Computer algebra independent integration tests

6-Hyperbolic-functions/6.6-Hyperbolic-cosecant/6.6.3-Hyperbolic-cosecant-functions

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July 25, 2021 Compiled on July 25, 2021 at 1:22am

Contents

Chapter 1

Introduction

This report gives the result of running the computer algebra independent integration problems. The listing of the problems are maintained by and can be downloaded from `https://rulebasedintegration.org`

The number of integrals in this report is [175]. This is test number [183].

1.1 Listing of CAS systems tested

The following systems were tested at this time.

1. Mathematica 12.3 (64 bit) on windows 10.
2. Rubi 4.16.1 in Mathematica 12.1 on windows 10.
3. Maple 2021.1 (64 bit) on windows 10.
4. Maxima 5.44 on Linux. (via sagemath 9.3)
5. Fricas 1.3.7 on Linux (via sagemath 9.3)
6. Giac/Xcas 1.7 on Linux. (via sagemath 9.3)
7. Sympy 1.8 under Python 3.8.8 using Anaconda distribution on Ubuntu.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 under windows 10 (64 bit)

Maxima, Fricas and Giac/Xcas were called from inside SageMath. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	solved	Failed
Rubi	% 100.00 (175)	% 0.00 (0)
Mathematica	% 100.00 (175)	% 0.00 (0)
Maple	% 77.71 (136)	% 22.29 (39)
Maxima	% 63.43 (111)	% 36.57 (64)
Fricas	% 75.43 (132)	% 24.57 (43)
Sympy	% 0.00 (0)	% 100.00 (175)
Giac	% 61.14 (107)	% 38.86 (68)
Mupad	% 52.00 (91)	% 48.00 (84)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

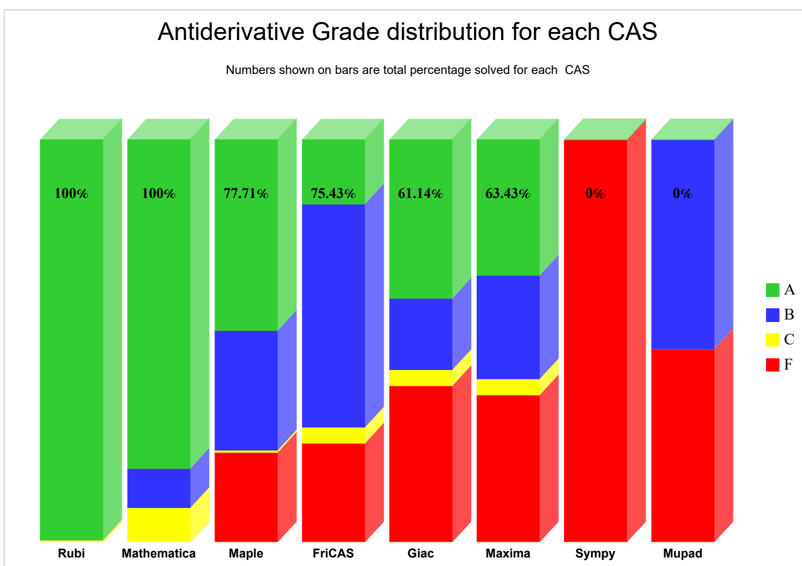
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

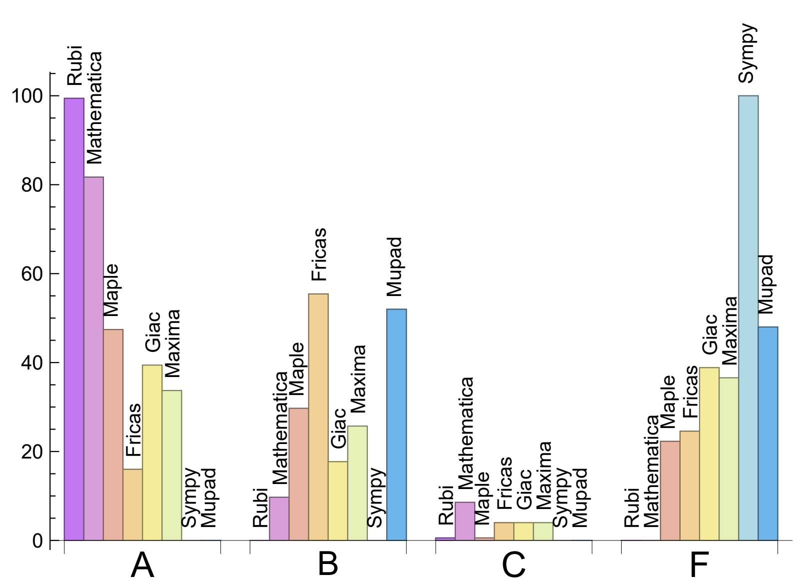
System	% A grade	% B grade	% C grade	% F grade
Rubi	99.43	0.00	0.57	0.00
Mathematica	81.71	9.71	8.57	0.00
Maple	47.43	29.71	0.57	22.29
Maxima	33.71	25.71	4.00	36.57
Fricas	16.00	55.43	4.00	24.57
Sympy	0.00	0.00	0.00	100.00
Giac	39.43	17.71	4.00	38.86
Mupad	0.00	52.00	0.00	48.00

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



The following table shows the distribution of the different types of failure for each CAS. There are 3 types of reasons why it can fail. The first is when CAS returns back the input within the time limit, which means it could not solve it. This the typical normal failure **F** .

The second is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned **F(-1)**.

The third is due to an exception generated. Assigned **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in CAS. This type of error requires more investigations to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	0	0.00 %	0.00 %	0.00 %
Mathematica	0	0.00 %	0.00 %	0.00 %
Maple	39	100.00 %	0.00 %	0.00 %
Maxima	64	100.00 %	0.00 %	0.00 %
Fricas	43	100.00 %	0.00 %	0.00 %
Sympy	175	99.43 %	0.57 %	0.00 %
Giac	68	73.53 %	20.59 %	5.88 %
Mupad	84	100.00 %	0.00 %	0.00 %

Table 1.4: Time and leaf size performance for each CAS

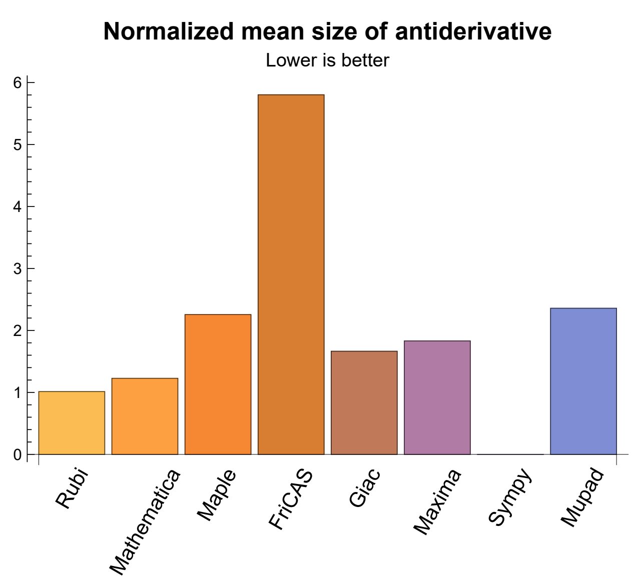
1.3 Performance

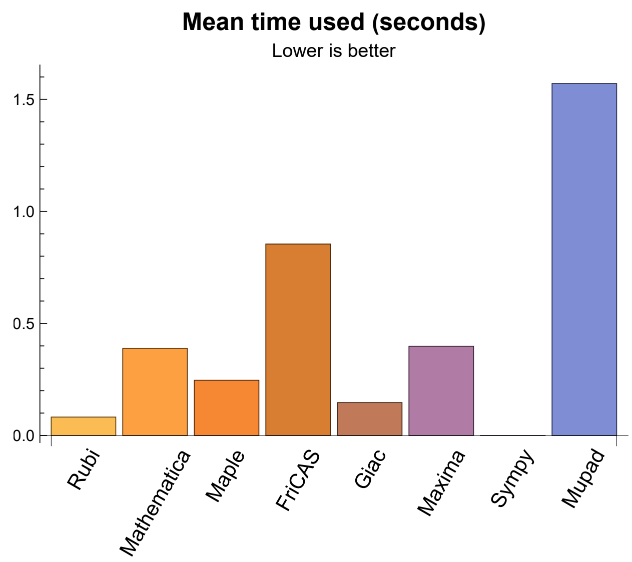
The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.08	68.71	1.01	62.00	1.00
Mathematica	0.39	71.59	1.23	61.00	1.00
Maple	0.25	143.17	2.26	99.00	1.65
Maxima	0.40	106.34	1.83	76.00	1.57
Fricas	0.85	466.33	5.80	155.00	3.60
Sympy	0.00	0.00	0.00	0.00	0.00
Giac	0.15	93.14	1.66	66.00	1.50
Mupad	1.57	151.23	2.36	81.00	2.16

Table 1.5: Time and leaf size performance for each CAS

The following are bar charts for the normalized leafsize and time used columns from the above table.





1.4 list of integrals that has no closed form antiderivative

{}

1.5 list of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {160}

Mathematica {157, 158, 159, 163}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

Mupad Verification phase not implemented yet.

1.7 Timing

The command `AboluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since these integrals are run in a batch mode, using an automated script, and by using sagemath (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 14 such integrals out of total 705, or about 2 percent. This pecrentage can be higher or lower depending on the specific input test file.

Such integrals can be indentified by looking at the output of the integration in each section for Maxima. The exception message will indicate of the error is due to the interactive question being asked or not.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
```

```
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima `abs_integrate` was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/ for reference.

1.9.2 Important note about FriCAS and Giac/XCAS results

There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error `Exception raised: NotImplementedError`

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi and Maple, the buildin system function `LeafSize` is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special buildin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy and Giac antiderivatives is determined using the following function, thanks to user `slelievre` at https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which is called directly from Python, the following code is used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
         leafCount =1
```

1.9.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative, Maple was used to determine the leaf size of Mupad output by post processing.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

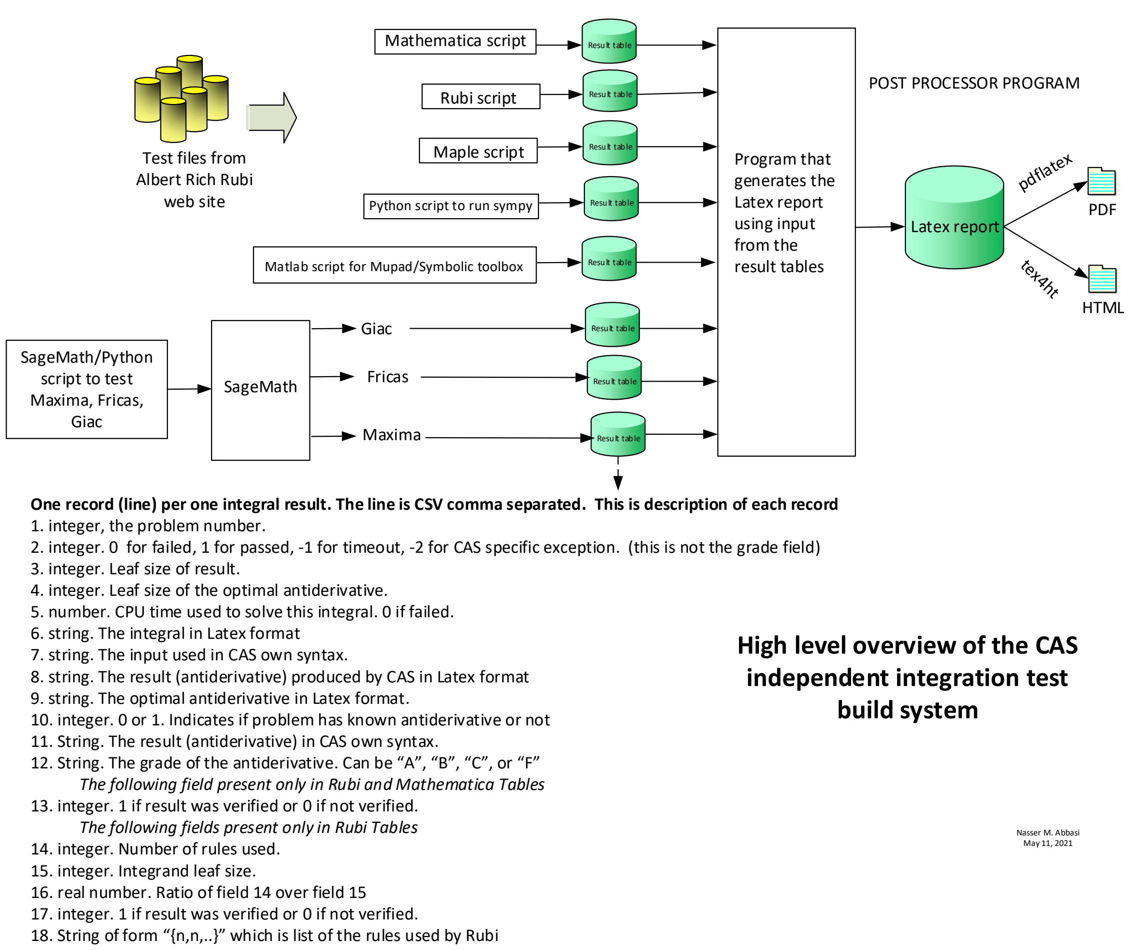
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



Chapter 2

detailed summary tables of results

2.1 List of integrals sorted by grade for each CAS

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175 }

B grade: { }

C grade: { 160 }

F grade: { }

2.1.2 Mathematica

A grade: { 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 71, 72, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 90, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 104, 105, 106, 107, 108, 109, 111, 114, 116, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 133, 135, 137, 138, 139, 141, 143, 145, 147, 149, 152, 153, 156, 157, 158, 160, 163, 164, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175 }

B grade: { 1, 24, 55, 67, 68, 69, 70, 73, 89, 91, 103, 110, 112, 159, 161, 162, 165 }

C grade: { 113, 115, 117, 132, 134, 136, 140, 142, 144, 146, 148, 150, 151, 154, 155 }

F grade: { }

2.1.3 Maple

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 42, 43, 44, 45, 49, 50, 66, 67, 68, 69, 70, 71, 72, 73, 74, 79, 80, 81, 82, 83, 85, 87, 96, 97, 98, 100, 102, 104, 106, 107, 109, 114, 116, 117, 118, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 138, 140, 141, 142, 143, 144, 145, 146, 147, 148, 150, 151, 152, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175 }

B grade: { 17, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 46, 47, 48, 62, 63, 64, 65, 75, 76, 77, 78, 84, 86, 88, 89, 90, 91, 92, 93, 94, 95, 99, 101, 103, 105, 108, 110, 111, 112, 113, 115, 119, 120, 121, 122, 123, 124 }

C grade: { 160 }

F grade: { 13, 14, 15, 16, 18, 19, 20, 21, 36, 37, 38, 39, 40, 41, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 137, 139, 149, 153, 154, 155, 156, 157, 158, 159, 161, 162, 163, 164 }

2.1.4 Maxima

A grade: { 1, 2, 29, 30, 31, 32, 33, 34, 35, 45, 46, 47, 48, 49, 50, 62, 63, 64, 65, 66, 67, 71, 72, 73, 74, 75, 77, 78, 79, 80, 81, 82, 84, 87, 93, 95, 97, 98, 105, 106, 107, 114, 115, 116, 117, 118, 119, 123, 126, 127, 128, 129, 130, 131, 133, 145, 162, 165, 166 }

B grade: { 3, 4, 5, 6, 42, 43, 44, 68, 69, 70, 76, 83, 85, 86, 88, 89, 90, 91, 92, 94, 96, 99, 100, 101, 102, 103, 104, 108, 109, 110, 111, 112, 113, 120, 121, 122, 124, 125, 141, 153, 160, 161, 167, 168, 169 }

C grade: { 22, 23, 24, 25, 26, 27, 28 }

F grade: { 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 36, 37, 38, 39, 40, 41, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 132, 134, 135, 136, 137, 138, 139, 140, 142, 143, 144, 146, 147, 148, 149, 150, 151, 152, 154, 155, 156, 157, 158, 159, 163, 164, 170, 171, 172, 173, 174, 175 }

2.1.5 FriCAS

A grade: { 49, 50, 62, 63, 66, 84, 97, 107, 108, 117, 118, 128, 129, 130, 131, 133, 135, 137, 139, 141, 143, 147, 149, 151, 153, 155, 161, 162 }

B grade: { 1, 2, 3, 4, 5, 6, 29, 30, 31, 32, 33, 34, 35, 42, 43, 44, 45, 46, 47, 48, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 64, 65, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 98, 99, 100, 101, 102, 103, 104, 105, 106, 109, 110, 111, 112, 113, 114, 115, 116, 119, 120, 121, 122, 123, 124, 125, 126, 127, 145, 160, 163, 164, 165, 166, 167, 168, 169 }

C grade: { 22, 23, 24, 25, 26, 27, 28 }

F grade: { 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 36, 37, 38, 39, 40, 41, 132, 134, 136, 138, 140, 142, 144, 146, 148, 150, 152, 154, 156, 157, 158, 159, 170, 171, 172, 173, 174, 175 }

2.1.6 Sympy

A grade: { }

B grade: { }

C grade: { }

F grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175 }

2.1.7 Giac

A grade: { 2, 4, 6, 29, 30, 31, 33, 34, 35, 42, 43, 44, 45, 46, 47, 48, 49, 50, 62, 63, 64, 65, 66, 67, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 93, 94, 95, 96, 97, 98, 100, 102, 103, 105, 107, 108, 114, 116, 117, 118, 119, 123, 125, 126, 127, 128, 129, 130, 131, 161, 162, 166, 168 }

B grade: { 1, 3, 5, 32, 68, 85, 86, 87, 88, 89, 90, 91, 92, 99, 101, 104, 106, 109, 110, 111, 112, 113, 115, 120, 121, 122, 124, 160, 165, 167, 169 }

C grade: { 22, 23, 24, 25, 26, 27, 28 }

F grade: { 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 36, 37, 38, 39, 40, 41, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 163, 164, 170, 171, 172, 173, 174, 175 }

2.1.8 Mupad

A grade: { }

B grade: { 1, 2, 3, 4, 5, 6, 25, 32, 42, 43, 44, 45, 49, 50, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 133, 141, 145, 153, 160, 161, 162, 165, 166, 167, 168, 169 }

C grade: { }

F grade: { 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 26, 27, 28, 29, 30, 31, 33, 34, 35, 36, 37, 38, 39, 40, 41, 46, 47, 48, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 76, 128, 129, 130, 131, 132, 134, 135, 136, 137, 138, 139, 140, 142, 143, 144, 146, 147, 148, 149, 150, 151, 152, 154, 155, 156, 157, 158, 159, 163, 164, 170, 171, 172, 173, 174, 175 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$

Problem 1	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	38	15	14	38	0	27	27
normalized size	1	1.00	3.17	1.25	1.17	3.17	0.00	2.25	2.25
time (sec)	N/A	0.006	0.016	0.017	0.309	0.505	0.000	0.128	0.111

Problem 2	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	12	18	43	0	18	18
normalized size	1	1.00	1.00	1.09	1.64	3.91	0.00	1.64	1.64
time (sec)	N/A	0.010	0.011	0.313	0.361	0.644	0.000	0.131	0.073

Problem 3	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	57	27	84	387	0	84	86
normalized size	1	1.00	1.68	0.79	2.47	11.38	0.00	2.47	2.53
time (sec)	N/A	0.022	0.013	0.322	0.332	0.585	0.000	0.120	1.403

Problem 4	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	35	23	90	164	0	31	31
normalized size	1	1.00	1.35	0.88	3.46	6.31	0.00	1.19	1.19
time (sec)	N/A	0.012	0.013	0.323	0.463	0.570	0.000	0.120	1.455

Problem 5	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	95	41	133	1114	0	110	193
normalized size	1	1.00	1.73	0.75	2.42	20.25	0.00	2.00	3.51
time (sec)	N/A	0.043	0.016	0.313	0.425	0.622	0.000	0.135	0.062

Problem 6	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	56	33	205	344	0	42	42
normalized size	1	1.00	1.33	0.79	4.88	8.19	0.00	1.00	1.00
time (sec)	N/A	0.015	0.017	0.291	0.319	1.200	0.000	0.142	0.077

Problem 7	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	61	101	0	0	0	0	-1
normalized size	1	1.00	0.76	1.26	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.037	0.126	0.418	0.000	0.521	0.000	0.000	0.000

Problem 8	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	57	154	0	0	0	0	-1
normalized size	1	1.00	0.75	2.03	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.030	0.222	0.375	0.000	1.105	0.000	0.000	0.000

Problem 9	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	48	87	0	0	0	0	-1
normalized size	1	1.00	0.89	1.61	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.020	0.193	0.239	0.000	1.093	0.000	0.000	0.000

Problem 10	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	50	108	0	0	0	0	-1
normalized size	1	1.00	0.93	2.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.020	0.037	0.362	0.000	1.604	0.000	0.000	0.000

Problem 11	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	63	100	0	0	0	0	-1
normalized size	1	1.00	0.79	1.25	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.032	0.068	0.407	0.000	0.941	0.000	0.000	0.000

Problem 12	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	67	164	0	0	0	0	-1
normalized size	1	1.00	0.84	2.05	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.032	0.130	0.389	0.000	0.470	0.000	0.000	0.000

Problem 13	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	79	0	0	0	0	0	-1
normalized size	1	1.00	0.68	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.061	0.160	0.299	0.000	0.565	0.000	0.000	0.000

Problem 14	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	66	0	0	0	0	0	-1
normalized size	1	1.00	0.75	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.039	0.112	0.291	0.000	0.934	0.000	0.000	0.000

Problem 15	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	60	0	0	0	0	0	-1
normalized size	1	1.00	0.71	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.038	0.061	0.245	0.000	2.145	0.000	0.000	0.000

Problem 16	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	54	0	0	0	0	0	-1
normalized size	1	1.00	0.96	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.022	0.036	0.351	0.000	0.505	0.000	0.000	0.000

Problem 17	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	52	227	0	0	0	0	-1
normalized size	1	1.00	0.93	4.05	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.023	0.046	0.329	0.000	1.400	0.000	0.000	0.000

Problem 18	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	73	0	0	0	0	0	-1
normalized size	1	1.00	0.81	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.039	0.091	0.208	0.000	0.613	0.000	0.000	0.000

Problem 19	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	68	0	0	0	0	0	-1
normalized size	1	1.00	0.76	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.038	0.136	0.211	0.000	0.447	0.000	0.000	0.000

Problem 20	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	80	0	0	0	0	0	-1
normalized size	1	1.00	0.68	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.056	0.162	0.208	0.000	0.591	0.000	0.000	0.000

Problem 21	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	67	0	0	0	0	0	-1
normalized size	1	1.00	0.91	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.035	0.104	0.447	0.000	0.731	0.000	0.000	0.000

Problem 22	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	C	C	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	41	114	74	115	0	72	-1
normalized size	1	1.00	1.02	2.85	1.85	2.88	0.00	1.80	-0.02
time (sec)	N/A	0.017	0.124	0.220	0.523	0.502	0.000	0.143	0.000

Problem 23	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	C	C	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	41	99	49	67	0	57	-1
normalized size	1	1.00	1.71	4.12	2.04	2.79	0.00	2.38	-0.04
time (sec)	N/A	0.012	0.063	0.189	0.633	0.563	0.000	0.140	0.000

Problem 24	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	C	C	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	3	3	20	67	19	15	0	27	-1
normalized size	1	1.00	6.67	22.33	6.33	5.00	0.00	9.00	-0.33
time (sec)	N/A	0.008	0.005	0.205	0.564	0.977	0.000	0.139	0.000

Problem 25	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	C	C	F	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	58	11	14	0	25	31
normalized size	1	1.00	1.00	4.46	0.85	1.08	0.00	1.92	2.38
time (sec)	N/A	0.010	0.005	0.195	0.495	1.047	0.000	0.126	1.715

Problem 26	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	C	C	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	27	118	23	26	0	50	-1
normalized size	1	1.00	0.82	3.58	0.70	0.79	0.00	1.52	-0.03
time (sec)	N/A	0.015	0.015	0.185	0.436	2.466	0.000	0.159	0.000

Problem 27	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	C	C	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	33	178	35	38	0	64	-1
normalized size	1	1.00	0.67	3.63	0.71	0.78	0.00	1.31	-0.02
time (sec)	N/A	0.020	0.025	0.184	0.459	1.452	0.000	0.168	0.000

Problem 28	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	C	C	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	39	238	47	50	0	76	-1
normalized size	1	1.00	0.60	3.66	0.72	0.77	0.00	1.17	-0.02
time (sec)	N/A	0.024	0.037	0.206	1.032	1.130	0.000	0.167	0.000

Problem 29	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	41	123	92	1128	0	75	-1
normalized size	1	1.00	0.63	1.89	1.42	17.35	0.00	1.15	-0.02
time (sec)	N/A	0.034	0.100	0.250	0.539	0.631	0.000	0.122	0.000

Problem 30	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	30	103	60	340	0	58	-1
normalized size	1	1.00	0.65	2.24	1.30	7.39	0.00	1.26	-0.02
time (sec)	N/A	0.025	0.068	0.206	0.474	0.567	0.000	0.118	0.000

Problem 31	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	20	67	24	97	0	29	-1
normalized size	1	1.00	0.77	2.58	0.92	3.73	0.00	1.12	-0.04
time (sec)	N/A	0.018	0.005	0.230	0.454	1.133	0.000	0.129	0.000

Problem 32	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	58	17	83	0	24	33
normalized size	1	1.00	1.00	4.46	1.31	6.38	0.00	1.85	2.54
time (sec)	N/A	0.014	0.006	0.213	0.446	2.481	0.000	0.131	1.603

Problem 33	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	27	130	35	285	0	54	-1
normalized size	1	1.00	0.75	3.61	0.97	7.92	0.00	1.50	-0.03
time (sec)	N/A	0.022	0.023	0.194	0.688	1.308	0.000	0.146	0.000

Problem 34	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	36	196	53	590	0	67	-1
normalized size	1	1.00	0.65	3.56	0.96	10.73	0.00	1.22	-0.02
time (sec)	N/A	0.030	0.028	0.202	0.534	0.598	0.000	0.163	0.000

Problem 35	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	42	262	71	984	0	80	-1
normalized size	1	1.00	0.57	3.54	0.96	13.30	0.00	1.08	-0.01
time (sec)	N/A	0.040	0.056	0.207	0.496	1.287	0.000	0.170	0.000

Problem 36	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	68	0	0	0	0	0	-1
normalized size	1	1.00	0.50	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.070	0.177	0.241	0.000	0.611	0.000	0.000	0.000

Problem 37	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	56	0	0	0	0	0	-1
normalized size	1	1.00	0.69	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.045	0.121	0.206	0.000	0.475	0.000	0.000	0.000

Problem 38	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	60	42	0	0	0	0	0	-1
normalized size	1	1.07	0.75	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.034	0.031	0.225	0.000	0.515	0.000	0.000	0.000

Problem 39	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	43	0	0	0	0	0	-1
normalized size	1	1.00	0.69	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.034	0.077	0.227	0.000	0.591	0.000	0.000	0.000

Problem 40	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	57	0	0	0	0	0	-1
normalized size	1	1.00	0.64	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.048	0.078	0.202	0.000	0.597	0.000	0.000	0.000

Problem 41	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	71	0	0	0	0	0	-1
normalized size	1	1.00	0.53	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.071	0.128	0.199	0.000	1.529	0.000	0.000	0.000

Problem 42	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	164	164	59	72	620	2825	0	51	498
normalized size	1	1.00	0.36	0.44	3.78	17.23	0.00	0.31	3.04
time (sec)	N/A	0.042	0.052	0.276	0.434	0.652	0.000	0.135	1.485

Problem 43	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	47	60	322	1493	0	39	356
normalized size	1	1.00	0.40	0.51	2.73	12.65	0.00	0.33	3.02
time (sec)	N/A	0.032	0.032	0.221	0.487	0.779	0.000	0.116	1.440

Problem 44	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	33	46	120	529	0	27	46
normalized size	1	1.00	0.53	0.74	1.94	8.53	0.00	0.44	0.74
time (sec)	N/A	0.023	0.019	0.204	0.511	1.571	0.000	0.130	1.457

Problem 45	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	29	13	81	0	13	71
normalized size	1	1.00	1.00	1.81	0.81	5.06	0.00	0.81	4.44
time (sec)	N/A	0.017	0.006	0.228	0.499	0.669	0.000	0.109	1.456

Problem 46	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	24	89	22	253	0	26	-1
normalized size	1	1.00	0.67	2.47	0.61	7.03	0.00	0.72	-0.03
time (sec)	N/A	0.016	0.025	0.249	0.440	0.651	0.000	0.111	0.000

Problem 47	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	38	230	46	1141	0	50	-1
normalized size	1	1.00	0.44	2.67	0.53	13.27	0.00	0.58	-0.01
time (sec)	N/A	0.033	0.040	0.209	0.465	0.811	0.000	0.110	0.000

Problem 48	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	132	55	362	72	2600	0	76	-1
normalized size	1	1.00	0.42	2.74	0.55	19.70	0.00	0.58	-0.01
time (sec)	N/A	0.051	0.116	0.219	0.581	0.742	0.000	0.131	0.000

Problem 49	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	54	63	35	32	0	29	26
normalized size	1	1.00	1.69	1.97	1.09	1.00	0.00	0.91	0.81
time (sec)	N/A	0.015	0.122	0.254	0.362	1.426	0.000	0.131	1.553

Problem 50	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	54	63	35	32	0	29	26
normalized size	1	1.00	1.69	1.97	1.09	1.00	0.00	0.91	0.81
time (sec)	N/A	0.015	0.108	0.260	0.343	0.753	0.000	0.140	1.501

Problem 51	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	136	0	0	569	0	0	-1
normalized size	1	1.00	1.27	0.00	0.00	5.32	0.00	0.00	-0.01
time (sec)	N/A	0.127	1.481	0.949	0.000	0.616	0.000	0.000	0.000

Problem 52	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	100	0	0	467	0	0	-1
normalized size	1	1.00	1.39	0.00	0.00	6.49	0.00	0.00	-0.01
time (sec)	N/A	0.042	1.247	0.698	0.000	0.579	0.000	0.000	0.000

Problem 53	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	80	0	0	386	0	0	-1
normalized size	1	1.00	2.00	0.00	0.00	9.65	0.00	0.00	-0.02
time (sec)	N/A	0.020	0.935	2.389	0.000	0.945	0.000	0.000	0.000

Problem 54	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	118	0	0	561	0	0	-1
normalized size	1	1.00	1.30	0.00	0.00	6.16	0.00	0.00	-0.01
time (sec)	N/A	0.087	1.120	0.821	0.000	1.497	0.000	0.000	0.000

Problem 55	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	327	0	0	886	0	0	-1
normalized size	1	1.00	2.66	0.00	0.00	7.20	0.00	0.00	-0.01
time (sec)	N/A	0.146	2.450	0.630	0.000	2.454	0.000	0.000	0.000

Problem 56	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	80	0	0	386	0	0	-1
normalized size	1	1.00	2.00	0.00	0.00	9.65	0.00	0.00	-0.02
time (sec)	N/A	0.023	1.009	2.784	0.000	0.582	0.000	0.000	0.000

Problem 57	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	117	0	0	561	0	0	-1
normalized size	1	1.00	1.29	0.00	0.00	6.16	0.00	0.00	-0.01
time (sec)	N/A	0.084	1.098	0.876	0.000	1.281	0.000	0.000	0.000

Problem 58	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	46	0	0	218	0	0	-1
normalized size	1	1.00	2.00	0.00	0.00	9.48	0.00	0.00	-0.04
time (sec)	N/A	0.017	0.736	0.829	0.000	0.460	0.000	0.000	0.000

Problem 59	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	46	0	0	218	0	0	-1
normalized size	1	1.00	2.00	0.00	0.00	9.48	0.00	0.00	-0.04
time (sec)	N/A	0.018	0.716	1.010	0.000	0.464	0.000	0.000	0.000

Problem 60	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	46	0	0	215	0	0	-1
normalized size	1	1.00	2.00	0.00	0.00	9.35	0.00	0.00	-0.04
time (sec)	N/A	0.016	0.689	0.565	0.000	0.432	0.000	0.000	0.000

Problem 61	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	46	0	0	215	0	0	-1
normalized size	1	1.00	2.00	0.00	0.00	9.35	0.00	0.00	-0.04
time (sec)	N/A	0.017	0.685	0.484	0.000	0.436	0.000	0.000	0.000

Problem 62	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	63	182	71	79	0	66	64
normalized size	1	1.00	1.09	3.14	1.22	1.36	0.00	1.14	1.10
time (sec)	N/A	0.071	0.150	0.210	0.313	0.468	0.000	0.143	1.589

Problem 63	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	56	137	59	67	0	50	52
normalized size	1	1.00	1.22	2.98	1.28	1.46	0.00	1.09	1.13
time (sec)	N/A	0.071	0.135	0.206	0.315	0.442	0.000	0.141	0.176

Problem 64	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	46	96	47	55	0	40	38
normalized size	1	1.00	1.28	2.67	1.31	1.53	0.00	1.11	1.06
time (sec)	N/A	0.060	0.127	0.212	0.309	0.411	0.000	0.126	1.474

Problem 65	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	35	51	31	40	0	26	24
normalized size	1	1.00	1.75	2.55	1.55	2.00	0.00	1.30	1.20
time (sec)	N/A	0.047	0.053	0.201	0.316	0.409	0.000	0.127	1.460

Problem 66	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	27	12	12	8	0	8	10
normalized size	1	1.00	1.93	0.86	0.86	0.57	0.00	0.57	0.71
time (sec)	N/A	0.021	0.019	0.116	0.312	0.394	0.000	0.129	0.066

Problem 67	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	37	19	29	31	0	22	26
normalized size	1	1.00	2.18	1.12	1.71	1.82	0.00	1.29	1.53
time (sec)	N/A	0.054	0.035	0.098	0.312	0.473	0.000	0.120	0.219

Problem 68	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	70	35	55	77	0	46	60
normalized size	1	1.00	2.69	1.35	2.12	2.96	0.00	1.77	2.31
time (sec)	N/A	0.084	0.115	0.110	0.316	0.420	0.000	0.122	1.645

Problem 69	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	81	53	81	130	0	50	63
normalized size	1	1.00	2.19	1.43	2.19	3.51	0.00	1.35	1.70
time (sec)	N/A	0.064	0.328	0.128	0.320	0.417	0.000	0.125	1.685

Problem 70	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	508	92	234	1440	0	169	239
normalized size	1	1.00	4.66	0.84	2.15	13.21	0.00	1.55	2.19
time (sec)	N/A	0.128	6.244	0.493	0.325	0.436	0.000	0.125	0.210

Problem 71	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	118	66	136	769	0	122	170
normalized size	1	1.00	1.57	0.88	1.81	10.25	0.00	1.63	2.27
time (sec)	N/A	0.051	0.872	0.414	0.327	0.422	0.000	0.139	0.150

Problem 72	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	61	37	44	222	0	59	74
normalized size	1	1.00	1.79	1.09	1.29	6.53	0.00	1.74	2.18
time (sec)	N/A	0.031	0.222	0.317	0.315	0.418	0.000	0.121	1.480

Problem 73	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	43	20	19	44	0	32	42
normalized size	1	1.00	2.53	1.18	1.12	2.59	0.00	1.88	2.47
time (sec)	N/A	0.010	0.011	0.023	0.312	0.406	0.000	0.112	0.066

Problem 74	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	64	87	85	186	0	84	121
normalized size	1	1.00	1.19	1.61	1.57	3.44	0.00	1.56	2.24
time (sec)	N/A	0.059	0.109	0.202	0.415	0.415	0.000	0.144	0.328

Problem 75	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	142	238	187	645	0	161	269
normalized size	1	1.00	1.41	2.36	1.85	6.39	0.00	1.59	2.66
time (sec)	N/A	0.158	0.394	0.321	0.408	0.422	0.000	0.155	1.948

Problem 76	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	163	163	213	822	373	2094	0	293	-1
normalized size	1	1.00	1.31	5.04	2.29	12.85	0.00	1.80	-0.01
time (sec)	N/A	0.316	0.930	0.342	0.419	0.459	0.000	0.161	0.000

Problem 77	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	104	262	157	807	0	155	199
normalized size	1	1.00	0.97	2.45	1.47	7.54	0.00	1.45	1.86
time (sec)	N/A	0.458	0.430	0.161	0.408	0.427	0.000	0.139	1.826

Problem 78	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	82	174	116	456	0	115	157
normalized size	1	1.00	1.02	2.18	1.45	5.70	0.00	1.44	1.96
time (sec)	N/A	0.288	0.132	0.171	0.405	0.422	0.000	0.135	1.668

Problem 79	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	61	92	84	238	0	86	129
normalized size	1	1.00	1.07	1.61	1.47	4.18	0.00	1.51	2.26
time (sec)	N/A	0.107	0.106	0.165	0.401	0.420	0.000	0.133	1.579

Problem 80	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	45	35	54	111	0	56	49
normalized size	1	1.00	1.22	0.95	1.46	3.00	0.00	1.51	1.32
time (sec)	N/A	0.066	0.026	0.084	0.403	0.396	0.000	0.136	1.516

Problem 81	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	58	49	83	156	0	82	287
normalized size	1	1.00	1.16	0.98	1.66	3.12	0.00	1.64	5.74
time (sec)	N/A	0.115	0.051	0.094	0.406	0.440	0.000	0.147	1.685

Problem 82	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	71	73	100	345	0	98	292
normalized size	1	1.00	1.20	1.24	1.69	5.85	0.00	1.66	4.95
time (sec)	N/A	0.160	0.289	0.098	0.422	0.434	0.000	0.150	1.728

Problem 83	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	124	108	158	947	0	141	617
normalized size	1	1.00	1.49	1.30	1.90	11.41	0.00	1.70	7.43
time (sec)	N/A	0.292	0.481	0.105	0.415	0.526	0.000	0.156	2.094

Problem 84	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	32	170	42	43	0	38	41
normalized size	1	1.00	0.84	4.47	1.11	1.13	0.00	1.00	1.08
time (sec)	N/A	0.128	0.034	0.165	0.304	0.410	0.000	0.138	1.551

Problem 85	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	15	39	36	0	35	39
normalized size	1	1.00	1.00	0.79	2.05	1.89	0.00	1.84	2.05
time (sec)	N/A	0.106	0.010	0.107	0.302	0.404	0.000	0.140	0.142

Problem 86	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	84	30	31	0	26	29
normalized size	1	1.00	1.00	4.20	1.50	1.55	0.00	1.30	1.45
time (sec)	N/A	0.092	0.031	0.120	0.301	0.464	0.000	0.142	1.463

Problem 87	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	20	21	28	0	25	12
normalized size	1	1.00	1.00	1.25	1.31	1.75	0.00	1.56	0.75
time (sec)	N/A	0.058	0.010	0.120	0.302	0.596	0.000	0.119	0.093

Problem 88	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	20	43	41	53	0	53	46
normalized size	1	1.00	0.71	1.54	1.46	1.89	0.00	1.89	1.64
time (sec)	N/A	0.080	0.029	0.182	0.303	0.405	0.000	0.137	0.229

Problem 89	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	64	49	81	33	0	27	31
normalized size	1	1.00	3.37	2.58	4.26	1.74	0.00	1.42	1.63
time (sec)	N/A	0.111	0.056	0.202	0.308	0.392	0.000	0.139	1.573

Problem 90	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	32	89	92	145	0	94	122
normalized size	1	1.00	0.80	2.22	2.30	3.62	0.00	2.35	3.05
time (sec)	N/A	0.130	0.062	0.205	0.307	0.412	0.000	0.125	1.950

Problem 91	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	96	93	257	69	0	55	207
normalized size	1	1.00	3.31	3.21	8.86	2.38	0.00	1.90	7.14
time (sec)	N/A	0.121	0.111	0.203	0.307	0.524	0.000	0.130	1.924

Problem 92	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	97	600	242	1398	0	194	228
normalized size	1	1.00	0.95	5.88	2.37	13.71	0.00	1.90	2.24
time (sec)	N/A	0.198	0.287	0.141	0.317	0.555	0.000	0.129	2.067

Problem 93	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	125	180	486	217	924	0	221	247
normalized size	1	1.00	1.44	3.89	1.74	7.39	0.00	1.77	1.98
time (sec)	N/A	0.378	1.207	0.149	0.409	0.590	0.000	0.136	2.024

Problem 94	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	56	274	127	476	0	97	121
normalized size	1	1.00	0.98	4.81	2.23	8.35	0.00	1.70	2.12
time (sec)	N/A	0.160	0.111	0.144	0.316	0.665	0.000	0.125	1.684

Problem 95	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	80	172	122	304	0	121	159
normalized size	1	1.00	1.04	2.23	1.58	3.95	0.00	1.57	2.06
time (sec)	N/A	0.206	0.223	0.136	0.403	0.764	0.000	0.135	1.693

Problem 96	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	19	31	48	80	0	39	20
normalized size	1	1.00	0.95	1.55	2.40	4.00	0.00	1.95	1.00
time (sec)	N/A	0.080	0.013	0.124	0.308	0.634	0.000	0.141	0.084

Problem 97	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	36	84	66	57	0	89	93
normalized size	1	1.00	0.56	1.31	1.03	0.89	0.00	1.39	1.45
time (sec)	N/A	0.111	0.059	0.149	0.404	0.565	0.000	0.145	2.357

Problem 98	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	67	81	91	256	0	85	133
normalized size	1	1.00	1.12	1.35	1.52	4.27	0.00	1.42	2.22
time (sec)	N/A	0.142	0.184	0.171	0.408	1.331	0.000	0.154	1.659

Problem 99	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	78	275	161	675	0	218	256
normalized size	1	1.00	0.82	2.89	1.69	7.11	0.00	2.29	2.69
time (sec)	N/A	0.220	0.156	0.194	0.411	0.640	0.000	0.138	2.994

Problem 100	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	114	170	226	1155	0	174	269
normalized size	1	1.00	1.10	1.63	2.17	11.11	0.00	1.67	2.59
time (sec)	N/A	0.270	0.629	0.180	0.418	1.555	0.000	0.176	1.793

Problem 101	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	149	138	1168	348	2778	0	374	513
normalized size	1	1.00	0.93	7.84	2.34	18.64	0.00	2.51	3.44
time (sec)	N/A	0.341	0.284	0.195	0.428	1.578	0.000	0.141	5.507

Problem 102	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	75	155	144	299	0	120	274
normalized size	1	1.00	0.69	1.42	1.32	2.74	0.00	1.10	2.51
time (sec)	N/A	0.086	0.203	0.251	0.328	1.372	0.000	0.148	4.105

Problem 103	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	126	99	96	123	0	62	237
normalized size	1	1.00	2.42	1.90	1.85	2.37	0.00	1.19	4.56
time (sec)	N/A	0.093	0.135	0.253	0.317	1.017	0.000	0.128	2.218

Problem 104	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	61	109	96	185	0	98	140
normalized size	1	1.00	0.79	1.42	1.25	2.40	0.00	1.27	1.82
time (sec)	N/A	0.067	0.124	0.250	0.317	1.974	0.000	0.124	0.577

Problem 105	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	71	67	42	51	0	38	85
normalized size	1	1.00	1.97	1.86	1.17	1.42	0.00	1.06	2.36
time (sec)	N/A	0.073	0.087	0.211	0.314	1.458	0.000	0.126	1.634

Problem 106	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	39	65	45	71	0	55	50
normalized size	1	1.00	0.87	1.44	1.00	1.58	0.00	1.22	1.11
time (sec)	N/A	0.048	0.040	0.209	0.311	1.217	0.000	0.123	0.225

Problem 107	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	17	15	11	0	13	14
normalized size	1	1.00	1.00	1.31	1.15	0.85	0.00	1.00	1.08
time (sec)	N/A	0.023	0.008	0.118	0.303	1.307	0.000	0.144	1.463

Problem 108	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	13	27	20	16	0	17	21
normalized size	1	1.00	1.18	2.45	1.82	1.45	0.00	1.55	1.91
time (sec)	N/A	0.038	0.036	0.116	0.310	1.970	0.000	0.122	0.178

Problem 109	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	12	36	40	0	38	27
normalized size	1	1.00	1.00	1.00	3.00	3.33	0.00	3.17	2.25
time (sec)	N/A	0.039	0.011	0.092	0.307	0.732	0.000	0.128	1.571

Problem 110	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	65	61	55	85	0	48	56
normalized size	1	1.00	2.41	2.26	2.04	3.15	0.00	1.78	2.07
time (sec)	N/A	0.056	0.037	0.182	0.309	1.062	0.000	0.130	1.640

Problem 111	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	78	75	97	0	68	81
normalized size	1	1.00	1.00	2.60	2.50	3.23	0.00	2.27	2.70
time (sec)	N/A	0.046	0.015	0.233	0.310	2.108	0.000	0.144	1.682

Problem 112	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	129	95	96	154	0	77	106
normalized size	1	1.00	3.00	2.21	2.23	3.58	0.00	1.79	2.47
time (sec)	N/A	0.075	0.042	0.247	0.309	0.572	0.000	0.126	1.862

Problem 113	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	194	194	253	1323	383	4025	0	432	611
normalized size	1	1.00	1.30	6.82	1.97	20.75	0.00	2.23	3.15
time (sec)	N/A	0.255	0.506	0.227	0.432	3.045	0.000	0.163	7.041

Problem 114	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	183	183	141	207	261	1746	0	215	707
normalized size	1	1.00	0.77	1.13	1.43	9.54	0.00	1.17	3.86
time (sec)	N/A	0.384	0.744	0.203	0.409	0.637	0.000	0.149	3.429

Problem 115	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	A	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	191	324	172	965	0	234	335
normalized size	1	1.00	1.69	2.87	1.52	8.54	0.00	2.07	2.96
time (sec)	N/A	0.163	0.201	0.189	0.419	0.867	0.000	0.130	4.011

Problem 116	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	82	95	108	349	0	102	376
normalized size	1	1.00	0.82	0.95	1.08	3.49	0.00	1.02	3.76
time (sec)	N/A	0.221	0.332	0.178	0.416	0.688	0.000	0.162	2.471

Problem 117	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	63	108	74	75	0	89	132
normalized size	1	1.00	1.03	1.77	1.21	1.23	0.00	1.46	2.16
time (sec)	N/A	0.099	0.062	0.180	0.409	1.261	0.000	0.128	2.526

Problem 118	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	11	21	28	27	0	22	25
normalized size	1	1.00	0.58	1.11	1.47	1.42	0.00	1.16	1.32
time (sec)	N/A	0.032	0.008	0.114	0.317	0.602	0.000	0.126	0.099

Problem 119	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	65	110	90	141	0	89	316
normalized size	1	1.00	1.14	1.93	1.58	2.47	0.00	1.56	5.54
time (sec)	N/A	0.178	0.085	0.126	0.415	0.629	0.000	0.158	0.344

Problem 120	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	37	106	82	199	0	80	261
normalized size	1	1.00	1.16	3.31	2.56	6.22	0.00	2.50	8.16
time (sec)	N/A	0.072	0.053	0.144	0.316	0.614	0.000	0.126	1.840

Problem 121	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	151	207	178	831	0	161	378
normalized size	1	1.00	1.72	2.35	2.02	9.44	0.00	1.83	4.30
time (sec)	N/A	0.332	0.588	0.148	0.428	1.719	0.000	0.151	2.623

Problem 122	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	83	219	190	1288	0	170	155
normalized size	1	1.00	1.19	3.13	2.71	18.40	0.00	2.43	2.21
time (sec)	N/A	0.094	0.135	0.153	0.327	0.887	0.000	0.128	1.975

Problem 123	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	183	183	269	360	330	3160	0	305	543
normalized size	1	1.00	1.47	1.97	1.80	17.27	0.00	1.67	2.97
time (sec)	N/A	0.337	1.579	0.167	0.418	2.180	0.000	0.142	2.898

Problem 124	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	130	388	364	4024	0	295	317
normalized size	1	1.00	1.09	3.26	3.06	33.82	0.00	2.48	2.66
time (sec)	N/A	0.141	0.259	0.161	0.334	0.714	0.000	0.153	2.182

Problem 125	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	199	199	84	91	386	592	0	90	413
normalized size	1	1.00	0.42	0.46	1.94	2.97	0.00	0.45	2.08
time (sec)	N/A	0.303	0.082	0.636	0.420	0.635	0.000	0.154	1.595

Problem 126	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	72	80	209	315	0	77	91
normalized size	1	1.00	0.49	0.54	1.42	2.14	0.00	0.52	0.62
time (sec)	N/A	0.171	0.067	0.607	0.416	1.977	0.000	0.149	1.530

Problem 127	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	56	69	84	121	0	64	78
normalized size	1	1.00	0.97	1.19	1.45	2.09	0.00	1.10	1.34
time (sec)	N/A	0.118	0.045	0.601	0.422	1.060	0.000	0.130	1.549

Problem 128	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	44	68	39	42	0	48	-1
normalized size	1	1.00	0.96	1.48	0.85	0.91	0.00	1.04	-0.02
time (sec)	N/A	0.091	0.037	0.704	0.415	0.451	0.000	0.156	0.000

Problem 129	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	48	106	36	66	0	71	-1
normalized size	1	1.00	0.65	1.43	0.49	0.89	0.00	0.96	-0.01
time (sec)	N/A	0.115	0.051	0.542	0.415	0.594	0.000	0.127	0.000

Problem 130	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	162	162	76	216	62	126	0	204	-1
normalized size	1	1.00	0.47	1.33	0.38	0.78	0.00	1.26	-0.01
time (sec)	N/A	0.153	0.066	0.540	0.419	0.609	0.000	0.146	0.000

Problem 131	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	250	250	106	326	90	218	0	278	-1
normalized size	1	1.00	0.42	1.30	0.36	0.87	0.00	1.11	-0.00
time (sec)	N/A	0.200	0.104	0.544	0.423	2.046	0.000	0.174	0.000

Problem 132	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	80	125	0	0	0	0	-1
normalized size	1	1.00	0.99	1.54	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.068	0.179	0.209	0.000	2.017	0.000	0.000	0.000

Problem 133	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	44	39	46	48	0	0	42
normalized size	1	1.00	1.47	1.30	1.53	1.60	0.00	0.00	1.40
time (sec)	N/A	0.045	0.050	0.145	0.475	1.213	0.000	0.000	2.153

Problem 134	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	60	127	0	0	0	0	-1
normalized size	1	1.00	0.50	1.07	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.086	0.116	0.158	0.000	1.990	0.000	0.000	0.000

Problem 135	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	74	97	0	92	0	0	-1
normalized size	1	1.00	1.07	1.41	0.00	1.33	0.00	0.00	-0.01
time (sec)	N/A	0.058	0.144	0.186	0.000	1.664	0.000	0.000	0.000

Problem 136	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	57	109	0	0	0	0	-1
normalized size	1	1.00	0.95	1.82	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.041	0.109	0.147	0.000	0.971	0.000	0.000	0.000

Problem 137	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	77	0	0	86	0	0	-1
normalized size	1	1.00	1.28	0.00	0.00	1.43	0.00	0.00	-0.02
time (sec)	N/A	0.035	0.090	0.143	0.000	0.709	0.000	0.000	0.000

Problem 138	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	43	90	0	0	0	0	-1
normalized size	1	1.00	0.93	1.96	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.032	0.075	0.287	0.000	0.607	0.000	0.000	0.000

Problem 139	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	54	0	0	43	0	0	-1
normalized size	1	1.00	1.32	0.00	0.00	1.05	0.00	0.00	-0.02
time (sec)	N/A	0.046	0.118	0.140	0.000	0.465	0.000	0.000	0.000

Problem 140	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	58	126	0	0	0	0	-1
normalized size	1	1.00	0.78	1.70	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.066	0.096	0.161	0.000	0.457	0.000	0.000	0.000

Problem 141	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	33	38	89	37	0	0	58
normalized size	1	1.00	1.32	1.52	3.56	1.48	0.00	0.00	2.32
time (sec)	N/A	0.041	0.040	0.136	0.428	0.461	0.000	0.000	1.471

Problem 142	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	60	112	0	0	0	0	-1
normalized size	1	1.00	0.94	1.75	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.055	0.099	0.137	0.000	0.593	0.000	0.000	0.000

Problem 143	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	128	95	121	0	110	0	0	-1
normalized size	1	1.00	0.74	0.95	0.00	0.86	0.00	0.00	-0.01
time (sec)	N/A	0.080	0.208	0.214	0.000	0.426	0.000	0.000	0.000

Problem 144	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	80	133	0	0	0	0	-1
normalized size	1	1.00	0.68	1.13	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.078	0.176	0.155	0.000	0.459	0.000	0.000	0.000

Problem 145	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	44	47	46	56	0	0	42
normalized size	1	1.00	1.47	1.57	1.53	1.87	0.00	0.00	1.40
time (sec)	N/A	0.041	0.052	0.136	0.467	1.028	0.000	0.000	1.555

Problem 146	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	162	162	63	140	0	0	0	0	-1
normalized size	1	1.00	0.39	0.86	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.100	0.127	0.150	0.000	0.539	0.000	0.000	0.000

Problem 147	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	87	113	0	102	0	0	-1
normalized size	1	1.00	0.91	1.18	0.00	1.06	0.00	0.00	-0.01
time (sec)	N/A	0.067	0.184	0.186	0.000	0.622	0.000	0.000	0.000

Problem 148	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	65	124	0	0	0	0	-1
normalized size	1	1.00	0.76	1.44	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.063	0.128	0.148	0.000	0.565	0.000	0.000	0.000

Problem 149	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	88	0	0	94	0	0	-1
normalized size	1	1.00	0.97	0.00	0.00	1.03	0.00	0.00	-0.01
time (sec)	N/A	0.073	0.162	0.149	0.000	0.658	0.000	0.000	0.000

Problem 150	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	60	152	0	0	0	0	-1
normalized size	1	1.00	0.46	1.17	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.073	0.115	0.159	0.000	0.429	0.000	0.000	0.000

Problem 151	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	63	130	0	106	0	0	-1
normalized size	1	1.00	0.66	1.35	0.00	1.10	0.00	0.00	-0.01
time (sec)	N/A	0.046	0.096	0.191	0.000	0.492	0.000	0.000	0.000

Problem 152	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	54	163	0	0	0	0	-1
normalized size	1	1.00	0.81	2.43	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.037	0.098	0.414	0.000	1.306	0.000	0.000	0.000

Problem 153	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	A	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	33	0	87	29	0	0	29
normalized size	1	1.00	1.22	0.00	3.22	1.07	0.00	0.00	1.07
time (sec)	N/A	0.039	0.035	0.152	0.464	0.473	0.000	0.000	1.474

Problem 154	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	66	0	0	0	0	0	-1
normalized size	1	1.00	0.96	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.054	0.108	0.138	0.000	0.491	0.000	0.000	0.000

Problem 155	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	A	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	53	0	0	78	0	0	-1
normalized size	1	1.00	0.77	0.00	0.00	1.13	0.00	0.00	-0.01
time (sec)	N/A	0.061	0.107	0.147	0.000	0.730	0.000	0.000	0.000

Problem 156	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	62	0	0	0	0	0	-1
normalized size	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.055	1.201	0.369	0.000	0.463	0.000	0.000	0.000

Problem 157	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	126	0	0	0	0	0	-1
normalized size	1	1.00	1.85	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.065	4.320	1.782	0.000	0.493	0.000	0.000	0.000

Problem 158	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	101	0	0	0	0	0	-1
normalized size	1	1.00	1.46	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.069	5.097	2.109	0.000	0.716	0.000	0.000	0.000

Problem 159	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	200	0	0	0	0	0	-1
normalized size	1	1.00	2.94	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.069	9.024	1.813	0.000	0.478	0.000	0.000	0.000

Problem 160	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	C	A	C	B	B	F	B	B
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	137	30	509	95	187	0	215	65
normalized size	1	3.26	0.71	12.12	2.26	4.45	0.00	5.12	1.55
time (sec)	N/A	0.135	0.424	1.109	0.706	1.576	0.000	0.548	1.520

Problem 161	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	62	0	76	48	0	38	48
normalized size	1	1.00	2.38	0.00	2.92	1.85	0.00	1.46	1.85
time (sec)	N/A	0.039	0.127	0.529	0.315	0.585	0.000	0.145	1.647

Problem 162	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	65	0	49	49	0	39	36
normalized size	1	1.00	2.50	0.00	1.88	1.88	0.00	1.50	1.38
time (sec)	N/A	0.046	0.112	0.728	0.354	0.409	0.000	0.111	1.592

Problem 163	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	115	0	0	475	0	0	-1
normalized size	1	1.00	1.28	0.00	0.00	5.28	0.00	0.00	-0.01
time (sec)	N/A	0.087	6.756	0.513	0.000	0.487	0.000	0.000	0.000

Problem 164	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	64	0	0	539	0	0	-1
normalized size	1	1.00	0.97	0.00	0.00	8.17	0.00	0.00	-0.02
time (sec)	N/A	0.075	0.918	0.513	0.000	1.133	0.000	0.000	0.000

Problem 165	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	54	23	22	65	0	145	43
normalized size	1	1.00	2.70	1.15	1.10	3.25	0.00	7.25	2.15
time (sec)	N/A	0.018	0.062	0.087	0.307	0.603	0.000	0.579	1.620

Problem 166	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	20	29	71	0	28	25
normalized size	1	1.00	1.00	1.05	1.53	3.74	0.00	1.47	1.32
time (sec)	N/A	0.027	0.072	0.327	0.326	0.677	0.000	0.155	1.438

Problem 167	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	81	51	150	643	0	210	140
normalized size	1	1.00	1.47	0.93	2.73	11.69	0.00	3.82	2.55
time (sec)	N/A	0.045	0.063	0.329	0.362	1.349	0.000	0.190	1.489

Problem 168	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	56	36	92	272	0	47	55
normalized size	1	1.00	1.33	0.86	2.19	6.48	0.00	1.12	1.31
time (sec)	N/A	0.033	0.068	0.302	0.335	0.441	0.000	0.161	1.454

Problem 169	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	135	84	232	1806	0	248	318
normalized size	1	1.00	1.52	0.94	2.61	20.29	0.00	2.79	3.57
time (sec)	N/A	0.070	0.063	0.313	0.353	0.513	0.000	0.223	1.477

Problem 170	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	84	144	0	0	0	0	-1
normalized size	1	1.00	0.76	1.30	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.068	0.223	0.485	0.000	0.408	0.000	0.000	0.000

Problem 171	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	80	212	0	0	0	0	-1
normalized size	1	1.00	0.75	1.98	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.063	0.115	0.550	0.000	0.478	0.000	0.000	0.000

Problem 172	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	66	120	0	0	0	0	-1
normalized size	1	1.00	0.92	1.67	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.043	0.097	0.344	0.000	0.432	0.000	0.000	0.000

Problem 173	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	68	146	0	0	0	0	-1
normalized size	1	1.00	0.94	2.03	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.044	0.082	0.587	0.000	0.460	0.000	0.000	0.000

Problem 174	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	86	143	0	0	0	0	-1
normalized size	1	1.00	0.77	1.29	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.060	0.135	0.452	0.000	0.448	0.000	0.000	0.000

Problem 175	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	95	227	0	0	0	0	-1
normalized size	1	1.00	0.86	2.05	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.061	0.190	0.688	0.000	0.453	0.000	0.000	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [114] had the largest ratio of [.6923]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	1	1	1.00	6	0.167
2	A	2	2	1.00	8	0.250
3	A	2	2	1.00	8	0.250
4	A	2	1	1.00	8	0.125
5	A	3	2	1.00	8	0.250
6	A	2	1	1.00	8	0.125
7	A	3	3	1.00	10	0.300
8	A	3	3	1.00	10	0.300
9	A	2	2	1.00	10	0.200
10	A	2	2	1.00	10	0.200
11	A	3	3	1.00	10	0.300
12	A	3	3	1.00	10	0.300
13	A	4	3	1.00	12	0.250

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
14	A	3	3	1.00	12	0.250
15	A	3	3	1.00	12	0.250
16	A	2	2	1.00	12	0.167
17	A	2	2	1.00	12	0.167
18	A	3	3	1.00	12	0.250
19	A	3	3	1.00	12	0.250
20	A	4	3	1.00	12	0.250
21	A	2	2	1.00	10	0.200
22	A	4	3	1.00	10	0.300
23	A	3	3	1.00	10	0.300
24	A	2	2	1.00	10	0.200
25	A	2	2	1.00	10	0.200
26	A	3	3	1.00	10	0.300
27	A	4	3	1.00	10	0.300
28	A	5	3	1.00	10	0.300
29	A	5	4	1.00	10	0.400
30	A	4	4	1.00	10	0.400
31	A	3	3	1.00	10	0.300
32	A	2	2	1.00	10	0.200
33	A	3	3	1.00	10	0.300
34	A	4	3	1.00	10	0.300
35	A	5	3	1.00	10	0.300
36	A	7	4	1.00	10	0.400
37	A	5	4	1.00	10	0.400
38	A	4	4	1.07	10	0.400
39	A	4	4	1.00	10	0.400
40	A	5	4	1.00	10	0.400
41	A	7	4	1.00	10	0.400
42	A	3	2	1.00	10	0.200
43	A	3	2	1.00	10	0.200
44	A	3	2	1.00	10	0.200
45	A	3	3	1.00	10	0.300
46	A	3	3	1.00	10	0.300
47	A	5	3	1.00	10	0.300
48	A	7	3	1.00	10	0.300
49	A	2	2	1.00	15	0.133

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
50	A	2	2	1.00	15	0.133
51	A	5	5	1.00	17	0.294
52	A	4	4	1.00	17	0.235
53	A	2	2	1.00	17	0.118
54	A	5	4	1.00	17	0.235
55	A	6	5	1.00	17	0.294
56	A	2	2	1.00	17	0.118
57	A	5	4	1.00	17	0.235
58	A	2	2	1.00	12	0.167
59	A	2	2	1.00	12	0.167
60	A	2	2	1.00	12	0.167
61	A	2	2	1.00	12	0.167
62	A	7	5	1.00	13	0.385
63	A	6	5	1.00	13	0.385
64	A	5	5	1.00	13	0.385
65	A	4	4	1.00	11	0.364
66	A	1	1	1.00	11	0.091
67	A	3	3	1.00	13	0.231
68	A	4	4	1.00	13	0.308
69	A	6	6	1.00	13	0.462
70	A	6	5	1.00	12	0.417
71	A	5	4	1.00	12	0.333
72	A	4	4	1.00	12	0.333
73	A	2	1	1.00	10	0.100
74	A	4	4	1.00	12	0.333
75	A	6	6	1.00	12	0.500
76	A	7	7	1.00	12	0.583
77	A	8	7	1.00	13	0.538
78	A	7	7	1.00	13	0.538
79	A	6	6	1.00	11	0.546
80	A	4	4	1.00	11	0.364
81	A	6	6	1.00	13	0.462
82	A	7	7	1.00	13	0.538
83	A	8	8	1.00	13	0.615
84	A	7	7	1.00	13	0.538
85	A	6	4	1.00	13	0.308

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
86	A	5	5	1.00	13	0.385
87	A	4	3	1.00	11	0.273
88	A	6	6	1.00	11	0.546
89	A	6	5	1.00	13	0.385
90	A	7	7	1.00	13	0.538
91	A	7	6	1.00	13	0.462
92	A	5	4	1.00	13	0.308
93	A	7	6	1.00	13	0.462
94	A	5	4	1.00	13	0.308
95	A	6	6	1.00	13	0.462
96	A	5	4	1.00	11	0.364
97	A	4	3	1.00	11	0.273
98	A	6	6	1.00	13	0.462
99	A	6	5	1.00	13	0.385
100	A	7	6	1.00	13	0.462
101	A	7	5	1.00	13	0.385
102	A	3	2	1.00	13	0.154
103	A	5	3	1.00	13	0.231
104	A	3	2	1.00	13	0.154
105	A	4	3	1.00	13	0.231
106	A	3	2	1.00	11	0.182
107	A	2	2	1.00	11	0.182
108	A	3	2	1.00	13	0.154
109	A	3	2	1.00	13	0.154
110	A	4	3	1.00	13	0.231
111	A	3	2	1.00	13	0.154
112	A	5	3	1.00	13	0.231
113	A	11	7	1.00	13	0.538
114	A	16	9	1.00	13	0.692
115	A	8	6	1.00	13	0.462
116	A	10	9	1.00	13	0.692
117	A	6	5	1.00	11	0.454
118	A	4	4	1.00	11	0.364
119	A	8	8	1.00	13	0.615
120	A	3	2	1.00	13	0.154
121	A	7	7	1.00	13	0.538

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
122	A	3	2	1.00	13	0.154
123	A	16	9	1.00	13	0.692
124	A	3	2	1.00	13	0.154
125	A	6	5	1.00	25	0.200
126	A	6	5	1.00	25	0.200
127	A	4	4	1.00	25	0.160
128	A	4	4	1.00	25	0.160
129	A	5	4	1.00	25	0.160
130	A	6	5	1.00	25	0.200
131	A	6	5	1.00	25	0.200
132	A	6	6	1.00	15	0.400
133	A	3	3	1.00	15	0.200
134	A	9	9	1.00	15	0.600
135	A	6	6	1.00	15	0.400
136	A	5	5	1.00	13	0.385
137	A	6	6	1.00	11	0.546
138	A	3	2	1.00	15	0.133
139	A	5	5	1.00	15	0.333
140	A	7	7	1.00	15	0.467
141	A	3	3	1.00	15	0.200
142	A	5	5	1.00	15	0.333
143	A	8	7	1.00	15	0.467
144	A	7	6	1.00	15	0.400
145	A	3	3	1.00	15	0.200
146	A	10	9	1.00	15	0.600
147	A	7	6	1.00	15	0.400
148	A	6	5	1.00	15	0.333
149	A	7	6	1.00	15	0.400
150	A	9	8	1.00	13	0.615
151	A	7	7	1.00	11	0.636
152	A	4	3	1.00	15	0.200
153	A	3	3	1.00	15	0.200
154	A	5	5	1.00	15	0.333
155	A	6	6	1.00	15	0.400
156	A	4	4	1.00	11	0.364
157	A	4	4	1.00	13	0.308

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
158	A	4	4	1.00	13	0.308
159	A	4	4	1.00	13	0.308
160	C	9	4	3.26	45	0.089
161	A	3	3	1.00	15	0.200
162	A	4	4	1.00	15	0.267
163	A	3	3	1.00	20	0.150
164	A	3	3	1.00	21	0.143
165	A	2	1	1.00	15	0.067
166	A	3	2	1.00	17	0.118
167	A	3	2	1.00	17	0.118
168	A	3	1	1.00	17	0.059
169	A	4	2	1.00	17	0.118
170	A	4	3	1.00	19	0.158
171	A	4	3	1.00	19	0.158
172	A	3	2	1.00	19	0.105
173	A	3	2	1.00	19	0.105
174	A	4	3	1.00	19	0.158
175	A	4	3	1.00	19	0.158

Chapter 3

Listing of integrals

3.1 $\int \operatorname{csch}(a + bx)\, dx$

Optimal. Leaf size=12

$$-\frac{\tanh^{-1}(\cosh(a+bx))}{b}$$

```
[Out] -arctanh(cosh(b*x+a))/b
```

Rubi [A] time = 0.01, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3770}

$$-\frac{\tanh^{-1}(\cosh(a+bx))}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[a + b*x],x]
```

```
[Out] -(ArcTanh[Cosh[a + b*x]]/b)
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \operatorname{csch}(a+bx)\, dx = -\frac{\tanh^{-1}(\cosh(a+bx))}{b}$$

Mathematica [B] time = 0.02, size = 38, normalized size = 3.17

$$\frac{\log\left(\sinh\left(\frac{a}{2}+\frac{bx}{2}\right)\right)}{b} - \frac{\log\left(\cosh\left(\frac{a}{2}+\frac{bx}{2}\right)\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[a + b*x],x]
```

```
[Out] -(Log[Cosh[a/2 + (b*x)/2]]/b) + Log[Sinh[a/2 + (b*x)/2]]/b
```

fricas [B] time = 0.51, size = 38, normalized size = 3.17

$$-\frac{\log\left(\cosh\left(bx+a\right)+\sinh\left(bx+a\right)+1\right)-\log\left(\cosh\left(bx+a\right)+\sinh\left(bx+a\right)-1\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a),x, algorithm="fricas")
```

```
[Out] -(log(cosh(b*x + a) + sinh(b*x + a) + 1) - log(cosh(b*x + a) + sinh(b*x + a
) - 1))/b
```

giac [B] time = 0.13, size = 27, normalized size = 2.25

$$-\frac{\log\left(e^{(bx+a)}+1\right)-\log\left(\left|e^{(bx+a)}-1\right|\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a),x, algorithm="giac")
```

```
[Out] -(log(e^(b*x + a) + 1) - log(abs(e^(b*x + a) - 1)))/b
```

maple [A] time = 0.02, size = 15, normalized size = 1.25

$$\frac{\ln\left(\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(b*x+a),x)
```

```
[Out] 1/b*ln(tanh(1/2*b*x+1/2*a))
```

maxima [A] time = 0.31, size = 14, normalized size = 1.17

$$\frac{\log\left(\tanh\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a),x, algorithm="maxima")
```

```
[Out] log(tanh(1/2*b*x + 1/2*a))/b
```

mupad [B] time = 0.11, size = 27, normalized size = 2.25

$$-\frac{2\,\mathrm{atan}\left(\frac{\mathrm{e}^{b\,x}\,\mathrm{e}^{a}\,\sqrt{-b^2}}{b}\right)}{\sqrt{-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sinh(a + b*x),x)
```

```
[Out] -(2*atan((exp(b*x)*exp(a)*(-b^2)^(1/2))/b))/(-b^2)^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{csch}\left(a+bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a),x)
```

```
[Out] Integral(csch(a + b*x), x)
```

3.2 $\int \operatorname{csch}^2(a+bx)\,dx$

Optimal. Leaf size=11

$$-\frac{\coth(a+bx)}{b}$$

```
[Out] -coth(b*x+a)/b
```

Rubi [A] time = 0.01, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3767, 8}

$$-\frac{\coth(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[a + b*x]^2,x]
```

```
[Out] -(Coth[a + b*x]/b)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}\int \operatorname{csch}^2(a+bx)\,dx &= -\frac{i\,\text{Subst}\left(\int 1\,dx, x, -i\coth(a+bx)\right)}{b}\\ &= -\frac{\coth(a+bx)}{b}\end{aligned}$$

Mathematica [A] time = 0.01, size = 11, normalized size = 1.00

$$-\frac{\coth(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[a + b*x]^2,x]
```

```
[Out] -(Coth[a + b*x]/b)
```

fricas [B] time = 0.64, size = 43, normalized size = 3.91

$$-\frac{2}{b\cosh\left(bx+a\right)^2+2\,b\cosh\left(bx+a\right)\sinh\left(bx+a\right)+b\sinh\left(bx+a\right)^2-b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -2/(b*cosh(b*x + a)^2 + 2*b*cosh(b*x + a)*sinh(b*x + a) + b*sinh(b*x + a)^2
 - b)
```

giac [A] time = 0.13, size = 18, normalized size = 1.64

$$-\frac{2}{b\left(e^{(2\,bx+2\,a)}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^2,x, algorithm="giac")
[Out] -2/(b*(e^(2*b*x + 2*a) - 1))
```

maple [A] time = 0.31, size = 12, normalized size = 1.09

$$-\frac{\coth\left(bx+a\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(b*x+a)^2,x)
[Out] -coth(b*x+a)/b
```

maxima [A] time = 0.36, size = 18, normalized size = 1.64

$$\frac{2}{b\left(e^{(-2\,bx-2\,a)}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^2,x, algorithm="maxima")
[Out] 2/(b*(e^(-2*b*x - 2*a) - 1))
```

mupad [B] time = 0.07, size = 18, normalized size = 1.64

$$-\frac{2}{b\,\left({\mathrm{e}}^{2\,a+2\,b\,x}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sinh(a + b*x)^2,x)
[Out] -2/(b*(exp(2*a + 2*b*x) - 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{csch}^{2}\left(a+bx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)**2,x)
[Out] Integral(csch(a + b*x)**2, x)
```

3.3 $\int \operatorname{csch}^3(a+bx)\,dx$

Optimal. Leaf size=34

$$\frac{\tanh^{-1}(\cosh(a+bx))}{2b}-\frac{\coth(a+bx)\operatorname{csch}(a+bx)}{2b}$$

```
[Out] 1/2*arctanh(cosh(b*x+a))/b-1/2*coth(b*x+a)*csch(b*x+a)/b
```

Rubi [A] time = 0.02, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3768, 3770}

$$\frac{\tanh^{-1}(\cosh(a+bx))}{2b}-\frac{\coth(a+bx)\operatorname{csch}(a+bx)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[a + b*x]^3,x]
```

```
[Out] ArcTanh[Cosh[a + b*x]]/(2*b) - (Coth[a + b*x]*Csch[a + b*x])/(2*b)
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \operatorname{csch}^3(a+bx)\,dx&=-\frac{\coth(a+bx)\operatorname{csch}(a+bx)}{2b}-\frac{1}{2}\int \operatorname{csch}(a+bx)\,dx\\&=\frac{\tanh^{-1}(\cosh(a+bx))}{2b}-\frac{\coth(a+bx)\operatorname{csch}(a+bx)}{2b}\end{aligned}$$

Mathematica [A] time = 0.01, size = 57, normalized size = 1.68

$$-\frac{\operatorname{csch}^2\left(\frac{1}{2}(a+bx)\right)}{8b}-\frac{\operatorname{sech}^2\left(\frac{1}{2}(a+bx)\right)}{8b}-\frac{\log\left(\tanh\left(\frac{1}{2}(a+bx)\right)\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[a + b*x]^3,x]
```

```
[Out] -1/8*Csch[(a + b*x)/2]^2/b - Log[Tanh[(a + b*x)/2]]/(2*b) - Sech[(a + b*x)/
2]^2/(8*b)
```

fricas [B] time = 0.59, size = 387, normalized size = 11.38

$$-\frac{2\cosh(bx+a)^3+6\cosh(bx+a)\sinh(bx+a)^2+2\sinh(bx+a)^3-\left(\cosh(bx+a)^4+4\cosh(bx+a)\sin\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -1/2*(2*cosh(b*x + a)^3 + 6*cosh(b*x + a)*sinh(b*x + a)^2 + 2*sinh(b*x + a)
^3 - (cosh(b*x + a)^4 + 4*cosh(b*x + a)*sinh(b*x + a)^3 + sinh(b*x + a)^4 +
 2*(3*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^2 - 2*cosh(b*x + a)^2 + 4*(cosh(b*
x + a)^3 - cosh(b*x + a))*sinh(b*x + a) + 1)*log(cosh(b*x + a) + sinh(b*x +
 a) + 1) + (cosh(b*x + a)^4 + 4*cosh(b*x + a)*sinh(b*x + a)^3 + sinh(b*x +
a)^4 + 2*(3*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^2 - 2*cosh(b*x + a)^2 + 4*(c
osh(b*x + a)^3 - cosh(b*x + a))*sinh(b*x + a) + 1)*log(cosh(b*x + a) + sinh
(b*x + a) - 1) + 2*(3*cosh(b*x + a)^2 + 1)*sinh(b*x + a) + 2*cosh(b*x + a))
/(b*cosh(b*x + a)^4 + 4*b*cosh(b*x + a)*sinh(b*x + a)^3 + b*sinh(b*x + a)^4
 - 2*b*cosh(b*x + a)^2 + 2*(3*b*cosh(b*x + a)^2 - b)*sinh(b*x + a)^2 + 4*(b
*cosh(b*x + a)^3 - b*cosh(b*x + a))*sinh(b*x + a) + b)
```

giac [B] time = 0.12, size = 84, normalized size = 2.47

$$-\frac{\frac{4\left(e^{(bx+a)}+e^{(-bx-a)}\right)}{\left(e^{(bx+a)}+e^{(-bx-a)}\right)^{2}-4}-\log\left(e^{(bx+a)}+e^{(-bx-a)}+2\right)+\log\left(e^{(bx+a)}+e^{(-bx-a)}-2\right)}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^3,x, algorithm="giac")
```

```
[Out] -1/4*(4*(e^(b*x + a) + e^(-b*x - a))/((e^(b*x + a) + e^(-b*x - a))^2 - 4) -
 log(e^(b*x + a) + e^(-b*x - a) + 2) + log(e^(b*x + a) + e^(-b*x - a) - 2))
/b
```

maple [A] time = 0.32, size = 27, normalized size = 0.79

$$\frac{-\frac{\operatorname{csch}(bx+a)\coth(bx+a)}{2}+\operatorname{arctanh}\left(e^{bx+a}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(b*x+a)^3,x)
```

```
[Out] 1/b*(-1/2*csch(b*x+a)*coth(b*x+a)+arctanh(exp(b*x+a)))
```

maxima [B] time = 0.33, size = 84, normalized size = 2.47

$$\frac{\log\left(e^{(-bx-a)}+1\right)}{2\,b}-\frac{\log\left(e^{(-bx-a)}-1\right)}{2\,b}+\frac{e^{(-bx-a)}+e^{(-3\,bx-3\,a)}}{b\left(2\,e^{(-2\,bx-2\,a)}-e^{(-4\,bx-4\,a)}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/2*log(e^(-b*x - a) + 1)/b - 1/2*log(e^(-b*x - a) - 1)/b + (e^(-b*x - a) +
 e^(-3*b*x - 3*a))/(b*(2*e^(-2*b*x - 2*a) - e^(-4*b*x - 4*a) - 1))
```

mupad [B] time = 1.40, size = 86, normalized size = 2.53

$$\frac{\operatorname{atan}\left(\frac{e^{b\,x}\,e^{a}\,\sqrt{-b^2}}{b}\right)}{\sqrt{-b^2}}-\frac{2\,e^{a+b\,x}}{b\,\left(e^{4\,a+4\,b\,x}-2\,e^{2\,a+2\,b\,x}+1\right)}-\frac{e^{a+b\,x}}{b\,\left(e^{2\,a+2\,b\,x}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sinh(a + b*x)^3,x)
```

```
[Out] atan((exp(b*x)*exp(a)*(-b^2)^(1/2))/b)/(-b^2)^(1/2) - (2*exp(a + b*x))/(b*(
exp(4*a + 4*b*x) - 2*exp(2*a + 2*b*x) + 1)) - exp(a + b*x)/(b*(exp(2*a + 2*
b*x) - 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{csch}^3(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)**3,x)
```

```
[Out] Integral(csch(a + b*x)**3, x)
```

3.4 $\int \operatorname{csch}^4(a+bx)\,dx$

Optimal. Leaf size=26

$$\frac{\coth(a+bx)}{b}-\frac{\coth^3(a+bx)}{3b}$$

```
[Out] coth(b*x+a)/b-1/3*coth(b*x+a)^3/b
```

Rubi [A] time = 0.01, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3767}

$$\frac{\coth(a+bx)}{b}-\frac{\coth^3(a+bx)}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[a + b*x]^4,x]
```

```
[Out] Coth[a + b*x]/b - Coth[a + b*x]^3/(3*b)
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}\int \operatorname{csch}^4(a+bx)\,dx &= \frac{i\,\text{Subst}\left(\int \left(1+x^2\right)\,dx,x,-i\coth(a+bx)\right)}{b}\\ &= \frac{\coth(a+bx)}{b}-\frac{\coth^3(a+bx)}{3b}\end{aligned}$$

Mathematica [A] time = 0.01, size = 35, normalized size = 1.35

$$\frac{2\coth(a+bx)}{3b}-\frac{\coth(a+bx)\operatorname{csch}^2(a+bx)}{3b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[a + b*x]^4,x]
```

```
[Out] (2*Coth[a + b*x])/(3*b) - (Coth[a + b*x]*Csch[a + b*x]^2)/(3*b)
```

fricas [B] time = 0.57, size = 164, normalized size = 6.31

$$-\frac{}{3\left(b\cosh\left(bx+a\right)^5+5\,b\cosh\left(bx+a\right)\sinh\left(bx+a\right)^4+b\sinh\left(bx+a\right)^5-3\,b\cosh\left(bx+a\right)^3+\left(10\,b\cosh\left(bx+\right.\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^4,x, algorithm="fricas")
```

```
[Out] -8/3*(cosh(b*x + a) + 2*sinh(b*x + a))/(b*cosh(b*x + a)^5 + 5*b*cosh(b*x +
a)*sinh(b*x + a)^4 + b*sinh(b*x + a)^5 - 3*b*cosh(b*x + a)^3 + (10*b*cosh(b
*x + a)^2 - 3*b)*sinh(b*x + a)^3 + (10*b*cosh(b*x + a)^3 - 9*b*cosh(b*x + a
))*sinh(b*x + a)^2 + 2*b*cosh(b*x + a) + (5*b*cosh(b*x + a)^4 - 9*b*cosh(b*
x + a)^2 + 4*b)*sinh(b*x + a))
```

giac [A] time = 0.12, size = 31, normalized size = 1.19

$$-\frac{4\left(3\,e^{(2\,bx+2\,a)}-1\right)}{3\,b\left(e^{(2\,bx+2\,a)}-1\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^4,x, algorithm="giac")
[Out] -4/3*(3*e^(2*b*x + 2*a) - 1)/(b*(e^(2*b*x + 2*a) - 1)^3)
```

maple [A] time = 0.32, size = 23, normalized size = 0.88

$$\frac{\left(\frac{2}{3}-\frac{\operatorname{csch}(bx+a)^{2}}{3}\right)\coth\left(bx+a\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(b*x+a)^4,x)
[Out] 1/b*(2/3-1/3*csch(b*x+a)^2)*coth(b*x+a)
```

maxima [B] time = 0.46, size = 90, normalized size = 3.46

$$\frac{4\,e^{(-2\,bx-2\,a)}}{b\left(3\,e^{(-2\,bx-2\,a)}-3\,e^{(-4\,bx-4\,a)}+e^{(-6\,bx-6\,a)}-1\right)}-\frac{4}{3\,b\left(3\,e^{(-2\,bx-2\,a)}-3\,e^{(-4\,bx-4\,a)}+e^{(-6\,bx-6\,a)}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^4,x, algorithm="maxima")
[Out] 4*e^(-2*b*x - 2*a)/(b*(3*e^(-2*b*x - 2*a) - 3*e^(-4*b*x - 4*a) + e^(-6*b*x
- 6*a) - 1)) - 4/3/(b*(3*e^(-2*b*x - 2*a) - 3*e^(-4*b*x - 4*a) + e^(-6*b*x
- 6*a) - 1))
```

mupad [B] time = 1.46, size = 31, normalized size = 1.19

$$-\frac{4\,\left(3\,\mathrm{e}^{2\,a+2\,b\,x}-1\right)}{3\,b\,\left(\mathrm{e}^{2\,a+2\,b\,x}-1\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sinh(a + b*x)^4,x)
[Out] -(4*(3*exp(2*a + 2*b*x) - 1))/(3*b*(exp(2*a + 2*b*x) - 1)^3)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{csch}^{4}\left(a+bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)**4,x)
[Out] Integral(csch(a + b*x)**4, x)
```

3.5 $\int \operatorname{csch}^5(a+bx)\,dx$

Optimal. Leaf size=55

$$-\frac{3\tanh^{-1}(\cosh(a+bx))}{8b}-\frac{\coth(a+bx)\operatorname{csch}^3(a+bx)}{4b}+\frac{3\coth(a+bx)\operatorname{csch}(a+bx)}{8b}$$

```
[Out] -3/8*arctanh(cosh(b*x+a))/b+3/8*coth(b*x+a)*csch(b*x+a)/b-1/4*coth(b*x+a)*c
sch(b*x+a)^3/b
```

Rubi [A] time = 0.04, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3768, 3770}

$$-\frac{3\tanh^{-1}(\cosh(a+bx))}{8b}-\frac{\coth(a+bx)\operatorname{csch}^3(a+bx)}{4b}+\frac{3\coth(a+bx)\operatorname{csch}(a+bx)}{8b}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[a + b*x]^5,x]
```

```
[Out] (-3*ArcTanh[Cosh[a + b*x]])/(8*b) + (3*Coth[a + b*x]*Csch[a + b*x])/(8*b) -
 (Coth[a + b*x]*Csch[a + b*x]^3)/(4*b)
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \operatorname{csch}^5(a+bx)\,dx &= -\frac{\coth(a+bx)\operatorname{csch}^3(a+bx)}{4b}-\frac{3}{4}\int \operatorname{csch}^3(a+bx)\,dx\\ &= \frac{3\coth(a+bx)\operatorname{csch}(a+bx)}{8b}-\frac{\coth(a+bx)\operatorname{csch}^3(a+bx)}{4b}+\frac{3}{8}\int \operatorname{csch}(a+bx)\,dx\\ &= -\frac{3\tanh^{-1}(\cosh(a+bx))}{8b}+\frac{3\coth(a+bx)\operatorname{csch}(a+bx)}{8b}-\frac{\coth(a+bx)\operatorname{csch}^3(a+bx)}{4b}\end{aligned}$$

Mathematica [A] time = 0.02, size = 95, normalized size = 1.73

$$-\frac{\operatorname{csch}^4\left(\frac{1}{2}(a+bx)\right)}{64b}+\frac{3\operatorname{csch}^2\left(\frac{1}{2}(a+bx)\right)}{32b}+\frac{\operatorname{sech}^4\left(\frac{1}{2}(a+bx)\right)}{64b}+\frac{3\operatorname{sech}^2\left(\frac{1}{2}(a+bx)\right)}{32b}+\frac{3\log\left(\tanh\left(\frac{1}{2}(a+bx)\right)\right)}{8b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[a + b*x]^5,x]
```

```
[Out] (3*Csch[(a + b*x)/2]^2)/(32*b) - Csch[(a + b*x)/2]^4/(64*b) + (3*Log[Tanh[(
a + b*x)/2]])/(8*b) + (3*Sech[(a + b*x)/2]^2)/(32*b) + Sech[(a + b*x)/2]^4/
(64*b)
```

fricas [B] time = 0.62, size = 1114, normalized size = 20.25

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^5,x, algorithm="fricas")
```

```
[Out] 1/8*(6*cosh(b*x + a)^7 + 42*cosh(b*x + a)*sinh(b*x + a)^6 + 6*sinh(b*x + a)
^7 + 2*(63*cosh(b*x + a)^2 - 11)*sinh(b*x + a)^5 - 22*cosh(b*x + a)^5 + 10*
(21*cosh(b*x + a)^3 - 11*cosh(b*x + a))*sinh(b*x + a)^4 + 2*(105*cosh(b*x +
 a)^4 - 110*cosh(b*x + a)^2 - 11)*sinh(b*x + a)^3 - 22*cosh(b*x + a)^3 + 2*
(63*cosh(b*x + a)^5 - 110*cosh(b*x + a)^3 - 33*cosh(b*x + a))*sinh(b*x + a)
^2 - 3*(cosh(b*x + a)^8 + 8*cosh(b*x + a)*sinh(b*x + a)^7 + sinh(b*x + a)^8
 + 4*(7*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^6 - 4*cosh(b*x + a)^6 + 8*(7*cos
h(b*x + a)^3 - 3*cosh(b*x + a))*sinh(b*x + a)^5 + 2*(35*cosh(b*x + a)^4 - 3
0*cosh(b*x + a)^2 + 3)*sinh(b*x + a)^4 + 6*cosh(b*x + a)^4 + 8*(7*cosh(b*x
+ a)^5 - 10*cosh(b*x + a)^3 + 3*cosh(b*x + a))*sinh(b*x + a)^3 + 4*(7*cosh(
b*x + a)^6 - 15*cosh(b*x + a)^4 + 9*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^2 -
4*cosh(b*x + a)^2 + 8*(cosh(b*x + a)^7 - 3*cosh(b*x + a)^5 + 3*cosh(b*x + a
)^3 - cosh(b*x + a))*sinh(b*x + a) + 1)*log(cosh(b*x + a) + sinh(b*x + a) +
 1) + 3*(cosh(b*x + a)^8 + 8*cosh(b*x + a)*sinh(b*x + a)^7 + sinh(b*x + a)^
8 + 4*(7*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^6 - 4*cosh(b*x + a)^6 + 8*(7*co
sh(b*x + a)^3 - 3*cosh(b*x + a))*sinh(b*x + a)^5 + 2*(35*cosh(b*x + a)^4 -
30*cosh(b*x + a)^2 + 3)*sinh(b*x + a)^4 + 6*cosh(b*x + a)^4 + 8*(7*cosh(b*x
 + a)^5 - 10*cosh(b*x + a)^3 + 3*cosh(b*x + a))*sinh(b*x + a)^3 + 4*(7*cosh
(b*x + a)^6 - 15*cosh(b*x + a)^4 + 9*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^2 -
 4*cosh(b*x + a)^2 + 8*(cosh(b*x + a)^7 - 3*cosh(b*x + a)^5 + 3*cosh(b*x +
a)^3 - cosh(b*x + a))*sinh(b*x + a) + 1)*log(cosh(b*x + a) + sinh(b*x + a)
- 1) + 2*(21*cosh(b*x + a)^6 - 55*cosh(b*x + a)^4 - 33*cosh(b*x + a)^2 + 3)
*sinh(b*x + a) + 6*cosh(b*x + a))/(b*cosh(b*x + a)^8 + 8*b*cosh(b*x + a)*si
nh(b*x + a)^7 + b*sinh(b*x + a)^8 - 4*b*cosh(b*x + a)^6 + 4*(7*b*cosh(b*x +
 a)^2 - b)*sinh(b*x + a)^6 + 8*(7*b*cosh(b*x + a)^3 - 3*b*cosh(b*x + a))*si
nh(b*x + a)^5 + 6*b*cosh(b*x + a)^4 + 2*(35*b*cosh(b*x + a)^4 - 30*b*cosh(b
*x + a)^2 + 3*b)*sinh(b*x + a)^4 + 8*(7*b*cosh(b*x + a)^5 - 10*b*cosh(b*x +
 a)^3 + 3*b*cosh(b*x + a))*sinh(b*x + a)^3 - 4*b*cosh(b*x + a)^2 + 4*(7*b*c
osh(b*x + a)^6 - 15*b*cosh(b*x + a)^4 + 9*b*cosh(b*x + a)^2 - b)*sinh(b*x +
 a)^2 + 8*(b*cosh(b*x + a)^7 - 3*b*cosh(b*x + a)^5 + 3*b*cosh(b*x + a)^3 -
b*cosh(b*x + a))*sinh(b*x + a) + b)
```

giac [B] time = 0.14, size = 110, normalized size = 2.00

$$\frac{\frac{4\left(3\left(e^{(bx+a)}+e^{(-bx-a)}\right)^3-20\,e^{(bx+a)}-20\,e^{(-bx-a)}\right)}{\left(\left(e^{(bx+a)}+e^{(-bx-a)}\right)^2-4\right)^2}-3\,\log\left(e^{(bx+a)}+e^{(-bx-a)}+2\right)+3\,\log\left(e^{(bx+a)}+e^{(-bx-a)}-2\right)}{16\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^5,x, algorithm="giac")
```

```
[Out] 1/16*(4*(3*(e^(b*x + a) + e^(-b*x - a))^3 - 20*e^(b*x + a) - 20*e^(-b*x - a
))/((e^(b*x + a) + e^(-b*x - a))^2 - 4)^2 - 3*log(e^(b*x + a) + e^(-b*x - a
) + 2) + 3*log(e^(b*x + a) + e^(-b*x - a) - 2))/b
```

maple [A] time = 0.31, size = 41, normalized size = 0.75

$$\frac{\left(-\frac{\mathrm{csch}(bx+a)^3}{4}+\frac{3\,\mathrm{csch}(bx+a)}{8}\right)\coth\left(bx+a\right)-\frac{3\,\mathrm{arctanh}\left(\mathrm{e}^{bx+a}\right)}{4}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(b*x+a)^5,x)
```

```
[Out] 1/b*((-1/4*csch(b*x+a)^3+3/8*csch(b*x+a))*coth(b*x+a)-3/4*arctanh(exp(b*x+a
)))
```

maxima [B] time = 0.43, size = 133, normalized size = 2.42

$$-\frac{3\,\log\left(e^{(-bx-a)}+1\right)}{8\,b}+\frac{3\,\log\left(e^{(-bx-a)}-1\right)}{8\,b}-\frac{3\,e^{(-bx-a)}-11\,e^{(-3\,bx-3\,a)}-11\,e^{(-5\,bx-5\,a)}+3\,e^{(-7\,bx-7\,a)}}{4\,b\left(4\,e^{(-2\,bx-2\,a)}-6\,e^{(-4\,bx-4\,a)}+4\,e^{(-6\,bx-6\,a)}-e^{(-8\,bx-8\,a)}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^5,x, algorithm="maxima")
```

```
[Out] -3/8*log(e^(-b*x - a) + 1)/b + 3/8*log(e^(-b*x - a) - 1)/b - 1/4*(3*e^(-b*x
 - a) - 11*e^(-3*b*x - 3*a) - 11*e^(-5*b*x - 5*a) + 3*e^(-7*b*x - 7*a))/(b*
(4*e^(-2*b*x - 2*a) - 6*e^(-4*b*x - 4*a) + 4*e^(-6*b*x - 6*a) - e^(-8*b*x -
 8*a) - 1))
```

mupad [B] time = 0.06, size = 193, normalized size = 3.51

$$\frac{3\,\mathrm{e}^{a+b\,x}}{4\,b\,\left(\mathrm{e}^{2\,a+2\,b\,x}-1\right)}-\frac{\mathrm{e}^{a+b\,x}}{2\,b\,\left(\mathrm{e}^{4\,a+4\,b\,x}-2\,\mathrm{e}^{2\,a+2\,b\,x}+1\right)}-\frac{2\,\mathrm{e}^{a+b\,x}}{b\,\left(3\,\mathrm{e}^{2\,a+2\,b\,x}-3\,\mathrm{e}^{4\,a+4\,b\,x}+\mathrm{e}^{6\,a+6\,b\,x}-1\right)}-\frac{}{b\,\left(6\,\mathrm{e}^{4\,a+4\,b\,x}-4\,\mathrm{e}^{2\,a}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sinh(a + b*x)^5,x)
```

```
[Out] (3*exp(a + b*x))/(4*b*(exp(2*a + 2*b*x) - 1)) - exp(a + b*x)/(2*b*(exp(4*a
+ 4*b*x) - 2*exp(2*a + 2*b*x) + 1)) - (2*exp(a + b*x))/(b*(3*exp(2*a + 2*b*
x) - 3*exp(4*a + 4*b*x) + exp(6*a + 6*b*x) - 1)) - (4*exp(3*a + 3*b*x))/(b*
(6*exp(4*a + 4*b*x) - 4*exp(2*a + 2*b*x) - 4*exp(6*a + 6*b*x) + exp(8*a + 8
*b*x) + 1)) - (3*atan((exp(b*x)*exp(a)*(-b^2)^(1/2))/b))/(4*(-b^2)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{csch}^{5}(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)**5,x)
```

```
[Out] Integral(csch(a + b*x)**5, x)
```

3.6 $\int \text{csch}^6(a+bx)\,dx$

Optimal. Leaf size=42

$$-\frac{\coth^5(a+bx)}{5b}+\frac{2\coth^3(a+bx)}{3b}-\frac{\coth(a+bx)}{b}$$

```
[Out] -coth(b*x+a)/b+2/3*coth(b*x+a)^3/b-1/5*coth(b*x+a)^5/b
```

Rubi [A] time = 0.02, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3767}

$$-\frac{\coth^5(a+bx)}{5b}+\frac{2\coth^3(a+bx)}{3b}-\frac{\coth(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[a + b*x]^6,x]
```

```
[Out] -(Coth[a + b*x]/b) + (2*Coth[a + b*x]^3)/(3*b) - Coth[a + b*x]^5/(5*b)
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}\int \text{csch}^6(a+bx)\,dx &= -\frac{i\,\text{Subst}\left(\int\left(1+2x^2+x^4\right)dx,x,-i\coth(a+bx)\right)}{b}\\ &= -\frac{\coth(a+bx)}{b}+\frac{2\coth^3(a+bx)}{3b}-\frac{\coth^5(a+bx)}{5b}\end{aligned}$$

Mathematica [A] time = 0.02, size = 56, normalized size = 1.33

$$-\frac{8\coth(a+bx)}{15b}-\frac{\coth(a+bx)\text{csch}^4(a+bx)}{5b}+\frac{4\coth(a+bx)\text{csch}^2(a+bx)}{15b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[a + b*x]^6,x]
```

```
[Out] (-8*Coth[a + b*x])/(15*b) + (4*Coth[a + b*x]*Csch[a + b*x]^2)/(15*b) - (Cot
h[a + b*x]*Csch[a + b*x]^4)/(5*b)
```

fricas [B] time = 1.20, size = 344, normalized size = 8.19

$$-\frac{}{15\left(b\cosh\left(bx+a\right)^8+8\,b\cosh\left(bx+a\right)\sinh\left(bx+a\right)^7+b\sinh\left(bx+a\right)^8-5\,b\cosh\left(bx+a\right)^6+\left(28\,b\cosh\left(b\right.\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^6,x, algorithm="fricas")
```

```
[Out] -16/15*(11*cosh(b*x + a)^2 + 18*cosh(b*x + a)*sinh(b*x + a) + 11*sinh(b*x +
 a)^2 - 5)/(b*cosh(b*x + a)^8 + 8*b*cosh(b*x + a)*sinh(b*x + a)^7 + b*sinh(
```

```
b*x + a)^8 - 5*b*cosh(b*x + a)^6 + (28*b*cosh(b*x + a)^2 - 5*b)*sinh(b*x +
a)^6 + 2*(28*b*cosh(b*x + a)^3 - 15*b*cosh(b*x + a))*sinh(b*x + a)^5 + 10*b
*cosh(b*x + a)^4 + 5*(14*b*cosh(b*x + a)^4 - 15*b*cosh(b*x + a)^2 + 2*b)*si
nh(b*x + a)^4 + 4*(14*b*cosh(b*x + a)^5 - 25*b*cosh(b*x + a)^3 + 10*b*cosh(
b*x + a))*sinh(b*x + a)^3 - 11*b*cosh(b*x + a)^2 + (28*b*cosh(b*x + a)^6 -
75*b*cosh(b*x + a)^4 + 60*b*cosh(b*x + a)^2 - 11*b)*sinh(b*x + a)^2 + 2*(4*
b*cosh(b*x + a)^7 - 15*b*cosh(b*x + a)^5 + 20*b*cosh(b*x + a)^3 - 9*b*cosh(
b*x + a))*sinh(b*x + a) + 5*b)
```

giac [A] time = 0.14, size = 42, normalized size = 1.00

$$-\frac{16\left(10\,e^{(4\,bx+4\,a)}-5\,e^{(2\,bx+2\,a)}+1\right)}{15\,b\left(e^{(2\,bx+2\,a)}-1\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^6,x, algorithm="giac")
```

```
[Out] -16/15*(10*e^(4*b*x + 4*a) - 5*e^(2*b*x + 2*a) + 1)/(b*(e^(2*b*x + 2*a) - 1
)^5)
```

maple [A] time = 0.29, size = 33, normalized size = 0.79

$$\frac{\left(-\frac{8}{15}-\frac{\mathrm{csch}(bx+a)^{4}}{5}+\frac{4\mathrm{csch}(bx+a)^{2}}{15}\right)\coth\left(bx+a\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(b*x+a)^6,x)
```

```
[Out] 1/b*(-8/15-1/5*csch(b*x+a)^4+4/15*csch(b*x+a)^2)*coth(b*x+a)
```

maxima [B] time = 0.32, size = 205, normalized size = 4.88

$$-\frac{16\,e^{(-2\,bx-2\,a)}}{3\,b\left(5\,e^{(-2\,bx-2\,a)}-10\,e^{(-4\,bx-4\,a)}+10\,e^{(-6\,bx-6\,a)}-5\,e^{(-8\,bx-8\,a)}+e^{(-10\,bx-10\,a)}-1\right)}+\frac{}{3\,b\left(5\,e^{(-2\,bx-2\,a)}-10\,e^{(-4\,bx-4\,a)}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^6,x, algorithm="maxima")
```

```
[Out] -16/3*e^(-2*b*x - 2*a)/(b*(5*e^(-2*b*x - 2*a) - 10*e^(-4*b*x - 4*a) + 10*e^
(-6*b*x - 6*a) - 5*e^(-8*b*x - 8*a) + e^(-10*b*x - 10*a) - 1)) + 32/3*e^(-4
*b*x - 4*a)/(b*(5*e^(-2*b*x - 2*a) - 10*e^(-4*b*x - 4*a) + 10*e^(-6*b*x - 6
*a) - 5*e^(-8*b*x - 8*a) + e^(-10*b*x - 10*a) - 1)) + 16/15/(b*(5*e^(-2*b*x
 - 2*a) - 10*e^(-4*b*x - 4*a) + 10*e^(-6*b*x - 6*a) - 5*e^(-8*b*x - 8*a) +
e^(-10*b*x - 10*a) - 1))
```

mupad [B] time = 0.08, size = 42, normalized size = 1.00

$$-\frac{16\,\left(10\,{\mathrm{e}}^{4\,a+4\,b\,x}-5\,{\mathrm{e}}^{2\,a+2\,b\,x}+1\right)}{15\,b\,{\left({\mathrm{e}}^{2\,a+2\,b\,x}-1\right)}^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sinh(a + b*x)^6,x)
```

```
[Out] -(16*(10*exp(4*a + 4*b*x) - 5*exp(2*a + 2*b*x) + 1))/(15*b*(exp(2*a + 2*b*x
) - 1)^5)
```

<u>**sympy**</u> **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{csch}^6(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)**6,x)
```

```
[Out] Integral(csch(a + b*x)**6, x)
```

3.7 $\int \operatorname{csch}^{\frac{5}{2}}(a+bx)\,dx$

Optimal. Leaf size=80

$$-\frac{2\cosh(a+bx)\operatorname{csch}^{\frac{3}{2}}(a+bx)}{3b}+\frac{2i\sqrt{i\sinh(a+bx)}\,\sqrt{\operatorname{csch}(a+bx)}\,F\left(\frac{1}{2}\left(ia+ibx-\frac{\pi}{2}\right)\middle|2\right)}{3b}$$

```
[Out] -2/3*cosh(b*x+a)*csch(b*x+a)^(3/2)/b-2/3*I*(sin(1/2*I*a+1/4*Pi+1/2*I*b*x)^2
)^(1/2)/sin(1/2*I*a+1/4*Pi+1/2*I*b*x)*EllipticF(cos(1/2*I*a+1/4*Pi+1/2*I*b*
x),2^(1/2))*csch(b*x+a)^(1/2)*(I*sinh(b*x+a))^(1/2)/b
```

Rubi [A] time = 0.04, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {3768, 3771, 2641}

$$-\frac{2\cosh(a+bx)\operatorname{csch}^{\frac{3}{2}}(a+bx)}{3b}+\frac{2i\sqrt{i\sinh(a+bx)}\,\sqrt{\operatorname{csch}(a+bx)}\,F\left(\frac{1}{2}\left(ia+ibx-\frac{\pi}{2}\right)\middle|2\right)}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[a + b*x]^(5/2),x]
```

```
[Out] (-2*Cosh[a + b*x]*Csch[a + b*x]^(3/2))/(3*b) + (((2*I)/3)*Sqrt[Csch[a + b*x
]]*EllipticF[(I*a - Pi/2 + I*b*x)/2, 2]*Sqrt[I*Sinh[a + b*x]])/b
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\begin{aligned}\int \operatorname{csch}^{\frac{5}{2}}(a+bx)\,dx &= -\frac{2\cosh(a+bx)\operatorname{csch}^{\frac{3}{2}}(a+bx)}{3b}-\frac{1}{3}\int \sqrt{\operatorname{csch}(a+bx)}\,dx\\ &= -\frac{2\cosh(a+bx)\operatorname{csch}^{\frac{3}{2}}(a+bx)}{3b}-\frac{1}{3}\left(\sqrt{\operatorname{csch}(a+bx)}\,\sqrt{i\sinh(a+bx)}\right)\int \frac{1}{\sqrt{i\sinh(a+bx)}}\,d\\ &= -\frac{2\cosh(a+bx)\operatorname{csch}^{\frac{3}{2}}(a+bx)}{3b}+\frac{2i\sqrt{\operatorname{csch}(a+bx)}\,F\left(\frac{1}{2}\left(ia-\frac{\pi}{2}+ibx\right)\middle|2\right)\sqrt{i\sinh(a+bx)}}{3b}\end{aligned}$$

Mathematica [A] time = 0.13, size = 61, normalized size = 0.76

$$-\frac{2\sqrt{\operatorname{csch}(a+bx)}\left(\coth(a+bx)+i\sqrt{i\sinh(a+bx)}\,F\left(\frac{1}{4}(-2ia-2ibx+\pi)\middle|2\right)\right)}{3b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[a + b*x]^(5/2),x]
```

```
[Out] (-2*Sqrt[Csch[a + b*x]]*(Coth[a + b*x] + I*EllipticF[((-2*I)*a + Pi - (2*I)
*b*x)/4, 2]*Sqrt[I*Sinh[a + b*x]]))/(3*b)
```

fricas [F] time = 0.52, size = 0, normalized size = 0.00

$$\text{integral}\left(\operatorname{csch}(bx+a)^{\frac{5}{2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(csch(b*x + a)^(5/2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{csch}(bx+a)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(csch(b*x + a)^(5/2), x)
```

maple [A] time = 0.42, size = 101, normalized size = 1.26

$$-\frac{i\sqrt{1-i\sinh(bx+a)}\,\sqrt{2}\,\sqrt{i\sinh(bx+a)+1}\,\sqrt{i\sinh(bx+a)}\,\operatorname{EllipticF}\left(\sqrt{1-i\sinh(bx+a)},\frac{\sqrt{2}}{2}\right)\sinh(b}{3\sinh(bx+a)^{\frac{3}{2}}\cosh(bx+a)\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(b*x+a)^(5/2),x)
```

```
[Out] -1/3/sinh(b*x+a)^(3/2)*(I*(1-I*sinh(b*x+a))^(1/2)*2^(1/2)*(I*sinh(b*x+a)+1)
^(1/2)*(I*sinh(b*x+a))^(1/2)*EllipticF((1-I*sinh(b*x+a))^(1/2),1/2*2^(1/2))
*sinh(b*x+a)+2*cosh(b*x+a)^2)/cosh(b*x+a)/b
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{csch}(bx+a)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(csch(b*x + a)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(\frac{1}{\sinh\left(a+b\,x\right)}\right)}^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/sinh(a + b*x))^(5/2),x)
```

```
[Out] int((1/sinh(a + b*x))^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{csch}^{\frac{5}{2}}(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)**(5/2),x)
```

```
[Out] Integral(csch(a + b*x)**(5/2), x)
```

3.8 $\int \operatorname{csch}^{\frac{3}{2}}(a+bx)\,dx$

Optimal. Leaf size=76

$$-\frac{2\cosh(a+bx)\sqrt{\operatorname{csch}(a+bx)}}{b}-\frac{2iE\left(\frac{1}{2}\left(ia+ibx-\frac{\pi}{2}\right)\middle|2\right)}{b\sqrt{i\sinh(a+bx)}\sqrt{\operatorname{csch}(a+bx)}}$$

```
[Out] -2*cosh(b*x+a)*csch(b*x+a)^(1/2)/b+2*I*(sin(1/2*I*a+1/4*Pi+1/2*I*b*x)^2)^(1
/2)/sin(1/2*I*a+1/4*Pi+1/2*I*b*x)*EllipticE(cos(1/2*I*a+1/4*Pi+1/2*I*b*x),2
^(1/2))/b/csch(b*x+a)^(1/2)/(I*sinh(b*x+a))^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {3768, 3771, 2639}

$$-\frac{2\cosh(a+bx)\sqrt{\operatorname{csch}(a+bx)}}{b}-\frac{2iE\left(\frac{1}{2}\left(ia+ibx-\frac{\pi}{2}\right)\middle|2\right)}{b\sqrt{i\sinh(a+bx)}\sqrt{\operatorname{csch}(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[a + b*x]^(3/2),x]
```

```
[Out] (-2*Cosh[a + b*x]*Sqrt[Csch[a + b*x]])/b - ((2*I)*EllipticE[(I*a - Pi/2 + I
*b*x)/2, 2])/(b*Sqrt[Csch[a + b*x]]*Sqrt[I*Sinh[a + b*x]])
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\begin{aligned}
\int \operatorname{csch}^{\frac{3}{2}}(a+bx)\,dx &= -\frac{2\cosh(a+bx)\sqrt{\operatorname{csch}(a+bx)}}{b}+\int \frac{1}{\sqrt{\operatorname{csch}(a+bx)}}\,dx\\
&= -\frac{2\cosh(a+bx)\sqrt{\operatorname{csch}(a+bx)}}{b}+\frac{\int \sqrt{i\sinh(a+bx)}\,dx}{\sqrt{\operatorname{csch}(a+bx)}\sqrt{i\sinh(a+bx)}}\\
&= -\frac{2\cosh(a+bx)\sqrt{\operatorname{csch}(a+bx)}}{b}-\frac{2iE\left(\frac{1}{2}\left(ia-\frac{\pi}{2}+ibx\right)\middle|2\right)}{b\sqrt{\operatorname{csch}(a+bx)}\sqrt{i\sinh(a+bx)}}
\end{aligned}$$

Mathematica [A] time = 0.22, size = 57, normalized size = 0.75

$$-\frac{2\sqrt{\operatorname{csch}(a+bx)}\left(\cosh(a+bx)-\sqrt{i\sinh(a+bx)}\,E\left(\frac{1}{4}(-2ia-2ibx+\pi)\middle|2\right)\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[a + b*x]^(3/2),x]
```

```
[Out] (-2*Sqrt[Csch[a + b*x]]*(Cosh[a + b*x] - EllipticE[((-2*I)*a + Pi - (2*I)*b
*x)/4, 2]*Sqrt[I*Sinh[a + b*x]]))/b
```

fricas [F] time = 1.11, size = 0, normalized size = 0.00

$$\text{integral}\left(\operatorname{csch}(bx+a)^{\frac{3}{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(csch(b*x + a)^(3/2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{csch}(bx+a)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(csch(b*x + a)^(3/2), x)
```

maple [A] time = 0.38, size = 154, normalized size = 2.03

$$\frac{2\sqrt{1-i\sinh(bx+a)}\,\sqrt{2}\,\sqrt{i\sinh(bx+a)+1}\,\sqrt{i\sinh(bx+a)}\,\operatorname{EllipticE}\left(\sqrt{1-i\sinh(bx+a)},\frac{\sqrt{2}}{2}\right)-\sqrt{1-i\sin}}{\cosh(bx+a)\sqrt{s}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(b*x+a)^(3/2),x)
```

```
[Out] (2*(1-I*sinh(b*x+a))^(1/2)*2^(1/2)*(I*sinh(b*x+a)+1)^(1/2)*(I*sinh(b*x+a))^
(1/2)*EllipticE((1-I*sinh(b*x+a))^(1/2),1/2*2^(1/2))-(1-I*sinh(b*x+a))^(1/2
)*2^(1/2)*(I*sinh(b*x+a)+1)^(1/2)*(I*sinh(b*x+a))^(1/2)*EllipticF((1-I*sinh
(b*x+a))^(1/2),1/2*2^(1/2))-2*cosh(b*x+a)^2)/cosh(b*x+a)/sinh(b*x+a)^(1/2)/
b
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{csch}(bx+a)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(csch(b*x + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(\frac{1}{\sinh(a+b\,x)}\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/sinh(a + b*x))^(3/2),x)
[Out] int((1/sinh(a + b*x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{csch}^{\frac{3}{2}}(a + bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)**(3/2),x)
[Out] Integral(csch(a + b*x)**(3/2), x)
```

3.9 $\int \sqrt{\operatorname{csch}(a+bx)}\,dx$

Optimal. Leaf size=54

$$-\frac{2i\sqrt{i\sinh(a+bx)}\sqrt{\operatorname{csch}(a+bx)}\,F\left(\frac{1}{2}\left(ia+ibx-\frac{\pi}{2}\right)\middle|2\right)}{b}$$

```
[Out] 2*I*(sin(1/2*I*a+1/4*Pi+1/2*I*b*x)^2)^(1/2)/sin(1/2*I*a+1/4*Pi+1/2*I*b*x)*E
llipticF(cos(1/2*I*a+1/4*Pi+1/2*I*b*x),2^(1/2))*csch(b*x+a)^(1/2)*(I*sinh(b
*x+a))^(1/2)/b
```

Rubi [A] time = 0.02, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3771, 2641}

$$-\frac{2i\sqrt{i\sinh(a+bx)}\sqrt{\operatorname{csch}(a+bx)}\,F\left(\frac{1}{2}\left(ia+ibx-\frac{\pi}{2}\right)\middle|2\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Csch[a + b*x]],x]
```

```
[Out] ((-2*I)*Sqrt[Csch[a + b*x]]*EllipticF[(I*a - Pi/2 + I*b*x)/2, 2]*Sqrt[I*Sin
h[a + b*x]])/b
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\begin{aligned}\int \sqrt{\operatorname{csch}(a+bx)}\,dx &= \left(\sqrt{\operatorname{csch}(a+bx)}\sqrt{i\sinh(a+bx)}\right)\int \frac{1}{\sqrt{i\sinh(a+bx)}}\,dx\\ &= -\frac{2i\sqrt{\operatorname{csch}(a+bx)}\,F\left(\frac{1}{2}\left(ia-\frac{\pi}{2}+ibx\right)\middle|2\right)\sqrt{i\sinh(a+bx)}}{b}\end{aligned}$$

Mathematica [A] time = 0.19, size = 48, normalized size = 0.89

$$\frac{2(i\sinh(a+bx))^{3/2}\operatorname{csch}^{\frac{3}{2}}(a+bx)F\left(\frac{1}{4}(-2ia-2ibx+\pi)\middle|2\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Csch[a + b*x]],x]
```

```
[Out] (2*Csch[a + b*x]^(3/2)*EllipticF[((-2*I)*a + Pi - (2*I)*b*x)/4, 2]*(I*Sinh[
a + b*x])^(3/2))/b
```

fricas [F] time = 1.09, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\sqrt{\mathrm{csch}\left(bx+a\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(csch(b*x + a)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\mathrm{csch}\left(bx+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(csch(b*x + a)), x)
```

maple [A] time = 0.24, size = 87, normalized size = 1.61

$$\frac{i\sqrt{-i\left(\sinh\left(bx+a\right)+i\right)}\,\sqrt{2}\,\sqrt{-i\left(-\sinh\left(bx+a\right)+i\right)}\,\sqrt{i\sinh\left(bx+a\right)}\ \mathrm{EllipticF}\left(\sqrt{-i\left(\sinh\left(bx+a\right)+i\right)},\frac{\sqrt{2}}{2}\right)}{\cosh\left(bx+a\right)\sqrt{\sinh\left(bx+a\right)}\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(b*x+a)^(1/2),x)
[Out] I*(-I*(sinh(b*x+a)+I))^(1/2)*2^(1/2)*(-I*(-sinh(b*x+a)+I))^(1/2)*(I*sinh(b*
x+a))^(1/2)*EllipticF((-I*(sinh(b*x+a)+I))^(1/2),1/2*2^(1/2))/cosh(b*x+a)/s
inh(b*x+a)^(1/2)/b
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\mathrm{csch}\left(bx+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(csch(b*x + a)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \sqrt{\frac{1}{\sinh\left(a+b\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/sinh(a + b*x))^(1/2),x)
[Out] int((1/sinh(a + b*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\mathrm{csch}\left(a+bx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)**(1/2),x)
[Out] Integral(sqrt(csch(a + b*x)), x)
```

3.10 $\int \frac{1}{\sqrt{\operatorname{csch}(a+bx)}}\,dx$

Optimal. Leaf size=54

$$-\frac{2iE\left(\frac{1}{2}\left(ia+ibx-\frac{\pi}{2}\right)\middle|2\right)}{b\sqrt{i\sinh(a+bx)}\sqrt{\operatorname{csch}(a+bx)}}$$

```
[Out] 2*I*(sin(1/2*I*a+1/4*Pi+1/2*I*b*x)^2)^(1/2)/sin(1/2*I*a+1/4*Pi+1/2*I*b*x)*E
llipticE(cos(1/2*I*a+1/4*Pi+1/2*I*b*x),2^(1/2))/b/csch(b*x+a)^(1/2)/(I*sinh
(b*x+a))^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3771, 2639}

$$-\frac{2iE\left(\frac{1}{2}\left(ia+ibx-\frac{\pi}{2}\right)\middle|2\right)}{b\sqrt{i\sinh(a+bx)}\sqrt{\operatorname{csch}(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[Csch[a + b*x]],x]
```

```
[Out] ((-2*I)*EllipticE[(I*a - Pi/2 + I*b*x)/2, 2])/(b*Sqrt[Csch[a + b*x]]*Sqrt[I
*Sinh[a + b*x]])
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sqrt{\operatorname{csch}(a+bx)}}\,dx &= \frac{\int \sqrt{i\sinh(a+bx)}\,dx}{\sqrt{\operatorname{csch}(a+bx)}\sqrt{i\sinh(a+bx)}}\\
&= -\frac{2iE\left(\frac{1}{2}\left(ia-\frac{\pi}{2}+ibx\right)\middle|2\right)}{b\sqrt{\operatorname{csch}(a+bx)}\sqrt{i\sinh(a+bx)}}
\end{aligned}$$

Mathematica [A] time = 0.04, size = 50, normalized size = 0.93

$$\frac{2\sqrt{i\sinh(a+bx)}\sqrt{\operatorname{csch}(a+bx)}\,E\left(\frac{1}{2}\left(\frac{\pi}{2}-i(a+bx)\right)\middle|2\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[Csch[a + b*x]],x]
```

```
[Out] (2*Sqrt[Csch[a + b*x]]*EllipticE[(Pi/2 - I*(a + b*x))/2, 2]*Sqrt[I*Sinh[a +
 b*x]])/b
```

fricas [F] time = 1.60, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{\sqrt{\text{csch}\,(bx+a)}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/csch(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(1/sqrt(csch(b*x + a)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{\text{csch}\,(bx+a)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/csch(b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(csch(b*x + a)), x)
```

maple [A] time = 0.36, size = 108, normalized size = 2.00

$$\frac{\sqrt{-i\left(\sinh\left(bx+a\right)+i\right)}\,\sqrt{2}\,\sqrt{-i\left(-\sinh\left(bx+a\right)+i\right)}\,\sqrt{i\sinh\left(bx+a\right)}\,\left(2\,\text{EllipticE}\left(\sqrt{1-i\sinh\left(bx+a\right)},\frac{\sqrt{2}}{2}\right)\right.}{\cosh\left(bx+a\right)\sqrt{\sinh\left(bx+a\right)}\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/csch(b*x+a)^(1/2),x)
```

```
[Out] (-I*(sinh(b*x+a)+I))^(1/2)*2^(1/2)*(-I*(-sinh(b*x+a)+I))^(1/2)*(I*sinh(b*x+
a))^(1/2)*(2*EllipticE((1-I*sinh(b*x+a))^(1/2),1/2*2^(1/2))-EllipticF((1-I*
sinh(b*x+a))^(1/2),1/2*2^(1/2)))/cosh(b*x+a)/sinh(b*x+a)^(1/2)/b
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{\text{csch}\,(bx+a)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/csch(b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(csch(b*x + a)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sqrt{\frac{1}{\sinh(a+b\,x)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1/sinh(a + b*x))^(1/2),x)
```

```
[Out] int(1/(1/sinh(a + b*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{\text{csch}\,(a+bx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/csch(b*x+a)**(1/2),x)
```

```
[Out] Integral(1/sqrt(csch(a + b*x)), x)
```

3.11 $\int \frac{1}{\operatorname{csch}^{\frac{3}{2}}(a+bx)}\,dx$

Optimal. Leaf size=80

$$\frac{2\cosh(a+bx)}{3b\sqrt{\operatorname{csch}(a+bx)}}+\frac{2i\sqrt{i\sinh(a+bx)}\,\sqrt{\operatorname{csch}(a+bx)}\,F\left(\frac{1}{2}\left(ia+ibx-\frac{\pi}{2}\right)\middle|2\right)}{3b}$$

```
[Out] 2/3*cosh(b*x+a)/b/csch(b*x+a)^(1/2)-2/3*I*(sin(1/2*I*a+1/4*Pi+1/2*I*b*x)^2)
^(1/2)/sin(1/2*I*a+1/4*Pi+1/2*I*b*x)*EllipticF(cos(1/2*I*a+1/4*Pi+1/2*I*b*x
),2^(1/2))*csch(b*x+a)^(1/2)*(I*sinh(b*x+a))^(1/2)/b
```

Rubi [A] time = 0.03, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {3769, 3771, 2641}

$$\frac{2\cosh(a+bx)}{3b\sqrt{\operatorname{csch}(a+bx)}}+\frac{2i\sqrt{i\sinh(a+bx)}\,\sqrt{\operatorname{csch}(a+bx)}\,F\left(\frac{1}{2}\left(ia+ibx-\frac{\pi}{2}\right)\middle|2\right)}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[a + b*x]^(-3/2),x]
```

```
[Out] (2*Cosh[a + b*x])/(3*b*Sqrt[Csch[a + b*x]]) + (((2*I)/3)*Sqrt[Csch[a + b*x]
]*EllipticF[(I*a - Pi/2 + I*b*x)/2, 2]*Sqrt[I*Sinh[a + b*x]])/b
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\operatorname{csch}^{\frac{3}{2}}(a+bx)}\,dx &= \frac{2\cosh(a+bx)}{3b\sqrt{\operatorname{csch}(a+bx)}}-\frac{1}{3}\int \sqrt{\operatorname{csch}(a+bx)}\,dx\\ &= \frac{2\cosh(a+bx)}{3b\sqrt{\operatorname{csch}(a+bx)}}-\frac{1}{3}\left(\sqrt{\operatorname{csch}(a+bx)}\,\sqrt{i\sinh(a+bx)}\right)\int \frac{1}{\sqrt{i\sinh(a+bx)}}\,dx\\ &= \frac{2\cosh(a+bx)}{3b\sqrt{\operatorname{csch}(a+bx)}}+\frac{2i\sqrt{\operatorname{csch}(a+bx)}\,F\left(\frac{1}{2}\left(ia-\frac{\pi}{2}+ibx\right)\middle|2\right)\sqrt{i\sinh(a+bx)}}{3b}\end{aligned}$$

Mathematica [A] time = 0.07, size = 63, normalized size = 0.79

$$\frac{\sqrt{\operatorname{csch}(a+bx)}\left(\sinh(2(a+bx))-2i\sqrt{i\sinh(a+bx)}\,F\left(\frac{1}{4}(-2ia-2ibx+\pi)\middle|2\right)\right)}{3b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[a + b*x]^(-3/2),x]
[Out] (Sqrt[Csch[a + b*x]]*((-2*I)*EllipticF[((-2*I)*a + Pi - (2*I)*b*x)/4, 2]*Sq
rt[I*Sinh[a + b*x]] + Sinh[2*(a + b*x)]))/(3*b)
```

fricas [F] time = 0.94, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{1}{\operatorname{csch}(bx+a)^{\frac{3}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/csch(b*x+a)^(3/2),x, algorithm="fricas")
[Out] integral(csch(b*x + a)^(-3/2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\operatorname{csch}(bx+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/csch(b*x+a)^(3/2),x, algorithm="giac")
[Out] integrate(csch(b*x + a)^(-3/2), x)
```

maple [A] time = 0.41, size = 100, normalized size = 1.25

$$\frac{-\frac{i\sqrt{1-i\sinh(bx+a)}\,\sqrt{2}\,\sqrt{i\sinh(bx+a)+1}\,\sqrt{i\sinh(bx+a)}\,\operatorname{EllipticF}\left(\sqrt{1-i\sinh(bx+a)},\frac{\sqrt{2}}{2}\right)}{3}+\frac{2\left(\cosh^2(bx+a)\right)\sinh(bx+a)}{3}}{\cosh(bx+a)\sqrt{\sinh(bx+a)}\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/csch(b*x+a)^(3/2),x)
[Out] (-1/3*I*(1-I*sinh(b*x+a))^(1/2)*2^(1/2)*(I*sinh(b*x+a)+1)^(1/2)*(I*sinh(b*x
+a))^(1/2)*EllipticF((1-I*sinh(b*x+a))^(1/2),1/2*2^(1/2))+2/3*cosh(b*x+a)^2
*sinh(b*x+a))/cosh(b*x+a)/sinh(b*x+a)^(1/2)/b
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\operatorname{csch}(bx+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/csch(b*x+a)^(3/2),x, algorithm="maxima")
[Out] integrate(csch(b*x + a)^(-3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(\frac{1}{\sinh(a+b\,x)}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1/sinh(a + b*x))^(3/2),x)
```

```
[Out] int(1/(1/sinh(a + b*x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\operatorname{csch}^{\frac{3}{2}}(a+bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/csch(b*x+a)**(3/2),x)
```

```
[Out] Integral(csch(a + b*x)**(-3/2), x)
```

3.12 $\int \frac{1}{\text{csch}^{\frac{5}{2}}(a+bx)}\,dx$

Optimal. Leaf size=80

$$\frac{2\cosh(a+bx)}{5b\text{csch}^{\frac{3}{2}}(a+bx)}+\frac{6iE\left(\frac{1}{2}\left(ia+ibx-\frac{\pi}{2}\right)\middle|2\right)}{5b\sqrt{i\sinh(a+bx)}\sqrt{\text{csch}(a+bx)}}$$

```
[Out] 2/5*cosh(b*x+a)/b/csch(b*x+a)^(3/2)-6/5*I*(sin(1/2*I*a+1/4*Pi+1/2*I*b*x)^2)
^(1/2)/sin(1/2*I*a+1/4*Pi+1/2*I*b*x)*EllipticE(cos(1/2*I*a+1/4*Pi+1/2*I*b*x
),2^(1/2))/b/csch(b*x+a)^(1/2)/(I*sinh(b*x+a))^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {3769, 3771, 2639}

$$\frac{2\cosh(a+bx)}{5b\text{csch}^{\frac{3}{2}}(a+bx)}+\frac{6iE\left(\frac{1}{2}\left(ia+ibx-\frac{\pi}{2}\right)\middle|2\right)}{5b\sqrt{i\sinh(a+bx)}\sqrt{\text{csch}(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[a + b*x]^(-5/2),x]
```

```
[Out] (2*Cosh[a + b*x])/(5*b*Csch[a + b*x]^(3/2)) + (((6*I)/5)*EllipticE[(I*a - P
i/2 + I*b*x)/2, 2])/(b*Sqrt[Csch[a + b*x]]*Sqrt[I*Sinh[a + b*x]])
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\text{csch}^{\frac{5}{2}}(a+bx)}\,dx &= \frac{2\cosh(a+bx)}{5b\text{csch}^{\frac{3}{2}}(a+bx)}-\frac{3}{5}\int \frac{1}{\sqrt{\text{csch}(a+bx)}}\,dx\\
&= \frac{2\cosh(a+bx)}{5b\text{csch}^{\frac{3}{2}}(a+bx)}-\frac{3\int \sqrt{i\sinh(a+bx)}\,dx}{5\sqrt{\text{csch}(a+bx)}\sqrt{i\sinh(a+bx)}}\\
&= \frac{2\cosh(a+bx)}{5b\text{csch}^{\frac{3}{2}}(a+bx)}+\frac{6iE\left(\frac{1}{2}\left(ia-\frac{\pi}{2}+ibx\right)\middle|2\right)}{5b\sqrt{\text{csch}(a+bx)}\sqrt{i\sinh(a+bx)}}
\end{aligned}$$

Mathematica [A] time = 0.13, size = 67, normalized size = 0.84

$$\frac{2\left(\cosh(a+bx)-3\sqrt{i\sinh(a+bx)}\operatorname{csch}^2(a+bx)E\left(\frac{1}{4}(-2ia-2ibx+\pi)\middle|2\right)\right)}{5b\operatorname{csch}^{\frac{3}{2}}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[a + b*x]^(-5/2),x]
```

```
[Out] (2*(Cosh[a + b*x] - 3*Csch[a + b*x]^2*EllipticE[((-2*I)*a + Pi - (2*I)*b*x)
/4, 2]*Sqrt[I*Sinh[a + b*x]]))/(5*b*Csch[a + b*x]^(3/2))
```

fricas [F] time = 0.47, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{\operatorname{csch}(bx+a)^{\frac{5}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/csch(b*x+a)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(csch(b*x + a)^(-5/2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\operatorname{csch}(bx+a)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/csch(b*x+a)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(csch(b*x + a)^(-5/2), x)
```

maple [A] time = 0.39, size = 164, normalized size = 2.05

$$\frac{-\frac{6\sqrt{1-i\sinh(bx+a)}\,\sqrt{2}\,\sqrt{i\sinh(bx+a)+1}\,\sqrt{i\sinh(bx+a)}\,\operatorname{EllipticE}\left(\sqrt{1-i\sinh(bx+a)},\frac{\sqrt{2}}{2}\right)}{5}+\frac{3\sqrt{1-i\sinh(bx+a)}\,\sqrt{2}\,\sqrt{i\sinh(bx+a)+1}\,\sqrt{i\sinh(bx+}}{5}}{\cosh(bx+a)\sqrt{\sinh(bx+a)}\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/csch(b*x+a)^(5/2),x)
```

```
[Out] (-6/5*(1-I*sinh(b*x+a))^(1/2)*2^(1/2)*(I*sinh(b*x+a)+1)^(1/2)*(I*sinh(b*x+a
))^(1/2)*EllipticE((1-I*sinh(b*x+a))^(1/2),1/2*2^(1/2))+3/5*(1-I*sinh(b*x+a
))^(1/2)*2^(1/2)*(I*sinh(b*x+a)+1)^(1/2)*(I*sinh(b*x+a))^(1/2)*EllipticF((1
-I*sinh(b*x+a))^(1/2),1/2*2^(1/2))+2/5*cosh(b*x+a)^4-2/5*cosh(b*x+a)^2)/cos
h(b*x+a)/sinh(b*x+a)^(1/2)/b
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\operatorname{csch}(bx+a)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/csch(b*x+a)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(csch(b*x + a)^(-5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(\frac{1}{\sinh(a+b\,x)}\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1/sinh(a + b*x))^(5/2),x)
[Out] int(1/(1/sinh(a + b*x))^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\operatorname{csch}^{\frac{5}{2}}(a+bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/csch(b*x+a)**(5/2),x)
[Out] Integral(csch(a + b*x)**(-5/2), x)
```

3.13 $\int (b\text{csch}(c+dx))^{7/2}\,dx$

Optimal. Leaf size=116

$$\frac{6ib^4E\left(\frac{1}{2}\left(ic+idx-\frac{\pi}{2}\right)\middle|2\right)}{5d\sqrt{i\sinh(c+dx)}\sqrt{b\text{csch}(c+dx)}}+\frac{6b^3\cosh(c+dx)\sqrt{b\text{csch}(c+dx)}}{5d}-\frac{2b\cosh(c+dx)(b\text{csch}(c+dx))^{5/2}}{5d}$$

```
[Out] -2/5*b*cosh(d*x+c)*(b*csch(d*x+c))^(5/2)/d+6/5*b^3*cosh(d*x+c)*(b*csch(d*x+
c))^(1/2)/d-6/5*I*b^4*(sin(1/2*I*c+1/4*Pi+1/2*I*d*x)^2)^(1/2)/sin(1/2*I*c+1
/4*Pi+1/2*I*d*x)*EllipticE(cos(1/2*I*c+1/4*Pi+1/2*I*d*x),2^(1/2))/d/(b*csch
(d*x+c))^(1/2)/(I*sinh(d*x+c))^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3768, 3771, 2639}

$$\frac{6b^3\cosh(c+dx)\sqrt{b\text{csch}(c+dx)}}{5d}+\frac{6ib^4E\left(\frac{1}{2}\left(ic+idx-\frac{\pi}{2}\right)\middle|2\right)}{5d\sqrt{i\sinh(c+dx)}\sqrt{b\text{csch}(c+dx)}}-\frac{2b\cosh(c+dx)(b\text{csch}(c+dx))^{5/2}}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Csch[c + d*x])^(7/2),x]
```

```
[Out] (6*b^3*Cosh[c + d*x]*Sqrt[b*Csch[c + d*x]])/(5*d) - (2*b*Cosh[c + d*x]*(b*C
sch[c + d*x])^(5/2))/(5*d) + (((6*I)/5)*b^4*EllipticE[(I*c - Pi/2 + I*d*x)/
2, 2])/(d*Sqrt[b*Csch[c + d*x]]*Sqrt[I*Sinh[c + d*x]])
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\begin{aligned}\int (b\text{csch}(c+dx))^{7/2}\,dx &= -\frac{2b\cosh(c+dx)(b\text{csch}(c+dx))^{5/2}}{5d}-\frac{1}{5}\left(3b^2\right)\int (b\text{csch}(c+dx))^{3/2}\,dx\\ &= \frac{6b^3\cosh(c+dx)\sqrt{b\text{csch}(c+dx)}}{5d}-\frac{2b\cosh(c+dx)(b\text{csch}(c+dx))^{5/2}}{5d}-\frac{1}{5}\left(3b^4\right)\int \frac{}{\sqrt{b}}\\ &= \frac{6b^3\cosh(c+dx)\sqrt{b\text{csch}(c+dx)}}{5d}-\frac{2b\cosh(c+dx)(b\text{csch}(c+dx))^{5/2}}{5d}-\frac{\left(3b^4\right)\int\sqrt{}}{5\sqrt{b\text{csch}(c+}}\\ &= \frac{6b^3\cosh(c+dx)\sqrt{b\text{csch}(c+dx)}}{5d}-\frac{2b\cosh(c+dx)(b\text{csch}(c+dx))^{5/2}}{5d}+\frac{6ib^4E\left(\frac{1}{2}\right.}{5d\sqrt{b\text{csch}(c-}}\end{aligned}$$

Mathematica [A] time = 0.16, size = 79, normalized size = 0.68

$$-\frac{2b^3\sqrt{b\text{csch}(c+dx)}\left(-3\cosh(c+dx)+\coth(c+dx)\text{csch}(c+dx)+3\sqrt{i\sinh(c+dx)}\,E\left(\frac{1}{4}(-2ic-2idx+\pi)\right|}{5d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Csch[c + d*x])^(7/2),x]
```

```
[Out] (-2*b^3*Sqrt[b*Csch[c + d*x]]*(-3*Cosh[c + d*x] + Coth[c + d*x]*Csch[c + d*
x] + 3*EllipticE[((-2*I)*c + Pi - (2*I)*d*x)/4, 2]*Sqrt[I*Sinh[c + d*x]]))/
(5*d)
```

fricas [F] time = 0.57, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{b\,\text{csch}\,(dx+c)}\,b^3\,\text{csch}\,(dx+c)^3\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*csch(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*csch(d*x + c))*b^3*csch(d*x + c)^3, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\,\text{csch}\,(dx+c))^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*csch(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] integrate((b*csch(d*x + c))^(7/2), x)
```

maple [F] time = 0.30, size = 0, normalized size = 0.00

$$\int (b\,\text{csch}\,(dx+c))^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*csch(d*x+c))^(7/2),x)
```

```
[Out] int((b*csch(d*x+c))^(7/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\,\text{csch}\,(dx+c))^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*csch(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((b*csch(d*x + c))^(7/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(\frac{b}{\sinh\left(c+d\,x\right)}\right)}^{7/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b/sinh(c + d*x))^(7/2),x)
```

```
[Out] int((b/sinh(c + d*x))^(7/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b \operatorname{csch}(c + dx))^{\frac{7}{2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*csch(d*x+c))**(7/2),x)
```

```
[Out] Integral((b*csch(c + d*x))**(7/2), x)
```

3.14 $\int (b\operatorname{csch}(c+dx))^{5/2}\,dx$

Optimal. Leaf size=88

$$-\frac{2b\cosh(c+dx)(b\operatorname{csch}(c+dx))^{3/2}}{3d}+\frac{2ib^2\sqrt{i\sinh(c+dx)}\,F\left(\frac{1}{2}\left(ic+idx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{b\operatorname{csch}(c+dx)}}{3d}$$

```
[Out] -2/3*b*cosh(d*x+c)*(b*csch(d*x+c))^(3/2)/d-2/3*I*b^2*(sin(1/2*I*c+1/4*Pi+1/
2*I*d*x)^2)^(1/2)/sin(1/2*I*c+1/4*Pi+1/2*I*d*x)*EllipticF(cos(1/2*I*c+1/4*P
i+1/2*I*d*x),2^(1/2))*(b*csch(d*x+c))^(1/2)*(I*sinh(d*x+c))^(1/2)/d
```

Rubi [A] time = 0.04, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3768, 3771, 2641}

$$-\frac{2b\cosh(c+dx)(b\operatorname{csch}(c+dx))^{3/2}}{3d}+\frac{2ib^2\sqrt{i\sinh(c+dx)}\,F\left(\frac{1}{2}\left(ic+idx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{b\operatorname{csch}(c+dx)}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Csch[c + d*x])^(5/2),x]
```

```
[Out] (-2*b*Cosh[c + d*x]*(b*Csch[c + d*x])^(3/2))/(3*d) + (((2*I)/3)*b^2*Sqrt[b*
Csch[c + d*x]]*EllipticF[(I*c - Pi/2 + I*d*x)/2, 2]*Sqrt[I*Sinh[c + d*x]])/
d
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\begin{aligned}\int (b\operatorname{csch}(c+dx))^{5/2}\,dx &= -\frac{2b\cosh(c+dx)(b\operatorname{csch}(c+dx))^{3/2}}{3d}-\frac{1}{3}b^2\int\sqrt{b\operatorname{csch}(c+dx)}\,dx\\ &= -\frac{2b\cosh(c+dx)(b\operatorname{csch}(c+dx))^{3/2}}{3d}-\frac{1}{3}\left(b^2\sqrt{b\operatorname{csch}(c+dx)}\sqrt{i\sinh(c+dx)}\right)\int\frac{}{\sqrt{i}}\\ &= -\frac{2b\cosh(c+dx)(b\operatorname{csch}(c+dx))^{3/2}}{3d}+\frac{2ib^2\sqrt{b\operatorname{csch}(c+dx)}\,F\left(\frac{1}{2}\left(ic-\frac{\pi}{2}+idx\right)\middle|2\right)\sqrt{}}{3d}\end{aligned}$$

Mathematica [A] time = 0.11, size = 66, normalized size = 0.75

$$-\frac{2b^2\sqrt{b\operatorname{csch}(c+dx)}\left(\coth(c+dx)+i\sqrt{i\sinh(c+dx)}\,F\left(\frac{1}{4}(-2ic-2idx+\pi)\middle|2\right)\right)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Csch[c + d*x])^(5/2),x]
```

```
[Out] (-2*b^2*Sqrt[b*Csch[c + d*x]]*(Coth[c + d*x] + I*EllipticF[((-2*I)*c + Pi -
 (2*I)*d*x)/4, 2]*Sqrt[I*Sinh[c + d*x]]))/(3*d)
```

fricas [F] time = 0.93, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{b\,\text{csch}\,(dx+c)}\,b^2\,\text{csch}\,(dx+c)^2\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*csch(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*csch(d*x + c))*b^2*csch(d*x + c)^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\,\text{csch}\,(dx+c))^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*csch(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*csch(d*x + c))^(5/2), x)
```

maple [F] time = 0.29, size = 0, normalized size = 0.00

$$\int (b\,\text{csch}\,(dx+c))^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*csch(d*x+c))^(5/2),x)
```

```
[Out] int((b*csch(d*x+c))^(5/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\,\text{csch}\,(dx+c))^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*csch(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*csch(d*x + c))^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(\frac{b}{\sinh\left(c+d\,x\right)}\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b/sinh(c + d*x))^(5/2),x)
```

```
[Out] int((b/sinh(c + d*x))^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\,\text{csch}\,(c+dx))^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*csch(d*x+c))**(5/2),x)
```

```
[Out] Integral((b*csch(c + d*x))**(5/2), x)
```

3.15 $\int (b\,\text{csch}(c+dx))^{3/2}\,dx$

Optimal. Leaf size=84

$$-\frac{2b\cosh(c+dx)\sqrt{b\,\text{csch}(c+dx)}}{d}-\frac{2ib^2E\left(\frac{1}{2}\left(ic+idx-\frac{\pi}{2}\right)\middle|2\right)}{d\sqrt{i\sinh(c+dx)}\sqrt{b\,\text{csch}(c+dx)}}$$

```
[Out] -2*b*cosh(d*x+c)*(b*csch(d*x+c))^(1/2)/d+2*I*b^2*(sin(1/2*I*c+1/4*Pi+1/2*I*
d*x)^2)^(1/2)/sin(1/2*I*c+1/4*Pi+1/2*I*d*x)*EllipticE(cos(1/2*I*c+1/4*Pi+1/
2*I*d*x),2^(1/2))/d/(b*csch(d*x+c))^(1/2)/(I*sinh(d*x+c))^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3768, 3771, 2639}

$$-\frac{2b\cosh(c+dx)\sqrt{b\,\text{csch}(c+dx)}}{d}-\frac{2ib^2E\left(\frac{1}{2}\left(ic+idx-\frac{\pi}{2}\right)\middle|2\right)}{d\sqrt{i\sinh(c+dx)}\sqrt{b\,\text{csch}(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Csch[c + d*x])^(3/2),x]
```

```
[Out] (-2*b*Cosh[c + d*x]*Sqrt[b*Csch[c + d*x]])/d - ((2*I)*b^2*EllipticE[(I*c -
Pi/2 + I*d*x)/2, 2])/(d*Sqrt[b*Csch[c + d*x]]*Sqrt[I*Sinh[c + d*x]])
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\begin{aligned}
\int (b\,\text{csch}(c+dx))^{3/2}\,dx &= -\frac{2b\cosh(c+dx)\sqrt{b\,\text{csch}(c+dx)}}{d}+b^2\int \frac{1}{\sqrt{b\,\text{csch}(c+dx)}}\,dx\\
&= -\frac{2b\cosh(c+dx)\sqrt{b\,\text{csch}(c+dx)}}{d}+\frac{b^2\int \sqrt{i\sinh(c+dx)}\,dx}{\sqrt{b\,\text{csch}(c+dx)}\sqrt{i\sinh(c+dx)}}\\
&= -\frac{2b\cosh(c+dx)\sqrt{b\,\text{csch}(c+dx)}}{d}-\frac{2ib^2E\left(\frac{1}{2}\left(ic-\frac{\pi}{2}+idx\right)\middle|2\right)}{d\sqrt{b\,\text{csch}(c+dx)}\sqrt{i\sinh(c+dx)}}
\end{aligned}$$

Mathematica [A] time = 0.06, size = 60, normalized size = 0.71

$$-\frac{2b\sqrt{b\,\text{csch}(c+dx)}\left(\cosh(c+dx)-\sqrt{i\sinh(c+dx)}\,E\left(\frac{1}{4}(-2ic-2idx+\pi)\middle|2\right)\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Csch[c + d*x])^(3/2),x]
```

```
[Out] (-2*b*Sqrt[b*Csch[c + d*x]]*(Cosh[c + d*x] - EllipticE[((-2*I)*c + Pi - (2*
I)*d*x)/4, 2]*Sqrt[I*Sinh[c + d*x]]))/d
```

fricas [F] time = 2.15, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{b\,\text{csch}\,(dx+c)}\,b\,\text{csch}\,(dx+c)\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*csch(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*csch(d*x + c))*b*csch(d*x + c), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\,\text{csch}\,(dx+c))^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*csch(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*csch(d*x + c))^(3/2), x)
```

maple [F] time = 0.24, size = 0, normalized size = 0.00

$$\int (b\,\text{csch}\,(dx+c))^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*csch(d*x+c))^(3/2),x)
```

```
[Out] int((b*csch(d*x+c))^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\,\text{csch}\,(dx+c))^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*csch(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*csch(d*x + c))^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(\frac{b}{\sinh\left(c+d\,x\right)}\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b/sinh(c + d*x))^(3/2),x)
```

```
[Out] int((b/sinh(c + d*x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\,\text{csch}\,(c+dx))^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*csch(d*x+c))**(3/2),x)
```

```
[Out] Integral((b*csch(c + d*x))**(3/2), x)
```

3.16 $\int \sqrt{b\text{csch}(c+dx)}\,dx$

Optimal. Leaf size=56

$$-\frac{2i\sqrt{i\sinh(c+dx)}\,F\left(\frac{1}{2}\left(ic+idx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{b\text{csch}(c+dx)}}{d}$$

```
[Out] 2*I*(sin(1/2*I*c+1/4*Pi+1/2*I*d*x)^2)^(1/2)/sin(1/2*I*c+1/4*Pi+1/2*I*d*x)*E
llipticF(cos(1/2*I*c+1/4*Pi+1/2*I*d*x),2^(1/2))*(b*csch(d*x+c))^(1/2)*(I*si
nh(d*x+c))^(1/2)/d
```

Rubi [A] time = 0.02, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3771, 2641}

$$-\frac{2i\sqrt{i\sinh(c+dx)}\,F\left(\frac{1}{2}\left(ic+idx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{b\text{csch}(c+dx)}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*Csch[c + d*x]],x]
```

```
[Out] ((-2*I)*Sqrt[b*Csch[c + d*x]]*EllipticF[(I*c - Pi/2 + I*d*x)/2, 2]*Sqrt[I*S
inh[c + d*x]])/d
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\begin{aligned}\int \sqrt{b\text{csch}(c+dx)}\,dx &= \left(\sqrt{b\text{csch}(c+dx)}\sqrt{i\sinh(c+dx)}\right)\int \frac{1}{\sqrt{i\sinh(c+dx)}}\,dx\\ &= -\frac{2i\sqrt{b\text{csch}(c+dx)}\,F\left(\frac{1}{2}\left(ic-\frac{\pi}{2}+idx\right)\middle|2\right)\sqrt{i\sinh(c+dx)}}{d}\end{aligned}$$

Mathematica [A] time = 0.04, size = 54, normalized size = 0.96

$$\frac{2i\sqrt{i\sinh(c+dx)}\,F\left(\frac{1}{2}\left(\frac{\pi}{2}-i(c+dx)\right)\middle|2\right)\sqrt{b\text{csch}(c+dx)}}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*Csch[c + d*x]],x]
```

```
[Out] ((2*I)*Sqrt[b*Csch[c + d*x]]*EllipticF[(Pi/2 - I*(c + d*x))/2, 2]*Sqrt[I*Si
nh[c + d*x]])/d
```

fricas [F] time = 0.50, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{b\,\text{csch}\,(dx+c)}\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*csch(d*x+c))^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(b*csch(d*x + c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b\,\text{csch}\,(dx+c)}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*csch(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(b*csch(d*x + c)), x)
```

maple [F] time = 0.35, size = 0, normalized size = 0.00

$$\int \sqrt{b\,\text{csch}\,(dx+c)}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*csch(d*x+c))^(1/2),x)
[Out] int((b*csch(d*x+c))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b\,\text{csch}\,(dx+c)}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*csch(d*x+c))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(b*csch(d*x + c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \sqrt{\frac{b}{\sinh\left(c+d\,x\right)}}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b/sinh(c + d*x))^(1/2),x)
[Out] int((b/sinh(c + d*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b\,\text{csch}\,(c+dx)}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*csch(d*x+c))**(1/2),x)
[Out] Integral(sqrt(b*csch(c + d*x)), x)
```

3.17 $\int \frac{1}{\sqrt{b\mathrm{csch}(c+dx)}}\,dx$

Optimal. Leaf size=56

$$-\frac{2iE\left(\frac{1}{2}\left(ic+idx-\frac{\pi}{2}\right)\middle|2\right)}{d\sqrt{i\sinh(c+dx)}\sqrt{b\mathrm{csch}(c+dx)}}$$

```
[Out] 2*I*(sin(1/2*I*c+1/4*Pi+1/2*I*d*x)^2)^(1/2)/sin(1/2*I*c+1/4*Pi+1/2*I*d*x)*E
llipticE(cos(1/2*I*c+1/4*Pi+1/2*I*d*x),2^(1/2))/d/(b*csch(d*x+c))^(1/2)/(I*
sinh(d*x+c))^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3771, 2639}

$$-\frac{2iE\left(\frac{1}{2}\left(ic+idx-\frac{\pi}{2}\right)\middle|2\right)}{d\sqrt{i\sinh(c+dx)}\sqrt{b\mathrm{csch}(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[b*Csch[c + d*x]],x]
```

```
[Out] ((-2*I)*EllipticE[(I*c - Pi/2 + I*d*x)/2, 2])/(d*Sqrt[b*Csch[c + d*x]]*Sqrt
[I*Sinh[c + d*x]])
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{b\mathrm{csch}(c+dx)}}\,dx &= \frac{\int \sqrt{i\sinh(c+dx)}\,dx}{\sqrt{b\mathrm{csch}(c+dx)}\sqrt{i\sinh(c+dx)}}\\ &= -\frac{2iE\left(\frac{1}{2}\left(ic-\frac{\pi}{2}+idx\right)\middle|2\right)}{d\sqrt{b\mathrm{csch}(c+dx)}\sqrt{i\sinh(c+dx)}}\end{aligned}$$

Mathematica [A] time = 0.05, size = 52, normalized size = 0.93

$$\frac{2iE\left(\frac{1}{4}(-2ic-2idx+\pi)\middle|2\right)}{d\sqrt{i\sinh(c+dx)}\sqrt{b\mathrm{csch}(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[b*Csch[c + d*x]],x]
```

```
[Out] ((2*I)*EllipticE[((-2*I)*c + Pi - (2*I)*d*x)/4, 2])/(d*Sqrt[b*Csch[c + d*x]
]*Sqrt[I*Sinh[c + d*x]])
```

fricas [F] time = 1.40, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{b\operatorname{csch}(dx+c)}}{b\operatorname{csch}(dx+c)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*csch(d*x+c))^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(b*csch(d*x + c))/(b*csch(d*x + c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{b\operatorname{csch}(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*csch(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate(1/sqrt(b*csch(d*x + c)), x)
```

maple [B] time = 0.33, size = 227, normalized size = 4.05

$$\frac{\sqrt{2}}{d\sqrt{\frac{b\,e^{dx+c}}{e^{2dx+2c}-1}}}-\frac{\left(\frac{2b\,e^{2dx+2c}-2b}{b\sqrt{e^{dx+c}\left(b\,e^{2dx+2c}-b\right)}}-\frac{\sqrt{e^{dx+c}+1}\,\sqrt{2-2\,e^{dx+c}}\,\sqrt{-e^{dx+c}}\left(-2\,\text{EllipticE}\left(\sqrt{e^{dx+c}+1},\frac{\sqrt{2}}{2}\right)+\text{EllipticF}\left(\sqrt{e^{dx+c}+1},\frac{\sqrt{2}}{2}\right)\right)}{\sqrt{e^{3dx+3c}b-b\,e^{dx+c}}}\right)\sqrt{2}\,\sqrt{b\,e^{dx+c}\left(e^{2dx+2c}-1\right)}}{d\sqrt{\frac{b\,e^{dx+c}}{e^{2dx+2c}-1}}\,\left(e^{2dx+2c}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*csch(d*x+c))^(1/2),x)
[Out] 1/d*2^(1/2)/(b*exp(d*x+c)/(exp(d*x+c)^2-1))^(1/2)-1/d*(2*(b*exp(d*x+c)^2-b)
/b/(exp(d*x+c)*(b*exp(d*x+c)^2-b))^(1/2)-(exp(d*x+c)+1)^(1/2)*(2-2*exp(d*x+
c))^(1/2)*(-exp(d*x+c))^(1/2)/(exp(d*x+c)^3*b-b*exp(d*x+c))^(1/2)*(-2*Ellip
ticE((exp(d*x+c)+1)^(1/2),1/2*2^(1/2))+EllipticF((exp(d*x+c)+1)^(1/2),1/2*2
^(1/2))))*2^(1/2)/(b*exp(d*x+c)/(exp(d*x+c)^2-1))^(1/2)*(b*exp(d*x+c)*(exp(
d*x+c)^2-1))^(1/2)/(exp(d*x+c)^2-1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{b\operatorname{csch}(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*csch(d*x+c))^(1/2),x, algorithm="maxima")
[Out] integrate(1/sqrt(b*csch(d*x + c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sqrt{\frac{b}{\sinh(c+d\,x)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b/sinh(c + d*x))^(1/2),x)
```

```
[Out] int(1/(b/sinh(c + d*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{b\operatorname{csch}(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*csch(d*x+c))**(1/2),x)
```

```
[Out] Integral(1/sqrt(b*csch(c + d*x)), x)
```

3.18 $\int \frac{1}{(b\,\mathbf{csch}(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=90

$$\frac{2\cosh(c+dx)}{3bd\sqrt{b\operatorname{csch}(c+dx)}}+\frac{2i\sqrt{i\sinh(c+dx)}\,F\left(\frac{1}{2}\left(ic+idx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{b\operatorname{csch}(c+dx)}}{3b^2d}$$

```
[Out] 2/3*cosh(d*x+c)/b/d/(b*csch(d*x+c))^(1/2)-2/3*I*(sin(1/2*I*c+1/4*Pi+1/2*I*d
*x)^2)^(1/2)/sin(1/2*I*c+1/4*Pi+1/2*I*d*x)*EllipticF(cos(1/2*I*c+1/4*Pi+1/2
*I*d*x),2^(1/2))*(b*csch(d*x+c))^(1/2)*(I*sinh(d*x+c))^(1/2)/b^2/d
```

Rubi [A] time = 0.04, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3769, 3771, 2641}

$$\frac{2\cosh(c+dx)}{3bd\sqrt{b\operatorname{csch}(c+dx)}}+\frac{2i\sqrt{i\sinh(c+dx)}\,F\left(\frac{1}{2}\left(ic+idx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{b\operatorname{csch}(c+dx)}}{3b^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Csch[c + d*x])^(-3/2),x]
```

```
[Out] (2*Cosh[c + d*x])/(3*b*d*Sqrt[b*Csch[c + d*x]]) + (((2*I)/3)*Sqrt[b*Csch[c
+ d*x]]*EllipticF[(I*c - Pi/2 + I*d*x)/2, 2]*Sqrt[I*Sinh[c + d*x]])/(b^2*d)
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(b\operatorname{csch}(c+dx))^{3/2}}\,dx &= \frac{2\cosh(c+dx)}{3bd\sqrt{b\operatorname{csch}(c+dx)}}-\frac{\int \sqrt{b\operatorname{csch}(c+dx)}\,dx}{3b^2}\\
&= \frac{2\cosh(c+dx)}{3bd\sqrt{b\operatorname{csch}(c+dx)}}-\frac{\left(\sqrt{b\operatorname{csch}(c+dx)}\sqrt{i\sinh(c+dx)}\right)\int \frac{1}{\sqrt{i\sinh(c+dx)}}\,dx}{3b^2}\\
&= \frac{2\cosh(c+dx)}{3bd\sqrt{b\operatorname{csch}(c+dx)}}+\frac{2i\sqrt{b\operatorname{csch}(c+dx)}\,F\left(\frac{1}{2}\left(ic-\frac{\pi}{2}+idx\right)\middle|2\right)\sqrt{i\sinh(c+dx)}}{3b^2d}
\end{aligned}$$

Mathematica [A] time = 0.09, size = 73, normalized size = 0.81

$$\frac{\operatorname{csch}^2(c+dx)\left(\sinh(2(c+dx))-2i\sqrt{i\sinh(c+dx)}F\left(\frac{1}{4}(-2ic-2idx+\pi)\middle|2\right)\right)}{3d(b\operatorname{csch}(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Csch[c + d*x])^(-3/2),x]
```

```
[Out] (Csch[c + d*x]^2*((-2*I)*EllipticF[((-2*I)*c + Pi - (2*I)*d*x)/4, 2]*Sqrt[I
*Sinh[c + d*x]] + Sinh[2*(c + d*x)]))/(3*d*(b*Csch[c + d*x])^(3/2))
```

fricas [F] time = 0.61, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{b\operatorname{csch}(dx+c)}}{b^2\operatorname{csch}(dx+c)^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*csch(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*csch(d*x + c))/(b^2*csch(d*x + c)^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b\operatorname{csch}(dx+c))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*csch(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*csch(d*x + c))^(-3/2), x)
```

maple [F] time = 0.21, size = 0, normalized size = 0.00

$$\int \frac{1}{(b\operatorname{csch}(dx+c))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*csch(d*x+c))^(3/2),x)
```

```
[Out] int(1/(b*csch(d*x+c))^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b\operatorname{csch}(dx+c))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*csch(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*csch(d*x + c))^(-3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(\frac{b}{\sinh(c+d\,x)}\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b/sinh(c + d*x))^(3/2),x)
```

```
[Out] int(1/(b/sinh(c + d*x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b\operatorname{csch}(c+dx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*csch(d*x+c))**(3/2),x)
```

```
[Out] Integral((b*csch(c + d*x))**(-3/2), x)
```

3.19 $\int \frac{1}{(b\text{csch}(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=90

$$\frac{2\cosh(c+dx)}{5bd(b\text{csch}(c+dx))^{3/2}}+\frac{6iE\left(\frac{1}{2}\left(ic+idx-\frac{\pi}{2}\right)\middle|2\right)}{5b^2d\sqrt{i\sinh(c+dx)}\sqrt{b\text{csch}(c+dx)}}$$

```
[Out] 2/5*cosh(d*x+c)/b/d/(b*csch(d*x+c))^(3/2)-6/5*I*(sin(1/2*I*c+1/4*Pi+1/2*I*d
*x)^2)^(1/2)/sin(1/2*I*c+1/4*Pi+1/2*I*d*x)*EllipticE(cos(1/2*I*c+1/4*Pi+1/2
*I*d*x),2^(1/2))/b^2/d/(b*csch(d*x+c))^(1/2)/(I*sinh(d*x+c))^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3769, 3771, 2639}

$$\frac{2\cosh(c+dx)}{5bd(b\text{csch}(c+dx))^{3/2}}+\frac{6iE\left(\frac{1}{2}\left(ic+idx-\frac{\pi}{2}\right)\middle|2\right)}{5b^2d\sqrt{i\sinh(c+dx)}\sqrt{b\text{csch}(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Csch[c + d*x])^(-5/2),x]
```

```
[Out] (2*Cosh[c + d*x])/(5*b*d*(b*Csch[c + d*x])^(3/2)) + (((6*I)/5)*EllipticE[(I
*c - Pi/2 + I*d*x)/2, 2])/(b^2*d*Sqrt[b*Csch[c + d*x]]*Sqrt[I*Sinh[c + d*x]
])
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(b\text{csch}(c+dx))^{5/2}}\,dx &= \frac{2\cosh(c+dx)}{5bd(b\text{csch}(c+dx))^{3/2}}-\frac{3\int \frac{1}{\sqrt{b\text{csch}(c+dx)}}\,dx}{5b^2}\\
&= \frac{2\cosh(c+dx)}{5bd(b\text{csch}(c+dx))^{3/2}}-\frac{3\int \sqrt{i\sinh(c+dx)}\,dx}{5b^2\sqrt{b\text{csch}(c+dx)}\sqrt{i\sinh(c+dx)}}\\
&= \frac{2\cosh(c+dx)}{5bd(b\text{csch}(c+dx))^{3/2}}+\frac{6iE\left(\frac{1}{2}\left(ic-\frac{\pi}{2}+idx\right)\middle|2\right)}{5b^2d\sqrt{b\text{csch}(c+dx)}\sqrt{i\sinh(c+dx)}}
\end{aligned}$$

Mathematica [A] time = 0.14, size = 68, normalized size = 0.76

$$\frac{\sinh(2(c+dx))-\frac{6iE\left(\frac{1}{4}(-2ic-2idx+\pi)\middle|2\right)}{\sqrt{i\sinh(c+dx)}}}{5b^2d\sqrt{b\text{csch}(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Csch[c + d*x])^(-5/2),x]
[Out] (((-6*I)*EllipticE[((-2*I)*c + Pi - (2*I)*d*x)/4, 2])/Sqrt[I*Sinh[c + d*x]]
 + Sinh[2*(c + d*x)])/(5*b^2*d*Sqrt[b*Csch[c + d*x]])
```

fricas [F] time = 0.45, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{b\,\text{csch}\,(dx+c)}}{b^3\,\text{csch}\,(dx+c)^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*csch(d*x+c))^(5/2),x, algorithm="fricas")
[Out] integral(sqrt(b*csch(d*x + c))/(b^3*csch(d*x + c)^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{(b\,\text{csch}\,(dx+c))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*csch(d*x+c))^(5/2),x, algorithm="giac")
[Out] integrate((b*csch(d*x + c))^(-5/2), x)
```

maple [F] time = 0.21, size = 0, normalized size = 0.00

$$\int\frac{1}{(b\,\text{csch}\,(dx+c))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*csch(d*x+c))^(5/2),x)
[Out] int(1/(b*csch(d*x+c))^(5/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{(b\,\text{csch}\,(dx+c))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*csch(d*x+c))^(5/2),x, algorithm="maxima")
[Out] integrate((b*csch(d*x + c))^(-5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{1}{\left(\frac{b}{\sinh(c+d\,x)}\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b/sinh(c + d*x))^(5/2),x)
```

```
[Out] int(1/(b/sinh(c + d*x))^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b\operatorname{csch}(c+dx))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*csch(d*x+c))**(5/2),x)
```

```
[Out] Integral((b*csch(c + d*x))**(-5/2), x)
```

3.20 $\int \frac{1}{(b\mathbf{csch}(c+dx))^{7/2}}\,dx$

Optimal. Leaf size=118

$$-\frac{10i\sqrt{i\sinh(c+dx)}\,F\left(\frac{1}{2}\left(ic+idx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{b\text{csch}(c+dx)}}{21b^4d}-\frac{10\cosh(c+dx)}{21b^3d\sqrt{b\text{csch}(c+dx)}}+\frac{2\cosh(c+dx)}{7bd(b\text{csch}(c+dx))^{5/2}}$$

```
[Out] 2/7*cosh(d*x+c)/b/d/(b*csch(d*x+c))^(5/2)-10/21*cosh(d*x+c)/b^3/d/(b*csch(d
*x+c))^(1/2)+10/21*I*(sin(1/2*I*c+1/4*Pi+1/2*I*d*x)^2)^(1/2)/sin(1/2*I*c+1/
4*Pi+1/2*I*d*x)*EllipticF(cos(1/2*I*c+1/4*Pi+1/2*I*d*x),2^(1/2))*(b*csch(d*
x+c))^(1/2)*(I*sinh(d*x+c))^(1/2)/b^4/d
```

Rubi [A] time = 0.06, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3769, 3771, 2641}

$$-\frac{10\cosh(c+dx)}{21b^3d\sqrt{b\text{csch}(c+dx)}}-\frac{10i\sqrt{i\sinh(c+dx)}\,F\left(\frac{1}{2}\left(ic+idx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{b\text{csch}(c+dx)}}{21b^4d}+\frac{2\cosh(c+dx)}{7bd(b\text{csch}(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Csch[c + d*x])^(-7/2),x]
```

```
[Out] (2*Cosh[c + d*x])/(7*b*d*(b*Csch[c + d*x])^(5/2)) - (10*Cosh[c + d*x])/(21*
b^3*d*Sqrt[b*Csch[c + d*x]]) - (((10*I)/21)*Sqrt[b*Csch[c + d*x]]*EllipticF
[(I*c - Pi/2 + I*d*x)/2, 2]*Sqrt[I*Sinh[c + d*x]])/(b^4*d)
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\int \frac{1}{(b\text{csch}(c+dx))^{7/2}}\,dx = \frac{2\cosh(c+dx)}{7bd(b\text{csch}(c+dx))^{5/2}} - \frac{5\int \frac{1}{(b\text{csch}(c+dx))^{3/2}}\,dx}{7b^2}$$
$$= \frac{2\cosh(c+dx)}{7bd(b\text{csch}(c+dx))^{5/2}} - \frac{10\cosh(c+dx)}{21b^3d\sqrt{b\text{csch}(c+dx)}} + \frac{5\int \sqrt{b\text{csch}(c+dx)}\,dx}{21b^4}$$
$$= \frac{2\cosh(c+dx)}{7bd(b\text{csch}(c+dx))^{5/2}} - \frac{10\cosh(c+dx)}{21b^3d\sqrt{b\text{csch}(c+dx)}} + \frac{\left(5\sqrt{b\text{csch}(c+dx)}\sqrt{i\sinh(c+d}\right.}{21b^4}$$
$$= \frac{2\cosh(c+dx)}{7bd(b\text{csch}(c+dx))^{5/2}} - \frac{10\cosh(c+dx)}{21b^3d\sqrt{b\text{csch}(c+dx)}} - \frac{10i\sqrt{b\text{csch}(c+dx)}F\left(\frac{1}{2}\left(ic-\frac{\pi}{2}+\right.\right.}{21b^4d}$$

Mathematica [A] time = 0.16, size = 80, normalized size = 0.68

$$\frac{\sqrt{b\text{csch}(c+dx)}\left(-26\sinh(2(c+dx))+3\sinh(4(c+dx))+40i\sqrt{i\sinh(c+dx)}F\left(\frac{1}{4}(-2ic-2idx+\pi)\middle|2\right)\right)}{84b^4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Csch[c + d*x])^(-7/2),x]
[Out] (Sqrt[b*Csch[c + d*x]]*((40*I)*EllipticF[((-2*I)*c + Pi - (2*I)*d*x)/4, 2]*
Sqrt[I*Sinh[c + d*x]] - 26*Sinh[2*(c + d*x)] + 3*Sinh[4*(c + d*x)]))/(84*b^
4*d)
```

fricas [F] time = 0.59, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{b\,\text{csch}\,(dx+c)}}{b^4\,\text{csch}\,(dx+c)^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*csch(d*x+c))^(7/2),x, algorithm="fricas")
[Out] integral(sqrt(b*csch(d*x + c))/(b^4*csch(d*x + c)^4), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b\,\text{csch}\,(dx+c))^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*csch(d*x+c))^(7/2),x, algorithm="giac")
[Out] integrate((b*csch(d*x + c))^(-7/2), x)
```

maple [F] time = 0.21, size = 0, normalized size = 0.00

$$\int \frac{1}{(b\,\text{csch}\,(dx+c))^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*csch(d*x+c))^(7/2),x)
[Out] int(1/(b*csch(d*x+c))^(7/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b\operatorname{csch}(dx+c))^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*csch(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((b*csch(d*x + c))^(-7/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(\frac{b}{\sinh(c+d\,x)}\right)^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b/sinh(c + d*x))^(7/2),x)
```

```
[Out] int(1/(b/sinh(c + d*x))^(7/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b\operatorname{csch}(c+dx))^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*csch(d*x+c))**(7/2),x)
```

```
[Out] Integral((b*csch(c + d*x))**(-7/2), x)
```

3.21 $\int (b\operatorname{csch}(c+dx))^n\,dx$

Optimal. Leaf size=74

$$\frac{b\cosh(c+dx)(b\operatorname{csch}(c+dx))^{n-1}\,{}_2F_1\left(\frac{1}{2},\frac{1-n}{2};\frac{3-n}{2};-\sinh^2(c+dx)\right)}{d(1-n)\sqrt{\cosh^2(c+dx)}}$$

```
[Out] b*cosh(d*x+c)*(b*csch(d*x+c))^(-1+n)*hypergeom([1/2, 1/2-1/2*n],[3/2-1/2*n]
,-sinh(d*x+c)^2)/d/(1-n)/(cosh(d*x+c)^2)^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3772, 2643}

$$\frac{b\cosh(c+dx)(b\operatorname{csch}(c+dx))^{n-1}\,{}_2F_1\left(\frac{1}{2},\frac{1-n}{2};\frac{3-n}{2};-\sinh^2(c+dx)\right)}{d(1-n)\sqrt{\cosh^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Csch[c + d*x])^n,x]
```

```
[Out] (b*Cosh[c + d*x]*(b*Csch[c + d*x])^(-1 + n)*Hypergeometric2F1[1/2, (1 - n)/
2, (3 - n)/2, -Sinh[c + d*x]^2])/(d*(1 - n)*Sqrt[Cosh[c + d*x]^2])
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 3772

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] && !IntegerQ[n]
```

Rubi steps

$$\begin{aligned}\int (b\operatorname{csch}(c+dx))^n\,dx &= (b\operatorname{csch}(c+dx))^n\left(\frac{\sinh(c+dx)}{b}\right)^n\int\left(\frac{\sinh(c+dx)}{b}\right)^{-n}dx\\ &= \frac{\cosh(c+dx)(b\operatorname{csch}(c+dx))^n\,{}_2F_1\left(\frac{1}{2},\frac{1-n}{2};\frac{3-n}{2};-\sinh^2(c+dx)\right)\sinh(c+dx)}{d(1-n)\sqrt{\cosh^2(c+dx)}}\end{aligned}$$

Mathematica [A] time = 0.10, size = 67, normalized size = 0.91

$$-\frac{\sinh(c+dx)\cosh(c+dx)\left(-\sinh^2(c+dx)\right)^{\frac{n-1}{2}}(b\operatorname{csch}(c+dx))^n\,{}_2F_1\left(\frac{1}{2},\frac{n+1}{2};\frac{3}{2};\cosh^2(c+dx)\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Csch[c + d*x])^n,x]
```

```
[Out] -((Cosh[c + d*x]*(b*Csch[c + d*x])^n*Hypergeometric2F1[1/2, (1 + n)/2, 3/2,
 Cosh[c + d*x]^2]*Sinh[c + d*x]*(-Sinh[c + d*x]^2)^((-1 + n)/2))/d)
```

fricas [F] time = 0.73, size = 0, normalized size = 0.00

$$\text{integral}\left((b\,\text{csch}\,(dx+c))^n\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*csch(d*x+c))^n,x, algorithm="fricas")
[Out] integral((b*csch(d*x + c))^n, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\,\text{csch}\,(dx+c))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*csch(d*x+c))^n,x, algorithm="giac")
[Out] integrate((b*csch(d*x + c))^n, x)
```

maple [F] time = 0.45, size = 0, normalized size = 0.00

$$\int (b\,\text{csch}\,(dx+c))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*csch(d*x+c))^n,x)
[Out] int((b*csch(d*x+c))^n,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\,\text{csch}\,(dx+c))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*csch(d*x+c))^n,x, algorithm="maxima")
[Out] integrate((b*csch(d*x + c))^n, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(\frac{b}{\mathrm{sinh}\left(c+d\,x\right)}\right)}^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b/sinh(c + d*x))^n,x)
[Out] int((b/sinh(c + d*x))^n, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\,\text{csch}\,(c+dx))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*csch(d*x+c))**n,x)
[Out] Integral((b*csch(c + d*x))**n, x)
```

3.22 $\int \left(-\operatorname{csch}^2(x)\right)^{5/2} \, dx$

Optimal. Leaf size=40

$$\frac{3}{8}\sin^{-1}(\coth(x))+\frac{1}{4}\coth(x)\left(-\operatorname{csch}^2(x)\right)^{3/2}+\frac{3}{8}\coth(x)\sqrt{-\operatorname{csch}^2(x)}$$

```
[Out] 3/8*arcsin(coth(x))+1/4*coth(x)*(-csch(x)^2)^(3/2)+3/8*coth(x)*(-csch(x)^2)
^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {4122, 195, 216}

$$\frac{3}{8}\sin^{-1}(\coth(x))+\frac{1}{4}\coth(x)\left(-\operatorname{csch}^2(x)\right)^{3/2}+\frac{3}{8}\coth(x)\sqrt{-\operatorname{csch}^2(x)}$$

Antiderivative was successfully verified.

