 +
2*(5*cosh(b*x + a)^2 + 7)*sinh(b*x + a)^3 + 14*cosh(b*x + a)^3 + 2*(5*cosh(
b*x + a)^3 + 21*cosh(b*x + a))*sinh(b*x + a)^2 - 12*(cosh(b*x + a)^2 + 2*co
sh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 - 1)*log(cosh(b*x + a) + sinh(b
*x + a) + 1) + 12*(cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b
*x + a)^2 - 1)*log(cosh(b*x + a) + sinh(b*x + a) - 1) + (5*cosh(b*x + a)^4
+ 42*cosh(b*x + a)^2 - 27)*sinh(b*x + a) - 27*cosh(b*x + a))/(b*cosh(b*x +
a)^2 + 2*b*cosh(b*x + a)*sinh(b*x + a) + b*sinh(b*x + a)^2 - b)
```

giac [A] time = 0.15, size = 75, normalized size = 1.03

$$\frac{\left(e^{(3\,bx+15\,a)} + 15\,e^{(bx+13\,a)}\right)e^{(-12\,a)} - \frac{12\,e^{(bx+a)}}{e^{(2\,bx+2\,a)}-1} - 12\,\log\left(e^{(bx+a)}+1\right) + 12\,\log\left(\left|e^{(bx+a)}-1\right|\right)}{6\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*b*x+2*a)*cosh(b*x+a)^3*csch(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/6*((e^(3*b*x + 15*a) + 15*e^(b*x + 13*a))*e^(-12*a) - 12*e^(b*x + a)/(e^(
2*b*x + 2*a) - 1) - 12*log(e^(b*x + a) + 1) + 12*log(abs(e^(b*x + a) - 1)))
/b
```

maple [A] time = 0.64, size = 79, normalized size = 1.08

$$\frac{e^{3bx+3a}}{6b} + \frac{5\,e^{bx+a}}{2b} - \frac{2\,e^{bx+a}}{b\left(e^{2bx+2a}-1\right)} - \frac{2\ln\left(1+e^{bx+a}\right)}{b} + \frac{2\ln\left(e^{bx+a}-1\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(2*b*x+2*a)*cosh(b*x+a)^3*csch(b*x+a)^2,x)
```

```
[Out] 1/6*exp(3*b*x+3*a)/b+5/2*exp(b*x+a)/b-2/b*exp(b*x+a)/(exp(2*b*x+2*a)-1)-2/b
*ln(1+exp(b*x+a))+2/b*ln(exp(b*x+a)-1)
```

maxima [A] time = 0.32, size = 87, normalized size = 1.19

$$-\frac{2\,\log\left(e^{(-bx-a)}+1\right)}{b}+\frac{2\,\log\left(e^{(-bx-a)}-1\right)}{b}+\frac{14\,e^{(-2\,bx-2\,a)}-27\,e^{(-4\,bx-4\,a)}+1}{6\,b\left(e^{(-3\,bx-3\,a)}-e^{(-5\,bx-5\,a)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*b*x+2*a)*cosh(b*x+a)^3*csch(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -2*log(e^(-b*x - a) + 1)/b + 2*log(e^(-b*x - a) - 1)/b + 1/6*(14*e^(-2*b*x
- 2*a) - 27*e^(-4*b*x - 4*a) + 1)/(b*(e^(-3*b*x - 3*a) - e^(-5*b*x - 5*a)))
```

mupad [B] time = 1.87, size = 77, normalized size = 1.05

$$\frac{5\,e^{a+b\,x}}{2\,b}-\frac{4\,\mathrm{atan}\left(\frac{e^{b\,x}\,e^{a}\,\sqrt{-b^2}}{b}\right)}{\sqrt{-b^2}}+\frac{e^{3\,a+3\,b\,x}}{6\,b}-\frac{2\,e^{a+b\,x}}{b\,\left(e^{2\,a+2\,b\,x}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cosh(a + b*x)^3*exp(2*a + 2*b*x))/sinh(a + b*x)^2,x)
```

```
[Out] (5*exp(a + b*x))/(2*b) - (4*atan((exp(b*x)*exp(a)*(-b^2)^(1/2))/b))/(-b^2)^
(1/2) + exp(3*a + 3*b*x)/(6*b) - (2*exp(a + b*x))/(b*(exp(2*a + 2*b*x) - 1)
)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*b*x+2*a)*cosh(b*x+a)**3*csch(b*x+a)**2,x)
```

```
[Out] Timed out
```

3.936 $\int e^{2(a+bx)} \coth^3(a+bx)\,dx$

Optimal. Leaf size=80

$$\frac{e^{2a+2bx}}{2b}+\frac{6}{b\left(1-e^{2a+2bx}\right)}-\frac{2}{b\left(1-e^{2a+2bx}\right)^2}+\frac{3\log\left(1-e^{2a+2bx}\right)}{b}$$

```
[Out] 1/2*exp(2*b*x+2*a)/b-2/b/(1-exp(2*b*x+2*a))^2+6/b/(1-exp(2*b*x+2*a))+3*ln(1
-exp(2*b*x+2*a))/b
```

Rubi [A] time = 0.06, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2282, 444, 43}

$$\frac{e^{2a+2bx}}{2b}+\frac{6}{b\left(1-e^{2a+2bx}\right)}-\frac{2}{b\left(1-e^{2a+2bx}\right)^2}+\frac{3\log\left(1-e^{2a+2bx}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*(a + b*x))*Coth[a + b*x]^3,x]
```

```
[Out] E^(2*a + 2*b*x)/(2*b) - 2/(b*(1 - E^(2*a + 2*b*x))^2) + 6/(b*(1 - E^(2*a +
2*b*x))) + (3*Log[1 - E^(2*a + 2*b*x)])/b
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
```

```
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$\int e^{2(a+bx)}\coth^3(a+bx)\,dx = \frac{\text{Subst}\left(\int \frac{x(1+x^2)^3}{(-1+x^2)^3}\,dx, x, e^{a+bx}\right)}{b}$$
$$= \frac{\text{Subst}\left(\int \frac{(1+x)^3}{(-1+x)^3}\,dx, x, e^{2a+2bx}\right)}{2b}$$
$$= \frac{\text{Subst}\left(\int \left(1+\frac{8}{(-1+x)^3}+\frac{12}{(-1+x)^2}+\frac{6}{-1+x}\right)dx, x, e^{2a+2bx}\right)}{2b}$$
$$= \frac{e^{2a+2bx}}{2b}-\frac{2}{b\left(1-e^{2a+2bx}\right)^2}+\frac{6}{b\left(1-e^{2a+2bx}\right)}+\frac{3\log\left(1-e^{2a+2bx}\right)}{b}$$

Mathematica [A] time = 0.12, size = 60, normalized size = 0.75

$$\frac{\frac{8-12e^{2(a+bx)}}{\left(e^{2(a+bx)}-1\right)^2}+e^{2(a+bx)}+6\log\left(1-e^{2(a+bx)}\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*(a + b*x))*Coth[a + b*x]^3,x]
```

```
[Out] (E^(2*(a + b*x)) + (8 - 12*E^(2*(a + b*x)))/(-1 + E^(2*(a + b*x)))^2 + 6*Lo
g[1 - E^(2*(a + b*x))])/(2*b)
```

fricas [B] time = 0.48, size = 398, normalized size = 4.98

$$\cosh(bx+a)^6+6\cosh(bx+a)\sinh(bx+a)^5+\sinh(bx+a)^6+\left(15\cosh(bx+a)^2-2\right)\sinh(bx+a)^4-2\,\mathrm{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*b*x+2*a)*cosh(b*x+a)^3*csch(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/2*(cosh(b*x + a)^6 + 6*cosh(b*x + a)*sinh(b*x + a)^5 + sinh(b*x + a)^6 +
(15*cosh(b*x + a)^2 - 2)*sinh(b*x + a)^4 - 2*cosh(b*x + a)^4 + 4*(5*cosh(b*
```

```
x + a)^3 - 2*cosh(b*x + a))*sinh(b*x + a)^3 + (15*cosh(b*x + a)^4 - 12*cosh
(b*x + a)^2 - 11)*sinh(b*x + a)^2 - 11*cosh(b*x + a)^2 + 6*(cosh(b*x + a)^4
 + 4*cosh(b*x + a)*sinh(b*x + a)^3 + sinh(b*x + a)^4 + 2*(3*cosh(b*x + a)^2
 - 1)*sinh(b*x + a)^2 - 2*cosh(b*x + a)^2 + 4*(cosh(b*x + a)^3 - cosh(b*x +
 a))*sinh(b*x + a) + 1)*log(2*sinh(b*x + a)/(cosh(b*x + a) - sinh(b*x + a))
) + 2*(3*cosh(b*x + a)^5 - 4*cosh(b*x + a)^3 - 11*cosh(b*x + a))*sinh(b*x +
 a) + 8)/(b*cosh(b*x + a)^4 + 4*b*cosh(b*x + a)*sinh(b*x + a)^3 + b*sinh(b*
x + a)^4 - 2*b*cosh(b*x + a)^2 + 2*(3*b*cosh(b*x + a)^2 - b)*sinh(b*x + a)^
2 + 4*(b*cosh(b*x + a)^3 - b*cosh(b*x + a))*sinh(b*x + a) + b)
```

giac [A] time = 0.14, size = 70, normalized size = 0.88

$$-\frac{\frac{9\,e^{(4\,bx+4\,a)}-6\,e^{(2\,bx+2\,a)}+1}{\left(e^{(2\,bx+2\,a)}-1\right)^2}-e^{(2\,bx+2\,a)}-6\,\log\left(\left|e^{(2\,bx+2\,a)}-1\right|\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*b*x+2*a)*cosh(b*x+a)^3*csch(b*x+a)^3,x, algorithm="giac")
```

```
[Out] -1/2*((9*e^(4*b*x + 4*a) - 6*e^(2*b*x + 2*a) + 1)/(e^(2*b*x + 2*a) - 1)^2 -
 e^(2*b*x + 2*a) - 6*log(abs(e^(2*b*x + 2*a) - 1)))/b
```

maple [A] time = 0.69, size = 70, normalized size = 0.88

$$\frac{e^{2bx+2a}}{2b}-\frac{6a}{b}-\frac{2\left(3\,e^{2bx+2a}-2\right)}{b\left(e^{2bx+2a}-1\right)^2}+\frac{3\ln\left(e^{2bx+2a}-1\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(2*b*x+2*a)*cosh(b*x+a)^3*csch(b*x+a)^3,x)
```

```
[Out] 1/2*exp(2*b*x+2*a)/b-6*a/b-2*(3*exp(2*b*x+2*a)-2)/b/(exp(2*b*x+2*a)-1)^2+3/
b*ln(exp(2*b*x+2*a)-1)
```

maxima [A] time = 0.32, size = 106, normalized size = 1.32

$$\frac{6\,(bx+a)}{b}+\frac{3\,\log\left(e^{(-bx-a)}+1\right)}{b}+\frac{3\,\log\left(e^{(-bx-a)}-1\right)}{b}-\frac{10\,e^{(-2\,bx-2\,a)}-5\,e^{(-4\,bx-4\,a)}-1}{2\,b\left(e^{(-2\,bx-2\,a)}-2\,e^{(-4\,bx-4\,a)}+e^{(-6\,bx-6\,a)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*b*x+2*a)*cosh(b*x+a)^3*csch(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 6*(b*x + a)/b + 3*log(e^(-b*x - a) + 1)/b + 3*log(e^(-b*x - a) - 1)/b - 1/2
*(10*e^(-2*b*x - 2*a) - 5*e^(-4*b*x - 4*a) - 1)/(b*(e^(-2*b*x - 2*a) - 2*e^
(-4*b*x - 4*a) + e^(-6*b*x - 6*a)))
```

mupad [B] time = 0.07, size = 80, normalized size = 1.00

$$\frac{3\ln\left(\mathrm{e}^{2a}\,\mathrm{e}^{2bx}-1\right)}{b}-\frac{6}{b\left(\mathrm{e}^{2a+2bx}-1\right)}-\frac{2}{b\left(\mathrm{e}^{4a+4bx}-2\,\mathrm{e}^{2a+2bx}+1\right)}+\frac{\mathrm{e}^{2a+2bx}}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cosh(a + b*x)^3*exp(2*a + 2*b*x))/sinh(a + b*x)^3,x)
```

```
[Out] (3*log(exp(2*a)*exp(2*b*x) - 1))/b - 6/(b*(exp(2*a + 2*b*x) - 1)) - 2/(b*(e
xp(4*a + 4*b*x) - 2*exp(2*a + 2*b*x) + 1)) + exp(2*a + 2*b*x)/(2*b)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*b*x+2*a)*cosh(b*x+a)**3*csch(b*x+a)**3,x)
```

```
[Out] Timed out
```

3.937 $\int e^x \mathbf{sech}(2x)\tanh(2x)\,dx$

Optimal. Leaf size=113

$$-\frac{e^{3x}}{e^{4x}+1}+\frac{\log\left(-\sqrt{2}e^x+e^{2x}+1\right)}{4\sqrt{2}}-\frac{\log\left(\sqrt{2}e^x+e^{2x}+1\right)}{4\sqrt{2}}-\frac{\tan^{-1}\left(1-\sqrt{2}e^x\right)}{2\sqrt{2}}+\frac{\tan^{-1}\left(\sqrt{2}e^x+1\right)}{2\sqrt{2}}$$

```
[Out] -exp(3*x)/(1+exp(4*x))+1/4*arctan(-1+exp(x)*2^(1/2))*2^(1/2)+1/4*arctan(1+e
xp(x)*2^(1/2))*2^(1/2)+1/8*ln(1+exp(2*x)-exp(x)*2^(1/2))*2^(1/2)-1/8*ln(1+e
xp(2*x)+exp(x)*2^(1/2))*2^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.750, Rules used = {2282, 12, 457, 297, 1162, 617, 204, 1165, 628}

$$-\frac{e^{3x}}{e^{4x}+1}+\frac{\log\left(-\sqrt{2}e^x+e^{2x}+1\right)}{4\sqrt{2}}-\frac{\log\left(\sqrt{2}e^x+e^{2x}+1\right)}{4\sqrt{2}}-\frac{\tan^{-1}\left(1-\sqrt{2}e^x\right)}{2\sqrt{2}}+\frac{\tan^{-1}\left(\sqrt{2}e^x+1\right)}{2\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^x*Sech[2*x]*Tanh[2*x],x]
```

```
[Out] -(E^(3*x)/(1 + E^(4*x))) - ArcTan[1 - Sqrt[2]*E^x]/(2*Sqrt[2]) + ArcTan[1 +
 Sqrt[2]*E^x]/(2*Sqrt[2]) + Log[1 - Sqrt[2]*E^x + E^(2*x)]/(4*Sqrt[2]) - Lo
g[1 + Sqrt[2]*E^x + E^(2*x)]/(4*Sqrt[2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
 + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
 n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$\int e^x \operatorname{sech}(2x)\tanh(2x)\,dx = \text{Subst}\left(\int \frac{2x^2\left(-1+x^4\right)}{\left(1+x^4\right)^2}\,dx,x,e^x\right)$$
$$= 2\,\text{Subst}\left(\int \frac{x^2\left(-1+x^4\right)}{\left(1+x^4\right)^2}\,dx,x,e^x\right)$$
$$= -\frac{e^{3x}}{1+e^{4x}} + \text{Subst}\left(\int \frac{x^2}{1+x^4}\,dx,x,e^x\right)$$
$$= -\frac{e^{3x}}{1+e^{4x}} - \frac{1}{2}\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx,x,e^x\right) + \frac{1}{2}\text{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx,x,e^x\right)$$
$$= -\frac{e^{3x}}{1+e^{4x}} + \frac{1}{4}\text{Subst}\left(\int \frac{1}{1-\sqrt{2}\,x+x^2}\,dx,x,e^x\right) + \frac{1}{4}\text{Subst}\left(\int \frac{1}{1+\sqrt{2}\,x+x^2}\,dx,x,\right.$$
$$= -\frac{e^{3x}}{1+e^{4x}} + \frac{\log\left(1-\sqrt{2}\,e^x+e^{2x}\right)}{4\sqrt{2}} - \frac{\log\left(1+\sqrt{2}\,e^x+e^{2x}\right)}{4\sqrt{2}} + \frac{\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx,x,1\right.}{2\sqrt{2}}$$
$$= -\frac{e^{3x}}{1+e^{4x}} - \frac{\tan^{-1}\left(1-\sqrt{2}\,e^x\right)}{2\sqrt{2}} + \frac{\tan^{-1}\left(1+\sqrt{2}\,e^x\right)}{2\sqrt{2}} + \frac{\log\left(1-\sqrt{2}\,e^x+e^{2x}\right)}{4\sqrt{2}} - \frac{\log\left(1\right.}{}$$

Mathematica [C] time = 0.03, size = 42, normalized size = 0.37

$$\frac{2}{3}e^{3x}\left({}_2F_1\left(\frac{3}{4},1;\frac{7}{4};-e^{4x}\right) - 2\,{}_2F_1\left(\frac{3}{4},2;\frac{7}{4};-e^{4x}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x*Sech[2*x]*Tanh[2*x],x]
```

```
[Out] (2*E^(3*x)*(Hypergeometric2F1[3/4, 1, 7/4, -E^(4*x)] - 2*Hypergeometric2F1[
3/4, 2, 7/4, -E^(4*x)]))/3
```

fricas [B] time = 0.51, size = 164, normalized size = 1.45

$$-\frac{4\left(\sqrt{2}\,e^{(4\,x)}+\sqrt{2}\right)\arctan\left(-\sqrt{2}\,e^x+\sqrt{2}\,\sqrt{\sqrt{2}\,e^x+e^{(2\,x)}+1}\,-1\right)+4\left(\sqrt{2}\,e^{(4\,x)}+\sqrt{2}\right)\arctan\left(-\sqrt{2}\,e^x+\frac{1}{2}\sqrt{2}\,\sqrt{\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sech(2*x)*tanh(2*x),x, algorithm="fricas")
```

```
[Out] -1/8*(4*(sqrt(2)*e^(4*x) + sqrt(2))*arctan(-sqrt(2)*e^x + sqrt(2)*sqrt(sqrt
(2)*e^x + e^(2*x) + 1) - 1) + 4*(sqrt(2)*e^(4*x) + sqrt(2))*arctan(-sqrt(2)
*e^x + 1/2*sqrt(2)*sqrt(-4*sqrt(2)*e^x + 4*e^(2*x) + 4) + 1) + (sqrt(2)*e^(
4*x) + sqrt(2))*log(4*sqrt(2)*e^x + 4*e^(2*x) + 4) - (sqrt(2)*e^(4*x) + sqr
t(2))*log(-4*sqrt(2)*e^x + 4*e^(2*x) + 4) + 8*e^(3*x))/(e^(4*x) + 1)
```

giac [A] time = 0.13, size = 90, normalized size = 0.80

$$\frac{1}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2e^{x}\right)\right)+\frac{1}{4}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2e^{x}\right)\right)-\frac{1}{8}\sqrt{2}\log\left(\sqrt{2}e^{x}+e^{(2x)}+1\right)+\frac{1}{8}\sqrt{2}\log$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sech(2*x)*tanh(2*x),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*e^x)) + 1/4*sqrt(2)*arctan(-1/2
*sqrt(2)*(sqrt(2) - 2*e^x)) - 1/8*sqrt(2)*log(sqrt(2)*e^x + e^(2*x) + 1) +
1/8*sqrt(2)*log(-sqrt(2)*e^x + e^(2*x) + 1) - e^(3*x)/(e^(4*x) + 1)
```

maple [C] time = 0.29, size = 40, normalized size = 0.35

$$-\frac{e^{3x}}{1+e^{4x}}+2\left(\sum_{\_R=\mathrm{RootOf}\left(4096\_Z^{4}+1\right)}\_R\ln\left(512\_R^{3}+e^{x}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)*sech(2*x)*tanh(2*x),x)
```

```
[Out] -exp(3*x)/(1+exp(4*x))+2*sum(_R*ln(512*_R^3+exp(x)),_R=RootOf(4096*_Z^4+1))
```

maxima [A] time = 0.42, size = 90, normalized size = 0.80

$$\frac{1}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2e^{x}\right)\right)+\frac{1}{4}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2e^{x}\right)\right)-\frac{1}{8}\sqrt{2}\log\left(\sqrt{2}e^{x}+e^{(2x)}+1\right)+\frac{1}{8}\sqrt{2}\log$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sech(2*x)*tanh(2*x),x, algorithm="maxima")
```

```
[Out] 1/4*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*e^x)) + 1/4*sqrt(2)*arctan(-1/2
*sqrt(2)*(sqrt(2) - 2*e^x)) - 1/8*sqrt(2)*log(sqrt(2)*e^x + e^(2*x) + 1) +
1/8*sqrt(2)*log(-sqrt(2)*e^x + e^(2*x) + 1) - e^(3*x)/(e^(4*x) + 1)
```

mupad [B] time = 0.28, size = 91, normalized size = 0.81

$$-\frac{e^{3x}}{e^{4x}+1}+\sqrt{2}\ln\left(1+\sqrt{2}e^{x}\left(-\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)\right)\left(\frac{1}{8}+\frac{1}{8}\mathrm{i}\right)+\sqrt{2}\ln\left(1+\sqrt{2}e^{x}\left(-\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)\right)\left(\frac{1}{8}-\frac{1}{8}\mathrm{i}\right)+\sqrt{2}\ln\left(1+\sqrt{2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((tanh(2*x)*exp(x))/cosh(2*x),x)
```

```
[Out] 2^(1/2)*log(1 - 2^(1/2)*exp(x)*(1/2 + 1i/2))*(1/8 + 1i/8) + 2^(1/2)*log(1 -
 2^(1/2)*exp(x)*(1/2 - 1i/2))*(1/8 - 1i/8) - 2^(1/2)*log(2^(1/2)*exp(x)*(1/
2 - 1i/2) + 1)*(1/8 - 1i/8) - 2^(1/2)*log(2^(1/2)*exp(x)*(1/2 + 1i/2) + 1)*
(1/8 + 1i/8) - exp(3*x)/(exp(4*x) + 1)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int e^{x} \tanh(2x) \operatorname{sech}(2x)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sech(2*x)*tanh(2*x),x)
```

```
[Out] Integral(exp(x)*tanh(2*x)*sech(2*x), x)
```

3.938 $\int e^x \operatorname{sech}^2(2x)\tanh(2x)\,dx$

Optimal. Leaf size=129

$$-\frac{e^x}{4\left(e^{4x}+1\right)}-\frac{e^{5x}}{\left(e^{4x}+1\right)^2}-\frac{\log\left(-\sqrt{2}e^x+e^{2x}+1\right)}{16\sqrt{2}}+\frac{\log\left(\sqrt{2}e^x+e^{2x}+1\right)}{16\sqrt{2}}-\frac{\tan^{-1}\left(1-\sqrt{2}e^x\right)}{8\sqrt{2}}+\frac{\tan^{-1}\left(\sqrt{2}e^x+1\right)}{8\sqrt{2}}$$

```
[Out] -exp(5*x)/(1+exp(4*x))^2-1/4*exp(x)/(1+exp(4*x))+1/16*arctan(-1+exp(x)*2^(1
/2))*2^(1/2)+1/16*arctan(1+exp(x)*2^(1/2))*2^(1/2)-1/32*ln(1+exp(2*x)-exp(x
)*2^(1/2))*2^(1/2)+1/32*ln(1+exp(2*x)+exp(x)*2^(1/2))*2^(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.714, Rules used = {2282, 12, 457, 288, 211, 1165, 628, 1162, 617, 204}

$$-\frac{e^x}{4\left(e^{4x}+1\right)}-\frac{e^{5x}}{\left(e^{4x}+1\right)^2}-\frac{\log\left(-\sqrt{2}e^x+e^{2x}+1\right)}{16\sqrt{2}}+\frac{\log\left(\sqrt{2}e^x+e^{2x}+1\right)}{16\sqrt{2}}-\frac{\tan^{-1}\left(1-\sqrt{2}e^x\right)}{8\sqrt{2}}+\frac{\tan^{-1}\left(\sqrt{2}e^x+1\right)}{8\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^x*Sech[2*x]^2*Tanh[2*x],x]
```

```
[Out] -(E^(5*x)/(1 + E^(4*x))^2) - E^x/(4*(1 + E^(4*x))) - ArcTan[1 - Sqrt[2]*E^x
]/(8*Sqrt[2]) + ArcTan[1 + Sqrt[2]*E^x]/(8*Sqrt[2]) - Log[1 - Sqrt[2]*E^x +
 E^(2*x)]/(16*Sqrt[2]) + Log[1 + Sqrt[2]*E^x + E^(2*x)]/(16*Sqrt[2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
```

```
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
 + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
 n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
```

```
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$\begin{aligned}
\int e^x \operatorname{sech}^2(2x)\tanh(2x)\,dx &= \operatorname{Subst}\left(\int \frac{4x^4\left(-1+x^4\right)}{\left(1+x^4\right)^3}\,dx, x, e^x\right)\\
&= 4\operatorname{Subst}\left(\int \frac{x^4\left(-1+x^4\right)}{\left(1+x^4\right)^3}\,dx, x, e^x\right)\\
&= -\frac{e^{5x}}{\left(1+e^{4x}\right)^2}+\operatorname{Subst}\left(\int \frac{x^4}{\left(1+x^4\right)^2}\,dx, x, e^x\right)\\
&= -\frac{e^{5x}}{\left(1+e^{4x}\right)^2}-\frac{e^x}{4\left(1+e^{4x}\right)}+\frac{1}{4}\operatorname{Subst}\left(\int \frac{1}{1+x^4}\,dx, x, e^x\right)\\
&= -\frac{e^{5x}}{\left(1+e^{4x}\right)^2}-\frac{e^x}{4\left(1+e^{4x}\right)}+\frac{1}{8}\operatorname{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, e^x\right)+\frac{1}{8}\operatorname{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx\right.\\
&= -\frac{e^{5x}}{\left(1+e^{4x}\right)^2}-\frac{e^x}{4\left(1+e^{4x}\right)}+\frac{1}{16}\operatorname{Subst}\left(\int \frac{1}{1-\sqrt{2}\,x+x^2}\,dx, x, e^x\right)+\frac{1}{16}\operatorname{Subst}\left(\int\right.\\
&= -\frac{e^{5x}}{\left(1+e^{4x}\right)^2}-\frac{e^x}{4\left(1+e^{4x}\right)}-\frac{\log\left(1-\sqrt{2}\,e^x+e^{2x}\right)}{16\sqrt{2}}+\frac{\log\left(1+\sqrt{2}\,e^x+e^{2x}\right)}{16\sqrt{2}}+\frac{\text{Sub}}{}\\
&= -\frac{e^{5x}}{\left(1+e^{4x}\right)^2}-\frac{e^x}{4\left(1+e^{4x}\right)}-\frac{\tan^{-1}\left(1-\sqrt{2}\,e^x\right)}{8\sqrt{2}}+\frac{\tan^{-1}\left(1+\sqrt{2}\,e^x\right)}{8\sqrt{2}}-\frac{\log\left(1-\sqrt{2}\right.}{16\sqrt{}}
\end{aligned}$$

Mathematica [A] time = 0.13, size = 120, normalized size = 0.93

$$\frac{1}{32}\left(-\frac{40e^x}{e^{4x}+1}+\frac{32e^x}{\left(e^{4x}+1\right)^2}-\sqrt{2}\log\left(-\sqrt{2}e^x+e^{2x}+1\right)+\sqrt{2}\log\left(\sqrt{2}e^x+e^{2x}+1\right)-2\sqrt{2}\tan^{-1}\left(1-\sqrt{2}e^x\right)+2\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x*Sech[2*x]^2*Tanh[2*x],x]
```

```
[Out] ((32*E^x)/(1 + E^(4*x))^2 - (40*E^x)/(1 + E^(4*x)) - 2*Sqrt[2]*ArcTan[1 - S
qrt[2]*E^x] + 2*Sqrt[2]*ArcTan[1 + Sqrt[2]*E^x] - Sqrt[2]*Log[1 - Sqrt[2]*E
^x + E^(2*x)] + Sqrt[2]*Log[1 + Sqrt[2]*E^x + E^(2*x)])/32
```

fricas [B] time = 0.51, size = 210, normalized size = 1.63

$$-\frac{4\left(\sqrt{2}e^{(8x)}+2\sqrt{2}e^{(4x)}+\sqrt{2}\right)\arctan\left(-\sqrt{2}e^x+\sqrt{2}\sqrt{\sqrt{2}e^x+e^{(2x)}+1}-1\right)+4\left(\sqrt{2}e^{(8x)}+2\sqrt{2}e^{(4x)}+\sqrt{2}\right)a}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sech(2*x)^2*tanh(2*x),x, algorithm="fricas")
```

```
[Out] -1/32*(4*(sqrt(2)*e^(8*x) + 2*sqrt(2)*e^(4*x) + sqrt(2))*arctan(-sqrt(2)*e^
x + sqrt(2)*sqrt(sqrt(2)*e^x + e^(2*x) + 1) - 1) + 4*(sqrt(2)*e^(8*x) + 2*s
qrt(2)*e^(4*x) + sqrt(2))*arctan(-sqrt(2)*e^x + 1/2*sqrt(2)*sqrt(-4*sqrt(2)
*e^x + 4*e^(2*x) + 4) + 1) - (sqrt(2)*e^(8*x) + 2*sqrt(2)*e^(4*x) + sqrt(2)
)*log(4*sqrt(2)*e^x + 4*e^(2*x) + 4) + (sqrt(2)*e^(8*x) + 2*sqrt(2)*e^(4*x)
 + sqrt(2))*log(-4*sqrt(2)*e^x + 4*e^(2*x) + 4) + 40*e^(5*x) + 8*e^x)/(e^(8
*x) + 2*e^(4*x) + 1)
```

giac [A] time = 0.14, size = 95, normalized size = 0.74

$$\frac{1}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2e^x\right)\right)+\frac{1}{16}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2e^x\right)\right)+\frac{1}{32}\sqrt{2}\log\left(\sqrt{2}e^x+e^{(2x)}+1\right)-\frac{1}{32}\sqrt{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sech(2*x)^2*tanh(2*x),x, algorithm="giac")
```

```
[Out] 1/16*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*e^x)) + 1/16*sqrt(2)*arctan(-1
/2*sqrt(2)*(sqrt(2) - 2*e^x)) + 1/32*sqrt(2)*log(sqrt(2)*e^x + e^(2*x) + 1)
 - 1/32*sqrt(2)*log(-sqrt(2)*e^x + e^(2*x) + 1) - 1/4*(5*e^(5*x) + e^x)/(e^
(4*x) + 1)^2
```

maple [C] time = 0.35, size = 44, normalized size = 0.34

$$-\frac{e^{x}\left(5\,e^{4x}+1\right)}{4\left(1+e^{4x}\right)^{2}}+4\left(\sum_{\_R=\mathrm{RootOf}\left(16777216\_Z^{4}+1\right)}\_R\ln\left(e^{x}+64\_R\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)*sech(2*x)^2*tanh(2*x),x)
```

```
[Out] -1/4*exp(x)*(5*exp(4*x)+1)/(1+exp(4*x))^2+4*sum(_R*ln(exp(x)+64*_R),_R=Root
Of(16777216*_Z^4+1))
```

maxima [A] time = 0.42, size = 101, normalized size = 0.78

$$\frac{1}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\,e^{x}\right)\right)+\frac{1}{16}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\,e^{x}\right)\right)+\frac{1}{32}\sqrt{2}\log\left(\sqrt{2}\,e^{x}+e^{(2\,x)}+1\right)-\frac{1}{32}\sqrt{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sech(2*x)^2*tanh(2*x),x, algorithm="maxima")
```

```
[Out] 1/16*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*e^x)) + 1/16*sqrt(2)*arctan(-1
/2*sqrt(2)*(sqrt(2) - 2*e^x)) + 1/32*sqrt(2)*log(sqrt(2)*e^x + e^(2*x) + 1)
 - 1/32*sqrt(2)*log(-sqrt(2)*e^x + e^(2*x) + 1) - 1/4*(5*e^(5*x) + e^x)/(e^
(8*x) + 2*e^(4*x) + 1)
```

mupad [B] time = 2.03, size = 122, normalized size = 0.95

$$-\frac{\frac{e^{5\,x}}{2}-\frac{e^{x}}{2}}{2\,e^{4\,x}+e^{8\,x}+1}-\frac{3\,e^{x}}{4\,\left(e^{4\,x}+1\right)}+\sqrt{2}\,\ln\left(-\frac{e^{x}}{4}+\sqrt{2}\,\left(-\frac{1}{8}-\frac{1}{8}i\right)\right)\,\left(\frac{1}{32}+\frac{1}{32}i\right)+\sqrt{2}\,\ln\left(-\frac{e^{x}}{4}+\sqrt{2}\,\left(-\frac{1}{8}+\frac{1}{8}i\right)\right)\,\left(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((tanh(2*x)*exp(x))/cosh(2*x)^2,x)
```

```
[Out] 2^(1/2)*log(- exp(x)/4 - 2^(1/2)*(1/8 + 1i/8))*(1/32 + 1i/32) - (3*exp(x))/
(4*(exp(4*x) + 1)) - (exp(5*x)/2 - exp(x)/2)/(2*exp(4*x) + exp(8*x) + 1) +
2^(1/2)*log(- exp(x)/4 - 2^(1/2)*(1/8 - 1i/8))*(1/32 - 1i/32) - 2^(1/2)*log
(2^(1/2)*(1/8 - 1i/8) - exp(x)/4)*(1/32 - 1i/32) - 2^(1/2)*log(2^(1/2)*(1/8
 + 1i/8) - exp(x)/4)*(1/32 + 1i/32)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int e^{x}\tanh\left(2x\right)\operatorname{sech}^{2}\left(2x\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sech(2*x)**2*tanh(2*x),x)
[Out] Integral(exp(x)*tanh(2*x)*sech(2*x)**2, x)
```

3.939 $\int e^x \mathbf{sech}(2x)\tanh^2(2x)\,dx$

Optimal. Leaf size=130

$$-\frac{3e^{3x}}{4\left(e^{4x}+1\right)}+\frac{e^{3x}}{\left(e^{4x}+1\right)^2}+\frac{5\log\left(-\sqrt{2}\,e^x+e^{2x}+1\right)}{16\sqrt{2}}-\frac{5\log\left(\sqrt{2}\,e^x+e^{2x}+1\right)}{16\sqrt{2}}-\frac{5\tan^{-1}\left(1-\sqrt{2}\,e^x\right)}{8\sqrt{2}}+\frac{5\tan^{-1}\left(\sqrt{2}\,e\right.}{8\sqrt{2}}$$

```
[Out] exp(3*x)/(1+exp(4*x))^2-3/4*exp(3*x)/(1+exp(4*x))+5/16*arctan(-1+exp(x)*2^(
1/2))*2^(1/2)+5/16*arctan(1+exp(x)*2^(1/2))*2^(1/2)+5/32*ln(1+exp(2*x)-exp(
x)*2^(1/2))*2^(1/2)-5/32*ln(1+exp(2*x)+exp(x)*2^(1/2))*2^(1/2)
```

Rubi [A] time = 0.11, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.714, Rules used = {2282, 12, 463, 457, 297, 1162, 617, 204, 1165, 628}

$$-\frac{3e^{3x}}{4\left(e^{4x}+1\right)}+\frac{e^{3x}}{\left(e^{4x}+1\right)^2}+\frac{5\log\left(-\sqrt{2}\,e^x+e^{2x}+1\right)}{16\sqrt{2}}-\frac{5\log\left(\sqrt{2}\,e^x+e^{2x}+1\right)}{16\sqrt{2}}-\frac{5\tan^{-1}\left(1-\sqrt{2}\,e^x\right)}{8\sqrt{2}}+\frac{5\tan^{-1}\left(\sqrt{2}\,e\right.}{8\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^x*Sech[2*x]*Tanh[2*x]^2,x]
```

```
[Out] E^(3*x)/(1 + E^(4*x))^2 - (3*E^(3*x))/(4*(1 + E^(4*x))) - (5*ArcTan[1 - Sqr
t[2]*E^x])/(8*Sqrt[2]) + (5*ArcTan[1 + Sqrt[2]*E^x])/(8*Sqrt[2]) + (5*Log[1
 - Sqrt[2]*E^x + E^(2*x)])/(16*Sqrt[2]) - (5*Log[1 + Sqrt[2]*E^x + E^(2*x)]
)/(16*Sqrt[2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
```

```
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
 + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
 n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

Rule 463

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^2, x_Symbol] :> -Simp[((b*c - a*d)^2*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/
(a*b^2*e*n*(p + 1)), x] + Dist[1/(a*b^2*n*(p + 1)), Int[(e*x)^m*(a + b*x^n)
^(p + 1)*Simp[(b*c - a*d)^2*(m + 1) + b^2*c^2*n*(p + 1) + a*b*d^2*n*(p + 1)
*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] &&
 IGtQ[n, 0] && LtQ[p, -1]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
```

```
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$\int e^x \operatorname{sech}(2x)\tanh^2(2x)\,dx = \operatorname{Subst}\left(\int \frac{2x^2\left(1-x^4\right)^2}{\left(1+x^4\right)^3}\,dx, x, e^x\right)$$

$$= 2\operatorname{Subst}\left(\int \frac{x^2\left(1-x^4\right)^2}{\left(1+x^4\right)^3}\,dx, x, e^x\right)$$

$$= \frac{e^{3x}}{\left(1+e^{4x}\right)^2} - \frac{1}{4}\operatorname{Subst}\left(\int \frac{x^2\left(4-8x^4\right)}{\left(1+x^4\right)^2}\,dx, x, e^x\right)$$

$$= \frac{e^{3x}}{\left(1+e^{4x}\right)^2} - \frac{3e^{3x}}{4\left(1+e^{4x}\right)} + \frac{5}{4}\operatorname{Subst}\left(\int \frac{x^2}{1+x^4}\,dx, x, e^x\right)$$

$$= \frac{e^{3x}}{\left(1+e^{4x}\right)^2} - \frac{3e^{3x}}{4\left(1+e^{4x}\right)} - \frac{5}{8}\operatorname{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, e^x\right) + \frac{5}{8}\operatorname{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx,\right.$$

$$= \frac{e^{3x}}{\left(1+e^{4x}\right)^2} - \frac{3e^{3x}}{4\left(1+e^{4x}\right)} + \frac{5}{16}\operatorname{Subst}\left(\int \frac{1}{1-\sqrt{2}\,x+x^2}\,dx, x, e^x\right) + \frac{5}{16}\operatorname{Subst}\left(\int\right.$$

$$= \frac{e^{3x}}{\left(1+e^{4x}\right)^2} - \frac{3e^{3x}}{4\left(1+e^{4x}\right)} + \frac{5\log\left(1-\sqrt{2}\,e^x+e^{2x}\right)}{16\sqrt{2}} - \frac{5\log\left(1+\sqrt{2}\,e^x+e^{2x}\right)}{16\sqrt{2}} +$$

$$= \frac{e^{3x}}{\left(1+e^{4x}\right)^2} - \frac{3e^{3x}}{4\left(1+e^{4x}\right)} - \frac{5\tan^{-1}\left(1-\sqrt{2}\,e^x\right)}{8\sqrt{2}} + \frac{5\tan^{-1}\left(1+\sqrt{2}\,e^x\right)}{8\sqrt{2}} + \frac{5\log\left(1-\right.}{}$$

Mathematica [C] time = 0.06, size = 58, normalized size = 0.45

$$\frac{e^{3x}-3e^{7x}}{4\left(e^{4x}+1\right)^2}-\frac{5}{16}\text{RootSum}\left[\#1^4+1\&,\frac{x-\log\left(e^x-\#1\right)}{\#1}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x*Sech[2*x]*Tanh[2*x]^2,x]
```

```
[Out] (E^(3*x) - 3*E^(7*x))/(4*(1 + E^(4*x))^2) - (5*RootSum[1 + #1^4 & , (x - Lo
g[E^x - #1])/#1 & ])/16
```

fricas [B] time = 0.51, size = 213, normalized size = 1.64

$$-\frac{20\left(\sqrt{2}e^{(8x)}+2\sqrt{2}e^{(4x)}+\sqrt{2}\right)\arctan\left(-\sqrt{2}e^x+\sqrt{2}\sqrt{\sqrt{2}e^x+e^{(2x)}+1}-1\right)+20\left(\sqrt{2}e^{(8x)}+2\sqrt{2}e^{(4x)}+\sqrt{2}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sech(2*x)*tanh(2*x)^2,x, algorithm="fricas")
```

```
[Out] -1/32*(20*(sqrt(2)*e^(8*x) + 2*sqrt(2)*e^(4*x) + sqrt(2))*arctan(-sqrt(2)*e
^x + sqrt(2)*sqrt(sqrt(2)*e^x + e^(2*x) + 1) - 1) + 20*(sqrt(2)*e^(8*x) + 2
*sqrt(2)*e^(4*x) + sqrt(2))*arctan(-sqrt(2)*e^x + 1/2*sqrt(2)*sqrt(-4*sqrt(
2)*e^x + 4*e^(2*x) + 4) + 1) + 5*(sqrt(2)*e^(8*x) + 2*sqrt(2)*e^(4*x) + sqr
t(2))*log(4*sqrt(2)*e^x + 4*e^(2*x) + 4) - 5*(sqrt(2)*e^(8*x) + 2*sqrt(2)*e
^(4*x) + sqrt(2))*log(-4*sqrt(2)*e^x + 4*e^(2*x) + 4) + 24*e^(7*x) - 8*e^(3
*x))/(e^(8*x) + 2*e^(4*x) + 1)
```

giac [A] time = 0.12, size = 99, normalized size = 0.76

$$\frac{5}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2e^x\right)\right)+\frac{5}{16}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2e^x\right)\right)-\frac{5}{32}\sqrt{2}\log\left(\sqrt{2}e^x+e^{(2x)}+1\right)+\frac{5}{32}\sqrt{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sech(2*x)*tanh(2*x)^2,x, algorithm="giac")
```

```
[Out] 5/16*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*e^x)) + 5/16*sqrt(2)*arctan(-1
/2*sqrt(2)*(sqrt(2) - 2*e^x)) - 5/32*sqrt(2)*log(sqrt(2)*e^x + e^(2*x) + 1)
 + 5/32*sqrt(2)*log(-sqrt(2)*e^x + e^(2*x) + 1) - 1/4*(3*e^(7*x) - e^(3*x))
/(e^(4*x) + 1)^2
```

maple [C] time = 0.35, size = 48, normalized size = 0.37

$$-\frac{e^{3x}\left(3\,e^{4x}-1\right)}{4\left(1+e^{4x}\right)^{2}}+2\left(\sum_{\_R=\mathrm{RootOf}\left(1048576\_Z^{4}+625\right)}\_R\ln\left(e^{x}+\frac{32768\_R^{3}}{125}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)*sech(2*x)*tanh(2*x)^2,x)
```

```
[Out] -1/4*exp(3*x)*(3*exp(4*x)-1)/(1+exp(4*x))^2+2*sum(_R*ln(exp(x)+32768/125*_R
^3),_R=RootOf(1048576*_Z^4+625))
```

maxima [A] time = 0.63, size = 105, normalized size = 0.81

$$\frac{5}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\,e^{x}\right)\right)+\frac{5}{16}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\,e^{x}\right)\right)-\frac{5}{32}\sqrt{2}\log\left(\sqrt{2}\,e^{x}+e^{(2x)}+1\right)+\frac{5}{32}\sqrt{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sech(2*x)*tanh(2*x)^2,x, algorithm="maxima")
```

```
[Out] 5/16*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*e^x)) + 5/16*sqrt(2)*arctan(-1
/2*sqrt(2)*(sqrt(2) - 2*e^x)) - 5/32*sqrt(2)*log(sqrt(2)*e^x + e^(2*x) + 1)
 + 5/32*sqrt(2)*log(-sqrt(2)*e^x + e^(2*x) + 1) - 1/4*(3*e^(7*x) - e^(3*x))
/(e^(8*x) + 2*e^(4*x) + 1)
```

mupad [B] time = 2.09, size = 112, normalized size = 0.86

$$\frac{e^{3\,x}}{2\,e^{4\,x}+e^{8\,x}+1}-\frac{3\,e^{3\,x}}{4\,\left(e^{4\,x}+1\right)}+\sqrt{2}\,\ln\left(\frac{25}{16}+\sqrt{2}\,e^{x}\,\left(-\frac{25}{32}-\frac{25}{32}\mathrm{i}\right)\right)\,\left(\frac{5}{32}+\frac{5}{32}\mathrm{i}\right)+\sqrt{2}\,\ln\left(\frac{25}{16}+\sqrt{2}\,e^{x}\,\left(-\frac{25}{32}+\frac{25}{32}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((tanh(2*x)^2*exp(x))/cosh(2*x),x)
```

```
[Out] 2^(1/2)*log(25/16 - 2^(1/2)*exp(x)*(25/32 + 25i/32))*(5/32 + 5i/32) + 2^(1/
2)*log(25/16 - 2^(1/2)*exp(x)*(25/32 - 25i/32))*(5/32 - 5i/32) - 2^(1/2)*lo
g(2^(1/2)*exp(x)*(25/32 - 25i/32) + 25/16)*(5/32 - 5i/32) - 2^(1/2)*log(2^(
1/2)*exp(x)*(25/32 + 25i/32) + 25/16)*(5/32 + 5i/32) + exp(3*x)/(2*exp(4*x)
 + exp(8*x) + 1) - (3*exp(3*x))/(4*(exp(4*x) + 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int e^{x}\tanh^{2}\left(2x\right)\operatorname{sech}\left(2x\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sech(2*x)*tanh(2*x)**2,x)
```

```
[Out] Integral(exp(x)*tanh(2*x)**2*sech(2*x), x)
```

3.940 $\int e^x \operatorname{sech}^2(2x) \tanh^2(2x) \, dx$

Optimal. Leaf size=149

$$-\frac{3e^x}{8\left(e^{4x}+1\right)}-\frac{5e^{5x}}{6\left(e^{4x}+1\right)^2}+\frac{4e^{5x}}{3\left(e^{4x}+1\right)^3}-\frac{3\log\left(-\sqrt{2}\,e^x+e^{2x}+1\right)}{32\sqrt{2}}+\frac{3\log\left(\sqrt{2}\,e^x+e^{2x}+1\right)}{32\sqrt{2}}-\frac{3\tan^{-1}\left(1-\sqrt{2}\,e^x\right)}{16\sqrt{2}}$$

```
[Out] 4/3*exp(5*x)/(1+exp(4*x))^3-5/6*exp(5*x)/(1+exp(4*x))^2-3/8*exp(x)/(1+exp(4
*x))+3/32*arctan(-1+exp(x)*2^(1/2))*2^(1/2)+3/32*arctan(1+exp(x)*2^(1/2))*2
^(1/2)-3/64*ln(1+exp(2*x)-exp(x)*2^(1/2))*2^(1/2)+3/64*ln(1+exp(2*x)+exp(x)
*2^(1/2))*2^(1/2)
```

Rubi [A] time = 0.13, antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 11, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.688, Rules used = {2282, 12, 463, 457, 288, 211, 1165, 628, 1162, 617, 204}

$$-\frac{3e^x}{8\left(e^{4x}+1\right)}-\frac{5e^{5x}}{6\left(e^{4x}+1\right)^2}+\frac{4e^{5x}}{3\left(e^{4x}+1\right)^3}-\frac{3\log\left(-\sqrt{2}\,e^x+e^{2x}+1\right)}{32\sqrt{2}}+\frac{3\log\left(\sqrt{2}\,e^x+e^{2x}+1\right)}{32\sqrt{2}}-\frac{3\tan^{-1}\left(1-\sqrt{2}\,e^x\right)}{16\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^x*Sech[2*x]^2*Tanh[2*x]^2,x]
```

```
[Out] (4*E^(5*x))/(3*(1 + E^(4*x))^3) - (5*E^(5*x))/(6*(1 + E^(4*x))^2) - (3*E^x)
/(8*(1 + E^(4*x))) - (3*ArcTan[1 - Sqrt[2]*E^x])/(16*Sqrt[2]) + (3*ArcTan[1
 + Sqrt[2]*E^x])/(16*Sqrt[2]) - (3*Log[1 - Sqrt[2]*E^x + E^(2*x)])/(32*Sqrt
[2]) + (3*Log[1 + Sqrt[2]*E^x + E^(2*x)])/(32*Sqrt[2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
```

```
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
 + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
 n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

Rule 463

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^2, x_Symbol] :> -Simp[((b*c - a*d)^2*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/
(a*b^2*e*n*(p + 1)), x] + Dist[1/(a*b^2*n*(p + 1)), Int[(e*x)^m*(a + b*x^n)
^(p + 1)*Simp[(b*c - a*d)^2*(m + 1) + b^2*c^2*n*(p + 1) + a*b*d^2*n*(p + 1)
*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] &&
 IGtQ[n, 0] && LtQ[p, -1]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$\int e^x \operatorname{sech}^2(2x)\tanh^2(2x)\,dx = \text{Subst}\left(\int \frac{4x^4\left(1-x^4\right)^2}{\left(1+x^4\right)^4}\,dx, x, e^x\right)$$
$$= 4\,\text{Subst}\left(\int \frac{x^4\left(1-x^4\right)^2}{\left(1+x^4\right)^4}\,dx, x, e^x\right)$$
$$= \frac{4e^{5x}}{3\left(1+e^{4x}\right)^3} - \frac{1}{3}\text{Subst}\left(\int \frac{x^4\left(8-12x^4\right)}{\left(1+x^4\right)^3}\,dx, x, e^x\right)$$
$$= \frac{4e^{5x}}{3\left(1+e^{4x}\right)^3} - \frac{5e^{5x}}{6\left(1+e^{4x}\right)^2} + \frac{3}{2}\text{Subst}\left(\int \frac{x^4}{\left(1+x^4\right)^2}\,dx, x, e^x\right)$$
$$= \frac{4e^{5x}}{3\left(1+e^{4x}\right)^3} - \frac{5e^{5x}}{6\left(1+e^{4x}\right)^2} - \frac{3e^x}{8\left(1+e^{4x}\right)} + \frac{3}{8}\text{Subst}\left(\int \frac{1}{1+x^4}\,dx, x, e^x\right)$$
$$= \frac{4e^{5x}}{3\left(1+e^{4x}\right)^3} - \frac{5e^{5x}}{6\left(1+e^{4x}\right)^2} - \frac{3e^x}{8\left(1+e^{4x}\right)} + \frac{3}{16}\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, e^x\right) + \frac{3}{16}\text{Su}$$
$$= \frac{4e^{5x}}{3\left(1+e^{4x}\right)^3} - \frac{5e^{5x}}{6\left(1+e^{4x}\right)^2} - \frac{3e^x}{8\left(1+e^{4x}\right)} + \frac{3}{32}\text{Subst}\left(\int \frac{1}{1-\sqrt{2}\,x+x^2}\,dx, x, e^x\right) +$$
$$= \frac{4e^{5x}}{3\left(1+e^{4x}\right)^3} - \frac{5e^{5x}}{6\left(1+e^{4x}\right)^2} - \frac{3e^x}{8\left(1+e^{4x}\right)} - \frac{3\log\left(1-\sqrt{2}\,e^x+e^{2x}\right)}{32\sqrt{2}} + \frac{3\log\left(1+\sqrt{}\right.}{32\sqrt{}}$$
$$= \frac{4e^{5x}}{3\left(1+e^{4x}\right)^3} - \frac{5e^{5x}}{6\left(1+e^{4x}\right)^2} - \frac{3e^x}{8\left(1+e^{4x}\right)} - \frac{3\tan^{-1}\left(1-\sqrt{2}\,e^x\right)}{16\sqrt{2}} + \frac{3\tan^{-1}\left(1+\sqrt{2}\right.}{16\sqrt{2}}$$

Mathematica [C] time = 0.07, size = 64, normalized size = 0.43

$$\frac{1}{96}\left(-9\text{RootSum}\left[\#1^4+1\&, \frac{x-\log\left(e^x-\#1\right)}{\#1^3}\&\right] - \frac{4e^x\left(6e^{4x}+29e^{8x}+9\right)}{\left(e^{4x}+1\right)^3}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x*Sech[2*x]^2*Tanh[2*x]^2,x]
```

```
[Out] ((-4*E^x*(9 + 6*E^(4*x) + 29*E^(8*x)))/(1 + E^(4*x))^3 - 9*RootSum[1 + #1^4
 & , (x - Log[E^x - #1])/#1^3 & ])/96
```

fricas [B] time = 0.51, size = 259, normalized size = 1.74

$$-\frac{36\left(\sqrt{2}\,e^{(12\,x)}+3\,\sqrt{2}\,e^{(8\,x)}+3\,\sqrt{2}\,e^{(4\,x)}+\sqrt{2}\right)\arctan\left(-\sqrt{2}\,e^{x}+\sqrt{2}\,\sqrt{\sqrt{2}\,e^{x}+e^{(2\,x)}+1}-1\right)+36\left(\sqrt{2}\,e^{(12\,x)}+\cdots\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sech(2*x)^2*tanh(2*x)^2,x, algorithm="fricas")
```

```
[Out] -1/192*(36*(sqrt(2)*e^(12*x) + 3*sqrt(2)*e^(8*x) + 3*sqrt(2)*e^(4*x) + sqrt
(2))*arctan(-sqrt(2)*e^x + sqrt(2)*sqrt(sqrt(2)*e^x + e^(2*x) + 1) - 1) + 3
6*(sqrt(2)*e^(12*x) + 3*sqrt(2)*e^(8*x) + 3*sqrt(2)*e^(4*x) + sqrt(2))*arct
an(-sqrt(2)*e^x + 1/2*sqrt(2)*sqrt(-4*sqrt(2)*e^x + 4*e^(2*x) + 4) + 1) - 9
*(sqrt(2)*e^(12*x) + 3*sqrt(2)*e^(8*x) + 3*sqrt(2)*e^(4*x) + sqrt(2))*log(4
*sqrt(2)*e^x + 4*e^(2*x) + 4) + 9*(sqrt(2)*e^(12*x) + 3*sqrt(2)*e^(8*x) + 3
*sqrt(2)*e^(4*x) + sqrt(2))*log(-4*sqrt(2)*e^x + 4*e^(2*x) + 4) + 232*e^(9*
x) + 48*e^(5*x) + 72*e^x)/(e^(12*x) + 3*e^(8*x) + 3*e^(4*x) + 1)
```

giac [A] time = 0.12, size = 103, normalized size = 0.69

$$\frac{3}{32}\,\sqrt{2}\,\arctan\left(\frac{1}{2}\,\sqrt{2}\left(\sqrt{2}+2\,e^{x}\right)\right)+\frac{3}{32}\,\sqrt{2}\,\arctan\left(-\frac{1}{2}\,\sqrt{2}\left(\sqrt{2}-2\,e^{x}\right)\right)+\frac{3}{64}\,\sqrt{2}\,\log\left(\sqrt{2}\,e^{x}+e^{(2\,x)}+1\right)-\frac{3}{64}\,\sqrt{2}\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sech(2*x)^2*tanh(2*x)^2,x, algorithm="giac")
```

```
[Out] 3/32*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*e^x)) + 3/32*sqrt(2)*arctan(-1
/2*sqrt(2)*(sqrt(2) - 2*e^x)) + 3/64*sqrt(2)*log(sqrt(2)*e^x + e^(2*x) + 1)
 - 3/64*sqrt(2)*log(-sqrt(2)*e^x + e^(2*x) + 1) - 1/24*(29*e^(9*x) + 6*e^(5
*x) + 9*e^x)/(e^(4*x) + 1)^3
```

maple [C] time = 0.37, size = 50, normalized size = 0.34

$$-\frac{e^{x}\left(29\,e^{8x}+6\,e^{4x}+9\right)}{24\left(1+e^{4x}\right)^{3}}+4\left(\sum_{\_R=\mathrm{RootOf}\left(268435456\_Z^{4}+81\right)}\_R\ln\left(e^{x}+\frac{128\_R}{3}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)*sech(2*x)^2*tanh(2*x)^2,x)
```

```
[Out] -1/24*exp(x)*(29*exp(8*x)+6*exp(4*x)+9)/(1+exp(4*x))^3+4*sum(_R*ln(exp(x)+1
28/3*_R),_R=RootOf(268435456*_Z^4+81))
```

maxima [A] time = 0.41, size = 115, normalized size = 0.77

$$\frac{3}{32}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2e^{x}\right)\right)+\frac{3}{32}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2e^{x}\right)\right)+\frac{3}{64}\sqrt{2}\log\left(\sqrt{2}e^{x}+e^{(2x)}+1\right)-\frac{3}{64}\sqrt{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sech(2*x)^2*tanh(2*x)^2,x, algorithm="maxima")
```

```
[Out] 3/32*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*e^x)) + 3/32*sqrt(2)*arctan(-1
/2*sqrt(2)*(sqrt(2) - 2*e^x)) + 3/64*sqrt(2)*log(sqrt(2)*e^x + e^(2*x) + 1)
 - 3/64*sqrt(2)*log(-sqrt(2)*e^x + e^(2*x) + 1) - 1/24*(29*e^(9*x) + 6*e^(5
*x) + 9*e^x)/(e^(12*x) + 3*e^(8*x) + 3*e^(4*x) + 1)
```

mupad [B] time = 2.17, size = 154, normalized size = 1.03

$$\frac{5\,e^{x}}{6\left(2\,e^{4x}+e^{8x}+1\right)}-\frac{\frac{e^{9x}}{3}-\frac{2\,e^{5x}}{3}+\frac{e^{x}}{3}}{3\,e^{4x}+3\,e^{8x}+e^{12x}+1}-\frac{7\,e^{x}}{8\left(e^{4x}+1\right)}+\sqrt{2}\,\ln\left(-\frac{3\,e^{x}}{8}+\sqrt{2}\left(-\frac{3}{16}-\frac{3}{16}i\right)\right)\left(\frac{3}{64}+\frac{3}{64}i\right)+\sqrt{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((tanh(2*x)^2*exp(x))/cosh(2*x)^2,x)
```

```
[Out] 2^(1/2)*log(- (3*exp(x))/8 - 2^(1/2)*(3/16 + 3i/16))*(3/64 + 3i/64) - (exp(
9*x)/3 - (2*exp(5*x))/3 + exp(x)/3)/(3*exp(4*x) + 3*exp(8*x) + exp(12*x) +
1) - (7*exp(x))/(8*(exp(4*x) + 1)) + 2^(1/2)*log(- (3*exp(x))/8 - 2^(1/2)*(
3/16 - 3i/16))*(3/64 - 3i/64) - 2^(1/2)*log(2^(1/2)*(3/16 - 3i/16) - (3*exp
(x))/8)*(3/64 - 3i/64) - 2^(1/2)*log(2^(1/2)*(3/16 + 3i/16) - (3*exp(x))/8)
*(3/64 + 3i/64) + (5*exp(x))/(6*(2*exp(4*x) + exp(8*x) + 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int e^{x}\tanh^{2}(2x)\,\mathrm{sech}^{2}(2x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sech(2*x)**2*tanh(2*x)**2,x)
```

```
[Out] Integral(exp(x)*tanh(2*x)**2*sech(2*x)**2, x)
```

3.941 $\int e^x \coth(2x)\mathbf{csch}(2x)\,dx$

Optimal. Leaf size=34

$$\frac{e^{3x}}{1-e^{4x}}+\frac{1}{2}\tan^{-1}\left(e^x\right)-\frac{1}{2}\tanh^{-1}\left(e^x\right)$$

```
[Out] exp(3*x)/(1-exp(4*x))+1/2*arctan(exp(x))-1/2*arctanh(exp(x))
```

Rubi [A] time = 0.03, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {2282, 12, 457, 298, 203, 206}

$$\frac{e^{3x}}{1-e^{4x}}+\frac{1}{2}\tan^{-1}\left(e^x\right)-\frac{1}{2}\tanh^{-1}\left(e^x\right)$$

Antiderivative was successfully verified.

```
[In] Int[E^x*Coth[2*x]*Csch[2*x],x]
```

```
[Out] E^(3*x)/(1 - E^(4*x)) + ArcTan[E^x]/2 - ArcTanh[E^x]/2
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
 + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
 n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$\begin{aligned}
\int e^x \coth(2x)\text{csch}(2x)\,dx &= \text{Subst}\left(\int \frac{2x^2\left(1+x^4\right)}{\left(1-x^4\right)^2}\,dx, x, e^x\right)\\
&= 2\,\text{Subst}\left(\int \frac{x^2\left(1+x^4\right)}{\left(1-x^4\right)^2}\,dx, x, e^x\right)\\
&= \frac{e^{3x}}{1-e^{4x}} - \text{Subst}\left(\int \frac{x^2}{1-x^4}\,dx, x, e^x\right)\\
&= \frac{e^{3x}}{1-e^{4x}} - \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, e^x\right) + \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, e^x\right)\\
&= \frac{e^{3x}}{1-e^{4x}} + \frac{1}{2}\tan^{-1}\left(e^x\right) - \frac{1}{2}\tanh^{-1}\left(e^x\right)
\end{aligned}$$

Mathematica [A] time = 0.06, size = 31, normalized size = 0.91

$$\frac{1}{2}\left(-\frac{2e^{3x}}{e^{4x}-1} + \tan^{-1}\left(e^x\right) - \tanh^{-1}\left(e^x\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x*Coth[2*x]*Csch[2*x],x]
```

```
[Out] ((-2*E^(3*x))/(-1 + E^(4*x)) + ArcTan[E^x] - ArcTanh[E^x])/2
```

fricas [B] time = 0.53, size = 202, normalized size = 5.94

$$-\frac{4\,\cosh(x)^3+12\,\cosh(x)^2\sinh(x)+12\,\cosh(x)\sinh(x)^2+4\,\sinh(x)^3-2\left(\cosh(x)^4+4\,\cosh(x)^3\sinh(x)+6\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*coth(2*x)*csch(2*x),x, algorithm="fricas")
```

```
[Out] -1/4*(4*cosh(x)^3 + 12*cosh(x)^2*sinh(x) + 12*cosh(x)*sinh(x)^2 + 4*sinh(x)
^3 - 2*(cosh(x)^4 + 4*cosh(x)^3*sinh(x) + 6*cosh(x)^2*sinh(x)^2 + 4*cosh(x)
*sinh(x)^3 + sinh(x)^4 - 1)*arctan(cosh(x) + sinh(x)) + (cosh(x)^4 + 4*cosh
(x)^3*sinh(x) + 6*cosh(x)^2*sinh(x)^2 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 - 1
)*log(cosh(x) + sinh(x) + 1) - (cosh(x)^4 + 4*cosh(x)^3*sinh(x) + 6*cosh(x)
^2*sinh(x)^2 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 - 1)*log(cosh(x) + sinh(x) -
 1))/(cosh(x)^4 + 4*cosh(x)^3*sinh(x) + 6*cosh(x)^2*sinh(x)^2 + 4*cosh(x)*s
inh(x)^3 + sinh(x)^4 - 1)
```

giac [A] time = 0.13, size = 35, normalized size = 1.03

$$-\frac{e^{(3\,x)}}{e^{(4\,x)}-1}+\frac{1}{2}\,\arctan\left(e^{x}\right)-\frac{1}{4}\,\log\left(e^{x}+1\right)+\frac{1}{4}\,\log\left(|e^{x}-1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*coth(2*x)*csch(2*x),x, algorithm="giac")
```

```
[Out] -e^(3*x)/(e^(4*x) - 1) + 1/2*arctan(e^x) - 1/4*log(e^x + 1) + 1/4*log(abs(e
^x - 1))
```

maple [C] time = 0.33, size = 48, normalized size = 1.41

$$-\frac{e^{3x}}{e^{4x}-1}-\frac{\ln\left(e^{x}+1\right)}{4}+\frac{\ln\left(e^{x}-1\right)}{4}+\frac{i\ln\left(e^{x}+i\right)}{4}-\frac{i\ln\left(e^{x}-i\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)*coth(2*x)*csch(2*x),x)
```

```
[Out] -exp(3*x)/(exp(4*x)-1)-1/4*ln(exp(x)+1)+1/4*ln(exp(x)-1)+1/4*I*ln(exp(x)+I)
-1/4*I*ln(exp(x)-I)
```

maxima [A] time = 0.41, size = 34, normalized size = 1.00

$$-\frac{e^{(3\,x)}}{e^{(4\,x)}-1}+\frac{1}{2}\,\arctan\left(e^{x}\right)-\frac{1}{4}\,\log\left(e^{x}+1\right)+\frac{1}{4}\,\log\left(e^{x}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*coth(2*x)*csch(2*x),x, algorithm="maxima")
```

```
[Out] -e^(3*x)/(e^(4*x) - 1) + 1/2*arctan(e^x) - 1/4*log(e^x + 1) + 1/4*log(e^x -
 1)
```

mupad [B] time = 0.20, size = 38, normalized size = 1.12

$$\frac{\ln\left(e^x-1\right)}{4}-\frac{\mathrm{atan}\left(e^{-x}\right)}{2}-\frac{\ln\left(-e^x-1\right)}{4}-\frac{e^{3x}}{e^{4x}-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((coth(2*x)*exp(x))/sinh(2*x),x)
```

```
[Out] log(exp(x) - 1)/4 - atan(exp(-x))/2 - log(- exp(x) - 1)/4 - exp(3*x)/(exp(4
*x) - 1)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int e^{x}\coth\left(2x\right)\operatorname{csch}\left(2x\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*coth(2*x)*csch(2*x),x)
```

```
[Out] Integral(exp(x)*coth(2*x)*csch(2*x), x)
```

3.942 $\int e^x \coth(2x)\mathbf{csch}^2(2x)\,dx$

Optimal. Leaf size=53

$$\frac{e^x}{4\left(1-e^{4x}\right)}-\frac{e^{5x}}{\left(1-e^{4x}\right)^2}-\frac{1}{8}\tan^{-1}\left(e^x\right)-\frac{1}{8}\tanh^{-1}\left(e^x\right)$$

```
[Out] -exp(5*x)/(1-exp(4*x))^2+1/4*exp(x)/(1-exp(4*x))-1/8*arctan(exp(x))-1/8*arc
tanh(exp(x))
```

Rubi [A] time = 0.04, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {2282, 12, 457, 288, 212, 206, 203}

$$\frac{e^x}{4\left(1-e^{4x}\right)}-\frac{e^{5x}}{\left(1-e^{4x}\right)^2}-\frac{1}{8}\tan^{-1}\left(e^x\right)-\frac{1}{8}\tanh^{-1}\left(e^x\right)$$

Antiderivative was successfully verified.

```
[In] Int[E^x*Coth[2*x]*Csch[2*x]^2,x]
```

```
[Out] -(E^(5*x)/(1 - E^(4*x))^2) + E^x/(4*(1 - E^(4*x))) - ArcTan[E^x]/8 - ArcTan
h[E^x]/8
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
```

```
x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
 + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
 n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$\int e^x \coth(2x)\operatorname{csch}^2(2x)\,dx = \text{Subst}\left(\int \frac{4x^4(-1-x^4)}{(1-x^4)^3}\,dx, x, e^x\right)$$
$$= 4\,\text{Subst}\left(\int \frac{x^4(-1-x^4)}{(1-x^4)^3}\,dx, x, e^x\right)$$
$$= -\frac{e^{5x}}{(1-e^{4x})^2} + \text{Subst}\left(\int \frac{x^4}{(1-x^4)^2}\,dx, x, e^x\right)$$
$$= -\frac{e^{5x}}{(1-e^{4x})^2} + \frac{e^x}{4(1-e^{4x})} - \frac{1}{4}\text{Subst}\left(\int \frac{1}{1-x^4}\,dx, x, e^x\right)$$
$$= -\frac{e^{5x}}{(1-e^{4x})^2} + \frac{e^x}{4(1-e^{4x})} - \frac{1}{8}\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, e^x\right) - \frac{1}{8}\text{Subst}\left(\int \frac{1}{1+x^2}\,dx\right.$$
$$= -\frac{e^{5x}}{(1-e^{4x})^2} + \frac{e^x}{4(1-e^{4x})} - \frac{1}{8}\tan^{-1}(e^x) - \frac{1}{8}\tanh^{-1}(e^x)$$

Mathematica [A] time = 0.10, size = 54, normalized size = 1.02

$$-\frac{-2e^x + 10e^{5x} + (e^{4x}-1)^2\tan^{-1}(e^x) + (e^{4x}-1)^2\tanh^{-1}(e^x)}{8(e^{4x}-1)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x*Coth[2*x]*Csch[2*x]^2,x]
```

```
[Out] -1/8*(-2*E^x + 10*E^(5*x) + (-1 + E^(4*x))^2*ArcTan[E^x] + (-1 + E^(4*x))^2
*ArcTanh[E^x])/(-1 + E^(4*x))^2
```

fricas [B] time = 0.54, size = 522, normalized size = 9.85

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*coth(2*x)*csch(2*x)^2,x, algorithm="fricas")
```

```
[Out] -1/16*(20*cosh(x)^5 + 200*cosh(x)^3*sinh(x)^2 + 200*cosh(x)^2*sinh(x)^3 + 1
00*cosh(x)*sinh(x)^4 + 20*sinh(x)^5 + 2*(cosh(x)^8 + 56*cosh(x)^3*sinh(x)^5
```

```
 + 28*cosh(x)^2*sinh(x)^6 + 8*cosh(x)*sinh(x)^7 + sinh(x)^8 + 2*(35*cosh(x)
^4 - 1)*sinh(x)^4 - 2*cosh(x)^4 + 8*(7*cosh(x)^5 - cosh(x))*sinh(x)^3 + 4*(
7*cosh(x)^6 - 3*cosh(x)^2)*sinh(x)^2 + 8*(cosh(x)^7 - cosh(x)^3)*sinh(x) +
1)*arctan(cosh(x) + sinh(x)) + (cosh(x)^8 + 56*cosh(x)^3*sinh(x)^5 + 28*cos
h(x)^2*sinh(x)^6 + 8*cosh(x)*sinh(x)^7 + sinh(x)^8 + 2*(35*cosh(x)^4 - 1)*s
inh(x)^4 - 2*cosh(x)^4 + 8*(7*cosh(x)^5 - cosh(x))*sinh(x)^3 + 4*(7*cosh(x)
^6 - 3*cosh(x)^2)*sinh(x)^2 + 8*(cosh(x)^7 - cosh(x)^3)*sinh(x) + 1)*log(co
sh(x) + sinh(x) + 1) - (cosh(x)^8 + 56*cosh(x)^3*sinh(x)^5 + 28*cosh(x)^2*s
inh(x)^6 + 8*cosh(x)*sinh(x)^7 + sinh(x)^8 + 2*(35*cosh(x)^4 - 1)*sinh(x)^4
 - 2*cosh(x)^4 + 8*(7*cosh(x)^5 - cosh(x))*sinh(x)^3 + 4*(7*cosh(x)^6 - 3*c
osh(x)^2)*sinh(x)^2 + 8*(cosh(x)^7 - cosh(x)^3)*sinh(x) + 1)*log(cosh(x) +
sinh(x) - 1) + 4*(25*cosh(x)^4 - 1)*sinh(x) - 4*cosh(x))/(cosh(x)^8 + 56*co
sh(x)^3*sinh(x)^5 + 28*cosh(x)^2*sinh(x)^6 + 8*cosh(x)*sinh(x)^7 + sinh(x)^
8 + 2*(35*cosh(x)^4 - 1)*sinh(x)^4 - 2*cosh(x)^4 + 8*(7*cosh(x)^5 - cosh(x)
)*sinh(x)^3 + 4*(7*cosh(x)^6 - 3*cosh(x)^2)*sinh(x)^2 + 8*(cosh(x)^7 - cosh
(x)^3)*sinh(x) + 1)
```

giac [A] time = 0.12, size = 42, normalized size = 0.79

$$-\frac{5 e^{(5x)} - e^{x}}{4 \left(e^{(4x)} - 1\right)^{2}} - \frac{1}{8} \arctan\left(e^{x}\right) - \frac{1}{16} \log\left(e^{x} + 1\right) + \frac{1}{16} \log\left(|e^{x} - 1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*coth(2*x)*csch(2*x)^2,x, algorithm="giac")
```

```
[Out] -1/4*(5*e^(5*x) - e^x)/(e^(4*x) - 1)^2 - 1/8*arctan(e^x) - 1/16*log(e^x + 1
) + 1/16*log(abs(e^x - 1))
```

maple [C] time = 0.38, size = 54, normalized size = 1.02

$$-\frac{e^{x} \left(5 e^{4x} - 1\right)}{4 \left(e^{4x} - 1\right)^{2}} + \frac{i \ln\left(e^{x} - i\right)}{16} - \frac{i \ln\left(e^{x} + i\right)}{16} - \frac{\ln\left(e^{x} + 1\right)}{16} + \frac{\ln\left(e^{x} - 1\right)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)*coth(2*x)*csch(2*x)^2,x)
```

```
[Out] -1/4*exp(x)*(5*exp(4*x)-1)/(exp(4*x)-1)^2+1/16*I*ln(exp(x)-I)-1/16*I*ln(exp
(x)+I)-1/16*ln(exp(x)+1)+1/16*ln(exp(x)-1)
```

maxima [A] time = 0.58, size = 47, normalized size = 0.89

$$-\frac{5 e^{(5x)} - e^{x}}{4 \left(e^{(8x)} - 2 e^{(4x)} + 1\right)} - \frac{1}{8} \arctan\left(e^{x}\right) - \frac{1}{16} \log\left(e^{x} + 1\right) + \frac{1}{16} \log\left(e^{x} - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*coth(2*x)*csch(2*x)^2,x, algorithm="maxima")
```

```
[Out] -1/4*(5*e^(5*x) - e^x)/(e^(8*x) - 2*e^(4*x) + 1) - 1/8*arctan(e^x) - 1/16*l
og(e^x + 1) + 1/16*log(e^x - 1)
```

mupad [B] time = 2.00, size = 80, normalized size = 1.51

$$\frac{\ln\left(\frac{1}{4}-\frac{e^x}{4}\right)}{16}-\frac{\ln\left(\frac{e^x}{4}+\frac{1}{4}\right)}{16}-\frac{\operatorname{atan}\left(e^x\right)}{8}-\frac{e^{5x}}{2\left(e^{8x}-2e^{4x}+1\right)}-\frac{3e^x}{4\left(e^{4x}-1\right)}-\frac{e^x}{2\left(e^{8x}-2e^{4x}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((coth(2*x)*exp(x))/sinh(2*x)^2,x)
```

```
[Out] log(1/4 - exp(x)/4)/16 - log(exp(x)/4 + 1/4)/16 - atan(exp(x))/8 - exp(5*x)
/(2*(exp(8*x) - 2*exp(4*x) + 1)) - (3*exp(x))/(4*(exp(4*x) - 1)) - exp(x)/(
2*(exp(8*x) - 2*exp(4*x) + 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int e^{x}\coth\left(2x\right)\operatorname{csch}^{2}\left(2x\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*coth(2*x)*csch(2*x)**2,x)
```

```
[Out] Integral(exp(x)*coth(2*x)*csch(2*x)**2, x)
```

3.943 $\int e^x \coth^2(2x)\mathbf{csch}(2x)\,dx$

Optimal. Leaf size=55

$$\frac{3e^{3x}}{4\left(1-e^{4x}\right)}-\frac{e^{3x}}{\left(1-e^{4x}\right)^2}+\frac{5}{8}\tan^{-1}\left(e^x\right)-\frac{5}{8}\tanh^{-1}\left(e^x\right)$$

```
[Out] -exp(3*x)/(1-exp(4*x))^2+3/4*exp(3*x)/(1-exp(4*x))+5/8*arctan(exp(x))-5/8*a
rctanh(exp(x))
```

Rubi [A] time = 0.05, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {2282, 12, 463, 457, 298, 203, 206}

$$\frac{3e^{3x}}{4\left(1-e^{4x}\right)}-\frac{e^{3x}}{\left(1-e^{4x}\right)^2}+\frac{5}{8}\tan^{-1}\left(e^x\right)-\frac{5}{8}\tanh^{-1}\left(e^x\right)$$

Antiderivative was successfully verified.

```
[In] Int[E^x*Coth[2*x]^2*Csch[2*x],x]
```

```
[Out] -(E^(3*x)/(1 - E^(4*x))^2) + (3*E^(3*x))/(4*(1 - E^(4*x))) + (5*ArcTan[E^x]
)/8 - (5*ArcTanh[E^x])/8
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
```

```
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
 + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
 n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

Rule 463

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^2, x_Symbol] :> -Simp[((b*c - a*d)^2*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/
(a*b^2*e*n*(p + 1)), x] + Dist[1/(a*b^2*n*(p + 1)), Int[(e*x)^m*(a + b*x^n)
^(p + 1)*Simp[(b*c - a*d)^2*(m + 1) + b^2*c^2*n*(p + 1) + a*b*d^2*n*(p + 1)
*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] &&
 IGtQ[n, 0] && LtQ[p, -1]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$\int e^x \coth^2(2x)\text{csch}(2x)\,dx = \text{Subst}\left(\int \frac{2x^2(1+x^4)^2}{(-1+x^4)^3}\,dx, x, e^x\right)$$
$$= 2\,\text{Subst}\left(\int \frac{x^2(1+x^4)^2}{(-1+x^4)^3}\,dx, x, e^x\right)$$
$$= -\frac{e^{3x}}{(1-e^{4x})^2} + \frac{1}{4}\text{Subst}\left(\int \frac{x^2(4+8x^4)}{(-1+x^4)^2}\,dx, x, e^x\right)$$
$$= -\frac{e^{3x}}{(1-e^{4x})^2} + \frac{3e^{3x}}{4(1-e^{4x})} + \frac{5}{4}\text{Subst}\left(\int \frac{x^2}{-1+x^4}\,dx, x, e^x\right)$$
$$= -\frac{e^{3x}}{(1-e^{4x})^2} + \frac{3e^{3x}}{4(1-e^{4x})} - \frac{5}{8}\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, e^x\right) + \frac{5}{8}\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x\right.$$
$$= -\frac{e^{3x}}{(1-e^{4x})^2} + \frac{3e^{3x}}{4(1-e^{4x})} + \frac{5}{8}\tan^{-1}(e^x) - \frac{5}{8}\tanh^{-1}(e^x)$$

Mathematica [C] time = 3.34, size = 161, normalized size = 2.93

$$-\frac{16e^{7x}(e^{4x}+1)^2\,{}_5F_4\left(\frac{7}{4},2,2,2,2;1,1,1,\frac{19}{4};e^{4x}\right)}{1155} - \frac{8e^{7x}(26e^{4x}+11e^{8x}+15)\,{}_4F_3\left(\frac{7}{4},2,2,2;1,1,\frac{19}{4};e^{4x}\right)}{1155} + \frac{e^{-5x}(-7}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^x*Coth[2*x]^2*Csch[2*x],x]
```

```
[Out] (177023 + 244931*E^(4*x) + 43161*E^(8*x) - 26091*E^(12*x) - 7*(25289 + 2415
2*E^(4*x) - 10058*E^(8*x) - 9048*E^(12*x) + 513*E^(16*x))*Hypergeometric2F1
[3/4, 1, 7/4, E^(4*x)])/(10752*E^(5*x)) - (8*E^(7*x)*(15 + 26*E^(4*x) + 11*
E^(8*x))*HypergeometricPFQ[{7/4, 2, 2, 2}, {1, 1, 19/4}, E^(4*x)])/1155 - (
16*E^(7*x)*(1 + E^(4*x))^2*HypergeometricPFQ[{7/4, 2, 2, 2, 2}, {1, 1, 1, 1
9/4}, E^(4*x)])/1155
```

fricas [B] time = 0.47, size = 557, normalized size = 10.13

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*coth(2*x)^2*csch(2*x),x, algorithm="fricas")
```

```
[Out] -1/16*(12*cosh(x)^7 + 420*cosh(x)^3*sinh(x)^4 + 252*cosh(x)^2*sinh(x)^5 + 8
4*cosh(x)*sinh(x)^6 + 12*sinh(x)^7 + 4*(105*cosh(x)^4 + 1)*sinh(x)^3 + 4*co
sh(x)^3 + 12*(21*cosh(x)^5 + cosh(x))*sinh(x)^2 - 10*(cosh(x)^8 + 56*cosh(x
)^3*sinh(x)^5 + 28*cosh(x)^2*sinh(x)^6 + 8*cosh(x)*sinh(x)^7 + sinh(x)^8 +
2*(35*cosh(x)^4 - 1)*sinh(x)^4 - 2*cosh(x)^4 + 8*(7*cosh(x)^5 - cosh(x))*si
nh(x)^3 + 4*(7*cosh(x)^6 - 3*cosh(x)^2)*sinh(x)^2 + 8*(cosh(x)^7 - cosh(x)^
3)*sinh(x) + 1)*arctan(cosh(x) + sinh(x)) + 5*(cosh(x)^8 + 56*cosh(x)^3*sin
h(x)^5 + 28*cosh(x)^2*sinh(x)^6 + 8*cosh(x)*sinh(x)^7 + sinh(x)^8 + 2*(35*c
osh(x)^4 - 1)*sinh(x)^4 - 2*cosh(x)^4 + 8*(7*cosh(x)^5 - cosh(x))*sinh(x)^3
 + 4*(7*cosh(x)^6 - 3*cosh(x)^2)*sinh(x)^2 + 8*(cosh(x)^7 - cosh(x)^3)*sinh
(x) + 1)*log(cosh(x) + sinh(x) + 1) - 5*(cosh(x)^8 + 56*cosh(x)^3*sinh(x)^5
 + 28*cosh(x)^2*sinh(x)^6 + 8*cosh(x)*sinh(x)^7 + sinh(x)^8 + 2*(35*cosh(x)
^4 - 1)*sinh(x)^4 - 2*cosh(x)^4 + 8*(7*cosh(x)^5 - cosh(x))*sinh(x)^3 + 4*(
7*cosh(x)^6 - 3*cosh(x)^2)*sinh(x)^2 + 8*(cosh(x)^7 - cosh(x)^3)*sinh(x) +
1)*log(cosh(x) + sinh(x) - 1) + 12*(7*cosh(x)^6 + cosh(x)^2)*sinh(x))/(cosh
(x)^8 + 56*cosh(x)^3*sinh(x)^5 + 28*cosh(x)^2*sinh(x)^6 + 8*cosh(x)*sinh(x)
^7 + sinh(x)^8 + 2*(35*cosh(x)^4 - 1)*sinh(x)^4 - 2*cosh(x)^4 + 8*(7*cosh(x
)^5 - cosh(x))*sinh(x)^3 + 4*(7*cosh(x)^6 - 3*cosh(x)^2)*sinh(x)^2 + 8*(cos
h(x)^7 - cosh(x)^3)*sinh(x) + 1)
```

giac [A] time = 0.12, size = 42, normalized size = 0.76

$$-\frac{3\,e^{(7\,x)}+e^{(3\,x)}}{4\,\left(e^{(4\,x)}-1\right)^{2}}+\frac{5}{8}\,\arctan\left(e^{x}\right)-\frac{5}{16}\,\log\left(e^{x}+1\right)+\frac{5}{16}\,\log\left(\left|e^{x}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*coth(2*x)^2*csch(2*x),x, algorithm="giac")
```

```
[Out] -1/4*(3*e^(7*x) + e^(3*x))/(e^(4*x) - 1)^2 + 5/8*arctan(e^x) - 5/16*log(e^x
 + 1) + 5/16*log(abs(e^x - 1))
```

maple [C] time = 0.45, size = 56, normalized size = 1.02

$$-\frac{e^{3x}\left(3\,e^{4x}+1\right)}{4\left(e^{4x}-1\right)^{2}}+\frac{5\ln\left(e^{x}-1\right)}{16}-\frac{5\ln\left(e^{x}+1\right)}{16}+\frac{5i\ln\left(e^{x}+i\right)}{16}-\frac{5i\ln\left(e^{x}-i\right)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)*coth(2*x)^2*csch(2*x),x)
```

```
[Out] -1/4*exp(3*x)*(3*exp(4*x)+1)/(exp(4*x)-1)^2+5/16*ln(exp(x)-1)-5/16*ln(exp(x
)+1)+5/16*I*ln(exp(x)+I)-5/16*I*ln(exp(x)-I)
```

maxima [A] time = 0.64, size = 47, normalized size = 0.85

$$-\frac{3\,e^{(7\,x)}+e^{(3\,x)}}{4\,\left(e^{(8\,x)}-2\,e^{(4\,x)}+1\right)}+\frac{5}{8}\,\arctan\left(e^{x}\right)-\frac{5}{16}\,\log\left(e^{x}+1\right)+\frac{5}{16}\,\log\left(e^{x}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*coth(2*x)^2*csch(2*x),x, algorithm="maxima")
```

```
[Out] -1/4*(3*e^(7*x) + e^(3*x))/(e^(8*x) - 2*e^(4*x) + 1) + 5/8*arctan(e^x) - 5/
16*log(e^x + 1) + 5/16*log(e^x - 1)
```

mupad [B] time = 1.87, size = 62, normalized size = 1.13

$$\frac{5\,\ln\left(\frac{25\,\mathrm{e}^{x}}{16}-\frac{25}{16}\right)}{16}-\frac{5\,\ln\left(\frac{25\,\mathrm{e}^{x}}{16}+\frac{25}{16}\right)}{16}-\frac{5\,\mathrm{atan}\left(\mathrm{e}^{-x}\right)}{8}-\frac{\mathrm{e}^{3\,x}}{\mathrm{e}^{8\,x}-2\,\mathrm{e}^{4\,x}+1}-\frac{3\,\mathrm{e}^{3\,x}}{4\,\left(\mathrm{e}^{4\,x}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((coth(2*x)^2*exp(x))/sinh(2*x),x)
```

```
[Out] (5*log((25*exp(x))/16 - 25/16))/16 - (5*log((25*exp(x))/16 + 25/16))/16 - (
5*atan(exp(-x)))/8 - exp(3*x)/(exp(8*x) - 2*exp(4*x) + 1) - (3*exp(3*x))/(4
*(exp(4*x) - 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int e^{x}\coth^{2}\left(2x\right)\operatorname{csch}\left(2x\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*coth(2*x)**2*csch(2*x),x)
```

```
[Out] Integral(exp(x)*coth(2*x)**2*csch(2*x), x)
```

3.944 $\int e^x \coth^2(2x)\mathbf{csch}^2(2x)\,dx$

Optimal. Leaf size=75

$$\frac{3e^x}{8(1-e^{4x})}-\frac{5e^{5x}}{6(1-e^{4x})^2}+\frac{4e^{5x}}{3(1-e^{4x})^3}-\frac{3}{16}\tan^{-1}(e^x)-\frac{3}{16}\tanh^{-1}(e^x)$$

```
[Out] 4/3*exp(5*x)/(1-exp(4*x))^3-5/6*exp(5*x)/(1-exp(4*x))^2+3/8*exp(x)/(1-exp(4
*x))-3/16*arctan(exp(x))-3/16*arctanh(exp(x))
```

Rubi [A] time = 0.07, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {2282, 12, 463, 457, 288, 212, 206, 203}

$$\frac{3e^x}{8(1-e^{4x})}-\frac{5e^{5x}}{6(1-e^{4x})^2}+\frac{4e^{5x}}{3(1-e^{4x})^3}-\frac{3}{16}\tan^{-1}(e^x)-\frac{3}{16}\tanh^{-1}(e^x)$$

Antiderivative was successfully verified.

```
[In] Int[E^x*Coth[2*x]^2*Csch[2*x]^2,x]
```

```
[Out] (4*E^(5*x))/(3*(1 - E^(4*x))^3) - (5*E^(5*x))/(6*(1 - E^(4*x))^2) + (3*E^x)
/(8*(1 - E^(4*x))) - (3*ArcTan[E^x])/16 - (3*ArcTanh[E^x])/16
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
```

```
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
 + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
 n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

Rule 463

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^2, x_Symbol] :> -Simp[((b*c - a*d)^2*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/
(a*b^2*e*n*(p + 1)), x] + Dist[1/(a*b^2*n*(p + 1)), Int[(e*x)^m*(a + b*x^n)
^(p + 1)*Simp[(b*c - a*d)^2*(m + 1) + b^2*c^2*n*(p + 1) + a*b*d^2*n*(p + 1)
*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] &&
 IGtQ[n, 0] && LtQ[p, -1]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$\int e^x \coth^2(2x)\operatorname{csch}^2(2x)\,dx = \text{Subst}\left(\int \frac{4x^4\left(1+x^4\right)^2}{\left(1-x^4\right)^4}\,dx, x, e^x\right)$$
$$= 4\,\text{Subst}\left(\int \frac{x^4\left(1+x^4\right)^2}{\left(1-x^4\right)^4}\,dx, x, e^x\right)$$
$$= \frac{4e^{5x}}{3\left(1-e^{4x}\right)^3} - \frac{1}{3}\text{Subst}\left(\int \frac{x^4\left(8+12x^4\right)}{\left(1-x^4\right)^3}\,dx, x, e^x\right)$$
$$= \frac{4e^{5x}}{3\left(1-e^{4x}\right)^3} - \frac{5e^{5x}}{6\left(1-e^{4x}\right)^2} + \frac{3}{2}\text{Subst}\left(\int \frac{x^4}{\left(1-x^4\right)^2}\,dx, x, e^x\right)$$
$$= \frac{4e^{5x}}{3\left(1-e^{4x}\right)^3} - \frac{5e^{5x}}{6\left(1-e^{4x}\right)^2} + \frac{3e^x}{8\left(1-e^{4x}\right)} - \frac{3}{8}\text{Subst}\left(\int \frac{1}{1-x^4}\,dx, x, e^x\right)$$
$$= \frac{4e^{5x}}{3\left(1-e^{4x}\right)^3} - \frac{5e^{5x}}{6\left(1-e^{4x}\right)^2} + \frac{3e^x}{8\left(1-e^{4x}\right)} - \frac{3}{16}\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, e^x\right) - \frac{3}{16}\text{Su}$$
$$= \frac{4e^{5x}}{3\left(1-e^{4x}\right)^3} - \frac{5e^{5x}}{6\left(1-e^{4x}\right)^2} + \frac{3e^x}{8\left(1-e^{4x}\right)} - \frac{3}{16}\tan^{-1}\left(e^x\right) - \frac{3}{16}\tanh^{-1}\left(e^x\right)$$

Mathematica [C] time = 5.54, size = 310, normalized size = 4.13

$$\frac{e^{-7x}\left(1280e^{16x}\left(1346e^{4x}+557e^{8x}+821\right)\,{}_4F_3\left(2,2,2,\frac{9}{4};1,1,\frac{21}{4};e^{4x}\right)+10240e^{16x}\left(42e^{4x}+19e^{8x}+23\right)\,{}_5F_4\left(2,2,\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^x*Coth[2*x]^2*Csch[2*x]^2,x]
```

```
[Out] (-1070609085 - 946471617*E^(4*x) + 369641285*E^(8*x) + 351173641*E^(12*x) -
 23818496*E^(16*x) + 1070609085*Hypergeometric2F1[1/4, 1, 5/4, E^(4*x)] + 7
32349800*E^(4*x)*Hypergeometric2F1[1/4, 1, 5/4, E^(4*x)] - 635067810*E^(8*x
)*Hypergeometric2F1[1/4, 1, 5/4, E^(4*x)] - 384831720*E^(12*x)*Hypergeometr
ic2F1[1/4, 1, 5/4, E^(4*x)] + 60913125*E^(16*x)*Hypergeometric2F1[1/4, 1, 5
/4, E^(4*x)] + 1280*E^(16*x)*(821 + 1346*E^(4*x) + 557*E^(8*x))*Hypergeomet
ricPFQ[{2, 2, 2, 9/4}, {1, 1, 21/4}, E^(4*x)] + 10240*E^(16*x)*(23 + 42*E^(
4*x) + 19*E^(8*x))*HypergeometricPFQ[{2, 2, 2, 2, 9/4}, {1, 1, 1, 21/4}, E^
(4*x)] + 20480*E^(16*x)*HypergeometricPFQ[{2, 2, 2, 2, 2, 9/4}, {1, 1, 1, 1
, 21/4}, E^(4*x)] + 40960*E^(20*x)*HypergeometricPFQ[{2, 2, 2, 2, 2, 9/4},
```

```
{1, 1, 1, 1, 21/4}, E^(4*x)] + 20480*E^(24*x)*HypergeometricPFQ[{2, 2, 2, 2
, 2, 9/4}, {1, 1, 1, 1, 21/4}, E^(4*x)])/(3818880*E^(7*x))
```

fricas [B] time = 0.53, size = 992, normalized size = 13.23

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*coth(2*x)^2*csch(2*x)^2,x, algorithm="fricas")
```

```
[Out] -1/96*(116*cosh(x)^9 + 9744*cosh(x)^3*sinh(x)^6 + 4176*cosh(x)^2*sinh(x)^7
+ 1044*cosh(x)*sinh(x)^8 + 116*sinh(x)^9 + 24*(609*cosh(x)^4 - 1)*sinh(x)^5
 - 24*cosh(x)^5 + 24*(609*cosh(x)^5 - 5*cosh(x))*sinh(x)^4 + 48*(203*cosh(x
)^6 - 5*cosh(x)^2)*sinh(x)^3 + 48*(87*cosh(x)^7 - 5*cosh(x)^3)*sinh(x)^2 +
18*(cosh(x)^12 + 220*cosh(x)^3*sinh(x)^9 + 66*cosh(x)^2*sinh(x)^10 + 12*cos
h(x)*sinh(x)^11 + sinh(x)^12 + 3*(165*cosh(x)^4 - 1)*sinh(x)^8 - 3*cosh(x)^
8 + 24*(33*cosh(x)^5 - cosh(x))*sinh(x)^7 + 84*(11*cosh(x)^6 - cosh(x)^2)*s
inh(x)^6 + 24*(33*cosh(x)^7 - 7*cosh(x)^3)*sinh(x)^5 + 3*(165*cosh(x)^8 - 7
0*cosh(x)^4 + 1)*sinh(x)^4 + 3*cosh(x)^4 + 4*(55*cosh(x)^9 - 42*cosh(x)^5 +
 3*cosh(x))*sinh(x)^3 + 6*(11*cosh(x)^10 - 14*cosh(x)^6 + 3*cosh(x)^2)*sinh
(x)^2 + 12*(cosh(x)^11 - 2*cosh(x)^7 + cosh(x)^3)*sinh(x) - 1)*arctan(cosh(
x) + sinh(x)) + 9*(cosh(x)^12 + 220*cosh(x)^3*sinh(x)^9 + 66*cosh(x)^2*sinh
(x)^10 + 12*cosh(x)*sinh(x)^11 + sinh(x)^12 + 3*(165*cosh(x)^4 - 1)*sinh(x)
^8 - 3*cosh(x)^8 + 24*(33*cosh(x)^5 - cosh(x))*sinh(x)^7 + 84*(11*cosh(x)^6
 - cosh(x)^2)*sinh(x)^6 + 24*(33*cosh(x)^7 - 7*cosh(x)^3)*sinh(x)^5 + 3*(16
5*cosh(x)^8 - 70*cosh(x)^4 + 1)*sinh(x)^4 + 3*cosh(x)^4 + 4*(55*cosh(x)^9 -
 42*cosh(x)^5 + 3*cosh(x))*sinh(x)^3 + 6*(11*cosh(x)^10 - 14*cosh(x)^6 + 3*
cosh(x)^2)*sinh(x)^2 + 12*(cosh(x)^11 - 2*cosh(x)^7 + cosh(x)^3)*sinh(x) -
1)*log(cosh(x) + sinh(x) + 1) - 9*(cosh(x)^12 + 220*cosh(x)^3*sinh(x)^9 + 6
6*cosh(x)^2*sinh(x)^10 + 12*cosh(x)*sinh(x)^11 + sinh(x)^12 + 3*(165*cosh(x
)^4 - 1)*sinh(x)^8 - 3*cosh(x)^8 + 24*(33*cosh(x)^5 - cosh(x))*sinh(x)^7 +
84*(11*cosh(x)^6 - cosh(x)^2)*sinh(x)^6 + 24*(33*cosh(x)^7 - 7*cosh(x)^3)*s
inh(x)^5 + 3*(165*cosh(x)^8 - 70*cosh(x)^4 + 1)*sinh(x)^4 + 3*cosh(x)^4 + 4
*(55*cosh(x)^9 - 42*cosh(x)^5 + 3*cosh(x))*sinh(x)^3 + 6*(11*cosh(x)^10 - 1
4*cosh(x)^6 + 3*cosh(x)^2)*sinh(x)^2 + 12*(cosh(x)^11 - 2*cosh(x)^7 + cosh(
x)^3)*sinh(x) - 1)*log(cosh(x) + sinh(x) - 1) + 12*(87*cosh(x)^8 - 10*cosh(
x)^4 + 3)*sinh(x) + 36*cosh(x))/(cosh(x)^12 + 220*cosh(x)^3*sinh(x)^9 + 66*
cosh(x)^2*sinh(x)^10 + 12*cosh(x)*sinh(x)^11 + sinh(x)^12 + 3*(165*cosh(x)^
4 - 1)*sinh(x)^8 - 3*cosh(x)^8 + 24*(33*cosh(x)^5 - cosh(x))*sinh(x)^7 + 84
*(11*cosh(x)^6 - cosh(x)^2)*sinh(x)^6 + 24*(33*cosh(x)^7 - 7*cosh(x)^3)*sin
h(x)^5 + 3*(165*cosh(x)^8 - 70*cosh(x)^4 + 1)*sinh(x)^4 + 3*cosh(x)^4 + 4*(
55*cosh(x)^9 - 42*cosh(x)^5 + 3*cosh(x))*sinh(x)^3 + 6*(11*cosh(x)^10 - 14*
cosh(x)^6 + 3*cosh(x)^2)*sinh(x)^2 + 12*(cosh(x)^11 - 2*cosh(x)^7 + cosh(x)
^3)*sinh(x) - 1)
```

giac [A] time = 0.12, size = 48, normalized size = 0.64

$$-\frac{29\,e^{(9\,x)} - 6\,e^{(5\,x)} + 9\,e^{x}}{24\,{\left(e^{(4\,x)} - 1\right)}^{3}} - \frac{3}{16}\,\arctan\left(e^{x}\right) - \frac{3}{32}\,\log\left(e^{x} + 1\right) + \frac{3}{32}\,\log\left({\left| e^{x} - 1 \right|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*coth(2*x)^2*csch(2*x)^2,x, algorithm="giac")
```

```
[Out] -1/24*(29*e^(9*x) - 6*e^(5*x) + 9*e^x)/(e^(4*x) - 1)^3 - 3/16*arctan(e^x) -
 3/32*log(e^x + 1) + 3/32*log(abs(e^x - 1))
```

maple [C] time = 0.46, size = 60, normalized size = 0.80

$$-\frac{{\mathrm e}^{x} \left(29\,{\mathrm e}^{8 x}-6\,{\mathrm e}^{4 x}+9\right)}{24 \left({\mathrm e}^{4 x}-1\right)^{3}}+\frac{3 i \ln \left({\mathrm e}^{x}-i\right)}{32}-\frac{3 i \ln \left({\mathrm e}^{x}+i\right)}{32}+\frac{3 \ln \left({\mathrm e}^{x}-1\right)}{32}-\frac{3 \ln \left({\mathrm e}^{x}+1\right)}{32}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)*coth(2*x)^2*csch(2*x)^2,x)
```

```
[Out] -1/24*exp(x)*(29*exp(8*x)-6*exp(4*x)+9)/(exp(4*x)-1)^3+3/32*I*ln(exp(x)-I)-
3/32*I*ln(exp(x)+I)+3/32*ln(exp(x)-1)-3/32*ln(exp(x)+1)
```

maxima [A] time = 0.44, size = 59, normalized size = 0.79

$$-\frac{29\,e^{(9\,x)} - 6\,e^{(5\,x)} + 9\,e^{x}}{24\,{\left(e^{(12\,x)} - 3\,e^{(8\,x)} + 3\,e^{(4\,x)} - 1\right)}} - \frac{3}{16}\,\arctan\left(e^{x}\right) - \frac{3}{32}\,\log\left(e^{x} + 1\right) + \frac{3}{32}\,\log\left(e^{x} - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*coth(2*x)^2*csch(2*x)^2,x, algorithm="maxima")
```

```
[Out] -1/24*(29*e^(9*x) - 6*e^(5*x) + 9*e^x)/(e^(12*x) - 3*e^(8*x) + 3*e^(4*x) -
1) - 3/16*arctan(e^x) - 3/32*log(e^x + 1) + 3/32*log(e^x - 1)
```

mupad [B] time = 2.18, size = 114, normalized size = 1.52

$$\frac{3\,\ln\left(\frac{3}{8}-\frac{3\,{\mathrm{e}}^x}{8}\right)}{32}-\frac{3\,\ln\left(-\frac{3\,{\mathrm{e}}^x}{8}-\frac{3}{8}\right)}{32}-\frac{7\,{\mathrm{e}}^x}{8\,\left({\mathrm{e}}^{4\,x}-1\right)}-\frac{\frac{2\,{\mathrm{e}}^{5\,x}}{3}+\frac{{\mathrm{e}}^{9\,x}}{3}+\frac{{\mathrm{e}}^x}{3}}{3\,{\mathrm{e}}^{4\,x}-3\,{\mathrm{e}}^{8\,x}+{\mathrm{e}}^{12\,x}-1}-\frac{5\,{\mathrm{e}}^x}{6\,\left({\mathrm{e}}^{8\,x}-2\,{\mathrm{e}}^{4\,x}+1\right)}-\frac{\ln\left(-\frac{3\,{\mathrm{e}}^x}{8}-\frac{3}{8}{}\mathrm{i}\right)\,3\mathrm{i}}{32}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((coth(2*x)^2*exp(x))/sinh(2*x)^2,x)
```

```
[Out] (3*log(3/8 - (3*exp(x))/8))/32 - (3*log(- (3*exp(x))/8 - 3/8))/32 - (log(-
(3*exp(x))/8 - 3i/8)*3i)/32 + (log(3i/8 - (3*exp(x))/8)*3i)/32 - (7*exp(x))
/(8*(exp(4*x) - 1)) - ((2*exp(5*x))/3 + exp(9*x)/3 + exp(x)/3)/(3*exp(4*x)
- 3*exp(8*x) + exp(12*x) - 1) - (5*exp(x))/(6*(exp(8*x) - 2*exp(4*x) + 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int e^x \coth^2(2x) \operatorname{csch}^2(2x)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*coth(2*x)**2*csch(2*x)**2,x)
```

```
[Out] Integral(exp(x)*coth(2*x)**2*csch(2*x)**2, x)
```

3.945 $\int e^{c+dx}\cosh(a+bx)\sinh^3(a+bx)\,dx$

Optimal. Leaf size=137

$$\frac{de^{c+dx}\sinh(2a+2bx)}{4(4b^2-d^2)}-\frac{de^{c+dx}\sinh(4a+4bx)}{8(16b^2-d^2)}-\frac{be^{c+dx}\cosh(2a+2bx)}{2(4b^2-d^2)}+\frac{be^{c+dx}\cosh(4a+4bx)}{2(16b^2-d^2)}$$

```
[Out] -1/2*b*exp(d*x+c)*cosh(2*b*x+2*a)/(4*b^2-d^2)+1/2*b*exp(d*x+c)*cosh(4*b*x+4
*a)/(16*b^2-d^2)+1/4*d*exp(d*x+c)*sinh(2*b*x+2*a)/(4*b^2-d^2)-1/8*d*exp(d*x
+c)*sinh(4*b*x+4*a)/(16*b^2-d^2)
```

Rubi [A] time = 0.10, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {5509, 5474}

$$\frac{de^{c+dx}\sinh(2a+2bx)}{4(4b^2-d^2)}-\frac{de^{c+dx}\sinh(4a+4bx)}{8(16b^2-d^2)}-\frac{be^{c+dx}\cosh(2a+2bx)}{2(4b^2-d^2)}+\frac{be^{c+dx}\cosh(4a+4bx)}{2(16b^2-d^2)}$$

Antiderivative was successfully verified.