```
[In] Int[(-Csch[x]^2)^(5/2),x]
```

```
[Out] (3*ArcSin[Coth[x]])/8 + (3*Coth[x]*Sqrt[-Csch[x]^2])/8 + (Coth[x]*(-Csch[x]
^2)^(3/2))/4
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 4122

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff = FreeFac
tors[Tan[e + f*x], x]}, Dist[(b*ff)/f, Subst[Int[(b + b*ff^2*x^2)^(p - 1),
x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{b, e, f, p}, x] && !IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\int \left(-\operatorname{csch}^2(x)\right)^{5/2} \, dx &= \text{Subst}\left(\int \left(1-x^2\right)^{3/2} \, dx, x, \coth(x)\right)\\
&= \frac{1}{4}\coth(x)\left(-\operatorname{csch}^2(x)\right)^{3/2}+\frac{3}{4}\text{Subst}\left(\int \sqrt{1-x^2} \, dx, x, \coth(x)\right)\\
&= \frac{3}{8}\coth(x)\sqrt{-\operatorname{csch}^2(x)}+\frac{1}{4}\coth(x)\left(-\operatorname{csch}^2(x)\right)^{3/2}+\frac{3}{8}\text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}} \, dx, x, \coth(\right.\\
&= \frac{3}{8}\sin^{-1}(\coth(x))+\frac{3}{8}\coth(x)\sqrt{-\operatorname{csch}^2(x)}+\frac{1}{4}\coth(x)\left(-\operatorname{csch}^2(x)\right)^{3/2}
\end{aligned}$$

Mathematica [A] time = 0.12, size = 41, normalized size = 1.02

$$\frac{1}{64}\sinh(x)\left(-\operatorname{csch}^2(x)\right)^{5/2}\left(6\left(\cosh(3x)+4\sinh^4(x)\log\left(\tanh\left(\frac{x}{2}\right)\right)\right)-22\cosh(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-Csch[x]^2)^(5/2),x]
```

```
[Out] ((-Csch[x]^2)^(5/2)*Sinh[x]*(-22*Cosh[x] + 6*(Cosh[3*x] + 4*Log[Tanh[x/2]]*
Sinh[x]^4)))/64
```

fricas [C] time = 0.50, size = 115, normalized size = 2.88

$$\frac{\left(-3i\,e^{(8\,x)}+12i\,e^{(6\,x)}-18i\,e^{(4\,x)}+12i\,e^{(2\,x)}-3i\right)\log\left(e^{x}+1\right)+\left(3i\,e^{(8\,x)}-12i\,e^{(6\,x)}+18i\,e^{(4\,x)}-12i\,e^{(2\,x)}+3i\right)\log\left(e\right.}{8\left(e^{(8\,x)}-4\,e^{(6\,x)}+6\,e^{(4\,x)}-4\,e^{(2\,x)}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-csch(x)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/8*((-3*I*e^(8*x) + 12*I*e^(6*x) - 18*I*e^(4*x) + 12*I*e^(2*x) - 3*I)*log(
e^x + 1) + (3*I*e^(8*x) - 12*I*e^(6*x) + 18*I*e^(4*x) - 12*I*e^(2*x) + 3*I)
*log(e^x - 1) + 6*I*e^(7*x) - 22*I*e^(5*x) - 22*I*e^(3*x) + 6*I*e^x)/(e^(8*
x) - 4*e^(6*x) + 6*e^(4*x) - 4*e^(2*x) + 1)
```

giac [C] time = 0.14, size = 72, normalized size = 1.80

$$-\frac{1}{16}\left(\frac{4i\left(3\left(e^{(-x)}+e^{x}\right)^{3}-20\,e^{(-x)}-20\,e^{x}\right)}{\left(\left(e^{(-x)}+e^{x}\right)^{2}-4\right)^{2}}-3i\,\log\left(e^{(-x)}+e^{x}+2\right)+3i\,\log\left(e^{(-x)}+e^{x}-2\right)\right)\mathrm{sgn}\left(-e^{(3\,x)}+e^{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-csch(x)^2)^(5/2),x, algorithm="giac")
```

```
[Out] -1/16*(4*I*(3*(e^(-x) + e^x)^3 - 20*e^(-x) - 20*e^x)/((e^(-x) + e^x)^2 - 4)
^2 - 3*I*log(e^(-x) + e^x + 2) + 3*I*log(e^(-x) + e^x - 2))*sgn(-e^(3*x) +
e^x)
```

maple [B] time = 0.22, size = 114, normalized size = 2.85

$$\frac{\sqrt{-\frac{e^{2x}}{\left(e^{2x}-1\right)^{2}}}\,\left(3\,e^{6x}-11\,e^{4x}-11\,e^{2x}+3\right)}{4\left(e^{2x}-1\right)^{3}}-\frac{3\,e^{-x}\left(e^{2x}-1\right)\sqrt{-\frac{e^{2x}}{\left(e^{2x}-1\right)^{2}}}\,\ln\left(e^{x}+1\right)}{8}+\frac{3\,e^{-x}\left(e^{2x}-1\right)\sqrt{-\frac{e^{2x}}{\left(e^{2x}-1\right)^{2}}}\,\ln\left(e\right.}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-csch(x)^2)^(5/2),x)
```

```
[Out] 1/4/(exp(2*x)-1)^3*(-exp(2*x)/(exp(2*x)-1)^2)^(1/2)*(3*exp(6*x)-11*exp(4*x)
-11*exp(2*x)+3)-3/8*exp(-x)*(exp(2*x)-1)*(-exp(2*x)/(exp(2*x)-1)^2)^(1/2)*l
n(exp(x)+1)+3/8*exp(-x)*(exp(2*x)-1)*(-exp(2*x)/(exp(2*x)-1)^2)^(1/2)*ln(ex
p(x)-1)
```

maxima [C] time = 0.52, size = 74, normalized size = 1.85

$$\frac{3i\,e^{(-x)}-11i\,e^{(-3\,x)}-11i\,e^{(-5\,x)}+3i\,e^{(-7\,x)}}{4\left(4\,e^{(-2\,x)}-6\,e^{(-4\,x)}+4\,e^{(-6\,x)}-e^{(-8\,x)}-1\right)}+\frac{3}{8}i\,\log\left(e^{(-x)}+1\right)-\frac{3}{8}i\,\log\left(e^{(-x)}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-csch(x)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/4*(3*I*e^(-x) - 11*I*e^(-3*x) - 11*I*e^(-5*x) + 3*I*e^(-7*x))/(4*e^(-2*x)
 - 6*e^(-4*x) + 4*e^(-6*x) - e^(-8*x) - 1) + 3/8*I*log(e^(-x) + 1) - 3/8*I*
log(e^(-x) - 1)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \left(-\frac{1}{\sinh(x)^2}\right)^{5/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-1/sinh(x)^2)^(5/2),x)
```

```
[Out] int((-1/sinh(x)^2)^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(-\operatorname{csch}^2(x)\right)^{\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-csch(x)**2)**(5/2),x)
```

```
[Out] Integral((-csch(x)**2)**(5/2), x)
```

3.23 $\int \left(-\text{csch}^2(x)\right)^{3/2}\,dx$

Optimal. Leaf size=24

$$\frac{1}{2}\sin^{-1}(\coth(x))+\frac{1}{2}\coth(x)\sqrt{-\text{csch}^2(x)}$$

```
[Out] 1/2*arcsin(coth(x))+1/2*coth(x)*(-csch(x)^2)^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {4122, 195, 216}

$$\frac{1}{2}\sin^{-1}(\coth(x))+\frac{1}{2}\coth(x)\sqrt{-\text{csch}^2(x)}$$

Antiderivative was successfully verified.

```
[In] Int[(-Csch[x]^2)^(3/2),x]
```

```
[Out] ArcSin[Coth[x]]/2 + (Coth[x]*Sqrt[-Csch[x]^2])/2
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 4122

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff = FreeFac
tors[Tan[e + f*x], x]}, Dist[(b*ff)/f, Subst[Int[(b + b*ff^2*x^2)^(p - 1),
x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{b, e, f, p}, x] &&  !IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \left(-\text{csch}^2(x)\right)^{3/2}\,dx &= \text{Subst}\left(\int \sqrt{1-x^2}\,dx,x,\coth(x)\right)\\ &= \frac{1}{2}\coth(x)\sqrt{-\text{csch}^2(x)}+\frac{1}{2}\text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}}\,dx,x,\coth(x)\right)\\ &= \frac{1}{2}\sin^{-1}(\coth(x))+\frac{1}{2}\coth(x)\sqrt{-\text{csch}^2(x)}\end{aligned}$$

Mathematica [A] time = 0.06, size = 41, normalized size = 1.71

$$\frac{1}{4}\text{csch}\left(\frac{x}{2}\right)\sqrt{-\text{csch}^2(x)}\,\text{sech}\left(\frac{x}{2}\right)\left(\cosh(x)+\sinh^2(x)\log\left(\tanh\left(\frac{x}{2}\right)\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-Csch[x]^2)^(3/2),x]
```

```
[Out] (Csch[x/2]*Sqrt[-Csch[x]^2]*Sech[x/2]*(Cosh[x] + Log[Tanh[x/2]]*Sinh[x]^2))
/4
```

fricas [C] time = 0.56, size = 67, normalized size = 2.79

$$\frac{\left(-i\,e^{(4\,x)}+2i\,e^{(2\,x)}-i\right)\log\left(e^{x}+1\right)+\left(i\,e^{(4\,x)}-2i\,e^{(2\,x)}+i\right)\log\left(e^{x}-1\right)+2i\,e^{(3\,x)}+2i\,e^{x}}{2\left(e^{(4\,x)}-2\,e^{(2\,x)}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-csch(x)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/2*((-I*e^(4*x) + 2*I*e^(2*x) - I)*log(e^x + 1) + (I*e^(4*x) - 2*I*e^(2*x)
 + I)*log(e^x - 1) + 2*I*e^(3*x) + 2*I*e^x)/(e^(4*x) - 2*e^(2*x) + 1)
```

giac [C] time = 0.14, size = 57, normalized size = 2.38

$$-\frac{1}{4}\left(\frac{4i\left(e^{(-x)}+e^{x}\right)}{\left(e^{(-x)}+e^{x}\right)^{2}-4}-i\,\log\left(e^{(-x)}+e^{x}+2\right)+i\,\log\left(e^{(-x)}+e^{x}-2\right)\right)\mathrm{sgn}\left(-e^{(3\,x)}+e^{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-csch(x)^2)^(3/2),x, algorithm="giac")
```

```
[Out] -1/4*(4*I*(e^(-x) + e^x)/((e^(-x) + e^x)^2 - 4) - I*log(e^(-x) + e^x + 2) +
 I*log(e^(-x) + e^x - 2))*sgn(-e^(3*x) + e^x)
```

maple [B] time = 0.19, size = 99, normalized size = 4.12

$$\frac{\sqrt{-\frac{\mathrm{e}^{2x}}{\left(\mathrm{e}^{2x}-1\right)^{2}}}\,\left(1+\mathrm{e}^{2x}\right)}{\mathrm{e}^{2x}-1}-\frac{\mathrm{e}^{-x}\left(\mathrm{e}^{2x}-1\right)\sqrt{-\frac{\mathrm{e}^{2x}}{\left(\mathrm{e}^{2x}-1\right)^{2}}}\,\ln\left(\mathrm{e}^{x}+1\right)}{2}+\frac{\mathrm{e}^{-x}\left(\mathrm{e}^{2x}-1\right)\sqrt{-\frac{\mathrm{e}^{2x}}{\left(\mathrm{e}^{2x}-1\right)^{2}}}\,\ln\left(\mathrm{e}^{x}-1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-csch(x)^2)^(3/2),x)
```

```
[Out] 1/(exp(2*x)-1)*(-exp(2*x)/(exp(2*x)-1)^2)^(1/2)*(1+exp(2*x))-1/2*exp(-x)*(e
xp(2*x)-1)*(-exp(2*x)/(exp(2*x)-1)^2)^(1/2)*ln(exp(x)+1)+1/2*exp(-x)*(exp(2
*x)-1)*(-exp(2*x)/(exp(2*x)-1)^2)^(1/2)*ln(exp(x)-1)
```

maxima [C] time = 0.63, size = 49, normalized size = 2.04

$$\frac{i\,e^{(-x)}+i\,e^{(-3\,x)}}{2\,e^{(-2\,x)}-e^{(-4\,x)}-1}+\frac{1}{2}i\,\log\left(e^{(-x)}+1\right)-\frac{1}{2}i\,\log\left(e^{(-x)}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-csch(x)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] (I*e^(-x) + I*e^(-3*x))/(2*e^(-2*x) - e^(-4*x) - 1) + 1/2*I*log(e^(-x) + 1)
 - 1/2*I*log(e^(-x) - 1)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.04

$$\int\left(-\frac{1}{\mathrm{sinh}(x)^{2}}\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-1/sinh(x)^2)^(3/2),x)
```

```
[Out] int((-1/sinh(x)^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(-\operatorname{csch}^2(x)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-csch(x)**2)**(3/2),x)
```

```
[Out] Integral((-csch(x)**2)**(3/2), x)
```

3.24 $\int \sqrt{-\text{csch}^2(x)}\, dx$

Optimal. Leaf size=3

$$\sin^{-1}(\coth(x))$$

```
[Out] arcsin(coth(x))
```

Rubi [A] time = 0.01, antiderivative size = 3, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {4122, 216}

$$\sin^{-1}(\coth(x))$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-Csch[x]^2],x]
```

```
[Out] ArcSin[Coth[x]]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 4122

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff = FreeFac
tors[Tan[e + f*x], x]}, Dist[(b*ff)/f, Subst[Int[(b + b*ff^2*x^2)^(p - 1),
x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{b, e, f, p}, x] && !IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \sqrt{-\text{csch}^2(x)}\, dx &= \text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}}\, dx, x, \coth(x)\right)\\ &= \sin^{-1}(\coth(x))\end{aligned}$$

Mathematica [B] time = 0.01, size = 20, normalized size = 6.67

$$\sinh(x)\sqrt{-\text{csch}^2(x)}\log\left(\tanh\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-Csch[x]^2],x]
```

```
[Out] Sqrt[-Csch[x]^2]*Log[Tanh[x/2]]*Sinh[x]
```

fricas [C] time = 0.98, size = 15, normalized size = 5.00

$$-i\log\left(e^x+1\right)+i\log\left(e^x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-csch(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -I*log(e^x + 1) + I*log(e^x - 1)
```

giac [C] time = 0.14, size = 27, normalized size = 9.00

$$\left(i\,\log\left(e^{x}+1\right)-i\,\log\left(\left|e^{x}-1\right|\right)\right)\mathrm{sgn}\left(-e^{(3\,x)}+e^{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-csch(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] (I*log(e^x + 1) - I*log(abs(e^x - 1)))*sgn(-e^(3*x) + e^x)
```

maple [B] time = 0.20, size = 67, normalized size = 22.33

$$\mathrm{e}^{-x}\left(\mathrm{e}^{2x}-1\right)\sqrt{-\frac{\mathrm{e}^{2x}}{\left(\mathrm{e}^{2x}-1\right)^{2}}}\,\ln\left(\mathrm{e}^{x}-1\right)-\mathrm{e}^{-x}\left(\mathrm{e}^{2x}-1\right)\sqrt{-\frac{\mathrm{e}^{2x}}{\left(\mathrm{e}^{2x}-1\right)^{2}}}\,\ln\left(\mathrm{e}^{x}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-csch(x)^2)^(1/2),x)
```

```
[Out] exp(-x)*(exp(2*x)-1)*(-exp(2*x)/(exp(2*x)-1)^2)^(1/2)*ln(exp(x)-1)-exp(-x)*
(exp(2*x)-1)*(-exp(2*x)/(exp(2*x)-1)^2)^(1/2)*ln(exp(x)+1)
```

maxima [C] time = 0.56, size = 19, normalized size = 6.33

$$i\,\log\left(e^{(-x)}+1\right)-i\,\log\left(e^{(-x)}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-csch(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] I*log(e^(-x) + 1) - I*log(e^(-x) - 1)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.33

$$\int \sqrt{-\frac{1}{\mathrm{sinh}(x)^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-1/sinh(x)^2)^(1/2),x)
```

```
[Out] int((-1/sinh(x)^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{-\operatorname{csch}^{2}(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-csch(x)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(-csch(x)**2), x)
```

3.25 $\int \frac{1}{\sqrt{-\mathrm{csch}^2(x)}}\,dx$

Optimal. Leaf size=13

$$\frac{\coth(x)}{\sqrt{-\mathrm{csch}^2(x)}}$$

```
[Out] coth(x)/(-csch(x)^2)^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {4122, 191}

$$\frac{\coth(x)}{\sqrt{-\mathrm{csch}^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[-Csch[x]^2],x]
```

```
[Out] Coth[x]/Sqrt[-Csch[x]^2]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 4122

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff = FreeFac
tors[Tan[e + f*x], x]}, Dist[(b*ff)/f, Subst[Int[(b + b*ff^2*x^2)^(p - 1),
x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{b, e, f, p}, x] && !IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{-\mathrm{csch}^2(x)}}\,dx &= \mathrm{Subst}\left(\int \frac{1}{\left(1-x^2\right)^{3/2}}\,dx, x, \coth(x)\right)\\ &= \frac{\coth(x)}{\sqrt{-\mathrm{csch}^2(x)}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 13, normalized size = 1.00

$$\frac{\coth(x)}{\sqrt{-\mathrm{csch}^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[-Csch[x]^2],x]
```

```
[Out] Coth[x]/Sqrt[-Csch[x]^2]
```

fricas [C] time = 1.05, size = 14, normalized size = 1.08

$$\frac{1}{2}\left(-i\,e^{(2\,x)} - i\right)e^{(-x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-csch(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*(-I*e^(2*x) - I)*e^(-x)
```

giac [C] time = 0.13, size = 25, normalized size = 1.92

$$-\frac{-i\,e^{(-x)} - i\,e^{x}}{2\,\mathrm{sgn}\left(-e^{(3\,x)} + e^{x}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-csch(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*(-I*e^(-x) - I*e^x)/sgn(-e^(3*x) + e^x)
```

maple [B] time = 0.20, size = 58, normalized size = 4.46

$$\frac{e^{2x}}{2\sqrt{-\frac{e^{2x}}{\left(e^{2x}-1\right)^{2}}}\,\left(e^{2x}-1\right)}+\frac{1}{2\left(e^{2x}-1\right)\sqrt{-\frac{e^{2x}}{\left(e^{2x}-1\right)^{2}}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-csch(x)^2)^(1/2),x)
```

```
[Out] 1/2/(-exp(2*x)/(exp(2*x)-1)^2)^(1/2)/(exp(2*x)-1)*exp(2*x)+1/2/(exp(2*x)-1)
/(-exp(2*x)/(exp(2*x)-1)^2)^(1/2)
```

maxima [C] time = 0.49, size = 11, normalized size = 0.85

$$\frac{1}{2}i\,e^{(-x)} + \frac{1}{2}i\,e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-csch(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*I*e^(-x) + 1/2*I*e^x
```

mupad [B] time = 1.72, size = 31, normalized size = 2.38

$$-e^{-2\,x}\,\sqrt{-\frac{1}{{\left(\frac{e^{-x}}{2}-\frac{e^{x}}{2}\right)}^2}}\,\left(\frac{e^{4\,x}}{4}-\frac{1}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-1/sinh(x)^2)^(1/2),x)
```

```
[Out] -exp(-2*x)*(-1/(exp(-x)/2 - exp(x)/2)^2)^(1/2)*(exp(4*x)/4 - 1/4)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-\operatorname{csch}^{2}{\left(x \right)}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-csch(x)**2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(-csch(x)**2), x)
```

3.26 $\int \frac{1}{\left(-\operatorname{csch}^2(x)\right)^{3/2}}\,dx$

Optimal. Leaf size=33

$$\frac{2\coth(x)}{3\sqrt{-\operatorname{csch}^2(x)}}+\frac{\coth(x)}{3\left(-\operatorname{csch}^2(x)\right)^{3/2}}$$

[Out] 1/3*coth(x)/(-csch(x)^2)^(3/2)+2/3*coth(x)/(-csch(x)^2)^(1/2)

Rubi [A] time = 0.01, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {4122, 192, 191}

$$\frac{2\coth(x)}{3\sqrt{-\operatorname{csch}^2(x)}}+\frac{\coth(x)}{3\left(-\operatorname{csch}^2(x)\right)^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[(-Csch[x]^2)^(-3/2),x]

[Out] Coth[x]/(3*(-Csch[x]^2)^(3/2)) + (2*Coth[x])/(3*Sqrt[-Csch[x]^2])

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 4122

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff = FreeFac
tors[Tan[e + f*x], x]}, Dist[(b*ff)/f, Subst[Int[(b + b*ff^2*x^2)^(p - 1),
x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{b, e, f, p}, x] &&  !IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\left(-\operatorname{csch}^2(x)\right)^{3/2}}\,dx &= \operatorname{Subst}\left(\int \frac{1}{\left(1-x^2\right)^{5/2}}\,dx,x,\coth(x)\right)\\
&= \frac{\coth(x)}{3\left(-\operatorname{csch}^2(x)\right)^{3/2}}+\frac{2}{3}\operatorname{Subst}\left(\int \frac{1}{\left(1-x^2\right)^{3/2}}\,dx,x,\coth(x)\right)\\
&= \frac{\coth(x)}{3\left(-\operatorname{csch}^2(x)\right)^{3/2}}+\frac{2\coth(x)}{3\sqrt{-\operatorname{csch}^2(x)}}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 27, normalized size = 0.82

$$\frac{9\coth(x)-\cosh(3x)\operatorname{csch}(x)}{12\sqrt{-\operatorname{csch}^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-Csch[x]^2)^(-3/2),x]
```

```
[Out] (9*Coth[x] - Cosh[3*x]*Csch[x])/(12*Sqrt[-Csch[x]^2])
```

fricas [C] time = 2.47, size = 26, normalized size = 0.79

$$\frac{1}{24}\left(i\,e^{(6\,x)}-9i\,e^{(4\,x)}-9i\,e^{(2\,x)}+i\right)e^{(-3\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-csch(x)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/24*(I*e^(6*x) - 9*I*e^(4*x) - 9*I*e^(2*x) + I)*e^(-3*x)
```

giac [C] time = 0.16, size = 50, normalized size = 1.52

$$\frac{i\left(9\,e^{(2\,x)}-1\right)e^{(-3\,x)}}{24\,\mathrm{sgn}\left(-e^{(3\,x)}+e^{x}\right)}-\frac{i\left(e^{(3\,x)}-9\,e^{x}\right)}{24\,\mathrm{sgn}\left(-e^{(3\,x)}+e^{x}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-csch(x)^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/24*I*(9*e^(2*x) - 1)*e^(-3*x)/sgn(-e^(3*x) + e^x) - 1/24*I*(e^(3*x) - 9*e
^x)/sgn(-e^(3*x) + e^x)
```

maple [B] time = 0.18, size = 118, normalized size = 3.58

$$-\frac{e^{4x}}{24\left(e^{2x}-1\right)\sqrt{-\frac{e^{2x}}{\left(e^{2x}-1\right)^{2}}}}+\frac{3\,e^{2x}}{8\sqrt{-\frac{e^{2x}}{\left(e^{2x}-1\right)^{2}}}\left(e^{2x}-1\right)}+\frac{3}{8\left(e^{2x}-1\right)\sqrt{-\frac{e^{2x}}{\left(e^{2x}-1\right)^{2}}}}-\frac{e^{-2x}}{24\left(e^{2x}-1\right)\sqrt{-\frac{e^{2x}}{\left(e^{2x}-1\right)^{2}}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-csch(x)^2)^(3/2),x)
```

```
[Out] -1/24*exp(4*x)/(exp(2*x)-1)/(-exp(2*x)/(exp(2*x)-1)^2)^(1/2)+3/8/(-exp(2*x)
/(exp(2*x)-1)^2)^(1/2)/(exp(2*x)-1)*exp(2*x)+3/8/(exp(2*x)-1)/(-exp(2*x)/(e
xp(2*x)-1)^2)^(1/2)-1/24*exp(-2*x)/(exp(2*x)-1)/(-exp(2*x)/(exp(2*x)-1)^2)^
(1/2)
```

maxima [C] time = 0.44, size = 23, normalized size = 0.70

$$-\frac{1}{24}i\,e^{(3\,x)}+\frac{3}{8}i\,e^{(-x)}-\frac{1}{24}i\,e^{(-3\,x)}+\frac{3}{8}i\,e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-csch(x)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/24*I*e^(3*x) + 3/8*I*e^(-x) - 1/24*I*e^(-3*x) + 3/8*I*e^x
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{\left(-\frac{1}{\sinh(x)^{2}}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-1/sinh(x)^2)^(3/2),x)
```

```
[Out] int(1/(-1/sinh(x)^2)^(3/2), x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-\operatorname{csch}^2(x)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-csch(x)**2)**(3/2),x)
```

```
[Out] Integral((-csch(x)**2)**(-3/2), x)
```

3.27 $\int \frac{1}{\left(-\mathrm{csch}^2(x)\right)^{5/2}}\,dx$

Optimal. Leaf size=49

$$\frac{8\coth(x)}{15\sqrt{-\mathrm{csch}^2(x)}}+\frac{4\coth(x)}{15\left(-\mathrm{csch}^2(x)\right)^{3/2}}+\frac{\coth(x)}{5\left(-\mathrm{csch}^2(x)\right)^{5/2}}$$

```
[Out] 1/5*coth(x)/(-csch(x)^2)^(5/2)+4/15*coth(x)/(-csch(x)^2)^(3/2)+8/15*coth(x)
/(-csch(x)^2)^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {4122, 192, 191}

$$\frac{8\coth(x)}{15\sqrt{-\mathrm{csch}^2(x)}}+\frac{4\coth(x)}{15\left(-\mathrm{csch}^2(x)\right)^{3/2}}+\frac{\coth(x)}{5\left(-\mathrm{csch}^2(x)\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(-Csch[x]^2)^(-5/2),x]
```

```
[Out] Coth[x]/(5*(-Csch[x]^2)^(5/2)) + (4*Coth[x])/(15*(-Csch[x]^2)^(3/2)) + (8*C
oth[x])/(15*Sqrt[-Csch[x]^2])
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 4122

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff = FreeFac
tors[Tan[e + f*x], x]}, Dist[(b*ff)/f, Subst[Int[(b + b*ff^2*x^2)^(p - 1),
x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{b, e, f, p}, x] &&  !IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\left(-\mathrm{csch}^2(x)\right)^{5/2}}\,dx &= \mathrm{Subst}\left(\int \frac{1}{\left(1-x^2\right)^{7/2}}\,dx,x,\coth(x)\right)\\
&= \frac{\coth(x)}{5\left(-\mathrm{csch}^2(x)\right)^{5/2}}+\frac{4}{5}\,\mathrm{Subst}\left(\int \frac{1}{\left(1-x^2\right)^{5/2}}\,dx,x,\coth(x)\right)\\
&= \frac{\coth(x)}{5\left(-\mathrm{csch}^2(x)\right)^{5/2}}+\frac{4\coth(x)}{15\left(-\mathrm{csch}^2(x)\right)^{3/2}}+\frac{8}{15}\,\mathrm{Subst}\left(\int \frac{1}{\left(1-x^2\right)^{3/2}}\,dx,x,\coth(x)\right)\\
&= \frac{\coth(x)}{5\left(-\mathrm{csch}^2(x)\right)^{5/2}}+\frac{4\coth(x)}{15\left(-\mathrm{csch}^2(x)\right)^{3/2}}+\frac{8\coth(x)}{15\sqrt{-\mathrm{csch}^2(x)}}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 33, normalized size = 0.67

$$\frac{(150\cosh(x)-25\cosh(3x)+3\cosh(5x))\operatorname{csch}(x)}{240\sqrt{-\operatorname{csch}^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-Csch[x]^2)^(-5/2),x]
```

```
[Out] ((150*Cosh[x] - 25*Cosh[3*x] + 3*Cosh[5*x])*Csch[x])/(240*Sqrt[-Csch[x]^2])
```

fricas [C] time = 1.45, size = 38, normalized size = 0.78

$$\frac{1}{480}\left(-3i\,e^{(10\,x)}+25i\,e^{(8\,x)}-150i\,e^{(6\,x)}-150i\,e^{(4\,x)}+25i\,e^{(2\,x)}-3i\right)e^{(-5\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-csch(x)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/480*(-3*I*e^(10*x) + 25*I*e^(8*x) - 150*I*e^(6*x) - 150*I*e^(4*x) + 25*I*
e^(2*x) - 3*I)*e^(-5*x)
```

giac [C] time = 0.17, size = 64, normalized size = 1.31

$$\frac{i\left(150\,e^{(4\,x)}-25\,e^{(2\,x)}+3\right)e^{(-5\,x)}}{480\,\operatorname{sgn}\left(-e^{(3\,x)}+e^{x}\right)}+\frac{i\left(3\,e^{(5\,x)}-25\,e^{(3\,x)}+150\,e^{x}\right)}{480\,\operatorname{sgn}\left(-e^{(3\,x)}+e^{x}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-csch(x)^2)^(5/2),x, algorithm="giac")
```

```
[Out] 1/480*I*(150*e^(4*x) - 25*e^(2*x) + 3)*e^(-5*x)/sgn(-e^(3*x) + e^x) + 1/480
*I*(3*e^(5*x) - 25*e^(3*x) + 150*e^x)/sgn(-e^(3*x) + e^x)
```

maple [B] time = 0.18, size = 178, normalized size = 3.63

$$\frac{e^{6x}}{160\left(e^{2x}-1\right)\sqrt{-\frac{e^{2x}}{\left(e^{2x}-1\right)^2}}}-\frac{5\,e^{4x}}{96\left(e^{2x}-1\right)\sqrt{-\frac{e^{2x}}{\left(e^{2x}-1\right)^2}}}+\frac{5\,e^{2x}}{16\sqrt{-\frac{e^{2x}}{\left(e^{2x}-1\right)^2}}\left(e^{2x}-1\right)}+\frac{5}{16\left(e^{2x}-1\right)\sqrt{-\frac{e^{2x}}{\left(e^{2x}-1\right)^2}}}-\frac{}{96\left(e^{2}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-csch(x)^2)^(5/2),x)
```

```
[Out] 1/160*exp(6*x)/(exp(2*x)-1)/(-exp(2*x)/(exp(2*x)-1)^2)^(1/2)-5/96*exp(4*x)/
(exp(2*x)-1)/(-exp(2*x)/(exp(2*x)-1)^2)^(1/2)+5/16/(-exp(2*x)/(exp(2*x)-1)^
2)^(1/2)/(exp(2*x)-1)*exp(2*x)+5/16/(exp(2*x)-1)/(-exp(2*x)/(exp(2*x)-1)^2)
^(1/2)-5/96*exp(-2*x)/(exp(2*x)-1)/(-exp(2*x)/(exp(2*x)-1)^2)^(1/2)+1/160*e
xp(-4*x)/(exp(2*x)-1)/(-exp(2*x)/(exp(2*x)-1)^2)^(1/2)
```

maxima [C] time = 0.46, size = 35, normalized size = 0.71

$$\frac{1}{160}i\,e^{(5\,x)}-\frac{5}{96}i\,e^{(3\,x)}+\frac{5}{16}i\,e^{(-x)}-\frac{5}{96}i\,e^{(-3\,x)}+\frac{1}{160}i\,e^{(-5\,x)}+\frac{5}{16}i\,e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-csch(x)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/160*I*e^(5*x) - 5/96*I*e^(3*x) + 5/16*I*e^(-x) - 5/96*I*e^(-3*x) + 1/160*
I*e^(-5*x) + 5/16*I*e^x
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\left(-\frac{1}{\sinh(x)^2}\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-1/sinh(x)^2)^(5/2),x)
[Out] int(1/(-1/sinh(x)^2)^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-\operatorname{csch}^2(x)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-csch(x)**2)**(5/2),x)
[Out] Integral((-csch(x)**2)**(-5/2), x)
```

3.28 $\int \frac{1}{\left(-\operatorname{csch}^2(x)\right)^{7/2}} \, dx$

Optimal. Leaf size=65

$$\frac{16 \coth(x)}{35\sqrt{-\operatorname{csch}^2(x)}}+\frac{8 \coth(x)}{35 \left(-\operatorname{csch}^2(x)\right)^{3/2}}+\frac{6 \coth(x)}{35 \left(-\operatorname{csch}^2(x)\right)^{5/2}}+\frac{\coth(x)}{7 \left(-\operatorname{csch}^2(x)\right)^{7/2}}$$

```
[Out] 1/7*coth(x)/(-csch(x)^2)^(7/2)+6/35*coth(x)/(-csch(x)^2)^(5/2)+8/35*coth(x)
/(-csch(x)^2)^(3/2)+16/35*coth(x)/(-csch(x)^2)^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {4122, 192, 191}

$$\frac{16 \coth(x)}{35\sqrt{-\operatorname{csch}^2(x)}}+\frac{8 \coth(x)}{35 \left(-\operatorname{csch}^2(x)\right)^{3/2}}+\frac{6 \coth(x)}{35 \left(-\operatorname{csch}^2(x)\right)^{5/2}}+\frac{\coth(x)}{7 \left(-\operatorname{csch}^2(x)\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(-Csch[x]^2)^(-7/2),x]
```

```
[Out] Coth[x]/(7*(-Csch[x]^2)^(7/2)) + (6*Coth[x])/(35*(-Csch[x]^2)^(5/2)) + (8*C
oth[x])/(35*(-Csch[x]^2)^(3/2)) + (16*Coth[x])/(35*Sqrt[-Csch[x]^2])
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 4122

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff = FreeFac
tors[Tan[e + f*x], x]}, Dist[(b*ff)/f, Subst[Int[(b + b*ff^2*x^2)^(p - 1),
x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{b, e, f, p}, x] &&  !IntegerQ[p]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{\left(-\operatorname{csch}^2(x)\right)^{7/2}}\,dx &= \text{Subst}\left(\int \frac{1}{\left(1-x^2\right)^{9/2}}\,dx, x, \coth(x)\right)\\
&= \frac{\coth(x)}{7\left(-\operatorname{csch}^2(x)\right)^{7/2}} + \frac{6}{7}\,\text{Subst}\left(\int \frac{1}{\left(1-x^2\right)^{7/2}}\,dx, x, \coth(x)\right)\\
&= \frac{\coth(x)}{7\left(-\operatorname{csch}^2(x)\right)^{7/2}} + \frac{6\coth(x)}{35\left(-\operatorname{csch}^2(x)\right)^{5/2}} + \frac{24}{35}\,\text{Subst}\left(\int \frac{1}{\left(1-x^2\right)^{5/2}}\,dx, x, \coth(x)\right)\\
&= \frac{\coth(x)}{7\left(-\operatorname{csch}^2(x)\right)^{7/2}} + \frac{6\coth(x)}{35\left(-\operatorname{csch}^2(x)\right)^{5/2}} + \frac{8\coth(x)}{35\left(-\operatorname{csch}^2(x)\right)^{3/2}} + \frac{16}{35}\,\text{Subst}\left(\int \frac{1}{\left(1-x^2\right)^{3/2}}\,dx\right.\\
&= \frac{\coth(x)}{7\left(-\operatorname{csch}^2(x)\right)^{7/2}} + \frac{6\coth(x)}{35\left(-\operatorname{csch}^2(x)\right)^{5/2}} + \frac{8\coth(x)}{35\left(-\operatorname{csch}^2(x)\right)^{3/2}} + \frac{16\coth(x)}{35\sqrt{-\operatorname{csch}^2(x)}}
\end{aligned}
$$

Mathematica [A] time = 0.04, size = 39, normalized size = 0.60

$$
\frac{(1225\cosh(x) - 245\cosh(3x) + 49\cosh(5x) - 5\cosh(7x))\operatorname{csch}(x)}{2240\sqrt{-\operatorname{csch}^2(x)}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(-Csch[x]^2)^(-7/2),x]
```

```
[Out] ((1225*Cosh[x] - 245*Cosh[3*x] + 49*Cosh[5*x] - 5*Cosh[7*x])*Csch[x])/(2240
*Sqrt[-Csch[x]^2])
```

fricas [C] time = 1.13, size = 50, normalized size = 0.77

$$
\frac{1}{4480}\left(5i\,e^{(14\,x)} - 49i\,e^{(12\,x)} + 245i\,e^{(10\,x)} - 1225i\,e^{(8\,x)} - 1225i\,e^{(6\,x)} + 245i\,e^{(4\,x)} - 49i\,e^{(2\,x)} + 5i\right)e^{(-7\,x)}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-csch(x)^2)^(7/2),x, algorithm="fricas")
```

```
[Out] 1/4480*(5*I*e^(14*x) - 49*I*e^(12*x) + 245*I*e^(10*x) - 1225*I*e^(8*x) - 12
25*I*e^(6*x) + 245*I*e^(4*x) - 49*I*e^(2*x) + 5*I)*e^(-7*x)
```

giac [C] time = 0.17, size = 76, normalized size = 1.17

$$
\frac{i\left(1225\,e^{(6\,x)} - 245\,e^{(4\,x)} + 49\,e^{(2\,x)} - 5\right)e^{(-7\,x)}}{4480\,\operatorname{sgn}\left(-e^{(3\,x)} + e^{x}\right)} - \frac{i\left(5\,e^{(7\,x)} - 49\,e^{(5\,x)} + 245\,e^{(3\,x)} - 1225\,e^{x}\right)}{4480\,\operatorname{sgn}\left(-e^{(3\,x)} + e^{x}\right)}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-csch(x)^2)^(7/2),x, algorithm="giac")
```

```
[Out] 1/4480*I*(1225*e^(6*x) - 245*e^(4*x) + 49*e^(2*x) - 5)*e^(-7*x)/sgn(-e^(3*x
) + e^x) - 1/4480*I*(5*e^(7*x) - 49*e^(5*x) + 245*e^(3*x) - 1225*e^x)/sgn(-
e^(3*x) + e^x)
```

maple [B] time = 0.21, size = 238, normalized size = 3.66

$$
-\frac{e^{8x}}{896\left(e^{2x}-1\right)\sqrt{-\frac{e^{2x}}{\left(e^{2x}-1\right)^2}}} + \frac{7\,e^{6x}}{640\left(e^{2x}-1\right)\sqrt{-\frac{e^{2x}}{\left(e^{2x}-1\right)^2}}} - \frac{7\,e^{4x}}{128\left(e^{2x}-1\right)\sqrt{-\frac{e^{2x}}{\left(e^{2x}-1\right)^2}}} + \frac{35\,e^{2x}}{128\sqrt{-\frac{e^{2x}}{\left(e^{2x}-1\right)^2}}\left(e^{2x}-1\right)} + \cdots
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-csch(x)^2)^(7/2),x)
```

```
[Out] -1/896*exp(8*x)/(exp(2*x)-1)/(-exp(2*x)/(exp(2*x)-1)^2)^(1/2)+7/640*exp(6*x
)/(exp(2*x)-1)/(-exp(2*x)/(exp(2*x)-1)^2)^(1/2)-7/128*exp(4*x)/(exp(2*x)-1)
/(-exp(2*x)/(exp(2*x)-1)^2)^(1/2)+35/128/(-exp(2*x)/(exp(2*x)-1)^2)^(1/2)/(
exp(2*x)-1)*exp(2*x)+35/128/(exp(2*x)-1)/(-exp(2*x)/(exp(2*x)-1)^2)^(1/2)-7
/128*exp(-2*x)/(exp(2*x)-1)/(-exp(2*x)/(exp(2*x)-1)^2)^(1/2)+7/640*exp(-4*x
)/(exp(2*x)-1)/(-exp(2*x)/(exp(2*x)-1)^2)^(1/2)-1/896*exp(-6*x)/(exp(2*x)-1
)/(-exp(2*x)/(exp(2*x)-1)^2)^(1/2)
```

maxima [C] time = 1.03, size = 47, normalized size = 0.72

$$-\frac{1}{896} i e^{(7x)} + \frac{7}{640} i e^{(5x)} - \frac{7}{128} i e^{(3x)} + \frac{35}{128} i e^{(-x)} - \frac{7}{128} i e^{(-3x)} + \frac{7}{640} i e^{(-5x)} - \frac{1}{896} i e^{(-7x)} + \frac{35}{128} i e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-csch(x)^2)^(7/2),x, algorithm="maxima")
```

```
[Out] -1/896*I*e^(7*x) + 7/640*I*e^(5*x) - 7/128*I*e^(3*x) + 35/128*I*e^(-x) - 7/
128*I*e^(-3*x) + 7/640*I*e^(-5*x) - 1/896*I*e^(-7*x) + 35/128*I*e^x
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\left(-\frac{1}{\sinh(x)^2}\right)^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-1/sinh(x)^2)^(7/2),x)
```

```
[Out] int(1/(-1/sinh(x)^2)^(7/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-\operatorname{csch}^{2}(x)\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-csch(x)**2)**(7/2),x)
```

```
[Out] Integral((-csch(x)**2)**(-7/2), x)
```

3.29 $\int \left(a\text{csch}^2(x)\right)^{5/2}\,dx$

Optimal. Leaf size=65

$$-\frac{3}{8}a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a}\coth(x)}{\sqrt{a\text{csch}^2(x)}}\right)+\frac{3}{8}a^2\coth(x)\sqrt{a\text{csch}^2(x)}-\frac{1}{4}a\coth(x)\left(a\text{csch}^2(x)\right)^{3/2}$$

```
[Out] -3/8*a^(5/2)*arctanh(coth(x)*a^(1/2)/(a*csch(x)^2)^(1/2))-1/4*a*coth(x)*(a*
csch(x)^2)^(3/2)+3/8*a^2*coth(x)*(a*csch(x)^2)^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {4122, 195, 217, 206}

$$\frac{3}{8}a^2\coth(x)\sqrt{a\text{csch}^2(x)}-\frac{3}{8}a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a}\coth(x)}{\sqrt{a\text{csch}^2(x)}}\right)-\frac{1}{4}a\coth(x)\left(a\text{csch}^2(x)\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Csch[x]^2)^(5/2),x]
```

```
[Out] (-3*a^(5/2)*ArcTanh[(Sqrt[a]*Coth[x])/Sqrt[a*Csch[x]^2]])/8 + (3*a^2*Coth[x
]*Sqrt[a*Csch[x]^2])/8 - (a*Coth[x]*(a*Csch[x]^2)^(3/2))/4
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 4122

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff = FreeFac
tors[Tan[e + f*x], x]}, Dist[(b*ff)/f, Subst[Int[(b + b*ff^2*x^2)^(p - 1),
x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{b, e, f, p}, x] &&  !IntegerQ[p]
```

Rubi steps

$$\int \left(a\text{csch}^2(x)\right)^{5/2} dx = -\left(a\,\text{Subst}\left(\int \left(-a+ax^2\right)^{3/2} dx, x, \coth(x)\right)\right)$$
$$= -\frac{1}{4}a\coth(x)\left(a\text{csch}^2(x)\right)^{3/2} + \frac{1}{4}\left(3a^2\right)\text{Subst}\left(\int \sqrt{-a+ax^2}\,dx, x, \coth(x)\right)$$
$$= \frac{3}{8}a^2\coth(x)\sqrt{a\text{csch}^2(x)} - \frac{1}{4}a\coth(x)\left(a\text{csch}^2(x)\right)^{3/2} - \frac{1}{8}\left(3a^3\right)\text{Subst}\left(\int \frac{1}{\sqrt{-a+ax^2}}\right.$$
$$= \frac{3}{8}a^2\coth(x)\sqrt{a\text{csch}^2(x)} - \frac{1}{4}a\coth(x)\left(a\text{csch}^2(x)\right)^{3/2} - \frac{1}{8}\left(3a^3\right)\text{Subst}\left(\int \frac{1}{1-ax^2}dx, x\right.$$
$$= -\frac{3}{8}a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a}\coth(x)}{\sqrt{a\text{csch}^2(x)}}\right) + \frac{3}{8}a^2\coth(x)\sqrt{a\text{csch}^2(x)} - \frac{1}{4}a\coth(x)\left(a\text{csch}^2(x)\right)^{3/2}$$

Mathematica [A] time = 0.10, size = 41, normalized size = 0.63

$$\frac{1}{64}\sinh(x)\left(a\text{csch}^2(x)\right)^{5/2}\left(6\left(\cosh(3x)+4\sinh^4(x)\log\left(\tanh\left(\frac{x}{2}\right)\right)\right)-22\cosh(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Csch[x]^2)^(5/2),x]
```

```
[Out] ((a*Csch[x]^2)^(5/2)*Sinh[x]*(-22*Cosh[x] + 6*(Cosh[3*x] + 4*Log[Tanh[x/2]]
*Sinh[x]^4)))/64
```

fricas [B] time = 0.63, size = 1128, normalized size = 17.35

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*csch(x)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/8*(6*a^2*cosh(x)^7 - 6*(a^2*e^(2*x) - a^2)*sinh(x)^7 - 22*a^2*cosh(x)^5
- 42*(a^2*cosh(x)*e^(2*x) - a^2*cosh(x))*sinh(x)^6 + 2*(63*a^2*cosh(x)^2 -
11*a^2 - (63*a^2*cosh(x)^2 - 11*a^2)*e^(2*x))*sinh(x)^5 - 22*a^2*cosh(x)^3
+ 10*(21*a^2*cosh(x)^3 - 11*a^2*cosh(x) - (21*a^2*cosh(x)^3 - 11*a^2*cosh(x
))*e^(2*x))*sinh(x)^4 + 2*(105*a^2*cosh(x)^4 - 110*a^2*cosh(x)^2 - 11*a^2 -
 (105*a^2*cosh(x)^4 - 110*a^2*cosh(x)^2 - 11*a^2)*e^(2*x))*sinh(x)^3 + 6*a^
2*cosh(x) + 2*(63*a^2*cosh(x)^5 - 110*a^2*cosh(x)^3 - 33*a^2*cosh(x) - (63*
a^2*cosh(x)^5 - 110*a^2*cosh(x)^3 - 33*a^2*cosh(x))*e^(2*x))*sinh(x)^2 - 2*
(3*a^2*cosh(x)^7 - 11*a^2*cosh(x)^5 - 11*a^2*cosh(x)^3 + 3*a^2*cosh(x))*e^(
2*x) + 3*(a^2*cosh(x)^8 - (a^2*e^(2*x) - a^2)*sinh(x)^8 - 4*a^2*cosh(x)^6 -
 8*(a^2*cosh(x)*e^(2*x) - a^2*cosh(x))*sinh(x)^7 + 4*(7*a^2*cosh(x)^2 - a^2
 - (7*a^2*cosh(x)^2 - a^2)*e^(2*x))*sinh(x)^6 + 6*a^2*cosh(x)^4 + 8*(7*a^2*
cosh(x)^3 - 3*a^2*cosh(x) - (7*a^2*cosh(x)^3 - 3*a^2*cosh(x))*e^(2*x))*sinh
(x)^5 + 2*(35*a^2*cosh(x)^4 - 30*a^2*cosh(x)^2 + 3*a^2 - (35*a^2*cosh(x)^4
- 30*a^2*cosh(x)^2 + 3*a^2)*e^(2*x))*sinh(x)^4 - 4*a^2*cosh(x)^2 + 8*(7*a^2
*cosh(x)^5 - 10*a^2*cosh(x)^3 + 3*a^2*cosh(x) - (7*a^2*cosh(x)^5 - 10*a^2*c
osh(x)^3 + 3*a^2*cosh(x))*e^(2*x))*sinh(x)^3 + 4*(7*a^2*cosh(x)^6 - 15*a^2*
cosh(x)^4 + 9*a^2*cosh(x)^2 - a^2 - (7*a^2*cosh(x)^6 - 15*a^2*cosh(x)^4 + 9
*a^2*cosh(x)^2 - a^2)*e^(2*x))*sinh(x)^2 + a^2 - (a^2*cosh(x)^8 - 4*a^2*cos
h(x)^6 + 6*a^2*cosh(x)^4 - 4*a^2*cosh(x)^2 + a^2)*e^(2*x) + 8*(a^2*cosh(x)^
7 - 3*a^2*cosh(x)^5 + 3*a^2*cosh(x)^3 - a^2*cosh(x) - (a^2*cosh(x)^7 - 3*a^
2*cosh(x)^5 + 3*a^2*cosh(x)^3 - a^2*cosh(x))*e^(2*x))*sinh(x))*log((cosh(x)
 + sinh(x) - 1)/(cosh(x) + sinh(x) + 1)) + 2*(21*a^2*cosh(x)^6 - 55*a^2*cos
h(x)^4 - 33*a^2*cosh(x)^2 + 3*a^2 - (21*a^2*cosh(x)^6 - 55*a^2*cosh(x)^4 -
```

```
33*a^2*cosh(x)^2 + 3*a^2)*e^(2*x))*sinh(x))*sqrt(a/(e^(4*x) - 2*e^(2*x) + 1
))*e^x/(8*cosh(x)*e^x*sinh(x)^7 + e^x*sinh(x)^8 + 4*(7*cosh(x)^2 - 1)*e^x*s
inh(x)^6 + 8*(7*cosh(x)^3 - 3*cosh(x))*e^x*sinh(x)^5 + 2*(35*cosh(x)^4 - 30
*cosh(x)^2 + 3)*e^x*sinh(x)^4 + 8*(7*cosh(x)^5 - 10*cosh(x)^3 + 3*cosh(x))*
e^x*sinh(x)^3 + 4*(7*cosh(x)^6 - 15*cosh(x)^4 + 9*cosh(x)^2 - 1)*e^x*sinh(x
)^2 + 8*(cosh(x)^7 - 3*cosh(x)^5 + 3*cosh(x)^3 - cosh(x))*e^x*sinh(x) + (co
sh(x)^8 - 4*cosh(x)^6 + 6*cosh(x)^4 - 4*cosh(x)^2 + 1)*e^x)
```

giac [A] time = 0.12, size = 75, normalized size = 1.15

$$\frac{1}{16}\,a^{\frac{5}{2}}\left(\frac{4\left(3\left(e^{(-x)}+e^{x}\right)^{3}-20\,e^{(-x)}-20\,e^{x}\right)}{\left(\left(e^{(-x)}+e^{x}\right)^{2}-4\right)^{2}}-3\,\log\left(e^{(-x)}+e^{x}+2\right)+3\,\log\left(e^{(-x)}+e^{x}-2\right)\right)\mathrm{sgn}\left(e^{(3\,x)}-e^{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*csch(x)^2)^(5/2),x, algorithm="giac")
```

```
[Out] 1/16*a^(5/2)*(4*(3*(e^(-x) + e^x)^3 - 20*e^(-x) - 20*e^x)/((e^(-x) + e^x)^2
 - 4)^2 - 3*log(e^(-x) + e^x + 2) + 3*log(e^(-x) + e^x - 2))*sgn(e^(3*x) -
e^x)
```

maple [B] time = 0.25, size = 123, normalized size = 1.89

$$\frac{a^{2}\sqrt{\frac{a\,\mathrm{e}^{2x}}{\left(\mathrm{e}^{2x}-1\right)^{2}}}\,\left(3\,\mathrm{e}^{6x}-11\,\mathrm{e}^{4x}-11\,\mathrm{e}^{2x}+3\right)}{4\left(\mathrm{e}^{2x}-1\right)^{3}}+\frac{3a^{2}\mathrm{e}^{-x}\left(\mathrm{e}^{2x}-1\right)\sqrt{\frac{a\,\mathrm{e}^{2x}}{\left(\mathrm{e}^{2x}-1\right)^{2}}}\,\ln\left(\mathrm{e}^{x}-1\right)}{8}-\frac{3a^{2}\mathrm{e}^{-x}\left(\mathrm{e}^{2x}-1\right)\sqrt{\frac{a\,\mathrm{e}^{2x}}{\left(\mathrm{e}^{2x}-1\right)^{2}}}\,\ln\left(\right.}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*csch(x)^2)^(5/2),x)
```

```
[Out] 1/4*a^2/(exp(2*x)-1)^3*(a*exp(2*x)/(exp(2*x)-1)^2)^(1/2)*(3*exp(6*x)-11*exp
(4*x)-11*exp(2*x)+3)+3/8*a^2*exp(-x)*(exp(2*x)-1)*(a*exp(2*x)/(exp(2*x)-1)^
2)^(1/2)*ln(exp(x)-1)-3/8*a^2*exp(-x)*(exp(2*x)-1)*(a*exp(2*x)/(exp(2*x)-1)
^2)^(1/2)*ln(exp(x)+1)
```

maxima [A] time = 0.54, size = 92, normalized size = 1.42

$$\frac{3}{8}\,a^{\frac{5}{2}}\log\left(e^{(-x)}+1\right)-\frac{3}{8}\,a^{\frac{5}{2}}\log\left(e^{(-x)}-1\right)+\frac{3\,a^{\frac{5}{2}}e^{(-x)}-11\,a^{\frac{5}{2}}e^{(-3\,x)}-11\,a^{\frac{5}{2}}e^{(-5\,x)}+3\,a^{\frac{5}{2}}e^{(-7\,x)}}{4\left(4\,e^{(-2\,x)}-6\,e^{(-4\,x)}+4\,e^{(-6\,x)}-e^{(-8\,x)}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*csch(x)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 3/8*a^(5/2)*log(e^(-x) + 1) - 3/8*a^(5/2)*log(e^(-x) - 1) + 1/4*(3*a^(5/2)*
e^(-x) - 11*a^(5/2)*e^(-3*x) - 11*a^(5/2)*e^(-5*x) + 3*a^(5/2)*e^(-7*x))/(4
*e^(-2*x) - 6*e^(-4*x) + 4*e^(-6*x) - e^(-8*x) - 1)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int\left(\frac{a}{\sinh(x)^{2}}\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a/sinh(x)^2)^(5/2),x)
```

```
[Out] int((a/sinh(x)^2)^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a \operatorname{csch}^2(x)\right)^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*csch(x)**2)**(5/2),x)
```

```
[Out] Integral((a*csch(x)**2)**(5/2), x)
```

3.30 $\int \left(a\text{csch}^2(x)\right)^{3/2}\,dx$

Optimal. Leaf size=46

$$\frac{1}{2}a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\coth(x)}{\sqrt{a\text{csch}^2(x)}}\right)-\frac{1}{2}a\coth(x)\sqrt{a\text{csch}^2(x)}$$

```
[Out] 1/2*a^(3/2)*arctanh(coth(x)*a^(1/2)/(a*csch(x)^2)^(1/2))-1/2*a*coth(x)*(a*c
sch(x)^2)^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {4122, 195, 217, 206}

$$\frac{1}{2}a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\coth(x)}{\sqrt{a\text{csch}^2(x)}}\right)-\frac{1}{2}a\coth(x)\sqrt{a\text{csch}^2(x)}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Csch[x]^2)^(3/2),x]
```

```
[Out] (a^(3/2)*ArcTanh[(Sqrt[a]*Coth[x])/Sqrt[a*Csch[x]^2]])/2 - (a*Coth[x]*Sqrt[
a*Csch[x]^2])/2
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 4122

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff = FreeFac
tors[Tan[e + f*x], x]}, Dist[(b*ff)/f, Subst[Int[(b + b*ff^2*x^2)^(p - 1),
x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{b, e, f, p}, x] &&  !IntegerQ[p]
```

Rubi steps

$$\int \left(a\text{csch}^2(x)\right)^{3/2} dx = -\left(a \,\text{Subst}\left(\int \sqrt{-a+ax^2}\, dx, x, \coth(x)\right)\right)$$
$$= -\frac{1}{2}a\coth(x)\sqrt{a\text{csch}^2(x)} + \frac{1}{2}a^2\,\text{Subst}\left(\int \frac{1}{\sqrt{-a+ax^2}}\, dx, x, \coth(x)\right)$$
$$= -\frac{1}{2}a\coth(x)\sqrt{a\text{csch}^2(x)} + \frac{1}{2}a^2\,\text{Subst}\left(\int \frac{1}{1-ax^2}\, dx, x, \frac{\coth(x)}{\sqrt{a\text{csch}^2(x)}}\right)$$
$$= \frac{1}{2}a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\coth(x)}{\sqrt{a\text{csch}^2(x)}}\right) - \frac{1}{2}a\coth(x)\sqrt{a\text{csch}^2(x)}$$

Mathematica [A] time = 0.07, size = 30, normalized size = 0.65

$$-\frac{1}{2}a\sinh(x)\sqrt{a\text{csch}^2(x)}\left(\log\left(\tanh\left(\frac{x}{2}\right)\right) + \coth(x)\text{csch}(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Csch[x]^2)^(3/2),x]
```

```
[Out] -1/2*(a*Sqrt[a*Csch[x]^2]*(Coth[x]*Csch[x] + Log[Tanh[x/2]])*Sinh[x])
```

fricas [B] time = 0.57, size = 340, normalized size = 7.39

$$\frac{\left(2\,a\cosh(x)^3 - 2\left(ae^{(2\,x)} - a\right)\sinh(x)^3 - 6\left(a\cosh(x)e^{(2\,x)} - a\cosh(x)\right)\sinh(x)^2 + 2\,a\cosh(x) - 2\left(a\cosh(x)^3\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*csch(x)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/2*(2*a*cosh(x)^3 - 2*(a*e^(2*x) - a)*sinh(x)^3 - 6*(a*cosh(x)*e^(2*x) - a
*cosh(x))*sinh(x)^2 + 2*a*cosh(x) - 2*(a*cosh(x)^3 + a*cosh(x))*e^(2*x) - (
a*cosh(x)^4 - (a*e^(2*x) - a)*sinh(x)^4 - 4*(a*cosh(x)*e^(2*x) - a*cosh(x))
*sinh(x)^3 - 2*a*cosh(x)^2 + 2*(3*a*cosh(x)^2 - (3*a*cosh(x)^2 - a)*e^(2*x)
 - a)*sinh(x)^2 - (a*cosh(x)^4 - 2*a*cosh(x)^2 + a)*e^(2*x) + 4*(a*cosh(x)^
3 - a*cosh(x) - (a*cosh(x)^3 - a*cosh(x))*e^(2*x))*sinh(x) + a)*log((cosh(x
) + sinh(x) + 1)/(cosh(x) + sinh(x) - 1)) + 2*(3*a*cosh(x)^2 - (3*a*cosh(x)
^2 + a)*e^(2*x) + a)*sinh(x))*sqrt(a/(e^(4*x) - 2*e^(2*x) + 1))*e^x/(4*cosh
(x)*e^x*sinh(x)^3 + e^x*sinh(x)^4 + 2*(3*cosh(x)^2 - 1)*e^x*sinh(x)^2 + 4*(
cosh(x)^3 - cosh(x))*e^x*sinh(x) + (cosh(x)^4 - 2*cosh(x)^2 + 1)*e^x)
```

giac [A] time = 0.12, size = 58, normalized size = 1.26

$$-\frac{1}{4}a^{\frac{3}{2}}\left(\frac{4\left(e^{(-x)} + e^x\right)}{\left(e^{(-x)} + e^x\right)^2 - 4} - \log\left(e^{(-x)} + e^x + 2\right) + \log\left(e^{(-x)} + e^x - 2\right)\right)\text{sgn}\left(e^{(3\,x)} - e^x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*csch(x)^2)^(3/2),x, algorithm="giac")
```

```
[Out] -1/4*a^(3/2)*(4*(e^(-x) + e^x)/((e^(-x) + e^x)^2 - 4) - log(e^(-x) + e^x +
2) + log(e^(-x) + e^x - 2))*sgn(e^(3*x) - e^x)
```

maple [B] time = 0.21, size = 103, normalized size = 2.24

$$-\frac{a\sqrt{\frac{a\,\mathrm{e}^{2x}}{\left(\mathrm{e}^{2x}-1\right)^{2}}}\,\left(1+\mathrm{e}^{2x}\right)}{\mathrm{e}^{2x}-1}-\frac{a\,\mathrm{e}^{-x}\left(\mathrm{e}^{2x}-1\right)\sqrt{\frac{a\,\mathrm{e}^{2x}}{\left(\mathrm{e}^{2x}-1\right)^{2}}}\,\ln\left(\mathrm{e}^{x}-1\right)}{2}+\frac{a\,\mathrm{e}^{-x}\left(\mathrm{e}^{2x}-1\right)\sqrt{\frac{a\,\mathrm{e}^{2x}}{\left(\mathrm{e}^{2x}-1\right)^{2}}}\,\ln\left(\mathrm{e}^{x}+1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*csch(x)^2)^(3/2),x)
```

```
[Out] -a/(exp(2*x)-1)*(a*exp(2*x)/(exp(2*x)-1)^2)^(1/2)*(1+exp(2*x))-1/2*a*exp(-x
)*(exp(2*x)-1)*(a*exp(2*x)/(exp(2*x)-1)^2)^(1/2)*ln(exp(x)-1)+1/2*a*exp(-x)
*(exp(2*x)-1)*(a*exp(2*x)/(exp(2*x)-1)^2)^(1/2)*ln(exp(x)+1)
```

maxima [A] time = 0.47, size = 60, normalized size = 1.30

$$-\frac{1}{2}\,a^{\frac{3}{2}}\log\left(e^{(-x)}+1\right)+\frac{1}{2}\,a^{\frac{3}{2}}\log\left(e^{(-x)}-1\right)-\frac{a^{\frac{3}{2}}e^{(-x)}+a^{\frac{3}{2}}e^{(-3\,x)}}{2\,e^{(-2\,x)}-e^{(-4\,x)}-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*csch(x)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/2*a^(3/2)*log(e^(-x) + 1) + 1/2*a^(3/2)*log(e^(-x) - 1) - (a^(3/2)*e^(-x
) + a^(3/2)*e^(-3*x))/(2*e^(-2*x) - e^(-4*x) - 1)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int {\left(\frac{a}{{\mathrm{sinh}\left(x\right)}^2}\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a/sinh(x)^2)^(3/2),x)
```

```
[Out] int((a/sinh(x)^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a \operatorname{csch}^{2}{\left(x \right)}\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*csch(x)**2)**(3/2),x)
```

```
[Out] Integral((a*csch(x)**2)**(3/2), x)
```

3.31 $\int \sqrt{a\text{csch}^2(x)}\,dx$

Optimal. Leaf size=26

$$-\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\coth(x)}{\sqrt{a\text{csch}^2(x)}}\right)$$

```
[Out] -arctanh(coth(x)*a^(1/2)/(a*csch(x)^2)^(1/2))*a^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {4122, 217, 206}

$$-\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\coth(x)}{\sqrt{a\text{csch}^2(x)}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*Csch[x]^2],x]
```

```
[Out] -(Sqrt[a]*ArcTanh[(Sqrt[a]*Coth[x])/Sqrt[a*Csch[x]^2]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 4122

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff = FreeFac
tors[Tan[e + f*x], x]}, Dist[(b*ff)/f, Subst[Int[(b + b*ff^2*x^2)^(p - 1),
x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{b, e, f, p}, x] && !IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\int \sqrt{a\text{csch}^2(x)}\,dx &= -\left(a\,\text{Subst}\left(\int \frac{1}{\sqrt{-a+ax^2}}\,dx, x, \coth(x)\right)\right)\\
&= -\left(a\,\text{Subst}\left(\int \frac{1}{1-ax^2}\,dx, x, \frac{\coth(x)}{\sqrt{a\text{csch}^2(x)}}\right)\right)\\
&= -\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\coth(x)}{\sqrt{a\text{csch}^2(x)}}\right)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 20, normalized size = 0.77

$$\sinh(x)\sqrt{a\text{csch}^2(x)}\,\log\left(\tanh\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*Csch[x]^2],x]
[Out] Sqrt[a*Csch[x]^2]*Log[Tanh[x/2]]*Sinh[x]
```

fricas [B] time = 1.13, size = 97, normalized size = 3.73

$$\left[\sqrt{\frac{a}{e^{(4x)} - 2e^{(2x)} + 1}}\left(e^{(2x)} - 1\right)\log\left(\frac{\cosh(x) + \sinh(x) - 1}{\cosh(x) + \sinh(x) + 1}\right), 2\sqrt{-a}\arctan\left(\frac{\sqrt{-a}\sqrt{\frac{a}{e^{(4x)}-2e^{(2x)}+1}}\left(e^{(2x)} - 1\right)e^{x}}{a\cosh(x)e^{x} + ae^{x}\sinh(x)}\right)\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*csch(x)^2)^(1/2),x, algorithm="fricas")
[Out] [sqrt(a/(e^(4*x) - 2*e^(2*x) + 1))*(e^(2*x) - 1)*log((cosh(x) + sinh(x) - 1
)/(cosh(x) + sinh(x) + 1)), 2*sqrt(-a)*arctan(sqrt(-a)*sqrt(a/(e^(4*x) - 2*
e^(2*x) + 1))*(e^(2*x) - 1)*e^x/(a*cosh(x)*e^x + a*e^x*sinh(x)))]
```

giac [A] time = 0.13, size = 29, normalized size = 1.12

$$-\sqrt{a}\left(\log\left(e^{x} + 1\right) - \log\left(|e^{x} - 1|\right)\right)\operatorname{sgn}\left(e^{(3x)} - e^{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*csch(x)^2)^(1/2),x, algorithm="giac")
[Out] -sqrt(a)*(log(e^x + 1) - log(abs(e^x - 1)))*sgn(e^(3*x) - e^x)
```

maple [B] time = 0.23, size = 67, normalized size = 2.58

$$\sqrt{\frac{a\,e^{2x}}{\left(e^{2x} - 1\right)^{2}}}\,e^{-x}\left(e^{2x} - 1\right)\ln\left(e^{x} - 1\right) - \sqrt{\frac{a\,e^{2x}}{\left(e^{2x} - 1\right)^{2}}}\,e^{-x}\left(e^{2x} - 1\right)\ln\left(e^{x} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*csch(x)^2)^(1/2),x)
[Out] (a*exp(2*x)/(exp(2*x)-1)^2)^(1/2)*exp(-x)*(exp(2*x)-1)*ln(exp(x)-1)-(a*exp(
2*x)/(exp(2*x)-1)^2)^(1/2)*exp(-x)*(exp(2*x)-1)*ln(exp(x)+1)
```

maxima [A] time = 0.45, size = 24, normalized size = 0.92

$$\sqrt{a}\log\left(e^{(-x)} + 1\right) - \sqrt{a}\log\left(e^{(-x)} - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*csch(x)^2)^(1/2),x, algorithm="maxima")
[Out] sqrt(a)*log(e^(-x) + 1) - sqrt(a)*log(e^(-x) - 1)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.04

$$\int \sqrt{\frac{a}{\sinh(x)^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a/sinh(x)^2)^(1/2),x)
[Out] int((a/sinh(x)^2)^(1/2), x)
```

<u>**sympy**</u> **[F]**   time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a \operatorname{csch}^2(x)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*csch(x)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(a*csch(x)**2), x)
```

3.32 $\int \frac{1}{\sqrt{a\mathbf{csch}^2(x)}}\,dx$

Optimal. Leaf size=13

$$\frac{\coth(x)}{\sqrt{a\operatorname{csch}^2(x)}}$$

```
[Out] coth(x)/(a*csch(x)^2)^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {4122, 191}

$$\frac{\coth(x)}{\sqrt{a\operatorname{csch}^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a*Csch[x]^2],x]
```

```
[Out] Coth[x]/Sqrt[a*Csch[x]^2]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 4122

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff = FreeFac
tors[Tan[e + f*x], x]}, Dist[(b*ff)/f, Subst[Int[(b + b*ff^2*x^2)^(p - 1),
x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{b, e, f, p}, x] && !IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{a\operatorname{csch}^2(x)}}\,dx &= -\left(a\,\text{Subst}\left(\int \frac{1}{\left(-a+ax^2\right)^{3/2}}\,dx, x, \coth(x)\right)\right)\\ &= \frac{\coth(x)}{\sqrt{a\operatorname{csch}^2(x)}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 13, normalized size = 1.00

$$\frac{\coth(x)}{\sqrt{a\operatorname{csch}^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a*Csch[x]^2],x]
```

```
[Out] Coth[x]/Sqrt[a*Csch[x]^2]
```

fricas [B] time = 2.48, size = 83, normalized size = 6.38

$$\frac{\left(\left(e^{(2x)}-1\right)\sinh(x)^2-\cosh(x)^2+\left(\cosh(x)^2+1\right)e^{(2x)}+2\left(\cosh(x)e^{(2x)}-\cosh(x)\right)\sinh(x)-1\right)\sqrt{\frac{a}{e^{(4x)}-2e^{(2x)}+1}}\,e^x}{2\left(a\cosh(x)e^x+ae^x\sinh(x)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*csch(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*((e^(2*x) - 1)*sinh(x)^2 - cosh(x)^2 + (cosh(x)^2 + 1)*e^(2*x) + 2*(cos
h(x)*e^(2*x) - cosh(x))*sinh(x) - 1)*sqrt(a/(e^(4*x) - 2*e^(2*x) + 1))*e^x/
(a*cosh(x)*e^x + a*e^x*sinh(x))
```

giac [B] time = 0.13, size = 24, normalized size = 1.85

$$\frac{e^{(-x)} + e^{x}}{2\,\sqrt{a}\,\mathrm{sgn}\left(e^{(3\,x)} - e^{x}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*csch(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*(e^(-x) + e^x)/(sqrt(a)*sgn(e^(3*x) - e^x))
```

maple [B] time = 0.21, size = 58, normalized size = 4.46

$$\frac{\mathrm{e}^{2x}}{2\sqrt{\frac{a\,\mathrm{e}^{2x}}{\left(\mathrm{e}^{2x}-1\right)^{2}}}\,\left(\mathrm{e}^{2x}-1\right)}+\frac{1}{2\left(\mathrm{e}^{2x}-1\right)\sqrt{\frac{a\,\mathrm{e}^{2x}}{\left(\mathrm{e}^{2x}-1\right)^{2}}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*csch(x)^2)^(1/2),x)
```

```
[Out] 1/2/(a*exp(2*x)/(exp(2*x)-1)^2)^(1/2)/(exp(2*x)-1)*exp(2*x)+1/2/(exp(2*x)-1
)/(a*exp(2*x)/(exp(2*x)-1)^2)^(1/2)
```

maxima [A] time = 0.45, size = 17, normalized size = 1.31

$$-\frac{e^{(-x)}}{2\,\sqrt{a}} - \frac{e^{x}}{2\,\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*csch(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/2*e^(-x)/sqrt(a) - 1/2*e^x/sqrt(a)
```

mupad [B] time = 1.60, size = 33, normalized size = 2.54

$$-\frac{\left(\frac{\mathrm{e}^{-2\,x}}{2}-\frac{\mathrm{e}^{2\,x}}{2}\right)\,\sqrt{\frac{1}{{\left(\frac{\mathrm{e}^{-x}}{2}-\frac{\mathrm{e}^{x}}{2}\right)}^2}}}{2\,\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a/sinh(x)^2)^(1/2),x)
```

```
[Out] -((exp(-2*x)/2 - exp(2*x)/2)*(1/(exp(-x)/2 - exp(x)/2)^2)^(1/2))/(2*a^(1/2)
)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a \operatorname{csch}^{2}{\left(x \right)}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*csch(x)**2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(a*csch(x)**2), x)
```

3.33 $\int \frac{1}{\left(a\operatorname{csch}^2(x)\right)^{3/2}}\,dx$

Optimal. Leaf size=36

$$\frac{\coth(x)}{3\left(a\operatorname{csch}^2(x)\right)^{3/2}}-\frac{2\coth(x)}{3a\sqrt{a\operatorname{csch}^2(x)}}$$

```
[Out] 1/3*coth(x)/(a*csch(x)^2)^(3/2)-2/3*coth(x)/a/(a*csch(x)^2)^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {4122, 192, 191}

$$\frac{\coth(x)}{3\left(a\operatorname{csch}^2(x)\right)^{3/2}}-\frac{2\coth(x)}{3a\sqrt{a\operatorname{csch}^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Csch[x]^2)^(-3/2),x]
```

```
[Out] Coth[x]/(3*(a*Csch[x]^2)^(3/2)) - (2*Coth[x])/(3*a*Sqrt[a*Csch[x]^2])
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 4122

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff = FreeFac
tors[Tan[e + f*x], x]}, Dist[(b*ff)/f, Subst[Int[(b + b*ff^2*x^2)^(p - 1),
x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{b, e, f, p}, x] && !IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\left(a\operatorname{csch}^2(x)\right)^{3/2}}\,dx &= -\left(a\operatorname{Subst}\left(\int \frac{1}{\left(-a+ax^2\right)^{5/2}}\,dx,x,\coth(x)\right)\right)\\
&= \frac{\coth(x)}{3\left(a\operatorname{csch}^2(x)\right)^{3/2}}+\frac{2}{3}\operatorname{Subst}\left(\int \frac{1}{\left(-a+ax^2\right)^{3/2}}\,dx,x,\coth(x)\right)\\
&= \frac{\coth(x)}{3\left(a\operatorname{csch}^2(x)\right)^{3/2}}-\frac{2\coth(x)}{3a\sqrt{a\operatorname{csch}^2(x)}}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 27, normalized size = 0.75

$$\frac{(\cosh(3x)-9\cosh(x))\operatorname{csch}^3(x)}{12\left(a\operatorname{csch}^2(x)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Csch[x]^2)^(-3/2),x]
```

```
[Out] ((-9*Cosh[x] + Cosh[3*x])*Csch[x]^3)/(12*(a*Csch[x]^2)^(3/2))
```

fricas [B] time = 1.31, size = 285, normalized size = 7.92

$$\frac{\left(\left(e^{(2x)}-1\right)\sinh(x)^6-\cosh(x)^6+6\left(\cosh(x)e^{(2x)}-\cosh(x)\right)\sinh(x)^5-3\left(5\cosh(x)^2-\left(5\cosh(x)^2-3\right)e^{(2x)}-\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*csch(x)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/24*((e^(2*x) - 1)*sinh(x)^6 - cosh(x)^6 + 6*(cosh(x)*e^(2*x) - cosh(x))*s
inh(x)^5 - 3*(5*cosh(x)^2 - (5*cosh(x)^2 - 3)*e^(2*x) - 3)*sinh(x)^4 + 9*co
sh(x)^4 - 4*(5*cosh(x)^3 - (5*cosh(x)^3 - 9*cosh(x))*e^(2*x) - 9*cosh(x))*s
inh(x)^3 - 3*(5*cosh(x)^4 - 18*cosh(x)^2 - (5*cosh(x)^4 - 18*cosh(x)^2 - 3)
*e^(2*x) - 3)*sinh(x)^2 + 9*cosh(x)^2 + (cosh(x)^6 - 9*cosh(x)^4 - 9*cosh(x
)^2 + 1)*e^(2*x) - 6*(cosh(x)^5 - 6*cosh(x)^3 - (cosh(x)^5 - 6*cosh(x)^3 -
3*cosh(x))*e^(2*x) - 3*cosh(x))*sinh(x) - 1)*sqrt(a/(e^(4*x) - 2*e^(2*x) +
1))*e^x/(a^2*cosh(x)^3*e^x + 3*a^2*cosh(x)^2*e^x*sinh(x) + 3*a^2*cosh(x)*e^
x*sinh(x)^2 + a^2*e^x*sinh(x)^3)
```

giac [A] time = 0.15, size = 54, normalized size = 1.50

$$-\frac{\frac{\left(9\,e^{(2x)}-1\right)e^{(-3x)}}{\mathrm{sgn}\left(e^{(3x)}-e^{x}\right)}-\frac{e^{(3x)}-9\,e^{x}}{\mathrm{sgn}\left(e^{(3x)}-e^{x}\right)}}{24\,a^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*csch(x)^2)^(3/2),x, algorithm="giac")
```

```
[Out] -1/24*((9*e^(2*x) - 1)*e^(-3*x)/sgn(e^(3*x) - e^x) - (e^(3*x) - 9*e^x)/sgn(
e^(3*x) - e^x))/a^(3/2)
```

maple [B] time = 0.19, size = 130, normalized size = 3.61

$$\frac{e^{4x}}{24a\left(e^{2x}-1\right)\sqrt{\frac{a\,e^{2x}}{\left(e^{2x}-1\right)^{2}}}}-\frac{3\,e^{2x}}{8a\left(e^{2x}-1\right)\sqrt{\frac{a\,e^{2x}}{\left(e^{2x}-1\right)^{2}}}}-\frac{3}{8\sqrt{\frac{a\,e^{2x}}{\left(e^{2x}-1\right)^{2}}}\,\left(e^{2x}-1\right)a}+\frac{e^{-2x}}{24a\left(e^{2x}-1\right)\sqrt{\frac{a\,e^{2x}}{\left(e^{2x}-1\right)^{2}}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*csch(x)^2)^(3/2),x)
```

```
[Out] 1/24/a*exp(4*x)/(exp(2*x)-1)/(a*exp(2*x)/(exp(2*x)-1)^2)^(1/2)-3/8/a*exp(2*
x)/(exp(2*x)-1)/(a*exp(2*x)/(exp(2*x)-1)^2)^(1/2)-3/8/(a*exp(2*x)/(exp(2*x)
-1)^2)^(1/2)/(exp(2*x)-1)/a+1/24/a*exp(-2*x)/(exp(2*x)-1)/(a*exp(2*x)/(exp(
2*x)-1)^2)^(1/2)
```

maxima [A] time = 0.69, size = 35, normalized size = 0.97

$$-\frac{e^{(3x)}}{24\,a^{\frac{3}{2}}}+\frac{3\,e^{(-x)}}{8\,a^{\frac{3}{2}}}-\frac{e^{(-3x)}}{24\,a^{\frac{3}{2}}}+\frac{3\,e^{x}}{8\,a^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*csch(x)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/24*e^(3*x)/a^(3/2) + 3/8*e^(-x)/a^(3/2) - 1/24*e^(-3*x)/a^(3/2) + 3/8*e^
x/a^(3/2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{\left(\frac{a}{\sinh(x)^2}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a/sinh(x)^2)^(3/2),x)
```

```
[Out] int(1/(a/sinh(x)^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a \operatorname{csch}^2(x)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*csch(x)**2)**(3/2),x)
```

```
[Out] Integral((a*csch(x)**2)**(-3/2), x)
```

3.34 $\int \frac{1}{\left(a\,\mathbf{csch}^2(x)\right)^{5/2}}\,dx$

Optimal. Leaf size=55

$$\frac{8\coth(x)}{15a^2\sqrt{a\operatorname{csch}^2(x)}}-\frac{4\coth(x)}{15a\left(a\operatorname{csch}^2(x)\right)^{3/2}}+\frac{\coth(x)}{5\left(a\operatorname{csch}^2(x)\right)^{5/2}}$$

```
[Out] 1/5*coth(x)/(a*csch(x)^2)^(5/2)-4/15*coth(x)/a/(a*csch(x)^2)^(3/2)+8/15*cot
h(x)/a^2/(a*csch(x)^2)^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {4122, 192, 191}

$$\frac{8\coth(x)}{15a^2\sqrt{a\operatorname{csch}^2(x)}}-\frac{4\coth(x)}{15a\left(a\operatorname{csch}^2(x)\right)^{3/2}}+\frac{\coth(x)}{5\left(a\operatorname{csch}^2(x)\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Csch[x]^2)^(-5/2),x]
```

```
[Out] Coth[x]/(5*(a*Csch[x]^2)^(5/2)) - (4*Coth[x])/(15*a*(a*Csch[x]^2)^(3/2)) +
(8*Coth[x])/(15*a^2*Sqrt[a*Csch[x]^2])
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 4122

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff = FreeFac
tors[Tan[e + f*x], x]}, Dist[(b*ff)/f, Subst[Int[(b + b*ff^2*x^2)^(p - 1),
x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{b, e, f, p}, x] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{1}{\left(a\operatorname{csch}^2(x)\right)^{5/2}}\,dx = -\left(a\,\text{Subst}\left(\int \frac{1}{\left(-a+ax^2\right)^{7/2}}\,dx, x, \coth(x)\right)\right)$$
$$= \frac{\coth(x)}{5\left(a\operatorname{csch}^2(x)\right)^{5/2}} + \frac{4}{5}\,\text{Subst}\left(\int \frac{1}{\left(-a+ax^2\right)^{5/2}}\,dx, x, \coth(x)\right)$$
$$= \frac{\coth(x)}{5\left(a\operatorname{csch}^2(x)\right)^{5/2}} - \frac{4\coth(x)}{15a\left(a\operatorname{csch}^2(x)\right)^{3/2}} - \frac{8\,\text{Subst}\left(\int \frac{1}{\left(-a+ax^2\right)^{3/2}}\,dx, x, \coth(x)\right)}{15a}$$
$$= \frac{\coth(x)}{5\left(a\operatorname{csch}^2(x)\right)^{5/2}} - \frac{4\coth(x)}{15a\left(a\operatorname{csch}^2(x)\right)^{3/2}} + \frac{8\coth(x)}{15a^2\sqrt{a\operatorname{csch}^2(x)}}$$

Mathematica [A] time = 0.03, size = 36, normalized size = 0.65

$$\frac{\sinh(x)(150\cosh(x) - 25\cosh(3x) + 3\cosh(5x))\sqrt{a\operatorname{csch}^2(x)}}{240a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Csch[x]^2)^(-5/2),x]
```

```
[Out] ((150*Cosh[x] - 25*Cosh[3*x] + 3*Cosh[5*x])*Sqrt[a*Csch[x]^2]*Sinh[x])/(240
*a^3)
```

fricas [B] time = 0.60, size = 590, normalized size = 10.73

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*csch(x)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/480*(3*(e^(2*x) - 1)*sinh(x)^10 - 3*cosh(x)^10 + 30*(cosh(x)*e^(2*x) - co
sh(x))*sinh(x)^9 - 5*(27*cosh(x)^2 - (27*cosh(x)^2 - 5)*e^(2*x) - 5)*sinh(x
)^8 + 25*cosh(x)^8 - 40*(9*cosh(x)^3 - (9*cosh(x)^3 - 5*cosh(x))*e^(2*x) -
5*cosh(x))*sinh(x)^7 - 10*(63*cosh(x)^4 - 70*cosh(x)^2 - (63*cosh(x)^4 - 70
*cosh(x)^2 + 15)*e^(2*x) + 15)*sinh(x)^6 - 150*cosh(x)^6 - 4*(189*cosh(x)^5
 - 350*cosh(x)^3 - (189*cosh(x)^5 - 350*cosh(x)^3 + 225*cosh(x))*e^(2*x) +
225*cosh(x))*sinh(x)^5 - 10*(63*cosh(x)^6 - 175*cosh(x)^4 + 225*cosh(x)^2 -
 (63*cosh(x)^6 - 175*cosh(x)^4 + 225*cosh(x)^2 + 15)*e^(2*x) + 15)*sinh(x)^
4 - 150*cosh(x)^4 - 40*(9*cosh(x)^7 - 35*cosh(x)^5 + 75*cosh(x)^3 - (9*cosh
(x)^7 - 35*cosh(x)^5 + 75*cosh(x)^3 + 15*cosh(x))*e^(2*x) + 15*cosh(x))*sin
h(x)^3 - 5*(27*cosh(x)^8 - 140*cosh(x)^6 + 450*cosh(x)^4 + 180*cosh(x)^2 -
(27*cosh(x)^8 - 140*cosh(x)^6 + 450*cosh(x)^4 + 180*cosh(x)^2 - 5)*e^(2*x)
- 5)*sinh(x)^2 + 25*cosh(x)^2 + (3*cosh(x)^10 - 25*cosh(x)^8 + 150*cosh(x)^
6 + 150*cosh(x)^4 - 25*cosh(x)^2 + 3)*e^(2*x) - 10*(3*cosh(x)^9 - 20*cosh(x
)^7 + 90*cosh(x)^5 + 60*cosh(x)^3 - (3*cosh(x)^9 - 20*cosh(x)^7 + 90*cosh(x
)^5 + 60*cosh(x)^3 - 5*cosh(x))*e^(2*x) - 5*cosh(x))*sinh(x) - 3)*sqrt(a/(e
^(4*x) - 2*e^(2*x) + 1))*e^x/(a^3*cosh(x)^5*e^x + 5*a^3*cosh(x)^4*e^x*sinh(
x) + 10*a^3*cosh(x)^3*e^x*sinh(x)^2 + 10*a^3*cosh(x)^2*e^x*sinh(x)^3 + 5*a^
3*cosh(x)*e^x*sinh(x)^4 + a^3*e^x*sinh(x)^5)
```

giac [A] time = 0.16, size = 67, normalized size = 1.22

$$\frac{\frac{\left(150\,e^{(4\,x)}-25\,e^{(2\,x)}+3\right)e^{(-5\,x)}}{\operatorname{sgn}\left(e^{(3\,x)}-e^{x}\right)} + \frac{3\,e^{(5\,x)}-25\,e^{(3\,x)}+150\,e^{x}}{\operatorname{sgn}\left(e^{(3\,x)}-e^{x}\right)}}{480\,a^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*csch(x)^2)^(5/2),x, algorithm="giac")
```

```
[Out] 1/480*((150*e^(4*x) - 25*e^(2*x) + 3)*e^(-5*x)/sgn(e^(3*x) - e^x) + (3*e^(5
*x) - 25*e^(3*x) + 150*e^x)/sgn(e^(3*x) - e^x))/a^(5/2)
```

maple [B] time = 0.20, size = 196, normalized size = 3.56

$$\frac{e^{6x}}{160a^2\left(e^{2x}-1\right)\sqrt{\frac{a\,e^{2x}}{\left(e^{2x}-1\right)^2}}}-\frac{5\,e^{4x}}{96a^2\left(e^{2x}-1\right)\sqrt{\frac{a\,e^{2x}}{\left(e^{2x}-1\right)^2}}}+\frac{5\,e^{2x}}{16a^2\left(e^{2x}-1\right)\sqrt{\frac{a\,e^{2x}}{\left(e^{2x}-1\right)^2}}}+\frac{5}{16\sqrt{\frac{a\,e^{2x}}{\left(e^{2x}-1\right)^2}}\left(e^{2x}-1\right)a^2}-\frac{}{96a^2 (}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*csch(x)^2)^(5/2),x)
```

```
[Out] 1/160/a^2*exp(6*x)/(exp(2*x)-1)/(a*exp(2*x)/(exp(2*x)-1)^2)^(1/2)-5/96/a^2*
exp(4*x)/(exp(2*x)-1)/(a*exp(2*x)/(exp(2*x)-1)^2)^(1/2)+5/16/a^2*exp(2*x)/(
exp(2*x)-1)/(a*exp(2*x)/(exp(2*x)-1)^2)^(1/2)+5/16/(a*exp(2*x)/(exp(2*x)-1)
^2)^(1/2)/(exp(2*x)-1)/a^2-5/96/a^2*exp(-2*x)/(exp(2*x)-1)/(a*exp(2*x)/(exp
(2*x)-1)^2)^(1/2)+1/160/a^2*exp(-4*x)/(exp(2*x)-1)/(a*exp(2*x)/(exp(2*x)-1)
^2)^(1/2)
```

maxima [A] time = 0.53, size = 53, normalized size = 0.96

$$-\frac{e^{(5\,x)}}{160\,a^{\frac{5}{2}}}+\frac{5\,e^{(3\,x)}}{96\,a^{\frac{5}{2}}}-\frac{5\,e^{(-x)}}{16\,a^{\frac{5}{2}}}+\frac{5\,e^{(-3\,x)}}{96\,a^{\frac{5}{2}}}-\frac{e^{(-5\,x)}}{160\,a^{\frac{5}{2}}}-\frac{5\,e^{x}}{16\,a^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*csch(x)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/160*e^(5*x)/a^(5/2) + 5/96*e^(3*x)/a^(5/2) - 5/16*e^(-x)/a^(5/2) + 5/96*
e^(-3*x)/a^(5/2) - 1/160*e^(-5*x)/a^(5/2) - 5/16*e^x/a^(5/2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{{\left(\frac{a}{{\mathrm{sinh}\left(x\right)}^2}\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a/sinh(x)^2)^(5/2),x)
```

```
[Out] int(1/(a/sinh(x)^2)^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a \operatorname{csch}^{2}{\left(x \right)}\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*csch(x)**2)**(5/2),x)
```

```
[Out] Integral((a*csch(x)**2)**(-5/2), x)
```

3.35 $\int \frac{1}{\left(a\text{csch}^2(x)\right)^{7/2}}\,dx$

Optimal. Leaf size=74

$$-\frac{16\coth(x)}{35a^3\sqrt{a\text{csch}^2(x)}}+\frac{8\coth(x)}{35a^2\left(a\text{csch}^2(x)\right)^{3/2}}-\frac{6\coth(x)}{35a\left(a\text{csch}^2(x)\right)^{5/2}}+\frac{\coth(x)}{7\left(a\text{csch}^2(x)\right)^{7/2}}$$

```
[Out] 1/7*coth(x)/(a*csch(x)^2)^(7/2)-6/35*coth(x)/a/(a*csch(x)^2)^(5/2)+8/35*cot
h(x)/a^2/(a*csch(x)^2)^(3/2)-16/35*coth(x)/a^3/(a*csch(x)^2)^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {4122, 192, 191}

$$-\frac{16\coth(x)}{35a^3\sqrt{a\text{csch}^2(x)}}+\frac{8\coth(x)}{35a^2\left(a\text{csch}^2(x)\right)^{3/2}}-\frac{6\coth(x)}{35a\left(a\text{csch}^2(x)\right)^{5/2}}+\frac{\coth(x)}{7\left(a\text{csch}^2(x)\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Csch[x]^2)^(-7/2),x]
```

```
[Out] Coth[x]/(7*(a*Csch[x]^2)^(7/2)) - (6*Coth[x])/(35*a*(a*Csch[x]^2)^(5/2)) +
(8*Coth[x])/(35*a^2*(a*Csch[x]^2)^(3/2)) - (16*Coth[x])/(35*a^3*Sqrt[a*Csch
[x]^2])
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 4122

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff = FreeFac
tors[Tan[e + f*x], x]}, Dist[(b*ff)/f, Subst[Int[(b + b*ff^2*x^2)^(p - 1),
x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{b, e, f, p}, x] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{1}{\left(a\operatorname{csch}^2(x)\right)^{7/2}}\,dx = -\left(a\,\text{Subst}\left(\int \frac{1}{\left(-a+ax^2\right)^{9/2}}\,dx, x, \coth(x)\right)\right)$$
$$= \frac{\coth(x)}{7\left(a\operatorname{csch}^2(x)\right)^{7/2}} + \frac{6}{7}\text{Subst}\left(\int \frac{1}{\left(-a+ax^2\right)^{7/2}}\,dx, x, \coth(x)\right)$$
$$= \frac{\coth(x)}{7\left(a\operatorname{csch}^2(x)\right)^{7/2}} - \frac{6\coth(x)}{35a\left(a\operatorname{csch}^2(x)\right)^{5/2}} - \frac{24\,\text{Subst}\left(\int \frac{1}{\left(-a+ax^2\right)^{5/2}}\,dx, x, \coth(x)\right)}{35a}$$
$$= \frac{\coth(x)}{7\left(a\operatorname{csch}^2(x)\right)^{7/2}} - \frac{6\coth(x)}{35a\left(a\operatorname{csch}^2(x)\right)^{5/2}} + \frac{8\coth(x)}{35a^2\left(a\operatorname{csch}^2(x)\right)^{3/2}} + \frac{16\,\text{Subst}\left(\int \frac{1}{\left(-a+ax^2\right)^{3/2}}\,dx,\right.}{35a^2}$$
$$= \frac{\coth(x)}{7\left(a\operatorname{csch}^2(x)\right)^{7/2}} - \frac{6\coth(x)}{35a\left(a\operatorname{csch}^2(x)\right)^{5/2}} + \frac{8\coth(x)}{35a^2\left(a\operatorname{csch}^2(x)\right)^{3/2}} - \frac{16\coth(x)}{35a^3\sqrt{a\operatorname{csch}^2(x)}}$$

Mathematica [**A**] time = 0.06, size = 42, normalized size = 0.57

$$\frac{\sinh(x)(-1225\cosh(x) + 245\cosh(3x) - 49\cosh(5x) + 5\cosh(7x))\sqrt{a\operatorname{csch}^2(x)}}{2240a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Csch[x]^2)^(-7/2),x]
```

```
[Out] ((-1225*Cosh[x] + 245*Cosh[3*x] - 49*Cosh[5*x] + 5*Cosh[7*x])*Sqrt[a*Csch[x
]^2]*Sinh[x])/(2240*a^4)
```

fricas [**B**] time = 1.29, size = 984, normalized size = 13.30

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*csch(x)^2)^(7/2),x, algorithm="fricas")
```

```
[Out] 1/4480*(5*(e^(2*x) - 1)*sinh(x)^14 - 5*cosh(x)^14 + 70*(cosh(x)*e^(2*x) - c
osh(x))*sinh(x)^13 - 7*(65*cosh(x)^2 - (65*cosh(x)^2 - 7)*e^(2*x) - 7)*sinh
(x)^12 + 49*cosh(x)^12 - 28*(65*cosh(x)^3 - (65*cosh(x)^3 - 21*cosh(x))*e^(
2*x) - 21*cosh(x))*sinh(x)^11 - 7*(715*cosh(x)^4 - 462*cosh(x)^2 - (715*cos
h(x)^4 - 462*cosh(x)^2 + 35)*e^(2*x) + 35)*sinh(x)^10 - 245*cosh(x)^10 - 70
*(143*cosh(x)^5 - 154*cosh(x)^3 - (143*cosh(x)^5 - 154*cosh(x)^3 + 35*cosh(
x))*e^(2*x) + 35*cosh(x))*sinh(x)^9 - 35*(429*cosh(x)^6 - 693*cosh(x)^4 + 3
15*cosh(x)^2 - (429*cosh(x)^6 - 693*cosh(x)^4 + 315*cosh(x)^2 - 35)*e^(2*x)
 - 35)*sinh(x)^8 + 1225*cosh(x)^8 - 8*(2145*cosh(x)^7 - 4851*cosh(x)^5 + 36
75*cosh(x)^3 - (2145*cosh(x)^7 - 4851*cosh(x)^5 + 3675*cosh(x)^3 - 1225*cos
h(x))*e^(2*x) - 1225*cosh(x))*sinh(x)^7 - 7*(2145*cosh(x)^8 - 6468*cosh(x)^
6 + 7350*cosh(x)^4 - 4900*cosh(x)^2 - (2145*cosh(x)^8 - 6468*cosh(x)^6 + 73
50*cosh(x)^4 - 4900*cosh(x)^2 - 175)*e^(2*x) - 175)*sinh(x)^6 + 1225*cosh(x
)^6 - 14*(715*cosh(x)^9 - 2772*cosh(x)^7 + 4410*cosh(x)^5 - 4900*cosh(x)^3
- (715*cosh(x)^9 - 2772*cosh(x)^7 + 4410*cosh(x)^5 - 4900*cosh(x)^3 - 525*c
osh(x))*e^(2*x) - 525*cosh(x))*sinh(x)^5 - 35*(143*cosh(x)^10 - 693*cosh(x)
^8 + 1470*cosh(x)^6 - 2450*cosh(x)^4 - 525*cosh(x)^2 - (143*cosh(x)^10 - 69
3*cosh(x)^8 + 1470*cosh(x)^6 - 2450*cosh(x)^4 - 525*cosh(x)^2 + 7)*e^(2*x)
```

```
+ 7)*sinh(x)^4 - 245*cosh(x)^4 - 140*(13*cosh(x)^11 - 77*cosh(x)^9 + 210*co
sh(x)^7 - 490*cosh(x)^5 - 175*cosh(x)^3 - (13*cosh(x)^11 - 77*cosh(x)^9 + 2
10*cosh(x)^7 - 490*cosh(x)^5 - 175*cosh(x)^3 + 7*cosh(x))*e^(2*x) + 7*cosh(
x))*sinh(x)^3 - 7*(65*cosh(x)^12 - 462*cosh(x)^10 + 1575*cosh(x)^8 - 4900*c
osh(x)^6 - 2625*cosh(x)^4 + 210*cosh(x)^2 - (65*cosh(x)^12 - 462*cosh(x)^10
 + 1575*cosh(x)^8 - 4900*cosh(x)^6 - 2625*cosh(x)^4 + 210*cosh(x)^2 - 7)*e^
(2*x) - 7)*sinh(x)^2 + 49*cosh(x)^2 + (5*cosh(x)^14 - 49*cosh(x)^12 + 245*c
osh(x)^10 - 1225*cosh(x)^8 - 1225*cosh(x)^6 + 245*cosh(x)^4 - 49*cosh(x)^2
+ 5)*e^(2*x) - 14*(5*cosh(x)^13 - 42*cosh(x)^11 + 175*cosh(x)^9 - 700*cosh(
x)^7 - 525*cosh(x)^5 + 70*cosh(x)^3 - (5*cosh(x)^13 - 42*cosh(x)^11 + 175*c
osh(x)^9 - 700*cosh(x)^7 - 525*cosh(x)^5 + 70*cosh(x)^3 - 7*cosh(x))*e^(2*x
) - 7*cosh(x))*sinh(x) - 5)*sqrt(a/(e^(4*x) - 2*e^(2*x) + 1))*e^x/(a^4*cosh
(x)^7*e^x + 7*a^4*cosh(x)^6*e^x*sinh(x) + 21*a^4*cosh(x)^5*e^x*sinh(x)^2 +
35*a^4*cosh(x)^4*e^x*sinh(x)^3 + 35*a^4*cosh(x)^3*e^x*sinh(x)^4 + 21*a^4*co
sh(x)^2*e^x*sinh(x)^5 + 7*a^4*cosh(x)*e^x*sinh(x)^6 + a^4*e^x*sinh(x)^7)
```

giac [A] time = 0.17, size = 80, normalized size = 1.08

$$-\frac{\frac{\left(1225\,e^{(6\,x)}-245\,e^{(4\,x)}+49\,e^{(2\,x)}-5\right)e^{(-7\,x)}}{\mathrm{sgn}\left(e^{(3\,x)}-e^{x}\right)}-\frac{5\,e^{(7\,x)}-49\,e^{(5\,x)}+245\,e^{(3\,x)}-1225\,e^{x}}{\mathrm{sgn}\left(e^{(3\,x)}-e^{x}\right)}}{4480\,a^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*csch(x)^2)^(7/2),x, algorithm="giac")
```

```
[Out] -1/4480*((1225*e^(6*x) - 245*e^(4*x) + 49*e^(2*x) - 5)*e^(-7*x)/sgn(e^(3*x)
 - e^x) - (5*e^(7*x) - 49*e^(5*x) + 245*e^(3*x) - 1225*e^x)/sgn(e^(3*x) - e
^x))/a^(7/2)
```

maple [B] time = 0.21, size = 262, normalized size = 3.54

$$\frac{e^{8x}}{896a^{3}\left(e^{2x}-1\right)\sqrt{\frac{a\,e^{2x}}{\left(e^{2x}-1\right)^{2}}}}-\frac{7\,e^{6x}}{640a^{3}\left(e^{2x}-1\right)\sqrt{\frac{a\,e^{2x}}{\left(e^{2x}-1\right)^{2}}}}+\frac{7\,e^{4x}}{128a^{3}\left(e^{2x}-1\right)\sqrt{\frac{a\,e^{2x}}{\left(e^{2x}-1\right)^{2}}}}-\frac{35\,e^{2x}}{128a^{3}\left(e^{2x}-1\right)\sqrt{\frac{a\,e^{2x}}{\left(e^{2x}-1\right)^{2}}}}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*csch(x)^2)^(7/2),x)
```

```
[Out] 1/896/a^3*exp(8*x)/(exp(2*x)-1)/(a*exp(2*x)/(exp(2*x)-1)^2)^(1/2)-7/640/a^3
*exp(6*x)/(exp(2*x)-1)/(a*exp(2*x)/(exp(2*x)-1)^2)^(1/2)+7/128/a^3*exp(4*x)
/(exp(2*x)-1)/(a*exp(2*x)/(exp(2*x)-1)^2)^(1/2)-35/128/a^3*exp(2*x)/(exp(2*
x)-1)/(a*exp(2*x)/(exp(2*x)-1)^2)^(1/2)-35/128/(a*exp(2*x)/(exp(2*x)-1)^2)^
(1/2)/(exp(2*x)-1)/a^3+7/128/a^3*exp(-2*x)/(exp(2*x)-1)/(a*exp(2*x)/(exp(2*
x)-1)^2)^(1/2)-7/640/a^3*exp(-4*x)/(exp(2*x)-1)/(a*exp(2*x)/(exp(2*x)-1)^2)
^(1/2)+1/896/a^3*exp(-6*x)/(exp(2*x)-1)/(a*exp(2*x)/(exp(2*x)-1)^2)^(1/2)
```

maxima [A] time = 0.50, size = 71, normalized size = 0.96

$$-\frac{e^{(7\,x)}}{896\,a^{\frac{7}{2}}}+\frac{7\,e^{(5\,x)}}{640\,a^{\frac{7}{2}}}-\frac{7\,e^{(3\,x)}}{128\,a^{\frac{7}{2}}}+\frac{35\,e^{(-x)}}{128\,a^{\frac{7}{2}}}-\frac{7\,e^{(-3\,x)}}{128\,a^{\frac{7}{2}}}+\frac{7\,e^{(-5\,x)}}{640\,a^{\frac{7}{2}}}-\frac{e^{(-7\,x)}}{896\,a^{\frac{7}{2}}}+\frac{35\,e^{x}}{128\,a^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*csch(x)^2)^(7/2),x, algorithm="maxima")
```

```
[Out] -1/896*e^(7*x)/a^(7/2) + 7/640*e^(5*x)/a^(7/2) - 7/128*e^(3*x)/a^(7/2) + 35
/128*e^(-x)/a^(7/2) - 7/128*e^(-3*x)/a^(7/2) + 7/640*e^(-5*x)/a^(7/2) - 1/8
96*e^(-7*x)/a^(7/2) + 35/128*e^x/a^(7/2)
```

mupad [F]   time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(\frac{a}{\sinh(x)^2}\right)^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a/sinh(x)^2)^(7/2),x)
[Out] int(1/(a/sinh(x)^2)^(7/2), x)
```

sympy [F]   time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a\operatorname{csch}^2(x)\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*csch(x)**2)**(7/2),x)
[Out] Integral((a*csch(x)**2)**(-7/2), x)
```

3.36 $\int \left(a\,\text{csch}^3(x)\right)^{5/2}\,dx$

Optimal. Leaf size=135

$$-\frac{154}{585}a^2\coth(x)\sqrt{a\,\text{csch}^3(x)}-\frac{2}{13}a^2\coth(x)\text{csch}^4(x)\sqrt{a\,\text{csch}^3(x)}+\frac{22}{117}a^2\coth(x)\text{csch}^2(x)\sqrt{a\,\text{csch}^3(x)}+\frac{154}{195}a^2$$

```
[Out] -154/585*a^2*coth(x)*(a*csch(x)^3)^(1/2)+22/117*a^2*coth(x)*csch(x)^2*(a*cs
ch(x)^3)^(1/2)-2/13*a^2*coth(x)*csch(x)^4*(a*csch(x)^3)^(1/2)+154/195*a^2*c
osh(x)*sinh(x)*(a*csch(x)^3)^(1/2)-154/195*I*a^2*(sin(1/4*Pi+1/2*I*x)^2)^(1
/2)/sin(1/4*Pi+1/2*I*x)*EllipticE(cos(1/4*Pi+1/2*I*x),2^(1/2))*sinh(x)^2*(a
*csch(x)^3)^(1/2)/(I*sinh(x))^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {4123, 3768, 3771, 2639}

$$-\frac{2}{13}a^2\coth(x)\text{csch}^4(x)\sqrt{a\,\text{csch}^3(x)}+\frac{22}{117}a^2\coth(x)\text{csch}^2(x)\sqrt{a\,\text{csch}^3(x)}-\frac{154}{585}a^2\coth(x)\sqrt{a\,\text{csch}^3(x)}+\frac{154}{195}a^2$$

Antiderivative was successfully verified.

```
[In] Int[(a*Csch[x]^3)^(5/2),x]
```

```
[Out] (-154*a^2*Coth[x]*Sqrt[a*Csch[x]^3])/585 + (22*a^2*Coth[x]*Csch[x]^2*Sqrt[a
*Csch[x]^3])/117 - (2*a^2*Coth[x]*Csch[x]^4*Sqrt[a*Csch[x]^3])/13 + (154*a^
2*Cosh[x]*Sqrt[a*Csch[x]^3]*Sinh[x])/195 - (((154*I)/195)*a^2*Sqrt[a*Csch[x
]^3]*EllipticE[Pi/4 - (I/2)*x, 2]*Sinh[x]^2)/Sqrt[I*Sinh[x]]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 4123

```
Int[((b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> Dist[(b
^IntPart[p]*(b*(c*Sec[e + f*x])^n)^FracPart[p])/(c*Sec[e + f*x])^(n*FracPar
t[p]), Int[(c*Sec[e + f*x])^(n*p), x], x] /; FreeQ[{b, c, e, f, n, p}, x] &
&  !IntegerQ[p]
```

Rubi steps

$$\int \left(a\text{csch}^3(x)\right)^{5/2} dx = -\frac{\left(a^2\sqrt{a\text{csch}^3(x)}\right)\int (i\text{csch}(x))^{15/2}\,dx}{(i\text{csch}(x))^{3/2}}$$

$$= -\frac{2}{13}a^2\coth(x)\text{csch}^4(x)\sqrt{a\text{csch}^3(x)} - \frac{\left(11a^2\sqrt{a\text{csch}^3(x)}\right)\int (i\text{csch}(x))^{11/2}\,dx}{13(i\text{csch}(x))^{3/2}}$$

$$= \frac{22}{117}a^2\coth(x)\text{csch}^2(x)\sqrt{a\text{csch}^3(x)} - \frac{2}{13}a^2\coth(x)\text{csch}^4(x)\sqrt{a\text{csch}^3(x)} - \frac{\left(77a^2\sqrt{a\text{csch}^3}\right.}{117(}$$

$$= -\frac{154}{585}a^2\coth(x)\sqrt{a\text{csch}^3(x)} + \frac{22}{117}a^2\coth(x)\text{csch}^2(x)\sqrt{a\text{csch}^3(x)} - \frac{2}{13}a^2\coth(x)\text{csch}^4(x$$

$$= -\frac{154}{585}a^2\coth(x)\sqrt{a\text{csch}^3(x)} + \frac{22}{117}a^2\coth(x)\text{csch}^2(x)\sqrt{a\text{csch}^3(x)} - \frac{2}{13}a^2\coth(x)\text{csch}^4(x$$

$$= -\frac{154}{585}a^2\coth(x)\sqrt{a\text{csch}^3(x)} + \frac{22}{117}a^2\coth(x)\text{csch}^2(x)\sqrt{a\text{csch}^3(x)} - \frac{2}{13}a^2\coth(x)\text{csch}^4(x$$

$$= -\frac{154}{585}a^2\coth(x)\sqrt{a\text{csch}^3(x)} + \frac{22}{117}a^2\coth(x)\text{csch}^2(x)\sqrt{a\text{csch}^3(x)} - \frac{2}{13}a^2\coth(x)\text{csch}^4(x$$

Mathematica [A] time = 0.18, size = 68, normalized size = 0.50

$$-\frac{2}{585}a^2\sinh(x)\sqrt{a\text{csch}^3(x)}\left(-231\cosh(x) + \coth(x)\text{csch}(x)\left(45\text{csch}^4(x) - 55\text{csch}^2(x) + 77\right) + 231\sqrt{i\sinh(x)}\,E\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Csch[x]^3)^(5/2),x]
```

```
[Out] (-2*a^2*Sqrt[a*Csch[x]^3]*(-231*Cosh[x] + Coth[x]*Csch[x]*(77 - 55*Csch[x]^
2 + 45*Csch[x]^4) + 231*EllipticE[(Pi - (2*I)*x)/4, 2]*Sqrt[I*Sinh[x]])*Sin
h[x])/585
```

fricas [F] time = 0.61, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{a\,\text{csch}(x)^3}\,a^2\,\text{csch}(x)^6, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*csch(x)^3)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a*csch(x)^3)*a^2*csch(x)^6, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\,\text{csch}(x)^3\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*csch(x)^3)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((a*csch(x)^3)^(5/2), x)
```

maple [F] time = 0.24, size = 0, normalized size = 0.00

$$\int \left(a\mathrm{csch}(x)^{3}\right)^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*csch(x)^3)^(5/2),x)
[Out] int((a*csch(x)^3)^(5/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\,\mathrm{csch}(x)^{3}\right)^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*csch(x)^3)^(5/2),x, algorithm="maxima")
[Out] integrate((a*csch(x)^3)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(\frac{a}{{\mathrm{sinh}\left(x\right)}^3}\right)}^{5/2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a/sinh(x)^3)^(5/2),x)
[Out] int((a/sinh(x)^3)^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a \operatorname{csch}^{3}{\left(x \right)}\right)^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*csch(x)**3)**(5/2),x)
[Out] Integral((a*csch(x)**3)**(5/2), x)
```

3.37 $\int \left(a\text{csch}^3(x)\right)^{3/2} dx$

Optimal. Leaf size=81

$$\frac{10}{21}a\cosh(x)\sqrt{a\text{csch}^3(x)}-\frac{2}{7}a\coth(x)\text{csch}(x)\sqrt{a\text{csch}^3(x)}+\frac{10}{21}ia\sqrt{i\sinh(x)}\,\sinh(x)F\left(\frac{\pi}{4}-\frac{ix}{2}\middle|2\right)\sqrt{a\text{csch}^3(x)}$$

```
[Out] 10/21*a*cosh(x)*(a*csch(x)^3)^(1/2)-2/7*a*coth(x)*csch(x)*(a*csch(x)^3)^(1/
2)+10/21*I*a*(sin(1/4*Pi+1/2*I*x)^2)^(1/2)/sin(1/4*Pi+1/2*I*x)*EllipticF(co
s(1/4*Pi+1/2*I*x),2^(1/2))*sinh(x)*(a*csch(x)^3)^(1/2)*(I*sinh(x))^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {4123, 3768, 3771, 2641}

$$\frac{10}{21}a\cosh(x)\sqrt{a\text{csch}^3(x)}-\frac{2}{7}a\coth(x)\text{csch}(x)\sqrt{a\text{csch}^3(x)}+\frac{10}{21}ia\sqrt{i\sinh(x)}\,\sinh(x)F\left(\frac{\pi}{4}-\frac{ix}{2}\middle|2\right)\sqrt{a\text{csch}^3(x)}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Csch[x]^3)^(3/2),x]
```

```
[Out] (10*a*Cosh[x]*Sqrt[a*Csch[x]^3])/21 - (2*a*Coth[x]*Csch[x]*Sqrt[a*Csch[x]^3
])/7 + ((10*I)/21)*a*Sqrt[a*Csch[x]^3]*EllipticF[Pi/4 - (I/2)*x, 2]*Sqrt[I*
Sinh[x]]*Sinh[x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 4123

```
Int[((b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> Dist[(b
^IntPart[p]*(b*(c*Sec[e + f*x])^n)^FracPart[p])/(c*Sec[e + f*x])^(n*FracPar
t[p]), Int[(c*Sec[e + f*x])^(n*p), x], x] /; FreeQ[{b, c, e, f, n, p}, x] &
&  !IntegerQ[p]
```

Rubi steps

$$\int \left(a\text{csch}^3(x)\right)^{3/2} dx = \frac{\left(ia\sqrt{a\text{csch}^3(x)}\right)\int (i\text{csch}(x))^{9/2} dx}{(i\text{csch}(x))^{3/2}}$$
$$= -\frac{2}{7}a\coth(x)\text{csch}(x)\sqrt{a\text{csch}^3(x)} + \frac{\left(5ia\sqrt{a\text{csch}^3(x)}\right)\int (i\text{csch}(x))^{5/2} dx}{7(i\text{csch}(x))^{3/2}}$$
$$= \frac{10}{21}a\cosh(x)\sqrt{a\text{csch}^3(x)} - \frac{2}{7}a\coth(x)\text{csch}(x)\sqrt{a\text{csch}^3(x)} + \frac{\left(5ia\sqrt{a\text{csch}^3(x)}\right)\int \sqrt{ics}}{21(i\text{csch}(x))^{3/2}}$$
$$= \frac{10}{21}a\cosh(x)\sqrt{a\text{csch}^3(x)} - \frac{2}{7}a\coth(x)\text{csch}(x)\sqrt{a\text{csch}^3(x)} + \frac{1}{21}\left(5a\sqrt{a\text{csch}^3(x)}\sqrt{i\sin}\right.$$
$$= \frac{10}{21}a\cosh(x)\sqrt{a\text{csch}^3(x)} - \frac{2}{7}a\coth(x)\text{csch}(x)\sqrt{a\text{csch}^3(x)} + \frac{10}{21}ia\sqrt{a\text{csch}^3(x)}F\left(\frac{\pi}{4} - \right.$$

Mathematica [A] time = 0.12, size = 56, normalized size = 0.69

$$-\frac{2}{21}a\sinh(x)\sqrt{a\text{csch}^3(x)}\left(\coth(x)\left(3\text{csch}^2(x)-5\right)-5i\sqrt{i\sinh(x)}F\left(\frac{1}{4}(\pi-2ix)\middle|2\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Csch[x]^3)^(3/2),x]
```

```
[Out] (-2*a*Sqrt[a*Csch[x]^3]*(Coth[x]*(-5 + 3*Csch[x]^2) - (5*I)*EllipticF[(Pi -
 (2*I)*x)/4, 2]*Sqrt[I*Sinh[x]])*Sinh[x])/21
```

fricas [F] time = 0.47, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{a\,\text{csch}(x)^3}\,a\,\text{csch}(x)^3, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*csch(x)^3)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a*csch(x)^3)*a*csch(x)^3, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\,\text{csch}(x)^3\right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*csch(x)^3)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a*csch(x)^3)^(3/2), x)
```

maple [F] time = 0.21, size = 0, normalized size = 0.00

$$\int \left(a\text{csch}(x)^3\right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*csch(x)^3)^(3/2),x)
```

```
[Out] int((a*csch(x)^3)^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a \operatorname{csch}(x)^3\right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*csch(x)^3)^(3/2),x, algorithm="maxima")
[Out] integrate((a*csch(x)^3)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(\frac{a}{\sinh(x)^3}\right)^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a/sinh(x)^3)^(3/2),x)
[Out] int((a/sinh(x)^3)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a \operatorname{csch}^3(x)\right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*csch(x)**3)**(3/2),x)
[Out] Integral((a*csch(x)**3)**(3/2), x)
```

3.38 $\int \sqrt{a\operatorname{csch}^3(x)}\,dx$

Optimal. Leaf size=56

$$-2\sinh(x)\cosh(x)\sqrt{a\operatorname{csch}^3(x)}-2i(i\sinh(x))^{3/2}E\left(\frac{\pi}{4}-\frac{ix}{2}\middle|2\right)\sqrt{a\operatorname{csch}^3(x)}$$

```
[Out] -2*I*(sin(1/4*Pi+1/2*I*x)^2)^(1/2)/sin(1/4*Pi+1/2*I*x)*EllipticE(cos(1/4*Pi
+1/2*I*x),2^(1/2))*(I*sinh(x))^(3/2)*(a*csch(x)^3)^(1/2)-2*cosh(x)*sinh(x)*
(a*csch(x)^3)^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 60, normalized size of antiderivative = 1.07, number of steps used = 4, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {4123, 3768, 3771, 2639}

$$-2\sinh(x)\cosh(x)\sqrt{a\operatorname{csch}^3(x)}+\frac{2i\sinh^2(x)E\left(\frac{\pi}{4}-\frac{ix}{2}\middle|2\right)\sqrt{a\operatorname{csch}^3(x)}}{\sqrt{i\sinh(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*Csch[x]^3],x]
```

```
[Out] -2*Cosh[x]*Sqrt[a*Csch[x]^3]*Sinh[x] + ((2*I)*Sqrt[a*Csch[x]^3]*EllipticE[P
i/4 - (I/2)*x, 2]*Sinh[x]^2)/Sqrt[I*Sinh[x]]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 4123

```
Int[((b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> Dist[(b
^IntPart[p]*(b*(c*Sec[e + f*x])^n)^FracPart[p])/(c*Sec[e + f*x])^(n*FracPar
t[p]), Int[(c*Sec[e + f*x])^(n*p), x], x] /; FreeQ[{b, c, e, f, n, p}, x] &
&  !IntegerQ[p]
```

Rubi steps

$$\int \sqrt{a\,\text{csch}^3(x)}\,dx = \frac{\sqrt{a\,\text{csch}^3(x)}\int (i\,\text{csch}(x))^{3/2}\,dx}{(i\,\text{csch}(x))^{3/2}}$$
$$= -2\cosh(x)\sqrt{a\,\text{csch}^3(x)}\,\sinh(x) - \frac{\sqrt{a\,\text{csch}^3(x)}\int \frac{1}{\sqrt{i\,\text{csch}(x)}}\,dx}{(i\,\text{csch}(x))^{3/2}}$$
$$= -2\cosh(x)\sqrt{a\,\text{csch}^3(x)}\,\sinh(x) + \frac{\left(\sqrt{a\,\text{csch}^3(x)}\,\sinh^2(x)\right)\int \sqrt{i\sinh(x)}\,dx}{\sqrt{i\sinh(x)}}$$
$$= -2\cosh(x)\sqrt{a\,\text{csch}^3(x)}\,\sinh(x) + \frac{2i\sqrt{a\,\text{csch}^3(x)}\,E\left(\frac{\pi}{4}-\frac{ix}{2}\middle|2\right)\sinh^2(x)}{\sqrt{i\sinh(x)}}$$

Mathematica [A] time = 0.03, size = 42, normalized size = 0.75

$$-2\sinh(x)\sqrt{a\,\text{csch}^3(x)}\left(\cosh(x)-\sqrt{i\sinh(x)}\,E\left(\frac{1}{4}(\pi-2ix)\middle|2\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*Csch[x]^3],x]
```

```
[Out] -2*Sqrt[a*Csch[x]^3]*(Cosh[x] - EllipticE[(Pi - (2*I)*x)/4, 2]*Sqrt[I*Sinh[
x]])*Sinh[x]
```

fricas [F] time = 0.51, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{a\,\text{csch}(x)^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*csch(x)^3)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a*csch(x)^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a\,\text{csch}(x)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*csch(x)^3)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a*csch(x)^3), x)
```

maple [F] time = 0.22, size = 0, normalized size = 0.00

$$\int \sqrt{a\,\text{csch}(x)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*csch(x)^3)^(1/2),x)
```

```
[Out] int((a*csch(x)^3)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a\,\text{csch}(x)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*csch(x)^3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a*csch(x)^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \sqrt{\frac{a}{\sinh(x)^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a/sinh(x)^3)^(1/2),x)
```

```
[Out] int((a/sinh(x)^3)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a \operatorname{csch}^3(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*csch(x)**3)**(1/2),x)
```

```
[Out] Integral(sqrt(a*csch(x)**3), x)
```

3.39 $\int \frac{1}{\sqrt{a\text{csch}^3(x)}}\,dx$

Optimal. Leaf size=62

$$\frac{2\coth(x)}{3\sqrt{a\text{csch}^3(x)}}-\frac{2i\sqrt{i\sinh(x)}\,\text{csch}^2(x)F\left(\frac{\pi}{4}-\frac{ix}{2}\middle|2\right)}{3\sqrt{a\text{csch}^3(x)}}$$

```
[Out] 2/3*coth(x)/(a*csch(x)^3)^(1/2)-2/3*I*csch(x)^2*(sin(1/4*Pi+1/2*I*x)^2)^(1/
2)/sin(1/4*Pi+1/2*I*x)*EllipticF(cos(1/4*Pi+1/2*I*x),2^(1/2))*(I*sinh(x))^(
1/2)/(a*csch(x)^3)^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {4123, 3769, 3771, 2641}

$$\frac{2\coth(x)}{3\sqrt{a\text{csch}^3(x)}}-\frac{2i\sqrt{i\sinh(x)}\,\text{csch}^2(x)F\left(\frac{\pi}{4}-\frac{ix}{2}\middle|2\right)}{3\sqrt{a\text{csch}^3(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a*Csch[x]^3],x]
```

```
[Out] (2*Coth[x])/(3*Sqrt[a*Csch[x]^3]) - (((2*I)/3)*Csch[x]^2*EllipticF[Pi/4 - (
I/2)*x, 2]*Sqrt[I*Sinh[x]])/Sqrt[a*Csch[x]^3]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 4123

```
Int[((b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> Dist[(b
^IntPart[p]*(b*(c*Sec[e + f*x])^n)^FracPart[p])/(c*Sec[e + f*x])^(n*FracPar
t[p]), Int[(c*Sec[e + f*x])^(n*p), x], x] /; FreeQ[{b, c, e, f, n, p}, x] &
&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{1}{\sqrt{a\text{csch}^3(x)}}\,dx = \frac{(i\text{csch}(x))^{3/2}\int \frac{1}{(i\text{csch}(x))^{3/2}}\,dx}{\sqrt{a\text{csch}^3(x)}}$$
$$= \frac{2\coth(x)}{3\sqrt{a\text{csch}^3(x)}} + \frac{(i\text{csch}(x))^{3/2}\int \sqrt{i\text{csch}(x)}\,dx}{3\sqrt{a\text{csch}^3(x)}}$$
$$= \frac{2\coth(x)}{3\sqrt{a\text{csch}^3(x)}} - \frac{\left(\text{csch}^2(x)\sqrt{i\sinh(x)}\right)\int \frac{1}{\sqrt{i\sinh(x)}}\,dx}{3\sqrt{a\text{csch}^3(x)}}$$
$$= \frac{2\coth(x)}{3\sqrt{a\text{csch}^3(x)}} - \frac{2i\text{csch}^2(x)F\left(\frac{\pi}{4}-\frac{ix}{2}\middle|2\right)\sqrt{i\sinh(x)}}{3\sqrt{a\text{csch}^3(x)}}$$

Mathematica [A] time = 0.08, size = 43, normalized size = 0.69

$$\frac{2\left(\coth(x)+\frac{\text{csch}(x)F\left(\frac{1}{4}(\pi-2ix)\middle|2\right)}{\sqrt{i\sinh(x)}}\right)}{3\sqrt{a\text{csch}^3(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a*Csch[x]^3],x]
[Out] (2*(Coth[x] + (Csch[x]*EllipticF[(Pi - (2*I)*x)/4, 2])/Sqrt[I*Sinh[x]]))/(3
*Sqrt[a*Csch[x]^3])
```

fricas [F] time = 0.59, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{a\,\text{csch}(x)^3}}{a\,\text{csch}(x)^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*csch(x)^3)^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(a*csch(x)^3)/(a*csch(x)^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a\,\text{csch}(x)^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*csch(x)^3)^(1/2),x, algorithm="giac")
[Out] integrate(1/sqrt(a*csch(x)^3), x)
```

maple [F] time = 0.23, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a\text{csch}(x)^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*csch(x)^3)^(1/2),x)
```

```
[Out] int(1/(a*csch(x)^3)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a\,\mathrm{csch}(x)^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*csch(x)^3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(a*csch(x)^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sqrt{\frac{a}{\sinh(x)^3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a/sinh(x)^3)^(1/2),x)
```

```
[Out] int(1/(a/sinh(x)^3)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a\,\mathrm{csch}^{3}(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*csch(x)**3)**(1/2),x)
```

```
[Out] Integral(1/sqrt(a*csch(x)**3), x)
```

3.40 $\int \frac{1}{\left(a\,\text{csch}^3(x)\right)^{3/2}}\,dx$

Optimal. Leaf size=89

$$-\frac{14\cosh(x)}{45a\sqrt{a\text{csch}^3(x)}}+\frac{2\sinh^2(x)\cosh(x)}{9a\sqrt{a\text{csch}^3(x)}}+\frac{14i\text{csch}(x)E\left(\frac{\pi}{4}-\frac{ix}{2}\middle|2\right)}{15a\sqrt{i\sinh(x)}\sqrt{a\text{csch}^3(x)}}$$

```
[Out] -14/45*cosh(x)/a/(a*csch(x)^3)^(1/2)+2/9*cosh(x)*sinh(x)^2/a/(a*csch(x)^3)^
(1/2)+14/15*I*csch(x)*(sin(1/4*Pi+1/2*I*x)^2)^(1/2)/sin(1/4*Pi+1/2*I*x)*Ell
ipticE(cos(1/4*Pi+1/2*I*x),2^(1/2))/a/(a*csch(x)^3)^(1/2)/(I*sinh(x))^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {4123, 3769, 3771, 2639}

$$-\frac{14\cosh(x)}{45a\sqrt{a\text{csch}^3(x)}}+\frac{2\sinh^2(x)\cosh(x)}{9a\sqrt{a\text{csch}^3(x)}}+\frac{14i\text{csch}(x)E\left(\frac{\pi}{4}-\frac{ix}{2}\middle|2\right)}{15a\sqrt{i\sinh(x)}\sqrt{a\text{csch}^3(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Csch[x]^3)^(-3/2),x]
```

```
[Out] (-14*Cosh[x])/(45*a*Sqrt[a*Csch[x]^3]) + (((14*I)/15)*Csch[x]*EllipticE[Pi/
4 - (I/2)*x, 2])/(a*Sqrt[a*Csch[x]^3]*Sqrt[I*Sinh[x]]) + (2*Cosh[x]*Sinh[x]
^2)/(9*a*Sqrt[a*Csch[x]^3])
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 4123

```
Int[((b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> Dist[(b
^IntPart[p]*(b*(c*Sec[e + f*x])^n)^FracPart[p])/(c*Sec[e + f*x])^(n*FracPar
t[p]), Int[(c*Sec[e + f*x])^(n*p), x], x] /; FreeQ[{b, c, e, f, n, p}, x] &
&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{1}{\left(a\operatorname{csch}^3(x)\right)^{3/2}}\,dx = -\frac{\left(i(i\operatorname{csch}(x))^{3/2}\right)\int \frac{1}{(i\operatorname{csch}(x))^{9/2}}\,dx}{a\sqrt{a\operatorname{csch}^3(x)}}$$
$$= \frac{2\cosh(x)\sinh^2(x)}{9a\sqrt{a\operatorname{csch}^3(x)}} - \frac{\left(7i(i\operatorname{csch}(x))^{3/2}\right)\int \frac{1}{(i\operatorname{csch}(x))^{5/2}}\,dx}{9a\sqrt{a\operatorname{csch}^3(x)}}$$
$$= -\frac{14\cosh(x)}{45a\sqrt{a\operatorname{csch}^3(x)}} + \frac{2\cosh(x)\sinh^2(x)}{9a\sqrt{a\operatorname{csch}^3(x)}} - \frac{\left(7i(i\operatorname{csch}(x))^{3/2}\right)\int \frac{1}{\sqrt{i\operatorname{csch}(x)}}\,dx}{15a\sqrt{a\operatorname{csch}^3(x)}}$$
$$= -\frac{14\cosh(x)}{45a\sqrt{a\operatorname{csch}^3(x)}} + \frac{2\cosh(x)\sinh^2(x)}{9a\sqrt{a\operatorname{csch}^3(x)}} + \frac{(7\operatorname{csch}(x))\int \sqrt{i\sinh(x)}\,dx}{15a\sqrt{a\operatorname{csch}^3(x)}\sqrt{i\sinh(x)}}$$
$$= -\frac{14\cosh(x)}{45a\sqrt{a\operatorname{csch}^3(x)}} + \frac{14i\operatorname{csch}(x)E\left(\frac{\pi}{4}-\frac{ix}{2}\middle|2\right)}{15a\sqrt{a\operatorname{csch}^3(x)}\sqrt{i\sinh(x)}} + \frac{2\cosh(x)\sinh^2(x)}{9a\sqrt{a\operatorname{csch}^3(x)}}$$

Mathematica [A] time = 0.08, size = 57, normalized size = 0.64

$$\frac{-33\cosh(x)+5\cosh(3x)+84\sqrt{i\sinh(x)}\operatorname{csch}^2(x)E\left(\frac{1}{4}(\pi-2ix)\middle|2\right)}{90a\sqrt{a\operatorname{csch}^3(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Csch[x]^3)^(-3/2),x]
[Out] (-33*Cosh[x] + 5*Cosh[3*x] + 84*Csch[x]^2*EllipticE[(Pi - (2*I)*x)/4, 2]*Sq
rt[I*Sinh[x]])/(90*a*Sqrt[a*Csch[x]^3])
```

fricas [F] time = 0.60, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{a\operatorname{csch}(x)^3}}{a^2\operatorname{csch}(x)^6}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*csch(x)^3)^(3/2),x, algorithm="fricas")
[Out] integral(sqrt(a*csch(x)^3)/(a^2*csch(x)^6), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a\operatorname{csch}(x)^3\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*csch(x)^3)^(3/2),x, algorithm="giac")
[Out] integrate((a*csch(x)^3)^(-3/2), x)
```

maple [F] time = 0.20, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a\operatorname{csch}(x)^3\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*csch(x)^3)^(3/2),x)
[Out] int(1/(a*csch(x)^3)^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a \operatorname{csch}(x)^3\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*csch(x)^3)^(3/2),x, algorithm="maxima")
[Out] integrate((a*csch(x)^3)^(-3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(\frac{a}{{\sinh(x)}^3}\right)}^{3/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a/sinh(x)^3)^(3/2),x)
[Out] int(1/(a/sinh(x)^3)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a \operatorname{csch}^{3}{\left(x \right)}\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*csch(x)**3)**(3/2),x)
[Out] Integral((a*csch(x)**3)**(-3/2), x)
```

3.41 $\int \frac{1}{\left(a\operatorname{csch}^3(x)\right)^{5/2}}\,dx$

Optimal. Leaf size=135

$$-\frac{26\coth(x)}{77a^2\sqrt{a\operatorname{csch}^3(x)}}+\frac{2\sinh^5(x)\cosh(x)}{15a^2\sqrt{a\operatorname{csch}^3(x)}}-\frac{26\sinh^3(x)\cosh(x)}{165a^2\sqrt{a\operatorname{csch}^3(x)}}+\frac{78\sinh(x)\cosh(x)}{385a^2\sqrt{a\operatorname{csch}^3(x)}}+\frac{26i\sqrt{i\sinh(x)}\operatorname{csch}^2(x)F\left(\frac{\pi}{4}\right.}{77a^2\sqrt{a\operatorname{csch}^3(x)}}$$

```
[Out] -26/77*coth(x)/a^2/(a*csch(x)^3)^(1/2)+78/385*cosh(x)*sinh(x)/a^2/(a*csch(x
)^3)^(1/2)-26/165*cosh(x)*sinh(x)^3/a^2/(a*csch(x)^3)^(1/2)+2/15*cosh(x)*si
nh(x)^5/a^2/(a*csch(x)^3)^(1/2)+26/77*I*csch(x)^2*(sin(1/4*Pi+1/2*I*x)^2)^(
1/2)/sin(1/4*Pi+1/2*I*x)*EllipticF(cos(1/4*Pi+1/2*I*x),2^(1/2))*(I*sinh(x))
^(1/2)/a^2/(a*csch(x)^3)^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {4123, 3769, 3771, 2641}

$$-\frac{26\coth(x)}{77a^2\sqrt{a\operatorname{csch}^3(x)}}+\frac{2\sinh^5(x)\cosh(x)}{15a^2\sqrt{a\operatorname{csch}^3(x)}}-\frac{26\sinh^3(x)\cosh(x)}{165a^2\sqrt{a\operatorname{csch}^3(x)}}+\frac{78\sinh(x)\cosh(x)}{385a^2\sqrt{a\operatorname{csch}^3(x)}}+\frac{26i\sqrt{i\sinh(x)}\operatorname{csch}^2(x)F\left(\frac{\pi}{4}\right.}{77a^2\sqrt{a\operatorname{csch}^3(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Csch[x]^3)^(-5/2),x]
```

```
[Out] (-26*Coth[x])/(77*a^2*Sqrt[a*Csch[x]^3]) + (((26*I)/77)*Csch[x]^2*EllipticF
[Pi/4 - (I/2)*x, 2]*Sqrt[I*Sinh[x]])/(a^2*Sqrt[a*Csch[x]^3]) + (78*Cosh[x]*
Sinh[x])/(385*a^2*Sqrt[a*Csch[x]^3]) - (26*Cosh[x]*Sinh[x]^3)/(165*a^2*Sqrt
[a*Csch[x]^3]) + (2*Cosh[x]*Sinh[x]^5)/(15*a^2*Sqrt[a*Csch[x]^3])
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 4123

```
Int[((b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> Dist[(b
^IntPart[p]*(b*(c*Sec[e + f*x])^n)^FracPart[p])/(c*Sec[e + f*x])^(n*FracPar
t[p]), Int[(c*Sec[e + f*x])^(n*p), x], x] /; FreeQ[{b, c, e, f, n, p}, x] &
&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{1}{\left(a\text{csch}^3(x)\right)^{5/2}}\,dx = -\frac{(i\text{csch}(x))^{3/2}\int \frac{1}{(i\text{csch}(x))^{15/2}}\,dx}{a^2\sqrt{a\text{csch}^3(x)}}$$

$$= \frac{2\cosh(x)\sinh^5(x)}{15a^2\sqrt{a\text{csch}^3(x)}} - \frac{\left(13(i\text{csch}(x))^{3/2}\right)\int \frac{1}{(i\text{csch}(x))^{11/2}}\,dx}{15a^2\sqrt{a\text{csch}^3(x)}}$$

$$= -\frac{26\cosh(x)\sinh^3(x)}{165a^2\sqrt{a\text{csch}^3(x)}} + \frac{2\cosh(x)\sinh^5(x)}{15a^2\sqrt{a\text{csch}^3(x)}} - \frac{\left(39(i\text{csch}(x))^{3/2}\right)\int \frac{1}{(i\text{csch}(x))^{7/2}}\,dx}{55a^2\sqrt{a\text{csch}^3(x)}}$$

$$= \frac{78\cosh(x)\sinh(x)}{385a^2\sqrt{a\text{csch}^3(x)}} - \frac{26\cosh(x)\sinh^3(x)}{165a^2\sqrt{a\text{csch}^3(x)}} + \frac{2\cosh(x)\sinh^5(x)}{15a^2\sqrt{a\text{csch}^3(x)}} - \frac{\left(39(i\text{csch}(x))^{3/2}\right)\int}{77a^2\sqrt{a\text{csc}}}$$

$$= -\frac{26\coth(x)}{77a^2\sqrt{a\text{csch}^3(x)}} + \frac{78\cosh(x)\sinh(x)}{385a^2\sqrt{a\text{csch}^3(x)}} - \frac{26\cosh(x)\sinh^3(x)}{165a^2\sqrt{a\text{csch}^3(x)}} + \frac{2\cosh(x)\sinh^5(x)}{15a^2\sqrt{a\text{csch}^3(x)}}$$

$$= -\frac{26\coth(x)}{77a^2\sqrt{a\text{csch}^3(x)}} + \frac{78\cosh(x)\sinh(x)}{385a^2\sqrt{a\text{csch}^3(x)}} - \frac{26\cosh(x)\sinh^3(x)}{165a^2\sqrt{a\text{csch}^3(x)}} + \frac{2\cosh(x)\sinh^5(x)}{15a^2\sqrt{a\text{csch}^3(x)}}$$

$$= -\frac{26\coth(x)}{77a^2\sqrt{a\text{csch}^3(x)}} + \frac{26i\text{csch}^2(x)F\left(\frac{\pi}{4}-\frac{ix}{2}\middle|2\right)\sqrt{i\sinh(x)}}{77a^2\sqrt{a\text{csch}^3(x)}} + \frac{78\cosh(x)\sinh(x)}{385a^2\sqrt{a\text{csch}^3(x)}} - \frac{26}{16}$$

Mathematica [A] time = 0.13, size = 71, normalized size = 0.53

$$\frac{\sinh(x)\sqrt{a\text{csch}^3(x)}\left(-19122\sinh(2x)+4406\sinh(4x)-826\sinh(6x)+77\sinh(8x)+24960i\sqrt{i\sinh(x)}\,F\left(\right.\right.}{73920a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Csch[x]^3)^(-5/2),x]
```

```
[Out] (Sqrt[a*Csch[x]^3]*Sinh[x]*((24960*I)*EllipticF[(Pi - (2*I)*x)/4, 2]*Sqrt[I
*Sinh[x]] - 19122*Sinh[2*x] + 4406*Sinh[4*x] - 826*Sinh[6*x] + 77*Sinh[8*x]
))/(73920*a^3)
```

fricas [F] time = 1.53, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{a\,\text{csch}(x)^3}}{a^3\,\text{csch}(x)^9},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*csch(x)^3)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a*csch(x)^3)/(a^3*csch(x)^9), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a\,\text{csch}(x)^3\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*csch(x)^3)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((a*csch(x)^3)^(-5/2), x)
```

maple [F] time = 0.20, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a\mathrm{csch}(x)^{3}\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*csch(x)^3)^(5/2),x)
```

```
[Out] int(1/(a*csch(x)^3)^(5/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a\,\mathrm{csch}(x)^{3}\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*csch(x)^3)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((a*csch(x)^3)^(-5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(\frac{a}{\sinh(x)^{3}}\right)^{5/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a/sinh(x)^3)^(5/2),x)
```

```
[Out] int(1/(a/sinh(x)^3)^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a\,\mathrm{csch}^{3}\,(x)\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*csch(x)**3)**(5/2),x)
```

```
[Out] Integral((a*csch(x)**3)**(-5/2), x)
```

3.42 $\int \left(a\text{csch}^4(x)\right)^{7/2} dx$

Optimal. Leaf size=164

$$-\frac{1}{13}a^3\cosh^2(x)\coth^{11}(x)\sqrt{a\text{csch}^4(x)}+\frac{6}{11}a^3\cosh^2(x)\coth^9(x)\sqrt{a\text{csch}^4(x)}-\frac{5}{3}a^3\cosh^2(x)\coth^7(x)\sqrt{a\text{csch}^4(x)}$$

```
[Out] 2*a^3*cosh(x)^2*coth(x)*(a*csch(x)^4)^(1/2)-3*a^3*cosh(x)^2*coth(x)^3*(a*cs
ch(x)^4)^(1/2)+20/7*a^3*cosh(x)^2*coth(x)^5*(a*csch(x)^4)^(1/2)-5/3*a^3*cos
h(x)^2*coth(x)^7*(a*csch(x)^4)^(1/2)+6/11*a^3*cosh(x)^2*coth(x)^9*(a*csch(x
)^4)^(1/2)-1/13*a^3*cosh(x)^2*coth(x)^11*(a*csch(x)^4)^(1/2)-a^3*cosh(x)*si
nh(x)*(a*csch(x)^4)^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 164, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {4123, 3767}

$$-\frac{1}{13}a^3\cosh^2(x)\coth^{11}(x)\sqrt{a\text{csch}^4(x)}+\frac{6}{11}a^3\cosh^2(x)\coth^9(x)\sqrt{a\text{csch}^4(x)}-\frac{5}{3}a^3\cosh^2(x)\coth^7(x)\sqrt{a\text{csch}^4(x)}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Csch[x]^4)^(7/2),x]
```

```
[Out] 2*a^3*Cosh[x]^2*Coth[x]*Sqrt[a*Csch[x]^4] - 3*a^3*Cosh[x]^2*Coth[x]^3*Sqrt[
a*Csch[x]^4] + (20*a^3*Cosh[x]^2*Coth[x]^5*Sqrt[a*Csch[x]^4])/7 - (5*a^3*Co
sh[x]^2*Coth[x]^7*Sqrt[a*Csch[x]^4])/3 + (6*a^3*Cosh[x]^2*Coth[x]^9*Sqrt[a*
Csch[x]^4])/11 - (a^3*Cosh[x]^2*Coth[x]^11*Sqrt[a*Csch[x]^4])/13 - a^3*Cosh
[x]*Sqrt[a*Csch[x]^4]*Sinh[x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 4123

```
Int[((b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> Dist[(b
^IntPart[p]*(b*(c*Sec[e + f*x])^n)^FracPart[p])/(c*Sec[e + f*x])^(n*FracPar
t[p]), Int[(c*Sec[e + f*x])^(n*p), x], x] /; FreeQ[{b, c, e, f, n, p}, x] &
&  !IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \left(a\text{csch}^4(x)\right)^{7/2} dx &= \left(a^3\sqrt{a\text{csch}^4(x)}\,\sinh^2(x)\right)\int \text{csch}^{14}(x)\,dx\\ &= -\left(\left(ia^3\sqrt{a\text{csch}^4(x)}\,\sinh^2(x)\right)\text{Subst}\left(\int \left(1+6x^2+15x^4+20x^6+15x^8+6x^{10}+x^{12}\right)\right.\right.\\ &= 2a^3\cosh^2(x)\coth(x)\sqrt{a\text{csch}^4(x)}\,-3a^3\cosh^2(x)\coth^3(x)\sqrt{a\text{csch}^4(x)}\,+\frac{20}{7}a^3\cosh^2(x\end{aligned}$$

Mathematica [A] time = 0.05, size = 59, normalized size = 0.36

$$-\frac{a^3\sinh(x)\cosh(x)\left(231\text{csch}^{12}(x)-252\text{csch}^{10}(x)+280\text{csch}^8(x)-320\text{csch}^6(x)+384\text{csch}^4(x)-512\text{csch}^2(x\right.}{3003}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Csch[x]^4)^(7/2),x]
```

```
[Out] -1/3003*(a^3*Cosh[x]*Sqrt[a*Csch[x]^4]*(1024 - 512*Csch[x]^2 + 384*Csch[x]^
4 - 320*Csch[x]^6 + 280*Csch[x]^8 - 252*Csch[x]^10 + 231*Csch[x]^12)*Sinh[x
])
```

fricas [B] time = 0.65, size = 2825, normalized size = 17.23

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*csch(x)^4)^(7/2),x, algorithm="fricas")
```

```
[Out] -2048/3003*(1716*a^3*cosh(x)^12 - 1287*a^3*cosh(x)^10 + 1716*(a^3*e^(4*x) -
 2*a^3*e^(2*x) + a^3)*sinh(x)^12 + 20592*(a^3*cosh(x)*e^(4*x) - 2*a^3*cosh(
x)*e^(2*x) + a^3*cosh(x))*sinh(x)^11 + 715*a^3*cosh(x)^8 + 1287*(88*a^3*cos
h(x)^2 - a^3 + (88*a^3*cosh(x)^2 - a^3)*e^(4*x) - 2*(88*a^3*cosh(x)^2 - a^3
)*e^(2*x))*sinh(x)^10 + 4290*(88*a^3*cosh(x)^3 - 3*a^3*cosh(x) + (88*a^3*co
sh(x)^3 - 3*a^3*cosh(x))*e^(4*x) - 2*(88*a^3*cosh(x)^3 - 3*a^3*cosh(x))*e^(
2*x))*sinh(x)^9 - 286*a^3*cosh(x)^6 + 715*(1188*a^3*cosh(x)^4 - 81*a^3*cosh
(x)^2 + a^3 + (1188*a^3*cosh(x)^4 - 81*a^3*cosh(x)^2 + a^3)*e^(4*x) - 2*(11
88*a^3*cosh(x)^4 - 81*a^3*cosh(x)^2 + a^3)*e^(2*x))*sinh(x)^8 + 1144*(1188*
a^3*cosh(x)^5 - 135*a^3*cosh(x)^3 + 5*a^3*cosh(x) + (1188*a^3*cosh(x)^5 - 1
35*a^3*cosh(x)^3 + 5*a^3*cosh(x))*e^(4*x) - 2*(1188*a^3*cosh(x)^5 - 135*a^3
*cosh(x)^3 + 5*a^3*cosh(x))*e^(2*x))*sinh(x)^7 + 78*a^3*cosh(x)^4 + 286*(55
44*a^3*cosh(x)^6 - 945*a^3*cosh(x)^4 + 70*a^3*cosh(x)^2 - a^3 + (5544*a^3*c
osh(x)^6 - 945*a^3*cosh(x)^4 + 70*a^3*cosh(x)^2 - a^3)*e^(4*x) - 2*(5544*a^
3*cosh(x)^6 - 945*a^3*cosh(x)^4 + 70*a^3*cosh(x)^2 - a^3)*e^(2*x))*sinh(x)^
6 + 572*(2376*a^3*cosh(x)^7 - 567*a^3*cosh(x)^5 + 70*a^3*cosh(x)^3 - 3*a^3*
cosh(x) + (2376*a^3*cosh(x)^7 - 567*a^3*cosh(x)^5 + 70*a^3*cosh(x)^3 - 3*a^
3*cosh(x))*e^(4*x) - 2*(2376*a^3*cosh(x)^7 - 567*a^3*cosh(x)^5 + 70*a^3*cos
h(x)^3 - 3*a^3*cosh(x))*e^(2*x))*sinh(x)^5 - 13*a^3*cosh(x)^2 + 26*(32670*a
^3*cosh(x)^8 - 10395*a^3*cosh(x)^6 + 1925*a^3*cosh(x)^4 - 165*a^3*cosh(x)^2
 + 3*a^3 + (32670*a^3*cosh(x)^8 - 10395*a^3*cosh(x)^6 + 1925*a^3*cosh(x)^4
- 165*a^3*cosh(x)^2 + 3*a^3)*e^(4*x) - 2*(32670*a^3*cosh(x)^8 - 10395*a^3*c
osh(x)^6 + 1925*a^3*cosh(x)^4 - 165*a^3*cosh(x)^2 + 3*a^3)*e^(2*x))*sinh(x)
^4 + 104*(3630*a^3*cosh(x)^9 - 1485*a^3*cosh(x)^7 + 385*a^3*cosh(x)^5 - 55*
a^3*cosh(x)^3 + 3*a^3*cosh(x) + (3630*a^3*cosh(x)^9 - 1485*a^3*cosh(x)^7 +
385*a^3*cosh(x)^5 - 55*a^3*cosh(x)^3 + 3*a^3*cosh(x))*e^(4*x) - 2*(3630*a^3
*cosh(x)^9 - 1485*a^3*cosh(x)^7 + 385*a^3*cosh(x)^5 - 55*a^3*cosh(x)^3 + 3*
a^3*cosh(x))*e^(2*x))*sinh(x)^3 + a^3 + 13*(8712*a^3*cosh(x)^10 - 4455*a^3*
cosh(x)^8 + 1540*a^3*cosh(x)^6 - 330*a^3*cosh(x)^4 + 36*a^3*cosh(x)^2 - a^3
 + (8712*a^3*cosh(x)^10 - 4455*a^3*cosh(x)^8 + 1540*a^3*cosh(x)^6 - 330*a^3
*cosh(x)^4 + 36*a^3*cosh(x)^2 - a^3)*e^(4*x) - 2*(8712*a^3*cosh(x)^10 - 445
5*a^3*cosh(x)^8 + 1540*a^3*cosh(x)^6 - 330*a^3*cosh(x)^4 + 36*a^3*cosh(x)^2
 - a^3)*e^(2*x))*sinh(x)^2 + (1716*a^3*cosh(x)^12 - 1287*a^3*cosh(x)^10 + 7
15*a^3*cosh(x)^8 - 286*a^3*cosh(x)^6 + 78*a^3*cosh(x)^4 - 13*a^3*cosh(x)^2
+ a^3)*e^(4*x) - 2*(1716*a^3*cosh(x)^12 - 1287*a^3*cosh(x)^10 + 715*a^3*cos
h(x)^8 - 286*a^3*cosh(x)^6 + 78*a^3*cosh(x)^4 - 13*a^3*cosh(x)^2 + a^3)*e^(
2*x) + 26*(792*a^3*cosh(x)^11 - 495*a^3*cosh(x)^9 + 220*a^3*cosh(x)^7 - 66*
a^3*cosh(x)^5 + 12*a^3*cosh(x)^3 - a^3*cosh(x) + (792*a^3*cosh(x)^11 - 495*
a^3*cosh(x)^9 + 220*a^3*cosh(x)^7 - 66*a^3*cosh(x)^5 + 12*a^3*cosh(x)^3 - a
^3*cosh(x))*e^(4*x) - 2*(792*a^3*cosh(x)^11 - 495*a^3*cosh(x)^9 + 220*a^3*c
osh(x)^7 - 66*a^3*cosh(x)^5 + 12*a^3*cosh(x)^3 - a^3*cosh(x))*e^(2*x))*sinh
(x))*sqrt(a/(e^(8*x) - 4*e^(6*x) + 6*e^(4*x) - 4*e^(2*x) + 1))*e^(2*x)/(26*
cosh(x)*e^(2*x)*sinh(x)^25 + e^(2*x)*sinh(x)^26 + 13*(25*cosh(x)^2 - 1)*e^(
2*x)*sinh(x)^24 + 104*(25*cosh(x)^3 - 3*cosh(x))*e^(2*x)*sinh(x)^23 + 26*(5
75*cosh(x)^4 - 138*cosh(x)^2 + 3)*e^(2*x)*sinh(x)^22 + 572*(115*cosh(x)^5 -
```

```
 46*cosh(x)^3 + 3*cosh(x))*e^(2*x)*sinh(x)^21 + 286*(805*cosh(x)^6 - 483*co
sh(x)^4 + 63*cosh(x)^2 - 1)*e^(2*x)*sinh(x)^20 + 1144*(575*cosh(x)^7 - 483*
cosh(x)^5 + 105*cosh(x)^3 - 5*cosh(x))*e^(2*x)*sinh(x)^19 + 143*(10925*cosh
(x)^8 - 12236*cosh(x)^6 + 3990*cosh(x)^4 - 380*cosh(x)^2 + 5)*e^(2*x)*sinh(
x)^18 + 286*(10925*cosh(x)^9 - 15732*cosh(x)^7 + 7182*cosh(x)^5 - 1140*cosh
(x)^3 + 45*cosh(x))*e^(2*x)*sinh(x)^17 + 143*(37145*cosh(x)^10 - 66861*cosh
(x)^8 + 40698*cosh(x)^6 - 9690*cosh(x)^4 + 765*cosh(x)^2 - 9)*e^(2*x)*sinh(
x)^16 + 208*(37145*cosh(x)^11 - 81719*cosh(x)^9 + 63954*cosh(x)^7 - 21318*c
osh(x)^5 + 2805*cosh(x)^3 - 99*cosh(x))*e^(2*x)*sinh(x)^15 + 52*(185725*cos
h(x)^12 - 490314*cosh(x)^10 + 479655*cosh(x)^8 - 213180*cosh(x)^6 + 42075*c
osh(x)^4 - 2970*cosh(x)^2 + 33)*e^(2*x)*sinh(x)^14 + 8*(1300075*cosh(x)^13
- 4056234*cosh(x)^11 + 4849845*cosh(x)^9 - 2771340*cosh(x)^7 + 765765*cosh(
x)^5 - 90090*cosh(x)^3 + 3003*cosh(x))*e^(2*x)*sinh(x)^13 + 52*(185725*cosh
(x)^14 - 676039*cosh(x)^12 + 969969*cosh(x)^10 - 692835*cosh(x)^8 + 255255*
cosh(x)^6 - 45045*cosh(x)^4 + 3003*cosh(x)^2 - 33)*e^(2*x)*sinh(x)^12 + 208
*(37145*cosh(x)^15 - 156009*cosh(x)^13 + 264537*cosh(x)^11 - 230945*cosh(x)
^9 + 109395*cosh(x)^7 - 27027*cosh(x)^5 + 3003*cosh(x)^3 - 99*cosh(x))*e^(2
*x)*sinh(x)^11 + 143*(37145*cosh(x)^16 - 178296*cosh(x)^14 + 352716*cosh(x)
^12 - 369512*cosh(x)^10 + 218790*cosh(x)^8 - 72072*cosh(x)^6 + 12012*cosh(x
)^4 - 792*cosh(x)^2 + 9)*e^(2*x)*sinh(x)^10 + 286*(10925*cosh(x)^17 - 59432
*cosh(x)^15 + 135660*cosh(x)^13 - 167960*cosh(x)^11 + 121550*cosh(x)^9 - 51
480*cosh(x)^7 + 12012*cosh(x)^5 - 1320*cosh(x)^3 + 45*cosh(x))*e^(2*x)*sinh
(x)^9 + 143*(10925*cosh(x)^18 - 66861*cosh(x)^16 + 174420*cosh(x)^14 - 2519
40*cosh(x)^12 + 218790*cosh(x)^10 - 115830*cosh(x)^8 + 36036*cosh(x)^6 - 59
40*cosh(x)^4 + 405*cosh(x)^2 - 5)*e^(2*x)*sinh(x)^8 + 1144*(575*cosh(x)^19
- 3933*cosh(x)^17 + 11628*cosh(x)^15 - 19380*cosh(x)^13 + 19890*cosh(x)^11
- 12870*cosh(x)^9 + 5148*cosh(x)^7 - 1188*cosh(x)^5 + 135*cosh(x)^3 - 5*cos
h(x))*e^(2*x)*sinh(x)^7 + 286*(805*cosh(x)^20 - 6118*cosh(x)^18 + 20349*cos
h(x)^16 - 38760*cosh(x)^14 + 46410*cosh(x)^12 - 36036*cosh(x)^10 + 18018*co
sh(x)^8 - 5544*cosh(x)^6 + 945*cosh(x)^4 - 70*cosh(x)^2 + 1)*e^(2*x)*sinh(x
)^6 + 572*(115*cosh(x)^21 - 966*cosh(x)^19 + 3591*cosh(x)^17 - 7752*cosh(x)
^15 + 10710*cosh(x)^13 - 9828*cosh(x)^11 + 6006*cosh(x)^9 - 2376*cosh(x)^7
+ 567*cosh(x)^5 - 70*cosh(x)^3 + 3*cosh(x))*e^(2*x)*sinh(x)^5 + 26*(575*cos
h(x)^22 - 5313*cosh(x)^20 + 21945*cosh(x)^18 - 53295*cosh(x)^16 + 84150*cos
h(x)^14 - 90090*cosh(x)^12 + 66066*cosh(x)^10 - 32670*cosh(x)^8 + 10395*cos
h(x)^6 - 1925*cosh(x)^4 + 165*cosh(x)^2 - 3)*e^(2*x)*sinh(x)^4 + 104*(25*co
sh(x)^23 - 253*cosh(x)^21 + 1155*cosh(x)^19 - 3135*cosh(x)^17 + 5610*cosh(x
)^15 - 6930*cosh(x)^13 + 6006*cosh(x)^11 - 3630*cosh(x)^9 + 1485*cosh(x)^7
- 385*cosh(x)^5 + 55*cosh(x)^3 - 3*cosh(x))*e^(2*x)*sinh(x)^3 + 13*(25*cosh
(x)^24 - 276*cosh(x)^22 + 1386*cosh(x)^20 - 4180*cosh(x)^18 + 8415*cosh(x)^
16 - 11880*cosh(x)^14 + 12012*cosh(x)^12 - 8712*cosh(x)^10 + 4455*cosh(x)^8
 - 1540*cosh(x)^6 + 330*cosh(x)^4 - 36*cosh(x)^2 + 1)*e^(2*x)*sinh(x)^2 + 2
6*(cosh(x)^25 - 12*cosh(x)^23 + 66*cosh(x)^21 - 220*cosh(x)^19 + 495*cosh(x
)^17 - 792*cosh(x)^15 + 924*cosh(x)^13 - 792*cosh(x)^11 + 495*cosh(x)^9 - 2
20*cosh(x)^7 + 66*cosh(x)^5 - 12*cosh(x)^3 + cosh(x))*e^(2*x)*sinh(x) + (co
sh(x)^26 - 13*cosh(x)^24 + 78*cosh(x)^22 - 286*cosh(x)^20 + 715*cosh(x)^18
- 1287*cosh(x)^16 + 1716*cosh(x)^14 - 1716*cosh(x)^12 + 1287*cosh(x)^10 - 7
15*cosh(x)^8 + 286*cosh(x)^6 - 78*cosh(x)^4 + 13*cosh(x)^2 - 1)*e^(2*x))
```

giac [A] time = 0.14, size = 51, normalized size = 0.31

$$-\frac{2048\, a^{\frac{7}{2}} \left(1716\, e^{(12\, x)} - 1287\, e^{(10\, x)} + 715\, e^{(8\, x)} - 286\, e^{(6\, x)} + 78\, e^{(4\, x)} - 13\, e^{(2\, x)} + 1\right)}{3003\, {\left(e^{(2\, x)} - 1\right)}^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*csch(x)^4)^(7/2),x, algorithm="giac")
```

```
[Out] -2048/3003*a^(7/2)*(1716*e^(12*x) - 1287*e^(10*x) + 715*e^(8*x) - 286*e^(6*
x) + 78*e^(4*x) - 13*e^(2*x) + 1)/(e^(2*x) - 1)^13
```

maple [A] time = 0.28, size = 72, normalized size = 0.44

$$-\frac{2048 a^{3} e^{-2 x} \sqrt{\frac{a\, e^{4 x}}{\left(e^{2 x}-1\right)^{4}}}\, \left(1716\, e^{12 x}-1287\, e^{10 x}+715\, e^{8 x}-286\, e^{6 x}+78\, e^{4 x}-13\, e^{2 x}+1\right)}{3003 \left(e^{2 x}-1\right)^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*csch(x)^4)^(7/2),x)
```

```
[Out] -2048/3003*a^3*exp(-2*x)/(exp(2*x)-1)^11*(a*exp(4*x)/(exp(2*x)-1)^4)^(1/2)*
(1716*exp(12*x)-1287*exp(10*x)+715*exp(8*x)-286*exp(6*x)+78*exp(4*x)-13*exp
(2*x)+1)
```

maxima [B] time = 0.43, size = 620, normalized size = 3.78

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*csch(x)^4)^(7/2),x, algorithm="maxima")
```

```
[Out] -2048/231*a^(7/2)*e^(-2*x)/(13*e^(-2*x) - 78*e^(-4*x) + 286*e^(-6*x) - 715*
e^(-8*x) + 1287*e^(-10*x) - 1716*e^(-12*x) + 1716*e^(-14*x) - 1287*e^(-16*x
) + 715*e^(-18*x) - 286*e^(-20*x) + 78*e^(-22*x) - 13*e^(-24*x) + e^(-26*x)
 - 1) + 4096/77*a^(7/2)*e^(-4*x)/(13*e^(-2*x) - 78*e^(-4*x) + 286*e^(-6*x)
- 715*e^(-8*x) + 1287*e^(-10*x) - 1716*e^(-12*x) + 1716*e^(-14*x) - 1287*e^
(-16*x) + 715*e^(-18*x) - 286*e^(-20*x) + 78*e^(-22*x) - 13*e^(-24*x) + e^(
-26*x) - 1) - 4096/21*a^(7/2)*e^(-6*x)/(13*e^(-2*x) - 78*e^(-4*x) + 286*e^(
-6*x) - 715*e^(-8*x) + 1287*e^(-10*x) - 1716*e^(-12*x) + 1716*e^(-14*x) - 1
287*e^(-16*x) + 715*e^(-18*x) - 286*e^(-20*x) + 78*e^(-22*x) - 13*e^(-24*x)
 + e^(-26*x) - 1) + 10240/21*a^(7/2)*e^(-8*x)/(13*e^(-2*x) - 78*e^(-4*x) +
286*e^(-6*x) - 715*e^(-8*x) + 1287*e^(-10*x) - 1716*e^(-12*x) + 1716*e^(-14
*x) - 1287*e^(-16*x) + 715*e^(-18*x) - 286*e^(-20*x) + 78*e^(-22*x) - 13*e^
(-24*x) + e^(-26*x) - 1) - 6144/7*a^(7/2)*e^(-10*x)/(13*e^(-2*x) - 78*e^(-4
*x) + 286*e^(-6*x) - 715*e^(-8*x) + 1287*e^(-10*x) - 1716*e^(-12*x) + 1716*
e^(-14*x) - 1287*e^(-16*x) + 715*e^(-18*x) - 286*e^(-20*x) + 78*e^(-22*x) -
 13*e^(-24*x) + e^(-26*x) - 1) + 8192/7*a^(7/2)*e^(-12*x)/(13*e^(-2*x) - 78
*e^(-4*x) + 286*e^(-6*x) - 715*e^(-8*x) + 1287*e^(-10*x) - 1716*e^(-12*x) +
 1716*e^(-14*x) - 1287*e^(-16*x) + 715*e^(-18*x) - 286*e^(-20*x) + 78*e^(-2
2*x) - 13*e^(-24*x) + e^(-26*x) - 1) + 2048/3003*a^(7/2)/(13*e^(-2*x) - 78*
e^(-4*x) + 286*e^(-6*x) - 715*e^(-8*x) + 1287*e^(-10*x) - 1716*e^(-12*x) +
1716*e^(-14*x) - 1287*e^(-16*x) + 715*e^(-18*x) - 286*e^(-20*x) + 78*e^(-22
*x) - 13*e^(-24*x) + e^(-26*x) - 1)
```

mupad [B] time = 1.48, size = 498, normalized size = 3.04

$$-\frac{2048\, a^{3} \sqrt{\frac{a}{\left(\frac{e^{-x}}{2}-\frac{e^{x}}{2}\right)^{4}}}\, \left(6\, e^{4 x}-4\, e^{2 x}-4\, e^{6 x}+e^{8 x}+1\right)}{7 \left(e^{2 x}-1\right)^{7} \left(e^{2 x}-2\, e^{4 x}+e^{6 x}\right)}-\frac{1536\, a^{3} \sqrt{\frac{a}{\left(\frac{e^{-x}}{2}-\frac{e^{x}}{2}\right)^{4}}}\, \left(6\, e^{4 x}-4\, e^{2 x}-4\, e^{6 x}+e^{8 x}+1\right)}{\left(e^{2 x}-1\right)^{8} \left(e^{2 x}-2\, e^{4 x}+e^{6 x}\right)}-1$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a/sinh(x)^4)^(7/2),x)
```

```
[Out] - (2048*a^3*(a/(exp(-x)/2 - exp(x)/2)^4)^(1/2)*(6*exp(4*x) - 4*exp(2*x) - 4
*exp(6*x) + exp(8*x) + 1))/(7*(exp(2*x) - 1)^7*(exp(2*x) - 2*exp(4*x) + exp
(6*x))) - (1536*a^3*(a/(exp(-x)/2 - exp(x)/2)^4)^(1/2)*(6*exp(4*x) - 4*exp(
2*x) - 4*exp(6*x) + exp(8*x) + 1))/((exp(2*x) - 1)^8*(exp(2*x) - 2*exp(4*x)
```

```
 + exp(6*x))) - (10240*a^3*(a/(exp(-x)/2 - exp(x)/2)^4)^(1/2)*(6*exp(4*x) -
 4*exp(2*x) - 4*exp(6*x) + exp(8*x) + 1))/(3*(exp(2*x) - 1)^9*(exp(2*x) - 2
*exp(4*x) + exp(6*x))) - (4096*a^3*(a/(exp(-x)/2 - exp(x)/2)^4)^(1/2)*(6*ex
p(4*x) - 4*exp(2*x) - 4*exp(6*x) + exp(8*x) + 1))/((exp(2*x) - 1)^10*(exp(2
*x) - 2*exp(4*x) + exp(6*x))) - (30720*a^3*(a/(exp(-x)/2 - exp(x)/2)^4)^(1/
2)*(6*exp(4*x) - 4*exp(2*x) - 4*exp(6*x) + exp(8*x) + 1))/(11*(exp(2*x) - 1
)^11*(exp(2*x) - 2*exp(4*x) + exp(6*x))) - (1024*a^3*(a/(exp(-x)/2 - exp(x)
/2)^4)^(1/2)*(6*exp(4*x) - 4*exp(2*x) - 4*exp(6*x) + exp(8*x) + 1))/((exp(2
*x) - 1)^12*(exp(2*x) - 2*exp(4*x) + exp(6*x))) - (2048*a^3*(a/(exp(-x)/2 -
 exp(x)/2)^4)^(1/2)*(6*exp(4*x) - 4*exp(2*x) - 4*exp(6*x) + exp(8*x) + 1))/
(13*(exp(2*x) - 1)^13*(exp(2*x) - 2*exp(4*x) + exp(6*x)))
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a \operatorname{csch}^4(x)\right)^{\frac{7}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*csch(x)**4)**(7/2),x)
```

```
[Out] Integral((a*csch(x)**4)**(7/2), x)
```

3.43 $\int \left(a\operatorname{csch}^4(x)\right)^{5/2}\,dx$

Optimal. Leaf size=118

$$-\frac{1}{9}a^2\cosh^2(x)\coth^7(x)\sqrt{a\operatorname{csch}^4(x)}+\frac{4}{7}a^2\cosh^2(x)\coth^5(x)\sqrt{a\operatorname{csch}^4(x)}-\frac{6}{5}a^2\cosh^2(x)\coth^3(x)\sqrt{a\operatorname{csch}^4(x)}+\frac{4}{3}a$$

```
[Out] 4/3*a^2*cosh(x)^2*coth(x)*(a*csch(x)^4)^(1/2)-6/5*a^2*cosh(x)^2*coth(x)^3*(
a*csch(x)^4)^(1/2)+4/7*a^2*cosh(x)^2*coth(x)^5*(a*csch(x)^4)^(1/2)-1/9*a^2*
cosh(x)^2*coth(x)^7*(a*csch(x)^4)^(1/2)-a^2*cosh(x)*sinh(x)*(a*csch(x)^4)^(
1/2)
```

Rubi [A] time = 0.03, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {4123, 3767}

$$-\frac{1}{9}a^2\cosh^2(x)\coth^7(x)\sqrt{a\operatorname{csch}^4(x)}+\frac{4}{7}a^2\cosh^2(x)\coth^5(x)\sqrt{a\operatorname{csch}^4(x)}-\frac{6}{5}a^2\cosh^2(x)\coth^3(x)\sqrt{a\operatorname{csch}^4(x)}+\frac{4}{3}a$$

Antiderivative was successfully verified.

```
[In] Int[(a*Csch[x]^4)^(5/2),x]
```

```
[Out] (4*a^2*Cosh[x]^2*Coth[x]*Sqrt[a*Csch[x]^4])/3 - (6*a^2*Cosh[x]^2*Coth[x]^3*
Sqrt[a*Csch[x]^4])/5 + (4*a^2*Cosh[x]^2*Coth[x]^5*Sqrt[a*Csch[x]^4])/7 - (a
^2*Cosh[x]^2*Coth[x]^7*Sqrt[a*Csch[x]^4])/9 - a^2*Cosh[x]*Sqrt[a*Csch[x]^4]
*Sinh[x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 4123

```
Int[((b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> Dist[(b
^IntPart[p]*(b*(c*Sec[e + f*x])^n)^FracPart[p])/(c*Sec[e + f*x])^(n*FracPar
t[p]), Int[(c*Sec[e + f*x])^(n*p), x], x] /; FreeQ[{b, c, e, f, n, p}, x] &
& !IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \left(a\operatorname{csch}^4(x)\right)^{5/2}\,dx &= \left(a^2\sqrt{a\operatorname{csch}^4(x)}\,\sinh^2(x)\right)\int \operatorname{csch}^{10}(x)\,dx\\ &= -\left(\left(ia^2\sqrt{a\operatorname{csch}^4(x)}\,\sinh^2(x)\right)\operatorname{Subst}\left(\int \left(1+4x^2+6x^4+4x^6+x^8\right)\,dx, x, -i\coth(x)\right)\right)\\ &= \frac{4}{3}a^2\cosh^2(x)\coth(x)\sqrt{a\operatorname{csch}^4(x)}\ -\frac{6}{5}a^2\cosh^2(x)\coth^3(x)\sqrt{a\operatorname{csch}^4(x)}\ +\frac{4}{7}a^2\cosh^2(x)\,\mathrm{co}\end{aligned}$$

Mathematica [A] time = 0.03, size = 47, normalized size = 0.40

$$-\frac{1}{315}a^2\sinh(x)\cosh(x)\left(35\operatorname{csch}^8(x)-40\operatorname{csch}^6(x)+48\operatorname{csch}^4(x)-64\operatorname{csch}^2(x)+128\right)\sqrt{a\operatorname{csch}^4(x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Csch[x]^4)^(5/2),x]
```

```
[Out] -1/315*(a^2*Cosh[x]*Sqrt[a*Csch[x]^4]*(128 - 64*Csch[x]^2 + 48*Csch[x]^4 -
40*Csch[x]^6 + 35*Csch[x]^8)*Sinh[x])
```

fricas [B] time = 0.78, size = 1493, normalized size = 12.65

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*csch(x)^4)^(5/2),x, algorithm="fricas")
```

```
[Out] -256/315*(126*a^2*cosh(x)^8 + 126*(a^2*e^(4*x) - 2*a^2*e^(2*x) + a^2)*sinh(
x)^8 - 84*a^2*cosh(x)^6 + 1008*(a^2*cosh(x)*e^(4*x) - 2*a^2*cosh(x)*e^(2*x)
 + a^2*cosh(x))*sinh(x)^7 + 84*(42*a^2*cosh(x)^2 - a^2 + (42*a^2*cosh(x)^2
- a^2)*e^(4*x) - 2*(42*a^2*cosh(x)^2 - a^2)*e^(2*x))*sinh(x)^6 + 36*a^2*cos
h(x)^4 + 504*(14*a^2*cosh(x)^3 - a^2*cosh(x) + (14*a^2*cosh(x)^3 - a^2*cosh
(x))*e^(4*x) - 2*(14*a^2*cosh(x)^3 - a^2*cosh(x))*e^(2*x))*sinh(x)^5 + 36*(
245*a^2*cosh(x)^4 - 35*a^2*cosh(x)^2 + a^2 + (245*a^2*cosh(x)^4 - 35*a^2*co
sh(x)^2 + a^2)*e^(4*x) - 2*(245*a^2*cosh(x)^4 - 35*a^2*cosh(x)^2 + a^2)*e^(
2*x))*sinh(x)^4 - 9*a^2*cosh(x)^2 + 48*(147*a^2*cosh(x)^5 - 35*a^2*cosh(x)^
3 + 3*a^2*cosh(x) + (147*a^2*cosh(x)^5 - 35*a^2*cosh(x)^3 + 3*a^2*cosh(x))*
e^(4*x) - 2*(147*a^2*cosh(x)^5 - 35*a^2*cosh(x)^3 + 3*a^2*cosh(x))*e^(2*x))
*sinh(x)^3 + 9*(392*a^2*cosh(x)^6 - 140*a^2*cosh(x)^4 + 24*a^2*cosh(x)^2 -
a^2 + (392*a^2*cosh(x)^6 - 140*a^2*cosh(x)^4 + 24*a^2*cosh(x)^2 - a^2)*e^(4
*x) - 2*(392*a^2*cosh(x)^6 - 140*a^2*cosh(x)^4 + 24*a^2*cosh(x)^2 - a^2)*e^
(2*x))*sinh(x)^2 + a^2 + (126*a^2*cosh(x)^8 - 84*a^2*cosh(x)^6 + 36*a^2*cos
h(x)^4 - 9*a^2*cosh(x)^2 + a^2)*e^(4*x) - 2*(126*a^2*cosh(x)^8 - 84*a^2*cos
h(x)^6 + 36*a^2*cosh(x)^4 - 9*a^2*cosh(x)^2 + a^2)*e^(2*x) + 18*(56*a^2*cos
h(x)^7 - 28*a^2*cosh(x)^5 + 8*a^2*cosh(x)^3 - a^2*cosh(x) + (56*a^2*cosh(x)
^7 - 28*a^2*cosh(x)^5 + 8*a^2*cosh(x)^3 - a^2*cosh(x))*e^(4*x) - 2*(56*a^2*
cosh(x)^7 - 28*a^2*cosh(x)^5 + 8*a^2*cosh(x)^3 - a^2*cosh(x))*e^(2*x))*sinh
(x))*sqrt(a/(e^(8*x) - 4*e^(6*x) + 6*e^(4*x) - 4*e^(2*x) + 1))*e^(2*x)/(18*
cosh(x)*e^(2*x)*sinh(x)^17 + e^(2*x)*sinh(x)^18 + 9*(17*cosh(x)^2 - 1)*e^(2
*x)*sinh(x)^16 + 48*(17*cosh(x)^3 - 3*cosh(x))*e^(2*x)*sinh(x)^15 + 36*(85*
cosh(x)^4 - 30*cosh(x)^2 + 1)*e^(2*x)*sinh(x)^14 + 504*(17*cosh(x)^5 - 10*c
osh(x)^3 + cosh(x))*e^(2*x)*sinh(x)^13 + 84*(221*cosh(x)^6 - 195*cosh(x)^4
+ 39*cosh(x)^2 - 1)*e^(2*x)*sinh(x)^12 + 144*(221*cosh(x)^7 - 273*cosh(x)^5
 + 91*cosh(x)^3 - 7*cosh(x))*e^(2*x)*sinh(x)^11 + 18*(2431*cosh(x)^8 - 4004
*cosh(x)^6 + 2002*cosh(x)^4 - 308*cosh(x)^2 + 7)*e^(2*x)*sinh(x)^10 + 4*(12
155*cosh(x)^9 - 25740*cosh(x)^7 + 18018*cosh(x)^5 - 4620*cosh(x)^3 + 315*co
sh(x))*e^(2*x)*sinh(x)^9 + 18*(2431*cosh(x)^10 - 6435*cosh(x)^8 + 6006*cosh
(x)^6 - 2310*cosh(x)^4 + 315*cosh(x)^2 - 7)*e^(2*x)*sinh(x)^8 + 144*(221*co
sh(x)^11 - 715*cosh(x)^9 + 858*cosh(x)^7 - 462*cosh(x)^5 + 105*cosh(x)^3 -
7*cosh(x))*e^(2*x)*sinh(x)^7 + 84*(221*cosh(x)^12 - 858*cosh(x)^10 + 1287*c
osh(x)^8 - 924*cosh(x)^6 + 315*cosh(x)^4 - 42*cosh(x)^2 + 1)*e^(2*x)*sinh(x
)^6 + 504*(17*cosh(x)^13 - 78*cosh(x)^11 + 143*cosh(x)^9 - 132*cosh(x)^7 +
63*cosh(x)^5 - 14*cosh(x)^3 + cosh(x))*e^(2*x)*sinh(x)^5 + 36*(85*cosh(x)^1
4 - 455*cosh(x)^12 + 1001*cosh(x)^10 - 1155*cosh(x)^8 + 735*cosh(x)^6 - 245
*cosh(x)^4 + 35*cosh(x)^2 - 1)*e^(2*x)*sinh(x)^4 + 48*(17*cosh(x)^15 - 105*
cosh(x)^13 + 273*cosh(x)^11 - 385*cosh(x)^9 + 315*cosh(x)^7 - 147*cosh(x)^5
 + 35*cosh(x)^3 - 3*cosh(x))*e^(2*x)*sinh(x)^3 + 9*(17*cosh(x)^16 - 120*cos
h(x)^14 + 364*cosh(x)^12 - 616*cosh(x)^10 + 630*cosh(x)^8 - 392*cosh(x)^6 +
 140*cosh(x)^4 - 24*cosh(x)^2 + 1)*e^(2*x)*sinh(x)^2 + 18*(cosh(x)^17 - 8*c
osh(x)^15 + 28*cosh(x)^13 - 56*cosh(x)^11 + 70*cosh(x)^9 - 56*cosh(x)^7 + 2
8*cosh(x)^5 - 8*cosh(x)^3 + cosh(x))*e^(2*x)*sinh(x) + (cosh(x)^18 - 9*cosh
(x)^16 + 36*cosh(x)^14 - 84*cosh(x)^12 + 126*cosh(x)^10 - 126*cosh(x)^8 + 8
4*cosh(x)^6 - 36*cosh(x)^4 + 9*cosh(x)^2 - 1)*e^(2*x))
```

giac [A] time = 0.12, size = 39, normalized size = 0.33

$$-\frac{256\,a^{\frac{5}{2}}\left(126\,e^{(8\,x)}-84\,e^{(6\,x)}+36\,e^{(4\,x)}-9\,e^{(2\,x)}+1\right)}{315\,\left(e^{(2\,x)}-1\right)^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*csch(x)^4)^(5/2),x, algorithm="giac")
```

```
[Out] -256/315*a^(5/2)*(126*e^(8*x) - 84*e^(6*x) + 36*e^(4*x) - 9*e^(2*x) + 1)/(e
^(2*x) - 1)^9
```

maple [A] time = 0.22, size = 60, normalized size = 0.51

$$-\frac{256a^{2}\mathrm{e}^{-2x}\sqrt{\frac{a\,\mathrm{e}^{4x}}{\left(\mathrm{e}^{2x}-1\right)^{4}}}\,\left(126\,\mathrm{e}^{8x}-84\,\mathrm{e}^{6x}+36\,\mathrm{e}^{4x}-9\,\mathrm{e}^{2x}+1\right)}{315\,\left(\mathrm{e}^{2x}-1\right)^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*csch(x)^4)^(5/2),x)
```

```
[Out] -256/315*a^2*exp(-2*x)/(exp(2*x)-1)^7*(a*exp(4*x)/(exp(2*x)-1)^4)^(1/2)*(12
6*exp(8*x)-84*exp(6*x)+36*exp(4*x)-9*exp(2*x)+1)
```

maxima [B] time = 0.49, size = 322, normalized size = 2.73

$$-\frac{256\,a^{\frac{5}{2}}e^{(-2\,x)}}{35\,\left(9\,e^{(-2\,x)}-36\,e^{(-4\,x)}+84\,e^{(-6\,x)}-126\,e^{(-8\,x)}+126\,e^{(-10\,x)}-84\,e^{(-12\,x)}+36\,e^{(-14\,x)}-9\,e^{(-16\,x)}+e^{(-18\,x)}-1\right)}+\ldots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*csch(x)^4)^(5/2),x, algorithm="maxima")
```

```
[Out] -256/35*a^(5/2)*e^(-2*x)/(9*e^(-2*x) - 36*e^(-4*x) + 84*e^(-6*x) - 126*e^(-
8*x) + 126*e^(-10*x) - 84*e^(-12*x) + 36*e^(-14*x) - 9*e^(-16*x) + e^(-18*x
) - 1) + 1024/35*a^(5/2)*e^(-4*x)/(9*e^(-2*x) - 36*e^(-4*x) + 84*e^(-6*x) -
 126*e^(-8*x) + 126*e^(-10*x) - 84*e^(-12*x) + 36*e^(-14*x) - 9*e^(-16*x) +
 e^(-18*x) - 1) - 1024/15*a^(5/2)*e^(-6*x)/(9*e^(-2*x) - 36*e^(-4*x) + 84*e
^(-6*x) - 126*e^(-8*x) + 126*e^(-10*x) - 84*e^(-12*x) + 36*e^(-14*x) - 9*e^
(-16*x) + e^(-18*x) - 1) + 512/5*a^(5/2)*e^(-8*x)/(9*e^(-2*x) - 36*e^(-4*x)
 + 84*e^(-6*x) - 126*e^(-8*x) + 126*e^(-10*x) - 84*e^(-12*x) + 36*e^(-14*x)
 - 9*e^(-16*x) + e^(-18*x) - 1) + 256/315*a^(5/2)/(9*e^(-2*x) - 36*e^(-4*x)
 + 84*e^(-6*x) - 126*e^(-8*x) + 126*e^(-10*x) - 84*e^(-12*x) + 36*e^(-14*x)
 - 9*e^(-16*x) + e^(-18*x) - 1)
```

mupad [B] time = 1.44, size = 356, normalized size = 3.02

$$-\frac{128\,a^{2}\,\sqrt{\frac{a}{{\left(\frac{\mathrm{e}^{-x}}{2}-\frac{\mathrm{e}^{x}}{2}\right)}^{4}}}\,\left(6\,\mathrm{e}^{4\,x}-4\,\mathrm{e}^{2\,x}-4\,\mathrm{e}^{6\,x}+\mathrm{e}^{8\,x}+1\right)}{5\,{\left(\mathrm{e}^{2\,x}-1\right)}^{5}\,\left(\mathrm{e}^{2\,x}-2\,\mathrm{e}^{4\,x}+\mathrm{e}^{6\,x}\right)}-\frac{256\,a^{2}\,\sqrt{\frac{a}{{\left(\frac{\mathrm{e}^{-x}}{2}-\frac{\mathrm{e}^{x}}{2}\right)}^{4}}}\,\left(6\,\mathrm{e}^{4\,x}-4\,\mathrm{e}^{2\,x}-4\,\mathrm{e}^{6\,x}+\mathrm{e}^{8\,x}+1\right)}{3\,{\left(\mathrm{e}^{2\,x}-1\right)}^{6}\,\left(\mathrm{e}^{2\,x}-2\,\mathrm{e}^{4\,x}+\mathrm{e}^{6\,x}\right)}-\ldots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a/sinh(x)^4)^(5/2),x)
```

```
[Out] - (128*a^2*(a/(exp(-x)/2 - exp(x)/2)^4)^(1/2)*(6*exp(4*x) - 4*exp(2*x) - 4*
exp(6*x) + exp(8*x) + 1))/(5*(exp(2*x) - 1)^5*(exp(2*x) - 2*exp(4*x) + exp(
```

```
6*x))) - (256*a^2*(a/(exp(-x)/2 - exp(x)/2)^4)^(1/2)*(6*exp(4*x) - 4*exp(2*
x) - 4*exp(6*x) + exp(8*x) + 1))/(3*(exp(2*x) - 1)^6*(exp(2*x) - 2*exp(4*x)
 + exp(6*x))) - (768*a^2*(a/(exp(-x)/2 - exp(x)/2)^4)^(1/2)*(6*exp(4*x) - 4
*exp(2*x) - 4*exp(6*x) + exp(8*x) + 1))/(7*(exp(2*x) - 1)^7*(exp(2*x) - 2*e
xp(4*x) + exp(6*x))) - (64*a^2*(a/(exp(-x)/2 - exp(x)/2)^4)^(1/2)*(6*exp(4*
x) - 4*exp(2*x) - 4*exp(6*x) + exp(8*x) + 1))/((exp(2*x) - 1)^8*(exp(2*x) -
 2*exp(4*x) + exp(6*x))) - (128*a^2*(a/(exp(-x)/2 - exp(x)/2)^4)^(1/2)*(6*e
xp(4*x) - 4*exp(2*x) - 4*exp(6*x) + exp(8*x) + 1))/(9*(exp(2*x) - 1)^9*(exp
(2*x) - 2*exp(4*x) + exp(6*x)))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a \operatorname{csch}^4(x)\right)^{\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*csch(x)**4)**(5/2),x)
[Out] Integral((a*csch(x)**4)**(5/2), x)
```

3.44 $\int \left(a\operatorname{csch}^4(x)\right)^{3/2}\,dx$

Optimal. Leaf size=62

$$-\frac{1}{5}a\cosh^2(x)\coth^3(x)\sqrt{a\operatorname{csch}^4(x)}+\frac{2}{3}a\cosh^2(x)\coth(x)\sqrt{a\operatorname{csch}^4(x)}-a\sinh(x)\cosh(x)\sqrt{a\operatorname{csch}^4(x)}$$

```
[Out] 2/3*a*cosh(x)^2*coth(x)*(a*csch(x)^4)^(1/2)-1/5*a*cosh(x)^2*coth(x)^3*(a*cs
ch(x)^4)^(1/2)-a*cosh(x)*sinh(x)*(a*csch(x)^4)^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {4123, 3767}

$$-\frac{1}{5}a\cosh^2(x)\coth^3(x)\sqrt{a\operatorname{csch}^4(x)}+\frac{2}{3}a\cosh^2(x)\coth(x)\sqrt{a\operatorname{csch}^4(x)}-a\sinh(x)\cosh(x)\sqrt{a\operatorname{csch}^4(x)}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Csch[x]^4)^(3/2),x]
```

```
[Out] (2*a*Cosh[x]^2*Coth[x]*Sqrt[a*Csch[x]^4])/3 - (a*Cosh[x]^2*Coth[x]^3*Sqrt[a
*Csch[x]^4])/5 - a*Cosh[x]*Sqrt[a*Csch[x]^4]*Sinh[x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 4123

```
Int[((b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> Dist[(b
^IntPart[p]*(b*(c*Sec[e + f*x])^n)^FracPart[p])/(c*Sec[e + f*x])^(n*FracPar
t[p]), Int[(c*Sec[e + f*x])^(n*p), x], x] /; FreeQ[{b, c, e, f, n, p}, x] &
& !IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \left(a\operatorname{csch}^4(x)\right)^{3/2}\,dx &= \left(a\sqrt{a\operatorname{csch}^4(x)}\,\sinh^2(x)\right)\int \operatorname{csch}^6(x)\,dx\\ &= -\left(\left(ia\sqrt{a\operatorname{csch}^4(x)}\,\sinh^2(x)\right)\operatorname{Subst}\left(\int \left(1+2x^2+x^4\right)\,dx, x, -i\coth(x)\right)\right)\\ &= \frac{2}{3}a\cosh^2(x)\coth(x)\sqrt{a\operatorname{csch}^4(x)} - \frac{1}{5}a\cosh^2(x)\coth^3(x)\sqrt{a\operatorname{csch}^4(x)} - a\cosh(x)\sqrt{a\operatorname{csch}\right.}\end{aligned}$$

Mathematica [A] time = 0.02, size = 33, normalized size = 0.53

$$-\frac{1}{15}a\sinh(x)\cosh(x)\left(3\operatorname{csch}^4(x)-4\operatorname{csch}^2(x)+8\right)\sqrt{a\operatorname{csch}^4(x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Csch[x]^4)^(3/2),x]
```

```
[Out] -1/15*(a*Cosh[x]*Sqrt[a*Csch[x]^4]*(8 - 4*Csch[x]^2 + 3*Csch[x]^4)*Sinh[x])
```

fricas [B] time = 1.57, size = 529, normalized size = 8.53

$$-\frac{16\left(10\,a\cosh(x)^4+10\left(\cdots\right.\right.}{15\left(10\,\cosh(x)e^{(2\,x)}\sinh(x)^9+e^{(2\,x)}\sinh(x)^{10}+5\left(9\,\cosh(x)^2-1\right)e^{(2\,x)}\sinh(x)^8+40\left(3\,\cosh(x)^3-\cosh(x)\cdots\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*csch(x)^4)^(3/2),x, algorithm="fricas")
```

```
[Out] -16/15*(10*a*cosh(x)^4 + 10*(a*e^(4*x) - 2*a*e^(2*x) + a)*sinh(x)^4 + 40*(a
*cosh(x)*e^(4*x) - 2*a*cosh(x)*e^(2*x) + a*cosh(x))*sinh(x)^3 - 5*a*cosh(x)
^2 + 5*(12*a*cosh(x)^2 + (12*a*cosh(x)^2 - a)*e^(4*x) - 2*(12*a*cosh(x)^2 -
 a)*e^(2*x) - a)*sinh(x)^2 + (10*a*cosh(x)^4 - 5*a*cosh(x)^2 + a)*e^(4*x) -
 2*(10*a*cosh(x)^4 - 5*a*cosh(x)^2 + a)*e^(2*x) + 10*(4*a*cosh(x)^3 - a*cos
h(x) + (4*a*cosh(x)^3 - a*cosh(x))*e^(4*x) - 2*(4*a*cosh(x)^3 - a*cosh(x))*
e^(2*x))*sinh(x) + a)*sqrt(a/(e^(8*x) - 4*e^(6*x) + 6*e^(4*x) - 4*e^(2*x) +
 1))*e^(2*x)/(10*cosh(x)*e^(2*x)*sinh(x)^9 + e^(2*x)*sinh(x)^10 + 5*(9*cosh
(x)^2 - 1)*e^(2*x)*sinh(x)^8 + 40*(3*cosh(x)^3 - cosh(x))*e^(2*x)*sinh(x)^7
 + 10*(21*cosh(x)^4 - 14*cosh(x)^2 + 1)*e^(2*x)*sinh(x)^6 + 4*(63*cosh(x)^5
 - 70*cosh(x)^3 + 15*cosh(x))*e^(2*x)*sinh(x)^5 + 10*(21*cosh(x)^6 - 35*cos
h(x)^4 + 15*cosh(x)^2 - 1)*e^(2*x)*sinh(x)^4 + 40*(3*cosh(x)^7 - 7*cosh(x)^
5 + 5*cosh(x)^3 - cosh(x))*e^(2*x)*sinh(x)^3 + 5*(9*cosh(x)^8 - 28*cosh(x)^
6 + 30*cosh(x)^4 - 12*cosh(x)^2 + 1)*e^(2*x)*sinh(x)^2 + 10*(cosh(x)^9 - 4*
cosh(x)^7 + 6*cosh(x)^5 - 4*cosh(x)^3 + cosh(x))*e^(2*x)*sinh(x) + (cosh(x)
^10 - 5*cosh(x)^8 + 10*cosh(x)^6 - 10*cosh(x)^4 + 5*cosh(x)^2 - 1)*e^(2*x))
```

giac [A] time = 0.13, size = 27, normalized size = 0.44

$$-\frac{16\,a^{\frac{3}{2}}\left(10\,e^{(4\,x)}-5\,e^{(2\,x)}+1\right)}{15\left(e^{(2\,x)}-1\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*csch(x)^4)^(3/2),x, algorithm="giac")
```

```
[Out] -16/15*a^(3/2)*(10*e^(4*x) - 5*e^(2*x) + 1)/(e^(2*x) - 1)^5
```

maple [A] time = 0.20, size = 46, normalized size = 0.74

$$-\frac{16a\,\mathrm{e}^{-2x}\sqrt{\frac{a\,\mathrm{e}^{4x}}{\left(\mathrm{e}^{2x}-1\right)^4}}\,\left(10\,\mathrm{e}^{4x}-5\,\mathrm{e}^{2x}+1\right)}{15\left(\mathrm{e}^{2x}-1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*csch(x)^4)^(3/2),x)
```

```
[Out] -16/15*a*exp(-2*x)/(exp(2*x)-1)^3*(a*exp(4*x)/(exp(2*x)-1)^4)^(1/2)*(10*exp
(4*x)-5*exp(2*x)+1)
```

maxima [B] time = 0.51, size = 120, normalized size = 1.94

$$-\frac{16\,a^{\frac{3}{2}}e^{(-2\,x)}}{3\left(5\,e^{(-2\,x)}-10\,e^{(-4\,x)}+10\,e^{(-6\,x)}-5\,e^{(-8\,x)}+e^{(-10\,x)}-1\right)}+\frac{32\,a^{\frac{3}{2}}e^{(-4\,x)}}{3\left(5\,e^{(-2\,x)}-10\,e^{(-4\,x)}+10\,e^{(-6\,x)}-5\,e^{(-8\,x)}+e^{(-10\,x)}\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*csch(x)^4)^(3/2),x, algorithm="maxima")
```

```
[Out] -16/3*a^(3/2)*e^(-2*x)/(5*e^(-2*x) - 10*e^(-4*x) + 10*e^(-6*x) - 5*e^(-8*x)
 + e^(-10*x) - 1) + 32/3*a^(3/2)*e^(-4*x)/(5*e^(-2*x) - 10*e^(-4*x) + 10*e^
(-6*x) - 5*e^(-8*x) + e^(-10*x) - 1) + 16/15*a^(3/2)/(5*e^(-2*x) - 10*e^(-4
*x) + 10*e^(-6*x) - 5*e^(-8*x) + e^(-10*x) - 1)
```

mupad [B] time = 1.46, size = 46, normalized size = 0.74

$$-\frac{4\,a\,e^{-2\,x}\,\sqrt{\frac{a}{{\left(\frac{e^{-x}}{2}-\frac{e^{x}}{2}\right)}^4}}\,\left(10\,e^{4\,x}-5\,e^{2\,x}+1\right)}{15\,{\left(e^{2\,x}-1\right)}^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a/sinh(x)^4)^(3/2),x)
```

```
[Out] -(4*a*exp(-2*x)*(a/(exp(-x)/2 - exp(x)/2)^4)^(1/2)*(10*exp(4*x) - 5*exp(2*x
) + 1))/(15*(exp(2*x) - 1)^3)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a \operatorname{csch}^{4}{\left(x \right)}\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*csch(x)**4)**(3/2),x)
```

```
[Out] Integral((a*csch(x)**4)**(3/2), x)
```

3.45 $\int \sqrt{a\text{csch}^4(x)}\, dx$

Optimal. Leaf size=16

$$\sinh(x)(-\cosh(x))\sqrt{a\text{csch}^4(x)}$$

```
[Out] -cosh(x)*sinh(x)*(a*csch(x)^4)^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {4123, 3767, 8}

$$\sinh(x)(-\cosh(x))\sqrt{a\text{csch}^4(x)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*Csch[x]^4],x]
```

```
[Out] -(Cosh[x]*Sqrt[a*Csch[x]^4]*Sinh[x])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 4123

```
Int[((b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> Dist[(b
^IntPart[p]*(b*(c*Sec[e + f*x])^n)^FracPart[p])/(c*Sec[e + f*x])^(n*FracPar
t[p]), Int[(c*Sec[e + f*x])^(n*p), x], x] /; FreeQ[{b, c, e, f, n, p}, x] &
&  !IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\int \sqrt{a\text{csch}^4(x)}\, dx &= \left(\sqrt{a\text{csch}^4(x)}\,\sinh^2(x)\right)\int \text{csch}^2(x)\,dx\\
&= -\left(\left(i\sqrt{a\text{csch}^4(x)}\,\sinh^2(x)\right)\text{Subst}\left(\int 1\,dx, x, -i\coth(x)\right)\right)\\
&= -\cosh(x)\sqrt{a\text{csch}^4(x)}\,\sinh(x)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 16, normalized size = 1.00

$$\sinh(x)(-\cosh(x))\sqrt{a\text{csch}^4(x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*Csch[x]^4],x]
```

```
[Out] -(Cosh[x]*Sqrt[a*Csch[x]^4]*Sinh[x])
```

fricas [B] time = 0.67, size = 81, normalized size = 5.06

$$-\frac{2\sqrt{\frac{a}{e^{(8x)}-4e^{(6x)}+6e^{(4x)}-4e^{(2x)}+1}}\left(e^{(4x)}-2e^{(2x)}+1\right)e^{(2x)}}{2\cosh(x)e^{(2x)}\sinh(x)+e^{(2x)}\sinh(x)^2+\left(\cosh(x)^2-1\right)e^{(2x)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*csch(x)^4)^(1/2),x, algorithm="fricas")
```

```
[Out] -2*sqrt(a/(e^(8*x) - 4*e^(6*x) + 6*e^(4*x) - 4*e^(2*x) + 1))*(e^(4*x) - 2*e
^(2*x) + 1)*e^(2*x)/(2*cosh(x)*e^(2*x)*sinh(x) + e^(2*x)*sinh(x)^2 + (cosh(
x)^2 - 1)*e^(2*x))
```

giac [A] time = 0.11, size = 13, normalized size = 0.81

$$-\frac{2\sqrt{a}}{e^{(2x)}-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*csch(x)^4)^(1/2),x, algorithm="giac")
```

```
[Out] -2*sqrt(a)/(e^(2*x) - 1)
```

maple [A] time = 0.23, size = 29, normalized size = 1.81

$$-2\sqrt{\frac{a\,e^{4x}}{\left(e^{2x}-1\right)^4}}\,e^{-2x}\left(e^{2x}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*csch(x)^4)^(1/2),x)
```

```
[Out] -2*(a*exp(4*x)/(exp(2*x)-1)^4)^(1/2)*exp(-2*x)*(exp(2*x)-1)
```

maxima [A] time = 0.50, size = 13, normalized size = 0.81

$$\frac{2\sqrt{a}}{e^{(-2x)}-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*csch(x)^4)^(1/2),x, algorithm="maxima")
```

```
[Out] 2*sqrt(a)/(e^(-2*x) - 1)
```

mupad [B] time = 1.46, size = 71, normalized size = 4.44

$$-\frac{\sqrt{a}\sqrt{\frac{1}{\left(\frac{e^{-x}}{2}-\frac{e^{x}}{2}\right)^4}}\left(3\,e^{4x}-2\,e^{2x}-2\,e^{6x}+\frac{e^{8x}}{2}+\frac{1}{2}\right)}{\left(e^{2x}-1\right)\left(e^{2x}-2\,e^{4x}+e^{6x}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a/sinh(x)^4)^(1/2),x)
```

```
[Out] -(a^(1/2)*(1/(exp(-x)/2 - exp(x)/2)^4)^(1/2)*(3*exp(4*x) - 2*exp(2*x) - 2*e
xp(6*x) + exp(8*x)/2 + 1/2))/((exp(2*x) - 1)*(exp(2*x) - 2*exp(4*x) + exp(6
*x)))
```

<u>**sympy**</u> **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a \operatorname{csch}^4(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*csch(x)**4)**(1/2),x)
```

```
[Out] Integral(sqrt(a*csch(x)**4), x)
```

3.46 $\int \frac{1}{\sqrt{a\,\mathrm{csch}^4(x)}}\,dx$

Optimal. Leaf size=36

$$\frac{\coth(x)}{2\sqrt{a\,\mathrm{csch}^4(x)}}-\frac{x\,\mathrm{csch}^2(x)}{2\sqrt{a\,\mathrm{csch}^4(x)}}$$

```
[Out] 1/2*coth(x)/(a*csch(x)^4)^(1/2)-1/2*x*csch(x)^2/(a*csch(x)^4)^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {4123, 2635, 8}

$$\frac{\coth(x)}{2\sqrt{a\,\mathrm{csch}^4(x)}}-\frac{x\,\mathrm{csch}^2(x)}{2\sqrt{a\,\mathrm{csch}^4(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a*Csch[x]^4],x]
```

```
[Out] Coth[x]/(2*Sqrt[a*Csch[x]^4]) - (x*Csch[x]^2)/(2*Sqrt[a*Csch[x]^4])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 4123

```
Int[((b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> Dist[(b
^IntPart[p]*(b*(c*Sec[e + f*x])^n)^FracPart[p])/(c*Sec[e + f*x])^(n*FracPar
t[p]), Int[(c*Sec[e + f*x])^(n*p), x], x] /; FreeQ[{b, c, e, f, n, p}, x] &
& !IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{a\,\mathrm{csch}^4(x)}}\,dx &= \frac{\mathrm{csch}^2(x)\int \sinh^2(x)\,dx}{\sqrt{a\,\mathrm{csch}^4(x)}}\\ &= \frac{\coth(x)}{2\sqrt{a\,\mathrm{csch}^4(x)}}-\frac{\mathrm{csch}^2(x)\int 1\,dx}{2\sqrt{a\,\mathrm{csch}^4(x)}}\\ &= \frac{\coth(x)}{2\sqrt{a\,\mathrm{csch}^4(x)}}-\frac{x\,\mathrm{csch}^2(x)}{2\sqrt{a\,\mathrm{csch}^4(x)}}\end{aligned}$$

Mathematica [A] time = 0.03, size = 24, normalized size = 0.67

$$\frac{\coth(x)-x\,\mathrm{csch}^2(x)}{2\sqrt{a\,\mathrm{csch}^4(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a*Csch[x]^4],x]
```

```
[Out] (Coth[x] - x*Csch[x]^2)/(2*Sqrt[a*Csch[x]^4])
```

fricas [B] time = 0.65, size = 253, normalized size = 7.03

$$\frac{\left(\left(e^{(4x)} - 2e^{(2x)} + 1\right)\sinh(x)^4 + \cosh(x)^4 + 4\left(\cosh(x)e^{(4x)} - 2\cosh(x)e^{(2x)} + \cosh(x)\right)\sinh(x)^3 - 4x\cosh(x)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*csch(x)^4)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/8*((e^(4*x) - 2*e^(2*x) + 1)*sinh(x)^4 + cosh(x)^4 + 4*(cosh(x)*e^(4*x) -
 2*cosh(x)*e^(2*x) + cosh(x))*sinh(x)^3 - 4*x*cosh(x)^2 + 2*(3*cosh(x)^2 +
(3*cosh(x)^2 - 2*x)*e^(4*x) - 2*(3*cosh(x)^2 - 2*x)*e^(2*x) - 2*x)*sinh(x)^
2 + (cosh(x)^4 - 4*x*cosh(x)^2 - 1)*e^(4*x) - 2*(cosh(x)^4 - 4*x*cosh(x)^2
- 1)*e^(2*x) + 4*(cosh(x)^3 - 2*x*cosh(x) + (cosh(x)^3 - 2*x*cosh(x))*e^(4*
x) - 2*(cosh(x)^3 - 2*x*cosh(x))*e^(2*x))*sinh(x) - 1)*sqrt(a/(e^(8*x) - 4*
e^(6*x) + 6*e^(4*x) - 4*e^(2*x) + 1))*e^(2*x)/(a*cosh(x)^2*e^(2*x) + 2*a*co
sh(x)*e^(2*x)*sinh(x) + a*e^(2*x)*sinh(x)^2)
```

giac [A] time = 0.11, size = 26, normalized size = 0.72

$$\frac{\left(2e^{(2x)} - 1\right)e^{(-2x)} - 4x + e^{(2x)}}{8\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*csch(x)^4)^(1/2),x, algorithm="giac")
```

```
[Out] 1/8*((2*e^(2*x) - 1)*e^(-2*x) - 4*x + e^(2*x))/sqrt(a)
```

maple [B] time = 0.25, size = 89, normalized size = 2.47

$$-\frac{e^{2x}x}{2\sqrt{\frac{a\,e^{4x}}{\left(e^{2x}-1\right)^4}}\left(e^{2x}-1\right)^2}+\frac{e^{4x}}{8\sqrt{\frac{a\,e^{4x}}{\left(e^{2x}-1\right)^4}}\left(e^{2x}-1\right)^2}-\frac{1}{8\left(e^{2x}-1\right)^2\sqrt{\frac{a\,e^{4x}}{\left(e^{2x}-1\right)^4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*csch(x)^4)^(1/2),x)
```

```
[Out] -1/2/(a*exp(4*x)/(exp(2*x)-1)^4)^(1/2)/(exp(2*x)-1)^2*exp(2*x)*x+1/8/(a*exp
(4*x)/(exp(2*x)-1)^4)^(1/2)/(exp(2*x)-1)^2*exp(4*x)-1/8/(exp(2*x)-1)^2/(a*e
xp(4*x)/(exp(2*x)-1)^4)^(1/2)
```

maxima [A] time = 0.44, size = 22, normalized size = 0.61

$$-\frac{\left(e^{(-4x)} - 1\right)e^{(2x)}}{8\sqrt{a}} - \frac{x}{2\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*csch(x)^4)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/8*(e^(-4*x) - 1)*e^(2*x)/sqrt(a) - 1/2*x/sqrt(a)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{\sqrt{\frac{a}{\sinh(x)^4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a/sinh(x)^4)^(1/2),x)
```

```
[Out] int(1/(a/sinh(x)^4)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a\,\mathrm{csch}^4\,(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*csch(x)**4)**(1/2),x)
```

```
[Out] Integral(1/sqrt(a*csch(x)**4), x)
```

3.47 $\int \frac{1}{\left(a\text{csch}^4(x)\right)^{3/2}}\,dx$

Optimal. Leaf size=86

$$-\frac{5x\text{csch}^2(x)}{16a\sqrt{a\text{csch}^4(x)}}+\frac{5\coth(x)}{16a\sqrt{a\text{csch}^4(x)}}+\frac{\sinh^3(x)\cosh(x)}{6a\sqrt{a\text{csch}^4(x)}}-\frac{5\sinh(x)\cosh(x)}{24a\sqrt{a\text{csch}^4(x)}}$$

```
[Out] 5/16*coth(x)/a/(a*csch(x)^4)^(1/2)-5/16*x*csch(x)^2/a/(a*csch(x)^4)^(1/2)-5
/24*cosh(x)*sinh(x)/a/(a*csch(x)^4)^(1/2)+1/6*cosh(x)*sinh(x)^3/a/(a*csch(x
)^4)^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {4123, 2635, 8}

$$-\frac{5x\text{csch}^2(x)}{16a\sqrt{a\text{csch}^4(x)}}+\frac{5\coth(x)}{16a\sqrt{a\text{csch}^4(x)}}+\frac{\sinh^3(x)\cosh(x)}{6a\sqrt{a\text{csch}^4(x)}}-\frac{5\sinh(x)\cosh(x)}{24a\sqrt{a\text{csch}^4(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Csch[x]^4)^(-3/2),x]
```

```
[Out] (5*Coth[x])/(16*a*Sqrt[a*Csch[x]^4]) - (5*x*Csch[x]^2)/(16*a*Sqrt[a*Csch[x]
^4]) - (5*Cosh[x]*Sinh[x])/(24*a*Sqrt[a*Csch[x]^4]) + (Cosh[x]*Sinh[x]^3)/(
6*a*Sqrt[a*Csch[x]^4])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 4123

```
Int[((b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> Dist[(b
^IntPart[p]*(b*(c*Sec[e + f*x])^n)^FracPart[p])/(c*Sec[e + f*x])^(n*FracPar
t[p]), Int[(c*Sec[e + f*x])^(n*p), x], x] /; FreeQ[{b, c, e, f, n, p}, x] &
&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{1}{\left(a\text{csch}^4(x)\right)^{3/2}}\,dx = \frac{\text{csch}^2(x)\int \sinh^6(x)\,dx}{a\sqrt{a\text{csch}^4(x)}}$$
$$= \frac{\cosh(x)\sinh^3(x)}{6a\sqrt{a\text{csch}^4(x)}} - \frac{\left(5\text{csch}^2(x)\right)\int \sinh^4(x)\,dx}{6a\sqrt{a\text{csch}^4(x)}}$$
$$= -\frac{5\cosh(x)\sinh(x)}{24a\sqrt{a\text{csch}^4(x)}} + \frac{\cosh(x)\sinh^3(x)}{6a\sqrt{a\text{csch}^4(x)}} + \frac{\left(5\text{csch}^2(x)\right)\int \sinh^2(x)\,dx}{8a\sqrt{a\text{csch}^4(x)}}$$
$$= \frac{5\coth(x)}{16a\sqrt{a\text{csch}^4(x)}} - \frac{5\cosh(x)\sinh(x)}{24a\sqrt{a\text{csch}^4(x)}} + \frac{\cosh(x)\sinh^3(x)}{6a\sqrt{a\text{csch}^4(x)}} - \frac{\left(5\text{csch}^2(x)\right)\int 1\,dx}{16a\sqrt{a\text{csch}^4(x)}}$$
$$= \frac{5\coth(x)}{16a\sqrt{a\text{csch}^4(x)}} - \frac{5x\text{csch}^2(x)}{16a\sqrt{a\text{csch}^4(x)}} - \frac{5\cosh(x)\sinh(x)}{24a\sqrt{a\text{csch}^4(x)}} + \frac{\cosh(x)\sinh^3(x)}{6a\sqrt{a\text{csch}^4(x)}}$$

Mathematica [A] time = 0.04, size = 38, normalized size = 0.44

$$\frac{(-60x + 45\sinh(2x) - 9\sinh(4x) + \sinh(6x))\text{csch}^6(x)}{192\left(a\text{csch}^4(x)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Csch[x]^4)^(-3/2),x]
```

```
[Out] (Csch[x]^6*(-60*x + 45*Sinh[2*x] - 9*Sinh[4*x] + Sinh[6*x]))/(192*(a*Csch[x
]^4)^(3/2))
```

fricas [B] time = 0.81, size = 1141, normalized size = 13.27

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*csch(x)^4)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/384*((e^(4*x) - 2*e^(2*x) + 1)*sinh(x)^12 + cosh(x)^12 + 12*(cosh(x)*e^(4
*x) - 2*cosh(x)*e^(2*x) + cosh(x))*sinh(x)^11 + 3*(22*cosh(x)^2 + (22*cosh(
x)^2 - 3)*e^(4*x) - 2*(22*cosh(x)^2 - 3)*e^(2*x) - 3)*sinh(x)^10 - 9*cosh(x
)^10 + 10*(22*cosh(x)^3 + (22*cosh(x)^3 - 9*cosh(x))*e^(4*x) - 2*(22*cosh(x
)^3 - 9*cosh(x))*e^(2*x) - 9*cosh(x))*sinh(x)^9 + 45*(11*cosh(x)^4 - 9*cosh
(x)^2 + (11*cosh(x)^4 - 9*cosh(x)^2 + 1)*e^(4*x) - 2*(11*cosh(x)^4 - 9*cosh
(x)^2 + 1)*e^(2*x) + 1)*sinh(x)^8 + 45*cosh(x)^8 + 72*(11*cosh(x)^5 - 15*co
sh(x)^3 + (11*cosh(x)^5 - 15*cosh(x)^3 + 5*cosh(x))*e^(4*x) - 2*(11*cosh(x)
^5 - 15*cosh(x)^3 + 5*cosh(x))*e^(2*x) + 5*cosh(x))*sinh(x)^7 - 120*x*cosh(
x)^6 + 6*(154*cosh(x)^6 - 315*cosh(x)^4 + 210*cosh(x)^2 + (154*cosh(x)^6 -
315*cosh(x)^4 + 210*cosh(x)^2 - 20*x)*e^(4*x) - 2*(154*cosh(x)^6 - 315*cosh
(x)^4 + 210*cosh(x)^2 - 20*x)*e^(2*x) - 20*x)*sinh(x)^6 + 36*(22*cosh(x)^7
- 63*cosh(x)^5 + 70*cosh(x)^3 - 20*x*cosh(x) + (22*cosh(x)^7 - 63*cosh(x)^5
 + 70*cosh(x)^3 - 20*x*cosh(x))*e^(4*x) - 2*(22*cosh(x)^7 - 63*cosh(x)^5 +
70*cosh(x)^3 - 20*x*cosh(x))*e^(2*x))*sinh(x)^5 + 45*(11*cosh(x)^8 - 42*cos
h(x)^6 + 70*cosh(x)^4 - 40*x*cosh(x)^2 + (11*cosh(x)^8 - 42*cosh(x)^6 + 70*
cosh(x)^4 - 40*x*cosh(x)^2 - 1)*e^(4*x) - 2*(11*cosh(x)^8 - 42*cosh(x)^6 +
70*cosh(x)^4 - 40*x*cosh(x)^2 - 1)*e^(2*x) - 1)*sinh(x)^4 - 45*cosh(x)^4 +
20*(11*cosh(x)^9 - 54*cosh(x)^7 + 126*cosh(x)^5 - 120*x*cosh(x)^3 + (11*cos
h(x)^9 - 54*cosh(x)^7 + 126*cosh(x)^5 - 120*x*cosh(x)^3 - 9*cosh(x))*e^(4*x
```

```
) - 2*(11*cosh(x)^9 - 54*cosh(x)^7 + 126*cosh(x)^5 - 120*x*cosh(x)^3 - 9*co
sh(x))*e^(2*x) - 9*cosh(x))*sinh(x)^3 + 3*(22*cosh(x)^10 - 135*cosh(x)^8 +
420*cosh(x)^6 - 600*x*cosh(x)^4 - 90*cosh(x)^2 + (22*cosh(x)^10 - 135*cosh(
x)^8 + 420*cosh(x)^6 - 600*x*cosh(x)^4 - 90*cosh(x)^2 + 3)*e^(4*x) - 2*(22*
cosh(x)^10 - 135*cosh(x)^8 + 420*cosh(x)^6 - 600*x*cosh(x)^4 - 90*cosh(x)^2
 + 3)*e^(2*x) + 3)*sinh(x)^2 + 9*cosh(x)^2 + (cosh(x)^12 - 9*cosh(x)^10 + 4
5*cosh(x)^8 - 120*x*cosh(x)^6 - 45*cosh(x)^4 + 9*cosh(x)^2 - 1)*e^(4*x) - 2
*(cosh(x)^12 - 9*cosh(x)^10 + 45*cosh(x)^8 - 120*x*cosh(x)^6 - 45*cosh(x)^4
 + 9*cosh(x)^2 - 1)*e^(2*x) + 6*(2*cosh(x)^11 - 15*cosh(x)^9 + 60*cosh(x)^7
 - 120*x*cosh(x)^5 - 30*cosh(x)^3 + (2*cosh(x)^11 - 15*cosh(x)^9 + 60*cosh(
x)^7 - 120*x*cosh(x)^5 - 30*cosh(x)^3 + 3*cosh(x))*e^(4*x) - 2*(2*cosh(x)^1
1 - 15*cosh(x)^9 + 60*cosh(x)^7 - 120*x*cosh(x)^5 - 30*cosh(x)^3 + 3*cosh(x
))*e^(2*x) + 3*cosh(x))*sinh(x) - 1)*sqrt(a/(e^(8*x) - 4*e^(6*x) + 6*e^(4*x
) - 4*e^(2*x) + 1))*e^(2*x)/(a^2*cosh(x)^6*e^(2*x) + 6*a^2*cosh(x)^5*e^(2*x
)*sinh(x) + 15*a^2*cosh(x)^4*e^(2*x)*sinh(x)^2 + 20*a^2*cosh(x)^3*e^(2*x)*s
inh(x)^3 + 15*a^2*cosh(x)^2*e^(2*x)*sinh(x)^4 + 6*a^2*cosh(x)*e^(2*x)*sinh(
x)^5 + a^2*e^(2*x)*sinh(x)^6)
```

giac [A] time = 0.11, size = 50, normalized size = 0.58

$$\frac{\left(110\,e^{(6\,x)}-45\,e^{(4\,x)}+9\,e^{(2\,x)}-1\right)e^{(-6\,x)}-120\,x+e^{(6\,x)}-9\,e^{(4\,x)}+45\,e^{(2\,x)}}{384\,a^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*csch(x)^4)^(3/2),x, algorithm="giac")
```

```
[Out] 1/384*((110*e^(6*x) - 45*e^(4*x) + 9*e^(2*x) - 1)*e^(-6*x) - 120*x + e^(6*x
) - 9*e^(4*x) + 45*e^(2*x))/a^(3/2)
```

maple [B] time = 0.21, size = 230, normalized size = 2.67

$$-\frac{5\,e^{2x}x}{16a\left(e^{2x}-1\right)^{2}\sqrt{\frac{a\,e^{4x}}{\left(e^{2x}-1\right)^{4}}}}+\frac{e^{8x}}{384a\left(e^{2x}-1\right)^{2}\sqrt{\frac{a\,e^{4x}}{\left(e^{2x}-1\right)^{4}}}}-\frac{3\,e^{6x}}{128a\left(e^{2x}-1\right)^{2}\sqrt{\frac{a\,e^{4x}}{\left(e^{2x}-1\right)^{4}}}}+\frac{15\,e^{4x}}{128a\left(e^{2x}-1\right)^{2}\sqrt{\frac{a\,e^{4x}}{\left(e^{2x}-1\right)^{4}}}}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*csch(x)^4)^(3/2),x)
```

```
[Out] -5/16/a*exp(2*x)/(exp(2*x)-1)^2/(a*exp(4*x)/(exp(2*x)-1)^4)^(1/2)*x+1/384/a
*exp(8*x)/(exp(2*x)-1)^2/(a*exp(4*x)/(exp(2*x)-1)^4)^(1/2)-3/128/a*exp(6*x)
/(exp(2*x)-1)^2/(a*exp(4*x)/(exp(2*x)-1)^4)^(1/2)+15/128/a*exp(4*x)/(exp(2*
x)-1)^2/(a*exp(4*x)/(exp(2*x)-1)^4)^(1/2)-15/128/(a*exp(4*x)/(exp(2*x)-1)^4
)^(1/2)/(exp(2*x)-1)^2/a+3/128/a*exp(-2*x)/(exp(2*x)-1)^2/(a*exp(4*x)/(exp(
2*x)-1)^4)^(1/2)-1/384/a*exp(-4*x)/(exp(2*x)-1)^2/(a*exp(4*x)/(exp(2*x)-1)^
4)^(1/2)
```

maxima [A] time = 0.46, size = 46, normalized size = 0.53

$$-\frac{\left(9\,e^{(-2\,x)}-45\,e^{(-4\,x)}+45\,e^{(-8\,x)}-9\,e^{(-10\,x)}+e^{(-12\,x)}-1\right)e^{(6\,x)}}{384\,a^{\frac{3}{2}}}-\frac{5\,x}{16\,a^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*csch(x)^4)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/384*(9*e^(-2*x) - 45*e^(-4*x) + 45*e^(-8*x) - 9*e^(-10*x) + e^(-12*x) -
1)*e^(6*x)/a^(3/2) - 5/16*x/a^(3/2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(\frac{a}{\sinh(x)^4}\right)^{3/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a/sinh(x)^4)^(3/2),x)
```

```
[Out] int(1/(a/sinh(x)^4)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a \operatorname{csch}^4(x)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*csch(x)**4)**(3/2),x)
```

```
[Out] Integral((a*csch(x)**4)**(-3/2), x)
```

3.48 $\int \frac{1}{\left(a\operatorname{csch}^4(x)\right)^{5/2}}\,dx$

Optimal. Leaf size=132

$$-\frac{63x\operatorname{csch}^2(x)}{256a^2\sqrt{a\operatorname{csch}^4(x)}}+\frac{63\coth(x)}{256a^2\sqrt{a\operatorname{csch}^4(x)}}+\frac{\sinh^7(x)\cosh(x)}{10a^2\sqrt{a\operatorname{csch}^4(x)}}-\frac{9\sinh^5(x)\cosh(x)}{80a^2\sqrt{a\operatorname{csch}^4(x)}}+\frac{21\sinh^3(x)\cosh(x)}{160a^2\sqrt{a\operatorname{csch}^4(x)}}-\frac{21\sin}{128a^2}$$

```
[Out] 63/256*coth(x)/a^2/(a*csch(x)^4)^(1/2)-63/256*x*csch(x)^2/a^2/(a*csch(x)^4)
^(1/2)-21/128*cosh(x)*sinh(x)/a^2/(a*csch(x)^4)^(1/2)+21/160*cosh(x)*sinh(x
)^3/a^2/(a*csch(x)^4)^(1/2)-9/80*cosh(x)*sinh(x)^5/a^2/(a*csch(x)^4)^(1/2)+
1/10*cosh(x)*sinh(x)^7/a^2/(a*csch(x)^4)^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {4123, 2635, 8}

$$-\frac{63x\operatorname{csch}^2(x)}{256a^2\sqrt{a\operatorname{csch}^4(x)}}+\frac{63\coth(x)}{256a^2\sqrt{a\operatorname{csch}^4(x)}}+\frac{\sinh^7(x)\cosh(x)}{10a^2\sqrt{a\operatorname{csch}^4(x)}}-\frac{9\sinh^5(x)\cosh(x)}{80a^2\sqrt{a\operatorname{csch}^4(x)}}+\frac{21\sinh^3(x)\cosh(x)}{160a^2\sqrt{a\operatorname{csch}^4(x)}}-\frac{21\sin}{128a^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Csch[x]^4)^(-5/2),x]
```

```
[Out] (63*Coth[x])/(256*a^2*Sqrt[a*Csch[x]^4]) - (63*x*Csch[x]^2)/(256*a^2*Sqrt[a
*Csch[x]^4]) - (21*Cosh[x]*Sinh[x])/(128*a^2*Sqrt[a*Csch[x]^4]) + (21*Cosh[
x]*Sinh[x]^3)/(160*a^2*Sqrt[a*Csch[x]^4]) - (9*Cosh[x]*Sinh[x]^5)/(80*a^2*S
qrt[a*Csch[x]^4]) + (Cosh[x]*Sinh[x]^7)/(10*a^2*Sqrt[a*Csch[x]^4])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 4123