```
[In] Int[E^(c + d*x)*Cosh[a + b*x]*Sinh[a + b*x]^3,x]
```

```
[Out] -(b*E^(c + d*x)*Cosh[2*a + 2*b*x])/(2*(4*b^2 - d^2)) + (b*E^(c + d*x)*Cosh[
4*a + 4*b*x])/(2*(16*b^2 - d^2)) + (d*E^(c + d*x)*Sinh[2*a + 2*b*x])/(4*(4*
b^2 - d^2)) - (d*E^(c + d*x)*Sinh[4*a + 4*b*x])/(8*(16*b^2 - d^2))
```

Rule 5474

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sinh[(d_.) + (e_.)*(x_)], x_Symbol] :
> -Simp[(b*c*Log[F]*F^(c*(a + b*x))*Sinh[d + e*x])/(e^2 - b^2*c^2*Log[F]^2)
, x] + Simp[(e*F^(c*(a + b*x))*Cosh[d + e*x])/(e^2 - b^2*c^2*Log[F]^2), x]
/; FreeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 - b^2*c^2*Log[F]^2, 0]
```

Rule 5509

```
Int[Cosh[(f_.) + (g_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sinh[(
d_.) + (e_.)*(x_)]^(m_.), x_Symbol] :> Int[ExpandTrigReduce[F^(c*(a + b*x))
, Sinh[d + e*x]^m*Cosh[f + g*x]^n, x], x] /; FreeQ[{F, a, b, c, d, e, f, g}
, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

Rubi steps

$$\int e^{c+dx}\cosh(a+bx)\sinh^3(a+bx)\,dx = \int\left(-\frac{1}{4}e^{c+dx}\sinh(2a+2bx)+\frac{1}{8}e^{c+dx}\sinh(4a+4bx)\right)dx$$
$$= \frac{1}{8}\int e^{c+dx}\sinh(4a+4bx)\,dx - \frac{1}{4}\int e^{c+dx}\sinh(2a+2bx)\,dx$$
$$= -\frac{be^{c+dx}\cosh(2a+2bx)}{2(4b^2-d^2)} + \frac{be^{c+dx}\cosh(4a+4bx)}{2(16b^2-d^2)} + \frac{de^{c+dx}\sinh(2a+2b}{4(4b^2-d^2)}$$

Mathematica [A] time = 1.21, size = 86, normalized size = 0.63

$$\frac{1}{8}e^{c+dx}\left(\frac{2d\sinh(2(a+bx))-4b\cosh(2(a+bx))}{4b^2-d^2}+\frac{4b\cosh(4(a+bx))-d\sinh(4(a+bx))}{16b^2-d^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(c + d*x)*Cosh[a + b*x]*Sinh[a + b*x]^3,x]
```

```
[Out] (E^(c + d*x)*((-4*b*Cosh[2*(a + b*x)] + 2*d*Sinh[2*(a + b*x)])/(4*b^2 - d^2
) + (4*b*Cosh[4*(a + b*x)] - d*Sinh[4*(a + b*x)])/(16*b^2 - d^2)))/8
```

fricas [B] time = 0.50, size = 505, normalized size = 3.69

$$-\frac{(4b^2d-d^3)\cosh(bx+a)\cosh(dx+c)\sinh(bx+a)^3-(4b^3-bd^2)\cosh(dx+c)\sinh(bx+a)^4+(16b^3-bd^2}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x+c)*cosh(b*x+a)*sinh(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -1/2*((4*b^2*d - d^3)*cosh(b*x + a)*cosh(d*x + c)*sinh(b*x + a)^3 - (4*b^3
- b*d^2)*cosh(d*x + c)*sinh(b*x + a)^4 + (16*b^3 - b*d^2 - 6*(4*b^3 - b*d^2
)*cosh(b*x + a)^2)*cosh(d*x + c)*sinh(b*x + a)^2 + ((4*b^2*d - d^3)*cosh(b*
x + a)^3 - (16*b^2*d - d^3)*cosh(b*x + a))*cosh(d*x + c)*sinh(b*x + a) - ((
4*b^3 - b*d^2)*cosh(b*x + a)^4 - (16*b^3 - b*d^2)*cosh(b*x + a)^2)*cosh(d*x
 + c) - ((4*b^3 - b*d^2)*cosh(b*x + a)^4 - (4*b^2*d - d^3)*cosh(b*x + a)*si
nh(b*x + a)^3 + (4*b^3 - b*d^2)*sinh(b*x + a)^4 - (16*b^3 - b*d^2)*cosh(b*x
 + a)^2 - (16*b^3 - b*d^2 - 6*(4*b^3 - b*d^2)*cosh(b*x + a)^2)*sinh(b*x + a
)^2 - ((4*b^2*d - d^3)*cosh(b*x + a)^3 - (16*b^2*d - d^3)*cosh(b*x + a))*si
nh(b*x + a))*sinh(d*x + c))/((64*b^4 - 20*b^2*d^2 + d^4)*cosh(b*x + a)^4 -
2*(64*b^4 - 20*b^2*d^2 + d^4)*cosh(b*x + a)^2*sinh(b*x + a)^2 + (64*b^4 - 2
0*b^2*d^2 + d^4)*sinh(b*x + a)^4)
```

giac [A] time = 0.14, size = 93, normalized size = 0.68

$$\frac{e^{(4\,bx+dx+4\,a+c)}}{16\,(4\,b+d)} - \frac{e^{(2\,bx+dx+2\,a+c)}}{8\,(2\,b+d)} - \frac{e^{(-2\,bx+dx-2\,a+c)}}{8\,(2\,b-d)} + \frac{e^{(-4\,bx+dx-4\,a+c)}}{16\,(4\,b-d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x+c)*cosh(b*x+a)*sinh(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/16*e^(4*b*x + d*x + 4*a + c)/(4*b + d) - 1/8*e^(2*b*x + d*x + 2*a + c)/(2
*b + d) - 1/8*e^(-2*b*x + d*x - 2*a + c)/(2*b - d) + 1/16*e^(-4*b*x + d*x -
 4*a + c)/(4*b - d)
```

maple [A] time = 0.36, size = 202, normalized size = 1.47

$$\frac{\sinh(2a-c+(2b-d)x)}{16b-8d} - \frac{\sinh(2a+c+(2b+d)x)}{8(2b+d)} - \frac{\sinh((4b-d)x+4a-c)}{16(4b-d)} + \frac{\sinh((4b+d)x+4a+c)}{64b+16d} - $$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(d*x+c)*cosh(b*x+a)*sinh(b*x+a)^3,x)
```

```
[Out] 1/8*sinh(2*a-c+(2*b-d)*x)/(2*b-d)-1/8*sinh(2*a+c+(2*b+d)*x)/(2*b+d)-1/16/(4
*b-d)*sinh((4*b-d)*x+4*a-c)+1/16/(4*b+d)*sinh((4*b+d)*x+4*a+c)-1/8*cosh(2*a
-c+(2*b-d)*x)/(2*b-d)-1/8*cosh(2*a+c+(2*b+d)*x)/(2*b+d)+1/16*cosh((4*b-d)*x
+4*a-c)/(4*b-d)+1/16*cosh((4*b+d)*x+4*a+c)/(4*b+d)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x+c)*cosh(b*x+a)*sinh(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(1-d/b>0)', see `assume?` for more d
etails)Is 1-d/b equal to -1?
```

mupad [B] time = 1.14, size = 228, normalized size = 1.66

$$\frac{b\,e^{c+d\,x}\sinh(a+b\,x)^4\,\left(10\,b^2-d^2\right)}{64\,b^4-20\,b^2\,d^2+d^4} - \frac{6\,b^3\cosh(a+b\,x)^4\,e^{c+d\,x}}{64\,b^4-20\,b^2\,d^2+d^4} + \frac{3\,b\cosh(a+b\,x)^2\,e^{c+d\,x}\sinh(a+b\,x)^2}{16\,b^2-d^2} - \frac{d\cosh}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a + b*x)*exp(c + d*x)*sinh(a + b*x)^3,x)
```

```
[Out] (b*exp(c + d*x)*sinh(a + b*x)^4*(10*b^2 - d^2))/(64*b^4 + d^4 - 20*b^2*d^2)
 - (6*b^3*cosh(a + b*x)^4*exp(c + d*x))/(64*b^4 + d^4 - 20*b^2*d^2) + (3*b*
cosh(a + b*x)^2*exp(c + d*x)*sinh(a + b*x)^2)/(16*b^2 - d^2) - (d*cosh(a +
b*x)*exp(c + d*x)*sinh(a + b*x)^3*(10*b^2 - d^2))/(64*b^4 + d^4 - 20*b^2*d^
2) + (6*b^2*d*cosh(a + b*x)^3*exp(c + d*x)*sinh(a + b*x))/((4*b^2 - d^2)*(1
6*b^2 - d^2))
```

sympy [A] time = 142.13, size = 1292, normalized size = 9.43

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x+c)*cosh(b*x+a)*sinh(b*x+a)**3,x)
```

```
[Out] Piecewise((x*exp(c)*sinh(a)**3*cosh(a), Eq(b, 0) & Eq(d, 0)), (x*exp(c)*exp
(d*x)*sinh(a - d*x/2)**4/8 + x*exp(c)*exp(d*x)*sinh(a - d*x/2)**3*cosh(a -
d*x/2)/4 - x*exp(c)*exp(d*x)*sinh(a - d*x/2)*cosh(a - d*x/2)**3/4 - x*exp(c
)*exp(d*x)*cosh(a - d*x/2)**4/8 - exp(c)*exp(d*x)*sinh(a - d*x/2)**4/(8*d)
- exp(c)*exp(d*x)*sinh(a - d*x/2)**2*cosh(a - d*x/2)**2/(2*d) - exp(c)*exp(
d*x)*sinh(a - d*x/2)*cosh(a - d*x/2)**3/(3*d) - exp(c)*exp(d*x)*cosh(a - d*
x/2)**4/(24*d), Eq(b, -d/2)), (x*exp(c)*exp(d*x)*sinh(a - d*x/4)**4/16 + x*
exp(c)*exp(d*x)*sinh(a - d*x/4)**3*cosh(a - d*x/4)/4 + 3*x*exp(c)*exp(d*x)*
sinh(a - d*x/4)**2*cosh(a - d*x/4)**2/8 + x*exp(c)*exp(d*x)*sinh(a - d*x/4)
*cosh(a - d*x/4)**3/4 + x*exp(c)*exp(d*x)*cosh(a - d*x/4)**4/16 + exp(c)*ex
p(d*x)*sinh(a - d*x/4)**4/(6*d) + 11*exp(c)*exp(d*x)*sinh(a - d*x/4)**3*cos
h(a - d*x/4)/(12*d) - 5*exp(c)*exp(d*x)*sinh(a - d*x/4)*cosh(a - d*x/4)**3/
(12*d) - exp(c)*exp(d*x)*cosh(a - d*x/4)**4/(6*d), Eq(b, -d/4)), (-x*exp(c)
*exp(d*x)*sinh(a + d*x/4)**4/16 + x*exp(c)*exp(d*x)*sinh(a + d*x/4)**3*cosh
(a + d*x/4)/4 - 3*x*exp(c)*exp(d*x)*sinh(a + d*x/4)**2*cosh(a + d*x/4)**2/8
 + x*exp(c)*exp(d*x)*sinh(a + d*x/4)*cosh(a + d*x/4)**3/4 - x*exp(c)*exp(d*
x)*cosh(a + d*x/4)**4/16 - exp(c)*exp(d*x)*sinh(a + d*x/4)**4/(6*d) + 11*ex
p(c)*exp(d*x)*sinh(a + d*x/4)**3*cosh(a + d*x/4)/(12*d) - 5*exp(c)*exp(d*x)
*sinh(a + d*x/4)*cosh(a + d*x/4)**3/(12*d) + exp(c)*exp(d*x)*cosh(a + d*x/4
)**4/(6*d), Eq(b, d/4)), (-x*exp(c)*exp(d*x)*sinh(a + d*x/2)**4/8 + x*exp(c
)*exp(d*x)*sinh(a + d*x/2)**3*cosh(a + d*x/2)/4 - x*exp(c)*exp(d*x)*sinh(a
+ d*x/2)*cosh(a + d*x/2)**3/4 + x*exp(c)*exp(d*x)*cosh(a + d*x/2)**4/8 + ex
p(c)*exp(d*x)*sinh(a + d*x/2)**4/(8*d) + exp(c)*exp(d*x)*sinh(a + d*x/2)**2
*cosh(a + d*x/2)**2/(2*d) - exp(c)*exp(d*x)*sinh(a + d*x/2)*cosh(a + d*x/2)
**3/(3*d) + exp(c)*exp(d*x)*cosh(a + d*x/2)**4/(24*d), Eq(b, d/2)), (10*b**
3*exp(c)*exp(d*x)*sinh(a + b*x)**4/(64*b**4 - 20*b**2*d**2 + d**4) + 12*b**
3*exp(c)*exp(d*x)*sinh(a + b*x)**2*cosh(a + b*x)**2/(64*b**4 - 20*b**2*d**2
 + d**4) - 6*b**3*exp(c)*exp(d*x)*cosh(a + b*x)**4/(64*b**4 - 20*b**2*d**2
+ d**4) - 10*b**2*d*exp(c)*exp(d*x)*sinh(a + b*x)**3*cosh(a + b*x)/(64*b**4
```

```
 - 20*b**2*d**2 + d**4) + 6*b**2*d*exp(c)*exp(d*x)*sinh(a + b*x)*cosh(a + b
*x)**3/(64*b**4 - 20*b**2*d**2 + d**4) - b*d**2*exp(c)*exp(d*x)*sinh(a + b*
x)**4/(64*b**4 - 20*b**2*d**2 + d**4) - 3*b*d**2*exp(c)*exp(d*x)*sinh(a + b
*x)**2*cosh(a + b*x)**2/(64*b**4 - 20*b**2*d**2 + d**4) + d**3*exp(c)*exp(d
*x)*sinh(a + b*x)**3*cosh(a + b*x)/(64*b**4 - 20*b**2*d**2 + d**4), True))
```

3.946 $\int e^{c+dx}\cosh(a+bx)\sinh^2(a+bx)\,dx$

Optimal. Leaf size=127

$$-\frac{be^{c+dx}\sinh(a+bx)}{4\left(b^2-d^2\right)}+\frac{3be^{c+dx}\sinh(3a+3bx)}{4\left(9b^2-d^2\right)}+\frac{de^{c+dx}\cosh(a+bx)}{4\left(b^2-d^2\right)}-\frac{de^{c+dx}\cosh(3a+3bx)}{4\left(9b^2-d^2\right)}$$

```
[Out] 1/4*d*exp(d*x+c)*cosh(b*x+a)/(b^2-d^2)-1/4*d*exp(d*x+c)*cosh(3*b*x+3*a)/(9*
b^2-d^2)-1/4*b*exp(d*x+c)*sinh(b*x+a)/(b^2-d^2)+3/4*b*exp(d*x+c)*sinh(3*b*x
+3*a)/(9*b^2-d^2)
```

Rubi [A] time = 0.09, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {5509, 5475}

$$-\frac{be^{c+dx}\sinh(a+bx)}{4\left(b^2-d^2\right)}+\frac{3be^{c+dx}\sinh(3a+3bx)}{4\left(9b^2-d^2\right)}+\frac{de^{c+dx}\cosh(a+bx)}{4\left(b^2-d^2\right)}-\frac{de^{c+dx}\cosh(3a+3bx)}{4\left(9b^2-d^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[E^(c + d*x)*Cosh[a + b*x]*Sinh[a + b*x]^2,x]
```

```
[Out] (d*E^(c + d*x)*Cosh[a + b*x])/(4*(b^2 - d^2)) - (d*E^(c + d*x)*Cosh[3*a + 3
*b*x])/(4*(9*b^2 - d^2)) - (b*E^(c + d*x)*Sinh[a + b*x])/(4*(b^2 - d^2)) +
(3*b*E^(c + d*x)*Sinh[3*a + 3*b*x])/(4*(9*b^2 - d^2))
```

Rule 5475

```
Int[Cosh[(d_.) + (e_.)*(x_)]*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symbol] :
> -Simp[(b*c*Log[F]*F^(c*(a + b*x))*Cosh[d + e*x])/(e^2 - b^2*c^2*Log[F]^2)
, x] + Simp[(e*F^(c*(a + b*x))*Sinh[d + e*x])/(e^2 - b^2*c^2*Log[F]^2), x]
/; FreeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 - b^2*c^2*Log[F]^2, 0]
```

Rule 5509

```
Int[Cosh[(f_.) + (g_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sinh[(
d_.) + (e_.)*(x_)]^(m_.), x_Symbol] :> Int[ExpandTrigReduce[F^(c*(a + b*x))
, Sinh[d + e*x]^m*Cosh[f + g*x]^n, x], x] /; FreeQ[{F, a, b, c, d, e, f, g}
, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

Rubi steps

$$\int e^{c+dx}\cosh(a+bx)\sinh^2(a+bx)\,dx = \int \left(-\frac{1}{4}e^{c+dx}\cosh(a+bx)+\frac{1}{4}e^{c+dx}\cosh(3a+3bx)\right)dx$$
$$= -\left(\frac{1}{4}\int e^{c+dx}\cosh(a+bx)\,dx\right)+\frac{1}{4}\int e^{c+dx}\cosh(3a+3bx)\,dx$$
$$= \frac{de^{c+dx}\cosh(a+bx)}{4\left(b^2-d^2\right)}-\frac{de^{c+dx}\cosh(3a+3bx)}{4\left(9b^2-d^2\right)}-\frac{be^{c+dx}\sinh(a+bx)}{4\left(b^2-d^2\right)}+$$

Mathematica [A] time = 0.99, size = 80, normalized size = 0.63

$$\frac{1}{4}e^{c+dx}\left(\frac{3b\sinh(3(a+bx))-d\cosh(3(a+bx))}{9b^2-d^2}+\frac{d\cosh(a+bx)-b\sinh(a+bx)}{(b-d)(b+d)}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(c + d*x)*Cosh[a + b*x]*Sinh[a + b*x]^2,x]
```

```
[Out] (E^(c + d*x)*((d*Cosh[a + b*x] - b*Sinh[a + b*x])/((b - d)*(b + d)) + (-(d*
Cosh[3*(a + b*x)]) + 3*b*Sinh[3*(a + b*x)])/(9*b^2 - d^2)))/4
```

fricas [B] time = 0.43, size = 379, normalized size = 2.98

$$-\frac{3\left(b^2d-d^3\right)\cosh\left(bx+a\right)\cosh\left(dx+c\right)\sinh\left(bx+a\right)^2-3\left(b^3-bd^2\right)\cosh\left(dx+c\right)\sinh\left(bx+a\right)^3+\left(9\,b^3-bd\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x+c)*cosh(b*x+a)*sinh(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/4*(3*(b^2*d - d^3)*cosh(b*x + a)*cosh(d*x + c)*sinh(b*x + a)^2 - 3*(b^3
- b*d^2)*cosh(d*x + c)*sinh(b*x + a)^3 + (9*b^3 - b*d^2 - 9*(b^3 - b*d^2)*c
osh(b*x + a)^2)*cosh(d*x + c)*sinh(b*x + a) + ((b^2*d - d^3)*cosh(b*x + a)^
3 - (9*b^2*d - d^3)*cosh(b*x + a))*cosh(d*x + c) + ((b^2*d - d^3)*cosh(b*x
+ a)^3 + 3*(b^2*d - d^3)*cosh(b*x + a)*sinh(b*x + a)^2 - 3*(b^3 - b*d^2)*si
nh(b*x + a)^3 - (9*b^2*d - d^3)*cosh(b*x + a) + (9*b^3 - b*d^2 - 9*(b^3 - b
*d^2)*cosh(b*x + a)^2)*sinh(b*x + a))*sinh(d*x + c))/((9*b^4 - 10*b^2*d^2 +
 d^4)*cosh(b*x + a)^4 - 2*(9*b^4 - 10*b^2*d^2 + d^4)*cosh(b*x + a)^2*sinh(b
*x + a)^2 + (9*b^4 - 10*b^2*d^2 + d^4)*sinh(b*x + a)^4)
```

giac [A] time = 0.14, size = 86, normalized size = 0.68

$$\frac{e^{(3\,bx+dx+3\,a+c)}}{8\,(3\,b+d)}-\frac{e^{(bx+dx+a+c)}}{8\,(b+d)}+\frac{e^{(-bx+dx-a+c)}}{8\,(b-d)}-\frac{e^{(-3\,bx+dx-3\,a+c)}}{8\,(3\,b-d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x+c)*cosh(b*x+a)*sinh(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/8*e^(3*b*x + d*x + 3*a + c)/(3*b + d) - 1/8*e^(b*x + d*x + a + c)/(b + d)
 + 1/8*e^(-b*x + d*x - a + c)/(b - d) - 1/8*e^(-3*b*x + d*x - 3*a + c)/(3*b
 - d)
```

maple [A] time = 0.35, size = 178, normalized size = 1.40

$$-\frac{\sinh\left(a-c+\left(b-d\right)x\right)}{8\left(b-d\right)}-\frac{\sinh\left(a+c+\left(b+d\right)x\right)}{8\left(b+d\right)}+\frac{\sinh\left(3a-c+\left(3b-d\right)x\right)}{24b-8d}+\frac{\sinh\left(3a+c+\left(3b+d\right)x\right)}{24b+8d}+\frac{\cosh\left(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(d*x+c)*cosh(b*x+a)*sinh(b*x+a)^2,x)
```

```
[Out] -1/8*sinh(a-c+(b-d)*x)/(b-d)-1/8*sinh(a+c+(b+d)*x)/(b+d)+1/8*sinh(3*a-c+(3*
b-d)*x)/(3*b-d)+1/8*sinh(3*a+c+(3*b+d)*x)/(3*b+d)+1/8*cosh(a-c+(b-d)*x)/(b-
d)-1/8*cosh(a+c+(b+d)*x)/(b+d)-1/8*cosh(3*a-c+(3*b-d)*x)/(3*b-d)+1/8*cosh(3
*a+c+(3*b+d)*x)/(3*b+d)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x+c)*cosh(b*x+a)*sinh(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(-d/b>0)', see `assume?` for more de
tails)Is -d/b equal to -1?
```

mupad [B] time = 2.27, size = 126, normalized size = 0.99

$$\frac{e^{c+d\,x}\,\left(3\,b^3\,\sinh\left(a+b\,x\right)^3+2\,b^2\,d\,\cosh\left(a+b\,x\right)^3-3\,b^2\,d\,\cosh\left(a+b\,x\right)\,\sinh\left(a+b\,x\right)^2-2\,b\,d^2\,\cosh\left(a+b\,x\right)^2\,\mathrm{si}\right.}{9\,b^4-10\,b^2\,d^2+d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a + b*x)*exp(c + d*x)*sinh(a + b*x)^2,x)
```

```
[Out] (exp(c + d*x)*(3*b^3*sinh(a + b*x)^3 + 2*b^2*d*cosh(a + b*x)^3 + d^3*cosh(a
 + b*x)*sinh(a + b*x)^2 - b*d^2*sinh(a + b*x)^3 - 2*b*d^2*cosh(a + b*x)^2*s
inh(a + b*x) - 3*b^2*d*cosh(a + b*x)*sinh(a + b*x)^2))/(9*b^4 + d^4 - 10*b^
2*d^2)
```

sympy [A] time = 43.50, size = 972, normalized size = 7.65

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x+c)*cosh(b*x+a)*sinh(b*x+a)**2,x)
```

```
[Out] Piecewise((x*exp(c)*sinh(a)**2*cosh(a), Eq(b, 0) & Eq(d, 0)), (x*exp(c)*exp
(d*x)*sinh(a - d*x)**3/8 + x*exp(c)*exp(d*x)*sinh(a - d*x)**2*cosh(a - d*x)
/8 - x*exp(c)*exp(d*x)*sinh(a - d*x)*cosh(a - d*x)**2/8 - x*exp(c)*exp(d*x)
*cosh(a - d*x)**3/8 - 3*exp(c)*exp(d*x)*sinh(a - d*x)**3/(8*d) - exp(c)*exp
(d*x)*sinh(a - d*x)**2*cosh(a - d*x)/(4*d) + exp(c)*exp(d*x)*cosh(a - d*x)*
*3/(8*d), Eq(b, -d)), (x*exp(c)*exp(d*x)*sinh(a - d*x/3)**3/8 + 3*x*exp(c)*
exp(d*x)*sinh(a - d*x/3)**2*cosh(a - d*x/3)/8 + 3*x*exp(c)*exp(d*x)*sinh(a
- d*x/3)*cosh(a - d*x/3)**2/8 + x*exp(c)*exp(d*x)*cosh(a - d*x/3)**3/8 - ex
p(c)*exp(d*x)*sinh(a - d*x/3)**3/(8*d) - 3*exp(c)*exp(d*x)*sinh(a - d*x/3)*
cosh(a - d*x/3)**2/(4*d) - 3*exp(c)*exp(d*x)*cosh(a - d*x/3)**3/(8*d), Eq(b
, -d/3)), (-x*exp(c)*exp(d*x)*sinh(a + d*x/3)**3/8 + 3*x*exp(c)*exp(d*x)*si
nh(a + d*x/3)**2*cosh(a + d*x/3)/8 - 3*x*exp(c)*exp(d*x)*sinh(a + d*x/3)*co
sh(a + d*x/3)**2/8 + x*exp(c)*exp(d*x)*cosh(a + d*x/3)**3/8 - exp(c)*exp(d*
x)*sinh(a + d*x/3)**3/(8*d) + 3*exp(c)*exp(d*x)*sinh(a + d*x/3)**2*cosh(a +
 d*x/3)/(4*d) - exp(c)*exp(d*x)*cosh(a + d*x/3)**3/(8*d), Eq(b, d/3)), (-x*
exp(c)*exp(d*x)*sinh(a + d*x)**3/8 + x*exp(c)*exp(d*x)*sinh(a + d*x)**2*cos
h(a + d*x)/8 + x*exp(c)*exp(d*x)*sinh(a + d*x)*cosh(a + d*x)**2/8 - x*exp(c
)*exp(d*x)*cosh(a + d*x)**3/8 + exp(c)*exp(d*x)*sinh(a + d*x)**3/(8*d) + ex
p(c)*exp(d*x)*sinh(a + d*x)*cosh(a + d*x)**2/(4*d) - exp(c)*exp(d*x)*cosh(a
 + d*x)**3/(8*d), Eq(b, d)), (3*b**3*exp(c)*exp(d*x)*sinh(a + b*x)**3/(9*b*
*4 - 10*b**2*d**2 + d**4) - 3*b**2*d*exp(c)*exp(d*x)*sinh(a + b*x)**2*cosh(
a + b*x)/(9*b**4 - 10*b**2*d**2 + d**4) + 2*b**2*d*exp(c)*exp(d*x)*cosh(a +
 b*x)**3/(9*b**4 - 10*b**2*d**2 + d**4) - b*d**2*exp(c)*exp(d*x)*sinh(a + b
*x)**3/(9*b**4 - 10*b**2*d**2 + d**4) - 2*b*d**2*exp(c)*exp(d*x)*sinh(a + b
*x)*cosh(a + b*x)**2/(9*b**4 - 10*b**2*d**2 + d**4) + d**3*exp(c)*exp(d*x)*
sinh(a + b*x)**2*cosh(a + b*x)/(9*b**4 - 10*b**2*d**2 + d**4), True))
```

3.947 $\int e^{c+dx}\cosh(a+bx)\sinh(a+bx)\,dx$

Optimal. Leaf size=66

$$\frac{be^{c+dx}\cosh(2a+2bx)}{4b^2-d^2}-\frac{de^{c+dx}\sinh(2a+2bx)}{2\left(4b^2-d^2\right)}$$

```
[Out] b*exp(d*x+c)*cosh(2*b*x+2*a)/(4*b^2-d^2)-1/2*d*exp(d*x+c)*sinh(2*b*x+2*a)/(
4*b^2-d^2)
```

Rubi [A] time = 0.05, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {5509, 12, 5474}

$$\frac{be^{c+dx}\cosh(2a+2bx)}{4b^2-d^2}-\frac{de^{c+dx}\sinh(2a+2bx)}{2\left(4b^2-d^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[E^(c + d*x)*Cosh[a + b*x]*Sinh[a + b*x],x]
```

```
[Out] (b*E^(c + d*x)*Cosh[2*a + 2*b*x])/(4*b^2 - d^2) - (d*E^(c + d*x)*Sinh[2*a +
 2*b*x])/(2*(4*b^2 - d^2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 5474

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sinh[(d_.) + (e_.)*(x_)], x_Symbol] :
> -Simp[(b*c*Log[F]*F^(c*(a + b*x))*Sinh[d + e*x])/(e^2 - b^2*c^2*Log[F]^2)
, x] + Simp[(e*F^(c*(a + b*x))*Cosh[d + e*x])/(e^2 - b^2*c^2*Log[F]^2), x]
/; FreeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 - b^2*c^2*Log[F]^2, 0]
```

Rule 5509

```
Int[Cosh[(f_.) + (g_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sinh[(
d_.) + (e_.)*(x_)]^(m_.), x_Symbol] :> Int[ExpandTrigReduce[F^(c*(a + b*x))
, Sinh[d + e*x]^m*Cosh[f + g*x]^n, x], x] /; FreeQ[{F, a, b, c, d, e, f, g}
, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

Rubi steps

$$\int e^{c+dx}\cosh(a+bx)\sinh(a+bx)\,dx=\int\frac{1}{2}e^{c+dx}\sinh(2a+2bx)\,dx$$
$$=\frac{1}{2}\int e^{c+dx}\sinh(2a+2bx)\,dx$$
$$=\frac{be^{c+dx}\cosh(2a+2bx)}{4b^2-d^2}-\frac{de^{c+dx}\sinh(2a+2bx)}{2\left(4b^2-d^2\right)}$$

Mathematica [A] time = 0.05, size = 47, normalized size = 0.71

$$\frac{e^{c+dx}(2b\cosh(2(a+bx))-d\sinh(2(a+bx)))}{2\left(4b^2-d^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(c + d*x)*Cosh[a + b*x]*Sinh[a + b*x],x]
```

```
[Out] (E^(c + d*x)*(2*b*Cosh[2*(a + b*x)] - d*Sinh[2*(a + b*x)]))/(2*(4*b^2 - d^2
))
```

fricas [B] time = 0.43, size = 142, normalized size = 2.15

$$\frac{b\cosh(bx+a)^2\cosh(dx+c)-d\cosh(bx+a)\cosh(dx+c)\sinh(bx+a)+b\cosh(dx+c)\sinh(bx+a)^2+(\ldots}{\left(4\,b^2-d^2\right)\cosh(bx+a)^2-\left(4\,b^2-d^2\right)\sin\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x+c)*cosh(b*x+a)*sinh(b*x+a),x, algorithm="fricas")
```

```
[Out] (b*cosh(b*x + a)^2*cosh(d*x + c) - d*cosh(b*x + a)*cosh(d*x + c)*sinh(b*x +
 a) + b*cosh(d*x + c)*sinh(b*x + a)^2 + (b*cosh(b*x + a)^2 - d*cosh(b*x + a
)*sinh(b*x + a) + b*sinh(b*x + a)^2)*sinh(d*x + c))/((4*b^2 - d^2)*cosh(b*x
 + a)^2 - (4*b^2 - d^2)*sinh(b*x + a)^2)
```

giac [A] time = 0.13, size = 47, normalized size = 0.71

$$\frac{e^{(2\,bx+dx+2\,a+c)}}{4\,(2\,b+d)}+\frac{e^{(-2\,bx+dx-2\,a+c)}}{4\,(2\,b-d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x+c)*cosh(b*x+a)*sinh(b*x+a),x, algorithm="giac")
```

```
[Out] 1/4*e^(2*b*x + d*x + 2*a + c)/(2*b + d) + 1/4*e^(-2*b*x + d*x - 2*a + c)/(2
*b - d)
```

maple [A] time = 0.20, size = 102, normalized size = 1.55

$$-\frac{\sinh\left(2a-c+(2b-d)\,x\right)}{4\,(2b-d)}+\frac{\sinh\left(2a+c+(2b+d)\,x\right)}{8b+4d}+\frac{\cosh\left(2a-c+(2b-d)\,x\right)}{8b-4d}+\frac{\cosh\left(2a+c+(2b+d)\,x\right)}{8b+4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(d*x+c)*cosh(b*x+a)*sinh(b*x+a),x)
```

```
[Out] -1/4*sinh(2*a-c+(2*b-d)*x)/(2*b-d)+1/4*sinh(2*a+c+(2*b+d)*x)/(2*b+d)+1/4*co
sh(2*a-c+(2*b-d)*x)/(2*b-d)+1/4*cosh(2*a+c+(2*b+d)*x)/(2*b+d)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x+c)*cosh(b*x+a)*sinh(b*x+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(1-d/b>0)', see `assume?` for more d
etails)Is 1-d/b equal to -1?
```

mupad [B] time = 1.91, size = 58, normalized size = 0.88

$$\frac{\mathrm{e}^{c+d\,x}\,\mathrm{e}^{-2\,a-2\,b\,x}\,\left(2\,b+d+2\,b\,\mathrm{e}^{4\,a+4\,b\,x}-d\,\mathrm{e}^{4\,a+4\,b\,x}\right)}{4\,\left(4\,b^2-d^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a + b*x)*exp(c + d*x)*sinh(a + b*x),x)
```

```
[Out] (exp(c + d*x)*exp(- 2*a - 2*b*x)*(2*b + d + 2*b*exp(4*a + 4*b*x) - d*exp(4*
a + 4*b*x)))/(4*(4*b^2 - d^2))
```

sympy [A] time = 8.45, size = 304, normalized size = 4.61

$$\begin{cases} xe^c\sinh(a)\cosh(a) & \text{for } b=0\wedge d=0 \\ \frac{xe^ce^{dx}\sinh^2\left(a-\frac{dx}{2}\right)}{4}+\frac{xe^ce^{dx}\sinh\left(a-\frac{dx}{2}\right)\cosh\left(a-\frac{dx}{2}\right)}{2}+\frac{xe^ce^{dx}\cosh^2\left(a-\frac{dx}{2}\right)}{4}+\frac{e^ce^{dx}\sinh\left(a-\frac{dx}{2}\right)\cosh\left(a-\frac{dx}{2}\right)}{2d} & \text{for } b=-\frac{d}{2} \\ -\frac{xe^ce^{dx}\sinh^2\left(a+\frac{dx}{2}\right)}{4}+\frac{xe^ce^{dx}\sinh\left(a+\frac{dx}{2}\right)\cosh\left(a+\frac{dx}{2}\right)}{2}-\frac{xe^ce^{dx}\cosh^2\left(a+\frac{dx}{2}\right)}{4}+\frac{e^ce^{dx}\sinh\left(a+\frac{dx}{2}\right)\cosh\left(a+\frac{dx}{2}\right)}{2d} & \text{for } b=\frac{d}{2} \\ \frac{be^ce^{dx}\sinh^2(a+bx)}{4b^2-d^2}+\frac{be^ce^{dx}\cosh^2(a+bx)}{4b^2-d^2}-\frac{de^ce^{dx}\sinh(a+bx)\cosh(a+bx)}{4b^2-d^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x+c)*cosh(b*x+a)*sinh(b*x+a),x)
```

```
[Out] Piecewise((x*exp(c)*sinh(a)*cosh(a), Eq(b, 0) & Eq(d, 0)), (x*exp(c)*exp(d*
x)*sinh(a - d*x/2)**2/4 + x*exp(c)*exp(d*x)*sinh(a - d*x/2)*cosh(a - d*x/2)
/2 + x*exp(c)*exp(d*x)*cosh(a - d*x/2)**2/4 + exp(c)*exp(d*x)*sinh(a - d*x/
2)*cosh(a - d*x/2)/(2*d), Eq(b, -d/2)), (-x*exp(c)*exp(d*x)*sinh(a + d*x/2)
**2/4 + x*exp(c)*exp(d*x)*sinh(a + d*x/2)*cosh(a + d*x/2)/2 - x*exp(c)*exp(
d*x)*cosh(a + d*x/2)**2/4 + exp(c)*exp(d*x)*sinh(a + d*x/2)*cosh(a + d*x/2)
/(2*d), Eq(b, d/2)), (b*exp(c)*exp(d*x)*sinh(a + b*x)**2/(4*b**2 - d**2) +
b*exp(c)*exp(d*x)*cosh(a + b*x)**2/(4*b**2 - d**2) - d*exp(c)*exp(d*x)*sinh
(a + b*x)*cosh(a + b*x)/(4*b**2 - d**2), True))
```

3.948 $\int e^{c+dx} \cosh(a+bx)\,dx$

Optimal. Leaf size=54

$$\frac{be^{c+dx}\sinh(a+bx)}{b^2-d^2}-\frac{de^{c+dx}\cosh(a+bx)}{b^2-d^2}$$

```
[Out] -d*exp(d*x+c)*cosh(b*x+a)/(b^2-d^2)+b*exp(d*x+c)*sinh(b*x+a)/(b^2-d^2)
```

Rubi [A] time = 0.02, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {5475}

$$\frac{be^{c+dx}\sinh(a+bx)}{b^2-d^2}-\frac{de^{c+dx}\cosh(a+bx)}{b^2-d^2}$$

Antiderivative was successfully verified.

```
[In] Int[E^(c + d*x)*Cosh[a + b*x],x]
```

```
[Out] -((d*E^(c + d*x)*Cosh[a + b*x])/(b^2 - d^2)) + (b*E^(c + d*x)*Sinh[a + b*x]
)/(b^2 - d^2)
```

Rule 5475

```
Int[Cosh[(d_.) + (e_.)*(x_)]*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symbol] :
> -Simp[(b*c*Log[F]*F^(c*(a + b*x))*Cosh[d + e*x])/(e^2 - b^2*c^2*Log[F]^2)
, x] + Simp[(e*F^(c*(a + b*x))*Sinh[d + e*x])/(e^2 - b^2*c^2*Log[F]^2), x]
/; FreeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 - b^2*c^2*Log[F]^2, 0]
```

Rubi steps

$$\int e^{c+dx}\cosh(a+bx)\,dx=-\frac{de^{c+dx}\cosh(a+bx)}{b^2-d^2}+\frac{be^{c+dx}\sinh(a+bx)}{b^2-d^2}$$

Mathematica [A] time = 0.05, size = 38, normalized size = 0.70

$$\frac{e^{c+dx}(b\sinh(a+bx)-d\cosh(a+bx))}{(b-d)(b+d)}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(c + d*x)*Cosh[a + b*x],x]
```

```
[Out] (E^(c + d*x)*(-(d*Cosh[a + b*x]) + b*Sinh[a + b*x]))/((b - d)*(b + d))
```

fricas [A] time = 0.48, size = 97, normalized size = 1.80

$$-\frac{d\cosh\left(bx+a\right)\cosh\left(dx+c\right)-b\cosh\left(dx+c\right)\sinh\left(bx+a\right)+\left(d\cosh\left(bx+a\right)-b\sinh\left(bx+a\right)\right)\sinh\left(dx+c\right)}{\left(b^2-d^2\right)\cosh\left(bx+a\right)^2-\left(b^2-d^2\right)\sinh\left(bx+a\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x+c)*cosh(b*x+a),x, algorithm="fricas")
```

```
[Out] -(d*cosh(b*x + a)*cosh(d*x + c) - b*cosh(d*x + c)*sinh(b*x + a) + (d*cosh(b
*x + a) - b*sinh(b*x + a))*sinh(d*x + c))/((b^2 - d^2)*cosh(b*x + a)^2 - (b
^2 - d^2)*sinh(b*x + a)^2)
```

giac [A] time = 0.13, size = 40, normalized size = 0.74

$$\frac{e^{(bx+dx+a+c)}}{2\left(b+d\right)}-\frac{e^{(-bx+dx-a+c)}}{2\left(b-d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x+c)*cosh(b*x+a),x, algorithm="giac")
```

```
[Out] 1/2*e^(b*x + d*x + a + c)/(b + d) - 1/2*e^(-b*x + d*x - a + c)/(b - d)
```

maple [A] time = 0.22, size = 78, normalized size = 1.44

$$\frac{\sinh\left(a-c+\left(b-d\right)x\right)}{2b-2d}+\frac{\sinh\left(a+c+\left(b+d\right)x\right)}{2b+2d}-\frac{\cosh\left(a-c+\left(b-d\right)x\right)}{2\left(b-d\right)}+\frac{\cosh\left(a+c+\left(b+d\right)x\right)}{2b+2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(d*x+c)*cosh(b*x+a),x)
```

```
[Out] 1/2*sinh(a-c+(b-d)*x)/(b-d)+1/2*sinh(a+c+(b+d)*x)/(b+d)-1/2*cosh(a-c+(b-d)*
x)/(b-d)+1/2*cosh(a+c+(b+d)*x)/(b+d)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x+c)*cosh(b*x+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
```

```
elp (example of legal syntax is 'assume(-d/b>0)', see `assume?` for more de
tails)Is -d/b equal to -1?
```

mupad [B] time = 1.75, size = 54, normalized size = 1.00

$$-\frac{e^{c-a-b\,x+d\,x}\,\left(b+d-b\,e^{2\,a+2\,b\,x}+d\,e^{2\,a+2\,b\,x}\right)}{2\,\left(b^2-d^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a + b*x)*exp(c + d*x),x)
```

```
[Out] -(exp(c - a - b*x + d*x)*(b + d - b*exp(2*a + 2*b*x) + d*exp(2*a + 2*b*x)))
/(2*(b^2 - d^2))
```

sympy [A] time = 2.10, size = 184, normalized size = 3.41

$$\begin{cases} xe^{c}\cosh{(a)} & \text{for}\: b=0 \wedge d=0 \\ \frac{xe^{c}e^{dx}\sinh{(a-dx)}}{2}+\frac{xe^{c}e^{dx}\cosh{(a-dx)}}{2}-\frac{e^{c}e^{dx}\sinh{(a-dx)}}{2d} & \text{for}\: b=-d \\ -\frac{xe^{c}e^{dx}\sinh{(a+dx)}}{2}+\frac{xe^{c}e^{dx}\cosh{(a+dx)}}{2}+\frac{e^{c}e^{dx}\sinh{(a+dx)}}{d}-\frac{e^{c}e^{dx}\cosh{(a+dx)}}{2d} & \text{for}\: b=d \\ \frac{be^{c}e^{dx}\sinh{(a+bx)}}{b^{2}-d^{2}}-\frac{de^{c}e^{dx}\cosh{(a+bx)}}{b^{2}-d^{2}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x+c)*cosh(b*x+a),x)
```

```
[Out] Piecewise((x*exp(c)*cosh(a), Eq(b, 0) & Eq(d, 0)), (x*exp(c)*exp(d*x)*sinh(
a - d*x)/2 + x*exp(c)*exp(d*x)*cosh(a - d*x)/2 - exp(c)*exp(d*x)*sinh(a - d
*x)/(2*d), Eq(b, -d)), (-x*exp(c)*exp(d*x)*sinh(a + d*x)/2 + x*exp(c)*exp(d
*x)*cosh(a + d*x)/2 + exp(c)*exp(d*x)*sinh(a + d*x)/d - exp(c)*exp(d*x)*cos
h(a + d*x)/(2*d), Eq(b, d)), (b*exp(c)*exp(d*x)*sinh(a + b*x)/(b**2 - d**2)
 - d*exp(c)*exp(d*x)*cosh(a + b*x)/(b**2 - d**2), True))
```

3.949 $\int e^{c+dx}\coth(a+bx)\,dx$

Optimal. Leaf size=53

$$\frac{e^{c+dx}}{d}-\frac{2e^{c+dx}\,{}_2F_1\left(1,\frac{d}{2b};\frac{d}{2b}+1;e^{2(a+bx)}\right)}{d}$$

```
[Out] exp(d*x+c)/d-2*exp(d*x+c)*hypergeom([1, 1/2*d/b],[1+1/2*d/b],exp(2*b*x+2*a)
)/d
```

Rubi [A] time = 0.06, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {5485, 2194, 2251}

$$\frac{e^{c+dx}}{d}-\frac{2e^{c+dx}\,{}_2F_1\left(1,\frac{d}{2b};\frac{d}{2b}+1;e^{2(a+bx)}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[E^(c + d*x)*Coth[a + b*x],x]
```

```
[Out] E^(c + d*x)/d - (2*E^(c + d*x)*Hypergeometric2F1[1, d/(2*b), 1 + d/(2*b), E
^(2*(a + b*x))])/d
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 2251

```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_)*(G_)^((h_.)*((f_.
) + (g_.)*(x_))), x_Symbol] :> Simp[(a^p*G^(h*(f + g*x))*Hypergeometric2F1[
-p, (g*h*Log[G])/(d*e*Log[F]), (g*h*Log[G])/(d*e*Log[F]) + 1, Simplify[-((b
*F^(e*(c + d*x)))/a)]])/(g*h*Log[G]), x] /; FreeQ[{F, G, a, b, c, d, e, f,
g, h, p}, x] && (ILtQ[p, 0] || GtQ[a, 0])
```

Rule 5485

```
Int[Coth[(d_.) + (e_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Sym
bol] :> Int[ExpandIntegrand[(F^(c*(a + b*x))*(1 + E^(2*(d + e*x)))^n)/(-1 +
 E^(2*(d + e*x)))^n, x], x] /; FreeQ[{F, a, b, c, d, e}, x] && IntegerQ[n]
```

Rubi steps

$$\int e^{c+dx}\coth(a+bx)\,dx = \int \left(e^{c+dx}+\frac{2e^{c+dx}}{-1+e^{2(a+bx)}}\right)dx$$
$$= 2\int \frac{e^{c+dx}}{-1+e^{2(a+bx)}}\,dx + \int e^{c+dx}\,dx$$
$$= \frac{e^{c+dx}}{d} - \frac{2e^{c+dx}\,{}_2F_1\left(1,\frac{d}{2b};1+\frac{d}{2b};e^{2(a+bx)}\right)}{d}$$

Mathematica [B] time = 1.94, size = 120, normalized size = 2.26

$$\frac{\coth(a)e^{c+dx}}{d} - \frac{2e^{2a+c}\left(\frac{e^{dx}\,{}_2F_1\left(1,\frac{d}{2b};\frac{d}{2b}+1;e^{2(a+bx)}\right)}{d} - \frac{e^{x(2b+d)}\,{}_2F_1\left(1,\frac{d}{2b}+1;\frac{d}{2b}+2;e^{2(a+bx)}\right)}{2b+d}\right)}{e^{2a}-1}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(c + d*x)*Coth[a + b*x],x]
```

```
[Out] (E^(c + d*x)*Coth[a])/d - (2*E^(2*a + c)*((E^(d*x)*Hypergeometric2F1[1, d/(
2*b), 1 + d/(2*b), E^(2*(a + b*x))])/d - (E^((2*b + d)*x)*Hypergeometric2F1
[1, 1 + d/(2*b), 2 + d/(2*b), E^(2*(a + b*x))])/(2*b + d)))/(-1 + E^(2*a))
```

fricas [F] time = 0.43, size = 0, normalized size = 0.00

$$\text{integral}\left(\cosh(bx+a)\,\text{csch}(bx+a)\,e^{(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x+c)*cosh(b*x+a)*csch(b*x+a),x, algorithm="fricas")
```

```
[Out] integral(cosh(b*x + a)*csch(b*x + a)*e^(d*x + c), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \cosh(bx+a)\,\text{csch}(bx+a)\,e^{(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x+c)*cosh(b*x+a)*csch(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(cosh(b*x + a)*csch(b*x + a)*e^(d*x + c), x)
```

maple [F] time = 0.30, size = 0, normalized size = 0.00

$$\int \mathrm{e}^{dx+c}\cosh\left(bx+a\right)\operatorname{csch}\left(bx+a\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(d*x+c)*cosh(b*x+a)*csch(b*x+a),x)
```

```
[Out] int(exp(d*x+c)*cosh(b*x+a)*csch(b*x+a),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-4\,b\int\frac{e^{(dx+c)}}{(2\,b-d)e^{(4\,bx+4\,a)}-2\,(2\,b-d)e^{(2\,bx+2\,a)}+2\,b-d}\,dx-\frac{\left((2\,be^{c}-de^{c})e^{(2\,bx+2\,a)}-2\,be^{c}-de^{c}\right)e^{(dx)}}{2\,bd-d^{2}-\left(2\,bd-d^{2}\right)e^{(2\,bx+2\,a)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x+c)*cosh(b*x+a)*csch(b*x+a),x, algorithm="maxima")
```

```
[Out] -4*b*integrate(e^(d*x + c)/((2*b - d)*e^(4*b*x + 4*a) - 2*(2*b - d)*e^(2*b*
x + 2*a) + 2*b - d), x) - ((2*b*e^c - d*e^c)*e^(2*b*x + 2*a) - 2*b*e^c - d*
e^c)*e^(d*x)/(2*b*d - d^2 - (2*b*d - d^2)*e^(2*b*x + 2*a))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\cosh\left(a+b\,x\right)\,\mathrm{e}^{c+d\,x}}{\sinh\left(a+b\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cosh(a + b*x)*exp(c + d*x))/sinh(a + b*x),x)
```

```
[Out] int((cosh(a + b*x)*exp(c + d*x))/sinh(a + b*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$e^{c}\int e^{dx}\cosh\left(a+bx\right)\operatorname{csch}\left(a+bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x+c)*cosh(b*x+a)*csch(b*x+a),x)
```

```
[Out] exp(c)*Integral(exp(d*x)*cosh(a + b*x)*csch(a + b*x), x)
```

3.950 $\int e^{c+dx}\coth(a+bx)\mathbf{csch}(a+bx)\,dx$

Optimal. Leaf size=101

$$\frac{4e^{a+x(b+d)+c}\,{}_2F_1\left(2,\frac{b+d}{2b};\frac{3b+d}{2b};e^{2(a+bx)}\right)}{b+d}-\frac{2e^{a+x(b+d)+c}\,{}_2F_1\left(1,\frac{b+d}{2b};\frac{3b+d}{2b};e^{2(a+bx)}\right)}{b+d}$$

```
[Out] -2*exp(a+c+(b+d)*x)*hypergeom([1, 1/2*(b+d)/b],[1/2*(3*b+d)/b],exp(2*b*x+2*
a))/(b+d)+4*exp(a+c+(b+d)*x)*hypergeom([2, 1/2*(b+d)/b],[1/2*(3*b+d)/b],exp
(2*b*x+2*a))/(b+d)
```

Rubi [A] time = 0.25, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {5511, 2251}

$$\frac{4e^{a+x(b+d)+c}\,{}_2F_1\left(2,\frac{b+d}{2b};\frac{3b+d}{2b};e^{2(a+bx)}\right)}{b+d}-\frac{2e^{a+x(b+d)+c}\,{}_2F_1\left(1,\frac{b+d}{2b};\frac{3b+d}{2b};e^{2(a+bx)}\right)}{b+d}$$

Antiderivative was successfully verified.

```
[In] Int[E^(c + d*x)*Coth[a + b*x]*Csch[a + b*x],x]
```

```
[Out] (-2*E^(a + c + (b + d)*x)*Hypergeometric2F1[1, (b + d)/(2*b), (3*b + d)/(2*
b), E^(2*(a + b*x))])/(b + d) + (4*E^(a + c + (b + d)*x)*Hypergeometric2F1[
2, (b + d)/(2*b), (3*b + d)/(2*b), E^(2*(a + b*x))])/(b + d)
```

Rule 2251

```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_)*(G_)^((h_.)*((f_.
) + (g_.)*(x_))), x_Symbol] :> Simp[(a^p*G^(h*(f + g*x))*Hypergeometric2F1[
-p, (g*h*Log[G])/(d*e*Log[F]), (g*h*Log[G])/(d*e*Log[F]) + 1, Simplify[-((b
*F^(e*(c + d*x)))/a)]])/(g*h*Log[G]), x] /; FreeQ[{F, G, a, b, c, d, e, f,
g, h, p}, x] && (ILtQ[p, 0] || GtQ[a, 0])
```

Rule 5511

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*(G_)[(d_.) + (e_.)*(x_)]^(m_.)*(H_)[(
d_.) + (e_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^(c*(a + b*x)),
 G[d + e*x]^m*H[d + e*x]^n, x], x] /; FreeQ[{F, a, b, c, d, e}, x] && IGtQ[
m, 0] && IGtQ[n, 0] && HyperbolicQ[G] && HyperbolicQ[H]
```

Rubi steps

$$\int e^{c+dx}\coth(a+bx)\text{csch}(a+bx)\,dx = \int\left(\frac{4e^{a+c+(b+d)x}}{\left(-1+e^{2(a+bx)}\right)^2}+\frac{2e^{a+c+(b+d)x}}{-1+e^{2(a+bx)}}\right)dx$$
$$= 2\int\frac{e^{a+c+(b+d)x}}{-1+e^{2(a+bx)}}\,dx+4\int\frac{e^{a+c+(b+d)x}}{\left(-1+e^{2(a+bx)}\right)^2}\,dx$$
$$= -\frac{2e^{a+c+(b+d)x}\,{}_2F_1\left(1,\frac{b+d}{2b};\frac{3b+d}{2b};e^{2(a+bx)}\right)}{b+d}+\frac{4e^{a+c+(b+d)x}\,{}_2F_1\left(2,\frac{b+d}{2b};\frac{3b+d}{2b};e\right.}{b+d}$$

Mathematica [A] time = 0.67, size = 92, normalized size = 0.91

$$\frac{e^c\text{csch}(a)\left(-2de^{x(b+d)}\,{}_2F_1\left(1,\frac{b+d}{2b};\frac{3b+d}{2b};e^{2bx}(\cosh(a)+\sinh(a))^2\right)-(b+d)e^{dx}(\cosh(a)-\sinh(a))\text{csch}(a+bx)\right)}{b(\coth(a)-1)(b+d)}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(c + d*x)*Coth[a + b*x]*Csch[a + b*x],x]
```

```
[Out] (E^c*Csch[a]*(-2*d*E^((b + d)*x)*Hypergeometric2F1[1, (b + d)/(2*b), (3*b +
 d)/(2*b), E^(2*b*x)*(Cosh[a] + Sinh[a])^2] - (b + d)*E^(d*x)*Csch[a + b*x]
*(Cosh[a] - Sinh[a])))/(b*(b + d)*(-1 + Coth[a]))
```

fricas [F] time = 0.44, size = 0, normalized size = 0.00

$$\text{integral}\left(\cosh\left(bx+a\right)\text{csch}\left(bx+a\right)^2e^{(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x+c)*cosh(b*x+a)*csch(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] integral(cosh(b*x + a)*csch(b*x + a)^2*e^(d*x + c), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\cosh\left(bx+a\right)\text{csch}\left(bx+a\right)^2e^{(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x+c)*cosh(b*x+a)*csch(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate(cosh(b*x + a)*csch(b*x + a)^2*e^(d*x + c), x)
```

maple [F] time = 0.29, size = 0, normalized size = 0.00

$$\int \mathrm{e}^{dx+c} \cosh \left(bx+a\right) \operatorname{csch}\left(bx+a\right)^{2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(d*x+c)*cosh(b*x+a)*csch(b*x+a)^2,x)
```

```
[Out] int(exp(d*x+c)*cosh(b*x+a)*csch(b*x+a)^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$16\,bd \int -\frac{e^{(bx+dx+a+c)}}{3\,b^{2}-4\,bd+d^{2}-\left(3\,b^{2}-4\,bd+d^{2}\right)e^{(6\,bx+6\,a)}+3\left(3\,b^{2}-4\,bd+d^{2}\right)e^{(4\,bx+4\,a)}-3\left(3\,b^{2}-4\,bd+d^{2}\right)e^{(2\,bx+}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x+c)*cosh(b*x+a)*csch(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 16*b*d*integrate(-e^(b*x + d*x + a + c)/(3*b^2 - 4*b*d + d^2 - (3*b^2 - 4*b
*d + d^2)*e^(6*b*x + 6*a) + 3*(3*b^2 - 4*b*d + d^2)*e^(4*b*x + 4*a) - 3*(3*
b^2 - 4*b*d + d^2)*e^(2*b*x + 2*a)), x) - 2*((3*b*e^c - d*e^c)*e^(3*b*x + 3
*a) - (3*b*e^c + d*e^c)*e^(b*x + a))*e^(d*x)/(3*b^2 - 4*b*d + d^2 + (3*b^2
- 4*b*d + d^2)*e^(4*b*x + 4*a) - 2*(3*b^2 - 4*b*d + d^2)*e^(2*b*x + 2*a))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cosh\left(a+b\,x\right)\,{\mathrm{e}}^{c+d\,x}}{{\mathrm{sinh}\left(a+b\,x\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cosh(a + b*x)*exp(c + d*x))/sinh(a + b*x)^2,x)
```

```
[Out] int((cosh(a + b*x)*exp(c + d*x))/sinh(a + b*x)^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x+c)*cosh(b*x+a)*csch(b*x+a)**2,x)
```

```
[Out] Timed out
```

3.951 $\int e^{c+dx} \coth(a+bx)\mathbf{csch}^2(a+bx)\,dx$

Optimal. Leaf size=113

$$\frac{4e^{2a+x(2b+d)+c}\,{}_2F_1\left(2,\frac{1}{2}\left(\frac{d}{b}+2\right);\frac{1}{2}\left(\frac{d}{b}+4\right);e^{2(a+bx)}\right)}{2b+d}-\frac{8e^{2a+x(2b+d)+c}\,{}_2F_1\left(3,\frac{1}{2}\left(\frac{d}{b}+2\right);\frac{1}{2}\left(\frac{d}{b}+4\right);e^{2(a+bx)}\right)}{2b+d}$$

```
[Out] 4*exp(2*a+c+(2*b+d)*x)*hypergeom([2, 1+1/2*d/b],[2+1/2*d/b],exp(2*b*x+2*a))
/(2*b+d)-8*exp(2*a+c+(2*b+d)*x)*hypergeom([3, 1+1/2*d/b],[2+1/2*d/b],exp(2*
b*x+2*a))/(2*b+d)
```

Rubi [A] time = 0.27, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {5511, 2251}

$$\frac{4e^{2a+x(2b+d)+c}\,{}_2F_1\left(2,\frac{1}{2}\left(\frac{d}{b}+2\right);\frac{1}{2}\left(\frac{d}{b}+4\right);e^{2(a+bx)}\right)}{2b+d}-\frac{8e^{2a+x(2b+d)+c}\,{}_2F_1\left(3,\frac{1}{2}\left(\frac{d}{b}+2\right);\frac{1}{2}\left(\frac{d}{b}+4\right);e^{2(a+bx)}\right)}{2b+d}$$

Antiderivative was successfully verified.

```
[In] Int[E^(c + d*x)*Coth[a + b*x]*Csch[a + b*x]^2,x]
```

```
[Out] (4*E^(2*a + c + (2*b + d)*x)*Hypergeometric2F1[2, (2 + d/b)/2, (4 + d/b)/2,
 E^(2*(a + b*x))])/(2*b + d) - (8*E^(2*a + c + (2*b + d)*x)*Hypergeometric2
F1[3, (2 + d/b)/2, (4 + d/b)/2, E^(2*(a + b*x))])/(2*b + d)
```

Rule 2251

```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_)*(G_)^((h_.)*((f_.
) + (g_.)*(x_))), x_Symbol] :> Simp[(a^p*G^(h*(f + g*x))*Hypergeometric2F1[
-p, (g*h*Log[G])/(d*e*Log[F]), (g*h*Log[G])/(d*e*Log[F]) + 1, Simplify[-((b
*F^(e*(c + d*x)))/a)]])/(g*h*Log[G]), x] /; FreeQ[{F, G, a, b, c, d, e, f,
g, h, p}, x] && (ILtQ[p, 0] || GtQ[a, 0])
```

Rule 5511

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*(G_)[(d_.) + (e_.)*(x_)]^(m_.)*(H_)[(
d_.) + (e_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^(c*(a + b*x)),
 G[d + e*x]^m*H[d + e*x]^n, x], x] /; FreeQ[{F, a, b, c, d, e}, x] && IGtQ[
m, 0] && IGtQ[n, 0] && HyperbolicQ[G] && HyperbolicQ[H]
```

Rubi steps

$$\int e^{c+dx}\coth(a+bx)\text{csch}^2(a+bx)\,dx = \int \left(\frac{8e^{2a+c+(2b+d)x}}{\left(-1+e^{2(a+bx)}\right)^3}+\frac{4e^{2a+c+(2b+d)x}}{\left(-1+e^{2(a+bx)}\right)^2}\right)dx$$
$$= 4\int \frac{e^{2a+c+(2b+d)x}}{\left(-1+e^{2(a+bx)}\right)^2}\,dx + 8\int \frac{e^{2a+c+(2b+d)x}}{\left(-1+e^{2(a+bx)}\right)^3}\,dx$$
$$= \frac{4e^{2a+c+(2b+d)x}\,{}_2F_1\left(2,\frac{1}{2}\left(2+\frac{d}{b}\right);\frac{1}{2}\left(4+\frac{d}{b}\right);e^{2(a+bx)}\right)}{2b+d} - \frac{8e^{2a+c+(2b+d)x}\,{}_2F_1\left(\right.}{}$$

Mathematica [A] time = 1.55, size = 159, normalized size = 1.41

$$-\frac{e^{c-\frac{ad}{b}}\left(d^2e^{\left(\frac{d}{b}+2\right)(a+bx)}\,{}_2F_1\left(1,\frac{d}{2b}+1;\frac{d}{2b}+2;e^{2(a+bx)}\right)+d(2b+d)e^{d\left(\frac{a}{b}+x\right)}\,{}_2F_1\left(1,\frac{d}{2b};\frac{d}{2b}+1;e^{2(a+bx)}\right)+(2b+d)e^{d\left(\frac{a}{b}+x\right.}\right.}{2b^2(2b+d)}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(c + d*x)*Coth[a + b*x]*Csch[a + b*x]^2,x]
```

```
[Out] -1/2*(E^(c - (a*d)/b)*((2*b + d)*E^(d*(a/b + x))*(d*Coth[a + b*x] + b*Csch[
a + b*x]^2) + d*(2*b + d)*E^(d*(a/b + x))*Hypergeometric2F1[1, d/(2*b), 1 +
 d/(2*b), E^(2*(a + b*x))] + d^2*E^((2 + d/b)*(a + b*x))*Hypergeometric2F1[
1, 1 + d/(2*b), 2 + d/(2*b), E^(2*(a + b*x))]))/(b^2*(2*b + d))
```

fricas [F] time = 0.46, size = 0, normalized size = 0.00

$$\text{integral}\left(\cosh\left(bx+a\right)\text{csch}\left(bx+a\right)^3 e^{(dx+c)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x+c)*cosh(b*x+a)*csch(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] integral(cosh(b*x + a)*csch(b*x + a)^3*e^(d*x + c), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \cosh\left(bx+a\right)\text{csch}\left(bx+a\right)^3 e^{(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x+c)*cosh(b*x+a)*csch(b*x+a)^3,x, algorithm="giac")
```

```
[Out] integrate(cosh(b*x + a)*csch(b*x + a)^3*e^(d*x + c), x)
```

maple [F] time = 0.36, size = 0, normalized size = 0.00

$$\int e^{dx+c} \cosh (bx+a) \operatorname{csch}(bx+a)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(d*x+c)*cosh(b*x+a)*csch(b*x+a)^3,x)
```

```
[Out] int(exp(d*x+c)*cosh(b*x+a)*csch(b*x+a)^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-48\,bd^2 \int \frac{e^{(dx+}}{48\,b^3-44\,b^2d+12\,bd^2-d^3+\left(48\,b^3-44\,b^2d+12\,bd^2-d^3\right)e^{(8\,bx+8\,a)}-4\left(48\,b^3-44\,b^2d+12\,bd^2-\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x+c)*cosh(b*x+a)*csch(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -48*b*d^2*integrate(e^(d*x + c)/(48*b^3 - 44*b^2*d + 12*b*d^2 - d^3 + (48*b
^3 - 44*b^2*d + 12*b*d^2 - d^3)*e^(8*b*x + 8*a) - 4*(48*b^3 - 44*b^2*d + 12
*b*d^2 - d^3)*e^(6*b*x + 6*a) + 6*(48*b^3 - 44*b^2*d + 12*b*d^2 - d^3)*e^(4
*b*x + 4*a) - 4*(48*b^3 - 44*b^2*d + 12*b*d^2 - d^3)*e^(2*b*x + 2*a)), x) +
 4*(12*b*d*e^c + (24*b^2*e^c - 10*b*d*e^c + d^2*e^c)*e^(4*b*x + 4*a) - (24*
b^2*e^c + 2*b*d*e^c - d^2*e^c)*e^(2*b*x + 2*a))*e^(d*x)/(48*b^3 - 44*b^2*d
+ 12*b*d^2 - d^3 - (48*b^3 - 44*b^2*d + 12*b*d^2 - d^3)*e^(6*b*x + 6*a) + 3
*(48*b^3 - 44*b^2*d + 12*b*d^2 - d^3)*e^(4*b*x + 4*a) - 3*(48*b^3 - 44*b^2*
d + 12*b*d^2 - d^3)*e^(2*b*x + 2*a))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cosh (a+b\,x)\,e^{c+d\,x}}{\sinh (a+b\,x)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cosh(a + b*x)*exp(c + d*x))/sinh(a + b*x)^3,x)
```

```
[Out] int((cosh(a + b*x)*exp(c + d*x))/sinh(a + b*x)^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x+c)*cosh(b*x+a)*csch(b*x+a)**3,x)
```

```
[Out] Timed out
```

3.952 $\int e^{c+dx} \cosh^2(a+bx) \sinh^3(a+bx) \, dx$

Optimal. Leaf size=195

$$\frac{de^{c+dx}\sinh(a+bx)}{8\left(b^2-d^2\right)}+\frac{de^{c+dx}\sinh(3a+3bx)}{16\left(9b^2-d^2\right)}-\frac{de^{c+dx}\sinh(5a+5bx)}{16\left(25b^2-d^2\right)}-\frac{be^{c+dx}\cosh(a+bx)}{8\left(b^2-d^2\right)}-\frac{3be^{c+dx}\cosh(3a+3b}{16\left(9b^2-d^2\right)}$$

```
[Out] -1/8*b*exp(d*x+c)*cosh(b*x+a)/(b^2-d^2)-3/16*b*exp(d*x+c)*cosh(3*b*x+3*a)/(
9*b^2-d^2)+5/16*b*exp(d*x+c)*cosh(5*b*x+5*a)/(25*b^2-d^2)+1/8*d*exp(d*x+c)*
sinh(b*x+a)/(b^2-d^2)+1/16*d*exp(d*x+c)*sinh(3*b*x+3*a)/(9*b^2-d^2)-1/16*d*
exp(d*x+c)*sinh(5*b*x+5*a)/(25*b^2-d^2)
```

Rubi [A] time = 0.14, antiderivative size = 195, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {5509, 5474}

$$\frac{de^{c+dx}\sinh(a+bx)}{8\left(b^2-d^2\right)}+\frac{de^{c+dx}\sinh(3a+3bx)}{16\left(9b^2-d^2\right)}-\frac{de^{c+dx}\sinh(5a+5bx)}{16\left(25b^2-d^2\right)}-\frac{be^{c+dx}\cosh(a+bx)}{8\left(b^2-d^2\right)}-\frac{3be^{c+dx}\cosh(3a+3b}{16\left(9b^2-d^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[E^(c + d*x)*Cosh[a + b*x]^2*Sinh[a + b*x]^3,x]
```

```
[Out] -(b*E^(c + d*x)*Cosh[a + b*x])/(8*(b^2 - d^2)) - (3*b*E^(c + d*x)*Cosh[3*a
+ 3*b*x])/(16*(9*b^2 - d^2)) + (5*b*E^(c + d*x)*Cosh[5*a + 5*b*x])/(16*(25*
b^2 - d^2)) + (d*E^(c + d*x)*Sinh[a + b*x])/(8*(b^2 - d^2)) + (d*E^(c + d*x
)*Sinh[3*a + 3*b*x])/(16*(9*b^2 - d^2)) - (d*E^(c + d*x)*Sinh[5*a + 5*b*x])
/(16*(25*b^2 - d^2))
```

Rule 5474

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sinh[(d_.) + (e_.)*(x_)], x_Symbol] :
> -Simp[(b*c*Log[F]*F^(c*(a + b*x))*Sinh[d + e*x])/(e^2 - b^2*c^2*Log[F]^2)
, x] + Simp[(e*F^(c*(a + b*x))*Cosh[d + e*x])/(e^2 - b^2*c^2*Log[F]^2), x]
/; FreeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 - b^2*c^2*Log[F]^2, 0]
```

Rule 5509

```
Int[Cosh[(f_.) + (g_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sinh[(
d_.) + (e_.)*(x_)]^(m_.), x_Symbol] :> Int[ExpandTrigReduce[F^(c*(a + b*x))
, Sinh[d + e*x]^m*Cosh[f + g*x]^n, x], x] /; FreeQ[{F, a, b, c, d, e, f, g}
, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

Rubi steps

$$\int e^{c+dx}\cosh^2(a+bx)\sinh^3(a+bx)\,dx = \int\left(-\frac{1}{8}e^{c+dx}\sinh(a+bx) - \frac{1}{16}e^{c+dx}\sinh(3a+3bx) + \frac{1}{16}e^{c+dx}\sinh(5a\right.$$
$$= -\left(\frac{1}{16}\int e^{c+dx}\sinh(3a+3bx)\,dx\right) + \frac{1}{16}\int e^{c+dx}\sinh(5a+5bx)\,dx - \frac{1}{8}$$
$$= -\frac{be^{c+dx}\cosh(a+bx)}{8\left(b^2-d^2\right)} - \frac{3be^{c+dx}\cosh(3a+3bx)}{16\left(9b^2-d^2\right)} + \frac{5be^{c+dx}\cosh(5a+5}{16\left(25b^2-d^2\right)}$$

Mathematica [A] time = 1.17, size = 117, normalized size = 0.60

$$\frac{1}{16}e^{c+dx}\left(\frac{d\sinh(3(a+bx)) - 3b\cosh(3(a+bx))}{9b^2-d^2} + \frac{5b\cosh(5(a+bx)) - d\sinh(5(a+bx))}{25b^2-d^2} + \frac{2d\sinh(a+bx) - 2}{(b-d)(b}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(c + d*x)*Cosh[a + b*x]^2*Sinh[a + b*x]^3,x]
```

```
[Out] (E^(c + d*x)*((-2*b*Cosh[a + b*x] + 2*d*Sinh[a + b*x])/((b - d)*(b + d)) +
(-3*b*Cosh[3*(a + b*x)] + d*Sinh[3*(a + b*x)])/(9*b^2 - d^2) + (5*b*Cosh[5*
(a + b*x)] - d*Sinh[5*(a + b*x)])/(25*b^2 - d^2)))/16
```

fricas [B] time = 0.45, size = 919, normalized size = 4.71

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x+c)*cosh(b*x+a)^2*sinh(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/16*(25*(9*b^5 - 10*b^3*d^2 + b*d^4)*cosh(b*x + a)*cosh(d*x + c)*sinh(b*x
+ a)^4 - (9*b^4*d - 10*b^2*d^3 + d^5)*cosh(d*x + c)*sinh(b*x + a)^5 + (25*b
^4*d - 26*b^2*d^3 + d^5 - 10*(9*b^4*d - 10*b^2*d^3 + d^5)*cosh(b*x + a)^2)*
cosh(d*x + c)*sinh(b*x + a)^3 + (50*(9*b^5 - 10*b^3*d^2 + b*d^4)*cosh(b*x +
 a)^3 - 9*(25*b^5 - 26*b^3*d^2 + b*d^4)*cosh(b*x + a))*cosh(d*x + c)*sinh(b
*x + a)^2 + (450*b^4*d - 68*b^2*d^3 + 2*d^5 - 5*(9*b^4*d - 10*b^2*d^3 + d^5
)*cosh(b*x + a)^4 + 3*(25*b^4*d - 26*b^2*d^3 + d^5)*cosh(b*x + a)^2)*cosh(d
*x + c)*sinh(b*x + a) + (5*(9*b^5 - 10*b^3*d^2 + b*d^4)*cosh(b*x + a)^5 - 3
*(25*b^5 - 26*b^3*d^2 + b*d^4)*cosh(b*x + a)^3 - 2*(225*b^5 - 34*b^3*d^2 +
b*d^4)*cosh(b*x + a))*cosh(d*x + c) + (5*(9*b^5 - 10*b^3*d^2 + b*d^4)*cosh(
b*x + a)^5 + 25*(9*b^5 - 10*b^3*d^2 + b*d^4)*cosh(b*x + a)*sinh(b*x + a)^4
- (9*b^4*d - 10*b^2*d^3 + d^5)*sinh(b*x + a)^5 - 3*(25*b^5 - 26*b^3*d^2 + b
*d^4)*cosh(b*x + a)^3 + (25*b^4*d - 26*b^2*d^3 + d^5 - 10*(9*b^4*d - 10*b^2
*d^3 + d^5)*cosh(b*x + a)^2)*sinh(b*x + a)^3 + (50*(9*b^5 - 10*b^3*d^2 + b*
```

```
d^4)*cosh(b*x + a)^3 - 9*(25*b^5 - 26*b^3*d^2 + b*d^4)*cosh(b*x + a))*sinh(
b*x + a)^2 - 2*(225*b^5 - 34*b^3*d^2 + b*d^4)*cosh(b*x + a) + (450*b^4*d -
68*b^2*d^3 + 2*d^5 - 5*(9*b^4*d - 10*b^2*d^3 + d^5)*cosh(b*x + a)^4 + 3*(25
*b^4*d - 26*b^2*d^3 + d^5)*cosh(b*x + a)^2)*sinh(b*x + a))*sinh(d*x + c))/(
(225*b^6 - 259*b^4*d^2 + 35*b^2*d^4 - d^6)*cosh(b*x + a)^6 - 3*(225*b^6 - 2
59*b^4*d^2 + 35*b^2*d^4 - d^6)*cosh(b*x + a)^4*sinh(b*x + a)^2 + 3*(225*b^6
 - 259*b^4*d^2 + 35*b^2*d^4 - d^6)*cosh(b*x + a)^2*sinh(b*x + a)^4 - (225*b
^6 - 259*b^4*d^2 + 35*b^2*d^4 - d^6)*sinh(b*x + a)^6)
```

giac [A] time = 0.15, size = 132, normalized size = 0.68

$$\frac{e^{(5\,bx+dx+5\,a+c)}}{32\,(5\,b+d)}-\frac{e^{(3\,bx+dx+3\,a+c)}}{32\,(3\,b+d)}-\frac{e^{(bx+dx+a+c)}}{16\,(b+d)}-\frac{e^{(-bx+dx-a+c)}}{16\,(b-d)}-\frac{e^{(-3\,bx+dx-3\,a+c)}}{32\,(3\,b-d)}+\frac{e^{(-5\,bx+dx-5\,a+c)}}{32\,(5\,b-d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x+c)*cosh(b*x+a)^2*sinh(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/32*e^(5*b*x + d*x + 5*a + c)/(5*b + d) - 1/32*e^(3*b*x + d*x + 3*a + c)/(
3*b + d) - 1/16*e^(b*x + d*x + a + c)/(b + d) - 1/16*e^(-b*x + d*x - a + c)
/(b - d) - 1/32*e^(-3*b*x + d*x - 3*a + c)/(3*b - d) + 1/32*e^(-5*b*x + d*x
 - 5*a + c)/(5*b - d)
```

maple [A] time = 0.52, size = 278, normalized size = 1.43

$$\frac{\sinh(a-c+(b-d)x)}{16b-16d}-\frac{\sinh(a+c+(b+d)x)}{16\,(b+d)}+\frac{\sinh(3a-c+(3b-d)x)}{96b-32d}-\frac{\sinh(3a+c+(3b+d)x)}{32\,(3b+d)}-\frac{\sinh((}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(d*x+c)*cosh(b*x+a)^2*sinh(b*x+a)^3,x)
```

```
[Out] 1/16*sinh(a-c+(b-d)*x)/(b-d)-1/16*sinh(a+c+(b+d)*x)/(b+d)+1/32*sinh(3*a-c+(
3*b-d)*x)/(3*b-d)-1/32*sinh(3*a+c+(3*b+d)*x)/(3*b+d)-1/32/(5*b-d)*sinh((5*b
-d)*x+5*a-c)+1/32/(5*b+d)*sinh((5*b+d)*x+5*a+c)-1/16*cosh(a-c+(b-d)*x)/(b-d
)-1/16*cosh(a+c+(b+d)*x)/(b+d)-1/32*cosh(3*a-c+(3*b-d)*x)/(3*b-d)-1/32*cosh
(3*a+c+(3*b+d)*x)/(3*b+d)+1/32*cosh((5*b-d)*x+5*a-c)/(5*b-d)+1/32*cosh((5*b
+d)*x+5*a+c)/(5*b+d)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x+c)*cosh(b*x+a)^2*sinh(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(-d/b>0)', see `assume?` for more de
tails)Is -d/b equal to -1?
```

mupad [B] time = 2.49, size = 395, normalized size = 2.03

$$\frac{3\cosh(a+bx)^3 e^{c+dx}\sinh(a+bx)^2\left(25b^5-10b^3d^2+bd^4\right)}{225b^6-259b^4d^2+35b^2d^4-d^6}-\frac{\cosh(a+bx)^5 e^{c+dx}\left(30b^5-6b^3d^2\right)}{225b^6-259b^4d^2+35b^2d^4-d^6}+\frac{6\cosh(a}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a + b*x)^2*exp(c + d*x)*sinh(a + b*x)^3,x)
```

```
[Out] (3*cosh(a + b*x)^3*exp(c + d*x)*sinh(a + b*x)^2*(b*d^4 + 25*b^5 - 10*b^3*d^
2))/(225*b^6 - d^6 + 35*b^2*d^4 - 259*b^4*d^2) - (cosh(a + b*x)^5*exp(c + d
*x)*(30*b^5 - 6*b^3*d^2))/(225*b^6 - d^6 + 35*b^2*d^4 - 259*b^4*d^2) + (6*c
osh(a + b*x)^4*exp(c + d*x)*sinh(a + b*x)*(5*b^4*d - b^2*d^3))/(225*b^6 - d
^6 + 35*b^2*d^4 - 259*b^4*d^2) - (cosh(a + b*x)^2*exp(c + d*x)*sinh(a + b*x
)^3*(65*b^4*d + d^5 - 18*b^2*d^3))/(225*b^6 - d^6 + 35*b^2*d^4 - 259*b^4*d^
2) + (2*b^2*d*exp(c + d*x)*sinh(a + b*x)^5*(13*b^2 - d^2))/(225*b^6 - d^6 +
 35*b^2*d^4 - 259*b^4*d^2) - (2*b*d^2*cosh(a + b*x)*exp(c + d*x)*sinh(a + b
*x)^4*(13*b^2 - d^2))/(225*b^6 - d^6 + 35*b^2*d^4 - 259*b^4*d^2)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x+c)*cosh(b*x+a)**2*sinh(b*x+a)**3,x)
```

```
[Out] Timed out
```

3.953 $\int e^{c+dx} \cosh^2(a+bx) \sinh^2(a+bx)\, dx$

Optimal. Leaf size=83

$$\frac{be^{c+dx}\sinh(4a+4bx)}{2(16b^2-d^2)}-\frac{de^{c+dx}\cosh(4a+4bx)}{8(16b^2-d^2)}-\frac{e^{c+dx}}{8d}$$

```
[Out] -1/8*exp(d*x+c)/d-1/8*d*exp(d*x+c)*cosh(4*b*x+4*a)/(16*b^2-d^2)+1/2*b*exp(d
*x+c)*sinh(4*b*x+4*a)/(16*b^2-d^2)
```

Rubi [A] time = 0.08, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {5509, 2194, 5475}

$$\frac{be^{c+dx}\sinh(4a+4bx)}{2(16b^2-d^2)}-\frac{de^{c+dx}\cosh(4a+4bx)}{8(16b^2-d^2)}-\frac{e^{c+dx}}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[E^(c + d*x)*Cosh[a + b*x]^2*Sinh[a + b*x]^2,x]
```

```
[Out] -E^(c + d*x)/(8*d) - (d*E^(c + d*x)*Cosh[4*a + 4*b*x])/(8*(16*b^2 - d^2)) +
 (b*E^(c + d*x)*Sinh[4*a + 4*b*x])/(2*(16*b^2 - d^2))
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 5475

```
Int[Cosh[(d_.) + (e_.)*(x_)]*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symbol] :
> -Simp[(b*c*Log[F]*F^(c*(a + b*x))*Cosh[d + e*x])/(e^2 - b^2*c^2*Log[F]^2)
, x] + Simp[(e*F^(c*(a + b*x))*Sinh[d + e*x])/(e^2 - b^2*c^2*Log[F]^2), x]
/; FreeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 - b^2*c^2*Log[F]^2, 0]
```

Rule 5509

```
Int[Cosh[(f_.) + (g_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sinh[(
d_.) + (e_.)*(x_)]^(m_.), x_Symbol] :> Int[ExpandTrigReduce[F^(c*(a + b*x))
, Sinh[d + e*x]^m*Cosh[f + g*x]^n, x], x] /; FreeQ[{F, a, b, c, d, e, f, g}
, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

Rubi steps

$$\int e^{c+dx}\cosh^2(a+bx)\sinh^2(a+bx)\,dx = \int\left(-\frac{1}{8}e^{c+dx}+\frac{1}{8}e^{c+dx}\cosh(4a+4bx)\right)dx$$
$$= -\left(\frac{1}{8}\int e^{c+dx}\,dx\right)+\frac{1}{8}\int e^{c+dx}\cosh(4a+4bx)\,dx$$
$$= -\frac{e^{c+dx}}{8d}-\frac{de^{c+dx}\cosh(4a+4bx)}{8\left(16b^2-d^2\right)}+\frac{be^{c+dx}\sinh(4a+4bx)}{2\left(16b^2-d^2\right)}$$

Mathematica [A] time = 0.39, size = 58, normalized size = 0.70

$$\frac{e^{c+dx}\left(d^2\cosh(4(a+bx))-4bd\sinh(4(a+bx))+16b^2-d^2\right)}{8\left(d^3-16b^2d\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(c + d*x)*Cosh[a + b*x]^2*Sinh[a + b*x]^2,x]
```

```
[Out] (E^(c + d*x)*(16*b^2 - d^2 + d^2*Cosh[4*(a + b*x)] - 4*b*d*Sinh[4*(a + b*x)
]))/(8*(-16*b^2*d + d^3))
```

fricas [B] time = 0.55, size = 303, normalized size = 3.65

$$\frac{16\,bd\cosh\left(bx+a\right)^3\cosh\left(dx+c\right)\sinh\left(bx+a\right)-6\,d^2\cosh\left(bx+a\right)^2\cosh\left(dx+c\right)\sinh\left(bx+a\right)^2+16\,bd\cosh\left(b\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x+c)*cosh(b*x+a)^2*sinh(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/8*(16*b*d*cosh(b*x + a)^3*cosh(d*x + c)*sinh(b*x + a) - 6*d^2*cosh(b*x +
a)^2*cosh(d*x + c)*sinh(b*x + a)^2 + 16*b*d*cosh(b*x + a)*cosh(d*x + c)*sin
h(b*x + a)^3 - d^2*cosh(d*x + c)*sinh(b*x + a)^4 - (d^2*cosh(b*x + a)^4 + 1
6*b^2 - d^2)*cosh(d*x + c) - (d^2*cosh(b*x + a)^4 - 16*b*d*cosh(b*x + a)^3*
sinh(b*x + a) + 6*d^2*cosh(b*x + a)^2*sinh(b*x + a)^2 - 16*b*d*cosh(b*x + a
)*sinh(b*x + a)^3 + d^2*sinh(b*x + a)^4 + 16*b^2 - d^2)*sinh(d*x + c))/((16
*b^2*d - d^3)*cosh(b*x + a)^4 - 2*(16*b^2*d - d^3)*cosh(b*x + a)^2*sinh(b*x
 + a)^2 + (16*b^2*d - d^3)*sinh(b*x + a)^4)
```

giac [A] time = 0.13, size = 58, normalized size = 0.70

$$\frac{e^{(4\,bx+dx+4\,a+c)}}{16\,(4\,b+d)}-\frac{e^{(-4\,bx+dx-4\,a+c)}}{16\,(4\,b-d)}-\frac{e^{(dx+c)}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x+c)*cosh(b*x+a)^2*sinh(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/16*e^(4*b*x + d*x + 4*a + c)/(4*b + d) - 1/16*e^(-4*b*x + d*x - 4*a + c)/
(4*b - d) - 1/8*e^(d*x + c)/d
```

maple [A] time = 0.29, size = 124, normalized size = 1.49

$$-\frac{\sinh(dx+c)}{8d}+\frac{\sinh((4b-d)x+4a-c)}{64b-16d}+\frac{\sinh((4b+d)x+4a+c)}{64b+16d}-\frac{\cosh(dx+c)}{8d}-\frac{\cosh((4b-d)x+4a-c)}{16(4b-d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(d*x+c)*cosh(b*x+a)^2*sinh(b*x+a)^2,x)
```

```
[Out] -1/8*sinh(d*x+c)/d+1/16/(4*b-d)*sinh((4*b-d)*x+4*a-c)+1/16/(4*b+d)*sinh((4*
b+d)*x+4*a+c)-1/8*cosh(d*x+c)/d-1/16*cosh((4*b-d)*x+4*a-c)/(4*b-d)+1/16*cos
h((4*b+d)*x+4*a+c)/(4*b+d)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x+c)*cosh(b*x+a)^2*sinh(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(3-d/b>0)', see `assume?` for more d
etails)Is 3-d/b equal to -1?
```

mupad [B] time = 2.73, size = 96, normalized size = 1.16

$$-\frac{\frac{d^2\,e^{c+d\,x}\,\left(\frac{e^{-4\,a-4\,b\,x}}{2}+\frac{e^{4\,a+4\,b\,x}}{2}\right)}{8}+\frac{b\,d\,e^{c+d\,x}\,\left(\frac{e^{-4\,a-4\,b\,x}}{2}-\frac{e^{4\,a+4\,b\,x}}{2}\right)}{2}}{16\,b^2\,d-d^3}-\frac{e^{c+d\,x}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a + b*x)^2*exp(c + d*x)*sinh(a + b*x)^2,x)
```

```
[Out] - ((d^2*exp(c + d*x)*(exp(- 4*a - 4*b*x)/2 + exp(4*a + 4*b*x)/2))/8 + (b*d*
exp(c + d*x)*(exp(- 4*a - 4*b*x)/2 - exp(4*a + 4*b*x)/2))/2)/(16*b^2*d - d^
3) - exp(c + d*x)/(8*d)
```

sympy [A] time = 88.85, size = 819, normalized size = 9.87

$$\begin{cases} xe^c \sinh^2(a)\cosh^2(a) \\ \frac{xe^ce^{dx}\sinh^4\left(a-\frac{dx}{4}\right)}{16} + \frac{xe^ce^{dx}\sinh^3\left(a-\frac{dx}{4}\right)\cosh\left(a-\frac{dx}{4}\right)}{4} + \frac{3xe^ce^{dx}\sinh^2\left(a-\frac{dx}{4}\right)\cosh^2\left(a-\frac{dx}{4}\right)}{8} + \frac{xe^ce^{dx}\sinh\left(a-\frac{dx}{4}\right)\cosh^3\left(a-\frac{dx}{4}\right)}{4} + \frac{xe^ce^{dx}\cos}{1} \\ \frac{xe^ce^{dx}\sinh^4\left(a+\frac{dx}{4}\right)}{16} - \frac{xe^ce^{dx}\sinh^3\left(a+\frac{dx}{4}\right)\cosh\left(a+\frac{dx}{4}\right)}{4} + \frac{3xe^ce^{dx}\sinh^2\left(a+\frac{dx}{4}\right)\cosh^2\left(a+\frac{dx}{4}\right)}{8} - \frac{xe^ce^{dx}\sinh\left(a+\frac{dx}{4}\right)\cosh^3\left(a+\frac{dx}{4}\right)}{4} + \frac{xe^ce^{dx}\text{co}}{} \\ \left(-\frac{x\sinh^4(a+bx)}{8} + \frac{x\sinh^2(a+bx)\cosh^2(a+bx)}{4} - \frac{x\cosh^4(a+bx)}{8} + \frac{\sinh^3(a+bx)\cosh(a+bx)}{8b} + \frac{\sinh(a+bx)\cosh^3(a+bx)}{8b}\right)e^c \\ -\frac{2b^2e^ce^{dx}\sinh^4(a+bx)}{16b^2d-d^3} + \frac{4b^2e^ce^{dx}\sinh^2(a+bx)\cosh^2(a+bx)}{16b^2d-d^3} - \frac{2b^2e^ce^{dx}\cosh^4(a+bx)}{16b^2d-d^3} + \frac{2bde^ce^{dx}\sinh^3(a+bx)\cosh(a+bx)}{16b^2d-d^3} + \frac{2bde^ce^{dx}\sinh(a-}{16b^2} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x+c)*cosh(b*x+a)**2*sinh(b*x+a)**2,x)
```

```
[Out] Piecewise((x*exp(c)*sinh(a)**2*cosh(a)**2, Eq(b, 0) & Eq(d, 0)), (x*exp(c)*
exp(d*x)*sinh(a - d*x/4)**4/16 + x*exp(c)*exp(d*x)*sinh(a - d*x/4)**3*cosh(
a - d*x/4)/4 + 3*x*exp(c)*exp(d*x)*sinh(a - d*x/4)**2*cosh(a - d*x/4)**2/8
+ x*exp(c)*exp(d*x)*sinh(a - d*x/4)*cosh(a - d*x/4)**3/4 + x*exp(c)*exp(d*x
)*cosh(a - d*x/4)**4/16 - exp(c)*exp(d*x)*sinh(a - d*x/4)**4/(6*d) - 5*exp(
c)*exp(d*x)*sinh(a - d*x/4)**3*cosh(a - d*x/4)/(12*d) - 5*exp(c)*exp(d*x)*s
inh(a - d*x/4)*cosh(a - d*x/4)**3/(12*d) - exp(c)*exp(d*x)*cosh(a - d*x/4)*
*4/(6*d), Eq(b, -d/4)), (x*exp(c)*exp(d*x)*sinh(a + d*x/4)**4/16 - x*exp(c)
*exp(d*x)*sinh(a + d*x/4)**3*cosh(a + d*x/4)/4 + 3*x*exp(c)*exp(d*x)*sinh(a
 + d*x/4)**2*cosh(a + d*x/4)**2/8 - x*exp(c)*exp(d*x)*sinh(a + d*x/4)*cosh(
a + d*x/4)**3/4 + x*exp(c)*exp(d*x)*cosh(a + d*x/4)**4/16 - exp(c)*exp(d*x)
*sinh(a + d*x/4)**4/(6*d) + 5*exp(c)*exp(d*x)*sinh(a + d*x/4)**3*cosh(a + d
*x/4)/(12*d) + 5*exp(c)*exp(d*x)*sinh(a + d*x/4)*cosh(a + d*x/4)**3/(12*d)
- exp(c)*exp(d*x)*cosh(a + d*x/4)**4/(6*d), Eq(b, d/4)), ((-x*sinh(a + b*x)
**4/8 + x*sinh(a + b*x)**2*cosh(a + b*x)**2/4 - x*cosh(a + b*x)**4/8 + sinh
(a + b*x)**3*cosh(a + b*x)/(8*b) + sinh(a + b*x)*cosh(a + b*x)**3/(8*b))*ex
p(c), Eq(d, 0)), (-2*b**2*exp(c)*exp(d*x)*sinh(a + b*x)**4/(16*b**2*d - d**
3) + 4*b**2*exp(c)*exp(d*x)*sinh(a + b*x)**2*cosh(a + b*x)**2/(16*b**2*d -
d**3) - 2*b**2*exp(c)*exp(d*x)*cosh(a + b*x)**4/(16*b**2*d - d**3) + 2*b*d*
exp(c)*exp(d*x)*sinh(a + b*x)**3*cosh(a + b*x)/(16*b**2*d - d**3) + 2*b*d*e
xp(c)*exp(d*x)*sinh(a + b*x)*cosh(a + b*x)**3/(16*b**2*d - d**3) - d**2*exp
(c)*exp(d*x)*sinh(a + b*x)**2*cosh(a + b*x)**2/(16*b**2*d - d**3), True))
```

3.954 $\int e^{c+dx}\cosh^2(a+bx)\sinh(a+bx)\,dx$

Optimal. Leaf size=127

$$-\frac{de^{c+dx}\sinh(a+bx)}{4\left(b^2-d^2\right)}-\frac{de^{c+dx}\sinh(3a+3bx)}{4\left(9b^2-d^2\right)}+\frac{be^{c+dx}\cosh(a+bx)}{4\left(b^2-d^2\right)}+\frac{3be^{c+dx}\cosh(3a+3bx)}{4\left(9b^2-d^2\right)}$$

```
[Out] 1/4*b*exp(d*x+c)*cosh(b*x+a)/(b^2-d^2)+3/4*b*exp(d*x+c)*cosh(3*b*x+3*a)/(9*
b^2-d^2)-1/4*d*exp(d*x+c)*sinh(b*x+a)/(b^2-d^2)-1/4*d*exp(d*x+c)*sinh(3*b*x
+3*a)/(9*b^2-d^2)
```

Rubi [A] time = 0.09, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {5509, 5474}

$$-\frac{de^{c+dx}\sinh(a+bx)}{4\left(b^2-d^2\right)}-\frac{de^{c+dx}\sinh(3a+3bx)}{4\left(9b^2-d^2\right)}+\frac{be^{c+dx}\cosh(a+bx)}{4\left(b^2-d^2\right)}+\frac{3be^{c+dx}\cosh(3a+3bx)}{4\left(9b^2-d^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[E^(c + d*x)*Cosh[a + b*x]^2*Sinh[a + b*x],x]
```

```
[Out] (b*E^(c + d*x)*Cosh[a + b*x])/(4*(b^2 - d^2)) + (3*b*E^(c + d*x)*Cosh[3*a +
 3*b*x])/(4*(9*b^2 - d^2)) - (d*E^(c + d*x)*Sinh[a + b*x])/(4*(b^2 - d^2))
- (d*E^(c + d*x)*Sinh[3*a + 3*b*x])/(4*(9*b^2 - d^2))
```

Rule 5474

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sinh[(d_.) + (e_.)*(x_)], x_Symbol] :
> -Simp[(b*c*Log[F]*F^(c*(a + b*x))*Sinh[d + e*x])/(e^2 - b^2*c^2*Log[F]^2)
, x] + Simp[(e*F^(c*(a + b*x))*Cosh[d + e*x])/(e^2 - b^2*c^2*Log[F]^2), x]
/; FreeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 - b^2*c^2*Log[F]^2, 0]
```

Rule 5509

```
Int[Cosh[(f_.) + (g_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sinh[(
d_.) + (e_.)*(x_)]^(m_.), x_Symbol] :> Int[ExpandTrigReduce[F^(c*(a + b*x))
, Sinh[d + e*x]^m*Cosh[f + g*x]^n, x], x] /; FreeQ[{F, a, b, c, d, e, f, g}
, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

Rubi steps

$$\int e^{c+dx}\cosh^2(a+bx)\sinh(a+bx)\,dx = \int\left(\frac{1}{4}e^{c+dx}\sinh(a+bx)+\frac{1}{4}e^{c+dx}\sinh(3a+3bx)\right)dx$$
$$=\frac{1}{4}\int e^{c+dx}\sinh(a+bx)\,dx+\frac{1}{4}\int e^{c+dx}\sinh(3a+3bx)\,dx$$
$$=\frac{be^{c+dx}\cosh(a+bx)}{4\left(b^2-d^2\right)}+\frac{3be^{c+dx}\cosh(3a+3bx)}{4\left(9b^2-d^2\right)}-\frac{de^{c+dx}\sinh(a+bx)}{4\left(b^2-d^2\right)}-$$

Mathematica [A] time = 0.56, size = 80, normalized size = 0.63

$$\frac{1}{4}e^{c+dx}\left(\frac{3b\cosh(3(a+bx))-d\sinh(3(a+bx))}{9b^2-d^2}+\frac{b\cosh(a+bx)-d\sinh(a+bx)}{(b-d)(b+d)}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(c + d*x)*Cosh[a + b*x]^2*Sinh[a + b*x],x]
```

```
[Out] (E^(c + d*x)*((b*Cosh[a + b*x] - d*Sinh[a + b*x])/((b - d)*(b + d)) + (3*b*
Cosh[3*(a + b*x)] - d*Sinh[3*(a + b*x)])/(9*b^2 - d^2)))/4
```

fricas [B] time = 0.45, size = 381, normalized size = 3.00

$$\frac{9\left(b^3-bd^2\right)\cosh\left(bx+a\right)\cosh\left(dx+c\right)\sinh\left(bx+a\right)^2-\left(b^2d-d^3\right)\cosh\left(dx+c\right)\sinh\left(bx+a\right)^3-\left(9\,b^2d-d^3+3\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x+c)*cosh(b*x+a)^2*sinh(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/4*(9*(b^3 - b*d^2)*cosh(b*x + a)*cosh(d*x + c)*sinh(b*x + a)^2 - (b^2*d -
 d^3)*cosh(d*x + c)*sinh(b*x + a)^3 - (9*b^2*d - d^3 + 3*(b^2*d - d^3)*cosh
(b*x + a)^2)*cosh(d*x + c)*sinh(b*x + a) + (3*(b^3 - b*d^2)*cosh(b*x + a)^3
 + (9*b^3 - b*d^2)*cosh(b*x + a))*cosh(d*x + c) + (3*(b^3 - b*d^2)*cosh(b*x
 + a)^3 + 9*(b^3 - b*d^2)*cosh(b*x + a)*sinh(b*x + a)^2 - (b^2*d - d^3)*sin
h(b*x + a)^3 + (9*b^3 - b*d^2)*cosh(b*x + a) - (9*b^2*d - d^3 + 3*(b^2*d -
d^3)*cosh(b*x + a)^2)*sinh(b*x + a))*sinh(d*x + c))/((9*b^4 - 10*b^2*d^2 +
d^4)*cosh(b*x + a)^4 - 2*(9*b^4 - 10*b^2*d^2 + d^4)*cosh(b*x + a)^2*sinh(b*
x + a)^2 + (9*b^4 - 10*b^2*d^2 + d^4)*sinh(b*x + a)^4)
```

giac [A] time = 0.12, size = 86, normalized size = 0.68

$$\frac{e^{(3\,bx+dx+3\,a+c)}}{8\,(3\,b+d)}+\frac{e^{(bx+dx+a+c)}}{8\,(b+d)}+\frac{e^{(-bx+dx-a+c)}}{8\,(b-d)}+\frac{e^{(-3\,bx+dx-3\,a+c)}}{8\,(3\,b-d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x+c)*cosh(b*x+a)^2*sinh(b*x+a),x, algorithm="giac")
```

```
[Out] 1/8*e^(3*b*x + d*x + 3*a + c)/(3*b + d) + 1/8*e^(b*x + d*x + a + c)/(b + d)
 + 1/8*e^(-b*x + d*x - a + c)/(b - d) + 1/8*e^(-3*b*x + d*x - 3*a + c)/(3*b
 - d)
```

maple [A] time = 0.36, size = 178, normalized size = 1.40

$$-\frac{\sinh(a-c+(b-d)x)}{8(b-d)}+\frac{\sinh(a+c+(b+d)x)}{8b+8d}-\frac{\sinh(3a-c+(3b-d)x)}{8(3b-d)}+\frac{\sinh(3a+c+(3b+d)x)}{24b+8d}+\frac{\cosh}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(d*x+c)*cosh(b*x+a)^2*sinh(b*x+a),x)
```

```
[Out] -1/8*sinh(a-c+(b-d)*x)/(b-d)+1/8*sinh(a+c+(b+d)*x)/(b+d)-1/8*sinh(3*a-c+(3*
b-d)*x)/(3*b-d)+1/8*sinh(3*a+c+(3*b+d)*x)/(3*b+d)+1/8*cosh(a-c+(b-d)*x)/(b-
d)+1/8*cosh(a+c+(b+d)*x)/(b+d)+1/8*cosh(3*a-c+(3*b-d)*x)/(3*b-d)+1/8*cosh(3
*a+c+(3*b+d)*x)/(3*b+d)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x+c)*cosh(b*x+a)^2*sinh(b*x+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(-d/b>0)', see `assume?` for more de
tails)Is -d/b equal to -1?
```

mupad [B] time = 2.18, size = 126, normalized size = 0.99

$$\frac{e^{c+d\,x}\,\left(3\,b^3\,\cosh\left(a+b\,x\right)^3-3\,b^2\,d\,\cosh\left(a+b\,x\right)^2\,\sinh\left(a+b\,x\right)+2\,b^2\,d\,\sinh\left(a+b\,x\right)^3-b\,d^2\,\cosh\left(a+b\,x\right)^3-\right.}{9\,b^4-10\,b^2\,d^2+d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a + b*x)^2*exp(c + d*x)*sinh(a + b*x),x)
```

```
[Out] (exp(c + d*x)*(3*b^3*cosh(a + b*x)^3 - b*d^2*cosh(a + b*x)^3 + d^3*cosh(a +
 b*x)^2*sinh(a + b*x) + 2*b^2*d*sinh(a + b*x)^3 - 2*b*d^2*cosh(a + b*x)*sin
h(a + b*x)^2 - 3*b^2*d*cosh(a + b*x)^2*sinh(a + b*x)))/(9*b^4 + d^4 - 10*b^
2*d^2)
```

sympy [A] time = 42.14, size = 972, normalized size = 7.65

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x+c)*cosh(b*x+a)**2*sinh(b*x+a),x)
```

```
[Out] Piecewise((x*exp(c)*sinh(a)*cosh(a)**2, Eq(b, 0) & Eq(d, 0)), (-x*exp(c)*ex
p(d*x)*sinh(a - d*x)**3/8 - x*exp(c)*exp(d*x)*sinh(a - d*x)**2*cosh(a - d*x
)/8 + x*exp(c)*exp(d*x)*sinh(a - d*x)*cosh(a - d*x)**2/8 + x*exp(c)*exp(d*x
)*cosh(a - d*x)**3/8 - exp(c)*exp(d*x)*sinh(a - d*x)**3/(8*d) - exp(c)*exp(
d*x)*sinh(a - d*x)**2*cosh(a - d*x)/(4*d) - exp(c)*exp(d*x)*cosh(a - d*x)**
3/(8*d), Eq(b, -d)), (x*exp(c)*exp(d*x)*sinh(a - d*x/3)**3/8 + 3*x*exp(c)*e
xp(d*x)*sinh(a - d*x/3)**2*cosh(a - d*x/3)/8 + 3*x*exp(c)*exp(d*x)*sinh(a -
 d*x/3)*cosh(a - d*x/3)**2/8 + x*exp(c)*exp(d*x)*cosh(a - d*x/3)**3/8 - exp
(c)*exp(d*x)*sinh(a - d*x/3)**3/(8*d) + 3*exp(c)*exp(d*x)*sinh(a - d*x/3)*c
osh(a - d*x/3)**2/(4*d) + exp(c)*exp(d*x)*cosh(a - d*x/3)**3/(8*d), Eq(b, -
d/3)), (x*exp(c)*exp(d*x)*sinh(a + d*x/3)**3/8 - 3*x*exp(c)*exp(d*x)*sinh(a
 + d*x/3)**2*cosh(a + d*x/3)/8 + 3*x*exp(c)*exp(d*x)*sinh(a + d*x/3)*cosh(a
 + d*x/3)**2/8 - x*exp(c)*exp(d*x)*cosh(a + d*x/3)**3/8 - 3*exp(c)*exp(d*x)
*sinh(a + d*x/3)**3/(8*d) + 3*exp(c)*exp(d*x)*sinh(a + d*x/3)**2*cosh(a + d
*x/3)/(4*d) + exp(c)*exp(d*x)*cosh(a + d*x/3)**3/(8*d), Eq(b, d/3)), (-x*ex
p(c)*exp(d*x)*sinh(a + d*x)**3/8 + x*exp(c)*exp(d*x)*sinh(a + d*x)**2*cosh(
a + d*x)/8 + x*exp(c)*exp(d*x)*sinh(a + d*x)*cosh(a + d*x)**2/8 - x*exp(c)*
exp(d*x)*cosh(a + d*x)**3/8 + exp(c)*exp(d*x)*sinh(a + d*x)**3/(8*d) - exp(
c)*exp(d*x)*sinh(a + d*x)*cosh(a + d*x)**2/(4*d) + 3*exp(c)*exp(d*x)*cosh(a
 + d*x)**3/(8*d), Eq(b, d)), (3*b**3*exp(c)*exp(d*x)*cosh(a + b*x)**3/(9*b*
*4 - 10*b**2*d**2 + d**4) + 2*b**2*d*exp(c)*exp(d*x)*sinh(a + b*x)**3/(9*b*
*4 - 10*b**2*d**2 + d**4) - 3*b**2*d*exp(c)*exp(d*x)*sinh(a + b*x)*cosh(a +
 b*x)**2/(9*b**4 - 10*b**2*d**2 + d**4) - 2*b*d**2*exp(c)*exp(d*x)*sinh(a +
 b*x)**2*cosh(a + b*x)/(9*b**4 - 10*b**2*d**2 + d**4) - b*d**2*exp(c)*exp(d
*x)*cosh(a + b*x)**3/(9*b**4 - 10*b**2*d**2 + d**4) + d**3*exp(c)*exp(d*x)*
sinh(a + b*x)*cosh(a + b*x)**2/(9*b**4 - 10*b**2*d**2 + d**4), True))
```

3.955 $\int e^{c+dx} \cosh^2(a+bx)\,dx$

Optimal. Leaf size=95

$$-\frac{de^{c+dx}\cosh^2(a+bx)}{4b^2-d^2}+\frac{2be^{c+dx}\sinh(a+bx)\cosh(a+bx)}{4b^2-d^2}+\frac{2b^2e^{c+dx}}{d\left(4b^2-d^2\right)}$$

```
[Out] 2*b^2*exp(d*x+c)/d/(4*b^2-d^2)-d*exp(d*x+c)*cosh(b*x+a)^2/(4*b^2-d^2)+2*b*e
xp(d*x+c)*cosh(b*x+a)*sinh(b*x+a)/(4*b^2-d^2)
```

Rubi [A] time = 0.04, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {5477, 2194}

$$-\frac{de^{c+dx}\cosh^2(a+bx)}{4b^2-d^2}+\frac{2be^{c+dx}\sinh(a+bx)\cosh(a+bx)}{4b^2-d^2}+\frac{2b^2e^{c+dx}}{d\left(4b^2-d^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[E^(c + d*x)*Cosh[a + b*x]^2,x]
```

```
[Out] (2*b^2*E^(c + d*x))/(d*(4*b^2 - d^2)) - (d*E^(c + d*x)*Cosh[a + b*x]^2)/(4*
b^2 - d^2) + (2*b*E^(c + d*x)*Cosh[a + b*x]*Sinh[a + b*x])/(4*b^2 - d^2)
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 5477

```
Int[Cosh[(d_.) + (e_.)*(x_)]^(n_)*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symb
ol] :> -Simp[(b*c*Log[F]*F^(c*(a + b*x))*Cosh[d + e*x]^n)/(e^2*n^2 - b^2*c^
2*Log[F]^2), x] + (Dist[(n*(n - 1)*e^2)/(e^2*n^2 - b^2*c^2*Log[F]^2), Int[F
^(c*(a + b*x))*Cosh[d + e*x]^(n - 2), x], x] + Simp[(e*n*F^(c*(a + b*x))*Si
nh[d + e*x]*Cosh[d + e*x]^(n - 1))/(e^2*n^2 - b^2*c^2*Log[F]^2), x]) /; Fre
eQ[{F, a, b, c, d, e}, x] && NeQ[e^2*n^2 - b^2*c^2*Log[F]^2, 0] && GtQ[n, 1
]
```

Rubi steps

$$\int e^{c+dx}\cosh^2(a+bx)\,dx = -\frac{de^{c+dx}\cosh^2(a+bx)}{4b^2-d^2} + \frac{2be^{c+dx}\cosh(a+bx)\sinh(a+bx)}{4b^2-d^2} + \frac{\left(2b^2\right)\int e^{c+dx}\,dx}{4b^2-d^2}$$
$$= \frac{2b^2e^{c+dx}}{d\left(4b^2-d^2\right)} - \frac{de^{c+dx}\cosh^2(a+bx)}{4b^2-d^2} + \frac{2be^{c+dx}\cosh(a+bx)\sinh(a+bx)}{4b^2-d^2}$$

Mathematica [**A**] time = 0.15, size = 55, normalized size = 0.58

$$\frac{e^{c+dx}\left(d^2\cosh(2(a+bx)) - 2bd\sinh(2(a+bx)) - 4b^2 + d^2\right)}{2d^3-8b^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(c + d*x)*Cosh[a + b*x]^2,x]
```

```
[Out] (E^(c + d*x)*(-4*b^2 + d^2 + d^2*Cosh[2*(a + b*x)] - 2*b*d*Sinh[2*(a + b*x)
]))/(-8*b^2*d + 2*d^3)
```

fricas [**A**] time = 0.45, size = 176, normalized size = 1.85

$$\frac{4\,bd\cosh\left(bx+a\right)\cosh\left(dx+c\right)\sinh\left(bx+a\right) - d^2\cosh\left(dx+c\right)\sinh\left(bx+a\right)^2 - \left(d^2\cosh\left(bx+a\right)^2 - 4\,b^2 + d^2\right)c}{2\left(\left(4\,b^2d - d^3\right)\cosh\left(bx+a\right)^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x+c)*cosh(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(4*b*d*cosh(b*x + a)*cosh(d*x + c)*sinh(b*x + a) - d^2*cosh(d*x + c)*si
nh(b*x + a)^2 - (d^2*cosh(b*x + a)^2 - 4*b^2 + d^2)*cosh(d*x + c) - (d^2*co
sh(b*x + a)^2 - 4*b*d*cosh(b*x + a)*sinh(b*x + a) + d^2*sinh(b*x + a)^2 - 4
*b^2 + d^2)*sinh(d*x + c))/((4*b^2*d - d^3)*cosh(b*x + a)^2 - (4*b^2*d - d^
3)*sinh(b*x + a)^2)
```

giac [**A**] time = 0.12, size = 58, normalized size = 0.61

$$\frac{e^{\left(2\,bx+dx+2\,a+c\right)}}{4\left(2\,b+d\right)} - \frac{e^{\left(-2\,bx+dx-2\,a+c\right)}}{4\left(2\,b-d\right)} + \frac{e^{\left(dx+c\right)}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x+c)*cosh(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/4*e^(2*b*x + d*x + 2*a + c)/(2*b + d) - 1/4*e^(-2*b*x + d*x - 2*a + c)/(2
*b - d) + 1/2*e^(d*x + c)/d
```

maple [A] time = 0.26, size = 124, normalized size = 1.31

$$\frac{\sinh(dx+c)}{2d}+\frac{\sinh(2a-c+(2b-d)x)}{8b-4d}+\frac{\sinh(2a+c+(2b+d)x)}{8b+4d}+\frac{\cosh(dx+c)}{2d}-\frac{\cosh(2a-c+(2b-d)x)}{4(2b-d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(d*x+c)*cosh(b*x+a)^2,x)
```

```
[Out] 1/2*sinh(d*x+c)/d+1/4*sinh(2*a-c+(2*b-d)*x)/(2*b-d)+1/4*sinh(2*a+c+(2*b+d)*
x)/(2*b+d)+1/2*cosh(d*x+c)/d-1/4*cosh(2*a-c+(2*b-d)*x)/(2*b-d)+1/4*cosh(2*a
+c+(2*b+d)*x)/(2*b+d)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x+c)*cosh(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(1-d/b>0)', see `assume?` for more d
etails)Is 1-d/b equal to -1?
```

mupad [B] time = 0.27, size = 68, normalized size = 0.72

$$\frac{2\,b^2\,\mathrm{e}^{c+d\,x}-d^2\cosh\left(a+b\,x\right)^2\,\mathrm{e}^{c+d\,x}+2\,b\,d\cosh\left(a+b\,x\right)\,\mathrm{e}^{c+d\,x}\sinh\left(a+b\,x\right)}{4\,b^2\,d-d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a + b*x)^2*exp(c + d*x),x)
```

```
[Out] (2*b^2*exp(c + d*x) - d^2*cosh(a + b*x)^2*exp(c + d*x) + 2*b*d*cosh(a + b*x
)*exp(c + d*x)*sinh(a + b*x))/(4*b^2*d - d^3)
```

sympy [A] time = 8.57, size = 432, normalized size = 4.55

$$\begin{cases} xe^{c}\cosh^{2}(a) & \text{for} \\ \frac{xe^{c}e^{dx}\sinh^{2}\left(a-\frac{dx}{2}\right)}{4}+\frac{xe^{c}e^{dx}\sinh\left(a-\frac{dx}{2}\right)\cosh\left(a-\frac{dx}{2}\right)}{2}+\frac{xe^{c}e^{dx}\cosh^{2}\left(a-\frac{dx}{2}\right)}{4}+\frac{e^{c}e^{dx}\sinh\left(a-\frac{dx}{2}\right)\cosh\left(a-\frac{dx}{2}\right)}{2d}+\frac{e^{c}e^{dx}\cosh^{2}\left(a-\frac{dx}{2}\right)}{d} & \text{for} \\ \frac{xe^{c}e^{dx}\sinh^{2}\left(a+\frac{dx}{2}\right)}{4}-\frac{xe^{c}e^{dx}\sinh\left(a+\frac{dx}{2}\right)\cosh\left(a+\frac{dx}{2}\right)}{2}+\frac{xe^{c}e^{dx}\cosh^{2}\left(a+\frac{dx}{2}\right)}{4}-\frac{e^{c}e^{dx}\sinh\left(a+\frac{dx}{2}\right)\cosh\left(a+\frac{dx}{2}\right)}{2d}+\frac{e^{c}e^{dx}\cosh^{2}\left(a+\frac{dx}{2}\right)}{d} & \text{for} \\ \left(-\frac{x\sinh^{2}(a+bx)}{2}+\frac{x\cosh^{2}(a+bx)}{2}+\frac{\sinh(a+bx)\cosh(a+bx)}{2b}\right)e^{c} & \text{for} \\ -\frac{2b^{2}e^{c}e^{dx}\sinh^{2}(a+bx)}{4b^{2}d-d^{3}}+\frac{2b^{2}e^{c}e^{dx}\cosh^{2}(a+bx)}{4b^{2}d-d^{3}}+\frac{2bde^{c}e^{dx}\sinh(a+bx)\cosh(a+bx)}{4b^{2}d-d^{3}}-\frac{d^{2}e^{c}e^{dx}\cosh^{2}(a+bx)}{4b^{2}d-d^{3}} & \text{oth} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x+c)*cosh(b*x+a)**2,x)
```

```
[Out] Piecewise((x*exp(c)*cosh(a)**2, Eq(b, 0) & Eq(d, 0)), (x*exp(c)*exp(d*x)*si
nh(a - d*x/2)**2/4 + x*exp(c)*exp(d*x)*sinh(a - d*x/2)*cosh(a - d*x/2)/2 +
x*exp(c)*exp(d*x)*cosh(a - d*x/2)**2/4 + exp(c)*exp(d*x)*sinh(a - d*x/2)*co
sh(a - d*x/2)/(2*d) + exp(c)*exp(d*x)*cosh(a - d*x/2)**2/d, Eq(b, -d/2)), (
x*exp(c)*exp(d*x)*sinh(a + d*x/2)**2/4 - x*exp(c)*exp(d*x)*sinh(a + d*x/2)*
cosh(a + d*x/2)/2 + x*exp(c)*exp(d*x)*cosh(a + d*x/2)**2/4 - exp(c)*exp(d*x
)*sinh(a + d*x/2)*cosh(a + d*x/2)/(2*d) + exp(c)*exp(d*x)*cosh(a + d*x/2)**
2/d, Eq(b, d/2)), ((-x*sinh(a + b*x)**2/2 + x*cosh(a + b*x)**2/2 + sinh(a +
 b*x)*cosh(a + b*x)/(2*b))*exp(c), Eq(d, 0)), (-2*b**2*exp(c)*exp(d*x)*sinh
(a + b*x)**2/(4*b**2*d - d**3) + 2*b**2*exp(c)*exp(d*x)*cosh(a + b*x)**2/(4
*b**2*d - d**3) + 2*b*d*exp(c)*exp(d*x)*sinh(a + b*x)*cosh(a + b*x)/(4*b**2
*d - d**3) - d**2*exp(c)*exp(d*x)*cosh(a + b*x)**2/(4*b**2*d - d**3), True)
)
```

3.956 $\int e^{c+dx}\cosh(a+bx)\coth(a+bx)\,dx$

Optimal. Leaf size=103

$$\frac{2e^{-a-x(b-d)+c}\,{}_2F_1\left(1,-\frac{b-d}{2b};\frac{b+d}{2b};e^{2(a+bx)}\right)}{b-d}-\frac{3e^{-a-x(b-d)+c}}{2(b-d)}+\frac{e^{a+x(b+d)+c}}{2(b+d)}$$

```
[Out] -3/2*exp(-a+c-(b-d)*x)/(b-d)+1/2*exp(a+c+(b+d)*x)/(b+d)+2*exp(-a+c-(b-d)*x)
*hypergeom([1, 1/2*(-b+d)/b],[1/2*(b+d)/b],exp(2*b*x+2*a))/(b-d)
```

Rubi [A] time = 0.21, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {5511, 2194, 2227, 2251}

$$\frac{2e^{-a-x(b-d)+c}\,{}_2F_1\left(1,-\frac{b-d}{2b};\frac{b+d}{2b};e^{2(a+bx)}\right)}{b-d}-\frac{3e^{-a-x(b-d)+c}}{2(b-d)}+\frac{e^{a+x(b+d)+c}}{2(b+d)}$$

Antiderivative was successfully verified.

```
[In] Int[E^(c + d*x)*Cosh[a + b*x]*Coth[a + b*x],x]
```

```
[Out] (-3*E^(-a + c - (b - d)*x))/(2*(b - d)) + E^(a + c + (b + d)*x)/(2*(b + d))
 + (2*E^(-a + c - (b - d)*x)*Hypergeometric2F1[1, -(b - d)/(2*b), (b + d)/(
2*b), E^(2*(a + b*x))])/(b - d)
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 2227

```
Int[(u_.)*(F_)^((a_.) + (b_.)*(v_)), x_Symbol] :> Int[u*F^(a + b*NormalizeP
owerOfLinear[v, x]), x] /; FreeQ[{F, a, b}, x] && PolynomialQ[u, x] && Powe
rOfLinearQ[v, x] &&  !PowerOfLinearMatchQ[v, x]
```

Rule 2251

```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_)*(G_)^((h_.)*((f_.
) + (g_.)*(x_))), x_Symbol] :> Simp[(a^p*G^(h*(f + g*x))*Hypergeometric2F1[
-p, (g*h*Log[G])/(d*e*Log[F]), (g*h*Log[G])/(d*e*Log[F]) + 1, Simplify[-((b
*F^(e*(c + d*x)))/a)]])/(g*h*Log[G]), x] /; FreeQ[{F, G, a, b, c, d, e, f,
g, h, p}, x] && (ILtQ[p, 0] || GtQ[a, 0])
```

Rule 5511

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*(G_)[(d_.) + (e_.)*(x_)]^(m_.)*(H_)[(
d_.) + (e_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^(c*(a + b*x)),
 G[d + e*x]^m*H[d + e*x]^n, x], x] /; FreeQ[{F, a, b, c, d, e}, x] && IGtQ[
m, 0] && IGtQ[n, 0] && HyperbolicQ[G] && HyperbolicQ[H]
```

Rubi steps

$$\begin{aligned}\int e^{c+dx}\cosh(a+bx)\coth(a+bx)\,dx &= \int\left(\frac{3}{2}e^{-a+c-(b-d)x}+\frac{1}{2}e^{-a+c-(b-d)x+2(a+bx)}+\frac{2e^{-a+c-(b-d)x}}{-1+e^{2(a+bx)}}\right)dx\\ &= \frac{1}{2}\int e^{-a+c-(b-d)x+2(a+bx)}\,dx+\frac{3}{2}\int e^{-a+c-(b-d)x}\,dx+2\int\frac{e^{-a+c-(b-d)x}}{-1+e^{2(a+bx)}}\,dx\\ &= -\frac{3e^{-a+c-(b-d)x}}{2(b-d)}+\frac{2e^{-a+c-(b-d)x}\,{}_2F_1\left(1,-\frac{b-d}{2b};\frac{b+d}{2b};e^{2(a+bx)}\right)}{b-d}+\frac{1}{2}\int e^{a+c+(b+d}\\ &= -\frac{3e^{-a+c-(b-d)x}}{2(b-d)}+\frac{e^{a+c+(b+d)x}}{2(b+d)}+\frac{2e^{-a+c-(b-d)x}\,{}_2F_1\left(1,-\frac{b-d}{2b};\frac{b+d}{2b};e^{2(a+bx)}\right)}{b-d}\end{aligned}$$

Mathematica [A] time = 0.58, size = 93, normalized size = 0.90

$$\frac{e^c\left(\frac{e^{dx}(b\cosh(a+bx)-d\sinh(a+bx))}{b-d}-2(\sinh(a)+\cosh(a))e^{x(b+d)}\,{}_2F_1\left(1,\frac{b+d}{2b};\frac{3b+d}{2b};e^{2bx}(\cosh(a)+\sinh(a))^2\right)\right)}{b+d}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(c + d*x)*Cosh[a + b*x]*Coth[a + b*x],x]
```

```
[Out] (E^c*(-2*E^((b + d)*x)*Hypergeometric2F1[1, (b + d)/(2*b), (3*b + d)/(2*b),
 E^(2*b*x)*(Cosh[a] + Sinh[a])^2]*(Cosh[a] + Sinh[a]) + (E^(d*x)*(b*Cosh[a
+ b*x] - d*Sinh[a + b*x]))/(b - d)))/(b + d)
```

fricas [F] time = 0.43, size = 0, normalized size = 0.00

$$\text{integral}\left(\cosh(bx+a)^2\operatorname{csch}(bx+a)\,e^{(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x+c)*cosh(b*x+a)^2*csch(b*x+a),x, algorithm="fricas")
```

```
[Out] integral(cosh(b*x + a)^2*csch(b*x + a)*e^(d*x + c), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \cosh\left(bx+a\right)^{2} \operatorname{csch}\left(bx+a\right) e^{(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x+c)*cosh(b*x+a)^2*csch(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(cosh(b*x + a)^2*csch(b*x + a)*e^(d*x + c), x)
```

maple [F] time = 0.65, size = 0, normalized size = 0.00

$$\int \mathrm{e}^{dx+c} \left(\cosh^{2}\left(bx+a\right)\right) \operatorname{csch}\left(bx+a\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(d*x+c)*cosh(b*x+a)^2*csch(b*x+a),x)
```

```
[Out] int(exp(d*x+c)*cosh(b*x+a)^2*csch(b*x+a),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-4\,b \int \frac{e^{(dx+c)}}{(3\,b-d)e^{(5\,bx+5\,a)} - 2\,(3\,b-d)e^{(3\,bx+3\,a)} + (3\,b-d)e^{(bx+a)}}\,dx + \frac{\left(5\,b^{2}e^{c} + 6\,bde^{c} + d^{2}e^{c} + \left(3\,b^{2}e^{c} - 4\,bde^{c} + \cdots\right.\right.}{2\left(\left(3\,b^{3} - b^{2}d - 3\,bd^{2} + d^{3}\right)e^{(3\cdots}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x+c)*cosh(b*x+a)^2*csch(b*x+a),x, algorithm="maxima")
```

```
[Out] -4*b*integrate(e^(d*x + c)/((3*b - d)*e^(5*b*x + 5*a) - 2*(3*b - d)*e^(3*b*
x + 3*a) + (3*b - d)*e^(b*x + a)), x) + 1/2*(5*b^2*e^c + 6*b*d*e^c + d^2*e^
c + (3*b^2*e^c - 4*b*d*e^c + d^2*e^c)*e^(4*b*x + 4*a) - 2*(6*b^2*e^c + b*d*
e^c - d^2*e^c)*e^(2*b*x + 2*a))*e^(d*x)/((3*b^3 - b^2*d - 3*b*d^2 + d^3)*e^
(3*b*x + 3*a) - (3*b^3 - b^2*d - 3*b*d^2 + d^3)*e^(b*x + a))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cosh\left(a+b\,x\right)^{2}\,\mathrm{e}^{c+d\,x}}{\sinh\left(a+b\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cosh(a + b*x)^2*exp(c + d*x))/sinh(a + b*x),x)
```

```
[Out] int((cosh(a + b*x)^2*exp(c + d*x))/sinh(a + b*x), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x+c)*cosh(b*x+a)**2*csch(b*x+a),x)
```

```
[Out] Timed out
```

3.957 $\int e^{c+dx}\coth^2(a+bx)\,dx$

Optimal. Leaf size=94

$$-\frac{4e^{c+dx}\,{}_2F_1\left(1,\frac{d}{2b};\frac{d}{2b}+1;e^{2(a+bx)}\right)}{d}+\frac{4e^{c+dx}\,{}_2F_1\left(2,\frac{d}{2b};\frac{d}{2b}+1;e^{2(a+bx)}\right)}{d}+\frac{e^{c+dx}}{d}$$

```
[Out] exp(d*x+c)/d-4*exp(d*x+c)*hypergeom([1, 1/2*d/b],[1+1/2*d/b],exp(2*b*x+2*a)
)/d+4*exp(d*x+c)*hypergeom([2, 1/2*d/b],[1+1/2*d/b],exp(2*b*x+2*a))/d
```

Rubi [A] time = 0.11, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {5485, 2194, 2251}

$$-\frac{4e^{c+dx}\,{}_2F_1\left(1,\frac{d}{2b};\frac{d}{2b}+1;e^{2(a+bx)}\right)}{d}+\frac{4e^{c+dx}\,{}_2F_1\left(2,\frac{d}{2b};\frac{d}{2b}+1;e^{2(a+bx)}\right)}{d}+\frac{e^{c+dx}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[E^(c + d*x)*Coth[a + b*x]^2,x]
```

```
[Out] E^(c + d*x)/d - (4*E^(c + d*x)*Hypergeometric2F1[1, d/(2*b), 1 + d/(2*b), E
^(2*(a + b*x))])/d + (4*E^(c + d*x)*Hypergeometric2F1[2, d/(2*b), 1 + d/(2*
b), E^(2*(a + b*x))])/d
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 2251

```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_)*(G_)^((h_.)*((f_.
) + (g_.)*(x_))), x_Symbol] :> Simp[(a^p*G^(h*(f + g*x))*Hypergeometric2F1[
-p, (g*h*Log[G])/(d*e*Log[F]), (g*h*Log[G])/(d*e*Log[F]) + 1, Simplify[-((b
*F^(e*(c + d*x)))/a)]])/(g*h*Log[G]), x] /; FreeQ[{F, G, a, b, c, d, e, f,
g, h, p}, x] && (ILtQ[p, 0] || GtQ[a, 0])
```

Rule 5485

```
Int[Coth[(d_.) + (e_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Sym
bol] :> Int[ExpandIntegrand[(F^(c*(a + b*x))*(1 + E^(2*(d + e*x)))^n)/(-1 +
 E^(2*(d + e*x)))^n, x], x] /; FreeQ[{F, a, b, c, d, e}, x] && IntegerQ[n]
```

Rubi steps

$$\int e^{c+dx}\coth^2(a+bx)\,dx=\int\left(e^{c+dx}+\frac{4e^{c+dx}}{\left(-1+e^{2(a+bx)}\right)^2}+\frac{4e^{c+dx}}{-1+e^{2(a+bx)}}\right)dx$$
$$=4\int\frac{e^{c+dx}}{\left(-1+e^{2(a+bx)}\right)^2}\,dx+4\int\frac{e^{c+dx}}{-1+e^{2(a+bx)}}\,dx+\int e^{c+dx}\,dx$$
$$=\frac{e^{c+dx}}{d}-\frac{4e^{c+dx}\,{}_2F_1\left(1,\frac{d}{2b};1+\frac{d}{2b};e^{2(a+bx)}\right)}{d}+\frac{4e^{c+dx}\,{}_2F_1\left(2,\frac{d}{2b};1+\frac{d}{2b};e^{2(a+bx)}\right)}{d}$$

Mathematica [A] time = 0.90, size = 145, normalized size = 1.54

$$-\frac{2de^{2a+c}\left(\frac{e^{dx}\,{}_2F_1\left(1,\frac{d}{2b};\frac{d}{2b}+1;e^{2(a+bx)}\right)}{d}-\frac{e^{x(2b+d)}\,{}_2F_1\left(1,\frac{d}{2b}+1;\frac{d}{2b}+2;e^{2(a+bx)}\right)}{2b+d}\right)}{\left(e^{2a}-1\right)b}+\frac{\operatorname{csch}(a)\sinh(bx)e^{c+dx}\operatorname{csch}(a+bx)}{b}+\frac{e^{c+dx}}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(c + d*x)*Coth[a + b*x]^2,x]
```

```
[Out] E^(c + d*x)/d - (2*d*E^(2*a + c)*((E^(d*x)*Hypergeometric2F1[1, d/(2*b), 1
+ d/(2*b), E^(2*(a + b*x))])/d - (E^((2*b + d)*x)*Hypergeometric2F1[1, 1 +
d/(2*b), 2 + d/(2*b), E^(2*(a + b*x))])/(2*b + d)))/(b*(-1 + E^(2*a))) + (E
^(c + d*x)*Csch[a]*Csch[a + b*x]*Sinh[b*x])/b
```

fricas [F] time = 0.50, size = 0, normalized size = 0.00

$$\text{integral}\left(\cosh\left(bx+a\right)^2\operatorname{csch}\left(bx+a\right)^2e^{(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x+c)*cosh(b*x+a)^2*csch(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] integral(cosh(b*x + a)^2*csch(b*x + a)^2*e^(d*x + c), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\cosh\left(bx+a\right)^2\operatorname{csch}\left(bx+a\right)^2e^{(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x+c)*cosh(b*x+a)^2*csch(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate(cosh(b*x + a)^2*csch(b*x + a)^2*e^(d*x + c), x)
```

maple [F] time = 0.68, size = 0, normalized size = 0.00

$$\int e^{dx+c} \left(\cosh^2(bx+a)\right) \operatorname{csch}(bx+a)^2\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(d*x+c)*cosh(b*x+a)^2*csch(b*x+a)^2,x)
```

```
[Out] int(exp(d*x+c)*cosh(b*x+a)^2*csch(b*x+a)^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$16\,bd \int -\frac{e^{(dx+c)}}{8\,b^2 - 6\,bd + d^2 - \left(8\,b^2 - 6\,bd + d^2\right)e^{(6\,bx+6\,a)} + 3\left(8\,b^2 - 6\,bd + d^2\right)e^{(4\,bx+4\,a)} - 3\left(8\,b^2 - 6\,bd + d^2\right)e^{(2\,bx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x+c)*cosh(b*x+a)^2*csch(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 16*b*d*integrate(-e^(d*x + c)/(8*b^2 - 6*b*d + d^2 - (8*b^2 - 6*b*d + d^2)*
e^(6*b*x + 6*a) + 3*(8*b^2 - 6*b*d + d^2)*e^(4*b*x + 4*a) - 3*(8*b^2 - 6*b*
d + d^2)*e^(2*b*x + 2*a)), x) + (8*b^2*e^c + 10*b*d*e^c + d^2*e^c + (8*b^2*
e^c - 6*b*d*e^c + d^2*e^c)*e^(4*b*x + 4*a) - 2*(8*b^2*e^c + 2*b*d*e^c - d^2
*e^c)*e^(2*b*x + 2*a))*e^(d*x)/(8*b^2*d - 6*b*d^2 + d^3 + (8*b^2*d - 6*b*d^
2 + d^3)*e^(4*b*x + 4*a) - 2*(8*b^2*d - 6*b*d^2 + d^3)*e^(2*b*x + 2*a))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cosh(a+b\,x)^2\,e^{c+d\,x}}{\sinh(a+b\,x)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cosh(a + b*x)^2*exp(c + d*x))/sinh(a + b*x)^2,x)
```

```
[Out] int((cosh(a + b*x)^2*exp(c + d*x))/sinh(a + b*x)^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x+c)*cosh(b*x+a)**2*csch(b*x+a)**2,x)
```

```
[Out] Timed out
```

3.958 $\int e^{c+dx}\coth^2(a+bx)\mathbf{csch}(a+bx)\,dx$

Optimal. Leaf size=151

$$-\frac{2e^{a+x(b+d)+c}\,{}_2F_1\left(1,\frac{b+d}{2b};\frac{3b+d}{2b};e^{2(a+bx)}\right)}{b+d}+\frac{8e^{a+x(b+d)+c}\,{}_2F_1\left(2,\frac{b+d}{2b};\frac{3b+d}{2b};e^{2(a+bx)}\right)}{b+d}-\frac{8e^{a+x(b+d)+c}\,{}_2F_1\left(3,\frac{b+d}{2b};\frac{3b+d}{2b};e^{2(a-}\right.}{b+d}$$

```
[Out] -2*exp(a+c+(b+d)*x)*hypergeom([1, 1/2*(b+d)/b],[1/2*(3*b+d)/b],exp(2*b*x+2*
a))/(b+d)+8*exp(a+c+(b+d)*x)*hypergeom([2, 1/2*(b+d)/b],[1/2*(3*b+d)/b],exp
(2*b*x+2*a))/(b+d)-8*exp(a+c+(b+d)*x)*hypergeom([3, 1/2*(b+d)/b],[1/2*(3*b+
d)/b],exp(2*b*x+2*a))/(b+d)
```

Rubi [A] time = 0.34, antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {5511, 2251}

$$-\frac{2e^{a+x(b+d)+c}\,{}_2F_1\left(1,\frac{b+d}{2b};\frac{3b+d}{2b};e^{2(a+bx)}\right)}{b+d}+\frac{8e^{a+x(b+d)+c}\,{}_2F_1\left(2,\frac{b+d}{2b};\frac{3b+d}{2b};e^{2(a+bx)}\right)}{b+d}-\frac{8e^{a+x(b+d)+c}\,{}_2F_1\left(3,\frac{b+d}{2b};\frac{3b+d}{2b};e^{2(a-}\right.}{b+d}$$