```
Int[((b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> Dist[(b
^IntPart[p]*(b*(c*Sec[e + f*x])^n)^FracPart[p])/(c*Sec[e + f*x])^(n*FracPar
t[p]), Int[(c*Sec[e + f*x])^(n*p), x], x] /; FreeQ[{b, c, e, f, n, p}, x] &
& !IntegerQ[p]
```

Rubi steps

$$\int \frac{1}{\left(a\text{csch}^4(x)\right)^{5/2}}\,dx = \frac{\text{csch}^2(x)\int \sinh^{10}(x)\,dx}{a^2\sqrt{a\text{csch}^4(x)}}$$
$$= \frac{\cosh(x)\sinh^7(x)}{10a^2\sqrt{a\text{csch}^4(x)}} - \frac{\left(9\text{csch}^2(x)\right)\int \sinh^8(x)\,dx}{10a^2\sqrt{a\text{csch}^4(x)}}$$
$$= -\frac{9\cosh(x)\sinh^5(x)}{80a^2\sqrt{a\text{csch}^4(x)}} + \frac{\cosh(x)\sinh^7(x)}{10a^2\sqrt{a\text{csch}^4(x)}} + \frac{\left(63\text{csch}^2(x)\right)\int \sinh^6(x)\,dx}{80a^2\sqrt{a\text{csch}^4(x)}}$$
$$= \frac{21\cosh(x)\sinh^3(x)}{160a^2\sqrt{a\text{csch}^4(x)}} - \frac{9\cosh(x)\sinh^5(x)}{80a^2\sqrt{a\text{csch}^4(x)}} + \frac{\cosh(x)\sinh^7(x)}{10a^2\sqrt{a\text{csch}^4(x)}} - \frac{\left(21\text{csch}^2(x)\right)\int \sinh^4(x)}{32a^2\sqrt{a\text{csch}^4(x)}}$$
$$= -\frac{21\cosh(x)\sinh(x)}{128a^2\sqrt{a\text{csch}^4(x)}} + \frac{21\cosh(x)\sinh^3(x)}{160a^2\sqrt{a\text{csch}^4(x)}} - \frac{9\cosh(x)\sinh^5(x)}{80a^2\sqrt{a\text{csch}^4(x)}} + \frac{\cosh(x)\sinh^7(x)}{10a^2\sqrt{a\text{csch}^4(x)}} +$$
$$= \frac{63\coth(x)}{256a^2\sqrt{a\text{csch}^4(x)}} - \frac{21\cosh(x)\sinh(x)}{128a^2\sqrt{a\text{csch}^4(x)}} + \frac{21\cosh(x)\sinh^3(x)}{160a^2\sqrt{a\text{csch}^4(x)}} - \frac{9\cosh(x)\sinh^5(x)}{80a^2\sqrt{a\text{csch}^4(x)}} +$$
$$= \frac{63\coth(x)}{256a^2\sqrt{a\text{csch}^4(x)}} - \frac{63x\text{csch}^2(x)}{256a^2\sqrt{a\text{csch}^4(x)}} - \frac{21\cosh(x)\sinh(x)}{128a^2\sqrt{a\text{csch}^4(x)}} + \frac{21\cosh(x)\sinh^3(x)}{160a^2\sqrt{a\text{csch}^4(x)}} -$$

Mathematica [A] time = 0.12, size = 55, normalized size = 0.42

$$\frac{\sinh^2(x)(-2520x + 2100\sinh(2x) - 600\sinh(4x) + 150\sinh(6x) - 25\sinh(8x) + 2\sinh(10x))\sqrt{a\text{csch}^4(x)}}{10240a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Csch[x]^4)^(-5/2),x]
```

```
[Out] (Sqrt[a*Csch[x]^4]*Sinh[x]^2*(-2520*x + 2100*Sinh[2*x] - 600*Sinh[4*x] + 15
0*Sinh[6*x] - 25*Sinh[8*x] + 2*Sinh[10*x]))/(10240*a^3)
```

fricas [B] time = 0.74, size = 2600, normalized size = 19.70

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*csch(x)^4)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/20480*(2*(e^(4*x) - 2*e^(2*x) + 1)*sinh(x)^20 + 2*cosh(x)^20 + 40*(cosh(x
)*e^(4*x) - 2*cosh(x)*e^(2*x) + cosh(x))*sinh(x)^19 + 5*(76*cosh(x)^2 + (76
*cosh(x)^2 - 5)*e^(4*x) - 2*(76*cosh(x)^2 - 5)*e^(2*x) - 5)*sinh(x)^18 - 25
*cosh(x)^18 + 30*(76*cosh(x)^3 + (76*cosh(x)^3 - 15*cosh(x))*e^(4*x) - 2*(7
6*cosh(x)^3 - 15*cosh(x))*e^(2*x) - 15*cosh(x))*sinh(x)^17 + 15*(646*cosh(x
)^4 - 255*cosh(x)^2 + (646*cosh(x)^4 - 255*cosh(x)^2 + 10)*e^(4*x) - 2*(646
*cosh(x)^4 - 255*cosh(x)^2 + 10)*e^(2*x) + 10)*sinh(x)^16 + 150*cosh(x)^16
+ 48*(646*cosh(x)^5 - 425*cosh(x)^3 + (646*cosh(x)^5 - 425*cosh(x)^3 + 50*c
osh(x))*e^(4*x) - 2*(646*cosh(x)^5 - 425*cosh(x)^3 + 50*cosh(x))*e^(2*x) +
50*cosh(x))*sinh(x)^15 + 60*(1292*cosh(x)^6 - 1275*cosh(x)^4 + 300*cosh(x)^
2 + (1292*cosh(x)^6 - 1275*cosh(x)^4 + 300*cosh(x)^2 - 10)*e^(4*x) - 2*(129
2*cosh(x)^6 - 1275*cosh(x)^4 + 300*cosh(x)^2 - 10)*e^(2*x) - 10)*sinh(x)^14
 - 600*cosh(x)^14 + 120*(1292*cosh(x)^7 - 1785*cosh(x)^5 + 700*cosh(x)^3 +
```

```
(1292*cosh(x)^7 - 1785*cosh(x)^5 + 700*cosh(x)^3 - 70*cosh(x))*e^(4*x) - 2*
(1292*cosh(x)^7 - 1785*cosh(x)^5 + 700*cosh(x)^3 - 70*cosh(x))*e^(2*x) - 70
*cosh(x))*sinh(x)^13 + 60*(4199*cosh(x)^8 - 7735*cosh(x)^6 + 4550*cosh(x)^4
 - 910*cosh(x)^2 + (4199*cosh(x)^8 - 7735*cosh(x)^6 + 4550*cosh(x)^4 - 910*
cosh(x)^2 + 35)*e^(4*x) - 2*(4199*cosh(x)^8 - 7735*cosh(x)^6 + 4550*cosh(x)
^4 - 910*cosh(x)^2 + 35)*e^(2*x) + 35)*sinh(x)^12 + 2100*cosh(x)^12 + 80*(4
199*cosh(x)^9 - 9945*cosh(x)^7 + 8190*cosh(x)^5 - 2730*cosh(x)^3 + (4199*co
sh(x)^9 - 9945*cosh(x)^7 + 8190*cosh(x)^5 - 2730*cosh(x)^3 + 315*cosh(x))*e
^(4*x) - 2*(4199*cosh(x)^9 - 9945*cosh(x)^7 + 8190*cosh(x)^5 - 2730*cosh(x)
^3 + 315*cosh(x))*e^(2*x) + 315*cosh(x))*sinh(x)^11 - 5040*x*cosh(x)^10 + 2
*(184756*cosh(x)^10 - 546975*cosh(x)^8 + 600600*cosh(x)^6 - 300300*cosh(x)^
4 + 69300*cosh(x)^2 + (184756*cosh(x)^10 - 546975*cosh(x)^8 + 600600*cosh(x
)^6 - 300300*cosh(x)^4 + 69300*cosh(x)^2 - 2520*x)*e^(4*x) - 2*(184756*cosh
(x)^10 - 546975*cosh(x)^8 + 600600*cosh(x)^6 - 300300*cosh(x)^4 + 69300*cos
h(x)^2 - 2520*x)*e^(2*x) - 2520*x)*sinh(x)^10 + 20*(16796*cosh(x)^11 - 6077
5*cosh(x)^9 + 85800*cosh(x)^7 - 60060*cosh(x)^5 + 23100*cosh(x)^3 - 2520*x*
cosh(x) + (16796*cosh(x)^11 - 60775*cosh(x)^9 + 85800*cosh(x)^7 - 60060*cos
h(x)^5 + 23100*cosh(x)^3 - 2520*x*cosh(x))*e^(4*x) - 2*(16796*cosh(x)^11 -
60775*cosh(x)^9 + 85800*cosh(x)^7 - 60060*cosh(x)^5 + 23100*cosh(x)^3 - 252
0*x*cosh(x))*e^(2*x))*sinh(x)^9 + 30*(8398*cosh(x)^12 - 36465*cosh(x)^10 +
64350*cosh(x)^8 - 60060*cosh(x)^6 + 34650*cosh(x)^4 - 7560*x*cosh(x)^2 + (8
398*cosh(x)^12 - 36465*cosh(x)^10 + 64350*cosh(x)^8 - 60060*cosh(x)^6 + 346
50*cosh(x)^4 - 7560*x*cosh(x)^2 - 70)*e^(4*x) - 2*(8398*cosh(x)^12 - 36465*
cosh(x)^10 + 64350*cosh(x)^8 - 60060*cosh(x)^6 + 34650*cosh(x)^4 - 7560*x*c
osh(x)^2 - 70)*e^(2*x) - 70)*sinh(x)^8 - 2100*cosh(x)^8 + 240*(646*cosh(x)^
13 - 3315*cosh(x)^11 + 7150*cosh(x)^9 - 8580*cosh(x)^7 + 6930*cosh(x)^5 - 2
520*x*cosh(x)^3 + (646*cosh(x)^13 - 3315*cosh(x)^11 + 7150*cosh(x)^9 - 8580
*cosh(x)^7 + 6930*cosh(x)^5 - 2520*x*cosh(x)^3 - 70*cosh(x))*e^(4*x) - 2*(6
46*cosh(x)^13 - 3315*cosh(x)^11 + 7150*cosh(x)^9 - 8580*cosh(x)^7 + 6930*co
sh(x)^5 - 2520*x*cosh(x)^3 - 70*cosh(x))*e^(2*x) - 70*cosh(x))*sinh(x)^7 +
60*(1292*cosh(x)^14 - 7735*cosh(x)^12 + 20020*cosh(x)^10 - 30030*cosh(x)^8
+ 32340*cosh(x)^6 - 17640*x*cosh(x)^4 - 980*cosh(x)^2 + (1292*cosh(x)^14 -
7735*cosh(x)^12 + 20020*cosh(x)^10 - 30030*cosh(x)^8 + 32340*cosh(x)^6 - 17
640*x*cosh(x)^4 - 980*cosh(x)^2 + 10)*e^(4*x) - 2*(1292*cosh(x)^14 - 7735*c
osh(x)^12 + 20020*cosh(x)^10 - 30030*cosh(x)^8 + 32340*cosh(x)^6 - 17640*x*
cosh(x)^4 - 980*cosh(x)^2 + 10)*e^(2*x) + 10)*sinh(x)^6 + 600*cosh(x)^6 + 2
4*(1292*cosh(x)^15 - 8925*cosh(x)^13 + 27300*cosh(x)^11 - 50050*cosh(x)^9 +
 69300*cosh(x)^7 - 52920*x*cosh(x)^5 - 4900*cosh(x)^3 + (1292*cosh(x)^15 -
8925*cosh(x)^13 + 27300*cosh(x)^11 - 50050*cosh(x)^9 + 69300*cosh(x)^7 - 52
920*x*cosh(x)^5 - 4900*cosh(x)^3 + 150*cosh(x))*e^(4*x) - 2*(1292*cosh(x)^1
5 - 8925*cosh(x)^13 + 27300*cosh(x)^11 - 50050*cosh(x)^9 + 69300*cosh(x)^7
- 52920*x*cosh(x)^5 - 4900*cosh(x)^3 + 150*cosh(x))*e^(2*x) + 150*cosh(x))*
sinh(x)^5 + 30*(323*cosh(x)^16 - 2550*cosh(x)^14 + 9100*cosh(x)^12 - 20020*
cosh(x)^10 + 34650*cosh(x)^8 - 35280*x*cosh(x)^6 - 4900*cosh(x)^4 + 300*cos
h(x)^2 + (323*cosh(x)^16 - 2550*cosh(x)^14 + 9100*cosh(x)^12 - 20020*cosh(x
)^10 + 34650*cosh(x)^8 - 35280*x*cosh(x)^6 - 4900*cosh(x)^4 + 300*cosh(x)^2
 - 5)*e^(4*x) - 2*(323*cosh(x)^16 - 2550*cosh(x)^14 + 9100*cosh(x)^12 - 200
20*cosh(x)^10 + 34650*cosh(x)^8 - 35280*x*cosh(x)^6 - 4900*cosh(x)^4 + 300*
cosh(x)^2 - 5)*e^(2*x) - 5)*sinh(x)^4 - 150*cosh(x)^4 + 120*(19*cosh(x)^17
- 170*cosh(x)^15 + 700*cosh(x)^13 - 1820*cosh(x)^11 + 3850*cosh(x)^9 - 5040
*x*cosh(x)^7 - 980*cosh(x)^5 + 100*cosh(x)^3 + (19*cosh(x)^17 - 170*cosh(x)
^15 + 700*cosh(x)^13 - 1820*cosh(x)^11 + 3850*cosh(x)^9 - 5040*x*cosh(x)^7
- 980*cosh(x)^5 + 100*cosh(x)^3 - 5*cosh(x))*e^(4*x) - 2*(19*cosh(x)^17 - 1
70*cosh(x)^15 + 700*cosh(x)^13 - 1820*cosh(x)^11 + 3850*cosh(x)^9 - 5040*x*
cosh(x)^7 - 980*cosh(x)^5 + 100*cosh(x)^3 - 5*cosh(x))*e^(2*x) - 5*cosh(x))
*sinh(x)^3 + 5*(76*cosh(x)^18 - 765*cosh(x)^16 + 3600*cosh(x)^14 - 10920*co
sh(x)^12 + 27720*cosh(x)^10 - 45360*x*cosh(x)^8 - 11760*cosh(x)^6 + 1800*co
sh(x)^4 - 180*cosh(x)^2 + (76*cosh(x)^18 - 765*cosh(x)^16 + 3600*cosh(x)^14
 - 10920*cosh(x)^12 + 27720*cosh(x)^10 - 45360*x*cosh(x)^8 - 11760*cosh(x)^
```

```
6 + 1800*cosh(x)^4 - 180*cosh(x)^2 + 5)*e^(4*x) - 2*(76*cosh(x)^18 - 765*co
sh(x)^16 + 3600*cosh(x)^14 - 10920*cosh(x)^12 + 27720*cosh(x)^10 - 45360*x*
cosh(x)^8 - 11760*cosh(x)^6 + 1800*cosh(x)^4 - 180*cosh(x)^2 + 5)*e^(2*x) +
 5)*sinh(x)^2 + 25*cosh(x)^2 + (2*cosh(x)^20 - 25*cosh(x)^18 + 150*cosh(x)^
16 - 600*cosh(x)^14 + 2100*cosh(x)^12 - 5040*x*cosh(x)^10 - 2100*cosh(x)^8
+ 600*cosh(x)^6 - 150*cosh(x)^4 + 25*cosh(x)^2 - 2)*e^(4*x) - 2*(2*cosh(x)^
20 - 25*cosh(x)^18 + 150*cosh(x)^16 - 600*cosh(x)^14 + 2100*cosh(x)^12 - 50
40*x*cosh(x)^10 - 2100*cosh(x)^8 + 600*cosh(x)^6 - 150*cosh(x)^4 + 25*cosh(
x)^2 - 2)*e^(2*x) + 10*(4*cosh(x)^19 - 45*cosh(x)^17 + 240*cosh(x)^15 - 840
*cosh(x)^13 + 2520*cosh(x)^11 - 5040*x*cosh(x)^9 - 1680*cosh(x)^7 + 360*cos
h(x)^5 - 60*cosh(x)^3 + (4*cosh(x)^19 - 45*cosh(x)^17 + 240*cosh(x)^15 - 84
0*cosh(x)^13 + 2520*cosh(x)^11 - 5040*x*cosh(x)^9 - 1680*cosh(x)^7 + 360*co
sh(x)^5 - 60*cosh(x)^3 + 5*cosh(x))*e^(4*x) - 2*(4*cosh(x)^19 - 45*cosh(x)^
17 + 240*cosh(x)^15 - 840*cosh(x)^13 + 2520*cosh(x)^11 - 5040*x*cosh(x)^9 -
 1680*cosh(x)^7 + 360*cosh(x)^5 - 60*cosh(x)^3 + 5*cosh(x))*e^(2*x) + 5*cos
h(x))*sinh(x) - 2)*sqrt(a/(e^(8*x) - 4*e^(6*x) + 6*e^(4*x) - 4*e^(2*x) + 1)
)*e^(2*x)/(a^3*cosh(x)^10*e^(2*x) + 10*a^3*cosh(x)^9*e^(2*x)*sinh(x) + 45*a
^3*cosh(x)^8*e^(2*x)*sinh(x)^2 + 120*a^3*cosh(x)^7*e^(2*x)*sinh(x)^3 + 210*
a^3*cosh(x)^6*e^(2*x)*sinh(x)^4 + 252*a^3*cosh(x)^5*e^(2*x)*sinh(x)^5 + 210
*a^3*cosh(x)^4*e^(2*x)*sinh(x)^6 + 120*a^3*cosh(x)^3*e^(2*x)*sinh(x)^7 + 45
*a^3*cosh(x)^2*e^(2*x)*sinh(x)^8 + 10*a^3*cosh(x)*e^(2*x)*sinh(x)^9 + a^3*e
^(2*x)*sinh(x)^10)
```

giac [A] time = 0.13, size = 76, normalized size = 0.58

$$\frac{\left(5754\,e^{(10\,x)} - 2100\,e^{(8\,x)} + 600\,e^{(6\,x)} - 150\,e^{(4\,x)} + 25\,e^{(2\,x)} - 2\right)e^{(-10\,x)} - 5040\,x + 2\,e^{(10\,x)} - 25\,e^{(8\,x)} + 150\,e^{(6\,x)} - 60}{20480\,a^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*csch(x)^4)^(5/2),x, algorithm="giac")
```

```
[Out] 1/20480*((5754*e^(10*x) - 2100*e^(8*x) + 600*e^(6*x) - 150*e^(4*x) + 25*e^(
2*x) - 2)*e^(-10*x) - 5040*x + 2*e^(10*x) - 25*e^(8*x) + 150*e^(6*x) - 600*
e^(4*x) + 2100*e^(2*x))/a^(5/2)
```

maple [B] time = 0.22, size = 362, normalized size = 2.74

$$-\frac{63\,e^{2x}x}{256a^2\left(e^{2x}-1\right)^2\sqrt{\frac{a\,e^{4x}}{\left(e^{2x}-1\right)^4}}}+\frac{e^{12x}}{10240a^2\left(e^{2x}-1\right)^2\sqrt{\frac{a\,e^{4x}}{\left(e^{2x}-1\right)^4}}}-\frac{5\,e^{10x}}{4096a^2\left(e^{2x}-1\right)^2\sqrt{\frac{a\,e^{4x}}{\left(e^{2x}-1\right)^4}}}+\frac{15\,e^{8x}}{2048a^2\left(e^{2x}-1\right)^2\sqrt{(}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*csch(x)^4)^(5/2),x)
```

```
[Out] -63/256/a^2*exp(2*x)/(exp(2*x)-1)^2/(a*exp(4*x)/(exp(2*x)-1)^4)^(1/2)*x+1/1
0240/a^2*exp(12*x)/(exp(2*x)-1)^2/(a*exp(4*x)/(exp(2*x)-1)^4)^(1/2)-5/4096/
a^2*exp(10*x)/(exp(2*x)-1)^2/(a*exp(4*x)/(exp(2*x)-1)^4)^(1/2)+15/2048/a^2*
exp(8*x)/(exp(2*x)-1)^2/(a*exp(4*x)/(exp(2*x)-1)^4)^(1/2)-15/512/a^2*exp(6*
x)/(exp(2*x)-1)^2/(a*exp(4*x)/(exp(2*x)-1)^4)^(1/2)+105/1024/a^2*exp(4*x)/(
exp(2*x)-1)^2/(a*exp(4*x)/(exp(2*x)-1)^4)^(1/2)-105/1024/(a*exp(4*x)/(exp(2
*x)-1)^4)^(1/2)/(exp(2*x)-1)^2/a^2+15/512/a^2*exp(-2*x)/(exp(2*x)-1)^2/(a*e
xp(4*x)/(exp(2*x)-1)^4)^(1/2)-15/2048/a^2*exp(-4*x)/(exp(2*x)-1)^2/(a*exp(4
*x)/(exp(2*x)-1)^4)^(1/2)+5/4096/a^2*exp(-6*x)/(exp(2*x)-1)^2/(a*exp(4*x)/(
exp(2*x)-1)^4)^(1/2)-1/10240/a^2*exp(-8*x)/(exp(2*x)-1)^2/(a*exp(4*x)/(exp(
2*x)-1)^4)^(1/2)
```

maxima [A] time = 0.58, size = 72, normalized size = 0.55

$$-\frac{\left(25\,e^{(-2\,x)} - 150\,e^{(-4\,x)} + 600\,e^{(-6\,x)} - 2100\,e^{(-8\,x)} + 2100\,e^{(-12\,x)} - 600\,e^{(-14\,x)} + 150\,e^{(-16\,x)} - 25\,e^{(-18\,x)} + 2\,e^{(-20\,x)}}{20480\,a^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*csch(x)^4)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/20480*(25*e^(-2*x) - 150*e^(-4*x) + 600*e^(-6*x) - 2100*e^(-8*x) + 2100*
e^(-12*x) - 600*e^(-14*x) + 150*e^(-16*x) - 25*e^(-18*x) + 2*e^(-20*x) - 2)
*e^(10*x)/a^(5/2) - 63/256*x/a^(5/2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(\frac{a}{\sinh(x)^4}\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a/sinh(x)^4)^(5/2),x)
```

```
[Out] int(1/(a/sinh(x)^4)^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a\operatorname{csch}^4(x)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*csch(x)**4)**(5/2),x)
```

```
[Out] Integral((a*csch(x)**4)**(-5/2), x)
```

3.49 $\int \frac{1}{a+ia\,\text{csch}(a+bx)}\,dx$

Optimal. Leaf size=32

$$\frac{x}{a}-\frac{\coth(a+bx)}{b(a+ia\,\text{csch}(a+bx))}$$

```
[Out] x/a-coth(b*x+a)/b/(a+I*a*csch(b*x+a))
```

Rubi [A] time = 0.01, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {3777, 8}

$$\frac{x}{a}-\frac{\coth(a+bx)}{b(a+ia\,\text{csch}(a+bx))}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Csch[a + b*x])^(-1),x]
```

```
[Out] x/a - Coth[a + b*x]/(b*(a + I*a*Csch[a + b*x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3777

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> -Simp[(Cot[c
+ d*x]*(a + b*Csc[c + d*x])^n)/(d*(2*n + 1)), x] + Dist[1/(a^2*(2*n + 1)),
Int[(a + b*Csc[c + d*x])^(n + 1)*(a*(2*n + 1) - b*(n + 1)*Csc[c + d*x]), x]
, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && LeQ[n, -1] && Intege
rQ[2*n]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{a+ia\,\text{csch}(a+bx)}\,dx &= -\frac{\coth(a+bx)}{b(a+ia\,\text{csch}(a+bx))}+\frac{\int a\,dx}{a^2}\\ &= \frac{x}{a}-\frac{\coth(a+bx)}{b(a+ia\,\text{csch}(a+bx))}\end{aligned}$$

Mathematica [A] time = 0.12, size = 54, normalized size = 1.69

$$-\frac{2\sinh\left(\frac{1}{2}(a+bx)\right)}{ab\left(\cosh\left(\frac{1}{2}(a+bx)\right)-i\sinh\left(\frac{1}{2}(a+bx)\right)\right)}+\frac{x}{a}+\frac{1}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Csch[a + b*x])^(-1),x]
```

```
[Out] b^(-1) + x/a - (2*Sinh[(a + b*x)/2])/(a*b*(Cosh[(a + b*x)/2] - I*Sinh[(a +
b*x)/2]))
```

fricas [A] time = 1.43, size = 32, normalized size = 1.00

$$\frac{bxe^{(bx+a)}+i\,bx+2i}{abe^{(bx+a)}+i\,ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*csch(b*x+a)),x, algorithm="fricas")
```

```
[Out] (b*x*e^(b*x + a) + I*b*x + 2*I)/(a*b*e^(b*x + a) + I*a*b)
```

giac [A] time = 0.13, size = 29, normalized size = 0.91

$$\frac{\frac{bx+a}{a} + \frac{2i}{a\left(e^{(bx+a)}+i\right)}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*csch(b*x+a)),x, algorithm="giac")
```

```
[Out] ((b*x + a)/a + 2*I/(a*(e^(b*x + a) + I)))/b
```

maple [A] time = 0.25, size = 63, normalized size = 1.97

$$-\frac{2}{ba\left(\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)+i\right)}-\frac{\ln\left(\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)}{ba}+\frac{\ln\left(\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)+1\right)}{ba}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+I*a*csch(b*x+a)),x)
```

```
[Out] -2/b/a/(tanh(1/2*b*x+1/2*a)+I)-1/b/a*ln(tanh(1/2*b*x+1/2*a)-1)+1/b/a*ln(tan
h(1/2*b*x+1/2*a)+1)
```

maxima [A] time = 0.36, size = 35, normalized size = 1.09

$$\frac{bx+a}{ab} + \frac{2i}{\left(ae^{(-bx-a)} - i\,a\right)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*csch(b*x+a)),x, algorithm="maxima")
```

```
[Out] (b*x + a)/(a*b) + 2*I/((a*e^(-b*x - a) - I*a)*b)
```

mupad [B] time = 1.55, size = 26, normalized size = 0.81

$$\frac{x}{a} + \frac{2\mathrm{i}}{a\,b\,\left(\mathrm{e}^{a+b\,x}+1\mathrm{i}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + (a*1i)/sinh(a + b*x)),x)
```

```
[Out] x/a + 2i/(a*b*(exp(a + b*x) + 1i))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{i\int\frac{1}{\operatorname{csch}(a+bx)-i}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*csch(b*x+a)),x)
```

```
[Out] -I*Integral(1/(csch(a + b*x) - I), x)/a
```

3.50 $\int \frac{1}{a-ia\,\mathbf{csch}(a+bx)}\,dx$

Optimal. Leaf size=32

$$\frac{x}{a}-\frac{\coth(a+bx)}{b(a-ia\operatorname{csch}(a+bx))}$$

```
[Out] x/a-coth(b*x+a)/b/(a-I*a*csch(b*x+a))
```

Rubi [A] time = 0.02, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {3777, 8}

$$\frac{x}{a}-\frac{\coth(a+bx)}{b(a-ia\operatorname{csch}(a+bx))}$$

Antiderivative was successfully verified.

```
[In] Int[(a - I*a*Csch[a + b*x])^(-1),x]
```

```
[Out] x/a - Coth[a + b*x]/(b*(a - I*a*Csch[a + b*x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3777

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> -Simp[(Cot[c
+ d*x]*(a + b*Csc[c + d*x])^n)/(d*(2*n + 1)), x] + Dist[1/(a^2*(2*n + 1)),
Int[(a + b*Csc[c + d*x])^(n + 1)*(a*(2*n + 1) - b*(n + 1)*Csc[c + d*x]), x]
, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && LeQ[n, -1] && Intege
rQ[2*n]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{a-ia\operatorname{csch}(a+bx)}\,dx &= -\frac{\coth(a+bx)}{b(a-ia\operatorname{csch}(a+bx))}+\frac{\int a\,dx}{a^2}\\ &= \frac{x}{a}-\frac{\coth(a+bx)}{b(a-ia\operatorname{csch}(a+bx))}\end{aligned}$$

Mathematica [A] time = 0.11, size = 54, normalized size = 1.69

$$-\frac{2\sinh\left(\frac{1}{2}(a+bx)\right)}{ab\left(\cosh\left(\frac{1}{2}(a+bx)\right)+i\sinh\left(\frac{1}{2}(a+bx)\right)\right)}+\frac{x}{a}+\frac{1}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - I*a*Csch[a + b*x])^(-1),x]
```

```
[Out] b^(-1) + x/a - (2*Sinh[(a + b*x)/2])/(a*b*(Cosh[(a + b*x)/2] + I*Sinh[(a +
b*x)/2]))
```

fricas [A] time = 0.75, size = 32, normalized size = 1.00

$$\frac{bxe^{(bx+a)}-i\,bx-2i}{abe^{(bx+a)}-i\,ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*csch(b*x+a)),x, algorithm="fricas")
[Out] (b*x*e^(b*x + a) - I*b*x - 2*I)/(a*b*e^(b*x + a) - I*a*b)
```

giac [A] time = 0.14, size = 29, normalized size = 0.91

$$\frac{\frac{bx+a}{a} - \frac{2i}{a\left(e^{(bx+a)}-i\right)}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*csch(b*x+a)),x, algorithm="giac")
[Out] ((b*x + a)/a - 2*I/(a*(e^(b*x + a) - I)))/b
```

maple [A] time = 0.26, size = 63, normalized size = 1.97

$$-\frac{\ln\left(\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)}{ba}+\frac{\ln\left(\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)+1\right)}{ba}-\frac{2}{ba\left(\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)-i\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a-I*a*csch(b*x+a)),x)
[Out] -1/b/a*ln(tanh(1/2*b*x+1/2*a)-1)+1/b/a*ln(tanh(1/2*b*x+1/2*a)+1)-2/b/a/(tan
h(1/2*b*x+1/2*a)-I)
```

maxima [A] time = 0.34, size = 35, normalized size = 1.09

$$\frac{bx+a}{ab} - \frac{2i}{\left(ae^{(-bx-a)}+i\,a\right)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*csch(b*x+a)),x, algorithm="maxima")
[Out] (b*x + a)/(a*b) - 2*I/((a*e^(-b*x - a) + I*a)*b)
```

mupad [B] time = 1.50, size = 26, normalized size = 0.81

$$\frac{x}{a} - \frac{2\mathrm{i}}{a\,b\,\left(\mathrm{e}^{a+b\,x}-\mathrm{i}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a - (a*1i)/sinh(a + b*x)),x)
[Out] x/a - 2i/(a*b*(exp(a + b*x) - 1i))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{i\int\frac{1}{\operatorname{csch}(a+bx)+i}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*csch(b*x+a)),x)
[Out] I*Integral(1/(csch(a + b*x) + I), x)/a
```

3.51 $\int (a+ia\,\mathbf{csch}(c+dx))^{5/2}\,dx$

Optimal. Leaf size=107

$$\frac{2a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a}\coth(c+dx)}{\sqrt{a+ia\text{csch}(c+dx)}}\right)}{d}+\frac{14a^3\coth(c+dx)}{3d\sqrt{a+ia\text{csch}(c+dx)}}+\frac{2a^2\coth(c+dx)\sqrt{a+ia\text{csch}(c+dx)}}{3d}$$

```
[Out] 2*a^(5/2)*arctanh(coth(d*x+c)*a^(1/2)/(a+I*a*csch(d*x+c))^(1/2))/d+14/3*a^3
*coth(d*x+c)/d/(a+I*a*csch(d*x+c))^(1/2)+2/3*a^2*coth(d*x+c)*(a+I*a*csch(d*
x+c))^(1/2)/d
```

Rubi [A] time = 0.13, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {3775, 3915, 3774, 203, 3792}

$$\frac{14a^3\coth(c+dx)}{3d\sqrt{a+ia\text{csch}(c+dx)}}+\frac{2a^2\coth(c+dx)\sqrt{a+ia\text{csch}(c+dx)}}{3d}+\frac{2a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a}\coth(c+dx)}{\sqrt{a+ia\text{csch}(c+dx)}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Csch[c + d*x])^(5/2),x]
```

```
[Out] (2*a^(5/2)*ArcTanh[(Sqrt[a]*Coth[c + d*x])/Sqrt[a + I*a*Csch[c + d*x]]])/d
+ (14*a^3*Coth[c + d*x])/(3*d*Sqrt[a + I*a*Csch[c + d*x]]) + (2*a^2*Coth[c
+ d*x]*Sqrt[a + I*a*Csch[c + d*x]])/(3*d)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3774

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[(-2*b)/d,
 Subst[Int[1/(a + x^2), x], x, (b*Cot[c + d*x])/Sqrt[a + b*Csc[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3775

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> -Simp[(b^2*Co
t[c + d*x]*(a + b*Csc[c + d*x])^(n - 2))/(d*(n - 1)), x] + Dist[a/(n - 1),
Int[(a + b*Csc[c + d*x])^(n - 2)*(a*(n - 1) + b*(3*n - 4)*Csc[c + d*x]), x]
, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && GtQ[n, 1] && Integer
Q[2*n]
```

Rule 3792

```
Int[csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*b*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]), x] /; Free
Q[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3915

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(d_
.) + (c_)), x_Symbol] :> Dist[c, Int[Sqrt[a + b*Csc[e + f*x]], x], x] + Dis
t[d, Int[Sqrt[a + b*Csc[e + f*x]]*Csc[e + f*x], x], x] /; FreeQ[{a, b, c, d
, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int (a+ia\text{csch}(c+dx))^{5/2}\,dx &= \frac{2a^2\coth(c+dx)\sqrt{a+ia\text{csch}(c+dx)}}{3d}+\frac{1}{3}(2a)\int\sqrt{a+ia\text{csch}(c+dx)}\left(\frac{3a}{2}+\right.\\ &= \frac{2a^2\coth(c+dx)\sqrt{a+ia\text{csch}(c+dx)}}{3d}+\frac{1}{3}\left(7ia^2\right)\int\text{csch}(c+dx)\sqrt{a+ia\text{csch}(\right.\\ &= \frac{14a^3\coth(c+dx)}{3d\sqrt{a+ia\text{csch}(c+dx)}}+\frac{2a^2\coth(c+dx)\sqrt{a+ia\text{csch}(c+dx)}}{3d}-\frac{\left(2ia^3\right)\text{Subs}}{}\\ &= \frac{2a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a}\coth(c+dx)}{\sqrt{a+ia\text{csch}(c+dx)}}\right)}{d}+\frac{14a^3\coth(c+dx)}{3d\sqrt{a+ia\text{csch}(c+dx)}}+\frac{2a^2\coth(c+dx)\sqrt{a}}{3d}\end{aligned}$$

Mathematica [A] time = 1.48, size = 136, normalized size = 1.27

$$\frac{2a^2\sqrt{a+ia\text{csch}(c+dx)}\left(\coth(c+dx)+\frac{14\sinh\left(\frac{1}{2}(c+dx)\right)}{\cosh\left(\frac{1}{2}(c+dx)\right)-i\sinh\left(\frac{1}{2}(c+dx)\right)}+\frac{3(-1)^{3/4}\coth(c+dx)\tanh^{-1}\left((-1)^{3/4}\sqrt{\text{csch}(c+dx)+i}\right)}{(\text{csch}(c+dx)-i)\sqrt{\text{csch}(c+dx)+i}}\right.}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Csch[c + d*x])^(5/2),x]
```

```
[Out] (2*a^2*Sqrt[a + I*a*Csch[c + d*x]]*(-7*I + Coth[c + d*x] + (3*(-1)^(3/4)*Ar
cTanh[(-1)^(3/4)*Sqrt[I + Csch[c + d*x]]]*Coth[c + d*x])/((-I + Csch[c + d*
x])*Sqrt[I + Csch[c + d*x]]) + (14*Sinh[(c + d*x)/2])/(Cosh[(c + d*x)/2] -
I*Sinh[(c + d*x)/2])))/(3*d)
```

fricas [B] time = 0.62, size = 569, normalized size = 5.32

$$\frac{6\sqrt{\frac{a^5}{d^2}}\left(de^{(2dx+2c)}-d\right)\log\left(\frac{\left(2a^3e^{(dx+c)}+2ia^3+2\sqrt{\frac{a^5}{d^2}}\left(de^{(2dx+2c)}-d\right)\sqrt{\frac{a}{e^{(2dx+2c)}-1}}\right)e^{(-dx-c)}}{d}\right)-6\sqrt{\frac{a^5}{d^2}}\left(de^{(2dx+2c)}-d\right)\log\left(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*csch(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/12*(6*sqrt(a^5/d^2)*(d*e^(2*d*x + 2*c) - d)*log((2*a^3*e^(d*x + c) + 2*I*
a^3 + 2*sqrt(a^5/d^2)*(d*e^(2*d*x + 2*c) - d)*sqrt(a/(e^(2*d*x + 2*c) - 1))
)*e^(-d*x - c)/d) - 6*sqrt(a^5/d^2)*(d*e^(2*d*x + 2*c) - d)*log((2*a^3*e^(d
*x + c) + 2*I*a^3 - 2*sqrt(a^5/d^2)*(d*e^(2*d*x + 2*c) - d)*sqrt(a/(e^(2*d*
x + 2*c) - 1)))*e^(-d*x - c)/d) + 6*sqrt(a^5/d^2)*(d*e^(2*d*x + 2*c) - d)*l
og((2*sqrt(a^5/d^2)*(a*d*e^(2*d*x + 2*c) - I*a*d*e^(d*x + c) - 2*a*d) + (2*
a^3*e^(3*d*x + 3*c) - 4*I*a^3*e^(2*d*x + 2*c) - 2*a^3*e^(d*x + c) + 4*I*a^3
)*sqrt(a/(e^(2*d*x + 2*c) - 1)))*e^(-2*d*x - 2*c)/d) - 6*sqrt(a^5/d^2)*(d*e
^(2*d*x + 2*c) - d)*log(-(2*sqrt(a^5/d^2)*(a*d*e^(2*d*x + 2*c) - I*a*d*e^(d
*x + c) - 2*a*d) - (2*a^3*e^(3*d*x + 3*c) - 4*I*a^3*e^(2*d*x + 2*c) - 2*a^3
*e^(d*x + c) + 4*I*a^3)*sqrt(a/(e^(2*d*x + 2*c) - 1)))*e^(-2*d*x - 2*c)/d)
+ (64*a^2*e^(3*d*x + 3*c) - 48*I*a^2*e^(2*d*x + 2*c) - 48*a^2*e^(d*x + c) +
 64*I*a^2)*sqrt(a/(e^(2*d*x + 2*c) - 1)))/(d*e^(2*d*x + 2*c) - d)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (ia\,\text{csch}(dx+c)+a)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*csch(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((I*a*csch(d*x + c) + a)^(5/2), x)
```

maple [F] time = 0.95, size = 0, normalized size = 0.00

$$\int (a + ia\,\mathrm{csch}\,(dx + c))^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*csch(d*x+c))^(5/2),x)
```

```
[Out] int((a+I*a*csch(d*x+c))^(5/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (i\,a\,\mathrm{csch}\,(dx + c) + a)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*csch(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((I*a*csch(d*x + c) + a)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(a + \frac{a\,1{}\mathrm{i}}{\sinh\left(c + d\,x\right)}\right)}^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + (a*1i)/sinh(c + d*x))^(5/2),x)
```

```
[Out] int((a + (a*1i)/sinh(c + d*x))^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (ia\,\mathrm{csch}\,(c + dx) + a)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*csch(d*x+c))**(5/2),x)
```

```
[Out] Integral((I*a*csch(c + d*x) + a)**(5/2), x)
```

3.52 $\int (a + ia\mathbf{csch}(c + dx))^{3/2}\, dx$

Optimal. Leaf size=72

$$\frac{2a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\coth(c+dx)}{\sqrt{a+ia\text{csch}(c+dx)}}\right)}{d}+\frac{2a^2\coth(c+dx)}{d\sqrt{a+ia\text{csch}(c+dx)}}$$

```
[Out] 2*a^(3/2)*arctanh(coth(d*x+c)*a^(1/2)/(a+I*a*csch(d*x+c))^(1/2))/d+2*a^2*co
th(d*x+c)/d/(a+I*a*csch(d*x+c))^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {3775, 21, 3774, 203}

$$\frac{2a^2\coth(c+dx)}{d\sqrt{a+ia\text{csch}(c+dx)}}+\frac{2a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\coth(c+dx)}{\sqrt{a+ia\text{csch}(c+dx)}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Csch[c + d*x])^(3/2),x]
```

```
[Out] (2*a^(3/2)*ArcTanh[(Sqrt[a]*Coth[c + d*x])/Sqrt[a + I*a*Csch[c + d*x]]])/d
+ (2*a^2*Coth[c + d*x])/(d*Sqrt[a + I*a*Csch[c + d*x]])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3774

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[(-2*b)/d,
 Subst[Int[1/(a + x^2), x], x, (b*Cot[c + d*x])/Sqrt[a + b*Csc[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3775

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> -Simp[(b^2*Co
t[c + d*x]*(a + b*Csc[c + d*x])^(n - 2))/(d*(n - 1)), x] + Dist[a/(n - 1),
Int[(a + b*Csc[c + d*x])^(n - 2)*(a*(n - 1) + b*(3*n - 4)*Csc[c + d*x]), x]
, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && GtQ[n, 1] && Integer
Q[2*n]
```

Rubi steps

$$\int (a+ia\text{csch}(c+dx))^{3/2}\,dx = \frac{2a^2\coth(c+dx)}{d\sqrt{a+ia\text{csch}(c+dx)}} + (2a)\int \frac{\frac{a}{2}+\frac{1}{2}ia\text{csch}(c+dx)}{\sqrt{a+ia\text{csch}(c+dx)}}\,dx$$
$$= \frac{2a^2\coth(c+dx)}{d\sqrt{a+ia\text{csch}(c+dx)}} + a\int \sqrt{a+ia\text{csch}(c+dx)}\,dx$$
$$= \frac{2a^2\coth(c+dx)}{d\sqrt{a+ia\text{csch}(c+dx)}} - \frac{(2ia^2)\,\text{Subst}\left(\int \frac{1}{a+x^2}\,dx, x, \frac{ia\coth(c+dx)}{\sqrt{a+ia\text{csch}(c+dx)}}\right)}{d}$$
$$= \frac{2a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\coth(c+dx)}{\sqrt{a+ia\text{csch}(c+dx)}}\right)}{d} + \frac{2a^2\coth(c+dx)}{d\sqrt{a+ia\text{csch}(c+dx)}}$$

Mathematica [A] time = 1.25, size = 100, normalized size = 1.39

$$-\frac{2ia\coth(c+dx)\sqrt{a+ia\text{csch}(c+dx)}\left(\sqrt{\text{csch}(c+dx)+i} - \sqrt[4]{-1}\tanh^{-1}\left((-1)^{3/4}\sqrt{\text{csch}(c+dx)+i}\right)\right)}{d(\text{csch}(c+dx)-i)\sqrt{\text{csch}(c+dx)+i}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Csch[c + d*x])^(3/2),x]
```

```
[Out] ((-2*I)*a*Coth[c + d*x]*Sqrt[a + I*a*Csch[c + d*x]]*(-((-1)^(1/4)*ArcTanh[(
-1)^(3/4)*Sqrt[I + Csch[c + d*x]]]) + Sqrt[I + Csch[c + d*x]]))/(d*(-I + Cs
ch[c + d*x])*Sqrt[I + Csch[c + d*x]])
```

fricas [B] time = 0.58, size = 467, normalized size = 6.49

$$2\sqrt{\frac{a^3}{d^2}}\,d\log\left(\frac{\left(2\,a^2e^{(dx+c)}+2i\,a^2+2\left(de^{(2\,dx+2\,c)}-d\right)\sqrt{\frac{a^3}{d^2}}\sqrt{\frac{a}{e^{(2\,dx+2\,c)}-1}}\right)e^{(-dx-c)}}{d}\right) - 2\sqrt{\frac{a^3}{d^2}}\,d\log\left(\frac{\left(2\,a^2e^{(dx+c)}+2i\,a^2-2\left(de^{(2\,dx+2\,c)}-d\right)\sqrt{\frac{a^3}{d^2}}\sqrt{\right.}}{d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*csch(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/4*(2*sqrt(a^3/d^2)*d*log((2*a^2*e^(d*x + c) + 2*I*a^2 + 2*(d*e^(2*d*x + 2
*c) - d)*sqrt(a^3/d^2)*sqrt(a/(e^(2*d*x + 2*c) - 1)))*e^(-d*x - c)/d) - 2*s
qrt(a^3/d^2)*d*log((2*a^2*e^(d*x + c) + 2*I*a^2 - 2*(d*e^(2*d*x + 2*c) - d)
*sqrt(a^3/d^2)*sqrt(a/(e^(2*d*x + 2*c) - 1)))*e^(-d*x - c)/d) + 2*sqrt(a^3/
d^2)*d*log((2*(a*d*e^(2*d*x + 2*c) - I*a*d*e^(d*x + c) - 2*a*d)*sqrt(a^3/d^
2) + (2*a^2*e^(3*d*x + 3*c) - 4*I*a^2*e^(2*d*x + 2*c) - 2*a^2*e^(d*x + c) +
 4*I*a^2)*sqrt(a/(e^(2*d*x + 2*c) - 1)))*e^(-2*d*x - 2*c)/d) - 2*sqrt(a^3/d
^2)*d*log(-(2*(a*d*e^(2*d*x + 2*c) - I*a*d*e^(d*x + c) - 2*a*d)*sqrt(a^3/d^
2) - (2*a^2*e^(3*d*x + 3*c) - 4*I*a^2*e^(2*d*x + 2*c) - 2*a^2*e^(d*x + c) +
 4*I*a^2)*sqrt(a/(e^(2*d*x + 2*c) - 1)))*e^(-2*d*x - 2*c)/d) + (8*a*e^(d*x
+ c) - 8*I*a)*sqrt(a/(e^(2*d*x + 2*c) - 1)))/d
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (i\,a\,\text{csch}\,(dx+c)+a)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*csch(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((I*a*csch(d*x + c) + a)^(3/2), x)
```

maple [F] time = 0.70, size = 0, normalized size = 0.00

$$\int (a+ia\,\mathrm{csch}\,(dx+c))^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*csch(d*x+c))^(3/2),x)
[Out] int((a+I*a*csch(d*x+c))^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (i\,a\,\mathrm{csch}\,(dx+c)+a)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*csch(d*x+c))^(3/2),x, algorithm="maxima")
[Out] integrate((I*a*csch(d*x + c) + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(a+\frac{a\,1\mathrm{i}}{\sinh\left(c+d\,x\right)}\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + (a*1i)/sinh(c + d*x))^(3/2),x)
[Out] int((a + (a*1i)/sinh(c + d*x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (ia\,\mathrm{csch}\,(c+dx)+a)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*csch(d*x+c))**(3/2),x)
[Out] Integral((I*a*csch(c + d*x) + a)**(3/2), x)
```

3.53 $\int \sqrt{a+ia\,\text{csch}(c+dx)}\,dx$

Optimal. Leaf size=40

$$\frac{2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\coth(c+dx)}{\sqrt{a+ia\text{csch}(c+dx)}}\right)}{d}$$

```
[Out] 2*arctanh(coth(d*x+c)*a^(1/2)/(a+I*a*csch(d*x+c))^(1/2))*a^(1/2)/d
```

Rubi [A] time = 0.02, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {3774, 203}

$$\frac{2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\coth(c+dx)}{\sqrt{a+ia\text{csch}(c+dx)}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + I*a*Csch[c + d*x]],x]
```

```
[Out] (2*Sqrt[a]*ArcTanh[(Sqrt[a]*Coth[c + d*x])/Sqrt[a + I*a*Csch[c + d*x]]])/d
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3774

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[(-2*b)/d,
 Subst[Int[1/(a + x^2), x], x, (b*Cot[c + d*x])/Sqrt[a + b*Csc[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \sqrt{a+ia\text{csch}(c+dx)}\,dx &= -\frac{(2ia)\,\text{Subst}\left(\int \frac{1}{a+x^2}\,dx, x, \frac{ia\coth(c+dx)}{\sqrt{a+ia\text{csch}(c+dx)}}\right)}{d}\\ &= \frac{2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\coth(c+dx)}{\sqrt{a+ia\text{csch}(c+dx)}}\right)}{d}\end{aligned}$$

Mathematica [A] time = 0.93, size = 80, normalized size = 2.00

$$\frac{2(-1)^{3/4}\coth(c+dx)\sqrt{a+ia\text{csch}(c+dx)}\tanh^{-1}\left((-1)^{3/4}\sqrt{\text{csch}(c+dx)+i}\right)}{d(\text{csch}(c+dx)-i)\sqrt{\text{csch}(c+dx)+i}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + I*a*Csch[c + d*x]],x]
```

```
[Out] (2*(-1)^(3/4)*ArcTanh[(-1)^(3/4)*Sqrt[I + Csch[c + d*x]]]*Coth[c + d*x]*Sqr
t[a + I*a*Csch[c + d*x]])/(d*(-I + Csch[c + d*x])*Sqrt[I + Csch[c + d*x]])
```

fricas [B] time = 0.95, size = 386, normalized size = 9.65

$$\frac{1}{2}\sqrt{\frac{a}{d^2}}\log\left(\frac{\left(2\left(de^{(2dx+2c)}-d\right)\sqrt{\frac{a}{e^{(2dx+2c)}-1}}\sqrt{\frac{a}{d^2}}+2ae^{(dx+c)}+2ia\right)e^{(-dx-c)}}{d}\right)-\frac{1}{2}\sqrt{\frac{a}{d^2}}\log\left(-\frac{\left(2\left(de^{(2dx+2c)}-d\right)\sqrt{\right.}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*csch(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(a/d^2)*log((2*(d*e^(2*d*x + 2*c) - d)*sqrt(a/(e^(2*d*x + 2*c) - 1)
)*sqrt(a/d^2) + 2*a*e^(d*x + c) + 2*I*a)*e^(-d*x - c)/d) - 1/2*sqrt(a/d^2)*
log(-(2*(d*e^(2*d*x + 2*c) - d)*sqrt(a/(e^(2*d*x + 2*c) - 1))*sqrt(a/d^2) -
 2*a*e^(d*x + c) - 2*I*a)*e^(-d*x - c)/d) + 1/2*sqrt(a/d^2)*log(((2*a*e^(3*
d*x + 3*c) - 4*I*a*e^(2*d*x + 2*c) - 2*a*e^(d*x + c) + 4*I*a)*sqrt(a/(e^(2*
d*x + 2*c) - 1)) + 2*(a*d*e^(2*d*x + 2*c) - I*a*d*e^(d*x + c) - 2*a*d)*sqrt
(a/d^2))*e^(-2*d*x - 2*c)/d) - 1/2*sqrt(a/d^2)*log(((2*a*e^(3*d*x + 3*c) -
4*I*a*e^(2*d*x + 2*c) - 2*a*e^(d*x + c) + 4*I*a)*sqrt(a/(e^(2*d*x + 2*c) -
1)) - 2*(a*d*e^(2*d*x + 2*c) - I*a*d*e^(d*x + c) - 2*a*d)*sqrt(a/d^2))*e^(-
2*d*x - 2*c)/d)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{i\,a\,\mathrm{csch}\left(dx+c\right)+a}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*csch(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(I*a*csch(d*x + c) + a), x)
```

maple [F] time = 2.39, size = 0, normalized size = 0.00

$$\int \sqrt{a+ia\,\mathrm{csch}\left(dx+c\right)}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*csch(d*x+c))^(1/2),x)
```

```
[Out] int((a+I*a*csch(d*x+c))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{i\,a\,\mathrm{csch}\left(dx+c\right)+a}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*csch(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(I*a*csch(d*x + c) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \sqrt{a+\frac{a\,1\mathrm{i}}{\mathrm{sinh}\left(c+d\,x\right)}}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + (a*1i)/sinh(c + d*x))^(1/2),x)
```

```
[Out] int((a + (a*1i)/sinh(c + d*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{ia\,\mathrm{csch}\left(c+dx\right)+a}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*csch(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(I*a*csch(c + d*x) + a), x)
```

3.54 $\int \frac{1}{\sqrt{a+ia\mathbf{csch}(c+dx)}}\, dx$

Optimal. Leaf size=91

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{a}\coth(c+dx)}{\sqrt{a+ia\text{csch}(c+dx)}}\right)}{\sqrt{a}\,d}-\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\coth(c+dx)}{\sqrt{2}\sqrt{a+ia\text{csch}(c+dx)}}\right)}{\sqrt{a}\,d}$$

```
[Out] 2*arctanh(coth(d*x+c)*a^(1/2)/(a+I*a*csch(d*x+c))^(1/2))/d/a^(1/2)-arctanh(
1/2*coth(d*x+c)*a^(1/2)*2^(1/2)/(a+I*a*csch(d*x+c))^(1/2))*2^(1/2)/d/a^(1/2
)
```

Rubi [A] time = 0.09, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {3776, 3774, 203, 3795}

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{a}\coth(c+dx)}{\sqrt{a+ia\text{csch}(c+dx)}}\right)}{\sqrt{a}\,d}-\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\coth(c+dx)}{\sqrt{2}\sqrt{a+ia\text{csch}(c+dx)}}\right)}{\sqrt{a}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a + I*a*Csch[c + d*x]],x]
```

```
[Out] (2*ArcTanh[(Sqrt[a]*Coth[c + d*x])/Sqrt[a + I*a*Csch[c + d*x]]])/(Sqrt[a]*d
) - (Sqrt[2]*ArcTanh[(Sqrt[a]*Coth[c + d*x])/(Sqrt[2]*Sqrt[a + I*a*Csch[c +
 d*x]])])/(Sqrt[a]*d)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3774

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[(-2*b)/d,
 Subst[Int[1/(a + x^2), x], x, (b*Cot[c + d*x])/Sqrt[a + b*Csc[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3776

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[1/a, In
t[Sqrt[a + b*Csc[c + d*x]], x], x] - Dist[b/a, Int[Csc[c + d*x]/Sqrt[a + b*
Csc[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3795

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, (b*Cot[e + f*x])/Sqrt[
a + b*Csc[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+iacsch(c+dx)}}\,dx = -\left(i\int \frac{csch(c+dx)}{\sqrt{a+iacsch(c+dx)}}\,dx\right) + \frac{\int \sqrt{a+iacsch(c+dx)}\,dx}{a}$$
$$= -\frac{(2i)\,\text{Subst}\left(\int \frac{1}{a+x^2}\,dx, x, \frac{ia\coth(c+dx)}{\sqrt{a+iacsch(c+dx)}}\right)}{d} + \frac{(2i)\,\text{Subst}\left(\int \frac{1}{2a+x^2}\,dx, x, \frac{ia\coth(c+dx)}{\sqrt{a+iacsch(c+dx)}}\right)}{d}$$
$$= \frac{2\tanh^{-1}\left(\frac{\sqrt{a}\coth(c+dx)}{\sqrt{a+iacsch(c+dx)}}\right)}{\sqrt{a}\,d} - \frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\coth(c+dx)}{\sqrt{2}\sqrt{a+iacsch(c+dx)}}\right)}{\sqrt{a}\,d}$$

Mathematica [A] time = 1.12, size = 118, normalized size = 1.30

$$\frac{\sqrt{a}\coth(c+dx)\left(2\tan^{-1}\left(\frac{\sqrt{ia(csch(c+dx)+i)}}{\sqrt{a}}\right) - \sqrt{2}\tan^{-1}\left(\frac{\sqrt{ia(csch(c+dx)+i)}}{\sqrt{2}\sqrt{a}}\right)\right)}{d\sqrt{ia(csch(c+dx)+i)}\sqrt{a+iacsch(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a + I*a*Csch[c + d*x]],x]
```

```
[Out] (Sqrt[a]*(2*ArcTan[Sqrt[I*a*(I + Csch[c + d*x])]/Sqrt[a]] - Sqrt[2]*ArcTan[
Sqrt[I*a*(I + Csch[c + d*x])]/(Sqrt[2]*Sqrt[a])])*Coth[c + d*x])/(d*Sqrt[I*
a*(I + Csch[c + d*x])]*Sqrt[a + I*a*Csch[c + d*x]])
```

fricas [B] time = 1.50, size = 561, normalized size = 6.16

$$-\frac{1}{2}\sqrt{2}\sqrt{\frac{1}{ad^2}}\log\left(\left(2\sqrt{2}\left(ade^{(2dx+2c)} - ad\right)\sqrt{\frac{a}{e^{(2dx+2c)}-1}}\sqrt{\frac{1}{ad^2}} + 2ae^{(dx+c)} - 2ia\right)e^{(-dx-c)}\right) + \frac{1}{2}\sqrt{2}\sqrt{\frac{1}{ad^2}}\,\text{lo}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*csch(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] -1/2*sqrt(2)*sqrt(1/(a*d^2))*log((2*sqrt(2)*(a*d*e^(2*d*x + 2*c) - a*d)*sqr
t(a/(e^(2*d*x + 2*c) - 1))*sqrt(1/(a*d^2)) + 2*a*e^(d*x + c) - 2*I*a)*e^(-d
*x - c)) + 1/2*sqrt(2)*sqrt(1/(a*d^2))*log(-(2*sqrt(2)*(a*d*e^(2*d*x + 2*c)
 - a*d)*sqrt(a/(e^(2*d*x + 2*c) - 1))*sqrt(1/(a*d^2)) - 2*a*e^(d*x + c) + 2
*I*a)*e^(-d*x - c)) + 1/2*sqrt(1/(a*d^2))*log((2*(d*e^(2*d*x + 2*c) - d)*sq
rt(a/(e^(2*d*x + 2*c) - 1))*sqrt(1/(a*d^2)) + 2*e^(d*x + c) + 2*I)*e^(-d*x
- c)/d) - 1/2*sqrt(1/(a*d^2))*log(-(2*(d*e^(2*d*x + 2*c) - d)*sqrt(a/(e^(2*
d*x + 2*c) - 1))*sqrt(1/(a*d^2)) - 2*e^(d*x + c) - 2*I)*e^(-d*x - c)/d) + 1
/2*sqrt(1/(a*d^2))*log(((2*a*d*e^(2*d*x + 2*c) - 2*I*a*d*e^(d*x + c) - 4*a*
d)*sqrt(1/(a*d^2)) + sqrt(a/(e^(2*d*x + 2*c) - 1))*(2*e^(3*d*x + 3*c) - 4*I
*e^(2*d*x + 2*c) - 2*e^(d*x + c) + 4*I))*e^(-2*d*x - 2*c)/d) - 1/2*sqrt(1/(
a*d^2))*log(-((2*a*d*e^(2*d*x + 2*c) - 2*I*a*d*e^(d*x + c) - 4*a*d)*sqrt(1/
(a*d^2)) - sqrt(a/(e^(2*d*x + 2*c) - 1))*(2*e^(3*d*x + 3*c) - 4*I*e^(2*d*x
+ 2*c) - 2*e^(d*x + c) + 4*I))*e^(-2*d*x - 2*c)/d)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{i\,a\,\text{csch}\,(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*csch(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(I*a*csch(d*x + c) + a), x)
```

maple [F] time = 0.82, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a + ia\,\mathrm{csch}\,(dx + c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+I*a*csch(d*x+c))^(1/2),x)
[Out] int(1/(a+I*a*csch(d*x+c))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{i\,a\,\mathrm{csch}\,(dx + c) + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*csch(d*x+c))^(1/2),x, algorithm="maxima")
[Out] integrate(1/sqrt(I*a*csch(d*x + c) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{a + \frac{a\,1\mathrm{i}}{\sinh(c+d\,x)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + (a*1i)/sinh(c + d*x))^(1/2),x)
[Out] int(1/(a + (a*1i)/sinh(c + d*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{ia\,\mathrm{csch}\,(c + dx) + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*csch(d*x+c))**(1/2),x)
[Out] Integral(1/sqrt(I*a*csch(c + d*x) + a), x)
```

3.55 $\int \frac{1}{(a+ia\,\mathbf{csch}(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=123

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{a}\coth(c+dx)}{\sqrt{a+ia\text{csch}(c+dx)}}\right)}{a^{3/2}d}-\frac{5\tanh^{-1}\left(\frac{\sqrt{a}\coth(c+dx)}{\sqrt{2}\sqrt{a+ia\text{csch}(c+dx)}}\right)}{2\sqrt{2}\,a^{3/2}d}-\frac{\coth(c+dx)}{2d(a+ia\text{csch}(c+dx))^{3/2}}$$

```
[Out] 2*arctanh(coth(d*x+c)*a^(1/2)/(a+I*a*csch(d*x+c))^(1/2))/a^(3/2)/d-1/2*coth
(d*x+c)/d/(a+I*a*csch(d*x+c))^(3/2)-5/4*arctanh(1/2*coth(d*x+c)*a^(1/2)*2^(
1/2)/(a+I*a*csch(d*x+c))^(1/2))/a^(3/2)/d*2^(1/2)
```

Rubi [A] time = 0.15, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {3777, 3920, 3774, 203, 3795}

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{a}\coth(c+dx)}{\sqrt{a+ia\text{csch}(c+dx)}}\right)}{a^{3/2}d}-\frac{5\tanh^{-1}\left(\frac{\sqrt{a}\coth(c+dx)}{\sqrt{2}\sqrt{a+ia\text{csch}(c+dx)}}\right)}{2\sqrt{2}\,a^{3/2}d}-\frac{\coth(c+dx)}{2d(a+ia\text{csch}(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Csch[c + d*x])^(-3/2),x]
```

```
[Out] (2*ArcTanh[(Sqrt[a]*Coth[c + d*x])/Sqrt[a + I*a*Csch[c + d*x]]])/(a^(3/2)*d
) - (5*ArcTanh[(Sqrt[a]*Coth[c + d*x])/(Sqrt[2]*Sqrt[a + I*a*Csch[c + d*x]]
)])/(2*Sqrt[2]*a^(3/2)*d) - Coth[c + d*x]/(2*d*(a + I*a*Csch[c + d*x])^(3/2
))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3774

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[(-2*b)/d,
 Subst[Int[1/(a + x^2), x], x, (b*Cot[c + d*x])/Sqrt[a + b*Csc[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3777

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> -Simp[(Cot[c
+ d*x]*(a + b*Csc[c + d*x])^n)/(d*(2*n + 1)), x] + Dist[1/(a^2*(2*n + 1)),
Int[(a + b*Csc[c + d*x])^(n + 1)*(a*(2*n + 1) - b*(n + 1)*Csc[c + d*x]), x]
, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && LeQ[n, -1] && Intege
rQ[2*n]
```

Rule 3795

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, (b*Cot[e + f*x])/Sqrt[
a + b*Csc[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3920

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c/a, Int[Sqrt[a + b*Csc[e + f*x]], x], x] - D
ist[(b*c - a*d)/a, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; Fre
```

```
eQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{1}{(a+ia\text{csch}(c+dx))^{3/2}}\,dx = -\frac{\coth(c+dx)}{2d(a+ia\text{csch}(c+dx))^{3/2}} - \frac{\int \frac{-2a+\frac{1}{2}ia\text{csch}(c+dx)}{\sqrt{a+ia\text{csch}(c+dx)}}\,dx}{2a^2}$$

$$= -\frac{\coth(c+dx)}{2d(a+ia\text{csch}(c+dx))^{3/2}} + \frac{\int \sqrt{a+ia\text{csch}(c+dx)}\,dx}{a^2} - \frac{(5i)\int \frac{\text{csch}(c+dx)}{\sqrt{a+ia\text{csch}(c+dx)}}\,d}{4a}$$

$$= -\frac{\coth(c+dx)}{2d(a+ia\text{csch}(c+dx))^{3/2}} - \frac{(2i)\,\text{Subst}\left(\int \frac{1}{a+x^2}\,dx, x, \frac{ia\coth(c+dx)}{\sqrt{a+ia\text{csch}(c+dx)}}\right)}{ad} + \frac{(5i)\,\text{Sub}}{}$$

$$= \frac{2\tanh^{-1}\left(\frac{\sqrt{a}\coth(c+dx)}{\sqrt{a+ia\text{csch}(c+dx)}}\right)}{a^{3/2}d} - \frac{5\tanh^{-1}\left(\frac{\sqrt{a}\coth(c+dx)}{\sqrt{2}\sqrt{a+ia\text{csch}(c+dx)}}\right)}{2\sqrt{2}\,a^{3/2}d} - \frac{\coth(c+dx)}{2d(a+ia\text{csch}(c+dx}$$

Mathematica [B] time = 2.45, size = 327, normalized size = 2.66

$$\frac{\left(\cosh\left(\frac{1}{2}(c+dx)\right)+i\sinh\left(\frac{1}{2}(c+dx)\right)\right)\left(-8\sqrt{ia(\text{csch}(c+dx)+i)}\tan^{-1}\left(\frac{\sqrt{ia(\text{csch}(c+dx)+i)}}{\sqrt{a}}\right)+i\text{csch}(c+dx)\left(-8\sqrt{i}\right.\right.}{4a^{3/2}d(\text{csch}(c+}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Csch[c + d*x])^(-3/2),x]
```

```
[Out] ((-2*Sqrt[a] - 8*ArcTan[Sqrt[I*a*(I + Csch[c + d*x])]/Sqrt[a]]*Sqrt[I*a*(I
+ Csch[c + d*x])] + 5*Sqrt[2]*ArcTan[Sqrt[I*a*(I + Csch[c + d*x])]/(Sqrt[2]
*Sqrt[a])]*Sqrt[I*a*(I + Csch[c + d*x])] + I*Csch[c + d*x]*(2*Sqrt[a] - 8*A
rcTan[Sqrt[I*a*(I + Csch[c + d*x])]/Sqrt[a]]*Sqrt[I*a*(I + Csch[c + d*x])]
+ 5*Sqrt[2]*ArcTan[Sqrt[I*a*(I + Csch[c + d*x])]/(Sqrt[2]*Sqrt[a])]*Sqrt[I*
a*(I + Csch[c + d*x])]))*(Cosh[(c + d*x)/2] + I*Sinh[(c + d*x)/2]))/(4*a^(3
/2)*d*(I + Csch[c + d*x])*Sqrt[a + I*a*Csch[c + d*x]]*(Cosh[(c + d*x)/2] -
I*Sinh[(c + d*x)/2]))
```

fricas [B] time = 2.45, size = 886, normalized size = 7.20

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*csch(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] -(sqrt(1/2)*(5*a^2*d*e^(2*d*x + 2*c) + 10*I*a^2*d*e^(d*x + c) - 5*a^2*d)*sq
rt(1/(a^3*d^2))*log((4*sqrt(1/2)*(a^2*d*e^(2*d*x + 2*c) - a^2*d)*sqrt(a/(e^
(2*d*x + 2*c) - 1))*sqrt(1/(a^3*d^2)) + 2*a*e^(d*x + c) - 2*I*a)*e^(-d*x -
c)) - sqrt(1/2)*(5*a^2*d*e^(2*d*x + 2*c) + 10*I*a^2*d*e^(d*x + c) - 5*a^2*d
)*sqrt(1/(a^3*d^2))*log(-(4*sqrt(1/2)*(a^2*d*e^(2*d*x + 2*c) - a^2*d)*sqrt(
a/(e^(2*d*x + 2*c) - 1))*sqrt(1/(a^3*d^2)) - 2*a*e^(d*x + c) + 2*I*a)*e^(-d
*x - c)) - (2*a^2*d*e^(2*d*x + 2*c) + 4*I*a^2*d*e^(d*x + c) - 2*a^2*d)*sqrt
(1/(a^3*d^2))*log((2*(a*d*e^(2*d*x + 2*c) - a*d)*sqrt(a/(e^(2*d*x + 2*c) -
1))*sqrt(1/(a^3*d^2)) + 2*e^(d*x + c) + 2*I)*e^(-d*x - c)/(a*d)) + (2*a^2*d
*e^(2*d*x + 2*c) + 4*I*a^2*d*e^(d*x + c) - 2*a^2*d)*sqrt(1/(a^3*d^2))*log(-
(2*(a*d*e^(2*d*x + 2*c) - a*d)*sqrt(a/(e^(2*d*x + 2*c) - 1))*sqrt(1/(a^3*d^
2)) - 2*e^(d*x + c) - 2*I)*e^(-d*x - c)/(a*d)) - (2*a^2*d*e^(2*d*x + 2*c) +
 4*I*a^2*d*e^(d*x + c) - 2*a^2*d)*sqrt(1/(a^3*d^2))*log(((2*a^2*d*e^(2*d*x
```

```
+ 2*c) - 2*I*a^2*d*e^(d*x + c) - 4*a^2*d)*sqrt(1/(a^3*d^2)) + sqrt(a/(e^(2*
d*x + 2*c) - 1))*(2*e^(3*d*x + 3*c) - 4*I*e^(2*d*x + 2*c) - 2*e^(d*x + c) +
 4*I))*e^(-2*d*x - 2*c)/(a*d)) + (2*a^2*d*e^(2*d*x + 2*c) + 4*I*a^2*d*e^(d*
x + c) - 2*a^2*d)*sqrt(1/(a^3*d^2))*log(-((2*a^2*d*e^(2*d*x + 2*c) - 2*I*a^
2*d*e^(d*x + c) - 4*a^2*d)*sqrt(1/(a^3*d^2)) - sqrt(a/(e^(2*d*x + 2*c) - 1)
)*(2*e^(3*d*x + 3*c) - 4*I*e^(2*d*x + 2*c) - 2*e^(d*x + c) + 4*I))*e^(-2*d*
x - 2*c)/(a*d)) + sqrt(a/(e^(2*d*x + 2*c) - 1))*(2*e^(3*d*x + 3*c) - 2*I*e^
(2*d*x + 2*c) - 2*e^(d*x + c) + 2*I))/(4*a^2*d*e^(2*d*x + 2*c) + 8*I*a^2*d*
e^(d*x + c) - 4*a^2*d)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(i\,a\operatorname{csch}(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*csch(d*x+c))^(3/2),x, algorithm="giac")
[Out] integrate((I*a*csch(d*x + c) + a)^(-3/2), x)
```

maple [F] time = 0.63, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+ia\operatorname{csch}(dx+c))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+I*a*csch(d*x+c))^(3/2),x)
[Out] int(1/(a+I*a*csch(d*x+c))^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(i\,a\operatorname{csch}(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*csch(d*x+c))^(3/2),x, algorithm="maxima")
[Out] integrate((I*a*csch(d*x + c) + a)^(-3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(a+\frac{a\,1\mathrm{i}}{\sinh(c+d\,x)}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + (a*1i)/sinh(c + d*x))^(3/2),x)
[Out] int(1/(a + (a*1i)/sinh(c + d*x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(ia\operatorname{csch}(c+dx)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*csch(d*x+c))**(3/2),x)
[Out] Integral((I*a*csch(c + d*x) + a)**(-3/2), x)
```

3.56 $\int \sqrt{a - ia\,\mathbf{csch}(c + dx)}\, dx$

Optimal. Leaf size=40

$$\frac{2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\coth(c+dx)}{\sqrt{a-ia\operatorname{csch}(c+dx)}}\right)}{d}$$

```
[Out] 2*arctanh(coth(d*x+c)*a^(1/2)/(a-I*a*csch(d*x+c))^(1/2))*a^(1/2)/d
```

Rubi [A] time = 0.02, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {3774, 203}

$$\frac{2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\coth(c+dx)}{\sqrt{a-ia\operatorname{csch}(c+dx)}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a - I*a*Csch[c + d*x]],x]
```

```
[Out] (2*Sqrt[a]*ArcTanh[(Sqrt[a]*Coth[c + d*x])/Sqrt[a - I*a*Csch[c + d*x]]])/d
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3774

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[(-2*b)/d,
 Subst[Int[1/(a + x^2), x], x, (b*Cot[c + d*x])/Sqrt[a + b*Csc[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \sqrt{a-ia\operatorname{csch}(c+dx)}\,dx &= \frac{(2ia)\operatorname{Subst}\left(\int \frac{1}{a+x^2}\,dx, x, -\frac{ia\coth(c+dx)}{\sqrt{a-ia\operatorname{csch}(c+dx)}}\right)}{d}\\ &= \frac{2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\coth(c+dx)}{\sqrt{a-ia\operatorname{csch}(c+dx)}}\right)}{d}\end{aligned}$$

Mathematica [A] time = 1.01, size = 80, normalized size = 2.00

$$-\frac{2(-1)^{3/4}\coth(c+dx)\sqrt{a-ia\operatorname{csch}(c+dx)}\tan^{-1}\left((-1)^{3/4}\sqrt{\operatorname{csch}(c+dx)-i}\right)}{d\sqrt{\operatorname{csch}(c+dx)-i}\,(\operatorname{csch}(c+dx)+i)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a - I*a*Csch[c + d*x]],x]
```

```
[Out] (-2*(-1)^(3/4)*ArcTan[(-1)^(3/4)*Sqrt[-I + Csch[c + d*x]]]*Coth[c + d*x]*Sq
rt[a - I*a*Csch[c + d*x]])/(d*Sqrt[-I + Csch[c + d*x]]*(I + Csch[c + d*x]))
```

fricas [B] time = 0.58, size = 386, normalized size = 9.65

$$\frac{1}{2}\sqrt{\frac{a}{d^2}}\log\left(\frac{\left(2\left(de^{(2\,dx+2\,c)}-d\right)\sqrt{\frac{a}{e^{(2\,dx+2\,c)}-1}}\sqrt{\frac{a}{d^2}}+2\,ae^{(dx+c)}-2i\,a\right)e^{(-dx-c)}}{d}\right)-\frac{1}{2}\sqrt{\frac{a}{d^2}}\log\left(-\frac{\left(2\left(de^{(2\,dx+2\,c)}-d\right)\sqrt{\right.}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-I*a*csch(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(a/d^2)*log((2*(d*e^(2*d*x + 2*c) - d)*sqrt(a/(e^(2*d*x + 2*c) - 1)
)*sqrt(a/d^2) + 2*a*e^(d*x + c) - 2*I*a)*e^(-d*x - c)/d) - 1/2*sqrt(a/d^2)*
log(-(2*(d*e^(2*d*x + 2*c) - d)*sqrt(a/(e^(2*d*x + 2*c) - 1))*sqrt(a/d^2) -
 2*a*e^(d*x + c) + 2*I*a)*e^(-d*x - c)/d) + 1/2*sqrt(a/d^2)*log(((2*a*e^(3*
d*x + 3*c) + 4*I*a*e^(2*d*x + 2*c) - 2*a*e^(d*x + c) - 4*I*a)*sqrt(a/(e^(2*
d*x + 2*c) - 1)) + 2*(a*d*e^(2*d*x + 2*c) + I*a*d*e^(d*x + c) - 2*a*d)*sqrt
(a/d^2))*e^(-2*d*x - 2*c)/d) - 1/2*sqrt(a/d^2)*log(((2*a*e^(3*d*x + 3*c) +
4*I*a*e^(2*d*x + 2*c) - 2*a*e^(d*x + c) - 4*I*a)*sqrt(a/(e^(2*d*x + 2*c) -
1)) - 2*(a*d*e^(2*d*x + 2*c) + I*a*d*e^(d*x + c) - 2*a*d)*sqrt(a/d^2))*e^(-
2*d*x - 2*c)/d)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{-i\, a \operatorname{csch}\left(dx + c\right) + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-I*a*csch(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-I*a*csch(d*x + c) + a), x)
```

maple [F] time = 2.78, size = 0, normalized size = 0.00

$$\int \sqrt{a - ia \operatorname{csch}\left(dx + c\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a-I*a*csch(d*x+c))^(1/2),x)
```

```
[Out] int((a-I*a*csch(d*x+c))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{-i\, a \operatorname{csch}\left(dx + c\right) + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-I*a*csch(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-I*a*csch(d*x + c) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \sqrt{a - \frac{a\,1{}\mathrm{i}}{\sinh\left(c + d\,x\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - (a*1i)/sinh(c + d*x))^(1/2),x)
```

```
[Out] int((a - (a*1i)/sinh(c + d*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{-ia \operatorname{csch}\left(c + dx\right) + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-I*a*csch(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(-I*a*csch(c + d*x) + a), x)
```

3.57 $\int \frac{1}{\sqrt{a-ia\,\mathbf{csch}(c+dx)}}\,dx$

Optimal. Leaf size=91

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{a}\coth(c+dx)}{\sqrt{a-ia\operatorname{csch}(c+dx)}}\right)}{\sqrt{a}\,d}-\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\coth(c+dx)}{\sqrt{2}\sqrt{a-ia\operatorname{csch}(c+dx)}}\right)}{\sqrt{a}\,d}$$

```
[Out] 2*arctanh(coth(d*x+c)*a^(1/2)/(a-I*a*csch(d*x+c))^(1/2))/d/a^(1/2)-arctanh(
1/2*coth(d*x+c)*a^(1/2)*2^(1/2)/(a-I*a*csch(d*x+c))^(1/2))*2^(1/2)/d/a^(1/2
)
```

Rubi [A] time = 0.08, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {3776, 3774, 203, 3795}

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{a}\coth(c+dx)}{\sqrt{a-ia\operatorname{csch}(c+dx)}}\right)}{\sqrt{a}\,d}-\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\coth(c+dx)}{\sqrt{2}\sqrt{a-ia\operatorname{csch}(c+dx)}}\right)}{\sqrt{a}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a - I*a*Csch[c + d*x]],x]
```

```
[Out] (2*ArcTanh[(Sqrt[a]*Coth[c + d*x])/Sqrt[a - I*a*Csch[c + d*x]]])/(Sqrt[a]*d
) - (Sqrt[2]*ArcTanh[(Sqrt[a]*Coth[c + d*x])/(Sqrt[2]*Sqrt[a - I*a*Csch[c +
 d*x]])])/(Sqrt[a]*d)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3774

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[(-2*b)/d,
 Subst[Int[1/(a + x^2), x], x, (b*Cot[c + d*x])/Sqrt[a + b*Csc[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3776

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[1/a, In
t[Sqrt[a + b*Csc[c + d*x]], x], x] - Dist[b/a, Int[Csc[c + d*x]/Sqrt[a + b*
Csc[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3795

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, (b*Cot[e + f*x])/Sqrt[
a + b*Csc[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{a-ia\text{csch}(c+dx)}}\,dx = i\int \frac{\text{csch}(c+dx)}{\sqrt{a-ia\text{csch}(c+dx)}}\,dx + \frac{\int \sqrt{a-ia\text{csch}(c+dx)}\,dx}{a}$$
$$= \frac{(2i)\,\text{Subst}\left(\int \frac{1}{a+x^2}\,dx, x, -\frac{ia\coth(c+dx)}{\sqrt{a-ia\text{csch}(c+dx)}}\right)}{d} - \frac{(2i)\,\text{Subst}\left(\int \frac{1}{2a+x^2}\,dx, x, -\frac{ia\coth(c+dx)}{\sqrt{a-ia\text{csch}(c+dx)}}\right)}{d}$$
$$= \frac{2\tanh^{-1}\left(\frac{\sqrt{a}\coth(c+dx)}{\sqrt{a-ia\text{csch}(c+dx)}}\right)}{\sqrt{a}\,d} - \frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\coth(c+dx)}{\sqrt{2}\sqrt{a-ia\text{csch}(c+dx)}}\right)}{\sqrt{a}\,d}$$

Mathematica [A] time = 1.10, size = 117, normalized size = 1.29

$$\frac{\sqrt{a}\coth(c+dx)\left(2\tan^{-1}\left(\frac{\sqrt{-ia(\text{csch}(c+dx)-i)}}{\sqrt{a}}\right) - \sqrt{2}\tan^{-1}\left(\frac{\sqrt{-ia(\text{csch}(c+dx)-i)}}{\sqrt{2}\sqrt{a}}\right)\right)}{d\sqrt{a(-1-i\text{csch}(c+dx))}\sqrt{a-ia\text{csch}(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a - I*a*Csch[c + d*x]],x]
```

```
[Out] (Sqrt[a]*(2*ArcTan[Sqrt[(-I)*a*(-I + Csch[c + d*x])]/Sqrt[a]] - Sqrt[2]*Arc
Tan[Sqrt[(-I)*a*(-I + Csch[c + d*x])]/(Sqrt[2]*Sqrt[a])])*Coth[c + d*x])/(d
*Sqrt[a*(-1 - I*Csch[c + d*x])]*Sqrt[a - I*a*Csch[c + d*x]])
```

fricas [B] time = 1.28, size = 561, normalized size = 6.16

$$-\frac{1}{2}\sqrt{2}\sqrt{\frac{1}{ad^2}}\log\left(\left(2\sqrt{2}\left(ade^{(2dx+2c)} - ad\right)\sqrt{\frac{a}{e^{(2dx+2c)}-1}}\sqrt{\frac{1}{ad^2}} + 2ae^{(dx+c)} + 2ia\right)e^{(-dx-c)}\right) + \frac{1}{2}\sqrt{2}\sqrt{\frac{1}{ad^2}}\log$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*csch(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] -1/2*sqrt(2)*sqrt(1/(a*d^2))*log((2*sqrt(2)*(a*d*e^(2*d*x + 2*c) - a*d)*sqr
t(a/(e^(2*d*x + 2*c) - 1))*sqrt(1/(a*d^2)) + 2*a*e^(d*x + c) + 2*I*a)*e^(-d
*x - c)) + 1/2*sqrt(2)*sqrt(1/(a*d^2))*log(-(2*sqrt(2)*(a*d*e^(2*d*x + 2*c)
 - a*d)*sqrt(a/(e^(2*d*x + 2*c) - 1))*sqrt(1/(a*d^2)) - 2*a*e^(d*x + c) - 2
*I*a)*e^(-d*x - c)) + 1/2*sqrt(1/(a*d^2))*log((2*(d*e^(2*d*x + 2*c) - d)*sq
rt(a/(e^(2*d*x + 2*c) - 1))*sqrt(1/(a*d^2)) + 2*e^(d*x + c) - 2*I)*e^(-d*x
- c)/d) - 1/2*sqrt(1/(a*d^2))*log(-(2*(d*e^(2*d*x + 2*c) - d)*sqrt(a/(e^(2*
d*x + 2*c) - 1))*sqrt(1/(a*d^2)) - 2*e^(d*x + c) + 2*I)*e^(-d*x - c)/d) + 1
/2*sqrt(1/(a*d^2))*log(((2*a*d*e^(2*d*x + 2*c) + 2*I*a*d*e^(d*x + c) - 4*a*
d)*sqrt(1/(a*d^2)) + sqrt(a/(e^(2*d*x + 2*c) - 1))*(2*e^(3*d*x + 3*c) + 4*I
*e^(2*d*x + 2*c) - 2*e^(d*x + c) - 4*I))*e^(-2*d*x - 2*c)/d) - 1/2*sqrt(1/(
a*d^2))*log(-((2*a*d*e^(2*d*x + 2*c) + 2*I*a*d*e^(d*x + c) - 4*a*d)*sqrt(1/
(a*d^2)) - sqrt(a/(e^(2*d*x + 2*c) - 1))*(2*e^(3*d*x + 3*c) + 4*I*e^(2*d*x
+ 2*c) - 2*e^(d*x + c) - 4*I))*e^(-2*d*x - 2*c)/d)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-i\,a\,\text{csch}\,(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*csch(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(-I*a*csch(d*x + c) + a), x)
```

maple [F] time = 0.88, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a - ia \operatorname{csch}(dx + c)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a-I*a*csch(d*x+c))^(1/2),x)
[Out] int(1/(a-I*a*csch(d*x+c))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-i\, a \operatorname{csch}(dx + c) + a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*csch(d*x+c))^(1/2),x, algorithm="maxima")
[Out] integrate(1/sqrt(-I*a*csch(d*x + c) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{a - \frac{a\,1\mathrm{i}}{\sinh(c+d\,x)}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a - (a*1i)/sinh(c + d*x))^(1/2),x)
[Out] int(1/(a - (a*1i)/sinh(c + d*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-ia \operatorname{csch}(c + dx) + a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*csch(d*x+c))**(1/2),x)
[Out] Integral(1/sqrt(-I*a*csch(c + d*x) + a), x)
```

3.58 $\int \sqrt{3+3i\text{csch}(x)}\,dx$

Optimal. Leaf size=23

$$2\sqrt{3}\tanh^{-1}\left(\frac{\coth(x)}{\sqrt{1+i\text{csch}(x)}}\right)$$

```
[Out] 2*arctanh(coth(x)/(1+I*csch(x))^(1/2))*3^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3774, 203}

$$2\sqrt{3}\tanh^{-1}\left(\frac{\coth(x)}{\sqrt{1+i\text{csch}(x)}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[3 + (3*I)*Csch[x]],x]
```

```
[Out] 2*Sqrt[3]*ArcTanh[Coth[x]/Sqrt[1 + I*Csch[x]]]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3774

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[(-2*b)/d,
 Subst[Int[1/(a + x^2), x], x, (b*Cot[c + d*x])/Sqrt[a + b*Csc[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \sqrt{3+3i\text{csch}(x)}\,dx &= -\left(6i\,\text{Subst}\left(\int \frac{1}{3+x^2}\,dx, x, \frac{3i\coth(x)}{\sqrt{3+3i\text{csch}(x)}}\right)\right)\\ &= 2\sqrt{3}\tanh^{-1}\left(\frac{\coth(x)}{\sqrt{1+i\text{csch}(x)}}\right)\end{aligned}$$

Mathematica [A] time = 0.74, size = 46, normalized size = 2.00

$$\frac{2\sqrt{3}\coth(x)\tan^{-1}\left(\sqrt{-1+i\text{csch}(x)}\right)}{\sqrt{-1+i\text{csch}(x)}\sqrt{1+i\text{csch}(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[3 + (3*I)*Csch[x]],x]
```

```
[Out] (2*Sqrt[3]*ArcTan[Sqrt[-1 + I*Csch[x]]]*Coth[x])/(Sqrt[-1 + I*Csch[x]]*Sqrt
[1 + I*Csch[x]])
```

fricas [B] time = 0.46, size = 218, normalized size = 9.48

$$\frac{1}{2}\sqrt{3}\log\left(\left(\frac{2\sqrt{3}\left(\sqrt{3}e^{(2x)}-\sqrt{3}\right)}{\sqrt{e^{(2x)}-1}}+6e^{x}+6i\right)e^{(-x)}\right)-\frac{1}{2}\sqrt{3}\log\left(-\left(\frac{2\sqrt{3}\left(\sqrt{3}e^{(2x)}-\sqrt{3}\right)}{\sqrt{e^{(2x)}-1}}-6e^{x}-6i\right)e^{(-x)}\right)+\frac{1}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+3*I*csch(x))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(3)*log((2*sqrt(3)*(sqrt(3)*e^(2*x) - sqrt(3))/sqrt(e^(2*x) - 1) +
6*e^x + 6*I)*e^(-x)) - 1/2*sqrt(3)*log(-(2*sqrt(3)*(sqrt(3)*e^(2*x) - sqrt(
3))/sqrt(e^(2*x) - 1) - 6*e^x - 6*I)*e^(-x)) + 1/2*sqrt(3)*log((6*sqrt(3)*e
^(2*x) - 6*I*sqrt(3)*e^x + sqrt(3)*(6*e^(3*x) - 12*I*e^(2*x) - 6*e^x + 12*I
)/sqrt(e^(2*x) - 1) - 12*sqrt(3))*e^(-2*x)) - 1/2*sqrt(3)*log(-(6*sqrt(3)*e
^(2*x) - 6*I*sqrt(3)*e^x - sqrt(3)*(6*e^(3*x) - 12*I*e^(2*x) - 6*e^x + 12*I
)/sqrt(e^(2*x) - 1) - 12*sqrt(3))*e^(-2*x))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{3i\ \mathrm{csch}(x) + 3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+3*I*csch(x))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(3*I*csch(x) + 3), x)
```

maple [F] time = 0.83, size = 0, normalized size = 0.00

$$\int \sqrt{3+3i\mathrm{csch}(x)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3+3*I*csch(x))^(1/2),x)
```

```
[Out] int((3+3*I*csch(x))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{3i\ \mathrm{csch}(x) + 3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+3*I*csch(x))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(3*I*csch(x) + 3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.04

$$\int \sqrt{3+\frac{3\mathrm{i}}{\sinh(x)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3i/sinh(x) + 3)^(1/2),x)
```

```
[Out] int((3i/sinh(x) + 3)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\sqrt{3}\int \sqrt{i\,\mathrm{csch}\,(x) + 1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+3*I*csch(x))**(1/2),x)
```

```
[Out] sqrt(3)*Integral(sqrt(I*csch(x) + 1), x)
```

3.59 $\int \sqrt{3-3 i \operatorname{csch}(x)}\,dx$

Optimal. Leaf size=23

$$2 \sqrt{3} \tanh^{-1}\left(\frac{\coth(x)}{\sqrt{1-i \operatorname{csch}(x)}}\right)$$

[Out] 2*arctanh(coth(x)/(1-I*csch(x))^(1/2))*3^(1/2)

Rubi [A] time = 0.02, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3774, 203}

$$2 \sqrt{3} \tanh^{-1}\left(\frac{\coth(x)}{\sqrt{1-i \operatorname{csch}(x)}}\right)$$

Antiderivative was successfully verified.

[In] Int[Sqrt[3 - (3*I)*Csch[x]],x]

[Out] 2*Sqrt[3]*ArcTanh[Coth[x]/Sqrt[1 - I*Csch[x]]]

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3774

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[(-2*b)/d,
 Subst[Int[1/(a + x^2), x], x, (b*Cot[c + d*x])/Sqrt[a + b*Csc[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}
\int \sqrt{3-3 i \operatorname{csch}(x)}\,dx &= 6 i \operatorname{Subst}\left(\int \frac{1}{3+x^2}\,dx, x, -\frac{3 i \coth(x)}{\sqrt{3-3 i \operatorname{csch}(x)}}\right)\\
&= 2 \sqrt{3} \tanh^{-1}\left(\frac{\coth(x)}{\sqrt{1-i \operatorname{csch}(x)}}\right)
\end{aligned}$$

Mathematica [A] time = 0.72, size = 46, normalized size = 2.00

$$\frac{2 \sqrt{3} \coth(x) \tan^{-1}\left(\sqrt{-1-i \operatorname{csch}(x)}\right)}{\sqrt{-1-i \operatorname{csch}(x)} \sqrt{1-i \operatorname{csch}(x)}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[3 - (3*I)*Csch[x]],x]

[Out] (2*Sqrt[3]*ArcTan[Sqrt[-1 - I*Csch[x]]]*Coth[x])/(Sqrt[-1 - I*Csch[x]]*Sqrt[1 - I*Csch[x]])

fricas [B] time = 0.46, size = 218, normalized size = 9.48

$$\frac{1}{2} \sqrt{3} \log\left(\left(\frac{2 \sqrt{3}\left(\sqrt{3} e^{(2 x)}-\sqrt{3}\right)}{\sqrt{e^{(2 x)}-1}}+6 e^{x}-6 i\right) e^{(-x)}\right)-\frac{1}{2} \sqrt{3} \log\left(-\left(\frac{2 \sqrt{3}\left(\sqrt{3} e^{(2 x)}-\sqrt{3}\right)}{\sqrt{e^{(2 x)}-1}}-6 e^{x}+6 i\right) e^{(-x)}\right)+\frac{1}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-3*I*csch(x))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(3)*log((2*sqrt(3)*(sqrt(3)*e^(2*x) - sqrt(3))/sqrt(e^(2*x) - 1) +
6*e^x - 6*I)*e^(-x)) - 1/2*sqrt(3)*log(-(2*sqrt(3)*(sqrt(3)*e^(2*x) - sqrt(
3))/sqrt(e^(2*x) - 1) - 6*e^x + 6*I)*e^(-x)) + 1/2*sqrt(3)*log((6*sqrt(3)*e
^(2*x) + 6*I*sqrt(3)*e^x + sqrt(3)*(6*e^(3*x) + 12*I*e^(2*x) - 6*e^x - 12*I
)/sqrt(e^(2*x) - 1) - 12*sqrt(3))*e^(-2*x)) - 1/2*sqrt(3)*log(-(6*sqrt(3)*e
^(2*x) + 6*I*sqrt(3)*e^x - sqrt(3)*(6*e^(3*x) + 12*I*e^(2*x) - 6*e^x - 12*I
)/sqrt(e^(2*x) - 1) - 12*sqrt(3))*e^(-2*x))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{-3i\,\operatorname{csch}(x) + 3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-3*I*csch(x))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-3*I*csch(x) + 3), x)
```

maple [F] time = 1.01, size = 0, normalized size = 0.00

$$\int \sqrt{3 - 3i\operatorname{csch}(x)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3-3*I*csch(x))^(1/2),x)
```

```
[Out] int((3-3*I*csch(x))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{-3i\,\operatorname{csch}(x) + 3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-3*I*csch(x))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-3*I*csch(x) + 3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.04

$$\int \sqrt{3 - \frac{3\mathrm{i}}{\sinh(x)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3 - 3i/sinh(x))^(1/2),x)
```

```
[Out] int((3 - 3i/sinh(x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\sqrt{3} \int \sqrt{-i\operatorname{csch}{(x)} + 1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-3*I*csch(x))**(1/2),x)
```

```
[Out] sqrt(3)*Integral(sqrt(-I*csch(x) + 1), x)
```

3.60 $\int \sqrt{-3+3i\operatorname{csch}(x)}\,dx$

Optimal. Leaf size=23

$$-2\sqrt{3}\tan^{-1}\left(\frac{\coth(x)}{\sqrt{-1+i\operatorname{csch}(x)}}\right)$$

```
[Out] -2*arctan(coth(x)/(-1+I*csch(x))^(1/2))*3^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3774, 207}

$$-2\sqrt{3}\tan^{-1}\left(\frac{\coth(x)}{\sqrt{-1+i\operatorname{csch}(x)}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-3 + (3*I)*Csch[x]],x]
```

```
[Out] -2*Sqrt[3]*ArcTan[Coth[x]/Sqrt[-1 + I*Csch[x]]]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 3774

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[(-2*b)/d,
 Subst[Int[1/(a + x^2), x], x, (b*Cot[c + d*x])/Sqrt[a + b*Csc[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \sqrt{-3+3i\operatorname{csch}(x)}\,dx &= -\left(6i\,\text{Subst}\left(\int \frac{1}{-3+x^2}\,dx, x, \frac{3i\coth(x)}{\sqrt{-3+3i\operatorname{csch}(x)}}\right)\right)\\ &= -2\sqrt{3}\tan^{-1}\left(\frac{\coth(x)}{\sqrt{-1+i\operatorname{csch}(x)}}\right)\end{aligned}$$

Mathematica [A] time = 0.69, size = 46, normalized size = 2.00

$$-\frac{2\sqrt{3}\coth(x)\tanh^{-1}\left(\sqrt{1+i\operatorname{csch}(x)}\right)}{\sqrt{-1+i\operatorname{csch}(x)}\sqrt{1+i\operatorname{csch}(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-3 + (3*I)*Csch[x]],x]
```

```
[Out] (-2*Sqrt[3]*ArcTanh[Sqrt[1 + I*Csch[x]]]*Coth[x])/(Sqrt[-1 + I*Csch[x]]*Sqr
t[1 + I*Csch[x]])
```

fricas [B] time = 0.43, size = 215, normalized size = 9.35

$$\frac{1}{2}i\sqrt{3}\log\left(\left(\frac{\sqrt{3}\left(2i\sqrt{3}e^{(2x)}-2i\sqrt{3}\right)}{\sqrt{e^{(2x)}-1}}+6ie^{x}+6\right)e^{(-x)}\right)-\frac{1}{2}i\sqrt{3}\log\left(\left(\frac{\sqrt{3}\left(-2i\sqrt{3}e^{(2x)}+2i\sqrt{3}\right)}{\sqrt{e^{(2x)}-1}}+6ie^{x}+6\right)e^{(}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3+3*I*csch(x))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*I*sqrt(3)*log((sqrt(3)*(2*I*sqrt(3)*e^(2*x) - 2*I*sqrt(3))/sqrt(e^(2*x)
 - 1) + 6*I*e^x + 6)*e^(-x)) - 1/2*I*sqrt(3)*log((sqrt(3)*(-2*I*sqrt(3)*e^(
2*x) + 2*I*sqrt(3))/sqrt(e^(2*x) - 1) + 6*I*e^x + 6)*e^(-x)) + 1/2*I*sqrt(3
)*log((6*I*sqrt(3)*e^(2*x) - 6*sqrt(3)*e^x + sqrt(3)*(6*I*e^(3*x) - 12*e^(2
*x) - 6*I*e^x + 12)/sqrt(e^(2*x) - 1) - 12*I*sqrt(3))*e^(-2*x)) - 1/2*I*sqr
t(3)*log((-6*I*sqrt(3)*e^(2*x) + 6*sqrt(3)*e^x + sqrt(3)*(6*I*e^(3*x) - 12*
e^(2*x) - 6*I*e^x + 12)/sqrt(e^(2*x) - 1) + 12*I*sqrt(3))*e^(-2*x))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{3i\,\operatorname{csch}(x) - 3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3+3*I*csch(x))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(3*I*csch(x) - 3), x)
```

maple [F] time = 0.56, size = 0, normalized size = 0.00

$$\int \sqrt{-3+3i\operatorname{csch}(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-3+3*I*csch(x))^(1/2),x)
```

```
[Out] int((-3+3*I*csch(x))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{3i\,\operatorname{csch}(x) - 3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3+3*I*csch(x))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(3*I*csch(x) - 3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.04

$$\int \sqrt{-3+\frac{3\mathrm{i}}{\sinh(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3i/sinh(x) - 3)^(1/2),x)
```

```
[Out] int((3i/sinh(x) - 3)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\sqrt{3}\int \sqrt{i\operatorname{csch}(x) - 1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3+3*I*csch(x))**(1/2),x)
```

```
[Out] sqrt(3)*Integral(sqrt(I*csch(x) - 1), x)
```

3.61 $\int \sqrt{-3-3i\mathbf{csch}(x)}\,dx$

Optimal. Leaf size=23

$$-2\sqrt{3}\tan^{-1}\left(\frac{\coth(x)}{\sqrt{-1-i\text{csch}(x)}}\right)$$

```
[Out] -2*arctan(coth(x)/(-1-I*csch(x))^(1/2))*3^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3774, 207}

$$-2\sqrt{3}\tan^{-1}\left(\frac{\coth(x)}{\sqrt{-1-i\text{csch}(x)}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-3 - (3*I)*Csch[x]],x]
```

```
[Out] -2*Sqrt[3]*ArcTan[Coth[x]/Sqrt[-1 - I*Csch[x]]]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 3774

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[(-2*b)/d,
 Subst[Int[1/(a + x^2), x], x, (b*Cot[c + d*x])/Sqrt[a + b*Csc[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \sqrt{-3-3i\text{csch}(x)}\,dx &= 6i\,\text{Subst}\left(\int \frac{1}{-3+x^2}\,dx, x, -\frac{3i\coth(x)}{\sqrt{-3-3i\text{csch}(x)}}\right)\\ &= -2\sqrt{3}\tan^{-1}\left(\frac{\coth(x)}{\sqrt{-1-i\text{csch}(x)}}\right)\end{aligned}$$

Mathematica [A] time = 0.69, size = 46, normalized size = 2.00

$$-\frac{2\sqrt{3}\coth(x)\tanh^{-1}\left(\sqrt{1-i\text{csch}(x)}\right)}{\sqrt{-1-i\text{csch}(x)}\sqrt{1-i\text{csch}(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-3 - (3*I)*Csch[x]],x]
```

```
[Out] (-2*Sqrt[3]*ArcTanh[Sqrt[1 - I*Csch[x]]]*Coth[x])/(Sqrt[-1 - I*Csch[x]]*Sqr
t[1 - I*Csch[x]])
```

fricas [B] time = 0.44, size = 215, normalized size = 9.35

$$\frac{1}{2}i\sqrt{3}\log\left(\left(\frac{\sqrt{3}\left(2i\sqrt{3}e^{(2x)}-2i\sqrt{3}\right)}{\sqrt{e^{(2x)}-1}}+6ie^{x}-6\right)e^{(-x)}\right)-\frac{1}{2}i\sqrt{3}\log\left(\left(\frac{\sqrt{3}\left(-2i\sqrt{3}e^{(2x)}+2i\sqrt{3}\right)}{\sqrt{e^{(2x)}-1}}+6ie^{x}-6\right)e^{(\cdot}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3-3*I*csch(x))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*I*sqrt(3)*log((sqrt(3)*(2*I*sqrt(3)*e^(2*x) - 2*I*sqrt(3))/sqrt(e^(2*x)
 - 1) + 6*I*e^x - 6)*e^(-x)) - 1/2*I*sqrt(3)*log((sqrt(3)*(-2*I*sqrt(3)*e^(
2*x) + 2*I*sqrt(3))/sqrt(e^(2*x) - 1) + 6*I*e^x - 6)*e^(-x)) + 1/2*I*sqrt(3
)*log((6*I*sqrt(3)*e^(2*x) + 6*sqrt(3)*e^x + sqrt(3)*(6*I*e^(3*x) + 12*e^(2
*x) - 6*I*e^x - 12)/sqrt(e^(2*x) - 1) - 12*I*sqrt(3))*e^(-2*x)) - 1/2*I*sqr
t(3)*log((-6*I*sqrt(3)*e^(2*x) - 6*sqrt(3)*e^x + sqrt(3)*(6*I*e^(3*x) + 12*
e^(2*x) - 6*I*e^x - 12)/sqrt(e^(2*x) - 1) + 12*I*sqrt(3))*e^(-2*x))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{-3i\,\mathrm{csch}(x) - 3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3-3*I*csch(x))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-3*I*csch(x) - 3), x)
```

maple [F] time = 0.48, size = 0, normalized size = 0.00

$$\int \sqrt{-3 - 3i\mathrm{csch}(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-3-3*I*csch(x))^(1/2),x)
```

```
[Out] int((-3-3*I*csch(x))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{-3i\,\mathrm{csch}(x) - 3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3-3*I*csch(x))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-3*I*csch(x) - 3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.04

$$\int \sqrt{-3 - \frac{3\mathrm{i}}{\sinh(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((- 3i/sinh(x) - 3)^(1/2),x)
```

```
[Out] int((- 3i/sinh(x) - 3)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\sqrt{3}\int \sqrt{-i\,\mathrm{csch}\,(x) - 1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3-3*I*csch(x))**(1/2),x)
```

```
[Out] sqrt(3)*Integral(sqrt(-I*csch(x) - 1), x)
```

3.62 $\int \frac{\sinh^4(x)}{i+\mathbf{csch}(x)} dx$

Optimal. Leaf size=58

$$-\frac{15ix}{8}+\frac{4\cosh^3(x)}{3}-4\cosh(x)-\frac{5}{4}i\sinh^3(x)\cosh(x)+\frac{15}{8}i\sinh(x)\cosh(x)-\frac{\sinh^3(x)\cosh(x)}{\operatorname{csch}(x)+i}$$

```
[Out] -15/8*I*x-4*cosh(x)+4/3*cosh(x)^3+15/8*I*cosh(x)*sinh(x)-5/4*I*cosh(x)*sinh
(x)^3-cosh(x)*sinh(x)^3/(I+csch(x))
```

Rubi [A] time = 0.07, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {3819, 3787, 2635, 8, 2633}

$$-\frac{15ix}{8}+\frac{4\cosh^3(x)}{3}-4\cosh(x)-\frac{5}{4}i\sinh^3(x)\cosh(x)+\frac{15}{8}i\sinh(x)\cosh(x)-\frac{\sinh^3(x)\cosh(x)}{\operatorname{csch}(x)+i}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[x]^4/(I + Csch[x]),x]
```

```
[Out] ((-15*I)/8)*x - 4*Cosh[x] + (4*Cosh[x]^3)/3 + ((15*I)/8)*Cosh[x]*Sinh[x] -
((5*I)/4)*Cosh[x]*Sinh[x]^3 - (Cosh[x]*Sinh[x]^3)/(I + Csch[x])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3819

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*(a + b*Csc[e +
 f*x])), x] - Dist[1/a^2, Int[(d*Csc[e + f*x])^n*(a*(n - 1) - b*n*Csc[e + f
*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[n, 0]
```

Rubi steps

$$\int \frac{\sinh^4(x)}{i+\operatorname{csch}(x)}\,dx = -\frac{\cosh(x)\sinh^3(x)}{i+\operatorname{csch}(x)} + \int (-5i+4\operatorname{csch}(x))\sinh^4(x)\,dx$$
$$= -\frac{\cosh(x)\sinh^3(x)}{i+\operatorname{csch}(x)} - 5i\int \sinh^4(x)\,dx + 4\int \sinh^3(x)\,dx$$
$$= -\frac{5}{4}i\cosh(x)\sinh^3(x) - \frac{\cosh(x)\sinh^3(x)}{i+\operatorname{csch}(x)} + \frac{15}{4}i\int \sinh^2(x)\,dx - 4\,\text{Subst}\left(\int \left(1-x^2\right)\,dx, x, c$$
$$= -4\cosh(x) + \frac{4\cosh^3(x)}{3} + \frac{15}{8}i\cosh(x)\sinh(x) - \frac{5}{4}i\cosh(x)\sinh^3(x) - \frac{\cosh(x)\sinh^3(x)}{i+\operatorname{csch}(x)} -$$
$$= -\frac{15ix}{8} - 4\cosh(x) + \frac{4\cosh^3(x)}{3} + \frac{15}{8}i\cosh(x)\sinh(x) - \frac{5}{4}i\cosh(x)\sinh^3(x) - \frac{\cosh(x)\sinh}{i+\operatorname{csch}(}$$

Mathematica [A] time = 0.15, size = 63, normalized size = 1.09

$$\frac{1}{96}\left(-180ix + 48i\sinh(2x) - 3i\sinh(4x) - 168\cosh(x) + 8\cosh(3x) + \frac{192\sinh\left(\frac{x}{2}\right)}{\sinh\left(\frac{x}{2}\right) - i\cosh\left(\frac{x}{2}\right)}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[x]^4/(I + Csch[x]),x]
```

```
[Out] ((-180*I)*x - 168*Cosh[x] + 8*Cosh[3*x] + (192*Sinh[x/2])/((-I)*Cosh[x/2] +
 Sinh[x/2]) + (48*I)*Sinh[2*x] - (3*I)*Sinh[4*x])/96
```

fricas [A] time = 0.47, size = 79, normalized size = 1.36

$$\frac{(-360i\,x + 168i)e^{(5\,x)} - 24\,(15\,x + 23)e^{(4\,x)} - 3i\,e^{(9\,x)} + 5\,e^{(8\,x)} + 40i\,e^{(7\,x)} - 120\,e^{(6\,x)} + 120i\,e^{(3\,x)} - 40\,e^{(2\,x)} - 5i\,e^{x} +}{192\,e^{(5\,x)} - 192i\,e^{(4\,x)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^4/(I+csch(x)),x, algorithm="fricas")
```

```
[Out] ((-360*I*x + 168*I)*e^(5*x) - 24*(15*x + 23)*e^(4*x) - 3*I*e^(9*x) + 5*e^(8
*x) + 40*I*e^(7*x) - 120*e^(6*x) + 120*I*e^(3*x) - 40*e^(2*x) - 5*I*e^x + 3
)/(192*e^(5*x) - 192*I*e^(4*x))
```

giac [A] time = 0.14, size = 66, normalized size = 1.14

$$-\frac{\left(552\,e^{(4\,x)} - 120i\,e^{(3\,x)} + 40\,e^{(2\,x)} + 5i\,e^{x} - 3\right)e^{(-4\,x)}}{192\,(e^{x} - i)} - \frac{1}{64}i\,e^{(4\,x)} + \frac{1}{24}\,e^{(3\,x)} + \frac{1}{4}i\,e^{(2\,x)} - \frac{7}{8}\,e^{x} - \frac{15}{8}i\,\log\left(i\,e^{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^4/(I+csch(x)),x, algorithm="giac")
```

```
[Out] -1/192*(552*e^(4*x) - 120*I*e^(3*x) + 40*e^(2*x) + 5*I*e^x - 3)*e^(-4*x)/(e
^x - I) - 1/64*I*e^(4*x) + 1/24*e^(3*x) + 1/4*I*e^(2*x) - 7/8*e^x - 15/8*I*
log(I*e^x)
```

maple [B] time = 0.21, size = 182, normalized size = 3.14

$$\frac{2i}{\tanh\left(\frac{x}{2}\right) - i} - \frac{i}{4\left(\tanh\left(\frac{x}{2}\right) - 1\right)^4} + \frac{3}{2\left(\tanh\left(\frac{x}{2}\right) - 1\right)} + \frac{7i}{8\left(\tanh\left(\frac{x}{2}\right) - 1\right)} - \frac{1}{2\left(\tanh\left(\frac{x}{2}\right) - 1\right)^2} + \frac{5i}{8\left(\tanh\left(\frac{x}{2}\right) - 1\right)^2} - \frac{}{3\,(\text{ta}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)^4/(I+csch(x)),x)
```

```
[Out] 2*I/(tanh(1/2*x)-I)-1/4*I/(tanh(1/2*x)-1)^4+3/2/(tanh(1/2*x)-1)+7/8*I/(tanh
(1/2*x)-1)-1/2/(tanh(1/2*x)-1)^2+5/8*I/(tanh(1/2*x)-1)^2-1/3/(tanh(1/2*x)-1
)^3-15/8*I*ln(tanh(1/2*x)+1)+15/8*I*ln(tanh(1/2*x)-1)+7/8*I/(tanh(1/2*x)+1)
+1/3/(tanh(1/2*x)+1)^3+1/4*I/(tanh(1/2*x)+1)^4-1/2/(tanh(1/2*x)+1)^2-5/8*I/
(tanh(1/2*x)+1)^2-3/2/(tanh(1/2*x)+1)-1/2*I/(tanh(1/2*x)+1)^3-1/2*I/(tanh(1
/2*x)-1)^3
```

maxima [A] time = 0.31, size = 71, normalized size = 1.22

$$-\frac{15}{8}ix-\frac{-5ie^{(-x)}+40e^{(-2x)}+120ie^{(-3x)}+552e^{(-4x)}-3}{16\left(12ie^{(-4x)}+12e^{(-5x)}\right)}-\frac{7}{8}e^{(-x)}-\frac{1}{4}ie^{(-2x)}+\frac{1}{24}e^{(-3x)}+\frac{1}{64}ie^{(-4x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^4/(I+csch(x)),x, algorithm="maxima")
```

```
[Out] -15/8*I*x - 1/16*(-5*I*e^(-x) + 40*e^(-2*x) + 120*I*e^(-3*x) + 552*e^(-4*x)
 - 3)/(12*I*e^(-4*x) + 12*e^(-5*x)) - 7/8*e^(-x) - 1/4*I*e^(-2*x) + 1/24*e^
(-3*x) + 1/64*I*e^(-4*x)
```

mupad [B] time = 1.59, size = 64, normalized size = 1.10

$$\frac{e^{-3x}}{24}-\frac{7e^{-x}}{8}-\frac{e^{-2x}1i}{4}+\frac{e^{2x}1i}{4}-\frac{x15i}{8}+\frac{e^{3x}}{24}+\frac{e^{-4x}1i}{64}-\frac{e^{4x}1i}{64}-\frac{7e^{x}}{8}-\frac{2}{e^{x}-i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)^4/(1/sinh(x) + 1i),x)
```

```
[Out] (exp(2*x)*1i)/4 - (7*exp(-x))/8 - (exp(-2*x)*1i)/4 - (x*15i)/8 + exp(-3*x)/
24 + exp(3*x)/24 + (exp(-4*x)*1i)/64 - (exp(4*x)*1i)/64 - (7*exp(x))/8 - 2/
(exp(x) - 1i)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sinh^{4}(x)}{\operatorname{csch}(x)+i}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)**4/(I+csch(x)),x)
```

```
[Out] Integral(sinh(x)**4/(csch(x) + I), x)
```

3.63 $\int \frac{\sinh^3(x)}{i+\text{csch}(x)} dx$

Optimal. Leaf size=46

$$-\frac{3x}{2}-\frac{4}{3}i\cosh^3(x)+4i\cosh(x)+\frac{3}{2}\sinh(x)\cosh(x)-\frac{\sinh^2(x)\cosh(x)}{\text{csch}(x)+i}$$

```
[Out] -3/2*x+4*I*cosh(x)-4/3*I*cosh(x)^3+3/2*cosh(x)*sinh(x)-cosh(x)*sinh(x)^2/(I
+csch(x))
```

Rubi [A] time = 0.07, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {3819, 3787, 2633, 2635, 8}

$$-\frac{3x}{2}-\frac{4}{3}i\cosh^3(x)+4i\cosh(x)+\frac{3}{2}\sinh(x)\cosh(x)-\frac{\sinh^2(x)\cosh(x)}{\text{csch}(x)+i}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[x]^3/(I + Csch[x]),x]
```

```
[Out] (-3*x)/2 + (4*I)*Cosh[x] - ((4*I)/3)*Cosh[x]^3 + (3*Cosh[x]*Sinh[x])/2 - (C
osh[x]*Sinh[x]^2)/(I + Csch[x])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3819

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*(a + b*Csc[e +
 f*x])), x] - Dist[1/a^2, Int[(d*Csc[e + f*x])^n*(a*(n - 1) - b*n*Csc[e + f
*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[n, 0]
```

Rubi steps

$$\int \frac{\sinh^3(x)}{i+\operatorname{csch}(x)}\,dx = -\frac{\cosh(x)\sinh^2(x)}{i+\operatorname{csch}(x)} + \int (-4i+3\operatorname{csch}(x))\sinh^3(x)\,dx$$
$$= -\frac{\cosh(x)\sinh^2(x)}{i+\operatorname{csch}(x)} - 4i\int \sinh^3(x)\,dx + 3\int \sinh^2(x)\,dx$$
$$= \frac{3}{2}\cosh(x)\sinh(x) - \frac{\cosh(x)\sinh^2(x)}{i+\operatorname{csch}(x)} + 4i\,\text{Subst}\left(\int \left(1-x^2\right)\,dx, x, \cosh(x)\right) - \frac{3\int 1\,dx}{2}$$
$$= -\frac{3x}{2} + 4i\cosh(x) - \frac{4}{3}i\cosh^3(x) + \frac{3}{2}\cosh(x)\sinh(x) - \frac{\cosh(x)\sinh^2(x)}{i+\operatorname{csch}(x)}$$

Mathematica [A] time = 0.13, size = 56, normalized size = 1.22

$$\frac{1}{12}\left(21i\cosh(x) - i\cosh(3x) + 3\left(-6x + \sinh(2x) + \frac{8\sinh\left(\frac{x}{2}\right)}{\cosh\left(\frac{x}{2}\right) + i\sinh\left(\frac{x}{2}\right)}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[x]^3/(I + Csch[x]),x]
```

```
[Out] ((21*I)*Cosh[x] - I*Cosh[3*x] + 3*(-6*x + (8*Sinh[x/2])/(Cosh[x/2] + I*Sinh
[x/2]) + Sinh[2*x]))/12
```

fricas [A] time = 0.44, size = 67, normalized size = 1.46

$$-\frac{3\,(12\,x-7)e^{(4\,x)} - (36i\,x+69i)e^{(3\,x)} + i\,e^{(7\,x)} - 2\,e^{(6\,x)} - 18i\,e^{(5\,x)} - 18\,e^{(2\,x)} - 2i\,e^{x} + 1}{24\left(e^{(4\,x)} - i\,e^{(3\,x)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^3/(I+csch(x)),x, algorithm="fricas")
```

```
[Out] -1/24*(3*(12*x - 7)*e^(4*x) - (36*I*x + 69*I)*e^(3*x) + I*e^(7*x) - 2*e^(6*
x) - 18*I*e^(5*x) - 18*e^(2*x) - 2*I*e^x + 1)/(e^(4*x) - I*e^(3*x))
```

giac [A] time = 0.14, size = 50, normalized size = 1.09

$$-\frac{3}{2}\,x + \frac{i\left(69\,e^{(3\,x)} - 18i\,e^{(2\,x)} + 2\,e^{x} + i\right)e^{(-3\,x)}}{24\,(e^{x} - i)} - \frac{1}{24}i\,e^{(3\,x)} + \frac{1}{8}\,e^{(2\,x)} + \frac{7}{8}i\,e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^3/(I+csch(x)),x, algorithm="giac")
```

```
[Out] -3/2*x + 1/24*I*(69*e^(3*x) - 18*I*e^(2*x) + 2*e^x + I)*e^(-3*x)/(e^x - I)
- 1/24*I*e^(3*x) + 1/8*e^(2*x) + 7/8*I*e^x
```

maple [B] time = 0.21, size = 137, normalized size = 2.98

$$\frac{i}{3\left(\tanh\left(\frac{x}{2}\right)-1\right)^3} + \frac{1}{2\left(\tanh\left(\frac{x}{2}\right)-1\right)^2} + \frac{i}{2\left(\tanh\left(\frac{x}{2}\right)-1\right)^2} + \frac{1}{2\tanh\left(\frac{x}{2}\right)-2} - \frac{3i}{2\left(\tanh\left(\frac{x}{2}\right)-1\right)} + \frac{3\ln\left(\tanh\left(\frac{x}{2}\right)-\right.}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)^3/(I+csch(x)),x)
```

```
[Out] 1/3*I/(tanh(1/2*x)-1)^3+1/2/(tanh(1/2*x)-1)^2+1/2*I/(tanh(1/2*x)-1)^2+1/2/(
tanh(1/2*x)-1)-3/2*I/(tanh(1/2*x)-1)+3/2*ln(tanh(1/2*x)-1)-1/3*I/(tanh(1/2*
```

```
x)+1)^3+1/2/(tanh(1/2*x)+1)+3/2*I/(tanh(1/2*x)+1)-1/2/(tanh(1/2*x)+1)^2+1/2
*I/(tanh(1/2*x)+1)^2-3/2*ln(tanh(1/2*x)+1)+2/(tanh(1/2*x)-I)
```

maxima [A] time = 0.32, size = 59, normalized size = 1.28

$$-\frac{3}{2}x + \frac{2i\,e^{(-x)} - 18\,e^{(-2\,x)} + 69i\,e^{(-3\,x)} + 1}{8\left(3i\,e^{(-3\,x)} + 3\,e^{(-4\,x)}\right)} + \frac{7}{8}i\,e^{(-x)} - \frac{1}{8}\,e^{(-2\,x)} - \frac{1}{24}i\,e^{(-3\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^3/(I+csch(x)),x, algorithm="maxima")
[Out] -3/2*x + 1/8*(2*I*e^(-x) - 18*e^(-2*x) + 69*I*e^(-3*x) + 1)/(3*I*e^(-3*x) +
 3*e^(-4*x)) + 7/8*I*e^(-x) - 1/8*e^(-2*x) - 1/24*I*e^(-3*x)
```

mupad [B] time = 0.18, size = 52, normalized size = 1.13

$$\frac{e^{2x}}{8} + \frac{e^{-x}\,7i}{8} - \frac{e^{-2x}}{8} - \frac{3x}{2} - \frac{e^{-3x}\,1i}{24} - \frac{e^{3x}\,1i}{24} + \frac{e^{x}\,7i}{8} + \frac{2i}{e^{x} - i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)^3/(1/sinh(x) + 1i),x)
[Out] (exp(-x)*7i)/8 - (3*x)/2 - exp(-2*x)/8 + exp(2*x)/8 - (exp(-3*x)*1i)/24 - (
exp(3*x)*1i)/24 + (exp(x)*7i)/8 + 2i/(exp(x) - 1i)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh^{3}(x)}{\operatorname{csch}(x) + i}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)**3/(I+csch(x)),x)
[Out] Integral(sinh(x)**3/(csch(x) + I), x)
```

3.64 $\int \frac{\sinh^2(x)}{i+\mathbf{csch}(x)} \, dx$

Optimal. Leaf size=36

$$\frac{3ix}{2} + 2\cosh(x) - \frac{3}{2}i\sinh(x)\cosh(x) - \frac{\sinh(x)\cosh(x)}{\operatorname{csch}(x) + i}$$

```
[Out] 3/2*I*x+2*cosh(x)-3/2*I*cosh(x)*sinh(x)-cosh(x)*sinh(x)/(I+csch(x))
```

Rubi [A] time = 0.06, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {3819, 3787, 2635, 8, 2638}

$$\frac{3ix}{2} + 2\cosh(x) - \frac{3}{2}i\sinh(x)\cosh(x) - \frac{\sinh(x)\cosh(x)}{\operatorname{csch}(x) + i}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[x]^2/(I + Csch[x]),x]
```

```
[Out] ((3*I)/2)*x + 2*Cosh[x] - ((3*I)/2)*Cosh[x]*Sinh[x] - (Cosh[x]*Sinh[x])/(I
+ Csch[x])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3819

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*(a + b*Csc[e +
 f*x])), x] - Dist[1/a^2, Int[(d*Csc[e + f*x])^n*(a*(n - 1) - b*n*Csc[e + f
*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sinh^2(x)}{i+\operatorname{csch}(x)}\,dx &= -\frac{\cosh(x)\sinh(x)}{i+\operatorname{csch}(x)} + \int (-3i+2\operatorname{csch}(x))\sinh^2(x)\,dx\\ &= -\frac{\cosh(x)\sinh(x)}{i+\operatorname{csch}(x)} - 3i\int \sinh^2(x)\,dx + 2\int \sinh(x)\,dx\\ &= 2\cosh(x) - \frac{3}{2}i\cosh(x)\sinh(x) - \frac{\cosh(x)\sinh(x)}{i+\operatorname{csch}(x)} + \frac{3}{2}i\int 1\,dx\\ &= \frac{3ix}{2} + 2\cosh(x) - \frac{3}{2}i\cosh(x)\sinh(x) - \frac{\cosh(x)\sinh(x)}{i+\operatorname{csch}(x)}\end{aligned}$$

Mathematica [A] time = 0.13, size = 46, normalized size = 1.28

$$\cosh(x) + \frac{1}{4}i\left(6x - \sinh(2x) - \frac{8\sinh\left(\frac{x}{2}\right)}{\cosh\left(\frac{x}{2}\right) + i\sinh\left(\frac{x}{2}\right)}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[x]^2/(I + Csch[x]),x]
```

```
[Out] Cosh[x] + (I/4)*(6*x - (8*Sinh[x/2])/(Cosh[x/2] + I*Sinh[x/2]) - Sinh[2*x])
```

fricas [B] time = 0.41, size = 55, normalized size = 1.53

$$\frac{(12i\,x-4i)e^{(3\,x)} + 4\,(3\,x+5)e^{(2\,x)} - i\,e^{(5\,x)} + 3\,e^{(4\,x)} - 3i\,e^{x} + 1}{8\,e^{(3\,x)} - 8i\,e^{(2\,x)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^2/(I+csch(x)),x, algorithm="fricas")
```

```
[Out] ((12*I*x - 4*I)*e^(3*x) + 4*(3*x + 5)*e^(2*x) - I*e^(5*x) + 3*e^(4*x) - 3*I
*e^x + 1)/(8*e^(3*x) - 8*I*e^(2*x))
```

giac [A] time = 0.13, size = 40, normalized size = 1.11

$$\frac{3}{2}i\,x + \frac{\left(-20i\,e^{(2\,x)} - 3\,e^{x} - i\right)e^{(-2\,x)}}{8\,(-i\,e^{x} - 1)} - \frac{1}{8}i\,e^{(2\,x)} + \frac{1}{2}\,e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^2/(I+csch(x)),x, algorithm="giac")
```

```
[Out] 3/2*I*x + 1/8*(-20*I*e^(2*x) - 3*e^x - I)*e^(-2*x)/(-I*e^x - 1) - 1/8*I*e^(
2*x) + 1/2*e^x
```

maple [B] time = 0.21, size = 96, normalized size = 2.67

$$-\frac{3i\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{2} - \frac{i}{2\left(\tanh\left(\frac{x}{2}\right)-1\right)^2} - \frac{1}{\tanh\left(\frac{x}{2}\right)-1} - \frac{i}{2\left(\tanh\left(\frac{x}{2}\right)-1\right)} + \frac{i}{2\left(\tanh\left(\frac{x}{2}\right)+1\right)^2} + \frac{3i\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)^2/(I+csch(x)),x)
```

```
[Out] -3/2*I*ln(tanh(1/2*x)-1)-1/2*I/(tanh(1/2*x)-1)^2-1/(tanh(1/2*x)-1)-1/2*I/(t
anh(1/2*x)-1)+1/2*I/(tanh(1/2*x)+1)^2+3/2*I*ln(tanh(1/2*x)+1)+1/(tanh(1/2*x
)+1)-1/2*I/(tanh(1/2*x)+1)-2*I/(tanh(1/2*x)-I)
```

maxima [A] time = 0.31, size = 47, normalized size = 1.31

$$\frac{3}{2} i x + \frac{3 i e^{(-x)} + 20 e^{(-2 x)} + 1}{4 \left(2 i e^{(-2 x)} + 2 e^{(-3 x)}\right)} + \frac{1}{2} e^{(-x)} + \frac{1}{8} i e^{(-2 x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^2/(I+csch(x)),x, algorithm="maxima")
```

```
[Out] 3/2*I*x + 1/4*(3*I*e^(-x) + 20*e^(-2*x) + 1)/(2*I*e^(-2*x) + 2*e^(-3*x)) +
1/2*e^(-x) + 1/8*I*e^(-2*x)
```

mupad [B] time = 1.47, size = 38, normalized size = 1.06

$$\frac{x\,3\mathrm{i}}{2} + \frac{\mathrm{e}^{-x}}{2} + \frac{\mathrm{e}^{-2\,x}\,1\mathrm{i}}{8} - \frac{\mathrm{e}^{2\,x}\,1\mathrm{i}}{8} + \frac{\mathrm{e}^{x}}{2} + \frac{2}{\mathrm{e}^{x} - \mathrm{i}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)^2/(1/sinh(x) + 1i),x)
```

```
[Out] (x*3i)/2 + exp(-x)/2 + (exp(-2*x)*1i)/8 - (exp(2*x)*1i)/8 + exp(x)/2 + 2/(e
xp(x) - 1i)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh^{2}(x)}{\operatorname{csch}(x) + i}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)**2/(I+csch(x)),x)
```

```
[Out] Integral(sinh(x)**2/(csch(x) + I), x)
```

3.65 $\int \frac{\sinh(x)}{i+\text{csch}(x)}\,dx$

Optimal. Leaf size=20

$$x-2i\cosh(x)-\frac{\cosh(x)}{\text{csch}(x)+i}$$

[Out] x-2*I*cosh(x)-cosh(x)/(I+csch(x))

Rubi [A] time = 0.05, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {3819, 3787, 2638, 8}

$$x-2i\cosh(x)-\frac{\cosh(x)}{\text{csch}(x)+i}$$

Antiderivative was successfully verified.

[In] Int[Sinh[x]/(I + Csch[x]),x]

[Out] x - (2*I)*Cosh[x] - Cosh[x]/(I + Csch[x])

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3819

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*(a + b*Csc[e +
 f*x])), x] - Dist[1/a^2, Int[(d*Csc[e + f*x])^n*(a*(n - 1) - b*n*Csc[e + f
*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sinh(x)}{i+\text{csch}(x)}\,dx &= -\frac{\cosh(x)}{i+\text{csch}(x)}+\int (-2i+\text{csch}(x))\sinh(x)\,dx\\ &= -\frac{\cosh(x)}{i+\text{csch}(x)}-2i\int \sinh(x)\,dx+\int 1\,dx\\ &= x-2i\cosh(x)-\frac{\cosh(x)}{i+\text{csch}(x)}\end{aligned}$$

Mathematica [A] time = 0.05, size = 35, normalized size = 1.75

$$x-i\cosh(x)-\frac{2\sinh\left(\frac{x}{2}\right)}{\cosh\left(\frac{x}{2}\right)+i\sinh\left(\frac{x}{2}\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[x]/(I + Csch[x]),x]
[Out] x - I*Cosh[x] - (2*Sinh[x/2])/(Cosh[x/2] + I*Sinh[x/2])
```

fricas [B] time = 0.41, size = 40, normalized size = 2.00

$$\frac{(2x-1)e^{(2x)} + (-2ix-5i)e^{x} - ie^{(3x)} - 1}{2e^{(2x)} - 2ie^{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)/(I+csch(x)),x, algorithm="fricas")
[Out] ((2*x - 1)*e^(2*x) + (-2*I*x - 5*I)*e^x - I*e^(3*x) - 1)/(2*e^(2*x) - 2*I*e
^x)
```

giac [A] time = 0.13, size = 26, normalized size = 1.30

$$x + \frac{(5e^{x} - i)e^{(-x)}}{2(ie^{x} + 1)} - \frac{1}{2}ie^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)/(I+csch(x)),x, algorithm="giac")
[Out] x + 1/2*(5*e^x - I)*e^(-x)/(I*e^x + 1) - 1/2*I*e^x
```

maple [B] time = 0.20, size = 51, normalized size = 2.55

$$\frac{i}{\tanh\left(\frac{x}{2}\right) - 1} - \ln\left(\tanh\left(\frac{x}{2}\right) - 1\right) - \frac{i}{\tanh\left(\frac{x}{2}\right) + 1} + \ln\left(\tanh\left(\frac{x}{2}\right) + 1\right) - \frac{2}{\tanh\left(\frac{x}{2}\right) - i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)/(I+csch(x)),x)
[Out] I/(tanh(1/2*x)-1)-ln(tanh(1/2*x)-1)-I/(tanh(1/2*x)+1)+ln(tanh(1/2*x)+1)-2/(
tanh(1/2*x)-I)
```

maxima [A] time = 0.32, size = 31, normalized size = 1.55

$$x - \frac{5ie^{(-x)} - 1}{2\left(ie^{(-x)} + e^{(-2x)}\right)} - \frac{1}{2}ie^{(-x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)/(I+csch(x)),x, algorithm="maxima")
[Out] x - 1/2*(5*I*e^(-x) - 1)/(I*e^(-x) + e^(-2*x)) - 1/2*I*e^(-x)
```

mupad [B] time = 1.46, size = 24, normalized size = 1.20

$$x - \frac{e^{-x}\,1i}{2} - \frac{e^{x}\,1i}{2} - \frac{2i}{e^{x} - i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)/(1/sinh(x) + 1i),x)
[Out] x - (exp(-x)*1i)/2 - (exp(x)*1i)/2 - 2i/(exp(x) - 1i)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh(x)}{\operatorname{csch}(x) + i}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)/(I+csch(x)),x)
```

```
[Out] Integral(sinh(x)/(csch(x) + I), x)
```

3.66 $\int \frac{\operatorname{csch}(x)}{i+\operatorname{csch}(x)}\,dx$

Optimal. Leaf size=14

$$\frac{i\coth(x)}{\operatorname{csch}(x)+i}$$

[Out] I*coth(x)/(I+csch(x))

Rubi [A] time = 0.02, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {3794}

$$\frac{i\coth(x)}{\operatorname{csch}(x)+i}$$

Antiderivative was successfully verified.

[In] Int[Csch[x]/(I + Csch[x]),x]

[Out] (I*Coth[x])/(I + Csch[x])

Rule 3794

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> -Simp[Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}
, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\operatorname{csch}(x)}{i+\operatorname{csch}(x)}\,dx = \frac{i\coth(x)}{i+\operatorname{csch}(x)}$$

Mathematica [A] time = 0.02, size = 27, normalized size = 1.93

$$\frac{2\sinh\left(\frac{x}{2}\right)}{\cosh\left(\frac{x}{2}\right)+i\sinh\left(\frac{x}{2}\right)}$$

Antiderivative was successfully verified.

[In] Integrate[Csch[x]/(I + Csch[x]),x]

[Out] (2*Sinh[x/2])/(Cosh[x/2] + I*Sinh[x/2])

fricas [A] time = 0.39, size = 8, normalized size = 0.57

$$\frac{2i}{e^x-i}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(x)/(I+csch(x)),x, algorithm="fricas")

[Out] 2*I/(e^x - I)

giac [A] time = 0.13, size = 8, normalized size = 0.57

$$\frac{2i}{e^x-i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)/(I+csch(x)),x, algorithm="giac")
```

```
[Out] 2*I/(e^x - I)
```

maple [A] time = 0.12, size = 12, normalized size = 0.86

$$\frac{2}{\tanh\left(\frac{x}{2}\right) - i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(x)/(I+csch(x)),x)
```

```
[Out] 2/(tanh(1/2*x)-I)
```

maxima [A] time = 0.31, size = 12, normalized size = 0.86

$$-\frac{2}{i\, e^{(-x)} - 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)/(I+csch(x)),x, algorithm="maxima")
```

```
[Out] -2/(I*e^(-x) - 1)
```

mupad [B] time = 0.07, size = 10, normalized size = 0.71

$$\frac{2\mathrm{i}}{\mathrm{e}^{x} - \mathrm{i}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sinh(x)*(1/sinh(x) + 1i)),x)
```

```
[Out] 2i/(exp(x) - 1i)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}(x)}{\operatorname{csch}(x) + i}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)/(I+csch(x)),x)
```

```
[Out] Integral(csch(x)/(csch(x) + I), x)
```

3.67 $\int \frac{\operatorname{csch}^2(x)}{i+\operatorname{csch}(x)} \, dx$

Optimal. Leaf size=17

$$-\tanh^{-1}(\cosh(x))+\frac{\coth(x)}{\operatorname{csch}(x)+i}$$

```
[Out] -arctanh(cosh(x))+coth(x)/(I+csch(x))
```

Rubi [A] time = 0.05, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {3789, 3770, 3794}

$$-\tanh^{-1}(\cosh(x))+\frac{\coth(x)}{\operatorname{csch}(x)+i}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[x]^2/(I + Csch[x]),x]
```

```
[Out] -ArcTanh[Cosh[x]] + Coth[x]/(I + Csch[x])
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3789

```
Int[csc[(e_.) + (f_.)*(x_)]^2/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Sym
bol] :> Dist[1/b, Int[Csc[e + f*x], x], x] - Dist[a/b, Int[Csc[e + f*x]/(a
+ b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 3794

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> -Simp[Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}
, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\operatorname{csch}^2(x)}{i+\operatorname{csch}(x)} \, dx &= -\left(i \int \frac{\operatorname{csch}(x)}{i+\operatorname{csch}(x)} \, dx\right)+\int \operatorname{csch}(x) \, dx\\ &= -\tanh^{-1}(\cosh(x))+\frac{\coth(x)}{i+\operatorname{csch}(x)}\end{aligned}$$

Mathematica [B] time = 0.03, size = 37, normalized size = 2.18

$$\log\left(\tanh\left(\frac{x}{2}\right)\right)-\frac{2 i \sinh\left(\frac{x}{2}\right)}{\cosh\left(\frac{x}{2}\right)+i \sinh\left(\frac{x}{2}\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[x]^2/(I + Csch[x]),x]
```

```
[Out] Log[Tanh[x/2]] - ((2*I)*Sinh[x/2])/(Cosh[x/2] + I*Sinh[x/2])
```

fricas [B] time = 0.47, size = 31, normalized size = 1.82

$$-\frac{(e^{x}-i)\log(e^{x}+1)-(e^{x}-i)\log(e^{x}-1)-2}{e^{x}-i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^2/(I+csch(x)),x, algorithm="fricas")
[Out] -((e^x - I)*log(e^x + 1) - (e^x - I)*log(e^x - 1) - 2)/(e^x - I)
```

giac [A] time = 0.12, size = 22, normalized size = 1.29

$$\frac{2}{e^{x}-i}-\log(e^{x}+1)+\log(|e^{x}-1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^2/(I+csch(x)),x, algorithm="giac")
[Out] 2/(e^x - I) - log(e^x + 1) + log(abs(e^x - 1))
```

maple [A] time = 0.10, size = 19, normalized size = 1.12

$$-\frac{2i}{\tanh\left(\frac{x}{2}\right)-i}+\ln\left(\tanh\left(\frac{x}{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(x)^2/(I+csch(x)),x)
[Out] -2*I/(tanh(1/2*x)-I)+ln(tanh(1/2*x))
```

maxima [A] time = 0.31, size = 29, normalized size = 1.71

$$\frac{4}{2e^{(-x)}+2i}-\log\left(e^{(-x)}+1\right)+\log\left(e^{(-x)}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^2/(I+csch(x)),x, algorithm="maxima")
[Out] 4/(2*e^(-x) + 2*I) - log(e^(-x) + 1) + log(e^(-x) - 1)
```

mupad [B] time = 0.22, size = 26, normalized size = 1.53

$$\ln(2-2e^{x})-\ln(-2e^{x}-2)+\frac{2}{e^{x}-i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sinh(x)^2*(1/sinh(x) + 1i)),x)
[Out] log(2 - 2*exp(x)) - log(- 2*exp(x) - 2) + 2/(exp(x) - 1i)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\operatorname{csch}^{2}(x)}{\operatorname{csch}(x)+i}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)**2/(I+csch(x)),x)
[Out] Integral(csch(x)**2/(csch(x) + I), x)
```

3.68 $\int \frac{\operatorname{csch}^3(x)}{i+\operatorname{csch}(x)} \, dx$

Optimal. Leaf size=26

$$-\coth(x)+i\tanh^{-1}(\cosh(x))-\frac{i\coth(x)}{\operatorname{csch}(x)+i}$$

```
[Out] I*arctanh(cosh(x))-coth(x)-I*coth(x)/(I+csch(x))
```

Rubi [A] time = 0.08, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {3790, 3789, 3770, 3794}

$$-\coth(x)+i\tanh^{-1}(\cosh(x))-\frac{i\coth(x)}{\operatorname{csch}(x)+i}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[x]^3/(I + Csch[x]),x]
```

```
[Out] I*ArcTanh[Cosh[x]] - Coth[x] - (I*Coth[x])/(I + Csch[x])
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3789

```
Int[csc[(e_.) + (f_.)*(x_)]^2/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Sym
bol] :> Dist[1/b, Int[Csc[e + f*x], x], x] - Dist[a/b, Int[Csc[e + f*x]/(a
+ b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 3790

```
Int[csc[(e_.) + (f_.)*(x_)]^3/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Sym
bol] :> -Simp[Cot[e + f*x]/(b*f), x] - Dist[a/b, Int[Csc[e + f*x]^2/(a + b*
Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 3794

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> -Simp[Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}
, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\operatorname{csch}^3(x)}{i+\operatorname{csch}(x)}\,dx &= -\coth(x)-i\int \frac{\operatorname{csch}^2(x)}{i+\operatorname{csch}(x)}\,dx\\ &= -\coth(x)-i\int \operatorname{csch}(x)\,dx-\int \frac{\operatorname{csch}(x)}{i+\operatorname{csch}(x)}\,dx\\ &= i\tanh^{-1}(\cosh(x))-\coth(x)-\frac{i\coth(x)}{i+\operatorname{csch}(x)}\end{aligned}$$

Mathematica [B] time = 0.11, size = 70, normalized size = 2.69

$$-\frac{1}{2}\tanh\left(\frac{x}{2}\right)-\frac{1}{2}\coth\left(\frac{x}{2}\right)-i\log\left(\sinh\left(\frac{x}{2}\right)\right)+i\log\left(\cosh\left(\frac{x}{2}\right)\right)-\frac{2\sinh\left(\frac{x}{2}\right)}{\cosh\left(\frac{x}{2}\right)+i\sinh\left(\frac{x}{2}\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[x]^3/(I + Csch[x]),x]
```

```
[Out] -1/2*Coth[x/2] + I*Log[Cosh[x/2]] - I*Log[Sinh[x/2]] - (2*Sinh[x/2])/(Cosh[
x/2] + I*Sinh[x/2]) - Tanh[x/2]/2
```

fricas [B] time = 0.42, size = 77, normalized size = 2.96

$$\frac{\left(i e^{(3x)} + e^{(2x)} - i e^{x} - 1\right) \log\left(e^{x} + 1\right) + \left(-i e^{(3x)} - e^{(2x)} + i e^{x} + 1\right) \log\left(e^{x} - 1\right) - 2i e^{(2x)} - 2 e^{x} + 4i}{e^{(3x)} - i e^{(2x)} - e^{x} + i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^3/(I+csch(x)),x, algorithm="fricas")
```

```
[Out] ((I*e^(3*x) + e^(2*x) - I*e^x - 1)*log(e^x + 1) + (-I*e^(3*x) - e^(2*x) + I
*e^x + 1)*log(e^x - 1) - 2*I*e^(2*x) - 2*e^x + 4*I)/(e^(3*x) - I*e^(2*x) -
e^x + I)
```

giac [B] time = 0.12, size = 46, normalized size = 1.77

$$\frac{2\left(e^{(2x)} - i e^{x} - 2\right)}{i e^{(3x)} + e^{(2x)} - i e^{x} - 1} + i \log\left(e^{x} + 1\right) - i \log\left(|e^{x} - 1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^3/(I+csch(x)),x, algorithm="giac")
```

```
[Out] 2*(e^(2*x) - I*e^x - 2)/(I*e^(3*x) + e^(2*x) - I*e^x - 1) + I*log(e^x + 1)
 - I*log(abs(e^x - 1))
```

maple [A] time = 0.11, size = 35, normalized size = 1.35

$$-\frac{\tanh\left(\frac{x}{2}\right)}{2} - \frac{2}{\tanh\left(\frac{x}{2}\right) - i} - \frac{1}{2 \tanh\left(\frac{x}{2}\right)} - i \ln\left(\tanh\left(\frac{x}{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(x)^3/(I+csch(x)),x)
```

```
[Out] -1/2*tanh(1/2*x)-2/(tanh(1/2*x)-I)-1/2/tanh(1/2*x)-I*ln(tanh(1/2*x))
```

maxima [B] time = 0.32, size = 55, normalized size = 2.12

$$-\frac{8\left(e^{(-x)} - i e^{(-2x)} + 2i\right)}{4 e^{(-x)} - 4i e^{(-2x)} - 4 e^{(-3x)} + 4i} + i \log\left(e^{(-x)} + 1\right) - i \log\left(e^{(-x)} - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^3/(I+csch(x)),x, algorithm="maxima")
```

```
[Out] -8*(e^(-x) - I*e^(-2*x) + 2*I)/(4*e^(-x) - 4*I*e^(-2*x) - 4*e^(-3*x) + 4*I)
 + I*log(e^(-x) + 1) - I*log(e^(-x) - 1)
```

mupad [B] time = 1.64, size = 60, normalized size = 2.31

$$-\ln\left(e^{x}\,2i - 2i\right)\,1i + \ln\left(e^{x}\,2i + 2i\right)\,1i + \frac{e^{2x}\,2i + 2\,e^{x} - 4i}{e^{2x}\,1i - e^{3x} + e^{x} - i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sinh(x)^3*(1/sinh(x) + 1i)),x)
```

```
[Out] log(exp(x)*2i + 2i)*1i - log(exp(x)*2i - 2i)*1i + (exp(2*x)*2i + 2*exp(x) -
 4i)/(exp(2*x)*1i - exp(3*x) + exp(x) - 1i)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}^3(x)}{\operatorname{csch}(x) + i}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)**3/(I+csch(x)),x)
```

```
[Out] Integral(csch(x)**3/(csch(x) + I), x)
```

3.69 $\int \frac{\text{csch}^4(x)}{i+\text{csch}(x)}\,dx$

Optimal. Leaf size=37

$$2i\coth(x)+\frac{3}{2}\tanh^{-1}(\cosh(x))+\frac{\coth(x)\text{csch}^2(x)}{\text{csch}(x)+i}-\frac{3}{2}\coth(x)\text{csch}(x)$$

```
[Out] 3/2*arctanh(cosh(x))+2*I*coth(x)-3/2*coth(x)*csch(x)+coth(x)*csch(x)^2/(I+c
sch(x))
```

Rubi [A] time = 0.06, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {3818, 3787, 3767, 8, 3768, 3770}

$$2i\coth(x)+\frac{3}{2}\tanh^{-1}(\cosh(x))+\frac{\coth(x)\text{csch}^2(x)}{\text{csch}(x)+i}-\frac{3}{2}\coth(x)\text{csch}(x)$$

Antiderivative was successfully verified.

```
[In] Int[Csch[x]^4/(I + Csch[x]),x]
```

```
[Out] (3*ArcTanh[Cosh[x]])/2 + (2*I)*Coth[x] - (3*Coth[x]*Csch[x])/2 + (Coth[x]*C
sch[x]^2)/(I + Csch[x])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3818

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(d^2*Cot[e + f*x]*(d*Csc[e + f*x])^(n - 2))/(f*(a +
 b*Csc[e + f*x])), x] - Dist[d^2/(a*b), Int[(d*Csc[e + f*x])^(n - 2)*(b*(n
- 2) - a*(n - 1)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[
a^2 - b^2, 0] && GtQ[n, 1]
```

Rubi steps

$$\int \frac{\text{csch}^4(x)}{i+\text{csch}(x)}\,dx = \frac{\coth(x)\text{csch}^2(x)}{i+\text{csch}(x)} - \int (2i-3\text{csch}(x))\text{csch}^2(x)\,dx$$
$$= \frac{\coth(x)\text{csch}^2(x)}{i+\text{csch}(x)} - 2i\int \text{csch}^2(x)\,dx + 3\int \text{csch}^3(x)\,dx$$
$$= -\frac{3}{2}\coth(x)\text{csch}(x) + \frac{\coth(x)\text{csch}^2(x)}{i+\text{csch}(x)} - \frac{3}{2}\int \text{csch}(x)\,dx - 2\,\text{Subst}(\int 1\,dx, x, -i\coth(x))$$
$$= \frac{3}{2}\tanh^{-1}(\cosh(x)) + 2i\coth(x) - \frac{3}{2}\coth(x)\text{csch}(x) + \frac{\coth(x)\text{csch}^2(x)}{i+\text{csch}(x)}$$

Mathematica [B] time = 0.33, size = 81, normalized size = 2.19

$$\frac{1}{8}\left(4i\tanh\left(\frac{x}{2}\right) + 4i\coth\left(\frac{x}{2}\right) - \text{csch}^2\left(\frac{x}{2}\right) - \text{sech}^2\left(\frac{x}{2}\right) - 12\log\left(\tanh\left(\frac{x}{2}\right)\right) + \frac{16\sinh\left(\frac{x}{2}\right)}{\sinh\left(\frac{x}{2}\right) - i\cosh\left(\frac{x}{2}\right)}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[x]^4/(I + Csch[x]),x]
```

```
[Out] ((4*I)*Coth[x/2] - Csch[x/2]^2 - 12*Log[Tanh[x/2]] - Sech[x/2]^2 + (16*Sinh
[x/2])/((-I)*Cosh[x/2] + Sinh[x/2]) + (4*I)*Tanh[x/2])/8
```

fricas [B] time = 0.42, size = 130, normalized size = 3.51

$$\frac{\left(3\,e^{(5\,x)} - 3i\,e^{(4\,x)} - 6\,e^{(3\,x)} + 6i\,e^{(2\,x)} + 3\,e^{x} - 3i\right)\log\left(e^{x}+1\right) - \left(3\,e^{(5\,x)} - 3i\,e^{(4\,x)} - 6\,e^{(3\,x)} + 6i\,e^{(2\,x)} + 3\,e^{x} - 3i\right)\text{lo}}{2\,e^{(5\,x)} - 2i\,e^{(4\,x)} - 4\,e^{(3\,x)} + 4i\,e^{(2\,x)} + 2\,e^{x} - 2i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^4/(I+csch(x)),x, algorithm="fricas")
```

```
[Out] ((3*e^(5*x) - 3*I*e^(4*x) - 6*e^(3*x) + 6*I*e^(2*x) + 3*e^x - 3*I)*log(e^x
+ 1) - (3*e^(5*x) - 3*I*e^(4*x) - 6*e^(3*x) + 6*I*e^(2*x) + 3*e^x - 3*I)*lo
g(e^x - 1) - 6*e^(4*x) + 6*I*e^(3*x) + 10*e^(2*x) - 2*I*e^x - 8)/(2*e^(5*x)
 - 2*I*e^(4*x) - 4*e^(3*x) + 4*I*e^(2*x) + 2*e^x - 2*I)
```

giac [A] time = 0.12, size = 50, normalized size = 1.35

$$-\frac{e^{(3\,x)} - 2i\,e^{(2\,x)} + e^{x} + 2i}{\left(e^{(2\,x)} - 1\right)^2} - \frac{2i}{i\,e^{x}+1} + \frac{3}{2}\log\left(e^{x}+1\right) - \frac{3}{2}\log\left(|e^{x}-1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^4/(I+csch(x)),x, algorithm="giac")
```

```
[Out] -(e^(3*x) - 2*I*e^(2*x) + e^x + 2*I)/(e^(2*x) - 1)^2 - 2*I/(I*e^x + 1) + 3/
2*log(e^x + 1) - 3/2*log(abs(e^x - 1))
```

maple [A] time = 0.13, size = 53, normalized size = 1.43

$$\frac{i\tanh\left(\frac{x}{2}\right)}{2} + \frac{\left(\tanh^2\left(\frac{x}{2}\right)\right)}{8} + \frac{2i}{\tanh\left(\frac{x}{2}\right) - i} - \frac{1}{8\tanh\left(\frac{x}{2}\right)^2} + \frac{i}{2\tanh\left(\frac{x}{2}\right)} - \frac{3\ln\left(\tanh\left(\frac{x}{2}\right)\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(x)^4/(I+csch(x)),x)
```

```
[Out] 1/2*I*tanh(1/2*x)+1/8*tanh(1/2*x)^2+2*I/(tanh(1/2*x)-I)-1/8/tanh(1/2*x)^2+1
/2*I/tanh(1/2*x)-3/2*ln(tanh(1/2*x))
```

maxima [B] time = 0.32, size = 81, normalized size = 2.19

$$-\frac{16\left(-i\,e^{(-x)} - 5\,e^{(-2\,x)} + 3i\,e^{(-3\,x)} + 3\,e^{(-4\,x)} + 4\right)}{16\,e^{(-x)} - 32i\,e^{(-2\,x)} - 32\,e^{(-3\,x)} + 16i\,e^{(-4\,x)} + 16\,e^{(-5\,x)} + 16i} + \frac{3}{2}\,\log\left(e^{(-x)} + 1\right) - \frac{3}{2}\,\log\left(e^{(-x)} - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^4/(I+csch(x)),x, algorithm="maxima")
```

```
[Out] -16*(-I*e^(-x) - 5*e^(-2*x) + 3*I*e^(-3*x) + 3*e^(-4*x) + 4)/(16*e^(-x) - 3
2*I*e^(-2*x) - 32*e^(-3*x) + 16*I*e^(-4*x) + 16*e^(-5*x) + 16*I) + 3/2*log(
e^(-x) + 1) - 3/2*log(e^(-x) - 1)
```

mupad [B] time = 1.69, size = 63, normalized size = 1.70

$$\frac{3\,\ln\left(3\,e^x + 3\right)}{2} - \frac{3\,\ln\left(3\,e^x - 3\right)}{2} - \frac{e^x}{e^{2\,x} - 1} - \frac{2\,e^x}{\left(e^{2\,x} - 1\right)^2} - \frac{2}{e^x - i} + \frac{2i}{e^{2\,x} - 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sinh(x)^4*(1/sinh(x) + 1i)),x)
```

```
[Out] (3*log(3*exp(x) + 3))/2 - (3*log(3*exp(x) - 3))/2 - exp(x)/(exp(2*x) - 1) -
 (2*exp(x))/(exp(2*x) - 1)^2 - 2/(exp(x) - 1i) + 2i/(exp(2*x) - 1)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}^{4}(x)}{\operatorname{csch}(x) + i}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)**4/(I+csch(x)),x)
```

```
[Out] Integral(csch(x)**4/(csch(x) + I), x)
```

3.70 $\int (a+b\,\text{csch}(c+dx))^4\,dx$

Optimal. Leaf size=109

$$a^4x-\frac{b^2\left(17a^2-2b^2\right)\coth(c+dx)}{3d}-\frac{2ab\left(2a^2-b^2\right)\tanh^{-1}(\cosh(c+dx))}{d}-\frac{4ab^3\coth(c+dx)\text{csch}(c+dx)}{3d}-\frac{b^2 c}{}$$

```
[Out] a^4*x-2*a*b*(2*a^2-b^2)*arctanh(cosh(d*x+c))/d-1/3*b^2*(17*a^2-2*b^2)*coth(
d*x+c)/d-4/3*a*b^3*coth(d*x+c)*csch(d*x+c)/d-1/3*b^2*coth(d*x+c)*(a+b*csch(
d*x+c))^2/d
```

Rubi [A] time = 0.13, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {3782, 4048, 3770, 3767, 8}

$$-\frac{b^2\left(17a^2-2b^2\right)\coth(c+dx)}{3d}-\frac{2ab\left(2a^2-b^2\right)\tanh^{-1}(\cosh(c+dx))}{d}+a^4x-\frac{4ab^3\coth(c+dx)\text{csch}(c+dx)}{3d}-\frac{b^2}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Csch[c + d*x])^4,x]
```

```
[Out] a^4*x - (2*a*b*(2*a^2 - b^2)*ArcTanh[Cosh[c + d*x]])/d - (b^2*(17*a^2 - 2*b
^2)*Coth[c + d*x])/(3*d) - (4*a*b^3*Coth[c + d*x]*Csch[c + d*x])/(3*d) - (b
^2*Coth[c + d*x]*(a + b*Csch[c + d*x])^2)/(3*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3782

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> -Simp[(b^2*Co
t[c + d*x]*(a + b*Csc[c + d*x])^(n - 2))/(d*(n - 1)), x] + Dist[1/(n - 1),
Int[(a + b*Csc[c + d*x])^(n - 3)*Simp[a^3*(n - 1) + (b*(b^2*(n - 2) + 3*a^2
*(n - 1)))*Csc[c + d*x] + (a*b^2*(3*n - 4))*Csc[c + d*x]^2, x], x], x] /; F
reeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[n, 2] && IntegerQ[2*n]
```

Rule 4048

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> -Simp[(b*C*Csc[e +
f*x]*Cot[e + f*x])/(2*f), x] + Dist[1/2, Int[Simp[2*A*a + (2*B*a + b*(2*A +
 C))*Csc[e + f*x] + 2*(a*C + B*b)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, e, f, A, B, C}, x]
```

Rubi steps

$$\int (a+b\text{csch}(c+dx))^4\,dx = -\frac{b^2\coth(c+dx)(a+b\text{csch}(c+dx))^2}{3d}+\frac{1}{3}\int (a+b\text{csch}(c+dx))\left(3a^3+b\left(9a^2-2b^2\right.\right.$$
$$= -\frac{4ab^3\coth(c+dx)\text{csch}(c+dx)}{3d}-\frac{b^2\coth(c+dx)(a+b\text{csch}(c+dx))^2}{3d}+\frac{1}{6}\int \left(6a^4\right.$$
$$= a^4x-\frac{4ab^3\coth(c+dx)\text{csch}(c+dx)}{3d}-\frac{b^2\coth(c+dx)(a+b\text{csch}(c+dx))^2}{3d}+\frac{1}{3}\left(b^2\right.$$
$$= a^4x-\frac{2ab\left(2a^2-b^2\right)\tanh^{-1}(\cosh(c+dx))}{d}-\frac{4ab^3\coth(c+dx)\text{csch}(c+dx)}{3d}-\frac{b^2\text{co}}{}$$
$$= a^4x-\frac{2ab\left(2a^2-b^2\right)\tanh^{-1}(\cosh(c+dx))}{d}-\frac{b^2\left(17a^2-2b^2\right)\coth(c+dx)}{3d}-\frac{4ab^3\text{co}}{}$$

Mathematica [B] time = 6.24, size = 508, normalized size = 4.66

$$\frac{a^4(c+dx)\sinh^4(c+dx)(a+b\text{csch}(c+dx))^4}{d(a\sinh(c+dx)+b)^4}+\frac{2ab\left(2a^2-b^2\right)\sinh^4(c+dx)\log\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right)(a+b\text{csch}(c\right.}{d(a\sinh(c+dx)+b)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Csch[c + d*x])^4,x]
```

```
[Out] (a^4*(c + d*x)*(a + b*Csch[c + d*x])^4*Sinh[c + d*x]^4)/(d*(b + a*Sinh[c +
d*x])^4) + ((-9*a^2*b^2*Cosh[(c + d*x)/2] + b^4*Cosh[(c + d*x)/2])*Csch[(c
+ d*x)/2]*(a + b*Csch[c + d*x])^4*Sinh[c + d*x]^4)/(3*d*(b + a*Sinh[c + d*x
])^4) - (a*b^3*Csch[(c + d*x)/2]^2*(a + b*Csch[c + d*x])^4*Sinh[c + d*x]^4)
/(2*d*(b + a*Sinh[c + d*x])^4) - (b^4*Coth[(c + d*x)/2]*Csch[(c + d*x)/2]^2
*(a + b*Csch[c + d*x])^4*Sinh[c + d*x]^4)/(24*d*(b + a*Sinh[c + d*x])^4) +
(2*a*b*(2*a^2 - b^2)*(a + b*Csch[c + d*x])^4*Log[Tanh[(c + d*x)/2]]*Sinh[c
+ d*x]^4)/(d*(b + a*Sinh[c + d*x])^4) - (a*b^3*(a + b*Csch[c + d*x])^4*Sech
[(c + d*x)/2]^2*Sinh[c + d*x]^4)/(2*d*(b + a*Sinh[c + d*x])^4) + ((a + b*Cs
ch[c + d*x])^4*Sech[(c + d*x)/2]*(-9*a^2*b^2*Sinh[(c + d*x)/2] + b^4*Sinh[(
c + d*x)/2])*Sinh[c + d*x]^4)/(3*d*(b + a*Sinh[c + d*x])^4) + (b^4*(a + b*C
sch[c + d*x])^4*Sech[(c + d*x)/2]^2*Sinh[c + d*x]^4*Tanh[(c + d*x)/2])/(24*
d*(b + a*Sinh[c + d*x])^4)
```

fricas [B] time = 0.44, size = 1440, normalized size = 13.21

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*csch(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/3*(3*a^4*d*x*cosh(d*x + c)^6 + 3*a^4*d*x*sinh(d*x + c)^6 - 12*a*b^3*cosh(
d*x + c)^5 - 3*a^4*d*x + 6*(3*a^4*d*x*cosh(d*x + c) - 2*a*b^3)*sinh(d*x + c
)^5 + 12*a*b^3*cosh(d*x + c) - 9*(a^4*d*x + 4*a^2*b^2)*cosh(d*x + c)^4 + 3*
(15*a^4*d*x*cosh(d*x + c)^2 - 3*a^4*d*x - 20*a*b^3*cosh(d*x + c) - 12*a^2*b
^2)*sinh(d*x + c)^4 - 36*a^2*b^2 + 4*b^4 + 12*(5*a^4*d*x*cosh(d*x + c)^3 -
10*a*b^3*cosh(d*x + c)^2 - 3*(a^4*d*x + 4*a^2*b^2)*cosh(d*x + c))*sinh(d*x
+ c)^3 + 3*(3*a^4*d*x + 24*a^2*b^2 - 4*b^4)*cosh(d*x + c)^2 + 3*(15*a^4*d*x
*cosh(d*x + c)^4 - 40*a*b^3*cosh(d*x + c)^3 + 3*a^4*d*x + 24*a^2*b^2 - 4*b^
4 - 18*(a^4*d*x + 4*a^2*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^2 - 6*((2*a^3*b
 - a*b^3)*cosh(d*x + c)^6 + 6*(2*a^3*b - a*b^3)*cosh(d*x + c)*sinh(d*x + c)
^5 + (2*a^3*b - a*b^3)*sinh(d*x + c)^6 - 3*(2*a^3*b - a*b^3)*cosh(d*x + c)^
4 - 3*(2*a^3*b - a*b^3 - 5*(2*a^3*b - a*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)
^4 - 2*a^3*b + a*b^3 + 4*(5*(2*a^3*b - a*b^3)*cosh(d*x + c)^3 - 3*(2*a^3*b
- a*b^3)*cosh(d*x + c))*sinh(d*x + c)^3 + 3*(2*a^3*b - a*b^3)*cosh(d*x + c)
```

```
^2 + 3*(5*(2*a^3*b - a*b^3)*cosh(d*x + c)^4 + 2*a^3*b - a*b^3 - 6*(2*a^3*b
- a*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 6*((2*a^3*b - a*b^3)*cosh(d*x +
 c)^5 - 2*(2*a^3*b - a*b^3)*cosh(d*x + c)^3 + (2*a^3*b - a*b^3)*cosh(d*x +
c))*sinh(d*x + c))*log(cosh(d*x + c) + sinh(d*x + c) + 1) + 6*((2*a^3*b - a
*b^3)*cosh(d*x + c)^6 + 6*(2*a^3*b - a*b^3)*cosh(d*x + c)*sinh(d*x + c)^5 +
 (2*a^3*b - a*b^3)*sinh(d*x + c)^6 - 3*(2*a^3*b - a*b^3)*cosh(d*x + c)^4 -
3*(2*a^3*b - a*b^3 - 5*(2*a^3*b - a*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^4 -
 2*a^3*b + a*b^3 + 4*(5*(2*a^3*b - a*b^3)*cosh(d*x + c)^3 - 3*(2*a^3*b - a*
b^3)*cosh(d*x + c))*sinh(d*x + c)^3 + 3*(2*a^3*b - a*b^3)*cosh(d*x + c)^2 +
 3*(5*(2*a^3*b - a*b^3)*cosh(d*x + c)^4 + 2*a^3*b - a*b^3 - 6*(2*a^3*b - a*
b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 6*((2*a^3*b - a*b^3)*cosh(d*x + c)^
5 - 2*(2*a^3*b - a*b^3)*cosh(d*x + c)^3 + (2*a^3*b - a*b^3)*cosh(d*x + c))*
sinh(d*x + c))*log(cosh(d*x + c) + sinh(d*x + c) - 1) + 6*(3*a^4*d*x*cosh(d
*x + c)^5 - 10*a*b^3*cosh(d*x + c)^4 + 2*a*b^3 - 6*(a^4*d*x + 4*a^2*b^2)*co
sh(d*x + c)^3 + (3*a^4*d*x + 24*a^2*b^2 - 4*b^4)*cosh(d*x + c))*sinh(d*x +
c))/(d*cosh(d*x + c)^6 + 6*d*cosh(d*x + c)*sinh(d*x + c)^5 + d*sinh(d*x + c
)^6 - 3*d*cosh(d*x + c)^4 + 3*(5*d*cosh(d*x + c)^2 - d)*sinh(d*x + c)^4 + 4
*(5*d*cosh(d*x + c)^3 - 3*d*cosh(d*x + c))*sinh(d*x + c)^3 + 3*d*cosh(d*x +
 c)^2 + 3*(5*d*cosh(d*x + c)^4 - 6*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^2 +
 6*(d*cosh(d*x + c)^5 - 2*d*cosh(d*x + c)^3 + d*cosh(d*x + c))*sinh(d*x + c
) - d)
```

giac [A] time = 0.13, size = 169, normalized size = 1.55

$$\frac{3\,(dx+c)a^4 - 6\left(2\,a^3b - ab^3\right)\log\left(e^{(dx+c)}+1\right) + 6\left(2\,a^3b - ab^3\right)\log\left(\left|e^{(dx+c)}-1\right|\right) - \frac{4\left(3\,ab^3e^{(5\,dx+5\,c)}+9\,a^2b^2e^{(4\,dx+4\,c)} - \cdots\right.}{\cdots}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*csch(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/3*(3*(d*x + c)*a^4 - 6*(2*a^3*b - a*b^3)*log(e^(d*x + c) + 1) + 6*(2*a^3*
b - a*b^3)*log(abs(e^(d*x + c) - 1)) - 4*(3*a*b^3*e^(5*d*x + 5*c) + 9*a^2*b
^2*e^(4*d*x + 4*c) - 18*a^2*b^2*e^(2*d*x + 2*c) + 3*b^4*e^(2*d*x + 2*c) - 3
*a*b^3*e^(d*x + c) + 9*a^2*b^2 - b^4)/(e^(2*d*x + 2*c) - 1)^3)/d
```

maple [A] time = 0.49, size = 92, normalized size = 0.84

$$\frac{a^4\,(dx+c) - 8a^3b\,\mathrm{arctanh}\left(e^{dx+c}\right) - 6a^2b^2\coth\,(dx+c) + 4a\,b^3\left(-\frac{\mathrm{csch}(dx+c)\coth(dx+c)}{2} + \mathrm{arctanh}\left(e^{dx+c}\right)\right) + b^4\,(\cdots}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*csch(d*x+c))^4,x)
```

```
[Out] 1/d*(a^4*(d*x+c)-8*a^3*b*arctanh(exp(d*x+c))-6*a^2*b^2*coth(d*x+c)+4*a*b^3*
(-1/2*csch(d*x+c)*coth(d*x+c)+arctanh(exp(d*x+c)))+b^4*(2/3-1/3*csch(d*x+c)
^2)*coth(d*x+c))
```

maxima [B] time = 0.32, size = 234, normalized size = 2.15

$$a^4x+2\,ab^3\left(\frac{\log\left(e^{(-dx-c)}+1\right)}{d} - \frac{\log\left(e^{(-dx-c)}-1\right)}{d} + \frac{2\left(e^{(-dx-c)}+e^{(-3\,dx-3\,c)}\right)}{d\left(2\,e^{(-2\,dx-2\,c)} - e^{(-4\,dx-4\,c)} - 1\right)}\right)+\frac{4}{3}\,b^4\left(\frac{3\,e^{(\cdots}}{d\left(3\,e^{(-2\,dx-2\,c)} - 3\,e^{(\cdots}\right.}\cdots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*csch(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] a^4*x + 2*a*b^3*(log(e^(-d*x - c) + 1)/d - log(e^(-d*x - c) - 1)/d + 2*(e^(
-d*x - c) + e^(-3*d*x - 3*c))/(d*(2*e^(-2*d*x - 2*c) - e^(-4*d*x - 4*c) - 1
```

```
))) + 4/3*b^4*(3*e^(-2*d*x - 2*c)/(d*(3*e^(-2*d*x - 2*c) - 3*e^(-4*d*x - 4*
c) + e^(-6*d*x - 6*c) - 1)) - 1/(d*(3*e^(-2*d*x - 2*c) - 3*e^(-4*d*x - 4*c)
 + e^(-6*d*x - 6*c) - 1))) + 4*a^3*b*log(tanh(1/2*d*x + 1/2*c))/d + 12*a^2*
b^2/(d*(e^(-2*d*x - 2*c) - 1))
```

mupad [B] time = 0.21, size = 239, normalized size = 2.19

$$a^4\,x-\frac{\frac{12\,a^2\,b^2}{d}+\frac{4\,a\,b^3\,e^{c+d\,x}}{d}}{e^{2\,c+2\,d\,x}-1}-\frac{\frac{4\,b^4}{d}+\frac{8\,a\,b^3\,e^{c+d\,x}}{d}}{e^{4\,c+4\,d\,x}-2\,e^{2\,c+2\,d\,x}+1}-\frac{8\,b^4}{3\,d\,\left(3\,e^{2\,c+2\,d\,x}-3\,e^{4\,c+4\,d\,x}+e^{6\,c+6\,d\,x}-1\right)}+\frac{4\,\mathrm{atan}\left(\frac{e^{d\,x}\,e^c\,\left(a\,b^3\ldots\right.}{d\,\sqrt{4\,a^6\,b\ldots}}\right.}{\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/sinh(c + d*x))^4,x)
```

```
[Out] a^4*x - ((12*a^2*b^2)/d + (4*a*b^3*exp(c + d*x))/d)/(exp(2*c + 2*d*x) - 1)
- ((4*b^4)/d + (8*a*b^3*exp(c + d*x))/d)/(exp(4*c + 4*d*x) - 2*exp(2*c + 2*
d*x) + 1) - (8*b^4)/(3*d*(3*exp(2*c + 2*d*x) - 3*exp(4*c + 4*d*x) + exp(6*c
 + 6*d*x) - 1)) + (4*atan((exp(d*x)*exp(c)*(a*b^3*(-d^2)^(1/2) - 2*a^3*b*(-
d^2)^(1/2)))/(d*(a^2*b^6 - 4*a^4*b^4 + 4*a^6*b^2)^(1/2)))*(a^2*b^6 - 4*a^4*
b^4 + 4*a^6*b^2)^(1/2))/(-d^2)^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b \operatorname{csch}\left(c + dx\right)\right)^{4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*csch(d*x+c))**4,x)
```

```
[Out] Integral((a + b*csch(c + d*x))**4, x)
```

3.71 $\int (a+b\,\mathrm{csch}(c+dx))^3\,dx$

Optimal. Leaf size=75

$$a^3x-\frac{b\left(6a^2-b^2\right)\tanh^{-1}(\cosh(c+dx))}{2d}-\frac{5ab^2\coth(c+dx)}{2d}-\frac{b^2\coth(c+dx)(a+b\mathrm{csch}(c+dx))}{2d}$$

```
[Out] a^3*x-1/2*b*(6*a^2-b^2)*arctanh(cosh(d*x+c))/d-5/2*a*b^2*coth(d*x+c)/d-1/2*
b^2*coth(d*x+c)*(a+b*csch(d*x+c))/d
```

Rubi [A] time = 0.05, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3782, 3770, 3767, 8}

$$-\frac{b\left(6a^2-b^2\right)\tanh^{-1}(\cosh(c+dx))}{2d}+a^3x-\frac{5ab^2\coth(c+dx)}{2d}-\frac{b^2\coth(c+dx)(a+b\mathrm{csch}(c+dx))}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Csch[c + d*x])^3,x]
```

```
[Out] a^3*x - (b*(6*a^2 - b^2)*ArcTanh[Cosh[c + d*x]])/(2*d) - (5*a*b^2*Coth[c +
d*x])/(2*d) - (b^2*Coth[c + d*x]*(a + b*Csch[c + d*x]))/(2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3782

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> -Simp[(b^2*Co
t[c + d*x]*(a + b*Csc[c + d*x])^(n - 2))/(d*(n - 1)), x] + Dist[1/(n - 1),
Int[(a + b*Csc[c + d*x])^(n - 3)*Simp[a^3*(n - 1) + (b*(b^2*(n - 2) + 3*a^2
*(n - 1)))*Csc[c + d*x] + (a*b^2*(3*n - 4))*Csc[c + d*x]^2, x], x], x] /; F
reeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[n, 2] && IntegerQ[2*n]
```

Rubi steps

$$\int (a+b\mathrm{csch}(c+dx))^3\,dx=-\frac{b^2\coth(c+dx)(a+b\mathrm{csch}(c+dx))}{2d}+\frac{1}{2}\int\left(2a^3+b\left(6a^2-b^2\right)\mathrm{csch}(c+dx)+5\right.$$
$$=a^3x-\frac{b^2\coth(c+dx)(a+b\mathrm{csch}(c+dx))}{2d}+\frac{1}{2}\left(5ab^2\right)\int\mathrm{csch}^2(c+dx)\,dx+\frac{1}{2}\left(b\right($$
$$=a^3x-\frac{b\left(6a^2-b^2\right)\tanh^{-1}(\cosh(c+dx))}{2d}-\frac{b^2\coth(c+dx)(a+b\mathrm{csch}(c+dx))}{2d}-$$
$$=a^3x-\frac{b\left(6a^2-b^2\right)\tanh^{-1}(\cosh(c+dx))}{2d}-\frac{5ab^2\coth(c+dx)}{2d}-\frac{b^2\coth(c+dx)(a}{2}$$

Mathematica [A] time = 0.87, size = 118, normalized size = 1.57

$$-\frac{-8a^3c - 8a^3dx - 24a^2b\log\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right) + 12ab^2\tanh\left(\frac{1}{2}(c+dx)\right) + 12ab^2\coth\left(\frac{1}{2}(c+dx)\right) + b^3\text{csch}^2\left(\frac{1}{2}\right.}{8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Csch[c + d*x])^3,x]
```

```
[Out] -1/8*(-8*a^3*c - 8*a^3*d*x + 12*a*b^2*Coth[(c + d*x)/2] + b^3*Csch[(c + d*x
)/2]^2 - 24*a^2*b*Log[Tanh[(c + d*x)/2]] + 4*b^3*Log[Tanh[(c + d*x)/2]] + b
^3*Sech[(c + d*x)/2]^2 + 12*a*b^2*Tanh[(c + d*x)/2])/d
```

fricas [B] time = 0.42, size = 769, normalized size = 10.25

$$\frac{2\,a^3dx\cosh\left(dx+c\right)^4 + 2\,a^3dx\sinh\left(dx+c\right)^4 - 2\,b^3\cosh\left(dx+c\right)^3 + 2\,a^3dx - 2\,b^3\cosh\left(dx+c\right) + 2\left(4\,a^3dx\cosh\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*csch(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/2*(2*a^3*d*x*cosh(d*x + c)^4 + 2*a^3*d*x*sinh(d*x + c)^4 - 2*b^3*cosh(d*x
 + c)^3 + 2*a^3*d*x - 2*b^3*cosh(d*x + c) + 2*(4*a^3*d*x*cosh(d*x + c) - b^
3)*sinh(d*x + c)^3 + 12*a*b^2 - 4*(a^3*d*x + 3*a*b^2)*cosh(d*x + c)^2 + 2*(
6*a^3*d*x*cosh(d*x + c)^2 - 2*a^3*d*x - 3*b^3*cosh(d*x + c) - 6*a*b^2)*sinh
(d*x + c)^2 - ((6*a^2*b - b^3)*cosh(d*x + c)^4 + 4*(6*a^2*b - b^3)*cosh(d*x
 + c)*sinh(d*x + c)^3 + (6*a^2*b - b^3)*sinh(d*x + c)^4 + 6*a^2*b - b^3 - 2
*(6*a^2*b - b^3)*cosh(d*x + c)^2 - 2*(6*a^2*b - b^3 - 3*(6*a^2*b - b^3)*cos
h(d*x + c)^2)*sinh(d*x + c)^2 + 4*((6*a^2*b - b^3)*cosh(d*x + c)^3 - (6*a^2
*b - b^3)*cosh(d*x + c))*sinh(d*x + c))*log(cosh(d*x + c) + sinh(d*x + c) +
 1) + ((6*a^2*b - b^3)*cosh(d*x + c)^4 + 4*(6*a^2*b - b^3)*cosh(d*x + c)*si
nh(d*x + c)^3 + (6*a^2*b - b^3)*sinh(d*x + c)^4 + 6*a^2*b - b^3 - 2*(6*a^2*
b - b^3)*cosh(d*x + c)^2 - 2*(6*a^2*b - b^3 - 3*(6*a^2*b - b^3)*cosh(d*x +
c)^2)*sinh(d*x + c)^2 + 4*((6*a^2*b - b^3)*cosh(d*x + c)^3 - (6*a^2*b - b^3
)*cosh(d*x + c))*sinh(d*x + c))*log(cosh(d*x + c) + sinh(d*x + c) - 1) + 2*
(4*a^3*d*x*cosh(d*x + c)^3 - 3*b^3*cosh(d*x + c)^2 - b^3 - 4*(a^3*d*x + 3*a
*b^2)*cosh(d*x + c))*sinh(d*x + c))/(d*cosh(d*x + c)^4 + 4*d*cosh(d*x + c)*
sinh(d*x + c)^3 + d*sinh(d*x + c)^4 - 2*d*cosh(d*x + c)^2 + 2*(3*d*cosh(d*x
 + c)^2 - d)*sinh(d*x + c)^2 + 4*(d*cosh(d*x + c)^3 - d*cosh(d*x + c))*sinh
(d*x + c) + d)
```

giac [A] time = 0.14, size = 122, normalized size = 1.63

$$\frac{2\,(dx+c)a^3 - \left(6\,a^2b - b^3\right)\log\left(e^{(dx+c)}+1\right) + \left(6\,a^2b - b^3\right)\log\left(\left|e^{(dx+c)}-1\right|\right) - \frac{2\left(b^3e^{(3\,dx+3\,c)} + 6\,ab^2e^{(2\,dx+2\,c)} + b^3e^{(dx+c)} - 6\,ab^2\right)}{\left(e^{(2\,dx+2\,c)}-1\right)^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*csch(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/2*(2*(d*x + c)*a^3 - (6*a^2*b - b^3)*log(e^(d*x + c) + 1) + (6*a^2*b - b^
3)*log(abs(e^(d*x + c) - 1)) - 2*(b^3*e^(3*d*x + 3*c) + 6*a*b^2*e^(2*d*x +
2*c) + b^3*e^(d*x + c) - 6*a*b^2)/(e^(2*d*x + 2*c) - 1)^2)/d
```

maple [A] time = 0.41, size = 66, normalized size = 0.88

$$\frac{a^3\,(dx+c) - 6a^2b\,\text{arctanh}\left(e^{dx+c}\right) - 3a\,b^2\coth\left(dx+c\right) + b^3\left(-\frac{\text{csch}(dx+c)\coth(dx+c)}{2} + \text{arctanh}\left(e^{dx+c}\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*csch(d*x+c))^3,x)
```

```
[Out] 1/d*(a^3*(d*x+c)-6*a^2*b*arctanh(exp(d*x+c))-3*a*b^2*coth(d*x+c)+b^3*(-1/2*
csch(d*x+c)*coth(d*x+c)+arctanh(exp(d*x+c))))
```

maxima [A] time = 0.33, size = 136, normalized size = 1.81

$$a^3x+\frac{1}{2}b^3\left(\frac{\log\left(e^{(-dx-c)}+1\right)}{d}-\frac{\log\left(e^{(-dx-c)}-1\right)}{d}+\frac{2\left(e^{(-dx-c)}+e^{(-3\,dx-3\,c)}\right)}{d\left(2\,e^{(-2\,dx-2\,c)}-e^{(-4\,dx-4\,c)}-1\right)}\right)+\frac{3\,a^2b\log\left(\tanh\left(\frac{1}{2}\,dx+\frac{1}{2}\right.\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*csch(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] a^3*x + 1/2*b^3*(log(e^(-d*x - c) + 1)/d - log(e^(-d*x - c) - 1)/d + 2*(e^(
-d*x - c) + e^(-3*d*x - 3*c))/(d*(2*e^(-2*d*x - 2*c) - e^(-4*d*x - 4*c) - 1
))) + 3*a^2*b*log(tanh(1/2*d*x + 1/2*c))/d + 6*a*b^2/(d*(e^(-2*d*x - 2*c) -
 1))
```

mupad [B] time = 0.15, size = 170, normalized size = 2.27

$$a^3\,x-\frac{\frac{6\,a\,b^2}{d}+\frac{b^3\,\mathrm{e}^{c+d\,x}}{d}}{\mathrm{e}^{2\,c+2\,d\,x}-1}+\frac{\mathrm{atan}\left(\frac{\mathrm{e}^{d\,x}\,\mathrm{e}^{c}\,\left(b^3\,\sqrt{-d^2}-6\,a^2\,b\,\sqrt{-d^2}\right)}{d\,\sqrt{36\,a^4\,b^2-12\,a^2\,b^4+b^6}}\right)\,\sqrt{36\,a^4\,b^2-12\,a^2\,b^4+b^6}}{\sqrt{-d^2}}-\frac{2\,b^3\,\mathrm{e}^{c+d\,x}}{d\,\left(\mathrm{e}^{4\,c+4\,d\,x}-2\,\mathrm{e}^{2\,c+2\,d\,x}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/sinh(c + d*x))^3,x)
```

```
[Out] a^3*x - ((6*a*b^2)/d + (b^3*exp(c + d*x))/d)/(exp(2*c + 2*d*x) - 1) + (atan
((exp(d*x)*exp(c)*(b^3*(-d^2)^(1/2) - 6*a^2*b*(-d^2)^(1/2)))/(d*(b^6 - 12*a
^2*b^4 + 36*a^4*b^2)^(1/2)))*(b^6 - 12*a^2*b^4 + 36*a^4*b^2)^(1/2))/(-d^2)^
(1/2) - (2*b^3*exp(c + d*x))/(d*(exp(4*c + 4*d*x) - 2*exp(2*c + 2*d*x) + 1)
)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\operatorname{csch}\left(c+dx\right)\right)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*csch(d*x+c))**3,x)
```

```
[Out] Integral((a + b*csch(c + d*x))**3, x)
```

3.72 $\int (a+b\,\mathrm{csch}(c+dx))^2\,dx$

Optimal. Leaf size=34

$$a^2x-\frac{2ab\tanh^{-1}(\cosh(c+dx))}{d}-\frac{b^2\coth(c+dx)}{d}$$

```
[Out] a^2*x-2*a*b*arctanh(cosh(d*x+c))/d-b^2*coth(d*x+c)/d
```

Rubi [A] time = 0.03, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3773, 3770, 3767, 8}

$$a^2x-\frac{2ab\tanh^{-1}(\cosh(c+dx))}{d}-\frac{b^2\coth(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Csch[c + d*x])^2,x]
```

```
[Out] a^2*x - (2*a*b*ArcTanh[Cosh[c + d*x]])/d - (b^2*Coth[c + d*x])/d
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3773

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^2, x_Symbol] :> Simp[a^2*x, x] +
 (Dist[2*a*b, Int[Csc[c + d*x], x], x] + Dist[b^2, Int[Csc[c + d*x]^2, x],
x]) /; FreeQ[{a, b, c, d}, x]
```

Rubi steps

$$\begin{aligned}\int (a+b\,\mathrm{csch}(c+dx))^2\,dx&=a^2x+(2ab)\int \mathrm{csch}(c+dx)\,dx+b^2\int \mathrm{csch}^2(c+dx)\,dx\\&=a^2x-\frac{2ab\tanh^{-1}(\cosh(c+dx))}{d}-\frac{\left(ib^2\right)\mathrm{Subst}\left(\int 1\,dx,x,-i\coth(c+dx)\right)}{d}\\&=a^2x-\frac{2ab\tanh^{-1}(\cosh(c+dx))}{d}-\frac{b^2\coth(c+dx)}{d}\end{aligned}$$

Mathematica [A] time = 0.22, size = 61, normalized size = 1.79

$$-\frac{-2a\left(ac+adx+2b\log\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right)\right)+b^2\tanh\left(\frac{1}{2}(c+dx)\right)+b^2\coth\left(\frac{1}{2}(c+dx)\right)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Csch[c + d*x])^2,x]
```

```
[Out] -1/2*(b^2*Coth[(c + d*x)/2] - 2*a*(a*c + a*d*x + 2*b*Log[Tanh[(c + d*x)/2]]
) + b^2*Tanh[(c + d*x)/2])/d
```

fricas [B] time = 0.42, size = 222, normalized size = 6.53

$$\frac{a^2dx\cosh(dx+c)^2 + 2a^2dx\cosh(dx+c)\sinh(dx+c) + a^2dx\sinh(dx+c)^2 - a^2dx - 2b^2 - 2\left(ab\cosh(dx\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*csch(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] (a^2*d*x*cosh(d*x + c)^2 + 2*a^2*d*x*cosh(d*x + c)*sinh(d*x + c) + a^2*d*x*
sinh(d*x + c)^2 - a^2*d*x - 2*b^2 - 2*(a*b*cosh(d*x + c)^2 + 2*a*b*cosh(d*x
 + c)*sinh(d*x + c) + a*b*sinh(d*x + c)^2 - a*b)*log(cosh(d*x + c) + sinh(d
*x + c) + 1) + 2*(a*b*cosh(d*x + c)^2 + 2*a*b*cosh(d*x + c)*sinh(d*x + c) +
 a*b*sinh(d*x + c)^2 - a*b)*log(cosh(d*x + c) + sinh(d*x + c) - 1))/(d*cosh
(d*x + c)^2 + 2*d*cosh(d*x + c)*sinh(d*x + c) + d*sinh(d*x + c)^2 - d)
```

giac [A] time = 0.12, size = 59, normalized size = 1.74

$$\frac{(dx+c)a^2 - 2ab\log\left(e^{(dx+c)}+1\right) + 2ab\log\left(\left|e^{(dx+c)}-1\right|\right) - \frac{2b^2}{e^{(2dx+2c)}-1}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*csch(d*x+c))^2,x, algorithm="giac")
```

```
[Out] ((d*x + c)*a^2 - 2*a*b*log(e^(d*x + c) + 1) + 2*a*b*log(abs(e^(d*x + c) - 1
)) - 2*b^2/(e^(2*d*x + 2*c) - 1))/d
```

maple [A] time = 0.32, size = 37, normalized size = 1.09

$$\frac{a^2(dx+c) - 4ab\,\mathrm{arctanh}\left(e^{dx+c}\right) - b^2\coth(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*csch(d*x+c))^2,x)
```

```
[Out] 1/d*(a^2*(d*x+c)-4*a*b*arctanh(exp(d*x+c))-b^2*coth(d*x+c))
```

maxima [A] time = 0.32, size = 44, normalized size = 1.29

$$a^2x + \frac{2ab\log\left(\tanh\left(\frac{1}{2}dx + \frac{1}{2}c\right)\right)}{d} + \frac{2b^2}{d\left(e^{(-2dx-2c)}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*csch(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] a^2*x + 2*a*b*log(tanh(1/2*d*x + 1/2*c))/d + 2*b^2/(d*(e^(-2*d*x - 2*c) - 1
))
```

mupad [B] time = 1.48, size = 74, normalized size = 2.18

$$a^2x - \frac{2b^2}{d\left(e^{2c+2dx}-1\right)} - \frac{4\,\mathrm{atan}\left(\frac{a\,b\,e^{dx}\,e^{c}\sqrt{-d^2}}{d\sqrt{a^2b^2}}\right)\sqrt{a^2b^2}}{\sqrt{-d^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/sinh(c + d*x))^2,x)
```

```
[Out] a^2*x - (2*b^2)/(d*(exp(2*c + 2*d*x) - 1)) - (4*atan((a*b*exp(d*x)*exp(c)*(
-d^2)^(1/2))/(d*(a^2*b^2)^(1/2)))*(a^2*b^2)^(1/2))/(-d^2)^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b\operatorname{csch}(c + dx))^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*csch(d*x+c))**2,x)
```

```
[Out] Integral((a + b*csch(c + d*x))**2, x)
```

3.73 $\int (a + b\operatorname{csch}(c + dx))\,dx$

Optimal. Leaf size=17

$$ax - \frac{b\tanh^{-1}(\cosh(c + dx))}{d}$$

```
[Out] a*x-b*arctanh(cosh(d*x+c))/d
```

Rubi [A] time = 0.01, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {3770}

$$ax - \frac{b\tanh^{-1}(\cosh(c + dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[a + b*Csch[c + d*x],x]
```

```
[Out] a*x - (b*ArcTanh[Cosh[c + d*x]])/d
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int (a + b\operatorname{csch}(c + dx))\,dx &= ax + b\int \operatorname{csch}(c + dx)\,dx\\ &= ax - \frac{b\tanh^{-1}(\cosh(c + dx))}{d}\end{aligned}$$

Mathematica [B] time = 0.01, size = 43, normalized size = 2.53

$$ax + \frac{b\log\left(\sinh\left(\frac{c}{2} + \frac{dx}{2}\right)\right)}{d} - \frac{b\log\left(\cosh\left(\frac{c}{2} + \frac{dx}{2}\right)\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[a + b*Csch[c + d*x],x]
```

```
[Out] a*x - (b*Log[Cosh[c/2 + (d*x)/2]])/d + (b*Log[Sinh[c/2 + (d*x)/2]])/d
```

fricas [B] time = 0.41, size = 44, normalized size = 2.59

$$\frac{adx - b\log\left(\cosh\left(dx + c\right) + \sinh\left(dx + c\right) + 1\right) + b\log\left(\cosh\left(dx + c\right) + \sinh\left(dx + c\right) - 1\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*csch(d*x+c),x, algorithm="fricas")
```

```
[Out] (a*d*x - b*log(cosh(d*x + c) + sinh(d*x + c) + 1) + b*log(cosh(d*x + c) + s
inh(d*x + c) - 1))/d
```

giac [A] time = 0.11, size = 32, normalized size = 1.88

$$ax - \frac{b\left(\log\left(e^{(dx+c)} + 1\right) - \log\left(\left|e^{(dx+c)} - 1\right|\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*csch(d*x+c),x, algorithm="giac")
```

```
[Out] a*x - b*(log(e^(d*x + c) + 1) - log(abs(e^(d*x + c) - 1)))/d
```

maple [A] time = 0.02, size = 20, normalized size = 1.18

$$ax+\frac{b\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a+b*csch(d*x+c),x)
```

```
[Out] a*x+b/d*ln(tanh(1/2*d*x+1/2*c))
```

maxima [A] time = 0.31, size = 19, normalized size = 1.12

$$ax+\frac{b\log\left(\tanh\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*csch(d*x+c),x, algorithm="maxima")
```

```
[Out] a*x + b*log(tanh(1/2*d*x + 1/2*c))/d
```

mupad [B] time = 0.07, size = 42, normalized size = 2.47

$$a\,x-\frac{2\,\mathrm{atan}\left(\frac{b\,\mathrm{e}^{d\,x}\,\mathrm{e}^{c}\,\sqrt{-d^2}}{d\,\sqrt{b^2}}\right)\,\sqrt{b^2}}{\sqrt{-d^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a + b/sinh(c + d*x),x)
```

```
[Out] a*x - (2*atan((b*exp(d*x)*exp(c)*(-d^2)^(1/2))/(d*(b^2)^(1/2)))*(b^2)^(1/2)
)/(-d^2)^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int(a+b\operatorname{csch}(c+dx))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*csch(d*x+c),x)
```

```
[Out] Integral(a + b*csch(c + d*x), x)
```

3.74 $\int \frac{1}{a+b\,\mathbf{csch}(c+dx)}\,dx$

Optimal. Leaf size=54

$$\frac{2b\tanh^{-1}\left(\frac{a-b\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{ad\sqrt{a^2+b^2}}+\frac{x}{a}$$

```
[Out] x/a+2*b*arctanh((a-b*tanh(1/2*d*x+1/2*c))/(a^2+b^2)^(1/2))/a/d/(a^2+b^2)^(1
/2)
```

Rubi [A] time = 0.06, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3783, 2660, 618, 204}

$$\frac{2b\tanh^{-1}\left(\frac{a-b\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{ad\sqrt{a^2+b^2}}+\frac{x}{a}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Csch[c + d*x])^(-1),x]
```

```
[Out] x/a + (2*b*ArcTanh[(a - b*Tanh[(c + d*x)/2])/Sqrt[a^2 + b^2]])/(a*Sqrt[a^2
+ b^2]*d)
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 3783

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(-1), x_Symbol] :> Simp[x/a, x]
- Dist[1/a, Int[1/(1 + (a*Sin[c + d*x])/b), x], x] /; FreeQ[{a, b, c, d}, x
] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{1}{a+b\operatorname{csch}(c+dx)}\,dx = \frac{x}{a} - \frac{\int \frac{1}{1+\frac{a\sinh(c+dx)}{b}}\,dx}{a}$$

$$= \frac{x}{a} + \frac{(2i)\,\text{Subst}\left(\int \frac{1}{1-\frac{2iax}{b}+x^2}\,dx, x, \tan\left(\frac{1}{2}(ic+idx)\right)\right)}{ad}$$

$$= \frac{x}{a} - \frac{(4i)\,\text{Subst}\left(\int \frac{1}{-4\left(1+\frac{a^2}{b^2}\right)-x^2}\,dx, x, -\frac{2ia}{b}+2\tan\left(\frac{1}{2}(ic+idx)\right)\right)}{ad}$$

$$= \frac{x}{a} + \frac{2b\tanh^{-1}\left(\frac{b\left(\frac{a}{b}-\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}\,d}$$

Mathematica [A] time = 0.11, size = 64, normalized size = 1.19

$$\frac{-\frac{2b\tan^{-1}\left(\frac{a-b\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a^2-b^2}}\right)}{d\sqrt{-a^2-b^2}}+\frac{c}{d}+x}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Csch[c + d*x])^(-1),x]
```

```
[Out] (c/d + x - (2*b*ArcTan[(a - b*Tanh[(c + d*x)/2])/Sqrt[-a^2 - b^2]])/(Sqrt[-
a^2 - b^2]*d))/a
```

fricas [B] time = 0.41, size = 186, normalized size = 3.44

$$\frac{\left(a^2+b^2\right)dx+\sqrt{a^2+b^2}\,b\log\left(\frac{a^2\cosh(dx+c)^2+a^2\sinh(dx+c)^2+2ab\cosh(dx+c)+a^2+2b^2+2\left(a^2\cosh(dx+c)+ab\right)\sinh(dx+c)+2\sqrt{a^2+b^2}\,(a\cosh\cdots}{a\cosh(dx+c)^2+a\sinh(dx+c)^2+2b\cosh(dx+c)+2\left(a\cosh(dx+c)+b\right)\sinh(dx+c)-a\cdots}\right.}{\left(a^3+ab^2\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*csch(d*x+c)),x, algorithm="fricas")
```

```
[Out] ((a^2 + b^2)*d*x + sqrt(a^2 + b^2)*b*log((a^2*cosh(d*x + c)^2 + a^2*sinh(d*
x + c)^2 + 2*a*b*cosh(d*x + c) + a^2 + 2*b^2 + 2*(a^2*cosh(d*x + c) + a*b)*
sinh(d*x + c) + 2*sqrt(a^2 + b^2)*(a*cosh(d*x + c) + a*sinh(d*x + c) + b))/
(a*cosh(d*x + c)^2 + a*sinh(d*x + c)^2 + 2*b*cosh(d*x + c) + 2*(a*cosh(d*x
+ c) + b)*sinh(d*x + c) - a)))/((a^3 + a*b^2)*d)
```

giac [A] time = 0.14, size = 84, normalized size = 1.56

$$-\frac{\frac{b\log\left(\frac{\left|2ae^{(dx+c)}+2b-2\sqrt{a^2+b^2}\right|}{\left|2ae^{(dx+c)}+2b+2\sqrt{a^2+b^2}\right|}\right)}{\sqrt{a^2+b^2}\,a}-\frac{dx+c}{a}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*csch(d*x+c)),x, algorithm="giac")
```

```
[Out] -(b*log(abs(2*a*e^(d*x + c) + 2*b - 2*sqrt(a^2 + b^2))/abs(2*a*e^(d*x + c)
+ 2*b + 2*sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*a) - (d*x + c)/a)/d
```

maple [A] time = 0.20, size = 87, normalized size = 1.61

$$-\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{da}+\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{da}-\frac{2b\,\mathrm{arctanh}\left(\frac{2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)b-2a}{2\sqrt{a^2+b^2}}\right)}{da\sqrt{a^2+b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*csch(d*x+c)),x)
```

```
[Out] -1/d/a*ln(tanh(1/2*d*x+1/2*c)-1)+1/d/a*ln(tanh(1/2*d*x+1/2*c)+1)-2/d/a*b/(a
^2+b^2)^(1/2)*arctanh(1/2*(2*tanh(1/2*d*x+1/2*c)*b-2*a)/(a^2+b^2)^(1/2))
```

maxima [A] time = 0.41, size = 85, normalized size = 1.57

$$-\frac{b\log\left(\frac{ae^{(-dx-c)}-b-\sqrt{a^2+b^2}}{ae^{(-dx-c)}-b+\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}\,ad}+\frac{dx+c}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*csch(d*x+c)),x, algorithm="maxima")
```

```
[Out] -b*log((a*e^(-d*x - c) - b - sqrt(a^2 + b^2))/(a*e^(-d*x - c) - b + sqrt(a^
2 + b^2)))/(sqrt(a^2 + b^2)*a*d) + (d*x + c)/(a*d)
```

mupad [B] time = 0.33, size = 121, normalized size = 2.24

$$\frac{x}{a}-\frac{b\,\ln\left(\frac{2\,b\,\mathrm{e}^{c+d\,x}}{a^2}-\frac{2\,b\,\left(a-b\,\mathrm{e}^{c+d\,x}\right)}{a^2\,\sqrt{a^2+b^2}}\right)}{a\,d\,\sqrt{a^2+b^2}}+\frac{b\,\ln\left(\frac{2\,b\,\mathrm{e}^{c+d\,x}}{a^2}+\frac{2\,b\,\left(a-b\,\mathrm{e}^{c+d\,x}\right)}{a^2\,\sqrt{a^2+b^2}}\right)}{a\,d\,\sqrt{a^2+b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b/sinh(c + d*x)),x)
```

```
[Out] x/a - (b*log((2*b*exp(c + d*x))/a^2 - (2*b*(a - b*exp(c + d*x)))/(a^2*(a^2
+ b^2)^(1/2))))/(a*d*(a^2 + b^2)^(1/2)) + (b*log((2*b*exp(c + d*x))/a^2 + (
2*b*(a - b*exp(c + d*x)))/(a^2*(a^2 + b^2)^(1/2))))/(a*d*(a^2 + b^2)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{a+b\,\mathrm{csch}\,(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*csch(d*x+c)),x)
```

```
[Out] Integral(1/(a + b*csch(c + d*x)), x)
```

3.75 $\int \frac{1}{(a+b\,\mathbf{csch}(c+dx))^2}\,dx$

Optimal. Leaf size=101

$$\frac{2b\left(2a^2+b^2\right)\tanh^{-1}\left(\frac{a-b\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{a^2d\left(a^2+b^2\right)^{3/2}}-\frac{b^2\coth(c+dx)}{ad\left(a^2+b^2\right)(a+b\operatorname{csch}(c+dx))}+\frac{x}{a^2}$$

```
[Out] x/a^2+2*b*(2*a^2+b^2)*arctanh((a-b*tanh(1/2*d*x+1/2*c))/(a^2+b^2)^(1/2))/a^
2/(a^2+b^2)^(3/2)/d-b^2*coth(d*x+c)/a/(a^2+b^2)/d/(a+b*csch(d*x+c))
```

Rubi [A] time = 0.16, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3785, 3919, 3831, 2660, 618, 204}

$$\frac{2b\left(2a^2+b^2\right)\tanh^{-1}\left(\frac{a-b\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{a^2d\left(a^2+b^2\right)^{3/2}}-\frac{b^2\coth(c+dx)}{ad\left(a^2+b^2\right)(a+b\operatorname{csch}(c+dx))}+\frac{x}{a^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Csch[c + d*x])^(-2),x]
```

```
[Out] x/a^2 + (2*b*(2*a^2 + b^2)*ArcTanh[(a - b*Tanh[(c + d*x)/2])/Sqrt[a^2 + b^2
]])/(a^2*(a^2 + b^2)^(3/2)*d) - (b^2*Coth[c + d*x])/(a*(a^2 + b^2)*d*(a + b
*Csch[c + d*x]))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 3785