Antiderivative was successfully verified.

```
[In] Int[E^(c + d*x)*Coth[a + b*x]^2*Csch[a + b*x],x]
```

```
[Out] (-2*E^(a + c + (b + d)*x)*Hypergeometric2F1[1, (b + d)/(2*b), (3*b + d)/(2*
b), E^(2*(a + b*x))])/(b + d) + (8*E^(a + c + (b + d)*x)*Hypergeometric2F1[
2, (b + d)/(2*b), (3*b + d)/(2*b), E^(2*(a + b*x))])/(b + d) - (8*E^(a + c
+ (b + d)*x)*Hypergeometric2F1[3, (b + d)/(2*b), (3*b + d)/(2*b), E^(2*(a +
 b*x))])/(b + d)
```

Rule 2251

```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_)*(G_)^((h_.)*((f_.
) + (g_.)*(x_))), x_Symbol] :> Simp[(a^p*G^(h*(f + g*x))*Hypergeometric2F1[
-p, (g*h*Log[G])/(d*e*Log[F]), (g*h*Log[G])/(d*e*Log[F]) + 1, Simplify[-((b
*F^(e*(c + d*x)))/a)]])/(g*h*Log[G]), x] /; FreeQ[{F, G, a, b, c, d, e, f,
g, h, p}, x] && (ILtQ[p, 0] || GtQ[a, 0])
```

Rule 5511

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*(G_)[(d_.) + (e_.)*(x_)]^(m_.)*(H_)[(
d_.) + (e_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^(c*(a + b*x)),
 G[d + e*x]^m*H[d + e*x]^n, x], x] /; FreeQ[{F, a, b, c, d, e}, x] && IGtQ[
m, 0] && IGtQ[n, 0] && HyperbolicQ[G] && HyperbolicQ[H]
```

Rubi steps

$$\int e^{c+dx}\coth^2(a+bx)\operatorname{csch}(a+bx)\,dx=\int\left(\frac{8e^{a+c+(b+d)x}}{\left(-1+e^{2(a+bx)}\right)^3}+\frac{8e^{a+c+(b+d)x}}{\left(-1+e^{2(a+bx)}\right)^2}+\frac{2e^{a+c+(b+d)x}}{-1+e^{2(a+bx)}}\right)dx$$
$$=2\int\frac{e^{a+c+(b+d)x}}{-1+e^{2(a+bx)}}\,dx+8\int\frac{e^{a+c+(b+d)x}}{\left(-1+e^{2(a+bx)}\right)^3}\,dx+8\int\frac{e^{a+c+(b+d)x}}{\left(-1+e^{2(a+bx)}\right)^2}\,d$$
$$=-\frac{2e^{a+c+(b+d)x}\,{}_2F_1\left(1,\frac{b+d}{2b};\frac{3b+d}{2b};e^{2(a+bx)}\right)}{b+d}+\frac{8e^{a+c+(b+d)x}\,{}_2F_1\left(2,\frac{b+d}{2b};\frac{3b+d}{2b};\right.}{b+d}$$

Mathematica [A] time = 1.33, size = 111, normalized size = 0.74

$$-\frac{e^{c-\frac{ad}{b}}\left(2\left(b^2+d^2\right)e^{\frac{(b+d)(a+bx)}{b}}\,{}_2F_1\left(1,\frac{b+d}{2b};\frac{3b+d}{2b};e^{2(a+bx)}\right)+(b+d)e^{d\left(\frac{a}{b}+x\right)}\operatorname{csch}(a+bx)(b\coth(a+bx)+d)\right)}{2b^2(b+d)}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(c + d*x)*Coth[a + b*x]^2*Csch[a + b*x],x]
```

```
[Out] -1/2*(E^(c - (a*d)/b)*((b + d)*E^(d*(a/b + x))*(d + b*Coth[a + b*x])*Csch[a
 + b*x] + 2*(b^2 + d^2)*E^(((b + d)*(a + b*x))/b)*Hypergeometric2F1[1, (b +
 d)/(2*b), (3*b + d)/(2*b), E^(2*(a + b*x))]))/(b^2*(b + d))
```

fricas [F] time = 0.52, size = 0, normalized size = 0.00

$$\text{integral}\left(\cosh\left(bx+a\right)^2\operatorname{csch}\left(bx+a\right)^3e^{(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x+c)*cosh(b*x+a)^2*csch(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] integral(cosh(b*x + a)^2*csch(b*x + a)^3*e^(d*x + c), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\cosh\left(bx+a\right)^2\operatorname{csch}\left(bx+a\right)^3e^{(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x+c)*cosh(b*x+a)^2*csch(b*x+a)^3,x, algorithm="giac")
```

```
[Out] integrate(cosh(b*x + a)^2*csch(b*x + a)^3*e^(d*x + c), x)
```

maple [F] time = 0.75, size = 0, normalized size = 0.00

$$\int e^{dx+c} \left(\cosh^2 (bx + a)\right) \operatorname{csch} (bx + a)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(d*x+c)*cosh(b*x+a)^2*csch(b*x+a)^3,x)
[Out] int(exp(d*x+c)*cosh(b*x+a)^2*csch(b*x+a)^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-48 \left(b^3 e^c + bd^2 e^c\right) \int \frac{}{15\, b^3 - 23\, b^2 d + 9\, bd^2 - d^3 + \left(15\, b^3 - 23\, b^2 d + 9\, bd^2 - d^3\right) e^{(8\, bx+8\, a)} - 4\left(15\, b^3 - 23\, b^2 d + 9\, b\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x+c)*cosh(b*x+a)^2*csch(b*x+a)^3,x, algorithm="maxima")
[Out] -48*(b^3*e^c + b*d^2*e^c)*integrate(e^(b*x + d*x + a)/(15*b^3 - 23*b^2*d +
9*b*d^2 - d^3 + (15*b^3 - 23*b^2*d + 9*b*d^2 - d^3)*e^(8*b*x + 8*a) - 4*(15
*b^3 - 23*b^2*d + 9*b*d^2 - d^3)*e^(6*b*x + 6*a) + 6*(15*b^3 - 23*b^2*d + 9
*b*d^2 - d^3)*e^(4*b*x + 4*a) - 4*(15*b^3 - 23*b^2*d + 9*b*d^2 - d^3)*e^(2*
b*x + 2*a)), x) + 2*((15*b^2*e^c - 8*b*d*e^c + d^2*e^c)*e^(5*b*x + 5*a) - 2
*(10*b^2*e^c + 3*b*d*e^c - d^2*e^c)*e^(3*b*x + 3*a) + (9*b^2*e^c + 14*b*d*e
^c + d^2*e^c)*e^(b*x + a))*e^(d*x)/(15*b^3 - 23*b^2*d + 9*b*d^2 - d^3 - (15
*b^3 - 23*b^2*d + 9*b*d^2 - d^3)*e^(6*b*x + 6*a) + 3*(15*b^3 - 23*b^2*d + 9
*b*d^2 - d^3)*e^(4*b*x + 4*a) - 3*(15*b^3 - 23*b^2*d + 9*b*d^2 - d^3)*e^(2*
b*x + 2*a))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cosh (a + b\, x)^2\, e^{c+d\, x}}{\sinh (a + b\, x)^3} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cosh(a + b*x)^2*exp(c + d*x))/sinh(a + b*x)^3,x)
[Out] int((cosh(a + b*x)^2*exp(c + d*x))/sinh(a + b*x)^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x+c)*cosh(b*x+a)**2*csch(b*x+a)**3,x)
```

```
[Out] Timed out
```

3.959 $\int e^{c+dx}\cosh^3(a+bx)\sinh^3(a+bx)\,dx$

Optimal. Leaf size=137

$$\frac{3de^{c+dx}\sinh(2a+2bx)}{32(4b^2-d^2)}-\frac{de^{c+dx}\sinh(6a+6bx)}{32(36b^2-d^2)}-\frac{3be^{c+dx}\cosh(2a+2bx)}{16(4b^2-d^2)}+\frac{3be^{c+dx}\cosh(6a+6bx)}{16(36b^2-d^2)}$$

```
[Out] -3/16*b*exp(d*x+c)*cosh(2*b*x+2*a)/(4*b^2-d^2)+3/16*b*exp(d*x+c)*cosh(6*b*x
+6*a)/(36*b^2-d^2)+3/32*d*exp(d*x+c)*sinh(2*b*x+2*a)/(4*b^2-d^2)-1/32*d*exp
(d*x+c)*sinh(6*b*x+6*a)/(36*b^2-d^2)
```

Rubi [A] time = 0.11, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {5509, 5474}

$$\frac{3de^{c+dx}\sinh(2a+2bx)}{32(4b^2-d^2)}-\frac{de^{c+dx}\sinh(6a+6bx)}{32(36b^2-d^2)}-\frac{3be^{c+dx}\cosh(2a+2bx)}{16(4b^2-d^2)}+\frac{3be^{c+dx}\cosh(6a+6bx)}{16(36b^2-d^2)}$$

Antiderivative was successfully verified.

```
[In] Int[E^(c + d*x)*Cosh[a + b*x]^3*Sinh[a + b*x]^3,x]
```

```
[Out] (-3*b*E^(c + d*x)*Cosh[2*a + 2*b*x])/(16*(4*b^2 - d^2)) + (3*b*E^(c + d*x)*
Cosh[6*a + 6*b*x])/(16*(36*b^2 - d^2)) + (3*d*E^(c + d*x)*Sinh[2*a + 2*b*x]
)/(32*(4*b^2 - d^2)) - (d*E^(c + d*x)*Sinh[6*a + 6*b*x])/(32*(36*b^2 - d^2)
)
```

Rule 5474

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sinh[(d_.) + (e_.)*(x_)], x_Symbol] :
> -Simp[(b*c*Log[F]*F^(c*(a + b*x))*Sinh[d + e*x])/(e^2 - b^2*c^2*Log[F]^2)
, x] + Simp[(e*F^(c*(a + b*x))*Cosh[d + e*x])/(e^2 - b^2*c^2*Log[F]^2), x]
/; FreeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 - b^2*c^2*Log[F]^2, 0]
```

Rule 5509

```
Int[Cosh[(f_.) + (g_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sinh[(
d_.) + (e_.)*(x_)]^(m_.), x_Symbol] :> Int[ExpandTrigReduce[F^(c*(a + b*x))
, Sinh[d + e*x]^m*Cosh[f + g*x]^n, x], x] /; FreeQ[{F, a, b, c, d, e, f, g}
, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

Rubi steps

$$\int e^{c+dx}\cosh^3(a+bx)\sinh^3(a+bx)\,dx = \int\left(-\frac{3}{32}e^{c+dx}\sinh(2a+2bx)+\frac{1}{32}e^{c+dx}\sinh(6a+6bx)\right)dx$$
$$= \frac{1}{32}\int e^{c+dx}\sinh(6a+6bx)\,dx - \frac{3}{32}\int e^{c+dx}\sinh(2a+2bx)\,dx$$
$$= -\frac{3be^{c+dx}\cosh(2a+2bx)}{16\left(4b^2-d^2\right)} + \frac{3be^{c+dx}\cosh(6a+6bx)}{16\left(36b^2-d^2\right)} + \frac{3de^{c+dx}\sinh(2}{32\left(4b^2-\right.}$$

Mathematica [A] time = 1.01, size = 113, normalized size = 0.82

$$\frac{e^{c+dx}\left(6\left(4b^3-bd^2\right)\cosh(6(a+bx))+6b\left(d^2-36b^2\right)\cosh(2(a+bx))+2d\sinh(2(a+bx))\left(\left(d^2-4b^2\right)\cosh(4(\right.\right.}{32\left(144b^4-40b^2d^2+d^4\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(c + d*x)*Cosh[a + b*x]^3*Sinh[a + b*x]^3,x]
```

```
[Out] (E^(c + d*x)*(6*b*(-36*b^2 + d^2)*Cosh[2*(a + b*x)] + 6*(4*b^3 - b*d^2)*Cos
h[6*(a + b*x)] + 2*d*(52*b^2 - d^2 + (-4*b^2 + d^2)*Cosh[4*(a + b*x)])*Sinh
[2*(a + b*x)]))/(32*(144*b^4 - 40*b^2*d^2 + d^4))
```

fricas [B] time = 0.48, size = 676, normalized size = 4.93

$$-\frac{10\left(4\,b^2d-d^3\right)\cosh\left(bx+a\right)^3\cosh\left(dx+c\right)\sinh\left(bx+a\right)^3-45\left(4\,b^3-bd^2\right)\cosh\left(bx+a\right)^2\cosh\left(dx+c\right)\sinh}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x+c)*cosh(b*x+a)^3*sinh(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -1/16*(10*(4*b^2*d - d^3)*cosh(b*x + a)^3*cosh(d*x + c)*sinh(b*x + a)^3 - 4
5*(4*b^3 - b*d^2)*cosh(b*x + a)^2*cosh(d*x + c)*sinh(b*x + a)^4 + 3*(4*b^2*
d - d^3)*cosh(b*x + a)*cosh(d*x + c)*sinh(b*x + a)^5 - 3*(4*b^3 - b*d^2)*co
sh(d*x + c)*sinh(b*x + a)^6 - 3*(15*(4*b^3 - b*d^2)*cosh(b*x + a)^4 - 36*b^
3 + b*d^2)*cosh(d*x + c)*sinh(b*x + a)^2 + 3*((4*b^2*d - d^3)*cosh(b*x + a)
^5 - (36*b^2*d - d^3)*cosh(b*x + a))*cosh(d*x + c)*sinh(b*x + a) - 3*((4*b^
3 - b*d^2)*cosh(b*x + a)^6 - (36*b^3 - b*d^2)*cosh(b*x + a)^2)*cosh(d*x + c
) - (3*(4*b^3 - b*d^2)*cosh(b*x + a)^6 - 10*(4*b^2*d - d^3)*cosh(b*x + a)^3
*sinh(b*x + a)^3 + 45*(4*b^3 - b*d^2)*cosh(b*x + a)^2*sinh(b*x + a)^4 - 3*(
4*b^2*d - d^3)*cosh(b*x + a)*sinh(b*x + a)^5 + 3*(4*b^3 - b*d^2)*sinh(b*x +
 a)^6 - 3*(36*b^3 - b*d^2)*cosh(b*x + a)^2 + 3*(15*(4*b^3 - b*d^2)*cosh(b*x
 + a)^4 - 36*b^3 + b*d^2)*sinh(b*x + a)^2 - 3*((4*b^2*d - d^3)*cosh(b*x + a
```

```
)^5 - (36*b^2*d - d^3)*cosh(b*x + a))*sinh(b*x + a))*sinh(d*x + c))/((144*b
^4 - 40*b^2*d^2 + d^4)*cosh(b*x + a)^6 - 3*(144*b^4 - 40*b^2*d^2 + d^4)*cos
h(b*x + a)^4*sinh(b*x + a)^2 + 3*(144*b^4 - 40*b^2*d^2 + d^4)*cosh(b*x + a)
^2*sinh(b*x + a)^4 - (144*b^4 - 40*b^2*d^2 + d^4)*sinh(b*x + a)^6)
```

giac [A] time = 0.13, size = 93, normalized size = 0.68

$$\frac{e^{(6\,bx+dx+6\,a+c)}}{64\,(6\,b+d)}-\frac{3\,e^{(2\,bx+dx+2\,a+c)}}{64\,(2\,b+d)}-\frac{3\,e^{(-2\,bx+dx-2\,a+c)}}{64\,(2\,b-d)}+\frac{e^{(-6\,bx+dx-6\,a+c)}}{64\,(6\,b-d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x+c)*cosh(b*x+a)^3*sinh(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/64*e^(6*b*x + d*x + 6*a + c)/(6*b + d) - 3/64*e^(2*b*x + d*x + 2*a + c)/(
2*b + d) - 3/64*e^(-2*b*x + d*x - 2*a + c)/(2*b - d) + 1/64*e^(-6*b*x + d*x
 - 6*a + c)/(6*b - d)
```

maple [A] time = 0.39, size = 202, normalized size = 1.47

$$\frac{3\sinh\left(2a-c+(2b-d)\,x\right)}{64\,(2b-d)}-\frac{3\sinh\left(2a+c+(2b+d)\,x\right)}{64\,(2b+d)}-\frac{\sinh\left((6b-d)\,x+6a-c\right)}{64\,(6b-d)}+\frac{\sinh\left((6b+d)\,x+6a+c\right)}{384b+64d}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(d*x+c)*cosh(b*x+a)^3*sinh(b*x+a)^3,x)
```

```
[Out] 3/64*sinh(2*a-c+(2*b-d)*x)/(2*b-d)-3/64*sinh(2*a+c+(2*b+d)*x)/(2*b+d)-1/64/
(6*b-d)*sinh((6*b-d)*x+6*a-c)+1/64/(6*b+d)*sinh((6*b+d)*x+6*a+c)-3/64*cosh(
2*a-c+(2*b-d)*x)/(2*b-d)-3/64*cosh(2*a+c+(2*b+d)*x)/(2*b+d)+1/64*cosh((6*b-
d)*x+6*a-c)/(6*b-d)+1/64*cosh((6*b+d)*x+6*a+c)/(6*b+d)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x+c)*cosh(b*x+a)^3*sinh(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(1-d/b>0)', see `assume?` for more d
etails)Is 1-d/b equal to -1?
```

mupad [B] time = 0.96, size = 182, normalized size = 1.33

$$-\frac{b^3\left(\frac{27\,\mathrm{e}^{c+d\,x}\,\cosh(2\,a+2\,b\,x)}{4}-\frac{3\,\mathrm{e}^{c+d\,x}\,\cosh(6\,a+6\,b\,x)}{4}\right)+d^3\left(\frac{3\,\mathrm{e}^{c+d\,x}\,\sinh(2\,a+2\,b\,x)}{32}-\frac{\mathrm{e}^{c+d\,x}\,\sinh(6\,a+6\,b\,x)}{32}\right)-b^2\,d\,\left(\frac{27\,\mathrm{e}^{c+d\,x}\,\sinh(2\,a+}{8}\right.}{144\,b^4-40\,b^2\,d^2+d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a + b*x)^3*exp(c + d*x)*sinh(a + b*x)^3,x)
```

```
[Out] -(b^3*((27*exp(c + d*x)*cosh(2*a + 2*b*x))/4 - (3*exp(c + d*x)*cosh(6*a + 6
*b*x))/4) + d^3*((3*exp(c + d*x)*sinh(2*a + 2*b*x))/32 - (exp(c + d*x)*sinh
(6*a + 6*b*x))/32) - b^2*d*((27*exp(c + d*x)*sinh(2*a + 2*b*x))/8 - (exp(c
+ d*x)*sinh(6*a + 6*b*x))/8) - b*d^2*((3*exp(c + d*x)*cosh(2*a + 2*b*x))/16
 - (3*exp(c + d*x)*cosh(6*a + 6*b*x))/16))/(144*b^4 + d^4 - 40*b^2*d^2)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x+c)*cosh(b*x+a)**3*sinh(b*x+a)**3,x)
```

```
[Out] Timed out
```

3.960 $\int e^{c+dx}\cosh^3(a+bx)\sinh^2(a+bx)\,dx$

Optimal. Leaf size=195

$$-\frac{be^{c+dx}\sinh(a+bx)}{8(b^2-d^2)}+\frac{3be^{c+dx}\sinh(3a+3bx)}{16(9b^2-d^2)}+\frac{5be^{c+dx}\sinh(5a+5bx)}{16(25b^2-d^2)}+\frac{de^{c+dx}\cosh(a+bx)}{8(b^2-d^2)}-\frac{de^{c+dx}\cosh(3a+3}{16(9b^2-d^2)}$$

```
[Out] 1/8*d*exp(d*x+c)*cosh(b*x+a)/(b^2-d^2)-1/16*d*exp(d*x+c)*cosh(3*b*x+3*a)/(9
*b^2-d^2)-1/16*d*exp(d*x+c)*cosh(5*b*x+5*a)/(25*b^2-d^2)-1/8*b*exp(d*x+c)*s
inh(b*x+a)/(b^2-d^2)+3/16*b*exp(d*x+c)*sinh(3*b*x+3*a)/(9*b^2-d^2)+5/16*b*e
xp(d*x+c)*sinh(5*b*x+5*a)/(25*b^2-d^2)
```

Rubi [A] time = 0.13, antiderivative size = 195, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {5509, 5475}

$$-\frac{be^{c+dx}\sinh(a+bx)}{8(b^2-d^2)}+\frac{3be^{c+dx}\sinh(3a+3bx)}{16(9b^2-d^2)}+\frac{5be^{c+dx}\sinh(5a+5bx)}{16(25b^2-d^2)}+\frac{de^{c+dx}\cosh(a+bx)}{8(b^2-d^2)}-\frac{de^{c+dx}\cosh(3a+3}{16(9b^2-d^2)}$$

Antiderivative was successfully verified.

```
[In] Int[E^(c + d*x)*Cosh[a + b*x]^3*Sinh[a + b*x]^2,x]
```

```
[Out] (d*E^(c + d*x)*Cosh[a + b*x])/(8*(b^2 - d^2)) - (d*E^(c + d*x)*Cosh[3*a + 3
*b*x])/(16*(9*b^2 - d^2)) - (d*E^(c + d*x)*Cosh[5*a + 5*b*x])/(16*(25*b^2 -
 d^2)) - (b*E^(c + d*x)*Sinh[a + b*x])/(8*(b^2 - d^2)) + (3*b*E^(c + d*x)*S
inh[3*a + 3*b*x])/(16*(9*b^2 - d^2)) + (5*b*E^(c + d*x)*Sinh[5*a + 5*b*x])/
(16*(25*b^2 - d^2))
```

Rule 5475

```
Int[Cosh[(d_.) + (e_.)*(x_)]*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symbol] :
> -Simp[(b*c*Log[F]*F^(c*(a + b*x))*Cosh[d + e*x])/(e^2 - b^2*c^2*Log[F]^2)
, x] + Simp[(e*F^(c*(a + b*x))*Sinh[d + e*x])/(e^2 - b^2*c^2*Log[F]^2), x]
/; FreeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 - b^2*c^2*Log[F]^2, 0]
```

Rule 5509

```
Int[Cosh[(f_.) + (g_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sinh[(
d_.) + (e_.)*(x_)]^(m_.), x_Symbol] :> Int[ExpandTrigReduce[F^(c*(a + b*x))
, Sinh[d + e*x]^m*Cosh[f + g*x]^n, x], x] /; FreeQ[{F, a, b, c, d, e, f, g}
, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

Rubi steps

$$\int e^{c+dx}\cosh^3(a+bx)\sinh^2(a+bx)\,dx = \int\left(-\frac{1}{8}e^{c+dx}\cosh(a+bx)+\frac{1}{16}e^{c+dx}\cosh(3a+3bx)+\frac{1}{16}e^{c+dx}\cosh(5\right.$$
$$= \frac{1}{16}\int e^{c+dx}\cosh(3a+3bx)\,dx+\frac{1}{16}\int e^{c+dx}\cosh(5a+5bx)\,dx-\frac{1}{8}\int$$
$$= \frac{de^{c+dx}\cosh(a+bx)}{8\left(b^2-d^2\right)}-\frac{de^{c+dx}\cosh(3a+3bx)}{16\left(9b^2-d^2\right)}-\frac{de^{c+dx}\cosh(5a+5bx}{16\left(25b^2-d^2\right)}$$

Mathematica [A] time = 1.29, size = 118, normalized size = 0.61

$$\frac{1}{16}e^{c+dx}\left(\frac{3b\sinh(3(a+bx))-d\cosh(3(a+bx))}{9b^2-d^2}+\frac{5b\sinh(5(a+bx))-d\cosh(5(a+bx))}{25b^2-d^2}+\frac{2d\cosh(a+bx)-}{(b-d)(}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(c + d*x)*Cosh[a + b*x]^3*Sinh[a + b*x]^2,x]
```

```
[Out] (E^(c + d*x)*((2*d*Cosh[a + b*x] - 2*b*Sinh[a + b*x])/((b - d)*(b + d)) + (
-(d*Cosh[3*(a + b*x)]) + 3*b*Sinh[3*(a + b*x)])/(9*b^2 - d^2) + (-(d*Cosh[5
*(a + b*x)]) + 5*b*Sinh[5*(a + b*x)])/(25*b^2 - d^2)))/16
```

fricas [B] time = 0.53, size = 917, normalized size = 4.70

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x+c)*cosh(b*x+a)^3*sinh(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/16*(5*(9*b^4*d - 10*b^2*d^3 + d^5)*cosh(b*x + a)*cosh(d*x + c)*sinh(b*x
 + a)^4 - 5*(9*b^5 - 10*b^3*d^2 + b*d^4)*cosh(d*x + c)*sinh(b*x + a)^5 - (75
*b^5 - 78*b^3*d^2 + 3*b*d^4 + 50*(9*b^5 - 10*b^3*d^2 + b*d^4)*cosh(b*x + a)
^2)*cosh(d*x + c)*sinh(b*x + a)^3 + (10*(9*b^4*d - 10*b^2*d^3 + d^5)*cosh(b
*x + a)^3 + 3*(25*b^4*d - 26*b^2*d^3 + d^5)*cosh(b*x + a))*cosh(d*x + c)*si
nh(b*x + a)^2 + (450*b^5 - 68*b^3*d^2 + 2*b*d^4 - 25*(9*b^5 - 10*b^3*d^2 +
b*d^4)*cosh(b*x + a)^4 - 9*(25*b^5 - 26*b^3*d^2 + b*d^4)*cosh(b*x + a)^2)*c
osh(d*x + c)*sinh(b*x + a) + ((9*b^4*d - 10*b^2*d^3 + d^5)*cosh(b*x + a)^5
 + (25*b^4*d - 26*b^2*d^3 + d^5)*cosh(b*x + a)^3 - 2*(225*b^4*d - 34*b^2*d^3
 + d^5)*cosh(b*x + a))*cosh(d*x + c) + ((9*b^4*d - 10*b^2*d^3 + d^5)*cosh(b
*x + a)^5 + 5*(9*b^4*d - 10*b^2*d^3 + d^5)*cosh(b*x + a)*sinh(b*x + a)^4 -
5*(9*b^5 - 10*b^3*d^2 + b*d^4)*sinh(b*x + a)^5 + (25*b^4*d - 26*b^2*d^3 + d
^5)*cosh(b*x + a)^3 - (75*b^5 - 78*b^3*d^2 + 3*b*d^4 + 50*(9*b^5 - 10*b^3*d
^2 + b*d^4)*cosh(b*x + a)^2)*sinh(b*x + a)^3 + (10*(9*b^4*d - 10*b^2*d^3 +
```

```
d^5)*cosh(b*x + a)^3 + 3*(25*b^4*d - 26*b^2*d^3 + d^5)*cosh(b*x + a))*sinh(
b*x + a)^2 - 2*(225*b^4*d - 34*b^2*d^3 + d^5)*cosh(b*x + a) + (450*b^5 - 68
*b^3*d^2 + 2*b*d^4 - 25*(9*b^5 - 10*b^3*d^2 + b*d^4)*cosh(b*x + a)^4 - 9*(2
5*b^5 - 26*b^3*d^2 + b*d^4)*cosh(b*x + a)^2)*sinh(b*x + a))*sinh(d*x + c))/
((225*b^6 - 259*b^4*d^2 + 35*b^2*d^4 - d^6)*cosh(b*x + a)^6 - 3*(225*b^6 -
259*b^4*d^2 + 35*b^2*d^4 - d^6)*cosh(b*x + a)^4*sinh(b*x + a)^2 + 3*(225*b^
6 - 259*b^4*d^2 + 35*b^2*d^4 - d^6)*cosh(b*x + a)^2*sinh(b*x + a)^4 - (225*
b^6 - 259*b^4*d^2 + 35*b^2*d^4 - d^6)*sinh(b*x + a)^6)
```

giac [A] time = 0.15, size = 132, normalized size = 0.68

$$\frac{e^{(5\,bx+dx+5\,a+c)}}{32\,(5\,b+d)}+\frac{e^{(3\,bx+dx+3\,a+c)}}{32\,(3\,b+d)}-\frac{e^{(bx+dx+a+c)}}{16\,(b+d)}+\frac{e^{(-bx+dx-a+c)}}{16\,(b-d)}-\frac{e^{(-3\,bx+dx-3\,a+c)}}{32\,(3\,b-d)}-\frac{e^{(-5\,bx+dx-5\,a+c)}}{32\,(5\,b-d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x+c)*cosh(b*x+a)^3*sinh(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/32*e^(5*b*x + d*x + 5*a + c)/(5*b + d) + 1/32*e^(3*b*x + d*x + 3*a + c)/(
3*b + d) - 1/16*e^(b*x + d*x + a + c)/(b + d) + 1/16*e^(-b*x + d*x - a + c)
/(b - d) - 1/32*e^(-3*b*x + d*x - 3*a + c)/(3*b - d) - 1/32*e^(-5*b*x + d*x
 - 5*a + c)/(5*b - d)
```

maple [A] time = 0.54, size = 278, normalized size = 1.43

$$-\frac{\sinh\left(a-c+(b-d)\,x\right)}{16\,(b-d)}-\frac{\sinh\left(a+c+(b+d)\,x\right)}{16\,(b+d)}+\frac{\sinh\left(3a-c+(3b-d)\,x\right)}{96b-32d}+\frac{\sinh\left(3a+c+(3b+d)\,x\right)}{96b+32d}+\frac{\sinh(}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(d*x+c)*cosh(b*x+a)^3*sinh(b*x+a)^2,x)
```

```
[Out] -1/16*sinh(a-c+(b-d)*x)/(b-d)-1/16*sinh(a+c+(b+d)*x)/(b+d)+1/32*sinh(3*a-c+
(3*b-d)*x)/(3*b-d)+1/32*sinh(3*a+c+(3*b+d)*x)/(3*b+d)+1/32/(5*b-d)*sinh((5*
b-d)*x+5*a-c)+1/32/(5*b+d)*sinh((5*b+d)*x+5*a+c)+1/16*cosh(a-c+(b-d)*x)/(b-
d)-1/16*cosh(a+c+(b+d)*x)/(b+d)-1/32*cosh(3*a-c+(3*b-d)*x)/(3*b-d)+1/32*cos
h(3*a+c+(3*b+d)*x)/(3*b+d)-1/32*cosh((5*b-d)*x+5*a-c)/(5*b-d)+1/32*cosh((5*
b+d)*x+5*a+c)/(5*b+d)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x+c)*cosh(b*x+a)^3*sinh(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(-d/b>0)', see `assume?` for more de
tails)Is -d/b equal to -1?
```

mupad [B] time = 2.42, size = 393, normalized size = 2.02

$$\frac{\cosh (a+b\,x)^5\,\mathrm{e}^{c+d\,x}\,\left(26\,b^4\,d-2\,b^2\,d^3\right)}{225\,b^6-259\,b^4\,d^2+35\,b^2\,d^4-d^6}+\frac{3\cosh (a+b\,x)^2\,\mathrm{e}^{c+d\,x}\,\sinh (a+b\,x)^3\,\left(25\,b^5-10\,b^3\,d^2+b\,d^4\right)}{225\,b^6-259\,b^4\,d^2+35\,b^2\,d^4-d^6}+\frac{2\cos}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a + b*x)^3*exp(c + d*x)*sinh(a + b*x)^2,x)
```

```
[Out] (cosh(a + b*x)^5*exp(c + d*x)*(26*b^4*d - 2*b^2*d^3))/(225*b^6 - d^6 + 35*b
^2*d^4 - 259*b^4*d^2) + (3*cosh(a + b*x)^2*exp(c + d*x)*sinh(a + b*x)^3*(b*
d^4 + 25*b^5 - 10*b^3*d^2))/(225*b^6 - d^6 + 35*b^2*d^4 - 259*b^4*d^2) + (2
*cosh(a + b*x)^4*exp(c + d*x)*sinh(a + b*x)*(b*d^4 - 13*b^3*d^2))/(225*b^6
- d^6 + 35*b^2*d^4 - 259*b^4*d^2) - (cosh(a + b*x)^3*exp(c + d*x)*sinh(a +
b*x)^2*(65*b^4*d + d^5 - 18*b^2*d^3))/(225*b^6 - d^6 + 35*b^2*d^4 - 259*b^4
*d^2) - (6*b^3*exp(c + d*x)*sinh(a + b*x)^5*(5*b^2 - d^2))/(225*b^6 - d^6 +
 35*b^2*d^4 - 259*b^4*d^2) + (6*b^2*d*cosh(a + b*x)*exp(c + d*x)*sinh(a + b
*x)^4*(5*b^2 - d^2))/(225*b^6 - d^6 + 35*b^2*d^4 - 259*b^4*d^2)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x+c)*cosh(b*x+a)**3*sinh(b*x+a)**2,x)
```

```
[Out] Timed out
```

3.961 $\int e^{c+dx}\cosh^3(a+bx)\sinh(a+bx)\,dx$

Optimal. Leaf size=137

$$-\frac{de^{c+dx}\sinh(2a+2bx)}{4(4b^2-d^2)}-\frac{de^{c+dx}\sinh(4a+4bx)}{8(16b^2-d^2)}+\frac{be^{c+dx}\cosh(2a+2bx)}{2(4b^2-d^2)}+\frac{be^{c+dx}\cosh(4a+4bx)}{2(16b^2-d^2)}$$

```
[Out] 1/2*b*exp(d*x+c)*cosh(2*b*x+2*a)/(4*b^2-d^2)+1/2*b*exp(d*x+c)*cosh(4*b*x+4*
a)/(16*b^2-d^2)-1/4*d*exp(d*x+c)*sinh(2*b*x+2*a)/(4*b^2-d^2)-1/8*d*exp(d*x+
c)*sinh(4*b*x+4*a)/(16*b^2-d^2)
```

Rubi [A] time = 0.09, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {5509, 5474}

$$-\frac{de^{c+dx}\sinh(2a+2bx)}{4(4b^2-d^2)}-\frac{de^{c+dx}\sinh(4a+4bx)}{8(16b^2-d^2)}+\frac{be^{c+dx}\cosh(2a+2bx)}{2(4b^2-d^2)}+\frac{be^{c+dx}\cosh(4a+4bx)}{2(16b^2-d^2)}$$

Antiderivative was successfully verified.

```
[In] Int[E^(c + d*x)*Cosh[a + b*x]^3*Sinh[a + b*x],x]
```

```
[Out] (b*E^(c + d*x)*Cosh[2*a + 2*b*x])/(2*(4*b^2 - d^2)) + (b*E^(c + d*x)*Cosh[4
*a + 4*b*x])/(2*(16*b^2 - d^2)) - (d*E^(c + d*x)*Sinh[2*a + 2*b*x])/(4*(4*b
^2 - d^2)) - (d*E^(c + d*x)*Sinh[4*a + 4*b*x])/(8*(16*b^2 - d^2))
```

Rule 5474

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sinh[(d_.) + (e_.)*(x_)], x_Symbol] :
> -Simp[(b*c*Log[F]*F^(c*(a + b*x))*Sinh[d + e*x])/(e^2 - b^2*c^2*Log[F]^2)
, x] + Simp[(e*F^(c*(a + b*x))*Cosh[d + e*x])/(e^2 - b^2*c^2*Log[F]^2), x]
/; FreeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 - b^2*c^2*Log[F]^2, 0]
```

Rule 5509

```
Int[Cosh[(f_.) + (g_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sinh[(
d_.) + (e_.)*(x_)]^(m_.), x_Symbol] :> Int[ExpandTrigReduce[F^(c*(a + b*x))
, Sinh[d + e*x]^m*Cosh[f + g*x]^n, x], x] /; FreeQ[{F, a, b, c, d, e, f, g}
, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

Rubi steps

$$\int e^{c+dx}\cosh^3(a+bx)\sinh(a+bx)\,dx = \int \left(\frac{1}{4}e^{c+dx}\sinh(2a+2bx)+\frac{1}{8}e^{c+dx}\sinh(4a+4bx)\right)dx$$
$$= \frac{1}{8}\int e^{c+dx}\sinh(4a+4bx)\,dx+\frac{1}{4}\int e^{c+dx}\sinh(2a+2bx)\,dx$$
$$= \frac{be^{c+dx}\cosh(2a+2bx)}{2\left(4b^2-d^2\right)}+\frac{be^{c+dx}\cosh(4a+4bx)}{2\left(16b^2-d^2\right)}-\frac{de^{c+dx}\sinh(2a+2b}{4\left(4b^2-d^2\right)}$$

Mathematica [A] time = 1.02, size = 86, normalized size = 0.63

$$\frac{1}{8}e^{c+dx}\left(\frac{4b\cosh(2(a+bx))-2d\sinh(2(a+bx))}{4b^2-d^2}+\frac{4b\cosh(4(a+bx))-d\sinh(4(a+bx))}{16b^2-d^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(c + d*x)*Cosh[a + b*x]^3*Sinh[a + b*x],x]
```

```
[Out] (E^(c + d*x)*((4*b*Cosh[2*(a + b*x)] - 2*d*Sinh[2*(a + b*x)])/(4*b^2 - d^2)
 + (4*b*Cosh[4*(a + b*x)] - d*Sinh[4*(a + b*x)])/(16*b^2 - d^2)))/8
```

fricas [B] time = 0.44, size = 501, normalized size = 3.66

$$-\frac{\left(4\,b^2d-d^3\right)\cosh\left(bx+a\right)\cosh\left(dx+c\right)\sinh\left(bx+a\right)^3-\left(4\,b^3-bd^2\right)\cosh\left(dx+c\right)\sinh\left(bx+a\right)^4-\left(16\,b^3-b\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x+c)*cosh(b*x+a)^3*sinh(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/2*((4*b^2*d - d^3)*cosh(b*x + a)*cosh(d*x + c)*sinh(b*x + a)^3 - (4*b^3
- b*d^2)*cosh(d*x + c)*sinh(b*x + a)^4 - (16*b^3 - b*d^2 + 6*(4*b^3 - b*d^2
)*cosh(b*x + a)^2)*cosh(d*x + c)*sinh(b*x + a)^2 + ((4*b^2*d - d^3)*cosh(b*
x + a)^3 + (16*b^2*d - d^3)*cosh(b*x + a))*cosh(d*x + c)*sinh(b*x + a) - ((
4*b^3 - b*d^2)*cosh(b*x + a)^4 + (16*b^3 - b*d^2)*cosh(b*x + a)^2)*cosh(d*x
 + c) - ((4*b^3 - b*d^2)*cosh(b*x + a)^4 - (4*b^2*d - d^3)*cosh(b*x + a)*si
nh(b*x + a)^3 + (4*b^3 - b*d^2)*sinh(b*x + a)^4 + (16*b^3 - b*d^2)*cosh(b*x
 + a)^2 + (16*b^3 - b*d^2 + 6*(4*b^3 - b*d^2)*cosh(b*x + a)^2)*sinh(b*x + a
)^2 - ((4*b^2*d - d^3)*cosh(b*x + a)^3 + (16*b^2*d - d^3)*cosh(b*x + a))*si
nh(b*x + a))*sinh(d*x + c))/((64*b^4 - 20*b^2*d^2 + d^4)*cosh(b*x + a)^4 -
2*(64*b^4 - 20*b^2*d^2 + d^4)*cosh(b*x + a)^2*sinh(b*x + a)^2 + (64*b^4 - 2
0*b^2*d^2 + d^4)*sinh(b*x + a)^4)
```

giac [**A**] time = 0.13, size = 93, normalized size = 0.68

$$\frac{e^{(4\,bx+dx+4\,a+c)}}{16\,(4\,b+d)}+\frac{e^{(2\,bx+dx+2\,a+c)}}{8\,(2\,b+d)}+\frac{e^{(-2\,bx+dx-2\,a+c)}}{8\,(2\,b-d)}+\frac{e^{(-4\,bx+dx-4\,a+c)}}{16\,(4\,b-d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x+c)*cosh(b*x+a)^3*sinh(b*x+a),x, algorithm="giac")
```

```
[Out] 1/16*e^(4*b*x + d*x + 4*a + c)/(4*b + d) + 1/8*e^(2*b*x + d*x + 2*a + c)/(2
*b + d) + 1/8*e^(-2*b*x + d*x - 2*a + c)/(2*b - d) + 1/16*e^(-4*b*x + d*x -
 4*a + c)/(4*b - d)
```

maple [**A**] time = 0.43, size = 202, normalized size = 1.47

$$-\frac{\sinh(2a-c+(2b-d)x)}{8(2b-d)}+\frac{\sinh(2a+c+(2b+d)x)}{16b+8d}-\frac{\sinh((4b-d)x+4a-c)}{16(4b-d)}+\frac{\sinh((4b+d)x+4a+c)}{64b+16d}+\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(d*x+c)*cosh(b*x+a)^3*sinh(b*x+a),x)
```

```
[Out] -1/8*sinh(2*a-c+(2*b-d)*x)/(2*b-d)+1/8*sinh(2*a+c+(2*b+d)*x)/(2*b+d)-1/16/(
4*b-d)*sinh((4*b-d)*x+4*a-c)+1/16/(4*b+d)*sinh((4*b+d)*x+4*a+c)+1/8*cosh(2*
a-c+(2*b-d)*x)/(2*b-d)+1/8*cosh(2*a+c+(2*b+d)*x)/(2*b+d)+1/16*cosh((4*b-d)*
x+4*a-c)/(4*b-d)+1/16*cosh((4*b+d)*x+4*a+c)/(4*b+d)
```

maxima [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x+c)*cosh(b*x+a)^3*sinh(b*x+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(1-d/b>0)', see `assume?` for more d
etails)Is 1-d/b equal to -1?
```

mupad [**B**] time = 2.68, size = 163, normalized size = 1.19

$$-\frac{b^3\left(6\,e^{c+d\,x}-16\cosh(a+b\,x)^4\,e^{c+d\,x}\right)+b^2\,d\left(4\,e^{c+d\,x}\sinh(a+b\,x)\cosh(a+b\,x)^3+6\,e^{c+d\,x}\sinh(a+b\,x)\cos\cdots\right.}{64\,b^4-20\,b\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a + b*x)^3*exp(c + d*x)*sinh(a + b*x),x)
```

```
[Out] -(b^3*(6*exp(c + d*x) - 16*cosh(a + b*x)^4*exp(c + d*x)) + b^2*d*(6*cosh(a
+ b*x)*exp(c + d*x)*sinh(a + b*x) + 4*cosh(a + b*x)^3*exp(c + d*x)*sinh(a +
 b*x)) - b*d^2*(3*cosh(a + b*x)^2*exp(c + d*x) - 4*cosh(a + b*x)^4*exp(c +
d*x)) - d^3*cosh(a + b*x)^3*exp(c + d*x)*sinh(a + b*x))/(64*b^4 + d^4 - 20*
b^2*d^2)
```

sympy [A] time = 143.71, size = 1295, normalized size = 9.45

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x+c)*cosh(b*x+a)**3*sinh(b*x+a),x)
```

```
[Out] Piecewise((x*exp(c)*sinh(a)*cosh(a)**3, Eq(b, 0) & Eq(d, 0)), (-x*exp(c)*ex
p(d*x)*sinh(a - d*x/2)**4/8 - x*exp(c)*exp(d*x)*sinh(a - d*x/2)**3*cosh(a -
 d*x/2)/4 + x*exp(c)*exp(d*x)*sinh(a - d*x/2)*cosh(a - d*x/2)**3/4 + x*exp(
c)*exp(d*x)*cosh(a - d*x/2)**4/8 + exp(c)*exp(d*x)*sinh(a - d*x/2)**4/(8*d)
 - exp(c)*exp(d*x)*sinh(a - d*x/2)**2*cosh(a - d*x/2)**2/(2*d) - exp(c)*exp
(d*x)*sinh(a - d*x/2)*cosh(a - d*x/2)**3/(3*d) - 7*exp(c)*exp(d*x)*cosh(a -
 d*x/2)**4/(24*d), Eq(b, -d/2)), (x*exp(c)*exp(d*x)*sinh(a - d*x/4)**4/16 +
 x*exp(c)*exp(d*x)*sinh(a - d*x/4)**3*cosh(a - d*x/4)/4 + 3*x*exp(c)*exp(d*
x)*sinh(a - d*x/4)**2*cosh(a - d*x/4)**2/8 + x*exp(c)*exp(d*x)*sinh(a - d*x
/4)*cosh(a - d*x/4)**3/4 + x*exp(c)*exp(d*x)*cosh(a - d*x/4)**4/16 - exp(c)
*exp(d*x)*sinh(a - d*x/4)**4/(6*d) - 5*exp(c)*exp(d*x)*sinh(a - d*x/4)**3*c
osh(a - d*x/4)/(12*d) + 11*exp(c)*exp(d*x)*sinh(a - d*x/4)*cosh(a - d*x/4)*
*3/(12*d) + exp(c)*exp(d*x)*cosh(a - d*x/4)**4/(6*d), Eq(b, -d/4)), (-x*exp
(c)*exp(d*x)*sinh(a + d*x/4)**4/16 + x*exp(c)*exp(d*x)*sinh(a + d*x/4)**3*c
osh(a + d*x/4)/4 - 3*x*exp(c)*exp(d*x)*sinh(a + d*x/4)**2*cosh(a + d*x/4)**
2/8 + x*exp(c)*exp(d*x)*sinh(a + d*x/4)*cosh(a + d*x/4)**3/4 - x*exp(c)*exp
(d*x)*cosh(a + d*x/4)**4/16 + exp(c)*exp(d*x)*sinh(a + d*x/4)**4/(6*d) - 5*
exp(c)*exp(d*x)*sinh(a + d*x/4)**3*cosh(a + d*x/4)/(12*d) + 11*exp(c)*exp(d
*x)*sinh(a + d*x/4)*cosh(a + d*x/4)**3/(12*d) - exp(c)*exp(d*x)*cosh(a + d*
x/4)**4/(6*d), Eq(b, d/4)), (x*exp(c)*exp(d*x)*sinh(a + d*x/2)**4/8 - x*exp
(c)*exp(d*x)*sinh(a + d*x/2)**3*cosh(a + d*x/2)/4 + x*exp(c)*exp(d*x)*sinh(
a + d*x/2)*cosh(a + d*x/2)**3/4 - x*exp(c)*exp(d*x)*cosh(a + d*x/2)**4/8 -
exp(c)*exp(d*x)*sinh(a + d*x/2)**4/(8*d) + exp(c)*exp(d*x)*sinh(a + d*x/2)*
*2*cosh(a + d*x/2)**2/(2*d) - exp(c)*exp(d*x)*sinh(a + d*x/2)*cosh(a + d*x/
2)**3/(3*d) + 7*exp(c)*exp(d*x)*cosh(a + d*x/2)**4/(24*d), Eq(b, d/2)), (-6
*b**3*exp(c)*exp(d*x)*sinh(a + b*x)**4/(64*b**4 - 20*b**2*d**2 + d**4) + 12
*b**3*exp(c)*exp(d*x)*sinh(a + b*x)**2*cosh(a + b*x)**2/(64*b**4 - 20*b**2*
d**2 + d**4) + 10*b**3*exp(c)*exp(d*x)*cosh(a + b*x)**4/(64*b**4 - 20*b**2*
d**2 + d**4) + 6*b**2*d*exp(c)*exp(d*x)*sinh(a + b*x)**3*cosh(a + b*x)/(64*
b**4 - 20*b**2*d**2 + d**4) - 10*b**2*d*exp(c)*exp(d*x)*sinh(a + b*x)*cosh(
```

```
a + b*x)**3/(64*b**4 - 20*b**2*d**2 + d**4) - 3*b*d**2*exp(c)*exp(d*x)*sinh
(a + b*x)**2*cosh(a + b*x)**2/(64*b**4 - 20*b**2*d**2 + d**4) - b*d**2*exp(
c)*exp(d*x)*cosh(a + b*x)**4/(64*b**4 - 20*b**2*d**2 + d**4) + d**3*exp(c)*
exp(d*x)*sinh(a + b*x)*cosh(a + b*x)**3/(64*b**4 - 20*b**2*d**2 + d**4), Tr
ue))
```

3.962 $\int e^{c+dx}\cosh^3(a+bx)\,dx$

Optimal. Leaf size=144

$$-\frac{de^{c+dx}\cosh^3(a+bx)}{9b^2-d^2}+\frac{3be^{c+dx}\sinh(a+bx)\cosh^2(a+bx)}{9b^2-d^2}-\frac{6b^2de^{c+dx}\cosh(a+bx)}{9b^4-10b^2d^2+d^4}+\frac{6b^3e^{c+dx}\sinh(a+bx)}{9b^4-10b^2d^2+d^4}$$

```
[Out] -6*b^2*d*exp(d*x+c)*cosh(b*x+a)/(9*b^4-10*b^2*d^2+d^4)-d*exp(d*x+c)*cosh(b*
x+a)^3/(9*b^2-d^2)+6*b^3*exp(d*x+c)*sinh(b*x+a)/(9*b^4-10*b^2*d^2+d^4)+3*b*
exp(d*x+c)*cosh(b*x+a)^2*sinh(b*x+a)/(9*b^2-d^2)
```

Rubi [A] time = 0.06, antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {5477, 5475}

$$\frac{6b^3e^{c+dx}\sinh(a+bx)}{-10b^2d^2+9b^4+d^4}-\frac{de^{c+dx}\cosh^3(a+bx)}{9b^2-d^2}-\frac{6b^2de^{c+dx}\cosh(a+bx)}{-10b^2d^2+9b^4+d^4}+\frac{3be^{c+dx}\sinh(a+bx)\cosh^2(a+bx)}{9b^2-d^2}$$

Antiderivative was successfully verified.

```
[In] Int[E^(c + d*x)*Cosh[a + b*x]^3,x]
```

```
[Out] (-6*b^2*d*E^(c + d*x)*Cosh[a + b*x])/(9*b^4 - 10*b^2*d^2 + d^4) - (d*E^(c +
 d*x)*Cosh[a + b*x]^3)/(9*b^2 - d^2) + (6*b^3*E^(c + d*x)*Sinh[a + b*x])/(9
*b^4 - 10*b^2*d^2 + d^4) + (3*b*E^(c + d*x)*Cosh[a + b*x]^2*Sinh[a + b*x])/
(9*b^2 - d^2)
```

Rule 5475

```
Int[Cosh[(d_.) + (e_.)*(x_)]*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symbol] :
> -Simp[(b*c*Log[F]*F^(c*(a + b*x))*Cosh[d + e*x])/(e^2 - b^2*c^2*Log[F]^2)
, x] + Simp[(e*F^(c*(a + b*x))*Sinh[d + e*x])/(e^2 - b^2*c^2*Log[F]^2), x]
/; FreeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 - b^2*c^2*Log[F]^2, 0]
```

Rule 5477

```
Int[Cosh[(d_.) + (e_.)*(x_)]^(n_)*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symb
ol] :> -Simp[(b*c*Log[F]*F^(c*(a + b*x))*Cosh[d + e*x]^n)/(e^2*n^2 - b^2*c^
2*Log[F]^2), x] + (Dist[(n*(n - 1)*e^2)/(e^2*n^2 - b^2*c^2*Log[F]^2), Int[F
^(c*(a + b*x))*Cosh[d + e*x]^(n - 2), x], x] + Simp[(e*n*F^(c*(a + b*x))*Si
nh[d + e*x]*Cosh[d + e*x]^(n - 1))/(e^2*n^2 - b^2*c^2*Log[F]^2), x]) /; Fre
eQ[{F, a, b, c, d, e}, x] && NeQ[e^2*n^2 - b^2*c^2*Log[F]^2, 0] && GtQ[n, 1
]
```

Rubi steps

$$\int e^{c+dx}\cosh^3(a+bx)\,dx = -\frac{de^{c+dx}\cosh^3(a+bx)}{9b^2-d^2} + \frac{3be^{c+dx}\cosh^2(a+bx)\sinh(a+bx)}{9b^2-d^2} + \frac{(6b^2)\int e^{c+dx}\cosh(a}{9b^2-d^2}$$

$$= -\frac{6b^2de^{c+dx}\cosh(a+bx)}{9b^4-10b^2d^2+d^4} - \frac{de^{c+dx}\cosh^3(a+bx)}{9b^2-d^2} + \frac{6b^3e^{c+dx}\sinh(a+bx)}{9b^4-10b^2d^2+d^4} + \frac{3be^{c+dx}}{}$$

Mathematica [A] time = 0.48, size = 106, normalized size = 0.74

$$\frac{e^{c+dx}\left(\left(d^3-b^2d\right)\cosh(3(a+bx)) + 3d\left(d^2-9b^2\right)\cosh(a+bx) + 6b\sinh(a+bx)\left(\left(b^2-d^2\right)\cosh(2(a+bx)) + 5b\right.\right.}{4\left(9b^4-10b^2d^2+d^4\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(c + d*x)*Cosh[a + b*x]^3,x]
```

```
[Out] (E^(c + d*x)*(3*d*(-9*b^2 + d^2)*Cosh[a + b*x] + (-(b^2*d) + d^3)*Cosh[3*(a
 + b*x)] + 6*b*(5*b^2 - d^2 + (b^2 - d^2)*Cosh[2*(a + b*x)])*Sinh[a + b*x])
)/(4*(9*b^4 - 10*b^2*d^2 + d^4))
```

fricas [B] time = 0.45, size = 381, normalized size = 2.65

$$-\frac{3\left(b^2d-d^3\right)\cosh(bx+a)\cosh(dx+c)\sinh(bx+a)^2 - 3\left(b^3-bd^2\right)\cosh(dx+c)\sinh(bx+a)^3 - 3\left(9\,b^3-bd\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x+c)*cosh(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -1/4*(3*(b^2*d - d^3)*cosh(b*x + a)*cosh(d*x + c)*sinh(b*x + a)^2 - 3*(b^3
- b*d^2)*cosh(d*x + c)*sinh(b*x + a)^3 - 3*(9*b^3 - b*d^2 + 3*(b^3 - b*d^2)
*cosh(b*x + a)^2)*cosh(d*x + c)*sinh(b*x + a) + ((b^2*d - d^3)*cosh(b*x + a
)^3 + 3*(9*b^2*d - d^3)*cosh(b*x + a))*cosh(d*x + c) + ((b^2*d - d^3)*cosh(
b*x + a)^3 + 3*(b^2*d - d^3)*cosh(b*x + a)*sinh(b*x + a)^2 - 3*(b^3 - b*d^2
)*sinh(b*x + a)^3 + 3*(9*b^2*d - d^3)*cosh(b*x + a) - 3*(9*b^3 - b*d^2 + 3*
(b^3 - b*d^2)*cosh(b*x + a)^2)*sinh(b*x + a))*sinh(d*x + c))/((9*b^4 - 10*b
^2*d^2 + d^4)*cosh(b*x + a)^4 - 2*(9*b^4 - 10*b^2*d^2 + d^4)*cosh(b*x + a)^
2*sinh(b*x + a)^2 + (9*b^4 - 10*b^2*d^2 + d^4)*sinh(b*x + a)^4)
```

giac [A] time = 0.13, size = 86, normalized size = 0.60

$$\frac{e^{(3\,bx+dx+3\,a+c)}}{8\,(3\,b+d)} + \frac{3\,e^{(bx+dx+a+c)}}{8\,(b+d)} - \frac{3\,e^{(-bx+dx-a+c)}}{8\,(b-d)} - \frac{e^{(-3\,bx+dx-3\,a+c)}}{8\,(3\,b-d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x+c)*cosh(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/8*e^(3*b*x + d*x + 3*a + c)/(3*b + d) + 3/8*e^(b*x + d*x + a + c)/(b + d)
 - 3/8*e^(-b*x + d*x - a + c)/(b - d) - 1/8*e^(-3*b*x + d*x - 3*a + c)/(3*b
 - d)
```

maple [A] time = 0.40, size = 178, normalized size = 1.24

$$\frac{3\sinh\left(a-c+(b-d)x\right)}{8(b-d)}+\frac{3\sinh\left(a+c+(b+d)x\right)}{8(b+d)}+\frac{\sinh\left(3a-c+(3b-d)x\right)}{24b-8d}+\frac{\sinh\left(3a+c+(3b+d)x\right)}{24b+8d}-\frac{3\,c}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(d*x+c)*cosh(b*x+a)^3,x)
```

```
[Out] 3/8*sinh(a-c+(b-d)*x)/(b-d)+3/8*sinh(a+c+(b+d)*x)/(b+d)+1/8*sinh(3*a-c+(3*b
-d)*x)/(3*b-d)+1/8*sinh(3*a+c+(3*b+d)*x)/(3*b+d)-3/8*cosh(a-c+(b-d)*x)/(b-d
)+3/8*cosh(a+c+(b+d)*x)/(b+d)-1/8*cosh(3*a-c+(3*b-d)*x)/(3*b-d)+1/8*cosh(3*
a+c+(3*b+d)*x)/(3*b+d)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x+c)*cosh(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(-d/b>0)', see `assume?` for more de
tails)Is -d/b equal to -1?
```

mupad [B] time = 2.20, size = 125, normalized size = 0.87

$$\frac{e^{c+d\,x}\,\left(9\,b^3\,\cosh\left(a+b\,x\right)^2\,\sinh\left(a+b\,x\right)-6\,b^3\,\sinh\left(a+b\,x\right)^3-7\,b^2\,d\,\cosh\left(a+b\,x\right)^3+6\,b^2\,d\,\cosh\left(a+b\,x\right)\,\sin}{9\,b^4-10\,b^2\,d^2+d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a + b*x)^3*exp(c + d*x),x)
```

```
[Out] (exp(c + d*x)*(d^3*cosh(a + b*x)^3 - 6*b^3*sinh(a + b*x)^3 - 7*b^2*d*cosh(a
 + b*x)^3 + 9*b^3*cosh(a + b*x)^2*sinh(a + b*x) - 3*b*d^2*cosh(a + b*x)^2*s
inh(a + b*x) + 6*b^2*d*cosh(a + b*x)*sinh(a + b*x)^2))/(9*b^4 + d^4 - 10*b^
2*d^2)
```

sympy [A] time = 42.81, size = 1046, normalized size = 7.26

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x+c)*cosh(b*x+a)**3,x)
```

```
[Out] Piecewise((x*exp(c)*cosh(a)**3, Eq(b, 0) & Eq(d, 0)), (-3*x*exp(c)*exp(d*x)
*sinh(a - d*x)**3/8 - 3*x*exp(c)*exp(d*x)*sinh(a - d*x)**2*cosh(a - d*x)/8
+ 3*x*exp(c)*exp(d*x)*sinh(a - d*x)*cosh(a - d*x)**2/8 + 3*x*exp(c)*exp(d*x
)*cosh(a - d*x)**3/8 + 5*exp(c)*exp(d*x)*sinh(a - d*x)**3/(8*d) + exp(c)*ex
p(d*x)*sinh(a - d*x)**2*cosh(a - d*x)/(4*d) - exp(c)*exp(d*x)*sinh(a - d*x)
*cosh(a - d*x)**2/d - 3*exp(c)*exp(d*x)*cosh(a - d*x)**3/(8*d), Eq(b, -d)),
 (x*exp(c)*exp(d*x)*sinh(a - d*x/3)**3/8 + 3*x*exp(c)*exp(d*x)*sinh(a - d*x
/3)**2*cosh(a - d*x/3)/8 + 3*x*exp(c)*exp(d*x)*sinh(a - d*x/3)*cosh(a - d*x
/3)**2/8 + x*exp(c)*exp(d*x)*cosh(a - d*x/3)**3/8 - exp(c)*exp(d*x)*sinh(a
- d*x/3)**3/(8*d) + 3*exp(c)*exp(d*x)*sinh(a - d*x/3)*cosh(a - d*x/3)**2/(4
*d) + 9*exp(c)*exp(d*x)*cosh(a - d*x/3)**3/(8*d), Eq(b, -d/3)), (-x*exp(c)*
exp(d*x)*sinh(a + d*x/3)**3/8 + 3*x*exp(c)*exp(d*x)*sinh(a + d*x/3)**2*cosh
(a + d*x/3)/8 - 3*x*exp(c)*exp(d*x)*sinh(a + d*x/3)*cosh(a + d*x/3)**2/8 +
x*exp(c)*exp(d*x)*cosh(a + d*x/3)**3/8 + 11*exp(c)*exp(d*x)*sinh(a + d*x/3)
**3/(8*d) - 15*exp(c)*exp(d*x)*sinh(a + d*x/3)**2*cosh(a + d*x/3)/(4*d) + 3
*exp(c)*exp(d*x)*sinh(a + d*x/3)*cosh(a + d*x/3)**2/d - exp(c)*exp(d*x)*cos
h(a + d*x/3)**3/(8*d), Eq(b, d/3)), (3*x*exp(c)*exp(d*x)*sinh(a + d*x)**3/8
 - 3*x*exp(c)*exp(d*x)*sinh(a + d*x)**2*cosh(a + d*x)/8 - 3*x*exp(c)*exp(d*
x)*sinh(a + d*x)*cosh(a + d*x)**2/8 + 3*x*exp(c)*exp(d*x)*cosh(a + d*x)**3/
8 - 3*exp(c)*exp(d*x)*sinh(a + d*x)**3/(8*d) + 3*exp(c)*exp(d*x)*sinh(a + d
*x)*cosh(a + d*x)**2/(4*d) - exp(c)*exp(d*x)*cosh(a + d*x)**3/(8*d), Eq(b,
d)), (-6*b**3*exp(c)*exp(d*x)*sinh(a + b*x)**3/(9*b**4 - 10*b**2*d**2 + d**
4) + 9*b**3*exp(c)*exp(d*x)*sinh(a + b*x)*cosh(a + b*x)**2/(9*b**4 - 10*b**
2*d**2 + d**4) + 6*b**2*d*exp(c)*exp(d*x)*sinh(a + b*x)**2*cosh(a + b*x)/(9
*b**4 - 10*b**2*d**2 + d**4) - 7*b**2*d*exp(c)*exp(d*x)*cosh(a + b*x)**3/(9
*b**4 - 10*b**2*d**2 + d**4) - 3*b*d**2*exp(c)*exp(d*x)*sinh(a + b*x)*cosh(
a + b*x)**2/(9*b**4 - 10*b**2*d**2 + d**4) + d**3*exp(c)*exp(d*x)*cosh(a +
b*x)**3/(9*b**4 - 10*b**2*d**2 + d**4), True))
```

3.963 $\int e^{c+dx}\cosh^2(a+bx)\coth(a+bx)\,dx$

Optimal. Leaf size=125

$$\frac{2e^{-2a-x(2b-d)+c}\,{}_2F_1\left(1,\frac{1}{2}\left(\frac{d}{b}-2\right);\frac{d}{2b};e^{2(a+bx)}\right)}{2b-d}-\frac{7e^{-2a-x(2b-d)+c}}{4(2b-d)}+\frac{e^{2a+x(2b+d)+c}}{4(2b+d)}+\frac{e^{c+dx}}{d}$$

```
[Out] -7/4*exp(-2*a+c-(2*b-d)*x)/(2*b-d)+exp(d*x+c)/d+1/4*exp(2*a+c+(2*b+d)*x)/(2
*b+d)+2*exp(-2*a+c-(2*b-d)*x)*hypergeom([1, -1+1/2*d/b],[1/2*d/b],exp(2*b*x
+2*a))/(2*b-d)
```

Rubi [A] time = 0.25, antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {5511, 2194, 2227, 2251}

$$\frac{2e^{-2a-x(2b-d)+c}\,{}_2F_1\left(1,\frac{1}{2}\left(\frac{d}{b}-2\right);\frac{d}{2b};e^{2(a+bx)}\right)}{2b-d}-\frac{7e^{-2a-x(2b-d)+c}}{4(2b-d)}+\frac{e^{2a+x(2b+d)+c}}{4(2b+d)}+\frac{e^{c+dx}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[E^(c + d*x)*Cosh[a + b*x]^2*Coth[a + b*x],x]
```

```
[Out] (-7*E^(-2*a + c - (2*b - d)*x))/(4*(2*b - d)) + E^(c + d*x)/d + E^(2*a + c
+ (2*b + d)*x)/(4*(2*b + d)) + (2*E^(-2*a + c - (2*b - d)*x)*Hypergeometric
2F1[1, (-2 + d/b)/2, d/(2*b), E^(2*(a + b*x))])/(2*b - d)
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 2227

```
Int[(u_.)*(F_)^((a_.) + (b_.)*(v_)), x_Symbol] :> Int[u*F^(a + b*NormalizeP
owerOfLinear[v, x]), x] /; FreeQ[{F, a, b}, x] && PolynomialQ[u, x] && Powe
rOfLinearQ[v, x] &&  !PowerOfLinearMatchQ[v, x]
```

Rule 2251

```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_)*(G_)^((h_.)*((f_.
) + (g_.)*(x_))), x_Symbol] :> Simp[(a^p*G^(h*(f + g*x))*Hypergeometric2F1[
-p, (g*h*Log[G])/(d*e*Log[F]), (g*h*Log[G])/(d*e*Log[F]) + 1, Simplify[-((b
*F^(e*(c + d*x)))/a)]])/(g*h*Log[G]), x] /; FreeQ[{F, G, a, b, c, d, e, f,
g, h, p}, x] && (ILtQ[p, 0] || GtQ[a, 0])
```

Rule 5511

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*(G_)[(d_.) + (e_.)*(x_)]^(m_.)*(H_)[(
d_.) + (e_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^(c*(a + b*x)),
 G[d + e*x]^m*H[d + e*x]^n, x], x] /; FreeQ[{F, a, b, c, d, e}, x] && IGtQ[
m, 0] && IGtQ[n, 0] && HyperbolicQ[G] && HyperbolicQ[H]
```

Rubi steps

$$\int e^{c+dx}\cosh^2(a+bx)\coth(a+bx)\,dx = \int\left(\frac{7}{4}e^{-2a+c-(2b-d)x}+e^{-2a+c-(2b-d)x+2(a+bx)}+\frac{1}{4}e^{-2a+c-(2b-d)x+4(a+bx)}+\frac{2e}{-}\right.$$

$$=\frac{1}{4}\int e^{-2a+c-(2b-d)x+4(a+bx)}\,dx+\frac{7}{4}\int e^{-2a+c-(2b-d)x}\,dx+2\int\frac{e^{-2a+c-(2b-d)}}{-1+e^{2(a+bx)}}$$

$$=-\frac{7e^{-2a+c-(2b-d)x}}{4(2b-d)}+\frac{2e^{-2a+c-(2b-d)x}\,{}_2F_1\left(1,\frac{1}{2}\left(-2+\frac{d}{b}\right);\frac{d}{2b};e^{2(a+bx)}\right)}{2b-d}+\frac{1}{4}\int$$

$$=-\frac{7e^{-2a+c-(2b-d)x}}{4(2b-d)}+\frac{e^{c+dx}}{d}+\frac{e^{2a+c+(2b+d)x}}{4(2b+d)}+\frac{2e^{-2a+c-(2b-d)x}\,{}_2F_1\left(1,\frac{1}{2}\left(-2+\right.\right.}{2b-d}$$

Mathematica [A] time = 1.10, size = 172, normalized size = 1.38

$$-\frac{e^{c-\frac{ad}{b}}\left(2\left(4b^2-d^2\right)e^{d\left(\frac{a}{b}+x\right)}\,{}_2F_1\left(1,\frac{d}{2b};\frac{d}{2b}+1;e^{2(a+bx)}\right)+2d(2b-d)e^{\left(\frac{d}{b}+2\right)(a+bx)}\,{}_2F_1\left(1,\frac{d}{2b}+1;\frac{d}{2b}+2;e^{2(a+bx)}\right)+de\right.}{8b^2d-2d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(c + d*x)*Cosh[a + b*x]^2*Coth[a + b*x],x]
```

```
[Out] -((E^(c - (a*d)/b)*(2*(4*b^2 - d^2)*E^(d*(a/b + x))*Hypergeometric2F1[1, d/
(2*b), 1 + d/(2*b), E^(2*(a + b*x))] + 2*(2*b - d)*d*E^((2 + d/b)*(a + b*x)
)*Hypergeometric2F1[1, 1 + d/(2*b), 2 + d/(2*b), E^(2*(a + b*x))] + d*E^(d*
(a/b + x))*(-2*b*Cosh[2*(a + b*x)] + d*Sinh[2*(a + b*x)])))/(8*b^2*d - 2*d^
3))
```

fricas [F] time = 0.44, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\cosh\left(bx+a\right)^3\operatorname{csch}\left(bx+a\right)e^{(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x+c)*cosh(b*x+a)^3*csch(b*x+a),x, algorithm="fricas")
```

```
[Out] integral(cosh(b*x + a)^3*csch(b*x + a)*e^(d*x + c), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \cosh(bx+a)^3 \operatorname{csch}(bx+a)e^{(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x+c)*cosh(b*x+a)^3*csch(b*x+a),x, algorithm="giac")
[Out] integrate(cosh(b*x + a)^3*csch(b*x + a)*e^(d*x + c), x)
```

maple [F] time = 0.83, size = 0, normalized size = 0.00

$$\int e^{dx+c}\left(\cosh^3(bx+a)\right)\operatorname{csch}(bx+a)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(d*x+c)*cosh(b*x+a)^3*csch(b*x+a),x)
[Out] int(exp(d*x+c)*cosh(b*x+a)^3*csch(b*x+a),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-4\,b\int\frac{e^{(dx+c)}}{(4\,b-d)e^{(6\,bx+6\,a)}-2\,(4\,b-d)e^{(4\,bx+4\,a)}+(4\,b-d)e^{(2\,bx+2\,a)}}\,dx+\frac{\left(24\,b^2de^c+14\,bd^2e^c+d^3e^c+\left(8\,b^2de^c-6\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x+c)*cosh(b*x+a)^3*csch(b*x+a),x, algorithm="maxima")
[Out] -4*b*integrate(e^(d*x + c)/((4*b - d)*e^(6*b*x + 6*a) - 2*(4*b - d)*e^(4*b*
x + 4*a) + (4*b - d)*e^(2*b*x + 2*a)), x) + 1/4*(24*b^2*d*e^c + 14*b*d^2*e^
c + d^3*e^c + (8*b^2*d*e^c - 6*b*d^2*e^c + d^3*e^c)*e^(6*b*x + 6*a) + (64*b
^3*e^c - 24*b^2*d*e^c - 10*b*d^2*e^c + 3*d^3*e^c)*e^(4*b*x + 4*a) - (64*b^3
*e^c + 40*b^2*d*e^c - 2*b*d^2*e^c - 3*d^3*e^c)*e^(2*b*x + 2*a))*e^(d*x)/((1
6*b^3*d - 4*b^2*d^2 - 4*b*d^3 + d^4)*e^(4*b*x + 4*a) - (16*b^3*d - 4*b^2*d^
2 - 4*b*d^3 + d^4)*e^(2*b*x + 2*a))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cosh(a+b\,x)^3\,e^{c+d\,x}}{\sinh(a+b\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cosh(a + b*x)^3*exp(c + d*x))/sinh(a + b*x),x)
```

```
[Out] int((cosh(a + b*x)^3*exp(c + d*x))/sinh(a + b*x), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x+c)*cosh(b*x+a)**3*csch(b*x+a),x)
```

```
[Out] Timed out
```

3.964 $\int e^{c+dx}\cosh(a+bx)\coth^2(a+bx)\,dx$

Optimal. Leaf size=160

$$\frac{6e^{-a-x(b-d)+c}\,{}_2F_1\left(1,-\frac{b-d}{2b};\frac{b+d}{2b};e^{2(a+bx)}\right)}{b-d}-\frac{4e^{-a-x(b-d)+c}\,{}_2F_1\left(2,-\frac{b-d}{2b};\frac{b+d}{2b};e^{2(a+bx)}\right)}{b-d}-\frac{5e^{-a-x(b-d)+c}}{2(b-d)}+\frac{e^{a+x(b+d)+c}}{2(b+d)}$$

```
[Out] -5/2*exp(-a+c-(b-d)*x)/(b-d)+1/2*exp(a+c+(b+d)*x)/(b+d)+6*exp(-a+c-(b-d)*x)
*hypergeom([1, 1/2*(-b+d)/b],[1/2*(b+d)/b],exp(2*b*x+2*a))/(b-d)-4*exp(-a+c
-(b-d)*x)*hypergeom([2, 1/2*(-b+d)/b],[1/2*(b+d)/b],exp(2*b*x+2*a))/(b-d)
```

Rubi [A] time = 0.31, antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {5511, 2194, 2227, 2251}

$$\frac{6e^{-a-x(b-d)+c}\,{}_2F_1\left(1,-\frac{b-d}{2b};\frac{b+d}{2b};e^{2(a+bx)}\right)}{b-d}-\frac{4e^{-a-x(b-d)+c}\,{}_2F_1\left(2,-\frac{b-d}{2b};\frac{b+d}{2b};e^{2(a+bx)}\right)}{b-d}-\frac{5e^{-a-x(b-d)+c}}{2(b-d)}+\frac{e^{a+x(b+d)+c}}{2(b+d)}$$

Antiderivative was successfully verified.

```
[In] Int[E^(c + d*x)*Cosh[a + b*x]*Coth[a + b*x]^2,x]
```

```
[Out] (-5*E^(-a + c - (b - d)*x))/(2*(b - d)) + E^(a + c + (b + d)*x)/(2*(b + d))
 + (6*E^(-a + c - (b - d)*x)*Hypergeometric2F1[1, -(b - d)/(2*b), (b + d)/(
2*b), E^(2*(a + b*x))])/(b - d) - (4*E^(-a + c - (b - d)*x)*Hypergeometric2
F1[2, -(b - d)/(2*b), (b + d)/(2*b), E^(2*(a + b*x))])/(b - d)
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 2227

```
Int[(u_.)*(F_)^((a_.) + (b_.)*(v_)), x_Symbol] :> Int[u*F^(a + b*NormalizeP
owerOfLinear[v, x]), x] /; FreeQ[{F, a, b}, x] && PolynomialQ[u, x] && Powe
rOfLinearQ[v, x] &&  !PowerOfLinearMatchQ[v, x]
```

Rule 2251

```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_)*(G_)^((h_.)*((f_.
) + (g_.)*(x_))), x_Symbol] :> Simp[(a^p*G^(h*(f + g*x))*Hypergeometric2F1[
-p, (g*h*Log[G])/(d*e*Log[F]), (g*h*Log[G])/(d*e*Log[F]) + 1, Simplify[-((b
*F^(e*(c + d*x)))/a)]])/(g*h*Log[G]), x] /; FreeQ[{F, G, a, b, c, d, e, f,
```

```
g, h, p}, x] && (ILtQ[p, 0] || GtQ[a, 0])
```

Rule 5511

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*(G_)[(d_.) + (e_.)*(x_)]^(m_.)*(H_)[(
d_.) + (e_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^(c*(a + b*x)),
 G[d + e*x]^m*H[d + e*x]^n, x], x] /; FreeQ[{F, a, b, c, d, e}, x] && IGtQ[
m, 0] && IGtQ[n, 0] && HyperbolicQ[G] && HyperbolicQ[H]
```

Rubi steps

$$\int e^{c+dx}\cosh(a+bx)\coth^2(a+bx)\,dx = \int\left(\frac{5}{2}e^{-a+c-(b-d)x}+\frac{1}{2}e^{-a+c-(b-d)x+2(a+bx)}+\frac{4e^{-a+c-(b-d)x}}{\left(-1+e^{2(a+bx)}\right)^2}+\frac{6e^{-a+c-(b-d}}{-1+e^{2(a+}}\right.$$

$$=\frac{1}{2}\int e^{-a+c-(b-d)x+2(a+bx)}\,dx+\frac{5}{2}\int e^{-a+c-(b-d)x}\,dx+4\int\frac{e^{-a+c-(b-d)x}}{\left(-1+e^{2(a+bx)}\right)^2}$$

$$=-\frac{5e^{-a+c-(b-d)x}}{2(b-d)}+\frac{6e^{-a+c-(b-d)x}\,{}_2F_1\left(1,-\frac{b-d}{2b};\frac{b+d}{2b};e^{2(a+bx)}\right)}{b-d}-\frac{4e^{-a+c-(b-d)x}}{}$$

$$=-\frac{5e^{-a+c-(b-d)x}}{2(b-d)}+\frac{e^{a+c+(b+d)x}}{2(b+d)}+\frac{6e^{-a+c-(b-d)x}\,{}_2F_1\left(1,-\frac{b-d}{2b};\frac{b+d}{2b};e^{2(a+bx)}\right)}{b-d}-$$

Mathematica [A] time = 1.26, size = 145, normalized size = 0.91

$$\frac{e^{c-\frac{ad}{b}}\operatorname{csch}(a+bx)\left(e^{d\left(\frac{a}{b}+x\right)}\left(b^2\cosh(2(a+bx))-bd\sinh(2(a+bx))-3b^2+2d^2\right)-4d(b-d)e^{\frac{(b+d)(a+bx)}{b}}\sinh(a+b\right.}{2b(b-d)(b+d)}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(c + d*x)*Cosh[a + b*x]*Coth[a + b*x]^2,x]
```

```
[Out] (E^(c - (a*d)/b)*Csch[a + b*x]*(-4*(b - d)*d*E^(((b + d)*(a + b*x))/b)*Hype
rgeometric2F1[1, (b + d)/(2*b), (3*b + d)/(2*b), E^(2*(a + b*x))]*Sinh[a +
b*x] + E^(d*(a/b + x))*(-3*b^2 + 2*d^2 + b^2*Cosh[2*(a + b*x)] - b*d*Sinh[2
*(a + b*x)])))/(2*b*(b - d)*(b + d))
```

fricas [F] time = 0.41, size = 0, normalized size = 0.00

$$\text{integral}\left(\cosh(bx+a)^3\operatorname{csch}(bx+a)^2e^{(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x+c)*cosh(b*x+a)^3*csch(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] integral(cosh(b*x + a)^3*csch(b*x + a)^2*e^(d*x + c), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \cosh\left(bx+a\right)^{3}\operatorname{csch}\left(bx+a\right)^{2} e^{(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x+c)*cosh(b*x+a)^3*csch(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate(cosh(b*x + a)^3*csch(b*x + a)^2*e^(d*x + c), x)
```

maple [F] time = 0.80, size = 0, normalized size = 0.00

$$\int {\mathrm e}^{dx+c} \left(\cosh^{3}\left(bx+a\right)\right) \operatorname{csch}\left(bx+a\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(d*x+c)*cosh(b*x+a)^3*csch(b*x+a)^2,x)
```

```
[Out] int(exp(d*x+c)*cosh(b*x+a)^3*csch(b*x+a)^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$16\,bd\int \frac{e^{(dx+c)}}{\left(15\,b^{2}-8\,bd+d^{2}\right)e^{(7\,bx+7\,a)}-3\left(15\,b^{2}-8\,bd+d^{2}\right)e^{(5\,bx+5\,a)}+3\left(15\,b^{2}-8\,bd+d^{2}\right)e^{(3\,bx+3\,a)}-\left(15\,b^{2}-8\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x+c)*cosh(b*x+a)^3*csch(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 16*b*d*integrate(e^(d*x + c)/((15*b^2 - 8*b*d + d^2)*e^(7*b*x + 7*a) - 3*(1
5*b^2 - 8*b*d + d^2)*e^(5*b*x + 5*a) + 3*(15*b^2 - 8*b*d + d^2)*e^(3*b*x +
3*a) - (15*b^2 - 8*b*d + d^2)*e^(b*x + a)), x) - 1/2*(15*b^3*e^c + 39*b^2*d
*e^c + 25*b*d^2*e^c + d^3*e^c - (15*b^3*e^c - 23*b^2*d*e^c + 9*b*d^2*e^c -
d^3*e^c)*e^(6*b*x + 6*a) + (105*b^3*e^c - 11*b^2*d*e^c - 17*b*d^2*e^c + 3*d
^3*e^c)*e^(4*b*x + 4*a) - (105*b^3*e^c + 59*b^2*d*e^c - b*d^2*e^c - 3*d^3*e
^c)*e^(2*b*x + 2*a))*e^(d*x)/((15*b^4 - 8*b^3*d - 14*b^2*d^2 + 8*b*d^3 - d^
4)*e^(5*b*x + 5*a) - 2*(15*b^4 - 8*b^3*d - 14*b^2*d^2 + 8*b*d^3 - d^4)*e^(3
*b*x + 3*a) + (15*b^4 - 8*b^3*d - 14*b^2*d^2 + 8*b*d^3 - d^4)*e^(b*x + a))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cosh (a+b\,x)^3\,\mathrm{e}^{c+d\,x}}{\sinh (a+b\,x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cosh(a + b*x)^3*exp(c + d*x))/sinh(a + b*x)^2,x)
```

```
[Out] int((cosh(a + b*x)^3*exp(c + d*x))/sinh(a + b*x)^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x+c)*cosh(b*x+a)**3*csch(b*x+a)**2,x)
```

```
[Out] Timed out
```

3.965 $\int e^{c+dx}\coth^3(a+bx)\,dx$

Optimal. Leaf size=135

$$-\frac{6e^{c+dx}\,{}_2F_1\left(1,\frac{d}{2b};\frac{d}{2b}+1;e^{2(a+bx)}\right)}{d}+\frac{12e^{c+dx}\,{}_2F_1\left(2,\frac{d}{2b};\frac{d}{2b}+1;e^{2(a+bx)}\right)}{d}-\frac{8e^{c+dx}\,{}_2F_1\left(3,\frac{d}{2b};\frac{d}{2b}+1;e^{2(a+bx)}\right)}{d}+\frac{e^{c+dx}}{d}$$

```
[Out] exp(d*x+c)/d-6*exp(d*x+c)*hypergeom([1, 1/2*d/b],[1+1/2*d/b],exp(2*b*x+2*a)
)/d+12*exp(d*x+c)*hypergeom([2, 1/2*d/b],[1+1/2*d/b],exp(2*b*x+2*a))/d-8*ex
p(d*x+c)*hypergeom([3, 1/2*d/b],[1+1/2*d/b],exp(2*b*x+2*a))/d
```

Rubi [A] time = 0.16, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {5485, 2194, 2251}

$$-\frac{6e^{c+dx}\,{}_2F_1\left(1,\frac{d}{2b};\frac{d}{2b}+1;e^{2(a+bx)}\right)}{d}+\frac{12e^{c+dx}\,{}_2F_1\left(2,\frac{d}{2b};\frac{d}{2b}+1;e^{2(a+bx)}\right)}{d}-\frac{8e^{c+dx}\,{}_2F_1\left(3,\frac{d}{2b};\frac{d}{2b}+1;e^{2(a+bx)}\right)}{d}+\frac{e^{c+dx}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[E^(c + d*x)*Coth[a + b*x]^3,x]
```

```
[Out] E^(c + d*x)/d - (6*E^(c + d*x)*Hypergeometric2F1[1, d/(2*b), 1 + d/(2*b), E
^(2*(a + b*x))])/d + (12*E^(c + d*x)*Hypergeometric2F1[2, d/(2*b), 1 + d/(2
*b), E^(2*(a + b*x))])/d - (8*E^(c + d*x)*Hypergeometric2F1[3, d/(2*b), 1 +
 d/(2*b), E^(2*(a + b*x))])/d
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 2251

```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_)*(G_)^((h_.)*((f_.
) + (g_.)*(x_))), x_Symbol] :> Simp[(a^p*G^(h*(f + g*x))*Hypergeometric2F1[
-p, (g*h*Log[G])/(d*e*Log[F]), (g*h*Log[G])/(d*e*Log[F]) + 1, Simplify[-((b
*F^(e*(c + d*x)))/a)]])/(g*h*Log[G]), x] /; FreeQ[{F, G, a, b, c, d, e, f,
g, h, p}, x] && (ILtQ[p, 0] || GtQ[a, 0])
```

Rule 5485

```
Int[Coth[(d_.) + (e_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Sym
bol] :> Int[ExpandIntegrand[(F^(c*(a + b*x))*(1 + E^(2*(d + e*x)))^n)/(-1 +
```

```
E^(2*(d + e*x)))^n, x], x] /; FreeQ[{F, a, b, c, d, e}, x] && IntegerQ[n]
```

Rubi steps

$$\int e^{c+dx}\coth^3(a+bx)\,dx = \int \left(e^{c+dx} + \frac{8e^{c+dx}}{\left(-1+e^{2(a+bx)}\right)^3} + \frac{12e^{c+dx}}{\left(-1+e^{2(a+bx)}\right)^2} + \frac{6e^{c+dx}}{-1+e^{2(a+bx)}}\right) dx$$
$$= 6\int \frac{e^{c+dx}}{-1+e^{2(a+bx)}}\,dx + 8\int \frac{e^{c+dx}}{\left(-1+e^{2(a+bx)}\right)^3}\,dx + 12\int \frac{e^{c+dx}}{\left(-1+e^{2(a+bx)}\right)^2}\,dx + \int e^{c+dx}$$
$$= \frac{e^{c+dx}}{d} - \frac{6e^{c+dx}\,{}_2F_1\left(1,\frac{d}{2b};1+\frac{d}{2b};e^{2(a+bx)}\right)}{d} + \frac{12e^{c+dx}\,{}_2F_1\left(2,\frac{d}{2b};1+\frac{d}{2b};e^{2(a+bx)}\right)}{d} - \frac{8e^{c+d}}{}$$

Mathematica [A] time = 3.81, size = 176, normalized size = 1.30

$$\frac{1}{2}e^c\left(-\frac{2e^{2a}\left(2b^2+d^2\right)\left(\frac{e^{dx}\,{}_2F_1\left(1,\frac{d}{2b};\frac{d}{2b}+1;e^{2(a+bx)}\right)}{d} - \frac{e^{x(2b+d)}\,{}_2F_1\left(1,\frac{d}{2b}+1;\frac{d}{2b}+2;e^{2(a+bx)}\right)}{2b+d}\right)}{\left(e^{2a}-1\right)b^2} + \frac{d\operatorname{csch}(a)e^{dx}\sinh(bx)\operatorname{csch}(a+bx)}{b^2}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(c + d*x)*Coth[a + b*x]^3,x]
```

```
[Out] (E^c*((2*E^(d*x)*Coth[a])/d - (E^(d*x)*Csch[a + b*x]^2)/b - (2*(2*b^2 + d^2
)*E^(2*a)*((E^(d*x)*Hypergeometric2F1[1, d/(2*b), 1 + d/(2*b), E^(2*(a + b*
x))])/d - (E^((2*b + d)*x)*Hypergeometric2F1[1, 1 + d/(2*b), 2 + d/(2*b), E
^(2*(a + b*x))])/(2*b + d)))/(b^2*(-1 + E^(2*a))) + (d*E^(d*x)*Csch[a]*Csch
[a + b*x]*Sinh[b*x])/b^2))/2
```

fricas [F] time = 0.49, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\cosh(bx+a)^3\operatorname{csch}(bx+a)^3 e^{(dx+c)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x+c)*cosh(b*x+a)^3*csch(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] integral(cosh(b*x + a)^3*csch(b*x + a)^3*e^(d*x + c), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \cosh(bx+a)^3 \operatorname{csch}(bx+a)^3 e^{(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x+c)*cosh(b*x+a)^3*csch(b*x+a)^3,x, algorithm="giac")
```

```
[Out] integrate(cosh(b*x + a)^3*csch(b*x + a)^3*e^(d*x + c), x)
```

maple [F] time = 0.88, size = 0, normalized size = 0.00

$$\int e^{dx+c}\left(\cosh^3(bx+a)\right)\operatorname{csch}(bx+a)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(d*x+c)*cosh(b*x+a)^3*csch(b*x+a)^3,x)
```

```
[Out] int(exp(d*x+c)*cosh(b*x+a)^3*csch(b*x+a)^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-48\left(2\,b^3e^c+bd^2e^c\right)\int \frac{}{48\,b^3-44\,b^2d+12\,bd^2-d^3+\left(48\,b^3-44\,b^2d+12\,bd^2-d^3\right)e^{(8\,bx+8\,a)}-4\left(48\,b^3-44\,b^2d\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x+c)*cosh(b*x+a)^3*csch(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -48*(2*b^3*e^c + b*d^2*e^c)*integrate(e^(d*x)/(48*b^3 - 44*b^2*d + 12*b*d^2
 - d^3 + (48*b^3 - 44*b^2*d + 12*b*d^2 - d^3)*e^(8*b*x + 8*a) - 4*(48*b^3 -
 44*b^2*d + 12*b*d^2 - d^3)*e^(6*b*x + 6*a) + 6*(48*b^3 - 44*b^2*d + 12*b*d
^2 - d^3)*e^(4*b*x + 4*a) - 4*(48*b^3 - 44*b^2*d + 12*b*d^2 - d^3)*e^(2*b*x
 + 2*a)), x) + (48*b^3*e^c + 44*b^2*d*e^c + 36*b*d^2*e^c + d^3*e^c - (48*b^
3*e^c - 44*b^2*d*e^c + 12*b*d^2*e^c - d^3*e^c)*e^(6*b*x + 6*a) + 3*(48*b^3*
e^c + 4*b^2*d*e^c - 8*b*d^2*e^c + d^3*e^c)*e^(4*b*x + 4*a) - 3*(48*b^3*e^c
+ 28*b^2*d*e^c - d^3*e^c)*e^(2*b*x + 2*a))*e^(d*x)/(48*b^3*d - 44*b^2*d^2 +
 12*b*d^3 - d^4 - (48*b^3*d - 44*b^2*d^2 + 12*b*d^3 - d^4)*e^(6*b*x + 6*a)
+ 3*(48*b^3*d - 44*b^2*d^2 + 12*b*d^3 - d^4)*e^(4*b*x + 4*a) - 3*(48*b^3*d
- 44*b^2*d^2 + 12*b*d^3 - d^4)*e^(2*b*x + 2*a))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cosh(a+b\,x)^3\,e^{c+d\,x}}{\sinh(a+b\,x)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cosh(a + b*x)^3*exp(c + d*x))/sinh(a + b*x)^3,x)
```

```
[Out] int((cosh(a + b*x)^3*exp(c + d*x))/sinh(a + b*x)^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(d*x+c)*cosh(b*x+a)**3*csch(b*x+a)**3,x)
```

```
[Out] Timed out
```

3.966 $\int \left(-\frac{3d^2 e^{a+bx}}{4\left(b^2-\frac{9d^2}{4}\right)\sqrt{\sinh(c+dx)}}+e^{a+bx}\sinh^{\frac{3}{2}}(c+dx)\right) dx$

Optimal. Leaf size=73

$$\frac{4be^{a+bx}\sinh^{\frac{3}{2}}(c+dx)}{4b^2-9d^2}-\frac{6de^{a+bx}\sqrt{\sinh(c+dx)}\cosh(c+dx)}{4b^2-9d^2}$$

```
[Out] 4*b*exp(b*x+a)*sinh(d*x+c)^(3/2)/(4*b^2-9*d^2)-6*d*exp(b*x+a)*cosh(d*x+c)*s
inh(d*x+c)^(1/2)/(4*b^2-9*d^2)
```

Rubi [A] time = 0.61, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 5, integrand size = 56, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.089, Rules used = {5482, 2253, 2252, 2251, 5476}

$$\frac{4be^{a+bx}\sinh^{\frac{3}{2}}(c+dx)}{4b^2-9d^2}-\frac{6de^{a+bx}\sqrt{\sinh(c+dx)}\cosh(c+dx)}{4b^2-9d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(-3*d^2*E^(a + b*x))/(4*(b^2 - (9*d^2)/4)*Sqrt[Sinh[c + d*x]]) + E^(a +
 b*x)*Sinh[c + d*x]^(3/2),x]
```

```
[Out] (-6*d*E^(a + b*x)*Cosh[c + d*x]*Sqrt[Sinh[c + d*x]])/(4*b^2 - 9*d^2) + (4*b
*E^(a + b*x)*Sinh[c + d*x]^(3/2))/(4*b^2 - 9*d^2)
```

Rule 2251

```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_)*(G_)^((h_.)*((f_.
) + (g_.)*(x_))), x_Symbol] :> Simp[(a^p*G^(h*(f + g*x))*Hypergeometric2F1[
-p, (g*h*Log[G])/(d*e*Log[F]), (g*h*Log[G])/(d*e*Log[F]) + 1, Simplify[-((b
*F^(e*(c + d*x)))/a)]])/(g*h*Log[G]), x] /; FreeQ[{F, G, a, b, c, d, e, f,
g, h, p}, x] && (ILtQ[p, 0] || GtQ[a, 0])
```

Rule 2252

```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_)*(G_)^((h_.)*((f_.
) + (g_.)*(x_))), x_Symbol] :> Dist[(a + b*F^(e*(c + d*x)))^p/(1 + (b/a)*F^
(e*(c + d*x)))^p, Int[G^(h*(f + g*x))*(1 + (b*F^(e*(c + d*x)))/a)^p, x], x]
 /; FreeQ[{F, G, a, b, c, d, e, f, g, h, p}, x] &&  !(ILtQ[p, 0] || GtQ[a,
0])
```

Rule 2253

```
Int[((a_) + (b_.)*(F_)^((e_.)*(v_)))^(p_)*(G_)^((h_.)*(u_)), x_Symbol] :> I
nt[G^(h*ExpandToSum[u, x])*(a + b*F^(e*ExpandToSum[v, x]))^p, x] /; FreeQ[{
F, G, a, b, e, h, p}, x] && LinearQ[{u, v}, x] &&  !LinearMatchQ[{u, v}, x]
```

Rule 5476

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sinh[(d_.) + (e_.)*(x_)]^(n_), x_Symb
ol] :> -Simp[(b*c*Log[F]*F^(c*(a + b*x))*Sinh[d + e*x]^n)/(e^2*n^2 - b^2*c^
2*Log[F]^2), x] + (-Dist[(n*(n - 1)*e^2)/(e^2*n^2 - b^2*c^2*Log[F]^2), Int[
F^(c*(a + b*x))*Sinh[d + e*x]^(n - 2), x], x] + Simp[(e*n*F^(c*(a + b*x))*C
osh[d + e*x]*Sinh[d + e*x]^(n - 1))/(e^2*n^2 - b^2*c^2*Log[F]^2), x]) /; Fr
eeQ[{F, a, b, c, d, e}, x] && NeQ[e^2*n^2 - b^2*c^2*Log[F]^2, 0] && GtQ[n,
1]
```

Rule 5482

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sinh[(d_.) + (e_.)*(x_)]^(n_), x_Symb
ol] :> Dist[(E^(n*(d + e*x))*Sinh[d + e*x]^n)/(-1 + E^(2*(d + e*x)))^n, Int
[(F^(c*(a + b*x))*(-1 + E^(2*(d + e*x)))^n)/E^(n*(d + e*x)), x], x] /; Free
Q[{F, a, b, c, d, e, n}, x] &&  !IntegerQ[n]
```

Rubi steps

$$\int\left(-\frac{3d^2e^{a+bx}}{4\left(b^2-\frac{9d^2}{4}\right)\sqrt{\sinh(c+dx)}}+e^{a+bx}\sinh^{\frac{3}{2}}(c+dx)\right)dx=-\frac{\left(3d^2\right)\int\frac{e^{a+bx}}{\sqrt{\sinh(c+dx)}}\,dx}{4b^2-9d^2}+\int e^{a+bx}\sinh^{\frac{3}{2}}(c+dx)$$

$$=-\frac{6de^{a+bx}\cosh(c+dx)\sqrt{\sinh(c+dx)}}{4b^2-9d^2}+\frac{4be^{a+bx}\,\mathrm{s}}{4b}$$

$$=-\frac{6de^{a+bx}\cosh(c+dx)\sqrt{\sinh(c+dx)}}{4b^2-9d^2}+\frac{4be^{a+bx}\,\mathrm{s}}{4b}$$

$$=-\frac{6de^{a+bx}\cosh(c+dx)\sqrt{\sinh(c+dx)}}{4b^2-9d^2}+\frac{4be^{a+bx}\,\mathrm{s}}{4b}$$

$$=-\frac{6d^2\exp\left(\frac{1}{2}(2a+c)+\frac{1}{2}(2b+d)x+\frac{1}{2}(-c-dx)\right)}{(2b+d)\left(4b^2-9d\right.}$$

$$=-\frac{6de^{a+bx}\cosh(c+dx)\sqrt{\sinh(c+dx)}}{4b^2-9d^2}+\frac{4be^{a+bx}\,\mathrm{s}}{4b}$$

Mathematica [C] time = 1.32, size = 155, normalized size = 2.12

$$\frac{2e^{a+bx}\left(e^{2(c+dx)}-1\right)\left(\left(4b^2+8bd+3d^2\right)\sinh^2(c+dx)\,{}_2F_1\left(1,\frac{1}{4}\left(\frac{2b}{d}+7\right);\frac{2b+d}{4d};e^{2(c+dx)}\right)-3d^2\,{}_2F_1\left(1,\frac{1}{4}\left(\frac{2b}{d}+3\right);\right.\right.}{(2b+d)(3d-2b)(2b+3d)\sqrt{\sinh(c+dx)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(-3*d^2*E^(a + b*x))/(4*(b^2 - (9*d^2)/4)*Sqrt[Sinh[c + d*x]]) +
E^(a + b*x)*Sinh[c + d*x]^(3/2),x]
```

```
[Out] (2*E^(a + b*x)*(-1 + E^(2*(c + d*x)))*(-3*d^2*Hypergeometric2F1[1, (3 + (2*
b)/d)/4, (5 + (2*b)/d)/4, E^(2*(c + d*x))] + (4*b^2 + 8*b*d + 3*d^2)*Hyperg
eometric2F1[1, (7 + (2*b)/d)/4, (2*b + d)/(4*d), E^(2*(c + d*x))]*Sinh[c +
d*x]^2))/((2*b + d)*(-2*b + 3*d)*(2*b + 3*d)*Sqrt[Sinh[c + d*x]])
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*sinh(d*x+c)^(3/2)-3/4*d^2*exp(b*x+a)/(b^2-9/4*d^2)/sin
h(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int e^{(bx+a)} \sinh (dx+c)^{\frac{3}{2}} - \frac{3\, d^2 e^{(bx+a)}}{\left(4\, b^2 - 9\, d^2\right)\sqrt{\sinh (dx+c)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*sinh(d*x+c)^(3/2)-3/4*d^2*exp(b*x+a)/(b^2-9/4*d^2)/sin
h(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(e^(b*x + a)*sinh(d*x + c)^(3/2) - 3*d^2*e^(b*x + a)/((4*b^2 - 9*d
^2)*sqrt(sinh(d*x + c))), x)
```

maple [F] time = 1.03, size = 0, normalized size = 0.00

$$\int e^{bx+a} \left(\sinh^{\frac{3}{2}} (dx+c)\right) - \frac{3 d^2 e^{bx+a}}{4 \left(b^2 - \frac{9 d^2}{4}\right) \sqrt{\sinh (dx+c)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(b*x+a)*sinh(d*x+c)^(3/2)-3/4*d^2*exp(b*x+a)/(b^2-9/4*d^2)/sinh(d*x+
c)^(1/2),x)
```

```
[Out] int(exp(b*x+a)*sinh(d*x+c)^(3/2)-3/4*d^2*exp(b*x+a)/(b^2-9/4*d^2)/sinh(d*x+
c)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int e^{(bx+a)} \sinh (dx+c)^{\frac{3}{2}} - \frac{3\, d^2 e^{(bx+a)}}{\left(4\, b^2 - 9\, d^2\right)\sqrt{\sinh (dx+c)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*sinh(d*x+c)^(3/2)-3/4*d^2*exp(b*x+a)/(b^2-9/4*d^2)/sin
h(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(e^(b*x + a)*sinh(d*x + c)^(3/2) - 3*d^2*e^(b*x + a)/((4*b^2 - 9*d
^2)*sqrt(sinh(d*x + c))), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int e^{a+b\,x}\,\sinh\left(c+d\,x\right)^{3/2}-\frac{3\,d^2\,e^{a+b\,x}}{4\,\sqrt{\sinh\left(c+d\,x\right)}\,\left(b^2-\frac{9\,d^2}{4}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(a + b*x)*sinh(c + d*x)^(3/2) - (3*d^2*exp(a + b*x))/(4*sinh(c + d*x
)^(1/2)*(b^2 - (9*d^2)/4)),x)
```

```
[Out] int(exp(a + b*x)*sinh(c + d*x)^(3/2) - (3*d^2*exp(a + b*x))/(4*sinh(c + d*x
)^(1/2)*(b^2 - (9*d^2)/4)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\left(\int 4b^2e^{bx}\sinh^{\frac{3}{2}}\left(c+dx\right)dx+\int\left(-\frac{3d^2e^{bx}}{\sqrt{\sinh\left(c+dx\right)}}\right)dx+\int\left(-9d^2e^{bx}\sinh^{\frac{3}{2}}\left(c+dx\right)\right)dx\right)e^a}{\left(2b-3d\right)\left(2b+3d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*sinh(d*x+c)**(3/2)-3/4*d**2*exp(b*x+a)/(b**2-9/4*d**2)
/sinh(d*x+c)**(1/2),x)
```

```
[Out] (Integral(4*b**2*exp(b*x)*sinh(c + d*x)**(3/2), x) + Integral(-3*d**2*exp(b
*x)/sqrt(sinh(c + d*x)), x) + Integral(-9*d**2*exp(b*x)*sinh(c + d*x)**(3/2
), x))*exp(a)/((2*b - 3*d)*(2*b + 3*d))
```

3.967 $\int e^{n\cosh(a+bx)}\sinh(a+bx)\,dx$

Optimal. Leaf size=17

$$\frac{e^{n\cosh(a+bx)}}{bn}$$

[Out] exp(n*cosh(b*x+a))/b/n

Rubi [A] time = 0.02, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {4337, 2194}

$$\frac{e^{n\cosh(a+bx)}}{bn}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*Cosh[a + b*x])*Sinh[a + b*x],x]
```

```
[Out] E^(n*Cosh[a + b*x])/(b*n)
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 4337

```
Int[(u_)*Sinh[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Cosh[c*(a + b*x)], x]}, Dist[d/(b*c), Subst[Int[SubstFor[1, Cosh[c*(a +
b*x)]/d, u, x], x], x, Cosh[c*(a + b*x)]/d], x] /; FunctionOfQ[Cosh[c*(a +
b*x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x]
```

Rubi steps

$$\begin{aligned}\int e^{n\cosh(a+bx)}\sinh(a+bx)\,dx &= \frac{\text{Subst}\left(\int e^{nx}\,dx, x, \cosh(a+bx)\right)}{b}\\ &= \frac{e^{n\cosh(a+bx)}}{bn}\end{aligned}$$

Mathematica [A] time = 0.04, size = 17, normalized size = 1.00

$$\frac{e^{n\cosh(a+bx)}}{bn}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*Cosh[a + b*x])*Sinh[a + b*x],x]
```

```
[Out] E^(n*Cosh[a + b*x])/(b*n)
```

fricas [A] time = 0.50, size = 26, normalized size = 1.53

$$\frac{\cosh\left(n\cosh\left(bx+a\right)\right)+\sinh\left(n\cosh\left(bx+a\right)\right)}{bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*cosh(b*x+a))*sinh(b*x+a),x, algorithm="fricas")
```

```
[Out] (cosh(n*cosh(b*x + a)) + sinh(n*cosh(b*x + a)))/(b*n)
```

giac [A] time = 0.14, size = 30, normalized size = 1.76

$$\frac{e^{\left(\frac{1}{2}\,ne^{\left(bx+a\right)}+\frac{1}{2}\,ne^{\left(-bx-a\right)}\right)}}{bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*cosh(b*x+a))*sinh(b*x+a),x, algorithm="giac")
```

```
[Out] e^(1/2*n*e^(b*x + a) + 1/2*n*e^(-b*x - a))/(b*n)
```

maple [A] time = 0.02, size = 17, normalized size = 1.00

$$\frac{{\mathrm e}^{n\cosh\left(bx+a\right)}}{bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*cosh(b*x+a))*sinh(b*x+a),x)
```

```
[Out] exp(n*cosh(b*x+a))/b/n
```

maxima [A] time = 0.31, size = 16, normalized size = 0.94

$$\frac{e^{\left(n\cosh\left(bx+a\right)\right)}}{bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*cosh(b*x+a))*sinh(b*x+a),x, algorithm="maxima")
```

```
[Out] e^(n*cosh(b*x + a))/(b*n)
```

mupad [B] time = 0.10, size = 16, normalized size = 0.94

$$\frac{\mathrm{e}^{n\cosh(a+b\,x)}}{b\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*cosh(a + b*x))*sinh(a + b*x),x)
```

```
[Out] exp(n*cosh(a + b*x))/(b*n)
```

sympy [A] time = 0.43, size = 36, normalized size = 2.12

$$\begin{cases} x\sinh(a) & \text{for}\ b = 0 \wedge n = 0 \\ xe^{n\cosh(a)}\sinh(a) & \text{for}\ b = 0 \\ \frac{\cosh(a+bx)}{b} & \text{for}\ n = 0 \\ \frac{e^{n\cosh(a+bx)}}{bn} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*cosh(b*x+a))*sinh(b*x+a),x)
```

```
[Out] Piecewise((x*sinh(a), Eq(b, 0) & Eq(n, 0)), (x*exp(n*cosh(a))*sinh(a), Eq(b
, 0)), (cosh(a + b*x)/b, Eq(n, 0)), (exp(n*cosh(a + b*x))/(b*n), True))
```

3.968 $\int e^{n\cosh(ac+bcx)}\sinh(c(a+bx))\,dx$

Optimal. Leaf size=22

$$\frac{e^{n\cosh(c(a+bx))}}{bcn}$$

```
[Out] exp(n*cosh(c*(b*x+a)))/b/c/n
```

Rubi [A] time = 0.02, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {4337, 2194}

$$\frac{e^{n\cosh(c(a+bx))}}{bcn}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*Cosh[a*c + b*c*x])*Sinh[c*(a + b*x)],x]
```

```
[Out] E^(n*Cosh[c*(a + b*x)])/(b*c*n)
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 4337

```
Int[(u_)*Sinh[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Cosh[c*(a + b*x)], x]}, Dist[d/(b*c), Subst[Int[SubstFor[1, Cosh[c*(a +
b*x)]/d, u, x], x], x, Cosh[c*(a + b*x)]/d], x] /; FunctionOfQ[Cosh[c*(a +
b*x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x]
```

Rubi steps

$$\begin{aligned}\int e^{n\cosh(ac+bcx)}\sinh(c(a+bx))\,dx &= \frac{\text{Subst}\left(\int e^{nx}\,dx, x, \cosh(c(a+bx))\right)}{bc}\\ &= \frac{e^{n\cosh(c(a+bx))}}{bcn}\end{aligned}$$

Mathematica [A] time = 0.22, size = 22, normalized size = 1.00

$$\frac{e^{n\cosh(c(a+bx))}}{bcn}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*Cosh[a*c + b*c*x])*Sinh[c*(a + b*x)],x]
```

```
[Out] E^(n*Cosh[c*(a + b*x)])/(b*c*n)
```

fricas [A]   time = 0.46, size = 35, normalized size = 1.59

$$\frac{\cosh\left(n\cosh\left(bcx+ac\right)\right)+\sinh\left(n\cosh\left(bcx+ac\right)\right)}{bcn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*cosh(b*c*x+a*c))*sinh(c*(b*x+a)),x, algorithm="fricas")
```

```
[Out] (cosh(n*cosh(b*c*x + a*c)) + sinh(n*cosh(b*c*x + a*c)))/(b*c*n)
```

giac [A]   time = 0.13, size = 38, normalized size = 1.73

$$\frac{e^{\left(\frac{1}{2}\,ne^{(bcx+ac)}+\frac{1}{2}\,ne^{(-bcx-ac)}\right)}}{bcn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*cosh(b*c*x+a*c))*sinh(c*(b*x+a)),x, algorithm="giac")
```

```
[Out] e^(1/2*n*e^(b*c*x + a*c) + 1/2*n*e^(-b*c*x - a*c))/(b*c*n)
```

maple [A]   time = 3.07, size = 39, normalized size = 1.77

$$\frac{\frac{\sinh(n\cosh(c(bx+a)))}{n}+\frac{\cosh(n\cosh(c(bx+a)))}{n}}{cb}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*cosh(b*c*x+a*c))*sinh(c*(b*x+a)),x)
```

```
[Out] 1/c/b*(1/n*sinh(n*cosh(c*(b*x+a)))+cosh(n*cosh(c*(b*x+a)))/n)
```

maxima [A]   time = 0.31, size = 22, normalized size = 1.00

$$\frac{e^{(n\cosh(bcx+ac))}}{bcn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*cosh(b*c*x+a*c))*sinh(c*(b*x+a)),x, algorithm="maxima")
```

```
[Out] e^(n*cosh(b*c*x + a*c))/(b*c*n)
```

mupad [B] time = 1.75, size = 38, normalized size = 1.73

$$\frac{\mathrm{e}^{\frac{n\,\mathrm{e}^{b\,c\,x}\,\mathrm{e}^{a\,c}}{2}}\,\mathrm{e}^{\frac{n\,\mathrm{e}^{-b\,c\,x}\,\mathrm{e}^{-a\,c}}{2}}}{b\,c\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(c*(a + b*x))*exp(n*cosh(a*c + b*c*x)),x)
[Out] (exp((n*exp(b*c*x)*exp(a*c))/2)*exp((n*exp(-b*c*x)*exp(-a*c))/2))/(b*c*n)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int e^{n\cosh{(ac+bcx)}}\sinh{(ac+bcx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*cosh(b*c*x+a*c))*sinh(c*(b*x+a)),x)
[Out] Integral(exp(n*cosh(a*c + b*c*x))*sinh(a*c + b*c*x), x)
```

3.969 $\int e^{n\cosh(c(a+bx))}\sinh(ac+bcx)\,dx$

Optimal. Leaf size=23

$$\frac{e^{n\cosh(ac+bcx)}}{bcn}$$

```
[Out] exp(n*cosh(b*c*x+a*c))/b/c/n
```

Rubi [A] time = 0.02, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {4337, 2194}

$$\frac{e^{n\cosh(ac+bcx)}}{bcn}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*Cosh[c*(a + b*x)])*Sinh[a*c + b*c*x],x]
```

```
[Out] E^(n*Cosh[a*c + b*c*x])/(b*c*n)
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 4337

```
Int[(u_)*Sinh[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Cosh[c*(a + b*x)], x]}, Dist[d/(b*c), Subst[Int[SubstFor[1, Cosh[c*(a +
b*x)]/d, u, x], x], x, Cosh[c*(a + b*x)]/d], x] /; FunctionOfQ[Cosh[c*(a +
b*x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x]
```

Rubi steps

$$\begin{aligned}\int e^{n\cosh(c(a+bx))}\sinh(ac+bcx)\,dx &= \frac{\text{Subst}\left(\int e^{nx}\,dx, x, \cosh(ac+bcx)\right)}{bc}\\ &= \frac{e^{n\cosh(ac+bcx)}}{bcn}\end{aligned}$$

Mathematica [A] time = 0.04, size = 22, normalized size = 0.96

$$\frac{e^{n\cosh(c(a+bx))}}{bcn}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*Cosh[c*(a + b*x)])*Sinh[a*c + b*c*x],x]
```

```
[Out] E^(n*Cosh[c*(a + b*x)])/(b*c*n)
```

fricas [A] time = 0.44, size = 35, normalized size = 1.52

$$\frac{\cosh\left(n\cosh\left(bcx+ac\right)\right)+\sinh\left(n\cosh\left(bcx+ac\right)\right)}{bcn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*cosh(c*(b*x+a)))*sinh(b*c*x+a*c),x, algorithm="fricas")
```

```
[Out] (cosh(n*cosh(b*c*x + a*c)) + sinh(n*cosh(b*c*x + a*c)))/(b*c*n)
```

giac [A] time = 0.13, size = 38, normalized size = 1.65

$$\frac{e^{\left(\frac{1}{2}\,ne^{\left(bcx+ac\right)}+\frac{1}{2}\,ne^{\left(-bcx-ac\right)}\right)}}{bcn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*cosh(c*(b*x+a)))*sinh(b*c*x+a*c),x, algorithm="giac")
```

```
[Out] e^(1/2*n*e^(b*c*x + a*c) + 1/2*n*e^(-b*c*x - a*c))/(b*c*n)
```

maple [A] time = 2.83, size = 39, normalized size = 1.70

$$\frac{\frac{\sinh(n\cosh(c(bx+a)))}{n}+\frac{\cosh(n\cosh(c(bx+a)))}{n}}{cb}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*cosh(c*(b*x+a)))*sinh(b*c*x+a*c),x)
```

```
[Out] 1/c/b*(1/n*sinh(n*cosh(c*(b*x+a)))+cosh(n*cosh(c*(b*x+a)))/n)
```

maxima [A] time = 0.30, size = 22, normalized size = 0.96

$$\frac{e^{\left(n\cosh(bcx+ac)\right)}}{bcn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*cosh(c*(b*x+a)))*sinh(b*c*x+a*c),x, algorithm="maxima")
```

```
[Out] e^(n*cosh(b*c*x + a*c))/(b*c*n)
```

mupad [B] time = 1.69, size = 38, normalized size = 1.65

$$\frac{\mathrm{e}^{\frac{n\,\mathrm{e}^{b\,c\,x}\,\mathrm{e}^{a\,c}}{2}}\,\mathrm{e}^{\frac{n\,\mathrm{e}^{-b\,c\,x}\,\mathrm{e}^{-a\,c}}{2}}}{b\,c\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*cosh(c*(a + b*x)))*sinh(a*c + b*c*x),x)
```

```
[Out] (exp((n*exp(b*c*x)*exp(a*c))/2)*exp((n*exp(-b*c*x)*exp(-a*c))/2))/(b*c*n)
```

sympy [A] time = 2.45, size = 51, normalized size = 2.22

$$\begin{cases} 0 & \text{for}\: c = 0 \wedge \left(b = 0 \vee c = 0\right) \wedge \left(c = 0 \vee n = 0\right) \\ x e^{n \cosh{\left(a c \right)}} \sinh{\left(a c \right)} & \text{for}\: b = 0 \\ \frac{\cosh{\left(a c + b c x \right)}}{b c} & \text{for}\: n = 0 \\ \frac{e^{n \cosh{\left(a c + b c x \right)}}}{b c n} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*cosh(c*(b*x+a)))*sinh(b*c*x+a*c),x)
```

```
[Out] Piecewise((0, Eq(c, 0) & (Eq(b, 0) | Eq(c, 0)) & (Eq(c, 0) | Eq(n, 0))), (x
*exp(n*cosh(a*c))*sinh(a*c), Eq(b, 0)), (cosh(a*c + b*c*x)/(b*c), Eq(n, 0))
, (exp(n*cosh(a*c + b*c*x))/(b*c*n), True))
```

3.970 $\int e^{n\cosh(a+bx)}\tanh(a+bx)\,dx$

Optimal. Leaf size=13

$$\frac{\text{Ei}(n\cosh(a+bx))}{b}$$

```
[Out] Ei(n*cosh(b*x+a))/b
```

Rubi [A] time = 0.02, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {4341, 2178}

$$\frac{\text{Ei}(n\cosh(a+bx))}{b}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*Cosh[a + b*x])*Tanh[a + b*x],x]
```

```
[Out] ExpIntegralEi[n*Cosh[a + b*x]]/b
```

Rule 2178

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - (c*f)/d))*ExpIntegralEi[(f*g*(c + d*x)*Log[F])/d])/d, x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 4341

```
Int[(u_)*Tanh[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Cosh[c*(a + b*x)], x]}, Dist[1/(b*c), Subst[Int[SubstFor[1/x, Cosh[c*(a
+ b*x)]/d, u, x], x], x, Cosh[c*(a + b*x)]/d], x] /; FunctionOfQ[Cosh[c*(a
+ b*x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x]
```

Rubi steps

$$\begin{aligned}\int e^{n\cosh(a+bx)}\tanh(a+bx)\,dx &= \frac{\text{Subst}\left(\int \frac{e^{nx}}{x}\,dx, x, \cosh(a+bx)\right)}{b}\\ &= \frac{\text{Ei}(n\cosh(a+bx))}{b}\end{aligned}$$

Mathematica [A] time = 0.06, size = 13, normalized size = 1.00

$$\frac{\text{Ei}(n\cosh(a+bx))}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*Cosh[a + b*x])*Tanh[a + b*x],x]
```

```
[Out] ExpIntegralEi[n*Cosh[a + b*x]]/b
```

fricas [A] time = 0.42, size = 13, normalized size = 1.00

$$\frac{\mathrm{Ei}\left(n\cosh\left(bx+a\right)\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*cosh(b*x+a))*tanh(b*x+a),x, algorithm="fricas")
```

```
[Out] Ei(n*cosh(b*x + a))/b
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int e^{\left(n\cosh\left(bx+a\right)\right)}\tanh\left(bx+a\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*cosh(b*x+a))*tanh(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(e^(n*cosh(b*x + a))*tanh(b*x + a), x)
```

maple [A] time = 0.07, size = 17, normalized size = 1.31

$$-\frac{\mathrm{Ei}\left(1,-n\cosh\left(bx+a\right)\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*cosh(b*x+a))*tanh(b*x+a),x)
```

```
[Out] -1/b*Ei(1,-n*cosh(b*x+a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int e^{\left(n\cosh\left(bx+a\right)\right)}\tanh\left(bx+a\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*cosh(b*x+a))*tanh(b*x+a),x, algorithm="maxima")
```

```
[Out] integrate(e^(n*cosh(b*x + a))*tanh(b*x + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.08

$$\int e^{n\cosh(a+b\,x)}\tanh\left(a+b\,x\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*cosh(a + b*x))*tanh(a + b*x),x)
```

```
[Out] int(exp(n*cosh(a + b*x))*tanh(a + b*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int e^{n\cosh(a+bx)}\tanh(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*cosh(b*x+a))*tanh(b*x+a),x)
```

```
[Out] Integral(exp(n*cosh(a + b*x))*tanh(a + b*x), x)
```

3.971 $\int e^{n\cosh(ac+bcx)}\tanh(c(a+bx))\,dx$

Optimal. Leaf size=18

$$\frac{\text{Ei}(n\cosh(c(a+bx)))}{bc}$$

```
[Out] Ei(n*cosh(c*(b*x+a)))/b/c
```

Rubi [A] time = 0.02, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {4341, 2178}

$$\frac{\text{Ei}(n\cosh(c(a+bx)))}{bc}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*Cosh[a*c + b*c*x])*Tanh[c*(a + b*x)],x]
```

```
[Out] ExpIntegralEi[n*Cosh[c*(a + b*x)]]/(b*c)
```

Rule 2178

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - (c*f)/d))*ExpIntegralEi[(f*g*(c + d*x)*Log[F])/d])/d, x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 4341

```
Int[(u_)*Tanh[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Cosh[c*(a + b*x)], x]}, Dist[1/(b*c), Subst[Int[SubstFor[1/x, Cosh[c*(a
+ b*x)]/d, u, x], x], x, Cosh[c*(a + b*x)]/d], x] /; FunctionOfQ[Cosh[c*(a
+ b*x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x]
```

Rubi steps

$$\begin{aligned}\int e^{n\cosh(ac+bcx)}\tanh(c(a+bx))\,dx &= \frac{\text{Subst}\left(\int \frac{e^{nx}}{x}\,dx, x, \cosh(c(a+bx))\right)}{bc}\\ &= \frac{\text{Ei}(n\cosh(c(a+bx)))}{bc}\end{aligned}$$

Mathematica [A] time = 0.18, size = 18, normalized size = 1.00

$$\frac{\text{Ei}(n\cosh(c(a+bx)))}{bc}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*Cosh[a*c + b*c*x])*Tanh[c*(a + b*x)],x]
```

```
[Out] ExpIntegralEi[n*Cosh[c*(a + b*x)]]/(b*c)
```

fricas [A] time = 0.51, size = 19, normalized size = 1.06

$$\frac{\operatorname{Ei}\left(n\cosh\left(bcx+ac\right)\right)}{bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*cosh(b*c*x+a*c))*tanh(c*(b*x+a)),x, algorithm="fricas")
```

```
[Out] Ei(n*cosh(b*c*x + a*c))/(b*c)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int e^{(n\cosh(bcx+ac))}\tanh\left((bx+a)c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*cosh(b*c*x+a*c))*tanh(c*(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(e^(n*cosh(b*c*x + a*c))*tanh((b*x + a)*c), x)
```

maple [A] time = 0.40, size = 31, normalized size = 1.72

$$\frac{\operatorname{Shi}\left(n\cosh\left(c\left(bx+a\right)\right)\right)+\mathrm{X}\left(n\cosh\left(c\left(bx+a\right)\right)\right)}{cb}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*cosh(b*c*x+a*c))*tanh(c*(b*x+a)),x)
```

```
[Out] 1/c/b*(Shi(n*cosh(c*(b*x+a)))+Chi(n*cosh(c*(b*x+a))))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int e^{(n\cosh(bcx+ac))}\tanh\left((bx+a)c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*cosh(b*c*x+a*c))*tanh(c*(b*x+a)),x, algorithm="maxima")
```

```
[Out] integrate(e^(n*cosh(b*c*x + a*c))*tanh((b*x + a)*c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.06

$$\int \tanh\left(c\,\left(a+b\,x\right)\right)\,{\mathrm{e}}^{n\,\cosh\left(a\,c+b\,c\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(c*(a + b*x))*exp(n*cosh(a*c + b*c*x)),x)
[Out] int(tanh(c*(a + b*x))*exp(n*cosh(a*c + b*c*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int e^{n\cosh\left(ac+bcx\right)}\tanh\left(ac+bcx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*cosh(b*c*x+a*c))*tanh(c*(b*x+a)),x)
[Out] Integral(exp(n*cosh(a*c + b*c*x))*tanh(a*c + b*c*x), x)
```

3.972 $\int e^{n\cosh(c(a+bx))}\tanh(ac+bcx)\,dx$

Optimal. Leaf size=19

$$\frac{\text{Ei}(n\cosh(ac+bxc))}{bc}$$

```
[Out] Ei(n*cosh(b*c*x+a*c))/b/c
```

Rubi [A] time = 0.02, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {4341, 2178}

$$\frac{\text{Ei}(n\cosh(ac+bxc))}{bc}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*Cosh[c*(a + b*x)])*Tanh[a*c + b*c*x],x]
```

```
[Out] ExpIntegralEi[n*Cosh[a*c + b*c*x]]/(b*c)
```

Rule 2178

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - (c*f)/d))*ExpIntegralEi[(f*g*(c + d*x)*Log[F])/d])/d, x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 4341

```
Int[(u_)*Tanh[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Cosh[c*(a + b*x)], x]}, Dist[1/(b*c), Subst[Int[SubstFor[1/x, Cosh[c*(a
+ b*x)]/d, u, x], x], x, Cosh[c*(a + b*x)]/d], x] /; FunctionOfQ[Cosh[c*(a
+ b*x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x]
```

Rubi steps

$$\begin{aligned}\int e^{n\cosh(c(a+bx))}\tanh(ac+bcx)\,dx &= \frac{\text{Subst}\left(\int \frac{e^{nx}}{x}\,dx, x, \cosh(ac+bcx)\right)}{bc}\\ &= \frac{\text{Ei}(n\cosh(ac+bcx))}{bc}\end{aligned}$$

Mathematica [A] time = 0.06, size = 18, normalized size = 0.95

$$\frac{\text{Ei}(n\cosh(c(a+bx)))}{bc}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*Cosh[c*(a + b*x)])*Tanh[a*c + b*c*x],x]
```

```
[Out] ExpIntegralEi[n*Cosh[c*(a + b*x)]]/(b*c)
```

fricas [A] time = 0.43, size = 19, normalized size = 1.00

$$\frac{\operatorname{Ei}\left(n\cosh\left(bcx+ac\right)\right)}{bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*cosh(c*(b*x+a)))*tanh(b*c*x+a*c),x, algorithm="fricas")
```

```
[Out] Ei(n*cosh(b*c*x + a*c))/(b*c)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int e^{\left(n\cosh\left((bx+a)c\right)\right)}\tanh\left(bcx+ac\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*cosh(c*(b*x+a)))*tanh(b*c*x+a*c),x, algorithm="giac")
```

```
[Out] integrate(e^(n*cosh((b*x + a)*c))*tanh(b*c*x + a*c), x)
```

maple [A] time = 0.34, size = 31, normalized size = 1.63

$$\frac{\operatorname{Shi}\left(n\cosh\left(c\left(bx+a\right)\right)\right)+\operatorname{X}\left(n\cosh\left(c\left(bx+a\right)\right)\right)}{cb}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*cosh(c*(b*x+a)))*tanh(b*c*x+a*c),x)
```

```
[Out] 1/c/b*(Shi(n*cosh(c*(b*x+a)))+Chi(n*cosh(c*(b*x+a))))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int e^{\left(n\cosh\left((bx+a)c\right)\right)}\tanh\left(bcx+ac\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*cosh(c*(b*x+a)))*tanh(b*c*x+a*c),x, algorithm="maxima")
```

```
[Out] integrate(e^(n*cosh((b*x + a)*c))*tanh(b*c*x + a*c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.05

$$\int \mathrm{e}^{n\cosh(c\,(a+b\,x))}\,\tanh\left(a\,c+b\,c\,x\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*cosh(c*(a + b*x)))*tanh(a*c + b*c*x),x)
[Out] int(exp(n*cosh(c*(a + b*x)))*tanh(a*c + b*c*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int e^{n\cosh\left(ac+bcx\right)}\tanh\left(ac+bcx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*cosh(c*(b*x+a)))*tanh(b*c*x+a*c),x)
[Out] Integral(exp(n*cosh(a*c + b*c*x))*tanh(a*c + b*c*x), x)
```

3.973 $\int e^{n\sinh(a+bx)}\cosh(a+bx)\,dx$

Optimal. Leaf size=17

$$\frac{e^{n\sinh(a+bx)}}{bn}$$

```
[Out] exp(n*sinh(b*x+a))/b/n
```

Rubi [A] time = 0.01, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {4336, 2194}

$$\frac{e^{n\sinh(a+bx)}}{bn}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*Sinh[a + b*x])*Cosh[a + b*x],x]
```

```
[Out] E^(n*Sinh[a + b*x])/(b*n)
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 4336

```
Int[Cosh[(c_.)*((a_.) + (b_.)*(x_))]*(u_), x_Symbol] :> With[{d = FreeFacto
rs[Sinh[c*(a + b*x)], x]}, Dist[d/(b*c), Subst[Int[SubstFor[1, Sinh[c*(a +
b*x)]/d, u, x], x], x, Sinh[c*(a + b*x)]/d], x] /; FunctionOfQ[Sinh[c*(a +
b*x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x]
```

Rubi steps

$$\begin{aligned}\int e^{n\sinh(a+bx)}\cosh(a+bx)\,dx &= \frac{\text{Subst}\left(\int e^{nx}\,dx, x, \sinh(a+bx)\right)}{b}\\ &= \frac{e^{n\sinh(a+bx)}}{bn}\end{aligned}$$

Mathematica [A] time = 0.02, size = 17, normalized size = 1.00

$$\frac{e^{n\sinh(a+bx)}}{bn}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*Sinh[a + b*x])*Cosh[a + b*x],x]
[Out] E^(n*Sinh[a + b*x])/(b*n)
```

fricas [A] time = 0.43, size = 26, normalized size = 1.53

$$\frac{\cosh\left(n\sinh\left(bx+a\right)\right)+\sinh\left(n\sinh\left(bx+a\right)\right)}{bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*sinh(b*x+a))*cosh(b*x+a),x, algorithm="fricas")
[Out] (cosh(n*sinh(b*x + a)) + sinh(n*sinh(b*x + a)))/(b*n)
```

giac [A] time = 0.15, size = 30, normalized size = 1.76

$$\frac{e^{\left(\frac{1}{2}\,ne^{\left(bx+a\right)}-\frac{1}{2}\,ne^{\left(-bx-a\right)}\right)}}{bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*sinh(b*x+a))*cosh(b*x+a),x, algorithm="giac")
[Out] e^(1/2*n*e^(b*x + a) - 1/2*n*e^(-b*x - a))/(b*n)
```

maple [A] time = 0.06, size = 17, normalized size = 1.00

$$\frac{\mathrm{e}^{n\sinh\left(bx+a\right)}}{bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*sinh(b*x+a))*cosh(b*x+a),x)
[Out] exp(n*sinh(b*x+a))/b/n
```

maxima [A] time = 0.30, size = 16, normalized size = 0.94

$$\frac{e^{\left(n\sinh\left(bx+a\right)\right)}}{bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*sinh(b*x+a))*cosh(b*x+a),x, algorithm="maxima")
[Out] e^(n*sinh(b*x + a))/(b*n)
```

mupad [B] time = 0.07, size = 16, normalized size = 0.94

$$\frac{e^{n\sinh(a+b\,x)}}{b\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a + b*x)*exp(n*sinh(a + b*x)),x)
```

```
[Out] exp(n*sinh(a + b*x))/(b*n)
```

sympy [A] time = 0.42, size = 36, normalized size = 2.12

$$\begin{cases} x\cosh(a) & \text{for } b = 0 \wedge n = 0 \\ xe^{n\sinh(a)}\cosh(a) & \text{for } b = 0 \\ \frac{\sinh(a+bx)}{b} & \text{for } n = 0 \\ \frac{e^{n\sinh(a+bx)}}{bn} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*sinh(b*x+a))*cosh(b*x+a),x)
```

```
[Out] Piecewise((x*cosh(a), Eq(b, 0) & Eq(n, 0)), (x*exp(n*sinh(a))*cosh(a), Eq(b
, 0)), (sinh(a + b*x)/b, Eq(n, 0)), (exp(n*sinh(a + b*x))/(b*n), True))
```

3.974 $\int e^{n\sinh(ac+bcx)}\cosh(c(a+bx))\,dx$

Optimal. Leaf size=22

$$\frac{e^{n\sinh(c(a+bx))}}{bcn}$$

[Out] exp(n*sinh(c*(b*x+a)))/b/c/n

Rubi [A] time = 0.01, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {4336, 2194}

$$\frac{e^{n\sinh(c(a+bx))}}{bcn}$$

Antiderivative was successfully verified.

[In] Int[E^(n*Sinh[a*c + b*c*x])*Cosh[c*(a + b*x)],x]

[Out] E^(n*Sinh[c*(a + b*x)])/(b*c*n)

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 4336

```
Int[Cosh[(c_.)*((a_.) + (b_.)*(x_))]*(u_), x_Symbol] :> With[{d = FreeFacto
rs[Sinh[c*(a + b*x)], x]}, Dist[d/(b*c), Subst[Int[SubstFor[1, Sinh[c*(a +
b*x)]/d, u, x], x], x, Sinh[c*(a + b*x)]/d], x] /; FunctionOfQ[Sinh[c*(a +
b*x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x]
```

Rubi steps

$$\begin{aligned}\int e^{n\sinh(ac+bcx)}\cosh(c(a+bx))\,dx &= \frac{\text{Subst}\left(\int e^{nx}\,dx, x, \sinh(c(a+bx))\right)}{bc}\\ &= \frac{e^{n\sinh(c(a+bx))}}{bcn}\end{aligned}$$

Mathematica [A] time = 0.13, size = 23, normalized size = 1.05

$$\frac{e^{n\sinh(ac+bcx)}}{bcn}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*Sinh[a*c + b*c*x])*Cosh[c*(a + b*x)],x]
[Out] E^(n*Sinh[a*c + b*c*x])/(b*c*n)
```

fricas [A] time = 0.48, size = 35, normalized size = 1.59

$$\frac{\cosh\left(n\sinh\left(bcx+ac\right)\right)+\sinh\left(n\sinh\left(bcx+ac\right)\right)}{bcn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*sinh(b*c*x+a*c))*cosh(c*(b*x+a)),x, algorithm="fricas")
[Out] (cosh(n*sinh(b*c*x + a*c)) + sinh(n*sinh(b*c*x + a*c)))/(b*c*n)
```

giac [A] time = 0.16, size = 38, normalized size = 1.73

$$\frac{e^{\left(\frac{1}{2}\,ne^{\left(bcx+ac\right)}-\frac{1}{2}\,ne^{\left(-bcx-ac\right)}\right)}}{bcn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*sinh(b*c*x+a*c))*cosh(c*(b*x+a)),x, algorithm="giac")
[Out] e^(1/2*n*e^(b*c*x + a*c) - 1/2*n*e^(-b*c*x - a*c))/(b*c*n)
```

maple [A] time = 2.84, size = 39, normalized size = 1.77

$$\frac{\frac{\sinh(n\sinh(c(bx+a)))}{n}+\frac{\cosh(n\sinh(c(bx+a)))}{n}}{cb}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*sinh(b*c*x+a*c))*cosh(c*(b*x+a)),x)
[Out] 1/c/b*(1/n*sinh(n*sinh(c*(b*x+a)))+cosh(n*sinh(c*(b*x+a)))/n)
```

maxima [A] time = 0.30, size = 22, normalized size = 1.00

$$\frac{e^{\left(n\sinh\left(bcx+ac\right)\right)}}{bcn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*sinh(b*c*x+a*c))*cosh(c*(b*x+a)),x, algorithm="maxima")
```

```
[Out] e^(n*sinh(b*c*x + a*c))/(b*c*n)
```

mupad [B] time = 1.71, size = 38, normalized size = 1.73

$$\frac{\mathrm{e}^{\frac{n\,\mathrm{e}^{b\,c\,x}\,\mathrm{e}^{a\,c}}{2}}\,\mathrm{e}^{-\frac{n\,\mathrm{e}^{-b\,c\,x}\,\mathrm{e}^{-a\,c}}{2}}}{b\,c\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(c*(a + b*x))*exp(n*sinh(a*c + b*c*x)),x)
[Out] (exp((n*exp(b*c*x)*exp(a*c))/2)*exp(-(n*exp(-b*c*x)*exp(-a*c))/2))/(b*c*n)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int e^{n\sinh(ac+bcx)}\cosh(ac+bcx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*sinh(b*c*x+a*c))*cosh(c*(b*x+a)),x)
[Out] Integral(exp(n*sinh(a*c + b*c*x))*cosh(a*c + b*c*x), x)
```

3.975 $\int e^{n\sinh(c(a+bx))}\cosh(ac+bcx)\,dx$

Optimal. Leaf size=23

$$\frac{e^{n\sinh(ac+bcx)}}{bcn}$$

```
[Out] exp(n*sinh(b*c*x+a*c))/b/c/n
```

Rubi [A] time = 0.01, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {4336, 2194}

$$\frac{e^{n\sinh(ac+bcx)}}{bcn}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*Sinh[c*(a + b*x)])*Cosh[a*c + b*c*x],x]
```

```
[Out] E^(n*Sinh[a*c + b*c*x])/(b*c*n)
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 4336

```
Int[Cosh[(c_.)*((a_.) + (b_.)*(x_))]*(u_), x_Symbol] :> With[{d = FreeFacto
rs[Sinh[c*(a + b*x)], x]}, Dist[d/(b*c), Subst[Int[SubstFor[1, Sinh[c*(a +
b*x)]/d, u, x], x], x, Sinh[c*(a + b*x)]/d], x] /; FunctionOfQ[Sinh[c*(a +
b*x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x]
```

Rubi steps

$$\begin{aligned}\int e^{n\sinh(c(a+bx))}\cosh(ac+bcx)\,dx &= \frac{\text{Subst}\left(\int e^{nx}\,dx, x, \sinh(ac+bcx)\right)}{bc}\\ &= \frac{e^{n\sinh(ac+bcx)}}{bcn}\end{aligned}$$

Mathematica [A] time = 0.04, size = 23, normalized size = 1.00

$$\frac{e^{n\sinh(ac+bcx)}}{bcn}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*Sinh[c*(a + b*x)])*Cosh[a*c + b*c*x],x]
[Out] E^(n*Sinh[a*c + b*c*x])/(b*c*n)
```

fricas [A] time = 0.42, size = 35, normalized size = 1.52

$$\frac{\cosh\left(n\sinh\left(bcx+ac\right)\right)+\sinh\left(n\sinh\left(bcx+ac\right)\right)}{bcn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*sinh(c*(b*x+a)))*cosh(b*c*x+a*c),x, algorithm="fricas")
[Out] (cosh(n*sinh(b*c*x + a*c)) + sinh(n*sinh(b*c*x + a*c)))/(b*c*n)
```

giac [A] time = 0.14, size = 38, normalized size = 1.65

$$\frac{e^{\left(\frac{1}{2}ne^{(bcx+ac)}-\frac{1}{2}ne^{(-bcx-ac)}\right)}}{bcn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*sinh(c*(b*x+a)))*cosh(b*c*x+a*c),x, algorithm="giac")
[Out] e^(1/2*n*e^(b*c*x + a*c) - 1/2*n*e^(-b*c*x - a*c))/(b*c*n)
```

maple [A] time = 2.68, size = 39, normalized size = 1.70

$$\frac{\frac{\sinh(n\sinh(c(bx+a)))}{n}+\frac{\cosh(n\sinh(c(bx+a)))}{n}}{cb}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*sinh(c*(b*x+a)))*cosh(b*c*x+a*c),x)
[Out] 1/c/b*(1/n*sinh(n*sinh(c*(b*x+a)))+cosh(n*sinh(c*(b*x+a)))/n)
```

maxima [A] time = 0.30, size = 22, normalized size = 0.96

$$\frac{e^{(n\sinh(bcx+ac))}}{bcn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*sinh(c*(b*x+a)))*cosh(b*c*x+a*c),x, algorithm="maxima")
```

```
[Out] e^(n*sinh(b*c*x + a*c))/(b*c*n)
```

mupad [B] time = 1.69, size = 38, normalized size = 1.65

$$\frac{e^{\frac{n\,e^{b\,c\,x}\,e^{a\,c}}{2}}\,e^{-\frac{n\,e^{-b\,c\,x}\,e^{-a\,c}}{2}}}{b\,c\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*sinh(c*(a + b*x)))*cosh(a*c + b*c*x),x)
[Out] (exp((n*exp(b*c*x)*exp(a*c))/2)*exp(-(n*exp(-b*c*x)*exp(-a*c))/2))/(b*c*n)
```

sympy [A] time = 2.27, size = 51, normalized size = 2.22

$$\begin{cases} x & \text{for}\: c = 0 \wedge \left(b = 0 \vee c = 0\right) \wedge \left(c = 0 \vee n = 0\right) \\ x e^{n \sinh{\left(a c \right)}} \cosh{\left(a c \right)} & \text{for}\: b = 0 \\ \frac{\sinh{\left(a c + b c x \right)}}{b c} & \text{for}\: n = 0 \\ \frac{e^{n \sinh{\left(a c + b c x \right)}}}{b c n} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*sinh(c*(b*x+a)))*cosh(b*c*x+a*c),x)
[Out] Piecewise((x, Eq(c, 0) & (Eq(b, 0) | Eq(c, 0)) & (Eq(c, 0) | Eq(n, 0))), (x
*exp(n*sinh(a*c))*cosh(a*c), Eq(b, 0)), (sinh(a*c + b*c*x)/(b*c), Eq(n, 0))
, (exp(n*sinh(a*c + b*c*x))/(b*c*n), True))
```

3.976 $\int e^{n\sinh(a+bx)}\coth(a+bx)\,dx$

Optimal. Leaf size=13

$$\frac{\mathrm{Ei}(n\sinh(a+bx))}{b}$$

```
[Out] Ei(n*sinh(b*x+a))/b
```

Rubi [A] time = 0.02, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {4340, 2178}

$$\frac{\mathrm{Ei}(n\sinh(a+bx))}{b}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*Sinh[a + b*x])*Coth[a + b*x],x]
```

```
[Out] ExpIntegralEi[n*Sinh[a + b*x]]/b
```

Rule 2178

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - (c*f)/d))*ExpIntegralEi[(f*g*(c + d*x)*Log[F])/d])/d, x] /; F
reeQ[{F, c, d, e, f, g}, x] && !$UseGamma === True
```

Rule 4340

```
Int[Coth[(c_.)*((a_.) + (b_.)*(x_))]*(u_), x_Symbol] :> With[{d = FreeFacto
rs[Sinh[c*(a + b*x)], x]}, Dist[1/(b*c), Subst[Int[SubstFor[1/x, Sinh[c*(a
+ b*x)]/d, u, x], x], x, Sinh[c*(a + b*x)]/d], x] /; FunctionOfQ[Sinh[c*(a
+ b*x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x]
```

Rubi steps

$$\begin{aligned}\int e^{n\sinh(a+bx)}\coth(a+bx)\,dx&=\frac{\mathrm{Subst}\left(\int\frac{e^{nx}}{x}\,dx,x,\sinh(a+bx)\right)}{b}\\&=\frac{\mathrm{Ei}(n\sinh(a+bx))}{b}\end{aligned}$$

Mathematica [A] time = 0.04, size = 13, normalized size = 1.00

$$\frac{\mathrm{Ei}(n\sinh(a+bx))}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*Sinh[a + b*x])*Coth[a + b*x],x]
```

```
[Out] ExpIntegralEi[n*Sinh[a + b*x]]/b
```

fricas [A] time = 0.45, size = 13, normalized size = 1.00

$$\frac{\mathrm{Ei}\left(n\sinh\left(bx+a\right)\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*sinh(b*x+a))*coth(b*x+a),x, algorithm="fricas")
```

```
[Out] Ei(n*sinh(b*x + a))/b
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \coth\left(bx+a\right)e^{\left(n\sinh\left(bx+a\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*sinh(b*x+a))*coth(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(coth(b*x + a)*e^(n*sinh(b*x + a)), x)
```

maple [A] time = 0.10, size = 17, normalized size = 1.31

$$-\frac{\mathrm{Ei}\left(1,-n\sinh\left(bx+a\right)\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*sinh(b*x+a))*coth(b*x+a),x)
```

```
[Out] -1/b*Ei(1,-n*sinh(b*x+a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \coth\left(bx+a\right)e^{\left(n\sinh\left(bx+a\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*sinh(b*x+a))*coth(b*x+a),x, algorithm="maxima")
```

```
[Out] integrate(coth(b*x + a)*e^(n*sinh(b*x + a)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.08

$$\int \coth(a+bx)\,\mathrm{e}^{n\sinh(a+bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(a + b*x)*exp(n*sinh(a + b*x)),x)
[Out] int(coth(a + b*x)*exp(n*sinh(a + b*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int e^{n\sinh(a+bx)}\coth(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*sinh(b*x+a))*coth(b*x+a),x)
[Out] Integral(exp(n*sinh(a + b*x))*coth(a + b*x), x)
```

3.977 $\int e^{n \sinh(ac+bcx)} \coth(c(a+bx))\,dx$

Optimal. Leaf size=18

$$\frac{\text{Ei}(n\sinh(c(a+bx)))}{bc}$$

```
[Out] Ei(n*sinh(c*(b*x+a)))/b/c
```

Rubi [A] time = 0.02, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {4340, 2178}

$$\frac{\text{Ei}(n\sinh(c(a+bx)))}{bc}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*Sinh[a*c + b*c*x])*Coth[c*(a + b*x)],x]
```

```
[Out] ExpIntegralEi[n*Sinh[c*(a + b*x)]]/(b*c)
```

Rule 2178

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - (c*f)/d))*ExpIntegralEi[(f*g*(c + d*x)*Log[F])/d])/d, x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 4340

```
Int[Coth[(c_.)*((a_.) + (b_.)*(x_))]*(u_), x_Symbol] :> With[{d = FreeFacto
rs[Sinh[c*(a + b*x)], x]}, Dist[1/(b*c), Subst[Int[SubstFor[1/x, Sinh[c*(a
+ b*x)]/d, u, x], x], x, Sinh[c*(a + b*x)]/d], x] /; FunctionOfQ[Sinh[c*(a
+ b*x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x]
```

Rubi steps

$$\begin{aligned}\int e^{n\sinh(ac+bcx)}\coth(c(a+bx))\,dx &= \frac{\text{Subst}\left(\int \frac{e^{nx}}{x}\,dx, x, \sinh(c(a+bx))\right)}{bc}\\ &= \frac{\text{Ei}(n\sinh(c(a+bx)))}{bc}\end{aligned}$$

Mathematica [A] time = 0.06, size = 18, normalized size = 1.00

$$\frac{\text{Ei}(n\sinh(c(a+bx)))}{bc}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*Sinh[a*c + b*c*x])*Coth[c*(a + b*x)],x]
```

```
[Out] ExpIntegralEi[n*Sinh[c*(a + b*x)]]/(b*c)
```

fricas [A] time = 0.52, size = 19, normalized size = 1.06

$$\frac{\operatorname{Ei}\left(n \sinh\left(bcx + ac\right)\right)}{bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*sinh(b*c*x+a*c))*coth(c*(b*x+a)),x, algorithm="fricas")
```

```
[Out] Ei(n*sinh(b*c*x + a*c))/(b*c)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \coth\left((bx + a)c\right) e^{\left(n \sinh\left(bcx + ac\right)\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*sinh(b*c*x+a*c))*coth(c*(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(coth((b*x + a)*c)*e^(n*sinh(b*c*x + a*c)), x)
```

maple [A] time = 0.40, size = 31, normalized size = 1.72

$$\frac{\operatorname{Shi}\left(n \sinh\left(c\left(bx + a\right)\right)\right) + \operatorname{X}\left(n \sinh\left(c\left(bx + a\right)\right)\right)}{cb}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*sinh(b*c*x+a*c))*coth(c*(b*x+a)),x)
```

```
[Out] 1/c/b*(Shi(n*sinh(c*(b*x+a)))+Chi(n*sinh(c*(b*x+a))))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \coth\left((bx + a)c\right) e^{\left(n \sinh\left(bcx + ac\right)\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*sinh(b*c*x+a*c))*coth(c*(b*x+a)),x, algorithm="maxima")
```

```
[Out] integrate(coth((b*x + a)*c)*e^(n*sinh(b*c*x + a*c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.06

$$\int \coth\left(c\ \left(a+b\,x\right)\right)\,{\mathrm{e}}^{n\,\mathrm{sinh}\left(a\,c+b\,c\,x\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(c*(a + b*x))*exp(n*sinh(a*c + b*c*x)),x)
[Out] int(coth(c*(a + b*x))*exp(n*sinh(a*c + b*c*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int e^{n \sinh{\left(ac+bcx \right)}} \coth{\left(ac+bcx \right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*sinh(b*c*x+a*c))*coth(c*(b*x+a)),x)
[Out] Integral(exp(n*sinh(a*c + b*c*x))*coth(a*c + b*c*x), x)
```

3.978 $\int e^{n\sinh(c(a+bx))}\coth(ac+bcx)\,dx$

Optimal. Leaf size=19

$$\frac{\text{Ei}(n\sinh(ac+bxc))}{bc}$$

```
[Out] Ei(n*sinh(b*c*x+a*c))/b/c
```

Rubi [A] time = 0.02, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {4340, 2178}

$$\frac{\text{Ei}(n\sinh(ac+bxc))}{bc}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*Sinh[c*(a + b*x)])*Coth[a*c + b*c*x],x]
```

```
[Out] ExpIntegralEi[n*Sinh[a*c + b*c*x]]/(b*c)
```

Rule 2178

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - (c*f)/d))*ExpIntegralEi[(f*g*(c + d*x)*Log[F])/d])/d, x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 4340

```
Int[Coth[(c_.)*((a_.) + (b_.)*(x_))]*(u_), x_Symbol] :> With[{d = FreeFacto
rs[Sinh[c*(a + b*x)], x]}, Dist[1/(b*c), Subst[Int[SubstFor[1/x, Sinh[c*(a
+ b*x)]/d, u, x], x], x, Sinh[c*(a + b*x)]/d], x] /; FunctionOfQ[Sinh[c*(a
+ b*x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x]
```

Rubi steps

$$\begin{aligned}\int e^{n\sinh(c(a+bx))}\coth(ac+bcx)\,dx &= \frac{\text{Subst}\left(\int \frac{e^{nx}}{x}\,dx, x, \sinh(ac+bcx)\right)}{bc}\\ &= \frac{\text{Ei}(n\sinh(ac+bcx))}{bc}\end{aligned}$$

Mathematica [A] time = 0.06, size = 18, normalized size = 0.95

$$\frac{\text{Ei}(n\sinh(c(a+bx)))}{bc}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*Sinh[c*(a + b*x)])*Coth[a*c + b*c*x],x]
```

```
[Out] ExpIntegralEi[n*Sinh[c*(a + b*x)]]/(b*c)
```

fricas [A] time = 0.51, size = 19, normalized size = 1.00

$$\frac{\operatorname{Ei}\left(n\sinh\left(bcx+ac\right)\right)}{bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*sinh(c*(b*x+a)))*coth(b*c*x+a*c),x, algorithm="fricas")
```

```
[Out] Ei(n*sinh(b*c*x + a*c))/(b*c)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \coth\left(bcx+ac\right)e^{\left(n\sinh\left(\left(bx+a\right)c\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*sinh(c*(b*x+a)))*coth(b*c*x+a*c),x, algorithm="giac")
```

```
[Out] integrate(coth(b*c*x + a*c)*e^(n*sinh((b*x + a)*c)), x)
```

maple [A] time = 0.40, size = 31, normalized size = 1.63

$$\frac{\operatorname{Shi}\left(n\sinh\left(c\left(bx+a\right)\right)\right)+\operatorname{X}\left(n\sinh\left(c\left(bx+a\right)\right)\right)}{cb}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*sinh(c*(b*x+a)))*coth(b*c*x+a*c),x)
```

```
[Out] 1/c/b*(Shi(n*sinh(c*(b*x+a)))+Chi(n*sinh(c*(b*x+a))))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \coth\left(bcx+ac\right)e^{\left(n\sinh\left(\left(bx+a\right)c\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*sinh(c*(b*x+a)))*coth(b*c*x+a*c),x, algorithm="maxima")
```

```
[Out] integrate(coth(b*c*x + a*c)*e^(n*sinh((b*x + a)*c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.05

$$\int \mathrm{e}^{n\sinh(c\,(a+b\,x))}\coth\left(a\,c+b\,c\,x\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*sinh(c*(a + b*x)))*coth(a*c + b*c*x),x)
[Out] int(exp(n*sinh(c*(a + b*x)))*coth(a*c + b*c*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int e^{n\sinh(ac+bcx)}\coth\left(ac+bcx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*sinh(c*(b*x+a)))*coth(b*c*x+a*c),x)
[Out] Integral(exp(n*sinh(a*c + b*c*x))*coth(a*c + b*c*x), x)
```

3.979 $\int \frac{\text{sech}^2(x)}{a+b\tanh(x)}\,dx$

Optimal. Leaf size=11

$$\frac{\log(a+b\tanh(x))}{b}$$

[Out] ln(a+b*tanh(x))/b

Rubi [A] time = 0.04, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3506, 31}

$$\frac{\log(a+b\tanh(x))}{b}$$

Antiderivative was successfully verified.

[In] Int[Sech[x]^2/(a + b*Tanh[x]),x]

[Out] Log[a + b*Tanh[x]]/b

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 3506

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(b*f), Subst[Int[(a + x)^n*(1 + x^2/b^2)^(m/2 - 1),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && NeQ[a^2 + b^2, 0
] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\text{sech}^2(x)}{a+b\tanh(x)}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{a+x}\,dx, x, b\tanh(x)\right)}{b}\\ &= \frac{\log(a+b\tanh(x))}{b}\end{aligned}$$

Mathematica [A] time = 0.06, size = 20, normalized size = 1.82

$$\frac{\log(a\cosh(x)+b\sinh(x))-\log(\cosh(x))}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[x]^2/(a + b*Tanh[x]),x]
```

```
[Out] (-Log[Cosh[x]] + Log[a*Cosh[x] + b*Sinh[x]])/b
```

fricas [B] time = 0.43, size = 42, normalized size = 3.82

$$\frac{\log\left(\frac{2\,(a\cosh(x)+b\sinh(x))}{\cosh(x)-\sinh(x)}\right)-\log\left(\frac{2\,\cosh(x)}{\cosh(x)-\sinh(x)}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^2/(a+b*tanh(x)),x, algorithm="fricas")
```

```
[Out] (log(2*(a*cosh(x) + b*sinh(x))/(cosh(x) - sinh(x))) - log(2*cosh(x)/(cosh(x
) - sinh(x))))/b
```

giac [B] time = 0.12, size = 45, normalized size = 4.09

$$\frac{(a+b)\log\left(\left|ae^{(2\,x)}+be^{(2\,x)}+a-b\right|\right)}{ab+b^2}-\frac{\log\left(e^{(2\,x)}+1\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^2/(a+b*tanh(x)),x, algorithm="giac")
```

```
[Out] (a + b)*log(abs(a*e^(2*x) + b*e^(2*x) + a - b))/(a*b + b^2) - log(e^(2*x) +
 1)/b
```

maple [A] time = 0.16, size = 12, normalized size = 1.09

$$\frac{\ln\left(a+b\tanh(x)\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(x)^2/(a+b*tanh(x)),x)
```

```
[Out] ln(a+b*tanh(x))/b
```

maxima [A] time = 0.30, size = 11, normalized size = 1.00

$$\frac{\log\left(b\tanh(x)+a\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^2/(a+b*tanh(x)),x, algorithm="maxima")
```

```
[Out] log(b*tanh(x) + a)/b
```

mupad [B] time = 1.77, size = 50, normalized size = 4.55

$$-\frac{2\,\mathrm{atan}\left(\frac{a\,\sqrt{-b^2}+a\,\mathrm{e}^{2\,x}\,\sqrt{-b^2}+b\,\mathrm{e}^{2\,x}\,\sqrt{-b^2}}{b^2}\right)}{\sqrt{-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(x)^2*(a + b*tanh(x))),x)
```

```
[Out] -(2*atan((a*(-b^2)^(1/2) + a*exp(2*x)*(-b^2)^(1/2) + b*exp(2*x)*(-b^2)^(1/2
))/b^2))/(-b^2)^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\mathrm{sech}^2\left(x\right)}{a+b\tanh\left(x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)**2/(a+b*tanh(x)),x)
```

```
[Out] Integral(sech(x)**2/(a + b*tanh(x)), x)
```

3.980 $\int \frac{\operatorname{sech}^2(x)}{1+\tanh^2(x)}\,dx$

Optimal. Leaf size=3

$$\tan^{-1}(\tanh(x))$$

```
[Out] arctan(tanh(x))
```

Rubi [A] time = 0.03, antiderivative size = 3, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3675, 203}

$$\tan^{-1}(\tanh(x))$$

Antiderivative was successfully verified.

```
[In] Int[Sech[x]^2/(1 + Tanh[x]^2),x]
```

```
[Out] ArcTan[Tanh[x]]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3675

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/(c^(m - 1)*f), Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^p
, x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && In
tegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
|| EqQ[n^2, 16])
```

Rubi steps

$$\begin{aligned}\int \frac{\operatorname{sech}^2(x)}{1+\tanh^2(x)}\,dx &= \text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \tanh(x)\right)\\ &= \tan^{-1}(\tanh(x))\end{aligned}$$

Mathematica [A] time = 0.00, size = 3, normalized size = 1.00

$$\tan^{-1}(\tanh(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[x]^2/(1 + Tanh[x]^2),x]
```

```
[Out] ArcTan[Tanh[x]]
```

fricas [B] time = 0.44, size = 19, normalized size = 6.33

$$-\arctan\left(-\frac{\cosh(x)+\sinh(x)}{\cosh(x)-\sinh(x)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^2/(1+tanh(x)^2),x, algorithm="fricas")
```

```
[Out] -arctan(-(cosh(x) + sinh(x))/(cosh(x) - sinh(x)))
```

giac [A] time = 0.12, size = 5, normalized size = 1.67

$$\arctan\left(e^{(2x)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^2/(1+tanh(x)^2),x, algorithm="giac")
```

```
[Out] arctan(e^(2*x))
```

maple [B] time = 0.23, size = 116, normalized size = 38.67

$$\frac{2\sqrt{2}\,\arctan\left(\frac{2\tanh\left(\frac{x}{2}\right)}{-2+2\sqrt{2}}\right)}{-2+2\sqrt{2}}-\frac{2\arctan\left(\frac{2\tanh\left(\frac{x}{2}\right)}{-2+2\sqrt{2}}\right)}{-2+2\sqrt{2}}-\frac{2\sqrt{2}\,\arctan\left(\frac{2\tanh\left(\frac{x}{2}\right)}{2+2\sqrt{2}}\right)}{2+2\sqrt{2}}-\frac{2\arctan\left(\frac{2\tanh\left(\frac{x}{2}\right)}{2+2\sqrt{2}}\right)}{2+2\sqrt{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(x)^2/(1+tanh(x)^2),x)
```

```
[Out] 2*2^(1/2)/(-2+2*2^(1/2))*arctan(2*tanh(1/2*x)/(-2+2*2^(1/2)))-2/(-2+2*2^(1/
2))*arctan(2*tanh(1/2*x)/(-2+2*2^(1/2)))-2*2^(1/2)/(2+2*2^(1/2))*arctan(2*t
anh(1/2*x)/(2+2*2^(1/2)))-2/(2+2*2^(1/2))*arctan(2*tanh(1/2*x)/(2+2*2^(1/2)
))
```

maxima [B] time = 0.40, size = 35, normalized size = 11.67

$$\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\,e^{(-x)}\right)\right)-\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\,e^{(-x)}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^2/(1+tanh(x)^2),x, algorithm="maxima")
```

```
[Out] arctan(1/2*sqrt(2)*(sqrt(2) + 2*e^(-x))) - arctan(-1/2*sqrt(2)*(sqrt(2) - 2
*e^(-x)))
```

mupad [B] time = 0.07, size = 5, normalized size = 1.67

$$\mathrm{atan}\left(\mathrm{e}^{2\,x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(x)^2*(tanh(x)^2 + 1)),x)
```

```
[Out] atan(exp(2*x))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}^{2}(x)}{\tanh^{2}(x)+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)**2/(1+tanh(x)**2),x)
```

```
[Out] Integral(sech(x)**2/(tanh(x)**2 + 1), x)
```

3.981 $\int \frac{\operatorname{sech}^2(x)}{9+\tanh^2(x)}\,dx$

Optimal. Leaf size=11

$$\frac{1}{3}\tan^{-1}\left(\frac{\tanh(x)}{3}\right)$$

```
[Out] 1/3*arctan(1/3*tanh(x))
```

Rubi [A] time = 0.03, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3675, 203}

$$\frac{1}{3}\tan^{-1}\left(\frac{\tanh(x)}{3}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sech[x]^2/(9 + Tanh[x]^2),x]
```

```
[Out] ArcTan[Tanh[x]/3]/3
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3675

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/(c^(m - 1)*f), Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^p
, x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && In
tegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
|| EqQ[n^2, 16])
```

Rubi steps

$$\begin{aligned}\int \frac{\operatorname{sech}^2(x)}{9+\tanh^2(x)}\,dx &= \text{Subst}\left(\int \frac{1}{9+x^2}\,dx, x, \tanh(x)\right)\\ &= \frac{1}{3}\tan^{-1}\left(\frac{\tanh(x)}{3}\right)\end{aligned}$$

Mathematica [F] time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}^2(x)}{9+\tanh^2(x)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sech[x]^2/(9 + Tanh[x]^2),x]
```

```
[Out] Integrate[Sech[x]^2/(9 + Tanh[x]^2), x]
```

fricas [B] time = 0.43, size = 21, normalized size = 1.91

$$-\frac{1}{3}\arctan\left(-\frac{9\cosh(x)+\sinh(x)}{3(\cosh(x)-\sinh(x))}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^2/(9+tanh(x)^2),x, algorithm="fricas")
```

```
[Out] -1/3*arctan(-1/3*(9*cosh(x) + sinh(x))/(cosh(x) - sinh(x)))
```

giac [A] time = 0.13, size = 11, normalized size = 1.00

$$\frac{1}{3}\arctan\left(\frac{5}{3}e^{(2x)}+\frac{4}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^2/(9+tanh(x)^2),x, algorithm="giac")
```

```
[Out] 1/3*arctan(5/3*e^(2*x) + 4/3)
```

maple [B] time = 0.30, size = 116, normalized size = 10.55

$$-\frac{2\sqrt{10}\,\arctan\left(\frac{18\tanh\left(\frac{x}{2}\right)}{6\sqrt{10}+6}\right)}{6\sqrt{10}+6}-\frac{2\arctan\left(\frac{18\tanh\left(\frac{x}{2}\right)}{6\sqrt{10}+6}\right)}{6\sqrt{10}+6}+\frac{2\sqrt{10}\,\arctan\left(\frac{18\tanh\left(\frac{x}{2}\right)}{6\sqrt{10}-6}\right)}{6\sqrt{10}-6}-\frac{2\arctan\left(\frac{18\tanh\left(\frac{x}{2}\right)}{6\sqrt{10}-6}\right)}{6\sqrt{10}-6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(x)^2/(9+tanh(x)^2),x)
```

```
[Out] -2*10^(1/2)/(6*10^(1/2)+6)*arctan(18*tanh(1/2*x)/(6*10^(1/2)+6))-2/(6*10^(1
/2)+6)*arctan(18*tanh(1/2*x)/(6*10^(1/2)+6))+2*10^(1/2)/(6*10^(1/2)-6)*arct
an(18*tanh(1/2*x)/(6*10^(1/2)-6))-2/(6*10^(1/2)-6)*arctan(18*tanh(1/2*x)/(6
*10^(1/2)-6))
```

maxima [A] time = 0.41, size = 11, normalized size = 1.00

$$-\frac{1}{3}\arctan\left(\frac{5}{3}e^{(-2x)} + \frac{4}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^2/(9+tanh(x)^2),x, algorithm="maxima")
[Out] -1/3*arctan(5/3*e^(-2*x) + 4/3)
```

mupad [B] time = 1.64, size = 11, normalized size = 1.00

$$\frac{\operatorname{atan}\left(\frac{5\,\mathrm{e}^{2\,x}}{3} + \frac{4}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(x)^2*(tanh(x)^2 + 9)),x)
[Out] atan((5*exp(2*x))/3 + 4/3)/3
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}^2(x)}{\tanh^2(x) + 9}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)**2/(9+tanh(x)**2),x)
[Out] Integral(sech(x)**2/(tanh(x)**2 + 9), x)
```

3.982 $\int \operatorname{sech}^2(x)(a+b\tanh(x))^n\,dx$

Optimal. Leaf size=19

$$\frac{(a+b\tanh(x))^{n+1}}{b(n+1)}$$

```
[Out] (a+b*tanh(x))^(1+n)/b/(1+n)
```

Rubi [A] time = 0.04, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3506, 32}

$$\frac{(a+b\tanh(x))^{n+1}}{b(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[x]^2*(a + b*Tanh[x])^n,x]
```

```
[Out] (a + b*Tanh[x])^(1 + n)/(b*(1 + n))
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 3506

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(b*f), Subst[Int[(a + x)^n*(1 + x^2/b^2)^(m/2 - 1),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && NeQ[a^2 + b^2, 0
] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}\int \operatorname{sech}^2(x)(a+b\tanh(x))^n\,dx &= \frac{\operatorname{Subst}\left(\int (a+x)^n\,dx, x, b\tanh(x)\right)}{b}\\ &= \frac{(a+b\tanh(x))^{1+n}}{b(1+n)}\end{aligned}$$

Mathematica [A] time = 0.19, size = 18, normalized size = 0.95

$$\frac{(a+b\tanh(x))^{n+1}}{bn+b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[x]^2*(a + b*Tanh[x])^n,x]
```

```
[Out] (a + b*Tanh[x])^(1 + n)/(b + b*n)
```

fricas [B] time = 0.43, size = 69, normalized size = 3.63

$$\frac{(a\cosh(x)+b\sinh(x))\cosh\left(n\log\left(\frac{a\cosh(x)+b\sinh(x)}{\cosh(x)}\right)\right)+(a\cosh(x)+b\sinh(x))\sinh\left(n\log\left(\frac{a\cosh(x)+b\sinh(x)}{\cosh(x)}\right)\right)}{(bn+b)\cosh(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^2*(a+b*tanh(x))^n,x, algorithm="fricas")
```

```
[Out] ((a*cosh(x) + b*sinh(x))*cosh(n*log((a*cosh(x) + b*sinh(x))/cosh(x))) + (a*
cosh(x) + b*sinh(x))*sinh(n*log((a*cosh(x) + b*sinh(x))/cosh(x))))/((b*n +
b)*cosh(x))
```

giac [B] time = 0.12, size = 39, normalized size = 2.05

$$\frac{\left(\frac{ae^{(2x)}+be^{(2x)}+a-b}{e^{(2x)}+1}\right)^{n+1}}{b(n+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^2*(a+b*tanh(x))^n,x, algorithm="giac")
```

```
[Out] ((a*e^(2*x) + b*e^(2*x) + a - b)/(e^(2*x) + 1))^(n + 1)/(b*(n + 1))
```

maple [A] time = 0.15, size = 20, normalized size = 1.05

$$\frac{(a+b\tanh(x))^{n+1}}{b\,(n+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(x)^2*(a+b*tanh(x))^n,x)
```

```
[Out] (a+b*tanh(x))^(n+1)/b/(n+1)
```

maxima [A] time = 0.30, size = 19, normalized size = 1.00

$$\frac{(b\tanh(x)+a)^{n+1}}{b(n+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^2*(a+b*tanh(x))^n,x, algorithm="maxima")
```

```
[Out] (b*tanh(x) + a)^(n + 1)/(b*(n + 1))
```

mupad [B] time = 1.79, size = 54, normalized size = 2.84

$$\frac{\left(a+\frac{b\left(e^{2x}-1\right)}{e^{2x}+1}\right)^n\left(a-b+a\,e^{2x}+b\,e^{2x}\right)}{b\left(e^{2x}+1\right)(n+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tanh(x))^n/cosh(x)^2,x)
```

```
[Out] ((a + (b*(exp(2*x) - 1))/(exp(2*x) + 1))^n*(a - b + a*exp(2*x) + b*exp(2*x)
))/(b*(exp(2*x) + 1)*(n + 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int(a+b\tanh(x))^n\operatorname{sech}^2(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)**2*(a+b*tanh(x))**n,x)
```

```
[Out] Integral((a + b*tanh(x))**n*sech(x)**2, x)
```

3.983 $\int \text{sech}^2(x)\left(1+\frac{1}{1-\tanh^2(x)}\right)\,dx$

Optimal. Leaf size=4

$$x+\tanh(x)$$

```
[Out] x+tanh(x)
```

Rubi [A] time = 0.05, antiderivative size = 4, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {206}

$$x+\tanh(x)$$

Antiderivative was successfully verified.

```
[In] Int[Sech[x]^2*(1 + (1 - Tanh[x]^2)^(-1)),x]
```

```
[Out] x + Tanh[x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \text{sech}^2(x)\left(1+\frac{1}{1-\tanh^2(x)}\right)\,dx &= \text{Subst}\left(\int \left(1+\frac{1}{1-x^2}\right)\,dx,x,\tanh(x)\right)\\ &= \tanh(x)+\text{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,\tanh(x)\right)\\ &= x+\tanh(x)\end{aligned}$$

Mathematica [A] time = 0.01, size = 4, normalized size = 1.00

$$x+\tanh(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[x]^2*(1 + (1 - Tanh[x]^2)^(-1)),x]
```

```
[Out] x + Tanh[x]
```

fricas [B] time = 0.42, size = 14, normalized size = 3.50

$$\frac{(x-1)\cosh(x)+\sinh(x)}{\cosh(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^2*(1+1/(1-tanh(x)^2)),x, algorithm="fricas")
[Out] ((x - 1)*cosh(x) + sinh(x))/cosh(x)
```

giac [B] time = 0.13, size = 12, normalized size = 3.00

$$x-\frac{2}{e^{(2x)}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^2*(1+1/(1-tanh(x)^2)),x, algorithm="giac")
[Out] x - 2/(e^(2*x) + 1)
```

maple [B] time = 0.17, size = 34, normalized size = 8.50

$$-\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)+\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)+\frac{2\tanh\left(\frac{x}{2}\right)}{\tanh^2\left(\frac{x}{2}\right)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(x)^2*(1+1/(1-tanh(x)^2)),x)
[Out] -ln(tanh(1/2*x)-1)+ln(tanh(1/2*x)+1)+2*tanh(1/2*x)/(tanh(1/2*x)^2+1)
```

maxima [B] time = 0.30, size = 12, normalized size = 3.00

$$x+\frac{2}{e^{(-2x)}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^2*(1+1/(1-tanh(x)^2)),x, algorithm="maxima")
[Out] x + 2/(e^(-2*x) + 1)
```

mupad [B] time = 1.72, size = 12, normalized size = 3.00

$$x-\frac{2}{\mathrm{e}^{2x}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(1/(tanh(x)^2 - 1) - 1)/cosh(x)^2,x)
```

```
[Out] x - 2/(exp(2*x) + 1)
```

sympy [B] time = 0.81, size = 29, normalized size = 7.25

$$-\frac{x\operatorname{sech}^{2}(x)}{\tanh^{2}(x)-1}-\frac{\tanh(x)\operatorname{sech}^{2}(x)}{\tanh^{2}(x)-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)**2*(1+1/(1-tanh(x)**2)),x)
```

```
[Out] -x*sech(x)**2/(tanh(x)**2 - 1) - tanh(x)*sech(x)**2/(tanh(x)**2 - 1)
```

3.984 $\int \frac{\operatorname{sech}^2(x)\left(2-\tanh^2(x)\right)}{1-\tanh^2(x)}\,dx$

Optimal. Leaf size=4

$$x+\tanh(x)$$

[Out] x+tanh(x)

Rubi [A] time = 0.07, antiderivative size = 4, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {3657, 3473, 8}

$$x+\tanh(x)$$

Antiderivative was successfully verified.

[In] Int[(Sech[x]^2*(2 - Tanh[x]^2))/(1 - Tanh[x]^2),x]

[Out] x + Tanh[x]

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3657

```
Int[(u_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*sec[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a, b]
```

Rubi steps

$$\int \frac{\operatorname{sech}^2(x)\left(2-\tanh^2(x)\right)}{1-\tanh^2(x)}\,dx = \int \left(2-\tanh^2(x)\right)\,dx$$
$$= 2x - \int \tanh^2(x)\,dx$$
$$= 2x + \tanh(x) - \int 1\,dx$$
$$= x + \tanh(x)$$

Mathematica [**A**] time = 0.00, size = 4, normalized size = 1.00

$$x + \tanh(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sech[x]^2*(2 - Tanh[x]^2))/(1 - Tanh[x]^2),x]
[Out] x + Tanh[x]
```

fricas [**B**] time = 0.43, size = 14, normalized size = 3.50

$$\frac{(x-1)\cosh(x) + \sinh(x)}{\cosh(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^2*(2-tanh(x)^2)/(1-tanh(x)^2),x, algorithm="fricas")
[Out] ((x - 1)*cosh(x) + sinh(x))/cosh(x)
```

giac [**B**] time = 0.13, size = 12, normalized size = 3.00

$$x - \frac{2}{e^{(2x)} + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^2*(2-tanh(x)^2)/(1-tanh(x)^2),x, algorithm="giac")
[Out] x - 2/(e^(2*x) + 1)
```

maple [**B**] time = 0.17, size = 34, normalized size = 8.50

$$-\ln\left(\tanh\left(\frac{x}{2}\right)-1\right) + \ln\left(\tanh\left(\frac{x}{2}\right)+1\right) + \frac{2\tanh\left(\frac{x}{2}\right)}{\tanh^2\left(\frac{x}{2}\right)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(x)^2*(2-tanh(x)^2)/(1-tanh(x)^2),x)
```

```
[Out] -ln(tanh(1/2*x)-1)+ln(tanh(1/2*x)+1)+2*tanh(1/2*x)/(tanh(1/2*x)^2+1)
```

maxima [B] time = 0.31, size = 12, normalized size = 3.00

$$x + \frac{2}{e^{(-2x)} + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^2*(2-tanh(x)^2)/(1-tanh(x)^2),x, algorithm="maxima")
```

```
[Out] x + 2/(e^(-2*x) + 1)
```

mupad [B] time = 1.66, size = 12, normalized size = 3.00

$$x - \frac{2}{e^{2x} + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((tanh(x)^2 - 2)/(cosh(x)^2*(tanh(x)^2 - 1)),x)
```

```
[Out] x - 2/(exp(2*x) + 1)
```

sympy [B] time = 0.81, size = 29, normalized size = 7.25

$$-\frac{x\operatorname{sech}^2(x)}{\tanh^2(x) - 1} - \frac{\tanh(x)\operatorname{sech}^2(x)}{\tanh^2(x) - 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)**2*(2-tanh(x)**2)/(1-tanh(x)**2),x)
```

```
[Out] -x*sech(x)**2/(tanh(x)**2 - 1) - tanh(x)*sech(x)**2/(tanh(x)**2 - 1)
```

3.985 $\int \frac{\mathrm{sech}^2(x)}{2+2\tanh(x)+\tanh^2(x)}\,dx$

Optimal. Leaf size=5

$$\tan^{-1}(\tanh(x)+1)$$

```
[Out] arctan(1+tanh(x))
```

Rubi [A] time = 0.05, antiderivative size = 5, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {4342, 617, 204}

$$\tan^{-1}(\tanh(x)+1)$$

Antiderivative was successfully verified.

```
[In] Int[Sech[x]^2/(2 + 2*Tanh[x] + Tanh[x]^2),x]
```

```
[Out] ArcTan[1 + Tanh[x]]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 4342

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^2, x_Symbol] :> With[{d = FreeFac
tors[Tan[c*(a + b*x)], x]}, Dist[d/(b*c), Subst[Int[SubstFor[1, Tan[c*(a +
b*x)]/d, u, x], x], x, Tan[c*(a + b*x)]/d], x] /; FunctionOfQ[Tan[c*(a + b*
x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && NonsumQ[u] && (EqQ[F, Sec] ||
 EqQ[F, sec])
```

Rubi steps

$$\int \frac{\operatorname{sech}^2(x)}{2+2\tanh(x)+\tanh^2(x)}\,dx = \text{Subst}\left(\int \frac{1}{2+2x+x^2}\,dx, x, \tanh(x)\right)$$
$$= -\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1+\tanh(x)\right)$$
$$= \tan^{-1}(1+\tanh(x))$$

Mathematica [F] time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}^2(x)}{2+2\tanh(x)+\tanh^2(x)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sech[x]^2/(2 + 2*Tanh[x] + Tanh[x]^2),x]
[Out] Integrate[Sech[x]^2/(2 + 2*Tanh[x] + Tanh[x]^2), x]
```

fricas [B] time = 0.43, size = 23, normalized size = 4.60

$$-\arctan\left(-\frac{3\cosh(x)+2\sinh(x)}{\cosh(x)-\sinh(x)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^2/(2+2*tanh(x)+tanh(x)^2),x, algorithm="fricas")
[Out] -arctan(-(3*cosh(x) + 2*sinh(x))/(cosh(x) - sinh(x)))
```

giac [A] time = 0.12, size = 9, normalized size = 1.80

$$\arctan\left(\frac{5}{2}e^{(2x)}+\frac{1}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^2/(2+2*tanh(x)+tanh(x)^2),x, algorithm="giac")
[Out] arctan(5/2*e^(2*x) + 1/2)
```

maple [C] time = 0.24, size = 42, normalized size = 8.40

$$\frac{i\ln\left(\tanh^2\left(\frac{x}{2}\right)+(1-i)\tanh\left(\frac{x}{2}\right)+1\right)}{2}-\frac{i\ln\left(\tanh^2\left(\frac{x}{2}\right)+(1+i)\tanh\left(\frac{x}{2}\right)+1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(x)^2/(2+2*tanh(x)+tanh(x)^2),x)
```

```
[Out] 1/2*I*ln(tanh(1/2*x)^2+(1-I)*tanh(1/2*x)+1)-1/2*I*ln(tanh(1/2*x)^2+(1+I)*ta
nh(1/2*x)+1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}(x)^2}{\tanh(x)^2 + 2\ \tanh(x) + 2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^2/(2+2*tanh(x)+tanh(x)^2),x, algorithm="maxima")
```

```
[Out] integrate(sech(x)^2/(tanh(x)^2 + 2*tanh(x) + 2), x)
```

mupad [B] time = 1.66, size = 9, normalized size = 1.80

$$\operatorname{atan}\left(\frac{5\,e^{2\,x}}{2}+\frac{1}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(x)^2*(2*tanh(x) + tanh(x)^2 + 2)),x)
```

```
[Out] atan((5*exp(2*x))/2 + 1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}^{2}(x)}{\tanh^{2}(x) + 2\tanh(x) + 2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)**2/(2+2*tanh(x)+tanh(x)**2),x)
```

```
[Out] Integral(sech(x)**2/(tanh(x)**2 + 2*tanh(x) + 2), x)
```

3.986 $\int \frac{\operatorname{sech}^2(x)}{\tanh^2(x)+\tanh^3(x)} \, dx$

Optimal. Leaf size=15

$$-\coth(x)-\log(\tanh(x))+\log(\tanh(x)+1)$$

```
[Out] -coth(x)-ln(tanh(x))+ln(1+tanh(x))
```

Rubi [A] time = 0.05, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {4342, 44}

$$-\coth(x)-\log(\tanh(x))+\log(\tanh(x)+1)$$

Antiderivative was successfully verified.

```
[In] Int[Sech[x]^2/(Tanh[x]^2 + Tanh[x]^3),x]
```

```
[Out] -Coth[x] - Log[Tanh[x]] + Log[1 + Tanh[x]]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 4342

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^2, x_Symbol] :> With[{d = FreeFac
tors[Tan[c*(a + b*x)], x]}, Dist[d/(b*c), Subst[Int[SubstFor[1, Tan[c*(a +
b*x)]/d, u, x], x], x, Tan[c*(a + b*x)]/d], x] /; FunctionOfQ[Tan[c*(a + b*
x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && NonsumQ[u] && (EqQ[F, Sec] ||
 EqQ[F, sec])
```

Rubi steps

$$\begin{aligned}
\int \frac{\operatorname{sech}^2(x)}{\tanh^2(x)+\tanh^3(x)} \, dx &= \operatorname{Subst}\left(\int \frac{1}{x^2(1+x)} \, dx, x, \tanh(x)\right)\\
&= \operatorname{Subst}\left(\int \left(\frac{1}{x^2}-\frac{1}{x}+\frac{1}{1+x}\right) \, dx, x, \tanh(x)\right)\\
&= -\coth(x)-\log(\tanh(x))+\log(1+\tanh(x))
\end{aligned}$$

Mathematica [A] time = 0.03, size = 11, normalized size = 0.73

$$x - \coth(x) - \log(\sinh(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[x]^2/(Tanh[x]^2 + Tanh[x]^3),x]
```

```
[Out] x - Coth[x] - Log[Sinh[x]]
```

fricas [B] time = 0.45, size = 77, normalized size = 5.13

$$\frac{2\,x\cosh(x)^2 + 4\,x\cosh(x)\sinh(x) + 2\,x\sinh(x)^2 - \left(\cosh(x)^2 + 2\,\cosh(x)\sinh(x) + \sinh(x)^2 - 1\right)\log\left(\frac{2\sinh(x)}{\cosh(x)-\sinh(x)}\right) - 2\,x - 2}{\cosh(x)^2 + 2\,\cosh(x)\sinh(x) + \sinh(x)^2 - 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^2/(tanh(x)^2+tanh(x)^3),x, algorithm="fricas")
```

```
[Out] (2*x*cosh(x)^2 + 4*x*cosh(x)*sinh(x) + 2*x*sinh(x)^2 - (cosh(x)^2 + 2*cosh(
x)*sinh(x) + sinh(x)^2 - 1)*log(2*sinh(x)/(cosh(x) - sinh(x))) - 2*x - 2)/(
cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)
```

giac [A] time = 0.12, size = 29, normalized size = 1.93

$$2\,x + \frac{e^{(2\,x)} - 3}{e^{(2\,x)} - 1} - \log\left(\left|e^{(2\,x)} - 1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^2/(tanh(x)^2+tanh(x)^3),x, algorithm="giac")
```

```
[Out] 2*x + (e^(2*x) - 3)/(e^(2*x) - 1) - log(abs(e^(2*x) - 1))
```

maple [B] time = 0.23, size = 32, normalized size = 2.13

$$-\frac{\tanh\left(\frac{x}{2}\right)}{2} + 2\ln\left(\tanh\left(\frac{x}{2}\right) + 1\right) - \frac{1}{2\tanh\left(\frac{x}{2}\right)} - \ln\left(\tanh\left(\frac{x}{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(x)^2/(tanh(x)^2+tanh(x)^3),x)
```

```
[Out] -1/2*tanh(1/2*x)+2*ln(tanh(1/2*x)+1)-1/2/tanh(1/2*x)-ln(tanh(1/2*x))
```

maxima [A] time = 0.31, size = 29, normalized size = 1.93

$$\frac{2}{e^{(-2\,x)} - 1} - \log\left(e^{(-x)} + 1\right) - \log\left(e^{(-x)} - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^2/(tanh(x)^2+tanh(x)^3),x, algorithm="maxima")
```

```
[Out] 2/(e^(-2*x) - 1) - log(e^(-x) + 1) - log(e^(-x) - 1)
```

mupad [B] time = 0.09, size = 23, normalized size = 1.53

$$2x - \ln\left(e^{2x} - 1\right) - \frac{2}{e^{2x} - 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(x)^2*(tanh(x)^2 + tanh(x)^3)),x)
```

```
[Out] 2*x - log(exp(2*x) - 1) - 2/(exp(2*x) - 1)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}^2(x)}{(\tanh(x) + 1)\tanh^2(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)**2/(tanh(x)**2+tanh(x)**3),x)
```

```
[Out] Integral(sech(x)**2/((tanh(x) + 1)*tanh(x)**2), x)
```

3.987 $\int \frac{\operatorname{sech}^2(x)}{-\tanh^2(x)+\tanh^3(x)}\,dx$

Optimal. Leaf size=15

$$\coth(x)+\log(1-\tanh(x))-\log(\tanh(x))$$

```
[Out] coth(x)+ln(1-tanh(x))-ln(tanh(x))
```

Rubi [A] time = 0.06, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {4342, 44}

$$\coth(x)+\log(1-\tanh(x))-\log(\tanh(x))$$

Antiderivative was successfully verified.

```
[In] Int[Sech[x]^2/(-Tanh[x]^2 + Tanh[x]^3),x]
```

```
[Out] Coth[x] + Log[1 - Tanh[x]] - Log[Tanh[x]]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 4342

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^2, x_Symbol] :> With[{d = FreeFac
tors[Tan[c*(a + b*x)], x]}, Dist[d/(b*c), Subst[Int[SubstFor[1, Tan[c*(a +
b*x)]/d, u, x], x], x, Tan[c*(a + b*x)]/d], x] /; FunctionOfQ[Tan[c*(a + b*
x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && NonsumQ[u] && (EqQ[F, Sec] ||
 EqQ[F, sec])
```

Rubi steps

$$\begin{aligned}
\int \frac{\operatorname{sech}^2(x)}{-\tanh^2(x)+\tanh^3(x)}\,dx &= \text{Subst}\left(\int \frac{1}{(-1+x)x^2}\,dx, x, \tanh(x)\right)\\
&= \text{Subst}\left(\int \left(\frac{1}{-1+x}-\frac{1}{x^2}-\frac{1}{x}\right)dx, x, \tanh(x)\right)\\
&= \coth(x)+\log(1-\tanh(x))-\log(\tanh(x))
\end{aligned}$$

Mathematica [A] time = 0.03, size = 11, normalized size = 0.73

$$-x+\coth(x)-\log(\sinh(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[x]^2/(-Tanh[x]^2 + Tanh[x]^3),x]
```

```
[Out] -x + Coth[x] - Log[Sinh[x]]
```

fricas [B] time = 0.42, size = 53, normalized size = 3.53

$$-\frac{\left(\cosh(x)^2+2\cosh(x)\sinh(x)+\sinh(x)^2-1\right)\log\left(\frac{2\sinh(x)}{\cosh(x)-\sinh(x)}\right)-2}{\cosh(x)^2+2\cosh(x)\sinh(x)+\sinh(x)^2-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^2/(-tanh(x)^2+tanh(x)^3),x, algorithm="fricas")
```

```
[Out] -((cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*log(2*sinh(x)/(cosh(x) -
sinh(x))) - 2)/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)
```

giac [A] time = 0.12, size = 26, normalized size = 1.73

$$\frac{e^{(2x)}+1}{e^{(2x)}-1}-\log\left(\left|e^{(2x)}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^2/(-tanh(x)^2+tanh(x)^3),x, algorithm="giac")
```

```
[Out] (e^(2*x) + 1)/(e^(2*x) - 1) - log(abs(e^(2*x) - 1))
```

maple [B] time = 0.24, size = 32, normalized size = 2.13

$$\frac{\tanh\left(\frac{x}{2}\right)}{2}+2\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)+\frac{1}{2\tanh\left(\frac{x}{2}\right)}-\ln\left(\tanh\left(\frac{x}{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(x)^2/(-tanh(x)^2+tanh(x)^3),x)
```

```
[Out] 1/2*tanh(1/2*x)+2*ln(tanh(1/2*x)-1)+1/2/tanh(1/2*x)-ln(tanh(1/2*x))
```

maxima [B] time = 0.30, size = 32, normalized size = 2.13

$$-2x-\frac{2}{e^{(-2x)}-1}-\log\left(e^{(-x)}+1\right)-\log\left(e^{(-x)}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^2/(-tanh(x)^2+tanh(x)^3),x, algorithm="maxima")
[Out] -2*x - 2/(e^(-2*x) - 1) - log(e^(-x) + 1) - log(e^(-x) - 1)
```

mupad [B] time = 1.66, size = 20, normalized size = 1.33

$$\frac{2}{e^{2x}-1}-\ln\left(e^{2x}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/(cosh(x)^2*(tanh(x)^2 - tanh(x)^3)),x)
[Out] 2/(exp(2*x) - 1) - log(exp(2*x) - 1)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}^{2}(x)}{(\tanh(x)-1)\tanh^{2}(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)**2/(-tanh(x)**2+tanh(x)**3),x)
[Out] Integral(sech(x)**2/((tanh(x) - 1)*tanh(x)**2), x)
```

3.988 $\int \frac{\text{sech}^2(x)}{3-4\tanh^3(x)}\,dx$

Optimal. Leaf size=102

$$\frac{\log\left(2\sqrt[3]{2}\,\tanh^2(x)+2^{2/3}\sqrt[3]{3}\,\tanh(x)+3^{2/3}\right)}{6\,6^{2/3}}-\frac{\log\left(\sqrt[3]{3}-2^{2/3}\tanh(x)\right)}{3\,6^{2/3}}+\frac{\tan^{-1}\left(\frac{2\,2^{2/3}\tanh(x)+\sqrt[3]{3}}{3^{5/6}}\right)}{3\,2^{2/3}\sqrt[6]{3}}$$

```
[Out] 1/18*arctan(1/3*(3^(1/3)+2*2^(2/3)*tanh(x))*3^(1/6))*2^(1/3)*3^(5/6)-1/18*l
n(3^(1/3)-2^(2/3)*tanh(x))*6^(1/3)+1/36*ln(3^(2/3)+2^(2/3)*3^(1/3)*tanh(x)+
2*2^(1/3)*tanh(x)^2)*6^(1/3)
```

Rubi [A] time = 0.11, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.467, Rules used = {3675, 200, 31, 634, 617, 204, 628}

$$\frac{\log\left(2\sqrt[3]{2}\,\tanh^2(x)+2^{2/3}\sqrt[3]{3}\,\tanh(x)+3^{2/3}\right)}{6\,6^{2/3}}-\frac{\log\left(\sqrt[3]{3}-2^{2/3}\tanh(x)\right)}{3\,6^{2/3}}+\frac{\tan^{-1}\left(\frac{2\,2^{2/3}\tanh(x)+\sqrt[3]{3}}{3^{5/6}}\right)}{3\,2^{2/3}\sqrt[6]{3}}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[x]^2/(3 - 4*Tanh[x]^3),x]
```

```
[Out] ArcTan[(3^(1/3) + 2*2^(2/3)*Tanh[x])/3^(5/6)]/(3*2^(2/3)*3^(1/6)) - Log[3^(
1/3) - 2^(2/3)*Tanh[x]]/(3*6^(2/3)) + Log[3^(2/3) + 2^(2/3)*3^(1/3)*Tanh[x]
 + 2*2^(1/3)*Tanh[x]^2]/(6*6^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
```

```
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 3675

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/(c^(m - 1)*f), Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^p
, x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && In
tegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
|| EqQ[n^2, 16])
```

Rubi steps

$$\int \frac{\operatorname{sech}^2(x)}{3-4\tanh^3(x)}\,dx = \operatorname{Subst}\left(\int \frac{1}{3-4x^3}\,dx, x, \tanh(x)\right)$$
$$= \frac{\operatorname{Subst}\left(\int \frac{1}{\sqrt[3]{3}-2^{2/3}x}\,dx, x, \tanh(x)\right)}{3\ 3^{2/3}} + \frac{\operatorname{Subst}\left(\int \frac{2\sqrt[3]{3}+2^{2/3}x}{3^{2/3}+2^{2/3}\sqrt[3]{3}x+2\sqrt[3]{2}x^2}\,dx, x, \tanh(x)\right)}{3\ 3^{2/3}}$$
$$= -\frac{\log\left(\sqrt[3]{3}-2^{2/3}\tanh(x)\right)}{3\ 6^{2/3}} + \frac{\operatorname{Subst}\left(\int \frac{1}{3^{2/3}+2^{2/3}\sqrt[3]{3}x+2\sqrt[3]{2}x^2}\,dx, x, \tanh(x)\right)}{2\sqrt[3]{3}} + \frac{\operatorname{Subst}\left(\int \frac{}{3^{2/}}\right.}{}$$
$$= -\frac{\log\left(\sqrt[3]{3}-2^{2/3}\tanh(x)\right)}{3\ 6^{2/3}} + \frac{\log\left(3^{2/3}+2^{2/3}\sqrt[3]{3}\tanh(x)+2\sqrt[3]{2}\tanh^2(x)\right)}{6\ 6^{2/3}} - \frac{\operatorname{Subst}\left(\int\right.}{}$$
$$= \frac{\tan^{-1}\left(\frac{3+2\ 6^{2/3}\tanh(x)}{3\sqrt{3}}\right)}{3\ 2^{2/3}\sqrt[6]{3}} - \frac{\log\left(\sqrt[3]{3}-2^{2/3}\tanh(x)\right)}{3\ 6^{2/3}} + \frac{\log\left(3^{2/3}+2^{2/3}\sqrt[3]{3}\tanh(x)+2\sqrt[3]{2}\ \mathrm{t}\right.}{6\ 6^{2/3}}$$

Mathematica [A] time = 0.11, size = 74, normalized size = 0.73

$$\frac{\log\left(2\sqrt[3]{6}\tanh^2(x)+6^{2/3}\tanh(x)+3\right)-2\log\left(3-6^{2/3}\tanh(x)\right)+2\sqrt{3}\tan^{-1}\left(\frac{2\ 6^{2/3}\tanh(x)+3}{3\sqrt{3}}\right)}{6\ 6^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[x]^2/(3 - 4*Tanh[x]^3),x]
```

```
[Out] (2*Sqrt[3]*ArcTan[(3 + 2*6^(2/3)*Tanh[x])/(3*Sqrt[3])] - 2*Log[3 - 6^(2/3)*
Tanh[x]] + Log[3 + 6^(2/3)*Tanh[x] + 2*6^(1/3)*Tanh[x]^2])/(6*6^(2/3))
```

fricas [B] time = 0.46, size = 309, normalized size = 3.03

$$-\frac{1}{18}\cdot 36^{\frac{1}{6}}\sqrt{3}\,(-1)^{\frac{1}{3}}\arctan\left(\frac{1}{54}\cdot 36^{\frac{1}{6}}\left(\left(36^{\frac{2}{3}}\sqrt{3}\,(-1)^{\frac{2}{3}}+3\cdot 36^{\frac{1}{3}}\sqrt{3}-9\sqrt{3}\,(-1)^{\frac{1}{3}}\right)\cosh(x)^2+2\left(36^{\frac{2}{3}}\sqrt{3}\,(-1)^{\frac{2}{3}}+3\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^2/(3-4*tanh(x)^3),x, algorithm="fricas")
```

```
[Out] -1/18*36^(1/6)*sqrt(3)*(-1)^(1/3)*arctan(1/54*36^(1/6)*((36^(2/3)*sqrt(3)*(
-1)^(2/3) + 3*36^(1/3)*sqrt(3) - 9*sqrt(3)*(-1)^(1/3))*cosh(x)^2 + 2*(36^(2
/3)*sqrt(3)*(-1)^(2/3) + 3*36^(1/3)*sqrt(3) - 9*sqrt(3)*(-1)^(1/3))*cosh(x)
```

```
*sinh(x) + (36^(2/3)*sqrt(3)*(-1)^(2/3) + 3*36^(1/3)*sqrt(3) - 9*sqrt(3)*(-
1)^(1/3))*sinh(x)^2 - 36^(2/3)*sqrt(3)*(-1)^(2/3) - 9*sqrt(3)*(-1)^(1/3)))
- 1/216*36^(2/3)*(-1)^(1/3)*log(2*((36^(2/3)*(-1)^(1/3) + 3*36^(1/3)*(-1)^(
2/3) + 3)*cosh(x)^2 - 2*(36^(2/3)*(-1)^(1/3) + 3*36^(1/3)*(-1)^(2/3))*cosh(
x)*sinh(x) + (36^(2/3)*(-1)^(1/3) + 3*36^(1/3)*(-1)^(2/3) + 3)*sinh(x)^2 -
36^(2/3)*(-1)^(1/3) + 3*36^(1/3)*(-1)^(2/3) - 21)/(cosh(x)^2 - 2*cosh(x)*si
nh(x) + sinh(x)^2)) + 1/108*36^(2/3)*(-1)^(1/3)*log(2*((36^(2/3)*(-1)^(1/3)
 - 3*36^(1/3)*(-1)^(2/3) - 9)*cosh(x) - (36^(2/3)*(-1)^(1/3) - 3*36^(1/3)*(
-1)^(2/3) - 12)*sinh(x))/(cosh(x) - sinh(x)))
```

giac [A] time = 0.14, size = 1, normalized size = 0.01

$$0$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^2/(3-4*tanh(x)^3),x, algorithm="giac")
```

```
[Out] 0
```

maple [C] time = 0.24, size = 34, normalized size = 0.33

$$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(36\_Z^3+1\right)}\_R\ln\left(-24\tanh\left(\frac{x}{2}\right)\_R^2+\tanh^2\left(\frac{x}{2}\right)+1\right)\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(x)^2/(3-4*tanh(x)^3),x)
```

```
[Out] 1/3*sum(_R*ln(-24*tanh(1/2*x)*_R^2+tanh(1/2*x)^2+1),_R=RootOf(36*_Z^3+1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\mathrm{sech}(x)^2}{4\,\tanh(x)^3-3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^2/(3-4*tanh(x)^3),x, algorithm="maxima")
```

```
[Out] -integrate(sech(x)^2/(4*tanh(x)^3 - 3), x)
```

mupad [B] time = 3.34, size = 169, normalized size = 1.66

$$-\frac{6^{1/3}\ln\left(\frac{6^{1/3}\left(29856\,e^{2x}-\frac{6^{1/3}\left(109440\,e^{2x}+153216\right)}{18}+672\right)}{18}-\frac{5696\,e^{2x}}{3}+\frac{4480}{3}\right)}{18}-\frac{6^{1/3}\ln\left(\frac{4480}{3}+\frac{6^{1/3}\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(29856\,e^{2x}-\frac{6^{1/3}\left(-\frac{1}{2}+\sqrt{\cdots}\right.}{\cdots}\right.}{18}\cdots\right.}{1\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/(cosh(x)^2*(4*tanh(x)^3 - 3)),x)
```

```
[Out] (6^(1/3)*log(4480/3 - (6^(1/3)*((3^(1/2)*1i)/2 + 1/2)*(29856*exp(2*x) + (6^
(1/3)*((3^(1/2)*1i)/2 + 1/2)*(109440*exp(2*x) + 153216))/18 + 672))/18 - (5
696*exp(2*x))/3)*((3^(1/2)*1i)/2 + 1/2))/18 - (6^(1/3)*log((6^(1/3)*((3^(1/
2)*1i)/2 - 1/2)*(29856*exp(2*x) - (6^(1/3)*((3^(1/2)*1i)/2 - 1/2)*(109440*e
xp(2*x) + 153216))/18 + 672))/18 - (5696*exp(2*x))/3 + 4480/3)*((3^(1/2)*1i
)/2 - 1/2))/18 - (6^(1/3)*log((6^(1/3)*(29856*exp(2*x) - (6^(1/3)*(109440*e
xp(2*x) + 153216))/18 + 672))/18 - (5696*exp(2*x))/3 + 4480/3))/18
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\operatorname{sech}^2(x)}{4\tanh^3(x)-3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)**2/(3-4*tanh(x)**3),x)
```

```
[Out] -Integral(sech(x)**2/(4*tanh(x)**3 - 3), x)
```

3.989 $\int \frac{\operatorname{sech}^2(x)}{11-5\tanh(x)+5\tanh^2(x)}\,dx$

Optimal. Leaf size=22

$$-\frac{2\tan^{-1}\left(\sqrt{\frac{5}{39}}\,(1-2\tanh(x))\right)}{\sqrt{195}}$$

[Out] -2/195*arctan(1/39*195^(1/2)*(1-2*tanh(x)))*195^(1/2)

Rubi [A] time = 0.07, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {4342, 618, 204}

$$-\frac{2\tan^{-1}\left(\sqrt{\frac{5}{39}}\,(1-2\tanh(x))\right)}{\sqrt{195}}$$

Antiderivative was successfully verified.

[In] Int[Sech[x]^2/(11 - 5*Tanh[x] + 5*Tanh[x]^2),x]

[Out] (-2*ArcTan[Sqrt[5/39]*(1 - 2*Tanh[x])])/Sqrt[195]

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 4342

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^2, x_Symbol] :> With[{d = FreeFac
tors[Tan[c*(a + b*x)], x]}, Dist[d/(b*c), Subst[Int[SubstFor[1, Tan[c*(a +
b*x)]/d, u, x], x], x, Tan[c*(a + b*x)]/d], x] /; FunctionOfQ[Tan[c*(a + b*
x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && NonsumQ[u] && (EqQ[F, Sec] ||
 EqQ[F, sec])
```

Rubi steps

$$\int \frac{\operatorname{sech}^2(x)}{11-5\tanh(x)+5\tanh^2(x)}\,dx = \text{Subst}\left(\int \frac{1}{11-5x+5x^2}\,dx, x, \tanh(x)\right)$$
$$= -\left(2\,\text{Subst}\left(\int \frac{1}{-195-x^2}\,dx, x, -5+10\tanh(x)\right)\right)$$
$$= -\frac{2\tan^{-1}\left(\sqrt{\frac{5}{39}}\,(1-2\tanh(x))\right)}{\sqrt{195}}$$

Mathematica [F] time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}^2(x)}{11-5\tanh(x)+5\tanh^2(x)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sech[x]^2/(11 - 5*Tanh[x] + 5*Tanh[x]^2),x]
```

```
[Out] Integrate[Sech[x]^2/(11 - 5*Tanh[x] + 5*Tanh[x]^2), x]
```

fricas [A] time = 0.48, size = 32, normalized size = 1.45

$$-\frac{2}{195}\,\sqrt{195}\,\arctan\left(-\frac{17\,\sqrt{195}\,\cosh(x)+5\,\sqrt{195}\,\sinh(x)}{195\,(\cosh(x)-\sinh(x))}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^2/(11-5*tanh(x)+5*tanh(x)^2),x, algorithm="fricas")
```

```
[Out] -2/195*sqrt(195)*arctan(-1/195*(17*sqrt(195)*cosh(x) + 5*sqrt(195)*sinh(x))
/(cosh(x) - sinh(x)))
```

giac [A] time = 0.13, size = 19, normalized size = 0.86

$$\frac{2}{195}\,\sqrt{195}\,\arctan\left(\frac{1}{195}\,\sqrt{195}\left(11\,e^{(2\,x)}+6\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^2/(11-5*tanh(x)+5*tanh(x)^2),x, algorithm="giac")
```

```
[Out] 2/195*sqrt(195)*arctan(1/195*sqrt(195)*(11*e^(2*x) + 6))
```

maple [C] time = 0.25, size = 62, normalized size = 2.82

$$\frac{i\sqrt{195}\,\ln\left(11\left(\tanh^2\left(\frac{x}{2}\right)\right)+\left(-i\sqrt{195}-5\right)\tanh\left(\frac{x}{2}\right)+11\right)}{195}-\frac{i\sqrt{195}\,\ln\left(11\left(\tanh^2\left(\frac{x}{2}\right)\right)+\left(i\sqrt{195}-5\right)\tanh\left(\frac{x}{2}\right)+1\right.}{195}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(x)^2/(11-5*tanh(x)+5*tanh(x)^2),x)
```

```
[Out] 1/195*I*195^(1/2)*ln(11*tanh(1/2*x)^2+(-I*195^(1/2)-5)*tanh(1/2*x)+11)-1/19
5*I*195^(1/2)*ln(11*tanh(1/2*x)^2+(I*195^(1/2)-5)*tanh(1/2*x)+11)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}(x)^2}{5\,\tanh(x)^2-5\,\tanh(x)+11}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^2/(11-5*tanh(x)+5*tanh(x)^2),x, algorithm="maxima")
```

```
[Out] integrate(sech(x)^2/(5*tanh(x)^2 - 5*tanh(x) + 11), x)
```

mupad [B] time = 0.11, size = 19, normalized size = 0.86

$$\frac{2\sqrt{195}\,\operatorname{atan}\left(\frac{\sqrt{195}\,\left(11\,e^{2\,x}+6\right)}{195}\right)}{195}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(x)^2*(5*tanh(x)^2 - 5*tanh(x) + 11)),x)
```

```
[Out] (2*195^(1/2)*atan((195^(1/2)*(11*exp(2*x) + 6))/195))/195
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}^2(x)}{5\tanh^2(x)-5\tanh(x)+11}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)**2/(11-5*tanh(x)+5*tanh(x)**2),x)
```

```
[Out] Integral(sech(x)**2/(5*tanh(x)**2 - 5*tanh(x) + 11), x)
```

3.990 $\int \frac{\text{sech}^2(x)(a+b\tanh(x))}{c+d\tanh(x)}\,dx$

Optimal. Leaf size=28

$$\frac{b\tanh(x)}{d}-\frac{(bc-ad)\log(c+d\tanh(x))}{d^2}$$

```
[Out] -(-a*d+b*c)*ln(c+d*tanh(x))/d^2+b*tanh(x)/d
```

Rubi [A] time = 0.10, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {4342, 43}

$$\frac{b\tanh(x)}{d}-\frac{(bc-ad)\log(c+d\tanh(x))}{d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Sech[x]^2*(a + b*Tanh[x]))/(c + d*Tanh[x]),x]
```

```
[Out] -(((b*c - a*d)*Log[c + d*Tanh[x]])/d^2) + (b*Tanh[x])/d
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 4342

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^2, x_Symbol] :> With[{d = FreeFac
tors[Tan[c*(a + b*x)], x]}, Dist[d/(b*c), Subst[Int[SubstFor[1, Tan[c*(a +
b*x)]/d, u, x], x], x, Tan[c*(a + b*x)]/d], x] /; FunctionOfQ[Tan[c*(a + b*
x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && NonsumQ[u] && (EqQ[F, Sec] ||
 EqQ[F, sec])
```

Rubi steps

$$\int \frac{\mathrm{sech}^2(x)(a+b\tanh(x))}{c+d\tanh(x)}\,dx = \mathrm{Subst}\left(\int \frac{a+bx}{c+dx}\,dx, x, \tanh(x)\right)$$
$$= \mathrm{Subst}\left(\int \left(\frac{b}{d}+\frac{-bc+ad}{d(c+dx)}\right)dx, x, \tanh(x)\right)$$
$$= -\frac{(bc-ad)\log(c+d\tanh(x))}{d^2}+\frac{b\tanh(x)}{d}$$

Mathematica [A] time = 0.36, size = 54, normalized size = 1.93

$$\frac{\cosh(x)(a+b\tanh(x))((bc-ad)(\log(\cosh(x))-\log(c\cosh(x)+d\sinh(x)))+bd\tanh(x))}{d^2(a\cosh(x)+b\sinh(x))}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sech[x]^2*(a + b*Tanh[x]))/(c + d*Tanh[x]),x]
```

```
[Out] (Cosh[x]*(a + b*Tanh[x])*((b*c - a*d)*(Log[Cosh[x]] - Log[c*Cosh[x] + d*Sin
h[x]]) + b*d*Tanh[x]))/(d^2*(a*Cosh[x] + b*Sinh[x]))
```

fricas [B] time = 0.44, size = 172, normalized size = 6.14

$$-\frac{2\,bd+\left((bc-ad)\cosh(x)^2+2\,(bc-ad)\cosh(x)\sinh(x)+(bc-ad)\sinh(x)^2+bc-ad\right)\log\left(\frac{2\,(c\cosh(x)+d\sinh(x))}{\cosh(x)-\sinh(x)}\right)}{d^2\cosh(x)^2+2\,d^2\cosh(x)\sinh}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^2*(a+b*tanh(x))/(c+d*tanh(x)),x, algorithm="fricas")
```

```
[Out] -(2*b*d + ((b*c - a*d)*cosh(x)^2 + 2*(b*c - a*d)*cosh(x)*sinh(x) + (b*c - a
*d)*sinh(x)^2 + b*c - a*d)*log(2*(c*cosh(x) + d*sinh(x))/(cosh(x) - sinh(x)
)) - ((b*c - a*d)*cosh(x)^2 + 2*(b*c - a*d)*cosh(x)*sinh(x) + (b*c - a*d)*s
inh(x)^2 + b*c - a*d)*log(2*cosh(x)/(cosh(x) - sinh(x))))/(d^2*cosh(x)^2 +
2*d^2*cosh(x)*sinh(x) + d^2*sinh(x)^2 + d^2)
```

giac [B] time = 0.13, size = 113, normalized size = 4.04

$$-\frac{\left(bc^2-acd+bcd-ad^2\right)\log\left(\left|ce^{(2\,x)}+de^{(2\,x)}+c-d\right|\right)}{cd^2+d^3}+\frac{(bc-ad)\log\left(e^{(2\,x)}+1\right)}{d^2}-\frac{bce^{(2\,x)}-ade^{(2\,x)}+bc-ad+2\,b}{d^2\left(e^{(2\,x)}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^2*(a+b*tanh(x))/(c+d*tanh(x)),x, algorithm="giac")
```

```
[Out] -(b*c^2 - a*c*d + b*c*d - a*d^2)*log(abs(c*e^(2*x) + d*e^(2*x) + c - d))/(c
*d^2 + d^3) + (b*c - a*d)*log(e^(2*x) + 1)/d^2 - (b*c*e^(2*x) - a*d*e^(2*x)
 + b*c - a*d + 2*b*d)/(d^2*(e^(2*x) + 1))
```

maple [B] time = 0.20, size = 100, normalized size = 3.57

$$\frac{\ln\left(\left(\tanh^2\left(\frac{x}{2}\right)\right)c+2\tanh\left(\frac{x}{2}\right)d+c\right)a}{d}-\frac{\ln\left(\left(\tanh^2\left(\frac{x}{2}\right)\right)c+2\tanh\left(\frac{x}{2}\right)d+c\right)cb}{d^2}+\frac{2\tanh\left(\frac{x}{2}\right)b}{d\left(\tanh^2\left(\frac{x}{2}\right)+1\right)}-\frac{\ln\left(\tanh^2\left(\right.\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(x)^2*(a+b*tanh(x))/(c+d*tanh(x)),x)
```

```
[Out] 1/d*ln(tanh(1/2*x)^2*c+2*tanh(1/2*x)*d+c)*a-1/d^2*ln(tanh(1/2*x)^2*c+2*tanh
(1/2*x)*d+c)*c*b+2/d*tanh(1/2*x)*b/(tanh(1/2*x)^2+1)-1/d*ln(tanh(1/2*x)^2+1
)*a+1/d^2*ln(tanh(1/2*x)^2+1)*c*b
```

maxima [B] time = 0.40, size = 66, normalized size = 2.36

$$-b\left(\frac{c\log\left(-(c-d)e^{(-2x)}-c-d\right)}{d^2}-\frac{c\log\left(e^{(-2x)}+1\right)}{d^2}-\frac{2}{de^{(-2x)}+d}\right)+\frac{a\log\left(d\tanh(x)+c\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^2*(a+b*tanh(x))/(c+d*tanh(x)),x, algorithm="maxima")
```

```
[Out] -b*(c*log(-(c - d)*e^(-2*x) - c - d)/d^2 - c*log(e^(-2*x) + 1)/d^2 - 2/(d*e
^(-2*x) + d)) + a*log(d*tanh(x) + c)/d
```

mupad [B] time = 2.15, size = 297, normalized size = 10.61

$$\frac{2\,\mathrm{atan}\left(\mathrm{e}^{2\,x}\left(\frac{4\left(a\,d\,\sqrt{-d^4}-b\,c\,\sqrt{-d^4}\right)}{d^2\,\sqrt{(a\,d-b\,c)^2}\,(c+d)\,(c-d)\,\sqrt{-d^4}}-\frac{4\,c^2\,\sqrt{a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2}}{d^4\,(c+d)\,(c-d)\,(a\,d-b\,c)}\right)\left(\frac{d^2\,\sqrt{-d^4}}{4}+\frac{c\,d\,\sqrt{-d^4}}{4}\right)+\frac{4\,c\left(d^2\,\sqrt{a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2}-c\,d\right.}{d^5\,(c+}\right.}{\sqrt{-d^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tanh(x))/(cosh(x)^2*(c + d*tanh(x))),x)
```

```
[Out] (2*atan(exp(2*x)*((4*(a*d*(-d^4)^(1/2) - b*c*(-d^4)^(1/2)))/(d^2*((a*d - b*
c)^2)^(1/2)*(c + d)*(c - d)*(-d^4)^(1/2)) - (4*c^2*(a^2*d^2 + b^2*c^2 - 2*a
*b*c*d)^(1/2))/(d^4*(c + d)*(c - d)*(a*d - b*c)))*((d^2*(-d^4)^(1/2))/4 + (
c*d*(-d^4)^(1/2))/4) + (4*c*(d^2*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)^(1/2) - c*
d*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)^(1/2))*((d^2*(-d^4)^(1/2))/4 + (c*d*(-d^4
```

```
)^(1/2))/4))/(d^5*(c + d)*(c - d)*(a*d - b*c)))*(a^2*d^2 + b^2*c^2 - 2*a*b*
c*d)^(1/2))/(-d^4)^(1/2) - (2*b)/(d*(exp(2*x) + 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b\tanh(x))\operatorname{sech}^2(x)}{c + d\tanh(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)**2*(a+b*tanh(x))/(c+d*tanh(x)),x)
```

```
[Out] Integral((a + b*tanh(x))*sech(x)**2/(c + d*tanh(x)), x)
```

3.991 $\int \frac{\text{sech}^2(x)(a+b\tanh(x))^2}{c+d\tanh(x)}\,dx$

Optimal. Leaf size=53

$$\frac{(bc-ad)^2\log(c+d\tanh(x))}{d^3}-\frac{b\tanh(x)(bc-ad)}{d^2}+\frac{(a+b\tanh(x))^2}{2d}$$

```
[Out] (-a*d+b*c)^2*ln(c+d*tanh(x))/d^3-b*(-a*d+b*c)*tanh(x)/d^2+1/2*(a+b*tanh(x))
^2/d
```

Rubi [A] time = 0.16, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {4342, 43}

$$-\frac{b\tanh(x)(bc-ad)}{d^2}+\frac{(bc-ad)^2\log(c+d\tanh(x))}{d^3}+\frac{(a+b\tanh(x))^2}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(Sech[x]^2*(a + b*Tanh[x])^2)/(c + d*Tanh[x]),x]
```

```
[Out] ((b*c - a*d)^2*Log[c + d*Tanh[x]])/d^3 - (b*(b*c - a*d)*Tanh[x])/d^2 + (a +
 b*Tanh[x])^2/(2*d)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 4342

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^2, x_Symbol] :> With[{d = FreeFac
tors[Tan[c*(a + b*x)], x]}, Dist[d/(b*c), Subst[Int[SubstFor[1, Tan[c*(a +
b*x)]/d, u, x], x], x, Tan[c*(a + b*x)]/d], x] /; FunctionOfQ[Tan[c*(a + b*
x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && NonsumQ[u] && (EqQ[F, Sec] ||
 EqQ[F, sec])
```

Rubi steps

$$\int \frac{\operatorname{sech}^2(x)(a+b\tanh(x))^2}{c+d\tanh(x)}\,dx = \operatorname{Subst}\left(\int \frac{(a+bx)^2}{c+dx}\,dx, x, \tanh(x)\right)$$
$$= \operatorname{Subst}\left(\int \left(-\frac{b(bc-ad)}{d^2}+\frac{b(a+bx)}{d}+\frac{(-bc+ad)^2}{d^2(c+dx)}\right)dx, x, \tanh(x)\right)$$
$$= \frac{(bc-ad)^2\log(c+d\tanh(x))}{d^3}-\frac{b(bc-ad)\tanh(x)}{d^2}+\frac{(a+b\tanh(x))^2}{2d}$$

Mathematica [A] time = 0.62, size = 61, normalized size = 1.15

$$-\frac{2bd\tanh(x)(bc-2ad)+2(bc-ad)^2(\log(\cosh(x))-\log(c\cosh(x)+d\sinh(x)))+b^2d^2\operatorname{sech}^2(x)}{2d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sech[x]^2*(a + b*Tanh[x])^2)/(c + d*Tanh[x]),x]
```

```
[Out] -1/2*(2*(b*c - a*d)^2*(Log[Cosh[x]] - Log[c*Cosh[x] + d*Sinh[x]]) + b^2*d^2
*Sech[x]^2 + 2*b*d*(b*c - 2*a*d)*Tanh[x])/d^3
```

fricas [B] time = 0.47, size = 688, normalized size = 12.98

$$\frac{2\,b^2cd-4\,abd^2+2\left(b^2cd-\left(2\,ab+b^2\right)d^2\right)\cosh(x)^2+4\left(b^2cd-\left(2\,ab+b^2\right)d^2\right)\cosh(x)\sinh(x)+2\left(b^2cd-\left(2\,ab\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^2*(a+b*tanh(x))^2/(c+d*tanh(x)),x, algorithm="fricas")
```

```
[Out] (2*b^2*c*d - 4*a*b*d^2 + 2*(b^2*c*d - (2*a*b + b^2)*d^2)*cosh(x)^2 + 4*(b^2
*c*d - (2*a*b + b^2)*d^2)*cosh(x)*sinh(x) + 2*(b^2*c*d - (2*a*b + b^2)*d^2)
*sinh(x)^2 + ((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*cosh(x)^4 + 4*(b^2*c^2 - 2*a*
b*c*d + a^2*d^2)*cosh(x)*sinh(x)^3 + (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sinh(x
)^4 + b^2*c^2 - 2*a*b*c*d + a^2*d^2 + 2*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*cos
h(x)^2 + 2*(b^2*c^2 - 2*a*b*c*d + a^2*d^2 + 3*(b^2*c^2 - 2*a*b*c*d + a^2*d^
2)*cosh(x)^2)*sinh(x)^2 + 4*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*cosh(x)^3 + (b
^2*c^2 - 2*a*b*c*d + a^2*d^2)*cosh(x))*sinh(x))*log(2*(c*cosh(x) + d*sinh(x
))/(cosh(x) - sinh(x))) - ((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*cosh(x)^4 + 4*(b
^2*c^2 - 2*a*b*c*d + a^2*d^2)*cosh(x)*sinh(x)^3 + (b^2*c^2 - 2*a*b*c*d + a^
2*d^2)*sinh(x)^4 + b^2*c^2 - 2*a*b*c*d + a^2*d^2 + 2*(b^2*c^2 - 2*a*b*c*d +
 a^2*d^2)*cosh(x)^2 + 2*(b^2*c^2 - 2*a*b*c*d + a^2*d^2 + 3*(b^2*c^2 - 2*a*b
*c*d + a^2*d^2)*cosh(x)^2)*sinh(x)^2 + 4*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*c
```

```
osh(x)^3 + (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*cosh(x))*sinh(x))*log(2*cosh(x)/
(cosh(x) - sinh(x))))/(d^3*cosh(x)^4 + 4*d^3*cosh(x)*sinh(x)^3 + d^3*sinh(x
)^4 + 2*d^3*cosh(x)^2 + d^3 + 2*(3*d^3*cosh(x)^2 + d^3)*sinh(x)^2 + 4*(d^3*
cosh(x)^3 + d^3*cosh(x))*sinh(x))
```

giac [B] time = 0.13, size = 264, normalized size = 4.98

$$\frac{\left(b^2c^3 - 2\,abc^2d + b^2c^2d + a^2cd^2 - 2\,abcd^2 + a^2d^3\right)\log\left(\left|ce^{(2\,x)} + de^{(2\,x)} + c - d\right|\right)}{cd^3 + d^4} - \frac{\left(b^2c^2 - 2\,abcd + a^2d^2\right)\log\left(e^{(2}\right.}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^2*(a+b*tanh(x))^2/(c+d*tanh(x)),x, algorithm="giac")
```

```
[Out] (b^2*c^3 - 2*a*b*c^2*d + b^2*c^2*d + a^2*c*d^2 - 2*a*b*c*d^2 + a^2*d^3)*log
(abs(c*e^(2*x) + d*e^(2*x) + c - d))/(c*d^3 + d^4) - (b^2*c^2 - 2*a*b*c*d +
 a^2*d^2)*log(e^(2*x) + 1)/d^3 + 1/2*(3*b^2*c^2*e^(4*x) - 6*a*b*c*d*e^(4*x)
 + 3*a^2*d^2*e^(4*x) + 6*b^2*c^2*e^(2*x) - 12*a*b*c*d*e^(2*x) + 4*b^2*c*d*e
^(2*x) + 6*a^2*d^2*e^(2*x) - 8*a*b*d^2*e^(2*x) - 4*b^2*d^2*e^(2*x) + 3*b^2*
c^2 - 6*a*b*c*d + 4*b^2*c*d + 3*a^2*d^2 - 8*a*b*d^2)/(d^3*(e^(2*x) + 1)^2)
```

maple [B] time = 0.23, size = 251, normalized size = 4.74

$$\frac{\ln\left(\left(\tanh^2\left(\frac{x}{2}\right)\right)c + 2\tanh\left(\frac{x}{2}\right)d + c\right)a^2}{d} - \frac{2\ln\left(\left(\tanh^2\left(\frac{x}{2}\right)\right)c + 2\tanh\left(\frac{x}{2}\right)d + c\right)cba}{d^2} + \frac{\ln\left(\left(\tanh^2\left(\frac{x}{2}\right)\right)c + 2\tanh\right.}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(x)^2*(a+b*tanh(x))^2/(c+d*tanh(x)),x)
```

```
[Out] 1/d*ln(tanh(1/2*x)^2*c+2*tanh(1/2*x)*d+c)*a^2-2/d^2*ln(tanh(1/2*x)^2*c+2*ta
nh(1/2*x)*d+c)*c*b*a+1/d^3*ln(tanh(1/2*x)^2*c+2*tanh(1/2*x)*d+c)*c^2*b^2+4/
d/(tanh(1/2*x)^2+1)^2*tanh(1/2*x)^3*a*b-2/d^2/(tanh(1/2*x)^2+1)^2*tanh(1/2*
x)^3*b^2*c+2/d/(tanh(1/2*x)^2+1)^2*b^2*tanh(1/2*x)^2+4/d/(tanh(1/2*x)^2+1)^
2*tanh(1/2*x)*a*b-2/d^2/(tanh(1/2*x)^2+1)^2*tanh(1/2*x)*b^2*c-1/d*ln(tanh(1
/2*x)^2+1)*a^2+2/d^2*ln(tanh(1/2*x)^2+1)*c*b*a-1/d^3*ln(tanh(1/2*x)^2+1)*c^
2*b^2
```

maxima [B] time = 0.42, size = 151, normalized size = 2.85

$$-b^2\left(\frac{2\left((c+d)e^{(-2\,x)} + c\right)}{2\,d^2e^{(-2\,x)} + d^2e^{(-4\,x)} + d^2} - \frac{c^2\log\left(-(c-d)e^{(-2\,x)} - c - d\right)}{d^3} + \frac{c^2\log\left(e^{(-2\,x)} + 1\right)}{d^3}\right) - 2\,ab\left(\frac{c\log\left(-(c-d)e^{(-2\,x)}\right.}{d^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^2*(a+b*tanh(x))^2/(c+d*tanh(x)),x, algorithm="maxima")
```

```
[Out] -b^2*(2*((c + d)*e^(-2*x) + c)/(2*d^2*e^(-2*x) + d^2*e^(-4*x) + d^2) - c^2*
log(-(c - d)*e^(-2*x) - c - d)/d^3 + c^2*log(e^(-2*x) + 1)/d^3) - 2*a*b*(c*
log(-(c - d)*e^(-2*x) - c - d)/d^2 - c*log(e^(-2*x) + 1)/d^2 - 2/(d*e^(-2*x
) + d)) + a^2*log(d*tanh(x) + c)/d
```

mupad [B] time = 2.13, size = 107, normalized size = 2.02

$$\frac{\ln\left(c-d+d\,e^{2\,x}+c\,e^{2\,x}\right)\,{\left(a\,d-b\,c\right)}^2}{d^3}-\frac{2\,\left(b^2\,d-b^2\,c+2\,a\,b\,d\right)}{d^2\,\left(e^{2\,x}+1\right)}-\frac{\ln\left(e^{2\,x}+1\right)\,{\left(a\,d-b\,c\right)}^2}{d^3}+\frac{2\,b^2}{d\,\left(2\,e^{2\,x}+e^{4\,x}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tanh(x))^2/(cosh(x)^2*(c + d*tanh(x))),x)
```

```
[Out] (log(c - d + d*exp(2*x) + c*exp(2*x))*(a*d - b*c)^2)/d^3 - (2*(b^2*d - b^2*
c + 2*a*b*d))/(d^2*(exp(2*x) + 1)) - (log(exp(2*x) + 1)*(a*d - b*c)^2)/d^3
+ (2*b^2)/(d*(2*exp(2*x) + exp(4*x) + 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + b\tanh\left(x\right)\right)^{2}\operatorname{sech}^{2}\left(x\right)}{c + d\tanh\left(x\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)**2*(a+b*tanh(x))**2/(c+d*tanh(x)),x)
```

```
[Out] Integral((a + b*tanh(x))**2*sech(x)**2/(c + d*tanh(x)), x)
```

3.992 $\int \frac{\text{sech}^2(x)(a+b\tanh(x))^3}{c+d\tanh(x)}\,dx$

Optimal. Leaf size=78

$$-\frac{(bc-ad)^3\log(c+d\tanh(x))}{d^4}+\frac{b\tanh(x)(bc-ad)^2}{d^3}-\frac{(bc-ad)(a+b\tanh(x))^2}{2d^2}+\frac{(a+b\tanh(x))^3}{3d}$$

```
[Out] -(-a*d+b*c)^3*ln(c+d*tanh(x))/d^4+b*(-a*d+b*c)^2*tanh(x)/d^3-1/2*(-a*d+b*c)
*(a+b*tanh(x))^2/d^2+1/3*(a+b*tanh(x))^3/d
```

Rubi [A] time = 0.16, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {4342, 43}

$$\frac{b\tanh(x)(bc-ad)^2}{d^3}-\frac{(bc-ad)(a+b\tanh(x))^2}{2d^2}-\frac{(bc-ad)^3\log(c+d\tanh(x))}{d^4}+\frac{(a+b\tanh(x))^3}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(Sech[x]^2*(a + b*Tanh[x])^3)/(c + d*Tanh[x]),x]
```

```
[Out] -(((b*c - a*d)^3*Log[c + d*Tanh[x]])/d^4) + (b*(b*c - a*d)^2*Tanh[x])/d^3 -
 ((b*c - a*d)*(a + b*Tanh[x])^2)/(2*d^2) + (a + b*Tanh[x])^3/(3*d)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 4342

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^2, x_Symbol] :> With[{d = FreeFac
tors[Tan[c*(a + b*x)], x]}, Dist[d/(b*c), Subst[Int[SubstFor[1, Tan[c*(a +
b*x)]/d, u, x], x], x, Tan[c*(a + b*x)]/d], x] /; FunctionOfQ[Tan[c*(a + b*
x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && NonsumQ[u] && (EqQ[F, Sec] ||
 EqQ[F, sec])
```

Rubi steps

$$\int \frac{\operatorname{sech}^2(x)(a+b\tanh(x))^3}{c+d\tanh(x)}\,dx = \text{Subst}\left(\int \frac{(a+bx)^3}{c+dx}\,dx, x, \tanh(x)\right)$$
$$= \text{Subst}\left(\int \left(\frac{b(bc-ad)^2}{d^3} - \frac{b(bc-ad)(a+bx)}{d^2} + \frac{b(a+bx)^2}{d} + \frac{(-bc+ad)^3}{d^3(c+dx)}\right) dx, x$$
$$= -\frac{(bc-ad)^3\log(c+d\tanh(x))}{d^4} + \frac{b(bc-ad)^2\tanh(x)}{d^3} - \frac{(bc-ad)(a+b\tanh(x)}{2d^2}$$

Mathematica [A] time = 0.87, size = 134, normalized size = 1.72

$$\frac{(a+b\tanh(x))^3(c\cosh(x)+d\sinh(x))\left(bd^2\left(\sinh(2x)\left(9a^2d-9abc+b^2d\right)+b(-9ad+3bc-2bd\tanh(x))\right)+6c\right.}{6d^4(c+d\tanh(x))(a\cosh(x)+b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sech[x]^2*(a + b*Tanh[x])^3)/(c + d*Tanh[x]),x]
```

```
[Out] ((c*Cosh[x] + d*Sinh[x])*(a + b*Tanh[x])^3*(6*(b*c - a*d)^3*Cosh[x]^2*(Log[
Cosh[x]] - Log[c*Cosh[x] + d*Sinh[x]]) + 6*b^3*c^2*d*Cosh[x]*Sinh[x] + b*d^
2*((-9*a*b*c + 9*a^2*d + b^2*d)*Sinh[2*x] + b*(3*b*c - 9*a*d - 2*b*d*Tanh[x
]))))/(6*d^4*(a*Cosh[x] + b*Sinh[x])^3*(c + d*Tanh[x]))
```

fricas [B] time = 0.55, size = 1975, normalized size = 25.32

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^2*(a+b*tanh(x))^3/(c+d*tanh(x)),x, algorithm="fricas")
```

```
[Out] -1/3*(6*b^3*c^2*d - 18*a*b^2*c*d^2 + 6*(b^3*c^2*d - (3*a*b^2 + b^3)*c*d^2 +
 (3*a^2*b + 3*a*b^2 + b^3)*d^3)*cosh(x)^4 + 24*(b^3*c^2*d - (3*a*b^2 + b^3)
*c*d^2 + (3*a^2*b + 3*a*b^2 + b^3)*d^3)*cosh(x)*sinh(x)^3 + 6*(b^3*c^2*d -
(3*a*b^2 + b^3)*c*d^2 + (3*a^2*b + 3*a*b^2 + b^3)*d^3)*sinh(x)^4 + 2*(9*a^2
*b + b^3)*d^3 + 6*(2*b^3*c^2*d - (6*a*b^2 + b^3)*c*d^2 + 3*(2*a^2*b + a*b^2
)*d^3)*cosh(x)^2 + 6*(2*b^3*c^2*d - (6*a*b^2 + b^3)*c*d^2 + 3*(2*a^2*b + a*
b^2)*d^3 + 6*(b^3*c^2*d - (3*a*b^2 + b^3)*c*d^2 + (3*a^2*b + 3*a*b^2 + b^3)
*d^3)*cosh(x)^2)*sinh(x)^2 + 3*((b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 -
a^3*d^3)*cosh(x)^6 + 6*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*
cosh(x)*sinh(x)^5 + (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*sin
h(x)^6 + b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3 + 3*(b^3*c^3 - 3
*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*cosh(x)^4 + 3*(b^3*c^3 - 3*a*b^2*c^
2*d + 3*a^2*b*c*d^2 - a^3*d^3 + 5*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2
```

```
- a^3*d^3)*cosh(x)^2)*sinh(x)^4 + 4*(5*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c
*d^2 - a^3*d^3)*cosh(x)^3 + 3*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^
3*d^3)*cosh(x))*sinh(x)^3 + 3*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^
3*d^3)*cosh(x)^2 + 3*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3 + 5
*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*cosh(x)^4 + 6*(b^3*c^3
 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*cosh(x)^2)*sinh(x)^2 + 6*((b^3*
c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*cosh(x)^5 + 2*(b^3*c^3 - 3*a
*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*cosh(x)^3 + (b^3*c^3 - 3*a*b^2*c^2*d
+ 3*a^2*b*c*d^2 - a^3*d^3)*cosh(x))*sinh(x))*log(2*(c*cosh(x) + d*sinh(x))/
(cosh(x) - sinh(x))) - 3*((b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^
3)*cosh(x)^6 + 6*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*cosh(x
)*sinh(x)^5 + (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*sinh(x)^6
 + b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3 + 3*(b^3*c^3 - 3*a*b^2
*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*cosh(x)^4 + 3*(b^3*c^3 - 3*a*b^2*c^2*d +
3*a^2*b*c*d^2 - a^3*d^3 + 5*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*
d^3)*cosh(x)^2)*sinh(x)^4 + 4*(5*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 -
 a^3*d^3)*cosh(x)^3 + 3*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)
*cosh(x))*sinh(x)^3 + 3*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)
*cosh(x)^2 + 3*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3 + 5*(b^3*
c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*cosh(x)^4 + 6*(b^3*c^3 - 3*a
*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*cosh(x)^2)*sinh(x)^2 + 6*((b^3*c^3 -
3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*cosh(x)^5 + 2*(b^3*c^3 - 3*a*b^2*c
^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*cosh(x)^3 + (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^
2*b*c*d^2 - a^3*d^3)*cosh(x))*sinh(x))*log(2*cosh(x)/(cosh(x) - sinh(x))) +
 12*(2*(b^3*c^2*d - (3*a*b^2 + b^3)*c*d^2 + (3*a^2*b + 3*a*b^2 + b^3)*d^3)*
cosh(x)^3 + (2*b^3*c^2*d - (6*a*b^2 + b^3)*c*d^2 + 3*(2*a^2*b + a*b^2)*d^3)
*cosh(x))*sinh(x))/(d^4*cosh(x)^6 + 6*d^4*cosh(x)*sinh(x)^5 + d^4*sinh(x)^6
 + 3*d^4*cosh(x)^4 + 3*d^4*cosh(x)^2 + 3*(5*d^4*cosh(x)^2 + d^4)*sinh(x)^4
+ d^4 + 4*(5*d^4*cosh(x)^3 + 3*d^4*cosh(x))*sinh(x)^3 + 3*(5*d^4*cosh(x)^4
+ 6*d^4*cosh(x)^2 + d^4)*sinh(x)^2 + 6*(d^4*cosh(x)^5 + 2*d^4*cosh(x)^3 + d
^4*cosh(x))*sinh(x))
```

giac [B] time = 0.15, size = 543, normalized size = 6.96

$$-\frac{\left(b^{3}c^{4}-3\,ab^{2}c^{3}d+b^{3}c^{3}d+3\,a^{2}bc^{2}d^{2}-3\,ab^{2}c^{2}d^{2}-a^{3}cd^{3}+3\,a^{2}bcd^{3}-a^{3}d^{4}\right)\log\left(\left|ce^{(2\,x)}+de^{(2\,x)}+c-d\right|\right)}{cd^{4}+d^{5}}+\frac{\left(b^{3}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^2*(a+b*tanh(x))^3/(c+d*tanh(x)),x, algorithm="giac")
```

```
[Out] -(b^3*c^4 - 3*a*b^2*c^3*d + b^3*c^3*d + 3*a^2*b*c^2*d^2 - 3*a*b^2*c^2*d^2 -
 a^3*c*d^3 + 3*a^2*b*c*d^3 - a^3*d^4)*log(abs(c*e^(2*x) + d*e^(2*x) + c - d
))/(c*d^4 + d^5) + (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*log(
e^(2*x) + 1)/d^4 - 1/6*(11*b^3*c^3*e^(6*x) - 33*a*b^2*c^2*d*e^(6*x) + 33*a^
```

```
2*b*c*d^2*e^(6*x) - 11*a^3*d^3*e^(6*x) + 33*b^3*c^3*e^(4*x) - 99*a*b^2*c^2*
d*e^(4*x) + 12*b^3*c^2*d*e^(4*x) + 99*a^2*b*c*d^2*e^(4*x) - 36*a*b^2*c*d^2*
e^(4*x) - 12*b^3*c*d^2*e^(4*x) - 33*a^3*d^3*e^(4*x) + 36*a^2*b*d^3*e^(4*x)
+ 36*a*b^2*d^3*e^(4*x) + 12*b^3*d^3*e^(4*x) + 33*b^3*c^3*e^(2*x) - 99*a*b^2
*c^2*d*e^(2*x) + 24*b^3*c^2*d*e^(2*x) + 99*a^2*b*c*d^2*e^(2*x) - 72*a*b^2*c
*d^2*e^(2*x) - 12*b^3*c*d^2*e^(2*x) - 33*a^3*d^3*e^(2*x) + 72*a^2*b*d^3*e^(
2*x) + 36*a*b^2*d^3*e^(2*x) + 11*b^3*c^3 - 33*a*b^2*c^2*d + 12*b^3*c^2*d +
33*a^2*b*c*d^2 - 36*a*b^2*c*d^2 - 11*a^3*d^3 + 36*a^2*b*d^3 + 4*b^3*d^3)/(d
^4*(e^(2*x) + 1)^3)
```

maple [B] time = 0.26, size = 542, normalized size = 6.95

$$\frac{\ln\left(\left(\tanh^2\left(\frac{x}{2}\right)\right)c+2\tanh\left(\frac{x}{2}\right)d+c\right)a^3}{d}-\frac{3\ln\left(\left(\tanh^2\left(\frac{x}{2}\right)\right)c+2\tanh\left(\frac{x}{2}\right)d+c\right)a^2bc}{d^2}+\frac{3\ln\left(\left(\tanh^2\left(\frac{x}{2}\right)\right)c+2\tan\right.}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(x)^2*(a+b*tanh(x))^3/(c+d*tanh(x)),x)
```

```
[Out] 1/d*ln(tanh(1/2*x)^2*c+2*tanh(1/2*x)*d+c)*a^3-3/d^2*ln(tanh(1/2*x)^2*c+2*ta
nh(1/2*x)*d+c)*a^2*b*c+3/d^3*ln(tanh(1/2*x)^2*c+2*tanh(1/2*x)*d+c)*c^2*b^2*
a-1/d^4*ln(tanh(1/2*x)^2*c+2*tanh(1/2*x)*d+c)*c^3*b^3+6/d/(tanh(1/2*x)^2+1)
^3*tanh(1/2*x)^5*a^2*b-6/d^2/(tanh(1/2*x)^2+1)^3*tanh(1/2*x)^5*a*b^2*c+2/d^
3/(tanh(1/2*x)^2+1)^3*tanh(1/2*x)^5*b^3*c^2+6/d/(tanh(1/2*x)^2+1)^3*tanh(1/
2*x)^4*a*b^2-2/d^2/(tanh(1/2*x)^2+1)^3*tanh(1/2*x)^4*b^3*c+12/d/(tanh(1/2*x
)^2+1)^3*tanh(1/2*x)^3*a^2*b-12/d^2/(tanh(1/2*x)^2+1)^3*tanh(1/2*x)^3*a*b^2
*c+4/d^3/(tanh(1/2*x)^2+1)^3*tanh(1/2*x)^3*b^3*c^2+8/3/d/(tanh(1/2*x)^2+1)^
3*tanh(1/2*x)^3*b^3+6/d/(tanh(1/2*x)^2+1)^3*tanh(1/2*x)^2*a*b^2-2/d^2/(tanh
(1/2*x)^2+1)^3*tanh(1/2*x)^2*b^3*c+6/d/(tanh(1/2*x)^2+1)^3*tanh(1/2*x)*a^2*
b-6/d^2/(tanh(1/2*x)^2+1)^3*tanh(1/2*x)*a*b^2*c+2/d^3/(tanh(1/2*x)^2+1)^3*t
anh(1/2*x)*b^3*c^2-1/d*ln(tanh(1/2*x)^2+1)*a^3+3/d^2*ln(tanh(1/2*x)^2+1)*a^
2*b*c-3/d^3*ln(tanh(1/2*x)^2+1)*c^2*b^2*a+1/d^4*ln(tanh(1/2*x)^2+1)*c^3*b^3
```

maxima [B] time = 0.42, size = 276, normalized size = 3.54

$$\frac{1}{3}b^3\left(\frac{2\left(3c^2+d^2+3\left(2c^2+cd\right)e^{(-2x)}+3\left(c^2+cd+d^2\right)e^{(-4x)}\right)}{3d^3e^{(-2x)}+3d^3e^{(-4x)}+d^3e^{(-6x)}+d^3}-\frac{3c^3\log\left(-(c-d)e^{(-2x)}-c-d\right)}{d^4}+\frac{3c^3\log\left(e^{(-2}\right.}{d^4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^2*(a+b*tanh(x))^3/(c+d*tanh(x)),x, algorithm="maxima")
```

```
[Out] 1/3*b^3*(2*(3*c^2 + d^2 + 3*(2*c^2 + c*d)*e^(-2*x) + 3*(c^2 + c*d + d^2)*e^
(-4*x))/(3*d^3*e^(-2*x) + 3*d^3*e^(-4*x) + d^3*e^(-6*x) + d^3) - 3*c^3*log(
-(c - d)*e^(-2*x) - c - d)/d^4 + 3*c^3*log(e^(-2*x) + 1)/d^4) - 3*a*b^2*(2*
```