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[(b^2*Cot
[c + d*x]*(a + b*Csc[c + d*x])^(n + 1))/(a*d*(n + 1)*(a^2 - b^2)), x] + Dis
t[1/(a*(n + 1)*(a^2 - b^2)), Int[(a + b*Csc[c + d*x])^(n + 1)*Simp[(a^2 - b
^2)*(n + 1) - a*b*(n + 1)*Csc[c + d*x] + b^2*(n + 2)*Csc[c + d*x]^2, x], x]
, x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && LtQ[n, -1] && Intege
rQ[2*n]
```

Rule 3831

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3919

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[(c*x)/a, x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(a+b\operatorname{csch}(c+dx))^2}\,dx &= -\frac{b^2\coth(c+dx)}{a\left(a^2+b^2\right)d(a+b\operatorname{csch}(c+dx))}-\frac{\int \frac{-a^2-b^2+ab\operatorname{csch}(c+dx)}{a+b\operatorname{csch}(c+dx)}\,dx}{a\left(a^2+b^2\right)}\\
&= \frac{x}{a^2}-\frac{b^2\coth(c+dx)}{a\left(a^2+b^2\right)d(a+b\operatorname{csch}(c+dx))}-\frac{\left(b\left(2a^2+b^2\right)\right)\int \frac{\operatorname{csch}(c+dx)}{a+b\operatorname{csch}(c+dx)}\,dx}{a^2\left(a^2+b^2\right)}\\
&= \frac{x}{a^2}-\frac{b^2\coth(c+dx)}{a\left(a^2+b^2\right)d(a+b\operatorname{csch}(c+dx))}-\frac{\left(2a^2+b^2\right)\int \frac{1}{1+\frac{a\sinh(c+dx)}{b}}\,dx}{a^2\left(a^2+b^2\right)}\\
&= \frac{x}{a^2}-\frac{b^2\coth(c+dx)}{a\left(a^2+b^2\right)d(a+b\operatorname{csch}(c+dx))}+\frac{\left(2i\left(2a^2+b^2\right)\right)\operatorname{Subst}\left(\int \frac{1}{1-\frac{2iax}{b}+x^2}\,dx,x,\right.}{a^2\left(a^2+b^2\right)d}\\
&= \frac{x}{a^2}-\frac{b^2\coth(c+dx)}{a\left(a^2+b^2\right)d(a+b\operatorname{csch}(c+dx))}-\frac{\left(4i\left(2a^2+b^2\right)\right)\operatorname{Subst}\left(\int \frac{1}{-4\left(1+\frac{a^2}{b^2}\right)-x^2}\,dx,\right.}{a^2\left(a^2+b^2\right)}\\
&= \frac{x}{a^2}+\frac{2b\left(2a^2+b^2\right)\tanh^{-1}\left(\frac{b\left(\frac{a}{b}-\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{\sqrt{a^2+b^2}}\right)}{a^2\left(a^2+b^2\right)^{3/2}d}-\frac{b^2\coth(c+dx)}{a\left(a^2+b^2\right)d(a+b\operatorname{csch}(c+dx))}\end{aligned}$$

Mathematica [A] time = 0.39, size = 142, normalized size = 1.41

$$\frac{\operatorname{csch}(c+dx)(a\sinh(c+dx)+b)\left(-\frac{ab^2\coth(c+dx)}{a^2+b^2}+\frac{2b\left(2a^2+b^2\right)(a+b\operatorname{csch}(c+dx))\tan^{-1}\left(\frac{a-b\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a^2-b^2}}\right)}{\left(-a^2-b^2\right)^{3/2}}+(c+dx)(a+b\operatorname{cs}\right.}{a^2d(a+b\operatorname{csch}(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Csch[c + d*x])^(-2),x]
```

```
[Out] (Csch[c + d*x]*(-((a*b^2*Coth[c + d*x])/(a^2 + b^2)) + (c + d*x)*(a + b*Csc
h[c + d*x]) + (2*b*(2*a^2 + b^2)*ArcTan[(a - b*Tanh[(c + d*x)/2])/Sqrt[-a^2
 - b^2]]*(a + b*Csch[c + d*x]))/(-a^2 - b^2)^(3/2))*(b + a*Sinh[c + d*x]))/
(a^2*d*(a + b*Csch[c + d*x])^2)
```

fricas [B] time = 0.42, size = 645, normalized size = 6.39

$$-\frac{2\,a^3b^2+2\,ab^4-\left(a^5+2\,a^3b^2+ab^4\right)dx\cosh\left(dx+c\right)^2-\left(a^5+2\,a^3b^2+ab^4\right)dx\sinh\left(dx+c\right)^2+\left(a^5+2\,a^3b^2+al\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*csch(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -(2*a^3*b^2 + 2*a*b^4 - (a^5 + 2*a^3*b^2 + a*b^4)*d*x*cosh(d*x + c)^2 - (a^
5 + 2*a^3*b^2 + a*b^4)*d*x*sinh(d*x + c)^2 + (a^5 + 2*a^3*b^2 + a*b^4)*d*x
+ (2*a^3*b + a*b^3 - (2*a^3*b + a*b^3)*cosh(d*x + c)^2 - (2*a^3*b + a*b^3)*
sinh(d*x + c)^2 - 2*(2*a^2*b^2 + b^4)*cosh(d*x + c) - 2*(2*a^2*b^2 + b^4 +
(2*a^3*b + a*b^3)*cosh(d*x + c))*sinh(d*x + c))*sqrt(a^2 + b^2)*log((a^2*co
sh(d*x + c)^2 + a^2*sinh(d*x + c)^2 + 2*a*b*cosh(d*x + c) + a^2 + 2*b^2 + 2
*(a^2*cosh(d*x + c) + a*b)*sinh(d*x + c) + 2*sqrt(a^2 + b^2)*(a*cosh(d*x +
c) + a*sinh(d*x + c) + b))/(a*cosh(d*x + c)^2 + a*sinh(d*x + c)^2 + 2*b*cos
h(d*x + c) + 2*(a*cosh(d*x + c) + b)*sinh(d*x + c) - a)) - 2*(a^2*b^3 + b^5
 + (a^4*b + 2*a^2*b^3 + b^5)*d*x)*cosh(d*x + c) - 2*(a^2*b^3 + b^5 + (a^5 +
 2*a^3*b^2 + a*b^4)*d*x*cosh(d*x + c) + (a^4*b + 2*a^2*b^3 + b^5)*d*x)*sinh
(d*x + c))/((a^7 + 2*a^5*b^2 + a^3*b^4)*d*cosh(d*x + c)^2 + (a^7 + 2*a^5*b^
2 + a^3*b^4)*d*sinh(d*x + c)^2 + 2*(a^6*b + 2*a^4*b^3 + a^2*b^5)*d*cosh(d*x
 + c) - (a^7 + 2*a^5*b^2 + a^3*b^4)*d + 2*((a^7 + 2*a^5*b^2 + a^3*b^4)*d*co
sh(d*x + c) + (a^6*b + 2*a^4*b^3 + a^2*b^5)*d)*sinh(d*x + c))
```

giac [A] time = 0.15, size = 161, normalized size = 1.59

$$-\frac{\frac{\left(2\,a^2b+b^3\right)\log\left(\frac{\left|2\,ae^{(dx+c)}+2\,b-2\,\sqrt{a^2+b^2}\right|}{\left|2\,ae^{(dx+c)}+2\,b+2\,\sqrt{a^2+b^2}\right|}\right)}{\left(a^4+a^2b^2\right)\sqrt{a^2+b^2}}-\frac{2\left(b^3e^{(dx+c)}-ab^2\right)}{\left(a^4+a^2b^2\right)\left(ae^{(2\,dx+2\,c)}+2\,be^{(dx+c)}-a\right)}-\frac{dx+c}{a^2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*csch(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -((2*a^2*b + b^3)*log(abs(2*a*e^(d*x + c) + 2*b - 2*sqrt(a^2 + b^2))/abs(2*
a*e^(d*x + c) + 2*b + 2*sqrt(a^2 + b^2)))/((a^4 + a^2*b^2)*sqrt(a^2 + b^2))
 - 2*(b^3*e^(d*x + c) - a*b^2)/((a^4 + a^2*b^2)*(a*e^(2*d*x + 2*c) + 2*b*e^
(d*x + c) - a)) - (d*x + c)/a^2)/d
```

maple [B] time = 0.32, size = 238, normalized size = 2.36

$$-\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{d\,a^2}+\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{d\,a^2}+\frac{2b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{d\left(\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b-2a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-b\right)\left(a^2+b^2\right)}+\frac{}{da\left(\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*csch(d*x+c))^2,x)
```

```
[Out] -1/d/a^2*ln(tanh(1/2*d*x+1/2*c)-1)+1/d/a^2*ln(tanh(1/2*d*x+1/2*c)+1)+2/d*b/
(tanh(1/2*d*x+1/2*c)^2*b-2*a*tanh(1/2*d*x+1/2*c)-b)/(a^2+b^2)*tanh(1/2*d*x+
1/2*c)+2/d/a*b^2/(tanh(1/2*d*x+1/2*c)^2*b-2*a*tanh(1/2*d*x+1/2*c)-b)/(a^2+b
^2)-4/d*b/(a^2+b^2)^(3/2)*arctanh(1/2*(2*tanh(1/2*d*x+1/2*c)*b-2*a)/(a^2+b^
2)^(1/2))-2/d/a^2*b^3/(a^2+b^2)^(3/2)*arctanh(1/2*(2*tanh(1/2*d*x+1/2*c)*b-
2*a)/(a^2+b^2)^(1/2))
```

maxima [A] time = 0.41, size = 187, normalized size = 1.85

$$-\frac{\left(2\,a^2b+b^3\right)\log\left(\frac{ae^{(-dx-c)}-b-\sqrt{a^2+b^2}}{ae^{(-dx-c)}-b+\sqrt{a^2+b^2}}\right)}{\left(a^4+a^2b^2\right)\sqrt{a^2+b^2}\,d}-\frac{2\left(b^3e^{(-dx-c)}+ab^2\right)}{\left(a^5+a^3b^2+2\left(a^4b+a^2b^3\right)e^{(-dx-c)}-\left(a^5+a^3b^2\right)e^{(-2\,dx-2\,c)}\right)d}+\frac{dx+c}{a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*csch(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -(2*a^2*b + b^3)*log((a*e^(-d*x - c) - b - sqrt(a^2 + b^2))/(a*e^(-d*x - c)
 - b + sqrt(a^2 + b^2)))/((a^4 + a^2*b^2)*sqrt(a^2 + b^2)*d) - 2*(b^3*e^(-d
*x - c) + a*b^2)/((a^5 + a^3*b^2 + 2*(a^4*b + a^2*b^3)*e^(-d*x - c) - (a^5
+ a^3*b^2)*e^(-2*d*x - 2*c))*d) + (d*x + c)/(a^2*d)
```

mupad [B] time = 1.95, size = 269, normalized size = 2.66

$$\frac{x}{a^2}-\frac{\frac{2\,b^2}{d\left(a^3+a\,b^2\right)}-\frac{2\,b^3\,e^{c+d\,x}}{a\,d\left(a^3+a\,b^2\right)}}{2\,b\,e^{c+d\,x}-a+a\,e^{2\,c+2\,d\,x}}-\frac{b\,\ln\left(\frac{2\,e^{c+d\,x}\left(2\,a^2\,b+b^3\right)}{a^3\left(a^2+b^2\right)}-\frac{2\,b\left(2\,a^2+b^2\right)\left(a-b\,e^{c+d\,x}\right)}{a^3\left(a^2+b^2\right)^{3/2}}\right)\left(2\,a^2+b^2\right)}{a^2\,d\left(a^2+b^2\right)^{3/2}}+\frac{b\,\ln\left(\frac{2\,e^{c+d\,x}\left(2\,a^2\,b+b^3\right)}{a^3\left(a^2+b^2\right)}\right.}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b/sinh(c + d*x))^2,x)
```

```
[Out] x/a^2 - ((2*b^2)/(d*(a*b^2 + a^3)) - (2*b^3*exp(c + d*x))/(a*d*(a*b^2 + a^3
)))/(2*b*exp(c + d*x) - a + a*exp(2*c + 2*d*x)) - (b*log((2*exp(c + d*x)*(2
*a^2*b + b^3))/(a^3*(a^2 + b^2)) - (2*b*(2*a^2 + b^2)*(a - b*exp(c + d*x)))
/(a^3*(a^2 + b^2)^(3/2)))*(2*a^2 + b^2))/(a^2*d*(a^2 + b^2)^(3/2)) + (b*log
((2*exp(c + d*x)*(2*a^2*b + b^3))/(a^3*(a^2 + b^2)) + (2*b*(2*a^2 + b^2)*(a
 - b*exp(c + d*x)))/(a^3*(a^2 + b^2)^(3/2)))*(2*a^2 + b^2))/(a^2*d*(a^2 + b
^2)^(3/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\left(a+b\operatorname{csch}\left(c+dx\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*csch(d*x+c))**2,x)
```

```
[Out] Integral((a + b*csch(c + d*x))**(-2), x)
```

3.76 $\int \frac{1}{(a+b\mathbf{csch}(c+dx))^3} dx$

Optimal. Leaf size=163

$$\frac{x}{a^3}-\frac{b^2\left(5a^2+2b^2\right)\coth(c+dx)}{2a^2d\left(a^2+b^2\right)^2(a+b\text{csch}(c+dx))}-\frac{b^2\coth(c+dx)}{2ad\left(a^2+b^2\right)(a+b\text{csch}(c+dx))^2}+\frac{b\left(6a^4+5a^2b^2+2b^4\right)\tanh^{-1}\left(\frac{a-b\,\text{ta}}{}\right.}{a^3d\left(a^2+b^2\right)^{5/2}}$$

```
[Out] x/a^3+b*(6*a^4+5*a^2*b^2+2*b^4)*arctanh((a-b*tanh(1/2*d*x+1/2*c))/(a^2+b^2)
^(1/2))/a^3/(a^2+b^2)^(5/2)/d-1/2*b^2*coth(d*x+c)/a/(a^2+b^2)/d/(a+b*csch(d
*x+c))^2-1/2*b^2*(5*a^2+2*b^2)*coth(d*x+c)/a^2/(a^2+b^2)^2/d/(a+b*csch(d*x+
c))
```

Rubi [A] time = 0.32, antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.583, Rules used = {3785, 4060, 3919, 3831, 2660, 618, 204}

$$\frac{b\left(5a^2b^2+6a^4+2b^4\right)\tanh^{-1}\left(\frac{a-b\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{a^3d\left(a^2+b^2\right)^{5/2}}-\frac{b^2\left(5a^2+2b^2\right)\coth(c+dx)}{2a^2d\left(a^2+b^2\right)^2(a+b\text{csch}(c+dx))}-\frac{b^2\coth(c+dx)}{2ad\left(a^2+b^2\right)(a+b\text{csch}(c}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Csch[c + d*x])^(-3),x]
```

```
[Out] x/a^3 + (b*(6*a^4 + 5*a^2*b^2 + 2*b^4)*ArcTanh[(a - b*Tanh[(c + d*x)/2])/Sq
rt[a^2 + b^2]])/(a^3*(a^2 + b^2)^(5/2)*d) - (b^2*Coth[c + d*x])/(2*a*(a^2 +
 b^2)*d*(a + b*Csch[c + d*x])^2) - (b^2*(5*a^2 + 2*b^2)*Coth[c + d*x])/(2*a
^2*(a^2 + b^2)^2*d*(a + b*Csch[c + d*x]))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 3785

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[(b^2*Cot
[c + d*x]*(a + b*Csc[c + d*x])^(n + 1))/(a*d*(n + 1)*(a^2 - b^2)), x] + Dis
t[1/(a*(n + 1)*(a^2 - b^2)), Int[(a + b*Csc[c + d*x])^(n + 1)*Simp[(a^2 - b
^2)*(n + 1) - a*b*(n + 1)*Csc[c + d*x] + b^2*(n + 2)*Csc[c + d*x]^2, x], x]
, x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && LtQ[n, -1] && Intege
rQ[2*n]
```

Rule 3831

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3919

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[(c*x)/a, x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 4060

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> Simp[((A*b^2 -
 a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(a*f*(m + 1)*(a^
2 - b^2)), x] + Dist[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m
 + 1)*Simp[A*(a^2 - b^2)*(m + 1) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x]
 + (A*b^2 - a*b*B + a^2*C)*(m + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
b, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{1}{(a+b\text{csch}(c+dx))^3}\,dx = -\frac{b^2\coth(c+dx)}{2a\left(a^2+b^2\right)d(a+b\text{csch}(c+dx))^2} - \frac{\int \frac{-2\left(a^2+b^2\right)+2ab\text{csch}(c+dx)-b^2\text{csch}^2(c+dx)}{(a+b\text{csch}(c+dx))^2}\,dx}{2a\left(a^2+b^2\right)}$$

$$= -\frac{b^2\coth(c+dx)}{2a\left(a^2+b^2\right)d(a+b\text{csch}(c+dx))^2} - \frac{b^2\left(5a^2+2b^2\right)\coth(c+dx)}{2a^2\left(a^2+b^2\right)^2 d(a+b\text{csch}(c+dx))} + \int$$

$$= \frac{x}{a^3} - \frac{b^2\coth(c+dx)}{2a\left(a^2+b^2\right)d(a+b\text{csch}(c+dx))^2} - \frac{b^2\left(5a^2+2b^2\right)\coth(c+dx)}{2a^2\left(a^2+b^2\right)^2 d(a+b\text{csch}(c+dx))} -$$

$$= \frac{x}{a^3} - \frac{b^2\coth(c+dx)}{2a\left(a^2+b^2\right)d(a+b\text{csch}(c+dx))^2} - \frac{b^2\left(5a^2+2b^2\right)\coth(c+dx)}{2a^2\left(a^2+b^2\right)^2 d(a+b\text{csch}(c+dx))} -$$

$$= \frac{x}{a^3} - \frac{b^2\coth(c+dx)}{2a\left(a^2+b^2\right)d(a+b\text{csch}(c+dx))^2} - \frac{b^2\left(5a^2+2b^2\right)\coth(c+dx)}{2a^2\left(a^2+b^2\right)^2 d(a+b\text{csch}(c+dx))} +$$

$$= \frac{x}{a^3} - \frac{b^2\coth(c+dx)}{2a\left(a^2+b^2\right)d(a+b\text{csch}(c+dx))^2} - \frac{b^2\left(5a^2+2b^2\right)\coth(c+dx)}{2a^2\left(a^2+b^2\right)^2 d(a+b\text{csch}(c+dx))} -$$

$$= \frac{x}{a^3} + \frac{b\left(6a^4+5a^2b^2+2b^4\right)\tanh^{-1}\left(\frac{b\left(\frac{a}{b}-\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{\sqrt{a^2+b^2}}\right)}{a^3\left(a^2+b^2\right)^{5/2} d} - \frac{b^2\coth(c+d}{2a\left(a^2+b^2\right)d(a+b\text{csc}}$$

Mathematica [A] time = 0.93, size = 213, normalized size = 1.31

$$\frac{\operatorname{csch}^2(c+dx)(a\sinh(c+dx)+b)\left(-\frac{3ab^2\left(2a^2+b^2\right)\coth(c+dx)(a\sinh(c+dx)+b)}{\left(a^2+b^2\right)^2}+\frac{ab^3\coth(c+dx)}{a^2+b^2}-\frac{2b\left(6a^4+5a^2b^2+2b^4\right)\operatorname{csch}(c+dx)(a\,\mathrm{s}\ldots}{(-a\ldots}\right.}{2a^3d(a+b\operatorname{csch}(c+dx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Csch[c + d*x])^(-3),x]
```

```
[Out] (Csch[c + d*x]^2*(b + a*Sinh[c + d*x])*((a*b^3*Coth[c + d*x])/(a^2 + b^2) -
 (3*a*b^2*(2*a^2 + b^2)*Coth[c + d*x]*(b + a*Sinh[c + d*x]))/(a^2 + b^2)^2
+ 2*(c + d*x)*Csch[c + d*x]*(b + a*Sinh[c + d*x])^2 - (2*b*(6*a^4 + 5*a^2*b
^2 + 2*b^4)*ArcTan[(a - b*Tanh[(c + d*x)/2])/Sqrt[-a^2 - b^2]]*Csch[c + d*x
]*(b + a*Sinh[c + d*x])^2)/(-a^2 - b^2)^(5/2)))/(2*a^3*d*(a + b*Csch[c + d*
x])^3)
```

fricas [B] time = 0.46, size = 2094, normalized size = 12.85

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*csch(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/2*(12*a^6*b^2 + 18*a^4*b^4 + 6*a^2*b^6 + 2*(a^8 + 3*a^6*b^2 + 3*a^4*b^4 +
 a^2*b^6)*d*x*cosh(d*x + c)^4 + 2*(a^8 + 3*a^6*b^2 + 3*a^4*b^4 + a^2*b^6)*d
*x*sinh(d*x + c)^4 + 2*(7*a^5*b^3 + 11*a^3*b^5 + 4*a*b^7 + 4*(a^7*b + 3*a^5
*b^3 + 3*a^3*b^5 + a*b^7)*d*x)*cosh(d*x + c)^3 + 2*(7*a^5*b^3 + 11*a^3*b^5
+ 4*a*b^7 + 4*(a^8 + 3*a^6*b^2 + 3*a^4*b^4 + a^2*b^6)*d*x*cosh(d*x + c) + 4
*(a^7*b + 3*a^5*b^3 + 3*a^3*b^5 + a*b^7)*d*x)*sinh(d*x + c)^3 + 2*(a^8 + 3*
a^6*b^2 + 3*a^4*b^4 + a^2*b^6)*d*x - 2*(6*a^6*b^2 - 3*a^4*b^4 - 15*a^2*b^6
- 6*b^8 + 2*(a^8 + a^6*b^2 - 3*a^4*b^4 - 5*a^2*b^6 - 2*b^8)*d*x)*cosh(d*x +
 c)^2 - 2*(6*a^6*b^2 - 3*a^4*b^4 - 15*a^2*b^6 - 6*b^8 - 6*(a^8 + 3*a^6*b^2
+ 3*a^4*b^4 + a^2*b^6)*d*x*cosh(d*x + c)^2 + 2*(a^8 + a^6*b^2 - 3*a^4*b^4 -
 5*a^2*b^6 - 2*b^8)*d*x - 3*(7*a^5*b^3 + 11*a^3*b^5 + 4*a*b^7 + 4*(a^7*b +
3*a^5*b^3 + 3*a^3*b^5 + a*b^7)*d*x)*cosh(d*x + c))*sinh(d*x + c)^2 + (6*a^6
*b + 5*a^4*b^3 + 2*a^2*b^5 + (6*a^6*b + 5*a^4*b^3 + 2*a^2*b^5)*cosh(d*x + c
)^4 + (6*a^6*b + 5*a^4*b^3 + 2*a^2*b^5)*sinh(d*x + c)^4 + 4*(6*a^5*b^2 + 5*
a^3*b^4 + 2*a*b^6)*cosh(d*x + c)^3 + 4*(6*a^5*b^2 + 5*a^3*b^4 + 2*a*b^6 + (
6*a^6*b + 5*a^4*b^3 + 2*a^2*b^5)*cosh(d*x + c))*sinh(d*x + c)^3 - 2*(6*a^6*
b - 7*a^4*b^3 - 8*a^2*b^5 - 4*b^7)*cosh(d*x + c)^2 - 2*(6*a^6*b - 7*a^4*b^3
 - 8*a^2*b^5 - 4*b^7 - 3*(6*a^6*b + 5*a^4*b^3 + 2*a^2*b^5)*cosh(d*x + c)^2
- 6*(6*a^5*b^2 + 5*a^3*b^4 + 2*a*b^6)*cosh(d*x + c))*sinh(d*x + c)^2 - 4*(6
*a^5*b^2 + 5*a^3*b^4 + 2*a*b^6)*cosh(d*x + c) - 4*(6*a^5*b^2 + 5*a^3*b^4 +
2*a*b^6 - (6*a^6*b + 5*a^4*b^3 + 2*a^2*b^5)*cosh(d*x + c)^3 - 3*(6*a^5*b^2
+ 5*a^3*b^4 + 2*a*b^6)*cosh(d*x + c)^2 + (6*a^6*b - 7*a^4*b^3 - 8*a^2*b^5 -
 4*b^7)*cosh(d*x + c))*sinh(d*x + c))*sqrt(a^2 + b^2)*log((a^2*cosh(d*x + c
)^2 + a^2*sinh(d*x + c)^2 + 2*a*b*cosh(d*x + c) + a^2 + 2*b^2 + 2*(a^2*cosh
(d*x + c) + a*b)*sinh(d*x + c) + 2*sqrt(a^2 + b^2)*(a*cosh(d*x + c) + a*sin
h(d*x + c) + b))/(a*cosh(d*x + c)^2 + a*sinh(d*x + c)^2 + 2*b*cosh(d*x + c)
 + 2*(a*cosh(d*x + c) + b)*sinh(d*x + c) - a)) - 2*(17*a^5*b^3 + 25*a^3*b^5
 + 8*a*b^7 + 4*(a^7*b + 3*a^5*b^3 + 3*a^3*b^5 + a*b^7)*d*x)*cosh(d*x + c) -
 2*(17*a^5*b^3 + 25*a^3*b^5 + 8*a*b^7 - 4*(a^8 + 3*a^6*b^2 + 3*a^4*b^4 + a^
2*b^6)*d*x*cosh(d*x + c)^3 + 4*(a^7*b + 3*a^5*b^3 + 3*a^3*b^5 + a*b^7)*d*x
- 3*(7*a^5*b^3 + 11*a^3*b^5 + 4*a*b^7 + 4*(a^7*b + 3*a^5*b^3 + 3*a^3*b^5 +
a*b^7)*d*x)*cosh(d*x + c)^2 + 2*(6*a^6*b^2 - 3*a^4*b^4 - 15*a^2*b^6 - 6*b^8
 + 2*(a^8 + a^6*b^2 - 3*a^4*b^4 - 5*a^2*b^6 - 2*b^8)*d*x)*cosh(d*x + c))*si
nh(d*x + c))/((a^11 + 3*a^9*b^2 + 3*a^7*b^4 + a^5*b^6)*d*cosh(d*x + c)^4 +
```

```
(a^11 + 3*a^9*b^2 + 3*a^7*b^4 + a^5*b^6)*d*sinh(d*x + c)^4 + 4*(a^10*b + 3*
a^8*b^3 + 3*a^6*b^5 + a^4*b^7)*d*cosh(d*x + c)^3 - 2*(a^11 + a^9*b^2 - 3*a^
7*b^4 - 5*a^5*b^6 - 2*a^3*b^8)*d*cosh(d*x + c)^2 + 4*((a^11 + 3*a^9*b^2 + 3
*a^7*b^4 + a^5*b^6)*d*cosh(d*x + c) + (a^10*b + 3*a^8*b^3 + 3*a^6*b^5 + a^4
*b^7)*d)*sinh(d*x + c)^3 - 4*(a^10*b + 3*a^8*b^3 + 3*a^6*b^5 + a^4*b^7)*d*c
osh(d*x + c) + 2*(3*(a^11 + 3*a^9*b^2 + 3*a^7*b^4 + a^5*b^6)*d*cosh(d*x + c
)^2 + 6*(a^10*b + 3*a^8*b^3 + 3*a^6*b^5 + a^4*b^7)*d*cosh(d*x + c) - (a^11
+ a^9*b^2 - 3*a^7*b^4 - 5*a^5*b^6 - 2*a^3*b^8)*d)*sinh(d*x + c)^2 + (a^11 +
 3*a^9*b^2 + 3*a^7*b^4 + a^5*b^6)*d + 4*((a^11 + 3*a^9*b^2 + 3*a^7*b^4 + a^
5*b^6)*d*cosh(d*x + c)^3 + 3*(a^10*b + 3*a^8*b^3 + 3*a^6*b^5 + a^4*b^7)*d*c
osh(d*x + c)^2 - (a^11 + a^9*b^2 - 3*a^7*b^4 - 5*a^5*b^6 - 2*a^3*b^8)*d*cos
h(d*x + c) - (a^10*b + 3*a^8*b^3 + 3*a^6*b^5 + a^4*b^7)*d)*sinh(d*x + c))
```

giac [**A**] time = 0.16, size = 293, normalized size = 1.80

$$-\frac{\frac{\left(6a^4b+5a^2b^3+2b^5\right)\log\left(\frac{\left|2ae^{(dx+c)}+2b-2\sqrt{a^2+b^2}\right|}{\left|2ae^{(dx+c)}+2b+2\sqrt{a^2+b^2}\right|}\right)}{\left(a^7+2a^5b^2+a^3b^4\right)\sqrt{a^2+b^2}}-\frac{2\left(7a^3b^3e^{(3dx+3c)}+4ab^5e^{(3dx+3c)}-6a^4b^2e^{(2dx+2c)}+9a^2b^4e^{(2dx+2c)}+6b^6e^{(2dx+2c)}-17a^3b\right.}{\left(a^7+2a^5b^2+a^3b^4\right)\left(ae^{(2dx+2c)}+2be^{(dx+c)}-a\right)^2}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*csch(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/2*((6*a^4*b + 5*a^2*b^3 + 2*b^5)*log(abs(2*a*e^(d*x + c) + 2*b - 2*sqrt(
a^2 + b^2))/abs(2*a*e^(d*x + c) + 2*b + 2*sqrt(a^2 + b^2)))/((a^7 + 2*a^5*b
^2 + a^3*b^4)*sqrt(a^2 + b^2)) - 2*(7*a^3*b^3*e^(3*d*x + 3*c) + 4*a*b^5*e^(
3*d*x + 3*c) - 6*a^4*b^2*e^(2*d*x + 2*c) + 9*a^2*b^4*e^(2*d*x + 2*c) + 6*b^
6*e^(2*d*x + 2*c) - 17*a^3*b^3*e^(d*x + c) - 8*a*b^5*e^(d*x + c) + 6*a^4*b^
2 + 3*a^2*b^4)/((a^7 + 2*a^5*b^2 + a^3*b^4)*(a*e^(2*d*x + 2*c) + 2*b*e^(d*x
 + c) - a)^2) - 2*(d*x + c)/a^3)/d
```

maple [**B**] time = 0.34, size = 822, normalized size = 5.04

$$-\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{d\,a^3}+\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{d\,a^3}+\frac{4a\,b^2\left(\tanh^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d\left(\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b-2a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-b\right)^2\left(a^4+2a^2b^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*csch(d*x+c))^3,x)
```

```
[Out] -1/d/a^3*ln(tanh(1/2*d*x+1/2*c)-1)+1/d/a^3*ln(tanh(1/2*d*x+1/2*c)+1)+4/d*a*
b^2/(tanh(1/2*d*x+1/2*c)^2*b-2*a*tanh(1/2*d*x+1/2*c)-b)^2/(a^4+2*a^2*b^2+b^
4)*tanh(1/2*d*x+1/2*c)^3+1/d/a*b^4/(tanh(1/2*d*x+1/2*c)^2*b-2*a*tanh(1/2*d*
x+1/2*c)-b)^2/(a^4+2*a^2*b^2+b^4)*tanh(1/2*d*x+1/2*c)^3-10/d*a^2*b/(tanh(1/
2*d*x+1/2*c)^2*b-2*a*tanh(1/2*d*x+1/2*c)-b)^2/(a^4+2*a^2*b^2+b^4)*tanh(1/2*
d*x+1/2*c)^2+1/d*b^3/(tanh(1/2*d*x+1/2*c)^2*b-2*a*tanh(1/2*d*x+1/2*c)-b)^2/
(a^4+2*a^2*b^2+b^4)*tanh(1/2*d*x+1/2*c)^2+2/d/a^2*b^5/(tanh(1/2*d*x+1/2*c)^
2*b-2*a*tanh(1/2*d*x+1/2*c)-b)^2/(a^4+2*a^2*b^2+b^4)*tanh(1/2*d*x+1/2*c)^2-
16/d*a*b^2/(tanh(1/2*d*x+1/2*c)^2*b-2*a*tanh(1/2*d*x+1/2*c)-b)^2/(a^4+2*a^2
*b^2+b^4)*tanh(1/2*d*x+1/2*c)-7/d/a*b^4/(tanh(1/2*d*x+1/2*c)^2*b-2*a*tanh(1
/2*d*x+1/2*c)-b)^2/(a^4+2*a^2*b^2+b^4)*tanh(1/2*d*x+1/2*c)-5/d*b^3/(tanh(1/
2*d*x+1/2*c)^2*b-2*a*tanh(1/2*d*x+1/2*c)-b)^2/(a^4+2*a^2*b^2+b^4)-2/d/a^2*b
^5/(tanh(1/2*d*x+1/2*c)^2*b-2*a*tanh(1/2*d*x+1/2*c)-b)^2/(a^4+2*a^2*b^2+b^4
)-6/d*a*b/(a^4+2*a^2*b^2+b^4)/(a^2+b^2)^(1/2)*arctanh(1/2*(2*tanh(1/2*d*x+1
/2*c)*b-2*a)/(a^2+b^2)^(1/2))-5/d/a*b^3/(a^4+2*a^2*b^2+b^4)/(a^2+b^2)^(1/2)
*arctanh(1/2*(2*tanh(1/2*d*x+1/2*c)*b-2*a)/(a^2+b^2)^(1/2))-2/d/a^3*b^5/(a^
4+2*a^2*b^2+b^4)/(a^2+b^2)^(1/2)*arctanh(1/2*(2*tanh(1/2*d*x+1/2*c)*b-2*a)/
(a^2+b^2)^(1/2))
```

maxima [B] time = 0.42, size = 373, normalized size = 2.29

$$-\frac{\left(6\,a^4b+5\,a^2b^3+2\,b^5\right)\log\left(\frac{ae^{(-dx-c)}-b-\sqrt{a^2+b^2}}{ae^{(-dx-c)}-b+\sqrt{a^2+b^2}}\right)}{2\left(a^7+2\,a^5b^2+a^3b^4\right)\sqrt{a^2+b^2}\,d}-\frac{6\,a^4b^2+3\,a^2b^4+\left(17\,a^3b^3+8\,ab^5\right)e^{(-d}}{\left(a^9+2\,a^7b^2+a^5b^4+4\left(a^8b+2\,a^6b^3+a^4b^5\right)e^{(-dx-c)}-2\left(a^9-3\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*csch(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/2*(6*a^4*b + 5*a^2*b^3 + 2*b^5)*log((a*e^(-d*x - c) - b - sqrt(a^2 + b^2
))/(a*e^(-d*x - c) - b + sqrt(a^2 + b^2)))/((a^7 + 2*a^5*b^2 + a^3*b^4)*sqr
t(a^2 + b^2)*d) - (6*a^4*b^2 + 3*a^2*b^4 + (17*a^3*b^3 + 8*a*b^5)*e^(-d*x -
 c) - 3*(2*a^4*b^2 - 3*a^2*b^4 - 2*b^6)*e^(-2*d*x - 2*c) - (7*a^3*b^3 + 4*a
*b^5)*e^(-3*d*x - 3*c))/((a^9 + 2*a^7*b^2 + a^5*b^4 + 4*(a^8*b + 2*a^6*b^3
+ a^4*b^5)*e^(-d*x - c) - 2*(a^9 - 3*a^5*b^4 - 2*a^3*b^6)*e^(-2*d*x - 2*c)
- 4*(a^8*b + 2*a^6*b^3 + a^4*b^5)*e^(-3*d*x - 3*c) + (a^9 + 2*a^7*b^2 + a^5
*b^4)*e^(-4*d*x - 4*c))*d) + (d*x + c)/(a^3*d)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(a+\frac{b}{\sinh(c+d\,x)}\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b/sinh(c + d*x))^3,x)
```

```
[Out] int(1/(a + b/sinh(c + d*x))^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+b\operatorname{csch}\left(c+dx\right)\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*csch(d*x+c))**3,x)
```

```
[Out] Integral((a + b*csch(c + d*x))**(-3), x)
```

3.77 $\int \frac{\sinh^3(x)}{a+b\,\mathbf{csch}(x)}\,dx$

Optimal. Leaf size=107

$$-\frac{b\sinh(x)\cosh(x)}{2a^2}+\frac{bx\left(a^2-2b^2\right)}{2a^4}-\frac{2b^4\tanh^{-1}\left(\frac{a-b\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{a^4\sqrt{a^2+b^2}}-\frac{\left(2a^2-3b^2\right)\cosh(x)}{3a^3}+\frac{\sinh^2(x)\cosh(x)}{3a}$$

```
[Out] 1/2*b*(a^2-2*b^2)*x/a^4-1/3*(2*a^2-3*b^2)*cosh(x)/a^3-1/2*b*cosh(x)*sinh(x)
/a^2+1/3*cosh(x)*sinh(x)^2/a-2*b^4*arctanh((a-b*tanh(1/2*x))/(a^2+b^2)^(1/2
))/a^4/(a^2+b^2)^(1/2)
```

Rubi [A] time = 0.46, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.538, Rules used = {3853, 4104, 3919, 3831, 2660, 618, 206}

$$\frac{bx\left(a^2-2b^2\right)}{2a^4}-\frac{\left(2a^2-3b^2\right)\cosh(x)}{3a^3}-\frac{2b^4\tanh^{-1}\left(\frac{a-b\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{a^4\sqrt{a^2+b^2}}-\frac{b\sinh(x)\cosh(x)}{2a^2}+\frac{\sinh^2(x)\cosh(x)}{3a}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[x]^3/(a + b*Csch[x]),x]
```

```
[Out] (b*(a^2 - 2*b^2)*x)/(2*a^4) - (2*b^4*ArcTanh[(a - b*Tanh[x/2])/Sqrt[a^2 + b
^2]])/(a^4*Sqrt[a^2 + b^2]) - ((2*a^2 - 3*b^2)*Cosh[x])/(3*a^3) - (b*Cosh[x
]*Sinh[x])/(2*a^2) + (Cosh[x]*Sinh[x]^2)/(3*a)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 3831

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3853

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(Cot[e + f*x]*(d*Csc[e + f*x])^n)/(a*f*n), x] - Dis
t[1/(a*d*n), Int[((d*Csc[e + f*x])^(n + 1)*Simp[b*n - a*(n + 1)*Csc[e + f*x
] - b*(n + 1)*Csc[e + f*x]^2, x])/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a,
```

```
b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1] && IntegerQ[2*n]
```

Rule 3919

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[(c*x)/a, x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 4104

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d
*Csc[e + f*x])^n)/(a*f*n), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rubi steps

$$\int \frac{\sinh^3(x)}{a+b\operatorname{csch}(x)}\,dx = \frac{\cosh(x)\sinh^2(x)}{3a} - \frac{i\int \frac{\left(-3ib-2ia\operatorname{csch}(x)-2ib\operatorname{csch}^2(x)\right)\sinh^2(x)}{a+b\operatorname{csch}(x)}\,dx}{3a}$$

$$= -\frac{b\cosh(x)\sinh(x)}{2a^2} + \frac{\cosh(x)\sinh^2(x)}{3a} + \frac{\int \frac{\left(-2\left(2a^2-3b^2\right)-ab\operatorname{csch}(x)+3b^2\operatorname{csch}^2(x)\right)\sinh(x)}{a+b\operatorname{csch}(x)}\,dx}{6a^2}$$

$$= -\frac{\left(2a^2-3b^2\right)\cosh(x)}{3a^3} - \frac{b\cosh(x)\sinh(x)}{2a^2} + \frac{\cosh(x)\sinh^2(x)}{3a} + \frac{i\int \frac{-3ib\left(a^2-2b^2\right)-3iab^2\operatorname{csch}(x)}{a+b\operatorname{csch}(x)}}{6a^3}$$

$$= \frac{b\left(a^2-2b^2\right)x}{2a^4} - \frac{\left(2a^2-3b^2\right)\cosh(x)}{3a^3} - \frac{b\cosh(x)\sinh(x)}{2a^2} + \frac{\cosh(x)\sinh^2(x)}{3a} + \frac{b^4\int \frac{\operatorname{csch}}{a+b\operatorname{csc}}}{a^4}$$

$$= \frac{b\left(a^2-2b^2\right)x}{2a^4} - \frac{\left(2a^2-3b^2\right)\cosh(x)}{3a^3} - \frac{b\cosh(x)\sinh(x)}{2a^2} + \frac{\cosh(x)\sinh^2(x)}{3a} + \frac{b^3\int \frac{1}{1+\frac{a\sin}{b}}}{a^4}$$

$$= \frac{b\left(a^2-2b^2\right)x}{2a^4} - \frac{\left(2a^2-3b^2\right)\cosh(x)}{3a^3} - \frac{b\cosh(x)\sinh(x)}{2a^2} + \frac{\cosh(x)\sinh^2(x)}{3a} + \frac{\left(2b^3\right)\operatorname{Sub}}{}$$

$$= \frac{b\left(a^2-2b^2\right)x}{2a^4} - \frac{\left(2a^2-3b^2\right)\cosh(x)}{3a^3} - \frac{b\cosh(x)\sinh(x)}{2a^2} + \frac{\cosh(x)\sinh^2(x)}{3a} - \frac{\left(4b^3\right)\operatorname{Sub}}{}$$

$$= \frac{b\left(a^2-2b^2\right)x}{2a^4} - \frac{2b^4\tanh^{-1}\left(\frac{b\left(\frac{a}{b}-\tanh\left(\frac{x}{2}\right)\right)}{\sqrt{a^2+b^2}}\right)}{a^4\sqrt{a^2+b^2}} - \frac{\left(2a^2-3b^2\right)\cosh(x)}{3a^3} - \frac{b\cosh(x)\sinh(x)}{2a^2} + \text{co}$$

Mathematica [A] time = 0.43, size = 104, normalized size = 0.97

$$\frac{\left(12ab^2-9a^3\right)\cosh(x)+a^3\cosh(3x)+3b\left(\frac{8b^3\tan^{-1}\left(\frac{a-b\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-a^2-b^2}}+2a^2x-a^2\sinh(2x)-4b^2x\right)}{12a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[x]^3/(a + b*Csch[x]),x]
```

```
[Out] ((-9*a^3 + 12*a*b^2)*Cosh[x] + a^3*Cosh[3*x] + 3*b*(2*a^2*x - 4*b^2*x + (8*
b^3*ArcTan[(a - b*Tanh[x/2])/Sqrt[-a^2 - b^2]])/Sqrt[-a^2 - b^2] - a^2*Sinh
[2*x]))/(12*a^4)
```

fricas [B] time = 0.43, size = 807, normalized size = 7.54

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^3/(a+b*csch(x)),x, algorithm="fricas")
```

```
[Out] 1/24*((a^5 + a^3*b^2)*cosh(x)^6 + (a^5 + a^3*b^2)*sinh(x)^6 - 3*(a^4*b + a^
2*b^3)*cosh(x)^5 - 3*(a^4*b + a^2*b^3 - 2*(a^5 + a^3*b^2)*cosh(x))*sinh(x)^
5 + a^5 + a^3*b^2 + 12*(a^4*b - a^2*b^3 - 2*b^5)*x*cosh(x)^3 - 3*(3*a^5 - a
^3*b^2 - 4*a*b^4)*cosh(x)^4 - 3*(3*a^5 - a^3*b^2 - 4*a*b^4 - 5*(a^5 + a^3*b
^2)*cosh(x)^2 + 5*(a^4*b + a^2*b^3)*cosh(x))*sinh(x)^4 + 2*(10*(a^5 + a^3*b
^2)*cosh(x)^3 - 15*(a^4*b + a^2*b^3)*cosh(x)^2 + 6*(a^4*b - a^2*b^3 - 2*b^5
)*x - 6*(3*a^5 - a^3*b^2 - 4*a*b^4)*cosh(x))*sinh(x)^3 - 3*(3*a^5 - a^3*b^2
 - 4*a*b^4)*cosh(x)^2 - 3*(3*a^5 - a^3*b^2 - 4*a*b^4 - 5*(a^5 + a^3*b^2)*co
sh(x)^4 + 10*(a^4*b + a^2*b^3)*cosh(x)^3 - 12*(a^4*b - a^2*b^3 - 2*b^5)*x*c
osh(x) + 6*(3*a^5 - a^3*b^2 - 4*a*b^4)*cosh(x)^2)*sinh(x)^2 + 24*(b^4*cosh(
x)^3 + 3*b^4*cosh(x)^2*sinh(x) + 3*b^4*cosh(x)*sinh(x)^2 + b^4*sinh(x)^3)*s
qrt(a^2 + b^2)*log((a^2*cosh(x)^2 + a^2*sinh(x)^2 + 2*a*b*cosh(x) + a^2 + 2
*b^2 + 2*(a^2*cosh(x) + a*b)*sinh(x) - 2*sqrt(a^2 + b^2)*(a*cosh(x) + a*sin
h(x) + b))/(a*cosh(x)^2 + a*sinh(x)^2 + 2*b*cosh(x) + 2*(a*cosh(x) + b)*sin
h(x) - a)) + 3*(a^4*b + a^2*b^3)*cosh(x) + 3*(2*(a^5 + a^3*b^2)*cosh(x)^5 +
 a^4*b + a^2*b^3 - 5*(a^4*b + a^2*b^3)*cosh(x)^4 + 12*(a^4*b - a^2*b^3 - 2*
b^5)*x*cosh(x)^2 - 4*(3*a^5 - a^3*b^2 - 4*a*b^4)*cosh(x)^3 - 2*(3*a^5 - a^3
*b^2 - 4*a*b^4)*cosh(x))*sinh(x))/((a^6 + a^4*b^2)*cosh(x)^3 + 3*(a^6 + a^4
*b^2)*cosh(x)^2*sinh(x) + 3*(a^6 + a^4*b^2)*cosh(x)*sinh(x)^2 + (a^6 + a^4*
b^2)*sinh(x)^3)
```

giac [A] time = 0.14, size = 155, normalized size = 1.45

$$\frac{b^4 \log\left(\frac{\left|2\,ae^x+2\,b-2\,\sqrt{a^2+b^2}\right|}{\left|2\,ae^x+2\,b+2\,\sqrt{a^2+b^2}\right|}\right)}{\sqrt{a^2+b^2}\,a^4}+\frac{a^2e^{(3\,x)}-3\,abe^{(2\,x)}-9\,a^2e^x+12\,b^2e^x}{24\,a^3}+\frac{\left(a^2b-2\,b^3\right)x}{2\,a^4}+\frac{\left(3\,a^2be^x+a^3-3\left(3\,a^3-4\,ab\right.\right.}{24\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^3/(a+b*csch(x)),x, algorithm="giac")
```

```
[Out] b^4*log(abs(2*a*e^x + 2*b - 2*sqrt(a^2 + b^2))/abs(2*a*e^x + 2*b + 2*sqrt(a
^2 + b^2)))/(sqrt(a^2 + b^2)*a^4) + 1/24*(a^2*e^(3*x) - 3*a*b*e^(2*x) - 9*a
^2*e^x + 12*b^2*e^x)/a^3 + 1/2*(a^2*b - 2*b^3)*x/a^4 + 1/24*(3*a^2*b*e^x +
a^3 - 3*(3*a^3 - 4*a*b^2)*e^(2*x))*e^(-3*x)/a^4
```

maple [B] time = 0.16, size = 262, normalized size = 2.45

$$-\frac{1}{3a\left(\tanh\left(\frac{x}{2}\right)-1\right)^3}-\frac{1}{2a\left(\tanh\left(\frac{x}{2}\right)-1\right)^2}-\frac{b}{2a^2\left(\tanh\left(\frac{x}{2}\right)-1\right)^2}+\frac{1}{2a\left(\tanh\left(\frac{x}{2}\right)-1\right)}-\frac{b}{2a^2\left(\tanh\left(\frac{x}{2}\right)-1\right)}-\frac{}{a^3\left(\tan\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)^3/(a+b*csch(x)),x)
```

```
[Out] -1/3/a/(tanh(1/2*x)-1)^3-1/2/a/(tanh(1/2*x)-1)^2-1/2/a^2/(tanh(1/2*x)-1)^2*
b+1/2/a/(tanh(1/2*x)-1)-1/2/a^2/(tanh(1/2*x)-1)*b-1/a^3/(tanh(1/2*x)-1)*b^2
```

```
-1/2*b/a^2*ln(tanh(1/2*x)-1)+b^3/a^4*ln(tanh(1/2*x)-1)+1/3/a/(tanh(1/2*x)+1
)^3-1/2/a/(tanh(1/2*x)+1)^2+1/2/a^2/(tanh(1/2*x)+1)^2*b-1/2/a/(tanh(1/2*x)+
1)-1/2/a^2/(tanh(1/2*x)+1)*b+1/a^3/(tanh(1/2*x)+1)*b^2+1/2*b/a^2*ln(tanh(1/
2*x)+1)-b^3/a^4*ln(tanh(1/2*x)+1)+2*b^4/a^4/(a^2+b^2)^(1/2)*arctanh(1/2*(2*
tanh(1/2*x)*b-2*a)/(a^2+b^2)^(1/2))
```

maxima [A] time = 0.41, size = 157, normalized size = 1.47

$$\frac{b^4 \log\left(\frac{ae^{(-x)}-b-\sqrt{a^2+b^2}}{ae^{(-x)}-b+\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}\,a^4} - \frac{\left(3\,abe^{(-x)} - a^2 + 3\left(3\,a^2 - 4\,b^2\right)e^{(-2\,x)}\right)e^{(3\,x)}}{24\,a^3} + \frac{3\,abe^{(-2\,x)} + a^2e^{(-3\,x)} - 3\left(3\,a^2 - 4\,b^2\right)e^{(-x)}}{24\,a^3} + \cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^3/(a+b*csch(x)),x, algorithm="maxima")
```

```
[Out] b^4*log((a*e^(-x) - b - sqrt(a^2 + b^2))/(a*e^(-x) - b + sqrt(a^2 + b^2)))/
(sqrt(a^2 + b^2)*a^4) - 1/24*(3*a*b*e^(-x) - a^2 + 3*(3*a^2 - 4*b^2)*e^(-2*
x))*e^(3*x)/a^3 + 1/24*(3*a*b*e^(-2*x) + a^2*e^(-3*x) - 3*(3*a^2 - 4*b^2)*e
^(-x))/a^3 + 1/2*(a^2*b - 2*b^3)*x/a^4
```

mupad [B] time = 1.83, size = 199, normalized size = 1.86

$$\frac{e^{-3\,x}}{24\,a} + \frac{e^{3\,x}}{24\,a} + \frac{x\,\left(a^2\,b - 2\,b^3\right)}{2\,a^4} - \frac{e^x\,\left(3\,a^2 - 4\,b^2\right)}{8\,a^3} + \frac{b\,e^{-2\,x}}{8\,a^2} - \frac{b\,e^{2\,x}}{8\,a^2} - \frac{e^{-x}\,\left(3\,a^2 - 4\,b^2\right)}{8\,a^3} - \frac{b^4\,\ln\left(-\frac{2\,b^4\,e^x}{a^5} - \frac{2\,b^4\,(a - b\,e^x)}{a^5\,\sqrt{a^2+b^2}}\right)}{a^4\,\sqrt{a^2+b^2}} + \frac{b^4 \cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)^3/(a + b/sinh(x)),x)
```

```
[Out] exp(-3*x)/(24*a) + exp(3*x)/(24*a) + (x*(a^2*b - 2*b^3))/(2*a^4) - (exp(x)*
(3*a^2 - 4*b^2))/(8*a^3) + (b*exp(-2*x))/(8*a^2) - (b*exp(2*x))/(8*a^2) - (
exp(-x)*(3*a^2 - 4*b^2))/(8*a^3) - (b^4*log(- (2*b^4*exp(x))/a^5 - (2*b^4*(
a - b*exp(x)))/(a^5*(a^2 + b^2)^(1/2))))/(a^4*(a^2 + b^2)^(1/2)) + (b^4*log
((2*b^4*(a - b*exp(x)))/(a^5*(a^2 + b^2)^(1/2)) - (2*b^4*exp(x))/a^5))/(a^4
*(a^2 + b^2)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh^3(x)}{a + b\operatorname{csch}(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)**3/(a+b*csch(x)),x)
```

```
[Out] Integral(sinh(x)**3/(a + b*csch(x)), x)
```

3.78 $\int \frac{\sinh^2(x)}{a+b\mathbf{csch}(x)}\,dx$

Optimal. Leaf size=80

$$-\frac{b\cosh(x)}{a^2}-\frac{x\left(a^2-2b^2\right)}{2a^3}+\frac{2b^3\tanh^{-1}\left(\frac{a-b\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{a^3\sqrt{a^2+b^2}}+\frac{\sinh(x)\cosh(x)}{2a}$$

```
[Out] -1/2*(a^2-2*b^2)*x/a^3-b*cosh(x)/a^2+1/2*cosh(x)*sinh(x)/a+2*b^3*arctanh((a
-b*tanh(1/2*x))/(a^2+b^2)^(1/2))/a^3/(a^2+b^2)^(1/2)
```

Rubi [A] time = 0.29, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.538, Rules used = {3853, 4104, 3919, 3831, 2660, 618, 206}

$$-\frac{x\left(a^2-2b^2\right)}{2a^3}+\frac{2b^3\tanh^{-1}\left(\frac{a-b\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{a^3\sqrt{a^2+b^2}}-\frac{b\cosh(x)}{a^2}+\frac{\sinh(x)\cosh(x)}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[x]^2/(a + b*Csch[x]),x]
```

```
[Out] -((a^2 - 2*b^2)*x)/(2*a^3) + (2*b^3*ArcTanh[(a - b*Tanh[x/2])/Sqrt[a^2 + b^
2]])/(a^3*Sqrt[a^2 + b^2]) - (b*Cosh[x])/a^2 + (Cosh[x]*Sinh[x])/(2*a)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 3831

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3853

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(Cot[e + f*x]*(d*Csc[e + f*x])^n)/(a*f*n), x] - Dis
t[1/(a*d*n), Int[((d*Csc[e + f*x])^(n + 1)*Simp[b*n - a*(n + 1)*Csc[e + f*x
] - b*(n + 1)*Csc[e + f*x]^2, x])/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1] && IntegerQ[2*n]
```

Rule 3919

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[(c*x)/a, x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 4104

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d
*Csc[e + f*x])^n)/(a*f*n), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sinh^2(x)}{a+b\operatorname{csch}(x)}\,dx &= \frac{\cosh(x)\sinh(x)}{2a} - \frac{i\int \frac{\left(-2ib-ia\operatorname{csch}(x)-ib\operatorname{csch}^2(x)\right)\sinh(x)}{a+b\operatorname{csch}(x)}\,dx}{2a}\\
&= -\frac{b\cosh(x)}{a^2} + \frac{\cosh(x)\sinh(x)}{2a} + \frac{\int \frac{-a^2+2b^2-ab\operatorname{csch}(x)}{a+b\operatorname{csch}(x)}\,dx}{2a^2}\\
&= -\frac{\left(a^2-2b^2\right)x}{2a^3} - \frac{b\cosh(x)}{a^2} + \frac{\cosh(x)\sinh(x)}{2a} - \frac{b^3\int \frac{\operatorname{csch}(x)}{a+b\operatorname{csch}(x)}\,dx}{a^3}\\
&= -\frac{\left(a^2-2b^2\right)x}{2a^3} - \frac{b\cosh(x)}{a^2} + \frac{\cosh(x)\sinh(x)}{2a} - \frac{b^2\int \frac{1}{1+\frac{a\sinh(x)}{b}}\,dx}{a^3}\\
&= -\frac{\left(a^2-2b^2\right)x}{2a^3} - \frac{b\cosh(x)}{a^2} + \frac{\cosh(x)\sinh(x)}{2a} - \frac{\left(2b^2\right)\operatorname{Subst}\left(\int \frac{1}{1+\frac{2ax}{b}-x^2}\,dx, x, \tanh\left(\frac{x}{2}\right)\right)}{a^3}\\
&= -\frac{\left(a^2-2b^2\right)x}{2a^3} - \frac{b\cosh(x)}{a^2} + \frac{\cosh(x)\sinh(x)}{2a} + \frac{\left(4b^2\right)\operatorname{Subst}\left(\int \frac{1}{4\left(1+\frac{a^2}{b^2}\right)-x^2}\,dx, x, \frac{2a}{b} - 2\tan\right.}{a^3}\\
&= -\frac{\left(a^2-2b^2\right)x}{2a^3} + \frac{2b^3\tanh^{-1}\left(\frac{b\left(\frac{a}{b}-\tanh\left(\frac{x}{2}\right)\right)}{\sqrt{a^2+b^2}}\right)}{a^3\sqrt{a^2+b^2}} - \frac{b\cosh(x)}{a^2} + \frac{\cosh(x)\sinh(x)}{2a}
\end{aligned}
$$

Mathematica [A] time = 0.13, size = 82, normalized size = 1.02

$$
\frac{-\frac{8b^3\tan^{-1}\left(\frac{a-b\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-a^2-b^2}} - 2a^2x + a^2\sinh(2x) - 4ab\cosh(x) + 4b^2x}{4a^3}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[x]^2/(a + b*Csch[x]),x]
```

```
[Out] (-2*a^2*x + 4*b^2*x - (8*b^3*ArcTan[(a - b*Tanh[x/2])/Sqrt[-a^2 - b^2]])/Sq
rt[-a^2 - b^2] - 4*a*b*Cosh[x] + a^2*Sinh[2*x])/(4*a^3)
```

fricas [B] time = 0.42, size = 456, normalized size = 5.70

$$\left(a^4 + a^2b^2\right)\cosh(x)^4 + \left(a^4 + a^2b^2\right)\sinh(x)^4 - a^4 - a^2b^2 - 4\left(a^4 - a^2b^2 - 2\,b^4\right)x\cosh(x)^2 - 4\left(a^3b + ab^3\right)\cosh$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^2/(a+b*csch(x)),x, algorithm="fricas")
```

```
[Out] 1/8*((a^4 + a^2*b^2)*cosh(x)^4 + (a^4 + a^2*b^2)*sinh(x)^4 - a^4 - a^2*b^2
- 4*(a^4 - a^2*b^2 - 2*b^4)*x*cosh(x)^2 - 4*(a^3*b + a*b^3)*cosh(x)^3 - 4*(
a^3*b + a*b^3 - (a^4 + a^2*b^2)*cosh(x))*sinh(x)^3 + 2*(3*(a^4 + a^2*b^2)*c
osh(x)^2 - 2*(a^4 - a^2*b^2 - 2*b^4)*x - 6*(a^3*b + a*b^3)*cosh(x))*sinh(x)
^2 + 8*(b^3*cosh(x)^2 + 2*b^3*cosh(x)*sinh(x) + b^3*sinh(x)^2)*sqrt(a^2 + b
^2)*log((a^2*cosh(x)^2 + a^2*sinh(x)^2 + 2*a*b*cosh(x) + a^2 + 2*b^2 + 2*(a
^2*cosh(x) + a*b)*sinh(x) + 2*sqrt(a^2 + b^2)*(a*cosh(x) + a*sinh(x) + b))/
(a*cosh(x)^2 + a*sinh(x)^2 + 2*b*cosh(x) + 2*(a*cosh(x) + b)*sinh(x) - a))
- 4*(a^3*b + a*b^3)*cosh(x) - 4*(a^3*b + a*b^3 - (a^4 + a^2*b^2)*cosh(x)^3
+ 2*(a^4 - a^2*b^2 - 2*b^4)*x*cosh(x) + 3*(a^3*b + a*b^3)*cosh(x)^2)*sinh(x
))/((a^5 + a^3*b^2)*cosh(x)^2 + 2*(a^5 + a^3*b^2)*cosh(x)*sinh(x) + (a^5 +
a^3*b^2)*sinh(x)^2)
```

giac [A] time = 0.13, size = 115, normalized size = 1.44

$$-\frac{b^3\log\left(\frac{\left|2\,ae^x+2\,b-2\,\sqrt{a^2+b^2}\right|}{\left|2\,ae^x+2\,b+2\,\sqrt{a^2+b^2}\right|}\right)}{\sqrt{a^2+b^2}\,a^3} + \frac{ae^{(2\,x)} - 4\,be^x}{8\,a^2} - \frac{\left(a^2 - 2\,b^2\right)x}{2\,a^3} - \frac{\left(4\,abe^x + a^2\right)e^{(-2\,x)}}{8\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^2/(a+b*csch(x)),x, algorithm="giac")
```

```
[Out] -b^3*log(abs(2*a*e^x + 2*b - 2*sqrt(a^2 + b^2))/abs(2*a*e^x + 2*b + 2*sqrt(
a^2 + b^2)))/(sqrt(a^2 + b^2)*a^3) + 1/8*(a*e^(2*x) - 4*b*e^x)/a^2 - 1/2*(a
^2 - 2*b^2)*x/a^3 - 1/8*(4*a*b*e^x + a^2)*e^(-2*x)/a^3
```

maple [B] time = 0.17, size = 174, normalized size = 2.18

$$\frac{1}{2a\left(\tanh\left(\frac{x}{2}\right)-1\right)^2}+\frac{1}{2a\left(\tanh\left(\frac{x}{2}\right)-1\right)}+\frac{b}{a^2\left(\tanh\left(\frac{x}{2}\right)-1\right)}+\frac{\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{2a}-\frac{\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)b^2}{a^3}-\frac{1}{2a\left(\tanh\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)^2/(a+b*csch(x)),x)
```

```
[Out] 1/2/a/(tanh(1/2*x)-1)^2+1/2/a/(tanh(1/2*x)-1)+1/a^2/(tanh(1/2*x)-1)*b+1/2/a
*ln(tanh(1/2*x)-1)-1/a^3*ln(tanh(1/2*x)-1)*b^2-1/2/a/(tanh(1/2*x)+1)^2+1/2/
a/(tanh(1/2*x)+1)-1/a^2/(tanh(1/2*x)+1)*b-1/2/a*ln(tanh(1/2*x)+1)+1/a^3*ln(
tanh(1/2*x)+1)*b^2-2*b^3/a^3/(a^2+b^2)^(1/2)*arctanh(1/2*(2*tanh(1/2*x)*b-2
*a)/(a^2+b^2)^(1/2))
```

maxima [A] time = 0.40, size = 116, normalized size = 1.45

$$-\frac{b^3\log\left(\frac{ae^{(-x)}-b-\sqrt{a^2+b^2}}{ae^{(-x)}-b+\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}\,a^3} - \frac{\left(4\,be^{(-x)} - a\right)e^{(2\,x)}}{8\,a^2} - \frac{4\,be^{(-x)} + ae^{(-2\,x)}}{8\,a^2} - \frac{\left(a^2 - 2\,b^2\right)x}{2\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^2/(a+b*csch(x)),x, algorithm="maxima")
```

```
[Out] -b^3*log((a*e^(-x) - b - sqrt(a^2 + b^2))/(a*e^(-x) - b + sqrt(a^2 + b^2)))
/(sqrt(a^2 + b^2)*a^3) - 1/8*(4*b*e^(-x) - a)*e^(2*x)/a^2 - 1/8*(4*b*e^(-x)
 + a*e^(-2*x))/a^2 - 1/2*(a^2 - 2*b^2)*x/a^3
```

mupad [B] time = 1.67, size = 157, normalized size = 1.96

$$\frac{e^{2x}}{8a}-\frac{e^{-2x}}{8a}-\frac{b\,e^{x}}{2a^2}-\frac{b\,e^{-x}}{2a^2}-\frac{x\left(a^2-2b^2\right)}{2a^3}-\frac{b^3\ln\left(\frac{2b^3\,e^{x}}{a^4}-\frac{2b^3\,(a-b\,e^{x})}{a^4\sqrt{a^2+b^2}}\right)}{a^3\sqrt{a^2+b^2}}+\frac{b^3\ln\left(\frac{2b^3\,e^{x}}{a^4}+\frac{2b^3\,(a-b\,e^{x})}{a^4\sqrt{a^2+b^2}}\right)}{a^3\sqrt{a^2+b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)^2/(a + b/sinh(x)),x)
```

```
[Out] exp(2*x)/(8*a) - exp(-2*x)/(8*a) - (b*exp(x))/(2*a^2) - (b*exp(-x))/(2*a^2)
 - (x*(a^2 - 2*b^2))/(2*a^3) - (b^3*log((2*b^3*exp(x))/a^4 - (2*b^3*(a - b*
exp(x)))/(a^4*(a^2 + b^2)^(1/2))))/(a^3*(a^2 + b^2)^(1/2)) + (b^3*log((2*b^
3*exp(x))/a^4 + (2*b^3*(a - b*exp(x)))/(a^4*(a^2 + b^2)^(1/2))))/(a^3*(a^2
+ b^2)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh^2(x)}{a+b\operatorname{csch}(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)**2/(a+b*csch(x)),x)
```

```
[Out] Integral(sinh(x)**2/(a + b*csch(x)), x)
```

3.79 $\int \frac{\sinh(x)}{a+b\mathbf{csch}(x)}\,dx$

Optimal. Leaf size=57

$$-\frac{2b^2\tanh^{-1}\left(\frac{a-b\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{a^2\sqrt{a^2+b^2}}-\frac{bx}{a^2}+\frac{\cosh(x)}{a}$$

```
[Out] -b*x/a^2+cosh(x)/a-2*b^2*arctanh((a-b*tanh(1/2*x))/(a^2+b^2)^(1/2))/a^2/(a^
2+b^2)^(1/2)
```

Rubi [A] time = 0.11, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.546, Rules used = {3853, 12, 3783, 2660, 618, 206}

$$-\frac{2b^2\tanh^{-1}\left(\frac{a-b\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{a^2\sqrt{a^2+b^2}}-\frac{bx}{a^2}+\frac{\cosh(x)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[x]/(a + b*Csch[x]),x]
```

```
[Out] -((b*x)/a^2) - (2*b^2*ArcTanh[(a - b*Tanh[x/2])/Sqrt[a^2 + b^2]])/(a^2*Sqrt
[a^2 + b^2]) + Cosh[x]/a
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 3783

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(-1), x_Symbol] :> Simp[x/a, x]
- Dist[1/a, Int[1/(1 + (a*Sin[c + d*x])/b), x], x] /; FreeQ[{a, b, c, d}, x
] && NeQ[a^2 - b^2, 0]
```

Rule 3853

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(Cot[e + f*x]*(d*Csc[e + f*x])^n)/(a*f*n), x] - Dis
```

```
t[1/(a*d*n), Int[((d*Csc[e + f*x])^(n + 1)*Simp[b*n - a*(n + 1)*Csc[e + f*x
] - b*(n + 1)*Csc[e + f*x]^2, x])/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1] && IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sinh(x)}{a+b\operatorname{csch}(x)}\,dx &= \frac{\cosh(x)}{a}-\frac{\int \frac{b}{a+b\operatorname{csch}(x)}\,dx}{a}\\
&= \frac{\cosh(x)}{a}-\frac{b\int \frac{1}{a+b\operatorname{csch}(x)}\,dx}{a}\\
&= -\frac{bx}{a^2}+\frac{\cosh(x)}{a}+\frac{b\int \frac{1}{1+\frac{a\sinh(x)}{b}}\,dx}{a^2}\\
&= -\frac{bx}{a^2}+\frac{\cosh(x)}{a}+\frac{(2b)\operatorname{Subst}\left(\int \frac{1}{1+\frac{2ax}{b}-x^2}\,dx,x,\tanh\left(\frac{x}{2}\right)\right)}{a^2}\\
&= -\frac{bx}{a^2}+\frac{\cosh(x)}{a}-\frac{(4b)\operatorname{Subst}\left(\int \frac{1}{4\left(1+\frac{a^2}{b^2}\right)-x^2}\,dx,x,\frac{2a}{b}-2\tanh\left(\frac{x}{2}\right)\right)}{a^2}\\
&= -\frac{bx}{a^2}-\frac{2b^2\tanh^{-1}\left(\frac{b\left(\frac{a}{b}-\tanh\left(\frac{x}{2}\right)\right)}{\sqrt{a^2+b^2}}\right)}{a^2\sqrt{a^2+b^2}}+\frac{\cosh(x)}{a}
\end{aligned}$$

Mathematica [A] time = 0.11, size = 61, normalized size = 1.07

$$\frac{b\left(\frac{2b\tan^{-1}\left(\frac{a-b\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-a^2-b^2}}-x\right)+a\cosh(x)}{a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[x]/(a + b*Csch[x]),x]
```

```
[Out] (b*(-x + (2*b*ArcTan[(a - b*Tanh[x/2])/Sqrt[-a^2 - b^2]])/Sqrt[-a^2 - b^2])
 + a*Cosh[x])/a^2
```

fricas [B] time = 0.42, size = 238, normalized size = 4.18

$$\frac{a^3+ab^2-2\left(a^2b+b^3\right)x\cosh(x)+\left(a^3+ab^2\right)\cosh(x)^2+\left(a^3+ab^2\right)\sinh(x)^2+2\left(b^2\cosh(x)+b^2\sinh(x)\right)\sqrt{a^2+}}{2\left(\left(a^4+\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)/(a+b*csch(x)),x, algorithm="fricas")
```

```
[Out] 1/2*(a^3 + a*b^2 - 2*(a^2*b + b^3)*x*cosh(x) + (a^3 + a*b^2)*cosh(x)^2 + (a
^3 + a*b^2)*sinh(x)^2 + 2*(b^2*cosh(x) + b^2*sinh(x))*sqrt(a^2 + b^2)*log((
a^2*cosh(x)^2 + a^2*sinh(x)^2 + 2*a*b*cosh(x) + a^2 + 2*b^2 + 2*(a^2*cosh(x
) + a*b)*sinh(x) - 2*sqrt(a^2 + b^2)*(a*cosh(x) + a*sinh(x) + b))/(a*cosh(x
)^2 + a*sinh(x)^2 + 2*b*cosh(x) + 2*(a*cosh(x) + b)*sinh(x) - a)) - 2*((a^2
```

```
*b + b^3)*x - (a^3 + a*b^2)*cosh(x))*sinh(x))/((a^4 + a^2*b^2)*cosh(x) + (a
^4 + a^2*b^2)*sinh(x))
```

giac [A] time = 0.13, size = 86, normalized size = 1.51

$$\frac{b^2\log\left(\frac{\left|2\,ae^x+2\,b-2\,\sqrt{a^2+b^2}\right|}{\left|2\,ae^x+2\,b+2\,\sqrt{a^2+b^2}\right|}\right)}{\sqrt{a^2+b^2}\,a^2}-\frac{bx}{a^2}+\frac{e^{(-x)}}{2\,a}+\frac{e^x}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)/(a+b*csch(x)),x, algorithm="giac")
```

```
[Out] b^2*log(abs(2*a*e^x + 2*b - 2*sqrt(a^2 + b^2))/abs(2*a*e^x + 2*b + 2*sqrt(a
^2 + b^2)))/(sqrt(a^2 + b^2)*a^2) - b*x/a^2 + 1/2*e^(-x)/a + 1/2*e^x/a
```

maple [A] time = 0.16, size = 92, normalized size = 1.61

$$-\frac{1}{a\left(\tanh\left(\frac{x}{2}\right)-1\right)}+\frac{b\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{a^2}+\frac{1}{a\left(\tanh\left(\frac{x}{2}\right)+1\right)}-\frac{b\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)}{a^2}+\frac{2b^2\,\mathrm{arctanh}\left(\frac{2\tanh\left(\frac{x}{2}\right)b-2a}{2\sqrt{a^2+b^2}}\right)}{a^2\sqrt{a^2+b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)/(a+b*csch(x)),x)
```

```
[Out] -1/a/(tanh(1/2*x)-1)+b/a^2*ln(tanh(1/2*x)-1)+1/a/(tanh(1/2*x)+1)-b/a^2*ln(t
anh(1/2*x)+1)+2*b^2/a^2/(a^2+b^2)^(1/2)*arctanh(1/2*(2*tanh(1/2*x)*b-2*a)/(
a^2+b^2)^(1/2))
```

maxima [A] time = 0.40, size = 84, normalized size = 1.47

$$\frac{b^2\log\left(\frac{ae^{(-x)}-b-\sqrt{a^2+b^2}}{ae^{(-x)}-b+\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}\,a^2}-\frac{bx}{a^2}+\frac{e^{(-x)}}{2\,a}+\frac{e^x}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)/(a+b*csch(x)),x, algorithm="maxima")
```

```
[Out] b^2*log((a*e^(-x) - b - sqrt(a^2 + b^2))/(a*e^(-x) - b + sqrt(a^2 + b^2)))/
(sqrt(a^2 + b^2)*a^2) - b*x/a^2 + 1/2*e^(-x)/a + 1/2*e^x/a
```

mupad [B] time = 1.58, size = 129, normalized size = 2.26

$$\frac{\mathrm{e}^{-x}}{2\,a}+\frac{\mathrm{e}^x}{2\,a}-\frac{b\,x}{a^2}-\frac{b^2\,\ln\left(-\frac{2\,b^2\,\mathrm{e}^x}{a^3}-\frac{2\,b^2\,\left(a-b\,\mathrm{e}^x\right)}{a^3\,\sqrt{a^2+b^2}}\right)}{a^2\,\sqrt{a^2+b^2}}+\frac{b^2\,\ln\left(\frac{2\,b^2\,\left(a-b\,\mathrm{e}^x\right)}{a^3\,\sqrt{a^2+b^2}}-\frac{2\,b^2\,\mathrm{e}^x}{a^3}\right)}{a^2\,\sqrt{a^2+b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)/(a + b/sinh(x)),x)
```

```
[Out] exp(-x)/(2*a) + exp(x)/(2*a) - (b*x)/a^2 - (b^2*log(- (2*b^2*exp(x))/a^3 -
(2*b^2*(a - b*exp(x)))/(a^3*(a^2 + b^2)^(1/2))))/(a^2*(a^2 + b^2)^(1/2)) +
(b^2*log((2*b^2*(a - b*exp(x)))/(a^3*(a^2 + b^2)^(1/2)) - (2*b^2*exp(x))/a^
3))/(a^2*(a^2 + b^2)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sinh(x)}{a+b\,\mathrm{csch}(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)/(a+b*csch(x)),x)
```

```
[Out] Integral(sinh(x)/(a + b*csch(x)), x)
```

3.80 $\int \frac{\operatorname{csch}(x)}{a+b\operatorname{csch}(x)}\,dx$

Optimal. Leaf size=37

$$-\frac{2\tanh^{-1}\left(\frac{a-b\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}}$$

```
[Out] -2*arctanh((a-b*tanh(1/2*x))/(a^2+b^2)^(1/2))/(a^2+b^2)^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {3831, 2660, 618, 206}

$$-\frac{2\tanh^{-1}\left(\frac{a-b\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[x]/(a + b*Csch[x]),x]
```

```
[Out] (-2*ArcTanh[(a - b*Tanh[x/2])/Sqrt[a^2 + b^2]])/Sqrt[a^2 + b^2]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 3831

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\text{csch}(x)}{a+b\,\text{csch}(x)}\,dx = \frac{\int \frac{1}{1+\frac{a\sinh(x)}{b}}\,dx}{b}$$
$$= \frac{2\,\text{Subst}\left(\int \frac{1}{1+\frac{2ax}{b}-x^2}\,dx, x, \tanh\left(\frac{x}{2}\right)\right)}{b}$$
$$= -\frac{4\,\text{Subst}\left(\int \frac{1}{4\left(1+\frac{a^2}{b^2}\right)-x^2}\,dx, x, \frac{2a}{b}-2\tanh\left(\frac{x}{2}\right)\right)}{b}$$
$$= -\frac{2\tanh^{-1}\left(\frac{b\left(\frac{a}{b}-\tanh\left(\frac{x}{2}\right)\right)}{\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}}$$

Mathematica [**A**] time = 0.03, size = 45, normalized size = 1.22

$$\frac{2\tan^{-1}\left(\frac{a-b\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-a^2-b^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[x]/(a + b*Csch[x]),x]
```

```
[Out] (2*ArcTan[(a - b*Tanh[x/2])/Sqrt[-a^2 - b^2]])/Sqrt[-a^2 - b^2]
```

fricas [**B**] time = 0.40, size = 111, normalized size = 3.00

$$\frac{\log\left(\frac{a^2\cosh(x)^2+a^2\sinh(x)^2+2\,ab\cosh(x)+a^2+2\,b^2+2\left(a^2\cosh(x)+ab\right)\sinh(x)-2\,\sqrt{a^2+b^2}\,(a\cosh(x)+a\sinh(x)+b)}{a\cosh(x)^2+a\sinh(x)^2+2\,b\cosh(x)+2\,(a\cosh(x)+b)\sinh(x)-a}\right)}{\sqrt{a^2+b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)/(a+b*csch(x)),x, algorithm="fricas")
```

```
[Out] log((a^2*cosh(x)^2 + a^2*sinh(x)^2 + 2*a*b*cosh(x) + a^2 + 2*b^2 + 2*(a^2*c
osh(x) + a*b)*sinh(x) - 2*sqrt(a^2 + b^2)*(a*cosh(x) + a*sinh(x) + b))/(a*c
osh(x)^2 + a*sinh(x)^2 + 2*b*cosh(x) + 2*(a*cosh(x) + b)*sinh(x) - a))/sqrt
(a^2 + b^2)
```

giac [**A**] time = 0.14, size = 56, normalized size = 1.51

$$\frac{\log\left(\frac{\left|2\,ae^x+2\,b-2\,\sqrt{a^2+b^2}\right|}{\left|2\,ae^x+2\,b+2\,\sqrt{a^2+b^2}\right|}\right)}{\sqrt{a^2+b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)/(a+b*csch(x)),x, algorithm="giac")
```

```
[Out] log(abs(2*a*e^x + 2*b - 2*sqrt(a^2 + b^2))/abs(2*a*e^x + 2*b + 2*sqrt(a^2 +
 b^2)))/sqrt(a^2 + b^2)
```

maple [**A**] time = 0.08, size = 35, normalized size = 0.95

$$\frac{2\,\text{arctanh}\left(\frac{2\tanh\left(\frac{x}{2}\right)b-2a}{2\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(x)/(a+b*csch(x)),x)
```

```
[Out] 2/(a^2+b^2)^(1/2)*arctanh(1/2*(2*tanh(1/2*x)*b-2*a)/(a^2+b^2)^(1/2))
```

maxima [A] time = 0.40, size = 54, normalized size = 1.46

$$\frac{\log\left(\frac{ae^{(-x)}-b-\sqrt{a^2+b^2}}{ae^{(-x)}-b+\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)/(a+b*csch(x)),x, algorithm="maxima")
```

```
[Out] log((a*e^(-x) - b - sqrt(a^2 + b^2))/(a*e^(-x) - b + sqrt(a^2 + b^2)))/sqrt
(a^2 + b^2)
```

mupad [B] time = 1.52, size = 49, normalized size = 1.32

$$\frac{2\,\mathrm{atan}\left(\frac{b}{\sqrt{-a^2-b^2}}+\frac{a\,\mathrm{e}^x}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-a^2-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sinh(x)*(a + b/sinh(x))),x)
```

```
[Out] (2*atan(b/(- a^2 - b^2)^(1/2) + (a*exp(x))/(- a^2 - b^2)^(1/2)))/(- a^2 - b
^2)^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}(x)}{a+b\operatorname{csch}(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)/(a+b*csch(x)),x)
```

```
[Out] Integral(csch(x)/(a + b*csch(x)), x)
```

3.81 $\int \frac{\text{csch}^2(x)}{a+b\text{csch}(x)}\,dx$

Optimal. Leaf size=50

$$\frac{2a\tanh^{-1}\left(\frac{a-b\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}}-\frac{\tanh^{-1}(\cosh(x))}{b}$$

```
[Out] -arctanh(cosh(x))/b+2*a*arctanh((a-b*tanh(1/2*x))/(a^2+b^2)^(1/2))/b/(a^2+b
^2)^(1/2)
```

Rubi [A] time = 0.11, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {3789, 3770, 3831, 2660, 618, 206}

$$\frac{2a\tanh^{-1}\left(\frac{a-b\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}}-\frac{\tanh^{-1}(\cosh(x))}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[x]^2/(a + b*Csch[x]),x]
```

```
[Out] -(ArcTanh[Cosh[x]]/b) + (2*a*ArcTanh[(a - b*Tanh[x/2])/Sqrt[a^2 + b^2]])/(b
*Sqrt[a^2 + b^2])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3789

```
Int[csc[(e_.) + (f_.)*(x_)]^2/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Sym
bol] :> Dist[1/b, Int[Csc[e + f*x], x], x] - Dist[a/b, Int[Csc[e + f*x]/(a
+ b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 3831

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f
```

```
}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\operatorname{csch}^2(x)}{a+b\operatorname{csch}(x)}\,dx &= \frac{\int \operatorname{csch}(x)\,dx}{b}-\frac{a\int \frac{\operatorname{csch}(x)}{a+b\operatorname{csch}(x)}\,dx}{b}\\ &= -\frac{\tanh^{-1}(\cosh(x))}{b}-\frac{a\int \frac{1}{1+\frac{a\sinh(x)}{b}}\,dx}{b^2}\\ &= -\frac{\tanh^{-1}(\cosh(x))}{b}-\frac{(2a)\operatorname{Subst}\left(\int \frac{1}{1+\frac{2ax}{b}-x^2}\,dx,x,\tanh\left(\frac{x}{2}\right)\right)}{b^2}\\ &= -\frac{\tanh^{-1}(\cosh(x))}{b}+\frac{(4a)\operatorname{Subst}\left(\int \frac{1}{4\left(1+\frac{a^2}{b^2}\right)-x^2}\,dx,x,\frac{2a}{b}-2\tanh\left(\frac{x}{2}\right)\right)}{b^2}\\ &= -\frac{\tanh^{-1}(\cosh(x))}{b}+\frac{2a\tanh^{-1}\left(\frac{b\left(\frac{a}{b}-\tanh\left(\frac{x}{2}\right)\right)}{\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}}\end{aligned}$$

Mathematica [A] time = 0.05, size = 58, normalized size = 1.16

$$\frac{\log\left(\tanh\left(\frac{x}{2}\right)\right)-\frac{2a\tan^{-1}\left(\frac{a-b\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-a^2-b^2}}}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[x]^2/(a + b*Csch[x]),x]
[Out] ((-2*a*ArcTan[(a - b*Tanh[x/2])/Sqrt[-a^2 - b^2]])/Sqrt[-a^2 - b^2] + Log[T
anh[x/2]])/b
```

fricas [B] time = 0.44, size = 156, normalized size = 3.12

$$\frac{\sqrt{a^2+b^2}\,a\log\left(\frac{a^2\cosh(x)^2+a^2\sinh(x)^2+2ab\cosh(x)+a^2+2b^2+2\left(a^2\cosh(x)+ab\right)\sinh(x)+2\sqrt{a^2+b^2}\left(a\cosh(x)+a\sinh(x)+b\right)}{a\cosh(x)^2+a\sinh(x)^2+2b\cosh(x)+2\left(a\cosh(x)+b\right)\sinh(x)-a}\right)-\left(a^2+b\right.}{a^2b+b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^2/(a+b*csch(x)),x, algorithm="fricas")
[Out] (sqrt(a^2 + b^2)*a*log((a^2*cosh(x)^2 + a^2*sinh(x)^2 + 2*a*b*cosh(x) + a^2
 + 2*b^2 + 2*(a^2*cosh(x) + a*b)*sinh(x) + 2*sqrt(a^2 + b^2)*(a*cosh(x) + a
*sinh(x) + b))/(a*cosh(x)^2 + a*sinh(x)^2 + 2*b*cosh(x) + 2*(a*cosh(x) + b)
*sinh(x) - a)) - (a^2 + b^2)*log(cosh(x) + sinh(x) + 1) + (a^2 + b^2)*log(c
osh(x) + sinh(x) - 1))/(a^2*b + b^3)
```

giac [A] time = 0.15, size = 82, normalized size = 1.64

$$-\frac{a\log\left(\frac{\left|2ae^x+2b-2\sqrt{a^2+b^2}\right|}{\left|2ae^x+2b+2\sqrt{a^2+b^2}\right|}\right)}{\sqrt{a^2+b^2}\,b}-\frac{\log\left(e^x+1\right)}{b}+\frac{\log\left(\left|e^x-1\right|\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^2/(a+b*csch(x)),x, algorithm="giac")
```

```
[Out] -a*log(abs(2*a*e^x + 2*b - 2*sqrt(a^2 + b^2))/abs(2*a*e^x + 2*b + 2*sqrt(a^
2 + b^2)))/(sqrt(a^2 + b^2)*b) - log(e^x + 1)/b + log(abs(e^x - 1))/b
```

maple [A] time = 0.09, size = 49, normalized size = 0.98

$$-\frac{2a\,\mathrm{arctanh}\left(\frac{2\tanh\left(\frac{x}{2}\right)b-2a}{2\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}}+\frac{\ln\left(\tanh\left(\frac{x}{2}\right)\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(x)^2/(a+b*csch(x)),x)
```

```
[Out] -2*a/b/(a^2+b^2)^(1/2)*arctanh(1/2*(2*tanh(1/2*x)*b-2*a)/(a^2+b^2)^(1/2))+1
/b*ln(tanh(1/2*x))
```

maxima [A] time = 0.41, size = 83, normalized size = 1.66

$$-\frac{a\log\left(\frac{ae^{(-x)}-b-\sqrt{a^2+b^2}}{ae^{(-x)}-b+\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}\,b}-\frac{\log\left(e^{(-x)}+1\right)}{b}+\frac{\log\left(e^{(-x)}-1\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^2/(a+b*csch(x)),x, algorithm="maxima")
```

```
[Out] -a*log((a*e^(-x) - b - sqrt(a^2 + b^2))/(a*e^(-x) - b + sqrt(a^2 + b^2)))/(
sqrt(a^2 + b^2)*b) - log(e^(-x) + 1)/b + log(e^(-x) - 1)/b
```

mupad [B] time = 1.68, size = 287, normalized size = 5.74

$$\frac{\ln\left(32\,b-32\,b\,e^x\right)}{b}-\frac{\ln\left(32\,b+32\,b\,e^x\right)}{b}+\frac{a\,\ln\left(128\,b^5\,e^x-64\,a^3\,b^2-64\,a\,b^4-128\,b^4\,e^x\,\sqrt{a^2+b^2}\,+32\,a^4\,b\,e^x+16\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sinh(x)^2*(a + b/sinh(x))),x)
```

```
[Out] log(32*b - 32*b*exp(x))/b - log(32*b + 32*b*exp(x))/b + (a*log(128*b^5*exp(
x) - 64*a^3*b^2 - 64*a*b^4 - 128*b^4*exp(x)*(a^2 + b^2)^(1/2) + 32*a^4*b*ex
p(x) + 160*a^2*b^3*exp(x) + 64*a*b^3*(a^2 + b^2)^(1/2) + 32*a^3*b*(a^2 + b^
2)^(1/2) - 96*a^2*b^2*exp(x)*(a^2 + b^2)^(1/2))*(a^2 + b^2)^(1/2))/(a^2*b +
 b^3) - (a*log(64*a*b^4 + 64*a^3*b^2 - 128*b^5*exp(x) - 128*b^4*exp(x)*(a^2
 + b^2)^(1/2) - 32*a^4*b*exp(x) - 160*a^2*b^3*exp(x) + 64*a*b^3*(a^2 + b^2)
^(1/2) + 32*a^3*b*(a^2 + b^2)^(1/2) - 96*a^2*b^2*exp(x)*(a^2 + b^2)^(1/2))*
(a^2 + b^2)^(1/2))/(a^2*b + b^3)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\mathrm{csch}^2(x)}{a+b\,\mathrm{csch}(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)**2/(a+b*csch(x)),x)
```

```
[Out] Integral(csch(x)**2/(a + b*csch(x)), x)
```

3.82 $\int \frac{\text{csch}^3(x)}{a+b\,\text{csch}(x)}\,dx$

Optimal. Leaf size=59

$$-\frac{2a^2\tanh^{-1}\left(\frac{a-b\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{b^2\sqrt{a^2+b^2}}+\frac{a\tanh^{-1}(\cosh(x))}{b^2}-\frac{\coth(x)}{b}$$

```
[Out] a*arctanh(cosh(x))/b^2-coth(x)/b-2*a^2*arctanh((a-b*tanh(1/2*x))/(a^2+b^2)^
(1/2))/b^2/(a^2+b^2)^(1/2)
```

Rubi [A] time = 0.16, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.538, Rules used = {3790, 3789, 3770, 3831, 2660, 618, 206}

$$-\frac{2a^2\tanh^{-1}\left(\frac{a-b\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{b^2\sqrt{a^2+b^2}}+\frac{a\tanh^{-1}(\cosh(x))}{b^2}-\frac{\coth(x)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[x]^3/(a + b*Csch[x]),x]
```

```
[Out] (a*ArcTanh[Cosh[x]])/b^2 - (2*a^2*ArcTanh[(a - b*Tanh[x/2])/Sqrt[a^2 + b^2]
])/(b^2*Sqrt[a^2 + b^2]) - Coth[x]/b
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3789

```
Int[csc[(e_.) + (f_.)*(x_)]^2/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Sym
bol] :> Dist[1/b, Int[Csc[e + f*x], x], x] - Dist[a/b, Int[Csc[e + f*x]/(a
+ b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 3790

```
Int[csc[(e_.) + (f_.)*(x_)]^3/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Sym
bol] :> -Simp[Cot[e + f*x]/(b*f), x] - Dist[a/b, Int[Csc[e + f*x]^2/(a + b*
```

```
Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 3831

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\text{csch}^3(x)}{a+b\text{csch}(x)}\,dx = -\frac{\coth(x)}{b} - \frac{a\int \frac{\text{csch}^2(x)}{a+b\text{csch}(x)}\,dx}{b}$$
$$= -\frac{\coth(x)}{b} - \frac{a\int \text{csch}(x)\,dx}{b^2} + \frac{a^2\int \frac{\text{csch}(x)}{a+b\text{csch}(x)}\,dx}{b^2}$$
$$= \frac{a\tanh^{-1}(\cosh(x))}{b^2} - \frac{\coth(x)}{b} + \frac{a^2\int \frac{1}{1+\frac{a\sinh(x)}{b}}\,dx}{b^3}$$
$$= \frac{a\tanh^{-1}(\cosh(x))}{b^2} - \frac{\coth(x)}{b} + \frac{\left(2a^2\right)\text{Subst}\left(\int \frac{1}{1+\frac{2ax}{b}-x^2}\,dx, x, \tanh\left(\frac{x}{2}\right)\right)}{b^3}$$
$$= \frac{a\tanh^{-1}(\cosh(x))}{b^2} - \frac{\coth(x)}{b} - \frac{\left(4a^2\right)\text{Subst}\left(\int \frac{1}{4\left(1+\frac{a^2}{b^2}\right)-x^2}\,dx, x, \frac{2a}{b}-2\tanh\left(\frac{x}{2}\right)\right)}{b^3}$$
$$= \frac{a\tanh^{-1}(\cosh(x))}{b^2} - \frac{2a^2\tanh^{-1}\left(\frac{b\left(\frac{a}{b}-\tanh\left(\frac{x}{2}\right)\right)}{\sqrt{a^2+b^2}}\right)}{b^2\sqrt{a^2+b^2}} - \frac{\coth(x)}{b}$$

Mathematica [A] time = 0.29, size = 71, normalized size = 1.20

$$\frac{\frac{4a^2\tan^{-1}\left(\frac{a-b\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-a^2-b^2}} - 2a\log\left(\tanh\left(\frac{x}{2}\right)\right) - 2b\coth(x)}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[x]^3/(a + b*Csch[x]),x]
```

```
[Out] ((4*a^2*ArcTan[(a - b*Tanh[x/2])/Sqrt[-a^2 - b^2]])/Sqrt[-a^2 - b^2] - 2*b*
Coth[x] - 2*a*Log[Tanh[x/2]])/(2*b^2)
```

fricas [B] time = 0.43, size = 345, normalized size = 5.85

$$\frac{2\,a^2b + 2\,b^3 - \left(a^2\cosh(x)^2 + 2\,a^2\cosh(x)\sinh(x) + a^2\sinh(x)^2 - a^2\right)\sqrt{a^2+b^2}\,\log\left(\frac{a^2\cosh(x)^2+a^2\sinh(x)^2+2\,ab\cosh(x}{a\cosh(x)^2+a}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^3/(a+b*csch(x)),x, algorithm="fricas")
```

```
[Out] (2*a^2*b + 2*b^3 - (a^2*cosh(x)^2 + 2*a^2*cosh(x)*sinh(x) + a^2*sinh(x)^2 -
 a^2)*sqrt(a^2 + b^2)*log((a^2*cosh(x)^2 + a^2*sinh(x)^2 + 2*a*b*cosh(x) +
```

```
a^2 + 2*b^2 + 2*(a^2*cosh(x) + a*b)*sinh(x) - 2*sqrt(a^2 + b^2)*(a*cosh(x)
+ a*sinh(x) + b))/(a*cosh(x)^2 + a*sinh(x)^2 + 2*b*cosh(x) + 2*(a*cosh(x) +
 b)*sinh(x) - a)) + (a^3 + a*b^2 - (a^3 + a*b^2)*cosh(x)^2 - 2*(a^3 + a*b^2
)*cosh(x)*sinh(x) - (a^3 + a*b^2)*sinh(x)^2)*log(cosh(x) + sinh(x) + 1) - (
a^3 + a*b^2 - (a^3 + a*b^2)*cosh(x)^2 - 2*(a^3 + a*b^2)*cosh(x)*sinh(x) - (
a^3 + a*b^2)*sinh(x)^2)*log(cosh(x) + sinh(x) - 1))/(a^2*b^2 + b^4 - (a^2*b
^2 + b^4)*cosh(x)^2 - 2*(a^2*b^2 + b^4)*cosh(x)*sinh(x) - (a^2*b^2 + b^4)*s
inh(x)^2)
```

giac [A] time = 0.15, size = 98, normalized size = 1.66

$$\frac{a^2\log\left(\frac{\left|2\,ae^x+2\,b-2\,\sqrt{a^2+b^2}\right|}{\left|2\,ae^x+2\,b+2\,\sqrt{a^2+b^2}\right|}\right)}{\sqrt{a^2+b^2}\,b^2}+\frac{a\log\left(e^x+1\right)}{b^2}-\frac{a\log\left(|e^x-1|\right)}{b^2}-\frac{2}{b\left(e^{(2\,x)}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^3/(a+b*csch(x)),x, algorithm="giac")
```

```
[Out] a^2*log(abs(2*a*e^x + 2*b - 2*sqrt(a^2 + b^2))/abs(2*a*e^x + 2*b + 2*sqrt(a
^2 + b^2)))/(sqrt(a^2 + b^2)*b^2) + a*log(e^x + 1)/b^2 - a*log(abs(e^x - 1)
)/b^2 - 2/(b*(e^(2*x) - 1))
```

maple [A] time = 0.10, size = 73, normalized size = 1.24

$$-\frac{\tanh\left(\frac{x}{2}\right)}{2b}+\frac{2a^2\operatorname{arctanh}\left(\frac{2\tanh\left(\frac{x}{2}\right)b-2a}{2\sqrt{a^2+b^2}}\right)}{b^2\sqrt{a^2+b^2}}-\frac{1}{2b\tanh\left(\frac{x}{2}\right)}-\frac{a\ln\left(\tanh\left(\frac{x}{2}\right)\right)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(x)^3/(a+b*csch(x)),x)
```

```
[Out] -1/2/b*tanh(1/2*x)+2*a^2/b^2/(a^2+b^2)^(1/2)*arctanh(1/2*(2*tanh(1/2*x)*b-2
*a)/(a^2+b^2)^(1/2))-1/2/b/tanh(1/2*x)-a/b^2*ln(tanh(1/2*x))
```

maxima [A] time = 0.42, size = 100, normalized size = 1.69

$$\frac{a^2\log\left(\frac{ae^{(-x)}-b-\sqrt{a^2+b^2}}{ae^{(-x)}-b+\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}\,b^2}+\frac{a\log\left(e^{(-x)}+1\right)}{b^2}-\frac{a\log\left(e^{(-x)}-1\right)}{b^2}+\frac{2}{be^{(-2\,x)}-b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^3/(a+b*csch(x)),x, algorithm="maxima")
```

```
[Out] a^2*log((a*e^(-x) - b - sqrt(a^2 + b^2))/(a*e^(-x) - b + sqrt(a^2 + b^2)))/
(sqrt(a^2 + b^2)*b^2) + a*log(e^(-x) + 1)/b^2 - a*log(e^(-x) - 1)/b^2 + 2/(
b*e^(-2*x) - b)
```

mupad [B] time = 1.73, size = 292, normalized size = 4.95

$$\frac{2}{b-b\,\mathrm{e}^{2\,x}}-\frac{a\,\ln\left(32\,\mathrm{e}^x-32\right)}{b^2}+\frac{a\,\ln\left(32\,\mathrm{e}^x+32\right)}{b^2}+\frac{a^2\,\ln\Big(32\,a^4\,\mathrm{e}^x-64\,a\,b^3-64\,a^3\,b-32\,a^3\,\sqrt{a^2+b^2}\,+128\,b^4\,\mathrm{e}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sinh(x)^3*(a + b/sinh(x))),x)
```

```
[Out] 2/(b - b*exp(2*x)) - (a*log(32*exp(x) - 32))/b^2 + (a*log(32*exp(x) + 32))/
b^2 + (a^2*log(32*a^4*exp(x) - 64*a*b^3 - 64*a^3*b - 32*a^3*(a^2 + b^2)^(1/
```

```
2) + 128*b^4*exp(x) + 128*b^3*exp(x)*(a^2 + b^2)^(1/2) + 160*a^2*b^2*exp(x)
 - 64*a*b^2*(a^2 + b^2)^(1/2) + 96*a^2*b*exp(x)*(a^2 + b^2)^(1/2))*(a^2 + b
^2)^(1/2))/(b^4 + a^2*b^2) - (a^2*log(32*a^3*(a^2 + b^2)^(1/2) - 64*a*b^3 -
 64*a^3*b + 32*a^4*exp(x) + 128*b^4*exp(x) - 128*b^3*exp(x)*(a^2 + b^2)^(1/
2) + 160*a^2*b^2*exp(x) + 64*a*b^2*(a^2 + b^2)^(1/2) - 96*a^2*b*exp(x)*(a^2
 + b^2)^(1/2))*(a^2 + b^2)^(1/2))/(b^4 + a^2*b^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}^3(x)}{a + b\operatorname{csch}(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)**3/(a+b*csch(x)),x)
```

```
[Out] Integral(csch(x)**3/(a + b*csch(x)), x)
```

3.83 $\int \frac{\operatorname{csch}^4(x)}{a+b\operatorname{csch}(x)} \, dx$

Optimal. Leaf size=83

$$-\frac{(2a^2-b^2)\tanh^{-1}(\cosh(x))}{2b^3}+\frac{2a^3\tanh^{-1}\left(\frac{a-b\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{b^3\sqrt{a^2+b^2}}+\frac{a\coth(x)}{b^2}-\frac{\coth(x)\operatorname{csch}(x)}{2b}$$

```
[Out] -1/2*(2*a^2-b^2)*arctanh(cosh(x))/b^3+a*coth(x)/b^2-1/2*coth(x)*csch(x)/b+2
*a^3*arctanh((a-b*tanh(1/2*x))/(a^2+b^2)^(1/2))/b^3/(a^2+b^2)^(1/2)
```

Rubi [A] time = 0.29, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.615, Rules used = {3851, 4082, 3998, 3770, 3831, 2660, 618, 206}

$$\frac{2a^3\tanh^{-1}\left(\frac{a-b\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{b^3\sqrt{a^2+b^2}}-\frac{(2a^2-b^2)\tanh^{-1}(\cosh(x))}{2b^3}+\frac{a\coth(x)}{b^2}-\frac{\coth(x)\operatorname{csch}(x)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[x]^4/(a + b*Csch[x]),x]
```

```
[Out] -((2*a^2 - b^2)*ArcTanh[Cosh[x]])/(2*b^3) + (2*a^3*ArcTanh[(a - b*Tanh[x/2]
)/Sqrt[a^2 + b^2]])/(b^3*Sqrt[a^2 + b^2]) + (a*Coth[x])/b^2 - (Coth[x]*Csch
[x])/(2*b)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3831

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3851

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> -Simp[(d^3*Cot[e + f*x]*(d*Csc[e + f*x])^(n - 3))/(b*f*(
n - 2)), x] + Dist[d^3/(b*(n - 2)), Int[((d*Csc[e + f*x])^(n - 3)*Simp[a*(n
 - 3) + b*(n - 3)*Csc[e + f*x] - a*(n - 2)*Csc[e + f*x]^2, x])/(a + b*Csc[e
 + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && GtQ[n
, 3]
```

Rule 3998

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/(csc[(
e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Dist[B/b, Int[Csc[e + f*x],
x], x] + Dist[(A*b - a*B)/b, Int[Csc[e + f*x]/(a + b*Csc[e + f*x]), x], x]
/; FreeQ[{a, b, e, f, A, B}, x] && NeQ[A*b - a*B, 0]
```

Rule 4082

```
Int[csc[(e_.) + (f_.)*(x_)]*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e
_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_S
ymbol] :> -Simp[(C*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2))
, x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b*A
*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Csc[e + f*x], x], x], x] /; Fr
eeQ[{a, b, e, f, A, B, C, m}, x] &&  !LtQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\operatorname{csch}^4(x)}{a+b\operatorname{csch}(x)}\,dx &= -\frac{\coth(x)\operatorname{csch}(x)}{2b} - \frac{\int \frac{\operatorname{csch}(x)\left(a+b\operatorname{csch}(x)+2a\operatorname{csch}^2(x)\right)}{a+b\operatorname{csch}(x)}\,dx}{2b} \\
&= \frac{a\coth(x)}{b^2} - \frac{\coth(x)\operatorname{csch}(x)}{2b} + \frac{i\int \frac{\operatorname{csch}(x)\left(iab-i\left(2a^2-b^2\right)\operatorname{csch}(x)\right)}{a+b\operatorname{csch}(x)}\,dx}{2b^2} \\
&= \frac{a\coth(x)}{b^2} - \frac{\coth(x)\operatorname{csch}(x)}{2b} - \frac{a^3\int \frac{\operatorname{csch}(x)}{a+b\operatorname{csch}(x)}\,dx}{b^3} + \frac{\left(2a^2-b^2\right)\int \operatorname{csch}(x)\,dx}{2b^3} \\
&= -\frac{\left(2a^2-b^2\right)\tanh^{-1}(\cosh(x))}{2b^3} + \frac{a\coth(x)}{b^2} - \frac{\coth(x)\operatorname{csch}(x)}{2b} - \frac{a^3\int \frac{1}{1+\frac{a\sinh(x)}{b}}\,dx}{b^4} \\
&= -\frac{\left(2a^2-b^2\right)\tanh^{-1}(\cosh(x))}{2b^3} + \frac{a\coth(x)}{b^2} - \frac{\coth(x)\operatorname{csch}(x)}{2b} - \frac{\left(2a^3\right)\operatorname{Subst}\left(\int \frac{1}{1+\frac{2ax}{b}-x^2}\,dx,\right.}{b^4} \\
&= -\frac{\left(2a^2-b^2\right)\tanh^{-1}(\cosh(x))}{2b^3} + \frac{a\coth(x)}{b^2} - \frac{\coth(x)\operatorname{csch}(x)}{2b} + \frac{\left(4a^3\right)\operatorname{Subst}\left(\int \frac{1}{4\left(1+\frac{a^2}{b^2}\right)-x^2}\,d\right.}{b^4} \\
&= -\frac{\left(2a^2-b^2\right)\tanh^{-1}(\cosh(x))}{2b^3} + \frac{2a^3\tanh^{-1}\left(\frac{b\left(\frac{a}{b}-\tanh\left(\frac{x}{2}\right)\right)}{\sqrt{a^2+b^2}}\right)}{b^3\sqrt{a^2+b^2}} + \frac{a\coth(x)}{b^2} - \frac{\coth(x)\operatorname{csch}(x)}{2b}
\end{aligned}
$$

Mathematica [A] time = 0.48, size = 124, normalized size = 1.49

$$
-\frac{-8a^2\log\left(\tanh\left(\frac{x}{2}\right)\right) + \frac{16a^3\tan^{-1}\left(\frac{a-b\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-a^2-b^2}} - 4ab\tanh\left(\frac{x}{2}\right) - 4ab\coth\left(\frac{x}{2}\right) + b^2\operatorname{csch}^2\left(\frac{x}{2}\right) + b^2\operatorname{sech}^2\left(\frac{x}{2}\right) + 4b^2\log}{8b^3}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[x]^4/(a + b*Csch[x]),x]
```

```
[Out] -1/8*((16*a^3*ArcTan[(a - b*Tanh[x/2])/Sqrt[-a^2 - b^2]])/Sqrt[-a^2 - b^2]
- 4*a*b*Coth[x/2] + b^2*Csch[x/2]^2 - 8*a^2*Log[Tanh[x/2]] + 4*b^2*Log[Tanh
[x/2]] + b^2*Sech[x/2]^2 - 4*a*b*Tanh[x/2])/b^3
```

fricas [B] time = 0.53, size = 947, normalized size = 11.41

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^4/(a+b*csch(x)),x, algorithm="fricas")
```

```
[Out] -1/2*(4*a^3*b + 4*a*b^3 + 2*(a^2*b^2 + b^4)*cosh(x)^3 + 2*(a^2*b^2 + b^4)*s
inh(x)^3 - 4*(a^3*b + a*b^3)*cosh(x)^2 - 2*(2*a^3*b + 2*a*b^3 - 3*(a^2*b^2
+ b^4)*cosh(x))*sinh(x)^2 - 2*(a^3*cosh(x)^4 + 4*a^3*cosh(x)*sinh(x)^3 + a^
3*sinh(x)^4 - 2*a^3*cosh(x)^2 + a^3 + 2*(3*a^3*cosh(x)^2 - a^3)*sinh(x)^2 +
 4*(a^3*cosh(x)^3 - a^3*cosh(x))*sinh(x))*sqrt(a^2 + b^2)*log((a^2*cosh(x)^
2 + a^2*sinh(x)^2 + 2*a*b*cosh(x) + a^2 + 2*b^2 + 2*(a^2*cosh(x) + a*b)*sin
h(x) + 2*sqrt(a^2 + b^2)*(a*cosh(x) + a*sinh(x) + b))/(a*cosh(x)^2 + a*sinh
(x)^2 + 2*b*cosh(x) + 2*(a*cosh(x) + b)*sinh(x) - a)) + 2*(a^2*b^2 + b^4)*c
osh(x) + ((2*a^4 + a^2*b^2 - b^4)*cosh(x)^4 + 4*(2*a^4 + a^2*b^2 - b^4)*cos
h(x)*sinh(x)^3 + (2*a^4 + a^2*b^2 - b^4)*sinh(x)^4 + 2*a^4 + a^2*b^2 - b^4
- 2*(2*a^4 + a^2*b^2 - b^4)*cosh(x)^2 - 2*(2*a^4 + a^2*b^2 - b^4 - 3*(2*a^4
 + a^2*b^2 - b^4)*cosh(x)^2)*sinh(x)^2 + 4*((2*a^4 + a^2*b^2 - b^4)*cosh(x)
^3 - (2*a^4 + a^2*b^2 - b^4)*cosh(x))*sinh(x))*log(cosh(x) + sinh(x) + 1) -
 ((2*a^4 + a^2*b^2 - b^4)*cosh(x)^4 + 4*(2*a^4 + a^2*b^2 - b^4)*cosh(x)*sin
h(x)^3 + (2*a^4 + a^2*b^2 - b^4)*sinh(x)^4 + 2*a^4 + a^2*b^2 - b^4 - 2*(2*a
^4 + a^2*b^2 - b^4)*cosh(x)^2 - 2*(2*a^4 + a^2*b^2 - b^4 - 3*(2*a^4 + a^2*b
^2 - b^4)*cosh(x)^2)*sinh(x)^2 + 4*((2*a^4 + a^2*b^2 - b^4)*cosh(x)^3 - (2*
a^4 + a^2*b^2 - b^4)*cosh(x))*sinh(x))*log(cosh(x) + sinh(x) - 1) + 2*(a^2*
b^2 + b^4 + 3*(a^2*b^2 + b^4)*cosh(x)^2 - 4*(a^3*b + a*b^3)*cosh(x))*sinh(x
))/(a^2*b^3 + b^5 + (a^2*b^3 + b^5)*cosh(x)^4 + 4*(a^2*b^3 + b^5)*cosh(x)*s
inh(x)^3 + (a^2*b^3 + b^5)*sinh(x)^4 - 2*(a^2*b^3 + b^5)*cosh(x)^2 - 2*(a^2
*b^3 + b^5 - 3*(a^2*b^3 + b^5)*cosh(x)^2)*sinh(x)^2 + 4*((a^2*b^3 + b^5)*co
sh(x)^3 - (a^2*b^3 + b^5)*cosh(x))*sinh(x))
```

giac [A] time = 0.16, size = 141, normalized size = 1.70

$$-\frac{a^{3} \log\left(\frac{\left|2\, a e^{x} + 2\, b - 2\, \sqrt{a^{2} + b^{2}}\right|}{\left|2\, a e^{x} + 2\, b + 2\, \sqrt{a^{2} + b^{2}}\right|}\right)}{\sqrt{a^{2} + b^{2}}\, b^{3}} - \frac{\left(2\, a^{2} - b^{2}\right) \log\left(e^{x} + 1\right)}{2\, b^{3}} + \frac{\left(2\, a^{2} - b^{2}\right) \log\left(\left|e^{x} - 1\right|\right)}{2\, b^{3}} - \frac{b e^{\left(3\, x\right)} - 2\, a e^{\left(2\, x\right)} + b e^{x} + 2\, a}{b^{2} \left(e^{\left(2\, x\right)} - 1\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^4/(a+b*csch(x)),x, algorithm="giac")
```

```
[Out] -a^3*log(abs(2*a*e^x + 2*b - 2*sqrt(a^2 + b^2))/abs(2*a*e^x + 2*b + 2*sqrt(
a^2 + b^2)))/(sqrt(a^2 + b^2)*b^3) - 1/2*(2*a^2 - b^2)*log(e^x + 1)/b^3 + 1
/2*(2*a^2 - b^2)*log(abs(e^x - 1))/b^3 - (b*e^(3*x) - 2*a*e^(2*x) + b*e^x +
 2*a)/(b^2*(e^(2*x) - 1)^2)
```

maple [A] time = 0.10, size = 108, normalized size = 1.30

$$\frac{\tanh^{2}\left(\frac{x}{2}\right)}{8b} + \frac{a \tanh\left(\frac{x}{2}\right)}{2b^{2}} - \frac{2a^{3} \operatorname{arctanh}\left(\frac{2 \tanh\left(\frac{x}{2}\right) b - 2a}{2\sqrt{a^{2}+b^{2}}}\right)}{b^{3}\sqrt{a^{2}+b^{2}}} - \frac{1}{8b \tanh\left(\frac{x}{2}\right)^{2}} + \frac{\ln\left(\tanh\left(\frac{x}{2}\right)\right) a^{2}}{b^{3}} - \frac{\ln\left(\tanh\left(\frac{x}{2}\right)\right)}{2b} + \frac{a}{2b^{2} \tanh$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(x)^4/(a+b*csch(x)),x)
```

```
[Out] 1/8/b*tanh(1/2*x)^2+1/2/b^2*a*tanh(1/2*x)-2/b^3*a^3/(a^2+b^2)^(1/2)*arctanh
(1/2*(2*tanh(1/2*x)*b-2*a)/(a^2+b^2)^(1/2))-1/8/b/tanh(1/2*x)^2+1/b^3*ln(ta
nh(1/2*x))*a^2-1/2/b*ln(tanh(1/2*x))+1/2*a/b^2/tanh(1/2*x)
```

maxima [B] time = 0.42, size = 158, normalized size = 1.90

$$-\frac{a^{3} \log\left(\frac{ae^{(-x)}-b-\sqrt{a^{2}+b^{2}}}{ae^{(-x)}-b+\sqrt{a^{2}+b^{2}}}\right)}{\sqrt{a^{2}+b^{2}}\,b^{3}}+\frac{be^{(-x)}+2\,ae^{(-2\,x)}+be^{(-3\,x)}-2\,a}{2\,b^{2}e^{(-2\,x)}-b^{2}e^{(-4\,x)}-b^{2}}-\frac{\left(2\,a^{2}-b^{2}\right)\log\left(e^{(-x)}+1\right)}{2\,b^{3}}+\frac{\left(2\,a^{2}-b^{2}\right)\log\left(e^{(-x)}-1\right)}{2\,b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^4/(a+b*csch(x)),x, algorithm="maxima")
```

```
[Out] -a^3*log((a*e^(-x) - b - sqrt(a^2 + b^2))/(a*e^(-x) - b + sqrt(a^2 + b^2)))
/(sqrt(a^2 + b^2)*b^3) + (b*e^(-x) + 2*a*e^(-2*x) + b*e^(-3*x) - 2*a)/(2*b^
2*e^(-2*x) - b^2*e^(-4*x) - b^2) - 1/2*(2*a^2 - b^2)*log(e^(-x) + 1)/b^3 +
1/2*(2*a^2 - b^2)*log(e^(-x) - 1)/b^3
```

mupad [B] time = 2.09, size = 617, normalized size = 7.43

$$\frac{e^{x}}{b-b\,e^{2\,x}}-\frac{2\,e^{x}}{b-2\,b\,e^{2\,x}+b\,e^{4\,x}}-\frac{\ln\left(24\,a^{4}+4\,b^{4}-20\,a^{2}\,b^{2}-24\,a^{4}\,e^{x}-4\,b^{4}\,e^{x}+20\,a^{2}\,b^{2}\,e^{x}\right)}{2\,b}+\frac{\ln\left(24\,a^{4}+4\,b^{4}-20\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sinh(x)^4*(a + b/sinh(x))),x)
```

```
[Out] exp(x)/(b - b*exp(2*x)) - (2*exp(x))/(b - 2*b*exp(2*x) + b*exp(4*x)) - log(
24*a^4 + 4*b^4 - 20*a^2*b^2 - 24*a^4*exp(x) - 4*b^4*exp(x) + 20*a^2*b^2*exp
(x))/(2*b) + log(24*a^4 + 4*b^4 - 20*a^2*b^2 + 24*a^4*exp(x) + 4*b^4*exp(x)
 - 20*a^2*b^2*exp(x))/(2*b) + (2*a)/(b^2*exp(2*x) - b^2) + (a^2*log(24*a^4
+ 4*b^4 - 20*a^2*b^2 - 24*a^4*exp(x) - 4*b^4*exp(x) + 20*a^2*b^2*exp(x)))/b
^3 - (a^2*log(24*a^4 + 4*b^4 - 20*a^2*b^2 + 24*a^4*exp(x) + 4*b^4*exp(x) -
20*a^2*b^2*exp(x)))/b^3 - (a^3*log(16*a*b^5 - 24*a^5*(a^2 + b^2)^(1/2) - 48
*a^5*b - 32*a^3*b^3 + 24*a^6*exp(x) - 32*b^6*exp(x) - 40*a^3*b^2*(a^2 + b^2
)^(1/2) - 32*b^5*exp(x)*(a^2 + b^2)^(1/2) + 56*a^2*b^4*exp(x) + 112*a^4*b^2
*exp(x) + 16*a*b^4*(a^2 + b^2)^(1/2) + 72*a^4*b*exp(x)*(a^2 + b^2)^(1/2) +
72*a^2*b^3*exp(x)*(a^2 + b^2)^(1/2))*(a^2 + b^2)^(1/2))/(b^5 + a^2*b^3) + (
a^3*log(24*a^5*(a^2 + b^2)^(1/2) + 16*a*b^5 - 48*a^5*b - 32*a^3*b^3 + 24*a^
6*exp(x) - 32*b^6*exp(x) + 40*a^3*b^2*(a^2 + b^2)^(1/2) + 32*b^5*exp(x)*(a^
2 + b^2)^(1/2) + 56*a^2*b^4*exp(x) + 112*a^4*b^2*exp(x) - 16*a*b^4*(a^2 + b
^2)^(1/2) - 72*a^4*b*exp(x)*(a^2 + b^2)^(1/2) - 72*a^2*b^3*exp(x)*(a^2 + b^
2)^(1/2))*(a^2 + b^2)^(1/2))/(b^5 + a^2*b^3)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}^{4}(x)}{a+b\operatorname{csch}(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)**4/(a+b*csch(x)),x)
```

```
[Out] Integral(csch(x)**4/(a + b*csch(x)), x)
```

3.84 $\int \frac{\cosh^4(x)}{i+\text{csch}(x)} \, dx$

Optimal. Leaf size=38

$$\frac{ix}{8}+\frac{\cosh^3(x)}{3}-\frac{1}{4}i\sinh(x)\cosh^3(x)+\frac{1}{8}i\sinh(x)\cosh(x)$$

```
[Out] 1/8*I*x+1/3*cosh(x)^3+1/8*I*cosh(x)*sinh(x)-1/4*I*cosh(x)^3*sinh(x)
```

Rubi [A] time = 0.13, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.538, Rules used = {3872, 2839, 2565, 30, 2568, 2635, 8}

$$\frac{ix}{8}+\frac{\cosh^3(x)}{3}-\frac{1}{4}i\sinh(x)\cosh^3(x)+\frac{1}{8}i\sinh(x)\cosh(x)$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]^4/(I + Csch[x]),x]
```

```
[Out] (I/8)*x + Cosh[x]^3/3 + (I/8)*Cosh[x]*Sinh[x] - (I/4)*Cosh[x]^3*Sinh[x]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2568

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m - 1)
)/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Cos[e + f*x])^n*(a
*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2839

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[
(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[g^2/(b*d), Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d,
```

```
e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cosh^4(x)}{i+\operatorname{csch}(x)}\,dx &= i\int \frac{\cosh^4(x)\sinh(x)}{i-\sinh(x)}\,dx\\
&= -\left(i\int \cosh^2(x)\sinh^2(x)\,dx\right)+\int \cosh^2(x)\sinh(x)\,dx\\
&= -\frac{1}{4}i\cosh^3(x)\sinh(x)+\frac{1}{4}i\int \cosh^2(x)\,dx+\operatorname{Subst}\left(\int x^2\,dx,x,\cosh(x)\right)\\
&= \frac{\cosh^3(x)}{3}+\frac{1}{8}i\cosh(x)\sinh(x)-\frac{1}{4}i\cosh^3(x)\sinh(x)+\frac{1}{8}i\int 1\,dx\\
&= \frac{ix}{8}+\frac{\cosh^3(x)}{3}+\frac{1}{8}i\cosh(x)\sinh(x)-\frac{1}{4}i\cosh^3(x)\sinh(x)
\end{aligned}
$$

Mathematica [A] time = 0.03, size = 32, normalized size = 0.84

$$\frac{ix}{8}-\frac{1}{32}i\sinh(4x)+\frac{\cosh(x)}{4}+\frac{1}{12}\cosh(3x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]^4/(I + Csch[x]),x]
```

```
[Out] (I/8)*x + Cosh[x]/4 + Cosh[3*x]/12 - (I/32)*Sinh[4*x]
```

fricas [A] time = 0.41, size = 43, normalized size = 1.13

$$\frac{1}{192}\left(24i\,xe^{(4\,x)}-3i\,e^{(8\,x)}+8\,e^{(7\,x)}+24\,e^{(5\,x)}+24\,e^{(3\,x)}+8\,e^{x}+3i\right)e^{(-4\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^4/(I+csch(x)),x, algorithm="fricas")
```

```
[Out] 1/192*(24*I*x*e^(4*x) - 3*I*e^(8*x) + 8*e^(7*x) + 24*e^(5*x) + 24*e^(3*x) +
 8*e^x + 3*I)*e^(-4*x)
```

giac [A] time = 0.14, size = 38, normalized size = 1.00

$$\frac{1}{192}\left(24\,e^{(3\,x)}+8\,e^{x}+3i\right)e^{(-4\,x)}+\frac{1}{8}i\,x-\frac{1}{64}i\,e^{(4\,x)}+\frac{1}{24}\,e^{(3\,x)}+\frac{1}{8}\,e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^4/(I+csch(x)),x, algorithm="giac")
```

```
[Out] 1/192*(24*e^(3*x) + 8*e^x + 3*I)*e^(-4*x) + 1/8*I*x - 1/64*I*e^(4*x) + 1/24
*e^(3*x) + 1/8*e^x
```

maple [B] time = 0.16, size = 170, normalized size = 4.47

$$-\frac{i}{8\left(\tanh\left(\frac{x}{2}\right)-1\right)}+\frac{i}{4\left(\tanh\left(\frac{x}{2}\right)+1\right)^4}-\frac{1}{3\left(\tanh\left(\frac{x}{2}\right)-1\right)^3}-\frac{i\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{8}-\frac{1}{2\left(\tanh\left(\frac{x}{2}\right)-1\right)^2}-\frac{3i}{8\left(\tanh\left(\frac{x}{2}\right)-1\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^4/(I+csch(x)),x)
```

```
[Out] -1/8*I/(tanh(1/2*x)-1)+1/4*I/(tanh(1/2*x)+1)^4-1/3/(tanh(1/2*x)-1)^3-1/8*I*
ln(tanh(1/2*x)-1)-1/2/(tanh(1/2*x)-1)^2-3/8*I/(tanh(1/2*x)-1)^2-1/2/(tanh(1
/2*x)-1)-1/8*I/(tanh(1/2*x)+1)+3/8*I/(tanh(1/2*x)+1)^2+1/8*I*ln(tanh(1/2*x)
+1)+1/3/(tanh(1/2*x)+1)^3-1/2*I/(tanh(1/2*x)+1)^3+1/2/(tanh(1/2*x)+1)-1/2*I
/(tanh(1/2*x)-1)^3-1/2/(tanh(1/2*x)+1)^2-1/4*I/(tanh(1/2*x)-1)^4
```

maxima [A] time = 0.30, size = 42, normalized size = 1.11

$$\frac{1}{192}\left(8\,e^{(-x)} + 24\,e^{(-3\,x)} - 3i\right)e^{(4\,x)} + \frac{1}{8}i\,x + \frac{1}{8}\,e^{(-x)} + \frac{1}{24}\,e^{(-3\,x)} + \frac{1}{64}i\,e^{(-4\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^4/(I+csch(x)),x, algorithm="maxima")
```

```
[Out] 1/192*(8*e^(-x) + 24*e^(-3*x) - 3*I)*e^(4*x) + 1/8*I*x + 1/8*e^(-x) + 1/24*
e^(-3*x) + 1/64*I*e^(-4*x)
```

mupad [B] time = 1.55, size = 41, normalized size = 1.08

$$\frac{\mathrm{e}^{-x}}{8} + \frac{\mathrm{e}^{-3\,x}}{24} + \frac{\mathrm{e}^{3\,x}}{24} + \frac{\mathrm{e}^{x}}{8} + \frac{x\,1\mathrm{i}}{8} + \frac{\mathrm{e}^{-4\,x}\,1\mathrm{i}}{64} - \frac{\mathrm{e}^{4\,x}\,1\mathrm{i}}{64}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^4/(1/sinh(x) + 1i),x)
```

```
[Out] (x*1i)/8 + exp(-x)/8 + exp(-3*x)/24 + exp(3*x)/24 + (exp(-4*x)*1i)/64 - (ex
p(4*x)*1i)/64 + exp(x)/8
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)**4/(I+csch(x)),x)
```

```
[Out] Timed out
```

3.85 $\int \frac{\cosh^3(x)}{i+\text{csch}(x)}\,dx$

Optimal. Leaf size=19

$$\frac{\sinh^2(x)}{2}-\frac{1}{3}i\sinh^3(x)$$

```
[Out] 1/2*sinh(x)^2-1/3*I*sinh(x)^3
```

Rubi [A] time = 0.11, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {3872, 2835, 2564, 30}

$$\frac{\sinh^2(x)}{2}-\frac{1}{3}i\sinh^3(x)$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]^3/(I + Csch[x]),x]
```

```
[Out] Sinh[x]^2/2 - (I/3)*Sinh[x]^3
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2835

```
Int[(cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.))/((
a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[Cos[e + f*
x]^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[1/(b*d), Int[Cos[e + f*x]^(p -
 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n, p}, x] &&
IntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ[n] && (LtQ[0, n, (p +
1)/2] || (LeQ[p, -n] && LtQ[-n, 2*p - 3]) || (GtQ[n, 0] && LeQ[n, -p]))
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cosh^3(x)}{i+\text{csch}(x)}\,dx &= i\int \frac{\cosh^3(x)\sinh(x)}{i-\sinh(x)}\,dx\\
&= -\left(i\int \cosh(x)\sinh^2(x)\,dx\right)+\int \cosh(x)\sinh(x)\,dx\\
&= -\text{Subst}\left(\int x\,dx,x,i\sinh(x)\right)+\text{Subst}\left(\int x^2\,dx,x,i\sinh(x)\right)\\
&= \frac{\sinh^2(x)}{2}-\frac{1}{3}i\sinh^3(x)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 19, normalized size = 1.00

$$\frac{\sinh^2(x)}{2} - \frac{1}{3} i \sinh^3(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]^3/(I + Csch[x]),x]
[Out] Sinh[x]^2/2 - (I/3)*Sinh[x]^3
```

fricas [B] time = 0.40, size = 36, normalized size = 1.89

$$\frac{1}{24} \left(-i e^{(6x)} + 3 e^{(5x)} + 3i e^{(4x)} - 3i e^{(2x)} + 3 e^{x} + i\right) e^{(-3x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^3/(I+csch(x)),x, algorithm="fricas")
[Out] 1/24*(-I*e^(6*x) + 3*e^(5*x) + 3*I*e^(4*x) - 3*I*e^(2*x) + 3*e^x + I)*e^(-3
*x)
```

giac [B] time = 0.14, size = 35, normalized size = 1.84

$$-\frac{1}{24} \left(3i e^{(2x)} - 3 e^{x} - i\right) e^{(-3x)} - \frac{1}{24} i e^{(3x)} + \frac{1}{8} e^{(2x)} + \frac{1}{8} i e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^3/(I+csch(x)),x, algorithm="giac")
[Out] -1/24*(3*I*e^(2*x) - 3*e^x - I)*e^(-3*x) - 1/24*I*e^(3*x) + 1/8*e^(2*x) + 1
/8*I*e^x
```

maple [A] time = 0.11, size = 15, normalized size = 0.79

$$-\frac{i}{3 \operatorname{csch}(x)^3} + \frac{1}{2 \operatorname{csch}(x)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^3/(I+csch(x)),x)
[Out] -1/3*I/csch(x)^3+1/2/csch(x)^2
```

maxima [B] time = 0.30, size = 39, normalized size = 2.05

$$\frac{1}{24} \left(3 e^{(-x)} + 3i e^{(-2x)} - i\right) e^{(3x)} - \frac{1}{8} i e^{(-x)} + \frac{1}{8} e^{(-2x)} + \frac{1}{24} i e^{(-3x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^3/(I+csch(x)),x, algorithm="maxima")
[Out] 1/24*(3*e^(-x) + 3*I*e^(-2*x) - I)*e^(3*x) - 1/8*I*e^(-x) + 1/8*e^(-2*x) +
1/24*I*e^(-3*x)
```

mupad [B] time = 0.14, size = 39, normalized size = 2.05

$$\frac{e^{-2x}}{8} - \frac{e^{-x}\, 1i}{8} + \frac{e^{2x}}{8} + \frac{e^{-3x}\, 1i}{24} - \frac{e^{3x}\, 1i}{24} + \frac{e^{x}\, 1i}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^3/(1/sinh(x) + 1i),x)
```

```
[Out] exp(-2*x)/8 - (exp(-x)*1i)/8 + exp(2*x)/8 + (exp(-3*x)*1i)/24 - (exp(3*x)*1
i)/24 + (exp(x)*1i)/8
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh^3(x)}{\operatorname{csch}(x)+i}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)**3/(I+csch(x)),x)
```

```
[Out] Integral(cosh(x)**3/(csch(x) + I), x)
```

3.86 $\int \frac{\cosh^2(x)}{i+\operatorname{csch}(x)}\,dx$

Optimal. Leaf size=20

$$\frac{ix}{2}+\cosh(x)-\frac{1}{2}i\sinh(x)\cosh(x)$$

```
[Out] 1/2*I*x+cosh(x)-1/2*I*cosh(x)*sinh(x)
```

Rubi [A] time = 0.09, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {3872, 2839, 2638, 2635, 8}

$$\frac{ix}{2}+\cosh(x)-\frac{1}{2}i\sinh(x)\cosh(x)$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]^2/(I + Csch[x]),x]
```

```
[Out] (I/2)*x + Cosh[x] - (I/2)*Cosh[x]*Sinh[x]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2839

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[
(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[g^2/(b*d), Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d,
 e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cosh^2(x)}{i+\operatorname{csch}(x)}\,dx &= i\int \frac{\cosh^2(x)\sinh(x)}{i-\sinh(x)}\,dx \\
&= -\left(i\int \sinh^2(x)\,dx\right)+\int \sinh(x)\,dx \\
&= \cosh(x)-\frac{1}{2}i\cosh(x)\sinh(x)+\frac{1}{2}i\int 1\,dx \\
&= \frac{ix}{2}+\cosh(x)-\frac{1}{2}i\cosh(x)\sinh(x)
\end{aligned}
$$

Mathematica [A] time = 0.03, size = 20, normalized size = 1.00

$$\frac{ix}{2}-\frac{1}{4}i\sinh(2x)+\cosh(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]^2/(I + Csch[x]),x]
```

```
[Out] (I/2)*x + Cosh[x] - (I/4)*Sinh[2*x]
```

fricas [B] time = 0.46, size = 31, normalized size = 1.55

$$\frac{1}{8}\left(4i\,xe^{(2\,x)}-i\,e^{(4\,x)}+4\,e^{(3\,x)}+4\,e^{x}+i\right)e^{(-2\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^2/(I+csch(x)),x, algorithm="fricas")
```

```
[Out] 1/8*(4*I*x*e^(2*x) - I*e^(4*x) + 4*e^(3*x) + 4*e^x + I)*e^(-2*x)
```

giac [B] time = 0.14, size = 26, normalized size = 1.30

$$\frac{1}{8}\,(4\,e^{x}+i)e^{(-2\,x)}+\frac{1}{2}i\,x-\frac{1}{8}i\,e^{(2\,x)}+\frac{1}{2}\,e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^2/(I+csch(x)),x, algorithm="giac")
```

```
[Out] 1/8*(4*e^x + I)*e^(-2*x) + 1/2*I*x - 1/8*I*e^(2*x) + 1/2*e^x
```

maple [B] time = 0.12, size = 84, normalized size = 4.20

$$-\frac{i\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{2}-\frac{i}{2\left(\tanh\left(\frac{x}{2}\right)-1\right)^{2}}-\frac{1}{\tanh\left(\frac{x}{2}\right)-1}-\frac{i}{2\left(\tanh\left(\frac{x}{2}\right)-1\right)}+\frac{i}{2\left(\tanh\left(\frac{x}{2}\right)+1\right)^{2}}+\frac{i\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)}{2}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^2/(I+csch(x)),x)
```

```
[Out] -1/2*I*ln(tanh(1/2*x)-1)-1/2*I/(tanh(1/2*x)-1)^2-1/(tanh(1/2*x)-1)-1/2*I/(t
anh(1/2*x)-1)+1/2*I/(tanh(1/2*x)+1)^2+1/2*I*ln(tanh(1/2*x)+1)+1/(tanh(1/2*x
)+1)-1/2*I/(tanh(1/2*x)+1)
```

maxima [B] time = 0.30, size = 30, normalized size = 1.50

$$\frac{1}{8}\left(4\,e^{(-x)}-i\right)e^{(2\,x)}+\frac{1}{2}i\,x+\frac{1}{2}\,e^{(-x)}+\frac{1}{8}i\,e^{(-2\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^2/(I+csch(x)),x, algorithm="maxima")
[Out] 1/8*(4*e^(-x) - I)*e^(2*x) + 1/2*I*x + 1/2*e^(-x) + 1/8*I*e^(-2*x)
```

mupad [B] time = 1.46, size = 29, normalized size = 1.45

$$\frac{e^{-x}}{2}+\frac{e^{x}}{2}+\frac{x\,1\mathrm{i}}{2}+\frac{e^{-2\,x}\,1\mathrm{i}}{8}-\frac{e^{2\,x}\,1\mathrm{i}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^2/(1/sinh(x) + 1i),x)
[Out] (x*1i)/2 + exp(-x)/2 + (exp(-2*x)*1i)/8 - (exp(2*x)*1i)/8 + exp(x)/2
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh^2(x)}{\operatorname{csch}(x)+i}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)**2/(I+csch(x)),x)
[Out] Integral(cosh(x)**2/(csch(x) + I), x)
```

3.87 $\int \frac{\cosh(x)}{i+\operatorname{csch}(x)}\,dx$

Optimal. Leaf size=16

$$\log(-\sinh(x)+i)-i\sinh(x)$$

```
[Out] ln(I-sinh(x))-I*sinh(x)
```

Rubi [A] time = 0.06, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {3872, 2833, 43}

$$\log(-\sinh(x)+i)-i\sinh(x)$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]/(I + Csch[x]),x]
```

```
[Out] Log[I - Sinh[x]] - I*Sinh[x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2833

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d*x)/b)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cosh(x)}{i+\operatorname{csch}(x)}\,dx &= i\int \frac{\cosh(x)\sinh(x)}{i-\sinh(x)}\,dx\\
&= i\,\text{Subst}\left(\int \frac{x}{i+x}\,dx, x, -\sinh(x)\right)\\
&= i\,\text{Subst}\left(\int \left(1-\frac{i}{i+x}\right)dx, x, -\sinh(x)\right)\\
&= \log(i-\sinh(x))-i\sinh(x)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 16, normalized size = 1.00

$$\log(-\sinh(x)+i)-i\sinh(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]/(I + Csch[x]),x]
```

```
[Out] Log[I - Sinh[x]] - I*Sinh[x]
```

fricas [B] time = 0.60, size = 28, normalized size = 1.75

$$-\frac{1}{2}\left(2\,xe^{x}-4\,e^{x}\log\left(e^{x}-i\right)+i\,e^{(2\,x)}-i\right)e^{(-x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)/(I+csch(x)),x, algorithm="fricas")
[Out] -1/2*(2*x*e^x - 4*e^x*log(e^x - I) + I*e^(2*x) - I)*e^(-x)
```

giac [B] time = 0.12, size = 25, normalized size = 1.56

$$\frac{1}{2}i\,e^{(-x)}-\frac{1}{2}i\,e^{x}-\log\left(-i\,e^{x}\right)+2\,\log\left(e^{x}-i\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)/(I+csch(x)),x, algorithm="giac")
[Out] 1/2*I*e^(-x) - 1/2*I*e^x - log(-I*e^x) + 2*log(e^x - I)
```

maple [A] time = 0.12, size = 20, normalized size = 1.25

$$\ln\left(i+\mathrm{csch}(x)\right)-\ln\left(\mathrm{csch}(x)\right)-\frac{i}{\mathrm{csch}(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)/(I+csch(x)),x)
[Out] ln(I+csch(x))-ln(csch(x))-I/csch(x)
```

maxima [A] time = 0.30, size = 21, normalized size = 1.31

$$x+\frac{1}{2}i\,e^{(-x)}-\frac{1}{2}i\,e^{x}+2\,\log\left(e^{(-x)}+i\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)/(I+csch(x)),x, algorithm="maxima")
[Out] x + 1/2*I*e^(-x) - 1/2*I*e^x + 2*log(e^(-x) + I)
```

mupad [B] time = 0.09, size = 12, normalized size = 0.75

$$\ln\left(\mathrm{sinh}(x)-\mathrm{i}\right)-\mathrm{sinh}(x)\,1\mathrm{i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)/(1/sinh(x) + 1i),x)
[Out] log(sinh(x) - 1i) - sinh(x)*1i
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\cosh\left(x\right)}{\mathrm{csch}\left(x\right)+i}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)/(I+csch(x)),x)
[Out] Integral(cosh(x)/(csch(x) + I), x)
```

3.88 $\int \frac{\text{sech}(x)}{i+\text{csch}(x)}\,dx$

Optimal. Leaf size=28

$$-\frac{1}{2}\text{sech}^2(x)-\frac{1}{2}i\tan^{-1}(\sinh(x))+\frac{1}{2}i\tanh(x)\text{sech}(x)$$

```
[Out] -1/2*I*arctan(sinh(x))-1/2*sech(x)^2+1/2*I*sech(x)*tanh(x)
```

Rubi [A] time = 0.08, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.546, Rules used = {3872, 2706, 2606, 30, 2611, 3770}

$$-\frac{1}{2}\text{sech}^2(x)-\frac{1}{2}i\tan^{-1}(\sinh(x))+\frac{1}{2}i\tanh(x)\text{sech}(x)$$

Antiderivative was successfully verified.

```
[In] Int[Sech[x]/(I + Csch[x]),x]
```

```
[Out] (-I/2)*ArcTan[Sinh[x]] - Sech[x]^2/2 + (I/2)*Sech[x]*Tanh[x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2706

```
Int[((g_.)*tan[(e_.) + (f_.)*(x_)])^(p_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Dist[1/a, Int[Sec[e + f*x]^2*(g*Tan[e + f*x])^p, x], x]
 - Dist[1/(b*g), Int[Sec[e + f*x]*(g*Tan[e + f*x])^(p + 1), x], x] /; FreeQ
[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && NeQ[p, -1]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int \frac{\operatorname{sech}(x)}{i+\operatorname{csch}(x)}\,dx &= i\int \frac{\tanh(x)}{i-\sinh(x)}\,dx\\ &= -\left(i\int \operatorname{sech}(x)\tanh^2(x)\,dx\right)+\int \operatorname{sech}^2(x)\tanh(x)\,dx\\ &= \frac{1}{2}i\operatorname{sech}(x)\tanh(x)-\frac{1}{2}i\int \operatorname{sech}(x)\,dx-\operatorname{Subst}\left(\int x\,dx,x,\operatorname{sech}(x)\right)\\ &= -\frac{1}{2}i\tan^{-1}(\sinh(x))-\frac{\operatorname{sech}^2(x)}{2}+\frac{1}{2}i\operatorname{sech}(x)\tanh(x)\end{aligned}$$

Mathematica [A] time = 0.03, size = 20, normalized size = 0.71

$$-\frac{1}{2}i\left(\tan^{-1}(\sinh(x))+\frac{1}{-\sinh(x)+i}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[x]/(I + Csch[x]),x]
```

```
[Out] (-1/2*I)*(ArcTan[Sinh[x]] + (I - Sinh[x])^(-1))
```

fricas [B] time = 0.40, size = 53, normalized size = 1.89

$$\frac{\left(e^{(2x)}-2ie^{x}-1\right)\log\left(e^{x}+i\right)-\left(e^{(2x)}-2ie^{x}-1\right)\log\left(e^{x}-i\right)+2ie^{x}}{2e^{(2x)}-4ie^{x}-2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)/(I+csch(x)),x, algorithm="fricas")
```

```
[Out] ((e^(2*x) - 2*I*e^x - 1)*log(e^x + I) - (e^(2*x) - 2*I*e^x - 1)*log(e^x - I
) + 2*I*e^x)/(2*e^(2*x) - 4*I*e^x - 2)
```

giac [B] time = 0.14, size = 53, normalized size = 1.89

$$\frac{e^{(-x)}-e^{x}-2i}{4\left(e^{(-x)}-e^{x}+2i\right)}+\frac{1}{4}\log\left(-ie^{(-x)}+ie^{x}-2\right)-\frac{1}{4}\log\left(-e^{(-x)}+e^{x}-2i\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)/(I+csch(x)),x, algorithm="giac")
```

```
[Out] 1/4*(e^(-x) - e^x - 2*I)/(e^(-x) - e^x + 2*I) + 1/4*log(-I*e^(-x) + I*e^x -
 2) - 1/4*log(-e^(-x) + e^x - 2*I)
```

maple [B] time = 0.18, size = 43, normalized size = 1.54

$$\frac{\ln\left(\tanh\left(\frac{x}{2}\right)+i\right)}{2}-\frac{i}{\tanh\left(\frac{x}{2}\right)-i}+\frac{1}{\left(\tanh\left(\frac{x}{2}\right)-i\right)^{2}}-\frac{\ln\left(\tanh\left(\frac{x}{2}\right)-i\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(x)/(I+csch(x)),x)
```

```
[Out] 1/2*ln(tanh(1/2*x)+I)-I/(tanh(1/2*x)-I)+1/(tanh(1/2*x)-I)^2-1/2*ln(tanh(1/2
*x)-I)
```

maxima [B] time = 0.30, size = 41, normalized size = 1.46

$$-\frac{2i\,e^{(-x)}}{4i\,e^{(-x)}+2\,e^{(-2\,x)}-2}-\frac{1}{2}\,\log\left(e^{(-x)}+i\right)+\frac{1}{2}\,\log\left(e^{(-x)}-i\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)/(I+csch(x)),x, algorithm="maxima")
```

```
[Out] -2*I*e^(-x)/(4*I*e^(-x) + 2*e^(-2*x) - 2) - 1/2*log(e^(-x) + I) + 1/2*log(e
^(-x) - I)
```

mupad [B] time = 0.23, size = 46, normalized size = 1.64

$$\frac{\ln\left(-1+\mathrm{e}^{x}\,1\mathrm{i}\right)}{2}-\frac{\ln\left(1+\mathrm{e}^{x}\,1\mathrm{i}\right)}{2}+\frac{1}{1-\mathrm{e}^{2\,x}+\mathrm{e}^{x}\,2\mathrm{i}}+\frac{1\mathrm{i}}{\mathrm{e}^{x}-\mathrm{i}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(x)*(1/sinh(x) + 1i)),x)
```

```
[Out] log(exp(x)*1i - 1)/2 - log(exp(x)*1i + 1)/2 + 1/(exp(x)*2i - exp(2*x) + 1)
+ 1i/(exp(x) - 1i)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\operatorname{sech}\left(x\right)}{\operatorname{csch}\left(x\right)+i}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)/(I+csch(x)),x)
```

```
[Out] Integral(sech(x)/(csch(x) + I), x)
```

3.89 $\int \frac{\text{sech}^2(x)}{i+\text{csch}(x)}\,dx$

Optimal. Leaf size=19

$$-\frac{1}{3}\text{sech}^3(x)-\frac{1}{3}i\tanh^3(x)$$

```
[Out] -1/3*sech(x)^3-1/3*I*tanh(x)^3
```

Rubi [A] time = 0.11, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {3872, 2839, 2606, 30, 2607}

$$-\frac{1}{3}\text{sech}^3(x)-\frac{1}{3}i\tanh^3(x)$$

Antiderivative was successfully verified.

```
[In] Int[Sech[x]^2/(I + Csch[x]),x]
```

```
[Out] -Sech[x]^3/3 - (I/3)*Tanh[x]^3
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2839

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[
(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[g^2/(b*d), Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d,
 e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\operatorname{sech}^2(x)}{i+\operatorname{csch}(x)}\,dx &= i\int \frac{\operatorname{sech}(x)\tanh(x)}{i-\sinh(x)}\,dx\\
&= -\left(i\int \operatorname{sech}^2(x)\tanh^2(x)\,dx\right)+\int \operatorname{sech}^3(x)\tanh(x)\,dx\\
&= -\operatorname{Subst}\left(\int x^2\,dx,x,\operatorname{sech}(x)\right)+\operatorname{Subst}\left(\int x^2\,dx,x,i\tanh(x)\right)\\
&= -\frac{1}{3}\operatorname{sech}^3(x)-\frac{1}{3}i\tanh^3(x)
\end{aligned}
$$

Mathematica [B] time = 0.06, size = 64, normalized size = 3.37

$$
\frac{-2i\sinh(x)+\cosh(x)+\cosh(2x)+i\sinh(x)\cosh(x)-3}{6\left(\cosh\left(\frac{x}{2}\right)-i\sinh\left(\frac{x}{2}\right)\right)\left(\cosh\left(\frac{x}{2}\right)+i\sinh\left(\frac{x}{2}\right)\right)^3}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[x]^2/(I + Csch[x]),x]
```

```
[Out] (-3 + Cosh[x] + Cosh[2*x] - (2*I)*Sinh[x] + I*Cosh[x]*Sinh[x])/(6*(Cosh[x/2
] - I*Sinh[x/2])*(Cosh[x/2] + I*Sinh[x/2])^3)
```

fricas [B] time = 0.39, size = 33, normalized size = 1.74

$$
\frac{6i\,e^{(2\,x)}+4\,e^{x}-2i}{3\,e^{(4\,x)}-6i\,e^{(3\,x)}-6i\,e^{x}-3}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^2/(I+csch(x)),x, algorithm="fricas")
```

```
[Out] (6*I*e^(2*x) + 4*e^x - 2*I)/(3*e^(4*x) - 6*I*e^(3*x) - 6*I*e^x - 3)
```

giac [B] time = 0.14, size = 27, normalized size = 1.42

$$
-\frac{i}{2\,(i\,e^{x}-1)}+\frac{3\,e^{(2\,x)}-1}{6\,(e^{x}-i)^3}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^2/(I+csch(x)),x, algorithm="giac")
```

```
[Out] -1/2*I/(I*e^x - 1) + 1/6*(3*e^(2*x) - 1)/(e^x - I)^3
```

maple [B] time = 0.20, size = 49, normalized size = 2.58

$$
-\frac{i}{2\left(\tanh\left(\frac{x}{2}\right)+i\right)}+\frac{i}{2\tanh\left(\frac{x}{2}\right)-2i}-\frac{2i}{3\left(\tanh\left(\frac{x}{2}\right)-i\right)^3}-\frac{1}{\left(\tanh\left(\frac{x}{2}\right)-i\right)^2}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(x)^2/(I+csch(x)),x)
```

```
[Out] -1/2*I/(tanh(1/2*x)+I)+1/2*I/(tanh(1/2*x)-I)-2/3*I/(tanh(1/2*x)-I)^3-1/(tan
h(1/2*x)-I)^2
```

maxima [B] time = 0.31, size = 81, normalized size = 4.26

$$
\frac{8\,e^{(-x)}}{12i\,e^{(-x)}+12i\,e^{(-3\,x)}+6\,e^{(-4\,x)}-6}-\frac{12i\,e^{(-2\,x)}}{12i\,e^{(-x)}+12i\,e^{(-3\,x)}+6\,e^{(-4\,x)}-6}+\frac{4i}{12i\,e^{(-x)}+12i\,e^{(-3\,x)}+6\,e^{(-4\,x)}-6}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^2/(I+csch(x)),x, algorithm="maxima")
```

```
[Out] 8*e^(-x)/(12*I*e^(-x) + 12*I*e^(-3*x) + 6*e^(-4*x) - 6) - 12*I*e^(-2*x)/(12
*I*e^(-x) + 12*I*e^(-3*x) + 6*e^(-4*x) - 6) + 4*I/(12*I*e^(-x) + 12*I*e^(-3
*x) + 6*e^(-4*x) - 6)
```

mupad [B] time = 1.57, size = 31, normalized size = 1.63

$$-\frac{\frac{2}{3}-2\,e^{2\,x}+\frac{e^{x}\,4i}{3}}{\left(e^{x}+1i\right)\,{\left(1+e^{x}\,1i\right)}^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(x)^2*(1/sinh(x) + 1i)),x)
```

```
[Out] -((exp(x)*4i)/3 - 2*exp(2*x) + 2/3)/((exp(x) + 1i)*(exp(x)*1i + 1)^3)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}^{2}(x)}{\operatorname{csch}(x)+i}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)**2/(I+csch(x)),x)
```

```
[Out] Integral(sech(x)**2/(csch(x) + I), x)
```

3.90 $\int \frac{\text{sech}^3(x)}{i+\text{csch}(x)}\,dx$

Optimal. Leaf size=40

$$-\frac{1}{4}\text{sech}^4(x)-\frac{1}{8}i\tan^{-1}(\sinh(x))+\frac{1}{4}i\tanh(x)\text{sech}^3(x)-\frac{1}{8}i\tanh(x)\text{sech}(x)$$

```
[Out] -1/8*I*arctan(sinh(x))-1/4*sech(x)^4-1/8*I*sech(x)*tanh(x)+1/4*I*sech(x)^3*
tanh(x)
```

Rubi [A] time = 0.13, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.538, Rules used = {3872, 2835, 2606, 30, 2611, 3768, 3770}

$$-\frac{1}{4}\text{sech}^4(x)-\frac{1}{8}i\tan^{-1}(\sinh(x))+\frac{1}{4}i\tanh(x)\text{sech}^3(x)-\frac{1}{8}i\tanh(x)\text{sech}(x)$$

Antiderivative was successfully verified.

```
[In] Int[Sech[x]^3/(I + Csch[x]),x]
```

```
[Out] (-I/8)*ArcTan[Sinh[x]] - Sech[x]^4/4 - (I/8)*Sech[x]*Tanh[x] + (I/4)*Sech[x
]^3*Tanh[x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2835

```
Int[(cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.))/((
a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[Cos[e + f*
x]^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[1/(b*d), Int[Cos[e + f*x]^(p -
 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n, p}, x] &&
IntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ[n] && (LtQ[0, n, (p +
1)/2] || (LeQ[p, -n] && LtQ[-n, 2*p - 3]) || (GtQ[n, 0] && LeQ[n, -p]))
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int \frac{\operatorname{sech}^3(x)}{i+\operatorname{csch}(x)}\,dx &= i\int \frac{\operatorname{sech}^2(x)\tanh(x)}{i-\sinh(x)}\,dx\\ &= -\left(i\int \operatorname{sech}^3(x)\tanh^2(x)\,dx\right)+\int \operatorname{sech}^4(x)\tanh(x)\,dx\\ &= \frac{1}{4}i\operatorname{sech}^3(x)\tanh(x)-\frac{1}{4}i\int \operatorname{sech}^3(x)\,dx-\operatorname{Subst}\left(\int x^3\,dx,x,\operatorname{sech}(x)\right)\\ &= -\frac{1}{4}\operatorname{sech}^4(x)-\frac{1}{8}i\operatorname{sech}(x)\tanh(x)+\frac{1}{4}i\operatorname{sech}^3(x)\tanh(x)-\frac{1}{8}i\int \operatorname{sech}(x)\,dx\\ &= -\frac{1}{8}i\tan^{-1}(\sinh(x))-\frac{\operatorname{sech}^4(x)}{4}-\frac{1}{8}i\operatorname{sech}(x)\tanh(x)+\frac{1}{4}i\operatorname{sech}^3(x)\tanh(x)\end{aligned}$$

Mathematica [A] time = 0.06, size = 32, normalized size = 0.80

$$\frac{1}{8}\left(-\frac{i}{\sinh(x)+i}+\frac{1}{(\sinh(x)-i)^2}-i\tan^{-1}(\sinh(x))\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[x]^3/(I + Csch[x]),x]
[Out] ((-I)*ArcTan[Sinh[x]] + (-I + Sinh[x])^(-2) - I/(I + Sinh[x]))/8
```

fricas [B] time = 0.41, size = 145, normalized size = 3.62

$$\frac{\left(e^{(6x)}-2i\,e^{(5x)}+e^{(4x)}-4i\,e^{(3x)}-e^{(2x)}-2i\,e^{x}-1\right)\log\left(e^{x}+i\right)-\left(e^{(6x)}-2i\,e^{(5x)}+e^{(4x)}-4i\,e^{(3x)}-e^{(2x)}-2i\,e^{x}\right.}{8\,e^{(6x)}-16i\,e^{(5x)}+8\,e^{(4x)}-32i\,e^{(3x)}-8\,e^{(2x)}-16i\,e^{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^3/(I+csch(x)),x, algorithm="fricas")
[Out] ((e^(6*x) - 2*I*e^(5*x) + e^(4*x) - 4*I*e^(3*x) - e^(2*x) - 2*I*e^x - 1)*lo
g(e^x + I) - (e^(6*x) - 2*I*e^(5*x) + e^(4*x) - 4*I*e^(3*x) - e^(2*x) - 2*I
*e^x - 1)*log(e^x - I) - 2*I*e^(5*x) - 4*e^(4*x) + 20*I*e^(3*x) + 4*e^(2*x)
 - 2*I*e^x)/(8*e^(6*x) - 16*I*e^(5*x) + 8*e^(4*x) - 32*I*e^(3*x) - 8*e^(2*x
) - 16*I*e^x - 8)
```

giac [B] time = 0.13, size = 94, normalized size = 2.35

$$-\frac{-i\,e^{(-x)}+i\,e^{x}-6}{16\left(-i\,e^{(-x)}+i\,e^{x}-2\right)}+\frac{3\left(e^{(-x)}-e^{x}\right)^2+12i\,e^{(-x)}-12i\,e^{x}+4}{32\left(e^{(-x)}-e^{x}+2i\right)^2}+\frac{1}{16}\log\left(-e^{(-x)}+e^{x}+2i\right)-\frac{1}{16}\log\left(-e^{(-x)}+e^{x}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^3/(I+csch(x)),x, algorithm="giac")
```

```
[Out] -1/16*(-I*e^(-x) + I*e^x - 6)/(-I*e^(-x) + I*e^x - 2) + 1/32*(3*(e^(-x) - e
^x)^2 + 12*I*e^(-x) - 12*I*e^x + 4)/(e^(-x) - e^x + 2*I)^2 + 1/16*log(-e^(-
x) + e^x + 2*I) - 1/16*log(-e^(-x) + e^x - 2*I)
```

maple [B] time = 0.20, size = 89, normalized size = 2.22

$$\frac{i}{4\tanh\left(\frac{x}{2}\right)+4i}+\frac{1}{4\left(\tanh\left(\frac{x}{2}\right)+i\right)^{2}}+\frac{\ln\left(\tanh\left(\frac{x}{2}\right)+i\right)}{8}+\frac{i}{\left(\tanh\left(\frac{x}{2}\right)-i\right)^{3}}-\frac{i}{2\left(\tanh\left(\frac{x}{2}\right)-i\right)}-\frac{1}{2\left(\tanh\left(\frac{x}{2}\right)-i\right)^{4}}+\frac{}{\left(\tan\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(x)^3/(I+csch(x)),x)
```

```
[Out] 1/4*I/(tanh(1/2*x)+I)+1/4/(tanh(1/2*x)+I)^2+1/8*ln(tanh(1/2*x)+I)+I/(tanh(1
/2*x)-I)^3-1/2*I/(tanh(1/2*x)-I)-1/2/(tanh(1/2*x)-I)^4+1/(tanh(1/2*x)-I)^2-
1/8*ln(tanh(1/2*x)-I)
```

maxima [B] time = 0.31, size = 92, normalized size = 2.30

$$\frac{8\left(i\,e^{(-x)}+2\,e^{(-2\,x)}-10i\,e^{(-3\,x)}-2\,e^{(-4\,x)}+i\,e^{(-5\,x)}\right)}{64i\,e^{(-x)}-32\,e^{(-2\,x)}+128i\,e^{(-3\,x)}+32\,e^{(-4\,x)}+64i\,e^{(-5\,x)}+32\,e^{(-6\,x)}-32}-\frac{1}{8}\log\left(e^{(-x)}+i\right)+\frac{1}{8}\log\left(e^{(-x)}-i\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^3/(I+csch(x)),x, algorithm="maxima")
```

```
[Out] 8*(I*e^(-x) + 2*e^(-2*x) - 10*I*e^(-3*x) - 2*e^(-4*x) + I*e^(-5*x))/(64*I*e
^(-x) - 32*e^(-2*x) + 128*I*e^(-3*x) + 32*e^(-4*x) + 64*I*e^(-5*x) + 32*e^(
-6*x) - 32) - 1/8*log(e^(-x) + I) + 1/8*log(e^(-x) - I)
```

mupad [B] time = 1.95, size = 122, normalized size = 3.05

$$\frac{\ln\left(-\frac{1}{4}+\frac{e^{x}\,1i}{4}\right)}{8}-\frac{\ln\left(\frac{1}{4}+\frac{e^{x}\,1i}{4}\right)}{8}-\frac{1i}{e^{2\,x}\,3i-e^{3\,x}+3\,e^{x}-i}-\frac{1}{4\,\left(e^{2\,x}-1+e^{x}\,2i\right)}-\frac{1}{2\,\left(e^{4\,x}-6\,e^{2\,x}+1-e^{3\,x}\,4i+e^{x}\,4i\right)}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(x)^3*(1/sinh(x) + 1i)),x)
```

```
[Out] log((exp(x)*1i)/4 - 1/4)/8 - log((exp(x)*1i)/4 + 1/4)/8 - 1i/(exp(2*x)*3i -
 exp(3*x) + 3*exp(x) - 1i) - 1/(4*(exp(2*x) + exp(x)*2i - 1)) - 1/(2*(exp(4
*x) - exp(3*x)*4i - 6*exp(2*x) + exp(x)*4i + 1)) - 1/(2*(exp(x)*2i - exp(2*
x) + 1)) - 1i/(4*(exp(x) + 1i))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\operatorname{sech}^{3}(x)}{\operatorname{csch}(x)+i}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)**3/(I+csch(x)),x)
```

```
[Out] Integral(sech(x)**3/(csch(x) + I), x)
```

3.91 $\int \frac{\textbf{sech}^4(x)}{i+\textbf{csch}(x)}\,dx$

Optimal. Leaf size=29

$$\frac{1}{5}i\tanh^5(x) - \frac{1}{3}i\tanh^3(x) - \frac{1}{5}\text{sech}^5(x)$$

```
[Out] -1/5*sech(x)^5-1/3*I*tanh(x)^3+1/5*I*tanh(x)^5
```

Rubi [A] time = 0.12, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.462$, Rules used = {3872, 2839, 2606, 30, 2607, 14}

$$\frac{1}{5}i\tanh^5(x) - \frac{1}{3}i\tanh^3(x) - \frac{1}{5}\text{sech}^5(x)$$

Antiderivative was successfully verified.

```
[In] Int[Sech[x]^4/(I + Csch[x]),x]
```

```
[Out] -Sech[x]^5/5 - (I/3)*Tanh[x]^3 + (I/5)*Tanh[x]^5
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2839

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[
(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[g^2/(b*d), Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d,
 e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int \frac{\operatorname{sech}^4(x)}{i+\operatorname{csch}(x)}\,dx &= i\int \frac{\operatorname{sech}^3(x)\tanh(x)}{i-\sinh(x)}\,dx\\ &= -\left(i\int \operatorname{sech}^4(x)\tanh^2(x)\,dx\right)+\int \operatorname{sech}^5(x)\tanh(x)\,dx\\ &= -\operatorname{Subst}\left(\int x^4\,dx,x,\operatorname{sech}(x)\right)+\operatorname{Subst}\left(\int x^2\left(1+x^2\right)\,dx,x,i\tanh(x)\right)\\ &= -\frac{1}{5}\operatorname{sech}^5(x)+\operatorname{Subst}\left(\int \left(x^2+x^4\right)\,dx,x,i\tanh(x)\right)\\ &= -\frac{1}{5}\operatorname{sech}^5(x)-\frac{1}{3}i\tanh^3(x)+\frac{1}{5}i\tanh^5(x)\end{aligned}$$

Mathematica [B] time = 0.11, size = 96, normalized size = 3.31

$$\frac{-96i\sinh(x)+18i\sinh(2x)-32i\sinh(3x)+9i\sinh(4x)+54\cosh(x)+32\cosh(2x)+18\cosh(3x)+16\cosh(4}{960\left(\cosh\left(\frac{x}{2}\right)-i\sinh\left(\frac{x}{2}\right)\right)^3\left(\cosh\left(\frac{x}{2}\right)+i\sinh\left(\frac{x}{2}\right)\right)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[x]^4/(I + Csch[x]),x]
```

```
[Out] (-240 + 54*Cosh[x] + 32*Cosh[2*x] + 18*Cosh[3*x] + 16*Cosh[4*x] - (96*I)*Si
nh[x] + (18*I)*Sinh[2*x] - (32*I)*Sinh[3*x] + (9*I)*Sinh[4*x])/(960*(Cosh[x
/2] - I*Sinh[x/2])^3*(Cosh[x/2] + I*Sinh[x/2])^5)
```

fricas [B] time = 0.52, size = 69, normalized size = 2.38

$$\frac{60i\,e^{(4x)}+24\,e^{(3x)}-8i\,e^{(2x)}+8\,e^{x}-4i}{15\,e^{(8x)}-30i\,e^{(7x)}+30\,e^{(6x)}-90i\,e^{(5x)}-90i\,e^{(3x)}-30\,e^{(2x)}-30i\,e^{x}-15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^4/(I+csch(x)),x, algorithm="fricas")
```

```
[Out] (60*I*e^(4*x) + 24*e^(3*x) - 8*I*e^(2*x) + 8*e^x - 4*I)/(15*e^(8*x) - 30*I*
e^(7*x) + 30*e^(6*x) - 90*I*e^(5*x) - 90*I*e^(3*x) - 30*e^(2*x) - 30*I*e^x
- 15)
```

giac [B] time = 0.13, size = 55, normalized size = 1.90

$$-\frac{-3i\,e^{(2x)}+12\,e^{x}+5i}{24\,(i\,e^{x}-1)^3}+\frac{15\,e^{(4x)}-60i\,e^{(3x)}-10\,e^{(2x)}+20i\,e^{x}+7}{120\,(e^{x}-i)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^4/(I+csch(x)),x, algorithm="giac")
```

```
[Out] -1/24*(-3*I*e^(2*x) + 12*e^x + 5*I)/(I*e^x - 1)^3 + 1/120*(15*e^(4*x) - 60*
I*e^(3*x) - 10*e^(2*x) + 20*I*e^x + 7)/(e^x - I)^5
```

maple [B] time = 0.20, size = 93, normalized size = 3.21

$$\frac{i}{6\left(\tanh\left(\frac{x}{2}\right)+i\right)^3}-\frac{3i}{8\left(\tanh\left(\frac{x}{2}\right)+i\right)}-\frac{1}{4\left(\tanh\left(\frac{x}{2}\right)+i\right)^2}-\frac{4i}{3\left(\tanh\left(\frac{x}{2}\right)-i\right)^3}+\frac{3i}{8\left(\tanh\left(\frac{x}{2}\right)-i\right)}+\frac{2i}{5\left(\tanh\left(\frac{x}{2}\right)-i\right)^5}+-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(x)^4/(I+csch(x)),x)
```

```
[Out] 1/6*I/(tanh(1/2*x)+I)^3-3/8*I/(tanh(1/2*x)+I)-1/4/(tanh(1/2*x)+I)^2-4/3*I/(
tanh(1/2*x)-I)^3+3/8*I/(tanh(1/2*x)-I)+2/5*I/(tanh(1/2*x)-I)^5+1/(tanh(1/2*
x)-I)^4-1/(tanh(1/2*x)-I)^2
```

maxima [B] time = 0.31, size = 257, normalized size = 8.86

$$\frac{32\,e^{(-x)}}{120i\,e^{(-x)}-120\,e^{(-2\,x)}+360i\,e^{(-3\,x)}+360i\,e^{(-5\,x)}+120\,e^{(-6\,x)}+120i\,e^{(-7\,x)}+60\,e^{(-8\,x)}-60}+\frac{}{120i\,e^{(-x)}-120\,e^{(-}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^4/(I+csch(x)),x, algorithm="maxima")
```

```
[Out] 32*e^(-x)/(120*I*e^(-x) - 120*e^(-2*x) + 360*I*e^(-3*x) + 360*I*e^(-5*x) +
120*e^(-6*x) + 120*I*e^(-7*x) + 60*e^(-8*x) - 60) + 32*I*e^(-2*x)/(120*I*e^
(-x) - 120*e^(-2*x) + 360*I*e^(-3*x) + 360*I*e^(-5*x) + 120*e^(-6*x) + 120*
I*e^(-7*x) + 60*e^(-8*x) - 60) + 96*e^(-3*x)/(120*I*e^(-x) - 120*e^(-2*x) +
 360*I*e^(-3*x) + 360*I*e^(-5*x) + 120*e^(-6*x) + 120*I*e^(-7*x) + 60*e^(-8
*x) - 60) - 240*I*e^(-4*x)/(120*I*e^(-x) - 120*e^(-2*x) + 360*I*e^(-3*x) +
360*I*e^(-5*x) + 120*e^(-6*x) + 120*I*e^(-7*x) + 60*e^(-8*x) - 60) + 16*I/(
120*I*e^(-x) - 120*e^(-2*x) + 360*I*e^(-3*x) + 360*I*e^(-5*x) + 120*e^(-6*x
) + 120*I*e^(-7*x) + 60*e^(-8*x) - 60)
```

mupad [B] time = 1.92, size = 207, normalized size = 7.14

$$-\frac{1\mathrm{i}}{4\,\left(\mathrm{e}^{2\,x}-1+\mathrm{e}^{x}\,2\mathrm{i}\right)}+\frac{1}{20\,\left(\mathrm{e}^{x}-\mathrm{i}\right)}-\frac{1}{8\,\left(\mathrm{e}^{x}+1\mathrm{i}\right)}+\frac{\frac{\mathrm{e}^{3\,x}}{40}-\frac{\mathrm{e}^{2\,x}\,3\mathrm{i}}{40}+\frac{\mathrm{e}^{x}}{8}+\frac{1}{40}\mathrm{i}}{\mathrm{e}^{4\,x}-6\,\mathrm{e}^{2\,x}+1-\mathrm{e}^{3\,x}\,4\mathrm{i}+\mathrm{e}^{x}\,4\mathrm{i}}-\frac{\frac{\mathrm{e}^{2\,x}}{40}+\frac{1}{24}-\frac{\mathrm{e}^{x}\,1\mathrm{i}}{20}}{\mathrm{e}^{2\,x}\,3\mathrm{i}-\mathrm{e}^{3\,x}+3\,\mathrm{e}^{x}-\mathrm{i}}-\frac{}{6\,\left(\mathrm{e}^{2}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(x)^4*(1/sinh(x) + 1i)),x)
```

```
[Out] 1/(20*(exp(x) - 1i)) - 1i/(4*(exp(2*x) + exp(x)*2i - 1)) - 1/(8*(exp(x) + 1
i)) + (exp(3*x)/40 - (exp(2*x)*3i)/40 + exp(x)/8 + 1i/40)/(exp(4*x) - exp(3
*x)*4i - 6*exp(2*x) + exp(x)*4i + 1) - (exp(2*x)/40 - (exp(x)*1i)/20 + 1/24
)/(exp(2*x)*3i - exp(3*x) + 3*exp(x) - 1i) - 1/(6*(exp(2*x)*3i + exp(3*x) -
 3*exp(x) - 1i)) + (exp(2*x)/4 - (exp(3*x)*1i)/10 + exp(4*x)/40 + (exp(x)*1
i)/10 + 1/40)/(exp(2*x)*10i - 10*exp(3*x) - exp(4*x)*5i + exp(5*x) + 5*exp(
x) - 1i)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}^{4}(x)}{\operatorname{csch}(x)+i}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)**4/(I+csch(x)),x)
```

```
[Out] Integral(sech(x)**4/(csch(x) + I), x)
```

3.92 $\int \frac{\cosh^5(x)}{a+b\,\mathbf{csch}(x)}\,dx$

Optimal. Leaf size=102

$$-\frac{b\sinh^4(x)}{4a^2}-\frac{b\left(a^2+b^2\right)^2\log(a\sinh(x)+b)}{a^6}+\frac{\left(a^2+b^2\right)^2\sinh(x)}{a^5}-\frac{b\left(2a^2+b^2\right)\sinh^2(x)}{2a^4}+\frac{\left(2a^2+b^2\right)\sinh^3(x)}{3a^3}+\frac{\text{si}}{}$$

```
[Out] -b*(a^2+b^2)^2*ln(b+a*sinh(x))/a^6+(a^2+b^2)^2*sinh(x)/a^5-1/2*b*(2*a^2+b^2
)*sinh(x)^2/a^4+1/3*(2*a^2+b^2)*sinh(x)^3/a^3-1/4*b*sinh(x)^4/a^2+1/5*sinh(
x)^5/a
```

Rubi [A] time = 0.20, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {3872, 2837, 12, 772}

$$\frac{\left(2a^2+b^2\right)\sinh^3(x)}{3a^3}-\frac{b\left(2a^2+b^2\right)\sinh^2(x)}{2a^4}+\frac{\left(a^2+b^2\right)^2\sinh(x)}{a^5}-\frac{b\left(a^2+b^2\right)^2\log(a\sinh(x)+b)}{a^6}-\frac{b\sinh^4(x)}{4a^2}+\frac{\text{sin}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]^5/(a + b*Csch[x]),x]
```

```
[Out] -((b*(a^2 + b^2)^2*Log[b + a*Sinh[x]])/a^6) + ((a^2 + b^2)^2*Sinh[x])/a^5 -
 (b*(2*a^2 + b^2)*Sinh[x]^2)/(2*a^4) + ((2*a^2 + b^2)*Sinh[x]^3)/(3*a^3) -
(b*Sinh[x]^4)/(4*a^2) + Sinh[x]^5/(5*a)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 772

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(
p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*(a + c*x^2)^p,
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IGtQ[p, 0]
```

Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\int \frac{\cosh^5(x)}{a+b\text{csch}(x)}\,dx = i\int \frac{\cosh^5(x)\sinh(x)}{ib+ia\sinh(x)}\,dx$$
$$= -\frac{i\,\text{Subst}\left(\int \frac{x\left(a^2-x^2\right)^2}{a(ib+x)}\,dx, x, ia\sinh(x)\right)}{a^5}$$
$$= -\frac{i\,\text{Subst}\left(\int \frac{x\left(a^2-x^2\right)^2}{ib+x}\,dx, x, ia\sinh(x)\right)}{a^6}$$
$$= -\frac{i\,\text{Subst}\left(\int \left(\left(a^2+b^2\right)^2 - \frac{b\left(a^2+b^2\right)^2}{b-ix} + ib\left(2a^2+b^2\right)x - \left(2a^2+b^2\right)x^2 - ibx^3 + x^4\right) dx, x, i}{a^6}$$
$$= -\frac{b\left(a^2+b^2\right)^2\log(b+a\sinh(x))}{a^6} + \frac{\left(a^2+b^2\right)^2\sinh(x)}{a^5} - \frac{b\left(2a^2+b^2\right)\sinh^2(x)}{2a^4} + \frac{\left(2a^2+\right.}{}$$

Mathematica [A] time = 0.29, size = 97, normalized size = 0.95

$$\frac{12a^5\sinh^5(x) - 15a^4b\sinh^4(x) - 30a^2b\left(2a^2+b^2\right)\sinh^2(x) + 60a\left(a^2+b^2\right)^2\sinh(x) - 60b\left(a^2+b^2\right)^2\log(a\,\text{si}}{60a^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]^5/(a + b*Csch[x]),x]
```

```
[Out] (-60*b*(a^2 + b^2)^2*Log[b + a*Sinh[x]] + 60*a*(a^2 + b^2)^2*Sinh[x] - 30*a
^2*b*(2*a^2 + b^2)*Sinh[x]^2 + 20*a^3*(2*a^2 + b^2)*Sinh[x]^3 - 15*a^4*b*Si
nh[x]^4 + 12*a^5*Sinh[x]^5)/(60*a^6)
```

fricas [B] time = 0.56, size = 1398, normalized size = 13.71

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^5/(a+b*csch(x)),x, algorithm="fricas")
```

```
[Out] 1/960*(6*a^5*cosh(x)^10 + 6*a^5*sinh(x)^10 - 15*a^4*b*cosh(x)^9 + 15*(4*a^5
*cosh(x) - a^4*b)*sinh(x)^9 + 10*(5*a^5 + 4*a^3*b^2)*cosh(x)^8 + 5*(54*a^5*
cosh(x)^2 - 27*a^4*b*cosh(x) + 10*a^5 + 8*a^3*b^2)*sinh(x)^8 - 60*(3*a^4*b
+ 2*a^2*b^3)*cosh(x)^7 + 20*(36*a^5*cosh(x)^3 - 27*a^4*b*cosh(x)^2 - 9*a^4*
b - 6*a^2*b^3 + 4*(5*a^5 + 4*a^3*b^2)*cosh(x))*sinh(x)^7 + 960*(a^4*b + 2*a
^2*b^3 + b^5)*x*cosh(x)^5 + 60*(5*a^5 + 14*a^3*b^2 + 8*a*b^4)*cosh(x)^6 + 2
0*(63*a^5*cosh(x)^4 - 63*a^4*b*cosh(x)^3 + 15*a^5 + 42*a^3*b^2 + 24*a*b^4 +
 14*(5*a^5 + 4*a^3*b^2)*cosh(x)^2 - 21*(3*a^4*b + 2*a^2*b^3)*cosh(x))*sinh(
x)^6 - 15*a^4*b*cosh(x) + 2*(756*a^5*cosh(x)^5 - 945*a^4*b*cosh(x)^4 + 280*
(5*a^5 + 4*a^3*b^2)*cosh(x)^3 - 630*(3*a^4*b + 2*a^2*b^3)*cosh(x)^2 + 480*(
a^4*b + 2*a^2*b^3 + b^5)*x + 180*(5*a^5 + 14*a^3*b^2 + 8*a*b^4)*cosh(x))*si
nh(x)^5 - 6*a^5 - 60*(5*a^5 + 14*a^3*b^2 + 8*a*b^4)*cosh(x)^4 + 10*(126*a^5
*cosh(x)^6 - 189*a^4*b*cosh(x)^5 - 30*a^5 - 84*a^3*b^2 - 48*a*b^4 + 70*(5*a
^5 + 4*a^3*b^2)*cosh(x)^4 - 210*(3*a^4*b + 2*a^2*b^3)*cosh(x)^3 + 480*(a^4*
b + 2*a^2*b^3 + b^5)*x*cosh(x) + 90*(5*a^5 + 14*a^3*b^2 + 8*a*b^4)*cosh(x)^
2)*sinh(x)^4 - 60*(3*a^4*b + 2*a^2*b^3)*cosh(x)^3 + 20*(36*a^5*cosh(x)^7 -
63*a^4*b*cosh(x)^6 + 28*(5*a^5 + 4*a^3*b^2)*cosh(x)^5 - 9*a^4*b - 6*a^2*b^3
 - 105*(3*a^4*b + 2*a^2*b^3)*cosh(x)^4 + 480*(a^4*b + 2*a^2*b^3 + b^5)*x*co
sh(x)^2 + 60*(5*a^5 + 14*a^3*b^2 + 8*a*b^4)*cosh(x)^3 - 12*(5*a^5 + 14*a^3*
b^2 + 8*a*b^4)*cosh(x))*sinh(x)^3 - 10*(5*a^5 + 4*a^3*b^2)*cosh(x)^2 + 10*(
```

```
27*a^5*cosh(x)^8 - 54*a^4*b*cosh(x)^7 + 28*(5*a^5 + 4*a^3*b^2)*cosh(x)^6 -
126*(3*a^4*b + 2*a^2*b^3)*cosh(x)^5 - 5*a^5 - 4*a^3*b^2 + 960*(a^4*b + 2*a^
2*b^3 + b^5)*x*cosh(x)^3 + 90*(5*a^5 + 14*a^3*b^2 + 8*a*b^4)*cosh(x)^4 - 36
*(5*a^5 + 14*a^3*b^2 + 8*a*b^4)*cosh(x)^2 - 18*(3*a^4*b + 2*a^2*b^3)*cosh(x
))*sinh(x)^2 - 960*((a^4*b + 2*a^2*b^3 + b^5)*cosh(x)^5 + 5*(a^4*b + 2*a^2*
b^3 + b^5)*cosh(x)^4*sinh(x) + 10*(a^4*b + 2*a^2*b^3 + b^5)*cosh(x)^3*sinh(
x)^2 + 10*(a^4*b + 2*a^2*b^3 + b^5)*cosh(x)^2*sinh(x)^3 + 5*(a^4*b + 2*a^2*
b^3 + b^5)*cosh(x)*sinh(x)^4 + (a^4*b + 2*a^2*b^3 + b^5)*sinh(x)^5)*log(2*(
a*sinh(x) + b)/(cosh(x) - sinh(x))) + 5*(12*a^5*cosh(x)^9 - 27*a^4*b*cosh(x
)^8 + 16*(5*a^5 + 4*a^3*b^2)*cosh(x)^7 - 84*(3*a^4*b + 2*a^2*b^3)*cosh(x)^6
 + 960*(a^4*b + 2*a^2*b^3 + b^5)*x*cosh(x)^4 + 72*(5*a^5 + 14*a^3*b^2 + 8*a
*b^4)*cosh(x)^5 - 3*a^4*b - 48*(5*a^5 + 14*a^3*b^2 + 8*a*b^4)*cosh(x)^3 - 3
6*(3*a^4*b + 2*a^2*b^3)*cosh(x)^2 - 4*(5*a^5 + 4*a^3*b^2)*cosh(x))*sinh(x))
/(a^6*cosh(x)^5 + 5*a^6*cosh(x)^4*sinh(x) + 10*a^6*cosh(x)^3*sinh(x)^2 + 10
*a^6*cosh(x)^2*sinh(x)^3 + 5*a^6*cosh(x)*sinh(x)^4 + a^6*sinh(x)^5)
```

giac [B] time = 0.13, size = 194, normalized size = 1.90

$$-\frac{6\,a^4\left(e^{(-x)}-e^x\right)^5+15\,a^3b\left(e^{(-x)}-e^x\right)^4+80\,a^4\left(e^{(-x)}-e^x\right)^3+40\,a^2b^2\left(e^{(-x)}-e^x\right)^3+240\,a^3b\left(e^{(-x)}-e^x\right)^2+120\,ab^3(}{960\,a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^5/(a+b*csch(x)),x, algorithm="giac")
```

```
[Out] -1/960*(6*a^4*(e^(-x) - e^x)^5 + 15*a^3*b*(e^(-x) - e^x)^4 + 80*a^4*(e^(-x)
 - e^x)^3 + 40*a^2*b^2*(e^(-x) - e^x)^3 + 240*a^3*b*(e^(-x) - e^x)^2 + 120*
a*b^3*(e^(-x) - e^x)^2 + 480*a^4*(e^(-x) - e^x) + 960*a^2*b^2*(e^(-x) - e^x
) + 480*b^4*(e^(-x) - e^x))/a^5 - (a^4*b + 2*a^2*b^3 + b^5)*log(abs(-a*(e^(
-x) - e^x) + 2*b))/a^6
```

maple [B] time = 0.14, size = 600, normalized size = 5.88

$$-\frac{7}{8a\left(\tanh\left(\frac{x}{2}\right)-1\right)^2}-\frac{1}{a\left(\tanh\left(\frac{x}{2}\right)-1\right)}+\frac{7}{8a\left(\tanh\left(\frac{x}{2}\right)+1\right)^2}-\frac{1}{a\left(\tanh\left(\frac{x}{2}\right)+1\right)}-\frac{1}{5a\left(\tanh\left(\frac{x}{2}\right)-1\right)^5}-\frac{1}{5a\left(\tanh\left(\frac{x}{2}\right)\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^5/(a+b*csch(x)),x)
```

```
[Out] -7/8/a/(tanh(1/2*x)-1)^2-1/a/(tanh(1/2*x)-1)+7/8/a/(tanh(1/2*x)+1)^2-1/a/(t
anh(1/2*x)+1)-1/5/a/(tanh(1/2*x)-1)^5-1/5/a/(tanh(1/2*x)+1)^5+1/2/a/(tanh(1
/2*x)+1)^4-1/2/a/(tanh(1/2*x)-1)^4-1/4/a^2/(tanh(1/2*x)-1)^4*b+1/2/a^4/(tan
h(1/2*x)+1)*b^3-1/a^5/(tanh(1/2*x)+1)*b^4+b^5/a^6*ln(tanh(1/2*x)+1)-1/a^2*b
*ln(tanh(1/2*x)^2*b-2*a*tanh(1/2*x)-b)-2/a^4*b^3*ln(tanh(1/2*x)^2*b-2*a*tan
h(1/2*x)-b)-1/a^6*b^5*ln(tanh(1/2*x)^2*b-2*a*tanh(1/2*x)-b)+1/2/a^2/(tanh(1
/2*x)+1)^3*b-1/3/a^3/(tanh(1/2*x)+1)^3*b^2+1/2/a^3/(tanh(1/2*x)+1)^2*b^2-1/
2/a^4/(tanh(1/2*x)+1)^2*b^3+b^5/a^6*ln(tanh(1/2*x)-1)-1/4/a^2/(tanh(1/2*x)+
1)^4*b-1/2/a^4/(tanh(1/2*x)-1)*b^3-1/a^5/(tanh(1/2*x)-1)*b^4-1/2/a^2/(tanh(
1/2*x)-1)^3*b-1/3/a^3/(tanh(1/2*x)-1)^3*b^2-1/2/a^3/(tanh(1/2*x)-1)^2*b^2-1
/2/a^4/(tanh(1/2*x)-1)^2*b^3-11/12/a/(tanh(1/2*x)-1)^3-11/12/a/(tanh(1/2*x)
+1)^3-9/8/a^2/(tanh(1/2*x)-1)^2*b-7/8/a^2/(tanh(1/2*x)-1)*b-2/a^3/(tanh(1/2
*x)-1)*b^2+b/a^2*ln(tanh(1/2*x)-1)+2*b^3/a^4*ln(tanh(1/2*x)-1)-9/8/a^2/(tan
h(1/2*x)+1)^2*b+7/8/a^2/(tanh(1/2*x)+1)*b-2/a^3/(tanh(1/2*x)+1)*b^2+b/a^2*l
n(tanh(1/2*x)+1)+2*b^3/a^4*ln(tanh(1/2*x)+1)
```

maxima [B] time = 0.32, size = 242, normalized size = 2.37

$$-\frac{\left(15\,a^3be^{(-x)}-6\,a^4-10\left(5\,a^4+4\,a^2b^2\right)e^{(-2\,x)}+60\left(3\,a^3b+2\,ab^3\right)e^{(-3\,x)}-60\left(5\,a^4+14\,a^2b^2+8\,b^4\right)e^{(-4\,x)}\right)e^{(5\,x)}\quad 1}{960\,a^5}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^5/(a+b*csch(x)),x, algorithm="maxima")
```

```
[Out] -1/960*(15*a^3*b*e^(-x) - 6*a^4 - 10*(5*a^4 + 4*a^2*b^2)*e^(-2*x) + 60*(3*a
^3*b + 2*a*b^3)*e^(-3*x) - 60*(5*a^4 + 14*a^2*b^2 + 8*b^4)*e^(-4*x))*e^(5*x
)/a^5 - 1/960*(15*a^3*b*e^(-4*x) + 6*a^4*e^(-5*x) + 60*(5*a^4 + 14*a^2*b^2
+ 8*b^4)*e^(-x) + 60*(3*a^3*b + 2*a*b^3)*e^(-2*x) + 10*(5*a^4 + 4*a^2*b^2)*
e^(-3*x))/a^5 - (a^4*b + 2*a^2*b^3 + b^5)*x/a^6 - (a^4*b + 2*a^2*b^3 + b^5)
*log(-2*b*e^(-x) + a*e^(-2*x) - a)/a^6
```

mupad [B] time = 2.07, size = 228, normalized size = 2.24

$$\frac{e^{5x}}{160a}-\frac{e^{-5x}}{160a}-\frac{e^{-2x}\left(3a^2b+2b^3\right)}{16a^4}-\frac{e^{2x}\left(3a^2b+2b^3\right)}{16a^4}+\frac{e^{x}\left(5a^4+14a^2b^2+8b^4\right)}{16a^5}-\frac{b\,e^{-4x}}{64a^2}-\frac{b\,e^{4x}}{64a^2}-\frac{\ln\left(2b\,e^{x}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^5/(a + b/sinh(x)),x)
```

```
[Out] exp(5*x)/(160*a) - exp(-5*x)/(160*a) - (exp(-2*x)*(3*a^2*b + 2*b^3))/(16*a^
4) - (exp(2*x)*(3*a^2*b + 2*b^3))/(16*a^4) + (exp(x)*(5*a^4 + 8*b^4 + 14*a^
2*b^2))/(16*a^5) - (b*exp(-4*x))/(64*a^2) - (b*exp(4*x))/(64*a^2) - (log(2*
b*exp(x) - a + a*exp(2*x))*(a^4*b + b^5 + 2*a^2*b^3))/a^6 - (exp(-x)*(5*a^4
 + 8*b^4 + 14*a^2*b^2))/(16*a^5) - (exp(-3*x)*(5*a^2 + 4*b^2))/(96*a^3) + (
exp(3*x)*(5*a^2 + 4*b^2))/(96*a^3) + (b*x*(a^2 + b^2)^2)/a^6
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\cosh^5(x)}{a+b\operatorname{csch}(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)**5/(a+b*csch(x)),x)
```

```
[Out] Integral(cosh(x)**5/(a + b*csch(x)), x)
```

3.93 $\int \frac{\cosh^4(x)}{a+b\mathbf{csch}(x)}\,dx$

Optimal. Leaf size=125

$$-\frac{\cosh^3(x)(4b-3a\sinh(x))}{12a^2}+\frac{2b\left(a^2+b^2\right)^{3/2}\tanh^{-1}\left(\frac{a-b\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{a^5}-\frac{\cosh(x)\left(8b\left(a^2+b^2\right)-a\left(3a^2+4b^2\right)\sinh(x)\right)}{8a^4}$$

```
[Out] 1/8*(3*a^4+12*a^2*b^2+8*b^4)*x/a^5+2*b*(a^2+b^2)^(3/2)*arctanh((a-b*tanh(1/
2*x))/(a^2+b^2)^(1/2))/a^5-1/12*cosh(x)^3*(4*b-3*a*sinh(x))/a^2-1/8*cosh(x)
*(8*b*(a^2+b^2)-a*(3*a^2+4*b^2)*sinh(x))/a^4
```

Rubi [A] time = 0.38, antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {3872, 2865, 2735, 2660, 618, 204}

$$\frac{x\left(12a^2b^2+3a^4+8b^4\right)}{8a^5}+\frac{2b\left(a^2+b^2\right)^{3/2}\tanh^{-1}\left(\frac{a-b\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{a^5}-\frac{\cosh(x)\left(8b\left(a^2+b^2\right)-a\left(3a^2+4b^2\right)\sinh(x)\right)}{8a^4}-\cos$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]^4/(a + b*Csch[x]),x]
```

```
[Out] ((3*a^4 + 12*a^2*b^2 + 8*b^4)*x)/(8*a^5) + (2*b*(a^2 + b^2)^(3/2)*ArcTanh[(
a - b*Tanh[x/2])/Sqrt[a^2 + b^2]])/a^5 - (Cosh[x]^3*(4*b - 3*a*Sinh[x]))/(1
2*a^2) - (Cosh[x]*(8*b*(a^2 + b^2) - a*(3*a^2 + 4*b^2)*Sinh[x]))/(8*a^4)
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2865

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(g*(g
*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*(b*c*(m + p + 1) - a*d*
p + b*d*(m + p)*Sin[e + f*x]))/(b^2*f*(m + p)*(m + p + 1)), x] + Dist[(g^2*
(p - 1))/(b^2*(m + p)*(m + p + 1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin
```

```
[e + f*x])^m*Simp[b*(a*d*m + b*c*(m + p + 1)) + (a*b*c*(m + p + 1) - d*(a^2
*p - b^2*(m + p)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p, 0] && NeQ[m + p + 1,
 0] && IntegerQ[2*m]
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\int \frac{\cosh^4(x)}{a+b\text{csch}(x)}\,dx = i\int \frac{\cosh^4(x)\sinh(x)}{ib+ia\sinh(x)}\,dx$$

$$= -\frac{\cosh^3(x)(4b-3a\sinh(x))}{12a^2} + \frac{\int \frac{\cosh^2(x)\left(-iab+i\left(3a^2+4b^2\right)\sinh(x)\right)}{ib+ia\sinh(x)}\,dx}{4a^2}$$

$$= -\frac{\cosh^3(x)(4b-3a\sinh(x))}{12a^2} - \frac{\cosh(x)\left(8b\left(a^2+b^2\right)-a\left(3a^2+4b^2\right)\sinh(x)\right)}{8a^4} + \frac{\int \frac{-iab(5}{}}{}$$

$$= \frac{\left(3a^4+12a^2b^2+8b^4\right)x}{8a^5} - \frac{\cosh^3(x)(4b-3a\sinh(x))}{12a^2} - \frac{\cosh(x)\left(8b\left(a^2+b^2\right)-a\left(3a^2+\right.\right.}{8a^4}$$

$$= \frac{\left(3a^4+12a^2b^2+8b^4\right)x}{8a^5} - \frac{\cosh^3(x)(4b-3a\sinh(x))}{12a^2} - \frac{\cosh(x)\left(8b\left(a^2+b^2\right)-a\left(3a^2+\right.\right.}{8a^4}$$

$$= \frac{\left(3a^4+12a^2b^2+8b^4\right)x}{8a^5} - \frac{\cosh^3(x)(4b-3a\sinh(x))}{12a^2} - \frac{\cosh(x)\left(8b\left(a^2+b^2\right)-a\left(3a^2+\right.\right.}{8a^4}$$

$$= \frac{\left(3a^4+12a^2b^2+8b^4\right)x}{8a^5} + \frac{2b\left(a^2+b^2\right)^{3/2}\tanh^{-1}\left(\frac{a-b\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{a^5} - \frac{\cosh^3(x)(4b-3a\sinh(}{12a^2}$$

Mathematica [A] time = 1.21, size = 180, normalized size = 1.44

$$\frac{-8a^3b\cosh(3x)-24ab\left(5a^2+4b^2\right)\cosh(x)+3\left(12a^4x+a^4\sinh(4x)+48a^2b^2x+8a^2\left(a^2+b^2\right)\sinh(2x)+64\right.}{96a^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]^4/(a + b*Csch[x]),x]
```

```
[Out] (-24*a*b*(5*a^2 + 4*b^2)*Cosh[x] - 8*a^3*b*Cosh[3*x] + 3*(12*a^4*x + 48*a^2
*b^2*x + 32*b^4*x + 64*a^2*b*Sqrt[-a^2 - b^2]*ArcTan[(a - b*Tanh[x/2])/Sqrt
[-a^2 - b^2]] + 64*b^3*Sqrt[-a^2 - b^2]*ArcTan[(a - b*Tanh[x/2])/Sqrt[-a^2
- b^2]] + 8*a^2*(a^2 + b^2)*Sinh[2*x] + a^4*Sinh[4*x]))/(96*a^5)
```

fricas [B] time = 0.59, size = 924, normalized size = 7.39

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^4/(a+b*csch(x)),x, algorithm="fricas")
```

```
[Out] 1/192*(3*a^4*cosh(x)^8 + 3*a^4*sinh(x)^8 - 8*a^3*b*cosh(x)^7 + 8*(3*a^4*cos
h(x) - a^3*b)*sinh(x)^7 + 24*(a^4 + a^2*b^2)*cosh(x)^6 + 4*(21*a^4*cosh(x)^
2 - 14*a^3*b*cosh(x) + 6*a^4 + 6*a^2*b^2)*sinh(x)^6 + 24*(3*a^4 + 12*a^2*b^
2 + 8*b^4)*x*cosh(x)^4 - 24*(5*a^3*b + 4*a*b^3)*cosh(x)^5 + 24*(7*a^4*cosh(
x)^3 - 7*a^3*b*cosh(x)^2 - 5*a^3*b - 4*a*b^3 + 6*(a^4 + a^2*b^2)*cosh(x))*s
inh(x)^5 - 8*a^3*b*cosh(x) + 2*(105*a^4*cosh(x)^4 - 140*a^3*b*cosh(x)^3 + 1
80*(a^4 + a^2*b^2)*cosh(x)^2 + 12*(3*a^4 + 12*a^2*b^2 + 8*b^4)*x - 60*(5*a^
3*b + 4*a*b^3)*cosh(x))*sinh(x)^4 - 3*a^4 - 24*(5*a^3*b + 4*a*b^3)*cosh(x)^
3 + 8*(21*a^4*cosh(x)^5 - 35*a^3*b*cosh(x)^4 - 15*a^3*b - 12*a*b^3 + 60*(a^
4 + a^2*b^2)*cosh(x)^3 + 12*(3*a^4 + 12*a^2*b^2 + 8*b^4)*x*cosh(x) - 30*(5*
a^3*b + 4*a*b^3)*cosh(x)^2)*sinh(x)^3 - 24*(a^4 + a^2*b^2)*cosh(x)^2 + 12*(
7*a^4*cosh(x)^6 - 14*a^3*b*cosh(x)^5 + 30*(a^4 + a^2*b^2)*cosh(x)^4 - 2*a^4
 - 2*a^2*b^2 + 12*(3*a^4 + 12*a^2*b^2 + 8*b^4)*x*cosh(x)^2 - 20*(5*a^3*b +
4*a*b^3)*cosh(x)^3 - 6*(5*a^3*b + 4*a*b^3)*cosh(x))*sinh(x)^2 + 192*((a^2*b
 + b^3)*cosh(x)^4 + 4*(a^2*b + b^3)*cosh(x)^3*sinh(x) + 6*(a^2*b + b^3)*cos
h(x)^2*sinh(x)^2 + 4*(a^2*b + b^3)*cosh(x)*sinh(x)^3 + (a^2*b + b^3)*sinh(x
)^4)*sqrt(a^2 + b^2)*log((a^2*cosh(x)^2 + a^2*sinh(x)^2 + 2*a*b*cosh(x) + a
^2 + 2*b^2 + 2*(a^2*cosh(x) + a*b)*sinh(x) + 2*sqrt(a^2 + b^2)*(a*cosh(x) +
 a*sinh(x) + b))/(a*cosh(x)^2 + a*sinh(x)^2 + 2*b*cosh(x) + 2*(a*cosh(x) +
b)*sinh(x) - a)) + 8*(3*a^4*cosh(x)^7 - 7*a^3*b*cosh(x)^6 + 18*(a^4 + a^2*b
^2)*cosh(x)^5 + 12*(3*a^4 + 12*a^2*b^2 + 8*b^4)*x*cosh(x)^3 - 15*(5*a^3*b +
 4*a*b^3)*cosh(x)^4 - a^3*b - 9*(5*a^3*b + 4*a*b^3)*cosh(x)^2 - 6*(a^4 + a^
2*b^2)*cosh(x))*sinh(x))/(a^5*cosh(x)^4 + 4*a^5*cosh(x)^3*sinh(x) + 6*a^5*c
osh(x)^2*sinh(x)^2 + 4*a^5*cosh(x)*sinh(x)^3 + a^5*sinh(x)^4)
```

giac [A] time = 0.14, size = 221, normalized size = 1.77

$$\frac{3\,a^3e^{(4\,x)} - 8\,a^2be^{(3\,x)} + 24\,a^3e^{(2\,x)} + 24\,ab^2e^{(2\,x)} - 120\,a^2be^x - 96\,b^3e^x}{192\,a^4} + \frac{\left(3\,a^4 + 12\,a^2b^2 + 8\,b^4\right)x}{8\,a^5} - \frac{\left(8\,a^3be^x + 3\,a^4 + \right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^4/(a+b*csch(x)),x, algorithm="giac")
```

```
[Out] 1/192*(3*a^3*e^(4*x) - 8*a^2*b*e^(3*x) + 24*a^3*e^(2*x) + 24*a*b^2*e^(2*x)
- 120*a^2*b*e^x - 96*b^3*e^x)/a^4 + 1/8*(3*a^4 + 12*a^2*b^2 + 8*b^4)*x/a^5
- 1/192*(8*a^3*b*e^x + 3*a^4 + 24*(5*a^3*b + 4*a*b^3)*e^(3*x) + 24*(a^4 + a
^2*b^2)*e^(2*x))*e^(-4*x)/a^5 - (a^4*b + 2*a^2*b^3 + b^5)*log(abs(2*a*e^x +
 2*b - 2*sqrt(a^2 + b^2))/abs(2*a*e^x + 2*b + 2*sqrt(a^2 + b^2)))/(sqrt(a^2
 + b^2)*a^5)
```

maple [B] time = 0.15, size = 486, normalized size = 3.89

$$\frac{3\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)}{8a} + \frac{7}{8a\left(\tanh\left(\frac{x}{2}\right)-1\right)^2} + \frac{5}{8a\left(\tanh\left(\frac{x}{2}\right)-1\right)} - \frac{3\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{8a} - \frac{7}{8a\left(\tanh\left(\frac{x}{2}\right)+1\right)^2} + \frac{5}{8a\left(\tanh\left(\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^4/(a+b*csch(x)),x)
```

```
[Out] 3/8/a*ln(tanh(1/2*x)+1)+7/8/a/(tanh(1/2*x)-1)^2+5/8/a/(tanh(1/2*x)-1)-3/8/a
*ln(tanh(1/2*x)-1)-7/8/a/(tanh(1/2*x)+1)^2+5/8/a/(tanh(1/2*x)+1)-2*b^5/a^5/
(a^2+b^2)^(1/2)*arctanh(1/2*(2*tanh(1/2*x)*b-2*a)/(a^2+b^2)^(1/2))-2/a*b/(a
^2+b^2)^(1/2)*arctanh(1/2*(2*tanh(1/2*x)*b-2*a)/(a^2+b^2)^(1/2))-4*b^3/a^3/
(a^2+b^2)^(1/2)*arctanh(1/2*(2*tanh(1/2*x)*b-2*a)/(a^2+b^2)^(1/2))-1/4/a/(t
anh(1/2*x)+1)^4+1/4/a/(tanh(1/2*x)-1)^4-1/a^4/(tanh(1/2*x)+1)*b^3-1/3/a^2/(
tanh(1/2*x)+1)^3*b-1/2/a^3/(tanh(1/2*x)+1)^2*b^2+1/a^4/(tanh(1/2*x)-1)*b^3+
1/3/a^2/(tanh(1/2*x)-1)^3*b+1/2/a^3/(tanh(1/2*x)-1)^2*b^2+1/a^5*ln(tanh(1/2
```