```
((c + d)*e^(-2*x) + c)/(2*d^2*e^(-2*x) + d^2*e^(-4*x) + d^2) - c^2*log(-(c
- d)*e^(-2*x) - c - d)/d^3 + c^2*log(e^(-2*x) + 1)/d^3) - 3*a^2*b*(c*log(-(
c - d)*e^(-2*x) - c - d)/d^2 - c*log(e^(-2*x) + 1)/d^2 - 2/(d*e^(-2*x) + d)
) + a^3*log(d*tanh(x) + c)/d
```

mupad [B] time = 2.73, size = 1347, normalized size = 17.27

$$\frac{2\left(2b^3d-b^3c+3ab^2d\right)}{d^2\left(2e^{2x}+e^{4x}+1\right)}-\frac{2\left(3a^2bd^2-3ab^2cd+3ab^2d^2+b^3c^2-b^3cd+b^3d^2\right)}{d^3\left(e^{2x}+1\right)}-\frac{8b^3}{3d\left(3e^{2x}+3e^{4x}+e^{6x}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tanh(x))^3/(cosh(x)^2*(c + d*tanh(x))),x)
```

```
[Out] (2*(2*b^3*d - b^3*c + 3*a*b^2*d))/(d^2*(2*exp(2*x) + exp(4*x) + 1)) - (2*(b
^3*c^2 + b^3*d^2 + 3*a*b^2*d^2 + 3*a^2*b*d^2 - b^3*c*d - 3*a*b^2*c*d))/(d^3
*(exp(2*x) + 1)) - (8*b^3)/(3*d*(3*exp(2*x) + 3*exp(4*x) + exp(6*x) + 1)) -
 (2*atan((((32*c*(a^6*d^6 + b^6*c^6 + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d
^3 + 15*a^4*b^2*c^2*d^4 - 6*a*b^5*c^5*d - 6*a^5*b*c*d^5)^(1/2)*(a^3*d^9 - a
^3*c*d^8 - b^3*c^3*d^6 + b^3*c^4*d^5 + 3*a*b^2*c^2*d^7 - 3*a*b^2*c^3*d^6 +
3*a^2*b*c^2*d^7 - 3*a^2*b*c*d^8))/(d^16*((a*d - b*c)^6)^(1/2)*(c + d)*(c -
d)^2*(2*c*d + c^2 + d^2)) - exp(2*x)*((32*c*(2*a^3*c*d^8 - 2*b^3*c^4*d^5 +
6*a*b^2*c^3*d^6 - 6*a^2*b*c^2*d^7)*(a^6*d^6 + b^6*c^6 + 15*a^2*b^4*c^4*d^2
- 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a*b^5*c^5*d - 6*a^5*b*c*d^5)^
(1/2))/(d^16*((a*d - b*c)^6)^(1/2)*(c + d)*(c - d)^2*(2*c*d + c^2 + d^2)) -
 (16*(c^2*(-d^8)^(1/2)*(a^6*d^6 + b^6*c^6 + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3
*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a*b^5*c^5*d - 6*a^5*b*c*d^5)^(1/2) + d^2*
(-d^8)^(1/2)*(a^6*d^6 + b^6*c^6 + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 +
 15*a^4*b^2*c^2*d^4 - 6*a*b^5*c^5*d - 6*a^5*b*c*d^5)^(1/2))*(c^2 + d^2)*(a^
6*d^6 + b^6*c^6 + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*
d^4 - 6*a*b^5*c^5*d - 6*a^5*b*c*d^5)^(1/2))/(d^13*(c + d)*(c - d)^2*(a*d -
b*c)^3*(-d^8)^(1/2)*(2*c*d + c^2 + d^2))) + (16*(c^2*(-d^8)^(1/2)*(a^6*d^6
+ b^6*c^6 + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 -
6*a*b^5*c^5*d - 6*a^5*b*c*d^5)^(1/2) - c*d*(-d^8)^(1/2)*(a^6*d^6 + b^6*c^6
+ 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a*b^5*c^
5*d - 6*a^5*b*c*d^5)^(1/2))*(c^2 + d^2)*(a^6*d^6 + b^6*c^6 + 15*a^2*b^4*c^4
*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a*b^5*c^5*d - 6*a^5*b*c*
d^5)^(1/2))/(d^13*(c + d)*(c - d)^2*(a*d - b*c)^3*(-d^8)^(1/2)*(2*c*d + c^2
 + d^2)))*(d^10*(-d^8)^(1/2) + 2*c*d^9*(-d^8)^(1/2) + c^2*d^8*(-d^8)^(1/2))
)/(16*(a^6*d^6 + b^6*c^6 + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4
*b^2*c^2*d^4 - 6*a*b^5*c^5*d - 6*a^5*b*c*d^5)^(1/2)))*(a^6*d^6 + b^6*c^6 +
```

```
15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a*b^5*c^5*
d - 6*a^5*b*c*d^5)^(1/2))/(-d^8)^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+b\tanh(x))^3 \operatorname{sech}^2(x)}{c+d\tanh(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)**2*(a+b*tanh(x))**3/(c+d*tanh(x)),x)
```

```
[Out] Integral((a + b*tanh(x))**3*sech(x)**2/(c + d*tanh(x)), x)
```

3.993 $\int \frac{\text{sech}^2(x)\tanh^2(x)}{\left(2+\tanh^3(x)\right)^2}\,dx$

Optimal. Leaf size=12

$$-\frac{1}{3\left(\tanh^3(x)+2\right)}$$

```
[Out] -1/3/(2+tanh(x)^3)
```

Rubi [A] time = 0.09, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {4342, 261}

$$-\frac{1}{3\left(\tanh^3(x)+2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(Sech[x]^2*Tanh[x]^2)/(2 + Tanh[x]^3)^2,x]
```

```
[Out] -1/(3*(2 + Tanh[x]^3))
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 4342

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^2, x_Symbol] :> With[{d = FreeFac
tors[Tan[c*(a + b*x)], x]}, Dist[d/(b*c), Subst[Int[SubstFor[1, Tan[c*(a +
b*x)]/d, u, x], x], x, Tan[c*(a + b*x)]/d], x] /; FunctionOfQ[Tan[c*(a + b*
x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && NonsumQ[u] && (EqQ[F, Sec] ||
 EqQ[F, sec])
```

Rubi steps

$$\int \frac{\operatorname{sech}^2(x)\tanh^2(x)}{\left(2+\tanh^3(x)\right)^2}\,dx = \text{Subst}\left(\int \frac{x^2}{\left(2+x^3\right)^2}\,dx, x, \tanh(x)\right)$$
$$= -\frac{1}{3\left(2+\tanh^3(x)\right)}$$

Mathematica [**A**] time = 0.04, size = 12, normalized size = 1.00

$$-\frac{1}{3\left(\tanh^3(x)+2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sech[x]^2*Tanh[x]^2)/(2 + Tanh[x]^3)^2,x]
```

```
[Out] -1/3*1/(2 + Tanh[x]^3)
```

fricas [**B**] time = 0.42, size = 73, normalized size = 6.08

$$-\frac{8\left(\cosh(x)^2+\cosh(x)\sinh(x)+\sinh(x)^2\right)}{9\left(3\cosh(x)^4+12\cosh(x)\sinh(x)^3+3\sinh(x)^4+2\left(9\cosh(x)^2+2\right)\sinh(x)^2+4\cosh(x)^2+4\left(3\cosh(x)^3\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^2*tanh(x)^2/(2+tanh(x)^3)^2,x, algorithm="fricas")
```

```
[Out] -8/9*(cosh(x)^2 + cosh(x)*sinh(x) + sinh(x)^2)/(3*cosh(x)^4 + 12*cosh(x)*si
nh(x)^3 + 3*sinh(x)^4 + 2*(9*cosh(x)^2 + 2)*sinh(x)^2 + 4*cosh(x)^2 + 4*(3*
cosh(x)^3 + cosh(x))*sinh(x) + 9)
```

giac [**B**] time = 0.12, size = 32, normalized size = 2.67

$$-\frac{2\left(3e^{(4x)}+1\right)}{9\left(3e^{(6x)}+3e^{(4x)}+9e^{(2x)}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^2*tanh(x)^2/(2+tanh(x)^3)^2,x, algorithm="giac")
```

```
[Out] -2/9*(3*e^(4*x) + 1)/(3*e^(6*x) + 3*e^(4*x) + 9*e^(2*x) + 1)
```

maple [A] time = 0.16, size = 11, normalized size = 0.92

$$-\frac{1}{3\left(2+\tanh^{3}(x)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(x)^2*tanh(x)^2/(2+tanh(x)^3)^2,x)
```

```
[Out] -1/3/(2+tanh(x)^3)
```

maxima [A] time = 0.30, size = 10, normalized size = 0.83

$$-\frac{1}{3\left(\tanh(x)^{3}+2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^2*tanh(x)^2/(2+tanh(x)^3)^2,x, algorithm="maxima")
```

```
[Out] -1/3/(tanh(x)^3 + 2)
```

mupad [B] time = 1.80, size = 32, normalized size = 2.67

$$-\frac{\frac{2\,e^{4\,x}}{3}+\frac{2}{9}}{9\,e^{2\,x}+3\,e^{4\,x}+3\,e^{6\,x}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^2/(cosh(x)^2*(tanh(x)^3 + 2)^2),x)
```

```
[Out] -((2*exp(4*x))/3 + 2/9)/(9*exp(2*x) + 3*exp(4*x) + 3*exp(6*x) + 1)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)**2*tanh(x)**2/(2+tanh(x)**3)**2,x)
```

```
[Out] Timed out
```

3.994 $\int \operatorname{sech}^2(x) \tanh^6(x) \left(1-\tanh^2(x)\right)^3 \, dx$

Optimal. Leaf size=33

$$-\frac{1}{13}\tanh^{13}(x)+\frac{3\tanh^{11}(x)}{11}-\frac{\tanh^9(x)}{3}+\frac{\tanh^7(x)}{7}$$

```
[Out] 1/7*tanh(x)^7-1/3*tanh(x)^9+3/11*tanh(x)^11-1/13*tanh(x)^13
```

Rubi [A] time = 0.11, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {3657, 2607, 270}

$$-\frac{1}{13}\tanh^{13}(x)+\frac{3\tanh^{11}(x)}{11}-\frac{\tanh^9(x)}{3}+\frac{\tanh^7(x)}{7}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[x]^2*Tanh[x]^6*(1 - Tanh[x]^2)^3,x]
```

```
[Out] Tanh[x]^7/7 - Tanh[x]^9/3 + (3*Tanh[x]^11)/11 - Tanh[x]^13/13
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 3657

```
Int[(u_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*sec[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a, b]
```

Rubi steps

$$\int \operatorname{sech}^2(x)\tanh^6(x)\left(1-\tanh^2(x)\right)^3\,dx = \int \operatorname{sech}^8(x)\tanh^6(x)\,dx$$
$$= i\,\mathrm{Subst}\left(\int x^6\left(1+x^2\right)^3\,dx, x, i\tanh(x)\right)$$
$$= i\,\mathrm{Subst}\left(\int \left(x^6+3x^8+3x^{10}+x^{12}\right)dx, x, i\tanh(x)\right)$$
$$= \frac{\tanh^7(x)}{7}-\frac{\tanh^9(x)}{3}+\frac{3\tanh^{11}(x)}{11}-\frac{\tanh^{13}(x)}{13}$$

Mathematica [B] time = 0.03, size = 67, normalized size = 2.03

$$\frac{16\tanh(x)}{3003}-\frac{1}{13}\tanh(x)\operatorname{sech}^{12}(x)+\frac{27}{143}\tanh(x)\operatorname{sech}^{10}(x)-\frac{53}{429}\tanh(x)\operatorname{sech}^{8}(x)+\frac{5\tanh(x)\operatorname{sech}^6(x)}{3003}+\frac{2\tanh(x)}{10}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[x]^2*Tanh[x]^6*(1 - Tanh[x]^2)^3,x]
```

```
[Out] (16*Tanh[x])/3003 + (8*Sech[x]^2*Tanh[x])/3003 + (2*Sech[x]^4*Tanh[x])/1001
 + (5*Sech[x]^6*Tanh[x])/3003 - (53*Sech[x]^8*Tanh[x])/429 + (27*Sech[x]^10
*Tanh[x])/143 - (Sech[x]^12*Tanh[x])/13
```

fricas [B] time = 0.44, size = 778, normalized size = 23.58

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^2*tanh(x)^6*(1-tanh(x)^2)^3,x, algorithm="fricas")
```

```
[Out] -64/3003*(1502*cosh(x)^9 + 13518*cosh(x)*sinh(x)^8 + 1501*sinh(x)^9 + (5403
6*cosh(x)^2 - 4511)*sinh(x)^7 - 4498*cosh(x)^7 + 14*(9012*cosh(x)^3 - 2249*
cosh(x))*sinh(x)^6 + 3*(63042*cosh(x)^4 - 31577*cosh(x)^2 + 2990)*sinh(x)^5
 + 9048*cosh(x)^5 + 2*(94626*cosh(x)^5 - 78715*cosh(x)^3 + 22620*cosh(x))*s
inh(x)^4 + (126084*cosh(x)^6 - 157885*cosh(x)^4 + 89700*cosh(x)^2 - 8294)*s
inh(x)^3 - 8008*cosh(x)^3 + 6*(9012*cosh(x)^7 - 15743*cosh(x)^5 + 15080*cos
h(x)^3 - 4004*cosh(x))*sinh(x)^2 + (13509*cosh(x)^8 - 31577*cosh(x)^6 + 448
50*cosh(x)^4 - 24882*cosh(x)^2 + 6292)*sinh(x) + 4004*cosh(x))/(cosh(x)^17
+ 17*cosh(x)*sinh(x)^16 + sinh(x)^17 + (136*cosh(x)^2 + 13)*sinh(x)^15 + 13
*cosh(x)^15 + 5*(136*cosh(x)^3 + 39*cosh(x))*sinh(x)^14 + (2380*cosh(x)^4 +
 1365*cosh(x)^2 + 78)*sinh(x)^13 + 78*cosh(x)^13 + 13*(476*cosh(x)^5 + 455*
cosh(x)^3 + 78*cosh(x))*sinh(x)^12 + 13*(952*cosh(x)^6 + 1365*cosh(x)^4 + 4
68*cosh(x)^2 + 22)*sinh(x)^11 + 286*cosh(x)^11 + 143*(136*cosh(x)^7 + 273*c
```

```
osh(x)^5 + 156*cosh(x)^3 + 22*cosh(x))*sinh(x)^10 + (24310*cosh(x)^8 + 6506
5*cosh(x)^6 + 55770*cosh(x)^4 + 15730*cosh(x)^2 + 714)*sinh(x)^9 + 716*cosh
(x)^9 + (24310*cosh(x)^9 + 83655*cosh(x)^7 + 100386*cosh(x)^5 + 47190*cosh(
x)^3 + 6444*cosh(x))*sinh(x)^8 + (19448*cosh(x)^10 + 83655*cosh(x)^8 + 1338
48*cosh(x)^6 + 94380*cosh(x)^4 + 25704*cosh(x)^2 + 1274)*sinh(x)^7 + 1300*c
osh(x)^7 + (12376*cosh(x)^11 + 65065*cosh(x)^9 + 133848*cosh(x)^7 + 132132*
cosh(x)^5 + 60144*cosh(x)^3 + 9100*cosh(x))*sinh(x)^6 + (6188*cosh(x)^12 +
39039*cosh(x)^10 + 100386*cosh(x)^8 + 132132*cosh(x)^6 + 89964*cosh(x)^4 +
26754*cosh(x)^2 + 1638)*sinh(x)^5 + 1794*cosh(x)^5 + (2380*cosh(x)^13 + 177
45*cosh(x)^11 + 55770*cosh(x)^9 + 94380*cosh(x)^7 + 90216*cosh(x)^5 + 45500
*cosh(x)^3 + 8970*cosh(x))*sinh(x)^4 + (680*cosh(x)^14 + 5915*cosh(x)^12 +
22308*cosh(x)^10 + 47190*cosh(x)^8 + 59976*cosh(x)^6 + 44590*cosh(x)^4 + 16
380*cosh(x)^2 + 1430)*sinh(x)^3 + 2002*cosh(x)^3 + (136*cosh(x)^15 + 1365*c
osh(x)^13 + 6084*cosh(x)^11 + 15730*cosh(x)^9 + 25776*cosh(x)^7 + 27300*cos
h(x)^5 + 17940*cosh(x)^3 + 6006*cosh(x))*sinh(x)^2 + (17*cosh(x)^16 + 195*c
osh(x)^14 + 1014*cosh(x)^12 + 3146*cosh(x)^10 + 6426*cosh(x)^8 + 8918*cosh(
x)^6 + 8190*cosh(x)^4 + 4290*cosh(x)^2 + 572)*sinh(x) + 2002*cosh(x))
```

giac [B] time = 0.16, size = 66, normalized size = 2.00

$$-\frac{32\left(3003\,e^{(18\,x)} - 9009\,e^{(16\,x)} + 18018\,e^{(14\,x)} - 16302\,e^{(12\,x)} + 10296\,e^{(10\,x)} - 2288\,e^{(8\,x)} + 286\,e^{(6\,x)} + 78\,e^{(4\,x)} + 13\,e^{(2\,x)} + 1\right)}{3003\left(e^{(2\,x)} + 1\right)^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^2*tanh(x)^6*(1-tanh(x)^2)^3,x, algorithm="giac")
```

```
[Out] -32/3003*(3003*e^(18*x) - 9009*e^(16*x) + 18018*e^(14*x) - 16302*e^(12*x) +
 10296*e^(10*x) - 2288*e^(8*x) + 286*e^(6*x) + 78*e^(4*x) + 13*e^(2*x) + 1)
/(e^(2*x) + 1)^13
```

maple [B] time = 0.64, size = 306, normalized size = 9.27

$$-\frac{\sinh^5(x)}{2\cosh(x)^7}-\frac{5\left(\sinh^3(x)\right)}{8\cosh(x)^7}-\frac{5\sinh(x)}{16\cosh(x)^7}+\frac{5\left(\frac{16}{35}+\frac{\operatorname{sech}(x)^6}{7}+\frac{6\operatorname{sech}(x)^4}{35}+\frac{8\operatorname{sech}(x)^2}{35}\right)\tanh(x)}{16}+\frac{3\left(\sinh^7(x)\right)}{2\cosh(x)^9}+\frac{21\left(\sinh\right.}{8\cosh(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(x)^2*tanh(x)^6*(1-tanh(x)^2)^3,x)
```

```
[Out] -1/2*sinh(x)^5/cosh(x)^7-5/8*sinh(x)^3/cosh(x)^7-5/16*sinh(x)/cosh(x)^7+5/1
6*(16/35+1/7*sech(x)^6+6/35*sech(x)^4+8/35*sech(x)^2)*tanh(x)+3/2*sinh(x)^7
/cosh(x)^9+21/8*sinh(x)^5/cosh(x)^9+35/16*sinh(x)^3/cosh(x)^9+105/128*sinh(
x)/cosh(x)^9-105/128*(128/315+1/9*sech(x)^8+8/63*sech(x)^6+16/105*sech(x)^4
+64/315*sech(x)^2)*tanh(x)-3/2*sinh(x)^9/cosh(x)^11-27/8*sinh(x)^7/cosh(x)^
```

```
11-63/16*sinh(x)^5/cosh(x)^11-315/128*sinh(x)^3/cosh(x)^11-189/256*sinh(x)/
cosh(x)^11+189/256*(256/693+1/11*sech(x)^10+10/99*sech(x)^8+80/693*sech(x)^
6+32/231*sech(x)^4+128/693*sech(x)^2)*tanh(x)+1/2*sinh(x)^11/cosh(x)^13+11/
8*sinh(x)^9/cosh(x)^13+33/16*sinh(x)^7/cosh(x)^13+231/128*sinh(x)^5/cosh(x)
^13+231/256*sinh(x)^3/cosh(x)^13+231/1024*sinh(x)/cosh(x)^13-231/1024*(1024
/3003+1/13*sech(x)^12+12/143*sech(x)^10+40/429*sech(x)^8+320/3003*sech(x)^6
+128/1001*sech(x)^4+512/3003*sech(x)^2)*tanh(x)
```

maxima [A] time = 0.30, size = 25, normalized size = 0.76

$$-\frac{1}{13}\tanh(x)^{13}+\frac{3}{11}\tanh(x)^{11}-\frac{1}{3}\tanh(x)^{9}+\frac{1}{7}\tanh(x)^{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^2*tanh(x)^6*(1-tanh(x)^2)^3,x, algorithm="maxima")
```

```
[Out] -1/13*tanh(x)^13 + 3/11*tanh(x)^11 - 1/3*tanh(x)^9 + 1/7*tanh(x)^7
```

mupad [B] time = 1.74, size = 820, normalized size = 24.85

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(tanh(x)^6*(tanh(x)^2 - 1)^3)/cosh(x)^2,x)
```

```
[Out] - ((64*exp(4*x))/143 - (256*exp(2*x))/429 + 80/429)/(6*exp(2*x) + 15*exp(4*
x) + 20*exp(6*x) + 15*exp(8*x) + 6*exp(10*x) + exp(12*x) + 1) - ((64*exp(2*
x))/143 - (768*exp(4*x))/143 + (3200*exp(6*x))/143 - (6400*exp(8*x))/143 +
(6720*exp(10*x))/143 - (3584*exp(12*x))/143 + (768*exp(14*x))/143)/(11*exp(
2*x) + 55*exp(4*x) + 165*exp(6*x) + 330*exp(8*x) + 462*exp(10*x) + 462*exp(
12*x) + 330*exp(14*x) + 165*exp(16*x) + 55*exp(18*x) + 11*exp(20*x) + exp(2
2*x) + 1) - ((160*exp(2*x))/143 - (256*exp(4*x))/143 + (128*exp(6*x))/143 -
 640/3003)/(7*exp(2*x) + 21*exp(4*x) + 35*exp(6*x) + 35*exp(8*x) + 21*exp(1
0*x) + 7*exp(12*x) + exp(14*x) + 1) - ((128*exp(6*x))/13 - (768*exp(8*x))/1
3 + (1920*exp(10*x))/13 - (2560*exp(12*x))/13 + (1920*exp(14*x))/13 - (768*
exp(16*x))/13 + (128*exp(18*x))/13)/(13*exp(2*x) + 78*exp(4*x) + 286*exp(6*
x) + 715*exp(8*x) + 1287*exp(10*x) + 1716*exp(12*x) + 1716*exp(14*x) + 1287
*exp(16*x) + 715*exp(18*x) + 286*exp(20*x) + 78*exp(22*x) + 13*exp(24*x) +
exp(26*x) + 1) - ((560*exp(4*x))/143 - (640*exp(2*x))/429 - (1792*exp(6*x))
/429 + (224*exp(8*x))/143 + 80/429)/(8*exp(2*x) + 28*exp(4*x) + 56*exp(6*x)
 + 70*exp(8*x) + 56*exp(10*x) + 28*exp(12*x) + 8*exp(14*x) + exp(16*x) + 1)
 - ((640*exp(2*x))/429 - (2560*exp(4*x))/429 + (4480*exp(6*x))/429 - (3584*
exp(8*x))/429 + (1792*exp(10*x))/715 - 256/2145)/(9*exp(2*x) + 36*exp(4*x)
+ 84*exp(6*x) + 126*exp(8*x) + 126*exp(10*x) + 84*exp(12*x) + 36*exp(14*x)
+ 9*exp(16*x) + exp(18*x) + 1) - ((32*exp(4*x))/13 - (256*exp(6*x))/13 + (8
```

```
00*exp(8*x))/13 - (1280*exp(10*x))/13 + (1120*exp(12*x))/13 - (512*exp(14*x
))/13 + (96*exp(16*x))/13)/(12*exp(2*x) + 66*exp(4*x) + 220*exp(6*x) + 495*
exp(8*x) + 792*exp(10*x) + 924*exp(12*x) + 792*exp(14*x) + 495*exp(16*x) +
220*exp(18*x) + 66*exp(20*x) + 12*exp(22*x) + exp(24*x) + 1) - ((128*exp(2*
x))/715 - 256/2145)/(5*exp(2*x) + 10*exp(4*x) + 10*exp(6*x) + 5*exp(8*x) +
exp(10*x) + 1) - 32/(715*(4*exp(2*x) + 6*exp(4*x) + 4*exp(6*x) + exp(8*x) +
 1)) - ((960*exp(4*x))/143 - (768*exp(2*x))/715 - (2560*exp(6*x))/143 + (33
60*exp(8*x))/143 - (10752*exp(10*x))/715 + (2688*exp(12*x))/715 + 32/715)/(
10*exp(2*x) + 45*exp(4*x) + 120*exp(6*x) + 210*exp(8*x) + 252*exp(10*x) + 2
10*exp(12*x) + 120*exp(14*x) + 45*exp(16*x) + 10*exp(18*x) + exp(20*x) + 1)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int\left(-\tanh^{6}(x)\operatorname{sech}^{2}(x)\right)dx-\int 3\tanh^{8}(x)\operatorname{sech}^{2}(x)\,dx-\int\left(-3\tanh^{10}(x)\operatorname{sech}^{2}(x)\right)dx-\int\tanh^{12}(x)\operatorname{sech}^{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)**2*tanh(x)**6*(1-tanh(x)**2)**3,x)
```

```
[Out] -Integral(-tanh(x)**6*sech(x)**2, x) - Integral(3*tanh(x)**8*sech(x)**2, x)
 - Integral(-3*tanh(x)**10*sech(x)**2, x) - Integral(tanh(x)**12*sech(x)**2
, x)
```

3.995 $\int \frac{\operatorname{sech}^2(x)\left(2+\tanh^2(x)\right)}{1+\tanh^3(x)} dx$

Optimal. Leaf size=26

$$\log(\tanh(x)+1)-\frac{2\tan^{-1}\left(\frac{1-2\tanh(x)}{\sqrt{3}}\right)}{\sqrt{3}}$$

```
[Out] ln(1+tanh(x))-2/3*arctan(1/3*(1-2*tanh(x))*3^(1/2))*3^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {4342, 1863, 31, 618, 204}

$$\log(\tanh(x)+1)-\frac{2\tan^{-1}\left(\frac{1-2\tanh(x)}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sech[x]^2*(2 + Tanh[x]^2))/(1 + Tanh[x]^3),x]
```

```
[Out] (-2*ArcTan[(1 - 2*Tanh[x])/Sqrt[3]])/Sqrt[3] + Log[1 + Tanh[x]]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1863

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, With[{q = a^(1/3)/b^(1/3)}, Dist[C
```

```
/b, Int[1/(q + x), x], x] + Dist[(B + C*q)/b, Int[1/(q^2 - q*x + x^2), x],
x]] /; EqQ[A*b^(2/3) - a^(1/3)*b^(1/3)*B - 2*a^(2/3)*C, 0]] /; FreeQ[{a, b}
, x] && PolyQ[P2, x, 2]
```

Rule 4342

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^2, x_Symbol] :> With[{d = FreeFac
tors[Tan[c*(a + b*x)], x]}, Dist[d/(b*c), Subst[Int[SubstFor[1, Tan[c*(a +
b*x)]/d, u, x], x], x, Tan[c*(a + b*x)]/d], x] /; FunctionOfQ[Tan[c*(a + b*
x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && NonsumQ[u] && (EqQ[F, Sec] ||
 EqQ[F, sec])
```

Rubi steps

$$\begin{aligned}\int \frac{\operatorname{sech}^2(x)\left(2+\tanh^2(x)\right)}{1+\tanh^3(x)}\,dx &= \operatorname{Subst}\left(\int \frac{2+x^2}{1+x^3}\,dx, x, \tanh(x)\right)\\ &= \operatorname{Subst}\left(\int \frac{1}{1+x}\,dx, x, \tanh(x)\right)+\operatorname{Subst}\left(\int \frac{1}{1-x+x^2}\,dx, x, \tanh(x)\right)\\ &= \log(1+\tanh(x))-2\operatorname{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, -1+2\tanh(x)\right)\\ &= -\frac{2\tan^{-1}\left(\frac{1-2\tanh(x)}{\sqrt{3}}\right)}{\sqrt{3}}+\log(1+\tanh(x))\end{aligned}$$

Mathematica [A] time = 0.22, size = 27, normalized size = 1.04

$$x+\frac{2\tan^{-1}\left(\frac{2\tanh(x)-1}{\sqrt{3}}\right)}{\sqrt{3}}-\log(\cosh(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sech[x]^2*(2 + Tanh[x]^2))/(1 + Tanh[x]^3),x]
[Out] x + (2*ArcTan[(-1 + 2*Tanh[x])/Sqrt[3]])/Sqrt[3] - Log[Cosh[x]]
```

fricas [B] time = 0.44, size = 50, normalized size = 1.92

$$-\frac{2}{3}\sqrt{3}\arctan\left(-\frac{\sqrt{3}\cosh(x)+\sqrt{3}\sinh(x)}{3\left(\cosh(x)-\sinh(x)\right)}\right)+2x-\log\left(\frac{2\cosh(x)}{\cosh(x)-\sinh(x)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^2*(2+tanh(x)^2)/(1+tanh(x)^3),x, algorithm="fricas")
```

```
[Out] -2/3*sqrt(3)*arctan(-1/3*(sqrt(3)*cosh(x) + sqrt(3)*sinh(x))/(cosh(x) - sin
h(x))) + 2*x - log(2*cosh(x)/(cosh(x) - sinh(x)))
```

giac [A] time = 0.14, size = 28, normalized size = 1.08

$$\frac{2}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}e^{(2x)}\right) + 2x - \log\left(e^{(2x)} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^2*(2+tanh(x)^2)/(1+tanh(x)^3),x, algorithm="giac")
```

```
[Out] 2/3*sqrt(3)*arctan(1/3*sqrt(3)*e^(2*x)) + 2*x - log(e^(2*x) + 1)
```

maple [C] time = 0.26, size = 78, normalized size = 3.00

$$\frac{i\sqrt{3}\ \ln\left(\tanh^2\left(\frac{x}{2}\right) + \left(-i\sqrt{3}\ -1\right)\tanh\left(\frac{x}{2}\right) + 1\right)}{3} - \frac{i\sqrt{3}\ \ln\left(\tanh^2\left(\frac{x}{2}\right) + \left(i\sqrt{3}\ -1\right)\tanh\left(\frac{x}{2}\right) + 1\right)}{3} + 2\ln\left(\tanh\left(\frac{x}{2}\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(x)^2*(2+tanh(x)^2)/(1+tanh(x)^3),x)
```

```
[Out] 1/3*I*3^(1/2)*ln(tanh(1/2*x)^2+(-I*3^(1/2)-1)*tanh(1/2*x)+1)-1/3*I*3^(1/2)*
ln(tanh(1/2*x)^2+(I*3^(1/2)-1)*tanh(1/2*x)+1)+2*ln(tanh(1/2*x)+1)-ln(tanh(1
/2*x)^2+1)
```

maxima [B] time = 0.41, size = 122, normalized size = 4.69

$$\frac{2}{3}\sqrt{3}\arctan\left(\frac{1}{6}\cdot 3^{\frac{3}{4}}\sqrt{2}\left(2\sqrt{3}e^{(-x)} + 3^{\frac{1}{4}}\sqrt{2}\right)\right) - \frac{2}{3}\sqrt{3}\arctan\left(\frac{1}{6}\cdot 3^{\frac{3}{4}}\sqrt{2}\left(2\sqrt{3}e^{(-x)} - 3^{\frac{1}{4}}\sqrt{2}\right)\right) + \frac{1}{3}\log\left(\tanh(x)^3\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^2*(2+tanh(x)^2)/(1+tanh(x)^3),x, algorithm="maxima")
```

```
[Out] 2/3*sqrt(3)*arctan(1/6*3^(3/4)*sqrt(2)*(2*sqrt(3)*e^(-x) + 3^(1/4)*sqrt(2))
) - 2/3*sqrt(3)*arctan(1/6*3^(3/4)*sqrt(2)*(2*sqrt(3)*e^(-x) - 3^(1/4)*sqrt
(2))) + 1/3*log(tanh(x)^3 + 1) - 1/3*log(3^(1/4)*sqrt(2)*e^(-x) + sqrt(3)*e
^(-2*x) + 1) - 1/3*log(-3^(1/4)*sqrt(2)*e^(-x) + sqrt(3)*e^(-2*x) + 1)
```

mupad [B] time = 0.50, size = 47, normalized size = 1.81

$$2x - \ln\left(768\,e^{2x} + 768\right) - \frac{2\sqrt{3}\ \mathrm{atan}\left(\frac{\frac{640\sqrt{3}}{3} - \frac{128\sqrt{3}\,e^{2x}}{3}}{\frac{640\,e^{2x}}{3} + 128}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((tanh(x)^2 + 2)/(cosh(x)^2*(tanh(x)^3 + 1)),x)
```

```
[Out] 2*x - log(768*exp(2*x) + 768) - (2*3^(1/2)*atan(((640*3^(1/2))/3 - (128*3^(
1/2)*exp(2*x))/3)/((640*exp(2*x))/3 + 128)))/3
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\tanh^{2}(x)+2\right)\operatorname{sech}^{2}(x)}{\left(\tanh(x)+1\right)\left(\tanh^{2}(x)-\tanh(x)+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)**2*(2+tanh(x)**2)/(1+tanh(x)**3),x)
```

```
[Out] Integral((tanh(x)**2 + 2)*sech(x)**2/((tanh(x) + 1)*(tanh(x)**2 - tanh(x) +
 1)), x)
```

3.996 $\int \left(1+\cosh^2(x)\right) \operatorname{sech}^2(x)\, dx$

Optimal. Leaf size=4

$$x+\tanh(x)$$

```
[Out] x+tanh(x)
```

Rubi [A] time = 0.02, antiderivative size = 4, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {3012, 8}

$$x+\tanh(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Cosh[x]^2)*Sech[x]^2,x]
```

```
[Out] x + Tanh[x]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3012

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x
_)]^2), x_Symbol] :> Simp[(A*Cos[e + f*x]*(b*Sin[e + f*x])^(m + 1))/(b*f*(m
 + 1)), x] + Dist[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1)), Int[(b*Sin[e + f*x
])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \left(1+\cosh^2(x)\right) \operatorname{sech}^2(x)\, dx &= \tanh(x)+\int 1\, dx\\ &= x+\tanh(x)\end{aligned}$$

Mathematica [A] time = 0.00, size = 4, normalized size = 1.00

$$x+\tanh(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Cosh[x]^2)*Sech[x]^2,x]
```

```
[Out] x + Tanh[x]
```

fricas [B] time = 0.47, size = 14, normalized size = 3.50

$$\frac{(x-1)\cosh(x)+\sinh(x)}{\cosh(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+cosh(x)^2)*sech(x)^2,x, algorithm="fricas")
[Out] ((x - 1)*cosh(x) + sinh(x))/cosh(x)
```

giac [B] time = 0.11, size = 12, normalized size = 3.00

$$x-\frac{2}{e^{(2x)}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+cosh(x)^2)*sech(x)^2,x, algorithm="giac")
[Out] x - 2/(e^(2*x) + 1)
```

maple [A] time = 0.37, size = 5, normalized size = 1.25

$$x+\tanh(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+cosh(x)^2)*sech(x)^2,x)
[Out] x+tanh(x)
```

maxima [B] time = 0.31, size = 12, normalized size = 3.00

$$x+\frac{2}{e^{(-2x)}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+cosh(x)^2)*sech(x)^2,x, algorithm="maxima")
[Out] x + 2/(e^(-2*x) + 1)
```

mupad [B] time = 1.65, size = 12, normalized size = 3.00

$$x-\frac{2}{e^{2x}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cosh(x)^2 + 1)/cosh(x)^2,x)
```

```
[Out] x - 2/(exp(2*x) + 1)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(\cosh^2(x)+1\right)\operatorname{sech}^2(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+cosh(x)**2)*sech(x)**2,x)
```

```
[Out] Integral((cosh(x)**2 + 1)*sech(x)**2, x)
```

3.997 $\int \frac{\text{sech}^2(x)}{1+\text{sech}^2(x)-3\tanh(x)}\,dx$

Optimal. Leaf size=20

$$\frac{2\tanh^{-1}\left(\frac{2\tanh(x)+3}{\sqrt{17}}\right)}{\sqrt{17}}$$

```
[Out] 2/17*arctanh(1/17*(3+2*tanh(x))*17^(1/2))*17^(1/2)
```

Rubi [A] time = 0.12, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {618, 206}

$$\frac{2\tanh^{-1}\left(\frac{2\tanh(x)+3}{\sqrt{17}}\right)}{\sqrt{17}}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[x]^2/(1 + Sech[x]^2 - 3*Tanh[x]),x]
```

```
[Out] (2*ArcTanh[(3 + 2*Tanh[x])/Sqrt[17]])/Sqrt[17]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{\operatorname{sech}^2(x)}{1+\operatorname{sech}^2(x)-3\tanh(x)}\,dx = \operatorname{Subst}\left(\int \frac{1}{2-3x-x^2}\,dx, x, \tanh(x)\right)$$
$$= -\left(2\operatorname{Subst}\left(\int \frac{1}{17-x^2}\,dx, x, -3-2\tanh(x)\right)\right)$$
$$= \frac{2\tanh^{-1}\left(\frac{3+2\tanh(x)}{\sqrt{17}}\right)}{\sqrt{17}}$$

Mathematica [F] time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}^2(x)}{1+\operatorname{sech}^2(x)-3\tanh(x)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sech[x]^2/(1 + Sech[x]^2 - 3*Tanh[x]),x]
[Out] Integrate[Sech[x]^2/(1 + Sech[x]^2 - 3*Tanh[x]), x]
```

fricas [B] time = 0.41, size = 67, normalized size = 3.35

$$\frac{1}{17}\sqrt{17}\log\left(\frac{3\left(\sqrt{17}-5\right)\cosh(x)^2-2\left(3\sqrt{17}-11\right)\cosh(x)\sinh(x)+3\left(\sqrt{17}-5\right)\sinh(x)^2-2\sqrt{17}+6}{\cosh(x)^2-6\cosh(x)\sinh(x)+\sinh(x)^2+3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^2/(1+sech(x)^2-3*tanh(x)),x, algorithm="fricas")
[Out] 1/17*sqrt(17)*log((3*(sqrt(17) - 5)*cosh(x)^2 - 2*(3*sqrt(17) - 11)*cosh(x)
*sinh(x) + 3*(sqrt(17) - 5)*sinh(x)^2 - 2*sqrt(17) + 6)/(cosh(x)^2 - 6*cosh
(x)*sinh(x) + sinh(x)^2 + 3))
```

giac [B] time = 0.12, size = 35, normalized size = 1.75

$$-\frac{1}{17}\sqrt{17}\log\left(\frac{\left|-\sqrt{17}+2e^{(2x)}-3\right|}{\left|\sqrt{17}+2e^{(2x)}-3\right|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^2/(1+sech(x)^2-3*tanh(x)),x, algorithm="giac")
```

```
[Out] -1/17*sqrt(17)*log(abs(-sqrt(17) + 2*e^(2*x) - 3)/abs(sqrt(17) + 2*e^(2*x)
- 3))
```

maple [B] time = 0.33, size = 63, normalized size = 3.15

$$-\frac{\sqrt{17}\,\ln\left(-\sqrt{17}\,\tanh\left(\frac{x}{2}\right)+2\left(\tanh^{2}\left(\frac{x}{2}\right)\right)-3\tanh\left(\frac{x}{2}\right)+2\right)}{17}+\frac{\sqrt{17}\,\ln\left(\sqrt{17}\,\tanh\left(\frac{x}{2}\right)+2\left(\tanh^{2}\left(\frac{x}{2}\right)\right)-3\tanh\right.}{17}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(x)^2/(1+sech(x)^2-3*tanh(x)),x)
```

```
[Out] -1/17*17^(1/2)*ln(-17^(1/2)*tanh(1/2*x)+2*tanh(1/2*x)^2-3*tanh(1/2*x)+2)+1/
17*17^(1/2)*ln(17^(1/2)*tanh(1/2*x)+2*tanh(1/2*x)^2-3*tanh(1/2*x)+2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}(x)^2}{\operatorname{sech}(x)^2-3\,\tanh(x)+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^2/(1+sech(x)^2-3*tanh(x)),x, algorithm="maxima")
```

```
[Out] integrate(sech(x)^2/(sech(x)^2 - 3*tanh(x) + 1), x)
```

mupad [B] time = 1.79, size = 50, normalized size = 2.50

$$-\frac{\sqrt{17}\,\left(\ln\left(2\,e^{2\,x}-\frac{\sqrt{17}\,\left(6\,e^{2\,x}+8\right)}{17}\right)-\ln\left(2\,e^{2\,x}+\frac{\sqrt{17}\,\left(6\,e^{2\,x}+8\right)}{17}\right)\right)}{17}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(x)^2*(1/cosh(x)^2 - 3*tanh(x) + 1)),x)
```

```
[Out] -(17^(1/2)*(log(2*exp(2*x) - (17^(1/2)*(6*exp(2*x) + 8))/17) - log(2*exp(2*
x) + (17^(1/2)*(6*exp(2*x) + 8))/17)))/17
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}^{2}(x)}{-3\tanh(x)+\operatorname{sech}^{2}(x)+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)**2/(1+sech(x)**2-3*tanh(x)),x)
```

```
[Out] Integral(sech(x)**2/(-3*tanh(x) + sech(x)**2 + 1), x)
```

3.998 $\int \frac{\text{sech}^2(x)}{\sqrt{4-\text{sech}^2(x)}}\,dx$

Optimal. Leaf size=9

$$\sinh^{-1}\left(\frac{\tanh(x)}{\sqrt{3}}\right)$$

```
[Out] arcsinh(1/3*tanh(x)*3^(1/2))
```

Rubi [A] time = 0.05, antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {4146, 215}

$$\sinh^{-1}\left(\frac{\tanh(x)}{\sqrt{3}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sech[x]^2/Sqrt[4 - Sech[x]^2],x]
```

```
[Out] ArcSinh[Tanh[x]/Sqrt[3]]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 4146

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, S
ubst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x
]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[
m/2] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned} \int \frac{\text{sech}^2(x)}{\sqrt{4-\text{sech}^2(x)}}\,dx &= \text{Subst}\left(\int \frac{1}{\sqrt{3+x^2}}\,dx, x, \tanh(x)\right) \\ &= \sinh^{-1}\left(\frac{\tanh(x)}{\sqrt{3}}\right) \end{aligned}$$

Mathematica [B] time = 0.04, size = 43, normalized size = 4.78

$$\frac{\sqrt{2\cosh(2x)+1}\operatorname{sech}(x)\tanh^{-1}\left(\frac{\sinh(x)}{\sqrt{4\sinh^2(x)+3}}\right)}{\sqrt{4-\operatorname{sech}^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[x]^2/Sqrt[4 - Sech[x]^2],x]
```

```
[Out] (ArcTanh[Sinh[x]/Sqrt[3 + 4*Sinh[x]^2]]*Sqrt[1 + 2*Cosh[2*x]]*Sech[x])/Sqrt
[4 - Sech[x]^2]
```

fricas [B] time = 0.42, size = 112, normalized size = 12.44

$$-\log\left(-\cosh(x)^2-2\cosh(x)\sinh(x)-\sinh(x)^2+\sqrt{\frac{2\cosh(x)^2+2\sinh(x)^2+1}{\cosh(x)^2-2\cosh(x)\sinh(x)+\sinh(x)^2}}\right)+\log\left(-\cosh(x\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^2/(4-sech(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -log(-cosh(x)^2 - 2*cosh(x)*sinh(x) - sinh(x)^2 + sqrt((2*cosh(x)^2 + 2*sin
h(x)^2 + 1)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))) + log(-cosh(x)^2
- 2*cosh(x)*sinh(x) - sinh(x)^2 + sqrt((2*cosh(x)^2 + 2*sinh(x)^2 + 1)/(cos
h(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) - 2)
```

giac [B] time = 0.15, size = 44, normalized size = 4.89

$$-\log\left(\sqrt{e^{(4x)}+e^{(2x)}+1}-e^{(2x)}\right)+\log\left(-\sqrt{e^{(4x)}+e^{(2x)}+1}+e^{(2x)}+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^2/(4-sech(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] -log(sqrt(e^(4*x) + e^(2*x) + 1) - e^(2*x)) + log(-sqrt(e^(4*x) + e^(2*x) +
 1) + e^(2*x) + 2)
```

maple [F] time = 0.39, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}(x)^2}{\sqrt{4-\operatorname{sech}(x)^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(x)^2/(4-sech(x)^2)^(1/2),x)
```

```
[Out] int(sech(x)^2/(4-sech(x)^2)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}(x)^2}{\sqrt{-\operatorname{sech}(x)^2+4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^2/(4-sech(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sech(x)^2/sqrt(-sech(x)^2 + 4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.11

$$\int \frac{1}{\cosh(x)^2\,\sqrt{4-\frac{1}{\cosh(x)^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(x)^2*(4 - 1/cosh(x)^2)^(1/2)),x)
```

```
[Out] int(1/(cosh(x)^2*(4 - 1/cosh(x)^2)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}^2(x)}{\sqrt{-(\operatorname{sech}(x)-2)(\operatorname{sech}(x)+2)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)**2/(4-sech(x)**2)**(1/2),x)
```

```
[Out] Integral(sech(x)**2/sqrt(-(sech(x) - 2)*(sech(x) + 2)), x)
```

3.999 $\int \frac{\operatorname{sech}^2(x)}{\sqrt{1-4\tanh^2(x)}}\,dx$

Optimal. Leaf size=9

$$\frac{1}{2}\sin^{-1}(2\tanh(x))$$

```
[Out] 1/2*arcsin(2*tanh(x))
```

Rubi [A] time = 0.05, antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {3675, 216}

$$\frac{1}{2}\sin^{-1}(2\tanh(x))$$

Antiderivative was successfully verified.

```
[In] Int[Sech[x]^2/Sqrt[1 - 4*Tanh[x]^2],x]
```

```
[Out] ArcSin[2*Tanh[x]]/2
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 3675

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/(c^(m - 1)*f), Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^p
, x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && In
tegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
|| EqQ[n^2, 16])
```

Rubi steps

$$\begin{aligned}\int \frac{\operatorname{sech}^2(x)}{\sqrt{1-4\tanh^2(x)}}\,dx &= \text{Subst}\left(\int \frac{1}{\sqrt{1-4x^2}}\,dx, x, \tanh(x)\right)\\ &= \frac{1}{2}\sin^{-1}(2\tanh(x))\end{aligned}$$

Mathematica [B] time = 0.06, size = 47, normalized size = 5.22

$$\frac{\sqrt{3\cosh(2x)-5}\,\mathrm{sech}(x)\tanh^{-1}\left(\frac{2\sinh(x)}{\sqrt{3\sinh^2(x)-1}}\right)}{2\sqrt{2-8\tanh^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[x]^2/Sqrt[1 - 4*Tanh[x]^2],x]
```

```
[Out] (ArcTanh[(2*Sinh[x])/Sqrt[-1 + 3*Sinh[x]^2]]*Sqrt[-5 + 3*Cosh[2*x]]*Sech[x]
)/(2*Sqrt[2 - 8*Tanh[x]^2])
```

fricas [B] time = 0.46, size = 118, normalized size = 13.11

$$-\frac{1}{2}\arctan\left(\frac{2\sqrt{2}\left(\cosh(x)^2+2\cosh(x)\sinh(x)+\sinh(x)^2-1\right)\sqrt{-\frac{3\cosh(x)^2+3\sin}{\cosh(x)^2-2\cosh(x)\sinh}}}{3\cosh(x)^4+12\cosh(x)\sinh(x)^3+3\sinh(x)^4+2\left(9\cosh(x)^2-5\right)\sinh(x)^2-10\cosh(x)^2+4\left(3\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^2/(1-4*tanh(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/2*arctan(2*sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*sqrt(
-(3*cosh(x)^2 + 3*sinh(x)^2 - 5)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2
))/(3*cosh(x)^4 + 12*cosh(x)*sinh(x)^3 + 3*sinh(x)^4 + 2*(9*cosh(x)^2 - 5)*
sinh(x)^2 - 10*cosh(x)^2 + 4*(3*cosh(x)^3 - 5*cosh(x))*sinh(x) + 3))
```

giac [B] time = 0.16, size = 44, normalized size = 4.89

$$-\arctan\left(\frac{1}{3}\sqrt{3}\left(\frac{2\left(\sqrt{3}\sqrt{-3e^{(4x)}+10e^{(2x)}-3}-4\right)}{3e^{(2x)}-5}-1\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^2/(1-4*tanh(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] -arctan(1/3*sqrt(3)*(2*(sqrt(3)*sqrt(-3*e^(4*x) + 10*e^(2*x) - 3) - 4)/(3*e
^(2*x) - 5) - 1))
```

maple [F] time = 0.52, size = 0, normalized size = 0.00

$$\int\frac{\mathrm{sech}(x)^2}{\sqrt{1-4\left(\tanh^2(x)\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(x)^2/(1-4*tanh(x)^2)^(1/2),x)
[Out] int(sech(x)^2/(1-4*tanh(x)^2)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}(x)^2}{\sqrt{-4 \tanh(x)^2 + 1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^2/(1-4*tanh(x)^2)^(1/2),x, algorithm="maxima")
[Out] integrate(sech(x)^2/sqrt(-4*tanh(x)^2 + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.11

$$\int \frac{1}{\cosh(x)^2 \sqrt{1 - 4\tanh(x)^2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(x)^2*(1 - 4*tanh(x)^2)^(1/2)),x)
[Out] int(1/(cosh(x)^2*(1 - 4*tanh(x)^2)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}^2(x)}{\sqrt{-(2\tanh(x) - 1)(2\tanh(x) + 1)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)**2/(1-4*tanh(x)**2)**(1/2),x)
[Out] Integral(sech(x)**2/sqrt(-(2*tanh(x) - 1)*(2*tanh(x) + 1)), x)
```

3.1000 $\int \frac{\text{sech}^2(x)}{\sqrt{-4+\tanh^2(x)}} \, dx$

Optimal. Leaf size=14

$$\tanh^{-1}\left(\frac{\tanh(x)}{\sqrt{\tanh^2(x)-4}}\right)$$

[Out] arctanh(tanh(x)/(-4+tanh(x)^2)^(1/2))

Rubi [A] time = 0.05, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3675, 217, 206}

$$\tanh^{-1}\left(\frac{\tanh(x)}{\sqrt{\tanh^2(x)-4}}\right)$$

Antiderivative was successfully verified.

[In] Int[Sech[x]^2/Sqrt[-4 + Tanh[x]^2],x]

[Out] ArcTanh[Tanh[x]/Sqrt[-4 + Tanh[x]^2]]

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 3675

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/(c^(m - 1)*f), Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^p
, x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && In
tegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
|| EqQ[n^2, 16])
```

Rubi steps

$$\int \frac{\operatorname{sech}^2(x)}{\sqrt{-4+\tanh^2(x)}}\,dx = \text{Subst}\left(\int \frac{1}{\sqrt{-4+x^2}}\,dx, x, \tanh(x)\right)$$
$$= \text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{\tanh(x)}{\sqrt{-4+\tanh^2(x)}}\right)$$
$$= \tanh^{-1}\left(\frac{\tanh(x)}{\sqrt{-4+\tanh^2(x)}}\right)$$

Mathematica [B] time = 0.04, size = 46, normalized size = 3.29

$$\frac{\sqrt{3\cosh(2x)+5}\,\operatorname{sech}(x)\tan^{-1}\left(\frac{\sinh(x)}{\sqrt{3\sinh^2(x)+4}}\right)}{\sqrt{2}\sqrt{\tanh^2(x)-4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[x]^2/Sqrt[-4 + Tanh[x]^2],x]
```

```
[Out] (ArcTan[Sinh[x]/Sqrt[4 + 3*Sinh[x]^2]]*Sqrt[5 + 3*Cosh[2*x]]*Sech[x])/(Sqrt
[2]*Sqrt[-4 + Tanh[x]^2])
```

fricas [A] time = 0.47, size = 1, normalized size = 0.07

$$0$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^2/(-4+tanh(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 0
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}(x)^2}{\sqrt{\tanh(x)^2-4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^2/(-4+tanh(x)^2)^(1/2),x, algorithm="giac")
[Out] integrate(sech(x)^2/sqrt(tanh(x)^2 - 4), x)
```

maple [F] time = 0.52, size = 0, normalized size = 0.00

$$\int \frac{\mathrm{sech}(x)^2}{\sqrt{-4+\tanh^2(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(x)^2/(-4+tanh(x)^2)^(1/2),x)
[Out] int(sech(x)^2/(-4+tanh(x)^2)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\mathrm{sech}(x)^2}{\sqrt{\tanh(x)^2-4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^2/(-4+tanh(x)^2)^(1/2),x, algorithm="maxima")
[Out] integrate(sech(x)^2/sqrt(tanh(x)^2 - 4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.07

$$\int \frac{1}{\cosh(x)^2\sqrt{\tanh(x)^2-4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(x)^2*(tanh(x)^2 - 4)^(1/2)),x)
[Out] int(1/(cosh(x)^2*(tanh(x)^2 - 4)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\mathrm{sech}^2(x)}{\sqrt{(\tanh(x)-2)(\tanh(x)+2)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)**2/(-4+tanh(x)**2)**(1/2),x)
[Out] Integral(sech(x)**2/sqrt((tanh(x) - 2)*(tanh(x) + 2)), x)
```

3.1001 $\int \sqrt{1+\coth^2(x)}\,\mathbf{sech}^2(x)\,dx$

Optimal. Leaf size=19

$$\tanh(x)\sqrt{\coth^2(x)+1} - \sinh^{-1}(\coth(x))$$

```
[Out] -arcsinh(coth(x))+(1+coth(x)^2)^(1/2)*tanh(x)
```

Rubi [A] time = 0.05, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3663, 277, 215}

$$\tanh(x)\sqrt{\coth^2(x)+1} - \sinh^{-1}(\coth(x))$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 + Coth[x]^2]*Sech[x]^2,x]
```

```
[Out] -ArcSinh[Coth[x]] + Sqrt[1 + Coth[x]^2]*Tanh[x]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 3663

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[(c*ff^(m + 1))/f, Subst[Int[(x^m*(a + b*(ff*x)^n)^p)/(c^2 + ff^2*x^2)^(m/
2 + 1), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[m/2]
```

Rubi steps

$$\int \sqrt{1+\coth^2(x)}\,\mathrm{sech}^2(x)\,dx = -\mathrm{Subst}\left(\int \frac{\sqrt{1+x^2}}{x^2}\,dx, x, \coth(x)\right)$$
$$= \sqrt{1+\coth^2(x)}\,\tanh(x) - \mathrm{Subst}\left(\int \frac{1}{\sqrt{1+x^2}}\,dx, x, \coth(x)\right)$$
$$= -\sinh^{-1}(\coth(x)) + \sqrt{1+\coth^2(x)}\,\tanh(x)$$

Mathematica [B] time = 0.23, size = 51, normalized size = 2.68

$$\sinh(x)\sqrt{\coth^2(x)+1}\,\mathrm{sech}(2x)\left(\cosh(x)+\sinh(x)\tanh(x)-\sqrt{-\cosh(2x)}\,\tan^{-1}\left(\frac{\cosh(x)}{\sqrt{-\cosh(2x)}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 + Coth[x]^2]*Sech[x]^2,x]
```

```
[Out] Sqrt[1 + Coth[x]^2]*Sech[2*x]*Sinh[x]*(Cosh[x] - ArcTan[Cosh[x]/Sqrt[-Cosh[
2*x]]]*Sqrt[-Cosh[2*x]] + Sinh[x]*Tanh[x])
```

fricas [B] time = 0.44, size = 219, normalized size = 11.53

$$-\frac{\left(\cosh(x)^2+2\,\cosh(x)\sinh(x)+\sinh(x)^2+1\right)\log\left(\frac{\cosh(x)^2+2\,\cosh(x)\sinh(x)+\sinh(x)^2+2\,\sqrt{\frac{\cosh(x)^2+\sinh(x)^2}{\cosh(x)^2-2\,\cosh(x)\sinh(x)+\sinh(x)^2}}}{\cosh(x)^2+2\,\cosh(x)\sinh(x)+\sinh(x)^2}\right.}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^2*(1+coth(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/2*((cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)*log((cosh(x)^2 + 2*co
sh(x)*sinh(x) + sinh(x)^2 + 2*sqrt((cosh(x)^2 + sinh(x)^2)/(cosh(x)^2 - 2*c
osh(x)*sinh(x) + sinh(x)^2)) + 1)/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^
2)) - (cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)*log((cosh(x)^2 + 2*co
sh(x)*sinh(x) + sinh(x)^2 - 2*sqrt((cosh(x)^2 + sinh(x)^2)/(cosh(x)^2 - 2*c
osh(x)*sinh(x) + sinh(x)^2)) + 1)/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^
2)) - 4*sqrt((cosh(x)^2 + sinh(x)^2)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(
x)^2)))/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)
```

giac [B] time = 0.15, size = 120, normalized size = 6.32

$$\frac{1}{2}\sqrt{2}\left(\sqrt{2}\log\left(\frac{\left|-2\sqrt{2}+2\sqrt{e^{(4x)}+1}-2\,e^{(2x)}+2\right|}{2\left(\sqrt{2}+\sqrt{e^{(4x)}+1}-e^{(2x)}+1\right)}\right)-\frac{4\left(\sqrt{e^{(4x)}+1}-e^{(2x)}+1\right)}{\left(\sqrt{e^{(4x)}+1}-e^{(2x)}\right)^2-2\sqrt{e^{(4x)}+1}+2\,e^{(2x)}-1}\right)\mathrm{sgn}\left(e\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^2*(1+coth(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(2)*(sqrt(2)*log(1/2*abs(-2*sqrt(2) + 2*sqrt(e^(4*x) + 1) - 2*e^(2*
x) + 2)/(sqrt(2) + sqrt(e^(4*x) + 1) - e^(2*x) + 1)) - 4*(sqrt(e^(4*x) + 1)
 - e^(2*x) + 1)/((sqrt(e^(4*x) + 1) - e^(2*x))^2 - 2*sqrt(e^(4*x) + 1) + 2*
e^(2*x) - 1))*sgn(e^(2*x) - 1)
```

maple [F] time = 0.65, size = 0, normalized size = 0.00

$$\int \operatorname{sech}(x)^2\sqrt{1+\coth^2(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(x)^2*(1+coth(x)^2)^(1/2),x)
```

```
[Out] int(sech(x)^2*(1+coth(x)^2)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\coth(x)^2+1}\,\operatorname{sech}(x)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^2*(1+coth(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(coth(x)^2 + 1)*sech(x)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{\sqrt{\coth(x)^2+1}}{\cosh(x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((coth(x)^2 + 1)^(1/2)/cosh(x)^2,x)
```

```
[Out] int((coth(x)^2 + 1)^(1/2)/cosh(x)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\coth^2(x)+1}\,\operatorname{sech}^2(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)**2*(1+coth(x)**2)**(1/2),x)
[Out] Integral(sqrt(coth(x)**2 + 1)*sech(x)**2, x)
```

3.1002 $\int \operatorname{sech}^2(x)\sqrt{1+\tanh^2(x)}\,dx$

Optimal. Leaf size=24

$$\frac{1}{2}\tanh(x)\sqrt{\tanh^2(x)+1}+\frac{1}{2}\sinh^{-1}(\tanh(x))$$

```
[Out] 1/2*arcsinh(tanh(x))+1/2*(1+tanh(x)^2)^(1/2)*tanh(x)
```

Rubi [A] time = 0.04, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3675, 195, 215}

$$\frac{1}{2}\tanh(x)\sqrt{\tanh^2(x)+1}+\frac{1}{2}\sinh^{-1}(\tanh(x))$$

Antiderivative was successfully verified.

```
[In] Int[Sech[x]^2*Sqrt[1 + Tanh[x]^2],x]
```

```
[Out] ArcSinh[Tanh[x]]/2 + (Tanh[x]*Sqrt[1 + Tanh[x]^2])/2
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 3675

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/(c^(m - 1)*f), Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^p
, x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && In
tegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
|| EqQ[n^2, 16])
```

Rubi steps

$$\int \operatorname{sech}^2(x)\sqrt{1+\tanh^2(x)}\,dx = \text{Subst}\left(\int \sqrt{1+x^2}\,dx, x, \tanh(x)\right)$$
$$= \frac{1}{2}\tanh(x)\sqrt{1+\tanh^2(x)} + \frac{1}{2}\text{Subst}\left(\int \frac{1}{\sqrt{1+x^2}}\,dx, x, \tanh(x)\right)$$
$$= \frac{1}{2}\sinh^{-1}(\tanh(x)) + \frac{1}{2}\tanh(x)\sqrt{1+\tanh^2(x)}$$

Mathematica [B] time = 0.10, size = 55, normalized size = 2.29

$$\frac{1}{4}\sqrt{\tanh^2(x)+1}\,\operatorname{sech}(x)\operatorname{sech}(2x)\left(-\sinh(x)+\sinh(3x)+2\sqrt{\cosh(2x)}\cosh^2(x)\tanh^{-1}\left(\frac{\sinh(x)}{\sqrt{\cosh(2x)}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[x]^2*Sqrt[1 + Tanh[x]^2],x]
```

```
[Out] (Sech[x]*Sech[2*x]*(2*ArcTanh[Sinh[x]/Sqrt[Cosh[2*x]]]*Cosh[x]^2*Sqrt[Cosh[
2*x]] - Sinh[x] + Sinh[3*x])*Sqrt[1 + Tanh[x]^2])/4
```

fricas [B] time = 0.54, size = 334, normalized size = 13.92

$$\frac{\left(\cosh(x)^4 + 4\cosh(x)\sinh(x)^3 + \sinh(x)^4 + 2\left(3\cosh(x)^2+1\right)\sinh(x)^2 + 2\cosh(x)^2 + 4\left(\cosh(x)^3 + \cosh(x\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^2*(1+tanh(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/4*((cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + 2*(3*cosh(x)^2 + 1)*sin
h(x)^2 + 2*cosh(x)^2 + 4*(cosh(x)^3 + cosh(x))*sinh(x) + 1)*log((cosh(x)^2
+ 2*cosh(x)*sinh(x) + sinh(x)^2 + 2*sqrt((cosh(x)^2 + sinh(x)^2)/(cosh(x)^2
 - 2*cosh(x)*sinh(x) + sinh(x)^2)) - 1)/(cosh(x)^2 + 2*cosh(x)*sinh(x) + si
nh(x)^2)) - (cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + 2*(3*cosh(x)^2 +
 1)*sinh(x)^2 + 2*cosh(x)^2 + 4*(cosh(x)^3 + cosh(x))*sinh(x) + 1)*log((cos
h(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 2*sqrt((cosh(x)^2 + sinh(x)^2)/(co
sh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) - 1)/(cosh(x)^2 + 2*cosh(x)*sinh(
x) + sinh(x)^2)) + 4*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*sqrt((
cosh(x)^2 + sinh(x)^2)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/(cosh(
x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + 2*(3*cosh(x)^2 + 1)*sinh(x)^2 + 2*
cosh(x)^2 + 4*(cosh(x)^3 + cosh(x))*sinh(x) + 1)
```

giac [B] time = 0.14, size = 145, normalized size = 6.04

$$\frac{1}{4}\sqrt{2}\left(\sqrt{2}\log\left(\frac{\sqrt{2}-\sqrt{e^{(4x)}+1}+e^{(2x)}+1}{\sqrt{2}+\sqrt{e^{(4x)}+1}-e^{(2x)}-1}\right)-\frac{4\left(3\left(\sqrt{e^{(4x)}+1}-e^{(2x)}\right)^{3}-\left(\sqrt{e^{(4x)}+1}-e^{(2x)}\right)^{2}-\sqrt{e^{(4x)}+1}+e^{(2x)}-1\right)}{\left(\left(\sqrt{e^{(4x)}+1}-e^{(2x)}\right)^{2}-2\sqrt{e^{(4x)}+1}+2e^{(2x)}-1\right)^{2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^2*(1+tanh(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(2)*(sqrt(2)*log((sqrt(2) - sqrt(e^(4*x) + 1) + e^(2*x) + 1)/(sqrt(
2) + sqrt(e^(4*x) + 1) - e^(2*x) - 1)) - 4*(3*(sqrt(e^(4*x) + 1) - e^(2*x))
^3 - (sqrt(e^(4*x) + 1) - e^(2*x))^2 - sqrt(e^(4*x) + 1) + e^(2*x) - 1)/((s
qrt(e^(4*x) + 1) - e^(2*x))^2 - 2*sqrt(e^(4*x) + 1) + 2*e^(2*x) - 1)^2)
```

maple [F] time = 0.47, size = 0, normalized size = 0.00

$$\int \mathrm{sech}(x)^{2}\sqrt{1+\tanh^{2}(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(x)^2*(1+tanh(x)^2)^(1/2),x)
```

```
[Out] int(sech(x)^2*(1+tanh(x)^2)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\tanh(x)^{2}+1}\,\mathrm{sech}(x)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^2*(1+tanh(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(tanh(x)^2 + 1)*sech(x)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\sqrt{\tanh(x)^{2}+1}}{\cosh(x)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((tanh(x)^2 + 1)^(1/2)/cosh(x)^2,x)
```

```
[Out] int((tanh(x)^2 + 1)^(1/2)/cosh(x)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\tanh^2(x) + 1} \operatorname{sech}^2(x) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)**2*(1+tanh(x)**2)**(1/2),x)
[Out] Integral(sqrt(tanh(x)**2 + 1)*sech(x)**2, x)
```

3.1003 $\int \operatorname{sech}^4(x) \left(-1+\operatorname{sech}^2(x)\right)^2 \tanh(x) \, dx$

Optimal. Leaf size=17

$$\frac{\tanh^6(x)}{6}-\frac{\tanh^8(x)}{8}$$

```
[Out] 1/6*tanh(x)^6-1/8*tanh(x)^8
```

Rubi [A] time = 0.08, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {4120, 2607, 14}

$$\frac{\tanh^6(x)}{6}-\frac{\tanh^8(x)}{8}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[x]^4*(-1 + Sech[x]^2)^2*Tanh[x],x]
```

```
[Out] Tanh[x]^6/6 - Tanh[x]^8/8
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 4120

```
Int[(u_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
b^p, Int[ActivateTrig[u*tan[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rubi steps

$$\int \operatorname{sech}^4(x)\left(-1+\operatorname{sech}^2(x)\right)^2\tanh(x)\,dx = \int \operatorname{sech}^4(x)\tanh^5(x)\,dx$$
$$= -\operatorname{Subst}\left(\int x^5\left(1+x^2\right)\,dx, x, i\tanh(x)\right)$$
$$= -\operatorname{Subst}\left(\int \left(x^5+x^7\right)\,dx, x, i\tanh(x)\right)$$
$$= \frac{\tanh^6(x)}{6}-\frac{\tanh^8(x)}{8}$$

Mathematica [A] time = 0.02, size = 25, normalized size = 1.47

$$-\frac{1}{8}\operatorname{sech}^8(x)+\frac{\operatorname{sech}^6(x)}{3}-\frac{\operatorname{sech}^4(x)}{4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[x]^4*(-1 + Sech[x]^2)^2*Tanh[x],x]
```

```
[Out] -1/4*Sech[x]^4 + Sech[x]^6/3 - Sech[x]^8/8
```

fricas [B] time = 0.42, size = 340, normalized size = 20.00

$$-\frac{}{3\left(\cosh(x)^{10}+10\cosh(x)\sinh(x)^9+\sinh(x)^{10}+\left(45\cosh(x)^2+8\right)\sinh(x)^8+8\cosh(x)^8+8\left(15\cosh(x)^3\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^4*(-1+sech(x)^2)^2*tanh(x),x, algorithm="fricas")
```

```
[Out] -4/3*(3*cosh(x)^6 + 18*cosh(x)*sinh(x)^5 + 3*sinh(x)^6 + (45*cosh(x)^2 - 4)
*sinh(x)^4 - 4*cosh(x)^4 + 4*(15*cosh(x)^3 - 4*cosh(x))*sinh(x)^3 + (45*cos
h(x)^4 - 24*cosh(x)^2 + 13)*sinh(x)^2 + 13*cosh(x)^2 + 2*(9*cosh(x)^5 - 8*c
osh(x)^3 + 7*cosh(x))*sinh(x) - 4)/(cosh(x)^10 + 10*cosh(x)*sinh(x)^9 + sin
h(x)^10 + (45*cosh(x)^2 + 8)*sinh(x)^8 + 8*cosh(x)^8 + 8*(15*cosh(x)^3 + 8*
cosh(x))*sinh(x)^7 + (210*cosh(x)^4 + 224*cosh(x)^2 + 29)*sinh(x)^6 + 29*co
sh(x)^6 + 2*(126*cosh(x)^5 + 224*cosh(x)^3 + 81*cosh(x))*sinh(x)^5 + (210*c
osh(x)^6 + 560*cosh(x)^4 + 435*cosh(x)^2 + 64)*sinh(x)^4 + 64*cosh(x)^4 + 4
*(30*cosh(x)^7 + 112*cosh(x)^5 + 135*cosh(x)^3 + 48*cosh(x))*sinh(x)^3 + (4
5*cosh(x)^8 + 224*cosh(x)^6 + 435*cosh(x)^4 + 384*cosh(x)^2 + 98)*sinh(x)^2
 + 98*cosh(x)^2 + 2*(5*cosh(x)^9 + 32*cosh(x)^7 + 81*cosh(x)^5 + 96*cosh(x)
^3 + 42*cosh(x))*sinh(x) + 56)
```

giac [B] time = 0.12, size = 41, normalized size = 2.41

$$-\frac{4\left(3\,e^{(12\,x)}-4\,e^{(10\,x)}+10\,e^{(8\,x)}-4\,e^{(6\,x)}+3\,e^{(4\,x)}\right)}{3\left(e^{(2\,x)}+1\right)^{8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^4*(-1+sech(x)^2)^2*tanh(x),x, algorithm="giac")
```

```
[Out] -4/3*(3*e^(12*x) - 4*e^(10*x) + 10*e^(8*x) - 4*e^(6*x) + 3*e^(4*x))/(e^(2*x
) + 1)^8
```

maple [A] time = 0.09, size = 20, normalized size = 1.18

$$-\frac{\operatorname{sech}(x)^{8}}{8}+\frac{\operatorname{sech}(x)^{6}}{3}-\frac{\operatorname{sech}(x)^{4}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(x)^4*(-1+sech(x)^2)^2*tanh(x),x)
```

```
[Out] -1/8*sech(x)^8+1/3*sech(x)^6-1/4*sech(x)^4
```

maxima [B] time = 0.30, size = 34, normalized size = 2.00

$$-\frac{4}{\left(e^{(-x)}+e^{x}\right)^{4}}+\frac{64}{3\left(e^{(-x)}+e^{x}\right)^{6}}-\frac{32}{\left(e^{(-x)}+e^{x}\right)^{8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^4*(-1+sech(x)^2)^2*tanh(x),x, algorithm="maxima")
```

```
[Out] -4/(e^(-x) + e^x)^4 + 64/3/(e^(-x) + e^x)^6 - 32/(e^(-x) + e^x)^8
```

mupad [B] time = 1.76, size = 375, normalized size = 22.06

$$\frac{\mathrm{e}^{2\,x}-5\,\mathrm{e}^{4\,x}+10\,\mathrm{e}^{6\,x}-10\,\mathrm{e}^{8\,x}+5\,\mathrm{e}^{10\,x}-\mathrm{e}^{12\,x}}{8\,\mathrm{e}^{2\,x}+28\,\mathrm{e}^{4\,x}+56\,\mathrm{e}^{6\,x}+70\,\mathrm{e}^{8\,x}+56\,\mathrm{e}^{10\,x}+28\,\mathrm{e}^{12\,x}+8\,\mathrm{e}^{14\,x}+\mathrm{e}^{16\,x}+1}-\frac{\frac{20\,\mathrm{e}^{4\,x}}{7}-\frac{10\,\mathrm{e}^{2\,x}}{7}-\frac{50\,\mathrm{e}^{6\,x}}{21}+\frac{5\,\mathrm{e}^{8\,x}}{7}}{6\,\mathrm{e}^{2\,x}+15\,\mathrm{e}^{4\,x}+20\,\mathrm{e}^{6\,x}+15\,\mathrm{e}^{8\,x}+6\,\mathrm{e}^{1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((tanh(x)*(1/cosh(x)^2 - 1)^2)/cosh(x)^4,x)
```

```
[Out] (exp(2*x) - 5*exp(4*x) + 10*exp(6*x) - 10*exp(8*x) + 5*exp(10*x) - exp(12*x
))/(8*exp(2*x) + 28*exp(4*x) + 56*exp(6*x) + 70*exp(8*x) + 56*exp(10*x) + 2
```

```
8*exp(12*x) + 8*exp(14*x) + exp(16*x) + 1) - ((20*exp(4*x))/7 - (10*exp(2*x
))/7 - (50*exp(6*x))/21 + (5*exp(8*x))/7 + 5/21)/(6*exp(2*x) + 15*exp(4*x)
+ 20*exp(6*x) + 15*exp(8*x) + 6*exp(10*x) + exp(12*x) + 1) - ((8*exp(2*x))/
7 - (10*exp(4*x))/7 + (4*exp(6*x))/7 - 2/7)/(5*exp(2*x) + 10*exp(4*x) + 10*
exp(6*x) + 5*exp(8*x) + exp(10*x) + 1) - ((10*exp(2*x))/7 - (30*exp(4*x))/7
 + (40*exp(6*x))/7 - (25*exp(8*x))/7 + (6*exp(10*x))/7 - 1/7)/(7*exp(2*x) +
 21*exp(4*x) + 35*exp(6*x) + 35*exp(8*x) + 21*exp(10*x) + 7*exp(12*x) + exp
(14*x) + 1) - ((2*exp(2*x))/7 - 5/21)/(3*exp(2*x) + 3*exp(4*x) + exp(6*x) +
 1) - 1/(7*(2*exp(2*x) + exp(4*x) + 1)) - ((3*exp(4*x))/7 - (5*exp(2*x))/7
+ 2/7)/(4*exp(2*x) + 6*exp(4*x) + 4*exp(6*x) + exp(8*x) + 1)
```

sympy [A] time = 6.58, size = 19, normalized size = 1.12

$$-\frac{\operatorname{sech}^{8}(x)}{8}+\frac{\operatorname{sech}^{6}(x)}{3}-\frac{\operatorname{sech}^{4}(x)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)**4*(-1+sech(x)**2)**2*tanh(x),x)
```

```
[Out] -sech(x)**8/8 + sech(x)**6/3 - sech(x)**4/4
```

3.1004 $\int e^{n\sinh(a+bx)}\sinh(2a+2bx)\,dx$

Optimal. Leaf size=43

$$\frac{2\sinh(a+bx)e^{n\sinh(a+bx)}}{bn}-\frac{2e^{n\sinh(a+bx)}}{bn^2}$$

```
[Out] -2*exp(n*sinh(b*x+a))/b/n^2+2*exp(n*sinh(b*x+a))*sinh(b*x+a)/b/n
```

Rubi [A] time = 0.04, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {12, 2176, 2194}

$$\frac{2\sinh(a+bx)e^{n\sinh(a+bx)}}{bn}-\frac{2e^{n\sinh(a+bx)}}{bn^2}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*Sinh[a + b*x])*Sinh[2*a + 2*b*x],x]
```

```
[Out] (-2*E^(n*Sinh[a + b*x]))/(b*n^2) + (2*E^(n*Sinh[a + b*x])*Sinh[a + b*x])/(b
*n)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2176

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_.), x_Symbol] :> Simp[((c + d*x)^m*(b*F^(g*(e + f*x)))^n)/(f*g*n*Log[F]),
x] - Dist[(d*m)/(f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^n
, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*m
] && !$UseGamma === True
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rubi steps

$$\int e^{n\sinh(a+bx)}\sinh(2a+2bx)\,dx = \frac{\text{Subst}\left(\int 2e^{nx}x\,dx,x,\sinh(a+bx)\right)}{b}$$
$$= \frac{2\,\text{Subst}\left(\int e^{nx}x\,dx,x,\sinh(a+bx)\right)}{b}$$
$$= \frac{2e^{n\sinh(a+bx)}\sinh(a+bx)}{bn} - \frac{2\,\text{Subst}\left(\int e^{nx}\,dx,x,\sinh(a+bx)\right)}{bn}$$
$$= -\frac{2e^{n\sinh(a+bx)}}{bn^2} + \frac{2e^{n\sinh(a+bx)}\sinh(a+bx)}{bn}$$

Mathematica [A] time = 0.06, size = 28, normalized size = 0.65

$$\frac{2e^{n\sinh(a+bx)}(n\sinh(a+bx)-1)}{bn^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*Sinh[a + b*x])*Sinh[2*a + 2*b*x],x]
```

```
[Out] (2*E^(n*Sinh[a + b*x])*(-1 + n*Sinh[a + b*x]))/(b*n^2)
```

fricas [A] time = 0.43, size = 73, normalized size = 1.70

$$\frac{2\left((n\sinh(bx+a)-1)\cosh(n\sinh(bx+a)) + (n\sinh(bx+a)-1)\sinh(n\sinh(bx+a))\right)}{bn^2\cosh(bx+a)^2 - bn^2\sinh(bx+a)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*sinh(b*x+a))*sinh(2*b*x+2*a),x, algorithm="fricas")
```

```
[Out] 2*((n*sinh(b*x + a) - 1)*cosh(n*sinh(b*x + a)) + (n*sinh(b*x + a) - 1)*sinh
(n*sinh(b*x + a)))/(b*n^2*cosh(b*x + a)^2 - b*n^2*sinh(b*x + a)^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int e^{(n\sinh(bx+a))}\sinh(2\,bx+2\,a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*sinh(b*x+a))*sinh(2*b*x+2*a),x, algorithm="giac")
```

```
[Out] integrate(e^(n*sinh(b*x + a))*sinh(2*b*x + 2*a), x)
```

maple [A] time = 0.37, size = 61, normalized size = 1.42

$$\frac{\left(\mathrm{e}^{2bx+2a}n-n-2\,\mathrm{e}^{bx+a}\right)\mathrm{e}^{-bx-a+\frac{n\,\mathrm{e}^{bx+a}}{2}-\frac{n\,\mathrm{e}^{-bx-a}}{2}}}{n^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*sinh(b*x+a))*sinh(2*b*x+2*a),x)
```

```
[Out] 1/n^2/b*(exp(2*b*x+2*a)*n-n-2*exp(b*x+a))*exp(-b*x-a+1/2*n*exp(b*x+a)-1/2*n
*exp(-b*x-a))
```

maxima [B] time = 0.40, size = 104, normalized size = 2.42

$$\frac{e^{\left(bx+\frac{1}{2}\,ne^{(bx+a)}-\frac{1}{2}\,ne^{(-bx-a)}+a\right)}}{bn}-\frac{e^{\left(-bx+\frac{1}{2}\,ne^{(bx+a)}-\frac{1}{2}\,ne^{(-bx-a)}-a\right)}}{bn}-\frac{2\,e^{\left(\frac{1}{2}\,ne^{(bx+a)}-\frac{1}{2}\,ne^{(-bx-a)}\right)}}{bn^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*sinh(b*x+a))*sinh(2*b*x+2*a),x, algorithm="maxima")
```

```
[Out] e^(b*x + 1/2*n*e^(b*x + a) - 1/2*n*e^(-b*x - a) + a)/(b*n) - e^(-b*x + 1/2*
n*e^(b*x + a) - 1/2*n*e^(-b*x - a) - a)/(b*n) - 2*e^(1/2*n*e^(b*x + a) - 1/
2*n*e^(-b*x - a))/(b*n^2)
```

mupad [B] time = 1.80, size = 108, normalized size = 2.51

$$\frac{\mathrm{e}^{\frac{n\,\mathrm{e}^{b\,x}\,\mathrm{e}^{a}}{2}}\,\mathrm{e}^{b\,x}\,\mathrm{e}^{a}\,\mathrm{e}^{-\frac{n\,\mathrm{e}^{-a}\,\mathrm{e}^{-b\,x}}{2}}}{b\,n}-\frac{\mathrm{e}^{-a}\,\mathrm{e}^{\frac{n\,\mathrm{e}^{b\,x}\,\mathrm{e}^{a}}{2}}\,\mathrm{e}^{-b\,x}\,\mathrm{e}^{-\frac{n\,\mathrm{e}^{-a}\,\mathrm{e}^{-b\,x}}{2}}}{b\,n}-\frac{2\,\mathrm{e}^{\frac{n\,\mathrm{e}^{b\,x}\,\mathrm{e}^{a}}{2}}\,\mathrm{e}^{-\frac{n\,\mathrm{e}^{-a}\,\mathrm{e}^{-b\,x}}{2}}}{b\,n^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*sinh(a + b*x))*sinh(2*a + 2*b*x),x)
```

```
[Out] (exp((n*exp(b*x)*exp(a))/2)*exp(b*x)*exp(a)*exp(-(n*exp(-a)*exp(-b*x))/2))/
(b*n) - (exp(-a)*exp((n*exp(b*x)*exp(a))/2)*exp(-b*x)*exp(-(n*exp(-a)*exp(-
b*x))/2))/(b*n) - (2*exp((n*exp(b*x)*exp(a))/2)*exp(-(n*exp(-a)*exp(-b*x))/
2))/(b*n^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int e^{n\sinh\left(a+bx\right)}\sinh\left(2a+2bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*sinh(b*x+a))*sinh(2*b*x+2*a),x)
```

```
[Out] Integral(exp(n*sinh(a + b*x))*sinh(2*a + 2*b*x), x)
```

3.1005 $\int e^{n\sinh(a+bx)}\sinh(2(a+bx))\,dx$

Optimal. Leaf size=43

$$\frac{2\sinh(a+bx)e^{n\sinh(a+bx)}}{bn}-\frac{2e^{n\sinh(a+bx)}}{bn^2}$$

```
[Out] -2*exp(n*sinh(b*x+a))/b/n^2+2*exp(n*sinh(b*x+a))*sinh(b*x+a)/b/n
```

Rubi [A] time = 0.04, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {12, 2176, 2194}

$$\frac{2\sinh(a+bx)e^{n\sinh(a+bx)}}{bn}-\frac{2e^{n\sinh(a+bx)}}{bn^2}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*Sinh[a + b*x])*Sinh[2*(a + b*x)],x]
```

```
[Out] (-2*E^(n*Sinh[a + b*x]))/(b*n^2) + (2*E^(n*Sinh[a + b*x])*Sinh[a + b*x])/(b
*n)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2176

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_.), x_Symbol] :> Simp[((c + d*x)^m*(b*F^(g*(e + f*x)))^n)/(f*g*n*Log[F]),
x] - Dist[(d*m)/(f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^n
, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*m
] && !$UseGamma === True
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rubi steps

$$\int e^{n\sinh(a+bx)}\sinh(2(a+bx))\,dx = \frac{\text{Subst}\left(\int 2e^{nx}x\,dx, x, \sinh(a+bx)\right)}{b}$$
$$= \frac{2\,\text{Subst}\left(\int e^{nx}x\,dx, x, \sinh(a+bx)\right)}{b}$$
$$= \frac{2e^{n\sinh(a+bx)}\sinh(a+bx)}{bn} - \frac{2\,\text{Subst}\left(\int e^{nx}\,dx, x, \sinh(a+bx)\right)}{bn}$$
$$= -\frac{2e^{n\sinh(a+bx)}}{bn^2} + \frac{2e^{n\sinh(a+bx)}\sinh(a+bx)}{bn}$$

Mathematica [A] time = 0.03, size = 28, normalized size = 0.65

$$\frac{2e^{n\sinh(a+bx)}(n\sinh(a+bx)-1)}{bn^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*Sinh[a + b*x])*Sinh[2*(a + b*x)],x]
```

```
[Out] (2*E^(n*Sinh[a + b*x])*(-1 + n*Sinh[a + b*x]))/(b*n^2)
```

fricas [A] time = 0.47, size = 73, normalized size = 1.70

$$\frac{2\left((n\sinh(bx+a)-1)\cosh(n\sinh(bx+a)) + (n\sinh(bx+a)-1)\sinh(n\sinh(bx+a))\right)}{bn^2\cosh(bx+a)^2 - bn^2\sinh(bx+a)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*sinh(b*x+a))*sinh(2*b*x+2*a),x, algorithm="fricas")
```

```
[Out] 2*((n*sinh(b*x + a) - 1)*cosh(n*sinh(b*x + a)) + (n*sinh(b*x + a) - 1)*sinh
(n*sinh(b*x + a)))/(b*n^2*cosh(b*x + a)^2 - b*n^2*sinh(b*x + a)^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int e^{(n\sinh(bx+a))}\sinh(2\,bx+2\,a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*sinh(b*x+a))*sinh(2*b*x+2*a),x, algorithm="giac")
```

```
[Out] integrate(e^(n*sinh(b*x + a))*sinh(2*b*x + 2*a), x)
```

maple [A] time = 0.22, size = 61, normalized size = 1.42

$$\frac{\left(\mathrm{e}^{2bx+2a}n-n-2\,\mathrm{e}^{bx+a}\right)\mathrm{e}^{-bx-a+\frac{n\,\mathrm{e}^{bx+a}}{2}-\frac{n\,\mathrm{e}^{-bx-a}}{2}}}{n^{2}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*sinh(b*x+a))*sinh(2*b*x+2*a),x)
```

```
[Out] 1/n^2/b*(exp(2*b*x+2*a)*n-n-2*exp(b*x+a))*exp(-b*x-a+1/2*n*exp(b*x+a)-1/2*n
*exp(-b*x-a))
```

maxima [B] time = 0.40, size = 104, normalized size = 2.42

$$\frac{e^{\left(bx+\frac{1}{2}\,ne^{(bx+a)}-\frac{1}{2}\,ne^{(-bx-a)}+a\right)}}{bn}-\frac{e^{\left(-bx+\frac{1}{2}\,ne^{(bx+a)}-\frac{1}{2}\,ne^{(-bx-a)}-a\right)}}{bn}-\frac{2\,e^{\left(\frac{1}{2}\,ne^{(bx+a)}-\frac{1}{2}\,ne^{(-bx-a)}\right)}}{bn^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*sinh(b*x+a))*sinh(2*b*x+2*a),x, algorithm="maxima")
```

```
[Out] e^(b*x + 1/2*n*e^(b*x + a) - 1/2*n*e^(-b*x - a) + a)/(b*n) - e^(-b*x + 1/2*
n*e^(b*x + a) - 1/2*n*e^(-b*x - a) - a)/(b*n) - 2*e^(1/2*n*e^(b*x + a) - 1/
2*n*e^(-b*x - a))/(b*n^2)
```

mupad [B] time = 0.00, size = 108, normalized size = 2.51

$$\frac{\mathrm{e}^{\frac{n\,\mathrm{e}^{b\,x}\,\mathrm{e}^{a}}{2}}\,\mathrm{e}^{b\,x}\,\mathrm{e}^{a}\,\mathrm{e}^{-\frac{n\,\mathrm{e}^{-a}\,\mathrm{e}^{-b\,x}}{2}}}{b\,n}-\frac{\mathrm{e}^{-a}\,\mathrm{e}^{\frac{n\,\mathrm{e}^{b\,x}\,\mathrm{e}^{a}}{2}}\,\mathrm{e}^{-b\,x}\,\mathrm{e}^{-\frac{n\,\mathrm{e}^{-a}\,\mathrm{e}^{-b\,x}}{2}}}{b\,n}-\frac{2\,\mathrm{e}^{\frac{n\,\mathrm{e}^{b\,x}\,\mathrm{e}^{a}}{2}}\,\mathrm{e}^{-\frac{n\,\mathrm{e}^{-a}\,\mathrm{e}^{-b\,x}}{2}}}{b\,n^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*sinh(a + b*x))*sinh(2*a + 2*b*x),x)
```

```
[Out] (exp((n*exp(b*x)*exp(a))/2)*exp(b*x)*exp(a)*exp(-(n*exp(-a)*exp(-b*x))/2))/
(b*n) - (exp(-a)*exp((n*exp(b*x)*exp(a))/2)*exp(-b*x)*exp(-(n*exp(-a)*exp(-
b*x))/2))/(b*n) - (2*exp((n*exp(b*x)*exp(a))/2)*exp(-(n*exp(-a)*exp(-b*x))/
2))/(b*n^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int e^{n\sinh{(a+bx)}}\sinh{(2a+2bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*sinh(b*x+a))*sinh(2*b*x+2*a),x)
```

```
[Out] Integral(exp(n*sinh(a + b*x))*sinh(2*a + 2*b*x), x)
```

3.1006 $\int e^{n\sinh\left(\frac{a}{2}+\frac{bx}{2}\right)}\sinh(a+bx)\,dx$

Optimal. Leaf size=64

$$\frac{4\sinh\left(\frac{a}{2}+\frac{bx}{2}\right)e^{n\sinh\left(\frac{a}{2}+\frac{bx}{2}\right)}}{bn}-\frac{4e^{n\sinh\left(\frac{a}{2}+\frac{bx}{2}\right)}}{bn^2}$$

```
[Out] -4*exp(n*sinh(1/2*a+1/2*b*x))/b/n^2+4*exp(n*sinh(1/2*a+1/2*b*x))*sinh(1/2*a
+1/2*b*x)/b/n
```

Rubi [A] time = 0.04, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {12, 2176, 2194}

$$\frac{4\sinh\left(\frac{a}{2}+\frac{bx}{2}\right)e^{n\sinh\left(\frac{a}{2}+\frac{bx}{2}\right)}}{bn}-\frac{4e^{n\sinh\left(\frac{a}{2}+\frac{bx}{2}\right)}}{bn^2}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*Sinh[a/2 + (b*x)/2])*Sinh[a + b*x],x]
```

```
[Out] (-4*E^(n*Sinh[a/2 + (b*x)/2]))/(b*n^2) + (4*E^(n*Sinh[a/2 + (b*x)/2])*Sinh[
a/2 + (b*x)/2])/(b*n)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2176

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_.), x_Symbol] :> Simp[((c + d*x)^m*(b*F^(g*(e + f*x)))^n)/(f*g*n*Log[F]),
x] - Dist[(d*m)/(f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^n
, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*m
] && !$UseGamma === True
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rubi steps

$$\int e^{n\sinh\left(\frac{a}{2}+\frac{bx}{2}\right)}\sinh(a+bx)\,dx = \frac{2\,\text{Subst}\left(\int 2e^{nx}x\,dx, x, \sinh\left(\frac{a}{2}+\frac{bx}{2}\right)\right)}{b}$$
$$= \frac{4\,\text{Subst}\left(\int e^{nx}x\,dx, x, \sinh\left(\frac{a}{2}+\frac{bx}{2}\right)\right)}{b}$$
$$= \frac{4e^{n\sinh\left(\frac{a}{2}+\frac{bx}{2}\right)}\sinh\left(\frac{a}{2}+\frac{bx}{2}\right)}{bn} - \frac{4\,\text{Subst}\left(\int e^{nx}\,dx, x, \sinh\left(\frac{a}{2}+\frac{bx}{2}\right)\right)}{bn}$$
$$= -\frac{4e^{n\sinh\left(\frac{a}{2}+\frac{bx}{2}\right)}}{bn^2} + \frac{4e^{n\sinh\left(\frac{a}{2}+\frac{bx}{2}\right)}\sinh\left(\frac{a}{2}+\frac{bx}{2}\right)}{bn}$$

Mathematica [A] time = 0.06, size = 36, normalized size = 0.56

$$\frac{4e^{n\sinh\left(\frac{1}{2}(a+bx)\right)}\left(n\sinh\left(\frac{1}{2}(a+bx)\right)-1\right)}{bn^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*Sinh[a/2 + (b*x)/2])*Sinh[a + b*x],x]
```

```
[Out] (4*E^(n*Sinh[(a + b*x)/2])*(-1 + n*Sinh[(a + b*x)/2]))/(b*n^2)
```

fricas [A] time = 0.44, size = 91, normalized size = 1.42

$$\frac{4\left(\left(n\sinh\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)-1\right)\cosh\left(n\sinh\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)\right)+\left(n\sinh\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)-1\right)\sinh\left(n\sinh\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)\right)\right)}{bn^2\cosh\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)^2-bn^2\sinh\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*sinh(1/2*a+1/2*b*x))*sinh(b*x+a),x, algorithm="fricas")
```

```
[Out] 4*((n*sinh(1/2*b*x + 1/2*a) - 1)*cosh(n*sinh(1/2*b*x + 1/2*a)) + (n*sinh(1/
2*b*x + 1/2*a) - 1)*sinh(n*sinh(1/2*b*x + 1/2*a)))/(b*n^2*cosh(1/2*b*x + 1/
2*a)^2 - b*n^2*sinh(1/2*b*x + 1/2*a)^2)
```

giac [B] time = 0.19, size = 255, normalized size = 3.98

$$2\left(ne^{\left(bx+\frac{1}{4}\left(2\,bxe^{\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)}+ne^{(bx+a)}-n\right)e^{\left(-\frac{1}{2}\,bx-\frac{1}{2}\,a\right)}-\frac{1}{4}\left(2\,bxe^{\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)}-ne^{(bx+a)}+n\right)e^{\left(-\frac{1}{2}\,bx-\frac{1}{2}\,a\right)}+a\right)} - ne^{\left(\frac{1}{4}\left(2\,bxe^{\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)}+ne^{(bx+a)}-n\right)e^{\left(-\frac{1}{2}\right.}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*sinh(1/2*a+1/2*b*x))*sinh(b*x+a),x, algorithm="giac")
```

```
[Out] 2*(n*e^(b*x + 1/4*(2*b*x*e^(1/2*b*x + 1/2*a) + n*e^(b*x + a) - n)*e^(-1/2*b
*x - 1/2*a) - 1/4*(2*b*x*e^(1/2*b*x + 1/2*a) - n*e^(b*x + a) + n)*e^(-1/2*b
*x - 1/2*a) + a) - n*e^(1/4*(2*b*x*e^(1/2*b*x + 1/2*a) + n*e^(b*x + a) - n)
*e^(-1/2*b*x - 1/2*a) - 1/4*(2*b*x*e^(1/2*b*x + 1/2*a) - n*e^(b*x + a) + n)
*e^(-1/2*b*x - 1/2*a)) - 2*e^(1/2*b*x + 1/4*(2*b*x*e^(1/2*b*x + 1/2*a) + n*
e^(b*x + a) - n)*e^(-1/2*b*x - 1/2*a) - 1/4*(2*b*x*e^(1/2*b*x + 1/2*a) - n*
e^(b*x + a) + n)*e^(-1/2*b*x - 1/2*a) + 1/2*a))*e^(-1/2*b*x - 1/2*a)/(b*n^2
)
```

maple [A] time = 0.34, size = 65, normalized size = 1.02

$$\frac{2\left(n\,e^{bx+a}-n-2\,e^{\frac{bx}{2}+\frac{a}{2}}\right)e^{-\frac{bx}{2}-\frac{a}{2}+\frac{n\,e^{\frac{bx}{2}+\frac{a}{2}}}{2}-\frac{n\,e^{-\frac{bx}{2}-\frac{a}{2}}}{2}}}{n^2 b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*sinh(1/2*b*x+1/2*a))*sinh(b*x+a),x)
```

```
[Out] 2/n^2/b*(n*exp(b*x+a)-n-2*exp(1/2*b*x+1/2*a))*exp(-1/2*b*x-1/2*a+1/2*n*exp(
1/2*b*x+1/2*a)-1/2*n*exp(-1/2*b*x-1/2*a))
```

maxima [B] time = 0.40, size = 117, normalized size = 1.83

$$\frac{2\,e^{\left(\frac{1}{2}\,bx+\frac{1}{2}\,ne^{\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)}-\frac{1}{2}\,ne^{\left(-\frac{1}{2}\,bx-\frac{1}{2}\,a\right)}+\frac{1}{2}\,a\right)}}{bn}-\frac{2\,e^{\left(-\frac{1}{2}\,bx+\frac{1}{2}\,ne^{\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)}-\frac{1}{2}\,ne^{\left(-\frac{1}{2}\,bx-\frac{1}{2}\,a\right)}-\frac{1}{2}\,a\right)}}{bn}-\frac{4\,e^{\left(\frac{1}{2}\,ne^{\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)}-\frac{1}{2}\,ne^{\left(-\frac{1}{2}\,bx-\frac{1}{2}\,a\right)}\right)}}{bn^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*sinh(1/2*a+1/2*b*x))*sinh(b*x+a),x, algorithm="maxima")
```

```
[Out] 2*e^(1/2*b*x + 1/2*n*e^(1/2*b*x + 1/2*a) - 1/2*n*e^(-1/2*b*x - 1/2*a) + 1/2
*a)/(b*n) - 2*e^(-1/2*b*x + 1/2*n*e^(1/2*b*x + 1/2*a) - 1/2*n*e^(-1/2*b*x -
 1/2*a) - 1/2*a)/(b*n) - 4*e^(1/2*n*e^(1/2*b*x + 1/2*a) - 1/2*n*e^(-1/2*b*x
 - 1/2*a))/(b*n^2)
```

mupad [B] time = 1.79, size = 127, normalized size = 1.98

$$\frac{2\,e^{-\frac{a}{2}}\,e^{b\,x}\,e^{-\frac{b\,x}{2}}\,e^{a}\,e^{-\frac{n\,e^{-\frac{a}{2}}\,e^{-\frac{b\,x}{2}}}{2}}\,e^{\frac{n\,e^{a/2}\,e^{\frac{b\,x}{2}}}{2}}}{b\,n}-\frac{2\,e^{-\frac{a}{2}}\,e^{-\frac{b\,x}{2}}\,e^{-\frac{n\,e^{-\frac{a}{2}}\,e^{-\frac{b\,x}{2}}}{2}}\,e^{\frac{n\,e^{a/2}\,e^{\frac{b\,x}{2}}}{2}}}{b\,n}-\frac{4\,e^{-\frac{n\,e^{-\frac{a}{2}}\,e^{-\frac{b\,x}{2}}}{2}}\,e^{\frac{n\,e^{a/2}\,e^{\frac{b\,x}{2}}}{2}}}{b\,n^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*sinh(a/2 + (b*x)/2))*sinh(a + b*x),x)
```

```
[Out] (2*exp(-a/2)*exp(b*x)*exp(-(b*x)/2)*exp(a)*exp(-(n*exp(-a/2)*exp(-(b*x)/2))
/2)*exp((n*exp(a/2)*exp((b*x)/2))/2))/(b*n) - (2*exp(-a/2)*exp(-(b*x)/2)*ex
p(-(n*exp(-a/2)*exp(-(b*x)/2))/2)*exp((n*exp(a/2)*exp((b*x)/2))/2))/(b*n) -
 (4*exp(-(n*exp(-a/2)*exp(-(b*x)/2))/2)*exp((n*exp(a/2)*exp((b*x)/2))/2))/(
b*n^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int e^{n\sinh\left(\frac{a}{2}+\frac{bx}{2}\right)}\sinh\left(a+bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*sinh(1/2*a+1/2*b*x))*sinh(b*x+a),x)
```

```
[Out] Integral(exp(n*sinh(a/2 + b*x/2))*sinh(a + b*x), x)
```

3.1007 $\int e^{n\sinh\left(\frac{1}{2}(a+bx)\right)}\sinh(a+bx)\,dx$

Optimal. Leaf size=64

$$\frac{4\sinh\left(\frac{a}{2}+\frac{bx}{2}\right)e^{n\sinh\left(\frac{a}{2}+\frac{bx}{2}\right)}}{bn}-\frac{4e^{n\sinh\left(\frac{a}{2}+\frac{bx}{2}\right)}}{bn^2}$$

```
[Out] -4*exp(n*sinh(1/2*a+1/2*b*x))/b/n^2+4*exp(n*sinh(1/2*a+1/2*b*x))*sinh(1/2*a
+1/2*b*x)/b/n
```

Rubi [A] time = 0.04, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {12, 2176, 2194}

$$\frac{4\sinh\left(\frac{a}{2}+\frac{bx}{2}\right)e^{n\sinh\left(\frac{a}{2}+\frac{bx}{2}\right)}}{bn}-\frac{4e^{n\sinh\left(\frac{a}{2}+\frac{bx}{2}\right)}}{bn^2}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*Sinh[(a + b*x)/2])*Sinh[a + b*x],x]
```

```
[Out] (-4*E^(n*Sinh[a/2 + (b*x)/2]))/(b*n^2) + (4*E^(n*Sinh[a/2 + (b*x)/2])*Sinh[
a/2 + (b*x)/2])/(b*n)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2176

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_.), x_Symbol] :> Simp[((c + d*x)^m*(b*F^(g*(e + f*x)))^n)/(f*g*n*Log[F]),
x] - Dist[(d*m)/(f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^n
, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*m
] &&  !$UseGamma === True
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rubi steps

$$\int e^{n\sinh\left(\frac{1}{2}(a+bx)\right)}\sinh(a+bx)\,dx = \frac{2\,\text{Subst}\left(\int 2e^{nx}x\,dx, x, \sinh\left(\frac{a}{2}+\frac{bx}{2}\right)\right)}{b}$$
$$= \frac{4\,\text{Subst}\left(\int e^{nx}x\,dx, x, \sinh\left(\frac{a}{2}+\frac{bx}{2}\right)\right)}{b}$$
$$= \frac{4e^{n\sinh\left(\frac{a}{2}+\frac{bx}{2}\right)}\sinh\left(\frac{a}{2}+\frac{bx}{2}\right)}{bn} - \frac{4\,\text{Subst}\left(\int e^{nx}\,dx, x, \sinh\left(\frac{a}{2}+\frac{bx}{2}\right)\right)}{bn}$$
$$= -\frac{4e^{n\sinh\left(\frac{a}{2}+\frac{bx}{2}\right)}}{bn^2} + \frac{4e^{n\sinh\left(\frac{a}{2}+\frac{bx}{2}\right)}\sinh\left(\frac{a}{2}+\frac{bx}{2}\right)}{bn}$$

Mathematica [A] time = 0.03, size = 36, normalized size = 0.56

$$\frac{4e^{n\sinh\left(\frac{1}{2}(a+bx)\right)}\left(n\sinh\left(\frac{1}{2}(a+bx)\right)-1\right)}{bn^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*Sinh[(a + b*x)/2])*Sinh[a + b*x],x]
```

```
[Out] (4*E^(n*Sinh[(a + b*x)/2])*(-1 + n*Sinh[(a + b*x)/2]))/(b*n^2)
```

fricas [A] time = 0.43, size = 91, normalized size = 1.42

$$\frac{4\left(\left(n\sinh\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)-1\right)\cosh\left(n\sinh\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)\right)+\left(n\sinh\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)-1\right)\sinh\left(n\sinh\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)\right)\right)}{bn^2\cosh\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)^2-bn^2\sinh\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*sinh(1/2*a+1/2*b*x))*sinh(b*x+a),x, algorithm="fricas")
```

```
[Out] 4*((n*sinh(1/2*b*x + 1/2*a) - 1)*cosh(n*sinh(1/2*b*x + 1/2*a)) + (n*sinh(1/
2*b*x + 1/2*a) - 1)*sinh(n*sinh(1/2*b*x + 1/2*a)))/(b*n^2*cosh(1/2*b*x + 1/
2*a)^2 - b*n^2*sinh(1/2*b*x + 1/2*a)^2)
```

giac [B] time = 0.19, size = 255, normalized size = 3.98

$$2\left(ne^{\left(bx+\frac{1}{4}\left(2\,bxe^{\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)}+ne^{(bx+a)}-n\right)e^{\left(-\frac{1}{2}\,bx-\frac{1}{2}\,a\right)}-\frac{1}{4}\left(2\,bxe^{\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)}-ne^{(bx+a)}+n\right)e^{\left(-\frac{1}{2}\,bx-\frac{1}{2}\,a\right)}+a\right)}-ne^{\left(\frac{1}{4}\left(2\,bxe^{\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)}+ne^{(bx+a)}-n\right)e^{\left(-\frac{1}{2}\right.}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*sinh(1/2*a+1/2*b*x))*sinh(b*x+a),x, algorithm="giac")
```

```
[Out] 2*(n*e^(b*x + 1/4*(2*b*x*e^(1/2*b*x + 1/2*a) + n*e^(b*x + a) - n)*e^(-1/2*b
*x - 1/2*a) - 1/4*(2*b*x*e^(1/2*b*x + 1/2*a) - n*e^(b*x + a) + n)*e^(-1/2*b
*x - 1/2*a) + a) - n*e^(1/4*(2*b*x*e^(1/2*b*x + 1/2*a) + n*e^(b*x + a) - n)
*e^(-1/2*b*x - 1/2*a) - 1/4*(2*b*x*e^(1/2*b*x + 1/2*a) - n*e^(b*x + a) + n)
*e^(-1/2*b*x - 1/2*a)) - 2*e^(1/2*b*x + 1/4*(2*b*x*e^(1/2*b*x + 1/2*a) + n*
e^(b*x + a) - n)*e^(-1/2*b*x - 1/2*a) - 1/4*(2*b*x*e^(1/2*b*x + 1/2*a) - n*
e^(b*x + a) + n)*e^(-1/2*b*x - 1/2*a) + 1/2*a))*e^(-1/2*b*x - 1/2*a)/(b*n^2
)
```

maple [A] time = 0.20, size = 65, normalized size = 1.02

$$\frac{2\left(n\,\mathrm{e}^{bx+a}-n-2\,\mathrm{e}^{\frac{bx}{2}+\frac{a}{2}}\right)\mathrm{e}^{-\frac{bx}{2}-\frac{a}{2}+\frac{n\,\mathrm{e}^{\frac{bx}{2}+\frac{a}{2}}}{2}-\frac{n\,\mathrm{e}^{-\frac{bx}{2}-\frac{a}{2}}}{2}}}{n^{2}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*sinh(1/2*b*x+1/2*a))*sinh(b*x+a),x)
```

```
[Out] 2/n^2/b*(n*exp(b*x+a)-n-2*exp(1/2*b*x+1/2*a))*exp(-1/2*b*x-1/2*a+1/2*n*exp(
1/2*b*x+1/2*a)-1/2*n*exp(-1/2*b*x-1/2*a))
```

maxima [B] time = 0.41, size = 117, normalized size = 1.83

$$\frac{2\,e^{\left(\frac{1}{2}\,bx+\frac{1}{2}\,ne^{\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)}-\frac{1}{2}\,ne^{\left(-\frac{1}{2}\,bx-\frac{1}{2}\,a\right)}+\frac{1}{2}\,a\right)}}{bn}-\frac{2\,e^{\left(-\frac{1}{2}\,bx+\frac{1}{2}\,ne^{\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)}-\frac{1}{2}\,ne^{\left(-\frac{1}{2}\,bx-\frac{1}{2}\,a\right)}-\frac{1}{2}\,a\right)}}{bn}-\frac{4\,e^{\left(\frac{1}{2}\,ne^{\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)}-\frac{1}{2}\,ne^{\left(-\frac{1}{2}\,bx-\frac{1}{2}\,a\right)}\right)}}{bn^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*sinh(1/2*a+1/2*b*x))*sinh(b*x+a),x, algorithm="maxima")
```

```
[Out] 2*e^(1/2*b*x + 1/2*n*e^(1/2*b*x + 1/2*a) - 1/2*n*e^(-1/2*b*x - 1/2*a) + 1/2
*a)/(b*n) - 2*e^(-1/2*b*x + 1/2*n*e^(1/2*b*x + 1/2*a) - 1/2*n*e^(-1/2*b*x -
 1/2*a) - 1/2*a)/(b*n) - 4*e^(1/2*n*e^(1/2*b*x + 1/2*a) - 1/2*n*e^(-1/2*b*x
 - 1/2*a))/(b*n^2)
```

mupad [B] time = 0.00, size = 127, normalized size = 1.98

$$\frac{2\,\mathrm{e}^{-\frac{a}{2}}\,\mathrm{e}^{b\,x}\,\mathrm{e}^{-\frac{b\,x}{2}}\,\mathrm{e}^{a}\,\mathrm{e}^{-\frac{n\,\mathrm{e}^{-\frac{a}{2}}\,\mathrm{e}^{-\frac{b\,x}{2}}}{2}}\,\mathrm{e}^{\frac{n\,\mathrm{e}^{a/2}\,\mathrm{e}^{\frac{b\,x}{2}}}{2}}}{b\,n}-\frac{2\,\mathrm{e}^{-\frac{a}{2}}\,\mathrm{e}^{-\frac{b\,x}{2}}\,\mathrm{e}^{-\frac{n\,\mathrm{e}^{-\frac{a}{2}}\,\mathrm{e}^{-\frac{b\,x}{2}}}{2}}\,\mathrm{e}^{\frac{n\,\mathrm{e}^{a/2}\,\mathrm{e}^{\frac{b\,x}{2}}}{2}}}{b\,n}-\frac{4\,\mathrm{e}^{-\frac{n\,\mathrm{e}^{-\frac{a}{2}}\,\mathrm{e}^{-\frac{b\,x}{2}}}{2}}\,\mathrm{e}^{\frac{n\,\mathrm{e}^{a/2}\,\mathrm{e}^{\frac{b\,x}{2}}}{2}}}{b\,n^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*sinh(a/2 + (b*x)/2))*sinh(a + b*x),x)
```

```
[Out] (2*exp(-a/2)*exp(b*x)*exp(-(b*x)/2)*exp(a)*exp(-(n*exp(-a/2)*exp(-(b*x)/2))
/2)*exp((n*exp(a/2)*exp((b*x)/2))/2))/(b*n) - (2*exp(-a/2)*exp(-(b*x)/2)*ex
p(-(n*exp(-a/2)*exp(-(b*x)/2))/2)*exp((n*exp(a/2)*exp((b*x)/2))/2))/(b*n) -
 (4*exp(-(n*exp(-a/2)*exp(-(b*x)/2))/2)*exp((n*exp(a/2)*exp((b*x)/2))/2))/(
b*n^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int e^{n\sinh\left(\frac{a}{2}+\frac{bx}{2}\right)}\sinh\left(a+bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*sinh(1/2*a+1/2*b*x))*sinh(b*x+a),x)
```

```
[Out] Integral(exp(n*sinh(a/2 + b*x/2))*sinh(a + b*x), x)
```

3.1008 $\int e^{n\cosh(a+bx)}\sinh(2a+2bx)\,dx$

Optimal. Leaf size=43

$$\frac{2\cosh(a+bx)e^{n\cosh(a+bx)}}{bn}-\frac{2e^{n\cosh(a+bx)}}{bn^2}$$

```
[Out] -2*exp(n*cosh(b*x+a))/b/n^2+2*exp(n*cosh(b*x+a))*cosh(b*x+a)/b/n
```

Rubi [A] time = 0.04, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {12, 2176, 2194}

$$\frac{2\cosh(a+bx)e^{n\cosh(a+bx)}}{bn}-\frac{2e^{n\cosh(a+bx)}}{bn^2}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*Cosh[a + b*x])*Sinh[2*a + 2*b*x],x]
```

```
[Out] (-2*E^(n*Cosh[a + b*x]))/(b*n^2) + (2*E^(n*Cosh[a + b*x])*Cosh[a + b*x])/(b
*n)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2176

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_.), x_Symbol] :> Simp[((c + d*x)^m*(b*F^(g*(e + f*x)))^n)/(f*g*n*Log[F]),
x] - Dist[(d*m)/(f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^n
, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*m
] && !$UseGamma === True
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rubi steps

$$\int e^{n\cosh(a+bx)}\sinh(2a+2bx)\,dx = \frac{i\,\text{Subst}\left(\int -2ie^{nx}x\,dx, x, \cosh(a+bx)\right)}{b}$$
$$= \frac{2\,\text{Subst}\left(\int e^{nx}x\,dx, x, \cosh(a+bx)\right)}{b}$$
$$= \frac{2e^{n\cosh(a+bx)}\cosh(a+bx)}{bn} - \frac{2\,\text{Subst}\left(\int e^{nx}\,dx, x, \cosh(a+bx)\right)}{bn}$$
$$= -\frac{2e^{n\cosh(a+bx)}}{bn^2} + \frac{2e^{n\cosh(a+bx)}\cosh(a+bx)}{bn}$$

Mathematica [A] time = 0.14, size = 28, normalized size = 0.65

$$\frac{2e^{n\cosh(a+bx)}(n\cosh(a+bx)-1)}{bn^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*Cosh[a + b*x])*Sinh[2*a + 2*b*x],x]
[Out] (2*E^(n*Cosh[a + b*x])*(-1 + n*Cosh[a + b*x]))/(b*n^2)
```

fricas [A] time = 0.42, size = 73, normalized size = 1.70

$$\frac{2\left(\left(n\cosh\left(bx+a\right)-1\right)\cosh\left(n\cosh\left(bx+a\right)\right)+\left(n\cosh\left(bx+a\right)-1\right)\sinh\left(n\cosh\left(bx+a\right)\right)\right)}{bn^2\cosh\left(bx+a\right)^2-bn^2\sinh\left(bx+a\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*cosh(b*x+a))*sinh(2*b*x+2*a),x, algorithm="fricas")
[Out] 2*((n*cosh(b*x + a) - 1)*cosh(n*cosh(b*x + a)) + (n*cosh(b*x + a) - 1)*sinh
(n*cosh(b*x + a)))/(b*n^2*cosh(b*x + a)^2 - b*n^2*sinh(b*x + a)^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int e^{\left(n\cosh\left(bx+a\right)\right)}\sinh\left(2\,bx+2\,a\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*cosh(b*x+a))*sinh(2*b*x+2*a),x, algorithm="giac")
[Out] integrate(e^(n*cosh(b*x + a))*sinh(2*b*x + 2*a), x)
```

maple [A] time = 0.47, size = 59, normalized size = 1.37

$$\frac{\left(\mathrm{e}^{2bx+2a}n+n-2\,\mathrm{e}^{bx+a}\right)\mathrm{e}^{-bx-a+\frac{n\,\mathrm{e}^{bx+a}}{2}+\frac{n\,\mathrm{e}^{-bx-a}}{2}}}{n^{2}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*cosh(b*x+a))*sinh(2*b*x+2*a),x)
```

```
[Out] 1/n^2/b*(exp(2*b*x+2*a)*n+n-2*exp(b*x+a))*exp(-b*x-a+1/2*n*exp(b*x+a)+1/2*n
*exp(-b*x-a))
```

maxima [B] time = 0.41, size = 103, normalized size = 2.40

$$\frac{e^{\left(bx+\frac{1}{2}\,ne^{(bx+a)}+\frac{1}{2}\,ne^{(-bx-a)}+a\right)}}{bn}+\frac{e^{\left(-bx+\frac{1}{2}\,ne^{(bx+a)}+\frac{1}{2}\,ne^{(-bx-a)}-a\right)}}{bn}-\frac{2\,e^{\left(\frac{1}{2}\,ne^{(bx+a)}+\frac{1}{2}\,ne^{(-bx-a)}\right)}}{bn^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*cosh(b*x+a))*sinh(2*b*x+2*a),x, algorithm="maxima")
```

```
[Out] e^(b*x + 1/2*n*e^(b*x + a) + 1/2*n*e^(-b*x - a) + a)/(b*n) + e^(-b*x + 1/2*
n*e^(b*x + a) + 1/2*n*e^(-b*x - a) - a)/(b*n) - 2*e^(1/2*n*e^(b*x + a) + 1/
2*n*e^(-b*x - a))/(b*n^2)
```

mupad [B] time = 1.76, size = 107, normalized size = 2.49

$$\frac{\mathrm{e}^{-a}\,\mathrm{e}^{\frac{n\,\mathrm{e}^{b\,x}\,\mathrm{e}^{a}}{2}}\,\mathrm{e}^{-b\,x}\,\mathrm{e}^{\frac{n\,\mathrm{e}^{-a}\,\mathrm{e}^{-b\,x}}{2}}}{b\,n}-\frac{2\,\mathrm{e}^{\frac{n\,\mathrm{e}^{b\,x}\,\mathrm{e}^{a}}{2}}\,\mathrm{e}^{\frac{n\,\mathrm{e}^{-a}\,\mathrm{e}^{-b\,x}}{2}}}{b\,n^{2}}+\frac{\mathrm{e}^{\frac{n\,\mathrm{e}^{b\,x}\,\mathrm{e}^{a}}{2}}\,\mathrm{e}^{b\,x}\,\mathrm{e}^{a}\,\mathrm{e}^{\frac{n\,\mathrm{e}^{-a}\,\mathrm{e}^{-b\,x}}{2}}}{b\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*cosh(a + b*x))*sinh(2*a + 2*b*x),x)
```

```
[Out] (exp(-a)*exp((n*exp(b*x)*exp(a))/2)*exp(-b*x)*exp((n*exp(-a)*exp(-b*x))/2))
/(b*n) - (2*exp((n*exp(b*x)*exp(a))/2)*exp((n*exp(-a)*exp(-b*x))/2))/(b*n^2
) + (exp((n*exp(b*x)*exp(a))/2)*exp(b*x)*exp(a)*exp((n*exp(-a)*exp(-b*x))/2
))/(b*n)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int e^{n\cosh{(a+bx)}}\sinh{(2a+2bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*cosh(b*x+a))*sinh(2*b*x+2*a),x)
```

```
[Out] Integral(exp(n*cosh(a + b*x))*sinh(2*a + 2*b*x), x)
```

3.1009 $\int e^{n\cosh(a+bx)}\sinh(2(a+bx))\,dx$

Optimal. Leaf size=43

$$\frac{2\cosh(a+bx)e^{n\cosh(a+bx)}}{bn}-\frac{2e^{n\cosh(a+bx)}}{bn^2}$$

```
[Out] -2*exp(n*cosh(b*x+a))/b/n^2+2*exp(n*cosh(b*x+a))*cosh(b*x+a)/b/n
```

Rubi [A] time = 0.04, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {12, 2176, 2194}

$$\frac{2\cosh(a+bx)e^{n\cosh(a+bx)}}{bn}-\frac{2e^{n\cosh(a+bx)}}{bn^2}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*Cosh[a + b*x])*Sinh[2*(a + b*x)],x]
```

```
[Out] (-2*E^(n*Cosh[a + b*x]))/(b*n^2) + (2*E^(n*Cosh[a + b*x])*Cosh[a + b*x])/(b
*n)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2176

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_.), x_Symbol] :> Simp[((c + d*x)^m*(b*F^(g*(e + f*x)))^n)/(f*g*n*Log[F]),
x] - Dist[(d*m)/(f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^n
, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*m
] &&  !$UseGamma === True
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rubi steps

$$\int e^{n\cosh(a+bx)}\sinh(2(a+bx))\,dx = \frac{i\,\text{Subst}\left(\int -2ie^{nx}x\,dx, x, \cosh(a+bx)\right)}{b}$$
$$= \frac{2\,\text{Subst}\left(\int e^{nx}x\,dx, x, \cosh(a+bx)\right)}{b}$$
$$= \frac{2e^{n\cosh(a+bx)}\cosh(a+bx)}{bn} - \frac{2\,\text{Subst}\left(\int e^{nx}\,dx, x, \cosh(a+bx)\right)}{bn}$$
$$= -\frac{2e^{n\cosh(a+bx)}}{bn^2} + \frac{2e^{n\cosh(a+bx)}\cosh(a+bx)}{bn}$$

Mathematica [A] time = 0.03, size = 28, normalized size = 0.65

$$\frac{2e^{n\cosh(a+bx)}(n\cosh(a+bx)-1)}{bn^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*Cosh[a + b*x])*Sinh[2*(a + b*x)],x]
```

```
[Out] (2*E^(n*Cosh[a + b*x])*(-1 + n*Cosh[a + b*x]))/(b*n^2)
```

fricas [A] time = 0.42, size = 73, normalized size = 1.70

$$\frac{2\left((n\cosh(bx+a)-1)\cosh(n\cosh(bx+a)) + (n\cosh(bx+a)-1)\sinh(n\cosh(bx+a))\right)}{bn^2\cosh(bx+a)^2 - bn^2\sinh(bx+a)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*cosh(b*x+a))*sinh(2*b*x+2*a),x, algorithm="fricas")
```

```
[Out] 2*((n*cosh(b*x + a) - 1)*cosh(n*cosh(b*x + a)) + (n*cosh(b*x + a) - 1)*sinh
(n*cosh(b*x + a)))/(b*n^2*cosh(b*x + a)^2 - b*n^2*sinh(b*x + a)^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int e^{(n\cosh(bx+a))}\sinh(2\,bx+2\,a)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*cosh(b*x+a))*sinh(2*b*x+2*a),x, algorithm="giac")
```

```
[Out] integrate(e^(n*cosh(b*x + a))*sinh(2*b*x + 2*a), x)
```

maple [A] time = 0.33, size = 59, normalized size = 1.37

$$\frac{\left(e^{2bx+2a}n+n-2\,e^{bx+a}\right)e^{-bx-a+\frac{n\,e^{bx+a}}{2}+\frac{n\,e^{-bx-a}}{2}}}{n^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*cosh(b*x+a))*sinh(2*b*x+2*a),x)
```

```
[Out] 1/n^2/b*(exp(2*b*x+2*a)*n+n-2*exp(b*x+a))*exp(-b*x-a+1/2*n*exp(b*x+a)+1/2*n
*exp(-b*x-a))
```

maxima [B] time = 0.40, size = 103, normalized size = 2.40

$$\frac{e^{\left(bx+\frac{1}{2}\,ne^{(bx+a)}+\frac{1}{2}\,ne^{(-bx-a)}+a\right)}}{bn}+\frac{e^{\left(-bx+\frac{1}{2}\,ne^{(bx+a)}+\frac{1}{2}\,ne^{(-bx-a)}-a\right)}}{bn}-\frac{2\,e^{\left(\frac{1}{2}\,ne^{(bx+a)}+\frac{1}{2}\,ne^{(-bx-a)}\right)}}{bn^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*cosh(b*x+a))*sinh(2*b*x+2*a),x, algorithm="maxima")
```

```
[Out] e^(b*x + 1/2*n*e^(b*x + a) + 1/2*n*e^(-b*x - a) + a)/(b*n) + e^(-b*x + 1/2*
n*e^(b*x + a) + 1/2*n*e^(-b*x - a) - a)/(b*n) - 2*e^(1/2*n*e^(b*x + a) + 1/
2*n*e^(-b*x - a))/(b*n^2)
```

mupad [B] time = 0.00, size = 107, normalized size = 2.49

$$\frac{e^{-a}\,e^{\frac{n\,e^{b\,x}\,e^{a}}{2}}\,e^{-b\,x}\,e^{\frac{n\,e^{-a}\,e^{-b\,x}}{2}}}{b\,n}-\frac{2\,e^{\frac{n\,e^{b\,x}\,e^{a}}{2}}\,e^{\frac{n\,e^{-a}\,e^{-b\,x}}{2}}}{b\,n^2}+\frac{e^{\frac{n\,e^{b\,x}\,e^{a}}{2}}\,e^{b\,x}\,e^{a}\,e^{\frac{n\,e^{-a}\,e^{-b\,x}}{2}}}{b\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*cosh(a + b*x))*sinh(2*a + 2*b*x),x)
```

```
[Out] (exp(-a)*exp((n*exp(b*x)*exp(a))/2)*exp(-b*x)*exp((n*exp(-a)*exp(-b*x))/2))
/(b*n) - (2*exp((n*exp(b*x)*exp(a))/2)*exp((n*exp(-a)*exp(-b*x))/2))/(b*n^2
) + (exp((n*exp(b*x)*exp(a))/2)*exp(b*x)*exp(a)*exp((n*exp(-a)*exp(-b*x))/2
))/(b*n)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int e^{n\cosh{(a+bx)}}\sinh{(2a+2bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*cosh(b*x+a))*sinh(2*b*x+2*a),x)
```

```
[Out] Integral(exp(n*cosh(a + b*x))*sinh(2*a + 2*b*x), x)
```

3.1010 $\int e^{n\cosh\left(\frac{a}{2}+\frac{bx}{2}\right)}\sinh(a+bx)\,dx$

Optimal. Leaf size=64

$$\frac{4\cosh\left(\frac{a}{2}+\frac{bx}{2}\right)e^{n\cosh\left(\frac{a}{2}+\frac{bx}{2}\right)}}{bn}-\frac{4e^{n\cosh\left(\frac{a}{2}+\frac{bx}{2}\right)}}{bn^2}$$

```
[Out] -4*exp(n*cosh(1/2*a+1/2*b*x))/b/n^2+4*exp(n*cosh(1/2*a+1/2*b*x))*cosh(1/2*a
+1/2*b*x)/b/n
```

Rubi [A] time = 0.05, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {12, 2176, 2194}

$$\frac{4\cosh\left(\frac{a}{2}+\frac{bx}{2}\right)e^{n\cosh\left(\frac{a}{2}+\frac{bx}{2}\right)}}{bn}-\frac{4e^{n\cosh\left(\frac{a}{2}+\frac{bx}{2}\right)}}{bn^2}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*Cosh[a/2 + (b*x)/2])*Sinh[a + b*x],x]
```

```
[Out] (-4*E^(n*Cosh[a/2 + (b*x)/2]))/(b*n^2) + (4*E^(n*Cosh[a/2 + (b*x)/2])*Cosh[
a/2 + (b*x)/2])/(b*n)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2176

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_.), x_Symbol] :> Simp[((c + d*x)^m*(b*F^(g*(e + f*x)))^n)/(f*g*n*Log[F]),
x] - Dist[(d*m)/(f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^n
, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*m
] &&  !$UseGamma === True
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rubi steps

$$\int e^{n\cosh\left(\frac{a}{2}+\frac{bx}{2}\right)}\sinh(a+bx)\,dx = \frac{(2i)\,\text{Subst}\left(\int -2ie^{nx}x\,dx, x, \cosh\left(\frac{a}{2}+\frac{bx}{2}\right)\right)}{b}$$
$$= \frac{4\,\text{Subst}\left(\int e^{nx}x\,dx, x, \cosh\left(\frac{a}{2}+\frac{bx}{2}\right)\right)}{b}$$
$$= \frac{4e^{n\cosh\left(\frac{a}{2}+\frac{bx}{2}\right)}\cosh\left(\frac{a}{2}+\frac{bx}{2}\right)}{bn} - \frac{4\,\text{Subst}\left(\int e^{nx}\,dx, x, \cosh\left(\frac{a}{2}+\frac{bx}{2}\right)\right)}{bn}$$
$$= -\frac{4e^{n\cosh\left(\frac{a}{2}+\frac{bx}{2}\right)}}{bn^2} + \frac{4e^{n\cosh\left(\frac{a}{2}+\frac{bx}{2}\right)}\cosh\left(\frac{a}{2}+\frac{bx}{2}\right)}{bn}$$

Mathematica [A] time = 0.17, size = 36, normalized size = 0.56

$$\frac{4e^{n\cosh\left(\frac{1}{2}(a+bx)\right)}\left(n\cosh\left(\frac{1}{2}(a+bx)\right)-1\right)}{bn^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*Cosh[a/2 + (b*x)/2])*Sinh[a + b*x],x]
```

```
[Out] (4*E^(n*Cosh[(a + b*x)/2])*(-1 + n*Cosh[(a + b*x)/2]))/(b*n^2)
```

fricas [A] time = 0.47, size = 91, normalized size = 1.42

$$\frac{4\left(\left(n\cosh\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)-1\right)\cosh\left(n\cosh\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)\right)+\left(n\cosh\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)-1\right)\sinh\left(n\cosh\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)\right)\right)}{bn^2\cosh\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)^2-bn^2\sinh\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*cosh(1/2*a+1/2*b*x))*sinh(b*x+a),x, algorithm="fricas")
```

```
[Out] 4*((n*cosh(1/2*b*x + 1/2*a) - 1)*cosh(n*cosh(1/2*b*x + 1/2*a)) + (n*cosh(1/
2*b*x + 1/2*a) - 1)*sinh(n*cosh(1/2*b*x + 1/2*a)))/(b*n^2*cosh(1/2*b*x + 1/
2*a)^2 - b*n^2*sinh(1/2*b*x + 1/2*a)^2)
```

giac [B] time = 0.18, size = 254, normalized size = 3.97

$$\frac{2\left(ne^{\left(bx+\frac{1}{4}\left(2\,bxe^{\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)}+ne^{(bx+a)}+n\right)e^{\left(-\frac{1}{2}\,bx-\frac{1}{2}\,a\right)}-\frac{1}{4}\left(2\,bxe^{\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)}-ne^{(bx+a)}-n\right)e^{\left(-\frac{1}{2}\,bx-\frac{1}{2}\,a\right)}+a\right)}+ne^{\left(\frac{1}{4}\left(2\,bxe^{\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)}+ne^{(bx+a)}+n\right)e^{\left(-\frac{1}{2}\right.}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*cosh(1/2*a+1/2*b*x))*sinh(b*x+a),x, algorithm="giac")
```

```
[Out] 2*(n*e^(b*x + 1/4*(2*b*x*e^(1/2*b*x + 1/2*a) + n*e^(b*x + a) + n)*e^(-1/2*b
*x - 1/2*a) - 1/4*(2*b*x*e^(1/2*b*x + 1/2*a) - n*e^(b*x + a) - n)*e^(-1/2*b
*x - 1/2*a) + a) + n*e^(1/4*(2*b*x*e^(1/2*b*x + 1/2*a) + n*e^(b*x + a) + n)
*e^(-1/2*b*x - 1/2*a) - 1/4*(2*b*x*e^(1/2*b*x + 1/2*a) - n*e^(b*x + a) - n)
*e^(-1/2*b*x - 1/2*a)) - 2*e^(1/2*b*x + 1/4*(2*b*x*e^(1/2*b*x + 1/2*a) + n*
e^(b*x + a) + n)*e^(-1/2*b*x - 1/2*a) - 1/4*(2*b*x*e^(1/2*b*x + 1/2*a) - n*
e^(b*x + a) - n)*e^(-1/2*b*x - 1/2*a) + 1/2*a))*e^(-1/2*b*x - 1/2*a)/(b*n^2
)
```

maple [A] time = 0.44, size = 63, normalized size = 0.98

$$\frac{2\left(n\,\mathrm{e}^{bx+a}+n-2\,\mathrm{e}^{\frac{bx}{2}+\frac{a}{2}}\right)\mathrm{e}^{-\frac{bx}{2}-\frac{a}{2}+\frac{n\,\mathrm{e}^{\frac{bx}{2}+\frac{a}{2}}}{2}+\frac{n\,\mathrm{e}^{-\frac{bx}{2}-\frac{a}{2}}}{2}}}{n^{2}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*cosh(1/2*b*x+1/2*a))*sinh(b*x+a),x)
```

```
[Out] 2/n^2/b*(n*exp(b*x+a)+n-2*exp(1/2*b*x+1/2*a))*exp(-1/2*b*x-1/2*a+1/2*n*exp(
1/2*b*x+1/2*a)+1/2*n*exp(-1/2*b*x-1/2*a))
```

maxima [B] time = 0.40, size = 117, normalized size = 1.83

$$\frac{2\,e^{\left(\frac{1}{2}\,bx+\frac{1}{2}\,ne^{\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)}+\frac{1}{2}\,ne^{\left(-\frac{1}{2}\,bx-\frac{1}{2}\,a\right)}+\frac{1}{2}\,a\right)}}{bn}+\frac{2\,e^{\left(-\frac{1}{2}\,bx+\frac{1}{2}\,ne^{\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)}+\frac{1}{2}\,ne^{\left(-\frac{1}{2}\,bx-\frac{1}{2}\,a\right)}-\frac{1}{2}\,a\right)}}{bn}-\frac{4\,e^{\left(\frac{1}{2}\,ne^{\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)}+\frac{1}{2}\,ne^{\left(-\frac{1}{2}\,bx-\frac{1}{2}\,a\right)}\right)}}{bn^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*cosh(1/2*a+1/2*b*x))*sinh(b*x+a),x, algorithm="maxima")
```

```
[Out] 2*e^(1/2*b*x + 1/2*n*e^(1/2*b*x + 1/2*a) + 1/2*n*e^(-1/2*b*x - 1/2*a) + 1/2
*a)/(b*n) + 2*e^(-1/2*b*x + 1/2*n*e^(1/2*b*x + 1/2*a) + 1/2*n*e^(-1/2*b*x -
 1/2*a) - 1/2*a)/(b*n) - 4*e^(1/2*n*e^(1/2*b*x + 1/2*a) + 1/2*n*e^(-1/2*b*x
 - 1/2*a))/(b*n^2)
```

mupad [B] time = 1.77, size = 127, normalized size = 1.98

$$\frac{2\,\mathrm{e}^{-\frac{a}{2}}\,\mathrm{e}^{-\frac{b\,x}{2}}\,\mathrm{e}^{\frac{n\,\mathrm{e}^{-\frac{a}{2}}\,\mathrm{e}^{-\frac{b\,x}{2}}}{2}}\,\mathrm{e}^{\frac{n\,\mathrm{e}^{a/2}\,\mathrm{e}^{\frac{b\,x}{2}}}{2}}}{b\,n}-\frac{4\,\mathrm{e}^{\frac{n\,\mathrm{e}^{-\frac{a}{2}}\,\mathrm{e}^{-\frac{b\,x}{2}}}{2}}\,\mathrm{e}^{\frac{n\,\mathrm{e}^{a/2}\,\mathrm{e}^{\frac{b\,x}{2}}}{2}}}{b\,n^{2}}+\frac{2\,\mathrm{e}^{-\frac{a}{2}}\,\mathrm{e}^{b\,x}\,\mathrm{e}^{-\frac{b\,x}{2}}\,\mathrm{e}^{a}\,\mathrm{e}^{\frac{n\,\mathrm{e}^{-\frac{a}{2}}\,\mathrm{e}^{-\frac{b\,x}{2}}}{2}}\,\mathrm{e}^{\frac{n\,\mathrm{e}^{a/2}\,\mathrm{e}^{\frac{b\,x}{2}}}{2}}}{b\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*cosh(a/2 + (b*x)/2))*sinh(a + b*x),x)
```

```
[Out] (2*exp(-a/2)*exp(-(b*x)/2)*exp((n*exp(-a/2)*exp(-(b*x)/2))/2)*exp((n*exp(a/
2)*exp((b*x)/2))/2))/(b*n) - (4*exp((n*exp(-a/2)*exp(-(b*x)/2))/2)*exp((n*e
xp(a/2)*exp((b*x)/2))/2))/(b*n^2) + (2*exp(-a/2)*exp(b*x)*exp(-(b*x)/2)*exp
(a)*exp((n*exp(-a/2)*exp(-(b*x)/2))/2)*exp((n*exp(a/2)*exp((b*x)/2))/2))/(b
*n)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int e^{n\cosh\left(\frac{a}{2}+\frac{bx}{2}\right)}\sinh\left(a+bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*cosh(1/2*a+1/2*b*x))*sinh(b*x+a),x)
```

```
[Out] Integral(exp(n*cosh(a/2 + b*x/2))*sinh(a + b*x), x)
```

3.1011 $\int e^{n\cosh\left(\frac{1}{2}(a+bx)\right)}\sinh(a+bx)\,dx$

Optimal. Leaf size=64

$$\frac{4\cosh\left(\frac{a}{2}+\frac{bx}{2}\right)e^{n\cosh\left(\frac{a}{2}+\frac{bx}{2}\right)}}{bn}-\frac{4e^{n\cosh\left(\frac{a}{2}+\frac{bx}{2}\right)}}{bn^2}$$

```
[Out] -4*exp(n*cosh(1/2*a+1/2*b*x))/b/n^2+4*exp(n*cosh(1/2*a+1/2*b*x))*cosh(1/2*a
+1/2*b*x)/b/n
```

Rubi [A] time = 0.04, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {12, 2176, 2194}

$$\frac{4\cosh\left(\frac{a}{2}+\frac{bx}{2}\right)e^{n\cosh\left(\frac{a}{2}+\frac{bx}{2}\right)}}{bn}-\frac{4e^{n\cosh\left(\frac{a}{2}+\frac{bx}{2}\right)}}{bn^2}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*Cosh[(a + b*x)/2])*Sinh[a + b*x],x]
```

```
[Out] (-4*E^(n*Cosh[a/2 + (b*x)/2]))/(b*n^2) + (4*E^(n*Cosh[a/2 + (b*x)/2])*Cosh[
a/2 + (b*x)/2])/(b*n)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2176

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_.), x_Symbol] :> Simp[((c + d*x)^m*(b*F^(g*(e + f*x)))^n)/(f*g*n*Log[F]),
x] - Dist[(d*m)/(f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^n
, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*m
] && !$UseGamma === True
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rubi steps

$$\int e^{n\cosh\left(\frac{1}{2}(a+bx)\right)}\sinh(a+bx)\,dx = \frac{(2i)\,\text{Subst}\left(\int -2ie^{nx}x\,dx, x, \cosh\left(\frac{a}{2}+\frac{bx}{2}\right)\right)}{b}$$
$$= \frac{4\,\text{Subst}\left(\int e^{nx}x\,dx, x, \cosh\left(\frac{a}{2}+\frac{bx}{2}\right)\right)}{b}$$
$$= \frac{4e^{n\cosh\left(\frac{a}{2}+\frac{bx}{2}\right)}\cosh\left(\frac{a}{2}+\frac{bx}{2}\right)}{bn} - \frac{4\,\text{Subst}\left(\int e^{nx}\,dx, x, \cosh\left(\frac{a}{2}+\frac{bx}{2}\right)\right)}{bn}$$
$$= -\frac{4e^{n\cosh\left(\frac{a}{2}+\frac{bx}{2}\right)}}{bn^2} + \frac{4e^{n\cosh\left(\frac{a}{2}+\frac{bx}{2}\right)}\cosh\left(\frac{a}{2}+\frac{bx}{2}\right)}{bn}$$

Mathematica [A] time = 0.04, size = 36, normalized size = 0.56

$$\frac{4e^{n\cosh\left(\frac{1}{2}(a+bx)\right)}\left(n\cosh\left(\frac{1}{2}(a+bx)\right)-1\right)}{bn^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*Cosh[(a + b*x)/2])*Sinh[a + b*x],x]
```

```
[Out] (4*E^(n*Cosh[(a + b*x)/2])*(-1 + n*Cosh[(a + b*x)/2]))/(b*n^2)
```

fricas [A] time = 0.46, size = 91, normalized size = 1.42

$$\frac{4\left(\left(n\cosh\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)-1\right)\cosh\left(n\cosh\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)\right)+\left(n\cosh\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)-1\right)\sinh\left(n\cosh\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)\right)\right)}{bn^2\cosh\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)^2-bn^2\sinh\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*cosh(1/2*a+1/2*b*x))*sinh(b*x+a),x, algorithm="fricas")
```

```
[Out] 4*((n*cosh(1/2*b*x + 1/2*a) - 1)*cosh(n*cosh(1/2*b*x + 1/2*a)) + (n*cosh(1/
2*b*x + 1/2*a) - 1)*sinh(n*cosh(1/2*b*x + 1/2*a)))/(b*n^2*cosh(1/2*b*x + 1/
2*a)^2 - b*n^2*sinh(1/2*b*x + 1/2*a)^2)
```

giac [B] time = 0.17, size = 254, normalized size = 3.97

$$\frac{2\left(ne^{\left(bx+\frac{1}{4}\left(2\,bxe^{\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)}+ne^{(bx+a)}+n\right)e^{\left(-\frac{1}{2}\,bx-\frac{1}{2}\,a\right)}-\frac{1}{4}\left(2\,bxe^{\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)}-ne^{(bx+a)}-n\right)e^{\left(-\frac{1}{2}\,bx-\frac{1}{2}\,a\right)}+a\right)}+ne^{\left(\frac{1}{4}\left(2\,bxe^{\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)}+ne^{(bx+a)}+n\right)e^{\left(-\frac{1}{2}\right.}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*cosh(1/2*a+1/2*b*x))*sinh(b*x+a),x, algorithm="giac")
```

```
[Out] 2*(n*e^(b*x + 1/4*(2*b*x*e^(1/2*b*x + 1/2*a) + n*e^(b*x + a) + n)*e^(-1/2*b
*x - 1/2*a) - 1/4*(2*b*x*e^(1/2*b*x + 1/2*a) - n*e^(b*x + a) - n)*e^(-1/2*b
*x - 1/2*a) + a) + n*e^(1/4*(2*b*x*e^(1/2*b*x + 1/2*a) + n*e^(b*x + a) + n)
*e^(-1/2*b*x - 1/2*a) - 1/4*(2*b*x*e^(1/2*b*x + 1/2*a) - n*e^(b*x + a) - n)
*e^(-1/2*b*x - 1/2*a)) - 2*e^(1/2*b*x + 1/4*(2*b*x*e^(1/2*b*x + 1/2*a) + n*
e^(b*x + a) + n)*e^(-1/2*b*x - 1/2*a) - 1/4*(2*b*x*e^(1/2*b*x + 1/2*a) - n*
e^(b*x + a) - n)*e^(-1/2*b*x - 1/2*a) + 1/2*a))*e^(-1/2*b*x - 1/2*a)/(b*n^2
)
```

maple [A] time = 0.31, size = 63, normalized size = 0.98

$$\frac{2\left(n\,\mathrm{e}^{bx+a}+n-2\,\mathrm{e}^{\frac{bx}{2}+\frac{a}{2}}\right)\mathrm{e}^{-\frac{bx}{2}-\frac{a}{2}+\frac{n\,\mathrm{e}^{\frac{bx}{2}+\frac{a}{2}}}{2}+\frac{n\,\mathrm{e}^{-\frac{bx}{2}-\frac{a}{2}}}{2}}}{n^2 b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*cosh(1/2*b*x+1/2*a))*sinh(b*x+a),x)
```

```
[Out] 2/n^2/b*(n*exp(b*x+a)+n-2*exp(1/2*b*x+1/2*a))*exp(-1/2*b*x-1/2*a+1/2*n*exp(
1/2*b*x+1/2*a)+1/2*n*exp(-1/2*b*x-1/2*a))
```

maxima [B] time = 0.41, size = 117, normalized size = 1.83

$$\frac{2\,e^{\left(\frac{1}{2}\,bx+\frac{1}{2}\,ne^{\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)}+\frac{1}{2}\,ne^{\left(-\frac{1}{2}\,bx-\frac{1}{2}\,a\right)}+\frac{1}{2}\,a\right)}}{bn}+\frac{2\,e^{\left(-\frac{1}{2}\,bx+\frac{1}{2}\,ne^{\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)}+\frac{1}{2}\,ne^{\left(-\frac{1}{2}\,bx-\frac{1}{2}\,a\right)}-\frac{1}{2}\,a\right)}}{bn}-\frac{4\,e^{\left(\frac{1}{2}\,ne^{\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)}+\frac{1}{2}\,ne^{\left(-\frac{1}{2}\,bx-\frac{1}{2}\,a\right)}\right)}}{bn^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*cosh(1/2*a+1/2*b*x))*sinh(b*x+a),x, algorithm="maxima")
```

```
[Out] 2*e^(1/2*b*x + 1/2*n*e^(1/2*b*x + 1/2*a) + 1/2*n*e^(-1/2*b*x - 1/2*a) + 1/2
*a)/(b*n) + 2*e^(-1/2*b*x + 1/2*n*e^(1/2*b*x + 1/2*a) + 1/2*n*e^(-1/2*b*x -
 1/2*a) - 1/2*a)/(b*n) - 4*e^(1/2*n*e^(1/2*b*x + 1/2*a) + 1/2*n*e^(-1/2*b*x
 - 1/2*a))/(b*n^2)
```

mupad [B] time = 0.00, size = 127, normalized size = 1.98

$$\frac{2\,\mathrm{e}^{-\frac{a}{2}}\,\mathrm{e}^{-\frac{b\,x}{2}}\,\mathrm{e}^{\frac{n\,\mathrm{e}^{-\frac{a}{2}}\,\mathrm{e}^{-\frac{b\,x}{2}}}{2}}\,\mathrm{e}^{\frac{n\,\mathrm{e}^{a/2}\,\mathrm{e}^{\frac{b\,x}{2}}}{2}}}{b\,n}-\frac{4\,\mathrm{e}^{\frac{n\,\mathrm{e}^{-\frac{a}{2}}\,\mathrm{e}^{-\frac{b\,x}{2}}}{2}}\,\mathrm{e}^{\frac{n\,\mathrm{e}^{a/2}\,\mathrm{e}^{\frac{b\,x}{2}}}{2}}}{b\,n^2}+\frac{2\,\mathrm{e}^{-\frac{a}{2}}\,\mathrm{e}^{b\,x}\,\mathrm{e}^{-\frac{b\,x}{2}}\,\mathrm{e}^{a}\,\mathrm{e}^{\frac{n\,\mathrm{e}^{-\frac{a}{2}}\,\mathrm{e}^{-\frac{b\,x}{2}}}{2}}\,\mathrm{e}^{\frac{n\,\mathrm{e}^{a/2}\,\mathrm{e}^{\frac{b\,x}{2}}}{2}}}{b\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(exp(n*cosh(a/2 + (b*x)/2))*sinh(a + b*x),x)

[Out] (2*exp(-a/2)*exp(-(b*x)/2)*exp((n*exp(-a/2)*exp(-(b*x)/2))/2)*exp((n*exp(a/2)*exp((b*x)/2))/2))/(b*n) - (4*exp((n*exp(-a/2)*exp(-(b*x)/2))/2)*exp((n*exp(a/2)*exp((b*x)/2))/2))/(b*n^2) + (2*exp(-a/2)*exp(b*x)*exp(-(b*x)/2)*exp(a)*exp((n*exp(-a/2)*exp(-(b*x)/2))/2)*exp((n*exp(a/2)*exp((b*x)/2))/2))/(b*n)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int e^{n\cosh\left(\frac{a}{2}+\frac{bx}{2}\right)}\sinh(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(n*cosh(1/2*a+1/2*b*x))*sinh(b*x+a),x)

[Out] Integral(exp(n*cosh(a/2 + b*x/2))*sinh(a + b*x), x)

3.1012 $\int \mathbf{csch}(x) \log(\tanh(x)) \mathbf{sech}(x) \, dx$

Optimal. Leaf size=9

$$\frac{1}{2} \log^2(\tanh(x))$$

```
[Out] 1/2*ln(tanh(x))^2
```

Rubi [A] time = 0.03, antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {2620, 29, 6686}

$$\frac{1}{2} \log^2(\tanh(x))$$

Antiderivative was successfully verified.

```
[In] Int[Csch[x]*Log[Tanh[x]]*Sech[x],x]
```

```
[Out] Log[Tanh[x]]^2/2
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 2620

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 6686

```
Int[(u_)*(y_)^(m_.), x_Symbol] :> With[{q = DerivativeDivides[y, u, x]}, Si
mp[(q*y^(m + 1))/(m + 1), x] /;  !FalseQ[q]] /; FreeQ[m, x] && NeQ[m, -1]
```

Rubi steps

$$\int \operatorname{csch}(x) \log(\tanh(x)) \operatorname{sech}(x) \, dx = \frac{1}{2} \log^2(\tanh(x))$$

Mathematica [A] time = 0.01, size = 9, normalized size = 1.00

$$\frac{1}{2} \log^2(\tanh(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[x]*Log[Tanh[x]]*Sech[x],x]
```

```
[Out] Log[Tanh[x]]^2/2
```

fricas [A] time = 0.42, size = 12, normalized size = 1.33

$$\frac{1}{2}\log\left(\frac{\sinh(x)}{\cosh(x)}\right)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)*log(tanh(x))*sech(x),x, algorithm="fricas")
```

```
[Out] 1/2*log(sinh(x)/cosh(x))^2
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{csch}(x)\log(\tanh(x))\operatorname{sech}(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)*log(tanh(x))*sech(x),x, algorithm="giac")
```

```
[Out] integrate(csch(x)*log(tanh(x))*sech(x), x)
```

maple [A] time = 0.17, size = 8, normalized size = 0.89

$$\frac{\ln(\tanh(x))^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(x)*ln(tanh(x))*sech(x),x)
```

```
[Out] 1/2*ln(tanh(x))^2
```

maxima [B] time = 1.48, size = 95, normalized size = 10.56

$$\left(\log(e^x+1)+\log(-e^x+1)\right)\log\left(e^{(2x)}+1\right)-\frac{1}{2}\log\left(e^{(2x)}+1\right)^2-\frac{1}{2}\log(e^x+1)^2-\log(e^x+1)\log(-e^x+1)-\frac{1}{2}\log$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)*log(tanh(x))*sech(x),x, algorithm="maxima")
```

```
[Out] (log(e^x + 1) + log(-e^x + 1))*log(e^(2*x) + 1) - 1/2*log(e^(2*x) + 1)^2 -
1/2*log(e^x + 1)^2 - log(e^x + 1)*log(-e^x + 1) - 1/2*log(-e^x + 1)^2 + (lo
g(e^(-x) + 1) + log(e^(-x) - 1) - log(e^(-2*x) + 1))*log(tanh(x))
```

mupad [B] time = 1.76, size = 21, normalized size = 2.33

$$\frac{\left(\ln\left(e^{2x}-1\right)-\ln\left(e^{2x}+1\right)\right)^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(tanh(x))/(cosh(x)*sinh(x)),x)
```

```
[Out] (log(exp(2*x) - 1) - log(exp(2*x) + 1))^2/2
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \log(\tanh(x))\operatorname{csch}(x)\operatorname{sech}(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)*ln(tanh(x))*sech(x),x)
```

```
[Out] Integral(log(tanh(x))*csch(x)*sech(x), x)
```

3.1013 $\int \operatorname{csch}(2x)\log(\tanh(x))\,dx$

Optimal. Leaf size=9

$$\frac{1}{4}\log^2(\tanh(x))$$

```
[Out] 1/4*ln(tanh(x))^2
```

Rubi [A] time = 0.02, antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3770, 6686}

$$\frac{1}{4}\log^2(\tanh(x))$$

Antiderivative was successfully verified.

```
[In] Int[Csch[2*x]*Log[Tanh[x]],x]
```

```
[Out] Log[Tanh[x]]^2/4
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 6686

```
Int[(u_)*(y_)^(m_.), x_Symbol] :> With[{q = DerivativeDivides[y, u, x]}, Si
mp[(q*y^(m + 1))/(m + 1), x] /;  !FalseQ[q]] /; FreeQ[m, x] && NeQ[m, -1]
```

Rubi steps

$$\int \operatorname{csch}(2x)\log(\tanh(x))\,dx=\frac{1}{4}\log^2(\tanh(x))$$

Mathematica [A] time = 0.01, size = 9, normalized size = 1.00

$$\frac{1}{4}\log^2(\tanh(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[2*x]*Log[Tanh[x]],x]
```

```
[Out] Log[Tanh[x]]^2/4
```

fricas [A] time = 0.42, size = 12, normalized size = 1.33

$$\frac{1}{4}\,\log\left(\frac{\sinh(x)}{\cosh(x)}\right)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(2*x)*log(tanh(x)),x, algorithm="fricas")
[Out] 1/4*log(sinh(x)/cosh(x))^2
```

giac [B] time = 0.13, size = 20, normalized size = 2.22

$$\frac{1}{4}\,\log\left(\frac{e^{(2\,x)}-1}{e^{(2\,x)}+1}\right)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(2*x)*log(tanh(x)),x, algorithm="giac")
[Out] 1/4*log((e^(2*x) - 1)/(e^(2*x) + 1))^2
```

maple [A] time = 0.12, size = 8, normalized size = 0.89

$$\frac{\ln\left(\tanh(x)\right)^2}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(2*x)*ln(tanh(x)),x)
[Out] 1/4*ln(tanh(x))^2
```

maxima [A] time = 0.30, size = 7, normalized size = 0.78

$$\frac{1}{4}\,\log\left(\tanh(x)\right)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(2*x)*log(tanh(x)),x, algorithm="maxima")
[Out] 1/4*log(tanh(x))^2
```

mupad [B] time = 1.67, size = 21, normalized size = 2.33

$$\frac{\left(\ln\left(\mathrm{e}^{2\,x}-1\right)-\ln\left(\mathrm{e}^{2\,x}+1\right)\right)^2}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(tanh(x))/sinh(2*x),x)
[Out] (log(exp(2*x) - 1) - log(exp(2*x) + 1))^2/4
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \log\left(\tanh\left(x\right)\right)\operatorname{csch}\left(2x\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(2*x)*ln(tanh(x)),x)
[Out] Integral(log(tanh(x))*csch(2*x), x)
```

3.1014 $\int \cosh(a+bx)F(c,d,\sinh(a+bx),r,s)\,dx$

Optimal. Leaf size=21

$$\text{Int}(\cosh(a+bx)F(c,d,\sinh(a+bx),r,s),x)$$

```
[Out] CannotIntegrate(cosh(b*x+a)*F(c,d,sinh(b*x+a),r,s),x)
```

Rubi [A] time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \cosh(a+bx)F(c,d,\sinh(a+bx),r,s)\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Cosh[a + b*x]*F[c, d, Sinh[a + b*x], r, s],x]
```

```
[Out] Defer[Subst][Defer[Int][F[c, d, x, r, s], x], x, Sinh[a + b*x]]/b
```

Rubi steps

$$\int \cosh(a+bx)F(c,d,\sinh(a+bx),r,s)\,dx = \frac{\text{Subst}(\int F(c,d,x,r,s)\,dx,x,\sinh(a+bx))}{b}$$

Mathematica [A] time = 0.04, size = 0, normalized size = 0.00

$$\int \cosh(a+bx)F(c,d,\sinh(a+bx),r,s)\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Cosh[a + b*x]*F[c, d, Sinh[a + b*x], r, s],x]
```

```
[Out] Integrate[Cosh[a + b*x]*F[c, d, Sinh[a + b*x], r, s], x]
```

fricas [A] time = 0.41, size = 0, normalized size = 0.00

$$\text{integral}\left(F\left(c,d,\sinh\left(bx+a\right),r,s\right)\cosh\left(bx+a\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*F(c,d,sinh(b*x+a),r,s),x, algorithm="fricas")
```

```
[Out] integral(F(c, d, sinh(b*x + a), r, s)*cosh(b*x + a), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int F(c,d,\sinh(bx+a),r,s)\cosh(bx+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*F(c,d,sinh(b*x+a),r,s),x, algorithm="giac")
[Out] integrate(F(c, d, sinh(b*x + a), r, s)*cosh(b*x + a), x)
```

maple [A] time = 0.11, size = 0, normalized size = 0.00

$$\int \cosh(bx+a)\,F(c,d,\sinh(bx+a),r,s)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)*F(c,d,sinh(b*x+a),r,s),x)
[Out] int(cosh(b*x+a)*F(c,d,sinh(b*x+a),r,s),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int F(c,d,\sinh(bx+a),r,s)\cosh(bx+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*F(c,d,sinh(b*x+a),r,s),x, algorithm="maxima")
[Out] integrate(F(c, d, sinh(b*x + a), r, s)*cosh(b*x + a), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.05

$$\int \cosh(a+b\,x)\,F(c,d,\sinh(a+b\,x),r,s)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a + b*x)*F(c, d, sinh(a + b*x), r, s),x)
[Out] int(cosh(a + b*x)*F(c, d, sinh(a + b*x), r, s), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int F(c,d,\sinh(a+bx),r,s)\cosh(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*F(c,d,sinh(b*x+a),r,s),x)
```

```
[Out] Integral(F(c, d, sinh(a + b*x), r, s)*cosh(a + b*x), x)
```

3.1015 $\int F(c,d,\cosh(a+bx),r,s)\sinh(a+bx)\,dx$

Optimal. Leaf size=21

$$\mathrm{Int}(\sinh(a+bx)F(c,d,\cosh(a+bx),r,s),x)$$

```
[Out] CannotIntegrate(F(c,d,cosh(b*x+a),r,s)*sinh(b*x+a),x)
```

Rubi [A] time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int F(c,d,\cosh(a+bx),r,s)\sinh(a+bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Int[F[c, d, Cosh[a + b*x], r, s]*Sinh[a + b*x],x]
[Out] Defer[Subst][Defer[Int][F[c, d, x, r, s], x], x, Cosh[a + b*x]]/b
```

Rubi steps

$$\int F(c,d,\cosh(a+bx),r,s)\sinh(a+bx)\,dx=\frac{\mathrm{Subst}(\int F(c,d,x,r,s)\,dx,x,\cosh(a+bx))}{b}$$

Mathematica [A] time = 0.04, size = 0, normalized size = 0.00

$$\int F(c,d,\cosh(a+bx),r,s)\sinh(a+bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[F[c, d, Cosh[a + b*x], r, s]*Sinh[a + b*x],x]
[Out] Integrate[F[c, d, Cosh[a + b*x], r, s]*Sinh[a + b*x], x]
```

fricas [A] time = 0.42, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(F\left(c,d,\cosh\left(bx+a\right),r,s\right)\sinh\left(bx+a\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F(c,d,cosh(b*x+a),r,s)*sinh(b*x+a),x, algorithm="fricas")
```

[Out] integral(F(c, d, cosh(b*x + a), r, s)*sinh(b*x + a), x)

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int F(c,d,\cosh(bx+a),r,s)\sinh(bx+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F(c,d,cosh(b*x+a),r,s)*sinh(b*x+a),x, algorithm="giac")

[Out] integrate(F(c, d, cosh(b*x + a), r, s)*sinh(b*x + a), x)

maple [A] time = 0.07, size = 0, normalized size = 0.00

$$\int F(c,d,\cosh(bx+a),r,s)\sinh(bx+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(F(c,d,cosh(b*x+a),r,s)*sinh(b*x+a),x)

[Out] int(F(c,d,cosh(b*x+a),r,s)*sinh(b*x+a),x)

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int F(c,d,\cosh(bx+a),r,s)\sinh(bx+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F(c,d,cosh(b*x+a),r,s)*sinh(b*x+a),x, algorithm="maxima")

[Out] integrate(F(c, d, cosh(b*x + a), r, s)*sinh(b*x + a), x)

mupad [A] time = 0.00, size = -1, normalized size = -0.05

$$\int \sinh(a+b\,x)\,F(c,d,\cosh(a+b\,x),r,s)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sinh(a + b*x)*F(c, d, cosh(a + b*x), r, s),x)

[Out] int(sinh(a + b*x)*F(c, d, cosh(a + b*x), r, s), x)

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int F(c,d,\cosh(a+bx),r,s)\sinh(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F(c,d,cosh(b*x+a),r,s)*sinh(b*x+a),x)
```

```
[Out] Integral(F(c, d, cosh(a + b*x), r, s)*sinh(a + b*x), x)
```

3.1016 $\int F(c,d,\tanh(a+bx),r,s)\mathbf{sech}^2(a+bx)\,dx$

Optimal. Leaf size=23

$$\text{Int}\left(\text{sech}^2(a+bx)F(c,d,\tanh(a+bx),r,s),x\right)$$

```
[Out] CannotIntegrate(F(c,d,tanh(b*x+a),r,s)*sech(b*x+a)^2,x)
```

Rubi [A] time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int F(c,d,\tanh(a+bx),r,s)\text{sech}^2(a+bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Int[F[c, d, Tanh[a + b*x], r, s]*Sech[a + b*x]^2,x]
[Out] Defer[Subst][Defer[Int][F[c, d, x, r, s], x], x, Tanh[a + b*x]]/b
```

Rubi steps

$$\int F(c,d,\tanh(a+bx),r,s)\text{sech}^2(a+bx)\,dx = \frac{\text{Subst}(\int F(c,d,x,r,s)\,dx, x, \tanh(a+bx))}{b}$$

Mathematica [A] time = 0.08, size = 0, normalized size = 0.00

$$\int F(c,d,\tanh(a+bx),r,s)\text{sech}^2(a+bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[F[c, d, Tanh[a + b*x], r, s]*Sech[a + b*x]^2,x]
[Out] Integrate[F[c, d, Tanh[a + b*x], r, s]*Sech[a + b*x]^2, x]
```

fricas [A] time = 0.41, size = 0, normalized size = 0.00

$$\text{integral}\left(F\left(c,d,\tanh\left(bx+a\right),r,s\right)\text{sech}\left(bx+a\right)^2,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F(c,d,tanh(b*x+a),r,s)*sech(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] integral(F(c, d, tanh(b*x + a), r, s)*sech(b*x + a)^2, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int F(c,d,\tanh(bx+a),r,s)\,\mathrm{sech}(bx+a)^2\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F(c,d,tanh(b*x+a),r,s)*sech(b*x+a)^2,x, algorithm="giac")
[Out] integrate(F(c, d, tanh(b*x + a), r, s)*sech(b*x + a)^2, x)
```

maple [A] time = 0.16, size = 0, normalized size = 0.00

$$\int F(c,d,\tanh(bx+a),r,s)\,\mathrm{sech}(bx+a)^2\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(F(c,d,tanh(b*x+a),r,s)*sech(b*x+a)^2,x)
[Out] int(F(c,d,tanh(b*x+a),r,s)*sech(b*x+a)^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int F(c,d,\tanh(bx+a),r,s)\,\mathrm{sech}(bx+a)^2\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F(c,d,tanh(b*x+a),r,s)*sech(b*x+a)^2,x, algorithm="maxima")
[Out] integrate(F(c, d, tanh(b*x + a), r, s)*sech(b*x + a)^2, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{F(c,d,\tanh(a+b\,x),r,s)}{\cosh(a+b\,x)^2}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(F(c, d, tanh(a + b*x), r, s)/cosh(a + b*x)^2,x)
[Out] int(F(c, d, tanh(a + b*x), r, s)/cosh(a + b*x)^2, x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int F(c,d,\tanh(a+bx),r,s)\,\mathrm{sech}^2(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F(c,d,tanh(b*x+a),r,s)*sech(b*x+a)**2,x)
```

```
[Out] Integral(F(c, d, tanh(a + b*x), r, s)*sech(a + b*x)**2, x)
```

3.1017 $\int \operatorname{csch}^2(a+bx)F(c,d,\coth(a+bx),r,s)\,dx$

Optimal. Leaf size=23

$$\operatorname{Int}\left(\operatorname{csch}^2(a+bx)F(c,d,\coth(a+bx),r,s),x\right)$$

```
[Out] CannotIntegrate(csch(b*x+a)^2*F(c,d,coth(b*x+a),r,s),x)
```

Rubi [A] time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \operatorname{csch}^2(a+bx)F(c,d,\coth(a+bx),r,s)\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Csch[a + b*x]^2*F[c, d, Coth[a + b*x], r, s],x]
[Out] -(Defer[Subst][Defer[Int][F[c, d, x, r, s], x], x, Coth[a + b*x]]/b)
```

Rubi steps

$$\int \operatorname{csch}^2(a+bx)F(c,d,\coth(a+bx),r,s)\,dx=-\frac{\operatorname{Subst}(\int F(c,d,x,r,s)\,dx,x,\coth(a+bx))}{b}$$

Mathematica [A] time = 0.08, size = 0, normalized size = 0.00

$$\int \operatorname{csch}^2(a+bx)F(c,d,\coth(a+bx),r,s)\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Csch[a + b*x]^2*F[c, d, Coth[a + b*x], r, s],x]
[Out] Integrate[Csch[a + b*x]^2*F[c, d, Coth[a + b*x], r, s], x]
```

fricas [A] time = 0.41, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(F\left(c,d,\coth\left(bx+a\right),r,s\right)\operatorname{csch}\left(bx+a\right)^{2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^2*F(c,d,coth(b*x+a),r,s),x, algorithm="fricas")
```

```
[Out] integral(F(c, d, coth(b*x + a), r, s)*csch(b*x + a)^2, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int F(c,d,\coth(bx+a),r,s)\operatorname{csch}(bx+a)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^2*F(c,d,coth(b*x+a),r,s),x, algorithm="giac")
[Out] integrate(F(c, d, coth(b*x + a), r, s)*csch(b*x + a)^2, x)
```

maple [A] time = 0.12, size = 0, normalized size = 0.00

$$\int \operatorname{csch}(bx+a)^2 F(c,d,\coth(bx+a),r,s)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(b*x+a)^2*F(c,d,coth(b*x+a),r,s),x)
[Out] int(csch(b*x+a)^2*F(c,d,coth(b*x+a),r,s),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int F(c,d,\coth(bx+a),r,s)\operatorname{csch}(bx+a)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^2*F(c,d,coth(b*x+a),r,s),x, algorithm="maxima")
[Out] integrate(F(c, d, coth(b*x + a), r, s)*csch(b*x + a)^2, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{F(c,d,\coth(a+b\,x),r,s)}{\sinh(a+b\,x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(F(c, d, coth(a + b*x), r, s)/sinh(a + b*x)^2,x)
[Out] int(F(c, d, coth(a + b*x), r, s)/sinh(a + b*x)^2, x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int F(c,d,\coth(a+bx),r,s)\operatorname{csch}^2(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)**2*F(c,d,coth(b*x+a),r,s),x)
[Out] Integral(F(c, d, coth(a + b*x), r, s)*csch(a + b*x)**2, x)
```

3.1018 $\int \operatorname{sech}(x)\left(5-11\operatorname{sech}^2(x)\right)\tanh(x)\,dx$

Optimal. Leaf size=13

$$\frac{11\operatorname{sech}^3(x)}{3}-5\operatorname{sech}(x)$$

```
[Out] -5*sech(x)+11/3*sech(x)^3
```

Rubi [A] time = 0.04, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {4339, 14}

$$\frac{11\operatorname{sech}^3(x)}{3}-5\operatorname{sech}(x)$$

Antiderivative was successfully verified.

```
[In] Int[Sech[x]*(5 - 11*Sech[x]^2)*Tanh[x],x]
```

```
[Out] -5*Sech[x] + (11*Sech[x]^3)/3
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 4339

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Cos[c*(a + b*x)], x]}, -Dist[(b*c)^(-1), Subst[Int[SubstFor[1/x, Cos[c*(
a + b*x)]/d, u, x], x], x, Cos[c*(a + b*x)]/d], x] /; FunctionOfQ[Cos[c*(a
+ b*x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Tan] || EqQ[F, ta
n])
```

Rubi steps

$$\begin{aligned}\int \operatorname{sech}(x)\left(5-11\operatorname{sech}^2(x)\right)\tanh(x)\,dx &= \operatorname{Subst}\left(\int \frac{-11+5x^2}{x^4}\,dx,x,\cosh(x)\right)\\ &= \operatorname{Subst}\left(\int \left(-\frac{11}{x^4}+\frac{5}{x^2}\right)dx,x,\cosh(x)\right)\\ &= -5\operatorname{sech}(x)+\frac{11\operatorname{sech}^3(x)}{3}\end{aligned}$$

Mathematica [A] time = 0.01, size = 13, normalized size = 1.00

$$\frac{11\text{sech}^3(x)}{3} - 5\text{sech}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[x]*(5 - 11*Sech[x]^2)*Tanh[x],x]
```

```
[Out] -5*Sech[x] + (11*Sech[x]^3)/3
```

fricas [B] time = 0.41, size = 87, normalized size = 6.69

$$-\frac{2\left(15\cosh(x)^3 + 45\cosh(x)\sinh(x)^2 + 15\sinh(x)^3 + \left(45\cosh(x)^2 - 29\right)\sinh(x) + \cosh(x\right.}{3\left(\cosh(x)^4 + 4\cosh(x)\sinh(x)^3 + \sinh(x)^4 + 2\left(3\cosh(x)^2 + 2\right)\sinh(x)^2 + 4\cosh(x)^2 + 4\left(\cosh(x)^3 + \cos\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)*(5-11*sech(x)^2)*tanh(x),x, algorithm="fricas")
```

```
[Out] -2/3*(15*cosh(x)^3 + 45*cosh(x)*sinh(x)^2 + 15*sinh(x)^3 + (45*cosh(x)^2 -
29)*sinh(x) + cosh(x))/(cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + 2*(3*
cosh(x)^2 + 2)*sinh(x)^2 + 4*cosh(x)^2 + 4*(cosh(x)^3 + cosh(x))*sinh(x) +
3)
```

giac [B] time = 0.13, size = 24, normalized size = 1.85

$$-\frac{2\left(15\left(e^{(-x)} + e^{x}\right)^2 - 44\right)}{3\left(e^{(-x)} + e^{x}\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)*(5-11*sech(x)^2)*tanh(x),x, algorithm="giac")
```

```
[Out] -2/3*(15*(e^(-x) + e^x)^2 - 44)/(e^(-x) + e^x)^3
```

maple [A] time = 0.09, size = 12, normalized size = 0.92

$$-5\,\text{sech}(x) + \frac{11\text{sech}(x)^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(x)*(5-11*sech(x)^2)*tanh(x),x)
```

```
[Out] -5*sech(x)+11/3*sech(x)^3
```

maxima [B] time = 0.31, size = 23, normalized size = 1.77

$$-\frac{10}{e^{(-x)} + e^{x}} + \frac{88}{3\left(e^{(-x)} + e^{x}\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)*(5-11*sech(x)^2)*tanh(x),x, algorithm="maxima")
[Out] -10/(e^(-x) + e^x) + 88/3/(e^(-x) + e^x)^3
```

mupad [B] time = 1.86, size = 26, normalized size = 2.00

$$-\frac{2\,e^{x}\,\left(15\,e^{4\,x}-14\,e^{2\,x}+15\right)}{3\,\left(e^{2\,x}+1\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(tanh(x)*(11/cosh(x)^2 - 5))/cosh(x),x)
[Out] -(2*exp(x)*(15*exp(4*x) - 14*exp(2*x) + 15))/(3*(exp(2*x) + 1)^3)
```

sympy [A] time = 0.50, size = 12, normalized size = 0.92

$$\frac{11\operatorname{sech}^{3}{\left(x \right)}}{3} - 5\operatorname{sech}{\left(x \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)*(5-11*sech(x)**2)*tanh(x),x)
[Out] 11*sech(x)**3/3 - 5*sech(x)
```

3.1019 $\int \frac{\operatorname{csch}^2(x)}{a+b\coth(x)}\,dx$

Optimal. Leaf size=12

$$-\frac{\log(a+b\coth(x))}{b}$$

```
[Out] -ln(a+b*coth(x))/b
```

Rubi [A] time = 0.05, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3506, 31}

$$-\frac{\log(a+b\coth(x))}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[x]^2/(a + b*Coth[x]),x]
```

```
[Out] -(Log[a + b*Coth[x]]/b)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 3506

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(b*f), Subst[Int[(a + x)^n*(1 + x^2/b^2)^(m/2 - 1),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && NeQ[a^2 + b^2, 0
] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\operatorname{csch}^2(x)}{a+b\coth(x)}\,dx &= -\frac{\operatorname{Subst}\left(\int \frac{1}{a+x}\,dx, x, b\coth(x)\right)}{b}\\ &= -\frac{\log(a+b\coth(x))}{b}\end{aligned}$$

Mathematica [A] time = 0.07, size = 20, normalized size = 1.67

$$\frac{\log(\sinh(x))-\log(a\sinh(x)+b\cosh(x))}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[x]^2/(a + b*Coth[x]),x]
```

```
[Out] (Log[Sinh[x]] - Log[b*Cosh[x] + a*Sinh[x]])/b
```

fricas [B] time = 0.47, size = 43, normalized size = 3.58

$$-\frac{\log\left(\frac{2\,(b\cosh(x)+a\sinh(x))}{\cosh(x)-\sinh(x)}\right)-\log\left(\frac{2\,\sinh(x)}{\cosh(x)-\sinh(x)}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^2/(a+b*coth(x)),x, algorithm="fricas")
```

```
[Out] -(log(2*(b*cosh(x) + a*sinh(x))/(cosh(x) - sinh(x))) - log(2*sinh(x)/(cosh(
x) - sinh(x))))/b
```

giac [B] time = 0.14, size = 46, normalized size = 3.83

$$-\frac{(a+b)\log\left(\left|ae^{(2\,x)}+be^{(2\,x)}-a+b\right|\right)}{ab+b^2}+\frac{\log\left(\left|e^{(2\,x)}-1\right|\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^2/(a+b*coth(x)),x, algorithm="giac")
```

```
[Out] -(a + b)*log(abs(a*e^(2*x) + b*e^(2*x) - a + b))/(a*b + b^2) + log(abs(e^(2
*x) - 1))/b
```

maple [A] time = 0.13, size = 13, normalized size = 1.08

$$-\frac{\ln\left(a+b\coth(x)\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(x)^2/(a+b*coth(x)),x)
```

```
[Out] -ln(a+b*coth(x))/b
```

maxima [A] time = 0.31, size = 12, normalized size = 1.00

$$-\frac{\log\left(b\coth(x)+a\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^2/(a+b*coth(x)),x, algorithm="maxima")
```

```
[Out] -log(b*coth(x) + a)/b
```

mupad [B] time = 0.22, size = 51, normalized size = 4.25

$$-\frac{2\,\mathrm{atan}\left(\frac{a\,\mathrm{e}^{2\,x}\,\sqrt{-b^2}-a\,\sqrt{-b^2}+b\,\mathrm{e}^{2\,x}\,\sqrt{-b^2}}{b^2}\right)}{\sqrt{-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sinh(x)^2*(a + b*coth(x))),x)
```

```
[Out] -(2*atan((a*exp(2*x)*(-b^2)^(1/2) - a*(-b^2)^(1/2) + b*exp(2*x)*(-b^2)^(1/2
))/b^2))/(-b^2)^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}^2(x)}{a+b\coth(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)**2/(a+b*coth(x)),x)
```

```
[Out] Integral(csch(x)**2/(a + b*coth(x)), x)
```

3.1020 $\int (a+b\coth(x))^n\text{csch}^2(x)\,dx$

Optimal. Leaf size=20

$$-\frac{(a+b\coth(x))^{n+1}}{b(n+1)}$$

```
[Out] -(a+b*coth(x))^(1+n)/b/(1+n)
```

Rubi [A] time = 0.05, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3506, 32}

$$-\frac{(a+b\coth(x))^{n+1}}{b(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Coth[x])^n*Csch[x]^2,x]
```

```
[Out] -((a + b*Coth[x])^(1 + n)/(b*(1 + n)))
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m + 1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 3506

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist[1/(b*f), Subst[Int[(a + x)^n*(1 + x^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && NeQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}\int (a+b\coth(x))^n\text{csch}^2(x)\,dx &= -\frac{\text{Subst}\left(\int (a+x)^n\,dx,x,b\coth(x)\right)}{b}\\ &= -\frac{(a+b\coth(x))^{1+n}}{b(1+n)}\end{aligned}$$

Mathematica [A] time = 0.24, size = 19, normalized size = 0.95

$$-\frac{(a+b\coth(x))^{n+1}}{bn+b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Coth[x])^n*Csch[x]^2,x]
```

```
[Out] -((a + b*Coth[x])^(1 + n)/(b + b*n))
```

fricas [B] time = 0.47, size = 70, normalized size = 3.50

$$-\frac{(b\cosh(x)+a\sinh(x))\cosh\left(n\log\left(\frac{b\cosh(x)+a\sinh(x)}{\sinh(x)}\right)\right)+(b\cosh(x)+a\sinh(x))\sinh\left(n\log\left(\frac{b\cosh(x)+a\sinh(x)}{\sinh(x)}\right)\right)}{(bn+b)\sinh(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*coth(x))^n*csch(x)^2,x, algorithm="fricas")
```

```
[Out] -((b*cosh(x) + a*sinh(x))*cosh(n*log((b*cosh(x) + a*sinh(x))/sinh(x))) + (b
*cosh(x) + a*sinh(x))*sinh(n*log((b*cosh(x) + a*sinh(x))/sinh(x))))/((b*n +
 b)*sinh(x))
```

giac [A] time = 0.14, size = 40, normalized size = 2.00

$$-\frac{\left(\frac{ae^{(2x)}+be^{(2x)}-a+b}{e^{(2x)}-1}\right)^{n+1}}{b(n+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*coth(x))^n*csch(x)^2,x, algorithm="giac")
```

```
[Out] -((a*e^(2*x) + b*e^(2*x) - a + b)/(e^(2*x) - 1))^(n + 1)/(b*(n + 1))
```

maple [A] time = 0.16, size = 21, normalized size = 1.05

$$-\frac{(a+b\coth(x))^{n+1}}{b\,(n+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*coth(x))^n*csch(x)^2,x)
```

```
[Out] -(a+b*coth(x))^(n+1)/b/(n+1)
```

maxima [A] time = 0.31, size = 20, normalized size = 1.00

$$-\frac{(b\coth(x)+a)^{n+1}}{b(n+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*coth(x))^n*csch(x)^2,x, algorithm="maxima")
```

```
[Out] -(b*coth(x) + a)^(n + 1)/(b*(n + 1))
```

mupad [B] time = 1.99, size = 55, normalized size = 2.75

$$-\frac{\left(a+\frac{b\left(e^{2x}+1\right)}{e^{2x}-1}\right)^n\left(b-a+a\,e^{2x}+b\,e^{2x}\right)}{b\left(e^{2x}-1\right)(n+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*coth(x))^n/sinh(x)^2,x)
```

```
[Out] -((a + (b*(exp(2*x) + 1))/(exp(2*x) - 1))^n*(b - a + a*exp(2*x) + b*exp(2*x
)))/(b*(exp(2*x) - 1)*(n + 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a+b\coth(x))^n\,\mathrm{csch}^2(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*coth(x))**n*csch(x)**2,x)
```

```
[Out] Integral((a + b*coth(x))**n*csch(x)**2, x)
```

3.1021 $\int \operatorname{csch}^2(x) \left(-1+\sinh^2(x)\right) \, dx$

Optimal. Leaf size=4

$$x+\coth(x)$$

```
[Out] x+coth(x)
```

Rubi [A] time = 0.02, antiderivative size = 4, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {3012, 8}

$$x+\coth(x)$$

Antiderivative was successfully verified.

```
[In] Int[Csch[x]^2*(-1 + Sinh[x]^2),x]
```

```
[Out] x + Coth[x]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3012

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x
_)]^2), x_Symbol] :> Simp[(A*Cos[e + f*x]*(b*Sin[e + f*x])^(m + 1))/(b*f*(m
 + 1)), x] + Dist[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1)), Int[(b*Sin[e + f*x
])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

Rubi steps

$$\begin{aligned} \int \operatorname{csch}^2(x) \left(-1+\sinh^2(x)\right) \, dx &= \coth(x)+\int 1 \, dx \\ &= x+\coth(x) \end{aligned}$$

Mathematica [A] time = 0.00, size = 4, normalized size = 1.00

$$x+\coth(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[x]^2*(-1 + Sinh[x]^2),x]
```

```
[Out] x + Coth[x]
```

fricas [**B**] time = 0.42, size = 14, normalized size = 3.50

$$\frac{(x-1)\sinh(x)+\cosh(x)}{\sinh(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^2*(-1+sinh(x)^2),x, algorithm="fricas")
[Out] ((x - 1)*sinh(x) + cosh(x))/sinh(x)
```

giac [**B**] time = 0.13, size = 12, normalized size = 3.00

$$x+\frac{2}{e^{(2x)}-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^2*(-1+sinh(x)^2),x, algorithm="giac")
[Out] x + 2/(e^(2*x) - 1)
```

maple [**A**] time = 0.38, size = 5, normalized size = 1.25

$$x+\coth(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(x)^2*(-1+sinh(x)^2),x)
[Out] x+coth(x)
```

maxima [**B**] time = 0.31, size = 12, normalized size = 3.00

$$x-\frac{2}{e^{(-2x)}-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^2*(-1+sinh(x)^2),x, algorithm="maxima")
[Out] x - 2/(e^(-2*x) - 1)
```

mupad [**B**] time = 1.72, size = 12, normalized size = 3.00

$$x+\frac{2}{e^{2x}-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sinh(x)^2 - 1)/sinh(x)^2,x)
```

```
[Out] x + 2/(exp(2*x) - 1)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (\sinh(x)-1)(\sinh(x)+1)\operatorname{csch}^2(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)**2*(-1+sinh(x)**2),x)
```

```
[Out] Integral((sinh(x) - 1)*(sinh(x) + 1)*csch(x)**2, x)
```

3.1022 $\int \left(-1-\frac{1}{1-\coth^2(x)}\right)\operatorname{csch}^2(x)\,dx$

Optimal. Leaf size=4

$$x+\coth(x)$$

[Out] x+coth(x)

Rubi [A] time = 0.06, antiderivative size = 4, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {453, 206}

$$x+\coth(x)$$

Antiderivative was successfully verified.

[In] Int[(-1 - (1 - Coth[x]^2)^(-1))*Csch[x]^2,x]

[Out] x + Coth[x]

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\int \left(-1-\frac{1}{1-\coth^2(x)}\right)\operatorname{csch}^2(x)\,dx &= -\text{Subst}\left(\int \frac{1-2x^2}{x^2\left(1-x^2\right)}\,dx,x,\tanh(x)\right)\\
&= \coth(x)+\text{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,\tanh(x)\right)\\
&= x+\coth(x)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 4, normalized size = 1.00

$$x + \coth(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 - (1 - Coth[x]^2)^(-1))*Csch[x]^2,x]
```

```
[Out] x + Coth[x]
```

fricas [B] time = 0.45, size = 14, normalized size = 3.50

$$\frac{(x-1)\sinh(x) + \cosh(x)}{\sinh(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1-1/(1-coth(x)^2))*csch(x)^2,x, algorithm="fricas")
```

```
[Out] ((x - 1)*sinh(x) + cosh(x))/sinh(x)
```

giac [B] time = 0.13, size = 12, normalized size = 3.00

$$x + \frac{2}{e^{(2x)} - 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1-1/(1-coth(x)^2))*csch(x)^2,x, algorithm="giac")
```

```
[Out] x + 2/(e^(2*x) - 1)
```

maple [B] time = 0.17, size = 32, normalized size = 8.00

$$\frac{\tanh\left(\frac{x}{2}\right)}{2} - \ln\left(\tanh\left(\frac{x}{2}\right) - 1\right) + \ln\left(\tanh\left(\frac{x}{2}\right) + 1\right) + \frac{1}{2\tanh\left(\frac{x}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-1-1/(1-coth(x)^2))*csch(x)^2,x)
```

```
[Out] 1/2*tanh(1/2*x)-ln(tanh(1/2*x)-1)+ln(tanh(1/2*x)+1)+1/2/tanh(1/2*x)
```

maxima [B] time = 0.31, size = 12, normalized size = 3.00

$$x - \frac{2}{e^{(-2x)} - 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1-1/(1-coth(x)^2))*csch(x)^2,x, algorithm="maxima")
```

```
[Out] x - 2/(e^(-2*x) - 1)
```

mupad [B] time = 1.83, size = 12, normalized size = 3.00

$$x+\frac{2}{e^{2x}-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/(coth(x)^2 - 1) - 1)/sinh(x)^2,x)
```

```
[Out] x + 2/(exp(2*x) - 1)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int\left(-\frac{2\operatorname{csch}^2(x)}{\coth^2(x)-1}\right)dx-\int\frac{\coth^2(x)\operatorname{csch}^2(x)}{\coth^2(x)-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1-1/(1-coth(x)**2))*csch(x)**2,x)
```

```
[Out] -Integral(-2*csch(x)**2/(coth(x)**2 - 1), x) - Integral(coth(x)**2*csch(x)*
*2/(coth(x)**2 - 1), x)
```

3.1023 $\int \frac{(a+b\coth(x))\mathrm{csch}^2(x)}{c+d\coth(x)}\,dx$

Optimal. Leaf size=28

$$\frac{(bc-ad)\log(c+d\coth(x))}{d^2}-\frac{b\coth(x)}{d}$$

```
[Out] -b*coth(x)/d+(-a*d+b*c)*ln(c+d*coth(x))/d^2
```

Rubi [A] time = 0.10, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {4344, 43}

$$\frac{(bc-ad)\log(c+d\coth(x))}{d^2}-\frac{b\coth(x)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Coth[x])*Csch[x]^2)/(c + d*Coth[x]),x]
```

```
[Out] -((b*Coth[x])/d) + ((b*c - a*d)*Log[c + d*Coth[x]])/d^2
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 4344

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^2, x_Symbol] :> With[{d = FreeFac
tors[Cot[c*(a + b*x)], x]}, -Dist[d/(b*c), Subst[Int[SubstFor[1, Cot[c*(a +
 b*x)]/d, u, x], x], x, Cot[c*(a + b*x)]/d], x] /; FunctionOfQ[Cot[c*(a + b
*x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && NonsumQ[u] && (EqQ[F, Csc] |
| EqQ[F, csc])
```

Rubi steps

$$\int \frac{(a+b\coth(x))\text{csch}^2(x)}{c+d\coth(x)}\,dx = -\text{Subst}\left(\int \frac{a+bx}{c+dx}\,dx, x, \coth(x)\right)$$
$$= -\text{Subst}\left(\int \left(\frac{b}{d}+\frac{-bc+ad}{d(c+dx)}\right)dx, x, \coth(x)\right)$$
$$= -\frac{b\coth(x)}{d}+\frac{(bc-ad)\log(c+d\coth(x))}{d^2}$$

Mathematica [A] time = 0.35, size = 56, normalized size = 2.00

$$\frac{\sinh(x)(a+b\coth(x))(-(bc-ad)(\log(\sinh(x))-\log(c\sinh(x)+d\cosh(x)))-bd\coth(x))}{d^2(a\sinh(x)+b\cosh(x))}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Coth[x])*Csch[x]^2)/(c + d*Coth[x]),x]
```

```
[Out] ((a + b*Coth[x])*(-(b*d*Coth[x]) - (b*c - a*d)*(Log[Sinh[x]] - Log[d*Cosh[x
] + c*Sinh[x]]))*Sinh[x])/(d^2*(b*Cosh[x] + a*Sinh[x]))
```

fricas [B] time = 0.48, size = 174, normalized size = 6.21

$$-\frac{2\,bd-\left((bc-ad)\cosh(x)^2+2\,(bc-ad)\cosh(x)\sinh(x)+(bc-ad)\sinh(x)^2-bc+ad\right)\log\left(\frac{2\,(d\cosh(x)+c\sinh(x))}{\cosh(x)-\sinh(x)}\right)}{d^2\cosh(x)^2+2\,d^2\cosh(x)\sinh}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*coth(x))*csch(x)^2/(c+d*coth(x)),x, algorithm="fricas")
```

```
[Out] -(2*b*d - ((b*c - a*d)*cosh(x)^2 + 2*(b*c - a*d)*cosh(x)*sinh(x) + (b*c - a
*d)*sinh(x)^2 - b*c + a*d)*log(2*(d*cosh(x) + c*sinh(x))/(cosh(x) - sinh(x)
)) + ((b*c - a*d)*cosh(x)^2 + 2*(b*c - a*d)*cosh(x)*sinh(x) + (b*c - a*d)*s
inh(x)^2 - b*c + a*d)*log(2*sinh(x)/(cosh(x) - sinh(x))))/(d^2*cosh(x)^2 +
2*d^2*cosh(x)*sinh(x) + d^2*sinh(x)^2 - d^2)
```

giac [B] time = 0.12, size = 113, normalized size = 4.04

$$\frac{\left(bc^2-acd+bcd-ad^2\right)\log\left(\left|ce^{(2\,x)}+de^{(2\,x)}-c+d\right|\right)}{cd^2+d^3}-\frac{(bc-ad)\log\left(\left|e^{(2\,x)}-1\right|\right)}{d^2}+\frac{bce^{(2\,x)}-ade^{(2\,x)}-bc+ad-2\,b}{d^2\left(e^{(2\,x)}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*coth(x))*csch(x)^2/(c+d*coth(x)),x, algorithm="giac")
```

```
[Out] (b*c^2 - a*c*d + b*c*d - a*d^2)*log(abs(c*e^(2*x) + d*e^(2*x) - c + d))/(c*
d^2 + d^3) - (b*c - a*d)*log(abs(e^(2*x) - 1))/d^2 + (b*c*e^(2*x) - a*d*e^(
2*x) - b*c + a*d - 2*b*d)/(d^2*(e^(2*x) - 1))
```

maple [B] time = 0.18, size = 94, normalized size = 3.36

$$-\frac{b\tanh\left(\frac{x}{2}\right)}{2d}-\frac{\ln\left(\left(\tanh^2\left(\frac{x}{2}\right)\right)d+2c\tanh\left(\frac{x}{2}\right)+d\right)a}{d}+\frac{\ln\left(\left(\tanh^2\left(\frac{x}{2}\right)\right)d+2c\tanh\left(\frac{x}{2}\right)+d\right)cb}{d^2}-\frac{b}{2d\tanh\left(\frac{x}{2}\right)}+\frac{\ln}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*coth(x))*csch(x)^2/(c+d*coth(x)),x)
```

```
[Out] -1/2*b/d*tanh(1/2*x)-1/d*ln(tanh(1/2*x)^2*d+2*c*tanh(1/2*x)+d)*a+1/d^2*ln(t
anh(1/2*x)^2*d+2*c*tanh(1/2*x)+d)*c*b-1/2*b/d/tanh(1/2*x)+1/d*ln(tanh(1/2*x
))*a-1/d^2*ln(tanh(1/2*x))*c*b
```

maxima [B] time = 0.32, size = 77, normalized size = 2.75

$$b\left(\frac{c\log\left(-(c-d)e^{(-2x)}+c+d\right)}{d^2}-\frac{c\log\left(e^{(-x)}+1\right)}{d^2}-\frac{c\log\left(e^{(-x)}-1\right)}{d^2}+\frac{2}{de^{(-2x)}-d}\right)-\frac{a\log\left(d\coth(x)+c\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*coth(x))*csch(x)^2/(c+d*coth(x)),x, algorithm="maxima")
```

```
[Out] b*(c*log(-(c - d)*e^(-2*x) + c + d)/d^2 - c*log(e^(-x) + 1)/d^2 - c*log(e^(
-x) - 1)/d^2 + 2/(d*e^(-2*x) - d)) - a*log(d*coth(x) + c)/d
```

mupad [B] time = 2.16, size = 297, normalized size = 10.61

$$\frac{2\,\mathrm{atan}\left(\mathrm{e}^{2\,x}\left(\frac{4\left(a\,d\,\sqrt{-d^4}-b\,c\,\sqrt{-d^4}\right)}{d^2\,\sqrt{{\left(a\,d-b\,c\right)}^2}\,\left(c+d\right)\,\left(c-d\right)\,\sqrt{-d^4}}-\frac{4\,c^2\,\sqrt{a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2}}{d^4\,\left(c+d\right)\,\left(c-d\right)\,\left(a\,d-b\,c\right)}\right)\left(\frac{d^2\,\sqrt{-d^4}}{4}+\frac{c\,d\,\sqrt{-d^4}}{4}\right)-\frac{4\,c\left(d^2\,\sqrt{a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2}-c\,d\right.}{d^5\,(c+}\right.}{\sqrt{-d^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*coth(x))/(sinh(x)^2*(c + d*coth(x))),x)
```

```
[Out] (2*atan(exp(2*x)*((4*(a*d*(-d^4)^(1/2) - b*c*(-d^4)^(1/2)))/(d^2*((a*d - b*
c)^2)^(1/2)*(c + d)*(c - d)*(-d^4)^(1/2)) - (4*c^2*(a^2*d^2 + b^2*c^2 - 2*a
*b*c*d)^(1/2))/(d^4*(c + d)*(c - d)*(a*d - b*c)))*((d^2*(-d^4)^(1/2))/4 + (
c*d*(-d^4)^(1/2))/4) - (4*c*(d^2*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)^(1/2) - c*
d*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)^(1/2))*((d^2*(-d^4)^(1/2))/4 + (c*d*(-d^4
```

```
)^(1/2))/4))/(d^5*(c + d)*(c - d)*(a*d - b*c)))*(a^2*d^2 + b^2*c^2 - 2*a*b*
c*d)^(1/2))/(-d^4)^(1/2) - (2*b)/(d*(exp(2*x) - 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b\coth(x))\operatorname{csch}^2(x)}{c + d\coth(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*coth(x))*csch(x)**2/(c+d*coth(x)),x)
```

```
[Out] Integral((a + b*coth(x))*csch(x)**2/(c + d*coth(x)), x)
```

3.1024 $\int \frac{(a+b\coth(x))^2\mathbf{csch}^2(x)}{c+d\coth(x)}\,dx$

Optimal. Leaf size=53

$$-\frac{(bc-ad)^2\log(c+d\coth(x))}{d^3}+\frac{b\coth(x)(bc-ad)}{d^2}-\frac{(a+b\coth(x))^2}{2d}$$

```
[Out] b*(-a*d+b*c)*coth(x)/d^2-1/2*(a+b*coth(x))^2/d-(-a*d+b*c)^2*ln(c+d*coth(x))
/d^3
```

Rubi [A] time = 0.15, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {4344, 43}

$$\frac{b\coth(x)(bc-ad)}{d^2}-\frac{(bc-ad)^2\log(c+d\coth(x))}{d^3}-\frac{(a+b\coth(x))^2}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Coth[x])^2*Csch[x]^2)/(c + d*Coth[x]),x]
```

```
[Out] (b*(b*c - a*d)*Coth[x])/d^2 - (a + b*Coth[x])^2/(2*d) - ((b*c - a*d)^2*Log[
c + d*Coth[x]])/d^3
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 4344

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^2, x_Symbol] :> With[{d = FreeFac
tors[Cot[c*(a + b*x)], x]}, -Dist[d/(b*c), Subst[Int[SubstFor[1, Cot[c*(a +
 b*x)]/d, u, x], x], x, Cot[c*(a + b*x)]/d], x] /; FunctionOfQ[Cot[c*(a + b
*x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && NonsumQ[u] && (EqQ[F, Csc] |
| EqQ[F, csc])
```

Rubi steps

$$\int \frac{(a+b\coth(x))^2\operatorname{csch}^2(x)}{c+d\coth(x)}\,dx = -\operatorname{Subst}\left(\int \frac{(a+bx)^2}{c+dx}\,dx, x, \coth(x)\right)$$
$$= -\operatorname{Subst}\left(\int \left(-\frac{b(bc-ad)}{d^2}+\frac{b(a+bx)}{d}+\frac{(-bc+ad)^2}{d^2(c+dx)}\right)dx, x, \coth(x)\right)$$
$$= \frac{b(bc-ad)\coth(x)}{d^2}-\frac{(a+b\coth(x))^2}{2d}-\frac{(bc-ad)^2\log(c+d\coth(x))}{d^3}$$

Mathematica [A] time = 0.53, size = 62, normalized size = 1.17

$$\frac{2bd\coth(x)(bc-2ad)+2(bc-ad)^2(\log(\sinh(x))-\log(c\sinh(x)+d\cosh(x)))-b^2d^2\operatorname{csch}^2(x)}{2d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Coth[x])^2*Csch[x]^2)/(c + d*Coth[x]),x]
```

```
[Out] (2*b*d*(b*c - 2*a*d)*Coth[x] - b^2*d^2*Csch[x]^2 + 2*(b*c - a*d)^2*(Log[Sin
h[x]] - Log[d*Cosh[x] + c*Sinh[x]]))/(2*d^3)
```

fricas [B] time = 0.49, size = 694, normalized size = 13.09

$$-\frac{2\,b^2cd-4\,abd^2-2\left(b^2cd-\left(2\,ab+b^2\right)d^2\right)\cosh(x)^2-4\left(b^2cd-\left(2\,ab+b^2\right)d^2\right)\cosh(x)\sinh(x)-2\left(b^2cd-\left(2\,ab\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*coth(x))^2*csch(x)^2/(c+d*coth(x)),x, algorithm="fricas")
```

```
[Out] -(2*b^2*c*d - 4*a*b*d^2 - 2*(b^2*c*d - (2*a*b + b^2)*d^2)*cosh(x)^2 - 4*(b^
2*c*d - (2*a*b + b^2)*d^2)*cosh(x)*sinh(x) - 2*(b^2*c*d - (2*a*b + b^2)*d^2
)*sinh(x)^2 + ((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*cosh(x)^4 + 4*(b^2*c^2 - 2*a
*b*c*d + a^2*d^2)*cosh(x)*sinh(x)^3 + (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sinh(
x)^4 + b^2*c^2 - 2*a*b*c*d + a^2*d^2 - 2*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*co
sh(x)^2 - 2*(b^2*c^2 - 2*a*b*c*d + a^2*d^2 - 3*(b^2*c^2 - 2*a*b*c*d + a^2*d
^2)*cosh(x)^2)*sinh(x)^2 + 4*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*cosh(x)^3 - (
b^2*c^2 - 2*a*b*c*d + a^2*d^2)*cosh(x))*sinh(x))*log(2*(d*cosh(x) + c*sinh(
x))/(cosh(x) - sinh(x))) - ((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*cosh(x)^4 + 4*(
b^2*c^2 - 2*a*b*c*d + a^2*d^2)*cosh(x)*sinh(x)^3 + (b^2*c^2 - 2*a*b*c*d + a
^2*d^2)*sinh(x)^4 + b^2*c^2 - 2*a*b*c*d + a^2*d^2 - 2*(b^2*c^2 - 2*a*b*c*d
+ a^2*d^2)*cosh(x)^2 - 2*(b^2*c^2 - 2*a*b*c*d + a^2*d^2 - 3*(b^2*c^2 - 2*a*
b*c*d + a^2*d^2)*cosh(x)^2)*sinh(x)^2 + 4*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*
```

```
cosh(x)^3 - (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*cosh(x))*sinh(x))*log(2*sinh(x)
/(cosh(x) - sinh(x))))/(d^3*cosh(x)^4 + 4*d^3*cosh(x)*sinh(x)^3 + d^3*sinh(
x)^4 - 2*d^3*cosh(x)^2 + d^3 + 2*(3*d^3*cosh(x)^2 - d^3)*sinh(x)^2 + 4*(d^3
*cosh(x)^3 - d^3*cosh(x))*sinh(x))
```

giac [B] time = 0.13, size = 265, normalized size = 5.00

$$-\frac{\left(b^2c^3-2\,abc^2d+b^2c^2d+a^2cd^2-2\,abcd^2+a^2d^3\right)\log\left(\left|ce^{(2\,x)}+de^{(2\,x)}-c+d\right|\right)}{cd^3+d^4}+\frac{\left(b^2c^2-2\,abcd+a^2d^2\right)\log\left(\left|e\right.\right.}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*coth(x))^2*csch(x)^2/(c+d*coth(x)),x, algorithm="giac")
```

```
[Out] -(b^2*c^3 - 2*a*b*c^2*d + b^2*c^2*d + a^2*c*d^2 - 2*a*b*c*d^2 + a^2*d^3)*lo
g(abs(c*e^(2*x) + d*e^(2*x) - c + d))/(c*d^3 + d^4) + (b^2*c^2 - 2*a*b*c*d
+ a^2*d^2)*log(abs(e^(2*x) - 1))/d^3 - 1/2*(3*b^2*c^2*e^(4*x) - 6*a*b*c*d*e
^(4*x) + 3*a^2*d^2*e^(4*x) - 6*b^2*c^2*e^(2*x) + 12*a*b*c*d*e^(2*x) - 4*b^2
*c*d*e^(2*x) - 6*a^2*d^2*e^(2*x) + 8*a*b*d^2*e^(2*x) + 4*b^2*d^2*e^(2*x) +
3*b^2*c^2 - 6*a*b*c*d + 4*b^2*c*d + 3*a^2*d^2 - 8*a*b*d^2)/(d^3*(e^(2*x) -
1)^2)
```

maple [B] time = 0.23, size = 203, normalized size = 3.83

$$-\frac{b^2\left(\tanh^2\left(\frac{x}{2}\right)\right)}{8d}-\frac{ba\tanh\left(\frac{x}{2}\right)}{d}+\frac{b^2\tanh\left(\frac{x}{2}\right)c}{2d^2}-\frac{\ln\left(\left(\tanh^2\left(\frac{x}{2}\right)\right)d+2c\tanh\left(\frac{x}{2}\right)+d\right)a^2}{d}+\frac{2\ln\left(\left(\tanh^2\left(\frac{x}{2}\right)\right)d+2c\right.}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*coth(x))^2*csch(x)^2/(c+d*coth(x)),x)
```

```
[Out] -1/8*b^2/d*tanh(1/2*x)^2-b/d*a*tanh(1/2*x)+1/2*b^2/d^2*tanh(1/2*x)*c-1/d*ln
(tanh(1/2*x)^2*d+2*c*tanh(1/2*x)+d)*a^2+2/d^2*ln(tanh(1/2*x)^2*d+2*c*tanh(1
/2*x)+d)*c*b*a-1/d^3*ln(tanh(1/2*x)^2*d+2*c*tanh(1/2*x)+d)*c^2*b^2-1/8*b^2/
d/tanh(1/2*x)^2+1/d*ln(tanh(1/2*x))*a^2-2/d^2*ln(tanh(1/2*x))*c*b*a+1/d^3*l
n(tanh(1/2*x))*c^2*b^2-b/d/tanh(1/2*x)*a+1/2*b^2/d^2/tanh(1/2*x)*c
```

maxima [B] time = 0.33, size = 177, normalized size = 3.34

$$b^2\left(\frac{2\left((c+d)e^{(-2\,x)}-c\right)}{2\,d^2e^{(-2\,x)}-d^2e^{(-4\,x)}-d^2}-\frac{c^2\log\left(-(c-d)e^{(-2\,x)}+c+d\right)}{d^3}+\frac{c^2\log\left(e^{(-x)}+1\right)}{d^3}+\frac{c^2\log\left(e^{(-x)}-1\right)}{d^3}\right)+2\,ab\left(\frac{c\,\mathrm{lo}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*coth(x))^2*csch(x)^2/(c+d*coth(x)),x, algorithm="maxima")
```

```
[Out] b^2*(2*((c + d)*e^(-2*x) - c)/(2*d^2*e^(-2*x) - d^2*e^(-4*x) - d^2) - c^2*l
og(-(c - d)*e^(-2*x) + c + d)/d^3 + c^2*log(e^(-x) + 1)/d^3 + c^2*log(e^(-x
) - 1)/d^3) + 2*a*b*(c*log(-(c - d)*e^(-2*x) + c + d)/d^2 - c*log(e^(-x) +
1)/d^2 - c*log(e^(-x) - 1)/d^2 + 2/(d*e^(-2*x) - d)) - a^2*log(d*coth(x) +
c)/d
```

mupad [B] time = 2.16, size = 107, normalized size = 2.02

$$\frac{\ln\left(e^{2x}-1\right)\,(a\,d-b\,c)^2}{d^3}-\frac{\ln\left(d-c+d\,e^{2x}+c\,e^{2x}\right)\,(a\,d-b\,c)^2}{d^3}-\frac{2\,\left(b^2\,d-b^2\,c+2\,a\,b\,d\right)}{d^2\,\left(e^{2x}-1\right)}-\frac{2\,b^2}{d\,\left(e^{4x}-2\,e^{2x}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*coth(x))^2/(sinh(x)^2*(c + d*coth(x))),x)
```

```
[Out] (log(exp(2*x) - 1)*(a*d - b*c)^2)/d^3 - (log(d - c + d*exp(2*x) + c*exp(2*x
))*(a*d - b*c)^2)/d^3 - (2*(b^2*d - b^2*c + 2*a*b*d))/(d^2*(exp(2*x) - 1))
 - (2*b^2)/(d*(exp(4*x) - 2*exp(2*x) + 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(a+b\coth(x))^2\operatorname{csch}^2(x)}{c+d\coth(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*coth(x))**2*csch(x)**2/(c+d*coth(x)),x)
```

```
[Out] Integral((a + b*coth(x))**2*csch(x)**2/(c + d*coth(x)), x)
```

3.1025 $\int \frac{(a+b\coth(x))^3\operatorname{csch}^2(x)}{c+d\coth(x)}\,dx$

Optimal. Leaf size=78

$$\frac{(bc-ad)^3\log(c+d\coth(x))}{d^4}-\frac{b\coth(x)(bc-ad)^2}{d^3}+\frac{(bc-ad)(a+b\coth(x))^2}{2d^2}-\frac{(a+b\coth(x))^3}{3d}$$

```
[Out] -b*(-a*d+b*c)^2*coth(x)/d^3+1/2*(-a*d+b*c)*(a+b*coth(x))^2/d^2-1/3*(a+b*cot
h(x))^3/d+(-a*d+b*c)^3*ln(c+d*coth(x))/d^4
```

Rubi [A] time = 0.15, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {4344, 43}

$$-\frac{b\coth(x)(bc-ad)^2}{d^3}+\frac{(bc-ad)(a+b\coth(x))^2}{2d^2}+\frac{(bc-ad)^3\log(c+d\coth(x))}{d^4}-\frac{(a+b\coth(x))^3}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Coth[x])^3*Csch[x]^2)/(c + d*Coth[x]),x]
```

```
[Out] -((b*(b*c - a*d)^2*Coth[x])/d^3) + ((b*c - a*d)*(a + b*Coth[x])^2)/(2*d^2)
- (a + b*Coth[x])^3/(3*d) + ((b*c - a*d)^3*Log[c + d*Coth[x]])/d^4
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 4344

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^2, x_Symbol] :> With[{d = FreeFac
tors[Cot[c*(a + b*x)], x]}, -Dist[d/(b*c), Subst[Int[SubstFor[1, Cot[c*(a +
 b*x)]/d, u, x], x], x, Cot[c*(a + b*x)]/d], x] /; FunctionOfQ[Cot[c*(a + b
*x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && NonsumQ[u] && (EqQ[F, Csc] |
| EqQ[F, csc])
```

Rubi steps

$$\int \frac{(a+b\coth(x))^3\text{csch}^2(x)}{c+d\coth(x)}\,dx = -\text{Subst}\left(\int \frac{(a+bx)^3}{c+dx}\,dx, x, \coth(x)\right)$$
$$= -\text{Subst}\left(\int \left(\frac{b(bc-ad)^2}{d^3} - \frac{b(bc-ad)(a+bx)}{d^2} + \frac{b(a+bx)^2}{d} + \frac{(-bc+ad)^3}{d^3(c+dx)}\right) dx\right.$$
$$= -\frac{b(bc-ad)^2\coth(x)}{d^3} + \frac{(bc-ad)(a+b\coth(x))^2}{2d^2} - \frac{(a+b\coth(x))^3}{3d} + \frac{(bc-ad}{}$$

Mathematica [A] time = 1.28, size = 136, normalized size = 1.74

$$\frac{(a+b\coth(x))^3(c\sinh(x)+d\cosh(x))\left(-bd\left(\sinh(2x)\left(9a^2d^2-9abcd+b^2\left(3c^2+d^2\right)\right)-3bd(bc-3ad)\right)-6\sinh}{6d^4(c+d\coth(x))(a\sinh(x)+b\cosh(x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Coth[x])^3*Csch[x]^2)/(c + d*Coth[x]),x]
```

```
[Out] ((a + b*Coth[x])^3*(d*Cosh[x] + c*Sinh[x])*(-2*b^3*d^3*Coth[x] - 6*(b*c - a
*d)^3*(Log[Sinh[x]] - Log[d*Cosh[x] + c*Sinh[x]])*Sinh[x]^2 - b*d*(-3*b*d*(
b*c - 3*a*d) + (-9*a*b*c*d + 9*a^2*d^2 + b^2*(3*c^2 + d^2))*Sinh[2*x])))/(6
*d^4*(c + d*Coth[x])*(b*Cosh[x] + a*Sinh[x])^3)
```

fricas [B] time = 0.50, size = 1980, normalized size = 25.38

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*coth(x))^3*csch(x)^2/(c+d*coth(x)),x, algorithm="fricas")
```

```
[Out] -1/3*(6*b^3*c^2*d - 18*a*b^2*c*d^2 + 6*(b^3*c^2*d - (3*a*b^2 + b^3)*c*d^2 +
 (3*a^2*b + 3*a*b^2 + b^3)*d^3)*cosh(x)^4 + 24*(b^3*c^2*d - (3*a*b^2 + b^3)
*c*d^2 + (3*a^2*b + 3*a*b^2 + b^3)*d^3)*cosh(x)*sinh(x)^3 + 6*(b^3*c^2*d -
(3*a*b^2 + b^3)*c*d^2 + (3*a^2*b + 3*a*b^2 + b^3)*d^3)*sinh(x)^4 + 2*(9*a^2
*b + b^3)*d^3 - 6*(2*b^3*c^2*d - (6*a*b^2 + b^3)*c*d^2 + 3*(2*a^2*b + a*b^2
)*d^3)*cosh(x)^2 - 6*(2*b^3*c^2*d - (6*a*b^2 + b^3)*c*d^2 + 3*(2*a^2*b + a*
b^2)*d^3 - 6*(b^3*c^2*d - (3*a*b^2 + b^3)*c*d^2 + (3*a^2*b + 3*a*b^2 + b^3)
*d^3)*cosh(x)^2)*sinh(x)^2 - 3*((b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 -
a^3*d^3)*cosh(x)^6 + 6*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*
cosh(x)*sinh(x)^5 + (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*sin
h(x)^6 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + a^3*d^3 - 3*(b^3*c^3 - 3
*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*cosh(x)^4 - 3*(b^3*c^3 - 3*a*b^2*c^
2*d + 3*a^2*b*c*d^2 - a^3*d^3 - 5*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2
```

```
- a^3*d^3)*cosh(x)^2)*sinh(x)^4 + 4*(5*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c
*d^2 - a^3*d^3)*cosh(x)^3 - 3*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^
3*d^3)*cosh(x))*sinh(x)^3 + 3*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^
3*d^3)*cosh(x)^2 + 3*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3 + 5
*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*cosh(x)^4 - 6*(b^3*c^3
 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*cosh(x)^2)*sinh(x)^2 + 6*((b^3*
c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*cosh(x)^5 - 2*(b^3*c^3 - 3*a
*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*cosh(x)^3 + (b^3*c^3 - 3*a*b^2*c^2*d
+ 3*a^2*b*c*d^2 - a^3*d^3)*cosh(x))*sinh(x))*log(2*(d*cosh(x) + c*sinh(x))/
(cosh(x) - sinh(x))) + 3*((b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^
3)*cosh(x)^6 + 6*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*cosh(x
)*sinh(x)^5 + (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*sinh(x)^6
 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + a^3*d^3 - 3*(b^3*c^3 - 3*a*b^2
*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*cosh(x)^4 - 3*(b^3*c^3 - 3*a*b^2*c^2*d +
3*a^2*b*c*d^2 - a^3*d^3 - 5*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*
d^3)*cosh(x)^2)*sinh(x)^4 + 4*(5*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 -
 a^3*d^3)*cosh(x)^3 - 3*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)
*cosh(x))*sinh(x)^3 + 3*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)
*cosh(x)^2 + 3*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3 + 5*(b^3*
c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*cosh(x)^4 - 6*(b^3*c^3 - 3*a
*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*cosh(x)^2)*sinh(x)^2 + 6*((b^3*c^3 -
3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*cosh(x)^5 - 2*(b^3*c^3 - 3*a*b^2*c
^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*cosh(x)^3 + (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^
2*b*c*d^2 - a^3*d^3)*cosh(x))*sinh(x))*log(2*sinh(x)/(cosh(x) - sinh(x))) +
 12*(2*(b^3*c^2*d - (3*a*b^2 + b^3)*c*d^2 + (3*a^2*b + 3*a*b^2 + b^3)*d^3)*
cosh(x)^3 - (2*b^3*c^2*d - (6*a*b^2 + b^3)*c*d^2 + 3*(2*a^2*b + a*b^2)*d^3)
*cosh(x))*sinh(x))/(d^4*cosh(x)^6 + 6*d^4*cosh(x)*sinh(x)^5 + d^4*sinh(x)^6
 - 3*d^4*cosh(x)^4 + 3*d^4*cosh(x)^2 + 3*(5*d^4*cosh(x)^2 - d^4)*sinh(x)^4
- d^4 + 4*(5*d^4*cosh(x)^3 - 3*d^4*cosh(x))*sinh(x)^3 + 3*(5*d^4*cosh(x)^4
- 6*d^4*cosh(x)^2 + d^4)*sinh(x)^2 + 6*(d^4*cosh(x)^5 - 2*d^4*cosh(x)^3 + d
^4*cosh(x))*sinh(x))
```

giac [B] time = 0.14, size = 544, normalized size = 6.97

$$\frac{\left(b^3c^4 - 3\,ab^2c^3d + b^3c^3d + 3\,a^2bc^2d^2 - 3\,ab^2c^2d^2 - a^3cd^3 + 3\,a^2bcd^3 - a^3d^4\right)\log\left(\left|ce^{(2\,x)} + de^{(2\,x)} - c + d\right|\right)}{cd^4 + d^5} - \frac{\left(b^3c^3\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*coth(x))^3*csch(x)^2/(c+d*coth(x)),x, algorithm="giac")
```

```
[Out] (b^3*c^4 - 3*a*b^2*c^3*d + b^3*c^3*d + 3*a^2*b*c^2*d^2 - 3*a*b^2*c^2*d^2 -
a^3*c*d^3 + 3*a^2*b*c*d^3 - a^3*d^4)*log(abs(c*e^(2*x) + d*e^(2*x) - c + d)
)/(c*d^4 + d^5) - (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*log(a
bs(e^(2*x) - 1))/d^4 + 1/6*(11*b^3*c^3*e^(6*x) - 33*a*b^2*c^2*d*e^(6*x) + 3
```

```
3*a^2*b*c*d^2*e^(6*x) - 11*a^3*d^3*e^(6*x) - 33*b^3*c^3*e^(4*x) + 99*a*b^2*
c^2*d*e^(4*x) - 12*b^3*c^2*d*e^(4*x) - 99*a^2*b*c*d^2*e^(4*x) + 36*a*b^2*c*
d^2*e^(4*x) + 12*b^3*c*d^2*e^(4*x) + 33*a^3*d^3*e^(4*x) - 36*a^2*b*d^3*e^(4
*x) - 36*a*b^2*d^3*e^(4*x) - 12*b^3*d^3*e^(4*x) + 33*b^3*c^3*e^(2*x) - 99*a
*b^2*c^2*d*e^(2*x) + 24*b^3*c^2*d*e^(2*x) + 99*a^2*b*c*d^2*e^(2*x) - 72*a*b
^2*c*d^2*e^(2*x) - 12*b^3*c*d^2*e^(2*x) - 33*a^3*d^3*e^(2*x) + 72*a^2*b*d^3
*e^(2*x) + 36*a*b^2*d^3*e^(2*x) - 11*b^3*c^3 + 33*a*b^2*c^2*d - 12*b^3*c^2*
d - 33*a^2*b*c*d^2 + 36*a*b^2*c*d^2 + 11*a^3*d^3 - 36*a^2*b*d^3 - 4*b^3*d^3
)/(d^4*(e^(2*x) - 1)^3)
```

maple [B] time = 0.24, size = 378, normalized size = 4.85

$$-\frac{b^3\left(\tanh^3\left(\frac{x}{2}\right)\right)}{24d}-\frac{3b^2\left(\tanh^2\left(\frac{x}{2}\right)\right)a}{8d}+\frac{b^3\left(\tanh^2\left(\frac{x}{2}\right)\right)c}{8d^2}-\frac{3b\,a^2\tanh\left(\frac{x}{2}\right)}{2d}+\frac{3b^2ca\tanh\left(\frac{x}{2}\right)}{2d^2}-\frac{b^3c^2\tanh\left(\frac{x}{2}\right)}{2d^3}-\frac{b^3\tanh\left(\right.}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*coth(x))^3*csch(x)^2/(c+d*coth(x)),x)
```

```
[Out] -1/24*b^3/d*tanh(1/2*x)^3-3/8*b^2/d*tanh(1/2*x)^2*a+1/8*b^3/d^2*tanh(1/2*x)
^2*c-3/2*b/d*a^2*tanh(1/2*x)+3/2*b^2/d^2*c*a*tanh(1/2*x)-1/2*b^3/d^3*c^2*ta
nh(1/2*x)-1/8*b^3/d*tanh(1/2*x)-1/d*ln(tanh(1/2*x)^2*d+2*c*tanh(1/2*x)+d)*a
^3+3/d^2*ln(tanh(1/2*x)^2*d+2*c*tanh(1/2*x)+d)*a^2*b*c-3/d^3*ln(tanh(1/2*x)
^2*d+2*c*tanh(1/2*x)+d)*c^2*b^2*a+1/d^4*ln(tanh(1/2*x)^2*d+2*c*tanh(1/2*x)+
d)*c^3*b^3-1/24*b^3/d/tanh(1/2*x)^3+1/d*ln(tanh(1/2*x))*a^3-3/d^2*ln(tanh(1
/2*x))*a^2*b*c+3/d^3*ln(tanh(1/2*x))*c^2*b^2*a-1/d^4*ln(tanh(1/2*x))*c^3*b^
3-3/2*b/d/tanh(1/2*x)*a^2+3/2*b^2/d^2/tanh(1/2*x)*c*a-1/2*b^3/d^3/tanh(1/2*
x)*c^2-1/8*b^3/d/tanh(1/2*x)-3/8*b^2/d/tanh(1/2*x)^2*a+1/8*b^3/d^2/tanh(1/2
*x)^2*c
```

maxima [B] time = 0.34, size = 316, normalized size = 4.05

$$\frac{1}{3}b^3\left(\frac{2\left(3c^2+d^2-3\left(2c^2+cd\right)e^{(-2x)}+3\left(c^2+cd+d^2\right)e^{(-4x)}\right)}{3d^3e^{(-2x)}-3d^3e^{(-4x)}+d^3e^{(-6x)}-d^3}+\frac{3c^3\log\left(-(c-d)e^{(-2x)}+c+d\right)}{d^4}-\frac{3c^3\log\left(e^{(-x}\right.}{d^4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*coth(x))^3*csch(x)^2/(c+d*coth(x)),x, algorithm="maxima")
```

```
[Out] 1/3*b^3*(2*(3*c^2 + d^2 - 3*(2*c^2 + c*d)*e^(-2*x) + 3*(c^2 + c*d + d^2)*e^
(-4*x))/(3*d^3*e^(-2*x) - 3*d^3*e^(-4*x) + d^3*e^(-6*x) - d^3) + 3*c^3*log(
-(c - d)*e^(-2*x) + c + d)/d^4 - 3*c^3*log(e^(-x) + 1)/d^4 - 3*c^3*log(e^(-
x) - 1)/d^4) + 3*a*b^2*(2*((c + d)*e^(-2*x) - c)/(2*d^2*e^(-2*x) - d^2*e^(-
4*x) - d^2) - c^2*log(-(c - d)*e^(-2*x) + c + d)/d^3 + c^2*log(e^(-x) + 1)/
d^3 + c^2*log(e^(-x) - 1)/d^3) + 3*a^2*b*(c*log(-(c - d)*e^(-2*x) + c + d)/
```

```
d^2 - c*log(e^(-x) + 1)/d^2 - c*log(e^(-x) - 1)/d^2 + 2/(d*e^(-2*x) - d)) -
 a^3*log(d*coth(x) + c)/d
```

mupad [B] time = 2.70, size = 1346, normalized size = 17.26

$$2\,\mathrm{atan}\left(\frac{\left(e^{2x}\left(\frac{32\,c\left(2\,a^3\,c\,d^8-6\,a^2\,b\,c^2\,d^7+6\,a\,b^2\,c^3\,d^6-2\,b^3\,c^4\,d^5\right)\sqrt{a^6\,d^6-6\,a^5\,b\,c\,d^5+15\,a^4\,b^2\,c^2\,d^4-20\,a^3\,b^3\,c^3\,d^3+15\,a^2\,b^4\,c^4\,d^2-6\,a\,b^5\,c^5\,d+b^6\,c^6}}{d^{16}\sqrt{(a\,d-b\,c)^6}\,(c+d)\,(c-d)^2\left(c^2+2\,c\,d+d^2\right)}-\frac{16\left(c^2\,\sqrt{-d^8}\,\sqrt{a^6\,d^6 \cdots}\right.}{\cdots}\right.\right.}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*coth(x))^3/(sinh(x)^2*(c + d*coth(x))),x)
```

```
[Out] (2*atan(((exp(2*x)*((32*c*(2*a^3*c*d^8 - 2*b^3*c^4*d^5 + 6*a*b^2*c^3*d^6 -
6*a^2*b*c^2*d^7)*(a^6*d^6 + b^6*c^6 + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d
^3 + 15*a^4*b^2*c^2*d^4 - 6*a*b^5*c^5*d - 6*a^5*b*c*d^5)^(1/2))/(d^16*((a*d
 - b*c)^6)^(1/2)*(c + d)*(c - d)^2*(2*c*d + c^2 + d^2)) - (16*(c^2*(-d^8)^(
1/2)*(a^6*d^6 + b^6*c^6 + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*
b^2*c^2*d^4 - 6*a*b^5*c^5*d - 6*a^5*b*c*d^5)^(1/2) + d^2*(-d^8)^(1/2)*(a^6*
d^6 + b^6*c^6 + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^
4 - 6*a*b^5*c^5*d - 6*a^5*b*c*d^5)^(1/2))*(c^2 + d^2)*(a^6*d^6 + b^6*c^6 +
15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a*b^5*c^5*
d - 6*a^5*b*c*d^5)^(1/2))/(d^13*(c + d)*(c - d)^2*(a*d - b*c)^3*(-d^8)^(1/2
)*(2*c*d + c^2 + d^2))) + (32*c*(a^6*d^6 + b^6*c^6 + 15*a^2*b^4*c^4*d^2 - 2
0*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a*b^5*c^5*d - 6*a^5*b*c*d^5)^(1/
2)*(a^3*d^9 - a^3*c*d^8 - b^3*c^3*d^6 + b^3*c^4*d^5 + 3*a*b^2*c^2*d^7 - 3*a
*b^2*c^3*d^6 + 3*a^2*b*c^2*d^7 - 3*a^2*b*c*d^8))/(d^16*((a*d - b*c)^6)^(1/2
)*(c + d)*(c - d)^2*(2*c*d + c^2 + d^2)) + (16*(c^2*(-d^8)^(1/2)*(a^6*d^6 +
 b^6*c^6 + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6
*a*b^5*c^5*d - 6*a^5*b*c*d^5)^(1/2) - c*d*(-d^8)^(1/2)*(a^6*d^6 + b^6*c^6 +
 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a*b^5*c^5
*d - 6*a^5*b*c*d^5)^(1/2))*(c^2 + d^2)*(a^6*d^6 + b^6*c^6 + 15*a^2*b^4*c^4*
d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a*b^5*c^5*d - 6*a^5*b*c*d
^5)^(1/2))/(d^13*(c + d)*(c - d)^2*(a*d - b*c)^3*(-d^8)^(1/2)*(2*c*d + c^2
+ d^2)))*(d^10*(-d^8)^(1/2) + 2*c*d^9*(-d^8)^(1/2) + c^2*d^8*(-d^8)^(1/2)))
/(16*(a^6*d^6 + b^6*c^6 + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*
b^2*c^2*d^4 - 6*a*b^5*c^5*d - 6*a^5*b*c*d^5)^(1/2)))*(a^6*d^6 + b^6*c^6 + 1
5*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a*b^5*c^5*d
 - 6*a^5*b*c*d^5)^(1/2))/(-d^8)^(1/2) - (2*(2*b^3*d - b^3*c + 3*a*b^2*d))/(
d^2*(exp(4*x) - 2*exp(2*x) + 1)) - (8*b^3)/(3*d*(3*exp(2*x) - 3*exp(4*x) +
exp(6*x) - 1)) - (2*(b^3*c^2 + b^3*d^2 + 3*a*b^2*d^2 + 3*a^2*b*d^2 - b^3*c*
d - 3*a*b^2*c*d))/(d^3*(exp(2*x) - 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b\coth(x))^3 \operatorname{csch}^2(x)}{c + d\coth(x)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*coth(x))**3*csch(x)**2/(c+d*coth(x)),x)
[Out] Integral((a + b*coth(x))**3*csch(x)**2/(c + d*coth(x)), x)
```

3.1026 $\int \cosh^3(x) \left(a + b\cosh^2(x)\right)^3 \sinh(x)\, dx$

Optimal. Leaf size=36

$$\frac{\left(a + b\cosh^2(x)\right)^5}{10b^2} - \frac{a\left(a + b\cosh^2(x)\right)^4}{8b^2}$$

```
[Out] -1/8*a*(a+b*cosh(x)^2)^4/b^2+1/10*(a+b*cosh(x)^2)^5/b^2
```

Rubi [A] time = 0.09, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {4335, 266, 43}

$$\frac{\left(a + b\cosh^2(x)\right)^5}{10b^2} - \frac{a\left(a + b\cosh^2(x)\right)^4}{8b^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]^3*(a + b*Cosh[x]^2)^3*Sinh[x],x]
```

```
[Out] -(a*(a + b*Cosh[x]^2)^4)/(8*b^2) + (a + b*Cosh[x]^2)^5/(10*b^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4335

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Cos[c*(a + b*x)], x]}, -Dist[d/(b*c), Subst[Int[SubstFor[1, Cos[c*(a + b
*x)]/d, u, x], x], x, Cos[c*(a + b*x)]/d], x] /; FunctionOfQ[Cos[c*(a + b*x
)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Sin] || EqQ[F, sin])
```

Rubi steps

$$\int \cosh^3(x)\left(a+b\cosh^2(x)\right)^3\sinh(x)\,dx = \text{Subst}\left(\int x^3\left(a+bx^2\right)^3\,dx,x,\cosh(x)\right)$$
$$=\frac{1}{2}\,\text{Subst}\left(\int x(a+bx)^3\,dx,x,\cosh^2(x)\right)$$
$$=\frac{1}{2}\,\text{Subst}\left(\int\left(-\frac{a(a+bx)^3}{b}+\frac{(a+bx)^4}{b}\right)dx,x,\cosh^2(x)\right)$$
$$=-\frac{a\left(a+b\cosh^2(x)\right)^4}{8b^2}+\frac{\left(a+b\cosh^2(x)\right)^5}{10b^2}$$

Mathematica [B] time = 0.26, size = 136, normalized size = 3.78

$$\frac{1}{32}\left(4a^3\cosh(2x)+a^3\cosh(4x)+12a^2b\cosh^4(x)+4a^2b\cosh(3x)\cosh^3(x)+8ab^2\cosh^6(x)+\frac{1}{32}ab^2(48\cosh(2\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]^3*(a + b*Cosh[x]^2)^3*Sinh[x],x]
```

```
[Out] (12*a^2*b*Cosh[x]^4 + 8*a*b^2*Cosh[x]^6 + 2*b^3*Cosh[x]^8 + 4*a^3*Cosh[2*x]
 + 4*a^2*b*Cosh[x]^3*Cosh[3*x] + a^3*Cosh[4*x] + (a*b^2*(48*Cosh[2*x] + 36*
Cosh[4*x] + 16*Cosh[6*x] + 3*Cosh[8*x]))/32 + (b^3*(140*Cosh[2*x] + 100*Cos
h[4*x] + 50*Cosh[6*x] + 15*Cosh[8*x] + 2*Cosh[10*x]))/320)/32
```

fricas [B] time = 0.43, size = 386, normalized size = 10.72

$$\frac{1}{5120}\,b^3\cosh(x)^{10}+\frac{1}{5120}\,b^3\sinh(x)^{10}+\frac{1}{1024}\left(3\,ab^2+2\,b^3\right)\cosh(x)^8+\frac{1}{1024}\left(9\,b^3\cosh(x)^2+3\,ab^2+2\,b^3\right)\sinh(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^3*(a+b*cosh(x)^2)^3*sinh(x),x, algorithm="fricas")
```

```
[Out] 1/5120*b^3*cosh(x)^10 + 1/5120*b^3*sinh(x)^10 + 1/1024*(3*a*b^2 + 2*b^3)*co
sh(x)^8 + 1/1024*(9*b^3*cosh(x)^2 + 3*a*b^2 + 2*b^3)*sinh(x)^8 + 1/1024*(16
*a^2*b + 24*a*b^2 + 9*b^3)*cosh(x)^6 + 1/1024*(42*b^3*cosh(x)^4 + 16*a^2*b
+ 24*a*b^2 + 9*b^3 + 28*(3*a*b^2 + 2*b^3)*cosh(x)^2)*sinh(x)^6 + 1/256*(8*a
^3 + 24*a^2*b + 21*a*b^2 + 6*b^3)*cosh(x)^4 + 1/1024*(42*b^3*cosh(x)^6 + 70
*(3*a*b^2 + 2*b^3)*cosh(x)^4 + 32*a^3 + 96*a^2*b + 84*a*b^2 + 24*b^3 + 15*(
16*a^2*b + 24*a*b^2 + 9*b^3)*cosh(x)^2)*sinh(x)^4 + 1/512*(64*a^3 + 120*a^2
*b + 84*a*b^2 + 21*b^3)*cosh(x)^2 + 1/1024*(9*b^3*cosh(x)^8 + 28*(3*a*b^2 +
 2*b^3)*cosh(x)^6 + 15*(16*a^2*b + 24*a*b^2 + 9*b^3)*cosh(x)^4 + 128*a^3 +
240*a^2*b + 168*a*b^2 + 42*b^3 + 24*(8*a^3 + 24*a^2*b + 21*a*b^2 + 6*b^3)*c
osh(x)^2)*sinh(x)^2
```

giac [B] time = 0.14, size = 224, normalized size = 6.22

$$\frac{1}{10240}\,b^3\Big(e^{(2\,x)}+e^{(-2\,x)}\Big)^5+\frac{3}{2048}\,ab^2\Big(e^{(2\,x)}+e^{(-2\,x)}\Big)^4+\frac{1}{1024}\,b^3\Big(e^{(2\,x)}+e^{(-2\,x)}\Big)^4+\frac{1}{128}\,a^2b\Big(e^{(2\,x)}+e^{(-2\,x)}\Big)^3+\frac{3}{256}\,ab^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^3*(a+b*cosh(x)^2)^3*sinh(x),x, algorithm="giac")
```

```
[Out] 1/10240*b^3*(e^(2*x) + e^(-2*x))^5 + 3/2048*a*b^2*(e^(2*x) + e^(-2*x))^4 +
1/1024*b^3*(e^(2*x) + e^(-2*x))^4 + 1/128*a^2*b*(e^(2*x) + e^(-2*x))^3 + 3/
256*a*b^2*(e^(2*x) + e^(-2*x))^3 + 1/256*b^3*(e^(2*x) + e^(-2*x))^3 + 1/64*
a^3*(e^(2*x) + e^(-2*x))^2 + 3/64*a^2*b*(e^(2*x) + e^(-2*x))^2 + 9/256*a*b^
2*(e^(2*x) + e^(-2*x))^2 + 1/128*b^3*(e^(2*x) + e^(-2*x))^2 + 1/16*a^3*(e^(
2*x) + e^(-2*x)) + 3/32*a^2*b*(e^(2*x) + e^(-2*x)) + 3/64*a*b^2*(e^(2*x) +
e^(-2*x)) + 1/128*b^3*(e^(2*x) + e^(-2*x))
```

maple [A] time = 0.03, size = 40, normalized size = 1.11

$$\frac{b^3\left(\cosh^{10}(x)\right)}{10}+\frac{3a\,b^2\left(\cosh^{8}(x)\right)}{8}+\frac{a^2b\left(\cosh^{6}(x)\right)}{2}+\frac{a^3\left(\cosh^{4}(x)\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^3*(a+b*cosh(x)^2)^3*sinh(x),x)
```

```
[Out] 1/10*b^3*cosh(x)^10+3/8*a*b^2*cosh(x)^8+1/2*a^2*b*cosh(x)^6+1/4*a^3*cosh(x)
^4
```

maxima [A] time = 0.32, size = 39, normalized size = 1.08

$$\frac{1}{10}\,b^3\cosh(x)^{10}+\frac{3}{8}\,ab^2\cosh(x)^8+\frac{1}{2}\,a^2b\cosh(x)^6+\frac{1}{4}\,a^3\cosh(x)^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^3*(a+b*cosh(x)^2)^3*sinh(x),x, algorithm="maxima")
```

```
[Out] 1/10*b^3*cosh(x)^10 + 3/8*a*b^2*cosh(x)^8 + 1/2*a^2*b*cosh(x)^6 + 1/4*a^3*c
osh(x)^4
```

mupad [B] time = 1.89, size = 39, normalized size = 1.08

$$\frac{a^3\cosh(x)^4}{4}+\frac{a^2\,b\cosh(x)^6}{2}+\frac{3\,a\,b^2\cosh(x)^8}{8}+\frac{b^3\cosh(x)^{10}}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^3*sinh(x)*(a + b*cosh(x)^2)^3,x)
```

```
[Out] (a^3*cosh(x)^4)/4 + (b^3*cosh(x)^10)/10 + (a^2*b*cosh(x)^6)/2 + (3*a*b^2*co
sh(x)^8)/8
```

sympy [A] time = 10.72, size = 44, normalized size = 1.22

$$\frac{a^3\cosh^4(x)}{4}+\frac{a^2b\cosh^6(x)}{2}+\frac{3ab^2\cosh^8(x)}{8}+\frac{b^3\cosh^{10}(x)}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)**3*(a+b*cosh(x)**2)**3*sinh(x),x)
```

```
[Out] a**3*cosh(x)**4/4 + a**2*b*cosh(x)**6/2 + 3*a*b**2*cosh(x)**8/8 + b**3*cosh
(x)**10/10
```

3.1027 $\int \cosh(x) \sinh^3(x) \left(a + b \sinh^2(x)\right)^3 \, dx$

Optimal. Leaf size=36

$$\frac{\left(a + b \sinh^2(x)\right)^5}{10b^2} - \frac{a \left(a + b \sinh^2(x)\right)^4}{8b^2}$$

```
[Out] -1/8*a*(a+b*sinh(x)^2)^4/b^2+1/10*(a+b*sinh(x)^2)^5/b^2
```

Rubi [A] time = 0.09, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {3198, 266, 43}

$$\frac{\left(a + b \sinh^2(x)\right)^5}{10b^2} - \frac{a \left(a + b \sinh^2(x)\right)^4}{8b^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]*Sinh[x]^3*(a + b*Sinh[x]^2)^3,x]
```

```
[Out] -(a*(a + b*Sinh[x]^2)^4)/(8*b^2) + (a + b*Sinh[x]^2)^5/(10*b^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 3198

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a
_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.), x_Symbol] :> With[{ff = FreeFa
ctors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(d*ff*x)^n*(1 - ff^2*x^2)^((m
 - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, d
, e, f, n, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \cosh(x)\sinh^3(x)\left(a+b\sinh^2(x)\right)^3\,dx = \text{Subst}\left(\int x^3\left(a+bx^2\right)^3\,dx,x,\sinh(x)\right)$$
$$=\frac{1}{2}\text{Subst}\left(\int x(a+bx)^3\,dx,x,\sinh^2(x)\right)$$
$$=\frac{1}{2}\text{Subst}\left(\int\left(-\frac{a(a+bx)^3}{b}+\frac{(a+bx)^4}{b}\right)dx,x,\sinh^2(x)\right)$$
$$=-\frac{a\left(a+b\sinh^2(x)\right)^4}{8b^2}+\frac{\left(a+b\sinh^2(x)\right)^5}{10b^2}$$

Mathematica [B] time = 0.62, size = 114, normalized size = 3.17

$$\frac{-20\left(64a^3+24ab^2-7b^3\right)\cosh(2x)+20\left(16a^3+18ab^2-5b^3\right)\cosh(4x)+b\left(320\sinh^6(x)\left((b-4a)^2-b^2\cosh(2x\right.\right.}{10240}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]*Sinh[x]^3*(a + b*Sinh[x]^2)^3,x]
```

```
[Out] (-20*(64*a^3 + 24*a*b^2 - 7*b^3)*Cosh[2*x] + 20*(16*a^3 + 18*a*b^2 - 5*b^3)
*Cosh[4*x] + b*(-10*(16*a - 5*b)*b*Cosh[6*x] + 15*(2*a - b)*b*Cosh[8*x] + 2
*b^2*Cosh[10*x] + 320*((-4*a + b)^2 - b^2*Cosh[2*x])*Sinh[x]^6))/10240
```

fricas [B] time = 0.40, size = 386, normalized size = 10.72

$$\frac{1}{5120}b^3\cosh(x)^{10}+\frac{1}{5120}b^3\sinh(x)^{10}+\frac{1}{1024}\left(3ab^2-2b^3\right)\cosh(x)^8+\frac{1}{1024}\left(9b^3\cosh(x)^2+3ab^2-2b^3\right)\sinh(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*sinh(x)^3*(a+b*sinh(x)^2)^3,x, algorithm="fricas")
```

```
[Out] 1/5120*b^3*cosh(x)^10 + 1/5120*b^3*sinh(x)^10 + 1/1024*(3*a*b^2 - 2*b^3)*co
sh(x)^8 + 1/1024*(9*b^3*cosh(x)^2 + 3*a*b^2 - 2*b^3)*sinh(x)^8 + 1/1024*(16
*a^2*b - 24*a*b^2 + 9*b^3)*cosh(x)^6 + 1/1024*(42*b^3*cosh(x)^4 + 16*a^2*b
- 24*a*b^2 + 9*b^3 + 28*(3*a*b^2 - 2*b^3)*cosh(x)^2)*sinh(x)^6 + 1/256*(8*a
^3 - 24*a^2*b + 21*a*b^2 - 6*b^3)*cosh(x)^4 + 1/1024*(42*b^3*cosh(x)^6 + 70
*(3*a*b^2 - 2*b^3)*cosh(x)^4 + 32*a^3 - 96*a^2*b + 84*a*b^2 - 24*b^3 + 15*(
16*a^2*b - 24*a*b^2 + 9*b^3)*cosh(x)^2)*sinh(x)^4 - 1/512*(64*a^3 - 120*a^2
*b + 84*a*b^2 - 21*b^3)*cosh(x)^2 + 1/1024*(9*b^3*cosh(x)^8 + 28*(3*a*b^2 -
 2*b^3)*cosh(x)^6 + 15*(16*a^2*b - 24*a*b^2 + 9*b^3)*cosh(x)^4 - 128*a^3 +
240*a^2*b - 168*a*b^2 + 42*b^3 + 24*(8*a^3 - 24*a^2*b + 21*a*b^2 - 6*b^3)*c
osh(x)^2)*sinh(x)^2
```

giac [B] time = 0.13, size = 224, normalized size = 6.22

$$\frac{1}{10240}\,b^3\Big(e^{(2\,x)}+e^{(-2\,x)}\Big)^5+\frac{3}{2048}\,ab^2\Big(e^{(2\,x)}+e^{(-2\,x)}\Big)^4-\frac{1}{1024}\,b^3\Big(e^{(2\,x)}+e^{(-2\,x)}\Big)^4+\frac{1}{128}\,a^2b\Big(e^{(2\,x)}+e^{(-2\,x)}\Big)^3-\frac{3}{256}\,ab^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*sinh(x)^3*(a+b*sinh(x)^2)^3,x, algorithm="giac")
```

```
[Out] 1/10240*b^3*(e^(2*x) + e^(-2*x))^5 + 3/2048*a*b^2*(e^(2*x) + e^(-2*x))^4 -
1/1024*b^3*(e^(2*x) + e^(-2*x))^4 + 1/128*a^2*b*(e^(2*x) + e^(-2*x))^3 - 3/
256*a*b^2*(e^(2*x) + e^(-2*x))^3 + 1/256*b^3*(e^(2*x) + e^(-2*x))^3 + 1/64*
a^3*(e^(2*x) + e^(-2*x))^2 - 3/64*a^2*b*(e^(2*x) + e^(-2*x))^2 + 9/256*a*b^
2*(e^(2*x) + e^(-2*x))^2 - 1/128*b^3*(e^(2*x) + e^(-2*x))^2 - 1/16*a^3*(e^(
2*x) + e^(-2*x)) + 3/32*a^2*b*(e^(2*x) + e^(-2*x)) - 3/64*a*b^2*(e^(2*x) +
e^(-2*x)) + 1/128*b^3*(e^(2*x) + e^(-2*x))
```

maple [A] time = 0.04, size = 40, normalized size = 1.11

$$\frac{b^3\left(\sinh^{10}(x)\right)}{10}+\frac{3a\,b^2\left(\sinh^{8}(x)\right)}{8}+\frac{a^2b\left(\sinh^{6}(x)\right)}{2}+\frac{a^3\left(\sinh^{4}(x)\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)*sinh(x)^3*(a+b*sinh(x)^2)^3,x)
```