```
*x)+1)*b^4-1/a^5*ln(tanh(1/2*x)-1)*b^4+1/2/a/(tanh(1/2*x)-1)^3+1/2/a/(tanh(
1/2*x)+1)^3+1/2/a^2/(tanh(1/2*x)-1)^2*b+3/2/a^2/(tanh(1/2*x)-1)*b+1/2/a^3/(
tanh(1/2*x)-1)*b^2+1/2/a^2/(tanh(1/2*x)+1)^2*b-3/2/a^2/(tanh(1/2*x)+1)*b+1/
2/a^3/(tanh(1/2*x)+1)*b^2-3/2/a^3*ln(tanh(1/2*x)-1)*b^2+3/2/a^3*ln(tanh(1/2
*x)+1)*b^2
```

maxima [A] time = 0.41, size = 217, normalized size = 1.74

$$-\frac{\left(8\,a^{2}be^{(-x)}-3\,a^{3}-24\,\left(a^{3}+ab^{2}\right)e^{(-2\,x)}+24\,\left(5\,a^{2}b+4\,b^{3}\right)e^{(-3\,x)}\right)e^{(4\,x)}}{192\,a^{4}}-\frac{8\,a^{2}be^{(-3\,x)}+3\,a^{3}e^{(-4\,x)}+24\,\left(5\,a^{2}b+4\right.}{192\,a^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^4/(a+b*csch(x)),x, algorithm="maxima")
```

```
[Out] -1/192*(8*a^2*b*e^(-x) - 3*a^3 - 24*(a^3 + a*b^2)*e^(-2*x) + 24*(5*a^2*b +
4*b^3)*e^(-3*x))*e^(4*x)/a^4 - 1/192*(8*a^2*b*e^(-3*x) + 3*a^3*e^(-4*x) + 2
4*(5*a^2*b + 4*b^3)*e^(-x) + 24*(a^3 + a*b^2)*e^(-2*x))/a^4 + 1/8*(3*a^4 +
12*a^2*b^2 + 8*b^4)*x/a^5 - (a^4*b + 2*a^2*b^3 + b^5)*log((a*e^(-x) - b - s
qrt(a^2 + b^2))/(a*e^(-x) - b + sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*a^5)
```

mupad [B] time = 2.02, size = 247, normalized size = 1.98

$$\frac{\mathrm{e}^{4\,x}}{64\,a}-\frac{\mathrm{e}^{-4\,x}}{64\,a}+\frac{x\,\left(3\,a^4+12\,a^2\,b^2+8\,b^4\right)}{8\,a^5}-\frac{\mathrm{e}^{-2\,x}\,\left(a^2+b^2\right)}{8\,a^3}+\frac{\mathrm{e}^{2\,x}\,\left(a^2+b^2\right)}{8\,a^3}-\frac{\mathrm{e}^{-x}\,\left(5\,a^2\,b+4\,b^3\right)}{8\,a^4}-\frac{b\,\mathrm{e}^{-3\,x}}{24\,a^2}-\frac{b\,\mathrm{e}^{3\,x}}{24\,a^2}-\frac{\mathrm{e}^{x}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^4/(a + b/sinh(x)),x)
```

```
[Out] exp(4*x)/(64*a) - exp(-4*x)/(64*a) + (x*(3*a^4 + 8*b^4 + 12*a^2*b^2))/(8*a^
5) - (exp(-2*x)*(a^2 + b^2))/(8*a^3) + (exp(2*x)*(a^2 + b^2))/(8*a^3) - (ex
p(-x)*(5*a^2*b + 4*b^3))/(8*a^4) - (b*exp(-3*x))/(24*a^2) - (b*exp(3*x))/(2
4*a^2) - (exp(x)*(5*a^2*b + 4*b^3))/(8*a^4) - (b*log((2*b*exp(x)*(a^2 + b^2
)^2)/a^6 - (2*b*(a - b*exp(x))*(a^2 + b^2)^(3/2))/a^6)*(a^2 + b^2)^(3/2))/a
^5 + (b*log((2*b*(a - b*exp(x))*(a^2 + b^2)^(3/2))/a^6 + (2*b*exp(x)*(a^2 +
 b^2)^2)/a^6)*(a^2 + b^2)^(3/2))/a^5
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh^{4}{\left(x \right)}}{a + b \operatorname{csch}{\left(x \right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)**4/(a+b*csch(x)),x)
```

```
[Out] Integral(cosh(x)**4/(a + b*csch(x)), x)
```

3.94 $\int \frac{\cosh^3(x)}{a+b\textbf{csch}(x)}\,dx$

Optimal. Leaf size=57

$$-\frac{b\sinh^2(x)}{2a^2}-\frac{b\left(a^2+b^2\right)\log(a\sinh(x)+b)}{a^4}+\frac{\left(a^2+b^2\right)\sinh(x)}{a^3}+\frac{\sinh^3(x)}{3a}$$

```
[Out] -b*(a^2+b^2)*ln(b+a*sinh(x))/a^4+(a^2+b^2)*sinh(x)/a^3-1/2*b*sinh(x)^2/a^2+
1/3*sinh(x)^3/a
```

Rubi [A] time = 0.16, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {3872, 2837, 12, 772}

$$\frac{\left(a^2+b^2\right)\sinh(x)}{a^3}-\frac{b\left(a^2+b^2\right)\log(a\sinh(x)+b)}{a^4}-\frac{b\sinh^2(x)}{2a^2}+\frac{\sinh^3(x)}{3a}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]^3/(a + b*Csch[x]),x]
```

```
[Out] -((b*(a^2 + b^2)*Log[b + a*Sinh[x]])/a^4) + ((a^2 + b^2)*Sinh[x])/a^3 - (b*
Sinh[x]^2)/(2*a^2) + Sinh[x]^3/(3*a)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 772

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(
p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*(a + c*x^2)^p,
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IGtQ[p, 0]
```

Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\int \frac{\cosh^3(x)}{a+b\text{csch}(x)}\,dx = i\int \frac{\cosh^3(x)\sinh(x)}{ib+ia\sinh(x)}\,dx$$
$$= -\frac{i\,\text{Subst}\left(\int \frac{x(a^2-x^2)}{a(ib+x)}\,dx, x, ia\sinh(x)\right)}{a^3}$$
$$= -\frac{i\,\text{Subst}\left(\int \frac{x(a^2-x^2)}{ib+x}\,dx, x, ia\sinh(x)\right)}{a^4}$$
$$= -\frac{i\,\text{Subst}\left(\int \left(a^2\left(1+\frac{b^2}{a^2}\right)-\frac{b(a^2+b^2)}{b-ix}+ibx-x^2\right)dx, x, ia\sinh(x)\right)}{a^4}$$
$$= -\frac{b\left(a^2+b^2\right)\log(b+a\sinh(x))}{a^4}+\frac{\left(a^2+b^2\right)\sinh(x)}{a^3}-\frac{b\sinh^2(x)}{2a^2}+\frac{\sinh^3(x)}{3a}$$

Mathematica [A] time = 0.11, size = 56, normalized size = 0.98

$$\frac{2a^3\sinh^3(x)+6a\left(a^2+b^2\right)\sinh(x)-6b\left(a^2+b^2\right)\log(a\sinh(x)+b)-3a^2b\sinh^2(x)}{6a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]^3/(a + b*Csch[x]),x]
```

```
[Out] (-6*b*(a^2 + b^2)*Log[b + a*Sinh[x]] + 6*a*(a^2 + b^2)*Sinh[x] - 3*a^2*b*Si
nh[x]^2 + 2*a^3*Sinh[x]^3)/(6*a^4)
```

fricas [B] time = 0.66, size = 476, normalized size = 8.35

$$\frac{a^3\cosh(x)^6+a^3\sinh(x)^6-3\,a^2b\cosh(x)^5+3\left(2\,a^3\cosh(x)-a^2b\right)\sinh(x)^5+24\left(a^2b+b^3\right)x\cosh(x)^3+3\left(3\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^3/(a+b*csch(x)),x, algorithm="fricas")
```

```
[Out] 1/24*(a^3*cosh(x)^6 + a^3*sinh(x)^6 - 3*a^2*b*cosh(x)^5 + 3*(2*a^3*cosh(x)
- a^2*b)*sinh(x)^5 + 24*(a^2*b + b^3)*x*cosh(x)^3 + 3*(3*a^3 + 4*a*b^2)*cos
h(x)^4 + 3*(5*a^3*cosh(x)^2 - 5*a^2*b*cosh(x) + 3*a^3 + 4*a*b^2)*sinh(x)^4
- 3*a^2*b*cosh(x) + 2*(10*a^3*cosh(x)^3 - 15*a^2*b*cosh(x)^2 + 12*(a^2*b +
b^3)*x + 6*(3*a^3 + 4*a*b^2)*cosh(x))*sinh(x)^3 - a^3 - 3*(3*a^3 + 4*a*b^2)
*cosh(x)^2 + 3*(5*a^3*cosh(x)^4 - 10*a^2*b*cosh(x)^3 - 3*a^3 - 4*a*b^2 + 24
*(a^2*b + b^3)*x*cosh(x) + 6*(3*a^3 + 4*a*b^2)*cosh(x)^2)*sinh(x)^2 - 24*((
a^2*b + b^3)*cosh(x)^3 + 3*(a^2*b + b^3)*cosh(x)^2*sinh(x) + 3*(a^2*b + b^3
)*cosh(x)*sinh(x)^2 + (a^2*b + b^3)*sinh(x)^3)*log(2*(a*sinh(x) + b)/(cosh(
x) - sinh(x))) + 3*(2*a^3*cosh(x)^5 - 5*a^2*b*cosh(x)^4 + 24*(a^2*b + b^3)*
x*cosh(x)^2 + 4*(3*a^3 + 4*a*b^2)*cosh(x)^3 - a^2*b - 2*(3*a^3 + 4*a*b^2)*c
osh(x))*sinh(x))/(a^4*cosh(x)^3 + 3*a^4*cosh(x)^2*sinh(x) + 3*a^4*cosh(x)*s
inh(x)^2 + a^4*sinh(x)^3)
```

giac [A] time = 0.12, size = 97, normalized size = 1.70

$$-\frac{a^2\left(e^{(-x)}-e^x\right)^3+3\,ab\left(e^{(-x)}-e^x\right)^2+12\,a^2\left(e^{(-x)}-e^x\right)+12\,b^2\left(e^{(-x)}-e^x\right)}{24\,a^3}-\frac{\left(a^2b+b^3\right)\log\left(\left|-a\left(e^{(-x)}-e^x\right)+2\,b\right|\right)}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^3/(a+b*csch(x)),x, algorithm="giac")
```

```
[Out] -1/24*(a^2*(e^(-x) - e^x)^3 + 3*a*b*(e^(-x) - e^x)^2 + 12*a^2*(e^(-x) - e^x
) + 12*b^2*(e^(-x) - e^x))/a^3 - (a^2*b + b^3)*log(abs(-a*(e^(-x) - e^x) +
2*b))/a^4
```

maple [B] time = 0.14, size = 274, normalized size = 4.81

$$-\frac{1}{3a\left(\tanh\left(\frac{x}{2}\right)-1\right)^3}-\frac{1}{2a\left(\tanh\left(\frac{x}{2}\right)-1\right)^2}-\frac{b}{2a^2\left(\tanh\left(\frac{x}{2}\right)-1\right)^2}-\frac{1}{a\left(\tanh\left(\frac{x}{2}\right)-1\right)}-\frac{b}{2a^2\left(\tanh\left(\frac{x}{2}\right)-1\right)}-\frac{b^2}{a^3\left(\tanh\left(\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^3/(a+b*csch(x)),x)
```

```
[Out] -1/3/a/(tanh(1/2*x)-1)^3-1/2/a/(tanh(1/2*x)-1)^2-1/2/a^2/(tanh(1/2*x)-1)^2*
b-1/a/(tanh(1/2*x)-1)-1/2/a^2/(tanh(1/2*x)-1)*b-1/a^3/(tanh(1/2*x)-1)*b^2+b
/a^2*ln(tanh(1/2*x)-1)+b^3/a^4*ln(tanh(1/2*x)-1)-1/3/a/(tanh(1/2*x)+1)^3+1/
2/a/(tanh(1/2*x)+1)^2-1/2/a^2/(tanh(1/2*x)+1)^2*b-1/a/(tanh(1/2*x)+1)+1/2/a
^2/(tanh(1/2*x)+1)*b-1/a^3/(tanh(1/2*x)+1)*b^2+b/a^2*ln(tanh(1/2*x)+1)+b^3/
a^4*ln(tanh(1/2*x)+1)-1/a^2*b*ln(tanh(1/2*x)^2*b-2*a*tanh(1/2*x)-b)-1/a^4*b
^3*ln(tanh(1/2*x)^2*b-2*a*tanh(1/2*x)-b)
```

maxima [B] time = 0.32, size = 127, normalized size = 2.23

$$-\frac{\left(3\,abe^{(-x)}-a^2-3\left(3\,a^2+4\,b^2\right)e^{(-2\,x)}\right)e^{(3\,x)}}{24\,a^3}-\frac{3\,abe^{(-2\,x)}+a^2e^{(-3\,x)}+3\left(3\,a^2+4\,b^2\right)e^{(-x)}}{24\,a^3}-\frac{\left(a^2b+b^3\right)x}{a^4}-\frac{\left(a^2b+b^3\right)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^3/(a+b*csch(x)),x, algorithm="maxima")
```

```
[Out] -1/24*(3*a*b*e^(-x) - a^2 - 3*(3*a^2 + 4*b^2)*e^(-2*x))*e^(3*x)/a^3 - 1/24*
(3*a*b*e^(-2*x) + a^2*e^(-3*x) + 3*(3*a^2 + 4*b^2)*e^(-x))/a^3 - (a^2*b + b
^3)*x/a^4 - (a^2*b + b^3)*log(-2*b*e^(-x) + a*e^(-2*x) - a)/a^4
```

mupad [B] time = 1.68, size = 121, normalized size = 2.12

$$\frac{e^{3\,x}}{24\,a}-\frac{e^{-3\,x}}{24\,a}+\frac{x\,\left(a^2\,b+b^3\right)}{a^4}+\frac{e^x\,\left(3\,a^2+4\,b^2\right)}{8\,a^3}-\frac{b\,e^{-2\,x}}{8\,a^2}-\frac{b\,e^{2\,x}}{8\,a^2}-\frac{\ln\left(2\,b\,e^x-a+a\,e^{2\,x}\right)\,\left(a^2\,b+b^3\right)}{a^4}-\frac{e^{-x}\,\left(3\,a^2+4\,b^2\right)}{8\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^3/(a + b/sinh(x)),x)
```

```
[Out] exp(3*x)/(24*a) - exp(-3*x)/(24*a) + (x*(a^2*b + b^3))/a^4 + (exp(x)*(3*a^2
 + 4*b^2))/(8*a^3) - (b*exp(-2*x))/(8*a^2) - (b*exp(2*x))/(8*a^2) - (log(2*
b*exp(x) - a + a*exp(2*x))*(a^2*b + b^3))/a^4 - (exp(-x)*(3*a^2 + 4*b^2))/(
8*a^3)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh^3(x)}{a+b\operatorname{csch}(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)**3/(a+b*csch(x)),x)
```

```
[Out] Integral(cosh(x)**3/(a + b*csch(x)), x)
```

3.95 $\int \frac{\cosh^2(x)}{a+b\operatorname{csch}(x)}\,dx$

Optimal. Leaf size=77

$$-\frac{\cosh(x)(2b-a\sinh(x))}{2a^2}+\frac{x\left(a^2+2b^2\right)}{2a^3}+\frac{2b\sqrt{a^2+b^2}\,\tanh^{-1}\left(\frac{a-b\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{a^3}$$

[Out] 1/2*(a^2+2*b^2)*x/a^3-1/2*cosh(x)*(2*b-a*sinh(x))/a^2+2*b*arctanh((a-b*tanh(1/2*x))/(a^2+b^2)^(1/2))*(a^2+b^2)^(1/2)/a^3

Rubi [A] time = 0.21, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {3872, 2865, 2735, 2660, 618, 204}

$$\frac{x\left(a^2+2b^2\right)}{2a^3}+\frac{2b\sqrt{a^2+b^2}\,\tanh^{-1}\left(\frac{a-b\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{a^3}-\frac{\cosh(x)(2b-a\sinh(x))}{2a^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]^2/(a + b*Csch[x]),x]
```

```
[Out] ((a^2 + 2*b^2)*x)/(2*a^3) + (2*b*Sqrt[a^2 + b^2]*ArcTanh[(a - b*Tanh[x/2])/
Sqrt[a^2 + b^2]])/a^3 - (Cosh[x]*(2*b - a*Sinh[x]))/(2*a^2)
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2865

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(g*(g
*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*(b*c*(m + p + 1) - a*d*
p + b*d*(m + p)*Sin[e + f*x]))/(b^2*f*(m + p)*(m + p + 1)), x] + Dist[(g^2*
(p - 1))/(b^2*(m + p)*(m + p + 1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin
[e + f*x])^m*Simp[b*(a*d*m + b*c*(m + p + 1)) + (a*b*c*(m + p + 1) - d*(a^2
*p - b^2*(m + p)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
```

```
m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p, 0] && NeQ[m + p + 1,
 0] && IntegerQ[2*m]
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cosh^2(x)}{a+b\operatorname{csch}(x)}\,dx &= i\int \frac{\cosh^2(x)\sinh(x)}{ib+ia\sinh(x)}\,dx\\
&= -\frac{\cosh(x)(2b-a\sinh(x))}{2a^2}+\frac{\int \frac{-iab+i\left(a^2+2b^2\right)\sinh(x)}{ib+ia\sinh(x)}\,dx}{2a^2}\\
&= \frac{\left(a^2+2b^2\right)x}{2a^3}-\frac{\cosh(x)(2b-a\sinh(x))}{2a^2}-\frac{\left(ib\left(a^2+b^2\right)\right)\int \frac{1}{ib+ia\sinh(x)}\,dx}{a^3}\\
&= \frac{\left(a^2+2b^2\right)x}{2a^3}-\frac{\cosh(x)(2b-a\sinh(x))}{2a^2}-\frac{\left(2ib\left(a^2+b^2\right)\right)\operatorname{Subst}\left(\int \frac{1}{ib+2iax-ibx^2}\,dx,x,\tanh\left(\frac{x}{2}\right)\right)}{a^3}\\
&= \frac{\left(a^2+2b^2\right)x}{2a^3}-\frac{\cosh(x)(2b-a\sinh(x))}{2a^2}+\frac{\left(4ib\left(a^2+b^2\right)\right)\operatorname{Subst}\left(\int \frac{1}{-4\left(a^2+b^2\right)-x^2}\,dx,x,2ia-\ldots\right)}{a^3}\\
&= \frac{\left(a^2+2b^2\right)x}{2a^3}+\frac{2b\sqrt{a^2+b^2}\,\tanh^{-1}\left(\frac{a-b\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{a^3}-\frac{\cosh(x)(2b-a\sinh(x))}{2a^2}
\end{aligned}
$$

Mathematica [A] time = 0.22, size = 80, normalized size = 1.04

$$
\frac{8b\sqrt{-a^2-b^2}\,\tan^{-1}\left(\frac{a-b\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2-b^2}}\right)+2a^2x+a^2\sinh(2x)-4ab\cosh(x)+4b^2x}{4a^3}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]^2/(a + b*Csch[x]),x]
```

```
[Out] (2*a^2*x + 4*b^2*x + 8*b*Sqrt[-a^2 - b^2]*ArcTan[(a - b*Tanh[x/2])/Sqrt[-a^
2 - b^2]] - 4*a*b*Cosh[x] + a^2*Sinh[2*x])/(4*a^3)
```

fricas [B] time = 0.76, size = 304, normalized size = 3.95

$$
\frac{a^2\cosh(x)^4+a^2\sinh(x)^4-4\,ab\cosh(x)^3+4\left(a^2+2\,b^2\right)x\cosh(x)^2+4\left(a^2\cosh(x)-ab\right)\sinh(x)^3-4\,ab\cosh(x\ldots}{\ldots}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^2/(a+b*csch(x)),x, algorithm="fricas")
```

```
[Out] 1/8*(a^2*cosh(x)^4 + a^2*sinh(x)^4 - 4*a*b*cosh(x)^3 + 4*(a^2 + 2*b^2)*x*co
sh(x)^2 + 4*(a^2*cosh(x) - a*b)*sinh(x)^3 - 4*a*b*cosh(x) + 2*(3*a^2*cosh(x
)^2 - 6*a*b*cosh(x) + 2*(a^2 + 2*b^2)*x)*sinh(x)^2 + 8*(b*cosh(x)^2 + 2*b*c
osh(x)*sinh(x) + b*sinh(x)^2)*sqrt(a^2 + b^2)*log((a^2*cosh(x)^2 + a^2*sinh
(x)^2 + 2*a*b*cosh(x) + a^2 + 2*b^2 + 2*(a^2*cosh(x) + a*b)*sinh(x) + 2*sqr
```

```
t(a^2 + b^2)*(a*cosh(x) + a*sinh(x) + b))/(a*cosh(x)^2 + a*sinh(x)^2 + 2*b*
cosh(x) + 2*(a*cosh(x) + b)*sinh(x) - a)) - a^2 + 4*(a^2*cosh(x)^3 - 3*a*b*
cosh(x)^2 + 2*(a^2 + 2*b^2)*x*cosh(x) - a*b)*sinh(x))/(a^3*cosh(x)^2 + 2*a^
3*cosh(x)*sinh(x) + a^3*sinh(x)^2)
```

giac [A] time = 0.14, size = 121, normalized size = 1.57

$$\frac{ae^{(2x)} - 4\,be^{x}}{8\,a^{2}} + \frac{\left(a^{2} + 2\,b^{2}\right)x}{2\,a^{3}} - \frac{\left(4\,abe^{x} + a^{2}\right)e^{(-2\,x)}}{8\,a^{3}} - \frac{\left(a^{2}b + b^{3}\right)\log\left(\frac{\left|2\,ae^{x}+2\,b-2\,\sqrt{a^{2}+b^{2}}\right|}{\left|2\,ae^{x}+2\,b+2\,\sqrt{a^{2}+b^{2}}\right|}\right)}{\sqrt{a^{2} + b^{2}}\,a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^2/(a+b*csch(x)),x, algorithm="giac")
```

```
[Out] 1/8*(a*e^(2*x) - 4*b*e^x)/a^2 + 1/2*(a^2 + 2*b^2)*x/a^3 - 1/8*(4*a*b*e^x +
a^2)*e^(-2*x)/a^3 - (a^2*b + b^3)*log(abs(2*a*e^x + 2*b - 2*sqrt(a^2 + b^2)
)/abs(2*a*e^x + 2*b + 2*sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*a^3)
```

maple [B] time = 0.14, size = 172, normalized size = 2.23

$$\frac{1}{2a\left(\tanh\left(\frac{x}{2}\right)-1\right)^{2}}+\frac{1}{2a\left(\tanh\left(\frac{x}{2}\right)-1\right)}+\frac{b}{a^{2}\left(\tanh\left(\frac{x}{2}\right)-1\right)}-\frac{\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{2a}-\frac{\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)b^{2}}{a^{3}}-\frac{1}{2a\left(\tanh\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^2/(a+b*csch(x)),x)
```

```
[Out] 1/2/a/(tanh(1/2*x)-1)^2+1/2/a/(tanh(1/2*x)-1)+1/a^2/(tanh(1/2*x)-1)*b-1/2/a
*ln(tanh(1/2*x)-1)-1/a^3*ln(tanh(1/2*x)-1)*b^2-1/2/a/(tanh(1/2*x)+1)^2+1/2/
a/(tanh(1/2*x)+1)-1/a^2/(tanh(1/2*x)+1)*b+1/2/a*ln(tanh(1/2*x)+1)+1/a^3*ln(
tanh(1/2*x)+1)*b^2-2*b*(a^2+b^2)^(1/2)/a^3*arctanh(1/2*(2*tanh(1/2*x)*b-2*a
)/(a^2+b^2)^(1/2))
```

maxima [A] time = 0.40, size = 122, normalized size = 1.58

$$-\frac{\left(4\,be^{(-x)} - a\right)e^{(2\,x)}}{8\,a^{2}} - \frac{4\,be^{(-x)} + ae^{(-2\,x)}}{8\,a^{2}} + \frac{\left(a^{2} + 2\,b^{2}\right)x}{2\,a^{3}} - \frac{\left(a^{2}b + b^{3}\right)\log\left(\frac{ae^{(-x)}-b-\sqrt{a^{2}+b^{2}}}{ae^{(-x)}-b+\sqrt{a^{2}+b^{2}}}\right)}{\sqrt{a^{2} + b^{2}}\,a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^2/(a+b*csch(x)),x, algorithm="maxima")
```

```
[Out] -1/8*(4*b*e^(-x) - a)*e^(2*x)/a^2 - 1/8*(4*b*e^(-x) + a*e^(-2*x))/a^2 + 1/2
*(a^2 + 2*b^2)*x/a^3 - (a^2*b + b^3)*log((a*e^(-x) - b - sqrt(a^2 + b^2))/(
a*e^(-x) - b + sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*a^3)
```

mupad [B] time = 1.69, size = 159, normalized size = 2.06

$$\frac{e^{2\,x}}{8\,a}-\frac{e^{-2\,x}}{8\,a}-\frac{b\,e^{x}}{2\,a^{2}}-\frac{b\,e^{-x}}{2\,a^{2}}+\frac{x\,\left(a^{2}+2\,b^{2}\right)}{2\,a^{3}}-\frac{b\,\ln\left(\frac{2\,b\,e^{x}\,\left(a^{2}+b^{2}\right)}{a^{4}}-\frac{2\,b\,\left(a-b\,e^{x}\right)\,\sqrt{a^{2}+b^{2}}}{a^{4}}\right)\,\sqrt{a^{2}+b^{2}}}{a^{3}}+\frac{b\,\ln\left(\frac{2\,b\,\left(a-b\,e^{x}\right)\,\sqrt{a^{2}+b^{2}}}{a^{4}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^2/(a + b/sinh(x)),x)
```

```
[Out] exp(2*x)/(8*a) - exp(-2*x)/(8*a) - (b*exp(x))/(2*a^2) - (b*exp(-x))/(2*a^2)
 + (x*(a^2 + 2*b^2))/(2*a^3) - (b*log((2*b*exp(x)*(a^2 + b^2))/a^4 - (2*b*(
```

```
a - b*exp(x))*(a^2 + b^2)^(1/2))/a^4)*(a^2 + b^2)^(1/2))/a^3 + (b*log((2*b*
(a - b*exp(x))*(a^2 + b^2)^(1/2))/a^4 + (2*b*exp(x)*(a^2 + b^2))/a^4)*(a^2
+ b^2)^(1/2))/a^3
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh^2(x)}{a + b\,\text{csch}(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)**2/(a+b*csch(x)),x)
```

```
[Out] Integral(cosh(x)**2/(a + b*csch(x)), x)
```

3.96 $\int \frac{\cosh(x)}{a+b\,\mathbf{csch}(x)}\,dx$

Optimal. Leaf size=20

$$\frac{\sinh(x)}{a} - \frac{b\log(a\sinh(x)+b)}{a^2}$$

```
[Out] -b*ln(b+a*sinh(x))/a^2+sinh(x)/a
```

Rubi [A] time = 0.08, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {3872, 2833, 12, 43}

$$\frac{\sinh(x)}{a} - \frac{b\log(a\sinh(x)+b)}{a^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]/(a + b*Csch[x]),x]
```

```
[Out] -((b*Log[b + a*Sinh[x]])/a^2) + Sinh[x]/a
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2833

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d*x)/b)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\int \frac{\cosh(x)}{a+b\text{csch}(x)}\,dx = i\int \frac{\cosh(x)\sinh(x)}{ib+ia\sinh(x)}\,dx$$
$$= -\frac{i\,\text{Subst}\left(\int \frac{x}{a(ib+x)}\,dx, x, ia\sinh(x)\right)}{a}$$
$$= -\frac{i\,\text{Subst}\left(\int \frac{x}{ib+x}\,dx, x, ia\sinh(x)\right)}{a^2}$$
$$= -\frac{i\,\text{Subst}\left(\int \left(1-\frac{b}{b-ix}\right)\,dx, x, ia\sinh(x)\right)}{a^2}$$
$$= -\frac{b\log(b+a\sinh(x))}{a^2}+\frac{\sinh(x)}{a}$$

Mathematica [A] time = 0.01, size = 19, normalized size = 0.95

$$\frac{a\sinh(x)-b\log(a\sinh(x)+b)}{a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]/(a + b*Csch[x]),x]
[Out] (-(b*Log[b + a*Sinh[x]]) + a*Sinh[x])/a^2
```

fricas [B] time = 0.63, size = 80, normalized size = 4.00

$$\frac{2\,bx\cosh(x)+a\cosh(x)^2+a\sinh(x)^2-2\,(b\cosh(x)+b\sinh(x))\log\left(\frac{2\,(a\sinh(x)+b)}{\cosh(x)-\sinh(x)}\right)+2\,(bx+a\cosh(x))\sinh(x}{2\left(a^2\cosh(x)+a^2\sinh(x)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)/(a+b*csch(x)),x, algorithm="fricas")
[Out] 1/2*(2*b*x*cosh(x) + a*cosh(x)^2 + a*sinh(x)^2 - 2*(b*cosh(x) + b*sinh(x))*
log(2*(a*sinh(x) + b)/(cosh(x) - sinh(x))) + 2*(b*x + a*cosh(x))*sinh(x) -
a)/(a^2*cosh(x) + a^2*sinh(x))
```

giac [A] time = 0.14, size = 39, normalized size = 1.95

$$-\frac{e^{(-x)}-e^x}{2\,a}-\frac{b\log\left(\left|-a\left(e^{(-x)}-e^x\right)+2\,b\right|\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)/(a+b*csch(x)),x, algorithm="giac")
[Out] -1/2*(e^(-x) - e^x)/a - b*log(abs(-a*(e^(-x) - e^x) + 2*b))/a^2
```

maple [A] time = 0.12, size = 31, normalized size = 1.55

$$-\frac{b\ln(a+b\,\text{csch}(x))}{a^2}+\frac{1}{a\,\text{csch}(x)}+\frac{b\ln(\text{csch}(x))}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)/(a+b*csch(x)),x)
[Out] -1/a^2*b*ln(a+b*csch(x))+1/a/csch(x)+1/a^2*b*ln(csch(x))
```

maxima [B] time = 0.31, size = 48, normalized size = 2.40

$$-\frac{bx}{a^2} - \frac{e^{(-x)}}{2\,a} + \frac{e^x}{2\,a} - \frac{b\log\left(-2\,be^{(-x)} + ae^{(-2\,x)} - a\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)/(a+b*csch(x)),x, algorithm="maxima")
```

```
[Out] -b*x/a^2 - 1/2*e^(-x)/a + 1/2*e^x/a - b*log(-2*b*e^(-x) + a*e^(-2*x) - a)/a
^2
```

mupad [B] time = 0.08, size = 20, normalized size = 1.00

$$\frac{\sinh(x)}{a} - \frac{b\,\ln\left(b + a\sinh(x)\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)/(a + b/sinh(x)),x)
```

```
[Out] sinh(x)/a - (b*log(b + a*sinh(x)))/a^2
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh\left(x\right)}{a + b\operatorname{csch}\left(x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)/(a+b*csch(x)),x)
```

```
[Out] Integral(cosh(x)/(a + b*csch(x)), x)
```

3.97 $\int \frac{\text{sech}(x)}{a+b\text{csch}(x)}\,dx$

Optimal. Leaf size=64

$$-\frac{b\log(a\sinh(x)+b)}{a^2+b^2}+\frac{\log(-\sinh(x)+i)}{2(b+ia)}-\frac{\log(\sinh(x)+i)}{2(-b+ia)}$$

```
[Out] 1/2*ln(I-sinh(x))/(I*a+b)-1/2*ln(I+sinh(x))/(I*a-b)-b*ln(b+a*sinh(x))/(a^2+
b^2)
```

Rubi [A] time = 0.11, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {3872, 2721, 801}

$$-\frac{b\log(a\sinh(x)+b)}{a^2+b^2}+\frac{\log(-\sinh(x)+i)}{2(b+ia)}-\frac{\log(\sinh(x)+i)}{2(-b+ia)}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[x]/(a + b*Csch[x]),x]
```

```
[Out] Log[I - Sinh[x]]/(2*(I*a + b)) - Log[I + Sinh[x]]/(2*(I*a - b)) - (b*Log[b
+ a*Sinh[x]])/(a^2 + b^2)
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 2721

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^
2, 0] && IntegerQ[(p + 1)/2]
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\int \frac{\text{sech}(x)}{a+b\text{csch}(x)}\,dx &= i\int \frac{\tanh(x)}{ib+ia\sinh(x)}\,dx\\
&= -\left(i\,\text{Subst}\left(\int \frac{x}{(ib+x)\left(a^2-x^2\right)}\,dx, x, ia\sinh(x)\right)\right)\\
&= -\left(i\,\text{Subst}\left(\int \left(\frac{1}{2(a+ib)(a-x)}-\frac{b}{\left(a^2+b^2\right)(b-ix)}+\frac{1}{2(a-ib)(a+x)}\right)dx, x, ia\sinh(x)\right)\right)\\
&= \frac{\log(i-\sinh(x))}{2(ia+b)}-\frac{\log(i+\sinh(x))}{2(ia-b)}-\frac{b\log(b+a\sinh(x))}{a^2+b^2}
\end{aligned}$$

Mathematica [A] time = 0.06, size = 36, normalized size = 0.56

$$\frac{-b\log(a\sinh(x)+b)+2a\tan^{-1}\left(\tanh\left(\frac{x}{2}\right)\right)+b\log(\cosh(x))}{a^2+b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[x]/(a + b*Csch[x]),x]
[Out] (2*a*ArcTan[Tanh[x/2]] + b*Log[Cosh[x]] - b*Log[b + a*Sinh[x]])/(a^2 + b^2)
```

fricas [A] time = 0.57, size = 57, normalized size = 0.89

$$\frac{2\,a\arctan\left(\cosh(x)+\sinh(x)\right)-b\log\left(\frac{2\,(a\sinh(x)+b)}{\cosh(x)-\sinh(x)}\right)+b\log\left(\frac{2\,\cosh(x)}{\cosh(x)-\sinh(x)}\right)}{a^2+b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)/(a+b*csch(x)),x, algorithm="fricas")
[Out] (2*a*arctan(cosh(x) + sinh(x)) - b*log(2*(a*sinh(x) + b)/(cosh(x) - sinh(x)
)) + b*log(2*cosh(x)/(cosh(x) - sinh(x))))/(a^2 + b^2)
```

giac [A] time = 0.15, size = 89, normalized size = 1.39

$$-\frac{ab\log\left(\left|-a\left(e^{(-x)}-e^x\right)+2\,b\right|\right)}{a^3+ab^2}+\frac{\left(\pi+2\,\arctan\left(\frac{1}{2}\left(e^{(2\,x)}-1\right)e^{(-x)}\right)\right)a}{2\left(a^2+b^2\right)}+\frac{b\log\left(\left(e^{(-x)}-e^x\right)^2+4\right)}{2\left(a^2+b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)/(a+b*csch(x)),x, algorithm="giac")
[Out] -a*b*log(abs(-a*(e^(-x) - e^x) + 2*b))/(a^3 + a*b^2) + 1/2*(pi + 2*arctan(1
/2*(e^(2*x) - 1)*e^(-x)))*a/(a^2 + b^2) + 1/2*b*log((e^(-x) - e^x)^2 + 4)/(
a^2 + b^2)
```

maple [A] time = 0.15, size = 84, normalized size = 1.31

$$-\frac{2b\ln\left(\left(\tanh^2\left(\frac{x}{2}\right)\right)b-2a\tanh\left(\frac{x}{2}\right)-b\right)}{2a^2+2b^2}+\frac{2b\ln\left(\tanh^2\left(\frac{x}{2}\right)+1\right)}{2a^2+2b^2}+\frac{4a\arctan\left(\tanh\left(\frac{x}{2}\right)\right)}{2a^2+2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(x)/(a+b*csch(x)),x)
[Out] -2*b/(2*a^2+2*b^2)*ln(tanh(1/2*x)^2*b-2*a*tanh(1/2*x)-b)+2/(2*a^2+2*b^2)*b*
ln(tanh(1/2*x)^2+1)+4/(2*a^2+2*b^2)*a*arctan(tanh(1/2*x))
```

maxima [A] time = 0.40, size = 66, normalized size = 1.03

$$-\frac{2\,a\arctan\left(e^{(-x)}\right)}{a^2+b^2}-\frac{b\log\left(-2\,be^{(-x)}+ae^{(-2\,x)}-a\right)}{a^2+b^2}+\frac{b\log\left(e^{(-2\,x)}+1\right)}{a^2+b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)/(a+b*csch(x)),x, algorithm="maxima")
[Out] -2*a*arctan(e^(-x))/(a^2 + b^2) - b*log(-2*b*e^(-x) + a*e^(-2*x) - a)/(a^2
+ b^2) + b*log(e^(-2*x) + 1)/(a^2 + b^2)
```

mupad [B] time = 2.36, size = 93, normalized size = 1.45

$$\frac{\ln\left(1+\mathrm{e}^x\,1\mathrm{i}\right)}{b+a\,1\mathrm{i}}-\frac{b\,\ln\left(a^3\,\mathrm{e}^{2\,x}-4\,a\,b^2-a^3+8\,b^3\,\mathrm{e}^x+2\,a^2\,b\,\mathrm{e}^x+4\,a\,b^2\,\mathrm{e}^{2\,x}\right)}{a^2+b^2}+\frac{\ln\left(\mathrm{e}^x+1\mathrm{i}\right)\,1\mathrm{i}}{a+b\,1\mathrm{i}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(x)*(a + b/sinh(x))),x)
```

```
[Out] log(exp(x)*1i + 1)/(a*1i + b) + (log(exp(x) + 1i)*1i)/(a + b*1i) - (b*log(a
^3*exp(2*x) - 4*a*b^2 - a^3 + 8*b^3*exp(x) + 2*a^2*b*exp(x) + 4*a*b^2*exp(2
*x)))/(a^2 + b^2)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}(x)}{a + b \operatorname{csch}(x)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)/(a+b*csch(x)),x)
```

```
[Out] Integral(sech(x)/(a + b*csch(x)), x)
```

3.98 $\int \frac{\text{sech}^2(x)}{a+b\,\text{csch}(x)}\,dx$

Optimal. Leaf size=60

$$\frac{2ab\tanh^{-1}\left(\frac{a-b\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{3/2}}-\frac{\text{sech}(x)(b-a\sinh(x))}{a^2+b^2}$$

```
[Out] 2*a*b*arctanh((a-b*tanh(1/2*x))/(a^2+b^2)^(1/2))/(a^2+b^2)^(3/2)-sech(x)*(b
-a*sinh(x))/(a^2+b^2)
```

Rubi [A] time = 0.14, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.462$, Rules used = {3872, 2866, 12, 2660, 618, 204}

$$\frac{2ab\tanh^{-1}\left(\frac{a-b\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{3/2}}-\frac{\text{sech}(x)(b-a\sinh(x))}{a^2+b^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[x]^2/(a + b*Csch[x]),x]
```

```
[Out] (2*a*b*ArcTanh[(a - b*Tanh[x/2])/Sqrt[a^2 + b^2]])/(a^2 + b^2)^(3/2) - (Sec
h[x]*(b - a*Sinh[x]))/(a^2 + b^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2866

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[((g*C
os[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m + 1)*(b*c - a*d - (a*c - b*d)*
Sin[e + f*x]))/(f*g*(a^2 - b^2)*(p + 1)), x] + Dist[1/(g^2*(a^2 - b^2)*(p +
 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*Simp[c*(a^2*(p +
2) - b^2*(m + p + 2)) + a*b*d*m + b*(a*c - b*d)*(m + p + 3)*Sin[e + f*x], x
```

```
], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ
[p, -1] && IntegerQ[2*m]
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int \frac{\mathrm{sech}^2(x)}{a+b\mathrm{csch}(x)}\,dx &= i\int \frac{\mathrm{sech}(x)\tanh(x)}{ib+ia\sinh(x)}\,dx\\
&= -\frac{\mathrm{sech}(x)(b-a\sinh(x))}{a^2+b^2}-\frac{i\int \frac{ab}{ib+ia\sinh(x)}\,dx}{a^2+b^2}\\
&= -\frac{\mathrm{sech}(x)(b-a\sinh(x))}{a^2+b^2}-\frac{(iab)\int \frac{1}{ib+ia\sinh(x)}\,dx}{a^2+b^2}\\
&= -\frac{\mathrm{sech}(x)(b-a\sinh(x))}{a^2+b^2}-\frac{(2iab)\,\mathrm{Subst}\left(\int \frac{1}{ib+2iax-ibx^2}\,dx,x,\tanh\left(\frac{x}{2}\right)\right)}{a^2+b^2}\\
&= -\frac{\mathrm{sech}(x)(b-a\sinh(x))}{a^2+b^2}+\frac{(4iab)\,\mathrm{Subst}\left(\int \frac{1}{-4(a^2+b^2)-x^2}\,dx,x,2ia-2ib\tanh\left(\frac{x}{2}\right)\right)}{a^2+b^2}\\
&= \frac{2ab\tanh^{-1}\left(\frac{a-b\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{3/2}}-\frac{\mathrm{sech}(x)(b-a\sinh(x))}{a^2+b^2}\end{aligned}$$

Mathematica [A] time = 0.18, size = 67, normalized size = 1.12

$$\frac{a\left(\tanh(x)-\frac{2b\tan^{-1}\left(\frac{a-b\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-a^2-b^2}}\right)-b\mathrm{sech}(x)}{a^2+b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[x]^2/(a + b*Csch[x]),x]
```

```
[Out] (-(b*Sech[x]) + a*((-2*b*ArcTan[(a - b*Tanh[x/2])/Sqrt[-a^2 - b^2]])/Sqrt[-
a^2 - b^2] + Tanh[x]))/(a^2 + b^2)
```

fricas [B] time = 1.33, size = 256, normalized size = 4.27

$$-\frac{2\,a^3+2\,ab^2-\left(ab\cosh(x)^2+2\,ab\cosh(x)\sinh(x)+ab\sinh(x)^2+ab\right)\sqrt{a^2+b^2}\,\log\left(\frac{a^2\cosh(x)^2+a^2\sinh(x)^2+2\,ab\cosh}{a\cosh(x)^2}\right.}{a^4+2\,a^2b^2+b^4+\left(a^4+2\,a^2b^2+b^4\right)\cosh(x)^2+2\left(a^4+2\,a^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^2/(a+b*csch(x)),x, algorithm="fricas")
```

```
[Out] -(2*a^3 + 2*a*b^2 - (a*b*cosh(x)^2 + 2*a*b*cosh(x)*sinh(x) + a*b*sinh(x)^2
+ a*b)*sqrt(a^2 + b^2)*log((a^2*cosh(x)^2 + a^2*sinh(x)^2 + 2*a*b*cosh(x) +
```

```
a^2 + 2*b^2 + 2*(a^2*cosh(x) + a*b)*sinh(x) + 2*sqrt(a^2 + b^2)*(a*cosh(x)
 + a*sinh(x) + b))/(a*cosh(x)^2 + a*sinh(x)^2 + 2*b*cosh(x) + 2*(a*cosh(x)
+ b)*sinh(x) - a)) + 2*(a^2*b + b^3)*cosh(x) + 2*(a^2*b + b^3)*sinh(x))/(a^
4 + 2*a^2*b^2 + b^4 + (a^4 + 2*a^2*b^2 + b^4)*cosh(x)^2 + 2*(a^4 + 2*a^2*b^
2 + b^4)*cosh(x)*sinh(x) + (a^4 + 2*a^2*b^2 + b^4)*sinh(x)^2)
```

giac [A] time = 0.15, size = 85, normalized size = 1.42

$$-\frac{ab\log\left(\frac{\left|2\,ae^{x}+2\,b-2\,\sqrt{a^2+b^2}\right|}{\left|2\,ae^{x}+2\,b+2\,\sqrt{a^2+b^2}\right|}\right)}{\left(a^2+b^2\right)^{\frac{3}{2}}}-\frac{2\,(be^{x}+a)}{\left(a^2+b^2\right)\left(e^{(2\,x)}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^2/(a+b*csch(x)),x, algorithm="giac")
```

```
[Out] -a*b*log(abs(2*a*e^x + 2*b - 2*sqrt(a^2 + b^2))/abs(2*a*e^x + 2*b + 2*sqrt(
a^2 + b^2)))/(a^2 + b^2)^(3/2) - 2*(b*e^x + a)/((a^2 + b^2)*(e^(2*x) + 1))
```

maple [A] time = 0.17, size = 81, normalized size = 1.35

$$-\frac{4ab\operatorname{arctanh}\left(\frac{2\tanh\left(\frac{x}{2}\right)b-2a}{2\sqrt{a^2+b^2}}\right)}{\left(2a^2+2b^2\right)\sqrt{a^2+b^2}}-\frac{2\left(b-a\tanh\left(\frac{x}{2}\right)\right)}{\left(a^2+b^2\right)\left(\tanh^2\left(\frac{x}{2}\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(x)^2/(a+b*csch(x)),x)
```

```
[Out] -4*a*b/(2*a^2+2*b^2)/(a^2+b^2)^(1/2)*arctanh(1/2*(2*tanh(1/2*x)*b-2*a)/(a^2
+b^2)^(1/2))-2/(a^2+b^2)*(b-a*tanh(1/2*x))/(tanh(1/2*x)^2+1)
```

maxima [A] time = 0.41, size = 91, normalized size = 1.52

$$-\frac{ab\log\left(\frac{ae^{(-x)}-b-\sqrt{a^2+b^2}}{ae^{(-x)}-b+\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{\frac{3}{2}}}-\frac{2\left(be^{(-x)}-a\right)}{a^2+b^2+\left(a^2+b^2\right)e^{(-2\,x)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^2/(a+b*csch(x)),x, algorithm="maxima")
```

```
[Out] -a*b*log((a*e^(-x) - b - sqrt(a^2 + b^2))/(a*e^(-x) - b + sqrt(a^2 + b^2)))
/(a^2 + b^2)^(3/2) - 2*(b*e^(-x) - a)/(a^2 + b^2 + (a^2 + b^2)*e^(-2*x))
```

mupad [B] time = 1.66, size = 133, normalized size = 2.22

$$\frac{a\,b\,\ln\left(\frac{2\,b\,(a-b\,\mathrm{e}^x)}{{\left(a^2+b^2\right)}^{3/2}}+\frac{2\,b\,\mathrm{e}^x}{a^2+b^2}\right)}{{\left(a^2+b^2\right)}^{3/2}}-\frac{a\,b\,\ln\left(\frac{2\,b\,\mathrm{e}^x}{a^2+b^2}-\frac{2\,b\,(a-b\,\mathrm{e}^x)}{{\left(a^2+b^2\right)}^{3/2}}\right)}{{\left(a^2+b^2\right)}^{3/2}}-\frac{\frac{2\,a}{a^2+b^2}+\frac{2\,b\,\mathrm{e}^x}{a^2+b^2}}{\mathrm{e}^{2\,x}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(x)^2*(a + b/sinh(x))),x)
```

```
[Out] (a*b*log((2*b*(a - b*exp(x)))/(a^2 + b^2)^(3/2) + (2*b*exp(x))/(a^2 + b^2))
)/(a^2 + b^2)^(3/2) - (a*b*log((2*b*exp(x))/(a^2 + b^2) - (2*b*(a - b*exp(x
)))/(a^2 + b^2)^(3/2)))/(a^2 + b^2)^(3/2) - ((2*a)/(a^2 + b^2) + (2*b*exp(x
))/(a^2 + b^2))/(exp(2*x) + 1)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}^2(x)}{a+b\operatorname{csch}(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)**2/(a+b*csch(x)),x)
[Out] Integral(sech(x)**2/(a + b*csch(x)), x)
```

3.99 $\int \frac{\textbf{sech}^3(x)}{a+b\textbf{csch}(x)}\,dx$

Optimal. Leaf size=95

$$-\frac{a^2b\log(a\sinh(x)+b)}{\left(a^2+b^2\right)^2}-\frac{\operatorname{sech}^2(x)(b-a\sinh(x))}{2\left(a^2+b^2\right)}-\frac{ia\log(-\sinh(x)+i)}{4(a-ib)^2}+\frac{ia\log(\sinh(x)+i)}{4(a+ib)^2}$$

```
[Out] -1/4*I*a*ln(I-sinh(x))/(a-I*b)^2+1/4*I*a*ln(I+sinh(x))/(a+I*b)^2-a^2*b*ln(b
+a*sinh(x))/(a^2+b^2)^2-1/2*sech(x)^2*(b-a*sinh(x))/(a^2+b^2)
```

Rubi [A] time = 0.22, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {3872, 2837, 12, 823, 801}

$$-\frac{a^2b\log(a\sinh(x)+b)}{\left(a^2+b^2\right)^2}-\frac{\operatorname{sech}^2(x)(b-a\sinh(x))}{2\left(a^2+b^2\right)}-\frac{ia\log(-\sinh(x)+i)}{4(a-ib)^2}+\frac{ia\log(\sinh(x)+i)}{4(a+ib)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[x]^3/(a + b*Csch[x]),x]
```

```
[Out] ((-I/4)*a*Log[I - Sinh[x]])/(a - I*b)^2 + ((I/4)*a*Log[I + Sinh[x]])/(a + I
*b)^2 - (a^2*b*Log[b + a*Sinh[x]])/(a^2 + b^2)^2 - (Sech[x]^2*(b - a*Sinh[x
]))/(2*(a^2 + b^2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 823

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(f*a*c*e - a*g*c*d + c*(c*d*f + a
*e*g)*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[1/
(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp[f
*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f + a
*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*
d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2
*m, 2*p])
```

Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
```

```
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\int \frac{\operatorname{sech}^3(x)}{a+b\operatorname{csch}(x)}\,dx = i\int \frac{\operatorname{sech}^2(x)\tanh(x)}{ib+ia\sinh(x)}\,dx$$
$$= -\left(\left(ia^3\right)\operatorname{Subst}\left(\int \frac{x}{a(ib+x)\left(a^2-x^2\right)^2}\,dx, x, ia\sinh(x)\right)\right)$$
$$= -\left(\left(ia^2\right)\operatorname{Subst}\left(\int \frac{x}{(ib+x)\left(a^2-x^2\right)^2}\,dx, x, ia\sinh(x)\right)\right)$$
$$= -\frac{\operatorname{sech}^2(x)(b-a\sinh(x))}{2\left(a^2+b^2\right)} - \frac{i\operatorname{Subst}\left(\int \frac{-ia^2b+a^2x}{(ib+x)\left(a^2-x^2\right)}\,dx, x, ia\sinh(x)\right)}{2\left(a^2+b^2\right)}$$
$$= -\frac{\operatorname{sech}^2(x)(b-a\sinh(x))}{2\left(a^2+b^2\right)} - \frac{i\operatorname{Subst}\left(\int \left(\frac{a(a-ib)}{2(a+ib)(a-x)} - \frac{2a^2b}{\left(a^2+b^2\right)(b-ix)} + \frac{a(a+ib)}{2(a-ib)(a+x)}\right)dx, x, ia\operatorname{sinh}\right.}{2\left(a^2+b^2\right)}$$
$$= -\frac{ia\log(i-\sinh(x))}{4(a-ib)^2} + \frac{ia\log(i+\sinh(x))}{4(a+ib)^2} - \frac{a^2b\log(b+a\sinh(x))}{\left(a^2+b^2\right)^2} - \frac{\operatorname{sech}^2(x)(b-a\sinh(}{2\left(a^2+b^2\right)}$$

Mathematica [A] time = 0.16, size = 78, normalized size = 0.82

$$\frac{-b\left(a^2+b^2\right)\operatorname{sech}^2(x) + a\left(a^2+b^2\right)\tanh(x)\operatorname{sech}(x) + 2a\left(\left(a^2-b^2\right)\tan^{-1}\left(\tanh\left(\frac{x}{2}\right)\right) + ab(\log(\cosh(x)) - \log(a\,\mathrm{s}\right.}{2\left(a^2+b^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[x]^3/(a + b*Csch[x]),x]
```

```
[Out] (2*a*((a^2 - b^2)*ArcTan[Tanh[x/2]] + a*b*(Log[Cosh[x]] - Log[b + a*Sinh[x]
])) - b*(a^2 + b^2)*Sech[x]^2 + a*(a^2 + b^2)*Sech[x]*Tanh[x])/(2*(a^2 + b^
2)^2)
```

fricas [B] time = 0.64, size = 675, normalized size = 7.11

$$\frac{\left(a^3+ab^2\right)\cosh(x)^3 + \left(a^3+ab^2\right)\sinh(x)^3 - 2\left(a^2b+b^3\right)\cosh(x)^2 - \left(2\,a^2b + 2\,b^3 - 3\left(a^3+ab^2\right)\cosh(x)\right)\sinh(x)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^3/(a+b*csch(x)),x, algorithm="fricas")
```

```
[Out] ((a^3 + a*b^2)*cosh(x)^3 + (a^3 + a*b^2)*sinh(x)^3 - 2*(a^2*b + b^3)*cosh(x
)^2 - (2*a^2*b + 2*b^3 - 3*(a^3 + a*b^2)*cosh(x))*sinh(x)^2 + ((a^3 - a*b^2
)*cosh(x)^4 + 4*(a^3 - a*b^2)*cosh(x)*sinh(x)^3 + (a^3 - a*b^2)*sinh(x)^4 +
 a^3 - a*b^2 + 2*(a^3 - a*b^2)*cosh(x)^2 + 2*(a^3 - a*b^2 + 3*(a^3 - a*b^2)
*cosh(x)^2)*sinh(x)^2 + 4*((a^3 - a*b^2)*cosh(x)^3 + (a^3 - a*b^2)*cosh(x))
*sinh(x))*arctan(cosh(x) + sinh(x)) - (a^3 + a*b^2)*cosh(x) - (a^2*b*cosh(x
)^4 + 4*a^2*b*cosh(x)*sinh(x)^3 + a^2*b*sinh(x)^4 + 2*a^2*b*cosh(x)^2 + a^2
*b + 2*(3*a^2*b*cosh(x)^2 + a^2*b)*sinh(x)^2 + 4*(a^2*b*cosh(x)^3 + a^2*b*c
osh(x))*sinh(x))*log(2*(a*sinh(x) + b)/(cosh(x) - sinh(x))) + (a^2*b*cosh(x
```

```
)^4 + 4*a^2*b*cosh(x)*sinh(x)^3 + a^2*b*sinh(x)^4 + 2*a^2*b*cosh(x)^2 + a^2
*b + 2*(3*a^2*b*cosh(x)^2 + a^2*b)*sinh(x)^2 + 4*(a^2*b*cosh(x)^3 + a^2*b*c
osh(x))*sinh(x))*log(2*cosh(x)/(cosh(x) - sinh(x))) - (a^3 + a*b^2 - 3*(a^3
 + a*b^2)*cosh(x)^2 + 4*(a^2*b + b^3)*cosh(x))*sinh(x))/((a^4 + 2*a^2*b^2 +
 b^4)*cosh(x)^4 + 4*(a^4 + 2*a^2*b^2 + b^4)*cosh(x)*sinh(x)^3 + (a^4 + 2*a^
2*b^2 + b^4)*sinh(x)^4 + a^4 + 2*a^2*b^2 + b^4 + 2*(a^4 + 2*a^2*b^2 + b^4)*
cosh(x)^2 + 2*(a^4 + 2*a^2*b^2 + b^4 + 3*(a^4 + 2*a^2*b^2 + b^4)*cosh(x)^2)
*sinh(x)^2 + 4*((a^4 + 2*a^2*b^2 + b^4)*cosh(x)^3 + (a^4 + 2*a^2*b^2 + b^4)
*cosh(x))*sinh(x))
```

giac [B] time = 0.14, size = 218, normalized size = 2.29

$$-\frac{a^{3} b \log\left(\left|-a\left(e^{(-x)}-e^{x}\right)+2 b\right|\right)}{a^{5}+2 a^{3} b^{2}+a b^{4}}+\frac{a^{2} b \log\left(\left(e^{(-x)}-e^{x}\right)^{2}+4\right)}{2\left(a^{4}+2 a^{2} b^{2}+b^{4}\right)}+\frac{\left(\pi+2 \arctan\left(\frac{1}{2}\left(e^{(2 x)}-1\right)e^{(-x)}\right)\right)\left(a^{3}-a b^{2}\right)}{4\left(a^{4}+2 a^{2} b^{2}+b^{4}\right)}-\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^3/(a+b*csch(x)),x, algorithm="giac")
```

```
[Out] -a^3*b*log(abs(-a*(e^(-x) - e^x) + 2*b))/(a^5 + 2*a^3*b^2 + a*b^4) + 1/2*a^
2*b*log((e^(-x) - e^x)^2 + 4)/(a^4 + 2*a^2*b^2 + b^4) + 1/4*(pi + 2*arctan(
1/2*(e^(2*x) - 1)*e^(-x)))*(a^3 - a*b^2)/(a^4 + 2*a^2*b^2 + b^4) - 1/2*(a^2
*b*(e^(-x) - e^x)^2 + 2*a^3*(e^(-x) - e^x) + 2*a*b^2*(e^(-x) - e^x) + 8*a^2
*b + 4*b^3)/((a^4 + 2*a^2*b^2 + b^4)*((e^(-x) - e^x)^2 + 4))
```

maple [B] time = 0.19, size = 275, normalized size = 2.89

$$-\frac{a^{2} b \ln\left(\left(\tanh^{2}\left(\frac{x}{2}\right)\right) b-2 a \tanh\left(\frac{x}{2}\right)-b\right)}{\left(a^{2}+b^{2}\right)^{2}}-\frac{\left(\tanh^{3}\left(\frac{x}{2}\right)\right) a^{3}}{\left(a^{2}+b^{2}\right)^{2}\left(\tanh^{2}\left(\frac{x}{2}\right)+1\right)^{2}}-\frac{\left(\tanh^{3}\left(\frac{x}{2}\right)\right) a b^{2}}{\left(a^{2}+b^{2}\right)^{2}\left(\tanh^{2}\left(\frac{x}{2}\right)+1\right)^{2}}+\frac{2(\cdots}{(a^{2}+b^{2}\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(x)^3/(a+b*csch(x)),x)
```

```
[Out] -a^2*b/(a^2+b^2)^2*ln(tanh(1/2*x)^2*b-2*a*tanh(1/2*x)-b)-1/(a^2+b^2)^2/(tan
h(1/2*x)^2+1)^2*tanh(1/2*x)^3*a^3-1/(a^2+b^2)^2/(tanh(1/2*x)^2+1)^2*tanh(1/
2*x)^3*a*b^2+2/(a^2+b^2)^2/(tanh(1/2*x)^2+1)^2*tanh(1/2*x)^2*a^2*b+2/(a^2+b
^2)^2/(tanh(1/2*x)^2+1)^2*tanh(1/2*x)^2*b^3+1/(a^2+b^2)^2/(tanh(1/2*x)^2+1)
^2*tanh(1/2*x)*a^3+1/(a^2+b^2)^2/(tanh(1/2*x)^2+1)^2*tanh(1/2*x)*a*b^2+1/(a
^2+b^2)^2*arctan(tanh(1/2*x))*a^3-1/(a^2+b^2)^2*arctan(tanh(1/2*x))*a*b^2+1
/(a^2+b^2)^2*ln(tanh(1/2*x)^2+1)*a^2*b
```

maxima [B] time = 0.41, size = 161, normalized size = 1.69

$$-\frac{a^{2} b \log\left(-2 b e^{(-x)}+a e^{(-2 x)}-a\right)}{a^{4}+2 a^{2} b^{2}+b^{4}}+\frac{a^{2} b \log\left(e^{(-2 x)}+1\right)}{a^{4}+2 a^{2} b^{2}+b^{4}}-\frac{\left(a^{3}-a b^{2}\right) \arctan\left(e^{(-x)}\right)}{a^{4}+2 a^{2} b^{2}+b^{4}}+\frac{a e^{(-x)}-2 b e^{(-2 x)}-\cdots}{a^{2}+b^{2}+2\left(a^{2}+b^{2}\right) e^{(-2 x)}\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^3/(a+b*csch(x)),x, algorithm="maxima")
```

```
[Out] -a^2*b*log(-2*b*e^(-x) + a*e^(-2*x) - a)/(a^4 + 2*a^2*b^2 + b^4) + a^2*b*lo
g(e^(-2*x) + 1)/(a^4 + 2*a^2*b^2 + b^4) - (a^3 - a*b^2)*arctan(e^(-x))/(a^4
 + 2*a^2*b^2 + b^4) + (a*e^(-x) - 2*b*e^(-2*x) - a*e^(-3*x))/(a^2 + b^2 + 2
*(a^2 + b^2)*e^(-2*x) + (a^2 + b^2)*e^(-4*x))
```

mupad [B] time = 2.99, size = 256, normalized size = 2.69

$$\frac{\frac{2 b}{a^{2}+b^{2}}-\frac{2 a e^{x}}{a^{2}+b^{2}}}{2 e^{2 x}+e^{4 x}+1}-\frac{\frac{2\left(a^{2} b+b^{3}\right)}{\left(a^{2}+b^{2}\right)^{2}}-\frac{e^{x}\left(a^{3}+a b^{2}\right)}{\left(a^{2}+b^{2}\right)^{2}}}{e^{2 x}+1}+\frac{a \ln\left(e^{x}+1 i\right)}{2\left(-a^{2} 1 i+2 a b+b^{2} 1 i\right)}-\frac{a^{2} b \ln\left(a^{6} e^{2 x}-a^{6}-a^{2} b^{4}-14 a^{4} b^{2}+a^{2} \cdots\right.}{a^{4}\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(x)^3*(a + b/sinh(x))),x)
```

```
[Out] ((2*b)/(a^2 + b^2) - (2*a*exp(x))/(a^2 + b^2))/(2*exp(2*x) + exp(4*x) + 1)
- ((2*(a^2*b + b^3))/(a^2 + b^2)^2 - (exp(x)*(a*b^2 + a^3))/(a^2 + b^2)^2)/
(exp(2*x) + 1) + (a*log(exp(x)*1i + 1)*1i)/(2*(a*b*2i - a^2 + b^2)) + (a*lo
g(exp(x) + 1i))/(2*(2*a*b - a^2*1i + b^2*1i)) - (a^2*b*log(a^6*exp(2*x) - a
^6 - a^2*b^4 - 14*a^4*b^2 + a^2*b^4*exp(2*x) + 14*a^4*b^2*exp(2*x) + 2*a*b^
5*exp(x) + 2*a^5*b*exp(x) + 28*a^3*b^3*exp(x)))/(a^4 + b^4 + 2*a^2*b^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}^3(x)}{a + b\operatorname{csch}(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)**3/(a+b*csch(x)),x)
```

```
[Out] Integral(sech(x)**3/(a + b*csch(x)), x)
```

3.100 $\int \frac{\operatorname{sech}^4(x)}{a+b\operatorname{csch}(x)}\,dx$

Optimal. Leaf size=104

$$-\frac{\operatorname{sech}^3(x)(b-a\sinh(x))}{3\left(a^2+b^2\right)}-\frac{\operatorname{sech}(x)\left(3a^2b-a\left(2a^2-b^2\right)\sinh(x)\right)}{3\left(a^2+b^2\right)^2}+\frac{2a^3b\tanh^{-1}\left(\frac{a-b\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{5/2}}$$

```
[Out] 2*a^3*b*arctanh((a-b*tanh(1/2*x))/(a^2+b^2)^(1/2))/(a^2+b^2)^(5/2)-1/3*sech
(x)^3*(b-a*sinh(x))/(a^2+b^2)-1/3*sech(x)*(3*a^2*b-a*(2*a^2-b^2)*sinh(x))/(
a^2+b^2)^2
```

Rubi [A] time = 0.27, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {3872, 2866, 12, 2660, 618, 204}

$$\frac{2a^3b\tanh^{-1}\left(\frac{a-b\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{5/2}}-\frac{\operatorname{sech}^3(x)(b-a\sinh(x))}{3\left(a^2+b^2\right)}-\frac{\operatorname{sech}(x)\left(3a^2b-a\left(2a^2-b^2\right)\sinh(x)\right)}{3\left(a^2+b^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[x]^4/(a + b*Csch[x]),x]
```

```
[Out] (2*a^3*b*ArcTanh[(a - b*Tanh[x/2])/Sqrt[a^2 + b^2]])/(a^2 + b^2)^(5/2) - (S
ech[x]^3*(b - a*Sinh[x]))/(3*(a^2 + b^2)) - (Sech[x]*(3*a^2*b - a*(2*a^2 -
b^2)*Sinh[x]))/(3*(a^2 + b^2)^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2866

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[((g*C
os[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m + 1)*(b*c - a*d - (a*c - b*d)*
Sin[e + f*x]))/(f*g*(a^2 - b^2)*(p + 1)), x] + Dist[1/(g^2*(a^2 - b^2)*(p +
```

```
1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*Simp[c*(a^2*(p +
2) - b^2*(m + p + 2)) + a*b*d*m + b*(a*c - b*d)*(m + p + 3)*Sin[e + f*x], x
], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ
[p, -1] && IntegerQ[2*m]
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\int \frac{\operatorname{sech}^4(x)}{a+b\operatorname{csch}(x)}\,dx = i\int \frac{\operatorname{sech}^3(x)\tanh(x)}{ib+ia\sinh(x)}\,dx$$

$$= -\frac{\operatorname{sech}^3(x)(b-a\sinh(x))}{3\left(a^2+b^2\right)} + \frac{\int \frac{\operatorname{sech}^2(x)\left(-iab+2ia^2\sinh(x)\right)}{ib+ia\sinh(x)}\,dx}{3\left(a^2+b^2\right)}$$

$$= -\frac{\operatorname{sech}^3(x)(b-a\sinh(x))}{3\left(a^2+b^2\right)} - \frac{\operatorname{sech}(x)\left(3a^2b-a\left(2a^2-b^2\right)\sinh(x)\right)}{3\left(a^2+b^2\right)^2} + \frac{\int -\frac{3ia^3b}{ib+ia\sinh(x)}\,dx}{3\left(a^2+b^2\right)^2}$$

$$= -\frac{\operatorname{sech}^3(x)(b-a\sinh(x))}{3\left(a^2+b^2\right)} - \frac{\operatorname{sech}(x)\left(3a^2b-a\left(2a^2-b^2\right)\sinh(x)\right)}{3\left(a^2+b^2\right)^2} - \frac{\left(ia^3b\right)\int \frac{1}{ib+ia\sinh(x)}\,dx}{\left(a^2+b^2\right)^2}$$

$$= -\frac{\operatorname{sech}^3(x)(b-a\sinh(x))}{3\left(a^2+b^2\right)} - \frac{\operatorname{sech}(x)\left(3a^2b-a\left(2a^2-b^2\right)\sinh(x)\right)}{3\left(a^2+b^2\right)^2} - \frac{\left(2ia^3b\right)\operatorname{Subst}\left(\int \frac{}{ib+2ia}\right.}{\left(a^2+\right.}$$

$$= -\frac{\operatorname{sech}^3(x)(b-a\sinh(x))}{3\left(a^2+b^2\right)} - \frac{\operatorname{sech}(x)\left(3a^2b-a\left(2a^2-b^2\right)\sinh(x)\right)}{3\left(a^2+b^2\right)^2} + \frac{\left(4ia^3b\right)\operatorname{Subst}\left(\int \frac{}{-4\left(a^2\right.}\right.}{}$$

$$= \frac{2a^3b\tanh^{-1}\left(\frac{a-b\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{5/2}} - \frac{\operatorname{sech}^3(x)(b-a\sinh(x))}{3\left(a^2+b^2\right)} - \frac{\operatorname{sech}(x)\left(3a^2b-a\left(2a^2-b^2\right)\sinh(x)\right)}{3\left(a^2+b^2\right)^2}$$

Mathematica [A] time = 0.63, size = 114, normalized size = 1.10

$$-\frac{\left(ab^2-2a^3\right)\tanh(x)+b\left(a^2+b^2\right)\operatorname{sech}^3(x)-a\left(a^2+b^2\right)\tanh(x)\operatorname{sech}^2(x)+3a^2b\operatorname{sech}(x)+\frac{6a^3b\tan^{-1}\left(\frac{a-b\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-a^2-b^2}}}{3\left(a^2+b^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[x]^4/(a + b*Csch[x]),x]
```

```
[Out] -1/3*((6*a^3*b*ArcTan[(a - b*Tanh[x/2])/Sqrt[-a^2 - b^2]])/Sqrt[-a^2 - b^2]
 + 3*a^2*b*Sech[x] + b*(a^2 + b^2)*Sech[x]^3 + (-2*a^3 + a*b^2)*Tanh[x] - a
*(a^2 + b^2)*Sech[x]^2*Tanh[x])/(a^2 + b^2)^2
```

fricas [B] time = 1.56, size = 1155, normalized size = 11.11

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^4/(a+b*csch(x)),x, algorithm="fricas")
```

```
[Out] -1/3*(6*(a^4*b + a^2*b^3)*cosh(x)^5 + 6*(a^4*b + a^2*b^3)*sinh(x)^5 + 4*a^5
 + 2*a^3*b^2 - 2*a*b^4 - 6*(a^3*b^2 + a*b^4)*cosh(x)^4 - 6*(a^3*b^2 + a*b^4
 - 5*(a^4*b + a^2*b^3)*cosh(x))*sinh(x)^4 + 4*(5*a^4*b + 7*a^2*b^3 + 2*b^5)
*cosh(x)^3 + 4*(5*a^4*b + 7*a^2*b^3 + 2*b^5 + 15*(a^4*b + a^2*b^3)*cosh(x)^
2 - 6*(a^3*b^2 + a*b^4)*cosh(x))*sinh(x)^3 + 12*(a^5 + a^3*b^2)*cosh(x)^2 +
 12*(a^5 + a^3*b^2 + 5*(a^4*b + a^2*b^3)*cosh(x)^3 - 3*(a^3*b^2 + a*b^4)*co
sh(x)^2 + (5*a^4*b + 7*a^2*b^3 + 2*b^5)*cosh(x))*sinh(x)^2 - 3*(a^3*b*cosh(
x)^6 + 6*a^3*b*cosh(x)*sinh(x)^5 + a^3*b*sinh(x)^6 + 3*a^3*b*cosh(x)^4 + 3*
a^3*b*cosh(x)^2 + 3*(5*a^3*b*cosh(x)^2 + a^3*b)*sinh(x)^4 + a^3*b + 4*(5*a^
3*b*cosh(x)^3 + 3*a^3*b*cosh(x))*sinh(x)^3 + 3*(5*a^3*b*cosh(x)^4 + 6*a^3*b
*cosh(x)^2 + a^3*b)*sinh(x)^2 + 6*(a^3*b*cosh(x)^5 + 2*a^3*b*cosh(x)^3 + a^
3*b*cosh(x))*sinh(x))*sqrt(a^2 + b^2)*log((a^2*cosh(x)^2 + a^2*sinh(x)^2 +
2*a*b*cosh(x) + a^2 + 2*b^2 + 2*(a^2*cosh(x) + a*b)*sinh(x) + 2*sqrt(a^2 +
b^2)*(a*cosh(x) + a*sinh(x) + b))/(a*cosh(x)^2 + a*sinh(x)^2 + 2*b*cosh(x)
+ 2*(a*cosh(x) + b)*sinh(x) - a)) + 6*(a^4*b + a^2*b^3)*cosh(x) + 6*(a^4*b
+ a^2*b^3 + 5*(a^4*b + a^2*b^3)*cosh(x)^4 - 4*(a^3*b^2 + a*b^4)*cosh(x)^3 +
 2*(5*a^4*b + 7*a^2*b^3 + 2*b^5)*cosh(x)^2 + 4*(a^5 + a^3*b^2)*cosh(x))*sin
h(x))/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*cosh(x)^6 + 6*(a^6 + 3*a^4*b^2 +
 3*a^2*b^4 + b^6)*cosh(x)*sinh(x)^5 + (a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*s
inh(x)^6 + a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6 + 3*(a^6 + 3*a^4*b^2 + 3*a^2*b
^4 + b^6)*cosh(x)^4 + 3*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6 + 5*(a^6 + 3*a^4
*b^2 + 3*a^2*b^4 + b^6)*cosh(x)^2)*sinh(x)^4 + 4*(5*(a^6 + 3*a^4*b^2 + 3*a^
2*b^4 + b^6)*cosh(x)^3 + 3*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*cosh(x))*sin
h(x)^3 + 3*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*cosh(x)^2 + 3*(a^6 + 3*a^4*b
^2 + 3*a^2*b^4 + b^6 + 5*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*cosh(x)^4 + 6*
(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*cosh(x)^2)*sinh(x)^2 + 6*((a^6 + 3*a^4*
b^2 + 3*a^2*b^4 + b^6)*cosh(x)^5 + 2*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*co
sh(x)^3 + (a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*cosh(x))*sinh(x))
```

giac [A] time = 0.18, size = 174, normalized size = 1.67

$$-\frac{a^{3} b \log\left(\frac{\left|2 \, a e^{x} + 2 \, b - 2 \, \sqrt{a^{2} + b^{2}}\right|}{\left|2 \, a e^{x} + 2 \, b + 2 \, \sqrt{a^{2} + b^{2}}\right|}\right)}{{\left(a^{4} + 2 \, a^{2} b^{2} + b^{4}\right)} \sqrt{a^{2} + b^{2}}} - \frac{2 \, {\left(3 \, a^{2} b e^{\left(5 \, x\right)} - 3 \, a b^{2} e^{\left(4 \, x\right)} + 10 \, a^{2} b e^{\left(3 \, x\right)} + 4 \, b^{3} e^{\left(3 \, x\right)} + 6 \, a^{3} e^{\left(2 \, x\right)} + 3 \, a^{2} b e^{x} + 2 \, a^{3} -}}{3 \, {\left(a^{4} + 2 \, a^{2} b^{2} + b^{4}\right)} {\left(e^{\left(2 \, x\right)} + 1\right)}^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^4/(a+b*csch(x)),x, algorithm="giac")
```

```
[Out] -a^3*b*log(abs(2*a*e^x + 2*b - 2*sqrt(a^2 + b^2))/abs(2*a*e^x + 2*b + 2*sqr
t(a^2 + b^2)))/((a^4 + 2*a^2*b^2 + b^4)*sqrt(a^2 + b^2)) - 2/3*(3*a^2*b*e^(
5*x) - 3*a*b^2*e^(4*x) + 10*a^2*b*e^(3*x) + 4*b^3*e^(3*x) + 6*a^3*e^(2*x) +
 3*a^2*b*e^x + 2*a^3 - a*b^2)/((a^4 + 2*a^2*b^2 + b^4)*(e^(2*x) + 1)^3)
```

maple [A] time = 0.18, size = 170, normalized size = 1.63

$$-\frac{4 a^{3} b \operatorname{arctanh}\left(\frac{2 \tanh\left(\frac{x}{2}\right) b - 2 a}{2 \sqrt{a^{2}+b^{2}}}\right)}{\left(2 a^{4}+4 a^{2} b^{2}+2 b^{4}\right) \sqrt{a^{2}+b^{2}}} - \frac{2 \left(-a^{3} \left(\tanh^{5}\left(\frac{x}{2}\right)\right) + \left(2 a^{2} b + b^{3}\right) \left(\tanh^{4}\left(\frac{x}{2}\right)\right) + \left(-\frac{2}{3} a^{3} + \frac{4}{3} a b^{2}\right) \left(\tanh^{3}\left(\frac{x}{2}\right)\right)}{\left(a^{4}+2 a^{2} b^{2}+b^{4}\right) \left(\tanh^{2}\left(\frac{x}{2}\right)+1\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(x)^4/(a+b*csch(x)),x)
```

```
[Out] -4*a^3*b/(2*a^4+4*a^2*b^2+2*b^4)/(a^2+b^2)^(1/2)*arctanh(1/2*(2*tanh(1/2*x)
*b-2*a)/(a^2+b^2)^(1/2))-2/(a^4+2*a^2*b^2+b^4)*(-a^3*tanh(1/2*x)^5+(2*a^2*b
```

```
+b^3)*tanh(1/2*x)^4+(-2/3*a^3+4/3*a*b^2)*tanh(1/2*x)^3+2*a^2*b*tanh(1/2*x)^
2-a^3*tanh(1/2*x)+4/3*a^2*b+1/3*b^3)/(tanh(1/2*x)^2+1)^3
```

maxima [B] time = 0.42, size = 226, normalized size = 2.17

$$-\frac{a^{3}b\log\left(\frac{ae^{(-x)}-b-\sqrt{a^{2}+b^{2}}}{ae^{(-x)}-b+\sqrt{a^{2}+b^{2}}}\right)}{\left(a^{4}+2\,a^{2}b^{2}+b^{4}\right)\sqrt{a^{2}+b^{2}}}-\frac{2\left(3\,a^{2}be^{(-x)}-6\,a^{3}e^{(-2\,x)}+3\,ab^{2}e^{(-4\,x)}+3\,a^{2}be^{(-5\,x)}-2\,a^{3}+ab^{2}+2\left(5\,a^{2}b\right.\right.}{3\left(a^{4}+2\,a^{2}b^{2}+b^{4}+3\left(a^{4}+2\,a^{2}b^{2}+b^{4}\right)e^{(-2\,x)}+3\left(a^{4}+2\,a^{2}b^{2}+b^{4}\right)e^{(-4\,x)}+\left(a^{4}\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^4/(a+b*csch(x)),x, algorithm="maxima")
```

```
[Out] -a^3*b*log((a*e^(-x) - b - sqrt(a^2 + b^2))/(a*e^(-x) - b + sqrt(a^2 + b^2)
))/((a^4 + 2*a^2*b^2 + b^4)*sqrt(a^2 + b^2)) - 2/3*(3*a^2*b*e^(-x) - 6*a^3*
e^(-2*x) + 3*a*b^2*e^(-4*x) + 3*a^2*b*e^(-5*x) - 2*a^3 + a*b^2 + 2*(5*a^2*b
 + 2*b^3)*e^(-3*x))/(a^4 + 2*a^2*b^2 + b^4 + 3*(a^4 + 2*a^2*b^2 + b^4)*e^(-
2*x) + 3*(a^4 + 2*a^2*b^2 + b^4)*e^(-4*x) + (a^4 + 2*a^2*b^2 + b^4)*e^(-6*x
))
```

mupad [B] time = 1.79, size = 269, normalized size = 2.59

$$\frac{\frac{2\,a\,b^2}{{\left(a^2+b^2\right)}^2}-\frac{2\,a^2\,b\,{\mathrm{e}}^x}{{\left(a^2+b^2\right)}^2}}{{\mathrm{e}}^{2\,x}+1}-\frac{\frac{4\,\left(a^3+a\,b^2\right)}{{\left(a^2+b^2\right)}^2}+\frac{8\,{\mathrm{e}}^x\,\left(a^2\,b+b^3\right)}{3\,{\left(a^2+b^2\right)}^2}}{2\,{\mathrm{e}}^{2\,x}+{\mathrm{e}}^{4\,x}+1}+\frac{\frac{8\,a}{3\,\left(a^2+b^2\right)}+\frac{8\,b\,{\mathrm{e}}^x}{3\,\left(a^2+b^2\right)}}{3\,{\mathrm{e}}^{2\,x}+3\,{\mathrm{e}}^{4\,x}+{\mathrm{e}}^{6\,x}+1}-\frac{a^3\,b\,\ln\left(\frac{2\,a^2\,b\,{\mathrm{e}}^x}{{\left(a^2+b^2\right)}^2}-\frac{2\,a^2\,b\,\left(a-b\,{\mathrm{e}}^x\right)}{{\left(a^2+b^2\right)}^{5/2}}\right)}{{\left(a^2+b^2\right)}^{5/2}}+\frac{a^3\,b\,\ln\left(\frac{2\,a^2\right.}{\left(a\right.}\right.}{\left(a\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(x)^4*(a + b/sinh(x))),x)
```

```
[Out] ((2*a*b^2)/(a^2 + b^2)^2 - (2*a^2*b*exp(x))/(a^2 + b^2)^2)/(exp(2*x) + 1) -
 ((4*(a*b^2 + a^3))/(a^2 + b^2)^2 + (8*exp(x)*(a^2*b + b^3))/(3*(a^2 + b^2)
^2))/(2*exp(2*x) + exp(4*x) + 1) + ((8*a)/(3*(a^2 + b^2)) + (8*b*exp(x))/(3
*(a^2 + b^2)))/(3*exp(2*x) + 3*exp(4*x) + exp(6*x) + 1) - (a^3*b*log((2*a^2
*b*exp(x))/(a^2 + b^2)^2 - (2*a^2*b*(a - b*exp(x)))/(a^2 + b^2)^(5/2)))/(a^
2 + b^2)^(5/2) + (a^3*b*log((2*a^2*b*(a - b*exp(x)))/(a^2 + b^2)^(5/2) + (2
*a^2*b*exp(x))/(a^2 + b^2)^2))/(a^2 + b^2)^(5/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}^{4}(x)}{a + b\operatorname{csch}(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)**4/(a+b*csch(x)),x)
```

```
[Out] Integral(sech(x)**4/(a + b*csch(x)), x)
```

3.101 $\int \frac{\mathrm{sech}^5(x)}{a+b\mathrm{csch}(x)}\,dx$

Optimal. Leaf size=149

$$-\frac{\mathrm{sech}^4(x)(b-a\sinh(x))}{4\left(a^2+b^2\right)}-\frac{\mathrm{sech}^2(x)\left(4a^2b-a\left(3a^2-b^2\right)\sinh(x)\right)}{8\left(a^2+b^2\right)^2}-\frac{a^4b\log(a\sinh(x)+b)}{\left(a^2+b^2\right)^3}-\frac{a(b+3ia)\log(-\mathrm{si}}{16(a-ib)}$$

```
[Out] -1/16*a*(3*I*a+b)*ln(I-sinh(x))/(a-I*b)^3+1/16*a*(3*a+I*b)*ln(I+sinh(x))/(I
*a-b)^3-a^4*b*ln(b+a*sinh(x))/(a^2+b^2)^3-1/4*sech(x)^4*(b-a*sinh(x))/(a^2+
b^2)-1/8*sech(x)^2*(4*a^2*b-a*(3*a^2-b^2)*sinh(x))/(a^2+b^2)^2
```

Rubi [A] time = 0.34, antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {3872, 2837, 12, 823, 801}

$$-\frac{a^4b\log(a\sinh(x)+b)}{\left(a^2+b^2\right)^3}-\frac{\mathrm{sech}^4(x)(b-a\sinh(x))}{4\left(a^2+b^2\right)}-\frac{\mathrm{sech}^2(x)\left(4a^2b-a\left(3a^2-b^2\right)\sinh(x)\right)}{8\left(a^2+b^2\right)^2}-\frac{a(b+3ia)\log(-\mathrm{si}}{16(a-ib)}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[x]^5/(a + b*Csch[x]),x]
```

```
[Out] -(a*((3*I)*a + b)*Log[I - Sinh[x]])/(16*(a - I*b)^3) + (a*(3*a + I*b)*Log[I
 + Sinh[x]])/(16*(I*a - b)^3) - (a^4*b*Log[b + a*Sinh[x]])/(a^2 + b^2)^3 -
(Sech[x]^4*(b - a*Sinh[x]))/(4*(a^2 + b^2)) - (Sech[x]^2*(4*a^2*b - a*(3*a^
2 - b^2)*Sinh[x]))/(8*(a^2 + b^2)^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 823

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(f*a*c*e - a*g*c*d + c*(c*d*f + a
*e*g)*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[1/
(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp[f
*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f + a
*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*
d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2
*m, 2*p])
```

Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\int \frac{\operatorname{sech}^5(x)}{a+b\operatorname{csch}(x)}\,dx = i\int \frac{\operatorname{sech}^4(x)\tanh(x)}{ib+ia\sinh(x)}\,dx$$

$$= -\left(\left(ia^5\right)\operatorname{Subst}\left(\int \frac{x}{a(ib+x)\left(a^2-x^2\right)^3}\,dx, x, ia\sinh(x)\right)\right)$$

$$= -\left(\left(ia^4\right)\operatorname{Subst}\left(\int \frac{x}{(ib+x)\left(a^2-x^2\right)^3}\,dx, x, ia\sinh(x)\right)\right)$$

$$= -\frac{\operatorname{sech}^4(x)(b-a\sinh(x))}{4\left(a^2+b^2\right)} - \frac{\left(ia^2\right)\operatorname{Subst}\left(\int \frac{-ia^2b+3a^2x}{(ib+x)\left(a^2-x^2\right)^2}\,dx, x, ia\sinh(x)\right)}{4\left(a^2+b^2\right)}$$

$$= -\frac{\operatorname{sech}^4(x)(b-a\sinh(x))}{4\left(a^2+b^2\right)} - \frac{\operatorname{sech}^2(x)\left(4a^2b-a\left(3a^2-b^2\right)\sinh(x)\right)}{8\left(a^2+b^2\right)^2} - \frac{i\operatorname{Subst}\left(\int \frac{-ia^2b\left(5a^2+b^2 \cdots\right.}{(ib+x) \cdots}\right.}{8 \cdots}$$

$$= -\frac{\operatorname{sech}^4(x)(b-a\sinh(x))}{4\left(a^2+b^2\right)} - \frac{\operatorname{sech}^2(x)\left(4a^2b-a\left(3a^2-b^2\right)\sinh(x)\right)}{8\left(a^2+b^2\right)^2} - \frac{i\operatorname{Subst}\left(\int \left(\frac{a(a-ib)^2(3a \cdots}{2(a+ib)(a- \cdots}\right.\right.}{\cdots}$$

$$= -\frac{a(3ia+b)\log(i-\sinh(x))}{16(a-ib)^3} + \frac{a(3a+ib)\log(i+\sinh(x))}{16(ia-b)^3} - \frac{a^4b\log(b+a\sinh(x))}{\left(a^2+b^2\right)^3} - \operatorname{sech} \cdots$$

Mathematica [A] time = 0.28, size = 138, normalized size = 0.93

$$\frac{8a^4b(\log(\cosh(x))-\log(a\sinh(x)+b)) - 2b\left(a^2+b^2\right)^2\operatorname{sech}^4(x) - 4a^2b\left(a^2+b^2\right)\operatorname{sech}^2(x) + 2a\left(a^2+b^2\right)^2\tanh(\cdots}{8\left(a^2+b^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[x]^5/(a + b*Csch[x]),x]
```

```
[Out] (2*a*(3*a^4 - 6*a^2*b^2 - b^4)*ArcTan[Tanh[x/2]] + 8*a^4*b*(Log[Cosh[x]] -
Log[b + a*Sinh[x]]) - 4*a^2*b*(a^2 + b^2)*Sech[x]^2 - 2*b*(a^2 + b^2)^2*Sec
h[x]^4 + a*(3*a^4 + 2*a^2*b^2 - b^4)*Sech[x]*Tanh[x] + 2*a*(a^2 + b^2)^2*Se
ch[x]^3*Tanh[x])/(8*(a^2 + b^2)^3)
```

fricas [B] time = 1.58, size = 2778, normalized size = 18.64

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^5/(a+b*csch(x)),x, algorithm="fricas")
```

```
[Out] 1/4*((3*a^5 + 2*a^3*b^2 - a*b^4)*cosh(x)^7 + (3*a^5 + 2*a^3*b^2 - a*b^4)*si
nh(x)^7 - 8*(a^4*b + a^2*b^3)*cosh(x)^6 - (8*a^4*b + 8*a^2*b^3 - 7*(3*a^5 +
 2*a^3*b^2 - a*b^4)*cosh(x))*sinh(x)^6 + (11*a^5 + 18*a^3*b^2 + 7*a*b^4)*co
sh(x)^5 + (11*a^5 + 18*a^3*b^2 + 7*a*b^4 + 21*(3*a^5 + 2*a^3*b^2 - a*b^4)*c
```

osh(x)^2 - 48*(a^4*b + a^2*b^3)*cosh(x))*sinh(x)^5 - 16*(2*a^4*b + 3*a^2*b^
3 + b^5)*cosh(x)^4 - (32*a^4*b + 48*a^2*b^3 + 16*b^5 - 35*(3*a^5 + 2*a^3*b^
2 - a*b^4)*cosh(x)^3 + 120*(a^4*b + a^2*b^3)*cosh(x)^2 - 5*(11*a^5 + 18*a^3
*b^2 + 7*a*b^4)*cosh(x))*sinh(x)^4 - (11*a^5 + 18*a^3*b^2 + 7*a*b^4)*cosh(x
)^3 - (11*a^5 + 18*a^3*b^2 + 7*a*b^4 - 35*(3*a^5 + 2*a^3*b^2 - a*b^4)*cosh(
x)^4 + 160*(a^4*b + a^2*b^3)*cosh(x)^3 - 10*(11*a^5 + 18*a^3*b^2 + 7*a*b^4)
cosh(x)^2 + 64(2*a^4*b + 3*a^2*b^3 + b^5)*cosh(x))*sinh(x)^3 - 8*(a^4*b +
 a^2*b^3)*cosh(x)^2 + (21*(3*a^5 + 2*a^3*b^2 - a*b^4)*cosh(x)^5 - 8*a^4*b -
 8*a^2*b^3 - 120*(a^4*b + a^2*b^3)*cosh(x)^4 + 10*(11*a^5 + 18*a^3*b^2 + 7*
a*b^4)*cosh(x)^3 - 96*(2*a^4*b + 3*a^2*b^3 + b^5)*cosh(x)^2 - 3*(11*a^5 + 1
8*a^3*b^2 + 7*a*b^4)*cosh(x))*sinh(x)^2 + ((3*a^5 - 6*a^3*b^2 - a*b^4)*cosh
(x)^8 + 8*(3*a^5 - 6*a^3*b^2 - a*b^4)*cosh(x)*sinh(x)^7 + (3*a^5 - 6*a^3*b^
2 - a*b^4)*sinh(x)^8 + 4*(3*a^5 - 6*a^3*b^2 - a*b^4)*cosh(x)^6 + 4*(3*a^5 -
 6*a^3*b^2 - a*b^4 + 7*(3*a^5 - 6*a^3*b^2 - a*b^4)*cosh(x)^2)*sinh(x)^6 + 8
(7(3*a^5 - 6*a^3*b^2 - a*b^4)*cosh(x)^3 + 3*(3*a^5 - 6*a^3*b^2 - a*b^4)*c
osh(x))*sinh(x)^5 + 3*a^5 - 6*a^3*b^2 - a*b^4 + 6*(3*a^5 - 6*a^3*b^2 - a*b^
4)*cosh(x)^4 + 2*(9*a^5 - 18*a^3*b^2 - 3*a*b^4 + 35*(3*a^5 - 6*a^3*b^2 - a*
b^4)*cosh(x)^4 + 30*(3*a^5 - 6*a^3*b^2 - a*b^4)*cosh(x)^2)*sinh(x)^4 + 8*(7
*(3*a^5 - 6*a^3*b^2 - a*b^4)*cosh(x)^5 + 10*(3*a^5 - 6*a^3*b^2 - a*b^4)*cos
h(x)^3 + 3*(3*a^5 - 6*a^3*b^2 - a*b^4)*cosh(x))*sinh(x)^3 + 4*(3*a^5 - 6*a^
3*b^2 - a*b^4)*cosh(x)^2 + 4*(7*(3*a^5 - 6*a^3*b^2 - a*b^4)*cosh(x)^6 + 3*a
^5 - 6*a^3*b^2 - a*b^4 + 15*(3*a^5 - 6*a^3*b^2 - a*b^4)*cosh(x)^4 + 9*(3*a^
5 - 6*a^3*b^2 - a*b^4)*cosh(x)^2)*sinh(x)^2 + 8*((3*a^5 - 6*a^3*b^2 - a*b^4
)*cosh(x)^7 + 3*(3*a^5 - 6*a^3*b^2 - a*b^4)*cosh(x)^5 + 3*(3*a^5 - 6*a^3*b^
2 - a*b^4)*cosh(x)^3 + (3*a^5 - 6*a^3*b^2 - a*b^4)*cosh(x))*sinh(x))*arctan
(cosh(x) + sinh(x)) - (3*a^5 + 2*a^3*b^2 - a*b^4)*cosh(x) - 4*(a^4*b*cosh(x
)^8 + 8*a^4*b*cosh(x)*sinh(x)^7 + a^4*b*sinh(x)^8 + 4*a^4*b*cosh(x)^6 + 6*a
^4*b*cosh(x)^4 + 4*a^4*b*cosh(x)^2 + 4*(7*a^4*b*cosh(x)^2 + a^4*b)*sinh(x)^
6 + 8*(7*a^4*b*cosh(x)^3 + 3*a^4*b*cosh(x))*sinh(x)^5 + a^4*b + 2*(35*a^4*b
*cosh(x)^4 + 30*a^4*b*cosh(x)^2 + 3*a^4*b)*sinh(x)^4 + 8*(7*a^4*b*cosh(x)^5
 + 10*a^4*b*cosh(x)^3 + 3*a^4*b*cosh(x))*sinh(x)^3 + 4*(7*a^4*b*cosh(x)^6 +
 15*a^4*b*cosh(x)^4 + 9*a^4*b*cosh(x)^2 + a^4*b)*sinh(x)^2 + 8*(a^4*b*cosh(
x)^7 + 3*a^4*b*cosh(x)^5 + 3*a^4*b*cosh(x)^3 + a^4*b*cosh(x))*sinh(x))*log(
2*(a*sinh(x) + b)/(cosh(x) - sinh(x))) + 4*(a^4*b*cosh(x)^8 + 8*a^4*b*cosh(
x)*sinh(x)^7 + a^4*b*sinh(x)^8 + 4*a^4*b*cosh(x)^6 + 6*a^4*b*cosh(x)^4 + 4*
a^4*b*cosh(x)^2 + 4*(7*a^4*b*cosh(x)^2 + a^4*b)*sinh(x)^6 + 8*(7*a^4*b*cosh
(x)^3 + 3*a^4*b*cosh(x))*sinh(x)^5 + a^4*b + 2*(35*a^4*b*cosh(x)^4 + 30*a^4
*b*cosh(x)^2 + 3*a^4*b)*sinh(x)^4 + 8*(7*a^4*b*cosh(x)^5 + 10*a^4*b*cosh(x)
^3 + 3*a^4*b*cosh(x))*sinh(x)^3 + 4*(7*a^4*b*cosh(x)^6 + 15*a^4*b*cosh(x)^4
 + 9*a^4*b*cosh(x)^2 + a^4*b)*sinh(x)^2 + 8*(a^4*b*cosh(x)^7 + 3*a^4*b*cosh
(x)^5 + 3*a^4*b*cosh(x)^3 + a^4*b*cosh(x))*sinh(x))*log(2*cosh(x)/(cosh(x)
- sinh(x))) + (7*(3*a^5 + 2*a^3*b^2 - a*b^4)*cosh(x)^6 - 48*(a^4*b + a^2*b^
3)*cosh(x)^5 - 3*a^5 - 2*a^3*b^2 + a*b^4 + 5*(11*a^5 + 18*a^3*b^2 + 7*a*b^4
)*cosh(x)^4 - 64*(2*a^4*b + 3*a^2*b^3 + b^5)*cosh(x)^3 - 3*(11*a^5 + 18*a^3
*b^2 + 7*a*b^4)*cosh(x)^2 - 16*(a^4*b + a^2*b^3)*cosh(x))*sinh(x))/((a^6 +
3*a^4*b^2 + 3*a^2*b^4 + b^6)*cosh(x)^8 + 8*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b
^6)*cosh(x)*sinh(x)^7 + (a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*sinh(x)^8 + 4*(
a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*cosh(x)^6 + 4*(a^6 + 3*a^4*b^2 + 3*a^2*b
^4 + b^6 + 7*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*cosh(x)^2)*sinh(x)^6 + a^6
 + 3*a^4*b^2 + 3*a^2*b^4 + b^6 + 8*(7*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*c
osh(x)^3 + 3*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*cosh(x))*sinh(x)^5 + 6*(a^
6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*cosh(x)^4 + 2*(3*a^6 + 9*a^4*b^2 + 9*a^2*b
^4 + 3*b^6 + 35*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*cosh(x)^4 + 30*(a^6 + 3
*a^4*b^2 + 3*a^2*b^4 + b^6)*cosh(x)^2)*sinh(x)^4 + 8*(7*(a^6 + 3*a^4*b^2 +
3*a^2*b^4 + b^6)*cosh(x)^5 + 10*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*cosh(x)
^3 + 3*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*cosh(x))*sinh(x)^3 + 4*(a^6 + 3*
a^4*b^2 + 3*a^2*b^4 + b^6)*cosh(x)^2 + 4*(7*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 +
b^6)*cosh(x)^6 + a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6 + 15*(a^6 + 3*a^4*b^2 +
3*a^2*b^4 + b^6)*cosh(x)^4 + 9*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*cosh(x)^

```
2)*sinh(x)^2 + 8*((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*cosh(x)^7 + 3*(a^6 +
3*a^4*b^2 + 3*a^2*b^4 + b^6)*cosh(x)^5 + 3*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b
^6)*cosh(x)^3 + (a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*cosh(x))*sinh(x))
```

giac [B] time = 0.14, size = 374, normalized size = 2.51

$$-\frac{a^5 b \log\left(\left|-a\left(e^{(-x)} - e^x\right) + 2b\right|\right)}{a^7 + 3a^5b^2 + 3a^3b^4 + ab^6} + \frac{a^4 b \log\left(\left(e^{(-x)} - e^x\right)^2 + 4\right)}{2\left(a^6 + 3a^4b^2 + 3a^2b^4 + b^6\right)} + \frac{\left(\pi + 2\arctan\left(\frac{1}{2}\left(e^{(2x)} - 1\right)e^{(-x)}\right)\right)\left(3a^5 - 6a^3b^2\right.}{16\left(a^6 + 3a^4b^2 + 3a^2b^4 + b^6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^5/(a+b*csch(x)),x, algorithm="giac")
```

```
[Out] -a^5*b*log(abs(-a*(e^(-x) - e^x) + 2*b))/(a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b
^6) + 1/2*a^4*b*log((e^(-x) - e^x)^2 + 4)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^
6) + 1/16*(pi + 2*arctan(1/2*(e^(2*x) - 1)*e^(-x)))*(3*a^5 - 6*a^3*b^2 - a*
b^4)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) - 1/4*(3*a^4*b*(e^(-x) - e^x)^4 +
3*a^5*(e^(-x) - e^x)^3 + 2*a^3*b^2*(e^(-x) - e^x)^3 - a*b^4*(e^(-x) - e^x)^
3 + 32*a^4*b*(e^(-x) - e^x)^2 + 8*a^2*b^3*(e^(-x) - e^x)^2 + 20*a^5*(e^(-x)
 - e^x) + 24*a^3*b^2*(e^(-x) - e^x) + 4*a*b^4*(e^(-x) - e^x) + 96*a^4*b + 6
4*a^2*b^3 + 16*b^5)/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*((e^(-x) - e^x)^2
+ 4)^2)
```

maple [B] time = 0.20, size = 1168, normalized size = 7.84

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(x)^5/(a+b*csch(x)),x)
```

```
[Out] 4/(a^4+2*a^2*b^2+b^4)/(a^2+b^2)/(tanh(1/2*x)^2+1)^4*tanh(1/2*x)^2*a^4*b+4/(
a^4+2*a^2*b^2+b^4)/(a^2+b^2)/(tanh(1/2*x)^2+1)^4*tanh(1/2*x)^4*a^2*b^3+3/4/
(a^4+2*a^2*b^2+b^4)/(a^2+b^2)*arctan(tanh(1/2*x))*a^5-5/4/(a^4+2*a^2*b^2+b^
4)/(a^2+b^2)/(tanh(1/2*x)^2+1)^4*tanh(1/2*x)^7*a^5+2/(a^4+2*a^2*b^2+b^4)/(a
^2+b^2)/(tanh(1/2*x)^2+1)^4*tanh(1/2*x)^6*b^5+3/4/(a^4+2*a^2*b^2+b^4)/(a^2+
b^2)/(tanh(1/2*x)^2+1)^4*tanh(1/2*x)^5*a^5-3/4/(a^4+2*a^2*b^2+b^4)/(a^2+b^2
)/(tanh(1/2*x)^2+1)^4*tanh(1/2*x)^3*a^5+2/(a^4+2*a^2*b^2+b^4)/(a^2+b^2)/(ta
nh(1/2*x)^2+1)^4*tanh(1/2*x)^2*b^5+5/4/(a^4+2*a^2*b^2+b^4)/(a^2+b^2)/(tanh(
1/2*x)^2+1)^4*tanh(1/2*x)*a^5+1/(a^4+2*a^2*b^2+b^4)/(a^2+b^2)*ln(tanh(1/2*x
)^2+1)*a^4*b-a^4*b/(a^4+2*a^2*b^2+b^4)/(a^2+b^2)*ln(tanh(1/2*x)^2*b-2*a*tan
h(1/2*x)-b)-3/2/(a^4+2*a^2*b^2+b^4)/(a^2+b^2)*arctan(tanh(1/2*x))*a^3*b^2-1
/4/(a^4+2*a^2*b^2+b^4)/(a^2+b^2)*arctan(tanh(1/2*x))*a*b^4-3/2/(a^4+2*a^2*b
^2+b^4)/(a^2+b^2)/(tanh(1/2*x)^2+1)^4*tanh(1/2*x)^7*a^3*b^2-1/4/(a^4+2*a^2*
b^2+b^4)/(a^2+b^2)/(tanh(1/2*x)^2+1)^4*tanh(1/2*x)^7*a*b^4+4/(a^4+2*a^2*b^2
+b^4)/(a^2+b^2)/(tanh(1/2*x)^2+1)^4*tanh(1/2*x)^6*a^4*b+7/4/(a^4+2*a^2*b^2+
b^4)/(a^2+b^2)/(tanh(1/2*x)^2+1)^4*tanh(1/2*x)^5*a*b^4+6/(a^4+2*a^2*b^2+b^4
)/(a^2+b^2)/(tanh(1/2*x)^2+1)^4*tanh(1/2*x)^6*a^2*b^3+6/(a^4+2*a^2*b^2+b^4)
/(a^2+b^2)/(tanh(1/2*x)^2+1)^4*tanh(1/2*x)^2*a^2*b^3+3/2/(a^4+2*a^2*b^2+b^4
)/(a^2+b^2)/(tanh(1/2*x)^2+1)^4*tanh(1/2*x)*a^3*b^2+1/4/(a^4+2*a^2*b^2+b^4)
/(a^2+b^2)/(tanh(1/2*x)^2+1)^4*tanh(1/2*x)*a*b^4+5/2/(a^4+2*a^2*b^2+b^4)/(a
^2+b^2)/(tanh(1/2*x)^2+1)^4*tanh(1/2*x)^5*a^3*b^2+4/(a^4+2*a^2*b^2+b^4)/(a^
2+b^2)/(tanh(1/2*x)^2+1)^4*tanh(1/2*x)^4*a^4*b-5/2/(a^4+2*a^2*b^2+b^4)/(a^2
+b^2)/(tanh(1/2*x)^2+1)^4*tanh(1/2*x)^3*a^3*b^2-7/4/(a^4+2*a^2*b^2+b^4)/(a^
2+b^2)/(tanh(1/2*x)^2+1)^4*tanh(1/2*x)^3*a*b^4
```

maxima [B] time = 0.43, size = 348, normalized size = 2.34

$$-\frac{a^4 b \log\left(-2be^{(-x)} + ae^{(-2x)} - a\right)}{a^6 + 3a^4b^2 + 3a^2b^4 + b^6} + \frac{a^4 b \log\left(e^{(-2x)} + 1\right)}{a^6 + 3a^4b^2 + 3a^2b^4 + b^6} - \frac{\left(3a^5 - 6a^3b^2 - ab^4\right)\arctan\left(e^{(-x)}\right)}{4\left(a^6 + 3a^4b^2 + 3a^2b^4 + b^6\right)} - \frac{8a^2be^{(-2x)} + 8}{4\left(a^4 + 2a^2b^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^5/(a+b*csch(x)),x, algorithm="maxima")
```

```
[Out] -a^4*b*log(-2*b*e^(-x) + a*e^(-2*x) - a)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6
) + a^4*b*log(e^(-2*x) + 1)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) - 1/4*(3*a^
5 - 6*a^3*b^2 - a*b^4)*arctan(e^(-x))/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) -
 1/4*(8*a^2*b*e^(-2*x) + 8*a^2*b*e^(-6*x) - (3*a^3 - a*b^2)*e^(-x) - (11*a^
3 + 7*a*b^2)*e^(-3*x) + 16*(2*a^2*b + b^3)*e^(-4*x) + (11*a^3 + 7*a*b^2)*e^
(-5*x) + (3*a^3 - a*b^2)*e^(-7*x))/(a^4 + 2*a^2*b^2 + b^4 + 4*(a^4 + 2*a^2*
b^2 + b^4)*e^(-2*x) + 6*(a^4 + 2*a^2*b^2 + b^4)*e^(-4*x) + 4*(a^4 + 2*a^2*b
^2 + b^4)*e^(-6*x) + (a^4 + 2*a^2*b^2 + b^4)*e^(-8*x))
```

mupad [B] time = 5.51, size = 513, normalized size = 3.44

$$\frac{\frac{8\left(a^2 b+b^3\right)}{\left(a^2+b^2\right)^2}-\frac{6\,e^x\left(a^3+a\,b^2\right)}{\left(a^2+b^2\right)^2}}{3\,e^{2x}+3\,e^{4x}+e^{6x}+1}-\frac{\frac{2\left(a^2 b+2\,b^3\right)}{\left(a^2+b^2\right)^2}-\frac{e^x\left(a^3+5\,a\,b^2\right)}{2\left(a^2+b^2\right)^2}}{2\,e^{2x}+e^{4x}+1}-\frac{\frac{4\,b}{a^2+b^2}-\frac{4\,a\,e^x}{a^2+b^2}}{4\,e^{2x}+6\,e^{4x}+4\,e^{6x}+e^{8x}+1}-\frac{\frac{2\left(a^4 b+a^2 b^3\right)}{\left(a^2+b^2\right)^3}-\frac{e^x\left(3\,a^5+2\,a^3 b^2-\right.}{4\left(a^2+b^2\right)^3}}{e^{2x}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(x)^5*(a + b/sinh(x))),x)
```

```
[Out] ((8*(a^2*b + b^3))/(a^2 + b^2)^2 - (6*exp(x)*(a*b^2 + a^3))/(a^2 + b^2)^2)/
(3*exp(2*x) + 3*exp(4*x) + exp(6*x) + 1) - ((2*(a^2*b + 2*b^3))/(a^2 + b^2)
^2 - (exp(x)*(5*a*b^2 + a^3))/(2*(a^2 + b^2)^2))/(2*exp(2*x) + exp(4*x) + 1
) - ((4*b)/(a^2 + b^2) - (4*a*exp(x))/(a^2 + b^2))/(4*exp(2*x) + 6*exp(4*x)
 + 4*exp(6*x) + exp(8*x) + 1) - ((2*(a^4*b + a^2*b^3))/(a^2 + b^2)^3 - (exp
(x)*(3*a^5 - a*b^4 + 2*a^3*b^2))/(4*(a^2 + b^2)^3))/(exp(2*x) + 1) + (log(e
xp(x) + 1i)*(a*b - a^2*3i))/(8*(3*a*b^2 - a^2*b*3i - a^3 + b^3*1i)) + (log(
exp(x)*1i + 1)*(a*b*1i - 3*a^2))/(8*(a*b^2*3i - 3*a^2*b - a^3*1i + b^3)) -
(a^4*b*log(9*a^10*exp(2*x) - 9*a^10 - a^2*b^8 - 12*a^4*b^6 - 30*a^6*b^4 - 2
20*a^8*b^2 + a^2*b^8*exp(2*x) + 12*a^4*b^6*exp(2*x) + 30*a^6*b^4*exp(2*x) +
 220*a^8*b^2*exp(2*x) + 2*a*b^9*exp(x) + 18*a^9*b*exp(x) + 24*a^3*b^7*exp(x
) + 60*a^5*b^5*exp(x) + 440*a^7*b^3*exp(x)))/(a^6 + b^6 + 3*a^2*b^4 + 3*a^4
*b^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}^{5}(x)}{a + b\operatorname{csch}(x)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)**5/(a+b*csch(x)),x)
```

```
[Out] Integral(sech(x)**5/(a + b*csch(x)), x)
```

3.102 $\int \frac{\tanh^5(x)}{i+\operatorname{csch}(x)} dx$

Optimal. Leaf size=109

$$-\frac{i}{4(1-i\sinh(x))}-\frac{15i}{16(1+i\sinh(x))}+\frac{i}{32(1-i\sinh(x))^2}+\frac{9i}{32(1+i\sinh(x))^2}-\frac{i}{24(1+i\sinh(x))^3}-\frac{21}{32}i\log(-\sinh($$

```
[Out] -21/32*I*ln(I-sinh(x))-11/32*I*ln(I+sinh(x))+1/32*I/(1-I*sinh(x))^2-1/4*I/(
1-I*sinh(x))-1/24*I/(1+I*sinh(x))^3+9/32*I/(1+I*sinh(x))^2-15/16*I/(1+I*sin
h(x))
```

Rubi [A] time = 0.09, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3879, 88}

$$-\frac{i}{4(1-i\sinh(x))}-\frac{15i}{16(1+i\sinh(x))}+\frac{i}{32(1-i\sinh(x))^2}+\frac{9i}{32(1+i\sinh(x))^2}-\frac{i}{24(1+i\sinh(x))^3}-\frac{21}{32}i\log(-\sinh($$

Antiderivative was successfully verified.

```
[In] Int[Tanh[x]^5/(I + Csch[x]),x]
```

```
[Out] ((-21*I)/32)*Log[I - Sinh[x]] - ((11*I)/32)*Log[I + Sinh[x]] + (I/32)/(1 -
I*Sinh[x])^2 - (I/4)/(1 - I*Sinh[x]) - (I/24)/(1 + I*Sinh[x])^3 + ((9*I)/32
)/(1 + I*Sinh[x])^2 - ((15*I)/16)/(1 + I*Sinh[x])
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 3879

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[1/(a^(m - n - 1)*b^n*d), Subst[Int[((a - b*x)^((m -
1)/2)*(a + b*x)^((m - 1)/2 + n))/x^(m + n), x], x, Sin[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ
[n]
```

Rubi steps

$$\begin{aligned}\int \frac{\tanh^5(x)}{i+\operatorname{csch}(x)} dx &= \text{Subst}\left(\int \frac{x^6}{(i-ix)^3(i+ix)^4} dx, x, i\sinh(x)\right)\\ &= \text{Subst}\left(\int \left(-\frac{i}{16(-1+x)^3}-\frac{i}{4(-1+x)^2}-\frac{11i}{32(-1+x)}+\frac{i}{8(1+x)^4}-\frac{9i}{16(1+x)^3}+\frac{15i}{16(1+x)^2}\right.\right.\\ &= -\frac{21}{32}i\log(i-\sinh(x))-\frac{11}{32}i\log(i+\sinh(x))+\frac{i}{32(1-i\sinh(x))^2}-\frac{i}{4(1-i\sinh(x))}-\frac{i}{24(1-}\end{aligned}$$

Mathematica [A] time = 0.20, size = 75, normalized size = 0.69

$$\frac{1}{96}\left(-\frac{2\left(33\sinh^4(x)+39i\sinh^3(x)+79\sinh^2(x)+29i\sinh(x)+44\right)}{(\sinh(x)-i)^3(\sinh(x)+i)^2}-63i\log(-\sinh(x)+i)-33i\log(\sinh(x)\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]^5/(I + Csch[x]),x]
```

```
[Out] ((-63*I)*Log[I - Sinh[x]] - (33*I)*Log[I + Sinh[x]] - (2*(44 + (29*I)*Sinh[
x] + 79*Sinh[x]^2 + (39*I)*Sinh[x]^3 + 33*Sinh[x]^4))/((-I + Sinh[x])^3*(I
+ Sinh[x])^2))/96
```

fricas [B] time = 1.37, size = 299, normalized size = 2.74

$$\frac{48i\,xe^{(10\,x)} + 6\,(16\,x - 11)e^{(9\,x)} + (144i\,x - 156i)e^{(8\,x)} + 16\,(24\,x - 23)e^{(7\,x)} + (96i\,x + 4i)e^{(6\,x)} + 36\,(16\,x - 15)e^{(5}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^5/(I+csch(x)),x, algorithm="fricas")
```

```
[Out] (48*I*x*e^(10*x) + 6*(16*x - 11)*e^(9*x) + (144*I*x - 156*I)*e^(8*x) + 16*(
24*x - 23)*e^(7*x) + (96*I*x + 4*I)*e^(6*x) + 36*(16*x - 15)*e^(5*x) + (-96
*I*x - 4*I)*e^(4*x) + 16*(24*x - 23)*e^(3*x) + (-144*I*x + 156*I)*e^(2*x) +
 6*(16*x - 11)*e^x + (-33*I*e^(10*x) - 66*e^(9*x) - 99*I*e^(8*x) - 264*e^(7
*x) - 66*I*e^(6*x) - 396*e^(5*x) + 66*I*e^(4*x) - 264*e^(3*x) + 99*I*e^(2*x
) - 66*e^x + 33*I)*log(e^x + I) + (-63*I*e^(10*x) - 126*e^(9*x) - 189*I*e^(
8*x) - 504*e^(7*x) - 126*I*e^(6*x) - 756*e^(5*x) + 126*I*e^(4*x) - 504*e^(3
*x) + 189*I*e^(2*x) - 126*e^x + 63*I)*log(e^x - I) - 48*I*x)/(48*e^(10*x) -
 96*I*e^(9*x) + 144*e^(8*x) - 384*I*e^(7*x) + 96*e^(6*x) - 576*I*e^(5*x) -
96*e^(4*x) - 384*I*e^(3*x) - 144*e^(2*x) - 96*I*e^x - 48)
```

giac [A] time = 0.15, size = 120, normalized size = 1.10

$$-\frac{33i\left(e^{(-x)} - e^{x}\right)^{2} + 100\,e^{(-x)} - 100\,e^{x} - 76i}{64\left(-i\,e^{(-x)} + i\,e^{x} - 2\right)^{2}} - \frac{-231i\left(e^{(-x)} - e^{x}\right)^{3} + 1026\left(e^{(-x)} - e^{x}\right)^{2} + 1548i\,e^{(-x)} - 1548i\,e^{x} - 7}{192\left(e^{(-x)} - e^{x} + 2i\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^5/(I+csch(x)),x, algorithm="giac")
```

```
[Out] -1/64*(33*I*(e^(-x) - e^x)^2 + 100*e^(-x) - 100*e^x - 76*I)/(-I*e^(-x) + I*
e^x - 2)^2 - 1/192*(-231*I*(e^(-x) - e^x)^3 + 1026*(e^(-x) - e^x)^2 + 1548*
I*e^(-x) - 1548*I*e^x - 776)/(e^(-x) - e^x + 2*I)^3 - 11/32*I*log(-e^(-x) +
 e^x + 2*I) - 21/32*I*log(-e^(-x) + e^x - 2*I)
```

maple [A] time = 0.25, size = 155, normalized size = 1.42

$$i\ln\left(\tanh\left(\frac{x}{2}\right) - 1\right) + i\ln\left(\tanh\left(\frac{x}{2}\right) + 1\right) - \frac{11i\ln\left(\tanh\left(\frac{x}{2}\right) + i\right)}{16} + \frac{i}{8\left(\tanh\left(\frac{x}{2}\right) + i\right)^{4}} + \frac{i}{4\left(\tanh\left(\frac{x}{2}\right) + i\right)^{2}} - \frac{}{4\left(\tanh\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^5/(I+csch(x)),x)
```

```
[Out] I*ln(tanh(1/2*x)-1)+I*ln(tanh(1/2*x)+1)-11/16*I*ln(tanh(1/2*x)+I)+1/8*I/(ta
nh(1/2*x)+I)^4+1/4*I/(tanh(1/2*x)+I)^2-1/4/(tanh(1/2*x)+I)^3-3/8/(tanh(1/2*
x)+I)-21/16*I*ln(tanh(1/2*x)-I)+3/8*I/(tanh(1/2*x)-I)^2+1/3*I/(tanh(1/2*x)-
I)^6-3/8*I/(tanh(1/2*x)-I)^4+1/(tanh(1/2*x)-I)^5+11/12/(tanh(1/2*x)-I)^3+1/
(tanh(1/2*x)-I)
```

maxima [B] time = 0.33, size = 144, normalized size = 1.32

$$-i\,x + \frac{33\,e^{(-x)} + 78i\,e^{(-2\,x)} + 184\,e^{(-3\,x)} - 2i\,e^{(-4\,x)} + 270\,e^{(-5\,x)} + 2i\,e^{(-6\,x)} + 184\,e^{(-7\,x)} - 78i\,e^{(-8\,x)} + 33}{48i\,e^{(-x)} - 72\,e^{(-2\,x)} + 192i\,e^{(-3\,x)} - 48\,e^{(-4\,x)} + 288i\,e^{(-5\,x)} + 48\,e^{(-6\,x)} + 192i\,e^{(-7\,x)} + 72\,e^{(-8\,x)} + 48i\,e^{(-9\,x)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^5/(I+csch(x)),x, algorithm="maxima")
```

```
[Out] -I*x + (33*e^(-x) + 78*I*e^(-2*x) + 184*e^(-3*x) - 2*I*e^(-4*x) + 270*e^(-5
*x) + 2*I*e^(-6*x) + 184*e^(-7*x) - 78*I*e^(-8*x) + 33*e^(-9*x))/(48*I*e^(-
x) - 72*e^(-2*x) + 192*I*e^(-3*x) - 48*e^(-4*x) + 288*I*e^(-5*x) + 48*e^(-6
*x) + 192*I*e^(-7*x) + 72*e^(-8*x) + 48*I*e^(-9*x) + 24*e^(-10*x) - 24) - 1
1/16*I*log(e^(-x) - I) - 21/16*I*log(I*e^(-x) - 1)
```

mupad [**B**] time = 4.11, size = 274, normalized size = 2.51

$$x\,1\mathrm{i}-\ln\left(\left(\frac{5\,\mathrm{e}^{x}}{8}-\frac{5}{8}\mathrm{i}\right)\left(\frac{5\,\mathrm{e}^{x}}{8}+\frac{5}{8}\mathrm{i}\right)\right)1\mathrm{i}+\frac{5\,\mathrm{atan}\left(\mathrm{e}^{x}\right)}{8}+\frac{1\mathrm{i}}{3\,\left(15\,\mathrm{e}^{4\,x}-15\,\mathrm{e}^{2\,x}-\mathrm{e}^{6\,x}+1-\mathrm{e}^{3\,x}\,20\mathrm{i}+\mathrm{e}^{5\,x}\,6\mathrm{i}+\mathrm{e}^{x}\,6\mathrm{i}\right)}-\frac{}{\mathrm{e}^{2\,x}\,1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^5/(1/sinh(x) + 1i),x)
```

```
[Out] x*1i - log(((5*exp(x))/8 - 5i/8)*((5*exp(x))/8 + 5i/8))*1i + (5*atan(exp(x)
))/8 + 1i/(3*(15*exp(4*x) - exp(3*x)*20i - 15*exp(2*x) + exp(5*x)*6i - exp(
6*x) + exp(x)*6i + 1)) - 1/(exp(2*x)*10i - 10*exp(3*x) - exp(4*x)*5i + exp(
5*x) + 5*exp(x) - 1i) - 31/(12*(exp(2*x)*3i - exp(3*x) + 3*exp(x) - 1i)) -
5i/(8*(exp(2*x) + exp(x)*2i - 1)) + 17i/(8*(exp(4*x) - exp(3*x)*4i - 6*exp(
2*x) + exp(x)*4i + 1)) + 1i/(8*(exp(3*x)*4i - 6*exp(2*x) + exp(4*x) - exp(x
)*4i + 1)) + 3i/(exp(x)*2i - exp(2*x) + 1) - 15/(8*(exp(x) - 1i)) + 1/(2*(e
xp(x) + 1i)) - 1/(4*(exp(2*x)*3i + exp(3*x) - 3*exp(x) - 1i))
```

sympy [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh^{5}{\left(x \right)}}{\operatorname{csch}{\left(x \right)} + i}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)**5/(I+csch(x)),x)
```

```
[Out] Integral(tanh(x)**5/(csch(x) + I), x)
```

3.103 $\int \frac{\tanh^4(x)}{i+\mathbf{csch}(x)}\,dx$

Optimal. Leaf size=52

$$-ix+\frac{1}{5}\tanh^5(x)(-\mathrm{csch}(x)+i)+\frac{1}{15}\tanh^3(x)(-4\mathrm{csch}(x)+5i)+\frac{1}{15}\tanh(x)(-8\mathrm{csch}(x)+15i)$$

```
[Out] -I*x+1/15*(15*I-8*csch(x))*tanh(x)+1/15*(5*I-4*csch(x))*tanh(x)^3+1/5*(I-cs
ch(x))*tanh(x)^5
```

Rubi [A] time = 0.09, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {3888, 3882, 8}

$$-ix+\frac{1}{5}\tanh^5(x)(-\mathrm{csch}(x)+i)+\frac{1}{15}\tanh^3(x)(-4\mathrm{csch}(x)+5i)+\frac{1}{15}\tanh(x)(-8\mathrm{csch}(x)+15i)$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[x]^4/(I + Csch[x]),x]
```

```
[Out] (-I)*x + ((15*I - 8*Csch[x])*Tanh[x])/15 + ((5*I - 4*Csch[x])*Tanh[x]^3)/15
 + ((I - Csch[x])*Tanh[x]^5)/5
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3882

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> -Simp[((e*Cot[c + d*x])^(m + 1)*(a + b*Csc[c + d*x]))/(d
*e*(m + 1)), x] - Dist[1/(e^2*(m + 1)), Int[(e*Cot[c + d*x])^(m + 2)*(a*(m
+ 1) + b*(m + 2)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && LtQ[
m, -1]
```

Rule 3888

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Dist[a^(2*n)/e^(2*n), Int[(e*Cot[c + d*x])^(m + 2*n
)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a^
2 - b^2, 0] && ILtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\tanh^4(x)}{i+\mathrm{csch}(x)}\,dx &= \int (-i+\mathrm{csch}(x))\tanh^6(x)\,dx\\
&= \frac{1}{5}(i-\mathrm{csch}(x))\tanh^5(x)-\frac{1}{5}\int (5i-4\mathrm{csch}(x))\tanh^4(x)\,dx\\
&= \frac{1}{15}(5i-4\mathrm{csch}(x))\tanh^3(x)+\frac{1}{5}(i-\mathrm{csch}(x))\tanh^5(x)+\frac{1}{15}\int (-15i+8\mathrm{csch}(x))\tanh^2(x)\\
&= \frac{1}{15}(15i-8\mathrm{csch}(x))\tanh(x)+\frac{1}{15}(5i-4\mathrm{csch}(x))\tanh^3(x)+\frac{1}{5}(i-\mathrm{csch}(x))\tanh^5(x)-\frac{1}{15}\\
&= -ix+\frac{1}{15}(15i-8\mathrm{csch}(x))\tanh(x)+\frac{1}{15}(5i-4\mathrm{csch}(x))\tanh^3(x)+\frac{1}{5}(i-\mathrm{csch}(x))\tanh^5(x
\end{aligned}$$

Mathematica [B] time = 0.14, size = 126, normalized size = 2.42

$$\frac{64i\sinh(x)+240x\sinh(2x)+178i\sinh(2x)+128i\sinh(3x)+120x\sinh(4x)+89i\sinh(4x)+6(89-120ix)\cos}{960\left(\cosh\left(\frac{x}{2}\right)-i\sinh\left(\frac{x}{2}\right)\right)^3\left(\cosh\left(\frac{x}{2}\right)+i\sin\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]^4/(I + Csch[x]),x]
```

```
[Out] (-200 + 6*(89 - (120*I)*x)*Cosh[x] - 128*Cosh[2*x] + 178*Cosh[3*x] - (240*I
)*x*Cosh[3*x] - 184*Cosh[4*x] + (64*I)*Sinh[x] + (178*I)*Sinh[2*x] + 240*x*
Sinh[2*x] + (128*I)*Sinh[3*x] + (89*I)*Sinh[4*x] + 120*x*Sinh[4*x])/(960*(C
osh[x/2] - I*Sinh[x/2])^3*(Cosh[x/2] + I*Sinh[x/2])^5)
```

fricas [B] time = 1.02, size = 123, normalized size = 2.37

$$\frac{-15i\,xe^{(8\,x)}-30\,(x+1)e^{(7\,x)}+(-30i\,x-30i)e^{(6\,x)}-10\,(9\,x+13)e^{(5\,x)}-2\,(45\,x+73)e^{(3\,x)}+(30i\,x+62i)e^{(2\,x)}-2\,(}{15\,e^{(8\,x)}-30i\,e^{(7\,x)}+30\,e^{(6\,x)}-90i\,e^{(5\,x)}-90i\,e^{(3\,x)}-30\,e^{(2\,x)}-30i\,e^{x}-15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^4/(I+csch(x)),x, algorithm="fricas")
```

```
[Out] (-15*I*x*e^(8*x) - 30*(x + 1)*e^(7*x) + (-30*I*x - 30*I)*e^(6*x) - 10*(9*x
+ 13)*e^(5*x) - 2*(45*x + 73)*e^(3*x) + (30*I*x + 62*I)*e^(2*x) - 2*(15*x +
 31)*e^x + 15*I*x + 50*I*e^(4*x) + 46*I)/(15*e^(8*x) - 30*I*e^(7*x) + 30*e^
(6*x) - 90*I*e^(5*x) - 90*I*e^(3*x) - 30*e^(2*x) - 30*I*e^x - 15)
```

giac [A] time = 0.13, size = 62, normalized size = 1.19

$$-\frac{21i\,e^{(2\,x)}-36\,e^{x}-19i}{24\,(i\,e^{x}-1)^3}-\frac{115\,e^{(4\,x)}-380i\,e^{(3\,x)}-530\,e^{(2\,x)}+340i\,e^{x}+91}{40\,(e^{x}-i)^5}-i\,\log\left(i\,e^{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^4/(I+csch(x)),x, algorithm="giac")
```

```
[Out] -1/24*(21*I*e^(2*x) - 36*e^x - 19*I)/(I*e^x - 1)^3 - 1/40*(115*e^(4*x) - 38
0*I*e^(3*x) - 530*e^(2*x) + 340*I*e^x + 91)/(e^x - I)^5 - I*log(I*e^x)
```

maple [B] time = 0.25, size = 99, normalized size = 1.90

$$i\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)-i\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)+\frac{5i}{8\left(\tanh\left(\frac{x}{2}\right)+i\right)}+\frac{i}{6\left(\tanh\left(\frac{x}{2}\right)+i\right)^3}-\frac{1}{4\left(\tanh\left(\frac{x}{2}\right)+i\right)^2}+\frac{11i}{8\left(\tanh\left(\frac{x}{2}\right)-i\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^4/(I+csch(x)),x)
```

```
[Out] I*ln(tanh(1/2*x)-1)-I*ln(tanh(1/2*x)+1)+5/8*I/(tanh(1/2*x)+I)+1/6*I/(tanh(1
/2*x)+I)^3-1/4/(tanh(1/2*x)+I)^2+11/8*I/(tanh(1/2*x)-I)+2/5*I/(tanh(1/2*x)-
I)^5+1/(tanh(1/2*x)-I)^4+1/(tanh(1/2*x)-I)^2
```

maxima [B] time = 0.32, size = 96, normalized size = 1.85

$$-i\,x-\frac{62\,e^{(-x)}+62i\,e^{(-2\,x)}+146\,e^{(-3\,x)}+50i\,e^{(-4\,x)}+130\,e^{(-5\,x)}-30i\,e^{(-6\,x)}+30\,e^{(-7\,x)}+46i}{30i\,e^{(-x)}-30\,e^{(-2\,x)}+90i\,e^{(-3\,x)}+90i\,e^{(-5\,x)}+30\,e^{(-6\,x)}+30i\,e^{(-7\,x)}+15\,e^{(-8\,x)}-15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^4/(I+csch(x)),x, algorithm="maxima")
```

```
[Out] -I*x - (62*e^(-x) + 62*I*e^(-2*x) + 146*e^(-3*x) + 50*I*e^(-4*x) + 130*e^(-
5*x) - 30*I*e^(-6*x) + 30*e^(-7*x) + 46*I)/(30*I*e^(-x) - 30*e^(-2*x) + 90*
I*e^(-3*x) + 90*I*e^(-5*x) + 30*e^(-6*x) + 30*I*e^(-7*x) + 15*e^(-8*x) - 15
)
```

mupad [B] time = 2.22, size = 237, normalized size = 4.56

$$-x\,1\mathrm{i}-\frac{1\mathrm{i}}{4\left(\mathrm{e}^{2x}-1+\mathrm{e}^{x}\,2\mathrm{i}\right)}+\frac{\frac{23\,\mathrm{e}^{x}}{40}-\frac{3}{8}\mathrm{i}}{1-\mathrm{e}^{2x}+\mathrm{e}^{x}\,2\mathrm{i}}-\frac{23}{40\,\left(\mathrm{e}^{x}-\mathrm{i}\right)}+\frac{7}{8\,\left(\mathrm{e}^{x}+1\mathrm{i}\right)}+\frac{\frac{\mathrm{e}^{2x}\,9\mathrm{i}}{8}-\frac{23\,\mathrm{e}^{3x}}{40}+\frac{9\,\mathrm{e}^{x}}{8}-\frac{3}{8}\mathrm{i}}{\mathrm{e}^{4x}-6\,\mathrm{e}^{2x}+1-\mathrm{e}^{3x}\,4\mathrm{i}+\mathrm{e}^{x}\,4\mathrm{i}}-\frac{\frac{3}{8}-}{\mathrm{e}^{2x}\,3\mathrm{i}-}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^4/(1/sinh(x) + 1i),x)
```

```
[Out] ((23*exp(x))/40 - 3i/8)/(exp(x)*2i - exp(2*x) + 1) - 1i/(4*(exp(2*x) + exp(
x)*2i - 1)) - x*1i - 23/(40*(exp(x) - 1i)) + 7/(8*(exp(x) + 1i)) + ((exp(2*
x)*9i)/8 - (23*exp(3*x))/40 + (9*exp(x))/8 - 3i/8)/(exp(4*x) - exp(3*x)*4i
- 6*exp(2*x) + exp(x)*4i + 1) - ((exp(x)*3i)/4 - (23*exp(2*x))/40 + 3/8)/(e
xp(2*x)*3i - exp(3*x) + 3*exp(x) - 1i) - 1/(6*(exp(2*x)*3i + exp(3*x) - 3*e
xp(x) - 1i)) - ((23*exp(4*x))/40 - (exp(3*x)*3i)/2 - (9*exp(2*x))/4 + (exp(
x)*3i)/2 + 23/40)/(exp(2*x)*10i - 10*exp(3*x) - exp(4*x)*5i + exp(5*x) + 5*
exp(x) - 1i)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\tanh^{4}(x)}{\operatorname{csch}(x)+i}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)**4/(I+csch(x)),x)
```

```
[Out] Integral(tanh(x)**4/(csch(x) + I), x)
```

3.104 $\int \frac{\tanh^3(x)}{i+\text{csch}(x)} \, dx$

Optimal. Leaf size=77

$$-\frac{i}{8(1-i\sinh(x))}-\frac{3i}{4(1+i\sinh(x))}+\frac{i}{8(1+i\sinh(x))^2}-\frac{11}{16}i\log(-\sinh(x)+i)-\frac{5}{16}i\log(\sinh(x)+i)$$

```
[Out] -11/16*I*ln(I-sinh(x))-5/16*I*ln(I+sinh(x))-1/8*I/(1-I*sinh(x))+1/8*I/(1+I*
sinh(x))^2-3/4*I/(1+I*sinh(x))
```

Rubi [A] time = 0.07, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3879, 88}

$$-\frac{i}{8(1-i\sinh(x))}-\frac{3i}{4(1+i\sinh(x))}+\frac{i}{8(1+i\sinh(x))^2}-\frac{11}{16}i\log(-\sinh(x)+i)-\frac{5}{16}i\log(\sinh(x)+i)$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[x]^3/(I + Csch[x]),x]
```

```
[Out] ((-11*I)/16)*Log[I - Sinh[x]] - ((5*I)/16)*Log[I + Sinh[x]] - (I/8)/(1 - I*
Sinh[x]) + (I/8)/(1 + I*Sinh[x])^2 - ((3*I)/4)/(1 + I*Sinh[x])
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 3879

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[1/(a^(m - n - 1)*b^n*d), Subst[Int[((a - b*x)^((m -
1)/2)*(a + b*x)^((m - 1)/2 + n))/x^(m + n), x], x, Sin[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ
[n]
```

Rubi steps

$$\begin{aligned}
\int \frac{\tanh^3(x)}{i+\text{csch}(x)} \, dx &= \text{Subst}\left(\int \frac{x^4}{(i-ix)^2(i+ix)^3} \, dx, x, i\sinh(x)\right)\\
&= \text{Subst}\left(\int \left(-\frac{i}{8(-1+x)^2}-\frac{5i}{16(-1+x)}-\frac{i}{4(1+x)^3}+\frac{3i}{4(1+x)^2}-\frac{11i}{16(1+x)}\right) dx, x, i\sinh(x)\right)\\
&= -\frac{11}{16}i\log(i-\sinh(x))-\frac{5}{16}i\log(i+\sinh(x))-\frac{i}{8(1-i\sinh(x))}+\frac{i}{8(1+i\sinh(x))^2}-\frac{3}{4(1+i
\end{aligned}$$

Mathematica [A] time = 0.12, size = 61, normalized size = 0.79

$$\frac{1}{16}\left(-\frac{2\left(5\sinh^2(x)+3i\sinh(x)+6\right)}{(\sinh(x)-i)^2(\sinh(x)+i)}-11i\log(-\sinh(x)+i)-5i\log(\sinh(x)+i)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]^3/(I + Csch[x]),x]
```

```
[Out] ((-11*I)*Log[I - Sinh[x]] - (5*I)*Log[I + Sinh[x]] - (2*(6 + (3*I)*Sinh[x]
+ 5*Sinh[x]^2))/((-I + Sinh[x])^2*(I + Sinh[x])))/16
```

fricas [B] time = 1.97, size = 185, normalized size = 2.40

$$\frac{8i\,xe^{(6\,x)} + 2\,(8\,x-5)e^{(5\,x)} + (8i\,x-12i)e^{(4\,x)} + 4\,(8\,x-7)e^{(3\,x)} + (-8i\,x+12i)e^{(2\,x)} + 2\,(8\,x-5)e^{x} + \left(-5i\,e^{(6\,x)} - \right.}{8\,e^{(6\,x)} - 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^3/(I+csch(x)),x, algorithm="fricas")
```

```
[Out] (8*I*x*e^(6*x) + 2*(8*x - 5)*e^(5*x) + (8*I*x - 12*I)*e^(4*x) + 4*(8*x - 7)
*e^(3*x) + (-8*I*x + 12*I)*e^(2*x) + 2*(8*x - 5)*e^x + (-5*I*e^(6*x) - 10*e
^(5*x) - 5*I*e^(4*x) - 20*e^(3*x) + 5*I*e^(2*x) - 10*e^x + 5*I)*log(e^x + I
) + (-11*I*e^(6*x) - 22*e^(5*x) - 11*I*e^(4*x) - 44*e^(3*x) + 11*I*e^(2*x)
- 22*e^x + 11*I)*log(e^x - I) - 8*I*x)/(8*e^(6*x) - 16*I*e^(5*x) + 8*e^(4*x
) - 32*I*e^(3*x) - 8*e^(2*x) - 16*I*e^x - 8)
```

giac [B] time = 0.12, size = 98, normalized size = 1.27

$$\frac{5\,e^{(-x)} - 5\,e^{x} - 6i}{16\left(-i\,e^{(-x)} + i\,e^{x} - 2\right)} + \frac{33i\left(e^{(-x)} - e^{x}\right)^{2} - 84\,e^{(-x)} + 84\,e^{x} - 52i}{32\left(e^{(-x)} - e^{x} + 2i\right)^{2}} - \frac{5}{16}i\,\log\left(i\,e^{(-x)} - i\,e^{x} + 2\right) - \frac{11}{16}i\,\log\left(i\,e^{(-x)} - i\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^3/(I+csch(x)),x, algorithm="giac")
```

```
[Out] 1/16*(5*e^(-x) - 5*e^x - 6*I)/(-I*e^(-x) + I*e^x - 2) + 1/32*(33*I*(e^(-x)
- e^x)^2 - 84*e^(-x) + 84*e^x - 52*I)/(e^(-x) - e^x + 2*I)^2 - 5/16*I*log(I
*e^(-x) - I*e^x + 2) - 11/16*I*log(I*e^(-x) - I*e^x - 2)
```

maple [A] time = 0.25, size = 109, normalized size = 1.42

$$i\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)+i\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)-\frac{5i\ln\left(\tanh\left(\frac{x}{2}\right)+i\right)}{8}+\frac{i}{4\left(\tanh\left(\frac{x}{2}\right)+i\right)^{2}}-\frac{1}{4\left(\tanh\left(\frac{x}{2}\right)+i\right)}-\frac{11i\ln\left(\tan\right.}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^3/(I+csch(x)),x)
```

```
[Out] I*ln(tanh(1/2*x)-1)+I*ln(tanh(1/2*x)+1)-5/8*I*ln(tanh(1/2*x)+I)+1/4*I/(tanh
(1/2*x)+I)^2-1/4/(tanh(1/2*x)+I)-11/8*I*ln(tanh(1/2*x)-I)+1/2*I/(tanh(1/2*x
)-I)^4+1/2*I/(tanh(1/2*x)-I)^2+1/(tanh(1/2*x)-I)^3+1/(tanh(1/2*x)-I)
```

maxima [B] time = 0.32, size = 96, normalized size = 1.25

$$-i\,x+\frac{5\,e^{(-x)} + 6i\,e^{(-2\,x)} + 14\,e^{(-3\,x)} - 6i\,e^{(-4\,x)} + 5\,e^{(-5\,x)}}{8i\,e^{(-x)} - 4\,e^{(-2\,x)} + 16i\,e^{(-3\,x)} + 4\,e^{(-4\,x)} + 8i\,e^{(-5\,x)} + 4\,e^{(-6\,x)} - 4} - \frac{5}{8}i\,\log\left(e^{(-x)} - i\right) - \frac{11}{8}i\,\log\left(i\,e^{(-x)} - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^3/(I+csch(x)),x, algorithm="maxima")
```

```
[Out] -I*x + (5*e^(-x) + 6*I*e^(-2*x) + 14*e^(-3*x) - 6*I*e^(-4*x) + 5*e^(-5*x))/
(8*I*e^(-x) - 4*e^(-2*x) + 16*I*e^(-3*x) + 4*e^(-4*x) + 8*I*e^(-5*x) + 4*e^
(-6*x) - 4) - 5/8*I*log(e^(-x) - I) - 11/8*I*log(I*e^(-x) - 1)
```

mupad [B] time = 0.58, size = 140, normalized size = 1.82

$$x\,1\mathrm{i}-\ln\left(\left(\frac{3\,\mathrm{e}^{x}}{4}-\frac{3}{4}\mathrm{i}\right)\left(\frac{3\,\mathrm{e}^{x}}{4}+\frac{3}{4}\mathrm{i}\right)\right)1\mathrm{i}+\frac{3\,\mathrm{atan}\left(\mathrm{e}^{x}\right)}{4}-\frac{1}{\mathrm{e}^{2\,x}\,3\mathrm{i}-\mathrm{e}^{3\,x}+3\,\mathrm{e}^{x}-\mathrm{i}}-\frac{1\mathrm{i}}{4\left(\mathrm{e}^{2\,x}-1+\mathrm{e}^{x}\,2\mathrm{i}\right)}+\frac{}{2\left(\mathrm{e}^{4\,x}-6\,\mathrm{e}^{2\,x}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^3/(1/sinh(x) + 1i),x)
```

```
[Out] x*1i - log(((3*exp(x))/4 - 3i/4)*((3*exp(x))/4 + 3i/4))*1i + (3*atan(exp(x)
))/4 - 1/(exp(2*x)*3i - exp(3*x) + 3*exp(x) - 1i) - 1i/(4*(exp(2*x) + exp(x
)*2i - 1)) + 1i/(2*(exp(4*x) - exp(3*x)*4i - 6*exp(2*x) + exp(x)*4i + 1)) +
 2i/(exp(x)*2i - exp(2*x) + 1) - 3/(2*(exp(x) - 1i)) + 1/(4*(exp(x) + 1i))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh^3(x)}{\operatorname{csch}(x) + i}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)**3/(I+csch(x)),x)
```

```
[Out] Integral(tanh(x)**3/(csch(x) + I), x)
```

3.105 $\int \frac{\tanh^2(x)}{i+\mathbf{csch}(x)} dx$

Optimal. Leaf size=36

$$-ix + \frac{1}{3}\tanh^3(x)(-\operatorname{csch}(x) + i) + \frac{1}{3}\tanh(x)(-2\operatorname{csch}(x) + 3i)$$

```
[Out] -I*x+1/3*(3*I-2*csch(x))*tanh(x)+1/3*(I-csch(x))*tanh(x)^3
```

Rubi [A] time = 0.07, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {3888, 3882, 8}

$$-ix + \frac{1}{3}\tanh^3(x)(-\operatorname{csch}(x) + i) + \frac{1}{3}\tanh(x)(-2\operatorname{csch}(x) + 3i)$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[x]^2/(I + Csch[x]),x]
```

```
[Out] (-I)*x + ((3*I - 2*Csch[x])*Tanh[x])/3 + ((I - Csch[x])*Tanh[x]^3)/3
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3882

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> -Simp[((e*Cot[c + d*x])^(m + 1)*(a + b*Csc[c + d*x]))/(d
*e*(m + 1)), x] - Dist[1/(e^2*(m + 1)), Int[(e*Cot[c + d*x])^(m + 2)*(a*(m
+ 1) + b*(m + 2)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && LtQ[
m, -1]
```

Rule 3888

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Dist[a^(2*n)/e^(2*n), Int[(e*Cot[c + d*x])^(m + 2*n
)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a^
2 - b^2, 0] && ILtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\tanh^2(x)}{i + \operatorname{csch}(x)} dx &= \int (-i + \operatorname{csch}(x)) \tanh^4(x) \, dx \\
&= \frac{1}{3}(i - \operatorname{csch}(x)) \tanh^3(x) - \frac{1}{3} \int (3i - 2\operatorname{csch}(x)) \tanh^2(x) \, dx \\
&= \frac{1}{3}(3i - 2\operatorname{csch}(x)) \tanh(x) + \frac{1}{3}(i - \operatorname{csch}(x)) \tanh^3(x) + \frac{1}{3} \int -3i \, dx \\
&= -ix + \frac{1}{3}(3i - 2\operatorname{csch}(x)) \tanh(x) + \frac{1}{3}(i - \operatorname{csch}(x)) \tanh^3(x)
\end{aligned}$$

Mathematica [A] time = 0.09, size = 71, normalized size = 1.97

$$\frac{2i\sinh(x) - 4\cosh(2x) + (6x + 5i)(\sinh(x) - i)\cosh(x)}{6\left(\cosh\left(\frac{x}{2}\right) - i\sinh\left(\frac{x}{2}\right)\right)\left(\cosh\left(\frac{x}{2}\right) + i\sinh\left(\frac{x}{2}\right)\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]^2/(I + Csch[x]),x]
```

```
[Out] (-4*Cosh[2*x] + (2*I)*Sinh[x] + (5*I + 6*x)*Cosh[x]*(-I + Sinh[x]))/(6*(Cos
h[x/2] - I*Sinh[x/2])*(Cosh[x/2] + I*Sinh[x/2])^3)
```

fricas [B] time = 1.46, size = 51, normalized size = 1.42

$$\frac{-3i\,xe^{(4\,x)} - 6\,(x+1)e^{(3\,x)} - 2\,(3\,x+5)e^{x} + 3i\,x + 8i}{3\,e^{(4\,x)} - 6i\,e^{(3\,x)} - 6i\,e^{x} - 3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^2/(I+csch(x)),x, algorithm="fricas")
```

```
[Out] (-3*I*x*e^(4*x) - 6*(x + 1)*e^(3*x) - 2*(3*x + 5)*e^x + 3*I*x + 8*I)/(3*e^(
4*x) - 6*I*e^(3*x) - 6*I*e^x - 3)
```

giac [A] time = 0.13, size = 38, normalized size = 1.06

$$\frac{i}{2\,(i\,e^{x}-1)} - \frac{15\,e^{(2\,x)} - 24i\,e^{x} - 13}{6\,(e^{x}-i)^{3}} - i\,\log\left(i\,e^{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^2/(I+csch(x)),x, algorithm="giac")
```

```
[Out] 1/2*I/(I*e^x - 1) - 1/6*(15*e^(2*x) - 24*I*e^x - 13)/(e^x - I)^3 - I*log(I*
e^x)
```

maple [B] time = 0.21, size = 67, normalized size = 1.86

$$i\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)-i\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)+\frac{i}{2\tanh\left(\frac{x}{2}\right)+2i}+\frac{3i}{2\left(\tanh\left(\frac{x}{2}\right)-i\right)}+\frac{2i}{3\left(\tanh\left(\frac{x}{2}\right)-i\right)^{3}}+\frac{1}{\left(\tanh\left(\frac{x}{2}\right)-i\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^2/(I+csch(x)),x)
```

```
[Out] I*ln(tanh(1/2*x)-1)-I*ln(tanh(1/2*x)+1)+1/2*I/(tanh(1/2*x)+I)+3/2*I/(tanh(1
/2*x)-I)+2/3*I/(tanh(1/2*x)-I)^3+1/(tanh(1/2*x)-I)^2
```

maxima [A] time = 0.31, size = 42, normalized size = 1.17

$$-i\,x - \frac{10\,e^{(-x)} + 6\,e^{(-3\,x)} + 8i}{6i\,e^{(-x)} + 6i\,e^{(-3\,x)} + 3\,e^{(-4\,x)} - 3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^2/(I+csch(x)),x, algorithm="maxima")
```

```
[Out] -I*x - (10*e^(-x) + 6*e^(-3*x) + 8*I)/(6*I*e^(-x) + 6*I*e^(-3*x) + 3*e^(-4*
x) - 3)
```

mupad [B] time = 1.63, size = 85, normalized size = 2.36

$$-x\,1\mathrm{i} + \frac{\frac{5\,\mathrm{e}^{x}}{6} - \frac{1}{2}\mathrm{i}}{1-\mathrm{e}^{2\,x}+\mathrm{e}^{x}\,2\mathrm{i}} - \frac{5}{6\,\left(\mathrm{e}^{x}-\mathrm{i}\right)} + \frac{1}{2\,\left(\mathrm{e}^{x}+1\mathrm{i}\right)} - \frac{\frac{5}{6}-\frac{5\,\mathrm{e}^{2\,x}}{6}+\mathrm{e}^{x}\,1\mathrm{i}}{\mathrm{e}^{2\,x}\,3\mathrm{i}-\mathrm{e}^{3\,x}+3\,\mathrm{e}^{x}-\mathrm{i}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^2/(1/sinh(x) + 1i),x)
```

```
[Out] ((5*exp(x))/6 - 1i/2)/(exp(x)*2i - exp(2*x) + 1) - x*1i - 5/(6*(exp(x) - 1i
)) + 1/(2*(exp(x) + 1i)) - (exp(x)*1i - (5*exp(2*x))/6 + 5/6)/(exp(2*x)*3i
- exp(3*x) + 3*exp(x) - 1i)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh^2(x)}{\operatorname{csch}(x)+i}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)**2/(I+csch(x)),x)
```

```
[Out] Integral(tanh(x)**2/(csch(x) + I), x)
```

3.106 $\int \frac{\tanh(x)}{i+\mathbf{csch}(x)}\,dx$

Optimal. Leaf size=45

$$-\frac{i}{2(1+i\sinh(x))}-\frac{3}{4}i\log(-\sinh(x)+i)-\frac{1}{4}i\log(\sinh(x)+i)$$

```
[Out] -3/4*I*ln(I-sinh(x))-1/4*I*ln(I+sinh(x))-1/2*I/(1+I*sinh(x))
```

Rubi [A] time = 0.05, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {3879, 88}

$$-\frac{i}{2(1+i\sinh(x))}-\frac{3}{4}i\log(-\sinh(x)+i)-\frac{1}{4}i\log(\sinh(x)+i)$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[x]/(I + Csch[x]),x]
```

```
[Out] ((-3*I)/4)*Log[I - Sinh[x]] - (I/4)*Log[I + Sinh[x]] - (I/2)/(1 + I*Sinh[x]
)
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 3879

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[1/(a^(m - n - 1)*b^n*d), Subst[Int[((a - b*x)^((m -
1)/2)*(a + b*x)^((m - 1)/2 + n))/x^(m + n), x], x, Sin[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ
[n]
```

Rubi steps

$$\begin{aligned}
\int \frac{\tanh(x)}{i+\operatorname{csch}(x)}\,dx &= \operatorname{Subst}\left(\int \frac{x^2}{(i-ix)(i+ix)^2}\,dx, x, i\sinh(x)\right)\\
&= \operatorname{Subst}\left(\int \left(-\frac{i}{4(-1+x)}+\frac{i}{2(1+x)^2}-\frac{3i}{4(1+x)}\right)dx, x, i\sinh(x)\right)\\
&= -\frac{3}{4}i\log(i-\sinh(x))-\frac{1}{4}i\log(i+\sinh(x))-\frac{i}{2(1+i\sinh(x))}
\end{aligned}$$

Mathematica [A] time = 0.04, size = 39, normalized size = 0.87

$$\frac{1}{4}\left(-\frac{2}{\sinh(x)-i}-3i\log(-\sinh(x)+i)-i\log(\sinh(x)+i)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]/(I + Csch[x]),x]
```

```
[Out] ((-3*I)*Log[I - Sinh[x]] - I*Log[I + Sinh[x]] - 2/(-I + Sinh[x]))/4
```

fricas [B] time = 1.22, size = 71, normalized size = 1.58

$$\frac{2i\,xe^{(2\,x)} + 2\,(2\,x-1)e^{x} + \left(-i\,e^{(2\,x)} - 2\,e^{x} + i\right)\log\left(e^{x}+i\right) + \left(-3i\,e^{(2\,x)} - 6\,e^{x} + 3i\right)\log\left(e^{x}-i\right) - 2i\,x}{2\,e^{(2\,x)} - 4i\,e^{x} - 2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/(I+csch(x)),x, algorithm="fricas")
```

```
[Out] (2*I*x*e^(2*x) + 2*(2*x - 1)*e^x + (-I*e^(2*x) - 2*e^x + I)*log(e^x + I) +
(-3*I*e^(2*x) - 6*e^x + 3*I)*log(e^x - I) - 2*I*x)/(2*e^(2*x) - 4*I*e^x - 2
)
```

giac [B] time = 0.12, size = 55, normalized size = 1.22

$$\frac{3i\,e^{(-x)} - 3i\,e^{x} - 2}{4\left(e^{(-x)} - e^{x} + 2i\right)} - \frac{1}{4}i\,\log\left(-i\,e^{(-x)} + i\,e^{x} - 2\right) - \frac{3}{4}i\,\log\left(-e^{(-x)} + e^{x} - 2i\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/(I+csch(x)),x, algorithm="giac")
```

```
[Out] 1/4*(3*I*e^(-x) - 3*I*e^x - 2)/(e^(-x) - e^x + 2*I) - 1/4*I*log(-I*e^(-x) +
 I*e^x - 2) - 3/4*I*log(-e^(-x) + e^x - 2*I)
```

maple [A] time = 0.21, size = 65, normalized size = 1.44

$$i\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)+i\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)-\frac{i\ln\left(\tanh\left(\frac{x}{2}\right)+i\right)}{2}-\frac{3i\ln\left(\tanh\left(\frac{x}{2}\right)-i\right)}{2}+\frac{i}{\left(\tanh\left(\frac{x}{2}\right)-i\right)^{2}}+\frac{1}{\tanh\left(\frac{x}{2}\right)-}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)/(I+csch(x)),x)
```

```
[Out] I*ln(tanh(1/2*x)-1)+I*ln(tanh(1/2*x)+1)-1/2*I*ln(tanh(1/2*x)+I)-3/2*I*ln(ta
nh(1/2*x)-I)+I/(tanh(1/2*x)-I)^2+1/(tanh(1/2*x)-I)
```

maxima [A] time = 0.31, size = 45, normalized size = 1.00

$$-i\,x + \frac{e^{(-x)}}{2i\,e^{(-x)} + e^{(-2\,x)} - 1} - \frac{1}{2}i\,\log\left(i\,e^{(-x)} + 1\right) - \frac{3}{2}i\,\log\left(i\,e^{(-x)} - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/(I+csch(x)),x, algorithm="maxima")
```

```
[Out] -I*x + e^(-x)/(2*I*e^(-x) + e^(-2*x) - 1) - 1/2*I*log(I*e^(-x) + 1) - 3/2*I
*log(I*e^(-x) - 1)
```

mupad [B] time = 0.23, size = 50, normalized size = 1.11

$$x\,1\mathrm{i} + \mathrm{atan}\left(\mathrm{e}^{x}\right) - \ln\left(\left(\mathrm{e}^{x}-\mathrm{i}\right)\,\left(\mathrm{e}^{x}+1\mathrm{i}\right)\right)\,1\mathrm{i} + \frac{1\mathrm{i}}{1-\mathrm{e}^{2\,x}+\mathrm{e}^{x}\,2\mathrm{i}} - \frac{1}{\mathrm{e}^{x}-\mathrm{i}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)/(1/sinh(x) + 1i),x)
```

```
[Out] x*1i + atan(exp(x)) - log((exp(x) - 1i)*(exp(x) + 1i))*1i + 1i/(exp(x)*2i -
 exp(2*x) + 1) - 1/(exp(x) - 1i)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh (x)}{\operatorname{csch}(x) + i} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/(I+csch(x)),x)
```

```
[Out] Integral(tanh(x)/(csch(x) + I), x)
```

3.107 $\int \frac{\coth(x)}{i+\mathbf{csch}(x)}\,dx$

Optimal. Leaf size=13

$$-i\log(-\sinh(x)+i)$$

```
[Out] -I*ln(I-sinh(x))
```

Rubi [A] time = 0.02, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {3879, 31}

$$-i\log(-\sinh(x)+i)$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]/(I + Csch[x]),x]
```

```
[Out] (-I)*Log[I - Sinh[x]]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 3879

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_.), x_Symbol] :> Dist[1/(a^(m - n - 1)*b^n*d), Subst[Int[((a - b*x)^((m - 1)/2)*(a + b*x)^((m - 1)/2 + n))/x^(m + n), x], x, Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ[n]
```

Rubi steps

$$\begin{aligned}\int \frac{\coth(x)}{i+\operatorname{csch}(x)}\,dx &= \operatorname{Subst}\left(\int \frac{1}{i+ix}\,dx, x, i\sinh(x)\right)\\ &= -i\log(i-\sinh(x))\end{aligned}$$

Mathematica [A] time = 0.01, size = 13, normalized size = 1.00

$$-i\log(-\sinh(x)+i)$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]/(I + Csch[x]),x]
```

```
[Out] (-I)*Log[I - Sinh[x]]
```

fricas [A] time = 1.31, size = 11, normalized size = 0.85

$$i\,x - 2i\,\log\left(e^{x} - i\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)/(I+csch(x)),x, algorithm="fricas")
```

```
[Out] I*x - 2*I*log(e^x - I)
```

giac [A] time = 0.14, size = 13, normalized size = 1.00

$$ix - 2i\,\log\left(i\,e^{x} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)/(I+csch(x)),x, algorithm="giac")
```

```
[Out] I*x - 2*I*log(I*e^x + 1)
```

maple [A] time = 0.12, size = 17, normalized size = 1.31

$$-i\ln\left(i + \operatorname{csch}(x)\right) + i\ln\left(\operatorname{csch}(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)/(I+csch(x)),x)
```

```
[Out] -I*ln(I+csch(x))+I*ln(csch(x))
```

maxima [A] time = 0.30, size = 15, normalized size = 1.15

$$-ix - 2i\,\log\left(i\,e^{(-x)} - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)/(I+csch(x)),x, algorithm="maxima")
```

```
[Out] -I*x - 2*I*log(I*e^(-x) - 1)
```

mupad [B] time = 1.46, size = 14, normalized size = 1.08

$$x\,1\mathrm{i} - \ln\left(\mathrm{e}^{x} - \mathrm{i}\right)\,2\mathrm{i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)/(1/sinh(x) + 1i),x)
```

```
[Out] x*1i - log(exp(x) - 1i)*2i
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\coth\left(x\right)}{\operatorname{csch}\left(x\right) + i}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)/(I+csch(x)),x)
```

```
[Out] Integral(coth(x)/(csch(x) + I), x)
```

3.108 $\int \frac{\coth^2(x)}{i+\mathbf{csch}(x)}\,dx$

Optimal. Leaf size=11

$$-\tanh^{-1}(\cosh(x)) - ix$$

```
[Out] -I*x-arctanh(cosh(x))
```

Rubi [A] time = 0.04, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3888, 3770}

$$-\tanh^{-1}(\cosh(x)) - ix$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]^2/(I + Csch[x]),x]
```

```
[Out] (-I)*x - ArcTanh[Cosh[x]]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3888

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Dist[a^(2*n)/e^(2*n), Int[(e*Cot[c + d*x])^(m + 2*n
)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a^
2 - b^2, 0] && ILtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\coth^2(x)}{i+\operatorname{csch}(x)}\,dx &= \int (-i+\operatorname{csch}(x))\,dx\\
&= -ix + \int \operatorname{csch}(x)\,dx\\
&= -ix - \tanh^{-1}(\cosh(x))
\end{aligned}$$

Mathematica [A] time = 0.04, size = 13, normalized size = 1.18

$$\log\left(\tanh\left(\frac{x}{2}\right)\right) - ix$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]^2/(I + Csch[x]),x]
```

```
[Out] (-I)*x + Log[Tanh[x/2]]
```

fricas [A] time = 1.97, size = 16, normalized size = 1.45

$$-i\,x - \log\left(e^{x} + 1\right) + \log\left(e^{x} - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^2/(I+csch(x)),x, algorithm="fricas")
```

```
[Out] -I*x - log(e^x + 1) + log(e^x - 1)
```

giac [A] time = 0.12, size = 17, normalized size = 1.55

$$-ix - \log(e^x + 1) + \log(|e^x - 1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^2/(I+csch(x)),x, algorithm="giac")
[Out] -I*x - log(e^x + 1) + log(abs(e^x - 1))
```

maple [B] time = 0.12, size = 27, normalized size = 2.45

$$i\ln\left(\tanh\left(\frac{x}{2}\right) - 1\right) - i\ln\left(\tanh\left(\frac{x}{2}\right) + 1\right) + \ln\left(\tanh\left(\frac{x}{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^2/(I+csch(x)),x)
[Out] I*ln(tanh(1/2*x)-1)-I*ln(tanh(1/2*x)+1)+ln(tanh(1/2*x))
```

maxima [B] time = 0.31, size = 20, normalized size = 1.82

$$-ix - \log\left(e^{(-x)} + 1\right) + \log\left(e^{(-x)} - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^2/(I+csch(x)),x, algorithm="maxima")
[Out] -I*x - log(e^(-x) + 1) + log(e^(-x) - 1)
```

mupad [B] time = 0.18, size = 21, normalized size = 1.91

$$\ln(2 - 2e^x) - \ln(-2e^x - 2) - x\,1\mathrm{i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^2/(1/sinh(x) + 1i),x)
[Out] log(2 - 2*exp(x)) - log(- 2*exp(x) - 2) - x*1i
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\coth^2(x)}{\operatorname{csch}(x) + i}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)**2/(I+csch(x)),x)
[Out] Integral(coth(x)**2/(csch(x) + I), x)
```

3.109 $\int \frac{\coth^3(x)}{i+\mathbf{csch}(x)}\,dx$

Optimal. Leaf size=12

$$-\operatorname{csch}(x) - i\log(\sinh(x))$$

```
[Out] -csch(x)-I*ln(sinh(x))
```

Rubi [A] time = 0.04, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3879, 43}

$$-\operatorname{csch}(x) - i\log(\sinh(x))$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]^3/(I + Csch[x]),x]
[Out] -Csch[x] - I*Log[Sinh[x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 3879

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[1/(a^(m - n - 1)*b^n*d), Subst[Int[((a - b*x)^((m -
1)/2)*(a + b*x)^((m - 1)/2 + n))/x^(m + n), x], x, Sin[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ
[n]
```

Rubi steps

$$\begin{aligned}
\int \frac{\coth^3(x)}{i+\operatorname{csch}(x)}\,dx &= \operatorname{Subst}\left(\int \frac{i-ix}{x^2}\,dx, x, i\sinh(x)\right)\\
&= \operatorname{Subst}\left(\int \left(\frac{i}{x^2}-\frac{i}{x}\right)dx, x, i\sinh(x)\right)\\
&= -\operatorname{csch}(x) - i\log(\sinh(x))
\end{aligned}$$

Mathematica [A] time = 0.01, size = 12, normalized size = 1.00

$$-\operatorname{csch}(x) - i\log(\sinh(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]^3/(I + Csch[x]),x]
[Out] -Csch[x] - I*Log[Sinh[x]]
```

fricas [B] time = 0.73, size = 40, normalized size = 3.33

$$\frac{i\,xe^{(2\,x)} + \left(-i\,e^{(2\,x)} + i\right)\log\left(e^{(2\,x)} - 1\right) - i\,x - 2\,e^{x}}{e^{(2\,x)} - 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^3/(I+csch(x)),x, algorithm="fricas")
```

```
[Out] (I*x*e^(2*x) + (-I*e^(2*x) + I)*log(e^(2*x) - 1) - I*x - 2*e^x)/(e^(2*x) -
1)
```

giac [B] time = 0.13, size = 38, normalized size = 3.17

$$\frac{i\,e^{(-x)} - i\,e^{x} + 2}{e^{(-x)} - e^{x}} - i\,\log\left(\left|-e^{(-x)} + e^{x}\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^3/(I+csch(x)),x, algorithm="giac")
```

```
[Out] (I*e^(-x) - I*e^x + 2)/(e^(-x) - e^x) - I*log(abs(-e^(-x) + e^x))
```

maple [A] time = 0.09, size = 12, normalized size = 1.00

$$-\mathrm{csch}(x) + i\ln\left(\mathrm{csch}(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^3/(I+csch(x)),x)
```

```
[Out] -csch(x)+I*ln(csch(x))
```

maxima [B] time = 0.31, size = 36, normalized size = 3.00

$$-i\,x + \frac{2\,e^{(-x)}}{e^{(-2\,x)} - 1} - i\,\log\left(e^{(-x)} + 1\right) - i\,\log\left(e^{(-x)} - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^3/(I+csch(x)),x, algorithm="maxima")
```

```
[Out] -I*x + 2*e^(-x)/(e^(-2*x) - 1) - I*log(e^(-x) + 1) - I*log(e^(-x) - 1)
```

mupad [B] time = 1.57, size = 27, normalized size = 2.25

$$-\frac{2\,\mathrm{e}^{x}}{\mathrm{e}^{2\,x} - 1} + x\,1\mathrm{i} - \ln\left(\mathrm{e}^{2\,x} - 1\right)\,1\mathrm{i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^3/(1/sinh(x) + 1i),x)
```

```
[Out] x*1i - log(exp(2*x) - 1)*1i - (2*exp(x))/(exp(2*x) - 1)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\coth^{3}(x)}{\mathrm{csch}\,(x) + i}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)**3/(I+csch(x)),x)
```

```
[Out] Integral(coth(x)**3/(csch(x) + I), x)
```

3.110 $\int \frac{\coth^4(x)}{i+\operatorname{csch}(x)} \, dx$

Optimal. Leaf size=27

$$-ix - \frac{1}{2}\tanh^{-1}(\cosh(x)) + \frac{1}{2}\coth(x)(-\operatorname{csch}(x) + 2i)$$

```
[Out] -I*x-1/2*arctanh(cosh(x))+1/2*coth(x)*(2*I-csch(x))
```

Rubi [A] time = 0.06, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {3888, 3881, 3770}

$$-ix - \frac{1}{2}\tanh^{-1}(\cosh(x)) + \frac{1}{2}\coth(x)(-\operatorname{csch}(x) + 2i)$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]^4/(I + Csch[x]),x]
```

```
[Out] (-I)*x - ArcTanh[Cosh[x]]/2 + (Coth[x]*(2*I - Csch[x]))/2
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3881

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> -Simp[(e*(e*Cot[c + d*x])^(m - 1)*(a*m + b*(m - 1)*Csc[c
 + d*x]))/(d*m*(m - 1)), x] - Dist[e^2/m, Int[(e*Cot[c + d*x])^(m - 2)*(a*m
 + b*(m - 1)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && GtQ[m, 1
]
```

Rule 3888

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Dist[a^(2*n)/e^(2*n), Int[(e*Cot[c + d*x])^(m + 2*n
)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a^
2 - b^2, 0] && ILtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\coth^4(x)}{i+\operatorname{csch}(x)} \, dx &= \int \coth^2(x)(-i+\operatorname{csch}(x)) \, dx \\
&= \frac{1}{2}\coth(x)(2i-\operatorname{csch}(x)) + \frac{1}{2}\int (-2i+\operatorname{csch}(x)) \, dx \\
&= -ix + \frac{1}{2}\coth(x)(2i-\operatorname{csch}(x)) + \frac{1}{2}\int \operatorname{csch}(x) \, dx \\
&= -ix - \frac{1}{2}\tanh^{-1}(\cosh(x)) + \frac{1}{2}\coth(x)(2i-\operatorname{csch}(x))
\end{aligned}$$

Mathematica [B] time = 0.04, size = 65, normalized size = 2.41

$$-ix + \frac{1}{2}i\tanh\left(\frac{x}{2}\right) + \frac{1}{2}i\coth\left(\frac{x}{2}\right) - \frac{1}{8}\operatorname{csch}^2\left(\frac{x}{2}\right) - \frac{1}{8}\operatorname{sech}^2\left(\frac{x}{2}\right) + \frac{1}{2}\log\left(\tanh\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]^4/(I + Csch[x]),x]
```

```
[Out] (-I)*x + (I/2)*Coth[x/2] - Csch[x/2]^2/8 + Log[Tanh[x/2]]/2 - Sech[x/2]^2/8
 + (I/2)*Tanh[x/2]
```

fricas [B] time = 1.06, size = 85, normalized size = 3.15

$$\frac{-2i\,xe^{(4\,x)} + (4i\,x + 4i)e^{(2\,x)} - \left(e^{(4\,x)} - 2\,e^{(2\,x)} + 1\right)\log\left(e^{x} + 1\right) + \left(e^{(4\,x)} - 2\,e^{(2\,x)} + 1\right)\log\left(e^{x} - 1\right) - 2i\,x - 2\,e^{(3\,x)} - 2}{2\left(e^{(4\,x)} - 2\,e^{(2\,x)} + 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^4/(I+csch(x)),x, algorithm="fricas")
```

```
[Out] 1/2*(-2*I*x*e^(4*x) + (4*I*x + 4*I)*e^(2*x) - (e^(4*x) - 2*e^(2*x) + 1)*log
(e^x + 1) + (e^(4*x) - 2*e^(2*x) + 1)*log(e^x - 1) - 2*I*x - 2*e^(3*x) - 2*
e^x - 4*I)/(e^(4*x) - 2*e^(2*x) + 1)
```

giac [B] time = 0.13, size = 48, normalized size = 1.78

$$\frac{e^{(3\,x)} - 2i\,e^{(2\,x)} + e^{x} + 2i}{\left(i\,e^{(2\,x)} - i\right)^{2}} - i\,\log\left(-i\,e^{x}\right) - \frac{1}{2}\,\log\left(e^{x} + 1\right) + \frac{1}{2}\,\log\left(|e^{x} - 1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^4/(I+csch(x)),x, algorithm="giac")
```

```
[Out] (e^(3*x) - 2*I*e^(2*x) + e^x + 2*I)/(I*e^(2*x) - I)^2 - I*log(-I*e^x) - 1/2
*log(e^x + 1) + 1/2*log(abs(e^x - 1))
```

maple [B] time = 0.18, size = 61, normalized size = 2.26

$$\frac{i\tanh\left(\frac{x}{2}\right)}{2}+\frac{\left(\tanh^{2}\left(\frac{x}{2}\right)\right)}{8}+i\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)-i\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)-\frac{1}{8\tanh\left(\frac{x}{2}\right)^{2}}+\frac{i}{2\tanh\left(\frac{x}{2}\right)}+\frac{\ln\left(\tanh\left(\frac{x}{2}\right)\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^4/(I+csch(x)),x)
```

```
[Out] 1/2*I*tanh(1/2*x)+1/8*tanh(1/2*x)^2+I*ln(tanh(1/2*x)-1)-I*ln(tanh(1/2*x)+1)
-1/8/tanh(1/2*x)^2+1/2*I/tanh(1/2*x)+1/2*ln(tanh(1/2*x))
```

maxima [B] time = 0.31, size = 55, normalized size = 2.04

$$-i\,x + \frac{e^{(-x)} + 2i\,e^{(-2\,x)} + e^{(-3\,x)} - 2i}{2\,e^{(-2\,x)} - e^{(-4\,x)} - 1} - \frac{1}{2}\,\log\left(e^{(-x)} + 1\right) + \frac{1}{2}\,\log\left(e^{(-x)} - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^4/(I+csch(x)),x, algorithm="maxima")
```

```
[Out] -I*x + (e^(-x) + 2*I*e^(-2*x) + e^(-3*x) - 2*I)/(2*e^(-2*x) - e^(-4*x) - 1)
 - 1/2*log(e^(-x) + 1) + 1/2*log(e^(-x) - 1)
```

mupad [B] time = 1.64, size = 56, normalized size = 2.07

$$\frac{\ln\left(1 - \mathrm{e}^{x}\right)}{2} - \frac{\ln\left(-\mathrm{e}^{x} - 1\right)}{2} - \frac{\mathrm{e}^{x} - 2\mathrm{i}}{\mathrm{e}^{2\,x} - 1} - \frac{2\,\mathrm{e}^{x}}{\mathrm{e}^{4\,x} - 2\,\mathrm{e}^{2\,x} + 1} - x\,1\mathrm{i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^4/(1/sinh(x) + 1i),x)
```

```
[Out] log(1 - exp(x))/2 - log(- exp(x) - 1)/2 - x*1i - (exp(x) - 2i)/(exp(2*x) -
1) - (2*exp(x))/(exp(4*x) - 2*exp(2*x) + 1)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\coth^4(x)}{\operatorname{csch}(x) + i}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)**4/(I+csch(x)),x)
```

```
[Out] Integral(coth(x)**4/(csch(x) + I), x)
```

3.111 $\int \frac{\coth^5(x)}{i+\mathbf{csch}(x)}\,dx$

Optimal. Leaf size=30

$$-\frac{1}{3}\text{csch}^3(x)+\frac{1}{2}i\text{csch}^2(x)-\text{csch}(x)-i\log(\sinh(x))$$

```
[Out] -csch(x)+1/2*I*csch(x)^2-1/3*csch(x)^3-I*ln(sinh(x))
```

Rubi [A] time = 0.05, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3879, 75}

$$-\frac{1}{3}\text{csch}^3(x)+\frac{1}{2}i\text{csch}^2(x)-\text{csch}(x)-i\log(\sinh(x))$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]^5/(I + Csch[x]),x]
[Out] -Csch[x] + (I/2)*Csch[x]^2 - Csch[x]^3/3 - I*Log[Sinh[x]]
```

Rule 75

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && EqQ[b*e + a*f, 0] &&  !(ILtQ[n + p
+ 2, 0] && GtQ[n + 2*p, 0])
```

Rule 3879

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[1/(a^(m - n - 1)*b^n*d), Subst[Int[((a - b*x)^((m -
1)/2)*(a + b*x)^((m - 1)/2 + n))/x^(m + n), x], x, Sin[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ
[n]
```

Rubi steps

$$\begin{aligned}\int \frac{\coth^5(x)}{i+\text{csch}(x)}\,dx &= \text{Subst}\left(\int \frac{(i-ix)^2(i+ix)}{x^4}\,dx, x, i\sinh(x)\right)\\ &= \text{Subst}\left(\int \left(-\frac{i}{x^4}+\frac{i}{x^3}+\frac{i}{x^2}-\frac{i}{x}\right)dx, x, i\sinh(x)\right)\\ &= -\text{csch}(x)+\frac{1}{2}i\text{csch}^2(x)-\frac{\text{csch}^3(x)}{3}-i\log(\sinh(x))\end{aligned}$$

Mathematica [A] time = 0.02, size = 30, normalized size = 1.00

$$-\frac{1}{3}\text{csch}^3(x)+\frac{1}{2}i\text{csch}^2(x)-\text{csch}(x)-i\log(\sinh(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]^5/(I + Csch[x]),x]
[Out] -Csch[x] + (I/2)*Csch[x]^2 - Csch[x]^3/3 - I*Log[Sinh[x]]
```

fricas [B] time = 2.11, size = 97, normalized size = 3.23

$$\frac{3i\,xe^{(6\,x)} + (-9i\,x + 6i)e^{(4\,x)} + (9i\,x - 6i)e^{(2\,x)} + \left(-3i\,e^{(6\,x)} + 9i\,e^{(4\,x)} - 9i\,e^{(2\,x)} + 3i\right)\log\left(e^{(2\,x)} - 1\right) - 3i\,x - 6\,e^{(5\,x)}}{3\left(e^{(6\,x)} - 3\,e^{(4\,x)} + 3\,e^{(2\,x)} - 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^5/(I+csch(x)),x, algorithm="fricas")
```

```
[Out] 1/3*(3*I*x*e^(6*x) + (-9*I*x + 6*I)*e^(4*x) + (9*I*x - 6*I)*e^(2*x) + (-3*I
*e^(6*x) + 9*I*e^(4*x) - 9*I*e^(2*x) + 3*I)*log(e^(2*x) - 1) - 3*I*x - 6*e^
(5*x) + 4*e^(3*x) - 6*e^x)/(e^(6*x) - 3*e^(4*x) + 3*e^(2*x) - 1)
```

giac [B] time = 0.14, size = 68, normalized size = 2.27

$$-\frac{11\left(e^{(-x)} - e^{x}\right)^{3} - 12i\left(e^{(-x)} - e^{x}\right)^{2} + 12\,e^{(-x)} - 12\,e^{x} - 16i}{6\left(-i\,e^{(-x)} + i\,e^{x}\right)^{3}} - i\,\log\left(-i\,e^{(-x)} + i\,e^{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^5/(I+csch(x)),x, algorithm="giac")
```

```
[Out] -1/6*(11*(e^(-x) - e^x)^3 - 12*I*(e^(-x) - e^x)^2 + 12*e^(-x) - 12*e^x - 16
*I)/(-I*e^(-x) + I*e^x)^3 - I*log(-I*e^(-x) + I*e^x)
```

maple [B] time = 0.23, size = 78, normalized size = 2.60

$$\frac{3\tanh\left(\frac{x}{2}\right)}{8} + \frac{\left(\tanh^{3}\left(\frac{x}{2}\right)\right)}{24} + \frac{i\left(\tanh^{2}\left(\frac{x}{2}\right)\right)}{8} + i\ln\left(\tanh\left(\frac{x}{2}\right) - 1\right) + i\ln\left(\tanh\left(\frac{x}{2}\right) + 1\right) - \frac{1}{24\tanh\left(\frac{x}{2}\right)^{3}} - i\ln\left(\tanh\left(\frac{x}{2}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^5/(I+csch(x)),x)
```

```
[Out] 3/8*tanh(1/2*x)+1/24*tanh(1/2*x)^3+1/8*I*tanh(1/2*x)^2+I*ln(tanh(1/2*x)-1)+
I*ln(tanh(1/2*x)+1)-1/24/tanh(1/2*x)^3-I*ln(tanh(1/2*x))+1/8*I/tanh(1/2*x)^
2-3/8/tanh(1/2*x)
```

maxima [B] time = 0.31, size = 75, normalized size = 2.50

$$-i\,x + \frac{6\,e^{(-x)} - 6i\,e^{(-2\,x)} - 4\,e^{(-3\,x)} + 6i\,e^{(-4\,x)} + 6\,e^{(-5\,x)}}{3\left(3\,e^{(-2\,x)} - 3\,e^{(-4\,x)} + e^{(-6\,x)} - 1\right)} - i\,\log\left(e^{(-x)} + 1\right) - i\,\log\left(e^{(-x)} - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^5/(I+csch(x)),x, algorithm="maxima")
```

```
[Out] -I*x + 1/3*(6*e^(-x) - 6*I*e^(-2*x) - 4*e^(-3*x) + 6*I*e^(-4*x) + 6*e^(-5*x
))/(3*e^(-2*x) - 3*e^(-4*x) + e^(-6*x) - 1) - I*log(e^(-x) + 1) - I*log(e^(
-x) - 1)
```

mupad [B] time = 1.68, size = 81, normalized size = 2.70

$$x\,1\mathrm{i} - \ln\left(\mathrm{e}^{2\,x} - 1\right)\,1\mathrm{i} - \frac{8\,\mathrm{e}^{x}}{3\,\left(3\,\mathrm{e}^{2\,x} - 3\,\mathrm{e}^{4\,x} + \mathrm{e}^{6\,x} - 1\right)} - \frac{\frac{8\,\mathrm{e}^{x}}{3} - 2\mathrm{i}}{\mathrm{e}^{4\,x} - 2\,\mathrm{e}^{2\,x} + 1} - \frac{2\,\mathrm{e}^{x} - 2\mathrm{i}}{\mathrm{e}^{2\,x} - 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^5/(1/sinh(x) + 1i),x)
```

```
[Out] x*1i - log(exp(2*x) - 1)*1i - (8*exp(x))/(3*(3*exp(2*x) - 3*exp(4*x) + exp(
6*x) - 1)) - ((8*exp(x))/3 - 2i)/(exp(4*x) - 2*exp(2*x) + 1) - (2*exp(x) -
2i)/(exp(2*x) - 1)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\coth^5(x)}{\operatorname{csch}(x) + i}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)**5/(I+csch(x)),x)
```

```
[Out] Integral(coth(x)**5/(csch(x) + I), x)
```

3.112 $\int \frac{\coth^6(x)}{i+\mathrm{csch}(x)}\,dx$

Optimal. Leaf size=43

$$-ix-\frac{3}{8}\tanh^{-1}(\cosh(x))+\frac{1}{12}\coth^3(x)(-3\mathrm{csch}(x)+4i)+\frac{1}{8}\coth(x)(-3\mathrm{csch}(x)+8i)$$

```
[Out] -I*x-3/8*arctanh(cosh(x))+1/12*coth(x)^3*(4*I-3*csch(x))+1/8*coth(x)*(8*I-3
*csch(x))
```

Rubi [A] time = 0.07, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {3888, 3881, 3770}

$$-ix-\frac{3}{8}\tanh^{-1}(\cosh(x))+\frac{1}{12}\coth^3(x)(-3\mathrm{csch}(x)+4i)+\frac{1}{8}\coth(x)(-3\mathrm{csch}(x)+8i)$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]^6/(I + Csch[x]),x]
```

```
[Out] (-I)*x - (3*ArcTanh[Cosh[x]])/8 + (Coth[x]^3*(4*I - 3*Csch[x]))/12 + (Coth[
x]*(8*I - 3*Csch[x]))/8
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3881

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> -Simp[(e*(e*Cot[c + d*x])^(m - 1)*(a*m + b*(m - 1)*Csc[c
 + d*x]))/(d*m*(m - 1)), x] - Dist[e^2/m, Int[(e*Cot[c + d*x])^(m - 2)*(a*m
 + b*(m - 1)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && GtQ[m, 1
]
```

Rule 3888

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Dist[a^(2*n)/e^(2*n), Int[(e*Cot[c + d*x])^(m + 2*n
)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a^
2 - b^2, 0] && ILtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\coth^6(x)}{i+\mathrm{csch}(x)}\,dx &= \int \coth^4(x)(-i+\mathrm{csch}(x))\,dx\\
&= \frac{1}{12}\coth^3(x)(4i-3\mathrm{csch}(x))+\frac{1}{4}\int \coth^2(x)(-4i+3\mathrm{csch}(x))\,dx\\
&= \frac{1}{12}\coth^3(x)(4i-3\mathrm{csch}(x))+\frac{1}{8}\coth(x)(8i-3\mathrm{csch}(x))+\frac{1}{8}\int(-8i+3\mathrm{csch}(x))\,dx\\
&= -ix+\frac{1}{12}\coth^3(x)(4i-3\mathrm{csch}(x))+\frac{1}{8}\coth(x)(8i-3\mathrm{csch}(x))+\frac{3}{8}\int \mathrm{csch}(x)\,dx\\
&= -ix-\frac{3}{8}\tanh^{-1}(\cosh(x))+\frac{1}{12}\coth^3(x)(4i-3\mathrm{csch}(x))+\frac{1}{8}\coth(x)(8i-3\mathrm{csch}(x))
\end{aligned}$$

Mathematica [B] time = 0.04, size = 129, normalized size = 3.00

$$-ix+\frac{2}{3}i\tanh\left(\frac{x}{2}\right)+\frac{2}{3}i\coth\left(\frac{x}{2}\right)-\frac{1}{64}\text{csch}^4\left(\frac{x}{2}\right)-\frac{5}{32}\text{csch}^2\left(\frac{x}{2}\right)+\frac{1}{64}\text{sech}^4\left(\frac{x}{2}\right)-\frac{5}{32}\text{sech}^2\left(\frac{x}{2}\right)+\frac{3}{8}\log\left(\tanh\left(\frac{x}{2}\right)\right)+\frac{1}{24}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]^6/(I + Csch[x]),x]
```

```
[Out] (-I)*x + ((2*I)/3)*Coth[x/2] - (5*Csch[x/2]^2)/32 + (I/24)*Coth[x/2]*Csch[x
/2]^2 - Csch[x/2]^4/64 + (3*Log[Tanh[x/2]])/8 - (5*Sech[x/2]^2)/32 + Sech[x
/2]^4/64 + ((2*I)/3)*Tanh[x/2] - (I/24)*Sech[x/2]^2*Tanh[x/2]
```

fricas [B] time = 0.57, size = 154, normalized size = 3.58

$$\frac{-24i\,xe^{(8\,x)}+(96i\,x+96i)e^{(6\,x)}+(-144i\,x-192i)e^{(4\,x)}+(96i\,x+160i)e^{(2\,x)}-9\left(e^{(8\,x)}-4\,e^{(6\,x)}+6\,e^{(4\,x)}-4\,e^{(2\,x)}+\right.}{24\left(e^{(8\,x)}-4\,e^{(6\,x)}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^6/(I+csch(x)),x, algorithm="fricas")
```

```
[Out] 1/24*(-24*I*x*e^(8*x) + (96*I*x + 96*I)*e^(6*x) + (-144*I*x - 192*I)*e^(4*x
) + (96*I*x + 160*I)*e^(2*x) - 9*(e^(8*x) - 4*e^(6*x) + 6*e^(4*x) - 4*e^(2*
x) + 1)*log(e^x + 1) + 9*(e^(8*x) - 4*e^(6*x) + 6*e^(4*x) - 4*e^(2*x) + 1)*
log(e^x - 1) - 24*I*x - 30*e^(7*x) - 18*e^(5*x) - 18*e^(3*x) - 30*e^x - 64*
I)/(e^(8*x) - 4*e^(6*x) + 6*e^(4*x) - 4*e^(2*x) + 1)
```

giac [B] time = 0.13, size = 77, normalized size = 1.79

$$-\frac{15\,e^{(7\,x)}-48i\,e^{(6\,x)}+9\,e^{(5\,x)}+96i\,e^{(4\,x)}+9\,e^{(3\,x)}-80i\,e^{(2\,x)}+15\,e^{x}+32i}{12\left(i\,e^{(2\,x)}-i\right)^4}-i\,\log\left(-i\,e^{x}\right)-\frac{3}{8}\,\log\left(e^{x}+1\right)+\frac{3}{8}\,\log\left(|e^{x}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^6/(I+csch(x)),x, algorithm="giac")
```

```
[Out] -1/12*(15*e^(7*x) - 48*I*e^(6*x) + 9*e^(5*x) + 96*I*e^(4*x) + 9*e^(3*x) - 8
0*I*e^(2*x) + 15*e^x + 32*I)/(I*e^(2*x) - I)^4 - I*log(-I*e^x) - 3/8*log(e^
x + 1) + 3/8*log(abs(e^x - 1))
```

maple [B] time = 0.25, size = 95, normalized size = 2.21

$$\frac{5i\tanh\left(\frac{x}{2}\right)}{8}+\frac{\left(\tanh^4\left(\frac{x}{2}\right)\right)}{64}+\frac{i\left(\tanh^3\left(\frac{x}{2}\right)\right)}{24}+\frac{\left(\tanh^2\left(\frac{x}{2}\right)\right)}{8}+i\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)-i\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)-\frac{1}{64\tanh\left(\frac{x}{2}\right)^4}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^6/(I+csch(x)),x)
```

```
[Out] 5/8*I*tanh(1/2*x)+1/64*tanh(1/2*x)^4+1/24*I*tanh(1/2*x)^3+1/8*tanh(1/2*x)^2
+I*ln(tanh(1/2*x)-1)-I*ln(tanh(1/2*x)+1)-1/64/tanh(1/2*x)^4+5/8*I/tanh(1/2*
x)+1/24*I/tanh(1/2*x)^3-1/8/tanh(1/2*x)^2+3/8*ln(tanh(1/2*x))
```

maxima [B] time = 0.31, size = 96, normalized size = 2.23

$$-i\,x+\frac{15\,e^{(-x)}+80i\,e^{(-2\,x)}+9\,e^{(-3\,x)}-96i\,e^{(-4\,x)}+9\,e^{(-5\,x)}+48i\,e^{(-6\,x)}+15\,e^{(-7\,x)}-32i}{12\left(4\,e^{(-2\,x)}-6\,e^{(-4\,x)}+4\,e^{(-6\,x)}-e^{(-8\,x)}-1\right)}-\frac{3}{8}\,\log\left(e^{(-x)}+1\right)+\frac{3}{8}\,\log\left(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^6/(I+csch(x)),x, algorithm="maxima")
```

```
[Out] -I*x + 1/12*(15*e^(-x) + 80*I*e^(-2*x) + 9*e^(-3*x) - 96*I*e^(-4*x) + 9*e^(
-5*x) + 48*I*e^(-6*x) + 15*e^(-7*x) - 32*I)/(4*e^(-2*x) - 6*e^(-4*x) + 4*e^
(-6*x) - e^(-8*x) - 1) - 3/8*log(e^(-x) + 1) + 3/8*log(e^(-x) - 1)
```

mupad [B] time = 1.86, size = 106, normalized size = 2.47

$$\frac{3\ln\left(\frac{3}{4}-\frac{3\,e^x}{4}\right)}{8}-x\,1i-\frac{3\ln\left(\frac{3\,e^x}{4}+\frac{3}{4}\right)}{8}-\frac{5\,e^x}{4\,\left(e^{2\,x}-1\right)}-\frac{9\,e^x}{2\,\left(e^{2\,x}-1\right)^2}-\frac{6\,e^x}{\left(e^{2\,x}-1\right)^3}-\frac{4\,e^x}{\left(e^{2\,x}-1\right)^4}+\frac{4i}{e^{2\,x}-1}+\frac{4i}{\left(e^{2\,x}-1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^6/(1/sinh(x) + 1i),x)
```

```
[Out] (3*log(3/4 - (3*exp(x))/4))/8 - x*1i - (3*log((3*exp(x))/4 + 3/4))/8 - (5*e
xp(x))/(4*(exp(2*x) - 1)) - (9*exp(x))/(2*(exp(2*x) - 1)^2) - (6*exp(x))/(e
xp(2*x) - 1)^3 - (4*exp(x))/(exp(2*x) - 1)^4 + 4i/(exp(2*x) - 1) + 4i/(exp(
2*x) - 1)^2 + 8i/(3*(exp(2*x) - 1)^3)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\coth^{6}{\left(x \right)}}{\operatorname{csch}{\left(x \right)} + i}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)**6/(I+csch(x)),x)
```

```
[Out] Integral(coth(x)**6/(csch(x) + I), x)
```

3.113 $\int \frac{\tanh^5(x)}{a+b\,\mathrm{csch}(x)}\,dx$

Optimal. Leaf size=194

$$-\frac{3b\tan^{-1}(\sinh(x))}{8(a^2+b^2)}-\frac{\tanh^4(x)(a-b\,\mathrm{csch}(x))}{4(a^2+b^2)}+\frac{3b\tanh(x)\mathrm{sech}(x)}{8(a^2+b^2)}+\frac{b^6\log(a+b\,\mathrm{csch}(x))}{a(a^2+b^2)^3}-\frac{b^5\tan^{-1}(\sinh(x))}{(a^2+b^2)^3}-\frac{b^3\,\mathrm{t}}{2}$$

```
[Out] -b^5*arctan(sinh(x))/(a^2+b^2)^3-1/2*b^3*arctan(sinh(x))/(a^2+b^2)^2-3/8*b*
arctan(sinh(x))/(a^2+b^2)+b^6*ln(a+b*csch(x))/a/(a^2+b^2)^3+ln(sinh(x))/a-a
*(a^4+3*a^2*b^2+3*b^4)*ln(tanh(x))/(a^2+b^2)^3+3/8*b*sech(x)*tanh(x)/(a^2+b
^2)-1/2*(a*(a^2+2*b^2)-b^3*csch(x))*tanh(x)^2/(a^2+b^2)^2-1/4*(a-b*csch(x))
*tanh(x)^4/(a^2+b^2)
```

Rubi [A] time = 0.25, antiderivative size = 194, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.538, Rules used = {3885, 894, 639, 199, 203, 635, 260}

$$-\frac{a(3a^2b^2+a^4+3b^4)\log(\tanh(x))}{(a^2+b^2)^3}-\frac{b^5\tan^{-1}(\sinh(x))}{(a^2+b^2)^3}-\frac{b^3\tan^{-1}(\sinh(x))}{2(a^2+b^2)^2}-\frac{3b\tan^{-1}(\sinh(x))}{8(a^2+b^2)}+\frac{b^6\log(a+b\,\mathrm{csch}}{a(a^2+b^2)^3}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[x]^5/(a + b*Csch[x]),x]
```

```
[Out] -((b^5*ArcTan[Sinh[x]])/(a^2 + b^2)^3) - (b^3*ArcTan[Sinh[x]])/(2*(a^2 + b^
2)^2) - (3*b*ArcTan[Sinh[x]])/(8*(a^2 + b^2)) + (b^6*Log[a + b*Csch[x]])/(a
*(a^2 + b^2)^3) + Log[Sinh[x]]/a - (a*(a^4 + 3*a^2*b^2 + 3*b^4)*Log[Tanh[x]
])/(a^2 + b^2)^3 + (3*b*Sech[x]*Tanh[x])/(8*(a^2 + b^2)) - ((a*(a^2 + 2*b^2
) - b^3*Csch[x])*Tanh[x]^2)/(2*(a^2 + b^2)^2) - ((a - b*Csch[x])*Tanh[x]^4)
/(4*(a^2 + b^2))
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 639

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((a*e
 - c*d*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[(d*(2*p + 3))/(2*
a*(p + 1)), Int[(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e}, x] && Lt
Q[p, -1] && NeQ[p, -3/2]
```

Rule 894

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 3885

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> -Dist[(-1)^((m - 1)/2)/(d*b^(m - 1)), Subst[Int[((b^2 - x^
2)^((m - 1)/2)*(a + x)^n)/x, x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\tanh^5(x)}{a+b\text{csch}(x)}\,dx = b^6\,\text{Subst}\left(\int \frac{1}{x(a+x)\left(-b^2-x^2\right)^3}\,dx, x, b\text{csch}(x)\right)$$

$$= b^6\,\text{Subst}\left(\int \left(-\frac{1}{ab^6x}+\frac{1}{a\left(a^2+b^2\right)^3(a+x)}+\frac{b^2+ax}{b^2\left(a^2+b^2\right)\left(b^2+x^2\right)^3}+\frac{b^4+a\left(a^2+2\right.}{b^4\left(a^2+b^2\right)^2\left(b^2\right.}\right.\right.$$

$$= \frac{b^6\log(a+b\text{csch}(x))}{a\left(a^2+b^2\right)^3}+\frac{\log(\sinh(x))}{a}+\frac{\text{Subst}\left(\int \frac{b^6+a\left(a^4+3a^2b^2+3b^4\right)x}{b^2+x^2}\,dx, x, b\text{csch}(x)\right)}{\left(a^2+b^2\right)^3}+\ldots$$

$$= \frac{b^6\log(a+b\text{csch}(x))}{a\left(a^2+b^2\right)^3}+\frac{\log(\sinh(x))}{a}-\frac{\left(a\left(a^2+2b^2\right)-b^3\text{csch}(x)\right)\tanh^2(x)}{2\left(a^2+b^2\right)^2}-\frac{(a-b\text{csc}}{4(}$$

$$= -\frac{b^5\tan^{-1}(\sinh(x))}{\left(a^2+b^2\right)^3}-\frac{b^3\tan^{-1}(\sinh(x))}{2\left(a^2+b^2\right)^2}+\frac{b^6\log(a+b\text{csch}(x))}{a\left(a^2+b^2\right)^3}+\frac{\log(\sinh(x))}{a}-\frac{a\left(a^4\right.}{}$$

$$= -\frac{b^5\tan^{-1}(\sinh(x))}{\left(a^2+b^2\right)^3}-\frac{b^3\tan^{-1}(\sinh(x))}{2\left(a^2+b^2\right)^2}-\frac{3b\tan^{-1}(\sinh(x))}{8\left(a^2+b^2\right)}+\frac{b^6\log(a+b\text{csch}(x))}{a\left(a^2+b^2\right)^3}+$$

Mathematica [C] time = 0.51, size = 253, normalized size = 1.30

$$\frac{-2a^2\left(a^2+b^2\right)^2\text{sech}^4(x)-2ab\left(a^2+b^2\right)^2\tanh(x)\text{sech}^3(x)+4a^2\left(2a^4+5a^2b^2+3b^4\right)\text{sech}^2(x)+ab\left(5a^4+14\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]^5/(a + b*Csch[x]),x]
```

```
[Out] (a*b*(5*a^4 + 14*a^2*b^2 + 9*b^4)*ArcTan[Sinh[x]] + 4*a*(a^5 + I*a^4*b + 3*
a^3*b^2 + (3*I)*a^2*b^3 + 3*a*b^4 + (3*I)*b^5)*Log[I - Sinh[x]] + 4*a*(a^5
```

```
- I*a^4*b + 3*a^3*b^2 - (3*I)*a^2*b^3 + 3*a*b^4 - (3*I)*b^5)*Log[I + Sinh[x
]] + 8*b^6*Log[b + a*Sinh[x]] + 4*a^2*(2*a^4 + 5*a^2*b^2 + 3*b^4)*Sech[x]^2
 - 2*a^2*(a^2 + b^2)^2*Sech[x]^4 + a*b*(5*a^4 + 14*a^2*b^2 + 9*b^4)*Sech[x]
*Tanh[x] - 2*a*b*(a^2 + b^2)^2*Sech[x]^3*Tanh[x])/(8*a*(a^2 + b^2)^3)
```

fricas [B] time = 3.05, size = 4025, normalized size = 20.75

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^5/(a+b*csch(x)),x, algorithm="fricas")
```

```
[Out] -1/4*(4*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*x*cosh(x)^8 + 4*(a^6 + 3*a^4*b^
2 + 3*a^2*b^4 + b^6)*x*sinh(x)^8 - (5*a^5*b + 14*a^3*b^3 + 9*a*b^5)*cosh(x)
^7 - (5*a^5*b + 14*a^3*b^3 + 9*a*b^5 - 32*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^
6)*x*cosh(x))*sinh(x)^7 - 8*(2*a^6 + 5*a^4*b^2 + 3*a^2*b^4 - 2*(a^6 + 3*a^4
*b^2 + 3*a^2*b^4 + b^6)*x)*cosh(x)^6 - (16*a^6 + 40*a^4*b^2 + 24*a^2*b^4 -
112*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*x*cosh(x)^2 - 16*(a^6 + 3*a^4*b^2 +
 3*a^2*b^4 + b^6)*x + 7*(5*a^5*b + 14*a^3*b^3 + 9*a*b^5)*cosh(x))*sinh(x)^6
 + (3*a^5*b + 2*a^3*b^3 - a*b^5)*cosh(x)^5 + (3*a^5*b + 2*a^3*b^3 - a*b^5 +
 224*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*x*cosh(x)^3 - 21*(5*a^5*b + 14*a^3
*b^3 + 9*a*b^5)*cosh(x)^2 - 48*(2*a^6 + 5*a^4*b^2 + 3*a^2*b^4 - 2*(a^6 + 3*
a^4*b^2 + 3*a^2*b^4 + b^6)*x)*cosh(x))*sinh(x)^5 - 8*(2*a^6 + 6*a^4*b^2 + 4
*a^2*b^4 - 3*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*x)*cosh(x)^4 - (16*a^6 + 4
8*a^4*b^2 + 32*a^2*b^4 - 280*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*x*cosh(x)^
4 + 35*(5*a^5*b + 14*a^3*b^3 + 9*a*b^5)*cosh(x)^3 + 120*(2*a^6 + 5*a^4*b^2
+ 3*a^2*b^4 - 2*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*x)*cosh(x)^2 - 24*(a^6
+ 3*a^4*b^2 + 3*a^2*b^4 + b^6)*x - 5*(3*a^5*b + 2*a^3*b^3 - a*b^5)*cosh(x))
*sinh(x)^4 - (3*a^5*b + 2*a^3*b^3 - a*b^5)*cosh(x)^3 + (224*(a^6 + 3*a^4*b^
2 + 3*a^2*b^4 + b^6)*x*cosh(x)^5 - 3*a^5*b - 2*a^3*b^3 + a*b^5 - 35*(5*a^5*
b + 14*a^3*b^3 + 9*a*b^5)*cosh(x)^4 - 160*(2*a^6 + 5*a^4*b^2 + 3*a^2*b^4 -
2*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*x)*cosh(x)^3 + 10*(3*a^5*b + 2*a^3*b^
3 - a*b^5)*cosh(x)^2 - 32*(2*a^6 + 6*a^4*b^2 + 4*a^2*b^4 - 3*(a^6 + 3*a^4*b
^2 + 3*a^2*b^4 + b^6)*x)*cosh(x))*sinh(x)^3 - 8*(2*a^6 + 5*a^4*b^2 + 3*a^2*
b^4 - 2*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*x)*cosh(x)^2 + (112*(a^6 + 3*a^
4*b^2 + 3*a^2*b^4 + b^6)*x*cosh(x)^6 - 16*a^6 - 40*a^4*b^2 - 24*a^2*b^4 - 2
1*(5*a^5*b + 14*a^3*b^3 + 9*a*b^5)*cosh(x)^5 - 120*(2*a^6 + 5*a^4*b^2 + 3*a
^2*b^4 - 2*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*x)*cosh(x)^4 + 10*(3*a^5*b +
 2*a^3*b^3 - a*b^5)*cosh(x)^3 - 48*(2*a^6 + 6*a^4*b^2 + 4*a^2*b^4 - 3*(a^6
+ 3*a^4*b^2 + 3*a^2*b^4 + b^6)*x)*cosh(x)^2 + 16*(a^6 + 3*a^4*b^2 + 3*a^2*b
^4 + b^6)*x - 3*(3*a^5*b + 2*a^3*b^3 - a*b^5)*cosh(x))*sinh(x)^2 + 4*(a^6 +
 3*a^4*b^2 + 3*a^2*b^4 + b^6)*x + ((3*a^5*b + 10*a^3*b^3 + 15*a*b^5)*cosh(x
)^8 + 8*(3*a^5*b + 10*a^3*b^3 + 15*a*b^5)*cosh(x)*sinh(x)^7 + (3*a^5*b + 10
*a^3*b^3 + 15*a*b^5)*sinh(x)^8 + 4*(3*a^5*b + 10*a^3*b^3 + 15*a*b^5)*cosh(x
)^6 + 4*(3*a^5*b + 10*a^3*b^3 + 15*a*b^5 + 7*(3*a^5*b + 10*a^3*b^3 + 15*a*b
^5)*cosh(x)^2)*sinh(x)^6 + 3*a^5*b + 10*a^3*b^3 + 15*a*b^5 + 8*(7*(3*a^5*b
+ 10*a^3*b^3 + 15*a*b^5)*cosh(x)^3 + 3*(3*a^5*b + 10*a^3*b^3 + 15*a*b^5)*co
sh(x))*sinh(x)^5 + 6*(3*a^5*b + 10*a^3*b^3 + 15*a*b^5)*cosh(x)^4 + 2*(9*a^5
*b + 30*a^3*b^3 + 45*a*b^5 + 35*(3*a^5*b + 10*a^3*b^3 + 15*a*b^5)*cosh(x)^4
 + 30*(3*a^5*b + 10*a^3*b^3 + 15*a*b^5)*cosh(x)^2)*sinh(x)^4 + 8*(7*(3*a^5*
b + 10*a^3*b^3 + 15*a*b^5)*cosh(x)^5 + 10*(3*a^5*b + 10*a^3*b^3 + 15*a*b^5)
*cosh(x)^3 + 3*(3*a^5*b + 10*a^3*b^3 + 15*a*b^5)*cosh(x))*sinh(x)^3 + 4*(3*
a^5*b + 10*a^3*b^3 + 15*a*b^5)*cosh(x)^2 + 4*(7*(3*a^5*b + 10*a^3*b^3 + 15*
a*b^5)*cosh(x)^6 + 3*a^5*b + 10*a^3*b^3 + 15*a*b^5 + 15*(3*a^5*b + 10*a^3*b
^3 + 15*a*b^5)*cosh(x)^4 + 9*(3*a^5*b + 10*a^3*b^3 + 15*a*b^5)*cosh(x)^2)*s
inh(x)^2 + 8*((3*a^5*b + 10*a^3*b^3 + 15*a*b^5)*cosh(x)^7 + 3*(3*a^5*b + 10
*a^3*b^3 + 15*a*b^5)*cosh(x)^5 + 3*(3*a^5*b + 10*a^3*b^3 + 15*a*b^5)*cosh(x
)^3 + (3*a^5*b + 10*a^3*b^3 + 15*a*b^5)*cosh(x))*sinh(x))*arctan(cosh(x) +
sinh(x)) + (5*a^5*b + 14*a^3*b^3 + 9*a*b^5)*cosh(x) - 4*(b^6*cosh(x)^8 + 8*
b^6*cosh(x)*sinh(x)^7 + b^6*sinh(x)^8 + 4*b^6*cosh(x)^6 + 6*b^6*cosh(x)^4 +
```

```
 4*b^6*cosh(x)^2 + 4*(7*b^6*cosh(x)^2 + b^6)*sinh(x)^6 + b^6 + 8*(7*b^6*cos
h(x)^3 + 3*b^6*cosh(x))*sinh(x)^5 + 2*(35*b^6*cosh(x)^4 + 30*b^6*cosh(x)^2
+ 3*b^6)*sinh(x)^4 + 8*(7*b^6*cosh(x)^5 + 10*b^6*cosh(x)^3 + 3*b^6*cosh(x))
*sinh(x)^3 + 4*(7*b^6*cosh(x)^6 + 15*b^6*cosh(x)^4 + 9*b^6*cosh(x)^2 + b^6)
*sinh(x)^2 + 8*(b^6*cosh(x)^7 + 3*b^6*cosh(x)^5 + 3*b^6*cosh(x)^3 + b^6*cos
h(x))*sinh(x))*log(2*(a*sinh(x) + b)/(cosh(x) - sinh(x))) - 4*((a^6 + 3*a^4
*b^2 + 3*a^2*b^4)*cosh(x)^8 + 8*(a^6 + 3*a^4*b^2 + 3*a^2*b^4)*cosh(x)*sinh(
x)^7 + (a^6 + 3*a^4*b^2 + 3*a^2*b^4)*sinh(x)^8 + 4*(a^6 + 3*a^4*b^2 + 3*a^2
*b^4)*cosh(x)^6 + 4*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + 7*(a^6 + 3*a^4*b^2 + 3*a
^2*b^4)*cosh(x)^2)*sinh(x)^6 + a^6 + 3*a^4*b^2 + 3*a^2*b^4 + 8*(7*(a^6 + 3*
a^4*b^2 + 3*a^2*b^4)*cosh(x)^3 + 3*(a^6 + 3*a^4*b^2 + 3*a^2*b^4)*cosh(x))*s
inh(x)^5 + 6*(a^6 + 3*a^4*b^2 + 3*a^2*b^4)*cosh(x)^4 + 2*(3*a^6 + 9*a^4*b^2
 + 9*a^2*b^4 + 35*(a^6 + 3*a^4*b^2 + 3*a^2*b^4)*cosh(x)^4 + 30*(a^6 + 3*a^4
*b^2 + 3*a^2*b^4)*cosh(x)^2)*sinh(x)^4 + 8*(7*(a^6 + 3*a^4*b^2 + 3*a^2*b^4)
*cosh(x)^5 + 10*(a^6 + 3*a^4*b^2 + 3*a^2*b^4)*cosh(x)^3 + 3*(a^6 + 3*a^4*b^
2 + 3*a^2*b^4)*cosh(x))*sinh(x)^3 + 4*(a^6 + 3*a^4*b^2 + 3*a^2*b^4)*cosh(x)
^2 + 4*(7*(a^6 + 3*a^4*b^2 + 3*a^2*b^4)*cosh(x)^6 + a^6 + 3*a^4*b^2 + 3*a^2
*b^4 + 15*(a^6 + 3*a^4*b^2 + 3*a^2*b^4)*cosh(x)^4 + 9*(a^6 + 3*a^4*b^2 + 3*
a^2*b^4)*cosh(x)^2)*sinh(x)^2 + 8*((a^6 + 3*a^4*b^2 + 3*a^2*b^4)*cosh(x)^7
+ 3*(a^6 + 3*a^4*b^2 + 3*a^2*b^4)*cosh(x)^5 + 3*(a^6 + 3*a^4*b^2 + 3*a^2*b^
4)*cosh(x)^3 + (a^6 + 3*a^4*b^2 + 3*a^2*b^4)*cosh(x))*sinh(x))*log(2*cosh(x
)/(cosh(x) - sinh(x))) + (32*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*x*cosh(x)^
7 - 7*(5*a^5*b + 14*a^3*b^3 + 9*a*b^5)*cosh(x)^6 + 5*a^5*b + 14*a^3*b^3 + 9
*a*b^5 - 48*(2*a^6 + 5*a^4*b^2 + 3*a^2*b^4 - 2*(a^6 + 3*a^4*b^2 + 3*a^2*b^4
 + b^6)*x)*cosh(x)^5 + 5*(3*a^5*b + 2*a^3*b^3 - a*b^5)*cosh(x)^4 - 32*(2*a^
6 + 6*a^4*b^2 + 4*a^2*b^4 - 3*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*x)*cosh(x
)^3 - 3*(3*a^5*b + 2*a^3*b^3 - a*b^5)*cosh(x)^2 - 16*(2*a^6 + 5*a^4*b^2 + 3
*a^2*b^4 - 2*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*x)*cosh(x))*sinh(x))/((a^7
 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6)*cosh(x)^8 + 8*(a^7 + 3*a^5*b^2 + 3*a^3*b^
4 + a*b^6)*cosh(x)*sinh(x)^7 + (a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6)*sinh(x
)^8 + a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6 + 4*(a^7 + 3*a^5*b^2 + 3*a^3*b^4
+ a*b^6)*cosh(x)^6 + 4*(a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6 + 7*(a^7 + 3*a^
5*b^2 + 3*a^3*b^4 + a*b^6)*cosh(x)^2)*sinh(x)^6 + 8*(7*(a^7 + 3*a^5*b^2 + 3
*a^3*b^4 + a*b^6)*cosh(x)^3 + 3*(a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6)*cosh(
x))*sinh(x)^5 + 6*(a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6)*cosh(x)^4 + 2*(3*a^
7 + 9*a^5*b^2 + 9*a^3*b^4 + 3*a*b^6 + 35*(a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b
^6)*cosh(x)^4 + 30*(a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6)*cosh(x)^2)*sinh(x)
^4 + 8*(7*(a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6)*cosh(x)^5 + 10*(a^7 + 3*a^5
*b^2 + 3*a^3*b^4 + a*b^6)*cosh(x)^3 + 3*(a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^
6)*cosh(x))*sinh(x)^3 + 4*(a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6)*cosh(x)^2 +
 4*(a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6 + 7*(a^7 + 3*a^5*b^2 + 3*a^3*b^4 +
a*b^6)*cosh(x)^6 + 15*(a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6)*cosh(x)^4 + 9*(
a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6)*cosh(x)^2)*sinh(x)^2 + 8*((a^7 + 3*a^5
*b^2 + 3*a^3*b^4 + a*b^6)*cosh(x)^7 + 3*(a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^
6)*cosh(x)^5 + 3*(a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6)*cosh(x)^3 + (a^7 + 3
*a^5*b^2 + 3*a^3*b^4 + a*b^6)*cosh(x))*sinh(x))
```

giac [B] time = 0.16, size = 432, normalized size = 2.23

$$\frac{b^6 \log\left(\left|-a\left(e^{(-x)} - e^x\right) + 2\,b\right|\right)}{a^7 + 3\,a^5 b^2 + 3\,a^3 b^4 + a b^6} - \frac{\left(\pi + 2\,\arctan\left(\frac{1}{2}\left(e^{(2\,x)} - 1\right)e^{(-x)}\right)\right)\left(3\,a^4 b + 10\,a^2 b^3 + 15\,b^5\right)}{16\left(a^6 + 3\,a^4 b^2 + 3\,a^2 b^4 + b^6\right)} + \frac{\left(a^5 + 3\,a^3 b^2 + 3\,a\right.}{2\left(a^6 + 3\,a\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^5/(a+b*csch(x)),x, algorithm="giac")
```

```
[Out] b^6*log(abs(-a*(e^(-x) - e^x) + 2*b))/(a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6)
 - 1/16*(pi + 2*arctan(1/2*(e^(2*x) - 1)*e^(-x)))*(3*a^4*b + 10*a^2*b^3 + 1
5*b^5)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) + 1/2*(a^5 + 3*a^3*b^2 + 3*a*b^4
```

```
)*log((e^(-x) - e^x)^2 + 4)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) - 1/4*(3*a^
5*(e^(-x) - e^x)^4 + 9*a^3*b^2*(e^(-x) - e^x)^4 + 9*a*b^4*(e^(-x) - e^x)^4
+ 5*a^4*b*(e^(-x) - e^x)^3 + 14*a^2*b^3*(e^(-x) - e^x)^3 + 9*b^5*(e^(-x) -
e^x)^3 + 8*a^5*(e^(-x) - e^x)^2 + 32*a^3*b^2*(e^(-x) - e^x)^2 + 48*a*b^4*(e
^(-x) - e^x)^2 + 12*a^4*b*(e^(-x) - e^x) + 40*a^2*b^3*(e^(-x) - e^x) + 28*b
^5*(e^(-x) - e^x) + 16*a^3*b^2 + 64*a*b^4)/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 +
b^6)*((e^(-x) - e^x)^2 + 4)^2)
```

maple [B] time = 0.23, size = 1323, normalized size = 6.82

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^5/(a+b*csch(x)),x)
```

```
[Out] -1/a*ln(tanh(1/2*x)+1)-1/a*ln(tanh(1/2*x)-1)-5/2/(a^4+2*a^2*b^2+b^4)/(a^2+b
^2)/(tanh(1/2*x)^2+1)^4*tanh(1/2*x)^7*a^2*b^3-6/(a^4+2*a^2*b^2+b^4)/(a^2+b^
2)/(tanh(1/2*x)^2+1)^4*tanh(1/2*x)^6*a^3*b^2-4/(a^4+2*a^2*b^2+b^4)/(a^2+b^2
)/(tanh(1/2*x)^2+1)^4*tanh(1/2*x)^6*a*b^4-20/(a^4+2*a^2*b^2+b^4)/(a^2+b^2)/
(tanh(1/2*x)^2+1)^4*tanh(1/2*x)^4*a^3*b^2-6/(a^4+2*a^2*b^2+b^4)/(a^2+b^2)/(
tanh(1/2*x)^2+1)^4*tanh(1/2*x)^2*a^3*b^2-4/(a^4+2*a^2*b^2+b^4)/(a^2+b^2)/(t
anh(1/2*x)^2+1)^4*tanh(1/2*x)^2*a*b^4-13/2/(a^4+2*a^2*b^2+b^4)/(a^2+b^2)/(t
anh(1/2*x)^2+1)^4*tanh(1/2*x)^5*a^2*b^3-11/4/(a^4+2*a^2*b^2+b^4)/(a^2+b^2)/
(tanh(1/2*x)^2+1)^4*tanh(1/2*x)^5*a^4*b+3/4/(a^4+2*a^2*b^2+b^4)/(a^2+b^2)/(
tanh(1/2*x)^2+1)^4*tanh(1/2*x)*a^4*b+b^6/(a^4+2*a^2*b^2+b^4)/(a^2+b^2)/a*ln
(tanh(1/2*x)^2*b-2*a*tanh(1/2*x)-b)-15/4/(a^4+2*a^2*b^2+b^4)/(a^2+b^2)/(tan
h(1/2*x)^2+1)^4*tanh(1/2*x)^5*b^5-15/4/(a^4+2*a^2*b^2+b^4)/(a^2+b^2)*arctan
(tanh(1/2*x))*b^5+1/(a^4+2*a^2*b^2+b^4)/(a^2+b^2)*ln(tanh(1/2*x)^2+1)*a^5-2
/(a^4+2*a^2*b^2+b^4)/(a^2+b^2)/(tanh(1/2*x)^2+1)^4*tanh(1/2*x)^2*a^5+7/4/(a
^4+2*a^2*b^2+b^4)/(a^2+b^2)/(tanh(1/2*x)^2+1)^4*tanh(1/2*x)*b^5+3/(a^4+2*a^
2*b^2+b^4)/(a^2+b^2)*ln(tanh(1/2*x)^2+1)*a^3*b^2+3/(a^4+2*a^2*b^2+b^4)/(a^2
+b^2)*ln(tanh(1/2*x)^2+1)*a*b^4-3/4/(a^4+2*a^2*b^2+b^4)/(a^2+b^2)*arctan(ta
nh(1/2*x))*a^4*b-5/2/(a^4+2*a^2*b^2+b^4)/(a^2+b^2)*arctan(tanh(1/2*x))*a^2*
b^3-7/4/(a^4+2*a^2*b^2+b^4)/(a^2+b^2)/(tanh(1/2*x)^2+1)^4*tanh(1/2*x)^7*b^5
-2/(a^4+2*a^2*b^2+b^4)/(a^2+b^2)/(tanh(1/2*x)^2+1)^4*tanh(1/2*x)^6*a^5+5/2/
(a^4+2*a^2*b^2+b^4)/(a^2+b^2)/(tanh(1/2*x)^2+1)^4*tanh(1/2*x)*a^2*b^3-3/4/(
a^4+2*a^2*b^2+b^4)/(a^2+b^2)/(tanh(1/2*x)^2+1)^4*tanh(1/2*x)^7*a^4*b-12/(a^
4+2*a^2*b^2+b^4)/(a^2+b^2)/(tanh(1/2*x)^2+1)^4*tanh(1/2*x)^4*a*b^4+13/2/(a^
4+2*a^2*b^2+b^4)/(a^2+b^2)/(tanh(1/2*x)^2+1)^4*tanh(1/2*x)^3*a^2*b^3+11/4/(
a^4+2*a^2*b^2+b^4)/(a^2+b^2)/(tanh(1/2*x)^2+1)^4*tanh(1/2*x)^3*a^4*b-8/(a^4
+2*a^2*b^2+b^4)/(a^2+b^2)/(tanh(1/2*x)^2+1)^4*tanh(1/2*x)^4*a^5+15/4/(a^4+2
*a^2*b^2+b^4)/(a^2+b^2)/(tanh(1/2*x)^2+1)^4*tanh(1/2*x)^3*b^5
```

maxima [B] time = 0.43, size = 383, normalized size = 1.97

$$\frac{b^6 \log\left(-2\,be^{(-x)} + ae^{(-2\,x)} - a\right)}{a^7 + 3\,a^5b^2 + 3\,a^3b^4 + ab^6} + \frac{\left(3\,a^4b + 10\,a^2b^3 + 15\,b^5\right)\arctan\left(e^{(-x)}\right)}{4\left(a^6 + 3\,a^4b^2 + 3\,a^2b^4 + b^6\right)} + \frac{\left(a^5 + 3\,a^3b^2 + 3\,ab^4\right)\log\left(e^{(-2\,x)} + 1\right)}{a^6 + 3\,a^4b^2 + 3\,a^2b^4 + b^6} + \cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^5/(a+b*csch(x)),x, algorithm="maxima")
```

```
[Out] b^6*log(-2*b*e^(-x) + a*e^(-2*x) - a)/(a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6)
 + 1/4*(3*a^4*b + 10*a^2*b^3 + 15*b^5)*arctan(e^(-x))/(a^6 + 3*a^4*b^2 + 3*
a^2*b^4 + b^6) + (a^5 + 3*a^3*b^2 + 3*a*b^4)*log(e^(-2*x) + 1)/(a^6 + 3*a^4
*b^2 + 3*a^2*b^4 + b^6) + 1/4*((5*a^2*b + 9*b^3)*e^(-x) + 8*(2*a^3 + 3*a*b^
2)*e^(-2*x) - (3*a^2*b - b^3)*e^(-3*x) + 16*(a^3 + 2*a*b^2)*e^(-4*x) + (3*a
^2*b - b^3)*e^(-5*x) + 8*(2*a^3 + 3*a*b^2)*e^(-6*x) - (5*a^2*b + 9*b^3)*e^(
-7*x))/(a^4 + 2*a^2*b^2 + b^4 + 4*(a^4 + 2*a^2*b^2 + b^4)*e^(-2*x) + 6*(a^4
```

```
 + 2*a^2*b^2 + b^4)*e^(-4*x) + 4*(a^4 + 2*a^2*b^2 + b^4)*e^(-6*x) + (a^4 +
2*a^2*b^2 + b^4)*e^(-8*x)) + x/a
```

mupad [B] time = 7.04, size = 611, normalized size = 3.15

$$\frac{\frac{6\,e^x\,\left(a^2\,b+b^3\right)}{{\left(a^2+b^2\right)}^2}+\frac{8\,\left(a^4+a^2\,b^2\right)}{a\,{\left(a^2+b^2\right)}^2}}{3\,e^{2\,x}+3\,e^{4\,x}+e^{6\,x}+1}-\frac{\frac{4\,a}{a^2+b^2}+\frac{4\,b\,e^x}{a^2+b^2}}{4\,e^{2\,x}+6\,e^{4\,x}+4\,e^{6\,x}+e^{8\,x}+1}-\frac{x}{a}-\frac{\frac{e^x\,\left(9\,a^2\,b+13\,b^3\right)}{2\,{\left(a^2+b^2\right)}^2}+\frac{2\,\left(4\,a^4+5\,a^2\,b^2\right)}{a\,{\left(a^2+b^2\right)}^2}}{2\,e^{2\,x}+e^{4\,x}+1}+\frac{\frac{e^x\,\left(5\,a^4\,b+14\,a^2\,b^3+9\,b^5 \cdots\right.}{4\,{\left(a^2+b^2\right)}^3}\cdots}{e^{2 \cdots}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^5/(a + b/sinh(x)),x)
```

```
[Out] ((6*exp(x)*(a^2*b + b^3))/(a^2 + b^2)^2 + (8*(a^4 + a^2*b^2))/(a*(a^2 + b^2
)^2))/(3*exp(2*x) + 3*exp(4*x) + exp(6*x) + 1) - ((4*a)/(a^2 + b^2) + (4*b*
exp(x))/(a^2 + b^2))/(4*exp(2*x) + 6*exp(4*x) + 4*exp(6*x) + exp(8*x) + 1)
- x/a - ((exp(x)*(9*a^2*b + 13*b^3))/(2*(a^2 + b^2)^2) + (2*(4*a^4 + 5*a^2*
b^2))/(a*(a^2 + b^2)^2))/(2*exp(2*x) + exp(4*x) + 1) + ((exp(x)*(5*a^4*b +
9*b^5 + 14*a^2*b^3))/(4*(a^2 + b^2)^3) + (2*(2*a^6 + 3*a^2*b^4 + 5*a^4*b^2)
)/(a*(a^2 + b^2)^3))/(exp(2*x) + 1) + (log(exp(x)*1i + 1)*(a*b*21i - 8*a^2
+ 15*b^2))/(8*(3*a*b^2 + a^2*b*3i - a^3 - b^3*1i)) + (b^6*log(64*a^13*exp(2
*x) - 64*a*b^12 - 64*a^13 + 159*a^3*b^10 - 492*a^5*b^8 - 1214*a^7*b^6 - 102
0*a^9*b^4 - 393*a^11*b^2 + 128*b^13*exp(x) - 159*a^3*b^10*exp(2*x) + 492*a^
5*b^8*exp(2*x) + 1214*a^7*b^6*exp(2*x) + 1020*a^9*b^4*exp(2*x) + 393*a^11*b
^2*exp(2*x) + 128*a^12*b*exp(x) + 64*a*b^12*exp(2*x) - 318*a^2*b^11*exp(x)
+ 984*a^4*b^9*exp(x) + 2428*a^6*b^7*exp(x) + 2040*a^8*b^5*exp(x) + 786*a^10
*b^3*exp(x)))/(a*b^6 + a^7 + 3*a^3*b^4 + 3*a^5*b^2) + (log(exp(x) + 1i)*(21
*a*b - a^2*8i + b^2*15i))/(8*(a*b^2*3i + 3*a^2*b - a^3*1i - b^3))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh^5(x)}{a+b\,\mathrm{csch}(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)**5/(a+b*csch(x)),x)
```

```
[Out] Integral(tanh(x)**5/(a + b*csch(x)), x)
```

3.114 $\int \frac{\tanh^4(x)}{a+b\mathbf{csch}(x)}\,dx$

Optimal. Leaf size=183

$$\frac{ab^2x}{\left(a^2+b^2\right)^2}+\frac{ax}{a^2+b^2}-\frac{a\tanh^3(x)}{3\left(a^2+b^2\right)}-\frac{ab^2\tanh(x)}{\left(a^2+b^2\right)^2}-\frac{a\tanh(x)}{a^2+b^2}-\frac{b\text{sech}^3(x)}{3\left(a^2+b^2\right)}+\frac{b\text{sech}(x)}{a^2+b^2}+\frac{2b^5\tanh^{-1}\left(\frac{a-b\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{a\left(a^2+b^2\right)^{5/2}}+\frac{\quad}{a\,(a}$$

```
[Out] a*b^2*x/(a^2+b^2)^2+b^4*x/a/(a^2+b^2)^2+a*x/(a^2+b^2)+2*b^5*arctanh((a-b*ta
nh(1/2*x))/(a^2+b^2)^(1/2))/a/(a^2+b^2)^(5/2)+b^3*sech(x)/(a^2+b^2)^2+b*sec
h(x)/(a^2+b^2)-1/3*b*sech(x)^3/(a^2+b^2)-a*b^2*tanh(x)/(a^2+b^2)^2-a*tanh(x
)/(a^2+b^2)-1/3*a*tanh(x)^3/(a^2+b^2)
```

Rubi [A] time = 0.38, antiderivative size = 183, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 9, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.692, Rules used = {3898, 2902, 2606, 3473, 8, 2735, 2660, 618, 204}

$$\frac{b^4x}{a\left(a^2+b^2\right)^2}+\frac{ab^2x}{\left(a^2+b^2\right)^2}+\frac{ax}{a^2+b^2}+\frac{2b^5\tanh^{-1}\left(\frac{a-b\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{a\left(a^2+b^2\right)^{5/2}}-\frac{a\tanh^3(x)}{3\left(a^2+b^2\right)}-\frac{ab^2\tanh(x)}{\left(a^2+b^2\right)^2}-\frac{a\tanh(x)}{a^2+b^2}-\frac{b\text{sech}^3(x)}{3\left(a^2+b^2\right)}+\cdots$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[x]^4/(a + b*Csch[x]),x]
```

```
[Out] (a*b^2*x)/(a^2 + b^2)^2 + (b^4*x)/(a*(a^2 + b^2)^2) + (a*x)/(a^2 + b^2) + (
2*b^5*ArcTanh[(a - b*Tanh[x/2])/Sqrt[a^2 + b^2]])/(a*(a^2 + b^2)^(5/2)) + (
b^3*Sech[x])/(a^2 + b^2)^2 + (b*Sech[x])/(a^2 + b^2) - (b*Sech[x]^3)/(3*(a^
2 + b^2)) - (a*b^2*Tanh[x])/(a^2 + b^2)^2 - (a*Tanh[x])/(a^2 + b^2) - (a*Ta
nh[x]^3)/(3*(a^2 + b^2))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 && !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
```

```
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2902

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[(a*d^2)/(a^2
 - b^2), Int[(g*Cos[e + f*x])^p*(d*Sin[e + f*x])^(n - 2), x], x] + (-Dist[(
b*d)/(a^2 - b^2), Int[(g*Cos[e + f*x])^p*(d*Sin[e + f*x])^(n - 1), x], x] -
 Dist[(a^2*d^2)/(g^2*(a^2 - b^2)), Int[((g*Cos[e + f*x])^(p + 2)*(d*Sin[e +
 f*x])^(n - 2))/(a + b*Sin[e + f*x]), x], x]) /; FreeQ[{a, b, d, e, f, g},
x] && NeQ[a^2 - b^2, 0] && IntegersQ[2*n, 2*p] && LtQ[p, -1] && GtQ[n, 1]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3898

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Int[(Cos[c + d*x]^m*(b + a*Sin[c + d*x])^n)/Sin[c + d*x]^(
m + n), x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[n] &&
 IntegerQ[m] && (IntegerQ[m/2] || LeQ[m, 1])
```

Rubi steps

$$\int \frac{\tanh^4(x)}{a+b\text{csch}(x)}\,dx = i\int \frac{\sinh(x)\tanh^4(x)}{ib+ia\sinh(x)}\,dx$$

$$= \frac{a\int \tanh^4(x)\,dx}{a^2+b^2} - \frac{b\int \text{sech}(x)\tanh^3(x)\,dx}{a^2+b^2} + \frac{(ib^2)\int \frac{\sinh(x)\tanh^2(x)}{ib+ia\sinh(x)}\,dx}{a^2+b^2}$$

$$= -\frac{a\tanh^3(x)}{3(a^2+b^2)} + \frac{(ab^2)\int \tanh^2(x)\,dx}{(a^2+b^2)^2} - \frac{b^3\int \text{sech}(x)\tanh(x)\,dx}{(a^2+b^2)^2} + \frac{(ib^4)\int \frac{\sinh(x)}{ib+ia\sinh(x)}\,dx}{(a^2+b^2)^2} + a$$

$$= \frac{b^4x}{a(a^2+b^2)^2} + \frac{b\text{sech}(x)}{a^2+b^2} - \frac{b\text{sech}^3(x)}{3(a^2+b^2)} - \frac{ab^2\tanh(x)}{(a^2+b^2)^2} - \frac{a\tanh(x)}{a^2+b^2} - \frac{a\tanh^3(x)}{3(a^2+b^2)} + \frac{(ab^2)\int}{(a^2+b}$$

$$= \frac{ab^2x}{(a^2+b^2)^2} + \frac{b^4x}{a(a^2+b^2)^2} + \frac{ax}{a^2+b^2} + \frac{b^3\text{sech}(x)}{(a^2+b^2)^2} + \frac{b\text{sech}(x)}{a^2+b^2} - \frac{b\text{sech}^3(x)}{3(a^2+b^2)} - \frac{ab^2\tanh(x)}{(a^2+b^2)^2}$$

$$= \frac{ab^2x}{(a^2+b^2)^2} + \frac{b^4x}{a(a^2+b^2)^2} + \frac{ax}{a^2+b^2} + \frac{b^3\text{sech}(x)}{(a^2+b^2)^2} + \frac{b\text{sech}(x)}{a^2+b^2} - \frac{b\text{sech}^3(x)}{3(a^2+b^2)} - \frac{ab^2\tanh(x)}{(a^2+b^2)^2}$$

$$= \frac{ab^2x}{(a^2+b^2)^2} + \frac{b^4x}{a(a^2+b^2)^2} + \frac{ax}{a^2+b^2} + \frac{2b^5\tanh^{-1}\left(\frac{a-b\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{a(a^2+b^2)^{5/2}} + \frac{b^3\text{sech}(x)}{(a^2+b^2)^2} + \frac{b\text{sech}(x)}{a^2+b^2} -$$

Mathematica [A] time = 0.74, size = 141, normalized size = 0.77

$$\frac{1}{3}\left(-\frac{a(4a^2+7b^2)\tanh(x)}{(a^2+b^2)^2} - \frac{b\text{sech}^3(x)}{a^2+b^2} + \frac{3b(a^2+2b^2)\text{sech}(x)}{(a^2+b^2)^2} + \frac{a\tanh(x)\text{sech}^2(x)}{a^2+b^2} + \frac{3\left(x - \frac{2b^5\tan^{-1}\left(\frac{a-b\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2-b^2}}\right)}{(-a^2-b^2)^{5/2}}\right.}{a}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]^4/(a + b*Csch[x]),x]
```

```
[Out] ((3*(x - (2*b^5*ArcTan[(a - b*Tanh[x/2])/Sqrt[-a^2 - b^2]])/(-a^2 - b^2)^(5
/2)))/a + (3*b*(a^2 + 2*b^2)*Sech[x])/(a^2 + b^2)^2 - (b*Sech[x]^3)/(a^2 +
b^2) - (a*(4*a^2 + 7*b^2)*Tanh[x])/(a^2 + b^2)^2 + (a*Sech[x]^2*Tanh[x])/(a
^2 + b^2))/3
```

fricas [B] time = 0.64, size = 1746, normalized size = 9.54

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^4/(a+b*csch(x)),x, algorithm="fricas")
```

```
[Out] 1/3*(3*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*x*cosh(x)^6 + 3*(a^6 + 3*a^4*b^2
 + 3*a^2*b^4 + b^6)*x*sinh(x)^6 + 8*a^6 + 22*a^4*b^2 + 14*a^2*b^4 + 6*(a^5*
b + 3*a^3*b^3 + 2*a*b^5)*cosh(x)^5 + 6*(a^5*b + 3*a^3*b^3 + 2*a*b^5 + 3*(a^
6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*x*cosh(x))*sinh(x)^5 + 3*(4*a^6 + 10*a^4*b
```

```
^2 + 6*a^2*b^4 + 3*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*x)*cosh(x)^4 + 3*(4*
a^6 + 10*a^4*b^2 + 6*a^2*b^4 + 15*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*x*cos
h(x)^2 + 3*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*x + 10*(a^5*b + 3*a^3*b^3 +
2*a*b^5)*cosh(x))*sinh(x)^4 + 4*(a^5*b + 5*a^3*b^3 + 4*a*b^5)*cosh(x)^3 + 4
*(a^5*b + 5*a^3*b^3 + 4*a*b^5 + 15*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*x*co
sh(x)^3 + 15*(a^5*b + 3*a^3*b^3 + 2*a*b^5)*cosh(x)^2 + 3*(4*a^6 + 10*a^4*b^
2 + 6*a^2*b^4 + 3*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*x)*cosh(x))*sinh(x)^3
 + 3*(4*a^6 + 12*a^4*b^2 + 8*a^2*b^4 + 3*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6
)*x)*cosh(x)^2 + 3*(4*a^6 + 12*a^4*b^2 + 8*a^2*b^4 + 15*(a^6 + 3*a^4*b^2 +
3*a^2*b^4 + b^6)*x*cosh(x)^4 + 20*(a^5*b + 3*a^3*b^3 + 2*a*b^5)*cosh(x)^3 +
 6*(4*a^6 + 10*a^4*b^2 + 6*a^2*b^4 + 3*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*
x)*cosh(x)^2 + 3*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*x + 4*(a^5*b + 5*a^3*b
^3 + 4*a*b^5)*cosh(x))*sinh(x)^2 + 3*(b^5*cosh(x)^6 + 6*b^5*cosh(x)*sinh(x)
^5 + b^5*sinh(x)^6 + 3*b^5*cosh(x)^4 + 3*b^5*cosh(x)^2 + b^5 + 3*(5*b^5*cos
h(x)^2 + b^5)*sinh(x)^4 + 4*(5*b^5*cosh(x)^3 + 3*b^5*cosh(x))*sinh(x)^3 + 3
*(5*b^5*cosh(x)^4 + 6*b^5*cosh(x)^2 + b^5)*sinh(x)^2 + 6*(b^5*cosh(x)^5 + 2
*b^5*cosh(x)^3 + b^5*cosh(x))*sinh(x))*sqrt(a^2 + b^2)*log((a^2*cosh(x)^2 +
 a^2*sinh(x)^2 + 2*a*b*cosh(x) + a^2 + 2*b^2 + 2*(a^2*cosh(x) + a*b)*sinh(x
) + 2*sqrt(a^2 + b^2)*(a*cosh(x) + a*sinh(x) + b))/(a*cosh(x)^2 + a*sinh(x)
^2 + 2*b*cosh(x) + 2*(a*cosh(x) + b)*sinh(x) - a)) + 3*(a^6 + 3*a^4*b^2 + 3
*a^2*b^4 + b^6)*x + 6*(a^5*b + 3*a^3*b^3 + 2*a*b^5)*cosh(x) + 6*(3*(a^6 + 3
*a^4*b^2 + 3*a^2*b^4 + b^6)*x*cosh(x)^5 + a^5*b + 3*a^3*b^3 + 2*a*b^5 + 5*(
a^5*b + 3*a^3*b^3 + 2*a*b^5)*cosh(x)^4 + 2*(4*a^6 + 10*a^4*b^2 + 6*a^2*b^4
+ 3*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*x)*cosh(x)^3 + 2*(a^5*b + 5*a^3*b^3
 + 4*a*b^5)*cosh(x)^2 + (4*a^6 + 12*a^4*b^2 + 8*a^2*b^4 + 3*(a^6 + 3*a^4*b^
2 + 3*a^2*b^4 + b^6)*x)*cosh(x))*sinh(x))/(a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*
b^6 + (a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6)*cosh(x)^6 + 6*(a^7 + 3*a^5*b^2
+ 3*a^3*b^4 + a*b^6)*cosh(x)*sinh(x)^5 + (a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b
^6)*sinh(x)^6 + 3*(a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6)*cosh(x)^4 + 3*(a^7
+ 3*a^5*b^2 + 3*a^3*b^4 + a*b^6 + 5*(a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6)*c
osh(x)^2)*sinh(x)^4 + 4*(5*(a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6)*cosh(x)^3
+ 3*(a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6)*cosh(x))*sinh(x)^3 + 3*(a^7 + 3*a
^5*b^2 + 3*a^3*b^4 + a*b^6)*cosh(x)^2 + 3*(a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*
b^6 + 5*(a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6)*cosh(x)^4 + 6*(a^7 + 3*a^5*b^
2 + 3*a^3*b^4 + a*b^6)*cosh(x)^2)*sinh(x)^2 + 6*((a^7 + 3*a^5*b^2 + 3*a^3*b
^4 + a*b^6)*cosh(x)^5 + 2*(a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6)*cosh(x)^3 +
 (a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6)*cosh(x))*sinh(x))
```

giac [A] time = 0.15, size = 215, normalized size = 1.17

$$-\frac{b^5\log\left(\frac{\left|2\,ae^x+2\,b-2\,\sqrt{a^2+b^2}\right|}{\left|2\,ae^x+2\,b+2\,\sqrt{a^2+b^2}\right|}\right)}{\left(a^5+2\,a^3b^2+ab^4\right)\sqrt{a^2+b^2}}+\frac{x}{a}+\frac{2\left(3\,a^2be^{(5\,x)}+6\,b^3e^{(5\,x)}+6\,a^3e^{(4\,x)}+9\,ab^2e^{(4\,x)}+2\,a^2be^{(3\,x)}+8\,b^3e^{(3\,x)}+6\right.}{3\left(a^4+2\,a^2b^2+b^4\right)\left(e^{(2\,x)}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^4/(a+b*csch(x)),x, algorithm="giac")
```

```
[Out] -b^5*log(abs(2*a*e^x + 2*b - 2*sqrt(a^2 + b^2))/abs(2*a*e^x + 2*b + 2*sqrt(
a^2 + b^2)))/((a^5 + 2*a^3*b^2 + a*b^4)*sqrt(a^2 + b^2)) + x/a + 2/3*(3*a^2
*b*e^(5*x) + 6*b^3*e^(5*x) + 6*a^3*e^(4*x) + 9*a*b^2*e^(4*x) + 2*a^2*b*e^(3
*x) + 8*b^3*e^(3*x) + 6*a^3*e^(2*x) + 12*a*b^2*e^(2*x) + 3*a^2*b*e^x + 6*b^
3*e^x + 4*a^3 + 7*a*b^2)/((a^4 + 2*a^2*b^2 + b^4)*(e^(2*x) + 1)^3)
```

maple [A] time = 0.20, size = 207, normalized size = 1.13

$$-\frac{\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{a}+\frac{\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)}{a}-\frac{2b^5\operatorname{arctanh}\left(\frac{2\tanh\left(\frac{x}{2}\right)b-2a}{2\sqrt{a^2+b^2}}\right)}{a\left(a^4+2a^2b^2+b^4\right)\sqrt{a^2+b^2}}+\frac{2\left(-a^3-2a\,b^2\right)\left(\tanh^5\left(\frac{x}{2}\right)\right)+2b^3\left(\tan\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^4/(a+b*csch(x)),x)
```

```
[Out] -1/a*ln(tanh(1/2*x)-1)+1/a*ln(tanh(1/2*x)+1)-2/a*b^5/(a^4+2*a^2*b^2+b^4)/(a
^2+b^2)^(1/2)*arctanh(1/2*(2*tanh(1/2*x)*b-2*a)/(a^2+b^2)^(1/2))+2/(a^4+2*a
^2*b^2+b^4)*((-a^3-2*a*b^2)*tanh(1/2*x)^5+b^3*tanh(1/2*x)^4+(-10/3*a^3-16/3
*a*b^2)*tanh(1/2*x)^3+(2*a^2*b+4*b^3)*tanh(1/2*x)^2+(-a^3-2*a*b^2)*tanh(1/2
*x)+2/3*a^2*b+5/3*b^3)/(tanh(1/2*x)^2+1)^3
```

maxima [A] time = 0.41, size = 261, normalized size = 1.43

$$-\frac{b^5\log\left(\frac{ae^{(-x)}-b-\sqrt{a^2+b^2}}{ae^{(-x)}-b+\sqrt{a^2+b^2}}\right)}{\left(a^5+2\,a^3b^2+ab^4\right)\sqrt{a^2+b^2}}-\frac{2\left(4\,a^3+7\,ab^2-3\left(a^2b+2\,b^3\right)e^{(-x)}+6\left(a^3+2\,ab^2\right)e^{(-2\,x)}-2\left(a^2b+4\,b^3\right)e^{(-3\,x)}+\right.}{3\left(a^4+2\,a^2b^2+b^4+3\left(a^4+2\,a^2b^2+b^4\right)e^{(-2\,x)}+3\left(a^4+2\,a^2b^2+b^4\right)\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^4/(a+b*csch(x)),x, algorithm="maxima")
```

```
[Out] -b^5*log((a*e^(-x) - b - sqrt(a^2 + b^2))/(a*e^(-x) - b + sqrt(a^2 + b^2)))
/((a^5 + 2*a^3*b^2 + a*b^4)*sqrt(a^2 + b^2)) - 2/3*(4*a^3 + 7*a*b^2 - 3*(a^
2*b + 2*b^3)*e^(-x) + 6*(a^3 + 2*a*b^2)*e^(-2*x) - 2*(a^2*b + 4*b^3)*e^(-3*
x) + 3*(2*a^3 + 3*a*b^2)*e^(-4*x) - 3*(a^2*b + 2*b^3)*e^(-5*x))/(a^4 + 2*a^
2*b^2 + b^4 + 3*(a^4 + 2*a^2*b^2 + b^4)*e^(-2*x) + 3*(a^4 + 2*a^2*b^2 + b^4
)*e^(-4*x) + (a^4 + 2*a^2*b^2 + b^4)*e^(-6*x)) + x/a
```

mupad [B] time = 3.43, size = 707, normalized size = 3.86

$$\frac{x}{a}+\frac{\frac{8\,a}{3\left(a^2+b^2\right)}+\frac{8\,b\,\mathrm{e}^x}{3\left(a^2+b^2\right)}}{3\,\mathrm{e}^{2\,x}+3\,\mathrm{e}^{4\,x}+\mathrm{e}^{6\,x}+1}-\frac{\frac{8\,\mathrm{e}^x\left(a^2\,b+b^3\right)}{3\left(a^2+b^2\right)^2}+\frac{4\left(a^4+a^2\,b^2\right)}{a\left(a^2+b^2\right)^2}}{2\,\mathrm{e}^{2\,x}+\mathrm{e}^{4\,x}+1}+\frac{\frac{2\,\mathrm{e}^x\left(a^2\,b+2\,b^3\right)}{\left(a^2+b^2\right)^2}+\frac{2\left(2\,a^4+3\,a^2\,b^2\right)}{a\left(a^2+b^2\right)^2}}{\mathrm{e}^{2\,x}+1}+\frac{2\,\mathrm{atan}\left(\left(\left(\mathrm{e}^x\left(\frac{2\,b^5}{a^3\,\sqrt{b^{10}}\,\left(a^2+b^2\right)^2\,\left(a^5\right.}\right.\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^4/(a + b/sinh(x)),x)
```

```
[Out] x/a + ((8*a)/(3*(a^2 + b^2)) + (8*b*exp(x))/(3*(a^2 + b^2)))/(3*exp(2*x) +
3*exp(4*x) + exp(6*x) + 1) - ((8*exp(x)*(a^2*b + b^3))/(3*(a^2 + b^2)^2) +
(4*(a^4 + a^2*b^2))/(a*(a^2 + b^2)^2))/(2*exp(2*x) + exp(4*x) + 1) + ((2*ex
p(x)*(a^2*b + 2*b^3))/(a^2 + b^2)^2 + (2*(2*a^4 + 3*a^2*b^2))/(a*(a^2 + b^2
)^2))/(exp(2*x) + 1) + (2*atan((exp(x)*((2*b^5)/(a^3*(b^10)^(1/2)*(a^2 + b^
2)^2*(a*b^4 + a^5 + 2*a^3*b^2)) + (2*(2*a^3*b^3*(b^10)^(1/2) + a*b^5*(b^10)
^(1/2) + a^5*b*(b^10)^(1/2)))/(a^2*b^4*(-a^2*(a^2 + b^2)^5)^(1/2)*(a*b^4 +
a^5 + 2*a^3*b^2)*(- a^12 - a^2*b^10 - 5*a^4*b^8 - 10*a^6*b^6 - 10*a^8*b^4 -
 5*a^10*b^2)^(1/2))) - (2*(a^6*(b^10)^(1/2) + a^2*b^4*(b^10)^(1/2) + 2*a^4*
b^2*(b^10)^(1/2)))/(a^2*b^4*(-a^2*(a^2 + b^2)^5)^(1/2)*(a*b^4 + a^5 + 2*a^3
*b^2)*(- a^12 - a^2*b^10 - 5*a^4*b^8 - 10*a^6*b^6 - 10*a^8*b^4 - 5*a^10*b^2
)^(1/2)))*((a^6*(- a^12 - a^2*b^10 - 5*a^4*b^8 - 10*a^6*b^6 - 10*a^8*b^4 -
5*a^10*b^2)^(1/2))/2 + (a^2*b^4*(- a^12 - a^2*b^10 - 5*a^4*b^8 - 10*a^6*b^6
 - 10*a^8*b^4 - 5*a^10*b^2)^(1/2))/2 + a^4*b^2*(- a^12 - a^2*b^10 - 5*a^4*b
^8 - 10*a^6*b^6 - 10*a^8*b^4 - 5*a^10*b^2)^(1/2)))*(b^10)^(1/2))/(- a^12 -
a^2*b^10 - 5*a^4*b^8 - 10*a^6*b^6 - 10*a^8*b^4 - 5*a^10*b^2)^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\tanh^4(x)}{a+b\operatorname{csch}(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)**4/(a+b*csch(x)),x)
[Out] Integral(tanh(x)**4/(a + b*csch(x)), x)
```

3.115 $\int \frac{\tanh^3(x)}{a+b\textbf{csch}(x)}\,dx$

Optimal. Leaf size=113

$$-\frac{a\left(a^2+2b^2\right)\log(\tanh(x))}{\left(a^2+b^2\right)^2}-\frac{b\tan^{-1}(\sinh(x))}{2\left(a^2+b^2\right)}-\frac{\tanh^2(x)(a-b\text{csch}(x))}{2\left(a^2+b^2\right)}+\frac{b^4\log(a+b\text{csch}(x))}{a\left(a^2+b^2\right)^2}-\frac{b^3\tan^{-1}(\sinh(x))}{\left(a^2+b^2\right)^2}$$

```
[Out] -b^3*arctan(sinh(x))/(a^2+b^2)^2-1/2*b*arctan(sinh(x))/(a^2+b^2)+b^4*ln(a+b
*csch(x))/a/(a^2+b^2)^2+ln(sinh(x))/a-a*(a^2+2*b^2)*ln(tanh(x))/(a^2+b^2)^2
-1/2*(a-b*csch(x))*tanh(x)^2/(a^2+b^2)
```

Rubi [A] time = 0.16, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {3885, 894, 639, 203, 635, 260}

$$-\frac{a\left(a^2+2b^2\right)\log(\tanh(x))}{\left(a^2+b^2\right)^2}-\frac{b^3\tan^{-1}(\sinh(x))}{\left(a^2+b^2\right)^2}-\frac{b\tan^{-1}(\sinh(x))}{2\left(a^2+b^2\right)}+\frac{b^4\log(a+b\text{csch}(x))}{a\left(a^2+b^2\right)^2}-\frac{\tanh^2(x)(a-b\text{csch}(x))}{2\left(a^2+b^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[x]^3/(a + b*Csch[x]),x]
```

```
[Out] -((b^3*ArcTan[Sinh[x]])/(a^2 + b^2)^2) - (b*ArcTan[Sinh[x]])/(2*(a^2 + b^2)
) + (b^4*Log[a + b*Csch[x]])/(a*(a^2 + b^2)^2) + Log[Sinh[x]]/a - (a*(a^2 +
 2*b^2)*Log[Tanh[x]])/(a^2 + b^2)^2 - ((a - b*Csch[x])*Tanh[x]^2)/(2*(a^2 +
 b^2))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 639

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((a*e
 - c*d*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[(d*(2*p + 3))/(2*
a*(p + 1)), Int[(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e}, x] && Lt
Q[p, -1] && NeQ[p, -3/2]
```

Rule 894

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
```

```
[m, 0] && ILtQ[n, 0]))
```

Rule 3885

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> -Dist[(-1)^((m - 1)/2)/(d*b^(m - 1)), Subst[Int[((b^2 - x^
2)^((m - 1)/2)*(a + x)^n)/x, x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\tanh^3(x)}{a+b\text{csch}(x)}\,dx = -\left(b^4 \text{Subst}\left(\int \frac{1}{x(a+x)\left(-b^2-x^2\right)^2}\,dx, x, b\text{csch}(x)\right)\right)$$

$$= -\left(b^4 \text{Subst}\left(\int \left(\frac{1}{ab^4x} - \frac{1}{a\left(a^2+b^2\right)^2(a+x)} + \frac{-b^2-ax}{b^2\left(a^2+b^2\right)\left(b^2+x^2\right)^2} + \frac{-b^4-a\left(a^2+\right.}{b^4\left(a^2+b^2\right)^2\left(\right.}\right.\right.\right.$$

$$= \frac{b^4\log(a+b\text{csch}(x))}{a\left(a^2+b^2\right)^2} + \frac{\log(\sinh(x))}{a} - \frac{\text{Subst}\left(\int \frac{-b^4-a\left(a^2+2b^2\right)x}{b^2+x^2}\,dx, x, b\text{csch}(x)\right)}{\left(a^2+b^2\right)^2} - \frac{b^2\,\text{Sul}}{}$$

$$= \frac{b^4\log(a+b\text{csch}(x))}{a\left(a^2+b^2\right)^2} + \frac{\log(\sinh(x))}{a} - \frac{(a-b\text{csch}(x))\tanh^2(x)}{2\left(a^2+b^2\right)} + \frac{b^4\,\text{Subst}\left(\int \frac{1}{b^2+x^2}\,dx,\right.}{\left(a^2+b^2\right)^{}}$$

$$= -\frac{b^3\tan^{-1}(\sinh(x))}{\left(a^2+b^2\right)^2} - \frac{b\tan^{-1}(\sinh(x))}{2\left(a^2+b^2\right)} + \frac{b^4\log(a+b\text{csch}(x))}{a\left(a^2+b^2\right)^2} + \frac{\log(\sinh(x))}{a} - \frac{a\left(a^2\right.}{}$$

Mathematica [C] time = 0.20, size = 191, normalized size = 1.69

$$\frac{a^4\log(-\sinh(x)+i) + a^4\log(\sinh(x)+i) + ia^3b\log(-\sinh(x)+i) - ia^3b\log(\sinh(x)+i) + a^2\left(a^2+b^2\right)\text{sec}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]^3/(a + b*Csch[x]),x]
```

```
[Out] (a*b*(a^2 + b^2)*ArcTan[Sinh[x]] + a^4*Log[I - Sinh[x]] + I*a^3*b*Log[I - S
inh[x]] + 2*a^2*b^2*Log[I - Sinh[x]] + (2*I)*a*b^3*Log[I - Sinh[x]] + a^4*L
og[I + Sinh[x]] - I*a^3*b*Log[I + Sinh[x]] + 2*a^2*b^2*Log[I + Sinh[x]] - (
2*I)*a*b^3*Log[I + Sinh[x]] + 2*b^4*Log[b + a*Sinh[x]] + a^2*(a^2 + b^2)*Se
ch[x]^2 + a*b*(a^2 + b^2)*Sech[x]*Tanh[x])/(2*a*(a^2 + b^2)^2)
```

fricas [B] time = 0.87, size = 965, normalized size = 8.54

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^3/(a+b*csch(x)),x, algorithm="fricas")
```

```
[Out] -((a^4 + 2*a^2*b^2 + b^4)*x*cosh(x)^4 + (a^4 + 2*a^2*b^2 + b^4)*x*sinh(x)^4
 - (a^3*b + a*b^3)*cosh(x)^3 - (a^3*b + a*b^3 - 4*(a^4 + 2*a^2*b^2 + b^4)*x
*cosh(x))*sinh(x)^3 - 2*(a^4 + a^2*b^2 - (a^4 + 2*a^2*b^2 + b^4)*x)*cosh(x)
^2 - (2*a^4 + 2*a^2*b^2 - 6*(a^4 + 2*a^2*b^2 + b^4)*x*cosh(x)^2 - 2*(a^4 +
2*a^2*b^2 + b^4)*x + 3*(a^3*b + a*b^3)*cosh(x))*sinh(x)^2 + (a^4 + 2*a^2*b^
```

```
2 + b^4)*x + ((a^3*b + 3*a*b^3)*cosh(x)^4 + 4*(a^3*b + 3*a*b^3)*cosh(x)*sin
h(x)^3 + (a^3*b + 3*a*b^3)*sinh(x)^4 + a^3*b + 3*a*b^3 + 2*(a^3*b + 3*a*b^3
)*cosh(x)^2 + 2*(a^3*b + 3*a*b^3 + 3*(a^3*b + 3*a*b^3)*cosh(x)^2)*sinh(x)^2
 + 4*((a^3*b + 3*a*b^3)*cosh(x)^3 + (a^3*b + 3*a*b^3)*cosh(x))*sinh(x))*arc
tan(cosh(x) + sinh(x)) + (a^3*b + a*b^3)*cosh(x) - (b^4*cosh(x)^4 + 4*b^4*c
osh(x)*sinh(x)^3 + b^4*sinh(x)^4 + 2*b^4*cosh(x)^2 + b^4 + 2*(3*b^4*cosh(x)
^2 + b^4)*sinh(x)^2 + 4*(b^4*cosh(x)^3 + b^4*cosh(x))*sinh(x))*log(2*(a*sin
h(x) + b)/(cosh(x) - sinh(x))) - ((a^4 + 2*a^2*b^2)*cosh(x)^4 + 4*(a^4 + 2*
a^2*b^2)*cosh(x)*sinh(x)^3 + (a^4 + 2*a^2*b^2)*sinh(x)^4 + a^4 + 2*a^2*b^2
+ 2*(a^4 + 2*a^2*b^2)*cosh(x)^2 + 2*(a^4 + 2*a^2*b^2 + 3*(a^4 + 2*a^2*b^2)*
cosh(x)^2)*sinh(x)^2 + 4*((a^4 + 2*a^2*b^2)*cosh(x)^3 + (a^4 + 2*a^2*b^2)*c
osh(x))*sinh(x))*log(2*cosh(x)/(cosh(x) - sinh(x))) + (4*(a^4 + 2*a^2*b^2 +
 b^4)*x*cosh(x)^3 + a^3*b + a*b^3 - 3*(a^3*b + a*b^3)*cosh(x)^2 - 4*(a^4 +
a^2*b^2 - (a^4 + 2*a^2*b^2 + b^4)*x)*cosh(x))*sinh(x))/(a^5 + 2*a^3*b^2 + a
*b^4 + (a^5 + 2*a^3*b^2 + a*b^4)*cosh(x)^4 + 4*(a^5 + 2*a^3*b^2 + a*b^4)*co
sh(x)*sinh(x)^3 + (a^5 + 2*a^3*b^2 + a*b^4)*sinh(x)^4 + 2*(a^5 + 2*a^3*b^2
+ a*b^4)*cosh(x)^2 + 2*(a^5 + 2*a^3*b^2 + a*b^4 + 3*(a^5 + 2*a^3*b^2 + a*b^
4)*cosh(x)^2)*sinh(x)^2 + 4*((a^5 + 2*a^3*b^2 + a*b^4)*cosh(x)^3 + (a^5 + 2
*a^3*b^2 + a*b^4)*cosh(x))*sinh(x))
```

giac [B] time = 0.13, size = 234, normalized size = 2.07

$$\frac{b^4\log\left(\left|-a\left(e^{(-x)}-e^x\right)+2\,b\right|\right)}{a^5+2\,a^3b^2+ab^4}-\frac{\left(\pi+2\,\arctan\left(\frac{1}{2}\left(e^{(2\,x)}-1\right)e^{(-x)}\right)\right)\left(a^2b+3\,b^3\right)}{4\left(a^4+2\,a^2b^2+b^4\right)}+\frac{\left(a^3+2\,ab^2\right)\log\left(\left(e^{(-x)}-e^x\right)^2+4\right.}{2\left(a^4+2\,a^2b^2+b^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^3/(a+b*csch(x)),x, algorithm="giac")
```

```
[Out] b^4*log(abs(-a*(e^(-x) - e^x) + 2*b))/(a^5 + 2*a^3*b^2 + a*b^4) - 1/4*(pi +
 2*arctan(1/2*(e^(2*x) - 1)*e^(-x)))*(a^2*b + 3*b^3)/(a^4 + 2*a^2*b^2 + b^4
) + 1/2*(a^3 + 2*a*b^2)*log((e^(-x) - e^x)^2 + 4)/(a^4 + 2*a^2*b^2 + b^4) -
 1/2*(a^3*(e^(-x) - e^x)^2 + 2*a*b^2*(e^(-x) - e^x)^2 + 2*a^2*b*(e^(-x) - e
^x) + 2*b^3*(e^(-x) - e^x) + 4*a*b^2)/((a^4 + 2*a^2*b^2 + b^4)*((e^(-x) - e
^x)^2 + 4))
```

maple [B] time = 0.19, size = 324, normalized size = 2.87

$$-\frac{\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{a}-\frac{\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)}{a}+\frac{b^4\ln\left(\left(\tanh^2\left(\frac{x}{2}\right)\right)b-2a\tanh\left(\frac{x}{2}\right)-b\right)}{a\left(a^2+b^2\right)^2}-\frac{\left(\tanh^3\left(\frac{x}{2}\right)\right)a^2b}{\left(a^2+b^2\right)^2\left(\tanh^2\left(\frac{x}{2}\right)+1\right)^2}-\frac{}{\left(a^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^3/(a+b*csch(x)),x)
```

```
[Out] -1/a*ln(tanh(1/2*x)-1)-1/a*ln(tanh(1/2*x)+1)+b^4/a/(a^2+b^2)^2*ln(tanh(1/2*
x)^2*b-2*a*tanh(1/2*x)-b)-1/(a^2+b^2)^2/(tanh(1/2*x)^2+1)^2*tanh(1/2*x)^3*a
^2*b-1/(a^2+b^2)^2/(tanh(1/2*x)^2+1)^2*tanh(1/2*x)^3*b^3-2/(a^2+b^2)^2/(tan
h(1/2*x)^2+1)^2*tanh(1/2*x)^2*a^3-2/(a^2+b^2)^2/(tanh(1/2*x)^2+1)^2*tanh(1/
2*x)^2*a*b^2+1/(a^2+b^2)^2/(tanh(1/2*x)^2+1)^2*tanh(1/2*x)*a^2*b+1/(a^2+b^2
)^2/(tanh(1/2*x)^2+1)^2*tanh(1/2*x)*b^3+1/(a^2+b^2)^2*ln(tanh(1/2*x)^2+1)*a
^3+2/(a^2+b^2)^2*ln(tanh(1/2*x)^2+1)*a*b^2-3/(a^2+b^2)^2*arctan(tanh(1/2*x)
)*b^3-1/(a^2+b^2)^2*arctan(tanh(1/2*x))*a^2*b
```

maxima [A] time = 0.42, size = 172, normalized size = 1.52

$$\frac{b^4\log\left(-2\,be^{(-x)}+ae^{(-2\,x)}-a\right)}{a^5+2\,a^3b^2+ab^4}+\frac{\left(a^2b+3\,b^3\right)\arctan\left(e^{(-x)}\right)}{a^4+2\,a^2b^2+b^4}+\frac{\left(a^3+2\,ab^2\right)\log\left(e^{(-2\,x)}+1\right)}{a^4+2\,a^2b^2+b^4}+\frac{be^{(-x)}+2\,ae^{(-}}{a^2+b^2+2\left(a^2+b^2\right)e^{(}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^3/(a+b*csch(x)),x, algorithm="maxima")
```

```
[Out] b^4*log(-2*b*e^(-x) + a*e^(-2*x) - a)/(a^5 + 2*a^3*b^2 + a*b^4) + (a^2*b +
3*b^3)*arctan(e^(-x))/(a^4 + 2*a^2*b^2 + b^4) + (a^3 + 2*a*b^2)*log(e^(-2*x
) + 1)/(a^4 + 2*a^2*b^2 + b^4) + (b*e^(-x) + 2*a*e^(-2*x) - b*e^(-3*x))/(a^
2 + b^2 + 2*(a^2 + b^2)*e^(-2*x) + (a^2 + b^2)*e^(-4*x)) + x/a
```

mupad [B] time = 4.01, size = 335, normalized size = 2.96

$$\frac{\frac{e^x\left(a^2 b+b^3\right)}{\left(a^2+b^2\right)^2}+\frac{2\left(a^4+a^2 b^2\right)}{a\left(a^2+b^2\right)^2}}{e^{2x}+1}-\frac{\frac{2a}{a^2+b^2}+\frac{2b\,e^x}{a^2+b^2}}{2\,e^{2x}+e^{4x}+1}-\frac{x}{a}+\frac{b^4\,\ln\left(4\,a^9\,e^{2x}-4\,a\,b^8-4\,a^9+7\,a^3\,b^6-14\,a^5\,b^4-17\,a^7\,b^2+8\,b^9\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^3/(a + b/sinh(x)),x)
```

```
[Out] ((exp(x)*(a^2*b + b^3))/(a^2 + b^2)^2 + (2*(a^4 + a^2*b^2))/(a*(a^2 + b^2)^
2))/(exp(2*x) + 1) - ((2*a)/(a^2 + b^2) + (2*b*exp(x))/(a^2 + b^2))/(2*exp(
2*x) + exp(4*x) + 1) - x/a + (b^4*log(4*a^9*exp(2*x) - 4*a*b^8 - 4*a^9 + 7*
a^3*b^6 - 14*a^5*b^4 - 17*a^7*b^2 + 8*b^9*exp(x) - 7*a^3*b^6*exp(2*x) + 14*
a^5*b^4*exp(2*x) + 17*a^7*b^2*exp(2*x) + 8*a^8*b*exp(x) + 4*a*b^8*exp(2*x)
- 14*a^2*b^7*exp(x) + 28*a^4*b^5*exp(x) + 34*a^6*b^3*exp(x)))/(a*b^4 + a^5
+ 2*a^3*b^2) + (log(exp(x)*1i + 1)*(a*2i + 3*b))/(2*(2*a*b + a^2*1i - b^2*1
i)) + (log(exp(x) + 1i)*(2*a + b*3i))/(2*(a*b*2i + a^2 - b^2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh^3(x)}{a+b\operatorname{csch}(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)**3/(a+b*csch(x)),x)
```

```
[Out] Integral(tanh(x)**3/(a + b*csch(x)), x)
```

3.116 $\int \frac{\tanh^2(x)}{a+b\mathbf{csch}(x)}\,dx$

Optimal. Leaf size=100

$$\frac{b^2x}{a\left(a^2+b^2\right)}+\frac{ax}{a^2+b^2}-\frac{a\tanh(x)}{a^2+b^2}+\frac{b\mathrm{sech}(x)}{a^2+b^2}+\frac{2b^3\tanh^{-1}\left(\frac{a-b\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{a\left(a^2+b^2\right)^{3/2}}$$

```
[Out] a*x/(a^2+b^2)+b^2*x/a/(a^2+b^2)+2*b^3*arctanh((a-b*tanh(1/2*x))/(a^2+b^2)^(
1/2))/a/(a^2+b^2)^(3/2)+b*sech(x)/(a^2+b^2)-a*tanh(x)/(a^2+b^2)
```

Rubi [A] time = 0.22, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.692, Rules used = {3898, 2902, 2606, 8, 3473, 2735, 2660, 618, 204}

$$\frac{b^2x}{a\left(a^2+b^2\right)}+\frac{ax}{a^2+b^2}+\frac{2b^3\tanh^{-1}\left(\frac{a-b\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{a\left(a^2+b^2\right)^{3/2}}-\frac{a\tanh(x)}{a^2+b^2}+\frac{b\mathrm{sech}(x)}{a^2+b^2}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[x]^2/(a + b*Csch[x]),x]
```

```
[Out] (a*x)/(a^2 + b^2) + (b^2*x)/(a*(a^2 + b^2)) + (2*b^3*ArcTanh[(a - b*Tanh[x/
2])/Sqrt[a^2 + b^2]])/(a*(a^2 + b^2)^(3/2)) + (b*Sech[x])/(a^2 + b^2) - (a*
Tanh[x])/(a^2 + b^2)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 && !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2902

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[(a*d^2)/(a^2
 - b^2), Int[(g*Cos[e + f*x])^p*(d*Sin[e + f*x])^(n - 2), x], x] + (-Dist[(
b*d)/(a^2 - b^2), Int[(g*Cos[e + f*x])^p*(d*Sin[e + f*x])^(n - 1), x], x] -
 Dist[(a^2*d^2)/(g^2*(a^2 - b^2)), Int[((g*Cos[e + f*x])^(p + 2)*(d*Sin[e +
 f*x])^(n - 2))/(a + b*Sin[e + f*x]), x], x]) /; FreeQ[{a, b, d, e, f, g},
x] && NeQ[a^2 - b^2, 0] && IntegersQ[2*n, 2*p] && LtQ[p, -1] && GtQ[n, 1]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3898

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Int[(Cos[c + d*x]^m*(b + a*Sin[c + d*x])^n)/Sin[c + d*x]^(
m + n), x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[n] &&
 IntegerQ[m] && (IntegerQ[m/2] || LeQ[m, 1])
```

Rubi steps

$$
\begin{aligned}
\int \frac{\tanh^2(x)}{a+b\text{csch}(x)}\,dx &= i\int \frac{\sinh(x)\tanh^2(x)}{ib+ia\sinh(x)}\,dx \\
&= \frac{a\int \tanh^2(x)\,dx}{a^2+b^2} - \frac{b\int \text{sech}(x)\tanh(x)\,dx}{a^2+b^2} + \frac{\left(ib^2\right)\int \frac{\sinh(x)}{ib+ia\sinh(x)}\,dx}{a^2+b^2} \\
&= \frac{b^2x}{a\left(a^2+b^2\right)} - \frac{a\tanh(x)}{a^2+b^2} + \frac{a\int 1\,dx}{a^2+b^2} + \frac{b\,\text{Subst}(\int 1\,dx, x, \text{sech}(x))}{a^2+b^2} - \frac{\left(ib^3\right)\int \frac{1}{ib+ia\sinh(x)}\,dx}{a\left(a^2+b^2\right)} \\
&= \frac{ax}{a^2+b^2} + \frac{b^2x}{a\left(a^2+b^2\right)} + \frac{b\text{sech}(x)}{a^2+b^2} - \frac{a\tanh(x)}{a^2+b^2} - \frac{\left(2ib^3\right)\text{Subst}\left(\int \frac{1}{ib+2iax-ibx^2}\,dx, x, \tanh\right.}{a\left(a^2+b^2\right)} \\
&= \frac{ax}{a^2+b^2} + \frac{b^2x}{a\left(a^2+b^2\right)} + \frac{b\text{sech}(x)}{a^2+b^2} - \frac{a\tanh(x)}{a^2+b^2} + \frac{\left(4ib^3\right)\text{Subst}\left(\int \frac{1}{-4\left(a^2+b^2\right)-x^2}\,dx, x, 2ia-\right.}{a\left(a^2+b^2\right)} \\
&= \frac{ax}{a^2+b^2} + \frac{b^2x}{a\left(a^2+b^2\right)} + \frac{2b^3\tanh^{-1}\left(\frac{a-b\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{a\left(a^2+b^2\right)^{3/2}} + \frac{b\text{sech}(x)}{a^2+b^2} - \frac{a\tanh(x)}{a^2+b^2}
\end{aligned}
$$

Mathematica [A] time = 0.33, size = 82, normalized size = 0.82

$$
-\frac{a\tanh(x)}{a^2+b^2} + \frac{b\text{sech}(x)}{a^2+b^2} + \frac{\frac{2b^3\tan^{-1}\left(\frac{a-b\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2-b^2}}\right)}{\left(-a^2-b^2\right)^{3/2}} + x}{a}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]^2/(a + b*Csch[x]),x]
```

```
[Out] (x + (2*b^3*ArcTan[(a - b*Tanh[x/2])/Sqrt[-a^2 - b^2]])/(-a^2 - b^2)^(3/2))
/a + (b*Sech[x])/(a^2 + b^2) - (a*Tanh[x])/(a^2 + b^2)
```

fricas [B] time = 0.69, size = 349, normalized size = 3.49

$$\frac{2\,a^4 + 2\,a^2b^2 + \left(a^4 + 2\,a^2b^2 + b^4\right)x\cosh(x)^2 + \left(a^4 + 2\,a^2b^2 + b^4\right)x\sinh(x)^2 + \left(b^3\cosh(x)^2 + 2\,b^3\cosh(x)\sinh(x\right.}{a^5 + 2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^2/(a+b*csch(x)),x, algorithm="fricas")
```

```
[Out] (2*a^4 + 2*a^2*b^2 + (a^4 + 2*a^2*b^2 + b^4)*x*cosh(x)^2 + (a^4 + 2*a^2*b^2
 + b^4)*x*sinh(x)^2 + (b^3*cosh(x)^2 + 2*b^3*cosh(x)*sinh(x) + b^3*sinh(x)^
2 + b^3)*sqrt(a^2 + b^2)*log((a^2*cosh(x)^2 + a^2*sinh(x)^2 + 2*a*b*cosh(x)
 + a^2 + 2*b^2 + 2*(a^2*cosh(x) + a*b)*sinh(x) + 2*sqrt(a^2 + b^2)*(a*cosh(
x) + a*sinh(x) + b))/(a*cosh(x)^2 + a*sinh(x)^2 + 2*b*cosh(x) + 2*(a*cosh(x
) + b)*sinh(x) - a)) + (a^4 + 2*a^2*b^2 + b^4)*x + 2*(a^3*b + a*b^3)*cosh(x
) + 2*(a^3*b + a*b^3 + (a^4 + 2*a^2*b^2 + b^4)*x*cosh(x))*sinh(x))/(a^5 + 2
*a^3*b^2 + a*b^4 + (a^5 + 2*a^3*b^2 + a*b^4)*cosh(x)^2 + 2*(a^5 + 2*a^3*b^2
 + a*b^4)*cosh(x)*sinh(x) + (a^5 + 2*a^3*b^2 + a*b^4)*sinh(x)^2)
```

giac [A] time = 0.16, size = 102, normalized size = 1.02

$$-\frac{b^3\log\left(\frac{\left|2\,ae^x+2\,b-2\,\sqrt{a^2+b^2}\right|}{\left|2\,ae^x+2\,b+2\,\sqrt{a^2+b^2}\right|}\right)}{\left(a^3+ab^2\right)\sqrt{a^2+b^2}} + \frac{x}{a} + \frac{2\,(be^x + a)}{\left(a^2+b^2\right)\left(e^{(2\,x)}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^2/(a+b*csch(x)),x, algorithm="giac")
```

```
[Out] -b^3*log(abs(2*a*e^x + 2*b - 2*sqrt(a^2 + b^2))/abs(2*a*e^x + 2*b + 2*sqrt(
a^2 + b^2)))/((a^3 + a*b^2)*sqrt(a^2 + b^2)) + x/a + 2*(b*e^x + a)/((a^2 +
b^2)*(e^(2*x) + 1))
```

maple [A] time = 0.18, size = 95, normalized size = 0.95

$$-\frac{\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{a}+\frac{\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)}{a}-\frac{2b^3\,\mathrm{arctanh}\left(\frac{2\tanh\left(\frac{x}{2}\right)b-2a}{2\sqrt{a^2+b^2}}\right)}{a\left(a^2+b^2\right)^{\frac{3}{2}}}+\frac{2b-2a\tanh\left(\frac{x}{2}\right)}{\left(a^2+b^2\right)\left(\tanh^2\left(\frac{x}{2}\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^2/(a+b*csch(x)),x)
```

```
[Out] -1/a*ln(tanh(1/2*x)-1)+1/a*ln(tanh(1/2*x)+1)-2/a*b^3/(a^2+b^2)^(3/2)*arctan
h(1/2*(2*tanh(1/2*x)*b-2*a)/(a^2+b^2)^(1/2))+2/(a^2+b^2)*(b-a*tanh(1/2*x))/
(tanh(1/2*x)^2+1)
```

maxima [A] time = 0.42, size = 108, normalized size = 1.08

$$-\frac{b^3\log\left(\frac{ae^{(-x)}-b-\sqrt{a^2+b^2}}{ae^{(-x)}-b+\sqrt{a^2+b^2}}\right)}{\left(a^3+ab^2\right)\sqrt{a^2+b^2}} + \frac{2\left(be^{(-x)}-a\right)}{a^2+b^2+\left(a^2+b^2\right)e^{(-2\,x)}} + \frac{x}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^2/(a+b*csch(x)),x, algorithm="maxima")
```

```
[Out] -b^3*log((a*e^(-x) - b - sqrt(a^2 + b^2))/(a*e^(-x) - b + sqrt(a^2 + b^2)))
/((a^3 + a*b^2)*sqrt(a^2 + b^2)) + 2*(b*e^(-x) - a)/(a^2 + b^2 + (a^2 + b^2
)*e^(-2*x)) + x/a
```

mupad [B] time = 2.47, size = 376, normalized size = 3.76

$$\frac{x}{a}+\frac{\frac{2a}{a^2+b^2}+\frac{2b\,e^x}{a^2+b^2}}{e^{2x}+1}+\frac{2\,\mathrm{atan}\left(\left(\frac{a^4\sqrt{-a^8-3a^6b^2-3a^4b^4-a^2b^6}}{2}+\frac{a^2b^2\sqrt{-a^8-3a^6b^2-3a^4b^4-a^2b^6}}{2}\right)\left(e^x\left(\frac{2b^3}{a^3\left(a^3+a\,b^2\right)\sqrt{b^6}\left(a^2+b^2\right)}+\right.\right.\right.}{\sqrt{-a^8-3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^2/(a + b/sinh(x)),x)
```

```
[Out] x/a + ((2*a)/(a^2 + b^2) + (2*b*exp(x))/(a^2 + b^2))/(exp(2*x) + 1) + (2*at
an(((a^4*(- a^8 - a^2*b^6 - 3*a^4*b^4 - 3*a^6*b^2)^(1/2))/2 + (a^2*b^2*(- a
^8 - a^2*b^6 - 3*a^4*b^4 - 3*a^6*b^2)^(1/2))/2)*(exp(x)*((2*b^3)/(a^3*(a*b^
2 + a^3)*(b^6)^(1/2)*(a^2 + b^2)) + (2*(a*b^3*(b^6)^(1/2) + a^3*b*(b^6)^(1/
2)))/(a^2*b^2*(-a^2*(a^2 + b^2)^3)^(1/2)*(a*b^2 + a^3)*(- a^8 - a^2*b^6 - 3
*a^4*b^4 - 3*a^6*b^2)^(1/2))) - (2*(a^4*(b^6)^(1/2) + a^2*b^2*(b^6)^(1/2)))
/(a^2*b^2*(-a^2*(a^2 + b^2)^3)^(1/2)*(a*b^2 + a^3)*(- a^8 - a^2*b^6 - 3*a^4
*b^4 - 3*a^6*b^2)^(1/2))))*(b^6)^(1/2))/(- a^8 - a^2*b^6 - 3*a^4*b^4 - 3*a^
6*b^2)^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh^2(x)}{a+b\,\mathrm{csch}\,(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)**2/(a+b*csch(x)),x)
```

```
[Out] Integral(tanh(x)**2/(a + b*csch(x)), x)
```

3.117 $\int \frac{\tanh(x)}{a+b\text{csch}(x)}\,dx$

Optimal. Leaf size=61

$$-\frac{a\log(\tanh(x))}{a^2+b^2}-\frac{b\tan^{-1}(\sinh(x))}{a^2+b^2}+\frac{b^2\log(a+b\text{csch}(x))}{a\left(a^2+b^2\right)}+\frac{\log(\sinh(x))}{a}$$

```
[Out] -b*arctan(sinh(x))/(a^2+b^2)+b^2*ln(a+b*csch(x))/a/(a^2+b^2)+ln(sinh(x))/a-
a*ln(tanh(x))/(a^2+b^2)
```

Rubi [A] time = 0.10, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.454, Rules used = {3885, 894, 635, 203, 260}

$$-\frac{a\log(\tanh(x))}{a^2+b^2}-\frac{b\tan^{-1}(\sinh(x))}{a^2+b^2}+\frac{b^2\log(a+b\text{csch}(x))}{a\left(a^2+b^2\right)}+\frac{\log(\sinh(x))}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[x]/(a + b*Csch[x]),x]
```

```
[Out] -((b*ArcTan[Sinh[x]])/(a^2 + b^2)) + (b^2*Log[a + b*Csch[x]])/(a*(a^2 + b^2
)) + Log[Sinh[x]]/a - (a*Log[Tanh[x]])/(a^2 + b^2)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 894

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 3885

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> -Dist[(-1)^((m - 1)/2)/(d*b^(m - 1)), Subst[Int[((b^2 - x^
2)^((m - 1)/2)*(a + x)^n)/x, x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\tanh(x)}{a+b\text{csch}(x)}\,dx = b^2\,\text{Subst}\left(\int \frac{1}{x(a+x)\left(-b^2-x^2\right)}\,dx, x, b\text{csch}(x)\right)$$
$$= b^2\,\text{Subst}\left(\int \left(-\frac{1}{ab^2x}+\frac{1}{a\left(a^2+b^2\right)(a+x)}+\frac{b^2+ax}{b^2\left(a^2+b^2\right)\left(b^2+x^2\right)}\right)dx, x, b\text{csch}(x)\right)$$
$$= \frac{b^2\log(a+b\text{csch}(x))}{a\left(a^2+b^2\right)}+\frac{\log(\sinh(x))}{a}+\frac{\text{Subst}\left(\int \frac{b^2+ax}{b^2+x^2}\,dx, x, b\text{csch}(x)\right)}{a^2+b^2}$$
$$= \frac{b^2\log(a+b\text{csch}(x))}{a\left(a^2+b^2\right)}+\frac{\log(\sinh(x))}{a}+\frac{a\,\text{Subst}\left(\int \frac{x}{b^2+x^2}\,dx, x, b\text{csch}(x)\right)}{a^2+b^2}+\frac{b^2\,\text{Subst}\left(\int\right.}{}$$
$$= -\frac{b\tan^{-1}(\sinh(x))}{a^2+b^2}+\frac{b^2\log(a+b\text{csch}(x))}{a\left(a^2+b^2\right)}+\frac{\log(\sinh(x))}{a}-\frac{a\log(\tanh(x))}{a^2+b^2}$$

Mathematica [C] time = 0.06, size = 63, normalized size = 1.03

$$\frac{2b^2\log(a\sinh(x)+b)+a(a+ib)\log(-\sinh(x)+i)+a(a-ib)\log(\sinh(x)+i)}{2a\left(a^2+b^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]/(a + b*Csch[x]),x]
```

```
[Out] (a*(a + I*b)*Log[I - Sinh[x]] + a*(a - I*b)*Log[I + Sinh[x]] + 2*b^2*Log[b
+ a*Sinh[x]])/(2*a*(a^2 + b^2))
```

fricas [A] time = 1.26, size = 75, normalized size = 1.23

$$-\frac{2\,ab\arctan\left(\cosh(x)+\sinh(x)\right)-b^2\log\left(\frac{2\,(a\sinh(x)+b)}{\cosh(x)-\sinh(x)}\right)-a^2\log\left(\frac{2\cosh(x)}{\cosh(x)-\sinh(x)}\right)+\left(a^2+b^2\right)x}{a^3+ab^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/(a+b*csch(x)),x, algorithm="fricas")
```

```
[Out] -(2*a*b*arctan(cosh(x) + sinh(x)) - b^2*log(2*(a*sinh(x) + b)/(cosh(x) - si
nh(x))) - a^2*log(2*cosh(x)/(cosh(x) - sinh(x))) + (a^2 + b^2)*x)/(a^3 + a*
b^2)
```

giac [A] time = 0.13, size = 89, normalized size = 1.46

$$\frac{b^2\log\left(\left|-a\left(e^{(-x)}-e^x\right)+2\,b\right|\right)}{a^3+ab^2}-\frac{\left(\pi+2\arctan\left(\frac{1}{2}\left(e^{(2\,x)}-1\right)e^{(-x)}\right)\right)b}{2\left(a^2+b^2\right)}+\frac{a\log\left(\left(e^{(-x)}-e^x\right)^2+4\right)}{2\left(a^2+b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/(a+b*csch(x)),x, algorithm="giac")
```

```
[Out] b^2*log(abs(-a*(e^(-x) - e^x) + 2*b))/(a^3 + a*b^2) - 1/2*(pi + 2*arctan(1/
2*(e^(2*x) - 1)*e^(-x)))*b/(a^2 + b^2) + 1/2*a*log((e^(-x) - e^x)^2 + 4)/(a
^2 + b^2)
```

maple [A] time = 0.18, size = 108, normalized size = 1.77

$$-\frac{\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{a}-\frac{\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)}{a}+\frac{b^2\ln\left(\left(\tanh^2\left(\frac{x}{2}\right)\right)b-2a\tanh\left(\frac{x}{2}\right)-b\right)}{a\left(a^2+b^2\right)}+\frac{4a\ln\left(\tanh^2\left(\frac{x}{2}\right)+1\right)}{4a^2+4b^2}-\frac{8b\,\text{arc}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)/(a+b*csch(x)),x)
```

```
[Out] -1/a*ln(tanh(1/2*x)-1)-1/a*ln(tanh(1/2*x)+1)+b^2/a/(a^2+b^2)*ln(tanh(1/2*x)
^2*b-2*a*tanh(1/2*x)-b)+4/(4*a^2+4*b^2)*a*ln(tanh(1/2*x)^2+1)-8/(4*a^2+4*b^
2)*b*arctan(tanh(1/2*x))
```

maxima [A] time = 0.41, size = 74, normalized size = 1.21

$$\frac{b^2 \log\left(-2\, be^{(-x)} + ae^{(-2\,x)} - a\right)}{a^3 + ab^2} + \frac{2\, b \arctan\left(e^{(-x)}\right)}{a^2 + b^2} + \frac{a \log\left(e^{(-2\,x)} + 1\right)}{a^2 + b^2} + \frac{x}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/(a+b*csch(x)),x, algorithm="maxima")
```

```
[Out] b^2*log(-2*b*e^(-x) + a*e^(-2*x) - a)/(a^3 + a*b^2) + 2*b*arctan(e^(-x))/(a
^2 + b^2) + a*log(e^(-2*x) + 1)/(a^2 + b^2) + x/a
```

mupad [B] time = 2.53, size = 132, normalized size = 2.16

$$\frac{\ln\left(1 + e^x\, 1i\right)}{a - b\, 1i} - \frac{x}{a} + \frac{b^2\, \ln\left(a^5\, e^{2\,x} - a\, b^4 - a^5 + a^3\, b^2 + 2\, b^5\, e^x - a^3\, b^2\, e^{2\,x} + 2\, a^4\, b\, e^x + a\, b^4\, e^{2\,x} - 2\, a^2\, b^3\, e^x\right)}{a^3 + a\, b^2} + \frac{\ln\left(e^x \cdots\right.}{-b + \cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)/(a + b/sinh(x)),x)
```

```
[Out] log(exp(x)*1i + 1)/(a - b*1i) + (log(exp(x) + 1i)*1i)/(a*1i - b) - x/a + (b
^2*log(a^5*exp(2*x) - a*b^4 - a^5 + a^3*b^2 + 2*b^5*exp(x) - a^3*b^2*exp(2*
x) + 2*a^4*b*exp(x) + a*b^4*exp(2*x) - 2*a^2*b^3*exp(x)))/(a*b^2 + a^3)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh(x)}{a + b \operatorname{csch}(x)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/(a+b*csch(x)),x)
```

```
[Out] Integral(tanh(x)/(a + b*csch(x)), x)
```

3.118 $\int \frac{\coth(x)}{a+b\operatorname{csch}(x)}\,dx$

Optimal. Leaf size=19

$$\frac{\log(a+b\operatorname{csch}(x))}{a}+\frac{\log(\sinh(x))}{a}$$

```
[Out] ln(a+b*csch(x))/a+ln(sinh(x))/a
```

Rubi [A] time = 0.03, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {3885, 36, 29, 31}

$$\frac{\log(a+b\operatorname{csch}(x))}{a}+\frac{\log(\sinh(x))}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]/(a + b*Csch[x]),x]
```

```
[Out] Log[a + b*Csch[x]]/a + Log[Sinh[x]]/a
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 3885

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> -Dist[(-1)^((m - 1)/2)/(d*b^(m - 1)), Subst[Int[((b^2 - x^
2)^((m - 1)/2)*(a + x)^n)/x, x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\coth(x)}{a+b\operatorname{csch}(x)}\,dx &= -\text{Subst}\left(\int \frac{1}{x(a+x)}\,dx,x,b\operatorname{csch}(x)\right)\\
&= -\frac{\text{Subst}\left(\int \frac{1}{x}\,dx,x,b\operatorname{csch}(x)\right)}{a}+\frac{\text{Subst}\left(\int \frac{1}{a+x}\,dx,x,b\operatorname{csch}(x)\right)}{a}\\
&= \frac{\log(a+b\operatorname{csch}(x))}{a}+\frac{\log(\sinh(x))}{a}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 11, normalized size = 0.58

$$\frac{\log(a\sinh(x)+b)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]/(a + b*Csch[x]),x]
```

```
[Out] Log[b + a*Sinh[x]]/a
```

fricas [A] time = 0.60, size = 27, normalized size = 1.42

$$-\frac{x-\log\left(\frac{2\,(a\sinh(x)+b)}{\cosh(x)-\sinh(x)}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)/(a+b*csch(x)),x, algorithm="fricas")
```

```
[Out] -(x - log(2*(a*sinh(x) + b)/(cosh(x) - sinh(x))))/a
```

giac [A] time = 0.13, size = 22, normalized size = 1.16

$$\frac{\log\left(\left|-a\left(e^{(-x)}-e^{x}\right)+2\,b\right|\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)/(a+b*csch(x)),x, algorithm="giac")
```

```
[Out] log(abs(-a*(e^(-x) - e^x) + 2*b))/a
```

maple [A] time = 0.11, size = 21, normalized size = 1.11

$$\frac{\ln\left(a+b\,\mathrm{csch}(x)\right)}{a}-\frac{\ln\left(\mathrm{csch}(x)\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)/(a+b*csch(x)),x)
```

```
[Out] ln(a+b*csch(x))/a-1/a*ln(csch(x))
```

maxima [A] time = 0.32, size = 28, normalized size = 1.47

$$\frac{x}{a}+\frac{\log\left(-2\,be^{(-x)}+ae^{(-2\,x)}-a\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)/(a+b*csch(x)),x, algorithm="maxima")
```

```
[Out] x/a + log(-2*b*e^(-x) + a*e^(-2*x) - a)/a
```

mupad [B] time = 0.10, size = 25, normalized size = 1.32

$$-\frac{x-\ln\left(2\,b\,\mathrm{e}^{x}-a+a\,\mathrm{e}^{2\,x}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)/(a + b/sinh(x)),x)
```

```
[Out] -(x - log(2*b*exp(x) - a + a*exp(2*x)))/a
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\coth\left(x\right)}{a+b\,\mathrm{csch}\left(x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)/(a+b*csch(x)),x)
```

```
[Out] Integral(coth(x)/(a + b*csch(x)), x)
```

3.119 $\int \frac{\coth^2(x)}{a+b\,\mathrm{csch}(x)}\,dx$

Optimal. Leaf size=57

$$\frac{2\sqrt{a^2+b^2}\tanh^{-1}\left(\frac{a-b\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{ab}+\frac{x}{a}-\frac{\tanh^{-1}(\cosh(x))}{b}$$

```
[Out] x/a-arctanh(cosh(x))/b+2*arctanh((a-b*tanh(1/2*x))/(a^2+b^2)^(1/2))*(a^2+b^
2)^(1/2)/a/b
```

Rubi [A] time = 0.18, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.615, Rules used = {3894, 4051, 3770, 3919, 3831, 2660, 618, 206}

$$\frac{2\sqrt{a^2+b^2}\tanh^{-1}\left(\frac{a-b\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{ab}+\frac{x}{a}-\frac{\tanh^{-1}(\cosh(x))}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]^2/(a + b*Csch[x]),x]
```

```
[Out] x/a - ArcTanh[Cosh[x]]/b + (2*Sqrt[a^2 + b^2]*ArcTanh[(a - b*Tanh[x/2])/Sqr
t[a^2 + b^2]])/(a*b)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3831

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3894

```
Int[cot[(c_.) + (d_.)*(x_)]^2*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_),
x_Symbol] :> Int[(-1 + Csc[c + d*x]^2)*(a + b*Csc[c + d*x])^n, x] /; FreeQ[
```

```
{a, b, c, d, n}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3919

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[(c*x)/a, x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 4051

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))/(csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)), x_Symbol] :> Dist[C/b, Int[Csc[e + f*x], x], x] + Dist[1/b, Int[
(A*b - a*C*Csc[e + f*x])/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f,
 A, C}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\coth^2(x)}{a+b\text{csch}(x)}\,dx &= -\int \frac{-1-\text{csch}^2(x)}{a+b\text{csch}(x)}\,dx \\
&= \frac{i\int \frac{-ib+ia\text{csch}(x)}{a+b\text{csch}(x)}\,dx}{b}+\frac{\int \text{csch}(x)\,dx}{b} \\
&= \frac{x}{a}-\frac{\tanh^{-1}(\cosh(x))}{b}-\frac{\left(a^2+b^2\right)\int \frac{\text{csch}(x)}{a+b\text{csch}(x)}\,dx}{ab} \\
&= \frac{x}{a}-\frac{\tanh^{-1}(\cosh(x))}{b}-\left(\frac{1}{a}+\frac{a}{b^2}\right)\int \frac{1}{1+\frac{a\sinh(x)}{b}}\,dx \\
&= \frac{x}{a}-\frac{\tanh^{-1}(\cosh(x))}{b}-\left(2\left(\frac{1}{a}+\frac{a}{b^2}\right)\right)\text{Subst}\left(\int \frac{1}{1+\frac{2ax}{b}-x^2}\,dx,x,\tanh\left(\frac{x}{2}\right)\right) \\
&= \frac{x}{a}-\frac{\tanh^{-1}(\cosh(x))}{b}+\left(4\left(\frac{1}{a}+\frac{a}{b^2}\right)\right)\text{Subst}\left(\int \frac{1}{4\left(1+\frac{a^2}{b^2}\right)-x^2}\,dx,x,\frac{2a}{b}-2\tanh\left(\frac{x}{2}\right)\right) \\
&= \frac{x}{a}-\frac{\tanh^{-1}(\cosh(x))}{b}+\frac{2\sqrt{a^2+b^2}\,\tanh^{-1}\left(\frac{b\left(\frac{a}{b}-\tanh\left(\frac{x}{2}\right)\right)}{\sqrt{a^2+b^2}}\right)}{ab}
\end{aligned}$$

Mathematica [A] time = 0.08, size = 65, normalized size = 1.14

$$\frac{2\sqrt{-a^2-b^2}\,\tan^{-1}\left(\frac{a-b\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2-b^2}}\right)+a\log\left(\tanh\left(\frac{x}{2}\right)\right)+bx}{ab}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]^2/(a + b*Csch[x]),x]
```

```
[Out] (b*x + 2*Sqrt[-a^2 - b^2]*ArcTan[(a - b*Tanh[x/2])/Sqrt[-a^2 - b^2]] + a*Lo
g[Tanh[x/2]])/(a*b)
```

fricas [B] time = 0.63, size = 141, normalized size = 2.47

$$\frac{bx-a\log(\cosh(x)+\sinh(x)+1)+a\log(\cosh(x)+\sinh(x)-1)+\sqrt{a^2+b^2}\,\log\left(\frac{a^2\cosh(x)^2+a^2\sinh(x)^2+2ab\cos\ldots}{a\cosh(x)\ldots}\right.}{ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^2/(a+b*csch(x)),x, algorithm="fricas")
```

```
[Out] (b*x - a*log(cosh(x) + sinh(x) + 1) + a*log(cosh(x) + sinh(x) - 1) + sqrt(a
^2 + b^2)*log((a^2*cosh(x)^2 + a^2*sinh(x)^2 + 2*a*b*cosh(x) + a^2 + 2*b^2
+ 2*(a^2*cosh(x) + a*b)*sinh(x) + 2*sqrt(a^2 + b^2)*(a*cosh(x) + a*sinh(x)
+ b))/(a*cosh(x)^2 + a*sinh(x)^2 + 2*b*cosh(x) + 2*(a*cosh(x) + b)*sinh(x)
- a)))/(a*b)
```

giac [A] time = 0.16, size = 89, normalized size = 1.56

$$\frac{x}{a} - \frac{\log\left(e^{x}+1\right)}{b} + \frac{\log\left(|e^{x}-1|\right)}{b} - \frac{\sqrt{a^{2}+b^{2}}\log\left(\frac{\left|2\,ae^{x}+2\,b-2\,\sqrt{a^{2}+b^{2}}\right|}{\left|2\,ae^{x}+2\,b+2\,\sqrt{a^{2}+b^{2}}\right|}\right)}{ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^2/(a+b*csch(x)),x, algorithm="giac")
```

```
[Out] x/a - log(e^x + 1)/b + log(abs(e^x - 1))/b - sqrt(a^2 + b^2)*log(abs(2*a*e^
x + 2*b - 2*sqrt(a^2 + b^2))/abs(2*a*e^x + 2*b + 2*sqrt(a^2 + b^2)))/(a*b)
```

maple [B] time = 0.13, size = 110, normalized size = 1.93

$$-\frac{\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{a}+\frac{\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)}{a}-\frac{2a\,\mathrm{arctanh}\left(\frac{2\tanh\left(\frac{x}{2}\right)b-2a}{2\sqrt{a^{2}+b^{2}}}\right)}{b\sqrt{a^{2}+b^{2}}}-\frac{2b\,\mathrm{arctanh}\left(\frac{2\tanh\left(\frac{x}{2}\right)b-2a}{2\sqrt{a^{2}+b^{2}}}\right)}{a\sqrt{a^{2}+b^{2}}}+\frac{\ln\left(\tanh\left(\frac{x}{2}\right)\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^2/(a+b*csch(x)),x)
```

```
[Out] -1/a*ln(tanh(1/2*x)-1)+1/a*ln(tanh(1/2*x)+1)-2*a/b/(a^2+b^2)^(1/2)*arctanh(
1/2*(2*tanh(1/2*x)*b-2*a)/(a^2+b^2)^(1/2))-2/a*b/(a^2+b^2)^(1/2)*arctanh(1/
2*(2*tanh(1/2*x)*b-2*a)/(a^2+b^2)^(1/2))+1/b*ln(tanh(1/2*x))
```

maxima [A] time = 0.42, size = 90, normalized size = 1.58

$$\frac{x}{a} - \frac{\log\left(e^{(-x)}+1\right)}{b} + \frac{\log\left(e^{(-x)}-1\right)}{b} - \frac{\sqrt{a^{2}+b^{2}}\log\left(\frac{ae^{(-x)}-b-\sqrt{a^{2}+b^{2}}}{ae^{(-x)}-b+\sqrt{a^{2}+b^{2}}}\right)}{ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^2/(a+b*csch(x)),x, algorithm="maxima")
```

```
[Out] x/a - log(e^(-x) + 1)/b + log(e^(-x) - 1)/b - sqrt(a^2 + b^2)*log((a*e^(-x)
 - b - sqrt(a^2 + b^2))/(a*e^(-x) - b + sqrt(a^2 + b^2)))/(a*b)
```

mupad [B] time = 0.34, size = 316, normalized size = 5.54

$$\frac{x}{a}+\frac{\ln\left(32\,a^{2}\,b+32\,b^{3}-32\,b^{3}\,\mathrm{e}^{x}-32\,a^{2}\,b\,\mathrm{e}^{x}\right)}{b}-\frac{\ln\left(32\,a^{2}\,b+32\,b^{3}+32\,b^{3}\,\mathrm{e}^{x}+32\,a^{2}\,b\,\mathrm{e}^{x}\right)}{b}+\frac{\ln\left(128\,b^{5}\,\mathrm{e}^{x}-64\,a^{3}\,b\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^2/(a + b/sinh(x)),x)
```

```
[Out] x/a + log(32*a^2*b + 32*b^3 - 32*b^3*exp(x) - 32*a^2*b*exp(x))/b - log(32*a
^2*b + 32*b^3 + 32*b^3*exp(x) + 32*a^2*b*exp(x))/b + (log(128*b^5*exp(x) -
64*a^3*b^2 - 64*a*b^4 - 128*b^4*exp(x)*(a^2 + b^2)^(1/2) + 32*a^4*b*exp(x)
```

```
+ 160*a^2*b^3*exp(x) + 64*a*b^3*(a^2 + b^2)^(1/2) + 32*a^3*b*(a^2 + b^2)^(1
/2) - 96*a^2*b^2*exp(x)*(a^2 + b^2)^(1/2))*(a^2 + b^2)^(1/2))/(a*b) - (log(
128*b^5*exp(x) - 64*a^3*b^2 - 64*a*b^4 + 128*b^4*exp(x)*(a^2 + b^2)^(1/2) +
 32*a^4*b*exp(x) + 160*a^2*b^3*exp(x) - 64*a*b^3*(a^2 + b^2)^(1/2) - 32*a^3
*b*(a^2 + b^2)^(1/2) + 96*a^2*b^2*exp(x)*(a^2 + b^2)^(1/2))*(a^2 + b^2)^(1/
2))/(a*b)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\coth^2(x)}{a + b\operatorname{csch}(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)**2/(a+b*csch(x)),x)
```

```
[Out] Integral(coth(x)**2/(a + b*csch(x)), x)
```

3.120 $\int \frac{\coth^3(x)}{a+b\mathrm{csch}(x)}\,dx$

Optimal. Leaf size=32

$$\left(\frac{a}{b^2}+\frac{1}{a}\right)\log(a+b\mathrm{csch}(x))+\frac{\log(\sinh(x))}{a}-\frac{\mathrm{csch}(x)}{b}$$

```
[Out] -csch(x)/b+(1/a+a/b^2)*ln(a+b*csch(x))+ln(sinh(x))/a
```

Rubi [A] time = 0.07, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3885, 894}

$$\left(\frac{a}{b^2}+\frac{1}{a}\right)\log(a+b\mathrm{csch}(x))+\frac{\log(\sinh(x))}{a}-\frac{\mathrm{csch}(x)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]^3/(a + b*Csch[x]),x]
```

```
[Out] -(Csch[x]/b) + (a^(-1) + a/b^2)*Log[a + b*Csch[x]] + Log[Sinh[x]]/a
```

Rule 894

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 3885

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> -Dist[(-1)^((m - 1)/2)/(d*b^(m - 1)), Subst[Int[((b^2 - x^
2)^((m - 1)/2)*(a + x)^n)/x, x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\coth^3(x)}{a+b\mathrm{csch}(x)}\,dx &= \frac{\mathrm{Subst}\left(\int \frac{-b^2-x^2}{x(a+x)}\,dx,x,b\mathrm{csch}(x)\right)}{b^2}\\ &= \frac{\mathrm{Subst}\left(\int \left(-1-\frac{b^2}{ax}+\frac{a^2+b^2}{a(a+x)}\right)dx,x,b\mathrm{csch}(x)\right)}{b^2}\\ &= -\frac{\mathrm{csch}(x)}{b}+\left(\frac{1}{a}+\frac{a}{b^2}\right)\log(a+b\mathrm{csch}(x))+\frac{\log(\sinh(x))}{a}\end{aligned}$$

Mathematica [A] time = 0.05, size = 37, normalized size = 1.16

$$\frac{\left(a^2+b^2\right)\log(a\sinh(x)+b)+a^2(-\log(\sinh(x)))-ab\mathrm{csch}(x)}{ab^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]^3/(a + b*Csch[x]),x]
```

```
[Out] (-(a*b*Csch[x]) - a^2*Log[Sinh[x]] + (a^2 + b^2)*Log[b + a*Sinh[x]])/(a*b^2
)
```

fricas [B] time = 0.61, size = 199, normalized size = 6.22

$$-\frac{b^2x\cosh(x)^2+b^2x\sinh(x)^2-b^2x+2\,ab\cosh(x)-\left(\left(a^2+b^2\right)\cosh(x)^2+2\left(a^2+b^2\right)\cosh(x)\sinh(x)+\left(a^2\right.\right.}{ab^2\cos}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^3/(a+b*csch(x)),x, algorithm="fricas")
```

```
[Out] -(b^2*x*cosh(x)^2 + b^2*x*sinh(x)^2 - b^2*x + 2*a*b*cosh(x) - ((a^2 + b^2)*
cosh(x)^2 + 2*(a^2 + b^2)*cosh(x)*sinh(x) + (a^2 + b^2)*sinh(x)^2 - a^2 - b
^2)*log(2*(a*sinh(x) + b)/(cosh(x) - sinh(x))) + (a^2*cosh(x)^2 + 2*a^2*cos
h(x)*sinh(x) + a^2*sinh(x)^2 - a^2)*log(2*sinh(x)/(cosh(x) - sinh(x))) + 2*
(b^2*x*cosh(x) + a*b)*sinh(x))/(a*b^2*cosh(x)^2 + 2*a*b^2*cosh(x)*sinh(x) +
 a*b^2*sinh(x)^2 - a*b^2)
```

giac [B] time = 0.13, size = 80, normalized size = 2.50

$$-\frac{a\log\left(\left|-e^{(-x)}+e^{x}\right|\right)}{b^2}+\frac{\left(a^2+b^2\right)\log\left(\left|-a\left(e^{(-x)}-e^{x}\right)+2\,b\right|\right)}{ab^2}+\frac{a\left(e^{(-x)}-e^{x}\right)+2\,b}{b^2\left(e^{(-x)}-e^{x}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^3/(a+b*csch(x)),x, algorithm="giac")
```

```
[Out] -a*log(abs(-e^(-x) + e^x))/b^2 + (a^2 + b^2)*log(abs(-a*(e^(-x) - e^x) + 2*
b))/(a*b^2) + (a*(e^(-x) - e^x) + 2*b)/(b^2*(e^(-x) - e^x))
```

maple [B] time = 0.14, size = 106, normalized size = 3.31

$$\frac{\tanh\left(\frac{x}{2}\right)}{2b}-\frac{\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{a}-\frac{\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)}{a}+\frac{a\ln\left(\left(\tanh^2\left(\frac{x}{2}\right)\right)b-2a\tanh\left(\frac{x}{2}\right)-b\right)}{b^2}+\frac{\ln\left(\left(\tanh^2\left(\frac{x}{2}\right)\right)b-2\right.}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^3/(a+b*csch(x)),x)
```

```
[Out] 1/2/b*tanh(1/2*x)-1/a*ln(tanh(1/2*x)-1)-1/a*ln(tanh(1/2*x)+1)+a/b^2*ln(tanh
(1/2*x)^2*b-2*a*tanh(1/2*x)-b)+1/a*ln(tanh(1/2*x)^2*b-2*a*tanh(1/2*x)-b)-1/
2/b/tanh(1/2*x)-a/b^2*ln(tanh(1/2*x))
```

maxima [B] time = 0.32, size = 82, normalized size = 2.56

$$\frac{x}{a}+\frac{2\,e^{(-x)}}{be^{(-2\,x)}-b}-\frac{a\log\left(e^{(-x)}+1\right)}{b^2}-\frac{a\log\left(e^{(-x)}-1\right)}{b^2}+\frac{\left(a^2+b^2\right)\log\left(-2\,be^{(-x)}+ae^{(-2\,x)}-a\right)}{ab^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^3/(a+b*csch(x)),x, algorithm="maxima")
```

```
[Out] x/a + 2*e^(-x)/(b*e^(-2*x) - b) - a*log(e^(-x) + 1)/b^2 - a*log(e^(-x) - 1)
/b^2 + (a^2 + b^2)*log(-2*b*e^(-x) + a*e^(-2*x) - a)/(a*b^2)
```

mupad [B] time = 1.84, size = 261, normalized size = 8.16

$$\frac{2\,e^x}{b-b\,e^{2\,x}}-\frac{x}{a}+\frac{\ln\left(16\,a^5\,e^{2\,x}-4\,a\,b^4-16\,a^5-16\,a^3\,b^2+8\,b^5\,e^x+16\,a^3\,b^2\,e^{2\,x}+32\,a^4\,b\,e^x+4\,a\,b^4\,e^{2\,x}+32\,a^2\,b\right.}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^3/(a + b/sinh(x)),x)
```

```
[Out] (2*exp(x))/(b - b*exp(2*x)) - x/a + log(16*a^5*exp(2*x) - 4*a*b^4 - 16*a^5
- 16*a^3*b^2 + 8*b^5*exp(x) + 16*a^3*b^2*exp(2*x) + 32*a^4*b*exp(x) + 4*a*b
^4*exp(2*x) + 32*a^2*b^3*exp(x))/a + (a*log(16*a^5*exp(2*x) - 4*a*b^4 - 16*
a^5 - 16*a^3*b^2 + 8*b^5*exp(x) + 16*a^3*b^2*exp(2*x) + 32*a^4*b*exp(x) + 4
*a*b^4*exp(2*x) + 32*a^2*b^3*exp(x)))/b^2 - (a*log(16*a^6*exp(2*x) + 4*b^6*
exp(2*x) - 16*a^6 - 4*b^6 - 20*a^2*b^4 - 32*a^4*b^2 + 20*a^2*b^4*exp(2*x) +
 32*a^4*b^2*exp(2*x)))/b^2
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\coth^3(x)}{a + b \operatorname{csch}(x)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)**3/(a+b*csch(x)),x)
```

```
[Out] Integral(coth(x)**3/(a + b*csch(x)), x)
```

3.121 $\int \frac{\coth^4(x)}{a+b\mathbf{csch}(x)}\,dx$

Optimal. Leaf size=88

$$\frac{2\left(a^2+b^2\right)^{3/2}\tanh^{-1}\left(\frac{a-b\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{ab^3}-\frac{\left(2a^2+3b^2\right)\tanh^{-1}(\cosh(x))}{2b^3}+\frac{a\coth(x)}{b^2}+\frac{x}{a}-\frac{\coth(x)\mathrm{csch}(x)}{2b}$$

```
[Out] x/a-1/2*(2*a^2+3*b^2)*arctanh(cosh(x))/b^3+2*(a^2+b^2)^(3/2)*arctanh((a-b*t
anh(1/2*x))/(a^2+b^2)^(1/2))/a/b^3+a*coth(x)/b^2-1/2*coth(x)*csch(x)/b
```

Rubi [A] time = 0.33, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.538, Rules used = {3898, 2893, 3057, 2660, 618, 204, 3770}

$$\frac{2\left(a^2+b^2\right)^{3/2}\tanh^{-1}\left(\frac{a-b\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{ab^3}-\frac{\left(2a^2+3b^2\right)\tanh^{-1}(\cosh(x))}{2b^3}+\frac{a\coth(x)}{b^2}+\frac{x}{a}-\frac{\coth(x)\mathrm{csch}(x)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]^4/(a + b*Csch[x]),x]
```

```
[Out] x/a - ((2*a^2 + 3*b^2)*ArcTanh[Cosh[x]])/(2*b^3) + (2*(a^2 + b^2)^(3/2)*Arc
Tanh[(a - b*Tanh[x/2])/Sqrt[a^2 + b^2]])/(a*b^3) + (a*Coth[x])/b^2 - (Coth[
x]*Csch[x])/(2*b)
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2893

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[(Cos[e + f*x]*(a + b
*Sin[e + f*x])^(m + 1)*(d*Sin[e + f*x])^(n + 1))/(a*d*f*(n + 1)), x] + (-Di
st[1/(a^2*d^2*(n + 1)*(n + 2)), Int[(a + b*Sin[e + f*x])^m*(d*Sin[e + f*x])
^(n + 2)*Simp[a^2*n*(n + 2) - b^2*(m + n + 2)*(m + n + 3) + a*b*m*Sin[e + f
*x] - (a^2*(n + 1)*(n + 2) - b^2*(m + n + 2)*(m + n + 4))*Sin[e + f*x]^2, x
], x], x] - Simp[(b*(m + n + 2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(
d*Sin[e + f*x])^(n + 2))/(a^2*d^2*f*(n + 1)*(n + 2)), x]) /; FreeQ[{a, b, d
, e, f, m}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || IntegersQ[2*m, 2*n])
&&  !m < -1 && LtQ[n, -1] && (LtQ[n, -2] || EqQ[m + n + 4, 0])
```

Rule 3057

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)])), x_Symbol] :> Simp[(C*x)/(b*d), x] + (Dist[(A*b^2 - a*b*B + a^2*C)
/(b*(b*c - a*d)), Int[1/(a + b*Sin[e + f*x]), x], x] - Dist[(c^2*C - B*c*d
+ A*d^2)/(d*(b*c - a*d)), Int[1/(c + d*Sin[e + f*x]), x], x]) /; FreeQ[{a,
b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && Ne
Q[c^2 - d^2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3898

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Int[(Cos[c + d*x]^m*(b + a*Sin[c + d*x])^n)/Sin[c + d*x]^(
m + n), x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[n] &&
 IntegerQ[m] && (IntegerQ[m/2] || LeQ[m, 1])
```

Rubi steps

$$\int \frac{\coth^4(x)}{a+b\text{csch}(x)}\,dx = i\int \frac{\cosh(x)\coth^3(x)}{ib+ia\sinh(x)}\,dx$$

$$= \frac{a\coth(x)}{b^2}-\frac{\coth(x)\text{csch}(x)}{2b}-\frac{i\int \frac{\text{csch}(x)\left(-2a^2-3b^2+ab\sinh(x)-2b^2\sinh^2(x)\right)}{ib+ia\sinh(x)}\,dx}{2b^2}$$

$$= \frac{x}{a}+\frac{a\coth(x)}{b^2}-\frac{\coth(x)\text{csch}(x)}{2b}-\frac{\left(i\left(a^2+b^2\right)^2\right)\int \frac{1}{ib+ia\sinh(x)}\,dx}{ab^3}+\frac{\left(2a^2+3b^2\right)\int \text{csch}(x)\,d}{2b^3}$$

$$= \frac{x}{a}-\frac{\left(2a^2+3b^2\right)\tanh^{-1}(\cosh(x))}{2b^3}+\frac{a\coth(x)}{b^2}-\frac{\coth(x)\text{csch}(x)}{2b}-\frac{\left(2i\left(a^2+b^2\right)^2\right)\text{Subst}\Big(}{}$$

$$= \frac{x}{a}-\frac{\left(2a^2+3b^2\right)\tanh^{-1}(\cosh(x))}{2b^3}+\frac{a\coth(x)}{b^2}-\frac{\coth(x)\text{csch}(x)}{2b}+\frac{\left(4i\left(a^2+b^2\right)^2\right)\text{Subst}\Big(}{}$$

$$= \frac{x}{a}-\frac{\left(2a^2+3b^2\right)\tanh^{-1}(\cosh(x))}{2b^3}+\frac{2\left(a^2+b^2\right)^{3/2}\tanh^{-1}\left(\frac{a-b\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{ab^3}+\frac{a\coth(x)}{b^2}-\text{coth(}$$

Mathematica [A] time = 0.59, size = 151, normalized size = 1.72

$$\frac{\text{csch}(x)(a\sinh(x)+b)\left(4a\left(2a^2+3b^2\right)\log\left(\tanh\left(\frac{x}{2}\right)\right)-16\left(-a^2-b^2\right)^{3/2}\tan^{-1}\left(\frac{a-b\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2-b^2}}\right)+4a^2b\tanh\left(\frac{x}{2}\right)+4a^2\right.}{8ab^3(a+b\text{csch}(x))}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]^4/(a + b*Csch[x]),x]
```

```
[Out] (Csch[x]*(b + a*Sinh[x])*(8*b^3*x - 16*(-a^2 - b^2)^(3/2)*ArcTan[(a - b*Tan
h[x/2])/Sqrt[-a^2 - b^2]] + 4*a^2*b*Coth[x/2] - a*b^2*Csch[x/2]^2 + 4*a*(2*
a^2 + 3*b^2)*Log[Tanh[x/2]] - a*b^2*Sech[x/2]^2 + 4*a^2*b*Tanh[x/2]))/(8*a*
b^3*(a + b*Csch[x]))
```

fricas [B] time = 1.72, size = 831, normalized size = 9.44

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^4/(a+b*csch(x)),x, algorithm="fricas")
```

```
[Out] 1/2*(2*b^3*x*cosh(x)^4 + 2*b^3*x*sinh(x)^4 - 2*a*b^2*cosh(x)^3 + 2*b^3*x -
2*a*b^2*cosh(x) + 2*(4*b^3*x*cosh(x) - a*b^2)*sinh(x)^3 - 4*a^2*b - 4*(b^3*
x - a^2*b)*cosh(x)^2 + 2*(6*b^3*x*cosh(x)^2 - 2*b^3*x - 3*a*b^2*cosh(x) + 2
*a^2*b)*sinh(x)^2 + 2*((a^2 + b^2)*cosh(x)^4 + 4*(a^2 + b^2)*cosh(x)*sinh(x
)^3 + (a^2 + b^2)*sinh(x)^4 - 2*(a^2 + b^2)*cosh(x)^2 + 2*(3*(a^2 + b^2)*co
sh(x)^2 - a^2 - b^2)*sinh(x)^2 + a^2 + b^2 + 4*((a^2 + b^2)*cosh(x)^3 - (a^
2 + b^2)*cosh(x))*sinh(x))*sqrt(a^2 + b^2)*log((a^2*cosh(x)^2 + a^2*sinh(x)
^2 + 2*a*b*cosh(x) + a^2 + 2*b^2 + 2*(a^2*cosh(x) + a*b)*sinh(x) + 2*sqrt(a
^2 + b^2)*(a*cosh(x) + a*sinh(x) + b))/(a*cosh(x)^2 + a*sinh(x)^2 + 2*b*cos
h(x) + 2*(a*cosh(x) + b)*sinh(x) - a)) - ((2*a^3 + 3*a*b^2)*cosh(x)^4 + 4*(
2*a^3 + 3*a*b^2)*cosh(x)*sinh(x)^3 + (2*a^3 + 3*a*b^2)*sinh(x)^4 + 2*a^3 +
3*a*b^2 - 2*(2*a^3 + 3*a*b^2)*cosh(x)^2 - 2*(2*a^3 + 3*a*b^2 - 3*(2*a^3 + 3
*a*b^2)*cosh(x)^2)*sinh(x)^2 + 4*((2*a^3 + 3*a*b^2)*cosh(x)^3 - (2*a^3 + 3*
a*b^2)*cosh(x))*sinh(x))*log(cosh(x) + sinh(x) + 1) + ((2*a^3 + 3*a*b^2)*co
sh(x)^4 + 4*(2*a^3 + 3*a*b^2)*cosh(x)*sinh(x)^3 + (2*a^3 + 3*a*b^2)*sinh(x)
^4 + 2*a^3 + 3*a*b^2 - 2*(2*a^3 + 3*a*b^2)*cosh(x)^2 - 2*(2*a^3 + 3*a*b^2 -
 3*(2*a^3 + 3*a*b^2)*cosh(x)^2)*sinh(x)^2 + 4*((2*a^3 + 3*a*b^2)*cosh(x)^3
- (2*a^3 + 3*a*b^2)*cosh(x))*sinh(x))*log(cosh(x) + sinh(x) - 1) + 2*(4*b^3
*x*cosh(x)^3 - 3*a*b^2*cosh(x)^2 - a*b^2 - 4*(b^3*x - a^2*b)*cosh(x))*sinh(
x))/(a*b^3*cosh(x)^4 + 4*a*b^3*cosh(x)*sinh(x)^3 + a*b^3*sinh(x)^4 - 2*a*b^
3*cosh(x)^2 + a*b^3 + 2*(3*a*b^3*cosh(x)^2 - a*b^3)*sinh(x)^2 + 4*(a*b^3*co
sh(x)^3 - a*b^3*cosh(x))*sinh(x))
```

giac [B] time = 0.15, size = 161, normalized size = 1.83

$$\frac{x}{a} - \frac{\left(2\,a^2 + 3\,b^2\right)\log\left(e^x + 1\right)}{2\,b^3} + \frac{\left(2\,a^2 + 3\,b^2\right)\log\left(\left|e^x - 1\right|\right)}{2\,b^3} - \frac{\left(a^4 + 2\,a^2b^2 + b^4\right)\log\left(\frac{\left|2\,ae^x + 2\,b - 2\,\sqrt{a^2+b^2}\right|}{\left|2\,ae^x + 2\,b + 2\,\sqrt{a^2+b^2}\right|}\right)}{\sqrt{a^2 + b^2}\,ab^3} - \frac{be^{(3\,x)} - 2}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^4/(a+b*csch(x)),x, algorithm="giac")
```

```
[Out] x/a - 1/2*(2*a^2 + 3*b^2)*log(e^x + 1)/b^3 + 1/2*(2*a^2 + 3*b^2)*log(abs(e^
x - 1))/b^3 - (a^4 + 2*a^2*b^2 + b^4)*log(abs(2*a*e^x + 2*b - 2*sqrt(a^2 +
b^2))/abs(2*a*e^x + 2*b + 2*sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*a*b^3) - (b*
e^(3*x) - 2*a*e^(2*x) + b*e^x + 2*a)/(b^2*(e^(2*x) - 1)^2)
```

maple [B] time = 0.15, size = 207, normalized size = 2.35

$$\frac{\tanh^2\left(\frac{x}{2}\right)}{8b} + \frac{a\tanh\left(\frac{x}{2}\right)}{2b^2} - \frac{\ln\left(\tanh\left(\frac{x}{2}\right) - 1\right)}{a} + \frac{\ln\left(\tanh\left(\frac{x}{2}\right) + 1\right)}{a} - \frac{2a^3\operatorname{arctanh}\left(\frac{2\tanh\left(\frac{x}{2}\right)b - 2a}{2\sqrt{a^2+b^2}}\right)}{b^3\sqrt{a^2 + b^2}} - \frac{4a\operatorname{arctanh}\left(\frac{2\tanh(}{2\sqrt{a}}\right.}{b\sqrt{a^2 + b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^4/(a+b*csch(x)),x)
```

```
[Out] 1/8/b*tanh(1/2*x)^2+1/2/b^2*a*tanh(1/2*x)-1/a*ln(tanh(1/2*x)-1)+1/a*ln(tanh
(1/2*x)+1)-2/b^3*a^3/(a^2+b^2)^(1/2)*arctanh(1/2*(2*tanh(1/2*x)*b-2*a)/(a^2
+b^2)^(1/2))-4*a/b/(a^2+b^2)^(1/2)*arctanh(1/2*(2*tanh(1/2*x)*b-2*a)/(a^2+b
^2)^(1/2))-2/a*b/(a^2+b^2)^(1/2)*arctanh(1/2*(2*tanh(1/2*x)*b-2*a)/(a^2+b^2
```

```
)^(1/2))-1/8/b/tanh(1/2*x)^2+1/b^3*ln(tanh(1/2*x))*a^2+3/2/b*ln(tanh(1/2*x)
)+1/2*a/b^2/tanh(1/2*x)
```

maxima [B] time = 0.43, size = 178, normalized size = 2.02

$$\frac{be^{(-x)} + 2\,ae^{(-2\,x)} + be^{(-3\,x)} - 2\,a}{2\,b^2e^{(-2\,x)} - b^2e^{(-4\,x)} - b^2} + \frac{x}{a} - \frac{\left(2\,a^2 + 3\,b^2\right)\log\left(e^{(-x)} + 1\right)}{2\,b^3} + \frac{\left(2\,a^2 + 3\,b^2\right)\log\left(e^{(-x)} - 1\right)}{2\,b^3} - \frac{\left(a^4 + 2\,a^2b^2 + b^4\right)}{\sqrt{a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^4/(a+b*csch(x)),x, algorithm="maxima")
```

```
[Out] (b*e^(-x) + 2*a*e^(-2*x) + b*e^(-3*x) - 2*a)/(2*b^2*e^(-2*x) - b^2*e^(-4*x)
 - b^2) + x/a - 1/2*(2*a^2 + 3*b^2)*log(e^(-x) + 1)/b^3 + 1/2*(2*a^2 + 3*b^
2)*log(e^(-x) - 1)/b^3 - (a^4 + 2*a^2*b^2 + b^4)*log((a*e^(-x) - b - sqrt(a
^2 + b^2))/(a*e^(-x) - b + sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*a*b^3)
```

mupad [B] time = 2.62, size = 378, normalized size = 4.30

$$\frac{\frac{2\,a}{b^2} - \frac{e^x}{b}}{e^{2\,x} - 1} + \frac{x}{a} + \frac{\ln\left(e^x - 1\right)\,\left(2\,a^2 + 3\,b^2\right)}{2\,b^3} - \frac{\ln\left(e^x + 1\right)\,\left(2\,a^2 + 3\,b^2\right)}{2\,b^3} - \frac{2\,e^x}{b\,\left(e^{4\,x} - 2\,e^{2\,x} + 1\right)} + \frac{\ln\left(a^3\,\sqrt{{\left(a^2 + b^2\right)}^3} - 2\,a^5\,b\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^4/(a + b/sinh(x)),x)
```

```
[Out] ((2*a)/b^2 - exp(x)/b)/(exp(2*x) - 1) + x/a + (log(exp(x) - 1)*(2*a^2 + 3*b
^2))/(2*b^3) - (log(exp(x) + 1)*(2*a^2 + 3*b^2))/(2*b^3) - (2*exp(x))/(b*(e
xp(4*x) - 2*exp(2*x) + 1)) + (log(a^3*((a^2 + b^2)^3)^(1/2) - 2*a^5*b - 2*a
*b^5 - 4*a^3*b^3 + a^6*exp(x) + 4*b^6*exp(x) + 2*a*b^2*((a^2 + b^2)^3)^(1/2
) - 4*b^3*exp(x)*((a^2 + b^2)^3)^(1/2) + 9*a^2*b^4*exp(x) + 6*a^4*b^2*exp(x
) - 3*a^2*b*exp(x)*((a^2 + b^2)^3)^(1/2))*((a^2 + b^2)^3)^(1/2))/(a*b^3) -
(log(a^6*exp(x) - 2*a^5*b - a^3*((a^2 + b^2)^3)^(1/2) - 4*a^3*b^3 - 2*a*b^5
 + 4*b^6*exp(x) - 2*a*b^2*((a^2 + b^2)^3)^(1/2) + 4*b^3*exp(x)*((a^2 + b^2)
^3)^(1/2) + 9*a^2*b^4*exp(x) + 6*a^4*b^2*exp(x) + 3*a^2*b*exp(x)*((a^2 + b^
2)^3)^(1/2))*((a^2 + b^2)^3)^(1/2))/(a*b^3)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\coth^4(x)}{a + b\,\mathrm{csch}\,(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)**4/(a+b*csch(x)),x)
```

```
[Out] Integral(coth(x)**4/(a + b*csch(x)), x)
```

3.122 $\int \frac{\coth^5(x)}{a+b\,\mathbf{csch}(x)}\,dx$

Optimal. Leaf size=70

$$\frac{\left(a^2+b^2\right)^2\log(a+b\operatorname{csch}(x))}{ab^4}-\frac{\left(a^2+2b^2\right)\operatorname{csch}(x)}{b^3}+\frac{a\operatorname{csch}^2(x)}{2b^2}+\frac{\log(\sinh(x))}{a}-\frac{\operatorname{csch}^3(x)}{3b}$$

```
[Out] -(a^2+2*b^2)*csch(x)/b^3+1/2*a*csch(x)^2/b^2-1/3*csch(x)^3/b+(a^2+b^2)^2*ln
(a+b*csch(x))/a/b^4+ln(sinh(x))/a
```

Rubi [A] time = 0.09, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3885, 894}

$$-\frac{\left(a^2+2b^2\right)\operatorname{csch}(x)}{b^3}+\frac{\left(a^2+b^2\right)^2\log(a+b\operatorname{csch}(x))}{ab^4}+\frac{a\operatorname{csch}^2(x)}{2b^2}+\frac{\log(\sinh(x))}{a}-\frac{\operatorname{csch}^3(x)}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]^5/(a + b*Csch[x]),x]
```

```
[Out] -(((a^2 + 2*b^2)*Csch[x])/b^3) + (a*Csch[x]^2)/(2*b^2) - Csch[x]^3/(3*b) +
((a^2 + b^2)^2*Log[a + b*Csch[x]])/(a*b^4) + Log[Sinh[x]]/a
```

Rule 894

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 3885

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> -Dist[(-1)^((m - 1)/2)/(d*b^(m - 1)), Subst[Int[((b^2 - x^
2)^((m - 1)/2)*(a + x)^n)/x, x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\coth^5(x)}{a+b\operatorname{csch}(x)}\,dx &= -\frac{\operatorname{Subst}\left(\int \frac{\left(-b^2-x^2\right)^2}{x(a+x)}\,dx,x,b\operatorname{csch}(x)\right)}{b^4}\\ &= -\frac{\operatorname{Subst}\left(\int \left(a^2\left(1+\frac{2b^2}{a^2}\right)+\frac{b^4}{ax}-ax+x^2-\frac{\left(a^2+b^2\right)^2}{a(a+x)}\right)dx,x,b\operatorname{csch}(x)\right)}{b^4}\\ &= -\frac{\left(a^2+2b^2\right)\operatorname{csch}(x)}{b^3}+\frac{a\operatorname{csch}^2(x)}{2b^2}-\frac{\operatorname{csch}^3(x)}{3b}+\frac{\left(a^2+b^2\right)^2\log(a+b\operatorname{csch}(x))}{ab^4}+\frac{\log(\sin}{a}\end{aligned}$$

Mathematica [A] time = 0.14, size = 83, normalized size = 1.19

$$\frac{3a^2b^2\operatorname{csch}^2(x)-6ab\left(a^2+2b^2\right)\operatorname{csch}(x)-6a^2\left(a^2+2b^2\right)\log(\sinh(x))+6\left(a^2+b^2\right)^2\log(a\sinh(x)+b)-2ab^3}{6ab^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]^5/(a + b*Csch[x]),x]
```

```
[Out] (-6*a*b*(a^2 + 2*b^2)*Csch[x] + 3*a^2*b^2*Csch[x]^2 - 2*a*b^3*Csch[x]^3 - 6
*a^2*(a^2 + 2*b^2)*Log[Sinh[x]] + 6*(a^2 + b^2)^2*Log[b + a*Sinh[x]])/(6*a*
b^4)
```

fricas [B] time = 0.89, size = 1288, normalized size = 18.40

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^5/(a+b*csch(x)),x, algorithm="fricas")
```

```
[Out] -1/3*(3*b^4*x*cosh(x)^6 + 3*b^4*x*sinh(x)^6 + 6*(a^3*b + 2*a*b^3)*cosh(x)^5
 + 6*(3*b^4*x*cosh(x) + a^3*b + 2*a*b^3)*sinh(x)^5 - 3*b^4*x - 3*(3*b^4*x +
 2*a^2*b^2)*cosh(x)^4 + 3*(15*b^4*x*cosh(x)^2 - 3*b^4*x - 2*a^2*b^2 + 10*(a
^3*b + 2*a*b^3)*cosh(x))*sinh(x)^4 - 4*(3*a^3*b + 4*a*b^3)*cosh(x)^3 + 4*(1
5*b^4*x*cosh(x)^3 - 3*a^3*b - 4*a*b^3 + 15*(a^3*b + 2*a*b^3)*cosh(x)^2 - 3*
(3*b^4*x + 2*a^2*b^2)*cosh(x))*sinh(x)^3 + 3*(3*b^4*x + 2*a^2*b^2)*cosh(x)^
2 + 3*(15*b^4*x*cosh(x)^4 + 3*b^4*x + 2*a^2*b^2 + 20*(a^3*b + 2*a*b^3)*cosh
(x)^3 - 6*(3*b^4*x + 2*a^2*b^2)*cosh(x)^2 - 4*(3*a^3*b + 4*a*b^3)*cosh(x))*
sinh(x)^2 + 6*(a^3*b + 2*a*b^3)*cosh(x) - 3*((a^4 + 2*a^2*b^2 + b^4)*cosh(x
)^6 + 6*(a^4 + 2*a^2*b^2 + b^4)*cosh(x)*sinh(x)^5 + (a^4 + 2*a^2*b^2 + b^4)
*sinh(x)^6 - 3*(a^4 + 2*a^2*b^2 + b^4)*cosh(x)^4 - 3*(a^4 + 2*a^2*b^2 + b^4
 - 5*(a^4 + 2*a^2*b^2 + b^4)*cosh(x)^2)*sinh(x)^4 - a^4 - 2*a^2*b^2 - b^4 +
 4*(5*(a^4 + 2*a^2*b^2 + b^4)*cosh(x)^3 - 3*(a^4 + 2*a^2*b^2 + b^4)*cosh(x)
)*sinh(x)^3 + 3*(a^4 + 2*a^2*b^2 + b^4)*cosh(x)^2 + 3*(5*(a^4 + 2*a^2*b^2 +
 b^4)*cosh(x)^4 + a^4 + 2*a^2*b^2 + b^4 - 6*(a^4 + 2*a^2*b^2 + b^4)*cosh(x)
^2)*sinh(x)^2 + 6*((a^4 + 2*a^2*b^2 + b^4)*cosh(x)^5 - 2*(a^4 + 2*a^2*b^2 +
 b^4)*cosh(x)^3 + (a^4 + 2*a^2*b^2 + b^4)*cosh(x))*sinh(x))*log(2*(a*sinh(x
) + b)/(cosh(x) - sinh(x))) + 3*((a^4 + 2*a^2*b^2)*cosh(x)^6 + 6*(a^4 + 2*a
^2*b^2)*cosh(x)*sinh(x)^5 + (a^4 + 2*a^2*b^2)*sinh(x)^6 - 3*(a^4 + 2*a^2*b^
2)*cosh(x)^4 - 3*(a^4 + 2*a^2*b^2 - 5*(a^4 + 2*a^2*b^2)*cosh(x)^2)*sinh(x)^
4 - a^4 - 2*a^2*b^2 + 4*(5*(a^4 + 2*a^2*b^2)*cosh(x)^3 - 3*(a^4 + 2*a^2*b^2
)*cosh(x))*sinh(x)^3 + 3*(a^4 + 2*a^2*b^2)*cosh(x)^2 + 3*(5*(a^4 + 2*a^2*b^
2)*cosh(x)^4 + a^4 + 2*a^2*b^2 - 6*(a^4 + 2*a^2*b^2)*cosh(x)^2)*sinh(x)^2 +
 6*((a^4 + 2*a^2*b^2)*cosh(x)^5 - 2*(a^4 + 2*a^2*b^2)*cosh(x)^3 + (a^4 + 2*
a^2*b^2)*cosh(x))*sinh(x))*log(2*sinh(x)/(cosh(x) - sinh(x))) + 6*(3*b^4*x*
cosh(x)^5 + 5*(a^3*b + 2*a*b^3)*cosh(x)^4 + a^3*b + 2*a*b^3 - 2*(3*b^4*x +
2*a^2*b^2)*cosh(x)^3 - 2*(3*a^3*b + 4*a*b^3)*cosh(x)^2 + (3*b^4*x + 2*a^2*b
^2)*cosh(x))*sinh(x))/(a*b^4*cosh(x)^6 + 6*a*b^4*cosh(x)*sinh(x)^5 + a*b^4*
sinh(x)^6 - 3*a*b^4*cosh(x)^4 + 3*a*b^4*cosh(x)^2 - a*b^4 + 3*(5*a*b^4*cosh
(x)^2 - a*b^4)*sinh(x)^4 + 4*(5*a*b^4*cosh(x)^3 - 3*a*b^4*cosh(x))*sinh(x)^
3 + 3*(5*a*b^4*cosh(x)^4 - 6*a*b^4*cosh(x)^2 + a*b^4)*sinh(x)^2 + 6*(a*b^4*
cosh(x)^5 - 2*a*b^4*cosh(x)^3 + a*b^4*cosh(x))*sinh(x))
```

giac [B] time = 0.13, size = 170, normalized size = 2.43

$$-\frac{\left(a^{3}+2\,ab^{2}\right)\log\left(\left|-e^{(-x)}+e^{x}\right|\right)}{b^{4}}+\frac{\left(a^{4}+2\,a^{2}b^{2}+b^{4}\right)\log\left(\left|-a\left(e^{(-x)}-e^{x}\right)+2\,b\right|\right)}{ab^{4}}+\frac{11\,a^{3}\left(e^{(-x)}-e^{x}\right)^{3}+22\,ab^{2}\left(e^{(-x)}\cdots\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^5/(a+b*csch(x)),x, algorithm="giac")
```

```
[Out] -(a^3 + 2*a*b^2)*log(abs(-e^(-x) + e^x))/b^4 + (a^4 + 2*a^2*b^2 + b^4)*log(
abs(-a*(e^(-x) - e^x) + 2*b))/(a*b^4) + 1/6*(11*a^3*(e^(-x) - e^x)^3 + 22*a
```

```
*b^2*(e^(-x) - e^x)^3 + 12*a^2*b*(e^(-x) - e^x)^2 + 24*b^3*(e^(-x) - e^x)^2
 + 12*a*b^2*(e^(-x) - e^x) + 16*b^3)/(b^4*(e^(-x) - e^x)^3)
```

maple [B] time = 0.15, size = 219, normalized size = 3.13

$$\frac{\tanh^3\left(\frac{x}{2}\right)}{24b}+\frac{a\left(\tanh^2\left(\frac{x}{2}\right)\right)}{8b^2}+\frac{a^2\tanh\left(\frac{x}{2}\right)}{2b^3}+\frac{7\tanh\left(\frac{x}{2}\right)}{8b}-\frac{\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{a}-\frac{\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)}{a}+\frac{a^3\ln\left(\left(\tanh^2\left(\frac{x}{2}\right)\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^5/(a+b*csch(x)),x)
```

```
[Out] 1/24/b*tanh(1/2*x)^3+1/8/b^2*a*tanh(1/2*x)^2+1/2/b^3*a^2*tanh(1/2*x)+7/8/b*
tanh(1/2*x)-1/a*ln(tanh(1/2*x)-1)-1/a*ln(tanh(1/2*x)+1)+a^3/b^4*ln(tanh(1/2
*x)^2*b-2*a*tanh(1/2*x)-b)+2*a/b^2*ln(tanh(1/2*x)^2*b-2*a*tanh(1/2*x)-b)+1/
a*ln(tanh(1/2*x)^2*b-2*a*tanh(1/2*x)-b)-1/24/b/tanh(1/2*x)^3-1/2/b^3/tanh(1
/2*x)*a^2-7/8/b/tanh(1/2*x)+1/8*a/b^2/tanh(1/2*x)^2-1/b^4*a^3*ln(tanh(1/2*x
))-2*a/b^2*ln(tanh(1/2*x))
```

maxima [B] time = 0.33, size = 190, normalized size = 2.71

$$-\frac{2\left(3\,abe^{(-2\,x)}-3\,abe^{(-4\,x)}-3\left(a^2+2\,b^2\right)e^{(-x)}+2\left(3\,a^2+4\,b^2\right)e^{(-3\,x)}-3\left(a^2+2\,b^2\right)e^{(-5\,x)}\right)}{3\left(3\,b^3e^{(-2\,x)}-3\,b^3e^{(-4\,x)}+b^3e^{(-6\,x)}-b^3\right)}+\frac{x}{a}-\frac{\left(a^3+2\,ab^2\right)\log}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^5/(a+b*csch(x)),x, algorithm="maxima")
```

```
[Out] -2/3*(3*a*b*e^(-2*x) - 3*a*b*e^(-4*x) - 3*(a^2 + 2*b^2)*e^(-x) + 2*(3*a^2 +
 4*b^2)*e^(-3*x) - 3*(a^2 + 2*b^2)*e^(-5*x))/(3*b^3*e^(-2*x) - 3*b^3*e^(-4*
x) + b^3*e^(-6*x) - b^3) + x/a - (a^3 + 2*a*b^2)*log(e^(-x) + 1)/b^4 - (a^3
 + 2*a*b^2)*log(e^(-x) - 1)/b^4 + (a^4 + 2*a^2*b^2 + b^4)*log(-2*b*e^(-x) +
 a*e^(-2*x) - a)/(a*b^4)
```

mupad [B] time = 1.97, size = 155, normalized size = 2.21

$$\frac{\frac{2\,a}{b^2}-\frac{2\,e^x\,\left(a^2+2\,b^2\right)}{b^3}}{e^{2\,x}-1}-\frac{x}{a}+\frac{\frac{2\,a}{b^2}-\frac{8\,e^x}{3\,b}}{e^{4\,x}-2\,e^{2\,x}+1}-\frac{8\,e^x}{3\,b\,\left(3\,e^{2\,x}-3\,e^{4\,x}+e^{6\,x}-1\right)}-\frac{\ln\left(e^{2\,x}-1\right)\,\left(a^3+2\,a\,b^2\right)}{b^4}+\frac{\ln\left(2\,b\,e^x-a\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^5/(a + b/sinh(x)),x)
```

```
[Out] ((2*a)/b^2 - (2*exp(x)*(a^2 + 2*b^2))/b^3)/(exp(2*x) - 1) - x/a + ((2*a)/b^
2 - (8*exp(x))/(3*b))/(exp(4*x) - 2*exp(2*x) + 1) - (8*exp(x))/(3*b*(3*exp(
2*x) - 3*exp(4*x) + exp(6*x) - 1)) - (log(exp(2*x) - 1)*(2*a*b^2 + a^3))/b^
4 + (log(2*b*exp(x) - a + a*exp(2*x))*(a^4 + b^4 + 2*a^2*b^2))/(a*b^4)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\coth^5(x)}{a+b\operatorname{csch}(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)**5/(a+b*csch(x)),x)
```

```
[Out] Integral(coth(x)**5/(a + b*csch(x)), x)
```

3.123 $\int \frac{\coth^6(x)}{a+b\mathbf{csch}(x)}\,dx$

Optimal. Leaf size=183

$$\frac{2\left(a^2+b^2\right)^{5/2}\tanh^{-1}\left(\frac{a-b\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{ab^5}+\frac{a\left(a^2+3b^2\right)\coth(x)}{b^4}+\frac{\left(a^2+3b^2\right)\tanh^{-1}(\cosh(x))}{2b^3}-\frac{\left(a^2+3b^2\right)\coth(x)\text{csch}(x}{2b^3}$$

```
[Out] x/a-3/8*arctanh(cosh(x))/b+1/2*(a^2+3*b^2)*arctanh(cosh(x))/b^3-(a^4+3*a^2*
b^2+3*b^4)*arctanh(cosh(x))/b^5+2*(a^2+b^2)^(5/2)*arctanh((a-b*tanh(1/2*x))
/(a^2+b^2)^(1/2))/a/b^5-a*coth(x)/b^2+a*(a^2+3*b^2)*coth(x)/b^4+1/3*a*coth(
x)^3/b^2+3/8*coth(x)*csch(x)/b-1/2*(a^2+3*b^2)*coth(x)*csch(x)/b^3-1/4*coth
(x)*csch(x)^3/b
```

Rubi [A] time = 0.34, antiderivative size = 183, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 9, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.692, Rules used = {3898, 2897, 3770, 3767, 8, 3768, 2660, 618, 204}

$$\frac{2\left(a^2+b^2\right)^{5/2}\tanh^{-1}\left(\frac{a-b\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{ab^5}+\frac{a\left(a^2+3b^2\right)\coth(x)}{b^4}+\frac{\left(a^2+3b^2\right)\tanh^{-1}(\cosh(x))}{2b^3}-\frac{\left(3a^2b^2+a^4+3b^4\right)\tanh^{-}}{b^5}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]^6/(a + b*Csch[x]),x]
```

```
[Out] x/a - (3*ArcTanh[Cosh[x]])/(8*b) + ((a^2 + 3*b^2)*ArcTanh[Cosh[x]])/(2*b^3)
 - ((a^4 + 3*a^2*b^2 + 3*b^4)*ArcTanh[Cosh[x]])/b^5 + (2*(a^2 + b^2)^(5/2)*
ArcTanh[(a - b*Tanh[x/2])/Sqrt[a^2 + b^2]])/(a*b^5) - (a*Coth[x])/b^2 + (a*
(a^2 + 3*b^2)*Coth[x])/b^4 + (a*Coth[x]^3)/(3*b^2) + (3*Coth[x]*Csch[x])/(8
*b) - ((a^2 + 3*b^2)*Coth[x]*Csch[x])/(2*b^3) - (Coth[x]*Csch[x]^3)/(4*b)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2897

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig[(d*sin[
e + f*x])^n*(a + b*sin[e + f*x])^m*(1 - sin[e + f*x]^2)^(p/2), x], x] /; Fr
```

```
eeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && IntegersQ[m, 2*n, p/2] && (
LtQ[m, -1] || (EqQ[m, -1] && GtQ[p, 0]))
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3898

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Int[(Cos[c + d*x]^m*(b + a*Sin[c + d*x])^n)/Sin[c + d*x]^(
m + n), x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[n] &&
 IntegerQ[m] && (IntegerQ[m/2] || LeQ[m, 1])
```

Rubi steps

$$\begin{aligned}
\int \frac{\coth^6(x)}{a+b\operatorname{csch}(x)}\,dx &= i\int \frac{\cosh(x)\coth^5(x)}{ib+ia\sinh(x)}\,dx \\
&= -\int \left(-\frac{1}{a}-\frac{\left(a^4+3a^2b^2+3b^4\right)\operatorname{csch}(x)}{b^5}-\frac{a\left(-a^2-3b^2\right)\operatorname{csch}^2(x)}{b^4}-\frac{\left(a^2+3b^2\right)\operatorname{csch}^3(x)}{b^3}\right. \\
&= \frac{x}{a}-\frac{a\int \operatorname{csch}^4(x)\,dx}{b^2}+\frac{\int \operatorname{csch}^5(x)\,dx}{b}-\frac{\left(i\left(a^2+b^2\right)^3\right)\int \frac{1}{ib+ia\sinh(x)}\,dx}{ab^5}-\frac{\left(a\left(a^2+3b^2\right)\right)\int}{b^4} \\
&= \frac{x}{a}-\frac{\left(a^4+3a^2b^2+3b^4\right)\tanh^{-1}(\cosh(x))}{b^5}-\frac{\left(a^2+3b^2\right)\coth(x)\operatorname{csch}(x)}{2b^3}-\frac{\coth(x)\operatorname{csch}^3(x)}{4b} \\
&= \frac{x}{a}+\frac{\left(a^2+3b^2\right)\tanh^{-1}(\cosh(x))}{2b^3}-\frac{\left(a^4+3a^2b^2+3b^4\right)\tanh^{-1}(\cosh(x))}{b^5}-\frac{a\coth(x)}{b^2}+ \\
&= \frac{x}{a}-\frac{3\tanh^{-1}(\cosh(x))}{8b}+\frac{\left(a^2+3b^2\right)\tanh^{-1}(\cosh(x))}{2b^3}-\frac{\left(a^4+3a^2b^2+3b^4\right)\tanh^{-1}(\cosh}{b^5}
\end{aligned}$$

Mathematica [A] time = 1.58, size = 269, normalized size = 1.47

$$\frac{\operatorname{csch}(x)(a\sinh(x)+b)\left(4a^2b^3\sinh(x)\operatorname{csch}^4\left(\frac{x}{2}\right)-64a^2b^3\sinh^4\left(\frac{x}{2}\right)\operatorname{csch}^3(x)+32a^2b\left(3a^2+7b^2\right)\tanh\left(\frac{x}{2}\right)+3\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]^6/(a + b*Csch[x]),x]
```

```
[Out] (Csch[x]*(b + a*Sinh[x])*(192*b^5*x + 384*(-a^2 - b^2)^(5/2)*ArcTan[(a - b*
Tanh[x/2])/Sqrt[-a^2 - b^2]] + 32*a^2*b*(3*a^2 + 7*b^2)*Coth[x/2] - 6*a*b^2
*(4*a^2 + 9*b^2)*Csch[x/2]^2 - 3*a*b^4*Csch[x/2]^4 + 24*a*(8*a^4 + 20*a^2*b
^2 + 15*b^4)*Log[Tanh[x/2]] - 6*a*b^2*(4*a^2 + 9*b^2)*Sech[x/2]^2 + 3*a*b^4
*Sech[x/2]^4 - 64*a^2*b^3*Csch[x]^3*Sinh[x/2]^4 + 4*a^2*b^3*Csch[x/2]^4*Sin
h[x] + 32*a^2*b*(3*a^2 + 7*b^2)*Tanh[x/2]))/(192*a*b^5*(a + b*Csch[x]))
```

fricas [B] time = 2.18, size = 3160, normalized size = 17.27

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^6/(a+b*csch(x)),x, algorithm="fricas")
```

```
[Out] 1/24*(24*b^5*x*cosh(x)^8 + 24*b^5*x*sinh(x)^8 - 6*(4*a^3*b^2 + 9*a*b^4)*cos
h(x)^7 + 6*(32*b^5*x*cosh(x) - 4*a^3*b^2 - 9*a*b^4)*sinh(x)^7 - 48*(2*b^5*x
 - a^4*b - 3*a^2*b^3)*cosh(x)^6 + 6*(112*b^5*x*cosh(x)^2 - 16*b^5*x + 8*a^4
*b + 24*a^2*b^3 - 7*(4*a^3*b^2 + 9*a*b^4)*cosh(x))*sinh(x)^6 + 24*b^5*x + 6
*(4*a^3*b^2 + a*b^4)*cosh(x)^5 + 6*(224*b^5*x*cosh(x)^3 + 4*a^3*b^2 + a*b^4
 - 21*(4*a^3*b^2 + 9*a*b^4)*cosh(x)^2 - 48*(2*b^5*x - a^4*b - 3*a^2*b^3)*co
sh(x))*sinh(x)^5 - 48*a^4*b - 112*a^2*b^3 + 48*(3*b^5*x - 3*a^4*b - 7*a^2*b
^3)*cosh(x)^4 + 6*(280*b^5*x*cosh(x)^4 + 24*b^5*x - 24*a^4*b - 56*a^2*b^3 -
 35*(4*a^3*b^2 + 9*a*b^4)*cosh(x)^3 - 120*(2*b^5*x - a^4*b - 3*a^2*b^3)*cos
h(x)^2 + 5*(4*a^3*b^2 + a*b^4)*cosh(x))*sinh(x)^4 + 6*(4*a^3*b^2 + a*b^4)*c
osh(x)^3 + 6*(224*b^5*x*cosh(x)^5 + 4*a^3*b^2 + a*b^4 - 35*(4*a^3*b^2 + 9*a
*b^4)*cosh(x)^4 - 160*(2*b^5*x - a^4*b - 3*a^2*b^3)*cosh(x)^3 + 10*(4*a^3*b
^2 + a*b^4)*cosh(x)^2 + 32*(3*b^5*x - 3*a^4*b - 7*a^2*b^3)*cosh(x))*sinh(x)
^3 - 16*(6*b^5*x - 9*a^4*b - 19*a^2*b^3)*cosh(x)^2 + 2*(336*b^5*x*cosh(x)^6
 - 48*b^5*x - 63*(4*a^3*b^2 + 9*a*b^4)*cosh(x)^5 + 72*a^4*b + 152*a^2*b^3 -
 360*(2*b^5*x - a^4*b - 3*a^2*b^3)*cosh(x)^4 + 30*(4*a^3*b^2 + a*b^4)*cosh(
x)^3 + 144*(3*b^5*x - 3*a^4*b - 7*a^2*b^3)*cosh(x)^2 + 9*(4*a^3*b^2 + a*b^4
)*cosh(x))*sinh(x)^2 + 24*((a^4 + 2*a^2*b^2 + b^4)*cosh(x)^8 + 8*(a^4 + 2*a
^2*b^2 + b^4)*cosh(x)*sinh(x)^7 + (a^4 + 2*a^2*b^2 + b^4)*sinh(x)^8 - 4*(a^
4 + 2*a^2*b^2 + b^4)*cosh(x)^6 - 4*(a^4 + 2*a^2*b^2 + b^4 - 7*(a^4 + 2*a^2*
b^2 + b^4)*cosh(x)^2)*sinh(x)^6 + 8*(7*(a^4 + 2*a^2*b^2 + b^4)*cosh(x)^3 -
3*(a^4 + 2*a^2*b^2 + b^4)*cosh(x))*sinh(x)^5 + 6*(a^4 + 2*a^2*b^2 + b^4)*co
sh(x)^4 + 2*(35*(a^4 + 2*a^2*b^2 + b^4)*cosh(x)^4 + 3*a^4 + 6*a^2*b^2 + 3*b
^4 - 30*(a^4 + 2*a^2*b^2 + b^4)*cosh(x)^2)*sinh(x)^4 + a^4 + 2*a^2*b^2 + b^
4 + 8*(7*(a^4 + 2*a^2*b^2 + b^4)*cosh(x)^5 - 10*(a^4 + 2*a^2*b^2 + b^4)*cos
h(x)^3 + 3*(a^4 + 2*a^2*b^2 + b^4)*cosh(x))*sinh(x)^3 - 4*(a^4 + 2*a^2*b^2
+ b^4)*cosh(x)^2 + 4*(7*(a^4 + 2*a^2*b^2 + b^4)*cosh(x)^6 - 15*(a^4 + 2*a^2
*b^2 + b^4)*cosh(x)^4 - a^4 - 2*a^2*b^2 - b^4 + 9*(a^4 + 2*a^2*b^2 + b^4)*c
osh(x)^2)*sinh(x)^2 + 8*((a^4 + 2*a^2*b^2 + b^4)*cosh(x)^7 - 3*(a^4 + 2*a^2
*b^2 + b^4)*cosh(x)^5 + 3*(a^4 + 2*a^2*b^2 + b^4)*cosh(x)^3 - (a^4 + 2*a^2*
b^2 + b^4)*cosh(x))*sinh(x))*sqrt(a^2 + b^2)*log((a^2*cosh(x)^2 + a^2*sinh(
x)^2 + 2*a*b*cosh(x) + a^2 + 2*b^2 + 2*(a^2*cosh(x) + a*b)*sinh(x) + 2*sqrt
(a^2 + b^2)*(a*cosh(x) + a*sinh(x) + b))/(a*cosh(x)^2 + a*sinh(x)^2 + 2*b*c
osh(x) + 2*(a*cosh(x) + b)*sinh(x) - a)) - 6*(4*a^3*b^2 + 9*a*b^4)*cosh(x)
- 3*((8*a^5 + 20*a^3*b^2 + 15*a*b^4)*cosh(x)^8 + 8*(8*a^5 + 20*a^3*b^2 + 15
*a*b^4)*cosh(x)*sinh(x)^7 + (8*a^5 + 20*a^3*b^2 + 15*a*b^4)*sinh(x)^8 - 4*(
8*a^5 + 20*a^3*b^2 + 15*a*b^4)*cosh(x)^6 - 4*(8*a^5 + 20*a^3*b^2 + 15*a*b^4
 - 7*(8*a^5 + 20*a^3*b^2 + 15*a*b^4)*cosh(x)^2)*sinh(x)^6 + 8*(7*(8*a^5 + 2
0*a^3*b^2 + 15*a*b^4)*cosh(x)^3 - 3*(8*a^5 + 20*a^3*b^2 + 15*a*b^4)*cosh(x)
)*sinh(x)^5 + 8*a^5 + 20*a^3*b^2 + 15*a*b^4 + 6*(8*a^5 + 20*a^3*b^2 + 15*a*
b^4)*cosh(x)^4 + 2*(24*a^5 + 60*a^3*b^2 + 45*a*b^4 + 35*(8*a^5 + 20*a^3*b^2
 + 15*a*b^4)*cosh(x)^4 - 30*(8*a^5 + 20*a^3*b^2 + 15*a*b^4)*cosh(x)^2)*sinh
(x)^4 + 8*(7*(8*a^5 + 20*a^3*b^2 + 15*a*b^4)*cosh(x)^5 - 10*(8*a^5 + 20*a^3
*b^2 + 15*a*b^4)*cosh(x)^3 + 3*(8*a^5 + 20*a^3*b^2 + 15*a*b^4)*cosh(x))*sin
```

```
h(x)^3 - 4*(8*a^5 + 20*a^3*b^2 + 15*a*b^4)*cosh(x)^2 + 4*(7*(8*a^5 + 20*a^3
*b^2 + 15*a*b^4)*cosh(x)^6 - 8*a^5 - 20*a^3*b^2 - 15*a*b^4 - 15*(8*a^5 + 20
*a^3*b^2 + 15*a*b^4)*cosh(x)^4 + 9*(8*a^5 + 20*a^3*b^2 + 15*a*b^4)*cosh(x)^
2)*sinh(x)^2 + 8*((8*a^5 + 20*a^3*b^2 + 15*a*b^4)*cosh(x)^7 - 3*(8*a^5 + 20
*a^3*b^2 + 15*a*b^4)*cosh(x)^5 + 3*(8*a^5 + 20*a^3*b^2 + 15*a*b^4)*cosh(x)^
3 - (8*a^5 + 20*a^3*b^2 + 15*a*b^4)*cosh(x))*sinh(x))*log(cosh(x) + sinh(x)
 + 1) + 3*((8*a^5 + 20*a^3*b^2 + 15*a*b^4)*cosh(x)^8 + 8*(8*a^5 + 20*a^3*b^
2 + 15*a*b^4)*cosh(x)*sinh(x)^7 + (8*a^5 + 20*a^3*b^2 + 15*a*b^4)*sinh(x)^8
 - 4*(8*a^5 + 20*a^3*b^2 + 15*a*b^4)*cosh(x)^6 - 4*(8*a^5 + 20*a^3*b^2 + 15
*a*b^4 - 7*(8*a^5 + 20*a^3*b^2 + 15*a*b^4)*cosh(x)^2)*sinh(x)^6 + 8*(7*(8*a
^5 + 20*a^3*b^2 + 15*a*b^4)*cosh(x)^3 - 3*(8*a^5 + 20*a^3*b^2 + 15*a*b^4)*c
osh(x))*sinh(x)^5 + 8*a^5 + 20*a^3*b^2 + 15*a*b^4 + 6*(8*a^5 + 20*a^3*b^2 +
 15*a*b^4)*cosh(x)^4 + 2*(24*a^5 + 60*a^3*b^2 + 45*a*b^4 + 35*(8*a^5 + 20*a
^3*b^2 + 15*a*b^4)*cosh(x)^4 - 30*(8*a^5 + 20*a^3*b^2 + 15*a*b^4)*cosh(x)^2
)*sinh(x)^4 + 8*(7*(8*a^5 + 20*a^3*b^2 + 15*a*b^4)*cosh(x)^5 - 10*(8*a^5 +
20*a^3*b^2 + 15*a*b^4)*cosh(x)^3 + 3*(8*a^5 + 20*a^3*b^2 + 15*a*b^4)*cosh(x
))*sinh(x)^3 - 4*(8*a^5 + 20*a^3*b^2 + 15*a*b^4)*cosh(x)^2 + 4*(7*(8*a^5 +
20*a^3*b^2 + 15*a*b^4)*cosh(x)^6 - 8*a^5 - 20*a^3*b^2 - 15*a*b^4 - 15*(8*a^
5 + 20*a^3*b^2 + 15*a*b^4)*cosh(x)^4 + 9*(8*a^5 + 20*a^3*b^2 + 15*a*b^4)*co
sh(x)^2)*sinh(x)^2 + 8*((8*a^5 + 20*a^3*b^2 + 15*a*b^4)*cosh(x)^7 - 3*(8*a^
5 + 20*a^3*b^2 + 15*a*b^4)*cosh(x)^5 + 3*(8*a^5 + 20*a^3*b^2 + 15*a*b^4)*co
sh(x)^3 - (8*a^5 + 20*a^3*b^2 + 15*a*b^4)*cosh(x))*sinh(x))*log(cosh(x) + s
inh(x) - 1) + 2*(96*b^5*x*cosh(x)^7 - 21*(4*a^3*b^2 + 9*a*b^4)*cosh(x)^6 -
144*(2*b^5*x - a^4*b - 3*a^2*b^3)*cosh(x)^5 - 12*a^3*b^2 - 27*a*b^4 + 15*(4
*a^3*b^2 + a*b^4)*cosh(x)^4 + 96*(3*b^5*x - 3*a^4*b - 7*a^2*b^3)*cosh(x)^3
+ 9*(4*a^3*b^2 + a*b^4)*cosh(x)^2 - 16*(6*b^5*x - 9*a^4*b - 19*a^2*b^3)*cos
h(x))*sinh(x))/(a*b^5*cosh(x)^8 + 8*a*b^5*cosh(x)*sinh(x)^7 + a*b^5*sinh(x)
^8 - 4*a*b^5*cosh(x)^6 + 6*a*b^5*cosh(x)^4 - 4*a*b^5*cosh(x)^2 + 4*(7*a*b^5
*cosh(x)^2 - a*b^5)*sinh(x)^6 + a*b^5 + 8*(7*a*b^5*cosh(x)^3 - 3*a*b^5*cosh
(x))*sinh(x)^5 + 2*(35*a*b^5*cosh(x)^4 - 30*a*b^5*cosh(x)^2 + 3*a*b^5)*sinh
(x)^4 + 8*(7*a*b^5*cosh(x)^5 - 10*a*b^5*cosh(x)^3 + 3*a*b^5*cosh(x))*sinh(x
)^3 + 4*(7*a*b^5*cosh(x)^6 - 15*a*b^5*cosh(x)^4 + 9*a*b^5*cosh(x)^2 - a*b^5
)*sinh(x)^2 + 8*(a*b^5*cosh(x)^7 - 3*a*b^5*cosh(x)^5 + 3*a*b^5*cosh(x)^3 -
a*b^5*cosh(x))*sinh(x))
```

giac [A] time = 0.14, size = 305, normalized size = 1.67

$$\frac{x}{a} - \frac{\left(8\,a^4 + 20\,a^2b^2 + 15\,b^4\right)\log\left(e^x+1\right)}{8\,b^5} + \frac{\left(8\,a^4 + 20\,a^2b^2 + 15\,b^4\right)\log\left(\left|e^x-1\right|\right)}{8\,b^5} - \frac{\left(a^6 + 3\,a^4b^2 + 3\,a^2b^4 + b^6\right)\log}{\sqrt{a^2+b^2}\,ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^6/(a+b*csch(x)),x, algorithm="giac")
```

```
[Out] x/a - 1/8*(8*a^4 + 20*a^2*b^2 + 15*b^4)*log(e^x + 1)/b^5 + 1/8*(8*a^4 + 20*
a^2*b^2 + 15*b^4)*log(abs(e^x - 1))/b^5 - (a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^
6)*log(abs(2*a*e^x + 2*b - 2*sqrt(a^2 + b^2))/abs(2*a*e^x + 2*b + 2*sqrt(a^
2 + b^2)))/(sqrt(a^2 + b^2)*a*b^5) - 1/12*(12*a^2*b*e^(7*x) + 27*b^3*e^(7*x
) - 24*a^3*e^(6*x) - 72*a*b^2*e^(6*x) - 12*a^2*b*e^(5*x) - 3*b^3*e^(5*x) +
72*a^3*e^(4*x) + 168*a*b^2*e^(4*x) - 12*a^2*b*e^(3*x) - 3*b^3*e^(3*x) - 72*
a^3*e^(2*x) - 152*a*b^2*e^(2*x) + 12*a^2*b*e^x + 27*b^3*e^x + 24*a^3 + 56*a
*b^2)/(b^4*(e^(2*x) - 1)^4)
```

maple [B] time = 0.17, size = 360, normalized size = 1.97

$$\frac{\tanh^4\left(\frac{x}{2}\right)}{64b} + \frac{a\left(\tanh^3\left(\frac{x}{2}\right)\right)}{24b^2} + \frac{a^2\left(\tanh^2\left(\frac{x}{2}\right)\right)}{8b^3} + \frac{\tanh^2\left(\frac{x}{2}\right)}{4b} + \frac{a^3\tanh\left(\frac{x}{2}\right)}{2b^4} + \frac{9a\tanh\left(\frac{x}{2}\right)}{8b^2} - \frac{\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{a} + \frac{\ln\left(\tan\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^6/(a+b*csch(x)),x)
```

```
[Out] 1/64/b*tanh(1/2*x)^4+1/24/b^2*a*tanh(1/2*x)^3+1/8/b^3*a^2*tanh(1/2*x)^2+1/4
/b*tanh(1/2*x)^2+1/2/b^4*a^3*tanh(1/2*x)+9/8/b^2*a*tanh(1/2*x)-1/a*ln(tanh(
1/2*x)-1)+1/a*ln(tanh(1/2*x)+1)-2*a^5/b^5/(a^2+b^2)^(1/2)*arctanh(1/2*(2*ta
nh(1/2*x)*b-2*a)/(a^2+b^2)^(1/2))-6/b^3*a^3/(a^2+b^2)^(1/2)*arctanh(1/2*(2*
tanh(1/2*x)*b-2*a)/(a^2+b^2)^(1/2))-6*a/b/(a^2+b^2)^(1/2)*arctanh(1/2*(2*ta
nh(1/2*x)*b-2*a)/(a^2+b^2)^(1/2))-2/a*b/(a^2+b^2)^(1/2)*arctanh(1/2*(2*tanh
(1/2*x)*b-2*a)/(a^2+b^2)^(1/2))-1/64/b/tanh(1/2*x)^4-1/8/b^3/tanh(1/2*x)^2*
a^2-1/4/b/tanh(1/2*x)^2+1/b^5*ln(tanh(1/2*x))*a^4+5/2/b^3*ln(tanh(1/2*x))*a
^2+15/8/b*ln(tanh(1/2*x))+1/24*a/b^2/tanh(1/2*x)^3+1/2*a^3/b^4/tanh(1/2*x)+
9/8*a/b^2/tanh(1/2*x)
```

maxima [A] time = 0.42, size = 330, normalized size = 1.80

$$-\frac{24\,a^{3}+56\,ab^{2}-3\left(4\,a^{2}b+9\,b^{3}\right)e^{(-x)}-8\left(9\,a^{3}+19\,ab^{2}\right)e^{(-2\,x)}+3\left(4\,a^{2}b+b^{3}\right)e^{(-3\,x)}+24\left(3\,a^{3}+7\,ab^{2}\right)e^{(-4\,x)}+3}{12\left(4\,b^{4}e^{(-2\,x)}-6\,b^{4}e^{(-4\,x)}+4\,b^{4}e^{(-6\,x)}-b^{4}e^{(-8\,x)}-\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^6/(a+b*csch(x)),x, algorithm="maxima")
```

```
[Out] -1/12*(24*a^3 + 56*a*b^2 - 3*(4*a^2*b + 9*b^3)*e^(-x) - 8*(9*a^3 + 19*a*b^2
)*e^(-2*x) + 3*(4*a^2*b + b^3)*e^(-3*x) + 24*(3*a^3 + 7*a*b^2)*e^(-4*x) + 3
*(4*a^2*b + b^3)*e^(-5*x) - 24*(a^3 + 3*a*b^2)*e^(-6*x) - 3*(4*a^2*b + 9*b^
3)*e^(-7*x))/(4*b^4*e^(-2*x) - 6*b^4*e^(-4*x) + 4*b^4*e^(-6*x) - b^4*e^(-8*
x) - b^4) + x/a - 1/8*(8*a^4 + 20*a^2*b^2 + 15*b^4)*log(e^(-x) + 1)/b^5 + 1
/8*(8*a^4 + 20*a^2*b^2 + 15*b^4)*log(e^(-x) - 1)/b^5 - (a^6 + 3*a^4*b^2 + 3
*a^2*b^4 + b^6)*log((a*e^(-x) - b - sqrt(a^2 + b^2))/(a*e^(-x) - b + sqrt(a
^2 + b^2)))/(sqrt(a^2 + b^2)*a*b^5)
```

mupad [B] time = 2.90, size = 543, normalized size = 2.97

$$\frac{\frac{8\,a}{3\,b^2}-\frac{6\,\mathrm{e}^x}{b}}{3\,\mathrm{e}^{2\,x}-3\,\mathrm{e}^{4\,x}+\mathrm{e}^{6\,x}-1}-\frac{\frac{\mathrm{e}^x\,\left(4\,a^2+9\,b^2\right)}{4\,b^3}-\frac{2\,\left(a^4+3\,a^2\,b^2\right)}{a\,b^4}}{\mathrm{e}^{2\,x}-1}+\frac{\frac{4\,a}{b^2}-\frac{\mathrm{e}^x\,\left(4\,a^2+13\,b^2\right)}{2\,b^3}}{\mathrm{e}^{4\,x}-2\,\mathrm{e}^{2\,x}+1}+\frac{x}{a}+\frac{\ln\left(\mathrm{e}^x-1\right)\,\left(8\,a^4+20\,a^2\,b^2+15\,b^4\right)}{8\,b^5}-\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^6/(a + b/sinh(x)),x)
```

```
[Out] ((8*a)/(3*b^2) - (6*exp(x))/b)/(3*exp(2*x) - 3*exp(4*x) + exp(6*x) - 1) - (
(exp(x)*(4*a^2 + 9*b^2))/(4*b^3) - (2*(a^4 + 3*a^2*b^2))/(a*b^4))/(exp(2*x)
 - 1) + ((4*a)/b^2 - (exp(x)*(4*a^2 + 13*b^2))/(2*b^3))/(exp(4*x) - 2*exp(2
*x) + 1) + x/a + (log(exp(x) - 1)*(8*a^4 + 15*b^4 + 20*a^2*b^2))/(8*b^5) -
(log(exp(x) + 1)*(8*a^4 + 15*b^4 + 20*a^2*b^2))/(8*b^5) - (4*exp(x))/(b*(6*
exp(4*x) - 4*exp(2*x) - 4*exp(6*x) + exp(8*x) + 1)) + (log(a^3*((a^2 + b^2)
^5)^(1/2) - 2*a^7*b - 2*a*b^7 - 6*a^3*b^5 - 6*a^5*b^3 + a^8*exp(x) + 4*b^8*
exp(x) + 2*a*b^2*((a^2 + b^2)^5)^(1/2) - 4*b^3*exp(x)*((a^2 + b^2)^5)^(1/2)
 + 13*a^2*b^6*exp(x) + 15*a^4*b^4*exp(x) + 7*a^6*b^2*exp(x) - 3*a^2*b*exp(x
)*((a^2 + b^2)^5)^(1/2))*((a^2 + b^2)^5)^(1/2))/(a*b^5) - (log(a^8*exp(x) -
 2*a^7*b - a^3*((a^2 + b^2)^5)^(1/2) - 6*a^3*b^5 - 6*a^5*b^3 - 2*a*b^7 + 4*
b^8*exp(x) - 2*a*b^2*((a^2 + b^2)^5)^(1/2) + 4*b^3*exp(x)*((a^2 + b^2)^5)^(
1/2) + 13*a^2*b^6*exp(x) + 15*a^4*b^4*exp(x) + 7*a^6*b^2*exp(x) + 3*a^2*b*e
xp(x)*((a^2 + b^2)^5)^(1/2))*((a^2 + b^2)^5)^(1/2))/(a*b^5)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\coth^{6}\left(x\right)}{a+b\,\mathrm{csch}\left(x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)**6/(a+b*csch(x)),x)
```

```
[Out] Integral(coth(x)**6/(a + b*csch(x)), x)
```

3.124 $\int \frac{\coth^7(x)}{a+b\operatorname{csch}(x)} dx$

Optimal. Leaf size=119

$$\frac{(a^2+b^2)^3 \log(a+b\operatorname{csch}(x))}{ab^6}+\frac{a(a^2+3b^2)\operatorname{csch}^2(x)}{2b^4}-\frac{(a^2+3b^2)\operatorname{csch}^3(x)}{3b^3}-\frac{(a^4+3a^2b^2+3b^4)\operatorname{csch}(x)}{b^5}+\frac{a\operatorname{csch}^4(x)}{4b^2}$$

```
[Out] -(a^4+3*a^2*b^2+3*b^4)*csch(x)/b^5+1/2*a*(a^2+3*b^2)*csch(x)^2/b^4-1/3*(a^2
+3*b^2)*csch(x)^3/b^3+1/4*a*csch(x)^4/b^2-1/5*csch(x)^5/b+(a^2+b^2)^3*ln(a+
b*csch(x))/a/b^6+ln(sinh(x))/a
```

Rubi [A] time = 0.14, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3885, 894}

$$-\frac{(a^2+3b^2)\operatorname{csch}^3(x)}{3b^3}+\frac{a(a^2+3b^2)\operatorname{csch}^2(x)}{2b^4}-\frac{(3a^2b^2+a^4+3b^4)\operatorname{csch}(x)}{b^5}+\frac{(a^2+b^2)^3 \log(a+b\operatorname{csch}(x))}{ab^6}+\frac{a\operatorname{csch}^4}{4b^2}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]^7/(a + b*Csch[x]),x]
```

```
[Out] -(((a^4 + 3*a^2*b^2 + 3*b^4)*Csch[x])/b^5) + (a*(a^2 + 3*b^2)*Csch[x]^2)/(2
*b^4) - ((a^2 + 3*b^2)*Csch[x]^3)/(3*b^3) + (a*Csch[x]^4)/(4*b^2) - Csch[x]
^5/(5*b) + ((a^2 + b^2)^3*Log[a + b*Csch[x]])/(a*b^6) + Log[Sinh[x]]/a
```

Rule 894

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 3885

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> -Dist[(-1)^((m - 1)/2)/(d*b^(m - 1)), Subst[Int[((b^2 - x^
2)^((m - 1)/2)*(a + x)^n)/x, x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\coth^7(x)}{a+b\operatorname{csch}(x)} dx &= \frac{\operatorname{Subst}\left(\int \frac{(-b^2-x^2)^3}{x(a+x)} dx, x, b\operatorname{csch}(x)\right)}{b^6}\\ &= \frac{\operatorname{Subst}\left(\int \left(-a^4\left(1+\frac{3b^2(a^2+b^2)}{a^4}\right)-\frac{b^6}{ax}+a(a^2+3b^2)x-(a^2+3b^2)x^2+ax^3-x^4+\frac{(a^2+b^2)^3}{a(a+x)}\right.\right.}{b^6}\\ &= -\frac{(a^4+3a^2b^2+3b^4)\operatorname{csch}(x)}{b^5}+\frac{a(a^2+3b^2)\operatorname{csch}^2(x)}{2b^4}-\frac{(a^2+3b^2)\operatorname{csch}^3(x)}{3b^3}+\frac{a\operatorname{csch}^4(x)}{4b^2}-\end{aligned}$$

Mathematica [A] time = 0.26, size = 130, normalized size = 1.09

$$\frac{30ab^2(a^2+3b^2)\operatorname{csch}^2(x)+\frac{60(a^2+b^2)^3\log(a\sinh(x)+b)}{a}-20b^3(a^2+3b^2)\operatorname{csch}^3(x)-60b(a^4+3a^2b^2+3b^4)\operatorname{csch}(x)-}{60b^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]^7/(a + b*Csch[x]),x]
```

```
[Out] (-60*b*(a^4 + 3*a^2*b^2 + 3*b^4)*Csch[x] + 30*a*b^2*(a^2 + 3*b^2)*Csch[x]^2
 - 20*b^3*(a^2 + 3*b^2)*Csch[x]^3 + 15*a*b^4*Csch[x]^4 - 12*b^5*Csch[x]^5 -
 60*a*(a^4 + 3*a^2*b^2 + 3*b^4)*Log[Sinh[x]] + (60*(a^2 + b^2)^3*Log[b + a*
Sinh[x]])/a)/(60*b^6)
```

fricas [B] time = 0.71, size = 4024, normalized size = 33.82

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^7/(a+b*csch(x)),x, algorithm="fricas")
```

```
[Out] -1/15*(15*b^6*x*cosh(x)^10 + 15*b^6*x*sinh(x)^10 + 30*(a^5*b + 3*a^3*b^3 +
3*a*b^5)*cosh(x)^9 + 30*(5*b^6*x*cosh(x) + a^5*b + 3*a^3*b^3 + 3*a*b^5)*sin
h(x)^9 - 15*(5*b^6*x + 2*a^4*b^2 + 6*a^2*b^4)*cosh(x)^8 + 15*(45*b^6*x*cosh
(x)^2 - 5*b^6*x - 2*a^4*b^2 - 6*a^2*b^4 + 18*(a^5*b + 3*a^3*b^3 + 3*a*b^5)*
cosh(x))*sinh(x)^8 - 40*(3*a^5*b + 8*a^3*b^3 + 6*a*b^5)*cosh(x)^7 + 40*(45*
b^6*x*cosh(x)^3 - 3*a^5*b - 8*a^3*b^3 - 6*a*b^5 + 27*(a^5*b + 3*a^3*b^3 + 3
*a*b^5)*cosh(x)^2 - 3*(5*b^6*x + 2*a^4*b^2 + 6*a^2*b^4)*cosh(x))*sinh(x)^7
- 15*b^6*x + 30*(5*b^6*x + 3*a^4*b^2 + 7*a^2*b^4)*cosh(x)^6 + 10*(315*b^6*x
*cosh(x)^4 + 15*b^6*x + 9*a^4*b^2 + 21*a^2*b^4 + 252*(a^5*b + 3*a^3*b^3 + 3
*a*b^5)*cosh(x)^3 - 42*(5*b^6*x + 2*a^4*b^2 + 6*a^2*b^4)*cosh(x)^2 - 28*(3*
a^5*b + 8*a^3*b^3 + 6*a*b^5)*cosh(x))*sinh(x)^6 + 4*(45*a^5*b + 115*a^3*b^3
 + 99*a*b^5)*cosh(x)^5 + 4*(945*b^6*x*cosh(x)^5 + 45*a^5*b + 115*a^3*b^3 +
99*a*b^5 + 945*(a^5*b + 3*a^3*b^3 + 3*a*b^5)*cosh(x)^4 - 210*(5*b^6*x + 2*a
^4*b^2 + 6*a^2*b^4)*cosh(x)^3 - 210*(3*a^5*b + 8*a^3*b^3 + 6*a*b^5)*cosh(x)
^2 + 45*(5*b^6*x + 3*a^4*b^2 + 7*a^2*b^4)*cosh(x))*sinh(x)^5 - 30*(5*b^6*x
+ 3*a^4*b^2 + 7*a^2*b^4)*cosh(x)^4 + 10*(315*b^6*x*cosh(x)^6 - 15*b^6*x - 9
*a^4*b^2 - 21*a^2*b^4 + 378*(a^5*b + 3*a^3*b^3 + 3*a*b^5)*cosh(x)^5 - 105*(
5*b^6*x + 2*a^4*b^2 + 6*a^2*b^4)*cosh(x)^4 - 140*(3*a^5*b + 8*a^3*b^3 + 6*a
*b^5)*cosh(x)^3 + 45*(5*b^6*x + 3*a^4*b^2 + 7*a^2*b^4)*cosh(x)^2 + 2*(45*a^
5*b + 115*a^3*b^3 + 99*a*b^5)*cosh(x))*sinh(x)^4 - 40*(3*a^5*b + 8*a^3*b^3
+ 6*a*b^5)*cosh(x)^3 + 40*(45*b^6*x*cosh(x)^7 + 63*(a^5*b + 3*a^3*b^3 + 3*a
*b^5)*cosh(x)^6 - 3*a^5*b - 8*a^3*b^3 - 6*a*b^5 - 21*(5*b^6*x + 2*a^4*b^2 +
 6*a^2*b^4)*cosh(x)^5 - 35*(3*a^5*b + 8*a^3*b^3 + 6*a*b^5)*cosh(x)^4 + 15*(
5*b^6*x + 3*a^4*b^2 + 7*a^2*b^4)*cosh(x)^3 + (45*a^5*b + 115*a^3*b^3 + 99*a
*b^5)*cosh(x)^2 - 3*(5*b^6*x + 3*a^4*b^2 + 7*a^2*b^4)*cosh(x))*sinh(x)^3 +
15*(5*b^6*x + 2*a^4*b^2 + 6*a^2*b^4)*cosh(x)^2 + 5*(135*b^6*x*cosh(x)^8 + 2
16*(a^5*b + 3*a^3*b^3 + 3*a*b^5)*cosh(x)^7 + 15*b^6*x - 84*(5*b^6*x + 2*a^4
*b^2 + 6*a^2*b^4)*cosh(x)^6 + 6*a^4*b^2 + 18*a^2*b^4 - 168*(3*a^5*b + 8*a^3
*b^3 + 6*a*b^5)*cosh(x)^5 + 90*(5*b^6*x + 3*a^4*b^2 + 7*a^2*b^4)*cosh(x)^4
+ 8*(45*a^5*b + 115*a^3*b^3 + 99*a*b^5)*cosh(x)^3 - 36*(5*b^6*x + 3*a^4*b^2
 + 7*a^2*b^4)*cosh(x)^2 - 24*(3*a^5*b + 8*a^3*b^3 + 6*a*b^5)*cosh(x))*sinh(
x)^2 + 30*(a^5*b + 3*a^3*b^3 + 3*a*b^5)*cosh(x) - 15*((a^6 + 3*a^4*b^2 + 3*
a^2*b^4 + b^6)*cosh(x)^10 + 10*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*cosh(x)*
sinh(x)^9 + (a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*sinh(x)^10 - 5*(a^6 + 3*a^4
*b^2 + 3*a^2*b^4 + b^6)*cosh(x)^8 - 5*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6 -
9*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*cosh(x)^2)*sinh(x)^8 + 40*(3*(a^6 + 3
*a^4*b^2 + 3*a^2*b^4 + b^6)*cosh(x)^3 - (a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)
*cosh(x))*sinh(x)^7 + 10*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*cosh(x)^6 + 10
*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6 + 21*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6
)*cosh(x)^4 - 14*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*cosh(x)^2)*sinh(x)^6 -
 a^6 - 3*a^4*b^2 - 3*a^2*b^4 - b^6 + 4*(63*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b
^6)*cosh(x)^5 - 70*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*cosh(x)^3 + 15*(a^6
+ 3*a^4*b^2 + 3*a^2*b^4 + b^6)*cosh(x))*sinh(x)^5 - 10*(a^6 + 3*a^4*b^2 + 3
*a^2*b^4 + b^6)*cosh(x)^4 + 10*(21*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*cosh
```

```
(x)^6 - a^6 - 3*a^4*b^2 - 3*a^2*b^4 - b^6 - 35*(a^6 + 3*a^4*b^2 + 3*a^2*b^4
 + b^6)*cosh(x)^4 + 15*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*cosh(x)^2)*sinh(
x)^4 + 40*(3*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*cosh(x)^7 - 7*(a^6 + 3*a^4
*b^2 + 3*a^2*b^4 + b^6)*cosh(x)^5 + 5*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*c
osh(x)^3 - (a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*cosh(x))*sinh(x)^3 + 5*(a^6
+ 3*a^4*b^2 + 3*a^2*b^4 + b^6)*cosh(x)^2 + 5*(9*(a^6 + 3*a^4*b^2 + 3*a^2*b^
4 + b^6)*cosh(x)^8 - 28*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*cosh(x)^6 + a^6
 + 3*a^4*b^2 + 3*a^2*b^4 + b^6 + 30*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*cos
h(x)^4 - 12*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*cosh(x)^2)*sinh(x)^2 + 10*(
(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*cosh(x)^9 - 4*(a^6 + 3*a^4*b^2 + 3*a^2*
b^4 + b^6)*cosh(x)^7 + 6*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*cosh(x)^5 - 4*
(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*cosh(x)^3 + (a^6 + 3*a^4*b^2 + 3*a^2*b^
4 + b^6)*cosh(x))*sinh(x))*log(2*(a*sinh(x) + b)/(cosh(x) - sinh(x))) + 15*
((a^6 + 3*a^4*b^2 + 3*a^2*b^4)*cosh(x)^10 + 10*(a^6 + 3*a^4*b^2 + 3*a^2*b^4
)*cosh(x)*sinh(x)^9 + (a^6 + 3*a^4*b^2 + 3*a^2*b^4)*sinh(x)^10 - 5*(a^6 + 3
*a^4*b^2 + 3*a^2*b^4)*cosh(x)^8 - 5*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 - 9*(a^6 +
 3*a^4*b^2 + 3*a^2*b^4)*cosh(x)^2)*sinh(x)^8 + 40*(3*(a^6 + 3*a^4*b^2 + 3*a
^2*b^4)*cosh(x)^3 - (a^6 + 3*a^4*b^2 + 3*a^2*b^4)*cosh(x))*sinh(x)^7 + 10*(
a^6 + 3*a^4*b^2 + 3*a^2*b^4)*cosh(x)^6 + 10*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 +
21*(a^6 + 3*a^4*b^2 + 3*a^2*b^4)*cosh(x)^4 - 14*(a^6 + 3*a^4*b^2 + 3*a^2*b^
4)*cosh(x)^2)*sinh(x)^6 - a^6 - 3*a^4*b^2 - 3*a^2*b^4 + 4*(63*(a^6 + 3*a^4*
b^2 + 3*a^2*b^4)*cosh(x)^5 - 70*(a^6 + 3*a^4*b^2 + 3*a^2*b^4)*cosh(x)^3 + 1
5*(a^6 + 3*a^4*b^2 + 3*a^2*b^4)*cosh(x))*sinh(x)^5 - 10*(a^6 + 3*a^4*b^2 +
3*a^2*b^4)*cosh(x)^4 + 10*(21*(a^6 + 3*a^4*b^2 + 3*a^2*b^4)*cosh(x)^6 - a^6
 - 3*a^4*b^2 - 3*a^2*b^4 - 35*(a^6 + 3*a^4*b^2 + 3*a^2*b^4)*cosh(x)^4 + 15*
(a^6 + 3*a^4*b^2 + 3*a^2*b^4)*cosh(x)^2)*sinh(x)^4 + 40*(3*(a^6 + 3*a^4*b^2
 + 3*a^2*b^4)*cosh(x)^7 - 7*(a^6 + 3*a^4*b^2 + 3*a^2*b^4)*cosh(x)^5 + 5*(a^
6 + 3*a^4*b^2 + 3*a^2*b^4)*cosh(x)^3 - (a^6 + 3*a^4*b^2 + 3*a^2*b^4)*cosh(x
))*sinh(x)^3 + 5*(a^6 + 3*a^4*b^2 + 3*a^2*b^4)*cosh(x)^2 + 5*(9*(a^6 + 3*a^
4*b^2 + 3*a^2*b^4)*cosh(x)^8 - 28*(a^6 + 3*a^4*b^2 + 3*a^2*b^4)*cosh(x)^6 +
 a^6 + 3*a^4*b^2 + 3*a^2*b^4 + 30*(a^6 + 3*a^4*b^2 + 3*a^2*b^4)*cosh(x)^4 -
 12*(a^6 + 3*a^4*b^2 + 3*a^2*b^4)*cosh(x)^2)*sinh(x)^2 + 10*((a^6 + 3*a^4*b
^2 + 3*a^2*b^4)*cosh(x)^9 - 4*(a^6 + 3*a^4*b^2 + 3*a^2*b^4)*cosh(x)^7 + 6*(
a^6 + 3*a^4*b^2 + 3*a^2*b^4)*cosh(x)^5 - 4*(a^6 + 3*a^4*b^2 + 3*a^2*b^4)*co
sh(x)^3 + (a^6 + 3*a^4*b^2 + 3*a^2*b^4)*cosh(x))*sinh(x))*log(2*sinh(x)/(co
sh(x) - sinh(x))) + 10*(15*b^6*x*cosh(x)^9 + 27*(a^5*b + 3*a^3*b^3 + 3*a*b^
5)*cosh(x)^8 - 12*(5*b^6*x + 2*a^4*b^2 + 6*a^2*b^4)*cosh(x)^7 - 28*(3*a^5*b
 + 8*a^3*b^3 + 6*a*b^5)*cosh(x)^6 + 3*a^5*b + 9*a^3*b^3 + 9*a*b^5 + 18*(5*b
^6*x + 3*a^4*b^2 + 7*a^2*b^4)*cosh(x)^5 + 2*(45*a^5*b + 115*a^3*b^3 + 99*a*
b^5)*cosh(x)^4 - 12*(5*b^6*x + 3*a^4*b^2 + 7*a^2*b^4)*cosh(x)^3 - 12*(3*a^5
*b + 8*a^3*b^3 + 6*a*b^5)*cosh(x)^2 + 3*(5*b^6*x + 2*a^4*b^2 + 6*a^2*b^4)*c
osh(x))*sinh(x))/(a*b^6*cosh(x)^10 + 10*a*b^6*cosh(x)*sinh(x)^9 + a*b^6*sin
h(x)^10 - 5*a*b^6*cosh(x)^8 + 10*a*b^6*cosh(x)^6 - 10*a*b^6*cosh(x)^4 + 5*a
*b^6*cosh(x)^2 + 5*(9*a*b^6*cosh(x)^2 - a*b^6)*sinh(x)^8 + 40*(3*a*b^6*cosh
(x)^3 - a*b^6*cosh(x))*sinh(x)^7 - a*b^6 + 10*(21*a*b^6*cosh(x)^4 - 14*a*b^
6*cosh(x)^2 + a*b^6)*sinh(x)^6 + 4*(63*a*b^6*cosh(x)^5 - 70*a*b^6*cosh(x)^3
 + 15*a*b^6*cosh(x))*sinh(x)^5 + 10*(21*a*b^6*cosh(x)^6 - 35*a*b^6*cosh(x)^
4 + 15*a*b^6*cosh(x)^2 - a*b^6)*sinh(x)^4 + 40*(3*a*b^6*cosh(x)^7 - 7*a*b^6
*cosh(x)^5 + 5*a*b^6*cosh(x)^3 - a*b^6*cosh(x))*sinh(x)^3 + 5*(9*a*b^6*cosh
(x)^8 - 28*a*b^6*cosh(x)^6 + 30*a*b^6*cosh(x)^4 - 12*a*b^6*cosh(x)^2 + a*b^
6)*sinh(x)^2 + 10*(a*b^6*cosh(x)^9 - 4*a*b^6*cosh(x)^7 + 6*a*b^6*cosh(x)^5
- 4*a*b^6*cosh(x)^3 + a*b^6*cosh(x))*sinh(x))
```

giac [B] time = 0.15, size = 295, normalized size = 2.48

$$-\frac{\left(a^5+3\,a^3b^2+3\,ab^4\right)\log\left(\left|-e^{(-x)}+e^x\right|\right)}{b^6}+\frac{\left(a^6+3\,a^4b^2+3\,a^2b^4+b^6\right)\log\left(\left|-a\left(e^{(-x)}-e^x\right)+2\,b\right|\right)}{ab^6}+\frac{137\,a^5\left(e^{(-x)}-\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^7/(a+b*csch(x)),x, algorithm="giac")
```

```
[Out] -(a^5 + 3*a^3*b^2 + 3*a*b^4)*log(abs(-e^(-x) + e^x))/b^6 + (a^6 + 3*a^4*b^2
 + 3*a^2*b^4 + b^6)*log(abs(-a*(e^(-x) - e^x) + 2*b))/(a*b^6) + 1/60*(137*a
^5*(e^(-x) - e^x)^5 + 411*a^3*b^2*(e^(-x) - e^x)^5 + 411*a*b^4*(e^(-x) - e^
x)^5 + 120*a^4*b*(e^(-x) - e^x)^4 + 360*a^2*b^3*(e^(-x) - e^x)^4 + 360*b^5*
(e^(-x) - e^x)^4 + 120*a^3*b^2*(e^(-x) - e^x)^3 + 360*a*b^4*(e^(-x) - e^x)^
3 + 160*a^2*b^3*(e^(-x) - e^x)^2 + 480*b^5*(e^(-x) - e^x)^2 + 240*a*b^4*(e^
(-x) - e^x) + 384*b^5)/(b^6*(e^(-x) - e^x)^5)
```

maple [B] time = 0.16, size = 388, normalized size = 3.26

$$-\frac{\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)}{a}-\frac{\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{a}+\frac{3a\ln\left(\left(\tanh^2\left(\frac{x}{2}\right)\right)b-2a\tanh\left(\frac{x}{2}\right)-b\right)}{b^2}-\frac{3a\ln\left(\tanh\left(\frac{x}{2}\right)\right)}{b^2}+\frac{19\tanh\left(\frac{x}{2}\right.}{16b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^7/(a+b*csch(x)),x)
```

```
[Out] -1/a*ln(tanh(1/2*x)+1)-1/a*ln(tanh(1/2*x)-1)-3*a/b^2*ln(tanh(1/2*x))+3*a/b^
2*ln(tanh(1/2*x)^2*b-2*a*tanh(1/2*x)-b)+19/16/b*tanh(1/2*x)-1/b^6*a^5*ln(ta
nh(1/2*x))+1/64/b^2*a*tanh(1/2*x)^4+1/24/b^3*a^2*tanh(1/2*x)^3+1/8/b^4*tanh
(1/2*x)^2*a^3+1/2/b^5*a^4*tanh(1/2*x)+1/64*a/b^2/tanh(1/2*x)^4-1/2/b^5/tanh
(1/2*x)*a^4+1/8*a^3/b^4/tanh(1/2*x)^2+a^5/b^6*ln(tanh(1/2*x)^2*b-2*a*tanh(1
/2*x)-b)-1/24/b^3/tanh(1/2*x)^3*a^2-1/160/b/tanh(1/2*x)^5+1/160/b*tanh(1/2*
x)^5+5/16/b^2*a*tanh(1/2*x)^2+11/8/b^3*a^2*tanh(1/2*x)+3*a^3/b^4*ln(tanh(1/
2*x)^2*b-2*a*tanh(1/2*x)-b)-11/8/b^3/tanh(1/2*x)*a^2+5/16*a/b^2/tanh(1/2*x)
^2-3/b^4*a^3*ln(tanh(1/2*x))+3/32/b*tanh(1/2*x)^3-3/32/b/tanh(1/2*x)^3+1/a*
ln(tanh(1/2*x)^2*b-2*a*tanh(1/2*x)-b)-19/16/b/tanh(1/2*x)
```

maxima [B] time = 0.33, size = 364, normalized size = 3.06

$$\frac{2\left(15\left(a^4+3a^2b^2+3b^4\right)e^{(-x)}-15\left(a^3b+3ab^3\right)e^{(-2x)}-20\left(3a^4+8a^2b^2+6b^4\right)e^{(-3x)}+15\left(3a^3b+7ab^3\right)e^{(-4\ldots}}{15\left(5b^5e^{(-2x)}\ldots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^7/(a+b*csch(x)),x, algorithm="maxima")
```

```
[Out] 2/15*(15*(a^4 + 3*a^2*b^2 + 3*b^4)*e^(-x) - 15*(a^3*b + 3*a*b^3)*e^(-2*x) -
 20*(3*a^4 + 8*a^2*b^2 + 6*b^4)*e^(-3*x) + 15*(3*a^3*b + 7*a*b^3)*e^(-4*x)
+ 2*(45*a^4 + 115*a^2*b^2 + 99*b^4)*e^(-5*x) - 15*(3*a^3*b + 7*a*b^3)*e^(-6
*x) - 20*(3*a^4 + 8*a^2*b^2 + 6*b^4)*e^(-7*x) + 15*(a^3*b + 3*a*b^3)*e^(-8*
x) + 15*(a^4 + 3*a^2*b^2 + 3*b^4)*e^(-9*x))/(5*b^5*e^(-2*x) - 10*b^5*e^(-4*
x) + 10*b^5*e^(-6*x) - 5*b^5*e^(-8*x) + b^5*e^(-10*x) - b^5) + x/a - (a^5 +
 3*a^3*b^2 + 3*a*b^4)*log(e^(-x) + 1)/b^6 - (a^5 + 3*a^3*b^2 + 3*a*b^4)*log
(e^(-x) - 1)/b^6 + (a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*log(-2*b*e^(-x) + a*
e^(-2*x) - a)/(a*b^6)
```

mupad [B] time = 2.18, size = 317, normalized size = 2.66

$$\frac{\frac{4a}{b^2}-\frac{64e^x}{5b}}{6e^{4x}-4e^{2x}-4e^{6x}+e^{8x}+1}+\frac{\frac{8a}{b^2}-\frac{8e^x\left(5a^2+27b^2\right)}{15b^3}}{3e^{2x}-3e^{4x}+e^{6x}-1}-\frac{\frac{8e^x\left(a^2+3b^2\right)}{3b^3}-\frac{2\left(a^4+5a^2b^2\right)}{ab^4}}{e^{4x}-2e^{2x}+1}-\frac{x}{a}-\frac{\frac{2e^x\left(a^4+3a^2b^2+3b^4\right)}{b^5}-\frac{2(\ldots}{}}{e^{2x}-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^7/(a + b/sinh(x)),x)
```

```
[Out] ((4*a)/b^2 - (64*exp(x))/(5*b))/(6*exp(4*x) - 4*exp(2*x) - 4*exp(6*x) + exp
(8*x) + 1) + ((8*a)/b^2 - (8*exp(x)*(5*a^2 + 27*b^2))/(15*b^3))/(3*exp(2*x)
```

```
 - 3*exp(4*x) + exp(6*x) - 1) - ((8*exp(x)*(a^2 + 3*b^2))/(3*b^3) - (2*(a^4
 + 5*a^2*b^2))/(a*b^4))/(exp(4*x) - 2*exp(2*x) + 1) - x/a - ((2*exp(x)*(a^4
 + 3*b^4 + 3*a^2*b^2))/b^5 - (2*(a^4 + 3*a^2*b^2))/(a*b^4))/(exp(2*x) - 1)
- (32*exp(x))/(5*b*(5*exp(2*x) - 10*exp(4*x) + 10*exp(6*x) - 5*exp(8*x) + e
xp(10*x) - 1)) - (log(exp(2*x) - 1)*(3*a*b^4 + a^5 + 3*a^3*b^2))/b^6 + (log
(2*b*exp(x) - a + a*exp(2*x))*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2))/(a*b^6)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\coth^7(x)}{a + b\operatorname{csch}(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)**7/(a+b*csch(x)),x)
```

```
[Out] Integral(coth(x)**7/(a + b*csch(x)), x)
```

3.125 $\int e^{c(a+bx)}\operatorname{csch}^2(ac+bcx)^{7/2}\,dx$

Optimal. Leaf size=199

$$\frac{64\sinh(ac+bcx)\sqrt{\operatorname{csch}^2(ac+bcx)}}{3bc\left(1-e^{2c(a+bx)}\right)^3}-\frac{48\sinh(ac+bcx)\sqrt{\operatorname{csch}^2(ac+bcx)}}{bc\left(1-e^{2c(a+bx)}\right)^4}+\frac{192\sinh(ac+bcx)\sqrt{\operatorname{csch}^2(ac+b}}{5bc\left(1-e^{2c(a+bx)}\right)^5}$$

```
[Out] -32/3*sinh(b*c*x+a*c)*(csch(b*c*x+a*c)^2)^(1/2)/b/c/(1-exp(2*c*(b*x+a)))^6+
192/5*sinh(b*c*x+a*c)*(csch(b*c*x+a*c)^2)^(1/2)/b/c/(1-exp(2*c*(b*x+a)))^5-
48*sinh(b*c*x+a*c)*(csch(b*c*x+a*c)^2)^(1/2)/b/c/(1-exp(2*c*(b*x+a)))^4+64/
3*sinh(b*c*x+a*c)*(csch(b*c*x+a*c)^2)^(1/2)/b/c/(1-exp(2*c*(b*x+a)))^3
```

Rubi [A] time = 0.30, antiderivative size = 199, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {6720, 2282, 12, 266, 43}

$$\frac{64\sinh(ac+bcx)\sqrt{\operatorname{csch}^2(ac+bcx)}}{3bc\left(1-e^{2c(a+bx)}\right)^3}-\frac{48\sinh(ac+bcx)\sqrt{\operatorname{csch}^2(ac+bcx)}}{bc\left(1-e^{2c(a+bx)}\right)^4}+\frac{192\sinh(ac+bcx)\sqrt{\operatorname{csch}^2(ac+b}}{5bc\left(1-e^{2c(a+bx)}\right)^5}$$

Antiderivative was successfully verified.

```
[In] Int[E^(c*(a + b*x))*(Csch[a*c + b*c*x]^2)^(7/2),x]
```

```
[Out] (-32*Sqrt[Csch[a*c + b*c*x]^2]*Sinh[a*c + b*c*x])/(3*b*c*(1 - E^(2*c*(a + b
*x)))^6) + (192*Sqrt[Csch[a*c + b*c*x]^2]*Sinh[a*c + b*c*x])/(5*b*c*(1 - E^
(2*c*(a + b*x)))^5) - (48*Sqrt[Csch[a*c + b*c*x]^2]*Sinh[a*c + b*c*x])/(b*c
*(1 - E^(2*c*(a + b*x)))^4) + (64*Sqrt[Csch[a*c + b*c*x]^2]*Sinh[a*c + b*c*
x])/(3*b*c*(1 - E^(2*c*(a + b*x)))^3)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6720

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a*v^m)^
FracPart[p])/v^(m*FracPart[p]), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
```

```
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rubi steps

$$\int e^{c(a+bx)}\text{csch}^2(ac+bcx)^{7/2}\,dx = \left(\sqrt{\text{csch}^2(ac+bcx)}\,\sinh(ac+bcx)\right)\int e^{c(a+bx)}\text{csch}^7(ac+bcx)\,dx$$
$$= \frac{\left(\sqrt{\text{csch}^2(ac+bcx)}\,\sinh(ac+bcx)\right)\text{Subst}\left(\int\frac{128x^7}{\left(-1+x^2\right)^7}\,dx,x,e^{c(a+bx)}\right)}{bc}$$
$$= \frac{\left(128\sqrt{\text{csch}^2(ac+bcx)}\,\sinh(ac+bcx)\right)\text{Subst}\left(\int\frac{x^7}{\left(-1+x^2\right)^7}\,dx,x,e^{c(a+bx)}\right)}{bc}$$
$$= \frac{\left(64\sqrt{\text{csch}^2(ac+bcx)}\,\sinh(ac+bcx)\right)\text{Subst}\left(\int\frac{x^3}{(-1+x)^7}\,dx,x,e^{2c(a+bx)}\right)}{bc}$$
$$= \frac{\left(64\sqrt{\text{csch}^2(ac+bcx)}\,\sinh(ac+bcx)\right)\text{Subst}\left(\int\left(\frac{1}{(-1+x)^7}+\frac{3}{(-1+x)^6}+\frac{3}{(-1+x)^5}+\frac{}{(}\right.\right.}{bc}$$
$$= -\frac{32\sqrt{\text{csch}^2(ac+bcx)}\,\sinh(ac+bcx)}{3bc\left(1-e^{2c(a+bx)}\right)^6}+\frac{192\sqrt{\text{csch}^2(ac+bcx)}\,\sinh(ac+bcx)}{5bc\left(1-e^{2c(a+bx)}\right)^5}$$

Mathematica [A] time = 0.08, size = 84, normalized size = 0.42

$$-\frac{16\left(6e^{2c(a+bx)}-15e^{4c(a+bx)}+20e^{6c(a+bx)}-1\right)\sinh(c(a+bx))\sqrt{\text{csch}^2(c(a+bx))}}{15bc\left(e^{2c(a+bx)}-1\right)^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(c*(a + b*x))*(Csch[a*c + b*c*x]^2)^(7/2),x]
```

```
[Out] (-16*(-1 + 6*E^(2*c*(a + b*x)) - 15*E^(4*c*(a + b*x)) + 20*E^(6*c*(a + b*x)
))*Sqrt[Csch[c*(a + b*x)]^2]*Sinh[c*(a + b*x)])/(15*b*c*(-1 + E^(2*c*(a + b
*x)))^6)
```

fricas [B] time = 0.64, size = 592, normalized size = 2.97

$$-\frac{}{15\left(bc\cosh\left(bcx+ac\right)^9+9\,bc\cosh\left(bcx+ac\right)\sinh\left(bcx+ac\right)^8+bc\sinh\left(bcx+ac\right)^9-6\,bc\cosh\left(bcx+ac\right)^7+6\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*(csch(b*c*x+a*c)^2)^(7/2),x, algorithm="fricas")
```

```
[Out] -16/15*(19*cosh(b*c*x + a*c)^3 + 57*cosh(b*c*x + a*c)*sinh(b*c*x + a*c)^2 +
 21*sinh(b*c*x + a*c)^3 + 21*(3*cosh(b*c*x + a*c)^2 - 1)*sinh(b*c*x + a*c)
- 9*cosh(b*c*x + a*c))/(b*c*cosh(b*c*x + a*c)^9 + 9*b*c*cosh(b*c*x + a*c)*s
inh(b*c*x + a*c)^8 + b*c*sinh(b*c*x + a*c)^9 - 6*b*c*cosh(b*c*x + a*c)^7 +
6*(6*b*c*cosh(b*c*x + a*c)^2 - b*c)*sinh(b*c*x + a*c)^7 + 15*b*c*cosh(b*c*x
 + a*c)^5 + 42*(2*b*c*cosh(b*c*x + a*c)^3 - b*c*cosh(b*c*x + a*c))*sinh(b*c
*x + a*c)^6 + 3*(42*b*c*cosh(b*c*x + a*c)^4 - 42*b*c*cosh(b*c*x + a*c)^2 +
5*b*c)*sinh(b*c*x + a*c)^5 - 19*b*c*cosh(b*c*x + a*c)^3 + 3*(42*b*c*cosh(b*
```

```
c*x + a*c)^5 - 70*b*c*cosh(b*c*x + a*c)^3 + 25*b*c*cosh(b*c*x + a*c))*sinh(
b*c*x + a*c)^4 + 3*(28*b*c*cosh(b*c*x + a*c)^6 - 70*b*c*cosh(b*c*x + a*c)^4
 + 50*b*c*cosh(b*c*x + a*c)^2 - 7*b*c)*sinh(b*c*x + a*c)^3 + 9*b*c*cosh(b*c
*x + a*c) + 3*(12*b*c*cosh(b*c*x + a*c)^7 - 42*b*c*cosh(b*c*x + a*c)^5 + 50
*b*c*cosh(b*c*x + a*c)^3 - 19*b*c*cosh(b*c*x + a*c))*sinh(b*c*x + a*c)^2 +
3*(3*b*c*cosh(b*c*x + a*c)^8 - 14*b*c*cosh(b*c*x + a*c)^6 + 25*b*c*cosh(b*c
*x + a*c)^4 - 21*b*c*cosh(b*c*x + a*c)^2 + 7*b*c)*sinh(b*c*x + a*c))
```

giac [A] time = 0.15, size = 90, normalized size = 0.45

$$-\frac{16\left(20\,e^{(6\,bcx+6\,ac)}-15\,e^{(4\,bcx+4\,ac)}+6\,e^{(2\,bcx+2\,ac)}-1\right)}{15\,bc\left(e^{(2\,bcx+2\,ac)}-1\right)^{6}\operatorname{sgn}\left(e^{(bcx+ac)}-e^{(-bcx-ac)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*(csch(b*c*x+a*c)^2)^(7/2),x, algorithm="giac")
```

```
[Out] -16/15*(20*e^(6*b*c*x + 6*a*c) - 15*e^(4*b*c*x + 4*a*c) + 6*e^(2*b*c*x + 2*
a*c) - 1)/(b*c*(e^(2*b*c*x + 2*a*c) - 1)^6*sgn(e^(b*c*x + a*c) - e^(-b*c*x
- a*c)))
```

maple [A] time = 0.64, size = 91, normalized size = 0.46

$$-\frac{16\left(20\,e^{6c(bx+a)}-15\,e^{4c(bx+a)}+6\,e^{2c(bx+a)}-1\right)\sqrt{\frac{e^{2c(bx+a)}}{\left(e^{2c(bx+a)}-1\right)^{2}}}\;e^{-c(bx+a)}}{15bc\left(e^{2c(bx+a)}-1\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(c*(b*x+a))*(csch(b*c*x+a*c)^2)^(7/2),x)
```

```
[Out] -16/15/b/c*(20*exp(6*c*(b*x+a))-15*exp(4*c*(b*x+a))+6*exp(2*c*(b*x+a))-1)*(
1/(exp(2*c*(b*x+a))-1)^2*exp(2*c*(b*x+a)))^(1/2)/(exp(2*c*(b*x+a))-1)^5*exp
(-c*(b*x+a))
```

maxima [B] time = 0.42, size = 386, normalized size = 1.94

$$-\frac{64\,e^{(6\,bcx+6\,ac)}}{3\,bc\left(e^{(12\,bcx+12\,ac)}-6\,e^{(10\,bcx+10\,ac)}+15\,e^{(8\,bcx+8\,ac)}-20\,e^{(6\,bcx+6\,ac)}+15\,e^{(4\,bcx+4\,ac)}-6\,e^{(2\,bcx+2\,ac)}+1\right)}+\frac{}{bc\left(e^{(12\,\ldots}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*(csch(b*c*x+a*c)^2)^(7/2),x, algorithm="maxima")
```

```
[Out] -64/3*e^(6*b*c*x + 6*a*c)/(b*c*(e^(12*b*c*x + 12*a*c) - 6*e^(10*b*c*x + 10*
a*c) + 15*e^(8*b*c*x + 8*a*c) - 20*e^(6*b*c*x + 6*a*c) + 15*e^(4*b*c*x + 4*
a*c) - 6*e^(2*b*c*x + 2*a*c) + 1)) + 16*e^(4*b*c*x + 4*a*c)/(b*c*(e^(12*b*c
*x + 12*a*c) - 6*e^(10*b*c*x + 10*a*c) + 15*e^(8*b*c*x + 8*a*c) - 20*e^(6*b
*c*x + 6*a*c) + 15*e^(4*b*c*x + 4*a*c) - 6*e^(2*b*c*x + 2*a*c) + 1)) - 32/5
*e^(2*b*c*x + 2*a*c)/(b*c*(e^(12*b*c*x + 12*a*c) - 6*e^(10*b*c*x + 10*a*c)
+ 15*e^(8*b*c*x + 8*a*c) - 20*e^(6*b*c*x + 6*a*c) + 15*e^(4*b*c*x + 4*a*c)
- 6*e^(2*b*c*x + 2*a*c) + 1)) + 16/15/(b*c*(e^(12*b*c*x + 12*a*c) - 6*e^(10
*b*c*x + 10*a*c) + 15*e^(8*b*c*x + 8*a*c) - 20*e^(6*b*c*x + 6*a*c) + 15*e^(
4*b*c*x + 4*a*c) - 6*e^(2*b*c*x + 2*a*c) + 1))
```

mupad [B] time = 1.60, size = 413, normalized size = 2.08

$$\frac{32\,\sqrt{\frac{1}{\left(\frac{e^{a\,c+b\,c\,x}}{2}-\frac{e^{-a\,c-b\,c\,x}}{2}\right)^2}}\,\left(e^{4\,a\,c+4\,b\,c\,x}-2\,e^{2\,a\,c+2\,b\,c\,x}+1\right)}{3\,b\,c\,\left(e^{a\,c+b\,c\,x}-e^{3\,a\,c+3\,b\,c\,x}\right)\,\left(e^{2\,a\,c+2\,b\,c\,x}-1\right)^3}+\frac{24\,\sqrt{\frac{1}{\left(\frac{e^{a\,c+b\,c\,x}}{2}-\frac{e^{-a\,c-b\,c\,x}}{2}\right)^2}}\,\left(e^{4\,a\,c+4\,b\,c\,x}-2\,e^{2\,a\,c+2\,b\,c\,x}+1\ldots\right.}{b\,c\,\left(e^{a\,c+b\,c\,x}-e^{3\,a\,c+3\,b\,c\,x}\right)\,\left(e^{2\,a\,c+2\,b\,c\,x}-1\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(c*(a + b*x))*(1/sinh(a*c + b*c*x)^2)^(7/2),x)
```

```
[Out] (32*(1/(exp(a*c + b*c*x)/2 - exp(- a*c - b*c*x)/2)^2)^(1/2)*(exp(4*a*c + 4*
b*c*x) - 2*exp(2*a*c + 2*b*c*x) + 1))/(3*b*c*(exp(a*c + b*c*x) - exp(3*a*c
+ 3*b*c*x))*(exp(2*a*c + 2*b*c*x) - 1)^3) + (24*(1/(exp(a*c + b*c*x)/2 - ex
p(- a*c - b*c*x)/2)^2)^(1/2)*(exp(4*a*c + 4*b*c*x) - 2*exp(2*a*c + 2*b*c*x)
 + 1))/(b*c*(exp(a*c + b*c*x) - exp(3*a*c + 3*b*c*x))*(exp(2*a*c + 2*b*c*x)
 - 1)^4) + (96*(1/(exp(a*c + b*c*x)/2 - exp(- a*c - b*c*x)/2)^2)^(1/2)*(exp
(4*a*c + 4*b*c*x) - 2*exp(2*a*c + 2*b*c*x) + 1))/(5*b*c*(exp(a*c + b*c*x) -
 exp(3*a*c + 3*b*c*x))*(exp(2*a*c + 2*b*c*x) - 1)^5) + (16*(1/(exp(a*c + b*
c*x)/2 - exp(- a*c - b*c*x)/2)^2)^(1/2)*(exp(4*a*c + 4*b*c*x) - 2*exp(2*a*c
 + 2*b*c*x) + 1))/(3*b*c*(exp(a*c + b*c*x) - exp(3*a*c + 3*b*c*x))*(exp(2*a
*c + 2*b*c*x) - 1)^6)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$e^{ac}\int\left(\operatorname{csch}^2\left(ac+bcx\right)\right)^{\frac{7}{2}}e^{bcx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*(csch(b*c*x+a*c)**2)**(7/2),x)
```

```
[Out] exp(a*c)*Integral((csch(a*c + b*c*x)**2)**(7/2)*exp(b*c*x), x)
```

3.126 $\int e^{c(a+bx)}\operatorname{csch}^2(ac+bcx)^{5/2}\,dx$

Optimal. Leaf size=147

$$-\frac{8\sinh(ac+bcx)\sqrt{\operatorname{csch}^2(ac+bcx)}}{bc\left(1-e^{2c(a+bx)}\right)^2}+\frac{32\sinh(ac+bcx)\sqrt{\operatorname{csch}^2(ac+bcx)}}{3bc\left(1-e^{2c(a+bx)}\right)^3}-\frac{4\sinh(ac+bcx)\sqrt{\operatorname{csch}^2(ac+bcx)}}{bc\left(1-e^{2c(a+bx)}\right)^4}$$

```
[Out] -4*sinh(b*c*x+a*c)*(csch(b*c*x+a*c)^2)^(1/2)/b/c/(1-exp(2*c*(b*x+a)))^4+32/
3*sinh(b*c*x+a*c)*(csch(b*c*x+a*c)^2)^(1/2)/b/c/(1-exp(2*c*(b*x+a)))^3-8*si
nh(b*c*x+a*c)*(csch(b*c*x+a*c)^2)^(1/2)/b/c/(1-exp(2*c*(b*x+a)))^2
```

Rubi [A] time = 0.17, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {6720, 2282, 12, 266, 43}

$$-\frac{8\sinh(ac+bcx)\sqrt{\operatorname{csch}^2(ac+bcx)}}{bc\left(1-e^{2c(a+bx)}\right)^2}+\frac{32\sinh(ac+bcx)\sqrt{\operatorname{csch}^2(ac+bcx)}}{3bc\left(1-e^{2c(a+bx)}\right)^3}-\frac{4\sinh(ac+bcx)\sqrt{\operatorname{csch}^2(ac+bcx)}}{bc\left(1-e^{2c(a+bx)}\right)^4}$$

Antiderivative was successfully verified.