```
[Out] 1/10*b^3*sinh(x)^10+3/8*a*b^2*sinh(x)^8+1/2*a^2*b*sinh(x)^6+1/4*a^3*sinh(x)
^4
```

maxima [A] time = 0.30, size = 39, normalized size = 1.08

$$\frac{1}{10}\,b^3\sinh(x)^{10}+\frac{3}{8}\,ab^2\sinh(x)^8+\frac{1}{2}\,a^2b\sinh(x)^6+\frac{1}{4}\,a^3\sinh(x)^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*sinh(x)^3*(a+b*sinh(x)^2)^3,x, algorithm="maxima")
```

```
[Out] 1/10*b^3*sinh(x)^10 + 3/8*a*b^2*sinh(x)^8 + 1/2*a^2*b*sinh(x)^6 + 1/4*a^3*s
inh(x)^4
```

mupad [B] time = 0.27, size = 39, normalized size = 1.08

$$\frac{a^3\sinh(x)^4}{4}+\frac{a^2\,b\sinh(x)^6}{2}+\frac{3\,a\,b^2\sinh(x)^8}{8}+\frac{b^3\sinh(x)^{10}}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)*sinh(x)^3*(a + b*sinh(x)^2)^3,x)
```

```
[Out] (a^3*sinh(x)^4)/4 + (b^3*sinh(x)^10)/10 + (a^2*b*sinh(x)^6)/2 + (3*a*b^2*si
nh(x)^8)/8
```

sympy [A] time = 10.84, size = 44, normalized size = 1.22

$$\frac{a^3\sinh^4(x)}{4}+\frac{a^2b\sinh^6(x)}{2}+\frac{3ab^2\sinh^8(x)}{8}+\frac{b^3\sinh^{10}(x)}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*sinh(x)**3*(a+b*sinh(x)**2)**3,x)
```

```
[Out] a**3*sinh(x)**4/4 + a**2*b*sinh(x)**6/2 + 3*a*b**2*sinh(x)**8/8 + b**3*sinh
(x)**10/10
```

3.1028 $\int \cosh(x)\sinh(x)\sqrt{a+b\sinh^2(x)}\,dx$

Optimal. Leaf size=19

$$\frac{\left(a+b\sinh^2(x)\right)^{3/2}}{3b}$$

```
[Out] 1/3*(a+b*sinh(x)^2)^(3/2)/b
```

Rubi [A] time = 0.06, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {3198, 261}

$$\frac{\left(a+b\sinh^2(x)\right)^{3/2}}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]*Sinh[x]*Sqrt[a + b*Sinh[x]^2],x]
```

```
[Out] (a + b*Sinh[x]^2)^(3/2)/(3*b)
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 3198

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a
_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.), x_Symbol] :> With[{ff = FreeFa
ctors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(d*ff*x)^n*(1 - ff^2*x^2)^((m
 - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, d
, e, f, n, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}\int \cosh(x)\sinh(x)\sqrt{a+b\sinh^2(x)}\,dx &= \text{Subst}\left(\int x\sqrt{a+bx^2}\,dx,x,\sinh(x)\right)\\ &= \frac{\left(a+b\sinh^2(x)\right)^{3/2}}{3b}\end{aligned}$$

Mathematica [A] time = 0.01, size = 19, normalized size = 1.00

$$\frac{\left(a+b\sinh^2(x)\right)^{3/2}}{3b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]*Sinh[x]*Sqrt[a + b*Sinh[x]^2],x]
```

```
[Out] (a + b*Sinh[x]^2)^(3/2)/(3*b)
```

fricas [B] time = 0.45, size = 154, normalized size = 8.11

$$\frac{\sqrt{2}\left(b\cosh(x)^4+4\,b\cosh(x)\sinh(x)^3+b\sinh(x)^4+2\,(2\,a-b)\cosh(x)^2+2\left(3\,b\cosh(x)^2+2\,a-b\right)\sinh(x)^2+\cdots\right.}{24\left(b\cosh(x)^3+3\,b\cosh(x)^2\sinh(x)+3\,b\cosh(x)\sin\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*sinh(x)*(a+b*sinh(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/24*sqrt(2)*(b*cosh(x)^4 + 4*b*cosh(x)*sinh(x)^3 + b*sinh(x)^4 + 2*(2*a -
b)*cosh(x)^2 + 2*(3*b*cosh(x)^2 + 2*a - b)*sinh(x)^2 + 4*(b*cosh(x)^3 + (2*
a - b)*cosh(x))*sinh(x) + b)*sqrt((b*cosh(x)^2 + b*sinh(x)^2 + 2*a - b)/(co
sh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/(b*cosh(x)^3 + 3*b*cosh(x)^2*sinh
(x) + 3*b*cosh(x)*sinh(x)^2 + b*sinh(x)^3)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{b\sinh(x)^2+a}\,\cosh(x)\sinh(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*sinh(x)*(a+b*sinh(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sinh(x)^2 + a)*cosh(x)*sinh(x), x)
```

maple [A] time = 0.05, size = 16, normalized size = 0.84

$$\frac{\left(a+b\left(\sinh^2(x)\right)\right)^{\frac{3}{2}}}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)*sinh(x)*(a+b*sinh(x)^2)^(1/2),x)
```

```
[Out] 1/3*(a+b*sinh(x)^2)^(3/2)/b
```

maxima [A] time = 0.30, size = 15, normalized size = 0.79

$$\frac{\left(b \sinh(x)^2 + a\right)^{\frac{3}{2}}}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*sinh(x)*(a+b*sinh(x)^2)^(1/2),x, algorithm="maxima")
[Out] 1/3*(b*sinh(x)^2 + a)^(3/2)/b
```

mupad [B] time = 1.87, size = 15, normalized size = 0.79

$$\frac{\left(b \sinh(x)^2 + a\right)^{3/2}}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)*sinh(x)*(a + b*sinh(x)^2)^(1/2),x)
[Out] (a + b*sinh(x)^2)^(3/2)/(3*b)
```

sympy [A] time = 1.32, size = 46, normalized size = 2.42

$$\begin{cases} \frac{a\sqrt{a+b\sinh^2(x)}}{3b} + \frac{\sqrt{a+b\sinh^2(x)}\,\sinh^2(x)}{3} & \text{for}\ b \neq 0 \\ \frac{\sqrt{a}\,\cosh^2(x)}{2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*sinh(x)*(a+b*sinh(x)**2)**(1/2),x)
[Out] Piecewise((a*sqrt(a + b*sinh(x)**2)/(3*b) + sqrt(a + b*sinh(x)**2)*sinh(x)*
*2/3, Ne(b, 0)), (sqrt(a)*cosh(x)**2/2, True))
```

3.1029 $\int \operatorname{csch}(x)\sqrt{1+\log^2(\coth(x))}\operatorname{sech}(x)\,dx$

Optimal. Leaf size=27

$$-\frac{1}{2}\log(\coth(x))\sqrt{\log^2(\coth(x))+1}-\frac{1}{2}\sinh^{-1}(\log(\coth(x)))$$

```
[Out] -1/2*arcsinh(ln(coth(x)))-1/2*ln(coth(x))*(1+ln(coth(x))^2)^(1/2)
```

Rubi [A] time = 0.17, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {6696, 195, 215}

$$-\frac{1}{2}\log(\coth(x))\sqrt{\log^2(\coth(x))+1}-\frac{1}{2}\sinh^{-1}(\log(\coth(x)))$$

Antiderivative was successfully verified.

```
[In] Int[Csch[x]*Sqrt[1 + Log[Coth[x]]^2]*Sech[x],x]
```

```
[Out] -ArcSinh[Log[Coth[x]]]/2 - (Log[Coth[x]]*Sqrt[1 + Log[Coth[x]]^2])/2
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 6696

```
Int[(u_.)*((a_.) + (b_.)*(y_)^(n_))^(p_), x_Symbol] :> With[{q = Derivative
Divides[y, u, x]}, Dist[q, Subst[Int[(a + b*x^n)^p, x], x, y], x] /;  !Fals
eQ[q]] /; FreeQ[{a, b, n, p}, x]
```

Rubi steps

$$\int \operatorname{csch}(x)\sqrt{1+\log^2(\coth(x))}\operatorname{sech}(x)\,dx = -\text{Subst}\left(\int \sqrt{1+x^2}\,dx, x, \log(\coth(x))\right)$$
$$= -\frac{1}{2}\log(\coth(x))\sqrt{1+\log^2(\coth(x))} - \frac{1}{2}\text{Subst}\left(\int \frac{1}{\sqrt{1+x^2}}\,dx, x,\right.$$
$$= -\frac{1}{2}\sinh^{-1}(\log(\coth(x))) - \frac{1}{2}\log(\coth(x))\sqrt{1+\log^2(\coth(x))}$$

Mathematica [A] time = 0.03, size = 27, normalized size = 1.00

$$-\frac{1}{2}\log(\coth(x))\sqrt{\log^2(\coth(x))+1} - \frac{1}{2}\sinh^{-1}(\log(\coth(x)))$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[x]*Sqrt[1 + Log[Coth[x]]^2]*Sech[x],x]
[Out] -1/2*ArcSinh[Log[Coth[x]]] - (Log[Coth[x]]*Sqrt[1 + Log[Coth[x]]^2])/2
```

fricas [B] time = 0.46, size = 53, normalized size = 1.96

$$-\frac{1}{2}\sqrt{\log\left(\frac{\cosh(x)}{\sinh(x)}\right)^2+1}\,\log\left(\frac{\cosh(x)}{\sinh(x)}\right) + \frac{1}{2}\log\left(\sqrt{\log\left(\frac{\cosh(x)}{\sinh(x)}\right)^2+1} - \log\left(\frac{\cosh(x)}{\sinh(x)}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)*sech(x)*(1+log(coth(x))^2)^(1/2),x, algorithm="fricas")
[Out] -1/2*sqrt(log(cosh(x)/sinh(x))^2 + 1)*log(cosh(x)/sinh(x)) + 1/2*log(sqrt(l
og(cosh(x)/sinh(x))^2 + 1) - log(cosh(x)/sinh(x)))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\log(\coth(x))^2+1}\,\operatorname{csch}(x)\operatorname{sech}(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)*sech(x)*(1+log(coth(x))^2)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(log(coth(x))^2 + 1)*csch(x)*sech(x), x)
```

maple [A] time = 0.24, size = 22, normalized size = 0.81

$$-\frac{\operatorname{arcsinh}(\ln(\coth(x)))}{2} - \frac{\ln(\coth(x))\sqrt{1+\ln(\coth(x))^2}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(x)*sech(x)*(1+ln(coth(x))^2)^(1/2),x)
[Out] -1/2*arcsinh(ln(coth(x)))-1/2*ln(coth(x))*(1+ln(coth(x))^2)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\log\left(\coth(x)\right)^2+1}\,\operatorname{csch}(x)\operatorname{sech}(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)*sech(x)*(1+log(coth(x))^2)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(log(coth(x))^2 + 1)*csch(x)*sech(x), x)
```

mupad [B] time = 1.88, size = 21, normalized size = 0.78

$$-\frac{\operatorname{asinh}\left(\ln\left(\coth(x)\right)\right)}{2}-\frac{\ln\left(\coth(x)\right)\sqrt{\ln\left(\coth(x)\right)^2+1}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(coth(x))^2 + 1)^(1/2)/(cosh(x)*sinh(x)),x)
[Out] - asinh(log(coth(x)))/2 - (log(coth(x))*(log(coth(x))^2 + 1)^(1/2))/2
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\log\left(\coth\left(x\right)\right)^2+1}\,\operatorname{csch}\left(x\right)\operatorname{sech}\left(x\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)*sech(x)*(1+ln(coth(x))**2)**(1/2),x)
[Out] Integral(sqrt(log(coth(x))**2 + 1)*csch(x)*sech(x), x)
```

3.1030 $\int \frac{\coth(\sqrt{x})\operatorname{csch}(\sqrt{x})}{\sqrt{x}}\,dx$

Optimal. Leaf size=8

$$-2\operatorname{csch}\left(\sqrt{x}\right)$$

```
[Out] -2*csch(x^(1/2))
```

Rubi [A] time = 0.19, antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6715, 2606, 8}

$$-2\operatorname{csch}\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(Coth[Sqrt[x]]*Csch[Sqrt[x]])/Sqrt[x],x]
```

```
[Out] -2*Csch[Sqrt[x]]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 6715

```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Dist[1/(m + 1), Subst[Int[SubstFor[x^(m +
 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && FunctionO
fQ[x^(m + 1), u, x]
```

Rubi steps

$$\int \frac{\coth\left(\sqrt{x}\right)\operatorname{csch}\left(\sqrt{x}\right)}{\sqrt{x}}\,dx = 2\,\text{Subst}\left(\int \coth(x)\operatorname{csch}(x)\,dx, x, \sqrt{x}\right)$$
$$= -\left(2i\,\text{Subst}\left(\int 1\,dx, x, -i\operatorname{csch}\left(\sqrt{x}\right)\right)\right)$$
$$= -2\operatorname{csch}\left(\sqrt{x}\right)$$

Mathematica [A] time = 0.02, size = 8, normalized size = 1.00

$$-2\operatorname{csch}\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Coth[Sqrt[x]]*Csch[Sqrt[x]])/Sqrt[x],x]
```

```
[Out] -2*Csch[Sqrt[x]]
```

fricas [B] time = 0.47, size = 37, normalized size = 4.62

$$-\frac{4\left(\cosh\left(\sqrt{x}\right) + \sinh\left(\sqrt{x}\right)\right)}{\cosh\left(\sqrt{x}\right)^2 + 2\cosh\left(\sqrt{x}\right)\sinh\left(\sqrt{x}\right) + \sinh\left(\sqrt{x}\right)^2 - 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x^(1/2))*csch(x^(1/2))/x^(1/2),x, algorithm="fricas")
```

```
[Out] -4*(cosh(sqrt(x)) + sinh(sqrt(x)))/(cosh(sqrt(x))^2 + 2*cosh(sqrt(x))*sinh(
sqrt(x)) + sinh(sqrt(x))^2 - 1)
```

giac [B] time = 0.13, size = 17, normalized size = 2.12

$$\frac{4}{e^{\left(-\sqrt{x}\right)} - e^{\sqrt{x}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x^(1/2))*csch(x^(1/2))/x^(1/2),x, algorithm="giac")
```

```
[Out] 4/(e^(-sqrt(x)) - e^sqrt(x))
```

maple [A] time = 0.16, size = 7, normalized size = 0.88

$$-2\operatorname{csch}\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x^(1/2))*csch(x^(1/2))/x^(1/2),x)
```

```
[Out] -2*csch(x^(1/2))
```

maxima [B] time = 0.31, size = 17, normalized size = 2.12

$$\frac{4}{e^{\left(-\sqrt{x}\right)} - e^{\sqrt{x}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x^(1/2))*csch(x^(1/2))/x^(1/2),x, algorithm="maxima")
```

```
[Out] 4/(e^(-sqrt(x)) - e^sqrt(x))
```

mupad [B] time = 1.75, size = 8, normalized size = 1.00

$$-\frac{2}{\sinh\left(\sqrt{x}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x^(1/2))/(x^(1/2)*sinh(x^(1/2))),x)
```

```
[Out] -2/sinh(x^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\coth\left(\sqrt{x}\right)\operatorname{csch}\left(\sqrt{x}\right)}{\sqrt{x}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x**(1/2))*csch(x**(1/2))/x**(1/2),x)
```

```
[Out] Integral(coth(sqrt(x))*csch(sqrt(x))/sqrt(x), x)
```

3.1031 $\int \frac{\cosh(\sqrt{x})\sinh(\sqrt{x})}{\sqrt{x}}\,dx$

Optimal. Leaf size=8

$$\sinh^2\left(\sqrt{x}\right)$$

```
[Out] sinh(x^(1/2))^2
```

Rubi [A] time = 0.01, antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {5370}

$$\sinh^2\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(Cosh[Sqrt[x]]*Sinh[Sqrt[x]])/Sqrt[x],x]
```

```
[Out] Sinh[Sqrt[x]]^2
```

Rule 5370

```
Int[Cosh[(a_.) + (b_.)*(x_)^(n_.)]*(x_)^(m_.)*Sinh[(a_.) + (b_.)*(x_)^(n_)]
^(p_.), x_Symbol] :> Simp[Sinh[a + b*x^n]^(p + 1)/(b*n*(p + 1)), x] /; Free
Q[{a, b, m, n, p}, x] && EqQ[m, n - 1] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{\cosh\left(\sqrt{x}\right)\sinh\left(\sqrt{x}\right)}{\sqrt{x}}\,dx = \sinh^2\left(\sqrt{x}\right)$$

Mathematica [A] time = 0.00, size = 12, normalized size = 1.50

$$\frac{1}{2}\cosh\left(2\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cosh[Sqrt[x]]*Sinh[Sqrt[x]])/Sqrt[x],x]
```

```
[Out] Cosh[2*Sqrt[x]]/2
```

fricas [B] time = 0.46, size = 17, normalized size = 2.12

$$\frac{1}{2}\,\cosh\left(\sqrt{x}\right)^2 + \frac{1}{2}\,\sinh\left(\sqrt{x}\right)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x^(1/2))*sinh(x^(1/2))/x^(1/2),x, algorithm="fricas")
[Out] 1/2*cosh(sqrt(x))^2 + 1/2*sinh(sqrt(x))^2
```

giac [B] time = 0.12, size = 17, normalized size = 2.12

$$\frac{1}{4}\,e^{\left(2\,\sqrt{x}\right)}+\frac{1}{4}\,e^{\left(-2\,\sqrt{x}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x^(1/2))*sinh(x^(1/2))/x^(1/2),x, algorithm="giac")
[Out] 1/4*e^(2*sqrt(x)) + 1/4*e^(-2*sqrt(x))
```

maple [A] time = 0.09, size = 7, normalized size = 0.88

$$\cosh^{2}\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x^(1/2))*sinh(x^(1/2))/x^(1/2),x)
[Out] cosh(x^(1/2))^2
```

maxima [A] time = 0.31, size = 6, normalized size = 0.75

$$\cosh\left(\sqrt{x}\right)^{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x^(1/2))*sinh(x^(1/2))/x^(1/2),x, algorithm="maxima")
[Out] cosh(sqrt(x))^2
```

mupad [B] time = 1.78, size = 6, normalized size = 0.75

$$\cosh\left(\sqrt{x}\right)^{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cosh(x^(1/2))*sinh(x^(1/2)))/x^(1/2),x)
[Out] cosh(x^(1/2))^2
```

sympy [A] time = 0.27, size = 7, normalized size = 0.88

$$\sinh^2\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x**(1/2))*sinh(x**(1/2))/x**(1/2),x)
```

```
[Out] sinh(sqrt(x))**2
```

3.1032 $\int \frac{\operatorname{sech}(\sqrt{x})\tanh(\sqrt{x})}{\sqrt{x}}\,dx$

Optimal. Leaf size=8

$$-2\operatorname{sech}\left(\sqrt{x}\right)$$

[Out] -2*sech(x^(1/2))

Rubi [A] time = 0.19, antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6715, 2606, 8}

$$-2\operatorname{sech}\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

[In] Int[(Sech[Sqrt[x]]*Tanh[Sqrt[x]])/Sqrt[x],x]

[Out] -2*Sech[Sqrt[x]]

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 && !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 6715

```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Dist[1/(m + 1), Subst[Int[SubstFor[x^(m +
 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && FunctionO
fQ[x^(m + 1), u, x]
```

Rubi steps

$$\int \frac{\operatorname{sech}\left(\sqrt{x}\right)\tanh\left(\sqrt{x}\right)}{\sqrt{x}}\,dx = 2\,\text{Subst}\left(\int \operatorname{sech}(x)\tanh(x)\,dx, x, \sqrt{x}\right)$$
$$= -\left(2\,\text{Subst}\left(\int 1\,dx, x, \operatorname{sech}\left(\sqrt{x}\right)\right)\right)$$
$$= -2\operatorname{sech}\left(\sqrt{x}\right)$$

Mathematica [A] time = 0.02, size = 8, normalized size = 1.00

$$-2\operatorname{sech}\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sech[Sqrt[x]]*Tanh[Sqrt[x]])/Sqrt[x],x]
```

```
[Out] -2*Sech[Sqrt[x]]
```

fricas [B] time = 0.42, size = 37, normalized size = 4.62

$$-\frac{4\left(\cosh\left(\sqrt{x}\right)+\sinh\left(\sqrt{x}\right)\right)}{\cosh\left(\sqrt{x}\right)^{2}+2\,\cosh\left(\sqrt{x}\right)\sinh\left(\sqrt{x}\right)+\sinh\left(\sqrt{x}\right)^{2}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x^(1/2))*tanh(x^(1/2))/x^(1/2),x, algorithm="fricas")
```

```
[Out] -4*(cosh(sqrt(x)) + sinh(sqrt(x)))/(cosh(sqrt(x))^2 + 2*cosh(sqrt(x))*sinh(
sqrt(x)) + sinh(sqrt(x))^2 + 1)
```

giac [B] time = 0.12, size = 15, normalized size = 1.88

$$-\frac{4}{e^{\left(-\sqrt{x}\right)}+e^{\sqrt{x}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x^(1/2))*tanh(x^(1/2))/x^(1/2),x, algorithm="giac")
```

```
[Out] -4/(e^(-sqrt(x)) + e^sqrt(x))
```

maple [A] time = 0.13, size = 7, normalized size = 0.88

$$-2\operatorname{sech}\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sech(x^(1/2))*tanh(x^(1/2))/x^(1/2),x)

[Out] -2*sech(x^(1/2))

maxima [B] time = 0.30, size = 15, normalized size = 1.88

$$-\frac{4}{e^{\left(-\sqrt{x}\right)}+e^{\sqrt{x}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(x^(1/2))*tanh(x^(1/2))/x^(1/2),x, algorithm="maxima")

[Out] -4/(e^(-sqrt(x)) + e^sqrt(x))

mupad [B] time = 1.73, size = 8, normalized size = 1.00

$$-\frac{2}{\cosh\left(\sqrt{x}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tanh(x^(1/2))/(x^(1/2)*cosh(x^(1/2))),x)

[Out] -2/cosh(x^(1/2))

sympy [A] time = 0.32, size = 8, normalized size = 1.00

$$-2\,\mathrm{sech}\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(x**(1/2))*tanh(x**(1/2))/x**(1/2),x)

[Out] -2*sech(sqrt(x))

3.1033 $\int \frac{\sinh^2(x)}{a+b\sinh(2x)}\,dx$

Optimal. Leaf size=52

$$\frac{\tanh^{-1}\left(\frac{b-a\tanh(x)}{\sqrt{a^2+b^2}}\right)}{2\sqrt{a^2+b^2}}+\frac{\log(a+b\sinh(2x))}{4b}$$

```
[Out] 1/4*ln(a+b*sinh(2*x))/b+1/2*arctanh((b-a*tanh(x))/(a^2+b^2)^(1/2))/(a^2+b^2
)^(1/2)
```

Rubi [A] time = 0.16, antiderivative size = 68, normalized size of antiderivative = 1.31, number of steps used = 9, number of rules used = 7, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.467, Rules used = {1075, 12, 634, 618, 206, 628, 260}

$$\frac{\tanh^{-1}\left(\frac{b-a\tanh(x)}{\sqrt{a^2+b^2}}\right)}{2\sqrt{a^2+b^2}}+\frac{\log\left(-a\tanh^2(x)+a+2b\tanh(x)\right)}{4b}+\frac{\log(\cosh(x))}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[x]^2/(a + b*Sinh[2*x]),x]
```

```
[Out] ArcTanh[(b - a*Tanh[x])/Sqrt[a^2 + b^2]]/(2*Sqrt[a^2 + b^2]) + Log[Cosh[x]]
/(2*b) + Log[a + 2*b*Tanh[x] - a*Tanh[x]^2]/(4*b)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1075

```
Int[((A_.) + (C_.)*(x_)^2)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*((d_) + (f_.
)*(x_)^2)), x_Symbol] :> With[{q = c^2*d^2 + b^2*d*f - 2*a*c*d*f + a^2*f^2}
, Dist[1/q, Int[(A*c^2*d - a*c*C*d + A*b^2*f - a*A*c*f + a^2*C*f + c*(-(b*C
*d) + A*b*f)*x)/(a + b*x + c*x^2), x], x] + Dist[1/q, Int[(c*C*d^2 - A*c*d*
f - a*C*d*f + a*A*f^2 - f*(-(b*C*d) + A*b*f)*x)/(d + f*x^2), x], x] /; NeQ[
q, 0]] /; FreeQ[{a, b, c, d, f, A, C}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{\sinh^2(x)}{a+b\sinh(2x)}\,dx = -\text{Subst}\left(\int \frac{x^2}{\left(-1+x^2\right)\left(a+2bx-ax^2\right)}\,dx, x, \tanh(x)\right)$$

$$= \frac{\text{Subst}\left(\int -\frac{2bx}{-1+x^2}\,dx, x, \tanh(x)\right)}{4b^2} + \frac{\text{Subst}\left(\int -\frac{2abx}{a+2bx-ax^2}\,dx, x, \tanh(x)\right)}{4b^2}$$

$$= -\frac{\text{Subst}\left(\int \frac{x}{-1+x^2}\,dx, x, \tanh(x)\right)}{2b} - \frac{a\,\text{Subst}\left(\int \frac{x}{a+2bx-ax^2}\,dx, x, \tanh(x)\right)}{2b}$$

$$= \frac{\log(\cosh(x))}{2b} - \frac{1}{2}\text{Subst}\left(\int \frac{1}{a+2bx-ax^2}\,dx, x, \tanh(x)\right) + \frac{\text{Subst}\left(\int \frac{2b-2ax}{a+2bx-ax^2}\,dx, x, \text{tar}\right.}{4b}$$

$$= \frac{\log(\cosh(x))}{2b} + \frac{\log\left(a+2b\tanh(x)-a\tanh^2(x)\right)}{4b} + \text{Subst}\left(\int \frac{1}{4\left(a^2+b^2\right)-x^2}\,dx, x, 2b\right.$$

$$= \frac{\tanh^{-1}\left(\frac{2b-2a\tanh(x)}{2\sqrt{a^2+b^2}}\right)}{2\sqrt{a^2+b^2}} + \frac{\log(\cosh(x))}{2b} + \frac{\log\left(a+2b\tanh(x)-a\tanh^2(x)\right)}{4b}$$

Mathematica [A] time = 0.09, size = 59, normalized size = 1.13

$$\frac{1}{4}\left(\frac{\log(a+b\sinh(2x))}{b} - \frac{2\tan^{-1}\left(\frac{b-a\tanh(x)}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-a^2-b^2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[x]^2/(a + b*Sinh[2*x]),x]
```

```
[Out] ((-2*ArcTan[(b - a*Tanh[x])/Sqrt[-a^2 - b^2]])/Sqrt[-a^2 - b^2] + Log[a + b
*Sinh[2*x]]/b)/4
```

fricas [B] time = 0.48, size = 251, normalized size = 4.83

$$\frac{\sqrt{a^2+b^2}\,b\log\left(\frac{b^2\cosh(x)^4+4\,b^2\cosh(x)\sinh(x)^3+b^2\sinh(x)^4+2\,ab\cosh(x)^2+2\left(3\,b^2\cosh(x)^2+ab\right)\sinh(x)^2+2\,a^2+b^2+4\left(b^2\cosh(x)^3+ab\cosh(x\right.}{b\cosh(x)^4+4\,b\cosh(x)\sinh(x)^3+b\sinh(x)^4+2\,a\cosh(x)^2+2\left(3\,b\cosh(x)^2+a\right)\sinh(x)^2+4\left(b\right.}\right.}{4\left(a^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^2/(a+b*sinh(2*x)),x, algorithm="fricas")
```

```
[Out] 1/4*(sqrt(a^2 + b^2)*b*log((b^2*cosh(x)^4 + 4*b^2*cosh(x)*sinh(x)^3 + b^2*s
inh(x)^4 + 2*a*b*cosh(x)^2 + 2*(3*b^2*cosh(x)^2 + a*b)*sinh(x)^2 + 2*a^2 +
b^2 + 4*(b^2*cosh(x)^3 + a*b*cosh(x))*sinh(x) + 2*(b*cosh(x)^2 + 2*b*cosh(x
)*sinh(x) + b*sinh(x)^2 + a)*sqrt(a^2 + b^2))/(b*cosh(x)^4 + 4*b*cosh(x)*si
nh(x)^3 + b*sinh(x)^4 + 2*a*cosh(x)^2 + 2*(3*b*cosh(x)^2 + a)*sinh(x)^2 + 4
*(b*cosh(x)^3 + a*cosh(x))*sinh(x) - b)) - 2*(a^2 + b^2)*x + (a^2 + b^2)*lo
g(2*(2*b*cosh(x)*sinh(x) + a)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))
/(a^2*b + b^3)
```

giac [A] time = 0.15, size = 92, normalized size = 1.77

$$-\frac{\log\left(\frac{\left|2\,be^{(2\,x)}+2\,a-2\,\sqrt{a^2+b^2}\right|}{\left|2\,be^{(2\,x)}+2\,a+2\,\sqrt{a^2+b^2}\right|}\right)}{4\,\sqrt{a^2+b^2}}-\frac{x}{2\,b}+\frac{\log\left(\left|be^{(4\,x)}+2\,ae^{(2\,x)}-b\right|\right)}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^2/(a+b*sinh(2*x)),x, algorithm="giac")
```

```
[Out] -1/4*log(abs(2*b*e^(2*x) + 2*a - 2*sqrt(a^2 + b^2))/abs(2*b*e^(2*x) + 2*a +
 2*sqrt(a^2 + b^2)))/sqrt(a^2 + b^2) - 1/2*x/b + 1/4*log(abs(b*e^(4*x) + 2*
a*e^(2*x) - b))/b
```

maple [A] time = 0.39, size = 75, normalized size = 1.44

$$-\frac{\ln\left(\tanh(x)-1\right)}{4b}-\frac{\ln\left(1+\tanh(x)\right)}{4b}+\frac{\ln\left(a\left(\tanh^2(x)\right)-2b\tanh(x)-a\right)}{4b}-\frac{\operatorname{arctanh}\left(\frac{2a\tanh(x)-2b}{2\sqrt{a^2+b^2}}\right)}{2\sqrt{a^2+b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)^2/(a+b*sinh(2*x)),x)
```

```
[Out] -1/4/b*ln(tanh(x)-1)-1/4/b*ln(1+tanh(x))+1/4/b*ln(a*tanh(x)^2-2*b*tanh(x)-a
)-1/2/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tanh(x)-2*b)/(a^2+b^2)^(1/2))
```

maxima [A] time = 0.42, size = 85, normalized size = 1.63

$$-\frac{\log\left(\frac{be^{(-2\,x)}-a-\sqrt{a^2+b^2}}{be^{(-2\,x)}-a+\sqrt{a^2+b^2}}\right)}{4\,\sqrt{a^2+b^2}}-\frac{x}{2\,b}+\frac{\log\left(be^{(4\,x)}+2\,ae^{(2\,x)}-b\right)}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^2/(a+b*sinh(2*x)),x, algorithm="maxima")
```

```
[Out] -1/4*log((b*e^(-2*x) - a - sqrt(a^2 + b^2))/(b*e^(-2*x) - a + sqrt(a^2 + b^
2)))/sqrt(a^2 + b^2) - 1/2*x/b + 1/4*log(b*e^(4*x) + 2*a*e^(2*x) - b)/b
```

mupad [B] time = 0.49, size = 273, normalized size = 5.25

$$\frac{\operatorname{atan}\left(\frac{a^7}{\left(-a^2-b^2\right)^{7/2}}+\frac{b^7\,\mathrm{e}^{2\,x}}{\left(-a^2-b^2\right)^{7/2}}+\frac{a\,b^6}{\left(-a^2-b^2\right)^{7/2}}+\frac{3\,a^3\,b^4}{\left(-a^2-b^2\right)^{7/2}}+\frac{3\,a^5\,b^2}{\left(-a^2-b^2\right)^{7/2}}+\frac{3\,a^2\,b^5\,\mathrm{e}^{2\,x}}{\left(-a^2-b^2\right)^{7/2}}+\frac{3\,a^4\,b^3\,\mathrm{e}^{2\,x}}{\left(-a^2-b^2\right)^{7/2}}+\frac{a^6\,b\,\mathrm{e}^{2\,x}}{\left(-a^2-b^2\right)^{7/2}}\right)}{2\,\sqrt{-a^2-b^2}}-\frac{x}{2\,b}+\frac{4\,b}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)^2/(a + b*sinh(2*x)),x)
```

```
[Out] atan(a^7/(- a^2 - b^2)^(7/2) + (b^7*exp(2*x))/(- a^2 - b^2)^(7/2) + (a*b^6)
/(- a^2 - b^2)^(7/2) + (3*a^3*b^4)/(- a^2 - b^2)^(7/2) + (3*a^5*b^2)/(- a^2
 - b^2)^(7/2) + (3*a^2*b^5*exp(2*x))/(- a^2 - b^2)^(7/2) + (3*a^4*b^3*exp(2
*x))/(- a^2 - b^2)^(7/2) + (a^6*b*exp(2*x))/(- a^2 - b^2)^(7/2))/(2*(- a^2
- b^2)^(1/2)) - x/(2*b) + (4*b^3*log(2*a*exp(2*x) - b + b*exp(4*x)))/(16*b^
4 + 16*a^2*b^2) + (4*a^2*b*log(2*a*exp(2*x) - b + b*exp(4*x)))/(16*b^4 + 16
*a^2*b^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh^2(x)}{a + b\sinh(2x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)**2/(a+b*sinh(2*x)),x)
```

```
[Out] Integral(sinh(x)**2/(a + b*sinh(2*x)), x)
```

3.1034 $\int \frac{\cosh^2(x)}{a+b\sinh(2x)}\,dx$

Optimal. Leaf size=52

$$\frac{\log(a+b\sinh(2x))}{4b}-\frac{\tanh^{-1}\left(\frac{b-a\tanh(x)}{\sqrt{a^2+b^2}}\right)}{2\sqrt{a^2+b^2}}$$

```
[Out] 1/4*ln(a+b*sinh(2*x))/b-1/2*arctanh((b-a*tanh(x))/(a^2+b^2)^(1/2))/(a^2+b^2
)^(1/2)
```

Rubi [A] time = 0.13, antiderivative size = 68, normalized size of antiderivative = 1.31, number of steps used = 8, number of rules used = 7, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.467, Rules used = {981, 634, 618, 206, 628, 12, 260}

$$-\frac{\tanh^{-1}\left(\frac{b-a\tanh(x)}{\sqrt{a^2+b^2}}\right)}{2\sqrt{a^2+b^2}}+\frac{\log\left(-a\tanh^2(x)+a+2b\tanh(x)\right)}{4b}+\frac{\log(\cosh(x))}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]^2/(a + b*Sinh[2*x]),x]
```

```
[Out] -ArcTanh[(b - a*Tanh[x])/Sqrt[a^2 + b^2]]/(2*Sqrt[a^2 + b^2]) + Log[Cosh[x]
]/(2*b) + Log[a + 2*b*Tanh[x] - a*Tanh[x]^2]/(4*b)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 981

```
Int[1/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*((d_) + (f_.)*(x_)^2)), x_Symbol]
 :> With[{q = c^2*d^2 + b^2*d*f - 2*a*c*d*f + a^2*f^2}, Dist[1/q, Int[(c^2*
d + b^2*f - a*c*f + b*c*f*x)/(a + b*x + c*x^2), x], x] - Dist[1/q, Int[(c*d
*f - a*f^2 + b*f^2*x)/(d + f*x^2), x], x] /; NeQ[q, 0]] /; FreeQ[{a, b, c,
d, f}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cosh^2(x)}{a+b\sinh(2x)}\,dx &= \text{Subst}\left(\int \frac{1}{\left(1-x^2\right)\left(a+2bx-ax^2\right)}\,dx, x, \tanh(x)\right)\\
&= \frac{\text{Subst}\left(\int \frac{2bx}{1-x^2}\,dx, x, \tanh(x)\right)}{4b^2} - \frac{\text{Subst}\left(\int \frac{-4b^2+2abx}{a+2bx-ax^2}\,dx, x, \tanh(x)\right)}{4b^2}\\
&= \frac{1}{2}\text{Subst}\left(\int \frac{1}{a+2bx-ax^2}\,dx, x, \tanh(x)\right) + \frac{\text{Subst}\left(\int \frac{2b-2ax}{a+2bx-ax^2}\,dx, x, \tanh(x)\right)}{4b} + \frac{\text{Subst}\left(\right.}{}\\
&= \frac{\log(\cosh(x))}{2b} + \frac{\log\left(a+2b\tanh(x)-a\tanh^2(x)\right)}{4b} - \text{Subst}\left(\int \frac{1}{4\left(a^2+b^2\right)-x^2}\,dx, x, 2b\right.\\
&= -\frac{\tanh^{-1}\left(\frac{2b-2a\tanh(x)}{2\sqrt{a^2+b^2}}\right)}{2\sqrt{a^2+b^2}} + \frac{\log(\cosh(x))}{2b} + \frac{\log\left(a+2b\tanh(x)-a\tanh^2(x)\right)}{4b}
\end{aligned}$$

Mathematica [A] time = 0.06, size = 59, normalized size = 1.13

$$\frac{1}{4}\left(\frac{2\tan^{-1}\left(\frac{b-a\tanh(x)}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-a^2-b^2}}+\frac{\log(a+b\sinh(2x))}{b}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]^2/(a + b*Sinh[2*x]),x]
```

```
[Out] ((2*ArcTan[(b - a*Tanh[x])/Sqrt[-a^2 - b^2]])/Sqrt[-a^2 - b^2] + Log[a + b*
Sinh[2*x]]/b)/4
```

fricas [B] time = 0.49, size = 251, normalized size = 4.83

$$\frac{\sqrt{a^2+b^2}\,b\log\left(\frac{b^2\cosh(x)^4+4\,b^2\cosh(x)\sinh(x)^3+b^2\sinh(x)^4+2\,ab\cosh(x)^2+2\left(3\,b^2\cosh(x)^2+ab\right)\sinh(x)^2+2\,a^2+b^2+4\left(b^2\cosh(x)^3+ab\cosh\right.}{b\cosh(x)^4+4\,b\cosh(x)\sinh(x)^3+b\sinh(x)^4+2\,a\cosh(x)^2+2\left(3\,b\cosh(x)^2+a\right)\sinh(x)^2+4\left(\right.}\right.}{4\left(a\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^2/(a+b*sinh(2*x)),x, algorithm="fricas")
```

```
[Out] 1/4*(sqrt(a^2 + b^2)*b*log((b^2*cosh(x)^4 + 4*b^2*cosh(x)*sinh(x)^3 + b^2*s
inh(x)^4 + 2*a*b*cosh(x)^2 + 2*(3*b^2*cosh(x)^2 + a*b)*sinh(x)^2 + 2*a^2 +
b^2 + 4*(b^2*cosh(x)^3 + a*b*cosh(x))*sinh(x) - 2*(b*cosh(x)^2 + 2*b*cosh(x
)*sinh(x) + b*sinh(x)^2 + a)*sqrt(a^2 + b^2))/(b*cosh(x)^4 + 4*b*cosh(x)*si
nh(x)^3 + b*sinh(x)^4 + 2*a*cosh(x)^2 + 2*(3*b*cosh(x)^2 + a)*sinh(x)^2 + 4
*(b*cosh(x)^3 + a*cosh(x))*sinh(x) - b)) - 2*(a^2 + b^2)*x + (a^2 + b^2)*lo
g(2*(2*b*cosh(x)*sinh(x) + a)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))
/(a^2*b + b^3)
```

giac [A] time = 0.14, size = 92, normalized size = 1.77

$$\frac{\log\left(\frac{\left|2\,be^{(2\,x)}+2\,a-2\,\sqrt{a^2+b^2}\right|}{\left|2\,be^{(2\,x)}+2\,a+2\,\sqrt{a^2+b^2}\right|}\right)}{4\,\sqrt{a^2+b^2}}-\frac{x}{2\,b}+\frac{\log\left(\left|be^{(4\,x)}+2\,ae^{(2\,x)}-b\right|\right)}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^2/(a+b*sinh(2*x)),x, algorithm="giac")
```

```
[Out] 1/4*log(abs(2*b*e^(2*x) + 2*a - 2*sqrt(a^2 + b^2))/abs(2*b*e^(2*x) + 2*a +
2*sqrt(a^2 + b^2)))/sqrt(a^2 + b^2) - 1/2*x/b + 1/4*log(abs(b*e^(4*x) + 2*a
*e^(2*x) - b))/b
```

maple [A] time = 0.34, size = 75, normalized size = 1.44

$$-\frac{\ln \left(\tanh (x)-1\right)}{4b}-\frac{\ln \left(1+\tanh (x)\right)}{4b}+\frac{\ln \left(a\left(\tanh ^{2}(x)\right)-2b\tanh (x)-a\right)}{4b}+\frac{\operatorname{arctanh}\left(\frac{2a\tanh (x)-2b}{2\sqrt{a^{2}+b^{2}}}\right)}{2\sqrt{a^{2}+b^{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^2/(a+b*sinh(2*x)),x)
```

```
[Out] -1/4/b*ln(tanh(x)-1)-1/4/b*ln(1+tanh(x))+1/4/b*ln(a*tanh(x)^2-2*b*tanh(x)-a
)+1/2/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tanh(x)-2*b)/(a^2+b^2)^(1/2))
```

maxima [A] time = 0.41, size = 85, normalized size = 1.63

$$\frac{\log \left(\frac{be^{(-2x)}-a-\sqrt{a^{2}+b^{2}}}{be^{(-2x)}-a+\sqrt{a^{2}+b^{2}}}\right)}{4\sqrt{a^{2}+b^{2}}}-\frac{x}{2b}+\frac{\log \left(be^{(4x)}+2ae^{(2x)}-b\right)}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^2/(a+b*sinh(2*x)),x, algorithm="maxima")
```

```
[Out] 1/4*log((b*e^(-2*x) - a - sqrt(a^2 + b^2))/(b*e^(-2*x) - a + sqrt(a^2 + b^2
)))/sqrt(a^2 + b^2) - 1/2*x/b + 1/4*log(b*e^(4*x) + 2*a*e^(2*x) - b)/b
```

mupad [B] time = 0.28, size = 137, normalized size = 2.63

$$\frac{\operatorname{atan}\left(\frac{a}{\sqrt{-a^{2}-b^{2}}}+\frac{b\,e^{2x}}{\sqrt{-a^{2}-b^{2}}}\right)}{2\sqrt{-a^{2}-b^{2}}}-\frac{x}{2b}+\frac{4b^{3}\,\ln \left(2\,a\,e^{2x}-b+b\,e^{4x}\right)}{16\,a^{2}\,b^{2}+16\,b^{4}}+\frac{4\,a^{2}\,b\,\ln \left(2\,a\,e^{2x}-b+b\,e^{4x}\right)}{16\,a^{2}\,b^{2}+16\,b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^2/(a + b*sinh(2*x)),x)
```

```
[Out] atan(a/(- a^2 - b^2)^(1/2) + (b*exp(2*x))/(- a^2 - b^2)^(1/2))/(2*(- a^2 -
b^2)^(1/2)) - x/(2*b) + (4*b^3*log(2*a*exp(2*x) - b + b*exp(4*x)))/(16*b^4
+ 16*a^2*b^2) + (4*a^2*b*log(2*a*exp(2*x) - b + b*exp(4*x)))/(16*b^4 + 16*a
^2*b^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh ^{2}(x)}{a+b\sinh (2x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)**2/(a+b*sinh(2*x)),x)
[Out] Integral(cosh(x)**2/(a + b*sinh(2*x)), x)
```

3.1035 $\int \frac{\sinh^2(x)}{a+b\cosh(2x)}\,dx$

Optimal. Leaf size=52

$$\frac{x}{2b}-\frac{\sqrt{a+b}\,\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tanh(x)}{\sqrt{a+b}}\right)}{2b\sqrt{a-b}}$$

```
[Out] 1/2*x/b-1/2*arctanh((a-b)^(1/2)*tanh(x)/(a+b)^(1/2))*(a+b)^(1/2)/b/(a-b)^(1
/2)
```

Rubi [A] time = 0.13, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {1130, 208}

$$\frac{x}{2b}-\frac{\sqrt{a+b}\,\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tanh(x)}{\sqrt{a+b}}\right)}{2b\sqrt{a-b}}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[x]^2/(a + b*Cosh[2*x]),x]
```

```
[Out] x/(2*b) - (Sqrt[a + b]*ArcTanh[(Sqrt[a - b]*Tanh[x])/Sqrt[a + b]])/(2*Sqrt[
a - b]*b)
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1130

```
Int[((d_.)*(x_))^(m_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Wi
th[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(d^2*(b/q + 1))/2, Int[(d*x)^(m - 2)/(b/2
 + q/2 + c*x^2), x], x] - Dist[(d^2*(b/q - 1))/2, Int[(d*x)^(m - 2)/(b/2 -
q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && G
eQ[m, 2]
```

Rubi steps

$$\int \frac{\sinh^2(x)}{a+b\cosh(2x)}\,dx = -\operatorname{Subst}\left(\int \frac{x^2}{-a-b+2ax^2+(-a+b)x^4}\,dx, x, \tanh(x)\right)$$
$$= \frac{1}{2}\left(-1+\frac{a}{b}\right)\operatorname{Subst}\left(\int \frac{1}{a-b+(-a+b)x^2}\,dx, x, \tanh(x)\right) - \frac{(a+b)\operatorname{Subst}\left(\int \frac{1}{a+b+(-a+b)x^2}\right.}{2b}$$
$$= \frac{x}{2b} - \frac{\sqrt{a+b}\,\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tanh(x)}{\sqrt{a+b}}\right)}{2\sqrt{a-b}\,b}$$

Mathematica [A] time = 0.08, size = 48, normalized size = 0.92

$$\frac{\frac{(a+b)\tan^{-1}\left(\frac{(a-b)\tanh(x)}{\sqrt{b^2-a^2}}\right)}{\sqrt{b^2-a^2}}+x}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[x]^2/(a + b*Cosh[2*x]),x]
```

```
[Out] (x + ((a + b)*ArcTan[((a - b)*Tanh[x])/Sqrt[-a^2 + b^2]])/Sqrt[-a^2 + b^2])
/(2*b)
```

fricas [A] time = 0.45, size = 303, normalized size = 5.83

$$\left[\frac{\sqrt{\frac{a+b}{a-b}}\log\left(\frac{b^2\cosh(x)^4+4b^2\cosh(x)\sinh(x)^3+b^2\sinh(x)^4+2ab\cosh(x)^2+2\left(3b^2\cosh(x)^2+ab\right)\sinh(x)^2+2a^2-b^2+4\left(b^2\cosh(x)^3+ab\cosh(x)\right)\mathrm{s}}{b\cosh(x)^4+4b\cosh(x)\sinh(x)^3+b\sinh(x)^4+2a\cosh(x)^2+2\left(3b\cosh(x)^2+a\right)\sinh(}\right.}{4b}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^2/(a+b*cosh(2*x)),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt((a + b)/(a - b))*log((b^2*cosh(x)^4 + 4*b^2*cosh(x)*sinh(x)^3 +
b^2*sinh(x)^4 + 2*a*b*cosh(x)^2 + 2*(3*b^2*cosh(x)^2 + a*b)*sinh(x)^2 + 2*a
^2 - b^2 + 4*(b^2*cosh(x)^3 + a*b*cosh(x))*sinh(x) + 2*((a*b - b^2)*cosh(x)
^2 + 2*(a*b - b^2)*cosh(x)*sinh(x) + (a*b - b^2)*sinh(x)^2 + a^2 - a*b)*sqr
t((a + b)/(a - b)))/(b*cosh(x)^4 + 4*b*cosh(x)*sinh(x)^3 + b*sinh(x)^4 + 2*
a*cosh(x)^2 + 2*(3*b*cosh(x)^2 + a)*sinh(x)^2 + 4*(b*cosh(x)^3 + a*cosh(x))
*sinh(x) + b)) + 2*x)/b, -1/2*(sqrt(-(a + b)/(a - b))*arctan((b*cosh(x)^2 +
 2*b*cosh(x)*sinh(x) + b*sinh(x)^2 + a)*sqrt(-(a + b)/(a - b))/(a + b)) - x
)/b]
```

giac [A] time = 0.14, size = 47, normalized size = 0.90

$$-\frac{(a+b)\arctan\left(\frac{be^{(2x)}+a}{\sqrt{-a^2+b^2}}\right)}{2\sqrt{-a^2+b^2}\,b}+\frac{x}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^2/(a+b*cosh(2*x)),x, algorithm="giac")
```

```
[Out] -1/2*(a + b)*arctan((b*e^(2*x) + a)/sqrt(-a^2 + b^2))/(sqrt(-a^2 + b^2)*b)
+ 1/2*x/b
```

maple [B] time = 0.28, size = 92, normalized size = 1.77

$$-\frac{\ln(\tanh(x)-1)}{4b}+\frac{\ln(1+\tanh(x))}{4b}-\frac{\operatorname{arctanh}\left(\frac{(a-b)\tanh(x)}{\sqrt{(a+b)(a-b)}}\right)a}{2b\sqrt{(a+b)(a-b)}}-\frac{\operatorname{arctanh}\left(\frac{(a-b)\tanh(x)}{\sqrt{(a+b)(a-b)}}\right)}{2\sqrt{(a+b)(a-b)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)^2/(a+b*cosh(2*x)),x)
```

```
[Out] -1/4/b*ln(tanh(x)-1)+1/4/b*ln(1+tanh(x))-1/2/b/((a+b)*(a-b))^(1/2)*arctanh(
(a-b)*tanh(x)/((a+b)*(a-b))^(1/2))*a-1/2/((a+b)*(a-b))^(1/2)*arctanh((a-b)*
tanh(x)/((a+b)*(a-b))^(1/2))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^2/(a+b*cosh(2*x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more details)Is 4*a^2-4*b^2 positive or negative?
```

mupad [B] time = 2.15, size = 153, normalized size = 2.94

$$\frac{x}{2b}-\frac{\ln\left(a\,b+2\,a^2\,e^{2\,x}-b^2\,e^{2\,x}+b\,\sqrt{a+b}\,\sqrt{a-b}+2\,a\,e^{2\,x}\,\sqrt{a+b}\,\sqrt{a-b}\right)\,\sqrt{a+b}}{4\,b\,\sqrt{a-b}}+\frac{\ln\left(b^2\,e^{2\,x}-2\,a^2\,e^{2\,x}-a\,b\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)^2/(a + b*cosh(2*x)),x)
```

```
[Out] x/(2*b) - (log(a*b + 2*a^2*exp(2*x) - b^2*exp(2*x) + b*(a + b)^(1/2)*(a - b
)^(1/2) + 2*a*exp(2*x)*(a + b)^(1/2)*(a - b)^(1/2))*(a + b)^(1/2))/(4*b*(a
- b)^(1/2)) + (log(b^2*exp(2*x) - 2*a^2*exp(2*x) - a*b + b*(a + b)^(1/2)*(a
 - b)^(1/2) + 2*a*exp(2*x)*(a + b)^(1/2)*(a - b)^(1/2))*(a + b)^(1/2))/(4*b
*(a - b)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh^2(x)}{a + b\cosh(2x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)**2/(a+b*cosh(2*x)),x)
```

```
[Out] Integral(sinh(x)**2/(a + b*cosh(2*x)), x)
```

3.1036 $\int \frac{\cosh^2(x)}{a+b\cosh(2x)}\,dx$

Optimal. Leaf size=52

$$\frac{x}{2b}-\frac{\sqrt{a-b}\,\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tanh(x)}{\sqrt{a+b}}\right)}{2b\sqrt{a+b}}$$

```
[Out] 1/2*x/b-1/2*arctanh((a-b)^(1/2)*tanh(x)/(a+b)^(1/2))*(a-b)^(1/2)/b/(a+b)^(1
/2)
```

Rubi [A] time = 0.11, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {1093, 208}

$$\frac{x}{2b}-\frac{\sqrt{a-b}\,\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tanh(x)}{\sqrt{a+b}}\right)}{2b\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]^2/(a + b*Cosh[2*x]),x]
```

```
[Out] x/(2*b) - (Sqrt[a - b]*ArcTanh[(Sqrt[a - b]*Tanh[x])/Sqrt[a + b]])/(2*b*Sqr
t[a + b])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1093

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^2), x], x] - Dist[c/q, Int
[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
0] && PosQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{\cosh^2(x)}{a+b\cosh(2x)}\,dx = \text{Subst}\left(\int \frac{1}{a+b-2ax^2+(a-b)x^4}\,dx, x, \tanh(x)\right)$$
$$= \frac{(a-b)\,\text{Subst}\left(\int \frac{1}{-a-b+(a-b)x^2}\,dx, x, \tanh(x)\right)}{2b} - \frac{(a-b)\,\text{Subst}\left(\int \frac{1}{-a+b+(a-b)x^2}\,dx, x, \tanh(\right.}{2b}$$
$$= \frac{x}{2b} - \frac{\sqrt{a-b}\,\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tanh(x)}{\sqrt{a+b}}\right)}{2b\sqrt{a+b}}$$

Mathematica [A] time = 0.06, size = 50, normalized size = 0.96

$$\frac{\frac{(a-b)\tan^{-1}\left(\frac{(a-b)\tanh(x)}{\sqrt{b^2-a^2}}\right)}{\sqrt{b^2-a^2}} + x}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]^2/(a + b*Cosh[2*x]),x]
[Out] (x + ((a - b)*ArcTan[((a - b)*Tanh[x])/Sqrt[-a^2 + b^2]])/Sqrt[-a^2 + b^2])
/(2*b)
```

fricas [A] time = 0.57, size = 297, normalized size = 5.71

$$\left[\frac{\sqrt{\frac{a-b}{a+b}}\,\log\left(\frac{b^2\cosh(x)^4+4\,b^2\cosh(x)\sinh(x)^3+b^2\sinh(x)^4+2\,ab\cosh(x)^2+2\left(3\,b^2\cosh(x)^2+ab\right)\sinh(x)^2+2\,a^2-b^2+4\left(b^2\cosh(x)^3+ab\cosh(x)\right)\text{s}}{b\cosh(x)^4+4\,b\cosh(x)\sinh(x)^3+b\sinh(x)^4+2\,a\cosh(x)^2+2\left(3\,b\cosh(x)^2+a\right)\sinh(}\right.}{4\,b}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^2/(a+b*cosh(2*x)),x, algorithm="fricas")
[Out] [1/4*(sqrt((a - b)/(a + b))*log((b^2*cosh(x)^4 + 4*b^2*cosh(x)*sinh(x)^3 +
b^2*sinh(x)^4 + 2*a*b*cosh(x)^2 + 2*(3*b^2*cosh(x)^2 + a*b)*sinh(x)^2 + 2*a
^2 - b^2 + 4*(b^2*cosh(x)^3 + a*b*cosh(x))*sinh(x) + 2*((a*b + b^2)*cosh(x)
^2 + 2*(a*b + b^2)*cosh(x)*sinh(x) + (a*b + b^2)*sinh(x)^2 + a^2 + a*b)*sqr
t((a - b)/(a + b)))/(b*cosh(x)^4 + 4*b*cosh(x)*sinh(x)^3 + b*sinh(x)^4 + 2*
a*cosh(x)^2 + 2*(3*b*cosh(x)^2 + a)*sinh(x)^2 + 4*(b*cosh(x)^3 + a*cosh(x))
*sinh(x) + b)) + 2*x)/b, 1/2*(sqrt(-(a - b)/(a + b))*arctan(-(b*cosh(x)^2 +
 2*b*cosh(x)*sinh(x) + b*sinh(x)^2 + a)*sqrt(-(a - b)/(a + b))/(a - b)) + x
)/b]
```

giac [A] time = 0.14, size = 49, normalized size = 0.94

$$-\frac{(a-b)\arctan\left(\frac{be^{(2x)}+a}{\sqrt{-a^2+b^2}}\right)}{2\sqrt{-a^2+b^2}\,b}+\frac{x}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^2/(a+b*cosh(2*x)),x, algorithm="giac")
```

```
[Out] -1/2*(a - b)*arctan((b*e^(2*x) + a)/sqrt(-a^2 + b^2))/(sqrt(-a^2 + b^2)*b)
+ 1/2*x/b
```

maple [B] time = 0.26, size = 92, normalized size = 1.77

$$-\frac{\ln(\tanh(x)-1)}{4b}+\frac{\ln(1+\tanh(x))}{4b}-\frac{\operatorname{arctanh}\left(\frac{(a-b)\tanh(x)}{\sqrt{(a+b)(a-b)}}\right)a}{2b\sqrt{(a+b)(a-b)}}+\frac{\operatorname{arctanh}\left(\frac{(a-b)\tanh(x)}{\sqrt{(a+b)(a-b)}}\right)}{2\sqrt{(a+b)(a-b)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^2/(a+b*cosh(2*x)),x)
```

```
[Out] -1/4/b*ln(tanh(x)-1)+1/4/b*ln(1+tanh(x))-1/2/b/((a+b)*(a-b))^(1/2)*arctanh(
(a-b)*tanh(x)/((a+b)*(a-b))^(1/2))*a+1/2/((a+b)*(a-b))^(1/2)*arctanh((a-b)*
tanh(x)/((a+b)*(a-b))^(1/2))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^2/(a+b*cosh(2*x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more details)Is 4*a^2-4*b^2 positive or negative?
```

mupad [B] time = 0.26, size = 120, normalized size = 2.31

$$\frac{x}{2\,b}-\frac{\ln\left(\frac{e^{2\,x}\,(a-b)}{b^2}+\frac{\sqrt{a-b}\,\left(b+a\,e^{2\,x}\right)}{b^2\,\sqrt{a+b}}\right)\sqrt{a-b}}{4\,b\,\sqrt{a+b}}+\frac{\ln\left(\frac{e^{2\,x}\,(a-b)}{b^2}-\frac{\sqrt{a-b}\,\left(b+a\,e^{2\,x}\right)}{b^2\,\sqrt{a+b}}\right)\sqrt{a-b}}{4\,b\,\sqrt{a+b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^2/(a + b*cosh(2*x)),x)
```

```
[Out] x/(2*b) - (log((exp(2*x)*(a - b))/b^2 + ((a - b)^(1/2)*(b + a*exp(2*x)))/(b
^2*(a + b)^(1/2)))*(a - b)^(1/2))/(4*b*(a + b)^(1/2)) + (log((exp(2*x)*(a -
 b))/b^2 - ((a - b)^(1/2)*(b + a*exp(2*x)))/(b^2*(a + b)^(1/2)))*(a - b)^(1
/2))/(4*b*(a + b)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh^2(x)}{a + b\cosh(2x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)**2/(a+b*cosh(2*x)),x)
```

```
[Out] Integral(cosh(x)**2/(a + b*cosh(2*x)), x)
```

3.1037 $\int \frac{\tanh(c+dx)}{\sqrt{a\sinh^2(c+dx)}}\,dx$

Optimal. Leaf size=30

$$\frac{\tan^{-1}\left(\frac{\sqrt{a\sinh^2(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}\,d}$$

[Out] arctan((a*sinh(d*x+c)^2)^(1/2)/a^(1/2))/d/a^(1/2)

Rubi [A] time = 0.04, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3205, 63, 203}

$$\frac{\tan^{-1}\left(\frac{\sqrt{a\sinh^2(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}\,d}$$

Antiderivative was successfully verified.

[In] Int[Tanh[c + d*x]/Sqrt[a*Sinh[c + d*x]^2],x]

[Out] ArcTan[Sqrt[a*Sinh[c + d*x]^2]/Sqrt[a]]/(Sqrt[a]*d)

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3205

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_)]^(m_.
), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m + 1
)/2)/(2*f), Subst[Int[(x^((m - 1)/2)*(b*ff^(n/2)*x^(n/2))^p)/(1 - ff*x)^((m
 + 1)/2), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{b, e, f, p}, x] && Integ
```

```
erQ[(m - 1)/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{\tanh(c+dx)}{\sqrt{a\sinh^2(c+dx)}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{\sqrt{ax}(1+x)}\,dx, x, \sinh^2(c+dx)\right)}{2d}$$
$$= \frac{\text{Subst}\left(\int \frac{1}{1+\frac{x^2}{a}}\,dx, x, \sqrt{a\sinh^2(c+dx)}\right)}{ad}$$
$$= \frac{\tan^{-1}\left(\frac{\sqrt{a\sinh^2(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}\,d}$$

Mathematica [A] time = 0.04, size = 31, normalized size = 1.03

$$\frac{\sinh(c+dx)\tan^{-1}(\sinh(c+dx))}{d\sqrt{a\sinh^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[c + d*x]/Sqrt[a*Sinh[c + d*x]^2],x]
[Out] (ArcTan[Sinh[c + d*x]]*Sinh[c + d*x])/(d*Sqrt[a*Sinh[c + d*x]^2])
```

fricas [B] time = 0.44, size = 335, normalized size = 11.17

$$\left[-\frac{\sqrt{-a}\,\log\left(-\frac{a\cosh(dx+c)^2+2\sqrt{ae^{(4dx+4c)}-2ae^{(2dx+2c)}+a}\left(\cosh(dx+c)e^{(dx+c)}+e^{(dx+c)}\sinh(dx+c)\right)\sqrt{-a}\,e^{(-dx-c)}-\left(ae^{(2dx+2c)}-a\right)\sinh(dx+c)^2-}{\left(e^{(2dx+2c)}-1\right)\sinh(dx+c)^2-\cosh(dx+c)^2+\left(\cosh(dx+c)^2+1\right)e^{(2dx+2c)}+2\left(\cosh(dx+\right.}\right.}{ad}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)/(a*sinh(d*x+c)^2)^(1/2),x, algorithm="fricas")
[Out] [-sqrt(-a)*log(-(a*cosh(d*x + c)^2 + 2*sqrt(a*e^(4*d*x + 4*c) - 2*a*e^(2*d*
x + 2*c) + a)*(cosh(d*x + c)*e^(d*x + c) + e^(d*x + c)*sinh(d*x + c))*sqrt(
-a)*e^(-d*x - c) - (a*e^(2*d*x + 2*c) - a)*sinh(d*x + c)^2 - (a*cosh(d*x +
```

```
c)^2 - a)*e^(2*d*x + 2*c) - 2*(a*cosh(d*x + c)*e^(2*d*x + 2*c) - a*cosh(d*x
 + c))*sinh(d*x + c) - a)/((e^(2*d*x + 2*c) - 1)*sinh(d*x + c)^2 - cosh(d*x
 + c)^2 + (cosh(d*x + c)^2 + 1)*e^(2*d*x + 2*c) + 2*(cosh(d*x + c)*e^(2*d*x
 + 2*c) - cosh(d*x + c))*sinh(d*x + c) - 1))/(a*d), 2*sqrt(a*e^(4*d*x + 4*c
) - 2*a*e^(2*d*x + 2*c) + a)*arctan(cosh(d*x + c) + sinh(d*x + c))/(a*d*e^(
2*d*x + 2*c) - a*d)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)/(a*sinh(d*x+c)^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(exp
(c)^3*t_nostep^3-exp(c)*t_nostep)]index.cc index_m operator + Error: Bad Ar
gument Value
```

maple [C] time = 0.36, size = 39, normalized size = 1.30

$$\frac{\text{`}\mathit{int/indef0}\text{`}\left(\frac{\sinh(dx+c)}{\cosh(dx+c)^{2}\sqrt{a\left(\sinh^{2}(dx+c)\right)}}, \sinh\left(dx+c\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(d*x+c)/(a*sinh(d*x+c)^2)^(1/2),x)
```

```
[Out] `int/indef0`(sinh(d*x+c)/cosh(d*x+c)^2/(a*sinh(d*x+c)^2)^(1/2),sinh(d*x+c))
/d
```

maxima [A] time = 0.44, size = 18, normalized size = 0.60

$$\frac{2\arctan\left(e^{(-dx-c)}\right)}{\sqrt{a}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)/(a*sinh(d*x+c)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 2*arctan(e^(-d*x - c))/(sqrt(a)*d)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\tanh\left(c+d\,x\right)}{\sqrt{a\,{\mathrm{sinh}\left(c+d\,x\right)}^2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(c + d*x)/(a*sinh(c + d*x)^2)^(1/2),x)
[Out] int(tanh(c + d*x)/(a*sinh(c + d*x)^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh\left(c + dx\right)}{\sqrt{a \sinh^{2}\left(c + dx\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)/(a*sinh(d*x+c)**2)**(1/2),x)
[Out] Integral(tanh(c + d*x)/sqrt(a*sinh(c + d*x)**2), x)
```

3.1038 $\int \frac{\coth(c+dx)}{\sqrt{a\cosh^2(c+dx)}}\,dx$

Optimal. Leaf size=31

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a\cosh^2(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}\,d}$$

[Out] -arctanh((a*cosh(d*x+c)^2)^(1/2)/a^(1/2))/d/a^(1/2)

Rubi [A] time = 0.04, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3205, 63, 206}

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a\cosh^2(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}\,d}$$

Antiderivative was successfully verified.

[In] Int[Coth[c + d*x]/Sqrt[a*Cosh[c + d*x]^2],x]

[Out] -(ArcTanh[Sqrt[a*Cosh[c + d*x]^2]/Sqrt[a]]/(Sqrt[a]*d))

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3205

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_)]^(m_.
), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m + 1
)/2)/(2*f), Subst[Int[(x^((m - 1)/2)*(b*ff^(n/2)*x^(n/2))^p)/(1 - ff*x)^((m
 + 1)/2), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{b, e, f, p}, x] && Integ
```

```
erQ[(m - 1)/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{\coth(c+dx)}{\sqrt{a\cosh^2(c+dx)}}\,dx = -\frac{\text{Subst}\left(\int \frac{1}{(1-x)\sqrt{ax}}\,dx, x, \cosh^2(c+dx)\right)}{2d}$$
$$= -\frac{\text{Subst}\left(\int \frac{1}{1-\frac{x^2}{a}}\,dx, x, \sqrt{a\cosh^2(c+dx)}\right)}{ad}$$
$$= -\frac{\tanh^{-1}\left(\frac{\sqrt{a\cosh^2(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}\,d}$$

Mathematica [A] time = 0.06, size = 49, normalized size = 1.58

$$\frac{\cosh(c+dx)\left(\log\left(\sinh\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\cosh\left(\frac{1}{2}(c+dx)\right)\right)\right)}{d\sqrt{a\cosh^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[c + d*x]/Sqrt[a*Cosh[c + d*x]^2],x]
[Out] (Cosh[c + d*x]*(-Log[Cosh[(c + d*x)/2]] + Log[Sinh[(c + d*x)/2]]))/(d*Sqrt[
a*Cosh[c + d*x]^2])
```

fricas [B] time = 0.43, size = 174, normalized size = 5.61

$$\left[\frac{\sqrt{ae^{(4\,dx+4\,c)}+2\,ae^{(2\,dx+2\,c)}+a}\,\log\left(\frac{\cosh(dx+c)+\sinh(dx+c)-1}{\cosh(dx+c)+\sinh(dx+c)+1}\right)}{ade^{(2\,dx+2\,c)}+ad}, \frac{2\sqrt{-a}\,\arctan\left(\frac{\sqrt{ae^{(4\,dx+4\,c)}+2\,ae^{(2\,dx+2\,c)}+a}\,\sqrt{-}}{a\cosh(dx+c)e^{(2\,dx+2\,c)}+a\cosh(dx+c)+\left(ae^{(2\,dx+2}\right.}\right.}{ad}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)/(a*cosh(d*x+c)^2)^(1/2),x, algorithm="fricas")
[Out] [sqrt(a*e^(4*d*x + 4*c) + 2*a*e^(2*d*x + 2*c) + a)*log((cosh(d*x + c) + sin
h(d*x + c) - 1)/(cosh(d*x + c) + sinh(d*x + c) + 1))/(a*d*e^(2*d*x + 2*c) +
```

```
a*d), 2*sqrt(-a)*arctan(sqrt(a*e^(4*d*x + 4*c) + 2*a*e^(2*d*x + 2*c) + a)*
sqrt(-a)/(a*cosh(d*x + c)*e^(2*d*x + 2*c) + a*cosh(d*x + c) + (a*e^(2*d*x +
2*c) + a)*sinh(d*x + c)))/(a*d)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)/(a*cosh(d*x+c)^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(exp
(c)^3*t_nostep^3+exp(c)*t_nostep)]index.cc index_m operator + Error: Bad Ar
gument Value
```

maple [A] time = 0.34, size = 31, normalized size = 1.00

$$-\frac{\cosh\left(dx+c\right)\operatorname{arctanh}\left(\cosh\left(dx+c\right)\right)}{\sqrt{a\left(\cosh^{2}\left(dx+c\right)\right)}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(d*x+c)/(a*cosh(d*x+c)^2)^(1/2),x)
```

```
[Out] -1/(a*cosh(d*x+c)^2)^(1/2)*cosh(d*x+c)*arctanh(cosh(d*x+c))/d
```

maxima [A] time = 0.46, size = 40, normalized size = 1.29

$$-\frac{\log\left(e^{(-dx-c)}+1\right)}{\sqrt{a}\,d}+\frac{\log\left(e^{(-dx-c)}-1\right)}{\sqrt{a}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)/(a*cosh(d*x+c)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -log(e^(-d*x - c) + 1)/(sqrt(a)*d) + log(e^(-d*x - c) - 1)/(sqrt(a)*d)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\coth\left(c+d\,x\right)}{\sqrt{a\,{\cosh\left(c+d\,x\right)}^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(c + d*x)/(a*cosh(c + d*x)^2)^(1/2),x)
```

```
[Out] int(coth(c + d*x)/(a*cosh(c + d*x)^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\coth(c+dx)}{\sqrt{a\cosh^2(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)/(a*cosh(d*x+c)**2)**(1/2),x)
```

```
[Out] Integral(coth(c + d*x)/sqrt(a*cosh(c + d*x)**2), x)
```

3.1039 $\int x\cosh(2x)\mathbf{sech}(x)\,dx$

Optimal. Leaf size=43

$$i\text{Li}_2\left(-ie^x\right)-i\text{Li}_2\left(ie^x\right)-2x\tan^{-1}\left(e^x\right)+2x\sinh(x)-2\cosh(x)$$

```
[Out] -2*x*arctan(exp(x))-2*cosh(x)+I*polylog(2,-I*exp(x))-I*polylog(2,I*exp(x))+
2*x*sinh(x)
```

Rubi [A] time = 0.07, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 7, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.875, Rules used = {5473, 3296, 2638, 5449, 4180, 2279, 2391}

$$i\text{PolyLog}\left(2,-ie^x\right)-i\text{PolyLog}\left(2,ie^x\right)-2x\tan^{-1}\left(e^x\right)+2x\sinh(x)-2\cosh(x)$$

Antiderivative was successfully verified.

```
[In] Int[x*Cosh[2*x]*Sech[x],x]
```

```
[Out] -2*x*ArcTan[E^x] - 2*Cosh[x] + I*PolyLog[2, (-I)*E^x] - I*PolyLog[2, I*E^x]
 + 2*x*Sinh[x]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5449

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) + (b_.)*(x_)]^(n_.)*Tanh[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> Int[(c + d*x)^m*Sinh[a + b*x]^n*Tanh[a + b*
x]^(p - 2), x] - Int[(c + d*x)^m*Sinh[a + b*x]^(n - 2)*Tanh[a + b*x]^p, x]
/; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 5473

```
Int[((e_.) + (f_.)*(x_))^(m_.)*(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) +
(d_.)*(x_)]^(q_.), x_Symbol] :> Int[ExpandTrigExpand[(e + f*x)^m*G[c + d*x]
^q, F, c + d*x, p, b/d, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && Member
Q[{Sinh, Cosh}, F] && MemberQ[{Sech, Csch}, G] && IGtQ[p, 0] && IGtQ[q, 0]
&& EqQ[b*c - a*d, 0] && IGtQ[b/d, 1]
```

Rubi steps

$$\begin{aligned}
\int x\cosh(2x)\text{sech}(x)\,dx &= \int (x\cosh(x)+x\sinh(x)\tanh(x))\,dx\\
&= \int x\cosh(x)\,dx+\int x\sinh(x)\tanh(x)\,dx\\
&= x\sinh(x)+\int x\cosh(x)\,dx-\int x\text{sech}(x)\,dx-\int \sinh(x)\,dx\\
&= -2x\tan^{-1}(e^x)-\cosh(x)+2x\sinh(x)+i\int \log\left(1-ie^x\right)\,dx-i\int \log\left(1+ie^x\right)\,dx-\\
&= -2x\tan^{-1}(e^x)-2\cosh(x)+2x\sinh(x)+i\,\text{Subst}\left(\int \frac{\log(1-ix)}{x}\,dx,x,e^x\right)-i\,\text{Subst}\\
&= -2x\tan^{-1}(e^x)-2\cosh(x)+i\text{Li}_2\left(-ie^x\right)-i\text{Li}_2\left(ie^x\right)+2x\sinh(x)
\end{aligned}$$

Mathematica [A] time = 0.03, size = 71, normalized size = 1.65

$$i\left(\text{Li}_2\left(-ie^{-x}\right)-\text{Li}_2\left(ie^{-x}\right)\right)+ix\left(\log\left(1-ie^{-x}\right)-\log\left(1+ie^{-x}\right)\right)+2x\sinh(x)-2\cosh(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Cosh[2*x]*Sech[x],x]
```

```
[Out] -2*Cosh[x] + I*x*(Log[1 - I/E^x] - Log[1 + I/E^x]) + I*(PolyLog[2, (-I)/E^x
] - PolyLog[2, I/E^x]) + 2*x*Sinh[x]
```

fricas [B] time = 0.44, size = 124, normalized size = 2.88

$$\frac{(x-1)\cosh(x)^2 + 2\,(x-1)\cosh(x)\sinh(x) + (x-1)\sinh(x)^2 + (-i\,\cosh(x) - i\,\sinh(x))\mathrm{Li}_2\,(i\,\cosh(x) + i\,\sinh(}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(2*x)*sech(x),x, algorithm="fricas")
```

```
[Out] ((x - 1)*cosh(x)^2 + 2*(x - 1)*cosh(x)*sinh(x) + (x - 1)*sinh(x)^2 + (-I*co
sh(x) - I*sinh(x))*dilog(I*cosh(x) + I*sinh(x)) + (I*cosh(x) + I*sinh(x))*d
ilog(-I*cosh(x) - I*sinh(x)) + (I*x*cosh(x) + I*x*sinh(x))*log(I*cosh(x) +
I*sinh(x) + 1) + (-I*x*cosh(x) - I*x*sinh(x))*log(-I*cosh(x) - I*sinh(x) +
1) - x - 1)/(cosh(x) + sinh(x))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\cosh(2\,x)\,\mathrm{sech}(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(2*x)*sech(x),x, algorithm="giac")
```

```
[Out] integrate(x*cosh(2*x)*sech(x), x)
```

maple [A] time = 0.25, size = 68, normalized size = 1.58

$$2\left(-\frac{1}{2}+\frac{x}{2}\right)\mathrm{e}^{x}+2\left(-\frac{1}{2}-\frac{x}{2}\right)\mathrm{e}^{-x}+ix\ln(1+i\mathrm{e}^{x})-ix\ln(1-i\mathrm{e}^{x})+i\,\mathrm{dilog}(1+i\mathrm{e}^{x})-i\,\mathrm{dilog}(1-i\mathrm{e}^{x})$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cosh(2*x)*sech(x),x)
```

```
[Out] 2*(-1/2+1/2*x)*exp(x)+2*(-1/2-1/2*x)*exp(-x)+I*x*ln(1+I*exp(x))-I*x*ln(1-I*
exp(x))+I*dilog(1+I*exp(x))-I*dilog(1-I*exp(x))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-(x+1)e^{(-x)} + (x-1)e^{x} - 2\int \frac{xe^{x}}{e^{(2\,x)}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(2*x)*sech(x),x, algorithm="maxima")
```

```
[Out] -(x + 1)*e^(-x) + (x - 1)*e^x - 2*integrate(x*e^x/(e^(2*x) + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x \cosh(2x)}{\cosh(x)} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*cosh(2*x))/cosh(x),x)
```

```
[Out] int((x*cosh(2*x))/cosh(x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x \cosh(2x) \operatorname{sech}(x) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(2*x)*sech(x),x)
```

```
[Out] Integral(x*cosh(2*x)*sech(x), x)
```

3.1040 $\int x \cosh(2x)\mathbf{sech}^2(x)\, dx$

Optimal. Leaf size=12

$$x^2 - x\tanh(x) + \log(\cosh(x))$$

```
[Out] x^2+ln(cosh(x))-x*tanh(x)
```

Rubi [A] time = 0.04, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {5473, 3720, 3475, 30}

$$x^2 - x\tanh(x) + \log(\cosh(x))$$

Antiderivative was successfully verified.

```
[In] Int[x*Cosh[2*x]*Sech[x]^2,x]
```

```
[Out] x^2 + Log[Cosh[x]] - x*Tanh[x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3720

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[(b*(c + d*x)^m*(b*Tan[e + f*x])^(n - 1))/(f*(n - 1)), x] + (-Di
st[(b*d*m)/(f*(n - 1)), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x],
 x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[
{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rule 5473

```
Int[((e_.) + (f_.)*(x_))^(m_.)*(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) +
(d_.)*(x_)]^(q_.), x_Symbol] :> Int[ExpandTrigExpand[(e + f*x)^m*G[c + d*x]
^q, F, c + d*x, p, b/d, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && Member
Q[{Sinh, Cosh}, F] && MemberQ[{Sech, Csch}, G] && IGtQ[p, 0] && IGtQ[q, 0]
&& EqQ[b*c - a*d, 0] && IGtQ[b/d, 1]
```

Rubi steps

$$
\begin{aligned}
\int x\cosh(2x)\operatorname{sech}^2(x)\,dx &= \int \left(x + x\tanh^2(x)\right)\,dx \\
&= \frac{x^2}{2} + \int x\tanh^2(x)\,dx \\
&= \frac{x^2}{2} - x\tanh(x) + \int x\,dx + \int \tanh(x)\,dx \\
&= x^2 + \log(\cosh(x)) - x\tanh(x)
\end{aligned}
$$

Mathematica [A] time = 0.02, size = 12, normalized size = 1.00

$$x^2 - x\tanh(x) + \log(\cosh(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Cosh[2*x]*Sech[x]^2,x]
```

```
[Out] x^2 + Log[Cosh[x]] - x*Tanh[x]
```

fricas [B] time = 0.42, size = 91, normalized size = 7.58

$$\frac{\left(x^2 - 2\,x\right)\cosh(x)^2 + 2\left(x^2 - 2\,x\right)\cosh(x)\sinh(x) + \left(x^2 - 2\,x\right)\sinh(x)^2 + x^2 + \left(\cosh(x)^2 + 2\,\cosh(x)\sinh(x) +\right.}{\cosh(x)^2 + 2\,\cosh(x)\sinh(x) + \sinh(x)^2 + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(2*x)*sech(x)^2,x, algorithm="fricas")
```

```
[Out] ((x^2 - 2*x)*cosh(x)^2 + 2*(x^2 - 2*x)*cosh(x)*sinh(x) + (x^2 - 2*x)*sinh(x
)^2 + x^2 + (cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)*log(2*cosh(x)/(
cosh(x) - sinh(x))))/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)
```

giac [B] time = 0.13, size = 47, normalized size = 3.92

$$\frac{x^2e^{(2\,x)} + x^2 - 2\,xe^{(2\,x)} + e^{(2\,x)}\log\left(e^{(2\,x)} + 1\right) + \log\left(e^{(2\,x)} + 1\right)}{e^{(2\,x)} + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(2*x)*sech(x)^2,x, algorithm="giac")
```

```
[Out] (x^2*e^(2*x) + x^2 - 2*x*e^(2*x) + e^(2*x)*log(e^(2*x) + 1) + log(e^(2*x) +
 1))/(e^(2*x) + 1)
```

maple [B] time = 0.22, size = 26, normalized size = 2.17

$$x^2 - 2x + \frac{2x}{1 + e^{2x}} + \ln\left(1 + e^{2x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cosh(2*x)*sech(x)^2,x)
```

```
[Out] x^2-2*x+2*x/(1+exp(2*x))+ln(1+exp(2*x))
```

maxima [B] time = 0.45, size = 33, normalized size = 2.75

$$\frac{x^2 + \left(x^2 - 2\,x\right)e^{(2\,x)}}{e^{(2\,x)} + 1} + \log\left(e^{(2\,x)} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(2*x)*sech(x)^2,x, algorithm="maxima")
```

```
[Out] (x^2 + (x^2 - 2*x)*e^(2*x))/(e^(2*x) + 1) + log(e^(2*x) + 1)
```

mupad [B] time = 1.74, size = 25, normalized size = 2.08

$$\ln\left(e^{2\,x} + 1\right) - 2\,x + \frac{2\,x}{e^{2\,x} + 1} + x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*cosh(2*x))/cosh(x)^2,x)
```

```
[Out] log(exp(2*x) + 1) - 2*x + (2*x)/(exp(2*x) + 1) + x^2
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\cosh\left(2x\right)\operatorname{sech}^2\left(x\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(2*x)*sech(x)**2,x)
```

```
[Out] Integral(x*cosh(2*x)*sech(x)**2, x)
```

3.1041 $\int x\cosh(2x)\mathbf{sech}^3(x)\,dx$

Optimal. Leaf size=53

$$-\frac{3}{2}i\text{Li}_2\left(-ie^x\right)+\frac{3}{2}i\text{Li}_2\left(ie^x\right)+3x\tan^{-1}\left(e^x\right)-\frac{\text{sech}(x)}{2}-\frac{1}{2}x\tanh(x)\text{sech}(x)$$

```
[Out] 3*x*arctan(exp(x))-3/2*I*polylog(2,-I*exp(x))+3/2*I*polylog(2,I*exp(x))-1/2
*sech(x)-1/2*x*sech(x)*tanh(x)
```

Rubi [A] time = 0.13, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {5473, 4180, 2279, 2391, 5455, 4185}

$$-\frac{3}{2}i\text{PolyLog}\left(2,-ie^x\right)+\frac{3}{2}i\text{PolyLog}\left(2,ie^x\right)+3x\tan^{-1}\left(e^x\right)-\frac{\text{sech}(x)}{2}-\frac{1}{2}x\tanh(x)\text{sech}(x)$$

Antiderivative was successfully verified.

```
[In] Int[x*Cosh[2*x]*Sech[x]^3,x]
```

```
[Out] 3*x*ArcTan[E^x] - ((3*I)/2)*PolyLog[2, (-I)*E^x] + ((3*I)/2)*PolyLog[2, I*E
^x] - Sech[x]/2 - (x*Sech[x]*Tanh[x])/2
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4185

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_)), x_Symbol] :>
 -Simp[(b^2*(c + d*x)*Cot[e + f*x]*(b*Csc[e + f*x])^(n - 2))/(f*(n - 1)), x
] + (Dist[(b^2*(n - 2))/(n - 1), Int[(c + d*x)*(b*Csc[e + f*x])^(n - 2), x]
, x] - Simp[(b^2*d*(b*Csc[e + f*x])^(n - 2))/(f^2*(n - 1)*(n - 2)), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2]
```

Rule 5455

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) + (b_.)*(x_)]*Tanh[(a_.) + (b_.)*
(x_)]^(p_), x_Symbol] :> Int[(c + d*x)^m*Sech[a + b*x]*Tanh[a + b*x]^(p - 2
), x] - Int[(c + d*x)^m*Sech[a + b*x]^3*Tanh[a + b*x]^(p - 2), x] /; FreeQ[
{a, b, c, d, m}, x] && IGtQ[p/2, 0]
```

Rule 5473

```
Int[((e_.) + (f_.)*(x_))^(m_.)*(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) +
(d_.)*(x_)]^(q_.), x_Symbol] :> Int[ExpandTrigExpand[(e + f*x)^m*G[c + d*x]
^q, F, c + d*x, p, b/d, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && Member
Q[{Sinh, Cosh}, F] && MemberQ[{Sech, Csch}, G] && IGtQ[p, 0] && IGtQ[q, 0]
&& EqQ[b*c - a*d, 0] && IGtQ[b/d, 1]
```

Rubi steps

$$\int x\cosh(2x)\text{sech}^3(x)\,dx = \int \left(x\text{sech}(x)+x\text{sech}(x)\tanh^2(x)\right)\,dx$$
$$= \int x\text{sech}(x)\,dx + \int x\text{sech}(x)\tanh^2(x)\,dx$$
$$= 2x\tan^{-1}(e^x) - i\int \log(1-ie^x)\,dx + i\int \log(1+ie^x)\,dx + \int x\text{sech}(x)\,dx - \int x\text{sec}$$
$$= 4x\tan^{-1}(e^x) - \frac{\text{sech}(x)}{2} - \frac{1}{2}x\text{sech}(x)\tanh(x) - i\int \log(1-ie^x)\,dx + i\int \log(1+ie$$
$$= 3x\tan^{-1}(e^x) - i\text{Li}_2(-ie^x) + i\text{Li}_2(ie^x) - \frac{\text{sech}(x)}{2} - \frac{1}{2}x\text{sech}(x)\tanh(x) + \frac{1}{2}i\int \log(1$$
$$= 3x\tan^{-1}(e^x) - 2i\text{Li}_2(-ie^x) + 2i\text{Li}_2(ie^x) - \frac{\text{sech}(x)}{2} - \frac{1}{2}x\text{sech}(x)\tanh(x) + \frac{1}{2}i\,\text{Subst}$$
$$= 3x\tan^{-1}(e^x) - \frac{3}{2}i\text{Li}_2(-ie^x) + \frac{3}{2}i\text{Li}_2(ie^x) - \frac{\text{sech}(x)}{2} - \frac{1}{2}x\text{sech}(x)\tanh(x)$$

Mathematica [A] time = 0.07, size = 78, normalized size = 1.47

$$-\frac{1}{2}i\left(3\text{Li}_2(-ie^{-x}) - 3\text{Li}_2(ie^{-x}) + 3x\log(1-ie^{-x}) - 3x\log(1+ie^{-x}) - i\text{sech}(x) - ix\tanh(x)\text{sech}(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Cosh[2*x]*Sech[x]^3,x]
```

```
[Out] (-1/2*I)*(3*x*Log[1 - I/E^x] - 3*x*Log[1 + I/E^x] + 3*PolyLog[2, (-I)/E^x]
- 3*PolyLog[2, I/E^x] - I*Sech[x] - I*x*Sech[x]*Tanh[x])
```

fricas [B] time = 0.44, size = 400, normalized size = 7.55

$$-\frac{2\,(x+1)\cosh(x)^{3}+6\,(x+1)\cosh(x)\sinh(x)^{2}+2\,(x+1)\sinh(x)^{3}-2\,(x-1)\cosh(x)-\left(3i\,\cosh(x)^{4}+12i\,\mathrm{co}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(2*x)*sech(x)^3,x, algorithm="fricas")
```

```
[Out] -1/2*(2*(x + 1)*cosh(x)^3 + 6*(x + 1)*cosh(x)*sinh(x)^2 + 2*(x + 1)*sinh(x)
^3 - 2*(x - 1)*cosh(x) - (3*I*cosh(x)^4 + 12*I*cosh(x)*sinh(x)^3 + 3*I*sinh
(x)^4 + (18*I*cosh(x)^2 + 6*I)*sinh(x)^2 + 6*I*cosh(x)^2 + (12*I*cosh(x)^3
+ 12*I*cosh(x))*sinh(x) + 3*I)*dilog(I*cosh(x) + I*sinh(x)) - (-3*I*cosh(x)
^4 - 12*I*cosh(x)*sinh(x)^3 - 3*I*sinh(x)^4 + (-18*I*cosh(x)^2 - 6*I)*sinh(
x)^2 - 6*I*cosh(x)^2 + (-12*I*cosh(x)^3 - 12*I*cosh(x))*sinh(x) - 3*I)*dilo
g(-I*cosh(x) - I*sinh(x)) - (-3*I*x*cosh(x)^4 - 12*I*x*cosh(x)*sinh(x)^3 -
3*I*x*sinh(x)^4 - 6*I*x*cosh(x)^2 + (-18*I*x*cosh(x)^2 - 6*I*x)*sinh(x)^2 +
 (-12*I*x*cosh(x)^3 - 12*I*x*cosh(x))*sinh(x) - 3*I*x)*log(I*cosh(x) + I*si
nh(x) + 1) - (3*I*x*cosh(x)^4 + 12*I*x*cosh(x)*sinh(x)^3 + 3*I*x*sinh(x)^4
+ 6*I*x*cosh(x)^2 + (18*I*x*cosh(x)^2 + 6*I*x)*sinh(x)^2 + (12*I*x*cosh(x)^
3 + 12*I*x*cosh(x))*sinh(x) + 3*I*x)*log(-I*cosh(x) - I*sinh(x) + 1) + 2*(3
*(x + 1)*cosh(x)^2 - x + 1)*sinh(x))/(cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + sin
h(x)^4 + 2*(3*cosh(x)^2 + 1)*sinh(x)^2 + 2*cosh(x)^2 + 4*(cosh(x)^3 + cosh(
x))*sinh(x) + 1)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\cosh\left(2\,x\right)\operatorname{sech}(x)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(2*x)*sech(x)^3,x, algorithm="giac")
```

```
[Out] integrate(x*cosh(2*x)*sech(x)^3, x)
```

maple [A] time = 0.27, size = 75, normalized size = 1.42

$$-\frac{e^{x}\left(x\,e^{2x}+e^{2x}-x+1\right)}{\left(1+e^{2x}\right)^{2}}-\frac{3ix\ln\left(1+ie^{x}\right)}{2}+\frac{3ix\ln\left(1-ie^{x}\right)}{2}-\frac{3i\,\mathrm{dilog}\left(1+ie^{x}\right)}{2}+\frac{3i\,\mathrm{dilog}\left(1-ie^{x}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cosh(2*x)*sech(x)^3,x)
```

```
[Out] -exp(x)*(x*exp(2*x)+exp(2*x)-x+1)/(1+exp(2*x))^2-3/2*I*x*ln(1+I*exp(x))+3/2
*I*x*ln(1-I*exp(x))-3/2*I*dilog(1+I*exp(x))+3/2*I*dilog(1-I*exp(x))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{(x+1)e^{(3\,x)}-(x-1)e^{x}}{e^{(4\,x)}+2\,e^{(2\,x)}+1}+12\int\frac{xe^{x}}{4\left(e^{(2\,x)}+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(2*x)*sech(x)^3,x, algorithm="maxima")
```

```
[Out] -((x + 1)*e^(3*x) - (x - 1)*e^x)/(e^(4*x) + 2*e^(2*x) + 1) + 12*integrate(1
/4*x*e^x/(e^(2*x) + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int\frac{x\cosh\left(2\,x\right)}{{\cosh(x)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*cosh(2*x))/cosh(x)^3,x)
```

```
[Out] int((x*cosh(2*x))/cosh(x)^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\cosh\left(2x\right)\operatorname{sech}^{3}\left(x\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(2*x)*sech(x)**3,x)
```

```
[Out] Integral(x*cosh(2*x)*sech(x)**3, x)
```

3.1042 $\int \sqrt{\operatorname{csch}(x)}\,(x\cosh(x)-4\operatorname{sech}(x)\tanh(x))\,dx$

Optimal. Leaf size=20

$$\frac{2x}{\sqrt{\operatorname{csch}(x)}}-\frac{4\operatorname{sech}(x)}{\operatorname{csch}^{\frac{3}{2}}(x)}$$

```
[Out] -4*sech(x)/csch(x)^(3/2)+2*x/csch(x)^(1/2)
```

Rubi [A] time = 0.17, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {6742, 5445, 3771, 2639, 2626}

$$\frac{2x}{\sqrt{\operatorname{csch}(x)}}-\frac{4\operatorname{sech}(x)}{\operatorname{csch}^{\frac{3}{2}}(x)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Csch[x]]*(x*Cosh[x] - 4*Sech[x]*Tanh[x]),x]
```

```
[Out] (2*x)/Sqrt[Csch[x]] - (4*Sech[x])/Csch[x]^(3/2)
```

Rule 2626

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*b*(a*Csc[e + f*x])^(m - 1)*(b*Sec[e + f*x])^(n -
1))/(f*(n - 1)), x] + Dist[(b^2*(m + n - 2))/(n - 1), Int[(a*Csc[e + f*x])^
m*(b*Sec[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1
] && IntegersQ[2*m, 2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 5445

```
Int[Cosh[(a_.) + (b_.)*(x_)^(n_.)]*Csch[(a_.) + (b_.)*(x_)^(n_.)]^(p_)*(x_)
^(m_.), x_Symbol] :> -Simp[(x^(m - n + 1)*Csch[a + b*x^n]^(p - 1))/(b*n*(p
- 1)), x] + Dist[(m - n + 1)/(b*n*(p - 1)), Int[x^(m - n)*Csch[a + b*x^n]^(
```

```
p - 1), x], x] /; FreeQ[{a, b, p}, x] && IntegerQ[n] && GeQ[m - n, 0] && Ne
Q[p, 1]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}\int \sqrt{\operatorname{csch}(x)}\,(x\cosh(x)-4\operatorname{sech}(x)\tanh(x))\,dx &= \int \left(x\cosh(x)\sqrt{\operatorname{csch}(x)}-\frac{4\operatorname{sech}^2(x)}{\sqrt{\operatorname{csch}(x)}}\right)dx\\ &= -\left(4\int \frac{\operatorname{sech}^2(x)}{\sqrt{\operatorname{csch}(x)}}\,dx\right)+\int x\cosh(x)\sqrt{\operatorname{csch}(x)}\,dx\\ &= \frac{2x}{\sqrt{\operatorname{csch}(x)}}-\frac{4\operatorname{sech}(x)}{\operatorname{csch}^{\frac{3}{2}}(x)}\end{aligned}$$

Mathematica [A] time = 1.14, size = 17, normalized size = 0.85

$$\frac{2(x\operatorname{csch}(x)-2\operatorname{sech}(x))}{\operatorname{csch}^{\frac{3}{2}}(x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Csch[x]]*(x*Cosh[x] - 4*Sech[x]*Tanh[x]),x]
[Out] (2*(x*Csch[x] - 2*Sech[x]))/Csch[x]^(3/2)
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^(1/2)*(x*cosh(x)-4*sech(x)*tanh(x)),x, algorithm="fricas"
)
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (x\cosh(x)-4\,\operatorname{sech}(x)\tanh(x))\sqrt{\operatorname{csch}(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^(1/2)*(x*cosh(x)-4*sech(x)*tanh(x)),x, algorithm="giac")
```

```
[Out] integrate((x*cosh(x) - 4*sech(x)*tanh(x))*sqrt(csch(x)), x)
```

maple [F] time = 0.84, size = 0, normalized size = 0.00

$$\int \sqrt{\operatorname{csch}(x)}\ (x\cosh(x) - 4\operatorname{sech}(x)\tanh(x))\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(x)^(1/2)*(x*cosh(x)-4*sech(x)*tanh(x)),x)
```

```
[Out] int(csch(x)^(1/2)*(x*cosh(x)-4*sech(x)*tanh(x)),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (x\cosh(x) - 4\ \operatorname{sech}(x)\tanh(x))\sqrt{\operatorname{csch}(x)}\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^(1/2)*(x*cosh(x)-4*sech(x)*tanh(x)),x, algorithm="maxima"
)
```

```
[Out] integrate((x*cosh(x) - 4*sech(x)*tanh(x))*sqrt(csch(x)), x)
```

mupad [B] time = 1.90, size = 51, normalized size = 2.55

$$\frac{e^{-x}\sqrt{-\frac{1}{\frac{e^{-x}}{2}-\frac{e^{x}}{2}}}\ \left(e^{2x}-1\right)\left(x-2\,e^{2x}+x\,e^{2x}+2\right)}{e^{2x}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(1/sinh(x))^(1/2)*((4*tanh(x))/cosh(x) - x*cosh(x)),x)
```

```
[Out] (exp(-x)*(-1/(exp(-x)/2 - exp(x)/2))^(1/2)*(exp(2*x) - 1)*(x - 2*exp(2*x) +
 x*exp(2*x) + 2))/(exp(2*x) + 1)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)**(1/2)*(x*cosh(x)-4*sech(x)*tanh(x)),x)
```

```
[Out] Timed out
```

3.1043 $\int \sinh(x)(\cosh(x) + \sinh(x))\,dx$

Optimal. Leaf size=22

$$-\frac{x}{2} + \frac{\sinh^2(x)}{2} + \frac{1}{2}\sinh(x)\cosh(x)$$

```
[Out] -1/2*x+1/2*cosh(x)*sinh(x)+1/2*sinh(x)^2
```

Rubi [A] time = 0.03, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {3089, 2564, 30, 2635, 8}

$$-\frac{x}{2} + \frac{\sinh^2(x)}{2} + \frac{1}{2}\sinh(x)\cosh(x)$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[x]*(Cosh[x] + Sinh[x]),x]
```

```
[Out] -x/2 + (Cosh[x]*Sinh[x])/2 + Sinh[x]^2/2
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 3089

```
Int[sin[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[ExpandTrig[sin[c + d*x]^m*(a
*cos[c + d*x] + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d}, x] && Inte
gerQ[m] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\int \sinh(x)(\cosh(x)+\sinh(x))\,dx &= -\left(i\int \left(i\cosh(x)\sinh(x)+i\sinh^2(x)\right)\,dx\right)\\ &= \int \cosh(x)\sinh(x)\,dx+\int \sinh^2(x)\,dx\\ &= \frac{1}{2}\cosh(x)\sinh(x)-\frac{\int 1\,dx}{2}-\text{Subst}(\int x\,dx,x,i\sinh(x))\\ &= -\frac{x}{2}+\frac{1}{2}\cosh(x)\sinh(x)+\frac{\sinh^2(x)}{2}\end{aligned}$$

Mathematica [A] time = 0.00, size = 22, normalized size = 1.00

$$-\frac{x}{2}+\frac{1}{4}\sinh(2x)+\frac{\cosh^2(x)}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[x]*(Cosh[x] + Sinh[x]),x]
[Out] -1/2*x + Cosh[x]^2/2 + Sinh[2*x]/4
```

fricas [A] time = 0.43, size = 29, normalized size = 1.32

$$-\frac{(2x-1)\cosh(x)-(2x+1)\sinh(x)}{4(\cosh(x)-\sinh(x))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)*(cosh(x)+sinh(x)),x, algorithm="fricas")
[Out] -1/4*((2*x - 1)*cosh(x) - (2*x + 1)*sinh(x))/(cosh(x) - sinh(x))
```

giac [A] time = 0.13, size = 10, normalized size = 0.45

$$-\frac{1}{2}x+\frac{1}{4}e^{(2x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)*(cosh(x)+sinh(x)),x, algorithm="giac")
```

```
[Out] -1/2*x + 1/4*e^(2*x)
```

maple [A] time = 0.06, size = 17, normalized size = 0.77

$$\frac{\cosh(x)\sinh(x)}{2} - \frac{x}{2} + \frac{\left(\cosh^2(x)\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)*(cosh(x)+sinh(x)),x)
```

```
[Out] 1/2*cosh(x)*sinh(x)-1/2*x+1/2*cosh(x)^2
```

maxima [A] time = 0.32, size = 22, normalized size = 1.00

$$\frac{1}{2}\cosh(x)^2 - \frac{1}{2}x + \frac{1}{8}e^{(2x)} - \frac{1}{8}e^{(-2x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)*(cosh(x)+sinh(x)),x, algorithm="maxima")
```

```
[Out] 1/2*cosh(x)^2 - 1/2*x + 1/8*e^(2*x) - 1/8*e^(-2*x)
```

mupad [B] time = 0.06, size = 10, normalized size = 0.45

$$\frac{e^{2x}}{4} - \frac{x}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)*(cosh(x) + sinh(x)),x)
```

```
[Out] exp(2*x)/4 - x/2
```

sympy [A] time = 0.17, size = 31, normalized size = 1.41

$$\frac{x\sinh^2(x)}{2} - \frac{x\cosh^2(x)}{2} + \frac{\sinh(x)\cosh(x)}{2} + \frac{\cosh^2(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)*(cosh(x)+sinh(x)),x)
```

```
[Out] x*sinh(x)**2/2 - x*cosh(x)**2/2 + sinh(x)*cosh(x)/2 + cosh(x)**2/2
```

3.1044 $\int \frac{1+\sinh^2(x)}{1+\cosh(x)+\sinh(x)}\,dx$

Optimal. Leaf size=69

$$\frac{1}{2\left(1-\tanh\left(\frac{x}{2}\right)\right)}+\frac{1}{\tanh\left(\frac{x}{2}\right)+1}-\frac{1}{2\left(\tanh\left(\frac{x}{2}\right)+1\right)^2}+\frac{1}{4}\log\left(1-\tanh\left(\frac{x}{2}\right)\right)+\frac{3}{4}\log\left(\tanh\left(\frac{x}{2}\right)+1\right)$$

```
[Out] 1/4*ln(1-tanh(1/2*x))+3/4*ln(1+tanh(1/2*x))+1/2/(1-tanh(1/2*x))-1/2/(1+tanh
(1/2*x))^2+1/(1+tanh(1/2*x))
```

Rubi [A] time = 0.20, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {4397, 12, 894}

$$\frac{1}{2\left(1-\tanh\left(\frac{x}{2}\right)\right)}+\frac{1}{\tanh\left(\frac{x}{2}\right)+1}-\frac{1}{2\left(\tanh\left(\frac{x}{2}\right)+1\right)^2}+\frac{1}{4}\log\left(1-\tanh\left(\frac{x}{2}\right)\right)+\frac{3}{4}\log\left(\tanh\left(\frac{x}{2}\right)+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Sinh[x]^2)/(1 + Cosh[x] + Sinh[x]),x]
```

```
[Out] Log[1 - Tanh[x/2]]/4 + (3*Log[1 + Tanh[x/2]])/4 + 1/(2*(1 - Tanh[x/2])) - 1
/(2*(1 + Tanh[x/2])^2) + (1 + Tanh[x/2])^(-1)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 894

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 4397

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rubi steps

$$\int \frac{1+\sinh^2(x)}{1+\cosh(x)+\sinh(x)}\,dx = \int \frac{\cosh^2(x)}{1+\cosh(x)+\sinh(x)}\,dx$$
$$= 2\,\text{Subst}\left(\int \frac{(1+x^2)^2}{2(1-x)^2(1+x)^3}\,dx, x, \tanh\left(\frac{x}{2}\right)\right)$$
$$= \text{Subst}\left(\int \frac{(1+x^2)^2}{(1-x)^2(1+x)^3}\,dx, x, \tanh\left(\frac{x}{2}\right)\right)$$
$$= \text{Subst}\left(\int \left(\frac{1}{2(-1+x)^2} + \frac{1}{4(-1+x)} + \frac{1}{(1+x)^3} - \frac{1}{(1+x)^2} + \frac{3}{4(1+x)}\right) dx, x, \tanh$$
$$= \frac{1}{4}\log\left(1-\tanh\left(\frac{x}{2}\right)\right) + \frac{3}{4}\log\left(1+\tanh\left(\frac{x}{2}\right)\right) + \frac{1}{2\left(1-\tanh\left(\frac{x}{2}\right)\right)} - \frac{1}{2\left(1+\tanh\left(\frac{x}{2}\right.\right.}$$

Mathematica [A] time = 0.03, size = 37, normalized size = 0.54

$$\frac{x}{4} + \frac{1}{8}\sinh(2x) + \frac{\cosh(x)}{2} - \frac{1}{8}\cosh(2x) - \log\left(\cosh\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Sinh[x]^2)/(1 + Cosh[x] + Sinh[x]),x]
```

```
[Out] x/4 + Cosh[x]/2 - Cosh[2*x]/8 - Log[Cosh[x/2]] + Sinh[2*x]/8
```

fricas [A] time = 0.43, size = 95, normalized size = 1.38

$$\frac{6\,x\cosh(x)^2 + 2\,\cosh(x)^3 + 6\,(x+\cosh(x))\sinh(x)^2 + 2\,\sinh(x)^3 - 8\left(\cosh(x)^2 + 2\,\cosh(x)\sinh(x) + \sinh(x)^2\right.}{8\left(\cosh(x)^2 + 2\,\cosh(x)\sinh(x) + \right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+sinh(x)^2)/(1+cosh(x)+sinh(x)),x, algorithm="fricas")
```

```
[Out] 1/8*(6*x*cosh(x)^2 + 2*cosh(x)^3 + 6*(x + cosh(x))*sinh(x)^2 + 2*sinh(x)^3
- 8*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2)*log(cosh(x) + sinh(x) + 1)
+ 2*(6*x*cosh(x) + 3*cosh(x)^2 + 1)*sinh(x) + 2*cosh(x) - 1)/(cosh(x)^2 + 2
*cosh(x)*sinh(x) + sinh(x)^2)
```

giac [A] time = 0.13, size = 27, normalized size = 0.39

$$\frac{1}{8}\,(2\,e^x - 1)e^{(-2\,x)} + \frac{3}{4}\,x + \frac{1}{4}\,e^x - \log\left(e^x + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+sinh(x)^2)/(1+cosh(x)+sinh(x)),x, algorithm="giac")
```

```
[Out] 1/8*(2*e^x - 1)*e^(-2*x) + 3/4*x + 1/4*e^x - log(e^x + 1)
```

maple [A] time = 0.21, size = 48, normalized size = 0.70

$$-\frac{1}{2\left(\tanh\left(\frac{x}{2}\right)-1\right)}+\frac{\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{4}-\frac{1}{2\left(\tanh\left(\frac{x}{2}\right)+1\right)^{2}}+\frac{1}{\tanh\left(\frac{x}{2}\right)+1}+\frac{3\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+sinh(x)^2)/(1+cosh(x)+sinh(x)),x)
```

```
[Out] -1/2/(tanh(1/2*x)-1)+1/4*ln(tanh(1/2*x)-1)-1/2/(tanh(1/2*x)+1)^2+1/(tanh(1/
2*x)+1)+3/4*ln(tanh(1/2*x)+1)
```

maxima [A] time = 0.31, size = 29, normalized size = 0.42

$$-\frac{1}{4}\,x+\frac{1}{4}\,e^{(-x)}-\frac{1}{8}\,e^{(-2\,x)}+\frac{1}{4}\,e^{x}-\log\left(e^{(-x)}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+sinh(x)^2)/(1+cosh(x)+sinh(x)),x, algorithm="maxima")
```

```
[Out] -1/4*x + 1/4*e^(-x) - 1/8*e^(-2*x) + 1/4*e^x - log(e^(-x) + 1)
```

mupad [B] time = 1.72, size = 27, normalized size = 0.39

$$\frac{3\,x}{4}+\frac{\mathrm{e}^{-x}}{4}-\frac{\mathrm{e}^{-2\,x}}{8}-\ln\left(\mathrm{e}^{x}+1\right)+\frac{\mathrm{e}^{x}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sinh(x)^2 + 1)/(cosh(x) + sinh(x) + 1),x)
```

```
[Out] (3*x)/4 + exp(-x)/4 - exp(-2*x)/8 - log(exp(x) + 1) + exp(x)/4
```

sympy [B] time = 1.18, size = 381, normalized size = 5.52

$$-\frac{x\tanh^{3}\left(\frac{x}{2}\right)}{4\tanh^{3}\left(\frac{x}{2}\right)+4\tanh^{2}\left(\frac{x}{2}\right)-4\tanh\left(\frac{x}{2}\right)-4}-\frac{x\tanh^{2}\left(\frac{x}{2}\right)}{4\tanh^{3}\left(\frac{x}{2}\right)+4\tanh^{2}\left(\frac{x}{2}\right)-4\tanh\left(\frac{x}{2}\right)-4}+\frac{x\tan}{4\tanh^{3}\left(\frac{x}{2}\right)+4\tanh}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+sinh(x)**2)/(1+cosh(x)+sinh(x)),x)

[Out] -x*tanh(x/2)**3/(4*tanh(x/2)**3 + 4*tanh(x/2)**2 - 4*tanh(x/2) - 4) - x*tan
h(x/2)**2/(4*tanh(x/2)**3 + 4*tanh(x/2)**2 - 4*tanh(x/2) - 4) + x*tanh(x/2)
/(4*tanh(x/2)**3 + 4*tanh(x/2)**2 - 4*tanh(x/2) - 4) + x/(4*tanh(x/2)**3 +
4*tanh(x/2)**2 - 4*tanh(x/2) - 4) + 4*log(tanh(x/2) + 1)*tanh(x/2)**3/(4*ta
nh(x/2)**3 + 4*tanh(x/2)**2 - 4*tanh(x/2) - 4) + 4*log(tanh(x/2) + 1)*tanh(
x/2)**2/(4*tanh(x/2)**3 + 4*tanh(x/2)**2 - 4*tanh(x/2) - 4) - 4*log(tanh(x/
2) + 1)*tanh(x/2)/(4*tanh(x/2)**3 + 4*tanh(x/2)**2 - 4*tanh(x/2) - 4) - 4*l
og(tanh(x/2) + 1)/(4*tanh(x/2)**3 + 4*tanh(x/2)**2 - 4*tanh(x/2) - 4) + 2*t
anh(x/2)**2/(4*tanh(x/2)**3 + 4*tanh(x/2)**2 - 4*tanh(x/2) - 4) - 6*tanh(x/
2)/(4*tanh(x/2)**3 + 4*tanh(x/2)**2 - 4*tanh(x/2) - 4) - 4/(4*tanh(x/2)**3
+ 4*tanh(x/2)**2 - 4*tanh(x/2) - 4)
```

3.1045 $\int x^5 \cosh^7\left(a+bx^3\right)\sinh\left(a+bx^3\right)\,dx$

Optimal. Leaf size=129

$$-\frac{\sinh\left(a+bx^3\right)\cosh^7\left(a+bx^3\right)}{192b^2}-\frac{7\sinh\left(a+bx^3\right)\cosh^5\left(a+bx^3\right)}{1152b^2}-\frac{35\sinh\left(a+bx^3\right)\cosh^3\left(a+bx^3\right)}{4608b^2}-\frac{35\sinh}{}$$

[Out] -35/3072*x^3/b+1/24*x^3*cosh(b*x^3+a)^8/b-35/3072*cosh(b*x^3+a)*sinh(b*x^3+a)/b^2-35/4608*cosh(b*x^3+a)^3*sinh(b*x^3+a)/b^2-7/1152*cosh(b*x^3+a)^5*sinh(b*x^3+a)/b^2-1/192*cosh(b*x^3+a)^7*sinh(b*x^3+a)/b^2

Rubi [A] time = 0.14, antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {5373, 5321, 2635, 8}

$$-\frac{\sinh\left(a+bx^3\right)\cosh^7\left(a+bx^3\right)}{192b^2}-\frac{7\sinh\left(a+bx^3\right)\cosh^5\left(a+bx^3\right)}{1152b^2}-\frac{35\sinh\left(a+bx^3\right)\cosh^3\left(a+bx^3\right)}{4608b^2}-\frac{35\sinh}{}$$

Antiderivative was successfully verified.

[In] Int[x^5*Cosh[a + b*x^3]^7*Sinh[a + b*x^3],x]

[Out] (-35*x^3)/(3072*b) + (x^3*Cosh[a + b*x^3]^8)/(24*b) - (35*Cosh[a + b*x^3]*Sinh[a + b*x^3])/(3072*b^2) - (35*Cosh[a + b*x^3]^3*Sinh[a + b*x^3])/(4608*b^2) - (7*Cosh[a + b*x^3]^5*Sinh[a + b*x^3])/(1152*b^2) - (Cosh[a + b*x^3]^7*Sinh[a + b*x^3])/(192*b^2)

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]
```

Rule 5321

```
Int[((a_.) + Cosh[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cosh[c + d*x])^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(m + 1)/n], 0]))
```

Rule 5373

```
Int[Cosh[(a_.) + (b_.)*(x_)^(n_.)]^(p_.)*(x_)^(m_.)*Sinh[(a_.) + (b_.)*(x_)
^(n_.)], x_Symbol] :> Simp[(x^(m - n + 1)*Cosh[a + b*x^n]^(p + 1))/(b*n*(p
+ 1)), x] - Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*Cosh[a + b*x^n]^(
p + 1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

Rubi steps

$$\int x^5 \cosh^7\left(a+bx^3\right)\sinh\left(a+bx^3\right)\,dx = \frac{x^3\cosh^8\left(a+bx^3\right)}{24b} - \frac{\int x^2\cosh^8\left(a+bx^3\right)\,dx}{8b}$$
$$= \frac{x^3\cosh^8\left(a+bx^3\right)}{24b} - \frac{\text{Subst}\left(\int \cosh^8(a+bx)\,dx,x,x^3\right)}{24b}$$
$$= \frac{x^3\cosh^8\left(a+bx^3\right)}{24b} - \frac{\cosh^7\left(a+bx^3\right)\sinh\left(a+bx^3\right)}{192b^2} - \frac{7\,\text{Subst}\left(\int \cos\right.}{}$$
$$= \frac{x^3\cosh^8\left(a+bx^3\right)}{24b} - \frac{7\cosh^5\left(a+bx^3\right)\sinh\left(a+bx^3\right)}{1152b^2} - \frac{\cosh^7\left(a+b\right.}{1}$$
$$= \frac{x^3\cosh^8\left(a+bx^3\right)}{24b} - \frac{35\cosh^3\left(a+bx^3\right)\sinh\left(a+bx^3\right)}{4608b^2} - \frac{7\cosh^5\left(a+\right.}{}$$
$$= \frac{x^3\cosh^8\left(a+bx^3\right)}{24b} - \frac{35\cosh\left(a+bx^3\right)\sinh\left(a+bx^3\right)}{3072b^2} - \frac{35\cosh^3\left(a\right.}{}$$
$$= -\frac{35x^3}{3072b} + \frac{x^3\cosh^8\left(a+bx^3\right)}{24b} - \frac{35\cosh\left(a+bx^3\right)\sinh\left(a+bx^3\right)}{3072b^2} - \frac{35}{}$$

Mathematica [A] time = 0.48, size = 120, normalized size = 0.93

$$\frac{-672\sinh\left(2\left(a+bx^3\right)\right)-168\sinh\left(4\left(a+bx^3\right)\right)-32\sinh\left(6\left(a+bx^3\right)\right)-3\sinh\left(8\left(a+bx^3\right)\right)+1344bx^3\cosh\left(\right.}{73728b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*Cosh[a + b*x^3]^7*Sinh[a + b*x^3],x]
```

```
[Out] (1344*b*x^3*Cosh[2*(a + b*x^3)] + 672*b*x^3*Cosh[4*(a + b*x^3)] + 192*b*x^3
*Cosh[6*(a + b*x^3)] + 24*b*x^3*Cosh[8*(a + b*x^3)] - 672*Sinh[2*(a + b*x^3
)] - 168*Sinh[4*(a + b*x^3)] - 32*Sinh[6*(a + b*x^3)] - 3*Sinh[8*(a + b*x^3
)])/(73728*b^2)
```

fricas [B] time = 0.47, size = 396, normalized size = 3.07

$$\frac{3\,bx^3\cosh\left(bx^3+a\right)^8+3\,bx^3\sinh\left(bx^3+a\right)^8+24\,bx^3\cosh\left(bx^3+a\right)^6+84\,bx^3\cosh\left(bx^3+a\right)^4-3\,\cosh\left(bx^3+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*cosh(b*x^3+a)^7*sinh(b*x^3+a),x, algorithm="fricas")
```

```
[Out] 1/9216*(3*b*x^3*cosh(b*x^3 + a)^8 + 3*b*x^3*sinh(b*x^3 + a)^8 + 24*b*x^3*co
sh(b*x^3 + a)^6 + 84*b*x^3*cosh(b*x^3 + a)^4 - 3*cosh(b*x^3 + a)*sinh(b*x^3
 + a)^7 + 12*(7*b*x^3*cosh(b*x^3 + a)^2 + 2*b*x^3)*sinh(b*x^3 + a)^6 + 168*
b*x^3*cosh(b*x^3 + a)^2 - 3*(7*cosh(b*x^3 + a)^3 + 8*cosh(b*x^3 + a))*sinh(
b*x^3 + a)^5 + 6*(35*b*x^3*cosh(b*x^3 + a)^4 + 60*b*x^3*cosh(b*x^3 + a)^2 +
 14*b*x^3)*sinh(b*x^3 + a)^4 - (21*cosh(b*x^3 + a)^5 + 80*cosh(b*x^3 + a)^3
 + 84*cosh(b*x^3 + a))*sinh(b*x^3 + a)^3 + 12*(7*b*x^3*cosh(b*x^3 + a)^6 +
30*b*x^3*cosh(b*x^3 + a)^4 + 42*b*x^3*cosh(b*x^3 + a)^2 + 14*b*x^3)*sinh(b*
x^3 + a)^2 - 3*(cosh(b*x^3 + a)^7 + 8*cosh(b*x^3 + a)^5 + 28*cosh(b*x^3 + a
)^3 + 56*cosh(b*x^3 + a))*sinh(b*x^3 + a))/b^2
```

giac [B] time = 0.14, size = 382, normalized size = 2.96

$$\frac{24\left(bx^3+a\right)e^{\left(8\,bx^3+8\,a\right)} - 24\,ae^{\left(8\,bx^3+8\,a\right)} + 192\left(bx^3+a\right)e^{\left(6\,bx^3+6\,a\right)} - 192\,ae^{\left(6\,bx^3+6\,a\right)} + 672\left(bx^3+a\right)e^{\left(4\,bx^3+4\,a\right)} - 6}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*cosh(b*x^3+a)^7*sinh(b*x^3+a),x, algorithm="giac")
```

```
[Out] 1/147456*(24*(b*x^3 + a)*e^(8*b*x^3 + 8*a) - 24*a*e^(8*b*x^3 + 8*a) + 192*(
b*x^3 + a)*e^(6*b*x^3 + 6*a) - 192*a*e^(6*b*x^3 + 6*a) + 672*(b*x^3 + a)*e^
(4*b*x^3 + 4*a) - 672*a*e^(4*b*x^3 + 4*a) + 1344*(b*x^3 + a)*e^(2*b*x^3 + 2
*a) - 1344*a*e^(2*b*x^3 + 2*a) + 1344*(b*x^3 + a)*e^(-2*b*x^3 - 2*a) - 1344
*a*e^(-2*b*x^3 - 2*a) + 672*(b*x^3 + a)*e^(-4*b*x^3 - 4*a) - 672*a*e^(-4*b*
x^3 - 4*a) + 192*(b*x^3 + a)*e^(-6*b*x^3 - 6*a) - 192*a*e^(-6*b*x^3 - 6*a)
+ 24*(b*x^3 + a)*e^(-8*b*x^3 - 8*a) - 24*a*e^(-8*b*x^3 - 8*a) - 3*e^(8*b*x^
3 + 8*a) - 32*e^(6*b*x^3 + 6*a) - 168*e^(4*b*x^3 + 4*a) - 672*e^(2*b*x^3 +
2*a) + 672*e^(-2*b*x^3 - 2*a) + 168*e^(-4*b*x^3 - 4*a) + 32*e^(-6*b*x^3 - 6
*a) + 3*e^(-8*b*x^3 - 8*a))/b^2
```

maple [A] time = 0.37, size = 194, normalized size = 1.50

$$\frac{\left(8b\,x^3-1\right)\mathrm{e}^{8b\,x^3+8a}}{49152b^2}+\frac{\left(6b\,x^3-1\right)\mathrm{e}^{6b\,x^3+6a}}{4608b^2}+\frac{7\left(4b\,x^3-1\right)\mathrm{e}^{4b\,x^3+4a}}{6144b^2}+\frac{7\left(2b\,x^3-1\right)\mathrm{e}^{2b\,x^3+2a}}{1536b^2}+\frac{7\left(2b\,x^3+1\right)\mathrm{e}^{-2b\,x^3-2a}}{1536b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*cosh(b*x^3+a)^7*sinh(b*x^3+a),x)
```

```
[Out] 1/49152*(8*b*x^3-1)/b^2*exp(8*b*x^3+8*a)+1/4608*(6*b*x^3-1)/b^2*exp(6*b*x^3
+6*a)+7/6144*(4*b*x^3-1)/b^2*exp(4*b*x^3+4*a)+7/1536*(2*b*x^3-1)/b^2*exp(2*
```

```
b*x^3+2*a)+7/1536*(2*b*x^3+1)/b^2*exp(-2*b*x^3-2*a)+7/6144*(4*b*x^3+1)/b^2*
exp(-4*b*x^3-4*a)+1/4608*(6*b*x^3+1)/b^2*exp(-6*b*x^3-6*a)+1/49152*(8*b*x^3
+1)/b^2*exp(-8*b*x^3-8*a)
```

maxima [A] time = 0.34, size = 213, normalized size = 1.65

$$\frac{\left(8\,bx^3e^{(8\,a)}-e^{(8\,a)}\right)e^{\left(8\,bx^3\right)}}{49152\,b^2}+\frac{\left(6\,bx^3e^{(6\,a)}-e^{(6\,a)}\right)e^{\left(6\,bx^3\right)}}{4608\,b^2}+\frac{7\left(4\,bx^3e^{(4\,a)}-e^{(4\,a)}\right)e^{\left(4\,bx^3\right)}}{6144\,b^2}+\frac{7\left(2\,bx^3e^{(2\,a)}-e^{(2\,a)}\right)e^{\left(2\,bx^3\right)}}{1536\,b^2}+\frac{7}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*cosh(b*x^3+a)^7*sinh(b*x^3+a),x, algorithm="maxima")
```

```
[Out] 1/49152*(8*b*x^3*e^(8*a) - e^(8*a))*e^(8*b*x^3)/b^2 + 1/4608*(6*b*x^3*e^(6*
a) - e^(6*a))*e^(6*b*x^3)/b^2 + 7/6144*(4*b*x^3*e^(4*a) - e^(4*a))*e^(4*b*x
^3)/b^2 + 7/1536*(2*b*x^3*e^(2*a) - e^(2*a))*e^(2*b*x^3)/b^2 + 7/1536*(2*b*
x^3 + 1)*e^(-2*b*x^3 - 2*a)/b^2 + 7/6144*(4*b*x^3 + 1)*e^(-4*b*x^3 - 4*a)/b
^2 + 1/4608*(6*b*x^3 + 1)*e^(-6*b*x^3 - 6*a)/b^2 + 1/49152*(8*b*x^3 + 1)*e^
(-8*b*x^3 - 8*a)/b^2
```

mupad [B] time = 0.28, size = 213, normalized size = 1.65

$$e^{-2\,b\,x^3-2\,a}\,\left(\frac{7}{1536\,b^2}+\frac{7\,x^3}{768\,b}\right)-e^{2\,b\,x^3+2\,a}\,\left(\frac{7}{1536\,b^2}-\frac{7\,x^3}{768\,b}\right)+e^{-6\,b\,x^3-6\,a}\,\left(\frac{1}{4608\,b^2}+\frac{x^3}{768\,b}\right)-e^{6\,b\,x^3+6\,a}\,\left(\frac{1}{4608\,b^2}-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*cosh(a + b*x^3)^7*sinh(a + b*x^3),x)
```

```
[Out] exp(- 2*a - 2*b*x^3)*(7/(1536*b^2) + (7*x^3)/(768*b)) - exp(2*a + 2*b*x^3)*
(7/(1536*b^2) - (7*x^3)/(768*b)) + exp(- 6*a - 6*b*x^3)*(1/(4608*b^2) + x^3
/(768*b)) - exp(6*a + 6*b*x^3)*(1/(4608*b^2) - x^3/(768*b)) + exp(- 4*a - 4
*b*x^3)*(7/(6144*b^2) + (7*x^3)/(1536*b)) - exp(4*a + 4*b*x^3)*(7/(6144*b^2
) - (7*x^3)/(1536*b)) + exp(- 8*a - 8*b*x^3)*(1/(49152*b^2) + x^3/(6144*b))
 - exp(8*a + 8*b*x^3)*(1/(49152*b^2) - x^3/(6144*b))
```

sympy [A] time = 68.21, size = 241, normalized size = 1.87

$$\begin{cases} -\frac{35x^3\sinh^8\left(a+bx^3\right)}{3072b}+\frac{35x^3\sinh^6\left(a+bx^3\right)\cosh^2\left(a+bx^3\right)}{768b}-\frac{35x^3\sinh^4\left(a+bx^3\right)\cosh^4\left(a+bx^3\right)}{512b}+\frac{35x^3\sinh^2\left(a+bx^3\right)\cosh^6\left(a+bx^3\right)}{768b}+\frac{31x^3 \mathrm{c}}{} \\ \frac{x^6\sinh(a)\cosh^7(a)}{6} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*cosh(b*x**3+a)**7*sinh(b*x**3+a),x)
```

```
[Out] Piecewise((-35*x**3*sinh(a + b*x**3)**8/(3072*b) + 35*x**3*sinh(a + b*x**3)
**6*cosh(a + b*x**3)**2/(768*b) - 35*x**3*sinh(a + b*x**3)**4*cosh(a + b*x*
*3)**4/(512*b) + 35*x**3*sinh(a + b*x**3)**2*cosh(a + b*x**3)**6/(768*b) +
31*x**3*cosh(a + b*x**3)**8/(1024*b) + 35*sinh(a + b*x**3)**7*cosh(a + b*x*
*3)/(3072*b**2) - 385*sinh(a + b*x**3)**5*cosh(a + b*x**3)**3/(9216*b**2) +
 511*sinh(a + b*x**3)**3*cosh(a + b*x**3)**5/(9216*b**2) - 31*sinh(a + b*x*
*3)*cosh(a + b*x**3)**7/(1024*b**2), Ne(b, 0)), (x**6*sinh(a)*cosh(a)**7/6,
 True))
```

3.1046 $\int \frac{\cosh^2(x)}{1+e^x} dx$

Optimal. Leaf size=39

$$\frac{3x}{4} - \frac{e^{-2x}}{8} + \frac{e^{-x}}{4} + \frac{e^x}{4} - \log\left(e^x + 1\right)$$

```
[Out] -1/8/exp(2*x)+1/4/exp(x)+1/4*exp(x)+3/4*x-ln(1+exp(x))
```

Rubi [A] time = 0.05, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2282, 12, 894}

$$\frac{3x}{4} - \frac{e^{-2x}}{8} + \frac{e^{-x}}{4} + \frac{e^x}{4} - \log\left(e^x + 1\right)$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]^2/(1 + E^x),x]
```

```
[Out] -1/(8*E^(2*x)) + 1/(4*E^x) + E^x/4 + (3*x)/4 - Log[1 + E^x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 894

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$\int \frac{\cosh^2(x)}{1+e^x}\,dx = \text{Subst}\left(\int \frac{\left(1+x^2\right)^2}{4x^3(1+x)}\,dx, x, e^x\right)$$
$$= \frac{1}{4}\text{Subst}\left(\int \frac{\left(1+x^2\right)^2}{x^3(1+x)}\,dx, x, e^x\right)$$
$$= \frac{1}{4}\text{Subst}\left(\int \left(1+\frac{1}{x^3}-\frac{1}{x^2}+\frac{3}{x}-\frac{4}{1+x}\right)dx, x, e^x\right)$$
$$= -\frac{1}{8}e^{-2x}+\frac{e^{-x}}{4}+\frac{e^x}{4}+\frac{3x}{4}-\log\left(1+e^x\right)$$

Mathematica [A] time = 0.03, size = 33, normalized size = 0.85

$$\frac{1}{4}\left(3x-\frac{e^{-2x}}{2}+e^{-x}+e^x-4\log\left(e^x+1\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]^2/(1 + E^x),x]
```

```
[Out] (-1/2*1/E^(2*x) + E^(-x) + E^x + 3*x - 4*Log[1 + E^x])/4
```

fricas [B] time = 0.42, size = 95, normalized size = 2.44

$$\frac{6\,x\cosh(x)^2+2\,\cosh(x)^3+6\,(x+\cosh(x))\sinh(x)^2+2\,\sinh(x)^3-8\left(\cosh(x)^2+2\,\cosh(x)\sinh(x)+\sinh(x\right.}{8\left(\cosh(x)^2+2\,\cosh(x)\sinh(x)\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^2/(1+exp(x)),x, algorithm="fricas")
```

```
[Out] 1/8*(6*x*cosh(x)^2 + 2*cosh(x)^3 + 6*(x + cosh(x))*sinh(x)^2 + 2*sinh(x)^3
- 8*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2)*log(cosh(x) + sinh(x) + 1)
+ 2*(6*x*cosh(x) + 3*cosh(x)^2 + 1)*sinh(x) + 2*cosh(x) - 1)/(cosh(x)^2 + 2
*cosh(x)*sinh(x) + sinh(x)^2)
```

giac [A] time = 0.11, size = 27, normalized size = 0.69

$$\frac{1}{8}\left(2\,e^x-1\right)e^{(-2\,x)}+\frac{3}{4}\,x+\frac{1}{4}\,e^x-\log\left(e^x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^2/(1+exp(x)),x, algorithm="giac")
```

```
[Out] 1/8*(2*e^x - 1)*e^(-2*x) + 3/4*x + 1/4*e^x - log(e^x + 1)
```

maple [A] time = 0.08, size = 48, normalized size = 1.23

$$-\frac{1}{2\left(\tanh\left(\frac{x}{2}\right)-1\right)}+\frac{\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{4}-\frac{1}{2\left(\tanh\left(\frac{x}{2}\right)+1\right)^{2}}+\frac{1}{\tanh\left(\frac{x}{2}\right)+1}+\frac{3\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^2/(exp(x)+1),x)
```

```
[Out] -1/2/(tanh(1/2*x)-1)+1/4*ln(tanh(1/2*x)-1)-1/2/(tanh(1/2*x)+1)^2+1/(tanh(1/
2*x)+1)+3/4*ln(tanh(1/2*x)+1)
```

maxima [A] time = 0.31, size = 27, normalized size = 0.69

$$\frac{1}{8}\,(2\,e^{x}-1)e^{(-2\,x)}+\frac{3}{4}\,x+\frac{1}{4}\,e^{x}-\log\left(e^{x}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^2/(1+exp(x)),x, algorithm="maxima")
```

```
[Out] 1/8*(2*e^x - 1)*e^(-2*x) + 3/4*x + 1/4*e^x - log(e^x + 1)
```

mupad [B] time = 0.06, size = 27, normalized size = 0.69

$$\frac{3\,x}{4}+\frac{\mathrm{e}^{-x}}{4}-\frac{\mathrm{e}^{-2\,x}}{8}-\ln\left(\mathrm{e}^{x}+1\right)+\frac{\mathrm{e}^{x}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^2/(exp(x) + 1),x)
```

```
[Out] (3*x)/4 + exp(-x)/4 - exp(-2*x)/8 - log(exp(x) + 1) + exp(x)/4
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\cosh^{2}\left(x\right)}{e^{x}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)**2/(1+exp(x)),x)
```

```
[Out] Integral(cosh(x)**2/(exp(x) + 1), x)
```

3.1047 $\int \operatorname{sech}(x)\sqrt{1+\operatorname{sech}(x)}\,\tanh^3(x)\,dx$

Optimal. Leaf size=25

$$\frac{2}{7}(\operatorname{sech}(x)+1)^{7/2}-\frac{4}{5}(\operatorname{sech}(x)+1)^{5/2}$$

```
[Out] -4/5*(1+sech(x))^(5/2)+2/7*(1+sech(x))^(7/2)
```

Rubi [A] time = 0.10, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {4373, 1570, 1469, 627, 43}

$$\frac{2}{7}(\operatorname{sech}(x)+1)^{7/2}-\frac{4}{5}(\operatorname{sech}(x)+1)^{5/2}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[x]*Sqrt[1 + Sech[x]]*Tanh[x]^3,x]
```

```
[Out] (-4*(1 + Sech[x])^(5/2))/5 + (2*(1 + Sech[x])^(7/2))/7
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 627

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 1469

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q
_.), x_Symbol] :> Dist[1/n, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^n
], x] /; FreeQ[{a, c, d, e, m, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[Simplify
[m - n + 1], 0]
```

Rule 1570

```
Int[(x_)^(m_.)*((a_.) + (c_.)*(x_)^(mn2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_.))
^(q_.), x_Symbol] :> Int[x^(m - 2*n*p)*(d + e*x^n)^q*(c + a*x^(2*n))^p, x]
```

```
/; FreeQ[{a, c, d, e, m, n, q}, x] && EqQ[mn2, -2*n] && IntegerQ[p]
```

Rule 4373

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^(n_), x_Symbol] :> With[{d = Free
Factors[Cos[c*(a + b*x)], x]}, -Dist[(b*c*d^(n - 1))^(-1), Subst[Int[SubstF
or[(1 - d^2*x^2)^((n - 1)/2)/x^n, Cos[c*(a + b*x)]/d, u, x], x], x, Cos[c*(
a + b*x)]/d], x] /; FunctionOfQ[Cos[c*(a + b*x)]/d, u, x]] /; FreeQ[{a, b,
c}, x] && IntegerQ[(n - 1)/2] && NonsumQ[u] && (EqQ[F, Tan] || EqQ[F, tan])
```

Rubi steps

$$\begin{aligned}\int \operatorname{sech}(x)\sqrt{1+\operatorname{sech}(x)}\tanh^3(x)\,dx &= -\operatorname{Subst}\left(\int \frac{\sqrt{1+\frac{1}{x}}\left(1-x^2\right)}{x^4}\,dx, x, \cosh(x)\right)\\ &= -\operatorname{Subst}\left(\int \frac{\left(-1+\frac{1}{x^2}\right)\sqrt{1+\frac{1}{x}}}{x^2}\,dx, x, \cosh(x)\right)\\ &= \operatorname{Subst}\left(\int \sqrt{1+x}\left(-1+x^2\right)dx, x, \operatorname{sech}(x)\right)\\ &= \operatorname{Subst}\left(\int (-1+x)(1+x)^{3/2}\,dx, x, \operatorname{sech}(x)\right)\\ &= \operatorname{Subst}\left(\int \left(-2(1+x)^{3/2}+(1+x)^{5/2}\right)dx, x, \operatorname{sech}(x)\right)\\ &= -\frac{4}{5}(1+\operatorname{sech}(x))^{5/2}+\frac{2}{7}(1+\operatorname{sech}(x))^{7/2}\end{aligned}$$

Mathematica [A] time = 0.19, size = 30, normalized size = 1.20

$$-\frac{8}{35}\cosh^4\left(\frac{x}{2}\right)(9\cosh(x)-5)\operatorname{sech}^3(x)\sqrt{\operatorname{sech}(x)+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[x]*Sqrt[1 + Sech[x]]*Tanh[x]^3,x]
[Out] (-8*Cosh[x/2]^4*(-5 + 9*Cosh[x])*Sech[x]^3*Sqrt[1 + Sech[x]])/35
```

fricas [B] time = 0.41, size = 431, normalized size = 17.24

$$-\frac{2\left(9\cosh(x)^6+54\cosh(x)\sinh(x)^5+9\sinh(x)^6+27\left(5\cosh(x)^2+1\right)\sinh(x)^4+27\cosh(x)^4+36\left(5\cosh(\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)*(1+sech(x))^(1/2)*tanh(x)^3,x, algorithm="fricas")
```

```
[Out] -2/35*(9*cosh(x)^6 + 54*cosh(x)*sinh(x)^5 + 9*sinh(x)^6 + 27*(5*cosh(x)^2 +
 1)*sinh(x)^4 + 27*cosh(x)^4 + 36*(5*cosh(x)^3 + 3*cosh(x))*sinh(x)^3 + 27*
(5*cosh(x)^4 + 6*cosh(x)^2 + 1)*sinh(x)^2 + 27*cosh(x)^2 + 54*(cosh(x)^5 +
2*cosh(x)^3 + cosh(x))*sinh(x) + (9*cosh(x)^7 + 7*(9*cosh(x) + 5)*sinh(x)^6
 + 9*sinh(x)^7 + 35*cosh(x)^6 + 7*(27*cosh(x)^2 + 30*cosh(x) + 7)*sinh(x)^5
 + 49*cosh(x)^5 + 35*(9*cosh(x)^3 + 15*cosh(x)^2 + 7*cosh(x) + 1)*sinh(x)^4
 + 35*cosh(x)^4 + 35*(9*cosh(x)^4 + 20*cosh(x)^3 + 14*cosh(x)^2 + 4*cosh(x)
 + 1)*sinh(x)^3 + 35*cosh(x)^3 + 7*(27*cosh(x)^5 + 75*cosh(x)^4 + 70*cosh(x
)^3 + 30*cosh(x)^2 + 15*cosh(x) + 7)*sinh(x)^2 + 49*cosh(x)^2 + 7*(9*cosh(x
)^6 + 30*cosh(x)^5 + 35*cosh(x)^4 + 20*cosh(x)^3 + 15*cosh(x)^2 + 14*cosh(x
) + 5)*sinh(x) + 35*cosh(x) + 9)/sqrt(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(
x)^2 + 1) + 9)/(cosh(x)^6 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6 + 3*(5*cosh(x)^
2 + 1)*sinh(x)^4 + 3*cosh(x)^4 + 4*(5*cosh(x)^3 + 3*cosh(x))*sinh(x)^3 + 3*
(5*cosh(x)^4 + 6*cosh(x)^2 + 1)*sinh(x)^2 + 3*cosh(x)^2 + 6*(cosh(x)^5 + 2*
cosh(x)^3 + cosh(x))*sinh(x) + 1)
```

giac [B] time = 0.15, size = 46, normalized size = 1.84

$$-\frac{2\,(((((((9\,e^{x}+35)e^{x}+49)e^{x}+35)e^{x}+35)e^{x}+49)e^{x}+35)e^{x}+9)}{35\left(e^{(2\,x)}+1\right)^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)*(1+sech(x))^(1/2)*tanh(x)^3,x, algorithm="giac")
```

```
[Out] -2/35*(((((((9*e^x + 35)*e^x + 49)*e^x + 35)*e^x + 35)*e^x + 49)*e^x + 35)*
e^x + 9)/(e^(2*x) + 1)^(7/2)
```

maple [F] time = 0.39, size = 0, normalized size = 0.00

$$\int \mathrm{sech}(x)\sqrt{1+\mathrm{sech}(x)}\,\left(\tanh^{3}(x)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(x)*(1+sech(x))^(1/2)*tanh(x)^3,x)
```

```
[Out] int(sech(x)*(1+sech(x))^(1/2)*tanh(x)^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\mathrm{sech}(x)+1}\,\mathrm{sech}(x)\tanh(x)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)*(1+sech(x))^(1/2)*tanh(x)^3,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(sech(x) + 1)*sech(x)*tanh(x)^3, x)
```

mupad [B] time = 2.04, size = 148, normalized size = 5.92

$$\frac{\left(\frac{72\,e^x}{35}-\frac{24}{5}\right)\sqrt{\frac{1}{\frac{e^{-x}}{2}+\frac{e^x}{2}}+1}}{\left(e^x+1\right)\,{\left(e^{2\,x}+1\right)}^2}-\frac{\left(\frac{16\,e^x}{7}-\frac{16}{7}\right)\sqrt{\frac{1}{\frac{e^{-x}}{2}+\frac{e^x}{2}}+1}}{\left(e^x+1\right)\,{\left(e^{2\,x}+1\right)}^3}-\frac{\left(\frac{44\,e^x}{35}-4\right)\sqrt{\frac{1}{\frac{e^{-x}}{2}+\frac{e^x}{2}}+1}}{\left(e^x+1\right)\,\left(e^{2\,x}+1\right)}-\frac{\left(\frac{18\,e^x}{35}+2\right)\sqrt{\frac{1}{\frac{e^{-x}}{2}+\frac{e^x}{2}}+1}}{e^x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((tanh(x)^3*(1/cosh(x) + 1)^(1/2))/cosh(x),x)
```

```
[Out] (((72*exp(x))/35 - 24/5)*(1/(exp(-x)/2 + exp(x)/2) + 1)^(1/2))/((exp(x) + 1
)*(exp(2*x) + 1)^2) - (((16*exp(x))/7 - 16/7)*(1/(exp(-x)/2 + exp(x)/2) + 1
)^(1/2))/((exp(x) + 1)*(exp(2*x) + 1)^3) - (((44*exp(x))/35 - 4)*(1/(exp(-x
)/2 + exp(x)/2) + 1)^(1/2))/((exp(x) + 1)*(exp(2*x) + 1)) - (((18*exp(x))/3
5 + 2)*(1/(exp(-x)/2 + exp(x)/2) + 1)^(1/2))/(exp(x) + 1)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\operatorname{sech}(x)+1}\,\tanh^3(x)\operatorname{sech}(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)*(1+sech(x))**(1/2)*tanh(x)**3,x)
```

```
[Out] Integral(sqrt(sech(x) + 1)*tanh(x)**3*sech(x), x)
```

3.1048 $\int \coth^3(x)\text{csch}(x)\sqrt{1+\text{csch}(x)}\,dx$

Optimal. Leaf size=37

$$-\frac{2}{7}(\text{csch}(x)+1)^{7/2}+\frac{4}{5}(\text{csch}(x)+1)^{5/2}-\frac{4}{3}(\text{csch}(x)+1)^{3/2}$$

```
[Out] -4/3*(1+csch(x))^(3/2)+4/5*(1+csch(x))^(5/2)-2/7*(1+csch(x))^(7/2)
```

Rubi [A] time = 0.11, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {4372, 1570, 1469, 697}

$$-\frac{2}{7}(\text{csch}(x)+1)^{7/2}+\frac{4}{5}(\text{csch}(x)+1)^{5/2}-\frac{4}{3}(\text{csch}(x)+1)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]^3*Csch[x]*Sqrt[1 + Csch[x]],x]
```

```
[Out] (-4*(1 + Csch[x])^(3/2))/3 + (4*(1 + Csch[x])^(5/2))/5 - (2*(1 + Csch[x])^(
7/2))/7
```

Rule 697

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rule 1469

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q
_.), x_Symbol] :> Dist[1/n, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^n
], x] /; FreeQ[{a, c, d, e, m, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[Simplify
[m - n + 1], 0]
```

Rule 1570

```
Int[(x_)^(m_.)*((a_.) + (c_.)*(x_)^(mn2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_.))
^(q_.), x_Symbol] :> Int[x^(m - 2*n*p)*(d + e*x^n)^q*(c + a*x^(2*n))^p, x]
/; FreeQ[{a, c, d, e, m, n, q}, x] && EqQ[mn2, -2*n] && IntegerQ[p]
```

Rule 4372

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^(n_), x_Symbol] :> With[{d = Free
Factors[Sin[c*(a + b*x)], x]}, Dist[1/(b*c*d^(n - 1)), Subst[Int[SubstFor[(
```

```
1 - d^2*x^2)^((n - 1)/2)/x^n, Sin[c*(a + b*x)]/d, u, x], x], x, Sin[c*(a +
b*x)]/d], x] /; FunctionOfQ[Sin[c*(a + b*x)]/d, u, x]] /; FreeQ[{a, b, c},
x] && IntegerQ[(n - 1)/2] && NonsumQ[u] && (EqQ[F, Cot] || EqQ[F, cot])
```

Rubi steps

$$\begin{aligned}\int \coth^3(x)\operatorname{csch}(x)\sqrt{1+\operatorname{csch}(x)}\,dx &= \operatorname{Subst}\left(\int \frac{\sqrt{1+\frac{1}{x}}\,\left(1+x^2\right)}{x^4}\,dx, x, \sinh(x)\right)\\ &= \operatorname{Subst}\left(\int \frac{\left(1+\frac{1}{x^2}\right)\sqrt{1+\frac{1}{x}}}{x^2}\,dx, x, \sinh(x)\right)\\ &= -\operatorname{Subst}\left(\int \sqrt{1+x}\,\left(1+x^2\right)\,dx, x, \operatorname{csch}(x)\right)\\ &= -\operatorname{Subst}\left(\int \left(2\sqrt{1+x}-2(1+x)^{3/2}+(1+x)^{5/2}\right)dx, x, \operatorname{csch}(x)\right)\\ &= -\frac{4}{3}(1+\operatorname{csch}(x))^{3/2}+\frac{4}{5}(1+\operatorname{csch}(x))^{5/2}-\frac{2}{7}(1+\operatorname{csch}(x))^{7/2}\end{aligned}$$

Mathematica [A] time = 0.05, size = 34, normalized size = 0.92

$$-\frac{1}{210}\operatorname{csch}^3(x)\sqrt{\operatorname{csch}(x)+1}\,(-117\sinh(x)+43\sinh(3x)+62\cosh(2x)-2)$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]^3*Csch[x]*Sqrt[1 + Csch[x]],x]
```

```
[Out] -1/210*(Csch[x]^3*Sqrt[1 + Csch[x]]*(-2 + 62*Cosh[2*x] - 117*Sinh[x] + 43*S
inh[3*x]))
```

fricas [B] time = 0.41, size = 271, normalized size = 7.32

$$-\frac{2\left(43\cosh(x)^6+2\,(129\cosh(x)+31)\sinh(x)^5+43\sinh(x)^6+62\cosh(x)^5+\left(645\cosh(x)^2+310\cosh(x)-\right.\right.}{105\left(\cosh(x\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^3*csch(x)*(1+csch(x))^(1/2),x, algorithm="fricas")
```

```
[Out] -2/105*(43*cosh(x)^6 + 2*(129*cosh(x) + 31)*sinh(x)^5 + 43*sinh(x)^6 + 62*c
osh(x)^5 + (645*cosh(x)^2 + 310*cosh(x) - 117)*sinh(x)^4 - 117*cosh(x)^4 +
4*(215*cosh(x)^3 + 155*cosh(x)^2 - 117*cosh(x) - 1)*sinh(x)^3 - 4*cosh(x)^3
 + (645*cosh(x)^4 + 620*cosh(x)^3 - 702*cosh(x)^2 - 12*cosh(x) + 117)*sinh(
x)^2 + 117*cosh(x)^2 + 2*(129*cosh(x)^5 + 155*cosh(x)^4 - 234*cosh(x)^3 - 6
*cosh(x)^2 + 117*cosh(x) + 31)*sinh(x) + 62*cosh(x) - 43)*sqrt((sinh(x) + 1
)/sinh(x))/(cosh(x)^6 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6 + 3*(5*cosh(x)^2 -
1)*sinh(x)^4 - 3*cosh(x)^4 + 4*(5*cosh(x)^3 - 3*cosh(x))*sinh(x)^3 + 3*(5*c
osh(x)^4 - 6*cosh(x)^2 + 1)*sinh(x)^2 + 3*cosh(x)^2 + 6*(cosh(x)^5 - 2*cosh
(x)^3 + cosh(x))*sinh(x) - 1)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\operatorname{csch}(x)+1}\,\coth(x)^3\operatorname{csch}(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^3*csch(x)*(1+csch(x))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(csch(x) + 1)*coth(x)^3*csch(x), x)
```

maple [F] time = 0.45, size = 0, normalized size = 0.00

$$\int \left(\coth^{3}(x)\right)\operatorname{csch}(x)\sqrt{1+\operatorname{csch}(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^3*csch(x)*(1+csch(x))^(1/2),x)
```

```
[Out] int(coth(x)^3*csch(x)*(1+csch(x))^(1/2),x)
```

maxima [B] time = 0.42, size = 389, normalized size = 10.51

$$\frac{124\sqrt{-2e^{(-x)}+e^{(-2x)}-1}\,e^{(-x)}}{105\sqrt{e^{(-x)}+1}\sqrt{e^{(-x)}-1}\left(3e^{(-2x)}-3e^{(-4x)}+e^{(-6x)}-1\right)}-\frac{78\sqrt{-2e^{(-x)}+e^{(-2x)}-1}\,e^{(-2x)}}{35\sqrt{e^{(-x)}+1}\sqrt{e^{(-x)}-1}\left(3e^{(-2x)}-3e^{(-4x)}+e^{(-6x)}-1\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^3*csch(x)*(1+csch(x))^(1/2),x, algorithm="maxima")
```

```
[Out] 124/105*sqrt(-2*e^(-x) + e^(-2*x) - 1)*e^(-x)/(sqrt(e^(-x) + 1)*sqrt(e^(-x)
 - 1)*(3*e^(-2*x) - 3*e^(-4*x) + e^(-6*x) - 1)) - 78/35*sqrt(-2*e^(-x) + e^
(-2*x) - 1)*e^(-2*x)/(sqrt(e^(-x) + 1)*sqrt(e^(-x) - 1)*(3*e^(-2*x) - 3*e^(
-4*x) + e^(-6*x) - 1)) - 8/105*sqrt(-2*e^(-x) + e^(-2*x) - 1)*e^(-3*x)/(sqr
t(e^(-x) + 1)*sqrt(e^(-x) - 1)*(3*e^(-2*x) - 3*e^(-4*x) + e^(-6*x) - 1)) +
```

```
78/35*sqrt(-2*e^(-x) + e^(-2*x) - 1)*e^(-4*x)/(sqrt(e^(-x) + 1)*sqrt(e^(-x)
 - 1)*(3*e^(-2*x) - 3*e^(-4*x) + e^(-6*x) - 1)) + 124/105*sqrt(-2*e^(-x) +
e^(-2*x) - 1)*e^(-5*x)/(sqrt(e^(-x) + 1)*sqrt(e^(-x) - 1)*(3*e^(-2*x) - 3*e
^(-4*x) + e^(-6*x) - 1)) - 86/105*sqrt(-2*e^(-x) + e^(-2*x) - 1)*e^(-6*x)/(
sqrt(e^(-x) + 1)*sqrt(e^(-x) - 1)*(3*e^(-2*x) - 3*e^(-4*x) + e^(-6*x) - 1))
 + 86/105*sqrt(-2*e^(-x) + e^(-2*x) - 1)/(sqrt(e^(-x) + 1)*sqrt(e^(-x) - 1)
*(3*e^(-2*x) - 3*e^(-4*x) + e^(-6*x) - 1))
```

mupad [B] time = 2.41, size = 207, normalized size = 5.59

$$-\frac{8\sqrt{1-\frac{1}{\frac{e^{-x}}{2}-\frac{e^{x}}{2}}}}{35\left(e^{2x}-1\right)}-\frac{8\sqrt{1-\frac{1}{\frac{e^{-x}}{2}-\frac{e^{x}}{2}}}}{35\left(e^{4x}-2e^{2x}+1\right)}-\frac{86\sqrt{1-\frac{1}{\frac{e^{-x}}{2}-\frac{e^{x}}{2}}}}{105}-\frac{16e^{x}\sqrt{1-\frac{1}{\frac{e^{-x}}{2}-\frac{e^{x}}{2}}}}{7\left(3e^{2x}-3e^{4x}+e^{6x}-1\right)}-\frac{16e^{x}\sqrt{1-\frac{1}{\frac{e^{-x}}{2}-\frac{e^{x}}{2}}}}{7\left(e^{4x}-2e^{2x}+1\right)}-\frac{124}{1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((coth(x)^3*(1/sinh(x) + 1)^(1/2))/sinh(x),x)
```

```
[Out] - (8*(1 - 1/(exp(-x)/2 - exp(x)/2))^(1/2))/(35*(exp(2*x) - 1)) - (8*(1 - 1/
(exp(-x)/2 - exp(x)/2))^(1/2))/(35*(exp(4*x) - 2*exp(2*x) + 1)) - (86*(1 -
1/(exp(-x)/2 - exp(x)/2))^(1/2))/105 - (16*exp(x)*(1 - 1/(exp(-x)/2 - exp(x
)/2))^(1/2))/(7*(3*exp(2*x) - 3*exp(4*x) + exp(6*x) - 1)) - (16*exp(x)*(1 -
 1/(exp(-x)/2 - exp(x)/2))^(1/2))/(7*(exp(4*x) - 2*exp(2*x) + 1)) - (124*ex
p(x)*(1 - 1/(exp(-x)/2 - exp(x)/2))^(1/2))/(105*(exp(2*x) - 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\operatorname{csch}(x)+1}\,\coth^{3}(x)\operatorname{csch}(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)**3*csch(x)*(1+csch(x))**(1/2),x)
```

```
[Out] Integral(sqrt(csch(x) + 1)*coth(x)**3*csch(x), x)
```

3.1049 $\int \cosh^x(x)(\log(\cosh(x)) + x\tanh(x))\,dx$

Optimal. Leaf size=4

$$\cosh^x(x)$$

```
[Out] cosh(x)^x
```

Rubi [A] time = 0.14, antiderivative size = 4, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {6742, 2553}

$$\cosh^x(x)$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]^x*(Log[Cosh[x]] + x*Tanh[x]),x]
```

```
[Out] Cosh[x]^x
```

Rule 2553

```
Int[Log[u_]*(u_)^((a_.)*(x_)), x_Symbol] :> Simp[u^(a*x)/a, x] - Int[Simpli
fyIntegrand[x*u^(a*x - 1)*D[u, x], x], x] /; FreeQ[a, x] && InverseFunction
FreeQ[u, x]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}
\int \cosh^x(x)(\log(\cosh(x)) + x\tanh(x))\,dx &= \int \left(\cosh^x(x)\log(\cosh(x)) + x\cosh^{-1+x}(x)\sinh(x)\right)dx \\
&= \int \cosh^x(x)\log(\cosh(x))\,dx + \int x\cosh^{-1+x}(x)\sinh(x)\,dx \\
&= \cosh^x(x)
\end{aligned}$$

Mathematica [A] time = 0.08, size = 4, normalized size = 1.00

$$\cosh^x(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]^x*(Log[Cosh[x]] + x*Tanh[x]),x]
[Out] Cosh[x]^x
```

fricas [B] time = 0.41, size = 13, normalized size = 3.25

$$\cosh\left(x\log\left(\cosh(x)\right)\right) + \sinh\left(x\log\left(\cosh(x)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^x*(log(cosh(x))+x*tanh(x)),x, algorithm="fricas")
[Out] cosh(x*log(cosh(x))) + sinh(x*log(cosh(x)))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(x\tanh(x) + \log\left(\cosh(x)\right)\right)\cosh(x)^{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^x*(log(cosh(x))+x*tanh(x)),x, algorithm="giac")
[Out] integrate((x*tanh(x) + log(cosh(x)))*cosh(x)^x, x)
```

maple [A] time = 0.14, size = 5, normalized size = 1.25

$$\cosh^{x}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^x*(ln(cosh(x))+x*tanh(x)),x)
[Out] cosh(x)^x
```

maxima [B] time = 0.62, size = 21, normalized size = 5.25

$$e^{\left(-x^{2}-x\log(2)+x\log\left(e^{(2\,x)}+1\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^x*(log(cosh(x))+x*tanh(x)),x, algorithm="maxima")
[Out] e^(-x^2 - x*log(2) + x*log(e^(2*x) + 1))
```

mupad [B] time = 1.95, size = 4, normalized size = 1.00

$$\cosh(x)^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^x*(log(cosh(x)) + x*tanh(x)),x)
```

```
[Out] cosh(x)^x
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(x\tanh\left(x\right)+\log\left(\cosh\left(x\right)\right)\right)\cosh^{x}\left(x\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)**x*(ln(cosh(x))+x*tanh(x)),x)
```

```
[Out] Integral((x*tanh(x) + log(cosh(x)))*cosh(x)**x, x)
```

3.1050 $\int F^{a+bx}(\cosh(c+dx)+\sinh(c+dx))^n\,dx$

Optimal. Leaf size=27

$$\frac{F^{a+bx}\left(e^{c+dx}\right)^n}{b\log(F)+dn}$$

```
[Out] exp(d*x+c)^n*F^(b*x+a)/(d*n+b*ln(F))
```

Rubi [A] time = 0.09, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {5648, 2281, 2287, 2194}

$$\frac{F^{a+bx}\left(e^{c+dx}\right)^n}{b\log(F)+dn}$$

Antiderivative was successfully verified.

```
[In] Int[F^(a + b*x)*(Cosh[c + d*x] + Sinh[c + d*x])^n,x]
```

```
[Out] ((E^(c + d*x))^n*F^(a + b*x))/(d*n + b*Log[F])
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 2281

```
Int[(u_.)*((a_.)*(F_)^(v_))^(n_), x_Symbol] :> Dist[(a*F^v)^n/F^(n*v), Int[
u*F^(n*v), x], x] /; FreeQ[{F, a, n}, x] &&  !IntegerQ[n]
```

Rule 2287

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 5648

```
Int[(u_.)*(Cosh[v_]*(a_.) + (b_.)*Sinh[v_])^(n_.), x_Symbol] :> Int[u*(a*E^
((a*v)/b))^n, x] /; FreeQ[{a, b, n}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int F^{a+bx}(\cosh(c+dx)+\sinh(c+dx))^n\,dx &= \int \left(e^{c+dx}\right)^n F^{a+bx}\,dx\\ &= \left(e^{-n(c+dx)}\left(e^{c+dx}\right)^n\right)\int e^{n(c+dx)}F^{a+bx}\,dx\\ &= \left(e^{-n(c+dx)}\left(e^{c+dx}\right)^n\right)\int e^{cn+a\log(F)+x(dn+b\log(F))}\,dx\\ &= \frac{\left(e^{c+dx}\right)^n F^{a+bx}}{dn+b\log(F)}\end{aligned}$$

Mathematica [A] time = 0.09, size = 33, normalized size = 1.22

$$\frac{F^{a+bx}(\sinh(c+dx)+\cosh(c+dx))^n}{b\log(F)+dn}$$

Antiderivative was successfully verified.

```
[In] Integrate[F^(a + b*x)*(Cosh[c + d*x] + Sinh[c + d*x])^n,x]
```

```
[Out] (F^(a + b*x)*(Cosh[c + d*x] + Sinh[c + d*x])^n)/(d*n + b*Log[F])
```

fricas [B] time = 0.41, size = 70, normalized size = 2.59

$$\frac{(\cosh(dnx+cn)+\sinh(dnx+cn))\cosh\left((bx+a)\log(F)\right)+(\cosh(dnx+cn)+\sinh(dnx+cn))\sinh\left((bx+a)\log(F)\right)}{dn+b\log(F)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(b*x+a)*(cosh(d*x+c)+sinh(d*x+c))^n,x, algorithm="fricas")
```

```
[Out] ((cosh(d*n*x + c*n) + sinh(d*n*x + c*n))*cosh((b*x + a)*log(F)) + (cosh(d*n
*x + c*n) + sinh(d*n*x + c*n))*sinh((b*x + a)*log(F)))/(d*n + b*log(F))
```

giac [C] time = 0.17, size = 274, normalized size = 10.15

$$2\left(\frac{2\left(dn+b\log\left(|F|\right)\right)\cos\left(-\frac{1}{2}\pi bx\mathrm{sgn}(F)+\frac{1}{2}\pi bx-\frac{1}{2}\pi a\mathrm{sgn}(F)+\frac{1}{2}\pi a\right)}{\left(\pi b\mathrm{sgn}(F)-\pi b\right)^2+4\left(dn+b\log\left(|F|\right)\right)^2}-\frac{\left(\pi b\mathrm{sgn}(F)-\pi b\right)\sin\left(-\frac{1}{2}\pi bx\mathrm{sgn}(F\right.}{\left(\pi b\mathrm{sgn}(F)-\pi b\right)^2+4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(b*x+a)*(cosh(d*x+c)+sinh(d*x+c))^n,x, algorithm="giac")
```

```
[Out] 2*(2*(d*n + b*log(abs(F)))*cos(-1/2*pi*b*x*sgn(F) + 1/2*pi*b*x - 1/2*pi*a*s
gn(F) + 1/2*pi*a)/((pi*b*sgn(F) - pi*b)^2 + 4*(d*n + b*log(abs(F)))^2) - (p
i*b*sgn(F) - pi*b)*sin(-1/2*pi*b*x*sgn(F) + 1/2*pi*b*x - 1/2*pi*a*sgn(F) +
1/2*pi*a)/((pi*b*sgn(F) - pi*b)^2 + 4*(d*n + b*log(abs(F)))^2))*e^(c*n + (d
*n + b*log(abs(F)))*x + a*log(abs(F))) - 1/2*I*(-2*I*e^(1/2*I*pi*b*x*sgn(F)
 - 1/2*I*pi*b*x + 1/2*I*pi*a*sgn(F) - 1/2*I*pi*a)/(I*pi*b*sgn(F) - I*pi*b +
 2*d*n + 2*b*log(abs(F))) + 2*I*e^(-1/2*I*pi*b*x*sgn(F) + 1/2*I*pi*b*x - 1/
2*I*pi*a*sgn(F) + 1/2*I*pi*a)/(-I*pi*b*sgn(F) + I*pi*b + 2*d*n + 2*b*log(ab
s(F))))*e^(c*n + (d*n + b*log(abs(F)))*x + a*log(abs(F)))
```

maple [A] time = 0.02, size = 34, normalized size = 1.26

$$\frac{F^{bx+a} \left(\cosh \left(dx+c\right)+\sinh \left(dx+c\right)\right)^{n}}{dn+b\ln(F)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(F^(b*x+a)*(cosh(d*x+c)+sinh(d*x+c))^n,x)
[Out] 1/(d*n+b*ln(F))*F^(b*x+a)*(cosh(d*x+c)+sinh(d*x+c))^n
```

maxima [A] time = 0.33, size = 28, normalized size = 1.04

$$\frac{F^{a} e^{\left(dnx+bx\log(F)+cn\right)}}{dn+b\log(F)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(b*x+a)*(cosh(d*x+c)+sinh(d*x+c))^n,x, algorithm="maxima")
[Out] F^a*e^(d*n*x + b*x*log(F) + c*n)/(d*n + b*log(F))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.04

$$\int F^{a+b\,x}\,{\left(\cosh\left(c+d\,x\right)+\sinh\left(c+d\,x\right)\right)}^n \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(F^(a + b*x)*(cosh(c + d*x) + sinh(c + d*x))^n,x)
[Out] int(F^(a + b*x)*(cosh(c + d*x) + sinh(c + d*x))^n, x)
```

sympy [A] time = 4.80, size = 94, normalized size = 3.48

$$\begin{cases} \frac{F^{a} F^{bx} \left(\sinh\left(c+dx\right)+\cosh\left(c+dx\right)\right)^{n}}{b\log\left(F\right)+dn} & \text{for}\: b \neq -\frac{dn}{\log\left(F\right)} \\ F^{a} x \left(\sinh\left(c+dx\right)+\cosh\left(c+dx\right)\right)^{n} e^{-dnx} + \frac{F^{a} \left(\sinh\left(c+dx\right)+\cosh\left(c+dx\right)\right)^{n} e^{-dnx}}{dn} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F**(b*x+a)*(cosh(d*x+c)+sinh(d*x+c))**n,x)
```

```
[Out] Piecewise((F**a*F**(b*x)*(sinh(c + d*x) + cosh(c + d*x))**n/(b*log(F) + d*n
), Ne(b, -d*n/log(F))), (F**a*x*(sinh(c + d*x) + cosh(c + d*x))**n*exp(-d*n
*x) + F**a*(sinh(c + d*x) + cosh(c + d*x))**n*exp(-d*n*x)/(d*n), True))
```

3.1051 $\int F^{a+bx}(\cosh(c+dx)-\sinh(c+dx))^n\,dx$

Optimal. Leaf size=32

$$-\frac{F^{a+bx}\left(e^{-c-dx}\right)^n}{dn-b\log(F)}$$

```
[Out] -exp(-d*x-c)^n*F^(b*x+a)/(d*n-b*ln(F))
```

Rubi [A] time = 0.09, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {5648, 2281, 2287, 2194}

$$-\frac{F^{a+bx}\left(e^{-c-dx}\right)^n}{dn-b\log(F)}$$

Antiderivative was successfully verified.

```
[In] Int[F^(a + b*x)*(Cosh[c + d*x] - Sinh[c + d*x])^n,x]
```

```
[Out] -(((E^(-c - d*x))^n*F^(a + b*x))/(d*n - b*Log[F]))
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 2281

```
Int[(u_.)*((a_.)*(F_)^(v_))^(n_), x_Symbol] :> Dist[(a*F^v)^n/F^(n*v), Int[
u*F^(n*v), x], x] /; FreeQ[{F, a, n}, x] &&  !IntegerQ[n]
```

Rule 2287

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 5648

```
Int[(u_.)*(Cosh[v_]*(a_.) + (b_.)*Sinh[v_])^(n_.), x_Symbol] :> Int[u*(a*E^
((a*v)/b))^n, x] /; FreeQ[{a, b, n}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int F^{a+bx}(\cosh(c+dx)-\sinh(c+dx))^n\,dx = \int \left(e^{-c-dx}\right)^n F^{a+bx}\,dx$$
$$= \left(e^{-n(-c-dx)}\left(e^{-c-dx}\right)^n\right)\int e^{n(-c-dx)}F^{a+bx}\,dx$$
$$= \left(e^{-n(-c-dx)}\left(e^{-c-dx}\right)^n\right)\int e^{-cn+a\log(F)-x(dn-b\log(F))}\,dx$$
$$= -\frac{\left(e^{-c-dx}\right)^n F^{a+bx}}{dn-b\log(F)}$$

Mathematica [A] time = 0.07, size = 37, normalized size = 1.16

$$-\frac{F^{a+bx}(\cosh(c+dx)-\sinh(c+dx))^n}{dn-b\log(F)}$$

Antiderivative was successfully verified.

```
[In] Integrate[F^(a + b*x)*(Cosh[c + d*x] - Sinh[c + d*x])^n,x]
```

```
[Out] -((F^(a + b*x)*(Cosh[c + d*x] - Sinh[c + d*x])^n)/(d*n - b*Log[F]))
```

fricas [B] time = 0.41, size = 76, normalized size = 2.38

$$-\frac{(\cosh(dnx+cn)-\sinh(dnx+cn))\cosh\left((bx+a)\log(F)\right)+(\cosh(dnx+cn)-\sinh(dnx+cn))\sinh\left((bx+\right.}{dn-b\log(F)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(b*x+a)*(cosh(d*x+c)-sinh(d*x+c))^n,x, algorithm="fricas")
```

```
[Out] -((cosh(d*n*x + c*n) - sinh(d*n*x + c*n))*cosh((b*x + a)*log(F)) + (cosh(d*
n*x + c*n) - sinh(d*n*x + c*n))*sinh((b*x + a)*log(F)))/(d*n - b*log(F))
```

giac [C] time = 0.17, size = 282, normalized size = 8.81

$$-2\left(\frac{2\left(dn-b\log(|F|)\right)\cos\left(-\frac{1}{2}\pi bx\mathrm{sgn}(F)+\frac{1}{2}\pi bx-\frac{1}{2}\pi a\mathrm{sgn}(F)+\frac{1}{2}\pi a\right)}{\left(\pi b\mathrm{sgn}(F)-\pi b\right)^2+4\left(dn-b\log(|F|)\right)^2}+\frac{\left(\pi b\mathrm{sgn}(F)-\pi b\right)\sin\left(-\frac{1}{2}\pi bx\mathrm{sgn}(\right.}{\left(\pi b\mathrm{sgn}(F)-\pi b\right)^2+}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(b*x+a)*(cosh(d*x+c)-sinh(d*x+c))^n,x, algorithm="giac")
```

```
[Out] -2*(2*(d*n - b*log(abs(F)))*cos(-1/2*pi*b*x*sgn(F) + 1/2*pi*b*x - 1/2*pi*a*
sgn(F) + 1/2*pi*a)/((pi*b*sgn(F) - pi*b)^2 + 4*(d*n - b*log(abs(F)))^2) + (
pi*b*sgn(F) - pi*b)*sin(-1/2*pi*b*x*sgn(F) + 1/2*pi*b*x - 1/2*pi*a*sgn(F) +
 1/2*pi*a)/((pi*b*sgn(F) - pi*b)^2 + 4*(d*n - b*log(abs(F)))^2))*e^(-c*n -
(d*n - b*log(abs(F)))*x + a*log(abs(F))) - 1/2*I*(-2*I*e^(1/2*I*pi*b*x*sgn(
F) - 1/2*I*pi*b*x + 1/2*I*pi*a*sgn(F) - 1/2*I*pi*a)/(I*pi*b*sgn(F) - I*pi*b
 - 2*d*n + 2*b*log(abs(F))) + 2*I*e^(-1/2*I*pi*b*x*sgn(F) + 1/2*I*pi*b*x -
1/2*I*pi*a*sgn(F) + 1/2*I*pi*a)/(-I*pi*b*sgn(F) + I*pi*b - 2*d*n + 2*b*log(
abs(F))))*e^(-c*n - (d*n - b*log(abs(F)))*x + a*log(abs(F)))
```

maple [A] time = 0.02, size = 37, normalized size = 1.16

$$\frac{F^{bx+a} \left(\cosh \left(dx+c\right)-\sinh \left(dx+c\right)\right)^{n}}{b \ln(F)-dn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(F^(b*x+a)*(cosh(d*x+c)-sinh(d*x+c))^n,x)
```

```
[Out] 1/(b*ln(F)-d*n)*F^(b*x+a)*(cosh(d*x+c)-sinh(d*x+c))^n
```

maxima [A] time = 0.34, size = 36, normalized size = 1.12

$$-\frac{F^{a} e^{\left(-dnx+bx\log(F)\right)}}{dne^{(cn)}-be^{(cn)}\log(F)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(b*x+a)*(cosh(d*x+c)-sinh(d*x+c))^n,x, algorithm="maxima")
```

```
[Out] -F^a*e^(-d*n*x + b*x*log(F))/(d*n*e^(c*n) - b*e^(c*n)*log(F))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int F^{a+b\,x}\,{\left(\cosh\left(c+d\,x\right)-\sinh\left(c+d\,x\right)\right)}^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(F^(a + b*x)*(cosh(c + d*x) - sinh(c + d*x))^n,x)
```

```
[Out] int(F^(a + b*x)*(cosh(c + d*x) - sinh(c + d*x))^n, x)
```

sympy [A] time = 5.00, size = 92, normalized size = 2.88

$$\begin{cases} \frac{F^{a} F^{bx} \left(-\sinh\left(c+dx\right)+\cosh\left(c+dx\right)\right)^{n}}{b\log\left(F\right)-dn} & \text{for}\: b \neq \frac{dn}{\log\left(F\right)} \\ F^{a} x \left(-\sinh\left(c+dx\right)+\cosh\left(c+dx\right)\right)^{n} e^{dnx} - \frac{F^{a} \left(-\sinh\left(c+dx\right)+\cosh\left(c+dx\right)\right)^{n} e^{dnx}}{dn} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F**(b*x+a)*(cosh(d*x+c)-sinh(d*x+c))**n,x)
```

```
[Out] Piecewise((F**a*F**(b*x)*(-sinh(c + d*x) + cosh(c + d*x))**n/(b*log(F) - d*
n), Ne(b, d*n/log(F))), (F**a*x*(-sinh(c + d*x) + cosh(c + d*x))**n*exp(d*n
*x) - F**a*(-sinh(c + d*x) + cosh(c + d*x))**n*exp(d*n*x)/(d*n), True))
```

3.1052 $\int \frac{\cosh^4(a+bx)-\sinh^4(a+bx)}{\cosh^4(a+bx)+\sinh^4(a+bx)}\,dx$

Optimal. Leaf size=51

$$\frac{\tan^{-1}\left(\sqrt{2}\tanh(a+bx)+1\right)}{\sqrt{2}\,b}-\frac{\tan^{-1}\left(1-\sqrt{2}\tanh(a+bx)\right)}{\sqrt{2}\,b}$$

```
[Out] 1/2*arctan(-1+2^(1/2)*tanh(b*x+a))/b*2^(1/2)+1/2*arctan(1+2^(1/2)*tanh(b*x+
a))/b*2^(1/2)
```

Rubi [A] time = 0.17, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1162, 617, 204}

$$\frac{\tan^{-1}\left(\sqrt{2}\tanh(a+bx)+1\right)}{\sqrt{2}\,b}-\frac{\tan^{-1}\left(1-\sqrt{2}\tanh(a+bx)\right)}{\sqrt{2}\,b}$$

Antiderivative was successfully verified.

```
[In] Int[(Cosh[a + b*x]^4 - Sinh[a + b*x]^4)/(Cosh[a + b*x]^4 + Sinh[a + b*x]^4)
,x]
```

```
[Out] -(ArcTan[1 - Sqrt[2]*Tanh[a + b*x]]/(Sqrt[2]*b)) + ArcTan[1 + Sqrt[2]*Tanh[
a + b*x]]/(Sqrt[2]*b)
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
```

```
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rubi steps

$$\int \frac{\cosh^4(a+bx)-\sinh^4(a+bx)}{\cosh^4(a+bx)+\sinh^4(a+bx)}\,dx = \frac{\text{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx,x,\tanh(a+bx)\right)}{b}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2}\,dx,x,\tanh(a+bx)\right)}{2b} + \frac{\text{Subst}\left(\int \frac{1}{1+\sqrt{2}x+x^2}\,dx,x,\tanh(a+bx)\right)}{2b}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx,x,1-\sqrt{2}\tanh(a+bx)\right)}{\sqrt{2}b} - \frac{\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx,x,1+\sqrt{2}\tanh(a+bx)\right)}{\sqrt{2}b}$$

$$= -\frac{\tan^{-1}\left(1-\sqrt{2}\tanh(a+bx)\right)}{\sqrt{2}b} + \frac{\tan^{-1}\left(1+\sqrt{2}\tanh(a+bx)\right)}{\sqrt{2}b}$$

Mathematica [A] time = 0.03, size = 25, normalized size = 0.49

$$\frac{\tan^{-1}\left(\frac{\sinh(2a+2bx)}{\sqrt{2}}\right)}{\sqrt{2}b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cosh[a + b*x]^4 - Sinh[a + b*x]^4)/(Cosh[a + b*x]^4 + Sinh[a + b
*x]^4),x]
```

```
[Out] ArcTan[Sinh[2*a + 2*b*x]/Sqrt[2]]/(Sqrt[2]*b)
```

fricas [B] time = 0.42, size = 192, normalized size = 3.76

$$-\frac{\sqrt{2}\arctan\left(-\frac{\sqrt{2}\cosh(bx+a)^3+3\sqrt{2}\cosh(bx+a)\sinh(bx+a)^2+\sqrt{2}\sinh(bx+a)^3+\left(3\sqrt{2}\cosh(bx+a)^2-7\sqrt{2}\right)\sinh(bx+a)+7\sqrt{2}\cosh(bx+a)}{4\left(\cosh(bx+a)^3-3\cosh(bx+a)^2\sinh(bx+a)+3\cosh(bx+a)\sinh(bx+a)^2-\sinh(bx+a)^3\right)}\right)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cosh(b*x+a)^4-sinh(b*x+a)^4)/(cosh(b*x+a)^4+sinh(b*x+a)^4),x, al
gorithm="fricas")
```

```
[Out] -1/2*(sqrt(2)*arctan(-1/4*(sqrt(2)*cosh(b*x + a)^3 + 3*sqrt(2)*cosh(b*x + a
)*sinh(b*x + a)^2 + sqrt(2)*sinh(b*x + a)^3 + (3*sqrt(2)*cosh(b*x + a)^2 -
```

```
7*sqrt(2))*sinh(b*x + a) + 7*sqrt(2)*cosh(b*x + a))/(cosh(b*x + a)^3 - 3*co
sh(b*x + a)^2*sinh(b*x + a) + 3*cosh(b*x + a)*sinh(b*x + a)^2 - sinh(b*x +
a)^3)) + sqrt(2)*arctan(-1/4*(sqrt(2)*cosh(b*x + a) + sqrt(2)*sinh(b*x + a)
)/(cosh(b*x + a) - sinh(b*x + a))))/b
```

giac [A] time = 0.77, size = 34, normalized size = 0.67

$$\frac{\sqrt{2}\arctan\left(\frac{1}{4}\sqrt{2}\left(e^{(4bx+4a)}-1\right)e^{(-2bx-2a)}\right)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cosh(b*x+a)^4-sinh(b*x+a)^4)/(cosh(b*x+a)^4+sinh(b*x+a)^4),x, al
gorithm="giac")
```

```
[Out] 1/2*sqrt(2)*arctan(1/4*sqrt(2)*(e^(4*b*x + 4*a) - 1)*e^(-2*b*x - 2*a))/b
```

maple [C] time = 0.78, size = 138, normalized size = 2.71

$$\frac{i\sqrt{2}\ \ln\left(-2i\sqrt{2}\ \left(\tanh^3\left(\frac{bx}{2}+\frac{a}{2}\right)\right)+\tanh^4\left(\frac{bx}{2}+\frac{a}{2}\right)-2i\sqrt{2}\ \tanh\left(\frac{bx}{2}+\frac{a}{2}\right)-2\left(\tanh^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)+1\right)}{4b}-\frac{i\sqrt{2}\ \ln\left(2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cosh(b*x+a)^4-sinh(b*x+a)^4)/(cosh(b*x+a)^4+sinh(b*x+a)^4),x)
```

```
[Out] 1/4*I/b*2^(1/2)*ln(-2*I*2^(1/2)*tanh(1/2*b*x+1/2*a)^3+tanh(1/2*b*x+1/2*a)^4
-2*I*2^(1/2)*tanh(1/2*b*x+1/2*a)-2*tanh(1/2*b*x+1/2*a)^2+1)-1/4*I/b*2^(1/2)
*ln(2*I*2^(1/2)*tanh(1/2*b*x+1/2*a)^3+tanh(1/2*b*x+1/2*a)^4+2*I*2^(1/2)*tan
h(1/2*b*x+1/2*a)-2*tanh(1/2*b*x+1/2*a)^2+1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$2\int\frac{\left(e^{(-bx-a)}+e^{(-5bx-5a)}\right)e^{(-bx-a)}}{6e^{(-4bx-4a)}+e^{(-8bx-8a)}+1}\,dx+2\int\frac{e^{(6bx+6a)}}{e^{(8bx+8a)}+6e^{(4bx+4a)}+1}\,dx+2\int\frac{e^{(-6bx-6a)}}{6e^{(-4bx-4a)}+e^{(-8bx-8a)}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cosh(b*x+a)^4-sinh(b*x+a)^4)/(cosh(b*x+a)^4+sinh(b*x+a)^4),x, al
gorithm="maxima")
```

```
[Out] 2*integrate((e^(-b*x - a) + e^(-5*b*x - 5*a))*e^(-b*x - a)/(6*e^(-4*b*x - 4
*a) + e^(-8*b*x - 8*a) + 1), x) + 2*integrate(e^(6*b*x + 6*a)/(e^(8*b*x + 8
*a) + 6*e^(4*b*x + 4*a) + 1), x) + 2*integrate(e^(-6*b*x - 6*a)/(6*e^(-4*b*
x - 4*a) + e^(-8*b*x - 8*a) + 1), x)
```

mupad [B] time = 0.25, size = 77, normalized size = 1.51

$$\frac{\sqrt{2}\left(\operatorname{atan}\left(\frac{\sqrt{2}\,e^{2a}\,e^{2bx}\,\sqrt{b^2}}{4b}\right)+\operatorname{atan}\left(\frac{\sqrt{b^2}\left(\frac{56\,\sqrt{2}\,e^{2a}\,e^{2bx}}{b}+\frac{8\,\sqrt{2}\,e^{6a}\,e^{6bx}}{b}\right)}{32}\right)\right)}{2\,\sqrt{b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cosh(a + b*x)^4 - sinh(a + b*x)^4)/(cosh(a + b*x)^4 + sinh(a + b*x)^4)
,x)
```

```
[Out] (2^(1/2)*(atan((2^(1/2)*exp(2*a)*exp(2*b*x)*(b^2)^(1/2))/(4*b)) + atan(((b^
2)^(1/2)*((56*2^(1/2)*exp(2*a)*exp(2*b*x))/b + (8*2^(1/2)*exp(6*a)*exp(6*b*
x))/b))/32)))/(2*(b^2)^(1/2))
```

sympy [A] time = 22.71, size = 100, normalized size = 1.96

$$\begin{cases} \frac{x\left(-\sinh^4(a)+\cosh^4(a)\right)}{\sinh^4(a)+\cosh^4(a)} & \text{for } b=0 \\ -x & \text{for } a=\log\left(-ie^{-bx}\right) \vee a=\log\left(ie^{-bx}\right) \\ \frac{\sqrt{2}\operatorname{atan}\left(\frac{\sqrt{2}\sinh(a+bx)}{\cosh(a+bx)}-1\right)}{2b}+\frac{\sqrt{2}\operatorname{atan}\left(\frac{\sqrt{2}\sinh(a+bx)}{\cosh(a+bx)}+1\right)}{2b} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cosh(b*x+a)**4-sinh(b*x+a)**4)/(cosh(b*x+a)**4+sinh(b*x+a)**4),x
)
```

```
[Out] Piecewise((x*(-sinh(a)**4 + cosh(a)**4)/(sinh(a)**4 + cosh(a)**4), Eq(b, 0)
), (-x, Eq(a, log(I*exp(-b*x))) | Eq(a, log(-I*exp(-b*x)))), (sqrt(2)*atan(
sqrt(2)*sinh(a + b*x)/cosh(a + b*x) - 1)/(2*b) + sqrt(2)*atan(sqrt(2)*sinh(
a + b*x)/cosh(a + b*x) + 1)/(2*b), True))
```

3.1053 $\int \frac{\cosh^3(a+bx)-\sinh^3(a+bx)}{\cosh^3(a+bx)+\sinh^3(a+bx)}\,dx$

Optimal. Leaf size=47

$$-\frac{1}{3b(\tanh(a+bx)+1)}-\frac{4\tan^{-1}\left(\frac{1-2\tanh(a+bx)}{\sqrt{3}}\right)}{3\sqrt{3}\,b}$$

```
[Out] -4/9*arctan(1/3*(1-2*tanh(b*x+a))*3^(1/2))/b*3^(1/2)-1/3/b/(1+tanh(b*x+a))
```

Rubi [A] time = 0.33, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {2074, 618, 204}

$$-\frac{1}{3b(\tanh(a+bx)+1)}-\frac{4\tan^{-1}\left(\frac{1-2\tanh(a+bx)}{\sqrt{3}}\right)}{3\sqrt{3}\,b}$$

Antiderivative was successfully verified.

```
[In] Int[(Cosh[a + b*x]^3 - Sinh[a + b*x]^3)/(Cosh[a + b*x]^3 + Sinh[a + b*x]^3)
,x]
```

```
[Out] (-4*ArcTan[(1 - 2*Tanh[a + b*x])/Sqrt[3]])/(3*Sqrt[3]*b) - 1/(3*b*(1 + Tanh
[a + b*x]))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2074

```
Int[(P_)^(p_)*(Q_)^(q_.), x_Symbol] :> With[{PP = Factor[P]}, Int[ExpandInt
egrand[PP^p*Q^q, x], x] /;  !SumQ[NonfreeFactors[PP, x]]] /; FreeQ[q, x] &&
 PolyQ[P, x] && PolyQ[Q, x] && IntegerQ[p] && NeQ[P, x]
```

Rubi steps

$$\int \frac{\cosh^3(a+bx)-\sinh^3(a+bx)}{\cosh^3(a+bx)+\sinh^3(a+bx)}\,dx = \frac{\text{Subst}\left(\int \frac{1+x+x^2}{1+x+x^3+x^4}\,dx,x,\tanh(a+bx)\right)}{b}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{1}{3(1+x)^2}+\frac{2}{3(1-x+x^2)}\right)dx,x,\tanh(a+bx)\right)}{b}$$

$$= -\frac{1}{3b(1+\tanh(a+bx))}+\frac{2\,\text{Subst}\left(\int \frac{1}{1-x+x^2}\,dx,x,\tanh(a+bx)\right)}{3b}$$

$$= -\frac{1}{3b(1+\tanh(a+bx))}-\frac{4\,\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,-1+2\tanh(a+bx)\right)}{3b}$$

$$= -\frac{4\tan^{-1}\left(\frac{1-2\tanh(a+bx)}{\sqrt{3}}\right)}{3\sqrt{3}\,b}-\frac{1}{3b(1+\tanh(a+bx))}$$

Mathematica [B] time = 1.35, size = 115, normalized size = 2.45

$$\frac{(\sinh(a+bx)-\cosh(a+bx))\left(\cosh(a+bx)\left(8\sqrt{3}\tan^{-1}\left(\frac{\text{sech}(bx)(\cosh(2a+bx)-2\sinh(2a+bx))}{\sqrt{3}}\right)+3\right)+\sinh(a+bx)\left(\right.\right.}{18b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cosh[a + b*x]^3 - Sinh[a + b*x]^3)/(Cosh[a + b*x]^3 + Sinh[a + b
*x]^3),x]
```

```
[Out] ((-Cosh[a + b*x] + Sinh[a + b*x])*((3 + 8*Sqrt[3]*ArcTan[(Sech[b*x]*(Cosh[2
*a + b*x] - 2*Sinh[2*a + b*x]))/Sqrt[3]])*Cosh[a + b*x] + (-3 + 8*Sqrt[3]*A
rcTan[(Sech[b*x]*(Cosh[2*a + b*x] - 2*Sinh[2*a + b*x]))/Sqrt[3]])*Sinh[a +
b*x]))/(18*b)
```

fricas [B] time = 0.45, size = 127, normalized size = 2.70

$$-\frac{8\left(\sqrt{3}\cosh(bx+a)^2+2\sqrt{3}\cosh(bx+a)\sinh(bx+a)+\sqrt{3}\sinh(bx+a)^2\right)\arctan\left(-\frac{\sqrt{3}\cosh(bx+a)+\sqrt{3}\sinh(b}{3(\cosh(bx+a)-\sinh(bx+}\right.}{18\left(b\cosh(bx+a)^2+2b\cosh(bx+a)\sinh(bx+a)+b\sinh(bx+a)^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cosh(b*x+a)^3-sinh(b*x+a)^3)/(cosh(b*x+a)^3+sinh(b*x+a)^3),x, al
gorithm="fricas")
```

```
[Out] -1/18*(8*(sqrt(3)*cosh(b*x + a)^2 + 2*sqrt(3)*cosh(b*x + a)*sinh(b*x + a) +
 sqrt(3)*sinh(b*x + a)^2)*arctan(-1/3*(sqrt(3)*cosh(b*x + a) + sqrt(3)*sinh
(b*x + a))/(cosh(b*x + a) - sinh(b*x + a))) + 3)/(b*cosh(b*x + a)^2 + 2*b*c
osh(b*x + a)*sinh(b*x + a) + b*sinh(b*x + a)^2)
```

giac [A] time = 0.22, size = 37, normalized size = 0.79

$$\frac{8\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}e^{(2bx+2a)}\right) - 3e^{(-2bx-2a)}}{18b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cosh(b*x+a)^3-sinh(b*x+a)^3)/(cosh(b*x+a)^3+sinh(b*x+a)^3),x, al
gorithm="giac")
```

```
[Out] 1/18*(8*sqrt(3)*arctan(1/3*sqrt(3)*e^(2*b*x + 2*a)) - 3*e^(-2*b*x - 2*a))/b
```

maple [C] time = 1.12, size = 120, normalized size = 2.55

$$\frac{2i\sqrt{3}\ \ln\left(\tanh^2\left(\frac{bx}{2}+\frac{a}{2}\right)+\left(-i\sqrt{3}\ -1\right)\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)+1\right)}{9b}-\frac{2i\sqrt{3}\ \ln\left(\tanh^2\left(\frac{bx}{2}+\frac{a}{2}\right)+\left(i\sqrt{3}\ -1\right)\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)\right.}{9b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cosh(b*x+a)^3-sinh(b*x+a)^3)/(cosh(b*x+a)^3+sinh(b*x+a)^3),x)
```

```
[Out] 2/9*I/b*3^(1/2)*ln(tanh(1/2*b*x+1/2*a)^2+(-I*3^(1/2)-1)*tanh(1/2*b*x+1/2*a)
+1)-2/9*I/b*3^(1/2)*ln(tanh(1/2*b*x+1/2*a)^2+(I*3^(1/2)-1)*tanh(1/2*b*x+1/2
*a)+1)-2/3/b/(tanh(1/2*b*x+1/2*a)+1)^2+2/3/b/(tanh(1/2*b*x+1/2*a)+1)
```

maxima [B] time = 0.46, size = 93, normalized size = 1.98

$$\frac{4\left(\sqrt{3}\arctan\left(\frac{1}{6}\cdot 3^{\frac{3}{4}}\sqrt{2}\left(2\sqrt{3}e^{(-bx-a)}+3^{\frac{1}{4}}\sqrt{2}\right)\right)-\sqrt{3}\arctan\left(\frac{1}{6}\cdot 3^{\frac{3}{4}}\sqrt{2}\left(2\sqrt{3}e^{(-bx-a)}-3^{\frac{1}{4}}\sqrt{2}\right)\right)\right)}{9b}-\frac{e^{(-2bx-2a)}}{6b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cosh(b*x+a)^3-sinh(b*x+a)^3)/(cosh(b*x+a)^3+sinh(b*x+a)^3),x, al
gorithm="maxima")
```

```
[Out] 4/9*(sqrt(3)*arctan(1/6*3^(3/4)*sqrt(2)*(2*sqrt(3)*e^(-b*x - a) + 3^(1/4)*s
qrt(2))) - sqrt(3)*arctan(1/6*3^(3/4)*sqrt(2)*(2*sqrt(3)*e^(-b*x - a) - 3^(
1/4)*sqrt(2))))/b - 1/6*e^(-2*b*x - 2*a)/b
```

mupad [B] time = 1.82, size = 48, normalized size = 1.02

$$\frac{4\sqrt{3}\,\mathrm{atan}\left(\frac{\sqrt{3}\,e^{2a}\,e^{2bx}\,\sqrt{b^2}}{3b}\right)}{9\sqrt{b^2}}-\frac{e^{-2a-2bx}}{6b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cosh(a + b*x)^3 - sinh(a + b*x)^3)/(cosh(a + b*x)^3 + sinh(a + b*x)^3)
,x)
```

```
[Out] (4*3^(1/2)*atan((3^(1/2)*exp(2*a)*exp(2*b*x)*(b^2)^(1/2))/(3*b)))/(9*(b^2)^
(1/2)) - exp(- 2*a - 2*b*x)/(6*b)
```

sympy [A] time = 15.24, size = 197, normalized size = 4.19

$$\begin{cases} -x & \text{for}\ a = \mathrm{lo} \\ \frac{x\left(-\sinh^3(a)+\cosh^3(a)\right)}{\sinh^3(a)+\cosh^3(a)} & \text{for}\ b = 0 \\ \frac{4\sqrt{3}\sinh(a+bx)\,\mathrm{atan}\left(\frac{2\sqrt{3}\sinh(a+bx)}{3\cosh(a+bx)}-\frac{\sqrt{3}}{3}\right)}{9b\sinh(a+bx)+9b\cosh(a+bx)}+\frac{3\sinh(a+bx)}{9b\sinh(a+bx)+9b\cosh(a+bx)}+\frac{4\sqrt{3}\cosh(a+bx)\,\mathrm{atan}\left(\frac{2\sqrt{3}\sinh(a+bx)}{3\cosh(a+bx)}-\frac{\sqrt{3}}{3}\right)}{9b\sinh(a+bx)+9b\cosh(a+bx)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cosh(b*x+a)**3-sinh(b*x+a)**3)/(cosh(b*x+a)**3+sinh(b*x+a)**3),x
)
```

```
[Out] Piecewise((-x, Eq(a, log(I*exp(-b*x))) | Eq(a, log(-I*exp(-b*x)))), (x*(-si
nh(a)**3 + cosh(a)**3)/(sinh(a)**3 + cosh(a)**3), Eq(b, 0)), (4*sqrt(3)*sin
h(a + b*x)*atan(2*sqrt(3)*sinh(a + b*x)/(3*cosh(a + b*x)) - sqrt(3)/3)/(9*b
*sinh(a + b*x) + 9*b*cosh(a + b*x)) + 3*sinh(a + b*x)/(9*b*sinh(a + b*x) +
9*b*cosh(a + b*x)) + 4*sqrt(3)*cosh(a + b*x)*atan(2*sqrt(3)*sinh(a + b*x)/(
3*cosh(a + b*x)) - sqrt(3)/3)/(9*b*sinh(a + b*x) + 9*b*cosh(a + b*x)), True
))
```

3.1054 $\int \frac{\cosh^2(a+bx)-\sinh^2(a+bx)}{\cosh^2(a+bx)+\sinh^2(a+bx)} \, dx$

Optimal. Leaf size=11

$$\frac{\tan^{-1}(\tanh(a+bx))}{b}$$

```
[Out] arctan(tanh(b*x+a))/b
```

Rubi [A] time = 0.06, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.051, Rules used = {4380, 203}

$$\frac{\tan^{-1}(\tanh(a+bx))}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(Cosh[a + b*x]^2 - Sinh[a + b*x]^2)/(Cosh[a + b*x]^2 + Sinh[a + b*x]^2)
,x]
```

```
[Out] ArcTan[Tanh[a + b*x]]/b
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 4380

```
Int[(u_.)*((a_.) + cos[(d_.) + (e_.)*(x_)]^2*(b_.) + (c_.)*sin[(d_.) + (e_.
)*(x_)]^2)^(p_.), x_Symbol] :> Dist[(a + c)^p, Int[ActivateTrig[u], x], x]
/; FreeQ[{a, b, c, d, e, p}, x] && EqQ[b - c, 0]
```

Rubi steps

$$\begin{aligned} \int \frac{\cosh^2(a+bx)-\sinh^2(a+bx)}{\cosh^2(a+bx)+\sinh^2(a+bx)} \, dx &= \int \frac{1}{\cosh^2(a+bx)+\sinh^2(a+bx)} \, dx \\ &= \frac{\text{Subst}\left(\int \frac{1}{1+x^2} \, dx, x, \tanh(a+bx)\right)}{b} \\ &= \frac{\tan^{-1}(\tanh(a+bx))}{b} \end{aligned}$$

Mathematica [A] time = 0.00, size = 17, normalized size = 1.55

$$\frac{\tan^{-1}(\sinh(2a+2bx))}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cosh[a + b*x]^2 - Sinh[a + b*x]^2)/(Cosh[a + b*x]^2 + Sinh[a + b
*x]^2),x]
```

```
[Out] ArcTan[Sinh[2*a + 2*b*x]]/(2*b)
```

fricas [B] time = 0.40, size = 38, normalized size = 3.45

$$-\frac{\arctan\left(-\frac{\cosh(bx+a)+\sinh(bx+a)}{\cosh(bx+a)-\sinh(bx+a)}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cosh(b*x+a)^2-sinh(b*x+a)^2)/(cosh(b*x+a)^2+sinh(b*x+a)^2),x, al
gorithm="fricas")
```

```
[Out] -arctan(-(cosh(b*x + a) + sinh(b*x + a))/(cosh(b*x + a) - sinh(b*x + a)))/b
```

giac [B] time = 0.16, size = 44, normalized size = 4.00

$$-\frac{\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\,e^{(bx+a)}\right)\right)-\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\,e^{(bx+a)}\right)\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cosh(b*x+a)^2-sinh(b*x+a)^2)/(cosh(b*x+a)^2+sinh(b*x+a)^2),x, al
gorithm="giac")
```

```
[Out] -(arctan(1/2*sqrt(2)*(sqrt(2) + 2*e^(b*x + a))) - arctan(-1/2*sqrt(2)*(sqrt
(2) - 2*e^(b*x + a))))/b
```

maple [B] time = 0.63, size = 148, normalized size = 13.45

$$\frac{2\sqrt{2}\ \arctan\left(\frac{2\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)}{-2+2\sqrt{2}}\right)}{b\left(-2+2\sqrt{2}\right)}-\frac{2\arctan\left(\frac{2\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)}{-2+2\sqrt{2}}\right)}{b\left(-2+2\sqrt{2}\right)}-\frac{2\sqrt{2}\ \arctan\left(\frac{2\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)}{2+2\sqrt{2}}\right)}{b\left(2+2\sqrt{2}\right)}-\frac{2\arctan\left(\frac{2\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)}{2+2\sqrt{2}}\right)}{b\left(2+2\sqrt{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cosh(b*x+a)^2-sinh(b*x+a)^2)/(cosh(b*x+a)^2+sinh(b*x+a)^2),x)
```

```
[Out] 2/b*2^(1/2)/(-2+2*2^(1/2))*arctan(2*tanh(1/2*b*x+1/2*a)/(-2+2*2^(1/2)))-2/b
/(-2+2*2^(1/2))*arctan(2*tanh(1/2*b*x+1/2*a)/(-2+2*2^(1/2)))-2/b*2^(1/2)/(2
+2*2^(1/2))*arctan(2*tanh(1/2*b*x+1/2*a)/(2+2*2^(1/2)))-2/b/(2+2*2^(1/2))*a
rctan(2*tanh(1/2*b*x+1/2*a)/(2+2*2^(1/2)))
```

maxima [B] time = 0.45, size = 49, normalized size = 4.45

$$\frac{\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2e^{(-bx-a)}\right)\right)-\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2e^{(-bx-a)}\right)\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cosh(b*x+a)^2-sinh(b*x+a)^2)/(cosh(b*x+a)^2+sinh(b*x+a)^2),x, al
gorithm="maxima")
```

```
[Out] (arctan(1/2*sqrt(2)*(sqrt(2) + 2*e^(-b*x - a))) - arctan(-1/2*sqrt(2)*(sqrt
(2) - 2*e^(-b*x - a))))/b
```

mupad [B] time = 0.09, size = 25, normalized size = 2.27

$$\frac{\operatorname{atan}\left(\frac{e^{2a}\,e^{2bx}\,\sqrt{b^2}}{b}\right)}{\sqrt{b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cosh(a + b*x)^2 - sinh(a + b*x)^2)/(cosh(a + b*x)^2 + sinh(a + b*x)^2)
,x)
```

```
[Out] atan((exp(2*a)*exp(2*b*x)*(b^2)^(1/2))/b)/(b^2)^(1/2)
```

sympy [A] time = 2.24, size = 56, normalized size = 5.09

$$\begin{cases} \frac{x\left(-\sinh^2(a)+\cosh^2(a)\right)}{\sinh^2(a)+\cosh^2(a)} & \text{for } b=0 \\ -x & \text{for } a=\log\left(-ie^{-bx}\right) \vee a=\log\left(ie^{-bx}\right) \\ \frac{\operatorname{atan}\left(\frac{\sinh(a+bx)}{\cosh(a+bx)}\right)}{b} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cosh(b*x+a)**2-sinh(b*x+a)**2)/(cosh(b*x+a)**2+sinh(b*x+a)**2),x
)
```

```
[Out] Piecewise((x*(-sinh(a)**2 + cosh(a)**2)/(sinh(a)**2 + cosh(a)**2), Eq(b, 0)
), (-x, Eq(a, log(I*exp(-b*x))) | Eq(a, log(-I*exp(-b*x)))), (atan(sinh(a +
 b*x)/cosh(a + b*x))/b, True))
```

3.1055 $\int \frac{\cosh(a+bx)-\sinh(a+bx)}{\cosh(a+bx)+\sinh(a+bx)}\,dx$

Optimal. Leaf size=22

$$-\frac{1}{2b(\sinh(a+bx)+\cosh(a+bx))^2}$$

```
[Out] -1/2/b/(cosh(b*x+a)+sinh(b*x+a))^2
```

Rubi [A] time = 0.05, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.032, Rules used = {4385}

$$-\frac{1}{2b(\sinh(a+bx)+\cosh(a+bx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Cosh[a + b*x] - Sinh[a + b*x])/(Cosh[a + b*x] + Sinh[a + b*x]),x]
```

```
[Out] -1/(2*b*(Cosh[a + b*x] + Sinh[a + b*x])^2)
```

Rule 4385

```
Int[(u_)*(y_)^(m_.), x_Symbol] :> With[{q = DerivativeDivides[ActivateTrig[
y], ActivateTrig[u], x]}, Simp[(q*ActivateTrig[y^(m + 1)])/(m + 1), x] /;
!FalseQ[q]] /; FreeQ[m, x] && NeQ[m, -1] &&  !InertTrigFreeQ[u]
```

Rubi steps

$$\int \frac{\cosh(a+bx)-\sinh(a+bx)}{\cosh(a+bx)+\sinh(a+bx)}\,dx = -\frac{1}{2b(\cosh(a+bx)+\sinh(a+bx))^2}$$

Mathematica [B] time = 0.02, size = 65, normalized size = 2.95

$$-\frac{\sinh(2a)\sinh(2bx)}{2b}-\frac{\cosh(2a)\cosh(2bx)}{2b}+\frac{\sinh(2a)\cosh(2bx)}{2b}+\frac{\cosh(2a)\sinh(2bx)}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cosh[a + b*x] - Sinh[a + b*x])/(Cosh[a + b*x] + Sinh[a + b*x]),x
]
```

```
[Out] -1/2*(Cosh[2*a]*Cosh[2*b*x])/b + (Cosh[2*b*x]*Sinh[2*a])/(2*b) + (Cosh[2*a]
*Sinh[2*b*x])/(2*b) - (Sinh[2*a]*Sinh[2*b*x])/(2*b)
```

fricas [A] time = 0.41, size = 40, normalized size = 1.82

$$-\frac{1}{2\left(b\cosh\left(bx+a\right)^{2}+2\,b\cosh\left(bx+a\right)\sinh\left(bx+a\right)+b\sinh\left(bx+a\right)^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cosh(b*x+a)-sinh(b*x+a))/(cosh(b*x+a)+sinh(b*x+a)),x, algorithm=
"fricas")
```

```
[Out] -1/2/(b*cosh(b*x + a)^2 + 2*b*cosh(b*x + a)*sinh(b*x + a) + b*sinh(b*x + a)
^2)
```

giac [A] time = 0.12, size = 14, normalized size = 0.64

$$-\frac{e^{(-2\,bx-2\,a)}}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cosh(b*x+a)-sinh(b*x+a))/(cosh(b*x+a)+sinh(b*x+a)),x, algorithm=
"giac")
```

```
[Out] -1/2*e^(-2*b*x - 2*a)/b
```

maple [A] time = 0.02, size = 36, normalized size = 1.64

$$-\frac{\cosh\left(bx+a\right)-\sinh\left(bx+a\right)}{2b\left(\cosh\left(bx+a\right)+\sinh\left(bx+a\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cosh(b*x+a)-sinh(b*x+a))/(cosh(b*x+a)+sinh(b*x+a)),x)
```

```
[Out] -1/2/b*(cosh(b*x+a)-sinh(b*x+a))/(cosh(b*x+a)+sinh(b*x+a))
```

maxima [A] time = 0.36, size = 14, normalized size = 0.64

$$-\frac{e^{(-2\,bx-2\,a)}}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cosh(b*x+a)-sinh(b*x+a))/(cosh(b*x+a)+sinh(b*x+a)),x, algorithm=
"maxima")
```

```
[Out] -1/2*e^(-2*b*x - 2*a)/b
```

mupad [B] time = 0.10, size = 14, normalized size = 0.64

$$-\frac{e^{-2\,a-2\,b\,x}}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cosh(a + b*x) - sinh(a + b*x))/(cosh(a + b*x) + sinh(a + b*x)),x)
```

```
[Out] -exp(- 2*a - 2*b*x)/(2*b)
```

sympy [A] time = 0.41, size = 37, normalized size = 1.68

$$\begin{cases} \frac{\sinh\left(a+bx\right)}{b\sinh\left(a+bx\right)+b\cosh\left(a+bx\right)} & \text{for}\ b \neq 0 \\ \frac{x(-\sinh\left(a\right)+\cosh\left(a\right))}{\sinh\left(a\right)+\cosh\left(a\right)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cosh(b*x+a)-sinh(b*x+a))/(cosh(b*x+a)+sinh(b*x+a)),x)
```

```
[Out] Piecewise((sinh(a + b*x)/(b*sinh(a + b*x) + b*cosh(a + b*x)), Ne(b, 0)), (x
*(-sinh(a) + cosh(a))/(sinh(a) + cosh(a)), True))
```

3.1056 $\int \frac{-\mathbf{csch}(a+bx)+\mathbf{sech}(a+bx)}{\mathbf{csch}(a+bx)+\mathbf{sech}(a+bx)}\,dx$

Optimal. Leaf size=14

$$\frac{1}{b(\tanh(a+bx)+1)}$$

```
[Out] 1/b/(1+tanh(b*x+a))
```

Rubi [A] time = 0.21, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.032, Rules used = {32}

$$\frac{1}{b(\tanh(a+bx)+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(-Csch[a + b*x] + Sech[a + b*x])/(Csch[a + b*x] + Sech[a + b*x]),x]
```

```
[Out] 1/(b*(1 + Tanh[a + b*x]))
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{-\operatorname{csch}(a+bx)+\operatorname{sech}(a+bx)}{\operatorname{csch}(a+bx)+\operatorname{sech}(a+bx)}\,dx &= -\frac{\operatorname{Subst}\left(\int \frac{1}{(1+x)^2}\,dx, x, \tanh(a+bx)\right)}{b}\\ &= \frac{1}{b(1+\tanh(a+bx))}\end{aligned}$$

Mathematica [B] time = 0.02, size = 65, normalized size = 4.64

$$\frac{\sinh(2a)\sinh(2bx)}{2b}+\frac{\cosh(2a)\cosh(2bx)}{2b}-\frac{\sinh(2a)\cosh(2bx)}{2b}-\frac{\cosh(2a)\sinh(2bx)}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-Csch[a + b*x] + Sech[a + b*x])/(Csch[a + b*x] + Sech[a + b*x]),
x]
```

```
[Out] (Cosh[2*a]*Cosh[2*b*x])/(2*b) - (Cosh[2*b*x]*Sinh[2*a])/(2*b) - (Cosh[2*a]*
Sinh[2*b*x])/(2*b) + (Sinh[2*a]*Sinh[2*b*x])/(2*b)
```

fricas [B] time = 0.41, size = 40, normalized size = 2.86

$$\frac{1}{2\left(b\cosh\left(bx+a\right)^{2}+2\,b\cosh\left(bx+a\right)\sinh\left(bx+a\right)+b\sinh\left(bx+a\right)^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-csch(b*x+a)+sech(b*x+a))/(csch(b*x+a)+sech(b*x+a)),x, algorithm
="fricas")
```

```
[Out] 1/2/(b*cosh(b*x + a)^2 + 2*b*cosh(b*x + a)*sinh(b*x + a) + b*sinh(b*x + a)^
2)
```

giac [A] time = 0.15, size = 14, normalized size = 1.00

$$\frac{e^{\left(-2\,bx-2\,a\right)}}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-csch(b*x+a)+sech(b*x+a))/(csch(b*x+a)+sech(b*x+a)),x, algorithm
="giac")
```

```
[Out] 1/2*e^(-2*b*x - 2*a)/b
```

maple [B] time = 1.08, size = 36, normalized size = 2.57

$$\frac{-\frac{2}{\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)+1}+\frac{2}{\left(\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)+1\right)^{2}}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-csch(b*x+a)+sech(b*x+a))/(csch(b*x+a)+sech(b*x+a)),x)
```

```
[Out] 1/b*(-2/(tanh(1/2*b*x+1/2*a)+1)+2/(tanh(1/2*b*x+1/2*a)+1)^2)
```

maxima [A] time = 0.35, size = 14, normalized size = 1.00

$$\frac{e^{\left(-2\,bx-2\,a\right)}}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-csch(b*x+a)+sech(b*x+a))/(csch(b*x+a)+sech(b*x+a)),x, algorithm
="maxima")
```

```
[Out] 1/2*e^(-2*b*x - 2*a)/b
```

mupad [B] time = 0.08, size = 14, normalized size = 1.00

$$\frac{e^{-2\,a-2\,b\,x}}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cosh(a + b*x) - 1/sinh(a + b*x))/(1/cosh(a + b*x) + 1/sinh(a + b*x))
,x)
```

```
[Out] exp(- 2*a - 2*b*x)/(2*b)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\operatorname{csch}{\left(a+bx\right)}}{\operatorname{csch}{\left(a+bx\right)}+\operatorname{sech}{\left(a+bx\right)}}\,dx-\int \left(-\frac{\operatorname{sech}{\left(a+bx\right)}}{\operatorname{csch}{\left(a+bx\right)}+\operatorname{sech}{\left(a+bx\right)}}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-csch(b*x+a)+sech(b*x+a))/(csch(b*x+a)+sech(b*x+a)),x)
```

```
[Out] -Integral(csch(a + b*x)/(csch(a + b*x) + sech(a + b*x)), x) - Integral(-sec
h(a + b*x)/(csch(a + b*x) + sech(a + b*x)), x)
```

3.1057 $\int \frac{-\mathrm{csch}^2(a+bx)+\mathrm{sech}^2(a+bx)}{\mathrm{csch}^2(a+bx)+\mathrm{sech}^2(a+bx)}\,dx$

Optimal. Leaf size=12

$$-\frac{\tan^{-1}(\tanh(a+bx))}{b}$$

```
[Out] -arctan(tanh(b*x+a))/b
```

Rubi [A] time = 0.26, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.026, Rules used = {204}

$$-\frac{\tan^{-1}(\tanh(a+bx))}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(-Csch[a + b*x]^2 + Sech[a + b*x]^2)/(Csch[a + b*x]^2 + Sech[a + b*x]^2
),x]
```

```
[Out] -(ArcTan[Tanh[a + b*x]]/b)
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{-\mathrm{csch}^2(a+bx)+\mathrm{sech}^2(a+bx)}{\mathrm{csch}^2(a+bx)+\mathrm{sech}^2(a+bx)}\,dx &= \frac{\mathrm{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, \tanh(a+bx)\right)}{b}\\ &= -\frac{\tan^{-1}(\tanh(a+bx))}{b}\end{aligned}$$

Mathematica [A] time = 0.01, size = 17, normalized size = 1.42

$$-\frac{\tan^{-1}(\sinh(2a+2bx))}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-Csch[a + b*x]^2 + Sech[a + b*x]^2)/(Csch[a + b*x]^2 + Sech[a +
b*x]^2),x]
```

```
[Out] -1/2*ArcTan[Sinh[2*a + 2*b*x]]/b
```

fricas [B] time = 0.43, size = 37, normalized size = 3.08

$$\frac{\arctan\left(-\frac{\cosh(bx+a)+\sinh(bx+a)}{\cosh(bx+a)-\sinh(bx+a)}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-csch(b*x+a)^2+sech(b*x+a)^2)/(csch(b*x+a)^2+sech(b*x+a)^2),x, a
lgorithm="fricas")
```

```
[Out] arctan(-(cosh(b*x + a) + sinh(b*x + a))/(cosh(b*x + a) - sinh(b*x + a)))/b
```

giac [B] time = 0.17, size = 43, normalized size = 3.58

$$\frac{\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2e^{(bx+a)}\right)\right)-\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2e^{(bx+a)}\right)\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-csch(b*x+a)^2+sech(b*x+a)^2)/(csch(b*x+a)^2+sech(b*x+a)^2),x, a
lgorithm="giac")
```

```
[Out] (arctan(1/2*sqrt(2)*(sqrt(2) + 2*e^(b*x + a))) - arctan(-1/2*sqrt(2)*(sqrt(
2) - 2*e^(b*x + a))))/b
```

maple [B] time = 0.80, size = 148, normalized size = 12.33

$$-\frac{2\sqrt{2}\arctan\left(\frac{2\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)}{-2+2\sqrt{2}}\right)}{b\left(-2+2\sqrt{2}\right)}+\frac{2\arctan\left(\frac{2\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)}{-2+2\sqrt{2}}\right)}{b\left(-2+2\sqrt{2}\right)}+\frac{2\sqrt{2}\arctan\left(\frac{2\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)}{2+2\sqrt{2}}\right)}{b\left(2+2\sqrt{2}\right)}+\frac{2\arctan\left(\frac{2\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)}{2+2\sqrt{2}}\right)}{b\left(2+2\sqrt{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-csch(b*x+a)^2+sech(b*x+a)^2)/(csch(b*x+a)^2+sech(b*x+a)^2),x)
```

```
[Out] -2/b*2^(1/2)/(-2+2*2^(1/2))*arctan(2*tanh(1/2*b*x+1/2*a)/(-2+2*2^(1/2)))+2/
b/(-2+2*2^(1/2))*arctan(2*tanh(1/2*b*x+1/2*a)/(-2+2*2^(1/2)))+2/b*2^(1/2)/(
2+2*2^(1/2))*arctan(2*tanh(1/2*b*x+1/2*a)/(2+2*2^(1/2)))+2/b/(2+2*2^(1/2))*
arctan(2*tanh(1/2*b*x+1/2*a)/(2+2*2^(1/2)))
```

maxima [B] time = 0.44, size = 50, normalized size = 4.17

$$-\frac{\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2e^{(-bx-a)}\right)\right)-\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2e^{(-bx-a)}\right)\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-csch(b*x+a)^2+sech(b*x+a)^2)/(csch(b*x+a)^2+sech(b*x+a)^2),x, a
lgorithm="maxima")
```

```
[Out] -(arctan(1/2*sqrt(2)*(sqrt(2) + 2*e^(-b*x - a))) - arctan(-1/2*sqrt(2)*(sqr
t(2) - 2*e^(-b*x - a))))/b
```

mupad [B] time = 1.96, size = 26, normalized size = 2.17

$$-\frac{\operatorname{atan}\left(\frac{e^{2a}e^{2bx}\sqrt{b^2}}{b}\right)}{\sqrt{b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cosh(a + b*x)^2 - 1/sinh(a + b*x)^2)/(1/cosh(a + b*x)^2 + 1/sinh(a +
 b*x)^2),x)
```

```
[Out] -atan((exp(2*a)*exp(2*b*x)*(b^2)^(1/2))/b)/(b^2)^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int\frac{\operatorname{csch}^2(a+bx)}{\operatorname{csch}^2(a+bx)+\operatorname{sech}^2(a+bx)}\,dx-\int\left(-\frac{\operatorname{sech}^2(a+bx)}{\operatorname{csch}^2(a+bx)+\operatorname{sech}^2(a+bx)}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-csch(b*x+a)**2+sech(b*x+a)**2)/(csch(b*x+a)**2+sech(b*x+a)**2),
x)
```

```
[Out] -Integral(csch(a + b*x)**2/(csch(a + b*x)**2 + sech(a + b*x)**2), x) - Inte
gral(-sech(a + b*x)**2/(csch(a + b*x)**2 + sech(a + b*x)**2), x)
```

3.1058 $\int \frac{-\operatorname{csch}^3(a+bx)+\operatorname{sech}^3(a+bx)}{\operatorname{csch}^3(a+bx)+\operatorname{sech}^3(a+bx)}\,dx$

Optimal. Leaf size=47

$$\frac{1}{3b(\tanh(a+bx)+1)}+\frac{4\tan^{-1}\left(\frac{1-2\tanh(a+bx)}{\sqrt{3}}\right)}{3\sqrt{3}\,b}$$

```
[Out] 4/9*arctan(1/3*(1-2*tanh(b*x+a))*3^(1/2))/b*3^(1/2)+1/3/b/(1+tanh(b*x+a))
```

Rubi [A] time = 0.41, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {2074, 618, 204}

$$\frac{1}{3b(\tanh(a+bx)+1)}+\frac{4\tan^{-1}\left(\frac{1-2\tanh(a+bx)}{\sqrt{3}}\right)}{3\sqrt{3}\,b}$$

Antiderivative was successfully verified.

```
[In] Int[(-Csch[a + b*x]^3 + Sech[a + b*x]^3)/(Csch[a + b*x]^3 + Sech[a + b*x]^3
),x]
```

```
[Out] (4*ArcTan[(1 - 2*Tanh[a + b*x])/Sqrt[3]])/(3*Sqrt[3]*b) + 1/(3*b*(1 + Tanh[
a + b*x]))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2074

```
Int[(P_)^(p_)*(Q_)^(q_.), x_Symbol] :> With[{PP = Factor[P]}, Int[ExpandInt
egrand[PP^p*Q^q, x], x] /;  !SumQ[NonfreeFactors[PP, x]]] /; FreeQ[q, x] &&
 PolyQ[P, x] && PolyQ[Q, x] && IntegerQ[p] && NeQ[P, x]
```

Rubi steps

$$\int \frac{-\text{csch}^3(a+bx)+\text{sech}^3(a+bx)}{\text{csch}^3(a+bx)+\text{sech}^3(a+bx)}\,dx = \frac{\text{Subst}\left(\int \frac{-1-x-x^2}{1+x+x^3+x^4}\,dx,x,\tanh(a+bx)\right)}{b}$$

$$= \frac{\text{Subst}\left(\int \left(-\frac{1}{3(1+x)^2}-\frac{2}{3\left(1-x+x^2\right)}\right)dx,x,\tanh(a+bx)\right)}{b}$$

$$= \frac{1}{3b(1+\tanh(a+bx))}-\frac{2\,\text{Subst}\left(\int \frac{1}{1-x+x^2}\,dx,x,\tanh(a+bx)\right)}{3b}$$

$$= \frac{1}{3b(1+\tanh(a+bx))}+\frac{4\,\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,-1+2\tanh(a+bx)\right)}{3b}$$

$$= \frac{4\tan^{-1}\left(\frac{1-2\tanh(a+bx)}{\sqrt{3}}\right)}{3\sqrt{3}\,b}+\frac{1}{3b(1+\tanh(a+bx))}$$

Mathematica [A] time = 0.33, size = 52, normalized size = 1.11

$$\frac{-3\sinh(2(a+bx))+3\cosh(2(a+bx))-8\sqrt{3}\,\tan^{-1}\left(\frac{2\tanh(a+bx)-1}{\sqrt{3}}\right)}{18b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-Csch[a + b*x]^3 + Sech[a + b*x]^3)/(Csch[a + b*x]^3 + Sech[a +
b*x]^3),x]
```

```
[Out] (-8*Sqrt[3]*ArcTan[(-1 + 2*Tanh[a + b*x])/Sqrt[3]] + 3*Cosh[2*(a + b*x)] -
3*Sinh[2*(a + b*x)])/(18*b)
```

fricas [B] time = 0.41, size = 127, normalized size = 2.70

$$\frac{8\left(\sqrt{3}\,\cosh\left(bx+a\right)^2+2\sqrt{3}\,\cosh\left(bx+a\right)\sinh\left(bx+a\right)+\sqrt{3}\,\sinh\left(bx+a\right)^2\right)\arctan\left(-\frac{\sqrt{3}\,\cosh(bx+a)+\sqrt{3}\,\sinh(bx+a}{3\,(\cosh(bx+a)-\sinh(bx+a))}\right.}{18\left(b\cosh\left(bx+a\right)^2+2\,b\cosh\left(bx+a\right)\sinh\left(bx+a\right)+b\sinh\left(bx+a\right)^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-csch(b*x+a)^3+sech(b*x+a)^3)/(csch(b*x+a)^3+sech(b*x+a)^3),x, a
lgorithm="fricas")
```

```
[Out] 1/18*(8*(sqrt(3)*cosh(b*x + a)^2 + 2*sqrt(3)*cosh(b*x + a)*sinh(b*x + a) +
sqrt(3)*sinh(b*x + a)^2)*arctan(-1/3*(sqrt(3)*cosh(b*x + a) + sqrt(3)*sinh(
```

```
b*x + a))/(cosh(b*x + a) - sinh(b*x + a))) + 3)/(b*cosh(b*x + a)^2 + 2*b*co
sh(b*x + a)*sinh(b*x + a) + b*sinh(b*x + a)^2)
```

giac [A] time = 0.22, size = 37, normalized size = 0.79

$$-\frac{8\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}e^{(2bx+2a)}\right) - 3e^{(-2bx-2a)}}{18b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-csch(b*x+a)^3+sech(b*x+a)^3)/(csch(b*x+a)^3+sech(b*x+a)^3),x, a
lgorithm="giac")
```

```
[Out] -1/18*(8*sqrt(3)*arctan(1/3*sqrt(3)*e^(2*b*x + 2*a)) - 3*e^(-2*b*x - 2*a))/
b
```

maple [C] time = 1.41, size = 120, normalized size = 2.55

$$\frac{2i\sqrt{3}\ \ln\left(\tanh^2\left(\frac{bx}{2}+\frac{a}{2}\right)+\left(i\sqrt{3}\ -1\right)\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)+1\right)}{9b}-\frac{2i\sqrt{3}\ \ln\left(\tanh^2\left(\frac{bx}{2}+\frac{a}{2}\right)+\left(-i\sqrt{3}\ -1\right)\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)+1\right)}{9b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-csch(b*x+a)^3+sech(b*x+a)^3)/(csch(b*x+a)^3+sech(b*x+a)^3),x)
```

```
[Out] 2/9*I/b*3^(1/2)*ln(tanh(1/2*b*x+1/2*a)^2+(I*3^(1/2)-1)*tanh(1/2*b*x+1/2*a)+
1)-2/9*I/b*3^(1/2)*ln(tanh(1/2*b*x+1/2*a)^2+(-I*3^(1/2)-1)*tanh(1/2*b*x+1/2
*a)+1)+2/3/b/(tanh(1/2*b*x+1/2*a)+1)^2-2/3/b/(tanh(1/2*b*x+1/2*a)+1)
```

maxima [B] time = 0.46, size = 93, normalized size = 1.98

$$-\frac{4\left(\sqrt{3}\arctan\left(\frac{1}{6}\cdot 3^{\frac{3}{4}}\sqrt{2}\left(2\sqrt{3}e^{(-bx-a)}+3^{\frac{1}{4}}\sqrt{2}\right)\right)-\sqrt{3}\arctan\left(\frac{1}{6}\cdot 3^{\frac{3}{4}}\sqrt{2}\left(2\sqrt{3}e^{(-bx-a)}-3^{\frac{1}{4}}\sqrt{2}\right)\right)\right)}{9b}+\frac{e^{(-2bx-2a)}}{6b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-csch(b*x+a)^3+sech(b*x+a)^3)/(csch(b*x+a)^3+sech(b*x+a)^3),x, a
lgorithm="maxima")
```

```
[Out] -4/9*(sqrt(3)*arctan(1/6*3^(3/4)*sqrt(2)*(2*sqrt(3)*e^(-b*x - a) + 3^(1/4)*
sqrt(2))) - sqrt(3)*arctan(1/6*3^(3/4)*sqrt(2)*(2*sqrt(3)*e^(-b*x - a) - 3^
(1/4)*sqrt(2))))/b + 1/6*e^(-2*b*x - 2*a)/b
```

mupad [B] time = 2.16, size = 48, normalized size = 1.02

$$\frac{e^{-2a-2bx}}{6b} - \frac{4\sqrt{3}\,\mathrm{atan}\left(\frac{\sqrt{3}\,e^{2a}\,e^{2bx}\,\sqrt{b^2}}{3b}\right)}{9\sqrt{b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cosh(a + b*x)^3 - 1/sinh(a + b*x)^3)/(1/cosh(a + b*x)^3 + 1/sinh(a +
 b*x)^3),x)
```

```
[Out] exp(- 2*a - 2*b*x)/(6*b) - (4*3^(1/2)*atan((3^(1/2)*exp(2*a)*exp(2*b*x)*(b^
2)^(1/2))/(3*b)))/(9*(b^2)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int\frac{\operatorname{csch}^3(a+bx)}{\operatorname{csch}^3(a+bx)+\operatorname{sech}^3(a+bx)}\,dx-\int\left(-\frac{\operatorname{sech}^3(a+bx)}{\operatorname{csch}^3(a+bx)+\operatorname{sech}^3(a+bx)}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-csch(b*x+a)**3+sech(b*x+a)**3)/(csch(b*x+a)**3+sech(b*x+a)**3),
x)
```

```
[Out] -Integral(csch(a + b*x)**3/(csch(a + b*x)**3 + sech(a + b*x)**3), x) - Inte
gral(-sech(a + b*x)**3/(csch(a + b*x)**3 + sech(a + b*x)**3), x)
```

3.1059 $\int \frac{-\mathrm{csch}^4(a+bx)+\mathrm{sech}^4(a+bx)}{\mathrm{csch}^4(a+bx)+\mathrm{sech}^4(a+bx)}\,dx$

Optimal. Leaf size=51

$$\frac{\tan^{-1}\left(1-\sqrt{2}\tanh(a+bx)\right)}{\sqrt{2}\,b}-\frac{\tan^{-1}\left(\sqrt{2}\tanh(a+bx)+1\right)}{\sqrt{2}\,b}$$

```
[Out] -1/2*arctan(-1+2^(1/2)*tanh(b*x+a))/b*2^(1/2)-1/2*arctan(1+2^(1/2)*tanh(b*x
+a))/b*2^(1/2)
```

Rubi [A] time = 1.42, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1162, 617, 204}

$$\frac{\tan^{-1}\left(1-\sqrt{2}\tanh(a+bx)\right)}{\sqrt{2}\,b}-\frac{\tan^{-1}\left(\sqrt{2}\tanh(a+bx)+1\right)}{\sqrt{2}\,b}$$

Antiderivative was successfully verified.

```
[In] Int[(-Csch[a + b*x]^4 + Sech[a + b*x]^4)/(Csch[a + b*x]^4 + Sech[a + b*x]^4
),x]
```

```
[Out] ArcTan[1 - Sqrt[2]*Tanh[a + b*x]]/(Sqrt[2]*b) - ArcTan[1 + Sqrt[2]*Tanh[a +
 b*x]]/(Sqrt[2]*b)
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
```

```
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rubi steps

$$\int \frac{-\text{csch}^4(a+bx)+\text{sech}^4(a+bx)}{\text{csch}^4(a+bx)+\text{sech}^4(a+bx)}\,dx = \frac{\text{Subst}\left(\int \frac{-1-x^2}{1+x^4}\,dx, x, \tanh(a+bx)\right)}{b}$$
$$= -\frac{\text{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2}\,dx, x, \tanh(a+bx)\right)}{2b} - \frac{\text{Subst}\left(\int \frac{1}{1+\sqrt{2}x+x^2}\,dx, x, \text{tar}\right.}{2b}$$
$$= -\frac{\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\sqrt{2}\tanh(a+bx)\right)}{\sqrt{2}\,b} + \frac{\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1\right.}{\sqrt{2}\,b}$$
$$= \frac{\tan^{-1}\left(1-\sqrt{2}\tanh(a+bx)\right)}{\sqrt{2}\,b} - \frac{\tan^{-1}\left(1+\sqrt{2}\tanh(a+bx)\right)}{\sqrt{2}\,b}$$

Mathematica [A] time = 0.03, size = 26, normalized size = 0.51

$$-\frac{\tan^{-1}\left(\frac{\sinh(2a+2bx)}{\sqrt{2}}\right)}{\sqrt{2}\,b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-Csch[a + b*x]^4 + Sech[a + b*x]^4)/(Csch[a + b*x]^4 + Sech[a + b*x]^4),x]
[Out] -(ArcTan[Sinh[2*a + 2*b*x]/Sqrt[2]]/(Sqrt[2]*b))
```

fricas [B] time = 0.41, size = 192, normalized size = 3.76

$$\frac{\sqrt{2}\arctan\left(-\frac{\sqrt{2}\cosh(bx+a)^3+3\sqrt{2}\cosh(bx+a)\sinh(bx+a)^2+\sqrt{2}\sinh(bx+a)^3+\left(3\sqrt{2}\cosh(bx+a)^2-7\sqrt{2}\right)\sinh(bx+a)+7\sqrt{2}\cosh(bx+a)}{4\left(\cosh(bx+a)^3-3\cosh(bx+a)^2\sinh(bx+a)+3\cosh(bx+a)\sinh(bx+a)^2-\sinh(bx+a)^3\right)}\right)+}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-csch(b*x+a)^4+sech(b*x+a)^4)/(csch(b*x+a)^4+sech(b*x+a)^4),x, a
lgorithm="fricas")
[Out] 1/2*(sqrt(2)*arctan(-1/4*(sqrt(2)*cosh(b*x + a)^3 + 3*sqrt(2)*cosh(b*x + a)
*sinh(b*x + a)^2 + sqrt(2)*sinh(b*x + a)^3 + (3*sqrt(2)*cosh(b*x + a)^2 - 7
```

```
*sqrt(2))*sinh(b*x + a) + 7*sqrt(2)*cosh(b*x + a))/(cosh(b*x + a)^3 - 3*cos
h(b*x + a)^2*sinh(b*x + a) + 3*cosh(b*x + a)*sinh(b*x + a)^2 - sinh(b*x + a
)^3)) + sqrt(2)*arctan(-1/4*(sqrt(2)*cosh(b*x + a) + sqrt(2)*sinh(b*x + a))
/(cosh(b*x + a) - sinh(b*x + a))))/b
```

giac [A] time = 0.35, size = 34, normalized size = 0.67

$$-\frac{\sqrt{2}\arctan\left(\frac{1}{4}\sqrt{2}\left(e^{(4\,bx+4\,a)}-1\right)e^{(-2\,bx-2\,a)}\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-csch(b*x+a)^4+sech(b*x+a)^4)/(csch(b*x+a)^4+sech(b*x+a)^4),x, a
lgorithm="giac")
```

```
[Out] -1/2*sqrt(2)*arctan(1/4*sqrt(2)*(e^(4*b*x + 4*a) - 1)*e^(-2*b*x - 2*a))/b
```

maple [C] time = 0.88, size = 138, normalized size = 2.71

$$\frac{i\sqrt{2}\,\ln\left(2i\sqrt{2}\,\left(\tanh^{3}\left(\frac{bx}{2}+\frac{a}{2}\right)\right)+\tanh^{4}\left(\frac{bx}{2}+\frac{a}{2}\right)+2i\sqrt{2}\,\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)-2\left(\tanh^{2}\left(\frac{bx}{2}+\frac{a}{2}\right)\right)+1\right)}{4b}-\frac{i\sqrt{2}\,\ln\left(-\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-csch(b*x+a)^4+sech(b*x+a)^4)/(csch(b*x+a)^4+sech(b*x+a)^4),x)
```

```
[Out] 1/4*I/b*2^(1/2)*ln(2*I*2^(1/2)*tanh(1/2*b*x+1/2*a)^3+tanh(1/2*b*x+1/2*a)^4+
2*I*2^(1/2)*tanh(1/2*b*x+1/2*a)-2*tanh(1/2*b*x+1/2*a)^2+1)-1/4*I/b*2^(1/2)*
ln(-2*I*2^(1/2)*tanh(1/2*b*x+1/2*a)^3+tanh(1/2*b*x+1/2*a)^4-2*I*2^(1/2)*tan
h(1/2*b*x+1/2*a)-2*tanh(1/2*b*x+1/2*a)^2+1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-2\int\frac{\left(e^{(-bx-a)}+e^{(-5\,bx-5\,a)}\right)e^{(-bx-a)}}{6\,e^{(-4\,bx-4\,a)}+e^{(-8\,bx-8\,a)}+1}\,dx-2\int\frac{\left(e^{(-4\,bx-4\,a)}+1\right)e^{(-2\,bx-2\,a)}}{6\,e^{(-4\,bx-4\,a)}+e^{(-8\,bx-8\,a)}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-csch(b*x+a)^4+sech(b*x+a)^4)/(csch(b*x+a)^4+sech(b*x+a)^4),x, a
lgorithm="maxima")
```

```
[Out] -2*integrate((e^(-b*x - a) + e^(-5*b*x - 5*a))*e^(-b*x - a)/(6*e^(-4*b*x -
4*a) + e^(-8*b*x - 8*a) + 1), x) - 2*integrate((e^(-4*b*x - 4*a) + 1)*e^(-2
*b*x - 2*a)/(6*e^(-4*b*x - 4*a) + e^(-8*b*x - 8*a) + 1), x)
```

mupad [B] time = 2.24, size = 77, normalized size = 1.51

$$-\frac{\sqrt{2}\left(\operatorname{atan}\left(\frac{\sqrt{2}\,e^{2a}\,e^{2bx}\,\sqrt{b^2}}{4b}\right)+\operatorname{atan}\left(\frac{\sqrt{b^2}\left(\frac{56\sqrt{2}\,e^{2a}\,e^{2bx}}{b}+\frac{8\sqrt{2}\,e^{6a}\,e^{6bx}}{b}\right)}{32}\right)\right)}{2\sqrt{b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cosh(a + b*x)^4 - 1/sinh(a + b*x)^4)/(1/cosh(a + b*x)^4 + 1/sinh(a +
 b*x)^4),x)
```

```
[Out] -(2^(1/2)*(atan((2^(1/2)*exp(2*a)*exp(2*b*x)*(b^2)^(1/2))/(4*b)) + atan(((b
^2)^(1/2)*((56*2^(1/2)*exp(2*a)*exp(2*b*x))/b + (8*2^(1/2)*exp(6*a)*exp(6*b
*x))/b))/32)))/(2*(b^2)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int\frac{\operatorname{csch}^4(a+bx)}{\operatorname{csch}^4(a+bx)+\operatorname{sech}^4(a+bx)}\,dx-\int\left(-\frac{\operatorname{sech}^4(a+bx)}{\operatorname{csch}^4(a+bx)+\operatorname{sech}^4(a+bx)}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-csch(b*x+a)**4+sech(b*x+a)**4)/(csch(b*x+a)**4+sech(b*x+a)**4),
x)
```

```
[Out] -Integral(csch(a + b*x)**4/(csch(a + b*x)**4 + sech(a + b*x)**4), x) - Inte
gral(-sech(a + b*x)**4/(csch(a + b*x)**4 + sech(a + b*x)**4), x)
```

Chapter 4

Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.0.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] :=
  If[ExpnType[result]<=ExpnType[optimal],
```

```
    If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
      If[LeafCount[result]<=2*LeafCount[optimal],
        "A",
      "B"],
    "C"],
  If[FreeQ[result,Integrate] && FreeQ[result,Int],
    "C",
  "F"]]


(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
```

```
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
        Exp,Log,
        Sin,Cos,Tan,Cot,Sec,Csc,
        ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
        Sinh,Cosh,Tanh,Coth,Sech,Csch,
        ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
        Erf, Erfc, Erfi,
        FresnelS, FresnelC,
        ExpIntegralE, ExpIntegralEi, LogIntegral,
        SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
        Gamma, LogGamma, PolyGamma,
        Zeta, PolyLog, ProductLog,
        EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.0.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
```

```
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems

GradeAntiderivative := proc(result,optimal)
local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal,
   debug:=false;

      leaf_count_result:=leafcount(result);
      #do NOT call ExpnType() if leaf size is too large. Recursion problem
      if leaf_count_result > 500000 then
          return "B";
      fi;

      leaf_count_optimal:=leafcount(optimal);

      ExpnType_result:=ExpnType(result);
      ExpnType_optimal:=ExpnType(optimal);

      if debug then
         print("ExpnType_result",ExpnType_result," ExpnType_optimal=",
   ExpnType_optimal);
      fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

  #This check below actually is not needed, since I only
  #call this grading only for passed integrals. i.e. I check
  #for "F" before calling this. But no harm of keeping it here.
  #just in case.


  if not type(result,freeof('int')) then
     return "F";
  end if;


  if ExpnType_result<=ExpnType_optimal then
      if debug then
           print("ExpnType_result<=ExpnType_optimal");
      fi;
```

```
      if is_contains_complex(result) then
         if is_contains_complex(optimal) then
            if debug then
               print("both result and optimal complex");
            fi;
            #both result and optimal complex
            if leaf_count_result<=2*leaf_count_optimal then
               return "A";
            else
               return "B";
            end if
         else #result contains complex but optimal is not
            if debug then
               print("result contains complex but optimal is not");
            fi;
            return "C";
         end if
      else # result do not contain complex
           # this assumes optimal do not as well
           if debug then
               print("result do not contain complex, this assumes optimal do not
    as well");
           fi;
           if leaf_count_result<=2*leaf_count_optimal then
              if debug then
                 print("leaf_count_result<=2*leaf_count_optimal");
              fi;
              return "A";
           else
              if debug then
                 print("leaf_count_result>2*leaf_count_optimal");
              fi;
              return "B";
           end if
      end if
  else #ExpnType(result) > ExpnType(optimal)
      if debug then
         print("ExpnType(result) > ExpnType(optimal)");
      fi;
      return "C";
  end if

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
```

```
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
```

```
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:
```

```
#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.0.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]
```

```
def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
           return True
        else:
           return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)
   ))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,
   ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type
   (expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
```

```
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,
    expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,
    Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        return "F"

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
    well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```

4.0.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
```

```
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    debug=False
    if debug: print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','
    sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e',
           'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U
    ']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in
    sagemath

def is_atom(expn):

    debug=False
    if debug: print ("Enter is_atom")


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-
    sagemath-equivalent-to-atomic-type-in-maple/
    try:
        if expn.parent() is SR:
```

```
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().
    gens()
        return False

    except AttributeError as error:
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],
    Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.
    args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
            return expnType(expn.operands()[0])   #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],
    Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],
    Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.
    args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()
    [1])) #max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #
    isinstance(expn,Add) or isinstance(expn,Mul)
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
```

```
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.
    func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(
    expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #
    is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    if debug: print ("Enter grade_antiderivative for sagemath")

    leaf_count_result  = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    if debug: print ("leaf_count_result=", leaf_count_result, "
    leaf_count_optimal=",leaf_count_optimal)


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",
    expnType_optimal)

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
```

```
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
        else: #result contains complex but optimal is not
            return "C"
    else: # result do not contain complex, this assumes optimal do not as
well
        if leaf_count_result <= 2*leaf_count_optimal:
            return "A"
        else:
            return "B"
else:
    return "C"
```