```
[In] Int[E^(c*(a + b*x))*(Csch[a*c + b*c*x]^2)^(5/2),x]
```

```
[Out] (-4*Sqrt[Csch[a*c + b*c*x]^2]*Sinh[a*c + b*c*x])/(b*c*(1 - E^(2*c*(a + b*x)
))^4) + (32*Sqrt[Csch[a*c + b*c*x]^2]*Sinh[a*c + b*c*x])/(3*b*c*(1 - E^(2*c
*(a + b*x)))^3) - (8*Sqrt[Csch[a*c + b*c*x]^2]*Sinh[a*c + b*c*x])/(b*c*(1 -
 E^(2*c*(a + b*x)))^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6720

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a*v^m)^
FracPart[p])/v^(m*FracPart[p]), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
```

```
[v, x] && EqQ[m, 1])
```

Rubi steps

$$\int e^{c(a+bx)}\text{csch}^2(ac+bcx)^{5/2}\,dx = \left(\sqrt{\text{csch}^2(ac+bcx)}\,\sinh(ac+bcx)\right)\int e^{c(a+bx)}\text{csch}^5(ac+bcx)\,dx$$
$$= \frac{\left(\sqrt{\text{csch}^2(ac+bcx)}\,\sinh(ac+bcx)\right)\text{Subst}\left(\int\frac{32x^5}{\left(-1+x^2\right)^5}\,dx,x,e^{c(a+bx)}\right)}{bc}$$
$$= \frac{\left(32\sqrt{\text{csch}^2(ac+bcx)}\,\sinh(ac+bcx)\right)\text{Subst}\left(\int\frac{x^5}{\left(-1+x^2\right)^5}\,dx,x,e^{c(a+bx)}\right)}{bc}$$
$$= \frac{\left(16\sqrt{\text{csch}^2(ac+bcx)}\,\sinh(ac+bcx)\right)\text{Subst}\left(\int\frac{x^2}{(-1+x)^5}\,dx,x,e^{2c(a+bx)}\right)}{bc}$$
$$= \frac{\left(16\sqrt{\text{csch}^2(ac+bcx)}\,\sinh(ac+bcx)\right)\text{Subst}\left(\int\left(\frac{1}{(-1+x)^5}+\frac{2}{(-1+x)^4}+\frac{1}{(-1+x)^3}\right)dx\right.}{bc}$$
$$= -\frac{4\sqrt{\text{csch}^2(ac+bcx)}\,\sinh(ac+bcx)}{bc\left(1-e^{2c(a+bx)}\right)^4}+\frac{32\sqrt{\text{csch}^2(ac+bcx)}\,\sinh(ac+bcx)}{3bc\left(1-e^{2c(a+bx)}\right)^3}-$$

Mathematica [A] time = 0.07, size = 72, normalized size = 0.49

$$-\frac{4\left(-4e^{2c(a+bx)}+6e^{4c(a+bx)}+1\right)\sinh(c(a+bx))\sqrt{\text{csch}^2(c(a+bx))}}{3bc\left(e^{2c(a+bx)}-1\right)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(c*(a + b*x))*(Csch[a*c + b*c*x]^2)^(5/2),x]
```

```
[Out] (-4*(1 - 4*E^(2*c*(a + b*x)) + 6*E^(4*c*(a + b*x)))*Sqrt[Csch[c*(a + b*x)]^
2]*Sinh[c*(a + b*x)])/(3*b*c*(-1 + E^(2*c*(a + b*x)))^4)
```

fricas [B] time = 1.98, size = 315, normalized size = 2.14

$$-\frac{}{3\left(bc\cosh\left(bcx+ac\right)^6+6\,bc\cosh\left(bcx+ac\right)\sinh\left(bcx+ac\right)^5+bc\sinh\left(bcx+ac\right)^6-4\,bc\cosh\left(bcx+ac\right)^4+\left(15\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*(csch(b*c*x+a*c)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] -4/3*(7*cosh(b*c*x + a*c)^2 + 10*cosh(b*c*x + a*c)*sinh(b*c*x + a*c) + 7*si
nh(b*c*x + a*c)^2 - 4)/(b*c*cosh(b*c*x + a*c)^6 + 6*b*c*cosh(b*c*x + a*c)*s
inh(b*c*x + a*c)^5 + b*c*sinh(b*c*x + a*c)^6 - 4*b*c*cosh(b*c*x + a*c)^4 +
(15*b*c*cosh(b*c*x + a*c)^2 - 4*b*c)*sinh(b*c*x + a*c)^4 + 7*b*c*cosh(b*c*x
 + a*c)^2 + 4*(5*b*c*cosh(b*c*x + a*c)^3 - 4*b*c*cosh(b*c*x + a*c))*sinh(b*
c*x + a*c)^3 + (15*b*c*cosh(b*c*x + a*c)^4 - 24*b*c*cosh(b*c*x + a*c)^2 + 7
*b*c)*sinh(b*c*x + a*c)^2 - 4*b*c + 2*(3*b*c*cosh(b*c*x + a*c)^5 - 8*b*c*co
sh(b*c*x + a*c)^3 + 5*b*c*cosh(b*c*x + a*c))*sinh(b*c*x + a*c))
```

giac [A] time = 0.15, size = 77, normalized size = 0.52

$$-\frac{4\left(6\,e^{(4\,bcx+4\,ac)}-4\,e^{(2\,bcx+2\,ac)}+1\right)}{3\,bc\left(e^{(2\,bcx+2\,ac)}-1\right)^4\text{sgn}\left(e^{(bcx+ac)}-e^{(-bcx-ac)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*(csch(b*c*x+a*c)^2)^(5/2),x, algorithm="giac")
```

```
[Out] -4/3*(6*e^(4*b*c*x + 4*a*c) - 4*e^(2*b*c*x + 2*a*c) + 1)/(b*c*(e^(2*b*c*x +
 2*a*c) - 1)^4*sgn(e^(b*c*x + a*c) - e^(-b*c*x - a*c)))
```

maple [A] time = 0.61, size = 80, normalized size = 0.54

$$-\frac{4\left(6\,\mathrm{e}^{4c(bx+a)}-4\,\mathrm{e}^{2c(bx+a)}+1\right)\sqrt{\frac{\mathrm{e}^{2c(bx+a)}}{\left(\mathrm{e}^{2c(bx+a)}-1\right)^{2}}}\;\mathrm{e}^{-c(bx+a)}}{3bc\left(\mathrm{e}^{2c(bx+a)}-1\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(c*(b*x+a))*(csch(b*c*x+a*c)^2)^(5/2),x)
```

```
[Out] -4/3/b/c*(6*exp(4*c*(b*x+a))-4*exp(2*c*(b*x+a))+1)*(1/(exp(2*c*(b*x+a))-1)^
2*exp(2*c*(b*x+a)))^(1/2)/(exp(2*c*(b*x+a))-1)^3*exp(-c*(b*x+a))
```

maxima [A] time = 0.42, size = 209, normalized size = 1.42

$$-\frac{8\,e^{(4\,bcx+4\,ac)}}{bc\left(e^{(8\,bcx+8\,ac)}-4\,e^{(6\,bcx+6\,ac)}+6\,e^{(4\,bcx+4\,ac)}-4\,e^{(2\,bcx+2\,ac)}+1\right)}+\frac{16\,e^{(2\,bcx+2\,ac)}}{3\,bc\left(e^{(8\,bcx+8\,ac)}-4\,e^{(6\,bcx+6\,ac)}+6\,e^{(4\,bcx+4\,ac)}-\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*(csch(b*c*x+a*c)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] -8*e^(4*b*c*x + 4*a*c)/(b*c*(e^(8*b*c*x + 8*a*c) - 4*e^(6*b*c*x + 6*a*c) +
6*e^(4*b*c*x + 4*a*c) - 4*e^(2*b*c*x + 2*a*c) + 1)) + 16/3*e^(2*b*c*x + 2*a
*c)/(b*c*(e^(8*b*c*x + 8*a*c) - 4*e^(6*b*c*x + 6*a*c) + 6*e^(4*b*c*x + 4*a*
c) - 4*e^(2*b*c*x + 2*a*c) + 1)) - 4/3/(b*c*(e^(8*b*c*x + 8*a*c) - 4*e^(6*b
*c*x + 6*a*c) + 6*e^(4*b*c*x + 4*a*c) - 4*e^(2*b*c*x + 2*a*c) + 1))
```

mupad [B] time = 1.53, size = 91, normalized size = 0.62

$$-\frac{2\,\mathrm{e}^{-a\,c-b\,c\,x}\,\sqrt{\frac{1}{{\left(\frac{\mathrm{e}^{a\,c+b\,c\,x}}{2}-\frac{\mathrm{e}^{-a\,c-b\,c\,x}}{2}\right)}^2}}\,\left(6\,\mathrm{e}^{4\,a\,c+4\,b\,c\,x}-4\,\mathrm{e}^{2\,a\,c+2\,b\,c\,x}+1\right)}{3\,b\,c\,{\left(\mathrm{e}^{2\,a\,c+2\,b\,c\,x}-1\right)}^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(c*(a + b*x))*(1/sinh(a*c + b*c*x)^2)^(5/2),x)
```

```
[Out] -(2*exp(- a*c - b*c*x)*(1/(exp(a*c + b*c*x)/2 - exp(- a*c - b*c*x)/2)^2)^(1
/2)*(6*exp(4*a*c + 4*b*c*x) - 4*exp(2*a*c + 2*b*c*x) + 1))/(3*b*c*(exp(2*a*
c + 2*b*c*x) - 1)^3)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$e^{ac}\int\left(\operatorname{csch}^{2}\left(ac+bcx\right)\right)^{\frac{5}{2}}e^{bcx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*(csch(b*c*x+a*c)**2)**(5/2),x)
```

```
[Out] exp(a*c)*Integral((csch(a*c + b*c*x)**2)**(5/2)*exp(b*c*x), x)
```

3.127 $\int e^{c(a+bx)} \operatorname{csch}^2(ac+bcx)^{3/2} \, dx$

Optimal. Leaf size=58

$$-\frac{2e^{4c(a+bx)} \sinh(ac+bcx)\sqrt{\operatorname{csch}^2(ac+bcx)}}{bc\left(1-e^{2c(a+bx)}\right)^2}$$

```
[Out] -2*exp(4*c*(b*x+a))*sinh(b*c*x+a*c)*(csch(b*c*x+a*c)^2)^(1/2)/b/c/(1-exp(2*
c*(b*x+a)))^2
```

Rubi [A] time = 0.12, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {6720, 2282, 12, 264}

$$-\frac{2e^{4c(a+bx)} \sinh(ac+bcx)\sqrt{\operatorname{csch}^2(ac+bcx)}}{bc\left(1-e^{2c(a+bx)}\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[E^(c*(a + b*x))*(Csch[a*c + b*c*x]^2)^(3/2),x]
```

```
[Out] (-2*E^(4*c*(a + b*x))*Sqrt[Csch[a*c + b*c*x]^2]*Sinh[a*c + b*c*x])/(b*c*(1
- E^(2*c*(a + b*x)))^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6720

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a*v^m)^
FracPart[p])/v^(m*FracPart[p]), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] &&  !IntegerQ[p] &&  !FreeQ[v, x] &&  !(EqQ[a, 1] && EqQ[m, 1]) &&  !(EqQ
[v, x] && EqQ[m, 1])
```

Rubi steps

$$\int e^{c(a+bx)}\text{csch}^2(ac+bcx)^{3/2}\,dx = \left(\sqrt{\text{csch}^2(ac+bcx)}\,\sinh(ac+bcx)\right)\int e^{c(a+bx)}\text{csch}^3(ac+bcx)\,dx$$
$$= \frac{\left(\sqrt{\text{csch}^2(ac+bcx)}\,\sinh(ac+bcx)\right)\text{Subst}\left(\int\frac{8x^3}{(-1+x^2)^3}\,dx,x,e^{c(a+bx)}\right)}{bc}$$
$$= \frac{\left(8\sqrt{\text{csch}^2(ac+bcx)}\,\sinh(ac+bcx)\right)\text{Subst}\left(\int\frac{x^3}{(-1+x^2)^3}\,dx,x,e^{c(a+bx)}\right)}{bc}$$
$$= -\frac{2e^{4c(a+bx)}\sqrt{\text{csch}^2(ac+bcx)}\,\sinh(ac+bcx)}{bc\left(1-e^{2c(a+bx)}\right)^2}$$

Mathematica [A] time = 0.04, size = 56, normalized size = 0.97

$$-\frac{2e^{4c(a+bx)}\sinh^3(c(a+bx))\text{csch}^2(c(a+bx))^{3/2}}{bc\left(e^{2c(a+bx)}-1\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(c*(a + b*x))*(Csch[a*c + b*c*x]^2)^(3/2),x]
```

```
[Out] (-2*E^(4*c*(a + b*x))*(Csch[c*(a + b*x)]^2)^(3/2)*Sinh[c*(a + b*x)]^3)/(b*c
*(-1 + E^(2*c*(a + b*x)))^2)
```

fricas [B] time = 1.06, size = 121, normalized size = 2.09

$$-\frac{2\left(\cosh\left(bcx+ac\right)+3\,\sinh\left(bcx+ac\right)\right)}{bc\cosh\left(bcx+ac\right)^3+3\,bc\cosh\left(bcx+ac\right)\sinh\left(bcx+ac\right)^2+bc\sinh\left(bcx+ac\right)^3-bc\cosh\left(bcx+ac\right)+3\left(bc\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*(csch(b*c*x+a*c)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -2*(cosh(b*c*x + a*c) + 3*sinh(b*c*x + a*c))/(b*c*cosh(b*c*x + a*c)^3 + 3*b
*c*cosh(b*c*x + a*c)*sinh(b*c*x + a*c)^2 + b*c*sinh(b*c*x + a*c)^3 - b*c*co
sh(b*c*x + a*c) + 3*(b*c*cosh(b*c*x + a*c)^2 - b*c)*sinh(b*c*x + a*c))
```

giac [A] time = 0.13, size = 64, normalized size = 1.10

$$-\frac{2\left(2\,e^{(2\,bcx+2\,ac)}-1\right)}{bc\left(e^{(2\,bcx+2\,ac)}-1\right)^2\text{sgn}\left(e^{(bcx+ac)}-e^{(-bcx-ac)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*(csch(b*c*x+a*c)^2)^(3/2),x, algorithm="giac")
```

```
[Out] -2*(2*e^(2*b*c*x + 2*a*c) - 1)/(b*c*(e^(2*b*c*x + 2*a*c) - 1)^2*sgn(e^(b*c*
x + a*c) - e^(-b*c*x - a*c)))
```

maple [A] time = 0.60, size = 69, normalized size = 1.19

$$-\frac{2\left(2\,\mathrm{e}^{2c(bx+a)}-1\right)\sqrt{\frac{\mathrm{e}^{2c(bx+a)}}{\left(\mathrm{e}^{2c(bx+a)}-1\right)^2}}\;\mathrm{e}^{-c(bx+a)}}{bc\left(\mathrm{e}^{2c(bx+a)}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(c*(b*x+a))*(csch(b*c*x+a*c)^2)^(3/2),x)
```

```
[Out] -2/b/c*(2*exp(2*c*(b*x+a))-1)*(1/(exp(2*c*(b*x+a))-1)^2*exp(2*c*(b*x+a)))^(
1/2)/(exp(2*c*(b*x+a))-1)*exp(-c*(b*x+a))
```

maxima [A] time = 0.42, size = 84, normalized size = 1.45

$$-\frac{4\,e^{(2\,bcx+2\,ac)}}{bc\left(e^{(4\,bcx+4\,ac)}-2\,e^{(2\,bcx+2\,ac)}+1\right)}+\frac{2}{bc\left(e^{(4\,bcx+4\,ac)}-2\,e^{(2\,bcx+2\,ac)}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*(csch(b*c*x+a*c)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -4*e^(2*b*c*x + 2*a*c)/(b*c*(e^(4*b*c*x + 4*a*c) - 2*e^(2*b*c*x + 2*a*c) +
1)) + 2/(b*c*(e^(4*b*c*x + 4*a*c) - 2*e^(2*b*c*x + 2*a*c) + 1))
```

mupad [B] time = 1.55, size = 78, normalized size = 1.34

$$-\frac{e^{-a\,c-b\,c\,x}\,\left(2\,e^{2\,a\,c+2\,b\,c\,x}-1\right)\,\sqrt{\frac{1}{{\left(\frac{e^{a\,c+b\,c\,x}}{2}-\frac{e^{-a\,c-b\,c\,x}}{2}\right)}^2}}}{b\,c\,\left(e^{2\,a\,c+2\,b\,c\,x}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(c*(a + b*x))*(1/sinh(a*c + b*c*x)^2)^(3/2),x)
```

```
[Out] -(exp(- a*c - b*c*x)*(2*exp(2*a*c + 2*b*c*x) - 1)*(1/(exp(a*c + b*c*x)/2 -
exp(- a*c - b*c*x)/2)^2)^(1/2))/(b*c*(exp(2*a*c + 2*b*c*x) - 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$e^{ac}\int\left(\operatorname{csch}^{2}\left(ac+bcx\right)\right)^{\frac{3}{2}}e^{bcx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*(csch(b*c*x+a*c)**2)**(3/2),x)
```

```
[Out] exp(a*c)*Integral((csch(a*c + b*c*x)**2)**(3/2)*exp(b*c*x), x)
```

3.128 $\int e^{c(a+bx)} \sqrt{\operatorname{csch}^2(ac+bcx)}\,dx$

Optimal. Leaf size=46

$$\frac{\log\left(1-e^{2c(a+bx)}\right)\sinh(ac+bcx)\sqrt{\operatorname{csch}^2(ac+bcx)}}{bc}$$

```
[Out] ln(1-exp(2*c*(b*x+a)))*sinh(b*c*x+a*c)*(csch(b*c*x+a*c)^2)^(1/2)/b/c
```

Rubi [A] time = 0.09, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {6720, 2282, 12, 260}

$$\frac{\log\left(1-e^{2c(a+bx)}\right)\sinh(ac+bcx)\sqrt{\operatorname{csch}^2(ac+bcx)}}{bc}$$

Antiderivative was successfully verified.

```
[In] Int[E^(c*(a + b*x))*Sqrt[Csch[a*c + b*c*x]^2],x]
```

```
[Out] (Sqrt[Csch[a*c + b*c*x]^2]*Log[1 - E^(2*c*(a + b*x))]*Sinh[a*c + b*c*x])/(b
*c)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6720

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a*v^m)^
FracPart[p])/v^(m*FracPart[p]), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] &&  !IntegerQ[p] &&  !FreeQ[v, x] &&  !(EqQ[a, 1] && EqQ[m, 1]) &&  !(EqQ
[v, x] && EqQ[m, 1])
```

Rubi steps

$$\int e^{c(a+bx)}\sqrt{\operatorname{csch}^2(ac+bcx)}\,dx = \left(\sqrt{\operatorname{csch}^2(ac+bcx)}\sinh(ac+bcx)\right)\int e^{c(a+bx)}\operatorname{csch}(ac+bcx)\,dx$$
$$= \frac{\left(\sqrt{\operatorname{csch}^2(ac+bcx)}\sinh(ac+bcx)\right)\operatorname{Subst}\left(\int \frac{2x}{-1+x^2}\,dx, x, e^{c(a+bx)}\right)}{bc}$$
$$= \frac{\left(2\sqrt{\operatorname{csch}^2(ac+bcx)}\sinh(ac+bcx)\right)\operatorname{Subst}\left(\int \frac{x}{-1+x^2}\,dx, x, e^{c(a+bx)}\right)}{bc}$$
$$= \frac{\sqrt{\operatorname{csch}^2(ac+bcx)}\log\left(1-e^{2c(a+bx)}\right)\sinh(ac+bcx)}{bc}$$

Mathematica [A] time = 0.04, size = 44, normalized size = 0.96

$$\frac{\log\left(1-e^{2c(a+bx)}\right)\sinh(c(a+bx))\sqrt{\operatorname{csch}^2(c(a+bx))}}{bc}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(c*(a + b*x))*Sqrt[Csch[a*c + b*c*x]^2],x]
[Out] (Sqrt[Csch[c*(a + b*x)]^2]*Log[1 - E^(2*c*(a + b*x))]*Sinh[c*(a + b*x)])/(b
*c)
```

fricas [A] time = 0.45, size = 42, normalized size = 0.91

$$\frac{\log\left(\frac{2\sinh(bcx+ac)}{\cosh(bcx+ac)-\sinh(bcx+ac)}\right)}{bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*(csch(b*c*x+a*c)^2)^(1/2),x, algorithm="fricas")
[Out] log(2*sinh(b*c*x + a*c)/(cosh(b*c*x + a*c) - sinh(b*c*x + a*c)))/(b*c)
```

giac [A] time = 0.16, size = 48, normalized size = 1.04

$$\frac{\log\left(\left|e^{(2\,bcx+2\,ac)}-1\right|\right)}{bc\operatorname{sgn}\left(e^{(bcx+ac)}-e^{(-bcx-ac)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*(csch(b*c*x+a*c)^2)^(1/2),x, algorithm="giac")
[Out] log(abs(e^(2*b*c*x + 2*a*c) - 1))/(b*c*sgn(e^(b*c*x + a*c) - e^(-b*c*x - a*
c)))
```

maple [A] time = 0.70, size = 68, normalized size = 1.48

$$\frac{\ln\left(e^{2bcx}-e^{-2ac}\right)\left(e^{2c(bx+a)}-1\right)\sqrt{\frac{e^{2c(bx+a)}}{\left(e^{2c(bx+a)}-1\right)^2}}\;e^{-c(bx+a)}}{bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(c*(b*x+a))*(csch(b*c*x+a*c)^2)^(1/2),x)
```

```
[Out] ln(exp(2*b*c*x)-exp(-2*a*c))/b/c*(exp(2*c*(b*x+a))-1)*(1/(exp(2*c*(b*x+a))-
1)^2*exp(2*c*(b*x+a)))^(1/2)*exp(-c*(b*x+a))
```

maxima [A] time = 0.42, size = 39, normalized size = 0.85

$$\frac{\log\left(e^{(bcx+ac)}+1\right)}{bc}+\frac{\log\left(e^{(bcx+ac)}-1\right)}{bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*(csch(b*c*x+a*c)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] log(e^(b*c*x + a*c) + 1)/(b*c) + log(e^(b*c*x + a*c) - 1)/(b*c)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int e^{c\,(a+b\,x)}\,\sqrt{\frac{1}{{\mathrm{sinh}\left(a\,c+b\,c\,x\right)}^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(c*(a + b*x))*(1/sinh(a*c + b*c*x)^2)^(1/2),x)
```

```
[Out] int(exp(c*(a + b*x))*(1/sinh(a*c + b*c*x)^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$e^{ac}\int\sqrt{\operatorname{csch}^{2}(ac+bcx)}\,e^{bcx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*(csch(b*c*x+a*c)**2)**(1/2),x)
```

```
[Out] exp(a*c)*Integral(sqrt(csch(a*c + b*c*x)**2)*exp(b*c*x), x)
```

3.129 $\int \frac{e^{c(a+bx)}}{\sqrt{\operatorname{csch}^2(ac+bcx)}} dx$

Optimal. Leaf size=74

$$\frac{e^{2c(a+bx)}\operatorname{csch}(ac+bcx)}{4bc\sqrt{\operatorname{csch}^2(ac+bcx)}} - \frac{x\operatorname{csch}(ac+bcx)}{2\sqrt{\operatorname{csch}^2(ac+bcx)}}$$

```
[Out] 1/4*exp(2*c*(b*x+a))*csch(b*c*x+a*c)/b/c/(csch(b*c*x+a*c)^2)^(1/2)-1/2*x*cs
ch(b*c*x+a*c)/(csch(b*c*x+a*c)^2)^(1/2)
```

Rubi [A] time = 0.11, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {6720, 2282, 12, 14}

$$\frac{e^{2c(a+bx)}\operatorname{csch}(ac+bcx)}{4bc\sqrt{\operatorname{csch}^2(ac+bcx)}} - \frac{x\operatorname{csch}(ac+bcx)}{2\sqrt{\operatorname{csch}^2(ac+bcx)}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(c*(a + b*x))/Sqrt[Csch[a*c + b*c*x]^2],x]
```

```
[Out] (E^(2*c*(a + b*x))*Csch[a*c + b*c*x])/(4*b*c*Sqrt[Csch[a*c + b*c*x]^2]) - (
x*Csch[a*c + b*c*x])/(2*Sqrt[Csch[a*c + b*c*x]^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6720

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a*v^m)^
FracPart[p])/v^(m*FracPart[p]), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] &&  !IntegerQ[p] &&  !FreeQ[v, x] &&  !(EqQ[a, 1] && EqQ[m, 1]) &&  !(EqQ
[v, x] && EqQ[m, 1])
```

Rubi steps

$$\int \frac{e^{c(a+bx)}}{\sqrt{\operatorname{csch}^2(ac+bcx)}}\,dx = \frac{\operatorname{csch}(ac+bcx)\int e^{c(a+bx)}\sinh(ac+bcx)\,dx}{\sqrt{\operatorname{csch}^2(ac+bcx)}}$$
$$= \frac{\operatorname{csch}(ac+bcx)\operatorname{Subst}\left(\int \frac{-1+x^2}{2x}\,dx, x, e^{c(a+bx)}\right)}{bc\sqrt{\operatorname{csch}^2(ac+bcx)}}$$
$$= \frac{\operatorname{csch}(ac+bcx)\operatorname{Subst}\left(\int \frac{-1+x^2}{x}\,dx, x, e^{c(a+bx)}\right)}{2bc\sqrt{\operatorname{csch}^2(ac+bcx)}}$$
$$= \frac{\operatorname{csch}(ac+bcx)\operatorname{Subst}\left(\int \left(-\frac{1}{x}+x\right)dx, x, e^{c(a+bx)}\right)}{2bc\sqrt{\operatorname{csch}^2(ac+bcx)}}$$
$$= \frac{e^{2c(a+bx)}\operatorname{csch}(ac+bcx)}{4bc\sqrt{\operatorname{csch}^2(ac+bcx)}} - \frac{x\operatorname{csch}(ac+bcx)}{2\sqrt{\operatorname{csch}^2(ac+bcx)}}$$

Mathematica [A] time = 0.05, size = 48, normalized size = 0.65

$$\frac{\left(e^{2c(a+bx)}-2bcx\right)\operatorname{csch}(c(a+bx))}{4bc\sqrt{\operatorname{csch}^2(c(a+bx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(c*(a + b*x))/Sqrt[Csch[a*c + b*c*x]^2],x]
```

```
[Out] ((E^(2*c*(a + b*x)) - 2*b*c*x)*Csch[c*(a + b*x)])/(4*b*c*Sqrt[Csch[c*(a + b
*x)]^2])
```

fricas [A] time = 0.59, size = 66, normalized size = 0.89

$$-\frac{(2\,bcx-1)\cosh\left(bcx+ac\right)-(2\,bcx+1)\sinh\left(bcx+ac\right)}{4\left(bc\cosh\left(bcx+ac\right)-bc\sinh\left(bcx+ac\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))/(csch(b*c*x+a*c)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/4*((2*b*c*x - 1)*cosh(b*c*x + a*c) - (2*b*c*x + 1)*sinh(b*c*x + a*c))/(b
*c*cosh(b*c*x + a*c) - b*c*sinh(b*c*x + a*c))
```

giac [A] time = 0.13, size = 71, normalized size = 0.96

$$-\frac{1}{2}\,x\operatorname{sgn}\left(e^{(bcx+ac)}-e^{(-bcx-ac)}\right)+\frac{e^{(2\,bcx+2\,ac)}\operatorname{sgn}\left(e^{(bcx+ac)}-e^{(-bcx-ac)}\right)}{4\,bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))/(csch(b*c*x+a*c)^2)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*x*sgn(e^(b*c*x + a*c) - e^(-b*c*x - a*c)) + 1/4*e^(2*b*c*x + 2*a*c)*sg
n(e^(b*c*x + a*c) - e^(-b*c*x - a*c))/(b*c)
```

maple [A] time = 0.54, size = 106, normalized size = 1.43

$$-\frac{x\,e^{c(bx+a)}}{2\left(e^{2c(bx+a)}-1\right)\sqrt{\frac{e^{2c(bx+a)}}{\left(e^{2c(bx+a)}-1\right)^2}}}+\frac{e^{3c(bx+a)}}{4bc\left(e^{2c(bx+a)}-1\right)\sqrt{\frac{e^{2c(bx+a)}}{\left(e^{2c(bx+a)}-1\right)^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(c*(b*x+a))/(csch(b*c*x+a*c)^2)^(1/2),x)
```

```
[Out] -1/2*x/(exp(2*c*(b*x+a))-1)/(1/(exp(2*c*(b*x+a))-1)^2*exp(2*c*(b*x+a)))^(1/
2)*exp(c*(b*x+a))+1/4/b/c/(exp(2*c*(b*x+a))-1)/(1/(exp(2*c*(b*x+a))-1)^2*ex
p(2*c*(b*x+a)))^(1/2)*exp(3*c*(b*x+a))
```

maxima [A] time = 0.41, size = 36, normalized size = 0.49

$$-\frac{bcx+ac}{2\,bc}+\frac{e^{(2\,bcx+2\,ac)}}{4\,bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))/(csch(b*c*x+a*c)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/2*(b*c*x + a*c)/(b*c) + 1/4*e^(2*b*c*x + 2*a*c)/(b*c)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\mathrm{e}^{c\,(a+b\,x)}}{\sqrt{\frac{1}{\mathrm{sinh}(a\,c+b\,c\,x)^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(c*(a + b*x))/(1/sinh(a*c + b*c*x)^2)^(1/2),x)
```

```
[Out] int(exp(c*(a + b*x))/(1/sinh(a*c + b*c*x)^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$e^{ac}\int \frac{e^{bcx}}{\sqrt{\operatorname{csch}^2(ac+bcx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))/(csch(b*c*x+a*c)**2)**(1/2),x)
```

```
[Out] exp(a*c)*Integral(exp(b*c*x)/sqrt(csch(a*c + b*c*x)**2), x)
```

3.130 $\int \frac{e^{c(a+bx)}}{\mathbf{csch}^2(ac+bcx)^{3/2}}\, dx$

Optimal. Leaf size=162

$$\frac{e^{-2c(a+bx)}\text{csch}(ac+bcx)}{16bc\sqrt{\text{csch}^2(ac+bcx)}}-\frac{3e^{2c(a+bx)}\text{csch}(ac+bcx)}{16bc\sqrt{\text{csch}^2(ac+bcx)}}+\frac{e^{4c(a+bx)}\text{csch}(ac+bcx)}{32bc\sqrt{\text{csch}^2(ac+bcx)}}+\frac{3x\text{csch}(ac+bcx)}{8\sqrt{\text{csch}^2(ac+bcx)}}$$

```
[Out] 1/16*csch(b*c*x+a*c)/b/c/exp(2*c*(b*x+a))/(csch(b*c*x+a*c)^2)^(1/2)-3/16*ex
p(2*c*(b*x+a))*csch(b*c*x+a*c)/b/c/(csch(b*c*x+a*c)^2)^(1/2)+1/32*exp(4*c*(
b*x+a))*csch(b*c*x+a*c)/b/c/(csch(b*c*x+a*c)^2)^(1/2)+3/8*x*csch(b*c*x+a*c)
/(csch(b*c*x+a*c)^2)^(1/2)
```

Rubi [A] time = 0.15, antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {6720, 2282, 12, 266, 43}

$$\frac{e^{-2c(a+bx)}\text{csch}(ac+bcx)}{16bc\sqrt{\text{csch}^2(ac+bcx)}}-\frac{3e^{2c(a+bx)}\text{csch}(ac+bcx)}{16bc\sqrt{\text{csch}^2(ac+bcx)}}+\frac{e^{4c(a+bx)}\text{csch}(ac+bcx)}{32bc\sqrt{\text{csch}^2(ac+bcx)}}+\frac{3x\text{csch}(ac+bcx)}{8\sqrt{\text{csch}^2(ac+bcx)}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(c*(a + b*x))/(Csch[a*c + b*c*x]^2)^(3/2),x]
```

```
[Out] Csch[a*c + b*c*x]/(16*b*c*E^(2*c*(a + b*x))*Sqrt[Csch[a*c + b*c*x]^2]) - (3
*E^(2*c*(a + b*x))*Csch[a*c + b*c*x])/(16*b*c*Sqrt[Csch[a*c + b*c*x]^2]) +
(E^(4*c*(a + b*x))*Csch[a*c + b*c*x])/(32*b*c*Sqrt[Csch[a*c + b*c*x]^2]) +
(3*x*Csch[a*c + b*c*x])/(8*Sqrt[Csch[a*c + b*c*x]^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6720

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a*v^m)^
FracPart[p])/v^(m*FracPart[p]), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
```

```
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{c(a+bx)}}{\operatorname{csch}^2(ac+bcx)^{3/2}}\,dx &= \frac{\operatorname{csch}(ac+bcx)\int e^{c(a+bx)}\sinh^3(ac+bcx)\,dx}{\sqrt{\operatorname{csch}^2(ac+bcx)}}\\
&= \frac{\operatorname{csch}(ac+bcx)\operatorname{Subst}\left(\int \frac{\left(-1+x^2\right)^3}{8x^3}\,dx,x,e^{c(a+bx)}\right)}{bc\sqrt{\operatorname{csch}^2(ac+bcx)}}\\
&= \frac{\operatorname{csch}(ac+bcx)\operatorname{Subst}\left(\int \frac{\left(-1+x^2\right)^3}{x^3}\,dx,x,e^{c(a+bx)}\right)}{8bc\sqrt{\operatorname{csch}^2(ac+bcx)}}\\
&= \frac{\operatorname{csch}(ac+bcx)\operatorname{Subst}\left(\int \frac{(-1+x)^3}{x^2}\,dx,x,e^{2c(a+bx)}\right)}{16bc\sqrt{\operatorname{csch}^2(ac+bcx)}}\\
&= \frac{\operatorname{csch}(ac+bcx)\operatorname{Subst}\left(\int \left(-3-\frac{1}{x^2}+\frac{3}{x}+x\right)dx,x,e^{2c(a+bx)}\right)}{16bc\sqrt{\operatorname{csch}^2(ac+bcx)}}\\
&= \frac{e^{-2c(a+bx)}\operatorname{csch}(ac+bcx)}{16bc\sqrt{\operatorname{csch}^2(ac+bcx)}}-\frac{3e^{2c(a+bx)}\operatorname{csch}(ac+bcx)}{16bc\sqrt{\operatorname{csch}^2(ac+bcx)}}+\frac{e^{4c(a+bx)}\operatorname{csch}(ac+bcx)}{32bc\sqrt{\operatorname{csch}^2(ac+bcx)}}+\frac{3xc}{8\sqrt{c}}
\end{aligned}$$

Mathematica [A] time = 0.07, size = 76, normalized size = 0.47

$$\frac{\left(e^{-2c(a+bx)}-3e^{2c(a+bx)}+\frac{1}{2}e^{4c(a+bx)}+6bcx\right)\operatorname{csch}^3(c(a+bx))}{16bc\operatorname{csch}^2(c(a+bx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(c*(a + b*x))/(Csch[a*c + b*c*x]^2)^(3/2),x]
```

```
[Out] ((E^(-2*c*(a + b*x)) - 3*E^(2*c*(a + b*x)) + E^(4*c*(a + b*x))/2 + 6*b*c*x)
*Csch[c*(a + b*x)]^3)/(16*b*c*(Csch[c*(a + b*x)]^2)^(3/2))
```

fricas [A] time = 0.61, size = 126, normalized size = 0.78

$$\frac{3\cosh(bcx+ac)^3+9\cosh(bcx+ac)\sinh(bcx+ac)^2-\sinh(bcx+ac)^3+6(2bcx-1)\cosh(bcx+ac)-3\left(4b\right.}{32(bc\cosh(bcx+ac)-bc\sinh(bcx+ac))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))/(csch(b*c*x+a*c)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/32*(3*cosh(b*c*x + a*c)^3 + 9*cosh(b*c*x + a*c)*sinh(b*c*x + a*c)^2 - sin
h(b*c*x + a*c)^3 + 6*(2*b*c*x - 1)*cosh(b*c*x + a*c) - 3*(4*b*c*x + cosh(b*
c*x + a*c)^2 + 2)*sinh(b*c*x + a*c))/(b*c*cosh(b*c*x + a*c) - b*c*sinh(b*c*
x + a*c))
```

giac [A] time = 0.15, size = 204, normalized size = 1.26

$$\frac{\left(12bcxe^{(-ac)}\operatorname{sgn}\left(e^{(bcx+ac)}-e^{(-bcx-ac)}\right)-2\left(3e^{(2bcx+2ac)}\operatorname{sgn}\left(e^{(bcx+ac)}-e^{(-bcx-ac)}\right)-\operatorname{sgn}\left(e^{(bcx+ac)}-e^{(-bcx-ac)}\right)\right)e^{(-2b}\right.}{32bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))/(csch(b*c*x+a*c)^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/32*(12*b*c*x*e^(-a*c)*sgn(e^(b*c*x + a*c) - e^(-b*c*x - a*c)) - 2*(3*e^(2
*b*c*x + 2*a*c)*sgn(e^(b*c*x + a*c) - e^(-b*c*x - a*c)) - sgn(e^(b*c*x + a*
c) - e^(-b*c*x - a*c)))*e^(-2*b*c*x - 3*a*c) + (e^(4*b*c*x + 9*a*c)*sgn(e^(
b*c*x + a*c) - e^(-b*c*x - a*c)) - 6*e^(2*b*c*x + 7*a*c)*sgn(e^(b*c*x + a*c
) - e^(-b*c*x - a*c)))*e^(-6*a*c))*e^(a*c)/(b*c)
```

maple [A] time = 0.54, size = 216, normalized size = 1.33

$$\frac{3x\,\mathrm{e}^{c(bx+a)}}{8\left(\mathrm{e}^{2c(bx+a)}-1\right)\sqrt{\frac{\mathrm{e}^{2c(bx+a)}}{\left(\mathrm{e}^{2c(bx+a)}-1\right)^{2}}}}+\frac{\mathrm{e}^{5c(bx+a)}}{32bc\left(\mathrm{e}^{2c(bx+a)}-1\right)\sqrt{\frac{\mathrm{e}^{2c(bx+a)}}{\left(\mathrm{e}^{2c(bx+a)}-1\right)^{2}}}}-\frac{3\,\mathrm{e}^{3c(bx+a)}}{16bc\left(\mathrm{e}^{2c(bx+a)}-1\right)\sqrt{\frac{\mathrm{e}^{2c(bx+a)}}{\left(\mathrm{e}^{2c(bx+a)}-1\right)^{2}}}}+\frac{}{16bc\left(\mathrm{e}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(c*(b*x+a))/(csch(b*c*x+a*c)^2)^(3/2),x)
```

```
[Out] 3/8*x/(exp(2*c*(b*x+a))-1)/(1/(exp(2*c*(b*x+a))-1)^2*exp(2*c*(b*x+a)))^(1/2
)*exp(c*(b*x+a))+1/32/b/c/(exp(2*c*(b*x+a))-1)/(1/(exp(2*c*(b*x+a))-1)^2*ex
p(2*c*(b*x+a)))^(1/2)*exp(5*c*(b*x+a))-3/16/b/c/(exp(2*c*(b*x+a))-1)/(1/(ex
p(2*c*(b*x+a))-1)^2*exp(2*c*(b*x+a)))^(1/2)*exp(3*c*(b*x+a))+1/16/b/c/(exp(
2*c*(b*x+a))-1)/(1/(exp(2*c*(b*x+a))-1)^2*exp(2*c*(b*x+a)))^(1/2)*exp(-c*(b
*x+a))
```

maxima [A] time = 0.42, size = 62, normalized size = 0.38

$$\frac{\left(e^{(6\,bcx+6\,ac)}-6\,e^{(4\,bcx+4\,ac)}+2\right)e^{(-2\,bcx-2\,ac)}}{32\,bc}+\frac{3\,(bcx+ac)}{8\,bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))/(csch(b*c*x+a*c)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/32*(e^(6*b*c*x + 6*a*c) - 6*e^(4*b*c*x + 4*a*c) + 2)*e^(-2*b*c*x - 2*a*c)
/(b*c) + 3/8*(b*c*x + a*c)/(b*c)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\mathrm{e}^{c\,(a+b\,x)}}{{\left(\frac{1}{{\mathrm{sinh}\left(a\,c+b\,c\,x\right)}^2}\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(c*(a + b*x))/(1/sinh(a*c + b*c*x)^2)^(3/2),x)
```

```
[Out] int(exp(c*(a + b*x))/(1/sinh(a*c + b*c*x)^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$e^{ac}\int \frac{e^{bcx}}{\left(\operatorname{csch}^{2}{\left(ac+bcx\right)}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))/(csch(b*c*x+a*c)**2)**(3/2),x)
```

```
[Out] exp(a*c)*Integral(exp(b*c*x)/(csch(a*c + b*c*x)**2)**(3/2), x)
```

3.131 $\int \frac{e^{c(a+bx)}}{\operatorname{csch}^2(ac+bcx)^{5/2}}\,dx$

Optimal. Leaf size=250

$$\frac{e^{-4c(a+bx)}\operatorname{csch}(ac+bcx)}{128bc\sqrt{\operatorname{csch}^2(ac+bcx)}}-\frac{5e^{-2c(a+bx)}\operatorname{csch}(ac+bcx)}{64bc\sqrt{\operatorname{csch}^2(ac+bcx)}}+\frac{5e^{2c(a+bx)}\operatorname{csch}(ac+bcx)}{32bc\sqrt{\operatorname{csch}^2(ac+bcx)}}-\frac{5e^{4c(a+bx)}\operatorname{csch}(ac+bcx)}{128bc\sqrt{\operatorname{csch}^2(ac+bcx)}}+\frac{e^{6c(a+b}}{192bc\sqrt{}}$$

```
[Out] 1/128*csch(b*c*x+a*c)/b/c/exp(4*c*(b*x+a))/(csch(b*c*x+a*c)^2)^(1/2)-5/64*c
sch(b*c*x+a*c)/b/c/exp(2*c*(b*x+a))/(csch(b*c*x+a*c)^2)^(1/2)+5/32*exp(2*c*
(b*x+a))*csch(b*c*x+a*c)/b/c/(csch(b*c*x+a*c)^2)^(1/2)-5/128*exp(4*c*(b*x+a
))*csch(b*c*x+a*c)/b/c/(csch(b*c*x+a*c)^2)^(1/2)+1/192*exp(6*c*(b*x+a))*csc
h(b*c*x+a*c)/b/c/(csch(b*c*x+a*c)^2)^(1/2)-5/16*x*csch(b*c*x+a*c)/(csch(b*c
*x+a*c)^2)^(1/2)
```

Rubi [A] time = 0.20, antiderivative size = 250, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {6720, 2282, 12, 266, 43}

$$\frac{e^{-4c(a+bx)}\operatorname{csch}(ac+bcx)}{128bc\sqrt{\operatorname{csch}^2(ac+bcx)}}-\frac{5e^{-2c(a+bx)}\operatorname{csch}(ac+bcx)}{64bc\sqrt{\operatorname{csch}^2(ac+bcx)}}+\frac{5e^{2c(a+bx)}\operatorname{csch}(ac+bcx)}{32bc\sqrt{\operatorname{csch}^2(ac+bcx)}}-\frac{5e^{4c(a+bx)}\operatorname{csch}(ac+bcx)}{128bc\sqrt{\operatorname{csch}^2(ac+bcx)}}+\frac{e^{6c(a+b}}{192bc\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(c*(a + b*x))/(Csch[a*c + b*c*x]^2)^(5/2),x]
```

```
[Out] Csch[a*c + b*c*x]/(128*b*c*E^(4*c*(a + b*x))*Sqrt[Csch[a*c + b*c*x]^2]) - (
5*Csch[a*c + b*c*x])/(64*b*c*E^(2*c*(a + b*x))*Sqrt[Csch[a*c + b*c*x]^2]) +
 (5*E^(2*c*(a + b*x))*Csch[a*c + b*c*x])/(32*b*c*Sqrt[Csch[a*c + b*c*x]^2])
 - (5*E^(4*c*(a + b*x))*Csch[a*c + b*c*x])/(128*b*c*Sqrt[Csch[a*c + b*c*x]^
2]) + (E^(6*c*(a + b*x))*Csch[a*c + b*c*x])/(192*b*c*Sqrt[Csch[a*c + b*c*x]
^2]) - (5*x*Csch[a*c + b*c*x])/(16*Sqrt[Csch[a*c + b*c*x]^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6720

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a*v^m)^
FracPart[p])/v^(m*FracPart[p]), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rubi steps

$$\int \frac{e^{c(a+bx)}}{\operatorname{csch}^2(ac+bcx)^{5/2}}\,dx = \frac{\operatorname{csch}(ac+bcx)\int e^{c(a+bx)}\sinh^5(ac+bcx)\,dx}{\sqrt{\operatorname{csch}^2(ac+bcx)}}$$

$$= \frac{\operatorname{csch}(ac+bcx)\operatorname{Subst}\left(\int \frac{(-1+x^2)^5}{32x^5}\,dx, x, e^{c(a+bx)}\right)}{bc\sqrt{\operatorname{csch}^2(ac+bcx)}}$$

$$= \frac{\operatorname{csch}(ac+bcx)\operatorname{Subst}\left(\int \frac{(-1+x^2)^5}{x^5}\,dx, x, e^{c(a+bx)}\right)}{32bc\sqrt{\operatorname{csch}^2(ac+bcx)}}$$

$$= \frac{\operatorname{csch}(ac+bcx)\operatorname{Subst}\left(\int \frac{(-1+x)^5}{x^3}\,dx, x, e^{2c(a+bx)}\right)}{64bc\sqrt{\operatorname{csch}^2(ac+bcx)}}$$

$$= \frac{\operatorname{csch}(ac+bcx)\operatorname{Subst}\left(\int \left(10-\frac{1}{x^3}+\frac{5}{x^2}-\frac{10}{x}-5x+x^2\right)dx, x, e^{2c(a+bx)}\right)}{64bc\sqrt{\operatorname{csch}^2(ac+bcx)}}$$

$$= \frac{e^{-4c(a+bx)}\operatorname{csch}(ac+bcx)}{128bc\sqrt{\operatorname{csch}^2(ac+bcx)}} - \frac{5e^{-2c(a+bx)}\operatorname{csch}(ac+bcx)}{64bc\sqrt{\operatorname{csch}^2(ac+bcx)}} + \frac{5e^{2c(a+bx)}\operatorname{csch}(ac+bcx)}{32bc\sqrt{\operatorname{csch}^2(ac+bcx)}} -$$

Mathematica [A] time = 0.10, size = 106, normalized size = 0.42

$$\frac{\left(\frac{1}{2}e^{-4c(a+bx)}-5e^{-2c(a+bx)}+10e^{2c(a+bx)}-\frac{5}{2}e^{4c(a+bx)}+\frac{1}{3}e^{6c(a+bx)}-20bcx\right)\operatorname{csch}^5(c(a+bx))}{64bc\operatorname{csch}^2(c(a+bx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(c*(a + b*x))/(Csch[a*c + b*c*x]^2)^(5/2),x]
```

```
[Out] ((1/(2*E^(4*c*(a + b*x))) - 5/E^(2*c*(a + b*x)) + 10*E^(2*c*(a + b*x)) - (5
*E^(4*c*(a + b*x)))/2 + E^(6*c*(a + b*x))/3 - 20*b*c*x)*Csch[c*(a + b*x)]^5
)/(64*b*c*(Csch[c*(a + b*x)]^2)^(5/2))
```

fricas [A] time = 2.05, size = 218, normalized size = 0.87

$$\frac{5\cosh(bcx+ac)^5+25\cosh(bcx+ac)\sinh(bcx+ac)^4-\sinh(bcx+ac)^5-5\left(2\cosh(bcx+ac)^2-3\right)\sinh}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))/(csch(b*c*x+a*c)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/384*(5*cosh(b*c*x + a*c)^5 + 25*cosh(b*c*x + a*c)*sinh(b*c*x + a*c)^4 - s
inh(b*c*x + a*c)^5 - 5*(2*cosh(b*c*x + a*c)^2 - 3)*sinh(b*c*x + a*c)^3 - 45
```

```
*cosh(b*c*x + a*c)^3 + 5*(10*cosh(b*c*x + a*c)^3 - 27*cosh(b*c*x + a*c))*si
nh(b*c*x + a*c)^2 - 60*(2*b*c*x - 1)*cosh(b*c*x + a*c) - 5*(cosh(b*c*x + a*
c)^4 - 24*b*c*x - 9*cosh(b*c*x + a*c)^2 - 12)*sinh(b*c*x + a*c))/(b*c*cosh(
b*c*x + a*c) - b*c*sinh(b*c*x + a*c))
```

giac [A] time = 0.17, size = 278, normalized size = 1.11

$$-\frac{\left(120\, bcxe^{(-ac)}\operatorname{sgn}\left(e^{(bcx+ac)}-e^{(-bcx-ac)}\right)-3\left(30\, e^{(4\,bcx+4\,ac)}\operatorname{sgn}\left(e^{(bcx+ac)}-e^{(-bcx-ac)}\right)-10\, e^{(2\,bcx+2\,ac)}\operatorname{sgn}\left(e^{(bcx+ac)}\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))/(csch(b*c*x+a*c)^2)^(5/2),x, algorithm="giac")
```

```
[Out] -1/384*(120*b*c*x*e^(-a*c)*sgn(e^(b*c*x + a*c) - e^(-b*c*x - a*c)) - 3*(30*
e^(4*b*c*x + 4*a*c)*sgn(e^(b*c*x + a*c) - e^(-b*c*x - a*c)) - 10*e^(2*b*c*x
 + 2*a*c)*sgn(e^(b*c*x + a*c) - e^(-b*c*x - a*c)) + sgn(e^(b*c*x + a*c) - e
^(-b*c*x - a*c)))*e^(-4*b*c*x - 5*a*c) - (2*e^(6*b*c*x + 20*a*c)*sgn(e^(b*c
*x + a*c) - e^(-b*c*x - a*c)) - 15*e^(4*b*c*x + 18*a*c)*sgn(e^(b*c*x + a*c)
 - e^(-b*c*x - a*c)) + 60*e^(2*b*c*x + 16*a*c)*sgn(e^(b*c*x + a*c) - e^(-b*
c*x - a*c)))*e^(-15*a*c))*e^(a*c)/(b*c)
```

maple [A] time = 0.54, size = 326, normalized size = 1.30

$$-\frac{5x\,\mathrm{e}^{c(bx+a)}}{16\left(\mathrm{e}^{2c(bx+a)}-1\right)\sqrt{\frac{\mathrm{e}^{2c(bx+a)}}{\left(\mathrm{e}^{2c(bx+a)}-1\right)^{2}}}}+\frac{\mathrm{e}^{7c(bx+a)}}{192bc\left(\mathrm{e}^{2c(bx+a)}-1\right)\sqrt{\frac{\mathrm{e}^{2c(bx+a)}}{\left(\mathrm{e}^{2c(bx+a)}-1\right)^{2}}}}-\frac{5\,\mathrm{e}^{5c(bx+a)}}{128bc\left(\mathrm{e}^{2c(bx+a)}-1\right)\sqrt{\frac{\mathrm{e}^{2c(bx+a)}}{\left(\mathrm{e}^{2c(bx+a)}-1\right)^{2}}}}+\frac{}{32bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(c*(b*x+a))/(csch(b*c*x+a*c)^2)^(5/2),x)
```

```
[Out] -5/16*x/(exp(2*c*(b*x+a))-1)/(1/(exp(2*c*(b*x+a))-1)^2*exp(2*c*(b*x+a)))^(1
/2)*exp(c*(b*x+a))+1/192/b/c/(exp(2*c*(b*x+a))-1)/(1/(exp(2*c*(b*x+a))-1)^2
*exp(2*c*(b*x+a)))^(1/2)*exp(7*c*(b*x+a))-5/128/b/c/(exp(2*c*(b*x+a))-1)/(1
/(exp(2*c*(b*x+a))-1)^2*exp(2*c*(b*x+a)))^(1/2)*exp(5*c*(b*x+a))+5/32/b/c/(
exp(2*c*(b*x+a))-1)/(1/(exp(2*c*(b*x+a))-1)^2*exp(2*c*(b*x+a)))^(1/2)*exp(3
*c*(b*x+a))-5/64/b/c/(exp(2*c*(b*x+a))-1)/(1/(exp(2*c*(b*x+a))-1)^2*exp(2*c
*(b*x+a)))^(1/2)*exp(-c*(b*x+a))+1/128/b/c/(exp(2*c*(b*x+a))-1)/(1/(exp(2*c
*(b*x+a))-1)^2*exp(2*c*(b*x+a)))^(1/2)*exp(-3*c*(b*x+a))
```

maxima [A] time = 0.42, size = 90, normalized size = 0.36

$$\frac{\left(2\, e^{(10\,bcx+10\,ac)}-15\, e^{(8\,bcx+8\,ac)}+60\, e^{(6\,bcx+6\,ac)}-30\, e^{(2\,bcx+2\,ac)}+3\right)e^{(-4\,bcx-4\,ac)}}{384\,bc}-\frac{5\,(bcx+ac)}{16\,bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))/(csch(b*c*x+a*c)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/384*(2*e^(10*b*c*x + 10*a*c) - 15*e^(8*b*c*x + 8*a*c) + 60*e^(6*b*c*x + 6
*a*c) - 30*e^(2*b*c*x + 2*a*c) + 3)*e^(-4*b*c*x - 4*a*c)/(b*c) - 5/16*(b*c*
x + a*c)/(b*c)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\mathrm{e}^{c\,(a+b\,x)}}{{\left(\frac{1}{{\mathrm{sinh}\left(a\,c+b\,c\,x\right)}^2}\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(c*(a + b*x))/(1/sinh(a*c + b*c*x)^2)^(5/2),x)
```

```
[Out] int(exp(c*(a + b*x))/(1/sinh(a*c + b*c*x)^2)^(5/2), x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$e^{ac}\int \frac{e^{bcx}}{\left(\operatorname{csch}^2\left(ac+bcx\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))/(csch(b*c*x+a*c)**2)**(5/2),x)
```

```
[Out] exp(a*c)*Integral(exp(b*c*x)/(csch(a*c + b*c*x)**2)**(5/2), x)
```

3.132 $\int \frac{x^5}{\sqrt{\mathbf{csch}(2\log(cx))}}\,dx$

Optimal. Leaf size=81

$$-\frac{2x^2}{21c^4\sqrt{\operatorname{csch}(2\log(cx))}}+\frac{2F\left(\csc^{-1}(cx)\middle|-1\right)}{21c^7x\sqrt{1-\frac{1}{c^4x^4}}\sqrt{\operatorname{csch}(2\log(cx))}}+\frac{x^6}{7\sqrt{\operatorname{csch}(2\log(cx))}}$$

```
[Out] -2/21*x^2/c^4/csch(2*ln(c*x))^(1/2)+1/7*x^6/csch(2*ln(c*x))^(1/2)+2/21*Elli
pticF(1/c/x,I)/c^7/x/(1-1/c^4/x^4)^(1/2)/csch(2*ln(c*x))^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {5552, 5550, 335, 277, 325, 221}

$$-\frac{2x^2}{21c^4\sqrt{\operatorname{csch}(2\log(cx))}}+\frac{2F\left(\csc^{-1}(cx)\middle|-1\right)}{21c^7x\sqrt{1-\frac{1}{c^4x^4}}\sqrt{\operatorname{csch}(2\log(cx))}}+\frac{x^6}{7\sqrt{\operatorname{csch}(2\log(cx))}}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/Sqrt[Csch[2*Log[c*x]]],x]
```

```
[Out] (-2*x^2)/(21*c^4*Sqrt[Csch[2*Log[c*x]]]) + x^6/(7*Sqrt[Csch[2*Log[c*x]]]) +
 (2*EllipticF[ArcCsc[c*x], -1])/(21*c^7*Sqrt[1 - 1/(c^4*x^4)]*x*Sqrt[Csch[2
*Log[c*x]]])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 5550

```
Int[Csch[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
 :> Dist[(Csch[d*(a + b*Log[x])]^p*(1 - 1/(E^(2*a*d)*x^(2*b*d)))^p)/x^(-(b*
d*p)), Int[(e*x)^m/(x^(b*d*p)*(1 - 1/(E^(2*a*d)*x^(2*b*d)))^p), x], x] /; F
```

```
reeQ[{a, b, d, e, m, p}, x] && !IntegerQ[p]
```

Rule 5552

```
Int[Csch[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m
_.), x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[x
^((m + 1)/n - 1)*Csch[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b
, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rubi steps

$$\begin{aligned}
\int \frac{x^5}{\sqrt{\operatorname{csch}(2\log(cx))}}\,dx &= \frac{\operatorname{Subst}\left(\int \frac{x^5}{\sqrt{\operatorname{csch}(2\log(x))}}\,dx, x, cx\right)}{c^6}\\
&= \frac{\operatorname{Subst}\left(\int \sqrt{1-\frac{1}{x^4}}\,x^6\,dx, x, cx\right)}{c^7\sqrt{1-\frac{1}{c^4x^4}}\,x\sqrt{\operatorname{csch}(2\log(cx))}}\\
&= -\frac{\operatorname{Subst}\left(\int \frac{\sqrt{1-x^4}}{x^8}\,dx, x, \frac{1}{cx}\right)}{c^7\sqrt{1-\frac{1}{c^4x^4}}\,x\sqrt{\operatorname{csch}(2\log(cx))}}\\
&= \frac{x^6}{7\sqrt{\operatorname{csch}(2\log(cx))}} + \frac{2\operatorname{Subst}\left(\int \frac{1}{x^4\sqrt{1-x^4}}\,dx, x, \frac{1}{cx}\right)}{7c^7\sqrt{1-\frac{1}{c^4x^4}}\,x\sqrt{\operatorname{csch}(2\log(cx))}}\\
&= -\frac{2x^2}{21c^4\sqrt{\operatorname{csch}(2\log(cx))}} + \frac{x^6}{7\sqrt{\operatorname{csch}(2\log(cx))}} + \frac{2\operatorname{Subst}\left(\int \frac{1}{\sqrt{1-x^4}}\,dx, x, \frac{1}{cx}\right)}{21c^7\sqrt{1-\frac{1}{c^4x^4}}\,x\sqrt{\operatorname{csch}(2\log(cx))}}\\
&= -\frac{2x^2}{21c^4\sqrt{\operatorname{csch}(2\log(cx))}} + \frac{x^6}{7\sqrt{\operatorname{csch}(2\log(cx))}} + \frac{2F\left(\csc^{-1}(cx)\middle|-1\right)}{21c^7\sqrt{1-\frac{1}{c^4x^4}}\,x\sqrt{\operatorname{csch}(2\log(cx))}}
\end{aligned}$$

Mathematica [C] time = 0.18, size = 80, normalized size = 0.99

$$\frac{x^2\left({}_2F_1\left(-\frac{1}{2},\frac{1}{4};\frac{5}{4};c^4x^4\right)-\left(1-c^4x^4\right)^{3/2}\right)}{7c^4\sqrt{2-2c^4x^4}\sqrt{\frac{c^2x^2}{c^4x^4-1}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/Sqrt[Csch[2*Log[c*x]]],x]
```

```
[Out] (x^2*(-(1 - c^4*x^4)^(3/2) + Hypergeometric2F1[-1/2, 1/4, 5/4, c^4*x^4]))/(
7*c^4*Sqrt[2 - 2*c^4*x^4]*Sqrt[(c^2*x^2)/(-1 + c^4*x^4)])
```

fricas [F] time = 2.02, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x^5}{\sqrt{\operatorname{csch}\left(2\,\log\left(cx\right)\right)}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/csch(2*log(c*x))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(x^5/sqrt(csch(2*log(c*x))), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5}{\sqrt{\operatorname{csch}\left(2\log\left(cx\right)\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/csch(2*log(c*x))^(1/2),x, algorithm="giac")
[Out] integrate(x^5/sqrt(csch(2*log(c*x))), x)
```

maple [A] time = 0.21, size = 125, normalized size = 1.54

$$\frac{x^2\left(3c^4x^4-2\right)\sqrt{2}}{42c^4\sqrt{\frac{c^2x^2}{c^4x^4-1}}}-\frac{\sqrt{c^2x^2+1}\,\sqrt{-c^2x^2+1}\,\operatorname{EllipticF}\left(x\sqrt{-c^2},i\right)\sqrt{2}\,x}{21c^4\sqrt{-c^2}\,\left(c^4x^4-1\right)\sqrt{\frac{c^2x^2}{c^4x^4-1}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/csch(2*ln(c*x))^(1/2),x)
[Out] 1/42*x^2*(3*c^4*x^4-2)/c^4*2^(1/2)/(c^2*x^2/(c^4*x^4-1))^(1/2)-1/21/c^4/(-c
^2)^(1/2)*(c^2*x^2+1)^(1/2)*(-c^2*x^2+1)^(1/2)/(c^4*x^4-1)*EllipticF(x*(-c^
2)^(1/2),I)*2^(1/2)*x/(c^2*x^2/(c^4*x^4-1))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5}{\sqrt{\operatorname{csch}\left(2\log\left(cx\right)\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/csch(2*log(c*x))^(1/2),x, algorithm="maxima")
[Out] integrate(x^5/sqrt(csch(2*log(c*x))), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^5}{\sqrt{\frac{1}{\sinh\left(2\ln\left(c\,x\right)\right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(1/sinh(2*log(c*x)))^(1/2),x)
[Out] int(x^5/(1/sinh(2*log(c*x)))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5}{\sqrt{\operatorname{csch}\left(2\log\left(cx\right)\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/csch(2*ln(c*x))**(1/2),x)
[Out] Integral(x**5/sqrt(csch(2*log(c*x))), x)
```

3.133 $\int \frac{x^4}{\sqrt{\operatorname{csch}(2\log(cx))}}\,dx$

Optimal. Leaf size=30

$$\frac{x^5\left(c^4-\frac{1}{x^4}\right)}{6c^4\sqrt{\operatorname{csch}(2\log(cx))}}$$

```
[Out] 1/6*(c^4-1/x^4)*x^5/c^4/csch(2*ln(c*x))^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {5552, 5550, 264}

$$\frac{x^5\left(c^4-\frac{1}{x^4}\right)}{6c^4\sqrt{\operatorname{csch}(2\log(cx))}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/Sqrt[Csch[2*Log[c*x]]],x]
```

```
[Out] ((c^4 - x^(-4))*x^5)/(6*c^4*Sqrt[Csch[2*Log[c*x]]])
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 5550

```
Int[Csch[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
 :> Dist[(Csch[d*(a + b*Log[x])]^p*(1 - 1/(E^(2*a*d)*x^(2*b*d)))^p)/x^(-(b*
d*p)), Int[(e*x)^m/(x^(b*d*p)*(1 - 1/(E^(2*a*d)*x^(2*b*d)))^p), x], x] /; F
reeQ[{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

Rule 5552

```
Int[Csch[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m
_.), x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[x
^((m + 1)/n - 1)*Csch[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b
, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rubi steps

$$\begin{aligned}
\int \frac{x^4}{\sqrt{\operatorname{csch}(2\log(cx))}}\,dx &= \frac{\operatorname{Subst}\left(\int \frac{x^4}{\sqrt{\operatorname{csch}(2\log(x))}}\,dx,x,cx\right)}{c^5}\\
&= \frac{\operatorname{Subst}\left(\int \sqrt{1-\frac{1}{x^4}}\,x^5\,dx,x,cx\right)}{c^6\sqrt{1-\frac{1}{c^4x^4}}\,x\sqrt{\operatorname{csch}(2\log(cx))}}\\
&= \frac{\left(c^4-\frac{1}{x^4}\right)x^5}{6c^4\sqrt{\operatorname{csch}(2\log(cx))}}
\end{aligned}$$

Mathematica [A] time = 0.05, size = 44, normalized size = 1.47

$$\frac{\left(c^4x^4-1\right)^2\sqrt{\frac{c^2x^2}{2c^4x^4-2}}}{6c^6x}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/Sqrt[Csch[2*Log[c*x]]],x]
[Out] ((-1 + c^4*x^4)^2*Sqrt[(c^2*x^2)/(-2 + 2*c^4*x^4)])/(6*c^6*x)
```

fricas [A] time = 1.21, size = 48, normalized size = 1.60

$$\frac{\sqrt{2}\left(c^8x^8-2\,c^4x^4+1\right)\sqrt{\frac{c^2x^2}{c^4x^4-1}}}{12\,c^6x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/csch(2*log(c*x))^(1/2),x, algorithm="fricas")
[Out] 1/12*sqrt(2)*(c^8*x^8 - 2*c^4*x^4 + 1)*sqrt(c^2*x^2/(c^4*x^4 - 1))/(c^6*x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\sqrt{\operatorname{csch}\left(2\,\log\left(cx\right)\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/csch(2*log(c*x))^(1/2),x, algorithm="giac")
[Out] integrate(x^4/sqrt(csch(2*log(c*x))), x)
```

maple [A] time = 0.14, size = 39, normalized size = 1.30

$$\frac{\sqrt{2}\,x\left(c^4x^4-1\right)}{12\sqrt{\frac{c^2x^2}{c^4x^4-1}}\;c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/csch(2*ln(c*x))^(1/2),x)
[Out] 1/12*2^(1/2)*x/(c^2*x^2/(c^4*x^4-1))^(1/2)*(c^4*x^4-1)/c^4
```

maxima [A] time = 0.47, size = 46, normalized size = 1.53

$$\frac{\left(\sqrt{2}\,c^4x^4-\sqrt{2}\right)\sqrt{c^2x^2+1}\,\sqrt{cx+1}\,\sqrt{cx-1}}{12\,c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/csch(2*log(c*x))^(1/2),x, algorithm="maxima")
[Out] 1/12*(sqrt(2)*c^4*x^4 - sqrt(2))*sqrt(c^2*x^2 + 1)*sqrt(c*x + 1)*sqrt(c*x -
 1)/c^5
```

mupad [B] time = 2.15, size = 42, normalized size = 1.40

$$\frac{\left(c^4\,x^4-1\right)^2\sqrt{\frac{2\,c^2\,x^2}{c^4\,x^4-1}}}{12\,c^6\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(1/sinh(2*log(c*x)))^(1/2),x)
[Out] ((c^4*x^4 - 1)^2*((2*c^2*x^2)/(c^4*x^4 - 1))^(1/2))/(12*c^6*x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\sqrt{\operatorname{csch}\left(2\log\left(cx\right)\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/csch(2*ln(c*x))**(1/2),x)
[Out] Integral(x**4/sqrt(csch(2*log(c*x))), x)
```

3.134 $\int \frac{x^3}{\sqrt{\operatorname{csch}(2\log(cx))}}\,dx$

Optimal. Leaf size=119

$$-\frac{2}{5c^4\sqrt{\operatorname{csch}(2\log(cx))}}+\frac{2F\left(\csc^{-1}(cx)\middle|-1\right)}{5c^5x\sqrt{1-\frac{1}{c^4x^4}}\sqrt{\operatorname{csch}(2\log(cx))}}-\frac{2E\left(\csc^{-1}(cx)\middle|-1\right)}{5c^5x\sqrt{1-\frac{1}{c^4x^4}}\sqrt{\operatorname{csch}(2\log(cx))}}+\frac{x^4}{5\sqrt{\operatorname{csch}(2\log(cx))}}$$

```
[Out] -2/5/c^4/csch(2*ln(c*x))^(1/2)+1/5*x^4/csch(2*ln(c*x))^(1/2)-2/5*EllipticE(
1/c/x,I)/c^5/x/(1-1/c^4/x^4)^(1/2)/csch(2*ln(c*x))^(1/2)+2/5*EllipticF(1/c/
x,I)/c^5/x/(1-1/c^4/x^4)^(1/2)/csch(2*ln(c*x))^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {5552, 5550, 335, 277, 325, 307, 221, 1181, 424}

$$\frac{2F\left(\csc^{-1}(cx)\middle|-1\right)}{5c^5x\sqrt{1-\frac{1}{c^4x^4}}\sqrt{\operatorname{csch}(2\log(cx))}}-\frac{2E\left(\csc^{-1}(cx)\middle|-1\right)}{5c^5x\sqrt{1-\frac{1}{c^4x^4}}\sqrt{\operatorname{csch}(2\log(cx))}}-\frac{2}{5c^4\sqrt{\operatorname{csch}(2\log(cx))}}+\frac{x^4}{5\sqrt{\operatorname{csch}(2\log(cx))}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/Sqrt[Csch[2*Log[c*x]]],x]
```

```
[Out] -2/(5*c^4*Sqrt[Csch[2*Log[c*x]]]) + x^4/(5*Sqrt[Csch[2*Log[c*x]]]) - (2*Ell
ipticE[ArcCsc[c*x], -1])/(5*c^5*Sqrt[1 - 1/(c^4*x^4)]*x*Sqrt[Csch[2*Log[c*x
]]]) + (2*EllipticF[ArcCsc[c*x], -1])/(5*c^5*Sqrt[1 - 1/(c^4*x^4)]*x*Sqrt[C
sch[2*Log[c*x]]])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 307

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-(b/a), 2]}
, -Dist[q^(-1), Int[1/Sqrt[a + b*x^4], x], x] + Dist[1/q, Int[(1 + q*x^2)/S
qrt[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 1181

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[-(a*c), 2]}, Dist[Sqrt[-c], Int[(d + e*x^2)/(Sqrt[q + c*x^2]*Sqrt[q - c
*x^2]), x], x]] /; FreeQ[{a, c, d, e}, x] && GtQ[a, 0] && LtQ[c, 0]
```

Rule 5550

```
Int[Csch[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
 :> Dist[(Csch[d*(a + b*Log[x])]^p*(1 - 1/(E^(2*a*d)*x^(2*b*d)))^p)/x^(-(b*
d*p)), Int[(e*x)^m/(x^(b*d*p)*(1 - 1/(E^(2*a*d)*x^(2*b*d)))^p), x], x] /; F
reeQ[{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

Rule 5552

```
Int[Csch[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m
_.), x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[x
^((m + 1)/n - 1)*Csch[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b
, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rubi steps

$$\int \frac{x^3}{\sqrt{\operatorname{csch}(2\log(cx))}}\,dx = \frac{\text{Subst}\left(\int \frac{x^3}{\sqrt{\operatorname{csch}(2\log(x))}}\,dx, x, cx\right)}{c^4}$$

$$= \frac{\text{Subst}\left(\int \sqrt{1-\frac{1}{x^4}}\,x^4\,dx, x, cx\right)}{c^5\sqrt{1-\frac{1}{c^4x^4}}\,x\sqrt{\operatorname{csch}(2\log(cx))}}$$

$$= -\frac{\text{Subst}\left(\int \frac{\sqrt{1-x^4}}{x^6}\,dx, x, \frac{1}{cx}\right)}{c^5\sqrt{1-\frac{1}{c^4x^4}}\,x\sqrt{\operatorname{csch}(2\log(cx))}}$$

$$= \frac{x^4}{5\sqrt{\operatorname{csch}(2\log(cx))}} + \frac{2\,\text{Subst}\left(\int \frac{1}{x^2\sqrt{1-x^4}}\,dx, x, \frac{1}{cx}\right)}{5c^5\sqrt{1-\frac{1}{c^4x^4}}\,x\sqrt{\operatorname{csch}(2\log(cx))}}$$

$$= -\frac{2}{5c^4\sqrt{\operatorname{csch}(2\log(cx))}} + \frac{x^4}{5\sqrt{\operatorname{csch}(2\log(cx))}} - \frac{2\,\text{Subst}\left(\int \frac{x^2}{\sqrt{1-x^4}}\,dx, x, \frac{1}{cx}\right)}{5c^5\sqrt{1-\frac{1}{c^4x^4}}\,x\sqrt{\operatorname{csch}(2\log(cx))}}$$

$$= -\frac{2}{5c^4\sqrt{\operatorname{csch}(2\log(cx))}} + \frac{x^4}{5\sqrt{\operatorname{csch}(2\log(cx))}} + \frac{2\,\text{Subst}\left(\int \frac{1}{\sqrt{1-x^4}}\,dx, x, \frac{1}{cx}\right)}{5c^5\sqrt{1-\frac{1}{c^4x^4}}\,x\sqrt{\operatorname{csch}(2\log(cx))}} - \frac{}{5c}$$

$$= -\frac{2}{5c^4\sqrt{\operatorname{csch}(2\log(cx))}} + \frac{x^4}{5\sqrt{\operatorname{csch}(2\log(cx))}} + \frac{2F\left(\csc^{-1}(cx)\middle|-1\right)}{5c^5\sqrt{1-\frac{1}{c^4x^4}}\,x\sqrt{\operatorname{csch}(2\log(cx))}} - \frac{}{5c}$$

$$= -\frac{2}{5c^4\sqrt{\operatorname{csch}(2\log(cx))}} + \frac{x^4}{5\sqrt{\operatorname{csch}(2\log(cx))}} - \frac{2E\left(\csc^{-1}(cx)\middle|-1\right)}{5c^5\sqrt{1-\frac{1}{c^4x^4}}\,x\sqrt{\operatorname{csch}(2\log(cx))}} + \frac{}{5c}$$

Mathematica [C] time = 0.12, size = 60, normalized size = 0.50

$$\frac{x^4\,{}_2F_1\left(-\frac{1}{2},\frac{3}{4};\frac{7}{4};c^4x^4\right)}{3\sqrt{2-2c^4x^4}\sqrt{\frac{c^2x^2}{c^4x^4-1}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/Sqrt[Csch[2*Log[c*x]]],x]
```

```
[Out] (x^4*Hypergeometric2F1[-1/2, 3/4, 7/4, c^4*x^4])/(3*Sqrt[2 - 2*c^4*x^4]*Sqr
t[(c^2*x^2)/(-1 + c^4*x^4)])
```

fricas [F] time = 1.99, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x^3}{\sqrt{\operatorname{csch}\left(2\,\log\left(cx\right)\right)}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/csch(2*log(c*x))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(x^3/sqrt(csch(2*log(c*x))), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\sqrt{\operatorname{csch}\left(2\,\log\left(cx\right)\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/csch(2*log(c*x))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^3/sqrt(csch(2*log(c*x))), x)
```

maple [A] time = 0.16, size = 127, normalized size = 1.07

$$\frac{x^4\sqrt{2}}{10\sqrt{\frac{c^2x^2}{c^4x^4-1}}}-\frac{\sqrt{c^2x^2+1}\,\sqrt{-c^2x^2+1}\,\left(\mathrm{EllipticF}\left(x\sqrt{-c^2}\,,i\right)-\mathrm{EllipticE}\left(x\sqrt{-c^2}\,,i\right)\right)\sqrt{2}\,x}{5\sqrt{-c^2}\,\left(c^4x^4-1\right)c^2\sqrt{\frac{c^2x^2}{c^4x^4-1}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/csch(2*ln(c*x))^(1/2),x)
```

```
[Out] 1/10*x^4*2^(1/2)/(c^2*x^2/(c^4*x^4-1))^(1/2)-1/5/(-c^2)^(1/2)*(c^2*x^2+1)^(
1/2)*(-c^2*x^2+1)^(1/2)/(c^4*x^4-1)/c^2*(EllipticF(x*(-c^2)^(1/2),I)-Ellipt
icE(x*(-c^2)^(1/2),I))*2^(1/2)*x/(c^2*x^2/(c^4*x^4-1))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\sqrt{\operatorname{csch}\left(2\,\log\left(cx\right)\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/csch(2*log(c*x))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^3/sqrt(csch(2*log(c*x))), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3}{\sqrt{\frac{1}{\sinh(2\,\ln(c\,x))}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(1/sinh(2*log(c*x)))^(1/2),x)
```

```
[Out] int(x^3/(1/sinh(2*log(c*x)))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\sqrt{\operatorname{csch}\left(2\log\left(cx\right)\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/csch(2*ln(c*x))**(1/2),x)
```

```
[Out] Integral(x**3/sqrt(csch(2*log(c*x))), x)
```

3.135 $\int \frac{x^2}{\sqrt{\operatorname{csch}(2\log(cx))}}\,dx$

Optimal. Leaf size=69

$$\frac{x^3}{4\sqrt{\operatorname{csch}(2\log(cx))}}-\frac{\tanh^{-1}\left(\sqrt{1-\frac{1}{c^4x^4}}\right)}{4c^4x\sqrt{1-\frac{1}{c^4x^4}}\sqrt{\operatorname{csch}(2\log(cx))}}$$

```
[Out] 1/4*x^3/csch(2*ln(c*x))^(1/2)-1/4*arctanh((1-1/c^4/x^4)^(1/2))/c^4/x/(1-1/c
^4/x^4)^(1/2)/csch(2*ln(c*x))^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {5552, 5550, 266, 47, 63, 206}

$$\frac{x^3}{4\sqrt{\operatorname{csch}(2\log(cx))}}-\frac{\tanh^{-1}\left(\sqrt{1-\frac{1}{c^4x^4}}\right)}{4c^4x\sqrt{1-\frac{1}{c^4x^4}}\sqrt{\operatorname{csch}(2\log(cx))}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/Sqrt[Csch[2*Log[c*x]]],x]
```

```
[Out] x^3/(4*Sqrt[Csch[2*Log[c*x]]]) - ArcTanh[Sqrt[1 - 1/(c^4*x^4)]]/(4*c^4*Sqrt
[1 - 1/(c^4*x^4)]*x*Sqrt[Csch[2*Log[c*x]]])
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5550

```
Int[Csch[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
 :> Dist[(Csch[d*(a + b*Log[x])]^p*(1 - 1/(E^(2*a*d)*x^(2*b*d)))^p)/x^(-(b*
```

```
d*p)), Int[(e*x)^m/(x^(b*d*p)*(1 - 1/(E^(2*a*d)*x^(2*b*d)))^p), x], x] /; F
reeQ[{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

Rule 5552

```
Int[Csch[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m
_.), x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[x
^((m + 1)/n - 1)*Csch[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b
, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^2}{\sqrt{\operatorname{csch}(2\log(cx))}}\,dx &= \frac{\operatorname{Subst}\left(\int \frac{x^2}{\sqrt{\operatorname{csch}(2\log(x))}}\,dx, x, cx\right)}{c^3}\\
&= \frac{\operatorname{Subst}\left(\int \sqrt{1-\frac{1}{x^4}}\,x^3\,dx, x, cx\right)}{c^4\sqrt{1-\frac{1}{c^4x^4}}\,x\sqrt{\operatorname{csch}(2\log(cx))}}\\
&= -\frac{\operatorname{Subst}\left(\int \frac{\sqrt{1-x}}{x^2}\,dx, x, \frac{1}{c^4x^4}\right)}{4c^4\sqrt{1-\frac{1}{c^4x^4}}\,x\sqrt{\operatorname{csch}(2\log(cx))}}\\
&= \frac{x^3}{4\sqrt{\operatorname{csch}(2\log(cx))}} + \frac{\operatorname{Subst}\left(\int \frac{1}{\sqrt{1-x}\,x}\,dx, x, \frac{1}{c^4x^4}\right)}{8c^4\sqrt{1-\frac{1}{c^4x^4}}\,x\sqrt{\operatorname{csch}(2\log(cx))}}\\
&= \frac{x^3}{4\sqrt{\operatorname{csch}(2\log(cx))}} - \frac{\operatorname{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sqrt{1-\frac{1}{c^4x^4}}\right)}{4c^4\sqrt{1-\frac{1}{c^4x^4}}\,x\sqrt{\operatorname{csch}(2\log(cx))}}\\
&= \frac{x^3}{4\sqrt{\operatorname{csch}(2\log(cx))}} - \frac{\tanh^{-1}\left(\sqrt{1-\frac{1}{c^4x^4}}\right)}{4c^4\sqrt{1-\frac{1}{c^4x^4}}\,x\sqrt{\operatorname{csch}(2\log(cx))}}
\end{aligned}
$$

Mathematica [A] time = 0.14, size = 74, normalized size = 1.07

$$
\frac{x\left(\sin^{-1}\left(c^2x^2\right)+c^2x^2\sqrt{1-c^4x^4}\right)}{4c^2\sqrt{2-2c^4x^4}\,\sqrt{\frac{c^2x^2}{c^4x^4-1}}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/Sqrt[Csch[2*Log[c*x]]],x]
```

```
[Out] (x*(c^2*x^2*Sqrt[1 - c^4*x^4] + ArcSin[c^2*x^2]))/(4*c^2*Sqrt[2 - 2*c^4*x^4
]*Sqrt[(c^2*x^2)/(-1 + c^4*x^4)])
```

fricas [A] time = 1.66, size = 92, normalized size = 1.33

$$
\frac{2\sqrt{2}\left(c^5x^5-cx\right)\sqrt{\frac{c^2x^2}{c^4x^4-1}}+\sqrt{2}\log\left(2\,c^4x^4-2\left(c^5x^5-cx\right)\sqrt{\frac{c^2x^2}{c^4x^4-1}}-1\right)}{16\,c^3}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/csch(2*log(c*x))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/16*(2*sqrt(2)*(c^5*x^5 - c*x)*sqrt(c^2*x^2/(c^4*x^4 - 1)) + sqrt(2)*log(2
*c^4*x^4 - 2*(c^5*x^5 - c*x)*sqrt(c^2*x^2/(c^4*x^4 - 1)) - 1))/c^3
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt{\operatorname{csch}\left(2 \log\left(cx\right)\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/csch(2*log(c*x))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^2/sqrt(csch(2*log(c*x))), x)
```

maple [A] time = 0.19, size = 97, normalized size = 1.41

$$\frac{x^3\sqrt{2}}{8\sqrt{\frac{c^2x^2}{c^4x^4-1}}} - \frac{\ln\left(\frac{c^4x^2}{\sqrt{c^4}} + \sqrt{c^4x^4-1}\right)\sqrt{2}\,x}{8\sqrt{c^4}\,\sqrt{\frac{c^2x^2}{c^4x^4-1}}\,\sqrt{c^4x^4-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/csch(2*ln(c*x))^(1/2),x)
```

```
[Out] 1/8*x^3*2^(1/2)/(c^2*x^2/(c^4*x^4-1))^(1/2)-1/8*ln(c^4*x^2/(c^4)^(1/2)+(c^4
*x^4-1)^(1/2))/(c^4)^(1/2)*2^(1/2)*x/(c^2*x^2/(c^4*x^4-1))^(1/2)/(c^4*x^4-1
)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt{\operatorname{csch}\left(2 \log\left(cx\right)\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/csch(2*log(c*x))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2/sqrt(csch(2*log(c*x))), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2}{\sqrt{\frac{1}{\sinh(2\,\ln(c\,x))}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(1/sinh(2*log(c*x)))^(1/2),x)
```

```
[Out] int(x^2/(1/sinh(2*log(c*x)))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt{\operatorname{csch}\left(2\log\left(cx\right)\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/csch(2*ln(c*x))**(1/2),x)
```

```
[Out] Integral(x**2/sqrt(csch(2*log(c*x))), x)
```

3.136 $\int \frac{x}{\sqrt{\operatorname{csch}(2\log(cx))}}\,dx$

Optimal. Leaf size=60

$$\frac{2F\left(\csc^{-1}(cx)\middle|-1\right)}{3c^3x\sqrt{1-\frac{1}{c^4x^4}}\sqrt{\operatorname{csch}(2\log(cx))}}+\frac{x^2}{3\sqrt{\operatorname{csch}(2\log(cx))}}$$

```
[Out] 1/3*x^2/csch(2*ln(c*x))^(1/2)+2/3*EllipticF(1/c/x,I)/c^3/x/(1-1/c^4/x^4)^(1
/2)/csch(2*ln(c*x))^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {5552, 5550, 335, 277, 221}

$$\frac{2F\left(\csc^{-1}(cx)\middle|-1\right)}{3c^3x\sqrt{1-\frac{1}{c^4x^4}}\sqrt{\operatorname{csch}(2\log(cx))}}+\frac{x^2}{3\sqrt{\operatorname{csch}(2\log(cx))}}$$

Antiderivative was successfully verified.

```
[In] Int[x/Sqrt[Csch[2*Log[c*x]]],x]
```

```
[Out] x^2/(3*Sqrt[Csch[2*Log[c*x]]]) + (2*EllipticF[ArcCsc[c*x], -1])/(3*c^3*Sqrt
[1 - 1/(c^4*x^4)]*x*Sqrt[Csch[2*Log[c*x]]])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] && !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 5550

```
Int[Csch[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
 :> Dist[(Csch[d*(a + b*Log[x])]^p*(1 - 1/(E^(2*a*d)*x^(2*b*d)))^p)/x^(-(b*
d*p)), Int[(e*x)^m/(x^(b*d*p)*(1 - 1/(E^(2*a*d)*x^(2*b*d)))^p), x], x] /; F
reeQ[{a, b, d, e, m, p}, x] && !IntegerQ[p]
```

Rule 5552

```
Int[Csch[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m
_.), x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[x
^((m + 1)/n - 1)*Csch[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b
, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rubi steps

$$\int \frac{x}{\sqrt{\operatorname{csch}(2\log(cx))}}\,dx = \frac{\operatorname{Subst}\left(\int \frac{x}{\sqrt{\operatorname{csch}(2\log(x))}}\,dx, x, cx\right)}{c^2}$$
$$= \frac{\operatorname{Subst}\left(\int \sqrt{1-\frac{1}{x^4}}\,x^2\,dx, x, cx\right)}{c^3\sqrt{1-\frac{1}{c^4x^4}}\,x\sqrt{\operatorname{csch}(2\log(cx))}}$$
$$= -\frac{\operatorname{Subst}\left(\int \frac{\sqrt{1-x^4}}{x^4}\,dx, x, \frac{1}{cx}\right)}{c^3\sqrt{1-\frac{1}{c^4x^4}}\,x\sqrt{\operatorname{csch}(2\log(cx))}}$$
$$= \frac{x^2}{3\sqrt{\operatorname{csch}(2\log(cx))}} + \frac{2\operatorname{Subst}\left(\int \frac{1}{\sqrt{1-x^4}}\,dx, x, \frac{1}{cx}\right)}{3c^3\sqrt{1-\frac{1}{c^4x^4}}\,x\sqrt{\operatorname{csch}(2\log(cx))}}$$
$$= \frac{x^2}{3\sqrt{\operatorname{csch}(2\log(cx))}} + \frac{2F\left(\csc^{-1}(cx)\middle|-1\right)}{3c^3\sqrt{1-\frac{1}{c^4x^4}}\,x\sqrt{\operatorname{csch}(2\log(cx))}}$$

Mathematica [C] time = 0.11, size = 57, normalized size = 0.95

$$\frac{x^2\,{}_2F_1\left(-\frac{1}{2},\frac{1}{4};\frac{5}{4};c^4x^4\right)}{\sqrt{2-2c^4x^4}\sqrt{\frac{c^2x^2}{c^4x^4-1}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/Sqrt[Csch[2*Log[c*x]]],x]
[Out] (x^2*Hypergeometric2F1[-1/2, 1/4, 5/4, c^4*x^4])/(Sqrt[2 - 2*c^4*x^4]*Sqrt[
(c^2*x^2)/(-1 + c^4*x^4)])
```

fricas [F] time = 0.97, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{x}{\sqrt{\operatorname{csch}\left(2\log(cx)\right)}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/csch(2*log(c*x))^(1/2),x, algorithm="fricas")
[Out] integral(x/sqrt(csch(2*log(c*x))), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{\operatorname{csch}\left(2\log(cx)\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/csch(2*log(c*x))^(1/2),x, algorithm="giac")
[Out] integrate(x/sqrt(csch(2*log(c*x))), x)
```

maple [A] time = 0.15, size = 109, normalized size = 1.82

$$\frac{x^2\sqrt{2}}{6\sqrt{\frac{c^2x^2}{c^4x^4-1}}} - \frac{\sqrt{c^2x^2+1}\,\sqrt{-c^2x^2+1}\ \mathrm{EllipticF}\left(x\sqrt{-c^2}, i\right)\sqrt{2}\,x}{3\sqrt{-c^2}\,\left(c^4x^4-1\right)\sqrt{\frac{c^2x^2}{c^4x^4-1}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/csch(2*ln(c*x))^(1/2),x)
```

```
[Out] 1/6*x^2*2^(1/2)/(c^2*x^2/(c^4*x^4-1))^(1/2)-1/3/(-c^2)^(1/2)*(c^2*x^2+1)^(1
/2)*(-c^2*x^2+1)^(1/2)/(c^4*x^4-1)*EllipticF(x*(-c^2)^(1/2),I)*2^(1/2)*x/(c
^2*x^2/(c^4*x^4-1))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{\operatorname{csch}\left(2\,\log\left(cx\right)\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/csch(2*log(c*x))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x/sqrt(csch(2*log(c*x))), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x}{\sqrt{\frac{1}{\sinh(2\,\ln(c\,x))}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(1/sinh(2*log(c*x)))^(1/2),x)
```

```
[Out] int(x/(1/sinh(2*log(c*x)))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{\operatorname{csch}\left(2\log\left(cx\right)\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/csch(2*ln(c*x))**(1/2),x)
```

```
[Out] Integral(x/sqrt(csch(2*log(c*x))), x)
```

3.137 $\int \frac{1}{\sqrt{\operatorname{csch}(2\log(cx))}}\,dx$

Optimal. Leaf size=60

$$\frac{\csc^{-1}\left(c^2x^2\right)}{2c^2x\sqrt{1-\frac{1}{c^4x^4}}\sqrt{\operatorname{csch}(2\log(cx))}}+\frac{x}{2\sqrt{\operatorname{csch}(2\log(cx))}}$$

```
[Out] 1/2*x/csch(2*ln(c*x))^(1/2)+1/2*arccsc(c^2*x^2)/c^2/x/(1-1/c^4/x^4)^(1/2)/c
sch(2*ln(c*x))^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.546, Rules used = {5546, 5544, 335, 275, 277, 216}

$$\frac{\csc^{-1}\left(c^2x^2\right)}{2c^2x\sqrt{1-\frac{1}{c^4x^4}}\sqrt{\operatorname{csch}(2\log(cx))}}+\frac{x}{2\sqrt{\operatorname{csch}(2\log(cx))}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[Csch[2*Log[c*x]]],x]
```

```
[Out] x/(2*Sqrt[Csch[2*Log[c*x]]]) + ArcCsc[c^2*x^2]/(2*c^2*Sqrt[1 - 1/(c^4*x^4)]
*x*Sqrt[Csch[2*Log[c*x]]])
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 5544

```
Int[Csch[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol] :> Dist[(Csch[d*(a
 + b*Log[x])]^p*(1 - 1/(E^(2*a*d)*x^(2*b*d)))^p)/x^(-(b*d*p)), Int[1/(x^(b*
d*p)*(1 - 1/(E^(2*a*d)*x^(2*b*d)))^p), x], x] /; FreeQ[{a, b, d, p}, x] &&
 !IntegerQ[p]
```

Rule 5546

```
Int[Csch[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.), x_Symbol] :> D
ist[x/(n*(c*x^n)^(1/n)), Subst[Int[x^(1/n - 1)*Csch[d*(a + b*Log[x])]^p, x]
, x, c*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{\sqrt{\operatorname{csch}(2\log(cx))}}\,dx &= \frac{\operatorname{Subst}\left(\int \frac{1}{\sqrt{\operatorname{csch}(2\log(x))}}\,dx, x, cx\right)}{c} \\
&= \frac{\operatorname{Subst}\left(\int \sqrt{1-\frac{1}{x^4}}\,x\,dx, x, cx\right)}{c^2\sqrt{1-\frac{1}{c^4x^4}}\,x\sqrt{\operatorname{csch}(2\log(cx))}} \\
&= -\frac{\operatorname{Subst}\left(\int \frac{\sqrt{1-x^4}}{x^3}\,dx, x, \frac{1}{cx}\right)}{c^2\sqrt{1-\frac{1}{c^4x^4}}\,x\sqrt{\operatorname{csch}(2\log(cx))}} \\
&= -\frac{\operatorname{Subst}\left(\int \frac{\sqrt{1-x^2}}{x^2}\,dx, x, \frac{1}{c^2x^2}\right)}{2c^2\sqrt{1-\frac{1}{c^4x^4}}\,x\sqrt{\operatorname{csch}(2\log(cx))}} \\
&= \frac{x}{2\sqrt{\operatorname{csch}(2\log(cx))}} + \frac{\operatorname{Subst}\left(\int \frac{1}{\sqrt{1-x^2}}\,dx, x, \frac{1}{c^2x^2}\right)}{2c^2\sqrt{1-\frac{1}{c^4x^4}}\,x\sqrt{\operatorname{csch}(2\log(cx))}} \\
&= \frac{x}{2\sqrt{\operatorname{csch}(2\log(cx))}} + \frac{\csc^{-1}\left(c^2x^2\right)}{2c^2\sqrt{1-\frac{1}{c^4x^4}}\,x\sqrt{\operatorname{csch}(2\log(cx))}}
\end{aligned}
$$

Mathematica [A] time = 0.09, size = 77, normalized size = 1.28

$$
\frac{x\left(2\sqrt{c^4x^4-1}-2\tan^{-1}\left(\sqrt{c^4x^4-1}\right)\right)}{4\sqrt{2}\sqrt{\frac{c^2x^2}{c^4x^4-1}}\sqrt{c^4x^4-1}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[Csch[2*Log[c*x]]],x]
```

```
[Out] (x*(2*Sqrt[-1 + c^4*x^4] - 2*ArcTan[Sqrt[-1 + c^4*x^4]]))/(4*Sqrt[2]*Sqrt[(
c^2*x^2)/(-1 + c^4*x^4)]*Sqrt[-1 + c^4*x^4])
```

fricas [A] time = 0.71, size = 86, normalized size = 1.43

$$
-\frac{\sqrt{2}\,cx\arctan\left(\frac{\left(c^4x^4-1\right)\sqrt{\frac{c^2x^2}{c^4x^4-1}}}{cx}\right)-\sqrt{2}\left(c^4x^4-1\right)\sqrt{\frac{c^2x^2}{c^4x^4-1}}}{4\,c^2x}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/csch(2*log(c*x))^(1/2),x, algorithm="fricas")
```

```
[Out] -1/4*(sqrt(2)*c*x*arctan((c^4*x^4 - 1)*sqrt(c^2*x^2/(c^4*x^4 - 1))/(c*x)) -
 sqrt(2)*(c^4*x^4 - 1)*sqrt(c^2*x^2/(c^4*x^4 - 1)))/(c^2*x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/csch(2*log(c*x))^(1/2),x, algorithm="giac")
[Out] Timed out
```

maple [F] time = 0.14, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{\operatorname{csch}\left(2 \ln \left(c x\right)\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/csch(2*ln(c*x))^(1/2),x)
[Out] int(1/csch(2*ln(c*x))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{\operatorname{csch}\left(2 \log\left(c x\right)\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/csch(2*log(c*x))^(1/2),x, algorithm="maxima")
[Out] integrate(1/sqrt(csch(2*log(c*x))), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sqrt{\frac{1}{\sinh(2\ \ln(c\,x))}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1/sinh(2*log(c*x)))^(1/2),x)
[Out] int(1/(1/sinh(2*log(c*x)))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{\operatorname{csch}\left(2 \log\left(c x\right)\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/csch(2*ln(c*x))**(1/2),x)
[Out] Integral(1/sqrt(csch(2*log(c*x))), x)
```

3.138 $\int \frac{\sqrt{\mathbf{csch}(2\log(cx))}}{x}\,dx$

Optimal. Leaf size=46

$$i\sqrt{i\sinh(2\log(cx))}\,\sqrt{\operatorname{csch}(2\log(cx))}\,F\left(\frac{\pi}{4}-i\log(cx)\middle|2\right)$$

```
[Out] I*(sin(1/4*Pi+I*ln(c*x))^2)^(1/2)/sin(1/4*Pi+I*ln(c*x))*EllipticF(cos(1/4*P
i+I*ln(c*x)),2^(1/2))*csch(2*ln(c*x))^(1/2)*(I*sinh(2*ln(c*x)))^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {3771, 2641}

$$i\sqrt{i\sinh(2\log(cx))}\,\sqrt{\operatorname{csch}(2\log(cx))}\,F\left(\frac{\pi}{4}-i\log(cx)\middle|2\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Csch[2*Log[c*x]]]/x,x]
```

```
[Out] I*Sqrt[Csch[2*Log[c*x]]]*EllipticF[Pi/4 - I*Log[c*x], 2]*Sqrt[I*Sinh[2*Log[
c*x]]]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{\operatorname{csch}(2\log(cx))}}{x}\,dx &= \operatorname{Subst}\left(\int \sqrt{\operatorname{csch}(2x)}\,dx, x, \log(cx)\right)\\ &= \left(\sqrt{\operatorname{csch}(2\log(cx))}\,\sqrt{i\sinh(2\log(cx))}\right)\operatorname{Subst}\left(\int \frac{1}{\sqrt{i\sinh(2x)}}\,dx, x, \log(cx)\right)\\ &= i\sqrt{\operatorname{csch}(2\log(cx))}\,F\left(\frac{\pi}{4}-i\log(cx)\middle|2\right)\sqrt{i\sinh(2\log(cx))}\end{aligned}$$

Mathematica [A] time = 0.07, size = 43, normalized size = 0.93

$$(i\sinh(2\log(cx)))^{3/2}\operatorname{csch}^{\frac{3}{2}}(2\log(cx))F\left(\frac{\pi}{4}-i\log(cx)\middle|2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Csch[2*Log[c*x]]]/x,x]
```

```
[Out] Csch[2*Log[c*x]]^(3/2)*EllipticF[Pi/4 - I*Log[c*x], 2]*(I*Sinh[2*Log[c*x]])
^(3/2)
```

fricas [F] time = 0.61, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{\operatorname{csch}\left(2\,\log\left(cx\right)\right)}}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(2*log(c*x))^(1/2)/x,x, algorithm="fricas")
[Out] integral(sqrt(csch(2*log(c*x)))/x, x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(2*log(c*x))^(1/2)/x,x, algorithm="giac")
[Out] Timed out
```

maple [A] time = 0.29, size = 90, normalized size = 1.96

$$\frac{i\sqrt{-i\left(\sinh\left(2\ln\left(cx\right)\right)+i\right)}\,\sqrt{2}\,\sqrt{-i\left(-\sinh\left(2\ln\left(cx\right)\right)+i\right)}\,\sqrt{i\sinh\left(2\ln\left(cx\right)\right)}\,\operatorname{EllipticF}\left(\sqrt{-i\left(\sinh\left(2\ln\left(cx\right)\right)+i\right)},\right.}{2\cosh\left(2\ln\left(cx\right)\right)\sqrt{\sinh\left(2\ln\left(cx\right)\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(2*ln(c*x))^(1/2)/x,x)
[Out] 1/2*I*(-I*(sinh(2*ln(c*x))+I))^(1/2)*2^(1/2)*(-I*(-sinh(2*ln(c*x))+I))^(1/2
)*(I*sinh(2*ln(c*x)))^(1/2)*EllipticF((-I*(sinh(2*ln(c*x))+I))^(1/2),1/2*2^
(1/2))/cosh(2*ln(c*x))/sinh(2*ln(c*x))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\operatorname{csch}\left(2\,\log\left(cx\right)\right)}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(2*log(c*x))^(1/2)/x,x, algorithm="maxima")
[Out] integrate(sqrt(csch(2*log(c*x)))/x, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{\frac{1}{\sinh\left(2\,\ln\left(c\,x\right)\right)}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/sinh(2*log(c*x)))^(1/2)/x,x)
[Out] int((1/sinh(2*log(c*x)))^(1/2)/x, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\operatorname{csch}\left(2\log\left(cx\right)\right)}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(2*ln(c*x))**(1/2)/x,x)
```

```
[Out] Integral(sqrt(csch(2*log(c*x)))/x, x)
```

3.139 $\int \frac{\sqrt{\mathbf{csch}(2\log(cx))}}{x^2}\,dx$

Optimal. Leaf size=41

$$-\frac{1}{2}c^2x\sqrt{1-\frac{1}{c^4x^4}}\,\csc^{-1}\left(c^2x^2\right)\sqrt{\mathrm{csch}(2\log(cx))}$$

```
[Out] -1/2*c^2*x*arccsc(c^2*x^2)*(1-1/c^4/x^4)^(1/2)*csch(2*ln(c*x))^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5552, 5550, 335, 275, 216}

$$-\frac{1}{2}c^2x\sqrt{1-\frac{1}{c^4x^4}}\,\csc^{-1}\left(c^2x^2\right)\sqrt{\mathrm{csch}(2\log(cx))}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Csch[2*Log[c*x]]]/x^2,x]
```

```
[Out] -(c^2*Sqrt[1 - 1/(c^4*x^4)]*x*ArcCsc[c^2*x^2]*Sqrt[Csch[2*Log[c*x]]])/2
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 5550

```
Int[Csch[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
 :> Dist[(Csch[d*(a + b*Log[x])]^p*(1 - 1/(E^(2*a*d)*x^(2*b*d)))^p)/x^(-(b*
d*p)), Int[(e*x)^m/(x^(b*d*p)*(1 - 1/(E^(2*a*d)*x^(2*b*d)))^p), x], x] /; F
reeQ[{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

Rule 5552

```
Int[Csch[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m
_.), x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[x
^((m + 1)/n - 1)*Csch[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b
, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rubi steps

$$\int \frac{\sqrt{\operatorname{csch}(2\log(cx))}}{x^2}\,dx = c\,\text{Subst}\left(\int \frac{\sqrt{\operatorname{csch}(2\log(x))}}{x^2}\,dx, x, cx\right)$$
$$= \left(c^2\sqrt{1-\frac{1}{c^4x^4}}\,x\sqrt{\operatorname{csch}(2\log(cx))}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{1}{x^4}}\,x^3}\,dx, x, cx\right)$$
$$= -\left(\left(c^2\sqrt{1-\frac{1}{c^4x^4}}\,x\sqrt{\operatorname{csch}(2\log(cx))}\right)\text{Subst}\left(\int \frac{x}{\sqrt{1-x^4}}\,dx, x, \frac{1}{cx}\right)\right)$$
$$= -\left(\frac{1}{2}\left(c^2\sqrt{1-\frac{1}{c^4x^4}}\,x\sqrt{\operatorname{csch}(2\log(cx))}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}}\,dx, x, \frac{1}{c^2x^2}\right)\right)$$
$$= -\frac{1}{2}c^2\sqrt{1-\frac{1}{c^4x^4}}\,x\csc^{-1}\left(c^2x^2\right)\sqrt{\operatorname{csch}(2\log(cx))}$$

Mathematica [A] time = 0.12, size = 54, normalized size = 1.32

$$\frac{\sqrt{c^4x^4-1}\sqrt{\frac{c^2x^2}{2c^4x^4-2}}\tan^{-1}\left(\sqrt{c^4x^4-1}\right)}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Csch[2*Log[c*x]]]/x^2,x]
```

```
[Out] (Sqrt[-1 + c^4*x^4]*Sqrt[(c^2*x^2)/(-2 + 2*c^4*x^4)]*ArcTan[Sqrt[-1 + c^4*x
^4]])/x
```

fricas [A] time = 0.46, size = 43, normalized size = 1.05

$$\frac{1}{2}\sqrt{2}\,c\arctan\left(\frac{\left(c^4x^4-1\right)\sqrt{\frac{c^2x^2}{c^4x^4-1}}}{cx}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(2*log(c*x))^(1/2)/x^2,x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(2)*c*arctan((c^4*x^4 - 1)*sqrt(c^2*x^2/(c^4*x^4 - 1))/(c*x))
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(2*log(c*x))^(1/2)/x^2,x, algorithm="giac")
```

```
[Out] Timed out
```

maple [F] time = 0.14, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\operatorname{csch}\left(2\ln\left(cx\right)\right)}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(2*ln(c*x))^(1/2)/x^2,x)
```

[Out] int(csch(2*ln(c*x))^(1/2)/x^2,x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\operatorname{csch}\left(2\,\log\left(cx\right)\right)}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(2*log(c*x))^(1/2)/x^2,x, algorithm="maxima")

[Out] integrate(sqrt(csch(2*log(c*x)))/x^2, x)

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{\frac{1}{\sinh(2\,\ln(c\,x))}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((1/sinh(2*log(c*x)))^(1/2)/x^2,x)

[Out] int((1/sinh(2*log(c*x)))^(1/2)/x^2, x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\operatorname{csch}\left(2\log\left(cx\right)\right)}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(2*ln(c*x))**(1/2)/x**2,x)

[Out] Integral(sqrt(csch(2*log(c*x)))/x**2, x)

3.140 $\int \frac{\sqrt{\operatorname{csch}(2\log(cx))}}{x^3}\,dx$

Optimal. Leaf size=74

$$c^3x\sqrt{1-\frac{1}{c^4x^4}}\,F\left(\csc^{-1}(cx)\middle|-1\right)\sqrt{\operatorname{csch}(2\log(cx))}-c^3x\sqrt{1-\frac{1}{c^4x^4}}\,E\left(\csc^{-1}(cx)\middle|-1\right)\sqrt{\operatorname{csch}(2\log(cx))}$$

```
[Out] -c^3*x*EllipticE(1/c/x,I)*(1-1/c^4/x^4)^(1/2)*csch(2*ln(c*x))^(1/2)+c^3*x*E
llipticF(1/c/x,I)*(1-1/c^4/x^4)^(1/2)*csch(2*ln(c*x))^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.467, Rules used = {5552, 5550, 335, 307, 221, 1181, 424}

$$c^3x\sqrt{1-\frac{1}{c^4x^4}}\,F\left(\csc^{-1}(cx)\middle|-1\right)\sqrt{\operatorname{csch}(2\log(cx))}-c^3x\sqrt{1-\frac{1}{c^4x^4}}\,E\left(\csc^{-1}(cx)\middle|-1\right)\sqrt{\operatorname{csch}(2\log(cx))}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Csch[2*Log[c*x]]]/x^3,x]
```

```
[Out] -(c^3*Sqrt[1 - 1/(c^4*x^4)]*x*Sqrt[Csch[2*Log[c*x]]]*EllipticE[ArcCsc[c*x],
 -1]) + c^3*Sqrt[1 - 1/(c^4*x^4)]*x*Sqrt[Csch[2*Log[c*x]]]*EllipticF[ArcCsc
[c*x], -1]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 307

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-(b/a), 2]}
, -Dist[q^(-1), Int[1/Sqrt[a + b*x^4], x], x] + Dist[1/q, Int[(1 + q*x^2)/S
qrt[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 1181

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[-(a*c), 2]}, Dist[Sqrt[-c], Int[(d + e*x^2)/(Sqrt[q + c*x^2]*Sqrt[q - c
*x^2]), x], x]] /; FreeQ[{a, c, d, e}, x] && GtQ[a, 0] && LtQ[c, 0]
```

Rule 5550

```
Int[Csch[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
 :> Dist[(Csch[d*(a + b*Log[x])]^p*(1 - 1/(E^(2*a*d)*x^(2*b*d)))^p)/x^(-(b*
d*p)), Int[(e*x)^m/(x^(b*d*p)*(1 - 1/(E^(2*a*d)*x^(2*b*d)))^p), x], x] /; F
```

```
reeQ[{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

Rule 5552

```
Int[Csch[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m
_.), x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[x
^((m + 1)/n - 1)*Csch[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b
, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rubi steps

$$\int \frac{\sqrt{\operatorname{csch}(2\log(cx))}}{x^3}\,dx = c^2\operatorname{Subst}\left(\int \frac{\sqrt{\operatorname{csch}(2\log(x))}}{x^3}\,dx, x, cx\right)$$
$$= \left(c^3\sqrt{1-\frac{1}{c^4x^4}}\,x\sqrt{\operatorname{csch}(2\log(cx))}\right)\operatorname{Subst}\left(\int \frac{1}{\sqrt{1-\frac{1}{x^4}}\,x^4}\,dx, x, cx\right)$$
$$= -\left(\left(c^3\sqrt{1-\frac{1}{c^4x^4}}\,x\sqrt{\operatorname{csch}(2\log(cx))}\right)\operatorname{Subst}\left(\int \frac{x^2}{\sqrt{1-x^4}}\,dx, x, \frac{1}{cx}\right)\right)$$
$$= \left(c^3\sqrt{1-\frac{1}{c^4x^4}}\,x\sqrt{\operatorname{csch}(2\log(cx))}\right)\operatorname{Subst}\left(\int \frac{1}{\sqrt{1-x^4}}\,dx, x, \frac{1}{cx}\right) - \left(c^3\sqrt{1-\frac{1}{c^4x^4}}\,x\sqrt{\mathrm{c}\right.$$
$$= c^3\sqrt{1-\frac{1}{c^4x^4}}\,x\sqrt{\operatorname{csch}(2\log(cx))}\,F\left(\csc^{-1}(cx)\middle|-1\right) - \left(c^3\sqrt{1-\frac{1}{c^4x^4}}\,x\sqrt{\operatorname{csch}(2\log(cx))}\right.$$
$$= -c^3\sqrt{1-\frac{1}{c^4x^4}}\,x\sqrt{\operatorname{csch}(2\log(cx))}\,E\left(\csc^{-1}(cx)\middle|-1\right) + c^3\sqrt{1-\frac{1}{c^4x^4}}\,x\sqrt{\operatorname{csch}(2\log(cx)}$$

Mathematica [C] time = 0.10, size = 58, normalized size = 0.78

$$-\frac{\sqrt{2-2c^4x^4}\sqrt{\frac{c^2x^2}{c^4x^4-1}}\,{}_2F_1\left(-\frac{1}{4},\frac{1}{2};\frac{3}{4};c^4x^4\right)}{x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Csch[2*Log[c*x]]]/x^3,x]
```

```
[Out] -((Sqrt[2 - 2*c^4*x^4]*Sqrt[(c^2*x^2)/(-1 + c^4*x^4)]*Hypergeometric2F1[-1/
4, 1/2, 3/4, c^4*x^4])/x^2)
```

fricas [F] time = 0.46, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{\sqrt{\operatorname{csch}\left(2\log(cx)\right)}}{x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(2*log(c*x))^(1/2)/x^3,x, algorithm="fricas")
```

```
[Out] integral(sqrt(csch(2*log(c*x)))/x^3, x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(2*log(c*x))^(1/2)/x^3,x, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.16, size = 126, normalized size = 1.70

$$\frac{\left(c^4x^4-1\right)\sqrt{2}\,\sqrt{\frac{c^2x^2}{c^4x^4-1}}}{x^2}-\frac{c^2\sqrt{c^2x^2+1}\,\sqrt{-c^2x^2+1}\,\left(\mathrm{EllipticF}\left(x\sqrt{-c^2}\,,i\right)-\mathrm{EllipticE}\left(x\sqrt{-c^2}\,,i\right)\right)\sqrt{2}\,\sqrt{\frac{c^2x^2}{c^4x^4-1}}}{\sqrt{-c^2}\;x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(2*ln(c*x))^(1/2)/x^3,x)
```

```
[Out] (c^4*x^4-1)/x^2*2^(1/2)*(c^2*x^2/(c^4*x^4-1))^(1/2)-c^2/(-c^2)^(1/2)*(c^2*x
^2+1)^(1/2)*(-c^2*x^2+1)^(1/2)*(EllipticF(x*(-c^2)^(1/2),I)-EllipticE(x*(-c
^2)^(1/2),I))*2^(1/2)*(c^2*x^2/(c^4*x^4-1))^(1/2)/x
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{\mathrm{csch}\left(2\,\log\left(cx\right)\right)}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(2*log(c*x))^(1/2)/x^3,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(csch(2*log(c*x)))/x^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{\sqrt{\frac{1}{\sinh(2\,\ln(c\,x))}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/sinh(2*log(c*x)))^(1/2)/x^3,x)
```

```
[Out] int((1/sinh(2*log(c*x)))^(1/2)/x^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{\mathrm{csch}\left(2\log\left(cx\right)\right)}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(2*ln(c*x))**(1/2)/x**3,x)
```

```
[Out] Integral(sqrt(csch(2*log(c*x)))/x**3, x)
```

3.141 $\int \frac{\sqrt{\operatorname{csch}(2\log(cx))}}{x^4}\,dx$

Optimal. Leaf size=25

$$\frac{1}{2}x\left(c^4-\frac{1}{x^4}\right)\sqrt{\operatorname{csch}(2\log(cx))}$$

```
[Out] 1/2*(c^4-1/x^4)*x*csch(2*ln(c*x))^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {5552, 5550, 261}

$$\frac{1}{2}x\left(c^4-\frac{1}{x^4}\right)\sqrt{\operatorname{csch}(2\log(cx))}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Csch[2*Log[c*x]]]/x^4,x]
```

```
[Out] ((c^4 - x^(-4))*x*Sqrt[Csch[2*Log[c*x]]])/2
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 5550

```
Int[Csch[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
 :> Dist[(Csch[d*(a + b*Log[x])]^p*(1 - 1/(E^(2*a*d)*x^(2*b*d)))^p)/x^(-(b*
d*p)), Int[(e*x)^m/(x^(b*d*p)*(1 - 1/(E^(2*a*d)*x^(2*b*d)))^p), x], x] /; F
reeQ[{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

Rule 5552

```
Int[Csch[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m
_.), x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[x
^((m + 1)/n - 1)*Csch[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b
, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{\operatorname{csch}(2\log(cx))}}{x^4}\,dx &= c^3\,\text{Subst}\left(\int \frac{\sqrt{\operatorname{csch}(2\log(x))}}{x^4}\,dx, x, cx\right)\\
&= \left(c^4\sqrt{1-\frac{1}{c^4x^4}}\,x\sqrt{\operatorname{csch}(2\log(cx))}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{1}{x^4}}\,x^5}\,dx, x, cx\right)\\
&= \frac{1}{2}\left(c^4-\frac{1}{x^4}\right)x\sqrt{\operatorname{csch}(2\log(cx))}
\end{aligned}$$

Mathematica [A] time = 0.04, size = 33, normalized size = 1.32

$$\frac{c^2}{2x\sqrt{\frac{c^2x^2}{2c^4x^4-2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Csch[2*Log[c*x]]]/x^4,x]
```

```
[Out] c^2/(2*x*Sqrt[(c^2*x^2)/(-2 + 2*c^4*x^4)])
```

fricas [A] time = 0.46, size = 37, normalized size = 1.48

$$\frac{\sqrt{2}\left(c^4x^4-1\right)\sqrt{\frac{c^2x^2}{c^4x^4-1}}}{2\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(2*log(c*x))^(1/2)/x^4,x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(2)*(c^4*x^4 - 1)*sqrt(c^2*x^2/(c^4*x^4 - 1))/x^3
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(2*log(c*x))^(1/2)/x^4,x, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.14, size = 38, normalized size = 1.52

$$\frac{\sqrt{2}\,\sqrt{\frac{c^2x^2}{c^4x^4-1}}\,\left(c^4x^4-1\right)}{2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(2*ln(c*x))^(1/2)/x^4,x)
```

```
[Out] 1/2*2^(1/2)*(c^2*x^2/(c^4*x^4-1))^(1/2)/x^3*(c^4*x^4-1)
```

maxima [B] time = 0.43, size = 89, normalized size = 3.56

$$\frac{1}{2}\,c^3\left(\frac{\sqrt{2}}{\sqrt{\frac{1}{cx}+1}\,\sqrt{-\frac{1}{cx}+1}\,\sqrt{\frac{1}{c^2x^2}+1}}-\frac{\sqrt{2}}{c^4x^4\sqrt{\frac{1}{cx}+1}\,\sqrt{-\frac{1}{cx}+1}\,\sqrt{\frac{1}{c^2x^2}+1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(2*log(c*x))^(1/2)/x^4,x, algorithm="maxima")
```

```
[Out] 1/2*c^3*(sqrt(2)/(sqrt(1/(c*x) + 1)*sqrt(-1/(c*x) + 1)*sqrt(1/(c^2*x^2) + 1
)) - sqrt(2)/(c^4*x^4*sqrt(1/(c*x) + 1)*sqrt(-1/(c*x) + 1)*sqrt(1/(c^2*x^2)
 + 1)))
```

mupad [B] time = 1.47, size = 58, normalized size = 2.32

$$\frac{c^4\,x\,\sqrt{\frac{2\,c^2\,x^2}{c^4\,x^4-1}}}{2}-\frac{\sqrt{\frac{2\,c^2\,x^2}{c^4\,x^4-1}}}{2\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/sinh(2*log(c*x)))^(1/2)/x^4,x)
```

```
[Out] (c^4*x*((2*c^2*x^2)/(c^4*x^4 - 1))^(1/2))/2 - ((2*c^2*x^2)/(c^4*x^4 - 1))^(
1/2)/(2*x^3)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\operatorname{csch}\left(2\log\left(cx\right)\right)}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(2*ln(c*x))**(1/2)/x**4,x)
```

```
[Out] Integral(sqrt(csch(2*log(c*x)))/x**4, x)
```

3.142 $\int \frac{\sqrt{\operatorname{csch}(2\log(cx))}}{x^5}\,dx$

Optimal. Leaf size=64

$$\frac{1}{3}\left(c^4-\frac{1}{x^4}\right)\sqrt{\operatorname{csch}(2\log(cx))}-\frac{1}{3}c^5x\sqrt{1-\frac{1}{c^4x^4}}\,F\left(\csc^{-1}(cx)\middle|-1\right)\sqrt{\operatorname{csch}(2\log(cx))}$$

```
[Out] 1/3*(c^4-1/x^4)*csch(2*ln(c*x))^(1/2)-1/3*c^5*x*EllipticF(1/c/x,I)*(1-1/c^4
/x^4)^(1/2)*csch(2*ln(c*x))^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5552, 5550, 335, 321, 221}

$$\frac{1}{3}\left(c^4-\frac{1}{x^4}\right)\sqrt{\operatorname{csch}(2\log(cx))}-\frac{1}{3}c^5x\sqrt{1-\frac{1}{c^4x^4}}\,F\left(\csc^{-1}(cx)\middle|-1\right)\sqrt{\operatorname{csch}(2\log(cx))}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Csch[2*Log[c*x]]]/x^5,x]
```

```
[Out] ((c^4 - x^(-4))*Sqrt[Csch[2*Log[c*x]]])/3 - (c^5*Sqrt[1 - 1/(c^4*x^4)]*x*Sq
rt[Csch[2*Log[c*x]]]*EllipticF[ArcCsc[c*x], -1])/3
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 5550

```
Int[Csch[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
 :> Dist[(Csch[d*(a + b*Log[x])]^p*(1 - 1/(E^(2*a*d)*x^(2*b*d)))^p)/x^(-(b*
d*p)), Int[(e*x)^m/(x^(b*d*p)*(1 - 1/(E^(2*a*d)*x^(2*b*d)))^p), x], x] /; F
reeQ[{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

Rule 5552

```
Int[Csch[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m
_.), x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[x
^((m + 1)/n - 1)*Csch[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b
, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rubi steps

$$\int \frac{\sqrt{\operatorname{csch}(2\log(cx))}}{x^5}\,dx = c^4\,\mathrm{Subst}\left(\int \frac{\sqrt{\operatorname{csch}(2\log(x))}}{x^5}\,dx, x, cx\right)$$
$$= \left(c^5\sqrt{1-\frac{1}{c^4x^4}}\,x\sqrt{\operatorname{csch}(2\log(cx))}\right)\mathrm{Subst}\left(\int \frac{1}{\sqrt{1-\frac{1}{x^4}}\,x^6}\,dx, x, cx\right)$$
$$= -\left(\left(c^5\sqrt{1-\frac{1}{c^4x^4}}\,x\sqrt{\operatorname{csch}(2\log(cx))}\right)\mathrm{Subst}\left(\int \frac{x^4}{\sqrt{1-x^4}}\,dx, x, \frac{1}{cx}\right)\right)$$
$$= \frac{1}{3}\left(c^4-\frac{1}{x^4}\right)\sqrt{\operatorname{csch}(2\log(cx))} - \frac{1}{3}\left(c^5\sqrt{1-\frac{1}{c^4x^4}}\,x\sqrt{\operatorname{csch}(2\log(cx))}\right)\mathrm{Subst}\left(\int \frac{1}{\sqrt{1-\right.\right.$$
$$= \frac{1}{3}\left(c^4-\frac{1}{x^4}\right)\sqrt{\operatorname{csch}(2\log(cx))} - \frac{1}{3}c^5\sqrt{1-\frac{1}{c^4x^4}}\,x\sqrt{\operatorname{csch}(2\log(cx))}\,F\left(\csc^{-1}(cx)\middle|-1\right)$$

Mathematica [C] time = 0.10, size = 60, normalized size = 0.94

$$-\frac{\sqrt{2-2c^4x^4}\sqrt{\frac{c^2x^2}{c^4x^4-1}}\,{}_2F_1\left(-\frac{3}{4},\frac{1}{2};\frac{1}{4};c^4x^4\right)}{3x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Csch[2*Log[c*x]]]/x^5,x]
```

```
[Out] -1/3*(Sqrt[2 - 2*c^4*x^4]*Sqrt[(c^2*x^2)/(-1 + c^4*x^4)]*Hypergeometric2F1[
-3/4, 1/2, 1/4, c^4*x^4])/x^4
```

fricas [F] time = 0.59, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\sqrt{\operatorname{csch}\left(2\,\log\left(cx\right)\right)}}{x^5}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(2*log(c*x))^(1/2)/x^5,x, algorithm="fricas")
```

```
[Out] integral(sqrt(csch(2*log(c*x)))/x^5, x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(2*log(c*x))^(1/2)/x^5,x, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.14, size = 112, normalized size = 1.75

$$\frac{\left(c^4x^4-1\right)\sqrt{2}\,\sqrt{\frac{c^2x^2}{c^4x^4-1}}}{3x^4} + \frac{c^4\sqrt{c^2x^2+1}\,\sqrt{-c^2x^2+1}\,\;\mathrm{EllipticF}\left(x\sqrt{-c^2}, i\right)\sqrt{2}\,\sqrt{\frac{c^2x^2}{c^4x^4-1}}}{3\sqrt{-c^2}\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(2*ln(c*x))^(1/2)/x^5,x)
```

```
[Out] 1/3*(c^4*x^4-1)/x^4*2^(1/2)*(c^2*x^2/(c^4*x^4-1))^(1/2)+1/3*c^4/(-c^2)^(1/2
)*(c^2*x^2+1)^(1/2)*(-c^2*x^2+1)^(1/2)*EllipticF(x*(-c^2)^(1/2),I)*2^(1/2)*
(c^2*x^2/(c^4*x^4-1))^(1/2)/x
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\operatorname{csch}\left(2 \log\left(cx\right)\right)}}{x^5}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(2*log(c*x))^(1/2)/x^5,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(csch(2*log(c*x)))/x^5, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{\frac{1}{\sinh(2\,\ln(c\,x))}}}{x^5}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/sinh(2*log(c*x)))^(1/2)/x^5,x)
```

```
[Out] int((1/sinh(2*log(c*x)))^(1/2)/x^5, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\operatorname{csch}\left(2\log\left(cx\right)\right)}}{x^5}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(2*ln(c*x))**(1/2)/x**5,x)
```

```
[Out] Integral(sqrt(csch(2*log(c*x)))/x**5, x)
```

3.143 $\int \frac{x^8}{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}\,dx$

Optimal. Leaf size=128

$$\frac{x}{32c^4\left(c^4-\frac{1}{x^4}\right)\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}-\frac{x^5}{16\left(c^4-\frac{1}{x^4}\right)\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}+\frac{\tanh^{-1}\left(\sqrt{1-\frac{1}{c^4x^4}}\right)}{32c^{12}x^3\left(1-\frac{1}{c^4x^4}\right)^{3/2}\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}+\frac{}{12\operatorname{csch}^{\frac{3}{2}}}$$

```
[Out] 1/32*x/c^4/(c^4-1/x^4)/csch(2*ln(c*x))^(3/2)-1/16*x^5/(c^4-1/x^4)/csch(2*ln
(c*x))^(3/2)+1/12*x^9/csch(2*ln(c*x))^(3/2)+1/32*arctanh((1-1/c^4/x^4)^(1/2
))/c^12/(1-1/c^4/x^4)^(3/2)/x^3/csch(2*ln(c*x))^(3/2)
```

Rubi [A] time = 0.08, antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.467, Rules used = {5552, 5550, 266, 47, 51, 63, 206}

$$-\frac{x^5}{16\left(c^4-\frac{1}{x^4}\right)\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}+\frac{x}{32c^4\left(c^4-\frac{1}{x^4}\right)\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}+\frac{\tanh^{-1}\left(\sqrt{1-\frac{1}{c^4x^4}}\right)}{32c^{12}x^3\left(1-\frac{1}{c^4x^4}\right)^{3/2}\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}+\frac{}{12\operatorname{csch}}$$

Antiderivative was successfully verified.

```
[In] Int[x^8/Csch[2*Log[c*x]]^(3/2),x]
```

```
[Out] x/(32*c^4*(c^4 - x^(-4))*Csch[2*Log[c*x]]^(3/2)) - x^5/(16*(c^4 - x^(-4))*C
sch[2*Log[c*x]]^(3/2)) + x^9/(12*Csch[2*Log[c*x]]^(3/2)) + ArcTanh[Sqrt[1 -
 1/(c^4*x^4)]]/(32*c^12*(1 - 1/(c^4*x^4))^(3/2)*x^3*Csch[2*Log[c*x]]^(3/2))
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5550

```
Int[Csch[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
 :> Dist[(Csch[d*(a + b*Log[x])]^p*(1 - 1/(E^(2*a*d)*x^(2*b*d)))^p)/x^(-(b*
d*p)), Int[(e*x)^m/(x^(b*d*p)*(1 - 1/(E^(2*a*d)*x^(2*b*d)))^p), x], x] /; F
reeQ[{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

Rule 5552

```
Int[Csch[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m
_.), x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[x
^((m + 1)/n - 1)*Csch[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b
, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rubi steps

$$\int \frac{x^8}{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}\,dx = \frac{\text{Subst}\left(\int \frac{x^8}{\operatorname{csch}^{\frac{3}{2}}(2\log(x))}\,dx, x, cx\right)}{c^9}$$

$$= \frac{\text{Subst}\left(\int \left(1-\frac{1}{x^4}\right)^{3/2} x^{11}\,dx, x, cx\right)}{c^{12}\left(1-\frac{1}{c^4x^4}\right)^{3/2} x^3 \operatorname{csch}^{\frac{3}{2}}(2\log(cx))}$$

$$= -\frac{\text{Subst}\left(\int \frac{(1-x)^{3/2}}{x^4}\,dx, x, \frac{1}{c^4x^4}\right)}{4c^{12}\left(1-\frac{1}{c^4x^4}\right)^{3/2} x^3 \operatorname{csch}^{\frac{3}{2}}(2\log(cx))}$$

$$= \frac{x^9}{12\operatorname{csch}^{\frac{3}{2}}(2\log(cx))} + \frac{\text{Subst}\left(\int \frac{\sqrt{1-x}}{x^3}\,dx, x, \frac{1}{c^4x^4}\right)}{8c^{12}\left(1-\frac{1}{c^4x^4}\right)^{3/2} x^3 \operatorname{csch}^{\frac{3}{2}}(2\log(cx))}$$

$$= -\frac{x^5}{16\left(c^4-\frac{1}{x^4}\right)\operatorname{csch}^{\frac{3}{2}}(2\log(cx))} + \frac{x^9}{12\operatorname{csch}^{\frac{3}{2}}(2\log(cx))} - \frac{\text{Subst}\left(\int \frac{1}{\sqrt{1-x}\,x^2}\,dx, x, \frac{1}{c^4x\ldots}\right.}{32c^{12}\left(1-\frac{1}{c^4x^4}\right)^{3/2} x^3 \operatorname{csch}^{\frac{3}{2}}(2\mathrm{l}\ldots}$$

$$= \frac{x}{32c^4\left(c^4-\frac{1}{x^4}\right)\operatorname{csch}^{\frac{3}{2}}(2\log(cx))} - \frac{x^5}{16\left(c^4-\frac{1}{x^4}\right)\operatorname{csch}^{\frac{3}{2}}(2\log(cx))} + \frac{x^9}{12\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}$$

$$= \frac{x}{32c^4\left(c^4-\frac{1}{x^4}\right)\operatorname{csch}^{\frac{3}{2}}(2\log(cx))} - \frac{x^5}{16\left(c^4-\frac{1}{x^4}\right)\operatorname{csch}^{\frac{3}{2}}(2\log(cx))} + \frac{x^9}{12\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}$$

$$= \frac{x}{32c^4\left(c^4-\frac{1}{x^4}\right)\operatorname{csch}^{\frac{3}{2}}(2\log(cx))} - \frac{x^5}{16\left(c^4-\frac{1}{x^4}\right)\operatorname{csch}^{\frac{3}{2}}(2\log(cx))} + \frac{x^9}{12\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}$$

Mathematica [A] time = 0.21, size = 95, normalized size = 0.74

$$\frac{c^3x^3\sqrt{1-c^4x^4}\left(8c^8x^8-14c^4x^4+3\right)-3cx\sin^{-1}\left(c^2x^2\right)}{192c^9\sqrt{2-2c^4x^4}\sqrt{\frac{c^2x^2}{c^4x^4-1}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8/Csch[2*Log[c*x]]^(3/2),x]
```

```
[Out] (c^3*x^3*Sqrt[1 - c^4*x^4]*(3 - 14*c^4*x^4 + 8*c^8*x^8) - 3*c*x*ArcSin[c^2*
x^2])/(192*c^9*Sqrt[2 - 2*c^4*x^4]*Sqrt[(c^2*x^2)/(-1 + c^4*x^4)])
```

fricas [A] time = 0.43, size = 110, normalized size = 0.86

$$\frac{2\sqrt{2}\left(8\,c^{13}x^{13}-22\,c^9x^9+17\,c^5x^5-3\,cx\right)\sqrt{\frac{c^2x^2}{c^4x^4-1}}+3\sqrt{2}\log\left(2\,c^4x^4+2\left(c^5x^5-cx\right)\sqrt{\frac{c^2x^2}{c^4x^4-1}}-1\right)}{768\,c^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/csch(2*log(c*x))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/768*(2*sqrt(2)*(8*c^13*x^13 - 22*c^9*x^9 + 17*c^5*x^5 - 3*c*x)*sqrt(c^2*x
^2/(c^4*x^4 - 1)) + 3*sqrt(2)*log(2*c^4*x^4 + 2*(c^5*x^5 - c*x)*sqrt(c^2*x^
2/(c^4*x^4 - 1)) - 1))/c^9
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/csch(2*log(c*x))^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:Warning, integration of abs or sign assumes constant sign
 by intervals (correct if the argument is real):Check [abs(t_nostep)]Warnin
g, integration of abs or sign assumes constant sign by intervals (correct i
f the argument is real):Check [abs(t_nostep)]Unable to divide, perhaps due
to rounding error%%%{1,[10,4,1,0]%%%}+%%%{-1,[6,0,1,0]%%%} / %%%{1,[0,2,0,1
]%%%} Error: Bad Argument Value
```

maple [A] time = 0.21, size = 121, normalized size = 0.95

$$\frac{x^{3} \left(8 c^{8} x^{8}-14 c^{4} x^{4}+3\right) \sqrt{2}}{384 c^{6} \sqrt{\frac{c^{2} x^{2}}{c^{4} x^{4}-1}}}+\frac{\ln \left(\frac{c^{4} x^{2}}{\sqrt{c^{4}}}+\sqrt{c^{4} x^{4}-1}\right) \sqrt{2}\, x}{128 c^{6} \sqrt{c^{4}}\, \sqrt{c^{4} x^{4}-1}\, \sqrt{\frac{c^{2} x^{2}}{c^{4} x^{4}-1}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/csch(2*ln(c*x))^(3/2),x)
```

```
[Out] 1/384*x^3*(8*c^8*x^8-14*c^4*x^4+3)/c^6*2^(1/2)/(c^2*x^2/(c^4*x^4-1))^(1/2)+
1/128/c^6*ln(c^4*x^2/(c^4)^(1/2)+(c^4*x^4-1)^(1/2))/(c^4)^(1/2)*2^(1/2)*x/(
c^4*x^4-1)^(1/2)/(c^2*x^2/(c^4*x^4-1))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{8}}{\operatorname{csch}\left(2 \, \log\left(c x\right)\right)^{\frac{3}{2}}}\,{d x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/csch(2*log(c*x))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^8/csch(2*log(c*x))^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^8}{{\left(\frac{1}{\mathrm{sinh}\left(2\,\ln\left(c\,x\right)\right)}\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(1/sinh(2*log(c*x)))^(3/2),x)
```

```
[Out] int(x^8/(1/sinh(2*log(c*x)))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{8}}{\operatorname{csch}^{\frac{3}{2}}{\left(2 \log{\left(c x \right)} \right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8/csch(2*ln(c*x))**(3/2),x)
```

```
[Out] Integral(x**8/csch(2*log(c*x))**(3/2), x)
```

3.144 $\int \frac{x^7}{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}\,dx$

Optimal. Leaf size=118

$$-\frac{6x^4}{77\left(c^4-\frac{1}{x^4}\right)\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}+\frac{4}{77c^4\left(c^4-\frac{1}{x^4}\right)\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}-\frac{4F\left(\csc^{-1}(cx)\middle|-1\right)}{77c^{11}x^3\left(1-\frac{1}{c^4x^4}\right)^{3/2}\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}+\frac{}{11cs}$$

```
[Out] 4/77/c^4/(c^4-1/x^4)/csch(2*ln(c*x))^(3/2)-6/77*x^4/(c^4-1/x^4)/csch(2*ln(c
*x))^(3/2)+1/11*x^8/csch(2*ln(c*x))^(3/2)-4/77*EllipticF(1/c/x,I)/c^11/(1-1
/c^4/x^4)^(3/2)/x^3/csch(2*ln(c*x))^(3/2)
```

Rubi [A] time = 0.08, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {5552, 5550, 335, 277, 325, 221}

$$-\frac{6x^4}{77\left(c^4-\frac{1}{x^4}\right)\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}+\frac{4}{77c^4\left(c^4-\frac{1}{x^4}\right)\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}-\frac{4F\left(\csc^{-1}(cx)\middle|-1\right)}{77c^{11}x^3\left(1-\frac{1}{c^4x^4}\right)^{3/2}\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}+\frac{}{11cs}$$

Antiderivative was successfully verified.

```
[In] Int[x^7/Csch[2*Log[c*x]]^(3/2),x]
```

```
[Out] 4/(77*c^4*(c^4 - x^(-4))*Csch[2*Log[c*x]]^(3/2)) - (6*x^4)/(77*(c^4 - x^(-4
))*Csch[2*Log[c*x]]^(3/2)) + x^8/(11*Csch[2*Log[c*x]]^(3/2)) - (4*EllipticF
[ArcCsc[c*x], -1])/(77*c^11*(1 - 1/(c^4*x^4))^(3/2)*x^3*Csch[2*Log[c*x]]^(3
/2))
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 5550

```
Int[Csch[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
 :> Dist[(Csch[d*(a + b*Log[x])]^p*(1 - 1/(E^(2*a*d)*x^(2*b*d)))^p)/x^(-(b*
d*p)), Int[(e*x)^m/(x^(b*d*p)*(1 - 1/(E^(2*a*d)*x^(2*b*d)))^p), x], x] /; F
reeQ[{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

Rule 5552

```
Int[Csch[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m
_.), x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[x
^((m + 1)/n - 1)*Csch[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b
, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rubi steps

$$\int \frac{x^7}{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}\,dx = \frac{\operatorname{Subst}\left(\int \frac{x^7}{\operatorname{csch}^{\frac{3}{2}}(2\log(x))}\,dx, x, cx\right)}{c^8}$$

$$= \frac{\operatorname{Subst}\left(\int \left(1-\frac{1}{x^4}\right)^{3/2} x^{10}\,dx, x, cx\right)}{c^{11}\left(1-\frac{1}{c^4x^4}\right)^{3/2} x^3\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}$$

$$= -\frac{\operatorname{Subst}\left(\int \frac{\left(1-x^4\right)^{3/2}}{x^{12}}\,dx, x, \frac{1}{cx}\right)}{c^{11}\left(1-\frac{1}{c^4x^4}\right)^{3/2} x^3\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}$$

$$= \frac{x^8}{11\operatorname{csch}^{\frac{3}{2}}(2\log(cx))} + \frac{6\operatorname{Subst}\left(\int \frac{\sqrt{1-x^4}}{x^8}\,dx, x, \frac{1}{cx}\right)}{11c^{11}\left(1-\frac{1}{c^4x^4}\right)^{3/2} x^3\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}$$

$$= -\frac{6x^4}{77\left(c^4-\frac{1}{x^4}\right)\operatorname{csch}^{\frac{3}{2}}(2\log(cx))} + \frac{x^8}{11\operatorname{csch}^{\frac{3}{2}}(2\log(cx))} - \frac{12\operatorname{Subst}\left(\int \frac{1}{x^4\sqrt{1-x^4}}\,dx, x,\right.}{77c^{11}\left(1-\frac{1}{c^4x^4}\right)^{3/2} x^3\operatorname{csch}^{\frac{3}{2}}(2\log}$$

$$= \frac{4}{77c^4\left(c^4-\frac{1}{x^4}\right)\operatorname{csch}^{\frac{3}{2}}(2\log(cx))} - \frac{6x^4}{77\left(c^4-\frac{1}{x^4}\right)\operatorname{csch}^{\frac{3}{2}}(2\log(cx))} + \frac{x^8}{11\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}$$

$$= \frac{4}{77c^4\left(c^4-\frac{1}{x^4}\right)\operatorname{csch}^{\frac{3}{2}}(2\log(cx))} - \frac{6x^4}{77\left(c^4-\frac{1}{x^4}\right)\operatorname{csch}^{\frac{3}{2}}(2\log(cx))} + \frac{x^8}{11\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}$$

Mathematica [C] time = 0.18, size = 80, normalized size = 0.68

$$\frac{x^2\left(\left(1-c^4x^4\right)^{5/2} - {}_2F_1\left(-\frac{3}{2},\frac{1}{4};\frac{5}{4};c^4x^4\right)\right)}{22c^6\sqrt{2-2c^4x^4}\sqrt{\frac{c^2x^2}{c^4x^4-1}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7/Csch[2*Log[c*x]]^(3/2),x]
```

```
[Out] (x^2*((1 - c^4*x^4)^(5/2) - Hypergeometric2F1[-3/2, 1/4, 5/4, c^4*x^4]))/(2
2*c^6*Sqrt[2 - 2*c^4*x^4]*Sqrt[(c^2*x^2)/(-1 + c^4*x^4)])
```

fricas [F] time = 0.46, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x^7}{\operatorname{csch}\left(2\,\log\left(cx\right)\right)^{\frac{3}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/csch(2*log(c*x))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(x^7/csch(2*log(c*x))^(3/2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^7}{\operatorname{csch}\left(2\,\log\left(cx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/csch(2*log(c*x))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^7/csch(2*log(c*x))^(3/2), x)
```

maple [A] time = 0.16, size = 133, normalized size = 1.13

$$\frac{x^2\left(7c^8x^8-13c^4x^4+4\right)\sqrt{2}}{308c^6\sqrt{\frac{c^2x^2}{c^4x^4-1}}}+\frac{\sqrt{c^2x^2+1}\,\sqrt{-c^2x^2+1}\,\operatorname{EllipticF}\left(x\sqrt{-c^2}, i\right)\sqrt{2}\,x}{77c^6\sqrt{-c^2}\,\left(c^4x^4-1\right)\sqrt{\frac{c^2x^2}{c^4x^4-1}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/csch(2*ln(c*x))^(3/2),x)
```

```
[Out] 1/308*x^2*(7*c^8*x^8-13*c^4*x^4+4)/c^6*2^(1/2)/(c^2*x^2/(c^4*x^4-1))^(1/2)+
1/77/c^6/(-c^2)^(1/2)*(c^2*x^2+1)^(1/2)*(-c^2*x^2+1)^(1/2)/(c^4*x^4-1)*Elli
pticF(x*(-c^2)^(1/2),I)*2^(1/2)*x/(c^2*x^2/(c^4*x^4-1))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^7}{\operatorname{csch}\left(2\,\log\left(cx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/csch(2*log(c*x))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^7/csch(2*log(c*x))^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^7}{{\left(\frac{1}{\sinh\left(2\,\ln\left(c\,x\right)\right)}\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(1/sinh(2*log(c*x)))^(3/2),x)
```

```
[Out] int(x^7/(1/sinh(2*log(c*x)))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^7}{\operatorname{csch}^{\frac{3}{2}}\left(2\log\left(cx\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/csch(2*ln(c*x))**(3/2),x)
```

```
[Out] Integral(x**7/csch(2*log(c*x))**(3/2), x)
```

3.145 $\int \frac{x^6}{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}\,dx$

Optimal. Leaf size=30

$$\frac{x^7\left(c^4-\frac{1}{x^4}\right)}{10c^4\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}$$

```
[Out] 1/10*(c^4-1/x^4)*x^7/c^4/csch(2*ln(c*x))^(3/2)
```

Rubi [A] time = 0.04, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {5552, 5550, 264}

$$\frac{x^7\left(c^4-\frac{1}{x^4}\right)}{10c^4\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/Csch[2*Log[c*x]]^(3/2),x]
```

```
[Out] ((c^4 - x^(-4))*x^7)/(10*c^4*Csch[2*Log[c*x]]^(3/2))
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 5550

```
Int[Csch[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
 :> Dist[(Csch[d*(a + b*Log[x])]^p*(1 - 1/(E^(2*a*d)*x^(2*b*d)))^p)/x^(-(b*
d*p)), Int[(e*x)^m/(x^(b*d*p)*(1 - 1/(E^(2*a*d)*x^(2*b*d)))^p), x], x] /; F
reeQ[{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

Rule 5552

```
Int[Csch[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m
_.), x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[x
^((m + 1)/n - 1)*Csch[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b
, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rubi steps

$$\int \frac{x^6}{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}\,dx = \frac{\operatorname{Subst}\left(\int \frac{x^6}{\operatorname{csch}^{\frac{3}{2}}(2\log(x))}\,dx, x, cx\right)}{c^7}$$

$$= \frac{\operatorname{Subst}\left(\int \left(1-\frac{1}{x^4}\right)^{3/2} x^9\,dx, x, cx\right)}{c^{10}\left(1-\frac{1}{c^4x^4}\right)^{3/2} x^3\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}$$

$$= \frac{\left(c^4-\frac{1}{x^4}\right)x^7}{10c^4\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}$$

Mathematica [**A**] time = 0.05, size = 44, normalized size = 1.47

$$\frac{\left(c^4x^4-1\right)^3\sqrt{\frac{c^2x^2}{2c^4x^4-2}}}{20c^8x}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/Csch[2*Log[c*x]]^(3/2),x]
```

```
[Out] ((-1 + c^4*x^4)^3*Sqrt[(c^2*x^2)/(-2 + 2*c^4*x^4)])/(20*c^8*x)
```

fricas [**B**] time = 1.03, size = 56, normalized size = 1.87

$$\frac{\sqrt{2}\left(c^{12}x^{12}-3\,c^8x^8+3\,c^4x^4-1\right)\sqrt{\frac{c^2x^2}{c^4x^4-1}}}{40\,c^8x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/csch(2*log(c*x))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/40*sqrt(2)*(c^12*x^12 - 3*c^8*x^8 + 3*c^4*x^4 - 1)*sqrt(c^2*x^2/(c^4*x^4
- 1))/(c^8*x)
```

giac [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/csch(2*log(c*x))^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:Warning, integration of abs or sign assumes constant sign
 by intervals (correct if the argument is real):Check [abs(t_nostep)]Warnin
g, integration of abs or sign assumes constant sign by intervals (correct i
f the argument is real):Check [abs(t_nostep)]Unable to divide, perhaps due
to rounding error%%%{1,[8,4,1,0]%%%}+%%%{-1,[4,0,1,0]%%%} / %%%{1,[0,2,0,1]
%%%} Error: Bad Argument Value
```

maple [**A**] time = 0.14, size = 47, normalized size = 1.57

$$\frac{\sqrt{2}\,x\left(c^8x^8-2c^4x^4+1\right)}{40c^6\sqrt{\frac{c^2x^2}{c^4x^4-1}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/csch(2*ln(c*x))^(3/2),x)
```

```
[Out] 1/40*2^(1/2)/c^6*x/(c^2*x^2/(c^4*x^4-1))^(1/2)*(c^8*x^8-2*c^4*x^4+1)
```

maxima [A] time = 0.47, size = 46, normalized size = 1.53

$$\frac{\left(\sqrt{2}\,c^4x^4-\sqrt{2}\right)\left(c^2x^2+1\right)^{\frac{3}{2}}(cx+1)^{\frac{3}{2}}(cx-1)^{\frac{3}{2}}}{40\,c^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/csch(2*log(c*x))^(3/2),x, algorithm="maxima")
```

```
[Out] 1/40*(sqrt(2)*c^4*x^4 - sqrt(2))*(c^2*x^2 + 1)^(3/2)*(c*x + 1)^(3/2)*(c*x -
1)^(3/2)/c^7
```

mupad [B] time = 1.56, size = 42, normalized size = 1.40

$$\frac{\left(c^4\,x^4-1\right)^3\,\sqrt{\frac{2\,c^2\,x^2}{c^4\,x^4-1}}}{40\,c^8\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(1/sinh(2*log(c*x)))^(3/2),x)
```

```
[Out] ((c^4*x^4 - 1)^3*((2*c^2*x^2)/(c^4*x^4 - 1))^(1/2))/(40*c^8*x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6}{\operatorname{csch}^{\frac{3}{2}}\left(2\log\left(cx\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/csch(2*ln(c*x))**(3/2),x)
```

```
[Out] Integral(x**6/csch(2*log(c*x))**(3/2), x)
```

3.146 $\int \frac{x^5}{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}\,dx$

Optimal. Leaf size=162

$$-\frac{2x^2}{15\left(c^4-\frac{1}{x^4}\right)\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}+\frac{4}{15c^4x^2\left(c^4-\frac{1}{x^4}\right)\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}-\frac{4F\left(\csc^{-1}(cx)\middle|-1\right)}{15c^9x^3\left(1-\frac{1}{c^4x^4}\right)^{3/2}\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}+\frac{}{15c^9x^3}$$

```
[Out] 4/15/c^4/(c^4-1/x^4)/x^2/csch(2*ln(c*x))^(3/2)-2/15*x^2/(c^4-1/x^4)/csch(2*
ln(c*x))^(3/2)+1/9*x^6/csch(2*ln(c*x))^(3/2)+4/15*EllipticE(1/c/x,I)/c^9/(1
-1/c^4/x^4)^(3/2)/x^3/csch(2*ln(c*x))^(3/2)-4/15*EllipticF(1/c/x,I)/c^9/(1-
1/c^4/x^4)^(3/2)/x^3/csch(2*ln(c*x))^(3/2)
```

Rubi [A] time = 0.10, antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {5552, 5550, 335, 277, 325, 307, 221, 1181, 424}

$$-\frac{2x^2}{15\left(c^4-\frac{1}{x^4}\right)\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}+\frac{4}{15c^4x^2\left(c^4-\frac{1}{x^4}\right)\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}-\frac{4F\left(\csc^{-1}(cx)\middle|-1\right)}{15c^9x^3\left(1-\frac{1}{c^4x^4}\right)^{3/2}\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}+\frac{}{15c^9x^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/Csch[2*Log[c*x]]^(3/2),x]
```

```
[Out] 4/(15*c^4*(c^4 - x^(-4))*x^2*Csch[2*Log[c*x]]^(3/2)) - (2*x^2)/(15*(c^4 - x
^(-4))*Csch[2*Log[c*x]]^(3/2)) + x^6/(9*Csch[2*Log[c*x]]^(3/2)) + (4*Ellipt
icE[ArcCsc[c*x], -1])/(15*c^9*(1 - 1/(c^4*x^4))^(3/2)*x^3*Csch[2*Log[c*x]]^
(3/2)) - (4*EllipticF[ArcCsc[c*x], -1])/(15*c^9*(1 - 1/(c^4*x^4))^(3/2)*x^3
*Csch[2*Log[c*x]]^(3/2))
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 307

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-(b/a), 2]}
, -Dist[q^(-1), Int[1/Sqrt[a + b*x^4], x], x] + Dist[1/q, Int[(1 + q*x^2)/S
qrt[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 1181

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[-(a*c), 2]}, Dist[Sqrt[-c], Int[(d + e*x^2)/(Sqrt[q + c*x^2]*Sqrt[q - c
*x^2]), x], x]] /; FreeQ[{a, c, d, e}, x] && GtQ[a, 0] && LtQ[c, 0]
```

Rule 5550

```
Int[Csch[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
 :> Dist[(Csch[d*(a + b*Log[x])]^p*(1 - 1/(E^(2*a*d)*x^(2*b*d)))^p)/x^(-(b*
d*p)), Int[(e*x)^m/(x^(b*d*p)*(1 - 1/(E^(2*a*d)*x^(2*b*d)))^p), x], x] /; F
reeQ[{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

Rule 5552

```
Int[Csch[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m
_.), x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[x
^((m + 1)/n - 1)*Csch[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b
, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rubi steps

$$\int \frac{x^5}{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}\,dx = \frac{\text{Subst}\left(\int \frac{x^5}{\operatorname{csch}^{\frac{3}{2}}(2\log(x))}\,dx, x, cx\right)}{c^6}$$

$$= \frac{\text{Subst}\left(\int \left(1-\frac{1}{x^4}\right)^{3/2} x^8\,dx, x, cx\right)}{c^9\left(1-\frac{1}{c^4x^4}\right)^{3/2} x^3\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}$$

$$= -\frac{\text{Subst}\left(\int \frac{(1-x^4)^{3/2}}{x^{10}}\,dx, x, \frac{1}{cx}\right)}{c^9\left(1-\frac{1}{c^4x^4}\right)^{3/2} x^3\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}$$

$$= \frac{x^6}{9\operatorname{csch}^{\frac{3}{2}}(2\log(cx))} + \frac{2\,\text{Subst}\left(\int \frac{\sqrt{1-x^4}}{x^6}\,dx, x, \frac{1}{cx}\right)}{3c^9\left(1-\frac{1}{c^4x^4}\right)^{3/2} x^3\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}$$

$$= -\frac{2x^2}{15\left(c^4-\frac{1}{x^4}\right)\operatorname{csch}^{\frac{3}{2}}(2\log(cx))} + \frac{x^6}{9\operatorname{csch}^{\frac{3}{2}}(2\log(cx))} - \frac{4\,\text{Subst}\left(\int \frac{1}{x^2\sqrt{1-x^4}}\,dx, x, \frac{1}{cx}\right)}{15c^9\left(1-\frac{1}{c^4x^4}\right)^{3/2} x^3\operatorname{csch}^{\frac{3}{2}}(2\log}$$

$$= \frac{4}{15c^4\left(c^4-\frac{1}{x^4}\right)x^2\operatorname{csch}^{\frac{3}{2}}(2\log(cx))} - \frac{2x^2}{15\left(c^4-\frac{1}{x^4}\right)\operatorname{csch}^{\frac{3}{2}}(2\log(cx))} + \frac{x^6}{9\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}$$

$$= \frac{4}{15c^4\left(c^4-\frac{1}{x^4}\right)x^2\operatorname{csch}^{\frac{3}{2}}(2\log(cx))} - \frac{2x^2}{15\left(c^4-\frac{1}{x^4}\right)\operatorname{csch}^{\frac{3}{2}}(2\log(cx))} + \frac{x^6}{9\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}$$

$$= \frac{4}{15c^4\left(c^4-\frac{1}{x^4}\right)x^2\operatorname{csch}^{\frac{3}{2}}(2\log(cx))} - \frac{2x^2}{15\left(c^4-\frac{1}{x^4}\right)\operatorname{csch}^{\frac{3}{2}}(2\log(cx))} + \frac{x^6}{9\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}$$

$$= \frac{4}{15c^4\left(c^4-\frac{1}{x^4}\right)x^2\operatorname{csch}^{\frac{3}{2}}(2\log(cx))} - \frac{2x^2}{15\left(c^4-\frac{1}{x^4}\right)\operatorname{csch}^{\frac{3}{2}}(2\log(cx))} + \frac{x^6}{9\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}$$

Mathematica [C] time = 0.13, size = 63, normalized size = 0.39

$$-\frac{x^4\,{}_2F_1\left(-\frac{3}{2},\frac{3}{4};\frac{7}{4};c^4x^4\right)}{6c^2\sqrt{2-2c^4x^4}\sqrt{\frac{c^2x^2}{c^4x^4-1}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/Csch[2*Log[c*x]]^(3/2),x]
```

```
[Out] -1/6*(x^4*Hypergeometric2F1[-3/2, 3/4, 7/4, c^4*x^4])/(c^2*Sqrt[2 - 2*c^4*x
^4]*Sqrt[(c^2*x^2)/(-1 + c^4*x^4)])
```

fricas [F] time = 0.54, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x^5}{\operatorname{csch}\left(2\,\log\left(cx\right)\right)^{\frac{3}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/csch(2*log(c*x))^(3/2),x, algorithm="fricas")
[Out] integral(x^5/csch(2*log(c*x))^(3/2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5}{\operatorname{csch}\left(2 \log(cx)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/csch(2*log(c*x))^(3/2),x, algorithm="giac")
[Out] integrate(x^5/csch(2*log(c*x))^(3/2), x)
```

maple [A] time = 0.15, size = 140, normalized size = 0.86

$$\frac{x^4\left(5c^4x^4-11\right)\sqrt{2}}{180c^2\sqrt{\frac{c^2x^2}{c^4x^4-1}}}+\frac{\sqrt{c^2x^2+1}\,\sqrt{-c^2x^2+1}\,\left(\mathrm{EllipticF}\left(x\sqrt{-c^2}, i\right)-\mathrm{EllipticE}\left(x\sqrt{-c^2}, i\right)\right)\sqrt{2}\,x}{15\sqrt{-c^2}\,\left(c^4x^4-1\right)c^4\sqrt{\frac{c^2x^2}{c^4x^4-1}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/csch(2*ln(c*x))^(3/2),x)
[Out] 1/180*x^4*(5*c^4*x^4-11)*2^(1/2)/c^2/(c^2*x^2/(c^4*x^4-1))^(1/2)+1/15/(-c^2
)^(1/2)*(c^2*x^2+1)^(1/2)*(-c^2*x^2+1)^(1/2)/(c^4*x^4-1)/c^4*(EllipticF(x*(
-c^2)^(1/2),I)-EllipticE(x*(-c^2)^(1/2),I))*2^(1/2)*x/(c^2*x^2/(c^4*x^4-1))
^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5}{\operatorname{csch}\left(2 \log(cx)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/csch(2*log(c*x))^(3/2),x, algorithm="maxima")
[Out] integrate(x^5/csch(2*log(c*x))^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^5}{{\left(\frac{1}{\sinh(2\ln(c\,x))}\right)}^{3/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(1/sinh(2*log(c*x)))^(3/2),x)
[Out] int(x^5/(1/sinh(2*log(c*x)))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5}{\operatorname{csch}^{\frac{3}{2}}\left(2\log(cx)\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/csch(2*ln(c*x))**(3/2),x)
[Out] Integral(x**5/csch(2*log(c*x))**(3/2), x)
```

3.147 $\int \frac{x^4}{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}\,dx$

Optimal. Leaf size=96

$$-\frac{3x}{16\left(c^4-\frac{1}{x^4}\right)\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}+\frac{3\tanh^{-1}\left(\sqrt{1-\frac{1}{c^4x^4}}\right)}{16c^8x^3\left(1-\frac{1}{c^4x^4}\right)^{3/2}\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}+\frac{x^5}{8\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}$$

```
[Out] -3/16*x/(c^4-1/x^4)/csch(2*ln(c*x))^(3/2)+1/8*x^5/csch(2*ln(c*x))^(3/2)+3/1
6*arctanh((1-1/c^4/x^4)^(1/2))/c^8/(1-1/c^4/x^4)^(3/2)/x^3/csch(2*ln(c*x))^
(3/2)
```

Rubi [A] time = 0.07, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {5552, 5550, 266, 47, 63, 206}

$$-\frac{3x}{16\left(c^4-\frac{1}{x^4}\right)\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}+\frac{3\tanh^{-1}\left(\sqrt{1-\frac{1}{c^4x^4}}\right)}{16c^8x^3\left(1-\frac{1}{c^4x^4}\right)^{3/2}\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}+\frac{x^5}{8\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/Csch[2*Log[c*x]]^(3/2),x]
```

```
[Out] (-3*x)/(16*(c^4 - x^(-4))*Csch[2*Log[c*x]]^(3/2)) + x^5/(8*Csch[2*Log[c*x]]
^(3/2)) + (3*ArcTanh[Sqrt[1 - 1/(c^4*x^4)]])/(16*c^8*(1 - 1/(c^4*x^4))^(3/2
)*x^3*Csch[2*Log[c*x]]^(3/2))
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5550

```
Int[Csch[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
 :> Dist[(Csch[d*(a + b*Log[x])]^p*(1 - 1/(E^(2*a*d)*x^(2*b*d)))^p)/x^(-(b*
d*p)), Int[(e*x)^m/(x^(b*d*p)*(1 - 1/(E^(2*a*d)*x^(2*b*d)))^p), x], x] /; F
reeQ[{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

Rule 5552

```
Int[Csch[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m
_.), x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[x
^((m + 1)/n - 1)*Csch[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b
, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rubi steps

$$\int \frac{x^4}{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}\,dx = \frac{\operatorname{Subst}\left(\int \frac{x^4}{\operatorname{csch}^{\frac{3}{2}}(2\log(x))}\,dx, x, cx\right)}{c^5}$$

$$= \frac{\operatorname{Subst}\left(\int \left(1-\frac{1}{x^4}\right)^{3/2} x^7\,dx, x, cx\right)}{c^8\left(1-\frac{1}{c^4x^4}\right)^{3/2} x^3\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}$$

$$= -\frac{\operatorname{Subst}\left(\int \frac{(1-x)^{3/2}}{x^3}\,dx, x, \frac{1}{c^4x^4}\right)}{4c^8\left(1-\frac{1}{c^4x^4}\right)^{3/2} x^3\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}$$

$$= \frac{x^5}{8\operatorname{csch}^{\frac{3}{2}}(2\log(cx))} + \frac{3\operatorname{Subst}\left(\int \frac{\sqrt{1-x}}{x^2}\,dx, x, \frac{1}{c^4x^4}\right)}{16c^8\left(1-\frac{1}{c^4x^4}\right)^{3/2} x^3\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}$$

$$= -\frac{3x}{16\left(c^4-\frac{1}{x^4}\right)\operatorname{csch}^{\frac{3}{2}}(2\log(cx))} + \frac{x^5}{8\operatorname{csch}^{\frac{3}{2}}(2\log(cx))} - \frac{3\operatorname{Subst}\left(\int \frac{1}{\sqrt{1-x}\,x}\,dx, x, \frac{1}{c^4x^4}\right)}{32c^8\left(1-\frac{1}{c^4x^4}\right)^{3/2} x^3\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}$$

$$= -\frac{3x}{16\left(c^4-\frac{1}{x^4}\right)\operatorname{csch}^{\frac{3}{2}}(2\log(cx))} + \frac{x^5}{8\operatorname{csch}^{\frac{3}{2}}(2\log(cx))} + \frac{3\operatorname{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sqrt{1-\frac{1}{c^4x^4}}\right)}{16c^8\left(1-\frac{1}{c^4x^4}\right)^{3/2} x^3\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}$$

$$= -\frac{3x}{16\left(c^4-\frac{1}{x^4}\right)\operatorname{csch}^{\frac{3}{2}}(2\log(cx))} + \frac{x^5}{8\operatorname{csch}^{\frac{3}{2}}(2\log(cx))} + \frac{3\tanh^{-1}\left(\sqrt{1-\frac{1}{c^4x^4}}\right)}{16c^8\left(1-\frac{1}{c^4x^4}\right)^{3/2} x^3\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}$$

Mathematica [A] time = 0.18, size = 87, normalized size = 0.91

$$\frac{c^3x^3\sqrt{1-c^4x^4}\left(2c^4x^4-5\right)-3cx\sin^{-1}\left(c^2x^2\right)}{32c^5\sqrt{2-2c^4x^4}\sqrt{\frac{c^2x^2}{c^4x^4-1}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/Csch[2*Log[c*x]]^(3/2),x]
```

```
[Out] (c^3*x^3*Sqrt[1 - c^4*x^4]*(-5 + 2*c^4*x^4) - 3*c*x*ArcSin[c^2*x^2])/(32*c^
5*Sqrt[2 - 2*c^4*x^4]*Sqrt[(c^2*x^2)/(-1 + c^4*x^4)])
```

fricas [A] time = 0.62, size = 102, normalized size = 1.06

$$\frac{2\sqrt{2}\left(2\,c^9x^9-7\,c^5x^5+5\,cx\right)\sqrt{\frac{c^2x^2}{c^4x^4-1}}+3\,\sqrt{2}\,\log\left(2\,c^4x^4+2\left(c^5x^5-cx\right)\sqrt{\frac{c^2x^2}{c^4x^4-1}}-1\right)}{128\,c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/csch(2*log(c*x))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/128*(2*sqrt(2)*(2*c^9*x^9 - 7*c^5*x^5 + 5*c*x)*sqrt(c^2*x^2/(c^4*x^4 - 1)
) + 3*sqrt(2)*log(2*c^4*x^4 + 2*(c^5*x^5 - c*x)*sqrt(c^2*x^2/(c^4*x^4 - 1))
 - 1))/c^5
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/csch(2*log(c*x))^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:Warning, integration of abs or sign assumes constant sign
 by intervals (correct if the argument is real):Check [abs(t_nostep)]Warnin
g, integration of abs or sign assumes constant sign by intervals (correct i
f the argument is real):Check [abs(t_nostep)]Unable to divide, perhaps due
to rounding error%%%{1,[6,4,1,0]%%%}+%%%{-1,[2,0,1,0]%%%} / %%%{1,[0,2,0,1]
%%%} Error: Bad Argument Value
```

maple [A] time = 0.19, size = 113, normalized size = 1.18

$$\frac{x^3\left(2c^4x^4-5\right)\sqrt{2}}{64c^2\sqrt{\frac{c^2x^2}{c^4x^4-1}}}+\frac{3\ln\left(\frac{c^4x^2}{\sqrt{c^4}}+\sqrt{c^4x^4-1}\right)\sqrt{2}\,x}{64\sqrt{c^4}\,c^2\sqrt{c^4x^4-1}\,\sqrt{\frac{c^2x^2}{c^4x^4-1}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/csch(2*ln(c*x))^(3/2),x)
```

```
[Out] 1/64*x^3*(2*c^4*x^4-5)*2^(1/2)/c^2/(c^2*x^2/(c^4*x^4-1))^(1/2)+3/64*ln(c^4*
x^2/(c^4)^(1/2)+(c^4*x^4-1)^(1/2))/(c^4)^(1/2)*2^(1/2)/c^2*x/(c^4*x^4-1)^(1
/2)/(c^2*x^2/(c^4*x^4-1))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\operatorname{csch}\left(2\,\log\left(cx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/csch(2*log(c*x))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^4/csch(2*log(c*x))^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^4}{{\left(\frac{1}{\sinh\left(2\,\ln\left(c\,x\right)\right)}\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(1/sinh(2*log(c*x)))^(3/2),x)
```

```
[Out] int(x^4/(1/sinh(2*log(c*x)))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\operatorname{csch}^{\frac{3}{2}}\left(2\log\left(cx\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/csch(2*ln(c*x))**(3/2),x)
```

```
[Out] Integral(x**4/csch(2*log(c*x))**(3/2), x)
```

3.148 $\int \frac{x^3}{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}\,dx$

Optimal. Leaf size=86

$$-\frac{2}{7\left(c^4-\frac{1}{x^4}\right)\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}-\frac{4F\left(\csc^{-1}(cx)\middle|-1\right)}{7c^7x^3\left(1-\frac{1}{c^4x^4}\right)^{3/2}\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}+\frac{x^4}{7\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}$$

```
[Out] -2/7/(c^4-1/x^4)/csch(2*ln(c*x))^(3/2)+1/7*x^4/csch(2*ln(c*x))^(3/2)-4/7*El
lipticF(1/c/x,I)/c^7/(1-1/c^4/x^4)^(3/2)/x^3/csch(2*ln(c*x))^(3/2)
```

Rubi [A] time = 0.06, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5552, 5550, 335, 277, 221}

$$-\frac{2}{7\left(c^4-\frac{1}{x^4}\right)\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}-\frac{4F\left(\csc^{-1}(cx)\middle|-1\right)}{7c^7x^3\left(1-\frac{1}{c^4x^4}\right)^{3/2}\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}+\frac{x^4}{7\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/Csch[2*Log[c*x]]^(3/2),x]
```

```
[Out] -2/(7*(c^4 - x^(-4))*Csch[2*Log[c*x]]^(3/2)) + x^4/(7*Csch[2*Log[c*x]]^(3/2
)) - (4*EllipticF[ArcCsc[c*x], -1])/(7*c^7*(1 - 1/(c^4*x^4))^(3/2)*x^3*Csch
[2*Log[c*x]]^(3/2))
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 5550

```
Int[Csch[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
 :> Dist[(Csch[d*(a + b*Log[x])]^p*(1 - 1/(E^(2*a*d)*x^(2*b*d)))^p)/x^(-(b*
d*p)), Int[(e*x)^m/(x^(b*d*p)*(1 - 1/(E^(2*a*d)*x^(2*b*d)))^p), x], x] /; F
reeQ[{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

Rule 5552

```
Int[Csch[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m
_.), x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[x
```

```
^((m + 1)/n - 1)*Csch[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b
, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rubi steps

$$\int \frac{x^3}{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}\,dx = \frac{\operatorname{Subst}\left(\int \frac{x^3}{\operatorname{csch}^{\frac{3}{2}}(2\log(x))}\,dx, x, cx\right)}{c^4}$$

$$= \frac{\operatorname{Subst}\left(\int \left(1-\frac{1}{x^4}\right)^{3/2} x^6\,dx, x, cx\right)}{c^7\left(1-\frac{1}{c^4x^4}\right)^{3/2} x^3\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}$$

$$= -\frac{\operatorname{Subst}\left(\int \frac{\left(1-x^4\right)^{3/2}}{x^8}\,dx, x, \frac{1}{cx}\right)}{c^7\left(1-\frac{1}{c^4x^4}\right)^{3/2} x^3\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}$$

$$= \frac{x^4}{7\operatorname{csch}^{\frac{3}{2}}(2\log(cx))} + \frac{6\operatorname{Subst}\left(\int \frac{\sqrt{1-x^4}}{x^4}\,dx, x, \frac{1}{cx}\right)}{7c^7\left(1-\frac{1}{c^4x^4}\right)^{3/2} x^3\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}$$

$$= -\frac{2}{7\left(c^4-\frac{1}{x^4}\right)\operatorname{csch}^{\frac{3}{2}}(2\log(cx))} + \frac{x^4}{7\operatorname{csch}^{\frac{3}{2}}(2\log(cx))} - \frac{4\operatorname{Subst}\left(\int \frac{1}{\sqrt{1-x^4}}\,dx, x, \frac{1}{cx}\right)}{7c^7\left(1-\frac{1}{c^4x^4}\right)^{3/2} x^3\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}$$

$$= -\frac{2}{7\left(c^4-\frac{1}{x^4}\right)\operatorname{csch}^{\frac{3}{2}}(2\log(cx))} + \frac{x^4}{7\operatorname{csch}^{\frac{3}{2}}(2\log(cx))} - \frac{4F\left(\csc^{-1}(cx)\middle|-1\right)}{7c^7\left(1-\frac{1}{c^4x^4}\right)^{3/2} x^3\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}$$

Mathematica [C] time = 0.13, size = 65, normalized size = 0.76

$$\frac{\sqrt{1-c^4x^4}\sqrt{\frac{c^2x^2}{c^4x^4-1}}\,{}_2F_1\left(-\frac{3}{2},\frac{1}{4};\frac{5}{4};c^4x^4\right)}{2\sqrt{2}\,c^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/Csch[2*Log[c*x]]^(3/2),x]
```

```
[Out] (Sqrt[1 - c^4*x^4]*Sqrt[(c^2*x^2)/(-1 + c^4*x^4)]*Hypergeometric2F1[-3/2, 1
/4, 5/4, c^4*x^4])/(2*Sqrt[2]*c^4)
```

fricas [F] time = 0.56, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{x^3}{\operatorname{csch}\left(2\log(cx)\right)^{\frac{3}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/csch(2*log(c*x))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(x^3/csch(2*log(c*x))^(3/2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\operatorname{csch}\left(2 \log\left(cx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/csch(2*log(c*x))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^3/csch(2*log(c*x))^(3/2), x)
```

maple [A] time = 0.15, size = 124, normalized size = 1.44

$$\frac{x^2\left(c^4x^4-3\right)\sqrt{2}}{28c^2\sqrt{\frac{c^2x^2}{c^4x^4-1}}}+\frac{\sqrt{c^2x^2+1}\,\sqrt{-c^2x^2+1}\,\operatorname{EllipticF}\left(x\sqrt{-c^2},i\right)\sqrt{2}\,x}{7\sqrt{-c^2}\,\left(c^4x^4-1\right)c^2\sqrt{\frac{c^2x^2}{c^4x^4-1}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/csch(2*ln(c*x))^(3/2),x)
```

```
[Out] 1/28*x^2*(c^4*x^4-3)*2^(1/2)/c^2/(c^2*x^2/(c^4*x^4-1))^(1/2)+1/7/(-c^2)^(1/
2)*(c^2*x^2+1)^(1/2)*(-c^2*x^2+1)^(1/2)/(c^4*x^4-1)*EllipticF(x*(-c^2)^(1/2
),I)*2^(1/2)/c^2*x/(c^2*x^2/(c^4*x^4-1))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\operatorname{csch}\left(2 \log\left(cx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/csch(2*log(c*x))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^3/csch(2*log(c*x))^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3}{{\left(\frac{1}{\sinh(2\,\ln(c\,x))}\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(1/sinh(2*log(c*x)))^(3/2),x)
```

```
[Out] int(x^3/(1/sinh(2*log(c*x)))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\operatorname{csch}^{\frac{3}{2}}\left(2\log\left(cx\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/csch(2*ln(c*x))**(3/2),x)
```

```
[Out] Integral(x**3/csch(2*log(c*x))**(3/2), x)
```

3.149 $\int \frac{x^2}{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}\,dx$

Optimal. Leaf size=91

$$-\frac{1}{2x\left(c^4-\frac{1}{x^4}\right)\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}-\frac{\csc^{-1}\left(c^2x^2\right)}{2c^6x^3\left(1-\frac{1}{c^4x^4}\right)^{3/2}\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}+\frac{x^3}{6\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}$$

```
[Out] -1/2/(c^4-1/x^4)/x/csch(2*ln(c*x))^(3/2)+1/6*x^3/csch(2*ln(c*x))^(3/2)-1/2*
arccsc(c^2*x^2)/c^6/(1-1/c^4/x^4)^(3/2)/x^3/csch(2*ln(c*x))^(3/2)
```

Rubi [A] time = 0.07, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {5552, 5550, 335, 275, 277, 216}

$$-\frac{1}{2x\left(c^4-\frac{1}{x^4}\right)\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}-\frac{\csc^{-1}\left(c^2x^2\right)}{2c^6x^3\left(1-\frac{1}{c^4x^4}\right)^{3/2}\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}+\frac{x^3}{6\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/Csch[2*Log[c*x]]^(3/2),x]
```

```
[Out] -1/(2*(c^4 - x^(-4))*x*Csch[2*Log[c*x]]^(3/2)) + x^3/(6*Csch[2*Log[c*x]]^(3
/2)) - ArcCsc[c^2*x^2]/(2*c^6*(1 - 1/(c^4*x^4))^(3/2)*x^3*Csch[2*Log[c*x]]^
(3/2))
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 5550

```
Int[Csch[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
 :> Dist[(Csch[d*(a + b*Log[x])]^p*(1 - 1/(E^(2*a*d)*x^(2*b*d)))^p)/x^(-(b*
d*p)), Int[(e*x)^m/(x^(b*d*p)*(1 - 1/(E^(2*a*d)*x^(2*b*d)))^p), x], x] /; F
reeQ[{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

Rule 5552

```
Int[Csch[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m
_.), x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[x
^((m + 1)/n - 1)*Csch[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b
, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rubi steps

$$\int \frac{x^2}{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}\,dx = \frac{\text{Subst}\left(\int \frac{x^2}{\operatorname{csch}^{\frac{3}{2}}(2\log(x))}\,dx, x, cx\right)}{c^3}$$
$$= \frac{\text{Subst}\left(\int \left(1-\frac{1}{x^4}\right)^{3/2} x^5\,dx, x, cx\right)}{c^6\left(1-\frac{1}{c^4x^4}\right)^{3/2} x^3\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}$$
$$= -\frac{\text{Subst}\left(\int \frac{\left(1-x^4\right)^{3/2}}{x^7}\,dx, x, \frac{1}{cx}\right)}{c^6\left(1-\frac{1}{c^4x^4}\right)^{3/2} x^3\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}$$
$$= -\frac{\text{Subst}\left(\int \frac{\left(1-x^2\right)^{3/2}}{x^4}\,dx, x, \frac{1}{c^2x^2}\right)}{2c^6\left(1-\frac{1}{c^4x^4}\right)^{3/2} x^3\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}$$
$$= \frac{x^3}{6\operatorname{csch}^{\frac{3}{2}}(2\log(cx))} + \frac{\text{Subst}\left(\int \frac{\sqrt{1-x^2}}{x^2}\,dx, x, \frac{1}{c^2x^2}\right)}{2c^6\left(1-\frac{1}{c^4x^4}\right)^{3/2} x^3\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}$$
$$= -\frac{1}{2\left(c^4-\frac{1}{x^4}\right)x\operatorname{csch}^{\frac{3}{2}}(2\log(cx))} + \frac{x^3}{6\operatorname{csch}^{\frac{3}{2}}(2\log(cx))} - \frac{\text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}}\,dx, x, \frac{1}{c^2x^2}\right)}{2c^6\left(1-\frac{1}{c^4x^4}\right)^{3/2} x^3\operatorname{csch}^{\frac{3}{2}}(2\log(}$$
$$= -\frac{1}{2\left(c^4-\frac{1}{x^4}\right)x\operatorname{csch}^{\frac{3}{2}}(2\log(cx))} + \frac{x^3}{6\operatorname{csch}^{\frac{3}{2}}(2\log(cx))} - \frac{\csc^{-1}\left(c^2x^2\right)}{2c^6\left(1-\frac{1}{c^4x^4}\right)^{3/2} x^3\operatorname{csch}^{\frac{3}{2}}(2\log(}$$

Mathematica [A] time = 0.16, size = 88, normalized size = 0.97

$$\frac{x\left(\sqrt{c^4x^4-1}\left(c^4x^4-4\right)+3\tan^{-1}\left(\sqrt{c^4x^4-1}\right)\right)}{12\sqrt{2}\,c^2\sqrt{\frac{c^2x^2}{c^4x^4-1}}\sqrt{c^4x^4-1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/Csch[2*Log[c*x]]^(3/2),x]
```

```
[Out] (x*((-4 + c^4*x^4)*Sqrt[-1 + c^4*x^4] + 3*ArcTan[Sqrt[-1 + c^4*x^4]]))/(12*
Sqrt[2]*c^2*Sqrt[(c^2*x^2)/(-1 + c^4*x^4)]*Sqrt[-1 + c^4*x^4])
```

fricas [A] time = 0.66, size = 94, normalized size = 1.03

$$\frac{3\sqrt{2}\,cx\arctan\left(\frac{\left(c^4x^4-1\right)\sqrt{\frac{c^2x^2}{c^4x^4-1}}}{cx}\right)+\sqrt{2}\left(c^8x^8-5\,c^4x^4+4\right)\sqrt{\frac{c^2x^2}{c^4x^4-1}}}{24\,c^4x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/csch(2*log(c*x))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/24*(3*sqrt(2)*c*x*arctan((c^4*x^4 - 1)*sqrt(c^2*x^2/(c^4*x^4 - 1))/(c*x))
 + sqrt(2)*(c^8*x^8 - 5*c^4*x^4 + 4)*sqrt(c^2*x^2/(c^4*x^4 - 1)))/(c^4*x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/csch(2*log(c*x))^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:Warning, integration of abs or sign assumes constant sign
 by intervals (correct if the argument is real):Check [abs(t_nostep)]Unable
 to divide, perhaps due to rounding error%%%{1,[4,4,1,0]%%%}+%%%{-1,[0,0,1,
0]%%%} / %%%{1,[0,2,0,1]%%%} Error: Bad Argument Value
```

maple [F] time = 0.15, size = 0, normalized size = 0.00

$$\int \frac{x^{2}}{\operatorname{csch}\left(2 \ln \left(c x\right)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/csch(2*ln(c*x))^(3/2),x)
```

```
[Out] int(x^2/csch(2*ln(c*x))^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{2}}{\operatorname{csch}\left(2 \, \log\left(c x\right)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/csch(2*log(c*x))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2/csch(2*log(c*x))^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2}{{\left(\frac{1}{\mathrm{sinh}\left(2\,\ln\left(c\,x\right)\right)}\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(1/sinh(2*log(c*x)))^(3/2),x)
```

```
[Out] int(x^2/(1/sinh(2*log(c*x)))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{2}}{\operatorname{csch}^{\frac{3}{2}}{\left(2 \log{\left(c x \right)} \right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/csch(2*ln(c*x))**(3/2),x)
```

```
[Out] Integral(x**2/csch(2*log(c*x))**(3/2), x)
```

3.150 $\int \frac{x}{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}\,dx$

Optimal. Leaf size=130

$$-\frac{6}{5x^2\left(c^4-\frac{1}{x^4}\right)\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}+\frac{12F\left(\csc^{-1}(cx)\middle|-1\right)}{5c^5x^3\left(1-\frac{1}{c^4x^4}\right)^{3/2}\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}-\frac{12E\left(\csc^{-1}(cx)\middle|-1\right)}{5c^5x^3\left(1-\frac{1}{c^4x^4}\right)^{3/2}\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}+\frac{}{5\mathrm{csc}}$$

```
[Out] -6/5/(c^4-1/x^4)/x^2/csch(2*ln(c*x))^(3/2)+1/5*x^2/csch(2*ln(c*x))^(3/2)-12
/5*EllipticE(1/c/x,I)/c^5/(1-1/c^4/x^4)^(3/2)/x^3/csch(2*ln(c*x))^(3/2)+12/
5*EllipticF(1/c/x,I)/c^5/(1-1/c^4/x^4)^(3/2)/x^3/csch(2*ln(c*x))^(3/2)
```

Rubi [A] time = 0.07, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.615, Rules used = {5552, 5550, 335, 277, 307, 221, 1181, 424}

$$-\frac{6}{5x^2\left(c^4-\frac{1}{x^4}\right)\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}+\frac{12F\left(\csc^{-1}(cx)\middle|-1\right)}{5c^5x^3\left(1-\frac{1}{c^4x^4}\right)^{3/2}\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}-\frac{12E\left(\csc^{-1}(cx)\middle|-1\right)}{5c^5x^3\left(1-\frac{1}{c^4x^4}\right)^{3/2}\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}+\frac{}{5\mathrm{csc}}$$

Antiderivative was successfully verified.

```
[In] Int[x/Csch[2*Log[c*x]]^(3/2),x]
```

```
[Out] -6/(5*(c^4 - x^(-4))*x^2*Csch[2*Log[c*x]]^(3/2)) + x^2/(5*Csch[2*Log[c*x]]^
(3/2)) - (12*EllipticE[ArcCsc[c*x], -1])/(5*c^5*(1 - 1/(c^4*x^4))^(3/2)*x^3
*Csch[2*Log[c*x]]^(3/2)) + (12*EllipticF[ArcCsc[c*x], -1])/(5*c^5*(1 - 1/(c
^4*x^4))^(3/2)*x^3*Csch[2*Log[c*x]]^(3/2))
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 307

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-(b/a), 2]}
, -Dist[q^(-1), Int[1/Sqrt[a + b*x^4], x], x] + Dist[1/q, Int[(1 + q*x^2)/S
qrt[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
```

```
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 1181

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[-(a*c), 2]}, Dist[Sqrt[-c], Int[(d + e*x^2)/(Sqrt[q + c*x^2]*Sqrt[q - c
*x^2]), x], x]] /; FreeQ[{a, c, d, e}, x] && GtQ[a, 0] && LtQ[c, 0]
```

Rule 5550

```
Int[Csch[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
 :> Dist[(Csch[d*(a + b*Log[x])]^p*(1 - 1/(E^(2*a*d)*x^(2*b*d)))^p)/x^(-(b*
d*p)), Int[(e*x)^m/(x^(b*d*p)*(1 - 1/(E^(2*a*d)*x^(2*b*d)))^p), x], x] /; F
reeQ[{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

Rule 5552

```
Int[Csch[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m
_.), x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[x
^((m + 1)/n - 1)*Csch[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b
, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rubi steps

$$
\begin{aligned}
\int \frac{x}{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}\,dx &= \frac{\text{Subst}\left(\int \frac{x}{\operatorname{csch}^{\frac{3}{2}}(2\log(x))}\,dx, x, cx\right)}{c^2} \\
&= \frac{\text{Subst}\left(\int \left(1-\frac{1}{x^4}\right)^{3/2} x^4\,dx, x, cx\right)}{c^5\left(1-\frac{1}{c^4x^4}\right)^{3/2} x^3\operatorname{csch}^{\frac{3}{2}}(2\log(cx))} \\
&= -\frac{\text{Subst}\left(\int \frac{\left(1-x^4\right)^{3/2}}{x^6}\,dx, x, \frac{1}{cx}\right)}{c^5\left(1-\frac{1}{c^4x^4}\right)^{3/2} x^3\operatorname{csch}^{\frac{3}{2}}(2\log(cx))} \\
&= \frac{x^2}{5\operatorname{csch}^{\frac{3}{2}}(2\log(cx))} + \frac{6\,\text{Subst}\left(\int \frac{\sqrt{1-x^4}}{x^2}\,dx, x, \frac{1}{cx}\right)}{5c^5\left(1-\frac{1}{c^4x^4}\right)^{3/2} x^3\operatorname{csch}^{\frac{3}{2}}(2\log(cx))} \\
&= -\frac{6}{5\left(c^4-\frac{1}{x^4}\right)x^2\operatorname{csch}^{\frac{3}{2}}(2\log(cx))} + \frac{x^2}{5\operatorname{csch}^{\frac{3}{2}}(2\log(cx))} - \frac{12\,\text{Subst}\left(\int \frac{x^2}{\sqrt{1-x^4}}\,dx, x, \frac{1}{cx}\right.}{5c^5\left(1-\frac{1}{c^4x^4}\right)^{3/2} x^3\operatorname{csch}^{\frac{3}{2}}(2\log} \\
&= -\frac{6}{5\left(c^4-\frac{1}{x^4}\right)x^2\operatorname{csch}^{\frac{3}{2}}(2\log(cx))} + \frac{x^2}{5\operatorname{csch}^{\frac{3}{2}}(2\log(cx))} + \frac{12\,\text{Subst}\left(\int \frac{1}{\sqrt{1-x^4}}\,dx, x, \frac{1}{cx}\right.}{5c^5\left(1-\frac{1}{c^4x^4}\right)^{3/2} x^3\operatorname{csch}^{\frac{3}{2}}(2\log} \\
&= -\frac{6}{5\left(c^4-\frac{1}{x^4}\right)x^2\operatorname{csch}^{\frac{3}{2}}(2\log(cx))} + \frac{x^2}{5\operatorname{csch}^{\frac{3}{2}}(2\log(cx))} + \frac{12F\left(\csc^{-1}(cx)\middle|-1\right)}{5c^5\left(1-\frac{1}{c^4x^4}\right)^{3/2} x^3\operatorname{csch}^{\frac{3}{2}}(2\log} \\
&= -\frac{6}{5\left(c^4-\frac{1}{x^4}\right)x^2\operatorname{csch}^{\frac{3}{2}}(2\log(cx))} + \frac{x^2}{5\operatorname{csch}^{\frac{3}{2}}(2\log(cx))} - \frac{12E\left(\csc^{-1}(cx)\middle|-1\right)}{5c^5\left(1-\frac{1}{c^4x^4}\right)^{3/2} x^3\operatorname{csch}^{\frac{3}{2}}(2\log}
\end{aligned}
$$

Mathematica [C] time = 0.11, size = 60, normalized size = 0.46

$$\frac{{}_2F_1\left(-\frac{3}{2},-\frac{1}{4};\frac{3}{4};c^4x^4\right)}{2c^2\sqrt{2-2c^4x^4}\sqrt{\frac{c^2x^2}{c^4x^4-1}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/Csch[2*Log[c*x]]^(3/2),x]
```

```
[Out] Hypergeometric2F1[-3/2, -1/4, 3/4, c^4*x^4]/(2*c^2*Sqrt[2 - 2*c^4*x^4]*Sqrt
[(c^2*x^2)/(-1 + c^4*x^4)])
```

fricas [F] time = 0.43, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x}{\operatorname{csch}\left(2\,\log\left(cx\right)\right)^{\frac{3}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/csch(2*log(c*x))^(3/2),x, algorithm="fricas")
[Out] integral(x/csch(2*log(c*x))^(3/2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\operatorname{csch}\left(2\,\log\left(cx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/csch(2*log(c*x))^(3/2),x, algorithm="giac")
[Out] integrate(x/csch(2*log(c*x))^(3/2), x)
```

maple [A] time = 0.16, size = 152, normalized size = 1.17

$$\frac{\left(c^{8} x^{8}+4 c^{4} x^{4}-5\right) \sqrt{2}}{20 \left(c^{4} x^{4}-1\right) c^{2} \sqrt{\frac{c^{2} x^{2}}{c^{4} x^{4}-1}}}-\frac{3 \sqrt{c^{2} x^{2}+1}\, \sqrt{-c^{2} x^{2}+1}\, \left(\operatorname{EllipticF}\left(x \sqrt{-c^{2}}, i\right)-\operatorname{EllipticE}\left(x \sqrt{-c^{2}}, i\right)\right) \sqrt{2}\, x}{5 \sqrt{-c^{2}}\, \left(c^{4} x^{4}-1\right) \sqrt{\frac{c^{2} x^{2}}{c^{4} x^{4}-1}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/csch(2*ln(c*x))^(3/2),x)
[Out] 1/20*(c^8*x^8+4*c^4*x^4-5)/(c^4*x^4-1)*2^(1/2)/c^2/(c^2*x^2/(c^4*x^4-1))^(1
/2)-3/5/(-c^2)^(1/2)*(c^2*x^2+1)^(1/2)*(-c^2*x^2+1)^(1/2)/(c^4*x^4-1)*(Elli
pticF(x*(-c^2)^(1/2),I)-EllipticE(x*(-c^2)^(1/2),I))*2^(1/2)*x/(c^2*x^2/(c^
4*x^4-1))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\operatorname{csch}\left(2\,\log\left(cx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/csch(2*log(c*x))^(3/2),x, algorithm="maxima")
[Out] integrate(x/csch(2*log(c*x))^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x}{{\left(\frac{1}{\sinh(2\,\ln(c\,x))}\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(1/sinh(2*log(c*x)))^(3/2),x)
[Out] int(x/(1/sinh(2*log(c*x)))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\operatorname{csch}^{\frac{3}{2}}{\left(2 \log{\left(cx \right)} \right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/csch(2*ln(c*x))**(3/2),x)
[Out] Integral(x/csch(2*log(c*x))**(3/2), x)
```

3.151 $\int \frac{1}{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}\,dx$

Optimal. Leaf size=96

$$\frac{3}{4x^3\left(c^4-\frac{1}{x^4}\right)\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}-\frac{3\tanh^{-1}\left(\sqrt{1-\frac{1}{c^4x^4}}\right)}{4c^4x^3\left(1-\frac{1}{c^4x^4}\right)^{3/2}\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}+\frac{x}{4\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}$$

```
[Out] 3/4/(c^4-1/x^4)/x^3/csch(2*ln(c*x))^(3/2)+1/4*x/csch(2*ln(c*x))^(3/2)-3/4*a
rctanh((1-1/c^4/x^4)^(1/2))/c^4/(1-1/c^4/x^4)^(3/2)/x^3/csch(2*ln(c*x))^(3/
2)
```

Rubi [A] time = 0.05, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.636, Rules used = {5546, 5544, 266, 47, 50, 63, 206}

$$\frac{3}{4x^3\left(c^4-\frac{1}{x^4}\right)\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}-\frac{3\tanh^{-1}\left(\sqrt{1-\frac{1}{c^4x^4}}\right)}{4c^4x^3\left(1-\frac{1}{c^4x^4}\right)^{3/2}\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}+\frac{x}{4\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[2*Log[c*x]]^(-3/2),x]
```

```
[Out] 3/(4*(c^4 - x^(-4))*x^3*Csch[2*Log[c*x]]^(3/2)) + x/(4*Csch[2*Log[c*x]]^(3/
2)) - (3*ArcTanh[Sqrt[1 - 1/(c^4*x^4)]])/(4*c^4*(1 - 1/(c^4*x^4))^(3/2)*x^3
*Csch[2*Log[c*x]]^(3/2))
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5544

```
Int[Csch[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol] :> Dist[(Csch[d*(a
 + b*Log[x])]^p*(1 - 1/(E^(2*a*d)*x^(2*b*d)))^p)/x^(-(b*d*p)), Int[1/(x^(b*
d*p)*(1 - 1/(E^(2*a*d)*x^(2*b*d)))^p), x], x] /; FreeQ[{a, b, d, p}, x] &&
 !IntegerQ[p]
```

Rule 5546

```
Int[Csch[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.), x_Symbol] :> D
ist[x/(n*(c*x^n)^(1/n)), Subst[Int[x^(1/n - 1)*Csch[d*(a + b*Log[x])]^p, x]
, x, c*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rubi steps

$$\int \frac{1}{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}\,dx = \frac{\operatorname{Subst}\left(\int \frac{1}{\operatorname{csch}^{\frac{3}{2}}(2\log(x))}\,dx, x, cx\right)}{c}$$

$$= \frac{\operatorname{Subst}\left(\int \left(1-\frac{1}{x^4}\right)^{3/2} x^3\,dx, x, cx\right)}{c^4\left(1-\frac{1}{c^4x^4}\right)^{3/2} x^3\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}$$

$$= -\frac{\operatorname{Subst}\left(\int \frac{(1-x)^{3/2}}{x^2}\,dx, x, \frac{1}{c^4x^4}\right)}{4c^4\left(1-\frac{1}{c^4x^4}\right)^{3/2} x^3\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}$$

$$= \frac{x}{4\operatorname{csch}^{\frac{3}{2}}(2\log(cx))} + \frac{3\operatorname{Subst}\left(\int \frac{\sqrt{1-x}}{x}\,dx, x, \frac{1}{c^4x^4}\right)}{8c^4\left(1-\frac{1}{c^4x^4}\right)^{3/2} x^3\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}$$

$$= \frac{3}{4\left(c^4-\frac{1}{x^4}\right)x^3\operatorname{csch}^{\frac{3}{2}}(2\log(cx))} + \frac{x}{4\operatorname{csch}^{\frac{3}{2}}(2\log(cx))} + \frac{3\operatorname{Subst}\left(\int \frac{1}{\sqrt{1-x}\,x}\,dx, x, \frac{1}{c^4\ldots}\right.}{8c^4\left(1-\frac{1}{c^4x^4}\right)^{3/2} x^3\operatorname{csch}^{\frac{3}{2}}(2\log\ldots}$$

$$= \frac{3}{4\left(c^4-\frac{1}{x^4}\right)x^3\operatorname{csch}^{\frac{3}{2}}(2\log(cx))} + \frac{x}{4\operatorname{csch}^{\frac{3}{2}}(2\log(cx))} - \frac{3\operatorname{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sqrt{1-\ldots}\right.}{4c^4\left(1-\frac{1}{c^4x^4}\right)^{3/2} x^3\operatorname{csch}^{\frac{3}{2}}(2\log\ldots}$$

$$= \frac{3}{4\left(c^4-\frac{1}{x^4}\right)x^3\operatorname{csch}^{\frac{3}{2}}(2\log(cx))} + \frac{x}{4\operatorname{csch}^{\frac{3}{2}}(2\log(cx))} - \frac{3\tanh^{-1}\left(\sqrt{1-\frac{1}{c^4x^4}}\right)\ldots}{4c^4\left(1-\frac{1}{c^4x^4}\right)^{3/2} x^3\operatorname{csch}^{\frac{3}{2}}(2\log\ldots}$$

Mathematica [C] time = 0.10, size = 63, normalized size = 0.66

$$\frac{{}_2F_1\left(-\frac{3}{2},-\frac{1}{2};\frac{1}{2};c^4x^4\right)}{4c^2x\sqrt{2-2c^4x^4}\sqrt{\frac{c^2x^2}{c^4x^4-1}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[2*Log[c*x]]^(-3/2),x]
```

```
[Out] Hypergeometric2F1[-3/2, -1/2, 1/2, c^4*x^4]/(4*c^2*x*Sqrt[2 - 2*c^4*x^4]*Sq
rt[(c^2*x^2)/(-1 + c^4*x^4)])
```

fricas [A] time = 0.49, size = 106, normalized size = 1.10

$$\frac{3\sqrt{2}\,c^3x^3\log\left(2\,c^4x^4-2\left(c^5x^5-cx\right)\sqrt{\frac{c^2x^2}{c^4x^4-1}}-1\right)+2\sqrt{2}\left(c^8x^8+c^4x^4-2\right)\sqrt{\frac{c^2x^2}{c^4x^4-1}}}{32\,c^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/csch(2*log(c*x))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/32*(3*sqrt(2)*c^3*x^3*log(2*c^4*x^4 - 2*(c^5*x^5 - c*x)*sqrt(c^2*x^2/(c^4
*x^4 - 1)) - 1) + 2*sqrt(2)*(c^8*x^8 + c^4*x^4 - 2)*sqrt(c^2*x^2/(c^4*x^4 -
 1)))/(c^4*x^3)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\operatorname{csch}\left(2\,\log\left(cx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/csch(2*log(c*x))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(csch(2*log(c*x))^(-3/2), x)
```

maple [A] time = 0.19, size = 130, normalized size = 1.35

$$\frac{\left(c^8x^8+c^4x^4-2\right)\sqrt{2}}{16x\left(c^4x^4-1\right)c^2\sqrt{\frac{c^2x^2}{c^4x^4-1}}}-\frac{3c^2\ln\left(\frac{c^4x^2}{\sqrt{c^4}}+\sqrt{c^4x^4-1}\right)\sqrt{2}\,x}{16\sqrt{c^4}\,\sqrt{c^4x^4-1}\,\sqrt{\frac{c^2x^2}{c^4x^4-1}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/csch(2*ln(c*x))^(3/2),x)
```

```
[Out] 1/16*(c^8*x^8+c^4*x^4-2)/x/(c^4*x^4-1)*2^(1/2)/c^2/(c^2*x^2/(c^4*x^4-1))^(1
/2)-3/16*c^2*ln(c^4*x^2/(c^4)^(1/2)+(c^4*x^4-1)^(1/2))/(c^4)^(1/2)*2^(1/2)*
x/(c^4*x^4-1)^(1/2)/(c^2*x^2/(c^4*x^4-1))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\operatorname{csch}\left(2\,\log\left(cx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/csch(2*log(c*x))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(csch(2*log(c*x))^(-3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(\frac{1}{\sinh(2\ln(c\,x))}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1/sinh(2*log(c*x)))^(3/2),x)
[Out] int(1/(1/sinh(2*log(c*x)))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\operatorname{csch}^{\frac{3}{2}}\left(2\log\left(cx\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/csch(2*ln(c*x))**(3/2),x)
[Out] Integral(csch(2*log(c*x))**(-3/2), x)
```

3.152 $\int \frac{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}{x}\,dx$

Optimal. Leaf size=67

$$-\cosh(2\log(cx))\sqrt{\operatorname{csch}(2\log(cx))}+\frac{iE\left(\frac{\pi}{4}-i\log(cx)\middle|2\right)}{\sqrt{i\sinh(2\log(cx))}\sqrt{\operatorname{csch}(2\log(cx))}}$$

```
[Out] -cosh(2*ln(c*x))*csch(2*ln(c*x))^(1/2)+I*(sin(1/4*Pi+I*ln(c*x))^2)^(1/2)/si
n(1/4*Pi+I*ln(c*x))*EllipticE(cos(1/4*Pi+I*ln(c*x)),2^(1/2))/csch(2*ln(c*x)
)^(1/2)/(I*sinh(2*ln(c*x)))^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3768, 3771, 2639}

$$-\cosh(2\log(cx))\sqrt{\operatorname{csch}(2\log(cx))}+\frac{iE\left(\frac{\pi}{4}-i\log(cx)\middle|2\right)}{\sqrt{i\sinh(2\log(cx))}\sqrt{\operatorname{csch}(2\log(cx))}}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[2*Log[c*x]]^(3/2)/x,x]
```

```
[Out] -(Cosh[2*Log[c*x]]*Sqrt[Csch[2*Log[c*x]]]) + (I*EllipticE[Pi/4 - I*Log[c*x]
, 2])/(Sqrt[Csch[2*Log[c*x]]]*Sqrt[I*Sinh[2*Log[c*x]]])
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\begin{aligned}
\int \frac{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}{x}\,dx &= \operatorname{Subst}\left(\int \operatorname{csch}^{\frac{3}{2}}(2x)\,dx, x, \log(cx)\right)\\
&= -\cosh(2\log(cx))\sqrt{\operatorname{csch}(2\log(cx))}+\operatorname{Subst}\left(\int \frac{1}{\sqrt{\operatorname{csch}(2x)}}\,dx, x, \log(cx)\right)\\
&= -\cosh(2\log(cx))\sqrt{\operatorname{csch}(2\log(cx))}+\frac{\operatorname{Subst}\left(\int \sqrt{i\sinh(2x)}\,dx, x, \log(cx)\right)}{\sqrt{\operatorname{csch}(2\log(cx))}\sqrt{i\sinh(2\log(cx))}}\\
&= -\cosh(2\log(cx))\sqrt{\operatorname{csch}(2\log(cx))}+\frac{iE\left(\frac{\pi}{4}-i\log(cx)\middle|2\right)}{\sqrt{\operatorname{csch}(2\log(cx))}\sqrt{i\sinh(2\log(cx))}}
\end{aligned}$$

Mathematica [A] time = 0.10, size = 54, normalized size = 0.81

$$\sqrt{\operatorname{csch}(2\log(cx))}\left(-\cosh(2\log(cx))+\sqrt{i\sinh(2\log(cx))}\,E\left(\frac{\pi}{4}-i\log(cx)\middle|2\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[2*Log[c*x]]^(3/2)/x,x]
```

```
[Out] Sqrt[Csch[2*Log[c*x]]]*(-Cosh[2*Log[c*x]] + EllipticE[Pi/4 - I*Log[c*x], 2]
*Sqrt[I*Sinh[2*Log[c*x]]])
```

fricas [F] time = 1.31, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{\operatorname{csch}\left(2\log(cx)\right)^{\frac{3}{2}}}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(2*log(c*x))^(3/2)/x,x, algorithm="fricas")
```

```
[Out] integral(csch(2*log(c*x))^(3/2)/x, x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(2*log(c*x))^(3/2)/x,x, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.41, size = 163, normalized size = 2.43

$$\frac{2\sqrt{1-i\sinh(2\ln(cx))}\,\sqrt{2}\,\sqrt{i\sinh(2\ln(cx))+1}\,\sqrt{i\sinh(2\ln(cx))}\,\operatorname{EllipticE}\left(\sqrt{1-i\sinh(2\ln(cx))},\frac{\sqrt{2}}{2}\right)-}{2\cosh}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(2*ln(c*x))^(3/2)/x,x)
```

```
[Out] 1/2*(2*(1-I*sinh(2*ln(c*x)))^(1/2)*2^(1/2)*(I*sinh(2*ln(c*x))+1)^(1/2)*(I*s
inh(2*ln(c*x)))^(1/2)*EllipticE((1-I*sinh(2*ln(c*x)))^(1/2),1/2*2^(1/2))-(1
-I*sinh(2*ln(c*x)))^(1/2)*2^(1/2)*(I*sinh(2*ln(c*x))+1)^(1/2)*(I*sinh(2*ln(
c*x)))^(1/2)*EllipticF((1-I*sinh(2*ln(c*x)))^(1/2),1/2*2^(1/2))-2*cosh(2*ln
(c*x))^2)/cosh(2*ln(c*x))/sinh(2*ln(c*x))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\operatorname{csch}\left(2\log(cx)\right)^{\frac{3}{2}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(2*log(c*x))^(3/2)/x,x, algorithm="maxima")
```

```
[Out] integrate(csch(2*log(c*x))^(3/2)/x, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(\frac{1}{\sinh(2\ln(c\,x))}\right)^{3/2}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/sinh(2*log(c*x)))^(3/2)/x,x)
```

```
[Out] int((1/sinh(2*log(c*x)))^(3/2)/x, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}^{\frac{3}{2}}\left(2\log\left(cx\right)\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(2*ln(c*x))**(3/2)/x,x)
```

```
[Out] Integral(csch(2*log(c*x))**(3/2)/x, x)
```

3.153 $\int \frac{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}{x^2}\,dx$

Optimal. Leaf size=27

$$-\frac{1}{2}x^3\left(c^4-\frac{1}{x^4}\right)\operatorname{csch}^{\frac{3}{2}}(2\log(cx))$$

```
[Out] -1/2*(c^4-1/x^4)*x^3*csch(2*ln(c*x))^(3/2)
```

Rubi [A] time = 0.04, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {5552, 5550, 261}

$$-\frac{1}{2}x^3\left(c^4-\frac{1}{x^4}\right)\operatorname{csch}^{\frac{3}{2}}(2\log(cx))$$

Antiderivative was successfully verified.

```
[In] Int[Csch[2*Log[c*x]]^(3/2)/x^2,x]
```

```
[Out] -((c^4 - x^(-4))*x^3*Csch[2*Log[c*x]]^(3/2))/2
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 5550

```
Int[Csch[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
 :> Dist[(Csch[d*(a + b*Log[x])]^p*(1 - 1/(E^(2*a*d)*x^(2*b*d)))^p)/x^(-(b*
d*p)), Int[(e*x)^m/(x^(b*d*p)*(1 - 1/(E^(2*a*d)*x^(2*b*d)))^p), x], x] /; F
reeQ[{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

Rule 5552

```
Int[Csch[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m
_.), x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[x
^((m + 1)/n - 1)*Csch[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b
, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rubi steps

$$\begin{aligned}
\int \frac{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}{x^2}\,dx &= c\,\text{Subst}\left(\int \frac{\operatorname{csch}^{\frac{3}{2}}(2\log(x))}{x^2}\,dx, x, cx\right)\\
&= \left(c^4\left(1-\frac{1}{c^4x^4}\right)^{3/2}x^3\operatorname{csch}^{\frac{3}{2}}(2\log(cx))\right)\text{Subst}\left(\int \frac{1}{\left(1-\frac{1}{x^4}\right)^{3/2}x^5}\,dx, x, cx\right)\\
&= -\frac{1}{2}\left(c^4-\frac{1}{x^4}\right)x^3\operatorname{csch}^{\frac{3}{2}}(2\log(cx))
\end{aligned}$$

Mathematica [A] time = 0.03, size = 33, normalized size = 1.22

$$-\sqrt{2}\,c^2x\sqrt{\frac{c^2x^2}{c^4x^4-1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[2*Log[c*x]]^(3/2)/x^2,x]
```

```
[Out] -(Sqrt[2]*c^2*x*Sqrt[(c^2*x^2)/(-1 + c^4*x^4)])
```

fricas [A] time = 0.47, size = 29, normalized size = 1.07

$$-\sqrt{2}\sqrt{\frac{c^2x^2}{c^4x^4-1}}\,c^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(2*log(c*x))^(3/2)/x^2,x, algorithm="fricas")
```

```
[Out] -sqrt(2)*sqrt(c^2*x^2/(c^4*x^4 - 1))*c^2*x
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(2*log(c*x))^(3/2)/x^2,x, algorithm="giac")
```

```
[Out] Timed out
```

maple [F] time = 0.15, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}\left(2\ln\left(cx\right)\right)^{\frac{3}{2}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(2*ln(c*x))^(3/2)/x^2,x)
```

```
[Out] int(csch(2*ln(c*x))^(3/2)/x^2,x)
```

maxima [B] time = 0.46, size = 87, normalized size = 3.22

$$-c\left(\frac{\sqrt{2}}{\left(\frac{1}{cx}+1\right)^{\frac{3}{2}}\left(-\frac{1}{cx}+1\right)^{\frac{3}{2}}\left(\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}}-\frac{\sqrt{2}}{c^4x^4\left(\frac{1}{cx}+1\right)^{\frac{3}{2}}\left(-\frac{1}{cx}+1\right)^{\frac{3}{2}}\left(\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(2*log(c*x))^(3/2)/x^2,x, algorithm="maxima")
```

```
[Out] -c*(sqrt(2)/((1/(c*x) + 1)^(3/2)*(-1/(c*x) + 1)^(3/2)*(1/(c^2*x^2) + 1)^(3/
2)) - sqrt(2)/(c^4*x^4*(1/(c*x) + 1)^(3/2)*(-1/(c*x) + 1)^(3/2)*(1/(c^2*x^2
) + 1)^(3/2)))
```

mupad [B] time = 1.47, size = 29, normalized size = 1.07

$$-c^2\,x\,\sqrt{\frac{2\,c^2\,x^2}{c^4\,x^4-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/sinh(2*log(c*x)))^(3/2)/x^2,x)
```

```
[Out] -c^2*x*((2*c^2*x^2)/(c^4*x^4 - 1))^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}^{\frac{3}{2}}\left(2\log\left(cx\right)\right)}{x^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(2*ln(c*x))**(3/2)/x**2,x)
```

```
[Out] Integral(csch(2*log(c*x))**(3/2)/x**2, x)
```

3.154 $\int \frac{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}{x^3}\,dx$

Optimal. Leaf size=69

$$\frac{1}{2}c^5x^3\left(1-\frac{1}{c^4x^4}\right)^{3/2}F\left(\csc^{-1}(cx)\middle|-1\right)\operatorname{csch}^{\frac{3}{2}}(2\log(cx))-\frac{1}{2}x^2\left(c^4-\frac{1}{x^4}\right)\operatorname{csch}^{\frac{3}{2}}(2\log(cx))$$

```
[Out] -1/2*(c^4-1/x^4)*x^2*csch(2*ln(c*x))^(3/2)+1/2*c^5*(1-1/c^4/x^4)^(3/2)*x^3*
csch(2*ln(c*x))^(3/2)*EllipticF(1/c/x,I)
```

Rubi [A] time = 0.05, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5552, 5550, 335, 288, 221}

$$\frac{1}{2}c^5x^3\left(1-\frac{1}{c^4x^4}\right)^{3/2}F\left(\csc^{-1}(cx)\middle|-1\right)\operatorname{csch}^{\frac{3}{2}}(2\log(cx))-\frac{1}{2}x^2\left(c^4-\frac{1}{x^4}\right)\operatorname{csch}^{\frac{3}{2}}(2\log(cx))$$

Antiderivative was successfully verified.

```
[In] Int[Csch[2*Log[c*x]]^(3/2)/x^3,x]
```

```
[Out] -((c^4 - x^(-4))*x^2*Csch[2*Log[c*x]]^(3/2))/2 + (c^5*(1 - 1/(c^4*x^4))^(3/
2)*x^3*Csch[2*Log[c*x]]^(3/2)*EllipticF[ArcCsc[c*x], -1])/2
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 5550

```
Int[Csch[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
 :> Dist[(Csch[d*(a + b*Log[x])]^p*(1 - 1/(E^(2*a*d)*x^(2*b*d)))^p)/x^(-(b*
d*p)), Int[(e*x)^m/(x^(b*d*p)*(1 - 1/(E^(2*a*d)*x^(2*b*d)))^p), x], x] /; F
reeQ[{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

Rule 5552

```
Int[Csch[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m
_.), x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[x
^((m + 1)/n - 1)*Csch[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b
, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rubi steps

$$\int \frac{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}{x^3}\,dx = c^2\,\text{Subst}\left(\int \frac{\operatorname{csch}^{\frac{3}{2}}(2\log(x))}{x^3}\,dx, x, cx\right)$$

$$= \left(c^5\left(1-\frac{1}{c^4x^4}\right)^{3/2} x^3\operatorname{csch}^{\frac{3}{2}}(2\log(cx))\right)\text{Subst}\left(\int \frac{1}{\left(1-\frac{1}{x^4}\right)^{3/2}x^6}\,dx, x, cx\right)$$

$$= -\left(\left(c^5\left(1-\frac{1}{c^4x^4}\right)^{3/2} x^3\operatorname{csch}^{\frac{3}{2}}(2\log(cx))\right)\text{Subst}\left(\int \frac{x^4}{\left(1-x^4\right)^{3/2}}\,dx, x, \frac{1}{cx}\right)\right)$$

$$= -\frac{1}{2}\left(c^4-\frac{1}{x^4}\right)x^2\operatorname{csch}^{\frac{3}{2}}(2\log(cx)) + \frac{1}{2}\left(c^5\left(1-\frac{1}{c^4x^4}\right)^{3/2} x^3\operatorname{csch}^{\frac{3}{2}}(2\log(cx))\right)\text{Subst}\left(\right.$$

$$= -\frac{1}{2}\left(c^4-\frac{1}{x^4}\right)x^2\operatorname{csch}^{\frac{3}{2}}(2\log(cx)) + \frac{1}{2}c^5\left(1-\frac{1}{c^4x^4}\right)^{3/2} x^3\operatorname{csch}^{\frac{3}{2}}(2\log(cx))F\left(\csc^{-1}(cx\right.$$

Mathematica [C] time = 0.11, size = 66, normalized size = 0.96

$$-\sqrt{2}\,c^2\sqrt{\frac{c^2x^2}{c^4x^4-1}}\left(\sqrt{1-c^4x^4}\;{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};c^4x^4\right)+1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[2*Log[c*x]]^(3/2)/x^3,x]
[Out] -(Sqrt[2]*c^2*Sqrt[(c^2*x^2)/(-1 + c^4*x^4)]*(1 + Sqrt[1 - c^4*x^4]*Hyperge
ometric2F1[1/4, 1/2, 5/4, c^4*x^4]))
```

fricas [F] time = 0.49, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\operatorname{csch}\left(2\,\log\left(cx\right)\right)^{\frac{3}{2}}}{x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(2*log(c*x))^(3/2)/x^3,x, algorithm="fricas")
[Out] integral(csch(2*log(c*x))^(3/2)/x^3, x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(2*log(c*x))^(3/2)/x^3,x, algorithm="giac")
[Out] Timed out
```

maple [F] time = 0.14, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}\left(2\ln\left(cx\right)\right)^{\frac{3}{2}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(2*ln(c*x))^(3/2)/x^3,x)
```

```
[Out] int(csch(2*ln(c*x))^(3/2)/x^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}\left(2\,\log\left(cx\right)\right)^{\frac{3}{2}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(2*log(c*x))^(3/2)/x^3,x, algorithm="maxima")
```

```
[Out] integrate(csch(2*log(c*x))^(3/2)/x^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(\frac{1}{\sinh(2\,\ln(c\,x))}\right)^{3/2}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/sinh(2*log(c*x)))^(3/2)/x^3,x)
```

```
[Out] int((1/sinh(2*log(c*x)))^(3/2)/x^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}^{\frac{3}{2}}\left(2\log\left(cx\right)\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(2*ln(c*x))**(3/2)/x**3,x)
```

```
[Out] Integral(csch(2*log(c*x))**(3/2)/x**3, x)
```

3.155 $\int \frac{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}{x^4} dx$

Optimal. Leaf size=69

$$\frac{1}{2}c^6x^3\left(1-\frac{1}{c^4x^4}\right)^{3/2}\csc^{-1}\left(c^2x^2\right)\operatorname{csch}^{\frac{3}{2}}(2\log(cx))-\frac{1}{2}x\left(c^4-\frac{1}{x^4}\right)\operatorname{csch}^{\frac{3}{2}}(2\log(cx))$$

```
[Out] -1/2*(c^4-1/x^4)*x*csch(2*ln(c*x))^(3/2)+1/2*c^6*(1-1/c^4/x^4)^(3/2)*x^3*ar
ccsc(c^2*x^2)*csch(2*ln(c*x))^(3/2)
```

Rubi [A] time = 0.06, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {5552, 5550, 335, 275, 288, 216}

$$\frac{1}{2}c^6x^3\left(1-\frac{1}{c^4x^4}\right)^{3/2}\csc^{-1}\left(c^2x^2\right)\operatorname{csch}^{\frac{3}{2}}(2\log(cx))-\frac{1}{2}x\left(c^4-\frac{1}{x^4}\right)\operatorname{csch}^{\frac{3}{2}}(2\log(cx))$$

Antiderivative was successfully verified.

```
[In] Int[Csch[2*Log[c*x]]^(3/2)/x^4,x]
```

```
[Out] -((c^4 - x^(-4))*x*Csch[2*Log[c*x]]^(3/2))/2 + (c^6*(1 - 1/(c^4*x^4))^(3/2)
*x^3*ArcCsc[c^2*x^2]*Csch[2*Log[c*x]]^(3/2))/2
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 5550

```
Int[Csch[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
 :> Dist[(Csch[d*(a + b*Log[x])]^p*(1 - 1/(E^(2*a*d)*x^(2*b*d)))^p)/x^(-(b*
d*p)), Int[(e*x)^m/(x^(b*d*p)*(1 - 1/(E^(2*a*d)*x^(2*b*d)))^p), x], x] /; F
reeQ[{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

Rule 5552

```
Int[Csch[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m
_.), x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[x
^((m + 1)/n - 1)*Csch[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b
, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rubi steps

$$\begin{aligned}
\int \frac{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}{x^4}\,dx &= c^3\,\text{Subst}\left(\int \frac{\operatorname{csch}^{\frac{3}{2}}(2\log(x))}{x^4}\,dx, x, cx\right)\\
&= \left(c^6\left(1-\frac{1}{c^4x^4}\right)^{3/2} x^3\operatorname{csch}^{\frac{3}{2}}(2\log(cx))\right)\text{Subst}\left(\int \frac{1}{\left(1-\frac{1}{x^4}\right)^{3/2} x^7}\,dx, x, cx\right)\\
&= -\left(\left(c^6\left(1-\frac{1}{c^4x^4}\right)^{3/2} x^3\operatorname{csch}^{\frac{3}{2}}(2\log(cx))\right)\text{Subst}\left(\int \frac{x^5}{\left(1-x^4\right)^{3/2}}\,dx, x, \frac{1}{cx}\right)\right)\\
&= -\left(\frac{1}{2}\left(c^6\left(1-\frac{1}{c^4x^4}\right)^{3/2} x^3\operatorname{csch}^{\frac{3}{2}}(2\log(cx))\right)\text{Subst}\left(\int \frac{x^2}{\left(1-x^2\right)^{3/2}}\,dx, x, \frac{1}{c^2x^2}\right)\right)\\
&= -\frac{1}{2}\left(c^4-\frac{1}{x^4}\right)x\operatorname{csch}^{\frac{3}{2}}(2\log(cx)) + \frac{1}{2}\left(c^6\left(1-\frac{1}{c^4x^4}\right)^{3/2} x^3\operatorname{csch}^{\frac{3}{2}}(2\log(cx))\right)\text{Subst}\left(\int \frac{}{\sqrt{}}\right.\\
&= -\frac{1}{2}\left(c^4-\frac{1}{x^4}\right)x\operatorname{csch}^{\frac{3}{2}}(2\log(cx)) + \frac{1}{2}c^6\left(1-\frac{1}{c^4x^4}\right)^{3/2} x^3\csc^{-1}\left(c^2x^2\right)\operatorname{csch}^{\frac{3}{2}}(2\log(cx))
\end{aligned}$$

Mathematica [C] time = 0.11, size = 53, normalized size = 0.77

$$-\frac{\sqrt{2}\,c^2\sqrt{\frac{c^2x^2}{c^4x^4-1}}\;{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};1-c^4x^4\right)}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[2*Log[c*x]]^(3/2)/x^4,x]
```

```
[Out] -((Sqrt[2]*c^2*Sqrt[(c^2*x^2)/(-1 + c^4*x^4)]*Hypergeometric2F1[-1/2, 1, 1/
2, 1 - c^4*x^4])/x)
```

fricas [A] time = 0.73, size = 78, normalized size = 1.13

$$-\frac{\sqrt{2}\,c^3x\arctan\left(\frac{\left(c^4x^4-1\right)\sqrt{\frac{c^2x^2}{c^4x^4-1}}}{cx}\right)+\sqrt{2}\sqrt{\frac{c^2x^2}{c^4x^4-1}}\,c^2}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(2*log(c*x))^(3/2)/x^4,x, algorithm="fricas")
```

```
[Out] -(sqrt(2)*c^3*x*arctan((c^4*x^4 - 1)*sqrt(c^2*x^2/(c^4*x^4 - 1))/(c*x)) + s
qrt(2)*sqrt(c^2*x^2/(c^4*x^4 - 1))*c^2)/x
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(2*log(c*x))^(3/2)/x^4,x, algorithm="giac")
[Out] Timed out
```

maple [F] time = 0.15, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}(2\ln(cx))^{\frac{3}{2}}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(2*ln(c*x))^(3/2)/x^4,x)
[Out] int(csch(2*ln(c*x))^(3/2)/x^4,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}\left(2\,\log(cx)\right)^{\frac{3}{2}}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(2*log(c*x))^(3/2)/x^4,x, algorithm="maxima")
[Out] integrate(csch(2*log(c*x))^(3/2)/x^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(\frac{1}{\sinh(2\,\ln(c\,x))}\right)^{3/2}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/sinh(2*log(c*x)))^(3/2)/x^4,x)
[Out] int((1/sinh(2*log(c*x)))^(3/2)/x^4, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}^{\frac{3}{2}}\left(2\log(cx)\right)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(2*ln(c*x))**(3/2)/x**4,x)
[Out] Integral(csch(2*log(c*x))**(3/2)/x**4, x)
```

3.156 $\int \operatorname{csch}\left(a+b\log\left(cx^n\right)\right)\,dx$

Optimal. Leaf size=62

$$-\frac{2e^a x\left(cx^n\right)^b\,{}_2F_1\left(1,\frac{b+\frac{1}{n}}{2b};\frac{1}{2}\left(3+\frac{1}{bn}\right);e^{2a}\left(cx^n\right)^{2b}\right)}{bn+1}$$

```
[Out] -2*exp(a)*x*(c*x^n)^b*hypergeom([1, 1/2*(b+1/n)/b],[3/2+1/2/b/n],exp(2*a)*(
c*x^n)^(2*b))/(b*n+1)
```

Rubi [A] time = 0.05, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {5546, 5548, 263, 364}

$$-\frac{2e^a x\left(cx^n\right)^b\,{}_2F_1\left(1,\frac{b+\frac{1}{n}}{2b};\frac{1}{2}\left(3+\frac{1}{bn}\right);e^{2a}\left(cx^n\right)^{2b}\right)}{bn+1}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[a + b*Log[c*x^n]],x]
```

```
[Out] (-2*E^a*x*(c*x^n)^b*Hypergeometric2F1[1, (b + n^(-1))/(2*b), (3 + 1/(b*n))/
2, E^(2*a)*(c*x^n)^(2*b)])/(1 + b*n)
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 5546

```
Int[Csch[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.), x_Symbol] :> D
ist[x/(n*(c*x^n)^(1/n)), Subst[Int[x^(1/n - 1)*Csch[d*(a + b*Log[x])]^p, x]
, x, c*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rule 5548

```
Int[Csch[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
 :> Dist[2^p/E^(a*d*p), Int[(e*x)^m/(x^(b*d*p)*(1 - 1/(E^(2*a*d)*x^(2*b*d))
)^p), x], x] /; FreeQ[{a, b, d, e, m}, x] && IntegerQ[p]
```

Rubi steps

$$\int \operatorname{csch}\left(a+b\log\left(cx^n\right)\right)dx = \frac{\left(x\left(cx^n\right)^{-1/n}\right)\operatorname{Subst}\left(\int x^{-1+\frac{1}{n}}\operatorname{csch}(a+b\log(x))\,dx,x,cx^n\right)}{n}$$
$$= \frac{\left(2e^{-a}x\left(cx^n\right)^{-1/n}\right)\operatorname{Subst}\left(\int \frac{x^{-1-b+\frac{1}{n}}}{1-e^{-2a}x^{-2b}}\,dx,x,cx^n\right)}{n}$$
$$= \frac{\left(2e^{-a}x\left(cx^n\right)^{-1/n}\right)\operatorname{Subst}\left(\int \frac{x^{-1+b+\frac{1}{n}}}{-e^{-2a}+x^{2b}}\,dx,x,cx^n\right)}{n}$$
$$= -\frac{2e^a x\left(cx^n\right)^b\ {}_2F_1\left(1,\frac{b+\frac{1}{n}}{2b};\frac{1}{2}\left(3+\frac{1}{bn}\right);e^{2a}\left(cx^n\right)^{2b}\right)}{1+bn}$$

Mathematica [A] time = 1.20, size = 62, normalized size = 1.00

$$-\frac{2e^a x\left(cx^n\right)^b\ {}_2F_1\left(1,\frac{1}{2}\left(1+\frac{1}{bn}\right);\frac{1}{2}\left(3+\frac{1}{bn}\right);e^{2\left(a+b\log\left(cx^n\right)\right)}\right)}{bn+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[a + b*Log[c*x^n]],x]
[Out] (-2*E^a*x*(c*x^n)^b*Hypergeometric2F1[1, (1 + 1/(b*n))/2, (3 + 1/(b*n))/2,
E^(2*(a + b*Log[c*x^n]))])/(1 + b*n)
```

fricas [F] time = 0.46, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\operatorname{csch}\left(b\log\left(cx^n\right)+a\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(a+b*log(c*x^n)),x, algorithm="fricas")
[Out] integral(csch(b*log(c*x^n) + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{csch}\left(b\log\left(cx^n\right)+a\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(a+b*log(c*x^n)),x, algorithm="giac")
[Out] integrate(csch(b*log(c*x^n) + a), x)
```

maple [F] time = 0.37, size = 0, normalized size = 0.00

$$\int \operatorname{csch}\left(a+b\ln\left(c\,x^n\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(a+b*ln(c*x^n)),x)
[Out] int(csch(a+b*ln(c*x^n)),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{csch}\left(b\log\left(cx^n\right) + a\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(a+b*log(c*x^n)),x, algorithm="maxima")
[Out] integrate(csch(b*log(c*x^n) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sinh\left(a + b\,\ln\left(c\,x^n\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sinh(a + b*log(c*x^n)),x)
[Out] int(1/sinh(a + b*log(c*x^n)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{csch}\left(a + b\log\left(cx^n\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(a+b*ln(c*x**n)),x)
[Out] Integral(csch(a + b*log(c*x**n)), x)
```

3.157 $\int \operatorname{csch}^2\left(a+b\log\left(cx^n\right)\right)\,dx$

Optimal. Leaf size=68

$$\frac{4e^{2a}x\left(cx^n\right)^{2b}\,{}_2F_1\left(2,\frac{1}{2}\left(2+\frac{1}{bn}\right);\frac{1}{2}\left(4+\frac{1}{bn}\right);e^{2a}\left(cx^n\right)^{2b}\right)}{2bn+1}$$

```
[Out] 4*exp(2*a)*x*(c*x^n)^(2*b)*hypergeom([2, 1+1/2/b/n],[2+1/2/b/n],exp(2*a)*(c
*x^n)^(2*b))/(2*b*n+1)
```

Rubi [A] time = 0.07, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {5546, 5548, 263, 364}

$$\frac{4e^{2a}x\left(cx^n\right)^{2b}\,{}_2F_1\left(2,\frac{1}{2}\left(2+\frac{1}{bn}\right);\frac{1}{2}\left(4+\frac{1}{bn}\right);e^{2a}\left(cx^n\right)^{2b}\right)}{2bn+1}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[a + b*Log[c*x^n]]^2,x]
```

```
[Out] (4*E^(2*a)*x*(c*x^n)^(2*b)*Hypergeometric2F1[2, (2 + 1/(b*n))/2, (4 + 1/(b*
n))/2, E^(2*a)*(c*x^n)^(2*b)])/(1 + 2*b*n)
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 5546

```
Int[Csch[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.), x_Symbol] :> D
ist[x/(n*(c*x^n)^(1/n)), Subst[Int[x^(1/n - 1)*Csch[d*(a + b*Log[x])]^p, x]
, x, c*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rule 5548

```
Int[Csch[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
 :> Dist[2^p/E^(a*d*p), Int[(e*x)^m/(x^(b*d*p)*(1 - 1/(E^(2*a*d)*x^(2*b*d))
)^p), x], x] /; FreeQ[{a, b, d, e, m}, x] && IntegerQ[p]
```

Rubi steps

$$\int \operatorname{csch}^2\left(a+b\log\left(cx^n\right)\right)\,dx = \frac{\left(x\left(cx^n\right)^{-1/n}\right)\operatorname{Subst}\left(\int x^{-1+\frac{1}{n}}\operatorname{csch}^2(a+b\log(x))\,dx,x,cx^n\right)}{n}$$
$$= \frac{\left(4e^{-2a}x\left(cx^n\right)^{-1/n}\right)\operatorname{Subst}\left(\int \frac{x^{-1-2b+\frac{1}{n}}}{\left(1-e^{-2a}x^{-2b}\right)^2}\,dx,x,cx^n\right)}{n}$$
$$= \frac{\left(4e^{-2a}x\left(cx^n\right)^{-1/n}\right)\operatorname{Subst}\left(\int \frac{x^{-1+2b+\frac{1}{n}}}{\left(-e^{-2a}+x^{2b}\right)^2}\,dx,x,cx^n\right)}{n}$$
$$= \frac{4e^{2a}x\left(cx^n\right)^{2b}\,{}_2F_1\left(2,\frac{1}{2}\left(2+\frac{1}{bn}\right);\frac{1}{2}\left(4+\frac{1}{bn}\right);e^{2a}\left(cx^n\right)^{2b}\right)}{1+2bn}$$

Mathematica [A] time = 4.32, size = 126, normalized size = 1.85

$$\frac{x\left(-\frac{e^{2a}\left(cx^n\right)^{2b}\,{}_2F_1\left(1,1+\frac{1}{2bn};2+\frac{1}{2bn};e^{2\left(a+b\log\left(cx^n\right)\right)}\right)}{2bn+1}-{}_2F_1\left(1,\frac{1}{2bn};1+\frac{1}{2bn};e^{2\left(a+b\log\left(cx^n\right)\right)}\right)-\coth\left(a+b\log\left(cx^n\right)\right)\right)}{bn}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Csch[a + b*Log[c*x^n]]^2,x]
```

```
[Out] (x*(-Coth[a + b*Log[c*x^n]] - (E^(2*a)*(c*x^n)^(2*b)*Hypergeometric2F1[1, 1
 + 1/(2*b*n), 2 + 1/(2*b*n), E^(2*(a + b*Log[c*x^n]))])/(1 + 2*b*n) - Hyper
geometric2F1[1, 1/(2*b*n), 1 + 1/(2*b*n), E^(2*(a + b*Log[c*x^n]))]))/(b*n)
```

fricas [F] time = 0.49, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\operatorname{csch}\left(b\log\left(cx^n\right)+a\right)^2,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(a+b*log(c*x^n))^2,x, algorithm="fricas")
```

```
[Out] integral(csch(b*log(c*x^n) + a)^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{csch}\left(b\log\left(cx^n\right)+a\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(a+b*log(c*x^n))^2,x, algorithm="giac")
```

```
[Out] integrate(csch(b*log(c*x^n) + a)^2, x)
```

maple [F] time = 1.78, size = 0, normalized size = 0.00

$$\int \operatorname{csch}\left(a+b\ln\left(c\,x^n\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(a+b*ln(c*x^n))^2,x)
```

```
[Out] int(csch(a+b*ln(c*x^n))^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{2\,x}{bc^{2\,b}ne^{\left(2\,b\log\left(x^{n}\right)+2\,a\right)}-bn}-4\int\frac{1}{4\left(bc^{b}ne^{\left(b\log\left(x^{n}\right)+a\right)}+bn\right)}\,dx+4\int\frac{1}{4\left(bc^{b}ne^{\left(b\log\left(x^{n}\right)+a\right)}-bn\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(a+b*log(c*x^n))^2,x, algorithm="maxima")
```

```
[Out] -2*x/(b*c^(2*b)*n*e^(2*b*log(x^n) + 2*a) - b*n) - 4*integrate(1/4/(b*c^b*n*
e^(b*log(x^n) + a) + b*n), x) + 4*integrate(1/4/(b*c^b*n*e^(b*log(x^n) + a)
 - b*n), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{1}{\sinh\left(a+b\,\ln\left(c\,x^{n}\right)\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sinh(a + b*log(c*x^n))^2,x)
```

```
[Out] int(1/sinh(a + b*log(c*x^n))^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\operatorname{csch}^{2}\left(a+b\log\left(cx^{n}\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(a+b*ln(c*x**n))**2,x)
```

```
[Out] Integral(csch(a + b*log(c*x**n))**2, x)
```

3.158 $\int \operatorname{csch}^3\left(a+b\log\left(cx^n\right)\right)\,dx$

Optimal. Leaf size=69

$$-\frac{8e^{3a}x\left(cx^n\right)^{3b}\,{}_2F_1\left(3,\frac{3b+\frac{1}{n}}{2b};\frac{1}{2}\left(5+\frac{1}{bn}\right);e^{2a}\left(cx^n\right)^{2b}\right)}{3bn+1}$$

```
[Out] -8*exp(3*a)*x*(c*x^n)^(3*b)*hypergeom([3, 1/2*(3*b+1/n)/b],[5/2+1/2/b/n],ex
p(2*a)*(c*x^n)^(2*b))/(3*b*n+1)
```

Rubi [A] time = 0.07, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {5546, 5548, 263, 364}

$$-\frac{8e^{3a}x\left(cx^n\right)^{3b}\,{}_2F_1\left(3,\frac{3b+\frac{1}{n}}{2b};\frac{1}{2}\left(5+\frac{1}{bn}\right);e^{2a}\left(cx^n\right)^{2b}\right)}{3bn+1}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[a + b*Log[c*x^n]]^3,x]
```

```
[Out] (-8*E^(3*a)*x*(c*x^n)^(3*b)*Hypergeometric2F1[3, (3*b + n^(-1))/(2*b), (5 +
 1/(b*n))/2, E^(2*a)*(c*x^n)^(2*b)])/(1 + 3*b*n)
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 5546

```
Int[Csch[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.), x_Symbol] :> D
ist[x/(n*(c*x^n)^(1/n)), Subst[Int[x^(1/n - 1)*Csch[d*(a + b*Log[x])]^p, x]
, x, c*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rule 5548

```
Int[Csch[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
 :> Dist[2^p/E^(a*d*p), Int[(e*x)^m/(x^(b*d*p)*(1 - 1/(E^(2*a*d)*x^(2*b*d))
)^p), x], x] /; FreeQ[{a, b, d, e, m}, x] && IntegerQ[p]
```

Rubi steps

$$\int \operatorname{csch}^3\left(a+b\log\left(cx^n\right)\right)\,dx = \frac{\left(x\left(cx^n\right)^{-1/n}\right)\operatorname{Subst}\left(\int x^{-1+\frac{1}{n}}\operatorname{csch}^3(a+b\log(x))\,dx, x, cx^n\right)}{n}$$

$$= \frac{\left(8e^{-3a}x\left(cx^n\right)^{-1/n}\right)\operatorname{Subst}\left(\int \frac{x^{-1-3b+\frac{1}{n}}}{\left(1-e^{-2a}x^{-2b}\right)^3}\,dx, x, cx^n\right)}{n}$$

$$= \frac{\left(8e^{-3a}x\left(cx^n\right)^{-1/n}\right)\operatorname{Subst}\left(\int \frac{x^{-1+3b+\frac{1}{n}}}{\left(-e^{-2a}+x^{2b}\right)^3}\,dx, x, cx^n\right)}{n}$$

$$= -\frac{8e^{3a}x\left(cx^n\right)^{3b}\ {}_2F_1\left(3, \frac{3b+\frac{1}{n}}{2b}; \frac{1}{2}\left(5+\frac{1}{bn}\right); e^{2a}\left(cx^n\right)^{2b}\right)}{1+3bn}$$

Mathematica [A] time = 5.10, size = 101, normalized size = 1.46

$$\frac{8e^a x(bn-1)\left(cx^n\right)^b\ {}_2F_1\left(1, \frac{1}{2}\left(1+\frac{1}{bn}\right); \frac{1}{2}\left(3+\frac{1}{bn}\right); e^{2\left(a+b\log\left(cx^n\right)\right)}\right) - 4x\left(bn\coth\left(a+b\log\left(cx^n\right)\right)+1\right)\operatorname{csch}\left(a+\right.}{8b^2n^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Csch[a + b*Log[c*x^n]]^3,x]
```

```
[Out] (-4*x*(1 + b*n*Coth[a + b*Log[c*x^n]])*Csch[a + b*Log[c*x^n]] + 8*E^a*(-1 +
 b*n)*x*(c*x^n)^b*Hypergeometric2F1[1, (1 + 1/(b*n))/2, (3 + 1/(b*n))/2, E^
(2*(a + b*Log[c*x^n]))])/(8*b^2*n^2)
```

fricas [F] time = 0.72, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\operatorname{csch}\left(b\log\left(cx^n\right)+a\right)^3, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(a+b*log(c*x^n))^3,x, algorithm="fricas")
```

```
[Out] integral(csch(b*log(c*x^n) + a)^3, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{csch}\left(b\log\left(cx^n\right)+a\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(a+b*log(c*x^n))^3,x, algorithm="giac")
```

```
[Out] integrate(csch(b*log(c*x^n) + a)^3, x)
```

maple [F] time = 2.11, size = 0, normalized size = 0.00

$$\int \operatorname{csch}\left(a+b\ln\left(c\,x^n\right)\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(a+b*ln(c*x^n))^3,x)
```

```
[Out] int(csch(a+b*ln(c*x^n))^3,x)
```

maxima [F]   time = 0.00, size = 0, normalized size = 0.00

$$-8\left(b^2n^2-1\right)\int\frac{1}{16\left(b^2c^bn^2e^{\left(b\log(x^n)+a\right)}+b^2n^2\right)}\,dx-8\left(b^2n^2-1\right)\int\frac{1}{16\left(b^2c^bn^2e^{\left(b\log(x^n)+a\right)}-b^2n^2\right)}\,dx-\frac{\left(bc^{3\,b}n+c\right.}{b^2c^{4\,b}n^2e^{(4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(a+b*log(c*x^n))^3,x, algorithm="maxima")
```

```
[Out] -8*(b^2*n^2 - 1)*integrate(1/16/(b^2*c^b*n^2*e^(b*log(x^n) + a) + b^2*n^2),
 x) - 8*(b^2*n^2 - 1)*integrate(1/16/(b^2*c^b*n^2*e^(b*log(x^n) + a) - b^2*
n^2), x) - ((b*c^(3*b)*n + c^(3*b))*x*e^(3*b*log(x^n) + 3*a) + (b*c^b*n - c
^b)*x*e^(b*log(x^n) + a))/(b^2*c^(4*b)*n^2*e^(4*b*log(x^n) + 4*a) - 2*b^2*c
^(2*b)*n^2*e^(2*b*log(x^n) + 2*a) + b^2*n^2)
```

mupad [F]   time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{1}{\sinh\left(a+b\,\ln\left(c\,x^n\right)\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sinh(a + b*log(c*x^n))^3,x)
```

```
[Out] int(1/sinh(a + b*log(c*x^n))^3, x)
```

sympy [F]   time = 0.00, size = 0, normalized size = 0.00

$$\int\operatorname{csch}^3\left(a+b\log\left(cx^n\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(a+b*ln(c*x**n))**3,x)
```

```
[Out] Integral(csch(a + b*log(c*x**n))**3, x)
```

3.159 $\int \operatorname{csch}^4\left(a+b\log\left(cx^n\right)\right)\,dx$

Optimal. Leaf size=68

$$\frac{16e^{4a}x\left(cx^n\right)^{4b}\,{}_2F_1\left(4,\frac{1}{2}\left(4+\frac{1}{bn}\right);\frac{1}{2}\left(6+\frac{1}{bn}\right);e^{2a}\left(cx^n\right)^{2b}\right)}{4bn+1}$$

```
[Out] 16*exp(4*a)*x*(c*x^n)^(4*b)*hypergeom([4, 2+1/2/b/n],[3+1/2/b/n],exp(2*a)*(
c*x^n)^(2*b))/(4*b*n+1)
```

Rubi [A] time = 0.07, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {5546, 5548, 263, 364}

$$\frac{16e^{4a}x\left(cx^n\right)^{4b}\,{}_2F_1\left(4,\frac{1}{2}\left(4+\frac{1}{bn}\right);\frac{1}{2}\left(6+\frac{1}{bn}\right);e^{2a}\left(cx^n\right)^{2b}\right)}{4bn+1}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[a + b*Log[c*x^n]]^4,x]
```

```
[Out] (16*E^(4*a)*x*(c*x^n)^(4*b)*Hypergeometric2F1[4, (4 + 1/(b*n))/2, (6 + 1/(b
*n))/2, E^(2*a)*(c*x^n)^(2*b)])/(1 + 4*b*n)
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 5546

```
Int[Csch[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.), x_Symbol] :> D
ist[x/(n*(c*x^n)^(1/n)), Subst[Int[x^(1/n - 1)*Csch[d*(a + b*Log[x])]^p, x]
, x, c*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rule 5548

```
Int[Csch[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
 :> Dist[2^p/E^(a*d*p), Int[(e*x)^m/(x^(b*d*p)*(1 - 1/(E^(2*a*d)*x^(2*b*d))
)^p), x], x] /; FreeQ[{a, b, d, e, m}, x] && IntegerQ[p]
```

Rubi steps

$$\int \operatorname{csch}^4\left(a+b\log\left(cx^n\right)\right)\,dx=\frac{\left(x\left(cx^n\right)^{-1/n}\right)\operatorname{Subst}\left(\int x^{-1+\frac{1}{n}}\operatorname{csch}^4(a+b\log(x))\,dx,x,cx^n\right)}{n}$$
$$=\frac{\left(16e^{-4a}x\left(cx^n\right)^{-1/n}\right)\operatorname{Subst}\left(\int\frac{x^{-1-4b+\frac{1}{n}}}{\left(1-e^{-2a}x^{-2b}\right)^4}\,dx,x,cx^n\right)}{n}$$
$$=\frac{\left(16e^{-4a}x\left(cx^n\right)^{-1/n}\right)\operatorname{Subst}\left(\int\frac{x^{-1+4b+\frac{1}{n}}}{\left(-e^{-2a}+x^{2b}\right)^4}\,dx,x,cx^n\right)}{n}$$
$$=\frac{16e^{4a}x\left(cx^n\right)^{4b}\,{}_2F_1\left(4,\frac{1}{2}\left(4+\frac{1}{bn}\right);\frac{1}{2}\left(6+\frac{1}{bn}\right);e^{2a}\left(cx^n\right)^{2b}\right)}{1+4bn}$$

Mathematica [B] time = 9.02, size = 200, normalized size = 2.94

$$\frac{x\left(4\left(4b^2n^2-1\right){}_2F_1\left(1,\frac{1}{2bn};1+\frac{1}{2bn};e^{2\left(a+b\log\left(cx^n\right)\right)}\right)+\operatorname{csch}^3\left(a+b\log\left(cx^n\right)\right)\left(\left(1-12b^2n^2\right)\cosh\left(a+b\log\left(cx^n\right)\right)\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Csch[a + b*Log[c*x^n]]^4,x]
```

```
[Out] (x*(4*E^(2*a)*(-1 + 2*b*n)*(c*x^n)^(2*b)*Hypergeometric2F1[1, 1 + 1/(2*b*n)
, 2 + 1/(2*b*n), E^(2*(a + b*Log[c*x^n]))] + 4*(-1 + 4*b^2*n^2)*Hypergeomet
ric2F1[1, 1/(2*b*n), 1 + 1/(2*b*n), E^(2*(a + b*Log[c*x^n]))] + Csch[a + b*
Log[c*x^n]]^3*((1 - 12*b^2*n^2)*Cosh[a + b*Log[c*x^n]] + (-1 + 4*b^2*n^2)*C
osh[3*(a + b*Log[c*x^n])] - 4*b*n*Sinh[a + b*Log[c*x^n]])))/(24*b^3*n^3)
```

fricas [F] time = 0.48, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\operatorname{csch}\left(b\log\left(cx^n\right)+a\right)^4,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(a+b*log(c*x^n))^4,x, algorithm="fricas")
```

```
[Out] integral(csch(b*log(c*x^n) + a)^4, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{csch}\left(b\log\left(cx^n\right)+a\right)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(a+b*log(c*x^n))^4,x, algorithm="giac")
```

```
[Out] integrate(csch(b*log(c*x^n) + a)^4, x)
```

maple [F] time = 1.81, size = 0, normalized size = 0.00

$$\int \operatorname{csch}\left(a+b\ln\left(c\,x^n\right)\right)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(a+b*ln(c*x^n))^4,x)
```

```
[Out] int(csch(a+b*ln(c*x^n))^4,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$16\left(4b^2n^2-1\right)\int\frac{1}{96\left(b^3c^bn^3e^{\left(b\log(x^n)+a\right)}+b^3n^3\right)}\,dx-16\left(4b^2n^2-1\right)\int\frac{1}{96\left(b^3c^bn^3e^{\left(b\log(x^n)+a\right)}-b^3n^3\right)}\,dx-\frac{(2b}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(a+b*log(c*x^n))^4,x, algorithm="maxima")
```

```
[Out] 16*(4*b^2*n^2 - 1)*integrate(1/96/(b^3*c^b*n^3*e^(b*log(x^n) + a) + b^3*n^3
), x) - 16*(4*b^2*n^2 - 1)*integrate(1/96/(b^3*c^b*n^3*e^(b*log(x^n) + a) -
 b^3*n^3), x) - 1/3*((2*b*c^(4*b)*n + c^(4*b))*x*e^(4*b*log(x^n) + 4*a) + 2
*(6*b^2*c^(2*b)*n^2 - b*c^(2*b)*n - c^(2*b))*x*e^(2*b*log(x^n) + 2*a) - (4*
b^2*n^2 - 1)*x)/(b^3*c^(6*b)*n^3*e^(6*b*log(x^n) + 6*a) - 3*b^3*c^(4*b)*n^3
*e^(4*b*log(x^n) + 4*a) + 3*b^3*c^(2*b)*n^3*e^(2*b*log(x^n) + 2*a) - b^3*n^
3)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{1}{\sinh\left(a+b\,\ln\left(c\,x^n\right)\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sinh(a + b*log(c*x^n))^4,x)
```

```
[Out] int(1/sinh(a + b*log(c*x^n))^4, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\operatorname{csch}^4\left(a+b\log\left(cx^n\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(a+b*ln(c*x**n))**4,x)
```

```
[Out] Integral(csch(a + b*log(c*x**n))**4, x)
```

3.160 $\int \left(-\left(\left(1-b^2n^2\right)\operatorname{csch}\left(a+b\log\left(cx^n\right)\right)\right)+2b^2n^2\operatorname{csch}^3\left(\right.\right.$

Optimal. Leaf size=42

$$-x\operatorname{csch}\left(a+b\log\left(cx^n\right)\right)-bnx\coth\left(a+b\log\left(cx^n\right)\right)\operatorname{csch}\left(a+b\log\left(cx^n\right)\right)$$

```
[Out] -x*csch(a+b*ln(c*x^n))-b*n*x*coth(a+b*ln(c*x^n))*csch(a+b*ln(c*x^n))
```

Rubi [C] time = 0.14, antiderivative size = 137, normalized size of antiderivative = 3.26, number of steps used = 9, number of rules used = 4, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.089, Rules used = {5546, 5548, 263, 364}

$$2e^a x(1-bn)\left(cx^n\right)^b\,{}_2F_1\left(1,\frac{b+\frac{1}{n}}{2b};\frac{1}{2}\left(3+\frac{1}{bn}\right);e^{2a}\left(cx^n\right)^{2b}\right)-\frac{16e^{3a}b^2n^2x\left(cx^n\right)^{3b}\,{}_2F_1\left(3,\frac{3b+\frac{1}{n}}{2b};\frac{1}{2}\left(5+\frac{1}{bn}\right);e^{2a}\left(cx^n\right)^{2b}\right)}{3bn+1}$$

Warning: Unable to verify antiderivative.

```
[In] Int[-((1 - b^2*n^2)*Csch[a + b*Log[c*x^n]]) + 2*b^2*n^2*Csch[a + b*Log[c*x^
n]]^3,x]
```

```
[Out] 2*E^a*(1 - b*n)*x*(c*x^n)^b*Hypergeometric2F1[1, (b + n^(-1))/(2*b), (3 + 1
/(b*n))/2, E^(2*a)*(c*x^n)^(2*b)] - (16*b^2*E^(3*a)*n^2*x*(c*x^n)^(3*b)*Hyp
ergeometric2F1[3, (3*b + n^(-1))/(2*b), (5 + 1/(b*n))/2, E^(2*a)*(c*x^n)^(2
*b)])/(1 + 3*b*n)
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 5546

```
Int[Csch[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.), x_Symbol] :> D
ist[x/(n*(c*x^n)^(1/n)), Subst[Int[x^(1/n - 1)*Csch[d*(a + b*Log[x])]^p, x]
, x, c*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rule 5548

```
Int[Csch[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
 :> Dist[2^p/E^(a*d*p), Int[(e*x)^m/(x^(b*d*p)*(1 - 1/(E^(2*a*d)*x^(2*b*d))
)^p), x], x] /; FreeQ[{a, b, d, e, m}, x] && IntegerQ[p]
```

Rubi steps

$$\int \left(-\left(1-b^2n^2\right)\text{csch}\left(a+b\log\left(cx^n\right)\right)+2b^2n^2\text{csch}^3\left(a+b\log\left(cx^n\right)\right)\right)dx = \left(2b^2n^2\right)\int \text{csch}^3\left(a+b\log\left(cx^n\right)\right)$$

$$= \left(2b^2nx\left(cx^n\right)^{-1/n}\right)\text{Subst}\left(\int x^{-1+\frac{1}{n}}\right.$$

$$= \left(16b^2e^{-3a}nx\left(cx^n\right)^{-1/n}\right)\text{Subst}\left(\int\right.$$

$$= \left(16b^2e^{-3a}nx\left(cx^n\right)^{-1/n}\right)\text{Subst}\left(\int\right.$$

$$= 2e^a(1-bn)x\left(cx^n\right)^b\ {}_2F_1\left(1,\frac{b+\frac{1}{n}}{2b};\right.$$

Mathematica [A] time = 0.42, size = 30, normalized size = 0.71

$$-x\left(bn\coth\left(a+b\log\left(cx^n\right)\right)+1\right)\text{csch}\left(a+b\log\left(cx^n\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[-((1 - b^2*n^2)*Csch[a + b*Log[c*x^n]]) + 2*b^2*n^2*Csch[a + b*Lo
g[c*x^n]]^3,x]
```

```
[Out] -(x*(1 + b*n*Coth[a + b*Log[c*x^n]])*Csch[a + b*Log[c*x^n]])
```

fricas [B] time = 1.58, size = 187, normalized size = 4.45

$$-\frac{2\left((bn+1)x\cosh\left(bn\log(x)+b\log(c)+a\right)^2+2\,(bn+1)x\cosh\left(bn\log(x)+\right.\right.}{\cosh\left(bn\log(x)+b\log(c)+a\right)^3+3\,\cosh\left(bn\log(x)+b\log(c)+a\right)\sinh\left(bn\log(x)+b\log(c)+a\right)^2+\sinh}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(-b^2*n^2+1)*csch(a+b*log(c*x^n))+2*b^2*n^2*csch(a+b*log(c*x^n))
^3,x, algorithm="fricas")
```

```
[Out] -2*((b*n + 1)*x*cosh(b*n*log(x) + b*log(c) + a)^2 + 2*(b*n + 1)*x*cosh(b*n*
log(x) + b*log(c) + a)*sinh(b*n*log(x) + b*log(c) + a) + (b*n + 1)*x*sinh(b
*n*log(x) + b*log(c) + a)^2 + (b*n - 1)*x)/(cosh(b*n*log(x) + b*log(c) + a)
^3 + 3*cosh(b*n*log(x) + b*log(c) + a)*sinh(b*n*log(x) + b*log(c) + a)^2 +
sinh(b*n*log(x) + b*log(c) + a)^3 + 3*(cosh(b*n*log(x) + b*log(c) + a)^2 -
1)*sinh(b*n*log(x) + b*log(c) + a) - cosh(b*n*log(x) + b*log(c) + a))
```

giac [B] time = 0.55, size = 215, normalized size = 5.12

$$-\frac{2\,bc^{3\,b}nxx^{3\,bn}e^{(3\,a)}}{c^{4\,b}x^{4\,bn}e^{(4\,a)}-2\,c^{2\,b}x^{2\,bn}e^{(2\,a)}+1}-\frac{2\,bc^{b}nxx^{bn}e^{a}}{c^{4\,b}x^{4\,bn}e^{(4\,a)}-2\,c^{2\,b}x^{2\,bn}e^{(2\,a)}+1}-\frac{2\,c^{3\,b}xx^{3\,bn}e^{(3\,a)}}{c^{4\,b}x^{4\,bn}e^{(4\,a)}-2\,c^{2\,b}x^{2\,bn}e^{(2\,a)}+1}+\frac{}{c^{4\,b}x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(-b^2*n^2+1)*csch(a+b*log(c*x^n))+2*b^2*n^2*csch(a+b*log(c*x^n))
^3,x, algorithm="giac")
```

```
[Out] -2*b*c^(3*b)*n*x*x^(3*b*n)*e^(3*a)/(c^(4*b)*x^(4*b*n)*e^(4*a) - 2*c^(2*b)*x
^(2*b*n)*e^(2*a) + 1) - 2*b*c^b*n*x*x^(b*n)*e^a/(c^(4*b)*x^(4*b*n)*e^(4*a)
- 2*c^(2*b)*x^(2*b*n)*e^(2*a) + 1) - 2*c^(3*b)*x*x^(3*b*n)*e^(3*a)/(c^(4*b)
*x^(4*b*n)*e^(4*a) - 2*c^(2*b)*x^(2*b*n)*e^(2*a) + 1) + 2*c^b*x*x^(b*n)*e^a
/(c^(4*b)*x^(4*b*n)*e^(4*a) - 2*c^(2*b)*x^(2*b*n)*e^(2*a) + 1)
```

maple [C] time = 1.11, size = 509, normalized size = 12.12

$$-\frac{2c^{b}\left(x^{n}\right)^{b}x\left(nb\left(x^{n}\right)^{2b}c^{2b}\mathrm{e}^{3a}\mathrm{e}^{-\frac{3ib\,\mathrm{csgn}\left(icx^{n}\right)^{3}\pi}{2}}\mathrm{e}^{\frac{3ib\,\mathrm{csgn}\left(icx^{n}\right)^{2}\mathrm{csgn}(ic)\pi}{2}}\mathrm{e}^{\frac{3ib\,\mathrm{csgn}\left(icx^{n}\right)^{2}\mathrm{csgn}\left(ix^{n}\right)\pi}{2}}\mathrm{e}^{-\frac{3ib\,\mathrm{csgn}\left(icx^{n}\right)\mathrm{csgn}(ic)\mathrm{csgn}\left(ix^{n}\right)\pi}{2}}+\mathrm{e}^{a}\mathrm{e}^{-\frac{ibcs}{}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(-b^2*n^2+1)*csch(a+b*ln(c*x^n))+2*b^2*n^2*csch(a+b*ln(c*x^n))^3,x)
```

```
[Out] -2*c^b*(x^n)^b*x/(((x^n)^b)^2*(c^b)^2*exp(2*a)*exp(-I*b*csgn(I*c*x^n)^3*Pi)
*exp(I*b*csgn(I*c*x^n)^2*csgn(I*c)*Pi)*exp(I*b*csgn(I*c*x^n)^2*csgn(I*x^n)*
Pi)*exp(-I*b*csgn(I*c*x^n)*csgn(I*c)*csgn(I*x^n)*Pi)-1)^2*(n*b*((x^n)^b)^2*
(c^b)^2*exp(3*a)*exp(-3/2*I*b*csgn(I*c*x^n)^3*Pi)*exp(3/2*I*b*csgn(I*c*x^n)
^2*csgn(I*c)*Pi)*exp(3/2*I*b*csgn(I*c*x^n)^2*csgn(I*x^n)*Pi)*exp(-3/2*I*b*c
sgn(I*c*x^n)*csgn(I*c)*csgn(I*x^n)*Pi)+exp(a)*exp(-1/2*I*b*csgn(I*c*x^n)^3*
Pi)*exp(1/2*I*b*csgn(I*c*x^n)^2*csgn(I*c)*Pi)*exp(1/2*I*b*csgn(I*c*x^n)^2*c
sgn(I*x^n)*Pi)*exp(-1/2*I*b*csgn(I*c*x^n)*csgn(I*c)*csgn(I*x^n)*Pi)*b*n+((x
^n)^b)^2*(c^b)^2*exp(3*a)*exp(-3/2*I*b*csgn(I*c*x^n)^3*Pi)*exp(3/2*I*b*csgn
(I*c*x^n)^2*csgn(I*c)*Pi)*exp(3/2*I*b*csgn(I*c*x^n)^2*csgn(I*x^n)*Pi)*exp(-
3/2*I*b*csgn(I*c*x^n)*csgn(I*c)*csgn(I*x^n)*Pi)-exp(a)*exp(-1/2*I*b*csgn(I*
c*x^n)^3*Pi)*exp(1/2*I*b*csgn(I*c*x^n)^2*csgn(I*c)*Pi)*exp(1/2*I*b*csgn(I*c
*x^n)^2*csgn(I*x^n)*Pi)*exp(-1/2*I*b*csgn(I*c*x^n)*csgn(I*c)*csgn(I*x^n)*Pi
))
```

maxima [B] time = 0.71, size = 95, normalized size = 2.26

$$-\frac{2\left(\left(bc^{3b}n+c^{3b}\right)xe^{\left(3\,b\log\left(x^{n}\right)+3\,a\right)}+\left(bc^{b}n-c^{b}\right)xe^{\left(b\log\left(x^{n}\right)+a\right)}\right)}{c^{4\,b}e^{\left(4\,b\log\left(x^{n}\right)+4\,a\right)}-2\,c^{2\,b}e^{\left(2\,b\log\left(x^{n}\right)+2\,a\right)}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(-b^2*n^2+1)*csch(a+b*log(c*x^n))+2*b^2*n^2*csch(a+b*log(c*x^n))
^3,x, algorithm="maxima")
```

```
[Out] -2*((b*c^(3*b)*n + c^(3*b))*x*e^(3*b*log(x^n) + 3*a) + (b*c^b*n - c^b)*x*e^
(b*log(x^n) + a))/(c^(4*b)*e^(4*b*log(x^n) + 4*a) - 2*c^(2*b)*e^(2*b*log(x^
n) + 2*a) + 1)
```

mupad [B] time = 1.52, size = 65, normalized size = 1.55

$$-\frac{2\,x\,\mathrm{e}^{a}\,{\left(c\,x^{n}\right)}^{b}\,\left(b\,n+\mathrm{e}^{2\,a}\,{\left(c\,x^{n}\right)}^{2\,b}+b\,n\,\mathrm{e}^{2\,a}\,{\left(c\,x^{n}\right)}^{2\,b}-1\right)}{{\left(\mathrm{e}^{2\,a}\,{\left(c\,x^{n}\right)}^{2\,b}-1\right)}^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*n^2 - 1)/sinh(a + b*log(c*x^n)) + (2*b^2*n^2)/sinh(a + b*log(c*x^n
))^3,x)
```

```
[Out] -(2*x*exp(a)*(c*x^n)^b*(b*n + exp(2*a)*(c*x^n)^(2*b) + b*n*exp(2*a)*(c*x^n)
^(2*b) - 1))/(exp(2*a)*(c*x^n)^(2*b) - 1)^2
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(2b^2n^2 \operatorname{csch}^2\left(a + b\log\left(cx^n\right)\right) + b^2n^2 - 1\right) \operatorname{csch}\left(a + b\log\left(cx^n\right)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(-b**2*n**2+1)*csch(a+b*ln(c*x**n))+2*b**2*n**2*csch(a+b*ln(c*x*
*n))**3,x)
```

```
[Out] Integral((2*b**2*n**2*csch(a + b*log(c*x**n))**2 + b**2*n**2 - 1)*csch(a +
b*log(c*x**n)), x)
```

3.161 $\int \text{csch}^3\left(a+2\log\left(c\sqrt{x}\right)\right)\,dx$

Optimal. Leaf size=26

$$-\frac{2e^{-a}c^6}{\left(c^4-\frac{e^{-2a}}{x^2}\right)^2}$$

```
[Out] -2*c^6/exp(a)/(c^4-1/exp(2*a)/x^2)^2
```

Rubi [A] time = 0.04, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {5546, 5548, 261}

$$-\frac{2e^{-a}c^6}{\left(c^4-\frac{e^{-2a}}{x^2}\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[a + 2*Log[c*Sqrt[x]]]^3,x]
```

```
[Out] (-2*c^6)/(E^a*(c^4 - 1/(E^(2*a)*x^2))^2)
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 5546

```
Int[Csch[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.), x_Symbol] :> D
ist[x/(n*(c*x^n)^(1/n)), Subst[Int[x^(1/n - 1)*Csch[d*(a + b*Log[x])]^p, x]
, x, c*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rule 5548

```
Int[Csch[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
 :> Dist[2^p/E^(a*d*p), Int[(e*x)^m/(x^(b*d*p)*(1 - 1/(E^(2*a*d)*x^(2*b*d))
)^p), x], x] /; FreeQ[{a, b, d, e, m}, x] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\int \text{csch}^3\left(a+2\log\left(c\sqrt{x}\right)\right)\,dx &= \frac{2\,\text{Subst}\left(\int x\text{csch}^3(a+2\log(x))\,dx,x,c\sqrt{x}\right)}{c^2}\\
&= \frac{\left(16e^{-3a}\right)\text{Subst}\left(\int \frac{1}{\left(1-\frac{e^{-2a}}{x^4}\right)^3x^5}\,dx,x,c\sqrt{x}\right)}{c^2}\\
&= -\frac{2c^6e^{-a}}{\left(c^4-\frac{e^{-2a}}{x^2}\right)^2}
\end{aligned}$$

Mathematica [B] time = 0.13, size = 62, normalized size = 2.38

$$\frac{2(\cosh(a)-\sinh(a))\left(\sinh^2(a)+\cosh^2(a)-2\sinh(a)\cosh(a)-2c^4x^2\right)}{c^2\left(\sinh(a)\left(c^4x^2+1\right)+\cosh(a)\left(c^4x^2-1\right)\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[a + 2*Log[c*Sqrt[x]]]^3,x]
```

```
[Out] (2*(Cosh[a] - Sinh[a])*(-2*c^4*x^2 + Cosh[a]^2 - 2*Cosh[a]*Sinh[a] + Sinh[a
]^2))/(c^2*((-1 + c^4*x^2)*Cosh[a] + (1 + c^4*x^2)*Sinh[a])^2)
```

fricas [A] time = 0.59, size = 48, normalized size = 1.85

$$-\frac{2\left(2\,c^4x^2e^{(2\,a)}-1\right)}{c^{10}x^4e^{(5\,a)}-2\,c^6x^2e^{(3\,a)}+c^2e^a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(a+2*log(c*x^(1/2)))^3,x, algorithm="fricas")
```

```
[Out] -2*(2*c^4*x^2*e^(2*a) - 1)/(c^10*x^4*e^(5*a) - 2*c^6*x^2*e^(3*a) + c^2*e^a)
```

giac [A] time = 0.15, size = 38, normalized size = 1.46

$$-\frac{2\left(2\,c^4x^2e^{(2\,a)}-1\right)e^{(-a)}}{\left(c^4x^2e^{(2\,a)}-1\right)^2c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(a+2*log(c*x^(1/2)))^3,x, algorithm="giac")
```

```
[Out] -2*(2*c^4*x^2*e^(2*a) - 1)*e^(-a)/((c^4*x^2*e^(2*a) - 1)^2*c^2)
```

maple [F] time = 0.53, size = 0, normalized size = 0.00

$$\int \operatorname{csch}\left(a+2\ln\left(c\sqrt{x}\right)\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(a+2*ln(c*x^(1/2)))^3,x)
```

```
[Out] int(csch(a+2*ln(c*x^(1/2)))^3,x)
```

maxima [B] time = 0.32, size = 76, normalized size = 2.92

$$-\frac{2\left(\frac{2\,c^4x^2e^{(2\,a)}}{c^8x^4e^{(5\,a)}-2\,c^4x^2e^{(3\,a)}+e^a}-\frac{1}{c^8x^4e^{(5\,a)}-2\,c^4x^2e^{(3\,a)}+e^a}\right)}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(a+2*log(c*x^(1/2)))^3,x, algorithm="maxima")
```

```
[Out] -2*(2*c^4*x^2*e^(2*a)/(c^8*x^4*e^(5*a) - 2*c^4*x^2*e^(3*a) + e^a) - 1/(c^8*
x^4*e^(5*a) - 2*c^4*x^2*e^(3*a) + e^a))/c^2
```

mupad [B] time = 1.65, size = 48, normalized size = 1.85

$$\frac{\frac{2\,\mathrm{e}^{-a}}{c^2}-4\,c^2\,x^2\,\mathrm{e}^a}{\mathrm{e}^{4\,a}\,c^8\,x^4-2\,\mathrm{e}^{2\,a}\,c^4\,x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sinh(a + 2*log(c*x^(1/2)))^3,x)
```

```
[Out] ((2*exp(-a))/c^2 - 4*c^2*x^2*exp(a))/(c^8*x^4*exp(4*a) - 2*c^4*x^2*exp(2*a)
 + 1)
```

<u>**sympy**</u> **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{csch}^3\left(a + 2\log\left(c\sqrt{x}\right)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(a+2*ln(c*x**(1/2)))**3,x)
```

```
[Out] Integral(csch(a + 2*log(c*sqrt(x)))**3, x)
```

3.162 $\int \operatorname{csch}^3\left(a+2\log\left(\frac{c}{\sqrt{x}}\right)\right)\,dx$

Optimal. Leaf size=26

$$\frac{2e^{-3a}c^2}{\left(e^{-2a}-\frac{c^4}{x^2}\right)^2}$$

```
[Out] 2*c^2/exp(3*a)/(exp(-2*a)-c^4/x^2)^2
```

Rubi [A] time = 0.05, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {5546, 5548, 263, 261}

$$\frac{2e^{-3a}c^2}{\left(e^{-2a}-\frac{c^4}{x^2}\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[a + 2*Log[c/Sqrt[x]]]^3,x]
```

```
[Out] (2*c^2)/(E^(3*a)*(E^(-2*a) - c^4/x^2)^2)
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 5546

```
Int[Csch[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.), x_Symbol] :> D
ist[x/(n*(c*x^n)^(1/n)), Subst[Int[x^(1/n - 1)*Csch[d*(a + b*Log[x])]^p, x]
, x, c*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rule 5548

```
Int[Csch[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
 :> Dist[2^p/E^(a*d*p), Int[(e*x)^m/(x^(b*d*p)*(1 - 1/(E^(2*a*d)*x^(2*b*d))
)^p), x], x] /; FreeQ[{a, b, d, e, m}, x] && IntegerQ[p]
```

Rubi steps

$$\int \operatorname{csch}^3\left(a+2\log\left(\frac{c}{\sqrt{x}}\right)\right)dx = -\left(\left(2c^2\right)\operatorname{Subst}\left(\int \frac{\operatorname{csch}^3(a+2\log(x))}{x^3}\,dx, x, \frac{c}{\sqrt{x}}\right)\right)$$
$$= -\left(\left(16c^2e^{-3a}\right)\operatorname{Subst}\left(\int \frac{1}{\left(1-\frac{e^{-2a}}{x^4}\right)^3 x^9}\,dx, x, \frac{c}{\sqrt{x}}\right)\right)$$
$$= -\left(\left(16c^2e^{-3a}\right)\operatorname{Subst}\left(\int \frac{x^3}{\left(-e^{-2a}+x^4\right)^3}\,dx, x, \frac{c}{\sqrt{x}}\right)\right)$$
$$= \frac{2c^2e^{-3a}}{\left(e^{-2a}-\frac{c^4}{x^2}\right)^2}$$

Mathematica [B] time = 0.11, size = 65, normalized size = 2.50

$$-\frac{2c^6(\sinh(2a)+\cosh(2a))\left(\sinh(a)\left(c^4+2x^2\right)+\cosh(a)\left(c^4-2x^2\right)\right)}{\left(\cosh(a)\left(x^2-c^4\right)-\sinh(a)\left(c^4+x^2\right)\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[a + 2*Log[c/Sqrt[x]]]^3,x]
```

```
[Out] (-2*c^6*((c^4 - 2*x^2)*Cosh[a] + (c^4 + 2*x^2)*Sinh[a])*(Cosh[2*a] + Sinh[2
*a]))/((-c^4 + x^2)*Cosh[a] - (c^4 + x^2)*Sinh[a])^2
```

fricas [A] time = 0.41, size = 49, normalized size = 1.88

$$-\frac{2\left(c^{10}e^{(5a)}-2c^6x^2e^{(3a)}\right)}{c^8e^{(4a)}-2c^4x^2e^{(2a)}+x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(a+2*log(c/x^(1/2)))^3,x, algorithm="fricas")
```

```
[Out] -2*(c^10*e^(5*a) - 2*c^6*x^2*e^(3*a))/(c^8*e^(4*a) - 2*c^4*x^2*e^(2*a) + x^
4)
```

giac [A] time = 0.11, size = 39, normalized size = 1.50

$$-\frac{2\left(c^{10}e^{(5a)}-2c^6x^2e^{(3a)}\right)}{\left(c^4e^{(2a)}-x^2\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(a+2*log(c/x^(1/2)))^3,x, algorithm="giac")
```

```
[Out] -2*(c^10*e^(5*a) - 2*c^6*x^2*e^(3*a))/(c^4*e^(2*a) - x^2)^2
```

maple [F] time = 0.73, size = 0, normalized size = 0.00

$$\int \operatorname{csch}\left(a+2\ln\left(\frac{c}{\sqrt{x}}\right)\right)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(a+2*ln(c/x^(1/2)))^3,x)
```

```
[Out] int(csch(a+2*ln(c/x^(1/2)))^3,x)
```

maxima [A] time = 0.35, size = 49, normalized size = 1.88

$$-\frac{2\left(c^{10}e^{(5a)} - 2c^6x^2e^{(3a)}\right)}{c^8e^{(4a)} - 2c^4x^2e^{(2a)} + x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(a+2*log(c/x^(1/2)))^3,x, algorithm="maxima")
```

```
[Out] -2*(c^10*e^(5*a) - 2*c^6*x^2*e^(3*a))/(c^8*e^(4*a) - 2*c^4*x^2*e^(2*a) + x^
4)
```

mupad [B] time = 1.59, size = 36, normalized size = 1.38

$$\frac{2c^2x^4e^a}{e^{4a}c^8 - 2e^{2a}c^4x^2 + x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sinh(a + 2*log(c/x^(1/2)))^3,x)
```

```
[Out] (2*c^2*x^4*exp(a))/(c^8*exp(4*a) + x^4 - 2*c^4*x^2*exp(2*a))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{csch}^3\left(a + 2\log\left(\frac{c}{\sqrt{x}}\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(a+2*ln(c/x**(1/2)))**3,x)
```

```
[Out] Integral(csch(a + 2*log(c/sqrt(x)))**3, x)
```

3.163 $\int \operatorname{csch}^p\left(a+\frac{\log(cx^n)}{n(-2+p)}\right)\,dx$

Optimal. Leaf size=90

$$-\frac{e^{2a}(2-p)x\left(cx^n\right)^{-\frac{2}{n(2-p)}}\left(1-e^{-2a}\left(cx^n\right)^{\frac{2}{n(2-p)}}\right)\operatorname{csch}^p\left(a-\frac{\log(cx^n)}{n(2-p)}\right)}{2(1-p)}$$

```
[Out] -1/2*exp(2*a)*(2-p)*x*(1-(c*x^n)^(2/n/(2-p))/exp(2*a))*csch(a-ln(c*x^n)/n/(
2-p))^p/(1-p)/((c*x^n)^(2/n/(2-p)))
```

Rubi [A] time = 0.09, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {5546, 5550, 261}

$$-\frac{e^{2a}(2-p)x\left(cx^n\right)^{-\frac{2}{n(2-p)}}\left(1-e^{-2a}\left(cx^n\right)^{\frac{2}{n(2-p)}}\right)\operatorname{csch}^p\left(a-\frac{\log(cx^n)}{n(2-p)}\right)}{2(1-p)}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[a + Log[c*x^n]/(n*(-2 + p))]^p,x]
```

```
[Out] -(E^(2*a)*(2 - p)*x*(1 - (c*x^n)^(2/(n*(2 - p)))/E^(2*a))*Csch[a - Log[c*x^
n]/(n*(2 - p))]^p)/(2*(1 - p)*(c*x^n)^(2/(n*(2 - p))))
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 5546

```
Int[Csch[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.), x_Symbol] :> D
ist[x/(n*(c*x^n)^(1/n)), Subst[Int[x^(1/n - 1)*Csch[d*(a + b*Log[x])]^p, x]
, x, c*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rule 5550

```
Int[Csch[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
 :> Dist[(Csch[d*(a + b*Log[x])]^p*(1 - 1/(E^(2*a*d)*x^(2*b*d)))^p)/x^(-(b*
d*p)), Int[(e*x)^m/(x^(b*d*p)*(1 - 1/(E^(2*a*d)*x^(2*b*d)))^p), x], x] /; F
reeQ[{a, b, d, e, m, p}, x] && !IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \operatorname{csch}^p\left(a+\frac{\log\left(cx^n\right)}{n(-2+p)}\right)dx &= \frac{\left(x\left(cx^n\right)^{-1/n}\right)\operatorname{Subst}\left(\int x^{-1+\frac{1}{n}}\operatorname{csch}^p\left(a+\frac{\log(x)}{n(-2+p)}\right)dx,x,cx^n\right)}{n}\\ &= \frac{\left(x\left(cx^n\right)^{-\frac{1}{n}+\frac{p}{n(-2+p)}}\left(1-e^{-2a}\left(cx^n\right)^{-\frac{2}{n(-2+p)}}\right)^p\operatorname{csch}^p\left(a+\frac{\log(cx^n)}{n(-2+p)}\right)\right)\operatorname{Subst}\left(\int x^{-1+\frac{1}{n}-\frac{}{n(}}\right.}{n}\\ &= -\frac{e^{2a}(2-p)x\left(cx^n\right)^{-\frac{2}{n(2-p)}}\left(1-e^{-2a}\left(cx^n\right)^{\frac{2}{n(2-p)}}\right)\operatorname{csch}^p\left(a-\frac{\log(cx^n)}{n(2-p)}\right)}{2(1-p)}\end{aligned}$$

Mathematica [A] time = 6.76, size = 115, normalized size = 1.28

$$\frac{2^{p-1}(p-2)x\left(\frac{e^a(cx^n)^{\frac{1}{n(p-2)}}}{e^{2a}(cx^n)^{\frac{2}{n(p-2)}}-1}\right)^p\left(e^{2a}\left(cx^n\right)^{\frac{2}{n(p-2)}}\left(\left(1-e^{-2a}\left(cx^n\right)^{-\frac{2}{n(p-2)}}\right)^p-1\right)+1\right)}{p-1}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Csch[a + Log[c*x^n]/(n*(-2 + p))]^p,x]
```

```
[Out] (2^(-1 + p)*(-2 + p)*x*((E^a*(c*x^n)^(1/(n*(-2 + p))))/(-1 + E^(2*a)*(c*x^n
)^(2/(n*(-2 + p)))))^p*(1 + E^(2*a)*(c*x^n)^(2/(n*(-2 + p)))*(-1 + (1 - 1/(
E^(2*a)*(c*x^n)^(2/(n*(-2 + p)))))^p)))/(-1 + p)
```

fricas [B] time = 0.49, size = 475, normalized size = 5.28

$$-\frac{\left(p-2\right)x\cosh\left(p\log\left(\frac{2\left(\cosh\left(\frac{anp-2\,an+n\log(x)+\log(c)}{np-2\,n}\right)+\sinh\left(\frac{anp-2\,an+n\log(x)+\log(c)}{np-2\,n}\right)\right)}{\cosh\left(\frac{anp-2\,an+n\log(x)+\log(c)}{np-2\,n}\right)^2+2\,\cosh\left(\frac{anp-2\,an+n\log(x)+\log(c)}{np-2\,n}\right)\sinh\left(\frac{anp-2\,an+n\log(x)+\log(c)}{np-2\,n}\right)+\sinh\left(\frac{anp-2\,an+n\log(\cdots}{np-2\,n}\right.}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(a+log(c*x^n)/n/(-2+p))^p,x, algorithm="fricas")
```

```
[Out] -((p - 2)*x*cosh(p*log(2*(cosh((a*n*p - 2*a*n + n*log(x) + log(c))/(n*p - 2
*n)) + sinh((a*n*p - 2*a*n + n*log(x) + log(c))/(n*p - 2*n)))/(cosh((a*n*p
- 2*a*n + n*log(x) + log(c))/(n*p - 2*n))^2 + 2*cosh((a*n*p - 2*a*n + n*log
(x) + log(c))/(n*p - 2*n))*sinh((a*n*p - 2*a*n + n*log(x) + log(c))/(n*p -
2*n)) + sinh((a*n*p - 2*a*n + n*log(x) + log(c))/(n*p - 2*n))^2 - 1)))*sinh
((a*n*p - 2*a*n + n*log(x) + log(c))/(n*p - 2*n)) + (p - 2)*x*sinh(p*log(2*
(cosh((a*n*p - 2*a*n + n*log(x) + log(c))/(n*p - 2*n)) + sinh((a*n*p - 2*a*
n + n*log(x) + log(c))/(n*p - 2*n)))/(cosh((a*n*p - 2*a*n + n*log(x) + log(
c))/(n*p - 2*n))^2 + 2*cosh((a*n*p - 2*a*n + n*log(x) + log(c))/(n*p - 2*n)
)*sinh((a*n*p - 2*a*n + n*log(x) + log(c))/(n*p - 2*n)) + sinh((a*n*p - 2*a
*n + n*log(x) + log(c))/(n*p - 2*n))^2 - 1)))*sinh((a*n*p - 2*a*n + n*log(x
) + log(c))/(n*p - 2*n)))/((p - 1)*cosh((a*n*p - 2*a*n + n*log(x) + log(c))
/(n*p - 2*n)) - (p - 1)*sinh((a*n*p - 2*a*n + n*log(x) + log(c))/(n*p - 2*n
)))
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(a+log(c*x^n)/n/(-2+p))^p,x, algorithm="giac")
```

```
[Out] Timed out
```

maple [F] time = 0.51, size = 0, normalized size = 0.00

$$\int \operatorname{csch}\left(a+\frac{\ln\left(c\,x^{n}\right)}{n\left(-2+p\right)}\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(a+ln(c*x^n)/n/(-2+p))^p,x)
```

```
[Out] int(csch(a+ln(c*x^n)/n/(-2+p))^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{csch}\left(a + \frac{\log\left(cx^{n}\right)}{n(p-2)}\right)^{p} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(a+log(c*x^n)/n/(-2+p))^p,x, algorithm="maxima")
[Out] integrate(csch(a + log(c*x^n)/(n*(p - 2)))^p, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(\frac{1}{\sinh\left(a + \frac{\ln(c\,x^{n})}{n\,(p-2)}\right)}\right)^{p} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/sinh(a + log(c*x^n)/(n*(p - 2))))^p,x)
[Out] int((1/sinh(a + log(c*x^n)/(n*(p - 2))))^p, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{csch}^{p}\left(a + \frac{\log\left(cx^{n}\right)}{n\left(p-2\right)}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(a+ln(c*x**n)/n/(-2+p))**p,x)
[Out] Integral(csch(a + log(c*x**n)/(n*(p - 2)))**p, x)
```

3.164 $\int \operatorname{csch}^p\left(a-\frac{\log(cx^n)}{n(-2+p)}\right)\,dx$

Optimal. Leaf size=66

$$\frac{(2-p)x\left(1-e^{-2a}\left(cx^n\right)^{-\frac{2}{n(2-p)}}\right)\operatorname{csch}^p\left(a+\frac{\log(cx^n)}{n(2-p)}\right)}{2(1-p)}$$

```
[Out] 1/2*(2-p)*x*(1-1/exp(2*a)/((c*x^n)^(2/n/(2-p))))*csch(a+ln(c*x^n)/n/(2-p))^
p/(1-p)
```

Rubi [A] time = 0.08, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {5546, 5550, 264}

$$\frac{(2-p)x\left(1-e^{-2a}\left(cx^n\right)^{-\frac{2}{n(2-p)}}\right)\operatorname{csch}^p\left(a+\frac{\log(cx^n)}{n(2-p)}\right)}{2(1-p)}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[a - Log[c*x^n]/(n*(-2 + p))]^p,x]
```

```
[Out] ((2 - p)*x*(1 - 1/(E^(2*a)*(c*x^n)^(2/(n*(2 - p)))))*Csch[a + Log[c*x^n]/(n
*(2 - p))]^p)/(2*(1 - p))
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 5546

```
Int[Csch[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.), x_Symbol] :> D
ist[x/(n*(c*x^n)^(1/n)), Subst[Int[x^(1/n - 1)*Csch[d*(a + b*Log[x])]^p, x]
, x, c*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rule 5550

```
Int[Csch[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
 :> Dist[(Csch[d*(a + b*Log[x])]^p*(1 - 1/(E^(2*a*d)*x^(2*b*d)))^p)/x^(-(b*
d*p)), Int[(e*x)^m/(x^(b*d*p)*(1 - 1/(E^(2*a*d)*x^(2*b*d)))^p), x], x] /; F
reeQ[{a, b, d, e, m, p}, x] && !IntegerQ[p]
```

Rubi steps

$$\int \operatorname{csch}^p\left(a-\frac{\log\left(cx^n\right)}{n(-2+p)}\right)dx = \frac{\left(x\left(cx^n\right)^{-1/n}\right)\operatorname{Subst}\left(\int x^{-1+\frac{1}{n}}\operatorname{csch}^p\left(a-\frac{\log(x)}{n(-2+p)}\right)dx,x,cx^n\right)}{n}$$

$$=\frac{\left(x\left(cx^n\right)^{-\frac{1}{n}-\frac{p}{n(-2+p)}}\left(1-e^{-2a}\left(cx^n\right)^{\frac{2}{n(-2+p)}}\right)^p\operatorname{csch}^p\left(a-\frac{\log(cx^n)}{n(-2+p)}\right)\right)\operatorname{Subst}\left(\int x^{-1+\frac{1}{n}+\cdots}\right.}{n}$$

$$=\frac{(2-p)x\left(1-e^{-2a}\left(cx^n\right)^{-\frac{2}{n(2-p)}}\right)\operatorname{csch}^p\left(a+\frac{\log(cx^n)}{n(2-p)}\right)}{2(1-p)}$$

Mathematica [A] time = 0.92, size = 64, normalized size = 0.97

$$\frac{e^{-2a}(p-2)x\left(e^{2a}-(cx^n)^{\frac{2}{n(p-2)}}\right)\operatorname{csch}^p\left(a+\frac{\log(cx^n)}{2n-np}\right)}{2(p-1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[a - Log[c*x^n]/(n*(-2 + p))]^p,x]
```

```
[Out] ((-2 + p)*x*(E^(2*a) - (c*x^n)^(2/(n*(-2 + p))))*Csch[a + Log[c*x^n]/(2*n -
 n*p)]^p)/(2*E^(2*a)*(-1 + p))
```

fricas [B] time = 1.13, size = 539, normalized size = 8.17

$$-\frac{\left(p-2\right)x\cosh\left(p\log\left(-\frac{2\left(\cosh\left(-\frac{anp-2\,an-n\log(x)-\log(c)}{np-2\,n}\right)+\sinh\left(-\frac{anp-2\,an-n\log(x)-\log(c)}{np-2\,n}\right)\right)}{\cosh\left(-\frac{anp-2\,an-n\log(x)-\log(c)}{np-2\,n}\right)^2+2\,\cosh\left(-\frac{anp-2\,an-n\log(x)-\log(c)}{np-2\,n}\right)\sinh\left(-\frac{anp-2\,an-n\log(x)-\log(c)}{np-2\,n}\right)+\sinh\left(-\frac{anp-2\,an-n\,l}{np-}\right.}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(a-log(c*x^n)/n/(-2+p))^p,x, algorithm="fricas")
```

```
[Out] -((p - 2)*x*cosh(p*log(-2*(cosh(-(a*n*p - 2*a*n - n*log(x) - log(c))/(n*p -
 2*n)) + sinh(-(a*n*p - 2*a*n - n*log(x) - log(c))/(n*p - 2*n)))/(cosh(-(a*
n*p - 2*a*n - n*log(x) - log(c))/(n*p - 2*n))^2 + 2*cosh(-(a*n*p - 2*a*n -
n*log(x) - log(c))/(n*p - 2*n))*sinh(-(a*n*p - 2*a*n - n*log(x) - log(c))/(
n*p - 2*n)) + sinh(-(a*n*p - 2*a*n - n*log(x) - log(c))/(n*p - 2*n))^2 - 1)
))*sinh(-(a*n*p - 2*a*n - n*log(x) - log(c))/(n*p - 2*n)) + (p - 2)*x*sinh(
p*log(-2*(cosh(-(a*n*p - 2*a*n - n*log(x) - log(c))/(n*p - 2*n)) + sinh(-(a
*n*p - 2*a*n - n*log(x) - log(c))/(n*p - 2*n)))/(cosh(-(a*n*p - 2*a*n - n*l
og(x) - log(c))/(n*p - 2*n))^2 + 2*cosh(-(a*n*p - 2*a*n - n*log(x) - log(c)
)/(n*p - 2*n))*sinh(-(a*n*p - 2*a*n - n*log(x) - log(c))/(n*p - 2*n)) + sin
h(-(a*n*p - 2*a*n - n*log(x) - log(c))/(n*p - 2*n))^2 - 1)))*sinh(-(a*n*p -
 2*a*n - n*log(x) - log(c))/(n*p - 2*n)))/((p - 1)*cosh(-(a*n*p - 2*a*n - n
*log(x) - log(c))/(n*p - 2*n)) - (p - 1)*sinh(-(a*n*p - 2*a*n - n*log(x) -
log(c))/(n*p - 2*n)))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{csch}\left(a-\frac{\log\left(cx^n\right)}{n\left(p-2\right)}\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(a-log(c*x^n)/n/(-2+p))^p,x, algorithm="giac")
```

```
[Out] integrate(csch(a - log(c*x^n)/(n*(p - 2)))^p, x)
```

maple [F] time = 0.51, size = 0, normalized size = 0.00

$$\int \operatorname{csch}\left(a-\frac{\ln\left(c\,x^n\right)}{n\left(-2+p\right)}\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(a-ln(c*x^n)/n/(-2+p))^p,x)
```

```
[Out] int(csch(a-ln(c*x^n)/n/(-2+p))^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(-\operatorname{csch}\left(-a + \frac{\log\left(cx^n\right)}{n(p-2)}\right)\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(a-log(c*x^n)/n/(-2+p))^p,x, algorithm="maxima")
[Out] integrate((-csch(-a + log(c*x^n)/(n*(p - 2))))^p, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \left(\frac{1}{\sinh\left(a - \frac{\ln(c\,x^n)}{n\,(p-2)}\right)}\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/sinh(a - log(c*x^n)/(n*(p - 2))))^p,x)
[Out] int((1/sinh(a - log(c*x^n)/(n*(p - 2))))^p, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{csch}^p\left(a - \frac{\log\left(cx^n\right)}{n\left(p-2\right)}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(a-ln(c*x**n)/n/(-2+p))**p,x)
[Out] Integral(csch(a - log(c*x**n)/(n*(p - 2)))**p, x)
```

3.165 $\int \frac{\operatorname{csch}\left(a+b\log(cx^n)\right)}{x}\,dx$

Optimal. Leaf size=20

$$-\frac{\tanh^{-1}\left(\cosh\left(a+b\log\left(cx^n\right)\right)\right)}{bn}$$

```
[Out] -arctanh(cosh(a+b*ln(c*x^n)))/b/n
```

Rubi [A] time = 0.02, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {3770}

$$-\frac{\tanh^{-1}\left(\cosh\left(a+b\log\left(cx^n\right)\right)\right)}{bn}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[a + b*Log[c*x^n]]/x,x]
```

```
[Out] -(ArcTanh[Cosh[a + b*Log[c*x^n]]]/(b*n))
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\operatorname{csch}\left(a+b\log\left(cx^n\right)\right)}{x}\,dx &= \frac{\operatorname{Subst}\left(\int \operatorname{csch}(a+bx)\,dx, x, \log\left(cx^n\right)\right)}{n}\\ &= -\frac{\tanh^{-1}\left(\cosh\left(a+b\log\left(cx^n\right)\right)\right)}{bn}\end{aligned}$$

Mathematica [B] time = 0.06, size = 54, normalized size = 2.70

$$\frac{\log\left(\sinh\left(\frac{a}{2}+\frac{1}{2}b\log\left(cx^n\right)\right)\right)}{bn}-\frac{\log\left(\cosh\left(\frac{a}{2}+\frac{1}{2}b\log\left(cx^n\right)\right)\right)}{bn}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[a + b*Log[c*x^n]]/x,x]
```

```
[Out] -(Log[Cosh[a/2 + (b*Log[c*x^n])/2]]/(b*n)) + Log[Sinh[a/2 + (b*Log[c*x^n])/
2]]/(b*n)
```

fricas [B] time = 0.60, size = 65, normalized size = 3.25

$$-\frac{\log\left(\cosh\left(bn\log(x)+b\log(c)+a\right)+\sinh\left(bn\log(x)+b\log(c)+a\right)+1\right)-\log\left(\cosh\left(bn\log(x)+b\log(c)+\right.\right.}{bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(a+b*log(c*x^n))/x,x, algorithm="fricas")
```

```
[Out] -(log(cosh(b*n*log(x) + b*log(c) + a) + sinh(b*n*log(x) + b*log(c) + a) + 1
) - log(cosh(b*n*log(x) + b*log(c) + a) + sinh(b*n*log(x) + b*log(c) + a) -
 1))/(b*n)
```

giac [B] time = 0.58, size = 145, normalized size = 7.25

$$-c^{b}\left(\frac{c^{b}e^{(-a)}\log\left(\sqrt{2\,x^{bn}|c|^{b}\cos\left(-\frac{1}{2}\,\pi b\mathrm{sgn}(c)+\frac{1}{2}\,\pi b\right)e^{a}+x^{2\,bn}|c|^{2\,b}e^{(2\,a)}+1}\right)}{bc^{2\,b}n}-\frac{c^{b}e^{(-a)}\log\left(\sqrt{-2\,x^{bn}|c|^{b}\cos\left(-\right.}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(a+b*log(c*x^n))/x,x, algorithm="giac")
```

```
[Out] -c^b*(c^b*e^(-a)*log(sqrt(2*x^(b*n)*abs(c)^b*cos(-1/2*pi*b*sgn(c) + 1/2*pi*
b)*e^a + x^(2*b*n)*abs(c)^(2*b)*e^(2*a) + 1))/(b*c^(2*b)*n) - c^b*e^(-a)*lo
g(sqrt(-2*x^(b*n)*abs(c)^b*cos(-1/2*pi*b*sgn(c) + 1/2*pi*b)*e^a + x^(2*b*n)
*abs(c)^(2*b)*e^(2*a) + 1))/(b*c^(2*b)*n))*e^a
```

maple [A] time = 0.09, size = 23, normalized size = 1.15

$$\frac{\ln\left(\tanh\left(\frac{a}{2}+\frac{b\ln(c\,x^{n})}{2}\right)\right)}{nb}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(a+b*ln(c*x^n))/x,x)
```

```
[Out] 1/n/b*ln(tanh(1/2*a+1/2*b*ln(c*x^n)))
```

maxima [A] time = 0.31, size = 22, normalized size = 1.10

$$\frac{\log\left(\tanh\left(\frac{1}{2}\,b\log\left(cx^{n}\right)+\frac{1}{2}\,a\right)\right)}{bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(a+b*log(c*x^n))/x,x, algorithm="maxima")
```

```
[Out] log(tanh(1/2*b*log(c*x^n) + 1/2*a))/(b*n)
```

mupad [B] time = 1.62, size = 43, normalized size = 2.15

$$-\frac{2\,\mathrm{atan}\left(\frac{\mathrm{e}^{-a}\,\sqrt{-b^2\,n^2}}{b\,n\,{\left(c\,x^n\right)}^b}\right)}{\sqrt{-b^2\,n^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*sinh(a + b*log(c*x^n))),x)
```

```
[Out] -(2*atan((exp(-a)*(-b^2*n^2)^(1/2))/(b*n*(c*x^n)^b)))/(-b^2*n^2)^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}\left(a+b\log\left(cx^{n}\right)\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(a+b*ln(c*x**n))/x,x)
```

```
[Out] Integral(csch(a + b*log(c*x**n))/x, x)
```

3.166 $\int \frac{\operatorname{csch}^2(a+b\log(cx^n))}{x}\,dx$

Optimal. Leaf size=19

$$-\frac{\coth\left(a+b\log\left(cx^n\right)\right)}{bn}$$

```
[Out] -coth(a+b*ln(c*x^n))/b/n
```

Rubi [A] time = 0.03, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {3767, 8}

$$-\frac{\coth\left(a+b\log\left(cx^n\right)\right)}{bn}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[a + b*Log[c*x^n]]^2/x,x]
```

```
[Out] -(Coth[a + b*Log[c*x^n]]/(b*n))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\operatorname{csch}^2\left(a+b\log\left(cx^n\right)\right)}{x}\,dx &= \frac{\operatorname{Subst}\left(\int \operatorname{csch}^2(a+bx)\,dx, x, \log\left(cx^n\right)\right)}{n}\\
&= -\frac{i\operatorname{Subst}\left(\int 1\,dx, x, -i\coth\left(a+b\log\left(cx^n\right)\right)\right)}{bn}\\
&= -\frac{\coth\left(a+b\log\left(cx^n\right)\right)}{bn}
\end{aligned}$$

Mathematica [A] time = 0.07, size = 19, normalized size = 1.00

$$-\frac{\coth\left(a+b\log\left(cx^n\right)\right)}{bn}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[a + b*Log[c*x^n]]^2/x,x]
```

```
[Out] -(Coth[a + b*Log[c*x^n]]/(b*n))
```

fricas [B] time = 0.68, size = 71, normalized size = 3.74

$$-\frac{2}{bn\cosh\left(bn\log(x)+b\log(c)+a\right)^2+2\,bn\cosh\left(bn\log(x)+b\log(c)+a\right)\sinh\left(bn\log(x)+b\log(c)+a\right)+bn\,s}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(a+b*log(c*x^n))^2/x,x, algorithm="fricas")
```

```
[Out] -2/(b*n*cosh(b*n*log(x) + b*log(c) + a)^2 + 2*b*n*cosh(b*n*log(x) + b*log(c
) + a)*sinh(b*n*log(x) + b*log(c) + a) + b*n*sinh(b*n*log(x) + b*log(c) + a
)^2 - b*n)
```

giac [A] time = 0.15, size = 28, normalized size = 1.47

$$-\frac{2}{\left(c^{2\,b}x^{2\,bn}e^{(2\,a)}-1\right)bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(a+b*log(c*x^n))^2/x,x, algorithm="giac")
```

```
[Out] -2/((c^(2*b)*x^(2*b*n)*e^(2*a) - 1)*b*n)
```

maple [A] time = 0.33, size = 20, normalized size = 1.05

$$-\frac{\coth\left(a+b\ln\left(c\,x^{n}\right)\right)}{bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(a+b*ln(c*x^n))^2/x,x)
```

```
[Out] -coth(a+b*ln(c*x^n))/b/n
```

maxima [A] time = 0.33, size = 29, normalized size = 1.53

$$-\frac{2}{bc^{2\,b}ne^{\left(2\,b\log\left(x^{n}\right)+2\,a\right)}-bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(a+b*log(c*x^n))^2/x,x, algorithm="maxima")
```

```
[Out] -2/(b*c^(2*b)*n*e^(2*b*log(x^n) + 2*a) - b*n)
```

mupad [B] time = 1.44, size = 25, normalized size = 1.32

$$\frac{2}{b\,n-b\,n\,\mathrm{e}^{2\,a}\,{\left(c\,x^{n}\right)}^{2\,b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*sinh(a + b*log(c*x^n))^2),x)
```

```
[Out] 2/(b*n - b*n*exp(2*a)*(c*x^n)^(2*b))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\operatorname{csch}^{2}\left(a+b\log\left(cx^{n}\right)\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(a+b*ln(c*x**n))**2/x,x)
```

```
[Out] Integral(csch(a + b*log(c*x**n))**2/x, x)
```

3.167 $\int \frac{\operatorname{csch}^3(a+b\log(cx^n))}{x}\,dx$

Optimal. Leaf size=55

$$\frac{\tanh^{-1}\left(\cosh\left(a+b\log\left(cx^n\right)\right)\right)}{2bn}-\frac{\coth\left(a+b\log\left(cx^n\right)\right)\operatorname{csch}\left(a+b\log\left(cx^n\right)\right)}{2bn}$$

```
[Out] 1/2*arctanh(cosh(a+b*ln(c*x^n)))/b/n-1/2*coth(a+b*ln(c*x^n))*csch(a+b*ln(c*
x^n))/b/n
```

Rubi [A] time = 0.04, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {3768, 3770}

$$\frac{\tanh^{-1}\left(\cosh\left(a+b\log\left(cx^n\right)\right)\right)}{2bn}-\frac{\coth\left(a+b\log\left(cx^n\right)\right)\operatorname{csch}\left(a+b\log\left(cx^n\right)\right)}{2bn}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[a + b*Log[c*x^n]]^3/x,x]
```

```
[Out] ArcTanh[Cosh[a + b*Log[c*x^n]]]/(2*b*n) - (Coth[a + b*Log[c*x^n]]*Csch[a +
b*Log[c*x^n]])/(2*b*n)
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\operatorname{csch}^3\left(a+b\log\left(cx^n\right)\right)}{x}\,dx &= \frac{\operatorname{Subst}\left(\int \operatorname{csch}^3(a+bx)\,dx,x,\log\left(cx^n\right)\right)}{n}\\ &= -\frac{\coth\left(a+b\log\left(cx^n\right)\right)\operatorname{csch}\left(a+b\log\left(cx^n\right)\right)}{2bn}-\frac{\operatorname{Subst}\left(\int \operatorname{csch}(a+bx)\,dx,x,\log\right.}{2n}\\ &= \frac{\tanh^{-1}\left(\cosh\left(a+b\log\left(cx^n\right)\right)\right)}{2bn}-\frac{\coth\left(a+b\log\left(cx^n\right)\right)\operatorname{csch}\left(a+b\log\left(cx^n\right)\right)}{2bn}\end{aligned}$$

Mathematica [A] time = 0.06, size = 81, normalized size = 1.47

$$-\frac{\log\left(\tanh\left(\frac{1}{2}\left(a+b\log\left(cx^n\right)\right)\right)\right)}{2bn}-\frac{\operatorname{sech}^2\left(\frac{1}{2}\left(a+b\log\left(cx^n\right)\right)\right)}{8bn}-\frac{\operatorname{csch}^2\left(\frac{1}{2}\left(a+b\log\left(cx^n\right)\right)\right)}{8bn}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[a + b*Log[c*x^n]]^3/x,x]
```

```
[Out] -1/8*Csch[(a + b*Log[c*x^n])/2]^2/(b*n) - Log[Tanh[(a + b*Log[c*x^n])/2]]/(
2*b*n) - Sech[(a + b*Log[c*x^n])/2]^2/(8*b*n)
```

fricas [B] time = 1.35, size = 643, normalized size = 11.69

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(a+b*log(c*x^n))^3/x,x, algorithm="fricas")
```

```
[Out] -1/2*(2*cosh(b*n*log(x) + b*log(c) + a)^3 + 6*cosh(b*n*log(x) + b*log(c) +
a)*sinh(b*n*log(x) + b*log(c) + a)^2 + 2*sinh(b*n*log(x) + b*log(c) + a)^3
- (cosh(b*n*log(x) + b*log(c) + a)^4 + 4*cosh(b*n*log(x) + b*log(c) + a)*si
nh(b*n*log(x) + b*log(c) + a)^3 + sinh(b*n*log(x) + b*log(c) + a)^4 + 2*(3*
cosh(b*n*log(x) + b*log(c) + a)^2 - 1)*sinh(b*n*log(x) + b*log(c) + a)^2 -
2*cosh(b*n*log(x) + b*log(c) + a)^2 + 4*(cosh(b*n*log(x) + b*log(c) + a)^3
- cosh(b*n*log(x) + b*log(c) + a))*sinh(b*n*log(x) + b*log(c) + a) + 1)*log
(cosh(b*n*log(x) + b*log(c) + a) + sinh(b*n*log(x) + b*log(c) + a) + 1) + (
cosh(b*n*log(x) + b*log(c) + a)^4 + 4*cosh(b*n*log(x) + b*log(c) + a)*sinh(
b*n*log(x) + b*log(c) + a)^3 + sinh(b*n*log(x) + b*log(c) + a)^4 + 2*(3*cos
h(b*n*log(x) + b*log(c) + a)^2 - 1)*sinh(b*n*log(x) + b*log(c) + a)^2 - 2*c
osh(b*n*log(x) + b*log(c) + a)^2 + 4*(cosh(b*n*log(x) + b*log(c) + a)^3 - c
osh(b*n*log(x) + b*log(c) + a))*sinh(b*n*log(x) + b*log(c) + a) + 1)*log(co
sh(b*n*log(x) + b*log(c) + a) + sinh(b*n*log(x) + b*log(c) + a) - 1) + 2*(3
*cosh(b*n*log(x) + b*log(c) + a)^2 + 1)*sinh(b*n*log(x) + b*log(c) + a) + 2
*cosh(b*n*log(x) + b*log(c) + a))/(b*n*cosh(b*n*log(x) + b*log(c) + a)^4 +
4*b*n*cosh(b*n*log(x) + b*log(c) + a)*sinh(b*n*log(x) + b*log(c) + a)^3 + b
*n*sinh(b*n*log(x) + b*log(c) + a)^4 - 2*b*n*cosh(b*n*log(x) + b*log(c) + a
)^2 + 2*(3*b*n*cosh(b*n*log(x) + b*log(c) + a)^2 - b*n)*sinh(b*n*log(x) + b
*log(c) + a)^2 + b*n + 4*(b*n*cosh(b*n*log(x) + b*log(c) + a)^3 - b*n*cosh(
b*n*log(x) + b*log(c) + a))*sinh(b*n*log(x) + b*log(c) + a))
```

giac [B] time = 0.19, size = 210, normalized size = 3.82

$$\frac{1}{2}\,c^{3\,b}\left(\frac{c^{b}e^{(-3\,a)}\log\left(\sqrt{2\,x^{bn}|c|^{b}\cos\left(-\frac{1}{2}\,\pi b\mathrm{sgn}(c)+\frac{1}{2}\,\pi b\right)e^{a}+x^{2\,bn}|c|^{2\,b}e^{(2\,a)}+1}\right)}{bc^{4\,b}n}-\frac{c^{b}e^{(-3\,a)}\log\left(\sqrt{-2\,x^{bn}|c|^{b}\,\mathrm{co}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(a+b*log(c*x^n))^3/x,x, algorithm="giac")
```

```
[Out] 1/2*c^(3*b)*(c^b*e^(-3*a)*log(sqrt(2*x^(b*n)*abs(c)^b*cos(-1/2*pi*b*sgn(c)
+ 1/2*pi*b)*e^a + x^(2*b*n)*abs(c)^(2*b)*e^(2*a) + 1))/(b*c^(4*b)*n) - c^b*
e^(-3*a)*log(sqrt(-2*x^(b*n)*abs(c)^b*cos(-1/2*pi*b*sgn(c) + 1/2*pi*b)*e^a
+ x^(2*b*n)*abs(c)^(2*b)*e^(2*a) + 1))/(b*c^(4*b)*n) - 2*(c^(2*b)*x^(3*b*n)
*e^(2*a) + x^(b*n))*e^(-2*a)/((c^(2*b)*x^(2*b*n)*e^(2*a) - 1)^2*b*c^(2*b)*n
))*e^(3*a)
```

maple [A] time = 0.33, size = 51, normalized size = 0.93

$$-\frac{\coth\left(a+b\ln\left(c\,x^{n}\right)\right)\operatorname{csch}\left(a+b\ln\left(c\,x^{n}\right)\right)}{2bn}+\frac{\operatorname{arctanh}\left(\mathrm{e}^{a+b\ln\left(c\,x^{n}\right)}\right)}{bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(a+b*ln(c*x^n))^3/x,x)
```

```
[Out] -1/2*coth(a+b*ln(c*x^n))*csch(a+b*ln(c*x^n))/b/n+1/b/n*arctanh(exp(a+b*ln(c
*x^n)))
```

maxima [B] time = 0.36, size = 150, normalized size = 2.73

$$-\frac{c^{3b}e^{(3b\log(x^n)+3a)}+c^be^{(b\log(x^n)+a)}}{bc^{4b}ne^{(4b\log(x^n)+4a)}-2bc^{2b}ne^{(2b\log(x^n)+2a)}+bn}+\frac{\log\left(\frac{\left(c^be^{(b\log(x^n)+a)}+1\right)e^{(-a)}}{c^b}\right)}{2bn}-\frac{\log\left(\frac{\left(c^be^{(b\log(x^n)+a)}-1\right)e^{(-a)}}{c^b}\right)}{2bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(a+b*log(c*x^n))^3/x,x, algorithm="maxima")
```

```
[Out] -(c^(3*b)*e^(3*b*log(x^n) + 3*a) + c^b*e^(b*log(x^n) + a))/(b*c^(4*b)*n*e^(
4*b*log(x^n) + 4*a) - 2*b*c^(2*b)*n*e^(2*b*log(x^n) + 2*a) + b*n) + 1/2*log
((c^b*e^(b*log(x^n) + a) + 1)*e^(-a)/c^b)/(b*n) - 1/2*log((c^b*e^(b*log(x^n
) + a) - 1)*e^(-a)/c^b)/(b*n)
```

mupad [B] time = 1.49, size = 140, normalized size = 2.55

$$\frac{\mathrm{atan}\left(\frac{e^{-a}\sqrt{-b^2n^2}}{b\,n\,(c\,x^n)^b}\right)}{\sqrt{-b^2n^2}}+\frac{e^{-a}}{(c\,x^n)^b\left(b\,n-\frac{b\,n\,e^{-2a}}{(c\,x^n)^{2b}}\right)}-\frac{2\,e^{-a}}{(c\,x^n)^b\left(b\,n-\frac{2\,b\,n\,e^{-2a}}{(c\,x^n)^{2b}}+\frac{b\,n\,e^{-4a}}{(c\,x^n)^{4b}}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*sinh(a + b*log(c*x^n))^3),x)
```

```
[Out] atan((exp(-a)*(-b^2*n^2)^(1/2))/(b*n*(c*x^n)^b))/(-b^2*n^2)^(1/2) + exp(-a)
/((c*x^n)^b*(b*n - (b*n*exp(-2*a))/(c*x^n)^(2*b))) - (2*exp(-a))/((c*x^n)^b
*(b*n - (2*b*n*exp(-2*a))/(c*x^n)^(2*b) + (b*n*exp(-4*a))/(c*x^n)^(4*b)))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\operatorname{csch}^3\left(a+b\log\left(cx^n\right)\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(a+b*ln(c*x**n))**3/x,x)
```

```
[Out] Integral(csch(a + b*log(c*x**n))**3/x, x)
```

3.168 $\int \frac{\text{csch}^4(a+b\log(cx^n))}{x}\,dx$

Optimal. Leaf size=42

$$\frac{\coth\left(a+b\log\left(cx^n\right)\right)}{bn}-\frac{\coth^3\left(a+b\log\left(cx^n\right)\right)}{3bn}$$

```
[Out] coth(a+b*ln(c*x^n))/b/n-1/3*coth(a+b*ln(c*x^n))^3/b/n
```

Rubi [A] time = 0.03, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {3767}

$$\frac{\coth\left(a+b\log\left(cx^n\right)\right)}{bn}-\frac{\coth^3\left(a+b\log\left(cx^n\right)\right)}{3bn}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[a + b*Log[c*x^n]]^4/x,x]
```

```
[Out] Coth[a + b*Log[c*x^n]]/(b*n) - Coth[a + b*Log[c*x^n]]^3/(3*b*n)
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\text{csch}^4\left(a+b\log\left(cx^n\right)\right)}{x}\,dx &= \frac{\text{Subst}\left(\int \text{csch}^4(a+bx)\,dx, x, \log\left(cx^n\right)\right)}{n}\\ &= \frac{i\,\text{Subst}\left(\int \left(1+x^2\right)\,dx, x, -i\coth\left(a+b\log\left(cx^n\right)\right)\right)}{bn}\\ &= \frac{\coth\left(a+b\log\left(cx^n\right)\right)}{bn}-\frac{\coth^3\left(a+b\log\left(cx^n\right)\right)}{3bn}\end{aligned}$$

Mathematica [A] time = 0.07, size = 56, normalized size = 1.33

$$\frac{2\coth\left(a+b\log\left(cx^n\right)\right)}{3bn}-\frac{\coth\left(a+b\log\left(cx^n\right)\right)\text{csch}^2\left(a+b\log\left(cx^n\right)\right)}{3bn}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[a + b*Log[c*x^n]]^4/x,x]
```

```
[Out] (2*Coth[a + b*Log[c*x^n]])/(3*b*n) - (Coth[a + b*Log[c*x^n]]*Csch[a + b*Log
[c*x^n]]^2)/(3*b*n)
```

fricas [B] time = 0.44, size = 272, normalized size = 6.48

$$-\frac{}{3\left(bn\cosh\left(bn\log(x)+b\log(c)+a\right)^5+5\,bn\cosh\left(bn\log(x)+b\log(c)+a\right)\sinh\left(bn\log(x)+b\log(c)+a\right)^4\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(a+b*log(c*x^n))^4/x,x, algorithm="fricas")
```

```
[Out] -8/3*(cosh(b*n*log(x) + b*log(c) + a) + 2*sinh(b*n*log(x) + b*log(c) + a))/
(b*n*cosh(b*n*log(x) + b*log(c) + a)^5 + 5*b*n*cosh(b*n*log(x) + b*log(c) +
 a)*sinh(b*n*log(x) + b*log(c) + a)^4 + b*n*sinh(b*n*log(x) + b*log(c) + a)
^5 - 3*b*n*cosh(b*n*log(x) + b*log(c) + a)^3 + (10*b*n*cosh(b*n*log(x) + b*
log(c) + a)^2 - 3*b*n)*sinh(b*n*log(x) + b*log(c) + a)^3 + 2*b*n*cosh(b*n*l
og(x) + b*log(c) + a) + (10*b*n*cosh(b*n*log(x) + b*log(c) + a)^3 - 9*b*n*c
osh(b*n*log(x) + b*log(c) + a))*sinh(b*n*log(x) + b*log(c) + a)^2 + (5*b*n*
cosh(b*n*log(x) + b*log(c) + a)^4 - 9*b*n*cosh(b*n*log(x) + b*log(c) + a)^2
 + 4*b*n)*sinh(b*n*log(x) + b*log(c) + a))
```

giac [**A**] time = 0.16, size = 47, normalized size = 1.12

$$-\frac{4\left(3\,c^{2\,b}x^{2\,bn}e^{(2\,a)}-1\right)}{3\left(c^{2\,b}x^{2\,bn}e^{(2\,a)}-1\right)^{3}bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(a+b*log(c*x^n))^4/x,x, algorithm="giac")
```

```
[Out] -4/3*(3*c^(2*b)*x^(2*b*n)*e^(2*a) - 1)/((c^(2*b)*x^(2*b*n)*e^(2*a) - 1)^3*b
*n)
```

maple [**A**] time = 0.30, size = 36, normalized size = 0.86

$$\frac{\left(\frac{2}{3}-\frac{\mathrm{csch}(a+b\ln(c\,x^{n}))^{2}}{3}\right)\coth\left(a+b\ln\left(c\,x^{n}\right)\right)}{nb}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(a+b*ln(c*x^n))^4/x,x)
```

```
[Out] 1/n/b*(2/3-1/3*csch(a+b*ln(c*x^n))^2)*coth(a+b*ln(c*x^n))
```

maxima [**B**] time = 0.34, size = 92, normalized size = 2.19

$$-\frac{4\left(3\,c^{2\,b}e^{\left(2\,b\log(x^{n})+2\,a\right)}-1\right)}{3\left(bc^{6\,b}ne^{\left(6\,b\log(x^{n})+6\,a\right)}-3\,bc^{4\,b}ne^{\left(4\,b\log(x^{n})+4\,a\right)}+3\,bc^{2\,b}ne^{\left(2\,b\log(x^{n})+2\,a\right)}-bn\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(a+b*log(c*x^n))^4/x,x, algorithm="maxima")
```

```
[Out] -4/3*(3*c^(2*b)*e^(2*b*log(x^n) + 2*a) - 1)/(b*c^(6*b)*n*e^(6*b*log(x^n) +
6*a) - 3*b*c^(4*b)*n*e^(4*b*log(x^n) + 4*a) + 3*b*c^(2*b)*n*e^(2*b*log(x^n)
 + 2*a) - b*n)
```

mupad [**B**] time = 1.45, size = 55, normalized size = 1.31

$$\frac{4\,\mathrm{e}^{4\,a}\,{\left(c\,x^{n}\right)}^{4\,b}\,\left(\mathrm{e}^{2\,a}\,{\left(c\,x^{n}\right)}^{2\,b}-3\right)}{3\,b\,n\,{\left(\mathrm{e}^{2\,a}\,{\left(c\,x^{n}\right)}^{2\,b}-1\right)}^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*sinh(a + b*log(c*x^n))^4),x)
```

```
[Out] (4*exp(4*a)*(c*x^n)^(4*b)*(exp(2*a)*(c*x^n)^(2*b) - 3))/(3*b*n*(exp(2*a)*(c
*x^n)^(2*b) - 1)^3)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}^4\left(a+b\log\left(cx^n\right)\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(a+b*ln(c*x**n))**4/x,x)
```

```
[Out] Integral(csch(a + b*log(c*x**n))**4/x, x)
```

3.169 $\int \frac{\operatorname{csch}^5(a+b\log(cx^n))}{x}\,dx$

Optimal. Leaf size=89

$$-\frac{3\tanh^{-1}\left(\cosh\left(a+b\log\left(cx^n\right)\right)\right)}{8bn}-\frac{\coth\left(a+b\log\left(cx^n\right)\right)\operatorname{csch}^3\left(a+b\log\left(cx^n\right)\right)}{4bn}+\frac{3\coth\left(a+b\log\left(cx^n\right)\right)\operatorname{csch}\left(\right.}{8bn}$$

```
[Out] -3/8*arctanh(cosh(a+b*ln(c*x^n)))/b/n+3/8*coth(a+b*ln(c*x^n))*csch(a+b*ln(c
*x^n))/b/n-1/4*coth(a+b*ln(c*x^n))*csch(a+b*ln(c*x^n))^3/b/n
```

Rubi [A] time = 0.07, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {3768, 3770}

$$-\frac{3\tanh^{-1}\left(\cosh\left(a+b\log\left(cx^n\right)\right)\right)}{8bn}-\frac{\coth\left(a+b\log\left(cx^n\right)\right)\operatorname{csch}^3\left(a+b\log\left(cx^n\right)\right)}{4bn}+\frac{3\coth\left(a+b\log\left(cx^n\right)\right)\operatorname{csch}\left(\right.}{8bn}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[a + b*Log[c*x^n]]^5/x,x]
```

```
[Out] (-3*ArcTanh[Cosh[a + b*Log[c*x^n]]])/(8*b*n) + (3*Coth[a + b*Log[c*x^n]]*Cs
ch[a + b*Log[c*x^n]])/(8*b*n) - (Coth[a + b*Log[c*x^n]]*Csch[a + b*Log[c*x^
n]]^3)/(4*b*n)
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\operatorname{csch}^5\left(a+b\log\left(cx^n\right)\right)}{x}\,dx &= \frac{\operatorname{Subst}\left(\int \operatorname{csch}^5(a+bx)\,dx,x,\log\left(cx^n\right)\right)}{n}\\
&= -\frac{\coth\left(a+b\log\left(cx^n\right)\right)\operatorname{csch}^3\left(a+b\log\left(cx^n\right)\right)}{4bn}-\frac{3\operatorname{Subst}\left(\int \operatorname{csch}^3(a+bx)\,dx,x,\right.}{4n}\\
&= \frac{3\coth\left(a+b\log\left(cx^n\right)\right)\operatorname{csch}\left(a+b\log\left(cx^n\right)\right)}{8bn}-\frac{\coth\left(a+b\log\left(cx^n\right)\right)\operatorname{csch}^3\left(a+\right.}{4bn}\\
&= -\frac{3\tanh^{-1}\left(\cosh\left(a+b\log\left(cx^n\right)\right)\right)}{8bn}+\frac{3\coth\left(a+b\log\left(cx^n\right)\right)\operatorname{csch}\left(a+b\log\left(cx\right.\right.}{8bn}
\end{aligned}$$

Mathematica [A] time = 0.06, size = 135, normalized size = 1.52

$$\frac{3\log\left(\tanh\left(\frac{1}{2}\left(a+b\log\left(cx^n\right)\right)\right)\right)}{8bn}+\frac{\operatorname{sech}^4\left(\frac{1}{2}\left(a+b\log\left(cx^n\right)\right)\right)}{64bn}+\frac{3\operatorname{sech}^2\left(\frac{1}{2}\left(a+b\log\left(cx^n\right)\right)\right)}{32bn}-\frac{\operatorname{csch}^4\left(\frac{1}{2}\left(a+b\log\right.\right.}{64bn}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[a + b*Log[c*x^n]]^5/x,x]
```

```
[Out] (3*Csch[(a + b*Log[c*x^n])/2]^2)/(32*b*n) - Csch[(a + b*Log[c*x^n])/2]^4/(6
4*b*n) + (3*Log[Tanh[(a + b*Log[c*x^n])/2]])/(8*b*n) + (3*Sech[(a + b*Log[c
*x^n])/2]^2)/(32*b*n) + Sech[(a + b*Log[c*x^n])/2]^4/(64*b*n)
```

fricas [B] time = 0.51, size = 1806, normalized size = 20.29

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(a+b*log(c*x^n))^5/x,x, algorithm="fricas")
```

```
[Out] 1/8*(6*cosh(b*n*log(x) + b*log(c) + a)^7 + 42*cosh(b*n*log(x) + b*log(c) +
a)*sinh(b*n*log(x) + b*log(c) + a)^6 + 6*sinh(b*n*log(x) + b*log(c) + a)^7
+ 2*(63*cosh(b*n*log(x) + b*log(c) + a)^2 - 11)*sinh(b*n*log(x) + b*log(c)
+ a)^5 - 22*cosh(b*n*log(x) + b*log(c) + a)^5 + 10*(21*cosh(b*n*log(x) + b*
log(c) + a)^3 - 11*cosh(b*n*log(x) + b*log(c) + a))*sinh(b*n*log(x) + b*log
(c) + a)^4 + 2*(105*cosh(b*n*log(x) + b*log(c) + a)^4 - 110*cosh(b*n*log(x)
 + b*log(c) + a)^2 - 11)*sinh(b*n*log(x) + b*log(c) + a)^3 - 22*cosh(b*n*lo
g(x) + b*log(c) + a)^3 + 2*(63*cosh(b*n*log(x) + b*log(c) + a)^5 - 110*cosh
(b*n*log(x) + b*log(c) + a)^3 - 33*cosh(b*n*log(x) + b*log(c) + a))*sinh(b*
n*log(x) + b*log(c) + a)^2 - 3*(cosh(b*n*log(x) + b*log(c) + a)^8 + 8*cosh(
b*n*log(x) + b*log(c) + a)*sinh(b*n*log(x) + b*log(c) + a)^7 + sinh(b*n*log
(x) + b*log(c) + a)^8 + 4*(7*cosh(b*n*log(x) + b*log(c) + a)^2 - 1)*sinh(b*
n*log(x) + b*log(c) + a)^6 - 4*cosh(b*n*log(x) + b*log(c) + a)^6 + 8*(7*cos
h(b*n*log(x) + b*log(c) + a)^3 - 3*cosh(b*n*log(x) + b*log(c) + a))*sinh(b*
n*log(x) + b*log(c) + a)^5 + 2*(35*cosh(b*n*log(x) + b*log(c) + a)^4 - 30*c
osh(b*n*log(x) + b*log(c) + a)^2 + 3)*sinh(b*n*log(x) + b*log(c) + a)^4 + 6
*cosh(b*n*log(x) + b*log(c) + a)^4 + 8*(7*cosh(b*n*log(x) + b*log(c) + a)^5
 - 10*cosh(b*n*log(x) + b*log(c) + a)^3 + 3*cosh(b*n*log(x) + b*log(c) + a)
)*sinh(b*n*log(x) + b*log(c) + a)^3 + 4*(7*cosh(b*n*log(x) + b*log(c) + a)^
6 - 15*cosh(b*n*log(x) + b*log(c) + a)^4 + 9*cosh(b*n*log(x) + b*log(c) + a
)^2 - 1)*sinh(b*n*log(x) + b*log(c) + a)^2 - 4*cosh(b*n*log(x) + b*log(c) +
 a)^2 + 8*(cosh(b*n*log(x) + b*log(c) + a)^7 - 3*cosh(b*n*log(x) + b*log(c)
 + a)^5 + 3*cosh(b*n*log(x) + b*log(c) + a)^3 - cosh(b*n*log(x) + b*log(c)
+ a))*sinh(b*n*log(x) + b*log(c) + a) + 1)*log(cosh(b*n*log(x) + b*log(c) +
 a) + sinh(b*n*log(x) + b*log(c) + a) + 1) + 3*(cosh(b*n*log(x) + b*log(c)
+ a)^8 + 8*cosh(b*n*log(x) + b*log(c) + a)*sinh(b*n*log(x) + b*log(c) + a)^
7 + sinh(b*n*log(x) + b*log(c) + a)^8 + 4*(7*cosh(b*n*log(x) + b*log(c) + a
)^2 - 1)*sinh(b*n*log(x) + b*log(c) + a)^6 - 4*cosh(b*n*log(x) + b*log(c) +
 a)^6 + 8*(7*cosh(b*n*log(x) + b*log(c) + a)^3 - 3*cosh(b*n*log(x) + b*log(
c) + a))*sinh(b*n*log(x) + b*log(c) + a)^5 + 2*(35*cosh(b*n*log(x) + b*log(
c) + a)^4 - 30*cosh(b*n*log(x) + b*log(c) + a)^2 + 3)*sinh(b*n*log(x) + b*l
og(c) + a)^4 + 6*cosh(b*n*log(x) + b*log(c) + a)^4 + 8*(7*cosh(b*n*log(x) +
 b*log(c) + a)^5 - 10*cosh(b*n*log(x) + b*log(c) + a)^3 + 3*cosh(b*n*log(x)
 + b*log(c) + a))*sinh(b*n*log(x) + b*log(c) + a)^3 + 4*(7*cosh(b*n*log(x)
+ b*log(c) + a)^6 - 15*cosh(b*n*log(x) + b*log(c) + a)^4 + 9*cosh(b*n*log(x
) + b*log(c) + a)^2 - 1)*sinh(b*n*log(x) + b*log(c) + a)^2 - 4*cosh(b*n*log
(x) + b*log(c) + a)^2 + 8*(cosh(b*n*log(x) + b*log(c) + a)^7 - 3*cosh(b*n*l
og(x) + b*log(c) + a)^5 + 3*cosh(b*n*log(x) + b*log(c) + a)^3 - cosh(b*n*lo
g(x) + b*log(c) + a))*sinh(b*n*log(x) + b*log(c) + a) + 1)*log(cosh(b*n*log
(x) + b*log(c) + a) + sinh(b*n*log(x) + b*log(c) + a) - 1) + 2*(21*cosh(b*n
*log(x) + b*log(c) + a)^6 - 55*cosh(b*n*log(x) + b*log(c) + a)^4 - 33*cosh(
b*n*log(x) + b*log(c) + a)^2 + 3)*sinh(b*n*log(x) + b*log(c) + a) + 6*cosh(
b*n*log(x) + b*log(c) + a))/(b*n*cosh(b*n*log(x) + b*log(c) + a)^8 + 8*b*n*
cosh(b*n*log(x) + b*log(c) + a)*sinh(b*n*log(x) + b*log(c) + a)^7 + b*n*sin
h(b*n*log(x) + b*log(c) + a)^8 - 4*b*n*cosh(b*n*log(x) + b*log(c) + a)^6 +
4*(7*b*n*cosh(b*n*log(x) + b*log(c) + a)^2 - b*n)*sinh(b*n*log(x) + b*log(c
) + a)^6 + 6*b*n*cosh(b*n*log(x) + b*log(c) + a)^4 + 8*(7*b*n*cosh(b*n*log(
```

```
x) + b*log(c) + a)^3 - 3*b*n*cosh(b*n*log(x) + b*log(c) + a))*sinh(b*n*log(
x) + b*log(c) + a)^5 + 2*(35*b*n*cosh(b*n*log(x) + b*log(c) + a)^4 - 30*b*n
*cosh(b*n*log(x) + b*log(c) + a)^2 + 3*b*n)*sinh(b*n*log(x) + b*log(c) + a)
^4 - 4*b*n*cosh(b*n*log(x) + b*log(c) + a)^2 + 8*(7*b*n*cosh(b*n*log(x) + b
*log(c) + a)^5 - 10*b*n*cosh(b*n*log(x) + b*log(c) + a)^3 + 3*b*n*cosh(b*n*
log(x) + b*log(c) + a))*sinh(b*n*log(x) + b*log(c) + a)^3 + 4*(7*b*n*cosh(b
*n*log(x) + b*log(c) + a)^6 - 15*b*n*cosh(b*n*log(x) + b*log(c) + a)^4 + 9*
b*n*cosh(b*n*log(x) + b*log(c) + a)^2 - b*n)*sinh(b*n*log(x) + b*log(c) + a
)^2 + b*n + 8*(b*n*cosh(b*n*log(x) + b*log(c) + a)^7 - 3*b*n*cosh(b*n*log(x
) + b*log(c) + a)^5 + 3*b*n*cosh(b*n*log(x) + b*log(c) + a)^3 - b*n*cosh(b*
n*log(x) + b*log(c) + a))*sinh(b*n*log(x) + b*log(c) + a))
```

giac [B] time = 0.22, size = 248, normalized size = 2.79

$$-\frac{1}{8}c^{5b}\left(\frac{3c^{b}e^{(-5a)}\log\left(\sqrt{2x^{bn}|c|^{b}\cos\left(-\frac{1}{2}\pi b\mathrm{sgn}(c)+\frac{1}{2}\pi b\right)e^{a}+x^{2bn}|c|^{2b}e^{(2a)}+1}\right)}{bc^{6b}n}-\frac{3c^{b}e^{(-5a)}\log\left(\sqrt{-2x^{bn}|c|^{b}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(a+b*log(c*x^n))^5/x,x, algorithm="giac")
```

```
[Out] -1/8*c^(5*b)*(3*c^b*e^(-5*a)*log(sqrt(2*x^(b*n)*abs(c)^b*cos(-1/2*pi*b*sgn(
c) + 1/2*pi*b)*e^a + x^(2*b*n)*abs(c)^(2*b)*e^(2*a) + 1))/(b*c^(6*b)*n) - 3
*c^b*e^(-5*a)*log(sqrt(-2*x^(b*n)*abs(c)^b*cos(-1/2*pi*b*sgn(c) + 1/2*pi*b)
*e^a + x^(2*b*n)*abs(c)^(2*b)*e^(2*a) + 1))/(b*c^(6*b)*n) - 2*(3*c^(6*b)*x^
(7*b*n)*e^(6*a) - 11*c^(4*b)*x^(5*b*n)*e^(4*a) - 11*c^(2*b)*x^(3*b*n)*e^(2*
a) + 3*x^(b*n))*e^(-4*a)/((c^(2*b)*x^(2*b*n)*e^(2*a) - 1)^4*b*c^(4*b)*n))*e
^(5*a)
```

maple [A] time = 0.31, size = 84, normalized size = 0.94

$$-\frac{\coth(a+b\ln(cx^{n}))\,\mathrm{csch}(a+b\ln(cx^{n}))^{3}}{4bn}+\frac{3\coth(a+b\ln(cx^{n}))\,\mathrm{csch}(a+b\ln(cx^{n}))}{8bn}-\frac{3\,\mathrm{arctanh}\left(e^{a+b\ln(cx^{n})}\right)}{4bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(a+b*ln(c*x^n))^5/x,x)
```

```
[Out] -1/4*coth(a+b*ln(c*x^n))*csch(a+b*ln(c*x^n))^3/b/n+3/8*coth(a+b*ln(c*x^n))*
csch(a+b*ln(c*x^n))/b/n-3/4/b/n*arctanh(exp(a+b*ln(c*x^n)))
```

maxima [B] time = 0.35, size = 232, normalized size = 2.61

$$\frac{3c^{7b}e^{\left(7b\log(x^{n})+7a\right)}-11c^{5b}e^{\left(5b\log(x^{n})+5a\right)}-11c^{3b}e^{\left(3b\log(x^{n})+3a\right)}+3c^{b}e^{\left(b\log(x^{n})+a\right)}}{4\left(bc^{8b}ne^{\left(8b\log(x^{n})+8a\right)}-4bc^{6b}ne^{\left(6b\log(x^{n})+6a\right)}+6bc^{4b}ne^{\left(4b\log(x^{n})+4a\right)}-4bc^{2b}ne^{\left(2b\log(x^{n})+2a\right)}+bn\right)}-\frac{3\log\left(\frac{\left(c^{b}e^{\cdots}\right.}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(a+b*log(c*x^n))^5/x,x, algorithm="maxima")
```

```
[Out] 1/4*(3*c^(7*b)*e^(7*b*log(x^n) + 7*a) - 11*c^(5*b)*e^(5*b*log(x^n) + 5*a) -
 11*c^(3*b)*e^(3*b*log(x^n) + 3*a) + 3*c^b*e^(b*log(x^n) + a))/(b*c^(8*b)*n
*e^(8*b*log(x^n) + 8*a) - 4*b*c^(6*b)*n*e^(6*b*log(x^n) + 6*a) + 6*b*c^(4*b
)*n*e^(4*b*log(x^n) + 4*a) - 4*b*c^(2*b)*n*e^(2*b*log(x^n) + 2*a) + b*n) -
3/8*log((c^b*e^(b*log(x^n) + a) + 1)*e^(-a)/c^b)/(b*n) + 3/8*log((c^b*e^(b*
log(x^n) + a) - 1)*e^(-a)/c^b)/(b*n)
```

mupad [B] time = 1.48, size = 318, normalized size = 3.57

$$\frac{2\,\mathrm{e}^{-a}}{{\left(c\,x^n\right)}^b\,\left(b\,n-\frac{3\,b\,n\,\mathrm{e}^{-2\,a}}{{\left(c\,x^n\right)}^{2\,b}}+\frac{3\,b\,n\,\mathrm{e}^{-4\,a}}{{\left(c\,x^n\right)}^{4\,b}}-\frac{b\,n\,\mathrm{e}^{-6\,a}}{{\left(c\,x^n\right)}^{6\,b}}\right)}-\frac{3\,\mathrm{atan}\left(\frac{\mathrm{e}^{-a}\,\sqrt{-b^2\,n^2}}{b\,n\,{\left(c\,x^n\right)}^b}\right)}{4\,\sqrt{-b^2\,n^2}}-\frac{3\,\mathrm{e}^{-a}}{4\,{\left(c\,x^n\right)}^b\,\left(b\,n-\frac{b\,n\,\mathrm{e}^{-2\,a}}{{\left(c\,x^n\right)}^{2\,b}}\right)}-\frac{}{{\left(c\,x^n\right)}^{3\,b}\,\left(b\,n-\frac{4\,b\,n}{(c\,x}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*sinh(a + b*log(c*x^n))^5),x)
```

```
[Out] (2*exp(-a))/((c*x^n)^b*(b*n - (3*b*n*exp(-2*a))/(c*x^n)^(2*b) + (3*b*n*exp(
-4*a))/(c*x^n)^(4*b) - (b*n*exp(-6*a))/(c*x^n)^(6*b))) - (3*atan((exp(-a)*(
-b^2*n^2)^(1/2))/(b*n*(c*x^n)^b)))/(4*(-b^2*n^2)^(1/2)) - (3*exp(-a))/(4*(c
*x^n)^b*(b*n - (b*n*exp(-2*a))/(c*x^n)^(2*b))) - (4*exp(-3*a))/((c*x^n)^(3*
b)*(b*n - (4*b*n*exp(-2*a))/(c*x^n)^(2*b) + (6*b*n*exp(-4*a))/(c*x^n)^(4*b)
 - (4*b*n*exp(-6*a))/(c*x^n)^(6*b) + (b*n*exp(-8*a))/(c*x^n)^(8*b))) - exp(
-a)/(2*(c*x^n)^b*(b*n - (2*b*n*exp(-2*a))/(c*x^n)^(2*b) + (b*n*exp(-4*a))/(
c*x^n)^(4*b)))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}^5\left(a+b\log\left(cx^n\right)\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(a+b*ln(c*x**n))**5/x,x)
```

```
[Out] Integral(csch(a + b*log(c*x**n))**5/x, x)
```

3.170 $\int \frac{\operatorname{csch}^{\frac{5}{2}}(a+b\log(cx^n))}{x}\,dx$

Optimal. Leaf size=111

$$-\frac{2\cosh\left(a+b\log\left(cx^n\right)\right)\operatorname{csch}^{\frac{3}{2}}\left(a+b\log\left(cx^n\right)\right)}{3bn}+\frac{2i\sqrt{i\sinh\left(a+b\log\left(cx^n\right)\right)}\sqrt{\operatorname{csch}\left(a+b\log\left(cx^n\right)\right)}F\left(\frac{1}{2}\left(ia+\right.\right.}{3bn}$$

```
[Out] -2/3*cosh(a+b*ln(c*x^n))*csch(a+b*ln(c*x^n))^(3/2)/b/n-2/3*I*(sin(1/2*I*a+1
/4*Pi+1/2*I*b*ln(c*x^n))^2)^(1/2)/sin(1/2*I*a+1/4*Pi+1/2*I*b*ln(c*x^n))*Ell
ipticF(cos(1/2*I*a+1/4*Pi+1/2*I*b*ln(c*x^n)),2^(1/2))*csch(a+b*ln(c*x^n))^(
1/2)*(I*sinh(a+b*ln(c*x^n)))^(1/2)/b/n
```

Rubi [A] time = 0.07, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {3768, 3771, 2641}

$$-\frac{2\cosh\left(a+b\log\left(cx^n\right)\right)\operatorname{csch}^{\frac{3}{2}}\left(a+b\log\left(cx^n\right)\right)}{3bn}+\frac{2i\sqrt{i\sinh\left(a+b\log\left(cx^n\right)\right)}\sqrt{\operatorname{csch}\left(a+b\log\left(cx^n\right)\right)}F\left(\frac{1}{2}\left(ia+\right.\right.}{3bn}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[a + b*Log[c*x^n]]^(5/2)/x,x]
```

```
[Out] (-2*Cosh[a + b*Log[c*x^n]]*Csch[a + b*Log[c*x^n]]^(3/2))/(3*b*n) + (((2*I)/
3)*Sqrt[Csch[a + b*Log[c*x^n]]]*EllipticF[(I*a - Pi/2 + I*b*Log[c*x^n])/2,
2]*Sqrt[I*Sinh[a + b*Log[c*x^n]]])/(b*n)
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\int \frac{\operatorname{csch}^{\frac{5}{2}}\left(a+b\log\left(cx^n\right)\right)}{x}\,dx = \frac{\operatorname{Subst}\left(\int \operatorname{csch}^{\frac{5}{2}}(a+bx)\,dx, x, \log\left(cx^n\right)\right)}{n}$$
$$= -\frac{2\cosh\left(a+b\log\left(cx^n\right)\right)\operatorname{csch}^{\frac{3}{2}}\left(a+b\log\left(cx^n\right)\right)}{3bn} - \frac{\operatorname{Subst}\left(\int \sqrt{\operatorname{csch}(a+bx)}\right.}{3n}$$
$$= -\frac{2\cosh\left(a+b\log\left(cx^n\right)\right)\operatorname{csch}^{\frac{3}{2}}\left(a+b\log\left(cx^n\right)\right)}{3bn} - \frac{\left(\sqrt{\operatorname{csch}\left(a+b\log\left(cx^n\right)\right)}\right.}{}$$
$$= -\frac{2\cosh\left(a+b\log\left(cx^n\right)\right)\operatorname{csch}^{\frac{3}{2}}\left(a+b\log\left(cx^n\right)\right)}{3bn} + \frac{2i\sqrt{\operatorname{csch}\left(a+b\log\left(cx^n\right)\right)}}{}$$

Mathematica [A] time = 0.22, size = 84, normalized size = 0.76

$$-\frac{2\sqrt{\operatorname{csch}\left(a+b\log\left(cx^n\right)\right)}\left(\coth\left(a+b\log\left(cx^n\right)\right)+i\sqrt{i\sinh\left(a+b\log\left(cx^n\right)\right)}F\left(\frac{1}{4}\left(-2ia-2ib\log\left(cx^n\right)+\pi\right)\right.\right.}{3bn}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[a + b*Log[c*x^n]]^(5/2)/x,x]
```

```
[Out] (-2*Sqrt[Csch[a + b*Log[c*x^n]]]*(Coth[a + b*Log[c*x^n]] + I*EllipticF[((-2
*I)*a + Pi - (2*I)*b*Log[c*x^n])/4, 2]*Sqrt[I*Sinh[a + b*Log[c*x^n]]]))/(3*
b*n)
```

fricas [F] time = 0.41, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{\operatorname{csch}\left(b\log\left(cx^n\right)+a\right)^{\frac{5}{2}}}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(a+b*log(c*x^n))^(5/2)/x,x, algorithm="fricas")
```

```
[Out] integral(csch(b*log(c*x^n) + a)^(5/2)/x, x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(a+b*log(c*x^n))^(5/2)/x,x, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.48, size = 144, normalized size = 1.30

$$-\frac{i\sqrt{1-i\sinh\left(a+b\ln\left(c\,x^n\right)\right)}\,\sqrt{2}\,\sqrt{i\sinh\left(a+b\ln\left(c\,x^n\right)\right)+1}\,\sqrt{i\sinh\left(a+b\ln\left(c\,x^n\right)\right)}\,\operatorname{EllipticF}\left(\sqrt{1-i\sinh\right.\right.}{3n\sinh\left(a+b\ln\left(c\,x^n\right)\right)^{\frac{3}{2}}\cosh\left(a+b\ln\left(c\,x\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(a+b*ln(c*x^n))^(5/2)/x,x)
```

```
[Out] -1/3/n/sinh(a+b*ln(c*x^n))^(3/2)*(I*(1-I*sinh(a+b*ln(c*x^n)))^(1/2)*2^(1/2)
*(I*sinh(a+b*ln(c*x^n))+1)^(1/2)*(I*sinh(a+b*ln(c*x^n)))^(1/2)*EllipticF((1
-I*sinh(a+b*ln(c*x^n)))^(1/2),1/2*2^(1/2))*sinh(a+b*ln(c*x^n))+2*cosh(a+b*l
n(c*x^n))^2)/cosh(a+b*ln(c*x^n))/b
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}\left(b\log\left(cx^n\right)+a\right)^{\frac{5}{2}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(a+b*log(c*x^n))^(5/2)/x,x, algorithm="maxima")
```

```
[Out] integrate(csch(b*log(c*x^n) + a)^(5/2)/x, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(\frac{1}{\sinh(a+b\,\ln(c\,x^n))}\right)^{5/2}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/sinh(a + b*log(c*x^n)))^(5/2)/x,x)
```

```
[Out] int((1/sinh(a + b*log(c*x^n)))^(5/2)/x, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}^{\frac{5}{2}}\left(a+b\log\left(cx^n\right)\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(a+b*ln(c*x**n))**(5/2)/x,x)
```

```
[Out] Integral(csch(a + b*log(c*x**n))**(5/2)/x, x)
```

3.171 $\int \frac{\operatorname{csch}^{\frac{3}{2}}(a+b\log(cx^n))}{x}\,dx$

Optimal. Leaf size=107

$$-\frac{2\cosh\left(a+b\log\left(cx^n\right)\right)\sqrt{\operatorname{csch}\left(a+b\log\left(cx^n\right)\right)}}{bn}-\frac{2iE\left(\frac{1}{2}\left(ia+ib\log\left(cx^n\right)-\frac{\pi}{2}\right)\middle|2\right)}{bn\sqrt{i\sinh\left(a+b\log\left(cx^n\right)\right)}\sqrt{\operatorname{csch}\left(a+b\log\left(cx^n\right)\right)}}$$

```
[Out] -2*cosh(a+b*ln(c*x^n))*csch(a+b*ln(c*x^n))^(1/2)/b/n+2*I*(sin(1/2*I*a+1/4*P
i+1/2*I*b*ln(c*x^n))^2)^(1/2)/sin(1/2*I*a+1/4*Pi+1/2*I*b*ln(c*x^n))*Ellipti
cE(cos(1/2*I*a+1/4*Pi+1/2*I*b*ln(c*x^n)),2^(1/2))/b/n/csch(a+b*ln(c*x^n))^(
1/2)/(I*sinh(a+b*ln(c*x^n)))^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {3768, 3771, 2639}

$$-\frac{2\cosh\left(a+b\log\left(cx^n\right)\right)\sqrt{\operatorname{csch}\left(a+b\log\left(cx^n\right)\right)}}{bn}-\frac{2iE\left(\frac{1}{2}\left(ia+ib\log\left(cx^n\right)-\frac{\pi}{2}\right)\middle|2\right)}{bn\sqrt{i\sinh\left(a+b\log\left(cx^n\right)\right)}\sqrt{\operatorname{csch}\left(a+b\log\left(cx^n\right)\right)}}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[a + b*Log[c*x^n]]^(3/2)/x,x]
```

```
[Out] (-2*Cosh[a + b*Log[c*x^n]]*Sqrt[Csch[a + b*Log[c*x^n]]])/(b*n) - ((2*I)*Ell
ipticE[(I*a - Pi/2 + I*b*Log[c*x^n])/2, 2])/(b*n*Sqrt[Csch[a + b*Log[c*x^n]
]]*Sqrt[I*Sinh[a + b*Log[c*x^n]]])
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\int \frac{\operatorname{csch}^{\frac{3}{2}}(a+b\log(cx^n))}{x}\,dx = \frac{\operatorname{Subst}\left(\int \operatorname{csch}^{\frac{3}{2}}(a+bx)\,dx, x, \log(cx^n)\right)}{n}$$
$$= -\frac{2\cosh(a+b\log(cx^n))\sqrt{\operatorname{csch}(a+b\log(cx^n))}}{bn} + \frac{\operatorname{Subst}\left(\int \frac{1}{\sqrt{\operatorname{csch}(a+bx)}}\,dx, x, 1\right.}{n}$$
$$= -\frac{2\cosh(a+b\log(cx^n))\sqrt{\operatorname{csch}(a+b\log(cx^n))}}{bn} + \frac{\operatorname{Subst}\left(\int \sqrt{i\sinh(a+b\right.}{n\sqrt{\operatorname{csch}(a+b\log(cx^n))}\sqrt{}}$$
$$= -\frac{2\cosh(a+b\log(cx^n))\sqrt{\operatorname{csch}(a+b\log(cx^n))}}{bn} - \frac{2iE\left(\frac{1}{2}\left(ia-\frac{\pi}{2}+i\right.\right.}{bn\sqrt{\operatorname{csch}(a+b\log(cx^n))}}$$

Mathematica [A] time = 0.12, size = 80, normalized size = 0.75

$$-\frac{2\sqrt{\operatorname{csch}(a+b\log(cx^n))}\left(\cosh(a+b\log(cx^n)) - \sqrt{i\sinh(a+b\log(cx^n))}\,E\left(\frac{1}{4}(-2ia-2ib\log(cx^n)+\pi)\middle|2\right)\right.}{bn}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[a + b*Log[c*x^n]]^(3/2)/x,x]
```

```
[Out] (-2*Sqrt[Csch[a + b*Log[c*x^n]]]*(Cosh[a + b*Log[c*x^n]] - EllipticE[((-2*I
)*a + Pi - (2*I)*b*Log[c*x^n])/4, 2]*Sqrt[I*Sinh[a + b*Log[c*x^n]]]))/(b*n)
```

fricas [F] time = 0.48, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{\operatorname{csch}(b\log(cx^n)+a)^{\frac{3}{2}}}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(a+b*log(c*x^n))^(3/2)/x,x, algorithm="fricas")
```

```
[Out] integral(csch(b*log(c*x^n) + a)^(3/2)/x, x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(a+b*log(c*x^n))^(3/2)/x,x, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.55, size = 212, normalized size = 1.98

$$\frac{2\sqrt{1-i\sinh(a+b\ln(c\,x^n))}\,\sqrt{2}\,\sqrt{i\sinh(a+b\ln(c\,x^n))+1}\,\sqrt{i\sinh(a+b\ln(c\,x^n))}\,\operatorname{EllipticE}\left(\sqrt{1-i\sinh(a+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(a+b*ln(c*x^n))^(3/2)/x,x)
```

```
[Out] 1/n*(2*(1-I*sinh(a+b*ln(c*x^n)))^(1/2)*2^(1/2)*(I*sinh(a+b*ln(c*x^n))+1)^(1
/2)*(I*sinh(a+b*ln(c*x^n)))^(1/2)*EllipticE((1-I*sinh(a+b*ln(c*x^n)))^(1/2)
,1/2*2^(1/2))-(1-I*sinh(a+b*ln(c*x^n)))^(1/2)*2^(1/2)*(I*sinh(a+b*ln(c*x^n)
)+1)^(1/2)*(I*sinh(a+b*ln(c*x^n)))^(1/2)*EllipticF((1-I*sinh(a+b*ln(c*x^n))
)^(1/2),1/2*2^(1/2))-2*cosh(a+b*ln(c*x^n))^2)/cosh(a+b*ln(c*x^n))/sinh(a+b*
ln(c*x^n))^(1/2)/b
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}\left(b\log\left(cx^{n}\right)+a\right)^{\frac{3}{2}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(a+b*log(c*x^n))^(3/2)/x,x, algorithm="maxima")
```

```
[Out] integrate(csch(b*log(c*x^n) + a)^(3/2)/x, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(\frac{1}{\sinh(a+b\,\ln(c\,x^n))}\right)^{3/2}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/sinh(a + b*log(c*x^n)))^(3/2)/x,x)
```

```
[Out] int((1/sinh(a + b*log(c*x^n)))^(3/2)/x, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}^{\frac{3}{2}}\left(a+b\log\left(cx^{n}\right)\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(a+b*ln(c*x**n))**(3/2)/x,x)
```

```
[Out] Integral(csch(a + b*log(c*x**n))**(3/2)/x, x)
```

3.172 $\int \frac{\sqrt{\operatorname{csch}(a+b\log(cx^n))}}{x}\,dx$

Optimal. Leaf size=72

$$-\frac{2i\sqrt{i\sinh\left(a+b\log\left(cx^n\right)\right)}\sqrt{\operatorname{csch}\left(a+b\log\left(cx^n\right)\right)}F\left(\frac{1}{2}\left(ia+ib\log\left(cx^n\right)-\frac{\pi}{2}\right)\middle|2\right)}{bn}$$

```
[Out] 2*I*(sin(1/2*I*a+1/4*Pi+1/2*I*b*ln(c*x^n))^2)^(1/2)/sin(1/2*I*a+1/4*Pi+1/2*
I*b*ln(c*x^n))*EllipticF(cos(1/2*I*a+1/4*Pi+1/2*I*b*ln(c*x^n)),2^(1/2))*csc
h(a+b*ln(c*x^n))^(1/2)*(I*sinh(a+b*ln(c*x^n)))^(1/2)/b/n
```

Rubi [A] time = 0.04, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {3771, 2641}

$$-\frac{2i\sqrt{i\sinh\left(a+b\log\left(cx^n\right)\right)}\sqrt{\operatorname{csch}\left(a+b\log\left(cx^n\right)\right)}F\left(\frac{1}{2}\left(ia+ib\log\left(cx^n\right)-\frac{\pi}{2}\right)\middle|2\right)}{bn}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Csch[a + b*Log[c*x^n]]]/x,x]
```

```
[Out] ((-2*I)*Sqrt[Csch[a + b*Log[c*x^n]]]*EllipticF[(I*a - Pi/2 + I*b*Log[c*x^n]
)/2, 2]*Sqrt[I*Sinh[a + b*Log[c*x^n]]])/(b*n)
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{\operatorname{csch}\left(a+b\log\left(cx^n\right)\right)}}{x}\,dx &= \frac{\operatorname{Subst}\left(\int \sqrt{\operatorname{csch}(a+bx)}\,dx, x, \log\left(cx^n\right)\right)}{n}\\
&= \frac{\left(\sqrt{\operatorname{csch}\left(a+b\log\left(cx^n\right)\right)}\sqrt{i\sinh\left(a+b\log\left(cx^n\right)\right)}\right)\operatorname{Subst}\left(\int \frac{1}{\sqrt{i\sinh(a+bx)}}\,dx, x,\right.}{n}\\
&= -\frac{2i\sqrt{\operatorname{csch}\left(a+b\log\left(cx^n\right)\right)}F\left(\frac{1}{2}\left(ia-\frac{\pi}{2}+ib\log\left(cx^n\right)\right)\middle|2\right)\sqrt{i\sinh\left(a+b\log\left(c\right.\right.}}{bn}
\end{aligned}$$

Mathematica [A] time = 0.10, size = 66, normalized size = 0.92

$$\frac{2\left(i\sinh\left(a+b\log\left(cx^n\right)\right)\right)^{3/2}\operatorname{csch}^{\frac{3}{2}}\left(a+b\log\left(cx^n\right)\right)F\left(\frac{1}{4}\left(-2ia-2ib\log\left(cx^n\right)+\pi\right)\middle|2\right)}{bn}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Csch[a + b*Log[c*x^n]]]/x,x]
```

```
[Out] (2*Csch[a + b*Log[c*x^n]]^(3/2)*EllipticF[((-2*I)*a + Pi - (2*I)*b*Log[c*x^
n])/4, 2]*(I*Sinh[a + b*Log[c*x^n]])^(3/2))/(b*n)
```

fricas [F] time = 0.43, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{\operatorname{csch}\left(b\log\left(cx^{n}\right)+a\right)}}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(a+b*log(c*x^n))^(1/2)/x,x, algorithm="fricas")
```

```
[Out] integral(sqrt(csch(b*log(c*x^n) + a))/x, x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(a+b*log(c*x^n))^(1/2)/x,x, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.34, size = 120, normalized size = 1.67

$$\frac{i\sqrt{-i\left(\sinh\left(a+b\ln\left(c\,x^{n}\right)\right)+i\right)}\,\sqrt{2}\,\sqrt{-i\left(-\sinh\left(a+b\ln\left(c\,x^{n}\right)\right)+i\right)}\,\sqrt{i\sinh\left(a+b\ln\left(c\,x^{n}\right)\right)}\,\operatorname{EllipticF}\left(\sqrt{-i\left(s\right.}\right.}{n\cosh\left(a+b\ln\left(c\,x^{n}\right)\right)\sqrt{\sinh\left(a+b\ln\left(c\,x^{n}\right)\right)}\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(a+b*ln(c*x^n))^(1/2)/x,x)
```

```
[Out] I/n*(-I*(sinh(a+b*ln(c*x^n))+I))^(1/2)*2^(1/2)*(-I*(-sinh(a+b*ln(c*x^n))+I)
)^(1/2)*(I*sinh(a+b*ln(c*x^n)))^(1/2)*EllipticF((-I*(sinh(a+b*ln(c*x^n))+I)
)^(1/2),1/2*2^(1/2))/cosh(a+b*ln(c*x^n))/sinh(a+b*ln(c*x^n))^(1/2)/b
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\operatorname{csch}\left(b\log\left(cx^{n}\right)+a\right)}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(a+b*log(c*x^n))^(1/2)/x,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(csch(b*log(c*x^n) + a))/x, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{\frac{1}{\sinh(a+b\,\ln(c\,x^{n}))}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/sinh(a + b*log(c*x^n)))^(1/2)/x,x)
```

```
[Out] int((1/sinh(a + b*log(c*x^n)))^(1/2)/x, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\operatorname{csch}\left(a+b\log\left(cx^{n}\right)\right)}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(a+b*ln(c*x**n))**(1/2)/x,x)
```

```
[Out] Integral(sqrt(csch(a + b*log(c*x**n)))/x, x)
```

3.173 $\int \frac{1}{x\sqrt{\operatorname{csch}(a+b\log(cx^n))}}\,dx$

Optimal. Leaf size=72

$$-\frac{2iE\left(\frac{1}{2}\left(ia+ib\log\left(cx^n\right)-\frac{\pi}{2}\right)\middle|2\right)}{bn\sqrt{i\sinh\left(a+b\log\left(cx^n\right)\right)}\sqrt{\operatorname{csch}\left(a+b\log\left(cx^n\right)\right)}}$$

```
[Out] 2*I*(sin(1/2*I*a+1/4*Pi+1/2*I*b*ln(c*x^n))^2)^(1/2)/sin(1/2*I*a+1/4*Pi+1/2*
I*b*ln(c*x^n))*EllipticE(cos(1/2*I*a+1/4*Pi+1/2*I*b*ln(c*x^n)),2^(1/2))/b/n
/csch(a+b*ln(c*x^n))^(1/2)/(I*sinh(a+b*ln(c*x^n)))^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {3771, 2639}

$$-\frac{2iE\left(\frac{1}{2}\left(ia+ib\log\left(cx^n\right)-\frac{\pi}{2}\right)\middle|2\right)}{bn\sqrt{i\sinh\left(a+b\log\left(cx^n\right)\right)}\sqrt{\operatorname{csch}\left(a+b\log\left(cx^n\right)\right)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[Csch[a + b*Log[c*x^n]]]),x]
```

```
[Out] ((-2*I)*EllipticE[(I*a - Pi/2 + I*b*Log[c*x^n])/2, 2])/(b*n*Sqrt[Csch[a + b
*Log[c*x^n]]]*Sqrt[I*Sinh[a + b*Log[c*x^n]]])
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x\sqrt{\operatorname{csch}\left(a+b\log\left(cx^n\right)\right)}}\,dx &= \frac{\operatorname{Subst}\left(\int \frac{1}{\sqrt{\operatorname{csch}(a+bx)}}\,dx,x,\log\left(cx^n\right)\right)}{n}\\
&= \frac{\operatorname{Subst}\left(\int \sqrt{i\sinh(a+bx)}\,dx,x,\log\left(cx^n\right)\right)}{n\sqrt{\operatorname{csch}\left(a+b\log\left(cx^n\right)\right)}\sqrt{i\sinh\left(a+b\log\left(cx^n\right)\right)}}\\
&= -\frac{2iE\left(\frac{1}{2}\left(ia-\frac{\pi}{2}+ib\log\left(cx^n\right)\right)\middle|2\right)}{bn\sqrt{\operatorname{csch}\left(a+b\log\left(cx^n\right)\right)}\sqrt{i\sinh\left(a+b\log\left(cx^n\right)\right)}}
\end{aligned}$$

Mathematica [A] time = 0.08, size = 68, normalized size = 0.94

$$\frac{2\sqrt{i\sinh\left(a+b\log\left(cx^n\right)\right)}\sqrt{\operatorname{csch}\left(a+b\log\left(cx^n\right)\right)}E\left(\frac{1}{2}\left(\frac{\pi}{2}-i\left(a+b\log\left(cx^n\right)\right)\right)\middle|2\right)}{bn}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[Csch[a + b*Log[c*x^n]]]),x]
```

```
[Out] (2*Sqrt[Csch[a + b*Log[c*x^n]]]*EllipticE[(Pi/2 - I*(a + b*Log[c*x^n]))/2,
2]*Sqrt[I*Sinh[a + b*Log[c*x^n]]])/(b*n)
```

fricas [F] time = 0.46, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{x\sqrt{\operatorname{csch}\left(b\log\left(cx^{n}\right)+a\right)}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/csch(a+b*log(c*x^n))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(1/(x*sqrt(csch(b*log(c*x^n) + a))), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\sqrt{\operatorname{csch}\left(b\log\left(cx^{n}\right)+a\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/csch(a+b*log(c*x^n))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(x*sqrt(csch(b*log(c*x^n) + a))), x)
```

maple [A] time = 0.59, size = 146, normalized size = 2.03

$$\frac{\sqrt{-i\left(\sinh\left(a+b\ln\left(c\,x^{n}\right)\right)+i\right)}\,\sqrt{2}\,\sqrt{-i\left(-\sinh\left(a+b\ln\left(c\,x^{n}\right)\right)+i\right)}\,\sqrt{i\sinh\left(a+b\ln\left(c\,x^{n}\right)\right)}\,\left(2\,\mathrm{EllipticE}\left(\sqrt{1-i\,}\right.\right.}{n\cosh\left(a+b\ln\left(c\,x^{n}\right)\right)\sqrt{\sinh\left(a+b\ln\left(c\,x^{n}\right.\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/csch(a+b*ln(c*x^n))^(1/2),x)
```

```
[Out] 1/n*(-I*(sinh(a+b*ln(c*x^n))+I))^(1/2)*2^(1/2)*(-I*(-sinh(a+b*ln(c*x^n))+I)
)^(1/2)*(I*sinh(a+b*ln(c*x^n)))^(1/2)*(2*EllipticE((1-I*sinh(a+b*ln(c*x^n))
)^(1/2),1/2*2^(1/2))-EllipticF((1-I*sinh(a+b*ln(c*x^n)))^(1/2),1/2*2^(1/2))
)/cosh(a+b*ln(c*x^n))/sinh(a+b*ln(c*x^n))^(1/2)/b
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\sqrt{\operatorname{csch}\left(b\log\left(cx^{n}\right)+a\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/csch(a+b*log(c*x^n))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(x*sqrt(csch(b*log(c*x^n) + a))), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x\,\sqrt{\frac{1}{\mathrm{sinh}\left(a+b\,\ln\left(c\,x^n\right)\right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(1/sinh(a + b*log(c*x^n)))^(1/2)),x)
[Out] int(1/(x*(1/sinh(a + b*log(c*x^n)))^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\sqrt{\operatorname{csch}\left(a+b\log\left(cx^{n}\right)\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/csch(a+b*ln(c*x**n))**(1/2),x)
[Out] Integral(1/(x*sqrt(csch(a + b*log(c*x**n)))), x)
```

3.174 $\int \frac{1}{x\,\mathrm{csch}^{\frac{3}{2}}(a+b\log(cx^n))}\,dx$

Optimal. Leaf size=111

$$\frac{2\cosh\left(a+b\log\left(cx^n\right)\right)}{3bn\sqrt{\operatorname{csch}\left(a+b\log\left(cx^n\right)\right)}}+\frac{2i\sqrt{i\sinh\left(a+b\log\left(cx^n\right)\right)}\sqrt{\operatorname{csch}\left(a+b\log\left(cx^n\right)\right)}F\left(\frac{1}{2}\left(ia+ib\log\left(cx^n\right)-\frac{\pi}{2}\right)\middle|2\right)}{3bn}$$

```
[Out] 2/3*cosh(a+b*ln(c*x^n))/b/n/csch(a+b*ln(c*x^n))^(1/2)-2/3*I*(sin(1/2*I*a+1/
4*Pi+1/2*I*b*ln(c*x^n))^2)^(1/2)/sin(1/2*I*a+1/4*Pi+1/2*I*b*ln(c*x^n))*Elli
pticF(cos(1/2*I*a+1/4*Pi+1/2*I*b*ln(c*x^n)),2^(1/2))*csch(a+b*ln(c*x^n))^(1
/2)*(I*sinh(a+b*ln(c*x^n)))^(1/2)/b/n
```

Rubi [A] time = 0.06, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {3769, 3771, 2641}

$$\frac{2\cosh\left(a+b\log\left(cx^n\right)\right)}{3bn\sqrt{\operatorname{csch}\left(a+b\log\left(cx^n\right)\right)}}+\frac{2i\sqrt{i\sinh\left(a+b\log\left(cx^n\right)\right)}\sqrt{\operatorname{csch}\left(a+b\log\left(cx^n\right)\right)}F\left(\frac{1}{2}\left(ia+ib\log\left(cx^n\right)-\frac{\pi}{2}\right)\middle|2\right)}{3bn}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Csch[a + b*Log[c*x^n]]^(3/2)),x]
```

```
[Out] (2*Cosh[a + b*Log[c*x^n]])/(3*b*n*Sqrt[Csch[a + b*Log[c*x^n]]]) + (((2*I)/3
)*Sqrt[Csch[a + b*Log[c*x^n]]]*EllipticF[(I*a - Pi/2 + I*b*Log[c*x^n])/2, 2
]*Sqrt[I*Sinh[a + b*Log[c*x^n]]])/(b*n)
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\int \frac{1}{x\text{csch}^{\frac{3}{2}}\left(a+b\log\left(cx^n\right)\right)}dx = \frac{\text{Subst}\left(\int \frac{1}{\text{csch}^{\frac{3}{2}}(a+bx)}dx, x, \log\left(cx^n\right)\right)}{n}$$
$$= \frac{2\cosh\left(a+b\log\left(cx^n\right)\right)}{3bn\sqrt{\text{csch}\left(a+b\log\left(cx^n\right)\right)}} - \frac{\text{Subst}\left(\int \sqrt{\text{csch}(a+bx)}\,dx, x, \log\left(cx^n\right)\right)}{3n}$$
$$= \frac{2\cosh\left(a+b\log\left(cx^n\right)\right)}{3bn\sqrt{\text{csch}\left(a+b\log\left(cx^n\right)\right)}} - \frac{\left(\sqrt{\text{csch}\left(a+b\log\left(cx^n\right)\right)}\sqrt{i\sinh\left(a+b\log\right.}\right.}{3}$$
$$= \frac{2\cosh\left(a+b\log\left(cx^n\right)\right)}{3bn\sqrt{\text{csch}\left(a+b\log\left(cx^n\right)\right)}} + \frac{2i\sqrt{\text{csch}\left(a+b\log\left(cx^n\right)\right)}F\left(\frac{1}{2}\left(ia-\frac{\pi}{2}+ib\log\right.\right.}{3bn}$$

Mathematica [A] time = 0.14, size = 86, normalized size = 0.77

$$\frac{\sqrt{\text{csch}\left(a+b\log\left(cx^n\right)\right)}\left(\sinh\left(2\left(a+b\log\left(cx^n\right)\right)\right)-2i\sqrt{i\sinh\left(a+b\log\left(cx^n\right)\right)}F\left(\frac{1}{4}\left(-2ia-2ib\log\left(cx^n\right)+\right.\right.\right.}{3bn}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Csch[a + b*Log[c*x^n]]^(3/2)),x]
```

```
[Out] (Sqrt[Csch[a + b*Log[c*x^n]]]*((-2*I)*EllipticF[((-2*I)*a + Pi - (2*I)*b*Lo
g[c*x^n])/4, 2]*Sqrt[I*Sinh[a + b*Log[c*x^n]]] + Sinh[2*(a + b*Log[c*x^n])]
))/(3*b*n)
```

fricas [F] time = 0.45, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{x\,\text{csch}\left(b\log\left(cx^n\right)+a\right)^{\frac{3}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/csch(a+b*log(c*x^n))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(1/(x*csch(b*log(c*x^n) + a)^(3/2)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\,\text{csch}\left(b\log\left(cx^n\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/csch(a+b*log(c*x^n))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/(x*csch(b*log(c*x^n) + a)^(3/2)), x)
```

maple [A] time = 0.45, size = 143, normalized size = 1.29

$$\frac{-\frac{i\sqrt{1-i\sinh(a+b\ln(c\,x^n))}\,\sqrt{2}\,\sqrt{i\sinh(a+b\ln(c\,x^n))+1}\,\sqrt{i\sinh(a+b\ln(c\,x^n))}\,\text{EllipticF}\left(\sqrt{1-i\sinh(a+b\ln(c\,x^n))},\frac{\sqrt{2}}{2}\right)}{3}+\frac{2\left(\cosh^2(a+b\ln(c\,x^n))\right)}{3}}{n\cosh\left(a+b\ln\left(c\,x^n\right)\right)\sqrt{\sinh\left(a+b\ln\left(c\,x^n\right)\right)}\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/csch(a+b*ln(c*x^n))^(3/2),x)
```

```
[Out] 1/n*(-1/3*I*(1-I*sinh(a+b*ln(c*x^n)))^(1/2)*2^(1/2)*(I*sinh(a+b*ln(c*x^n))+
1)^(1/2)*(I*sinh(a+b*ln(c*x^n)))^(1/2)*EllipticF((1-I*sinh(a+b*ln(c*x^n)))^
(1/2),1/2*2^(1/2))+2/3*cosh(a+b*ln(c*x^n))^2*sinh(a+b*ln(c*x^n)))/cosh(a+b*
ln(c*x^n))/sinh(a+b*ln(c*x^n))^(1/2)/b
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x \operatorname{csch}\left(b \log\left(cx^{n}\right) + a\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/csch(a+b*log(c*x^n))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(x*csch(b*log(c*x^n) + a)^(3/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x\,{\left(\frac{1}{\mathrm{sinh}\left(a+b\,\ln\left(c\,x^n\right)\right)}\right)}^{3/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(1/sinh(a + b*log(c*x^n)))^(3/2)),x)
```

```
[Out] int(1/(x*(1/sinh(a + b*log(c*x^n)))^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x \operatorname{csch}^{\frac{3}{2}}{\left(a + b \log{\left(c x^{n} \right)} \right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/csch(a+b*ln(c*x**n))**(3/2),x)
```

```
[Out] Integral(1/(x*csch(a + b*log(c*x**n))**(3/2)), x)
```

3.175 $\int \frac{1}{x\mathbf{csch}^{\frac{5}{2}}(a+b\log(cx^n))}\,dx$

Optimal. Leaf size=111

$$\frac{2\cosh\left(a+b\log\left(cx^n\right)\right)}{5bn\text{csch}^{\frac{3}{2}}\left(a+b\log\left(cx^n\right)\right)}+\frac{6iE\left(\frac{1}{2}\left(ia+ib\log\left(cx^n\right)-\frac{\pi}{2}\right)\middle|2\right)}{5bn\sqrt{i\sinh\left(a+b\log\left(cx^n\right)\right)}\sqrt{\text{csch}\left(a+b\log\left(cx^n\right)\right)}}$$

```
[Out] 2/5*cosh(a+b*ln(c*x^n))/b/n/csch(a+b*ln(c*x^n))^(3/2)-6/5*I*(sin(1/2*I*a+1/
4*Pi+1/2*I*b*ln(c*x^n))^2)^(1/2)/sin(1/2*I*a+1/4*Pi+1/2*I*b*ln(c*x^n))*Elli
pticE(cos(1/2*I*a+1/4*Pi+1/2*I*b*ln(c*x^n)),2^(1/2))/b/n/csch(a+b*ln(c*x^n)
)^(1/2)/(I*sinh(a+b*ln(c*x^n)))^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {3769, 3771, 2639}

$$\frac{2\cosh\left(a+b\log\left(cx^n\right)\right)}{5bn\text{csch}^{\frac{3}{2}}\left(a+b\log\left(cx^n\right)\right)}+\frac{6iE\left(\frac{1}{2}\left(ia+ib\log\left(cx^n\right)-\frac{\pi}{2}\right)\middle|2\right)}{5bn\sqrt{i\sinh\left(a+b\log\left(cx^n\right)\right)}\sqrt{\text{csch}\left(a+b\log\left(cx^n\right)\right)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Csch[a + b*Log[c*x^n]]^(5/2)),x]
```

```
[Out] (2*Cosh[a + b*Log[c*x^n]])/(5*b*n*Csch[a + b*Log[c*x^n]]^(3/2)) + (((6*I)/5
)*EllipticE[(I*a - Pi/2 + I*b*Log[c*x^n])/2, 2])/(b*n*Sqrt[Csch[a + b*Log[c
*x^n]]]*Sqrt[I*Sinh[a + b*Log[c*x^n]]])
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\int \frac{1}{x\text{csch}^{\frac{5}{2}}\left(a+b\log\left(cx^n\right)\right)}\,dx = \frac{\text{Subst}\left(\int \frac{1}{\text{csch}^{\frac{5}{2}}(a+bx)}\,dx, x, \log\left(cx^n\right)\right)}{n}$$
$$= \frac{2\cosh\left(a+b\log\left(cx^n\right)\right)}{5bn\text{csch}^{\frac{3}{2}}\left(a+b\log\left(cx^n\right)\right)} - \frac{3\,\text{Subst}\left(\int \frac{1}{\sqrt{\text{csch}(a+bx)}}\,dx, x, \log\left(cx^n\right)\right)}{5n}$$
$$= \frac{2\cosh\left(a+b\log\left(cx^n\right)\right)}{5bn\text{csch}^{\frac{3}{2}}\left(a+b\log\left(cx^n\right)\right)} - \frac{3\,\text{Subst}\left(\int \sqrt{i\sinh(a+bx)}\,dx, x, \log\left(cx^n\right)\right)}{5n\sqrt{\text{csch}\left(a+b\log\left(cx^n\right)\right)}\sqrt{i\sinh\left(a+b\log\left(cx^n\right)\right)}}$$
$$= \frac{2\cosh\left(a+b\log\left(cx^n\right)\right)}{5bn\text{csch}^{\frac{3}{2}}\left(a+b\log\left(cx^n\right)\right)} + \frac{6iE\left(\frac{1}{2}\left(ia-\frac{\pi}{2}+ib\log\left(cx^n\right)\right)\middle|2\right)}{5bn\sqrt{\text{csch}\left(a+b\log\left(cx^n\right)\right)}\sqrt{i\sinh\left(a+b\log\left(cx^n\right)\right)}}$$

Mathematica [A] time = 0.19, size = 95, normalized size = 0.86

$$\frac{2\left(\cosh\left(a+b\log\left(cx^n\right)\right)-3\sqrt{i\sinh\left(a+b\log\left(cx^n\right)\right)}\text{csch}^2\left(a+b\log\left(cx^n\right)\right)E\left(\frac{1}{4}\left(-2ia-2ib\log\left(cx^n\right)+\pi\right)\middle|2\right)\right)}{5bn\text{csch}^{\frac{3}{2}}\left(a+b\log\left(cx^n\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Csch[a + b*Log[c*x^n]]^(5/2)),x]
```

```
[Out] (2*(Cosh[a + b*Log[c*x^n]] - 3*Csch[a + b*Log[c*x^n]]^2*EllipticE[((-2*I)*a
 + Pi - (2*I)*b*Log[c*x^n])/4, 2]*Sqrt[I*Sinh[a + b*Log[c*x^n]]]))/(5*b*n*C
sch[a + b*Log[c*x^n]]^(3/2))
```

fricas [F] time = 0.45, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{x\,\text{csch}\left(b\log\left(cx^n\right)+a\right)^{\frac{5}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/csch(a+b*log(c*x^n))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(1/(x*csch(b*log(c*x^n) + a)^(5/2)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\,\text{csch}\left(b\log\left(cx^n\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/csch(a+b*log(c*x^n))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(1/(x*csch(b*log(c*x^n) + a)^(5/2)), x)
```

maple [A] time = 0.69, size = 227, normalized size = 2.05

$$\frac{-\frac{6\sqrt{1-i\sinh(a+b\ln(cx^n))}\sqrt{2}\sqrt{i\sinh(a+b\ln(cx^n))+1}\sqrt{i\sinh(a+b\ln(cx^n))}\,\text{EllipticE}\left(\sqrt{1-i\sinh(a+b\ln(cx^n))},\frac{\sqrt{2}}{2}\right)}{5}+\frac{3\sqrt{1-i\sinh(a+b\ln(cx^n))}}{}}{n\cosh\left(a+b\ln\left(cx^n\right)\right)\sqrt{s}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/csch(a+b*ln(c*x^n))^(5/2),x)
```

```
[Out] 1/n*(-6/5*(1-I*sinh(a+b*ln(c*x^n)))^(1/2)*2^(1/2)*(I*sinh(a+b*ln(c*x^n))+1)
^(1/2)*(I*sinh(a+b*ln(c*x^n)))^(1/2)*EllipticE((1-I*sinh(a+b*ln(c*x^n)))^(1
/2),1/2*2^(1/2))+3/5*(1-I*sinh(a+b*ln(c*x^n)))^(1/2)*2^(1/2)*(I*sinh(a+b*ln
(c*x^n))+1)^(1/2)*(I*sinh(a+b*ln(c*x^n)))^(1/2)*EllipticF((1-I*sinh(a+b*ln(
c*x^n)))^(1/2),1/2*2^(1/2))+2/5*cosh(a+b*ln(c*x^n))^4-2/5*cosh(a+b*ln(c*x^n
))^2)/cosh(a+b*ln(c*x^n))/sinh(a+b*ln(c*x^n))^(1/2)/b
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x \operatorname{csch}\left(b \log\left(cx^{n}\right) + a\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/csch(a+b*log(c*x^n))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(x*csch(b*log(c*x^n) + a)^(5/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x\,{\left(\frac{1}{\mathrm{sinh}\left(a+b\,\ln\left(c\,x^n\right)\right)}\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(1/sinh(a + b*log(c*x^n)))^(5/2)),x)
```

```
[Out] int(1/(x*(1/sinh(a + b*log(c*x^n)))^(5/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x \operatorname{csch}^{\frac{5}{2}}{\left(a + b \log{\left(c x^{n} \right)} \right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/csch(a+b*ln(c*x**n))**(5/2),x)
```

```
[Out] Integral(1/(x*csch(a + b*log(c*x**n))**(5/2)), x)
```

Chapter 4

Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.0.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*          GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] :=
  If[ExpnType[result]<=ExpnType[optimal],
    If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
      If[LeafCount[result]<=2*LeafCount[optimal],
        "A",
      "B"],
    "C"],
  If[FreeQ[result,Integrate] && FreeQ[result,Int],
    "C",
  "F"]]


(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
```

```
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
        Exp,Log,
        Sin,Cos,Tan,Cot,Sec,Csc,
        ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
        Sinh,Cosh,Tanh,Coth,Sech,Csch,
        ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
        Erf, Erfc, Erfi,
        FresnelS, FresnelC,
        ExpIntegralE, ExpIntegralEi, LogIntegral,
        SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
        Gamma, LogGamma, PolyGamma,
        Zeta, PolyLog, ProductLog,
        EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]
```

```
AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.0.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems

GradeAntiderivative := proc(result,optimal)
local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal,
   debug:=false;

      leaf_count_result:=leafcount(result);
      #do NOT call ExpnType() if leaf size is too large. Recursion problem
      if leaf_count_result > 500000 then
          return "B";
      fi;

      leaf_count_optimal:=leafcount(optimal);

      ExpnType_result:=ExpnType(result);
      ExpnType_optimal:=ExpnType(optimal);

      if debug then
         print("ExpnType_result",ExpnType_result," ExpnType_optimal=",
   ExpnType_optimal);
      fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

  #This check below actually is not needed, since I only
  #call this grading only for passed integrals. i.e. I check
  #for "F" before calling this. But no harm of keeping it here.
  #just in case.


  if not type(result,freeof('int')) then
     return "F";
  end if;


  if ExpnType_result<=ExpnType_optimal then
      if debug then
          print("ExpnType_result<=ExpnType_optimal");
      fi;
```

```
      if is_contains_complex(result) then
         if is_contains_complex(optimal) then
            if debug then
               print("both result and optimal complex");
            fi;
            #both result and optimal complex
            if leaf_count_result<=2*leaf_count_optimal then
               return "A";
            else
               return "B";
            end if
         else #result contains complex but optimal is not
            if debug then
               print("result contains complex but optimal is not");
            fi;
            return "C";
         end if
      else # result do not contain complex
           # this assumes optimal do not as well
           if debug then
               print("result do not contain complex, this assumes optimal do
   not as well");
           fi;
           if leaf_count_result<=2*leaf_count_optimal then
              if debug then
                 print("leaf_count_result<=2*leaf_count_optimal");
              fi;
              return "A";
           else
              if debug then
                 print("leaf_count_result>2*leaf_count_optimal");
              fi;
              return "B";
           end if
      end if
  else #ExpnType(result) > ExpnType(optimal)
      if debug then
         print("ExpnType(result) > ExpnType(optimal)");
      fi;
      return "C";
  end if

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
```

```
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:
```

```
AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.0.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]
```

```
def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
           return True
        else:
           return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn
   )))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,
   ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or
   type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,
   expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType
   ,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9
```

```
#main function
def grade_antiderivative(result,optimal):

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        return "F"

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
   well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```

4.0.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
```

```
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    debug=False
    if debug: print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','
   sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e',
           'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','
   hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in
   sagemath

def is_atom(expn):

    debug=False
    if debug: print ("Enter is_atom")


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-
   sagemath-equivalent-to-atomic-type-in-maple/
    try:
        if expn.parent() is SR:
```

```
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens")
    :
            return expn in expn.parent().base_ring() or expn in expn.parent().
    gens()
        return False

    except AttributeError as error:
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],
    Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.
    args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer
    )
            return expnType(expn.operands()[0])   #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],
    Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],
    Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(
    expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()
    [1])) #max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #
    isinstance(expn,Add) or isinstance(expn,Mul)
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn
    .func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(
    expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #
    is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
```

```
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    if debug: print ("Enter grade_antiderivative for sagemath")

    leaf_count_result  = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    if debug: print ("leaf_count_result=", leaf_count_result, "
    leaf_count_optimal=",leaf_count_optimal)


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",
    expnType_optimal)

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
    well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```